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1970
THE ATTACK OF GROUND TARGETS
BY ARTILLERY FIRE:
A BAYESIAN APPLICATION

DISSERTATION

Presented in Partial Fulfillment of the Requirements
for the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

by

Robert Warren Blum, B.S., M.Sc., A.M.
The Ohio State University
1969

Approved by

[Signature]
Adviser
Department of Industrial Engineering
To
Saint Barbara
of the Artillerymen
Preface

The purpose of this effort is to contribute to the establishment of a sound theoretical foundation for that body of technical military knowledge encompassing the attack of ground targets by observed artillery fires. The presence of a theoretical structure within which to evaluate and compare doctrine and doctrinal procedures could be of inestimable value to the Profession of Arms in general and to the Field Artillery in particular. To provide such structurings is, to me, the raison d'etre of Military Operations Research.

While the inquisitive scholar who might on occasion turn these pages will feel some strangeness initially with the jargon of the field artilleryman: the battery center, the fall of shot, fire for effect, etc. - so will the Gunner be uncomfortable at first meeting with the unfamiliar terms of the Bayesian: the prior distribution, the loss function, optimum economic sample size, and so forth. But from a studied attempt at mutual understanding the strangeness will pass and the unfamiliar can be coaxed to life.

From my brothers-in-arms I ask your sincere attendance to the arguments presented here. Do not just taste of them to sample their flavor. Their worth, if they have any worth, will be in their total digestion. You are cautioned not to let yourselves be stereotyped with those to whom Bradshaw must have been referring when
he said, "Some managers would rather live with a problem they can't solve than use a solution they don't understand." Some of the concepts which you may long have held or phenomena with which you may already have become acquainted will be rediscovered here. My hope is that this rediscovery will occur from the unassailable logic of the argument. Its reasoning can be reinforced by, and in turn can reinforce, your own visceral feelings and intuitive beliefs. Only in the presence of a strong complementarity between the theoretic and the pragmatic can arguments vast and small be advanced and examined within the context of the whole.

The text is not well suited to selective or sporadic study. Chapter I is a brief description of the major technical problems which an observed fire attack situation presents to the field artilleryman. Chapter II serves to introduce our interpretation of Bayesian Decision Theory. Chapter III develops the geometry of the field artillery attack and from it formulates a measure of effectiveness, ammunition expenditure, for the attack. Chapters IV through VI present the theoretical arguments of the Bayesian application. The mathematics are by no means formidable if the reader is familiar with basic probability theory and the integral calculus. Even then, a deliberate attempt has been made to sprinkle liberally the mathematical arguments with meaningful interpretive comments. In
Chapter VII the current doctrine and doctrinal procedures are examined for theoretical discrepancies. Some special applications of the theory are also advanced there.

Without the good offices of many persons and agencies this effort would never have reached maturity. My appreciation is unbounded for this educational opportunity made possible by the United States Army and, in particular, its Field Artillery branch. To each of my professors I give heartfelt thanks for his generous sharing of knowledge and understanding. But to my academic mentor, Professor William T. Morris, I owe particular thanks for his encouragement and patient guidance. He taught me much. Of the many others who have helped me on my way I wish especially to acknowledge Colonel Andrew C. Anderson, Director of the Gunnery Department at the United States Army Field Artillery School, and his staff for their cooperation and ready assistance in arranging for the experimental phase of this investigation. Finally, to each of you belongs some measure of whatever good might come of this effort; however to me alone belongs any criticism for its - and thus my own - inadequacies.

Columbus, Ohio
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Studies in Inventory Theory. Professor Walter C. Giffin

Studies in Queueing Theory. Professor Walter C. Giffin

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CHAPTER I

THE FIELD ARTILLERY GUNNERY PROBLEM

The Gunnery Problem

The gunnery problem inherent to the employment of field artillery is described succinctly in the following quotation.

Field artillery weapons normally are emplaced in defilade to conceal them from the enemy. For the vast majority of targets, placing pieces in defilade precludes sighting the weapon directly at the target (direct fire). Consequently, indirect fire must be employed to attack the targets. The gunnery problem is primarily the problem of indirect fire. The solution of this problem requires weapon and ammunition settings which, when applied to the piece and the ammunition, will cause the projectile to burst on, or at a proper height above, the target. [3, pg. 1-1]

The Accuracy of Fire

Accurate indirect fire generally implies the use of an observer positioned so as to obtain line-of-sight observation into the target area. From the point of vantage the observer carries out his three primary tasks: detecting and locating targets suitable for attack by artillery weapons, initiating calls for fire on targets selected for attack, and, when necessary, adjusting the fires of the battery through his observation of the fall of shot with relation to the target being attacked. The observer communicates by wire or radio with the fire direction center at the battery position where his reported observations are converted into settings to be applied to the weapons and ammunition.
Accurate fire is also dependent on the availability of survey information on the precise location of the guns to include a line of known direction by which they can be oriented to the battlemap. The field artillery organization structure provides this survey capability. Further, it is necessary to locate by survey (triangulation) one or more identifiable points in the target area on which the guns can be registered\(^1\) so as to adjust the mean point of impact of the fall of shot onto a point of known location, thus obtaining registration corrections which can be applied to the standard weapon and fuze settings (chart data) to account for non-standard ballistic and meteorological conditions\(^2\).

As we have just stated, survey is extended into the target area by triangulation from a surveyed base established within friendly lines. The end points of this base are obviously vantage points which meet all the requirements of an observation post. Consequently it is not uncommon for an observer to be positioned at a surveyed location established in conjunction with the target area survey or even adjunctive to it.

\(^1\)"Registration" is a technical term used by field artillery for the more generally used term "calibration". However, "calibration" is reserved by field artillery for the measurement of velocity error.

\(^2\)Non-standard conditions of projectile weight, velocity error, air temperature, propellant temperature, air density, wind direction and speed, projectile drift and rotation of the earth can be accounted for by registration.
Figure 1. A typical orientation of the Observer, the Target and the Guns.
The Determination of Firing Data

If we assume, as we shall throughout this investigation, that the locations of both the observer and the guns are known, then the location by the observer of any target with respect to his own position also locates that target with respect to the guns. Figure 1 illustrates the situation. The vector GO is known if the points G and O are known. The determination by the observer of the vector OT immediately determines the vector GT. The vector GT together with the current registration corrections is converted by the fire direction center to appropriate settings for the guns. As we can see, the observer's location of the target at point T is in essence a survey whose accuracy is somewhat suspect since he does not have any assistance in measuring the distance to the target (the magnitude of OT) beyond a battle-map and his own prowess at range estimation. He does, however, possess reasonably accurate direction finding instruments and angle measuring devices with which he can determine the direction of the OT vector.

Adjustment of Fire

If the observer correctly assesses the vector OT, both in distance and direction, and if valid registration corrections exist, then point T is the expected point of impact of rounds fired from point G.
in response to the observer’s request. The evaluation of the accuracy of the vector OT is a subjective process on the part of the observer. If he is confident that OT is correctly described and if registration corrections are available, then his call for fire requests the guns immediately to "Fire for Effect", meaning that no adjustment of the mean point of impact of the fall of shot is necessary and all guns should engage the target. On the other hand, lack of confidence by the observer in the accuracy of his target location, or lack of registration corrections for the guns, indicates uncertainty about the mean point of impact of the fall of shot. The probable need for adjustment of this mean point of impact is communicated by the observer through his call for fire to the battery fire direction center. The message "Adjust Fire" requests the guns to fire sampling rounds which the observer can adjust onto the target. These adjusting rounds are fired in pairs, generally by the two guns nearest the geometrical center (point G) of the battery position. Through appropriate corrections to the observed mean point of impact of each pair of adjusting rounds, the observer attempts to bracket the target, the bracket being successively decreased in size until the target is seen.

\[\text{Vector GT is assumed here to lie in a horizontal plane, as is vector OT. The difference in altitude between points G and T is determined from the contour lines of the battlemap when vector OT becomes known. Corrections can be applied to the guns to compensate for differences in altitude between guns and target.}\]
to lie between the mean points of impact of two pairs of adjusting rounds whose nominal separation distance is 100 meters. Splitting this 100 meter bracket is doctrinal evidence [3, pg. 10-7] that the mean point of impact is within 50 meters of the target and Fire for Effect is now warranted. We shall examine the efficacy of this doctrinal approach in some detail.

The Fall of Shot Distributions

Cannon fire is known to be imprecise. It is very unlikely that any two rounds fired in time proximity from the same gun at identical settings will impact at the same point in the target area. Rather, a long series of rounds will be distributed about a mean point of impact. Two independent distributions describe theoretically the dispersion of the fall of shot about the mean point of impact. These are the range and deflection (cross-range) dispersion patterns which are well represented by the normal or Gaussian distributions [3, pg. 2-23]. The range distribution is oriented along the gun-target line GT when its mean is adjusted to point T. The deflection distribution is oriented orthogonally to line GT and its mean follows the mean of the range distribution, otherwise the distribution of the fall of shot along the orthogonal axis is independent of the distribution in range.

The variances of these two distributions are functions of the angle of departure (in the vertical plane) of the projectile from the gun tube and the velocity of the shell as it leaves the muzzle. Muzzle
Table 1

Range and deflection probable errors$^a$ in meters about the mean point of impact of the fall of shot distributions for a 105-mm Howitzer. [4]

<table>
<thead>
<tr>
<th>Range to Target (meters)</th>
<th>Propelling Charge</th>
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<tr>
<td></td>
<td>(Charge Number and Muzzle Velocity in meters/second)</td>
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<td></td>
<td>1 (195)</td>
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<tr>
<td>1,000</td>
<td>4</td>
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<td>1</td>
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$^a$The largest value in each block is the range probable error. A probable error is defined as 0.6745 standard deviations.
Table 2

Range and deflection probable errors in meters about the mean point of impact of the fall of shot distributions for a 155-mm Howitzer. [5]

<table>
<thead>
<tr>
<th>Range to Target (meters)</th>
<th>Propelling Charge (Charge Number and Muzzle Velocity in meters/second)</th>
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*The largest value in each block is the range probable error.*
velocity in turn is a function of the weight and type of propellant used while the angle of departure determines the expected range to the impact point under standard ballistic and meteorological conditions.

These variances are well known for all US Army field artillery weapon types. Firing data tables contain this information in terms of the probable errors PE equal to .6745 standard deviations. Tables 1 and 2 illustrate the accuracy of two typical American artillery weapons in their tabulation of the range and deflection probable errors at selected mean target ranges (distance GT) for the muzzle velocity associated with each of the propelling charges available. Notice that the deflection probable errors are significantly smaller than the corresponding range probable errors.

The Target Location

In a previous section we asserted the observer's ability to determine accurately the direction to his target. Let us discuss how this is done. Referring to Figure 2, assume that the observer is located at point 0 atop Hill 925 overlooking the valley to the north. Using his direction finding equipment, or perhaps having been assisted by a survey party, he determines that the direction to the bridge across the river to his left front is 6185 mils, the clockwise

\[ 6400 \text{ mils} = 360 \text{ degrees or } 17.78 \text{ mils is about 1 degree.} \] For small angles, 1 mil subtends a chord of 1 meter on a radius of 1000 meters.
Figure 2: The use of the battlemap in determining the location of a target.
Knowing his own location on the map, he uses a protractor to lay off a fan of radial lines at 100 mil increments across his sector of responsibility. He then proceeds to study the terrain, determining the directions to reference points easily identified both on the map and on the ground, such as Hill 530 at direction 690 mils. His binoculars contain a reticled optic by which he can measure angular deviations. Suppose a target appears at point T short of the stream bed to the northeast. Measuring through his binoculars the angle to the target as 60 mils clockwise from Hill 530, he quickly determines that the map location of the target lies along the radial line whose direction is 750 mils.

Knowing the direction of the vector OT, the observer's uncertainty about the target location is now reduced only to uncertainty as to its distance, the magnitude of OT. He then estimates roughly the distance to the target and, searching his map along the radial line at direction 750 mils, he attempts to refine that estimate by matching the graphical representation on the map with his observation of the terrain in the vicinity of the target. Having decided the distance to the target he initiates a call for fire to the battery fire direction.

Grid north (0 mils direction) is the Y-reference orientation of the XY grid superimposed on battlemaps (generally Universal Transverse Mercator grid projections to scale 1:50,000). This XY grid allows map locations to be described in a manner which is universally understood.
The location of the target may be identified to the fire direction center by its polar plot from the observer's known position or by its grid coordinates. For our purposes the polar plot method is more accurate than location by grid coordinates because it retains better the direction of the OT vector. Obviously the polar plot method is not feasible when the observer's location is in doubt.

**Conclusion**

We shall be concerned in this investigation with the observer's estimation of the distance to the target (the magnitude of the OT vector) and the quantitative assimilation of his uncertainty about that distance into an optimum procedure for the attack of the target. Our principal assumptions are

1. The location of the guns and the observer (the vector GO) on a common grid are known.
2. The direction of the vector OT is devoid of doubt.
3. The guns have developed registration corrections valid throughout the target area so that the point on the ground at which the guns are directed to fire is the expected point of impact of the fall of shot.

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6 The time interval required is quite short for this entire procedure. An observer is considered proficient if the time does not exceed 2.5 minutes between detection of the target and the last word of his call for fire message [3, pg. 28-6].
CHAPTER II

THE DECISION PROBLEM

The Doctrine

In Chapter I we alluded to the degree of confidence by the observer in his location of the target as the factor determining whether or not adjustment of the mean point of impact of the fall of shot onto the target is appropriate prior to entering Fire for Effect. The current US Army doctrine is as follows:

When the observer cannot locate the target with sufficient accuracy to warrant firing for effect, he will conduct an adjustment. Lack of accuracy in the location may be the result of poor visibility, deceptive terrain, poor maps, or difficulty on the part of the observer in pinpointing the target. If, in his opinion, fire for effect can be delivered on the basis of target location and surprise is desired, he will request FIRE FOR EFFECT in his call for fire. If registration has not been accomplished recently, adjustment may be directed by the S3[the unit's Operations Officer; author] regardless of the accuracy of the target location. [3, pg. 10-1]

The Accuracy of the Target Location

As we can see, the accuracy of the target location is a subjective determination on the part of the observer. It is not in general a determinate quantity which can be measured a priori to the attack for then there would be perfect foreknowledge and no decision problem would exist in the context of our investigation. However the accuracy must be assessed in some form in order for the observer to make the decision as to whether or not Fire for Effect is warranted. A
convenient method to express this accuracy or lack thereof is with a probability distribution. We shall see in Chapter IV how this may be accomplished and in subsequent chapters how it may be used in the decision problem.

The Loss Function

Even so, simply describing the accuracy of the target location is still not adequate to the problem solution. Suppose, for example, that the observer can say that his target location is accurate within $\pm 150$ meters along the OT vector. Is this sufficiently accurate? If not, what about $\pm 100$ meters or $\pm 85$ meters? Obviously more information is required. One of the first issues to be resolved is the cost of being in error when entering Fire for Effect. This cost for any given error will be shown in Chapter III to be a function of the vulnerability of the target to attack by the particular weapons system being employed, the angle between the vectors OT and GT, the variances of the fall of shot distributions and the assurance of success demanded from the attack. This loss function is necessary to the analysis and its presence in the analytical arena with the probability function describing the error which the observer believes to be contained in his location of the target is sufficient to guide us, now acting the part of the observer, analytically to an intuitively appealing decision consistent with the information available.
The Optimum Sample Size

If the decision is not to enter Fire for Effect but first to sample the fall of shot, the analysis will lead us to the optimum sample size from which to assess the error with respect to the target in the observed mean point of impact and hence in the observer's target location. The optimum sample size (the number of adjusting rounds) will be achieved when the cost of sampling just balances the expected cost reduction to be attained from the resulting decrease in the observer's uncertainty as to the error in the location of the target.

Bayes' Theorem

The analysis is based on an application of Bayes' Theorem simply derived from equivalent statements of the joint probability of two events A and B.

\[ P(AB) = P(A|B) P(B) = P(B|A) P(A) \]

which yields immediately

\[ P(A|B) = \frac{P(B|A) P(A)}{P(B)} \]

If A is a state of nature, which for our purposes might be that the observer's error in determining the magnitude of the OT vector is less than some particular value, and B is some report such as a sample mean which we might receive containing information about A, then the probability of A holding on condition that the report B is received is given by Bayes' Theorem above.
Because of its dependence on Bayes' Theorem, the decision process which we describe here is termed Bayesian and its underlying structure is then Bayesian Decision Theory.
CHAPTER III

THE LOSS FUNCTION

The Measure of Effectiveness

As the measure of effectiveness on which to optimize the attack, we might consider dollar cost, degree of surprise achieved, amount of time required to engage the target, ammunition expenditure, accuracy, level of damage, unwanted side effects, and so forth. We elect ammunition expenditure as the phenomenon to be minimized subject to the attainment of some required level of damage. The optimization of this choice for the measure of effectiveness also tends to force other major candidates (cost, surprise, time, accuracy) in their respective directions of optimality.

The Parameters of the Loss Function

Given any particular combination of target, attacking weapon and observer, the combination to include their geographical orientation, we assume that the expected amount of ammunition to be expended in the attack is a function of four parameters of which only one is unknown and later will be considered a random variable. The parameters assumed as known are the vulnerability of a given target to the type of attacking weapon, the assurance of success demanded from the attack, and the distribution of the shot pattern from the attacking weapon at target range. The unknown parameter
is the accuracy of the observer's target location.

The Vulnerability Criterion

The vulnerability criterion is most conveniently defined for analytical purposes as the maximum allowable miss distance, at a given direction from the target center, from which the requisite damage effects from a single bursting round will be presented at the target. A concrete bunker would likely require a direct hit; therefore its vulnerability is described by its physical dimensions. A standing man might suffer lethal effects from a burst as much as 50 meters away. His envelope of vulnerability is a circle 100 meters in diameter with himself at its center. Vehicles dispersed rather evenly within a truck park of dimensions say 175 x 250 meters might be assigned a vulnerability envelope of 150 x 225 meters in order to obtain adequate coverage.

The assurance of success demanded from the attack can be made explicit by defining the minimum probability for achieving at least a given number of bursts within the vulnerability envelope. For example an attack on the truck park described above might be undertaken with a goal of placing at least six bursts within the 150 x 225 meter envelope with probability .90. In the judgment of the commander ordering the attack on this facility, it might be that an equivalent assurance of success can be achieved by demanding only
a single burst be placed with probability .90 within a reduced envelope of say 75 x 100 meters about the center of the park, thereby intimating that a sufficiency of those rounds which fall outside the reduced envelope are expected still to burst within the target perimeter and produce the necessary damage. The list of equivalent combinations of vulnerability envelope and assurance of success is virtually endless for any given attack situation.

The Single Shot Hit Probability $P_{SSH}$

The vulnerability criterion, the accuracy of the target location, and the shot pattern interact to form a composite parameter which we shall call the single shot hit probability $P_{SSH}$, the probability that a single round will fall within the vulnerability envelope.

If we insist, for analytical convenience, that the vulnerability envelope be defined as a rectangle with one of its two orthogonal axes parallel to the line of fire (GD or GD' of Figure 3), then

$$P_{SSH} = P_r P_d$$

where $P_r$ is the probability of placing a shot within the range limits and $P_d$ the probability of placing a shot within the deflection limits of the target when the mean point of impact is at $D$ or $D'$ depending on the direction of the error. The assumption of such a target orientation is not uncommon; conversion factors for various target bias angles (TGD or TGD') are given on page 2-29.
Figure 3: The geometry of an attack situation when the observer mislocates the target on the OT line.
of Artillery Cannon Gunnery [3] and their use in reorienting a target is thoroughly explained. Briefly, a rectangular vulnerability envelope may be reoriented from its true axis orthogonally to any line of fire and retain its original dimensions if the bias angle does not exceed 400 mils (22.5 degrees). The bias angles with which we shall be dealing will not exceed 100 mils and will seldom exceed 50 mils.

Recall from Chapter I that the geographical positions of the observer and the guns are known as is the direction of the observer-target vector OT. The error in the target location is then an error in magnitude only of the observer-target vector. In order to lessen any possible confusion in subsequent terminology, we shall call this error an error in the observer-target distance. Then not only the magnitude but also the direction of the error (e.g., the error might be 200 meters short of the target on the OT vector) will affect the range and deflection limits taken from the expected point of impact of the bursts and hence will affect the product

\[ P_{SSH} (\pm) = P_{r(\pm)} P_{d(\pm)} \]

where the subscripted signs indicate the direction of the error and the basic symbols remain as previously defined. Consider the diagrams of Figure 3.

For any given angle OTG, the angle included between the observer-target line and the gun-target line, let us determine
\( P_r(+) \) and \( P_d(+) \) if the observer mislocates the target at point \( D \) behind the target as seen by the observer, and also \( P_r(-) \) and \( P_d(-) \) if the observer mislocates the target at point \( D' \) in front of the target as seen by the observer. We shall find that for any two unique points \( D \) and \( D' \), equidistant from the center of the target at point \( T \) on the line \( OT \) extended, that \( P_{SSH(+) \text{ and } P_{SSH(-)} \) do not have necessarily the same value. The single shot hit probabilities \( P_{SSH} \) will be transformed into the loss functions for ammunition expenditure. If \( P_{SSH(+) \text{ and } P_{SSH(-)} \) have different values for the same magnitude of error in the target location, then it is apparent that an error in one direction will cost less than the same error in the other direction.

Figure 4 serves as the reference diagram for the derivation of the expressions for \( P_r(+) \) and \( P_r(-) \), while Figure 5 serves the same purpose for \( P_d(+) \) and \( P_d(-) \), respectively. Referring to Figure 4 and assuming that the observer mislocates the target at \( D \), it is apparent that the random variable \( r \) defining the range component of the burst of a shell fired to achieve an expected range \( GD = \mu_r \) must be restricted to the range interval (whose reflection on \( GD \) is \( EF \)) of the vulnerability envelope if a target hit is to be obtained. Therefore
Figure 4: The geometry for determining the probability $P$ that the fall of shot will impact within the range interval of the vulnerability envelope reflected onto the apparent gun-target line GD or GD'.

- OBSERVER
- VULNERABILITY ENVELOPE (POINT T MISLOCATED AT $D'$)
- VULNERABILITY ENVELOPE (POINT T MISLOCATED AT $D$)
- GUNS
\[ P_{r(+)} = \text{PROB} \left[ \mu_r - (DC + CF) \leq r \leq \mu_r - (DC - EC) \right] \]

\[ = \text{PROB} \left[ \left( \frac{DC - EC}{\sigma_r} \right) \leq Z \leq \left( \frac{DC + CF}{\sigma_r} \right) \right] \quad (3-1) \]

where \( \sigma_r^2 \) is the variance of the fall of shot range distribution at mean point of impact \( \mu_r \) and \( Z \) is the standardized random variable of the normal distribution.

Similarly we find, now taking \( \mu_r = GD \),

\[ P_{r(-)} = \text{PROB} \left[ \mu_r + (D'J - HJ) \leq r \leq \mu_r + (D'J + JL) \right] \]

\[ = \text{PROB} \left[ \left( \frac{D'J - HJ}{\sigma_r} \right) \leq Z \leq \left( \frac{D'J + JL}{\sigma_r} \right) \right] \quad (3-2) \]

We must find identities for the line segments in the expressions for \( P_{r(+)} \) and \( P_{r(-)} \) in terms of the known quantities \( GT \) (the gun-target range), \( \tau \) (the angle OTG), the one dimensional error vector \( TD \) or \( TD' \), and the dimensions \( a \times b \) of the assigned vulnerability envelope.

Given that \( PE, TC \) and \( SF \) are each perpendicular to \( GD \), we have immediately that \( EC = CF = 1/2 b \). We next determine an expression for the bias angle \( \alpha \). The angles \( \gamma, \tau, \) and \( \alpha \) are related by

\[ \gamma = \tau - \alpha \]

By the Law of Sines

\[ \frac{\sin \alpha}{\sin \gamma} = \frac{TD}{GT} = \frac{\sin \alpha}{\sin (\tau - \alpha)} \]
Letting $TD/GT = 1/K$, then

$$\sin (\tau - \alpha) = K \sin \alpha$$

Substituting the trigonometric identity for the sine of the difference of two angles,

$$\sin \tau \cos \alpha - \cos \tau \sin \alpha = K \sin \alpha$$

$$\sin \tau - \cos \tau \tan \alpha = K \tan \alpha$$

which yields

$$\alpha = \tan^{-1} \left[ (\sin \tau)/(K + \cos \tau) \right] \quad (3-3)$$

Similarly we find an expression for the bias angle $\beta$, first recognizing that if PL, TJ and SH are each perpendicular to GD, then HJ = JL = 1/2 b with the vulnerability envelope reoriented to the new axis GD'.

The angles $\gamma'$, $\tau$, and $\beta$ are related by

$$\gamma' = 180^0 - (\tau + \beta)$$

and so

$$\sin (\gamma') = \sin (\tau + \beta)$$

Again by the Law of Sines

$$\frac{\sin \beta}{\sin \gamma'} = \frac{TD'}{GT} = \frac{\sin \beta}{\sin (\tau + \beta)}$$

Since we are interested in the case where $TD' = TD$, we have

$$TD'/GT = TD/GT = 1/K \quad (3-4)$$
Then

\[ \sin (\tau + \beta) = K \sin \beta \]
\[ \sin \tau \cos \beta + \cos \tau \sin \beta = K \sin \beta \]
\[ \sin \tau + \cos \tau \tan \beta = K \tan \beta \]

which yields

\[ \beta = \tan^{-1} \left( \frac{\sin \tau}{K - \cos \tau} \right) \] \hspace{1cm} (3-5)

Referring still to Figure 4,

\[ DC = TD \cos (\tau - \alpha) \]

and

\[ D'J = TD' \cos (\tau + \beta) \]

If we let \( TD = TD' = \delta \), the magnitude of the observer's error, then substitution of the derived expressions in Eqn (3-1) and (3-2) yields

\[ P_{r(\tau)} = \text{PROB} \left[ \frac{\delta \cos(\tau - \alpha) - 1/2b}{\sigma_r} \leq Z \leq \frac{\delta \cos(\tau - \alpha) + 1/2b}{\sigma_r} \right] \] \hspace{1cm} (3-6)

\[ P_{r(-)} = \text{PROB} \left[ \frac{\delta \cos(\tau + \alpha) - 1/2b}{\sigma_r} \leq Z \leq \frac{\delta \cos(\tau + \alpha) + 1/2b}{\sigma_r} \right] \] \hspace{1cm} (3-7)

where \( \alpha \) and \( \beta \) are as previously defined by Eqns (3-3) and (3-5).

We now proceed to derive similar expressions for \( P_{d(+)} \) and \( P_{d(-)} \).

Given that UC is perpendicular to GD in Figure 5b, the situation where the observer mislocates the target at point D over the actual
Figure 5: The geometry for determining the probability $P$ that the fall of shot will impact within the deflection interval of the vulnerability envelope.
location, and realizing that the vulnerability envelope is again oriented to GD, we have

\[ TC = TD \sin \gamma = TD \sin (\tau - a) \]

Now

\[ TR = TU \quad 1/2a \]

so that

\[ RC = TC - TR \]

\[ = TD \sin (\tau - a) - 1/2 a \]

while

\[ UC = TC + TU \]

\[ = TD \sin (\tau - a) + 1/2 a \]

As before \( TD = \delta \), the magnitude of the observer's error, and \( \mu_d = 0 \) is the mean point of impact of the deflection distribution perpendicular to GD about D. Therefore, letting \( \sigma^2 \) be the variance of the deflection distribution of the random variable \( d \)

\[ P_{d(+)} = \text{PROB} \left[ \mu_d - UC \leq d \leq \mu_d - RC \right] \]

\[ = \text{PROB} \left[ RC/\sigma \leq Z \leq UC/\sigma \right] \]

Substituting for \( RC \) and \( UC \),

\[ P_{d(+)} = \text{PROB} \left[ \frac{\delta \sin (\tau - a) - 1/2 a}{\sigma_d} \leq Z \leq \frac{\delta \sin (\tau - a) + 1/2 a}{\sigma_d} \right] \]

(3-8)

Referring now to Figure 5a illustrating the geometry of the situation where the observer mislocates the target at point \( D' \) short
of the true location, we derive a similar expression for $P_d(\cdot)$.

Now with the vulnerability envelope reoriented to GD' and
given that WC' is perpendicular to GD' extended, we have

$$\text{TC}' = D' T \sin (180^\circ - \delta') = \delta \sin (\tau + \beta)$$

Also

$$TX = TW = 1/2 a$$

so that

$$XC' = \text{TC}' - TX$$

$$= \delta \sin (\tau + \beta) - 1/2 a$$

while

$$WC' = \text{TC}' + TW$$

$$= \delta \sin (\tau + \beta) + 1/2 a$$

and we have

$$P_d(\cdot) = \text{PROB}[\mu_d + XC' \leq d \leq \mu_d + WC']$$

$$= \text{PROB}[XC'/\sigma_d \leq Z \leq WC'/\sigma_d]$$

Substituting for XC' and WC',

$$P_d(-) = \text{PROB}\left[\frac{\delta \cos (\tau + \beta) - 1/2 b}{\sigma_d} \leq Z \leq \frac{\delta \sin (\tau + \beta) + 1/2 a}{\sigma_d}\right]$$

(3-9)

Summarizing:

$$P_{r(+)} = \text{PROB}\left[\frac{\delta \cos (\tau - a) - 1/2 b}{\sigma_r} \leq Z \leq \frac{\delta \cos (\tau - a) + 1/2 b}{\sigma_r}\right]$$

(3-10)

$$P_{d(+)} = \text{PROB}\left[\frac{\delta \sin (\tau - a) - 1/2 a}{\sigma_d} \leq Z \leq \frac{\delta \sin (\tau - a) + 1/2 a}{\sigma_d}\right]$$

(3-11)
\[ P_r(-) = \text{PROB} \left[ \frac{\delta \cos (\tau + \beta) - 1/2 b}{\sigma_r} < Z < \frac{\delta \sin (\tau + \beta) + 1/2 b}{\sigma_r} \right] \] (3-12)

\[ P_d(-) = \text{PROB} \left[ \frac{\delta \sin (\tau + \beta) - 1/2 a}{\sigma_d} < Z < \frac{\delta \sin (\tau + \beta) + 1/2 a}{\sigma_d} \right] \] (3-13)

where

\[ a = \tan^{-1} \left[ \frac{\sin \varphi}{K + \cos \tau} \right] \] (3-15)

\[ \beta = \tan^{-1} \left[ \frac{\sin \varphi}{K - \cos \tau} \right] \] (3-16)

\[ K = \frac{GT}{D^T} \tau = \frac{GT}{DT} \] (3-17)

We now have the necessary expressions with which to formulate the binomial equation which describes the number of rounds \( N \) to fire in order to attain some required level of assurance \( P_k \) that at least \( n \) of those rounds will fall within the vulnerability envelope \( a \times b \). It should be understood here that \( N \) is fixed prior to the attack and the rounds are fired "blind" in that no feedback or damage assessment is reported until after all rounds have burst in the target area.

\[ P_k = \sum_{i=n}^{N} \binom{N}{i} P_{SSH}^i (1 - P_{SSH})^{N-i} \] (3-18)

Unless \( n \leq 1 \), this binomial expansion is transcendental in \( N \). This transcendentalism prevents us obtaining a closed form expression for \( N \) although \( N \) can obviously be determined by numerical methods for any given combination of values \( P_{SSH} \) and \( P_k \). But this is not the only difficulty. American artillery is customarily employed in
batteries of two to six guns depending on the size of the weapons.

Our light and medium artillery batteries contain six guns each.

A target is attacked by all guns in the battery firing volley fire;
a volley being constituted by one round from each gun simultaneously.

Therefore, we are not concerned so much with the number of rounds
as with the number of volleys we need to fire.

The Single Volley Hit Probability $P_{SVH}$

A six gun battery is emplaced typically as shown in Figure 6.
The battery is illustrated as firing a "parallel sheaf" in which the
lines of fire from all guns are parallel. This is the simplest sheaf
to fire since all gun settings are identical. A "closed sheaf" im-
plies that the lines of fire intersect at the target while an "open
sheaf" diverges to some pre-selected width at target range.

Guns #1 and #2 comprise the right platoon, Guns #3 and #4 the
center platoon, and Guns #5 and #6 the left platoon. The battery
center, that point from which all firing data is computed, is
doctrinally [3, pg. 3-2] the geometric center of the battery. In
practice it might be anywhere within the area of the center platoon
as designated by the battery commander. We shall show shortly the
effect on expected ammunition expenditure if the battery center is
displaced by 20 meters from the geometric center. The breadth of
the battery is about 200 meters with 40 meters between guns, and
Figure 6: A representation of a six-gun battery firing a parallel sheaf.
the depth is about 50 meters. Obviously the burst point of each of the six rounds in any given volley is independent of the other five rounds. Based on the geometrical configuration of the battery and the location of the battery center, we can compute a single volley hit probability, \( P_{SVH} \), the probability that at least \( n \) rounds from a single volley will fall within the vulnerability envelope. If we make the assumption \( n = 1 \), then

\[
P_{SVH} = 1 - \prod_{i=1}^{6} (1 - P_{r_i} P_{d_i}) = 1 - \prod_{i=1}^{6} (1 - P_{SSH_i})
\]

(3-19)

where \( P_{r_i} \) is the probability of a round from Gun \( i \) falling within the range limits of the vulnerability envelope and similarly for \( P_{d_i} \).

The frustration encountered in Eqn (3-18) in obtaining a closed form expression for \( N \) if \( n > 1 \) is even more disheartening when dealing with the term \( P_{SVH} \) in Eqn (3-19). We choose to avoid this problem by assuming that the required assurance of success for the attack and the vulnerability criteria together can be expressed by the commander as a vulnerability envelope in which it is required at least one round burst with probability \( P_k \). We might justify this assumption by conceiving a vulnerability envelope sufficiently smaller than the target perimeter such that a single burst within the envelope implies some sufficiency of the remaining bursts outside the envelope but within some larger envelope. We asserted near the
beginning of this chapter that a vulnerability envelope existed for 
n = 1 which would produce equivalent damage to that expected from 
an envelope based on some n > 1.

Now if \( P_k \) is taken to be the probability that at least one vol­
ley will yield at least one hit within the vulnerability envelope, then 
the number of volleys \( N \) required is obtained by solution of the 
binomial probability Eqn (3-18) with \( P_{SSH} \) replaced by \( P_{SVH} \):

\[
\begin{align*}
P_k &= 1 - (1 - P_{SVH})^N \\
or \\
(1 - P_{SVH})^N &= 1 - P_k
\end{align*}
\]

This formulation for \( (1 - P_k) \) is only valid when \( P_{SVH} \) is known 
unconditionally. Since \( P_{SVH} \) as we have developed it is conditioned 
on a knowledge of the error \( \delta \), it is properly written \( P_{SVH} \mid \delta \).

We shall see in the next chapter that \( \delta \) can be considered a random 
variable with a probability density function which we symbolize here 
temporarily as \( f(\delta) \). Then in order to express \( (1 - P_k) \) as an un-
conditional function of \( \delta \) we must write, for the continuous case

\[
1 - P_k = \int_{\delta} (1 - P_{SVH} \mid \delta)^N f(\delta) \, d\delta \quad (3-20)
\]

The loss function \( N \) (which we recall represents the amount of 
ammunition to expend in Fire for Effect and which we desire to 
minimize subject to \( P_k \) being at least as great as that demanded 
by the commander's guidance) appears to be an implicit function
from a cursory examination of Eqn. (3-20). However since

\[ P_{SVH} | \delta \leq 1 \]

we can use the Binomial Theorem to expand

\[(1 - P_{SVH} | \delta)^N\]

to a polynomial of degree \(N\). For temporary convenience of notation let us symbolize \(P_{SVH} | \delta\) as \(p\). Then

\[
(1 - p)^N = 1 - Np + \frac{N(N - 1)p^2}{2!} - \frac{N(N - 1)(N - 2)p^3}{3!} + \ldots
\]

\[
= 1 - Np + \frac{N(N - 1)p^2}{2!} - R(p)
\]

where

\[0 < R(p) < \frac{N(N - 1)p^2}{2!}\]

Therefore, returning to the notation \(P_{SVH} | \delta\), we see that

\[(1 - P_{SVH} | \delta)^N \geq 1 - N(P_{SVH} | \delta)\] \hspace{1cm} (3-21)

the expression to the right of this inequality being a pessimistic approximation of \((1 - P_{SVH} | \delta)^N\). We now express Eqn. (3-20) as an inequality

\[1 - P_k \geq \int \left[ 1 - N(P_{SVH} | \delta) \right] f(\delta) \, d\delta\] \hspace{1cm} (3-22)

or, since the integral over all \(\delta\) of \(f(\delta)\) is identically unity, \(f(\delta)\) being a proper probability density function,

\[P_k \leq N \int \delta \left( P_{SVH} | \delta \right) f(\delta) \, d\delta\] \hspace{1cm} (3-23)

We now force this inequality to be an equality, thus forcing \(N\) to its lowest value which will still yield the desired \(P_k\).
The logic of this argument may seem difficult to grasp at first and the reader is cautioned to reflect on it carefully in order to achieve full understanding. \( P_k \) is fixed by the commander's guidance. The expression to the right of the inequality in Eqn. (3-23) will never be less than this value for \( P_k \). But since we do not wish unnecessarily to expend ammunition, we choose that value for \( N \) which will just meet the guidance. Therefore we desire the value of the integral multiplier of \( N \) to be as great as possible. This value will then force \( N \) to its minimum acceptable level yet still be in accord with the guidance establishing the value for \( P_k \). The manipulation of the integral multiplier in Eqn. (3-23) to yield its maximum value is the subject of Chapter VI.

Continuing our analytical argument, we have

\[
P_k = N \int_{\delta} (P_{SVH} | \delta) f(\delta) \, d\delta \quad (3-24)
\]

or, upon solving for \( N \),

\[
N = \frac{P_k}{\int_{\delta} (P_{SVH} | \delta) f(\delta) \, d\delta} \quad (3-25)
\]

Eqn. (3-25) is the loss function \( N \) for the attack. Its development has been the purpose of this chapter. It is the number of volleys to expend in the attack given an immediate call for Fire for Effect. \( P_k \) and \( P_{SVH} | \delta \) together contain all of the physical parameters of the attack situation and the commander's guidance,
while \( f(\delta) \) contains all of the currently available information on the location of the target as is presently known.

Our next consideration is the expression of \( P_{SVH,\delta} \) in a mathematical form amenable to convenient integration. We illustrate this technique with an example attack situation.

**An Example**

Some interesting insights can be gained by solving an example problem at various values of the parameters \( \tau \) and \( \delta \). Consider a 155 mm Howitzer battery (six guns) firing high explosive shells at a target at range 8000 meters with propelling charge \( 6 \). From Table 2 in Chapter I we obtain \( \sigma_r = 27/\cdot 6745 = 40.03 \) meters and \( \sigma_d = 4/\cdot 6745 = 5.93 \) meters. Assume that the battery is configured as shown in Figure 6 but that the depth of the battery is insignificant so that all guns are essentially on a line perpendicular to the line of fire. (The disposition of the guns along the battery depth dimension is arbitrary, while the lateral displacement is generally fixed at about 40 meters. Consequently, the assumption of no longitudinal displacement follows.) The distance between guns is taken as \( 7 \sigma_d = 40 \) meters. If the target is assigned a vulnerability envelope of 150 x 100 meters and \( P_k = 0.90 \), Eqn. (3-19) yields the values for \( P_{SVH} \) given the location of the battery center, the angle \( \tau \), and the observer's error \( \delta \).
Figure 7: The single volley hit probabilities $P_{SVH} \mid \delta$ conditioned on a knowledge of the observer's error $\delta$ in the location of the target for the following attack situations of the example problem:

(a) = 0 mils  
(b) = 400 mils  
(c) = 800 mils  
(d) = 1200 mils  
(e) = 1600 mils

LEGEND

□ Battery center at Gun #3
○ Battery center at Gun #4
● Data points from curve fitted to the data points of Table 5
Figures 7 graph $P_{SVH} | \delta$ as a function of the observer's error $\delta$ for values of $\tau$ at 0 mils, 400 mils ($=22.5^0$), 800 mils ($=45^0$), 1200 mils ($=67.5^0$) and 1600 mils ($=90^0$).

The data points graphed in Figures 7 are summarized in Tables 3 and 4. Notice the effect on $P_{SVH} | \delta$ of changes in the angle $\tau$. Also for any given angle $\tau$ it seems that $P_{SVH} | \delta$ is affected significantly at the larger values of $\delta$ by the choice of location for battery center. In fact the asymmetry of the curves is reversed when the battery center is changed from Gun #3 to Gun #4. This asymmetry is directly a result of taking the battery center coincident with one of the guns of the center platoon. With the center at Gun #3, a positive error $\delta$ allows the sheaf of Guns #4 through #6 to pass to the left of the plotted target location at point D, while a corresponding negative error $\delta$ allows the sheaf of only two guns, #1 and #2, to pass to the right of the plotted location at $D'$. The explanation for the lack of a mirror image between the data points $P_{SVH} | \delta$ when the battery center is shifted to Gun #4 lies in the fact that, except for angles $\tau$ of 0 mils and 1600 mils, the bias angles $\alpha$ and $\beta$ are unequal for positive and negative errors of the same magnitude. This leads us to conclude that the geometric center of the battery is not necessarily the point from which to realize minimum asymmetry in the data points $P_{SVH} | \delta$. Depending on the
Table 3

The single volley hit probabilities $P_{SVH}^{\delta}$ for the attacks described by the example situations if the error in the target location is $\delta$ meters and the battery center is located at Gun #4.

<table>
<thead>
<tr>
<th>Error $\delta$ (meters)</th>
<th>Angle $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 mils</td>
</tr>
<tr>
<td>+200</td>
<td>0.0003</td>
</tr>
<tr>
<td>+175</td>
<td>0.0029</td>
</tr>
<tr>
<td>+150</td>
<td>0.0196</td>
</tr>
<tr>
<td>+125</td>
<td>0.0936</td>
</tr>
<tr>
<td>+100</td>
<td>0.2983</td>
</tr>
<tr>
<td>+ 75</td>
<td>0.6230</td>
</tr>
<tr>
<td>+ 50</td>
<td>0.8813</td>
</tr>
<tr>
<td>+ 25</td>
<td>0.9770</td>
</tr>
<tr>
<td>0</td>
<td>0.9918</td>
</tr>
<tr>
<td>- 25</td>
<td>0.9770</td>
</tr>
<tr>
<td>- 50</td>
<td>0.8813</td>
</tr>
<tr>
<td>- 75</td>
<td>0.6230</td>
</tr>
<tr>
<td>-100</td>
<td>0.2983</td>
</tr>
<tr>
<td>-125</td>
<td>0.0936</td>
</tr>
<tr>
<td>-150</td>
<td>0.0196</td>
</tr>
<tr>
<td>-175</td>
<td>0.0029</td>
</tr>
<tr>
<td>-200</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
The single volley hit probabilities $P_{SVH}$ for the attacks described by the example situations if the error in the target location is 6 meters and the battery center is located at Gun #4.

<table>
<thead>
<tr>
<th>Error $\delta$</th>
<th>Angle $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 mils</td>
</tr>
<tr>
<td>+200</td>
<td>.0003</td>
</tr>
<tr>
<td>+175</td>
<td>.0029</td>
</tr>
<tr>
<td>+150</td>
<td>.0196</td>
</tr>
<tr>
<td>+125</td>
<td>.0936</td>
</tr>
<tr>
<td>+100</td>
<td>.2983</td>
</tr>
<tr>
<td>+ 75</td>
<td>.6230</td>
</tr>
<tr>
<td>+ 50</td>
<td>.8813</td>
</tr>
<tr>
<td>+ 25</td>
<td>.9770</td>
</tr>
<tr>
<td>0</td>
<td>.9918</td>
</tr>
<tr>
<td>- 25</td>
<td>.9770</td>
</tr>
<tr>
<td>- 50</td>
<td>.8813</td>
</tr>
<tr>
<td>- 75</td>
<td>.6230</td>
</tr>
<tr>
<td>-100</td>
<td>.2983</td>
</tr>
<tr>
<td>-125</td>
<td>.0936</td>
</tr>
<tr>
<td>-150</td>
<td>.0196</td>
</tr>
<tr>
<td>-175</td>
<td>.0029</td>
</tr>
<tr>
<td>-200</td>
<td>.0003</td>
</tr>
</tbody>
</table>
quadrant of the angle $\tau$, minimum asymmetry will be achieved when the battery center is biased to the right or left of the geometric center along the axis of the battery. This is not a point which need be emphasized further; the imponderables of practical situations seem to mitigate against attempting to determine some optimum non-geometric center for the battery. Since each battery commonly answers calls for fire from a number of observers we shall take arbitrarily the geometric center as optimum and later force symmetry on $P_{SVH}$ | $\delta$. (See Appendix A for the values of $P_{ri}$ and $P_{di}$ used in Eqn. (3-19) for computing the data points of Figures 7 and Tables 2 and 3.)

From a study of these data points some inferences might be made as to the optimum value or values of the angle $\tau$. If we could be assured that the magnitude of the observer’s error $\delta$ would not exceed 150 meters as he enters Fire for Effect in this example attack situation, then it would seem we should favor the larger angles of 1200 to 1600 mils. However, at these angles the observer is at a decided disadvantage during the sampling process in that the larger range dispersion of the shot pattern might be observed as significant deviations from the OT line even when the mean point of impact is on that line. Consequently the observer finds it difficult to sense sample deviation components along the OT line when the deflection
deviation components are large. In practice, this phenomenon is observable for angles \( \tau \) as low as 500 mils [3, pg. 18-8]. Since the function \( P_{SVH} | \delta \) falls most steeply and therefore is most undesirable for angle \( \tau = 0 \) where deviations are most easily observed, it would appear that a trade-off exists between costs and ease of sample assessment. We make no attempt here to resolve this trade-off if indeed it does exist; its psychophysical aspects are beyond the scope of this investigation. However in Chapter VII we shall obtain some hints as to where to look for the best trade-off from a strictly technical viewpoint.

It is appropriate to note that Figures 7 and Tables 3 and 4 hold for angles \( \hat{\tau} = \tau + 3200 \text{ mils} \) if the signs of the arguments \( \delta \) are reversed. They hold for angles \( \hat{\tau} = 1600 \text{ mils} - \tau \) if both the signs of the arguments \( \delta \) and the locations of the battery center are reversed, and if \( \hat{\tau} = -\tau \) they hold if just the locations of the battery center are reversed.

Fitting the Function \( P_{SVH} | \delta \)

From such data points we can fit a continuous function to approximate \( P_{SVH} | \delta \) for use in the integral in the denominator of Eqn. (3-25), given any set of values for the terms

a. \( \sigma_r = \) gun-target range standard deviation for the chosen propelling charge
b. \( \sigma_d \) = gun-target deflection standard deviation for the chosen propelling charge

c. \( \tau \) = angle OTG, the horizontal clockwise angle between the gun-target line and the observer-target line measured from the gun-target line

d. \( \delta \) = observer's target location error vector

e. \( a \times b \) = target vulnerability envelope in meters where \( a \) is the deflection spread and \( b \) the range spread as viewed from the guns.

f. \( P_k \) = the demanded assurance for success

g. \( \mu_r \) = the apparent gun-target range GD or GD'

and the physical disposition of the guns in the battery position area.

We shall find it analytically convenient if the continuous fitted function \( P_{SVN} \) is symmetrical about \( \delta = 0 \). Since the skewedness reverses direction with a reversal of the location of battery center between the extreme points at Gun #3 and Gun #4, it must be as we stated previously that minimum asymmetry is obtained at some intermediate location. We assume this location is known, and taking it as the battery geometric center, force symmetry on the loss functions obtained when the battery center is taken at Gun #3 and Gun #4 respectively.

Arbitrarily, we determine the symmetrical data points at \( \pm \delta \)
Table 5
The single volley hit probabilities $P_{SVH} | \delta$ obtained from forcing symmetry on the combined data points of Tables 3 and 4 such that the battery center is at or near the geometrical center.

<table>
<thead>
<tr>
<th>Error</th>
<th>Angle $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 mils</td>
</tr>
<tr>
<td>200</td>
<td>.0003</td>
</tr>
<tr>
<td>175</td>
<td>.0029</td>
</tr>
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<td>.6230</td>
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<td>50</td>
<td>.8813</td>
</tr>
<tr>
<td>25</td>
<td>.9970</td>
</tr>
<tr>
<td>0</td>
<td>.9918</td>
</tr>
</tbody>
</table>
by averaging the values in Tables 3 and 4. The resulting data points are shown in Table 5. The data points at $\delta = 200$ meters for angles $\tau$ of 1200 mils and 1600 mils have been omitted in favor of the extrapolated values to be obtained from the fitted function.

Exponential functions are fitted to the data points of Table 5; the exponential form being chosen because of its compatibility with the mathematics of Chapter VI. By standard regression techniques, it is first determined that a function of the form $ae^{-b\delta^2}$, where $a$ and $b$ both are positive, represents the best fit. However, the least squares estimate of the best fit forces the constant $a$ to be greater than unity for each value of the angle $\tau$. Since for $\delta = 0$, the single volley hit probability $P_{SVH}\delta = ae^{-b\delta^2}$ would be equal to $a$ which is greater than unity and thus incompatible with our notions of probabilities, we fix the constant $a$ at some more reasonable value (the choice here was $a = 0.992$ which approximates $P_{SVH}\delta$ for $\delta = 0$ in Table 5) and proceed to determine the constant $b$ alone by least squares estimation, having previously fixed $a$. The form $ae^{-b\delta^2}$ for $P_{SVH}\delta$ also serves to smooth out the "W-effect" in the interval $-50 \leq \delta \leq 50$ meters for the larger values of the angle $\tau$. This "W-effect" results from a unique combination of target vulnerability envelope dimension, the fall of shot deflection standard deviation $\sigma_d$, and the distance separating the guns. It seems to be a
transient phenomenon whose interpretation is of no great importance.

The fitted functions are made explicit below for our example attack situations. The fitted data points are, in addition, superimposed on the graphs of Figures 7.

a. For $t = 0$:

$$P_{SVH} \quad \delta = 0.992 \exp [0.000188 \delta^2]$$

b. For $t = 400$ mils:

$$P_{SVH} \quad \delta = 0.992 \exp [0.000156 \delta^2]$$

c. For $t = 800$ mils:

$$P_{SVH} \quad \delta = 0.992 \exp [0.0000940 \delta^2]$$

d. For $t = 1200$ mils:

$$P_{SVH} \quad \delta = 0.992 \exp [0.0000348 \delta^2]$$

e. For $t = 1600$ mils:

$$P_{SVH} \quad \delta = 0.992 \exp [0.0000188 \delta^2]$$

Summary

We have seen how a function $P_{SVH} \mid \delta$ might be developed so as to relate explicitly the ammunition expenditure $N$ to the commander's demanded assurance of success for the attack $P_k$, the physical parameters of the attack situation and the probability density function for the error $\delta$ in the magnitude of the vector $OT$. This fitted
function $P_{SVH} \mid \delta$ has been shown to be essentially exponential in form with the exponent being quadratic in the error $\delta$, thereby making the natural logarithm of this function parabolic with its vertex at $\delta = 0$. The focus of the parabola is particularly sensitive to the angle $\tau$. The loss function is then defined as

$$N(\tau) = \frac{P_k}{\int_{\delta} e^{-b \delta^2} f(\delta) d\delta}$$

(3-26)

where we have identified $N$ as being functionally dependent on the angle $\tau$. Hereafter we will refer to the loss function as $N$, implicitly understanding the functional dependence.

The usefulness of this function was intimated in Chapter II. As a by-product of its development here we have gained some insight into the interactions among the various parameters of the field artillery gunnery problem. We leave the loss function now until Chapter VI where its specific use in the economic analysis will be made clear.
CHAPTER IV

THE PRIOR DISTRIBUTION

Introduction

One of the functions of an artillery observer is to detect and locate targets in his sector of responsibility which are appropriate for attack by field artillery. As the observer detects such a target he immediately begins to process all available information for an accurate location. This information includes the direction and approximate distance to the target, its proximity to known points such as road and stream junctions or previous target locations and his experience as an observer and his familiarity with the terrain in the tactical area. With such information he proceeds to refine the target location as best he can within the time permitted by the tactical situation. Quite soon he will have settled upon some initial location, identified by its map coordinates or a polar plot from a known point, which he communicates to the firing battery as part of his Call for Fire. This initial target location is the observer's estimate at this time of the most likely location of the target. If he is sufficiently confident of the accuracy of this estimate he will so indicate in the Call for Fire by immediately requesting Fire for Effect. Otherwise he will call for Adjust Fire so as to sample the fall of shot of a portion of the battery, usually the guns.
of the center platoon, for their mean point of impact with which he

can then revise his initial target location until he has sufficient

certainty in his data to enter Fire for Effect with the entire

battery.

The salient feature of the above description is the uncertainty

on the part of the observer as to the true location of the target. Let

us make some simplifying but hopefully realistic assumptions which

will allow us to examine the phenomenon of uncertainty on a quanti- 

tatively descriptive level. First we suppose that the observer can
determine the direction to the target at least to an accuracy of 10
mils so that the deflection error in his initial target location does
not exceed 10 meters per 1000 meters of distance to the target.

From this we henceforth neglect the deflection error. Next we

assume that the location of the observer himself is known so that if
we invoke both of these assumptions together, the observer’s

uncertainty about the true location of the target is reduced to

uncertainty about only the true distance to the target. In Chapter I

we showed that given these two assumptions it was necessary only
to assess accurately the magnitude of the observer-target vector $OT$
in order properly to orient the three major elements of the firing
problem: the guns, the target and the observer.

Because of his uncertainty about the true distance to the target,
we shall consider the observer's estimate of that distance for any given target to be a random variable. The distribution of that estimate as a random variable is the subject of this chapter.

The Prior Distribution of the Distance $R$

If we can elicit from the observer his subjective probability distribution function for the distance $R$ to the target (the magnitude of the vector $OT$ and true target location), we have the classical function known in Bayesian Decision Theory as the prior distribution of $R$, it being made explicit prior to receiving active sample information such as the fall of shot of the adjusting rounds. The density function describing this prior distribution of $R$ we symbolize as

$$PR(R) = \mathcal{G}(\mu_{pr}, V_{pr})$$  \hspace{1cm} (4-1)$$

where $\mathcal{G}$ represents the form of the distribution having prior mean $\mu_{pr}$ and prior variance $V_{pr}$. If we choose to code the distance $R$ by subtracting from it some known constant, say $\omega$, then letting $m = R - \omega$ the distribution of $m$ has the same form as that of $R$. Specifically,

$$E[m] = E[R - \omega] = \mu_{pr} - \omega = m_{pr}$$  \hspace{1cm} (4-2)$$

and

$$V[m] = V[R - \omega] = V[R] = V_{pr}$$  \hspace{1cm} (4-3)$$
Therefore we write

\[ PR(m) = \mathcal{F}(m_{pr}, V_{pr}) \]  \hspace{1cm} (4-4)

We shall see that such coding is quite convenient when we take \( \omega \) as the observer's prior estimate of the distance to the target which is reported to the fire direction center in the Call for Fire. With this in mind it is helpful to realize that

\[ -m = \omega - R = \delta \]  \hspace{1cm} (4-5)

and the expectation

\[ -m_{pr} = E[\omega - R] = E[\delta] \]  \hspace{1cm} (4-6)

where \( E[\delta] \) is the expected value of the observer's error \( \delta \), the variable in the loss function of Chapter III. The distribution of \( \delta \) is of paramount importance to our analysis and will be discussed fully later in this chapter.

**Eliciting the Prior Distribution**

As part of this investigation, a field experiment was conducted at the US Army Field Artillery School, Fort Sill, Oklahoma, from 24 February 1969 through 1 March 1969. Twenty-seven field artillery observers were individually interviewed in an attempt to elicit their subjective probability distributions \( PR(m) \) for the location of actual ground targets.

The interviews were conducted on the firing range and the subjects were student officers or officer candidates being trained in
observed fire techniques. Each student was equipped with binoculars and a battlemap of scale 1:25,000 on a Universal Tranverse Mercator Grid on which was plotted his own position (the observation post) and radial direction lines at 100 mil increments across the target area as explained in Chapter I. The interviews were obtained in conjunction with scheduled firing exercises during which the students were graded on their conduct of assigned observed fire problems. The instructor identified a particular target to all students by its description (white car body, e.g.) and its angular displacement from a reference marker at known direction. Having given the students a few minutes to locate the target and prepare their Calls for Fire, the instructor then selected one student to conduct the mission. If no student was being interviewed at the time, the student selected to conduct the fire mission was directed to the interviewer while the class continued with another problem until the interview was completed, at which time he resumed his place in the normal sequence of events to demonstrate his firing ability. All targets were located by their UTM grid coordinates.

During each interview the subject was guided carefully to an assessment of the cumulative distribution function of PR(m). He was cautioned repeatedly to answer no question he did not fully understand. The 1, 25, 50, 75 and 99 per cent fractiles of PR(m) were
elicited. The questionnaire used during the interviews is reproduced in Appendix C as also are the response sheets prepared on each subject. The questionnaire is adapted from Winkler [18] who demonstrated the feasibility of attempting to extract from an individual a subjective prior probability distribution.

Winkler reported that many of his subjects showed an affinity for the normal distribution (in estimating the parameter of a Bernoulli process). Significantly or no, our subjects showed a marked preference for the uniform distribution even though the interview was intended assiduously to avoid biasing the subject in favor of any one distribution or even suggesting, however indirectly, the form of an "acceptable" distribution. The reason for this propensity to the uniform distribution is as yet not fully explained. However, reflection by this investigator on his own experiences in estimating target distances and similar phenomena indicates that the mental process through which an observer refines his estimate of the location of a target (which he must report as a point estimate to the fire direction center) can be described as a series of decisions which successively reduce the span of the interval on the OT vector within which the observer believes the target to be located. Eliciting the prior distribution is then a reversal of this process in which weights are assigned to each of the successively smaller sub-intervals. It is
obvious that the larger the interval the more at ease is the observer (the decision maker) in asserting that it contains the target. It could be then that any decrease in the size of the parent interval [the range of the random variable \( m \) of \( PR(m) \)] is accompanied by a proportional increase in the uncertainty on the part of the observer that the target is included in the sub-interval. The proportionality of this increase in uncertainty would then account for the uniformity of \( PR(m) \).

Raw graphs of each subject's cumulative prior probability distribution are presented in Appendix D together with its implied distribution function determined by a least squares estimate of the best fit. These fitted functions are summarized in Table 6.

Of the 27 subjects interviewed, six were disqualified: one because of an inability to respond to the questionnaire, another because it was later determined that the true location of the target was questionable, and the remainder because they apparently had grossly misoriented their maps and were attempting to locate their assigned targets along direction lines whose errors could not be attributed reasonably to errors of interpolation or minor misorientations, hence their initial estimates of the target locations were judged to be invalid for the purposes of this experiment.

During that portion of the interview concerned with the initial
Table 6
Summary of fitted prior distributions from field experiment of 24 February through 1 March 1969. The origin is taken at that point on the OT vector corresponding to the observer's estimate of the distance to the target.

<table>
<thead>
<tr>
<th>Subject Number</th>
<th>PR(m)$^a$</th>
<th>Subject Number</th>
<th>PR(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>U(-656.8, 636.9)</td>
<td>15</td>
<td>U(-720.5, 440.5)</td>
</tr>
<tr>
<td>2</td>
<td>U(-215.2, 215.2)</td>
<td>16</td>
<td>U(-361.0, 321.0)</td>
</tr>
<tr>
<td>3</td>
<td>U(-105.3, 67.3)</td>
<td>17</td>
<td>U(-151.9, 101.9)</td>
</tr>
<tr>
<td>4</td>
<td>Disqualified</td>
<td>18</td>
<td>U(-125.2, 145.2)</td>
</tr>
<tr>
<td>5</td>
<td>U(-183.7, 127.7)</td>
<td>19</td>
<td>Disqualified</td>
</tr>
<tr>
<td>6</td>
<td>U(-230.6, 230.6)</td>
<td>20</td>
<td>U(-417.9, 437.9)</td>
</tr>
<tr>
<td>7</td>
<td>U(-400.2, 340.2)</td>
<td>21</td>
<td>U(-511.2, 591.2)</td>
</tr>
<tr>
<td>8</td>
<td>N(0, 1361.89)</td>
<td>22</td>
<td>Disqualified</td>
</tr>
<tr>
<td>9</td>
<td>Disqualified</td>
<td>23</td>
<td>U(-487.6, 599.6)</td>
</tr>
<tr>
<td>10</td>
<td>U(-659.9, 659.9)</td>
<td>24</td>
<td>Disqualified</td>
</tr>
<tr>
<td>11</td>
<td>U(-615.8, 935.8)</td>
<td>25</td>
<td>U(-966.4, 1326.4)</td>
</tr>
<tr>
<td>12</td>
<td>U(-419.0, 459.0)</td>
<td>26</td>
<td>U(-837.1, 797.1)</td>
</tr>
<tr>
<td>13</td>
<td>Disqualified</td>
<td>27</td>
<td>U(-296.7, 288.7)</td>
</tr>
<tr>
<td>14</td>
<td>U(-195.1, 175.1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$U(a, b) symbolizes that m is uniformly distributed on the interval (a, b); N(m$^{pr}$, V$^{pr}$) symbolizes that m is normally distributed with mean m$^{pr}$ and variance V$^{pr}$. 
estimate of the target location, each subject was cautioned repeatedly to think only of his uncertainty about the true location of the target and to divorce from his mind any consideration of his uncertainty about where the initial sampling rounds would fall. However, in preparing the questionnaire, it was anticipated that some subjects would bias their estimates as a result of previous experiences from which they derived, so they felt, a tendency to over or underestimate distances. A subterfuge was employed to reveal such tendencies by way of asking the subject to anticipate the observer-target distance component of the mean point of impact of the initial two sampling rounds. Since in all but one case the mean point of impact of the fall of shot had been previously adjusted to near (50 meters) coincidence with a known point in the target area (this was essentially a crude form of the precision registration procedure), it could be assumed that the expected mean point of impact of these initial sampling rounds was the observer's estimate of the initial target location. Surprisingly only 7 of the 21 qualifying subjects were indifferent as to whether the sample rounds would fall over or short of the target as they viewed it. Data reduction showed that of the 14 subjects who revealed a bias, only 8 were biased in the proper direction. The correct interpretation of this revelation is unknown; therefore no attempt has been made to adjust the assessed distributions for bias of this type.
Appendix B contains statistical analyses of the estimating errors in the observer's initial target locations. Of particular interest to our present discussion is the analysis of data in Table 20, the angular error in the observer's estimate. Our assumption has been that the observer can determine accurately (± 10 mils) the direction to the target, confining his uncertainty to the distance to the target. This results in a polar plot of \((\phi, R)\) where \(\phi\) has known value and \(R\) is the unknown random variable to be estimated.

Unfortunately, in the environment of our field experiment, the students were required in this phase of their instruction to locate the targets by their UTM grid coordinates. This seems to have resulted in an excessive dispersion of the angular error from what might be expected using the polar plot method for locating targets. The observed standard deviation was 31.4 mils implying a probable deflection error in the target location of approximately 42 meters for targets at 2000 meters distance. The mean angular error is most assuredly zero. That \(\phi\) may be considered a known constant in any polar plot estimate of target location is retained as a viable assumption from the practical experience of this author. The above results are provided here for the reader's own evaluation of the validity of this assumption.
The Prior Distribution of the Error

To this point we have dealt with the prior distribution of the coded distance \( m = R - \omega \) to the target along the OT vector. The loss function of Chapter III is defined in terms of the error \( \delta \) in the observer's estimate of \( m \). The random variable \( \delta \) is much more convenient to work with than is \( m \). Therefore we would like to determine the prior distribution of \( \delta \) or \( \text{PR}(\delta) \). Figure 8 illustrates the simple relationship between \( \text{PR}(m) \) and \( \text{PR}(\delta) \). The transformation to \( \text{PR}(\delta) \) is particularly easy if \( \text{PR}(m) \) is symmetrical about the mean \( m_{\text{pr}} \) as is the case with the uniform and normal forms of \( \text{PR}(m) \) obtained from our experiment which were summarized in Table 6. When we can claim symmetry, the expression for \( \text{PR}(\delta) \) is identical to that for \( \text{PR}(m) \). If \( \omega \) is taken as the observer's prior estimate of the distance \( R \) to the target, then \( m_{\text{pr}} = \mathbb{E}[R] - \omega \) from Eqn. (4-2). But \( \omega - R \) patently represents the error \( \delta \) contained in that estimate. Since the coded estimate \( m = R - \omega \), we see directly that

\[
\text{PR}(\delta) = \text{PR}(-m) \quad (4-7)
\]

as we intimated in Eqn. (4-5). Then if we claim symmetry for \( \text{PR}(m) \)

\[
\text{PR}(\delta) = \mathcal{F}(-m_{\text{pr}}, V_{\text{pr}}) \quad (4-8)
\]

where \( \mathcal{F} \) symbolizes the form of \( \text{PR}(m) \).
Figure 8: Relationships between the prior probability density functions of $R$, $m$, and $\delta$. 

\[
E[R] = \mu_{pr}
\]
We shall find this transformation extremely useful to our purposes.

**A Revised Form of PR(δ)**

If we can show later that the form of the posterior distribution of the error δ as a result of the infusion of sample information is relatively insensitive to the assumption that our prior distributions are uniform, we may wish to revise our assumption on the form of PR(δ).

Suppose the revised form of PR(δ) is the normal distribution. We might transform the old PR(δ) obtained from our field experiment into the revised form by retaining the mean (-m<sub>pr</sub>) and imputing a revised variance V<sub>pr</sub> using the 1% and 99% fractile limits of the uniform distribution as the ± 3 - sigma units of the revised normal distribution. Specifically we set

\[ \delta \sigma = 0.98 (b - a) \] (4-9)

where (b - a) is the span of the uniform distribution. This yields

\[ V_{pr} = [16.7(b - a)]^2 \] (4-10)

and our revised PR(δ) becomes

\[ PR(\delta) = N[-m_{pr}, V_{pr}] \] (4-11)

where V<sub>pr</sub> is now given by Eqn. (4-10). If for convenience we let

\[ M_{pr} = -m_{pr} \]

we can write

\[ PR(\delta) = N[M_{pr}, V_{pr}] \] (4-12)
Table 7 summarizes the transformation of the prior distributions from our field experiment into the normal form of Eqn. (4-12).
Table 7
Summary of prior distribution function parameters after transformation of PR(δ) to the normal form. The origin is taken at that point on the OT vector corresponding to the observer's estimate of the distance to the target.

<table>
<thead>
<tr>
<th>Subject Number</th>
<th>PR(δ) = N[M_{pr}, V_{pr}]</th>
<th>Subject Number</th>
<th>PR(δ) = N[M_{pr}, V_{pr}]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M_{pr}</td>
<td>V_{pr}</td>
<td>M_{pr}</td>
</tr>
<tr>
<td>1</td>
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<td>44652.14</td>
<td>15</td>
</tr>
<tr>
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<td>0</td>
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<td>16</td>
</tr>
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<td>3</td>
<td>19</td>
<td>795.04</td>
<td>17</td>
</tr>
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<td>5</td>
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<td>2587.42</td>
<td>18</td>
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<tr>
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</tr>
<tr>
<td>14</td>
<td>10</td>
<td>3656.20</td>
<td></td>
</tr>
</tbody>
</table>

b Missing numbers identify subjects disqualified from the experiment.
CHAPTER V

THE PREPOSTERIOR AND POSTERIOR DISTRIBUTIONS

Definitions

The preposterior distribution is defined as the prior probability function of the mean of the posterior probability density function of the error $\delta$ or $\text{PR}(M_{p0})$. The posterior distribution of the error $\delta$, given sample information, we symbolize as $\text{PO}(\delta \mid m_s)$ where $m_s$ is the sample mean.

The Posterior Distribution of the Error

Our first concern is with the transformation of the prior distribution $\text{PR}(\delta)$ into the posterior distribution $\text{PO}(\delta \mid m_s)$ as a result of having observed the mean $m_s$ of a sample of size $n$. It is important here for us to recognize that the distribution of the sample mean $m_s$ is controlled by the distribution of the fall of shot, a process which in actuality is completely independent from any uncertainty about the target location. However as we shall consider them, the distribution of the mean point of impact of the fall of shot and the random variable $\delta$ are mathematically interrelated.

Consider the distribution of the fall of shot along the observer-target line, given its mean point of impact at the origin of a rectangular grid oriented so that the Y-axis is along the apparent gun target line (GD or GD'). Figure 9 illustrates the problem; the points $O$, $D$, $G$ and the angle $\tau$ retain their meanings from Chapter III. If $x$ is
Figure 9: Rotation of the axes $X', Y'$ to $XY$ such that $x = x \cos (90^\circ - \gamma) + y \sin (90^\circ - \gamma) = x \sin \gamma + y \cos \gamma$. 

\[ x = x \cos (90^\circ - \gamma) + y \sin (90^\circ - \gamma) = x \sin \gamma + y \cos \gamma. \]
a random variable distributed according to the deflection dispersion of the fall of shot and $y'$ is a random variable distributed according to the range dispersion of the fall of shot, then the random variable $x = x' \sin \gamma + y' \cos \gamma$ is distributed along the observer-target line about the apparent target location $D$ (or $D'$).

Since $x'$ and $y'$ are each normally distributed on orthogonal axes through the origin at $D$ such that

\begin{align*}
    x' &\approx N(0, \sigma_d^2) \\
    y' &\approx N(0, \sigma_r^2)
\end{align*}

then

\[ x = x' \sin \gamma + y' \cos \gamma \] (5-3)

is normally distributed along the observer-target line with mean zero (at the point $D$) and variance $(\sigma_d^2 \sin^2 \gamma) + (\sigma_r^2 \cos^2 \gamma)$, or

\[ x \approx N[0, (\sigma_d^2 \sin^2 \gamma) + (\sigma_r^2 \cos^2 \gamma)] \] (5-4)

However, if the observer makes an error $\delta$ in his estimate of the distance to the target, then letting $X = x + \delta$ we have\(^1\)

\[ X \approx N[\delta, (\sigma_d^2 \sin^2 \gamma) + (\sigma_r^2 \cos^2 \gamma)] \] (5-6)

It is well known that the mean of a sample of $n$ observations of the normally distributed variable $X$, where

\[ m_s = \frac{1}{n} \sum_{i=1}^{n} X_i \] (5-7)

\(^1\)The angle $\angle DOG = \gamma$ can be measured by the fire direction center. It approximates the angle $\angle DOT = \tau$ used in the loss function. In practice, therefore the loss function would be chosen using $\gamma$ as the "best estimate" of $\tau$. 

is also distributed normally. For the case here under discussion

\[ m_s \approx N \left[ \delta, \frac{(\sigma_\delta \sin \gamma)^2 + (\sigma_r \cos \gamma)^2}{n} \right] \]  

(5-8)

From Eqn. (5-8) we have the likelihood of the sample mean given the error \( \delta \), it being the probability density function of \( m_s \) which we write as \( LK(m_s | \delta) \).

According to the logic of Bayes' Theorem [Chapter II], the posterior distribution of the error \( \delta \) is obtained from the relationship

\[ P_O(\delta | m_s) = \frac{LK(m_s | \delta)PR(\delta)}{\int_\delta LK(m_s | \delta)PR(\delta) d\delta} \]  

(5-9)

Following Morris [12, pg. 73], we examine Eqn. (5-9) to determine the form of the probability density function \( P_O(\delta | m_s) \).

Letting \( V_{m_s} = \left[ (\sigma_\delta \sin \gamma)^2 + (\sigma_r \cos \gamma)^2 \right] /n \),

\[ P_O(\delta | m_s) = \frac{\exp \left[ -\frac{1}{2V_{m_s}} (\delta - m_s)^2 \right]}{\sqrt{2\pi V_{m_s}}(b - a) \int_\delta LK(m_s | \delta)PR(\delta) d\delta} \]  

(5-10)

for \( \delta \) distributed uniformly in the interval \([a, b]\). The integral of \( P_O(\delta | m_s) \) over the range of \( \delta \) must be unity. Hence the integral in the denominator must have the value

\[ \int_\delta LK(m_s | \delta)PR(\delta) d\delta = \frac{1}{b - a} \left[ F_N(B) - F_N(A) \right] \]  

(5-11)

where
\[ F_N(x) = \text{standardized cumulative normal distribution function evaluated at } x \]

\[ A = \frac{a - m_s}{V_{m_s}} \]

\[ B = \frac{b - m_s}{V_{m_s}} \]

The posterior distribution of the observer's error given sample information \( PO(\delta | m_s) \) is now exposed as a doubly truncated normal distribution arising from a uniform prior distribution and a normal sample distribution. With this illumination we might be amenable to a revision of the form of \( PR(\delta) \) from the uniform distribution to a normal distribution about the mean \( M_{pr} \) but with variance \( V_{pr} \) obtained as set forth in the last section of Chapter IV.

We must now revise Eqn. (5-10) for the posterior distribution \( PO(\delta | m_s) \) using our altered prior

\[ PR(\delta) = N(M_{pr}, V_{pr}) \]

We get

\[ PO(\delta | m_s) = \frac{\exp \left[ -\frac{(m_s - \delta)^2}{2V_{m_s}} - \frac{(\delta - M_{pr})^2}{2V_{pr}} \right]}{2\pi \sqrt{V_{m_s} V_{pr}} \int LK(m_s | \delta) PR(\delta) \, d\delta} \]

(5-13)

If we let \( V_{m_s} = kV_{pr}/n \) such that the ratio \( k/n \) is the constant of proportionality relating the variance of the sample mean and the
prior variance, then it has been shown [12, pg. 74] that Eqn. (5-13) yields

$$\text{PO}(\delta | m_s) = A e^{-\left[\delta - \frac{M_{pr} + (n/k) m_s}{1 + n/k}\right]^{2} \left[\frac{1 + n/k}{2V_{pr}}\right]}$$  \hspace{1cm} (5-14)

where A is evaluated by requiring the integral of \(\text{PO}(\delta | m_s)\) for \(\delta\) in \(( - \infty , \infty )\) to be unity, thereby explicitly describing \(\text{PO}(\delta | m_s)\) in the form of a probability density function. We obtain

$$A = \frac{1}{\sqrt{2\pi V_{pr}(1 + n/k)}}$$  \hspace{1cm} (5-15)

Therefore, we have

$$\text{PO}(\delta | m_s) = N(M_{po}, V_{po})$$  \hspace{1cm} (5-16)

where

$$M_{po} = \frac{M_{pr} + (n/k) m_s}{1 + n/k} = \frac{k M_{pr} + n m_s}{k + n}$$  \hspace{1cm} (5-17)

and

$$V_{po} = \frac{V_{pr}}{1 + n/k} = \frac{n V_{m_s}}{k + n}$$  \hspace{1cm} (5-18)

Notice that the rate of divergence from unity of the ratio \(V_{pr}/V_{po}\) taken as a function of the sample size \(n\) is controlled by the slope \(1/k\).

The maximum value of the posterior variance \(V_{po}\) is the prior variance \(V_{pr}\). Its minimum value approaches zero as \(n\) approaches infinity.

For \(k \ll 1\), meaning that the variance of the fall of shot random
variable $X$ is significantly smaller than the prior variance, the posterior variance declines rapidly from its maximum value as the sample size increases. On the other hand if $k >> 1$ (which is unlikely because of the tightness of the fall of shot distributions), the posterior variance will decline quite slowly from $V_{pr}$ as $n$ increases. In Eqn. (5-17), $k$ is shown also to be the weight assigned to $M_{pr}$ in the determination of the posterior mean. For very small $k$, the value of the sample mean $m_s$ will dominate $M_{po}$ since very little weight is awarded $M_{pr}$. Therefore in those instances where the prior variance is significantly larger than the variance of the fall of shot distribution (as was the case for most of the subjects in our field experiment) such that $k << 1$, then the posterior mean of the error in the observer's target location is weighted heavily in favor of the sample mean and therefore relatively insensitive to the prior mean. We shall show in the following chapter that $M_{po}$ is the optimum posterior estimate of the error $\delta$ and is therefore the negative of the correction factor to be applied to the initial estimate $\omega$ of the distance $OT$ in order to shift the mean point of impact onto the target [from point $D$ (or $D'$) to point $T$, in the context of Chapter III].

The Prior Distribution of the Mean of the Posterior Distribution

Our next major concern is to develop $PR(M_{po})$, the prior distribution of the posterior mean whose expected value we use in the
economic analysis for the optimum sampling procedure discussed in
the following chapter.

Morris [12] has developed the prior distribution of $M_{po}$ when
all the distributions are normal, as ours have been shown effectively
to be. Summarizing his development, we have

$$M_{po} = \frac{k M_{pr} + n m_s}{k + n}$$  \hfill (5-19)

whose expected value is

$$E_{pr}[M_{po}] = \frac{k M_{pr} + n E_{pr}[m_s]}{k + n}$$  \hfill (5-20)

But the prior expected value of $m_s$ is $E[\delta] = M_{pr}$. By substitution,
then

$$E_{pr}[M_{po}] = M_{pr}$$  \hfill (5-21)

From Eqn.(5-19) we have also that

$$V_{pr}(M_{po}) = \left[\frac{n}{(k + n)}\right]^2 V_{pr}(m_s)$$  \hfill (5-22)

where

$$V_{pr}(m_s) = E[m_s - M_{pr}]^2$$

$$= E[m_s - \delta + (\delta - M_{pr})]^2$$

$$= V_{m_s} + V_{pr}$$
or
\[ V_{pr} (m_s) = \frac{k + n}{n} \]  

Therefore, substituting the derived expression Eqn (5-23), for the term \( V_{pr} (m_s) \) of Eqn. (5-22),

\[ V_{pr} (M_{po}) = \left[ \frac{n}{k + n} \right] V_{pr} = V_{pr} - V_{po} \] (5-26)

Since we have determined previously that \( V_{po} \) ranges from zero to a maximum of \( V_{pr} \), we see from Eqn. (5-26) that the extrema of the prior variance of the posterior mean are the extrema of the posterior variance, but are of opposite character (maximum vis-à-vis minimum). This says simply that our prior uncertainty about \( M_{po} \) increases with an increase in the anticipated sample size.

Summary

Before embarking on a discussion in Chapter VI of the preregister posterior economic analysis, we pause to summarize in Table 8 the distributions with which we have been dealing.
**Table 8**

**Summary of the Distributions**

1. PR(δ): The prior distribution of the observer's error δ.
   \[
   \text{PR}(\delta) = N[M_{\text{pr}}, V_{\text{pr}}]
   \]
2. PR(R): The prior distribution of the observer's estimate of the distance R to the target.
3. PR(m): The prior distribution of the coded estimate \( m = R - \omega \), where \( \omega \) is the distance OD or OD' reported by the observer to the fire direction center. Also \( \delta = \omega - R \).
4. LK(m_s | δ): The likelihood of observing a sample mean \( m_s \) of the fall of shot given the observer's error δ.
   \[
   \text{LK}(m_s | \delta) = N[\delta, \text{kV}_{\text{pr}}/n]
   \]
   where \( \text{kV}_{\text{pr}} = (\sigma_d \sin \gamma)^2 + (\sigma_r \cos \gamma)^2 \)
   in which \( \sigma_d^2 \) and \( \sigma_r^2 \) are, respectively, the variances of the fall of shot deflection and range distributions. The sample size is n.
5. PO(δ | m_s): The posterior distribution of the error δ after receipt of sample information.
   \[
   \text{PO}(\delta | m_s) = N[M_{\text{po}}, V_{\text{po}}]
   \]
   where
   \[
   M_{\text{po}} = \frac{kM_{\text{pr}} + nm_s}{k + n}
   \]
   and
   \[
   V_{\text{po}} = \frac{kV_{\text{pr}}}{k + n}
   \]
6. PR(M_{po}): The prior distribution of the mean of the posterior distribution.
   \[
   \text{PR}(M_{\text{po}}) = N[M_{\text{pr}}, V_{\text{pr}} - V_{\text{po}}]
   \]
CHAPTER VI
THE ECONOMIC ANALYSIS

The Economic Implication of Uncertainty

The decision maker, the observer, has been supposed uncertain about the distance $R$ to the target. He has expressed this uncertainty by $\text{PR}(\delta)$, the prior distribution of the error discussed in Chapter IV. A penalty described by the loss function $N$ of Chapter III is exacted according to the uncertainty, since the greater the uncertainty the smaller will be $P_{SVH}$ the unconditional single volley hit probability. The decision maker naturally wishes to avoid incurring any larger than necessary penalty in ammunition expenditure to destroy or successfully damage the target, so he may elect to sample the error $\delta$ by observing the mean point of impact with respect to the target of the sample fall of shot. This additional information will encourage him to reduce his uncertainty about the error $\delta$ which in turn will decrease the attendant penalty. The question to be answered is classic. What is the optimum size of the sample from which to obtain additional information on the error $\delta$ prior to calling for Fire for Effect? Intuitively the optimum sample size depends on the marginal decrease in the penalty cost for a given sampling cost. The answer to this question is the goal of the economic analysis.
Minimizing the Penalty Cost

Recall Eqn. (3-25) which described the loss function $N$ in terms of the commander's guidance $P_k$ (the demanded assurance of success) and the unconditional single volley hit probability $P_{SVH}$, the latter term being defined as an integral. We restate here this loss function, but now explicitly identify the probability density function for the error $\delta$ as $PR(\delta)$ from Chapter IV rather than the general symbolism $f(\delta)$ which we used in Eqn. (3-25).

$$N = \frac{P_k}{\int_\delta (P_{SVH} | \delta) PR(\delta) d\delta} \quad (6-1)$$

Since $P_k$ is fixed as the commander's guidance, our only avenue to the minimization of the required ammunition expenditure $N$ is through the maximization of the integral $P_{SVH}$ in the denominator of $N$. It is thoroughly reasonable that the greater the probability of hitting the target on any given try, the smaller will be the number of tries required to attain some given probability of success.

We found also in Chapter III that we could fit an exponential function to the conditional probability $P_{SVH} | \delta$ such that

$$P_{SVH} | \delta = ae^{-b \delta^2} \quad a, b \geq 0 \quad (6-2)$$

which further defines the integral in the denominator of $N$ as

$$\int_\delta ae^{-b \delta^2} PR(\delta) d\delta \quad (6-3)$$
which we wish to minimize with respect to some parameter not yet defined.

In Chapter IV we developed the three companion prior distributions \( PR(R) \), \( PR(m) \) and \( PR(\delta) \). \( PR(m) \) served us as a means for transforming \( PR(R) \) to \( PR(\delta) \), where we remember that \( R \) is the random variable representing the distance to the target about which the observer is uncertain. We defined the parameter \( \omega \) as the observer's point estimate of the distance to the target which he communicated to the fire direction center to identify the tentative location of the target. The random variables \( \delta \) and \( R \) are thus related by the coding parameter \( \omega \) as

\[
\delta = \omega - R
\]

Intuitively, \( \omega \) is a decision variable and it is with respect to that decision variable that the integral defining \( P_{SVH} \) can be maximized. We transform Eqn. (6-3) using the relationship of Eqn. (6-4) into

\[
\max_{\omega} \int_{R} -b(\omega - R)^2 \cdot \omega \cdot ae \cdot PR(R) \, dR
\]

which we have indicated is to be maximized with respect to the estimate \( \omega \). The value of \( \omega \) which forces Eqn. (6-5) to its greatest value is then the optimum estimate of the distance between the observer and the target in the light of the observer's stated uncertainty.
PR(R) of that distance. It is the point estimate which should be reported to the fire direction center and the resulting value for \( N \) is the number of volleys which should be expended in Fire for Effect to meet the commander's guidance if no further information comes available. Since we shall be dealing frequently with expressions of the form of Eqn. (6-5) we approach their optimization through a lemma as follows.

A Lemma

If \( f(x) \) is the probability density function for the normally distributed random variable \( x \), and if the function \( L(s, x) \) is of the form \( a \exp \left[ -b(s-x)^2 \right] \), \( a > 0, \ b > 0 \); then the function \( I(s) = \int x L(s, x)f(x)dx \) is a global maximum at the point \( s^* = F[x] \).

Proof: Setting the partial derivative with respect to \( s \) of the function \( I(s) \) to zero and recognizing that \( \frac{\partial L}{\partial s} < 0 \) for \( s > x \) and \( \frac{\partial L}{\partial s} > 0 \) for \( s < x \), we have

\[
\int_{-\infty}^{s} \frac{\partial L}{\partial s} f(x)dx + \int_{s}^{\infty} \frac{\partial L}{\partial s} f(x)dx = 0 \quad (6-6)
\]

Since

\[
\left. -\frac{\partial L}{\partial s} \right|_{x = s - k} = \left. \frac{\partial L}{\partial s} \right|_{x = s + k} \quad (6-7)
\]

it is obvious that Eqn. (6-6) holds only if \( s \) is taken such that \( s^* = F[x] \).

To determine the nature of the stationary point \( s^* = F[x] \) we make the conventional test on the sign of
Upon writing out the integral on the right hand side of Eqn. (6-9) and making the substitution $y^2 = (b + 1/2V)(x - E \{x\})^2$, where $V$ is the variance of $x$, we obtain an expression of the form

$$K \left\{ \frac{2b}{b + 1/2V} \right\} \int_0^\infty y^2 e^{-y^2} dy - \int_0^\infty e^{-y^2} dy$$

where $K$ is a positive constant.

It is apparent that the sign of this last expression is determined by the sign of that portion which is contained within the braces. Evaluating the integrals within the braces, we have the expression

$$\frac{2b}{b + 1/2V} \left( \frac{\sqrt{\pi}}{4} - \frac{\sqrt{\pi}}{2} \right)$$

(6-10)

Since the variance $V$ is always positive we see that the sign of Eqn. (6-10) is negative, implying that $s^*$ is indeed an extremum which yields a maximum $I(s)$. Thus the proof is established.

The Best Prior Estimate and the Best Prior Act

We see that the Lemma exactly describes the conditions of Eqn. (6 - 5). The optimizing value $s^* = E \{x\}$ will also hold whenever the probability density function $f(x)$ - or PR(R) - is symmetrical about the mean $E \{x\}$ and monotone non-increasing for larger $x$ such as is the case when $x$ or, more precisely, when $R$ is uniformly
or normally distributed, as we have described it in Chapter IV. The optimum value of the prior estimate \( \omega \) is then \( \omega^* = E[R] = \mu_{pr} \). So as to minimize the ammunition expenditure \( N \). Substituting this value for \( \omega \) in Eqn. (6-5), the minimum prior value for \( N \) becomes

\[
N^* = \frac{\mathcal{P}_k}{\int \frac{ae^{-b(\mu_{pr} - R)^2}}{PR(\theta)} d\theta}
\]

(6-11)

However the random variable with which we have been most concerned in this analysis is \( \delta \). In the explanation of Figure 8 in Chapter IV we defined the origin from which to measure the error \( \delta \) as that point along the OT line corresponding to the observer's estimate \( \omega \). If, as the Lemma suggests, the observer takes for this estimate \( \omega^* \), then we have defined the origin for \( \delta \). Since \( PR(R) \) is symmetrical about \( \mu_{pr} \) then \( PR(\delta) \) is symmetrical about \( E[\delta] = M_{pr} = 0 \). We can now properly perform the inverse transformation of Eqn. (6-11) to \( \delta \) as the variable of integration such that

\[
N^* = \frac{\mathcal{P}_k}{\int \frac{ae^{-b\delta^2}}{PR(\delta)} d\delta}
\]

(6-12)

where \( \delta \) is now distributed with zero mean as

\[
PR(\delta) = \text{Normal } (0, V_{pr})
\]

(6-13)

The evaluation of the integral in the denominator of Eqn. (6-12) is straightforward. This integral, we recall, represents the greatest
value for the single volley hit probability which we can obtain with
the information currently available. Writing out PR(δ) we have

\[
\text{PR(H)} = \frac{a}{\sqrt{2\pi V_{pr}}} \int_{-\infty}^{\infty} e^{-\frac{(\delta-a)^2}{2V_{pr}}} d\delta
\]

(6-14)

where

\[
g(\delta) = \frac{[2bV_{pr} + 1]}{2V_{pr}} \delta^2
\]

which yields

\[
\text{PR(H)} = \frac{a}{\sqrt{2bV_{pr} + 1}}
\]

(6-15)

so that we have \(N_{pr}^*\) immediately as

\[
N_{pr}^* = \sqrt{2bV_{pr} + 1} \left(\frac{P_k}{a}\right)
\]

(6-16)

explicitly defining what Bayesian Decision Theory knows as the "best prior act" companion to \(\omega^* = \mu_{pr}\) which is the "best prior estimate".

In introducing this chapter we described the penalty \(N\) as being
exacted according to the observer's uncertainty about the distance to
the target. We now see from Eqn. (6-16) that the magnitude of the
variance \(V_{pr}\) is sufficient to describe that uncertainty. The remain-
ing symbols in Eqn. (6-16) - \(b\), \(P_k\) and \(a\) - are either composite
physical parameters of the attack situation or the commander's
guidance for the attack. Notice that if \(V_{pr} = 0\) which we interpret
as absolute certainty of the value of the no-longer-random variables
R and δ - or equivalently as perfect prior information, then \( \text{N}^*_{pr} = \frac{P_k}{a} \) where a is the value of the single volley hit probability

\[ PR_{SVH} | \delta \text{ when } \delta \text{ is zero.} \]

The Sensitivity of \( \text{N}^*_{pr} \) to the Form of \( PR(\delta) \)

We have deliberately delayed until now a discussion of the sensitivity of the best prior act to the form of the probability density function \( PR(\delta) \). We agreed in Chapter V to revise \( PR(\delta) \) from the uniform to a normal or Gaussian distribution because the posterior distribution \( PO(\delta | m_s) \) would be a doubly truncated normal distribution if \( PR(\delta) \) were uniform and a conventional normal distribution if it were Gaussian. Since \( PO(\delta | m_s) \) approached normality it seemed reasonable to agree to the revision. However, the best justification for such a revision would be if we could show that the best prior act \( \text{N}^*_{pr} \) was insensitive to the change in the form of \( PR(\delta) \).

There is no doubt but that the best prior estimate \( \omega^* = \mu_{pr} \) is unaffected by the change from the uniform to the Gaussian form. The Lemma assures us of this. But what of the value of \( \text{N}^*_{pr} \) which proceeds from that estimate?

Had we insisted on \( PR(\delta) \) being a uniform distribution, we would have found ourselves eventually in an analytical morass. It is unlikely that any great good can come here from a retracing of
this investigator's path into that swamp. Suffice it to say that since
the variance of the uniformly distributed random variable $\delta$ is
greater than the variance of that random variable when normally
distributed, as we have related the variances in Eqn. (4-10), then
the best prior act $N^*_p$ proceeding from the normal distribution is
generally less than its value if based on the uniform distribution.

When the prior variance is quite small approaching zero uncertainty,
the two values are in substantial agreement. However as $V_{pr}$
on values more in accord with those observed in our field experi­
ment, $N^*_p$ may be as little as 30%, or even less, of its value
resulting from a uniformly distributed error if the numerical
calculations by this author on a few selected situations hold in gen­
eral. We cannot therefore justify the revision on the basis of insensi­
tivity of $N^*_p$ to the form of the prior distribution.

We maintain, however, that the revision was wise, and not
solely because it yielded an analytically tractable model, certainly
an important consideration in itself. If we accept that the subjects in
our field experiment were representative of the population of field
artillery observers, then the fact that 25 of the 26 subjects who
were able to voice a prior distribution on the distance $R$ described
a uniform distribution indicates perhaps that somehow they may have
been trained to think about observer-target distance estimates in
this fashion. Since from the Bayesian logic this judgment pattern
is costly in terms of ammunition expenditure, it is suggested that observers be deliberately trained to think about their uncertain estimates of observer-target distances and distances in general as normally distributed random variables. Then the results of our analysis will be in accord with the actual judgment patterns of the observers and, in addition, we can point to a resulting savings in ammunition.

**The Best Posterior Estimate and the Best Posterior Act**

We know that the amount of ammunition to expend in the attack can be reduced from \( N^* \) if we obtain additional information which will serve to reduce the variance \( V \). Suppose we elect not to enter Fire for Effect with \( N^* \) but to sample first for the mean point of impact of the sample fall of shot. We know from Chapter V that this information will alter \( P_R(\delta) \) to the posterior distribution

\[
P_O(\delta | m_s)
\]

where \( m_s \) is the mean of the sample fall of shot. The loss function of Eqn. (6-1) then becomes

\[
N = \frac{P_k}{\int_{\delta} (P_{SVH} | \delta) P_O(\delta | m_s) \, d\delta}
\]

or equivalently, in terms of the random variable \( R \),

\[
N = \frac{P_k}{\int_{R} (P_{SVH} | \omega - R) P_O(R | m_s) \, dR}
\]
where we wish to maximize with respect to the estimate \( \omega \) the value of the integral in the denominator of \( N \) in order to minimize \( N \).

Since \( P_O(R \mid m_s) \) is normally distributed, it is symmetrical about its mean which we shall call \( \mu_{\text{po}} \). Then the Lemma assures us that the best posterior estimate \( \omega^* \) as a result of the sample information is \( \mu_{\text{po}} \). This tells us to fix the origin at \( M \) for the measurement of the error \( \delta \) for the posterior distribution \( P_O(\delta \mid m_s) \), which we recall from Chapter V had mean

\[
M_{\text{po}} = \frac{kM_{\text{pr}} + n(m_s)}{k + n} \tag{6-19}
\]

but since \( M_{\text{pr}} \) was zero from the best prior estimate

\[
M_{\text{po}} = \frac{n(m_s)}{k + n} \tag{6-20}
\]

while the variance is

\[
V_{\text{po}} = \frac{kV_{\text{pr}}}{k + n} \tag{6-21}
\]

where \( n \) is the number of rounds in the sample.

We should take Eqn. (6-20) as telling us that the best estimate of the error after receiving the sample information is \( M_{\text{po}} \) and that the Fire for Effect now should be directed at that point on the ground corresponding to \( \mu_{\text{pr}} - M \). Further the prior expected value of \( M \) is \( M \) from the mean of the preposterior distribution of Chapter \( V \). This tells us that the sampling rounds should have been directed at the point on the ground corresponding to \( \mu_{\text{pr}} \), the best prior
estimate \( \omega^* \). Upon reflection, we might say that we knew this all along; but it is comforting to find that the analysis and our intuition remain in accord.

The best posterior estimate has been identified as \( \omega^* = \mu_{\text{post}} \) which leads immediately to the best posterior act \( N^*_{\text{post}} \). Fixing the origin for the posterior error \( \delta \) at \( M \), the posterior distribution \( \text{post} \) for the error is now distributed as

\[
\text{post}(\delta \mid m_s) = \text{Normal}(0, V_{\text{post}})
\]

(6-22)

from which proceeds the argument leading to \( N^*_{\text{post}} \) precisely parallel to that which produced Eqn. (6-16) from Eqn. (6-13). \( N^*_{\text{post}} \) is then formulated from the expression for \( N^*_{\text{pr}} \) if we replace \( V_{\text{pr}} \) by \( V_{\text{post}} \), or

\[
N^*_{\text{post}} = \sqrt{2bV_{\text{post}}} + 1 \quad (P_k/a)
\]

(6-23)

The Value of Information

The difference \( N^*_{\text{pr}} - N^*_{\text{post}} \) represents the number of volleys which has been saved as a result of the information obtained from the mean point of impact of the fall of shot of \( n \) sample rounds. This difference obviously represents the expected value of the sample information which we choose to symbolize as EVSI.

\[
\text{EVSI} = N^*_{\text{pr}} - N^*_{\text{post}}
\]

(6-24)

\[
= (P_k/a) \left[ \sqrt{2bV_{\text{pr}}} + 1 - \sqrt{2bV_{\text{post}}} + 1 \right]
\]

(6-25)
If we allow the sample size \( n \) in the expression for \( V \) in Eqn. (6-25) to approach infinity, we have what Bayesian theorists construe as perfect information, since there would be then no uncertainty about \( \delta \) as measured by the posterior variance. The expected value of perfect information \( \text{EVPI} \) is then the upper bound of \( \text{EVSI} \) of

\[
\lim_{n \to \infty} \text{EVSI} = \text{EVPI} = N^* - \left( P_k / a \right) \text{pr}
\]

The lower bound of \( \text{EVSI} \) is obviously zero as it should be in the absence of sample information, \( n = 0 \).

**The Optimum Sample Size**

The sampling program will obviously impose its own inherent cost, thus reducing the gain expected from the sample information. For our purpose, the cost of the sampling program can be taken as being directly proportional to the number of rounds which comprise the sample. We represent this sampling cost as \( \lambda n \) where \( \lambda \) is the constant of proportionality, a predetermined parameter of the problem. For example, since the \( N^* \) are expressed in units of volleys which we shall take as six rounds, we might take \( \lambda = 1/6 \) so that \( \lambda n \) also has volleys as its unit of measure. The expected net gain from sample information is then expressed as the difference between the expected value of sample information and the cost of obtaining the information.
Figure 10: The expected net gain from sample information ENGSI plotted as a function of the sample size $n$. The expected value of perfect information EVPI is the asymptote for ENGSI if sample information is free from cost.
ENGSI = EVSI - λn

= N* - N* - λn
   pr   po

(6-27)

We wish to determine the optimum value n* for the sample size n which maximizes ENGSI. The problem is graphically displayed in Figure 10. We might logically determine that the optimum value for n is zero, meaning that the single volley hit probability PSYH and the tightness of the observer's prior distribution PR(δ) together recommend that the attack be initiated in the Fire for Effect phase since the expected reduction in the magnitude of N* pr is insufficient to offset its cost λn for any n > 0.

We attempt to extract the optimum value for n by the conventional method of the Calculus, setting the partial derivative of ENGSI with respect to n at zero. The necessary condition for n to be an optimum value is that

\[ \frac{\partial EVSI}{\partial n} - \lambda = 0 \]  

(6-28)

but N* pr is not a function of n so Eqn. (6-28) becomes

\[ - \frac{\partial N* po}{\partial n} - \lambda = 0 \]  

(6-29)

where N* is defined by Eqn. (6-23) and Eqn. (6-21). Performing the partial differentiation yields the optimum sample size n* as the value of n which satisfies
When both members of Eqn. (6-30) are squared to remove the radical we are left with a quartic in \( n \), effectively dissuading us from any attempt to find \( n^* \) as an explicit function of the remaining parameters. But the analysis is far from stymied since numerical procedures, such as the optimum Fibonacci search [14], can be employed to find the stationary point \( n^* \) of ENGSI.

This optimum sample size \( n^* \) is then the size of the sample from which we can extract the maximum information value considering the cost of the information.

**Sequential Sampling**

Our discussion so far of an information collection program has assumed a fixed plan; fixed in that the entire sample is observed before any mean is assessed or posterior distribution developed. Suppose that we were to fire these \( n \) rounds sequentially, developing a new posterior distribution \( P(\delta | m) \) after each impact is observed. The posterior distribution arising from the observation of the impact of any single round or group of rounds comprising a stage in the sequence becomes the prior distribution for the subsequent stage.

For simplicity let us divide the sample size into two sub-samples \( n_1 \) and \( n_2 \) so that \( n_1 + n_2 = n \), \( n_1 \) comprising the first stage and \( n_2 \) the
second. Then the posterior distribution from the first stage observation is

$$ PO(\delta \mid m_{s_1}) = \text{Normal}(M_{p01}, V_{p01}) \quad (6-31) $$

where

$$ M_{p01} = \frac{kM_{pr} + n_1(m_{s_1})}{k + n_1} \quad (6-32) $$

and

$$ V_{p01} = \frac{kV_{pr}}{k + n_1} \quad (6-33) $$

$PO(\delta \mid m_{s_1})$ then becomes the second stage prior distribution on which its posterior distribution $PO(\delta \mid m_{s_2})$ is developed.

$$ PO(\delta \mid m_{s_2}) = \text{Normal}(M_{p02}, V_{p02}) \quad (6-34) $$

where

$$ M_{p02} = \frac{(k + n_1)M_{p01} + n_2(m_{s_2})}{(k + n_1) + n_2} \quad (6-35) $$

since the second stage prior variance $V_{pr2} = V_{p01}$ is related to the variance $kV_{pr}$ of the fall of shot by the multiplier $(k + n_1)$. Eqn. (6-35) reduces to

$$ M_{p02} = \frac{kM_{pr} + n_1(m_{s_1}) + n_2(m_{s_2})}{k + n_1 + n_2} \quad (6-36) $$

or

$$ M_{p02} = \frac{kM_{pr}}{k + n} \quad (6-37) $$
which is identically $M_{po}^*$, the posterior mean from the fixed sample plan. The second stage posterior variance is similarly shown to be

$$V_{po_2} = \frac{(k + n_1)}{(k + n_1) + n_2} \cdot V_{po_1}$$

$$= \frac{kV_{pr}}{k + n}$$

Therefore the results of a sequential sampling plan have been shown to be consistent with those of a fixed plan. However, there are some subtleties in the sequential sampling program which are not contained in the fixed program.

After each stage of the sequential program is evaluated for its posterior distribution, the prior distribution for the next subsequent stage, it is appropriate to make a new best prior estimate of the distance $\omega$ to the target. By extension of our discussion of sequential programs to multi-stage programs and from our development of the best prior estimate nearer the beginning of this Chapter, we know that the best prior estimate $\omega^*_i$ entering the $i$th stage of a sequential program is $\omega^*_i = \mu_{pr} - M_{po_{i-1}}$, where $\mu_{pr}$ is the mean of the first stage prior $PR(R)$. Since the mean of the preposterior distribution tells us to direct the $i$th stage sampling rounds at that point on the ground corresponding to $\omega^*_i$, the variance of the fall of shot can change at each stage of the program if there is a change in the
apparent gun-target range. This phenomenon was explained in Chapter I. Further, the fall of shot variance which we can call $V_f$ and the prior variance were related through the parameter $k$ at the beginning of the first stage such that

$$V_f = kV_{pr} \quad (6-39)$$

Therefore, it is necessary at the beginning of each stage to recompute the new value $k_i$, now a stage parameter, based on the current fall of shot variance $V_{f_i}$ and the first stage prior variance $V_{pr}$, or

$$k_i = \frac{V_{f_i}}{V_{pr}} \quad (6-40)$$

Because $k_i$ may change at each stage, the optimum economic sample size for the subsequent stages as determined by Eqn. (6-30) may also change. Intuitively, $n^*$ for the remaining stages will increase as $V_{f_i}$ decreases and vice-versa; this is also what a mathematical analysis of the results of the change would tell us.

The other subtlety in the sequential sampling program is a sophistication over the fixed program. When the sample size is fixed at any $n$, we assume for analytical purposes that the entire $n$ rounds are fired to impact simultaneously so that the observer acts only as a sensor to assess the mean point of impact with respect to the target of those $n$ sample rounds. He thus has no opportunity to take advantage of any "bonus" information which may become
available during the sampling process which might cause him to revise substantially his prior judgment on the distance to the target. This bonus information might be presented in the form of a burst from a sample round directing the attention of the observer to a fold in the terrain which he can now relate to the position of the target and further identify on his battlemap, having been previously unable to identify that particular terrain feature on the ground. With this information he may then be able to reduce significantly his uncertainty about the distance to the target.

In a sequential sampling program, the observer can take advantage of such bonus information and indeed he should. At any stage in the sequence the observer may revise his prior distribution beyond the extent implied solely from the "standard" information contained in the sample fall of shot. From this revised prior distribution resulting from the bonus information a new solution \((\omega^*, N^*_{pr}, N^*_{po}, \text{and } n^*)\) is computed and the problem continues to its ultimate conclusion.

**Summary**

In this Chapter we have merged the subjective probability distributions developed in Chapters IV and V with the loss function of Chapter III into a mathematical model of the attack situation. The solution to this mathematical formulation answers the critical
questions:

1. How much ammunition should be expended in the Fire for Effect Phase of the attack?

2. Should Fire for Effect be entered immediately or should it be preceded by a Sampling or Adjustment Phase?

3. At what point on the ground should the sampling rounds be directed and at what point should the Fire for Effect be directed?

4. What is the optimum number of rounds to expend in the Sampling Phase?

The answer to each of these questions has been predicated on the observer's ability to communicate to the fire direction center his uncertainty about the location of the target along the observer-target line in the form of a probability distribution PR(R).
CHAPTER VII
SOME COMMENTS AND APPLICATIONS

Comments on Current Doctrine and Procedures

The purpose of this investigation has been to establish a theoretical foundation for the doctrine governing observed artillery fire in the attack on ground targets.

The Field Artillery of the United States Army with doctrinal procedures developed from the experiences of World War II has earned an enviable reputation for its expertise both from its allies and its antagonists. However to our knowledge no explicit theory encompassing the entire spectrum of the observed fire attack has been available to guide the development of its doctrine. Consequently it is not surprising that, from the decision theoretic viewpoint presented here, some shortcomings in our current doctrine have been illuminated. Certain of these failings have been long recognized and discussed informally among those knowledgeable in field artillery tactics and techniques, but a theoretical framework has not been available within which to analyze the potential effects of procedural changes which these discussions have continually and inevitably suggested. It is hoped that our interpretation of Bayesian Decision Theory can provide the nucleus of a mature theoretical structure within which pertinent doctrine and doctrinal procedures can evolve
in a consistent and purposeful manner.

The following shortcomings are proposed as areas in which current doctrine is considered either remiss or with which it is unable to cope in an explicit quantitative fashion.

1. Doctrinal procedures are deterministic while the processes to which those procedures are addressed are probabilistic.

For example, doctrine prescribes that Fire for Effect be entered with the weapon and ammunition settings established when the observed mean point of impact of the rounds (a single round from one gun or one round each from a pair of guns fired simultaneously) in any sub-sample of the adjusting rounds is at the target. [3, pgs. 10-7, 12-2]. This procedural situation corresponds somewhat to the receipt of bonus information discussed in the previous chapter. However, this doctrine forces the observer to revise his prior distribution after receiving such information so as to place its mean (μ or M depending on which coordinate system one prefers) at the mean point of impact of those particular sub-sample rounds. Doctrinal insistence of this sort militates against the effective use of an on-the-spot observer to assimilate all of the information available to him which alters his uncertainty about the target location.

2. In the absence of a target hit, doctrinal procedures advocate entry into Fire for Effect when the difference between the expected points of impact of the rounds in
two sub-samples, taken separately, is 100 meters and the target was observed to have been bracketed within the interval described on the observer-target line by the bursts of the two sub-samples, one sub-sample having burst beyond the target and the other short of the target. Fire for Effect is then entered at the mean of these two expected points of impact, termed "splitting a 100-meter bracket". [3, pg. 10-7]

Although this is perhaps a continuation of the first citation, we give it separate status because of the dominance of this procedure in the determination of Fire for Effect weapon and ammunition settings. Notice that the apparent error in the mean of each sub-sample assessment is designated only as OVER or SHORT of the target on the OT vector; its magnitude, believed less than the nominal 100 meters, is ignored.

Ignoring the information available from the fall of shot of sample rounds directed outside of this 100-meter bracket, we restrict our discussion to the two pairs of rounds which are conventionally used to establish the bracket.

For purposes of argument we assume that the observer's prior distribution PR(R) immediately preceding the firing of these bracketing rounds has its mean at the center of the 100-meter bracket, at which point doctrine prescribes the Fire for Effect be directed if the bracket is subsequently realized. Simply verifying that the observed bracket includes the target is equivalent to assuming that the target will be always observed equidistant from
the mean points of impact of the two pairs of sub-sample rounds, or

\[ M_{po} = 0, \]  

surely an invalid assumption for general use.

From our discussion in Chapter VI of the best posterior estimate of the distance to the target, we know that the Fire for Effect should be directed at the point corresponding to \( \mu_{pr} - M_{po} \), the observer having assessed the relative position of the target with respect to the center of the observed bracket as \( -M_{po} \) (assuming that the value of the parameter \( k \) in Eqn. (6-20) is insignificant compared to \( n = 4 \)).

There is always the possibility that the 100-meter bracket established at the fire direction center does not contain the target which was observed to have been bracketed by the fall of shot. This results from the fire direction center using expected values for the end-points of the bracket while the observer can use only observed values. As the displacement of the target from the midpoint of the observed bracket increases toward one of the end-points, the probability that the target is not contained in the fire direction center's bracket also increases. This probability is the probability that the error in Fire for Effect will be at least 50 meters, which can be quite costly as the single volley hit probabilities of Table 3 and 4 tend to indicate. The best posterior estimate from the Bayesian point of view will take such displacements into account,
significantly reducing the Fire for Effect error and thus increasing the probability of a successful attack.

3. The doctrine already credits the observer with prior judgment on the accuracy of his initial target location as the following quotation amply demonstrates.

If, in his [the observer's] opinion, fire for effect can be delivered on the basis of target location and surprise is desired, he will request FIRE FOR EFFECT in his call for fire. [3, pg. 10-1]

However, there is no doctrinal guidance to assist the observer in evaluating his subjective prior judgment to determine if sufficient accuracy does exist to support an immediate call for Fire for Effect.

This is a major argument underlying the purpose of our thesis.

The quantitative description of the guidance which could be used is embodied in Eqn. (6-30) which contains the optimum sample size \( n^* \) as an implicit function. If for \( n = 0 \) the left hand expression of that equation is non-positive, the best decision for the observer (in the context of our mathematical model) is to call for Fire for Effect because the variance \( V_{pr} \) in his subjective prior probability distribution of the error in his initial target location is sufficiently small to abstain from sampling in an adjustment phase of the attack.

Table 9 illustrates the upper limit on the prior variances in the example targeting situations of Chapter III which would lead to
Table 9

The maximum variance \( V_{\text{fr}} \) which can be tolerated and yet permit entry into the Fire for Effect phase of the attack at the optimum prior estimate of the distance \( R \) to the target without first sampling for the mean point of impact of the fall of shot. Data is taken from the example targeting situations presented in Chapter III.

<table>
<thead>
<tr>
<th>Angle OTG ( \tau ) (mils)</th>
<th>Variance of the Fall of Shot ( V_{f_r} )</th>
<th>Demanded Assurance of Success ( P_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a )</td>
<td>(.80)</td>
</tr>
<tr>
<td>0</td>
<td>1602.37</td>
<td>1482.71</td>
</tr>
<tr>
<td>400</td>
<td>1372.86</td>
<td>1482.57</td>
</tr>
<tr>
<td>800</td>
<td>818.77</td>
<td>1423.95</td>
</tr>
<tr>
<td>1200</td>
<td>264.68</td>
<td>1278.65</td>
</tr>
<tr>
<td>1600</td>
<td>35.17</td>
<td>628.04</td>
</tr>
</tbody>
</table>

\( aV_{f_r} = (\sigma_d \sin \tau)^2 + (\sigma_r \cos \tau)^2 \)
immediate Fire for Effect as the best economic decision.\(^1\) An appreciation for the strictness which Table 9 places on the observer's prior distribution of the error in his target location can be gained from a determination of the maximum (mathematical) range of the error \(\delta\) considering \(\text{PR}(\delta)\) as a uniform distribution. Recall Eqn. (4-9), the mathematical source of the prior variances from the subjects in our field experiment. We rewrite it here as

\[
\delta \sigma = 0.98L
\]  \hspace{1cm} (7-1)

where \(L\) is the span of the interval defining the range of \(\delta\) in \(\text{PR}(\delta)\) and \(\sigma\) is the standard deviation. Then

\[
L = 6.12\sigma = 6.12 \sqrt{V_{\text{pr}}} \]  \hspace{1cm} (7-2)

Taking \(V_{\text{pr}} = 1389.98\) from the Table at an angle \(\tau\) of zero mils and \(P = 0.90\) we determine \(L \approx 228.17\) meters. This tells us that the prior error \(\delta\), distributed uniformly is restricted in this example situation to the interval defined by its mean \(M_{\text{pr}} + 114\) meters if immediate Fire for Effect is to be warranted.

A further study of Table 9 indicates that the \(V_{\text{pr}}\) at \(\tau = 0\) and \(\tau = 400\) mils have approximately the same values from which they decline at an increasing rate for the larger angles. From this we might infer that \(V_{\text{pr}}\) is likely a unimodal function of \(\tau\) and its

\(^{1}\text{The mathematical analysis which facilitates the development of Table 9 is provided in a later section of this chapter.}\)
maximum occurs between $\tau = 0$ and $\tau = 400$ mils, perhaps near $\tau = 300$ mils.

The greater the maximum allowable variance $\bar{V}_{pr}$, the more desirable is the situation because the "tightness" of $\text{PR}(\delta)$ is relaxed to include more instances in which sampling is not required. Also as $\bar{V}_{pr}$ increases the optimum sample size decreases for any given $V_{pr}$ when sampling is required to reduce the prior variance to a posterior variance of magnitude $\bar{V}_{pr}$. From this we can conclude that angle $\tau = 300$ mils is somewhere near an optimal prescription for the orientation of the observer, the target and the guns. It also assists us in the resolution of the trade-off available between ammunition expenditure and ease of sampling assessment as a function of the angle $\tau$ which was postulated in Chapter III.

We can also reason, with this knowledge of the effect of the angle $\tau$ on $\bar{V}_{pr}$, to priorities in the suppression of enemy artillery observation capabilities. If our combat intelligence efforts can locate enemy gun positions and observation posts (often included in the list of "Essential Elements of Information") then our field artillerymen can infer from the angle $\tau$ which of the possible combinations of enemy observers and guns could do us the most harm by bringing surprise Fire for Effect on our critical positions and installations.
4. The doctrine is not specific in guiding the Operations Officer to a determination of the amount of ammunition which should be expended in Fire for Effect.

The following guidance is provided.

The mission, description of target, batteries and ammunition available, and pertinent orders from higher headquarters govern the number of rounds to be fired in fire for effect. [3, pg. 18-5]

While it is readily admitted that the qualitative guidance cited here gives full reign to the Operations Officer's practiced ability to assimilate each of the factors into an intuitive judgment of their aggregate effect from which he may then prescribe the intensity of the fires for effect, it is unreasonable to expect, even given his ability to weight each factor individually, that his determination of their aggregate effect would be consistent over a number of situations.

Man's ability to decide by weighing single attributes between alternatives is good. But when a number of attributes (here the factors which affect the ammunition expenditure in Fire for Effect) must be weighted or traded-off, his ability is "less precise" [17]. Shepard is quite insistent on this point (which is applicable to the entire problem considered here, not just the determination of the intensity of the attack) as he states

... that limitation on our analytical powers often prevent us from seeing the consequences
of our actions - even when the relevant information is fully available. [17, pg. 259]

and further on

In many practical decision problems, then, to collapse the various component dimensions into one over-all evaluative dimension may not in itself be amiss. The point at issue, however, is whether people perform this dimensional compression in any optimum way. An examination of the pertinent experimental literature suggests that they do not. [17, pg. 264]

There is no claim here that our analysis provides the Operations Officer with a neat mathematical formulation to interrelate all the factors which influence his decision on ammunition expenditure. However we can claim that the ammunition expenditure implied by the technical factors impinging on his decision, coupled with the observer's expression of his uncertainty about the location of the target, is available from our model. The amount of ammunition which should be expended in Fire for Effect, from the decision theoretic approach presented here, is that value of \( N^* \) in Eqn. (6-16) obtained from the final stage of the sequential sampling phase of the attack, considering the Fire for Effect phase as just an additional stage appended to the sampling phase.

Then in any situation where time is of the essence and immediate Fire for Effect must be directed at the target without regard to the economic advantages of further sampling, this value
$N^*$ is the best decision on the intensity of the attack with the information already available.

As a general rule $N^*$ from Eqn. (6-16) will not be an integer but will have some value expressed by a mixed decimal. Since it is not the habit of the Field Artillery to fire portions of volleys, we leave the "rounding-off" of $N^*$ to an appropriate integer to the Operations Officer's adjustment of $N^*$ for the intangibles of the attack situation.

A Procedural Approach

It is only fitting that we should suggest an attack procedure, compatible with the decision theoretic analysis presented here, to overcome the major theoretical deficiencies in current procedures. It would be most desirable if this could be accomplished within the basic structure of present procedures, making a minimum of changes so as to avoid undue perturbations in the training status of affected personnel. The procedural approach about to be suggested is an attempt at such a Utopian ideal.

It has been apparent throughout this analysis that a general purpose computational device, a computer, is necessary to any field application. Such a device is available to field artillery units in the Gun Direction Computer M18 (FADAC) which "will solve any computational task assigned for which a program has been written"
within the limitations of its magnetic disc memory capacity of about 8,000 bits. We shall assume that sufficient excess capacity is available from its primary task of solving ballistic trajectories with which to perform the calculations attendant to these procedures.

In addition the use of a computer assists in the division of labor within the attack problem. Grayson [9] is particularly succinct on this subject and we quote him here, applying a rather liberal interpretation of his argument to the situations of present concern to us.

Let the operator, or a specialist, bring to bear all his judgment and experience on the forecast of prices - a task which he is best suited to do - and leave the memory and calculation procedures to other devices (mathematical models and computers) by which he can be excelled in this regard. [9, pg. 116]

The following broad procedures are suggested, assuming that the reader is familiar with the entire analytical argument to this point.

1. The observer reports to the fire direction center the direction to the target, its description (to include its approximate dimensions) and pertinent information from which the form and parameters of a tentative prior distribution $PR(R)$ of the target distance can be determined.

2. The Operations Officer, having been provided guidance
by the commander on acceptable vulnerability criteria, selects an appropriate vulnerability envelope for the target and the demanded assurance of success for the attack as input to the computer.

3. With additional information on the geometry, ballistics and fall of shot distributions in the attack the computer is directed to determine:

   a. An appropriate loss function (which could be conceivably selected from among stored candidates).

   b. The explicit parameters of the prior distribution \( PR(R) \) and firing data to place the mean point of impact of the fall of shot at a point corresponding to the distance \( \omega^* = \mu_{pr} \) (the prior expected value of \( R \)) from the observer on the OT line.

   c. The optimum prior value of the loss function \( N^*_{pr} \).

   d. The optimum sample size \( n^*_{pr} \). (If \( n^*_{pr} \) is less than unity, the prior variance \( V_{pr} \) is sufficiently small to warrant immediate entry into Fire for Effect.)

4. If the attack situation is sufficiently critical to demand immediate Fire for Effect, that fire should be directed at the point corresponding to \( \mu_{pr} \) with an intensity \( N^*_{pr} \) modified appropriately by the Operations Officer's assessment of the intangibles of the current tactical situation.
5. If the computation of \( n^* \) from 3d above yields a value greater than or equal to unity, \( V \) is too great from strictly economic considerations to warrant immediate entry into Fire for Effect, then an information collection program is initiated using the sequential sampling model of Chapter VI. After each stage of the sequence the observer reports:

a. The observed mean point of impact with respect to the target of the sample fall of shot in that stage.

b. If any "bonus" information was presented to cause him to revise significantly his uncertainty about the target location; and if so, the description of his revised prior distribution.

c. Any change in the vulnerability posture of the target which might affect the loss function \( N \).

6. With this updated information the cycle is re-entered, iteration continuing until either the tactical or economic analysis warrants attacking the target with Fire for Effect.

These broad procedures incorporate the Bayesian technique and surmount the theoretical deficiencies in current doctrinal procedures yet hopefully they yield to the pragmatic considerations militating against massive or complex procedural revisions.

**On the Determination of Equipment Specifications**

Let us now hypothesize the obvious: a development effort for a range finding device to assist the observer in determining
target locations. For such a device to be useful the three basic assumptions of our investigation as set forth in Chapter I must hold.

The first two of these assumptions are necessary in order to translate the target location reference point from the observer's position to the firing battery position. Therefore we have to assume that the location of the guns and the observer on a common grid are known and that the direction of the observer-target line also is known.

The principal purpose of a range finder is to locate targets with sufficient accuracy so that the Fire for Effect phase of the attack can be entered immediately without any preliminary sampling or adjustment of the mean point of impact of the fall of shot. In order for this purpose to be fulfilled it is obvious that our third assumption must hold, to wit: the guns have developed registration corrections valid throughout the target area so that the point on the ground at which the guns are directed to fire is the expected point of impact of the fall of shot.

It is characteristic of any measuring device to contain a random error in its readout and the distribution of such a random error often can be assumed Gaussian in form. If we take the probability distribution of this random error as \( \text{PR}(\delta) \) which we developed in Chapter IV as the observer's subjective prior probability distribution,
then the analysis of this investigation is entirely applicable to the
inanimate distance finding device.

Eqn. (6-30) provides us a means for determining the maximum
variance of the random error characteristic to the device which can
be tolerated so that the measured distance R is sufficiently accurate
to allow immediate Fire for Effect on the target. We restate Eqn.
(6-30) here as

\[ \frac{b k V_{P/A}}{\sqrt{2 b k V_{P/A} + 1}} = \lambda \quad (7-3) \]

Evaluating Eqn. (7-3) at \( n = 0 \), thereby anticipating the optimum
economic decision against sampling, we obtain the equation

\[ \frac{b k V_{P/A}}{\sqrt{(2 b V_{P/A} + 1) k^2}} = \lambda \quad (7-4) \]

Now \( k V_{P/A} \) is implied from the discussion introducing Eqn. (5-14)
as the variance of the fall of shot distribution along the observer-
target line. Letting \( V_f \) symbolize this fall of shot variance, its
substitution for \( k V_{P/A} \) in Eqn. (7-4) followed by some rearrangement
of terms yields

\[ \left( \frac{b V_f (P_k / a)}{\lambda} \right)^2 = k^3 (2 b V_f + k) \quad (7-5) \]
For any given attack situation every parameter in Eqn. (7-5) but $k$ is of known value. The determination of that value for $k$ which satisfies Eqn. (7-5), which is again of necessity a numerical procedure because Eqn. (7-5) is a quartic in $k$, immediately determines the maximum variance $\overline{V}_{pr}$ which can be tolerated in the measured distance $R$, the output of the range finder. Table 9, preceding summarizes the calculations for $\overline{V}_{pr}$ for the example attack situations of Chapter III.

Our determination in Chapter VI of the best prior decision

$$\omega^* = \mu_{pr}$$

as the point estimate of the distance $R$ to the target is equally valid here. It tells us the obvious. First that the mean of the distribution of the random error should be calibrated at zero in the readout. Then the reading $R$ obtained during any given target locating attempt has expected value $\mu_{pr}$, as does the mean of any group of $v$ independent readings and thus should be accepted as the best prior estimate of $R$. Further, since $\overline{V}_{pr}$ is predicated on a single reading by the range finder in order to estimate $R$ and since the maximum variance is not a constant (as we can see from Table 9) but varies with each attack situation, it would not be practicable to take the worst case condition $\min (\overline{V}_{pr})$ as the upper limit on the random error variance of the device. Rather $\overline{V}_{pr}$ over a spectrum of plausible target attack situations may be determined and a specified
maximum variance $V_{\text{max}}$ selected which the developer can meet within the budget restrictions of the development effort. Then, assuming $V_{\text{max}} \geq \overline{V}_{\text{pr}}$ for a given attack situation,

$$\nu = \frac{V_{\text{max}}}{\overline{V}_{\text{pr}}}$$

is the minimum number of independent readings from the range finder of the distance $R$ which must be averaged to insure that the variance of the average reading is not greater than the allowable maximum $\overline{V}_{\text{pr}}$.

We now have a fair amount of leeway within which the designer can balance trade-offs between an economically attainable variance in the equipment and the number of procedural repititions allowable in field usage. With such information reasonable military specifications for the equipment can be written with confidence that devices meeting those specifications can fulfill their intended purpose. It also provides the basic information from which to prescribe field calibration and periodic test procedures for the equipment.
CHAPTER VIII
SUMMARY AND CONCLUSIONS

We have interpreted Bayesian Decision Theory to apply to the attack of ground targets by observed artillery fire. This interpretation is anticipated to serve as the nucleus of a more extensive theoretical interpretation of the entire field artillery gunnery problem within which doctrine and doctrinal procedures can be structured so as to be complementary.

The effect on ammunition expenditure from uncertainty about the error in the target location during Fire for Effect has been quantified. We have demonstrated that ammunition expenditure is particularly sensitive to the angle included between the observer-target line and the gun-target line. As a function of this angle a trade-off has been shown to exist between the ammunition cost of an attack and the ease of assessment by the observer of the fall of shot. Disregarding the human limitations of the observer in assessing accurate deviations of the fall of shot from the target, we have suggested to future investigators that the optimum trade-off may be when the angle is near 16.9 degrees or 300 mils.

The results of a field experiment to elicit subjective probability distributions from field artillery observers supported Winkler's findings [18] on the feasibility of attempting to extract from an
individual such subjective utterances. These results showed also that artillery observers tend toward interval estimates of the location of a target and that their subjective locating errors are distributed uniformly within those intervals. We indicated this to be a costly judgment pattern in terms of ammunition expenditure and suggested that observers be trained deliberately to think of their target locating errors as normally distributed random variables rather than being uniformly distributed as presently seems to be the case.

Reasoning in a manner consistent with Bayesian Decision Theory, we illustrated the method for determining the best estimate of the target location at various phases of the attack sequence. An analytical argument was presented which would determine if immediate Fire for Effect should be directed at the best estimate of the target location or if adjustment of that estimate would be economical by sampling the error in the mean point of impact of the fall of shot. If sampling was required, we provided the means for deciding the optimum economic sample size. A means for determining the ammunition expenditure in Fire for Effect was also developed.

Current field artillery doctrine and doctrinal procedures were examined for decision theoretic deficiencies. We commented in four areas where we considered the doctrine to be either remiss or
inadequate in an explicit quantitative fashion. The intent of these comments was summarized in a broad listing of suggested procedures for solution to the observed fire problem which incorporated the Bayesian approach but attempted to avoid total disruption of the present concepts by massive and complex revisions.

Finally we proposed a unique application of the optimum economic sample size to the determination of specifications on the accuracy of range finding equipment to permit surprise Fire for Effect on targets located by these devices.

The purpose of this investigative effort has been fulfilled. Bayesian Decision Theory can be applied quite rationally to the field artillery gunnery problem. This interpretation provides now a means to evaluate the theoretical efficacy of doctrine and doctrinal procedures in this area.
APPENDIX A

CALCULATING $P_{SVH} \mid \delta$

The following tables illustrate the calculation of the single volley hit probability $P_{SVH} \mid \delta$ in accordance with the formulation of Eqn. (3-19). The resulting data points are summarized in Tables 3 and 4 from which we derive Table 5 and the continuous fitted function $P_{SVH} \mid \delta$ for use in the loss function $N$.

The example targeting situations of Chapter III are used to illustrate the calculations. The six guns of the battery are assumed to be colinear, this common line being always perpendicular to the apparent gun-target line GD or GD'. This assumption accounts for the single entry $P_{r}$ for all guns for any given error $\delta$ and angle $\tau$.

The constant parameters of the problems are:

a. The target vulnerability envelope is 150 meters (front) by 100 meters (depth).

b. $GT = 8000$ meters

c. $\sigma_r = 40$ meters

d. $\sigma_d = 6$ meters

e. The separation distance between guns is $7\sigma_d$. 

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Table 10

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Angle $τ = 0$; Battery Center at Gun #4
Table II

Angle \( \tau = 400 \) mils; Battery Center at Gun #3

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Table 12

Angle \( \tau = 400 \) mils; Battery Center at Gun #4

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Table 13

Angle $\tau = 800$ mils; Battery Center at Gun #3

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Table 14

Angle $\tau = 800$ mils; Battery Center at Gun #4

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<td>$P_d$ for Gun #:</td>
<td>$P_{SVH}^{1/6}$</td>
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<td>----------------</td>
<td>-----------------</td>
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Table 17

Angle $\tau = 1600$ mils;  Battery Center at Gun #3

<table>
<thead>
<tr>
<th>Error $\delta$</th>
<th>$P_r$</th>
<th>$P_d$ for Gun #</th>
<th>$P_{SVH}$</th>
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<tr>
<td>200</td>
<td>0.7849</td>
<td>0.4725 0 0 0 0 0</td>
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<td>0.7864</td>
<td>1 0.0021 0 0 0 0</td>
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<td>0.7873</td>
<td>1 0.9126 0 0 0 0</td>
<td>0.9401</td>
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<td>0.7878</td>
<td>1 1 0.0765 0 0 0</td>
<td>0.9577</td>
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<tr>
<td>100</td>
<td>0.7881</td>
<td>1 1 0.9974 0 0 0</td>
<td>0.9904</td>
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<td>1 1 1 0.4976 0 0</td>
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<td>0.5315 1 1 1 0.0027 0</td>
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<td>0 0 0.9239 1 1</td>
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<td>Error $\delta$</td>
<td>$P_r$</td>
<td>$P_d$ for Gun #4</td>
<td>$P_{SVH}$</td>
</tr>
<tr>
<td>--------------</td>
<td>-------</td>
<td>-----------------</td>
<td>---------</td>
</tr>
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<td>.7884</td>
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APPENDIX B

SOME STATISTICAL ANALYSES ON THE EXPERIMENTAL DATA
Table 19
Target locating errors by student observer subjects in field experiment of 24 February 1969 through 1 March 1969.

<table>
<thead>
<tr>
<th>Class of Observer Subjects</th>
<th>Subject Number</th>
<th>Obs-Tgt Distance (meters)</th>
<th>Locating Error (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Deflection</td>
</tr>
<tr>
<td>FAOCC&lt;sup&gt;a&lt;/sup&gt; 7-69</td>
<td>1</td>
<td>2755</td>
<td>118.8(R)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1165</td>
<td>36.4(L)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2871</td>
<td>49.2(R)</td>
</tr>
<tr>
<td></td>
<td>4&lt;sup&gt;c&lt;/sup&gt;</td>
<td>Subject not responsive</td>
<td></td>
</tr>
<tr>
<td>FAOBC&lt;sup&gt;b&lt;/sup&gt; 9-69</td>
<td>5</td>
<td>1729</td>
<td>31.1(L)</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>2791</td>
<td>58.5(L)</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>1546</td>
<td>69.2(R)</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>3114</td>
<td>3.2(L)</td>
</tr>
<tr>
<td></td>
<td>9&lt;sup&gt;c&lt;/sup&gt;</td>
<td>2791</td>
<td>330.3(L)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>2341</td>
<td>58.6(R)</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>2440</td>
<td>64.7(L)</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>2506</td>
<td>99.0(L)</td>
</tr>
<tr>
<td></td>
<td>13&lt;sup&gt;c&lt;/sup&gt;</td>
<td>2822</td>
<td>206.4(L)</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>1043</td>
<td>48.0(R)</td>
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<tr>
<td></td>
<td>15</td>
<td>2087</td>
<td>94.0(R)</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>2880</td>
<td>196.5(R)</td>
</tr>
</tbody>
</table>

<sup>a</sup>Field Artillery Officer Candidate Course
<sup>b</sup>Field Artillery Officer Basic Course
<sup>c</sup>Subject Disqualified
<table>
<thead>
<tr>
<th>Class of Observer Subjects</th>
<th>Subject Number</th>
<th>Obs-Tgt Distance (meters)</th>
<th>Locating Error (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
<tr>
<td>FAOBC 9-69</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(continued)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>1717</td>
<td>67.8(L)</td>
<td>471.6(+)</td>
</tr>
<tr>
<td>18</td>
<td>1494</td>
<td>91.8(L)</td>
<td>787.2(+)</td>
</tr>
<tr>
<td>19c True target location questionable</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>2262</td>
<td>66.5(L)</td>
<td>270.3(+)</td>
</tr>
<tr>
<td>21</td>
<td>2400</td>
<td>10.5(L)</td>
<td>516.1(-)</td>
</tr>
<tr>
<td>22c</td>
<td>2218</td>
<td>215.4(L)</td>
<td>141.5(+)</td>
</tr>
<tr>
<td>FAOCC 9-69</td>
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<td></td>
</tr>
<tr>
<td>23</td>
<td>1652</td>
<td>73.9(L)</td>
<td>106.7(+)</td>
</tr>
<tr>
<td>24c</td>
<td>1640</td>
<td>258.5(L)</td>
<td>66.4(+)</td>
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<tr>
<td>25</td>
<td>1721</td>
<td>26.2(R)</td>
<td>43.7(-)</td>
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<tr>
<td>26</td>
<td>1402</td>
<td>24.2(L)</td>
<td>287.7(+)</td>
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<tr>
<td>27</td>
<td>2803</td>
<td>13.9(R)</td>
<td>113.8(-)</td>
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Table 20
An analysis of the angular errors in target locations.

<table>
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<tr>
<th>Subject Number</th>
<th>Direction OT (mils)</th>
<th>Direction OD (mils)</th>
<th>Angular Error ( \mu )</th>
<th>Remarks</th>
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<td>19.7 R</td>
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<td>4</td>
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<td>N/A</td>
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<td>#4 disqualified, Not responsive.</td>
</tr>
<tr>
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<td>1756.2</td>
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<tr>
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<td>5509.3</td>
<td>5483.5</td>
<td>25.8 L</td>
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<td>5503.0</td>
<td>5531.7</td>
<td>28.7 R</td>
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<td>8</td>
<td>6014.5</td>
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<td>1.2 L</td>
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<tr>
<td>9</td>
<td>5509.3</td>
<td>5376.5</td>
<td>(132.8 L)</td>
<td>#9 disqualified. Gross error in OD.</td>
</tr>
<tr>
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<td>5633.2</td>
<td>5657.1</td>
<td>23.9 R</td>
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<td>5519.9</td>
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<td>38.5 L</td>
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<td>5705.9</td>
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<td>5.6 L</td>
<td></td>
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<tr>
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<td>5368.7</td>
<td>(92.8 L)</td>
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<td>42.8 L</td>
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<td>6227.8</td>
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<td>(153.1 L)</td>
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<td>273.4</td>
<td>14.6 L</td>
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<tr>
<td>27</td>
<td>16.7</td>
<td>22.0</td>
<td>5.3 R</td>
<td></td>
</tr>
</tbody>
</table>

Statistics: \( \Sigma n\bar{m} = 25.1 \text{ L}; \Sigma m^2 = 19743.68; \ n = 21 \)

Assumption: \( \{m\} \approx \text{NID}(\mu, \sigma^2) \)

Sample Variance: \( s^2 = 985.96 \)

Test: \( H_0: \mu = 0 \) 
Significance level (Type I error) = .10

\( H_1: \mu \neq 0 \)

\[
t = \left| \frac{\bar{m} - \mu_0}{s} \right| \sqrt{n} = .174; \ t < t_{.05, 20} = 1.725
\]

Therefore Accept \( H_0 = \mu = 0 \).
Test for Independence
Between OT Range Component Errors
and Deflection Component Errors
in Target Locations

1. Data is taken from Table 19, omitting disqualified subjects.

2. Let: $e_r =$ range component error
   $e_d =$ deflection component error

3. Assumption:
   \[
   \{e_r, e_d\} \sim \text{Bivariate Normal} [\mu_r, \mu_d; \sigma \text{V}_r, \sigma \text{V}_d; \theta]
   \]

4. Statistics:
   \[
   n = 21
   \]
   \[
   \Sigma_{e_r} = 3,327.6; \quad \Sigma_{e_r}^2 = 3,646,098.24
   \]
   \[
   \Sigma_{e_d} = 46.8; \quad \Sigma_{e_d}^2 = 118,703.76
   \]

5. Covariance Matrix:
   \[
   S^2 = \frac{1}{n-1} \begin{bmatrix}
   a_{11} & a_{12} \\
   a_{21} & a_{22}
   \end{bmatrix} = \frac{1}{20} \begin{bmatrix}
   3,645,939.78 & 25,459.37 \\
   25,459.37 & 118,701.53
   \end{bmatrix}
   \]
   where
   \[
   a_{11} = \Sigma_{e_r}^2 - (\Sigma_{e_r})^2/n
   \]
   \[
   a_{22} = \Sigma_{e_d}^2 - (\Sigma_{e_d})^2/n
   \]
   \[
   a_{12} = a_{21} = \Sigma_{e_r e_d} - (\Sigma_{e_r} \Sigma_{e_d})/n
   \]
6. Sample Correlation Coefficient:

\[ r = \frac{a_{12}}{\sqrt{a_{11} a_{22}}} = .0387 \]

7. Student-Fisher t-Test on the Correlation Coefficient [13, pg. 99]:

- \( H_0: \rho = 0 \); Significance level (Type I Error) = .10;
- \( H_1: \rho \neq 0 \);

\[ t = \left| r \right| \sqrt{\frac{n - 2}{1 - r^2}} = .1688 \]

\[ t < t_{0.05, 19} = 1.729 \]

Therefore accept \( H_0: \rho = 0 \), a necessary and sufficient condition for the independence of bivariate normal random variables.
Tests for Means of OT

Range and Deflection Component Errors
in Target Locations

1. Data is taken from Table 19, omitting disqualified subjects.

2. Let: $e_r = \text{range component error}$
   $e_d = \text{deflection component error}$

3. Assumptions:
   $\{e_r\} \sim \text{Normal} (\mu_r, \sigma_r)$
   $\{e_d\} \sim \text{Normal} (\mu_d, \sigma_d)$

4. Statistics (see Test for Independence, proceeding):
   $S_r^2 = 182,296.99; \quad S_d^2 = 5,935.08$

5. Student t-Test on $\mu_r$:

   $H_0: \mu_r = 0; \quad \text{Significance level (Type I Error)} = .10$
   $H_1: \mu_r > 0; \quad \text{Significance level (Type I Error)} = .10$

   $t = \frac{\overline{e_r} - \mu_r}{S_r} \sqrt{\frac{1}{n}} = 35.715$
   $t_{.10, 20} = 1.325$

   $t > t_{.10, 20}; \text{therefore accept } H_1: \mu_r > 0,$ implying that the observed tendency was for Observer-Subjects to overestimate distances.

6. Student t-Test on $\mu_d$:

   $H_0: \mu_d = 0; \quad \text{Significance level (Type I Error)} = .10$
   $H_1: \mu_d \neq 0; \quad \text{Significance level (Type I Error)} = .10$
\[ t = \left| \frac{(e_d - \mu_0)}{S_d} \right| \sqrt{n} = 1.487 \]

\[ t_{.05, 20} = 1.725 \]

\[ t > t_{.05, 20}; \text{ therefore accept } H_o: \mu_d = 0. \]
Tests for Equality of Means and Variances of OT Range Component Errors Between Officer Candidate Course and Officer Basic Course Students

1. Data is taken from Table 19, omitting disqualified subjects.

2. Let: $e_1 =$ range component error for Officer Candidate Course student Observer-Subject.

   $e_2 =$ range component error for Officer Basic Course student Observer-Subject.

3. Assumptions:

   \[ \{e_1\} \sim \text{Normal} (\mu_1, \sigma_1^2) \]
   \[ \{e_2\} \sim \text{Normal} (\mu_2, \sigma_2^2) \]

4. Statistics:

   \[ \Sigma e_1 = 716.0; \quad \Sigma e_2 = 526,907.46; \quad n_1 = 8 \]
   \[ \Sigma e_2 = 2,611.6; \quad \Sigma e_2 = 3,119,190.78; \quad n_2 = 13 \]
   \[ S_1^2 = 75,259.71; \quad S_2^2 = 259,915.82 \]

5. F-Test on Equality of Variances:

   \[ H_0 : \sigma_1^2 = \sigma_2^2 \quad \text{Significance level (Type I Error)} = .10 \]

   \[ H_1 : \sigma_1^2 < \sigma_2^2 \]

   \[ F = \frac{S_2^2}{S_1^2} = 3.454 \]

   \[ F_{.10(12, 7)} = 2.67 \]
\[ F > F_{10}^{12, 7} \]; therefore accept \( H_0: \mu_1 = \mu_2 \).

6. Student's t-Test on Equality of Means:

\[ H_0: \mu_1 = \mu_2; \quad \text{Significance level (Type I Error)} = .10 \]

\[ H_1: \mu_1 \neq \mu_2; \]

\[
t = \frac{\left( \bar{c}_1 - \bar{c}_2 \right)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = .650
\]

\[
t = 1.729
\]

\[ .05, 19 \]

\[ t < t_{.05, 19}; \text{therefore accept } H_0: \mu_1 = \mu_2. \]
APPENDIX C

THE INTERVIEW QUESTIONNAIRE AND RESPONSES

The following questionnaire, liberally adapted and modified from that suggested by Winkler [18], was used to elicit the subjective prior probability distributions PR(m). The responses are included immediately following.

Questionnaire: [The parenthetical notes are instructions to the Interviewer.]

1. SUBJECT: [See Questionnaire Response Sheet. Obtain personal data from Subject and attempt to put him at ease.]
2. EXPERIMENT: [Enter experimental conditions and known data as listed on the Response Sheet.]
3. INTERVIEW:
   a. [Explain to the Subject that the questions he shall be asked are statistically motivated and concern his judgment pattern in arriving at a decision on the location of the target. Caution him to refrain from answering any question he does not fully understand; the question will be amplified until he does understand.]
   b. Where do you think the target is most likely located? What location do you intend reporting to the fire direction center?
   c. If the initial adjusting rounds fired at your reported location are more than say 50 meters from the target to which one of
the following three causes would you attribute the error?

(1) An error at the gun position, either in the fire direction center or the firing battery. A misplot for instance, or an error in the Quadrant or Deflection settings.

(2) The natural dispersion pattern of the fall of shot.

(3) A mislocation of the target by you, the observer.

d. Are you in the least uncertain about the location of the target? If you are not in any way uncertain about that location, and considering that the guns have been registered, you should be willing to call for immediate Fire for Effect.

e. Is your uncertainty primarily from having to choose between two or more "sure" locations (discrete) or does it arise from having to select a point from within an interval in which you believe the target to be located?

f. [Explain to the Subject that the following questions are designed to reconstruct his decision pattern in locating this target. Caution him to disregard any consideration of the fall of shot dispersion in answering these questions. We are concerned here only with his uncertainty about the true location of the target.]

Using the initial target location (ITL) as a reference point (that location which you say is to be reported to the fire direction center):
(1) How far **short** of the **ITL** do you think the target conceivably might be located - where there is in your mind not more than 1 chance in 100 (1%) that the true location of the target, if I were to read it to you from the Trig List, would plot closer to your position than that point?

(2) Now consider only the area on your battlemap beyond the plot of your **ITL**. How far **beyond** the **ITL** do you think the target conceivably might be located - again where there is in your mind not more than 1 chance in 100 (1%) that the true location of the target would prove to be farther from your position than is that point?

(3) Again considering the area on your battlemap this side of the **ITL** plot, how far **short** of the **ITL** would you place the point where you think there is not more than 1 chance in 4 (1 to 3 odds) (25%) that the target will prove to be closer to you than is that point? [If necessary draw a sketch and explain that this point must be "inside" the 1% point.]

(4) Now returning to the area beyond the **ITL** plot, how far **beyond** the **ITL** would you say is the point where you think there is not more than 1 chance in 4 that the target will prove to be farther from you than is that point?

(5) You have described on the **OT** line an interval in which you think the target definitely to be located. Where in that
interval do you think there are equal chances that the target will prove to be either beyond or short of that point? (50%) (odds of 1 to 1). Where is that point with relation to the ITL? Is it perhaps the ITL?

4. FEEDBACK: [Explain to the Subject any unusual implications of his responses. Does he wish to change any of his responses? Has he any qualifications to his responses?]

5. PREDICTION OF THE FALL OF SHOT OF THE INITIAL ROUNDS IN ADJUSTMENT.

a. [Explain to the Subject that he is now to consider together the dispersion pattern of the fall of shot and his uncertainty (doubt) about the accuracy of his reported ITL. Set up for him the following betting situation.]

b. Assume that we are both willing to enter into a bet as to the location, OVER or SHORT, with respect to the target of the observed mean point of impact of the first two adjusting rounds fired at your reported ITL. Assume also that the guns are correctly registered and that there is no error either at the fire direction center or in the firing battery itself. One of us gets to choose first, OVER or SHORT. The bet is sizable. Does it matter to you which of us chooses first, or will you bet only if you have first choice?

c. [If the Subject does not demand first choice he is assumed
indifferent for purposes of this experiment.]

d. Since you have demanded first choice, which direction do you select - OVER or SHORT? To satisfy my curiosity, can you tell me how far OVER/SHORT you think the mean point of impact will be? How far?

6. This concludes the interview. Your instructor will take over from here.
SUBJECT:

Name: ___________________________ Subject Nr: 1
Grade: OC
Length of Service: 9 mos.
Approx. nr. of mans. fired: 6
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 251325 Feb 69 Visibility: Ex
Observation Post:
Description: FP 304S Grid: 59033-37451
Has registration been conducted?
YES: NO: 
Formal: x
Informal: 
Target:
Description: #315 Grid: 61399-36039

INTERVIEW:

Most likely location of target: 61400-35900
If in error, the error is:
An error at the Guns or in the FDC: 
A result of the fall of shot dispersion: 
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: 
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: x?
An interval estimate:
Fractiles of the CDF using the ITL as the origin:
1%: 700 meters short of ITL
25%: 250 meters short of ITL
75%: 300 meters over the ITL
99%: 600 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT:OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by ________ meters. SHORT of the
ITL by 200 meters. INDIFFERENT: ___________
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ____________________________________ Subject Nr: ________
Grade: OC
Length of Service: 10 mos.
Approx. nr. of men fired: ______
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 251345 Feb 69 Visibility: Ex
Observation Post:
   Description: FP 3045 Grid: 59033-37451
Has registration been conducted?
   YES: ________ NO: ________
      Formal: x
      Informal: ________
Target:
   Description: #324 Grid: 59996-36795

INTERVIEW:

Most likely location of target: 60200-36700
If in error, the error is:
   An error at the Guns or in the FDC:
   A result of the fall of shot dispersion: x
   In the estimate of the target location: x
Observer is uncertain about the location of the target:
   YES: x ________ NO: ________
Observer's selection of initial target location (ITL) was from:
   Two or more sure locations at discrete points:
   An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
   1%: 200 meters short of ITL
   25%: 125 meters short of ITL
   75%: 125 meters over the ITL
   99%: 200 meters over the ITL
   50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT:OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by ________ meters. SHORT of the
ITL by ________ meters. INDIFFERENT: x ________
SUBJECT:

Name: ___________________________ Subject Nr: ___ 3 ___
Grade: ___ OC ___
Length of Service: ___ 10 mos. ___
Approx. nr. of mans. fired: ___ 7 ___
Statistical training: ___ none ___

EXPERIMENTAL ENVIRONMENT:

Date/Time: ___ 251450 Feb 69 ___ Visibility: ___ Ex ___
Observation Post:
Description: ___ FP 304S ___ Grid: ___ 59033-37451 ___
Has registration been conducted?
YES: ___ NO: ___
Formal: ___ x ___
Informal: ___
Target:
Description: ___ #337 ___ Grid: ___ 61877-37054 ___

INTERVIEW:

Most likely location of target: ___ 61550-37050 ___
If in error, the error is:
An error at the Guns or in the FDC: ___
A result of the fall of shot dispersion: ___ X ___
In the estimate of the target location: ___ X ___
Observer is uncertain about the location of the target:
YES: ___ NO: ___
Observer’s selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: ___
An interval estimate: ___ X ___
Fractiles of the CDF using the ITL as the origin:
1%: ___ 100 ___ meters short of ITL
25%: ___ 75 ___ meters short of ITL
75%: ___ 30 ___ meters over the ITL
99%: ___ 50 ___ meters over the ITL
50%: ___ 0 ___ meters over/short of the ITL

FEEDBACK COMMENTS:
The disparity between the 1% and 25% points was pointed out and the
1% point changed accordingly.
PREDICTION OF FALL OF SHOT OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _____ meters. SHORT of the
ITL by _____ 40 ___ meters. INDIFFERENT: _____
SUBJECT:

Name: ___________________________ Subject Nr: __4__
Grade: OC
Length of Service: 9 mos.
Approx. nr. of mens. fired: __10__
Statistical training: __none__

EXPERIMENTAL ENVIRONMENT:

Date/Time: 251515 Feb 69
Visibility: Ex
Observation Post:
Description: FP 3045 Grid: 59033-37451
Has registration been conducted?
YES: x NO: ________
Formal: ________
Informal: ________
Target:
Description: Unk Grid: not determined

INTERVIEW:

Most likely location of target: __61500-36660__
If in error, the error is:
An error at the Guns or in the FDC: ________
A result of the fall of shot dispersion: ________
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: ________
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: ________
An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
1%: 200 meters short of ITL
25%: 100 meters short of ITL
75%: 100 meters over the ITL
99%: 200 meters over the ITL
50%: 200 meters over/short of the ITL

FEEDBACK COMMENTS:
The subject is incapable of considering the decision in a probability environment.

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by ________ meters. SHORT of the
ITL by ________ meters. INDIFFERENT: ____________
SUBJECT:

Name: _____________________________ Subject Nr: 5
Grade: OC
Length of Service: 9 mos.
Approx. nr. of mens. fired: 5
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 251550 Feb 69 Visibility: Ex
Observation Post:
Description: FP 304S Grid: 59033-37451
Has registration been conducted?
YES: X NO: ________
Formal: x Informal: ________
Target:
Description: #302 Grid: 60742-37187

INTERVIEW:

Most likely location of target: 61250-37140
If in error, the error is:
An error at the Guns or in the FDC: x
A result of the fall of shot dispersion: ________
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: ________
Observer’s selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: ________
An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
1%: 200 meters short of ITL
25%: 90 meters short of ITL
75%: 50 meters over the ITL
99%: 100 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by ________ meters. SHORT of the
ITL by ________ meters. INDIFFERENT: ________
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ___________________________ Subject Nr: 6
Grade: 2LT
Length of Service: 3 mos.
Approx. nr. of mans. fired: 2

EXPERIMENTAL ENVIRONMENT:

Date/Time: 260925 Feb 69 Visibility: Ex
Observation Post:
  Description: FP 438 Grid: 64044-83988
Has registration been conducted?
  YES: x
  NO: 
Target:
  Description: #472 Grid: 61903-41778

INTERVIEW:

Most likely location of target: 62240-41420
If in error, the error is:
  An error at the Guns or in the FDC: x
  A result of the fall of shot dispersion: 
  In the estimate of the target location: x
Observer is uncertain about the location of the target:
  YES: x
  NO:
Observer's selection of initial target location (ITL) was from:
  Two or more sure locations at discrete points: 
  An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
  1%: 200 meters short of ITL
  25%: 150 meters short of ITL
  75%: 150 meters over the ITL
  99%: 200 meters over the ITL
  50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:
Subject first described a skewed distribution but retracted it in favor of a symmetrical pdf.
PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _______ meters. SHORT of the ITL by _______ meters. INDIFFERENT: _______
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ___________________________ Subject Nr: __7____
Grade: ______L T____
Length of Service: __8 mos.____
Approx. nr. of msns. fired: ___2___
Statistical training: ______none________

EXPERIMENTAL ENVIRONMENT:

Date/Time: 261000 Feb 60 Visibility: Ex
Observation Post:
   Description: FP 438 Grid: 64044-83988
Has registration been conducted?
   YES:   ___ NO:   ___
   Formal:    __________ Informal:   x
Target:
   Description: #426 Grid: 62852-40972

INTERVIEW:

Most likely location of target: 62200-41600
If in error, the error is:
   An error at the Guns or in the FDC: __________
   A result of the fall of shot dispersion:   x
   In the estimate of the target location:  x
Observer is uncertain about the location of the target:
   YES:     x NO:   __________
Observer's selection of initial target location (ITL) was from:
   Two or more sure locations at discrete points:   x?
An interval estimate:
Fractiles of the CDF using the ITL as the origin:
1%:  400 meters short of ITL
25%:  200 meters short of ITL
75%:  200 meters over the ITL
99%:  300 meters over the ITL
50%:  50 meters over/short of the ITL

FEEDBACK COMMENTS:
The pdf was deliberately skewed by the subject. "If I'm off, I think I'll be over."

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _______ meters. SHORT of the
ITL by ______200____ meters. INDIFFERENT: _____________
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ___________________________ Subject Nr: 8
Grade: 2 LT
Length of Service: 6 mos.
Approx. nr. of mans. fired: 7
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 261030 Feb 69 Visibility: Ex
Observation Post:
Description: FP 438 Grid: 64044-83988
Has registration been conducted?
YES: NO: ___________
Formal: ___________
Informal: x
Target:
Description: #458 Grid: 62894-42881

INTERVIEW:

Most likely location of target: 63050-42480
If in error, the error is:
An error at the Guns or in the FDC: ___________
A result of the fall of shot dispersion: ___________
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: ___________
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: ___________
An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
1%: 350 meters short of ITL
25%: 50 meters short of ITL
75%: 50 meters over the ITL
99%: 350 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _________ meters. SHORT of the
ITL by _______ meters. INDIFFERENT: _________
SUBJECT:

Name: ___________________________ Subject Nr: 9
Grade: _______ 2LT _______
Length of Service: _______ 3 mos. _______
Approx. nr. of mns. fired: _______ 3 _______
Statistical training: _______ none _______

EXPERIMENTAL ENVIRONMENT:

Date/Time: 261305 Feb 69 Visibility: _______ Ex _______
Observation Post:
Description: _______ FP 438 _______
Grid: _______ 64044-83988 _______
Has registration been conducted?
YES: _______ NO: _______
Formal: _______
Informal: _______ x _______
Target:
Description: _______ #472 _______
Grid: _______ 61903-41778 _______

INTERVIEW:

Most likely location of target: _______ 61900-41350 _______
If in error, the error is:
   An error at the Guns or in the FDC: _______
   A result of the fall of shot dispersion: _______ x _______
   In the estimate of the target location: _______ x _______
Observer is uncertain about the location of the target:
YES: _______ x _______
NO: _______
Observer's selection of initial target location (ITL) was from:
   Two or more sure locations at discrete points: _______
   An interval estimate: _______ x _______
Fractiles of the CDF using the ITL as the origin:
1%: _______ 500 _______ meters short of ITL
25%: _______ 400 _______ meters short of ITL
75%: _______ 600 _______ meters over the ITL
99%: _______ 800 _______ meters over the ITL
50%: _______ 100 _______ meters over short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _______ meters. SHORT of the
ITL by _______ 200 _______ meters. INDIFFERENT: _______
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ___________________________________ Subject Nr: 10
Grade: 2LT
Length of Service: 4 mos.
Approx. nr. of mans. fired: 3

EXPERIMENTAL ENVIRONMENT:

Date/Time: 26/1325 Feb 69 Visibility: Ex
Observation Post:
Description: FP 438 Grid: 64044-83988
Has registration been conducted?
YES: NO: X
Target:
Description: #462 Grid: 62444-41696

INTERVIEW:

Most likely location of target: 62380-41850
If in error, the error is:
An error at the Guns or in the FDC: 
A result of the fall of shot dispersion: x
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: X NO: 
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points:
An interval estimate: X
Fractiles of the CDF using the ITL as the origin:
1%: 400 600 meters short of ITL
25%: 400 meters short of ITL
75%: 400 meters over the ITL
99%: 600 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT:OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _______ meters. SHORT of the
ITL by _______ meters. INDIFFERENT: X
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ___________________________ Subject Nr: 11
Grade: 2LT
Length of Service: 2 yrs.
Approx. nr. of mens. fired: 3

EXPERIMENTAL ENVIRONMENT:

Date/Time: 261350 Feb 69 Visibility: Ex
Observation Post:
Description: FP 438 Grid: 64044-83988
Has registration been conducted?
YES: ____________ NO: ____________
Formal: ____________ Informal: x
Target:
Description: #419 Grid: 62189-41572

INTERVIEW:

Most likely location of target: 62080-41580
If in error, the error is:
An error at the Guns or in the FDC:
A result of the fall of shot dispersion:
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: ____________
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points:
An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
1%: 500 meters short of ITL
25%: 200 meters short of ITL
75%: 500 meters over the ITL
99%: 1000 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by 100 meters. SHORT of the
ITL by _______ meters. INDIFFERENT: ____________
SUBJECT:

Name: ___________________________ Subject Nr: 12
Grade: 2LT
Length of Service: 4 1/2 yrs.
Approx. nr. of men. fired: 100
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 26/1420 Feb 69 Visibility: Ex
Observation Post:
Description: FP 438
Grid: 64044-83988
Has registration been conducted?
YES: _____ NO: _____
Formal: _____ Informal: x
Target:
Description: Jones
Grid: 62643-42065

INTERVIEW:

Most likely location of target: 62500-42100
If in error, the error is:
An error at the Guns or in the FDC: _____
A result of the fall of shot dispersion:
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: _____
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: _____
An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
1%: 400 meters short of ITL
25%: 150 meters short of ITL
75%: 150 meters over the ITL
99%: 500 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by 200 meters. SHORT of the
ITL by _____ meters. INDIFFERENT: _______________
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ___________________________ Subject Nr: 13
Grade: 2LT
Length of Service: 1 yr.
Approx. nr. of mans. fired: 3

EXPERIMENTAL ENVIRONMENT:

Date/Time: 261550 Feb 69 Visibility: Ex
Observation Post:
Description: FP 438 Grid: 64044-83988
Has registration been conducted?
YES: NO: __________
Formal: ___________________________ Informal: x
Target:
Description: #485 Grid: 62267-42179

INTERVIEW:

Most likely location of target: 62260-41860
If in error, the error is:
   An error at the Guns or in the FDC: __________
   A result of the fall of shot dispersion: __________
   In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: __________
Observer's selection of initial target location (ITL) was from:
   Two or more sure locations at discrete points: _______
   An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
   1%: 350 meters short of ITL
   25%: 300 meters short of ITL
   75%: 300 meters over the ITL
   99%: 350 meters over the ITL
   50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by 150 meters. SHORT of the
ITL by ______ meters. INDIFFERENT: __________
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ___________________________ Subject Nr: 14
Grade: ___________ 2LT ______
Length of Service: 4 mos. ______
Approx. nr. of msns. fired: 5 ______
Statistical training: as part of course in Psych. ______

EXPERIMENTAL ENVIRONMENT:

Date/Time: 280815 Feb 69  Visiblity: Ex ______
Observation Post:
Description: FP 443  Grid: 64109-40165 ______
Has registration been conducted?
YES: NO: ______
   Formal: ______
   Informal: x ______
Target:
Description: Tank  Grid: 63471-40990 ______

INTERVIEW:

Most likely location of target: 63230-41380 ______
If in error, the error is:
   An error at the Guns or in the FDC: ______
   A result of the fall of shot dispersion: ______
   In the estimate of the target location: x ______
Observer is uncertain about the location of the target:
YES: x NO: ______
Observer's selection of initial target location (ITL) was from:
   Two or more sure locations at discrete points: ______
   An interval estimate: x ______
Fractiles of the CDF using the ITL as the origin:
1%: 200 meters short of ITL ______
25%: 100 meters short of ITL ______
75%: 100 meters over the ITL ______
99%: 150 meters over the ITL ______
50%: 0 meters over/short of the ITL ______

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT:OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by ______ meters. SHORT of the
ITL by ______ meters. INDIFFERENT: x ______
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ____________ Subject Nr: 15
Grade: 2LT
Length of Service: 4 mos.
Approx. nr. of msns. fired: 2
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 280850 Feb 69 Visibility: Ex
Observation Post:
  Description: FP 443
  Grid: 64109-40165

Has registration been conducted?
YES: NO:
  Formal: _________
  Informal: x

Target:
  Description: #442
  Grid: 62327-41251

INTERVIEW:

Most likely location of target: 61820-41670

If in error, the error is:
  An error at the Guns or in the FDC: _________
  A result of the fall of shot dispersion: _________
  In the estimate of the target location: x

Observer is uncertain about the location of the target:
YES: x NO:

Observer's selection of initial target location (ITL) was from:
  Two or more sure locations at discrete points: _________
  An interval estimate: x

Fractiles of the CDF using the ITL as the origin:
  1%: 700 meters short of ITL
  25%: 500 meters short of ITL
  75%: 200 meters over the ITL
  99%: 500-300 meters over the ITL
  50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT OF THE INITIAL ROUNDS IN ADJUSTMENT: OVER the ITL by 300 meters. SHORT of the ITL by _______ meters. INDIFFERENT: ____________
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ______________________ Subject Nr: __________
Grade: 2LT
Length of Service: 3 mos.
Approx. nr. of mns. fired: 2
Statistical training: one credit crse in Engr. Stat.

EXPERIMENTAL ENVIRONMENT:

Date/Time: 280935 Feb 69 Visibility: Ex
Observation Post:
  Description: FP 443
  Grid: 64109-40165
Has registration been conducted?
  YES:
  NO: _____
    Formal:
    Informal: x
Target:
  Description: #423
  Grid: 63026-42833

INTERVIEW:

Most likely location of target: 63150-43050
If in error, the error is:
  An error at the Guns or in the FDC:
  A result of the fall of shot dispersion: x
  In the estimate of the target location: x
Observer is uncertain about the location of the target:
  YES: x
  NO: 
Observer's selection of initial target location (ITL) was from:
  Two or more sure locations at discrete points:
  An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
  1%: 400 meters short of ITL
  25%: 100 meters short of ITL
  75%: 100 meters over the ITL
  99%: 300 meters over the ITL
  50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _______ meters. SHORT of the
ITL by _______ meters. INDIFFERENT: x
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: __________________________ Subject Nr: 17
Grade: 2LT
Length of Service: 3 mos.
Approx. nr. of men. fired: 2

EXPERIMENTAL ENVIRONMENT:

Date/Time: 281000 Feb 69 Visibility: Ex
Observation Post:
Description: FP 443 Grid: 64109-40165
Has registration been conducted?
YES: NO: ____________
Formal: 
Informal: 
Target:
Description: #422 Grid: 62884-41368

INTERVIEW:

Most likely location of target: 62500-41650
If in error, the error is:
An error at the Guns or in the FDC: ____________
A result of the fall of shot dispersion: ____________
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: 
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points:
An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
1%: 150 meters short of ITL
25%: 100 meters short of ITL
75%: 50 meters over the ITL
99%: 75 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN ADJUSTMENT: OVER the ITL by _____ meters. SHORT of the ITL by _____ meters. INDIFFERENT: ________
QUESTIONNAIRE RESPONSE SHEET

SUBJECT:

Name: ________________________ Subject Nr: 18
Grade: 2LT
Length of Service: 7 mos.
Approx. nr. of mens. fired: 4
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 281040 Feb 69 Visibility: Ex
Observation Post:
Description: FP 443 Grid: 64109-40165
Has registration been conducted?
YES: No: ___________
Formal: ___________
Informal: x
Target:
Description: #426 Grid: 62852-40972

INTERVIEW:

Most likely location of target: 62140-41320
If in error, the error is:
An error at the Guns or in the FDC: ___________
A result of the fall of shot dispersion: ___________
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: ___________
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: _______
An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
1%: 100 meters short of ITL
25%: 75 meters short of ITL
75%: 75 meters over the ITL
99%: 150 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by 200 meters. SHORT of the
ITL by ______ meters. INDIFFERENT: __________
SUBJECT:

Name: ___________________________ Subject Nr: __________
Grade: 2Lt
Length of Service: 4 mos.
Approx. nr. of mans. fired: 2
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 28/3/69 Feb 69 Visibility: Ex
Observation Post:
Description: FP 443 Grid: 64109-40165
Has registration been conducted?
YES: NO:__________
Formal: ___________
Informal: _______
Target:
Description: (#459?) Grid: Unknown

INTERVIEW:

Most likely location of target: 63150-42100
If in error, the error is:
An error at the Guns or in the FDC: _____________
A result of the fall of shot dispersion: ___________
In the estimate of the target location: _______
Observer is uncertain about the location of the target:
YES: NO:__________
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: ___________
An interval estimate: ______
Fractiles of the CDF using the ITL as the origin:
1%: 700 meters short of ITL
25%: 300 meters short of ITL
75%: 200 meters over the ITL
99%: 500 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _______ meters. SHORT of the
ITL by _____ meters. INDIFFERENT: _________
SUBJECT:

Name: ___________________________ Subject Nr: 20
Grade: 2LT
Length of Service: 2 yrs
Approx. nr. of mens. fired: 4
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 281415 Feb 69 Visibility: Ex
Observation Post:
Description: FP 443 Grid: 64109-40165
Has registration been conducted?
YES: _______________ NO: _______________
Formal: _______________ Informal: x
Target:
Description: #462 Grid: 62444-41696

INTERVIEW:

Most likely location of target: 62200-41830
If in error, the error is:
An error at the Guns or in the FDC: _______________
A result of the fall of shot dispersion: _______________
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: _______________
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: _______________
An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
1%: 400 meters short of ITL
25%: 200 meters short of ITL
75%: 200 meters over the ITL
99%: 450 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:
Subject retracted initial distribution after being shown the graph of the cdf and having its meaning explained.
PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _______ meters. SHORT of the ITL by _______ meters. INDIFFERENT: x
SUBJECT:

Name: ____________________________ Subject Nr: 21
Grade: 2LT
Length of Service: 1 yr.
Approx. nr. of mens. fired: 3
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 281535 Feb 69 Visibility: Ex
Observation Post:
Description: FP 443 Grid: 64109-40165
Has registration been conducted?
YES: NO: _______
Formal: _______
Informal: _______x
Target:
Description: Jones Grid: 62643-42065

INTERVIEW:

Most likely location of target: 62950-41650
If in error, the error is:
An error at the Guns or in the FDC: _______
A result of the fall of shot dispersion: _______
In the estimate of the target location: _______x
Observer is uncertain about the location of the target:
YES: _______x NO: _______
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: _______
An interval estimate: _______x
Fracctiles of the CDF using the ITL as the origin:
1%: 500 meters short of ITL
25%: 200 meters short of ITL
75%: 300 meters over the ITL
99%: 600 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _______ meters. SHORT of the
ITL by _______ meters. INDIFFERENT: _______x
SUBJECT:

Name: ___________________________  Subject Nr:  22
Grade:  2LT
Length of Service:  4 mos.
Approx. nr. of mens. fired:  7

EXPERIMENTAL ENVIRONMENT:

Date/Time:  281600  Feb 69  Visibility:  Ex
Observation Post:
  Description:  FP 443  Grid:  64109-40165
Has registration been conducted?
  YES:  NO: ___________
  Formal:  ___________
  Informal:  x
Target:
  Description:  #443  Grid:  62343-41506

INTERVIEW:

Most likely location of target:  62100-41420
If in error, the error is:
  An error at the Guns or in the FDC:  ___________
  A result of the fall of shot dispersion:  ___________
  In the estimate of the target location:  x
Observer is uncertain about the location of the target:
  YES:  x  NO: ___________
Observer's selection of initial target location (ITL) was from:
  Two or more sure locations at discrete points:  ___________
  An interval estimate:  x
Fractiles of the CDF using the ITL as the origin:
  1%:  100  meters short of ITL
  25%:  50  meters short of ITL
  75%:  50  meters over the ITL
  99%:  100  meters over the ITL
  50%:  0  meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by __________ meters. SHORT of the
ITL by __________ meters. INDIFFERENT: ___________
SUBJECT:

Name: ____________________________ Subject Nr: __23____
Grade: __QC____________________
Length of Service: __9 mos.________
Approx. nr. of mens. fired: __1_____ Statistical training: __in Psych. and Econ. crses.

EXPERIMENTAL ENVIRONMENT:

Date/Time: __010805 Mar 69__ Visibility: __Ex_________
Observation Post:
Description: __FP_165__________ Grid: __48192-34811__
Has registration been conducted?
YES: ______________ NO: __x________
Formal: ____________ Informal: __________
Target:
Description: __#6_________ Grid: __48781-36354___

INTERVIEW:

Most likely location of target: __48750-36480____
If in error, the error is:
   An error at the Guns or in the FDC: __x__________
   A result of the fall of shot dispersion: ____________
   In the estimate of the target location: ____________
Observer is uncertain about the location of the target:
YES: __x________________ NO: _________________
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: __________
An interval estimate: __x_______________________
Fractiles of the CDF using the ITL as the origin:
1%: __500_meters short of ITL
25%: __100_meters short of ITL
75%: __300_meters over the ITL
99%: __600_meters over the ITL
50%: __20_meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT:OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by ___________meters. SHORT of the
ITL by __100__ meters. INDIFFERENT: _____________
SUBJECT:

Name: ____________________________ Subject Nr: 24
Grade: OC
Length of Service: 8 mos.
Approx. nr. of msns. fired: 1
Statistical training: none

EXPERIMENTAL ENVIRONMENT:

Date/Time: 010850 Mar 69 Visibility: Ex
Observation Post:
Description: FP 165 Grid: 48192-34811
Has registration been conducted?
YES: NO: x
Formal: Informal: x
Target:
Description: #82 Grid: 47916-36428

INTERVIEW:

Most likely location of target: 47650-36450
If in error, the error is:
   An error at the Guns or in the FDC: x
   A result of the fall of shot dispersion: x
   In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: x
Observer's selection of initial target location (ITL) was from:
   Two or more sure locations at discrete points: x
   An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
   1%: 200 meters short of ITL
   25%: 100 meters short of ITL
   75%: 150 meters over the ITL
   99%: 400 meters over the ITL
   50%: 75 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by _______ meters. SHORT of the ITL by _______ meters. INDIFFERENT: _______
SUBJECT:

Name: ______________________ Subject Nr: 25
Grade: OC
Length of Service: 8 mos.
Approx. nr. of mns. fired: 2

EXPERIMENTAL ENVIRONMENT:

Date/Time: 010925 Mar 69 Visibility: Ex
Observation Post:
Description: FP 165 Grid: 48192-34811
Has registration been conducted?
YES: NO: ____________
Formal: ____________
Informal: x
Target:
Description: #4 Grid: 48533-36498

INTERVIEW:

Most likely location of target: 48550-36450
If in error, the error is:
An error at the Guns or in the FDC: ____________
A result of the fall of shot dispersion: ____________
In the estimate of the target location: x
Observer is uncertain about the location of the target:
YES: x NO: ____________
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: ____________
An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
1%: 800 meters short of ITL
25%: 200 meters short of ITL
75%: 400 meters over the ITL
99%: 1500 meters over the ITL
50%: 0 meters over/short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by ________ meters. SHORT of the
ITL by ________ meters. INDIFFERENT: x

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by ________ meters. SHORT of the
ITL by ________ meters. INDIFFERENT: x
SUBJECT:

Name: ____________________________ Subject Nr: 26
Grade: ____________________________
Length of Service: 9 mos.
Approx. nr. of msns. fired: 2
Statistical training: Math, Stat. in Physics Major

EXPERIMENTAL ENVIRONMENT:

Date/Time: 011010 Mar 69 Visibility: Ex
Observation Post:
Description: FP 165 Grid: 48192-34811
Has registration been conducted?
YES: NO: ____________
Formal: ____________
Informal: x
Target:
Description: #92 Grid: 48583-36157

INTERVIEW:

Most likely location of target: 48640-36440
If in error, the error is:
An error at the Guns or in the FDC: ______
A result of the fall of shot dispersion: x
In the estimate of the target location: ______
Observer is uncertain about the location of the target:
YES: x NO: ______
Observer's selection of initial target location (ITL) was from:
Two or more sure locations at discrete points: ______
An interval estimate: x
Fractiles of the CDF using the ITL as the origin:
1%: 800 meters short of ITL
25%: 400 meters short of ITL
75%: 400 meters over the ITL
99%: 800 meters over the ITL
50%: 100 meters over short of the ITL

FEEDBACK COMMENTS:

PREDICTION OF FALL OF SHOT: OF THE INITIAL ROUNDS IN
ADJUSTMENT: OVER the ITL by ______ meters. SHORT of the
ITL by 150 meters. INDIFFERENT: ____________
### QUESTIONNAIRE RESPONSE SHEET

**SUBJECT:**

Name: ___________________________  Subject Nr: 27  
Grade: __________  OCS  
Length of Service: 9 mos.  
Approx. nr. of mans. fired: 1  

**EXPERIMENTAL ENVIRONMENT:**

Date/Time: 011040 Mar 69  
Visibility: Ex  
Observation Post:  
Description: FP 165  
Grid: 48192-34811  
Has registration been conducted?  
YES:  
NO:  
Formal:  
Informal: X  
Target:  
Description:  
Grid: 48238-37614

**INTERVIEW:**

Most likely location of target: 48250-37500  
If in error, the error is:  
An error at the Guns or in the FDC:  
A result of the fall of shot dispersion:  
In the estimate of the target location: X  
Observer is uncertain about the location of the target:  
YES: X  
NO:  
Observer's selection of initial target location (ITL) was from:  
Two or more sure locations at discrete points:  
An interval estimate: X  
Fractiles of the CDF using the ITL as the origin:  
1%: 300 meters short of ITL  
25%: 120 meters short of ITL  
75%: 100 meters over the ITL  
99%: 300 meters over the ITL  
50%: 0 meters over/short of the ITL

**FEEDBACK COMMENTS:**

**PREDICTION OF FALL OF SHOT:** OF THE INITIAL ROUNDS IN  
ADJUSTMENT: OVER the ITL by _______ meters. SHORT of the  
ITL by _______ meters. INDIFFERENT: __________
APPENDIX D

GRAPHS OF THE PRIOR DISTRIBUTION FUNCTIONS
Figure 11: Graphs of the cumulative distribution functions from which to infer the prior probability density functions PR(m) of the subjects interviewed during the field experiment. The linear functions shown are fitted by regression techniques to the elicited data points plotted on each graph.

(a) Observer #1  (n) Observer #15
(b) Observer #2  (o) Observer #16
(c) Observer #3  (p) Observer #17
(d) Observer #5  (q) Observer #18
(e) Observer #6  (r) Observer #19
(f) Observer #7  (s) Observer #20
(g) Observer #8  (t) Observer #21
(h) Observer #9  (u) Observer #22
(i) Observer #10 (v) Observer #23
(j) Observer #11 (w) Observer #24
(k) Observer #12 (x) Observer #25
(l) Observer #13 (y) Observer #26
(m) Observer #14 (z) Observer #27

LEGEND

Observer's Estimate of Target Location Along OT Line
Actual Target Location Along OT Line
OBS 1
FIG. II(a)

OBS 2
FIG. II(b)
FIG. 11(c) OBS 3
FIG. 11(d) OBS 5
OBS 6
FIG. II(e)

CDF of PR(m)
(hundreds of meters)

OBS 7
FIG. II(f)
CDF of \( PR(m) \)

(hundreds of meters)

CDF of \( PR(m) \)

(hundreds of meters)

OBS 10

FIG. II(i)

OBS II

FIG. II(j)
OBS 12
FIG. II(k)

(hundreds of meters)

OBS 13
FIG. II(l)
OBS 14
FIG. II(m)

CDF of PR(m)

(hundreds of meters)

OBS 15
FIG. II(n)
OBS 22
FIG. II(u)

OBS 23
FIG. II(v)
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11. ___________ "Request for Data for the Study - 'Accuracy Requirements for the Target Acquisition Component of the Artillery Weapons System', " Memorandum from Director of Gunnery, USAAMS, to Director of Operations and Plans, USAAMS, Fort Sill, Okla., 27 July 1964.


