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A COMPARATIVE STUDY OF TEACHING STRATEGIES INVOLVING CLOSED-CIRCUIT TELEVISION AND PROGRAMMED INSTRUCTION

DISSERTATION
Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By
Frederick Nicholson Moore, B.Sc., M.Sc.

* * * * * * * *

The Ohio State University 1969

Approved by
[Signature]
Advisor
College of Education
I wish to depart from the normal format for acknowledgments and first of all express my deep appreciation to my wife, Sandra, without whose continued support and inspiration this dissertation would never have been written. To my adviser, Professor Harold C. Trimble, I wish to express my appreciation for his support, his helpful suggestions, and for his ability to make smooth a path that abounds in rough spots. Also, I wish to thank the other members of my reading committee, Professor John Hiner and Professor Robert J. Fisher, for their valuable guidance and help, not only in the reading of the manuscript, but also in the conducting of the experiment. To Professor Arnold S. Ross, Chairman of the Department of Mathematics at The Ohio State University, special thanks is due for providing the course and the means for the large scale experiment performed in this study. Special thanks are also due Mr. Robert Dudgeon of The Ohio State University Statistics Laboratory, for helping process the statistical data.
VITA

August 14, 1935

1958

1958 - 1960

1960 - 1964

1964

1964 - 1967

1967 - 1969

Born - Cincinnati, Ohio

B.Sc., Education, Wittenberg University, Springfield, Ohio

Teacher of Mathematics, Springfield High School, Springfield, Ohio

Teacher of Mathematics, Upper Arlington High School, Columbus, Ohio

M.Sc., Mathematics, The Ohio State University, Columbus, Ohio

Teaching Assistant in Mathematics, The Ohio State University, Columbus, Ohio

Counselor and Instructor, Department of Mathematics, The Ohio State University, Columbus, Ohio

FIELDS OF STUDY

Studies in Mathematics Education . . . Professor Harold C. Trimble

Studies in Mathematics

Algebra . . . . . . . . . . . . . . . . Professor Alan Woods

Analysis . . . . . . . . . . . . . . Professor Ranko Bojanic

Topology . . . . . . . . . . . . . Professor Norman Levine

Studies in Teacher Education . . . . . Professor L. O. Andrews

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<td>13</td>
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<td>14</td>
<td>Unadjusted Means for Pre-Attitude, Post-Attitude, and Achievement</td>
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<tr>
<td>15</td>
<td>Adjusted Means for Post-Attitude and Achievement</td>
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CHAPTER I

INTRODUCTION AND BACKGROUND

Among the many problems facing American universities and colleges is the problem of large enrollment increases. As early as 1957, Rickover warned that:

"Within the next fifteen years, six million youngsters will clamor for admission to our institutions of higher learning. Colleges and universities are now being exhorted to prepare for the flood . . . (17, 91)"

With this projected growth in enrollment now a reality, colleges and universities are being called upon to devise teaching strategies which try to deal effectively with large numbers. As Hartnett and Stewart report, "Rising enrollments have caused college administrators to search for new approaches to undergraduate education. Televised instruction, team teaching, programmed teaching, and large lecture sections are just a few of the many contrivances hopefully attempted in a growing number of schools." (10, 356) But multiplied enrollments have meant multiplied problems, and there does not appear to be a single best strategy for providing for large group instruction.

This problem has been of particular concern to the Mathematics Department at The Ohio State University. It has found that,
in addition to the increased numbers, many of the entering freshman simply do not have the mathematical background to compete successfully in the traditional college mathematics courses. (College algebra and above).

All students who enter The Ohio State University are administered a set of placement examinations by the testing center of the university. In particular, an examination in mathematics is given for the purpose of placing students in their initial mathematics course. The placement levels are indicated in the following schedule:

Placement Level I. A student wanting or needing calculus enrolls in Mathematics 151 (Analytic Geometry and Calculus) or Mathematics 121 (Calculus and Statistics). Students who do not want calculus enroll in Mathematics 116.

Placement Level II. A student wanting or needing calculus enrolls in Mathematics 150 (Algebra and Trigonometry) or Mathematics 121. Other students enroll in Mathematics 116.

Placement Level III. A student wanting or needing calculus enrolls in Mathematics 101. Other students enroll in Mathematics 116.

Placement Level IV. A student who places in this level and who needs college level mathematics must first take Mathematics 101.

Placement Level V. Students placing in this level are not permitted to schedule any mathematics at Ohio State. They must
retake the placement examination and place in level IV or higher before scheduling any mathematics course.

A brief description of the above mentioned courses can be found in appendix A.

The following table shows the percent of students placing in the various levels for the past four years. (9)

<p>| TABLE 1 |
|---|---|---|---|---|---|
| | MATHEMATICS PLACEMENT LEVEL (Percent) |</p>
<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>65-66</td>
<td>12</td>
<td>42</td>
<td>24</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>66-67</td>
<td>11</td>
<td>36</td>
<td>25</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>67-68</td>
<td>9</td>
<td>35</td>
<td>24</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>68-69</td>
<td>3</td>
<td>34</td>
<td>23</td>
<td>17</td>
<td>18</td>
</tr>
</tbody>
</table>

This table indicates that in 1963-69, thirty-five percent of the entering freshman rank at a level below that of college algebra. (level 4 and 5).

In an attempt to remedy the problem of inadequate mathematical background, The Ohio State Mathematics Department has offered a so-called remedial course (presently titled Mathematics 101) consisting basically of high school algebra. Since virtually all Ohio
State students have at least one mathematics course to complete in order to meet graduation requirements, Mathematics 101 is offered in order to provide the needed background for the particular college-required sequence. The table below shows the breakdown, by college, of the mathematics requirement. (21)

TABLE 2
MATHEMATICS REQUIREMENT BY COLLEGE

<table>
<thead>
<tr>
<th>College of</th>
<th>Hours of required mathematics beyond Mathematics 101</th>
</tr>
</thead>
<tbody>
<tr>
<td>Administrative Science</td>
<td>15</td>
</tr>
<tr>
<td>Agriculture and Home Economics</td>
<td>5 - 15</td>
</tr>
<tr>
<td>Arts</td>
<td>10</td>
</tr>
<tr>
<td>Biological Sciences</td>
<td>10</td>
</tr>
<tr>
<td>Education</td>
<td>5 (Elementary)</td>
</tr>
<tr>
<td>Engineering</td>
<td>10 and up</td>
</tr>
<tr>
<td>Humanities</td>
<td>10</td>
</tr>
<tr>
<td>Mathematics and Physical Sciences</td>
<td>10 and up</td>
</tr>
<tr>
<td>Optometry</td>
<td>10</td>
</tr>
<tr>
<td>Pharmacy</td>
<td>15</td>
</tr>
<tr>
<td>Social and Behavioral Sciences</td>
<td>10</td>
</tr>
</tbody>
</table>

The number of students involved in this review of algebra is perhaps significant. The fact is that Mathematics 101 is the
largest enrollment course taught by the Department. The figures for the last three years are given in table 3.

<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>Number</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965-66</td>
<td>Autumn</td>
<td>722</td>
<td>1294</td>
</tr>
<tr>
<td></td>
<td>Spring</td>
<td>315</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Summer</td>
<td>257</td>
<td></td>
</tr>
<tr>
<td>1966-67</td>
<td>Autumn</td>
<td>1609</td>
<td>2326</td>
</tr>
<tr>
<td></td>
<td>Spring</td>
<td>495</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Summer</td>
<td>222</td>
<td></td>
</tr>
<tr>
<td>1967-68</td>
<td>Autumn</td>
<td>194</td>
<td>2908</td>
</tr>
<tr>
<td></td>
<td>Spring</td>
<td>730</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Summer</td>
<td>234</td>
<td></td>
</tr>
</tbody>
</table>

It is estimated that the 1968-69 enrollment total will be 3300.

It is the purpose of this investigation to compare two different teaching methods as they apply to Mathematics 101. One method is that used by the Department in the past, namely, teaching via closed-circuit television. It has been the feeling that, due to the numbers involved and the resources available, this is the best method of instruction. Television has been used primarily in the autumn quarter when enrollment is maximum. The usual technique has been to provide three televised lectures per week. These have been viewed in large classrooms (maximum size 250) at designated
hours throughout the day. The remaining two days are used for recitation. The student attends a small class (30-40), with a graduate student or teaching assistant in charge, where he may ask questions and receive general assistance.

The second method is to provide for instruction by using programmed materials. It is not the purpose of this study, however, to test the general effectiveness of programmed learning, but rather the method of using it. As Coulson reports, "Recent research suggests that programmed instruction is far from the self-contained, self-instructional vehicle it is often assumed to be. The effectiveness of even the most carefully prepared program is governed by factors external to the program itself . . ." (7, 3)

This study focuses upon factors which relate to student responsibility for learning. The general strategy is to offer the student a variety of resources to augment a programmed text. The extent to which these resources are utilized is determined by the student.

The problem under investigation includes the following questions:

1. Would allowing students to regulate and evaluate their own progress with the program be any less effective than requiring a steady rate of progress by giving periodic examinations?

2. Does a teaching strategy which provides for individual responsibility have a positive effect on the students' attitude toward mathematics?
The general hypotheses tested in this study are these:

1. There is no significant difference in achievement between those taught by closed-circuit television and those taught by programmed learning.

2. Students using the programmed method will have a greater positive attitude change toward mathematics than those taught by television.

In terms of future plans for meeting the growing educational demands of a large university, the Department is moving toward more individualized instruction. Such a move will naturally increase the responsibility of the student for learning. Among the many questions which first will need to be answered are the following: "To what extent will the student assume a greater responsibility for learning, given sufficient resources?" "How can resources peculiar to The Ohio State University Mathematics Department best be utilized in providing for individualized instruction?" It is the hope of this investigator that this study will prove to be of value to the Department in its attempt to find answers to these questions.

Chapter II deals with a survey of the literature pertinent to this study. The following chapter is devoted to the methods and procedures employed. In Chapter IV, the statistical results are presented and analyzed, and the last chapter includes a summary of the study and the recommendations made on the basis of the results.
CHAPTER II

SURVEY OF THE RELATED LITERATURE

This chapter deals with two aspects of the literature germane to this study. First, selected background reading pertaining to the definition of the problem and the establishment of a theoretical base for the particular strategy employed in this study is presented. The second part of the chapter deals with a report of related studies.

Background. Since this investigation deals, in part, with the concept of individual responsibility for learning, it seems appropriate to make some mention of the nature of the audience and the course for which this study was designed. The content of Mathematics 101 has been considered as remedial, since intermediate algebra is a subject usually reserved for the domain of the high school curriculum. However, it is not the purpose here to discuss the philosophy behind the offering of such work in institutions of higher education. The literature is abundant with references, pro and con, to this issue. The fact is that most state universities and colleges do maintain an open admissions policy. Thus, the departmental position has been that shared by
Hughes; namely, that there must be "a realization that the universities and colleges have, in fact, an obligation to those whom they admit." (13, 169) As Cameron remarked in the American Mathematical Monthly, (3, 155)

State universities have their own peculiar problems. Many of them are due to a heterogeneous student body, representing an incredibly wide range of ability, preparation, and interest . . . in state universities with relatively unselected freshman classes, differentiated programs of study appear to be the only feasible way of providing the type of education appropriate to the various levels of ability.

This study is, then, one attempt to deal with this "incredibly wide range of ability"; viz, it is a strategy for providing an opportunity for college students to learn remedial mathematics. In fact, the degree to which this strategy proves successful is the central concern.

It is not the intention of this study to defend programmed instruction. There is a wealth of data that exists to attest to the general effectiveness of programmed learning. The reader is referred to what Silberman (19, 1) has called "the most comprehensive source of information now available in this field: Teaching Machines and Programmed Learning. (11) This is a comprehensive source document and provides a reasonably representative picture of past and current developments. For those interested in current research in programmed instruction in mathematics, Zoll has recently reviewed the field, reported his findings, and provided
an extensive bibliography in *The Mathematics Teacher*, February, 1969. (24)

As previously mentioned, it is the factors external to the program itself that come under investigation in this study. In particular, to what extent will the student be responsible for learning, given a particular set of resources? Central to this concern is the theoretical base underlying the strategy. It is the position of the investigator that the learner will succeed in learning to the extent that he spends the amount of time he needs to learn the task. This position is substantiated by the work of Bloom (2) and Carroll (4). This time-rate is the key to what Bloom calls mastery learning. "Given sufficient time and appropriate types of help, ninety-five percent of students can learn a subject up to a high level of mastery." (2, 4) In Carroll's model, he identifies the essential components of this rate process as (1) Determinants of time needed for learning, and (2) Determinants of time spent on learning. The later is a function of two variables: the time allowed for learning, and what he terms perseverance, the amount of time the learner is willing to spend in learning.

In the strategy reported in this study, it has been the intention of the investigator to focus upon this last aspect of Carroll's model by attempting to maximize these last two rate factors. In the case of time allowed for learning, the formal lecture technique has been replaced with a programmed-tutor approach.
This approach is designed to provide the student with more time for doing mathematics, with less time spent in hearing about mathematics. In terms of perseverance, it is the contention of the investigator that the learner is willing to spend time in learning to the extent to which he experiences success in learning. Remedial students have a particularly significant need to achieve some self-confidence which comes through success. Programmed materials provide an immediate reinforcement, and a tutor provides individual approval. Theoretically, then, the strategy employed allows for an individual to work at his own rate, with the expectation that the psychological rewards of approval and success will increase his perseverance.

Related Studies. Although the majority of research in this area suggests little or no difference between the performance of students working independently and those working in formally-organized classes, there is some evidence to suggest that the unstructured approach is more effective. The findings of Hartnett and Stewart (10) support the view that for certain students, the routine of attending classes, taking notes, and writing examinations may not be the most meaningful process of learning. Students taking college courses in the traditional fashion were paired with students of equal ability who took the same course on an individual study program. Using scores on the common final examination as the criteria of achievement, in all six courses the mean performance was higher
for those who worked independently, and in two of the six, the
differences were large enough to be significant at the .05 level
of confidence. It should be noted that even though the two groups
were matched according to ability, student by student, those in-
volved in this study were of high ability. The question of inde-
pendent work at other ability levels was not investigated.

In a study comparing independent work to a teacher-
oriented approach in reading improvement, Miller (15) found that
in terms of reading rate, there is a significant difference at the
five percent level favoring those who worked independently.

In a college remedial course in intermediate algebra (very
similar to Mathematics 101), Yesslerman (23) compared gains made by
students using programmed materials under three conditions of super-
vision: (i) none (ii) moderate and (iii) great. The students
who had no supervision did the program at home at their own rate,
and came to class only to take unit tests. These tests were taken
on the students' own schedule. Those in the moderate group had
to follow a unit completion schedule, and were required to attend
lectures and to take one midterm examination. The requirements for
group (iii) were essentially the same as those for (ii) with the
exception that subsequent units could be obtained only after satis-
factory completion of the previous unit. The results of this in-
vestigation showed that varying amounts of supervision did not
significantly affect learning from a program.
Hennemann (12) investigated the use of a programmed text as a supplement to a course similar in scope and nature to Mathematics 101. One aspect of his experiment was to determine the effectiveness of independent work as compared with the standard technique. All students were required to complete an introductory calculus text as a term project. The control or structured group was required to meet three deadlines; two midterm examinations and the final examination. Students in the permissive or experimental group were allowed to pace themselves in completing the program and preparing for the final examination. They were given "problem sets" similar to the midterms, but these were neither turned in nor graded. The results showed that the structured group received slightly higher grades on the final examination, but not enough to reject the hypothesis that there is no difference in the treatments. The students in the permissive groups rated the programmed text higher with regard to its effectiveness as a teaching device. This last result is suggestive of a more positive attitude toward mathematics as a result of method. In terms of recommendations, Hennemann suggested that some modification of the permissive treatment was needed in order to provide an external check on the student.

Perhaps the research most relevant to this study is that reported by Bartz and Darby. (1) Their study was designed to investigate the effects of a programmed text under traditional and independent study techniques. Students were allowed to use either a programmed or a non-programmed text under one of three instructional
techniques for a semester course in college algebra. The three instructional techniques investigated differed essentially in the amount of contact with an instructor, and are summarized as follows:

(1) Students were required to attend lectures, and were required to take five unit examinations. This is the traditional technique for teaching the course.

(2) Students were required to visit the instructor once per week, but were excused from regular class attendance. This is the tutored technique. These students were required to take the five unit examinations.

(3) Students were not required to attend class, nor were they required to have a weekly meeting with the instructor. The only test that was required was the final examination.

In terms of the three techniques, the results showed that students under the traditional instructional technique performed significantly higher (five percent level) on the achievement test than did students under individual study. In comparing materials, students using the non-programmed text performed significantly higher (five percent level) than did students using the programmed text. Graphically, the results for performance are shown below.
Perhaps the most striking finding in this research was that the students under the individual techniques (2 and 3) did better using the non-programmed materials. Also, this seems to challenge the classical assumption that the best method of using programmed materials is that of self-pacing and individualized study. It would seem, therefore, that in order to take advantage of the individual rate idea of the programmed text, the student needs more supervision.

In summary, the majority of the evidence seems to indicate that there is no significant difference between a permissive and a structured instructional technique. It is perhaps important to note that most of the findings reported involved a relatively small population. In particular, Miller's work at Oklahoma State University involved 142 students; Yesselman at the University of New Mexico used 176 students; Hennemann tested 164 students in his Cornell study; Bartz and Darby, working at Purdue, used 147
students. Although the present study involves a sample of considerable size, the results, as reported in Chapter IV, tend to confirm the findings of Bartz and Darby.
CHAPTER III

METHODS AND PROCEDURES

This chapter presents a description of the design of the experiment, a discussion of the instruments used, and a brief summary of the philosophical basis for the choice of treatment applied to each of the two groups.

It should be kept in mind that a necessary condition for this experiment was that it be carried out within the existing framework determined by the Department of Mathematics. Thus, while it might have been more desirable to have more student-instructor contact hours, the reality of large numbers and available funds and personnel dictated the boundary conditions.

Selection of Subjects. The subjects in this study are the 1630 students who completed Mathematics 101 during the fall quarter, 1968. The majority were first quarter freshmen (88.6 percent) who had placed in level III or IV on the placement examination. The division of these students into groups A and B was made arbitrarily in the following manner. Of the seven pre-determined hours available for Mathematics 101 instruction, three were chosen as programmed hours and four were designated as television hours. The students
were assigned to an hour in a random fashion by the registrar. Using this method of determining the groups produced sixty-four sections (classes) ranging in size from twenty-two to thirty-nine students per section. These sixty-four sections were equally divided between group A (the television sections) and group B (the programmed sections). By actual count, 746 students were taught by closed circuit television and 831 were taught using the programmed materials.

Instruments Used in the Study. Since selection of content is of utmost importance in evaluating achievement, a review of the appropriate commercial materials was made during the summer, 1963. A list of the texts reviewed appears in appendix B. The fundamental purpose of the study dictated the decision that identical materials be used in both groups. The text selection was Algebra Review Manual, A Program for Self-Instruction by Reigh and Hauck. The publisher is McGraw-Hill. (16)

Of the various criteria used in making this choice, two deserve special mention:

(1) It has been the departmental philosophy that the main emphasis of Mathematics 101 be on the development of manipulative skills. The material chosen must stress computational skills and must contain an abundance of drill problems.
Because of the variety of background and range of training of the students in Mathematics 101, the level of sophistication of the content should be on the order of what has traditionally been labeled as "intermediate algebra".

The topics to be covered during the quarter were:

(1) Polynomial products
(2) Factoring
(3) Algebraic Fractions
(4) Linear and Fractional Equations and Inequalities
(5) Plane Coordinates and Graphing
(6) Systems of Linear Equations
(7) Exponents and Radicals
(8) Quadratic Equations

To measure attitude, the instrument chosen was one developed by Dr. Stephen S. Shatkin. This is an opinionnaire consisting of twenty-two items, with each item designed to elicit information on some specific feeling the student has with regard to mathematics. For each item there are five responses: Strongly Agree, Agree, Undecided, Disagree, Strongly Disagree. By assigning the numbers from one to five to each response, a quantitative measure of the students' attitude is obtained. The opinionnaire appears in appendix F.
The Plan for the Television Sections. The students in group A attended three forty-eight minute television lectures per week. The general procedure for presentation was as follows. At 11:00 a.m., 1:00 p.m., 6:00 p.m., and 7:00 p.m. the lectures were viewed by students of the several sections in large television rooms. The table below shows the class size for the television audience.

### TABLE 4

**CLASS SIZE FOR TELEVISION LECTURES**

<table>
<thead>
<tr>
<th>Time</th>
<th>Room</th>
<th>Number of Assigned Students</th>
<th>Room Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:00</td>
<td>HF 206</td>
<td>203</td>
<td>210</td>
</tr>
<tr>
<td></td>
<td>BZ 100</td>
<td>104</td>
<td>185</td>
</tr>
<tr>
<td>1:00</td>
<td>BZ 100</td>
<td>114</td>
<td>185</td>
</tr>
<tr>
<td>6:00</td>
<td>CH 200</td>
<td>173</td>
<td>320</td>
</tr>
<tr>
<td></td>
<td>HF 206</td>
<td>174</td>
<td>210</td>
</tr>
<tr>
<td>7:00</td>
<td>HF 206</td>
<td>184</td>
<td>210</td>
</tr>
</tbody>
</table>

The lectures, presented by the investigator, provided an organized supplement to the programmed text. Content was explained, examples and drill problems were presented, and points of particular significance or difficulty were elaborated upon. Specific problems were assigned as homework from the text.

On Tuesday and Thursday the students met in individual classrooms for recitation. A teaching assistant from the Department
was assigned to each section. This time was generally spent as a question and answer period, and it was the responsibility of the teaching assistant to provide whatever help the students needed.

**Attitude.** The attitude opinionnaire was administered on the first day of classes and again at the end of the quarter, and the pre- and post-scores were recorded.

**Philosophy.** A basic assumption of television instruction is that the pace and direction of the course is set by the instructor in charge. Thus, the students are asked to learn at a rate predetermined by the teacher, are required to take periodic mid-term examinations, and are urged to meet their homework responsibility on a daily basis. This method of organizing a course can be characterized as structured, directed, and regulated by the instructor. This standardization has certain advantages when dealing with large group instruction, and it is the method employed by the Mathematics Department in the majority of all freshman level courses.

**Plan for the Programmed Sections.** It was expected that both students and instructors (henceforth to be called tutors) in Group B would be unfamiliar with the use of programmed materials. Hence, orientations were planned for the purpose of:

1. Bringing into focus the role of both the teacher and the student in using programmed materials.
(2) Discussing the nature and purpose of this study. This discussion involved only the tutors.

(3) Outlining course procedures.

(4) Identifying the responsibilities of the student and the tutor.

The orientation of the tutors took place prior to the opening of the quarter. In the case of the students, three large group meetings of approximately three hundred and twenty each were conducted by the investigator on the first day of classes.

Attitude. As in the case of the students in Group A, the attitude opinionnaire was administered on the first day of the quarter and again at the conclusion of the course.

The Operation of the Course. The plan for Group B included no lecturing in the traditional sense. Rather, each student was to proceed through the programmed text at his own rate. If he encountered any difficulties, he would seek help from his tutor. The following general procedure was used. Two days a week (either Monday and Wednesday or Tuesday and Thursday) the students met with a tutor in an assigned room. The class size ranged from twenty-seven to thirty-nine. During this time the student could seek assistance from the tutor, work together with his classmates in small groups, read the programmed text, or take unit tests over completed work. (These unit tests will be discussed later) The
The pattern of class activity was not standardized, and a great deal of freedom was given to the tutors in terms of classroom management. The remaining three days were spent as follows. On Tuesday and Thursday (or Monday and Wednesday, depending upon which two days were spent with the tutor) it was suggested to the student that he go to his assigned room for individual work. There was no supervision in this room, since the tutor was assigned to one group of students on Monday and Wednesday and another group on Tuesday and Thursday. It was intended that, through the encouragement of the tutor, small groups of students would work together during this time on content of common concern. Thus, for example, a student who met with his tutor on Monday and Wednesday could use the Tuesday and Thursday class hours as he saw fit.

On Friday a unique tutoring plan was used. At each hour of the day beginning at 9:00 a.m. and continuing through 3:00 p.m., rooms were reserved in various buildings throughout the campus. Each room was staffed by a set of tutors, usually two or three, with each tutor on duty for one hour. These tutoring rooms were available to any 101 student who wanted help with mathematics. Students were encouraged to come to any room at any time at their convenience, and could stay as long as they wished.

To review, then, the traditional "five classes per week" schedule was distributed for the student as follows:

Two days were spent in class with the regular tutor.
Two days were spent in individual work.
One day was used as a special help day.

For the tutor, the distribution was:

Monday and Wednesday were spent in
class with one section.
Tuesday and Thursday were spent in
class with another section.
Friday was used as a special help day.

Philosophy. The basic idea behind this particular use of program­med instruction was that the pace of course work would be set by
the student. The mathematical backgrounds of those taking this
course are very diverse, and experience has shown that the range
of training extends from no formal work in algebra or geometry
to four years of secondary school mathematics. With such a variety
of training and experience, it would seem that an instructional
method which provides a high degree of flexibility would be de­
sirable. Furthermore, the fact that a student places in Mathem­
atics 101 indicates in many cases that he has experienced learning
difficulties with mathematics. If this be the case, then he needs
to be provided with a type of instruction which attempts to diag­
nose specific difficulties and correct them.

Three features of the programmed technique should be noted.
(1) The student was to work at his own rate, spending additional
time on those topics with which he had difficulty, and moving more
rapidly through those areas which were more familiar to him. This was one way to provide for the diversity in mathematical background of the students.

(2) The students needing it were provided with more individual attention than those in the television sections.

(3) The students were to be responsible for covering the required material. Tests were designed (see p. 29) to help the student evaluate his own progress, but they were not required. It was suggested that the students use two days a week for individual study, but again, this was not required. No formal homework assignments were given; rather, it was up to the student to decide his own homework schedule. The responsibility was clearly with the student to work at his own rate and to finish as much of the course as he was able to within the ten week quarter.

Generally speaking then, the course of a study for Group B could be characterized as informal, permissive, and flexible.

Provisions for Testing: Television Sections. The conventional method used for testing in large enrollment freshman courses which has been used by the Department in the past has been to give three common mid-terms and a common final examination. Following this pattern three mid-terms were administered to the students in group A. Since all students, regardless of class hour, took the same examination, it was considered necessary to give the test in the evening in order to protect the validity of the results. The
television lecture prior to an examination was spent in review. In addition, a review sheet in the form of a sample examination was given to all students, and the recitation hour immediately preceding the mid-term was used to discuss this sheet. The classroom instructors were free to administer periodic quizzes as they deemed advisable.

Provisions for Testing: Programmed Sections. In keeping with the philosophy of allowing each individual student to assume his own responsibility for learning, no mid-terms were required of the students in group B. In place of formal testing the following procedure was used. Upon completing a unit of work the student was encouraged to request a test from the tutor over that particular unit. This test was taken during a regular class period, graded by the tutor, and returned to the student. Depending upon the test results, the tutor discussed those items which were missed, prescribed additional drill work, or suggested that the student continue on with the program. If, after additional work, the student wanted to take a re-test over a particular unit, he could do so. This testing procedure was designed to help the student evaluate his own progress, and was entirely voluntary. It was intended that the threat of grade, which is traditionally present in normal testing, would be removed.

The tests over the various units of work were designed by the investigator prior to the study. For the purpose of re-testing,
two forms of each examination were provided the tutor. A list of these tests by topic appears below.

1. Polynomial Products and Factoring
2. Algebraic Fractions
3. Linear Equations and Inequalities
4. Graphing and Absolute Value
5. Systems of Linear Equations and Inequalities

Final Exam. A common final exam was given to all students in both groups. Students from both groups were given a review sheet in the form of a sample test, and the last week of class was spent in discussing this review sheet. The examination appears in appendix E.

Teaching Personnel. The instructors and tutors used in this study were chosen from the teaching assistants and teaching associates employed by the mathematics department. A five hour teaching assignment consisted of being responsible for two sections. There were 21 teachers involved, of whom 8 were responsible for four sections each. A breakdown of the teaching assignments show that:

- 9 teachers taught 2 sections of Group B only.
- 7 teachers taught 2 sections of Group B and 2 sections of Group A.
- 7 teachers taught 2 sections of Group A only.
- 1 teacher taught 4 sections of Group A only.

Classification of Teaching Personnel.
First year graduate students in mathematics 4
Further Classification of Group B. In an attempt to determine the extent to which students are self-responsible for learning, Group B was further classified into three groups:

- B₁ Those students who intensively used the learning resources.
- B₂ Those students who moderately used the learning resources.
- B₃ Those students who made virtually no use of the learning resources.

This classification was made by the tutors at the conclusion of the quarter, and essentially three criteria were used:

1. The daily work of each student as observed by the tutor.
2. The number of tests taken over the assigned material.
3. Student progress in completing the program.

During each class session the students were asked to report their progress on a prescribed form. Thus, for those students attending
and reporting, the tutor could determine at any given time how much of the program the student had completed.

The tutors were given specific directions that in no way should this classification be a function of achievement. Thus it would be quite possible for one student in group B₁ to fail the course, and another in group B₃ to receive an A or a B.

Gathering of Data on Punch Cards. At the conclusion of the quarter, the following data was gathered and punched onto I.B.M. cards:

1. Student name
2. Student identification number
3. College
4. Year
5. Instructor
6. Initial attitude score
7. Final attitude score
8. Final examination score
9. Method (Group A or Group B)
10. Sub-classification (B₁, B₂, or B₃)

An analysis of these statistics and the results are reported in Chapter IV.

Basic Assumptions. The following are the four basic assumptions underlying this study.
1. The relative effectiveness of a course in intermediate algebra taught by closed-circuit television and by programmed instruction can be determined on the basis of the scores made by the groups on a final examination at the end of the quarter.

2. The reactions and attitudes of both groups of students relative to mathematics can be determined by the Shatkin opinionnaire.

3. Attendance, test taking, and progress charts do, in fact, determine intensive, moderate, and non-users of the learning resources.

4. The two groups were divided equally in terms of native mathematical ability and previous mathematical experience.
CHAPTER IV

STATISTICAL RESULTS AND CONCLUSIONS

Introduction. The statistical results presented in this chapter deal with some 1630 of the students who completed Mathematics 101 during the fall quarter, 1968. Of the total of 2042 students who received a grade in the course, 412 students were not included in the study for one or more of the following reasons:

(1) Work incomplete. Because of illness or personal tragedy some students were given a grade of incomplete in the course. They subsequently took a make-up examination which was different from that administered at the end of the course. (51)

(2) Transfer of section. In order to accommodate students' employment schedules, some transfers of sections were made. Although every effort was taken to avoid a transfer to a different method, this was not always possible. (77)

(3) Incomplete data: attitude. Students who did not complete a pre- or post-attitude opinionaire could not be statistically analyzed. (181)

(4) Incomplete data: achievement. Students who did not complete the final examination could not be statistically analyzed. (99)
(5) Cheating on the final examination. In a few cases, students were discovered cheating on the examination. They received a failing grade in the course, and their statistics were deleted from the study. (4)

Statistical Results. An analysis of covariance computer program, written by the University of Miami Biometric Laboratory, (5) was used on the following data collected for each of the 1630 students completing Mathematics 101, fall quarter, 1968.

(1) Subject's pre-attitude score.
(2) Subject's post-attitude score.
(3) Subject's recitation teacher or tutor.
(4) Subject's final examination score.

In the case of the students from Group B, the sub-classification into groups $B_1$, $B_2$, and $B_3$ was included. The analysis of covariance model is given by the equation,

$$ Y_{i,j,k} = \mu + \alpha_i + \beta_j + \delta x_{ij} + \epsilon_{i,j,k} $$

where $Y_{i,j,k}$ is the response of the $k^{th}$ student, with instructor $i$ and method $j$, and where

(1) $\mu$ is an overall mean. (attitude or achievement)
(2) $\alpha_i$ is the effect for the $i^{th}$ method.
   ($i = 1, 2, 3, 4$)
(3) $\beta_j$ is the effect for the $j^{th}$ instructor.
   ($j = 1, 2, \ldots, 24$)
(4) B is the regression coefficient of x.

(5) \( x_{i,j,k} \) is the pre-attitude score for the \( k^{th} \) student in method j with instructor i.

(6) \( \epsilon_{i,j,k} \) is the error adjustment for the \( k^{th} \) student in method j with instructor i.

The variables \( \alpha_1, \alpha_2, \alpha_3, \) and \( \alpha_4 \) (effect for method) correspond to groups \( B_1, B_2, B_3, \) and A respectively, where

- \( B_1 \) is the set of students who intensively used the learning resources.
- \( B_2 \) is the set of students who moderately used the learning resources.
- \( B_3 \) is the set of students who made virtually no use of the learning resources.

The principal assumption in the use of this model is that the error adjustments, \( \epsilon_{i,j,k} \) are independent and identically distributed with normal, \( N(0,\sigma^2) \), distribution. Other assumptions, implicit in the model, are discussed in Winer's, "Statistical Principles in Experimental Design." (22).

The first test involved the determination of the significance of the regression coefficient B. A t-test was used, where t is defined by

\[
    t = \frac{\hat{B}}{\hat{\sigma}_B}
\]

Specifically, the hypothesis tested was the \( B = 0 \). The data resulting from this test is given in Table 5.
This table clearly shows that at the .01 significance level, the hypothesis that \( B = 0 \) must be rejected. Hence, the student's pre-attitude score is a valid covariate and should remain in the model.

Next, an analysis of covariance for the effects of instructors and methods was made, making adjustments for pre-attitude. The specific hypotheses tested were:

Hypothesis (1) \( \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0 \) (with respect to attitude)

Hypothesis (2) \( \beta_1 = \beta_2 = \ldots = \beta_k = 0 \)

(\( \alpha_i \) and \( \beta_j \) are the effects for method and instructor, respectively, in the model.)

The data resulting from testing these hypotheses is given in Table 6.
TABLE 6
ANALYSIS OF COVARIANCE FOR POST-ATTITUDE

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method ((\alpha))</td>
<td>3</td>
<td>9255.027</td>
<td>3085.342</td>
<td>17.578</td>
</tr>
<tr>
<td>Instructor ((\beta_1))</td>
<td>23</td>
<td>5106.871</td>
<td>222.038</td>
<td>1.265</td>
</tr>
<tr>
<td>Error</td>
<td>1602</td>
<td>281163.2</td>
<td>175.508</td>
<td></td>
</tr>
</tbody>
</table>

The critical values for F are given in Table 7.

TABLE 7
CRITICAL F VALUES

<table>
<thead>
<tr>
<th>df</th>
<th>.10</th>
<th>.05</th>
<th>.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3, (\infty))</td>
<td>2.08</td>
<td>2.60</td>
<td>3.78</td>
</tr>
<tr>
<td>(20, (\infty))</td>
<td>1.42</td>
<td>1.57</td>
<td>1.88</td>
</tr>
</tbody>
</table>

From Tables 6 and 7 it is clear that the F statistic for testing the effect for instructor on post-attitude is not significant at the .10 level. Hence, we fail to reject the hypothesis that \(\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0\). However, the F statistic for testing the effects for method on post-attitude is significant at the .01 level. Hence we reject the hypothesis that \(\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0\).
The analysis of covariance model was next applied to test the effect of instructors and methods on achievement. The specific hypotheses were:

Hypothesis (3) \( \alpha_i = \beta_j = \alpha_3 = \alpha_4 = 0 \) (with respect to achievement)

Hypothesis (4) \( \beta_i = \beta_{i-1} = \ldots = \beta_n = 0 \)

(\( \alpha_i \) and \( \beta_j \) are the effects for method and instructor, respectively, in the model.)

The data resulting from testing these hypotheses is given in Table 8.

### Table 8

**ANALYSIS OF COVARIANCE FOR ACHIEVEMENT**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1</td>
<td>6120.7</td>
<td>6120.7</td>
<td></td>
</tr>
<tr>
<td>Method (( \alpha_1 ))</td>
<td>3</td>
<td>14069.9</td>
<td>4690.3</td>
<td>127.19</td>
</tr>
<tr>
<td>Instructor (( \beta_1 ))</td>
<td>23</td>
<td>53678.97</td>
<td>2336.85</td>
<td>2.306</td>
</tr>
<tr>
<td>Error</td>
<td>1602</td>
<td>17721.4</td>
<td>11.1</td>
<td></td>
</tr>
</tbody>
</table>

The \( F \) statistic testing the effect for instructor was 2.306. Using Table 7 we conclude that this statistic is significant at the .01 level, and we must reject hypothesis (4). The \( F \) statistic testing the effect for method was 127.19, a figure which is clearly significant at the .01 level. Hence, hypothesis (3) that \( \alpha_1 = \beta_2 = \alpha_3 = \alpha_4 = 0 \) is rejected.
Table 9 gives the unadjusted means for the pre-attitude scores, the post-attitude scores, and the final examination. The range of possible scores for attitude was from 22 to 110. The highest grade possible on the final examination was 200.

<table>
<thead>
<tr>
<th>Group</th>
<th>Pre-attitude</th>
<th>Post-attitude</th>
<th>Achievement</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>68.2602</td>
<td>60.6463</td>
<td>85.7226</td>
<td>246</td>
</tr>
<tr>
<td>B2</td>
<td>68.7169</td>
<td>65.5687</td>
<td>113.0899</td>
<td>378</td>
</tr>
<tr>
<td>B3</td>
<td>70.2076</td>
<td>69.5961</td>
<td>134.5730</td>
<td>260</td>
</tr>
<tr>
<td>S</td>
<td>65.6943</td>
<td>66.1340</td>
<td>136.1758</td>
<td>746</td>
</tr>
<tr>
<td>Overall</td>
<td>67.5020</td>
<td>65.7269</td>
<td>123.0901</td>
<td>1630</td>
</tr>
</tbody>
</table>

Since it has been shown that the pre-attitude covariate should be included in the model, a computation of adjusted means was made using the following formula

\[ \bar{Y}_j' = \bar{Y}_j - B(\bar{X}_j - \bar{X}), \]

\( \bar{Y}_j' \) is the adjusted mean (attitude or achievement) for group j.

\( \bar{Y}_j \) is the unadjusted mean (attitude or achievement) for group j.
$B$ is the regression coefficient ($0.8037$).

$\bar{X}_j$ is the pre-attitude mean for group $j$.

$\bar{X}$ is the overall mean for pre-attitude.

The results of this computation are given in Table 10.

### TABLE 10

**ADJUSTED GROUP MEANS FOR POST-ATTITUDE**

AND ACHIEVEMENT

<table>
<thead>
<tr>
<th>Group</th>
<th>Post-attitude</th>
<th>Achievement</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>60.0369</td>
<td>85.1182</td>
<td>246</td>
</tr>
<tr>
<td>$B_2$</td>
<td>64.5922</td>
<td>112.1134</td>
<td>378</td>
</tr>
<tr>
<td>$B_3$</td>
<td>67.4213</td>
<td>132.3902</td>
<td>260</td>
</tr>
<tr>
<td>$A$</td>
<td>67.5871</td>
<td>135.0227</td>
<td>746</td>
</tr>
</tbody>
</table>

A test for comparing any two adjusted means in an analysis of covariance model was used to compare group $B_1$ with $A$, $B_2$ with $A$, and $B_3$ with $A$. The $F$ statistic involved in this test is given by

$$F = \frac{\left[ \bar{Y}_i' - \bar{Y}_j' \right]^2}{MSc \left[ \frac{1}{n_i} + \frac{1}{n_j} + \frac{(\bar{Y}_i - \bar{X})^2}{E} \right]}$$

where

1. $\bar{Y}_i'$ and $\bar{Y}_j'$ are the adjusted means of the two groups to be compared.
(2) MSo is the mean square for error in the analysis of covariance table.

(3) $n_i$ and $n_j$ are the two group sizes.

(4) $\bar{X}_i$ and $\bar{X}_j$ are the pre-attitude means for the two groups to be compared.

(5) $E$ is given by the formula

$$E = \sum_{i=1}^{4} (\bar{X}_i - \bar{X})^2 \cdot n_i$$

($\bar{X}_i$ is the pre-attitude mean for group $i$)

($\bar{X}$ is the overall attitude mean)

The data in Tables 11 and 12 summarizes results of applying this test to the four groups.

**TABLE 11**

<table>
<thead>
<tr>
<th>Comparing Methods</th>
<th>Hypothesis</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$ and $A$</td>
<td>$\alpha_1 = \alpha_4$</td>
<td>47.7653</td>
</tr>
<tr>
<td>$B_2$ and $A$</td>
<td>$\alpha_2 = \alpha_4$</td>
<td>8.5175</td>
</tr>
<tr>
<td>$B_3$ and $A$</td>
<td>$\alpha_3 = \alpha_4$</td>
<td>0.0163</td>
</tr>
</tbody>
</table>
TABLE 12
F TEST DATA COMPARING EFFECTS FOR METHOD ON ACHIEVEMENT

<table>
<thead>
<tr>
<th>Comparing Methods</th>
<th>Hypothesis</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>B₁ and A</td>
<td>$\alpha_1 = \alpha_4$</td>
<td>331.1428</td>
</tr>
<tr>
<td>B₂ and A</td>
<td>$\alpha_2 = \alpha_4$</td>
<td>79.0892</td>
</tr>
<tr>
<td>B₃ and A</td>
<td>$\alpha_3 = \alpha_4$</td>
<td>.6487</td>
</tr>
</tbody>
</table>

The critical values for this F statistic are given in Table 13.

<table>
<thead>
<tr>
<th>df</th>
<th>.10</th>
<th>.05</th>
<th>.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,∞)</td>
<td>2.71</td>
<td>3.84</td>
<td>6.63</td>
</tr>
</tbody>
</table>

From Tables 11, 12 and 13 the obtained F statistic testing the hypothesis that $\alpha_1 = \alpha_4$ indicates that with respect to both attitude and achievement, this hypothesis must be rejected at the .01 significance level. Consequently, there is a significant difference in both attitude and achievement for the student who makes virtually no use of the resources available in the programmed method when compared to the student using the television method.
Similarly, the hypothesis that $\alpha_3 = \alpha_4$ must be rejected at the .01 significance level, and we conclude that there is a significant difference in both attitude and achievement for the student who makes moderate use of the resources available in the programmed method when compared to the student using the television method.

However, the $F$ statistic testing the hypothesis that $\alpha_3 = \alpha_4$ indicates that with respect to both attitude and achievement, this hypothesis can not be rejected at the .10 level. Thus there is no significant difference in attitude or achievement for those students who intensively use the resources in the programmed method when compared to the student using the television method.

In order to make an overall comparison between the two treatments (programmed and television), groups $B_1$, $B_2$, and $B_3$ were combined and compared with group $A$. The resulting $F$ statistic for attitude was 14.4 and the $F$ statistic for achievement was 38.96. Using Table 13 it was observed that these $F$ values are significant at the .01 level. Hence, we conclude that there is a significant difference for method with respect to both attitude and achievement for the student in the programmed treatment when compared with the student in the television treatment. The unadjusted means for these two groups appear in Table 14.

Finally, since it has been shown that the pre-attitude covariate should be included in the model, a computation of adjusted
means was made. The formula used was that given on p. 37, and the results of this computation appear in Table 15.

### Table 11

**Unadjusted Means for Pre-Attitude, Post-Attitude, and Achievement**

<table>
<thead>
<tr>
<th>Method</th>
<th>Pre-attitude</th>
<th>Post-attitude</th>
<th>Achievement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Programmed</td>
<td>69.0282</td>
<td>65.3835</td>
<td>111.7941</td>
</tr>
<tr>
<td>Television</td>
<td>65.6943</td>
<td>66.1340</td>
<td>136.4758</td>
</tr>
</tbody>
</table>

### Table 15

**Adjusted Means for Post-Attitude and Achievement**

<table>
<thead>
<tr>
<th>Method</th>
<th>Post-Attitude</th>
<th>Achievement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Programmed</td>
<td>64.1569</td>
<td>110.5675</td>
</tr>
<tr>
<td>Television</td>
<td>67.5871</td>
<td>135.0227</td>
</tr>
</tbody>
</table>

Conclusions. The following conclusions are based on the statistical analysis of the data collected during the experiment. Relative to teaching remedial mathematics (intermediate algebra) at The Ohio State University, it can be concluded that
(1) The television method produces a significant positive difference on students' attitude when compared to the programmed method. (p < .01)

(2) The television method produces a significant positive difference on students' achievement when compared to the programmed method. (p < .01)

(3) For students who intensively use the programmed method, there is no significant difference in either attitude or achievement when compared with the students using the television method.

These findings indicate that the hypotheses stated in Chapter I must be rejected. These hypotheses were:

(1) There is no significant difference in achievement between those taught by closed-circuit television and those taught by programmed learning.

(2) Students using the programmed method will have a greater positive attitude change toward mathematics than those taught by television. In fact, the anticipated result that the programmed treatment would produce a more positive effect on student attitude was actually reversed. In terms of change in mean score, it was found that the television sections went from a pre-attitude score of 65.69 to a post-attitude score of 66.13, while the programmed sections decreased from a pre-score of 69.028 to a post-score of 65.3835. The conclusions of this study, however, must be considered in terms of certain limitations which are presented next.
Limitations. The limitations of this study can be classified into three groups: (1) subjects, (2) teaching personnel, and (3) instruments.

(1) Limitations of the subjects. This study involves basically first quarter freshman at a large state university. Furthermore, the fact that the course content is remedial indicates that the students involved have either a limited high school mathematical background or have achieved little success in high school mathematics. It is a possibility, therefore, that the results might differ if this experiment were performed using a group of students of different training, experience, and maturity. In addition, it was a stated assumption in Chapter III that the two groups were divided equally in terms of native mathematical ability and previous mathematical experience. This assumption was considered reasonable in view of the large number of students involved in the study. It is possible, however, that this assumption was in error.

(2) Limitations of teaching personnel. A five-hour teaching assignment for a given tutor in the programmed sections was the responsibility for two sections of students. Therefore, the number of student-tutor contact hours ranged from two to three per week. Due to the availability of funds and personnel, this method of scheduling was an administrative necessity. If the number of student-tutor contact hours had been increased, it is a
possibility that the results of this study would be different. Furthermore, it is a possibility that the results of the television treatment would be different if the same students were taught by a lecturer with a different teaching style and personality from that of the investigator.

(3) Limitations of the instruments. It is a possibility that the final examination did not measure accurately changes in achievement due to the method of instruction.
CHAPTER V

SUMMARY AND RECOMMENDATIONS

This investigation was concerned primarily with testing the effectiveness of programmed instruction as used in a teaching strategy which attempts to maximize student responsibility for learning. In Chapter I the question was clearly stated: "To what extent will the student be responsible for learning, given sufficient resources." The results of the statistics analyzed are clearly in favor of the more structured and supervised method, closed-circuit television. However, it is the view of the investigator that these results must be interpreted relative to the role which both the teacher and the student were asked to assume. In the case of the television treatment, the instructor was the dominant figure, with the student assuming the traditional (but not necessarily the most desirable) passive role. This method is the familiar one, and the adjustments necessary on the part of both the instructor and the student were minimal. However, both student and teacher were asked to assume an entirely new role in the programmed treatment. The student was now asked to make decisions that, in his previous experience, had been imposed upon him; in effect, he was asked to be responsible for his own learning.
It was perhaps too ambitious to expect students in remedial mathematics to assume this responsibility. The teachers were asked to assume the role of tutor and diagnostician. Because this task is an unfamiliar one, the programmed treatment undoubtedly did not make full use of their potential teaching capacities. As Heimer has remarked in a recent issue of *The Mathematics Teacher* (11, 110) "... very few teachers have learned to make really effective use of programmed materials." In reporting on the non-program variables in the application of programmed instruction, Coulson et al. report that "It appears that teachers can play an effective part in programmed instruction only when their role has been empirically developed as an integral, interacting part of the total instructional system including the students, the program, and any other classroom resources." (7, 3) In providing programmed instruction for large numbers of students at a large university such as Ohio State, it is not yet known how to involve the teacher (in the present case, the graduate teaching assistant) to such a degree. Therefore, in terms of the conclusions of this study, it is important to keep these considerations in mind lest one be tempted to make sweeping generalizations regarding the effectiveness of television instruction. A classical assumption of programmed learning is that the best method of using the materials is that of self-pacing and individualized study. It would appear from the results of this study that this assumption is valid only
when the student is given sufficient supervision and direction. Hennemann (12) has suggested in a similar study that external checks are necessary. This writer defines external checks as the total of the teacher-initiated activities; supervision, lectures, work schedules, testing procedures, etc. Bartz and Darby (1) found that programmed materials were more effective when used in a traditional teaching situation, a result which is confirmed by the findings of this study. This has a very practical implication in terms of future instructional innovations in the Mathematics Department at The Ohio State University. Future plans for individualizing instruction should include certain external checks previously mentioned. The assumption should not be made that the student will assume responsibility as a matter of course. This is not to say that new instructional programs should be rigid and inflexible. Rather, it says that our responsibility to the student is to help him assume a responsible role as a learner.

**Recommendations for Further Study.** On the basis of the results of this investigation, the writer recommends the following:

1. A course of study using the programmed learning strategy should be offered in Mathematics 101 as an elective to the traditionally taught course. During the course of this research, the investigator formed certain opinions regarding the effectiveness of the programmed treatment. From various conversations with both students and teachers, it was observed that many
participants were strongly in favor of this method, while others were strongly opposed. Students who had developed a more mature outlook toward education commended this technique. Those who were less motivated preferred to be externally directed. Similar remarks apply to the teachers involved. One group favored the individualized approach, while another preferred the traditional teacher-oriented approach. Although these intuitive conclusions are not the result of an organized investigation, they do suggest that the programmed method should be offered on an elective basis, both in terms of student scheduling and in terms of teaching assignments.

(2) This study should be replicated using students different from those enrolled in a remedial course. The implication is that students with additional background, sharper mathematical skills, and generally speaking, more native intelligence will respond in a more positive fashion to the individualized approach.

(3) An individual study approach to Mathematics 101 should be developed which provides

(a) more individual student-tutor contact hours.
(b) closer supervision of students.
(c) frequent examinations.

In this respect, a follow-up study to the present one has been initiated at The Ohio State University during the Spring Quarter,
1969. Since the enrollment during this quarter is less than one-half that of the fall, and since teaching personnel is now available, it has been possible to increase the number of student-tutor contact hours to five per week. A programmed text is being used, and the students are to work in much the same manner as described in this study. Based on the findings of this investigation, the following additional procedures have been incorporated:

(i) Each student has received a work schedule suggesting a realistic pace for covering the required material.

(ii) Daily attendance is kept.

(iii) Midterm examinations are required of all students every two weeks.

There is no control group being used in this follow-up study. Rather, the Shatkin opinionnaire is being used to determine pre- and post-attitude, and the results will be compared with the programmed group in the present study. Furthermore, the same final examination (with minor modifications) as used in the present study will be administered at the end of the quarter. These results will also be compared with the data reported in the present study.

By a process of continual modification and innovation, it is the hope of this investigator and other concerned colleagues that we are moving toward an optimal strategy for providing effective instruction for large numbers.
APPENDIX A

Descriptions of Freshman Mathematics Courses
at The Ohio State University
The following is a brief description of the non-honors courses available to freshman at The Ohio State University.

Mathematics 105  Principles of Mathematics. The course develops basic ideas on arithmetic, algebra, and geometry through the study of the structure of selected mathematical systems. (This course is for elementary education majors)

Mathematics 116  Mathematics for the Behavioral, Economic, and Social Sciences I. The sequence 116, 117 treats topics in mathematics with applications to the non-physical sciences. Topics include analytic geometry, calculus, linear algebra, linear programming and graph theory, applications.

Mathematics 117  Mathematics for the Behavioral, Economic, and Social Sciences II. This course is a continuation of Mathematics 116.

Mathematics 121  Mathematics for the Business, Social, and Biological Sciences I. The sequence 121, 122, and 123 is designed to introduce the student to calculus, probability, and statistics.

Mathematics 122  Mathematics for the Business, Social, and Biological Sciences II. This course is a continuation of Mathematics 121.

Mathematics 123  Mathematics for the Business, Social, and Biological Sciences III. This course is a continuation of Mathematics 122.
Mathematics 125  Elementary Mathematical Statistics. Elementary principles of probability and an introduction to the use of the binomial and normal distributions.

Mathematics 150  Algebra and Trigonometry. Inequalities, functions, graphs, exponential, logarithmic, and trigonometric functions and their graphs, complex numbers, and inverse functions.

Mathematics 151  Calculus and Analytic Geometry. Lines, slopes, derivatives, limits, differentiation rules, the mean-value theorem, applications of derivatives to: curve sketching, maxima and minima, linear motion, related rates, approximations, and a study of the conic sections.
APPENDIX B

Commercial Texts Available for Intermediate Algebra at the College Level


APPENDIX C

The Midterm Examinations
Find the indicated products in problems 1 - 4.
1. \(3x \ y^2 \ (5xy - 2x^3 y^2) = \)
2. \((6x - y)^2 = \)
3. \((4x + 7y) \ (x - 3y) = \)
4. \((x + 2) \ (x^2 + 2x + 4) = \)

5. Factor into primes: \(1008 = \)
6. The least common multiple of 35, 63, and 75 is

7. For what value(s) of \(x\), if any, is the fraction \(\frac{x-1}{x-2}\) undefined?
8. The set of numbers: \(\{... -3, -2, -1, 0, 1, 2, 3, ...\}\) is called the set of

Factor as completely as possible in problems 9 - 14.
9. \(1 - 4x^2 = \)
10. \(8x^3 + y^3 = \)
11. \(x^2 - 3x - 28 = \)
12. \(x^3 - 2x^2 - 9x + 18 = \)
13. \(20x^2 - 3xy - 35y^2 = \)
14. \(9(x-1)^2 - y^2 = \)

15. Reduce \(\frac{1 - 4x^2}{6x^2 - x - 1}\)

Perform the indicated operations in problems 16 - 20.
16. \(\frac{4x^2 - y^2}{x + 2y} \cdot \frac{xy}{4x^2 - 2xy} = \)
17. \(\frac{x}{x + 1} - \frac{x + 1}{x + 2} = \)
18. \(\frac{3}{(x-y)(x+y)} - \frac{3}{(x+2y)(x-y)} + \frac{1}{(x+y)(x+2y)} = \)
19. \(\frac{x - x^2}{x + 1} \cdot \frac{x^2 - 1}{x^2 - 1} = \)
20. \(\frac{3x - 2}{x^2 - 4} + \frac{2x + 1}{x^2 + x - 2} - \frac{2x - 3}{x - 3x + 2} = \)
Math 101
Midterm 2
November 5, 1968

Solve each of the following:
1. $4(x + 5) = 3(1 - x) - 4$
2. $4(1 - x) \leq x - 6$
3. $|x - 2| = 7$
4. $\frac{4x + 3}{6} - \frac{4x - 1}{9} = \frac{1}{2}$
5. $\frac{1}{x + 3} = \frac{x}{x - 2} - 1$
6. $\frac{2x}{2x + 3} - \frac{2x}{2x - 3} = \frac{2x + 28}{4x^2 - 9}$
7. $(x + 6)(x + 1) \leq 0$
8. $\frac{(x-1)(x+2)}{(x - 6)} \geq 0$
9. $x^2 - 3x < x$
10. $\frac{3x}{x + 2} \geq 1$

11. Let $A = \{x | x$ is an integer and $|x| < x \leq 4\}$
and $B = \{x | x$ is a real number and $0 < x < 3\}$.

Then: (a) $\frac{3}{2} \in A$ and $\frac{3}{2} \notin B$
(b) $\frac{3}{2} \in B$ and $\frac{3}{2} \notin A$
(c) $\frac{3}{2} \in A$ and $\frac{3}{2} \notin B$
(d) $\frac{3}{2} \notin A$ and $\frac{3}{2} \in B$
(e) None of the above

12. Let $E = \{o, e, t, v\}$. Arrange the letters in the set $E$ so that they spell a common word associated with today ________________.

Sketch the following sets on the number line:

13. $\{x | 1 \leq 3x + 4 \leq 16\}$
14. $\{x | |x| \leq 2\}$. 

[Number line diagrams]
Graph the following sets.

15. \((x,y) \mid 2x - 5y = 10\)  
16. \((x,y) \mid y \leq 2x + 1\)

For questions 17-20, you have been provided with a sheet containing graphs. Match each set below with the letter which corresponds to the correct graph of that set.

17. \((x,y) \mid y \geq |2 - x| \)  
18. \((x,y) \mid |y| = |x|\)
19. \((x,y) \mid |x| \leq 1 \text{ and } |y| \leq 1\)  
20. \((x,y) \mid y = |x| + |x-1|\)
Math 101
Midterm 3
December 2, 1968

1. Find the value of:
   
   (a) \(16^{5/4}\) =  
   (b) \((\frac{9}{25})^{-1/2}\) =  
   (c) \(\frac{2^4 \cdot 4^3}{\sqrt[4]{45}}\) =

2. Simplify:
   
   (a) \(\sqrt{72}\) =  
   (b) \(\sqrt{12x^5 y^8}\) =

3. Simplify by applying the laws of exponents. State all answers without the use of negative exponents.
   
   (a) \((2x^{-1}y^2)^{-2}\) =  
   (b) \(\frac{a^{-3}b^{-4}}{a^{-5}b^{-2}}\) =  
   (c) \(\left(\frac{125a^6b^3}{64c^6}\right)^{2/3}\) =

4. Rationalize the denominator and simplify:
   
   (a) \(\frac{5}{\sqrt{18}}\) =  
   (b) \(\frac{\sqrt[3]{5x^2}}{\sqrt[3]{9y^4z^2}}\) =  
   (c) \(\frac{3}{\sqrt{3} - 2}\) =

5. Perform the indicated operations and simplify:
   
   (a) \(\frac{\sqrt{27} \cdot \sqrt{18}}{\sqrt{6}}\) =  
   (b) \(\sqrt{x} \cdot \sqrt[4]{3}\) =  
   (c) \(3\sqrt[3]{6x^{2/3}y^{4}} \cdot 3\sqrt[3]{4xy^2}\) =
   
   (d) \((2\sqrt{5} - \sqrt{3}) (\sqrt{5} + 3\sqrt{3})\) =  
   (e) \(\sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}} + \sqrt{\frac{1}{6}}\) =

6. Answer true or false. (Consider \(a, b, \) and \(x\) as real numbers, with \(a \geq 0\) and \(b \geq 0\)).

   Circle your answer

   (a) \(\sqrt{a+b} = \sqrt{a} + \sqrt{b}\) 
   T \quad F

   (b) \(\sqrt[n]{\sqrt{a}} = \sqrt{n} \sqrt{a}\) 
   T \quad F

   (c) \(\sqrt[2]{x^2} = x\) 
   T \quad F

   (d) \(\sqrt{a} \cdot \sqrt{a} = r+\sqrt{a}\) 
   T \quad F
In problems 7-10, solve by any method. Simplify any radical answers.

7. $3x^2 - 5x = 12 \quad x = \quad 8. \quad x^2 - 2x - 4 = 0 \quad x =$

9. \[ \begin{align*}
    x - 6y &= -3 \\
    2x + y &= 7
\end{align*} \quad x = ____ \quad y = ____ \quad 10. \quad \begin{align*}
    \frac{2}{x} + \frac{1}{y} &= 3 \\
    \frac{1}{x} - \frac{2}{y} &= -1
\end{align*} \quad x = ____ \quad y = ____

Graph the sets $R$ and $T$.

11. $R = \{(x,y) \mid x + y \leq 2 \text{ and } y \geq 2x - 1\}$

12. $T = \{(x,y) \mid x - 2y \geq -3 \text{ and } |x| \leq 1 \text{ and } y \geq 0\}$
APPENDIX D

Tests for the Programmed Sections
Find each of the indicated products.

1. \(-4ab(6x^2b + 2yab^2 - c^2) =\)
2. \((7x - 2y)(7x + 2y) =\)
3. \((5r - s)^2 =\)
4. \((10x - 3)(4x + 2) =\)
5. \((1/4a - 2/3b)^2 =\)
6. \((3x + y + 1)(x - 2y + 2) =\)
7. \((3x - 2)(9x^2 + 4x + 4) =\)
8. \((x + y - 3)(x + y + 3) =\)
9. \((a + b + c)^2 =\)
10. \((m + n - r - 2s)(m + n + r + 2s) =\)
11. \(4x^2 - 25y^2 =\)
12. \(12m^2 - 11mn + 2n^2 =\)
13. \(12b^3 - 32b^2 - 12b =\)
14. \(27x^3 - 1 =\)
15. \((x + 3y)^2 + 6(x + 3y) + 9 =\)
16. \(81y^4 - 1 =\)
17. \(3mc - 5nc - 9md + 15nd =\)
18. \(4x^2 - 9(y - 1)^2 =\)
19. \(r^2 + 2rs - rt - 2r - 4s + 2t =\)
20. \((3x + 5y)^3 + 64 =\)
Find each of the indicated products.

1. \(-3xy(5axy^2 + 3bx^2y - b^2) = \)
2. \((3x - 5y)(3x + 5y) = \)
3. \((4x - y)^2 = \)
4. \((8x - 5)(4x + 7) = \)
5. \((2/3x + 1/4y)^2 = \)
6. \((2x + y + 1)(x - y + 2) = \)
7. \((2x - 3)(4x^2 + 4x + 9) = \)
8. \((r + s - 1)(r + s + 1) = \)
9. \((x + y + z)^2 = \)
10. \((x + y - 2a - b)(x + y + 2a + b) = \)

Factor each of the following completely:

11. \(16x^2 - 49y^2 = \)
12. \(6a^2 - ab - 2b^2 = \)
13. \(20p^3q + 7p^2q^2 - 3pq^3 = \)
14. \(8x^3 + 27 = \)
15. \((3x - y)^2 + 4(3x - y) + 4 = \)
16. \(16x^4 - 1 = \)
17. \(12rt + 3st - 4ru - su = \)
18. \(x^2 - 36(y + 2)^2 = \)
19. \(a^2 - ab - ac + a - b - c = \)
20. \((3x - y)^3 + 27 = \)
1. (a) List all the prime factors of 84.

   (b) Find the least common multiple of 84, 147 and 14.

2. Simplify: \( \frac{x^3 - x^2y - 12xy^2}{x^2y - 3xy^2 - 4y^3} \)

Perform the indicated operations in problems 3 - 7.

3. \( \frac{5a - 5b}{a + 2b} \cdot \frac{2a + 4b}{a - b} = \)

4. \( \frac{y^2 - x^2}{xy^2} \cdot \frac{x^2}{xy + y^2} \cdot \frac{y^4}{x^2 - xy} = \)

5. \( \frac{x}{6yz} + \frac{3y}{10xz} - \frac{2z}{15xy} = \)

6. \( \frac{3}{(a - b)(a + b)} - \frac{3}{(a + 2b)(a - b)} + \frac{1}{(a + b)(a + 2b)} = \)

7. \( \frac{3x - 2y}{x^2 - 4y^2} + \frac{2x + y}{x^2 + xy - 2y^2} - \frac{2x - 3y}{x^2 - 3xy + 2y^2} = \)

8. Simplify: \( \frac{w - \frac{w^2}{w + 1}}{w^2 - 1} + 1 = \)
Solve each of the following for $x$.

1. $5(8 + 2x) - 3(3x + 9) = 7$  
   $x =$

2. $\frac{3x - 2}{4} + 1 = \frac{2x + 3}{3}$  
   $x =$

3. $4x + 3 > 6x - 5$

4. $\frac{6}{2x - 3} + \frac{2x}{x + 3} = 2$  
   $x =$

5. $\frac{3}{x - 2} - \frac{1}{x - 5} = \frac{1}{x^2 - 7x + 10}$  
   $x =$

6. $\frac{7 - 5x}{3 - 2x} - \frac{14x^2 - 29x - 28}{4x^2 - 16x + 15} + 1 = 0$  
   $x =$

7. $\frac{10}{x} > \frac{4}{x} + 3$

8. $\frac{6}{x - 2} + \frac{5}{x} < \frac{-21}{x^2 - 2x}$

9. $x^2 + 5x > 2x$

10. The expression $\frac{x - \frac{1}{2}}{x - 2}$ is not a real number if $x =$

11. Solve for $l$ if $s = \frac{n}{2} (a + 1)$  
    $l =$

12. Sketch on the number line below the set of points satisfying
    $3 \leq 2x + 7 < 11$
Math 101
Section 4
(Form M)  

Name
Date

Solve each of the following for \( x \).

1. \( 5(3x - 10) + 13 = 7(2x - 5) \)  \( x = \)

2. \( \frac{3x - 5}{2} + 3 = \frac{3x + 2}{4} \)  \( x = \)

3. \( 5x - 8 > 7x - 2 \)

4. \( \frac{x}{x + 2} + \frac{3}{x + 1} = 1 \)  \( x = \)

5. \( \frac{2}{x - 2} - \frac{3}{x + 1} = \frac{3}{x^2 - x - 2} \)  \( x = \)

6. \( \frac{3}{x - 2} - \frac{1}{x - 5} = \frac{1}{x^2 - 7x + 10} \)  \( x = \)

7. \( \frac{14}{x} > \frac{4}{x} + 5 \)

8. \( y \leq |x| + 1 \)

9. \( x^2 - 3x > x \)

10. The expression \( \frac{x - 2}{x - 1} \) is not a real number if  \( x = \)

11. Solve for  \( p \) is \( s = \frac{c}{1 - p} \)  \( p = \)

12. Sketch on the number line below the set of points satisfying

\( 1 \leq 2x - 3 < 5 \).
1. Define: $|x| =$

2 - 3. Which of the following are true for all real numbers $x$ and $y$.

Circle the correct response.

(a) $|x| + |y| = |x + y|$  
(b) $|x\cdot y| = |x| \cdot |y|$  
(c) $|x - y| = |y - x|$  
(d) $x < |x|$

Sketch the graphs of each of the following:

4. $x = 4$  
5. $|y| = 3$

6. $3x - 2y = 6$  
7. $y \leq x - 4$

8. $y \leq |x| + 1$  
9. $y = |2x - 7|$

10. $|x - 1| + |y - 2| \leq 0$
1. Define: \(|x| = \)

2 - 3. Which of the following are true for all real numbers \(x\) and \(y\). Circle the correct response.

(a) \(|x| - |y| = |x - y|\) \(\text{T} \quad \text{F}\)

(b) \(\frac{|x|}{y} = \frac{|x|}{y}\) \(\text{T} \quad \text{F}\)

(c) \(|x - y| = |y - x|\) \(\text{T} \quad \text{F}\)

(d) \(|x^2| = x^2\) \(\text{T} \quad \text{F}\)

Sketch the graphs of each of the following:

4. \(y = 3\)  
5. \(|x| = 2\)

6. \(2x + 3y = 6\)  
7. \(y \geq x + 1\)

8. \(y \geq |x| - 2\)  
9. \(y = |3x + 5|\)

10. \(|x| + |y - 1| \leq 0\)
In problems 1 - 3, solve for $x$ and $y$ by any method.

1. \[
\begin{cases}
3x - 2y = -16 \\
5x + 6y = -8
\end{cases}
\]

\[x = ____ \quad y = ____\]

2. \[
\begin{cases}
\frac{1}{x} + \frac{2}{y} = 1 \\
\frac{2}{x} + \frac{3}{y} = 3
\end{cases}
\]

\[x = ____ \quad y = ____\]

3. \[
\begin{cases}
bx + ay = a^2 \\
x - y = b
\end{cases}
\]

\[x = ____ \quad y = ____\]

In 4 - 5, solve by graphing, shading the common solution.

4. \[
\begin{cases}
y < x + 1 \\
y > -x + 3
\end{cases}
\]

5. \[
\begin{cases}
y \leq x \\
|x| \leq 2
\end{cases}
\]
APPENDIX E

The Final Examination
1. Find the products:
   (a) \((3x - 4y)^2 = \)
   (b) \((x^2 - y^2)(x^2 + y^2) = \)

2. Factor completely:
   (a) \(9x^2 - y^2 = \)
   (b) \(x^3 + 64 = \)

3. Solve for \(x\):
   (a) \(\frac{2}{1-x} = 5\) \(x = \)
   (b) \(|2x - 1| = 5\) \(x = \)
   (c) \(-3x + 11 < 5\)

4. Find the value of
   (a) \(27^{2/3} = \)
   (b) \((\frac{4}{89})^{-1/2} = \)
   (c) \(\frac{12}{\sqrt{16}} = \)

5. Simplify:
   (a) \(\sqrt{48} = \)
   (b) \(\sqrt[3]{80} x^9 y^{16} = \)

6. Rationalize the denominator: \(\sqrt{\frac{2}{7}} = \)

7. Reduce: \(\frac{x^2 - y^2}{x - y} = \)

8. Find the value of
   \(|2x| + |x-4|\) if \(x = 1\)

9. For what value(s) of \(x\), if any, is the fraction
   \(\frac{x-2}{x^2 + 4x + 3} \) undefined?
10. Graph each of the following:
   (a) $4x - 3y = 12$
   (b) $x - 2y \geq 2$

11. Find the products:
   (a) $(x - 4)(x^2 - 4x + 4)$
   (b) $(4x + 7y)(5x - y)$

12. Factor completely:
   (a) $8x^2 - 10x - 15$
   (b) $x^4 + 8x^2 - 9$

13. Find:
   \[
   \frac{x^2 - x - 2}{x + 1} \div \frac{x^2 + x - 6}{x + 3} =
   \]

14. Solve for $x$ if:
   (a) $\frac{4x - 11}{6} = \frac{x - 5}{2} + \frac{7}{2}$
   (b) $3x^2 - 6x - 2 = 0$
   (c) $3(1+x) + 4 > x+5$
   (d) $(x-3)\cdot |x+2| > 0$

15. Solve by any method:
   \[
   \begin{align*}
   4x + y &= 5 \\
   x - 5y &= 17
   \end{align*}
   \]

16. Perform the indicated operations and simplify:
   (a) $\sqrt{7} \cdot \sqrt{21} =$
   (b) $2 \cdot \frac{3\sqrt{x}}{\sqrt{x}} \cdot 3 \cdot \frac{4\sqrt{x}}{\sqrt{x}} =$
   (c) $\sqrt{5} \cdot \sqrt{20} \cdot \sqrt{5} =$
   (d) $\frac{\sqrt{32x^7y^5}}{\sqrt{2x^3z^3} \cdot \sqrt{2yz^5}} =$
   (e) $(2\sqrt{5} - 3\sqrt{2})(\sqrt{5} + 2\sqrt{2}) =$

17. Rationalize the denominator:
   \[
   \frac{3}{\sqrt{7} + 2} =
   \]

18. Factor completely:
   (a) $6x^4y + 3x^3y^2 - 3x^2y^3 =$
   (b) $x^3 - 4x^2 - 4x + 16 =$
19. Perform the indicated operations. Reduce all answers when possible.

(a) \[
\frac{5}{4x - x^2} + \frac{10}{3x^2 - 48} =
\]

(b) \[
\frac{3x - 2}{y} =
\]

(c) \[
\frac{x}{x^2 - 16} - \frac{x + 1}{x^2 - 5x + 4} =
\]

20. Solve for \(x\):

(a) \[
\frac{x}{x-2} = 1 - \frac{4}{x+2} + \frac{3-x}{x^2-4} =
\]

(b) \[
\frac{8}{x} - \frac{5}{x+3} = 3
\]

21. Sketch on the number line below all numbers \(x\) which satisfy the inequality.

\[3 \leq 2x + 5 \leq 15\]

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

22. Graph

(a) \[
\begin{align*}
\frac{x}{y} & \leq 6 \\
y & \leq 3
\end{align*}
\]

(b) \[
|x| + |y| \geq 0
\]

23. Perform the indicated operations and simplify:

(a) \[
\sqrt{\frac{5}{6}} + \sqrt{\frac{6}{5}} =
\]

(b) \[
9\sqrt{8} - \sqrt{72} + \sqrt{98} =
\]

(c) \[
\frac{8\sqrt{2}x}{16^{1/4}} =
\]

24. Rationalize the denominator and simplify:

\[
\frac{x^2 - 1}{\sqrt{x} - 1} =
\]

25. Solve for \(v\) if \(a = \frac{b}{t} - \frac{v}{t}\)

\(v = \)
APPENDIX F

The Attitude Opinionnaire
MATHMATICS OPINIONAIRE

Directions: Each statement below expresses a feeling which a particular person has toward mathematics. You are asked to express the extent to which you personally agree or disagree with the opinion stated, on a 5-point scale: SA (Strongly Agree), A (Agree), U (Undecided), D (Disagree), and SD (Strongly Disagree). Fill in the circle with the feeling expressed.

SA A U D SD

1. I feel at ease with mathematics. 0 0 0 0 0

2. When I hear the word mathematics, I have a distinct feeling of dislike. 0 0 0 0 0

3. I do not feel sure of myself in mathematics. 0 0 0 0 0

4. Mathematics is a subject I feel I can sink my teeth into. 0 0 0 0 0

5. Mathematics makes me feel uncomfortable, uneasy, irritable and impatient. 0 0 0 0 0

6. Mathematics is something which I enjoy doing a great deal. 0 0 0 0 0

7. Mathematics is fascinating and fun for me. 0 0 0 0 0

8. I enjoy the challenge of mathematics problems. 0 0 0 0 0

9. I feel under a great strain in a mathematics class. 0 0 0 0 0

10. I approach mathematics with a feeling of hesitation. 0 0 0 0 0

11. Mathematics is stimulating to me. 0 0 0 0 0

12. Mathematics is my most dreaded subject. 0 0 0 0 0
<table>
<thead>
<tr>
<th></th>
<th>Question</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.</td>
<td>I have a definite favorable reaction to math: it's enjoyable.</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>14.</td>
<td>Working with mathematics is fun.</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>15.</td>
<td>It scares me to have to take mathematics.</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>16.</td>
<td>At present, I would rate my general attitude toward math as favorable.</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>17.</td>
<td>Mathematics is very interesting to me.</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>18.</td>
<td>When I approach my mathematics work, I experience a sense of fear of not being able to do it.</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>19.</td>
<td>I have a feeling of insecurity when attempting mathematics.</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>20.</td>
<td>Mathematics is a subject in school which I have liked and enjoyed studying.</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>21.</td>
<td>The feeling I have toward math is a positive feeling.</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>22.</td>
<td>Math makes me feel as though I'm lost in a jungle and can't find my way out.</td>
<td>0 0 0 0 0</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


21. *The Ohio State University Bulletin, College Bulletins, LXXII, Books, 2, 3, 4, 5, 7, 8, 10, 12, 14, 15, 16, Columbus, Ohio: The Ohio State University, 1968.*

