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PRESENTATION OF DISCRETE SOURCES AND THE
GALACTIC SPUR IN THE 20-40 MHZ RANGE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * * *

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ACKNOWLEDGMENTS

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CHAPTER I

INTRODUCTION

In the years preceding 1964 (the year the author began his research), much work had been accomplished at frequencies of 38 MHz and above with regard to the determination of the flux densities and spectra of discrete radio sources and the mapping of extended regions of radio emission. Surveys of the flux densities of large numbers of discrete radio sources had been made at many frequencies by various observers; for example, at 81 MHz by Shakeshaft et al. (1955), at 139 MHz by Westerbou (1958), at 159 MHz by Edge et al. (1959), at 950 MHz by Harris and Roberts (1960), at 178 MHz by Bennett (1962), and at 408 MHz by Long, Haseler, and Elsmore (1963). A compilation of the spectral indexes of 160 discrete radio sources in the 38 to 3200 MHz frequency range based on flux density measurements at individual frequencies by various observers was made by Conway, Kellermann, and Long (1963). Surveys of the galactic background emission had also been made by various observers; for example, at 600 MHz by Piddington and Trent (1956), at 38 MHz by Blythe (1957), at 250 MHz by Ko and Kraus (1957), at 400 MHz by Seeger, Stumpers, and van Hurck (1960), and at 178 MHz by Turtle and Baldwin (1962).
Based on the author's review in early 1964 of the literature then available, the frequency region below 40 MHz had the potential of being a source of important new data in the investigation of radio sources. Therefore, the author decided to make observations at frequencies below 40 MHz. It was hoped that some unusual features in the spectra or in the spatial distribution of various sources of radio emission might be found at lower frequencies. It was also hoped that the measurements of any unusual spectral or spatial features might be combined with higher radio-frequency or optical measurements to obtain information about the origin of the radio source emission.

The instrument chosen for the author's observations was the 1000-foot diameter radio telescope of the Arecibo Ionospheric Observatory, Arecibo, Puerto Rico. The observatory is located at 18°21'N latitude and 66°45'W longitude. The radio telescope is fully steerable from zenith to 20° from zenith, permitting observations within the declination range -1.6° to +38.4°. In terms of aperture area (73,000 square meters) it is the world's largest steerable radio telescope. See Figure 1. In contrast to interferometric arrays, which usually must be connected to view a particular declination while a 24-hour drift is taken, the Arecibo radio telescope permitted the author to observe many sources (at all different declinations) in one night's program.

To minimize problems caused by the earth's atmosphere the author confined his observations, for the most part, to the 2300 to 0600 local time period. Because of the poorer resolution (telescope beamwidth increases with decreasing frequency) and the greater concentration of
interfering radio stations below 20 MHz, the author found that meaningful measurements below 20 MHz were not possible at Arecibo and, therefore, confined his observations to the 20-40 MHz range.

The lack of extensive published data on the flux densities of radio sources below 38 MHz before 1964 was noted by other radio astronomers. Since then, measurements have been made by several observers (Roger, Costain, and Purton, 1965; Bazelyan et al., 1965; Erickson and Cronyn, 1965) at frequencies below 40 MHz using various types of antennas. It should be of interest to compare the author's measurements using a steerable, filled-aperture pencil beam with the other low-frequency measurements, which used a T-shaped Mills cross or spaced interferometric arrays. Many radio sources measured in these other surveys were not measured by the author. The measurements on these additional sources are quite valuable in supporting his findings concerning the relationship between dekameter spectral characteristics and the type of radio source.

The Galactic Spur, also referred to as the North Polar Spur is a region of relatively high radio brightness which appeared to emerge from the galactic plane at roughly $\lambda = 30^\circ$ and extend toward the North Galactic Pole. The author's interest in the Spur was first aroused by a paper by Hanbury-Brown, Davies, and Hazard (1960), which discussed several aspects of this curious feature of the radio sky (particularly the theories of the Spur's origin). In a later paper, Davies (1964) described his observations of the Galactic Spur at 237 MHz and the experimental results obtained relevant to the origin of
the radiation from the Spur. The author decided that low frequency (20-40 MHz) observations could make a definite contribution to the body of knowledge about the Spur and, therefore, undertook his investigation of the Spur. It was realized (and accepted) that, because of the zenith angle limitation of the Arecibo telescope, the observational results would be confined to the northern celestial hemisphere.

In the author's overall investigation of radio emission in the 20-40 MHz range, there were two distinct astronomical problems and an associated technical (engineering) problem. One astronomical problem was concerned with the measurement and the interpretation of the spectra of the various discrete radio sources. The other was concerned with the mapping and the interpretation of the radio brightness distribution of the Galactic Spur region. The associated engineering problem was the determination of the antenna parameters of the Arecibo radio telescope.

The technical (electrical engineering) objectives of the author's work were:

(1) To determine experimentally the effective area (a function of zenith angle) and the beamwidth of the radio telescope at the four operating frequencies;

(2) To determine theoretically (based on the antenna patterns of the feed and the feed-reflector geometry) the aperture distribution, the feed efficiency, and the main beam efficiency as a function of the zenith angle.
The scientific (astronomical and astrophysical) objectives of the author's investigation were:

(1) To measure in the 20-40 MHz range the flux density and spectral index of as many discrete radio sources as possible within the constraints imposed by the frequency range, the radio telescope, and the observing time assigned;

(2) To attempt to show some relationship between the shapes of the low frequency radio spectra and the types of radio sources;

(3) To map at several frequencies in the 20-40 MHz range as much of the Galactic Spur as is possible within the declination limits imposed by the radio telescope;

(4) To use the data obtained from the observations of the Galactic Spur to support a theory of the Spur's origin.

The Arecibo radio telescope and the 20-40 MHz instrumentation used by the author are described in Part A of Chapter II. The 1000-foot diameter spherical reflector was illuminated by a log-periodic feed designed to cover the 20-40 MHz range. A four-way power-divider was used to couple the feed to four independent radiometers. Their operating frequencies, chosen as a result of an on-site survey of radio station interference, were 38.75, 33.45, 26.70 and 22.30 MHz.

The flux densities of the various discrete radio sources were determined from antenna temperature measurements using an expression involving the effective area (a parameter of the radio telescope), which was calibrated experimentally using the radio source Taurus A, whose flux density at the operating frequencies was assumed (based on
exaggeration of its spectra above 38 MHz to the 20-40 MHz range). For the contour maps of the Galactic Spur, the temperature scale is established, to a first approximation, by comparison to the noise temperature output of a calibrated noise generator. Additionally, the proper scaling required correction for cable and feed VSWR losses and adjustments involving feed efficiency and main beam efficiency (other telescope parameters). Detailed analyses of the various antenna parameters of the radio telescope have been carried out by the author and are presented in Part B of Chapter II.

The observations were made during four periods: October-November 1964, April 1965, March 1966 and March-April 1967. The first two periods were used for the observations of discrete radio sources and the last two periods were used for the observation of the Galactic Spur. A discussion of the various observations made at the Arecibo Ionospheric Observatory is presented in Chapter III.

The flux densities of eight discrete radio sources were measured relative to the calibration source Taurus A. Differences in the shape of the spectra in the 20-40-MHz-range were found for the various radio sources observed. A discussion of the experimental results of the discrete source observations, including an error analysis of the flux density measurements and a theoretical interpretation of the spectra found for the various radio sources, is presented in Chapter IV.

The brightness distribution of the Galactic Spur region was measured at the four operating frequencies. The celestial area covered
was between $12^h$ to $18^h$ right ascension and $0^\circ$ to $+35^\circ$ declination. A discussion of the experimental results of the Galactic Spur observations, including contour maps of the brightness distribution and a theoretical interpretation of the ridge structure found in these maps, is presented in Chapter V.
A. DESCRIPTION OF FACILITY AND INSTRUMENTATION

1. The Arecibo Radio Telescope

The radio telescope of the Arecibo Ionospheric Observatory consists of a circular reflector which is a section of a spherical surface and a large central feed structure suspended about 500 feet above it. The servo-controlled mechanical operation of this central feed structure accomplishes the steering of the telescope beam in azimuth and elevation.

The radius of curvature of the reflector (a spherical cap) is 870 feet. The diameter of the aperture is 1000 feet. The reflector is made of one-half inch galvanized steel mesh and is supported and stabilized by sets of steel cables suspended between anchors embedded in concrete at the rim of the reflector. The surface tolerance of the spherical reflector, as found by photogrammetric means, is 0.1 foot rms.

The massive (550 tons) central feed structure consists of a triangular platform, a horizontal circular azimuth track, a large rotating feed arm, a vertical zenith track and two carriage houses. See
Figure 2. The central feed structure is held in place by three sets of stranded steel cables (the kind used for long-span suspension bridges) running from three reinforced concrete towers to the triangular platform. Access to the central feed structure is provided by a catwalk beneath the 430 MHz waveguide run coming from a nearby hill and by a cable car which runs from a small building near the rim of the bowl to the triangular platform.

The triangular platform consists of three rectangular steel trusses (each 216 feet long) bolted together to form a triangular-shaped structure. The cables supporting the whole central feed structure are connected to the corners of the triangular platform. The azimuth rail track is bolted to the bottom of the triangular platform and forms a horizontal circle 132 feet in diameter. This track supports the feed arm and allows it to rotate with the help of the center bearing assembly.

The feed arm is a 304-foot long steel truss in the form of a segment of a circle which is concentric with the center of curvature of the reflector. The feed arm is mounted below the triangular platform and is driven along the horizontal circular azimuth track by the azimuth servo system. On the curved underside of the feed arm is another set of rails on which the two carriage houses run.

The two carriage houses, which internally are about the size of a small room, run along below the feed arm and are independently controlled by separate servo systems for each carriage house. As they
Figure 2. The central feed structure.
move along the zenith track under the feed arm, the bottom of the carriage houses describe a circle, concentric with the center of curvature of the reflector surface, having a radius of 435 feet (one-half the radius of curvature of the reflector). Under the carriage houses are mounted the feeds which illuminate the 1000-foot diameter reflector. The position of a carriage house (and thereby of the feed beneath it) along the zenith track determine the zenith angle of the radio telescope beam. The zenith angle for each carriage house can be varied from 0° (when the carriage house is at the center of the feed arm) up to a maximum of 20° (when the carriage house is out at the end of the feed arm).

Carriage House 1 (CH1) with its 96-foot long line-source feed is used mainly for ionospheric and planetary radar work. Carriage House 2 (CH2) is used mainly for radio astronomy. Since it is relatively easy to change feeds on CH2, quite a variety of radio astronomy programs are conducted using CH2.

Receivers are mounted right inside the carriage house to minimize the length of cable from the feed. A temperature stabilized refrigerated enclosure is available in CH2 for those receivers needing such environmental control. Since there is ample room for equipment, the incoming signals are usually processed in the carriage house to a stage where they are ready to be put onto magnetic tape or to drive a pen on a chart recorder. The processed signals are brought down on shielded cables to the main building where they are recorded or further processed.
The radio telescope is operated from the control room (a part of the main building) from which one can look out at the reflector and feed structure. Usually, the recording equipment for an experiment is also in the control room. The three panels on the left side of the telescope control console (see Figure 3) contain the controls for the hydraulic and mechanical operation of Carriage House 1, Carriage House 2, and the azimuth drive system. The fourth panel contains the controls for putting in the beam-pointing coordinates — zenith angle (CH1 or CH2) and azimuth. One may set in the desired azimuth and zenith angle either by hand on this same panel (manual mode) or with the aid of a digital computer (computer mode). The feed arm, CH1 and CH2 are driven to the commanded positions by closed-loop servo systems. The right-hand panel of the console is used for computer operation of the telescope. The chief function of the computer is coordinate conversion. An observer can run his program entirely in terms of celestial coordinates, including right ascension and declination scans at any rates (limited only by the ability of the telescope's drive systems to keep up with the rapidly changing zenith angle and azimuth coordinates commanded by the computer).

2. Instrumentation for the 20-40 MHz Observations

A schematic diagram of the instrumentation used in the 20-40 MHz observations at the Arecibo Ionospheric Observatory is shown in Figure 4. The same basic instrumentation was used for both the discrete source observations (October-November 1964 and April 1965) and the Galactic Spur observations (March 1966 and March-April 1967).
Figure 3. The telescope control console in the control room. Looking out the windows one can see the central feed structure.
Figure 4. Instrumentation for 20-40 MHz observations at the Arecibo Ionospheric Observatory.
The log-periodic feed consists of two coaxial sets of twelve dipoles, each set in a similar log-periodic configuration. The two dipole sets are isolated from one another and have their planes of polarization at right angles to each other. See Figure 5. By introducing a relative phase delay of 90° into the output of either one or the other dipole set, one could obtain either right- or left-handed circular polarization with this feed. Since polarization measurements were not involved in either the discrete source or the Galactic Spur work, one of the dipole sets was arbitrarily chosen for all observations. The log-periodic feed was mounted to CH2 in such a way that the dipoles extended out from the corners of the carriage house. Thus, the dipole sets were mounted with their planes of polarization at a 45° angle to the plane of motion of CH2 along the zenith track. See Figure 6.

The signal from the log-periodic feed comes in on RG-9 cable through a coaxial switch (used for antenna temperature calibration) to an isolator-power divider. The feed terminal of the log-periodic is at the tip of its 30-foot mast. The length of cable from the feed terminal to the coaxial switch is about 40 feet, 30 feet inside the mast and about 10 feet inside the carriage house. The outputs of the four-way power divider go into four separate, fixed-frequency radiometers mounted in a rack inside CH2. See Figure 7. The high isolation (30 dB minimum) between the output terminals of the power divider precludes any interaction between radiometers.

The radiometers used in the Arecibo observations were originally designed and manufactured for the measurement of ionospheric
Figure 5. The log-periodic feed being hoisted up to CH 2.
Figure 6. The 20–40 MHz log-periodic feed mounted under Carriage House 2.
Figure 7. The portion of the 20–40 MHz instrumentation installed in CH 2. Shown are the four radiometers and the noise generator.
absorption, particularly the changes in absorption caused by solar bursts, and were called riometers (radiometric ionospheric opacity meters). Their operation is based on the servoed null-comparison concept of Ryle and Vonberg (1948). In these radiometers the input noise from the antenna is compared, at a 340 Hz switching rate, with the noise emitted by a thermionic diode. The diode is operated in the temperature limited mode (in contrast to the usual space charge limited mode) and the output noise power of the diode is linearly proportional to its d.c. plate current. The frequency bandwidth and the sampling period for the antenna noise and the comparison noise source (the diode) are equal. The current through the diode is controlled by means of a servo loop whose error signal is generated by the difference between the antenna noise and the comparison diode noise. The system is driven to a null, such that the noise in the diode line and the antenna line are equalized. Since the noise power output of the diode is proportional to its current, the diode current is then a direct measure of the noise power in the antenna line. (To obtain the noise power incident upon the antenna, cable and impedance mismatch losses must be taken into account.)

In the radio frequency range noise power and equivalent (black-body) temperature are related by the simple expression $P = k T \Delta f$, where $P$ is the power, $k$ is Boltzmann's constant, $T$ is the equivalent noise temperature, and $\Delta f$ is the bandwidth. Thus, when the system is servoed to a null, the diode plate current is directly proportional to antenna temperature.
Various bandwidths and integrating time constants can be obtained by means of selector switches on the front panel of the radiometers. Normally, a bandwidth of 30 KHz (determined by a crystal filter in the i.f. amplifier portion of the radiometer) and a time constant of ten seconds were used for the observations.

The four particular frequencies chosen in the 20 to 40 MHz range were determined as a result of an on-site radio interference survey made by the author during a two week period in July 1964. The log-periodic antenna was erected, pointing vertically, on a hill near one of the three towers which support the central feed structure. An SP-600 receiver was connected to the log-periodic antenna and swept in frequency from about 19 to 42 MHz. In analyzing the radio station interference data the frequency range was divided into 0.1 MHz intervals and a mark set down in each interval for each time any interfering signal was found in that interval. With about seven days and nights of frequency sweeps (hundreds of sweeps), certain frequency regions showed a repetitive presence of interfering radio signals, while other regions were relatively clear, especially at night. In general, there was much more interference during the day than at night, and for both day and night the interference was much more frequent and more densely packed at the 20 MHz end than at the 40 MHz end. Based on the criteria of minimum interference and an even spread of operating frequencies throughout the 20-40 MHz range, the following four operating frequencies were selected: 22.30 MHz, 26.70 MHz, 33.45 MHz, and 38.75 MHz.
The output of each radiometer is taken as the d.c. voltage across an external resistor connected into the plate circuit of the comparison noise diode. This voltage is directly proportional to antenna temperature measured by each radiometer at its particular operating frequency. The antenna temperature consists of contributions due to discrete sources and to sky background, with sky background being the dominant contribution, except for the strongest radio sources. The sky background temperature is a highly dependent function (inversely proportional to the 2.5 power) of frequency. External resistors of different values were used on the outputs of the four radiometers to keep their output voltage level about the same order. Their resistance values were in the range of 1000-4000 ohms. The radiometer output voltages are brought down from CH2 on shielded cables to the control room where they are fed into an eight-channel Sanborn recorder. Large (500 μf) smoothing capacitors were connected across the radiometer output voltages to filter out any stray 60 Hz voltage picked up along the long length (~2000 feet) of cable from CH2 to the control room. These capacitors were found necessary to prevent jittering of the recorder pen, which otherwise would have broadened the lines on the observational records.

The use of an eight-channel recorder permits the recording of each radiometer output in two slightly different formats. On one channel, absolute antenna temperature is recorded. No zero-offset bias is used; zero on the record (the right hand margin of the grid for that particular channel) corresponds to zero antenna temperature. On the other
channel zero-offset bias is used to remove most of the radiometer voltage due to the sky background. The d.c. bias voltage is supplied by mercury cells and a potentiometer. The employment of zero-offset bias permits the recording of the radiometer output with a more sensitive scale (i.e., more scale divisions on the grid for a given change in antenna temperature) and thus improves the measurability of a discrete source or feature of a contour map. Adjustment of the low-resistance potentiometer of the zero-offset network has no effect on the overall gain of the channel; it merely changes the baseline position on the grid. The input resistance of each channel of the Sanborn recorder is very high (about one megohm); therefore, there is no interaction between two channels connected to the same radiometer output when one channel is zero-offset biased and the other is not.

The instrumentation is calibrated in terms of uncorrected antenna temperature (uncorrected for feed impedance mismatch and cable losses) several times during an observing session by the use of a very simple technique. First, all zero-offset bias voltages were removed. Then, by means of a remotely controlled (from the control room) coaxial switch, the antenna is removed from the input to the four-way power divider and in its place is substituted a calibrated noise source. The noise generator (calibrated noise source) is connected to the coaxial switch by a negligibly short cable. By means of a dial setting on its face (a one-time adjustment), an equivalent noise temperature of 11,600°K is obtained from the noise generator. The impedance of all components other than the feed are all 50 ohms in the 20-40 MHz
frequency range, so there are no other impedance mismatches. Hence, the calibration signal from the noise generator corresponds directly to an uncorrected antenna temperature of 11,600°K.
B. DETERMINATION OF THE TELESCOPE PARAMETERS

1. Feed VSWR and Cable Losses

In the measurement of the flux density of discrete radio sources relative to a calibration source (Taurus A), it was not necessary to determine the losses resulting from imperfect impedance matching at the feed or from the attenuation in the cable between the feed and the input to the radiometers since exactly the same losses occur in both the measurements of the effective area using the calibration source and in the flux density measurements of the other discrete sources. For the discrete source work, it is simpler and more direct to draw up a set of curves of the ratio of uncorrected antenna temperature (i.e., uncorrected for the impedance mismatch and cable losses) to flux density as a function of zenith angle for the four frequencies, rather than curves of effective area. In the mapping work, however, the absolute temperature scale is established not by reference to some astronomical object but by comparison to the noise temperature of the calibration noise generator. Hence, the losses between the feed and coaxial switch greater than those between the noise generator and the coaxial switch must be measured. Further losses beyond the junction point (the coaxial switch) are of no consequence since they are mutual. See Figure 4.

The voltage standing wave ratio (VSWR) of the random noise generator (50-ohm nominal output impedance) was less than 1.05 throughout the 20-40 MHz range and the length of cable (RG-9) between the noise generator and the coaxial switch was less than 2 feet. Hence, losses
in the noise generator line are negligible.

The feed terminal is at the tip of the 30-foot mast of the log-periodic feed which is attached upside down to the bottom of Carriage House 2. The length of cable (RG-9) between the feed terminal and the coaxial switch is roughly 40 feet, 30 feet inside the mast of the log-periodic feed and about 10 feet inside the carriage house. The attenuation for RG-9 is frequency dependent and is given in a chart in terms of dB per 100 feet as a function of frequency. Table 1 gives the cable losses and the required correction factors for the four operating frequencies. The correction factor obviously is the reciprocal of the attenuation factor.

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>RG-9 ATTENUATION (dB/100 ft)</th>
<th>ATTENUATION IN 40 FT. CABLE (dB)</th>
<th>CABLE LOSS CORRECTION FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>0.86</td>
<td>0.34</td>
<td>0.925</td>
</tr>
<tr>
<td>26.70</td>
<td>0.95</td>
<td>0.38</td>
<td>0.917</td>
</tr>
<tr>
<td>33.45</td>
<td>1.07</td>
<td>0.43</td>
<td>0.906</td>
</tr>
<tr>
<td>38.75</td>
<td>1.18</td>
<td>0.47</td>
<td>0.898</td>
</tr>
</tbody>
</table>

The VSWR of the log-periodic feed as a function of frequency is shown in Figure 8. Since the feed in its original form consisted of two sets of dipoles orthogonally-polarized with respect to each other, there is a VSWR curve for each set. The VSWR at the four operating frequencies is roughly the same for either feed. The ratio of reflected
Figure 8. The voltage standing wave ratio of the two dipole sets of the log-periodic feed. The nominal impedance is 50 ohms.
to incident power is related to VSWR by the following expression:

\[
\frac{P_r}{P_i} = |K|^2 = \left(\frac{\text{VSWR} - 1}{\text{VSWR} + 1}\right)^2
\]

where \(|K|\) is the absolute value of the reflection coefficient. The attenuation factor is given by:

\[
\text{attenuation} = \frac{P_i - P_r}{P_i} = 1 - |K|^2.
\]

Table 2 shows the VSWR, reflected power, and correction factors for the four operating frequencies.

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>VSWR</th>
<th>REFLECTED POWER (proportion)</th>
<th>ATTENUATION FACTOR</th>
<th>VSWR CORRECTION FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>1.25</td>
<td>0.012</td>
<td>0.988</td>
<td>1.01</td>
</tr>
<tr>
<td>26.70</td>
<td>1.40</td>
<td>0.028</td>
<td>0.972</td>
<td>1.03</td>
</tr>
<tr>
<td>33.45</td>
<td>1.25</td>
<td>0.012</td>
<td>0.988</td>
<td>1.01</td>
</tr>
<tr>
<td>38.75</td>
<td>1.60</td>
<td>0.053</td>
<td>0.947</td>
<td>1.06</td>
</tr>
</tbody>
</table>

The total correction factor for the VSWR loss and the cable loss is the product of the two individual correction factors. These total correction factors, which are given for the four operating frequencies in Table 3 below, will be used to correct "uncorrected antenna temperature" (i.e., equivalent antenna temperature measured at the coaxial switch) to "corrected antenna temperature" (i.e., true antenna
temperature incident upon the feed).

TABLE 3. Total Correction Factors for Converting to Corrected Antenna Temperature

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>TOTAL CORRECTION FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>1.10</td>
</tr>
<tr>
<td>26.70</td>
<td>1.12</td>
</tr>
<tr>
<td>33.45</td>
<td>1.12</td>
</tr>
<tr>
<td>38.75</td>
<td>1.18</td>
</tr>
</tbody>
</table>

2. Aperture Distribution

Preliminary to any theoretical calculation of radio telescope beamwidth or effective area, it is necessary to determine the field and power distribution across the aperture. The aperture of the Arecibo radio telescope is circular with a diameter of 1000 feet (305 meters). Circular symmetry in the radio telescope geometry is present only in the case of 0° zenith angle (feed pointing straight down).

For zenith angles other than 0° it is very difficult to make purely theoretical calculations of the antenna parameters. On the Arecibo radio telescope a change of zenith angle involves not merely a change in the pointing direction of the feed but actual lateral motion of this feed relative to the reflector. For zenith angles other than 0° all circular symmetry in the geometry is lost and strict analytic calculations become overly complex. Calculations of the effective area were made over the whole range of zenith angles, but these were based upon measurements of an astronomical object rather than upon
theoretical considerations. In this section and the following sections of this chapter, detailed theoretical analyses will be confined to the 0° zenith angle case. In some cases, extension of the results to zenith angles other than 0° will be made by estimation or by using simplifications.

Let us consider the aperture distribution for 0° zenith angle. The aperture distribution is determined by the illumination pattern of the log-periodic feed. The E-plane and H-plane feed patterns (for each of the cross-polarized dipole sets) of the log-periodic antenna have been measured for 30 MHz. They were given in terms of dB below the maximum value as a function of angle from the feed axis. (The maximum directivity is along the feed axis, which points straight down in the case of 0° zenith angle.) These feed patterns do not change appreciably for the other frequencies within the feed's operating range (20-40 MHz). For any frequency the elements of the feed determining the radiation pattern are in scale (proportional to wavelength). The location of the center of focus shifts with increasing frequency toward the shorter dipole end of the feed. With a spherical reflector, unlike a parabolic dish, the distance from the center of focus to the reflector is not critical. The mast of the log-periodic feed is 30 feet long. The distance from the bottom of the carriage house to the bottom of the bowl is 435 feet. To keep the following analysis simple it will be assumed that the center of focus is at the same point for all frequencies — at the center of the mast of the log-periodic feed (15 feet below the bottom of the carriage house, 420 feet above the bottom of
the reflector. The angle, measured at the center of focus, from the vertical axis to the rim of the dish is found from geometry to be equal to:

\[
\text{arc cot} \left( \frac{420 - 158}{500} \right) = 62.4^\circ.
\]

The geometry relating the angles from the feed axis to the various aperture radii is illustrated in Figure 9. Tables 4 and 5 give the values of the E-plane and the H-plane feed patterns for angles from the feed axis corresponding to each hundred feet of aperture radius.

TABLE 4. Values of the E-plane Pattern for Various Angles from the Feed Axis and Corresponding Aperture Radii

<table>
<thead>
<tr>
<th>ANGLE FROM FEED AXIS (°)</th>
<th>APERTURE RADIUS (feet)</th>
<th>E-PLANE FEED PATTERN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
<td>0 dB</td>
</tr>
<tr>
<td>13.6°</td>
<td>100</td>
<td>0.5</td>
</tr>
<tr>
<td>26.8°</td>
<td>200</td>
<td>2.3</td>
</tr>
<tr>
<td>39.3°</td>
<td>300</td>
<td>4.7</td>
</tr>
<tr>
<td>51.2°</td>
<td>400</td>
<td>8.4</td>
</tr>
<tr>
<td>62.4°</td>
<td>500</td>
<td>13.0</td>
</tr>
</tbody>
</table>
Figure 9. Geometric relationship between the angles from the feed axis and the various aperture radii.
TABLE 5. Values of the H-plane Pattern for Various Angles from the Feed Axis and Corresponding Aperture Radii

<table>
<thead>
<tr>
<th>ANGLE FROM FEED AXIS</th>
<th>APERTURE RADIUS (feet)</th>
<th>H-PLANE FEED PATTERN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
<td>0 dB 1.00 1.00</td>
</tr>
<tr>
<td>13.6°</td>
<td>100</td>
<td>0.1 dB .99 .98</td>
</tr>
<tr>
<td>26.8°</td>
<td>200</td>
<td>0.5 dB .94 .89</td>
</tr>
<tr>
<td>39.3°</td>
<td>300</td>
<td>1.7 dB .82 .68</td>
</tr>
<tr>
<td>51.2°</td>
<td>400</td>
<td>3.2 dB .69 .48</td>
</tr>
<tr>
<td>62.4°</td>
<td>500</td>
<td>5.0 dB .56 .32</td>
</tr>
</tbody>
</table>

The aperture amplitude distribution as a function of radius, $A(r)$, is normalized to the zero radius in the same way as the feed field pattern is normalized to the 0° nadir angle (along the feed axis). Hence, by definition, $A(0) = 1$. We will also define an aperture power distribution $P(r) = [A(r)]^2$, which is also normalized to zero radius. This aperture power distribution corresponds to the power pattern of the feed in the same way as aperture amplitude distribution corresponds to the field pattern of the feed. Curves of the amplitude and power distribution across the aperture as a function of radius for the E-plane and H-plane (defined by the plane of the dipole set of the log-periodic feed that is being used) are shown in Figure 10.

Although for a radio telescope zenith angle of 0° the aperture distribution is symmetric across the vertical axis for any diameter (the E-plane and the H-plane define two such diameters), the aperture distribution is not circularly symmetric, since it depends on azimuth...
Figure 10. Amplitude and power distributions across the aperture for the E-plane and the H-plane.
angle (symbolized by $\phi$ in the following discussion). The illumination of the feed and the resulting aperture distribution is elliptical. From the point of view of simplifying later calculations, it is advantageous to establish, as an alternative model to the actual elliptical distribution, some sort of circular aperture distribution that is independent of the azimuth angle, so that later we might use $A(r)$ and $P(r)$ rather than $A(r,\phi)$ and $P(r,\phi)$. In a following section on feed efficiency and spillover, this same reasoning is found applicable to the beam pattern of the feed.

To set up the discussion we will assume that the E-plane is defined by $\phi = 0$ and the H-plane by $\phi = \frac{\pi}{2}$. Hence, $A_E(r) = A(r,\phi=0)$, $A_H(r) = A(r,\phi=\frac{\pi}{2})$, $P_E(r) = P(r,\phi=0)$, and $P_H = P(r,\phi=\frac{\pi}{2})$. We define a composite aperture amplitude distribution $A_C(r)$ as the geometric mean of the E-plane and H-plane amplitude distributions, that is, $A_C(r) = \sqrt{A_E(r)A_H(r)}$. Similarly, we define a composite power aperture distribution $P_C(r)$ as the geometric mean of $P_E(r)$ and $P_H(r)$. Table 6 gives values of the composite aperture distribution functions $P_C(r)$ and $A_C(r)$, calculated from their definitions, for various values of aperture radius. Figure 11 shows the composite amplitude and power distributions as a function of the aperture radius.
Figure II. Composite amplitude and power distributions as functions of aperture radius.
TABLE 6. Values of the Composite Aperture Distribution for Various Aperture Radii

<table>
<thead>
<tr>
<th>APERTURE RADIUS (feet)</th>
<th>COMPOSITE APERTURE DISTRIBUTION power</th>
<th>amplitude</th>
<th>dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>.93</td>
<td>.97</td>
<td>0.3</td>
</tr>
<tr>
<td>200</td>
<td>.72</td>
<td>.85</td>
<td>1.4</td>
</tr>
<tr>
<td>300</td>
<td>.49</td>
<td>.70</td>
<td>3.1</td>
</tr>
<tr>
<td>400</td>
<td>.26</td>
<td>.51</td>
<td>5.8</td>
</tr>
<tr>
<td>500</td>
<td>.12</td>
<td>.35</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Based on analysis in later sections of this chapter, it is found that the composite aperture distribution can be taken as roughly the distribution in what we will call the cross-plane (the plane bisecting the right angle formed by the E-plane and H-plane. Thus, we have $A_c(r) = A(r, \phi=\frac{\pi}{4})$ and $P_c(r) = P(r, \phi=\frac{\pi}{4})$. Also, the feed pattern of the cross-plane should correspond to this cross-plane aperture distribution in the same way as the feed patterns of the E-plane and H-plane correspond to their aperture distributions, since for the cross-plane case the geometry relating the angle from the feed axis to the aperture radius is exactly the same. This fact will be useful in later sections on feed efficiency and spillover and on radio telescope beamwidth.

3. Feed Efficiency and Spillover

The basic equation relating brightness and antenna temperature is:

$$T_A(\theta, \phi) = n_M T_b(\theta, \phi) + n_{SL} \langle T_b \rangle_{SL} + n_{SP} \langle T_b \rangle_{SP}.$$
where $\eta_{SP}$ is the spillover (the portion of the feed illumination that misses the reflector), $\eta_M$ is the main beam efficiency, $\eta_{SL}$ is the side lobe term excluding feed spillover (i.e., $\eta_{SL} = 1 - \eta_M - \eta_{SP}$), $T_A(0,\phi)$ is the antenna temperature, $T_b(0,\phi)$ is the average brightness temperature within the main beam, $\langle T_b \rangle_{SL}$ is the average brightness temperature in the sidelobes, and $\langle T_b \rangle_{SP}$ is the temperature seen by the spillover.

The spillover coefficient $\eta_{SP}$ is equal to $1 - \varepsilon_f$, where $\varepsilon_f$ is the feed efficiency of the log-periodic feed. The illumination of the reflector by the feed is rather efficient, especially for small zenith angles, since the shape of its forward pattern is well matched to the reflector. The front-to-back ratio of the log-periodic feed is very high; less than one percent is radiated into the back hemisphere.

Let us first consider the situation for a radio telescope zenith angle of 0° (feed axis pointing straight down). The feed patterns at 30 MHz have been measured and, for a log-periodic antenna, these do not change appreciably for the other frequencies within its operating range (20 to 40 MHz), so the one analysis here will cover all four operating frequencies. The E-plane and H-plane power patterns of the log-periodic feed are shown in linear scale in Figure 12. The illumination of the spherical reflector by the log-periodic feed from the viewpoint of the center of focus of the feed is shown in the (a) portion of Figure 13.
Figure 12. Illumination of the reflector surface by the E-plane and the H-plane power patterns of the log-periodic feed for 0° zenith angle. Half power beamwidths are 64° for $P_E(\theta)$ and 98° for $P_H(\theta)$. The angle from the beam axis to the rim of the reflector surface is 62.4°.
Figure 13. Illumination of the reflector for 0° radio telescope zenith angle
(a) by the actual feed pattern,
(b) by the composite feed pattern $P_c(\theta)$ having circular symmetry. The
illumination of the reflector shows azimuthal symmetry.
To continue the analysis for determining the feed efficiency, some simplifications must be made. Instead of an E-plane power pattern \( P_E(\theta) = P(\theta, \phi=0) \) and an H-plane power pattern \( P_H(\theta) = P(\theta, \phi=\frac{\pi}{2}) \), it would be simpler in the analysis to work with a single composite power pattern \( P_C(\theta) \) with azimuthal symmetry; i.e., \( P_C(\theta, \phi) = P_C(\theta) \). We will take \( P_C(\theta) \) as the geometric mean of \( P_E(\theta) \) and \( P_H(\theta) \); i.e., \( P_C(\theta) = \sqrt{P_E(\theta) P_H(\theta)} \). The illumination on the reflector by the composite pattern will be circular, instead of elliptical, as shown in the (b) portion of Figure 13.

For a feed pattern with negligible radiation into the back hemisphere, the efficiency is given by:

\[
\epsilon_f = \frac{\int_0^{2\pi} \int_0^\rho f_n(\theta, \phi) \sin \theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\frac{\pi}{2}} f_n(\theta, \phi) \sin \theta \, d\theta \, d\phi}
\]

For azimuthal symmetry we have:

\[
\epsilon_f = \frac{\int_0^\rho P_C(\theta) \sin \theta \, d\theta}{\int_0^{\frac{\pi}{2}} P_C(\theta) \sin \theta \, d\theta}
\]

where \( \rho \) is the angle from the feed axis to the rim of the reflector.

As a next step, it is necessary to approximate the power pattern of our composite beam by an analytic function. It can be observed from Figure 12 that the E-plane power pattern of the feed has roughly a \( \cos^4 \theta \) shape while the H-plane power pattern has roughly a \( \cos^2 \theta \) shape.
Thus, the composite beam should be approximated with a $\cos^3 \theta$ pattern. Note the following comparative table.

<table>
<thead>
<tr>
<th>POWER PATTERN</th>
<th>LOG-PERIODIC BEAM HPBW</th>
<th>COSINE APPROXIMATION</th>
<th>LOG-PERIODIC BEAM HPBW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_E(0)$</td>
<td>64°</td>
<td>$\cos^4 \theta$</td>
<td>$\cos^2 \theta$</td>
</tr>
<tr>
<td>$P_H(0)$</td>
<td>98°</td>
<td>$\cos^2 \theta$</td>
<td>$\cos \theta$</td>
</tr>
<tr>
<td>$P_C(0)$</td>
<td>78°</td>
<td>$\cos^3 \theta$</td>
<td>$\cos^3 \theta$</td>
</tr>
</tbody>
</table>

Obviously, with a sin $\theta$ in the integral, $\cos^n \theta$ is a wise choice for approximating $P(\theta)$. Thus, for a vertical feed axis and a $\cos^n \theta$ power pattern, we have:

$$\varepsilon_f = 1 - \cos^{n+1} \rho$$

For the Arecibo radio telescope the angle, measured at the center of focus, from the vertical axis to the rim of the dish is 62.4°. The center of focus varies slightly with frequency; however, to keep the analysis simple, it is assumed that the center of focus for all the operating frequencies is at the center of the mast of the log-periodic feed (15 feet below the bottom of the carriage house). Table 8 shows the theoretical feed efficiencies (at 0° radio telescope zenith angle) for $\cos^n \theta$ power patterns for various values of $n$. 


TABLE 8. Feed Efficiencies of Various Cosine Power Patterns for a Reflector Whose Rim Subtends an Angle of 62.4° from the Feed Axis

<table>
<thead>
<tr>
<th>BEAM SHAPE P(θ)</th>
<th>HPBW</th>
<th>EFFICIENCY (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cos θ</td>
<td>120°</td>
<td>78.7</td>
</tr>
<tr>
<td>cos²θ</td>
<td>90°</td>
<td>90.1</td>
</tr>
<tr>
<td>cos³θ</td>
<td>76°</td>
<td>95.4</td>
</tr>
<tr>
<td>cos⁴θ</td>
<td>66°</td>
<td>97.9</td>
</tr>
</tbody>
</table>

With the Arecibo radio telescope, a change in zenith angle (pointing angle of the radio telescope beam in space measured from the local vertical) entails more than just changing the pointing direction of the feed axis with respect to the vertical; the center of focus of the feed is actually moved laterally. The log-periodic feed is rigidly attached to the bottom of Carriage House 2. In changing zenith angles, the carriage house runs along a zenith angle track; the center of focus of the feed follows the carriage house. This is illustrated in Figure 14; for simplicity, the figure shows the composite feed pattern \( P_C(θ) \). The log-periodic feed was mounted to the bottom of Carriage House 2 with its E-plane and H-plane at a 45° angle to the plane of the zenith angle track. Figure 15 shows the illumination of the reflector for a zenith angle (\( ζ \)) of 15° by the actual feed pattern and by the composite pattern \( P_C(θ) \). The composite pattern \( P_C(θ) \) closely describes the feed pattern along the plane of the zenith angle track. For all further analysis we will consider only the composite plane (i.e., the
Figure 14. Illumination of reflector by the composite power pattern of the beam. \( P_c(\theta) = \sqrt{P_E(\theta) \cdot P_H(\theta)} \). This is roughly the power pattern in a plane 45° to the \( E \) and \( H \)-planes. Half power beamwidth is 78°. Solid curve is the beam at 0° zenith angle; broken curve shows direction of this same pattern at 15° zenith angle. Angles from beam axis to rim of the dish in the plane of the zenith angle track are shown.
Figure 15. Illumination of the reflector for 15° radio telescope zenith angle
(a) by the actual feed pattern,
(b) by the composite feed pattern $P_c(\theta)$ having circular symmetry.
Azimuthal symmetry is lost in illuminating the reflector.
\[ \phi = \frac{\pi}{4} \text{ plane) power pattern } P_C(0). \] We will further assume that \( P_C(0) = \cos^3 \theta. \)

For a given zenith angle, the only simple thing one can do is to measure geometrically the angle from the feed axis (tilted and offset) to the rim of the reflector for various azimuthal directions. Four such directions are defined by the angles to the rim in the plane of the zenith angle track and in the plane perpendicular to it. Note Figure 14 for the angles to the rim in the plane of the zenith angle track for \( \xi = 15^\circ. \) Rough measurements of the angles to the rim of the reflector in the four directions were made for zenith angles of 5°, 10°, 15°, and 20° based on the reflector-feed geometry. Since \( P(0) = \cos^3 \theta, \) we have \( \epsilon_f = \text{avg} (1 - \cos^4 \phi) \) or

\[ \epsilon_f = \frac{1}{4} (4 - \cos^4 \rho_i - \cos^4 \rho_{\|} - 2 \cos^4 \rho_{\perp}), \]

where \( \rho_i \) and \( \rho_{\|} \) are the angles from the feed axis to the rim in the plane of the zenith angle track and \( \rho_{\perp} \) is the angle to the rim in the perpendicular plane (the two angles in this plane are always equal). The feed efficiencies for various zenith angles are given in Table 9. A graph of feed efficiency \( \epsilon_f \) and spillover \( \eta_{SP} \) versus radio telescope zenith angle is given in Figure 16.
Figure 16. Feed efficiency $\varepsilon_f$ and spillover $\eta_{SP}$ as a function of radio telescope zenith angle.
TABLE 9. Feed Efficiency as a Function of Zenith Angle for the Log-Periodic Feed Using the \( P_c(\theta) = \cos^3 \theta \) Pattern Approximation

<table>
<thead>
<tr>
<th>ZENITH ANGLE</th>
<th>FEED EFFICIENCY (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>95.4</td>
</tr>
<tr>
<td>5°</td>
<td>94.6</td>
</tr>
<tr>
<td>10°</td>
<td>92.3</td>
</tr>
<tr>
<td>15°</td>
<td>88.4</td>
</tr>
<tr>
<td>20°</td>
<td>82.4</td>
</tr>
</tbody>
</table>

4. Radio Telescope Beamwidth

The radio telescope beamwidth for the four operating frequencies was experimentally determined at a zenith angle of 4.8° using the radio source 3C157 (IC443). This source has an angular diameter of 29 minutes of arc (Bennett, 1962), which is large enough that it does not scintillate (Aarons and Guidice, 1966). Therefore, a fairly accurate measurement (± 5 percent) of the half-power width of the response to this source is possible even though 3C157 has a much lower flux density than the strong sources Taurus A and Virgo A. See Figure 17. The declination scan in this figure, which is part of an actual record taken at AIO, was made at a scan rate of 30° per hour; therefore, each minute of time corresponds to 0.5°. The peak and half-power levels are shown on the odd channels of the record (the upper grids in the figure) for each operating frequency. The odd channels used offset biasing to eliminate most of the background temperature, permitting a better display of the variation in the antenna response.
Figure 17. Declination cut of the discrete source 3C 157 at a rate of 30°/hr. Declination scan was made at a rate of 30° per hour; thus one minute of time is equivalent to 0.5°. Half-power levels are shown.
Because the angular diameter of 3C157 is not negligibly small compared to the beamwidth of the radio telescope, the measured response of the radio telescope to this source is wider than the radio telescope beamwidth (i.e., the radio telescope's response to a "point source"). The convolution of a Gaussian antenna beam with a Gaussian-distributed source (3C157) results in a response which is also Gaussian in which $\sigma_R^2 = \sigma_M^2 + \sigma_S^2$, where $\sigma_R$, $\sigma_M$, and $\sigma_S$ are the standard deviations of the response, the beam, and the source. For any Gaussian curve, the width between half-levels is directly proportional to the standard deviation; therefore, we may also write:

$$\hat{\theta}_R^2 = \hat{\theta}_M^2 + \hat{\theta}_S^2,$$

where $\hat{\theta}_R$ is the half-power width of the response, $\hat{\theta}_M$ is the half-power beamwidth of the radio telescope, and $\hat{\theta}_S$ is the half-intensity width of the source. (The symbol * in all cases will stand for half-power or half-intensity width.) Having measured the half-power width of the response, we can obtain the actual half-power beamwidth using the expression:

$$\hat{\theta}_M = \left(1 - \frac{\hat{\theta}_S^2}{\hat{\theta}_R^2}\right)^{\frac{1}{2}} \hat{\theta}_R.$$

This "correction" to the half-power width of the response is small; it amounts to roughly three percent for the 38.75 MHz beam and one percent for the 22.30 MHz beam. Table 10 gives the half-power response to
3C157 and the half-power beamwidth of the radio telescope for the four operating frequencies.


<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>HALF-POWER WIDTH OF RESPONSE TO 3C157</th>
<th>HALF-POWER BEAMWIDTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>3.33°</td>
<td>3.30°</td>
</tr>
<tr>
<td>26.70</td>
<td>2.75°</td>
<td>2.71°</td>
</tr>
<tr>
<td>33.45</td>
<td>2.33°</td>
<td>2.28°</td>
</tr>
<tr>
<td>38.75</td>
<td>2.08°</td>
<td>2.02°</td>
</tr>
</tbody>
</table>

Since the aperture distribution in the E-plane (defined by the plane of the dipole set of the log-periodic feed) is more tapered than that of the H-plane, the radio telescope beam is slightly wider in the E-plane than in the H-plane. The declination cut of Figure 17 was taken in such a way that the beam plane through which the source passed was roughly the plane at 45° to the E- and H-planes, which we have called the cross-plane (X-plane). From ellipse geometry, the width across the 45° plane, \( \hat{\theta}_X \), is given exactly by \( \sqrt{2} \frac{\theta_E \theta_H}{\sqrt{\theta_E^2 + \theta_H^2}} \) where \( \theta_E \) and \( \theta_H \) are the widths across the E- and H-planes.

For the theoretical derivation of half-power beamwidth from aperture distribution, we will consider only the 0° zenith angle case. To be able to compare the calculated beamwidth to the measured beamwidth, which was that of the cross-plane, we will derive the theoretical
beamwidth for the cross-plane. The first step is to ascertain the aperture distribution of the cross-plane. Since for $\zeta = 0^\circ$ the aperture diameter is uniform, the ellipticity of the radio telescope beam will be very small, even though the aperture illumination by the feed is distinctively elliptical. The ratio of $\hat{\theta}_E$ to $\hat{\theta}_H$ is less than 1.2 (demonstrated later in the section). Hence, the exact formulation for $\hat{\theta}_X$ from elliptical geometry can be reduced to the simpler form $\sqrt{\hat{\theta}_E \hat{\theta}_H}$ with an error of less than one percent. Since it has been thus shown that $\hat{\theta}_X$ can be represented as the geometric mean of $\hat{\theta}_E$ and $\hat{\theta}_H$, the aperture distribution correspondent to $\hat{\theta}_X$ must be the composite distribution $A_C(r)$ of Figure 11, which was defined as the geometric mean of $A_E(r)$ and $A_H(r)$.

To simplify the problem, let us hypothesize a circularly symmetric beam of half-power beamwidth $\hat{\theta}_X$. Associated with this beam is a circularly symmetric aperture amplitude distribution $A_C(r)$. It is advantageous to put $A_C(r)$ into the form:

$$A(r_n) = 1 - T r_n^2,$$

where $T$ is the taper ($0 \leq T \leq 1$), and $r_n$ is the normalized radius of the aperture (i.e., $r_n = r/500$ ft.). To obtain the half-power beamwidth from the aperture amplitude distribution we start with a table from page 195 of Silver (1949). Unfortunately, this table is a distribution function $A(r_n) = (1 - r_n^2)^p$ which corresponds to the $1 - T r_n^2$ distribution only for $p = 0$ and $p = 1$. For $p = 0$ we have zero taper and $A(r_n) = 1$; for $p = 1$ we have full taper ($T = 1$) and
A(r_n) = 1 - r_n^2. On page 187 of the same text there is a table on linear apertures that does involve a distribution of the form 1 - T \chi_n^2. This table was used to interpolate values for T = 0.5 for the circular aperture case. The following table was then obtained.

**TABLE 11. Pattern Characteristics Produced by a 1 - Tr_n^2 Distribution over a Circular Aperture**

<table>
<thead>
<tr>
<th>ILLUMINATION</th>
<th>TAPER</th>
<th>HALF-POWER BEAMWIDTH</th>
<th>SIDE LOBE LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>T = 0</td>
<td>1.00\frac{\lambda}{D}</td>
<td>-17.6 dB</td>
</tr>
<tr>
<td>Tapered to 0.5 at rim</td>
<td>T = 0.5</td>
<td>1.11\frac{\lambda}{D}</td>
<td>-21.3 dB</td>
</tr>
<tr>
<td>Tapered to zero at rim</td>
<td>T = 1</td>
<td>1.27\frac{\lambda}{D}</td>
<td>-24.6 dB</td>
</tr>
</tbody>
</table>

Since A_E(r) tapers to 0.05 and A_H(r) tapers to 0.56 at the rim, from the above table one finds that the ratio of \hbar_E to \hbar_H is 1.25/1.09 = 1.15 (less than 1.2).

From Figure 11 or Table 6, it can be seen that the composite aperture distribution A_C(r) tapers to a value of 0.35 (9 dB) at the edge of the aperture (T = 0.65). Therefore, \hbar_C is given by 1.16 \lambda/D where \lambda is the operating wavelength and D is the aperture diameter (305 meters). Table 12 gives the theoretically-derived half-power beamwidths for the four operating frequencies.
TABLE 12. Cross-Plane Half-Power Beamwidth Calculated for 0° Zenith Angle from the Expression $\theta_X = 1.16 \frac{\lambda}{D}$

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>WAVELENGTH (meters)</th>
<th>$\frac{\lambda}{D}$</th>
<th>$\theta_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>13.45</td>
<td>0.0442</td>
<td>2.94°</td>
</tr>
<tr>
<td>26.70</td>
<td>11.23</td>
<td>0.0368</td>
<td>2.43°</td>
</tr>
<tr>
<td>33.45</td>
<td>8.97</td>
<td>0.0294</td>
<td>1.96°</td>
</tr>
<tr>
<td>38.75</td>
<td>7.75</td>
<td>0.0254</td>
<td>1.70°</td>
</tr>
</tbody>
</table>

The difference between the theoretically derived $\theta_X$ for $\zeta = 0^\circ$ calculated by $1.16 \frac{\lambda}{D}$ and the measured $\theta_X$ for $\zeta = 4.8^\circ$ is about 10 percent for the lower frequencies and 16 percent for 38.75 MHz. This difference can in no way be explained by the difference in zenith angles. The half-power beamwidths should be roughly equal for zenith angles of 0° and 5° and should not be much greater even for 10°. From a rough assessment of some records of Virgo A (in which, unfortunately, scintillation is present) over various zenith angles, it would appear that even for zenith angles approaching 20° the beamwidth does not increase by more than 10 percent.

To achieve more accurate theoretical calculations, we must now regard a point that we have not thus far considered. The reflector of the Arecibo radio telescope is a spherical cap and not a true parabolic surface. For the aperture out to a radius of 300 feet, the difference between the spherical reflector surface and a corresponding parabolic surface is very small (the deviation at 300-foot radius is less than one-half foot). For radii beyond 300 feet, however, the
deviation begins to increase. See Figure 18. The parabola drawn has an f/D ratio of 0.42, since the distance from the assumed center of focus of the log-periodic feed (at the center of its 30-foot mast) to the bottom of the bowl (vertex of the parabola) is 420 feet. The equation for this parabola is $r^2 = 4 f z$, where $r$ is the radius, $z$ is the vertical height and $f$ is the focal distance (420 feet). For $r = 400$ feet, the deviation is about 2 feet. At the edge of the reflector the deviation is about 9 feet (2.7 meters).

At higher frequencies, if one wants to make use of the full reflector aperture and not limit the aperture illumination to less than 600 feet in diameter, it is necessary to use a line source feed to correct for the spherical aberration. As wavelength is increased, the spherical-to-parabolic deviations (which have a fixed value in terms of linear dimensions) become a smaller fractional portion of the wavelength; hence, the phase deviation decreases for decreasing frequency. At lower frequencies one may use a "point source" feed and still illuminate the whole reflector, especially if the illumination is well tapered. The log-periodic feed is effectively a "point source" feed at any one frequency.

Let us examine the phase deviation of the spherical surface from a true parabolic surface for the aperture distribution at the rim of the reflector. Values are given in Table 13. The linear deviation $\Delta z$ in the vertical direction (we are considering only the $0^\circ$ zenith angle case) is taken as 2.7 meters. The phase angle is $2\pi(\Delta z/\lambda)$ radians or $360 (\Delta z/\lambda)$ degrees.
Spherical Equation
\[ z = P - (P^2 - r^2)^{1/2} \quad P = 870 \text{ FT.} \]

Parabolic Equation
\[ r^2 - 4fz \quad f = 420 \text{ FT.} \]

Figure 18. Geometry illustrating the deviation of the spherical reflector surface from a true parabolic surface as a function of radius.
TABLE 13. Phase Deviation as a Function of Frequency for a Linear Deviation of 2.7 Meters

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>(\lambda) (m)</th>
<th>(\Delta\ell/\lambda)</th>
<th>(\delta)</th>
<th>(\cos \delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>13.45</td>
<td>0.20</td>
<td>72°</td>
<td>0.31</td>
</tr>
<tr>
<td>26.70</td>
<td>11.23</td>
<td>0.24</td>
<td>86°</td>
<td>0.06</td>
</tr>
<tr>
<td>33.45</td>
<td>8.97</td>
<td>0.30</td>
<td>108°</td>
<td>-0.31</td>
</tr>
<tr>
<td>38.75</td>
<td>7.75</td>
<td>0.35</td>
<td>126°</td>
<td>-0.59</td>
</tr>
</tbody>
</table>

Except for the region near the rim, the aperture distribution is in phase. For the amplitude distribution (a complex quantity) over the whole aperture, the real part is enormously larger than the imaginary part; hence, the quadrature component out near the rim is of no significance, while the in-phase component should be included along with the other in-phase (real) portions of the aperture distribution. When the phase deviation is greater than 90° the real part is then negative; hence, the aperture distribution is "tapered" to a value less than zero at the rim, further widening the beam because of the negative aperture distribution.

To keep this problem within reasonable bounds of simplicity, we will assume that the aperture distribution is circularly symmetric and that its amplitude, exclusive of the phase deviation, is given by the composite amplitude distribution \(A_c(r)\). At the rim we will assume that the real part (the only significant part) of the distribution is given by \(A_c(r) \cos \delta\). Although it is obviously not exact, we will
simply extrapolate Table 11 to cover those frequencies for which the
taper $T$ becomes greater than one at the rim, i.e., where $A_C(r) \cos \delta$
becomes negative. Table 14 gives the improved formulas (in this case,
frequency dependent) for the half-power beamwidth $\hat{\theta}_X$ and the resultant
theoretical values of $\hat{\theta}_X$ for $0^\circ$ zenith angle, based on consideration
of the spherical-to-parabolic phase deviation for the outer portion of
the reflector surface.

**TABLE 14. In-Phase Aperture Distribution at the Rim Allowing
for Phase Deviation and the Corrected Theoretical
Half-Power Beamwidth**

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>$A_C(r) \cos \delta$ (r = 500 ft)</th>
<th>$\hat{\theta}_X$ (formula)</th>
<th>$\hat{\theta}_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>0.11</td>
<td>1.23 $\frac{\lambda}{D}$</td>
<td>3.13°</td>
</tr>
<tr>
<td>26.70</td>
<td>0.02</td>
<td>1.26 $\frac{\lambda}{D}$</td>
<td>2.60°</td>
</tr>
<tr>
<td>33.45</td>
<td>-0.11</td>
<td>1.30 $\frac{\lambda}{D}$</td>
<td>2.19°</td>
</tr>
<tr>
<td>38.75</td>
<td>-0.21</td>
<td>1.33 $\frac{\lambda}{D}$</td>
<td>1.94°</td>
</tr>
</tbody>
</table>

With this additional correction the theoretically calculated
values of $\hat{\theta}_X$ (for $0^\circ$ zenith angle) agree with the measured values of
$\hat{\theta}_X$ (at $4.8^\circ$ zenith angle) to within about 4 percent (see Table 15).
This is certainly well within the errors of the experimental measure-
ments and the extremely simplified theoretical calculations.
5. Effective Area

The determination of the effective area of the radio telescope as a function of zenith angle for the four operating frequencies was accomplished experimentally by tracking the calibration source, Taurus A, over a range of zenith angles and measuring the antenna temperature contribution of this source as a function of zenith angle.

Taurus A was chosen as the calibration source for several important reasons. Of the four strongest discrete radio sources, only Taurus A and Virgo A pass within the viewing range of the Arecibo radio telescope. The declination of Taurus A (+21°59') is closer to the latitude of A10 (18°21'N) than the declination of Virgo A (+12°40'). Therefore, the lowest zenith angle (the nearest approach at meridian transit) is 3°38' for Taurus A and 5°41' for Virgo A. Using Taurus A the effective area could be experimentally determined over the greatest range of zenith angles, 3.63° to 20°. Finally, the spectrum of Taurus A is one of the best defined of all the radio

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**TABLE 15. Comparison of the Cross-Plane Half-Power Beamwidths Measured at ζ = 4.8° and Calculated Theoretically for ζ = 0°**

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>MEASURED θ_X</th>
<th>THEORETICAL θ_X</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>3.30°</td>
<td>3.13°</td>
</tr>
<tr>
<td>26.70</td>
<td>2.71°</td>
<td>2.60°</td>
</tr>
<tr>
<td>33.45</td>
<td>2.28°</td>
<td>2.19°</td>
</tr>
<tr>
<td>38.75</td>
<td>2.02°</td>
<td>1.94°</td>
</tr>
</tbody>
</table>
sources, closely following a simple power law over a wide range of frequencies (38 to 10,000 MHz). Based on the work of Conway, Kellermann and Long (1963), Taurus A was assumed to have a flux density of $2300 \times 10^{-26}$ W m$^{-2}$ Hz$^{-1}$ at 38 MHz and a spectral index $\alpha$ of 0.27.

(In this paper the spectral index $\alpha$ will always be defined by the relationship $S \propto \lambda^\alpha \propto \nu^{-\alpha}$.) The Taurus flux densities assumed in the derivation of radio telescope effective area are given in Table 16.

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>FLUX DENSITY $(10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>2670</td>
</tr>
<tr>
<td>26.70</td>
<td>2550</td>
</tr>
<tr>
<td>33.45</td>
<td>2410</td>
</tr>
<tr>
<td>38.75</td>
<td>2300</td>
</tr>
</tbody>
</table>

The effective area $A_e$ was determined from the expression $A_e = \frac{2}{k} \frac{T_A}{S}$, where $k$ is Boltzmann's constant, $T_A$ is the antenna temperature of Taurus A (exclusive of any antenna temperature due to the background) and $S$ is the flux density of Taurus A. The antenna temperature in this expression is that incident on the feed; the corrections for cable and feed VSWR losses discussed in a previous section of this chapter must be applied.

Effective area calibration measurements were made by tracking Taurus A from 20° zenith angle to 3.6° (meridian transit) and then out to 20° again. To be able (in later analysis) to separate the antenna
It was necessary to obtain a measurement of antenna temperature due to the background surrounding Taurus A. At the beginning of the calibration run the radio telescope was held at 20° zenith angle while Taurus A drifted into the beam (tracking was begun when the source reached the center of the beam). At the end of the run the radio telescope was again held at 20° zenith angle while Taurus A drifted out of the beam. At each frequency, the antenna temperature (at 20° zenith angle) when Taurus A was in the center of the beam and when it was completely out of the beam was measured. The difference between the antenna temperature under these two conditions is the antenna temperature due to Taurus A alone. The portion of the antenna temperature remaining (that portion not due to Taurus A) is due to the sky background.

At the beginning of the calibration run, the beam (looking at the sky) drifted in from a region of lower right ascension than Taurus A; at the end the beam drifted out through a region of higher right ascension. The residual antenna temperature measured in the "drift-in" and "drift-out" cases could be somewhat different if the brightness temperature of the sky background were different on opposite sides of Taurus A. However, the residual antenna temperatures in both cases were not very different and the average value of the "drift-in" and "drift-out" residual measurements was taken as the antenna temperature due to the background.
The 20° zenith angle value of background-induced antenna temperature (one value for each frequency) was subtracted from the total antenna temperature values measured at all the various zenith angles, yielding $T_A$ due to Taurus A alone as a function of zenith angle. From these values of $T_A$, values for effective area were calculated from the expression $A_e = 2kT_A/S$. Initial curves of effective areas as a function of zenith angle were drawn; these are shown in Figure 19. These curves are somewhat in error; the needed corrections are discussed in the following paragraphs.

In looking over the initial effective area curves, it was noted that at low zenith angles the effective area was quite a bit higher for the lower frequencies. As a result of the analysis of feed efficiency and spillover, an explanation (or at least a partial explanation) for this low-frequency effective area enhancement at low zenith angles became apparent, based on the fact that the sky brightness temperature is so much greater at lower frequencies. The error (the apparent increase in effective area) came about because of an oversimplification of the value of antenna temperature due to the background. For each frequency one constant value, the 20° zenith angle value, was used for all zenith angles. However, because of the change in feed spillover with zenith angle, the antenna temperature obtained from the same sky background also changes as a function of zenith angle. For a uniform background region, we have:

$$T_A = (\eta_M + \eta_{SL}) T_b + \eta_{SP} T_{SP} \quad \text{with} \quad \eta_M + \eta_{SL} + \eta_{SP} = 1,$$
Figure 19. Initial curves of effective area without correction for change in the background-induced antenna temperature caused by the change of feed efficiency with zenith angle.
where \( T_b \) is the brightness temperature of the uniform region, \( \eta_M \) and \( \eta_{SL} \) are the main beam and side lobe efficiencies (for a uniform region both the main beam and the side lobes are assumed to see the same brightness temperature), and \( \eta_{SP} \) is the spillover. (A fuller development of beam efficiency is presented in the following section.) The temperature \( T_{SP} \) seen by the spillover term is that of the earth which is negligibly small compared to the sky temperature at these frequencies. Thus, we may write:

\[
T_A = (\eta_M + \eta_{SL}) T_b = (1 - \eta_{SP}) T_b.
\]

At low zenith angles \( 1 - \eta_{SP} \) is equal to 0.95, while at 20° zenith angle \( 1 - \eta_{SP} \) is equal to 0.82 (see Table 10). Hence, the antenna temperature contribution from the same background is 0.13 \( T_b \) greater at low zenith angles than at 20°. Since for each frequency only the constant value for 20° zenith angle was subtracted throughout (to obtain the value of \( T_A \) supposed due to Taurus A alone at the various zenith angles), additional subtraction will be required. This will amount to 0.13 \( T_b \) at the lower zenith angles and progressively less at higher angles — according to the difference between the value of \( (1 - \eta_{SP}) \) at any zenith angle and 0.82, the value of \( (1 - \eta_{SP}) \) at 20°. For a curve of this correction factor as a function of zenith angle, see Figure 20.

While the percentage correction is not a function of frequency, the numerical value of the correction factor (in terms of °K of antenna temperature) certainly is, since the brightness temperature is
\[ \frac{T_A}{T_b} = 1 - \eta_{SP}(\xi) = \varepsilon_f(\xi) \]

Figure 20. Correction factor to account for the greater antenna temperature contribution of the background at zenith angles below 20°. Also shown is the antenna to brightness temperature ratio for a uniform region as a function of zenith angle.
strongly frequency dependent. In particular, $T_b \propto \nu^{-\beta}$, where the brightness temperature index $\beta$ has a value in the order of 2.5 in the 20-40 MHz range. The brightness temperature for the region surrounding Taurus A is roughly 10,000°K at 38.75 MHz, 14,500°K at 33.45 MHz, 25,000°K at 26.70 MHz, and 40,000°K at 22.30 MHz. These values were extrapolated based on the temperatures shown for the region around Taurus A in surveys of galactic radio emission at higher frequencies by various observers (Turtle and Baldwin, 1962; Ko and Kraus, 1957; Seeger et al., 1965).

The additional background-induced antenna temperature for zenith angles lower than 20° was greater at the lower frequencies; hence, the error in effective area introduced by the improper simple subtraction was thereby greater for the lower frequencies. The correction applied will, therefore, be greater at the lower frequencies (proportional to $T_b$). One might go back and correct the antenna temperature at each experimental point. However, an easier way is to calculate an effective area correction $\delta A_e$ for each experimental point from the expression $\delta A_e = 2 k(\delta T_A)/S$, where $\delta T_A$ is the antenna temperature correction and $S$ is the assumed flux density for Taurus A for the particular operating frequency (see Table 16). After making this correction at each experimental point, the corrected effective area curves were drawn for each frequency. These are presented in Figure 21.

Even with the correction for the variation in the antenna temperature due to the background, the effective area for 38.75 MHz is lower than the effective area for 33.45 MHz, which is in turn lower than that
Figure 21. Corrected effective area curves for the four operating frequencies.
for 26.70 MHz. The lower effective area of the higher frequencies can be explained by the phase error effects resulting from the divergence of the spherical reflector from a true parabolic surface. (This topic was covered in the previous section of this chapter.) For the same linear deviation, the phase deviation (or phase error) introduced is proportionally greater for the shorter wavelengths (higher frequencies). Hence, the deterioration in effective area caused by phase error will be greater for the higher frequencies.

The effective area for 22.30 MHz should be larger or at least as large as the effective area for 26.70 MHz. However, the effective area curve for 22.30 MHz is a bit irregular and wanders between the 26.70 and 33.45 MHz curves. The problem at 22.30 MHz was reading accurately the values of antenna temperature on the Taurus A tracking records. Unfortunately, there was a large amount of radio station interference on the 22.30 MHz channel during the time of the calibration runs. The combination of radio interference and the scintillation of the source (greatest for the lowest frequency) made the 22.30 MHz portion of the $A_e$ calibration records difficult to read. For any zenith angle the effective area for 22.30 MHz may be in error by five to ten percent. However, over a wide range of zenith angles the average effective area error for 22.30 MHz should be less than five percent.

The aperture efficiency of the radio telescope is given by:

$$\eta_a = \frac{A_e}{A_p}.$$
where \( A_p \) is the physical (geometric) area of the collecting aperture. For the AIO radio telescope we have \( A_p = \frac{\pi}{4} \cdot 305^2 = 73,000 \text{ m}^2 \).

Curves of aperture efficiency as a function of zenith angle for the four operating frequencies are given in Figure 22. For low zenith angles the measured aperture efficiency ranges from 45 to 51 percent. For the higher zenith angles aperture efficiency decreases and at 20° is down by about 2.4 dB (see Table 17). By way of comparison, in planetary radar measurements at 430 MHz using the line source feed on Carriage House 1, Pettengill, Dyce, and Campbell (1967) found the reduction in one-way gain from zenith to 20° zenith angle to be about 2.4 dB.

**TABLE 17. Reduction in Aperture Efficiency from Low Zenith Angles to 20° Zenith Angle**

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>( n_{a}(\zeta = 20°)/n_{a}(\zeta = \text{low}) ) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>-2.5</td>
</tr>
<tr>
<td>26.70</td>
<td>-2.4</td>
</tr>
<tr>
<td>33.45</td>
<td>-2.4</td>
</tr>
<tr>
<td>38.75</td>
<td>-2.3</td>
</tr>
</tbody>
</table>

6. Main Beam Efficiency

Assuming the radiation efficiency equal to unity, the general expression for antenna temperature may be written as:

\[
T_A(\theta, \phi) = \frac{1}{\Omega_A} \int_{4\pi} F_n(\theta, \phi) T_b(\theta, \phi) \, d\Omega ,
\]
Figure 22. Aperture efficiency as a function of zenith angle for the four operating frequencies.
where $T_A(\theta, \phi)$ is the antenna temperature observed (corrected for any feed VSWR or cabling losses); $\Omega_A$ is the antenna solid angle, equal to 

$$\int_0^{4\pi} F_n(\theta, \phi) \, d\Omega,$$

the integral of the antenna pattern over the whole sphere; $F_n(\theta, \phi)$ is the normalized power pattern of the antenna; and $T_b(\theta, \phi)$ is the brightness temperature viewed by the antenna. The expression may be expanded to:

$$T_A(\theta, \phi) = \frac{1}{\Omega_A} \int_M F_n(\theta, \phi) \, T_b(\theta, \phi) \, d\Omega + \frac{1}{\Omega_A} \int_{4\pi-M} F_n(\theta, \phi) \, T_b(\theta, \phi) \, d\Omega,$$

where $M$ stands for the main beam and $4\pi-M$ for that portion outside the main beam (Ko, 1964).

Let us denote the average brightness temperature seen by the main beam as $\langle T_b \rangle$, which will be a function of the pointing direction of the main beam. Let us assume an average brightness temperature outside the main beam and denote it as $\langle T_{b-M} \rangle$. The expression for antenna temperature then becomes:

$$T_A(\theta, \phi) = \frac{\langle T_b \rangle}{\Omega_A} \int_M F_n(\theta, \phi) \, d\Omega + \frac{\langle T_{b-M} \rangle}{\Omega_A} \int_{4\pi-M} F_n(\theta, \phi) \, d\Omega,$$

$$= \frac{\Omega_M}{\Omega_A} \langle T_b \rangle + (1 - \frac{\Omega_M}{\Omega_A}) \langle T_{b-M} \rangle,$$

where $\Omega_M$ is the main solid angle, $\eta_M$ is the main beam efficiency and $(1 - \eta_M)$ is called the stray factor (Seeger et al., 1965).
The second term is not as simple as it looks, since distinctly different temperatures are seen by the portions of the antenna pattern outside the main beam. Consider the feed illuminating the 1000 foot reflector. We denote that portion of the power radiated by the feed which hits the reflector surface as $\varepsilon_f$, and that portion of it which misses the reflector surface as the spillover $n_{SP} = 1 - \varepsilon_f$. Also, we denote that portion of the power radiated by the feed which hits the reflecting surface, but does not go into the main beam by $n_{SL}$. Since $\varepsilon_f = n_M + n_{SL} + n_{SB}$, and $n_M + n_{SL} + n_{SP} = 1$, the expression for antenna temperature then becomes:

$$T_A(\theta, \phi) = n_M T_b(\theta, \phi) + n_{SL} \langle T_b \rangle_{SL} + n_{SP} \langle T_b \rangle_{SP}.$$  

The temperature $\langle T_b \rangle_{SL}$ associated with $n_{SL}$ is the brightness temperature of the sky in the general forward direction (roughly the forward hemisphere) of the radio telescope. Since the sidelobes nearest the main beam are stronger, the $n_{SL} \langle T_b \rangle_{SL}$ term will be more influenced by the sky temperature of the region within 10 beamwidths of the main beam's pointing direction.

Consider next the temperature seen by the spillover term. In the annulus between the rim of the dish and the horizon, the feed sees the earth, whose temperature ($\approx 290^\circ$) is very small compared to the sky. For angles from the horizon to about $5^\circ$ above, the feed sees sky temperature heavily attenuated by the long ionospheric path near the horizon. Beyond this, the feed would see sky temperature; however, with its high front-to-back ratio the feed's rear hemisphere radiation
contribution is negligible. Thus, for practical purposes, we may consider:

$$\eta_{sp} \langle T_b \rangle_{sp} \approx \eta_{sp} T_{\text{earth}} \approx 0,$$

since $T_{\text{earth}}$ is less than four percent of sky brightness temperature at 38.75 MHz and less than one percent at 22.30 MHz. The expression for antenna temperature may then be written as:

$$T_A(\theta,\phi) = n_M \frac{T_b}{\eta_{sc}} + n_{SL} \langle T_b \rangle_{SL},$$

where $n_M + n_{SL} = 1 - \eta_{sp}$.

If the main lobe and side lobes looked at sky having a uniform brightness temperature $T_b$, then $T_A$ would equal $(1 - \eta_{sp}) T_b$. If the brightness temperature is fairly small outside the main beam, one could write as an approximation $T_A(\theta,\phi) = n_M T_b(\theta,\phi)$; the antenna temperature would be related to brightness temperature by a simple linking parameter, the antenna main beam efficiency.

The most direct way to determine the main beam efficiency of an antenna is simply to map the complete pattern of the antenna down to at least $-30 \text{ dB}$ using a strong discrete radio source. By straightforward graphical analysis, the main beam solid angle and the antenna solid angle can be measured, yielding main beam efficiency. However, there are some qualifications. The flux density of the radio source must be sufficient that the antenna temperature due to the source is at least 30 dB greater than the antenna temperature due to the sky background. The half-intensity width of the source must be small (but not
necessarily negligibly small) compared to the half-power beamwidth of the antenna. Finally, the antenna must be sufficiently steerable relative to the source to be able to accomplish the mapping of the pattern. For the 25-meter Dwingeloo radio telescope, Seeger et al. (1965) using the sun (whose radio brightness temperature was greater than $10^4$ times the background sky temperature) as the radio source at 400 MHz mapped the complete (two-dimensional) antenna pattern down to $-33$ dB and from the complete pattern determined $\Omega_M$, $\Omega_A$, and $\eta_M$.

Operating with the Arecibo radio telescope at 20-40 MHz, the antenna temperature due to the strongest available discrete radio sources (Taurus A, Virgo A) is less than 10 dB greater than the sky background temperature at 20-40 MHz. In addition, there is the problem of scintillation. The antenna temperature due to the (quiet) sun is in the order of or less than the sky background temperature at 20-40 MHz. The Arecibo radio telescope cannot be steered beyond 20° zenith angle. Obviously, to find beam efficiency of the Arecibo radio telescope at 20-40 MHz some other, less direct technique had to be employed.

The antenna solid angle of the Arecibo radio telescope was determined from the expression:

$$\Omega_A = \frac{\lambda^2}{A_e^2}.$$  

The radiation efficiency is assumed equal to unity. The effective area of the radio telescope as a function of frequency and zenith angle is
available from Figure 21. The antenna solid angle at $0^\circ$, $5^\circ$, $10^\circ$, $15^\circ$, and $20^\circ$ zenith angles for the four operating frequencies was calculated. The values are presented in Table 18. The effective area for $0^\circ$ zenith angle is assumed to be equal to the effective area for $5^\circ$ zenith angle.

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>ANTENNA SOLID ANGLE (square degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta = 0^\circ$</td>
<td>$\zeta = 5^\circ$</td>
</tr>
<tr>
<td>22.30</td>
<td>16.6</td>
</tr>
<tr>
<td>26.70</td>
<td>11.2</td>
</tr>
<tr>
<td>33.45</td>
<td>7.6</td>
</tr>
<tr>
<td>38.75</td>
<td>6.06</td>
</tr>
</tbody>
</table>

The main beam solid angle of the Arecibo radio telescope was evaluated using the following expression:

$$\Omega_M = k_d \hat{\theta}_E \hat{\theta}_H = k_d \hat{\theta}_X^2,$$

where $k_d$ is a factor depending on the shape of the power pattern of the radio telescope (hence, on the aperture distribution), $\hat{\theta}_E$ and $\hat{\theta}_H$ are the half-power beamwidths in the E- and H-planes, and $\hat{\theta}_X$ is the half-power beamwidth in the cross-plane (the plane at $45^\circ$ to the E- and H-planes). It has been shown that, to a good approximation, $\hat{\theta}_X^2 = \hat{\theta}_E \hat{\theta}_H$.

Consider first the case of $0^\circ$ radio telescope zenith angle. For a beam with circular symmetry, i.e., $P(0,\phi) = P(0)$, the factor $k_d$ has
a value between 1.01 for a uniform distribution to 1.13 for a highly tapered distribution, as shown in Table 19 (Ko, 1964). Based on the aperture distribution a value for $k_d$ of 1.09 was chosen.

**TABLE 19. Expressions for Main Beam Solid Angle from Half-Power Beamwidth for Two Types of Circularly Symmetric Distribution**

<table>
<thead>
<tr>
<th>APERTURE DISTRIBUTION</th>
<th>BEAM POWER PATTERN</th>
<th>MAIN BEAM SOLID ANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>Gaussian $[e^{-\psi^2}]$</td>
<td>$\Omega_M = 1.133 \hat{\psi}^2$</td>
</tr>
<tr>
<td>Uniform</td>
<td>Bessel func$[2J_1(\psi)/\psi]^2$</td>
<td>$\Omega_M = 1.008 \hat{\psi}^2$</td>
</tr>
</tbody>
</table>

For increasing zenith angle the aperture illumination becomes more asymmetric and the antenna pattern deteriorates (gain decreases, sidelobes increase, etc.). Since effective area was determined from measurements over a whole range of zenith angles, $\Omega_A$ is known as a function of zenith angle. However, since the measurements of half-power beamwidth were limited, $\Omega_M$ is not precisely known as a function of zenith angle. The half-power beamwidth $\hat{\theta}_X$ was precisely measured only at zenith angle of 4.8° (using the non-scintillating source IC443). From a casual inspection of the records of the stronger sources, including many records in which scintillation was present, it appears that the half-power beamwidth increases slightly with increasing zenith angle. For the analysis here, we must obtain values of $\Omega_M$ at 0°, 5°, 10°, 15° and 20° to complement the values of $\Omega_A$ determined at these zenith angles (Table 18). The half-power beamwidths at 5° will be taken as the measured values of $\hat{\theta}_X$ (Table 15) for $\zeta = 4.8°$. 
We will assume (based on the inspection of the records) that the half-power beamwidths for all frequencies increase 2 percent for $\zeta = 10^\circ$, 5 percent for $\zeta = 15^\circ$, and 10 percent for $\zeta = 20^\circ$ above the $\zeta = 5^\circ$ values. The half-power beamwidths at $\zeta = 0^\circ$ will be assumed equal to the $\zeta = 5^\circ$ values.

Using the expression $\Omega_M = 1.09 \delta_X^2$, the main beam solid angles have been calculated for the four operating frequencies at the various zenith angles. The values are given in Table 20.

**TABLE 20. Main Beam Solid Angle as a Function of Zenith Angle and Frequency**

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>$\zeta = 0^\circ$</th>
<th>$\zeta = 5^\circ$</th>
<th>$\zeta = 10^\circ$</th>
<th>$\zeta = 15^\circ$</th>
<th>$\zeta = 20^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>11.90</td>
<td>11.90</td>
<td>12.40</td>
<td>13.15</td>
<td>14.35</td>
</tr>
<tr>
<td>26.70</td>
<td>8.02</td>
<td>8.02</td>
<td>8.30</td>
<td>8.80</td>
<td>9.68</td>
</tr>
<tr>
<td>33.45</td>
<td>5.66</td>
<td>5.66</td>
<td>5.86</td>
<td>6.21</td>
<td>6.86</td>
</tr>
<tr>
<td>38.75</td>
<td>4.46</td>
<td>4.46</td>
<td>4.62</td>
<td>4.89</td>
<td>5.37</td>
</tr>
</tbody>
</table>

The main beam efficiencies have been calculated from the values of $\Omega_M$ in Table 20 and $\Omega_A$ in Table 18 and are shown in Table 21. The accuracy of these values of $\eta_M$ obviously depend upon the accuracy of the calculations of $\Omega_M$ and $\Omega_A$. The accuracy of $\Omega_M$, which depends upon the measurement of $\delta_X$ at low zenith angles (and the assumption that $\delta_X$ increases slightly with zenith angle), should be about 3 percent. The accuracy of $\Omega_A$, which depends directly upon the accuracy of the $\delta_X$ measurements, should also be about 3 percent.
TABLE 21. Main Beam Efficiency as a Function of Zenith Angle and Frequency

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>$\zeta = 0^\circ$</th>
<th>$\zeta = 5^\circ$</th>
<th>$\zeta = 10^\circ$</th>
<th>$\zeta = 15^\circ$</th>
<th>$\zeta = 20^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>72</td>
<td>72</td>
<td>69</td>
<td>64</td>
<td>51</td>
</tr>
<tr>
<td>26.70</td>
<td>73</td>
<td>73</td>
<td>70</td>
<td>65</td>
<td>51</td>
</tr>
<tr>
<td>33.45</td>
<td>75</td>
<td>75</td>
<td>71</td>
<td>64</td>
<td>55</td>
</tr>
<tr>
<td>38.75</td>
<td>74</td>
<td>74</td>
<td>70</td>
<td>63</td>
<td>54</td>
</tr>
</tbody>
</table>

From Table 21 the dependency of the main beam efficiency on zenith angle is clearly established, varying in a way that would be expected for a fixed reflector-moving feed system (main beam efficiency decreases for increasing zenith angle). From the values obtained in the numerical calculations (Table 21) it can be seen that there is some dependence on frequency, but it is smaller and without significant functional relationship. In the analysis involved in the establishing of the temperature scale for non-uniform regions, it will be much simpler to use a main beam efficiency dependent on zenith angle only. Therefore, for the various zenith angles a representative value of $\eta_M$ (the average value for the four frequencies at each zenith angle) will be used, together with an error bound (see Table 22).

A curve of the main beam efficiency as a function of zenith angle is presented in Figure 23. Included for comparison is a curve of aperture efficiency. The aperture efficiency curve represents the average of the curves for the four operating frequencies (shown in Figure 22).
TABLE 22. Average Main Beam Efficiency as a Function of Zenith Angle

<table>
<thead>
<tr>
<th>ZENITH ANGLE</th>
<th>MAIN BEAM EFFICIENCY (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>74 ± 2</td>
</tr>
<tr>
<td>5°</td>
<td>74 ± 2</td>
</tr>
<tr>
<td>10°</td>
<td>70 ± 2</td>
</tr>
<tr>
<td>15°</td>
<td>64 ± 2</td>
</tr>
<tr>
<td>20°</td>
<td>53 ± 2</td>
</tr>
</tbody>
</table>
Figure 23. Main beam efficiency $\eta_M$ and aperture efficiency $\eta_a$ as a function of zenith angle.
A. DISCRETE SOURCE OBSERVATIONS

1. October-November 1964 Observations

The first part of the author's investigation of discrete radio sources at the Arecibo Ionospheric Observatory was accomplished during the months of October and November 1964. The receiving and recording equipment (shipped to Ramey Air Force Base, Puerto Rico) was picked up by the author and brought to AIO several days before the start of the observations. The 20-40 MHz log-periodic feed was already at AIO; it had been brought there in July 1964 for use in the on-site radio interference survey.

The log-periodic feed was assembled at the bottom of the bowl and hoisted up to Carriage House 2 through an open area in the reflector mesh. See Figure 24. Both orthogonally-polarized dipole sets of the feed were installed. In addition to the author's observations, the log-periodic feed was used during this same October-November 1964 period for a program conducted by a visiting scientific group from the High Altitude Observatory, Boulder, Colorado. This program, which was concerned with the polarization of the fine structure of Jupiter radio bursts (22-36 MHz, swept frequency), required the use of both dipole
Figure 24. Assembling of the 20–40 MHz log-periodic feed at the bottom of the bowl under the reflector mesh.
sets to obtain the right-handed and left-handed circular polarization needed to investigate this Jupiter radiation. Since no polarization work was involved in the discrete source flux density measurements, the author simply used one of the dipole sets.

In the October-November 1964 period, the time that Jupiter was in the limited viewing range of the A10 radio telescope was also at night; hence, the Jupiter observations competed directly with the discrete source program for the available observing time at night (midnight to 0500 local time). In addition to these radio astronomy programs, there were also competing ionospheric and planetary radar programs. For the October portion of the period, the Jupiter program was assigned the prime night time available to radio astronomy, while the discrete source program was assigned various pieces of observing time around the Jupiter program.

Beginning in November, the 20-40 MHz discrete source program received a greater share of the observing time in the 2300 to 0500 local time period. In general, satisfactory results were obtained over the next two weeks. On at least one night, however, the discrete source observations were disturbed in part by strong Jupiter burst radiation!

During the third week of November the author completed the portion of the discrete source measurements, using the last three observing nights to obtain the data for the determination of the telescope's effective area as a function of zenith angle and operating
frequency. These measurements were important — a mistake in an obser-
vation of a discrete source would cause problems only for that parti-
cular source; however, a mistake in the effective area calibration
could effect the flux density determinations of all the sources. The
effective area calibration runs were, therefore, put off until the
end of the October-November period — until experience with the radio
telescope and the 20-40 MHz instrumentation had been obtained and many
little troublesome problems had been cleared up.

In the calibration runs for effective area determination, the
source Taurus A was tracked from 20° zenith angle to 3.6° and then out
to 20° again. To separate the antenna temperature contributed by
Taurus A from that contributed by the background (at 20° zenith angle),
Taurus A was allowed to "drift in" before tracking was begun and to
"drift out" of the telescope beam after tracking was ended. Additional
corrections based on other calculated telescope parameters had to be
made to account for the change in background-induced antenna tempera-
ture with zenith angle before the final effective area curves could be
drawn. (For a more detailed discussion, see Section B 5 of Chapter II.)

Instead of simply tracking Taurus A, one could have scanned back
and forth across the source many times during the calibration run, and
measured the difference between the peak antenna temperature when
Taurus A was in the center of the beam (noting the zenith angle for
each peak) and the antenna temperature when Taurus A was completely
out of the beam immediately on either side of each peak. The antenna
temperature contribution of Taurus A at the zenith angles of the
various peaks could then have been extracted directly for each scan across Taurus A, without need for any later adjusted-subtraction of the background antenna temperature. However, with the large telescope beamwidth at these low frequencies, it was necessary to scan well away from Taurus A to get the source out of the main beam. During the October-November 1964 period, the mechanical condition of the radio telescope was such that the maximum scanning rate was about 0.5° per minute. If scanning back and forth across Taurus A had been employed instead of straight tracking, only about eight zenith angle data points could have been obtained on one night's calibration run. With the higher zenith angles there is also the problem that one cannot scan too far off a source in certain directions when the source is at a zenith angle of 15° or greater without exceeding the telescope limit. (On the Arecibo radio telescope, when a limit was overridden, triggering the limit switches, these could not be reset by the observer, and operation of the telescope was usually ended for the night.) Because of these problems with regard to telescope operation, it was decided simply to track Taurus A over the zenith angle range for the effective area calibration runs.

To improve the accuracy in separating the antenna temperature contribution of a discrete source from that of the background, both "right-ascension cuts" and "declination cuts" were used on many sources instead of, or in addition to, the usual "drift" observations. For the October-November 1964 and April 1965 periods the fastest declination scan that could be used was about 30° per hour and,
correspondingly, the fastest right-ascension scan rate was about $2^\text{h}$ per hour. If one tried to use a much faster rate, it was found that in certain instances either the azimuth drive or the carriage house (zenith angle) drive would lag behind in achieving the position commanded by the computer.

In making a right-ascension or declination cut of a source, the author knew (from the programmed scan rates he set in the computer) the time of the peak (the passage of the source through the center of the telescope beam) and always marked down on the recording chart the value of the zenith angle (taken from the computer display) at the time of the peak.

Examples of records of right-ascension and declination cuts are shown in Figures 25, 26, and 27. The sources 3C47 and 3C123 have angular diameters of 1.5 arc-minutes or less, while the source IC443 (3C157) has an angular diameter of 29 arc-minutes. Note the scintillation of the sources 3C47 and 3C123 (Figures 25 and 26) and the lack of scintillation for 3C157 (Figure 27). In all the 20-40 MHz discrete source records this phenomenon was apparent; sources of small angular diameter (less than 5 arc-minutes) scintillate while sources of large angular diameter (greater than 15 arc-minutes) do not. This experimentally observed relationship between ionospheric scintillation and radio source angular diameter and its application in the determination of the size of nighttime ionospheric irregularities has been the subject of a paper by Aarons and Guidice (1966).
RA RATE = +2 HR/HR  
TIME OF PEAK: 2310 AST

ZENITH ANGLE AT PEAK = 3.44°

NOV. 5, 1964  
2300 to 2320 AST

Figure 25. Right ascension cut of the source 3C47.
Figure 26. Declination cut of the source 3C 123.
DEC. RATE = 30°/HR  TIME OF PEAK 0331 AST
ZENITH ANGLE AT PEAK = 4.83°

NOV. 6, 1964  0321 to 0341 AST

38.75 MHz
33.75 MHz
26.70 MHz
22.30 MHz

Figure 27. Declination cut of the source IC 443 (3C 157).
The author's observations in the October-November 1964 period were concentrated on the twelve radio sources shown in Table 23. Among the extragalactic radio sources observed were quasi-stellar sources (quasars) and sources associated with peculiar galaxies — double radio sources and core-and-halo sources. (Peculiar galaxies are galaxies having strong radio emission, in contrast to normal galaxies which are weak radio emitters.) Among the galactic sources observed were supernova remnants and thermal sources (ionized hydrogen nebulae). In the notation of Figure 23, QSS stands for quasi-stellar source, DS for double source, C&H for core-and-halo source, SNR for supernova remnant and T for thermal source. A few observations of sources other than those listed in Table 23 were made, but these observations yielded no significant results. The records taken permitted the determination of the flux densities for six of the twelve sources concentrated on in the observations.
<table>
<thead>
<tr>
<th>SOURCE</th>
<th>RIGHT ASCENSION</th>
<th>DECLINATION</th>
<th>TYPE</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C33</td>
<td>01h06.2m</td>
<td>+13°03'</td>
<td>DS</td>
<td>Measured</td>
</tr>
<tr>
<td>3C47</td>
<td>01h33.6m</td>
<td>+20°42'</td>
<td>QSS, C&amp;H</td>
<td>Measured</td>
</tr>
<tr>
<td>3C48</td>
<td>01h34.8m</td>
<td>+32°54'</td>
<td>QSS</td>
<td></td>
</tr>
<tr>
<td>3C75</td>
<td>02h55.0m</td>
<td>+5°32'</td>
<td>DS</td>
<td></td>
</tr>
<tr>
<td>3C79</td>
<td>03h07.2m</td>
<td>+16°55'</td>
<td>DS</td>
<td></td>
</tr>
<tr>
<td>3C111</td>
<td>03h56.0m</td>
<td>+37°53'</td>
<td>DS</td>
<td></td>
</tr>
<tr>
<td>3C123</td>
<td>04h33.9m</td>
<td>+29°34'</td>
<td>DS</td>
<td>Measured</td>
</tr>
<tr>
<td>3C134</td>
<td>05h01.3m</td>
<td>+38°02'</td>
<td>DS</td>
<td>Measured</td>
</tr>
<tr>
<td>Taurus A</td>
<td>05h31.5m</td>
<td>+21°59'</td>
<td>SNR</td>
<td>Measured</td>
</tr>
<tr>
<td>IC443</td>
<td>06h14.1m</td>
<td>+22°34'</td>
<td>SNR</td>
<td>Measured</td>
</tr>
<tr>
<td>Rosette Nebula</td>
<td>06h29m</td>
<td>+5°12'</td>
<td>T</td>
<td></td>
</tr>
</tbody>
</table>

Of the six sources measured, three (3C33, 3C98, and 3C123) are double radio sources, two (IC443, Taurus A) are supernova remnants, and one (3C47) is a core-and-halo source. The source 3C47 is also known to be a quasi-stellar radio source; however, the contribution of the quasi-stellar portion is not significant at these low frequencies. The observations taken on Taurus A as a discrete source obviously do not tell us anything about its flux density; however, they are useful in confirming the consistency of the effective area calibration technique.
The observations used to determine effective area were made by tracking Taurus A as the zenith angle continuously changed. For the observations used to calculate flux density, the modes of operation were right ascension and declination cuts (mainly) and some drifts. A question might arise on whether the effective area determination is consistent with the discrete source measurements. To check the consistency experimentally, the author took right ascension and declination cuts on Taurus A in exactly the same way as he did for the other discrete sources. Taking ten discrete source flux density measurements of Taurus A having a representative spread of zenith angles the author compared the flux density values measured at each frequency with the assumed Taurus A values used in obtaining the effective area curves. The rms. difference was less than four percent, thereby demonstrating the consistency between the effective area calibration technique and the discrete source observations.

For six sources in Table 23, the author was unable to obtain positive results. The quasi-stellar source 3C48, although moderately strong at meter and centimeter wavelengths, has a spectrum which curves down sharply at dekameter wavelengths; its flux density in the 20-40 MHz range was below the limits of the measurements. The sources 3C75 and 3C79 were known to be rather weak; measurement of their flux densities was expected to be marginal. In the case of 3C79, some results were obtained (for low zenith angles), but not enough to publish any flux density values. The sources 3C111 and 3C134 are moderately strong, but both have zenith angles greater than 19.5° at closest approach.
(meridian transit); hence, only a few drifts could be taken on them.
The stronger source, 3C134, is near the galactic plane ($b_{II} = -2^\circ$ for the source) and appeared to be confusion limited on the records. The Rosette Nebula (3C163) is a thermal source of large extent; its brightness temperature in the 20–40 MHz range is in the order of the sky background temperature, making it a poor source to try to detect.

2. April 1965 Observations

In April 1965, the author returned to AIO for the second part of his discrete source measurement program. The difficulties this time were quite different from those encountered in the October-November 1964 observing period. The competition among programs for available radio telescope time at night was minimal; the radio telescope was available for discrete source observations almost every night during the two week period that the log-periodic feed was mounted under Carriage House 2. The problems this time were mechanical difficulties with the radio telescope and radio interference.

On many of the nights assigned to the discrete source program, repetitive scanning operations were made impossible because of some mechanical malfunction of the radio telescope. During the nights that the telescope would operate, the operation of the planned program (scanning across the various sources) was always hampered by the sluggish "uphill" motion of CH2. In fairness to the staff of the Ionospheric Observatory, the conditions prevailing at the site during this period must be noted. In response to information that certain Puerto
Rican extremist elements might attempt to sabotage the radio telescope, extraordinary security precautions were put into effect to guard key points of the observatory. The AIO personnel normally assigned to the maintenance and repair of the radio telescope were required to share in the guard duty; thus severely limiting the amount of radio telescope repair work that could be accomplished during this period. The conditions in and around the control room at night during observations resembled those of a military headquarters in a war zone. The April 1965 observing sessions were a unique experience for the author!

In addition, there was the severe problem of radio interference. The operation of citizen's band walkie-talkies (27 MHz) by the guards around the site during this maximum security period produced prominent interference on the records, particularly on the 26.70 MHz operating frequency. However, this interference occurred only over short, well-defined periods and was readily traceable to this cause. Another type of interference caused a more serious disturbance. Spread throughout the records taken during April 1965 were periods of massive interference; sometimes on only one or two frequencies, other times on all frequencies. At the time when the April 1965 data was being analyzed, the author assumed that this interference was caused by radio broadcasting stations or by severe distant thunderstorms. (Such interference effects were much more prominent in the daytime and, combined with the uncertainty in daytime ionospheric absorption, precluded reliable daytime measurements for frequencies below 40 MHz.) However, it is possible that some of the interference on the April 1965 records
may have come from sparking across insulators on the high voltage line near the site. This power-line radiation was found to be directly responsible for the interference which so severely limited the author's March 1966 Galactic Spur observations.

The author's observations in the April 1965 period were concentrated on the nine radio sources shown in Table 24. The records taken permitted the determination of the flux densities for three radio sources. Figure 28 shows a declination cut of Virgo A (3C274). Virgo A is a core-and-halo source, while Hercules A and 3C353 are double radio sources. The records from two of the other sources yielded antenna temperatures strong enough to be measured, but these sources lie in regions of strong background variation. Separation from the confusing background becomes a rather subjective process; therefore, flux density determinations were not made for these sources. Although the flux densities of only three sources were determined, the three are very important sources, likely to be used for comparison (or calibration) in other dekameter surveys.

Of the six radio sources whose flux density was not determined, the sources 3C227, 3C234, and 3C327 were known to be weak, and measurements were thus expected to be marginal. In the case of 3C310, some results were obtained, but there were not enough good records to determine flux density with sufficient accuracy and reliability to justify publication. The strong quasi-stellar source 3C273 was confusion-limited; it could not be separated from the surrounding background, part of which is associated with the Galactic Spur.
DEC. RATE = -30°/HR.  
TIME OF PEAK 2348 AST  
ZENITH ANGLE AT PEAK = 16.40°  
APRIL 24, 1965  
2338 to 2358 AST  

Figure 28. Declination cut of the source Virgo A (3C 274).
### TABLE 24. Discrete Radio Sources Whose Measurement Was Attempted in the April 1965 Period

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>RIGHT ASCENSION</th>
<th>DECLINATION</th>
<th>TYPE</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C227</td>
<td>09h45.2m</td>
<td>+ 7° 39'</td>
<td>DS</td>
<td></td>
</tr>
<tr>
<td>3C234</td>
<td>09h59.0m</td>
<td>+29° 01'</td>
<td>DS</td>
<td></td>
</tr>
<tr>
<td>3C273</td>
<td>12h26.6m</td>
<td>+ 2° 21'</td>
<td>QSS</td>
<td></td>
</tr>
<tr>
<td>Virgo A</td>
<td>12h28.3m</td>
<td>+12° 40'</td>
<td>C&amp;H</td>
<td>Measured</td>
</tr>
<tr>
<td>3C310</td>
<td>15h02.8m</td>
<td>+26° 12'</td>
<td>DS</td>
<td></td>
</tr>
<tr>
<td>3C327</td>
<td>16h00.3m</td>
<td>+ 2° 03'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Herc A</td>
<td>16h48.7m</td>
<td>+ 5° 05'</td>
<td>DS</td>
<td>Measured</td>
</tr>
<tr>
<td>3C353</td>
<td>17h17.9m</td>
<td>- 0° 56'</td>
<td>DS</td>
<td>Measured</td>
</tr>
<tr>
<td>3C392</td>
<td>18h53.6m</td>
<td>+ 1° 14'</td>
<td>SNR</td>
<td></td>
</tr>
</tbody>
</table>

The very strong supernova remnant 3C392 was also observed. The records for this source, which has an angular diameter of 16 arcminutes (Bennett, 1962), show no scintillation, quite similar in this respect to the records for the source IC443. Figure 29 shows a drift through 3C392 with the telescope at 20° zenith angle. (On the night this record was taken, CH2’s pump-actuator connector arm locked in the "uphill" driving position and CH2 ran into the 20° zenith angle limit, so only 20° zenith angle drifts could be taken.) Unfortunately, 3C392 is right on the galactic plane ($b^{II} = 0°$) near a region of strong galactic radiation. Even though the source is very strong, there is
Figure 29. Drift through the source 3C 392 (W44).
no way with a telescope beamwidth of two to three degrees to separate
the antenna temperature contribution of 3C392 from the antenna tem­
perature contribution of the galactic plane at this galactic longi­
tude ($\ell^I = 35^\circ$).

It was originally the plan of the author to accomplish the third
and final part of the 20–40 MHz discrete source investigation at the
Arecibo Ionospheric Observatory in July 1965. However, this could
not be arranged, so the April 1965 observations concluded the author's
discrete source measurements.

Because of the resolution and declination limitations imposed on
the measurements by the telescope beamwidth and zenith angle limit,
emphasis has been placed on accuracy and reliability rather than on
the number of sources. The author has attempted to put out, at
several frequencies in the 20–40 MHz range, reliable measurements of
the flux densities of some of the stronger radio sources that may be
used with confidence by other observers.

In the author's measurement work there are no determinations of
flux density based on one small bump in the midst of large background
variations or radio interference on a single day-long record. (All
determinations are based on at least five different measurements at
each operating frequency; most sources measured have at least eight.)
If a source was definitely observed but could not be clearly separated
from the background, flux density values are not quoted. It is felt
that these careful multi-frequency measurements on a small number of sources represents a definite contribution to the study of low-frequency radio source emission.
B. GALACTIC SPUR OBSERVATIONS

1. March 1966 Observations

The author's first attempt to obtain observations of the Galactic Spur region at the Arecibo Ionospheric Observatory was made in March 1966. The instrumentation used in these observations was roughly the same as that used for the discrete source investigation. The same four frequencies in the 20-40 MHz range were used. In addition, an attempt was made to obtain observations of the Spur at 10 MHz.

The log-periodic feed mast was mounted under Carriage House 2 in the same way as it was for the discrete source observations. However, only one set of log-periodic dipoles was installed. In the plane orthogonal to the 20-40 MHz log-periodic configuration, a 10 MHz two-element Yagi antenna (consisting of a folded dipole driven element and a linear dipole reflector) was installed on the 30-foot mast. The long, somewhat drooping dipoles of the 10 MHz feed were supported by nylon ropes under tension. (The linear dipole of the reflector element was almost 50 feet long.) The 10 MHz portion of the feed was electrically independent of the 20-40 MHz log-periodic part. This composite feed was mounted to CH2 in such a way that 20-40 MHz log-periodic elements and the 10 MHz elements extended out from the corners of the carriage house. See Figure 30.

Because of interfering radio stations which would not stay out of the guard band at 10,000 MHz, the 10 MHz observations were found to be of little value. Realizing this after a few nights of observation,
the author agreed to remove the 10 MHz portion of the composite feed, which then permitted AIO radio astronomers to install small high-frequency feeds on the 1.5° offset position on Carriage House 2. The Galactic Spur observations were then continued using only the one-plane 20-40 MHz log-periodic feed.

Successful ground-based observations of galactic radio emission have been made by astronomers at, and even considerably below, 10 MHz. The antennas used were large arrays of dipoles made up of wire strung between wooden poles. The site location had to be particularly isolated from interference. Special circuitry in the receiver for band-sweeping and recording of minimum level was required to fight interference. Observations had to be confined to nighttime minimum-absorption period and to sunspot minimum years. For frequencies well below 10 MHz, a site having especially favorable ionospheric conditions had to be used. From observations at Tasmania, Australia taken mainly during 1963-1964, surveys of the galactic emission were made by Ellis and Hamilton (1966) at 4.7 MHz and by Reber (1968) at 2.1 MHz.

In the 20-40 MHz observations of March 1966, the region mapped was between 12h to 16h right ascension and +2° to +32° declination. (The range of declination at AIO over which it is possible to observe is -1.5° to +38.5°, set by the 20° zenith angle limit of the telescope.) To avoid the problems caused by ionospheric absorption and radio station interference in the 20-40 MHz frequency range, it was necessary to make observations during the midnight to 0600 local time period. This limitation confined the observation of the 12h to 16h right
ascension region of the sky at Arecibo to roughly the month of March.

In the region of the observations the brightness temperature contours of the Galactic Spur run roughly east-west. To make the scans perpendicular to the expected contours, it was decided to map the Spur region by means of a series of constant right-ascension scans from +32° to +2° or +2° to +32° declination, each successive scan reversed in direction. Since the beamwidth of the radio telescope was roughly 2° at the highest observational frequency and roughly 3.3° at the lowest, an overall scan spacing of 1.5° was used for the map. To cover the 60° of right ascension (12h to 16h) with a scan spacing of 1.5°, forty scans were needed. One night’s observations consisted of ten scans (declination cuts at a rate of 1° per minute) spaced uniformly over the Spur region with 6° (24min of right ascension) separation between successive scans. Sets of scans from four nights of observation had to be taken, each with uniform 24min scan spacing but each displaced 6min in right ascension from every other night’s scan. The four sets of scans would then be interlaced to form the complete map. To prevent erroneous mapping of any short-term radio interference as a real brightness temperature feature, each of the four different sets of scans would be observed redundantly. Thus, eight good nights of observations were required for the overall program.

Since the constant right ascension scans from +32° to +2° (or from +2° to +32°) were taken at a 1° per minute declination rate starting each of the scans on an integral minute, each minute time mark on the record corresponds directly to an integral degree of declination.
On a large piece of grid paper the antenna temperature at each degree of declination was simply marked down along the lines of constant right ascension. Contours connecting points of equal antenna temperature could then be drawn.

Unfortunately, because of broadband power-line interference, the results of the March 1966 observations of the Galactic Spur at AIO were rather limited. Since interference from power lines can often become, if not corrected, the limiting factor in the reception of low-level radio signals, it is perhaps worth-while to describe the author's experience with this problem at AIO. Corrective measures applicable to the type of power-line interference experienced at AIO have been described by Hoglund and Sullivan (1965).

The problem was caused by sparking across old, corroded insulators on the poles (located on the tops of nearby hills) of a high voltage transmission line that passes within two miles of the site. Because of the height of these poles above the radio telescope, the r.f. interference from the arcing across the insulators radiated line-of-sight into the log-periodic feed suspended over the 1000-foot spherical reflector. Unless this insulator arcing was suppressed in some way, the broadband power-line interference made observations below 200 MHz virtually impossible at AIO. As far as the March 1966 observations were concerned, the best way seemed to be heavy rainfall! During heavy rain, the water on the corroded insulator strings would conduct and bypass the intermittent metallic connection and thus stop the sparking; hence, the absence of radiating interference. For example, during the
night of March 11-12, it rained heavily throughout the observations and the records were perfect, remarkably free of interference.

On the next few observing nights the weather was clear, and power-line interference ruined the records. About March 15 some new "bonded" insulators with radio-frequency interference suppression supplied by AIO were installed by the power company on several of the power-line poles nearest the site. This replacement with the new bonded insulators resulted in the reduction of the radiated interference to a more tolerable level, making the records taken after March 15 marginally usable. Unfortunately, there were no other long periods of heavy rainfall during the observations taken after March 15, and the records from those observations were of poor quality and reliability. Later in 1966 (after the author's visit), all the insulators on all the poles within several miles of the site were replaced by these new bonded insulators; this measure successfully cleared up the power-line interference problem at AIO.

In processing the data for making contour maps, at first only the one night's data (March 11-12) unspoiled by the broadband power-line interference was used. Two of the records from the March 11-12 observations, scans in declination (between +2° and +32°) at constant right ascensions of 13h06m and 14h48m are shown in Figures 31 and 32. Later, some additional data points from a few of the nights' observations damaged by the interference were used, but only in a relative way, to fill in some of the detail missing between the points of the March 11-12 observations. This permitted the author to obtain information on
RASTER "A", RUN 4, R.A.: 13 h 12 m
11-12 MARCH 1966
DEC.: +2° to +32°
TIME: 0151 - 0221 AST
DEC. RATE: +1°/MIN.

Figure 31. Declination scan along 13 h 12 m right ascension on the night of 11-12 March 1966.
The heavy rain occurring this night effectively silenced the broadband power-line interference.
In the scan note that integral minutes of time correspond to integral degrees of declination.
Figure 32. Declination scan along 14h 48m right ascension on the night of 11-12 March 1966. As in Run #4, the continuing heavy rain silenced the power-line interference. In the scan note that integral minutes of time correspond to integral degrees of declination.
certain portions of the map that would have otherwise been too incomplete to be useful.

The contour maps of the Galactic Spur based primarily on the one perfect night's data turned out remarkably well, with consistent results for all four frequencies. One problem, however, was that east-west detail was lost; it is as if an antenna having a $2^\circ$ or $3^\circ$ beamwidth in the north-south direction and a $12^\circ$ beamwidth in the east-west direction were used. This does not make the contour maps invalid; it means only that the resolution along the contours (which run mainly east-west) is poor. Fortunately, to investigate the ridge structure of the Spur requires resolution across the contour ridges, and this was not lost. From his analysis of these contour maps, the author was able to obtain significant results. Evidence of shell structure in the Galactic Spur — important support for the supernova remnant theory of the Spur's origin — was found (Guidice, 1967a).

Because of the great dependency of his results on so little data (basically, only one night's observations) the author, after learning from the AIO staff of their success in clearing up the power-line interference, decided to return to AIO to complete his 20-40 MHz investigation of the Galactic Spur. For the 1967 observations it was decided to extend the region under investigation to $18^h$ right ascension (the bounds in March 1966 were $12^h - 16^h$). To avoid the problems of ionospheric absorption and radio station interference in the 20-40 MHz range, it was again necessary to restrict observations to the midnight-0600 local time period. As a result of this requirement, the
1967 observations were confined to the March-April period.

2. March-April 1967 Observations

The 1967 observations at the Arecibo Ionospheric Observatory were carried out over the four week interval from March 21 to April 16. The author's 20-40 MHz Galactic Spur program was assigned observing time roughly two-thirds of the nights during this period. In general, satisfactory data was collected on these nights over the midnight to 0600 local time period, although radio station interference on the lower operating frequencies was often a problem in the early part of the night (before 0130 local standard time). In a few instances, there were problems with the 20-40 MHz instrumentation (the 26.7 MHz radiometer and the eight-channel recorder) or with the radio telescope operation; but, in general, the observational program was carried out smoothly.

The instrumentation for the March-April 1967 observations was basically the same as for the March 1966 program. Only one set of 20-40 MHz log-periodic dipoles was installed on the mast of the feed. The dipoles in the orthonogonal plane were simply left off. Unlike 1966, no attempt was made to set up any antenna system for a different frequency range in the orthonogonal plane of polarization.

As the nucleus of his mapping plan, the author decided to use the same basic mapping arrangement that had been used in the 1966 observations — sets of ten precisely displaced declination cuts, each set taken on a different night, to cover the declination range $+2^\circ$ to $+32^\circ$. 
Since the survey plan called for a declination range of 0° to +35°, several additional nights would be used to take drifts covering the areas between 0° to +2° and +32° to +35°.

The rasters (the ten-scan sets) from the four nights' observations, each displaced 6 m in right ascension from the other nights' rasters, are interlaced to form a map (with 1.5° east-west resolution) covering 4 h of right ascension. However, the right ascension range to be covered in the survey extended from 12 h to 18 h. Hence, two of these 4 h maps were required to cover the 12 h to 18 h range of right ascension and provide some overlap, which is necessary to check the consistency of the survey.

One choice for mapping coverage would be one four raster map covering 12 h to 16 h and another covering 16 h to 18 h, giving 2 h of overlap. However, the previous year's observations show that there is little contour information pertaining to the Spur in the 12 h to 13 h right ascension range. What little there is in this range (the area $\alpha = 12^h$ to $13^h$, $\delta = 0^\circ$ to $+10^\circ$) would be best mapped with right ascension cuts rather than declination cuts, since the contours in this area run north-south rather than east-west.

The author chose to take one group of observations for a four raster map covering 13 h to 17 h and another group for a four raster map covering 14 h to 18 h. The 3 h overlap proved to be most beneficial since, with the radio interference at the lower frequencies, as much duplicate coverage as possible was needed. The areas of interest in the 12 h-13 h range were covered by right ascension scans in the early
portion of the night before the main mapping program for that night was undertaken.

To facilitate the comparison of declination scans within a group, a notation system was set up for the rasters. (A raster is defined here as a set of ten +2° to +32° declination scans, each separated 24m in right ascension, taken in one night's program.) The right ascension region 14h00m to 14h18m is common to the 13h-17h and the 14h-18h ranges of the 1967 observations and is also common to the 12h-16h coverage of the 1966 observations. Therefore, the raster including a declination scan along 14h00m was designated as an "A" raster. Those rasters including a declination scan along 14h06m, 14h12m, or 14h18m were designated as "B", "C", or "D" rasters, respectively. In the 1966 Arecibo program, it was an "A" raster that turned out to be free from power-line interference (see Figures 31 and 32) and therefore became the basis for the limited-resolution, multifrequency maps that resulted from this program.

For convenience of description, the 1967 observations may be divided into three segments. In the March 21-31 segment, the author used the six nights assigned to the Galactic Spur program to make the standard ten-scan sets across the α = 13h to 17h, δ = +2° to +32° region. Because of an instability problem in the eight channel recorder, one whole night's records were ruined. (The problem was corrected and did not bother the observations for the remainder of the 1967 program.) The five good sets of scans contained one "A", "B", and "D" rasters and two "C" rasters. The correlation between the data found
in the duplicate "C" rasters was good. On all records, however, there was radio station interference in the early portions of the night, particularly on the lower frequencies.

During the second segment (April 1-9) of the 1967 observations, right ascension cuts were made across areas containing features whose contours run north-south to some extent. These areas, which were found by inspection of the multifrequency maps of the Galactic Spur resulting from the 1966 observations at AIO, include: \( \alpha = 12^h00^m \) to \( 13^h20^m \), \( \delta = 0^\circ \) to \( +10^\circ \); \( \alpha = 13^h \) to \( 14^h \), \( \delta = +10^\circ \) to \( +20^\circ \); and \( \alpha = 14^h40^m \) to \( 15^h40^m \), \( \delta = 0^\circ \) to \( +10^\circ \). Scanning rates for the right ascension cuts were between \( +4^m \) per minute (scanning with the benefit of the earth's rotation of \( +1^m \) per minute) and \( -2^m \) per minute (scanning contrary to the earth's rotation). Also during this segment, several nights were devoted to taking long drifts (some over the whole \( 12^h \) to \( 18^h \) range) at declinations of \( +35^\circ \), \( +34^\circ \), \( +33^\circ \), \( +1^\circ \), and \( 0^\circ \) to fill in the full declination range of the survey.

In the April 10-16 segment, four nights (one each for "A", "B", "C" and "D" rasters) were devoted to obtaining the standard ten-scan sets across the \( \alpha = 14^h \) to \( 18^h \), \( \delta = +2^\circ \) to \( +32^\circ \) region. Unfortunately, because of late starts and early evening interference, much of the data in the \( 14^h-15^h \) range was lost. Also, because of problems with the 26.7 MHz radiometer, no 26.7 MHz data was obtained for the "A" and the "C" rasters. On two nights when the telescope had to remain in a fixed position for the night, long drifts were taken at \( +32^\circ \) and \( +2^\circ \). The outputs from these drifts were used to tie together the data points at
the top and bottom of the various raster scans.

Figures 33 and 34 give examples of the data gathered during the March-April 1967 observations. Figure 33 shows a "A" raster scan of \( \alpha = 16^h 24^m \) in the 13\(^h\)-17\(^h\) survey, taken on the night of March 26-27. Figure 34 shows a "D" raster scan of \( \alpha = 17^h 30^m \) in the 14\(^h\)-18\(^h\) survey, taken on the night of April 13-14.

Because of the zenith angle limitation, the author's observations of the Galactic Spur were confined to the northern celestial hemisphere. Because of the resolution limitation due to beamwidth, the observations cannot fully show the sharpness of the rise in the ridge structure. In spite of these limitations, however, the author believes that his observations of the Spur have produced significant results.

The multi-frequency contour maps of the Spur at frequencies below 40 MHz are a new contribution to the experimental body of knowledge concerning the Spur. The "fall-out" from the observational results — the geometry of the Spur's bright radiation belt, its spectrum at low frequencies, and the shell source characteristics found in its ridge structure — should lead to a further understanding of the origin of the Spur's radiation.

It is felt the observational results in the 20-40 MHz range and the interpretation of these results in terms of evidence for the supernova remnant theory of the Spur's origin represents a definite contribution to the study of low-frequency radio source emission.
Figure 33. Declination scan along 16\textsuperscript{h} 24\textsuperscript{m} right ascension on the night of 26-27 March 1967. This record shows a "A" raster scan of the 13\textsuperscript{h} - 17\textsuperscript{h} region made during the first segment of the March - April 1967 observations.
RASTER "D", RUN #9
13-14 APRIL 1967
R.A.: 17 h 30 m
DEC.: +32° to +2°
TIME: 0148 - 0518 AST.
DEC. RATE: -1°/MIN.

+32° 39.75 MHz
+32° 33.45 MHz
+32° 26.70 MHz
+32° 22.30 MHz

Figure 34. Declination scan along 17° 30' right ascension on the night of 13-14 APRIL 1967. This record shows a "D" raster scan of the 14° - 19° region made during the third segment of the March - April 1967 observations.
CHAPTER IV
DISCRETE SOURCES

A. OBSERVATIONAL RESULTS

1. Flux Density Determinations

In the process of determining flux density the initial quantity measured is uncorrected antenna temperature. To obtain flux density one possible approach might be to correct each measured value of uncorrected antenna temperature using the appropriate factor from Table 3. Flux density would then be determined from the expression \( S = 2k \frac{T_A}{A_e} \), with the value of \( A_e \) taken from Figure 21 (\( T_A \) is corrected antenna temperature). However, in the final analysis, what is being measured is nothing more than the flux density of the source relative to the assumed flux density of Taurus A. The same data from the tracking of Taurus A used for the effective area curves can be used in a much simpler way (without applying the correction factors of Table 3) to obtain a set of curves directly relating uncorrected antenna temperature to flux density as a function of zenith angle for the four operating frequencies. Such a set of curves is given in Figure 35. This relationship, denoted by \( Z \), is given in terms of °K of uncorrected antenna temperature per flux unit. Given an
Figure 35. Curves showing °K of uncorrected antenna temperature obtained per flux unit of flux density as a function of zenith angle.

\( Z \) is the ratio of uncorrected antenna temperature (in °K) to flux density (in flux units).

1 flux unit = \( 10^{-26} \) Wm\(^{-2}\) Hz\(^{-1}\)
uncorrected antenna temperature measurement of a source and the zenith angle at peak, one finds the flux density simply by dividing the measured value by the value of Z obtained from the set of curves.

Individual determinations of the flux density of each source at each operating frequency were averaged to obtain the final values of flux density. The number of records (right ascension cuts, declination cuts, drifts) used in obtaining these final values depended upon the particular source and, to some extent, upon the operating frequency.

The nearer the declination of a source to the latitude of AIO, the greater the viewing time available each night, and in general, the greater the number of records. Also, the nearer the source to AIO's latitude, the more freedom in types of scan used and the greater the variety of the zenith angles at peak. For sources which pass just inside AIO's 20° zenith angle viewing cone, only drifts can be taken and usually only one drift per night.

Many useful records did not have uniformly good results on all operating frequencies. Because of radio station interference, scintillation (more troublesome at the lower frequencies), or improper scale settings that caused the pen trace to go off-scale, there were some records where measurements could not be made on all the operating frequencies. However, in no case were individual flux density determinations from any record used unless consistent flux density
determinations on at least two frequencies could be obtained from that record.

For sources where a large number of good records were available, a selection of the best records was made. In these cases, usually not more than ten or twelve measurements were used to obtain the final values of flux density. For cases where the number of records was limited, one's choice of the "best" records became more restricted. However, in no case were determinations made where the record showed any inconsistency or had scintillation or interference of such a nature that the peak or the baseline could not be clearly defined.

As a criterion for producing a reportable final flux density determination, the author required at least five consistent individual flux density determinations at each frequency. For several sources (3C79, 3C134, 3C310) even though several individual determinations were made, not enough were made to insure reliability; hence, no flux density values were published for these sources. Certain other sources, 3C273 and 3C392, were certainly strong enough to detect, and sufficient unspoiled records were available. (Records of 3C392 were all free of scintillation.) However, the sources could not be reliably separated from their confusing backgrounds with the beam-widths available; hence, flux densities were not reported.

For the sources whose flux density values are quoted (Guidice, 1966), the number of individual determinations at each frequency is given in Table 25. The ten "determinations" at each frequency made
from the Taurus A observations did not measure its flux density (assumed), but verified the consistency of the effective area calibration involving the continuous tracking of Taurus A.

### TABLE 25. Number of Records Which Were Averaged to Determine Flux Density at Each Frequency for Each Source Measured

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>22.30 MHz</th>
<th>26.70 MHz</th>
<th>33.45 MHz</th>
<th>38.75 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C33</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>3C47</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>3C98</td>
<td>5</td>
<td>5</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>3C123</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Taurus A</td>
<td>10(^a)</td>
<td>10(^a)</td>
<td>10(^a)</td>
<td>10(^a)</td>
</tr>
<tr>
<td>IC443</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>Virgo A</td>
<td>11</td>
<td>9</td>
<td>12</td>
<td>—</td>
</tr>
<tr>
<td>Herc A</td>
<td>10</td>
<td>12</td>
<td>9</td>
<td>—</td>
</tr>
<tr>
<td>3C353</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>—</td>
</tr>
</tbody>
</table>

\(^a\)Used to establish consistency of effective area calibration.

The flux density of IC443 (3C157) at each frequency was corrected for the effect of the angular size of the source. The apparent flux density \( S_A \) of a source whose angular diameter is not small compared to the half-power beamwidth of the antenna is:

\[
S_A = \frac{2 k T_A}{A_e}
\]
while the true flux density of such a source is:

\[ S = \frac{2 k T_A}{A_e} K = K S_A. \]

The correction factor \( K \) is given by:

\[ K = \frac{\Omega_s}{\sum_{\text{source}} F_n(\theta, \phi) d\Omega} , \]

where \( \Omega_s \) is the solid angle subtended by the radio source and \( F_n(\theta, \phi) \) is the normalized power pattern of the antenna. For half-power beamwidths much greater than the angular diameter of the source, \( K \) approaches unity and the apparent flux density is essentially the true flux density. For further detail, see Guidice (1967b). For a radio source having circular symmetric gaussian distribution with half-intensity width \( \theta_S \) and an antenna with a gaussian main beam having circular symmetry and half-power beamwidth \( \theta_H \), the correction factor \( K \) is given by:

\[ K = \left( 1 + \frac{\theta_S^2}{\theta_H^2} \right) . \]

The half-intensity of IC443 is taken as 29 arc-minutes (Bennett, 1962), and the half-power beamwidth for the four operating frequencies is obtained from Table 15. The results are shown in Table 26. For the other sources whose flux density was measured by the author, their angular diameters are 5 arc-minutes or less and thereby negligibly small compared to the antenna half-power beamwidth. Therefore,
for these sources $K$ is essentially equal to one, and no corrections are necessary.

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>BEAMWIDTH °</th>
<th>APPARENT FLUX DENSITY MEASURED (flux units)</th>
<th>CORRECTION FACTOR</th>
<th>TRUE FLUX DENSITY (flux units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.30</td>
<td>3.30</td>
<td>521</td>
<td>1.021</td>
<td>532</td>
</tr>
<tr>
<td>26.70</td>
<td>2.71</td>
<td>544</td>
<td>1.032</td>
<td>561</td>
</tr>
<tr>
<td>33.45</td>
<td>2.28</td>
<td>560</td>
<td>1.044</td>
<td>584</td>
</tr>
<tr>
<td>38.75</td>
<td>2.02</td>
<td>517</td>
<td>1.058</td>
<td>547</td>
</tr>
</tbody>
</table>

As a result of the October-November 1964 observations, the flux densities of 3C33, 3C47, 3C98, 3C123, and IC443 (3C157) were measured at 22.30, 26.70, 33.45, and 38.75 MHz relative to the calibration source Taurus A (3C144). As a result of the April 1965 observations, the flux densities of Virgo A (3C274), Hercules A (3C348), and 3C353 were measured, again relative to Taurus A, at 22.30, 26.70, and 33.45 MHz. Table 27 shows the results of the measurements (Guidice, 1966). Because of irregularities (instability) in the 38.75 MHz radiometer used in the April measurements, the 38.75 MHz measurements from the April 1965 observations were found to be unreliable and were therefore discarded. (Although the radiometers used in the April observations were the same kind as those used in the October-November observations, different individual radiometers were used on all operating frequencies.)
The spectra of the nine sources observed at Arecibo are presented in Figure 36.

TABLE 27. Flux Densities of the Radio Sources

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>FLUX DENSITY (W m(^{-2}) Hz(^{-1}) x 10(^{-26}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>22.30 MHz</td>
</tr>
<tr>
<td>3C33</td>
<td>259</td>
</tr>
<tr>
<td>3C47</td>
<td>225</td>
</tr>
<tr>
<td>3C98</td>
<td>264</td>
</tr>
<tr>
<td>3C123</td>
<td>795</td>
</tr>
<tr>
<td>Taurus A</td>
<td>2670(^a)</td>
</tr>
<tr>
<td>IC443</td>
<td>532</td>
</tr>
<tr>
<td>Virgo A</td>
<td>5060</td>
</tr>
<tr>
<td>Herc A</td>
<td>2393</td>
</tr>
<tr>
<td>3C353</td>
<td>865</td>
</tr>
</tbody>
</table>

\(^a\)Assumed flux density

Among the many limitations to the 20-40 MHz measurement work were ionospheric absorption and scintillation. To minimize both absorption and scintillation, one should observe through the least amount of ionosphere (i.e., avoid observations near the horizon). The Arecibo radio telescope itself takes care of this, limiting observations to a maximum of 20° from zenith.

The strongest ionospheric absorption takes place in the D-region (about 80 km altitude), which is present only in the daylight hours.
Figure 36. Radio spectra of nine discrete sources observed at Arecibo. Experimental values are shown as open circles. For calibration purposes, flux density of Taurus A was assumed.
This absorption is proportional to \((f + f_L)^{-2}\), where \(f\) is the radio frequency of operation and \(f_L\) is the longitudinal component of the electron gyro frequency (Whitehead, 1959). Absorption in the E-region (100-110 km altitude), which is also present in the daytime, is proportional to \(H f_o\), where \(H\) is the scale height and \(f_o\) is the electron collision frequency at the height of maximum electron density. Absorption in the F-region (complex, 200-400 km altitude), which is present at night as well as in the day, is proportional to \(f_o (dN/dh)^{-1}\), where \(f_o\) is the collision frequency and \((dN/dh)\) is the electron density gradient \((f_o \ll 20 \text{ MHz})\).

For lowest ionospheric absorption, one should observe during a period of minimum solar activity. The year 1964 was a sunspot minimum year. Ionospheric absorption is minimum around midnight to 0600, local time. The 20-40 MHz observations were taken mainly in the 2200-0500 period, Atlantic Standard Time (the local time zone). Under these somewhat ideal conditions, the ionospheric absorption for the 20-40 MHz observations were estimated to be in the order of 0.2 dB or less (for all operating frequencies). Since due to ionospheric absorption both the discrete source antenna temperature and the effective area (also determined from antenna temperature measurements) would be similarly low, the flux densities determined should be unaffected if the absorption is the same. At night the absorption itself is small, and any difference can be taken as negligibly small (as far as making corrections to the flux density values is concerned). Ionospheric absorption, however, is considered in the error analysis.
Ionospheric scintillation depends on several factors: the ionospheric path length (minimized at AIO by telescope's zenith angle limit), the angular extent of the source, the latitude of the observatory, the time of day, and the operating wavelength. Scintillation amplitude decreases rapidly with increasing angular diameter beyond a certain angular diameter (around five to ten arc-minutes) because of the decorrelation of the scintillation over the extent of the source (Aarons and Guidice, 1966). However, except for IC443, all the sources whose flux densities were measured in this investigation have angular diameters of 5 arc-minutes or less and thus scintillate.

With regard to latitudinal variation the highest scintillations are found at auroral and at equatorial latitudes (Koster, 1963). Hence, low-latitude observations such as these made at Arecibo tend to minimize ionospheric scintillation. With regard to the diurnal variation of ionospheric scintillation, a nighttime maximum is observed. Unfortunately, because of the higher (and more variable) ionospheric absorption, the more frequent occurrence of thunderstorms, and the much greater radio station interference during the day, the use of daytime observations for the 20-40 MHz investigation was found to be impractical.

Regarding wavelength dependence, as long as the scintillation results from weak scattering, the scintillation index (a measure of the relative magnitude of the scintillation) is proportional to $\lambda^\xi$, with $1 < \xi < 2$. Scintillation index is defined by the expression
\[ \frac{T_A^{(\text{max})} - T_A^{(\text{min})}}{T_A^{(\text{max})} + T_A^{(\text{min})}} \], where \( T_A \) is the antenna temperature of the source (above the background). For weak scattering the average phase change is less than one radian — in contrast to strong scattering, for which the average phase change is greater than one radian. For these low-latitude 20-40 MHz observations, weak scattering is generally a valid assumption (Aarons and Guidice, 1966). The greater scintillation at the lower operating frequencies (longer wavelengths) is quite apparent in all the records of the sources that scintillate.

Besides calibration errors and errors caused by the ionospheric effects, the only other cause of possibly serious error is confusion with other sources or the background. The low sidelobe level (especially the very low level beyond a couple of beamwidths from the beam axis) achieved with the AI0 "filled aperture" pencil beam prevents improper measurements resulting from a very strong source in a sidelobe, which may plague observations using interferometric or synthetic aperture techniques. To avoid erroneous flux density measurements caused by confusion within the main beam, determinations of flux density were made only on those sources where the records were consistent and clear separation from the confusion could be made (see Chapter III). From Table 27, none of the six strong sources measured (flux densities greater than 500 flux units) is confusion limited. As for the three smaller sources, 3C33 and 3C47 appear to be rather isolated and are not felt to be confusion limited, while 3C93 might be confusion limited.
2. Error Analysis

To start the error analysis, let us first separate the errors into two distinct categories: random measurement errors and precalculated systematic errors. The random measurement error depends only upon the statistical distribution of the individual numerical values of flux density measured at each operating frequency; it is independent of any considerations of the instrumentation used or the environment encountered in making the measurements. On the other hand, the precalculated systematic error is determined (or estimated) by the system through which the measurements are made; it is independent of the dispersion of the experimentally measured data points. The word "systematic" is used here in a broader sense than instrumentation; it includes the effects of the source's surroundings (confusion error) and the intervening medium (ionospheric errors).

The anticipated error, or total error, is taken as the sum of the random measurement error and the precalculated systematic error. This approach is in consonance with the error analysis used by Bazelyan et al. (1963) on their low frequency measurements of the very strong discrete sources Cassiopeia A, Cygnus A, Virgo A, and Taurus A. A random measurement error, a precalculated systematic error, and a total anticipated error have been calculated for each flux density value presented in Table 27 (except for Taurus A, whose flux density values are assumed).

Random Measurement Error: The determination of the random measurement error is based on the statistical approach suggested in
Menzel (1960). We assume that the set of flux density measurements of each source at each frequency constitutes a randomly distributed sample. A normal (gaussian) distribution of individual measurements \(x_i\) in the sample is further assumed. The distribution of the experimental values is centered around a sample mean, which is taken as the average value of the individual measurements. It is given symbolically by:

\[
\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i ,
\]

where \(n\) is the number of individual measurements in the sample.

As the number of individual measurements goes to infinity, the sample mean approaches the population mean; i.e., as \(n \to \infty\), \(\overline{x} \to \mu\). The population mean \((\mu)\) is defined as the average value of an infinite number of normally distributed individual measurements of the parameter. The parameter (flux density of a source at a given frequency) is assumed to be invariant with respect to time.

The average standard deviation of individual measurements is given by:

\[
s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n - 1}} .
\]

The standard deviation of the sample mean from the population (or true) mean is given by:

\[
s_{\overline{x}} = \frac{s}{\sqrt{n}} .
\]
For the purposes of this analysis, we take the random measurement error ($\Delta_r$) as equal to the standard deviation of the sample mean. Thus, in general, increasing the number of individual measurements in the sample tends to decrease the random measurement error.

For each flux density value quoted in Table 27 (excluding Taurus A), a random measurement error has been calculated using the expression:

$$\Delta_r = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n(n-1)}}$$

These errors are given in Table 28, both in terms of flux units and as a percentage of the average measured flux density ($\bar{x}$).

**TABLE 28. Random Measurement Errors in Terms of Flux Units and as a Percentage of Average Measured Flux Density**

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>22.30 MHz f.u.</th>
<th>22.30 MHz percent</th>
<th>26.70 MHz f.u.</th>
<th>26.70 MHz percent</th>
<th>33.45 MHz f.u.</th>
<th>33.45 MHz percent</th>
<th>38.75 MHz f.u.</th>
<th>38.75 MHz percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C33</td>
<td>10</td>
<td>3.9</td>
<td>10</td>
<td>4.3</td>
<td>5</td>
<td>2.6</td>
<td>6</td>
<td>3.6</td>
</tr>
<tr>
<td>3C47</td>
<td>7</td>
<td>3.2</td>
<td>5</td>
<td>2.5</td>
<td>4</td>
<td>2.5</td>
<td>6</td>
<td>4.3</td>
</tr>
<tr>
<td>3C98</td>
<td>12</td>
<td>4.5</td>
<td>13</td>
<td>5.4</td>
<td>8</td>
<td>3.7</td>
<td>7</td>
<td>3.8</td>
</tr>
<tr>
<td>3C123</td>
<td>13</td>
<td>1.6</td>
<td>13</td>
<td>1.7</td>
<td>12</td>
<td>1.8</td>
<td>12</td>
<td>1.9</td>
</tr>
<tr>
<td>IC443</td>
<td>5</td>
<td>1.0</td>
<td>6</td>
<td>1.1</td>
<td>6</td>
<td>1.1</td>
<td>5</td>
<td>1.0</td>
</tr>
<tr>
<td>Virgo A</td>
<td>115</td>
<td>2.3</td>
<td>61</td>
<td>1.4</td>
<td>72</td>
<td>1.9</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Herc A</td>
<td>61</td>
<td>2.5</td>
<td>48</td>
<td>2.3</td>
<td>47</td>
<td>2.6</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>3C353</td>
<td>21</td>
<td>2.4</td>
<td>24</td>
<td>2.9</td>
<td>27</td>
<td>3.5</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
Precalculated Systematic Error: In the precalculated systematic error, account is taken of the various statistically independent errors resulting from calibration, confusion, and ionospheric effects. The systematic error is taken as the square root of the sum of the squares of the various estimated errors. The errors included in this analysis are the noise generator calibration error ($E_{\text{gen}}$), the effective area calibration error ($E_{\text{effA}}$), the confusion error ($E_{\text{conf}}$), the ionospheric absorption error ($E_{\text{abs}}$), and the ionospheric scintillation error ($E_{\text{scin}}$).

The flux density measurements are acknowledged to be relative to Taurus A; therefore, a calibration error due to inaccuracy in the Taurus flux density assumptions need not be considered here. The extent of the possible deviation of Taurus A from the assumed straight line spectrum is discussed in Part B of this chapter. In any case, the deviation from the assumed flux densities should be less than five percent.

The maximum variation in the output of the random noise generator operated within rated conditions (100-130 volts input, -20°C to + 50°C environmental temperature) is 0.1 dB. The long term stability of the output of the noise generator is also 0.1 dB. Therefore, we take 0.1 dB or 2.3 percent as the noise generator calibration error for all frequencies.

Next, we consider the effective area calibration error. The root mean square variation between the flux densities found from the
ordinary observations of Taurus A and the assumed flux densities used in the effective area calibration (involving the tracking of Taurus A) was 3.3 percent. We take this 3.3 percent as the effective area calibration error for all frequencies.

The fluctuation in radiometer output resulting from the noise-like nature of the input signal is directly proportional to the total system noise temperature and inversely proportional to the square root of the product of the input bandwidth and the output integration time. The largest contributors to system noise temperature for frequencies below 40 MHz are the source itself and the sky background. The receiver noise temperature, which is in the order of 600°K or less, is negligible. The greatest system noise temperature occurs at the lower frequencies, and this results chiefly from the sky background noise having $T_B \propto v^{-\beta}$ with $\beta \approx 2.5$ for the 20-40 MHz range. For the regions surrounding the measured sources, the brightness temperature of the background at 38.75 MHz is in the 8000 to 20,000°K range. For 22.30 MHz, $T_B^{\text{bgd}}$ is in the 32,000 to 80,000°K range. Fortunately, for the regions surrounding the weakest measured sources (3C33, 3C47, 3C98), the background temperature is near the lower end of these ranges.

For all frequencies the radiometer bandwidth ($B$) was 30,000 Hz and the post-detection time constant ($\tau$) was 10 seconds. At 22.30 MHz (the worst case), $\Delta T(\text{rms}) = \frac{T_{\text{sys}}}{\sqrt{B\tau}} \approx \frac{T_B^{\text{bgd}}}{\sqrt{B\tau}} = 60-150$°K. For the higher operating frequencies, $\Delta T(\text{rms})$ is much lower since the values of $T_{\text{sys}}$ are much less. For 22.30 MHz, a $\Delta T(\text{rms})$ fluctuation of
60°K corresponds to about 5 to 9 flux units of flux density (depending on telescope zenith angle).

In determining the antenna temperature due to the source, the author used at least a minute's worth of record at the peak of the scan and at the background level on either side of the peak. Hence, for any assignment of error due to fluctuation, one would have to consider this one minute as the integrating time rather than the 10 second output time constant of the radiometer. Hence, the resultant errors due to fluctuation would have values of only 0.4 times those values discussed in the previous paragraph.

It is obvious that the fluctuation error, even at the lower frequencies, is rather small in terms of flux units. Since the confusion error (discussed next) is significantly larger than any error due to fluctuation, the precalculated systematic error will not include fluctuation error among its considerations.

The confusion error results from the inability to determine exactly what part of the antenna temperature measured is due to the flux density of the particular discrete source scanned and what part is due to other sources or background variations in the main beam or in the sidelobes. Because of the good sidelobe characteristics and the high main beam efficiency of the pencil beam used in these observations and the strength of the discrete sources measured, confusion due to other sources or background variations outside the main beam can be neglected. With regard to sources within the main beam, the
particular discrete sources measured in this investigation are by far the strongest within the celestial area encompassed by the beamwidth of the antenna; the flux density of any confusion source coming within the extent of the main beam is probably less than 20 percent of the flux density of the discrete source intended to be measured. (This might not quite be true for 3C98.) It should be noted that the relative antenna temperature contribution from another (confusing) source is never as great as from the nominally measured source (the confusing source does not go through the center of the beam). In most instances, the antenna temperature contribution from a confusing source becomes merely part of the background or can be separated from that due to the nominally measured source. For most of the discrete sources measured, both right ascension and declination cuts were taken to help further eliminate any confusion.

The most unknown and undoubtedly the largest contribution to the confusion error is the confusion due to the background variations within the main beam. Fortunately, the background brightness temperature and the complexity of the areas surrounding the sources whose flux densities are presented in Table 27 are relatively low, except for Hercules A and 3C353. (Background $T_b \approx 8000^\circ K - 13,000^\circ K$ at 38.75 MHz; for Hercules A and 3C353, background $T_b \approx 18,000^\circ K - 22,000^\circ K$.)

The confusion error is proportional to beamwidth squared. The beamwidths for the four operating frequencies have been determined in the radio telescope parameter analysis of Chapter II and are given in Table 15. Once a value for the confusion error is established at one
operating frequency, the values at the other operating frequencies can be determined by scaling proportional to the square of the relative beamwidths.

Since it has the widest beamwidth, the 22.30 MHz operating frequency will have the largest confusion error. For these observations at Arecibo, the minimum accurately measurable flux density at 22.30 MHz is in the order of 250 flux units. (Note that this is not the same as the minimum detectable flux density, which at 22.30 MHz might be as low as 50 flux units.) It is estimated that for the relatively confusion-clear, minimum background areas surrounding most of the measured sources the confusion error on the average should be about 10 percent of this 250 flux unit value. Therefore, we arbitrarily take 25 flux units as the confusion error at 22.30 MHz for the sources in the relatively unconfused regions; for Hercules A and 3C353 we take the confusion error at 22.30 MHz as 50 flux units. Scaling the confusion errors for the other three operating frequencies, we obtain Table 29. For simplicity, the confusion error is rounded off to whole flux units.
TABLE 29. Confusion Errors for the Four Operating Frequencies

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>BEAMWIDTH</th>
<th>CONFUSION ERROR (flux units)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3C33, 3C47, 3C98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3C123, IC443, Virgo A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hercules A and 3C353</td>
</tr>
<tr>
<td>22.30</td>
<td>3.30°</td>
<td>25</td>
</tr>
<tr>
<td>26.70</td>
<td>2.71°</td>
<td>17</td>
</tr>
<tr>
<td>33.45</td>
<td>2.28°</td>
<td>12</td>
</tr>
<tr>
<td>38.75</td>
<td>2.02°</td>
<td>9</td>
</tr>
</tbody>
</table>

The ionospheric absorption error is due to the difference in ionospheric absorption present during the measurements of the various discrete sources (measured for the most part in the 2200-0500 local time period) and that present during the tracking runs on the calibration source Taurus A (measured in the 0030-0330 local time period). Since the nighttime ionospheric absorption, which takes place in the F-region (mainly in the F₂ region), is deviative absorption, in contrast to the non-deviative absorption of the daytime D-region, the dependence upon frequency is not easy to define. If the operating frequency is several times higher than the critical frequency, \( f_0^F_2 \), the absorption for near zenith observations should be small (\( f_0^F_2 \) is the frequency below which a vertically-travelling wave will not penetrate the ionosphere's \( F_2 \) layer). Since the Arecibo observations in October-November 1964 and April 1965 were made near the minimum in the 11-year sunspot cycle, it would be unusual for the nighttime low-latitude \( f_0^F_2 \) during these observations to be greater than 5 MHz. Measurements using
riometers at 30 MHz by AFCRL in Hawaii (a low latitude site, comparable to Arecibo) showed that the zenith absorption was in the order of 0.2 dB for the midnight-0600 local time period and somewhat higher before midnight. Since it is only the difference in absorption that causes the error, we will take the ionospheric absorption error as 0.1 dB (2.3 percent).

The scintillation error results from the distortion in the antenna pattern response caused by the passage of the signal from the source through the "screen" of ionospheric irregularities. This distortion makes it difficult to determine accurately the peak antenna temperature. For these low-latitude 20-40 MHz observations, weak scattering is assumed. Therefore, the temperature on the average will be the same as if the screen of ionospheric irregularities were not present; the screen simply increases the variation in the signal level. Unfortunately, the periods of these scintillations is usually in the order of 0.5 to 1.5 minutes, thus extending over a significant portion of the antenna pattern, making the author's averaging out (by "eyeball") a rather imprecise and subjective process.

For weak scattering the scintillation index (the relative magnitude of the scintillations) is proportional to $\lambda^\xi$, with $1 < \xi < 2$. For the purpose of this error analysis we will assume $\xi = 1.5$. By establishing this wavelength (frequency) dependence, once the scintillation error is determined at one frequency, the values at the other operating frequencies can be found by scaling proportional to $\lambda^{1.5}$. 
Since the variance caused by the scintillation does not change the average intensity of the signal (weak scattering case), and the author was careful not to "bias" his determination of peak antenna temperature to the high side or the low, the introduced error (hopefully) should be "random". Therefore, the resultant error in a group of measurements averaged to obtain a final value should be less than the possible error in any individual measurement. In particular, the error for the averaged value of flux density should be decreased by roughly a factor of \(1/\sqrt{n}\) over that expected for an individual measurement (\(n\) is the number of observations of the source). In calculating errors that go into the precalculated systematic error, we endeavor to tie the errors to the system and try to avoid relationships strongly dependent upon numerical values associated with the observations. For most of the sources the average number of observations at each operating frequency is about 8-11, except for 3C98 and 3C353 where the number is 5. For the sake of simplicity, let us assume \(n = 9\) for the former case, and \(n = 5\) for the latter. Thus, the error for individual measurements will be reduced by a factor of 1/3 for most sources and by 1/2.24 for 3C98 and 3C353.

Scintillation is the greatest at the lowest frequency, 22.30 MHz. It is estimated that due to scintillation an individual measurement of antenna temperature (and, therefore, of flux density) at 22.30 MHz could be in error by as much as 20 percent. With the appropriate reduction due to the number of observations, the scintillation error at
22.30 MHz becomes 6.7 percent for 3C33, 3C47, 3C123, Virgo A and Hercules A and 8.9 percent for 3C98 and 3C353. As noted in previous discussions, the source IC443 (3C157) does not scintillate because of its large angular diameter; hence, no scintillation error is assigned. The scintillation errors for the three other operating frequencies are scaled using the factor \((\frac{\lambda}{11.30})^{1.5}\), where 11.30 meters is the wavelength corresponding to 22.30 MHz. The final results for the ionospheric scintillation error are presented in Table 30.

### Table 30. Ionospheric Scintillation Errors for the Four Operating Frequencies

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>SCINTILLATION ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3C33, 3C47, 3C123, Virgo A and Hercules A</td>
</tr>
<tr>
<td>22.30</td>
<td>6.7</td>
</tr>
<tr>
<td>26.70</td>
<td>5.1</td>
</tr>
<tr>
<td>33.45</td>
<td>3.6</td>
</tr>
<tr>
<td>38.75</td>
<td>2.9</td>
</tr>
</tbody>
</table>

The precalculated systematic error is calculated for each source at each operating frequency from the expression:

\[ \Delta_s = \sqrt{(E_{\text{gen}})^2 + (E_{\text{effA}})^2 + (E_{\text{conf}})^2 + (E_{\text{abs}})^2 + (E_{\text{scin}})^2} \]

The results of these calculations in terms of flux units and as a percentage of the measured flux density are shown in Table 31.
### TABLE 31. Precalculated Systematic Error in Terms of Flux Units and as a Percentage of the Measured Flux Density

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>PRECALCULATED SYSTEMATIC ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>22.30 MHz f.u. percent</td>
</tr>
<tr>
<td>3C33</td>
<td>33 12.7 24 10.2 17 8.6 13 7.7</td>
</tr>
<tr>
<td>3C47</td>
<td>31 13.7 22 11.0 16 9.5 12 8.6</td>
</tr>
<tr>
<td>3C98</td>
<td>36 13.6 26 10.8 19 8.9 13 7.0</td>
</tr>
<tr>
<td>3C123</td>
<td>69 8.7 54 7.3 41 6.0 35 5.7</td>
</tr>
<tr>
<td>IC443</td>
<td>35 6.6 31 5.5 29 5.0 27 4.9</td>
</tr>
<tr>
<td>Virgo A</td>
<td>412 8.1 312 6.8 227 5.9 — —</td>
</tr>
<tr>
<td>Herc A</td>
<td>191 8.0 150 7.0 111 6.0 — —</td>
</tr>
<tr>
<td>3C353</td>
<td>100 11.6 76 9.3 57 7.5 — —</td>
</tr>
</tbody>
</table>

**Total Anticipated Error:** The anticipated or total error $\Delta_a$ is obtained from the following expression:

$$\Delta_a = \Delta_r + \Delta_s$$

The anticipated errors for each source at each operating frequency are shown in Table 32.
TABLE 32. Total Anticipated Error in Terms of Flux Units and as a Percentage of the Measured Flux Density

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>TOTAL ANTICIPATED ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>22.30 MHz f.u. percent</td>
</tr>
<tr>
<td>3C33</td>
<td>43 16.6</td>
</tr>
<tr>
<td>3C47</td>
<td>38 16.9</td>
</tr>
<tr>
<td>3C98</td>
<td>48 18.1</td>
</tr>
<tr>
<td>3C123</td>
<td>82 10.3</td>
</tr>
<tr>
<td>IC443</td>
<td>40 7.6</td>
</tr>
<tr>
<td>Virgo A</td>
<td>526 10.4</td>
</tr>
<tr>
<td>Herc A</td>
<td>252 10.5</td>
</tr>
<tr>
<td>3C353</td>
<td>121 14.0</td>
</tr>
</tbody>
</table>
B. THEORETICAL INTERPRETATION

1. Supernova Remnants

All of the sources given in Table 27 are non-thermal. The radio emission from these sources results from synchrotron radiation coming from relativistic electrons moving in a magnetic field. Of the sources in Table 27, only Taurus A and IC443 are galactic; the remainder are extragalactic. In addition, Taurus A and IC443 are the only supernova remnants in this group.

**Taurus A:** The strong radio source Taurus A was first identified as being associated with the Crab Nebula by Bolton (1948). The Crab Nebula is believed to be the remnant of a supernova explosion that occurred in 1054 A.D. Over the frequency range 38-3200 MHz, Conway, Kellermann, and Long (1963) found the spectrum of Taurus A to be linear with a spectral index of 0.27 (S ∝ λ^{0.27} ∝ ν^{-0.27}). Flux density measurements of Taurus A have been made over an even wider frequency range than this, to frequencies as high as 35 GHz (Tolbert and Straiton, 1965) and as low as 12.5 MHz (Bazelyan et al., 1963). Even over this wider range, the spectrum appears to be linear, with roughly the same spectral index value.

Observations made at Cambridge University using lunar occultation and interferometric techniques have uncovered in the Crab Nebula the presence of a component of very small angular diameter with enhanced emission at lower frequencies (Hewish and Okoye, 1965). The spectrum of this so-called "compact" source in the Crab Nebula is quite steep,
having a spectral index of roughly 1.2. The compact source accounts for about 20 percent of the total flux density of Taurus A at 38 MHz. From later work involving sources of very small angular diameter (such as quasars) and interplanetary scintillation, the compact source in the Crab Nebula was found to have an angular diameter in the order of 0.1 arc-second.

The apparent linearity of the spectrum of the total emission from the Crab Nebula down to frequencies as low as 12.5 MHz, in spite of the compact source with its enhanced low frequency emission (its steep spectrum), may be accounted for by several possible explanations. One simple explanation might be that the low frequency measurements of the flux density of Taurus A were not sufficiently accurate or reliable to show with certainty any deviation from the linear radio spectrum that was usually assumed for Taurus A (until the compact source discovery). In the dekameter range the total flux density of Taurus A is less than the flux density of Virgo A and only in the order of one-tenth of the flux density of Cassiopeia A or Cygnus A. The total anticipated error quoted for the flux density measurements of Taurus A by Bazelyan et al. (1962) is 40 percent at 12.5 and 16.7 MHz and 30 percent for frequencies in the 20-40 MHz range. However, a significant portion of the error allocated was due to uncertainty concerning their calibration source Cassiopeia A; this uncertainty has been reduced by recent improved measurements on Cassiopeia A at 10 MHz (Bridle, 1966). Hence, a total anticipated error of about 20 percent would give a more
reasonable estimate of the absolute accuracy of the Taurus A measurements of Bazelyan et al.

Another explanation may be that the low-frequency enhancement (above the expected linear spectrum) due to the compact source may be counterbalanced to some degree by an effect which tends to decrease the total emission received from the Crab Nebula at lower frequencies. Such an effect, galactic H II absorption, is found in this investigation for the source IC443. There are several similarities between Taurus A and IC443. Both are galactic supernova remnants with similar distances — 1100 parsecs for Taurus A, in the range 800–2000 parsecs (according to various astronomers) for IC443. Their high frequency spectral indexes are similar — 0.27 for Taurus A, about 0.35 for IC443. IC443 has no enhanced low-frequency compact source associated with it; the spectrum of its total emission curves downward at lower frequencies (because of galactic H II absorption).

The quoted accuracies on Taurus A for the 38 MHz measurements of Conway, Kellerman, and Long (1963) and the 26.3 MHz measurements of Erickson and Cronyn (1965) are 10 percent. The flux densities of Taurus A from these two references and those from Bazelyan et al. (1963) are plotted in Figure 37, together with a best fit straight line. Except for Bazelyan's 35 MHz value (which is about 12 percent high), all the experimental values are within 8 percent of a straight line spectrum; the rms. deviation of the experimental points from the straight line is about 6 percent. Hence, it may be concluded that, whatever mechanisms are involved, the spectrum of the total emission
Figure 37. Measured flux density of the total emission from the Crab Nebula (Taurus A) at various frequencies by several observers. Also included is the best fit straight line spectrum.
from the Crab Nebula in the 20-40 MHz range is not appreciably dif­ferent from a straight line.

**IC443:** Of all the sources measured in the author's 20-40 MHz investigation, IC443 (3C157) gave the most consistent results. Its random measurement error for each frequency was only about one per­cent. IC443 also has the most noticeable feature of any of the sources observed; its spectrum reaches a turnaround point (a maximum) at about 30 MHz. Because of the outstanding consistency of the experimental results, the author feels that this maximum found in the IC443 spectrum is definitely real. Moreover, it can be explained quite satisfactorily by the process of galactic H II absorption.

The coefficient of absorption for radio waves passing through a plasma of ionized hydrogen (H II region) is given by Ginzburg and Syrovatskii (1965) as:

\[
\mu = \frac{10^{-2} N_e^2}{T_e^2 \nu^2} \left[ 17.7 + \ln \frac{T_e^2}{\nu} \right],
\]

where \( \mu \) is the absorption coefficient (in \( \text{cm}^{-1} \)), \( N_e \) is the electron density (in \( \text{cm}^{-3} \), i.e., electrons per \( \text{cm}^3 \)), \( T_e \) is the electron temperature (in °K) and \( \nu \) is the frequency (in Hz).

Knowing the absorption coefficient \( \mu \), one can compute the optical thickness of the gas (in the direction through the gas that one sees the source) from the expression:

\[
\tau = \int \mu \, dl,
\]
where $\tau$ is the optical thickness (dimensionless), and $dl$ is an element of the path length (in cm) through the gas. The integration is carried out over the length of region in which absorption takes place. Combining the two expressions, we have:

$$\tau = \frac{10^{-2}}{\frac{T_e^{\frac{3}{2}}}{v^2}} \left[ 17.7 + \ln \frac{T_e^{\frac{3}{2}}}{v} \right] \int N_e^2 \, dl .$$

We now define the term emission measure, $\mathcal{M}$, by the following expression:

$$\mathcal{M} = \int N_e^2 \, dl \quad \text{(cm}^{-5}) .$$

In astronomy, emission measure is usually expressed in units of cm$^{-6}$ pc. Since 1 parsec $= 3.08 \times 10^{18}$ cm, we have:

$$\mathcal{M} = 3.08 \times 10^{18} \int N_e^2 \, dl \quad \text{(cm}^{-6}\text{pc}).$$

Substituting this expression into the expression for optical thickness, we have:

$$\tau = 3.08 \times 10^{16} \frac{T_e^{\frac{3}{2}}}{v^2} \mathcal{M} \left[ 17.7 + \ln \frac{T_e^{\frac{3}{2}}}{v} \right] .$$

The logarithmic term within the brackets varies very slowly. An order of magnitude change in $T_e^{\frac{3}{2}}$ or $v$ causes a change of only about 10-15 percent in the bracketed factor. Since for H II regions $T_e \approx 10,000^\circ\text{K}$, and since the frequencies of interest in this analysis will be in the 20-40 MHz range, we will assume for the logarithmic term that $T_e = \ldots$
10^4 \, ^\circ K \text{ and } v = 30 \times 10^6 \, \text{Hz}. \text{ Thus simplifying, we have:}

\[ \tau = \tau(v) = 4.4 \times 10^{17} \frac{T_e^3}{M} \nu^{-2} = \xi \nu^{-2}. \]

Over the 22.3 to 38.75 MHz range, based on the value we find for \( M \), \( \tau \) is in the range 0.10 to 0.32.

The brightness temperature observed when an intervening absorption region (an ionised hydrogen plasma region, for example) lies between the source and the observer is given by:

\[ T(v) \text{ observed} = T_s(v) e^{-\tau(v)} + T_c(1 - e^{-\tau(v)}), \]

where \( T_s \) is the brightness temperature of the (unattenuated) source and \( T_c \) is the emission temperature of the intervening attenuating region. Let us consider the relative magnitudes of \( T_s \) and \( T_c \) at 30 MHz, the frequency at which the spectrum of IC443 reaches its maximum.

The flux density and brightness temperature of a radio source are related by the familiar expression:

\[ S_s(v) = \frac{2k}{\lambda^2} T_s(v) \nu \Omega_s = \frac{2k}{c^2} \nu^2 T_s(v) \nu \Omega_s, \]

where \( \Omega_s \) is the solid angle of the source. For IC443, the angular diameter of the source is 20 arc-minutes or 0.84 \times 10^{-2} \text{ radians}. Making the rough assumption that \( \Omega_s \approx \theta_s^2 \), we have \( \Omega_s = 0.70 \times 10^{-4} \text{ steradians} \). For \( v = 30 \text{ MHz} \), the wavelength \( \lambda \) is 10 meters. To obtain the unattenuated term \( T_s(v) \), we must find and use the corresponding unattenuated flux density, \( S_s(v) \), at 30 MHz. By extrapolation from higher
frequencies, \( S_s(v) = 700 \times 10^{-26} \text{ Wm}^{-2} \text{ Hz}^{-1} \) at 30 MHz. Solving for \( T_s(v) \) and substituting in the numerical values, we obtain:

\[
T_s(v) = \frac{\lambda^2 S_s(v)}{2k N} = 3.6 \times 10^5 \text{ °K}.
\]

The emission temperature of the intervening ionized hydrogen plasma region is, at most, the assumed electron temperature of the region (10,000 °K). Hence, at 30 MHz \( T_s \) is at least 36 times greater than \( T_e \). We will therefore neglect the second term in the observed brightness temperature equation, leaving:

\[
T(v) \text{ observed} = T_s(v) e^{-\tau(v)}.
\]

For simplicity, let us go to the flux density form of this expression:

\[
S(v) \text{ observed} = S_s(v) e^{-\tau(v)}.
\]

If there were no H II absorption region in front of IC443, the flux density as a function of frequency would be given simply by:

\[
S(v) = S_s(v) = \Lambda v^{-\alpha},
\]

where \( \alpha \) is the spectral index of IC443 found at higher frequencies where the effects of galactic H II absorption are negligible. A straight line spectrum for IC443 at higher frequencies is assumed; this has been found experimentally for almost all supernova remnants. Slightly different numerical values for IC443's spectral index are quoted by various authors: 0.28 by Conway, Kellermann, and Long (1963); 0.37 by Harris (1962); 0.40 by Hogg (1964). We will use the value 0.35 (the average of these three) for the spectral index of
IC443 in the calculations to follow. The intensity of a radio wave passing through an absorbing medium with optical depth $\tau$ is attenuated by the factor $e^{-\tau}$. Hence, for low frequencies, the flux density expression becomes:

$$S(v) = [A v^{-\alpha}] e^{-\tau(v)} = A v^{-\alpha} e^{-\xi v^{-2}}.$$

At the turnaround frequency $v_o$ (the frequency at which the flux density reaches its maximum value), the first derivative is equal to zero. Thus, we have:

$$\frac{dS(v)}{dv} = A v^{-\alpha} e^{-\xi v^{-2}} (-\alpha v^{-1} + 2\xi v^{-3}) = 0 \text{ at } v = v_o.$$

Hence, $-\alpha v_o^{-1} + 2\xi v_o^{-3} = 0$ and thus $\xi = \frac{1}{2} \alpha v_o^2$.

Knowing the turnaround frequency for IC443 (about 30 MHz according to the author's measured spectrum) and the spectral index in the absence of H II absorption (the high frequency spectral index, 0.35), one can find the emission measure of the absorbing H II region, if one assumes the value of electron temperature.

$$\mathcal{M} = 2.28 \times 10^{-18} \frac{T_e^{3/2}}{v_o} \xi = 1.14 \times 10^{-18} \frac{T_e^{3/2}}{v_o^2} \alpha v_o^2.$$

Assuming $T_e = 10^4$ °K we have:

$$\mathcal{M} = 1.14 \times 10^{-12} \alpha v_o^2 = 360 \text{ cm}^{-6} \text{ pc}.$$

Figure 38 shows the experimentally measured spectrum of IC443 together with a theoretical curve based on the effect of absorption by
Figure 38. Low-frequency spectrum of the galactic supernova remnant IC 443. Shown for comparison is the theoretical curve for a source whose low-frequency spectrum is altered by attenuation by an ionized hydrogen plasma region with an emission measure \( \int N_e^2 \, d\ell \) of 360 cm\(^{-6}\) pc.

- Measured flux density values. Error bar indicates total anticipated error.
- Spectrum of IC 443 based on measured values of flux density.
- Theoretical spectrum, \( S(\nu) = A \nu^{-\alpha} e^{-E \nu^{-2}} \)
  where \( \alpha = 0.35 \), \( T_e = 10^4 \) K, \( M_\text{e} = 360 \text{ cm}^{-6} \) pc
  and \( E = 0.44 \times 10^{17} T_e^{-3/2} M_\text{e} = 158 \times 10^{12} \)
- Expected spectrum of IC 443 in the absence of low-frequency attenuation.
an ionized hydrogen plasma. For the source a spectral index (excluding the effect of H II absorption) of 0.35 is assumed, and for the H II region an electron temperature of 10,000°K and an emission measure \( \int N_e^2 \, d\ell \) of 360 cm\(^{-6}\) pc are assumed. It is easily seen that the theoretical curve fits well within the total-anticipated-error bounds of the measured flux density values.

2. Core-and-Halo Sources

Of the seven extragalactic radio sources whose 20–40 MHz flux densities were measured by the author (Taurus A and IC443 are galactic sources), five have spectra which show some degree of downward curvature toward lower frequencies, while two have spectra which appear to remain linear, or possibly even have slight upward curvature. The two sources whose spectra do not show evidence of downward curvature in the 20–40 MHz range are 3C47 and Virgo A (3C274). Of all the sources whose flux densities were measured by the author, 3C47 and Virgo A are the only core-and-halo sources.

In general, extragalactic core-and-halo sources consist of a small central core having an angular diameter in the order of several arc-seconds (depending on their distances) and an extended diffuse halo with an angular diameter much larger than that of the core (perhaps 10 to 100 times greater). The core portion itself may be double or even more complex. The spectrum of the halo is much steeper (has a greater spectral index) than that of the core; hence, at lower frequencies the emission from the halo predominates. The low-frequency
spectral character of a particular core-and-halo source depends on the relative intensities and spectral indexes of the core and the halo.

The source 3C47, a quasi-stellar radio source, has a core with an angular diameter of 2-10 arc-seconds and a halo with angular diameter ≳ 1 arc-minute (Anderson et al., 1965). The core of Virgo A (∼ 0.5 arc-min) is associated with the optically visible "jet" of M87 and is double (Lequeux, 1962). Its halo has an angular diameter in the order of 5 arc-minutes. Another important core-and-halo source is 3C84 (associated with the Seyfert galaxy NGC1275), whose flux density at centimeter wavelengths has been found to be variable (Dent, 1966). Unfortunately, because of its declination (+41°20'), 3C84 could not be observed with the Arecibo telescope.

In examining the halo predominance effect on the low-frequency spectral shape of a core-and-halo source, it is best to begin by using 3C84 (NGC1275) as the example, since the effect on this source is most striking (Roger, Costain, and Purton, 1965). See Figure 39. The measured decameter flux density of 3C84 rises to a value several times greater than that which would be predicted by linear extrapolation of its meter wavelength flux density to lower frequencies.

The spectral index of 3C84 (total source) over the meter and decimeter range — given as 0.70 by Conway, Kellermann, and Long (1963) — is determined to a large extent by the radiation from the core, which has a spectral index of 0.62 (Kellermann, 1964). However, as one goes to lower frequencies, the radiation from the halo (because of
Figure 39. Spectra of the core-and-halo sources 3C47 and Virgo A, whose 20-40 MHz flux density was measured by the author. Also shown for purposes of comparison is the spectrum of the core-and-halo source 3C84 (NGC 1275).
its very high spectral index) has increased greatly, becoming the dominant influence in the radiation from the total source at dekameter wavelengths. Thus, the slope of 3C84's spectrum changes from that determined by the combination of the emission from the core (mainly) and the halo, and curves upward at lower frequencies, asymptotically approaching the spectral index (slope) of the halo alone. Estimations of the halo's spectral index are 1.25 (Kellermann, 1964) and ~2.0 (Roger, Costain, and Purton, 1965).

**Virgo A:** With regard to the spectrum of Virgo A, the change in slope as one goes to lower frequencies (dekameter wavelengths) is hardly noticeable compared to that exhibited by 3C84. The reason for this lack of abrupt spectral change in Virgo A may be explained by the fact that in the meter and decimeter wavelength region the emission from the halo of Virgo A makes up a much larger proportion of the total source emission than it does in the case of 3C84. At 400 MHz the flux density of 3C84's core is 1.8 times greater than the flux density of its halo; however, for Virgo A at 400 MHz the flux density of the halo is 1.43 times greater than the flux density of the core (Kellermann, 1964). Hence, for Virgo A the halo exerts a significant influence on the total-source spectrum well into the meter and even the decimeter range. (In the decimeter range, it appears that the spectrum of the core of Virgo A becomes steeper at higher frequencies.) In the meter-decimeter range, Virgo A already has a steep spectrum; the total-source spectral index (Conway, Kellermann, and Long, 1963) is 0.83, influenced to a significant extent by the spectral index of
the halo — given as 1.02 by Kellermann (1964). Hence, in going from meter to dekameter wavelengths the transition in the spectrum as the influence of the halo predominates should be slight, smooth, and gradual — perhaps, not even clearly measurable, considering the much larger errors inherent in dekameter measurements.

The author's results on Virgo A are consistent with the measurement of Erickson and Cronyn (1965) at 26.3 MHz (4990 ± 930 flux units) and with the measurement of Roger, Costain, and Purton (1965) at 22.25 MHz (5300 ± 810 flux units). However, these results are not consistent with the multifrequency (12.5 to 40 MHz) results of Bazelyan et al. (1963), which show the spectrum of Virgo A to curve downward to lower frequencies. The author believes these latter results to be in error.

3C47: With regard to 3C47, no abrupt change in its spectrum is found. As can be seen in Figure 39, its spectral behavior at lower frequencies is similar to that of Virgo A rather than to that of 3C84. This is to be expected, since its spectral index and relative halo-to-core intensity at meter and decimeter wavelengths are similar to those of Virgo A. The spectral index of 3C47 in the meter–decimeter range is 0.87 (Conway, Kellermann, and Long, 1963). At 408 MHz the flux density of the halo is 70 percent of the flux density of the total source (Anderson et al., 1965). Unlike 3C84 and Virgo A, no separate estimates of the spectral indexes for the halo and the core are available for 3C47. (In general, much less experimental data is available for 3C47 than there is for 3C84 or Virgo A.)
The author's results on 3C47 are consistent (within the limits of each party's total anticipated error) with the measurement of Erickson and Cronyn at 26.3 MHz (178 ± 18 flux units). The single frequency measurement of Erickson and Cronyn by itself is inconclusive in indicating any trend in 3C47's low-frequency spectrum. No measurements on 3C47 are given by Bazelyan et al. or by Roger, Costain, and Purton.

One simple explanation for the spectrum of the halo being steeper than that of the core is based on the premise that the halo is much older than the core. The theory holds that the relativistic electrons responsible for the halo's radiation resulted from much earlier outbursts in the source and have, in the course of time, spread out in space and become what we now see as the halo. Because of the greater escape of the more energetic relativistic electrons from the halo region and effects such as the inverse-Compton effect (loss of energy by relativistic electrons due to interaction with photons less energetic than the electrons), the energy spectrum of the halo's relativistic electrons is steepened with time. The number of electrons \( N(E) \) having energies equal to \( E \) (within some small unit energy bandwidth \( \Delta E \)) is a function of the energy \( E \). If the electron energy spectrum is of the form \( N(E) \propto E^{-\gamma} \), the synchrotron radiation is of the form \( S \propto v^{-\alpha} \), with the two exponents (spectral indexes) related by the expression \( \alpha = \frac{1}{2} (\gamma - 1) \). Hence, a steepening of the energy spectrum (an increase in the slope \( \gamma \)) in the course of time would lead to a steepening of the radiation spectrum (i.e., cause the slope \( \alpha \) to become greater). The relativistic electrons responsible for the core's
radiation were generated at a later time. Thus, the effects which
tend to steepen the electron energy spectrum (and, thereby, the radia-
tion spectrum) have not had as long a time to work on the core elec-
trons; hence, the core has a flatter electron energy spectrum and a
flatter synchrotron radiation spectrum than the halo.

3. Double Radio Sources

Of the extragalactic sources whose 20-40 MHz flux densities were
measured by the author, the five that show evidence of downward curva-
ture at lower frequencies are all double radio sources. These sources
are 3C33, 3C98, 3C123, Hercules A (3C348), and 3C353. Based on inter-
erferometric measurements of these sources at higher frequencies by
various observers, none of these five sources have been found to have
any significant radio halo. With regard to optical identification,
all five of these radio sources have been identified with galaxies.
The characteristics of these five radio sources, based mainly on data
tabulated by Howard and Maran (1965), are shown in Table 33.

The components of the radio emission from these double radio
sources are usually located on opposite sides of the associated gal-
axy, sometimes at rather large angular distances from the galaxy in
comparison to the component size (3C33 and 3C98, for example). The
radio intensity is not particularly correlated with the optical in-
tensity of the associated galaxy. The strongest radio source, Her-
cules A, is identified with a 19th magnitude galaxy, while 3C33 and
3C98 are identified with 15th magnitude galaxies.
TABLE 33. Characteristics of Five Double Radio Sources

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SEPARATION</th>
<th>COMPONENT SIZE</th>
<th>OPTICAL IDENTIFICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C33</td>
<td>3.8'</td>
<td>d ≤ 20&quot;</td>
<td>galaxy</td>
</tr>
<tr>
<td>3C98</td>
<td>3.4'</td>
<td>d ≤ 30&quot;</td>
<td>E galaxy</td>
</tr>
<tr>
<td>3C123</td>
<td>12.5&quot;</td>
<td>d = 5&quot;</td>
<td>galaxy</td>
</tr>
<tr>
<td>Hercules A</td>
<td>1.95'</td>
<td>d = 0.75'</td>
<td>El or SO galaxy</td>
</tr>
<tr>
<td>(3C348)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3C353</td>
<td>2.5'</td>
<td>d = 1.4'</td>
<td>galaxy</td>
</tr>
</tbody>
</table>

The downward curvature at lower frequencies exhibited by these extragalactic double radio sources may be caused by several possible attenuating effects. These include synchrotron self-absorption, the internal refractive index effect (the effect on energy propagation out of the source by a medium having a refractive index less than one), and absorption by galactic ionized hydrogen plasma regions. It will be shown that synchrotron self-absorption, the effect customarily associated with quasi-stellar radio sources, is unlikely to be the cause of the low-frequency downward curvature found for the five double radio sources observed by the author, with the possible exception of the source 3C123.

Using ordinary assumptions concerning electron densities and magnetic fields in a source, the frequency below which the internal refractive index effect starts to become important can occur in the dekameter wavelength range. (At higher electron energies and,
consequently, higher radiating frequencies the effect of the medium of the source on the \textit{magnetobremssstrahlung} is negligible.) It has already been shown earlier in this chapter that absorption by galactic H II can be a cause of increasing attenuation at lower frequencies. Unfortunately, because of the limited accuracy of the flux density measurements and the fact that the maxima in the spectra of the five double sources observed all occur below the frequency range of the observations (20-40 MHz), the author is unable to distinguish between these latter two effects.

The synchrotron self-absorption phenomenon is associated with sources of extremely high surface brightness (Williams, 1963). The quasi-stellar radio sources have moderate flux densities but extremely narrow angular diameters (<1 arc-sec) and, therefore, extremely high source brightness temperatures. It will be shown that, considering their flux densities, the five double radio sources (with the possible exception of 3C123) do not have sufficiently small angular diameters to have the high brightness temperatures required for the occurrence of the synchrotron self-absorption effect.

The flux density of a discrete radio source is given by:

\[ S = \frac{2 k}{\lambda^2} \int_{\text{source}} T_b(\Omega) \, d\Omega = \frac{2 k}{v^2} \frac{c^2}{v^2} T_b \, \Omega_s, \]

where \( T_b \) is the (average) brightness temperature over the source and
$\Omega_s$ is the solid angle of the source. Rewriting in terms of $T_b$, we have:

$$T_b = \frac{c^2}{2k} \frac{S}{\nu^2 \Omega_s} \approx \frac{c^2}{2k} \frac{S}{\nu^2 \phi^2},$$

assuming $\phi^2 \approx \Omega_s$, where $\phi$ is the angular diameter of the source. (Simplified approximations will be used throughout this derivation, which is intended to show only the rough order of the parameters involved.)

The power spectral density of the total radiation from a single electron (Ginzburg and Syrovatskii, 1965) has a maximum at the frequency:

$$v_m = 0.29 \nu_c = 0.29 \frac{3cH_\perp}{4\pi m c} \left(\frac{E}{mc^2}\right)^2$$

$$= 1.8 \times 10^{18} H_\perp E^2,$$

where $\nu_c$ is the characteristic frequency of the magnetobremsstrahlung, $m$ is the electron mass (in grams), $e$ is the electron charge (in esu), $H_\perp$ is the magnetic field intensity (in oersteds) perpendicular to the direction of motion of the electron, and $E$ is the electron energy (in ergs). The electron energy for which the emission maximum occurs at a frequency $v_m$ is:

$$E = 7.5 \times 10^{-10} \nu_m^{\frac{1}{2}} H_\perp^{-\frac{1}{2}} \quad (E \text{ in ergs})$$

$$E = 7.5 \times 10^{-17} \nu_m^{\frac{1}{2}} H_\perp^{-\frac{1}{2}} \quad (E \text{ in joules}).$$
According to Williams (1963), synchrotron self-absorption occurs if the brightness temperature of emission is of the same order as the mean kinetic temperature of the relevant relativistic electrons. Thus, for synchrotron self-absorption to take place,

$$ T_b = \frac{E}{2k} $$

at frequencies $v = v_c$. Substituting into the brightness temperature equation above we have:

$$ \frac{E}{2k} = \frac{c^2}{2k} \frac{S}{v_c^2 \phi^2} $$

Note that to keep $S$ in the customary flux density units of $W \, m^{-2} \, Hz^{-1}$ (= watts $m^{-2}$ sec), we must have $E$ in joules (= watt-sec) and $c$ in $m/sec$. Substituting the expression for electron energy, we have:

$$ 7.5 \times 10^{-17} v_m^{\frac{1}{2}} H_\perp^{\frac{1}{2}} = c^2 \frac{S}{v_c^2 \phi^2} $$

Since $v_m = 0.29 \, v_c$, we have:

$$ 7.5 \times 10^{-17} H_\perp^{\frac{1}{2}} = \frac{9 \times 10^{16} S}{(0.29)^\frac{1}{2} v_c^{\frac{1}{2}} \phi^2} $$

Solving for $\phi^2$, we obtain:

$$ \phi^2 = 2.25 \times 10^{33} \frac{S H_\perp^{\frac{1}{2}}}{v_c^{\frac{5}{2}}} \quad (rad^2) $$

where $S$ is in $W \, m^{-2} \, Hz^{-1}$ and $v$ is in Hz. Note that $S = S_e$, the extrapolated flux density at the frequency $v = v_c$; i.e., the flux density
that would be observed in the absence of absorption. For flux density in flux units \((10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1})\) and frequency in MHz instead of Hz, we have:

\[
\phi^2 = 2.25 \times 10^{-8} \, S_e \, H_\perp^{1/2} \, \nu_c^{-5/2} \, (\text{rad}^2) .
\]

Finally, for \(\phi^2\) in arc-sec\(^2\) instead of radians\(^2\), we have:

\[
\phi^2 = 960 \, S_e \, H_\perp^{1/2} \, \nu_c^{-5/2} \, (\text{arc-sec}^2) .
\]

As a check, we compare this result with that given for synchrotron self-absorption by Kellermann (1966), equation (13):

\[
f_c = 16 \, B^{5/8} \, S_e^{1/8} \, \nu_c^{5/8} \, (\text{MHz}) ,
\]

where \(f_c\) is the frequency (MHz) where the optical depth is equal to unity, \(S_e\) is the extrapolated flux density in flux units, \(\theta\) is the source angular diameter in arc-seconds, and \(B\) is the magnetic flux density in gauss (B in gauss = H\perp in oersted). Solving for \(\theta^2\) we have:

\[
\theta^2 = 16^{5/2} \, B^{5/2} \, S_e \, f_c^{-5/2} = 1024 \, S_e \, B^2 \, f_c^{-5/2} .
\]

Thus, the result of the author's derivation is in close agreement with Kellermann's result (closer even than needed for the purpose intended).

Returning to the author's derivation and solving for \(\phi\), we have:

\[
\phi = 31 \, S_e^{1/2} \, H_\perp^{1/4} \, \nu_c^{-5/4} \, (\text{arc-sec}) .
\]
We will assume that $v_c$ is equal to the turnaround frequency (i.e., the maximum in the radiation spectrum of the source). For all five double radio sources, the turnaround frequency occurs below the frequency range of the author's observations; hence, the turnaround frequency will have to be estimated (roughly) from the shape of the spectrum of the sources (see Figure 36). It will also be assumed that $H_\perp \approx 10^{-4}$ oersted ($B \approx 10^{-4}$ gauss), a customary assumption in similar astrophysical problems. (With $\phi \propto H_\perp^{1/2}$, the dependence on $H_\perp$ is not that critical.) The value of the extrapolated flux density (in the absence of any absorption) at the turnaround frequency must also be estimated from Figure 36. The estimated values of these parameters for the five double radio sources and the resulting angular diameters necessary for synchrotron self absorption to be the cause of the observed downward curvature are shown in Table 34. Note that for self-absorption to be the cause, the actual angular diameters of the components of the double sources should be approximately equal to or less than the angular diameters characteristic of self-absorption shown in this table.
TABLE 34. Angular Diameters of the Five Sources Necessary for Synchrotron Self-Absorption To Be the Cause of the Downward Curvature at Low Frequencies

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>TURNAROUND FREQUENCY (MHz)</th>
<th>EXTRAPOLATED FLUX DENSITY AT $v_c$ (flux units)</th>
<th>ANGULAR DIAMETER (arc-sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C33</td>
<td>15</td>
<td>400</td>
<td>2.1</td>
</tr>
<tr>
<td>3C98</td>
<td>20</td>
<td>350</td>
<td>1.4</td>
</tr>
<tr>
<td>3C123</td>
<td>15</td>
<td>1200</td>
<td>3.6</td>
</tr>
<tr>
<td>Herc A</td>
<td>15</td>
<td>4000</td>
<td>6.7</td>
</tr>
<tr>
<td>3C353</td>
<td>20</td>
<td>1200</td>
<td>2.5</td>
</tr>
</tbody>
</table>

*Flux density that would be observed in the absence of any absorption

Comparing the actual angular diameters of the components of the five double radio sources (Table 33) with those of Table 34, we see that only 3C123 is even close to having a sufficiently small component angular diameter (i.e., a sufficiently high component surface brightness temperature) for self-absorption to cause the observed low frequency attenuation. The angular diameters of the components of the four other double radio sources are all at least a factor of seven too high (have surface brightness temperatures at least a factor of 50 too low). Hence, except possibly for 3C123, we can rule out synchrotron self-absorption as the cause of the low-frequency downward curvature of the double radio sources.

Next, we turn our attention to the internal refractive index effect, sometimes called the Razin effect (Razin, 1960). According to
Ginzburg and Syrovatskii (1965), the plasma in a synchrotron radiation source has a strong influence on the radiation only under the condition:

\[(1 - n^2) \left( \frac{E}{mc^2} \right)^2 \geq 1 .\]

The refractive index \( n \) is a function of frequency and is given by:

\[n = n(v) = \left( 1 - \frac{e^2 N_e}{\pi m v^2} \right)^{\frac{1}{2}},\]

where \( N_e \) is the electron concentration (electrons per cm\(^3\)) in the plasma. Substituting into the previous expression we have:

\[v^2 \leq \frac{e^2 N_e}{\pi m} \left( \frac{E}{mc^2} \right)^2 .\]

In a manner similar to the derivation concerning self-absorption, we define the characteristic frequency of the magnetobremsstrahlung by the expression:

\[\nu_c = \frac{3 e H_\perp}{4 \pi m c} \left( \frac{E}{mc^2} \right)^2 ,\]

and again substituting, we have:

\[v^2 \leq \frac{4}{3} e c \frac{N_e}{H_\perp} \nu_c \quad \text{(all in cgs units).}\]
Since $e = 4.80 \times 10^{-10}$ esu and $c = 3 \times 10^{10}$ cm/sec, we have:

$$v^2 \lesssim 19.2 \frac{N_e}{H_\perp} v_c.$$  

We are interested in frequencies over which the synchrotron radiation is taking place; namely, $v = v_c$. Thus, we have:

$$v \lesssim 20 \frac{N_e}{H_\perp} \text{ (Hz)}$$

as the condition for the refractive index of the medium (the plasma) having an effect on the synchrotron radiation. For frequencies $v \gg 20 \frac{N_e}{H_\perp}$, the effect of the plasma is negligible.

The values of $N_e$ or $H_\perp$ for the five double radio sources are not known. Let us consider values of $N_e$ and $H_\perp$ of 10 electrons per cm$^3$ and $10^{-4}$ oersted ($B = 10^{-4}$ gauss). The value for $N_e$ may seem somewhat large for the average electron density in a radio source, but if we consider it to be the electron concentration in the widely separated compact components of the double radio source, it might be taken as a reasonable value. This justification will not hold for 3C353, which has a component separation of 2.5 arc-min and component diameters of 1.4 arc-min. (The downward curvature of this source is almost surely not caused by the Razin effect.) The use of the above values of $N_e$ and $H_\perp$ yields a value of $v \sim 2 \times 10^6$ Hz as the frequency below which the internal refractive index effect becomes important. This frequency is much too low to explain the observed spectra of the double radio sources. If, however, $H_\perp$ were of the order of $10^{-5}$ oersted
(certainly, a possible value, we would have \( v \sim 20 \text{ MHz} \) as the frequency below which the Razin effect becomes important — a value more compatible with observational results if the Razin effect is to be held responsible for the low-frequency alteration of the spectra. Therefore, it is possible for the internal refractive index effect to be a cause of (but not necessarily the exclusive cause of) the downward curvature at lower frequencies.

The third effect, which must be considered as the most probable cause of the downward curvature in the observed spectra of the five double radio sources, is absorption by H II regions in our own galaxy. The parametric relationships involved in this absorption mechanism have already been discussed in connection with the supernova remnant IC443. The spectral range below 40 MHz is where one expects H II absorption to begin to exert a significant attenuative effect. Unfortunately, we have no specific data on the emission measure of the galactic H II in the general direction of these radio sources.

If galactic H II absorption exerted the major influence on the spectra of the double radio sources, one might (very generally) expect to see the downward curvature start at somewhat higher frequencies for the sources with lower galactic latitudes, since one would expect the emission measure of the H II to be greater for lines of sight nearer to the galactic plane. For example, the galactic latitude \( b^{II} \) of the supernova remnant IC443 is +3°, and the downward curvature of its spectrum is caused by H II absorption. The core-and-halo sources 3C47 and Virgo A are far-removed from the galactic plane,
having galactic latitudes of -41° and +75° respectively. The galactic latitudes and longitudes (new galactic coordinate system) of the five double radio sources are given in Table 35. Comparing Tables 34 and 35, there is no definite correlation between estimated turnaround frequencies (admittedly, very speculative estimates) and the galactic latitudes of the five double radio sources. In any case, the data sample — five sources — is not large enough to give very meaningful results.

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>NEW GALACTIC COORDINATES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>longitude ((\ell^\Pi))</td>
</tr>
<tr>
<td>3C33</td>
<td>129°</td>
</tr>
<tr>
<td>3C98</td>
<td>180°</td>
</tr>
<tr>
<td>3C123</td>
<td>171°</td>
</tr>
<tr>
<td>Herc A</td>
<td>23°</td>
</tr>
<tr>
<td>3C353</td>
<td>21°</td>
</tr>
</tbody>
</table>

It may be concluded that the low-frequency downward curvature of the spectra of the extragalactic double radio sources is probably not the result of synchrotron self-absorption, but is more likely to be the result of galactic H II absorption (most likely) or of the Razin effect, or a combination of both. For the source 3C353, the downward curvature is most likely caused by galactic H II absorption.
CHAPTER V
GALACTIC SPUR

A. OBSERVATIONAL RESULTS

1. Corrected Antenna Temperature Maps

The data from the March 1966 observations have already been used to draw contour maps for the Galactic Spur region bounded by \( \alpha = 12^h \) to \( 16^h \), \( \delta = +2^\circ \) to \( +32^\circ \) (Guidice, 1967a). However, these maps (one for each operating frequency) were limited in east-west resolution; they are in effect one raster maps. The only clearly reliable data was in the "A" raster taken on the night of March 11-12.

The contour maps presented in this paper will be based on the data taken during the March-April 1967 observations. Compared to the 1966 results, the data from the 1967 observations are much more extensive. The data point density is 4 times greater (in general, data points every \( 6^m \) of right ascension instead of every \( 24^m \)); the right ascension range is 50 percent greater (\( 6^h \) instead of \( 4^h \)); the declination range is 17 percent greater (\( 35^\circ \) instead of \( 30^\circ \)). Overall, there should be seven times as many potential data points in the 1967 survey as in the one-raster results of the 1966 Galactic Spur work. A potential data point is defined as a point on the mapped area...
(α = 12h to 18h, δ = +35° to 0°) to which there should be assigned a value of antenna temperature. By reason of the scanning and data reduction techniques used, there is a potential data point every 6" of right ascension and every 1° of declination. Because of radio station interference, equipment problems, etc., all potential data points do not have actual data associated with them for all operating frequencies. Because of overlap, some potential data points may have as many as three measurements of antenna temperature associated with them for some or possibly all operating frequencies. For example, a potential data point in the α = 14h to 17h, δ = +2° to +32° might have an antenna temperature measurement from the 13h-17h raster-scan survey, another from the 14h-18h raster-scan survey, and possibly another from a drift or separate right ascension scan (from the April 1-9 segment of the 1967 observations).

The extent of the lack of data at potential data points depended upon the region of the map and upon the operating frequency. Some areas in the 12h-13h range, where no contour information relating to the Spur was found in the results of the 1966 observations, were not covered in the 1967 observations. For the δ = +2° to +32° raster scan surveys covering 13h-17h and 14h-18h, late starts or early evening interference limited the results for the first hour of right ascension. However, the loss of the 14h to 15h range of the 14h-18h survey was adequately covered by the 13h-17h survey, so only the 13h-14h range was weak in data. In general, the right ascension range 14h to 18h was well covered, including not only the +2° to +32° portion, but the
0° to +2° and the +32° to +35° portions as well (data from the drifts were quite good). With regard to operating frequency, 22.30 MHz was most bothered by radio station interference; 38.75 MHz was bothered the least. There was also a problem with the 26.70 MHz radiometer which prevented data acquisition from the "A" and "C" raster scans of the 14^h-18^h survey and from some drifts. Overall, data was obtained for about 70 percent of the potential data points on the 22.30 and 26.70 MHz operating frequencies and for about 80 percent on the 33.45 and 38.75 MHz operating frequencies.

In processing the data for any given night, the amplitude data from the record were taken, adjusted for any instrumental baseline drift (usually zero, and very small if present at all), and converted to antenna temperatures by means of the temperature calibrations made at the beginning, middle and end of that night. The measured quantity here is uncorrected antenna temperature, since the calibration does not take into account the feed VSWR and cable losses in the antenna line which are not present in the noise generator line. The additional loss factors can be calculated; see Section B 1 of Chapter II. They are independent of radio telescope zenith angle and depend only on operating frequency.

To begin the data reduction process, the records from four nights of the 13^h-17^h raster-scan survey were reduced. Each night covered a different raster. (The record of a fifth night in the first segment of the 1967 observations giving a duplicate "C" raster was not reduced in detail.) Next, the records from the four nights (each covering a
separate raster) of the 14^h^-18^h raster-scan survey from the third
segment of the 1967 observations were reduced and the data were com­
piled. Lastly, the records from the various right ascension scans
and drifts from the second segment of the observations were reduced.
The data from all segments of the 1967 observations were then con­
verted from uncorrected to corrected antenna temperature.

For each of the four operating frequencies, all the data were
plotted on a large sheet of paper (about 5 feet by 2 feet) having a
right ascension and declination grid covering the range 12^h^00^m to
18^h^00^m and 0° to +35°. The grid resolution (i.e., the distance be­
tween grid lines) was 0.5° in declination and 2^m in right ascension.
(Potential data points occurred every 1° of declination and every 6^m
of right ascension.) For many data points, more than one value of
corrected antenna temperature was available from the different ob­servational segments. To obtain the single value of corrected an­
tenna temperature to be assigned to any potential data point, usually
the average of the values from the various segments was taken, after
eliminating—any—obviously erroneous data. All erratically high an­
tenna temperatures (perhaps caused by radio interference) were ex­
cluded. Finally, contours connecting points of equal antenna tem­
perature were then drawn for each large sheet.

Where there was uncertainty or confusion about the contour struc­
ture on one of the sheets (i.e., for one of the operating frequencies),
it was often possible to look to the other sheets for help in making
a proper judgment on the contour structure to be drawn for that sheet.
In almost all cases, information from the higher frequencies (38.75 or 33.45 MHz) was used to help out at the lower frequencies (26.70 or 22.30 MHz).

The contour intervals used for each operating frequency are based on the criterion that the density of the contours should be roughly the same for the four maps. Hence, the contour interval should be proportional to corrected antenna temperature, which is in turn proportional to brightness temperature. As a starting point, the contour interval of the 38.75 MHz map is chosen to be 1000°K. Since \( T_A = T_B = \nu^{-\beta} \) with the brightness temperature index \( \beta \) equal to about 2.5 in the 20-40 MHz range for the region of the Galactic Spur (including the North Galactic Pole region), we construct Table 36. With 1000°K as the 38.75 MHz contour interval, the choices of contour intervals for the remaining frequencies became rather evident.

**TABLE 36. The Contour Intervals for the Corrected Antenna Temperature Maps at the Four Operating Frequencies**

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>( \left( \frac{\nu}{38.75} \right)^{-2.5} )</th>
<th>CONTOUR INTERVAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.75</td>
<td>1.00</td>
<td>1000°K</td>
</tr>
<tr>
<td>33.45</td>
<td>1.45</td>
<td>1500°K</td>
</tr>
<tr>
<td>26.70</td>
<td>2.53</td>
<td>2500°K</td>
</tr>
<tr>
<td>22.30</td>
<td>3.98</td>
<td>4000°K</td>
</tr>
</tbody>
</table>
The corrected antenna temperature contour maps for the four operating frequencies resulting from the March–April 1967 observations are presented in Figures 40 through 43. The contour intervals for each frequency are those specified in Table 36. In comparing these new maps with the maps resulting from the March 1966 observations (GUIDICE, 1967a), the most striking feature is how much the vast, sweeping, circular arc of the Spur stands out from the background. Much of the credit for this advance comes from the increased data point density of the new maps. However, an equally important factor in bringing out the extent of the "circular arc" geometry of the Galactic Spur in the northern celestial hemisphere was the expansion of the 1967 survey to $18^\text{h}$ right ascension. The decision to extend the March–April 1967 observations to $18^\text{h}$ was based on the suggestion of the author's dissertation adviser, Professor Ko.

To verify the internal consistency of the temperature scale of the maps (Figures 40 through 43) over the 20–40 MHz range, one must: (1) find, within the area covered by the survey, a region having uniform brightness temperature; (2) determine the relationship between antenna (or brightness) temperature and frequency over this uniform region; and (3) compare this temperature–frequency relationship with absolute calibrations of brightness temperature vs. frequency made over the same frequency range on the same (or similar) region of the celestial sky using antennas with precisely calculable antenna parameters.
Figure 40. Corrected antenna temperature contour map of the Galactic Spur region for 38.75 MHz.
Figure 41. Corrected antenna temperature contour map of the Galactic Spur region for 33.45 MHz.
Figure 42. Corrected antenna temperature contour map of the Galactic Spur region for 26.70 MHz.
Figure 43. Corrected antenna temperature contour map of the Galactic Spur region for 22.30 MHz.
From Section B 6 of Chapter II, we have:

$$T_A(\theta,\phi) = \eta_M \hat{T}_b(\theta,\phi) + \eta_{SL} \langle T_b \rangle_{SL} + \eta_{SP} \langle T_b \rangle_{SP},$$

with $\eta_M + \eta_{SL} + \eta_{SP} = 1$. The symbols are defined in Chapter II. (In this paper the symbol $T_A$ and the term antenna temperature always denote corrected antenna temperature, unless specified otherwise.)

Since $\langle T_b \rangle_{SP} \approx T_{\text{earth}}$, which is negligibly small compared to celestial brightness temperatures at these frequencies, the last term in the above expression may be dropped, yielding:

$$T_A(\theta,\phi) = \eta_M \hat{T}_b(\theta,\phi) + \eta_{SL} \langle T_b \rangle_{SL},$$

where $\eta_M + \eta_{SL} = 1 - \eta_{SP} = \varepsilon_f$. In a uniform brightness region it may be assumed that $\hat{T}_b(\theta,\phi) = \langle T_b \rangle_{SL}$ and that, except for variations in feed efficiency due to zenith angle differences, the antenna temperature would be constant and directly proportional to brightness temperature. There is a uniform brightness region present in the area of the author's maps; it is the region near the North Galactic Pole $(\alpha = 12^h 49^m, \delta = +27.4^\circ)$. At any of the operating frequencies, the variation in antenna temperature measured in this region was found to be less than seven percent (Guidice, 1967a). For the March-April observations, the average values of (corrected) antenna temperature for this region are given in Table 37.

The antenna temperatures for the North Galactic Pole region are plotted against frequency in Figure 44. The values at the four operating frequencies form a very good fit (on the log-log plot) to a
Corrected antenna temperature (°K) of region near North Galactic Pole

Figure 44. The corrected antenna temperature measured by observations of the region near the North Galactic Pole.
straight line with a slope of 2.45, where the slope is the brightness
temperature spectral index $\beta$ ($T_A \propto T_b \propto v^{-\beta}$).

TABLE 37. Antenna Temperatures at the Four Operating
Frequencies for the Uniform Brightness Region
around the North Galactic Pole

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>ANTENNA TEMPERATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.75</td>
<td>7,700°K</td>
</tr>
<tr>
<td>33.45</td>
<td>11,100°K</td>
</tr>
<tr>
<td>26.70</td>
<td>19,300°K</td>
</tr>
<tr>
<td>22.30</td>
<td>29,700°K</td>
</tr>
</tbody>
</table>

Measurements at frequencies in the 14 to 85 MHz range were made
by Yates and Wielebinski (1966) using low resolution aerials (wave-
length-scaled dipole-and-reflecting screen) for which the beam para-
eters can be calculated quite accurately. Their measurements in-
cluded a region near the South Galactic Pole, a region also having
uniform brightness temperature. Over the 20-40 MHz frequency range
they found $\beta$ in the range of 2.4 to 2.5.

It is also of interest to investigate the consistency of the re-
sults of the March-April 1967 observations (all the maps together)
with maps made by other observers at frequencies well outside the 20-
40 MHz range. Here a difficulty arises in that maps at higher fre-
quencies tend to concentrate on a region of high brightness tempera-
ture or a feature of unusual brightness distribution, rather than on
low brightness uniform regions, such as the galactic poles. Many
observers, making a high resolution map of some feature, measure only relative brightness and arbitrarily take the temperature of the region outside the lowest contour of the feature equal to zero. Low resolution measurements of the spectral characteristics of sky brightness using simple antennas (for which very accurate calibration is possible) seem to have been made mainly in the southern hemisphere. In surveys at 960 MHz by Wilson and Bolton (1960) and at 1390 MHz by Westerhout (1958), observations are limited to bright regions near the galactic plane. In surveys by various observers at frequencies between 64 and 910 MHz, presented by Ko (1958), emphasis is centered on the galactic plane, but some of the maps presented do give an indication of the brightness temperature of the Galactic Spur region for right ascensions of $16^h$ or greater. However, in the region near the North Galactic Pole, no contours are shown.

Fortunately, two surveys, one at 178 MHz by Turtle and Baldwin (1962) and the other at 400 MHz by Seeger et al. (1965), do encompass the region of the North Galactic Pole. In both these surveys, particular attention has been given to the problem of scaling absolute brightness temperature. The final temperature scale of the 178 MHz survey is correct to within 10 percent and the zero level is known to within $15^\circ$K. The temperature scale of the 400 MHz survey is correct to $\pm 5$ percent and the background level (zero level) is accurate to $\pm 6^\circ$K. A careful estimation was made of the average brightness temperature at 178 MHz and 400 MHz over the same region near the North Galactic Pole as was used in the 20-40 MHz determination. For 178 MHz
the average brightness temperature (corrected aerial temperature) is 140°K. For the 400 MHz survey, in which the brightness temperature for large-scale, low-intensity regions is given by $T_b = 0.9$ (units + 14)°K, the average intensity was 8 units, giving a brightness temperature of 20°K. For the author's 20-40 MHz observations, the average of the feed efficiency $\varepsilon_f$ over the NGP regions was 0.91. Since $\varepsilon_f = 1 - n_{SP} = n_M + n_{SL}$, for this uniform region we have $T_A = (n_M + n_{SL}) T_b = 0.91 T_b$. The brightness temperatures for the four frequencies are taken as $(0.91)^{-1}$ or 1.10 times the antenna temperatures given in Table 37.

The various brightness temperatures found for the NGP region are plotted as a function of frequency in Figure 45. The average brightness temperature (8100°K) of this region found in the 38 MHz survey by Kenderdine (1963) has also been included in this figure, although the determination of absolute temperature was not as well established in the survey as in the others. The best fitting straight line has a slope (brightness temperature index $\beta$) of 2.53. The fact that this is slightly steeper than the brightness temperature index (2.45) of the observations in the 20 to 40 MHz range alone is not unexpected, since it is well known that the slope of the cosmic noise background does become less steep as one goes to the lower frequencies.

2. Brightness Temperature Scaling

The final problem in mapping a region is to relate the antenna temperature (a quantity inherently dependent upon the radio telescope)
Figure 45. The brightness temperature as a function of frequency for the region near the North Galactic Pole based on the work of various observers. The brightness temperatures for the 20-40 MHz observations are scaled from the corrected antenna temperature measurements.
to the brightness temperature (a quantity independent of the measurement system). The same type of problem exists in discrete source observations in relating the antenna temperature difference of a source above the background level to the flux density. In the discrete source case the linking parameter is the antenna effective area; in the mapping case the linking parameter is the antenna beam efficiency.

For the 20-40 MHz Arecibo observations the antenna temperature for any celestial region of uniform or very slowly changing brightness is related to brightness temperature by the feed efficiency \( \varepsilon_f \), since in this case \( \langle T_b \rangle_{SL} = \hat{T}_b(\theta,\phi) \) and, therefore, \( T_A(\theta,\phi) = \varepsilon_f \hat{T}_b(\theta,\phi) \). Values of \( \varepsilon_f \) as a function of zenith angle can be obtained from Figure 16. (Within an accuracy commensurate to the temperature scale accuracy of the maps, \( \varepsilon_f \) is independent of frequency.)

The above statements are reasonably true even for a region of uniformly changing brightness. Note the simplified example (a) of Figure 46. The main beam sees a brightness temperature of 12 (arbitrary units) giving a contribution to antenna temperature of \( \eta_M \hat{T}_b(\theta,\phi) = \eta_M \times 12 \). The sidelobes also see an average brightness temperature \( \langle T_b \rangle_{SL} \) of 12; i.e., 13 in the upper quadrant, 11 in the lower quadrant, and 12 in the two lateral quadrants. Hence, the sidelobes give a contribution of \( \eta_{SL} \times 12 \); and \( T_A = \eta_M \times 12 + \eta_{SL} \times 12 = (1 - \eta_{SF}) \times 12 = \varepsilon_f \times 12 \).

However, a problem does arise in the case of a peak in a ridge structure (a type of brightness temperature distribution often present
Figure 46. Representation of the brightness temperatures (contours) seen by the main beam and sidelobes for the case of (a) a region of uniformly varying brightness and (b) a peak in the ridge structure. Contour units are arbitrary, to be used for illustrative calculations.
in the Galactic Spur). Note example (b) of Figure 46. The main beam contribution to antenna temperature is again $n_M \times 12$, but the sidelobe contribution is $n_{SL} \times 11.5$, since the average brightness temperature seen by the sidelobes is 11.5 (12 in the two lateral quadrants, but only 11 in the upper and lower quadrants).

In the case of a small region (angular size in the order of the beamwidth) significantly brighter than the surrounding region, the problem is worse, since all the quadrants of the sidelobes see a brightness temperature lower than the $\nB(\theta, \phi)$ seen by the main beam.

For these latter two cases, it is necessary to use the main beam efficiency $n_M$ itself, rather than the combination $n_M + n_{SL}$. (Once we have $n_M$ directly, $n_{SL}$ is then obtainable since $n_{SL} = 1 - n_{SP} - n_M = \varepsilon_f - n_M$.) The main beam efficiency of the Arecibo radio telescope is analyzed in Section B 6 of Chapter II; a curve of $n_M$ as a function of zenith angle is given in Figure 23. Again, within the accuracy of the temperature scale of the maps, $n_M$ is independent of frequency.

For the simplest approach (fully adequate for regions of uniform or very slowly changing brightness), the actual brightness temperature values could be scaled from the measured values of antenna temperature using the single-parameter expression $T_b = \frac{1}{\varepsilon_f} T_A$, where $\varepsilon_f$ is a known function of telescope zenith angle (see Figure 16). Since during all observations, the value of zenith angle was noted on the eight channel recording paper (see Figures 33 and 34, for example), the zenith angles associated with the antenna temperatures at all the potential
data points are known. The question that arises is how much of an error in scaled brightness temperature results if this simplified scaling approach is used over the whole map.

Let us rewrite the antenna temperature-brightness temperature equation in the form:

$$T_A(\theta, \phi) = n_M \hat{T}_b(\theta, \phi) + n_{SL} \frac{1}{R} \hat{T}_b(\theta, \phi) = (n_M + \frac{1}{R} n_{SL}) \hat{T}_b(\theta, \phi),$$

where $R$ is the ratio of $\hat{T}_b(\theta, \phi)$ to $\langle T_b \rangle_{SL}$. (The sidelobe brightness temperature $\langle T_b \rangle_{SL}$ is weighted toward the temperature close to the main beam since that is where the sidelobe response is strongest.)

With this new equation, the above question has been simplified to the question of when is $n_M + \frac{1}{R} n_{SL}$ significantly different from $n_M + n_{SL}$, which is equal to $\varepsilon_f$. Obviously, the larger the value of $R$ (i.e., the higher the temperature of the bright region in the main beam is above the surrounding temperature), the greater the difference between $n_M + \frac{1}{R} n_{SL}$ and $\varepsilon_f$. Also, and not as obvious, $n_M + \frac{1}{R} n_{SL}$ becomes more significantly different from $\varepsilon_f$ for higher zenith angles, since $n_{SL}$ increases with increasing zenith angle while $n_M$ decreases.

To see this, note Figures 16 (for $\varepsilon_f$) and 23 (for $n_M$). Although both $\varepsilon_f$ and $n_M$ decrease with increasing zenith angle, $n_M$ decreases more rapidly. Hence $n_{SL}$, which is equal to $\varepsilon_f - n_M$, increases with increasing zenith angle. Then, for $R > 1$ in the factor $(n_M + \frac{1}{R} n_{SL})$, the larger $n_{SL}$ becomes in relation to $n_M$ the greater the influence of the $\frac{1}{R} n_{SL}$ term and, hence, the greater the divergence of the value of
Scale factors for values of $R$ and zenith angle ($\xi$) are given in Table 38. To avoid ambiguity, the term scale factor is defined as the ratio of brightness temperature to antenna temperature. Hence, for the 20-40 MHz Arecibo measurements, the scale factors will be greater than one. For $R = 1$ in Table 38, the scale factor is simply equal to the reciprocal of the feed efficiency.

**TABLE 38. Scale Factors for Converting the Maps from Antenna Temperature to Brightness Temperature**

<table>
<thead>
<tr>
<th>ZENITH ANGLE</th>
<th>SCALE FACTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R = 1$</td>
</tr>
<tr>
<td>0°</td>
<td>1.05</td>
</tr>
<tr>
<td>5°</td>
<td>1.06</td>
</tr>
<tr>
<td>10°</td>
<td>1.08</td>
</tr>
<tr>
<td>15°</td>
<td>1.13</td>
</tr>
<tr>
<td>20°</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Let us now discuss in a general way how the antenna temperature maps were scaled to brightness temperature. The technique was the same for all four maps, since frequency was not a consideration in the antenna parameters involved in the scaling.

For portions of the map with fairly constant or very slowly changing brightness, the simple scale factor $1/\varepsilon_f$ was used. Since, as
shown in the discussion involving example (a) of Figure 46, this simple scaling technique is also valid for regions of regularly changing brightness, such as the sides of the ridge structure of the Spur, the \( \frac{1}{\epsilon_f} \) scale factor was used for these portions of the map as well. (This simple technique is not quite valid for the area of the peak in the ridge structure.) Zones of zenith angles (integral degrees) were laid out over all the data points. Where there was more than one zenith angle associated with a data point (because of multiple measurements of antenna temperature from different observational segments), an average zenith angle was used (if it made any significant difference).

Since it made no sense to try to achieve more accuracy in brightness temperature scaling than was actually present in the antenna temperature maps themselves, it was decided to simplify the scaling technique even further. From Figure 16 it may be seen that for zenith angles less than 13°, we have \( 0.95 \leq \epsilon_f \leq 0.90 \); therefore, a scale factor of 1.08 is accurate to within ±0.03 for \( \xi < 13° \) (see also Table 38, \( R = 1 \)). Attempts at scaling accuracy of better than ±3 percent are not justified. Hence, the scale factor 1.08 was used for 0° to 12° zenith angle. Other zenith angles were also lumped together to minimize the number of different scale factors but still keep the scale factor accuracy of ±0.03. The scale factors used for the various zenith angles are shown in Table 39.

Let us now discuss the scale factor for the peaks in the ridge structure. It is found that, except for the bright regions at the
extremes of the Spur's circular arc on the 20-40 MHz maps, all the 
zenith angles for the data points associated with the ridge structure 
peaks are less than 13° and almost all are less than 10°. (The two 
bright regions will be discussed later in the section.)

TABLE 39. Actual Scale Factors Used to Convert to Brightness 
Temperature for Regions of Uniform, Slowly Varying or 
Uniformly Varying Brightness

<table>
<thead>
<tr>
<th>ZENITH ANGLE</th>
<th>SCALE FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0° to 12°</td>
<td>1.08</td>
</tr>
<tr>
<td>13°, 14°</td>
<td>1.11</td>
</tr>
<tr>
<td>15°, 16°</td>
<td>1.14</td>
</tr>
<tr>
<td>17°, 18°</td>
<td>1.17</td>
</tr>
<tr>
<td>19°</td>
<td>1.20</td>
</tr>
</tbody>
</table>

The lower the zenith angle, the less the true scale factor 
for the ridge peak regions will be different from the simple $1/\varepsilon_f$ 
scale factor. Let us use example (b) of Figure 46. Since $\zeta \leq 13°$ for 
all ridge peak points considered here, we take $\zeta = 13°$ as the limiting 

case. For $\zeta = 13°$, $\varepsilon_f = 0.90$, $\eta_M = 0.67$, so that $\eta_{SL} = \varepsilon_f - \eta_M = 0.23$. 
For the contours of Figure 46 (b), an antenna temperature of 10.7 (ar-
bitrary units) would be measured:

$$T_A(\theta, \phi) = \eta_M \, T_b(\theta, \phi) + \eta_{SL} \, \langle T_b \rangle_{SL} = 0.67 \times 12 + 0.23 \times 11.5 = 10.7.$$ 

Hence, the proper scale factor, i.e., $\frac{T_b(\theta, \phi)}{T_A(\theta, \phi)}$, would be 
$\frac{12}{10.7} = 1.12$. The simple scale factor is $1/\varepsilon_f = (0.90)^{-1} = 1.11$. 
Thus, even for the worst zenith angle, the true scale factor is only
one percent higher than the $1/e_f$ scale factor.

One might argue that example (b) of Figure 46 is a favorable situation; for example, for wider ridge peaks and steep ridge contours, the difference between $\frac{N}{D}(\theta, \phi)$ and $\langle T_D \rangle_{SL}$ might be greater and the variance between the proper and the simple scale factor might be more. The author considered other situations more analogous to the maps; even in these cases the variance between the two scale factors was always less than three percent. (It must also be remembered that, for most ridge peak data points, $\zeta < 10^\circ$.)

Since the possible minor improvement in scaling accuracy would not justify the additional work (considering the other sources of inaccuracy in the maps), the simplified scaling technique using Table 39 was also used for the ridge peak regions. It might also be noted that $1.08$, the scale factor of Table 39 used to cover the whole $0^\circ$ to $12^\circ$ range, is one or two percent higher than the actual value of $1/e_f$ for low zenith angles ($< 9^\circ$). Hence, being one or two percent too high for the other types of regions just examined, the scale factor 1.08 is about exactly correct for the ridge structure peak regions (where $\zeta$ was almost always $< 10^\circ$).

The only important portions of the antenna temperature maps that contain small regions significantly brighter than the surrounding area and that also have high zenith angles associated with them are found in the lower left and lower right hand portions of the maps. To warrant special scaling treatment, high zenith angle ($> 10^\circ$) is also a
requirement since, for low zenith angles, the proper scaling factor for the bright region is greater but not drastically greater than the value (1.08) called for by Table 39. The proper brightness temperatures for these two regions were scaled along the lines of the approach used in obtaining Table 38. Although beamwidth varies with frequency, for the purposes of scaling the extent of the main beam was considered to be 3° (circular) for all frequencies and the sidelobes were considered to be contained in a circle about 15° in diameter (excluding the main beam) centered on the beam axis.

The bright region centered around $\alpha = 17^h 40^m$, $\delta = +7^o$ seems to be definitely associated with the Spur. As for the region around $\alpha = 12^h 50^m$, $\delta = 0^o$, it would appear that some of the contours are associated with the Spur, but the central portion (at $\alpha = 12^h 30^m$, $\delta = +2^o$) is probably attributable solely to the extragalactic discrete source 3C273.

Obviously, the very strong discrete radio sources, such as Virgo A or Hercules A, cause the area within a beamwidth or so to be an area of great brightness. However, the positions of these strong sources are well known and the areas surrounding these sources are not included in the mapping process (for either the antenna temperature or the brightness temperature maps). These strong sources are in no way connected with the Spur.

After scaling all the data points to brightness temperature, the new scaled data for each operating frequency was again plotted on a separate large grid sheet and contours connecting points of equal
brightness temperature drawn. Since the scale factor for most portions of the maps is not much greater than one, the same contour intervals (see Table 36) used for the antenna temperature maps were also used for the brightness temperature maps. The brightness temperature maps for the four operating frequencies are presented in Figures 47 through 50. Since the parameters involved in the scaling are not dependent upon frequency, the temperature relationship between maps of different operating frequencies are the same as in the antenna temperature maps.

Regarding the absolute brightness temperature accuracy of the maps, it is believed that the background level is correct to within ± one-half a contour interval (±500°K for 38.75 MHz, ±750°K for 33.45 MHz, ±1250°K for 26.70 MHz, ±2000°K for 22.30 MHz) and that the final brightness temperature scale is correct to within ±7 percent.

The Galactic Spur results should be considered absolute measurements because, with direct reading null-balancing radiometers and with all extraneous noise temperatures negligibly small compared to the sky temperature in this frequency range, zero antenna temperature can be directly related to zero brightness temperature. For both sets of maps, the results are internally consistent; i.e., the temperature levels on the maps are proper relative to each other over the frequency range. This is shown by the demonstration that for a region of uniform brightness, such that \( T_A = \text{constant} \times T_b \), the antenna temperatures determined at the four operating frequencies closely follow a power law relationship \( T_A \propto T_b \propto \lambda^\beta \) with \( \beta = 2.45 \). This value of \( \beta \) agrees very well with the \( \beta \) found for the South Galactic Pole region over the same
Figure 47. Brightness temperature contour map of the Galactic Spur region for 38.75 MHz.
Figure 48. Brightness temperature contour map of the Galactic Spur region for 33.45 MHz.
Figure 49. Brightness temperature contour map of the Galactic Spur region for 26.70 MHz.
Figure 50. Brightness temperature contour map of the Galactic Spur region for 22.30 MHz.
frequency range by experimenters using simple wavelength-scaled antennas having precisely determinable beam characteristics.
B. THEORETICAL INTERPRETATION

1. Theories of the Spur's Origin

While the existence of the bright band of radiation emerging from the galactic plane at $\lambda^{\text{II}} = 30^\circ$ has been known for years, no conclusive determination of its origin has been made. So far, no one has been able to identify this unusual radio feature with anything observed optically. Various theories have been advanced; the three most relevant ones are discussed in the following paragraphs.

Rainbow Theory: One theory of the Galactic Spur is that the radiation is caused by directional emission of relativistic electrons spiraling around a galactic magnetic field which is aligned along a spiral arm (Tunmer, 1958; 1959). In this type of "rainbow" effect, the bright emission depends on direction and does not necessitate a large volume emissivity. Assuming random motion of the relativistic electrons, those with velocities along the spiral arm would diffuse quickly and soon be lost. Those whose velocities are nearly perpendicular to the field would diffuse slowly and form a large population of electrons moving in nearly circular paths around the magnetic field in the spiral arm. The emission (synchrotron radiation) from these electrons would be perpendicular to the spiral arm. An observer inside the spiral arm would see a bright ring whose axis is the direction of the magnetic field. The bright band of the Spur is taken to be part of a great circle (in galactic coordinates) crossing the galactic plane around $\lambda^{\text{II}} = 30^\circ$ and $\lambda^{\text{II}} = 180^\circ$ and running close to the galactic poles.
Strong objections to this theory are made on several counts. The theory is not consistent with the results of Mills (1959), which show that radiation from a spiral arm is roughly isotropic. The distribution of relativistic electrons is likely to be substantially isotropic, similar to that found for cosmic rays. Also, the location of the Spur does not agree with the direction of the local spiral arm. Finally, the geometry of the Spur's radiation belt does not support the assumption of its being a great circle. Based on the results of several surveys, the Spur appears to be a patchy loop, in the form of a small circle (subtending an angle of about 90°) which lies mostly above the galactic plane.

Minor Spiral Arm Theory: Another theory of the Galactic Spur is that the radiation originates in the spiral structure of the Galaxy and that the Spur corresponds to a minor arm or inter-arm link extending from a (major) spiral arm lying near the solar system (Hanbury-Brown, Davies, and Hazard, 1960). The presence of this type or irregularity in the spiral structure — short minor arms extending from the major spiral arms — was studied by Gum (1955), who referred to these features as "inter-arm links" or, when they are short, as "fins".

There are several strong objections to this theory. Any nearby feature such as a minor spiral arm should also be readily observable in the existing optical and neutral hydrogen surveys; however, an examination of the distribution of stars and H II regions does not give any evidence of a minor arm in the position required by this theory. The
distribution of radiation across a spiral arm (major or minor) should be symmetrical on the average (i.e., the average along the arm). However, Davies (1964) reports that his observed isophotes at 237 MHz do not agree with the "symmetrical" bright-band distribution expected theoretically for a minor spiral arm. Also, in theory the length of any minor spiral arm (inter-arm link or fin) should be at most, a 90° arc. However, it can be shown that the Spur is shaped into at least a semicircle (Large, Quigley, and Haslam, 1962), and possibly covers about three-quarters of a circle.

**Supernova Remnant Theory:** The third theory of the Spur's origin (and the one which at present seems to be most acceptable) is that the Spur is a supernova remnant whose center is at a relatively close distance (30 to 50 parsecs) from the sun (Hanbury-Brown, Davies, and Hazard, 1960; Davies, 1964). In contrast to the "rainbow" or "minor arm" theories, which consider the Spur an integral part of the structure of the Galaxy, the supernova theory considers the Spur to be a superimposed or foreground object.

The radio emission from the remnants of a supernova (exploding star) is caused by synchrotron radiation from electrons with relativistic velocities that have been hurled a great distance away from the center of the original explosion of the star. One of the important characteristics of supernova remnant radio sources is that they are shell sources; the emission comes from relativistic electrons in a shell, more or less spherical, which may be quite thin compared to the
radius. (There may be multiple shells, an indication of multiple explosions in the original star.) In an explosion, the ejection of matter is not uniform. Hence, the distribution of electrons across the shell is not uniform and the synchrotron radiation emitted from the spherical shell is patchy.

The theory that the Spur is a supernova remnant is based on experimental evidence showing that the brightness distribution, spectra and polarization characteristics of the Spur are similar to those found for other supernova remnants (shell sources). To understand how the radiation from a spherical shell gives rise to a bright ring intensity distribution, see Figure 51. For simplicity, a shell of uniform thickness and electron density and constant emissivity is assumed. For all frequencies above a few MHz, the tenuous ionized shell is optically thin. Therefore, the observed brightness temperature will be proportional to the optical depth of the intervening medium; i.e., proportional to the line-of-sight path length through the shell. For the model shown in Figure 51, the observed intensity distribution represents the bright limb of the disk presented to an observer on the sun (or the earth) by a spherical shell source. Figure 51 shows the spherical shell source at such a distance, relative to its radius, that the bright ring subtends an angle of roughly 90° as viewed from the sun, in accordance with observational findings. In reality, the electron distribution and, therefore, the radiation from the Spur's shell is patchy; hence, in actual observation, the bright ring appears broken and incomplete.
Figure 51. Left, a shell source model at such a distance (relative to its radius) from the sun that it subtends an angle of roughly 90°. Right, the theoretical diametric (limb to limb) profile of such a source as seen from the sun (assuming the source is optically thin).
Davies (1964) found the spectral characteristics of the Spur to be similar to those of the Cygnus Loop, a Type II supernova remnant with a diameter of about 40 parsecs at a distance of about 770 parsecs (Minkowski, 1959). Both objects have similar brightness temperatures at 1415 MHz and 237 MHz. Davies obtained a brightness temperature spectral index $\beta$ of roughly 2.55 for the Spur, quite similar to the $\beta$ of 2.45 of the Cygnus Loop. Measurements of the linearly polarized component of galactic background emission at 408 MHz by Westerhout et al. (1962) have shown polarization in the position of the outer edges and ridges of the Spur. The spectrum and the observation of polarization for the Spur indicate that the emission comes from synchrotron radiation.

A serious weak point of the supernova remnant theory of the Spur's origin is the lack of any corresponding optical emission. In the case of other supernova remnants such as Cassiopeia A, IC443, and the Cygnus Loop, corresponding filamentary emission has been found at optical wavelengths. In the Cygnus Loop, the thin filaments are optically bright and the optical emission is roughly correlated in position with the radio emission. In IC443, the maximum of the radio emission coincides with the strongest optical feature (Hogg, 1964). As for the Spur, no corresponding object has been found in the Palomar Sky Atlas. A search using a fast camera and narrowband filters (Davies, Hanbury-Brown, and Meaburn, 1963) failed to show any signs of an optical remnant.
2. Geometry of the Spur

Maps of galactic radio emission in the northern hemisphere have shown an unusual protrusion or peninsula that appeared to emerge from the galactic plane at a galactic longitude (l°) of about 30° and to extend from the plane toward the general direction of the North Galactic Pole. Because of the shape of this prominence in the celestial sky as seen in the maps of the pre-1960 observations, the feature was called a "spur". Because of its proximity to the North Galactic Pole, this "spur" became known as the North Polar Spur.

Because of limited resolution at lower frequencies and limited sensitivity at the higher frequencies, the actual shape of the North Polar Spur was not well defined beyond (east of) $\alpha = 16^h$, $\delta = +20^\circ$ in the early investigations. These investigations included work at 81 MHz by Baldwin (1955), at 250 MHz by Ko and Kraus (1957), at 600 MHz by Piddington and Trent (1956), and at 910 MHz by Denisse, Leroux, and Steinberg (1955). The results (including maps) of these and several other early investigators of the galactic background radiation were summarized in a paper by Ko (1958).

Hanbury-Brown, Davies, and Hazard (1960) suggested that the Spur was more than just a small protrusion jutting out of the galactic plane, and was actually part of a bright, circular, somewhat broken ring of large angular extent. Later surveys by Turtle and Baldwin (1962) at 178 MHz and by Seeger et al. (1965) at 400 MHz have shown the great extent of the Spur. The Spur does not end at $16^h$ right ascension; indeed, after reaching $\alpha = 14^h$ around $\delta = +18^\circ$, it curves back toward $\delta = 0^\circ$. 
at $\alpha = 12^h30^m$, as shown in this paper (see Figures 47-50). Davies (1964) extended observations of the Spur into the southern hemisphere and found it continued beyond $\alpha = 12^h30^m$, $\delta = 0^\circ$. Since the Spur clearly curved away from (and was in no way associated with) the North Galactic Pole, he referred to it instead as the Galactic Spur.

In Figure 52, the author has drawn on the 38.75 MHz antenna temperature map a cross-hatched band covering the ridge peak of the circumferential arc extending from $\alpha = 18^h$, $\delta = 0^\circ$ around to $\alpha = 12^h30^m$, $\delta = 0^\circ$. This band will be taken as the northern hemispheric portion of the bright ring that will be used in establishing the geometry of the Spur.

To show the geometry of the Spur, the map from the 600 MHz survey of the cosmic radio emission by Piddington and Trent (1956) will be used. See Figure 53. This contour map is to be used only for geometric purposes; it should not be used to infer spectral information. The temperatures shown are not absolute but are temperatures above a minimum value persisting in the cold parts of the celestial sky. Also, the quantity shown in the contours is "beam temperature" obtained from "effective aerial temperature" (corrected antenna temperature) by using the scale factor $1/0.65$, the reciprocal of main beam efficiency. The use of $1/\eta_M$ as the scale factor assumes that the effective sky temperature seen by the sidelobes ($\left\langle T_B \right\rangle_{SL}$ in the author's notation) is equal to zero. Although this assumption may be more permissible for higher frequencies, it would still limit the temperature scaling accuracy at 600 MHz.
Figure 52. Cross-hatched band showing the ridge peak of the Spur in the form of a circular arc. In the background is a corrected antenna temperature contour map (38.75 MHz) showing the brightness distribution of the Galactic Spur region in the northern hemisphere.
Figure 53. The bright ring of the Spur drawn as a continuous band, as envisioned by the author. The contour map in the background is from the 600 MHz survey of Piddington and Trent (1956). Areas where supplementary surveys were used are shown in marked off rectangles.
On the Piddington and Trent map, a continuous band has been drawn to show the shape of the bright ring of the Galactic Spur, as envisioned by the author. The portion of the band above 0° declination comes from the 20-40 MHz maps of this paper. The remainder of the circular ring is based on the temperature contours of the Piddington and Trent map, and on those from the maps of Davies (1964) at 237 MHz and Seeger et al. (1965) at 400 MHz. The latter survey was particularly helpful for covering the region around α = 12h30m from δ = +10° to δ = -35°. For the small area around α = 13h30m, δ = -45°, a map of the Centaurus A region at 960 MHz by Bolton and Clark (1960) was of use. The areas where these additional surveys were used to supplement the Piddington and Trent map are shown by the marked off rectangles in Figure 53.

The bright ring of the Spur in Figure 53 does not appear to be particularly circular. The asymmetry is exaggerated by the Mercator projection of the map. For an arc running along roughly constant declination, the actual length in degrees is equal to 15 (° of R.A.) cos δ, while on the Mercator projection it is shown as 15 (° of R.A.). Hence, the actual arc length of the portion of the ring between α = 19h and α = 15h (-55° to -65° declination) is only one-half that shown in the Mercator projection of Figure 53 (cos δ ≈ cos 60° = 0.5).

If the circular arc of the Spur did not extend over such a large range of right ascension (~ 7°), a sinusoidal equal-area projection would be the best to show the shape of the Galactic Spur. However, this type of projection, which was used by Seeger et al. (1965) for the
presentation of the maps from their 400 MHz survey, is practical over only about 4\(^h\) of right ascension. If the right ascension range covered in Figure 53 is divided into two portions, one for 16\(^h\) to 20\(^h\) and the other for 12\(^h\) to 16\(^h\), this type of projection can be utilized as shown in Figure 54. To make it simple, the contours of the Piddington and Trent map were not transferred to Figure 54; only the envisioned bright ring of the Spur has been drawn in the new projection.

The finding that the Spur's arc of radiation is somewhat elliptical and/or asymmetric rather than perfectly circular does not particularly weaken the argument for the supernova remnant theory of the Spur's origin. In any explosion (a supernova, for example) it would be highly unlikely that all matter would be ejected with uniform distribution and velocity. The incompleteness of the ring of the Spur itself, or of the Cygnus Loop, attests to the lack of uniform distribution of the relativistic electrons in the expanding shell. As for the lack of uniform expansion velocity of the shell (i.e., asymmetry in the shape of the bright limb of the shell source), Hogg (1964) found such asymmetry in the supernova remnant IC443.

To investigate further the geometry of the Spur's arc, the author made use of a stereographic projection of the celestial sphere. A stereographic projection is a map projection showing the celestial sphere's lines of right ascension and declination projected onto a tangent plane by radials from a point on the surface of the sphere opposite to the point of tangency. For example, if the point of tangency of the tangent plane is one of the celestial poles, then the origin of the radials
Figure 54. Sinusoidal equal-area projection showing the bright arc of the Spur. This type of projection eliminates the distortion at large declinations occurring in a Mercator projection.
is the other celestial pole. This type of projection provides an important tool for investigating large circular arcs; it distinguishes between great circles and large small circles. A great circle projects on the stereogram as a circular arc (or straight line) which intersects the primitive (the projection of the equator) at diametrically opposite points. However, a small circle projects as a circle on the stereogram and does not intersect the primitive at diametrically opposite points.

At the center of the author's stereogram, shown in Figure 55, is the south celestial pole. For reference, the galactic plane has been drawn on the stereogram; note that it intersects the celestial equator ($\delta = 0^\circ$) at diametrically opposite points. The thick part of the Spur's ring (from $\alpha = 18^h, \delta = 0^\circ$ to $\alpha = 12^h30^m, \delta = 0^\circ$) comes from the author's 20-40 MHz maps; the remainder of the ring (the thinner part) comes from the surveys of Piddington and Trent, Seeger et al., Turtle and Baldwin, etc. (see Figure 54). By means of this stereographic projection, the radiation from the Spur is shown to be in the form of a small circle.

Large, Quigley, and Haslam (1962) used a stereographic projection to investigate the Cetus Arc, another unusual feature of the radio sky. For comparison purposes in the same paper, the North Polar Spur was shown on another stereographic projection of the celestial sphere. As far as the geometry of the Spur is concerned, there are some slight differences between their view of the shape of the Spur's bright band and the author's view, but these differences are not significant. The Spur in the stereographic projection of Large, Quigley, and Haslam is also clearly part of a small circle.
Figure 55. Stereographic projection of the celestial sphere showing the Galactic Spur. The south celestial pole is at the center of the stereogram. The thick line shows the bright arc of the Spur in the northern hemisphere (from this paper's maps). The thin line shows the continuation of the Spur into the southern hemisphere based on other surveys. The thin dashed line is the remainder of a circle best fitting the northern hemispheric arc of the Spur.
The demonstration that the Spur's radiation is not in the form of a great circle constitutes important evidence against the rainbow theory of Tunmer (1958). The finding that the radiation from the Spur can be traced over such a large arc-length of the small circle, (i.e., nearly three-quarters of the circumference) constitutes evidence against the Spur being a minor spiral arm (an inter-arm link or fin).

The galactic coordinates of the direction to the center of the Spur (i.e., the center of the small circle) are roughly $l = 345^\circ$ and $b = +33^\circ$. Thus, the center of the Spur is well above the galactic plane. This finding constitutes evidence that the Spur is not a part of the galactic spiral structure, but is a local phenomenon; i.e., the Spur is a foreground object. This, of course, supports the nearby supernova remnant theory of the Spur's origin.

From Figure 55 (or Figure 53 after compensating for the distortion), the celestial coordinates for the center of the Galactic Spur are found to be $\alpha = 15^h10^m$, $\delta = -17^\circ$. Hanbury-Brown, Davies, and Hazard (1960) gave the direction to the Spur's center as roughly $\alpha = 15^h$, $\delta = -20^\circ$. Large, Quigley, and Haslam (1962) located the center of the North Polar Spur (the whole feature including the continuation into the southern hemisphere) at $\alpha = 15^h00^m \pm 10^m$, $\delta = -15^\circ \pm 2^\circ$. The coordinates for the Spur's center found in this section will be used in the remaining parts of this chapter in the processes of obtaining radial profiles across the northern hemispheric portion of the Spur's ridge structure and in reconstructing a simplified shell source model of the Spur based on the 20-40 MHz brightness temperature maps.
3. Spectrum of the Spur

Over the whole region of the author's maps it was found that the brightness temperature spectral index of the Spur, in general, is not very much different from that of the North Galactic Pole region (where $\beta$ is equal to 2.45). To obtain a more specific result, the author decided to measure the spectral index on the ridge structure peak for twelve coordinate points spaced out along the bright circumferential arc of the Galactic Spur. The results of these determinations of spectral index are shown in Table 40. The accuracy of the measurement of $\beta$, not including the inaccuracies in the contour maps from which the values of brightness temperature at the four operating frequencies were taken, is about 0.1.

For the brighter parts of the Spur region, $\beta = 2.5$ at the ridge peaks. For the areas of the maps where the contours along the ridge peak are lower (i.e., east of $a = 14^h$), $\beta$ for the Spur seems to be a little less. The accuracy of the maps and of the $\beta$ determinations does not warrant any strong conclusions here. The result might be explained if, accepting the supernova remnant theory, one assumes that the $\beta$ of the foreground object (the shell source) is very slightly higher than the $\beta$ of the background. Then, where the contour levels are low, the background brightness temperature predominates, and the $\beta$ measured is the $\beta$ of the background; for example, the $\beta = 2.45$ of the North Galactic Pole region. Where the contour levels are high, the emission of the Spur predominates and, hence, the $\beta$ measured is the $\beta$ of the shell source emission. Considering all the possible inaccuracies involved, the author determines the brightness temperature spectral index of the
Galactic Spur in the 20-40 MHz range to be equal to 2.50 ± 0.15.

TABLE 40. Brightness Temperature Spectral Index for Twelve Points on the Ridge-Structure Peak along the Spur's Bright Arc

<table>
<thead>
<tr>
<th>COORDINATES</th>
<th>BRIGHTNESS TEMP SPECTRAL INDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rt. Ascension Declination</td>
<td></td>
</tr>
<tr>
<td>17°42' +07°</td>
<td>2.5</td>
</tr>
<tr>
<td>17°24' +10°</td>
<td>2.5</td>
</tr>
<tr>
<td>17°06' +12°</td>
<td>2.5</td>
</tr>
<tr>
<td>16°54' +15°</td>
<td>2.5</td>
</tr>
<tr>
<td>16°30' +16°</td>
<td>2.5</td>
</tr>
<tr>
<td>16°12' +18°</td>
<td>2.5</td>
</tr>
<tr>
<td>15°54' +20°</td>
<td>2.5</td>
</tr>
<tr>
<td>15°24' +20°</td>
<td>2.5</td>
</tr>
<tr>
<td>14°36' +20°</td>
<td>2.4</td>
</tr>
<tr>
<td>14°06' +18°</td>
<td>2.45</td>
</tr>
<tr>
<td>13°24' +16°</td>
<td>2.45</td>
</tr>
<tr>
<td>12°36' +03°</td>
<td>2.45</td>
</tr>
</tbody>
</table>

It is interesting to compare the spectrum of the Galactic Spur with the spectra of other celestial objects known (or thought) to be supernova remnants. First, however, we must clarify the term "spectral index". The brightness temperature spectral index is defined by $T_b \propto \lambda^{-\beta} \propto \nu^{-\beta}$. However, most supernova remnant radio sources, although certainly not point sources, have angular diameters which are usually smaller than the telescope beamwidth. Hence, the spectral index quoted
is usually the flux density spectral index $\alpha$, defined by $S \propto \lambda^\alpha \propto \nu^{-\alpha}$. The relation between the two indexes is $\alpha = \beta - 2$. Thus, for the Galactic Spur, $\alpha$ is equal to 0.50 for the 20–40 MHz range.

Table 41 gives the flux density spectral indexes for some known and suspected supernova remnants. Unless noted otherwise, the spectral indexes quoted are for the range 100 to 1500 MHz. The spectral indexes for supernova remnants are usually constant over a wide frequency range, often to frequencies beyond 10 GHz.

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kepler's Supernova (CTA78)</td>
<td>0.65</td>
</tr>
<tr>
<td>Puppis A (CTA36)</td>
<td>0.61</td>
</tr>
<tr>
<td>IC443 (3C157, CTA41)</td>
<td>0.35</td>
</tr>
<tr>
<td>HB21 (CTA91)</td>
<td>0.34</td>
</tr>
<tr>
<td>Tycho's Supernova (CTA2, 3C10)</td>
<td>0.67</td>
</tr>
<tr>
<td>W44 (3C392, CTA83)</td>
<td>0.56</td>
</tr>
<tr>
<td>Crab Nebula (3C144, CTA36)</td>
<td>0.27</td>
</tr>
<tr>
<td>Cassiopeia A (3C461, CTA105)</td>
<td>0.77</td>
</tr>
<tr>
<td>Cygnus Loop (CTA93)</td>
<td>0.45</td>
</tr>
</tbody>
</table>
| Galactic Spur (North Polar Spur) | 0.55$^a$

$^a$Davies (1964), 237 to 1415 MHz

The spectral index of the Galactic Spur is right in the middle of the range of spectral indexes for other known supernova remnants. This
finding supports the supernova remnant theory of the Spur's origin, but it constitutes no proof of that theory. On the other hand, if the spectral index of the Spur lay outside the range of most supernova remnants (for example, if \( \alpha \) were less than 0.1 or greater than 1.0), this would have to be construed as important evidence against the supernova remnant theory. The spectral index of the Spur, together with the observations of polarization (Wielebinski, Shakeshaft, and Pauliny-Toth, 1962; Westerhout et al., 1962), does clearly support the idea that the emission mechanism of the Spur is synchrotron radiation; however, this idea, which is compatible with most of the theories of the Spur's origin, was not usually in dispute.

The Spur's spectral index at low frequencies (\( \beta = 2.50 \)) is roughly the same as at higher frequencies (\( \beta = 2.55 \) over the 237-1415 MHz range). Also, no downward curvature towards lower frequencies was found over the 20-40 MHz range in the spectra drawn for any of the points along the Spur's bright arc (Table 40). The most likely cause of any downward curvature, if it were to exist, would be galactic H II absorption. The finding that no downward curvature exists in the 20-40 MHz range shows that the emission measure of any intervening ionized hydrogen plasma (between the sun and the Spur) must be low. This can be interpreted as indicating that the distance between the sun and the Spur is small; hence, the Spur is a "local" object. This interpretation would, of course, support the nearby supernova remnant of the Spur's origin.
4. Radial Profile

To show that the contour structure of the Spur displayed in the 20–40 MHz maps is indicative of a shell source, a radial profile (brightness temperature vs. central angle, similar to Figure 51) is most effective. First, one must choose a celestial coordinate point through which the radial profile will be drawn. (For a radial profile, the other point defining the line on the map obviously must be the coordinate point of the center of the Spur.)

We should like to obtain a radial profile for which the edge of the Spur is well defined; i.e., such that beyond the edge of the Spur the radial profile line goes off into an unconfused area, such as the uniform-brightness North Galactic Pole region. The immediate choice that comes to mind is a radial profile through the North Galactic Pole itself ($\alpha = 12^h49^m, \delta = +27.4^\circ$). However, this radial line would cut over a part of the Spur where the contour levels are rather low. It would be better if it cut the ridge of the Spur further to the west, where the contour levels are higher. A radial profile line through the point $\alpha = 14^h, \delta = +20^\circ$ would cut through higher contour levels of the Spur and still continue off into a low temperature unconfused region. We will therefore choose this line as the radial profile line on the maps. The radial profile chosen here is the same as the one used by the author to analyze the maps resulting from the March 1966 observations (Guidice, 1967a).

With $\alpha = 15^h10^m, \delta = -17^\circ$ established as the celestial coordinates of the direction to the Spur's center, the point $\alpha = 14^h, \delta = +20^\circ$
corresponds to a central angle of 41°. The central angle to a point on
the radial profile is defined as the angle measured from the sun be-
tween the line joining the sun and this point and the line joining the
sun and the Spur's center (see Figure 51). The brightness temperature
at various central angles along the radial profile are obtained (at
2.5° intervals) from the brightness temperature maps (Figures 47 through
50). Because of the southern boundary (δ = 0°) of these maps, bright-
ess temperature values are available only for central angles of 20° or
greater.

For the investigation of the ridge structure of the Spur we are
interested in spatial rather than spectral characteristics; hence, in-
stead of a separate radial profile for each frequency, it was decided
to obtain one composite radial profile utilizing the data from all four
frequencies. The data used for this composite profile should be com-
posed of equally weighted data from the four maps. However, since
brightness temperature is highly frequency dependent, for equal weight-
ing one must first normalize the brightness temperatures obtained from
the maps. (Without normalization, any resulting composite profile
would be overly weighted toward the low frequency results.) For each
operating frequency, all the brightness temperatures will be normalized
to the value of the North Galactic Pole region. By definition then,

\[ T_n(\theta, \phi) = \frac{T_b(\theta, \phi)}{T_b(bgd)} \]

and \( T_n(bgd) = 1 \) for each operating frequency, where

\( T_n \) is the normalized temperature (dimensionless) and "bgd" indicates
the background or North Galactic Pole region. The brightness tempera-
tures of the NGP region are shown in Figure 46; they were obtained by
multiplying the antenna temperature values of Table 37 by the scale factor 1.10.

Table 42 gives the average normalized temperature (average of the four frequencies) at the various central angles. The composite radial profile determined from the 20-40 MHz maps is shown in Figure 56.

TABLE 42. Average Normalized Temperatures for Various Central Angles for a Radial Profile through the Point \( \alpha = 14^\circ, \delta = +20^\circ \)

<table>
<thead>
<tr>
<th>CENTRAL ANGLE</th>
<th>( T_n ) (avg. of four freq.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;50°</td>
<td>~1.00</td>
</tr>
<tr>
<td>47.5°</td>
<td>1.12</td>
</tr>
<tr>
<td>45°</td>
<td>1.43</td>
</tr>
<tr>
<td>42.5°</td>
<td>1.74</td>
</tr>
<tr>
<td>40°</td>
<td>1.99</td>
</tr>
<tr>
<td>37.5°</td>
<td>1.85</td>
</tr>
<tr>
<td>35°</td>
<td>1.66</td>
</tr>
<tr>
<td>32.5°</td>
<td>1.51</td>
</tr>
<tr>
<td>30°</td>
<td>1.42</td>
</tr>
<tr>
<td>27.5°</td>
<td>1.37</td>
</tr>
<tr>
<td>25°</td>
<td>1.37</td>
</tr>
<tr>
<td>22.5°</td>
<td>1.39</td>
</tr>
<tr>
<td>20°</td>
<td>1.37</td>
</tr>
</tbody>
</table>

The similarity between the experimentally determined radial profile of the Spur (Figure 56) and the theoretical shell-source radial
Figure 56. Experimentally determined radial profile through the coordinate point $\alpha = 14^h$, $\delta = +20^\circ$. 
profile (half the diametric profile of Figure 51) is quite apparent. There are two important aspects of the experimental profile, both of which are indicative of a shell source. First, the brightness temperature inside the Spur (i.e., south of the peak in the east-west ridge structure) is greater than the brightness temperature outside. Secondly, the outside edge of the radial profile is definitely steeper than the inside edge. From the peak at 40° central angle the normalized temperature of the profile has decreased to roughly the same value at 45° (on the outside edge) as it has at 30° (on the inside edge). This steeper outside edge, as noted by Davies (1964), is definitely indicative of a shell source. The same indications of shell source structure in the Spur were also found in the radial profile constructed from the maps of the March 1966 observations (Guidice, 1967a).

Because of the beamwidths (2° to 3.3°) used in 20-40 MHz observations, it is probable that the observed profile is somewhat broadened or smoothed out compared to the true brightness distribution. However, this fact does not weaken the evidence for the Spur's being a shell source presented in the previous paragraph. The smoothing effect of non-negligible beamwidth is proportionately much greater to a region of steep rise than to a region of shallow rise. (On an extended flat region, it would have no effect.) If the distribution smoothed by convolution with an antenna beam shows the slope of the steeper edge to be two times the slope of the more gradual edge, the true (unsmoothed) distribution will show an even higher ratio. For example, if because of beamwidth smoothing the bright limb of the Spur were broadened by 23
percent, the steep edge would have been broadened by 41 percent and the shallower edge by 15 percent. The ratio of the slope of the steep rise to the shallower rise for the true (unsmoothed) distribution would be 2.45. Thus, for the radial profile examined in this paper, it may be stated that the outside edge of the Spur is at least a factor of two steeper than the inside edge.

It is difficult to take a radial profile for the whole range of central angles through the Spur west of 14$^h$ right ascension without including some confusing background regions which have no relation to the Spur. Since the background is certainly not uniform in the areas west of 14$^h$, statements about the temperature inside the Spur's bright ring versus the temperature outside are not too meaningful for these areas. However, information associated with the immediate areas of the Spur's ridge structure might still be relevant.

Let us take some radial profiles with limited range of central angles through various portions of the Spur's bright ring west of 14$^h$; for example, through the points $\alpha = 16^h, \delta = +20^\circ$; $\alpha = 17^h, \delta = +20^\circ$; and $\alpha = 18^h, \delta = +10^\circ$. (We choose $\delta = +10^\circ$ instead of $+20^\circ$ for the last case because a profile through $\alpha = 18^h, \delta = +20^\circ$ would be badly confused by Hercules A.)

With $\alpha = 15^h10^m, \delta = -17^\circ$ (the direction to the Spur's center) as the 0$^\circ$ central angle, composite radial profiles through the above three coordinate points were constructed over a range of central angles limited to the area of ridge structure peak. These are shown in Figure
57. The average of the four normalized brightness temperatures for the four operating frequencies were used to obtain the experimental values (open circles) shown in the figure.

It can be readily seen from these additional experimental radial profiles that the slope of the outside edge of the Spur's bright ring is in all cases about a factor of two steeper than the inside edge. Thus, along a great length of the Spur's circumferential arc, the slope of the outside edge is steeper than the inside.

The experimental results obtained from the various radial profiles (Figures 56 and 57) constitute important evidence in support of the supernova remnant (i.e., shell source) theory of the Spur's origin. Negatively, these results constitute important evidence against the minor spiral arm theory of the Spur's origin. The bright band of the Spur has been shown to be definitely not symmetric, even on average, over an extensive portion of the bright band.

5. Shell Thickness

If one accepts the shell source concept of the Spur's brightness distribution, a radial brightness-temperature profile can be used to reconstruct the shell structure emitting the radiation observed. To make it a reasonable reconstruction problem, quite a few simplifying assumptions have to be made. From the experimentally determined, simplified model of the shell, the thickness of the shell can be determined.
Figure 57. Limited central-angle radial profiles through the Spur west of $\alpha = 14^h$. Although the intensity of the ridge peaks is much higher than for the radial profile through $\alpha = 14^h, \delta = +20^\circ$, the background is confused.
For the sake of simplicity, spherical symmetry is assumed over the section through the Spur made by the radial profile. Also, uniform relativistic electron density inside of the shell is assumed. (The source of the radio emission is synchrotron radiation, magneto-\textit{bremsstrahlung},.)

For the purposes of this problem, the Spur is considered to be a hollow shell in front of a uniform background. To obtain a meaningful result, we must use a radial profile through the Spur for which the background is unconfused, because one of the steps in the reconstruction involves subtracting out the background. The radial profiles west of 14 h are therefore unsuitable. Hence, we will use the radial profile through the point \( \alpha = 14^h, \delta = +20^\circ \) for the shell source reconstruction.

The distance from the sun to the center of the Spur is given as 50 parsecs by Hanbury-Brown, Davies, and Hazard (1960) and as 30 parsecs by Davies (1964). For the purposes of the problem, a distance of 40 parsecs will be assumed.

For the radiation coming from the direction of the Spur we have:

\[
T_{\text{obs}} = T_s (1 - e^{-\tau}) + T_{\text{bgd}} e^{-\tau},
\]

where \( T_{\text{obs}} \) is the observed brightness temperature, \( T_s \) is the equivalent temperature of the Spur (the temperature that would be observed if the Spur were optically thick), \( \tau \) is the optical depth of the Spur, and \( T_{\text{bgd}} \) is the temperature of the background radiation behind the Spur.
Making the very reasonable assumption that the Spur is optically thin (i.e., $\tau \ll 1$), we can make the small $\tau$ approximations: $1 - e^{-\tau} = \tau$ and $e^{-\tau} = 1 - \tau \approx 1$. Then, we have:

$$T_{\text{obs}} - T_{\text{bgd}} = \tau T_s.$$ 

Thus, the observed brightness temperature in the direction of the Spur above the background temperature is proportional to the optical depth.

Rather than reconstructing an experimental model for each operating frequency, it is better to employ the same approach used for the radial profile and obtain one composite shell source model using average normalized temperatures. (The shell source reconstruction is concerned with spatial rather than spectral characteristics.)

The brightness temperature at the North Galactic Pole region is used as the subtracted background temperature; it is assumed that the background temperature behind the Spur for all points along the radial profile through $\alpha = 14^h$, $\delta = +20^\circ$ is roughly the same as in the NGP region. In terms of normalized temperature $T_{\text{bgd}}$, is equal to one for all frequencies, since brightness temperatures for the radial profile were normalized to their NGP region values. Obviously, the average normalized $T_{\text{bgd}}$ (average of the four frequencies) is also equal to one; hence, for any central angle of the radial profile, $T_{\text{obs}} - T_{\text{bgd}}$ in terms of average normalized brightness temperature is equal to the value in Table 42 minus one.
The spherically-symmetrical model is assumed to have uniform relativistic electron density in its shell (whose thickness will be determined) and nothing in the interior. The observed temperature above the background is proportional to the length of the line-of-sight path through the portion of the source (the shell) containing the relativistic electrons. The emission path length is a function of central angle. The path having the greatest length within the electron-filled medium of the shell (and, hence, the maximum emission) is the single-entry path that grazes the inside edge of the shell. Paths having a central angle greater than this maximum-emission path are also single-entry but the length of their paths within the electron medium of the shell is shorter. Paths having a central angle smaller than the maximum-emission path run through the empty interior of the sources and, hence, go through the electron-filled shell twice, but the total length of any double-entry path within the electron-filled medium is always less than the length of the maximum-emission path.

As a first step in model fitting, the outside edge of the shell is positioned nearly tangent to the 47.5° central-angle line. Because of the width of the main beam used in the 20-40 MHz observations, there will be a response to the Spur (i.e., a measured antenna temperature) at central angles slightly beyond the outside "edge" of the Spur. This is shown by the very small value (0.12) of normalized background-subtracted temperature found for the 47.5° central angle. (The outside "edge" of the Spur for the model under construction is found to occur at about a 47° central angle.)
The maximum emission path-length, which corresponds to the maximum normalized background-subtracted temperature, occurs at 40° central angle. This path passes near the inside edge of the shell. The length on the 40° central-angle line between its entry into and its exit from the shell is measured on the model (see Figure 58) and found equal to 28.0 parsecs. (The scale of length in the model is set by the assumption of 40 parsecs as the sun-to-Spur center distance.) This path length through the electron-filled medium corresponds to a normalized background-subtracted temperature of 0.99 (dimensionless); i.e., normalized temperature value at 40° central angle in Table 42 minus one. Using this proportionality of emission path-length to normalized background-subtracted temperature (28.0 parsecs to 0.99), the emission path-lengths for the other central angles are scaled from their normalized background-subtracted brightness temperatures. The emission path-lengths for various central angles of the radial profile are given in Table 43 along with the corresponding normalized \( T_{\text{obs}} - T_{\text{bgd}} \) values.

Along the sighting line of each central angle, the emission path-length (scaled from the temperatures) is laid out, as shown in Figure 58. The outside edge of the shell is adjusted for best fit to the emission path-lengths of the 47.5°, 45°, 42.5° and 40° central angles. For 37.5° and lower central angles, the emission length must be divided into two equal parts since the sighting line goes through the shell twice. The inside edge of the spherical shell (the dashed line in Figure 58) is drawn, concentric with the outside edge, as the best fit to the interior ends of the emission path-lengths for the central
Figure 58. Reconstruction of the shell source from the radial profile of brightness temperature (normalized, background-subtracted, average of four frequencies) through the coordinate point $\alpha = 14^h, \delta = +20^\circ$. The shell is assumed to be spherical, of uniform density, and optically thin. The temperature measured (above background) is proportional to the line-of-sight path length through the shell.
angles of 37.5° and lower and grazing incidence of the 40° central-angle line. For clarity in the figure, the paths for central angles between 30° and 20° have not been included.

TABLE 43. Average Normalized T_{obs} - T_{bgd} Values and Emission Path-Lengths for the Various Central Angles, Assuming a Sun-to-Spur Center Distance of 40 Parsecs

<table>
<thead>
<tr>
<th>CENTRAL ANGLE</th>
<th>AVERAGE NORMALIZED BACKGROUND-SUBTRACTED TEMPERATURE</th>
<th>EMISSION PATH-LENGTH (parsecs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;50°</td>
<td>~0</td>
<td>0</td>
</tr>
<tr>
<td>47.5°</td>
<td>0.12</td>
<td>3.4</td>
</tr>
<tr>
<td>45°</td>
<td>0.43</td>
<td>12.3</td>
</tr>
<tr>
<td>42.5°</td>
<td>0.74</td>
<td>20.9</td>
</tr>
<tr>
<td>40°</td>
<td>0.99^a</td>
<td>28.0^a</td>
</tr>
<tr>
<td>37.5°</td>
<td>0.85</td>
<td>24.0</td>
</tr>
<tr>
<td>35°</td>
<td>0.66</td>
<td>18.6</td>
</tr>
<tr>
<td>32.5°</td>
<td>0.51</td>
<td>14.4</td>
</tr>
<tr>
<td>30°</td>
<td>0.42</td>
<td>11.9</td>
</tr>
<tr>
<td>27.5°</td>
<td>0.37</td>
<td>10.4</td>
</tr>
<tr>
<td>25°</td>
<td>0.37</td>
<td>10.4</td>
</tr>
<tr>
<td>22.5°</td>
<td>0.39</td>
<td>10.9</td>
</tr>
<tr>
<td>20°</td>
<td>0.37</td>
<td>10.4</td>
</tr>
</tbody>
</table>

^a Correspondence from which the other emission path-lengths are scaled.
The thickness of the shell, which is equivalent to one-half the emission path-length of the 0° central angle, is measured on the figure and found equal to 3.7 parsecs. The radius of the shell, measured to the center of the shell, is 27.4 parsecs. This gives a ratio of shell thickness to radius of 0.135 or roughly 1 to 7.5.

The numerical value found for the shell’s thickness for the radial profile through $\alpha = 14^h$, $\delta = +20^\circ$ is dependent upon the choice of 40 parsecs as the distance to the Spur’s center. By themselves, the calculations and construction based on the 20-40 MHz brightness temperature maps yield the shell thickness only in proportion to this assumed distance. The numerical value of the radius of the shell is likewise directly dependent upon this assumption. The estimate of this distance (no measurement has been made) is based on general geometric considerations, the observance of polarization, and the supposition that the Spur is a foreground object superimposed on the Galaxy. For numerical values of the Spur’s shell thickness and radius based on different assumptions of distance to the Spur’s center, see Table 44.

The values of shell thickness for the various assumed distances to the Spur’s center should be considered as maximum values. The antenna beamwidths used in the author’s observations are not negligibly small compared to the steepness of the rise in the ridge structure of the Spur. The average beamwidth of the 20-40 MHz observations was 2.6° (2.0° at 38.75 MHz, 3.3° at 22.30 MHz). Because of the smoothing effect of the non-negligible beamwidth on the contours, the shell thickness found through reconstruction (Figure 58) from the 20-40 MHz maps
is probably somewhat greater than the "true" shell thickness; i.e., the thickness that would have been determined from observations using a negligibly small beamwidth. The "true" shell thickness could very well be 15 to 25 percent less. Assuming 30 parsecs as the distance to the Spur's center, Davies (1964) from his 237 MHz observations with a 1.1° beamwidth found a shell thickness of about 2 parsecs. This value is in good agreement with that obtained from the wider-beamwidth 20-40 MHz measurements (see Table 44).

TABLE 44. Numerical Values for Shell Thickness and Radius Derived from the 20-40 MHz Maps for the Radial Profile through α = 14\textdegree, δ = +20\textdegree for Various Assumed Distances to the Spur's Center

<table>
<thead>
<tr>
<th>ASSUMED DISTANCE TO SPUR'S CENTER (parsecs)</th>
<th>RADIUS TO SHELL CENTER (parsecs)</th>
<th>SHELL THICKNESS (parsecs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>13.7</td>
<td>1.85</td>
</tr>
<tr>
<td>30</td>
<td>20.6</td>
<td>2.8</td>
</tr>
<tr>
<td>40</td>
<td>27.4</td>
<td>3.7</td>
</tr>
<tr>
<td>50</td>
<td>34.3</td>
<td>4.6</td>
</tr>
<tr>
<td>75</td>
<td>51.4</td>
<td>6.9</td>
</tr>
</tbody>
</table>
The author's program for the investigation of radio emission in the 20-40 MHz range using the Arecibo radio telescope involved three essential parts. The first part was technical (electrical engineering) and was concerned with the experimental and theoretical determination of the antenna parameters. The second and third parts were scientific (astronomy and physics) and were concerned with the investigation and interpretation of the spectra of discrete radio sources and the brightness temperature distribution of the Galactic Spur region. The results obtained and conclusions reached in each of these areas are summarized in this chapter.

With regard to the experimental and theoretical determination of the antenna parameters of the Arecibo radio telescope, the following results were obtained:

1. The beamwidths ($\theta_X$) of the four operating frequencies were measured using the non-scintillating source IC443 (small corrections were made for non-negligible source width) and were found to be consistent with the reflector diameter and the aperture distribution (analyzed theoretically). The beamwidth was $2.0^\circ$ at 38.75 MHz and $3.3^\circ$ at 22.30 MHz.
(2) The effective areas ($A_e$) for the four operating frequencies were measured as a function of zenith angle ($3.5^\circ$ to $20^\circ$) using Taurus A as the calibrating source. The aperture efficiency $\eta_a$ at low zenith angles was between 45 and 51 percent, depending on frequency. Because the phase deviation caused by the reflector being a spherical cap rather than a true parabola is greater for higher frequencies, $A_e$ and $\eta_a$ are thereby smaller. $A_e$ and $\eta_a$ decrease with increasing zenith angle, being about 2.4 dB below their low zenith-angle values at $\zeta = 20^\circ$.

(3) From the antenna patterns of the log-periodic feed and the feed structure-reflector geometry, the feed efficiency ($\varepsilon_f$) as a function of zenith angle was derived theoretically. Over $0^\circ$ to $20^\circ$ zenith angle, $\varepsilon_f$ varies from 0.95 to 0.82 (independent of frequency).

(4) The main beam efficiency $\eta_M$ (equal to $\Omega_M/\Omega_A$) was derived theoretically as a function of zenith angle using the measured antenna parameters $\hat{\theta}_X$ and $A_e$ ($\Omega_M = k_d \hat{\theta}_X^2$; $\Omega_A = \lambda^2/A_e$). At low zenith angles, $\eta_M = 0.74$, decreasing to roughly 1.5 dB below this value at $\zeta = 20^\circ$ (it is substantially independent of frequency).

It is the general conclusion of the author that it is possible to determine accurately, by experimental and theoretical means, the antenna parameters of a variable aperture geometry radio telescope and to use these determinations in the calibration of astronomical observations (the brightness temperature scaling of the Galactic Spur region, for example).

With regard to the spectra of the discrete sources measured by
the author, the following results were obtained:

(1) The spectrum of the supernova remnant IC443 reaches a turnaround point (i.e., a flux density maximum) at about 30 MHz. This spectral behavior is explained most satisfactorily by galactic H II absorption, with an emission measure of $360 \text{ cm}^{-6} \text{ pc}$ for the absorbing ionized hydrogen plasma region.

(2) The spectra of the core-and-halo sources 3C47 and Virgo A show no evidence of downward curvature and may even have slight upward curvature. In contrast, the core-and-halo source 3C84 (not observed by the author) has a very pronounced upward curvature at dekameter wavelengths caused by the halo predominance effect. In the case of 3C47 and Virgo A, the spectral index of the halo is not drastically different from that of the core, and the emission of the halo makes up a large portion of the total source emission even in the decimeter region. Hence, in going from meter to dekameter wavelengths, the transition in their spectra, as the influence of the halo predominates, is smooth and gradual with no sharp upward curvature.

(3) The spectra of five extragalactic double radio sources (3C33, 3C98, 3C123, Hercules A, and 3C353) show some slight downward curvature in the 20-40 MHz range, but reach their flux density maximums at frequencies below the 20-40 MHz range. The downward curvature is probably not caused by synchrotron self-absorption (with the possible exception of 3C123), but is likely to be caused by galactic H II absorption, or by the internal refractive index effect (Razin effect), or by a combination of both. For 3C353, galactic H II absorption is the most likely cause.
Based on his own results with a very limited number of sources and on the results of other low-frequency observers, it is the general conclusion of the author that it is possible to show a causal relationship between the observed low-frequency spectrum and the type of radio source.

With regard to the origin of the radiation from the Galactic Spur, the three most plausible theories (discussed in Section B 1 of Chapter V) are the rainbow theory, the minor spiral arm theory, and the supernova remnant theory. From the author's 20-40 MHz maps of the brightness distribution of the Galactic Spur region in the northern hemisphere, the following relevant pieces of evidence were obtained:

(1) The geometry of the Spur, especially as shown by means of a stereographic projection, indicates that the Spur's radiation belt is roughly in the form of a small circle and definitely not part of a great circle, contrary to the great circle assumption of the rainbow theory.

(2) The finding that the Spur's radiation belt can be traced over such a large arc-length of a small circle (nearly 270°) is contrary to the minor spiral arm theory, which would hold that the Spur's arc-length should be, at most, 90°.

(3) The brightness temperature spectral index $\beta$ was measured in the 20-40 MHz range and found equal to $2.50 \pm 0.15$. The flux density spectral index $\alpha$ is then equal to $0.50 \pm 0.15$, which is right in the middle of the range of spectral indexes of other supernova remnants (for the others, $0.27 \leq \alpha \leq 0.77$). This shows that the spectrum of the Spur is compatible with its being a supernova remnant.
(4) By means of a radial profile across the Spur (through the point \( \alpha = 14^h, \delta = +20^\circ \)) at a place where the background is not confused, it was found that the brightness temperature inside the Spur (i.e., south of the peak in the ridge structure) is greater than the brightness temperature outside. This is indicative of a shell source and, hence, supports the supernova remnant theory.

(5) From several radial profiles across the Spur (including the one through \( \alpha = 14^h, \delta = +20^\circ \)) along the length of its bright arc, it was found that the slope of the outside edge is steeper, by at least a factor of two, than the inside edge. This steeper edge is highly indicative of a shell source, thus supporting the supernova remnant theory. These results also constitute evidence against the minor spiral arm theory, since the Spur's bright radiation belt has been shown to be definitely not symmetric, even on average, over an extensive portion of the Spur's arc.

Based on the evidence obtained, it is the general conclusion of the author that the supernova remnant theory is the most likely explanation of the Spur's origin. This theory holds that the Spur's radiation is the limb-brightened emission from an optically thin source, which is in the form of a hollow shell, roughly spherical in shape, subtending a very large solid angle because of its relative proximity to the sun. The distribution of emission from the shell is patchy; hence, the bright ring is not complete. The Spur is a foreground object whose radiation is superimposed on the galactic background. Its emission mechanism is magnetobremstrahlung.
magnetic braking radiation (also called synchrotron radiation), from relativistic electrons spiralling in the enhanced magnetic fields associated with the expanding shock front resulting from the explosion of the star.

If one accepts the supernova remnant of the Spur's origin and the shell source concept associated with it, then it is possible to reconstruct a simplified model of the Spur from a radial profile through a portion of the Spur where the background is unconfused. From such a reconstruction, it is found that the thickness of the shell is less than $1/7.5$ of its radius. The actual numerical value of the thickness depends upon the assumed value of the sun-to-Spur center distance. For the usual assumptions of this distance (30-50 parsecs), the shell thickness is in the order of 2-4 parsecs.
REFERENCES


Bridle, A.H. (1967). Flux densities of Cassiopeia A and Cygnus A at 10.05 MHz, Observatory, 87: 60-63.


