EXTINCTION AND BACKSCATTER OF VISIBLE AND INFRARED LASER RADIATION BY ATMOSPHERIC AEROSOLS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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*****

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CHAPTER I
INTRODUCTION

The use of laser sources for atmospheric communications and ranging requires quantitative information if system performances are to be evaluated. Effects such as molecular absorption and signal extinction by Rayleigh and aerosol particles play major roles in determining the performance of the system. Which of the above constituents dominates under given conditions depends largely upon what wavelength is used. Wavelengths in the ultraviolet region will be affected by all three, while visible wavelengths are mainly affected by aerosol scattering. The development of a variety of laser sources covering a wide spectrum of wavelengths, has caused new interests in understanding how the optical properties of these constituents vary with wavelength. Thus the purpose of this study is to determine how atmospheric aerosol particles (including precipitation and fog particles) affect the two optical properties, signal transmittance and signal backscatter for selected wavelengths between 0.34 and 10.6 microns.

Atmospheric aerosol particles which affect the optical properties of the atmosphere range in size from $10^{-2}$ micron radius for small continental particles to several thousand microns for precipitation.
particles. Composition varies from a mixture of dry salts to pure liquid water.

Wavelength dependence for aerosol scattering depends largely on the particle size density distribution. For the wavelength region covered in this study, the wavelength dependence is approximately $1/\lambda$ for average continental aerosol conditions. For heavy fog conditions, scattering becomes less wavelength dependent and is completely independent of wavelength for precipitation conditions if particle forward scattering is neglected.

The characteristics of aerosols are discussed briefly in Chapter II. Their structure, composition and size density distributions are included for continental, fog and precipitation aerosols.

Chapter III discusses the characteristics of the aerosol extinction and backscatter coefficients for a medium containing only one single particle and one containing many particles. Also included are some definitions of terminology and an approximation method for computing the backscattering amplitude function for large spherical particles.

Chapter IV gives the calculated results for the aerosol extinction and backscatter coefficients using the aerosol models given in Chapter III.

Chapter V describes the outdoor molecular absorption measurements of the $\text{CO}_2\ P(20)$, 10.59 micron, line. Included is a discussion of
a similar experiment performed at The Ohio State University using an indoor multipass absorption cell.

Chapter VI describes the measurements of the aerosol extinction and backscatter coefficients during continental, fog and precipitation conditions for 0.6328 and 10.59 microns.
CHAPTER II
CHARACTERISTICS OF AEROSOLS

A. Introduction

Atmospheric aerosols are not only interesting in themselves but they determine among various other properties the optical properties of the atmosphere (e.g., scattering and attenuation of radiation). In order to study the optical properties, it is first necessary to understand aerosol particles. It is not the purpose of this chapter to investigate in detail the complex and numerous areas dealing with aerosols, but rather to discuss generally the following three groups:

1. Structure
2. Chemical composition
3. Size distribution.

B. Structure of Aerosols

Large particles in the atmosphere such as fog and precipitation are of course largely pure liquid water droplets. But for the smaller suspended continental particles the structure is more complex.

There is direct evidence from electron microscope pictures which demonstrates the great variability of these aerosols. [2] The physical constitution of aerosol particles in the atmosphere varies between the
extremes of a dry insoluble dust particle and a clear salt solution particle. The dry dust particles of course come from arid soil and the salt particles produced by industrial processes or ocean sea spray represent dissolved droplets. Over wide continental areas, aerosol particles consist of mixtures of insoluble and soluble materials. Composition can vary greatly depending on geographic location, e.g., inland near cities or near country areas.

Formation of mixed particles can occur by coagulation of aerosols produced by industrial and other combustion processes. This process occurs in cloud free air by Brownian motion and occurs in clouds by numerous condensation and evaporation cycles acting upon condensation nuclei. [3] Thence a mixed particle or nucleus is the most general concept for an atmospheric continental aerosol particle.

C. General Composition

Of all the parameters describing continental aerosols, composition is the least understood. But for a broad classification, aerosols may be divided into two groups: soluble and insoluble.

Growth curves versus relative humidity for mixed particles, pure droplets of solution and dry particles have been computed. These growth curves have been confirmed by measurements of Junge,[2] Average measured growth curves for large and small Central European continental aerosols have been compared to computed growth
curves and found to agree fairly well with curves for mixed particles containing about 25 per cent soluble material. For higher relative humidities, indications are that the mixed particles behave more and more like droplets of salt solutions.

Therefore most investigators use the assumption of Gibbons[4] who states that the index of refraction of uncontaminated continental aerosol particles will be expected to be between that of NaCl and that of water. The index of refraction for dry NaCl is approximately 1.5, while the real part of the index of refraction for water is approximately 1.34. Hence, the index of refraction for continental aerosol particles of moist NaCl will be expected to be somewhere between these limits and to be generally smaller for a higher relative humidity.

The effect that an aerosol particle has on propagating radiation in the atmosphere depends in part, on its composition, or in other words its index of refraction (m = n₁ - jn₂).

Figure 1 gives the results of Centeno[5] and Bayly[6] who measured the index of refraction for pure water in the visible and infrared regions. The figure is essentially Centeno's results except for the imaginary index of refraction in the 2.8μ region where the more recent results of Bayly have been used.

Unfortunately Bayly's results are not published in terms of n₂ and must therefore be converted. A brief discussion showing the method of conversion is given in Appendix A.
Fig. 1.--Index of refraction \((n_1 - jn_2)\) for liquid water.

All calculations presented in this study will assume that the aerosols are pure water particles. This assumption appears to be a fairly good approximation for relative humidities above 70 per cent.

D. Size Distribution

1. Continental and maritime aerosols

Atmospheric aerosol particles range in size from clusters of a few molecules to particles near 20\(\mu\) radius. The smallest aerosols consist of small ions held together by electrical charge. Their mass concentration is several orders of magnitude smaller than all the other
aerosols and therefore are of no importance to aerosol optical properties. The next group of particles in the size distribution are called Aitken particles \((R \leq 0.1\mu)\). Several workers have found a structured spectra with a lower size limit of \(4.0 \times 10^{-3}\mu\) for these particles. Junge[2] has compiled and averaged these particles along with other particle size measurements from impactor and sedimentation results and presented the average model size distribution for the atmospheric continental aerosols in Fig. 2. The size distribution is the number of particles per cm\(^3\) with radii between \(R\) and \(R + \Delta R\) and is plotted as a function of \(R\) in microns. The calculated growth rate versus relative humidity for these particles is also given. Curve 3 in Fig. 2 is the average sea spray (chloride particles) size distribution resulting from various independent measurements in maritime areas not subject to continental particle contamination. It can be seen that the maritime aerosol particles have a much smaller particle concentration in comparison to the continental aerosols.

The main features of the continental model are an apparent lower limit, a peak concentration around \(0.03\mu\) and a long uniform slope from \(0.1\) to \(~20\mu\) which is given by the power law

\[
\eta(R) = \frac{C}{2.3} R^{-\nu - 1}
\]

with \(\nu\) between 2.5 and 4.0.
Model size distributions for atmospheric aerosols. Curve 1, continent. Curve 2, ocean. The part of the curve below 1 μ is estimated. Curve 3, sea-spray component of the maritime aerosol. The hatched area between curves 2 and 3 represents the non-sea spray component over the ocean. The straight lines indicate the shift of the continental log radius number distributions curve due to particle growth by humidity for mixed particles of about 20% soluble material.

Fig. 2.--Comparison between continental and sea aerosol distributions.
It must be remembered that this distribution represents the average of many distributions. Measurements taken by Goetz[3] and Fenn[7] for instance, found considerable structure in the distribution for $R > 0.1\mu$ and often no power law relation. Their results are still being evaluated by many investigators.

2. Fog particle aerosols

Unlike continental and maritime aerosols, experimental measurements support the view that fog particles do not in general have a power law size distribution but because experimental difficulties do not allow accurate measurements of particle diameters below $1.0\mu$, published fog size distributions have been inconsistent.

The following distribution was proposed by Deirmendjian[8] after a survey of various measurements of water clouds and aerosols. The distribution is

\begin{equation}
\eta(R) = C R^\alpha e^{-bR^\gamma},
\end{equation}

where $\eta(R)$ is the volume concentration at radius $R$ and $C$, $\alpha$, $b$ and $\gamma$ are constants. Chu and Hogg[9] introduced the terms $R_m$ and $a$ into Eq. (2) with $a = R/R_m$. Equation (2) then becomes

\begin{equation}
\eta(a) = C a^\alpha e^{-ba^\gamma},
\end{equation}
where $R_m$ is the radius of the drops with the maximum number density. The constant $C$ can be determined from the integral

\begin{equation}
R_m \int_0^\infty \eta(a)da = N = \text{Total particulate concentration} = \frac{R_mC}{\gamma} b^{-\left(\frac{\alpha+1}{\gamma}\right)} \Gamma\left(\frac{\alpha+1}{\gamma}\right),
\end{equation}

where $\Gamma$ represents the gamma function. $b$ can be determined from the derivative

\begin{equation}
\frac{d\eta(a)}{da} \bigg|_{a=1} = Ca^{\alpha-1} e^{-ba^\gamma} (a^{-\gamma} ba^\gamma) = 0
\end{equation}

where

$$b = \frac{\alpha}{\gamma}.$$

This form of the size distribution offers advantages in comparison to the power law distribution of Junge for describing the aerosol size distribution. It has the ability to vary both the lower and upper end of the size distribution whereas the power law distribution describes only the decreasing number density for the larger particles. It also has the ability to vary the maximum number density denoted by $R_m$ in Eq. (3). This shift in maximum number density resulting from changes in meteorological conditions has been reported by several investigators. [10, 11]
In Fig. 3, five plots of Eq. (3) are given for several combinations of parameters. The three curves having the smallest slopes compare with measurement made by Eldridge[12] and Harris,[10] while the steeper sloped curves ($\alpha=6$) reproduce some of the distributions found by Durbin[13] for cumulus clouds having visibilities between 230 and 2100 meters. Figure 4 shows a comparison between the size distribution measured by Harris during a fog having a visibility of 50 meters and the fog model distribution $\alpha=1$, $\gamma=1$. It can be seen that the general shape of the distribution is closely approximated by the aerosol model.

It is of course impossible to provide a model for every possible fog distribution, but it is felt that this model provides sufficient flexibility to describe atmospheric fog well enough for the experimental accuracy required by most investigators.

3. Precipitation droplet model

Precipitation particles are not in the true sense of the word aerosol particles because they are not suspended in the atmosphere. Nevertheless, they present similar optical properties to radiation as do the previously discussed aerosols. Therefore the precipitation particle will be referred to as an aerosol particle in this study.

Laws and Parsons,[14] using the flour-method of particle sizing, found that the particle size distribution varied with rain rate. Their results for three rain rates are given in Fig. 5. The size distribution
Fig. 3.--Fog model size distribution $\eta(a)$. 
Fig. 4.—Comparison of fog size distribution model
\( \alpha = 1, \gamma = 1, R_m = 0.6 \mu \) and measurements
by Harris [10].

is plotted as a function of particle diameter in millimeters. Instead of
the size density distribution that the previous distribution models used,
this distribution is tabulated in percentage of total volume for 0.25 mm
intervals. An expression relating the two distributions will be given later.
In order to facilitate calculation, empirical expressions are needed to describe the model. The tabulated results in Fig. 5 were fitted to normalized Gaussian distributions having mean values and variances which are functions of rain rate. The precipitation model found is then expressed as

\[
m(R, r) = \frac{2}{\sqrt{\pi}} \frac{0.4}{\sigma(r)} \exp\left(-\frac{(R-R_0(r))^2}{\sigma^2(r)}\right),
\]
where $m(R, r)$ is the normalized percentage distribution, $r$ is the rain rate, $R_0$ is the mean particle radius in cm, $\sigma$ is the variance of the distribution in cm and 0.4 is the normalizing constant. The expressions found for the mean radius and the variance are, respectively

$$R_0(r) = 0.045 \log_{10}(r) + 0.05 \text{ cm}, \quad (7)$$

and

$$\sigma(r) = 0.05 \log_{10}(r) + 0.05 \text{ cm}, \quad (8)$$

where $r$ is the rain rate in mm/HR.

All the above expressions and constants were fitted to the curves in Fig. 5.

It must be remembered that all the above distributions are models representing average findings and do not describe variations from these averages which obviously occur. Fortunately exactness is not needed for most laser propagation applications and satisfactory results can be obtained using aerosol models approximating typical atmospheric conditions.
CHAPTER III
AEROSOL SCATTERING THEORY

A. Introduction

It is well known that electromagnetic radiation suffers attenuation while propagating through the atmosphere. This total attenuation is a result of molecular and aerosol scattering and absorption. It is the purpose of this chapter to develop the information and expressions necessary to determine theoretically the expected atmospheric aerosol attenuation contribution to the total attenuation for visible and infrared radiation. Also included is an approximate solution for determining the backscatter amplitude of large ($R > > \lambda$) aerosol particles. The backscatter amplitude function is used to determine the ratio of energy scattered from a particle back toward the energy source to the energy incident upon it. The interesting effects of multiple scattering, non-linearities caused by high energy sources and fluctuations caused by turbulence will not be discussed.

B. Aerosol Extinction Coefficient

1. Single particle scattering

Let a fixed particle of arbitrary shape and composition be illuminated by a monochromatic collimated beam whose electric field is given by
where $k$ is the propagation constant for the medium surrounding the particle. The scattered far field wave is a spherical, outgoing wave written as

$$U = \frac{S(\theta, \phi)}{jkr} e^{\mathbf{U}_0 - jkr + jkZ},$$

where $S(\theta, \phi)$ is the scatter amplitude function.\[15]

The addition of the amplitude of the incident and scattered wave in the forward direction gives

$$U_0 + U = U_0 \left(1 + \frac{S(\theta, \phi)}{jkZ} e^{-jk(X^2 + Y^2)/2Z}\right),$$

where $X$ and $Y$ are located in a plane at $Z$ and have amplitude much less than $Z$. Squaring the modulus of Eq. (11) and integrating over an area $A$ in the plane located at $Z$, the total intensity becomes

$$A - C.$$

The results may be interpreted as a reduction in total light intensity from that which would have resulted if no particle were present. The reduction is equivalent to covering up a portion of the area $A$. Thus, $C$ has dimensions of area and may be written as $C_{\text{EXT}}$. Completing the two Fresnel integrals in Eq. (11), the fundamental expression for $C_{\text{EXT}}$ may be written as
a. Spherical particles with index of refraction near one

Up to this point in the discussion the particle shape and composition have been arbitrary. In order to be able to make calculations it is necessary to remove these generalizations. Because it is not unreasonable and because of the relative ease of calculations due to symmetry, a spherical shaped particle will be used for all calculations. The particle composition will be given by its index of refraction \( m = n_1 - jn_2 \).

Making further assumptions gives a relatively simple expression for \( C_{\text{EXT}} \).

Van de Hulst[16] used a combination of geometric optics and Huygen's principle to obtain the following approximation formula for the extinction cross section

\[
C_{\text{EXT}} = \pi R^2 Q_{\text{EXT}} = \pi R^2 \left( 2 - 4 e^{-\rho \tan \beta} \left( \frac{\cos \beta}{\rho} \right) \sin(\rho - \beta) \right.
\]

\[ - 4 e^{-\rho \tan \beta} \left( \frac{\cos \beta}{\rho} \right)^2 \cos(\rho - 2\beta) + 4 \left( \frac{\cos \beta}{\rho} \right)^2 \cos 2\beta \]

where \( \rho = \frac{4\pi R}{\lambda} (n_1 - 1) \), \( R \) = particle radius, \( \tan \beta = \frac{n_2}{n_1} \), and \( Q_{\text{EXT}} \) is defined as the efficiency factor of extinction. The above expression was derived with the assumptions that \( (n_1 - 1) \ll 1 \) and \( \frac{2\pi R}{\lambda} \gg 1 \).
$C_{\text{EXT}}$ is the sum of two cross sections. The relation is written as

\begin{equation}
C_{\text{EXT}} = C_{\text{SCA}} + C_{\text{ABS}},
\end{equation}

where $C_{\text{SCA}}$ and $C_{\text{ABS}}$ are the scatter and absorption cross sections, respectively. For lossy particles ($n_2 \neq 0$), $C_{\text{ABS}}$ is nonzero. Using the same assumption used in deriving the approximate expression for $C_{\text{EXT}}$, Van de Hulst gave the following expression for $C_{\text{ABS}}$,

\begin{equation}
C_{\text{ABS}} = \pi R^2 Q_{\text{ABS}} = \pi R^2 \left( 1 + \frac{e^{-4n_2x}}{2n_2} + \frac{e^{-4n_2x}}{8x^2n_2^2} \right),
\end{equation}

where $x = \frac{2\pi R}{\lambda}$ and $Q_{\text{ABS}}$ is the efficiency factor for absorption. It should be remembered that Eqs. (14) and (16) are approximations and not exact expressions.

Pluss[17] has made exact calculations using the classical boundary value theory of Mie,[15] and has made comparisons with the approximate solutions for various indices of refraction. The results of the exact and approximate calculations for the absorption efficiency cross sections are given in Fig. 6 by the solid and dashed curves, respectively.

The discrepancies between the two curves depend on the value of $n_2$. For $n_2 = 10$ the approximation fails entirely. For $n_2$ between $1 \times 10^{-2}$ and $1 \times 10^6$ the approximation reproduces the exact solution with some discrepancy.
The value reached for $Q_{\text{ABS}}$ for a nonzero value of $n_2$ if $x \to \infty$ is 1.

Figure 7 shows the results of the exact and approximate calculations for the extinction efficiency cross section. It can be seen that unlike $Q_{\text{ABS}}$, $Q_{\text{EXT}}$ has resonant peaks for $n_2 < 1.0$. These are caused by interference between the transmitted and scattered waves. The maxima and minima are dependent upon the value of $n_2$. The larger $n_2$ the less resonant structure in $Q_{\text{EXT}}$. $Q_{\text{EXT}}$ also differs from $Q_{\text{ABS}}$ in its asymptotic value for large $x$. The value reached by $Q_{\text{EXT}}$ for $x \to \infty$ is 2.

The tendency for the approximation to fall short of the exact Mie values is also evident in Fig. 7. Deirmendjian[18] has proposed an empirical formula which corrects this deficiency. This correction factor modifies Eq. (14) so that is accurately describes the actual values of the maxima and minima of $Q_{\text{EXT}}$. His formula has been used in the calculations which are discussed here, but because of the complexity of the expressions the reader is referred to the original paper for details.

For completeness, $Q_{\text{SCA}}$ is given in Fig. 8. It should be pointed out that $Q_{\text{SCA}}$ can be calculated by substituting $Q_{\text{EXT}}$ and $Q_{\text{ABS}}$ into Eq. (15).
Fig. 6.--Efficiency factor for absorption as a function of $x = 2\pi R/\lambda$.

Fig. 7.--Efficiency factor for extinction as a function of $x = 2\pi R/\lambda$. 
Fig. 8. -- Efficiency factor for scattering as a function of $x = 2\pi R/\lambda$.

### 2. Many particle scattering

The scattering theory presented above for a single scatterer may be applied to a medium containing many scatterers if it is assumed that the particles are independent scatterers. That is, if the scattering theory has to investigate in detail the phase relation between the waves scattered by neighboring particles, then the particles are called dependent scatterers and will not be treated here. Van de Hulst[16] estimated that particles can be considered independent if they have a separation of 3 times the radii of the drops. He also estimated that for extreme conditions such as fog with an optical depth of 10 meters,
particle separation would be approximately 20 times the radii of the drops. Therefore all calculations that were made in this study assumed that the scattering medium consisted of independent scatterers. Adding further to these assumptions by assuming that all the scatterers have the same composition (same index of refraction), permits an extinction coefficient for the medium to be defined.

It will be assumed that the extinction of a monochromatic beam propagating in a scattering medium is given by Lambert's law. The change in intensity in traversing an infinitesimal path $dl$ in the direction of propagation is then written as

\begin{equation}
\frac{dI_\lambda}{I_\lambda} = - B_{\text{EXT}}(\lambda, m) I_\lambda \, da
\end{equation}

where $da$ is the amount of scatterers in a volume of unit cross sectional area and length $dl$. The proportionality constant $B_{\text{EXT}}(\lambda, m)$ is the extinction coefficient. Figure 9 shows the relationship between some of the physical properties involved.

When the scatter volume is used to specify the amount of scatterers as opposed to, for example, scatter mass, $B_{\text{EXT}}$ is defined as the volume extinction coefficient.

Earlier it was shown that the extinction cross section of a particle has physical units of areas. Thus the volume extinction coefficient will have physical units $\text{cm}^{-1}$ when the extinction cross sections
Fig. 9.—Relationships between the physical properties of a scattering medium.

of all the independent particles are summed. The expression for the volume extinction coefficient may then be written as

\[
B_{\text{EXT}}(\lambda, m) = \pi \int_{R_1}^{R_2} Q_{\text{EXT}}(\lambda, m) \eta(R) R^2 \, dR \text{ cm}^{-1}
\]

where \(Q_{\text{EXT}}(\lambda, m)\) is the efficiency factor for extinction of a spherical particle with index of refraction \(m\), \(\eta(R)\) is the size density distribution, \(R_1\) and \(R_2\) are the lower and upper limits of the size distribution, respectively and \(\lambda\) is the wavelength. Similar expressions for \(B_{\text{SCA}}\) and \(B_{\text{ABS}}\) can be obtained by replacing \(Q_{\text{EXT}}\) in Eq. (18) by \(Q_{\text{SCA}}\) and \(Q_{\text{ABS}}\), respectively.
The transmittance $T_\lambda$ through a scatter medium of amount $a$ is defined as the ratio of the intensity after passing through the medium to the intensity before entering. Equation (17) may be integrated to give the transmittance as

\begin{equation}
T = \frac{I_\lambda}{I_\lambda^0} = e^{-\text{BEXT}(\lambda, m)a}
\end{equation}

3. Energy transfer equations

Having defined the volume extinction and scattering coefficients and transmittance of a medium containing many scatterers, it is now possible to determine the amount of radiation incident upon a scattering medium that is scattered in a direction other than $\theta = 0^\circ$.

Assume that $I_{\lambda SC}$ in Fig. 9 is scattered in the direction of an observer or unpolarized detector and that $I_\lambda = T_1 I_{\lambda S}$, where $T_1$ is the transmittance of the medium between the source and the scatter volume. The energy transfer equations relating the source energy to the received detector energy are:

a. Source to scattering volume

\[ I_\lambda = I_{\lambda S} T_1 = W A, \]

where

\[ I_{\lambda S} = \text{total energy of transmitter source (watts)} \]

\[ T_1 = \text{transmittance between source and scatter volume} \]
A = area of scatter volume perpendicular to propagation path

W = irradiance incident on area A (watts/cm²)

b. Scatter volume

\[ I_{\lambda}(m, \theta) = W A AL BSCA(m, \lambda) \frac{P(m, \lambda, \theta)}{4\pi} \]

where

\[ \theta = \text{scattering angle (180° for backscatter)} \]

\[ I_{\lambda}(m, \theta) = \text{intensity of scattered radiation (watts/steradians)} \]

\[ BSCA = \text{atmospheric volume scattering coefficient (cm⁻¹)} \]

\[ \frac{P(\theta)}{4\pi} = \text{normalized angular scattering function (steradians⁻¹)} \]

c. Scattering volume to detector or observer

\[ I_{\lambda_{R}}(\theta) = I_{\lambda}(m, \theta) T_{2} \omega \]

where

\[ T_{2} = \text{transmittance between scatter volume and detector} \]

\[ \omega = \text{solid angle subtended by detector (steradians)} \]

\[ I_{\lambda_{R}} = \text{received energy at detector (watts)} \]

The amount of energy received at the detector is related to the volume scatter coefficient as would be expected (i.e., the larger the value BSCA the more energy that is scattered in all directions), but it is also related to the scattering angle \( \theta \). The normalized angular scatter function \( \frac{P(\theta)}{4\pi} \) expresses the angular functional dependence of the scattering volume. \( \frac{P(\theta)}{4\pi} \) is a function of the scatter amplitude function and is written as
\[
\frac{P(m, \lambda, \theta)}{4\pi} = \frac{\int_{R_1}^{R_2} |S(m, \lambda, \theta)|^2 \eta(R) dR}{k^2 \pi \int_{R_1}^{R_2} Q_{SCA}(m, \lambda) \eta(R) R^2 dR} = \frac{1}{k^2 B_{SCA}} \int_{R_1}^{R_2} |S(m, \lambda, \theta)|^2 \eta(R) dR \text{ (steradian)}^{-1},
\]

where \(k\) is the free space propagation constant.

C. **Approximation Method to Determine the Backscatter Amplitude Function**

In the previous section it was shown that the amplitude of the far field in the forward direction (\(\theta = 0^\circ\)) is expressed in terms of \(S(0^\circ)\). In this section an approximate solution for the scatter amplitude function at \(\theta = 180^\circ\) will be given.

An exact solution may be obtained for \(S(\theta)\) based on the classical boundary value method of Mie.[15] An infinite set of eigenfunctions are used to represent the scattered field. For particles large in comparison to wavelength (\(R > > \lambda\)), the series converges very slowly and makes computations very lengthy. For example, the number of terms necessary for convergence is given by \(1.2x + 4.0\), where \(x = \frac{2\pi R}{\lambda}\) is the size parameter. For visible wavelengths, rain particles can make \(x\) as large as 600. Thus over 700 terms would have to be added before convergence took place. Also, if it were necessary to sum over a size distribution of particles as in the previous section, it is easily seen...
that computations get very lengthy. A modified geometric optics solution proposed by Peters, et al[19, 20] for scattering by dielectric bodies has been used in this study to reduce the formidable calculations necessary for $R \gg \lambda$. The method for approximating $S(180^\circ)$ will be briefly presented here.

Geometrical optics is based upon Fermat's principle, which states that for the isotropic, homogeneous media to be considered here, optical ray paths follow straight lines. Also, at the boundary surface between two media the direction of the incident ray is altered according to the well known laws of reflection and refraction. Using these two laws permits the ray paths to be traced through a dielectric body (a sphere in this case) so that the emergent rays which contribute to the backscatter can be found.

Figure 10 shows the possible backscatter rays in a dielectric sphere having one or two internal reflections. For a sphere, glory ray 3 with one internal reflection is predicted by ray tracing for index of refraction in the range $1.41 < n_i < 2.0$. Similarly, a twice internally reflected glory ray is predicted for $n_i < 1.15$. As was previously seen, water has an index of refraction $n_i \sim 1.33$ for visible and infrared wavelengths. Since $n_i \sim 1.33$ does not satisfy either of the above ranges required for glory rays, they will not have to be considered in the glory ray contribution to the backscatter amplitude function.
Fig. 10.---Possible backscatter rays in a dielectric sphere having one or two internal reflections.

Rays 1 and 2 may be described in a general manner by the field

\[ U(t) = A_0 e^{i\phi_0} F(t) e^{-jk\ell}, \]

where

- \( A_0 \) is the amplitude at a reference point,
- \( \phi_0 \) is the phase at a reference point,
- \( F(t) \) is the spatial attenuation factor,
- \( e^{-jk\ell} \) is the spatial phase delay factor, and
- \( \ell \) is the distance along the ray from the reference point.
F(\ell) for the two rays is given as:

Ray 1 (scattered from front interface)  \( F(\ell) = \frac{a}{2\ell} \)

Ray 2 (scattered from rear interface)  \( F(\ell) = \frac{a}{2\ell} \frac{1}{2/d\theta_1/d\theta_o - 1} \)

where \( a \) is the radius of the sphere and \( d\theta_1/d\theta_o \) is the derivative of the refracted angle with respect to the incident angle. For a lossy sphere such as a water particle illuminated by infrared radiation, Snell's law of refraction becomes rather complex but remains valid. Stratton[14] has given the following form of Snell's law for lossy media as,

\[
(22) \quad \tan(\theta_1) = \frac{\sin(\theta_o)}{q(\theta_o)} ,
\]

where

\[
q^2(\theta_o) = \frac{1}{2} [ n_1^2 - n_2^2 - \sin^2(\theta_o) + \sqrt{4n_1^2n_2^2 + (n_1^2 - n_2^2 - \sin^2(\theta_o))^2} ] .
\]

Equation (22) can be differentiated and \( d\theta_1/d\theta_o \) solved for in the limit as \( \theta_1 \) and \( \theta_o \to 0 \) (on axis scattering). For lossless dielectric spheres \( \frac{d\theta_1}{d\theta_o} \) reduces to \( n_1 \).

The fields of Ray 1 and 2 may be added to give the total backscatter field \( \overline{E_S} \). For large \( \ell \) \( \overline{E_S} \) is written as

\[
(23) \quad \overline{E_S} = \frac{|E_1|}{\ell} e^{j k_0 \ell} \frac{a}{2} e^{-j 2k \cdot c \cdot a} \begin{bmatrix} R_{12}(0) + \frac{T_{12}(0)T_{21}(0)R_{21}(0) e^{j(4ka - \pi)}}{2/d\theta_1/d\theta_o - 1} \\ 0 \end{bmatrix} ,
\]
where $|E_i|$ is the amplitude of the incident wave, $R$, $T$ are the reflection and transmission coefficients with $R_{12}(0^\circ)$ indicating the reflection coefficient at the front interface with normal incidence, $k = \frac{2\pi}{\lambda}$ is the propagation constant inside the spherical particle and $k_o$ is the propagation constant outside the particle. A comparison of the exact boundary value solution for the backscattering function with the two component ray scattering approximation having $n_1 = 1.23$ is shown in Fig. 11. It is apparent from the exact backscattering curve that there are rays present not previously considered. Examining the emergent rays Kouyoumjian, et al[20] have shown that a stationary ray exists at an angle near the backscatter. They have shown that the results of the spatial attenuation $F(f)$ for this ray is

$$F(f) = \frac{4\pi R_o}{\lambda f} S |A_i(Z)|$$

(24) where $A_i(Z)$ is the Airy Integral defined as

$$A_i(Z) = \frac{1}{2} \int_{-\infty}^{\infty} e^{-j(Zt-t^3/3)} dt$$

$$S = \left( \frac{a}{k_o h} \right)^{\frac{1}{3}}$$

$$h = \frac{8 \tan(\theta_0')}{9 \cos^2(\theta_0')},$$

$$Z = k_o S(-\theta), \quad \theta = -6\theta_1' + 2\theta_0'.$$
\( R_0 \) is the radius of the circle of stationary rays emerging in the back-scatter direction, \( \theta \) is the angle between the backscatter direction and the stationary ray.

Determining \( \theta_i' \) and \( \theta_o' \) is somewhat lengthy for lossy particles because of the complexity of Snell's law, but the method is straightforward. Using Snell's law as given by Stratton, taking its derivative and noting that \( \frac{d\theta_i'}{d\theta_o'} = \frac{1}{3} \) for two internally reflected stationary rays (singly internally reflected stationary rays are diverging rays for this region of \( n_1 \) and therefore can be neglected) will give the necessary information to calculate \( \theta_i' \) and \( \theta_o' \). The following expressions can be solved for \( \theta_i' \) and \( \theta_o' \)

\[
(25) \quad \frac{\sin(2\theta_i')}{2} = \frac{q(\theta_o') \sin(\theta_o')}{(q^2(\theta_o') + \sin^2(\theta_o'))}
\]

and

\[
(25a) \quad \cos(2\theta_i') \frac{d\theta_i'}{d\theta_o'} = \frac{d}{d\theta_o'} \left[ \frac{q'(\theta_o') \sin(\theta_o')}{(q^2(\theta_o') + \sin^2(\theta_o'))} \right]
\]

where

\[
\frac{d\theta_i'}{d\theta_o'} = \frac{1}{3}
\]

The scattered field for the stationary ray can then be written as

\[
(26) \quad E^S = \frac{e^{-jk_o\ell}}{\ell} A_0 e^{j\phi_0} \frac{4\pi R_0}{\lambda} S |A_i(Z) |
\]

where
\[ A_o = \left| E_i \right| \frac{R_{21}^2(\theta_1')}{T_{12}(\theta_o')} \frac{T_{12}(\theta_1')} {T_{21}(\theta_1')} \left(1 - \frac{6d\theta_1'} {d\theta_o'}\right)^{-\frac{1}{2}}, \]

\[ \phi_o = k_0a(1 - \cos\theta_o') + 3ka(2\cos\theta_1') + k_0a(1 - \cos\gamma) - \pi, \]

and

\[ \gamma = \pi + \theta_o' - 6\theta_1'. \]

\( \overline{R} \) and \( \overline{T} \) are weighted average reflection and transmission coefficients, respectively. Thus

\[ \frac{R_{21}^2(\theta_1')} {T_{12}(\theta_o')} \frac{T_{12}(\theta_1')} {T_{21}(\theta_1')} = \frac{1}{2} \left[R_{21}^{2\parallel} T_{12}^{2\parallel} T_{21}^{2\parallel} - R_{21}^{2\perp} T_{12}^{2\perp} T_{21}^{2\perp}\right]. \]

The square of the backscatter amplitude function for a sphere with \( 1.15 < n_1 < 1.41 \) is

\[ \left| S(180) \right|^2 = \frac{\pi^2 a^2}{\lambda^2} \left[ R_{12}(0) + \frac{T_{12}(0)R_{12}(0)T_{21}(0)}{\left(\frac{2}{d\theta_1} - 1\right)} e^{i(4ka - \pi)} + \frac{4\pi R_o S}{a} \left| A_i(Z) \right| \right. \]

\[ \cdot 2 \frac{T_{12}(\theta_o')}{\lambda} \frac{T_{12}(\theta_1')}{T_{21}(\theta_1')} \left(1 - \frac{d\theta_1'}{d\theta_o'}\right)^{-\frac{1}{2}} e^{i\phi_o} \left. \right|^2. \]

The comparison of the improved three component backscatter amplitude function with the exact curve is shown in Fig. 12. The approximate curve fits well to the exact curve for values of \( x > 3 \). Further checks
of the accuracy were made for values of \( x \) up to 45. The results showed that the error in approximating the exact backscatter amplitude function is within \( \pm 5 \) decibels. Therefore, calculations using this approximation will have an accuracy dependent upon this error.
Fig. 11.—Comparison of exact and approximate solutions of $S(180^\circ)$ of a dielectric sphere with $n_1 = 1.23$.
(First approximation).

Fig. 12.—Comparison of exact and approximate solutions of $S(180^\circ)$ of a dielectric sphere with $n_1 = 1.23$.
(Second approximation).
CHAPTER IV
CALCULATIONS USING AEROSOL MODELS

A. Introduction

Having described the nature of the aerosol particles and formulated the expressions for the extinction and backscatter coefficients in the previous chapter, this chapter will present the calculations for the two coefficients using the three aerosol models given in Chapter II. Also included are discussions of results found by other investigators.

B. Aerosol Extinction Coefficient

1. Continental aerosol model

To compute the volume extinction coefficient $B_{EXT}$ given in Eq. (18), it is necessary to fit a model to the aerosol size distribution given in Fig. 2. Figure 13 shows two models similar to curve 1 in the figure. The model represent the distribution for 90 and 95 per cent relative humidities. They are plotted for several power law distribution slopes $v$ between 2.5 and 4.0 and are normalized to a total aerosol concentration of $10^2$ per cm$^3$.

Calculations of $B_{EXT}$ were made using models similar to those in Fig. 13, the only difference being in the choice of relative humidity and concentration. Substituting the results of Eq. (14) and Fig. 1 into
Eq. (18) and numerically integrating over the particle radius gives the results for $B_{E X T}$ found in Figs. 14 and 15. The calculations were made for selected wavelengths between 0.34 and 10.6$\mu$ and a total particle concentration of $10^3$ per cm$^3$.

The decrease in extinction coefficient for infrared wavelengths is apparent from the figures. For $\nu = 3$ the wavelength dependence is approximately a $1/\lambda$ function. But it is obvious that this function is very general. It doesn't, for example, explain the three extinction windows located at 2.0, 3.7 and 10.6$\mu$ for $\nu \geq 3$. These windows have been observed by several investigators[21, 22, 23] and are due to variations in the imaginary index of refraction term for water.

For $\nu < 3$ it can be seen that the approximation is no longer valid. As $\nu$ decreases, the distribution contains a larger portion of large radii particles and the scattering becomes wavelength independent.

In order to estimate the volume extinction coefficient for continental areas from these figures, it is necessary to know the total particle concentration. Integrating the distribution given in Fig. 2 for relative humidities < 90% gives an approximate concentration of $5 \times 10^3$. Since the models are normalized to $10^3$ the multiplicity factor is 5. Thus for an uncontaminated continental atmosphere having a relative humidity < 90%, $B_{E X T}$ is estimated from curve $\nu = 3$ to be $1.6 \times 10^{-1}$Km$^{-1}$ for $\lambda = 0.55\mu$ and $5.3 \times 10^{-3}$Km$^{-1}$ for $\lambda = 10.6\mu$. 
Fig. 13.--Continental aerosol size distribution model for ≤ 90 and 95 percent relative humidity.
Fig. 14.—Extinction coefficient vs. wavelength for continental aerosol model with relative humidity ≤ 90%.
Fig. 15.--Extinction coefficient vs. wavelength for continental aerosol model with relative humidity = 95%.
It must be remembered that \( \text{BEXT} \) does not include molecular absorption.

The above estimates for \( \text{BEXT} \) were made using an average particle concentration and \( \nu = 3 \). Of course these conditions do not apply at all times. To accurately predict \( \text{BEXT} \) using Fig. 14 and 15, an investigator would have to accurately measure the particle size distribution and concentration. This of course is not possible in most cases. Often the investigator has only the means to measure meteorological conditions and visibility (visibility = \( \frac{3.92}{\text{BEXT}(\lambda=0.55\mu)} \)). See Chapter VI for derivation). Therefore the best that the investigator can do is to determine upper and lower limits for \( \text{BEXT} \) for given atmospheric conditions. This, it is felt, can be done with reasonable accuracy for continental and light haze conditions by placing the \( \nu = 2.25 \) or \( 4.0 \) curve (depending on whether the upper or lower limit is wanted), corrected for relative humidity, on the corresponding \( \text{BEXT}(0.55\mu) \) obtained from visibility measurements. Thus the value of \( \text{BEXT} \) for wavelengths other than 0.55\( \mu \) will be expected to fall between the two curves \( \nu = 2.25 \) and \( \nu = 4.0 \).

From published experimental results near ocean areas the multiplicity factor for Figs. 14 and 15 was estimated to be one rather than the five used for continental areas.[21,24] Entering the curves for \( \nu = 3 \) and relative humidities < 90\%, the ocean \( \text{BEXT} \) is estimated to be \( 3.5 \times 10^{-2} \text{ Km}^{-1} \) at \( \lambda = 0.55\mu \) (visibility = 110 Km). This
corresponds to the high visibilities usually present near ocean areas. A procedure similar to the one just described for the continental aerosols can be used to find the upper and lower limits for $B_{EXT}$ at other wavelengths in ocean areas.

2. Fog droplet model

Calculations similar to those in the previous section will be presented here. The difference is in the choice of the aerosol model, all the calculations in this study will use the fog model in Eq. (3).

In the previous section $B_{EXT}$ for the continental aerosol was proportional to the total particle concentration. This parameter, as it was explained, is difficult for most investigators to measure. Therefore in this section $B_{EXT}$ will be normalized to the fog liquid water density because it is somewhat easier to measure.

The total fog droplet volume is written as

$$V_W = \int_0^{\infty} \eta(R) \frac{4\pi}{3} R^3 dR.$$  

Letting $a = R/R_m$ and using Eq. (4), Eq. (29) may be rewritten as

$$V_W = \frac{4\pi}{3} \left( \frac{\gamma}{\alpha} \right)^3 \frac{\Gamma\left(\frac{\alpha+4}{\gamma}\right)}{\Gamma\left(\frac{\alpha+1}{\gamma}\right)} R_m^3 N.$$
where $\Gamma$ represents the gamma function and $N$ is the total fog particle concentration. Assuming the density of water to be a constant $1 \text{ gm/cm}^3$, $B_{\text{EXT}}$ may be written

$$B_{\text{EXT}} = \frac{4.34\pi \int_0^\infty \eta(a) Q_{\text{EXT}}(m, a) a^2 da}{R_m \frac{4\pi}{3} \int_0^\infty \eta(a) a^3 da} \left( \frac{\text{dB}}{\text{Km}} \left| \frac{\text{mg}}{\text{m}^3} \right) \right),$$

where $R_m$ has dimensions of microns.

The results of Eq. (31) are plotted in Figs. 16a to 20a for several combinations of the model parameters and for several wavelengths between 0.34 and 10.6$\mu$. Associated with these figures are Figs. 16b to 20b giving the fog model water density content versus visibility ($\lambda = 0.55\mu$) for selected values of $R_m$.

There are several interesting features that should be noted.

For values of $R_m > 5\mu$ the extinction coefficient becomes wavelength independent. That is, a fog meeting these conditions would give the same value of $B_{\text{EXT}}$ for wavelengths between 0.34 and 10.6$\mu$. For $R_m < 5.0$, infrared wavelengths do not necessarily guarantee smaller values of $B_{\text{EXT}}$ compared to the shorter wavelengths. For instance, for the fog model $\sigma = 1$, $\gamma = 1.0$ and $R_m < 0.5\mu$, $B_{\text{EXT}}(3.9)/B_{\text{EXT}}(0.7)$ is less than one. But for $R_m = 0.7\mu$, $B_{\text{EXT}}(3.9)/B_{\text{EXT}}(0.7)$ is greater than one. Thus it is seen that relatively small changes in $R_m$ can change the wavelength dependence considerably.
Fig. 16(a, b).—Extinction coefficient and water density content for fog model $\alpha = 1$, $\gamma = 1$. 

(a) Total Extinction (dB km$^{-1}$) vs. $R_m$ (microns) for various particle sizes. 

(b) Water content vs. visibility for fog model $\alpha = 1.0$, $\gamma = 1.0$. 

- $R_m$ (microns): Radius of particles with maximum number density.
Fig. 17(a, b).--Extinction coefficient and water density content for fog model $\alpha = 2.0$, $\gamma = 0.5$. 
Fig. 18(a, b).--Extinction coefficient and water density content for fog model $\alpha = 3.0$, $\gamma = 0.5$. 
Fig. 19(a, b)-Extinction coefficient and water density content for fog model $\alpha = 6.0$, $\gamma = 1.0$. 

\[ \text{TOTAL EXTINCTION (m}^{-1} \text{)} \]

\[ \text{RADIUS OF PARTICLES WITH MAXIMUM NUMBER DENSITY (MICRONS)} \]

\[ \alpha = 6.0 \quad \gamma = 1.0 \]
Fig. 20 (a, b).--Extinction coefficient and water density content for fog model \( \alpha = 6.0, \gamma = 0.5 \).
Also from observing the various models for $\lambda = 10.6\mu$ it would appear that this wavelength would make a good source wavelength to measure the liquid water density for a fog. B_{EXT} for all models and for $R_m$ between 0.3 and 2.0$\mu$ is approximately a constant

$$4.0 \frac{\text{dB}}{\text{Km}} \left| \frac{\text{mg}}{\text{m}^3} \right| + 1.5 \frac{\text{dB}}{\text{Km}} \left| \frac{\text{mg}}{\text{m}^3} \right|.$$ 

Thus by measuring $B_{EXT}(10.6)$, the water density can be determined.

If an investigator wishes to determine the upper and lower bounds for $B_{EXT}$, it is necessary to place limits on $R_m$ and on the fog model. From the results of Chu and Hogg[9] it is apparent that one fog model and one $R_m$ cannot describe every real fog size distribution. For instance, a plot of a percentage time distribution for the ratio $B_{EXT}(3.5)/B_{EXT}(0.63)$ during various fogs indicated a range from 0.2 to 2.0 with a median value 0.6. The two extreme ratios can be explained by entering Fig. 20a at $R_m = 0.3\mu$ and Fig. 17a at $R_m = 0.8\mu$ or Fig. 20a at $R_m = 1.0\mu$, respectively. They also made simultaneous measurements of $B_{EXT}(0.63)$, $B_{EXT}(10.6)$ and the water density during a dense fog. The fog water density was determined by measuring the attenuation of 3mm waves. At the start of the experiment the two signals could not be detected and the 3mm data indicated a fog density of 300 mg/m$^3$. If Fig. 20a is entered at $R_m = 0.7\mu$, the results for $B_{EXT}(0.3)$ and $B_{EXT}(10.6)$ is 1200 and 150 dB/Km, respectively for a water density of 300 mg/m$^3$. These results certainly verify why they were unable to detect any signal. As the fog lifted and the signal
became detectable, the ratio \( \frac{B_{\text{EXT}}(0.63)}{B_{\text{EXT}}(10.6)} \) was approximately 8. Entering Fig. 20a at \( R_m = 0.7 \mu \) will give this same ratio. Thus from these results and the results of other investigators,\[10, 24\] it appears that for experimental accuracy, a lower and upper limit of 0.3 and 1.0 \( \mu \) may be placed on \( R_m \), respectively.

It should be cautioned that these values appear to be able to explain present experimental results but should not be taken as exact values.

3. Precipitation model

Water density is not a conveniently measurable parameter of precipitation. The only such parameter is the rate of precipitation or in other words the volume of water reaching the ground per unit time. The rate can be expressed as

\[
(32) \quad r = 15.1 \int_0^\infty \eta(R)V(R)R^3 \, dR \text{ mm/HR},
\]

where \( r \) is the rain rate, \( V(R) \) is the terminal velocity in m/sec of the drops with radius \( R \). \( \eta(R) \) and \( R \) have dimensions of m\(^{-3}\) and cm, respectively.\[25\]

In order to facilitate calculations of the rain rate an empirical expression for \( V(R) \) was derived from experimental measurements given by Goldstein.\[25\] The expression is
(33) \[ V(R) = 9.6 (1 - e^{-11R}) \text{ m/sec}, \]

where \( R \) has dimensions of \( \text{cm} \).

In order to normalize \( B_{\text{EXT}} \) to rain rate, it is first necessary to express the percentage size density distribution of Laws and Parsons to a size density distribution. The two distributions are related by

\[
m(R, r) = \frac{\eta(R) \frac{4\pi}{3} R^3 V(R) dR}{\int_0^\infty \eta(R) \frac{4\pi}{3} R^3 V(R) dR} = \frac{15.1 \eta(R)V(R)R^3 dR}{r}.
\]

Next it is necessary to introduce a new function \( g(R, \lambda) \) before \( B_{\text{EXT}} \) can be defined. \( g(R, \lambda) \) is given as

\[
g(R, \lambda) = \frac{Q_{\text{EXT}}(R, \lambda)}{15.1 V(R)R^3}.
\]

Using the results of Eq. (18), Eq. (34) and Eq. (35) \( B_{\text{EXT}} \) can be written as

\[
B_{\text{EXT}} = 0.434 r \int_0^\infty m(R, r)g(R, \lambda) dR \text{ dB/Km}.
\]

The calculations for \( B_{\text{EXT}} \) can be made somewhat easier by noting that \( Q_{\text{EXT}} \) can be approximated by its asymptotic value of 2.0 for wavelengths between 0.33 and 11.0. Equation (36) can also be solved for \( B_{\text{ABS}} \) by replacing 2.0 with 1.0, the asymptotic value of \( Q_{\text{ABS}} \) for \( n_2 \neq 0 \).
Figure 21 shows the plotted results of $B_{EXT}$ and $B_{ABS}$. The coefficients are plotted as functions of rain rate for wavelengths between 0.3 and 11.0 $\mu$.

It can be seen that for low rain rates the coefficients are not linear functions of rain rate, although for rain rates > 40 mm/Hr, linearity is approached. For very large rain rates of 90 to 100 mm/HR, $B_{EXT}$ is approximately 26 dB/Km. Although this is a sizable attenuation coefficient, it is considerably smaller than the values encountered by Chu and Hogg[9] for dense fog conditions.

Thus, dense fogs are expected to provide much higher attenuations than the heaviest rain storms.

C. Aerosol Backscatter Coefficient

1. Continental aerosol model

It was shown in Chapter III that an illuminated aerosol scatter volume scatters part of the incident radiation backward toward the radiation source ($\theta = 180^\circ$). This scattered radiation was shown to be proportional to the product of the normalized angular scatter function evaluated at $\theta = 180^\circ$ and the volume scatter coefficient. This product, $\frac{P(180)}{4\pi} B_{SCA}$, will be referred to as the volume backscatter coefficient.

Equation (20) can be solved for the backscatter coefficient which is written as
Fig. 21.--Attenuation coefficient for precipitation vs. rain rate.
\begin{equation}
\frac{P(180)}{4\pi} \text{BSCA} = \frac{1}{k^2} \int_{R_1}^{R_2} \left| S(m, \lambda, 180) \right|^2 \eta(R) \, dR, \text{ (steradian cm)}^{-1}.
\end{equation}

Calculations were made using the exact theory of Mie and using the aerosol continental models shown in Fig. 13 for relative humidities < 90%. \( R_1 \) and \( R_2 \) were 0.1\( \mu \) (R.H. < 90%) and 10.0\( \mu \), respectively. Simpson's approximation method for integration was used.

The results of the calculations are shown in Fig. 22 for various size distribution slopes \( v \) between 2.5 and 4.0 and for selected wavelengths between 0.6 and 10.6\( \mu \). The total aerosol particle concentration is \( 10^3 \) per cm\(^3 \).

It is interesting to compare the wavelength dependence of these curves with the extinction coefficient curves in Fig. 14. The \( v = 3 \) curve of \( \text{BEXT} \), for example, has an approximate \( 1/\lambda \) dependence while the \( v = 3 \) curve of \( \frac{P(180)}{4\pi} \text{BSCA} \) has an approximate \( 1/\lambda^2 \) wavelength dependence. Therefore if the extinction coefficient windows of \( \text{BEXT} \) and atmospheric absorption are not taken into account, a laser ranging system would have a received signal to continental aerosol backscatter noise ratio dependence of \( 1/\lambda^3 \). This example has not taken into account the wavelength dependence of the scattering target.

It should also be pointed out in Fig. 22 that as \( v \) decreases the backscatter curve becomes more structured and the \( 1/\lambda^2 \) dependence no longer holds.
Fig. 22.--Backscatter coefficients vs. wavelength for continental aerosol model.
2. **Fog droplet model**

The backscatter coefficient expressed in Eq. (20) was calculated for the size distributions given by Eq. (3). Calculations were made for various combinations of model parameters, for selected wavelengths in the range $0.7 < \lambda < 10.6\mu$ and for $R_m = 0.2, 0.7, 1.2, 3.2$ and $4.2\mu$.

In order to make the calculations it was necessary to use both the exact and approximate solutions for $S(m, \lambda, 180^\circ)$. For wavelengths $10.6, 8.0$, and $5.3\mu$, the exact solution for $S(m, \lambda, 180^\circ)$ was used. For wavelengths $3.5, 2.8, 1.06$ and $0.7\mu$, the integration in Eq. (20) was divided into two parts. For $x < 3.0$ the exact solution for $S(m, \lambda, 180^\circ)$ was used; for $x > 3.0$ the approximate solution for $S(m, \lambda, 180^\circ)$ was used. The results were normalized to the fog liquid water density given in Eq. (30) and plotted in Figs. 23 to 27 as a function of $R_m$ and $\lambda$.

In order to check the accuracy of the approximate solution, exact calculations were made for $R_m = 0.2\mu$ and $\lambda = 0.7, 1.06, 2.8$ and $3.5\mu$. The results are indicated in the figures. It can be seen that there is some disagreement, but it must be remembered that the approximation was estimated to be good to only $\pm 5$ decibels. The difference is within this error. It is felt that the approximate curves give the correct variation of the backscatter coefficient for variations of $R_m$. Therefore if the approximate curves are displaced vertically to match the
exact values at $R_m = 0.2 \mu$, the backscatter curves should give the correct results for larger values of $R_m$.

There are very few backscatter experimental measurements by other investigators to compare with these calculated results. However, previous discussions about the fog model BEXT will make it possible to estimate a range for the backscatter coefficient.

It was previously shown that the experimental measurements of fog BEXT could be explained using Figs. 16a to 20a with a $R_m$ between 0.3 and 1.0$\mu$. For a given $R_m$ in this range, it is apparent that there is at most only a 6 decibel difference in the backscatter coefficients for all fog models. Therefore the upper and lower limits are predictable to within 6 decibels.

It is again apparent that 10.6$\mu$ radiation offers the lowest scattering coefficient. But unlike the extinction coefficient curves, the 10.6$\mu$ backscatter curve does not approach the shorter wavelength curves for $R_m > 5.0 \mu$. The backscatter coefficient remains wavelength dependent for all values of $R_m$. It is also interesting to note the exceptionally low results for $\lambda = 2.8 \mu$. This result is undoubtedly due to the high BABS for water in this wavelength region. The investigator need not concern himself with the backscattering at 2.8$\mu$ because high atmospheric water vapor absorption in this region prohibits its use as a laser source for atmospheric propagation.
Fig. 23.--Backscatter coefficients vs. wavelength for fog model $\alpha = 1$, $\gamma = 1$. 
Fig. 24.—Backscatter coefficients vs. wavelength for fog model $\alpha = 2$, $\gamma = 0.5$. 
Fig. 25. -- Backscatter coefficients vs. wavelength for fog model $\alpha = 3$, $\gamma = 0.5$. 
Fig. 26.--Backscatter coefficients vs. wavelength for fog model $\alpha = 6$, $\gamma = 0.5$. 

[Graph showing backscatter coefficients vs. wavelength for different radii of particles in microns.]
Fig. 27. -- Backscatter coefficients vs. wavelength for fog model $\alpha = 6$, $\gamma = 0.5$. 
3. Precipitation model

The function \( g(R, \lambda) \) must again be introduced because the calculations of the normalized angular scatter function \( \frac{P(180)}{4\pi} \) involves the percentage density distribution of Laws and Parsons. \( g(R, \lambda) \) for the backscatter model may be written as

\[
g(R, \lambda) = \frac{|S(m, \lambda, 180)|^2}{15.1 \ V(R) R^3},
\]

where \( R \) has dimensions cm and \( V(R) \) has dimensions m/sec. \( \frac{P(180)}{4\pi} \) may be written as

\[
\frac{P(180)}{4\pi} = \frac{10^{43} R}{B_{SCA} k^2} \int_0^\infty m(R) g(R, \lambda) dR \ (\text{steradian})^{-1},
\]

where \( B_{SCA} \) has dimensions \( \text{Km}^{-1} \), \( k \) has dimensions \( \text{m}^{-1} \) and \( r \) has dimensions mm/HR.

The backscatter coefficient was calculated using Eq. (39) and is plotted in Fig. 28 as a function of rain rate. Several curves are given for selected wavelengths in the range \( 0.34 < \lambda < 10.6 \mu \). The modified geometric optics approximation for \( S(m, \lambda, 180^\circ) \) was used in the calculations.

The curves of the backscatter coefficients show clearly that the function is not a linear function of rain rate. For rain rates greater than 25 mm/HR the backscatter coefficient varies exponentially with rain rate. It should also be pointed out that although \( B_{EXT} \) is wavelength
independent for rain, $\frac{P(180)}{4\pi} BSCA$ is not. For rain rates above 25 mm/HR the calculated backscatter coefficient at $\lambda = 10.6\mu$ is approximately 7 dB below those values for $\lambda = 0.7\mu$. Here again 10.6\mu results in the lowest scattering coefficient.

The results of Fig. 28 indicate that the largest coefficient is for $\lambda = 1.06\mu$. This result appears to be in error and is probably due to the error in the approximate method of $S(1.06, 180^\circ)$.

It is interesting to compare the calculated backscatter coefficients for fog and rain at 0.63\mu. For very heavy rain rates of 100 mm/HR, the coefficient is estimated to be $2.0 \times 10^{-1}$ (Km steradian)$^{-1}$, while for a dense fog having a water density of 300 mg/m$^3$ and a fog model $\alpha = 6.0$, $\gamma = 0.5$ at $R_m = 0.7\mu$, the estimate is $9.0 \times 10^{-1}$ (Km steradian)$^{-1}$. This difference is considerably smaller than that found for the extinction coefficients under the same conditions.

D. Atmospheric Slant Paths

1. Integrated aerosol extinction coefficient

In the preceding sections the atmospheric aerosol distribution was assumed to be homogeneous over the entire path. Such an assumption is not valid for atmospheric slant paths. In this case the transmittance for a path of length $L$ is given by

$$T = e^{-\int_0^L B_{EXT}(\ell) \, d\ell}$$

(40)
Fig. 28. -- Calculated backscatter coefficients vs. rain rate for precipitation aerosol model.
where $\text{BE}_\text{XT}$ includes only aerosol extinction losses and does not include molecular absorption or other losses.

An integrated extinction coefficient can be defined as

$$(41) \quad \text{BE}_\text{XT}(L) = \int_0^L \text{BE}_\text{XT}(t) \, dt.$$  

For zenith paths the incremental path length and incremental change in altitude are equal and Eq. (41) becomes

$$(42) \quad \text{BE}_\text{XT}(L) = \text{BE}_\text{XT}(H) = \int_0^H \text{BE}_\text{XT}(h) \, dh ,$$

where $H$ is the terminating altitude above the path origin.

For zenith angles less than $80^\circ$ the path length along the slant path is related by the sine of the elevation angle $\phi$ by

$$(43) \quad h = l \sin \phi .$$

The integrated extinction coefficient becomes

$$(44) \quad \text{BE}_\text{XT}'(L) = \text{BE}_\text{XT}'(H) = \int_0^H \text{BE}_\text{XT}(h) \csc(\phi) \, dh = \csc(\phi) \text{BE}_\text{XT}(H).$$

It is seen that the extinction coefficient along a slant path is the product of the cosecant of the elevation angle and the integrated extinction coefficient. The slant path transmittance can then be obtained by raising the zenith transmittance to the $n$th power, where $n$ is the cosecant of the elevation angle.
2. **Slant path extinction coefficient for continental aerosol at λ = 0.55 microns**

The extinction coefficient given in Eq. (18) involves both the aerosol size distribution and the total aerosol concentration. To determine the exact transmittance requires the knowledge of both of these variables at each point along the path. Fortunately such exactness is not required for most laser propagation applications and satisfactory results can be obtained using a model which approximates average atmospheric conditions.

Elterman[26] has compiled an extensive atlas of vertical profiles for aerosol extinction coefficients at λ = 0.55μ. The profiles are a result of measurements made using searchlight optical probing methods.

No one aerosol profile describes all atmospheric aerosol distributions. One model is used for this study. It is based on the mean vertical profile compiled by Elterman from 105 profiles representing measurements made during December 1963 to December 1964 over New Mexico. Figure 29 shows the aerosol extinction coefficient as a function of the altitude. B_{EXT}(h) in Eq. (44) can be found directly from the mean vertical profile for 2.5 Km ≤ h ≤ 35 Km.

No one aerosol model could represent the average aerosol extinction coefficient for altitudes below 2.5 Km because of the large day to day variations. Therefore, for this study a linear function was used to describe the extinction coefficient for 0 Km < h < 2.5 Km.
Fig. 29.--Computed mean aerosol extinction coefficients from 105 vertical profiles.[26]
A straight line is fitted to the mean data at $h = 2.5 \text{ Km}$ and extended to $B_{EXT}(h=0)$. The resulting linear function is given as

$$B_{EXT}(h) = -h \frac{(B_{EXT}(h=0) - 6.5 \times 10^{-3})}{2.5} + B_{EXT}(h=0)$$

where $h$ is the altitude in Km and $B_{EXT}(h=0)$ is the aerosol extinction coefficient at ground level. Because of the irregular shape of the mean vertical profile, an approximation formula cannot be easily described for $B_{EXT}(h)$ with $2.5 \text{ Km} < h < 35 \text{ Km}$.

It is necessary therefore to compute the integrated extinction coefficient in Eq. (41) by numerical integration techniques. After the integration, $B_{EXT}(H)$ was then converted into transmittance and the results shown in Fig. 30. The results were plotted as a function of horizontal daylight visibility rather than $B_{EXT}(h=0)$.

The aerosol extinction coefficient for nonzenith paths traversing 34 Km is obtained by multiplying the zenith extinction coefficient by the cosecant of the elevation angle. A plot of the total extinction versus elevation angles is shown in Fig. 31.

In these calculations day to day variations in the aerosol profiles have been neglected. Undoubtedly the transmittance in certain geographical areas is subject to aerosol variations unique to that region. Therefore these calculated results should be considered as approximations and not as the exact extinction.
Fig. 30. -- Calculated 0.55 micron transmittance for mean aerosol and Rayleigh particle concentration in a zenith path from ground level vs. the terminus altitude for several ground level visibilities.

3. **Slant path extinction coefficient for continental aerosols at other wavelengths**

The slant path calculations in the previous section are only correct for wavelengths near 0.55 microns because the experimentally determined extinction coefficient profiles were made at $\lambda = 0.55$ microns. In order to make slant path calculations at other wavelengths it is necessary to correct the profile data in Fig. 29. Figure 14 will be used in correcting the profile data for the appropriate wavelength.

First, enter the figure at the desired wavelength $\lambda_d$ on curve $\nu = 3$, and determine $B_{\lambda_d}^{\text{EXT}}(h=0)$. Next enter at $\lambda = 0.55\mu$ and
Fig. 31. -- Calculated 0.55 micron attenuation for mean aerosol and Rayleigh particle concentration vs. the elevation angle for propagation paths traversing a slant path terminating at 34 Km above ground level.
determine $B_{\text{EXT}}^{\lambda=0.55\mu}(h=0)$. Then the integrated extinction coefficient in Eq. (42) can be written as

$$B_{\text{EXT}}^{\lambda}(H) = \int_{0}^{H} \frac{B_{\text{EXT}}^{\lambda}(h)dh}{B_{\text{EXT}}^{\lambda=0.55\mu}(h)}$$

$$= \frac{B_{\text{EXT}}^{\lambda=0.55\mu}(h=0)}{B_{\text{EXT}}^{\lambda=0.55\mu}(h=0)}$$

where $B_{\text{EXT}}^{\lambda=0.55\mu}(H)$ is the integrated extinction coefficient at the desired wavelength $\lambda_d$. The zenith or slant path transmittance can then be calculated by raising the transmittance at $\lambda = 0.55$ microns to the $m$(th) power, where $m$ is the ratio $B_{\text{EXT}}^{\lambda=0.55\mu}(h=0)/B_{\text{EXT}}^{\lambda=0.55\mu}(h=0)$ determined from Fig. 14. For example, suppose the transmittance is desired at $10.6\mu$ for a zenith path having a terminus range of 6 Km and a ground level visibility of 19 Km. From Fig. 14 it is seen that $B_{\text{EXT}}^{\lambda=0.55\mu}(h=0)$ and $B_{\text{EXT}}^{\lambda=0.55\mu}(h=0)$ are $1.6 \times 10^{-1}$ and $5.3 \times 10^{-3}$ Km$^{-1}$, respectively. The ratio $B_{\text{EXT}}^{\lambda_d=0.55\mu}(h=0)/B_{\text{EXT}}^{\lambda=0.55\mu}(h=0)$ is then $3.3 \times 10^{-2}$. The transmittance at $\lambda = 0.55\mu$ is found in Fig. 29 to be 70%. Therefore the transmittance at $10.6\mu$ is expected to be $(70)^{3.3 \times 10^{-2}}$ or approximately 98.5%.
CHAPTER V
OUTDOOR MEASUREMENTS OF WATER VAPOR
AND CARBON DIOXIDE ABSORPTION AT 10.59 MICRONS

A. Introduction

The measured extinction coefficient at visible wavelengths is caused only by aerosol scattering. This is because there is negligible aerosol and molecular absorption in this wavelength region. But for infrared wavelengths such as 10.6μ, it is not possible to attribute the total extinction coefficient to scattering alone. Most infrared wavelengths are partially absorbed by aerosols and molecules and unless it is possible to determine the molecular absorption contribution, it is not possible to determine what part aerosol scattering contributes to the total measured extinction coefficient.

B. Results of Indoor Measurements of Absorption By Water Vapor and Water Vapor-Air Mixture

McCoy[1] has made pure water vapor and water vapor-air mixture absorption measurements of the CO₂ P(20) line using an indoor 980 meter multipass absorption cell.

At a temperature of 25°C, McCoy measured the difference in transmittance of the P(20) line for the evacuated cell and for the cell
filled with water vapor at various pressures. He used the following transmittance expression

\[(47) \quad T_\lambda = e^{-K(\lambda)L},\]

where \(K(\lambda)\) is the extinction coefficient due to water vapor absorption at wavelength \(\lambda\). Equation (47) can be solved for the extinction coefficient

\[(48) \quad K(\lambda) = -\frac{\ln T}{L}.\]

He made absorption measurements for a water vapor-air mixture by first filling the cell with pure water vapor and then adding manufactured "air" consisting of 80% nitrogen and 20% oxygen until the total pressure was 700 torr. Taking into account self broadening and broadening from neutral gases his results are expressed in the following empirical equation

\[(49) \quad \text{Loss} = 1.43 \times 10^{-2} p + 3.62 \times 10^{-3} p^2 \text{ dB Km}^{-1}\]

(water vapor-air mixture).

McCoy has also determined the loss of radiation oscillating on the \(P(20)\) line from atmospheric carbon dioxide. He estimated the carbon dioxide content of a sea level atmosphere to be 330 parts per million. This results in a partial pressure of about 0.255 torr or \(3.29 \times 10^{-4}\)
atmospheres. The transmittance for this partial pressure of carbon dioxide is

\[ T(10.59) = e^{-k(10.59)(3.29 \times 10^{-4})L} \]

where \( k(10.59) \) is the Lorentzian extinction coefficient at line center.

From transmittance measurements using an indoor multipass absorption cell,[1] he found the following extinction coefficient

\[ k(10.59) = 2.43 \times 10^{-2} \text{ atm}^{-1} \text{ Km}^{-1} \]

With this value the transmittance from Eq. (50) is

\[ T(10.59) = e^{-0.080L} \]

where \( L \) is in kilometers.

In units of decibels, the absorption loss is

\[ \text{Loss} = 0.347 \text{ dB Km}^{-1} \text{ (carbon dioxide - 330 parts per million).} \]

C. Outdoor Measurements of the Molecular Absorption at 10.59 Microns

1. Calibration methods

After the 10.59\( \mu \) laser was allowed to warm up and maximum thermal stability was achieved, the signal beam detector was calibrated against the reference detector. (The reader is referred to Appendix B for details of the experimental apparatus.)
The output of the laser was tuned to the center frequency of the P(20) line by adjusting the cavity length for maximum voltage output from the reference detector. Frequent checks were made with a spectrometer to be certain that the laser was oscillating only on the P(20) line. A twenty-four inch diameter parabola which fed the signal beam detector was placed in front of the collimating telescope to collect the 10.59μ collimated beam. The voltage output from the thermopile signal detector and the reference detector were set in one to one correspondence. After the extinction coefficient measurement the calibration process was repeated. If the two output voltages were within ± 5% the results of the extinction coefficient were recorded. If not, the whole process was repeated.

Before each run the temperature and relative humidity were recorded. The relative humidity was measured at both ends of the propagation range with sling psychrometers and the average reading of the two was recorded with an estimated error of ± 3%.

2. **Absorption measurements at 10.59μ**

After the thermopile detector was calibrated against the reference detector, the thermopile was removed to the remote site and placed at the focal point of the twenty-four inch Stellite parabola. The visible slave laser beam was pointed into the upper half of the parabola and focused onto the aperture of the detector. Fine alignment was
accomplished by moving the detector until maximum output voltage was obtained. Because the beam width at the remote site assured complete beam collection, the one-way path was chosen over the two-way path.

The transmittance was determined from the ratio of the signal detector voltage and the reference detector voltage. Equation (48) was then used to determine the extinction coefficient.

The recorded results were the average of several measured ratios over a period of several minutes. All measurements were made during visibilities greater than 8 km. Measurements were made during daylight and night time hours. The temperature and relative humidity range covered was 64 to 87°F and 50 to 90%, respectively. The attenuation measurements are shown as the solid curve drawn through the measured points in Fig. 32. The calculated attenuation using Eqs. (49) and (53) is shown as the dotted curve. The measured outdoor absorption results clearly verify the accuracy of Eq. (49).

If a 0.47 dB km⁻¹ loss is attributed to carbon dioxide the calculated curve agrees with the solid curve in Fig. 32. A 0.47 dB km⁻¹ loss indicates a carbon dioxide concentration of approximately 445 ppm. This result is in agreement with measurements of the 10.59µ P(20) line center transmittance using the indoor multipass absorption cell filled with dried air from a well ventilated laboratory room.
Further indications that the carbon dioxide concentration may be higher than usually assumed results from preliminary night time measurements over the outdoor path before and after maturing wheat surrounding the path was cut. Differences as great as 1 decibel per kilometer were found. More measurements are needed to verify these results. Independent measurements of the carbon dioxide concentration should also be a part of the program.

Fig. 32. --Measured outdoor attenuation at 10.59 microns.
CHAPTER VI
AEROSOL EXTINCTION AND
BACKSCATTER MEASUREMENTS

A. Introduction

The aerosol extinction and backscatter measurements using the outdoor propagation path may be divided into the following groups:

1. The aerosol extinction coefficient
   a. Continental conditions
   b. Light fog conditions
   c. Precipitation conditions

2. The aerosol backscatter coefficient
   a. Continental conditions
   b. Light fog conditions
   c. Precipitation conditions

This chapter discusses the measurements in each of these groups.

B. Aerosol Extinction Coefficient Measurements

1. Visibility approximation

The term V denotes the horizontal visibility which is defined to be the distance at which one can see a large black object in daylight against the sky near the horizon. The relation between the horizontal visibility
and the extinction coefficient, given originally by Koschmieder and described by Middleton[27] is

\[ V = \frac{1}{B_{\text{EXT}}} \ln \frac{1}{T}, \]  

where \( T \) is the threshold value of brightness contrast of the eye, and \( B_{\text{EXT}} \) is the average extinction coefficient for daylight as seen by the human eye. Numerically \( B_{\text{EXT}} \) is approximately equal to the spectral extinction coefficient \( B_{\text{EXT}}(0.55) \). For daylight and large black objects it is customary to take \( T = 0.02 \), hence Eq. (54) becomes

\[ V = \frac{3.92}{B_{\text{EXT}}(0.55)}. \]  

For visibilities reported in this chapter, \( B_{\text{EXT}} \) will be assigned the value of the extinction coefficient measured at 0.6328\( \mu \).

2. **Local continental aerosol conditions at 0.6328 microns**

Extinction measurements at 0.6328\( \mu \) are not in themselves significant. That is, the extinction coefficient can be reasonably determined by visually estimating the visibility and substituting the estimate into Eq. (55). Therefore it is obvious that for the great range of visibilities that can and do occur in everyday weather, the extinction coefficient at 0.6328\( \mu \) covers a corresponding range. What is significant about these extinction measurements is the upper limit to the visibility or the corresponding lower limit found for the 0.6328\( \mu \) extinction coefficient.
If continental aerosols were not present, the extinction coefficient at 0.6328 μ would be determined by Rayleigh scattering.[28] Figure 33 shows how the Rayleigh extinction coefficient varies with wavelength for dry and humid air. For λ = 0.6328 μ, the extinction coefficient is approximately $6.0 \times 10^{-3}$ Km$^{-1}$ with a corresponding visibility of 655 Km. It is interesting to compare this result with that found for an average continental aerosol model.

The extinction coefficient for the average continental aerosol model given in Fig. 2 is determined by entering Fig. 14 at λ = 0.6328 μ and on ν = 3.0. Since the total particle concentration for Fig. 2 is approximately $5 \times 10^3$ per cm$^3$, the results found in Fig. 14 must be multiplied by five. The result for the extinction coefficient is $5.0 \times 3.0 \times 10^{-2}$ Km$^{-1}$ or $1.5 \times 10^{-1}$ Km$^{-1}$ with a corresponding visibility of 26 Km. Comparing the two extinction coefficients shows that Rayleigh scattering can be neglected for visible and longer wavelengths. Therefore, visibility upper limits are determined by the presence of continental aerosols. This conclusion is strongly supported by the experimental findings of this study. The maximum visibility measured was 23 Km and 20 to 22 Km was often the range of visibility measured on clear nights. From these results it appears that this locale has a continental aerosol concentration close to that given in Fig. 2.
Fig. 33.--Rayleigh extinction coefficient for dry and very moist air.
3. **Light fog conditions at 0.6328 and 10.59 microns**

In this section the results of simultaneous transmittance measurements at 0.6328 and 10.59\(\mu\) during light fog will be presented. Unfortunately heavier fogs were not studied because of the locale. Of the three fogs studied, the lowest visibility recorded was 3.8 Km. Even though heavier fogs were not studied, the results from the light fogs studied definitely showed that the reduction in scattering loss at 10.59\(\mu\) is considerable when compared to 0.6328\(\mu\).

The computed water vapor and carbon dioxide absorption for 10.59\(\mu\) is plotted along with the total measured extinction coefficients in Figs. 34 and 35. The aerosol scattering contribution at 10.59\(\mu\) is the difference in the measured extinction and the calculated molecular absorption.

It can be seen that for the September 23 fog, \(B_{\text{EXT}}(0.63)\) was approximately equal to 2.5 dB/Km while \(B_{\text{EXT}}(10.59)\) was approximately equal to 0.3 dB/Km, a reduction of almost ten to one. Even with the molecular absorption included at 10.59\(\mu\), the total extinction coefficient was less than \(B_{\text{EXT}}(0.63)\).

Similar results were found during the October 15 fog except for a larger reduction factor. At several times during the measurement it can be seen that the total measured extinction was less than the computed molecular absorption. Of course this condition cannot exist because it requires that the aerosol extinction coefficient be
negative. The negative results are undoubtedly due to the values of the aerosol extinction coefficient which approach the experimental accuracy of the system. The accuracy of the empirical equation for water vapor and carbon dioxide absorption is estimated to be ± 10%, while the accuracy in measuring the total attenuation at 10.59 microns is estimated to be ± 5%.

Fig. 34.—Aerosol extinction coefficient for 10.59 and 0.6328 microns during light fog October 15, 1968.
Fig. 35.—Aerosol extinction coefficient for 10.59 and 0.6328 microns during light fog September 23, 1968.
Results for measurements made on August 22 are given in Fig. 36. The molecular absorption has been subtracted from the measured 10.59 micron attenuation and the resulting aerosol extinction coefficient plotted against the measured aerosol extinction coefficient at 0.6328 microns. It can be seen that for this light fog, 10.59 microns again offers a significant reduction in losses caused by fog aerosols.

The results found for the September and August fog can be explained by entering Fig. 17a at $R_m = 0.3\mu$ while the 4:00 A.M. October 15 results can be explained by entering Fig. 20a at $R_m = 0.3\mu$.

![Graph showing aerosol extinction coefficient for 10.59 microns vs. 0.6328 microns during a light fog on August 22, 1968.](image)
The improvement found for $B_{EXT}(10.59)$ compared to $B_{EXT}(0.63)$ should not be taken as a conclusive result. Heavier fogs having larger $R_m$ will reduce the difference between the two coefficients. But for heavier fog, $B_{EXT}(10.59)$ will only approach $B_{EXT}(0.63)$, never exceed it.

4. Precipitation conditions at 0.6328 and 10.59 microns

In the theoretical calculations of the precipitation extinction coefficient, it was shown that the coefficient is wavelength independent for visible and infrared wavelengths. Simultaneous measurements at 0.6328 and 10.59 microns during two rain storms support this fact.

During the rain storms, the 10.59$\mu$ transmittance was measured over a one-way path terminating at the remote site. The temperature and relative humidity were measured and used to calculate the molecular absorption due to water vapor and carbon dioxide. The molecular absorption was then subtracted from the total attenuation measured during the rain storm and the difference attenuation was used to compute the precipitation extinction coefficient. The 0.6328$\mu$ transmittance was simultaneously measured over a two-way path terminating at the laboratory. The scattering loss due to aerosol particles other than precipitation particles was determined from transmittance measurements before the rain started. This attenuation was subtracted from the total attenuation measured during the rain storm and the difference
attenuation was used to compute the precipitation extinction coefficient at 0.6328\(\mu\). A comparison of the two measured extinction coefficients are given in Figs. 37a and b. The dots represent the measured data and the solid curve represents a one to one correspondence between the two coefficients. It is seen that most of the data are slightly above the solid curve and would appear that rain was slightly wavelength dependent. It will be shown that this apparent wavelength dependence is caused by part of the scattered beam being collected by the receiver.

Because a rain drop has a strong forward scattered beam (angles near \(\theta = 0^\circ\)) and because receiving optics are finite in size, part of the scattered beam is collected at angles near \(\theta = 0^\circ\). The definition of the extinction coefficient was defined in Chapter III for \(\theta = 0^\circ\). Therefore the additional collected signal received from the forward scattered beam causes an apparent increase in the measured transmittance or correspondingly a lower extinction coefficient. Chu and Hogg\([9]\) have proposed a simplified rain forward scattering model which offers an explanation for the apparently lower \(B_{\text{EXT}}(0.63)\). They derived a reduction factor \(B\) from the forward scattering model which is a function of physical parameters such as beam diameter, rain rate, wavelength, beam divergence, etc., and which is subtracted from the direct beam extinction coefficient for \(\theta = 0^\circ\). The apparent accuracy of the model is indicated in Fig. 38 where the measured points are compared with the curve representing the direct beam extinction coefficient corrected for forward scattering.
Fig. 37(a, b).--Rain extinction coefficient for 0.6328 and 10.59 microns.
Therefore, theoretical calculations show that visible and infrared radiation have equal transmittance through rain, but strong forward scattering by rain and finite receiver optics result in an apparent increase in transmittance for shorter wavelengths.

During rain storms, rain rates can be very nonhomogeneous. The accuracy of transmission measurements versus rain rate therefore
depends on the homogeneity of the rain and/or on the accuracy of the average rain rate data along the propagation path. Since only one rain rate gauge was available for these measurements it was necessary to pick a rain storm that appeared to be homogeneous in rainfall over the entire propagation path. The rain storm on October 2, 1968 met these requirements. The rainfall was geographically widespread and the storm produced several periods of steady rainfall rates. A co-worker at the remote site verified increases and decreases in rain rates measured by the gauge located on the roof of the laboratory. The measured rainfall rates for this storm varied from near zero to a very high 90 mm/HR.

The results vs. rain rate on October 2, 1968 found for the precipitation $B_{EXT}(0.63)$ are represented by the solid dots in Fig. 39. The solid curve represents the direct beam extinction coefficient given by Eq. (36) corrected for forward scattering. The corresponding $B_{EXT}(10.59)$ can be found by referring to Fig. 37a.

It can be seen that the data is scattered, but this is to be expected since only one rain gauge was used. Although the points are scattered they tend to follow the calculated curve. For the low rain rates they definitely fall above the calculated curve and for the higher rain rates they approach the predicted values.
The results of these measurements give a reasonable indication of the accuracy of the theoretical calculations. Undoubtedly these results would have been more conclusive had more rain rate gauges been employed.

Fig. 39.--Measured extinction coefficient vs. rain rate at 0.6328 microns during a rain storm. October 2, 1968.
C. Aerosol Backscatter Measurements at 0.6328 Microns

1. Integrated backscatter coefficient

In most laser radar applications, the receiving optics of the system are pointed along the propagation path toward some distant target. When the target is illuminated by the transmitted beam, part of the incident beam is scattered back toward the receiver.

At the same time that the target is being illuminated so are atmospheric aerosol particles in the propagation path. The scattered signal from the aerosols for angles near 180 degrees is also directed toward the receiver. It is this scattered signal which is of interest here.

In Chapter III, the energy transfer equations that gave the energy scattered toward a receiver from a unit scattering volume were derived. The total energy received by the receiver is the summation of these contributing scattering volumes. The summation gives the following total energy transfer equation,

\[
I_{\lambda R} = \int_{L_1}^{L_2} I_\lambda(180) T_2 \omega \, dR = \frac{BSCAP(180)A}{4\pi} \int_{L_1}^{L_2} W T_2 \omega \, dl \quad \text{watts},
\]

where \(L_1\) is the distance between the source and the first scattering volume in the receiver's field of view and \(L_2\) is the distance to the last
scattering volume in the field of view measured along the propagation path. All other notation from Chapter III remains unchanged.

\[ W \text{ is related to the transmitter source power by the relation} \]

\[ WA = I\lambda_S \, T_1 = I\lambda_S \, e^{-B_{EXT}\ell} \text{ watts}, \]

where \( \ell \) is the distance between the source and the scattering volume measured along the propagation path. \( T_2 \) is assumed to be equal to \( T_1 \) and \( \omega \) is given by

\[ \omega = \frac{\pi D^2}{4\ell^2} \text{ steradians}, \]

where \( D \) is the effective diameter of the receiving optics. Equations (57) and (58) may be used to rewrite Eq. (56). Making these substitutions,

\[ I\lambda_R = I\lambda_S \times \text{BSCA} \frac{P(180)}{4\pi} \frac{\pi D^2}{4} \int_{L_1}^{L_2} \frac{e^{-2B_{EXT}\ell}}{\ell^2} \, d\ell \text{ watts}. \]

The backscatter optics for this study were placed directly below the transmitted beam. A diagram showing the physical layout is shown in Fig. 40.

For obvious reasons the aerosol scattered signal at \( \theta = 180^\circ \) could not be measured, but by placing the backscatter receiver as close to the transmitted beam as possible without causing interference, the angle could be closely approximated.
With the receiver setting below the transmitted beam it was necessary to have the backscatter receiver's field of view intersect the beam close to the transmitter. This was accomplished by using a detector having a 2 inch diameter aperture. Using this detector aperture and a 19 inch focal length lens, the measured acceptance angle for the receiver and the computed intersection length $L_1$ were $1.5^\circ$ and $6.2 \times 10^{-3}$ Km, respectively.

Making the following change of variables in Eq. (59),

$$\ell = \frac{X}{6.2 \times 10^{-3}},$$

the ratio of the received backscatter power to the transmitted power is expressed as
It is seen that the received backscatter power is directly proportional to the backscatter coefficient and an exponential integral involving $B_{\text{EXT}}$. For values of $B_{\text{EXT}} < 4 \text{ Km}^{-1}$ (corresponding to visibilities $> 0.98 \text{ Km}$), the exponential integral varies between 0.9 and 1.0 with the latter corresponding to low values of $B_{\text{EXT}}$. Therefore, for $B_{\text{EXT}} < 4 \text{ Km}^{-1}$, Eq. (61) may be reasonably approximated by

\begin{equation}
\frac{I_R}{I_S} = 2.04 \times 10^{-6} \frac{P(180)}{4\pi} \text{BSCA} \int_{1}^{\infty} e^{-1.2 \times 10^{-2} B_{\text{EXT}} X^2} \, dX,
\end{equation}

where $L_2$ has been set equal to infinity.

It is important to remember that the multiplicity constant in Eq. (62) results from the physical parameters used in this study, i.e., diameter of receiving optics, receiver field of view, etc.

2. Measured 0.63$\mu$ backscatter power

a. Continental atmospheric conditions

The backscatter power was measured along with the transmittance.

The procedure was to raise the beam after making the transmittance measurements so that it passed just above the remote site. This provided an unobstructed propagation path for the aerosol backscatter measurements.
It was shown earlier that the lower limit for atmospheric scattering in continental areas is determined by the presence of continental aerosols. For this locale it was shown that the average continental aerosol model predicted fairly well the lower limit for $B_{\text{EXT}}(\lambda = 0.63)$ or correspondingly the upper limit for the visibility at $\lambda = 0.63\mu$. Therefore, the average continental aerosol model should give good agreement between the measured and calculated backscatter results.

For the 20 to 23 Km continental visibilities measured in this locale, the corresponding backscatter coefficients can be found in Fig. 21. Entering the figure at $\lambda = 0.63\mu$ and using $v = 3.0$, the calculated value for the continental backscatter coefficient can be found. Making this substitution into Eq. (62) and converting to decibels, the ratio of the calculated backscatter power to the transmitted power for this locale is $-81$ decibels.

The measured results were consistent with calculated values. The values varied between $-82.5$ and $-81.5$ decibels with an average value of $-82$ decibels.

Because most of the measurements were made during the summer and fall seasons, haze conditions prevailed which limited the number of days that were indicative of continental aerosol conditions. Therefore these measured results should be considered as average backscatter values when continental conditions prevailed. Backscatter at $10.6\mu$
under continental conditions was below the measurement threshold.
However from previous calculations it can be estimated as -105 decibels.

b. Light fog conditions

The same experimental procedure that was used for the continental aerosol measurements was followed in this study. That is, after making the transmittance measurements at 0.63μ through the fog or haze condition, the beam was raised and a backscatter measurement taken. The results found are given in Fig. 41 for $B_{EXT}$ between 0.25 and 1.0 Km$^{-1}$ or corresponding visibilities between 15.5 and 3.9 Km. The backscatter power is given in decibels below the 0.6328μ laser source power.

Plotted in the same figure are two calculated backscatter curves for fog models $\alpha = 1.0$, $\gamma = 1.0$ and $\alpha = 2.0$, $\gamma = 0.5$ both at $R_m = 0.3μ$. These models and $R_m$ were picked because they approximately explained the fog $B_{EXT}(10.59)$ and $B_{EXT}(0.63)$ measurements in Figs. 34-36. The calculated values for the two models were made using Eq. (62) and Figs. 23 and 24. The corrected backscatter coefficients in these figures were used.

It can be seen that the two models bracket the measured results. For low values of $B_{EXT}$ the theoretical curves approach the continental aerosol conditions, i.e., $\approx -81$ decibels and for $B_{EXT} > 0.7$ Km$^{-1}$ the backscatter power approaches a linear function of $B_{EXT}$. Although measurements were not made during conditions for $B_{EXT} > 1.0$ Km$^{-1}$,
Fig. 41.—Measured backscatter power at 0.6328 microns for haze and light fog.
it is felt that extrapolating the calculated curves into this linear region should give experimentally reliable results.

To determine the expected backscatter power at other wavelengths, all that the investigator has to do is to find the backscatter coefficient in Figs. 23-27 for the wavelength of interest. For example, using the fog model $\alpha = 1.0$, $\gamma = 1.0$ and $R_m = 0.3\mu$, the ratio $\frac{P(180)}{4\pi} B_{SCA}(10.6)/\frac{P(180)}{4\pi} B_{SCA}(0.63)$ is approximately $1.6 \times 10^{-2}$ or -18 decibels. Thus the expected backscatter power for 10.6$\mu$ would be approximately 18 decibels below the experimental values found at 0.6328$\mu$ in Fig. 41.

c. Precipitation conditions

Backscatter power measurements at 0.63$\mu$ were also taken during the October 2nd rain storm. Measurements were taken during periods of heavy rain rate when the transmitted signal fell below the receiver noise level and also after sufficient data was gathered for the transmittance information. The measured and calculated results are given in Fig. 42.

The received backscatter power has been plotted as decibels below the transmitted power versus measured rain rate. The measured data is given by the solid dots and the calculated results by the upper solid curve. The lower calculated curve is explained later. The calculated results were made by simply substituting the results of Fig. 28 into Eq. (62). It was necessary to correct for the continental aerosol
conditions that existed during the rain, which from the zero rain rate position on the curve is seen to be -81.5 decibels.

The measured results fell below the noncorrected curve for all rain rates measured, but the general trend of the curve closely approximates the measured values. The difference is again probably due to the approximate solution of \( S(\lambda, 180^\circ) \). But unlike the fog model where the approximate solution was 5 decibels below the exact solution, the rain calculated results are approximately 6.5 decibels too high. Keeping this in mind, it seems justifiable to lower the curves in Fig. 28 by 6.5 decibels. The corrected curve has been plotted in Fig. 42. It can be seen that this curve closely approximates the measured results.

Backscatter measurements at 10.59 microns were not made because the thermocouple signal detector used in the experimental set-up was not able to detect the small signal levels scattered by the atmosphere. The use of a pulsed or Q-switched laser and a cooled detector would provide sufficient system sensitivity to measure atmospheric backscatter at 10.59 microns under continental aerosol, precipitation, and fog conditions.
Fig. 42.--Measured backscatter power vs. rain rate for 0.6328 microns during rain storm October 2, 1968.
CHAPTER VII
SUMMARY

Three general aerosol models describing continental, fog and precipitation conditions have been used to calculate atmospheric extinction and backscatter coefficients. Experimental measurements of the two coefficients at 0.6328 and 10.59 microns have been made using a 1950 meter outdoor propagation path. Also an empirical formula has been given relating the atmospheric molecular absorption at 10.59 microns to the atmospheric water vapor pressure.

Calculations of the extinction and backscatter coefficients were made using aerosol models which were found to agree with average experimental measurements. The constituents of the models were assumed to be spherical water particles. Calculations were made for wavelengths between 0.34 and 10.6 microns.

The extinction coefficient was generally found to be wavelength dependent for aerosol models having a maximum number density radius \( R_m \) less than 5.0 microns. For precipitation conditions it was seen that the coefficient became completely wavelength independent for the wavelength range given above. The coefficient for dense fog was shown to be much higher than for the heaviest rain rates. And for 10.6 microns
it was shown how the coefficient could be used to determine the liquid water density of fog.

The experimental measurements of the extinction coefficients at 0.6328 and 10.59 microns were explained by the aerosol model calculations. The continental and precipitation models gave close agreement while the light fog measurements were found to agree with the fog models having a $R_m = 0.3$ microns. From these measurements and the results of other investigators it was found that for $R_m$ between 0.3 and 1.0 microns most experimental results could be explained.

An approximate solution was given for computing the backscatter coefficient of large spherical particles. This solution along with an exact solution for smaller particles was used to compute the coefficient for the three general aerosol models. From the results it was shown that large aerosol particle sizes did not cause the backscatter coefficient to become wavelength independent. The experimental measurements of the coefficient were found to closely agree with the continental and fog model. Although the precipitation model gave results which were too large, they were still within the predicted error of the approximation solution.

The difference in the backscattered power for dense fog and heavy rain was shown to be much less than the differences found for the two extinction coefficients.
From measurements of the molecular transmittance at 10.59 microns, the extinction was found to vary as a function of water vapor pressure. This was shown to be in agreement with similar indoor measurements.

This study has shown that it is possible to predict meaningful and reliable atmospheric aerosol extinction and backscatter coefficients for visible and infrared radiation.
APPENDIX A
A CONVERSION METHOD FOR BAYLY'S
H₂O INDEX OF REFRACTION MEASUREMENTS

To convert Bayly's results into \( n_2 \) it is necessary to assume that the extinction of electromagnetic radiation through an absorption cell obeys Lambert's law.

His published results are given in terms of \( \frac{t}{\log_{10} \frac{I_o}{I}} \) where \( t \) is his absorption cell thickness and \( I_o \) and \( I \) are the incident and transmitted intensities for the absorption cell, respectively. Given Lambert's law

\[
\frac{I_o}{I} = e^{kt}
\]

where

\[
k = \frac{2\pi n_2}{\lambda}
\]

then

\[
\log_{10} \frac{I_o}{I} = kt \log_{10} e = 0.434t \quad n_2 \quad \frac{2\pi}{\lambda}.
\]

Thus for a given wavelength the expression for \( n_2 \) may be written as

\[
n_2 = \frac{\log_{10} \frac{I_o}{I}}{t} \quad \frac{\lambda}{2\pi} \quad \frac{1}{0.434}.
\]
APPENDIX B
EXPERIMENTAL APPARATUS

A. Introduction

The apparatus used for the outdoor measurements may be divided into the following three groups and subgroups:

1. The single frequency 10.59 micron laser source
2. The 0.6328 micron laser source
3. The associated experimental equipment and facilities
   a. outdoor propagation path
   b. collecting optics
   c. detectors

This Appendix discusses the equipment in each of these groups.

B. Design And Construction Of A Single Frequency Carbon Dioxide Laser

1. Discussion

The carbon dioxide laser is perhaps the simplest to construct and the most efficient of many laser sources. But like most lasers special precautions must be taken in its design if it is to be a single mode and single frequency amplitude stabilized laser.
2. Single frequency and single mode characteristics and operation

The 10.6μ CO₂ laser can oscillate at a number of frequencies near 10.6μ on the rotational transitions of the \( \Sigma^+_u - \Sigma^+_g \) 00'1 - 10'0 vibrational band. The strongest oscillations usually occur on the P branch transition \( \Delta J = -1 \) where \( J \) denotes the rotational quantum number and

\[ P \text{ branch transition is } J_{\text{upper}} \rightarrow J_{\text{lower}} = -1. \]

For example, the laser transition identified as P(22) is the transition from the \( \Sigma^+_u \) state with \( J = 21 \) to the \( \Sigma^+_g \) state with \( J = 22 \). The \( J \) value denotes the lower state.

Multifrequency operation occurs because of strong competition among the upper rotational levels. Patel[29] has shown that the optical gain per pass through a CO₂ gas medium is a function of the \( J \) rotational levels. For a given vibrational level and temperature the optical gain function has a peak value for \( J_{\text{peak}} \) and drops off for values of \( J \) on either side of \( J_{\text{peak}} \). Under normal operating conditions \( J_{\text{peak}} = 20 \). Therefore if the input power or pump power is sufficiently large so that the gain per-pass exceeds the loss per-pass for several \( J \) numbers other than just \( J_{\text{peak}} \), the output will be multifrequency. McCoy[1] and others have shown that for a 50 cm near hemispherical resonator that is is possible to operate on one \( J \) transition.
McCoy has assumed in his calculations that for pressures of 10 to 20 torr the CO₂ Doppler and collision broadened line width is 75 MHz. He also assumed that a 60 MHz separation between an oscillating line center and the nearest cavity resonance frequency is sufficient for line suppression. The two following equations were used in determining his results

\[ d = \left[ q_{20} + \frac{1}{2\pi} \cos^{-1}\left(1 - \frac{2d}{b}\right)\right] \frac{\lambda_{20}}{2} \]

and

\[ d = \left[ q_{i} + \frac{1}{2\pi} \cos^{-1}\left(1 - \frac{2d}{b}\right) \pm \frac{60}{C/2d}\right] \frac{\lambda_{i}}{2} \]

where \( q \) is an integer, \( C \) is the speed of propagation in the cavity, \( d \) is the mirror separation, \( b \) is the mirror radius of curvature and \( \lambda \) is the wavelength. His computer calculations indicated that there are no lengths which would be resonant for the P(20) line and simultaneously suppress all other P lines. When the number of lines to be suppressed was reduced to only the 8 strongest P lines observed, several cavity lengths were found. These results indicate that it is possible to operate a near hemispherical cavity in single frequency operation.

With a near hemispherical resonator, higher order transverse modes oscillate readily. Because of diffraction losses the higher order modes have increasingly larger losses in comparison to the fundamental
mode. By introducing sufficient loss into the cavity it is possible for the higher order modes to be eliminated.

3. 10.59 micron laser construction

a. Plasma tube

Figure 43 is a photograph of the 10.6μ laser. The plasma tube is water cooled and is terminated at one end with a NaCl window mounted at the Brewster angle. The window is held in place with silicone rubber cement. A continuous gas flow system was used. Pre-mixed carbon dioxide-nitrogen-helium in a 10-10-80% mixture was leaked into the plasma tube through a needle type control valve. Two anti-sputtering type neon sign electrodes were excited by the rectified output from two parallel 12-kilovolt, 30-ma air core transformers. No special mechanical or thermal stabilization was incorporated in mounting the plasma tube.

b. Cavity resonator construction

A one inch diameter flat Irtran II mirror, aluminum coated for 80% reflectance at 10.6μ, was mounted into a specially designed internal metallic mirror mount. To provide electrical insulation from the plasma discharge, parts of the mirror mount were milled from a dielectric material, Lavite. Air-tight seals and flexibility for cavity alignment is possible due to a silicone rubber compound used to bind the dielectric
Fig. 43. - 10.59 micron laser source.

to the metallic surfaces. Mirror adjustments are provided by eight interlocking corner mounted screws.

In order to be able to operate at only P(20) by using cavity length control, the mirror mount was placed on a longitudinally adjustable flat base located at the NaCl window end of the plasma tube. The mirror mount in this case is not used to seal off the plasma tube. Fine tuning
of the cavity length was accomplished by tuning an adjustable screw which exerts pressure through a lever arm to the flat base mounted above a fixed base by four fairly rigid aluminum braces. About 5 microns of travel was provided by 6 revolutions of the screw.

The second resonator mirror was a one inch diameter, aluminum-coated convex mirror with 90-cm radius of curvature mounted in a mirror mount similar to the one just described. The mirror mount is used to form an air-tight seal at the end of the plasma tube.

c. 10.59 micron laser operation

The higher order transverse modes of the near hemispherical resonator were suppressed by placing a small iris in the cavity near the flat mirror. With an aperture size necessary to provide single transverse mode operation, the output power was approximately 0.5 watts. When the cavity was properly aligned and the iris removed the laser was capable of 11.0 watts.

When operating on a stable resonance the laser could be tuned to line center, as indicated by maximum output. Since no special care was taken to reduce thermal and mechanical effects on the laser, long periods of output amplitude stability were not possible. The output amplitude would remain constant within 10% for periods up to 15 minutes under good laboratory conditions.
C. 0.6328 Micron Laser Source and Collimator

The 1 milliwatt 0.6328\(\mu\) source used in this study is shown in Fig. 44. It is a Spectra Physics 112 He-Ne gas laser operated in a near hemispherical mode. The output from the flat mirror is used for monitoring laser power output. The output from the spherical mirror is directed through various neutral density filters into a collimating telescope. The telescope is a Tropel model 1557 Laser Collimator which expands the beam to approximately 3 inches diameter with variable divergence rates.

Fig. 44.--0.6328 micron laser source.
D. Associated Experimental Equipment
And Facilities

1. Outdoor propagation path

The description of the location of the experimental site is important to an investigator in interpreting experimental results. The location of the propagation path used was one of availability and economics.

The laser outdoor propagation path is located 4 miles northwest of downtown Columbus, Ohio and is shown by the dotted line on the terrain map in Fig. 45. The path originates from a 2nd floor corner laboratory room, 15 feet above ground level. The path terminates at the remote site 1.95 Km down range and approximately 30 feet above ground level. The terrain below the path is mainly flat farm land with one four-lane highway crossing perpendicular to it. There is no heavy industry in the immediate area which could cause heavy local atmospheric contamination.

2. The remote site

The remote site shown in Fig. 46 consists of a three foot square limestone table upon which a twelve inch diameter flat mirror is mounted. The stone table is mounted on a fourteen inch diameter utility pole which extends approximately thirty feet above the ground. The utility pole has a plywood shield surrounding it to provide protection from wind induced vibrations and the weather. A small building rests
Fig. 45.--Terrain map showing propagation path.
upon four utility poles to provide protection for experimenters and equipment. The building also houses a polished Stellite twenty-four inch diameter, eight inch focal length parabola.
3. **10.59 micron auxiliary optics and pointing methods**

Figure 47 shows the combination of optics used to collimate the diverging 10.59μm beam. The laser output is reflected from an Irtran II dichroic beamsplitter with 60% reflectance into a collimating telescope with a one inch diameter, ten inch focal length eyepiece mirror and an eight inch diameter, seventy-two inch focal length objective mirror. The measured transmitted beam diameter and divergence from the telescope is approximately 2.5 inches and 0.08 milliradians, respectively.

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**Fig. 47.** -- Diagram of the auxiliary optics and collimating telescope for the 10.59 micron laser.
Pointing the 10.6μ laser beam down range to the remote site is accomplished by slaving the beam collinearly with a visible He-Ne laser beam. This is accomplished by pointing the 0.6328μ source (not the slave visible source) toward the 12 inch diameter flat mirror located at the remote site. The flat mirror is adjusted so that the beam is returned to the 2nd floor room and illuminates the 8 inch objective mirror of the telescope. The objective, eyepiece beamsplitter and beam steering mirrors are adjusted so that the returned beam is directed to the output cavity mirrors of the 10.6μ and slave laser. The incident beam to the 10.6μ laser is used to align the resonator cavity. This procedure aligns the two beams close enough so that only fine adjustment of the beamsplitter is needed to bring the two beams into optimal alignment.

The two lasers, telescope and auxiliary optics are mounted on a limestone table which is supported at one end by a milling table. The milling table has up-down and transverse adjustments for pointing the collinear beams at the remote site.

4. Collecting optics and detector systems
   a. Visible

Three detectors were used in making measurements at 0.6328μ. One was used to measure the 0.6328μ laser output from the flat mirror of the hemispherical resonator. This detector will be referred to as
the reference beam detector. The second detector was used to measure the intensity of the beam after propagating over the outdoor range. The third detector is used to measure the intensity of the backscatter beam. The last two detectors will be referred to as the signal beam and backscatter detectors, respectively.

The reference beam of the 0.6328μm source was fed through a red bandpass filter and into the 929 phototube reference detector. The output of the detector was measured with a dc voltmeter. The output voltage was monitored during measurements to detect changes in the output power of the laser.

The output beam from the spherical mirror end of the 0.6328μm laser was chopped at 13 Hz before entering the neutral density filters and the collimator. The output from the collimator was pointed down range to the 12 inch flat mirror at the remote site, reflected back into the room and collected by a twenty-four inch diameter, eight inch focal length polished Stellite parabola. The beam falls completely on the flat mirror except for momentary steering during periods of high turbulence. The beam has a measured beam divergence of 0.1 milliradians.

The collected beam was focused onto a RCA 7265 photomultiplier tube. The two inch aperture of this signal detector was covered with a red plastic low pass filter at 6000 Angstroms. The output of the
A photomultiplier was fed into an Infrared Industries model 601 tunable microvolt meter. The system's measuring range was 55 dB for non-daylight hours.

The returned signal caused by atmospheric backscatter was collected by a five inch diameter, nineteen inch focal length lens. A baffle was used to reduce the amount of unwanted scattered light reaching the detector. The collected beam focused onto a RCA 7265 photomultiplier tube. The output from the backscatter detector was fed into a microvolt meter similar to the one used for the signal beam. The system's measuring range for non-daylight hours was 86 dB. A photograph of the collecting optics is shown in Fig. 48.

Fig. 48.--Collecting optics for the signal and backscatter beams.
b. **Infrared**

The reference beam for the P(20) 10.59μ laser was supplied by the transmitted portion of the beam from the Irtran II beamsplitter. It was fed into a Perkin-Elmer model 112G double pass monochromator, chopped at 13 Hz and focused onto a thermocouple. The thermocouple signal was fed to an Infrared Industries model 601 tunable microvolt meter. Fixed slits 0.1 mm wide and a 75 line per mm grating provided more resolution than needed for identification of the rotational lines in the laser output.

The twenty-four inch diameter parabola located at the remote site was used to feed an Eppley eight junction bismuth-silver thermopile signal beam detector. The linearity of the thermopile and reference detector was checked simultaneously over the intensity range used and found to agree within 3%. The large thermopile detector area (∼3/8") made alignment easy and assured complete detection of the focused signal beam.

c. **Rain gauge system**

The rain gauge used was a continuous measuring rain rate device. This type of gauge was used because of the inherent errors in the standard tipping bucket rain gauge and the formidable task of analyzing data from several hundred tipplings.
The continuously measuring rainfall rate gauge was patterned after a similar gauge designed by Semplak.[30] The gauge utilizes the high dielectric constant of water to change the frequency of an oscillator.

The rainfall is collected by a 12 inch diameter funnel and directed to an inclined plane set at an angle of 45°. The plane contains two parallel 15 inch 10 gauge wires coated with five coats of clear lacquer and separated by a groove 1/8" by 1/8". These parallel wires provide the capacitance for a relaxation oscillator. The relaxation oscillator diagram and associated electronic circuit is shown in Fig. 49.

![Rain gauge electronic circuit](image)

**Fig. 49.** Rain gauge electronic circuit.
The output of the relaxation oscillator is fed into two 914 gates which give a constant amplitude and constant pulse width output. A dc voltmeter is used to measure the average voltage of the output signal. Figure 50 shows the calibration curve for the rain gauge.

This type of gauge proved to be a reliable and convenient method of measuring rain rate. It was found that the range of highest sensitivity for the gauge could be varied by placing solid rods of variable lengths in the slotted groove. This caused the gauge to become more sensitive to low rainfall rates.

![Calibration curve for continuously measuring rain rate gauge.](image-url)
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