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GEYER, Manvel Allan, 1930-
DEADBEAT RESPONSE OF A BRUSHLESS ALTERNATOR.

The Ohio State University, Ph.D., 1969
Engineering, electrical

University Microfilms, Inc., Ann Arbor, Michigan
DEADBEAT RESPONSE OF A
BRUSHLESS ALTERNATOR

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * * *

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1969

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ACKNOWLEDGMENTS

The assistance of the technical staff and the management of the Aerospace Electrical Division, Westinghouse Electric Corporation, is gratefully acknowledged. The necessary records, test data, test equipment and hardware were furnished with all possible dispatch.

Acknowledgment must also go to my adviser, Professor F.C. Weimer, and to the other members of my reading committee, Professor N.A. Smith and Professor R.B. Lackey.
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"Simulation of a Silicon Controlled Rectifier", Simulation, Vol. 9, No. 4, pp. 166-167, October, 1967
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LIST OF SYMBOLS

$ed$ = output of exciter rectifier load
$ed'$ = exciter direct-axis transient internal voltage
$es$ = amount of exciter saturation excitation
$et$ = exciter terminal voltage
$ex$ = voltage applied to field of generator
$ia$ = exciter armature current
$if$ = exciter field current
$m$ = exciter field voltage
$ra$ = exciter armature resistance
$td'$ = exciter closed-circuit time constant
$tdo'$ = exciter open-circuit time constant
$xd$ = exciter direct-axis synchronous reactance
$xd'$ = exciter direct-axis transient synchronous reactance
$xq$ = exciter quadrature axis synchronous reactance
$Ed$ = generator direct-axis synchronous internal voltage
$Ed'$ = generator direct-axis transient internal voltage
$Es$ = amount of generator saturation excitation
$Et$ = terminal voltage of generator
$I$ = generator load current
If = generator field current
Ke = rectifier voltage conversion ratio
Ki = rectifier current conversion ratio
L = digital signal to return regulator to linear mode
L_f = generator field circuit inductance in henries
M = digital signal to apply regulator voltage
N = digital signal to reverse regulator voltage
R_a = generator armature resistance
R_f = generator field circuit resistance in ohms
T_d' = generator closed-circuit time constant
T_d_0' = generator open-circuit time constant
X_d = generator direct-axis synchronous reactance
X_d' = generator direct-axis transient synchronous reactance
X_q = generator quadrature-axis synchronous reactance
\theta = lagging phase angle of I
\lambda = a switching interval
CHAPTER 1

INTRODUCTION

In modern aircraft, constant-speed alternators are used to supply the required electrical power.\textsuperscript{1,2,3} In the original system developed for the B-36\textsuperscript{4,5} the alternators were built in much the same manner as ground power stations. That is, a d-c machine was used as an exciter for the a-c machine. The B-52 pushed this type of system to its limits. The high altitude attained caused extremely short brush life and unreliable operation.\textsuperscript{6,7} The brushless alternator was thus developed and first used on the Boeing 707\textsuperscript{8,9} and KC-135 tanker. A schematic diagram of a brushless alternator is shown in Figure 1.

The brushless alternator, while curing mechanical problems, has created some control problems.\textsuperscript{10} The exciter output voltage is no longer available as a feedback signal, nor can it be reversed in polarity. The removal of the commutator caused dramatic decrease in alternator weight, and the competition between suppliers caused less and less iron to be used in the stack. Today, one pound of alternator for each kilowatt of output power is a feasible reality. The machine, however, operates
I. All on Alternator Shaft

Figure 1

Schematic Diagram of a Brushless Alternator
almost continuously in a saturated region of the iron. Thus, an electrical power system consisting of two non-linear machines, two input leads and two output leads has been developed.

The problem is now one of controlling the minimums and maximums of the output voltage under severe load-switching transients. A typical transient limit curve is shown in Figure 2. By per unit is meant a dimensionless system of units. For example, a 120 KVA, 120 volt system would be defined

\[
\begin{align*}
120 \text{ KVA} &= 1 \text{ p.u. base} \\
120 \text{ volts} &= 1 \text{ p.u. base} \\
1000 \text{ amps} &= 1 \text{ p.u. base}
\end{align*}
\]

and all voltages and currents would be divided by these base units to convert them into per unit quantities.

When further requirements, such as supplying a 2.00 per unit load current with only a 0.02 per unit droop in voltage for five seconds and supplying a 3.00 per unit short circuit current for five seconds and demanding that upon removal of these load conditions the voltage stay within the bounds of Figure 2, are specified, the control problem becomes almost impossible by conventional, classical methods.

Further complicating this picture is the desire of the equipment designer utilizing this a-c power to have the transient envelope reduced both in magnitude
Figure 2

Typical Transient Limits
and in time. Based upon weight and reliability, his desires are usually considered above those of the power system designer.

One control criterion, deadbeat response,* will satisfy most of these problems. A typical deadbeat response is shown in Figure 3. Therefore, a deadbeat response controller will be developed for a brushless alternator power system. Because the system is so non-linear, state variable techniques will be used.

*By "deadbeat response" it is meant that a system responds to a stepwise input in the quickest manner without overshoot.
Figure 3

Typical Deadbeat Response
CHAPTER 2

DEADBEAT PERFORMANCE

The concept of state variable is that the value of all the variables which describe the system at some time $t_0$ is observed. Thus a system is given a set of initial conditions and specific inputs and is allowed to respond until time $t_0$. At this time $t_0$, the values of all the variables describing the system are observed. This set of data is the state of the system at time $t_0$. Since $t_0$ can be any value, it can be considered to vary continuously from some initial point to some final point. These variables are thus the "state" variables of the system.

Consider a mass positioning system with no springs or damping, given an actuating force $F$ and the requirement of deadbeat response. The following relations hold during the application of $F$:

$$\dot{x}(t) = \frac{1}{m} \int F \, dt = \frac{Ft}{m}$$

$$\ddot{x}(t) = \int \dot{x} \, dt = \frac{Ft^2}{2m}$$

The plot of $\dot{x}(t)$ and $x(t)$ for a properly selected $F(t)$ is shown in Figure 4.
Figure 4
Plot of Variables in Second-Order Deadbeat Response
The phase plane plot of this system is shown in Figure 5. This is a plot of \( \dot{x}(t) \) versus \( x(t) \). The trajectory for this is readily obtained by eliminating \( t \) in the above equations:

\[
\dot{x^2}(t) = \frac{2F}{m} x(t) \quad 0 < t < t_1
\]

This is actually in terms of state variables if \( x(t) \) and \( \dot{x}(t) \) are chosen as the state variables describing the system. In terms of state variables:

\[
\begin{align*}
\dot{x_1} &= x_2 \\
\dot{x_2} &= \frac{F}{m}
\end{align*}
\]

To make the plot of \( x_2 \) versus \( x_1 \) more standard, an initial condition of \( \frac{-F t_1^2}{m} \) will be assumed for \( x_1 \). The state variable plot equivalent to the phase plane plot of Figure 5 is shown in Figure 6. It should be noted that the curves are identical except for variable names and origin location. Thus state variables are a means of analyzing and designing in the phase plane.

Several important points about deadbeat response can be observed from this very simple example:

1. The output arrives tangentially at steady state on the time plot (Figure 4) with zero velocity. At this time the actuating force is removed. Tou\textsuperscript{12} shows this to also be a minimum-time trajectory for these initial
Figure 5
Phase Plane Plot of Figure 4
Figure 6

State Variable Plot for Figure 4
12

conditions. This not to imply that all deadbeat response trajectories are minimum-time trajectories.

2. The control problem is one of predicting the time \( t_1 \) at which the actuating force, \( F \), must be reversed in polarity.

3. Either a small dead zone or a linear non-switching controller is needed at the origin of Figure 6 to prevent limit cycling.

Deadbeat performance is very well discussed by Tou. However, he discussed it in terms of a sampled-data system. He shows that a controller with a variable gain can be used to settle a nth-order linear system with deadbeat response in \( n \) sampling periods. As will be shown, the brushless alternator is a second-order continuous system with a fixed maximum gain. Thus the problem becomes twofold.

First, the system must be made a switching system during the transient period. This is a matter of sensing when the transient occurs and of changing the mode of the regulator at that time. This seems to be a simple engineering problem and will be deferred until the actual hardware is constructed or simulated.

Second, the actual time of switching must be determined. Moreover, at the switching instant, the regulator must be reversed in polarity. At the instant of load application, the regulator goes full on and remains
saturated until the alternator voltage gets within two percent - the dashed lines of Figure 7 - of the regulation point. The exciter thus has too much stored energy, and the voltage overshoots. For deadbeat response, a point, A, (see Figure 7) must be determined where the regulator will be switched to a maximum negative value so as to prevent overshoot but to give minimum time of response. Another point must be determined on the resulting curve where the regulator will be returned to continuous control. A reverse argument holds for load removal. Since the response curves are functions of the load current, the switching points are also expected to be functions of the load current.

Thus, a mathematical description of the brushless alternator, which is both complete enough to be accurate during the necessary transient period and also simple enough to be used, must be determined. By using state variables, a curve such as Figure 8 will be generated for determining the switching point, A. Each load current may generate a different curve. If it does, the curve may become four-dimensional. That is, the actual shape of the switching boundary may be a function of the magnitude and phase angle of the current.
Figure 7

Typical Load Application Transient Envelope
Figure 8

Typical Switching Curve for Second-Order System
MATHEMATICAL DESCRIPTION OF A BRUSHLESS ALTERNATOR

A brushless alternator is essentially two salient pole a-c machines, the rectified output of one supplying d-c excitation for the other. A complete mathematical description such as derived by Concordia\textsuperscript{13}, Riaz and Smith\textsuperscript{14}, or Boffi and Haas\textsuperscript{15} is very complex and nonlinear. Such descriptions give very good results in system simulations, but it must be remembered that such things as paralleling, load division, stability, and regulation are of no interest in this switching problem. Any set of equations accurately describing the response in the neighborhood of point A of Figure 7 will be sufficient for load on. That is, the initial dip and maximum droop of the voltage envelope need not be determined accurately as long as the curve from point A to rated voltage is determined accurately.
With this in mind, the following equations are presented.

A. Application of Load

1. Generator

   (1) \( e_x = R_f I_f + L_f \left( \frac{dI_f}{dt} \right) \) volts

Where \( L_f \) = field circuit inductance in henries
\( R_f \) = field circuit resistance in ohms
\( I_f \) = field current in amperes
\( e_x \) = exciter voltage in volts

Let \( T_{do}' = \frac{L_f}{R_f} \), the open-circuit time constant.

Then:

   (2) \( \frac{e_x}{R_f} = I_f + T_{do}' \left( \frac{dI_f}{dt} \right) \) amperes

Let \( I_{fb} \) be a base field current which is the amount of field current required to produce rated voltage on the no-load air-gap line. Then \( I_f \) in per unit becomes equal to \( E_d \) in per unit, the direct-axis synchronous internal voltage. \(^{13}\) Likewise, let \( e_{xb} \) be a base exciter voltage such that:

   \( \frac{e_{xb}}{R_f} = I_{fb} \) amperes

Then equation 2, in per unit, becomes:

   (3) \( e_x = E_d + T_{do}' \frac{dE_d}{dt} \) p.u.
In a salient pole generator the one variable that does not change instantaneously during a transition from one operating state to another is $E_d'$, the direct axis transient internal voltage. This variable is proportional to the net flux linkages with the field winding, which cannot change instantaneously. Such a variable makes a convenient handle for this problem.

The initial value of $E_d'$ can be calculated from the original operating state and this initial value then used as a boundary condition for the solution of the equation for $E_d'$ after a load application. The terminal voltage can then be calculated from $E_d'$.

According to Wagner, the relation between $E_d$ and $E_d'$ under closed-circuit armature conditions is:

$$E_d = E_d' \frac{R_t^2 + X_{dt}X_{qt}}{R_t^2 + X_{dt}'X_{qt}} = K_1E_d' \text{ p.u.}$$

In this equation the subscript $t$ means the sum of a machine constant and an external load quantity. That is:

$$X_{dt}' = X_d' + X_{ext} \text{ p.u.}$$

$$R_t = R_a + R_{ext} \text{ p.u.}$$

$$X_{qt} = X_q + X_{ext} \text{ p.u.}$$

$$X_{dt} = X_d + X_{ext} \text{ p.u.}$$
Obviously, the time constant of the machine will change because of the mutual effects between the armature and field circuits. Wagner\(^1\) has shown that the equivalent closed-circuit time constant, \(T_d'\), is:

\[
T_d' = \frac{R_t^2 + X_{dt}\'X_{qt}}{R_t^2 + X_{dt}X_{qt}} \quad T_{do}' = \frac{T_{do}}{K_1}
\]

Thus, for closed-circuit armature conditions, equation 3 becomes

\[
ed' = K_1Ed' + T_d' \frac{dK_1Ed'}{dt} \quad \text{p.u.}
\]

It should be pointed out that saturation has been neglected in the derivation of equation 6. This equation could be corrected for saturation by subtracting from \(e_x\) the amount of per-unit excitation corresponding to that necessary to overcome saturation. Harder and Cheek\(^18,19\) and Holloway\(^20\) have shown this correction to be essentially constant for load-on applications. Therefore, saturation will be included at this time by subtracting from \(e_x\) the per-unit amount of excitation, \(E_s\), necessary to overcome saturation at normal no load voltage. Then equation 6 becomes:

\[
ed' = E_s + K_1Ed' + T_{do}' \frac{dEd'}{dt} \quad \text{p.u.}
\]
The initial value of $E_d'$ can be derived to be:

$$E_{dI}' = \frac{1 + (X_d' + X_q)\sin\theta + X_qX_d'P^2 I^2}{\sqrt{1 + 2X_q\sin\theta + X_q^2 P^2 I^2}} \text{ p.u.} \tag{8}$$

Where $I$ is the initial per-unit load current and $\theta$ is its lagging phase angle. The relation between $E_d'$ and the terminal voltage $E_t$ can be derived as:

$$E_t = \frac{\sqrt{(X_{qt}^2 + R_t^2)(X_{ext}^2 + R_{ext}^2)}}{X_{dt}'X_{qt} + R_t^2} E_{dI}'$$

$$= K_2E_{dI}' \text{ p.u.} \tag{9}$$

Equation 7 now becomes:

$$e_x = E_s + \frac{K_1}{K_2} E_t + \frac{T_{do}}{K_2} \frac{dE_t}{dt} \text{ p.u.} \tag{10}$$

In terms of state variables:

$$x_1 = E_t \text{ p.u.}$$
$$x_2 = e_x \text{ p.u.}$$

Equation 10 becomes:

$$x_2 = E_s + \frac{K_1}{K_2} x_1 + \frac{T_{do}}{K_2} \frac{dE_t}{dt} \text{ p.u.}$$

$$\dot{x}_1 = \frac{K_2}{T_{do}} x_2 - \frac{K_2}{T_{do}} E_s - \frac{K_1}{T_{do}} x_1 \text{ p.u./sec.} \tag{11}$$
The initial condition for \( x_1 \) is given by:

\[
(12) \quad x_1(0) = k_2 E_d I_1 \quad \text{p.u.}
\]

2. **Exciter**

The a-c exciter mode of operation presents complications if a rigorous mathematical approach is pursued. These additional complications are caused by the non-sinusoidal exciter armature currents due to the rectifier circuit and the inductive load of the main a-c alternator field circuit. These difficulties can be circumvented by some facts and assumptions which have little influence upon the usefulness of the results.

(a) \( x_d = x_q \)

(b) Subtransient effects are neglected

(c) Rectifier voltage drops are neglected

(d) The alternator field and rectifier circuit combination is a resistive load of the exciter.

Then:

\[
e_{d-c} = K_e e_t = e_x \quad \text{volts} \quad K_e = \begin{cases} 
1.17 \text{ half wave} \\
2.34 \text{ full wave}
\end{cases}
\]

where \( e_{d-c} \) is the output of the rectifier and \( e_t \) is the terminal voltage of the exciter. Thus:

\[
e_x = K_e e_t \quad \text{volts}
\]
Under the above assumptions, the same equations apply for both generator and exciter. Let the regulator saturation level be denoted m. The problem of a per-unit system now arises. When \( e_x \) is one per-unit, then \( e_t \) should also be one per-unit. Therefore:

\[
K_e \ e_{tb} = e_{xb} \text{ volts}
\]

and in per-unit:

\[
e_x = e_t \text{ p.u.}
\]

Now let \( i_{f_b} \) be a base field current which is the amount of field current required to produce \( e_{tb} \) on the no-load air-gap line. The load is the generator field resistance multiplied by the following derived constant:

\[
i_a = K_I I_f \text{ amperes} \quad K_I = \begin{cases} 0.577 & \text{half wave} \\ 0.816 & \text{full wave} \end{cases}
\]

\[
K_e e_t = e_x \text{ volts}
\]

\[
\frac{e_x}{K_i I_f} = \frac{K_e e_t}{i_a} \text{ ohms}
\]

\[
e_t = \frac{e_x i_a}{I_f K_i K_e} = \frac{i_a R_f}{K_i K_e} \text{ volts}
\]

\[
e_t = i_ar_a \text{ volts}
\]

\[
r_a = K_I R_f \text{ ohms} \quad K_I = \begin{cases} 1.485 & \text{half wave} \\ 0.523 & \text{full wave} \end{cases}
\]
But one per-unit generator field current is also one per-unit load current on the exciter. Therefore:

(13) \( i_{ab} = K_i I_{fb} \) amperes

and one per-unit ohms is:

(14) \( r_b = K_f R_f \) ohms

in the exciter field circuit:

(15) \( m_b = r_f i_{fb} \) volts

Then:

(16) \( m = e_d + t_{do} \frac{de_d}{dt} \) (in per-unit quantities)

(17) \( e_d = e_d' \frac{r_a^2 + x_{d'q}}{r_a^2 + x_{d'q}} = K_3 e_d' \) p.u.

(18) \( t_{d'} = \frac{r_a^2 + x_{d'q}}{r_a^2 + x_{d'q}} t_{do} = \frac{t_{do}}{K_3} \) seconds

(19) \( m = K_3 e_d' + t_{do} \frac{de_d'}{dt} \) p.u.

(20) \( m = e_s + K_3 e_d' + t_{do} \frac{de_d'}{dt} \) p.u.

(21) \( e_t = \frac{\sqrt{(x_{q}^2 + r_a^2)^2 r_a^2}}{x_{d'q} + r_a^2} e_d' = K_4 e_d' \) p.u.

(22) \( m = e_s + \frac{K_3}{K_4} e_t + \frac{t_{do}}{K_4} \frac{de_t}{dt} \) p.u.
Since \( e_x = e_t \) p.u. and \( x_2 = e_t \):

\[
(23) \quad \dot{x}_2 = \frac{K_4 m}{t_{do}} - \frac{K_4}{t_{do}} - \frac{K_3 x_2}{t_{do}} \quad \text{p.u./second.}
\]

Now the initial condition for \( x_2 \) must be determined. This can be obtained from equation 8 by setting the power factor to unity. Thus:

\[
e_{dI} = \frac{1 + x_q x_d' I_f^2}{\sqrt{1 + x_q^2 I_f^2}} \quad \text{p.u.}
\]

But \( I_f = E_d + E_s \) in per-unit.

\[
(24) \quad e_{dI} = \frac{1 + x_q x_d' (E_d I_{k1} + E_s)^2}{\sqrt{1 + x_q^2 (E_d I_{k1} + E_s)^2}}
\]

\[
e_{tI} = K_4 e_{dI} \quad \text{p.u.}
\]

\[
(25) \quad x_2(0) = K_4 e_{dI} \quad \text{p.u.}
\]

In summary, the state variable equations for load application are:

\[
(11) \quad \dot{x}_1 = \frac{K_2}{T_{do}} (x_2 - E_s) - \frac{K_1}{T_{do}} x_1 \quad \text{p.u./second}
\]

\[
(23) \quad \dot{x}_2 = \frac{K_4}{t_{do}} (m - e_s) - \frac{K_3}{t_{do}} x_2 \quad \text{p.u./second}
\]

\[
(12) \quad x_1(0) = K_2 E_{dI} \quad \text{p.u.}
\]
(25) \( x_2(0) = k_4e^{d_1} \) p.u.

3. Verification of Equations

It would be foolish to go further without first verifying equations 11, 12, 23, and 25. Test data were readily available on a system with the parameters of Table 1 and Figures 9 and 10.

Military classification prevents any further identification. The test oscillogram is shown in Figure 11. The state variable versus time curves read from this oscillogram are shown in Figure 12.

Inspection of Figure 11 shows the exciter field voltage to be a pulse-width modulated voltage. Unfortunately, the pulse repetition rate is too high to allow the widths to be clearly photographed. Therefore, the exciter equations cannot be verified directly.

The alternator field voltage, however, can be easily read and the alternator equations can be verified. This has been done by a simple computer program in Appendix 1. The results are plotted in Figure 12. During the critical rise between 0.90 p.u. and 1.0 p.u. the error does not exceed 2.0 per-cent. This is within the accuracy of reading the oscillogram. The alternator equations are thus verified. Since the exciter equations are identical to the alternator equations, it is not unreasonable to assume the exciter equations as accurate.
TABLE I

PARAMETERS OF TEST SYSTEM

**Generator**
- \( X_q = 0.740 \) per-unit
- \( T_d = 0.227 \) seconds
- \( R_f = 0.4 \) ohms @ 25°C
- \( X_d' = 0.256 \) per-unit
- \( X_d = 2.017 \) per-unit
- \( E_b = 118.0 \) volts

**Exciter**
- \( x_q = x_d = 1.770 \) ohms
- \( t_d = 0.092 \) seconds
- \( r_f = 6.3 \) ohms @ 110°C
- \( x_d' = 0.395 \) ohms

**Half wave rectifier**
Figure 9
Alternator No-Load Saturation Curve
Figure 10

Exciter No-Load Saturation Curve
118 volts Alternator Line-Neutral Voltage

Exciter Field Amperes

Exciter Field Volts

Alternator Load Current

9.1 volts Alternator Field Volts

Alternator Field Amperes

Figure 11

100%, 0.75 Lagging Power Factor Load Application
Alternator Volts in
Equation 11 Test

20
10

100^ 0,75 P.F. Load Application

Exciter Volts

Figure 12

100% 0.75 P.F. Load Application
B. Removal of Load

As can be observed from the previous discussion, the exciter is never unloaded. Therefore, only the generator will be considered here.

The test oscillogram for load removal is shown in Figure 13. The state variable versus time curves read from this oscillogram are shown in Figure 14.

At this point, the first thought is to try the equations 11, 12, 23 and 25 previously developed. This was done in Appendix 2. The results are plotted in Figure 14. Quite obviously, equation 11 is not valid.

The primary reason that equation 11 is not valid is that it assumes a constant field current required for alternator saturation. Inspection of Figure 9 shows that while this assumption is not too inaccurate for load application and consequent voltage dip, it is very inaccurate for load removal and consequent voltage rise. The amount of field current required for saturation versus terminal voltage is plotted in Figure 15. This curve is obtained from Figure 9 by plotting the difference in amperes between the air-gap line (the straight line) and the actual field current required for the alternator output voltage (the curved line). This difference at no-load is due to saturation of the iron in the machine.
118 volts
Alternator Line-Neutral Voltage

Exciter Field Amperes

Exciter Field Volts

Alternator Load Current

Alternator Field Volts

Alternator Field Amperes

Figure 13

100%, 0.75 Lagging Power Factor Load Removal
Figure 14

100% 0.75 P.F. Load Removal
Figure 15

Alternator Saturation Field Current
Thus, equation 11 must be modified to properly present saturation during load removal. To do this, the dashed straight line was drawn in Figure 15 to approximate the saturation over the area of interest. Thus, the following equations are presented.

$$I_s = (x_1 - 112.5)(0.41) \text{ amperes}$$

In per-unit quantities:

$$I_s = (x - 112.5)\left(\frac{0.41}{16.9}\right) = E_s \text{ p.u.}$$

But $x_1$ must be expressed in per-unit.

$$E_s = 118.0(x_1 - \frac{112.5}{118.0})(0.0243) \text{ p.u.}$$

$$E_s = 118.0(x_1 - 0.954)(0.243) \text{ p.u.}$$

$$E_s = 2.87x_1 - 2.73 = K_x x_1 - K_c \text{ p.u.}$$

Then equation 11 becomes:

$$\frac{x}{1} = \frac{x_2 - (K_x + 1)x_1 + K_c}{T \text{ do}} \text{ p.u./second}$$

The response using equation 26 is calculated in Appendix 3 and also plotted in Figure 14. This is very accurate between the points of interest. Thus, equation 26 is verified for load removal. It should be noted that in equation 26 new constants must be derived for each alternator.
CHAPTER 4

DETERMINATION OF SWITCHING SURFACES

A. Application of Load

Let
\[ A_1 = \frac{K_2}{T_{do}'} \]
\[ B_1 = \frac{K_1}{T_{do}'} \]
\[ C_1 = \frac{K_4}{t_{do}'} \]
\[ D_1 = \frac{K_3}{t_{do}'} \]

Then equations 11 and 23 become:
\[ (27) \dot{x}_1 = A_1 x_2 - B_1 x_1 - A_1 E \quad \text{p.u.} \]
\[ (28) \dot{x}_2 = -D_1 x_2 + C_1 (m - e_s) \quad \text{p.u.} \]

A state variable diagram is included as Figure 16 to aid the reader in following the rest of this chapter. Let \( \mathbf{v} \) be the state vector. That is:
\[ (29) \mathbf{v} = \begin{bmatrix} x_1 \\ x_2 \\ m_1 \\ m_2 \end{bmatrix} \quad \text{p.u.} \]
Figure 16

State Variable Diagram

\[ m_2 = m - e_s \quad 0 < t < \lambda_1 \]
\[ m_2 = -(n + e_s) \quad \lambda_1 < t < \lambda_2 \]
Where:

\[ m_1 = E_s \]
\[ m_2 = (m-e_s) \quad 0 < t < \lambda_1 \]
\[ m_2 = -(n+e_s) \quad \lambda_1 < t < \lambda_2 \]

Then the initial-state vectors are:

\[
v(0) = \begin{bmatrix}
1 \\
B + E_s \\
A \\
E_s \\
D (B + E_s)
\end{bmatrix}
p.u.
\]

and

\[
(30) \ v(0^+) = \begin{bmatrix}
K_2 E d t' \\
B + E_s \\
E_s \\
(m - e_s)
\end{bmatrix} = \begin{bmatrix}
N \\
P \\
Q \\
R
\end{bmatrix}
\]

A, B, C and D are \( A_1 \), \( B_1 \), \( C_1 \) and \( D_1 \) calculated for the moment prior to load application. Define a matrix \( T \) such that \( \dot{v} = T \ v \). The solution to this equation is:

\[
(31) \ v(t) = \theta(t) \ v(0^+)
\]

where the overall transition matrix \( \theta(t) \) is given by

\[
(32) \ \theta(t) = e^{Tt} = L^{-1} \left\{ S I - T \right\}^{-1}
\]
Then:

\[
T = \begin{bmatrix}
-B_1 & A_1 & -A_1 & 0 \\
0 & -D_1 & 0 & C_1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

and:

\[
\Psi(t) = L^{-1} \begin{bmatrix}
[S+B_1 & -A_1 & A_1 & 0] \\
0 & S+D_1 & 0 & -C_1 \\
0 & 0 & S & 0 \\
0 & 0 & 0 & S
\end{bmatrix}^{-1}
\]

The solution is:

\[
\Psi(\ ) = \begin{bmatrix}
V & W & X & Z \\
0 & Y & 0 & U \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where:

\[
V = e^{-B_1 \lambda}
\]

\[
W = \frac{A_1}{D_1-B_1}(e^{-B_1 \lambda} - e^{-D_1 \lambda})
\]

\[
X = \frac{A_1}{B_1}(e^{-B_1 \lambda} - 1)
\]

\[
Y = e^{-D_1 \lambda}
\]

\[
U = \frac{C_1(1 - e^{-D_1 \lambda})}{D_1}
\]

\[
Z = A_1 C_1 \left( \frac{1}{B_1 D_1} - \frac{e^{-B_1 \lambda}}{B_1(D_1-B_1)} + \frac{e^{-D_1 \lambda}}{D_1(D_1-B_1)} \right)
\]
In equation 32, \( t \) can only run until point \( \lambda_1 \) of Figure 7 is reached. At that time the regulator is switched to some maximum negative value \( n \) p.u. Call this time \( \lambda_1 \). Then equation 31 becomes:

\[
(35) \quad v(\lambda_1) = \Theta(\lambda_1) \cdot v(0^+) 
\]

From this time \( \lambda_1 \) to the time \( \lambda_2 \) when the regulator is returned to the linear mode, the same differential equation applies, but with a new initial-state vector \( v(\lambda_1^+) \). The relationship between \( v(\lambda_1) \) and \( v(\lambda_1^+) \) is the \( \mathbf{W} \) matrix. That is:

\[
(36) \quad v(\lambda_1^+) = \mathbf{W} \cdot v(\lambda_1) 
\]

In this case:

\[
(37) \quad \mathbf{W} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -n-e_s/m-e_s \end{bmatrix} 
\]

Thus

\[
(38) \quad v(\lambda_1 + \lambda_2) = \Theta(\lambda_2) \cdot \mathbf{W} \cdot v(\lambda_1) 
\]

Since deadbeat response is required:

\[
(39) \quad x_1(\lambda_1 + \lambda_2) = 1.0 
\]

\[
(40) \quad x_2(\lambda_1 + \lambda_2) = E_s + \frac{B_1}{A_1} 
\]
This will give two equations in two unknowns, assuming A, B, C and D are known. The time \( \lambda_1 \) for switching can be determined for each load condition.

The problem now is the actual manipulation of equation 33. To facilitate this, the matrix 34 will be rewritten.

\[
\theta(\lambda_1) = \begin{bmatrix}
A & B & D & E \\
0 & C & 0 & F \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\( \lambda = \lambda_1 \)

\[
\theta(\lambda_2) = \begin{bmatrix}
G & H & K & L \\
0 & J & 0 & M \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\( \lambda = \lambda_2 \)

Then equation 38 becomes:

\[
\begin{bmatrix}
G & H & K & L \\
0 & J & 0 & M \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
A & B & D & E \\
0 & C & 0 & F \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
N \\
P \\
Q \\
R
\end{bmatrix}
\]

\( (41) \)

When expanded, the results for \( x_1(\lambda_1+\lambda_2) \) and \( x_2(\lambda_1+\lambda_2) \) are:
\[ x_1(\lambda_1+\lambda_2) = A_2 e^{-B_1 \lambda_1 \lambda_2} + B_2 e^{+B_1 \lambda_2} + C_2 e^{-D_1 \lambda_1 \lambda_2} \]

\[ x_2(\lambda_1+\lambda_2) = \frac{(D_1-B_1)}{A_1 C_2 e^{-D_1 \lambda_1 \lambda_2}} - \frac{(D_1-B_1)}{A_1 D_2 e} \]

Where:

\[ A_2 = K_2 E_1 \left( +\frac{(B_1 + E_S)}{A_1 C_1 (m+n)} \right) \]

\[ B_2 = \frac{A_1 C_1 (m+n)}{B_1 (D_1-B_1)} \]

\[ C_2 = \frac{A_1}{(D_1-B_1)} \frac{C_1}{D_1 (m-e_S)} - \frac{(B_1 + E_S)}{A_1} \]

\[ D_2 = \frac{-B_1 B_2}{D_1} \]

Let

\[ A_3 = 1.0 + \frac{A_1 E_S}{B_1} + (n+e_S) \frac{A_1 C_1}{B_1 D_1} \]

\[ A_4 = -\frac{B_1}{D_1-B_1} + \frac{A_1 E_S}{D_1-B_1} + \frac{A_1 C_1 (n+e_S)}{D_1(D_1-B_1)} \]

After inserting equations 39 and 40 into equations 42 and 43:

\[ A_3 = A_2 e^{-B_1 \lambda_1 \lambda_2} + B_2 e^{+B_1 \lambda_2} + C_2 e^{-D_1 \lambda_1 \lambda_2} + D_2 e^{-D_1 \lambda_1 \lambda_2} \]

\[ A_4 = +C_2 e^{+D_2 e} \]
The state variable $x_1(t)$ can be calculated for $\lambda_1$ by using equation 35. The corresponding value of $I_1$ can readily be calculated from Ohm's law. This has been done in Appendix 5 for various loads, using the parameters of Table 2 and Figures 17 and 18. This system was chosen because of its availability for testing. The system is the one used on the Boeing 707 aircraft. The results are plotted in Figure 19. Leading power factors are not considered because they are unrealistic loads. The switching surface is plotted in Figure 20.

B. Removal of Load

Let

\[ A_1 = \frac{1}{T_{do'}} \]

\[ B_1 = \frac{K_x + 1}{T_{do'}} \]

\[ C_1 = \frac{K_4}{t_{do'}} \]

\[ D_1 = \frac{K_3}{t_{do'}} \]

\[ E_s = -K_c \]

\[ E_{s1} = E_s \text{ prior to load removal} \]

Then equations 26 and 23 become the same as equations 27 and 28.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Generator</strong></td>
<td></td>
</tr>
<tr>
<td>$X_q$</td>
<td>0.735 per-unit</td>
</tr>
<tr>
<td>$T_{do'}$</td>
<td>0.086 seconds</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.316 ohms @ 25°C</td>
</tr>
<tr>
<td>$X_d'$</td>
<td>0.301 per-unit</td>
</tr>
<tr>
<td>$X_d$</td>
<td>1.66 per-unit</td>
</tr>
<tr>
<td>$E_b$</td>
<td>118.0 volts</td>
</tr>
<tr>
<td><strong>Exciter</strong></td>
<td></td>
</tr>
<tr>
<td>$X_d = X_q$</td>
<td>0.509</td>
</tr>
<tr>
<td>$t_{do'}$</td>
<td>0.0855</td>
</tr>
<tr>
<td>$r_f$</td>
<td>8.58 ohms @ 110°C</td>
</tr>
<tr>
<td>$X_d'$</td>
<td>0.118</td>
</tr>
<tr>
<td><strong>Full wave rectifier</strong></td>
<td></td>
</tr>
<tr>
<td>$K_x$</td>
<td>3.50</td>
</tr>
<tr>
<td>$K_c$</td>
<td>3.34</td>
</tr>
</tbody>
</table>
Figure 17

Boeing 707 Alternator N-L Saturation Curve
Figure 18

Boeing 707 Exciter Saturation Curves
Figure 19

Load-On Switching Curves

The diagram shows the load-on switching curves with $E_t$ on the vertical axis and $I_1$ (p.u.) on the horizontal axis. The curves illustrate the behavior of the linear controller above the specified line for different angles (40°, 30°, 20°, and 10°).
Figure 20

Load-On Switching Surface
Equation 51 can be directly substituted into equation 50.

\[ -B_1(\lambda_1 + \lambda_2) - B_1 \lambda_2 \]

(52) \[ A_3 - A_4 = A_2 e + B_2 e \]

\[ -B_1 \lambda_2 - B_1 \lambda_1 \]

(53) \[ A_3 - A_4 = e (A_2 e + B_2) \]

Factor equation 51:

\[ -D_1 \lambda_2 - D_1 \lambda_1 \]

(54) \[ A_4 = e (C_2 e + D_2) \]

Raise equation 53 to the \( D_1 \) power, raise equation 54 to the \( B_1 \) power and divide the resulting equations:

\[
\frac{D_1}{A_4} \cdot \frac{A_3 - A_4}{B_1} = \frac{-B_1 \lambda_1}{B_1} \cdot \frac{D_1}{(C_2 e + D_2)}
\]

(55) \[ \frac{(A_3 - A_4)}{B_1} = \frac{-B_1 \lambda_1}{D_1} \cdot \frac{D_1}{(C_2 e + D_2)} \]

The time variable \( \lambda_2 \) has now been eliminated from the equations and \( \lambda_1 \) can be solved for directly. This can be done by re-arranging equation 55 and using any root-finding method.

\[
-D_1 \lambda_1 - (A_2 e + B_2) D_1 / B_1 = 0.
\]

(56) \[ \frac{C_2 e}{A_4} + D_2 - \frac{A_2 e}{A_3 - A_4} \cdot \frac{A_3 - A_4}{B_1} = 0. \]
This is implemented in Appendix 4.

One of the problems unique to a brushless alternator now becomes apparent. The state variable \( x_2 \) is unobservable. Some other measurable variable must be used to determine the switching instant in the physical system. The load current and its phase angle as well as \( x_1 \), the terminal voltage, are measurable. Thus these will be used.

The initial state vector is:

\[
(57) \quad \mathbf{v}(0^+) = 
\begin{bmatrix}
E_{d1'} \\
\frac{B}{A} + E_{si} \\
E_s \\
-(n + e_s)
\end{bmatrix} = 
\begin{bmatrix}
N \\
P \\
Q \\
R
\end{bmatrix}
\]
where $A$ and $B$ are $A_1$ and $B_1$ calculated for the instant prior to load removal. The $T$ matrix is still the same as equation 33. Therefore the $\Theta(\lambda)$ matrix is the same as equation 34. Thus equations 35 and 36 are still valid. Equation 37, however, is changed.

\[
(58) \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & \frac{m-es}{n-es}
\end{bmatrix}
\]

With this new matrix, $W$, equations 38, 39, and 40 are still valid. Now equation 38 becomes:

\[
(59) \begin{bmatrix}
G & H & K & L \\
0 & J & 0 & M \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & \frac{m-es}{n-es}
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
A & B & D & E \\
0 & C & 0 & F \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
N \\
P \\
Q \\
R
\end{bmatrix}
\]

(60) $v(\lambda_1+\lambda_2)$ = $G(AN+BP+DQ+ER)+H(CP+FR)+KQ+LR(\frac{m-es}{n-es})$

When expanded, the results for $x_1(\lambda_1+\lambda_2)$ and $x_2(\lambda_1+\lambda_2)$ are:

\[
(61) x_1(\lambda_1+\lambda_2) = A_2e + B_1\lambda_2 - B_1(\lambda_1+\lambda_2) + D_1(\lambda_1+\lambda_2) + D_1\lambda_2 + \frac{AC}{B_1D_1} (m-es) - \frac{A_1E_s}{B_1}
\]

$$+ D_2 e + \frac{AC}{B_1D_1} (m-es) - \frac{A_1E_s}{B_1}$$
\[(62) x_2(\lambda_1+\lambda_2) = -\frac{(D_1-B_1)}{A_1} C_2 e^{\frac{-D_1(\lambda_1+\lambda_2)}{A_1}} - \frac{(D_1-B_1)}{A_1} D_2 e^{\frac{-D_1\lambda_2}{A_1}}
\]
\[+ \frac{C_1}{D_1} (m-e_s)\]

Where:
\[(63) A_2 = E_d I + \frac{A_1}{(B+1) (D_1-B_1)} + \frac{A_1 E_s (n+e_s) A_1 C_1}{B_1 + B_1 (D_1-B_1)}\]
\[(64) B_2 = -\frac{A_1 C_1 (m+n)}{B_1 (D_1-B_1)}\]
\[(65) C_2 = -(B+1) \frac{A_1}{D_1-B_1} - \frac{A_1 C_1 (n+e_s)}{D_1 (D_1-B_1)}\]
\[(66) D_2 = -\frac{B_1 B_2}{D_1}\]

Since deadbeat response is required:
\[(67) x_1(\lambda_1+\lambda_2) = 1.0\]
\[(68) x_2(\lambda_1+\lambda_2) = 1.0 + E_s I\]

Let
\[(69) A_3 = 1.0 + \frac{A_1 E_s}{B_1} - (m-e_s) \frac{A_1 C_1}{B_1 D_1}\]
\[(70) A_4 = -(1.0+E_s I) \frac{A_1}{(D_1-B_1)} - \frac{A_1 C_1 (m-e_s)}{D_1 (D_1-B_1)}\]

Then for deadbeat response:
\[(71) A_3 = A_2 e^{-B_1(\lambda_1+\lambda_2)} + B_2 e^{-B_1\lambda_2} + C_2 e^{-D_1(\lambda_1+\lambda_2)} + D_2 e^{-D_1\lambda_2}\]
\[(72) A_4 = C_2 e^{-D_1(\lambda_1+\lambda_2)} + D_2 e^{-D_1\lambda_2}\]
Equations 71 and 72 are identical to equations 50 and 51. Therefore the same method of solution will work.

Here again the characteristics of the brushless alternator must be recognized. The output of the exciter, $x_2$, cannot be driven negative. Therefore if the solution of equations 71 and 72 indicate a negative value of $x_2$ at time $\lambda_1$, the equations must be changed to allow $x_2$ to become only zero. If $\lambda_0$ is the time when $x_2$ goes to zero:

\begin{align*}
(73) \quad & v(\lambda_0) = \Theta(\lambda_0) v(0^+) \\
(74) \quad & v(\lambda_0^+) = \mathbf{V} v(\lambda_0) \\
(75) \quad & \mathbf{V} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
(76) \quad & v(\lambda_1+\lambda_0) = \Theta(\lambda_1) \mathbf{V} v(\lambda_0) = \Theta(\lambda_1) \mathbf{V} \Theta(\lambda_0) v(0^+) \\
(77) \quad & v(\lambda_1+\lambda_0^+) = \mathbf{W} v(\lambda_1+\lambda_0) \\
(78) \quad & \mathbf{W} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & m-es & 0 \end{bmatrix} \\
\end{align*}

\begin{align*}
& v(\lambda_2+\lambda_1+\lambda_0) = \Theta(\lambda_2) \mathbf{W} v(\lambda_1+\lambda_0^+) \\
(79) \quad & v(\lambda_2+\lambda_1+\lambda_0) = \Theta(\lambda_2) \mathbf{W} \Theta(\lambda_1) \mathbf{V} \Theta(\lambda_0) v(0^+) 
\end{align*}
(80) \( \nu(\lambda^0) = \begin{bmatrix} \frac{e^{-B_1\lambda_0} - e^{-D_1\lambda_0}}{D_1-B_1} \frac{e^{-B_1\lambda_0} - e^{-D_1\lambda_0}}{D_1-B_1} \\ \frac{1}{B_1D_1} \frac{e^{-B_1\lambda_0} - e^{-D_1\lambda_0}}{D_1-B_1} \\ -(n+e_s)A_1C_1 \frac{e^{-D_1\lambda_0}}{D_1(D_1-B_1)} \\ 0 \\ E_s \\ -(n+e_s) \end{bmatrix} \)

(81) \( \nu(\lambda_2 + \lambda_1 + \lambda_0) = \Theta(\lambda_2) \otimes \Theta(\lambda_1) \nu(\lambda^0) \)

(82) \( \nu(\lambda_1 + \lambda_0) = \begin{bmatrix} x_1(\lambda_0) e^{-B_1\lambda_1} + E_s A_1 \left(e^{-B_1\lambda_1} - 1\right) \\ 0 \\ E_s \\ 0 \end{bmatrix} \)
(83) \( \nu(\lambda_1+\lambda_0^+) = \begin{bmatrix}
\nu_1(\lambda_0)e^{-B_1\lambda_1} + C_1 \frac{A_1}{B_1}(e^{-B_1\lambda_1} - 1).
\nu_2(\lambda_0)e^{A_1\lambda_1}
\nu_3(\lambda_0)e^{A_1\lambda_1}
\nu_4(\lambda_0)e^{A_1\lambda_1}
\end{bmatrix} \)

(84) \( x_1(\lambda_2+\lambda_1+\lambda_0) = \begin{bmatrix}
x_1(\lambda_0)e^{-B_1\lambda_1} + C_1 \frac{A_1}{B_1}(e^{-B_1\lambda_1} - 1).
x_2(\lambda_0)e^{A_1\lambda_1}
x_3(\lambda_0)e^{A_1\lambda_1}
x_4(\lambda_0)e^{A_1\lambda_1}
\end{bmatrix} e^{-B_1\lambda_2}

\begin{align*}
&+ C_1 \frac{A_1}{B_1}(e^{-B_1\lambda_1} - 1.)
&+ (m-e_s)A_1C_1 \left[ \frac{1 - e}{B_1D_1(B_1-B_1)} + e \frac{B_1-B_1}{D_1(D_1-B_1)} \right]
\end{align*}

(85) \( x_2(\lambda_2+\lambda_1+\lambda_0) = (m-e_s)^{C_1}(1.-e^{-D_1\lambda_2}) \)

Inserting equations 67 and 68 into 84 and 85 respectively:

(86) \( 1. + \frac{A_1}{B_1}(m-e_s)A_1C_1 \frac{A_1}{B_1D_1} = e^{-B_1(\lambda_1+\lambda_2)}(x_1(\lambda_0) + \frac{A_1e_s}{B_1}) \)

\begin{align*}
&-e^{-B_1\lambda_2}(m-e_s) \frac{A_1C_1}{B_1(D_1-B_1)}
&+e^{-D_1\lambda_2}(m-e_s) \frac{A_1C_1}{D_1(D_1-B_1)}
\end{align*}
Equation 87 can be solved for $\lambda_2$ directly.

$$e^{-D_1\lambda_2} = \frac{-D_1}{C_1(m-e_s)}(1 + E_{SI}) + 1.$$  

$$-D_1\lambda_2 = \ln 1 - \frac{D_1(1 + E_{SI})}{C_1(m-e_s)}$$

(88) \hspace{1cm} \lambda_2 = \frac{\ln(1 - \frac{D_1(1 + E_{SI})}{C_1(m-e_s)})}{-D_1}

From equation 73:

(89) \hspace{1cm} x_2(\lambda_0) = \frac{B + E_{SI}}{A} e^{-D_1\lambda_0} (n + e_s) \frac{C_1}{D_1} (1 - e^{-D_1\lambda_0}) = 0.

This can be solved for $\lambda_0$ directly.

$$\lambda_0 = \frac{\ln \left( \frac{(n+e_s)C_1}{D_1} \right)}{-D_1} \frac{B + E_{SI} + \frac{C_1}{D_1}(n+e_s)}{A}$$

(90) \hspace{1cm} \lambda_0 = \frac{\ln \left( \frac{(n+e_s)C_1}{D_1} \right)}{-D_1} \frac{B + E_{SI} + \frac{C_1}{D_1}(n+e_s)}{A}

Now that $\lambda_0$ and $\lambda_2$ are known, equation 79 can be evaluated and then equation 86 can be solved for $\lambda_1$. 
Define:

\[ 1 + \frac{A_1 E_s}{B_1} - (m - e_s) \frac{A_1 C_1}{B_1 D_1} e^{-D_1 \lambda_2 (m - e_s)} \frac{A_1 C_1}{D_1 (D_1 - B_1)} \]

(91) \[ A_4 = \frac{-B_1 \lambda_2}{e} \frac{A_1 C_1}{B_1 (D_1 - B_1)} \frac{A_1}{(x_1(\lambda_0 + B_1 E_s))} \]

Then:

(92) \[ \lambda_1 = \frac{\ln A_4}{-B_1} \]

Equations 71, 72, 79, 83, 88, 90, 91, and 92 are solved in Appendix 6 for various loads. The results are plotted in Figure 21. The coordinates of current and phase angle refer to values immediately prior to load removal. The switching surface is for time \( \lambda_1 \). The switching at time \( \lambda_0 \) will be accomplished by sensing the exciter field current.

If the \( 0=10^\circ \) curve is ignored as being a very unrealistic power factor angle, the switching surface is seen to be a plane at \( E_t = 1.009 \text{ p.u.} \) with a slight dimple at the origin. If it is further realized that the transient overshoot caused by the removal of such small load currents is within the range of the linear controller, the dimple at the origin can also be ignored.
Figure 21

Load Removal Switching Curves
C. Build-Up

Build-up is similar to a load application in the sense of the sequence of the application of \( m \) and \( n \). However, no load current flows. This information can be used to sense the difference and apply the proper switching point. Thus equations 27 and 28 would apply with the proper values of the constants.

Under no-load conditions, \( E_t \) equals \( E_d' \). Therefore \( K_2 \) equals one. Inspection of Figure 16 shows that \( K_1 \) also equals one. Since a very large amount of the transient occurs in the unsaturated region of the iron, \( E_s \) will be arbitrarily reduced 50% to more nearly approximate the proper total time. The variables \( D_1 \) and \( C_1 \) are not changed.

The initial-state vector is:

\[
\mathbf{v}(0^+) = \begin{bmatrix}
0 \\
0 \\
\frac{E_s}{2} \\
m
\end{bmatrix} \text{ p.u.}
\]

The final-state vector is:

\[
\mathbf{v}(\lambda_1+\lambda_2) = \begin{bmatrix}
1. \\
1 + E_s \\
E_s \\
0
\end{bmatrix}
\]
Thus:

\[ A_1 = B_1 = \frac{1}{T_{do}} \]

\[ C_1 = \frac{K_4}{T_{do}} \]

\[ D_1 = \frac{K_3}{T_{do}} \]

(93) \[ x_1 = A_1 x_2 - A_1 x_1 - \frac{A_1 E_S}{2} \text{ p.u.} \]

(94) \[ x_2 = -D_1 x_2 + C_1 m \text{ p.u.} \]

The matrix \( T \) is now:

\[
T = \begin{bmatrix}
-A_1 & A_1 & -A_1 & 0 \\
0 & -D_1 & 0 & C_1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(95) This is the same as matrix 33 if \( A_1 \) is substituted for \( B_1 \). State vector 41 then becomes:

\[
\nu(\lambda_1 + \lambda_2) = \begin{bmatrix}
G(DQ+ER)+HFR+KQ+LR(\frac{-R}{m}) \\
JFR+MR(\frac{-R}{m}) \\
Q \\
R(\frac{-n}{m})
\end{bmatrix}
\]

(96)
When expanded, the results for $x_1(\lambda_1+\lambda_2)$ and $x_2(\lambda_1+\lambda_2)$ are:

\begin{equation}
(97) \quad x_1(\lambda_1+\lambda_2) = A_2 e^{-D_1\lambda_2} - A_1 \lambda_2 - D_1(\lambda_1+\lambda_2)
\end{equation}

\begin{equation}
+ B_2 e^{-D_1\lambda_2} - \left(-\frac{E_s}{2} + \frac{nC_1}{D_1}\right)
\end{equation}

\begin{equation}
(98) \quad x_2(\lambda_1+\lambda_2) = -\frac{(D_1-A_1)}{A_1} C_2 e^{-D_1(\lambda_1+\lambda_2)} - \frac{nC_1}{D_1}
\end{equation}

\begin{equation}
\quad - \frac{(D_1-A_1)}{A_1} D_2 e^{-D_1\lambda_2}
\end{equation}

Where:

\begin{equation}
(99) \quad A_2 = \frac{E_s}{2} - \frac{mC_1}{D_1-A_1}
\end{equation}

\begin{equation}
(100) \quad B_2 = \frac{C_1(m+n)}{D_1-A_1}
\end{equation}

\begin{equation}
(101) \quad C_2 = \frac{A_1 C_1 m}{D_1(D_1-A_1)}
\end{equation}

\begin{equation}
(102) \quad D_2 = -\frac{A_1 B_2}{D_1}
\end{equation}

Let:

\begin{equation}
(103) \quad A_3 = 1 + \frac{E_s}{2} + \frac{nC_1}{D_1}
\end{equation}
\( A_4 = -(1. + E_s + \frac{nC_1}{D_1}) \frac{A_1}{D_1-A_1} \)

Then:

\[
\begin{align*}
(105) & \quad A_3 = A_2e + B_2e + C_2e + D_2e \\
& \quad -A_1(\lambda_1+\lambda_2) -A_1\lambda_2 -D_1(\lambda_1+\lambda_2) -D_1\lambda_2 \\
(106) & \quad A_4 = C_2e + D_2e \\
& \quad -D_1(\lambda_1+\lambda_2) -D_1\lambda_2
\end{align*}
\]

Equation 106 can be directly substituted into equation 105.

\[
\begin{align*}
(107) & \quad A_3-A_4 = A_2e + B_2e \\
& \quad -A_1(\lambda_1+\lambda_2) -A_1\lambda_2 \\
(108) & \quad A_3-A_4 = e (A_2e + B_2)
\end{align*}
\]

Factor equation 106:

\[
\begin{align*}
(109) & \quad A_4 = e (C_2e + D_2) \\
& \quad -D_1\lambda_2 -D_1\lambda_1
\end{align*}
\]

Raise equation 108 to the \( D_1 \) power, raise equation 109 to the \( A_1 \) power and divide the resulting equations.

\[
\begin{align*}
(110) & \quad \frac{D_1}{B_1} \frac{(A_3-A_4)}{A_4} = \frac{-A_1\lambda_1}{-D_1\lambda_1} \frac{D_1}{B_1} \\
& \quad \frac{(A_2e + B_2)}{(C_2e + D_2)} \\
& \quad \frac{B_1}{B_1}
\end{align*}
\]
Equation 111 is identical to equation 56. The same technique will therefore be used to find \( \lambda_1 \) and \( x_1(\lambda_1) \).

\[
(111) \quad \frac{-D_1\lambda_1}{C_2e} + D_2 - \left[ \frac{-A_1\lambda_1}{A_2e} + B_2 \right] \frac{D_1}{B_1} = 0.
\]

Equations 111 and 112 have been solved in Appendix 7 for the Boeing 707 system. The results are:

\[
\begin{align*}
\lambda_1 &= 0.04041 \text{ seconds} \\
x_1(\lambda_1) &= 0.792 \text{ p.u.} \\
\lambda_2 &= 0.01428 \text{ seconds} \\
x_1(\lambda_1 + \lambda_2) &= 1.0 \text{ p.u.}
\end{align*}
\]

D. **Short-Circuit Removal**

Short-circuit removal is similar to build-up, with the exception of the initial condition of \( e_t \). The field current in all real systems is limited, so \( e_t \) is also limited. Call this voltage limit \( e_{t1} \). The initial condition of \( e_t \) is thus \( e_{t1} \).
The initial condition of $E_t$ must be zero, since $E_d'$ was zero the previous instant. (Equation 8 does not apply because the terminal voltage was not regulated to 1.0 per unit.) $E_s$ will again be reduced 50%.

The initial-state vector is:

$$v(0^+) = \begin{bmatrix} 0 \\ etl \\ Es/2 \\ m-es \end{bmatrix} \text{ p.u.}$$

The final-state vector is:

$$v(\lambda_1+\lambda_2) = \begin{bmatrix} 1. \\ 1+E_s \\ Es \\ 0 \end{bmatrix}$$

Thus $A_l$, $B_l$, $C_l$, $D_l$ and the matrix $T$ are the same as for build-up. State vector $4l$ then becomes:

$$v(\lambda_1+\lambda_2) = \begin{bmatrix} G(BP+DQ+ER)+H(CP+FR)+KQ+LR\frac{-n-esa}{m-es} \\ J(CP+FR)+MR\frac{-n-esa}{m-es} \\ Q \\ R\frac{-n-esa}{m-es} \end{bmatrix}$$
When expanded, the results for $x_1(\lambda_1+\lambda_2)$ and $x_2(\lambda_1+\lambda_2)$ are:

$$
(114) \quad x_1(\lambda_1+\lambda_2) = A_2e^{-D_1\lambda_2} + B_2e^{-D_1\lambda_2} + C_2e^{-D_1\lambda_2}
$$

$$
+ D_2e^{-D_1\lambda_2} \left[ \frac{E_s}{2} + \frac{(n+e_s)C_1}{D_1} \right]
$$

$$
(115) \quad x_2(\lambda_1+\lambda_2) = -\frac{(D_1-A_1)}{A_1} C_2e^{-D_1\lambda_2} - \frac{nC_1}{D_1}
$$

$$
- \frac{(D_1-A_1)}{A_1} D_2e^{-D_1\lambda_2}
$$

Where:

$$
(116) \quad A_2 = \frac{E_s}{2} + \frac{A_1e_t1-C_1(m-e_s)}{D_1-A_1}
$$

$$
(117) \quad B_2 = \frac{C_1(m+n)}{D_1-A_1}
$$

$$
(118) \quad C_2 = \frac{A_1}{D_1-A_1} \left[ \frac{(m-e_s)C_1}{D_1} - e_t1 \right]
$$

$$
(119) \quad D_2 = \frac{-A_1B_2}{D_1}
$$

Let:

$$
(120) \quad A_3 = 1.0 + \frac{E_s}{2} + \frac{(n+e_s)C_1}{D_1}
$$
\begin{align}
(121) \quad A_4 &= -(1. + E_s + \frac{(n+e_s)C_1}{D_1}) \frac{A_1}{D_1-A_1} \\
\text{Then:} & \\
-A_1(\lambda_1+\lambda_2) &-A_1\lambda_2 &-D_1(\lambda_1+\lambda_2) &-D_1\lambda_2 \\
(122) \quad A_3 &= A_2e + B_2e + C_2e + D_2e \\
-D_1(\lambda_1+\lambda_2) &-D_1\lambda_2 \\
(123) \quad A_4 &= C_2e + D_2e \\
\end{align}

The last two equations are the same as equations 105 and 106. Thus equation 111 is valid and:

\begin{align}
(124) \quad x_1(\lambda_1) &= \frac{et_1A_1}{D_1-A_1}(e^{-A_1\lambda_1} - e^{-D_1\lambda_1}) + \frac{E_s}{2}(e^{-A_1\lambda_1} - 1.) + \\
& \frac{m-e_s}{D_1} \left( \frac{C_1}{D_1} \frac{C_1e}{D_1-A_1} + \frac{A_1C_1e}{D_1(D_1-A_1)} \right) \\
\end{align}

Equations 111 and 124 are solved for short-circuit removal in Appendix 8. The value of et_1 is read from the load saturation curve of the exciter, Figure 18. For this test the exciter field current was limited to three amperes. Thus et_1 equals 20.0 volts, or 8.1 per unit. The results are:

\[
\begin{align*}
\lambda_1 &= 0.005 \text{ seconds} \\
x_1(\lambda_1) &= 0.49 \text{ p.u.} \\
\lambda_2 &= 0.012 \text{ seconds}
\end{align*}
\]
CHAPTER 5

SYNTHESIS OF THE CONTROLLER

A. Control of the Regulator

Figure 22 is a simplified schematic of the transistorized regulator used. When the input to the comparator is greater than the reference, the output of the comparator is at ground potential, shutting the regulator output transistor off. A positive signal at pin 9 of the comparator also puts the output of the comparator at ground potential. Since a signal of only a few volts at the output of the comparator turns the regulator output transistor on, full external control of the regulator can be accomplished by making the output of the comparator into an OR gate and also using pin 9 of the comparator. The output of the regulator is a pulse width-modulated signal, the modulation coming from the ripple on the input signal.

This easily accomplishes the application of the control signals if turning off the regulator can be made to apply the negative field forcing and the voltage \( m \) is the applied field voltage when the regulator is full on. A
Figure 22
Simplified Schematic of a Typical Regulator
solution is shown in Figure 23. In this figure, $N$ is a digital signal to turn the negative voltage $n$ on; $M$ is the digital signal to turn the positive voltage $m$ on; and $L$ is the digital signal to return the regulator to the linear mode. When the signal $L$ is positive, $Q_7$ is turned on, also turning $Q_8$ on and shorting out the zener diode $Z$, which has a zener voltage $n$. This is necessary because—the regulator is a pulse-width modulated regulator. When $N$ or $M$ is on, $L$ is off and the zener diode $Z$ is operative. This scheme for applying the negative voltage $n$ has the distinct advantage that $n$ is applied only so long as the exciter field current is flowing. There is no possibility of a negative field current. The value of $n$ need not have any relation to the value of $m$, either.

B. Switching Surfaces

1. Load Removal

As was pointed out in Section B of Chapter 4, the switching surface for load removal is a plane parallel to the $I - 0$ surface and perpendicular to the $E_t$ axis. This fact, along with the zener diode of Section A of this chapter, reduces the load removal plane to a point on the $E_t$ axis. That is, $m$ must be switched on when $E_t$ equals 1.009 p.u.

2. Load Application

The surface of Figure 20 should be describable with a mathematical equation in terms of $E_t$, $I$ and $0$. The
Figure 23
Regulator Schematic for Optimal Switching
exact form is a matter of conjecture. To be realistic, the equation must use forms of the variables which can be quickly found. The response time of any sensor must be a few milliseconds at the most. The variable $E_t$ can be measured with a three-phase full-wave bridge rectifier and filtered successfully with a response time of only a few milliseconds. The variable $\theta$ can be most easily (quickly) measured in combination with $I$. Figure 24 shows a method of measuring $I \sin \theta$. To be quickest, all three phases will be sensed and summed to reduce the response time by a factor of three.

The output of the circuit of Figure 24 is a square wave in phase with a line to line voltage and modulated by a sine wave in phase with and proportional to a current leading by $90^\circ$ the respective line to line voltage. The dc output is:

$$E_0 = \frac{2}{\pi} I_m \sin \theta \text{ volts}$$

With this in mind, Figure 20 is approximated by the equation:

$$E_t = A(I \sin \theta)^2 + B(I \sin \theta) + C \text{ p.u.}$$

The coefficients are found by using Program 039, Least Squares Fit for Several Variables, Westinghouse Aerospace Equipment Division computer library. The program is much too lengthy to list here, but is available upon request.
Figure 24

A Circuit for Sensing Ising
The coefficients for the Boeing 707 system are:

\[ A = 0.0255 \]
\[ B = -0.0693 \]
\[ C = 0.997 \]

The rms error is 0.265%, a very acceptable approximation.

3. Build-Up

Since no current is flowing during build-up, the switching surface reduces to a point on the \( E_t \) axis. In this case, \( m \) is applied until \( E_t \) equals 0.792 p.u., and then \( n \) is applied until \( E_t \) equals 1.0 p.u.

4. Short-Circuit Removal

Short-circuit removal is also a point on the \( E_t \) axis. Load current can be sensed to distinguish between this condition and build-up. The voltage \( n \) will be applied when \( E_t \) rises to 0.49 p.u.

C. Logic

The logic of this controller is quite complicated for such a seemingly simple task. This is because four distinct types of operation must be controlled. To facilitate this logic, nine digital signals will be developed by comparators. They are:

\[ A = 1 \quad E_t < 0.95 \text{ p.u.} \]
\[ B = 1 \quad E_t < 0.995 \text{ p.u.} \]
\[ C = 1 \quad \text{Switch to } n \text{ (output of load-application surface comparator)} \]
\[ D = 1 \quad E_t > 1.005 \text{ p.u.} \]
E = 1  \( E_t > 1.05 \) p.u.
F = 1  Switch to m (output of load-removal surface comparator)
G = 1  Switch to n (output of short-circuit comparator)
H = 1  \( I_L < 0.10 \) p.u.
J = 1  Switch to n (output of build-up comparator)

The output of the logic will be:

K = 1  Controller is not in short-circuit removal mode
L = 1  Put regulator in linear mode
M = 1  Turn m on
N = 1  Turn n on
P = 1  Controller is not in build-up mode

With the exception of the surface comparators, the numbers were picked to allow the regulator to react linearly to small load changes. Any load changes large enough to cause a five percent change in the terminal voltage will turn the deadbeat response controller on. The logic will return the system to the linear control of the regulator when the terminal voltage is within one half of one percent of the rated value. This is to allow for small variations as a result of temperature changes in the alternator.
1. Build-Up

The value of H will be determined from some sort of memory. That is because it is necessary to know why $E_t$ is less than 0.2 $E_{tB}$. With this acknowledged, the logic can be broken into four sections corresponding to the types of operation.

The logic flow-diagram for build-up is shown in Figure 25. The only signals of importance are A, B, G, H and J. The output is L, M, N and P. The binary numbers in the squares are the output signals L, M, N and P in that order. The decimal numbers in the squares are state assignments. The binary numbers on the connecting lines are the input signals A, B, G, H and J in that order. A flow table for Figure 25 is shown in Table 3. The logic output equations are:

$$L = (\overline{B} + \overline{A} F_2) \quad P = M \overline{N}$$
$$M = (\overline{J} + A F_1) \quad P = J(A + \overline{F}_1) + P$$
$$N = BP (AF_2 + JF_1) \equiv B + P + (A + \overline{F}_2)(\overline{J} + F_1)$$
$$P = \overline{B} + \overline{H} = \overline{BH}$$

The philosophy behind the minimization is that a flip-flop will be set to 1 by P and reset to 0 by GH. Thus most of the squares of the Karnaugh maps are "don't-care" conditions. The output of the flip-flop is the actual P signal to the rest of the logic.
Figure 25

Logic Flow-Diagram for Build-Up
### TABLE 3
FLOW TABLE FOR FIGURE 25

<table>
<thead>
<tr>
<th>ABGHJ</th>
<th>Input</th>
<th>Output</th>
<th>LMNP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11110</td>
<td>11010</td>
<td>1100</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>6</td>
<td>4</td>
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</tbody>
</table>
2. Short-Circuit Removal

The logic flow-diagram for short-circuit removal is shown in Figure 26. The only signals of importance are A, B, G and H. The output is L, M, N, and K. The philosophy here is that K is the output of a flip-flop which is set to 1 by GH or the K output of the logic and is reset to 0 by GH. A flow table for Figure 26 is shown in Table 4. The logic output equations are:

\[ L = \overline{A}F_1PK \text{ or } L = \overline{M} \overline{N} \]
\[ M = (GF_1F_2 + AGF_1) PK \]
\[ N = G(F_1 + F_2) PK \]
\[ K = B \]

3. Load Application

The output of this logic must have PK as a product in all its terms. With that understanding, the only signals of importance are A, B and C. The output is L, M, and N. The logic flow-diagram for load application is shown in Figure 27. A flow table for Figure 27 is shown in Table 5. A merged and encoded flow table is shown in Table 6. The logic output equations are:

\[ L = (\overline{F_1}F_4 + F_2F_3 + \overline{F_2}F_3F_4) PK \text{ or } L = \overline{M} \overline{N} \]
\[ M = (F_1F_4 + \overline{F}_2F_4 + F_1F_2F_3) PK \]
\[ N = (F_1F_2\overline{F}_3F_4 + \overline{F}_1F_2F_4) PK \]
Figure 26
Logic Flow-Diagram
for Short-Circuit Removal
### TABLE 4

**FLOW TABLE FOR FIGURE 26**

<table>
<thead>
<tr>
<th>ABGH</th>
<th>Input</th>
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<td>10</td>
<td>15</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ABGH</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
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<td>12</td>
</tr>
<tr>
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</tr>
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</tr>
</tbody>
</table>
Figure 27

Logic Flow-Diagram for Load Application
### TABLE 5

**FLOW TABLE FOR FIGURE 27**

<table>
<thead>
<tr>
<th>ABC</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
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<td>111</td>
</tr>
</tbody>
</table>
### TABLE 6

**MERGED AND ENCODED FLOW TABLE FOR FIGURE 27**

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<td>22</td>
<td>1001</td>
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</tr>
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</table>
4. Load Removal

The logic of load removal is seen to be the mirror image of load application. That is, if D, E and F are substituted for A, B and C respectively in Figure 27, the logic of load application will work for load removal. The outputs will be L, N, and M instead of L, M and N, however.

A little contemplation at this point suggests that one logic package be shared for load application and load removal. By using two flip-flops, one set by A and reset by L and another set by E and reset by L, and a few logic gates, one logic package can be shared. The results, using NAND gates, are shown in Figure 28.

5. Implementation of Logic

The implementation, using NAND gates, of the logic is shown in Figures 29, 30, 31, and 32. The statement \( L = \overline{M} \overline{N} \) has been chosen for ease of implementation.

D. Sensing

1. Comparators

As noted in Part C of this chapter, nine comparators are needed. The requirements are fast response and high sensitivity for the lowest cost. The circuit of Figure 33, using a Westinghouse WC306D integrated circuit operational amplifier, was chosen. This amplifier in
Figure 28

Combination Load Application
and Load Removal Logic
Figure 29

Logic Implementation for Build-Up
Figure 30

Logic Implementation

for Short-Circuit Removal
Figure 31

Logic Implementation for Load Rem
Figure 31

Implementation for Load Removal and Application
Figure 32

Logic System for Deadbeat

Response of a Brushless Alternator
Figure 33

Comparator Circuit

$X > Y \Rightarrow E_o = 0.24\,\text{v}$

$X < Y \Rightarrow E_o = 6.0\,\text{v}$
this configuration has a minimum sensitivity of 1 mv and a maximum response time of 300 nanoseconds. Hysterisis has been added to prevent chatter.

2. Terminal Voltage

The circuit for sensing terminal voltage is shown in Figure 34. The operational amplifier network is a low-pass filter with 40 db per decade roll-off and a -3db point of 360 Hz. The filter is thus 40 db down at 2400 Hz, the fundamental ripple frequency. The restraint ripple is 0.04% E, quite an acceptable figure. The frequency compensation network for the operational amplifier is not shown, but it is always used for all amplifiers except comparators and is the standard network recommended by Westinghouse.

3. Load Current, Phase Angle Product

This circuit, Figure 24, was explained previously. All that is added is a three-input summer and a low pass filter with a roll-off at 180 Hz. The total circuit is shown in Figure 35. The current transformer turns ratio is 500:5. The potentiometer setting is so chosen that the output of the filter is one volt when I is one p.u. and 0 is 90'.

4. Load Current for Logic Signal H

As was noted in Section C, Part 1 of this chapter, the logic signal H must be determined from some sort of
Figure 34

Terminal Voltage Sensing Circuit
Figure 35

Circuit for Sensing $\text{Isin} \theta$

1.0 p.u. $\text{Isin} \theta = 1.0 \text{v}$

$\theta = 90$ degrees
memory. The exact value of $I_L$ for which $H$ is 1 is not too important. It is only necessary to determine if the exceptionally low voltage is due to a short-circuit or a normal build-up. With this in mind, the circuit of Figure 36 will do quite well.

E. Computing

The computing will be done on an analog basis. The determination of all surfaces except that of load application can be done using the terminal voltage sensing circuit and comparators. The load application surface requires somewhat more sophistication.

As was noted in Section B, Part 2, of this chapter, the describing equation for the load application surface requires a squared term of the variable. At the low voltages required of integrated circuits, a diode quarter-square multiplier is unworkable. Therefore, the circuit of Figure 37 has been developed. It is a time sharing multiplier. The load application surface can now be generated and the terminal voltage compared to it. The complete sensing schematic is shown in Figure 38. The calculations for the resistances of the load application surface comparator are done in Appendix 9.
1.0 p.u. $I=3.6v$

Figure 36
Load Current Sensing
for Logic Signal H
\[ (\text{Isin} \theta)^2 \]

1.0 p.u. \( \frac{(\text{Isin} \theta)^2}{2} = 0.500V \)

Figure 37

Circuit for Computing \((\text{Isin} \theta)^2\)
Figure 38
Complete Sensing Schematic
Chapter 6

RESULTS, CONCLUSIONS AND RECOMMENDATIONS

A. Build-Up

Polaroid pictures taken from a memory oscilloscope for both the normal regulator and the deadbeat response controller are shown in Figure 39. These are pictures of the line-to-neutral voltage of the alternator. The characteristics of the memory oscilloscope cause the center portion of the sine waves not to be recorded. The results match the calculations. The conclusion is that the equations for build-up are accurate.

B. Short-Circuit Removal

Polaroid pictures of a 315 ampere (2.84 p.u.) short-circuit removal are shown in Figure 40. Again the results match the calculations. The conclusion is that the equations for short-circuit removal are accurate.

C. Load Application

The load application transient is small with respect to the previous two transients. The oscilloscope pictures were therefore taken from the output of the voltage sensing circuit, Figure 34. This allowed for a larger
With Normal Regulator    With Deadbeat Response Controller

All units are centimeters
20ms/cm horizontal scale
0.5 p.u./cm vertical scale

Figure 39

Build-Up of Boeing 707 System
With Normal Regulator

With Deadbeat Response Controller

All units are centimeters
20ms/cm horizontal scale
0.5p.u./cm vertical scale

Figure 40

315 Ampere Short-Circuit Removal on Boeing 707 System
per unit scale. The memory oscilloscope cannot stand signals exceeding the maximum screen dimensions. Any signal exceeding these dimensions causes a "splash" on the screen which completely obliterates the desired picture.

Polaroid pictures of a 26.9 kva 0.696 p.f. load application are shown in Figure 41. The results and the calculations coincide. This was the only load bank available, but it is sufficient for proof.

The deadbeat response system shows only slight improvement over the normal system. This was expected for this transient condition.

D. Load Removal

Polaroid pictures of a 26.9 kva 0.686 p.f. load removal are shown in Figure 42. Again only slight improvement was gained over the normal system. Part of this was due to the load level. The system itself, however, is an excellent system.

E. Recommendations

The deadbeat response controller gives excellent and dramatic improvement to the build-up and short-circuit removal transients, while only slight improvements are achieved in load application and removal transients. The majority of the hardware is involved in the load removal and application part of the controller. It would seem
With Normal Regulator

With Deadbeat Response Controller

All units are centimeters
20ms/cm horizontal scale
0.2 p.u./cm vertical scale

Figure 41

26.9 Kva 0.687 p.f. Load Application on Boeing 707 System
With Normal Regulator

With Deadbeat Response Controller

All units are centimeters
20ms/cm horizontal scale
0.2 p.u./cm vertical scale

Figure 42

26.9 Kva 0.687 p.f. Load Removal on Boeing 707 System
logical, then, to include only the build-up and short-circuit removal circuits in all future brushless-alternator systems.
APPENDIX 1

VERIFICATION OF ALTERNATOR EQUATIONS

\[ R_f \text{ @ } 110^\circ \text{C} = \frac{(0.4)(234 + 110)}{259} = 0.531 \text{ ohms} \]

From Figure 9:

\[ I_{fb} = 16.9 \text{ amperes} \]
\[ I_s = 2.3 \text{ amperes} \]
\[ e_{xb} = (0.531)(16.9) = 8.98 \text{ volts} \]
\[ E_s = \frac{I_s}{I_{fb}} = \frac{2.3}{16.9} = 0.136 \text{ p.u.} \]
VERIFICATION OF EQUATIONS 11, 12, 23, 25

DIMENSION X2(200), X1(200), DT(200)

ZL1 = 1. FOR NO LOAD INITIALLY

REAL(5, 100) XDP, XQ, XD, ZL, PF, ZLI, PFI, ES, XN, TDOP, X2B

N = XN + 1

X2(I) MUST BE READ AT MILLISEcond INTERVALS

READ(5, 100) (X2(I), I=1, N)

X2(I) IS READ IN VOLTS

DO 6 I = 1, N
  X2(I) = X2(I) / X2B
6  CONTINUE

THETA = ARCCOS(PF)
XE = ZL * SIN(THETA)
RE = ZL * PF
XDPT = XDP + XE
XQT = XQ + XE
XD = XD + XE
R2 = RE ** 2
XK1 = (R2 + XDPT * XQT) / (R2 + XDPT * XQT)
XK2 = (SQRT((XQT ** 2 + R2) * (XE ** 2 + R2)) / (XDPT * XQT + R2)
BETA = ARCCOS(PFI)
AI = 1. / ZLI
IF(ZLI = 0.11, 1, 2
2  EDPI = (1. + (XDP + XQ) * AI * SIN(BETA) + XQ * XDP * (AI ** 2)) / SQRT(1. + 2. * XQ * AI * SIN(BETA) + (XQ ** 2) * (AI ** 2))
  GO TO 3
1  EDPI = 1.
3  XI = XK2 * EDPI
  XI(1) = XI
  DT(1) = 0.
  DO 4 I = 2, N
  J = I - 1
  DX1 = (XK2 * (X2(J) - ES) - XK1 * X1(J)) / (1000. * TDOP)
  X1(I) = X1(J) + DX1
  X2(J) = X2(J) * X2B
4 DT(I) = DT(J) + 1.
   X2(N) = X2(N) * X2B
   WRITE(6, 102)
   WRITE(6, 101)(X2(I), X1(I), DT(I), I = 1, N)
GO TO 5
100 FORMAT(7F10.4)
101 FORMAT(6X1F5.1, 5X1F5.3, 4X1F4.0)
102 FORMAT(1H17X2HX27X2HX16X4HTIME/7X5HVCLTS5X2HPU7X2HMS)
END
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APPENDIX 2

VERIFICATION OF EQUATIONS 11, 12
VERIFICATION OF EQUATIONS 11, 12 FOR LOAD REMOVAL
DIMENSION X2(200), XI(200), DT(200)
5 READ(5, 100) XDP, XC, XD, ZLI, PFI, ES, TDOP, XN, X2B
N=XN+1
C X2(I) MUST BE READ AT 5 MILISECOND INTERVALS
READ(5, 100)(X2(I), I=1, N)
C X2(I) IS READ IN VOLTS
DO 6 I=1, N
X2(I)=X2(I)/X2B
C X2(I) MUST BE READ AT 5 MILISECOND INTERVALS
6 CONTINUE
BETA=ACOS(PFI)
AI=1./ZLI
EDPI=(1.+(XDP+XQ)*AI*SI+BETA)+XG*XDP*(AI*2))/$3RT(1.+2.*XC*AI*SI)
BETA+(XQ*2)*(AI*2))
XI(I)=EDPI
DT(I)=0.
DO 4 I=2, N
J=I-1
DX1=(X2(J)-ES-XI(J))/200.*TDOP
XI(I)=XI(J)+DX1
X2(J)=X2(J)*X2B
4 DT(I)=DT(J)+1.
X2(N)=X2(N)*X2B
WRITE(6, 102)
WRITE(6, 101)(X2(I), XI(I), DT(I), I=1, N)
GO TO 5
100 FORMAT(7F10.4)
101 FORMAT(6X1F5.1, 5X1F5.3, 4X1F4.0)
102 FORMAT(1H17X2H27X2H16X4H11HE7XS5VOLTS5X2HPU7X2HMS)
END
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APPENDIX 3

VERIFICATION OF EQUATION 26
VERIFICATION OF EQUATIONS 26 FOR LOAD REMOVAL

DIMENSION X2(2CC), X1(200), DT(200)

READ(5,100) XDP, XQ, XD, ZLI, PFI, ES, TDCP, XN, X2B

N = XN + 1

X2(I) MUST BE READ AT 5 MILLISECOND INTERVALS

READ(5,100) (X2(I), I = 1, N)

X2(I) IS READ IN VOLTS

DO 6 I = 1, N

X2(I) = X2(I) / X2B

6 CONTINUE

BETA = ARCCOS (PFI)

AI = 1 / ZLI

EDPI = (1 + (XDP + XQ) * AI * SIN(BETA) + XQ * XDP * (AI * 2)) / SQRT(1 + 2 * XQ * AI * SIN(BETA) + (XQ * 2) * (AI * 2))

X1(I) = EDPI

DT(1) = 0.

DO 4 I = 2, N

J = I - 1

DX1 = (X2(J) - 3.87 * X1(J) + 2.73) / (200 * TDCP)

X1(I) = X1(J) + DX1

X2(J) = X2(J) * X2B

4 DT(I) = DT(J) + 5.

X2(N) = X2(N) * X2B

WRITE(6, 102)

WRITE(6, 101) (X2(I), X1(I), DT(I), I = 1, N)

GO TO 5

100 FORMAT (7F10.4)

101 FORMAT (6X1F5.1, 5X1F5.3, 4X1F4.0)

102 FORMAT (1H17X2HX2HX2HX16X4HTIME/7X5HVCLTS5X2HPU7X2HMS)

END
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<td>4.8</td>
<td>1.099</td>
<td>65.</td>
</tr>
<tr>
<td>3.9</td>
<td>1.078</td>
<td>70.</td>
</tr>
<tr>
<td>2.6</td>
<td>1.055</td>
<td>75.</td>
</tr>
<tr>
<td>1.2</td>
<td>1.032</td>
<td>80.</td>
</tr>
<tr>
<td>0.2</td>
<td>1.007</td>
<td>85.</td>
</tr>
<tr>
<td>0.</td>
<td>0.982</td>
<td>90.</td>
</tr>
</tbody>
</table>
APPENDIX 4

IMPLEMENTATION OF EQUATIONS 56 AND 57

Let $W = e^{\lambda_1}$. Then equation 56 becomes:

$$f(W) = \frac{D_1}{C_2W + D_2} - \left[ \frac{B_1}{A_2W + B_2} \right]^{D_1/B_1}$$

The time $\lambda_2$ can now be determined from equation 51:

$$\lambda_2 = \frac{\ln \left\{ \frac{A_4}{-D_1\lambda_1} \right\}}{\frac{C_2e^{\lambda_1} + D_2}{-D_1}}$$
APPENDIX 5

PROGRAM FOR LOAD-ON SWITCHING SURFACE
THIS PROGRAM FOR LOAD-ON SWITCHING SURFACE

5 WRITE(3,102)

7 READ(1,100)Xw,XDP,XO,TDCP,KF,ES

6 READ(1,100)XL,XLDP,TLDCP,XME,ELS

XL = 523*RF

XL2 = XL/RL

XLDP = XLDP/RLB

XLL = XL3

XL3 = (1. + XLDP*XL2)/(1. + XLDP*XL2)

XL4 = (SQR2(XL2**2 + 1.))/(XLDP*XL2 + 1.)

CI = XLI/TLDCP

DI = XLI/TLDCP

5 READ(1,100)XM,XN

XM = XM/XMB

XN = XN/XMB

4 READ(1,100)ZL,PF,ZLI,PFI,ZZ

LZ = ZZ + 1.

THETA = ARCCOS(PF)

XE = ZL*SIN(THETA)

KE = ZL*PF

XDT = XD + XE

XDPT = XDP + XE

XQT = XQ + XE

R2 = XE**2

XL1 = (R2 + XDT*XU1)/(R2 + XDPT*XQT)

XL2 = (SQR2((XLI**2 + R2) * (XE**2 + R2)))/(XDPT*XQT + R2)

AI = XL2/TDCP

BI = XL1/TDCP.

ZLI = -1. FOR NO LOAD INITIALLY

IF(ZLI = 0.,1,1,2

2 BETA = ARCCOS(PFI)

AI = 1./ZLI

RI = ZLI*PFI

XI = ZLI*SIN(BETA)

XD1I = XD + XI

XDPTI = XDP + XI.
Aw I I I X>, + X I
321 =3
1 * * 2
X M [ = { 32I+XDTi*XQTI / ( R2I+XDPTI *XQTI ) ) ) / ( XDPTI*XQTI+R2I)
XK2I_=( Sw«Tl ( Xi,I **2 + K2I )*(Xl **2 + K2I )  )  )/( XDPTI*XQTI+R2I)
4 = XK 217 TO - j P *
b  = X K 1 1 / T D o P
~Et'pT=

T I V +
(XDPT+X UV a I
* S L M
B E T M  +  X^VxDP* U I **2")')/ SGRTTiY+2•*XQ*A I*S'I

b i  J
T
j

A6 = I  ./TDCP
A = I  ./TDCP
A1 =XK2*EDPI+ (d/A+ES)*D3+Al*ES/b1-(XM-ELS)*D3*C1/B1
A2=XK2*EDPI+(b/A+ES)*D3+A1*ES/b1-(XM-ELS)*D3*C1/B1
B2=D3*C1*(XM+X i )/B1
S2=D3*(C1/D1*(XM-ELS)-(b/A+ES))
D2=-b1*82/01
A4=-(B1/(D1-B1)+D3*ES+D3*C1*(XN+ELS)/D1)
SAVE=(C2+D2/4/(A2+B2)/(A3-A4))**((D1/B1)
TIM1=0.1
DT=0.1
22 DC 10 I=1,10
w=EXP(-TIM1)
F=(C2+m**D1+D2)/A4-((A2*w**B1+b2)/(A3-A4))**((D1/B1)
Z1=ABS(F+SAVE)
Z2=ABS(F-SAVE)
IF(Z2-Z2)*8,6,9
9 SAVE=F
TIM1=TIM1+DT
10 CONTINUE
WRITE(3,104)ZL,PF,ZLI,PF1
GO TO 4
8 IF(DT=0.002)20,20,21
21 TIM1=TIM1-DT
DT=DT/10.
118
Tfin = Tim - DT

C THIS IS VALUE OF LAMDA 1 FOR INITIATION OF NEWTONS METHOD

DO 13 I = 1, 10

F = (C2 * W**D1 + D2) / A4 - ((A2 * W**B1 + e2) / (A3 - A4)) ** (C1 / B1)
FP = C2 * D1 / A4 * W**D1 - A2 * D1 / (A3 - A4) * (W**B1 + B2) / (A3 - A4)**(D1 + 1.1)

wl = w - F / FP

Tfin = -ALOG(w)

TIM2 = -ALOG(w)

Y2 = ABS(TIM2 - TIM1) / ABS(TIM2)

IF (Y2 < 0.001) 11, 11, 12

11 CONTINUE

12 W = W * D1

13 CONTINUE

14 ZLI = 0.

15 WRITE (3, 104), ZLI, PF, ZLI, PFI

A6 = EXP(-B1 * TIM1)
A7 = EXP(-D1 * TIM1)


TIM2 = (ALOG(A4/(C2+A7+D2))) / (-D1)

WRITE (3, 103), TIM1, TIM2, F, Y2, 1

XII = X1 / ZL

WRITE (3, 106)

WRITE (3, 105) X1, XII

GO TO 10 (4, 5, 6, 7), 12

100 FORMAT (7F10.4)
102 FORMAT (11H1)

103 FORMAT (/9X7HLAMDA, 13X7HLAMDA, 24X1HF9X2HY27X1H1/L6X, 41X1F9.5, 2X112 1/.)
APPENDI

PROGRAM FOR LOAD REMOVA
104 FORMAT(12X2HZL5X2HPF6X3H2L15X3HPFI/9X1F6.2,3X1F4,2,3X1F6.2,3X1F4.
12)
105 FORMAT(15X1F5.3,3X1F5.3///)
106 FORMAT(17X2MX16X2H1L)
END
APPENDIX 6

PROGRAM FOR LOAD REMOVAL SWITCHING SURFACE
C THIS PROGRAM FOR LOAD REMOVAL SWITCHING SURFACE

WRITE(3,102)
7 REAL(1,100)XG,XDP,XD,TDUP,RF,CS
6 REAL(1,100)XLG,XLDP,TLDCP,AMB,ELS
ESi=ES
FS=CS
RLB=.523*RF
XLG=XLG/RLB
XLDP=XLDP/RLB
XLDP=XLD
XLDP=XLD
XLDP=XLD
XK3=(1.+XLD*XLQ)/(1.+XLDP*XLQ)
XK4=(SQRTI(XLQ**2+1.))/(XLDP*XLQ+1.)
CL=XK4/TLDCP
DL=XK3/TLDCP
A1=1./TDUP
B1=4.*G/TCOP
ES=-3.34
5 READ(1,100)XM,XN
XM=XM/AMB
XN=XN/AMB
4 READ(1,100)ZL,PF,ZLI,PF,ZZ
ZLI=ZL
PFI=PF
LZ=ZZ+.1
2 BETA=ARCOS(PFI)
AI=1./ZLI
RI=ZLI*PF
XI=ZLI*SIN(BETA)
XCTI=AX+XI
XOPTI=AXD+XI
XQTI=AY+XI
R2I=K1**2
XR2I=(R2I+XCTI*XQTI)/(R2I+XOPTI*XQTI)
XR2I=(SQRTI(XQTI**2+R2I)*(XI**2+R2I))/(XOPTI*XQTI+R2I)
A=XK2I/TDUP
B=XKII/TDUP
\[
E_{\text{OP1}} = \frac{(1 + (\text{ALP} + \text{XQ}) \times \text{AI} \times \sin(\text{BETA}) + \text{XQ} \times \text{XDP} \times (\text{AI} \times 2))}{\sqrt{\text{SQR}((1 + 2 \times \text{XQ} \times \text{AI} \times \sin(\text{BETA}) + (\text{XQ} \times \text{XDP}) \times (\text{AI} \times 2))}}
\]

3

\[
D_3 = \frac{\text{AI}}{\text{C1} - \text{B1}}
\]

- \[
A_2 = \frac{E_{\text{OP1}} + (\text{B/A} + \text{FS}) \times D_3 + \text{AI} \times \text{ES} / \text{B1} + (\text{XN} + \text{ELS}) \times \text{C1} \times D_3 / \text{B1}
\]

- \[
B_2 = -\frac{\text{C1} \times D_3 \times (\text{XM} + \text{IN}) / \text{B1}}{\text{C1} \times D_3 \times (\text{XM} + \text{ELS}) / \text{D1}}
\]

- \[
C_2 = -\frac{(\text{B/A} + \text{FS}) \times \text{C3} - \text{C1} \times D_3 \times (\text{XM} + \text{ELS}) / \text{D1}}{\text{B1} \times \text{B2} / \text{D1}}
\]

- \[
A_3 = -\text{C1} \times \text{ES} / \text{B1} - (\text{XM} - \text{ELS}) \times \text{C1} \times \text{B1} / \text{D1}
\]

- \[
A_4 = -\frac{(1 + \text{ES}) \times \text{C1} \times (\text{XM} - \text{ELS}) / \text{D1} \times \text{D3}}{\text{C1} \times \text{ES} / \text{B1}
\]

- \[
\text{SAVE} = (\text{C2} \times \text{C1} \times (\text{A4} - ((\text{A2} + \text{B2}) / (\text{A3} - \text{A4}) \times (\text{D1} / \text{B1})
\]

10 CONTINUE

- \[
\text{IF}(\text{FT} - 0.002) = 20, 20, 21
\]

21 \[
\text{TIM1} = \text{TIM1} - \text{DT}
\]

- \[
\text{DT} = \text{LT} / 10.
\]

- \[
\text{TIM1} = \text{TIM1} + \text{DT}
\]

22 \[
\text{IF}(\text{FT} - 0.002) = 20, 20, 21
\]

C

- THIS IS VALUE OF LAMDA 1 FOR INITIATION OF NEWTONS METHOD

DO 13 1 = 1, 10

- \[
F = (\text{C2} \times w / \text{D1} + \text{D2}) / \text{A4} - (((\text{A2} \times w / \text{B1} + \text{B2}) / (\text{A3} - \text{A4}))) \times (\text{D1} / \text{B1})
\]

- \[
\text{FP} = \text{C2} \times \text{C1} / (\text{A4} \times w / \text{D1} - 1.) - 2 \times \text{D1} / (\text{A3} - \text{A4}) 
\]

- \[
\text{W1} = \text{w} / \text{FP}
\]

- \[
\text{TIM1} = -\text{ALOG}(\text{w})
\]
TIM2=-ALOG(W1)
Y2=ABS(TIM2-TIM1)/ABS(TIM2)
IF(Y2<0.001)11,11,12
12 W=W1
13 CONTINUE
11 IF(ZLI<0.)14,14,15
14 ZLI=0.

15 WRITE(3,104)ZL,PF,ZLI,PFI
A6=EXP(-B1*TIM1)
A7=EXP(-D1*TIM1)
X1=A6*(EDP/E03*(B/A+FS)+A1*ES/B1+(XN+ELS)*C1*03/B1)-A7*(D3*(B/A+FS)
TIM2=(ALOG(A4/((C2*A7+D2))/(-D1))))
X2=A7*(B/A+FS)-C1*(1.-A7)*(XN+ELS)/D1
IF(X2<0.)30,31,31
30 TIMC=(ALOG(C1*(XN+ELS)/D1/(B/A+ESI+C1*(XN+ELS)/D1)))/(-D1)
A6=EXP(-B1*TIM0)
A7=EXP(-D1*TIM0)
X0=A6*(EDP/E03*(B/A+FS)+A1*ES/D1+(XN+ELS)*C1*03/B1)-A7*(D3*(B/A+FS)
TIM2=(ALOG((1.-D1)*(1.+ESI)/C1/(XM-ELS)))/(-D1)
A6=EXP(-B1*TIM2)
A7=EXP(-D1*TIM2)
TIMI=(ALOG(A4))/(-B1)
A6=EXP(-B1*TIM1)
X1=X0*A6+ES*A1*(A6-1.)/B1
X2=0.
WRITE(3,108)TIM0,TIM1,TIM2
WRITE(3,107)X0,X1,X2
GO TO (4,5,6,7),LZ
100 FORMAT(7F10.4)
102 FORMAT(1H1)
103 FORMAT(/9X7HLAMDA 13X7HLAMDA 24X1HF9X2HY27X1HI76X,4(1X1F9.5),2X1I2)
104 FORMAT(/12X2HZL5X2HPF6X3HLI6X3HPF1/9X1F6.2,3X1F4.2,3X1F6.2,3X1F4.2)
105 FORMAT(15X1F5.3,3X1F5.3///)
109 FORMAT(17X2HX10X2HA2)
107 FORMAT(17X2HX08X2HX18X2HX2/12X,3(3X1F7.3))
108 FORMAT(16X4HTIM06X4HTIM16X4HTIM2/11X,3(3X1F7.4))
END
APPENDIX 7

PROGRAM FOR BUILD-UP SWITCHING POINT
C

THIS PROGRAM FOR BUILD UP SWITCHING POINT

WRITE(3,102)

READ(1,100)XC,XDP,XD,TDOP,RF,ES

READ(1,100)XLQ,XLDP,TLDP,XMB,ELS

RLB=523*RF

ALU=XLQ/RLB

XLDP=XLDP/RLB

XLQ=XLQ

XK3=(1.+XLQ*XLQ)/(1.+XLQ*XLQ)

XK4=(SQRT(XLQ**2+1.))/(XLQ*XLQ+1.)

C1=XK4/TLDP

D1=XK3/TLDP

READ(1,100)XM,XN

XM=XM/XMB

XA=XN/XMB

A1=I./TDOP

A1=A1

A2=ES/2.*XM*C1/(D1-A1)

B2=C1*(XM+XA)/(D1-A1)

C2=A1*C1*XM/(D1-A1)

D2=-A1*B2/D1

A3=1.+ES/2.*XM*C1/D1

A4=-((1.+ES*XM*C1/D1)*A1/(D1-A1))**D2

SAVE=(C2+D2)/A4-((A2+B2)/(A3-A4))**D1/D1

TIM1=0.1

DT=0.1

22 GO TO 10 I=1,10

W=EXP(-TIM1)

F=(C2*W**D1+D2)/A4-((A2*W**B1+B2)/(A3-A4))**D1/D1

Z1=ABS(F-SAVE)

Z2=ABS(F-SAVE)

IF(Z1-Z2)<18.9

SAVE=F

TIM1=TIM1+DT

CONTINUE

GO TO 4
8 IF(DT-0.002) .GE. 20,20,21
21 TIM1=TIM1+DT
   DT=DT/10.
   TIM1=TIM1+DT
   GO TO 22
20 TIM1=TIM1-DT
C THIS IS VALUE OF LAMDA 1 FOR INITIATION OF NEWTONS METHOD
DO 13 I=1,10
   F=(C2*w**D1+D2)/A4-((A2*w**B1+B2)/(A3-A4))**(D1/B1)
   FP=C2*D1/A4*w**(D1-1.0)-A2*D1/(A3-A4)*w**(B1-1.0)*((A2*w**B1+B2)/(
   1A3-A4))**(D1/B1-1.0)
   W1=w-F/FP
   TIM1=-ALOG(W)
   TIM2=-ALOG(W1)
   Y2=ABS(TIM2-TIM1)/ABS(TIM2)
   IF(Y2-0.001),11,12
12 W=W1
13 CONTINUE
   A6=EXP(-B1*TIM1)
   A7=EXP(-D1*TIM1)
   X1=(ES/2.)*((A6-1.0)+XM*(C1/D1-C1*A6/(D1-A1)+A1*C1*A7/D1/(D1-A1))
   TIM2=(ALOG(A4/(C2*A7+D2)))/(-D1)
   WRITE(3,103)TIM1,TIM2,F,Y2,I
   WRITE(3,106)
4 WRITE(3,105)X1
   GO TO 7
100 FORMAT(7F10.4)
102 FORMAT(1H1)
103 FORMAT(/9X7HLAMDA 13X7HLAMDA 24X1HF9X2HY27X1H1/6X,4(1X1F9.5),2X1I2
1/)
105 FORMAT(15X1F5.3//://)
106 FORMAT(17X2HX1)
END
APPENDIX 8

PROGRAM FOR SHORT-CIRCUIT SWITCHING POINT
C THIS PROGRAM FOR SHORT CIRCUIT SWITCHING POINT

WRITE(3,102)
7 READ(1,100)XQ,XDP,XD,TLCP,RF,ES
6 READ(1,100)XLQ,TLDOP,XMB,ELS,ETL
RLB=.523*RF
XLQ=XLQ/RLB
XLD=XLQ
XLG=XLC/RLB
XLC—XLD
XK3=(1.+XLC*XLQ)/(1.+XLCP*XLQ)
XK4=(SQR(XLQ**2+1.))/(XLDOP*XLQ+1.)
C1=XK4/TLDOP
D1=XK3/TLDOP
5 READ(1,100)XM,XN
XM=XM/XMB
XM=XM/XMB
A1=1./TLDOP
B1=A1
B2=C1*(XM+XN)/(C1-A1)
D2=-A1*B2/C1
A3=1.+ES/2.*XN*C1/D1
A4=-1.+ES*XN*C1/C1)**A1/(C1-A1)
SAVE=(C2+D2)*A4-((A2+B2)/(A3-A4)**D1/B1)
TIM1=0.1
DT=0.1
.22 DC LC_10_1=1,10
W=EXP(-TIM1)
F=(C2+D2)/A4-(A2+B2)/(A3-A4)**D1/B1)
Z1=ABS(F+SAVE)
Z2=ABS(F-SAVE)
IF(Z1—Z2)<8,9
9 SAVE=F
TIM1=TIM1+CT
10 CONTINUE
GC TO 4
8 IF(LT=Q.002)20,20,21
21 TIM1=TIM1-CT
   DT=DT/10.
   TIM1=TIM1+DT
   GC TO 22
20 TIM1=TIM1-CT
C THIS IS VALUE OF LAMDA 1 FOR INITIATION OF NEWTONS METHOD
CC 13 I=1,1C
   F=(C2*w**D1*D2)/A4-((A2*w**B1+B2)/(A3-A4))**(D1/B1)
   FP=(C2*D1/A4*w**(D1-1.))-A2*D1/(A3-A4)*(w**(B1-1.))*((A2*w**B1+B2)/
      (A3-A4))**(D1/B1-1.)
   W=(w-F/FP)
   TIM1=-ALOG(W)
   TIM2=-ALOG(W1)
   Y2=ABS(TIM2-TIM1)/ABS(TIM2)
   IF(Y2<0.001)11,11,12
12 W=W1
13 CONTINUE
11 A6=EXP(-B1*TIM1)
   A7=EXP(-D1*TIM1)
   T1=A1*(A6-A7)/(D1-A1)
   TIM2=(-ALOG(A4/(C2*A7+D2)))/(D1)
   WRITE(3,103)TIM1,TIM2,F,Y2,I
   WRITE(3,106)
4 WRITE(3,105)X1
   GC TO 7
100 FCRMAT(7F10.4)
102 FCRMAT(1H1)
103 FCRMAT(/9X7HLMADA 13X7HLMADA 24X1HF9X2HYV27X1HT/6X,4(1X1F9.5),2X112
1/
105 FCRMAT(15X1F5.3////////)
106 FCRMAT(17X2F+X1)
END
APPENDIX 9

CALCULATION FOR RESISTANCES OF FIGURE 38

Refer to Figure 43.

\[ I_4 = I_2 + I_3 \]

\[ I_1 = \frac{(E_0 - E_F)}{R_1} = \frac{(E_F - E_1)}{R_2} \]

\[ E_0 - E_F = \frac{R_1}{R_2} E_F - \frac{R_1}{R_2} E_1 \]

\[ E_0 = E_F R_1 \left( \frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{R_1}{R_2} E_1 \]

\[ \frac{(E_2 - E_F)}{R_3} + \frac{(E_3 - E_F)}{R_4} = \frac{E_F}{R_5} \]

\[ \frac{R_5}{R_3} E_2 - \frac{R_5}{R_3} E_F + \frac{R_5}{R_4} E_3 - \frac{R_5}{R_4} E_F = E_F \]

\[ E_F R_5 \left( \frac{1}{R_5} + \frac{1}{R_4} + \frac{1}{R_3} \right) = R_5 \left( \frac{E_2}{R_3} + \frac{E_3}{R_4} \right) \]

\[ E_F = \frac{\frac{E_2}{R_3} + \frac{E_3}{R_4}}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}} \]

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Figure 43
Summer for Figure 38
\[ E_0 = \left( \frac{E_2 + E_3}{R_3 R_4} \right) \left( \frac{R_3 R_4 R_5}{R_4 R_5 + R_3 R_5 + R_3 R_4} \right) \left( \frac{R_1 + R_2}{R_2} \right) - \frac{R_1}{R_2} E_1 \]

\[ E_0 = \frac{E_2 R_4 R_5 (R_1 + R_2) + E_3 R_3 R_5 (R_1 + R_2)}{(R_4 R_5 + R_3 R_5 + R_3 R_4) R_2} - \frac{R_1}{R_2} E_1 \]

\[ E_t = 0.0255 x^2 - 0.0693x + .997 \]

\[ \frac{(3E_g)}{3} = 0.0255 \left( \frac{x^2}{2} \right)^2 - 0.0693x + .997 \]

\[ (3E_g) = 0.153 \left( \frac{x^2}{2} \right) - 0.2088x + 2.991 \]

\[ E_0 = 0.153 E_2 - 0.2088E_1 + E_3 \]

Let \( \frac{R_1}{R_2} = 0.2088 \)

Let \( R_1 = 1K \)

Then \( R_2 = 4790 \)

Then \( \frac{(R_1 + R_2)}{R_2} = 1.21 \)

Let \( R_5 = 100K \)
Then \[
\frac{10^5 R_4 \times 1.21}{10^5 R_4 + 10^5 R_3 + R_3 R_4} = 0.153
\]

and \[
\frac{10^5 R_3 (1.21)}{10^5 R_4 + 10^5 R_3 - R_3 R_4} = 1.0
\]

Let \( R_3 = nR_4 \)

\[
\frac{10^5 nR_4 \times (1.21)}{10^5 R_4 + 10^5 nR_4 + nR_4^2} = 1.
\]

\( nR_4 = 10^5 (.21n = 1.00) \)

\[
10^5 R_4 (1.21) = (.153)10^5 R_4 + (.153) (10^5 nR_4) + (.153) nR_4^2
\]

\( n = 6.54 \)

\[
R_4 = 10^5 \left( .21 \times \frac{6.54}{6.54} - 1.00 \right) = 4900
\]

\( R_3 = 32000 \)
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