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PERFORMANCE CHARACTERISTICS OF COMMUNICATION SYSTEMS EMPLOYING NEAR-SYNCHRONOUS (IDCSP) SATELLITES

DISSEPTION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By


****

The Ohio State University
1968

Approved by

[Signature]

Adviser
Department of Electrical Engineering
ACKNOWLEDGMENTS

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...and to my friend Professor Buck Walter I am most grateful.
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Signal-to-Noise Ratio Improvements in an Adaptively Phased Array. First IEEE Annual Communications Convention, 7-9 June, 1965, Boulder, Colorado. (Co-Author)


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CHAPTER 1
INTRODUCTION

For proper utilization and evaluation of communication satellites it is desirable to know their performance characteristics. These, together with the characteristics of the user station determine the total system performance.

The principal parameters of the satellite which characterize its performance are the maximum effective radiated power (ERP), the power transfer and the bandwidth. The maximum ERP can be determined by measuring the received coherent signal strengths while a CW tone is being transmitted to the satellite with sufficient strength so that its output amplifier is saturated. The power transfer and the bandwidth are determined by measuring the ERP as the input power and frequency are varied, respectively.

In order to measure the ERP, the satellite signal must be received in a well-calibrated receiving system. Because the signal is weak, a fairly large antenna must be employed. From the measured signal, the known antenna gain, and a knowledge of the propagation path the ERP can be calculated. Most of the uncertainties of this type of calculation can be traced back to uncertainties of the antenna gain since gain of large antennas is difficult
to measure. Thus, the approach to the task was to instrument several experiments for accurately calibrating a 30-foot paraboloid and then using this well-calibrated antenna for determining the ERP, power transfer, and bandwidth of IDCSP satellites.

A commonly accepted procedure for determining the gain of a large antenna is the gain-comparison method, whereby the received signal strength is measured with an antenna whose absolute gain is accurately known and with the large antenna under test, both being connected to identical receiving systems. The difference in the signal strength is then the difference in gain between the two antennas. The antenna used for comparison is usually a standard-gain horn, since its gain can be rigorously calculated and precisely measured. Unfortunately, the commercially available standard-gain horns have some 40 dB less gain than the 30-foot paraboloid: thus they are not suitable for direct comparison. To solve this difficulty and yet retain the accuracy afforded by the use of standard gain horns, an identical pair of these was calibrated by the method of absolute gain of identical antennas; then an identical pair of 3-foot dishes were calibrated by the same technique; the 3-foot dishes were then calibrated against the standard gain horns. Finally, one of the 3-foot dishes was mounted on the 30-foot paraboloid and the signal strength received by the precisely calibrated 3-foot dish and by the large antenna was accurately measured. From this measurement
the gain of the large antenna was obtained. An ideally suited coherent signal source for this measurement was any one of the IDCSP satellites.

Another technique for antenna gain calibration is to measure the received noise power from radio stars. Basically, this technique is also a comparison method; however, the standard used is, in effect, the antenna which was used to measure the flux of the radio star. Thus one can calibrate the system against any system used for radio stars; the most accurate data available was used. Radiation from the stars is steady, their angular sizes are quite small, and their fluxes are published. Their small but finite angular extent, the source brightness distribution and the partial polarization of the flux require careful evaluation to determine their effects on experimental data. The particularly appealing feature of the radio-star technique is that it uses directly the most accurate standard values that are presently available.

The radio-star measurement is, of course, quite independent of the coherent measurement approach. The intent was specifically to use several independent techniques and to show agreement in order to establish confidence in the precision of the results.

As a byproduct of these measurements it was possible to obtain also a knowledge of the normalized antenna pattern. This was simply measured by recording the available power from the antenna as a source swept through the pattern. A well-suited source
for the purpose is one of the IDCSP satellites, their maximum coherent power output as observed in very-narrow band receiver allowed accurate pattern measurements beyond the -30 dB level. From such patterns the antenna directivity was calculated. While it was not possible to obtain enough pattern information for a high-precision gain computation, the agreement was a further contribution to the confidence level of the system calibration.

Another useful by-product was a measurement of the antenna noise temperature which influences the sensitivity of the communication system. Antenna noise temperature varies with changes in atmospheric conditions, with time, and with antenna positions. Apart from the effects of strong, discrete noise sources such as the sun, moon, and radio stars, the changes in antenna noise temperature cannot be predicted, but must be measured. The measurement was carried out with the same radiometer used for the calibration with the radio star.

After the antenna parameters have been determined precisely, the ERP's of IDCSP satellites were measured. The ERP of the satellite (and its variation as a function of time, of input power, and of frequency) characterize the communication performance of the link.

The maximum ERP was measured with a coherent signal. In this mode a sufficiently strong CW tone is transmitted to the
satellite so that its output amplifier is saturated, and the return signal strength is carefully measured. The satellites were at high elevation angles to minimize atmospheric absorption effects, and they were not in the vicinity of strong sources (radio-stars) to avoid sensitivity degradation. The power transfer and bandwidth was then measured by measuring the variations in the ERP as the input power to the satellite and the frequency were varied, respectively. In order to be able to evaluate accurately the ERP data, the polarization patterns of the received signals need to be known. These were measured and the measurements were referred to the satellites. Additionally, it was necessary to calculate the pattern angle and the apparent rotation of the tilt angle of the polarization pattern caused by changes in path geometry.

The power spectral density curves of the amplitude scintillations of the received signal indicate how the power is distributed as a function of frequency. Amplitude scintillations is a very small problem on a communication link using the IDCSP satellites, but it nevertheless exists to some degree. The PSD curves were computed for the satellites and the effect of scintillations were evaluated.

The IDCSP satellites have a beacon on-board for identification and radio location purposes. The ERP on the beacon frequency is some 10 dB below the power output on the communication channel.
and varies as much as 6 dB as a function of satellite input power.
This ERP and its variations as a function of input power to the
communication channel were also measured.

Access to the satellites depends upon the availability of
accurate look angles data for them. Look angles, range, and
range rate data were normally provided by the U.S. Air Force;
additionally a simple technique for calculating approximate satellite
positions was developed and used successfully. It is also necessary
to be able to determine the signal strengths and signal-to-noise ratios
on a typical communication link.

In high precision measurements an error analysis is in-
dispensable. To find the total error the following guiding principles
were used: a) it was attempted to include all errors; b) the errors
included were estimated at their approximately correct value, over-
estimates and underestimates were carefully avoided; c) the random
and systematic errors were separated; d) when high levels of confi-
dence were claimed, large confidence intervals (on the order of 3σ)
were used; and e) the most pessimistic combination of errors were
assumed.

To recapitulate, for the proper design of multiple-access com-
munication systems using the IDCSP satellites it is necessary to
know the performance characteristics of the user stations and of
the satellite. In this study, experiments have been carried out for
accurately calibrating an antenna by precisely measuring its gain, power pattern, and noise temperature and then using this accurately calibrated antenna for measuring the maximum ERP, power transfer and band-shape of IDCSP satellites.

In order to have good confidence in the accuracy of the results, independent measurement approaches were used. The antenna gain was measured by direct comparison with a gain-standard antenna using satellites as sources; it was also measured by comparison with well-calibrated astronomy antennas through use of a radio star source; and it was checked by use of experimentally generated antenna patterns. The maximum ERP was measured as a saturating CW tone was transmitted to the satellite. The power transfer and the band-shape of the satellite were obtained by measuring the variations in ERP as a function of input power and frequency, respectively. Additionally, the beacon ERP and its variation as a function of input power were measured. A careful analysis was made of the effects of errors on the precision of the measurements.
A. The Method of Absolute Gain Calibration of Antennas

The usual figure of merit of an antenna is its gain. Antenna gain is defined as the ratio of the maximum radiation intensity to the maximum radiation intensity of some reference antenna with the same power input. [1] When the reference is an isotropic antenna then the gain figure is an absolute one and can be stated as

\[ G_{\text{abs}} = \frac{P_1}{P_i}, \]

where

- \( G_{\text{abs}} \) = absolute gain of antenna under test,
- \( P_1 \) = power received with antenna under test, and
- \( P_i \) = power received with an isotropic antenna.

In practical situations, no isotropic antenna is available for comparing the received powers, and the absolute gain method refers to power measurements involving two identical antennas. This method was first published by Cutler, et al. [2]. Two identical antennas are arranged in free space as shown in Fig. 1.
Applying the Friis transmission formula,[3] the received power, \( P_r \), is

\[
(2) \quad P_r = P_t \frac{A_{er} A_{et}}{\lambda^2 R^2}
\]

where

\( P_t \) = transmitted power,

\( A_{er} \) = maximum effective aperture of receiving antenna,

\( A_{et} \) = maximum effective aperture of transmitting antenna,

\( \lambda \) = wavelength, and

\( R \) = range between antennas, as shown in Fig. 1.

For the purposes of gain measurements \( P_t \) is the power flowing into the antenna terminals measured at the surface defined by the input flange of the transmitting horn when the impedance of the generator delivering the power is matched to the transmitting...
horn, i. e., the impedance of the generator is the complex conjugate of the impedance of the transmitting horn. Likewise, $P_r$ is the power flowing from the antenna terminals measured at the surface defined by the output flange of the receiving horn when the impedance of the detector is matched to that of the receiving horn.

The relationship between gain and antenna aperture is

\begin{equation}
A_{er} = \frac{G_{or} \lambda^2}{4\pi},
\end{equation}

where $G_{or}$ is the absolute gain of the receiver antenna. Substituting Eq. (3) into Eq. (2) one obtains

\begin{equation}
P_r = P_t \frac{G_{or} \lambda^2 G_{ot}^2}{(4\pi)^2 \lambda^2 R^2} = P_t G_{or} G_{ot} \left( \frac{\lambda}{4\pi R} \right)^2.
\end{equation}

For identical antennas, $A_{er} = A_{et}$ and $G_{or} = G_{ot}$; thus the expression for the absolute gain of one of the antennas is

\begin{equation}
G_o = \frac{4\pi R}{\lambda} \sqrt{\frac{P_r}{P_t}}.
\end{equation}

It is difficult to make two identical antennas. In practice $G_{or} \neq G_{ot}$; the difference between them is usually a few per cent. Thus in a practical absolute gain calibration experiment the measured quantity is $G_o = (G_{or} G_{ot})^{\frac{1}{2}}$. To resolve this difficulty an additional experiment is usually performed by comparing the two antennas to a third antenna whose gain need not be known.
Transmitting with the third antenna and receiving first with the original receiving antenna, one measures a received power $P_1$. Next the original transmitting antenna is used for reception and the received power, $P_2$, is noted. The ratio of the two powers is the same as the ratio of the gains: $P_1/P_2 = G_{or}/G_{ot}$, and it follows that the individual gains are

$$G_{or} = \frac{P_1}{P_2} G_o \quad \text{and} \quad G_{ot} = \frac{P_2}{P_1} G_o.$$  

B. The Absolute Gain of a Pyramidal Horn

1. Calculated gain

The theoretical gain of an electromagnetic horn has been calculated by Schelkunoff,[4] According to Silver[5] comparisons have been made between gain standards and pyramidal horns and the calculated values have agreed with the comparison values within five per cent. Silver recommended the use of horns as highly accurate secondary gain standards. Schelkunoff derived expressions for the gain of an electromagnetic horn which is flared out in either the $E$ or $H$ plane, and for the case when the horn flares out in both $E$ and $H$ planes (i.e., for a pyramidal horn). For this latter case the expression for gain is
\[ G_0 = \frac{8\pi l_al_b}{ab} \left[ C^2 \left( \frac{b}{\sqrt{2\lambda l_b}} \right) + S^2 \left( \frac{b}{\sqrt{2\lambda l_b}} \right) \right] \]

\[ \cdot \left\{ [C(u) - C(v)]^2 + [S(u) - S(v)]^2 \right\} \]

where

\[ C = \text{Fresnel cosine integral} \ C(x) = \int_{0}^{x} \cos \left( \frac{\pi z^2}{2} \right) \, dz; \]

\[ S = \text{Fresnel sine integral} \ S(x) = \int_{0}^{x} \sin \left( \frac{\pi z^2}{2} \right) \, dz; \]

\[ u = \frac{1}{\sqrt{2}} \left[ \sqrt{\frac{\lambda l_a}{a}} + \frac{a}{\sqrt{\lambda l_a}} \right]; \]

\[ v = \frac{1}{\sqrt{2}} \left[ \sqrt{\frac{\lambda l_a}{a}} - \frac{a}{\sqrt{\lambda l_a}} \right]; \] and

\[ a, b, l_a, \text{ and } l_b \text{ are dimensions of the pyramidal horn as shown in Fig. 2.} \]

Two pyramidal horns (Wave line Model 599) have been used to obtain numerical results. The physical dimensions of these were

\[ l_a = 2.641''; \]
\[ l_b = 2.981''; \]
\[ a = 3.531''; \text{ and} \]
\[ b = 2.641''. \]
Fig. 2--The pyramidal horn and its various dimensions.

Using these values in Eq. (7) the gain was calculated in 200 MHz increments from 7.0 to 8.0 GHz. The results of these computations are given in Table I.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Numeric</th>
<th>Gain</th>
<th>dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0 GHz</td>
<td>22.336</td>
<td>13.58</td>
<td></td>
</tr>
<tr>
<td>7.2</td>
<td>23.881</td>
<td>13.78</td>
<td></td>
</tr>
<tr>
<td>7.4</td>
<td>25.222</td>
<td>14.02</td>
<td></td>
</tr>
<tr>
<td>7.6</td>
<td>26.897</td>
<td>14.28</td>
<td></td>
</tr>
<tr>
<td>7.8</td>
<td>28.710</td>
<td>14.58</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td>30.717</td>
<td>14.88</td>
<td></td>
</tr>
</tbody>
</table>
2. Measured gain
   
a. Manufacturer's values

The manufacturer (Waveline, Inc.) supplies a calibration curve with units sold; the curve is shown in Fig. 3. The accuracy of the gain variation curve is $\pm 0.3$ dB over its entire range, and $\pm 0.2$ dB at the frequency of calibration, 8.80 GHz. [6] The circles in Fig. 3 represent the theoretical calculations shown in Table I; the curve represents the manufacturer's calibration by the absolute gain method described in Section A. The variations in gain from one horn to the next was stated to be much less than $\pm 0.1$ dB, and this good consistency was attributed to the very close mechanical tolerances maintained while manufacturing the horns.

![Graph showing gain variation vs. frequency](image.png)

Fig. 3—Gain variation vs. frequency (calibration frequency 8.80 GHz) Waveline Model 599 horn.
b. Experimental results

i. equipment setup and method of measurements

The experiments were carried out in an anechoic chamber. The horns were mounted on a platform 2.5 meters high. The platform and the concrete floor were covered with absorbers; additionally there was a hairflex baffle around each horn. The side walls, covered with the absorber, were 1.6 meters from the centerline, and the height of the ceiling was about 7 meters. No cross-polarized component could be measured within the range of the power meter (25 dB). The antennas were individually leveled and adjusted about three axes. The initial alignment was carried out optically, this optical alignment was found to correspond to the position of maximum received power. The distance between the apertures of the horn was 3 meters, measured to an accuracy of 5 millimeters. Before and after each group of about 20 measurements the transmitting frequency, and the VSWR's of the transmitter horn, receiver horn, and detector were measured and the detector was calibrated against the precision attenuator. The measurements themselves consisted of readings of received power levels; the antennas were alternately adjusted to vertical and horizontal polarization. After the initial alignment of the antennas was verified
experimentally to correspond to peak received power, the transmitter antenna was fixed except for rotational motion. This rotational motion, controlling the polarization of the transmitted wave, could readily be adjusted and accurately repeated by using level indicators mounted on the flat parts of the horn. The receiver antenna was adjusted for each measurement about two axes (polarization and azimuth); the third axis (elevation) was fixed after the optical alignment was proved experimentally. After the receiving antenna was adjusted for peak deflection on the power meter, the meter monitoring the outgoing power and the received power meter were adjusted for null and zeroed on the appropriate scales. The outgoing power was ascertained to be of a set value, and then the received power was read as that corresponding to a certain meter deflection. This meter deflection was reproduced by attaching the detector to the flange of the coupler nearest to the horn and noting the setting of the precision attenuator. This setting corresponded to the insertion loss of the two horns, the E-H tuner, and the intervening space. The block diagram of the equipment is shown in Fig. 4.
Fig. 4--Block diagram of equipment for absolute gain measurements of pyramidal horns
(a) transmitter, (b) receiver.
One hundred measurements were taken in each polarization at 7.3 GHz, and 93 measurements were taken in each polarization at 7.95 GHz. The distributions of all the measured gains in the form of histograms are shown in Figs. 5 and 6. The arithmetic means and the rms values, as well as the standard deviations, are given directly on the histograms. If these distributions were to closely resemble some known statistical distribution, one could readily set a given per cent confidence interval for the mean value. Since these distributions do not clearly resemble any commonly known distribution, no such fine statistical points should be directly applied to them. However, it is possible to calculate a definite lower and a probable upper bound on the confidence level for the measured data to be within a given range from the mean. The procedure is carried out in Appendix I. On basis of the considerations presented in Appendix I one can tabulate the values shown in Table II for the gain of the pyramidal horn Waveline Model 599, at the frequencies of 7.3 and 7.95 GHz. The confidence level for the tabulated values is 99.0 per cent.
TABLE II
Measured Gains of Waveline Model 599 Pyramidal Horn

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Vertical Polarization (dB)</th>
<th>Gain Horizontal Polarization (dB)</th>
<th>Both Polarizations (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3</td>
<td>13.96 ± 0.088</td>
<td>14.00 ± 0.044</td>
<td>13.98 ± 0.110</td>
</tr>
<tr>
<td>7.95</td>
<td>14.52 ± 0.114</td>
<td>14.52 ± 0.114</td>
<td>14.52 ± 0.114</td>
</tr>
</tbody>
</table>

C. Accuracy Considerations and Corrections

1. Sources of uncertainties and corrections

   a. Frequency

Since the gain is directly proportional to frequency (see Eq. (5)), variations in frequency will introduce variations in the measured gain. To reduce this possible source of errors, a Dymec oscillator synchronizer was used to control the frequency of the klystron. The frequency of operation, $f_o$, for the experimental setup shown in Fig. 4 is

$$f_o = N f_{\text{crystal}} + f_{\text{IF}},$$

where

- $N$ is an integer,
- $f_{\text{crystal}} = 120,003,197 \text{ Hz}$, and
- $f_{\text{IF}} = 3 \times [10 \text{ MHz reference crystal}] = 3 \times [10,000,012 \text{ Hz}].$
Fig. 5--Histogram of gain measurements, pyramidal horn, 7.3 GHz.
Fig. 6--Histogram of gain measurements, pyramidal horn, 7.95 GHz.
The frequencies $f_{\text{crystal}}$ and $f_{\text{IF}}$ were measured with a counter (accuracy $\pm 1$ count), which in turn was synchronized to a rubidium vapor frequency standard (accuracy 1 part in $10^{11}$). Evaluating Eq. (8) for the two frequencies used,

$$f_{\text{o}}\bigg|_{7.3 \text{ GHz}} = 7,290,194,981 \pm 64 \text{ Hz}$$

and

$$f_{\text{o}}\bigg|_{7.95 \text{ GHz}} = 7,950,211,048 \pm 69 \text{ Hz}.$$ 

The power spectral density of the output of a klystron oscillator controlled with a Dymec synchronizer using crystal references is known to be much less than the above 69 Hz uncertainties. Temperature changes resulting from cycling of the crystal ovens cause a frequency shift of about 30 Hz. The transmitted power was cw; no modulation of any kind was used. It can be stated that the frequency was controlled to the nearest 100 Hz or to an accuracy of 1 part in $10^8$. The effect of this very small frequency uncertainty is not measurable and will not be considered in the measured gain figure.
b. Separation between antennas

For accurate results the separation between antennas should be such that they are in the far-field of each other, the field is uniform in amplitude and in phase over the receiving aperture; and the scattered fields, whether scattered by the antennas, or by surrounding objects, should be not measurable. Obviously, there should be no obstacles in the propagation path. Furthermore, the propagation path should be in "free-space."

Air is known to approximate ideal "free space" conditions (lossless, isotropic); thus nothing further will be said on this requirement. The separation between the antennas was three meters (varied a few centimeters with each group of measurements) and this distance was determined to the nearest 0.5 cm. This inaccuracy in the value for range results in $\pm 0.007$ dB in the value for the gain.

It was not feasible to directly verify the absence of scattered fields and the uniformity of the field over the receiving aperture. However, indirect evidence that the scattered fields were negligible was obtained by observing that the cross polarized component was at least 25 dB below the principal polarization. Similarly, small lateral or up-and-down motion of the receiver antenna produced no
measurable changes in meter deflection, indicating that the field was, in all probability, uniform over the receiving aperture. An accurate indication for the phase uniformity of the field can be obtained by calculating the differential path length between a point in the center of the transmitting horn and a point at its farthest edge, as measured at a range, R, and then converting this path difference to phase difference. It can be shown[7] that the range is

\[ R = d^2 \frac{d}{8\delta} , \]

where

\[ d = \text{largest dimension of antenna}, \]
\[ \delta = \text{differential path length}, \]

and the phase difference is

\[ \text{Phase Difference} = \frac{360^\circ}{\lambda} \delta . \]

Substituting appropriate values for R, d, and \( \lambda \) into Eqs. (10) and (11), a phase difference of 4° is calculated at the receiving aperture. Based on this calculation and on the experimental measurements, the field at the aperture of the receiving horn will be taken to be uniform in amplitude and in phase and thus no correction to the gain figure results from this source.
The range between the antennas is commonly measured perpendicularly from the plane of the aperture of the transmitting horn to that of the receiving horn, although other points, such as the apex of the horn, may be used. The range in the present case was three meters, which closely corresponds to $10 \frac{d^2}{\lambda}$. At this distance the range is probably adequately well defined, regardless of reference point. To be precise it is stated that in this experiment the range $R$ was measured perpendicularly from the plane of the aperture of the transmitting horn to the plane of the aperture of the receiving horn in the manner employed by Chu and Semplak[13]

It was intended to remove all obstructions from the propagation path. Unfortunately, this intention may not have been quite realized at 7.3 GHz. Looking at Fig. 5 one can see that the mean value of vertical data and of horizontal data differ by 0.041 dB, with that of the vertical being lower. However, no such difference exists at 7.95 GHz. The explanation for this discrepancy at 7.3 GHz is most likely in the fact that the side-walls of the enclosure where the experiments were carried out may have been a trifle too close. The requirements for adequate height or width can be calculated from the consideration that free-space conditions should prevail in the solid angle of the main beam; i.e., obstructions which potentially interfere with propagation either by
reflection or absorption should be outside of the main beam. The beamwidth between first fulls BWBN, is approximately\[8\]

\[
(12) \quad BWBN|_{E\text{-plane}} = \frac{115}{A_{E\lambda}} \text{ degrees}, \quad BWBN|_{H\text{-plane}} = \frac{172}{A_{H\lambda}} \text{ degrees},
\]

where

\[A_{E\lambda}, A_{H\lambda} = \text{ aperture in the respective plane in free-space wavelength.}\]

Using Eq. (12) the following values are calculated for the angle between the peak and the first null:

\[
E \text{ plane }_{7.3 \text{ GHz}} = 35.8^\circ, \quad H \text{ plane }_{7.3 \text{ GHz}} = 39.5^\circ
\]

(13) and

\[
E \text{ plane }_{7.95 \text{ GHz}} = 32.4^\circ, \quad H \text{ plane }_{7.95 \text{ GHz}} = 36.3^\circ.
\]

The measured values were found to be about 5° more in E plane and about 2.5° more in H plane. For the obstruction to be outside of the main beam the following relationship should hold:

\[
\tan \frac{BWBN}{2} = \frac{2h}{R} \quad \text{for the E plane}
\]

(14) and

\[
\tan \frac{BWBN}{2} = \frac{2w}{R} \quad \text{for the H plane},
\]

where
\[ h = \text{vertical distance of antennas from obstruction}, \]
\[ w = \text{horizontal distance of antennas from obstruction}, \]
\[ R = \text{range}. \]

Substituting appropriate values into Eq. (14) the following values are calculated for \( h \) and \( w \) at the two frequencies:

\[ h\big|_{7.3 \text{ GHz}} = 1.15 \text{ m}, \quad w\big|_{7.3 \text{ GHz}} = 1.3 \text{ m} \]

(15) and

\[ h\big|_{7.95 \text{ GHz}} = 1.06 \text{ m}, \quad w\big|_{7.95 \text{ GHz}} = 1.15 \text{ m}. \]

Since the minimum vertical distance \( h \) was 2.5 meters, it was most likely quite adequate. The minimum horizontal distance \( w \), however, was only 1.6 meters and it may not have been adequate at the lower frequency. On the basis of the above considerations as well as the highly consistent data on the higher frequency, the measured gain of the horn at 7.3 GHz (see Table II) will be taken to be the mean value obtained from the measurements on horizontal polarization with the given random error. A possible systematic error of 0.04 dB is assigned to the vertical polarization measurements.
c. Identicalness

The horns used in these experiments were identical as nearly as could be determined. The manufacturer stated[6] that variations in gain from horn-to-horn should be less than \( \pm 0.1 \text{ dB} \). It was confirmed by measurements that if there was any difference in the levels of the power received by the antennas this difference must have been smaller than 0.05 dB. Using Eq. (6), one can see that the correction from this source is at most \( \pm 0.025 \text{ dB} \), thus it will be treated as an uncertainty rather than a correction.

d. Mismatch

A basic principle of power transfer is that for maximum delivery of power to a load the impedance of the load must be the complex conjugate of the impedance of the source. Any other value of load impedance results in the delivery of less than the maximum available power or "mismatch loss." (A more accurate term is conjugate mismatch loss, since other bases of comparison are possible.[9] ) The conjugate mismatch loss, \( M_c \), is given by

\[
M_c = 10 \log_{10} \frac{|1-\Gamma_G \Gamma_L|^2}{(1-|\Gamma_G|^2)(1-|\Gamma_L|^2)} \text{ dB},
\]

where
\[ \Gamma_G = \text{reflection coefficient of generator, and} \]
\[ \Gamma_L = \text{reflection coefficient of load.} \]

The magnitude of reflection coefficients can be obtained from standing wave ratio measurements;

\[ |\Gamma| = \frac{\text{SWR} - 1}{\text{SWR} + 1}. \]  \hfill (17)

Equation (16) gives the difference between the amount of power actually delivered to the load and the maximum available from the source. There are charts available[10] for convenient and accurate evaluation of \( M_c \) for minimum and maximum possible loss for various combinations of source and load SWR's.

The SWR's of the horns, detector, and source were carefully measured before, during, and after the measurements. These measured values were used with Reference 10 to evaluate the limits of conjugate mismatch losses; the results are given in Table III.

The average mismatch loss between receiver horn and detector (from Table III) is 0.027 ± 0.007 dB at 7.3 GHz and it is 0.039 ± 0.020 dB at 7.95 GHz. The corresponding corrections to the gain figures are half that much; i.e., 0.014 ± 0.004 dB and 0.020 ± 0.010 dB, respectively. The average mismatch loss between transmitter horn and source, and detector and source are 0.020 ± 0.008 dB, and 0.020 ± 0.006 dB at 7.3 GHz, respectively; the
corresponding figures at 7.95 GHz are $0.121 \pm 0.021$ dB and $0.125 \pm 0.035$ dB. There is no difference between the average mismatch losses between the transmitter horn and source and detector and source on 7.3 GHz; there is a $0.004$ dB larger average loss between source and detector on 7.95 GHz than between source and transmitter horn. Thus, the figure for gain at 7.95 GHz should be decreased by $0.002$ dB. The uncertainties in the measurements of gain should be increased by $\pm 0.004$ dB and $\pm 0.018$ dB at the low and higher frequencies, respectively.

**TABLE III**

Standing Wave Ratios and Corresponding Conjugate Mismatch Losses

<table>
<thead>
<tr>
<th>Frequency</th>
<th>SWR</th>
<th>Conjugate Mismatch Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter Horn Source</td>
<td>1.03, 1.14</td>
<td>0.028 to 0.012 dB</td>
</tr>
<tr>
<td>Detector Source</td>
<td>7.3 GHz</td>
<td>1.02, 1.14</td>
</tr>
<tr>
<td>Receiver Horn Detector</td>
<td>1.17, 1.02</td>
<td>0.034 to 0.020 dB</td>
</tr>
<tr>
<td>Transmitter Horn Source</td>
<td>1.03, 1.40</td>
<td>0.142 to 0.100 dB</td>
</tr>
<tr>
<td>Detector Source</td>
<td>7.95 GHz</td>
<td>1.05, 1.40</td>
</tr>
<tr>
<td>Receiver Horn Detector</td>
<td>1.20, 1.05</td>
<td>0.058 to 0.019 dB</td>
</tr>
</tbody>
</table>
e. Insertion Loss

The insertion losses of the horn and E-H tuner configuration and of the horn alone were measured by determining the SWR's of these when their outputs were short circuited. The insertion loss of the horn-tuner combination was 0.250 dB and of the horn alone was 0.180 dB. To compensate for the insertion loss of the tuner the gain figures should be increased by \((0.250 - 0.180)/2 = 0.035\) dB.

f. Near Field

Based on experimental evidence, it has been claimed\(^{11}\) that the measured gain of pyramidal horns may be considerably in error when the measurements are carried out at short distance and the aperture-to-aperture separation is used in Eq. (5). The magnitude of the error may be as large as one dB at \(2D^2/\lambda\), where \(D\) is the larger horn dimension, and true far-field conditions may not be realized until the separation is several times \(2D^2/\lambda\). Thus, it is necessary in high precision measurements to use relatively large separations and/or apply corrections to the measured gains.

The earlier works of Jakes\(^{12}\) and Braun\(^{11}\) have recently been updated by Chu and Semplak,\(^{13}\) who give accurate tabulated corrections to be used with the measured gain values. It is to be noted that the near field corrections of Reference 13 apply to measured gain values as defined by Eq. (5) where \(R\) is the distance measured perpendicularly between the planes of the apertures of the horns. Using
the Tables I and II of Reference 13 one obtains the following values for the M, H, and N, P parameters:

\[
\begin{align*}
M &= 8\lambda \ell_E/b^2 = 5.50 \\
H &= 8\lambda R/b^2 = 219 \\
N &= 8\lambda \ell_H/a^2 = 2.75 \\
P &= 8\lambda R/a^2 = 22
\end{align*}
\]

at 7.3 GHz,

\[
\begin{align*}
M &= 5.04 \\
H &= 201 \\
N &= 2.52 \\
P &= 112
\end{align*}
\]

at 7.95 GHz;

and obtains the following "near-field" corrections:

\[
\begin{align*}
(20) & \quad E\, \text{plane} \quad |_{7.3\, \text{GHz}} = 0.030 \, \text{dB}, \quad H\, \text{plane} \quad |_{7.3\, \text{GHz}} = 0.043 \, \text{dB}, \\
(21) & \quad E\, \text{plane} \quad |_{7.95\, \text{GHz}} = 0.036 \, \text{dB}, \quad H\, \text{plane} \quad |_{7.95\, \text{GHz}} = 0.055 \, \text{dB},
\end{align*}
\]
Thus the correction due to "near-field" conditions is 0.073 dB at 7.3 GHz, and 0.091 dB at 7.95 GHz.

g. Interaction

The effects of interaction between antennas which results from multiple re-radiation has been analyzed by Silver. The power received in the absolute gain measurement experiment with identical antennas is shown by him to vary between the limits for a displacement of a quarter wavelength in distance.

\[
\frac{P_r}{P_t} = \left(\frac{G_o \lambda}{4 \pi R}\right)^2 \left(1 \pm \frac{1}{\frac{A_s^2}{\lambda^2 R^2}}\right)^2
\]

where \(A_s\) = scattering cross section, assumed to be one-half of physical cross section. For use in Eq. (5) the square root of the ratio of received to transmitted power is required; thus Eq. (22) becomes

\[
\sqrt{\frac{P_r}{P_t}} = \frac{G_o \lambda}{4 \pi R} \left(1 \pm \frac{1}{\frac{A_o^2}{4 \lambda^2 R^2}}\right) = \frac{G_o \lambda}{4 \pi R} \left(1 \mp \frac{A_o^2}{4 \lambda^2 R^2}\right).
\]

Substituting appropriate numerical values into Eq. (23) one obtains:

\[
\sqrt{\frac{P_r}{P_t}} \pm \frac{G_o \lambda}{4 \pi R} (1 \mp 10^{-5})
\]
As can be seen the interaction effects resulting from multiple re-radiation by the horns were immeasurably small and thus could be neglected.

h. Calibration

The power levels were monitored with precision power meters (HP 431B and HP 430C). The deflections of these meters in turn were converted into attenuator settings, to the nearest full dB mark, of an HP 832A variable attenuator. This precision attenuator was calibrated by the Hewlett-Packard Company, by the use of a parallel IF substitution system with a piston attenuator, traceable to the National Bureau of Standards, as the standard. On basis of the Hewlett-Packard Company's Measurement Standards Calibration Report and Test Data the following correction and uncertainties will be applied to the measured gain figures:

Incremental attenuation correction

\[ 7.3 \text{ GHz}: \frac{-(31.19 - 31.00)}{2} = -0.095 \text{ dB} \]
\[ 7.95 \text{ GHz}: \frac{-(31.18 - 31.00)}{2} = -0.090 \text{ dB} \]

Uncertainties

Incremental attenuation

\[ \pm (0.02 + 3 \times 0.02/10) \text{ dB} = \]
\[ = \pm (0.02 + 0.062) = \pm 0.082 \text{ dB} \]

Standing Wave Ratio \( \pm 0.015 \text{ dB} \).
i. Miscellaneous

There may have been other sources of inaccuracies, such as system instabilities and errors in readings of the precision attenuator. The system instabilities, especially drift in level of transmitted power were checked repeatedly with a meter that was carefully null-ed and zeroed prior to each test and checked after each test. It was found that power level drifts during each measurement were less than \( \pm 0.02 \) dB, contributing a maximum uncertainty of \( \pm 0.01 \) dB to the measured gain figures.

The attenuator was always set to the nearest full dB mark (usually 31.0 dB). It is estimated that the hairlines could be aligned to an accuracy of \( \pm 0.1 \) dB. This contributes an uncertainty of \( \pm 0.005 \) dB to the gain. Fractional dB values were calculated by expressing in dB the ratio of actual meter deflection to deflection at the nearest full dB mark. This method produced a very accurate vernier scale.

In addition to the above random uncertainties, there may be an absolute error hidden in the precision attenuator. However, this possible absolute error could only be traced if there were other calibrating facilities available using standards traceable to primary attenuation standards other than those of the National Bureau of Standards. Thus, it will be assumed that the calibration of the
precision attenuator by the Hewlett-Packard Company was correct in the absolute sense, within the uncertainties quoted in Sec. H.

2. **Interpretation of uncertainties and corrections**

In order to find the absolute gain of the horn antenna certain measurements have been carried out. It has been found that the measured data are scattered about a mean value. Setting up calculations similar to those in Appendix I one can find the probability that a measured data point is within \( \sigma \) distance from the mean value calculated from the measured data. Additionally, the absolute value of this mean is not known with absolute precision since in the calculations fixed quantities - range, wavelength, and power ratio - were used which themselves are subject to some uncertainty. It is safe to state that the uncertainty which is due to randomness could be reduced by obtaining additional data points; while the uncertainties which may cause a shift in the mean value could be reduced by such an expedient as improved precision in instrumentation (see entry Calibration in Table IV).

The above uncertainties and corrections for the measured gain of a pyramidal horn are tabulated below.

The sum of all corrections is +0.027 dB at 7.3 GHz, and +0.054 dB at 7.95 GHz. Some of the uncertainties listed in Table IV contributed to the spread of the measured data. In the opinion of the author, the entries labeled Mismatch and Miscellaneous were such
uncertainties. The spread of the data, calculated with certain level
of confidence (see Appendix I) on basis of actual measurements, is
given by the entry labeled Randomness. This entry, since it is based
on actual measurements, is regarded as substantially more signif-
icant than some value obtained by combining in some arbitrary
fashion all the uncertainties that one could think of. The other
entries, namely: Separation, Identicalness, and Calibration do not
contribute to the spread of the data but may shift the mean value
itself.

**TABLE IV**
Uncertainties and Corrections

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Separation</td>
<td>$\pm 0.160%$</td>
<td>-</td>
</tr>
<tr>
<td>Identicalness</td>
<td>$\pm 0.574$</td>
<td>-</td>
</tr>
<tr>
<td>Mismatch</td>
<td>$\pm 0.184 (7.3\ \text{GHz})$</td>
<td>$+0.014\ \text{dB (7.3 GHz)}$</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.642 (7.95\ \text{GHz})$</td>
<td>$+0.018 (7.95\ \text{GHz})$</td>
</tr>
<tr>
<td>Insertion loss</td>
<td>-</td>
<td>$+0.035 (7.3\ \text{and 7.95 GHz})$</td>
</tr>
<tr>
<td>Near-field</td>
<td>-</td>
<td>$+0.073 (7.3\ \text{GHz})$</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>$+0.091 (7.95\ \text{GHz})$</td>
</tr>
<tr>
<td>Interaction</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Calibration</td>
<td>$\pm 2.208$</td>
<td>$-0.095 (7.3\ \text{GHz})$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-0.090 (7.95\ \text{GHz})$</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>$\pm 0.345$</td>
<td>-</td>
</tr>
<tr>
<td>Randomness</td>
<td>$\pm 2.00 (7.3\ \text{GHz})$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\pm 2.60 (7.95\ \text{GHz})$</td>
<td>-</td>
</tr>
</tbody>
</table>

The effect of the uncertainties which are not due to randomness
will be evaluated next. In solving Eq. (5), fixed numbers were used
for the variables like \( R, \lambda, \) and power ratio \( P_r/P_t \), yet from Table IV one reads that these fixed numbers themselves were subject to uncertainties. These uncertainties were maximum possible errors in the given parameter. To evaluate their effects Eq. (5) will be solved as follows

\[
G_o = \frac{4\pi R}{\lambda} \sqrt{\frac{P_r}{P_t}} = \frac{4\pi R}{\lambda (P_t/P_r)^{1/2}} = \frac{4\pi R}{\lambda P^{1/2}}
\]

Equation (25) is Eq. (5) in slightly different form. In a practical absolute gain experiment the measured quantity is \( G_o = (G_{or} G_{ot})^{1/2} \):

\[
G_o = (G_{or} G_{ot})^{1/2} = (G_{or} (G_{or} + \Delta G_{or}))^{1/2}
\]

\[
= G_{or} (1 + (\Delta G_{or}/G_{or}))^{1/2}
\]

Combining Eqs. (25) and (26), denoting the fixed quantities used in the calculation of the gain with zero subscripts and the maximum uncertainties in the fixed quantities by the prefix \( \Delta \), one obtains

\[
G_{or} = \frac{4\pi}{c} \frac{(R_o \pm \Delta R)(f_o \pm \Delta f)}{(P_o \pm \Delta P)^{1/2} \left(1 \pm (\Delta G_{or}/G_{or})\right)^{1/2}}
\]

\[
= \left(4\pi \frac{R_o f_o}{c P_o^{1/2}}\right) \frac{(1 \pm (\Delta R)/R_o)(1 \pm (\Delta f)/f_o)}{\left(1 \pm (\Delta P)/P_o\right) \left(1 \pm (\Delta G_{or}/G_{or})\right)^{1/2}}
\]
The first term in Eq. (27) containing the error term whose effect is the shifting of the mean value of the measured data. The maximum amount of this possible shift is evaluated by substituting the maximum uncertainties in the fixed quantities into the second term of Eq. (27)

$$G_{or} = \frac{4\pi R_o f_o}{c P_o^{\frac{1}{2}}} \left( \frac{1 \pm (\Delta R)/R_o \pm (\Delta f)/f_o \pm (\Delta R)/(R_o f_o)}{(1 \pm (\Delta P)/P_o \pm (\Delta G_{or})/G_{or} \pm (\Delta P)(\Delta G_{or})/P_o G_{or})^{\frac{1}{2}}} \right)$$  

(28)

$$G_{or} = \frac{4\pi R_o f_o}{c P_o^{\frac{1}{2}}} \left( \frac{1 \pm 0.0016}{(1 \pm 0.03)^{\frac{1}{2}}} \right)$$  

(29)

In arriving at Eq. (29) from Eq. (28) the terms which were several orders of magnitude smaller than the largest term were dropped.

Equation (29) is evaluated as

$$G_{or} = \frac{4\pi R_o f_o}{c P_o^{\frac{1}{2}}} \left\{ \begin{array}{c} +1.7 \% \\\ -1.6 \% \end{array} \right.$$

(30)

On the basis of Eq. (30) the maximum possible calibration errors do not exceed $\pm 1.7$ percent $(\pm 0.07 \text{ dB})$. 
D. Conclusions

It is concluded that the mean value of the measured data points, when the above corrections are applied to it, is the gain of the horns. The maximum error in this gain which may arise in the process of calculating the gain from the measured data with fixed quantities in Eq. (5) when these quantities themselves may be subject to errors (such as calibration) will be taken to be $\pm 1.7$ per cent on basis of Eq. (30). Additionally the measured data scatters randomly about a mean value. One expects to find 99 percent of the measured data points within two percent of this mean value.

The calculated gain values, the values quoted by the manufacturer, and the experimentally determined (with 99.7 per cent confidence) values are tabulated below for convenient comparison.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Calculated (Eq. 7)</th>
<th>Manufacturer's Value</th>
<th>Measured Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3 GHz</td>
<td>13.90 dB</td>
<td>13.75 $\pm$ 0.30 dB</td>
<td>14.03 $\pm$ 0.08 dB</td>
</tr>
<tr>
<td>7.95 GHz</td>
<td>14.81</td>
<td>14.31 $\pm$ 0.30</td>
<td>14.57 $\pm$ 0.08</td>
</tr>
</tbody>
</table>

In Table V the tabulated uncertainties in the measured values are the sum of those due to randomness (see Appendix I) and to calibration.
The calculated and measured values are in excellent agreement. Furthermore, the quoted and measured values are also in close agreement. In conclusion, the absolute gain of the small pyramidal horns is estimated to be 14.03 dB at 7.290 GHz, and 14.57 dB at 7.950 GHz; the uncertainties due to randomness in these figures are ±0.006 dB and ±0.007 dB respectively; the confidence level in this estimate is 99.7 per cent. The maximum possible calibration errors may cause a shift in the true gain not exceeding ±1.7 per cent (±0.07 dB). The total errors are less than ±1.8 per cent (±0.08 dB).
CHAPTER III
A GAIN-CALIBRATION STANDARD ANTENNA FOR
THE OSU SATELLITE COMMUNICATION FACILITY

A. Introduction

Evaluation of the 30-foot-diameter paraboloidal reflector antennas of the Satellite Communication Facility, The Ohio State University ElectroScience Laboratory, for communication and possible radio astronomy applications requires the accurate determination of their gains. The technique of absolute gain measurement is not feasible because it would require another 30-foot reflector antenna in the far-field zone. The gain could be calculated and the basic technique of such calculations is straightforward; however, complete measurements of the primary feed pattern are needed, and estimates must be made of the effects of aperture blockage, feed and antenna efficiency, reflector surface irregularities, and large-scale reflector deformation. Although it may be possible to estimate these effects fairly accurately, in the final result the calculated gain may differ from the actual gain by as much as 10 percent. Other techniques, such as direct substitution of a standard gain horn, are impractical because of the large minimum range required for the transmitter-to-antenna distance. A commonly accepted criterion for the minimum separation R between the source antenna and the antenna under test is $R = \frac{2D^2}{\lambda}$, where $\lambda$ is the operating
wavelength, and $D$ is the largest linear dimension of the antenna ($D$ is the aperture diameter for a paraboloidal reflector). For a 30-foot antenna operating at $\lambda = 1.5$ inches, the distance $R$ is 2.7 miles. A reliable test site at this range which would permit the use of sufficiently high elevation angles is very nearly impossible. Even if such a site were available, it would be useful for determining the gain at only one antenna position where the large-scale reflector deformation caused by sag may affect the gain figure unduly pessimistically.

Placing a source in a helicopter or aircraft would permit gain measurements at various antenna positions; it would also create difficult tracking problems, require that the pilot maintain extremely stable position and course, and the procedure would be extremely costly.

A convenient source for use in performing substitution gain measurements is one of the active communication satellites. Since standard gain horns are relatively low-gain devices, they cannot be used directly to determine the gain of the antennas at the OSU Satellite Communication Facility. A reasonable approach is to calibrate another antenna of sufficient gain to be useful for directly detecting satellite signals and yet small enough so that precise gain measurements could be made on an antenna test range.

For this purpose, a 3-foot-diameter paraboloidal reflector antenna was obtained for use as the gain calibration standard. The gain of this antenna was measured at 7.3 GHz and at 7.95 GHz by the absolute gain measurement technique and by the technique of gain
comparison. Descriptions are given of the antenna, the method of measurements, and their results. The accuracies involved in taking the data are carefully evaluated and appropriate corrections are applied when necessary. The final results obtained with the two techniques are compared.

B. The Gain-Calibration Standard Antenna

A three-foot aperture-diameter, 12-3/16 inches focal length, \((f/D = 0.339)\), spun-aluminum paraboloidal reflector was equipped with a "shepherd's crook" waveguide feed. The measured VSWR at the output flange of the antenna feed was 1.25 at 7.3 GHz and 1.13 at 8.0 GHz. The flange of the waveguide feed of the antenna defines the surface where the gain of the antenna is measured. The power at this surface is measured under matched conditions. The measured far-field power patterns are shown in Figs. 7 and 8. A high-resolution far-field pattern showing only the main beam is given in Fig. 9; a complete 360° pattern in the azimuth plane is shown in Fig. 10. The half-power beamwidth is 2.9 degrees at 7.95 GHz, slightly larger at 7.3 GHz. The near side-lobes (angular distance from on-axis less than 30 degrees) are at least 23 dB down from the peak; the far side-lobes (angular distance greater than 30 degrees) are below the -35 dB level. The antenna is linearly polarized in the on-axis direction. This fact was ascertained by
transmitting to it with a known linearly polarized antenna, rotating
the standard antenna about its principal axis, and recording the
resultant power pattern. These polarization patterns are shown in
Figs. 11 and 12. Also plotted on these patterns is the calculated
values of the square of the cosine of the angular distance between
the planes of polarization of transmitter and standard antenna. Con-
sidering the accuracy of the recording equipment (estimated to be
± 1/2 to ±1 dB) and that of the electro-mechanical rotator (about
±2°), it can be said that the calculated and measured values are in
close agreement. The standard antenna may be regarded as a truly
linearly polarized one with negligibly small cross-polarized
response. This statement applies to the on-axis direction where
the measurements were carried out. It is probably true for the en-
tire main beam. The intended use for the antenna is only in the on-
axis direction. The measurements provided adequate information
for this intended use.
Fig. 7. Measured far-field power pattern of the standard antenna.
Fig. 8--Measured far-field power pattern of the standard antenna.
Fig. 9--Measured far-field power pattern in the main beam region of the standard antenna.
Fig. 10—Measured far-field power pattern of the standard antenna 360°.
Fig. 11--Measured polarization pattern of the standard antenna.
Fig. 12--Measured polarization pattern of the standard antenna.
C. The Gain of the Standard Antenna by the Absolute-Gain Technique

1. Experimental setup, method of measurements

The experiments were carried out on an outdoor antenna test range. A three-foot paraboloid, practically identical to the standard antenna, was used for transmission. The transmitter was mounted on an easily adjustable platform located on top of a building some 30 feet above the ground. The standard antenna was mounted on a special tower that permitted quick and easy adjustment about three axes; the height of the feed above ground was some 25 feet. There were no obstructions within 500 feet of the line of sight between the transmitter and receiver; the ground was covered with grass and weeds, that in turn was covered with light snow on some occasions. The range between the antennas was 1200 feet. The measured cross-polarized component was -30 dB relative to the principal polarization. Because of the long range, relatively low sensitivity of the receiving equipment, and low level of transmitted power, it was not feasible to measure the field variations in the vicinity of the receiving antenna. However, no fluctuations in received power level could be observed by moving the receiving antenna over a distance greater than one diameter either vertically or horizontally; this was taken as a valid indication that the field over the aperture was fairly uniform (further considerations are presented in Appendix II). Before and after each
group of about 20 measurements, the transmitting frequency, the VSWR's of the transmitter antenna, the standard antenna, and of the detector were measured, and the detector was calibrated against the precision attenuator. The measurements themselves consisted of readings of received power levels; the antennas were alternately adjusted to vertical and horizontal polarization. During measurements the standard antenna was first adjusted about three axes (polarization, elevation, and azimuth) for peak received power. Next the transmitter was adjusted about its three axes, also for peak received power. Then the power meters monitoring the transmitted and received powers were nulled and zeroed on the appropriate scales. The transmitted power was then adjusted (if necessary) to a fixed value, the pointing of the standard antenna was once more adjusted for peak response, and then the received power was read as that corresponding to a certain meter deflection. This meter deflection was reproduced by attaching the waveguide-to-coax adaptor of the detector system to the flange of the coupler nearest to the antenna and noting the setting of the precision attenuator. This setting then corresponds to the insertion loss of the two antennas, the E-H tuner, and the intervening space. The block diagram of the equipment is shown in Fig. 13.
Fig. 13—Block diagram of equipment for absolute gain measurement of the standard antenna; (a) Transmitter, (b) Receiver.
2. Results of measurements

Of the 173 measurements taken at 7.3 GHz 85 were in horizontal and 88 were in vertical polarization. The distributions of all the measured gains in the form of histograms are shown in Figs. 14-16. The arithmetic means, the rms values, the ranges, and the standard deviations are given directly on the histograms. A definite lower bound and a probable upper bound on the confidence level for the random variable - in this case the measured gain of the antenna is regarded as the random variable - to be within a given range from the sample mean are calculated in Appendix III. There are also presented in this appendix some statistical considerations about the population distribution of the random variable on the basis of the sample (the measured data points).

Because of the experience gained during the measurements at 7.3 GHz, it was decided that it would not be necessary to take as many measurements at 7.95 GHz as were taken previously. Furthermore, the frequency band of principal interest is the 7.3 GHz region. There were about 20 measurements made at 7.95 GHz; these were consistently within less than 0.5 percent from their average value; these experimental results seem to justify the above decision. The values of measured gain for the standard antenna, with 90.0 percent confidence level, without any corrections and without any accuracy considerations other than the random variations, are tabulated below.
The uncertainties due to random variations are tabulated for the
values at 7.3 GHz; no such uncertainties are tabulated for the values
at 7.95 GHz because the smaller number of data points did not permit
statistical analysis.

### Table VI

**Measured Gains of Gain-Calibration Standard Antenna**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Vertical Polarization</th>
<th>Horizontal Polarization</th>
<th>Both Polarizations</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3 GHz</td>
<td>34.68 ± 0.14 dB</td>
<td>34.74 ± 0.11 dB</td>
<td>34.71 ± 0.15 dB</td>
</tr>
<tr>
<td>7.95 GHz</td>
<td>35.35</td>
<td>35.35</td>
<td>35.35</td>
</tr>
</tbody>
</table>

3. **Accuracy considerations and corrections**

a. **Frequency**

Since the gain of an antenna is directly proportional to the
square of the frequency, variations in frequency will introduce
variations in the measured gain. To eliminate this source of error,
a Dymec oscillator synchronizer was used to control the frequency
of the klystron (see Fig. 13a). The klystron was operating at the
following precisely determined frequencies:
\[ f_0 \mid_{7.3 \text{ GHz}} = 7,290,194,981 \pm 63 \text{ Hz} \]

(31) and

\[ f_0 \mid_{7.95 \text{ GHz}} = 7,950,211,048 \pm 69 \text{ Hz}. \]

In addition to the above uncertainty of \( \pm 63 \text{ Hz} \), one must consider the uncertainties resulting from temperature changes (such as produced by the cycling of the crystal ovens), and from the width of the power spectrum of the output of the klystron. The combined effect of these latter sources of uncertainties is known to be less than 40 Hz. (This is based on several years of experimental evidence involving such critical applications of Dymec-controlled oscillators as the determination of the frequency spectrum of lunar-reflected signals to an accuracy of 1 part in \( 10^7 \).) Thus, it may be stated that the frequency was controlled to an accuracy of 1 part in \( 10^8 \). The transmitter power was cw; no modulation of any kind was used. The effect of this very small frequency uncertainty is immeasurable and will not be considered in the measured gain figure.

b. Separation between antennas

For accurate results the separation between antennas should be such that they are in the far fields of each other, the field is uniform in amplitude and in phase over the receiving aperture and the scattered fields are negligibly small. The slant range between
the antennas was determined by surveying techniques and was found to be 1197.4 feet. Each angular measurement was repeated at least three times and in each case all three measurements were found to be within less than one scale division (20 seconds of arc on the azimuth scale and 1 minute of arc on the elevation scale). Where possible, redundant measurements (e.g., measuring all three angles of a triangle) were used. The survey was greatly facilitated by the availability of a base-line with a length which was accurately known: the side of a building 306' - 7-5/8" long. A Guerley precision telescope was used for the survey. Assuming, rather pessimistically, that the error was three times the smallest scale division, it can be calculated that the accuracy in the determined slant range was better than 2 feet. The 2 feet inaccuracy in the value for the range produces an inaccuracy in the gain figure of less than \( \pm 0.007 \text{ dB} \).

A commonly accepted criterion designating the beginning of the far-field zone is that \( R \geq 2 D^2 / \lambda \). In the present case \( R = 1197.4 \text{ feet} \) corresponds to \( 8.3 \left( 2 D^2 / \lambda \right) \). An accurate indication for the phase uniformity of the field can be obtained by calculating the differential path length between the center of the transmitter and a point at its farthest edge, as measured at a range, \( R \), and then converting this path difference to phase difference. When this calculation is carried out, it is found that the
phase difference was less than $3^\circ$ at the receiving aperture. Based on this calculation and on the experimental measurements of the amplitude variations, the field at the receiving aperture will be taken to be uniform in amplitude and in phase and thus no correction to the gain figure results from this source.

Fig. 14--Histogram of gain measurements vertical polarization.
Fig. 15--Histogram of gain measurements horizontal polarization.

Total: 85
Mean: 34.744 dB
R. M. S.: 34.747
σ ~ s: 0.0466
Range: 0.35
Fig. 16--Histogram of gain measurements both polarizations.
c. Identicalness

Absolute gain measurements are usually made between antennas whose gains are identical. In the present case it was intended to duplicate the standard antenna and use this duplicate as the transmitter. With the aid of a third antenna it was determined that the gain of the duplicate antenna was 0.33 dB lower than that of the standard. Additionally, an E-H tuner was inserted between the source and the transmitter antenna for matching purpose. The insertion loss of this device was measured to be 0.07 dB. To compensate for this insertion loss and for the difference in gain between the antennas the gain figures should be increased by 

\[(0.330 + 0.07) \div 2 = 0.20 \text{ dB}\].

The maximum uncertainty in the gain figure from this source is \(\pm 0.025 \text{ dB}\).

d. Mismatch

When the impedance of the load is not the complex conjugate of the impedance of the source, the power delivered to the load is less than the maximum available by the amount called the conjugate mismatch loss. The minimum and maximum possible values of mismatch losses can be evaluated accurately, provided source and load standing wave ratios are precisely measured. The SWR's of the antennas, detector, and source were carefully measured before, during, and after the measurements. These measured values and the calculated limits of conjugate mismatch losses are given in Table VII.
The average mismatch loss between receiver and detector from Table VII is $0.060 \pm 0.030$ dB at 7.3 GHz and $0.021 \pm 0.017$ dB at 7.95 GHz. The corresponding corrections to the gain figures are half that much: $0.030 \pm 0.015$ dB and $0.011 \pm 0.009$ dB, respectively. The average mismatch loss between transmitter and source, and detector and source are practically the same on both frequencies (the difference in average values is $0.001$ dB on either frequency), thus there is no correction to be applied to the gain figure; however, the uncertainty should be increased by $\pm 0.010$ dB on the 7.3 GHz data and by $\pm 0.0025$ dB on the 7.95 GHz data.

e. Interaction

The effects of interaction between antennas because of multiple re-radiation was calculated by the method described by Silver.[14] This effect was found to cause an uncertainty in the gain figures of less than $0.0002$ dB. This amount of uncertainty is negligibly small; no correction from this source will be applied to the gain figures.

f. Calibration

All power level measurements during the experiments were converted into dial readings of an HP 832A precision variable attenuator. On the basis of the Hewlett-Packard Company's Measurements Standards Calibration Report and Test Data the
following corrections and uncertainties will be applied to the measured gain figures:

Incremental Attenuation Correction:

\[ 7.3 \text{ GHz: } -(31.19 - 31.00) \div 2 = -0.095 \text{ dB} \]
\[ 7.95 \text{ GHz: } -(31.18 - 31.00) \div 2 = -0.090 \text{ dB} \]

Uncertainties:

Incremental Attenuation:

\[ \pm (0.02 + 31 \times 0.02/10) \text{ dB} = \pm (0.02 + 0.062) = \pm 0.082 \text{ dB} \]

Standing Wave Ratio: \( \pm 0.015 \text{ dB} \).

g. Miscellaneous

Other sources, not mentioned above, may also have contributed to the inaccuracies in the measurements. These sources are system instabilities, especially drift in level of transmitted power, and the repeatability of the precision attenuator settings. It was found that power level drifts during each measurement were at most \( \pm 0.02 \text{ dB} \), contributing a maximum uncertainty of \( \pm 0.01 \text{ dB} \). It is estimated that the precision attenuator settings could be repeated to an accuracy of \( \pm 0.01 \text{ dB} \); this contributes an uncertainty of \( \pm 0.005 \text{ dB} \) to the gain figure.

Calibration of the precision attenuator against a primary standard other than the one used by the Hewlett-Packard Company may reveal an absolute error in the precision attenuator. This
TABLE VII
Standing Wave Ratios and Corresponding Conjugate Mismatch Losses

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>SWR</th>
<th>Conjugate Mismatch Losses</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Maximum</td>
<td>Minimum</td>
<td>Average</td>
<td></td>
</tr>
<tr>
<td>Transmitter</td>
<td></td>
<td>1.07</td>
<td>0.042 dB</td>
<td>0.004 dB</td>
<td>0.023 ± 0.019 dB</td>
<td></td>
</tr>
<tr>
<td>Source</td>
<td></td>
<td>1.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detector</td>
<td>7.3 GHz</td>
<td>1.06</td>
<td>0.038</td>
<td>0.006</td>
<td>0.022 ± 0.016</td>
<td></td>
</tr>
<tr>
<td>Source</td>
<td></td>
<td>1.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Receiver</td>
<td></td>
<td>1.25</td>
<td>0.090</td>
<td>0.030</td>
<td>0.060 ± 0.030</td>
<td></td>
</tr>
<tr>
<td>Detector</td>
<td></td>
<td>1.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transmitter</td>
<td></td>
<td>1.07</td>
<td>0.178</td>
<td>0.078</td>
<td>0.128 ± 0.050</td>
<td></td>
</tr>
<tr>
<td>Source</td>
<td></td>
<td>1.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detector</td>
<td>7.95 GHz</td>
<td>1.06</td>
<td>0.170</td>
<td>0.085</td>
<td>0.127 ± 0.043</td>
<td></td>
</tr>
<tr>
<td>Source</td>
<td></td>
<td>1.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Receiver</td>
<td></td>
<td>1.13</td>
<td>0.038</td>
<td>0.004</td>
<td>0.021 ± 0.017</td>
<td></td>
</tr>
<tr>
<td>Detector</td>
<td></td>
<td>1.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
additional calibration was not carried out. It will be assumed that
the calibration of the precision attenuator by the Hewlett-Packard
Company was correct in the absolute sense within the uncertainties
quoted in Section f above.

4. Interpretation of uncertainties
and corrections

The considerations set forth in Chapter II, Section C:2
apply to the measurements evaluated in this section. In order to
find the absolute gain of the gain-calibration standard antenna the
absolute-gain technique was used. Again, it has been found that
the data points are scattered about a mean value. Calculations,
similar to the ones carried out in Appendix I, were worked out in
Appendix III for the gain-calibration standard data. From these
calculations one can readily find the probability that a measured
data point is within \( k = h \sigma \) distance from the mean value which was
determined from the measured data points. Additionally, the
absolute value of this mean is not known with absolute precision; it
may be subject to uncertainties which may cause a shift in its value.

The above uncertainties and corrections for the measured
gain of the standard antenna, as determined by the absolute gain
measurement technique, are given in Table VIII. The sum of all
corrections is +0.135 dB at 7.3 GHz, and +0.121 dB at 7.95 GHz.
The uncertainties listed in Table VIII may be divided into two
categories: those uncertainties which contributed to the spread of the data; and those uncertainties which may cause a shift in the mean value itself. The former will be designated as random errors, the latter will be designated as calibration errors. In the opinion of the author, the entries labeled mismatch and miscellaneous belong to the first category. The spread of the data calculated with certain level of confidence (see Appendix III) on basis of actual measurements is given by the entry labeled Randomness. The other entries in the Uncertainty column may cause a shift in the mean value; their possible maximum effect will be calculated next.

TABLE VIII
Uncertainties and Corrections
(Absolute-Gain Measurements)

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty</th>
<th>Corrections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Separation</td>
<td>± 0.007 dB</td>
<td>---</td>
</tr>
<tr>
<td>Identicalness</td>
<td>± 0.025</td>
<td>+ 0.200 dB</td>
</tr>
<tr>
<td>Mismatch</td>
<td>± 0.025 (7.3 GHz)</td>
<td>+ 0.030 (7.3 GHz)</td>
</tr>
<tr>
<td></td>
<td>± 0.034 (7.95 GHz)</td>
<td>+ 0.011 (7.95 GHz)</td>
</tr>
<tr>
<td>Interaction</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Calibration</td>
<td>± 0.097</td>
<td>- 0.095 (7.3 GHz)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.090 (7.95 GHz)</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>± 0.015</td>
<td>---</td>
</tr>
<tr>
<td>Randomness</td>
<td>± 0.132 (7.3 GHz)</td>
<td>---</td>
</tr>
</tbody>
</table>
In order to evaluate the possible maximum effect of the uncertainties which are calibration errors one can proceed as in Chapter II, Section C:2. Since the measurement technique employed in Chapter II was identical to the one used here, Eq. (28) applies here also. Evaluating Eq. (28) for the possible maximum uncertainties which apply here one obtains

\[
G_{or} = \frac{4 \pi R f_o}{c P_o} \frac{1 \pm 0.00167}{(1 \pm 0.0297)^{\frac{1}{2}}}
\]

Equation (32) is practically identical to Eq. (29). The reason for this fact is that in obtaining the measurements of the pyramidal horn and of the gain-calibration standard antenna the same instruments were used. The largest source of uncertainty is the precision with which the calibration of the instruments were carried out. The other parameters were either the same in both experiments (for example: frequency) or measured with the same relative precision (for example: range). Equation (32) is evaluated as

\[
G_{or} = \frac{4 \pi R f_o}{c P_o} \left\{ +1.7\% \right. \left. -1.6\% \right\}
\]

On the basis of Eq. (33) the maximum possible calibration errors do not exceed $\pm 1.7$ percent ($\pm 0.07$ dB).
D. The Gain of the Standard Antenna by the Gain-Comparison Technique

1. Experimental setup, method of measurements

A pyramidal horn, the gain of which has been accurately determined by the absolute-gain technique, [16], and the 3-foot paraboloid were mounted on the special tower described in Section C-1. The horn was mounted behind the dish; a hairflex baffle larger than the dish was placed between the two antennas; the antennas were turned alternately toward the transmitter. The horn was matched to the detector with an E-H tuner. The paraboloid was equipped with a coupler (nominal coupling factor: -20 dB) and no matching was necessary here. The same arrangement was used for transmission as during the absolute gain measurement. The range was 550 feet; no cross-polarized components and no field variations could be measured in the near-vicinity of the receiving setup. Initially the paraboloid was directed toward the transmitter and adjusted about three axes for peak received power. Next the transmitter was likewise adjusted and fixed in position; and finally the receiving paraboloid was adjusted again. It was found that this final adjustment produced an almost negligibly small increase in the received power level thus assuring accurate alignment of the antennas. Because of the much broader beamwidth of the horn, the precision alignment of the paraboloid served...
adequately well for the horn also. Both vertical and horizontal polarizations were used, and the receiving antennas were rotated by 180°. (Naturally, each change in polarization required the re-alignment of the transmitter and receivers.) The measurements themselves consisted of alternately directing the horn and the paraboloid toward the transmitter, adjusting their positions for peak received power, and observing this power level on an HP-431B precision power meter which was kept carefully nulled and zeroed. The coupler on the output of the paraboloid brought the level of the power received by the dish to within a few percent to that received by the horn. The diagram of the receiving equipment is shown in Fig. 17; for the diagram of the transmitting equipment see Fig. 13a.

2. Results of measurements

A total of 146 points was obtained at 7.3 GHz and 151 at 7.95 GHz. The distributions of the data points in the form of histograms are shown in Figs. 18 and 19. The arithmetic means, the quadratic means, the ranges, and the standard deviations are given directly on the histograms. The bounds on the confidence level for the various ranges from the mean are determined in Appendix III. The values of the incremental gain determined for the standard antenna with 99.0 percent confidence level, without any corrections and without any accuracy considerations other than the random errors in the measurements, are tabulated below.
TABLE IX
Incremental Gain of Standard Antenna Relative to the Gain of a Pyramidal Horn

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Incremental Gain $^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3 GHz</td>
<td>20.721 ± 0.315 dB</td>
</tr>
<tr>
<td>7.95 GHz</td>
<td>20.953 ± 0.264 dB</td>
</tr>
</tbody>
</table>

$^1$ The tabulated uncertainties apply to the individual samples shown in Figs. 13 and 19.

3. Accuracy considerations and corrections

The various sources of inaccuracies and the resulting corrections as they apply to the absolute gain measurements have been considered in Section III-C. These considerations will be repeated here to the extent that they apply to the gain comparison method.

The frequency of the transmitter was controlled with a stabilizer to the same accuracy as before, hence no corrections and no measurable inaccuracies will be assigned because of the minuscule variations in frequency.

The separation between antennas was 550 feet which corresponds to about $4D^2/ \lambda$ (where $D = 3$ feet, the diameter of the paraboloid). At this range the ratio of the measured gain to that at $R = \infty$ is essentially unity. Furthermore, this method of gain measurement is independent of the range (provided, of course, that the far-field criterion is adequately satisfied), hence no correction
and no inaccuracies result from this possible source. It is assumed that reflections were negligibly small. The validity of this assumption is shown in Appendix II.

The identicalness of the antennas is a problem peculiar to the absolute gain method and it does not enter in measurements of gain by the comparison technique.

The mismatches between the antenna and the detector caused losses, thus corrections will have to be applied to the measured incremental gain figures and the magnitude of inaccuracies introduced will have to be determined. The pertinent values of the accurately measured SWR's and the calculated limits of conjugate mismatch losses are given in Table X. The mismatch losses at 7.3 GHz between either of the antennas and the detector were the same. Since the incremental gain of the paraboloid over the horn was sought, no correction is necessary when the gain of each antenna differs from the true figure by the same amount. The inaccuracies in the measurements, however, may increase by as much as the sum of the inaccuracies; i.e., $\pm 0.024$ dB. At 7.95 GHz the incremental gain will be reduced by $0.039 - 0.007 = 0.032$ dB, since this is the amount by which the average mismatch loss of the horn was higher than that of the paraboloid. The inaccuracies will be increased by $(\pm 0.022) + (\pm .002) = \pm 0.024$ dB.
Fig. 17--Block diagram of equipment for gain measurement by the gain-comparison technique.
Fig. 18--Histogram of data points - comparison of gain of pyramidal horn and three-foot paraboloid on 7.3 GHz.
Fig. 19--Histogram of data points - comparison of gain of pyramidal horn and three-foot paraboloid on 7.95 GHz.
Interaction between antennas is an effect which is inversely proportional to range. This effect was found to cause an uncertainty of less than 0.0002 dB when the range was 1200 feet. At half this distance, although this effect causes an uncertainty in the gain of the paraboloid which is four times the above figure, the uncertainty attributed to this source is still negligibly small.

The -20 dB coupler used with the paraboloid was calibrated by using the precision attenuator whose calibration accuracy is given in Section C-3:f. The uncertainties due to calibration inaccuracies are ± 0.077 dB.

Miscellaneous sources of errors (see Section C-3:g) may have contributed as much as ± 0.015 dB uncertainty to the measurements.

Finally, since the gain-comparison measurements were based on the gain of the pyramidal horn, the accuracy with which this gain figure is known must be considered in evaluating the accuracy with which the gain of the gain-calibration standard is determined. For the purposes here the maximum uncertainty in the gain of the horn is sought. This maximum uncertainty is the sum of the uncertainties due to random errors and to calibration errors. This approach is probably over-conservative. Thus, the maximum uncertainty from this source is ± 0.08 dB at the frequencies of 7.3 and 7.95 GHz.
<table>
<thead>
<tr>
<th>Frequency</th>
<th>SWR</th>
<th>Conjugate Mismatch Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Maximum</td>
</tr>
<tr>
<td>Horn</td>
<td>1.12</td>
<td>0.032</td>
</tr>
<tr>
<td>Detector</td>
<td>7.3 GHz</td>
<td>1.06</td>
</tr>
<tr>
<td>Paraboloid (de-coupled output)</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td>Horn</td>
<td>1.20</td>
<td>0.061</td>
</tr>
<tr>
<td>Detector</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>Paraboloid (de-coupled output)</td>
<td>7.95 GHz</td>
<td>1.01</td>
</tr>
<tr>
<td>Detector</td>
<td>1.08</td>
<td></td>
</tr>
</tbody>
</table>
4. **Interpretation of uncertainties and corrections**

Similar considerations set forth previously in regard to interpreting uncertainties and corrections will be applied here also.

The above uncertainties and corrections for the measured gain (as determined by the gain-comparison technique) of the standard antenna are tabulated below.

**TABLE XI**
Uncertainties and Corrections
(Gain-Comparison Measurements)

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Separation</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Mismatch</td>
<td>± 0.024 dB (7.3 GHz)</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>± 0.024 (7.95 GHz)</td>
<td>-0.032 dB (7.95 GHz)</td>
</tr>
<tr>
<td>Interaction</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Calibration</td>
<td>± 0.077</td>
<td>---</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>± 0.015</td>
<td>---</td>
</tr>
<tr>
<td>Gain of Horn</td>
<td>± 0.160 (7.3 GHz)</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>± 0.180 (7.95 GHz)</td>
<td>---</td>
</tr>
<tr>
<td>Randomness</td>
<td>± 0.315 (7.3 GHz)</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>± 0.264 (7.95 GHz)</td>
<td>---</td>
</tr>
</tbody>
</table>
The only correction from Table XI is \(-0.03\) dB to be applied to the 7.95 GHz data. Adding the appropriate gains of the horn-antennas from Chapter II to the incremental gain figures listed in Table IX and applying the correction from Table XI, one obtains 34.75 dB at 7.3 GHz and 35.49 dB at 7.95 GHz. In the opinion of the author the entries labeled Mismatch and Miscellaneous contributed to the spread of the data. The spread of the data calculated with certain level of confidence (see Appendix III) on basis of actual measurements is given by the entry labeled Randomness. The entries labeled Calibration and Gain of Horn may cause a shift in the mean value; their possible maximum effect will be calculated next.

The gain of the gain-calibration standard antenna as measured in a gain comparison experiment is given by the following expression,

\[ G_3^* = \left( G_{\text{horn}} \right) \left( \frac{M_3^*}{M_{\text{horn}}} \right) \frac{C}{L} \]

where \( G_{\text{horn}} \) = absolute power gain of the horn
\( M_3^* \), \( M_{\text{horn}} \) = deflections of the power meter when detector is connected to 3-foot dish and horn respectively
\( C \) = coupling factor of 20 dB coupler
\( L \) = insertion loss of tuner.
All entries in Eq. (34) are to be made in numeric. Denoting the fixed quantities used in calculating the gain with zero subscripts and the maximum uncertainties in the fixed quantities by the prefix $\Delta$, one obtains

$$G_3' \left(1 \pm \Delta \frac{G_3'}{G_3}\right) = \left(G_{\text{horn} o} \pm \Delta G_{\text{horn}}\right) \left(M_3' / M_{\text{horn}}\right) (C_o + \Delta C) / L$$

Since the meter deflections were almost the same up-scale values regardless which antenna was connected to the detector, the uncertainties in these quantities need not be considered. The insertion loss of the tuner was quite small, the uncertainty in its value was small enough to be negligible here. Equation (35) is evaluated as follows

$$G_3' \left(1 \pm \Delta \frac{G_3'}{G_3}\right) = \left(G_{\text{horn} o}\right) \left(M_3' / M_{\text{horn}}\right) (C_o / L) \cdot$$

$$\left(1 \pm \Delta G_{\text{horn}} / G_{\text{horn} o}\right) \left(1 \pm \Delta C / C_o\right)$$

$$G_3' \left(1 \pm \Delta \frac{G_3'}{G_3}\right) = \frac{G_{\text{horn} o} M_3' C_o}{M_{\text{horn}} L} \cdot$$

$$\left(1 \pm \frac{\Delta G_{\text{horn}}}{G_{\text{horn} o}} \pm \frac{\Delta C}{C_o} \pm \frac{\Delta C \Delta G_{\text{horn}}}{C_o G_{\text{horn} o}}\right)$$
The quantities with the \( \Delta \) prefix in the parenthesis in Eq. (37) constitute the error term whose effect is the possible shifting of the mean value of the measured data. The maximum amount of this possible shifting is evaluated by substituting the maximum uncertainties in the fixed quantities into Eq. (37).

\[
G_3' (1 \pm \Delta G_3'/G_3') = \frac{G_{\text{horn}} o M_3' C_o}{M_{\text{horn}} L} \cdot \left(1 \pm \frac{1.8}{100} \pm \frac{1.8}{100} \pm \frac{(3.7)(1.8)}{2500}\right)
\]

\[
G_3' (1 \pm \Delta G_3'/G_3') = \left(\frac{G_{\text{horn}} o M_3' C_o}{M_{\text{horn}} L}\right) (1 \pm 0.036) .
\]

On the basis of Eq. (39) the maximum possible calibration errors do not exceed \( \pm 3.6 \) per cent (\( \pm 0.15 \) dB).
E. Conclusions

Considering first the results of the absolute gain technique experiment, it is concluded that the mean value of the measured data points, when the corrections listed in Table VIII are applied to it, is the gain of the standard antenna. The maximum error in this gain which may arise in the process of calculating the gain from the measured data with fixed quantities when these quantities themselves may be subject to calibration errors is $\pm 1.7$ per cent (see Eq. (33)). Additionally, the measured data points scatter randomly about a mean value. One expects to find 90 per cent of the measured data points within 3 per cent of this mean value. The mean of the sample is an unbiased estimate of the mean of the population; the 99.7 per cent confidence interval is given by

$$\pm 3\sigma / \sqrt{n} = \pm 3 (0.063 \, \text{dB}) / \sqrt{173} = \pm 3 (1.46 \, \%)/\sqrt{173} = \pm 0.33 \, \text{per cent} (\pm 0.01 \, \text{dB}).$$

Considering next the results of the gain-comparison technique experiment, it is concluded that the mean value of the measured data points, when the correction listed in Table XI are applied to it, is the gain of the standard antenna. The maximum possible error in this gain which may arise in the process of calculating the gain from the measured data with fixed quantities when these quantities themselves may be subject to calibration errors (such as the maximum error - the sum of uncertainties due to calibration and to randomness - in the gain of the horn antenna) is $\pm 3.6$ per cent (see Eq. (39)).
Additionally, the measured data scatter about a mean value. As one of the results of the considerations presented in Appendix III it is estimated with 99.7 per cent confidence that the true mean is within ± 0.03 dB of the sample mean.

The gain of the gain-calibration standard antenna has been determined by two independent methods: absolute-gain and gain-comparison methods. The results of the two methods are given in Table XII.

The gain figure of principal interest is the one at 7.3 GHz. The two measurements yielded two figures which are within 0.10 dB; this is remarkably close. Summing the two sources of errors an upper limit on all uncertainties may be obtained for each of the two measurements. If one were to plot the mean values and the upper limit on all uncertainties that go with each value it could be seen that the gain-comparison data stretch between the values 34.57 and 34.93 dB; (34.75 ± 0.03 ± 0.15 dB); the absolute-gain data stretch between the values 34.77 and 34.93 dB (34.85 ± 0.01 ± 0.07 dB).

The region of overlap is 34.77 to 34.93 dB. Since this region of overlap encompasses all of the uncertainties of the absolute-gain data, its significance is interpreted as a valid argument for discarding values for the gain which could be obtained through some application of uncertainties to the mean value when these resultant gain
<table>
<thead>
<tr>
<th>Frequency</th>
<th>Absolute-Gain Technique</th>
<th>Gain-Comparison Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Errors</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Random</td>
<td>Confidence Level</td>
</tr>
<tr>
<td>7.3 GHz</td>
<td>34.85 dB ±0.01 dB</td>
<td>99.7%</td>
</tr>
<tr>
<td>7.95 GHz</td>
<td>35.47 dB</td>
<td>---</td>
</tr>
</tbody>
</table>

¹ Maximum possible errors (see Eq. (33))

² Maximum possible errors (see Eq. (39))
values are outside this region of overlap. The actual gain value will be taken to be the mid value between the gain entries of Table XII.

The gain of the standard antenna will be taken as $34.80 \pm 0.15$ dB at 7.3 GHz and $35.48 \pm 0.18$ dB at 7.95 GHz. The uncertainties of $\pm 0.15$ and $\pm 0.18$ dB are to be regarded as upper limits.
CHAPTER IV
CALIBRATION OF A THIRTY-FOOT PARABOLOIDAL REFLECTOR

A. Introduction

Utilization of a large ground terminal, such as the Satellite Communication Facility at The Ohio State University ElectroScience Laboratory, for certain communication and radio astronomy applications requires accurate knowledge of system parameters. The subject of this chapter is the determination of those parameters which characterize the performance of the thirty-foot paraboloidal reflector antenna: gain, power pattern, and noise temperature.

The antenna gain is the parameter which normally has the greatest practical significance. (For a definition of antenna gain see Chapter II.) Unfortunately, it is very difficult to measure the gain of a large antenna with great precision. There are various methods in common use which yield fairly accurate gain figures and which may require elaborate technical preparations. Some of these methods are: gain comparison, pattern integration, extrapolation of near-field measurements, radiometric measurements on celestial sources of known fluxes, and measurement of received power level from a known source located in the far-field (on a collimation tower,
in an aircraft or helicopter, in a satellite, or even on a tethered balloon). A private survey [17-24] has been conducted for the purpose of ascertaining the methods used for gain-calibration and the resultant accuracies on some of the installations employing high-gain antennas. The results of this survey together with the results of some of the antenna calibrations reported in the literature, [25-30] are given in Table XIII.

In general, much higher accuracies are claimed for the gains of horn-reflector antennas than for the gains of paraboloids. This is probably due to the fact that the aperture distribution of a horn reflector can be calculated quite accurately from the field distribution in the waveguide. Knowledge of the aperture distribution allows the calculation of the far-field pattern by Fourier transformation, which in turn leads to the antenna-gain, provided losses are known. For paraboloidal reflectors there is no method available to calculate the gain directly and to have the calculated and measured gains agree with much less than ten percent difference. Thus, the gains of large paraboloids must be measured, and the resultant accuracy is determined by the amount of care exercised in carrying out the measurements. For greater accuracy, and especially for greater confidence in the measurements, normally more than one technique is employed. On the basis of the survey, it may be said that a carefully executed set of gain measurement with
TABLE XIII
Results of Survey of Antenna Gain Calibration

<table>
<thead>
<tr>
<th>Antenna Description</th>
<th>Method of Calibration</th>
<th>Claimed Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paraboloid, $D = 85'$</td>
<td>gain comparison collimation tower</td>
<td>$\pm 0.3 - 1.0$ dB</td>
</tr>
<tr>
<td>&quot;</td>
<td>gain comparison radio stars</td>
<td>$\pm 0.5$</td>
</tr>
<tr>
<td>&quot;</td>
<td>collimation tower aircraft</td>
<td>$\pm 1$</td>
</tr>
<tr>
<td>&quot;</td>
<td>radio stars</td>
<td>$\pm 0.3$</td>
</tr>
<tr>
<td>Horn-Reflector</td>
<td>near field measurements gain comparison</td>
<td>$\pm 0.04$</td>
</tr>
<tr>
<td>&quot;</td>
<td>pattern integration gain comparison</td>
<td>$\pm 0.4$</td>
</tr>
<tr>
<td>&quot;</td>
<td>gain comparison radio stars</td>
<td>$\pm 0.1 - 0.3$</td>
</tr>
<tr>
<td>Paraboloid, $D = 15'$</td>
<td>collimation tower radio stars</td>
<td>$\pm 0.44$</td>
</tr>
<tr>
<td>&quot;</td>
<td>collimation tower tethered balloon</td>
<td>$\pm 0.5$</td>
</tr>
<tr>
<td>&quot;</td>
<td>gain comparison collimation tower</td>
<td>$\pm 0.3$</td>
</tr>
<tr>
<td>&quot;</td>
<td>radio stars</td>
<td>$\pm 0.1$</td>
</tr>
<tr>
<td>&quot;</td>
<td>gain comparison pattern integration radio stars</td>
<td>$\pm 0.1 - 1.0$</td>
</tr>
<tr>
<td>&quot;</td>
<td>radio stars aircraft</td>
<td>$\pm 0.5$</td>
</tr>
</tbody>
</table>
at least two independent techniques can yield an accuracy of 0.5 to 1 dB.

The survey made it apparent that the gain measurements reported in Table XIII were carried on for various purposes and with various degrees of diligence and certitude. In some cases the accuracy estimates would, in the opinion of the author, be difficult to substantiate. For example, in the 11th entry in Table XIII an accuracy of 0.1 dB was claimed because three successive measurements on a collimation tower miles away, at nearly zero degree elevation angle, yielded gain figures which were within 0.1 dB. Perhaps the details of this chapter, showing the laboriousness of a precise gain measurement of a 30-foot paraboloid, will illustrate for the reader the difficulty of exact measurements of this type with accurately known limits, and will serve to show that reported gain figures for large antennas are likely to be uncertain unless a great deal of time and care was taken in the measurements; in other words unless an accurate knowledge of gain was of prime importance in one of the major missions of the installation.

Based partly on the results of the above survey, partly on technical and practical considerations, a decision was made to calibrate the gain of the paraboloid by the gain-comparison technique and by radio-astronomical measurements. Gain-comparison is a method whereby received signal strength is measured both with
an antenna whose absolute gain is accurately known and with the antenna under test, both being connected to identical receiving systems. The difference in the signal strength is then a measure of the difference in gain of the two antennas. The antenna used for comparison was a precisely calibrated 3-foot dish,[31] the signal sources used were the various IDCSP satellites.

Radio-astronomical measurements involve the measurements of noise power received from celestial sources such as radio stars and the moon. Basically, this technique is also a comparison method; however, the standard used is, in effect, the antenna which was used to measure the flux of the radio source. Thus, one can calibrate the system against any system used for radio astronomy; naturally, the most accurate data available should be used. The particularly appealing feature of the radio-astronomy technique is that it allows the direct use of the most accurate standard values available. Of course, the radio-astronomy technique is quite independent of the gain-comparison technique. The interest is specifically to use several independent techniques and to show agreement in order to establish confidence in the precision of the results.

In addition to the determination of the absolute gain of a large paraboloid, the normalized antenna power patterns along orthogonal "cuts", and variations in the antenna noise temperature versus elevation and versus azimuth angles are parameters of importance
which were included in the measurements.

B. Absolute Gain

1. Comparison measurements with a gain-calibration standard antenna

a. Results of measurements

The simplified block diagram of the system used for collecting the data for the gain-comparison experiment is shown in Fig. 20. The procedure consisted of alternately switching the receiver to the 30-foot paraboloid and to the 3-foot gain-calibration standard antenna. In coincidence with the switching action, the precision IF attenuator was adjusted to produce a nearly identical up-scale deflection regardless of the position of the RF switch. The output of the RMS voltmeter was recorded on magnetic tape for analysis by a computer and on paper chart by visual analysis. The setting of the IF attenuator was recorded manually.

Fig. 20--Simplified block diagram of receiving system.
The signal source in each measurement was one of the IDCSP satellites. The satellite was caused to produce its saturated output by a locally controlled transmitter the frequency and power level of which were extremely stable. The frequency of the transmitter was known within a few Hz and this accuracy was maintained throughout the experiments. The variations in the output power level, as was confirmed by actual measurements, if these existed, they were less than 0.1 dB, thus the output of the satellite was always the saturated level. This arrangement provided sufficiently strong (signal-plus-noise to noise ratios in the 3-foot dish channel were about +13 dB) and stable signals. The elevation angles, mostly above 30°, were high enough to eliminate the usual near-horizon measurement problems, such as structural obstructions, atmospheric scintillations, and multipath effects. The 35,000 kilometer average range eliminated the near-field problems. The incoming signals were left-hand elliptically polarized; so was the 30-foot antenna. The polarization pattern and the orientation of the major axis of the incoming signal was measured several times for each satellite with the linearly polarized 3-foot antenna.

The gain of the 30-foot antenna as measured at the flange of the feed-horn (refer to Fig. 20) is given by

\[
G_{30} \text{ (numeric)} = \frac{G_3 \Delta L \left( \frac{M_{30}'}{M_3} \right) P_{L3'} \left( \frac{(S+N)}{N} \right)_3}{L \text{ guide } P_{L3'}}
\]
where \( G_{30'} \) = absolute power gain of the large antenna, numeric
\( G_{3'} \) = absolute power gain of the gain standard,
\( \Delta L \) = differential value of IF attenuator setting,
\( M_{30'}, M_{3'} \) = deflections of the rms voltmeter when receiver is connected to the 30' and 3' dish respectively,
\( L_{\text{guide}} \) = incremental path loss,
\( P_{L_{30'}}, P_{L_{3'}} \) = losses due to polarization mismatch between incoming signals and 30' and 3' dishes respectively,
\( \left( \frac{S+N}{N} \right)_{3'} \) = correction factor for low signal levels in 3' channel.

More than 300 data points were collected over a three month period utilizing at least seven different satellites as signal sources. Data points from this total which could not be directly referred to a discrete setting of the precision IF attenuator, i.e., which were dependent on meter deflections occurring on more than one meter range, or data points collected during marginal operating conditions, e.g.: manual positioning of the antennas, were discarded. Thus, there remained 294 data points which are free from known experimental errors. It is assumed that these data points are statistically independent. The distributions of all the measured gains is shown in Fig. 21 in the form of a histogram. The arithmetic mean of the 294
points is 53.165 dB, the quadratic mean is 53.168 dB, and the standard deviation is 0.178 dB. A Gaussian density function, properly scaled for the values of the histogram, is shown superimposed on the experimental data. As can be seen, the normal density function provides a good fit. For normal distribution, 68.3 percent of all points fall within one standard deviation, and 95.5 percent fall within two standard deviations. The corresponding percentages for the measured points are 69.7 percent and 96 percent, respectively.

![Histogram](image)

Fig. 21--Histogram of gain-comparison measurements, 30-foot paraboloid.

The confidence levels versus the ranges from the mean of the sample whose histogram is shown in Fig. 21 are plotted in Fig. 22. The Tchebycheff inequality evaluated for arbitrary density function is shown in this latter figure. One can readily ascertain that the lower bounds on the confidence levels for the ranges of 6.6 and 10.5
percent are 62 and 85 percent, respectively. That is to say, the probability that a measured data point is within a range of 10.5 percent (of the mean) from the mean value of the sample is at least 0.85. Because of the relatively small range, less than one dB, within which the experimental points fall, Tchebycheff's inequality evaluated for arbitrary density function should not be applied to values of the range from the mean which exceed the 20 percent value of the mean (shown by dotted line on Fig. 22).

Because of the remarkably close conformance of the measured data points to the normal distribution, it is stated that the Tchebycheff inequality solved for normal density function is applicable to the sample whose histogram is shown in Fig. 21. It can be readily ascertained from Fig. 22 that the ranges for the 90 and 99 percent confidence levels are 6.6 and 10.5 percent respectively. As far as the sample itself is concerned, these are the confidence levels and ranges which should be used, rather than those that may be obtained from Tchebyscheff's inequality for arbitrary density functions.

The statements made in Appendix III in regard to samples which have been drawn from a normal population also apply to the sample whose histogram is shown in Fig. 21. An unbiased estimate of the population mean \( \mu \), is the mean of the sample, \( \bar{x} \). An unbiased estimate of the population standard deviation \( \sigma \) is \( s(n/(n-1))^{\frac{1}{2}} \).
where \( s \) is the standard deviation of the sample and \( n = 294 \) for Fig. 21; thus \( \sigma = s \) for all practical purposes.

In consequence of these assertions the true mean may be estimated to be the mean of the measured sample. The confidence level in the correctness of this estimate is 99.7 percent when the range from the mean is less than \( 3\sigma / \sqrt{n} \). Specifically, for the histogram of Fig. 21 the value of the \( 3\sigma / \sqrt{n} \) range that goes with the 99.7 percent confidence level is evaluated as

\[
3\sigma / \sqrt{n} = \pm 3 \times \frac{4.2\%}{\sqrt{294}} = \pm 12.6 / 17.15 = \pm 0.73 \text{ percent.}
\]

Additionally, at this point it is of particular interest to invoke the Central-limit Theorem in the sense stated in Reference 32:

"If an arbitrary population distribution has a mean \( \mu \) and a finite variance \( \sigma^2 \), then the distribution of the sample mean approaches the normal distribution with mean \( \mu \) and variance \( \sigma^2 / n \) as the sample size \( n \) increases".

Because of the applicability of this very strong theorem to the measured data, and because of the above assertions the range of \( \pm 0.73 \) percent (\( \pm 0.032 \) dB) with 99.7 percent confidence level will be used for the uncertainty due to randomness in evaluating the gain of the 30-foot paraboloid from the gain-comparison data.

Another method for measurements of this type has been proposed by Giger. [33] An extension of the technique described in
Ref. 33 is to make four measurements of the total received power at IF in rapid succession: first with the antenna under test, then with the gain-standard antenna while receiving a signal, and then repeat these two measurements without signal input. The total power in the first case consists of the satellite carrier and system noise:

\[(41) \quad P_I = g_1 [ \text{ERP} \times \text{Path Loss} \times G_{30'} + k T_{\text{sys}_{30'}} B] \]

where

- \( g_1 \) = receiver gain when antenna under test is connected to receiver
- \( \text{ERP} \) = effective radiated power of satellite
- \( G_{30'} \) = power gain of antenna under test
- \( k T_{\text{sys}_{30'}} B \) = noise power of system when 30' antenna is used.

In the second case Eq. (41) reads:

\[(42) \quad P_{\text{II}} = g_2 [ \text{ERP} \times \text{Path Loss} \times G_{3} + k T_{\text{sys}_{3}} B] \]

where

- \( g_2 \) = receiver gain when gain-standard antenna is connected to receiver
- \( G_{3} \) = power gain of standard antenna
- \( k T_{\text{sys}_{3}} B \) = noise power of system when gain-standard antenna is used.
Fig. 22—Lower and upper bounds on the confidence level for the measured gain to be within a given range from the mean value of the sample.
In the last two cases:

(43) \[ P_{III} = g_3 k T_{sys,3} B, \] and

(44) \[ P_{IV} = g_4 k T_{sys,3} B. \]

These last two measurements may be carried out by either pointing the antenna away from the satellite by a few degrees or simply turning off the transmitter, since the unexcited satellite output is 20 dB below the receiving sensitivity level when the 30-foot antenna is used. Equations (41) to (44) form a system of four equations with four unknowns: gain, ERP, and system noise powers. While it may be desirable to measure the four power levels in absolute terms, it is not necessary. One may obtain an expression for the gain of the large antenna which involves only the ratios of these powers and does not contain ERP and system noise power terms explicitly. This latter feature is very desirable since these parameters are difficult to measure accurately. From Eq. (42)

(45) \[ \text{ERP} \times P_{L.} = \frac{P_{II} / g_2 - k T_{sys,3} B}{G_3^t}. \]

Substituting Eq. (45) into Eq. (41)

(46) \[ \frac{P_{I}}{g_1} = \frac{P_{II} / g_2 - k T_{sys,3} B}{G_3^t} G_{30}^t P_{III} / g_3^t. \]
From Eq. (46)

\[
G'_{30} = \frac{P_{I}/g_1 - P_{III}/g_3}{P_{II}/g_2 - P_{IV}/g_4} \cdot G_3'
\]

\[
= G_3' \frac{P_{I}}{P_{II}} \frac{g_2}{g_1} \left[ \frac{1 - (P_{III}/P_{I})(g_1/g_3)}{1 - (P_{IV}/P_{II})(g_2/g_4)} \right].
\]

Let

\[
\frac{P_{I}}{P_{II}} = \alpha, \quad \frac{P_{I}}{P_{III}} = \beta, \quad \text{and} \quad \frac{P_{II}}{P_{IV}} = \gamma,
\]

then

\[
G'_{30} = G_3' \frac{g_2}{g_1} \frac{\alpha \gamma}{\beta} \frac{1}{\gamma - g_2/g_4}. \]

Assumptions inherent in arriving at Eq. (48) are that the ERP and Path Loss remain constant during the time necessary to complete the first two measurements; these are certainly valid assumptions for time intervals on the order of one minute. Based on experimental evidence it may be further assumed that the gain of the receiver also remains essentially constant, i.e.: \( g_1 = g_2 = g_3 = g_4 \), during one set of measurements. With these simplifying assumptions

Eq. (48) becomes

\[
G'_{30} = G_3' \frac{\alpha \gamma}{\beta} \frac{\beta - 1}{\gamma - 1}.
\]
The majority of the data was taken in a form suitable for the first technique of gain comparison measurement. On one occasion, data suitable for use with Eq. (49) were taken exclusively resulting in some 25 points. The mean value of these points is $53.01 \pm 0.45$ dB. This figure compares very well with the mean value established by the previous technique, the results of which are statistically more significant because of the several times larger number of samples.

b. Accuracy considerations and corrections

The above results obtained from the two gain-comparison techniques were based on the gain of the gain-standard antenna. This gain is $34.80 \pm 0.15$ dB [38], as has been verified recently (see footnote). Thus, any measurement with this antenna may contain a maximum uncertainty not exceeding $\pm 0.15$ dB.

The IF attenuator was an AIL Type 30 Precision IF Attenuator. The differential value of the setting of the precision IF attenuator was usually 22 dB. The incremental attenuation is claimed by the manufacturer to be accurate within $0.005$ dB per dB of indicated scale reading. [34] Thus, the measurements uncertainty from this source is $\pm 0.110$ dB.

The gain of gain-standard antenna was independently confirmed by Mr. R.C. Taylor of this laboratory on December 15, 1967 (after all satellite measurements were completed). He compared the signal from the 3-foot gain-standard with that from the standard gain horn (whose gain measurements are described in Chapter II) while the 3-foot gain-standard was in the configuration used to obtain satellite data and gain-comparison data. The average value of his measurements yields a gain of $34.84 \pm 0.22$ dB.
The voltmeter used was an Hewlett-Packard 3400A true rms voltmeter. The accuracy of this device at the 455 KHz IF frequency is ±1% of full-scale reading. Since the meter deflections were almost identical up-scale values regardless which antenna was connected to the receiver, the uncertainties in the meter deflections need not be considered. The recorders were calibrated before and after the experiments, the calibrating voltages could be referred to a standard cell. The effect between the record and playback processes was properly compensated for.

The incremental path loss in the gain-standard antenna was measured and found to be 1.87 dB. The uncertainty in this figure is ±0.02 dB.

The power loss, $P_L$, due to mismatch between the polarization patterns of the incoming signal and that of the receiving antenna can be calculated from [35]

$$P_L \text{ (dB)} = 10 \log_{10} \frac{(R_1^2 + 1)(R_2^2 + 1)}{(R_1 R_2 + 1)^2 \cos^2 \theta + (R_1 \pm R_2)^2 \sin^2 \theta}$$

where

- $R =$ voltage axial ratio, the ratio of the two orthogonal field vectors: $E_x / E_y$,
- $\theta =$ polarization mismatch angle, the angle between the major axes of the polarization ellipses, $0^\circ < \theta < 90^\circ$. 
In Eq. (50) the plus signs apply when the polarizations are in the same screw-sense, and the minus sign applies when the polarizations are in the opposite screw-sense, Equation (50) applies in the case when both polarizations are elliptical; when one of the polarizations is linear, Eq. (50) becomes

\[ P_L (dB) = 10 \log_{10} \frac{2 (R^2 + 1)}{(R^2 + 1) + (R^2 - 1) \cos 2\theta}. \]

The power loss due to polarization mismatch between the linearly polarized gain-standard antenna and elliptically polarized incoming signals of axial ratios between 1.10 and 1.30 is shown in Fig. 23. The axial ratios of the incoming signals were mostly 1.2, this could be measured with an accuracy of about 1%; the orientation of the major axis of the polarization pattern could be ascertained within about 5 degrees. The combined uncertainties in \( P_L \), resulting from the inaccuracies in the measurements of \( R \) and \( \theta \) are ±0.08 dB.

The power loss between the incoming signal and the large antenna due to polarization mismatch was always less than 0.15 dB, typically on the order of 0.05 dB. Figure 24 shows the calculated results of Eq. (50) for a particular satellite track. The calculations were repeated for every occasion when gain comparison data have been collected, Fig. 24 shows typical results of these calculations. The center field of this figure shows the results of the calculations.
based on Eq. (50) while $R_1$ was maintained constant at its measured value of 1.15, and $\theta$ and $R_2$ were varied about their measured values of $40^\circ$ and 1.18, respectively. From Fig. 24 at $R_1 = 1.15$, $R_2 = 1.18$, and $\theta = 40^\circ$ (this point is indicated by a star) the polarization mismatch loss is read as 0.041 dB. The values shown in the left and right fields of Fig. 24 were obtained similarly while $R_1$ was maintained constant at -0.05 and +0.05 unit respectively from its measured value. From Fig. 24 the typical value of the polarization mismatch loss can be evaluated and the magnitude of the effect of variations in axial ratios $R_1$, and $R_2$, and in angles between major axes, about the typical measured values can be ascertained. Since the polarization mismatch losses were repeated for every occasion when gain-comparison data have been collected, the actual calculated values and not typical ones were used in reducing gain and other measurements. Based on these calculations and on plots similar to Fig. 24, the uncertainty in the final gain figure from this source is conservatively set at $\pm 0.01$ dB. The polarization pattern of the 30-foot paraboloid is shown in Fig. 25.

The signals received by the gain-standard antenna were typically 13 dB above the noise level. This slight amount of contamination of the signal by the noise contributed to the spread in the measurements. For accurate results the measured signal-plus-noise to noise ratios had to be converted to signal-to-noise ratios. The
First ratio was measured to an accuracy of at least $\pm \frac{1}{2} \text{dB}$. The corrections applied to this ratio to convert it into the second one was on the order of one-fourth of a dB with an accuracy of $\pm 0.025 \text{dB}$.

Fig. 23--Power loss due to polarization mismatch between linearly polarized gain-standard antenna and elliptically polarized incoming signals for axial ratios between 1.10 and 1.30.

The SWR's of the gain-standard antenna and its waveguide connections, and the feed-horn of the large antenna and its waveguide connection were carefully measured. The conjugate mismatch losses based on these measurements are given in Table XIV.

Since the average conjugate mismatch loss is larger for the feed-horn of the large antenna than for the gain-standard antenna, the gain of the large antenna has to be corrected by $+0.079 \text{dB}$. The
inaccuracies resulting from these mismatch losses are taken to be
± 0.04 dB.

TABLE XIV
Standing Wave Ratios and Corresponding Conjugate Mismatch Losses

<table>
<thead>
<tr>
<th>SWR</th>
<th>Conjugate Mismatch Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
</tr>
<tr>
<td>Gain-Standard Antenna Waveguide</td>
<td>1.110</td>
</tr>
<tr>
<td>Feed-Horn Waveguide</td>
<td>1.138</td>
</tr>
</tbody>
</table>

The gain-standard antenna has a 3 dB beamwidth of 3 degrees, the beamwidth of the large antenna is one-tenth as large. The tracking of the satellites was automatic, using a sum-difference monopulse system. The estimated accuracy of tracking was taken to be one-tenth of the 3 dB beamwidth of the large antenna; this estimate was based on actual tracking experience. It is reasonable to assume normal distribution for the tracking process:

\[
(52) \quad P(\theta) = \left( \frac{1}{\sigma \sqrt{2\pi}} \right) \exp \left( -\frac{\theta^2}{2\sigma^2} \right)
\]

where
\[
\theta = \text{pointing angle}
\]
\[
\sigma = \text{rms tracking error}
\]
Fig. 24--Power loss due to polarization mismatch between elliptically polarized receiving antenna and elliptically polarized incoming signals.

Let the rms tracking error be defined as

\[(53) \quad \sigma = k \Omega \]

where

\[ k = \text{a numeric, } 0 < k < 0.3 \]

\[ \Omega = 3 \text{ dB beamwidth} \]
Equation (52) now becomes

\[ P(\theta) = \frac{1}{(2\pi k^2 \Omega^2)^{1/2}} \exp\left(-\theta^2/(2k^2\Omega^2)\right). \]  

It will be shown below that the far-field pattern, \( E(\theta) \), can be very closely approximated by the \((\sin x)/x\) function, thus the gain may be written as:

\[ G(\theta) = G_0 \left((\sin x)/x\right)^2. \]  

The function \((\sin x)/x\) equals one-half when \( x = 1.392 \) radians; let \( x = \alpha \), then

\[ a = x/\theta = 1.392/(\Omega/2) = 2.784/\Omega \]

Thus, Eq. (55) becomes

\[ G(\theta) = G_0 \left((\sin 2.784\theta/\Omega)/(2.784\theta/\Omega)\right)^2 \]

The expected value of the gain, \( E\{G(\theta)\} \), is defined by the integral [36]

\[ E\{G(\theta)\} = \int_{-\infty}^{\infty} G(\theta) \, p(\theta) \, d\theta \]

\[ = G_0 \left( \int_{-\pi/2}^{\pi/2} \frac{\exp(-\theta^2/(2k^2\Omega^2)) \cdot ((\sin 2.784\theta/\Omega)/(2.784\theta/\Omega))^2}{(2\pi k^2 \Omega^2)^{1/2}} \right) d\theta \]
The variance of the gain, \( \sigma^2 \), from the expected value is given by:

\[
\sigma^2 = E \left\{ \left[ G(\theta) - E \{ G(\theta) \} \right]^2 \right\} = E \left\{ G^2(\theta) \right\} - E^2 \left\{ G(\theta) \right\},
\]

where

\[
E \left\{ G^2(\theta) \right\} = \int_{-\infty}^{\infty} G^2(\theta) p(\theta) \, d\theta
\]

\[
= G_0^2 \int_{-\pi/2}^{\pi/2} \exp \left( \frac{-\theta^2}{(2 \, k^2 \, \Omega^2)} \right) \frac{1}{(2 \pi \, k^2 \, \Omega^2)^{1/4}} \, d\theta \cdot \left( (\sin 2.784 \, \theta/\Omega)/(2.784 \, \theta/\Omega) \right)^4 \, d\theta.
\]

In Eqs. (58) and (60) the infinite limits follow from the definition; these have been replaced with finite limits consistent with the practical values of \( \theta \). The positive square-root of the variance is the standard deviation \( \sigma \). Thus, to find the average value of the gain under the assumed conditions that the tracking errors have a normal distribution and that the far-field pattern is well approximated by the \((\sin x)/x\) function it is necessary to evaluate the integral of Eq. (58). The difference between the results and \( G_0 \) may be called as the expected gain-loss due to tracking inaccuracies.

To find the standard deviation about the average value, Eq. (59) has to be evaluated. This can be done by inserting the results of Eqs. (58) and (60) into Eq. (59). A computer program was written to
evaluate these integrals numerically. For simplicity in the computations, the following notation was adopted: \( \theta \Omega = kz \), \( d\theta = k\Omega \, dz \), and

\[
\frac{E \{ G(\theta) \}}{G(\theta)} = \sqrt{\frac{2}{\pi}} \int_0^\infty \left( \frac{\sin (2.784kz)}{2.784 \, kz} \right)^2 \, e^{-z^2/2} \, dz,
\]

\[
\frac{E \{ G^2(\theta) \}}{G^2(\theta)} = \sqrt{\frac{2}{\pi}} \int_0^\infty \left( \frac{\sin (2.784kz)}{2.784 \, kz} \right)^4 \, e^{-z^2/2} \, dz.
\]

The computations were truncated after an increase in the upper limit of the integration produced negligible change in the result. The results of the computations are plotted in Fig. 26. The horizontal scale is the ratio of the tracking error to the half-power beamwidth, the vertical scale is gain-loss in decibels. From the curves one can determine the average gain-loss and the one and two standard deviations from the expected value of the gain. Thus, for a tracking error not exceeding one-tenth of the 3 dB beamwidth, (i.e., 2 minutes of arc) the average gain-loss does not exceed 0.11 dB, the magnitude of one standard deviation is 0.15 dB and that of two standard deviations is 0.31 dB. On basis of the results of these calculations, the gain of the large antenna should be increased by + 0.11 dB.
Fig. 25--Measured polarization pattern of 30-foot paraboloid.

The above uncertainties and corrections for the measured gain of the large antenna, as determined from the comparison measurements with the gain-standard antenna, are given in Table XV. The sum of the corrections is +0.19 dB. The absolute gain of the 30-foot paraboloid based on the gain-comparison measurement technique is 53.35 dB. The first seven entries are, in the opinion of the author, calibration errors; these may cause a shift in the
Fig. 26--Average gain-loss and standard deviations from the expected value of the gain as function of tracking error to half-power beamwidth ratio.
mean value. The next three entries are random errors; these contribute to the spread of the data. The spread of the data calculated with a very high level of confidence on the basis of actual measurements is given by the last entry labeled Randomness. The possible maximum effect of the calibration errors will be calculated next.

**TABLE XV**

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain Standard</td>
<td>± 0.15 dB</td>
<td>-</td>
</tr>
<tr>
<td>IF Attenuator</td>
<td>± 0.11</td>
<td>-</td>
</tr>
<tr>
<td>Meter Deflections</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Path Differential</td>
<td>± 0.02</td>
<td>-</td>
</tr>
<tr>
<td>Polarization Mismatch, Standard Antenna</td>
<td>± 0.08</td>
<td>-</td>
</tr>
<tr>
<td>Polarization Mismatch, Large Antenna</td>
<td>± 0.01</td>
<td>-</td>
</tr>
<tr>
<td>Signal-to-Noise-Ratio Correction</td>
<td>± 0.03</td>
<td>-</td>
</tr>
<tr>
<td>Conjugate Mismatch Losses</td>
<td>± 0.04</td>
<td>+ 0.079</td>
</tr>
<tr>
<td>Tracking</td>
<td>± 0.15</td>
<td>+ 0.11</td>
</tr>
<tr>
<td>Noise Contamination</td>
<td>± 0.05</td>
<td>-</td>
</tr>
<tr>
<td>Randomness</td>
<td>± 0.03</td>
<td>-</td>
</tr>
</tbody>
</table>
Equation (40) may be rewritten as

\[
G_{30}'(1 \pm \Delta G_{30}^{'}/G_{30}') = \frac{(G_3^{'1} \pm \Delta G_3^{'1})(\Delta L \pm \Delta(\Delta L))(M_{30}'/M_3^{'})}{(P_{L3}' \pm \Delta P_{L3}^{'})} \cdot (L_{\text{guide}} \pm \Delta L_{\text{guide}}) \cdot \left(\frac{S+N}{N}\right)^{3'}
\]

Substituting the maximum possible uncertainties due to calibration errors from Table XV in place of the quantities with \( \Delta \) prefixes in Eq. (63), carrying out the indicated multiplications, and neglecting terms which are orders of magnitude smaller than others, Eq. (63) reduces to

\[
G_{30}'(1 \pm \Delta G_{30}^{'}/G_{30}') = \frac{G_3^{'1} \Delta L (M_{30}'/M_3^{'}) P_{L30}^{'((S+N)/N)}_{3'}}{L_{\text{guide}} P_{L3}^{'}}
\]

\[
(1 \pm \Delta G_3^{'}/G_3^{'}) = \frac{(1 \pm \Delta L_{\text{guide}}/L_{\text{guide}} \pm \Delta P_{L3}'/P_{L3}^{'})}{\Delta ((S+N)/N)_{3'}}
\]

\[
G_{30}'(1 \pm \Delta G_{30}^{'}/G_{30}') = \frac{G_3^{'1} \Delta L(M_{30}'/M_3^{'}) P_{L30}^{'((S+N)/N)}_{3'}}{L_{\text{guide}} P_{L3}^{'}} = (1 \pm 0.95)
\]

On the basis of Eq. (65) the maximum possible calibration errors do not exceed \( \pm 9.5 \) per cent (\( \pm 0.39 \) dB).
2. **Radio-astronomical measurements**

a. **Results of measurements**

i. **Cassiopeia-A**

Radio-astronomical methods are well accepted as means for calibrating large antennas,[37] and radio-stars and the moon provide attractive sources for the measurements. These measurements are also comparative ones, the standard of the comparison is the antenna system with which the source was calibrated originally. From the many radio-star sources available for observations -- see for example the third Cambridge Catalog for a dependable listing -- [38] Cassiopeia-A (3C461) was selected for the measurements. The physical properties of the source are well known,[39] excluding the sun, this radio-star is the strongest at X-band frequencies; its angular size is only four minutes of arc between half-intensity points; it is symmetrical about its center; it is randomly polarized; and it is observable at any time of the day from the OSU antenna site. In addition to the above physical properties, the accurately known flux density [40] and spectrum [41] of the source made it a natural choice for observation.

The block diagram of the system used for making radio-astronomical measurements is shown in Fig. 27. It is a Dicke-type [42] radiometer operated under nearly balanced conditions. The balancing is achieved by inserting some known amount of noise
Fig. 27—Block diagram of radiometer.
power into the antenna line. [43, 44] Balancing is very desirable from the standpoint of reducing the threshold due to gain fluctuations and thus improving stability. The method chosen for balancing does away with the need for cooling the comparison-load to the extent that its temperature is the same as that of the antenna-line, at the expense of slightly reducing the sensitivity of the system. Since the equivalent system noise temperature is about 630°K, this method was more practical. The theoretical sensitivity, ΔT min, of the radiometer is given by [45, 46]

\[ \Delta T_{\text{min}} = \frac{(\pi / \sqrt{2}) (T_A + T_R)}{(B \tau)^{\frac{1}{2}}} \]

where

- \( T_A \) = effective antenna-line noise temperature \( \sim 290°K \)
- \( T_R \) = noise temperature of the receiver \( \sim 630°K \)
- \( B \) = pre-detection noise bandwidth \( \sim 7.549 \) MHz
- \( \tau \) = equivalent integration time constant of low pass filter, \( \tau_{RC} \approx 1.2 \) sec, \( \tau = 2 \tau_{RC} \)

which yields a minimum detectable temperature of 0.48°K. In practice, Eq. (66) is that increase in deflection which is equal to the rms value of the fluctuations. The measured value of the rms fluctuations is twice as large as that calculated from Eq. (66), thus the sensitivity is about 1°K. The linearity of the system was
measured and found to be excellent, the linear range extended over 
$\pm 100^\circ K$ from the usual point of operation. The stability of the 
system, when balanced, was better than $0.5^\circ K/hr$.

When the radio-star is in the antenna beam, there results a 
change, $\Delta T$, in the antenna temperature. The change in noise power, 
$\Delta P$, is proportional to $\Delta T$:

\[ \Delta P = \bar{t} k \Delta T B, \]

where

$\bar{t}$ = power transmission coefficient due to losses in the 
waveguide (and in the atmosphere), and

$k = 1.38 \times 10^{-23}$ joules/$^\circ K$

The change, $\Delta P$, is directly proportional to the flux density of the 
star

\[ \Delta P = \frac{1}{2} S A_e B \]

where

$\frac{1}{2}$ = correction factor due to the fact that incoming 
radiation is randomly polarized and reception is 
discretely polarized (left-hand circular)

$S$ = flux density of star, watts/m$^2$/Hz,

$A_e$ = effective aperture of antenna.

The relationship between antenna gain, $G$, and effective aperture is

\[ G = 4\pi A_e / \lambda^2 \]
where

\[ \lambda = \text{the wavelength corresponding to the frequency of operation.} \]

Combining Eqs. (67), (68), and (69), the antenna gain is

\[ G = \frac{8 \pi k}{S \lambda^2} \cdot \tau \Delta T, \]

\[ = \frac{8 \times 3.141593 \times 1.38046 \times 10^{23} \times (7.276704 \times 10^9)^2}{0.680 \times 10^{-23} \times (2.997930 \times 10^8)^2} \cdot \tau \Delta T, \]

\[ = 3.005 \times 10^4 \cdot \tau \Delta T. \]

where the flux density \( S (S = 1.086 \times 10^{-23} \text{ watts/m}^2/\text{Hz}, \text{ frequency } = 4080 \text{ MHz, Epoch } = 1964.7) \) as reported in reference 40 has been corrected for the date (Epoch: 1967.8, secular decrease: 1.1 percent per annum) and the frequency of observation (\( f = 7276.704 \text{ MHz, spectral index } = -0.75 \)), and the resulting flux density of \( 0.680 \times 10^{-23} \text{ watts/m}^2/\text{Hz} \) has been used.

Most of the data were taken in the form of drift curves. The antenna was positioned ahead of the expected position of the star and the output of the radiometer was recorded as the star passed through the beam. For calculation of look angles for a celestial body see Appendix IV. Two typical drift curves are shown in Fig. 28. From the drift curves and from the numerous efforts of manually peaking
the antenna a total of 48 records were collected. For each of these records it was verified that the star was unmistakably at, or very nearly at, the peak response of the antenna pattern. These records were divided into intervals of five seconds duration. The average value of the deflection in each interval was taken as one independent data point. The assumption of statistical independence is based upon the careful evaluation of the effects of the apparent maximum velocity of the star, the beamwidth, the smooth shape of the pattern, and of the effects of the integration time constant on the height and shape of the deflection.[37] In this manner some 177 assumed to be statistically independent data points were collected; the distribution of these points is shown as a histogram in Fig. 29. The arithmetic mean of the data is $5.38^\circ K$, the standard deviation is $0.45^\circ K$. Since most of the data were collected at near upper culmination (upper culmination is $71.0^\circ$), the value of $t$ in Eq. (67) and subsequent ones depends almost exclusively on the waveguide circuits between the output of the feed-horn and the input into the tunnel diode amplifier. This path loss has been very accurately determined and was found to be $1.12 \pm 0.02$ dB. Combining these values with Eq. (70) one finds that the absolute gain, $G_0$, of the antenna as determined by radiometric measurements on Cassiopeia is $53.20$ dB.
To the above figure of 53.20 dB for the absolute gain of the antenna the following corrections apply. The compensation for the small but finite size of the source relative to the half-power beam-width of the antenna is +1.6 per cent. [39, 40] The uncertainty in this correction figure is ±0.16 per cent, and it is a calibration error.

The compensation for the reduction of the peak deflection due to the integration time is 1.0 per cent. [37] The uncertainty in this correction figure is ±0.1 per cent and it is a calibration error. The sum of the corrections is +2.6 per cent (+0.11 dB). Applying these corrections to the gain, the absolute gain is 53.31 dB.

The uncertainty due to randomness will be taken as equal to

\[ \pm \frac{3\sigma}{\sqrt{n}} = \pm 3 \times 0.45/\sqrt{177} = \pm 0.11^\circ \text{K} = \pm 2 \text{ per cent} \ (\pm 0.086 \text{ dB}) \]

with 99.7 per cent confidence. The other uncertainties enumerated in the following are all calibration errors. The uncertainty in the attenuation between the feed-horn and the input to the tunnel diode amplifier is 0.5 per cent. The flux density of Cassiopeia-A is reported [40] with a probable error of approximately 2 per cent based upon a 6.3 per cent limit for 99 per cent level of confidence, hence the uncertainty from this source is ±2.0 per cent. The probable uncertainty in the value of the spectral index is 0.01 unit, the uncertainty from this source is ±1 per cent. The probable error in the correction for the secular decrease of the flux density of the star is ±0.47 per cent.
In order to evaluate the maximum possible effects of the calibration errors, one may start with Eq. (70) and proceed as in Eqs. (63), (65). It may be shown that when the above enumerated maximum possible uncertainties - except that which is due to randomness - is substituted into Eq. (70) in the manner shown in Eq. (63) the result is

\[ G \left(1 \pm \frac{\Delta G}{G}\right) = G(1 \pm 0.044). \]

On the basis of Eq. (71) the maximum possible calibration errors do not exceed \( \pm 4.4 \) percent (\( \pm 0.187 \) dB) in the radio star data.

Additionally, the data points are spread about a mean value due to randomness. The random error is 2 percent, the confidence level is 99.7 percent.

ii. Moon

The moon is another convenient extra-terrestrial source that may be used for antenna calibration. The same radiometer which was used for the radio-star was also used for measuring the maximum change in antenna temperature when the moon was in the center of the beam. The method of collecting data was again to pre-position the antenna and to record the radiometer output as the moon swept through the beam, although at times it was possible to track it automatically with the sum-difference monopulse system. Two consecutive drift curves are shown in Fig. 30. These particular
Fig. 28--Two drift curves of Cassiopeia-A through the beam of the antenna.

Fig. 29--Histogram of gain measurements with the radio star Cassiopeia-A.
curves are typical ones; they were obtained by pre-positioning the
elevation axis while the azimuth axis was controlled by the monopulse
tracking.

Fig. 30--Two drift curves of the Moon through the beam of the antenna.

The brightness temperature, $T_b$, of the moon and the energy
flux at its surface, assuming black-body radiation, are given by
Planck's law

$$S = \frac{2 \pi f^2}{c^2} \frac{hf}{e^{hf/kT_b} - 1}$$

where

$S$ = flux density, watts/m$^2$/Hz

$f$ = frequency, Hz

$c$ = speed of light, $2.99793 \times 10^8$ m/sec

$h$ = Planck's constant $6.6254 \times 10^{-34}$ watt-sec$^2$

$k$ = Boltzmann's constant, $1.38046 \times 10^{-23}$ watt-sec/°K

$T$ = brightness temperature, °K

For $hf < kT_b$ Planck's law of thermal radiation from an ideal black
body is sufficiently well approximated by the Rayleigh-Jeans formula
Far away from the black body, the flux is reduced by a factor of \((r_1/r_2)^2\), where \(r_1\) = radius of the black body sphere, and \(r_2\) = distance of the observer from the center of the black body. For the moon as observed from the earth Eq. (73) becomes

\[
S = 2\pi k T / \lambda^2 \cdot \frac{r_1}{r_2}^2.
\]

The best available measurements \([37, 47]\) list the brightness temperature of the moon at X-band frequencies as consisting of a steady component, \(T_{eo}\), and a sinusoidal one of amplitude \(T_{e1}\)

\[
T_b = T_{eo} + T_{e1} \cos (\Omega t - \xi)
\]

where

\[
T_{eo} = 216^\circ K\]
\[
T_{e1} = 16^\circ K\]

\(\Omega\) = angular velocity of rotation, \(2\pi / t_0\) where

\(t_0\) = period of rotation

\(t\) = time

\(\xi\) = phase lag ~ 40°

Combining Eqs. (74), (75), and (70) and using the value of \(\bar{t} = 1.12\) dB given above, the gain of the antenna is

\[
G = \frac{2.53 \times 10^5 \Delta T}{216 + 16 \cos (\Omega t - 40^\circ)} \cdot \frac{\Omega_s}{\Omega_s^*}
\]
where

$$\frac{\Omega_s}{\Omega_s'} = \text{correction factor for the finite size of the source.}$$

Using the notation of Ref. [41], the source solid angle, $\Omega_s$, is defined as

$$(77) \quad \Omega_s = \int \int_{\text{source}} \psi(\theta, \phi) \sin \theta \, d\theta \, d\phi,$$

and the effective source solid angle, $\Omega_s'$, as

$$(73) \quad \Omega_s' = \int \int_{\text{source}} \psi(\theta, \phi) \, P(\theta, \phi) \sin \theta \, d\theta \, d\phi$$

where

$$\psi(\theta, \phi) = \text{normalized distribution function of the brightness temperature of the source, } \psi_{\text{max}} = 1,$$

$$P(\theta, \phi) = \text{normalized power pattern of the antenna, } P_{\text{max}} = 1.$$

The correction factor is greater than unity for sources of finite size, it is very nearly equal to unity for sources which are essentially point sources when their angular diameter is compared to the half-power beamwidth of the antenna. The distribution function for the moon is normally taken to be a constant: $T_b$. The antenna power pattern over solid angle subtended by the moon is very closely given by the function $(\sin^2 x)/x^2$, furthermore, it is circularly symmetrical. (It is possible that because of circular symmetry the function
(J_1(x)/x)^2 provides a better fit to the pattern than the ((sin x)/x)^2 function. However, for the purposes here the latter provided a good enough fit, what is more; the computations were greatly facilitated by this choice.) Evaluating Eq. (77) one obtains for the circularly symmetrical moon

\[
(79) \quad \Omega_s = \iint \psi(\theta, \phi) \sin \theta \, d\theta \, d\phi \cdot T_b \sin \theta \, d\theta \, d\phi
\]

\[
\leq 2\pi T_b \int_0^{\theta_{\text{max}}} \theta \, d\theta = \pi T_b (\theta_{\text{max}})^2
\]

where

\[2 \theta_{\text{max}} = \text{angular diameter of the moon}, \sim 30^\circ.\]

Similarly evaluating Eq. (78) one obtains

\[
(80) \quad \Omega'_s = \iint \psi(\theta, \phi) P(\theta, \phi) \sin \theta \, d\theta \, d\phi
\]

\[
\leq 2\pi T_b \int_0^{\theta_{\text{max}}} \frac{\sin^2 x}{x^2} \theta \, d\theta
\]

where

\[x = \frac{2.274}{\theta_{\text{max}}} \theta, \theta \text{ is given in radians},\]

\[d\theta = dx/(2.274/\theta_{\text{max}})\] and the limits are:

\[\theta = 0, x = 0; \theta = \theta_{\text{max}}, x = 2.274\]
Substituting these into Eq. (80) one obtains

\[ \Omega_s' = 2\pi T_b \int_0^{2.274} \frac{\sin^2 x}{x^2} \frac{x}{(2.274/\theta_{\text{max}})(2.274/\theta_{\text{max}})} \, dx \]

This latter equation can be rapidly and accurately evaluated by replacing the \( \cos 2x \) term by the first few terms in its power series expansion. Using the first eight terms, the value of the integral is 1.1437344. Combining this result with the above computations, the correction factor is

\[ \frac{\Omega_s}{\Omega_s'} = \frac{\pi T_b (\theta_{\text{max}})^2}{2\pi T_b (\theta_{\text{max}}/2.274)^2 (1.14373)} = \frac{(2.274)^2}{2.28746} = 2.26. \]

Substituting this value into Eq. (76), the expression for the gain becomes

\[ G = 5.71 \times 10^5 \frac{\Delta T}{216 \div 16 \cos (\Omega t - 40^\circ)} \]

With the aid of the American Ephemeris and Nautical Almanac, the denominator in Eq. (83) is evaluated for the date of observation (October 27, 1967) as 216 - 7 = 209 K. The average value of \( \Delta T \) based on some 60 data points is 76.5 K; the computed standard
deviation is $\pm 4.1^o$K. Substituting these values into Eq. (83), the absolute gain, $G_0$, of the 30 foot paraboloidal reflector as determined by radiometric measurements on the moon is 53.2 dB. Because of the inaccurate knowledge about the brightness temperature and its distribution, because of the large temporal and insolation dependent variations in the brightness temperature, and because of the simplifications assumed in the corrections for the size of the source, a probable upper limit of $\pm 1$ dB uncertainty is suggested for the gain figure obtained by radiometric measurements using the moon for source.

C. Antenna Patterns

The shape of the far-field power pattern of the antenna is of great practical concern since it is the spatial distribution of radiated energy. On three separate occasions, using three different satellites as far-field sources, three sets of two orthogonal (elevation and azimuth) pattern cuts were generated. The experimental procedure consisted of tracking the satellite with the monopulse system, while the motion of the antenna under test was controlled by the servo-loop connected to the tracking antenna. Accurately controlled error-voltages were inserted into the servo-loop, which produced incremental offsets of two minutes of arc in the position of the antenna. The antenna was thus moved off target by one degree in about 25
increments. The deflection of the true rms voltmeter was recorded, this deflection was maintained about the same throughout the measurement by adjusting the precision IF attenuator (see Fig. 20). The meter deflection and the attenuator settings were converted into received power readings, normalized to unity when the target was on axis. The received power readings together with the known offset voltages, converted into angular increments, yield the far-field pattern of the antenna. The pattern cut along the elevation axis is a true cut; it is taken along a great circle. The cut along the azimuth axis, however, is not taken along a great circle and thus it is not a true cut. To correct the azimuth pattern, the measured angular effects must be multiplied by the cosine of the elevation angle. This correction is adequate for the small (less than one degree) incremental azimuth motions involved in taking the patterns. The measured elevation and azimuth pattern cuts are shown in Figures 31 and 32 respectively. The data points have been corrected for the effects of the receiver noise at low signal-plus-noise to noise ratios. Superimposed upon part of the elevation pattern is the function \(\frac{\sin^2 x}{x^2}\). It can be seen that this function provides a very good fit for the section of the pattern between the "10 dB down" points. The 3 dB beamwidth is taken to be 20 minutes of arc. The azimuth pattern cut corrected for the effect of nonzero elevation angle is replotted in Fig. 33. The scales on Fig. 33 are the same as those of Fig. 31. The beamwidth
Fig. 31--Measured antenna power patterns, elevation cut.

between first nulls in the azimuth plane is about 45 minutes of arc, the first side lobes are 21 dB below the peak of the main beam.

Integration of the measured antenna power pattern could, in principle, be used to obtain the gain of the antenna if the losses were accurately known, and if accurate patterns were available over a solid angle of $4\pi$. It is not practical to attempt to fulfill either of these conditions for large antennas. Although the losses are not
Fig. 32--Measured antenna power patterns, azimuth cut.

known in this case, and the pattern is known accurately only over ±1 degree from on-axis, it is useful to carry out the integration since it yields a figure for the directivity which is an upper limit on the gain. The directivity of the antenna for the principal polarization is [48]
Fig. 33--Measured antenna power patterns, azimuth cut, corrected for elevation angles.

$$D = \frac{4\pi P(\theta = 0, \phi = 0)}{2\pi \int \int P(\theta, \phi) \sin \theta \, d\theta \, d\phi}.$$  

Normalizing by $P_{\text{max}} = P(\theta=0, \phi=0)$, assuming circular symmetry of $P(\theta, \phi)$, as well as negligibly small cross polarized response, and replacing $\sin \theta$ by $\theta$, Eq. (84) becomes
Rewriting the integral in Eq. (85)

\[
D = \frac{2\pi}{\left( P(0,0) \right)_{\theta=0}^{\theta=58^\circ}} \int_{0}^{\pi} \left( \frac{(\sin k\theta)/(k\theta)^2}{(P(\theta,\phi)/P(0,0))_{\theta=0}^{\theta=58^\circ}} \right) \theta d\theta
\]

The integrands for the first two integrals of Eq. (86) are plotted in Fig. 34. It appears from this figure that the principal contribution to the gain is from the main lobe. The first integral is not unlike Eq. (81), its value is \(1,1437 \, k^2 = 5.1985 \times 10^{-6}\), where \(k\) arises from the horizontal coordinate conversion in fitting the \((\sin^2 x)/x^2\) function to the pattern. The second integral was evaluated by obtaining the values for the integrand from Fig. 31. Its value is \(1.5772 \times 10^{-6}\). In the third integrand \(\sin \theta\) has not been replaced by \(\theta\) for obvious reasons, and an average steady level of amplitude \(K\) relative to the pattern maximum has been assumed. The value of this integral is \(1.9986 K\). Thus, the directivity is
Fig. 34--The value of the function $(P(\theta, \phi)/P(\theta = 0, \phi = 0)) \cdot \theta$ versus the pattern angle $\theta$.

(87) \[ D = 2/((5.1985 + 1.5772) \times 10^{-6} + 1.9986 K) \, . \]

For negligibly small $K$, $D$ is $2.96 \times 10^{-5} = 54.7 \text{ dB}$. The effect of the magnitude of $K$ on the directivity is plotted in Fig. 35. Since
the measured gain was 53.3 dB, it appears that the magnitude of $K$ is about -59 dB (6 dB below isotropic level). From the results of Fig. 35, it may also be said that for accurate calculated gain the pattern should be measured precisely down to about 10 dB below the isotropic level. In addition to the information obtained about the spatial distribution of radiated energy, on the basis of the results presented in this section it may be concluded that the gain of the antenna must be less than 54.7 dB; alternately, the maximum efficiency does not exceed 65 per cent.
D. Antenna Noise Temperature

The antenna noise temperature, $T_A$, for a lossless antenna, when the antenna is pointed to a specific direction $\theta_o, \phi_o$, is the weighted average of the temperature of the radiation impinging on the antenna. [49]

\begin{equation}
T_A(\theta_o, \phi_o) = \frac{\int_{\Omega} T_p(\theta, \phi) P_p(\theta, \phi) d\Omega}{\int_{\Omega} P_p(\theta, \phi) d\Omega} + \frac{\int_{\Omega} T_c(\theta, \phi) P_c(\theta, \phi) d\Omega}{\int_{\Omega} P_c(\theta, \phi) d\Omega}
\end{equation}

where

$T(\theta, \phi)$ = temperature distribution on the sphere surrounding the antenna

$P(\theta, \phi)$ = weighting function, antenna power pattern

p, c = subscripts denoting principal and cross polarization

(left-hand circular polarization and right-hand circular polarization, respectively)

$d\Omega = \text{element of solid angle, } \sin \theta d\theta d\phi$.

It is normally assumed that the cross-polarized response of the antenna is negligible in comparison with the response on principal polarization. The numerator in Eq. (88) is evaluated with the aid of Eq. (84).
\[ \int_{\Omega} P(\theta, \phi) \, d\Omega = (4\pi/D) \, P(\theta = 0, \phi = 0). \]

Substituting Eq. (89) into Eq. (88) and replacing \( D \, P(\theta, \phi)/P(\theta=0,\phi=0) \) with \( G(\theta, \phi) \) one obtains the commonly used form:

\[ T_A = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} T(\theta, \phi) \, G(\theta, \phi) \, \sin\theta \, d\theta \, d\phi. \]

Thus, if the distribution function \( T(\theta, \phi) \), of the brightness temperature on the sphere surrounding the antenna were known, together with the spatial power response, \( G(\theta, \phi) \), of the antenna one could, at least in principle, find the antenna temperature. The distribution function, however, at X-band frequencies is dependent on a host of variables, some of these are: time of the day, season of the year, atmospheric conditions, and weather. A more practical approach is to simply measure the antenna temperature and its variations as function of elevation and azimuth angles. The results of the measurements of the changes in antenna noise temperature as function of elevation angle are shown in Fig. 36. The changes are shown relative to the temperature of the antenna at zenith. The absolute magnitude of \( T_A \) at zenith, calculated from radiometric measurements corrected for waveguide circuit losses, and for noise power coupled into the antenna line from a noise source, at the time of observation is about \( 10^0K \).
Fig. 36--Changes in antenna temperature as function of elevation angle.

The variations in the antenna noise temperature as function of azimuth angle measured at elevation angles of 0.5 and 2 degrees is shown in Figures 37 and 38, respectively.
Fig. 37--Antenna noise temperature as function of azimuth angle. Elevation angle = 0.5°.

Fig. 38--Antenna noise temperature as function of azimuth angle. Elevation angle = 2.0°.
E. Conclusions

The absolute gain of the 30-foot paraboloid has been measured by gain comparison techniques, by radiometric measurement on Cassiopeia-A, and by radiometric measurements on the Moon. The directivity has been calculated by integrating the measured power pattern of the antenna. This calculation yields an upper limit on the gain. The results are given in Table XVI.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Absolute Gain</th>
<th>Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Random dB</td>
</tr>
<tr>
<td>Gain Comparison (Eq. 40)</td>
<td>53.35 dB</td>
<td>± 0.03 dB</td>
</tr>
<tr>
<td>Gain Comparison (Eq. 49)</td>
<td>53.01 ± 0.45</td>
<td>---</td>
</tr>
<tr>
<td>Radiometric, Cassiopeia-A</td>
<td>53.31 ± 0.09</td>
<td>.997 %</td>
</tr>
<tr>
<td>Radiometric, Moon</td>
<td>53.2 ± 1</td>
<td>---</td>
</tr>
<tr>
<td>Pattern Integration</td>
<td>&lt;54.7</td>
<td>---</td>
</tr>
</tbody>
</table>

1 Maximum possible errors (see Eqs. (65), and (71)).
The gain of the thirty-foot paraboloid has been determined by five methods. Of the results of these five methods the statistically most significant results, and also the statistically independent results, are the first and third entries in Table XVI: the result of the gain comparison technique and the result of the radiometric technique. These measurements yielded two figures which are within 0.04 dB; this is phenomenal. In both instances the random errors were successfully reduced to relatively small values with very high confidence levels through the application of the Central-limit Theorem and Statistical Estimation Theory. The maximum possible calibration errors are twice as large in the gain-comparison experiment than in the radiometric measurements. Summing the two sources of errors a definite upper limit on all uncertainties may be obtained for each of the two measurements: gain-comparison:

$$53.35 \pm 0.42 \text{ dB;}$$ radiometric, Cassiopeia-A: $$53.31 \pm 0.28 \text{ dB.}$$ It is proposed that the actual gain value is the mid-value: $$53.31 + (53.35 - 53.31)/2 = 53.33 \text{ dB.}$$ It is also proposed that the maximum possible uncertainty is the mutual region of overlap of the summed uncertainties in the two measurements: $$\pm 0.30 \text{ dB (±7.2 per cent).}$$

The gain of the thirty-foot paraboloid will be taken as 53.33 $$\pm 0.30 \text{ dB at 7.3 GHz.}$$ The uncertainty of $$\pm 0.30 \text{ dB is to be regarded as an upper limit.}$$
The measured polarization pattern of the antenna is shown in Fig. 25. The axial ratio is 1.16; the orientation of the major axis is 35 degrees, measured clock-wise from the local horizontal.

Tracking inaccuracies not exceeding one-tenth of the beam-width result in gain-loss that is on the order of one-tenth of a dB.

The far-field power patterns of the antenna are shown in Figs. 31 and 33. The pattern is essentially symmetric; the half-power beamwidth is 20 minutes of arc; the beamwidth between first nulls is about 45 minutes of arc; the first sidelobes are 21 dB below the on-axis peak; the pattern is accurately approximated between the -10 dB points by the function \((\sin x)/x^2\). The average level of the far-out side lobes is about 6 dB below isotropic level.

The antenna noise temperature at zenith is about 100 K. The variations in antenna temperature as function of elevation and azimuth angles (at 0.5 and 2 degree elevation angles) are shown in Figs. 36, 37 and 38.
CHAPTER V
SATELLITE PERFORMANCE CHARACTERISTICS

A. Introduction

For proper utilization and evaluation of communication satellites it is necessary to know their performance characteristics. These, together with the characteristics of the user stations, determine the total communication system performance. The other chapters of this study deal with the characteristics of the station; the purpose of this chapter is to present measurements of the parameters of the satellites.

The principal parameters of the satellite which characterize its performance are the maximum effective radiated power (ERP), the power transfer \( P_T = \frac{P_{out}}{P_{in}} \), where \( P_{out} = \text{ERP} \), and \( P_{in} \) = incident power density at the satellite times the effective aperture of the receiving antenna), and the bandwidth. Additionally, it is desirable to know the polarization pattern, the beacon suppression, and the frequency and amplitude stability of the satellite signal. In order to establish these parameters the performance characteristics of seven of the near-synchronous IDCSP (Interim Defense Communication Satellite Program) satellites were measured: three were from the first launch, designated as I-1, I-2, and I-4, and
four from the second (successful) launch: II-1, II-2, II-4, and II-5.
The catalogue of satellite trackings and data collected is shown in Table XVII.

In Table XVII the entry "Gain" refers to gain comparison experiments to determine the gain of the 30 foot paraboloidal receiving antenna; "Polarization" refers to measurements of the polarization pattern of the received signal; "ERP" refers to measurements of the effective radiated power; "P_T" refers to measurements of the power transfer curve; "Bandwidth" refers to measurements of the bandwidth; and "Pattern" refers to measurements of the pattern of the receiving antenna. Subscripts "1" indicate measurements carried out on the communication channel frequencies (7276.7 MHz); subscripts "2" indicate measurements on the beacon channel frequency (7299.5 MHz).

Additionally, as a matter of routine, the position of the satellite, the absolute frequency, the amplitude scintillations of the received signal, and the calibration of the system were recorded on most trackings.

B. Satellite Parameters

1. Effective Radiated Power

Perhaps the most important parameter that characterizes the performance of a communication system employing the near-synchronous IDCSP satellites is their effective radiated power. In
TABLE XVII
Catalogue of Satellite Trackings and Data Collection

| I-1  | 7-14-67 | Polarization₁; ERP₁, 2 |
|-------------------------------|-----------------|
| I-1  | 7-27-67 | Cancelled due to storm |
| I-2  | 7-17-67 | Gain; Polarization₁; ERP₁, 2 ; Pₜ₁, 2 |
| I-2  | 7-31-67 | Gain; Polarization₁, 2 ; ERP₁, 2 ; Pₜ₁, 2 ; Bandwidth |
| I-4  | 7-18-67 | Gain; Polarization₁; ERP₁, 2 ; Pₜ₁, 2 ; Pattern |
| I-4  | 8-1-67  | Polarization₁; ERP₁, 2 ; Pₜ₁, 2 ; Bandwidth |
| II-1 | 7-10-67 | Gain; ERP₁ |
| II-1 | 7-24-67 | Polarization₁; ERP₁, 2 ; Pₜ₁, 2 |
| II-1 | 8-4-67  | Gain; Polarization₁, 2 ; ERP₁, 2 ; Pₜ₁, 2 ; Bandwidth |
| II-1 | 9-13-67 | Gain; Polarization₁; ERP₁, 2 ; Pₜ₁, 2 |
| II-1 | 10-20-67 | Cancelled due to strong winds |
| II-2 | 7-12-67 | Polarization₁; ERP₁, 2 ; Pₜ₁, 2 |
| II-2 | 7-24-67 | Gain; Polarization₁; ERP₁, 2 ; Pₜ₁, 2 |
| II-2 | 8-7-67  | Cancelled |
| II-4 | 7-17-67 | Polarization₁; ERP₁, 2 ; Pₜ₁, 2 |
| II-4 | 8-11-67 | Gain; Polarization₁; ERP₁; Pₜ₁ ; Bandwidth; Pattern |
| II-5 | 7-18-67 | Gain; Polarization₁; ERP₁, 2 ; Pₜ₁, 2 |
| II-5 | 8-2-67  | Gain; Polarization₁; ERP₁; Pₜ₁ ; Bandwidth; Pattern |
| II-5 | 8-17-67 | Gain; Polarization₁; ERP₁, 2 ; Pₜ₁, 2 |
| II-5 | 9-12-67 | Gain; Polarization₁; ERP₁, 2 ; Pₜ₁, 2 |

principle, this parameter should be quite easy to measure; in practice, especially if good accuracy is desired, the task might prove to be Sisyphean. The purpose of this section is to present the results of the theoretical investigations and of the practical measurements, together with their uncertainties, in order to establish the
accurate value of the total ERP of seven of the IDCSP near-
synchronous satellites.

The total ERP of the satellites in terms of quantities which can
be measured and/or calculated is given by the following expression

\[
(91) \quad \text{ERP (dBm)} = \text{Path Loss} + \text{Atmospheric Absorption}
+ \text{Pattern Angle Loss} + \text{Polarization}
+ \text{Mismatch Loss} + \text{Waveguide Losses}
+ \text{Tracking Errors} + \text{Amplitude Scintillations}
- \text{Antenna Gain} + \text{Sensitivity (dBm)}
- \text{Signal-plus-Noise to Noise Ratio .}
\]

In Eq. (91) all entries are in dB, except as noted; the losses are to
be introduced as positive numbers. Some of the entries in Eq. (91),
e.g., Antenna Gain, make up some of the other principal sections of
this study; some of the other entries, e.g., Atmospheric Absorp-
tion,[50,51] are the fruits of painstaking research of others. In
addition to the entries given in Eq. (91), one may wish to account
for the fact that the transmitted ERP to the satellites might not have
been quite sufficient to fully saturate them and as a result there
might have been some small additional power loss.

It is of crucial importance to establish the accuracy of these
measurements and to state the uncertainty of the results. Toward
this purpose the entries of Eq. (91) will next be scrutinized. The
Path Loss was calculated in each case based on the predicted ranges. The predictions were assumed to be accurate, the calculations based on the predictions are believed to be accurate to within $\pm 0.01$ dB. The Atmospheric Absorption was determined from Reference 50. An uncertainty of $\pm 0.02$ dB is attached to this entry. The Pattern Angle Loss was calculated on basis of predicted look angles. This entry is believed to be accurate within $\pm 0.01$ dB. The Polarization Mismatch Losses were calculated from the measured polarization patterns; the uncertainty in this entry is $\pm 0.01$ dB. The Waveguide Losses were determined accurately by VSWR measurements of the shorted waveguide and by inserting the noise from a waveguide noise source at the flange of the antenna feed-horn and at the flange of the receiver tunnel diode amplifier input. The final results agreed within $\pm 0.02$ dB and this figure will be taken as the uncertainty in this entry. The Amplitude Scintillations, whether these are due to the receiving system or to the incoming signal, constitute a correction rather than an uncertainty. The residual uncertainty from this entry and from the Tracking Errors entry is adequately represented in the standard deviation entry for the measured signal-to-noise ratios. The uncertainty in the Antenna Gain entry is $\pm 0.30$ dB (see Chapter IV). The System Sensitivity is based upon the measurements carried out with a waveguide noise-source calibrated by the National Bureau of Standards and by Signalite, Inc. The uncertainty from this
source is ± 0.07 dB. The measured signal-plus-noise to noise ratios contain uncertainties due to the equipment and also due to the randomness of the measurements themselves. The uncertainties due to the equipment are those due to the precision IF attenuator: ±0.17 dB, and those due to the RMS Voltmeter: ±0.05 dB.

The uncertainties due to randomness will be taken as one standard deviation as given in Table XVIII.

The results are given in Table XVIII. The column labeled "Effective Radiated Power, dBm, f = 7,2767.704 MHz" is based on Eq. (91), so is the column labeled "Beacon ERP dBm". The rms uncertainty calculations do not apply to the beacon ERP values. The total ERP column is the result of correcting the communication channel ERP by adding to it the suppressed beacon ERP (the source of power is common to the two frequency channels) and the measured amplitude scintillations, when available. The Maximum ERP is the result of adding the ideal hard limiter loss to the total ERP.

2. Polarization Patterns

For accurate determination of received signal strength it is necessary to know the magnitude of the losses due to polarization mismatch. These losses can be readily calculated if the axial ratios and the angle between the major axes of the polarization ellipses are known. (See Eqs. (50), (51)) In order to be able to calculate these losses accurately, the polarization patterns of the satellites have
TABLE XVIII
Effective Radiated Powers of Near-Synchronous IDCSP Satellites

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Date</th>
<th>Hour (EDT)</th>
<th>Elevation (Degrees)</th>
<th>Range (Kilometers)</th>
<th>ERP of Transmitter (dB)</th>
<th>Path Loss (dB)</th>
<th>Waveguide Loss (dB)</th>
<th>Atmospheric Absorption (dB)</th>
<th>Tracking Errors (dB)</th>
<th>System Sensitivity (dBm)</th>
<th>Number of Measurements</th>
<th>Maximum ERP (dBm)</th>
<th>Minimum ERP (dBm)</th>
<th>Standard Deviation (dB)</th>
<th>Effective Radiated Power (dBm)</th>
<th>Beacon ERP (dBm)</th>
<th>Expected Beacon ERP (dBm)</th>
<th>Amplitude Shift (dB)</th>
<th>Total ERP (dBm)</th>
<th>RMS Uncertainty (dB)</th>
<th>Ideal Hard Limit Loss (dB)</th>
<th>Maximum ERP (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>7-14</td>
<td>16</td>
<td>44</td>
<td>177</td>
<td>35,323</td>
<td>0.09</td>
<td>0.46</td>
<td>200.65</td>
<td>1,12</td>
<td>-147.38</td>
<td>28</td>
<td>+0.45/-4.08</td>
<td>34.62</td>
<td></td>
<td>36.27</td>
<td>***</td>
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<td>***</td>
<td>10.57</td>
<td>0.52</td>
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<td></td>
</tr>
<tr>
<td>1-2</td>
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<td>02</td>
<td>41</td>
<td>202</td>
<td>35,745</td>
<td>0.01</td>
<td>0.36</td>
<td>200.76</td>
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<td>27</td>
<td>+4.3/-10.16</td>
<td>34.33</td>
<td></td>
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<td>36.70</td>
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<td>07</td>
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<td>182</td>
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<td>36.86</td>
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\( f = 7,268 \), 280 MHz

** Assumed polarization mismatch loss; 0.14 dB

*** There is one successful measurement available which indicates that Beacon ERP is on the order of +24 dBm. Assuming 4 dB beacon suppression and no amplitude scintillations, the total ERP is 36.4 dBm.

N/A = Not Available
been measured with the linearly polarized gain-standard antenna described in Chapter III. The polarization pattern measurements for the various satellites were normally carried out on the communication channel frequency (7276.7 MHz). Additionally, it was feasible to obtain the polarization patterns on the beacon frequency (7299.5 MHz) of satellites I-2 and II-1.

While the data in its measured form is all that is necessary for calculating the polarization mismatch losses for the OSU site, it should be made universally useful. To make the measured data available to users at other geographic locations the measured quantities should be referred to the satellites. Thus, it is necessary to calculate the pattern angle and the apparent rotation of the tilt angle of the polarization pattern caused by changes in path geometry.[52]

The transmit-receive site is at a known location (latitude and longitude of the site is known); the geometry is shown in Fig. 39. Let spherical unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_\phi$ be defined in the conventional sense as shown in Fig. 39. It follows that elevation angles are measured from the $\mathbf{e}_\theta$, $\mathbf{e}_\phi$ plane toward the $\mathbf{e}_r$ direction and azimuth angles are measured clockwise in the $\mathbf{e}_\theta$, $\mathbf{e}_\phi$ plane from the $-\mathbf{e}_\theta$ direction; $\mathbf{e}_\phi$ points toward east, $\mathbf{e}_\theta$ toward south, and $\mathbf{e}_r$ toward zenith. Define the vectors $\mathbf{R}$ and $\mathbf{V}$ which are unit vectors in the direction of the position of the satellite and in the direction of the
linear (vertical) polarization of the receiving antenna. Then from Fig. 40 the following equations can readily be obtained

\[ V = \cos(\text{el}) \, \hat{e}_x + \sin(\text{el}) \cos(\text{az}) \, \hat{e}_\theta - \sin(\text{el}) \sin(\text{az}) \, \hat{e}_\phi, \]

and

\[ R = \sin(\text{el}) \, \hat{e}_x - \cos(\text{el}) \cos(\text{az}) \, \hat{e}_\theta + \cos(\text{el}) \sin(\text{az}) \, \hat{e}_\phi, \]

where el, az, refer to elevation and azimuth directions. V and R can be expressed in the stationary frame 0X₁X₂X₃ by resolving \( \hat{e}_x \), \( \hat{e}_\theta \), \( \hat{e}_\phi \) along \( \hat{E}_1 \), \( \hat{E}_2 \), and \( \hat{E}_3 \), which are unit vectors in that frame:

\[ V = \cos(\text{el}) \left[ \sin \phi \sin \theta \hat{E}_1 + \sin \phi \sin \theta \hat{E}_2 + \cos \theta \hat{E}_3 \right] \]

\[ + \sin(\text{el}) \cos(\text{az}) \left[ \cos \phi \cos \theta \hat{E}_1 + \cos \phi \sin \theta \hat{E}_2 - \sin \theta \hat{E}_3 \right] \]

\[ - \sin(\text{el}) \sin(\text{az}) \left[ - \sin \phi \hat{E}_1 + \cos \phi \hat{E}_2 \right], \]

and

\[ R = \sin \phi \sin \theta \hat{E}_1 - \cos(\text{el}) \cos(\text{az}) \left[ \hat{E}_1 + \cos(\text{el}) \sin(\text{az}) \hat{E}_1 \right], \]

where \( \theta \) is the co-latitude of the station, \( \phi \) is related to its longitude in a manner to be specified below, and the brackets in Eq. (95) contain the same quantities as the brackets in Eq. (94).

Expanding Eqs. (94) and (95) and rearranging, one obtains
\begin{align*}
\mathbf{V} &= E_1 \left\{ \cos(\theta) \cos \phi + \sin(\theta) \cos(\alpha) \cos \theta \cos \phi \\
&\quad + \sin(\theta) \sin(\alpha) \sin \phi \right\} \\
&\quad + E_2 \left\{ \cos(\theta) \sin \phi + \sin(\theta) \cos(\alpha) \cos \theta \cos \phi \\
&\quad - \sin(\theta) \sin(\alpha) \cos \phi \right\} \\
&\quad + E_3 \left\{ \cos(\theta) \cos \phi - \sin(\theta) \cos(\alpha) \sin \phi \right\}
\end{align*}

and

\begin{align*}
\mathbf{R} &= E_1 \left\{ \sin(\theta) \sin \phi \cos \phi - \cos(\theta) \cos(\alpha) \cos \theta \cos \phi \\
&\quad - \cos(\theta) \sin(\alpha) \sin \phi \right\} \\
&\quad + E_2 \left\{ \sin(\theta) \sin \phi \cos \phi - \cos(\theta) \cos(\alpha) \cos \theta \sin \phi \\
&\quad + \cos(\theta) \sin(\alpha) \cos \phi \right\} \\
&\quad + E_3 \left\{ \sin(\theta) \cos \phi + \cos(\theta) \cos(\alpha) \sin \phi \right\}.
\end{align*}

In Eqs. (92), (94), and (96) and subsequent ones, vertical antenna polarizations are considered since this choice of polarization was used. It is, however, a simple matter to generalize the calculations to include horizontal polarization. Consider the $\mathbf{H}$ vector shown in Fig. 40 designating the direction of horizontal polarization. It follows from the geometry that for the horizontally polarized case Eqs. (92), (94), and (96) should read:
\[
H = \cos(az) \begin{pmatrix} e_\theta \\ e_\phi \end{pmatrix} + \sin(az) \begin{pmatrix} e_\phi \\ -e_\theta \end{pmatrix}
\]

\[
H = \cos(az) \begin{pmatrix} \cos \theta \cos \phi & \sin \theta \sin \phi \\ -\sin \theta \cos \phi & \cos \theta \sin \phi \end{pmatrix} E_1 + \sin(az) \begin{pmatrix} \sin \phi & \cos \phi \end{pmatrix} E_2
\]

\[
H = \begin{pmatrix} \cos(az) \cos \theta \cos \phi - \sin(az) \sin \phi \\ \sin(az) \cos \phi + \cos(az) \cos \theta \sin \phi \end{pmatrix} + \begin{pmatrix} \sin(az) \cos \phi \cos \theta \\ \cos(az) \sin \theta \sin \phi \end{pmatrix} + \begin{pmatrix} \cos(az) \cos \phi \sin \theta \\ \sin(az) \sin \phi \cos \theta \end{pmatrix} - \begin{pmatrix} \cos(az) \sin \theta \cos \phi \\ \sin(az) \cos \theta \sin \phi \end{pmatrix}
\]

Of course, Eqs. (93), (95) and (97) remain unaltered.

The geographical location of the Satellite Communications Facility, ElectroScience Laboratory, The Ohio State University is 083° 02' 30" W and 40° 00' 10" N. Let the \(X_1 - X_3\) plane of Fig. 39 be rotated around axis \(0X_3\) such that the meridian plane of OSU is the \(X_1 - X_3\) plane. Then, for OSU \(\theta = 49° 59' 50"\), and \(\phi = 0\).

Substituting the various sines and cosines and their products into Eqs. (96) and (97), one can readily obtain an expression for the polarization vector \(V\) and range vector \(R\) as functions of elevation, and azimuth angles. All vectors are referred to the stationary reference frame \(0X_1X_2X_3\) whose \(0X_3\) axis is coincident with the meridian plane of the site. The vectors \(V\) and \(R\) are found to be
Fig. 39--Reference frame and transmitter-receiver station geometry.
Fig. 40--Directions of propagation and of polarization.
The satellite is in equatorial orbit ($\theta = 90^\circ$) and it is spin stabilized, the spin axis is aligned with the $0X_3$ axis. The pattern angle, $\alpha$, is given by the relationship

$$\alpha = \cos^{-1} \left[ \mathbf{E}_3 \cdot \mathbf{R} \right] - 90^\circ$$

$$= \cos^{-1} \left\{ 0.6428277 \sin(\text{el}) + 0.7660108 \cos(\text{el}) \cos(\text{az}) \right\} - 90^\circ.$$

Equation (100) has been solved for satellite II-5 for the entire visible portion of its orbit relative to the Satellite Communication Facility (some 136 hours) which occurred between July 4 through July 10, 1967. The results are shown in Fig. 41; for convenience the predicted look angles are also plotted on this figure. The period of oscillations apparent in the pattern angle curve is the anomalistic period of the satellite. (Anomaly, in the astronomical sense of the word, is an
Fig. 41—Calculated values of the pattern angle, $\alpha$; power loss, $P$; also, predicted look angles. Satellite II-5, July 4-10, 1967. (All calculations are relative to OSU.)
angular quantity used in defining the position of a point in orbit; hence, anomalistic period is the average time between consecutive passages of the satellite through the same point in its orbit, this point is normally the perigee (closest point of orbit to earth).) Since the orbital parameters of the satellites are very nearly identical, the calculated values of the pattern angle are applicable to all satellites.

The loss of power, $P_a$, as function of the pattern angle, is[53]

$$(101) \quad P_a = 12 \left(\frac{\alpha}{36}\right)^2, \text{ decibels},$$

the graph of Eq. (101) for $\alpha$ and $P_a$ ranging over practical values is shown in Fig. 42. This figure is universally useful if $\alpha$ is known; for the specific shape of $P_a$ as a function of look angles see Fig. 41.

The polarization patterns of the satellites were measured with a linearly polarized (vertical polarization) receiving antenna by rotating the measuring antenna about $\mathbf{R}$ as an axis in a plane perpendicular to $\mathbf{R}$. It can be seen from Fig. 43 that because of the orbital motion of the satellite an apparent rotation of the polarization pattern will take place. The magnitude of this apparent rotation is largest when the satellite is rising (due west), or setting (due east), and it is zero when the satellite is on the local meridian. From the geometry of Fig. 43 it can be seen that the angular difference, $\Delta r$, between the vector $\mathbf{V}$ and the direction corresponding to the local vertical polarization in the pattern of the satellite antenna
Fig. 42—Graph of the function $P_\alpha$. 

$$P_\alpha = 12 \left( \frac{\alpha}{36} \right)^2, \text{ dB}$$
Fig. 43--Geometry of apparent rotation of polarization tilt angle.
that is, the amount by which the tilt angle of the polarization pattern of the satellite antenna appears to rotate as result of changing path geometry) is

\[ \Delta \tau = \cos^{-1} \left[ \frac{R \times V}{|R \times V|} \cdot \frac{R \times E_3}{|R \times E_3|} \right] \]  

making use of the fact that \( V \cdot R = 0 \), and of the vector identity 

\[ (A \times B) \cdot (C \times D) = (A \cdot C) (B \cdot D) - (A \cdot D) (B \cdot C) \]  

Eq. (102) reduces to

\[ \Delta \tau = \cos^{-1} \frac{V \cdot E_3}{|R \times V| |R \times E_3|} \]

which when expanded yields

\[ \tau = \cos^{-1} \left\{ \frac{0.6428277 \cos(\text{el}) - 0.7660108 \sin(\text{el}) \cos(\text{az})}{(R_1^2 + R_2^2)} \right\} 
\]

\[ \left[ (R_3 V_3 - R_2 V_2)^2 + (R_3 V_1 - R_1 V_3)^2 + (R_1 V_2 - R_2 V_1)^2 \right] \]

where

\[ R_1 = \{0, 7660108 \sin(\text{el}) - 0.6428277 \cos(\text{el}) \cos(\text{az})\} \]

\[ V_1 = \{0, 7660108 \cos(\text{el}) + 0.6428277 \sin(\text{el}) \cos(\text{az})\} \]

Equation (104) has been solved for the specific look angles when polarization data were collected; additionally, it has been solved for the entire visible portion (relative to OSU) of the same orbit for which the pattern angle calculations were completed. The results of the computations are shown in Fig. 44. Again, elevation and azimuth
Fig. 44--Magnitude of apparent rotation of polarization pattern tilt angle due to changes in path geometry.
Fig. 45--Polarization pattern of received signal.
Fig. 46--Polarization pattern of received signal.
Fig. 47--Polarization pattern of received signal.
Fig. 48--Polarization pattern of received signal.
Fig. 49--Polarization pattern of received signal.
Fig. 50--Polarization pattern of received signal.
Fig. 51--Polarization pattern of received signal.
Fig. 52--Polarization pattern of received signal.
Fig. 53--Polarization pattern of received signal.
Fig. 54--Polarization pattern of received signal.
Fig. 55--Polarization pattern of received signal.
Fig. 56--Polarization pattern of received signal.
Fig. 57--Polarization pattern of received signal.
Fig. 58--Polarization pattern of received signal.
Fig. 59--Polarization pattern of received signal.
Fig. 60--Polarization pattern of received signal.
angles are also plotted for convenience. From Fig. 43 it is obvious that the effect of the apparent rotation of the tilt angle is such as to tend to make the measured polarization patterns appear to rotate clockwise when the satellite is East of the local meridian, and opposite to that rotation when the satellite is in the western skies.

The experimentally determined polarization patterns are shown in Figs. 45-60. The axial ratios based on these patterns are all approximately 1.2. The smallest axial ratio (1.17) is that of satellite I-2 and the largest (1.24) are those of satellite II-1 and I-1. On Figures 45-60 the measured tilt angle, $\tau$, is shown relative to the horizontal of the receiver site. Also shown on those figures is the value of $\Delta \tau$, and the direction of satellite vertical at the measured pattern angle. Applying the correct value - in magnitude and sign - of $\Delta \tau$ to the patterns, one obtains the polarization pattern of the radiation pattern of the satellite at the measured pattern angle, $\alpha$.

To establish a suitable reference direction, the vertical direction for the satellite is taken to be in the direction of the spin axis when $\alpha$ is zero. The results of the polarization pattern measurements are tabulated below. In Table XIX, the sixth entry - orientation of the major axis - is the orientation of the major axis of the polarization pattern of the satellite measured in a plane perpendicular to the pattern angle, $\alpha$, and the orientation is measured relative to the above defined reference vertical direction.
<table>
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<th>Azimuth, Degrees</th>
<th>Frequency, MHz</th>
<th>Orientation of Major Axis, Degrees</th>
<th>Pattern Angle, Degrees</th>
<th>P, dB</th>
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3. Bandwidth

For intelligent utilization of a satellite communication channel it is essential that the bandwidth of the transponder and the bandshape be known accurately. Knowledge about the bandwidth and its shape is also required for passive (radiometric) measurements of the effective radiated power; it may also be necessary for experimental evaluation of satellite performance such as analyzing causes of signal drop-outs.

The band-shape of the satellites were measured by simultaneously shifting the exciter of the transmitter and the stabilized local oscillator of the receiver in discrete frequency increments of 1.872 MHz, and monitoring the receiver IF power. Each power level measurement at a given frequency was accompanied by a measurement of the receiver noise level at the same frequency, so that corrections could be made for the receiver gain fluctuations due to frequency changes. The noise figure of the first stage of the receiver (tunnel diode amplifier) was constant over the frequency band used during the measurements (7.295 GHz - 7.258 GHz, about 40 MHz is the total extent of frequencies used; the bandwidth of tunnel diode amplifier is greater than 500 MHz). The results of the measurements are shown in Figs. 61-65. The
horizontal axis of these figures is the received frequency; the
vertical axis is the received power level normalized to that level
which was measured on the most frequently used frequency:
7,276.704 MHz. In general, the measured "3 dB down" bandwidth
of the satellites is about 30 MHz; the tops of the curves are smooth
across the band with fluctuations of about 1 dB. Also shown on
Figs. 61-65 are the measured values of the receiver noise level
at each frequency. The same vertical dB scale is to be used for
these curves, however, the noise level is some -35 dB relative
to the 0 dB reference level of the band-shapes. The noise level
measurements are normalized to the noise level at 7,276.704
MHz.

4. Power Transfer

The IDCSP communication satellites are of the hard-limiting,
nonlinear type. There is always an output from the satellite; when
it is unexcited, the energy is spread over the bandwidth; when it
is saturated, all of the energy is concentrated in the bandwidth
corresponding to the coherent input frequency; when the satellite
is excited but not saturated, part of the energy is in the bandwidth
corresponding to the coherent input frequency; when the satellite
RECEIVED POWER (dB)

MEASURED CURVE
CURVE CORRECTED FOR GAIN FLUCTUATIONS

Satellite I-2
July 31, 1967

RECEIVED FREQUENCY (MHz)

Noise Level Reference (About-35 dB)

Fig. 61--Band-shape of satellite.
Fig. 62--Band-shape of satellite.
Fig. 63--Band-shape of satellite.
Fig. 64--Band-shape of satellite.
Fig. 65--Band-shape of satellite.
is excited but not saturated, part of the energy is in the bandwidth corresponding to the coherent input frequency and part of it is radiated incoherently over the frequencies in the bandwidth of the satellites. It is therefore of interest to determine the power transfer characteristics of the satellites, i.e., to determine the amplitude variations in the coherent output power as a function of input power changes.

The experimentally determined power transfer characteristics of the satellites are shown in Figs. 66-71. The horizontal axis is in terms of the incident power density at the satellite times the effective aperture of the receiving antenna of the satellite. The up-link frequency was 8 GHz, the incident power density was calculated on this frequency. The gain of the receiving antenna of the satellite was taken to be unity. The vertical axis is the coherent satellite output power on the down-link frequency of 7.3 GHz, normalized to that coherent output which is available from the satellite when the input power is about -83 dBm. This figure results from the addition of the ERP of the ground transmitter (+119 dBm), the path loss (-201 dB), the gain of the receiving antenna of the satellite (+0 dB), and other losses (about -1 dB).
The curves are normalized to this output level taken as unity. The experimental determination of the maximum satellite output is the subject of Section 1. The curves of these figures are based on about ten data points; a probable error not exceeding one half dB is suggested for the shapes of these curves - this suggestion seems to be well substantiated by the close repeatability of the data points, some of which were taken two months apart. Because of the very close resemblance in the shapes of the curves, it may be said that the power transfer characteristics of the satellites are very similar.

There are plotted on Figs. 66-72, in addition to the experimentally determined transfer curves, curves based on the calculations whose results are tabulated in Table XX of Section 8. (The calculated curves were properly adjusted for the fact that 4KW transmit power may not fully saturate the satellites). It may also be said that the experimentally determined power transfer curves of the satellites are very much like the curves calculated on basis of the signal-to-noise ratio in ideal hard limiters.
Fig. 66--Power transfer curve of the satellite.
Fig. 67--Power transfer curve of the satellite.
Fig. 68--Power transfer curve of the satellite.
Fig. 69--Power transfer curve of the satellite.
Fig. 70--Power transfer curve of the satellite.
Fig. 71--Power transfer curve of the satellite.
4. Beacon Suppression

For location, identification, and tracking purposes the satellites have a beacon transmitter on board. The output power of this transmitter decreases as the input power on the communication channel frequency is increased. One of the objectives of the measurement program was to determine this output power variation. It has been found that the beacon power level decreases typically by about 4 dB as the satellite is saturated. Exceptions are satellites II-1 and II-4 in which cases the decrease is about 2 and 3 dB, respectively. The measured beacon power suppression curves are shown in Figs. 72-77. The horizontal axis is in terms of input power levels into the satellite on the communication channel frequency; the vertical axis is satellite beacon output power level in dB relative to its peak level. Since the measurement of the absolute value of this peak level is treated in Section 1 the purpose here is simply to show the variations in the output level. Since each of the curves is based on about ten data points, a probable error not exceeding one half dB is estimated for these relative curves.

6. Amplitude Scintillations

Scintillations in the amplitude of the received signal are highly undesirable and designers of communication systems go to extreme lengths to eliminate or at least to minimize them. In some satellite
Fig. 72—Beacon power suppression curve.
Fig. 73--Beacon power suppression curve.
Fig. 74--Beacon power suppression curve.
Fig. 75--Beacon power suppression curve.
Fig. 76--Beacon power suppression curve.
Fig. 77--Beacon power suppression curve.
communication systems, such as the ones employing passive reflectors like Echo I and Echo II, the problem of scintillations may be extreme[52,54,55] thereby making the communication system only marginally useful.[56,57] In systems employing the near-synchronous IDCSP satellites the problem is small when compared to systems using Echo type satellites, but the problem nevertheless exists to some degree.

The subject of scintillations has occupied the attention of this writer for some time.[58] The mathematical approaches and methods of analysis in dealing with the phenomenon of scintillations, as it occurs in communication systems employing near-synchronous satellites, are those used by the author in his previous work in this field and the results presented here are directly based upon this work.[59-64]

In analyzing scintillating signals one is interested in how the power is distributed as a function of frequency. The method of power spectral density analysis[65,66,67] is a very powerful and widely used method for obtaining this information. The power spectrum and its transform, the autocorrelation function, are based on Fourier analysis. The transform pair is given as

\[
\Phi(\tau) = \int_{-\infty}^{\infty} P(f) \cos \omega \tau \, df, \text{ and}
\]
where

\[ \phi(\tau) = \text{the autocorrelation function of the function } f(t) \]

\[ P(f) = \text{the power spectrum of the function } f(t) \].

The definition of the autocorrelation function \( \phi(\tau) \) of the function \( f(t) \) is

\[
\phi(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t) f(t+\tau) \, dt.
\]

where

\[ \tau = \text{the amount of delay by which the function } f(t) \text{ is delayed relative to itself} \].

The signals received from the IDCSP satellites were normally continuous waves (cw). They were produced by exciting the satellite with a clean carrier of sufficient amplitude so that its output was very nearly the saturated output. The measured amplitude variations in the transmitted carrier were normally less than 0.05 dB; the frequency of the carrier was derived from a rubidium-vapor standard whose stability was 5 parts in \( 10^{11} \) per year. The frequency of the returned signal was stable to at least within five parts in \( 10^9 \). [68] Experiments with the satellites using two transmitters, such that the sum of the transmitted energies did not produce saturation, operating
at frequencies derived from the same rubidium-vapor standard indicated periodic enhancement and cancellation of the signal.\[69\] The explanation given for this phenomenon was that the signals from the two transmitters arrive at the satellite alternately in-and out-of-phase as the differential path length between the antennas changed. From the fact that this phenomenon could be observed on the returned signal it was concluded that the frequency and phase stability of the entire link (transmitter, satellite, receiver, and propagating medium) was excellent.

Signals received from the satellites were demodulated with phase-lock demodulators which are linear detectors. The AM, PM, and FM outputs of the demodulators are proportional to the received signal-plus-noise level, phase, and frequency, respectively. From the early experiments it was obvious that the received signals were stable. Frequency and phase variations were smaller than could be conveniently measured. Frequency variations, if they were present, were less than one cycle; the PM output of the demodulator was noise, indicating that if phase variations were present these must have been on the order of at most a few degrees. Amplitude scintillations of the received signal-plus-noise level were less than $\pm 1$ dB, and usually sinusoidal.
The AM output of the phase-lock demodulator was recorded on paper chart and on magnetic tape. An example of the chart recordings is shown in Fig. 78. From this figure it can be seen that the signal scintillates in amplitude. The rate is 80 peaks in a period of 31 seconds or 2.6 cycles per second. The peak amplitude is about 0.3 dB. The analog magnetic tape recordings were converted into digital recordings with an analog-to-digital converter and the sampled data were analyzed with a computer. The adaptation of the data to analysis by a computer was done as follows. \[70\] The autocovariance function was computed first. (The autocovariance function is the autocorrelation function computed without the average value of \(f(t)\).) The discrete form of the autocovariance function is

\[
C_m(\tau) = \frac{1}{N-m} \sum_{j=0}^{N-m} S_j S_{j+m}, \quad m = 0, 1, \ldots, M, \tag{108}
\]

where

\[
S_i = \text{discrete data points, } i = 0, 1, 2, \ldots, N
\]

\[
N = \text{total number of data points less one}
\]

\[
M = \text{total number of covariance values less one}
\]

For sufficiently rapid sampling rates and for a large enough number of samples Eq. (108) is the autocovariance function with an error that is vanishingly small. The sample interval, \(\Delta \tau\), equals the interval between adjacent autocovariance lag terms. The delay,
\( \tau \), corresponding to \( C_m \) is \( m\Delta t \). The maximum covariance lag should not exceed one-tenth of the record length.

![AM OUTPUT OF PHASE-LOCK DEMODULATOR (VOLTS)]

\[ \text{TIME} \]

![SATELLITE II-5 AUGUST 2, 1967 7,276.704 MHz]

Fig. 78--The output of the linear detector showing amplitude scintillations.

Equation (108) was programmed on an IBM 7094 computer, and the autocovariance function was computed. The power spectrum could then be readily obtained by Fourier transform. Since the purpose here is to show the energy content of the received signal as a function of frequency, and since the autocovariance function does not present extra information over what is available from the spectral density function, only the Power Spectral Density, PSD, curves were produced from the data.

The narrowest bandwidth in the receiving system was that of the magnetic tape recorder: approximately 1 KHz. The original data were sampled at the rate of 500 per second; this sampling rate produces a folding frequency at 250 Hz. The sample length was normally
30 seconds, yielding a total of some 15,000 points. The maximum lag in computing the covariance function was 2.0 seconds. This lag sets the limit on the frequency resolution at 0.5 Hz. The data points were summed and their arithmetic average was found. This average value was subtracted from all points, thus in the subsequent computing process the covariance function rather than the correlation function was found. The data were digitally filtered, smoothed and quintimated: the process took the equally weighted average of five consecutive data points, then took the equally weighted average of four consecutive data points, and then took every fifth sample of the smoothed data. The shape of the digital filter function is shown in Fig. 79 (Reproduced through courtesy of Mr. R. Turpin). The horizontal axis is in terms of frequency normalized to the original folding frequency designated as \( f_N \); in this case \( f_N = 250 \) Hz. The new folding frequency is 50 Hz. The resulting spectrum, obtained from the covariance function through Fourier transformation, is considered accurate to 60 per cent of this folding frequency.[66]

In order to ascertain the general shape of the spectrum of the received signal, the first PSD analysis was carried out to 250 Hz at the relatively low frequency resolution of 2 Hz. The reliable portion of the spectrum -- between 0 Hz and 0.6 \( \times 250 \) or 150 Hz -- is shown in Fig. 80. The computing process includes a normalizing procedure: it searches for the strongest component and divides all computed
Fig. 79—Transfer function for $S_4 S_5$.
(Reproduced from R. Turpin: "Notes on Digital Spectral Analysis"; reproduced by permission of author.)
points by that value. The strongest component occurs at 60 Hz, which is the power line frequency. Elaborate attempts were made to eliminate this spurious result; however, these attempts were only moderately successful: the 60 Hz component could only be reduced. (It may sound somewhat pessimistic, but perhaps the only solution would be to convert the entire facility to dc operation!) The second strongest component occurs at approximately 2.5 Hz, the source of this component, discussed below, is believed to be the satellite. The entire spectrum, with the exception of the above two strong lines, is essentially flat. From these features one may deduce that the original signal -- the output of the linear detector consisting of received signal plus noise -- contained energies at 0 Hz (the average, or dc, level of the output which was removed from the data), at 60 Hz, at 2.5 Hz, and the rest of the energy was distributed uniformly over the entire bandwidth (the spectrum of the noise is Gaussian).

The PSD curves obtained from satellite I-2 on 7,276.704 MHz and on 7,299.500 MHz are shown in Figs. 81 and 82, respectively. Again, only the reliable portion, between dc and 30 Hz, of the spectrum is shown. The strongest spectral component on the communication channel occurs at dc; on the beacon channel it occurs at 7.5 Hz. These components as well as the one at 8.75 Hz in Fig. 81 are probably due to very slow gain fluctuations and/or to the presence
of spurious interfering signals - such as the one produced by the residual of the local oscillator signal - in the receiving system.

Strong spectral components observable in both Figs. 81 and 82 occur at identical frequencies. These components are at 2.75, 5.50, 11.00 and 21.75 Hz. (There is a weak but discernible component at 16.50 Hz in Fig. 81, but not in Fig. 82. The amplitude of this component is about the same as its neighboring points and its possible existence was not suspected until after the existence of the above spectral components at even multiples of 2.75 Hz was noted. If odd multiples of the principal component exist, these are on the same level as the noise.) The origin of these components is extraneous to the ground station. It is possible, but highly improbable, that the intervening medium -- the ionosphere and/or the troposphere -- could be responsible for these very regular amplitude scintillations. It appears more likely that the origin of these regular scintillations is in the satellite itself. From a knowledge of the average value of the data, the normalizing factor used in computing the PSD curves, and the amplitudes of the various spectral lines relative to the strongest one, the amplitudes of the signals occurring at the various frequencies relative to the average value of the data can be computed. Considering the strongest components on the communication channel at dc, 2.75, 8.75, and 11.0 Hz, the result is ± 0.24 dB; 0.17 dB is due to the strongest components probably caused by the receiver itself,
and 0.07 dB is due to the strongest components extraneous to the system and occurring at 2.75 and at 11.0 Hz. All other components contribute negligibly, less than 0.01 dB, to the amplitude scintillations. The strongest components on the beacon channel occur at 7.5 and at 2.75 Hz. These components cause an amplitude scintillation of ±0.22 dB; 0.16 dB is due to the 7.5 Hz component and 0.06 dB to the next strongest component at 2.75 Hz; the effects of the other components are negligible.

The PSD curves for Satellite I-4 are shown in Figs. 83 and 84. The strongest components on the communication channel (see Fig. 83) occur at dc, 1.75 - 2.50, and at 4.0 Hz. On the beacon channel the strongest components are at 9.25, 2.50, and at 4.25 Hz. The two low frequency components are most likely extraneous to the system; the components at dc and at 9.25 Hz are probably caused by the receiver itself. The effect of the strongest components is calculated to be ±0.08 dB on the communication channel; 0.02 dB of which is probably due to the satellite signal. The effect on the beacon channel is ±0.82 dB; 0.10 dB of which is likely to be due to the satellite signal.

The PSD curves for satellite II-1 are shown in Figs. 85 and 86, obtained on the communication channel and on the beacon channel respectively. The strongest components occurring on both channels are at or near dc and at 2.75 Hz; additionally there is a strong
component at 8.25 Hz occurring on the beacon channel only. Higher frequency components are not readily discernible. It is suggested that with the exception of the 2.75 Hz components, the origin of these components is in the receiver; the origin of the 2.75 Hz component is extraneous to the system, it is probable that the satellite itself generates this component.

A suggested and possible mechanism for the generation of the 2.75 Hz component is the rotation of the satellite resulting from the spin-stabilization process. Should this be indeed correct, then the satellite is probably rotating at 2.75 revolutions per second, or at 165 rpm. The effect of the strongest components is calculated to be $\pm 0.11$ dB on the communication channel; 0.03 dB of which is probably due to the signal from the satellite. The effect on the beacon channel is $\pm 1.13$ dB; 0.03 dB of which is assigned to the satellite signal.

The PSD curves for satellite II-2 are shown in Figs. 87 and 88. The strongest components occurring on both channels are at dc, 2.75, and at 5.50 Hz. The component at dc is probably due to the receiver; the components at 2.75 and at 5.50 Hz are most likely to originate in the satellite. The effects of these strongest components on amplitude scintillations are $\pm 0.16$ dB on the communication channel; 0.05 dB of which is probably due to the satellite signal. The effect on the
beacon channel is $\pm 0.46$ dB; 0.08 dB of which is likely to be due to the satellite signal.

The PSD curves for satellite II-4 on the communication channel are shown in Figs. 89 and 90. Strong spectral components occurring coincidentally on both of these figures can be seen at dc, 2.75, 5.50, and at 11.0 Hz. Additionally, strong components at 7.0 and at 21.5 Hz may be observed on the July 17th data. The resultant amplitude scintillations on the July 17th data are calculated to be $\pm 0.51$ dB; 0.17 dB of which is likely to be due to the satellite signal. The effect on the August 11th data is calculated to be $\pm 0.09$ dB; 0.02 dB of which is probably due to the satellite signal.

One of the most successful attempts at obtaining PSD curves of the amplitude scintillations of the received signal were carried out with satellite II-5. The scintillations on the received signal are readily observable (see Fig. 78). The principal component of the spectrum, regardless of date or of frequency occurs at 2.50 Hz. The second, fourth and sixth harmonic of the fundamental are clearly discernible on Fig. 91; and Fig. 92 the second harmonic is observable; on Fig. 93 the fundamental and its fourth harmonic are dominant. The effects of the strongest components are calculated to be $\pm 0.22$ dB for Fig. 91, 0.02 dB of which is likely to be due to the receiver; $\pm 0.36$ dB for Fig. 92, 0.17 dB of which is likely to be
due to the receiver; and ± 0.55 dB for Fig. 93, 0.17 dB of which is likely to be due to the receiver.

In order to ascertain the performance of the receiving and recording systems as well as that of the playback and analog-to-digital conversion processes the PSD curves shown in Figs. 94-96 were prepared. The PSD curve of the noise due to the analog tape recorder, analog play-back, and analog-to-digital converter is shown in Fig. 94. The input signal was 1.0 V dc, obtained from a dry cell, connected directly to the input of the analog tape recorder. The peak fluctuations are 43 micro volts. The fluctuations are 43.7 dB below the average 1.0 V value, they occur at or very close to 0.00 Hz. The mean level of the PSD curve (the noise level) is 49 dB below the average level of 1.0 V dc.

Figure 95 shows a PSD curve obtained when a locally generated, spectrally pure RF signal is inserted into the antenna line and the stabilized local oscillator is offset from the center frequency of this RF signal by 9.75 Hz. The signal-plus-noise to noise ratio was +20 dB. The strongest component occurs at 3.00 Hz, although almost equally strong components are seen at 5.25, 8.25 and, most significantly, at 9.75 Hz. The peak amplitude of the strongest component is 25.2 dB below the average level due to the signal-plus-noise. This is negligibly small.
The PSD curve shown in Fig. 96 was obtained from a noise output recording of the receiver system while the stabilized local oscillator was left at 9.75 Hz from center frequency, like during the time when the data for Fig. 95 was taken. The maximum occurs at dc, the amplitude of this maximum is 18.4 dB below the average level of the recorded noise output. The next maximum is at 9.75 Hz, this is due to the local oscillator signal. Such an accurate knowledge about the frequency of the local oscillator (and that of the transmitter also) was the principal reason why the spectral lines occurring in Figs. 80-93 could be assigned either to the satellite or to the ground station. The peak of the line due to the local oscillator is -21.4 dB relative to the average level. The mean level of the PSD curve is about -28 dB relative to the average level. This level is essentially flat for all frequencies, indicating that with the exception of the very low frequency fluctuations and those due to the local oscillator the spectrum is flat, i.e., it is due to Gaussian noise.

From the length of the data used (30 seconds) in computing the PSD curves, the frequency resolution obtained (0.5 Hz), and the number of data pieces available, with the aid of the ready formulas given in Ref. 67, the stabilities (accuracies) of the PSD curves shown in Figs. 80-96 may be computed. The results of these computations are shown graphically in Fig. 97. From Fig. 97 one
Figs. 80-81--Power spectral density curves of amplitude scintillations.
Figs. 82-83—Power spectral density curves of amplitude scintillations.
Figs. 84-85—Power spectral density curves of amplitude scintillations.
Figs. 86-87—Power spectral density curves of amplitude scintillations.
SATELLITE, II-2
DATE JULY 24, 1967
FREQUENCY 7,299.500 MHz
SAMPLING RATE 500/SEC
LENGTH OF DATA 30.0 SEC
MAX COVARIANCE LAG 2.0 SEC
RESOLUTION 0.5 Hz

Figs. 88-89—Power spectral density curves of amplitude scintillations.
Fig. 90 -- Power spectral density curves of amplitude scintillations.
Fig. 91--Power spectral density curves of amplitude scintillations.
Fig. 92--Power spectral density curves of amplitude scintillations.
Figs. 93-94--Power spectral density curves of amplitude scintillations.
Figs. 95-96--Power spectral density curves of amplitude scintillations.
Fig. 97--Graphical representation of the relationship between the confidence level and the computed spectrum; for the latter to be within a given range from the true spectrum.
can readily find the range, at any given frequency, within which the computed points in the power spectral density curve fall from the value of the true spectrum at that same frequency, versus the confidence level.

7. Look Angles

Access to the satellites depends upon the availability of accurate look angles data for them. Look angles, range, and range rate data were provided by the Satellite Test Center (USAF), Sunnyvale, California. Such data, however, were not usually available immediately following a launching, and a technique for calculating approximate satellite positions was developed and used successfully.

It is shown in Appendix IV that it is a relatively simple task to compute the look-angles for a celestial body. If the latitude and longitude of the point of observation, the accurate time, and the position of the object on the celestial sphere (right ascension and declination) are known, look-angles can be calculated from the relations

\[
\text{Elevation} = \sin^{-1}\left[ \sin L \sin D + \cos L \cos D \cos \text{LHA} \right],
\]

and

\[
\text{Azimuth} = \cos^{-1}\left[ \frac{\sin D}{\cos L \cos \text{EL}} - \tan L \tan \text{EL} \right],
\]
where

\[ L = \text{Latitude}, \]
\[ D = \text{Declination}, \]
\[ \text{EL} = \text{Elevation}, \]
\[ \text{LHA} = \text{Local Hour Angle}. \]

These formulas result from solving the astronomical triangle by means of spherical trigonometry. For bodies with orbits in the Earth's equatorial plane (declination = 0°, inclination = 0°, eccentricity of orbit = 0) -- these conditions are very closely approximated by the IDCSP-I, and -II satellites (but not the IDCSP-III satellites) -- the formulas further reduce to

\[
\begin{align*}
(111) & \quad \text{Elevation} = \sin^{-1}[\cos L \cos \text{LHA}] \\
(112) & \quad \text{Azimuth} = \cos^{-1}[\tan L \tan \text{EL}] .
\end{align*}
\]

In Eq. (111), the LHA is taken as the angular distance between the longitudes of the observer and of the satellite. Of course, the LHA must be known at the time of observation.

Communication satellites are much closer to the Earth than the nearest celestial body, the Moon (a natural satellite of the Earth). This closeness results in large values of Horizontal Parallax. The magnitude of this parallax is simply
(113) \[ \text{Horizontal Parallax} = \tan^{-1} \left[ \frac{R_e \sin L}{h - R_e \cos L} \right] \]

where

\( R_e \) = radius of Earth, and
\( h \) = radius of orbit.

The magnitude of Parallax Correction is

(114) \[ \text{Parallax Correction} = (\text{Horizontal Parallax}) \cdot (\cos \text{ine of calculated elevation}). \]

The elevation angle of a satellite in an ideal equatorial orbit is then

(115) \[ \text{Satellite Elevation} = \text{Calculated Elevation (for a celestial body)} - \text{Parallax Correction}. \]

Azimuth calculations require no correction.

The above considerations, together with launch information available in the news, have enabled The Satellite Communication Facility to locate the IDCSP-I satellites long before look angle predictions became available. The first contact was made on July 17, 1966 at about 6:00 P.M., EST. These considerations were again used successfully in locating the IDCSP-II satellites within one hour of their insertion into orbit on January 18, 1967 at 4:00 P.M., EST.
A typical comparison between experimentally measured look-angles and look-angles provided by The Satellite Test Center is shown in Fig. 98. From this figure it can be seen that the values provided are accurate except for a time delay of about one hour; the measured position at a given time equaling the predicted position at a time one hour later. This time offset does not significantly affect initial acquisition since the time rate of change of position is very slow for near-synchronous satellites (less than 2°/hour for either axis). The operator simply inserts a bias determined by recent tracking experience in the manual antenna position controls.

Fig. 98--Measured and predicted satellite look-angles.
8. Signal Strength and Signal-to-Noise Ratios

The purpose of this section is to determine the signal strengths and signal-to-noise ratios on a typical satellite communication link.

The one-way path loss, $P_L$, between two isotropic antennas is simply the ratio of transmitted to received power

$$ P_L = \frac{(\text{Transmit power})}{(\text{Received power})} = \left( \frac{4\pi d}{\lambda} \right)^2, \text{ or} $$

$$ P_L(\text{dB}) = 32.45 + 20 \log_{10} f + 20 \log_{10} d $$

where $d$ is distance (km), $f$ is frequency (MHz), and $\lambda$ is the wavelength. For the frequencies and distances encountered with the satellites, it is simple to construct a graph based on Eq. (116). This graph is shown in Fig. 99. It can be seen that the minimum value for path loss is 200 dB, while the maximum is slightly less than 206 dB. This last statement is valid, of course, only for a station at latitude 40°N, the latitude of the OSU Satellite Communication Facility.

Next, a typical link calculation is included.
Up-Link

Power output  + 66 dBm
Antenna gain, transmitter (approximate)  + 53 dB
Effective transmitted power  +119 dBm
Path loss (from Fig. 99, average)  -201.5 dB
Atmospheric attenuation  - 0.2 dB
Tracking error  - 0.1 dB
Polarization mismatch (estimated)  - 0.1 dB
Incident power  - 82.9 dBm
Antenna gain, satellite (assumed)  + 3.0 dB
Effective coherent power input to satellite  - 79.9 dBm

If the satellite input signal-to-noise ratio, \((S/N)_\text{in}\) is known, Davenport's [72] results can be used to determine the signal-to-noise ratio at the output, \((S/N)_\text{out}\), and the coherent power output of a hard-limiting satellite. For the limiting cases of \((S/N)_\text{in} \to 0\), and \((S/N)_\text{in} \to \infty\), the ratios of \((S/N)_\text{out}\) to \((S/N)_\text{in}\) approach -1 dB and +3 dB respectively. That is, for low \((S/N)_\text{in}\) there results a degradation and for high \((S/N)_\text{in}\) there results an improvement; the cross-over point, i.e., \((S/N)_\text{out} = (S/N)_\text{in}\), occurring for \((S/N)_\text{in} \approx 0\) dB.
FREE SPACE PATH LOSS, $P_L$, IS GIVEN BY $P_L = 198 + ΔL_f + ΔL_d$ (dB)

EXAMPLE: DISTANCE = 40,000 km
FREQUENCY = 8000 MHz
$P_L = 198 + 2.16 + 2.41 = 202.57$ (dB)

Fig. 99--Graph of calculated free-space path loss versus frequency and distance for IDCSP satellite applications.
The noise figure of the satellite is taken as 10 dB. Converting this to noise power density, \( P_N \), gives

\[
(118) \quad P_N = k(F-l) T = 1.38 \times 10^{-23} \times (10-l) 290
\]

\[= -194.3 \text{ dBW/cps} = -164.3 \text{ dBm/cps}.\]

At the Satellite

| Effective coherent power input to satellite | -79.9 dBm |
| Satellite noise power density | -164.3 dBm/cps |
| Satellite noise bandwidth (\( \sim 30 \) MHz) | +74.8 dB |
| Satellite noise power | -89.5 dB |
| Signal-to-noise ratio, input | +9.6 dB |

Down Link

| Power output (design) | +34 dBm |
| Antenna gain, satellite (design) | +3 dB |
| Effective radiated power (design) | +37 dBm |
| Path loss, average | -200.5 dB |
| Atmospheric attenuation | -0.2 |
| Tracking errors | -0.1 |
| Antenna gain, receiver | +53.4 |
| Plumbing losses | -1.1 |
| Polarization mismatch | -0.1 |
| Received power level | -111.6 dBm |

Using Davenport's results, entries to Table XX are computed for the average figures given in the up-link computation above.
<table>
<thead>
<tr>
<th>Transmitter Power Output</th>
<th>Satellite (S/N) Input</th>
<th>Satellite (S/N) Output</th>
<th>Satellite Coherent Power Output</th>
<th>Per cent of Maximum</th>
<th>Decibels Relative to Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00 KW</td>
<td>+ 9.6 dB</td>
<td>12.2 dB</td>
<td></td>
<td>94.4</td>
<td>- .24</td>
</tr>
<tr>
<td>2.00</td>
<td>6.6</td>
<td>8.8</td>
<td></td>
<td>88.5</td>
<td>- .52</td>
</tr>
<tr>
<td>1.60</td>
<td>5.6</td>
<td>7.6</td>
<td></td>
<td>85.1</td>
<td>- .70</td>
</tr>
<tr>
<td>1.00</td>
<td>3.6</td>
<td>5.0</td>
<td></td>
<td>76.0</td>
<td>- 1.20</td>
</tr>
<tr>
<td>800 W</td>
<td>2.6</td>
<td>3.7</td>
<td></td>
<td>70.0</td>
<td>- 1.54</td>
</tr>
<tr>
<td>500</td>
<td>0.6</td>
<td>1.1</td>
<td></td>
<td>56.2</td>
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</tr>
<tr>
<td>400</td>
<td>- 0.4</td>
<td>0.0</td>
<td></td>
<td>50.0</td>
<td>- 3.01</td>
</tr>
<tr>
<td>250</td>
<td>- 2.4</td>
<td>- 2.3</td>
<td></td>
<td>37.1</td>
<td>- 4.30</td>
</tr>
<tr>
<td>200</td>
<td>- 3.4</td>
<td>- 3.4</td>
<td></td>
<td>31.4</td>
<td>- 5.03</td>
</tr>
<tr>
<td>125</td>
<td>- 5.4</td>
<td>- 5.6</td>
<td></td>
<td>21.6</td>
<td>- 6.66</td>
</tr>
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<td>100</td>
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<td>11.7</td>
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<td></td>
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<tr>
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<td>-12.1</td>
<td></td>
<td>5.8</td>
<td>-12.36</td>
</tr>
</tbody>
</table>
The above calculated figure of \(-111.6\,\text{dBm}\) for received power level can be converted to received signal-to-noise ratio at the ground terminal by determining the noise threshold level of the receiver system. This noise power, \(P_{\text{noise}}\), is given by

\[
P_{\text{noise}} = k T_e B = k \left[ (F-1) \frac{290 + T_a}{290} + T_l \right] * B_n,
\]

where \(k\) is Boltzman's constant, \(F\) is the receiver noise figure (numeric), \(T_a\) is the antenna temperature, and \(B_n\) is the IF noise bandwidth. For the case of interest,

\[
P_{\text{noise}} = 1.38 \times 10^{-23} \times (700) \times 187 = -177.4\,\text{dBw} = -147.4\,\text{dBm}.
\]

Using the values of calculated received power level and noise threshold level, a typical figure for the received signal-to-noise ratio on a communication satellite link for the Satellite Communication Facility is calculated below:

\[
(\text{Received} \ \frac{S}{N})_{\text{calculated}} = (-147.4 + 11.6)\,\text{dB} = +35.8\,\text{dB}.
\]
CHAPTER VI
GROUND STATION PARAMETERS

A. Description of the Antenna Array Facility [69, 72]

The purpose of this section is to give a brief description of the ground station facility. Many of its aspects, some of which are unique, e.g., the adaptively phased array feature, are treated in detail elsewhere. [73-86] The four-element adaptively phased array of The ElectroScience Laboratory (formerly Antenna Laboratory), The Ohio State University, is located in Columbus, Ohio. The installation consists of four paraboloidal reflectors 30-feet in diameter located on the corners of a square with sides 60 feet long whose diagonals are oriented in the principal directions of the compass. A modern, new building located in the center of the array houses the control equipment, the receivers, the data collecting, and some of the data-reducing equipment. The reflectors are of solid surface and they can be operated at frequencies greater than 15 GHz. Present frequencies of operation are in the S-band (receive only) and in the X-band (receive and transmit) regions. The paraboloids are focal point fed at S-band, while cassegrainian feed is employed on X-band. The measured absolute gain at 2,270 MHz is 43 dB; it is
53 dB at 7,277 MHz. Polarization diversity is available: VP, HP, RCP, LCP. The method of tracking is sum-difference monopulse. One local oscillator is common to all four elements; this STALO can be made to track the frequency and/or the phase of the incoming signal. The signals are amplified at the feeds, down-converted to the first IF and brought to the centrally located control building. Parametric amplifiers are used on S-band and tunnel diode amplifiers on X-band. The individual signals of the four antennas, through the utilization of various phase-locked loops, can be added coherently resulting in 6 dB enhancement in the signal-to-noise ratio in the sum channel relative to the individual channels.[87] Thus the installation, through signal processing, can be made to be the equivalent of a sixty-foot reflector. In addition to the phased array operation each antenna and receiver can be operated as an independent system. Data reducing, processing, and analyzing equipment both analog (Technical Products Company statistical analyzing equipment: probability density, power spectrum, and correlation functions) and high-speed digital (IBM 7094, 1620) are readily available.

The guiding principle in designing, developing and instrumenting the system was to obtain a versatile and flexible research tool in the fields of active satellite communication, propagation, and large array antennas. As an example of versatility the antennas can be
used on any frequency, restricted only by manufacturing tolerances, merely by changing the front RF section and STALO. Recent achievements with the installation include the demonstration of the feasibility of the adaptively phased array principle, the use of the array for passive (Echo I, Echo II, and the Moon) and active (IDCSP-I, -II, -III) satellite communications, phase-coherent transmission, and multiple access techniques.

The specifications of the system are summarized below:

**Location:**
- 08° 02’ 30” West Longitude
- 40° 00’ 10” North Latitude
- Center of radiation - 830 feet above MSL

**Antennas:**
- Number - 4
- Aperture diameter: 30 feet
- Type: Paraboloid, solid surface, 0.040” rms, f/D=0.416
- Mounting: AZ/EL
- Limits of Movements: ±185° in Azimuth
- 0° to 92° in Elevation
- Velocity Limits: 2°/sec both Azimuth and Elevation
- Acceleration Limits: 3°/sec² both Azimuth and Elevation
- Pointing Accuracy: ±1/10th of 3 dB Beamwidth
- Beamwidth: 1° at 2270 MHz, 0.3° at 8000 MHz
- Polarization: Linear or Circular (S-band) RCP or LCP (X-band)
- Gain: (measured) 43 ± 0.5 dB at 2270 MHz, 53.4 ± 0.3 dB at 7,277 MHz
- Feeds: Focal point (S-band), Cassegrainian (X-band)

**Receivers:**
- (RF Section)
- Frequency: 2260 - 2395 MHz
- Parametric Amplifiers: Noise figure: 3 dB
- Gain: 20 dB
- Bandwidth: 30 MHz (3 dB points)
- Down-converters: Noise figure: 8.5 dB
- Gain: 40 dB
- Bandwidth: 8 MHz (3 dB points)
- First IF: 30 MHz
Frequency: 7.2 - 8.4 GHz (This range of frequencies is covered by two tunnel diode amplifiers operating from a diplexer)

Tunnel diode amplifier: Noise figure: 5 dB max
Gain: 15 dB
Bandwidth: 650 MHz

Down-converters: Noise figure: 8 dB
Gain: 40 dB
Bandwidth: 8 MHz
First IF: 30 MHz

Transmitters:
Frequency: 7.9 - 8.4 GHz
Polarization: LCP or RCP
Stability: 1 part in $10^{10}$/sec
Power: 10 KW, CW in each antenna (4 VA 885 B Klystrons)
Modulation: Pulses AM, FM, PM

Communications receivers:
Type: Single conversion from 30 MHz (input) to 5 MHz (2nd IF), phase-lock loop operating at 5 MHz, phase-locked demodulators.

RF Characteristics:
1. Gain: Typically 68 dB from 30 MHz signal input to 5 MHz IF output
2. Gain control: Manual, or automatic
3. IF bandwidth (at 5 MHz): Switchable from 12 KHz to 300 KHz with possibility of 1 KHz with proper filter.

4. Input impedance: 50Ω
Output impedance: 50Ω
5. Linear dynamic range (input): -78 dBm to -40 dBm

Phase-lock loop characteristics
1. Type of filter: Lag (passive)
2. Loop bandwidth: Switchable, 2, 10, 50, 100 Hz
3. Limiting: Limiting occurs on noise for all IF bandwidths

Phase-locked demodulators:
Inputs (five)
1. Frequency: 5 MHz, nominal
2. Limiter Range: -30 to +20 dBm signal and noise
3. Modulation: CW, AM, FM, PM
4. IF bandwidth: 300 KHz
Outputs (five)

1. AM detector
   a.) Video bandwidth: 3, 10, 30, or 100 KHz (selectable)
   b.) Linear range (± 1 dB): 30 dB

2. FM detector
   a.) Max. deviation, two ranges: ± 15 or ± 150 KHz
   b.) Video bandwidth, selectable: 3, 10, 30 or 100 KHz
   c.) Linearity: ± 1% of full scale

3. PM detector
   a.) Modulation index: 1 radian maximum
   b.) Video frequency response: 500 Hz to 3, 10, 30 KHz

4. Loop bandwidth, selectable: 250 Hz, 500 Hz, 3 KHz, 30 KHz, or 100 KHz

Stabilized local oscillator:
Input frequency: 7.2 - 8.4 GHz
Tracking mode: Frequency-lock, and/or phase-lock
IF frequencies: 30 MHz, 3.0 MHz, 455 KHz
Bandwidths:
   IF: 8 MHz at 30 MHz, narrowed to 6 KHz, 2 KHz, or 187 Hz at 455 KHz
Loop BW: Variable from 2 Hz to 12 Hz
First local oscillator (doppler) offset range: ± 100 KHz about center frequency

Radiometric receiver:
Type: Dicke, balanced by adding noise to antenna line
Switching rate: 1000 Hz
Frequency: 7.2 - 7.76 GHz
Pre-detection bandwidth: 7.549 MHz
Pre-detection gain: 80 dB
Detector: Square-law
Post detection filter and amplifier: Tuned precisely to switching frequency; 3 db bandwidth is 3 Hz; voltage gain up to 70 dB.
Integration: Selectable low pass RC filters, time constants of 0, 12, 0.3, 1.2, 3, 0, and 5.2 seconds
Sensitivity: 1°K (1 sec. integration)
Linearity: Measured linear range exceeds ± 100°K from point of operation
Stability: Measured stability is better than 0.5°K/hr.
A block diagram of the radiometric receiver is shown in Fig. 27.
Tape recorders, magnetic:
Model: Ampex
Channels: 7, direct or FM
Tape: \( \frac{1}{2} \) inch
Frequency range 0.1 to 12.5 KHz direct
0 to 1.25 KHz FM

Model: Sangamo
Channels: 14 direct or FM
Tape: \( \frac{1}{2} \) inch or 1 inch
Frequency response: dc to 600 KHz
Tape recorder is equipped with loop capability for correlation type analyses

Tape recorders, paper:
Models: 1 Sanborn 150 (2)
1 Sanborn 350
Channels: 4 + 8, total of 12
Speeds: \( \frac{1}{2} \) to 100 mm/sec
Digital paper-tape recorder and high-speed oscillographic recorder are also available.

Station frequency standard:
X 4700 A Rubidium Vapor Frequency Standard
(Varian Associates)

The overall view of the Antenna Array Facility is shown in Figs. 100 and 103. The block diagrams of the system for S-band operation are shown in Figs. 101 and 102. The block diagrams of the system for X-band operation are shown in Figs. 104-108 and in Fig. 110. The monopulse tracking receiver[76] whose block diagram is shown in Fig. 109, and the phase-locked demodulators whose block diagrams are shown in Fig. 111 are used with operations on either S-band or X-band frequencies. The figures 100-111 are self-explanatory.
Fig. 100--Antenna array facility at The Ohio State University
S-band operation on three antennas,
West antenna operating on X-band.
Fig. 101--Block diagram of S-band feed system.
Fig. 102--Signal cohering and combining receiver (used principally on S-band).
Fig. 103--Antenna array facility at The Ohio State University
X-band operation on all four antennas.
Fig. 104--System block diagram.
(Note: there are three slave antennas.)
Fig. 105--Slave feed system (X-band).
Fig. 106--Monopulse feed system.
Fig. 107 -- X-band phase-locked local oscillator.
Fig. 108 -- Power amplifier exciter.
Fig. 109--Monopulse tracking receiver.
Fig. 110--Signal combining and cohering receiver (used principally on X-band).
Fig. 111--Phase locked demodulators.
B. Receiving System Parameters

1. Sensitivity

In a receiving system the parameter of greatest importance is the sensitivity. The sensitivity, $P_n$, for the purposes here may be defined as that input power level which is equal to the equivalent thermal noise power of the system

\[
P_n = k T_{\text{sys}} B
\]

where

- $k =$ Boltzman's constant
- $T_{\text{sys}} =$ effective system noise temperature
- $B =$ narrowest bandwidth in the system (187 Hz)

The sensitivity of the receiving system employed for measuring the satellite parameters was determined as follows. As a preparatory step, the insertion loss of the waveguide circuits between the output flange of the feed-horn and the input flange of the tunnel-diode amplifier was measured, employing first a coherent signal source and then a wide-band noise source. The final result of these measurements is that the insertion loss of the waveguide circuits is $1.12 \pm 0.02$ dB. Next a precisely calibrated noise source was attached to the input flange of the system where ordinarily the feed-horn is attached. The calibration of the noise source was done by Signalite, Inc. and by the National Bureau of Standards against the
National Standard of Reference; the corresponding Certificate of Calibration and Report of Calibration are included in Appendix VI.

The noise source was alternately turned on and off, and the IF noise level was monitored. When the noise source is turned on, the IF noise power consists of

\[(123) \quad P_{IF} = (T_{\text{noise source}} + T_{\text{receiver}} + T_{\text{waveguide losses}} + T_{\text{coupled in}}) \ \text{kB}\]

where

\[T_{\text{noise source}} = 11,110 \pm 175^\circ\text{K as per Certificate of Calibration by the National Bureau of Standards and Official Correspondence from Mr. C.K.S. Miller of NBS.}\]

\[T_{\text{receiver}} = \text{Receiver noise temperature}\]

\[T_{\text{waveguide losses}} = \text{Equivalent noise temperature of lossy waveguide circuits}\]

\[T_{\text{coupled in}} = \text{Equivalent noise temperature coupled into the antenna line by the waveguide couplers.}\]

When the noise source is turned off, the IF noise power consists of

\[(124) \quad P_{IF} = (T_{\text{load}} + T_{\text{receiver}} + T_{\text{waveguide losses}} + T_{\text{coupled in}}) \ \text{kB}\]
where

\[ T_{\text{load}} = \text{equivalent noise temperature of the noise source} \]

in its unfired condition.

Forming the ratio, \( R \), of Eq. (123) and (124), knowing the insertion loss \( \alpha \) of the waveguide circuits, having measured the physical temperature of the waveguide circuit and of the coupler terminations, and knowing the coupling factors, it may be shown that for the particular experimental conditions the receiver noise temperature is given by the following expression

\[
(125) \quad T_{\text{receiver}} = \frac{\alpha T_{\text{noise source}} + (1.11 - \alpha) \cdot 277 - R(307)}{R - 1}, \quad ^{\circ}\text{K}.
\]

The result is \( T_{\text{receiver}} = 632.5 \pm 19.2^\circ\text{K} \).

The corresponding noise figure, NF, follows from

\[
(126) \quad (\text{NF} - 1) \cdot 290 = 632.5\,^\circ\text{K},
\]

which yields

\[
(127) \quad \text{NF} = 5.02 \text{ dB}.
\]

The system noise temperature, \( T_{\text{sys}} \) is given by

\[
(128) \quad T_{\text{sys}} = T_{\text{receiver}} + T_{\text{waveguide losses}} + T_{\text{antenna}} + T_{\text{coupled in}}.
\]
Using the above determined figure for $T_{\text{receiver}}$ and the previously determined figures for $T_{\text{waveguide losses}}$ and $T_{\text{antenna}}$, one obtains the following value

\begin{equation}
T_{\text{sys}} = 632.5 + 66 + 7.7 + 33 = 739.2 \, ^{\circ}\text{K}
\end{equation}

This is the system noise temperature when the antenna is at zenith, the receiver frequency is 7,276.704 MHz, and normal atmospheric conditions prevail. The uncertainty in this figure is $\pm 23.1 \, ^{\circ}\text{K}$ or 3.1%. The sensitivity, $P_n$, of the system is

\begin{equation}
P_n = k T_{\text{sys}} B = 1.38 \times 10^{-20} \times 739.2 \times 187
\end{equation}

\begin{equation}
f = 7,276.704 \, \text{MHz}
\end{equation}

\begin{equation}
= 1.908 \times 10^{-15} \, \text{milliwatts}, \text{ or}
\end{equation}

\begin{equation}
P_n(\text{dBm}) = 10 \log_{10} (k T_{\text{sys}} B) = -147.2
\end{equation}

\begin{equation}
f = 7,276.704 \, \text{MHz}
\end{equation}

The noise power density, $S = P_n / B$, is

\begin{equation}
S = P_n / B = -169.9 \, \text{dBm/Hz}
\end{equation}

Similar measurements on the beacon frequency of the satellite ($f = 7,299.500 \, \text{MHz}$), in the same 187 Hz bandwidth yield the following values.
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(133) \( T_{\text{receiver}} = 1393.3^\circ K \) 

(134) \( \text{NF} = 7.64 \text{ dB} \) 

(135) \( T_{\text{sys}} = 1500^\circ K \) 

\( f = 7,299,500 \text{ MHz} \) 

(136) \( P_n = -144.1 \text{ dBm} \) 

(137) \( S = -166.8 \text{ dBm/Hz} \) 

2. Bandwidth

For calculation of the sensitivity and of the noise power of the system it is necessary to know the narrowest bandwidth in the system. This narrowest bandwidth occurs at the 455 KHz IF frequency. The measured band-shape is shown in Fig. 112. The integrated bandwidth is 187 Hz.

3. Linearity

The system was calibrated as a matter of routine prior to and after each satellite tracking. Typical results are shown in Figs. 113 and 114 collected on the two frequencies of principal interest. The system is essentially linear. It is known that the RF section of the receiver suffers no gain compression nor saturation effects until the level of -40 dBm is reached. Furthermore, it has been measured and found that the IF section of the receiver is absolutely linear over a 41 dB range from the noise level (satellite signal-plus-noise to noise ratios did not exceed +36 dB).
Fig. 112--Band-shape of the receiver
Fig. 113--Graph of the receiver output power as a function of input power showing linearity.
Fig. 114--Graph of the receiver output power as a function of input power showing linearity.
4. Radiometer

The radiometer is a Dicke type device normally operated at or near balance by adding noise to the antenna line. This procedure results in good stability at the expense of a slight loss in sensitivity. The switch is a ferrite device operating at RF, located directly ahead of the tunnel diode amplifier. It is driven at 1000 Hz, and it provides a minimum isolation of 35 dB between ports. The pre-detection circuitry is a conventional mixer - IF amplifier combination and it is normally used for communication purposes. An additional IF amplifier in the radiometer channel provides some 40 dB more gain over that normally utilized in the communication channel. This gain is necessary to provide a useable output level from the detector. The detector is a crystal, operated in its square-law region. (Since the radiometer is usually balanced before experimentation this point of square-law detection is not as important as it is with other types of radiometers which operate over wide ranges of input power levels.)

The sensitivity of the radiometer - that change in the antenna temperature which causes a change in the output level equal to the rms value of the fluctuations - was measured and found to be 2.9, 1.9, 0.9, 0.6, and 0.5 K when the time constant of the low-pass RC filter was 0.1, 0.3, 1.2, 3.0, and 5.0 seconds respectively.
The measured predetection bandwidth of the radiometer is shown in Fig. 115. The integrated noise bandwidth is 7.549 MHz.

The stability of the radiometer is excellent, as it is expected from design consideration. The measured drift over approximately a one hour interval was $3.5^0 K$. The measured change in the temperature of the load used for comparison was $3.8^0 K$ over the same time interval. It was concluded that the drift in the output of the radiometer did not exceed $0.5^0 K$ per hour.

The linearity of the radiometer was tested under various operating conditions; the result of one such test is shown in Fig. 116. The experimental procedure consisted of varying the amount of noise coupled into the antenna line. The noise generator was the same one calibrated by the National Bureau of Standards, its output was controlled by a precision variable attenuator: Hewlett-Packard Model H382A, serial number 1603. The attenuated output was coupled into the antenna line through an accurately calibrated coupler: Hewlett-Packard Model H752C, serial number 2499. The Calibration Reports of these devices are included in Appendix VI. The radiometer is linear over a range of $+100^0 K$ from the point of operation.
NOISE BANDWIDTH: 7.549 MHz

Fig. 115--Pre-detection band-shape of the radiometer.
Fig. 116--Radiometer output versus changes in equivalent input temperature.
4. Data Reduction

Because of the large volume of data collected and because of the desire for good accuracy in the results, most of the data reduction was done with the aid of an IBM 7094 Computer. The data were collected in analog form; they were in most instances recorded on paper charts for direct reduction and "quick look" analysis, and on magnetic tape for accurate processing. The analog data from the magnetic tape was transcribed to a digital tape with the aid of an Analog-to-Digital converter. These digital tapes were in turn processed with the computer by appropriate programs. The data collection and reduction procedure is illustrated graphically in Fig. 117; the various programs and their flow charts are given in Appendix V.
Fig. 117--Graphical illustration of data collection and reduction procedures.
APPENDIX I

The purpose of this appendix is to present some statistical considerations and to use these to evaluate the data presented in Chapter II. Specifically, a definite lower bound and a probable upper bound will be calculated for the confidence level and the corresponding ranges of uncertainties due to randomness that go with these levels.

When the probability density function of the data is unknown one can apply the Tchebycheff inequality as a first order statistical analysis.[88]

\[(138) \quad P \{|x - \bar{x}| \geq h \sigma\} \leq \frac{1}{h^2},\]

where

- \(x\) = any random variable with arbitrary probability density function and finite variance,
- \(\bar{x}\) = mean of the random variable \(x\),
- \(\sigma\) = standard deviation, and
- \(h\) = a numeric.
This inequality yields a lower bound on the confidence level for the random variable \( x \) to be within a given range from the mean.

(Usually, both the confidence level and the range are expressed in per cent; thus, a probability of 0.90 is taken as 90\% confidence level; a range of \( (x - \overline{x})/\overline{x} = 0.01 \) is taken as 1\%.) Dividing the left side of expression (138) by \( \overline{x}(x \neq 0) \) does not alter the inequality; further, substituting into it \( k = h\sigma/\overline{x} \), one obtains

\[
(139) \quad P\left\{ \left| \frac{x - \overline{x}}{\overline{x}} \right| \geq k \right\} \leq \frac{1}{k^2} \left( \frac{\sigma}{\overline{x}} \right)^2.
\]

Expression (139), together with the values of the standard deviations and the means given in Figs. 5 and 6 was used to generate the curves A through D shown in Fig. 118. The procedure was as follows: for a 90\% confidence level for the range to be less than one per cent one requires that the probability that \( \left| x - \overline{x} \right| / \overline{x} \cdot 100 \geq 1.0 \) be less than 0.1. Under these conditions \( k = 0.01 \) and \( (1/k^2) (\sigma/\overline{x})^2 \leq 0.1 \).

Substituting values for \( \sigma \) and \( \overline{x} \), and choosing values for \( k \) one can readily plot the curves of Fig. 118. From this figure the various combinations of confidence levels and ranges can be ascertained; e.g., one can say with at least 90\% confidence that the measured gain of the pyramidal horn at 7.95 GHz is within 2.25\% of the mean (worst case in Fig. 118); and that at the 90\% confidence limit the
Fig. 118--Lower and upper bounds on the confidence level for the measured gain (regarded as the random variable) to be within a given range from the mean value of the sample.
measured gain value at 7.3 GHz and horizontal polarization is within 0.93% of the mean value (best case in Fig. 118). To find the true gain one must apply the corrections determined in Chapter II.

The Tchebycheff inequality is quite useful because it provides a lower bound on the confidence level regardless of the density function; however, this lower bound is quite pessimistic for small values of \( h \). To obtain a more optimistic estimate on the confidence interval, the Tchebycheff inequality will be evaluated with the assumption that the density function of the data is normal,

\[
(140) \quad f(x) \mid_{\text{normal}} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2} \left(\frac{x - \bar{x}}{\sigma}\right)^2}.
\]

The general shape of \( f(x) \) is shown in Fig. 119; the shaded areas are where \( |x - \bar{x}| > h\sigma \). The probability that \( |x - \bar{x}| \) is greater than \( h\sigma \) is given by the integral of \( f(x) \) over the shaded regions;

\[
(141) \quad P \{ |x - \bar{x}| > h\sigma \} = \int_{-\infty}^{\bar{x} - h\sigma} f(x) \, dx + \int_{\bar{x} + h\sigma}^{\infty} f(x) \, dx
\]

\[
= 1 - \int_{\bar{x} - h\sigma}^{\bar{x} + h\sigma} f(x) \, dx
\]
Fig. 119--Gaussian probability density function.
The left side of Eq. (141) may be rewritten

\[
(142) \quad P\left( \frac{x - \bar{x}}{\bar{x}} > \frac{\sigma}{\bar{x}} \right) = 1 - \int_{\bar{x} - \sigma}^{\infty} e^{\frac{-(x - \bar{x})^2}{2\sigma^2}} \, dx.
\]

Let \((x - \bar{x})/\sigma = u\), then \(dx = \sigma \, du\) and the limits of integration become

\(\pm \sigma\). Equation (142) becomes

\[
(143) \quad P\left( \frac{x - \bar{x}}{\bar{x}} > \frac{\sigma}{\bar{x}} \right) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{h} e^{-\frac{u^2}{2}} \, du;
\]

alternately

\[
(144) \quad P\left( \frac{x - \bar{x}}{\bar{x}} < \frac{\sigma}{\bar{x}} \right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{h} e^{-\frac{u^2}{2}} \, du.
\]

Substituting \(k = \sigma/\bar{x}\) into Eq. (144) one obtains

\[
(145) \quad P\left( \frac{x - \bar{x}}{\bar{x}} < k \right) = \frac{1}{2\pi} \int_{-\infty}^{k/(\sigma/\bar{x})} e^{-\frac{u^2}{2}} \, du.
\]

The right side of Eq. (145) is available in the tables of Ref. 89.

Reference 89 was used to generate the curves A' - D'. Obviously,
the confidence levels are much higher and the ranges from the mean are significantly smaller.

Now that there are two sets of confidence level curves available, it is necessary to find the error in the assumption that the data had a normal density function. To remain on the conservative side in the analysis, the magnitude of the maximum error in approximating the density function of the measured data with that of a gaussian distribution will next be determined. The error $\epsilon(x)$ is the difference between the actual density function $f(x)$ and the normal density function

$$
(146) \quad \epsilon(x) = f(x) - \frac{1}{\sqrt{2\pi \sigma^2}} \ e^{-\frac{1}{2} \left( \frac{x - \bar{x}}{\sigma} \right)^2}.
$$

Expanding Eq. (146) as a series in terms of Hermite Polynomials, evaluating the coefficients in terms of the moments $m_1$ of $f(x)$, one obtains the following first order correction:

$$
(147) \quad f(x) = \frac{1}{\sqrt{2\pi \sigma^2}} \ e^{-\frac{x^2}{2\sigma^2}} \left[ 1 + \frac{m_3}{3! \sigma^3} \left( \frac{x^3}{\sigma^3} - \frac{3x}{\sigma} \right) \right],
$$

where $\bar{x} = 0$ was assumed. The maximum error can be found as described below. First one finds the values of $x$ where $d\epsilon(x)/dx = 0$. 

(148) \[
\frac{d\varepsilon(x)}{dx} = \frac{d}{dx} \left\{ \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \frac{m_3}{3!\sigma^3} \left( \frac{x^3}{\sigma^3} - 3\frac{x}{\sigma}\right) \right\}.
\]

From Eq. (148) there results a biquadratic equation,

(149) \[x^4 - 6\sigma^2 x^2 + 3\sigma^4 = 0,\]

whose four roots are

(150) \[x_{1,2} = \pm 2.3344\sigma, \text{ and } x_{3,4} = \pm 0.74196\sigma,\]

as can be verified by substitution.

Next, one evaluates the second derivative of \(\varepsilon(x)\) with respect to \(x\)

(151) \[
\frac{d^2\varepsilon(x)}{dx^2} = \frac{d}{dx} \left\{ -\frac{x^2}{\sigma^2} e^{\frac{2\sigma^2}{\sigma^2}} \frac{m_3}{3!\sigma^3} \left( \frac{x^3}{\sigma^3} - 3\frac{x}{\sigma} \right) \right\} + \frac{1}{\sigma\sqrt{2\pi}} \frac{x^2}{2\sigma^2} \frac{m_3}{3!\sigma^3} \left( \frac{3x^2}{\sigma^3} - \frac{3}{\sigma} \right) \right\} + \frac{m_3}{3!\sigma^5} \frac{e^{2\sigma^2}}{\sigma^2 \sqrt{2\pi}} \frac{x^4}{\sigma^4} \left( \frac{10x^2}{\sigma^2} + 15 \right).
\]

By trial and error it is found that

\[\frac{d^2\varepsilon(x)}{dx^2} < 0\]
when \( x = + 2.344 \sigma \) and when \( x = - 0.74196 \sigma \); hence maximum errors occur at these values. The magnitudes of these errors can be found by evaluating

\[
\epsilon(x) = \frac{e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi}} \frac{m_3}{3! \sigma^3} \left( \frac{x^3}{\sigma^3} - \frac{3x}{\sigma} \right).
\]

Equation (152) evaluated for the above values of \( x \) is

\[
\begin{align*}
(153) \quad \epsilon(x) &= 0.02489 \frac{m_3}{\sigma^4} \quad \text{for } x = 2.3344 \sigma, \quad \text{and} \\
\epsilon(x) &= 0.09196 \frac{m_3}{\sigma^4} \quad \text{for } x = -0.74196 \sigma.
\end{align*}
\]

It can be seen that an absolute maximum occurs at \( x = -0.74196 \sigma \), and there is a local maximum at \( x = 2.3344 \sigma \). The third moments of the data were evaluated by

\[
(154) \quad m_3 = \frac{1}{N} \sum_{j=1}^{N} (x_j - \bar{x})^3.
\]

The results of Eq. (154) were

\[
\begin{align*}
\text{Vertical Polarization} & \quad m_3 = -1.0796 \times 10^{-3} \\
\text{Horizontal Polarization} & \quad m_3 = 5.1203 \times 10^{-3} \quad \text{for } f = 7.3 \text{ GHz}, \\
\text{Both Polarizations} & \quad m_3 = 2.2992 \times 10^{-3}
\end{align*}
\]

(155) and

\[
\begin{align*}
\text{Vertical Polarization} & \quad m_3 = -6.2629 \times 10^{-3} \\
\text{Horizontal Polarization} & \quad m_3 = -3.9302 \times 10^{-3} \quad \text{for } f = 7.95 \text{ GHz}, \\
\text{Both Polarizations} & \quad m_3 = -5.0934 \times 10^{-3}
\end{align*}
\]
Substituting the values of $m_3$ from Eq. (155), and those of $\sigma$ from Figs. 5 and 6 into Eq. (153) the absolute maximum errors in approximating the density function of the measured data with that of a Gaussian distribution are

\begin{equation}
\epsilon(x) = \begin{cases} 
-39.5\% & \text{VP} \\
+15.6 & \text{HP} \\
\text{abs. max} & -37.0 & \text{both VP and HP data}
\end{cases}
\end{equation}

for $f = 7.3$ GHz, and

\begin{equation}
\epsilon(x) = \begin{cases} 
-35.3\% & \text{VP} \\
-21.8 & \text{HP} \\
\text{abs. max} & -28.5 & \text{both VP and HP data}
\end{cases}
\end{equation}

for $f = 7.95$ GHz. The errors occurring at the local maximum ($x = 2.3344\sigma$) are about one-fourth of the absolute maximum errors given by Eqs. (156) and (157). Because of these errors it would be unduly optimistic to use the primed curves of Fig. 118 in establishing confidence intervals; likewise, it would be unnecessarily pessimistic to use the curves obtained by Tchebycheff's inequality. These two sets of curves then serve as upper and lower bounds, respectively, on the confidence levels. From Fig. 118 the following ranges are taken for the 99.0 per cent confidence level:

\begin{align}
(158) & \quad \text{Vertical Polarization} & 2.0\% & \text{range} \\
& \quad \text{Horizontal Polarization} & 1.0 & \text{for } f = 7.3 \text{ GHz} \\
& \quad \text{Both Polarizations} & 2.5 & \\
\end{align}

Vertical, Horizontal, and Both Polarizations

\begin{align}
& 2.6\% & \text{for } f = 7.95 \text{ GHz}.
\end{align}
In conclusion, a range of 2.0 per cent (0.088 dB) for the 7.3 GHz data and a range of 2.6 per cent (0.114 dB) for the 7.95 GHz data will be taken with a confidence level of 99.0 per cent. These are the values used in Tables II and IV.

Ultimately, however, one is not so much concerned with the sample itself (see the histograms of Figs. 5 and 6), nor is he concerned with the mean of the sample; rather one is concerned more with estimating the parameters of the population based on the parameters of the sample. The particular parameter of interest here is the mean of the population, i.e., the gain of each of the horns. One is justified in invoking the Central-limit[32] Theorem and assert that the mean of the sample is an unbiased estimator of the mean of the population, the error due to randomness in this assertion is \(3\sigma / \sqrt{n}\) when the confidence level is 99.7 per cent.

Specifically, for the 7.3 GHz data (see Fig. 5):
\[
3\sigma / \sqrt{n} = 3 (0.68\%) / \sqrt{200} = 0.14\ per\ cent\ (0.006\ dB); \quad \text{and for the}
\]
7.95 GHz data (see Fig. 6): \(3\sigma / \sqrt{n} = 3 (0.72\%) / \sqrt{186} = 0.16\ per\ cent\ (0.007\ dB).\)
APPENDIX II

The purpose of this Appendix is to calculate the magnitude of the error in the gain measurements due to ground-reflected waves. The problem of arriving at an accurate estimate of the error in the gain measurements caused by waves reflected from the ground has been solved recently by Ruze. His solution will be used here to determine the magnitudes of the errors involved in the gain measurements by the gain-comparison and absolute gain techniques.

Ruze considers the vertical field, \( e(y) \), at the receiving aperture to be the sum of (a) a plane wave at normal incidence, and (b) a ground-reflected wave of amplitude \( a \), relative phase \( \phi \), and angle of incidence \( \alpha \):

\[
e(y) = 1 + a \left( \frac{2\pi}{\lambda} y \sin \alpha + \phi \right)
\]

(159)

The experimental setup is usually adjusted for maximum field intensity at the center of the test antenna, i.e., for \( \phi = 0 \). The standard-gain horn used in the gain comparison measurement receives a power proportional to

\[
P_{\text{horn}} = G_{\text{horn}} (1 + a)^2
\]

(160)
where

\[ G_{\text{horn}} = \text{power gain of the standard gain horn}. \]

The power received by the test antenna is proportional to

\[
P_T = G_T \frac{\left[ \int_{-y_o}^{y_o} \left[ 1 + e^{j \left( \frac{2\pi}{\lambda} y \sin \alpha \right)} \right] \cdot \cos \frac{\pi}{2} \frac{y}{y_o} \cdot dy \right]^2}{\int_{-y_o}^{y_o} \cos \frac{\pi}{2} \frac{y}{y_o} \cdot dy}
\]

(161)

where

\[ G_T = \text{power gain of the test antenna} \]

\[ \pm y_o = \text{vertical extent of the aperture} \]

\[ \cos \frac{\pi}{2} \frac{y}{y_o} = \text{feed pattern, test antenna}. \]

The assumption that the aperture illumination is cosinusoidal is very accurate in this case. Integrating Eq. (161) and taking the ratio of Eq. (161) to Eq. (160) one obtains

\* The form of Eq. (162) appearing in Ref. 91 is believed to be in error.
Equation (162) can be evaluated to accurately estimate the magnitude of the error due to ground reflections. In the gain comparison measurements the range was 550 feet, the source was about 30 feet above ground, thus $\alpha = 5.7$. The half-power beamwidth of the source was 2.9; referring to Figs. 7-10 the magnitude of $a$ is estimated to be $10^{-3}$. Using $\lambda = 4.1$ cm and substituting the above values into Eq. (162)

\[
\frac{P_T}{P_{\text{horn}}} = \frac{G_T}{G_{\text{horn}}} \left[ 1 + a \left( \frac{\pi}{2} \right)^2 \frac{\cos \left( \frac{2\pi}{\lambda} y_0 \sin \alpha \right)}{\left( \frac{\pi}{2} \right)^2 - \left( \frac{2\pi}{\lambda} y_0 \sin \alpha \right)^2} \right].
\]

For the case of absolute gain measurements $\alpha = 2.9$ which closely corresponds to the first null in the pattern of the receiving and transmitting antennas. Therefore, it is not necessary to consider the ground reflections in this case. In conclusion, the possible errors introduced in the gain measurements by the ground reflections are negligibly small.
APPENDIX III

The purpose of this Appendix is to calculate confidence levels for the random variable - the measured gain of the gain-calibration standard antenna - to be within a given range from the mean value determined from the data, and to test the hypothesis that the measured data are a sample of a random variable with normal distribution. Certain statistical inferences will be made on basis of these calculations and tests.

When the probability density function of the data is unknown one may apply the Tchebycheff inequality as a first order statistical analysis.[88] This inequality can be written as

\[
P\left\{ \left[ \frac{|x - \overline{x}|}{|\overline{x}|} \right] > k \right\} \leq \frac{1}{k^2} \left( \frac{\sigma}{\overline{x}} \right)^2
\]

where

- \( x \) = any random variable with arbitrary probability density function and finite variance.
- \( \overline{x} \) = mean of the random variable \( x \),
- \( \sigma \) = standard deviation, and
- \( k \) = a numeric.
This inequality yields a lower bound on the confidence level for the random variable \( x \) to be within a given range from the mean.

Substituting the values of \( \sigma \) and \( x \) from Figs. 14-16 and Figs. 18 and 19 into expression (164) and assigning values of \( k \), a set of curves can be generated from which the various combinations of confidence levels and ranges can be ascertained (see Fig. 122); e.g., from curve E of Fig. 122 for a range of 3.3% the confidence level is at least 90%, and from curve A of Fig. 122 for a range of 9.3% the confidence level is again at least 90%.

To obtain a probable upper bound on the confidence level for the random variable \( x \) to be within a given range from the mean the Tchebycheff inequality is evaluated under the assumption that the density function of the data was Gaussian. With this assumption expression (164) becomes

\[
P \left\{ \left| \frac{x - \bar{x}}{\sigma} \right| < k \right\} = \frac{1}{\sqrt{2\pi}} \int_{-k}^{k} e^{-\frac{u^2}{2}} du.
\]  

(165)

Proceeding as above the primed set of curves of Fig. 122 were generated. From Curve E' of Fig. 122 for a range of 1.7% the confidence level is at most 90%, and from curve A' of Fig. 122 for a range of 5.8% the confidence level is again at most 90%.

Using the mid-values between the matching primed and un-primed
Fig. 122--Lower and upper bounds on the confidence level for the measured gain to be within a given range from the mean value of the data.
curves of Fig. 122 the following probable ranges are established as corresponding to a 90.0 per cent confidence level.

<table>
<thead>
<tr>
<th>Polarization</th>
<th>Percentage Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Polarization</td>
<td>3.3% range</td>
</tr>
<tr>
<td>Horizontal Polarization</td>
<td>2.5</td>
</tr>
<tr>
<td>Both Polarizations</td>
<td>3.5</td>
</tr>
<tr>
<td>Both Polarizations</td>
<td>7</td>
</tr>
<tr>
<td>Both Polarizations</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Absolute gain measurement $f = 7.3\,\text{GHz}$

Gain comparison measurement $f = 7.3\,\text{GHz}$

Gain comparison measurement $f = 7.95\,\text{GHz}$.

On the basis of the above calculations, a probable range of 3.0 per cent (0.132 dB) for the absolute gain measurements, a probable range of 7 per cent (0.315 dB) for $f = 7.3\,\text{GHz}$ gain comparison measurement, and a probable range of 5.9 per cent (0.264 dB) for the $f = 7.95\,\text{GHz}$ gain comparison measurement may be taken as corresponding to a confidence level of 90.0 per cent.

The above calculations of confidence levels were applicable to the individual samples. On the basis of these samples it is possible to draw inferences about the population. These inferences, of course, are subject to some uncertainty, unless the sample consists of the entire population. If it can be shown that the sample has been drawn from a normal population, Statistical Estimation Theory permits one to reach certain conclusions about the population mean $\mu$ and standard deviation $\sigma$. Thus, it is of great practical interest to test the hypothesis whether a particular sample has been drawn from a normal population. The three
samples one wishes to test are depicted in Figs. 16, 18, and 19.

The test that will be applied is the $\chi^2$-test.\[92\]

The hypothesis $H$ is: the data $x_1, \ldots, x_n$ are a sample of a random variable $X$ with normal distribution. The purpose of the test is to determine whether the data $x_1, \ldots, x_n$ may be considered as consistent with hypothesis $H$. To carry out the test one proceeds as follows: Each histogram will be divided into $r$ class intervals such that each interval will contain at least 10 data points. The class intervals need not be of uniform length; the requirement that they contain at least 10 data points may be achieved by grouping of class intervals, especially towards the tails of the histograms. Let the sample frequency in class interval $i$ be $v_i$. The number of values $x$ from a sample of size $n$ which are expected to fall in the $i^{th}$ interval is

\begin{equation}
np_i = \left( \frac{n}{\sigma \sqrt{2\pi}} \right) \int_{I_i} \exp \left( -\frac{[x - x]^2}{2\sigma^2} \right) dx
\end{equation}

where the integration is to be carried out over the $i^{th}$ interval. Next, one computes $\chi^2$ from the following relationship:

\begin{equation}
\chi^2 = \sum_{i=1}^{r} \frac{(v_i - np_i)^2}{np_i}.
\end{equation}
The number of degrees of freedom \( m = r - 1 - b \), where

\[ b = 2 \] since two of the parameters, mean value and standard

deviation, in the population distribution are determined from the

sample. Selecting a level of significance \( \epsilon \), from tables of the

\( \chi^2 \)-distribution for a given degree of freedom \( m \), one determines

\( \chi^2_o \) such that

\[
(168) \quad P (\chi^2 \geq \chi^2_o) = \epsilon
\]

Interpretation from the test: If in the sample a value \( \chi^2 \leq \chi^2_o \)
is found, the sample \( x_1, \ldots, x_n \) is considered as consistent with

the hypothesis \( H \); otherwise, i.e., if \( \chi^2 > \chi^2_o \), the sample \( x_1, \ldots, x_n \)
shows a significant deviation from \( H \), and the hypothesis \( H \) is re-

jected at the \( \epsilon \) level of significance.

The results of the \( \chi^2 \)-test indicate that when this test is

applied to the sample shown in Fig. 16, the hypothesis \( H \) is to be

rejected at the \( \epsilon = 0.05 \) significance level (the hypothesis \( H 
\)
could be accepted at the \( \epsilon = 0.01 \) significance level). Similar test

applied to the data of Fig. 18 shows that the sample may be consider-

ed as consistent with the hypothesis \( H \) at the \( \epsilon = 0.10 \) significance

level. (The actual value of \( \epsilon \), as found by linear extrapolation of

tabulated values, is \( \epsilon = 0.17 \).) Likewise, it could be shown that

the sample whose histogram is shown in Fig. 19 is also consistent
with the hypothesis $H$ at about the same significance level as
was found for Fig. 18.

As an immediate consequence of these results in evaluating
the confidence levels from Fig. 122 the curves based on Tcheby-
cheff's inequality solved for normal density function will be used
in conjunction with the gain-comparison data. Thus the following
ranges are established as corresponding to a 99.0 per cent confi-
dence level.

- Both polarizations: 7.5% gain comparison measurement
  \[ f = 7.3 \text{ GHz} \]

- Both polarizations: 6.3% gain comparison measurement
  \[ f = 7.95 \text{ GHz} \]

These are the values used in Table IX.

Furthermore, it is asserted that the samples whose histo-
grams are shown in Figs. 18 and 19 have been drawn from a
normal population. An unbiased estimate of $\mu$ is $\bar{x}$. The pre-
cision of this estimate is expressed by the fact that the 99.7 per
cent confidence interval is $\bar{x} \pm (3\sigma /\sqrt{n})$. An unbiased estimate of
$\sigma$ is $s(n/(n-1))^{1/2}$.

In consequence of the above assertions, one is well justified
in estimating the true mean as the mean of the measured sample;
furthermore, the confidence level in the correctness of this esti-
mate is 99.7 per cent when the range from the mean is less than
$3\sigma/\sqrt{n}$. Specifically, for the gain-comparison measurements at
\( f = 7.3 \text{ GHz} \) the value of \( 3\sigma / \sqrt{n} \) is equal to: 
\[
\frac{3 \times 3\%}{\sqrt{146}} = 0.75\% = 0.033 \text{ dB};
\]
for the gain-comparison measurements at 
\( f = 7.95 \text{ GHz} \) the value of \( 3\sigma / \sqrt{n} \) is equal to 
\[
\frac{3 \times 2.8\%}{\sqrt{151}} = 0.68\% = 0.029 \text{ dB}. 
\]
These values will be used for the uncertainty due to randomness at the 99.7 per cent confidence level.
APPENDIX IV

The purpose of this appendix is to present a simple method of computing look angles for a celestial body, that is to say, to present the necessary angular relationship enabling one to point the antenna, whose geographic location is known, toward an object in space whose location on the celestial sphere is known at a given instant of time.

The problem of pointing an antenna toward a celestial object is essentially the reverse of the sailor's problem. In the present case, it is assumed that the geographical location of the antenna site is well known and a particular pointing direction is sought, whereas in the case of the navigator, the pointing is read from the scales of the sextant and the geographical location at the time of observation is sought. The basis of all calculations of this type is a solution to the astronomical triangle which is formed by arcs of great circles passing through the pole, the zenith, and the celestial object (Fig. 123c). The point directly above the point of observation is zenith; by drawing a great circle through the pole and the zenith one obtains the great circle labeled observer's meridian in
Fig. 123a, b--Components of the astronomical triangle.
Fig. 123a. On it the latitude of the observer is measured. Next a great circle is drawn through the observer's zenith and the celestial object; the elevation of the object is measured on it. Perpendicular to the latter and to the observer's meridian is the observer's horizon circle. For clarity these three great circles, or arcs of them are shown in Fig. 123a. Figure 123b shows the observer's meridians again; a portion of the great circle

![Diagram showing astronomical triangle](c)

Fig. 123c--The astronomical triangle.
through the pole and the object S on which the object's declination is measured; and the equatorial great circle, perpendicular to the pole-object great circle, on which the right ascension and hour angle are measured. By superimposing Fig. 123a on Fig. 123b one obtains the astronomical triangle shown in heavy lines in Fig. 123c. Azimuth is the arc measured on the horizon circle, in the clockwise sense, from the point closest to the pole where the horizon circle and the observer's meridian intersect to the intersection of the horizon circle and the zenith-object great circle. Elevation is the arc measured from this last point along the zenith-object great circle toward zenith and up to the location of the object S. Hour angle is the arc measured on the equator in the clockwise sense from the observer's meridian to the meridian of the object (the great circle through the pole and the object). Hour angle can be expressed in time measure (e.g., rising sun: +18 hours; setting sun: +6 hours) or in angular measure ($90^\circ$ and $270^\circ$, respectively). Declination is the arc measured from this last point along the meridian of the object toward the pole and up to the location of the object S. Right ascension is the arc measured on the equatorial circle in a counter-clockwise sense from a point called the Vernal Equinox, V, which is a reference on the celestial sphere just like
the meridian passing through Greenwich is a reference on the globe. Latitude is the arc measured on the observer's meridian from the intersection of this with the equatorial circle toward the pole and up to the observer's zenith. Complementary arc segments, e.g., co-latitude are self-explanatory.

From spherical trigonometry[ 93] one obtains the following basic formula:

\[ \cos \alpha = \cos \beta \cos \gamma + \sin \beta \sin \gamma \cos \delta. \]  

Let \( \alpha = \) Local Hour Angle, LHA; \( \beta = 90^\circ - EL \) (EL = elevation of object); \( \gamma = 90^\circ - L \) (L = latitude of observer); \( \delta = 90^\circ - D \) (D = declination of object S), and substitute these into Eq. (169) obtain

\[ \cos (90^\circ - EL) = \cos (90^\circ - L) \cos (90^\circ - D) + \sin (90^\circ - L) \sin (90^\circ - D) \cos LHA, \]

which reduces to

\[ \sin EL = \sin L \sin D + \cos L \cos D \cos LHA. \]

Equation (171) gives the elevation of the object, as seen from a given geographical location, in terms of the observer's latitude and the declination and Local Hour Angle of the object:

\[ \text{Elevation} = \text{Arc sine} \left[ \sin L \sin D + \cos L \cos D \cos LHA \right]. \]
Now let $a = \text{Azimuth, } AZ$; $a = 90^\circ - D$; $\beta = 90^\circ - L$; $\gamma = 90^\circ - EL$; and substitute into Eq. (169) to obtain

\begin{equation}
\cos (90^\circ - D) = \cos (90^\circ - L) \cos (90^\circ - EL) + \sin (90^\circ - L) \sin (90^\circ - EL) \cos AZ ,
\end{equation}

which reduces to

\begin{equation}
\cos AZ = \frac{\sin D}{\cos L \cos EL} - \tan L \tan EL .
\end{equation}

Equation (174) yields the azimuth of the object, as seen from a given geographical location, in terms of the observer's latitude and the elevation and declination of the object:

\begin{equation}
\text{Azimuth} = \text{Arc cosine} \left[ \frac{\sin D}{\cos L \cos EL} - \tan L \tan EL \right] .
\end{equation}

Equations (172) and (175) are the ones used for obtaining look angles for a celestial body. The location of the Satellite Communication Facility of The Ohio State University ElectroScience Laboratory is $40^\circ 00' 10''$ North Latitude, $083^\circ 02' 30''$ West Longitude. The position of Cassiopeia-A on the celestial sphere,\[39,94,95\] corrected for Epoch 1967.0 (Right Ascension increment = $2^\circ 70$/year, Declination increment = $19'7$/year), is 23 hours, 22 minutes, 04 seconds Right Ascension, and $58^\circ 42' 14''$ Declination (northerly). The position of the moon is tabulated in the Ephemeris\[96\] and the Almanac.\[97\] The Local Hour Angle, Right Ascension, and Sidereal
Time are related by the formula

\[ LHA = ST - RA \]

The Sidereal Time is readily computed for any given local civil time with the aid of the explanatory section in the Ephemeris. When the Greenwich Hour Angle of the object is known, LHA is simply found by the formula

\[ LHA = GHA \pm \text{Longitude} \]

The minus sign in Eq. (12) goes with westerly and the plus sign with easterly longitudes.

Equations (172) and (175) produce pointing information referred to the center of the earth, whereas the observations are to be made from the surface of the earth. These two references are called geocentric and geodetic, respectively. The difference between the true elevation and the observed elevation is immediately obvious from Fig. 5. It can also be seen from Fig. 124 that the maximum parallax occurs when nearby objects are on the horizon, hence the descriptive name, horizontal parallax. The effect of the parallax is to decrease the elevation of the body, that is, the geocentric (true) and geodetic (observed) elevation angles are related by the following equation:
**Fig. 124—Horizontal parallax.**
(178) \[ \text{True Elevation - Parallax Correction} = \text{Observed Elevation}. \]

From elementary trigonometry and Fig. 124 the parallax correction is:

(179) \[ \text{Parallax Correction} = (\text{Horizontal Parallax}) \times (\text{Cosine of true elevation}). \]

The Horizontal Parallax presents a vexing problem in the case of the moon but it is negligible in case of Cassiopeia-A.

The position of Cassiopeia-A has been computed with the aid of Eqs. (172) and (175). These calculations were experimentally verified through the radiometric measurements. The locus of apparent trajectory of the radio star Cassiopeia-A is plotted in Fig. 125. The maximum tracking velocities in elevation (occurring at easterly or westerly elongation) and in azimuth (occurring at maximum elevation angles) are also shown on this figure. The combined effects of the maximum integration time constant of five seconds and target velocity of 24 minutes of arc per one minute of time are a negligible (less than 2%) reduction in the peak of the response of the radiometer and a time-displacement of the peak by an amount equal to the time constant. At the normally used integration constants of 1 and 3 seconds these effects were practically negligible.

At low elevation angles corrections have to be made for atmospheric refraction. This may be accomplished with the aid of Figs. 126 and 127, which are based on references 97 and 98.
Fig. 125--Locus of apparent trajectory of the radio star Cassiopeia-A.
Fig. 126--Refraction errors at low elevation angles.
X = 1.9 cm (REFERENCE 97)

TO FIND APPARENT POSITION (WHERE ANTENNA IS POINTING AT) ADD CORRECTIONS TO CALCULATED POSITION.

Fig. 127--Refraction errors for elevation angles greater than 10 degrees.
APPENDIX V

The purpose of this appendix is to present the various programs and their flow charts which were employed for facilitating the data reduction and data analysis processes.

In order to obtain many of the final results presented in previous chapters, e.g.: ERP, Gain, Power Transfer, etc., most of the time it is necessary to know the average deflection of the RMS voltmeter for a specific configuration of the equipment used in the experiment. It was also desirable to know the standard deviation about this average and the dB value of the average. For this purpose a computer program was developed by Mr. Hugh Geary, which computes these quantities from the input data. The program and its flow chart is given below.
Calculation of Average Deflection, Standard Deviation, and dB Values

**Tape**

**First**

**FMT1**

(1H+SUB#2,7/Rcords, 7X7/AV;R;AGE,12X10/HAN, DEVP=20X

**FMT2**

(INTEGERS(X;SUM1,X;SUM2,X;SUM3,X;SUM4,X;SUM1,X;SUM2,X;SUM3,X;SUM4

*X*600),Y*600,TIME=)

**FMT3**

(CALL SUBROUTINE X=1=12CLOCK(),)

**FILT**

FILE;IS;ITAPE5;71;ITAPI;E5;

**ATTACH**

FILES: POOL;#;TAPE5;71;

**OPEN**

REWIND; TAPE5;71;

**R**

THIS DECK PROCESSES RECORDS OF LENGTH 192, 7094 WORDS-

**C**

TVPRPC IS 1/TOTAL NUMBER OF VALUES PER RECORD PER CHANNEL-

**C**

SCALE IS 1/71448 CS 1=142147; 2048 AND RL IS RECORD LENGTH-

**F**

SCALE=0.0006935340E-

RL=576-

TVPRPC=.027777777-

K=0-

X;SUM1=0-

X;SUM1=0-

**F**

READ DECIMAL; TAPE5;71;Z20=FMT2 (NUM;+I(x;y);J=0;I;y;L;RL=)

READ DECIMAL; TAPE5;71;Z20=FMT2 (NUM;+I(x;y);J=0;I;y;L;RL=)

**F**

Z20

TRANSFER(Z210;Z20;Z30) PROVIDED(NUM=)-

Z30

DO THROUGH(Z40)-; J=0;15;J;L;RL=-

X;SUM1=X;SUM1+Y(x;J)-

Z40-

X;SUM1=X;SUM1+Y(x;J)-

Y=T+1-

Z50

READ DECIMAL; TAPE5;71;Z20=FMT2 (NUM;+I(x;y);J=0;I;y;L;RL=)

TRANSFER(Z210;Z35;Z60) PROVIDED(NUM=)-

Z60

DO THROUGH(Z70)-; J=0;15;J;L;RL=-

X;SUM1=X;SUM1+Y(x;J)-

Z70

X;SUM1=X;SUM1+Y(x;J)-

Z75

READ DECIMAL; TAPE5;71;Z20=FMT2 (NUM;+I(x;y);J=0;15;J;L;RL=)

TRANSFER(Z230)-

Z130

TRANSFER(Z210;Z25;Z130) PROVIDED(I=)-

Z20

TRANSFER(Z210;Z25;Z20) PROVIDED(I=)-

Z90

I=0-

Z100

XAVE1=X;SUM1*TVPRPC/T-

XAVE1=X;SUM1*TVPRPC/T-

XAVE1=X;SUM1*TVPRPC/T-

ST=YAVE1*SCALE=SORT(*XAVE1*XAVE1)-

ST=YAVE1*SCALE=SORT(*XAVE1*XAVE1)-

ST=YAVE1*SCALE=SORT(*XAVE1*XAVE1)-

ST=YAVE1*SCALE=SORT(*XAVE1*XAVE1)-

ST=YAVE1*SCALE=SORT(*XAVE1*XAVE1)-

ST=YAVE1*SCALE=SORT(*XAVE1*XAVE1)-

ST=YAVE1*SCALE=SORT(*XAVE1*XAVE1)-

ST=YAVE1*SCALE=SORT(*XAVE1*XAVE1)-

ST=YAVE1*SCALE=SORT(*XAVE1*XAVE1)-

**F**

WRITE; OUTPUT; FMT4; (X;I;1*XAVE1;I;T*D8

**F**

TRANSFER(Z220;Z25;Z75) PROVIDED(I=)-

Z200

I=1-

TRANSFER(Z210;Z22;Z21;I=1)

**F**

WRITE; OUTPUT; FMT6;

**F**

T794; EITHER THE RECORD NUMBER OR THE TONE BURST NUMBER IS NE

GATIVE. THIS IS NOT IN-

Z220

CLOSE; UNLOAD; TAPE5;
Calculation of Average Deflection, Standard Deviation, and dB Values

WRITE OUTPUT, FORMAT 1 (X*F+T) -

F FORMAT (12H END OF FILE), 3X, SHA, EQUALS, 15, 3X, 8MT, EQUALS, F9.0) -
CALL SUBROUTINE (TIME), 3X, 8CL, 3X -
WRITE OUTPUT, FORMAT (TIME) -
F FORMAT (-3PJ10.3) -
*** END PROGRAM (FIRST) -
*** DATA
Calculation of Average Deflection, Standard Deviation, and dB Values.
Calculation of Average Deflection, Standard Deviation, and dB Values.
To find the gain of the 30-foot antenna, the average deflections when the 30-foot antenna and when the gain-standard antenna was connected to the receiver had to be compared. The average values were obtained from the program given above from which the differential gain of the large antenna over the standard antenna were obtained. These differential gain numbers could be obtained most conveniently in dB. A program was written by Mr. James Reed which converts these dB values into numeric, then adds them to the numerical gain of the standard antenna properly compensated for the various losses (e.g.: polarization mismatch, insertion losses) and computes a numerical gain for the 30-foot antenna. The program then finds the arithmetic average of the gain numbers, the quadratic mean of the same, and the standard deviation about the mean (see Fig. 21). The program and the flow chart is given below.
$EXECUTE  IBJOB
$SUBJOG  GO*MAP
$INFTC SATELL NODECK
C DATA CARDS FOR THIS PROGRAM ARE ORDERED AS FOLLOWS: CARD 1:
C COL 5 PUNCH 0, COLS 6-9 PASS IDENTIFICATION IE, SATELLITE
C 1-1 JULY 14, 1967 CARD 2 COLUMNS 1 THRU 10 NUMBER OF GAIN COMPAR
C ISN INPUTS COLS 11 THRU 20 PASS CONSTANT THESE 2 CARDS ARE FOL
C LOWED BY THE INPUT VALUES, FOLLOW THESE CARDS BY ANY NUMBER OF
C SETS OF CARDS IN THE ORDER DESCRIBED ABOVE, WITH THE EXCEPTION
C THAT THE LAST PASS IDENT CARD HAS A 1 IN COLUMN 5
C
DIMENSION XP(100), X(500)
N = 0,
N1 = 1
1 READ(5+2) IND
2 FORMAT(1X*I4,4OH
WRITE(6+2) IND
3 FORMAT(I10,F10.2.
N = N + NP
READ(5+3) NP, CONST
4 FORMAT(7F1.2)
WRITE(6+7)
5 CONTINUE
WRITE(6+6) X(X), K = N1* N
6 FORMAT(1X*10F11.2)
N1 = N1 + NP
1 IF(IND.EQ.0) GO TO 1
A = 0.0
Q = 0.0
S = 0.0
DO 8 I = 1, N
A = A + X(I)
Q = Q + X(I)*X(I)
8 CONTINUE
FN = N
A = A/FN
Q = SORT(Q/FN)
DO 9 I = 1, N
S = S + (X(I) - A)**2
9 CONTINUE
S = SORT(S/FN)
WRITE(6+10) A, Q, S
10 FORMAT(1X*3HA =E15.7,10X*3HE =E15.7,10X*3HS =E15.7)
STOP
END
$DATA
0 SATELLITE 2-1, SEPTEMBER 13, 1967
23 31, 224
22.426 22.452 22.559 22.125 21.984 22.382 22.289
22.150 22.274 22.214 22.317 22.300 22.179 22.414
22.417 22.184 22.266 22.350 22.412 22.540
0 SATELLITE 2-5, SEPTEMBER 12, 1967
20 31, 183
22.458 22.376 22.420 22.595 22.361 22.501
22.481 22.485 22.293 22.520 22.478 22.300 22.681
22.236 22.419 22.276 22.236 22.306
1 SATELLITE 2-5, OCTOBER 6, 1967
21 31.222
Gain Differential Calculation.
As a matter of routine, prior to and after each satellite tracking the receiving system was calibrated by inserting an RF signal into the antenna. The input power level was controlled by an Hewlett-Packard Model H382A (serial number 34) variable attenuator; the Calibration Report of this device is included in Appendix VI. As the input power level was varied, the output level, as indicated by the RMS voltmeter, varied also. The average value of the meter deflection was obtained with the first program described above; these averages were further processed with the program given below (created by Mr. James Reed) whose flow chart is also included. This program corrects the input data (the average levels) for the differences in the center frequencies and for the differences in amplitude responses of the record and playback processes. The first difference results in a non-zero output from the playback process when in actuality the input of the recording amplifier was short-circuited; the second difference results in an output which is greater or smaller than what actually was recorded, e.g.: a recorded +1.00 volt dc may be played back as +0.985 v dc. After correcting the input data the program multiplies the data with the correct multiplying factor to account for the fact that the RMS voltmeter was kept on the same scale by changing the settings of the precision IF attenuator. This last procedure produces the true rms volts value of the IF frequency of 455 KHz.
From this result, the changes in the signal-plus-noise to noise ratio and the changes in signal-to-noise ratio is computed for each setting of the variable attenuator in the input power line.
Calculation of Calibration Curve

REAL MD(40), MF(40), VRMS(40), DB(40), A(40), B(40)

DIMENSION MD(40), MF(40), VRMS(40), DB(40), A(40), B(40)

REAL MD(40), MF(40), VRMS(40), DB(40), A(40), B(40)

EXECUTE IBJOB

DATA MD, MF, VRMS, DB, A, B

GO TO MAP

DATA 0.0

Calculation of Calibration Curve

REAL MD(40), MF(40), VRMS(40), DB(40), A(40), B(40)

DIMENSION MD(40), MF(40), VRMS(40), DB(40), A(40), B(40)

REAL MD(40), MF(40), VRMS(40), DB(40), A(40), B(40)

EXECUTE IBJOB

DATA MD, MF, VRMS, DB, A, B

GO TO MAP

DATA 0.0
Calculation of Calibration Curve.
The program and flow chart which computes the polarization mismatch losses is given below. These were also created by Mr. James Reed. For the mathematical formulas applied in this program see Chapter IV.
EXECUTE IBJOB  
$IBFTC CALCPL NODECK
C  
*CALCULATION OF PL(POLARIZATION MISMATCH LOSSES,LINEAR,ELLIPITCAL)-*
DIMENSION R1(10),R2(10),DELR2(10),THETA(100),DELT(20),TR(100),
2R2(100),THETAM(100)
PI=3.14159265
CONST=PI/180.0
READ(5,1) N
READ(5,1) M
READ(5,2) (R1(I),I=1,N)
READ(5,1) L
READ(5,2) (RA2(I),I=1,M)
READ(5,2) (DELR2(I),I=1,L)
READ(5,2) (THETA(I),I=1,J)
1 FORMAT(I10)
2 FORMAT(7F10.2)
DO 3 MN=1,N
DO 3 NA=1,L
WRITE(6,5) R1(MN)
5 FORMAT(1H1,9X,4HR1 =,F5.2,,8X,2HR2,7X,5HTHETA,11X,2HPL)
DO 3 LN=1,N
R2(LN)=RA2(LN)+DELR2(NA)
THETA(LN)=THETA(LN)+DELT(NA)
TR(LN)=THETA(LN)*CONST
CT=COS(TR(LN))
ST=SIN(TR(LN))
A=((R1(MN)*R1(MN)+1.0)*(R2(LN)*R2(LN)+1.0))/(((R1(MN)*R2(LN)+1.0)
2**2)*CT*CT+((R1(MN)+R2(LN))**2)*ST*ST)
PL=10.0*ALOG10(A)
WRITE(6,6) R2(LN),THETA(LN),PL
6 FORMAT(F11.2,E13.5)
3 CONTINUE
READ(5,7) NT,THL,THI
READ(5,7) NR,RL,RI
7 FORMAT(110,2F10.2)
TH=THL
DO 11 NC=1,NT
THR=TH*CONST
WRITE(6,8) TH
11 CONTINUE
8 FORMAT(1H1,9X,7HTHETA =,F5.2,,8X,1HR13X,2HPL)
R=RL
DO 9 ND=1,NR
RSP=RSP+R
RSP=RSP+(RSP-1.0)*COS(2.0*THR))
WRITE(6,10) R,PL
10 FORMAT(F11.2,E19.5)
9 R=R+RI
11 TH=TH+THI
STOP
END
#DATA
7
1.10 1.15 1.20 1.25 1.30 1.35 1.40
17
1.190 1.200 1.210 1.220 1.210 1.220 1.220
1.130 1.240 1.240 1.240 1.210 1.190 1.190
1.180 1.170 1.240
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<th>30.00</th>
<th>60.00</th>
<th>65.00</th>
<th>55.00</th>
<th>50.00</th>
<th>40.00</th>
<th>30.00</th>
<th>25.00</th>
<th>30.00</th>
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<td>5.00</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>7</td>
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<td></td>
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<td></td>
</tr>
<tr>
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<td>3.000</td>
<td>6.000</td>
<td>10.00</td>
<td>-3.00</td>
<td>-6.00</td>
<td>-10.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
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<td>1.000</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>21</td>
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<td>0.010</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Polarization Mismatch Losses Calculation.
The program, and its flow chart, which computes the pattern angle and power loss due to pattern angle; and the program, complete with its flow chart, which computes the apparent rotation of the tilt angle of the polarization pattern caused by changes in path geometry are given below. These were created by Mr. James Reed. The mathematical formulas computed with the aid of these programs are given in Chapter IV.
**Pattern Angle and Power Loss Due to Pattern Angle Calculation**

```fortran
PI = 3.1415927
DTR = PI / 180.0
RTD = 1.0 / PI
READ(5,1) N
1 FORMAT(I5)
WRITE (5,4)
FORMAT(3X,9HELEVATION,2X,7-AZI=TH,5X,5H,ALPHA,8X,7-HPA,ALPHA)
DO 5 I = 1, N
READ(5,2) AZ, EL
2 FORMAT(2F10.2)
AZ = AZ * DTR
EL = EL * DTR
ALPHA = ARCOS(0.6423277 * SIN(FLH) - 0.764010 * COS(FLH) * COS(AZR)) * RTD
P = 90.0
PA = 12.0 * (ALPHA / 36.0)**2
WRITE(6,3) FL, AZ, ALPHA, PA
3 FORMAT(2F10.2,F16.7,F15.7)
CONTINUE
5 CONTINUE
STOP
END
```

*SDATA*
Pattern Angle and Power Loss Due to Pattern Angle Calculation.
Calculation of Apparent Rotation of Tilt Angle of Polarization Pattern Caused by Changes in Path Geometry

```plaintext
*EXECUTE 19JOB
*MAIN
R1=3.1415927
DTR=R1/180.0
RTD=180.0/R1
READ(S1) N
1 FORMAT(I10)
WRITE(6,2)
2 FORMAT(3X,'SHELEVATION',2X,'ZIMUTH',3X,'11DELTA THETA',4X,'9NUMERATO'
     3R,6X,'11DENOMINATOR')
3 FORMAT(2F10.2,F14.7)
4 READ(S4) AZ,EL
5 FORMAT(2F10.2)
ELR=EL*DTR
SEL=SIGN(ELR)
CEL=COS(ELR)
SAZ=SIGN(AZR)
CAZ=COS(AZR)
CC=CEL*CAZ
SC=SEL*CAZ
R1=0.7660108*SEL-0.6428277*CC
R2=CEL*SAZ
R3=0.6428277*SEL+0.7660108*CC
V1=0.7660108*CEL+0.6428277*SC
V2=SEL*SAZ
V3=0.6428277*CEL-0.7660108*SC
DEN=SORT((R2*R2+R1*R1)*(R2+V3-R3*V2)**2+(R3*V1-R1*V3)**2+(R1*V2-
     2*R2*V1)**2))
DT=RTD*ARCOS(V3/DEN)
WRITE(6,3) EL,AZ,DT,V3,DEN
STOP
END
```

 Calculation of Apparent Rotation of Tilt Angle of Polarization Pattern Caused by Changes in Path Geometry.
The program which computes and plots the power spectral density of the amplitude scintillations was written by Mr. Richard Turpin. It is reproduced here with his permission and through his courtesy. The flow chart was added by Mr. James Reed.
PARAMETERS CARD TO CHANGE DIMENSION AND INPUT BUFFER SIZE

PARAMETERS CARD TO CHANGE DIMENSION AND INPUT BUFFER SIZE

PARAMETERS CARD TO CHANGE DIMENSION AND INPUT BUFFER SIZE

PARAMETERS CARD TO CHANGE DIMENSION AND INPUT BUFFER SIZE

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PARAMETERS CARD TO CHANGE DIMENSION AND INPUT BUFFER SIZE

PARAMETERS CARD TO CHANGE DIMENSION AND INPUT BUFFER SIZE

PARAMETERS CARD TO CHANGE DIMENSION AND INPUT BUFFER SIZE
Power Spectral Density Calculation

F FILE
(9H FILE NO. + 12) -
DEF RINE POOL.POOI*2 + RECORD -
FILIS FILE LIST (A*5 INPUTS) -
ATTACH FILES + POOL*1 -
READ READ DECIMAL + ASKIP*9 -
TRANSFER(READ) -

F SKIP1
CONTINUE -
WRITE OUTPUT + FILE + (1) -

F FILE
(9H FILE NO. + 12) -
NRSKIP = TSKIP*NSPS + NCHS/(3*NR) + 5 -
DO THROUGH(SKIP) + I = 1 + 1 + L + NF -

F SKIP2
READ DECIMAL + A*EOF*9 -
NRDATA = TN*NSPS + NCHS/(3*NR) + 5 -
NJ = 3*NR/NCHS -
K = 0 -
DO THROUGH(DATAIN) + I = 1 + 1 + L + NRDATA -
READ DECIMAL + A*EOF*FMT*IA + IB + IC + ID + (IX(J) + J = K) + J + L + K + NJ) -
K = K + NJ -
ND = NRDATA*NJ =
ND = (ND - 3)/5 -
DO THROUGH(DECIM) + I = 1 + 1 + L + ND -

F DECIM
IX(1) = IX(5*1 - 1) + 2*(IX(5*1 - 2) + IX(5*1 + 2)) + 3*(IX(5*1 - 2) + IX(5*1 + 2)) + 4*(IX(5*1 - 1) + IX(5*1) + IX(5*1 + 1)) + IX(5*1 + 3) -
NSPS = NSPS/5 -
NC = TN*NSPS + 0001 -
TDATA = 1 + ND/NSPS -
WRITE OUTPUT + FTD ("DAT A") -

F FTD
(1H TDATA = 9F631 -
UNDERWRI TE + MEAN + EOF + TRANSFER TO (ACF) -
DO THROUGH(AVG) + I = 1 + 1 + L + ND -
AVG = AVG + IX(I) -
AVG = AVG/(ND + 1) -
WRITE OUTPUT + FMTB (IAVS) -

F FMTB
(3X + 9H AVERAGE = 115) -
DO THROUGH(DEBIAS) + I = 0 + 1 + 1 + L + ND -
DEBIA S = IX(1) + IX(1) -
ACF = IX(1) + IX(1) -
I W(J) = 0 -
DO THROUGH(ACF1) + J = 0 + 1 + J + LE + NC -
ACF = IX(1) + IX(1) -
I W(J) = 1 + I W(J) + (ND - J) -
MINF = 2*NC*FMIN/NSPS + 301 -
MAXF = 2*NC*FMAX/NSPS + 301 -
NP = MAXF - MINF -
RAWPSD = IX(1) + IX(1) + IX(1) -
DO THROUGH(PSD1) + J = 0 + 1 + J + LE + NP -
PSD1 = IX(1) + IX(1) + IX(1) -
PSD2 = IX(1) + IX(1) + IX(1) -
SWPSD = IX(1) + IX(1) + IX(1) -
SMOOTH = IX(1) + IX(1) + IX(1) -
I W(NP) = (IX(NP) - 1 + IX(NP)) + 2 -
DO THROUGH(TRY) + J = 0 + 1 + J + LE + NP -
K = 1 -
Power Spectral Density Calculation

KINC
K=K+1
TRANSFER TO (MAXM) PROVIDED(K.E.NP+1)
TRANSFER TO (KINC) PROVIDED(IW(J), GE:IW(K))

TRY
CONTINUE
MAX=IW(J)
WRITE OUTPUT+FMTA(4+MAX)

F MAXM
(I+SHMAX =15)
PROVIDED(NOPLOT,E:1), X:E=1
WRITE OUTPUT+FFPSD=((E*(J+MINF)/TM+1)*IW(I)/MAX+1=0..1..LE.
F+NP))

F FFPSD
(364POWER SPECTRUV (HANNING SMOOTHING) // (5(F10.2,PE12.
3)))

F FMAXM
(134HMAX, PSD AT +FS+2+3M HZ )
PROVIDED(CARDS.E:2+1), TRANSFER TO (NOCARD)

PUNCH CARDS, E(NP)

F FFC
(4(F8+2,9),PE10.31)

NOCARD
PROVIDED(NPLOT,E:2), TRANSFER TO (END)

CALL SUBROUTINE()=LOT.(DATA+44+0)

DEL=FFMAX-MFIM/SIZAXS

CALL SUBROUTINE()=LOT.(0.0+4FAXIS-15,SIZAXS+0,FFMIN,DELFL,
1+3)

F FAXIS
FREQUENCY (HZ)

DBINC= SIZDB/10

CALL SUBROUTINE()=LOT.(0..0..FFRD+9,10..90..5+SIZDB+DBINC,
1+3)

F FORO
PSD(.DB)

CALL SUBROUTINE()=LOT.(0..10..3)

CALL SUBROUTINE()=LOT.(SIZAXS+10,2)

CALL SUBROUTINE()=LOT.(SIZAXS+0,2)

CALL SUBROUTINE()=LOT.(SIZAXS+10,10+10*LOG*(1+4IW(NP)/MAX)/

DBINC+3)

DEL=SIZAXS/5P

DO THROUGH(PLOTPS),I=NP=-1+-1+1*GE:0

PLOTPS
CALL SUBROUTINE()=LOT.(1+4DEL+4+10*LOG*(1+4IW(I)/MAX)/

DBINC+2)

S=SIZAXS-4

FR=1./(TM+4DEL)

CALL SUBROUTINE()=LOT.(X,1..3)

CALL SUBROUTINE()=LOT.(X+1..25+2)

CALL SUBROUTINE()=LOT.(X+FFR+1.25+3)

CALL SUBROUTINE()=LOT.(X+FFR+1.21)

CALL SUBROUTINE()=LOT.(X+25+1..125+3)

CALL SUBROUTINE()=LOT.(X+1..125+2)

CALL SUBROUTINE()=LOT.(X+1.11075+2)

CALL SUBROUTINE()=LOT.(X+1.1175+2)

CALL SUBROUTINE()=LOT.(X+FFR+11.125+3)

CALL SUBROUTINE()=LOT.(X+FFR+11075+2)

CALL SUBROUTINE()=LOT.(X+FFR+1175+2)

CALL SUBROUTINE()=LOT.(X+FFR+11.25+2)

CALL SUBROUTINE()=LOT.(X+FFR+11.125+3)

CALL SUBROUTINE()=LOT.(X+25+FFR+11.25+2)

CALL SUBROUTINE()=LOT.(X+25+FFR+1.125+2)

CALL SUBROUTINE()=LOT.(X+FFR+1.06+4+FFR+0.120)

F FFPR
FREQUENCY RESOLUTION-

CALL SUBROUTINE()=LOT.E(1)

TRANFER(END)

EOF
WRITE OUTPUT+EOF

F FFEOF
(19H END OF FILE)

END
CLOSE UNLOAD+A 1

CALL SUBROUTINE()=END OF (1)

END PROGRAM (START)
### Power Spectral Density Calculation

<table>
<thead>
<tr>
<th>*** DATA</th>
<th>166</th>
<th>500</th>
<th>250</th>
<th>1</th>
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<td>2.0</td>
<td>0.0</td>
<td>50.0</td>
<td>1.0</td>
<td>25.0</td>
<td>20.0</td>
<td></td>
</tr>
</tbody>
</table>

(N18, 3N6, 75°C12)
Power Spectral Density Calculation.
Power Spectral Density Calculation.
Power Spectral Density Calculation.
Power Spectral Density Calculation.
APPENDIX VI

The followings are a Certificate of Calibration of Signalite, Incorporated and a Report of Calibration, U.S. Department of Commerce National Bureau of Standards Institute for Basic Standards on the waveguide noise source employed for determining the sensitivity of the receiving system. Additionally, the Calibration Reports by the Hewlett-Packard Company on a Waveguide Directional Coupler and two Variable Attenuators are also included.
CERTIFICATE OF CALIBRATION

Test Date * January 18, 1967

Noise Source Type TD-11 _____________ Serial No. 7A006 _____________

Waveguide Band 7.05 - 10.00 GHz Calibration No. 59 _____________

Calibration Data (Note 1)

<table>
<thead>
<tr>
<th>f (GHz)</th>
<th>Ib (mAdc)</th>
<th>Nr-1 (Tube-in-mount)</th>
<th>L</th>
<th>L-1</th>
<th>Nr-1 (Note 2) (Tube only)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.30</td>
<td>200</td>
<td>15.49 + .15</td>
<td>db</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.00</td>
<td>200</td>
<td>15.49 + .15</td>
<td>db</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Noise Generator Hewlett Packard H347, 7A007

Tested By: Certified By:
Development Engineer Manager, Special Products Div.
The Calibration Standards are traceable to the National Bureau of Standards.

Note 1: For explanation of symbols and insertion loss correction, see p. 2.

Note 2: In addition to the calibration measurement tolerances indicated, the absolute center values of Nr-1 of the standards are reported by NBS to ±.1 db. This uncertainty in absolute value is not included in the measurement tolerance on Nr-1 given above.

*This certificate is valid for one (1) year from test date.
I SYMBOLS:
1. $c$: calibration frequency
2. $b$: tube current
3. $N_{r-1}$: excess noise ratio, defined as $10 \log \left( \frac{T_e}{290} - 1 \right)$, where $T_e$ is the effective electron temperature of the discharge.
4. $L$: insertion loss of the operating tube-in-mount. (Note 3)
5. $\frac{L}{L-1}$: the correction to be added to the excess noise ratio obtained from tube-in-mount to obtain the excess noise ratio of the tube only.

II INSERTION LOSS CORRECTION:* Not Applicable

The noise stamped on the calibrated tube is the excess noise ratio of the unmounted tube at its rated current. To calculate the noise-in-mount, it is necessary to make an insertion loss correction. This correction should be subtracted from the noise stamped on the tube.

Loss Correction: Determine the hot insertion loss in db of the tube-in-mount at the rated tube current. This number, $L_{db}$, is used as follows:

Let $L_{db} = 10 \log L'$, where $L'$ is the actual power ratio represented by $L_{db}$. The value of the correction, $\Delta$, is $\frac{L}{L-1}$, expressed in db or may be calculated from $L_{db} - (L-1)_{db} = \Delta_{db}$.

Note 3: To insure accuracy in making loss measurements, and in using the tube-in-mount for noise measurements, a tube-in-mount should be operated for at least 10 minutes prior to use so the mount can reach a stable temperature.

*If a waveguide mount has lossy material added inside the waveguide-beyond-cutoff region of the tube holder, or in series with the tube, in the case of a coaxial mount, this correction procedure is not appropriate.

Also if the cold insertion loss in db of the tube-in-mount is greater than 1% of the hot insertion loss in db, the above correction procedure becomes inaccurate.
REPORT OF CALIBRATION
WAVEGUIDE NOISE SOURCE
Hewlett-Packard Company
Model H347A, Serial No. 105
Noise Source
Signalite, Incorporated
Model TD-I1
Argon Gas Tube
Submitted by:
The Ohio State University
Columbus, Ohio

In addition to the components listed above, the assembly includes a
Hewlett-Packard Company, Model H910A fixed waveguide termination.

The measurements were performed using a WR112-to-WR90 transition
which has subsequently been removed.

The measurements were performed under ambient conditions of approxi­
mately 23°C and 40 percent relative humidity. The d-c current through the
noise tube was maintained constant at 200 ma to ± 0.25 ma during the measure­
ment. The reflection coefficient magnitude of the radiometer input port was
less than 0.048. The center calibration frequency was accurate to ± 0.1 percent.

The effective noise temperature, $T_{ne}$, is proportional to the power emerging
from the output port of the waveguide noise source when it is connected to a re­
flectionless load. The excess noise ratio expressed in decibels is

$$ENR = 10 \log_{10} \frac{T_{ne} - T_0}{T_0}$$

where $T_{ne}$ is in degrees Kelvin and $T_0$ is 290°K.

Page 1 of 2
Test No. 70298
Date of Calibration: December 15, 1967
The waveguide noise source assembly must remain intact for the calibration to be valid.

<table>
<thead>
<tr>
<th>Center Calibration Frequency (GHz)</th>
<th>$T_{ne} (^\circ$K)</th>
<th>ENR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.277</td>
<td>11,130</td>
<td>15.73</td>
</tr>
</tbody>
</table>

The estimated limits of error of the effective noise temperature measurement are ±175^\circ$K; the corresponding limits of error of the excess noise ratio are ±0.07 decibel.

For the Director,
Institute for Basic Standards

Roy E. Larson, Chief
Microwave Calibration Services
Engineering Division
Radio Standards Laboratory

Page 2 of 2
Test No. 70298
Date of Calibration: December 15, 1967
Reference: P. O. No. P 09533-A
Mr. Stephen L. Zolnay  
Electro Science Lab  
Ohio State University  
Columbus, Ohio 43210

Reference: NBS Test No. 70298

Dear Mr. Zolnay:

We recently calibrated a WR112 Waveguide Noise Source for you at 7.277 GHz. The item was measured using our WR90 waveguide radiometer and the NBS Reference Standard in WR90 waveguide.

To make the requested measurements of the effective noise temperature of the WR112 waveguide noise source, it was necessary to employ a WR112 to WR90 transition onto the WR112 waveguide noise source. The measurements made actually measure the effective noise temperature of the combination (i.e., the noise source which is comprised of the transition attached to the WR112 waveguide noise source). The value of the effective noise temperature obtained for the combination was 11,050 K. Since you had requested the value measured for the effective noise temperature of the WR112 waveguide noise source alone, the effects of the transition had to be accounted for so that when removed a value of the effective noise temperature of the WR112 waveguide noise source could be arrived at.

The process of determining the effect of the transition is fairly involved and requires some additional measurements. The overriding effect of attaching a transition to a noise generator is to attenuate the output of the noise generator, consequently, removing the transition will result in an increase of the noise output of the generator. In the Report of Calibration returned with the item calibrated, the value of the effective noise temperature of the WR112 waveguide noise source reported was 11,130 K, which implies the
transition had an attenuating effect of 80°K. Inadvertently, however, one term of the equation used in calculating the effect of the transition was omitted in the calculation and the results reported neglected the effect of this term. Including this omitted term, the attenuating effect is correctly calculated as 60°K and the effective noise temperature output of the WR112 waveguide noise source should be corrected to read 11,110°K (the corresponding value of the excess noise ratio will be 15.72 dB).

The estimated limits of error of the effective noise temperature measurement were not affected. It should be mentioned that the reported value was well within the limits of error quoted.

Very truly yours,

Charles K. S. Miller
Microwave Calibration Services
Engineering Division
Radio Standards Laboratory
MEASUREMENT STANDARDS

CALIBRATION REPORT

Date January 5, 1967  Calibration No. U-2220

Item Waveguide Directional Coupler, Model H752C

Ident. Serial No. 2499  Mfr. Hewlett-Packard

Submitted by  Ohio State University

Ambient conditions: - 0°C  -  % R.H.  Due

This Directional Coupler was checked by means of square-law detection, using a standard attenuator traceable to NBS. Source and load SWR’s were less than 1.02. The unused port was terminated with a load of SWR less than 1.01. SWR was measured with equipment traceable to NBS through length and attenuation standards.

<table>
<thead>
<tr>
<th>FREQUENCY-GHz</th>
<th>COUPLING-DB</th>
<th>SWR</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.300</td>
<td>9.92</td>
<td>1.02</td>
</tr>
<tr>
<td>8.000</td>
<td>10.28</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Measurement Uncertainty:

Coupling Factor: ± .03 db
SWR: ± .01

B. P. Hand, Manager
MEASUREMENT STANDARDS

CALIBRATION REPORT

Date: January 31, 1967  Calibration No.: U-2266

Item: Variable Attenuator, Model H382A

Ident. Serial No.: 1603  Mfr.: Hewlett-Packard

Submitted by: Ohio State

Ambient conditions: _° C. _% R.H. Due

This Attenuator was checked through the use of a parallel IF substitution system, with a piston attenuator traceable to NBS as the standard. Source and load SWR's seen by the Attenuator were less than 1.05.

SWR measurements were made on a slotted line and are traceable to NBS through length and attenuation standards.

B. P. Hand, Manager
**Measurement Standards**

**Test Data**

**Date:** January 31, 1967  
**Calibration No.:** U-2266

**Item:** Variable Attenuator, H382A  
**Ident. Ser. No.:** 1603

<table>
<thead>
<tr>
<th>Setting</th>
<th>Incremental Attenuation-DB &amp; SWR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.30 GHz</td>
</tr>
<tr>
<td></td>
<td>8.00 GHz</td>
</tr>
<tr>
<td>L</td>
<td>R</td>
</tr>
<tr>
<td>0</td>
<td>Ref. 1.01 1.02</td>
</tr>
<tr>
<td>1</td>
<td>1.00 1.01</td>
</tr>
<tr>
<td>3</td>
<td>3.00 &quot;</td>
</tr>
<tr>
<td>5</td>
<td>5.00 &quot;</td>
</tr>
<tr>
<td>7</td>
<td>7.02 1.03</td>
</tr>
<tr>
<td>9</td>
<td>9.01 &quot;</td>
</tr>
<tr>
<td>11</td>
<td>11.01 &quot;</td>
</tr>
<tr>
<td>13</td>
<td>13.01 &quot;</td>
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<td>15</td>
<td>15.02 &quot;</td>
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<tr>
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<td>19.03 &quot;</td>
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<td>27</td>
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<td>41</td>
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</tr>
<tr>
<td>43</td>
<td>43.00 &quot;</td>
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</tbody>
</table>

**Residual Attenuation-DB**

\[
\text{Residual: } \pm 0.02 \text{ db} \\
\text{Incremental: } \pm (0.02 + 0.02/10) \text{ db} \\
\text{SWR: } \pm 0.015
\]

**Measurement Uncertainty:**

\[
\text{Residual: } \pm 0.02 \text{ db} \\
\text{Incremental: } \pm (0.02 + 0.02/10) \text{ db} \\
\text{SWR: } \pm 0.015
\]
MEASUREMENT STANDARDS

CALIBRATION REPORT

Date: May 1, 1967 Calibration No.: U-2464

Item: Variable Attenuator, Model H382A

Ident. Serial No.: 34 Mfr.: Hewlett-Packard

Submitted by: Ohio State

Ambient conditions: - °C - % R.H. Due: __________

This Attenuator was checked by means of parallel-IF substitution, using a standard piston attenuator traceable to NBS. Source and load SWR's were less than 1.03. SWR measurements were made on a slotted line and are traceable to NBS through length and attenuation standards.

Measurement Uncertainty: ±(.02 + .02/10)db

B. P. Hand, Manager
### TEST DATA

**Date:** May 1, 1967  
**Calibration No.:** U-2464

**Item:** Variable Attenuator, H382A  
**Ident. Serial No.:** 34

<table>
<thead>
<tr>
<th>SETTING</th>
<th><strong>7.30 GHz</strong></th>
<th></th>
<th><strong>8.00 GHz</strong></th>
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<tbody>
<tr>
<td></td>
<td><strong>Left</strong></td>
<td><strong>Right</strong></td>
<td><strong>Left</strong></td>
</tr>
<tr>
<td>0</td>
<td>Ref.</td>
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<td>1.10</td>
</tr>
<tr>
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<td>1.09</td>
</tr>
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<td>3.01</td>
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<tr>
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</tr>
</tbody>
</table>

**RESIDUAL ATTENUATION-DB**

<table>
<thead>
<tr>
<th>SETTING</th>
<th><strong>Left</strong></th>
<th><strong>Right</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.27</td>
<td>0.11</td>
</tr>
</tbody>
</table>
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June 7-9, 1965.


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