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A STUDY OF LOW FREQUENCY OSCILLATIONS
IN A RAREFIED MAGNETOPLASMA

DISSERTATION

Presented in Partial Fulfillment of the Requirements
for the Degree Doctor of Philosophy
in the Graduate School of
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By

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1968

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FIELD OF STUDY

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INTRODUCTION

Interest in the containment of plasmas by magnetic fields has been stimulated in recent years by the effort to design a controlled thermonuclear reactor. One of the prominent aspects of this program has been the study of instabilities associated with various configurations in both hot plasma experiments and smaller scale cold plasma ones. A body of theoretical knowledge of instabilities has grown in the course of this work, much of which is applicable to cold, or non-thermonuclear plasmas.

Among the most important instabilities studied have been those which manifest themselves in the form of low frequency potential fluctuations which propagate across magnetic lines of induction. These can be divided into two categories: macroscopic, or hydrodynamic instabilities, which can be described in terms of a fluid model of the plasma, and microscopic instabilities in which kinetic theory is involved. These, in turn, are divided into two subclasses: electrostatic modes in which electric fields are derivable from a potential function, and electromagnetic modes which involve magnetic field fluctuations. We shall be concerned with an electrostatic microinstability.

The distinction between macroinstabilities and microinstabilities cannot be sharply made. One of the earliest hydrodynamic instabilities discovered was the flute (or interchange) instability, which is closely related to the Rayleigh-Taylor instability of the interface of a heavy fluid supported against gravity by a lighter one. In a cylindrical
plasma column flutes form on the surface, thus giving the phenomenon its name. It was shown by Kruskal and Schwartzchild\(^1\) (1954) and Longmire and Rosenbluth\(^2\) (1956) that for magnetic fields which have gradients in the same direction as the plasma density gradient, a charge separation occurs within perturbations of the plasma surface. However, it was shown by Krall, Rosenbluth and Rostoker\(^3\) (1962) that a complete understanding of the flute instability requires the inclusion of the stabilizing effect of finite Larmor radii. Thus, kinetic theory cannot be avoided.

It was shown by Rudakov and Sagdeev\(^4\) (1961) that a rarefied, non-uniform plasma is unstable to waves of length longer than the ion cyclotron frequency, in the absence of longitudinal electric fields. These so-called drift waves had been predicted in 1957 by Tserkovnikov.\(^5\) Kadomtsev and Timofeev\(^6\) (1963) extended this work to show the destabilizing influence of finite gyroradius (or ion inertia). Further studies by Galeev, Oraevskii, and Sagdeev\(^7\) indicated that a rarefied plasma confined by a magnetic field is "universally unstable" to local short wave disturbances for any ratio between space gradients of density and temperature. Thus, the name "universal instability" became attached to this phenomenon.

In the paper of Kadomtsev and Timofeev, it was pointed out that the disturbance will only grow in a plasma whose length exceeds the transverse dimension by an order of magnitude. Rosenbluth and Krall\(^8\) (1965) greatly extended this work by calculating the dependence of length criteria on plasma parameters for non-isothermal and rotating plasmas.
Mikhailovski and Timofeev\(^9\) (1963) showed that a non-uniform rarefied plasma may be unstable to wavelengths shorter than the ion Larmor radius at frequencies that are close to the ion cyclotron frequency or multiples of that frequency. This is another branch of the universal instability.

D'Angelo and Motley\(^10\) (1963) detected low frequency waves in a thermally ionized potassium plasma, which were later identified as drift waves. Buchelnikova\(^11\) (1964) and Buchelnikova et al.\(^12\) (1965) observed both drift waves and a higher frequency wave that appeared when the ion cyclotron radius became larger than the dimensions of the plasma column. These observations were made in a thermally ionized potassium plasma. Lashinsky\(^13,14\) (1964, 1965) reported observations of both damped and growing drift waves in a thermally ionized plasma. However, Chen\(^15,16\) (1965) has pointed out that the drift waves observed by D'Angelo and Motley and Lashinsky were not of the collisionless universal type, but of the drift dissipative type. The growth of this over-stability is associated with the finite resistivity of the plasma.

The experiment to be discussed here is performed on a plasma that is clearly collisionless, and, in this respect, resembles most closely the lower density parts of the experiments of Buchelnikova. However, the present experiment involves a rotating plasma that is not isothermal, and the measurements are extended into the region in which the wave frequency is the same magnitude as the ion cyclotron frequency.

Particle transport across magnetic field lines is also discussed. The magnitude of this transport is found to be several orders of mag-
mitude greater than that predicted by collisional diffusion.
CHAPTER I

THEORY

The Motion of Guiding Centers

Consider a particle with charge \( q \) and mass \( m \) acted upon by a constant force \( F \) in a uniform magnetic field. The equation of motion is

\[
\frac{dv}{dt} = \frac{q}{m} \left( v \times B \right) + \frac{F}{m}
\]  

(1)

which splits into component equations parallel and perpendicular to the magnetic field:

\[
\frac{dv}{dt} = \frac{F}{m}
\]  

(2)

\[
\frac{dv}{dt} = \frac{q}{m} \left( v \times B \right) + \frac{F}{m}
\]  

(3)

Expression 2 represents a constant acceleration along field lines under the force \( F \). We introduce a drift velocity \( \nu_d \) to solve Eq. 3, so that

\[
v = \nu_d + \nu
\]  

(4)

If we set

\[
\nu_d = \frac{1}{q} \frac{F \times B}{B^2}
\]  

(5)

Eq. 3 reduces to

\[
\frac{d\nu}{dt} = \frac{q}{m} \left( \nu \times B \right)
\]  

(6)

Therefore, the chosen value of \( \nu_d \) transforms away the force term and an observer moving at this velocity will see the particle motion entirely
governed by the magnetic field, i.e., he will see particles moving in circular paths. The constant drift velocity superimposed upon this motion yields a cycloidal path, shown in Fig. 1.

The motion of the particle can, therefore, be described as a superposition of a drift velocity \( v_d \) and a gyration around a point called the guiding center. The motion of the guiding center does not follow the laws of particle dynamics.

In the present case, the plasma is immersed in an inwardly directed, radial electric field and an axial B field. Eq. 5 predicts a drift velocity for both ions and electrons in the same azimuthal direction, with magnitude

\[
    v_d = \frac{(E \times B)}{B^2}
\]

This results in a steady rotation. In the next section we will show that an inhomogeneous plasma is subject also to an oscillating drift motion.

The Propagation of Drift Waves

In an inhomogeneous plasma it is possible to have charge currents in the absence of net particle motion. They are associated with the plasma density gradient and are analogous to the surface currents of magnetization in a magnetized solid. These currents, generally referred to as drift currents, flow transversely to the plasma density gradient. We will show that under certain conditions drift currents are unstable to small fluctuations in density.

We first consider the case of an infinite plasma slab suspended in a uniform magnetic field in the z direction. The slab is non-uniform in the x direction, with a constant density gradient, dn/dx. For the
Fig. 1.—The top figure (A) illustrates the path of an ion with zero initial velocity in perpendicular electric and magnetic fields. In the bottom figure the ion has an initial y component of velocity.
present, we assume that no zero order electric fields are present and that the ions are cold. To get a physical picture of the behavior of a disturbance in this plasma we will follow the treatment of Chen\textsuperscript{15} for a resistive instability, and eliminate features that are not applicable to a non-resistive plasma.

Assume density and potential perturbations of the form

\[ n = n_0 \nu e^{i(k_y y + k_z z - \omega t)} \]
\[ \varphi = \varphi_0 e^{i(k_y y + k_z z - \omega t)} \]  
(8)
as illustrated in Fig. 2. With zero resistivity electrons can flow freely along B and the small perturbation in potential arises from the disparity in the velocities with which ions and electrons diffuse out of a volume of perturbed plasma density. With zero resistivity electrons can flow freely along B and will be in equilibrium along each line of force, yielding

\[ n_e = n_0 e^{\varphi/kT} \]  
(9)
where \( \varphi \) is the electrostatic potential at any point. If we expand \( n \) to first order

\[ n_e = n_0 \left(1 + \frac{e\varphi}{kT}\right) \]  
(10)
From Eqs. 8 and 10

\[ \nu = \frac{e\varphi_0}{kT} \]  
(11)
Because of the initial density gradient, we can draw the typical line of constant density labeled "isobar" in Fig. 2. The equipotentials will coincide with the isobars. This distribution of potential gives rise to electric field \( E_y \) in the figure, which results in a drift velocity
$E_y/B$ of electron and ion guiding centers in the $x$ direction. At position 2 in Fig. 2, $v_e$ is a maximum and $n_1 = 0$. Some time later, after $v_e$ has brought particles from a dense area in the initial distribution, $n_1$ will be positive at 2. At position 4 the particles drift in from the right, where the density is low, and therefore $n_1$ will decrease. The perturbation, in this way, moves upward on the diagram. To find the magnitude of the phase velocity of the disturbance, we observe that at any given value of $x$ the rate at which the drift changes the density is

$$\frac{\partial n_e}{\partial t} = -i\omega n = v_e \frac{dn_e}{dx} \tag{12}$$

Using the fact that $\phi$ varies in the same way as $n_1$, we have

$$E_y = -\frac{\partial \phi}{\partial y} = -ik_\perp \phi \tag{13}$$

Since, from Eq. 11

$$\phi_o = \frac{kT}{e} \nu = \frac{kT}{e} \frac{n_1}{n_o} e^{-i(k_y y + k_z z - \omega t)}$$

$$\phi = \frac{kT}{e} \frac{n_1}{n_o} \tag{14}$$

we have

$$v_e = \frac{E}{B} = -ik_\perp \frac{kT}{eB} \frac{n_1}{n_o} \tag{15}$$

and from 12 and 15

$$-\omega n_1 = -k_\perp \frac{kT}{eB} \frac{n_1}{n_o} n'_o \tag{16}$$

$$\omega = k_\perp \frac{kT}{eB} \frac{n'_o}{n_o}$$

The wave is stable since $v_e$ is always $90^\circ$ out of phase with the density perturbation.

We next examine the conditions under which this disturbance can
Fig. 2.—Schematic of the propagation of a drift wave in the y direction.
grow. In our treatment we have thus far ignored the difference in mass between ions and electrons and assumed that both particles move with the same drift velocity $v_\text{e}$. The ions, because of their greater inertia, acquire a substantial $y$ component of velocity from $E_y$. This can be seen from the consideration of the motion of a particle in a static magnetic field with an electric field oscillating at a frequency lower than the cyclotron frequency of the particle. The drift velocity of the particle is given by

$$v_\text{D} = \frac{E \times B}{B^2} \frac{m_\text{e}}{eB^2}$$  \hspace{1cm} (17)

The second term in this expression is considerably larger in the case of ions than it is for the less massive electrons. It contributes to the ion motion a velocity component in the $y$ direction out of phase with $E_y$ by $90^\circ$. At position 2, $E_y$ is at its maximum and therefore, $E_y = 0$. Since $v_y$ is $90^\circ$ out of phase with $E_y$, at position 2, $v_y = 0$. At position 3, $E_y$ and $v_{ix}$ vanish, but $v_{iy}$ is maximum. As charges drift from the dense part of the plasma at position 2, the ions are depleted by $v_{iy}$, resulting in a net negative charge buildup at position 2. In like manner a positive net charge builds up at position 4. A downward shift in the potential distribution results, so that it no longer coincides with $n_1$. This is depicted in Fig. 3. As a result, where $n_1$ is positive there is also a component of $E_y$ which brings in particles from the left where the plasma is dense, thus increasing the density. Where $n_1$ is negative, the drift is from the right where the plasma is rarefied and thus the density in this region is lowered. In this manner the wave grows.
Fig. 3.—Schematic of a growing drift wave showing a shift in the phase of the potential distribution caused by ion inertia.
For application to a cylindrical plasma configuration, with the magnetic field parallel to the axis, the perturbed density takes the form

\[ n_1 = n_0 v_e e^{i (m \varphi + k_z z - \omega t)} \]  

(18)

The function must be single valued in \( \varphi \) and consequently \( m \) is an integer.

The Gravitational Instability

We have, thus far, provided a picture of a growing drift wave in the absence of external forces. We will briefly consider the effect on an inhomogeneous plasma of an outward gravitational acceleration. This acceleration is usually simulated by field line curvature or plasma rotation.

In Fig. 4, \( g \) is directed in the positive \( x \) direction. This causes an upward drift of electrons causing a buildup of negative charge at position 2, and positive charge at position 4, a result identical to that of ion inertia. The resulting \( E_y \) causes the particles to drift radially (positive \( x \) direction in the figure). Since \( v_e \) is in phase with the density perturbation \( n_1 \), the density at position 1 increases and that at position 3 decreases, resulting in growth.

In conclusion, an outward gravitational force can only serve as a destabilizing influence on a plasma with a negative density gradient.
Fig. 4.—A schematic of a drift wave growing under the influence of a gravitational acceleration $g$. 

$\phi$ $V_c$ $V_e$

$\nabla n$ $
abla n$

$B$ $B$

$\nabla n$ $\nabla n$
Stability Criteria

The conditions necessary for stable plasma configurations have been of fundamental interest in plasma confinement research. At first the universal instability was thought to be a property of all plasmas exhibiting finite density gradients, but later studies revealed that certain aspects of the problem had been neglected. The most important of these was the damping effects of particles streaming along magnetic lines of induction.

To see how this occurs we first consider the mechanism responsible for the initial charge separation. Suppose that the plasma density increases within a small filament in the plasma column. The fast moving electrons will diffuse out of this volume along the magnetic field lines, forming a positively charged filament. However, an electrostatic field parallel to the magnetic field lines will result from this charge separation and will act as a restoring force. It is quite clear that if we assume infinite mobility for electrons flowing along field lines the charge separation would be neutralized before the drift wave could grow. There are three mechanisms which can limit electron mobility along magnetic field lines:

1) electron inertia
2) collisions with ions (finite resistivity)
3) the acceleration of resonating electrons by waves.

Since the frequency of the wave propagating along the magnetic field lines must be equal to that of the wave propagating across these
lines, a minimum wavelength corresponding to the minimum phase velocity must be a precondition for the instability to occur. The most complete calculation of the length criteria under a number of different conditions is that of Krall and Rosenbluth. The calculations are long and complex and will not be discussed here. We will consider only the results and the relevant conditions.

Krall and Rosenbluth consider an infinite plasma immersed in a magnetic field which for present purposes will be assumed to have only a z component but can be a function of x. Provision is made for a small zero order density gradient in the x direction. An electrostatic disturbance of the form:

$$E_1 = -\nabla \varphi_1 = -\nabla \varphi(x) e^{i ky} e^{i K z} e^{i \omega t}$$

(19)

K $\ll$ k is assumed. Under the simplifying assumption that $\omega$ is smaller than the ion cyclotron frequency, a dispersion equation can be derived. From the dispersion relation, growing waves are seen to result provided they are sufficiently long. In a finite plasma the maximum wavelength along the field lines that is possible is twice the length of the plasma column. The stability conditions should, therefore, be expressible in terms of this length. From our consideration of the frequency and growth mechanisms, a dependence upon ion inertia (or ion gyroradius) and the radial density gradient would also be anticipated. In fact, Krall and Rosenbluth have found that the stability condition is a function of three natural parameters:

1) $\lambda \equiv \frac{\omega}{K v_i}$ where $v_i$ is the ion thermal velocity

2) $b \equiv \frac{1}{2} k^2 R_i^2$ where $R_i$ is the ion gyroradius

3) and $n'/Kn$. 
The significance of these parameters is expressed in the marginal stability curves of Fig. 5, which are taken from the paper of Krall and Rosenbluth. The maximum length for a plasma column that is stable to a wave with a given value of $k$ is, assuming $T_i = T_e$,

$$L_{\text{max}}(T_e = T_i) \leq 20.6 \left( \frac{1}{n} \frac{dn}{dx} \right)^{-1} + 1 + \frac{1}{2} k^2 \lambda_D^2$$

(20)

in the gaussian system, where $\lambda_D = kT_i/4\pi e^2 n$, the Debye length for ions. For the case $T_e \neq T_i$,

$$L \leq L_{\text{max}}(T_e = T_i)^{\frac{3}{2}}(1 + T_i/T_e + k^2 \lambda_D^2)$$

(21)

Mikhailovski and Timofeev\textsuperscript{9} have extended the instability theory to the ion cyclotron frequency, for wavelengths shorter than the ion gyroradius. This was done by determining the conditions necessary for the intersection of the drift and ion cyclotron branches of the dispersion equation used by Krall and Rosenbluth. The condition for instability obtained under the simplifying assumption of zero electron temperature is:

$$\frac{1}{n} \frac{dn}{dx} R_i \geq 2 \left( 1 + \frac{\omega_e^2}{2 \omega_{oe}} \right)^{\frac{1}{2}}$$

(22)

where $\omega_{oe} = e^2 n/m_e c$ in MKS units.
Fig. 5.—Marginal Stability curves given by Krall and Rosenbluth. The length of the plasma column is related to $\lambda$ through $k$, the wave number in the $z$ direction.
CHAPTER II
EXPERIMENTAL APPARATUS

Vacuum Chamber

The plasma chamber (Fig. 6) consists of cylindrical stainless steel enclosure in which is mounted axially a three electrode electron gun. The end walls of the chamber are shielded from the central region by conducting plates formed of five concentric rings, each 1/4 inch wide, made of type 304 stainless steel. These rings are alternately offset as shown in the figure and mounted on ceramic insulators so that each ring is electrically isolated. The shields at each end are completed by a large outer ring of 3.25 inches inner diameter and 6.0 inches outer diameter. The shielding system on the side opposite the electron gun has a stainless steel disc at the center of the first ring which reflects electrons passing along the axis. The central region of the chamber bounded by the shields is 14.5 inches long and the entire vacuum chamber is 36 inches long and 8.4 inches in diameter.

Provision is made for leads to be brought out through vacuum wall leadthroughs soldered into small plates which may be affixed over apertures in the end plates with CVC con-o-ring seals. The half inch thick plates which form the end walls are provided with demountable viton o-ring seals which rest in grooved flanges. Vacuum leadthroughs are also soldered to various parts of the end plates and sidewalls and are sealed with solder when not in use.
Fig. 6.—Diagram of the plasma chamber.
A push-pull vacuum sealed shaft is mounted in an opening in the sidewall at the central plane of the chamber by means of a viton o-ring set into a grooved flange. The shaft itself is sealed by a vacuum grease reservoir and has a 5 inch stroke which permits a probe to be moved to any radial position.

The pumping system is a CVC PMC-4B oil diffusion pump with an ambient cooled chevron baffle backed by a Kinney 3 CFM forepump. The system is capable of achieving pressures as low as $5 \times 10^{-7}$ torr. For operation in this lowest range the walls are baked out for several days by means of heating tapes wrapped around the sidewalls.

The pressure was monitored by a Bayard-Alpert type hot cathode ionization gauge. The circuit used to control this gauge is shown in Fig. 7. The gauge tube mount is a Veeco "quick-connect" type, soldered onto a stainless steel tubulation which in turn is welded to the chamber sidewall. For the detection of large leaks provision is also made for the mounting of an NRC thermocouple gauge.

**Probes**

The probes used in these studies consisted of tungsten wires sheathed in glass except for exposed sections at the tips which served as charge collecting areas. The glass was melted around the wire to form a coating about 1/32 inch in thickness. In the case of shielded probes a jacket of 1/8 inch stainless steel tubing covered the insulated section. This shield was held in place by epoxy resin and was generally AC grounded through a capacitor of a few microfarads.

The gauge of the tungsten wire varied from 5 mils to 20 mils, de-
Fig. 7.—Wiring diagram of the ionization gauge control. Points labeled by the same letters are connected to each other.
pending on the measurements involved. Flat tungsten ribbon, 15 mils wide and 1 mil thick, was also used. The exposed tips of the probes used as Langmuir or current collecting probes were cleaned, first in a boiling solution of NaOH and Na₃PO₄ and then electrolytically in a sodium nitrite solution. This removes the dark gray oxide that forms on tungsten heated in the atmosphere. The last process was also used to remove deposits which form on the probe tips after extended use.

The Electron Gun

The electron gun, which was mounted behind the shielding rings at the west end of the chamber, is sketched in Fig. 8. The cathode is a one inch length of 10 mil diameter thoriated tungsten wire, bent into a 1/4 inch diameter hairpin loop which is oriented so that the plane of the loop is perpendicular to the axis of the system. The ends of the filament are held by stainless steel clamps which are secured to ceramic offsets by stainless steel machine screws. By this same means the offsets are secured to a molybdenum supporting plate.

The electrodes of the gun are molybdenum plates, 3 inches in diameter, with 1/4 inch holes for passage of the beam. The electrodes are spaced with ceramic supports as shown in the figure. The relatively simple design of the gun is made possible by the focusing provided by the external magnetic field.

The cathode is heated by the passage of a current of several amperes from a direct current supply (Fig. 9). The cathode is biased at -300 volts relative to the accelerator electrode. The third electrode is a decelerator biased at the same potential as the cathode.
Fig. 8.—Diagram of the electron gun.
Fig. 9.--Wiring diagram of the filament power supply.
The bias source is a Power Designs model s-305 regulated power supply. A second bias source is connected to the positive terminal of the cathode bias source. This power supply is continuously variable and allows the electron gun as a whole to be biased at selected potentials relative to the chamber walls.

The current drawn from the filament is a function of the pressure in the chamber, varying from over 10 ma at $10^{-4}$ torr to 5 ma at $10^{-6}$ torr. Most of the emitted electrons are collected by the accelerator electrode; generally, less than one milliampere passes into the plasma chamber.

The Magnetic Fields

A cylindrically symmetric magnetic field was provided by the positioning of two sets of aluminum coils, three coils in each set, around the vacuum chamber. The coils are installed in a manner permitting changes of position. The separate coils of each set were connected in series, but the sets themselves were connected in parallel to minimize resistance to match the available power supply.

The magnet current supply was a three phase selenium rectifier bridge with capacitative filtering which could provide as much as 200 amperes without undue heating. The current was adjusted by a ganged set of auto-transformers installed in the 3-phase input line.

Two magnetic field configurations were used in these studies. The first was a gentle mirror configuration of mirror ratio 1.22, the central axis profile of which is plotted in Fig. 11. The second configuration was the relatively uniform one plotted in Fig. 12 (which
Fig. 10.—Wiring diagram of the circuit used to bias the electron gun relative to the walls of the vacuum chamber.
forms a diverging profile near the end shields). The interaction of the plasma with the second configuration is in theory less complicated, a factor responsible for the decision to take the bulk of the measurements in this regime. The off axis plots in Fig. 12 display a gentle increase in field strength with increasing distance from the axis. All magnetic flux measurements were taken with an Empire model 900 gaussmeter.

The Production of Ions

The electron gun is designed to inject electrons into the plasma chamber at low velocity. The function of the gun can be illustrated in terms of the potential well shown in Fig. 13. Electrons from the filament with total energies less than 300 ev will be trapped in region A while the rest will pass into region B (the plasma chamber). Electrons which lose energy by collisions with neutrals in region B become trapped in this region resulting in a buildup of negative charge. This space charge increases until equilibrium is achieved between the rates at which electrons enter and escape. The rate of escape increases as the potential well in the chamber is filled.

The ionization rate within both the gun and the chamber increases with background pressure. Ions drawn into the plasma chamber from the gun region raise the potential in the chamber by compensating the electron space charge. The background gas pressures in this experiment were varied from less than 10⁻⁶ torr to 10⁻⁴ torr and for most measurements the gas was air. Ionization occurs primarily by collisions between neutral gas molecules and energetic electrons. In the case of
air, mainly singly ionized N\textsubscript{2} molecules are expected to be produced. Some observations were also made with argon as the background gas in order to observe the dependence of collective behavior on ion mass or ionization cross section.

Ion densities are difficult to measure, but can be estimated by the use of suitable assumptions. The ionization efficiency, se, for an electron moving through a neutral gas is defined as the number of electron-ion pairs per centimeter of path-length produced by the electron for a gas pressure of one torr. A plot of the ionization efficiency against electron energy for N\textsubscript{2}, O\textsubscript{2}, and argon appears in Fig. 14. If plotted on a linear scale, the values would rise linearly to a maximum and then decrease almost hyperbolically at high energies. The ionization efficiency is numerically equal to the ionization cross section and, like the cross section, is directly proportional to pressure for a given electron energy.

To estimate the ion density it is necessary to equate the ion production and loss rates. Two mechanisms are responsible for the loss of ions: volume recombination and wall recombination at the end plates. Volume recombination is negligible in all but highly dense plasmas and can be ignored in the present case. A rough approximation of the wall recombination rate can be made by assuming that ions drift to the end shields with the velocity distribution of neutrals and recombine there. At 300°K the ion energy would be 0.03 ev and the component of velocity along the magnetic field would average 3 \times 10^4 \text{ cm/sec}. The mean distance travelled by ions is half the length of the plasma column, or 18 cm, resulting in a mean ion lifetime of 10^{-3} \text{ seconds}. Measurements
Fig. 11.—Plot of magnetic induction on the axis of the chamber for the mirror field.

$I_m = 210$ amperes
Fig. 12.—Plot of magnetic induction on and off axis for the uniform field.
Fig. 13.—The top diagram shows the potential well in the absence of space charge which is formed by the electrodes within the electron gun and plasma chamber.
Fig. 14.—Ionization efficiency as a function of electron energy for argon, O₂ and N₂ at 0°C, as given by A. von Engel, Ionized Gases, p. 63, London: Oxford University Press 1965.
of the electron density indicate a value of roughly $10^7$/cm$^3$ in the pressure ranges of interest with electron temperatures centering around 20 ev in these same pressure ranges ($10^{-6}$ torr to $10^{-4}$ torr). Equating the ionization and recombination rates we have:

$$\bar{\nu}_e e^* P N_e = N_i / t_i$$

where $s_e$ = the ionization efficiency

$\bar{\nu}_e$ = the average electron velocity

$P$ = the pressure in torr

$N_e$ = the electron density

$N_i$ = the ion density

$t_i$ = the mean ion lifetime

At an electron temperature of 20 ev, $\bar{\nu}_e$ is $2.7 \times 10^8$ cm/sec and $s_e$ is 1/cm-torr. Using these values, $N_i$ as predicted by this expression is $10^6$/cm$^3$ at $10^{-6}$ torr and $10^8$/cm$^3$ at $10^{-4}$ torr.

We have considered only ions produced in the plasma chamber and have ignored ions which enter the chamber from the electron gun. As these ions increase with higher pressure they compensate the negative space charge within the chamber. At a suitably high pressure a neutral plasma can be anticipated.

**The Modes of Operation**

The electron gun is designed to inject low velocity electrons into the chamber. The accelerating electrode removes electrons from the space charge surrounding the filament and accelerates them into the space between the accelerating and decelerating electrodes. Those electrons whose velocities are directed toward the aperture in the deceler-
ating electrode and whose energies are great enough to overcome the opposing potential gradient will pass into the plasma chamber through the half inch aperture in the shielding at the gun end.

These electrons form a column 0.6 cm in diameter along the axis chamber. They are constrained by the longitudinal magnetic field to revolve in small orbits of gyroradius \( r_c \), where

\[
r_c = \frac{m_e v}{e B}
\]

in MKS units, with \( m_e \) the electron mass, \( e \) the electron charge and \( v \) the component of velocity perpendicular to the field. For a 1 ev electron in a \( 2 \times 10^{-2} \text{W/m}^2 \) field, \( r_c \) is \( 10^{-2} \text{ mm} \), which means that in the absence of collisions electrons could be expected to follow magnetic induction lines. This condition is found to be substantially modified by the presence of collective oscillations, with greatly enhanced radial transport resulting.

The shielding plates (rings) at each end, which serve as reflectors of the electrons, are connected to leads which are individually brought out of the vacuum chamber. This permits operation of the device in several modes, each characterized by the manner in which these leads are terminated. The present studies are focused primarily on two modes: one in which the shielding rings are individually brought to ground through 2.3 megohm resistors and another in which the rings are permitted to float electrostatically. The behavior of the observed instability was essentially the same in each case, but increased operating stability was exhibited in the finite resistance mode.

A third mode that is achieved by biasing the reflecting rings
(east end only) at the potential of the decelerator gun electrode was studied briefly and was found to be identical to the floating mode as far as the stability was concerned.
CHAPTER III

PROBE THEORY

The Probe Characteristic

Basically, an electrostatic probe is a small conducting electrode inserted into a plasma. It is biased at potentials positive and negative with respect to the plasma and the current collected by the probe is then used to acquire information about plasma conditions. The interpretation of the current-voltage characteristic of an electrostatic probe is rigorously possible only under conditions which ensure that the disturbance caused by the probe is localized. A degree of ambiguity is introduced in the absence of these conditions.

A typical probe characteristic for a nonmagnetic plasma appears in Fig. 15. At point $V_s$, generally called the space potential, the probe is at the same potential as the surrounding plasma. The current reaching the probe at this point is predominantly electron current since electrons tend to have much higher velocities than ions because of their small mass. Particles reach the probe by virtue of their thermal velocities. As the probe is made positive relative to $V_s$, electrons entering the charge sheath surrounding the probe are accelerated toward the probe and the small ion current is still further diminished. A layer of negative charge builds up around the probe until the total negative charge in the layer equals the positive charge on the probe. As a result, most of the potential drop occurs between the
charge sheath and the probe and little is sustained by the rest of the plasma. If the area of the sheath is constant as the potential is increased, the probe characteristic will flatten out in this region as the current to the probe remains almost constant.

As the probe potential is made negative relative to $V_s$ we begin to repel electrons and attract ions. This is region B in Fig. 15, usually called the transition region. If the electron distribution were Maxwellian, the shape of the curve in this region would be exponential, since the ion current is practically negligible compared to the electron current throughout most of this region. As the probe is biased more negatively, the net current collected decreases to zero at the point at which electron and ion currents are equal. This point, known as the floating potential, is the potential an isolated electrode would assume.

As $V_p$ is made still more negative, nearly all electrons are repelled and an ion saturation current sets in with an ion sheath surrounding the probe (region C). The disparity in the absolute magnitudes of the ion and electron currents is a direct consequence not only of the difference in mass between the two particles, but also of the difference in temperature and a consequent difference in sheath formation. Also, in the presence of a magnetic field the motion of the electrons is much more strongly influenced by the field than is the motion of the ions.

In the next section we consider the formation of sheaths and the collection of particles in detail.
Fig. 15.—A typical probe characteristic for a non-magnetized plasma.
Consider a plasma of dimensions $R$ into which is introduced at some point a potential $V$. Then, Poisson's equation is

$$\nabla^2 V = \frac{-e}{\varepsilon_0} (n_i - n_e) \tag{23}$$

where $n_i$ and $n_e$ are the ion and electron densities respectively. For simplicity, we treat the problem in one dimension, so that Poisson's equation becomes

$$\frac{d^2 V}{dx^2} = \frac{-e}{\varepsilon_0} (n_i - n_e) \tag{24}$$

We normalize $V$, $n_i$, $n_e$ and $x$ as follows:

$$\eta = \frac{-eV}{kT_e} \quad \nu_i = \frac{n_i}{n_o} \quad \nu_e = \frac{n_e}{n_o} \quad \xi = x/R$$

where $n_o$ is the undisturbed plasma density. Equation 24 becomes

$$\frac{\lambda_D^2}{R^2} \frac{d^2 \eta}{d\xi^2} = \nu_i(\eta) - \nu_e(\eta) \tag{25}$$

where $\lambda_D = (\varepsilon_o kT_e / e^2 n_o)^{\frac{1}{2}}$. The electrons can be reasonably assumed to be in thermal equilibrium, so that

$$n_e = n_o e^{-\eta}.$$ 

Ions are usually not in thermal equilibrium, especially in tenuous plasmas, so we will assume, for simplicity, that $n_i$ is a constant, as would be true for ions of infinite mass. Eq. 25 takes the form

$$\frac{d^2 \eta}{d(x/\lambda_D)^2} = 1 - e^{-\eta} \approx \eta$$
for small values of $\eta$. Thus

$$V = V_p e^{-x/\lambda_D}$$

(26)

and the potential drop from $V$ to $V_p$ is sustained largely over a distance $\lambda_D$ which is called the Debye shielding length. Beyond this distance the plasma is essentially undisturbed by the externally introduced potential.

For positive $V_p$, an insufficiency in the ion density will only enhance the shielding effect by diminishing the effective shielding length. When $V_p$ is negative, care must be taken in treating the large discrepancy between the velocities with which electrons and ions stream into the sheath. We consider this in the next section.

The Bohm Sheath Criterion

Eq. 23 is not soluble in closed form for any but the simplest charge distributions and geometries. Exact solutions have been obtained only for the sheath associated with an infinite plane conductor in an infinite plasma. No rigorous treatment of the modifications of a sheath by a magnetic field has been developed. We will consider the case of an infinite plane sheath in the absence of a magnetic field.

Consider two infinite planes, labeled A and B in Fig. 16. We will let surface A be the sheath-plasma boundary, and surface B the conductor surface. The conductor is sufficiently negative so that ions are attracted and electrons repelled. If surface A is to represent the sheath edge, the electric field of the sheath and its derivatives must
vanish at this surface to ensure a smooth transition to the plasma solution.

For simplicity, we shall give the ions only a drift velocity at the sheath edge so that the ion distribution function is

\[ f_i(0,v) = n_o \delta(v-v_o) \]

\[ f_i(x,v) = n_o \delta[(v^2 + 2eV/m_i)^{1/2} - v_o] \] (27)

If nearly all electrons are repelled, their distribution will be Maxwellian:

\[ f_e = n_o (m_e/2\pi kT_e)^{1/2} e^{-m_e(v+2ev/m_e)/2kT_e} \] (28)

The densities are given by

\[ n_i = \int_{-\infty}^{+\infty} f_i dv = n_o \int_{-\infty}^{+\infty} \delta[(v^2 + 2eV/m_i)^{1/2} - v_o] dv \]

\[ = n_o (1 + 2ev/m_i v_o^2)^{-1/2} \]

\[ n_e = \int_{-\infty}^{+\infty} f_e dv = n_o e^{-eV/kT_e} \] (29)

Poisson's equation becomes

\[ \frac{d^2V}{dx^2} = \frac{en_o}{\varepsilon_o} \left[ (1 + 2ev/m_i v_o^2)^{-1/2} - e^{eV/kT_e} \right] \] (30)

Setting \( V(0) = V_o = 0 \) and integrating from 0 to \( x \) with \( dV/dx \) as an integrating factor:

\[ \frac{1}{2}(V^2 - V_o^2) = \frac{n_o}{\varepsilon_o} \left[ m_i v_o^2 [(1 + 2ev/m_i v_o^2)^{1/2} -1] kT_e (e^{-eV/kT_e} -1) \right] \]
In practice $V'_0$ has a small value and can therefore be ignored. In this case the left side of equation the equation is positive, which means that

$$m_i v_0^2 \frac{[(1 - 2eV/m_i v_0^2)^{1/2} - 1]}{k T_e} > 1 - e^{-eV/kT_e}$$

Upon expanding $V$ around $x = 0$ where $V$ is small,

$$m_i v_0^2 \frac{[-eV/m_i v_0^2 - \frac{1}{2}(eV/m_i v_0^2)^2]}{k T_e} > [-eV/kT_e - \frac{1}{2}(eV/kT_e)^2] (31)$$

which reduces to

$$m_i v_0^2/2kT_e > \frac{1}{2} (32)$$

which is the Bohm sheath criterion. It states that in order for the sheath equation to have a solution under the conditions we have assumed, the ion kinetic energy near the sheath edge must be greater than the electron kinetic energy. Because of the cold ion assumption $T_i \ll T_e$, the ions must gain energy before entering the sheath, which means that the conductor is incompletely shielded by the space charge in the sheath and the field which penetrates into the plasma serves to accelerate the ions. Therefore, in part C of the probe characteristic (Fig. 15) the ions stream in with a velocity much greater than their random velocities, with an unknown velocity distribution at the sheath edge. However, if the shape of the probe is planar, the ion current density is roughly $n_0$ times the critical velocity of the ions, which is

$$V_c = (kT_e/m_i)^{1/2}. (33)$$
Fig. 16.—Diagram of a sheath-conductor interface.
Electron Collection

On the basis of the measured electron density, the Debye length of the present plasma is on the order of a centimeter, which implies that thick sheaths surround the probes, as one would expect for a tenuous plasma. The mean free path of a particle is several times the dimensions of the plasma, allowing collisions within the sheath to be neglected. The original probe theory of Langmuir\(^{17}\) (1926) applies to a plasma under these conditions in the absence of a magnetic field. In the absence of a rigorous theory for the magnetic field case, the theory of Langmuir will be treated first and then features which carry over to the magnetic field case will be considered.

Consider a cylindrical probe, the length of which is sufficiently greater than its diameter so that end effects may be neglected. Let the radii of the probe and the sheath be \(a\) and \(s\) respectively, with \(a \ll s\) to satisfy the thick sheath assumption. We consider individual particle orbits within the sheath boundary.

We assume first an attractive force field of cylindrical symmetry. Let

\[
\begin{align*}
  u_a &:= \text{radial velocity of a particle at the probe surface} \\
  v_a &:= \text{tangential velocity of particle at the probe surface} \\
  u_s &:= \text{radial velocity of particle at sheath edge} \\
  v_s &:= \text{tangential velocity of the particle at the sheath edge}
\end{align*}
\]

From the conservation of energy and angular momentum:

\[ u_s^2 + v_s^2 = u_a^2 + v_a^2 + 2eV/m \]

\[ sv_s = a v_a \] (34)
Solving for $u_a$,

$$u_a^2 = u_s^2 + v_s^2 \left( 1 - s^2/a^2 \right) - 2eV/m \quad (35)$$

A necessary condition for the particle to strike the probe is

$$u_a \geq 0$$

This condition imposes the following condition on $v_s$:

$$v_s^2 \leq (u_s^2 - 2eV/m)(s^2/a^2 -1) \equiv v*^2 \quad (36)$$

If $f(u_s, v_s)$ is the distribution function at $s$, the current density is given by the integral of $n_0uf(u,v)$ taken over all $u$ from 0 to and over $v$ from $-v_s^*$ to $v_s^*$:

$$j = n_0 \int_0^\infty udu \int_{-v_s^*}^{v_s^*} (u,v) \, dv \quad (37)$$

and

$$I = j A_s$$

where $A_s$ is the probe surface area.

In the case of Maxwell's distribution

$$f(u,v) = \frac{m}{2\pi kT} e^{-m(u^2 + v^2)/2kT}$$

The integration of Eq. 37, using this distribution function appears in Langmuir and Mott-Smith$^{18}$. For a repulsive potential

$$I = e A_a n_o \left( kT/2\pi m \right)^{1/2} e^{eV/kT} \quad (38)$$

a value independent of the sheath radius. This relation also holds for a flat probe, providing end effects are ignored. In fact, Langmuir has shown that the shape of the I-V curve in this region (region B) is independent of the probe geometry.
Langmuir's solution for the attractive potential is

\[ I = \frac{1}{2} n_0 (2kT/\pi m)^{3/2} \frac{s}{a} \text{erf} \left( \frac{\varphi^{1/2}}{2} \right) \left( 1 - \text{erf} (\eta + \varphi)^{1/2} \right) \]

where \( \text{erf} \)

\[ x = (2/\pi) \int_{0}^{x} e^{-t^2} \, dt \]

\[ \varphi = \frac{a^2 \eta}{(s^2 - a^2)} \]

\[ \eta = eV/kT. \]

For \( s >> a \), \( \text{erf} \sim 2x/\sqrt{\pi} \).

For \( \eta >> 1 \),

\[ I = \frac{1}{2} n_0 (2kT/\pi m)^{3/2} (2/\sqrt{\pi}) (\eta + 1)^{1/2} \]

for a cylinder.

We would therefore expect the electron current to the probe to vary exponentially with \( V \) in region B for a Maxwellian distribution, and to change at the space potential to an \( I^2 \) form. Langmuir gave plots of \( I \) vs. \( V \) for a cylindrical and a flat probe of the same area, using the thick sheath collisionless approximation. These appear in Fig. 17.

The Interpretation of Probe Data

The presence of a magnetic field changes the collected electron currents by altering the dynamic behavior of the electrons. The plasma becomes anisotropic in that the mean free path of electrons perpendicular to the field is of the order of a gyroradius. The saturation electron current is consequently reduced. The magnitude of the elec-
Fig. 17.—Characteristics of cylindrical and plane probes according to the Langmuir theory.
tron current near the space potential has been estimated by Bohm et al. However, the behavior at the space potential is sufficiently obscure so as to make uncertain the point on the characteristic which corresponds to the space potential. Electron saturation is usually destroyed by the magnetic field; that is, the current collected by the probe continues to increase with voltage.

The transition should still be exponential in form for a Maxwellian electron distribution. Since $T_e$ is calculated from the slope of the $\ln I$ vs. $V_p$ plot, uncertainty in the absolute value of $I$ is not critical.

In the absence of saturation, the space potential cannot be clearly defined. Flat probes occasionally display a break in their characteristics which might be loosely interpreted as the point marking the space potential. These discontinuities are not always present unambiguously, and in the absence of an adequate theory, the space potential remains a questionable quantity. Since the plasma density is calculated from the magnitude of the current collected by a probe biased at the space potential, only a crude estimate of this quantity can be made. This estimate will in general be lower than the actual value, because of the reduction in the collected current by the magnetic field.

The floating potential is an easily measured quantity, and can be related through the Langmuir theory to the space potential. The current densities to a probe are given by

$$j_i = \frac{1}{2} n_i (kT_e/m_i)^{1/2}$$

$$j_e = \frac{1}{2} n_e (2kT_e/m_e)^{1/2} e^{eV/kT_e}$$
Setting $j_1 = j_e$ when $V = V_f$, 

$$V_f = \left(\frac{kT_e}{2e}\right) \ln \left(\frac{nm_n^2}{2m_i n_e}\right)$$  \hspace{1cm} (41)

This expression is no more exact for the magnetic field case than the expression for $j_e$, but it does supply an estimate which can be checked for consistency against other data. For the measured temperatures of about 20 ev, prevailing in this experiment, $V_f$, as given by Eq. 41 is about -70 volts for $N_2$.

The influence of a biased probe on the behavior of the plasma is a measure of the reliability of probe data at points far from the floating potential. Fig. 18 shows oscilloscope traces detected on a probe biased at the indicated potentials. The horizontal graticule lines serve as ground references for the signals which are detected across a 45 ohm resistor. The dc floating potential is about -180 volts, and no substantial change in frequency is noted for probe biases more negative than this. However, at $V_p = -100$ volts, the frequency has diminished by 10%. The probe is starting to influence the behavior of the plasma. This is probably in the neighborhood of the space potential.

**Probe Measurements**

Langmuir probe measurements were made at various radii in the central plane by means of two methods illustrated in the circuit of Figs. 19 and 20. The first of these (Fig. 19) is the slow point-by-point measurement of collected current for various probe bias values, a complete set for each radial position. The relatively stable opera-
Fig. 18.—Oscillations detected on a biased probe showing the influence of probe bias potential on oscillation frequency.
tion of the device permits this time consuming, though accurate, method to be used. To minimize uncertainties, a large number of measurements was made over a period of several weeks. All probe measurements were made in the uniform field, with $I_m = 50$ amperes for most cases.

The primary purpose of the probe measurements was to obtain estimates of the electron temperature, a parameter that appears in the expression for the frequency of drift waves. Three different probes were used: a shielded probe of 20 mil round tungsten wire with collecting area $20 \times 10^{-6} \text{m}^2$, a non-shielded probe of 10 mil tungsten wire, $7.6 \times 10^{-7} \text{m}^2$ in area, and a non-shielded probe of flat 15 mil wide tungsten of area $7.2 \times 10^{-6} \text{m}^2$. The electron temperatures obtained were found to be independent of the type of probe used, as one would expect for valid probe measurements.

Fig. 18 shows some typical curves taken at different radii for pressures just below $10^{-6}$ torr. In Fig. 22 the bias potentials for each curve are plotted against the corresponding in $I$ giving linear plots, the inverse slopes of which are proportional to the temperature, as given by Eq. 39. Table 1 summarizes the results of several sets of measurements in this low pressure region. Fluctuations in these results are probably attributable to uncontrollable changes in pressure and outgassing conditions that occur from day to day.
Fig. 19.—Circuit for the point by point observation of probe curves.
Fig. 20.—Circuit for the display of probe characteristic on an oscilloscope screen.
Fig. 21.—A set of probe characteristics taken at various distances from the central axis. All were measured in the central plane of the chamber.
Fig. 22.—Plots of $\ln I$ against $v_p$ for the probe data of Fig. 20. The slopes of these curves yield the indicated electron temperatures.
A Summary of the Results of Measurements of Electron Temperatures at Different Radial Positions

<table>
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<th>0.5 cm</th>
<th>1.0 cm</th>
<th>1.5 cm</th>
<th>2.0 cm</th>
<th>3.0 cm</th>
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<td>18 ev</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>19</td>
<td>16</td>
<td>22</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>12</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No electron current saturation was evident in any of these curves, a factor which complicates the interpretation. Eq. 42 predicted that for an electron temperature of 20 ev, the space potential would be about 70 volts more positive than the floating potential. The floating potentials generally fell between -200 volts and -150 volts relative to the walls of the chamber. This would place the space potentials between -130 and -80 volts. This is also roughly the region in which the ln vs. v plots tend to become non-linear.

Frequently, in the absence of a saturation region, a discontinuity in the probe characteristic is taken to mark the onset of elec-
Fig. 23.—Probe characteristics at various radii as displayed on an oscilloscope screen. The signals are taken across a 120K resistor at 1 V/cm deflection. The top sweep in each set is the potential applied to the probe as measured from the top line of the graticule at 50 V/cm deflection. This sweep is delayed with respect to the others by about one graticule division.
tron collection, although no clear theoretical scheme exists for this interpretation. To detect a break in the characteristic, it is convenient to display it as an oscillogram, as is provided for in the circuit of Fig. 20. This method was found feasible only at pressures around $10^{-5}$ torr and at high filament temperatures. At lower plasma densities stray capacitance obscures the collected current. Unfortunately, at these higher pressures and filament temperatures the oscillations under study are irregular and the electron temperature is elevated to about 50 ev.

Fig. 23 shows a typical set of characteristics for the radial positions indicated. The upper trace is the bias sweep at 50V/cm with zero at the top graticule line. The lower trace is the current signal across 120 kΩ at 1V/cm. The bias signal is delayed approximately one centimeter by the capacitance of the system. The probe is the flat one.

No clear break is evident in these curves, although those at 2.0 cm and 3.0 cm show a change in shape at -30 volts and -10 volts respectively. The curves indicate clearly that the electron density falls off to negligibility somewhere between 3 and 4 cm.

An estimate of the magnitude of the electron density can be made using Eq. 28, providing the current collected at the space potential is known. Uncertainty in this value and in the validity of Eq. 28 in the presence of magnetic field, restricts the accuracy of this estimate to an order of magnitude. Fig. 24 is a plot of the radial density profile for the low pressure region. The space potential was taken to be the point at which the probe characteristic deviates from the ex-
potential shape. The plot is based on several sets of measurements, all of which yield density values of approximately $10^7$/cm$^3$.

Poisson's equation can be used to check the consistency of the measured space potentials and density. For simplicity, we consider a long cylinder of charge of radius $r_a$ and charge density $\rho$, suspended coaxially inside a grounded conducting cylinder of radius $r_c$. Under these conditions Poisson's equation predicts a potential function given by:

$$U(r) = -\frac{k}{4\varepsilon_0} \left[ r^2 - r_a^2 \left( 1 - 2 \ln \frac{r}{r_c} \right) \right]$$

For our particular case

$$r_a \approx 3 \times 10^{-2} \text{ m}$$
$$r_c = 10.7 \times 10^{-2} \text{ m}$$
$$\rho = n_e e \text{ and } n_e \approx 10^7/\text{cm}^3 = 10^{13}/\text{m}^3$$

and the potential at the axis should be -150 volts, which is reasonably close to the range of values indicated by the probes. In this calculation the charge compensation of ions was ignored.

**Axial Potential Measurements**

Floating potentials were measured along the axis with seven probes placed along the chamber and projecting to within one centimeter of the axis. The results of these measurements are plotted in Fig. 25. The spacing of the probes is indicated by the points on the plot. The results show a constant floating potential along most of the length of the column with sharp potential drops at the ends.

The two end probes which exhibit the marked falls in potential
Fig. 24.--Measured electron density plotted against distance from the central axis of the chamber.
are within the sheaths associated with the plasma end-shield interface. This region sustains the potential drop between the main body of the plasma and the end shields. The potential within this region must be more negative than that of the plasma, since it is this potential which must reflect electrons from the end shields. The relationship between this potential and the floating potential of a probe placed within the sheath is not embraced by the Langmuir theory.

**Ion Collection**

Ion currents, which are much smaller than electron currents in systems such as this, were collected on the flat end rings. The modification of the ion orbits by the magnetic field at low field values is relatively small compared to the dimensions involved, i.e. the widths of the rings. The Langmuir theory is, therefore, applicable, with the rings serving as plane probes.

At sufficiently negative biases (around -300 volts) the end rings collect ions in the saturation region. The magnitudes of these currents were measured with a Hewlett-Packard model 425A microammeter (at 10^-6 torr pressure) and are recorded in Table 2. The "corrected" current densities are those for this adjustment was made for the shielding of the recessed rings by the others. The magnitude of this adjustment was determined by interchanging the recessed and non-recessed rings.
Fig. 25.—Floating potentials measured along the axis.
### TABLE 2

<table>
<thead>
<tr>
<th>Ring</th>
<th>Inner Radius</th>
<th>Current</th>
<th>j (Measured)</th>
<th>j (Corrected)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0 cm</td>
<td>0.80 µA</td>
<td>0.63 µA/cm³</td>
<td>0.63 µA/cm³</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>0.11</td>
<td>0.0289</td>
<td>0.072</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
<td>0.45</td>
<td>0.059</td>
<td>0.059</td>
</tr>
<tr>
<td>4</td>
<td>1.4</td>
<td>0.25</td>
<td>0.0196</td>
<td>0.049</td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
<td>0.65</td>
<td>0.0339</td>
<td>0.034</td>
</tr>
<tr>
<td>6</td>
<td>3.1</td>
<td>0.30</td>
<td>0.0112</td>
<td>0.011</td>
</tr>
</tbody>
</table>

The restriction on the collection of slow-moving particles, imposed by the Bohm sheath criterion, renders the determination of the ion velocity distribution impossible. However, we have for the ion current density

\[ j_i = \frac{1}{2} (kT_e/M)^{1/2} \cdot \]

For \( N_2 \) ions with \((3/2)kT_e = 20 \text{ ev}\), the value of \( n_i \) is \( 10^7/\text{cm}^3 \) at the center, the same order of magnitude as the electron density. Table 2 also shows that the density falls to 5% of maximum value at the sixth ring, whose inner diameter was 3.15 cm. This agrees with the radial electron density profile.
CHAPTER IV
STUDIES OF OSCILLATIONS

Oscillations in the Mirror Field

Oscillations of potential, such as those in the oscillogram of Fig. 26 are first detected in the course of operating in the mirror field configuration at a pressure of $10^{-5}$ torr. These oscillations were detected on a probe in the central plane of the chamber. The frequencies were strongly dependent on magnetic field strength, ranging from 140 kHz at 60 gauss to 14 kHz at 310 gauss.

Fig. 26 depicts relative amplitudes of the oscillations picked up at various positions within 5 cm of the central axis. The amplitude is maximum near the center and decreases steadily with distance from the axis. Beyond the 5 cm point, the amplitude diminishes sharply into negligibility near the chamber wall. Changes in signal shape indicate that those detected near the chamber walls were mainly induced potentials. A shielded probe was used in these observations.

Two other probes were installed in the central plane, spaced azimuthally from the original probe by 100° and 120° as shown in Fig. 27. The signals detected, shown in the chopped mode display in Fig. 27 exhibit phase shifts equivalent to the angular spacing of the probes. The disturbance is seen to rotate clockwise on Fig. 27, which is viewed from the west end of the device with magnetic field direction pointing east. Since there is a radial electric field
Fig. 26.—Oscillations detected on a floating probe at increasing distances from the axis. The deflection scale is 5 V/div with a time base of 20 µsec/div.
Fig. 27.—The top diagram illustrates the probe positions in the central plane of the chamber as viewed from the west end. The signals detected on these probes and their relative phases are shown below.
pointing toward the axis, the direction of rotation of the disturbance is the vector $\mathbf{E} \times \mathbf{B}$. To check phase relationships at points along the axis a set of four probes, spaced at 2 centimeter intervals, was placed parallel to the axis, with the first probe 4 centimeter from the gun end rings. No measurable phase shifts could be detected on these probes, an indication that the propagation velocity along magnetic field lines greatly exceeds that transverse to the field lines. No signals could be detected on probes placed outside the end shields, an indication that the oscillations do not originate within the electron gun.

The probe data suggests a rotating vane of charge or an azimuthally propagating wave. The radial extent of the disturbance was determined by collecting electrons from it with a positively biased probe. The collected current is drawn through a resistor on the ground side of a battery bias source, with an oscilloscope displaying the variation of potential across the resistor. At each radial position the probe bias was varied from zero to 100 volts and the increase in oscillation amplitude was noted. For a 250 gauss mirror field (measured at center) the increase in amplitude was less than 25% for radial positions beyond 2 centimeters. At the 2 centimeter position the change in amplitude rose sharply to 100% and then to several hundred present as the probe was moved inward. The position at which the dramatic growth in collected current commenced varied somewhat with changes in magnetic field, but most changes occurred with uncontrollable variations in pressure which occurred from day to day. Measurements over a period of one week indicated that the disturbance was confined to a cylindrical volume, between 2 and 3 cm in radius. In these measure-
ments (which served as a preliminary survey) the field strength was 250 gauss.

The variation of rotational frequency with magnetic field strength was determined by reading frequencies from oscillograms. In Fig. 27 the frequency is plotted against $100/I_m$ where $I_m$ is the magnetic coil current in amperes. At magnetic field values above 330 gauss the plot is linear and passes through the origin. A transitional region occurs between 330 and 200 gauss in which the frequency falls somewhat as the magnetic field decreases and then at 200 gauss continues to rise linearly with a slope about one half that in the high field region. The frequency axis intercept of this line is at 20 kHz.

This data is consistent with the expectation that at least part of the rotational frequency is associated with the electric field drift of guiding centers in the inwardly directed radial electric field caused by negative space charge. The velocity associated with each guiding center is

$$v_d = \frac{E \times B}{B^2}$$

and the linearity of our curves suggest that $E$ remains nearly constant as $B$ is varied. This has not been observed in dense plasma experiments primarily because they are collision dominated with transport coefficients varying as $1/B^2$. The electric field is a function of the radial transport rate. In a later section it is shown that the radial current in this experiment is essentially constant over the range of magnetic field strengths used in these studies.

The relationship between the potential applied to the decelerator
electrode and the rotation frequency is plotted in Fig. 28 for a 700 gauss magnetic field. For these measurements the differences of potential between the various electrodes in the electron gun were kept constant as the entire gun was biased with respect to the walls of the vacuum chamber. The decrease in the decelerator potential should result in an increased electron density within the chamber and a consequent change in the electric field, as the data seems to indicate.

The growth rates and propagation velocities of density and potential perturbations, such as the ones responsible for the signals detected here, are of primary interest for comparisons with theory. It is necessary to separate this propagation velocity from the drift motion of the plasma as a whole and to determine the dependence of the growth of oscillations on the plasma dimensions and magnetic field configuration. In the next section we consider these problems.

Oscillations in the Uniform Field Configuration

Plasma instabilities are frequently associated with certain magnetic field shapes. In particular, the mirror field is conducive to the interchange (flute) type of instability by virtue of the outward curvature of the field lines. The coil configuration was therefore altered to provide the field plotted in Fig. 12. This field is hydrodynamically stable in that the field lines are essentially straight over a relatively large portion of space and diverge at the ends of the chamber. The slight increase in B with distance from the axis contributes further to the hydrodynamic stability. For these reasons, most measurements were made in this configuration.
Fig. 28.—Frequency as a function of inverse $B$ for oscillations of a plasma in the mirror field.
Fig. 29.—The variation of frequency with bias applied to the decelerating electrode.
A survey of the oscillations at different pressures was made by admitting air through a thermally controlled leak device. While this device did not permit the selection of arbitrary stable pressures, in conjunction with a valve, a sample of pressures over a wide range could be achieved and maintained long enough to make quick observations. At pressures above $5 \times 10^{-6}$ torr, stable oscillations tend to disappear when the filament current is raised beyond a certain point. When this occurs a random "hash" signal sets in. The stable oscillations can be restored by turning the filament current down to zero and then up again to a point below this critical current. At about $6 \times 10^{-4}$ torr, this random signal is accompanied by an ion current to the walls of the chamber which indicates that the inward radial electrostatic field has vanished. The stable rotary signal can be observed clearly only at very low filament currents, which restrict the potential of the reflector plates to -50 volts or less.

Fig. 30 is a plot of frequency vs pressure for $I_m = 100$ amperes. Two points not shown are 140 kHz at $3 \times 10^{-7}$ torr and 14 kHz at $6 \times 10^{-4}$ torr. The general fall in frequency with increasing pressure is understandable in terms of the increase in ion density with pressure and the consequent reduction of the radial E field. The rotational rate should fall linearly as the ion density rises linearly with pressure. The reasons for the nearly hyperbolic, rather than linear form, for the curve in Fig. 30 are not obvious. The most likely cause is a possible change in density gradients with pressure.

The region above $2 \times 10^{-4}$ torr is of interest. The frequency of rotation was here independent of pressure and a small ion current was
Fig. 30.—The variation of frequency with pressure.
detectable on a negatively biased probe near the walls of the chamber. The escape of ions signifies that the inward radial electrostatic field has vanished and consequently the frequency no longer has a component attributable to body rotation of the plasma. The residual frequency of 14 kHz represents the drift wave frequency at these higher pressures.

The variation of frequency with magnetic field was investigated for several different pressure values. These plots of frequency vs $100/I_m$ are given in Figs. 31, 32 and 33. At the lower pressures ($3 \times 10^{-7}$ torr and $4 \times 10^{-6}$ torr) the results are similar to the mirror field case. At magnetic fields above 400 gauss the frequency is directly proportional to $1/B$ and approaches zero as $B$ becomes infinite. At 400 gauss an abrupt decrease in slope sets in. As the pressure is raised the curves continue to change behavior at about the same point, but slope in the high field region is reduced and the curves can no longer be extrapolated through the origin. The reduction in slope is very likely a result of the reduction of the radial $E$ field as the pressure rises.

The sharp discontinuities in the curve suggest a cyclotron frequency resonance, since the other plasma parameters are energy dependent and would exhibit a diffuse influence, characteristic of an energy distribution. The break in the $f$ vs $1/B$ curves occurs near the value $I_m = 100$ amperes ($B = 430$ gauss). The $N_2$ ion cyclotron frequency at this point is 23.4 kHz, approximately the value of the drift wave frequency as deduced from data presented in the next section.

The above measurements were repeated with argon as the background
gas. The argon was admitted through the same leak system that was used to vary the air pressure within the system. The background air pressure was below $10^{-6}$ torr and the argon was admitted at a rate sufficient to bring the pressure to $10^{-4}$ torr. A small correction in the pressure readings was made to compensate for the fact that the ionization efficiency of argon exceeds that of air by a factor of 1.6 at the electron energies that prevail in hot cathode ionization gauges.

Fig. 34 shows a plot of $100/I_m$ vs frequency with argon present at $1 \times 10^{-4}$ torr. The point of discontinuity in the curve is at $I_m = 140$ amperes ($B = 600$ gauss). Since the ratio of the mass of the argon ion to that of $N_2$ is 1.43, the break in this curve occurs when the ion cyclotron frequency reaches the same value as was recorded for $N_2$. We would expect the drift eave frequency to remain the same for both gases, since the frequency of the wave is a function of electron parameters. The argon data, then, lend further support to the hypothesis that the break in the $f$ vs $1/B$ curves represents a point of ion cyclotron resonance.

Fig. 35 is a plot of the relative oscillation amplitudes for various values of $B$ with a background of $8.5 \times 10^{-5}$ torr of $N_2$. The pattern is the same at other pressures. The amplitude is constant as long as the magnetic field is considerably higher than 400 gauss, but falls off as the 400 gauss point is approached, and falls off rapidly as the magnetic field is further reduced.

There is presently no theory for disturbances at frequencies greater than the ion cyclotron frequency. The persistence of a linear $f$ vs $1/B$ characteristic indicates that the mechanism of propagation is prob-
ably similar to that assumed in the theory of drift waves. The attenuation of the wave for frequencies higher than the ion cyclotron frequency suggests that the instability is largely confined to the lower frequencies.

Care should be exercised in the application of the collisionless theory to the cases in which the background pressures were in the $10^{-4}$ torr range. Morse\textsuperscript{19} has found that a hollow cathode discharge in argon at $10^{-4}$ torr is subject to a similar type of instability which may be explained in terms of a collisional model.

The Radial Electrostatic Field

The radial electric field can be varied by changing the bias on the decelerator electrode, which is in contact with the plasma column. To do this without changing the interelectrode fields within the gun, the entire gun was biased as was done for a similar measurement in the mirror field case. The mirror field result indicated that the frequency varied linearly when potentials lower than -500 volts were applied. We are interested here in the extrapolation of the $f$ vs. applied bias curve to zero applied bias. For this reason, a positive bias was applied to the gun electrodes so that the decelerator potential could be varied between -300 volts and ground (wall potential).

Fig. 36 (top) shows $f$ vs. decelerator bias for several values of $I_m$ at a pressure of $5 \times 10^{-5}$ torr. At this pressure a signal was detectable down to -50 volts and each curve in the figure is extrapolated to the zero volt point to obtain the frequency axis intercept. Since the radial field must vanish when the decelerator is at wall
Fig. 31.—The variation of frequency with inverse $B$ at low pressure.
Fig. 32.—Variation of frequency with inverse B at intermediate pressures.
Fig. 33.—Variation of frequency with inverse B at high pressures.
Pressure $8.5 \times 10^{-5}$ torr

Pressure $1.5 \times 10^{-4}$ torr
Fig. 34.—Variation of frequency with inverse B with argon as the background gas.
Fig. 35.—Amplitude as a function of magnetic induction.
Fig. 36.—The top figure shows the variation of oscillation frequency as the bias potential applied to the electron gun is changed. In the bottom graph, the frequency axis intercepts of the top plot are graphed against inverse $B$. 
Fig. 37.—The variation of oscillation frequency with electron gun bias at low pressure.
potential, the frequency axis intercepts represent the frequency of the drift wave. These intercept values are plotted against $100/I_m$ in Fig. 36 (bottom). The variation of $f$ with $B$ agrees with the theoretical model.

The curves of Fig. 36 show that even at as low a pressure as $5 \times 10^{-5}$ torr, the space charge is compensated sufficiently to make the frequency of rotation small relative to the drift wave frequency. For instance, at $I_m = 100$ amperes, the drift wave frequency is 23 kHz and the frequency of rotation at $V = -300$ volts is about 8 kHz. Similar curves for data taken at $7 \times 10^{-7}$ torr (Fig. 37) show a steeper slope. The drift wave frequency is approximately 20 kHz and the plasma rotation rate at $V = -300$ volts is 80 kHz. This supports the contention that the sharp drop in frequency of the signal as the pressure is raised from $10^{-7}$ torr to $10^{-4}$ torr is due largely to a drop in plasma rotation rate. Final evidence of this was seen when the decelerator bias was varied in the higher pressure region at about $3 \times 10^{-4}$ torr. The signal frequency remained essentially constant at 15 kHz as the bias was varied between -300 volts and -150 volts.

**The Influence of Plasma Length**

The theory we have discussed predicts that the stability of the plasma to different azimuthal wavelengths is a function of the length of the plasma column. This dependence was observed by placing a stainless steel reflecting sheet at various positions along the axis, thereby limiting the plasma to the volume between this reflecter and the electron gun. The results are displayed in Table 3. All values were
observed with $I_m$ at 100 amperes. The amplitudes are those for signals detected on a floating probe.

**TABLE 3**

<table>
<thead>
<tr>
<th>Length</th>
<th>f</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>36 cm</td>
<td>86</td>
<td>$5v$</td>
</tr>
<tr>
<td>25</td>
<td>87</td>
<td>$2v$</td>
</tr>
<tr>
<td>22</td>
<td>116</td>
<td>$1v$</td>
</tr>
<tr>
<td>18</td>
<td>150</td>
<td>.03v</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>.02v</td>
</tr>
<tr>
<td>10</td>
<td>Not Detectable</td>
<td></td>
</tr>
</tbody>
</table>

In the case of the 18 cm column length, the signal is small and unsteady with both $m = 1$ and $m = 2$ modes present. These modes were of about the same amplitude. This agrees with the calculations of Krall and Rosenbluth\(^8\) which predict the attenuation of long wavelengths as the column is shortened. The waves with higher azimuthal numbers are generally obscured by the $m = 1$ wave, which in the longer column is much greater in amplitude.

The increase in the frequency of the $m = 1$ mode that occurs as the column is shortened is probably attributable to the increased rotational rates caused by the reduction in radial transport as the wave attenuates. In another section it is shown that electrons are transported radially at an anomalous rate by the oscillations.
The Agreement of Experimental Results with Theory

We have referred to several points of agreement between the behavior of the observed instability and the qualitative behavior predicted by the theory of the universal instability. The quantitative aspects present some interpretive difficulties and are, therefore, treated separately here.

The expression we have derived for the drift wave frequency is

\[ \omega = k \left( \frac{kT_e}{eB} \right) \left( \frac{n'}{n} \right) \]

where we have assumed that the disturbance was sufficiently localized to be associated with a plasma density gradient at some point in space. The experimental conditions involve a disturbance which encompasses most of the volume of a cylinder approximately 3 centimeters in radius. In this case some sort of average must replace the localized gradient. For simplicity, we will assume that \( n'/n \) can be approximated by \( 1/r \), where \( r \) is the plasma radius. Using the following values:

\[ r = 3 \times 10^{-2} \text{ m} \]
\[ \frac{3}{2} kT_e = 20 \text{ ev} \]
\[ k = \frac{2\pi}{\lambda} = \frac{2\pi}{2\pi r} = 1/r \]
\[ B = 4.3 \times 10^{-2} \text{ w/m}^2 \quad (I_m = 100 \text{ amperes}) \]

Eq. 16 predicts a frequency of 50 kHz. In view of the approximations made, this compares favorably with the observed value of about 20 kHz.

The stability conditions for the plasma column are given by equa-
tions 20 and 21. If we assume the following values for the \( m = 1 \) wave:

\[
\frac{n'}{n} = \frac{1}{3} \text{ cm}
\]
\[
k = \frac{1}{3} \text{ cm}
\]
\[
\lambda_o = 1 \text{ cm}
\]
\[
\frac{T_i}{T_e} \text{ is negligible}
\]

the longest stable machine is predicted to 31 cm long. This compares with the experimental value of 18 cm, at which the \( m = 1 \) wave was still barely detectable. Again, in view of the crude approximations made, the agreement between theory and experiment is fairly good.

The additional stability condition at the ion cyclotron frequency is given by

\[
(n'/n)R_i > 2(1 + \frac{\omega_e^2}{\omega_{oe}^2})^{\frac{1}{2}}
\]

which for our case \( (\omega_{oe} < \omega_e) \) reduces to

\[
R > 2n/n'(B\varepsilon_o/en)
\]

For \( B = 4.3 \times 10^{-2} \text{ W/m}^2 \)

\[
\frac{n'}{n} = \frac{1}{3} \times 10^{-2} \text{ /m}
\]
\[
n_e = 10^{13} /\text{m}^3
\]

the condition reduces to

\[
R_i > 5 \times 10^{-5} \text{ m} = 5 \times 10^{-3} \text{ cm}
\]

This condition is easily fulfilled in the present case since even a room temperature ion \( \text{N}_2 \) ion has a gyroradius of 0.1 cm. In a radial electric field energies of several electron volts are possible, with gyroradii of several centimeters.
Particle Transport Across Magnetic Field Lines

The subject of particle transport in magnetized plasmas has been of paramount interest, especially in high temperature plasma experiments. Interest in transport rates in lower energy discharges was stimulated by the discovery by Bohm et al.\(^{20}\) that the diffusion of the plasma created by an arc was much higher than that expected from collisions along, and furthermore, it seemed to vary as \(1/B\) instead of the classically anticipated \(1/B^2\). Simon,\(^{21}\) however, showed that in systems with conducting end plates that were not floating, an enhanced radial transport results from the tendency of the plates to short out electric fields caused by the separation of ions and electrons. Simon and Neidigh\(^{22}\) showed that the transport rate across magnetic induction lines in the type of experiment performed by Bohm's group is explainable in terms of end plate effects if the data is correctly interpreted.

The theory developed by Bohm to explain the anomalous rate of transport was based on the presence of fluctuating electrostatic fields. The expression derived by Bohm was justified largely on the basis of arbitrary dimensional considerations, but has recently been applied to a variety of experiments with surprising success. Enhanced plasma drain, attributable to oscillations, has been observed by many workers, including Golant,\(^{23}\) Lenhnert and Hoh,\(^{24}\) and Thomassen.\(^{25}\) In these experiments the mechanism suggested by Simon could not be operative.

The two possible sources of transport for charged particles across magnetic induction lines are diffusion and drift. We will consider the diffusion process first.
Diffusion of charged particles in a uniform magnetic field is dependent upon collisions. In the present experiment electrons diffuse out of the central column by collisions with neutrals, each collision resulting in a random step of about one gyroradius in length. Since the motion is random, with net transport resulting from density gradients, the process is governed by Fick's Law:

\[ \n = -D \n \]

where \( n \) is the mean velocity of particle motion, \( n \), the local density, and \( D \), the diffusion constant. Townshend and Gill have rigorously derived an expression for the diffusion constant of electrons in a magnetic field:

\[ D = D_0 /[1 + (\omega \tau)^2] \]

where \( \omega \) is the electron cyclotron frequency given by

\[ \omega = \frac{eB}{m_e} \]

and \( \tau \) is the collision time. In the present case \( \omega \tau \gg 1 \), making the diffusion constant a function of \( 1/B^2 \). \( D_0 \) is the field free diffusion constant given by:

\[ D_c = \frac{v_e \lambda}{3} \]

Where \( \lambda_e \) is the mean free path. The mean free path for a 20 ev electron in \( N_2 \) at \( 10^{-6} \) torr is about 200 meters, as calculated from data given by Landolt-Börnstein Tabellen.\(^27\)

We want to apply the diffusion equation to a uniform cylinder of electrons, of density \( n_1 \) and radius \( r_1 \), suspended in a magnetic field which is taken to be uniform and parallel to the cylinder axis. We consider only radial diffusion, so that in cylindrical coordinates
the diffusion equation reduces to:

\[ nv = -D \frac{dn}{dr} \]

If we consider a cylindrical surface of radius \( r \), where \( r > r_1 \) then the current through a meter long section of this surface is

\[ I = JA = nev (2\pi r) = 2\pi neD \frac{dn}{dr} r \quad (42) \]

For equilibrium, the current \( I \) must be independent of \( r \). Solving under the boundary condition \( n = n_1 \) when \( r = r_1 \), we get

\[ n = -(I/2\pi eD) \ln \left( \frac{r}{r'} \right) + n_1 \quad (43) \]

For a given value of \( n_1 \) the current \( I \) is determined by the value of \( r \) at which \( n = 0 \), i.e. the position at which the current is collected and measured.

For the mode in which the end rings were permitted to float, the current to the outermost ring was measured. This ring had an inner radius of 8.3 cm, which is taken as \( r \) in Eq. 43. The radius of the initial plasma column was 0.3 cm, the radius of the electron gun aperture, and the electron density in the column was \( 10^7/cm^3 \). The insertion of these values into Eq. 43 yields the values listed in the third column of Table 4. The measured values, which appear in the second column, are greater by three orders of magnitude and show no dependence on \( B \).
The currents predicted on the basis of Bohm's diffusion constant are listed in the last column of Table 4. Bohm's diffusion constant is given by

\[ D_B = \alpha \frac{kT_e}{dB} \]

where \( \alpha \) is a constant smaller than unity, in this case taken as 1/32, following Bohm. The Bohm expression is generally interpreted as describing a drift motion in oscillating electrostatic fields, the effective amplitudes of which are functions of \( kT_e \). A serious objection to the application of this expression to cases such as the present one lies in the fact that it assumes a random walk process, an assumption inherent in Fick's law. The oscillations observed in the
present experiment are correlated with density fluctuations and cannot serve as a mechanism for random transport. Surprisingly, however, the Bohm expression predicts transport rate of the same order of magnitude as that observed in this experiment.

The independence of the radial current of magnetic field strength is not a unique observation, but is surprising in view of the model we have used which ascribed radial currents to \( E \times B/B^2 \) drifting of particles in an oscillating azimuthal electric field. In the present experiment, analysis of the changes in oscillation amplitude with changes in \( B \) (Fig. 34) removes the contradiction. In the low magnetic field region where drift velocities are most sensitive to changes in \( B \), the oscillation amplitude steadily declines as \( B \) is decreased. The reduction in the effective azimuthal electric field probably masks the true dependence of the radial current on \( B \).

When small currents are drawn by the end rings through 2.3 megohm resistors to ground, the oscillation amplitudes remain practically constant over a wide range of values of \( B \). The reduction in oscillation amplitude, as \( I_m \) is reduced from 170 amperes to 50 amperes, is less than 10%. For this reason it is instructive to observe the currents collected on each ring as \( B \) changes. The results appear in Table 5 with the same currents normalized to 100 at each value of \( B \) appearing in Table 6. Table 6 clearly shows a contraction of the plasma column, with current collected on the innermost rings increasing as the field increases.
TABLE 5

Currents Collected at End Rings at Different Values of B.
The Outer Diameter of Each Ring in cm
Appears in Parenthesis

<table>
<thead>
<tr>
<th>I_m</th>
<th>I(0.63 cm)</th>
<th>2(1.26)</th>
<th>3(1.89)</th>
<th>4(2.52)</th>
<th>5(3.15)</th>
<th>6(3.78)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>58.9 µa</td>
<td>64.2 µa</td>
<td>67.0 µa</td>
<td>44.3 µa</td>
<td>30.7 µa</td>
<td>11.8 µa</td>
</tr>
<tr>
<td>80</td>
<td>85.0</td>
<td>91.4</td>
<td>92.8</td>
<td>77.0</td>
<td>60.6</td>
<td>1.7</td>
</tr>
<tr>
<td>100</td>
<td>91.4</td>
<td>113.5</td>
<td>95.0</td>
<td>77.0</td>
<td>58.0</td>
<td>.82</td>
</tr>
<tr>
<td>150</td>
<td>117.0</td>
<td>128.0</td>
<td>101.0</td>
<td>67.0</td>
<td>11.8</td>
<td>.15</td>
</tr>
<tr>
<td>170</td>
<td>123.0</td>
<td>122.0</td>
<td>53.4</td>
<td>34.8</td>
<td>4.1</td>
<td>.1</td>
</tr>
</tbody>
</table>

TABLE 6

Values in Table 4 Normalized to 100 for Each Value of B

<table>
<thead>
<tr>
<th>I_m</th>
<th>I(0.63 cm)</th>
<th>2(1.26)</th>
<th>3(1.89)</th>
<th>4(2.52)</th>
<th>5(3.15)</th>
<th>6(3.78)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>21.2 µa</td>
<td>23.2 µa</td>
<td>24.2 µa</td>
<td>16.0 µa</td>
<td>11.1 µa</td>
<td>4.3 µa</td>
</tr>
<tr>
<td>80</td>
<td>20.9</td>
<td>22.4</td>
<td>22.7</td>
<td>18.8</td>
<td>14.8</td>
<td>.4</td>
</tr>
<tr>
<td>100</td>
<td>21.5</td>
<td>24.4</td>
<td>22.3</td>
<td>18.1</td>
<td>13.5</td>
<td>.2</td>
</tr>
<tr>
<td>150</td>
<td>26.4</td>
<td>29.1</td>
<td>25.0</td>
<td>16.6</td>
<td>2.9</td>
<td>0</td>
</tr>
<tr>
<td>170</td>
<td>35.5</td>
<td>35.4</td>
<td>16.8</td>
<td>11.0</td>
<td>1.3</td>
<td>0</td>
</tr>
</tbody>
</table>

The lack of a rigorous theory to describe the enhanced radial drift points up a defect that presently characterizes all plasma instability theories: the theories are all linear. In the treatment
of the universal instability it was assumed that the disturbance was radially localized and could be described in terms of a small perturbation within a constant density gradient. Implicit in this is the assumption that the wave disturbance is small enough so as to not substantially modify this gradient. In experiments it is a large signal that is observed, which substantially influences the plasma distribution. This must be considered in the search for agreement between theory and experiment.

Conclusions

The present work extends the original observations of drift waves in a collisionless plasma into the ion cyclotron frequency region. The plasma was found to be unstable to drift waves at the ion cyclotron frequency as predicted by the treatment of Mikhailovskii and Timoffeev. The experiment also represents the first test of the length criterion in the collisionless regime. The plasma was found to approach quiescence as its length was reduced.

Measurements of the electron radial transport rate revealed it to be in excess of that predicted by collisional diffusion by several orders of magnitude. The order of magnitude of the observed diffusion rate was in agreement with that calculated on the basis of the Bohm diffusion constant. Similar observations had been heretofore made only in dense plasma experiments.
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