TENSILE STRENGTH OF LIQUID HELIUM

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * * * *

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CHAPTER I

INTRODUCTION

The tensile strength of a liquid is defined as the maximum stress, the magnitude of the negative pressure in the liquid, that can be attained before the liquid spontaneously, and immediately, changes from an isothermal metastable state into a liquid-vapor equilibrium state. It will be seen later that, given a set negative pressure of magnitude less than the tensile strength, it is possible for certain events, such as the passage of a cosmic ray through the liquid, to promote the transition into the liquid-vapor state. In this context the term "rupture pressure" will be used to denote the magnitude of the ambient pressure at the time of such an event. The term "threshold pressure" will denote that stress below which the metastable state persists. There will be a threshold, if one exists, for each type of event which initiates nucleation in the liquid. The distinction between the terms "tensile strength", "rupture pressure", and "threshold pressure" must be kept in mind throughout this dissertation.

1.1 Background

Liquid tensile strength measurements are generally notoriously non-reproducible (1) and are not understood in terms of any current theory.
Experiments performed on water, for example, have produced tensile strengths ranging from about 1.5 atm to about 300 atm (Table 1). As can be seen from the table, the same methods do not reliably give even similar results.

**TABLE 1**

TENSILE STRENGTH OF WATER

<table>
<thead>
<tr>
<th>Method</th>
<th>Tension (atm)</th>
<th>Observer</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berthelot</td>
<td>50</td>
<td>Berthelot (2), 1850</td>
<td>Small sealed capsule filled and then water stressed by cooling</td>
</tr>
<tr>
<td>Berthelot</td>
<td>150</td>
<td>Dixon (2), 1895</td>
<td></td>
</tr>
<tr>
<td>Vincent</td>
<td>157</td>
<td>Vincent (2), 1940</td>
<td>157 atm is largest value received. Filled under vacuum</td>
</tr>
<tr>
<td>Vincent</td>
<td>1.5</td>
<td>Scott (3), 1941</td>
<td></td>
</tr>
<tr>
<td>Reynolds</td>
<td>4.93</td>
<td>Reynolds (2), 1878</td>
<td>Centrifugal U-tube</td>
</tr>
<tr>
<td>Berthelot</td>
<td>34</td>
<td>Meyer (2), 1911</td>
<td></td>
</tr>
<tr>
<td>Briggs</td>
<td>277</td>
<td>Briggs (4), 1950</td>
<td>Spinning capillary</td>
</tr>
</tbody>
</table>

Tensile strength measurements can be affected by impurities and surface effects, both of which may cause the liquid to rupture at smaller stresses than expected for a pure liquid. It is especially difficult to prepare a liquid which is free of dissolved gases, so that when a stress is applied the dissolved gas forms small bubbles which grow as the pressure is increased until they cause rupture. Thus tensile strength measurements can depend upon the amount and type of dissolved gases, producing a wide array of measured values.
Liquid tensile strengths should also be reduced if the adhesion between the liquid and a surface is smaller than the cohesion between the molecules of the liquid, i.e., the contact angle between the liquid and the surface is $> 90^\circ$. It is then easier to pull the liquid from the surface and the tensile strength thus becomes a property of the liquid-solid interface rather than a property of the liquid. No distinction is made between surfaces of colloidal particles and container surfaces (5).

If, in addition, there exist small cracks or cavities in the surface, then the rupture pressure of a non-wetting liquid (contact angle $> 90^\circ$) will be further reduced by surface tension forces (6). If the cavity is not completely filled, for example, the region of vapor will grow and cause rupture as soon as the pressure in the liquid becomes less than the vapor pressure. Even if the cavity is completely filled, the rupture process will be aided by the curvature of the liquid-solid interface. This occurs because the free energy of a non-wetting liquid is reduced by decreasing the surface area. If the rupture pressure is then determined by the cavity with the greatest curvature, it is not surprising that the tensile strength measurements of normal liquids are extremely non-reproducible.

Measurements of the tensile strength of liquid helium, however, should not be influenced by any of these effects. Due to its very low temperature there can be no dissolved gases in the liquid; at most there will be only suspended colloidal particles. The rupture process should not be influenced by surface effects like those described above since liquid helium
seems to be a universal wetting agent (contact angle < 90°). If a liquid wets the surface, then even the presence of cracks and cavities will have no influence on the tensile strength measurements and the measured values should reflect only the properties of the bulk liquid.

Support for the universality of the wetting nature of liquid helium comes from several sources. First it is shown by S. Franchetti (7) that the van der Waals' attraction between helium and several types of surfaces is greater than the attraction between helium molecules. Supporting this conclusion is the fact that there is no surface known over which the helium film will not spread. Evidence for the wetting of solid impurity particles comes from (a) the increase in the helium film flow rate when the surface is contaminated with substances such as solid air (8) and (b) the increase in the thickness of the helium film when the surface is contaminated with a thin layer of solid air (9).

1.2 Previous Measurements

A. Misener and Hébert

The first measurements of the tensile strength of liquid helium were in 1956 by A. D. Misener and G. R. Hébert (10). Their method consisted of filling a bellows with liquid helium and producing the tension by pulling on the ends of the bellows. Although this method proved valuable for measuring rupture pressures of other liquids (2, 11), it was not sensitive enough for
liquid helium. They were able to deduce only that the tensile strength of liquid helium II was less than 0.3 atm.

B. Beams

Later the same year J. W. Beams (12) reported measurements on helium taken by a spinning capillary method. The apparatus consisted of a cylindrical block of copper with a hollow space in its center and two narrow passageways to the outside of the block. These passageways sloped upwards so that the interior space could be completely filled. The block was then spun about its axis with a high angular velocity, producing a tension in the liquid. When the liquid ruptured, it escaped through the passageways and was detected by a carbon resistor at the end of one passageway. The results obtained were 0.14 ± 0.02 atm at 1.8°K. Beams later confirmed these results using an accelerating U-tube to produce rupture (13).

C. Finch et al.

Recently ultrasonic cavitation techniques have been used by R. D. Finch et al. (14) to measure the tensile strength of helium below 2.17°K. A ceramic piezoelectric transducer, driven at 47 kc, produced pressure fluctuations large enough to cause cavitation of the liquid. A threshold of about 0.0012 atm was determined by the onset of a hissing noise, detected by an identical transducer mounted beneath the first. The noise was assumed to be caused by the growth and collapse of small bubbles.
R. D. Finch later (15) repeated his measurements, but this time in a cylindrical resonance cavity, and extended them to higher temperatures. Below 2°K a driving voltage of 10V again produced the threshold characterized by the hissing noise. It was not until the driving voltage was increased to 100V that bubbles could be seen visually. Above 2.2°K, however, both thresholds appeared at about 10V driving voltage. These measurements do not give a clear indication of the size of the tensile strength.

The measurements of Finch et al. have recently been criticized by D. Y. Hsieh (16) as showing a property of the ultrasonic technique rather than a general property of the fluid. For this reason a comparison with their results may not be meaningful and no significance should be placed on such comparisons.

1.3 Theories of the Tensile Strength

A. Introduction

There are two properties that theories of the rupture of a liquid must contain. First, it is necessary that the theory have a mechanism by which small nuclear bubbles can be formed. Secondly, once such a bubble is created, there must be a means by which it can grow. It is the growth of this bubble that separates the liquid into a liquid-vapor state.

The minimum work required to form a bubble of radius $r$ is

$$ W = 4\pi r^2 \sigma + \frac{4\pi}{3} r^3 (P_L - P_V) $$

(1)
where $\sigma$ is the surface tension, $P_L$ is the pressure in the liquid, and $P_V$ is the vapor pressure. This work attains a maximum value of

$$W_{\text{max}} = \frac{16\pi\sigma^3}{3(P_L - P_V)^2}$$  \hspace{1cm} (2)

at a critical radius $r^* = -2\sigma/(P_L - P_V)$. Since the system will try to minimize its free energy, a bubble with $r < r^*$ will collapse while one with $r > r^*$ will grow. The catastrophic growth of the bubble in this latter case is observed as a rupture of the liquid. For the measurements on liquid helium, for which $\sigma = 0.35 \text{ ergs/cm}^2$ (17), described in this dissertation, $P_V \ll P_L$ and the relative sizes of $r^*$ and $W_{\text{max}}$ as a function of $P_L$ are shown in Table 2.

**TABLE 2**

PRESSURE DEPENDENCE OF CRITICAL BUBBLE PARAMETERS

<table>
<thead>
<tr>
<th>$P$ (atm)</th>
<th>$r^*$ (cm)</th>
<th>$W_{\text{max}}$(erg)</th>
<th>$W_{\text{max}}$(eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>$7.0 \times 10^{-8}$</td>
<td>$7.2 \times 10^{-15}$</td>
<td>$4.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>-5</td>
<td>$1.4 \times 10^{-7}$</td>
<td>$2.9 \times 10^{-14}$</td>
<td>$1.8 \times 10^{-2}$</td>
</tr>
<tr>
<td>-1</td>
<td>$7.0 \times 10^{-7}$</td>
<td>$7.2 \times 10^{-13}$</td>
<td>$4.5 \times 10^{-1}$</td>
</tr>
<tr>
<td>-0.1</td>
<td>$7.0 \times 10^{-6}$</td>
<td>$7.2 \times 10^{-11}$</td>
<td>$4.5 \times 10$</td>
</tr>
<tr>
<td>-0.01</td>
<td>$7.0 \times 10^{-5}$</td>
<td>$7.2 \times 10^{-9}$</td>
<td>$4.5 \times 10^3$</td>
</tr>
<tr>
<td>-0.001</td>
<td>$7.0 \times 10^{-4}$</td>
<td>$7.2 \times 10^{-7}$</td>
<td>$4.5 \times 10^5$</td>
</tr>
</tbody>
</table>
Starting from an analysis by Plesset and Zwick (18), Seitz (19) has shown that in normal liquids the time required for a bubble to grow to macroscopic size is typically of order of magnitude 1 microsecond. From this analysis, since liquid helium II is characterized by a very high thermal conductivity combined with a low density and a low viscosity, it is clear that the growth rate of a bubble in helium will be even larger. The rate of growth of a bubble thus is of no consequence to these measurements.

B. Van der Waals' process

If the liquid is treated as a continuum, one method of calculating the tensile strength comes from an extension of van der Waals' equation of state. This treatment does not require pre-existing nuclear bubbles and is thus a special case of the preceding discussion (i.e., \( r^* = 0 \)). When applied to liquids at their boiling points, van der Waals' equation of state gives a relatively good account of such quantities as the heat of vaporization, the coefficient of thermal expansion, and the isothermal bulk modulus (20). Thus extending the theory to pressures below the saturated vapor pressure of the liquid should yield a theoretical maximum tensile strength that can be attained.

The reduced form of van der Waals' equation is

\[
\pi = \frac{8\Theta}{3\phi - 1} - \frac{3}{\phi^2}
\]  

where \( \pi \) is the reduced pressure \( P/P_c \), \( \phi \) is the reduced volume \( V/V_c \), and \( \Theta \) is the reduced temperature \( T/T_c \). The condition for instability at constant
temperature, \( \frac{\partial \tau}{\partial \varphi} \theta = 0 \), yields the following relationship between \( \varphi \) and \( \theta \) at \( \pi_{\text{min}} \), the minimum reduced pressure,

\[
\varphi^3 - \frac{9\varphi^2}{4\theta} + \frac{3\varphi}{2\theta} - \frac{1}{4\theta} = 0.
\] (4)

Solving this for \( \theta \) and substituting back into Eq. 3 gives the minimum reduced pressure, and thus the rupture pressure,

\[
\pi_{\text{min}} = \frac{P_{\text{rup}}}{P_c} = \frac{3}{\varphi^2} - \frac{2}{\varphi^3}.
\] (5)

Tensile strengths calculated from these expressions are invariably much larger than the measured values (see Table 3). Helium, which has \( T_c = 5.26^\circ K \) and \( P_c = 2.26 \text{ atm} \) at \( 1^\circ K \), for example, has a calculated tensile strength of 25.2 atm, while the largest measured value is only 0.14 atm.

**TABLE 3**

**THEORETICAL VS EXPERIMENTAL TENSILE STRENGTHS**

<table>
<thead>
<tr>
<th>Substance</th>
<th>Maximum Experimental P (atm)</th>
<th>Van der Waals P (atm)</th>
<th>Density Fluctuations P (atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>-300</td>
<td>-996</td>
<td>-1320</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>-39</td>
<td>-179</td>
<td>-223</td>
</tr>
<tr>
<td>Helium (1°K)</td>
<td>-0.14</td>
<td>-25.2</td>
<td>-6.4</td>
</tr>
</tbody>
</table>
C. Nucleation theory

J. C. Fisher (6), by a method similar to one employed by Landau and Lifshitz (21), has calculated tensile strengths from a consideration of statistical density fluctuations. These density fluctuations can be large enough in localized regions to create vapor bubbles such as described in Section 1.3A. The approach followed by Fisher, which is an adaptation of the Arrhenius form of chemical reaction rates, gives a rate of formation of bubbles of the critical radius, per mole,

\[
\frac{dn}{dt} = \frac{NkT}{h} \exp \left[ -\frac{(\Delta f_0 + \frac{16\pi\sigma^3}{3(P_L - P_V)^2})}{kT} \right],
\]

where \(\Delta f_0\) is the activation energy of a molecule on the bubble surface.

For liquid helium the term \(\Delta f_0\) can be ignored when compared to \(\frac{16\pi\sigma^3}{3(P_L - P_V)^2}\), except in the limit of high negative pressures, since the activation energy cannot be greater than the van der Waals' energy of attraction between liquid atoms. For helium this is about \(1.5 \times 10^{-15}\) ergs, so that the nucleation rate cannot be affected by \(\Delta f_0\) unless the energy required for the formation of the bubble of critical radius is less than \(10^{-14}\) ergs. But from Table 3 it is seen that a bubble of this energy cannot nucleate rupture unless \(P < -5\) atm. Thus for the measurements described in this dissertation \(\Delta f_0\) can always be neglected.

In Eq. 6, \(dn/dt\) can be replaced by \(1/t\), since the first such bubble initiates rupture, yielding the rupture pressure,
\[ P_L = P_V \left[ \frac{16\pi}{3} \frac{\sigma^3}{kT \ln(NkTt/h) - \Delta f_0} \right]^{1/2}. \]  

This function is extremely insensitive to the time \( t \) and calculations are based on one bubble per mole per second. Table 3 shows that this theory also predicts tensile strengths which are much too large.

D. Cosmic radiation

Even if all dissolved gases, impurity particles, surface effects, etc., could be eliminated, the liquid in question still could not be considered pure in that it is continually bombarded by cosmic rays. Indeed, there is strong evidence that the tensile strength of normal liquids may be limited by mechanisms involving cosmic radiation (1). In particular it has been found that many liquids have their tensile strengths reduced considerably when subjected to various energetic nuclear particles (22, 23, 24, 25). Also, even though the stresses involved in bubble chambers are considerably larger than the tensile strength of the liquid, the fact that bubbles do form along the paths of nuclear particles must be taken as evidence that cosmic radiation may place a limit on the tensile strength. Therefore, since the tensile strength of liquid helium should be a property only of the bulk liquid, helium provides a very good test of the theory that cosmic radiation can cause rupture of the liquid.

The theory advanced by F. Seitz (19) considers that the necessary localized energy (see Sec. 1.3A) comes from the thermalization of secondary electrons (\( \delta \) rays) produced by the incident nuclear particle. The thermalization
arises from the transfer of energy from the $\delta$ ray to the liquid through subsequent Rutherford scattering, which also tends to keep the $\delta$ ray localized. The $\delta$ ray must be localized to the extent that enough collisions occur, within a region corresponding to the critical bubble size, to yield the necessary localized energy to form a bubble.

The proposal by Seitz has been criticized by A. G. Tenner (26) as not localizing enough energy to nucleate a bubble of the critical size. Instead he suggests that the process proceeds in a more complicated manner. Each $\delta$ ray is assumed to excite and ionize many atoms along its path. The energy for the formation of a bubble then comes from de-excitation of the electronic structure and from the recombination of positive and negative ions. He concludes that one $\delta$ ray is not sufficient; but rather that each $\delta$ ray itself causes a cascade of secondary electrons. It is the presence of these in a small region which causes the bubble formation.

In neither theory is it clearly shown how secondary electrons can lose enough energy in a localized region to form a bubble of the critical radius. There are, however, other methods by which incident nuclear particles can lose large amounts of energy in small regions. One of these processes is the collision of the incident particle with the nucleus of a helium atom, or with the nucleus of an impurity atom such as oxygen or nitrogen. Although these events occur very infrequently, large amounts of energy are released during each collision.
1.4 Theories Pertinent Only to Liquid He II

Liquid helium II has properties which make it unique among liquids [see, for example, Atkins (17)]. There are in particular two intrinsic sources of nuclear bubbles arising from the presence of negative ions and from the cores of vortex structures in the liquid. The effect of these pre-existing defects will be discussed here and again in more detail in Chapter V.

A. Ions

There are two methods by which ions can form small nuclear bubbles in liquid helium. It is known that free electrons in the liquid are localized within small "holes" in the liquid. These regions, which are void of helium atoms, have a diameter of about 25\(\AA\) (27, 28). That these regions behave like "bubbles" in a macroscopic sense is shown by the experiments of Springett and Donnelly (29), where the dependence of the bubble size was determined as a function of a positive external pressure. The stress necessary to cause such a bubble to rupture the liquid is \(P = 2\sigma/r = 2.5\) atm, which is considerably larger than the observed pressures.

The second mechanism involves the recombination of positive and negative ions in the liquid. In liquid helium positive ions are known to attract helium atoms, forming a cluster with an effective mass of about 100 atoms and a diameter of about 12\(\AA\) (30). Such clustering should have little effect, however, on the recombination energy (31) and this energy should thus be
about $4 \times 10^{-11}$ ergs. From Table 2 this energy is seen to correspond to a critical bubble radius of about $15^\circ$ and a tensile strength of about 5 atm.

B. Vortices

It is also known that vortex structures--lines and rings--can exist in liquid He II [see Brewer, Quantum Fluids (32)] and that they have core diameters estimated to be about $5^\circ$ (33). For a liquid with density $\rho$ and surface tension $\sigma$, a vortex will have an energy per unit length

$$\epsilon = 2\pi\sigma a + \pi Pa^2 + \frac{\rho gh^2}{4\pi m^2} \ln \frac{R}{a}$$

where $a$ is the vortex core radius, $P$ is the pressure applied to the liquid, and $R$ is a dimension of order of the container radius. This results from consideration of a classical vortex line with quantized circulation $\kappa = n\hbar/m$.

It was suggested as early as 1944 by R. B. Dean (34) that rupture in superfluid helium could be caused by the growth of a vortex core as the stress in the liquid increased. In support of this, it was found by Edwards, Cleary and Fairbank (35) that the nucleation of bubbles in a helium bubble chamber, operated below 2.17°K, could be enhanced by artificially produced vortex rings. Recently Finch (36), using his ultrasonic cavitation technique, has observed that the rupture pressure decreases sharply when vortex lines of $n = 1$ (and also perhaps $n = 2, 3$) are introduced via a rotating shaft. However, according to Eq. 8 the tensile strength should be about 20 atm.
While this pressure is attainable in bubble chambers, the necessary pressure for rupture is much too large for this process to account for the measured tensile strengths. Finch requires a hypothesis that the periodicity of the sound waves in his chamber cause a pulsation of the vortex core radius. However, in the experiments by Beams and in those described in this dissertation there is no way that a large pulsation of the core radius can occur.

There is a mechanism involving vortices which can explain the observed rupture pressures. It is necessary to postulate, however, that the vortex line, which has energy $\sim 1 \times 10^{-7}$ ergs/cm, have a length $\geq 300 \mu$ convoluted into a spherical region of $1 \mu$ diameter (the critical bubble diameter). Only by such an unlikely vortex configuration can enough energy be localized to account for the rupture pressure of $\sim 0.014$ atm characteristic of the experiments described in this dissertation. The annihilation of a suitably convoluted vortex line, or ring, can thus cause the liquid to rupture. These processes will be considered in more detail in Chapter V.

1.5 Method of Measurement

The experiments described in this dissertation were done below the helium $\lambda$ temperature (2.17\textdegree K) so that use could be made of some of the properties of superfluid helium. An increase of temperature $dT$ at one end of a superleak (a narrow constriction through which only the superfluid component of helium can pass) produces a corresponding pressure difference $dT = \rho sdT$, \[\newpage\]
where $\rho$ is the mass density and $s$ is the entropy per unit mass of the helium. This is the well-known "fountain," or thermo-hydrodynamical, effect.

The apparatus, which will be described later, consists essentially of a capsule connected to a helium bath by a superleak. A vacuum space around the capsule and superleak provide thermal isolation from the helium bath (Figure 1). If the bath is warmed relative to the capsule, or the capsule cooled relative to the bath, the pressure inside the capsule decreases and is given by:

$$P = -\int_{T_1}^{T_2} \rho s dT + \rho g \Delta h + P_{\text{vap}}.$$  \hspace{1cm} (9)

$P_{\text{vap}}$ is the vapor pressure of the bath, $T_1$ is the capsule temperature, $T_2$ is the bath temperature, and $\rho g \Delta h$ is the hydrostatic pressure difference since $\rho$ can be treated as a constant.

The "fountain" pressures were obtained by numerical integration of $\rho s dT$. The specific entropy was measured by Botts and Gorter (37), and $\rho$ was assumed to be constant ($\rho = 0.145$). The estimated error in this integration is about 1%.

There are several advantages to this method of measurement. Foremost among these is that the tensile strength is determined under static conditions, which other authors (12,14) have not been able to do. The use of the "fountain" effect provides a very simple method of producing tension in the liquid. It is also very easy to recycle the system after each rupture so that
Figure 1

Basic Probe for Tensile Strength Measurements
each measurement can be repeated many times. Superfluid, having zero entropy, also provided indirectly a convenient method of detecting rupture. This will be described later.
CHAPTER 2

PRELIMINARY MEASUREMENTS

The measurements taken on the first experimental run were in partnership with F. W. Duncan, whose unpublished report on the measurements was used to partially fulfill the requirements for an M.S. degree. The He\textsuperscript{4} cryostat was of standard design and will not be described here.

2.1 Apparatus

The experimental apparatus, made entirely of glass, is shown by the diagram in Figure 2. The capsule is enclosed in a vacuum space and communicates with the helium bath through the superleak. The superleak was constructed of jeweler's rouge (\text{Fe}_2\text{O}_3) packed tightly into a channel between the capsule and the bath.

The temperature of the capsule was measured by a carbon resistor. This resistor, a 100 ohm Allen-Bradley hot composition carbon resistor, was coated with a silicone vacuum grease and placed into the well in the capsule wall. The grease both held the resistor in place and insured good thermal contact between the resistor and the capsule. Calibration of the thermometer was achieved by thermally linking the capsule to the helium bath, accomplished by putting a small quantity of helium gas into the vacuum space.
Figure 2

Probe for He\textsuperscript{4} Cryostat
- Charcoal cage
- Rupture chamber
- Carbon resistor
- Vertical side-tube
- Vacuum jacket
- Superleak
- Resistor leads
A small cage, made from a small mesh copper screen, filled with activated charcoal was used to pump helium gas from the vacuum space. This cage, located in the vacuum space side tube, was attached by a thin rod to a steel cylinder above the top flange of the cryostat. The cage could then be moved vertically by a magnet outside the adsorption pump tube. When the cage was below the helium bath level, a high vacuum was attained by adsorption of the gas by the charcoal.

2.2 Procedure

The following procedure was repeated for each measurement of the tensile strength. A small quantity of helium gas was put into the vacuum space so that the capsule could be cooled to the desired temperature by pumping the He\textsuperscript{4} bath to that temperature. Since both the capsule and the bath were at the same temperature, the capsule was filled by the hydrostatic head pressure. The charcoal cage was then lowered to pump the helium gas from the vacuum space, thus thermally isolating the capsule from the He\textsuperscript{4} bath.

The bath was then allowed to warm until the resulting fountain pressure was sufficient to cause rupture of the liquid. The capsule temperature at the moment of rupture was determined by the carbon thermometer, and the temperature of the bath was determined from its vapor pressure, which was measured by an oil manometer (butyl phthalate). The bath temperature changed slowly enough so that the calculated fountain pressures should be correct to within one or two millimeters of Hg.
The apparatus was constructed of glass so that visual observation of
the liquid rupture could be made, and for the first measurements the rupture
was determined visually. However, it was soon discovered that the instant
the liquid ruptured there was an immediate temperature rise in the capsule.
This temperature rise was seen as a sudden change in the resistance of the
carbon thermometer mounted on the capsule. After this discovery visual
observation was no longer used since monitoring the temperature proved to be
much more convenient.

The temperature rise is due to the increase of specific entropy in the
capsule as the superfluid, with zero entropy, flows out to the He\textsuperscript{4} bath.
Between 0.6°K and 2.0°K the increase in entropy per cm\textsuperscript{3} of the liquid (31) in
the capsule is at least 7.5 times larger than the entropy per cm\textsuperscript{3} needed to
form the vapor with the percentage difference decreasing as the temperature
increases (see Table 4).

<table>
<thead>
<tr>
<th>TABLE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPECIFIC ENTROPY (ergs/°K cm\textsuperscript{3}) OF He\textsuperscript{4}</td>
</tr>
<tr>
<td>AS A FUNCTION OF TEMPERATURE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T(°K)</th>
<th>0.6</th>
<th>0.85</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid</td>
<td>2.5 x 10\textsuperscript{3}</td>
<td>9.4 x 10\textsuperscript{3}</td>
<td>2.4 x 10\textsuperscript{4}</td>
<td>2.9 x 10\textsuperscript{5}</td>
<td>1.3 x 10\textsuperscript{6}</td>
</tr>
<tr>
<td>Vapor</td>
<td>6.7</td>
<td>3.7 x 10\textsuperscript{2}</td>
<td>1.7 x 10\textsuperscript{3}</td>
<td>3.4 x 10\textsuperscript{4}</td>
<td>1.7 x 10\textsuperscript{5}</td>
</tr>
</tbody>
</table>
2.3 Experimental Results

The experimental results (see Figure 3) show a large amount of scatter, with all rupture pressures lying between 0.006 atm and 0.030 atm. These values lie between those of Beams (12) (0.14 atm) and those of Finch (14) (0.0012 atm) and fail to show a definite tensile strength. This is in contradiction with the measurements of both Beams and Finch who report definite tensile strengths. The results are in agreement, however, in both magnitude and amount of scatter with the measurements by Wilson, Edwards and Tough (38) received under static conditions from measurements of the osmotic pressure of He³ in He⁴.

The group of points in the lower left corner of the figure were produced by tapping the top flange of the cryostat. Relatively gentle taps were sufficient to initiate rupture at this pressure, with stronger taps having no additional influence. For example, a 10 gram weight dropped one centimeter had the same effect as the same weight dropped 10 centimeters. Thus the possibility existed that even under normal conditions the rupture could have been initiated by mechanical vibrations.

The scatter of the results indicates that the mechanism responsible for rupture may be due to events occurring statistically in time. Thus there exists the possibility of a threshold pressure, below which rupture never occurs, and an average waiting time needed for rupture at pressures greater than the threshold. Information concerning these hypotheses, which could
Figure 3

Rupture Pressure vs Temperature
prove valuable for the determination of the rupture mechanism, could not be obtained from this cryostat. A new cryostat was used for this reason and to try to eliminate mechanical vibrations.
CHAPTER III

EXPERIMENTAL APPARATUS AND PROCEDURE

The requirements that must be met by the cryostat are the ability to decrease the temperature of the capsule relative to the He\textsuperscript{4} bath and to maintain the temperatures of the capsule and the bath to a high degree of precision. With this apparatus the capsule is cooled by means of a thermal connection to a He\textsuperscript{3} reservoir, while the He\textsuperscript{4} bath temperature remains constant. The He\textsuperscript{3} reservoir thus has the dual responsibility of cooling the capsule and of maintaining the capsule at a constant temperature. The main features of the cryostat are shown in Figure 4.

3.1 Experimental Apparatus

A. Dewar assembly

Concentric with the probe there are two glass dewar vessels. The inner vessel contains liquid helium, which is both a refrigerant and the means by which the fountain pressure is generated. This He\textsuperscript{4} bath can be pumped to about 1.2\textdegree{}K and its temperature regulated, by three parallel valves in the pumping line, to within 5 millidegrees over the necessary temperature range (1.25\textdegree{}K to 1.55\textdegree{}K). The outer vessel, which is filled with liquid nitrogen, serves to reduce the heat influx to the liquid helium.
Figure 4

Experiment Space for He$^3$ Cryostat
B. Vacuum jacket

The purpose of the vacuum jacket is to provide thermal insulation between the He\textsuperscript{4} bath and the experiment space. The stainless steel tube \textit{L} (0.375" o.d. x 0.010" wall) leads to a vacuum system which is capable of pumping the space to a pressure of about 10\textsuperscript{-5} torr. The electrical leads pass through the stainless steel tube \textit{M} (0.375" o.d. x 0.010" wall), which opens directly into the vacuum jacket, to a junction box at the top of the cryostat.

C. He\textsuperscript{3} reservoir

The He\textsuperscript{3} reservoir \textit{B} is constructed of copper and has an internal volume of about 20 cm\textsuperscript{3}. There is a 1/4" hollow copper tube \textit{A} through the center of the reservoir. He\textsuperscript{3} is condensed into the reservoir and subsequently pumped from the reservoir, through the german silver tube \textit{D} (12 mm o.d. x 0.2 mm wall) and the stainless steel tube \textit{E} (1" o.d. x 0.010" wall). The tube \textit{D} also serves the purpose of providing thermal insulation between the copper flange \textit{F} and the He\textsuperscript{3} reservoir.

The tube \textit{E} is in direct contact with the He\textsuperscript{4} bath, which reduces the heat flow down the tube and also cools the incoming He\textsuperscript{3} gas so that it will condense. The system contained 10 liter-atmospheres of He\textsuperscript{3} which, due to a large heat influx into the reservoir, was only sufficient to maintain the reservoir at 0.85\textdegree{}K for approximately two hours with the He\textsuperscript{4} bath at 1.4\textdegree{}K.
D. Thermal link

The stainless steel tube N (0.125" o. d. x 0.010" wall) provides the means by which the capsule is cooled below the He\text{4} bath temperature. A small amount of He\text{4} placed in this tube, via the small german silver tube G (2 mm x 0.1 mm wall), gives a weak thermal connection between the He\text{3} reservoir and the capsule. This thermal connection arises from the conduction of heat by the distillation of helium from the capsule to the He\text{3} reservoir. The thermal link was sufficiently large for the capsule to cool from 1.4°K to 0.85°K in about one minute, but was small enough that a heater on the capsule could warm the capsule to the He\text{4} bath temperature without appreciably increasing the rate of evaporation from the He\text{3} reservoir.

E. Capsule

Each capsule C, which was made of copper, was constructed in two parts. The top is a solid cap attached directly to the thermal link, while the bottom is a hollow cylindrical chamber connected to the superleak by a Wood's metal solder joint. The two parts of the capsule are joined together by another Wood's metal solder joint. Since the assembled capsule is nearly a closed system, it was necessary to exercise care in making this solder joint. There were five capsules of volume \(\sim 1\ \text{cm}^3\) and one capsule of volume \(\sim 0.05\ \text{cm}^3\) used throughout the series of experiments.

In one of the experiments the capsule was placed in a vacuum chamber, pumped clean, and then sealed with an atmosphere of clean helium.
inside. The capsule had a bushing on its side with a threaded hole which opened into the capsule. Mated to this bushing was a screw which had an indium metal O-ring under its head. The vacuum chamber had a side part with a movable cylindrical rod through it. The vacuum seal around the rod was made with a double rubber O-ring seal. Once the chamber was pumped to a sufficiently low pressure, helium gas was introduced and the capsule sealed by tightening the screw. As an added precaution, the screw head was then soldered to the bushing with Wood's metal.

Attached to the capsule are a carbon resistor T for thermometry purposes (to be described later) and a heater H. The heater consists of six inches of one mil Evanohm wire, with a resistance of 60 ohms, wound around a small copper cylinder. Both the resistor and the heater were tied to the capsule with a piece of thread, with Apiezon N grease providing the thermal connection. The electrical leads of both the resistor and the heater were 3 mil copper wires, heat sunk to both the He\textsuperscript{4} bath and to the He\textsuperscript{3} reservoir.

F. Superleak

There were two different types of superleaks S used. The first consisted of a 4 cm stainless steel tube with diameter 1/8" (0.010" wall) packed tightly with jeweler's rouge powder (Fe\textsubscript{2}O\textsubscript{3}). The channels through the rouge are estimated to be about 0.1 microns in diameter. The first such superleaks were packed with the use of a small rod in a drill press. It was
found, however, that the rouge could be packed tighter by tapping it into the tube with a hammer.

The other superleak was constructed of porous Vycor glass, which was held by epoxy (Stycast 2850GT) in a 2 cm length of 1/8" diameter stainless steel tube. Care was taken that the epoxy completely covered all but the ends of the glass and filled the space between the glass and the tube. The channels in the Vycor glass have diameters of about 40Å.

Connection between the superleak and the He⁴ bath was made by a small diameter stainless steel tube K. The tube fits through a small hole in the cap at the bottom of the vacuum jacket, with a Wood's metal solder joint then sealing the gap. The tube has a small bend in it to allow for differential contraction during the cooling process.

G. Vibration mounting

Since it was observed that mechanical vibrations can affect the tensile strength measurements, considerable care was taken to minimize such vibrations. With the exception of the rotary vacuum pumps, all of the cryostat is mounted on a massive steel frame which is suspended at each corner by a large spring in series with rubber shock absorbers. This method of support should reduce both high and low frequency mechanical vibrations.

The 3" pumping line for the He⁴ bath is isolated from its pump by two sections of flexible tubing. However, since this tubing must withstand pressures up to one atmosphere, it is fairly rigid and transmits the major
source of vibration to the cryostat. Each small rotary pump is isolated by two sections of rubber tubing which are joined by a section of copper tubing imbedded in a concrete block.

3.2 Thermometry

A. He⁴ bath temperature

The temperature of the He⁴ bath was determined from its vapor pressure, where use was made of the T₅₈ vapor pressure tables (39). The vapor pressure was measured by a system of mercury and oil manometers, with the oil manometer (butyl phthalate) calibrated against the mercury manometer. The connection to the manometers was from the cryostat side of the regulating valves in the pumping line through a 1/4" i.d. copper tube.

Since the fountain pressure increases rapidly with temperature, it is necessary to measure the He⁴ bath temperature very accurately. For this purpose the level difference of the manometers was measured with a Wild cathetometer, which has a resolution of better than 0.1 mm. The He⁴ bath temperature was thus measurable to within 1 or 2 millidegrees. To insure that the bath temperature remained constant to within 10 millidegrees, readings of the manometer levels were taken every ten minutes and the temperature was continually regulated. The error in the fountain pressure was less than 1 mm Hg.

Since the pressure in the capsule is dependent upon the helium level in the dewar, through the hydrostatic pressure difference, the fountain pressure
was correspondingly reduced as the helium level fell. This was accomplished by a decrease in the He\textsuperscript{4} bath temperature. This correction was very small; if it had not been made the pressure in the capsule would be in error by no more than 0.3 mm Hg and is thus only a minor source of error.

B. Capsule temperature

The temperature of the capsule was monitored by an Allen-Bradley Ohmite carbon resistor with nominal resistance of 50 ohms. After removing most of the insulation, the resistor was coated with vacuum grease and pressed into a copper sheath. The assembly was then tied to the capsule, with Apiezon N grease providing the thermal contact. This procedure yielded good thermal connection between the carbon resistor and the capsule. The resistance of the thermometer was measured by an AC resistance bridge (Cryotronics Model ML-155AC) and an accompanying bridge monitor (Cryotronics Model ML-500BM).

C. He\textsuperscript{3} reservoir temperature

The temperature of the He\textsuperscript{3} reservoir is determined by measurement of the vapor pressure of He\textsuperscript{3}, which is in a system independent of the He\textsuperscript{3} in the reservoir. Normally the He\textsuperscript{3} is stored at room temperature in a 1/2 liter bottle which is connected from one end to a system of mercury and oil manometers such as just described in Section A. Also included in this measuring system is a McLeod gauge (CVC Type 100A). The other end of the storage bottle leads to a bulb in the He\textsuperscript{3} reservoir via a 3/16" i.d. copper
tube to the top of the cryostat and a 4 mm o.d. stainless steel tube (0.010" wall) to the bulb. The time required for the pressure to equilibrate was only a few minutes, even for temperatures as low as 0.5°K.

D. Calibration of carbon thermometer

To calibrate the carbon resistor it is first necessary to thermally anchor the capsule to the He³ reservoir. This is accomplished by putting enough He⁴ in the tube N (Figure 4) so that there is bulk liquid extending part way through the He³ reservoir. The high thermal conductivity of bulk superfluid helium ensures that the capsule and the He³ reservoir remain at the same temperature. The temperature was regulated by manipulation of the pumping speed of the He³.

3.3 Experimental Procedure

A. Preliminary steps

After the system has been precooled to about 80°K, usually by leaving the nitrogen dewar filled overnight, He⁴ is transferred into the helium dewar. Once the system reaches 4.2°K, and exchange gas has been pumped from the vacuum jacket, the He⁴ bath is cooled to about 2°K where the oil manometer is calibrated against the mercury manometer. Then the He⁴ bath is pumped to ~ 1.2°K, the He³ is condensed into the reservoir, and a small amount of helium is put into the tube between the capsule and the He³ reservoir. After cooling the He³ reservoir to slightly below the desired capsule temperature and warming the He⁴ bath to the appropriate temperature, the system is
B. Observation of rupture

As was discussed in Chapter II, the rupture of the helium is accompanied by a sharply defined rise in the capsule temperature. In these experiments it was found that the condition of the liquid in the capsule could be determined at all times by monitoring the temperature. The rectified off-balance signal from the resistance bridge was displayed on a chart recorder (Barber Colman Model 8000-16000). A sample trace is given in Figure 5.

With the capsule empty at 0.85°K (A, Figure 5), the power into the heater is increased (B) so that the capsule warms until it reaches the He\textsuperscript{4} bath temperature (C). The capsule then begins to fill and, since the incoming superfluid has zero entropy, the power from the heater goes entirely into heating the superfluid. The capsule was always made to fill at a rate such that the superfluid flowed through the superleak at less than the critical velocity. Thus the temperature remains constant until the capsule is filled (D). Once the capsule is filled, it then warms steadily until the heater is turned off (E).

When the capsule cools to 0.85°K, the heater is again turned on (F), but with reduced power, so as to maintain the temperature until rupture occurs (G). As the capsule empties, the increase of specific entropy is
Figure 5

Capsule Temperature as Seen on Chart Recorder
quickly offset by the heat conduction to the He\textsuperscript{3} reservoir and the temperature remains constant until the capsule has emptied (H) (see Appendix A). At this time the capsule has returned to the initial state (A) and the cycle can be repeated. The waiting time $\tau$ required for the rupture to occur is illustrated in the figure. It should be noted that the time chosen for the beginning of the interval $\tau$ is arbitrary—provided it is chosen before rupture occurs.
4.1 **Introduction**

To provide a conceptual framework for discussion of the experiments, some general observations concerning the rupture of a liquid under stress will be presented here. A stress large enough to cause the liquid to rupture does not necessarily do so immediately; and types of rupture processes can be conveniently described by the relationship between the applied stress $P$ and the average waiting time $\langle \tau \rangle$ needed for the rupture to occur.

A. **Simple rupture**

A liquid with a well-defined tensile strength can be described by curve (a) in Figure 6. For any stress less than the tensile strength $P_0$, the liquid is stable, i.e., all waiting times $\tau$ are infinite. When the pressure $P_0$ is reached, however, the liquid will immediately rupture. This, for instance, is the result expected from a van der Waals' rupture process (Section 1.3A). A well-defined tensile strength should also result from rupture by uniform nucleation centers, such as holes created by negative ions or vortex cores.
Figure 6

$P_{rup} \text{ vs } <r>^{-1}$ for Simple and Statistical Rupture Processes
B. Waiting times

At a given stress the occurrence of rupture may depend upon events of a statistical nature, such as the formation of nucleating centers or the activation of nucleating sites by internal or external mechanisms. The probability that rupture will be initiated within a given time will in general depend upon the amount of stress, since the critical bubble size is dependent upon the pressure in the liquid (Section 1.3A). If an external statistical mechanism is responsible for rupture, however, then it is possible for $\langle \tau \rangle$ to be independent of $P$.

The waiting times observed at a given pressure will display a statistical distribution about some mean waiting time $\langle \tau \rangle$. The shape of the distribution, the form of the dependence of $\langle \tau \rangle$ on $P$, and the value of the threshold pressure $P_0$ at which $\langle \tau \rangle$ becomes infinite (Figure 5, curve B) will depend on, and may be useful in identifying, the nucleation process.

C. Non-stationary distribution

If $\langle \tau \rangle$ depends upon any parameter other than $P$, then the $\langle \tau \rangle$ vs $P$ relationship may well not be unique. In particular, if $\langle \tau \rangle$ depends upon the number of nucleation centers present in the liquid, a separate $\langle \tau \rangle$ vs $P$ curve would exist for each number of centers. These curves should all have the same threshold, however. A non-stationary distribution of waiting times occurs when the number of centers changes, as they may well do, during the
course of the measurements. Several mechanisms for changing the number of centers will be discussed in the next chapter.

There are, of course, many possible rupture processes which have even more complicated properties than those just described. These include such processes as those in which $<\tau>$ depends upon many parameters and those which have centers of non-equal nucleation probabilities. These will not be discussed due to the prohibitively complicated analysis involved.

4.2 Time Dependence of $<\tau>$

A. First measurements

Some measurements were taken with the present apparatus before it was moved to its final location and vibration mounted. These results immediately showed that for each pressure there is a distribution of the waiting times $\tau$ required for rupture. A sample distribution is shown by the histogram in Figure 7, where the height of each block corresponds to the number of waiting times in that $<\tau>/2$ interval. A rupture process which occurs randomly in time has a probability distribution given by $P(t) = (1/<\tau>)e^{-t/<\tau>}$, which gives the distribution indicated by the dashed lines on the histogram. The agreement is remarkable and indicated strongly that the rupture is due to events which are random in time.

The statistical error in the determination of $<\tau>$ is estimated by the standard deviation of the waiting times. Since the events seem to occur randomly in time, the standard deviation is taken as $N^{-1/2}<\tau>$, where $N$ is
Figure 7

Distribution of Waiting Times
OBSERVATIONS OF RUPTURE

\[ \langle \tau \rangle \]

\[ \langle \tau^2 \rangle \]

\[ \text{P = 28 MM HG} \]

\[ \langle \tau \rangle = 116 \text{ SEC} \]
the number of ruptures. This method of estimating the error is useful, although not quite correct since a standard deviation has meaning only for a symmetrical distribution.

As can be seen from Figure 8, the statistical error of $<\tau>$ is so large that the $<\tau>^{-1}$ vs $P$ dependence cannot be clearly defined. The solid curve represents a possible best fit to the data points, although the dashed straight line also fits within the experimental error. The dashed line indicates that there is a threshold at about 9 mm Hg but that no definite tensile strength can be ascertained.

The solid curve is suggestive of a $P = P^*(1 - e^{-\lambda/<\tau>})$ relationship, where $\lambda$ is a constant. Such a rupture pressure dependence gives a tensile strength, as defined in the introduction, equal to $P^*$, but does not give a non-zero threshold pressure. Figure 8 then indicates a tensile strength of about 35 mm Hg from such a relationship. If there is a definite tensile strength $P^*$, then this information could be very useful in determining the mechanism responsible for the rupture in liquid He II. The problem of determining whether $P^*$ actually exists and, if it does, how well it can be measured will be taken up later.

Subsequent close examination of these data and data taken after the cryostat was moved and vibration mounted indicates the possibility of a time dependence of the waiting times at constant pressure. In most cases this dependence is small and its existence uncertain due to the statistical uncertainty of $<\tau>$ and to the relatively short running time between helium
Figure 8

$P_{rup} \ vs \ <\tau>^{-1}$
transfers. Because of the possibility of this time dependence, a more
detailed analysis of the data could not be made.

B. Longer running times

To establish clearly whether the time effect does exist, a larger set
of dewars was obtained so that the running time between helium transfers
could be extended. The usable running time was increased from three or
four hours to between twelve and eighteen hours, which proved to be sufficient.

Before the following discussion of results, it should be mentioned that
runs 35, 36 and 41 extended for several days each. In each run there were
two time periods when the system was at 4.2°K for about 12 hours. The
transfer of helium into the dewar occurred at the beginning and end of each of
these time periods. In runs 36 and 41 there was an additional helium transfer
which was followed by an extended period of time with the system left below
the helium λ point. For example, the complete history of one of the long runs
is presented in Figure 9. The vertical scale is <τ> and the horizontal scale
is the total number of hours since the capsule first filled with liquid helium.
Each point represents the average of ten to fifteen rupture observations.

In view of this time dependence of <τ>, the problem of finding a
tensile strength for liquid helium through the relationship \( P = P_0(1-e^{-\nu/<\tau>}) \)
is made more complicated even though the time dependence of <τ> does not
affect the pressure \( P_0 \); it only alters the shape of the \( P \) vs \( <\tau>^{-1} \) curves.
To obtain a \( P \) vs \( <\tau>^{-1} \) curve it is necessary to take several measurements
Figure 9

Sample Run
of $<\tau>$ at different pressures, all at the same time $t$. The time dependence of $<\tau>$ is so large, however, that only two curves of $<\tau>$ vs $t$ at constant $P$ can be obtained at the same time (see Figure 9). Thus, without extrapolating $<\tau>$ vs $t$ curves for long distances, it is possible only to obtain two such $P$ vs $<\tau>^{-1}$ points at any time $t$. Due to the uncertainty of the $<\tau>$ vs $t$ curves, such extrapolation is not feasible. Thus it is not possible to obtain the data necessary to determine whether such a $P_0$ actually exists.

C. Existence of time dependence

In Figure 10 samples of data are presented to illustrate the time dependence of $<\tau>$ for fixed pressures. Each point of Figure 10(a) represents about ten consecutive rupture observations, and each point of Figure 10(b) represents about sixteen consecutive observations. From this diagram it is seen that, within experimental error, $<\tau>$ is a linear function of time. A complete analysis of all the data shows that the time dependence may not be exactly linear, but that it is more nearly linear than exponential. However, the rate of change of $<\tau>$ is not always the same and can change even within a run (see Figure 9).

Sets of points at constant pressures taken between helium transfers form, again within experimental error, parallel lines. This is demonstrated in Figure 10 where the lines through the points for each run are drawn parallel. However, the lines through the points for run 36 are not parallel to those of run 42, although in this instance they differ only slightly.
Figure 10

Time Dependence of $\langle \tau \rangle$
RUN 36

O = P = 11.5 mm Hg
Δ P = 15.9 mm Hg

RUN 42

O = P = 12.0 mm Hg
Δ P = 20.8 mm Hg
D. Significance of the time dependence

The existence of the time dependence indicates that the rupture process is characterized by a non-stationary distribution of waiting times. Although this time dependence is an additional complication which must be sorted from the data, it provides a very important clue to the determination of the rupture mechanism. For example, the rupture process must depend upon some characteristic of the liquid sample. It is difficult to imagine an external source, such as cosmic radiation or mechanical vibrations, which changes in such a regular manner. The tensile strength must depend, rather, on mechanisms involving nucleating centers.

4.3 Time Dependence of Number of Nucleating Sites

It can easily be shown by a calculation identical to that used to determine molecular mean free paths that $<\tau>$ is inversely proportional to the number ($n$) of nucleation sites. Clearly, if in the time interval $t$ to $t + dt$ the probability of a rupture occurring is given by $g(P)n\cdot dt$, then $<\tau> = 1/g(P)n\alpha$ where $g(P)$ contains the pressure dependence and $\alpha$ is a constant factor. Thus the shape of the $<\tau>$ vs time (since the capsule was first filled) curves should give information on the manner in which the number of nucleation sites decreases.
A. Decay of sites proportional to the number of ruptures

There are two reasons for concluding that emptying or filling the capsule does not reduce the number of nucleating sites. The first is that, in graphs of $<\tau>$ vs the number of ruptures, some of the resulting curves at constant pressure will cross one another if they are extrapolated a short distance. Since these curves clearly cannot intersect, then either the relationship between $<\tau>$ and the number of ruptures is very strange or $<\tau>$ is not directly related to the number of ruptures.

The second reason is that $<\tau>$ continues to increase even when the capsule is quiescent (Figure 11). In run 41 the time dependence was too small to be reliably extracted from the statistical noise, but run 36 shows clearly that the decay of nucleating centers continues as a function of time. Thus the sites must disappear either spontaneously or by action with the container walls. The possibility that rupture does act as a small source of nucleating sites will be considered later.

B. $\frac{dn}{dt} \propto n^2$

The experimental data (Figure 10) indicate that the $<\tau>$ vs $t$ curves for constant $P$ are parallel straight lines. Since $<\tau> \propto n^{-1}$ it follows that the number of sites must decrease at a rate given by $\frac{dn}{dt} = -An^2$, where $A$ is a positive constant. This is consistent with the idea of sites decaying by collisions with each other. Thus the number of sites decreases according to
Figure 11

$\langle \tau \rangle$ vs Time - $\langle \tau \rangle$ Independent of the Number of Measurements
RUN 36

\[ P = 25.7 \text{ mm Hg} \]

RUN 41

\[ P = 20.0 \text{ mm Hg} \]
\(1/n = At + 1/n_0\), where \(n_0\) is the number of sites at \(t = 0\). Within this framework the pressure dependence can be of two types.

The first possibility is that \(\langle \tau \rangle = 1/g(P)n_0\), which says that the number of sites is pressure independent but that the probability of a site causing rupture is proportional to some function \(g(P)\) of \(P\). Thus

\[
\langle \tau \rangle = \frac{At}{g(P)\alpha} + \frac{1}{g(P)\alpha n_0}.
\] (10)

The family of curves arising from this equation is shown in part (a) of Figure 12.

For each pressure, \(\langle \tau \rangle\) vs \(t\) is linear with the slope of the line a function of the pressure. All the lines have a common origin at \(\langle \tau \rangle = 0\) and \(t_0 = -1/A\alpha n_0\), which is always negative. The experimental results shown in Figure 9 are not compatible with these features.

The other possibility is that a change in the pressure changes the number of nucleation sites. This is possible if at each pressure there are sites which are unaccessible to the rupture process. Increasing the pressure then increases the number of accessible sites.

Assuming that the pressure can only change the number of sites, then

\[
\langle \tau \rangle = At + \frac{1}{n_0(P)\alpha}
\] (11)

where \(A\) is a positive constant and \(n_0(P)\) is the number of sites at time \(t = 0\) for the pressure \(P\). This family of curves, as shown in part (b) of Figure 12, consists of parallel straight lines which all cross the \(\langle \tau \rangle\) axis at positive
Figure 12

$\langle \tau \rangle$ vs Time - $dn/dt \propto n^2$
values. The line which begins at the origin corresponds to the limit $n_0(P) = \infty$. Although these lines are parallel and straight, they do not approach the positive $t$ axis as the experimental results indicate.

C. $\frac{dn}{dt} \propto n$

If the number of possible nucleation sites is reduced by spontaneous annihilation or by collisions with the walls of the chamber, for example, then the rate of change of the sites is given by $\frac{dn}{dt} = -An$, where $A$ is a positive constant. Thus $n = n_0 e^{-At}$, where $n_0$ is the number of sites at $t = 0$.

It then follows that:

$$<\tau> = \frac{De^{At}}{g(P)}$$

(12)

where $g(P)$ contains the pressure dependence and $D$ is a constant. Simple differentiation of $<\tau>$ yields:

$$\frac{d<\tau>}{dt} = A<\tau>$$

(13)

which says that the slope of a curve depends only on $<\tau>$ and not on $P$. This family of curves is shown by Figure 13.

The fact that $d<\tau>/dt$ depends on $<\tau>$ and not on $P$ suggests that decay processes of this kind are worth investigating, even though the experimental results fit straight lines better than they fit exponentials. There also is no provision for different runs to show a different slope in the $<\tau>$ vs $t$ results.
Figure 13

$<\tau> \text{ vs Time} - \frac{dn}{dt} \propto n$
4.4 Modification of Exponential Decay

The discussion of Section 4.3C assumes that there are initially $n_0$ sites which then decay with a rate proportional to the number of sites $n$. In this section the influence of a small source of sites on the $<\tau> vs t$ dependence will be investigated. Two general types of source will be considered.

A. Constant source

If the nucleation sites are produced at a constant rate $c$, for example, by cosmic radiation, the number of sites changes at a rate $\frac{dn}{dt} = -An + c$. Thus $n$ is given by:

$$n = (n_0 - \frac{c}{A})e^{-At} + \frac{c}{A}.$$  \hspace{1cm} (14)

Since the source is considered as a perturbation to the number of sites, it is assumed that $n_0 >> c/A$ and $<\tau>$ is approximated by:

$$<\tau> = \frac{1}{\alpha g(P)[n_0e^{-At} + (c/A)]}.$$  \hspace{1cm} (15)

For $n_0e^{-At} >> c/A$, the source has little influence on the $<\tau> vs t$ curves. However, when $n_0e^{-At}$ and $c/A$ become comparable in magnitude, the measurements of $<\tau>$ become successingly lower than would be expected if the source were not present. As $c/A$ begins to dominate, the $<\tau> vs t$ curve approaches a constant value for $<\tau>$, and since $<\tau> = 1/\alpha g(P)n$ the limit that $<\tau>$ approaches is a function of the pressure. The presence of the
c/A term also causes the slopes at given \(<\tau>\) to decrease as the pressure increases, as can be seen in Figure 14.

Even though the experimental measurements are limited to values of less than about 1000 seconds, it is clear that the family of curves shown in the diagram is not compatible with the experimental data. The \(<\tau>\) vs \(t\) curves obtained from the experiments do not have slopes dependent upon the pressure and \(<\tau>\) shows no sign of ever approaching a limiting value.

B. Source as a function of the number of ruptures

If equal numbers of new nucleation sites are produced each time the capsule empties or fills, then the rate of production of sites is roughly proportional to \(<\tau>^{-1}\). The time required to fill the capsule and to cool it again will be called \(\delta\), which is about two or three minutes. Thus the rate of change of the number of nucleation sites is approximately given by

\[
\frac{dn}{dt} = -An + \frac{c}{\delta + <\tau>}
\]  

(16)

where \(A\) and \(c\) are positive constants. Since \(<\tau> \propto 1/n\), the solution to this equation is quite complicated and, since it is transcendental, cannot be put into a simple form. Thus the treatment of the problem will be qualitative instead of quantitative.

The following discussion considers the effect of the source of nucleating sites on the \(<\tau>\) vs \(t\) dependence at one pressure. The sites will be divided into two categories: (1) the number \(n_1\) of sites left from those
Figure 14

$\langle \tau \rangle$ vs Time - Small Constant Source of Sites
present at the beginning of the measurements, and (2) the number \( n_2 \) which are produced when the capsule empties and fills. Since \( <\tau> \) increases with time and \( \frac{dn_2}{dt} = -An_2 + c/\delta + <\tau> \), the dependence of \( n_2 \) on time is shown in part (a) of Figure 15.

The total number of nucleating sites is illustrated by the solid curve in part (b) of the diagram. The dashed line represents the number of sites \( n_1 \) that would exist if there was no source. Clearly \( n \) must be a monotonically decreasing function of time. Finally, the solid curve in Figure 16 displays the shape of the \( <\tau> \) vs \( t \) dependence which results from this total number of sites, whereas the dashed curve represents the dependence if the source were not present. Thus for \( <\tau> \) greater than some \( <\tau>_1 \), the time dependence can be expected to be more nearly linear than a pure exponential. For \( <\tau> \) less than \( <\tau>_1 \), however, the curve bends rather sharply towards \( <\tau>_0 \).

The above discussion is designed to illustrate only a possible change in the \( <\tau> \) vs \( t \) dependence. There are several factors which can additionally affect the results. As the number of sites decreases, the source must have an increasing effect on \( <\tau> \). The shape of the curve also depends upon the initial value of \( <\tau> \) since starting at large \( <\tau> \) will have little influence on the measurements. Also, since the actual measurements are occurring statistically in time, the detailed effect of the source can be quite complicated. However, the general features of the above result must be maintained.
Figure 15

\[ N \text{ vs Time - Source of Sites } \propto (\delta + \langle r \rangle)^{-1} \]
Figure 16

$<\tau>$ vs Time - Source of Sites $\propto (\delta + <\tau>)^{-1}$
4.5 Other Experimental Results

A. \( \tau \) distribution

Because of the time dependence of the waiting times, it is not possible to obtain their probability distribution directly. However, since this time dependence is approximately linear, it is possible to scale the waiting times so as to compensate for the time effect.

This was done for two of the curves exhibited in Figure 10. In each case the data were scaled to an average waiting time of 100 seconds. The actual waiting time \( \tau \) for each data point, of which there were 10-15 for each of the points in Figure 9, was multiplied by \( 100/\tau' \) where \( \tau' \) is the expected waiting time for that point as determined by the straight line fit.

The histograms in Figure 17 illustrate the distributions of the waiting times. The height of each block is the number of rupture observations which occurred in that \(<\tau>/2\) interval. The solid lines represent the experimental data, while the dashed lines give the distribution expected if the rupture process is due to events occurring randomly in time but with an average waiting time equal to 100 seconds.

Even though both histograms demonstrate that the rupture process is random in time, it is not possible to obtain this result for all the curves. For example, the \(<\tau> vs t\) curve for \( P = 12 \) mm Hg of run 42, as seen in Figure 10, is an exception. For this set of points the best straight line fit extrapolates to \(<\tau> = 0\) at a time which occurs after the first data points were
Figure 17

Normalized Distributions of Waiting Times
taken. The curve through the points must bend sharply toward the origin at small values of $<\tau>$. This is in agreement with the discussion in Section 4.4B of this chapter.

B. $\alpha$ source

A natural $\alpha$ emitter was inserted into the capsule to test the hypothesis that rupture of the liquid is initiated by cosmic radiation interactions. The $\alpha$ source was thorianite (Th-U oxide) and emitted three particles per ten seconds. The energy range of the particles was 0-7 MeV with a broad peak at about 3.5 MeV. The range of an $\alpha$ particle in liquid helium is only about 0.3 mm, so that any effect it may have must occur in a rather small region.

The results obtained with the $\alpha$ emitter in the capsule are embodied in Figure 18, where $<\tau>^{-1}$ is plotted as a function of pressure. The measurements were taken on several different days and with the use of the smaller dewar set. Thus there were several helium transfers during each run.

It is immediately apparent that the presence of the $\alpha$ source does influence the experimental results. The $<\tau>^{-1}$ vs P curve is shifted, when compared to Figure 8, to much lower pressures and is considerably more steep. Close examination of the data from these experiments does not reveal any time dependence of $<\tau>$. The reproducibility of the data also is in agreement with this conclusion. It cannot be determined, however, whether there is a non-zero threshold pressure, although the results follow very well a $<\tau>^{-1} = cP^8$ dependence.
Figure 18

\[ P_{rup} \text{ vs } \tau^{-1} \text{ with } \alpha \text{ Emitter} \]
\[ \langle t^{-1} \rangle \times 10^2 \text{ (SEC}^{-1}) \]

\[ \langle r^{-1} \rangle = (1.2 \times 10^6)p^8 \]
C. Small capsule

To further test the hypothesis that the rupture of the liquid is initiated by cosmic radiation, the volume of the capsule was reduced from 1 cm$^3$ to about 0.05 cm$^3$. The small capsule was cylindrical with a diameter of about 3 mm and with a length of about 5 mm. The Vycor glass superleak was the same one used in runs 36 and 41.

Since both the volume and the surface area of the capsule have been considerably reduced, it is expected that, for a given pressure, the waiting times should be much larger. The increase in waiting time should be at least in proportion to the ratio of the surface areas, which means an increase by a factor of about 8. Figure 10 compares the results obtained from run 36, with a capsule of volume 1 cm$^3$, to the results from run 42, with the capsule of volume 0.05 cm$^3$. The differences between the two runs are not of a kind that can be accounted for by the difference in capsule size.

Thus it must be concluded from this, and from other data, that, even though an $\alpha$ emitter does influence the rupture process, cosmic radiation cannot be the primary cause of rupture. The $\alpha$ particles either initiate rupture by a mechanism different from that for a normal rupture or the particles somehow create nucleation sites which are the same as those for normal rupture.

D. Clean capsule

An experiment with a 1 cm$^3$ capsule was performed in an attempt to determine whether impurities in the liquid were affecting the results. To
ensure a drastic diminution of impurities, the capsule was placed in a vacuum chamber and pumped to a pressure of less than $10^{-3}$ mm Hg. The capsule was then flushed twice with helium vapor from a storage dewar, and then sealed while in this helium atmosphere by means of an indium metal O-ring. Table 5 shows the change in the estimated impurity concentration and the amount of solid each impurity can form. There are other less abundant impurities not included in the table.

**TABLE 5**

**CAPSULE IMPURITIES**

<table>
<thead>
<tr>
<th>Substance</th>
<th>Capsule filled with air</th>
<th>Capsule evacuated and filled with He$^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of molecules</td>
<td>Volume of Solid (cc)</td>
</tr>
<tr>
<td>N$_2$</td>
<td>$2 \times 10^{19}$</td>
<td>$8 \times 10^{-4}$</td>
</tr>
<tr>
<td>O$_2$</td>
<td>$5 \times 10^{18}$</td>
<td>$3 \times 10^{-4}$</td>
</tr>
<tr>
<td>A</td>
<td>$2 \times 10^{17}$</td>
<td>$4 \times 10^{-6}$</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>$8 \times 10^{16}$</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>$8 \times 10^{15}$</td>
<td>$4 \times 10^{-7}$</td>
</tr>
<tr>
<td>Ne</td>
<td>$5 \times 10^{14}$</td>
<td>$2 \times 10^{-8}$</td>
</tr>
<tr>
<td>H$_2$</td>
<td>$1 \times 10^{13}$</td>
<td>$6 \times 10^{-10}$</td>
</tr>
<tr>
<td>He$^3$</td>
<td>$1 \times 10^{7}$</td>
<td></td>
</tr>
</tbody>
</table>
The results of this experiment, which was run 41, have been shown in Figure 9. There is no appreciable difference between the results of this run and any other run, and it must be concluded that impurity particles from air contamination of the capsule do not affect the measurements.

E. Memory effects

There are two curious effects which have been observed that must be explainable by any mechanism postulated for the rupture process. During the course of an experimental run there were usually two time periods of about 12 hours duration when the capsule is left at 4.2°K. Upon the resumption of measurements, the obtainable pressures and waiting times were found to depend upon the state of the capsule during the period at 4.2°K.

If the capsule is warmed to 4.2°K in such a manner that it is full of liquid helium, then the system behaves as if, during the period at 4.2°K, nucleation sites are regenerated, i.e., the \( \langle \tau \rangle \) vs \( t \) curves (at constant \( P \)) are shifted towards smaller \( P \) when the measurements are resumed. This effect is documented by runs 36 and 41 in Figure 19.

Before the break at 4.2°K in run 36 the measurements at \( P = 16 \) mm Hg yielded a \( \langle \tau \rangle \) greater than 500 seconds, which could be reduced to 225 seconds by increasing the pressure to \( P = 26 \) mm Hg. Upon resumption of measurements \( \langle \tau \rangle \) had gone back to 120 seconds at \( P = 16 \) mm Hg as late as two hours after the \( \lambda \) temperature was again reached. Run 41 shows a similar result in that before the break a \( \langle \tau \rangle \) vs time curve at \( P = 27.5 \)
Figure 19

$\langle \tau \rangle$ vs Time – Effect of Full Capsule at 4.2°C
RUN 36

(a)

O $P = 16$ mm Hg
$\Delta P = 26$ mm Hg

12 hours at 4.2°K

RUN 41

(b)

O $P = 27.5$ mm Hg
$\Delta P = 20.0$ mm Hg

12 hours at 4.2°K
mm Hg was being obtained. After the period at 4.2°K the liquid always ruptured before the pressure \( P = 27.5 \) mm Hg could be attained. Two hours after the \( \lambda \) point was reached, the pressure \( P = 20 \) mm Hg gave \( \langle \tau \rangle = 90 \) seconds.

If, however, the capsule is warmed to 4.2°K such that the capsule is empty, then the results obtained after the warm period are consistent with the maintenance of the number of nucleation sites. Within experimental error the \( \langle \tau \rangle \) vs time curves for the same pressures begin again at the upper value of \( \langle \tau \rangle \) attained immediately before the break. This effect is illustrated in Figure 20.

A third effect which concerns a memory during a quiescent period has been mentioned in Section 4.3A. If the capsule is left full while its temperature is below the \( \lambda \) point for an extended period of time during which no measurements are taken, then upon resumption of measurement it is found that \( \langle \tau \rangle \) is larger than before the quiescent period (see Figure 11). The increase of \( \langle \tau \rangle \) is consistent with the number of nucleating sites continuing to decay at the same rate as the decay of sites during periods when measurements are being taken.
Figure 20

$<\tau>$ vs Time - Effect of Empty Capsule at 4.2°K
RUN 36

\[ P = 20 \text{ mm Hg} \]

RUN 41

\[ P = 9 \text{ mm Hg} \]
CHAPTER V

DISCUSSION OF MECHANISMS

5.1 Characteristics of the Rupture Mechanism

It is clear from the experimental results that liquid helium does not have a well-defined tensile strength. Rupture of the liquid is due to events which occur randomly in time, but which also decrease in frequency as a function of time. The rupture process is pressure dependent and there is an approximately linear dependence of $\langle \tau \rangle$ on time. These are the main features that any mechanism postulated for the rupture process must contain.

The discussion of the previous chapter shows that these features of the data can be described by a mechanism containing nucleating sites as the cause of rupture, especially when the act of rupture is considered as a small source of nucleating sites. In addition to describing these gross characteristics of the data, the mechanism must also describe how enough energy can be localized to form a bubble of the critical size. The discussion of this chapter will consider the problem of finding a mechanism which has all of these features.
5.2 Localized Energy Fluctuations

It has been shown by Fisher (Section 1.3C) that the rate (number per mole) at which bubbles of the critical radius are formed is

$$\frac{dn}{dt} = \frac{NkT}{h} \exp \left[ -\left( \frac{\Delta f_0}{kT} + \frac{16\pi^3}{3(P_i - P_v)^2kT} \right) \right]. \quad (17)$$

With the pressure in the capsule at -0.013 atm, the energy required to form such a bubble is about $3 \times 10^{-9}$ ergs and the bubble radius is 5000 Å. Thus, since $\Delta f_0 \sim 0$ for liquid helium, the frequency at which localized energy fluctuations as large as $3 \times 10^{-9}$ ergs occur in a spherical region of 5000 Å radius is $\sim 10^{10}$ per second. Obviously, mechanisms which are dependent upon energy fluctuations of this type cannot describe the experimental results.

5.3 Nuclear Radiation

A. Cosmic rays

Since cosmic rays are energetic and appear randomly in time, their interaction with the liquid must be considered as a possible rupture mechanism. This mechanism, although proposed by both Beams and Finch, was never tested for liquid helium. The necessary localized energy may come from: (1) the interaction of secondary particles, electrons or α particles, with the liquid; (2) the interaction of a cosmic ray itself with the liquid; or (3) the collision of a cosmic ray or secondary particle with impurity atoms. Although it was found (Section 4.5B) that the addition of a weak α particle emitter did influence the tensile strength measurements, the
following discussion should verify that cosmic radiation is not responsible for the rupture process in these experiments.

It is expected that reducing the size of the capsule should increase the average waiting times at each pressure. If the interaction occurs near the capsule walls, then $\langle \tau \rangle$ should increase inversely as the ratio of the surface areas; and interactions occurring uniformly through the liquid should increase $\langle \tau \rangle$ inversely as the ratio of the volumes. It was found (Section 4.5C) that decreasing the volume by a factor of 20 (and the surface area by a factor of 8) had no appreciable effect on the results.

It was also discovered that the cosmic ray flux is too small to account for the short waiting times which were observed. Measurements of the flux (40) indicate a rate of 1 or 2 particles per minute through the $1 \text{ cm}^3$ capsule and about 1 particle per 10 minutes for the $0.05 \text{ cm}^3$ capsule. Several succeeding waiting times of less than 10 seconds have been observed, and could always be obtained by increasing the pressure, for both capsules.

The only cosmic ray mechanism that can yield the observed time dependence is the collision of cosmic rays with impurity particles which are numerically decreasing in an approximately exponential manner. It will be shown in a later section that, although there are many such particles (see Section 4.5D), such a decrease is unlikely.
B. Neutrons

Since the room where the cryostat is now located also contains some shielded neutron emitters, the interaction of the resulting small neutron flux with the liquid must be considered. The arguments presented against cosmic ray interactions are also valid for neutron interactions, with the exception that the neutron flux may be as large as 1 particle per 10 seconds through the 1 cm$^3$ capsule. As further evidence, the experimental results did not change appreciably when the cryostat was moved from a room far from the neutron emitters to its present site, whereas the change should have been very large if the neutrons are responsible for the rupture process, since the neutron flux is very different.

The collision cross-sections between neutrons and the liquid are much too small to account for the experimental results. Since the cross-section between neutrons and He$^4$ atoms is 0.8 barns (41), the total cross-section is 0.02 cm$^2$ for the 1 cm$^3$ capsule, and 0.001 cm$^2$ for the 0.05 cm$^3$ capsule. Thus the collision rates between neutrons and He$^4$ atoms are about 1 collision per 20 minutes and 1 collision per 7 hours respectively. The cross-sections of neutron-oxygen and neutron-nitrogen collisions are less than 10 barns (41) and, since the number of such atoms is much less than the number of He$^4$ atoms, these collisions are unimportant. Although the n-He$^3$ cross-section is very large (5437 barns) (41), there must be $\sim 2 \times 10^{19}$ He$^3$ atoms in a 1 cm$^3$ capsule to yield 1 collision per 10 minutes. However, there can be at most
such atoms in the capsule (Table 4, Section 4.5D), and the interaction can also be ignored.

5.4 Impurities

Impurity particles in liquid helium can be of two types. All liquid helium contains some He\(^3\) atoms in solution with a natural abundance of about 1 part in 10\(^8\) (17). This concentration varies slightly according to the source of the helium. Impurities such as air can exist as small colloidal particles which will remain in suspension until they migrate to the container walls.

Even though there are copious quantities of impurities in the capsule (Table 4, Section 4.5D), it is not clear how they can influence the rupture process. Both positive and negative ions, with the exception of a free electron, attract helium atoms and thus cannot even form bubbles in the liquid, especially since helium adheres to all known surfaces. He\(^3\) atoms, especially in the small concentrations of these experiments, cannot by themselves influence the rupture process. The only way enough energy can be localized at an impurity site is through the interaction of energetic nuclear particles with an impurity atom; and this has been eliminated as a rupture mechanism.

Even if a mechanism involving impurity particles could be found, the observed time dependence of \(<\tau>\) has to be explained. If the time dependence is due to the disappearance of impurities from the system, then the tensile strength measurements should have changed considerably after the capsule was
evacuated and filled with clean helium gas. However, no significant change was noticed (Section 4.5D).

From the shape of the time dependence of \( \langle \tau \rangle \), it has been inferred (Sections 4.4, 4.3) that the nucleating sites must disappear in an almost exponential manner, and that they decrease their number neither by coalescence nor by escape through the superleak when the capsule empties. Since the impurities are particles, spontaneous annihilation must be ruled out, which leaves only diffusion to the walls as the means by which they leave the system. But if the particles are removed from consideration by interaction with the walls, then emptying the capsule would rid the system of all the impurities. It is thus difficult to reconcile mechanisms involving impurity particles with the experimental results.

5.5 Ions

It was seen in Chapter I that there are two ways that ions can serve as a mechanism for the rupture of liquid helium. The first comes from simple recombination of a free electron with a \( \text{He}^+ \) ion. The energy released is only about \( 4 \times 10^{-11} \) ergs, whereas \( 3 \times 10^{-9} \) ergs are required to form a bubble of the critical radius. Since the ions are formed by the interaction of cosmic radiation with the liquid, the number of such ions must be essentially constant. Thus this mechanism cannot explain the time dependence of \( \langle \tau \rangle \).

The second mechanism consists of the blow-up of the pre-existing holes which form around free electrons. Although the detailed structure of these
holes is not well understood, it is known that they have a diameter of about
25Å (see Section 1.4A). The pressure necessary to cause this bubble to
rupture the liquid is $P = \frac{2\sigma}{\tau} = 2.5$ atm. This mechanism also is unable to
account for the time dependence of $<\tau>$ and it must be concluded that ions are
not responsible for the measured tensile strength.

5.6 Preliminary Summary

Table 6 summarizes those features which the mechanisms considered
so far are unable to describe. It is seen that all of these mechanisms can be
criticized on rather serious grounds and can be eliminated from further
consideration. Not included in Table 6 are the mechanisms involving energy
release due to the annihilation of a vortex structure. Because of their
importance they will be considered in the next two sections of this chapter.

5.7 Vortex Lines

It was suggested earlier (Section 1.4B) that rupture in liquid He II
could be initiated by the annihilation of an $n = 1$ vortex line (which has energy
per length $\sim 1 \times 10^{-7}$ ergs/cm) if a length 300μ or more is convoluted into a
spherical region of 1μ diameter. This process yields sufficient localized
energy to form a bubble of the critical radius at a stress of 0.014 atm. It
then remains to be shown that the experimental results can be explained by
this mechanism.
<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van der Waals' process</td>
<td>1. Predicts a tensile strength much too large.</td>
</tr>
<tr>
<td></td>
<td>2. There is no provision for a time dependence.</td>
</tr>
<tr>
<td></td>
<td>3. The rupture should not occur statistically in time.</td>
</tr>
<tr>
<td>Nucleation theory</td>
<td>1. The localized energy fluctuations are too small.</td>
</tr>
<tr>
<td></td>
<td>2. There is no provision for a time dependence of $&lt;\tau&gt;$.</td>
</tr>
<tr>
<td>Cosmic radiation</td>
<td>1. The cosmic ray flux is too small.</td>
</tr>
<tr>
<td></td>
<td>2. The time dependence of $&lt;\tau&gt;$ can only come from a time dependence of the number of impurity atoms—but this time dependence cannot be of the proper form.</td>
</tr>
<tr>
<td>Neutrons</td>
<td>1. Collision cross-sections are too small.</td>
</tr>
<tr>
<td></td>
<td>2. The time dependence of $&lt;\tau&gt;$ can only come from a time dependence of the number of impurity atoms.</td>
</tr>
<tr>
<td>Ions</td>
<td>1. The blow-up of the hole around a free electron requires a stress which is too large.</td>
</tr>
<tr>
<td></td>
<td>2. The recombination energy is too small.</td>
</tr>
<tr>
<td></td>
<td>3. The number of ions is essentially constant—no time dependence of $&lt;\tau&gt;$.</td>
</tr>
<tr>
<td>The blow-up of a vortex core (Section 1.4B)</td>
<td>1. Predicts a tensile strength much too large.</td>
</tr>
</tbody>
</table>
To account for the time dependence of $<\tau>$, it is first necessary that there initially be a large number of vortex lines in the capsule and that these vortices be sufficiently long that such a convolution is probable. In Appendix B it is calculated that, when the capsule fills the first time, there are $\sim 10^{10}$ vortex lines created and each has a length of about 200 cm. Thus there certainly are enough vortex lines and their lengths are sufficient to permit convolution caused by the interactions between them.

The time dependence of $<\tau>$ then follows from the spontaneous decay of the number of vortex lines. The perturbation to this decay (see Section 4.4B) comes from a small increase in the lengths of some of the vortex lines each time the capsule is filled. Since the pores in the Vycor glass are not uniform and since the critical velocity is dependent upon the pore diameter, the critical velocity is probably exceeded in many of the pores even though care was taken that the capsule fill at less than the critical rate. Exceeding the critical velocity then manifests itself as an increase either in the number of vortex lines or as an increase in the lengths of existing lines.

The observed distribution of waiting times is due to the probability that a vortex line will have a region which is suitably convoluted at the instant of annihilation. This probability will be random in time with $<\tau>$ dependent upon the total length of line existing in the capsule.

It has been assumed throughout this discussion that $n = 1$ vortex lines are unstable and that they annihilate spontaneously. The stability of vortex lines has not been unambiguously determined, although Careri (42) has
measured a half-life of about ten minutes for vortex lines rotating in a cylindrical annulus at 0.8°K. The annihilation cannot occur by interaction with the container walls since emptying the capsule would then greatly enhance this annihilation. The decay of vortex lines also cannot come from the mutual annihilation of two vortex lines with opposite circulation, since this would not yield the proper time dependence.

Thus the main features of the experimental data can be produced if \( n = 1 \) vortex lines are sufficiently convoluted and if these vortex lines annihilate spontaneously with a measured half life of 2 or 3 hours. The experiments with the \( \alpha \) source, however, must then be taken as representing an entirely different rupture process. Since positive ions exceeding a critical velocity of about 5 m/sec (30) are known to produce vortex rings (43,44), a mechanism involving annihilation of vortex rings will be considered in the next section.

5.8 Vortex Rings

The rupture mechanism arising from the annihilation of vortex rings, instead of vortex lines, contains many of the features concerning the production of vortex lines in the capsule. It is again necessary that a large number of rather long vortex lines be created when the capsule is initially filled (see Appendix A) and that, each time the capsule is filled thereafter, an increase in either the number of vortex lines or in the lengths of existing vortex lines is experienced.
The vortex rings are then continually created from the vortex lines in the capsule. The production of these rings occurs when a vortex line becomes convoluted in such a manner that it becomes kinked and a closed loop results. This loop can then break away from the line in the form of a vortex ring (45). Thus the rate of production of these rings depends upon the total length of vortex line in excess of the shortest possible length, and each time a ring is formed this length decreases by the length necessary to form the ring.

Since a vortex ring of 1μ diameter, which is the critical bubble diameter for these experiments, has an energy of only about 3 x 10^{-11} ergs, enough energy can be localized to cause rupture only if the vortex ring is sufficiently convoluted. Since it is already necessary to assume that the interactions between vortex lines cause the lines to become convoluted, these same interactions will cause the vortex rings to also be convoluted. Vortex rings have a fairly large velocity (31) (> 1 cm/sec for R < 10μ). The annihilation should thus occur by interaction with the container walls and the bubbles should always form there. This is consistent with the observations of Edwards et al. (35). It should be mentioned that vortex lines are hydrodynamically stable at surfaces, whereas vortex rings are not.

The observed approximately exponential time dependence of <τ> is then due to the decrease in the rate of production of vortex rings as the total length of the vortex lines in the capsule decreases, either by spontaneous annihilation or by the production of the vortex rings themselves. Similarly
the observed distribution of waiting times comes from the probability that a vortex ring will be suitably convoluted at the instant of annihilation.

This rupture mechanism, although more complicated, contains all the features of the mechanism involving the annihilation of convoluted vortex lines. There are, however, advantages that this mechanism contains. First, it is not necessary that the vortex lines be unstable, although they may be. The time dependence of $\langle \tau \rangle$ can arise from the shortening of the vortex lines as vortex rings are produced.

More important, however, the results of the experiments with the $\alpha$ emitter (Section 4.5B) indicate that the rupture process can be attributed to the annihilation of vortex rings. The $\alpha$ particles in these experiments must produce vortex rings in such quantity as to predominate over the production of rings from vortex lines, which explains the lack of a time dependence of $\langle \tau \rangle$ and the relatively low rupture pressures.

5.9 Summary

Rupture mechanisms involving the convolution of vortex lines or vortex rings are the only ones which can produce the main features of the experimental data and also contain enough localized energy to cause rupture of the liquid. Of these two theories only the one concerning the annihilation of vortex rings can account for the experimental results obtained with the $\alpha$ source. It is concluded therefore that the rupture in liquid He II is caused by this annihilation of vortex rings.
There are, however, experimental data which seem to contradict any process involving vortices. It is not clear, for example, how the curious results obtained after the system was at 4.2°K (Section 4.5E) are consistent with this scheme. Similarly, it was found that momentarily warming the capsule above the $\chi$ temperature had no noticeable effect on the measurements.
APPENDIX A

T(t) AS THE CAPSULE EMPTIES

The liquid in the capsule will initially be considered to have a volume \( V_0 \), a temperature \( T_0 \), and a specific entropy \( s_0 \). When the liquid ruptures, superfluid helium, having zero entropy, flows from the capsule at a constant rate \( V_0/t' \), where \( t' \) is the time required for the capsule to empty. Thus as the volume \( V \) of the liquid decreases, the specific entropy \( s \) increases. This produces an effective heat input:

\[
\dot{q}_1 = \frac{TV(s - s_0)}{t},
\]

(18)

or

\[
\dot{q}_1 = \frac{T s_0 V_0}{t'}. \tag{19}
\]

Since \( t' \) is normally about 50 seconds and \( V_0 \sim 1 \text{ cm}^3 \), \( s_0 = 9.4 \times 10^3 \text{ ergs/°Kcm}^3 \) and \( T_0 = 0.85°\text{K} \), then \( \dot{q}_1 = 150 \times 10^{-6} \) (watts).

The dominant heat conduction through the thermal connection to the \( \text{He}^3 \) reservoir is through distillation of the liquid helium from the top of the capsule to the reservoir. The rate at which this distillation process can conduct heat is given by

\[
\dot{q}_2 = \dot{L}h = \frac{PVL}{RT}, \tag{20}
\]

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where $L$ is the latent heat, $P$ is the pressure, and $\dot{V}$ is the volume rate of flow of the vapor. Since $P = 2\mu$ at $0.85^\circ K$ and the diameter of the stainless steel tube of the thermal connection is about $3$ mm, the gaseous flow through the tube is almost molecular. Thus the conductance $C$ of the tube, defined by $C = \frac{PV}{\Delta P}$, is given (46) by:

$$C = 3.6KA \left( \frac{T}{M} \right)^{1/2} \text{ l/sec},$$

where $A$ is the cross-sectional area of the tube in cm$^2$ and $K$ is a factor which is a function of the ratio of the length of the tube to its radius. Since the tube is 4 cm long, then $C \approx 170$ cm$^3$/sec. Using the expansion $\Delta P = (dP/dT)\Delta T$, the resulting $q_2 = (6.9 \times 10^{-2})\Delta T$ (watts/°K) is obtained.

Equating $q_2$ to $q_1$ results in $\Delta T = 2.2 \times 10^{-3}$°K, which is about half as large as the observed $\Delta T$. An error of a factor of 1/2 is not surprising since the conditions for molecular flow are not quite satisfied. The shape of the emptying curve of Figure 5 can now be explained in the following manner.

The bulk of the constant heat influx $q_1 = 150 \times 10^{-6}$ (watts) due to the increase of specific entropy is carried away by heat conduction $q_2 = 6.9 \times 10^{-2}\Delta T$ (watts) through the thermal connection between the capsule and the He$^3$ reservoir. This produces a temperature rise $\Delta T = 2.2 \times 10^{-3}$°K.

The heat needed to increase the temperature of the liquid, with a heat capacity $C(J/g^\circ K)$, by $\Delta T$ is given by $\rho CV\Delta T$, where $\rho$ is the density (g/cm$^3$) and $V$ is the volume of the liquid. Thus the time required to increase the temperature by an amount $\Delta T$ is
(\dot{q}_1 - \dot{q}_2) t = \rho CV \Delta T. \quad (22)

As an upper limit the volume can be taken as 1 cm$^3$. Thus,

\[
t = \frac{(5 \times 10^{-3}) \Delta T}{(150 \times 10^{-6} - 6.9 \times 10^{-2} \Delta T)} \text{ seconds.} \tag{23}
\]

The time required to increase the temperature by $2.0 \times 10^{-3}$°K is then about 1 second, which is much less than the time required for the capsule to empty. The net result is that the top of the emptying curve will appear to be flat.
APPENDIX B

THE INITIAL NUMBER AND LENGTHS OF VORTEX LINES

Since the capsule and bath temperature are very different when the capsule is first filled, the helium flow into the capsule will take place at the critical rate and a large initial number of vortex lines will be formed. The following calculations will be based on the Vycor glass superleak, although similar results should be obtained from consideration of a superleak of packed jeweler's rouge.

The diameter of the Vycor glass superleak is about 1/16" and thus the area of the end of the superleak is about $2 \times 10^{-2} \text{ cm}^2$. Since the average channel diameter is $40\AA$ and the glass is about 30% void, there are of order $10^{10}$ channels opening into the capsule. Following a model by Feynman (45), it will be assumed that, when the liquid reaches the critical velocity, a vortex line will be spun out in a helical shape from each channel. Thus there will be $\sim 10^{10}$ vortex lines in the capsule as a result of the initial filling.

When the capsule begins to fill the bath is at 1.2°K and the capsule at about 1.8°K, the $\lambda$ transition temperature for Vycor glass. Thus there is a pressure drop across the superleak of about $\Delta P = 150 \text{ mm Hg}$. Since the critical velocity is about 25 cm/sec (47), the volume rate of flow is given by
\[ \dot{V} = v_c A = 2.5 \times 10^{-12} \text{ cm}^3/\text{sec} \text{ through each channel.} \] The volume of the capsule is \( \sim 1 \text{ cm}^3 \) which indicates that the capsule should fill in about 40 seconds. Thus the total energy available per channel is \( \Delta P \dot{V} t = 2 \times 10^{-5} \text{ ergs.} \) Assuming this all goes into creating a vortex line at each channel, the length of each vortex line, with energy per unit length \( \epsilon \sim 1 \times 10^{-7} \text{ ergs/cm,} \) is about 200 cm.
BIBLIOGRAPHY


23. B. Hahn, Lettere Alla Redazione, p. 4350.


