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A GRAPH THEORY INTERPRETATION OF
DISTRIBUTION CHANNEL STRUCTURE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the
Degree Doctor of Philosophy in the Graduate School of
The Ohio State University

By

Lynn Edward Gill, B.S., M.B.A.

* * * * * * *

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1968

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CHAPTER I

INTRODUCTION

The marketing, or distributive, system has been described as a network of firms and trading relations among firms.\(^1\) Graph theory has been successfully applied in studying aspects of network structure, for example, in studying electrical, transportation, and sociometric networks.\(^2\) It appears probable that graph theory might also have applicability in analyzing the structure of

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distributive networks. Graph theory offers concepts, methods, and theorems appropriate to the analysis of structure \(^3\) and thereby provides an analytic tool of promise for analysis of distribution channel structure. As Balderston has stated,

> Analytical tools of great power for some situations are becoming available to us. Unless some of us learn these tools and experiment with their empirical application, we shall be missing out on an important avenue of improvement both in the scientific-descriptive theory of marketing channel behavior and in the provision of normative rules for the firm to follow.\(^4\)

**Objective**

The objective of this dissertation is to determine whether distribution channel structure can be analyzed using graph theory concepts, such that assumptions and implications of theories about structure may be made explicit for examination, refinement, and possible empirical

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application. Because it is not possible, within dissertation time and energy constraints, to explore all possible applications of graph theory for analysis of all theories about distribution channel structure, it is necessary to limit the scope of the inquiry. A theory about distribution channel structure is selected for analysis, based upon the criteria that it must embody elements which could be receptive to graph theory analysis. Graph theory is based upon combinatorial and topological (geometric) principles. Therefore, as a necessary condition for a coordination of the elements of the distribution channel theory with the elements of graph theory, the selected theory must have combinatorial or topological attributes. Similarly, graph theory concepts which will be used for analysis must be limited. If an appropriate coordination can be made between the selected distribution channel structure concepts and the selected graph theory concepts, it may be possible to utilize the logical structure and precise terminology of graph theory for analysis of distribution channel structure.

For purposes of preliminary discussion, a distribution channel may be viewed as the sequence of marketing institutions that convey a product and its title from point of production to point of use or consumption. The concept
of structure in distribution channels assumes that the vertical arrangements and patterns of institutions that evolve in distribution networks are not random, but some semblance of order, or structure, can be determined. Structure denotes arrangement of the components of distribution channels into an integrated whole, or some totality that has been built up from component parts and arranged so as to function in some sensible, purposeful manner. Structure embodies physically identifiable elements, such as business entities, which have some role in the distribution process. None of these business entities is self-sufficient, so that structure must also be perceived to include the links or interactions which permit the flow of products and their titles through the system, from production to consumption.

The term structure, as applied to distribution

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8 Ibid.
channels, therefore assumes a nonrandom patterning of business entities and interactions among business entities. The entire structure may be viewed as a system, with outputs deriving from the marketing activities of member units and the relationships these activities have to one another and to the system as a whole. ⁹ Despite the use of structural concepts in the study of distribution channels, formal analysis of structure is relatively underdeveloped. The technical terminology employed in describing structures is meager, and few concepts are defined rigorously. As a consequence, analysis of structural properties tends to be couched in ambiguous terminology, and detailed studies of structure are rare. ¹⁰

The distributive network, which encompasses all distribution channels, may be viewed as a system of firms, and interactions, or trading relationships, between firms. If the complex of firms and interactions can be systematized, the amorphous mass of marketing phenomena reveals structure, which may be receptive to analysis with graph theory

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¹⁰ A notable exception is found in Baligh and Richartz.
concepts. In order to gain an understanding of the functioning of the distributive network as a total system, it is first necessary to break the whole down into manageable parts for analysis and study. In order to achieve the primary objective of this dissertation, then, it is necessary to develop a conceptual framework for decoupling the distributive network, in terms of distribution channels, which will allow graph theory analysis. To summarize, this inquiry depends upon:

1. Development of a conceptual framework and particular viewpoint through which the distributive network may be decoupled into comprehensive units for analysis, in terms of distribution channels.

2. Identification of a theory about distribution channel structure which has attributes which may be receptive to graph theory analysis.

3. Determination of particular concepts of graph theory which are developed sufficiently to allow fair probability of application.

4. Determination by means of conceptual analysis whether graph theory provides an effectual tool for analysis of distribution channel structure.

\[\text{Beckman and Davidson, loc. cit.}\]
Rationale

General

The distribution channel, as a unit of study, has received increased recent attention in the marketing literature. Although Professor Ralph Breyer was early to recognize the need for studying marketing in terms of marketing systems, instead of in terms of individual firms, development of a literature of distribution channels is largely a recent phenomenon.

There are two logical approaches to the study of distribution channels— that of studying the behavioral relationships of the members within the channel, and that of studying the structure and design of the channel. The behavioral approach assumes a given structure and is concerned with the economic, social, and political interactions between the channel members. The study of distribution channel structure is concerned with the vertical arrangement


14 Ibid.
and pattern of the channel, or the arrangement of the components into an integrated whole. The second approach is the one taken in this dissertation.

An understanding of distribution channel structure is important for the following reasons:

1. Performance of functions in a distribution channel is related to, and dependent upon, its structure.

2. Structure must be understood to locate the decision-making center, or centralizing organization, which is vital to understanding channel behavior.

3. Channel objectives are attainable only through the structural elements, i.e., through cooperation or interaction among the business entities in the structure.15

Analysis of structure involves an understanding of both the components and the interrelations between the components. Traditional marketing study has concentrated primarily on the components, or the marketing institutions, and little on the analysis of both the components and their interrelations.16 "The institutional parts of the channel, for example, wholesalers and retailers, have received considerable attention in the literature. However, the

15 Bell, op. cit., p. 79.
16 Ibid., p. 78.
marketing channel as such has not received the intensive study that it warrants."

Among the reasons that have been advanced to explain this paucity of channel analysis is the hypothesis that the complexity of distribution channels discourages investigators. "Channels, in a very real sense, are among the most complicated phenomena encountered in an advanced economy." Complexity, however, should not be sufficient to dissuade one from attempting to develop a conceptual tool for dealing with this complexity. An area of greatest need for study appears to be an adequate development of concepts, theorems, and methods appropriate to analysis of distribution channel structure. Breyer; Vaile, Grether and Cox; and

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17 Mallen, op. cit., p. vii.


19 Breyer (1949), op. cit.

and Revzan (among others), have developed analytical systems which, although helpful, do not cope completely with the complexity of empirical distribution channels. Interaction patterns are for the most part described verbally, not with analytical rigor and preciseness; and relationships between concepts of structure are not clearly indicated.

A new approach is required to deal effectively with the complexity of empirical, or extant, distribution channels. Graph theory, the mathematics concerned with the abstract notion of structure, has been recognized by several scholars as having potential for this purpose. The rationale for using graph theory for analysis of distribution channel structure is discussed in the following sections.

21 Revzan, op. cit., pp. 112-115.

Explication of Theories about Distribution Channel Structure

By interpreting the theories of distribution channel structure in a logically consistent manner, otherwise obscure assumptions or relationships may become explicit. Debate may then be focused on particular assumptions and relationships. Advantages of using a symbolic form, which also may accrue to an analysis of distribution channel structure, are:

First, the rigors of translating from verbal to symbolic form requires much clarification (and generalization) and frequently a great deal of fuzziness is rubbed off in the translation. This may result in immediate qualitative gains in understanding. Second, even within a single social science (and to a high degree among social sciences), the specialized sub-disciplines have jargons of their own that reduce the interchange of ideas. Reduction of all to a common language may reveal interrelations previously not known to any of them. Third, verbal theorizing is much more at the mercy of intuition than is mathematical analysis. Finally, mathematics is tremendously more efficient than prose theorizing. Verbal theory is pick and shovel; mathematics is steam shovel. The superiority in efficiency is so great that the difference ceases to become one of degree and becomes one of kind.23

Of the available systems of logical and symbolic form, graph theory appears to be especially applicable to the study of distribution channel structure, in that it is specifically developed to deal with structural concepts. Three principal benefits may be gained from employing graph theory in the study of structure, which might also accrue in the study of distribution channel structure.

First, the vocabulary for describing structure is enriched by useful new terms having precise meanings. The language of graph theory contains a large number of concepts which refer to complex structural properties. In addition to this, graph theory provides a common language with which to interpret diverse structural concepts, which may be couched in ambiguous verbal terminology, and not rigorously defined or defended. Propositions and theories about structure can be related to each other and examined to determine if they are consistent.

Second, graph theory and associated branches of mathematics provide techniques of computation and formulas for calculating certain quantitative features of empirical

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structures. Matrix algebra has a close relation to graph theory, and the manipulative advantages of matrix algebra are therefore implicit in graph theory. It is also possible to develop readily comprehensible summary indices which relate to structure; much as it is possible to develop summary statistics, such as the mean and variance, to describe large systems of numerical data. This is perhaps a most promising attribute, as traditional attempts to chart or "graph" distribution channels quickly become too complex for comprehension. 25

The third benefit of the use of graph theory stems from its theorems. The axioms of graph theory lead to an extensive body of logically derived statements. Provided a coordination can be made between the elements of structure and the elements of graph theory, each of these statements or theorems becomes a valid assertion about structural propositions that satisfy the axioms of graph theory. This offers a means to validate the internal consistency and logic of theories about distribution channel structure. Statements logically derived from propositions about

25 For example, see Vaile, Grether and Cox, op. cit., p. 133, Figure 10.
properties of structure must also have empirical meaning, or be true statements, in order for the original proposition to be valid.

Eventual Usefulness

When theories and assumptions are stated explicitly, they are susceptible to cumulative refinement. Through research efforts and debate, the theories and methods explored here may be sufficiently developed to provide a means for empirical study of distribution channels. This is discussed more completely in the section of the final chapter entitled, "Implications for Future Research."

Indirect Benefits

Interpreting theories of distribution channel structure may result in the following by-products. First, a careful analysis of existing theories within a logically consistent framework tends to clarify what is known and to identify what remains unknown. This can be likened to the development of the periodic chart of the elements, which led to discovery of several new elements by defining gaps in the existing theory. One scholar has described this aspect of theoretical analysis as "building islands of theory by
defining relationships between variables." In this manner new hypotheses may be stimulated which could contribute to development of a more comprehensive theory.

Second, the conceptual framework developed for the study of distribution channel structure, and insights provided by graph theory analysis, may provide useful tools for teaching. In recent years, distribution channels courses have appeared in the marketing curriculums at several universities. This study may provide material useful for teaching such a course.

Background

The Channel Dimension

It is not surprising that in the study of distribution channels—a dynamic area with an increasing literature—there is not a single, precise definition of the

26 Harold Guetzkow, "Influences of Intellectual Milieux in the Development of a Model; An Autobiographical Essay" (address presented at the Symposium on the Process of Model Building in the Behavioral Sciences, Ohio State University, April 20, 1967).

distribution channel. However, there are several attributes of distribution channels about which there is general agreement:

1. The distribution channel in its vertical dimension spans the economic gap between production and consumption.

2. The distribution channel transfers some aspect of a product or group of products across this gap.

3. Included in the distribution channel are a set of business entities which perform the activities necessary to effect this transfer.

Differences in the definition of the distribution channel focus on specific aspects, e.g.: Does the marketing channel include the consumer, or only the producer and intervening intermediaries? Which aspects of product are transferred by the channel—only ownership; ownership and physical possession; or a combination of flows such as ownership, physical possession, promotion, negotiation, financing, risking, ordering, and payment? What is the product definition—a generic product such as steel, a particular product such as refrigerators, or a specific product such as a Westinghouse refrigerator? Which business organizations comprise the proper elements of study—aggregates of institutional types such as wholesalers, brokers, and retailers; or specific individual businesses?
Which organizations are included—only trading concerns such as producers, wholesalers, retailers, brokers, and agents; or also non-trading concerns such as transportation and storage companies, insurance companies, and commercial banks? Several researchers have analyzed and compared these differing concepts and definitions, and this analysis is not repeated here. A workable definition for a distribution channel is, however, adopted in Chapter III.

This diversity in concepts and definitions has no doubt arisen due to the enormous complexity of activities that characterize the distributive network, and the need of the researcher to study only a part of the distributive network at any one time. The distributive network is synonymous with what Howard has called "the structure of distribution," which includes all marketing channels in use by all companies at any given time. Therefore, the distributive network may be viewed as including all of the

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29 Breyer (1949), op. cit., p. 279.
trading and non-trading business entities engaged in marketing, linked together by their business interconnections.30 This can be construed broadly enough to include the marketing activities of producers, such as manufacturers, farmers, and extractive firms; and to include consumers in a limited sense as the terminus of the channel.

The distributive network, as a system, is interconnected in such a manner that a change in one relationship will affect all other relationships throughout the network. Thus, as Alderson puts it, "We must look at the whole system to learn about any of its parts. This would appear to be a counsel of despair telling us that we must know everything in order to learn anything."31 As it is not possible to "know everything," Alderson points out that "one escape from having to study everything at once is to decouple the operating groups which make up a system and study certain aspects of the interactions among subsystems or the processes which are going on within a subsystem."32 It is this


32 Ibid.
decoupling procedure which allows the study of distribution channels. This dissertation is concerned with providing the conceptual framework that allows decoupling of the distributive network, and based upon this framework, determining whether graph theory provides an effectual means to analyze the structure of the resultant subsystems.

Graph Theory

Graph theory is a branch of the abstract mathematics of combinatorial topology. It combines combinatorial analysis—the analysis of discrete arrangement of elements, with topology—the properties of geometric figures. Graph theory therefore deals with patterns of relationships among sets of elements. It provides an appropriate language, and accessible and powerful mathematical tools, for analyzing systems of elements and their relationships which comprise structure.

Based on the simple idea of points interconnected by lines, graph theory provides a body of logically derived statements which allow conclusions to be drawn about properties of a structure from other properties. It has the potential to serve as a mathematical model of the structural properties of any empirical systems consisting of
relationships among sets of elements. If an appropriate co-
ordination is made so that each element of an empirical sys-
tem is identified with a point, and each relationship is
identified with a line, then for all true statements about
structural properties of the resultant graph there are cor-
responding true statements about structural properties of
the empirical system.

Distribution channels can be represented by graphs,
with the business entities, and relationships between the
entities, represented respectively by points and lines. It
seems natural, therefore, to take advantage of the abstract
concepts of mathematics available in graph theory to analyze
distribution channel structure. Concepts of structure de-
rived from the mathematical theory of graphs should be help-
ful in clarifying intuitive notions of structure as they
apply to distribution channels.

The author is properly cautioned by Halbert, who
notes, "Borrowed, adopted, and refined techniques, no matter
how glamorous or powerful they may be in their own disci-
plines, offer no panacea for marketing problems. . . . They
offer, at best, an additional analytic device whose value
depends upon the skill of the marketing analyst and upon the
clarity with which problems can be stated and analyzed." Graph theory may provide a useful tool to interpret and analyze theories about distribution channel structure. It is not here implied that graph theory will provide the ultimate means to an understanding of distribution channels, only that it may provide an additional means for analysis to supplement other approaches to the study of distribution channels.

CHAPTER II

THE RESEARCH PLAN

Methodology

General

The method of research used in this dissertation is primarily exploratory and theoretical. It is theoretical in that it is produced introspectively, with deductive logic. There are two required processes involved in performing the dissertation research. The first is a thorough review of the literature to identify, analyze, and interpret relevant concepts. Two bodies of literature are reviewed, i.e., the distribution channel literature and the graph theory literature. The distribution channel literature is studied to provide the necessary background for developing the conceptual framework through which the distributive network may be decoupled into comprehensible units. It is also reviewed to identify a theory of distribution channel structure which
may be analyzed using graph theory concepts. Basically, such a theory will be based upon combinatorial principles, or upon geometric interpretations of points and lines, consistent with the capabilities of graph theory as a branch of the mathematics of combinatorial topology.

Halbert has cautioned that in comparison to the natural and physical sciences, "marketing has no theory that is defensible on the grounds of logical consistency, philosophic adequacy, or experimental foundation." Since it is therefore unlikely that an existing theory will provide complete internal consistency, an additional criterion for selection of a theory is that it be sufficiently developed and comprehensive to allow the possibility for interpretation and analysis by means of the symbolic logic and axiom system of graph theory.

The graph theory literature also must be surveyed, to identify concepts capable of being developed within the time and energy constraints of the dissertation. A large number of graph theory concepts may be found in the mathematical

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literature, and often are highly abstract and theoretical. Therefore, the graph theory concepts to be used in this inquiry must be limited to the relatively few that have been sufficiently developed to merit inclusion in graph theory texts, or have found successful application in analysis of other empirical networks.

The second research process of the dissertation is conceptual, and is conducted in two stages. In the first stage, a particular viewpoint and conceptual framework is developed which may make distribution channel structure receptive to graph theory analysis. In general, the conceptual process involves breaking down the distributive network into sets of business entities by specifying rules governing their selection, and decoupling the resultant network segments on a functional basis. Based upon this conceptual framework, the second stage is to attempt to determine whether selected concepts of graph theory provide an effective means for analysis of distribution channel structure. For this purpose, the selected theory of distribution channel structure is subjected to analysis using graph theory concepts as tools. The objective is to interpret the theory and examine its implications in graph theory terms. Provided such an interpretation and examination results in
improved theoretical understanding, or indicates an insightful application to the study of extant distribution channels, it could be concluded, on a prima facie basis, that distribution channel structure can be effectually analyzed using graph theory concepts.

**Graph Theory Analysis**

The strategy for application of graph theory concepts is primarily molar as opposed to atomistic. Using a molar strategy, meaningful relationships are set up intuitively, then the chain of reasoning is traced back far enough to validate the propositions.\(^2\) This approach is particularly well-suited to the graph theory method of analysis, and offers a means to validate logic and theoretical consistency. The axioms of graph theory lead to an extensive body of logically derived statements. Each of these statements, when propositions about graph theory structure are stated that satisfy the axioms of graph theory, becomes a valid assertion about distribution channel structure. If, when

examined, these logically derived statements prove to have intuitive validity, the theory may be concluded to be consistent.

The examples used in the dissertation to represent distribution channel structure are devised rather than empirical. These examples are designed to be simple, yet plausible, and to embody the salient ingredients necessary for interpretation of distribution channel structure. However, any conclusions of this inquiry must therefore be viewed as hypotheses, to be verified by future empirical testing. The concepts developed in the dissertation form the conceptual groundwork for future empirical work.

**Limitations**

In order to specify an area researchable within the time and energy constraints of the dissertation, it has been necessary to rigorously define and delimit the area of study. The result of this study is not, therefore, an integrated, comprehensive theory. Omissions must be recognized in order to allow meaningful conclusions about this research and to designate limitations of application. A specific theory about distribution channel structure is analyzed, using only a few of the many possible graph theory concepts.
However, through a process of cumulative refinement, a more integrated theory may in time be developed from this beginning.

This study, like any conceptual, exploratory work, is susceptible to subjectivity. The necessity for narrowing the field of study and of selecting only certain concepts from the literature for analysis required judgment on the part of the writer. The writer has guarded, however, against introduction of unnecessary subjectivity.

The conclusions of the study are limited in that they are not based upon empirical enterprise channel groups. The study is nonsituational and general, using representative structural patterns to probe and analyze distribution channel structure, and to demonstrate the applicability of the graph theory method of analysis. Treatment of each concept is sufficient to fix the point of view and to indicate the major relationships. Emphasis is upon relationships between the elements of the system. Temporal and spatial concepts of distance are not explicitly considered, but are treated implicitly as factors contributing to the notion of economic distance. Other elements of the total distribution effort (marketing research, product planning, physical availability of product, advertising and sales promotion, personal
selling, and pricing) are included only to the extent that they affect the structure of distribution channels. Form-changing activities are excluded from the analysis, which ignores possible trade-offs between marketing and form-changing activities.

The study is at the level of static structure, or at the level of frameworks. "The accurate description of these frameworks is the beginning of organized theoretical knowledge in almost any field, for without accuracy in this description of static relationships no accurate functional or dynamic theory is possible." It should be recognized that although this inquiry is a necessary first step, it is limited in that it does not explicitly consider dynamic aspects of distribution channels.

There is a certain information loss which must be recognized due to the process of abstracting the channel to its graph or matrix representation, and from the process of

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developing summary indices to represent structural characteristics. Regardless, aside from purely educational objectives, the purpose of information is to assist in formulating conclusions and in the making of decisions. Graph theory may provide a research model for use in exploring the capabilities or limitations of existing distribution channel structures; however, such a model cannot reproduce all the details of reality.

Finally, the graph theory applications and measures developed do not have general applicability without modification in other areas of analysis. They are techniques borrowed from the abstract mathematics of graph theory, which are specifically applied to analysis of a specific aspect of distribution channel structure.
CHAPTER III

A CONCEPTUAL FRAMEWORK FOR GRAPH THEORY ANALYSIS
OF DISTRIBUTION CHANNEL STRUCTURE

Introduction

The distributive network, as a system, is interconnected such that a change in one relationship will affect all other relationships throughout the network. Analysis of any system requires, as a prerequisite, a logically consistent framework for decoupling the system for analysis. Such a framework must preserve the relationships within the subsystems, among the subsystems, and between each subsystem and the system as a whole.

The distributive network has as components business entities engaged in the process of distribution, and trading relationships among the business entities. A conceptual framework for decoupling the system must therefore contain rules for selecting sets of business entities from the network, and must make explicit the nature of the trading
relations among the sets of business entities. Such a framework requires:

1. A working definition of what is meant by a distribution channel.

2. A knowledge of the nature of the distribution network.

3. Relation of distribution channels to the distributive network: determination of how distribution channels exist in the distributive network, and specification of rules which govern how they may be extracted.

4. Selection of a logical subset of business entities for analyzing structure of distribution channels.

5. Explication of the relationships among business entities in the selected subsets.

A primary concern of this chapter is the manner in which sets of business entities, and sets of interconnections between business entities, may be selected from the distributive network. Set notation provides, therefore, a useful means to discuss this decoupling process.

**A Distribution Channel Definition**

A distribution channel is defined by Bucklin as being composed of that set of business entities which perform all the activities (functions) utilized to move a product and
its title from production to consumption.¹ Bucklin further defines an extant distribution channel as a set of business entities, constituting a distribution channel, at some point in time and through which some product actually has flowed; which continues in existence as long as trading relationships are maintained.² The product moves through the channel in essentially unaltered form, i.e., when the form of the product is changed, or the product is consumed, the distribution channel for that product ends. The development of a means to analyze the structure of extant, or real world, distribution channels is of primary interest in this dissertation.

It is important to note that these definitions do not restrict the institutions in the channel³ to those that take physical possession of the goods. As Davidson points out, "the general tendency is for the physical flow of


²Ibid.

³To avoid the necessity of repeating "extant distribution channels" throughout the dissertation, channels are understood to be extant distribution channels, as defined, unless otherwise specified.
merchandise to accompany the route of exchange. This is not, however, universally the case. . . ." Business entities which could be included in a channel are all those which perform functions, or arrange for performance of functions, necessary to move or facilitate the movement of the product and its title from production to consumption.

Bucklin's definition of a distribution channel has three elements which must be further defined in order to unambiguously discuss distribution phenomena: the set of business entities, the product, and the set of functions performed by the channel. Because these terms may be defined in a number of ways by channel analysts, there are many possible ways that subsets of channel dimension can be extracted from the distributive network. Depending upon how the set of business entities and product are defined, and depending upon which activities (functions) are considered pertinent, the number of channels which may be selected are infinite. Distribution channels are embedded everywhere in the distributive network, connected directly and indirectly


5 Bucklin (1966), loc. cit.
together, overlapping and intertwining in a bewildering array. In order to begin an analysis of the relation of distribution channels to the distributive network, and to specify rules for their extraction, it is necessary first to delineate the nature of the distributive network. Then the three terms in the definition of a distribution channel must be further limited to allow unambiguous analysis, and to indicate the viewpoint taken in this inquiry.

**The Distributive Network**

The distributive network may be viewed as including all of the trading and non-trading business entities engaged in marketing, linked together by their trading relationships. The business entities in the distributive network comprise the elements of the universal set \( U \) from which all other subsets may be selected, for example, a subset making up a channel of distribution. An example may clarify this concept.

**A Distributive Network for a Simple Economy**

Assume a simple economy made up of a farmer \((F)\), a miner \((E)\), a manufacturer \((M)\), a wholesaler \((W)\), and a retailer \((R)\). They are the elements of the universal set
for this simple economy, that is, $U = (F, E, M, W, R)$.

Each of the members of the simple economy specializes functionally. Only the farmer farms, only the manufacturer manufactures, and the miner alone mines. The wholesaler performs only wholesaling functions, and the retailer performs only retailing functions. Of course, each of the five individuals is also a consumer in the simple economy.

The farmer sells food and other farm products to the manufacturer for processing; to the wholesaler and retailer for resale; and to the miner, wholesaler, and retailer for their own use. These trading relationships are represented in Figure 1A by lines from the farmer to the other members of the simple economy. Similarly, the manufacturer sells his products to the wholesaler and retailer for resale, and to the farmer and miner for their own use. These trading relationships and similar relationships for the miner, wholesaler, and retailer are included with those of the farmer in Figure 1B. Figure 1B is the distributive network which includes all business entities in the simple economy engaged in marketing, linked together by their trading relationships.
A. Direct Trading Relationships from Farmer to the Other Members of a Simple Economy

B. Network of all Trading Relationships Among the Members of a Simple Economy--A Distributive Network

Figure 1
DISTRIBUTIVE NETWORK FOR A SIMPLE ECONOMY
Distribution Channels in the Distributive Network

It is illustrative at this point to see how distribution channels might exist in, and be extracted from, the distributive network. Assume that distribution channels are defined as having one business entity at each level, and that the product and its title is conveyed through the channel from production to point of use or consumption. When the product changes form or is consumed, the distribution channel for that product terminates. In order to differentiate the two possible terminations of a channel, the final business unit is subscripted. The subscript i indicates that the product is used in an intermediate sense, as in the case of a manufacturer who incorporates it into a new product. The subscript c indicates that the individual consumes the product at that point, in the sense that each of the business entities in the simple economy is also an individual consumer.

The manufacturer sells his products to F, E, W, and R, who consume them. These channels could be listed: \( M-F_c \), \( M-E_c \), \( M-W_c \), and \( M-R_c \), and summarized as \( M-C \) (or manufacturer M to consumer C channels), since the product is consumed at the termini. M also sells to R, who sells to F, who
consumes the product, which is a M-R-F_C (M-R-C) channel. Similarly, M sells to W, who sells to R, who sells to F, who consumes the product, which is a M-W-R-F_C (M-W-R-C) channel of distribution. Table 1 lists the channels in the simple distributive network which originate at M and end with a consumer.

TABLE 1

DISTRIBUTION CHANNELS ORIGINATING FROM THE MANUFACTURER AND ENDING WITH A CONSUMER IN A SIMPLE DISTRIBUTIVE NETWORK

<table>
<thead>
<tr>
<th></th>
<th>Channel Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M-C</td>
</tr>
<tr>
<td>M-R-F_C</td>
<td></td>
</tr>
<tr>
<td>M-E_C</td>
<td></td>
</tr>
<tr>
<td>M-R-C</td>
<td></td>
</tr>
<tr>
<td>M-W_C</td>
<td></td>
</tr>
</tbody>
</table>

It should be noted that M-W-C channels are not possible, since the wholesaler performs only wholesaling functions, and a W-C link would indicate a retail sale.

The manufacturer may also sell products in the simple economy which the others use to produce their own products;
for example, a tractor to the farmer, a conveyer to the miner, store furnishings to the retailer, and delivery trucks to the wholesaler. He may sell to them directly, which constitutes a M-I (manufacturer to intermediate user) channel; or he may sell to them through the wholesaler, which constitutes a M-W-I channel. The channels which originate with the manufacturer and end with an intermediate user are listed in Table 2. It should be noted that M-R-I or M-W-R-I channels are not possible, since the retailer does not perform wholesaling functions (in the simple economy, at least).

**TABLE 2**

**DISTRIBUTION CHANNELS ORIGINATING FROM THE MANUFACTURER AND ENDING WITH AN INTERMEDIATE USER IN A SIMPLE DISTRIBUTIVE NETWORK**

<table>
<thead>
<tr>
<th>Channel Type</th>
<th>Channel Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-I</td>
<td>M-W-I</td>
</tr>
<tr>
<td>M-F_i</td>
<td>M-W-F_i</td>
</tr>
<tr>
<td>M-E_i</td>
<td>M-W-E_i</td>
</tr>
<tr>
<td>M-R_i</td>
<td>M-W-R_i</td>
</tr>
<tr>
<td>M-W_i</td>
<td>X</td>
</tr>
</tbody>
</table>
Similarly, the channels may be listed that originate from the farmer and from the miner. For example, the farmer sells his products to \( W \), \( M \), \( R \), and \( E \), who consume them. He also sells his products to \( W \), who sells to \( R \), who sells to either \( E \) or \( M \). By this process of analysis, all possible channels originating from the farmer and miner can be listed.

It is not possible for channels to begin with \( R \) or \( W \), since the vertical dimension of channels is from production to intermediate use or consumption. All of the possible channels in the simple distributive network, including those listed in Tables 1 and 2, are listed in Table 3. Each of the channels constitutes a proper subset of the universal set \( U \) of business entities included in the distributive network.

In the simple distributive network in Figure 1B, there are connections to each member from every other member. If an additional member of the same type as one of the original members is added to the simple economy, there will not necessarily be a trading relationship, or connection, between each pair of members. Assume that a second manufacturer \( M_1 \) enters the simple economy, and decides to process only extractive products furnished by the miner, while the
TABLE 3
DISTRIBUTION CHANNELS IN A SIMPLE DISTRIBUTIVE NETWORK

<table>
<thead>
<tr>
<th>CHANNELS ORIGINATING FROM FARMER (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F-C</strong></td>
</tr>
<tr>
<td><strong>F-M_C</strong></td>
</tr>
<tr>
<td><strong>F-E_C</strong></td>
</tr>
<tr>
<td><strong>F-R_C</strong></td>
</tr>
<tr>
<td><strong>F-W_C</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHANNELS ORIGINATING FROM MINER (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>E-C</strong></td>
</tr>
<tr>
<td><strong>E-F_C</strong></td>
</tr>
<tr>
<td><strong>E-M_C</strong></td>
</tr>
<tr>
<td><strong>E-R_C</strong></td>
</tr>
<tr>
<td><strong>E-W_C</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHANNELS ORIGINATING FROM MANUFACTURER (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M-C</strong></td>
</tr>
<tr>
<td><strong>M-F_C</strong></td>
</tr>
<tr>
<td><strong>M-E_C</strong></td>
</tr>
<tr>
<td><strong>M-R_C</strong></td>
</tr>
<tr>
<td><strong>M-W_C</strong></td>
</tr>
</tbody>
</table>
original manufacturer, M, decides to specialize in processing only farm products. This means that there is no trading relationship from F to \( M_1 \), or from E to M. This is illustrated in Figure 2, where there are no lines going from F to \( M_1 \) or from E to M. It is evident that as more business entities are included in a distributive network, there will be more and more elements that are not directly connected. At the same time, the distributive network becomes increasingly complex.

**FIGURE 2**

A DISTRIBUTIVE NETWORK OF A SIMPLE ECONOMY WITH TWO MANUFACTURERS
Elements of the universal set. The basic business entity in a distributive network is the establishment, which is defined as a single or separate place of business.\textsuperscript{6} The place of business need not be an entire building, but may be part of a building from which business is regularly transacted. An establishment may also encompass more than one building, for example, a lumber yard or petroleum bulk station consisting of fenced yards enclosing several buildings and physical facilities.

An establishment is not synonymous with a company, a corporation, or other business organization under one ownership, unless the latter consists of a single place of business. A business unit organized under one ownership is an enterprise, which may consist of several establishments.\textsuperscript{7}

For example, a chain store enterprise may consist of many


retail stores, warehouses, and regional offices, each of which constitutes an establishment. In the simple distributive network described here, each establishment is also an enterprise, but this is not necessarily so in more complex distributive networks.

The distributive network from which extant channels of distribution are selected. In the distributive network of the United States, there are over 1.7 million retail establishments of all types, such as grocery, department, furniture, and hardware stores. Also included are over 300,000 wholesaling establishments, including merchant and functional middlemen, manufacturer's sale branches, and so forth. In addition to trading establishments, thousands of non-trading establishments that facilitate the marketing process could be included in the distributive network. Examples of these non-trading establishments are commercial

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8 Censuses for Wholesale and Retail Trade, Office of Business Economics, United States Department of Commerce, 1963.

9 Ibid.

banks, transporation and storage companies, and insurance companies. All of these trading and non-trading business entities are linked together in the distributive network of the United States by the trading relationships between them. The business entities in the distributive network of the United States are the elements of the universal set U from which all subsets may be selected for analysis.

**Subsets of Channel Dimension in the Distributive Network**

Having delineated the nature of the distributive network (the universal set), it is necessary, in this context, to discuss significant distinct subsets of the distributive network, each representing unique groups of enterprises with particular product assemblies, common interest, sizes of concerns, and so forth. Each of these groups constitutes a distinct subsystem which could be studied as an individual operating unit of the distributive network. Included among such subsystems are distribution channels, which in general are those subsystems which span the economic gap between production and consumption.

The most basic business entity in the distributive network has been defined as the business establishment. Not
all analysts, however, will desire to study channels in terms of business establishments. For example, many business establishments may be under one ownership and management; therefore, many interactions among business establishments are internal to a specific firm. If what is of interest is not internal aspects of firms, but only interrelations between firms, a different definition of business entities is required. Other specific interests might also lead to different definitions of the business entities in a channel, or in a subset of less than channel dimension.

Depending upon the definitions formulated by an analyst for the set of business entities, for the product, and for functions performed in a distribution channel (the terms in the distribution channel definition), many subsystems of channel dimension can be selected from the distributive network. Breyer has analyzed the many ways that subsets of channel dimension can be selected from the distributive network, and has named the various types of channels.\(^{11}\) The study of these empirical, or extant, channels

\(^{11}\) Breyer (1949), *op. cit.*, pp. 1-216.
is called the systemic approach. In the following sections, several subsets of business entities are discussed to show their relation to the subset defined for analysis in this inquiry.

Assume that some reasonable criteria for selecting a set of business entities is adopted by a channel analyst; such as a set which includes a group of producers with high product cross elasticities of demand, similar production characteristics, or similar product physical characteristics; their customers; and the business entities which perform or arrange for performance of the functions required to market their products. These business entities could be listed in an array such as the following.

\[
\begin{array}{cccccc}
 b_1 & b_2 & b_3 & b_4 & b_5 \\
 b_6 & b_7 & b_8 & b_9 & b_{10} \\
 b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\
 b_{16} & b_{17} & b_{18} & b_{19} & b_{20} \\
 b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \\
\end{array}
\]

\[
U = (b_1, b_2, b_3, \ldots b_n)
\]

where \( n \) is finite

\[
B = (b_1, b_2, b_3, \ldots b_{25})
\]

**FIGURE 3**

A SET OF BUSINESS ESTABLISHMENTS SELECTED FROM THE UNIVERSAL SET \( U \)

\[12\text{Ibid.},\ p.\ 279.]
Each of the $b_i$ is a business entity selected from the distributive network. The $b_i$ from $b_1$ to $b_{25}$ are the elements of the set $B$, which is a subset of $U$, i.e., $B \subseteq U$.

**Business Unit Distribution Channels**

Several distribution channels could be specified by determining trading relations between the elements of the set $B$, and by defining a distribution channel as having one business establishment at each level. A distribution channel formed in this manner is called a **business unit distribution channel**.\(^{13}\) For example, if $b_1$ is a producing establishment, $b_8$ a wholesaling establishment, $b_{12}$ a retailing establishment, and $b_{19}$ a customer,\(^{14}\) the channel which includes these specific business units is a business unit distribution channel.

An important reason for studying business unit channels would be if a channel were fully integrated, that is, with all business establishments under one ownership. In this case, all relationships in the channel would be intra-

\(^{13}\)Ibid., p. 38.

\(^{14}\)A customer is considered a business entity, consistent with the definition of a distribution channel proposed by Bucklin (1966), op. cit., p. 3.
organizational, as there is only one enterprise in the channel. It may also be desirable to select business establishments as elements of the set in some studies if a detailed analysis, including interfirm interactions, is required.

**Type Distribution Channels**

Figure 4 is an illustration of the kind of distribution channel which McCammon and Little have found to be specified in most discussions of distribution channels.  

![Diagram of Type Channel of Distribution]

**FIGURE 4**

**TYPE CHANNEL OF DISTRIBUTION**

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In this concept, the box labeled "Manufacturers" might represent all the elements of B which are manufacturing establishments, and the other boxes might represent the other establishments classified as to type in a similar fashion. The specific establishments through which some aspect of the product flow passes are not considered individually, only the classification of the enterprise. Such a marketing channel is called a type channel of distribution. This particular kind of channel of distribution would be called the manufacturer-wholesaler-retailer-consumer channel (M-W-R-C), based upon the types of establishments at each level through which the product flows. The type channel of distribution is concerned with the various levels, or types of institutions, through which some aspect of the product (generally the title) passes from the source to destination. The varieties of type channels are illustrated in marketing texts similarly to Figure 5.

A type channel is useful for summarizing a particular distribution structure. It is inadequate, however, for detailed analysis of structure, as the individual establishments in the distribution channels lose visibility when they

16Breyer (1949), op. cit., p. 281.
FIGURE 5

TYPICAL ILLUSTRATION OF TYPE DISTRIBUTION CHANNELS
are aggregated by type. For example, the channel of a manufacturer selling to one retailer is represented in the same manner as a manufacturer selling to 100 retailers, i.e., by a M-R-C type channel. Similarly, in Table 3 all of the listed business unit distribution channels would be represented by the type distribution channels in the column headings. The business unit distribution channel, on the other hand, focuses attention on the individual establishments in the channel.

**Enterprise Distribution Channels**

The previous sections have shown how sets of establishments may be selected from the elements of the universal set U, which includes all elements in the distributive network. It was also shown how traditional channel definitions aggregate establishments into type channels of distribution. A third alternative is to form a partition of the elements in U prior to selection of elements for analysis, e.g., if interorganizational relationships are to be studied, the elements in the universal set U might first be grouped according to ownership into enterprises. For example, in Figure 3, business establishments b₁ and b₂ may be manufacturer establishments under common ownership by M₁,
while $b_3$, $b_4$, $b_5$ are manufacturing establishments under common ownership by $M_2$. In a similar fashion, the other establishments in a distributive network may be grouped by ownership. The universal set, then, is defined in terms of enterprises instead of establishments, and the elements selected from the universal set will therefore be enterprises. The universal set defined in terms of enterprises is subscripted $U_E$ to differentiate it from that defined in terms of establishments.

An analyst may be interested in the set of enterprises which is coextensive with the set of establishments listed in Figure 3, which deal in a specifically defined product. In this case, the analyst would select from $U_E$ the subset of enterprises that deal in this product, shown in Figure 6. It is selected from a distributive network similar to those illustrated in Figures 1B and 2, but more complex, with many firms included at each level. In set notation $E \subseteq U_E$, that is, the set of enterprises $E$ shown in Figure 6 is a subset of the universal set $U_E$ of enterprises included in the distributive network.
Figure 7 shows the enterprises selected from the distributive network, with the trading relationships (not further defined at this point) between the enterprises indicated by lines. For example, $M_1$ has trading relationships with $W_1$, $R_1$, and $W_2$; $W_3$ has trading relationships with $M_2$, $R_2$, $R_3$, $R_4$, and $R_5$, and so forth.
FIGURE 7

SEGMENT OF A DISTRIBUTIVE NETWORK

It can be seen that there are many routes through the enterprises by which the product and its title are transferred from producer to consumer: $M_1-W_1-R_1-C_1$, $M_1-W_2-R_3-C_5$, etc. Each one of these routes, which includes at each level one specific enterprise which has trading relationships for the good, can be considered a separate channel of distribution. This type of distribution channel is called an enterprise distribution channel.\(^\text{17}\)

Figure 8 on the following page lists all of the enterprise distribution channels in the distributive network which include $M_1$. Those which include $M_2$ could similarly be listed.

\(^{17}\) Breyer (1949), op. cit., p. 279.
It can be seen that, depending upon the viewpoint of the analyst, many other concepts for channels of distribution could be defined. For example, the above examples include only sets of trading business entities; however, in some views, non-trading business entities such as transportation and storage companies could be included in the channel. Trading relationships specified could be limited to title flow; or several flows could be included. Firms which change the form of the product could be included in the grouping of institutions, in addition to firms which are specifically involved in performing marketing
functions. 18

The Enterprise Channel Group

Introduction

The enterprise channel group is one of the possible real world, or extant, sets of enterprises which may be selected from the distributive network. An enterprise channel is defined as being made up of specific, independently owned business entities. 19 For example, one of the enterprises in the channel might be a specific retailing enterprise, made up of many retail stores, warehouses, and offices under common ownership. A single enterprise channel has one enterprise at each level, or vertical position. 20 Figure 8 illustrates several enterprise channels extracted from a distributive network.

18 For examples of analysts who take this approach, see Bucklin (1966), op. cit., p. 7; and Wroe Alderson, Dynamic Marketing Behavior (Homewood, Ill.: R. D. Irwin, Inc., 1965), p. 86.

19 Breyer (1949), loc. cit.

An enterprise channel group is composed of two or more enterprise channels, short of the entire distributive network. To qualify as an enterprise channel group, at least one enterprise must have established trading relations with a minimum of two other enterprises, both of which are at either a higher or lower level. \(^{21}\) Figure 9 is an illustration of an enterprise channel group, made up of the enterprise channels listed in Figure 8. Figure 7 is a larger enterprise channel group which includes as a subgroup the enterprise channel group in Figure 9.

\[
\begin{align*}
M_1 & \rightarrow W_1 \\
M_1 & \rightarrow W_2 \\
R_1 & \rightarrow C_1 \\
R_1 & \rightarrow C_2 \\
R_2 & \rightarrow C_3 \\
R_3 & \rightarrow C_4 \\
R_3 & \rightarrow C_5 \\
R_4 & \rightarrow C_6 \\
\end{align*}
\]

\text{FIGURE 9}

\text{ENTERPRISE CHANNEL GROUP}

A particular enterprise channel group may be selected from the distributive network on the basis of a common

\(^{21}\text{Ibid.}\)
characteristic, or interest, of a set of trading enterprises. For example, an enterprise channel group may include a specific manufacturer and the intermediaries in all the channels through which he markets his product (e.g., Figure 9). Similarly, several producers of a product or product group and their intermediaries can be included in an enterprise channel group (e.g., Figure 7). Still larger channel groups can be selected, such as the group of manufacturers in an industry and all enterprises which market the industry product.

Selection of enterprise channel groups has the advantage of allowing analysis in a systematic manner at increasing levels of complexity. First, the enterprise channels that make up an enterprise channel group for a specific manufacturer's product can be extracted from the distribution network. Second, enterprise channel groups can be determined for other manufacturers of the same product. Then, the channel groups for each manufacturer can be combined to form a larger channel group that includes all manufacturers of the product, and all intermediaries who market or facilitate marketing of the product. In other words, if A is the set of enterprises for the first manufacturer, and B and C are the sets of enterprises for other
manufacturers of the same product, the enterprise channel group \( E \) formed from \( A, B, \) and \( C \) is \( E = A \cup B \cup C \).

The Unit of Analysis

The enterprise channel group which includes all enterprises dealing in a specific product is utilized as the unit of analysis in this dissertation. This enterprise channel group has most of the interesting characteristics of more complex groupings of institutions, having both vertical and horizontal dimensions. It provides a compromise between being so complex that the theory would be obscured by minutiae, and so simple as to be trivial.

The enterprise channel group has the property of focusing attention on interorganizational relationships, since the elements in the included channels are defined as separately owned business enterprises. The fully integrated channel is an exception to this, and such channels are excluded from analysis here, as are partially integrated channels. Non-trading enterprises—such as commercial banks, transportation and storage companies, and insurance companies—are not included in this analysis. Although the method of analysis is sufficiently general to cope with inclusion of non-trading concerns, it is felt that their
inclusion would add unnecessary complexity. In any case, the non-trading concern acts only as an agent for one or the other of the concerns between which it functions. For example, when a shipper arranges transportation for a product, he is responsible for the performance of the shipping function. If the terms of sale are FOB the shipper's establishment, the receiver is responsible for performing the shipping function. In either case, the carrier acts only as an agent to perform the actual shipping function.

The product associated with the enterprise channel group is not defined as a specific product, except that services are excluded; and the product may be defined as homogeneous or heterogeneous, as the analysis requires. The general method developed in the dissertation is applicable to any product or product class that fits the needs of the analyst. For example, a channel group could be defined for a particular Westinghouse refrigerator model, all Westinghouse refrigerators, all refrigerators, and even more general product classes such as all household appliances.

The method of structural analysis in this inquiry may be applicable to any subsets of enterprises that the analyst requires; however, for sake of clarity and comparability of analysis, this particular enterprise channel group
is selected. It should be recognized, therefore, that any conclusions drawn apply only to the enterprise channel group defined in this way, although it may be possible to modify them for other subsets.

**The Functional Channel Concept**

**Introduction**

In the previous sections, a workable definition for a distribution channel was presented, and the set of business entities and product defined to indicate the manner in which the distributive network may be decoupled into sets of enterprises in an enterprise channel group. A system, however, is comprised of not only the set of elements in the system, but the set of interconnections between elements and some motivating force which causes the system to function. The distribution channel has been described by Alderson as an organized behavior system, defined in primitive terms of theoretical language, or "primitives," namely, sets, behavior, and expectations. A set is a collection of definite and well distinguished elements which in a behavior

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22 Alderson, *op. cit.*, p. 44.

23 Ibid., p. 25.
system are persons or aggregates of persons, \textsuperscript{24} defined for this inquiry as a set of enterprises in an enterprise channel group. The study of the structure of distribution channels is concerned with the vertical dimensions and patterns of these structural arrangements in the distributive network. \textsuperscript{25}

Expectations may be viewed as a factor which causes enterprises to organize themselves into certain structural arrangements, and to cooperate in performing marketing activities. As long as expectations remain constant, the structure will remain constant. Expectations, therefore, are assumed constant, and structure is analyzed at a given point in time.

In order for a system to be specified, the elements of the set must be in some way related to one another and to the system as a whole. Behavior may be viewed as the activities performed by the elements of the set, which enable the product and its title to be conveyed through the distribution channel. Performance of activities, then, provides

\textsuperscript{24}Ibid., p. 47.

a means for analyzing the interactions among elements of the set of business entities. The interactions, which define the boundaries of the set, may be viewed in terms of the activities, or functions, that the enterprises perform in order to accomplish the distribution process.

**Functions in Distribution Channels**

A function is a major and distinctive economic activity which is inherent in the marketing process, pervades it throughout, and which tends to become specialized. The nature of marketing functions is that they are major activities which must be performed in the marketing of all products.

The channel of distribution is primarily an institutional arrangement for organizing work necessary to market a product. Marketing functions, which describe work, tend to be performed by firms who can perform them most economically. That is, firms tend to specialize in performing certain functions. The concept of the division of labor within a firm, or intraorganizational specialization, has long been

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26 Beckman and Davidson, *op. cit.*, p. 423.
recognized. Specialization, however, may also occur interorganizationally, for example when enterprises specialize in performing a specific function or group of functions.\(^{27}\) For example, a manufacturer's agent generally specializes in a specific function, the selling function. A service wholesaler typically specializes in a group of functions, such as buying, selling, storing, and risk bearing, in the sense that it performs these functions and not others.

Each enterprise in the channel need not perform all the functions; indeed, it is unlikely that they would do so. Also, each function may be performed several times by different enterprises in the channel before the product reaches the consumer. Although each individual enterprise will not perform every function, all functions must be performed by the channel system.

**Flows and Related Functions**

**General.** Recognizing that distribution through the channel entails a continuous process, Breyer\(^{28}\) first, and

\(^{27}\)Ibid.

later others such as Vaile, Grether, and Cox,\(^{29}\) conceptualized the performance of the marketing functions in terms of a series of flows through the channel. For example, the latter include among those flows physical possession, title, promotion, ordering, payment, risk taking, financing, and negotiation.\(^{30}\) Others list a lesser number of flows;\(^{31}\) indeed, the specific flows which should be included in a marketing channel system, and how marketing activities should be classified into functions, remain unsettled and controversial questions in marketing thought. Almost every marketing scholar can produce and defend a different list of functions (or flows). Whichever functions and flows are used by the analyst, all of the functions must be performed by the marketing system. Therefore, each of the marketing functions must be associated with one of the flows. In order for a flow to occur, the functions closely associated


\(^{30}\) Ibid.

with that flow must be performed. Since it is beyond the
scope of this dissertation to enter into the controversy
concerning marketing functions, a well-known and widely used
list is adopted for this inquiry.

1. Buying
2. Selling
3. Transportation
4. Storage
5. Standardization and grading
6. Financing
7. Risk bearing
8. Marketing information

The flows which are transmitted through the distributive
network are limited to the tangible flows of: title, prod­
uct (logistics), and payment.

**Title flow.** It is possible to visualize ownership of
a product, represented by its title, as flowing from level
to level through the network from producer to consumer.
This flow may be called the *title flow*. It is clear that
performance of the buying and selling functions is prereq-
uisite to the flow of title. That is, title cannot be
transferred from one firm to another without the performance
of buying on the part of one and selling on the part of the
other. Buying and selling are composites of many specific

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32 Beckman and Davidson, *op. cit.*., p. 424.
but related activities performed by the buyer and seller (or their representatives) leading up to the actual transaction, at which time the flow is initiated.

Similarly, in order for the title to flow, the buyer and seller must previously have performed marketing information activities. The seller must identify prospective customers, ascertain their needs and wants, and plan product availability before a sale, or title flow, is possible. The buyer must recognize a discrepancy in his product assortment; and search, by seeking marketing information, for a seller who can satisfy his requirement. The seller must determine the shape or form in which a product is to be placed on the market. The functions of establishing and maintaining standards, or standardizing; and grading, which tests the conformity of commodities to standards; are important prerequisites to the flow of title.

Risk bearing is closely related to the title flow, as risk normally passes along with ownership. The owner of the good always assumes the risk of loss because of such factors as price changes, theft, deterioration, or obsolescence. 33 It is possible, however, to shift performance of

33 Ibid., p. 126.
risk bearing to institutions outside of the channel (non-trading firms), such as insurance companies. In this case, the non-trading firm acts as an agent for the trading firm in performing this function, as previously discussed.

**Logistics flow.** A second flow between production and consumption is that of the physical product, which may be called the logistics flow. Functions associated with the flow of the physical product are transportation and storage. The transportation function is necessary to move the physical product between distribution channel members. Storage involves an interruption in the flow of goods at facilities of certain channel members, who perform, therefore, the storage function.

**Payment flow.** A third flow through the marketing channel system is that of payment, termed the payment flow. Associated with the payment flow is the financing function, e.g., a retailer may pay a manufacturer prior to receiving payment from his own customer, which requires a financing arrangement.

The marketing functions, which must be performed by any marketing channel system, have each been associated with one of the flows through the system: title, logistic, and
payment flows. Table 4 summarizes the flows and the functions that are associated with each flow. It should be re-emphasized that in any discussion of flows through the system, the functions are implicit as a basis for these flows.

**TABLE 4**

FLOWS THROUGH A MARKETING NETWORK AND ASSOCIATED MARKETING FUNCTIONS

<table>
<thead>
<tr>
<th>Flows Through a Marketing Network</th>
<th>Marketing Functions Associated with Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Title flow</td>
<td>Buying, selling, standardization and grading, risk bearing, marketing information</td>
</tr>
<tr>
<td>Logistics flow</td>
<td>Transportation and storage</td>
</tr>
<tr>
<td>Payment flow</td>
<td>Financing</td>
</tr>
</tbody>
</table>

**Network Interconnections**

In order for a flow to occur, there must be an interrelation, or network connection, over which the flow is transmitted. This dissertation deals with the network of these interconnections which, along with the enterprises included in the channel, constitutes the structure of the
distribution channel. In the previous sections, the enterprises in the distributive network, enterprise channel, and enterprise channel group were shown as linked together by unspecified network connections. For example, in Figure 9 (see page 58 above), the enterprise channel group is depicted as being connected by lines which represent those unspecified network connections.

In terms of the functional channel concept, each of these network connections could transmit at least one, and more often, several of the flows. For example, the network connection between M₁ and W₁ in Figure 9, could be viewed as transmitting title, logistics, and payment flows. However, it is also possible to decouple the enterprise channels and enterprise channel groups on the basis of each individual flow.

Flow Networks

An enterprise logistics network, enterprise title network, and enterprise payment network can be specified, each of which would have as one connection a link between M₁ and W₁ in Figure 9. To further illustrate this concept, a segment of the enterprise channel group shown in Figure 9 is reproduced in Figure 10.
The network connections between $M_1$ and $W_2$, and between $W_2$ and $R_2$, transmit title and payment flows. The network connection between $M_1$ and $R_2$ transmits the logistics flow. This represents a distribution arrangement where a wholesaler stocks samples of a product and takes orders, and the manufacturer ships directly to the retailer. The title and payment of the products passes through the wholesaler, but the physical product passes directly from manufacturer to retailer. Figure 11 illustrates the segment of the enterprise channel group decoupled into its flow network links.
In this manner, the enterprise channel group can be expressed in terms of flow networks, each of which transmits one of the flows. Since each enterprise in the channel group does not perform each function, different sets of enterprises and sets of connections will generally be included in each of the flow networks. The union of these sets of enterprises and sets of connections will comprise the enterprise channel group as a whole. Decoupling the
enterprise channel group into flow networks allows the interactions to be analyzed in terms of the specific flows. Several flows could also be included on a single graph. The latter graph is, of course, the complete graph of the enterprise channel group.

Recapitulation

In this section, a workable definition for a distribution channel was stated, and terms incorporated in the definition—the set of business entities, the product, and the functions performed by the channel—were defined more completely to indicate the nature of the subset selected for analysis in this inquiry. The nature of the distributive network as an interrelated system was also delineated.

Basically, the process for selecting a subset decouples the distributive network into enterprise channel groups by selecting sets of enterprises, and then decouples the resultant enterprise channel groups on a functional basis into flow networks.

The basic business entities in the distributive network are establishments, which are separate places of business. This dissertation is primarily concerned with inter-organizational aspects of distribution channel structure,
that is, the relationships between operating organizations. Therefore, the establishments are grouped according to ownership into enterprises, which are viewed as the elements of the universal set $U_E$ from which all subsets, including distribution channels, may be selected from the distributive network. Enterprises are linked in the distributive network by enterprise distribution channels, which span the vertical dimension from production to consumption, and have one enterprise at each vertical level.

These enterprise channels of distribution exist as components of larger aggregations, called enterprise channel groups, composed of two or more enterprise channels of distribution. Enterprise channel groups are the resultant of commonality of interests of the enterprises included in the enterprise channel group, for example, a common product, common production characteristics, or a similar criterion for defining the boundaries of the enterprise channel group. Several enterprise channel groups may be included in a larger enterprise channel group. For example, an enterprise channel group may be defined for all the enterprise distribution channels linking one manufacturer to his customers. This enterprise channel group is part of an enterprise channel group defined for all manufacturers of the product
and their associated enterprise distribution channels. Although still larger enterprise channel groups may be defined, such as for all manufacturers of heterogeneous products in an industry, for clarity of exposition the latter enterprise channel group is selected as the unit of analysis in this inquiry.

A system is defined by not only a set of elements, but the set of relationships between them. These relationships are defined by the manner in which the work of distribution is organized, in terms of functions, by the enterprises in the set which comprises the enterprise channel group. The interconnections between the set of enterprises are viewed as a set of links capable of transmitting title, logistics, and payment flows, which are initiated by performance of the closely associated marketing functions. The links which transmit a specific flow could be used to decouple the enterprise channel group into the title flow, logistics flow, and payment flow networks for specific analysis.
CHAPTER IV

BASIC CONCEPTS FOR A GRAPH THEORY INTERPRETATION
OF DISTRIBUTION CHANNEL STRUCTURE

Introduction

Review and analysis of the graph theory literature indicates certain concepts have the likelihood of being receptive to development within the time and energy constraints of the dissertation. Basically, they are those concepts which have been sufficiently developed in the literature to merit their inclusion in contemporary graph theory texts, or which have found successful application in analysis of other empirical networks.¹ Theoretical

presentation of these concepts is not intended to be exhaustive, but sufficient to indicate the graph theory method of analysis, and to designate areas of future research potential.

Graph theory has considerable advantage as an analytical tool in that complex relationships may be developed beginning with simple, basic concepts. This section provides a foundation of these basic concepts. The method of presentation is intended to convey a primarily expository and intuitive, as opposed to rigorously mathematical, knowledge of graph theory to the reader. Mathematical sophistication and elegance has been sacrificed to some extent to make the concepts more readily accessible to readers of varying mathematical background. Terminology, mathematical symbols, and axioms which are essential to development of the primary concepts are introduced, while other interesting but nonessential concepts are not introduced, to avoid unnecessary complexity.

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Definition of a Graph

A graph $G$ consists of a nonempty set of points $V$, a (possibly empty) set of lines $E$, and a function $\Gamma$ which assigns lines to the points in the graph. The points are called vertices, designated by $v$, and the lines are called edges, designated by $e$. In Figure 12, $V = (v_1, v_2, v_3, v_4)$, and $E = (e_1, e_2, e_3, e_4, e_5)$. Graph theory focuses on the associations between vertices and edges, therefore, a graph can be expressed abstractly by enumerating edges and vertices, as in Table 5, which contains all essential information concerning the graph shown in Figure 12.
### TABLE 5

**EDGES AND VERTICES IN A SIMPLE GRAPH**

<table>
<thead>
<tr>
<th>Edges</th>
<th>Corresponding Vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_1$</td>
<td>$v_1$ &amp; $v_2$</td>
</tr>
<tr>
<td>$e_2$</td>
<td>$v_1$ &amp; $v_4$</td>
</tr>
<tr>
<td>$e_3$</td>
<td>$v_1$ &amp; $v_3$</td>
</tr>
<tr>
<td>$e_4$</td>
<td>$v_3$ &amp; $v_4$</td>
</tr>
<tr>
<td>$e_5$</td>
<td>$v_2$ &amp; $v_4$</td>
</tr>
</tbody>
</table>

A vertex, by definition, is the end point of an edge. The function $\Gamma$ formally maps $E$ into $V$, that is, $\Gamma$ specifies which vertices are *incident with* (are the end points of) which edges. The edges and vertices listed in Table 5 may be formally related; for example, $e_1 \sim (v_1 \& v_2)$, which indicates that vertices $v_1$ and $v_2$ are incident with edge $e_1$. It may also be said that edge $e_1$ *joins* vertices $v_1$ and $v_2$. Two vertices that are joined by an edge are *adjacent* vertices. The incidence relation, or $\Gamma$ function, although
a fundamental concept of graph theory, is seldom explicitly stated.

Interpretation of a Graph

Graph theory, per se, deals with abstract relationships among sets of vertices and sets of edges. However, if an appropriate coordination is made so that each entity of an empirical system is identified with a vertex and each relationship among entities is identified with an edge, then for all true statements about structural properties of the resultant graph there are corresponding true statements about structural properties of the empirical system.\(^2\) It is this ability to coordinate empirical terms to vertices and edges that makes available a formal theory of structure and provides conceptual tools for analysis of distribution channel structures. The set of enterprises in a distributive network may be abstracted as vertices in a graph, and the set of trading relations among the enterprises may be abstracted as edges. This coordination of empirical elements to the vertices and edges of a graph \(G\) constitutes an interpretation of the graph.

\(^2\)Harary, Norman, and Cartwright, op. cit., p. 22.
The set of edges $E$ in $G$ is defined as being a possibly empty, or null, set. A set of enterprises selected from the distributive network, the elements of which do not trade together, is an interpretation of a graph with no edges. A graph with no edges ($E = 0$) is said to be degenerate.

Classification of Graphs

Graphs can be classified on the basis of structural features, in accordance with the axiom system defined for graph theory. If $V$ and $E$ are both finite sets (the empty set is considered a finite set), $G$ is called a finite graph. If either set is not finite, $G$ is, of course, an infinite graph. Since sets of enterprises and sets of trading relations among enterprises in the distributive network are finite, the inquiry in this dissertation is in the realm of finite graphs.

Graphs may be either planar or non-planar. A graph is planar if it can be represented such that all vertices are distinct points and no two edges intersect at any point other than a vertex, i.e., if it can be represented in two-dimensional euclidean space ($\mathbb{R}^2$). A system of highways may, therefore, generally be idealized as a planar graph. A
non-planar graph cannot be represented in $\varepsilon^2$; for example, airline routes generally tend to be non-planar. Figure 12 is a planar graph, and Figure 1B (page 36 above) is a non-planar graph. Note that it is not possible to redraw Figure 1B so that its edges do not intersect at points other than vertices.

A graph is said to be simple (or ordinary) if it has no loops and no parallel edges. An edge is called a loop if it is incident with only one vertex. Parallel edges are two or more edges having as end points the same vertices. Finally, graphs may be directed or undirected. The directed graph, or digraph, has edges which have orientation, or direction. Figure 13 is a digraph, in which the edges are represented as arrows to indicate the direction of the edge.

FIGURE 13

A DIRECTED GRAPH (DIGRAPH)
On the other hand, an undirected graph, for example Figure 12, has edges which are not specified as to beginning and ending vertices. Both digraphs and undirected graphs may be finite or infinite, planar or non-planar and simple or non-simple.
CHAPTER V

THE THEORY OF CONTACT ADVANTAGE

Introduction

Review of the available literature of distribution structure has identified the Theory of Contact Advantage as having characteristics which may make it receptive to interpretation, analysis and examination for theoretical implications by means of graph theory. Basically, the Theory of Contact Advantage is based upon combinatorial principles. Graph theory is a branch of the mathematics of combinatorial topology, and if an appropriate coordination can be made so that elements of distribution channel structure in the Theory of Contact Advantage can be identified with elements of graph theory, then the concepts and axiom systems of graph theory may become available for analysis of distribution channel structure.

The Theory of Contact Advantage has been conceptualized, defined, and developed by Alderson over a period of
years beginning in 1949.\(^1\) Other analysts have debated and further developed the theory, and it remains a topic of current interest in recent publications.\(^2\) Because of this continuity of development, the "theory" in the Theory of Contact Advantage is interpreted in its connotation of a body of related concepts, as contrasted to a singular theory.

The Theory of Contact Advantage attempts to explain why intermediaries arise in the process of exchange, and how independent but coordinated agencies known as marketing


channels contribute to economic efficiency. Basically, the theory postulates that intermediaries exist because of the cost savings which can be achieved through an intermediate sort, that is, a sort between producers and customers. Alderson, in 1949, differentiated between four varieties of sorting: sorting out, assorting, apportioning, and collecting. Sorting out means to take a collection of objects differing in kind and separate them into groups that are each of a similar grade or quality. For example, a tobacco producer may sort out tobacco leaves into several marketable grades. Opposite to sorting out is assorting, which is the process of making up assortments. For example, a drug wholesaler makes up an assortment of products in filling a druggist's order. Apportioning refers to breaking down large quantities of a good into smaller quantities, and collecting refers to making up large quantities of a good from smaller quantities.

Intermediaries arise in the process of exchange, the Theory of Contact Advantage postulates, because of the inherent advantage of performing an intermediate sort. The advantage of including an intermediary in a distributive

\[Alderson (1949), \textit{op. cit.}, pp. 150-151.\]
network is shown by a simple example. Assume that producers and customers are located in close proximity, and that it is feasible for the producer to deliver directly to the customer. If there are five producers and five customers and each customer buys all of the products handled, twenty-five transactions are required to effect the exchange. A "transaction" in this analysis simultaneously effects the title, payment, and logistic flows. Figure 14A illustrates this pattern of decentralized exchange.

With one intermediary located between the producers and customers to perform an intermediary sort, the requirements of the customers might be satisfied by means of ten transactions, as illustrated in Figure 14B. The intermediary creates possession utility by facilitating the transfer of goods from producer to consumer, and by reducing the effort involved in the act of exchange.

The Index of Sorting Balance

To provide a means for measuring the increase in efficiency due to inclusion of one intermediary in a distribution structure, on the basis of transactions required to effect exchange, Alderson proposed an index of sorting
A. DISTRIBUTION STRUCTURE EXHIBITING PATTERN OF DECENTRALIZED EXCHANGE

B. DISTRIBUTION STRUCTURE EXHIBITING PATTERN OF CENTRALIZED EXCHANGE

FIGURE 14

CENTRALIZED AND DECENTRALIZED EXCHANGE IN A DISTRIBUTIVE NETWORK SEGMENT
balance. The index is defined as the ratio of item flow, measured by the number of invoice lines on orders, to the total of the producers (S) who supply the intermediary, and the retailers (C) that are customers of the intermediary.

\[
\text{Index of Sorting Balance} = \frac{\text{Item Flow}}{S + C}
\]

Item flow is a measure of the individual transactions which might be required if producers and customers were to deal together directly, shown in Figure 14A, while \( S + C \) is the number of transactions required if an intermediary is included, shown in Figure 14B. The higher the ratio, Alderson theorized, the more advantageous is it for an intermediary to act between the producers and the retailers.

The index is a function of the number of products handled by the wholesaler, and the frequency of order by the average customer. It is also affected by the number of suppliers and customers, consequently, a chief objection to its use is that it is ambiguous unless the factors are known to vary in some consistent manner. Otherwise, various

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\(^4\) Ibid., pp. 153-154.

\(^5\) Banks, op. cit., p. 332.
combinations of factors might result in the same index value.

The Index of Transactional Efficiency

To remedy objections to Alderson's index, Seymour Banks proposed an index of transactional efficiency, which also measures the reduction in the number of transactions due to the inclusion of one intermediary. If there are C retailers and S producers and each retailer stocks each producer's product, C x S transactions are necessary to completely effect exchange. Thus, in Figure 14A, C = 5 and S = 5, and CS = 25 transactions are required to effect exchange. When an intermediary is included in the network, as in Figure 14B, C orders are placed with him, and he places S orders with the producers. In centralized exchange, therefore, C + S = 10 transactions are required to effect exchange.

The index may be stated:

\[
\text{Index of Transactional Efficiency} = \frac{CS}{C + S}
\]

and measures the relationship of a network of decentralized

\(^6\text{Ibid.}\)
exchange to the same network with one intermediary included, or centralized exchange. In the example of Figure 14, the index of transactional efficiency is \( \frac{5 \times 5}{5 + 5} = 2.5 \), which indicates that the one intermediary reduces by 2.5 the number of transactions required for exchange. The magnitude of the index, however, increases directly with the number of producers and retailers, and therefore does not have an upper bound.

**The Index of Contact Advantage**

In 1954, Alderson further refined the concept of transactional efficiency by proposing an index of contact advantage. To illustrate this index, he assumes a simple exchange economy consisting of five households, each of which produces a surplus of some article used by all five. In order to exchange the surplus goods, ten separate transactions are required for decentralized exchange, as illustrated in Figure 15A. When the pattern of decentralized exchange

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A. DISTRIBUTION STRUCTURE EXHIBITING PATTERN OF DECENTRALIZED EXCHANGE

B. DISTRIBUTION STRUCTURE EXHIBITING PATTERN OF CENTRALIZED EXCHANGE

FIGURE 15
CENTRALIZED AND DECENTRALIZED EXCHANGE IN A DISTRIBUTIVE NETWORK
exchange is replaced by that which includes one intermediary, five transactions are required instead of ten, as illustrated in Figure 15B.

The number of transactions required to carry out decentralized exchange is \( S(S - 1)/2 \), where \( S = v \) is the number of producers. Thus, in Figure 15A, \( 5(5 - 1)/2 = 10 \) separate transactions are required. In the centralized case, however, only \( S = 5 \) transactions are required, where \( S = v - 1 \) and \( v \) is the number of participants, including the intermediary, in the network. The index of contact advantage is the ratio of transactions in a distribution network with no intermediary to transactions in that same distribution network with one intermediary included, and is stated:

\[
\text{Index of Contact Advantage} = \frac{S(S - 1)/2}{S} \quad \text{or} \quad \frac{(S - 1)}{2}
\]

The index increases as a function of the number of producers in the distribution network.

Exchange through intermediaries, it is postulated, arises out of considerations of efficiency in the process of exchange. By reducing the number of transactions required, and therefore the effort associated with exchange, the
intermediary frees the producer to perform his main endeavor of production more efficiently. It is from increased production due to increased transactional efficiency, Alderson theorizes, that the intermediary derives payment for his services.  

Contact Costs

Three studies have further developed the theory of contact advantage, by dealing with contact costs and the nature of payments to intermediaries. Balderston, in 1958, analyzed establishment of contacts and negotiation of transactions, with the purpose of theorizing about the adjustment of the number and types of intermediary agencies to the number and types of supplier and customers, and marketing tasks required by the characteristics of the commodity.  

Baligh and Richartz, in 1967, extended the Balderston analysis to include additional variables in the analysis of vertical market structures. Bucklin, in 1966, developed

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8 Alderson (1967), op. cit., p. 36.


10 Baligh and Richartz, loc. cit.
economic cost curves to reflect economies of scale in contact activities and effects of market decentralization.\textsuperscript{11}

The Balderston and the Baligh and Richartz studies are summarized in part in this section, as they are extensions of the theory of contact advantage which utilize primarily combinatorial aspects and are consistent, therefore, with the present inquiry. The Bucklin study utilizes partial equilibria economic analysis and provides several useful insights, which are detailed in a later section.

Balderston's study, and subsequently the study by Baligh and Richartz, are based upon a set of assumptions:

1. The distribution structure is defined for one homogeneous product, in which every supplier and every customer deals exclusively.

2. Only the generating of information leading to transaction decisions is considered. Intermediaries (W) carry no inventory or are otherwise involved in the logistics flow; their activities are confined to establishment and maintenance of contacts. Shipments are made directly from suppliers to customers.

3. The total product flow through the distribution structure is exogenously determined.

4. The number of suppliers (S) and customers (C) is exogenously determined.

\textsuperscript{11}Bucklin, loc. cit.
5. The cost \( q \) of transmitting information is independent of the amount of information or number of messages, and is constant per time period.

6. The second and subsequent intermediary to enter the distribution system replicates the network of the first. There are no barriers to entry.

7. An "adequate-bargaining network" is defined such that each of the \( C \) customers must contact each of the \( S \) suppliers, in order to determine each supplier's price.

In the case of decentralized exchange, illustrated in Figure 14A (page 89 above), \( S \times C \) contacts are required in an adequate-bargaining network. On the other hand, \( S + C \) contacts are required in the case of centralized exchange, which is provided by inclusion of one intermediary, as illustrated in Figure 14B. The cost of a contact is \( q \), therefore the total cost for decentralized exchange is \( T_1 = qSC \), and the total cost for centralized exchange with one intermediary is \( T_2 = q(S + C) \). Since each additional intermediary entering the system replicates this latter network, i.e., duplicates the network shown in Figure 14B (page 89 above), the total cost for centralized exchange with \( W \) intermediaries is \( T_3 = qW(S + C) \).

In this view, intermediaries can enter the system until the total cost of centralized exchange \( (T_3) \) is equal to that of decentralized exchange \( (T_1) \), at which point
suppliers and customers are indifferent to dealing direct or through intermediaries, as their costs are the same in either case. They of course will not incur costs greater than those due to direct exchange. The maximum number of intermediaries at the first level of intermediaries \( W \) between \( S \) and \( C \) is therefore achieved when \( T_1 = T_3 \), i.e.:

\[
q_{SC} = q_W (S + C)
\]

or, solving for \( W \)

\[
W^* = \frac{SC}{S + C}
\]

Where \( W^* \) is the maximum number of intermediaries possible at that level.

The first intermediary in the network can make a pure profit \( M \) of up to the difference between the cost for a network without an intermediary and for that same network with an intermediary, that is:

\[
M = T_1 - T_2 = q_{SC} - q(S + C)
\]

or

\[
M = q[SC - (S + C)]
\]

On the other hand, when \( W^* \) intermediaries are in the
network, pure profits are equal to zero, as defined above. 

Pure profits $M$, therefore, are a function of the number of 
wholesalers $W$ in the system, or $M = f(W)$. The sum of costs 
of the network with intermediaries and pure profits accruing 
to intermediaries, $T_4$, is:

$$T_4 = T_3 + M$$

or

$$T_4 = qW(S + C) + f(W)$$

This function may be differentiated to determine the $W$ that 
minimizes $T_4$, i.e., $\frac{d}{dW} (T_4) = 0$. The function $M = f(W)$ 
has a negative slope, with maximum potential pure profit at 
zero intermediaries, and minimum pure profit at $W^*$ interme­
diaries, as illustrated in Figure 16. Since $(S + C)$ is 
exogenously determined, total costs, $T_3 = qW(S + C)$, 
increase with $W$, that is, costs are a function of $W$ with a 
positive slope. Therefore, $T_4$, the sum of total costs plus 
pure profits, is minimized when $W_e$ intermediaries, the 
equilibrium number of intermediaries, are in the network at 
the first level.

Based upon this analysis, Baligh and Richartz assume 
that the maximum number of intermediaries $W_1^*$ at the first 
level is the equilibrium number $W_e$, or $W_1^* = W_e = SC/(S + C)$. 


They note that two additional levels of intermediaries can be formed in the system:

- \[ W_2^* = \frac{W_1^* S}{W_1^* + S} \]
- \[ W_3^* = \frac{W_1^* C}{W_1^* + C} \]

where \( W_2^* \) are between the suppliers and the \( W_1^* \) level, and \( W_3^* \) are between the \( W_1^* \) level and the customers. The second and third levels of intermediaries in turn allow additional
levels of intermediaries to evolve, and the process of new levels evolving between existing levels can continue so long as the product of the number of sellers in the level above the new level and the number of buyers in the level below the new level is greater than or equal to the sum of participants in those levels. That is,

\[ \frac{ab}{a + b} \geq 1 \]

where

a = participants in the level above the level to be inserted

b = participants in the level below the level to be inserted

If it is assumed that no fractional middlemen exist, the total number of middlemen and total number of levels must converge to some finite number for any finite number of suppliers and customers.

Summary

The Theory of Contact Advantage postulates that intermediaries may exist in a distribution system because of cost savings which can be achieved through an intermediate sort. The intermediate sort provided by an intermediary reduces the number of transactions required to effect
exchange in the distribution system, that is, an intermediary reduces the number of connections required between the participants in a distribution system.

Scholars have proposed indices of sorting balance, transactional efficiency, and contact advantage as measures of increased efficiency, on the basis of transactions required to effect exchange, due to inclusion of one intermediary in a distribution system. By making assumptions for the distribution structure and the nature of contact costs, the Theory of Contact Advantage has been extended to consider the question of the number of intermediaries which may exist in a distribution system.
CHAPTER VI

GRAPH THEORY INTERPRETATION OF THE
THEORY OF CONTACT ADVANTAGE

Introduction

In order to interpret the theory of contact advantage and determine its implications, it is necessary to introduce several graph theory concepts. These, and the basic concepts contained in Chapter IV, are used in this analysis and interpretation. The distributive network for which the index of contact advantage is derived has been represented in Figure 15A by a graph which is reproduced in Figure 17A. The producers in the network are coordinated with the vertices, and the transactions among producers are coordinated with edges. As defined in Chapter IV, this graph is a

A. DISTRIBUTION STRUCTURE EXHIBITING PATTERN OF DECENTRALIZED EXCHANGE

B. DISTRIBUTION STRUCTURE EXHIBITING PATTERN OF CENTRALIZED EXCHANGE

FIGURE 17

CENTRALIZED AND DECENTRALIZED EXCHANGE IN A DISTRIBUTIVE NETWORK
finite graph, in accordance with the axioms of graph theory, since \( V = (v_1, v_2, v_3, v_4, v_5) \) and \( E = (e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}) \) are both finite sets. It is defined as an ordinary graph, since it has no loops or parallel edges, and it is defined as non-planar since it must be represented in \( \varepsilon^n \) where \( n > 2 \). It should be remembered that the graph theory measures discussed below are governed by the axioms of graph theory which apply to this kind of a graph, and may or may not apply to other kinds of graphs.

**Edge Progressions**

One can visualize starting at a given vertex in a graph, traversing a set of edges in a continuous manner, and ultimately arriving at a specified vertex without traversing any one edge more than once. If the final vertex is different from the first, such an edge progression is called a **chain**. If the final vertex is the same vertex as the beginning vertex, the edge progression is known as a **circuit**. Thus, in Figure 17A, a finite sequence of edges \( e_1, e_2, e_7, e_{10} \) constitutes a chain, while the finite sequence \( e_1, e_2, e_3 \) constitutes a circuit. Such sequences of edges
which form continuous routes play a fundamental role in graph theory.  

Connectedness

A graph is connected if there exists at least one chain between every pair of its vertices. Other graphs are disconnected. Figures 17A and 17B are both connected graphs, since any vertex can be reached from any other vertex. The length of a chain is defined as the number of edges contained in the chain. The distance between any two vertices in a connected graph is the length of any shortest chain joining them, i.e., the number of edges in the chain. In Figure 17A, the distance between any two vertices is one, as they are directly connected. In Figure 17B, each vertex is of distance two from every other vertex with the exception of $v_6$, and $v_6$ is distance one from the other vertices.

It is evident that although the graphs in Figures 17A and 17B are both connected graphs, the former is connected to a greater degree than the latter. In fact, Figure 17A is connected to the greatest degree possible,

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since every vertex has an edge connecting it to each other vertex. Such a graph is completely connected, or more simply, a complete graph. On the other hand, the graph in Figure 17B is a minimally connected graph. Such a graph, in which every pair of distinct vertices are joined by one and only one chain, is called a tree.

It is also possible for a tree $T$ to be a subgraph of a graph $G$. A graph $G_1$ is a subgraph of a graph $G$ if and only if the sets of vertices and edges in $G_1$ are contained in the sets of vertices and edges in $G$, and the incidence among vertices is maintained. For example, Figure 18 is a tree which is a subgraph of the graph of Figure 17A, since each pair of distinct edges are joined by precisely one chain. There are several other subgraphs of Figure 17A which are trees, for example, edges $e_1$, $e_2$, $e_9$, and $e_{10}$. The edges of $G$ which appear in $T$ are called branches relative to $T$, and the edges not included in $T$ but contained in $G$ are called chords relative to $T$. For example, edges $e_1$, $e_2$, $e_4$, and $e_5$ are branches of the tree shown in Figure 18, and, because this tree is a subgraph of Figure 17A, the edges $e_3$, $e_6$, $e_7$, $e_8$, $e_9$, and $e_{10}$ in Figure 17A are chords relative to the tree. The removal of any one edge from a tree results in a disconnected graph. For example,
removal of $e_1$ from Figure 18 results in $v_1$ being disconnected from the graph. Such a vertex, which is not joined with any other vertex, is called an isolated vertex.

The Relationship of Vertices and Edges

The number of edges incident with a vertex $v$ is called the degree of $v$. Thus, in Figure 18, the degree of $v_2$ is four, while the degree of all other vertices is one. The number of edges in a graph is related to the degree of the vertices in the graph. Since every edge has as
endpoints two vertices, it contributes to the degree of both vertices. For example, in Figure 18, edge $e_2$ contributes one to the degree of $v_2$ and one to the degree of $v_3$; hence, one edge contributes two to the total degree of a graph. Therefore, the number of edges in a graph is equal to one-half of the sum of the degrees of all vertices in the graph, that is:

$$E = \frac{1}{2} \sum \delta$$

where $\sum \delta = \delta_1 + \delta_2 + \delta_3 + \ldots + \delta_n$, and $n$ is the number of vertices in the graph. In very simple graphs, it is easy to count the number of edges; however, in more complex graphs it is more efficient to calculate the number of edges from knowledge of degree of the vertices, as will be illustrated in Chapter VIII.

It has already been noted that the simple exchange economy graphed in Figure 17A constitutes a complete graph. It is reproduced, along with several other complete graphs, in Figure 19. The degree of each vertex is indicated in parentheses. It is evident that all vertices of a complete graph have the same degree, which is equal to the number of vertices $v$ in the graph minus one, or $v - 1$. Therefore, the sum of the degrees of all vertices in a complete graph is
the number of vertices $v$, times the degree of each vertex $(v - 1)$, that is:

$$\sum \delta = v(v - 1)$$

The number of edges in a graph is equal to one-half of the sum of degrees of all vertices; therefore, the theorem in graph theory which specifies the number of edges in a complete graph results in:

$$\text{Edges (E)} = \frac{v(v - 1)}{2}$$

For example, in Figure 17A, $E = 5(5 - 1)/2 = 10$ edges.
The Indices of Contact Advantage and Transaction Efficiency in Terms of Connectivity

Interpretation

Alderson found that the number of transactions required to carry out decentralized exchange (Figure 17A) is \( S(S - 1)/2 \), where \( S \) is the number of producers. This formula, which is the numerator of the index of contact advantage, corresponds to the theorem in graph theory which specifies the number of edges in a complete graph, as elaborated in the previous sections.

The graph representing the simple distributive network with one intermediary, which allows centralized exchange (Figure 17B), has been shown to be a tree. It is a general theorem of graph theory that every tree with \( v \) vertices has precisely \( v - 1 \) edges. This corresponds to the formula proposed by Alderson for the number of transactions in a distributive network with one intermediary (centralized exchange), which is the denominator of the formula for the index of contact advantage, with \( S = v - 1 \) due to the inclusion of one intermediary. Recognizing these relationships, the index of contact advantage may be restated as follows.
Index of Contact Advantage = $\frac{S(S - 1)/2}{S}$

$= \frac{\text{Edges in complete graph of } S \text{ vertices}}{\text{Edges in a tree graph of } S + 1 \text{ vertices}}$

$= \frac{v(v - 1)/2}{v - 1}$

where there is one more vertex in the tree graph due to the addition of an intermediary. As before, the index of contact advantage has a lower limit of zero and an unbounded upper limit which increases directly with the number of vertices.

**Implications for Distribution Structure Analysis**

Analysis of extant distribution channel structure. The index of contact advantage was intended by its originator to indicate the advantage of a distribution structure with one intermediary (which may be represented by a tree graph) over that same distribution structure without an intermediary (which may be represented by a graph which is complete). As such, the index compares two specific structures—a tree graph and a complete graph—and is therefore not suited for analysis of extant distribution structures, which are likely to vary from these two specific structures.
In addition, the index allows for only one intermediary in the structure, and is not bounded with an upper limit, which makes comparison of index values for different structures difficult. Similar disadvantages can be stated for the index of transaction efficiency.

For these reasons, it may be useful to state, in graph theory terms, the theoretical implications of the two indices, and to examine these implications in terms of distribution channel structure. In this way, it may be possible to define indices which are applicable to extant distribution channel structure.

**Theoretical implications.** The index of contact advantage compares a distribution structure with no intermediary, represented by a complete graph, to the same distribution structure with one intermediary, represented by a graph which is a tree. Similarly, the index of transaction efficiency compares a distribution structure with no intermediary to the same distribution structure with one intermediary, which also may be represented by a graph which is a tree. This is intuitively consistent with the definition of a tree, which specifies that in a tree there is one and only one chain or "path" between pairs of vertices. A
distribution structure which may be represented by a tree, therefore, effects complete exchange in the structure with the minimum number of transactions. The implication of the indices of transactional efficiency and contact advantage may be stated more formally in the form of a proposition:

Distribution structures which are most efficient on the basis of transactions required to effect exchange may generally be represented by graphs which are trees.

A corollary, however, must be noted:

All graphs which are trees do not necessarily represent the most efficient distribution structures on the basis of transactions required to effect exchange.

**Distribution Structures Represented by Graphs Which Are Trees**

**Introduction**

The analysis of contact costs by Balderston, and the subsequent study by Baligh and Richartz, assumed that inclusion of the first intermediary in a distribution

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structure would result in a structure with a graph similar to that in Figure 14B (page 89 above). The graph of this distribution structure is a tree. It was also assumed, in order to allow a determinate solution for the number of intermediaries in the network, that the second intermediary and subsequent intermediaries entering the distribution structure would replicate the network of the first intermediary. The graph of this resultant structure, however, is not a tree. Figure 20 is a graph for a distribution structure with two intermediaries, where the second replicates the network of the first, in which there is more than one chain between every producer and customer.

FIGURE 20
DISTRIBUTION STRUCTURE WITH A REPLICATED INTERMEDIARY NETWORK
The assumption that the second and subsequent intermediaries replicate the network of the first is seen to automatically build redundancy in the form of unnecessary contacts into the structure, as multiple chains (paths) are provided between each producer and retailer. Since each chain represents one contact between a producer and an intermediary, and a second contact between an intermediary and a customer, and each contact has a constant cost of $q$ in the Balderston and the Richartz and Baligh analyses, each redundant chain incurs a cost for the system of $2q$. It might be desirable, therefore, to relax the assumption of the replicated network, and examine an alternative explanation for evolution of intermediaries in distribution structures.

Assumptions

The following are assumed for this inquiry.

1. The distribution structure is defined for one homogeneous product, in which every supplier and every customer deals exclusively.

2. A transaction simultaneously effects the title, logistics, and payment flows.

3. The total product flow through the distribution structure is exogenously determined.

4. The number of suppliers ($S$) and customers ($C$) is exogenously determined.
5. There are increasing economies of scale present in effecting transactions. The costs of effecting a transaction decrease with increasing order size and with decreasing economic distance. Both are subject to diminishing returns. Economic distance includes such factors as spatial and temporal distance, and information discrepancies.

6. The price of a product at each producer is constant, and known to all customers. The price of the product delivered to the customer is, therefore, the price at the producer's location plus the transaction cost.

7. Supply equals demand, that is, demand clears the market.

8. The firms in the network behave in accordance with the profit motive.

Intermediaries at the First Level

Since transaction costs vary with order size, each customer will purchase the product from as few sources as required to fill his product need. Similarly, each supplier will sell to as few customers as required to market his

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5 A complete analysis of economies of scale in contact activities may be found in Louis P. Bucklin, "A Theory of Distribution Channel Structure" (Institute of Business and Economic Research, University of California, Berkeley, California, 1966), pp. 46-69.

supply of product. In the simplest distribution example that can be stated, suppliers (S) equal consumers (C), and each S produces one unit of product, while each C demands one unit of product. The distribution structure for this example is graphed in Figure 21A. Each supplier sells his entire output of one unit to one customer, therefore no intermediary may be justified on the basis of transactions required to effect exchange. The order sizes are indicated by a number associated with each edge in the graph. A real number associated with an edge in this manner is called its length.

In a slightly more complex distribution system, $S = C$ and the S produce different amounts of product, while the C each demand the same amount. Assume that the odd-numbered suppliers produce 1-1/2 units of product, while the even-numbered suppliers produce 1/2 unit of product. The resulting distribution structure is graphed in Figure 21B. The supplier $S_1$ sells one unit to $C_1$ and 1/2 unit to $C_2$. The customer $C_2$, to satisfy his product requirement, must purchase 1/2 unit from both $S_1$ and $S_2$. Since in order to increase average order size and reduce transaction costs, $S_1$ and $C_2$ prefer to make only one transaction instead of two, it is possible for an intermediary to enter the system
A. DISTRIBUTION STRUCTURE WITH $S = C$ AND SUPPLY AND DEMAND ARE EQUAL FOR EACH $S$ AND $C$

B. DISTRIBUTION STRUCTURE WITH $S = C$ AND SUPPLY = 1 1/2 UNIT FOR $S_1$ AND $S_3$ AND = 1/2 UNIT FOR $S_2$ AND $S_4$. EACH $C$ DEMANDS 1 UNIT

C. DISTRIBUTION STRUCTURE B WITH INTERMEDIARIES

FIGURE 21

DISTRIBUTION STRUCTURES ILLUSTRATING EVOLUTION OF INTERMEDIARIES IN TREE PATTERNS
D. DISTRIBUTION STRUCTURE WITH $C > S$, AND SUPPLY = 2 UNITS FOR $S_1$ AND $S_2$.
EACH $C$ DEMANDS 1 UNIT

E. DISTRIBUTION STRUCTURE D WITH INTERMEDIARIES

F. DISTRIBUTION STRUCTURE C WITH NONESSENTIAL INTERMEDIARY ($W_3$)

FIGURE 21 (continued)
to reduce their required transactions. The average order size in Figure 21B, computed by summing the orders indicated on the edges and dividing by the number of edges, is

\[
\frac{1 + \frac{1}{2} + \frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{2}}{6} \text{ Units} = \frac{2}{3} \text{ Units/Transaction}
\]

while in Figure 21C, the same structure with intermediaries included at the first level, the average order size is

\[
\frac{1 - \frac{1}{2} + \frac{1}{2} + 1 + 1 - \frac{1}{2} + \frac{1}{2} + 1 + 1}{8} = \frac{1}{1} \text{ Unit/Transaction}
\]

Two further simple distribution systems in which \( S = C \) could be analyzed, with similar results. The customers could have different demands while each supplier has the same supply, or the suppliers and customers could both have different supplies and demands. Both of these situations would enable evolution of intermediaries, similarly to the example discussed above.

If \( S > C \) or \( C > S \), based upon the assumptions of this analysis, intermediaries are always justified in the network on the basis of transactions required to effect exchange. For example, \( C > S \) and each \( S \) produces 2 units while each \( C \)...
demands 1 unit in Figure 21D. Introduction of intermedi­aries into this structure, shown in Figure 21E, results in average order size being increased from

\[
\frac{1 + 1 + 1 + 1}{4} = 1 \text{ Unit/Transaction}
\]

to

\[
\frac{2 + 1 + 1 + 2 + 1 + 1}{6} = \frac{4}{3} \text{ Units/Transaction}
\]

Scale Economies

As the analysis in the previous section indicates, inclusion of intermediaries in a distribution structure not only reduces the number of transactions required to effect exchange, but as a result increases average order size. Because order size increases with inclusion of an intermediary, by assumption costs Q associated with ordering are reduced. However, it should be noted that there is a limit to the number of intermediaries which may be included at any level. For example, in Figure 21F, addition of a nonessential intermediary \( W_3 \) to the distribution structure in Graph C reduces average order size from 1 to

\[
\frac{1-1/2 + 1/2 + 1 + 1 + 1-1/2 + 1 + 1/2 + 1/2 + 1/2}{9} = \frac{8}{9} \text{ Units/Transaction}
\]
The cost curve $Q = f(W)$ associated with order size will therefore be U-shaped, as indicated in Figure 22. The number of intermediaries which will minimize order costs is indicated by $W^*$. 

![FIGURE 22
TRANSACTION COSTS DUE TO INCLUSION OF INTERMEDIARIES](image)

Adding intermediaries to a distributive network will generally result in trading centers which reduce spatial distance between producers and consumers,\(^7\) and therefore an aspect of economic distance. The first intermediary in a

\[^7\text{Bucklin, op. cit., p. 65.}\]
system may reduce the spatial component of economic distance greatly; however, subsequent intermediaries reduce economic distance proportionately less, because the spatial distances involved become less and less as intermediaries are added. Also, at some point the effect of factors which decrease costs inherent in economic distance as intermediaries are added, such as information costs, are outweighed by factors which increase costs inherent in economic distance as intermediaries are added, such as extra handling costs. Therefore, inclusion of additional intermediaries past this point increases total economic distance (cost) \( D = f(W) \). The cost curve associated with economic distance is, as indicated by this analysis, U-shaped, as shown in Figure 22. The cost curve which is the summation of the order and economic distance cost curves, which represents total transaction costs \( T \), will also have the characteristic shape. The minimum point of the total transaction cost curve corresponds to \( W_e \), the equilibrium number of intermediaries.

**Evolution of Distribution Structures with Several Levels of Intermediaries**

In any distribution structure of \( S \) suppliers and \( C \) customers, there are a limited number of tree patterns which
may evolve. The distribution structure in Figure 21C (see page 119 above) is a tree pattern with average order size equal to one. This pattern and six other structural patterns are illustrated in Figure 23 for this distribution structure, along with the average order size for each tree structure.

Figure 23A may evolve due to inclusion of one intermediary, I₁, at the first level, and has an average order size of one. Figure 23B, with the same average order size, may evolve due to inclusion of a second intermediary, I₂, at the first level. Any additional intermediaries at the first level for this structure will be nonessential, as their inclusion would reduce average order size and thereby increase transaction costs. Figure 23C, which increases average order size to 1.2, may evolve from Figure 23B by inclusion of I₃, at the second level. Similarly, Figure 23D may evolve by inclusion of a second intermediary, I₄, at the second level.

Figure 23E may evolve from Figure 23C due to the inclusion of I₄ at the third level. Intermediaries I₁ and I₂, however, become nonessential in Figure 23E, since I₃ and I₄ could be directly linked, which would eliminate unnecessary intermediate handling of goods at I₁ and I₂. Note that
A. AVERAGE ORDER SIZE = 1

B. AVERAGE ORDER SIZE = 1

C. AVERAGE ORDER SIZE = 1.2

FIGURE 23
TREES IN A DISTRIBUTION STRUCTURE
D. Average Order Size = 1.2

E. Average order size = 1.33

FIGURE 23 (continued)
Average order size = 1.33

G. Average order size = 1.54

FIGURE 23 (continued)
Figure 23E is not a tree, since there are redundant paths between \(I_3\) and \(I_4\). The redundant paths between \(I_3\) and \(I_4\) in Figure 23E may be "collapsed," in effect, or reduced into one redundant path without nonessential intermediaries. When the nonessential intermediaries are eliminated in this manner from Figure 23E, a tree graph is again formed, as shown in Figure 23F.

Finally, Figure 23G may evolve from Figure 23F, due to inclusion of intermediaries \(I_5\) and \(I_6\) at the fourth level, and \(I_7\) and \(I_8\) at the fifth level. Figure 23G (assuming that \(I_1\) and \(I_2\) are still included) contains the maximum number of intermediaries for this distribution structure. Any additional levels will result in redundant paths which cannot be reduced into nonredundant paths without resulting in a previous structure. Figure 23G also has the maximum average order size, and therefore contains the number of intermediaries which corresponds to \(W^*\) in Figure 22 (see page 123 above), the number of intermediaries that minimizes ordering costs. The equilibrium number \(W_e\) of intermediaries, which minimizes total transaction costs, will depend

\[\text{This will be elaborated in an example which follows.}\]
upon the relation of the order cost curve and the economic distance cost curve in Figure 22, but will generally be less than \( W^* \), due to the opportunity for reducing redundant paths, which eliminates nonessential intermediaries. The equilibrium number of intermediaries \( W_e \) will be contained in one of the possible tree structures for the distribution network, since trees provide the minimum number of transactions required to effect exchange, and therefore minimum ordering costs.

The maximum number of intermediaries at each level may be determined by Table 6, in which it is assumed that no partial intermediaries exist. For example, in Figure 23E, up to \( W_1^* = \frac{1}{2} S (\geq C) = 2 \) intermediaries could evolve at the first level, as \( S \geq W_1^{**} = 4 \geq 2 \). Similarly, second and third levels of one intermediary each may be inserted, for example, \( W_2^* = \frac{1}{2} W_1^* (< S) = 1, \) as \( W_1^* \geq W_2^* = 2 \geq 1 \). The levels may be inserted which include \( I_5 \) and \( I_6 \), and \( I_7 \) and \( I_8 \), which terminates the process. The graph which results, if nonessential intermediaries are eliminated, is Figure 23G. Any additional levels will cause the distribution structure represented by this graph to revert to the distribution structure corresponding to Figure 23F, when redundant paths are eliminated.
<table>
<thead>
<tr>
<th>Level</th>
<th>Intermediaries at Level $n =$</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Greatest Integer Less Than</td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td>$W_1^* = \frac{1}{2} S$</td>
<td>If $S \geq C$ and $S \geq W_1^{<strong>}$, otherwise $= \frac{1}{2} C$ If $C &gt; S$ and $C \geq W_1^{</strong>}$, otherwise $W_1^* = 0$</td>
</tr>
<tr>
<td>2d</td>
<td>$W_2^* = \frac{1}{2} S$</td>
<td>If $S \leq W_1^<em>$ and $S \geq W_2^{**}$, otherwise $= \frac{1}{2} W_1^</em>$ If $W_1^* &lt; S$ and $W_1^* \geq W_2^{**}$, otherwise $W_2^* = 0$</td>
</tr>
<tr>
<td>3d</td>
<td>$W_3^* = \frac{1}{2} C$</td>
<td>If $C \leq W_1^<em>$ and $C \geq W_3^{**}$, otherwise $= \frac{1}{2} W_1^</em>$ If $W_1^* &lt; C$ and $W_1^* \geq W_3^{<strong>}$, otherwise $W_3^{</strong>} = 0$</td>
</tr>
<tr>
<td>nth</td>
<td>$W_n^* = \frac{1}{2} a$</td>
<td>If $a \geq b$ and $a \geq W_n^{<strong>}$, otherwise $= \frac{1}{2} b$ If $b &gt; a$ and $b \geq W_n^{</strong>}$, otherwise $W_n^* = 0$</td>
</tr>
</tbody>
</table>
where:

\[ W_n^* = n^{th} \text{ level of intermediaries and} \]
\[ n = 1, 2, 3, \ldots i \]
\[ S = \text{number of original producers} \]
\[ C = \text{number of original customers} \]
\[ W_n^{**} = \text{computed value for } n^{th} \text{ level intermediaries} \]
\[ a = \text{participants in level above level to be inserted} \]
\[ b = \text{participants in level below level to be inserted} \]

and:

Process continues until insertion of next level will result in a graph which is a previous structure when nonessential intermediaries are eliminated by collapsing redundant paths.

\(^a\)The analysis in this table utilizes logic which may be readily programmed for computer determination of \( W_n^* \).
Implications

In this analysis, each of the distribution structures that are most efficient on the basis of transactions required to effect exchange, and with lowest ordering costs, are represented by graphs which are trees. This result, arrived at through a molar analysis, tends to support the proposition that distribution structures efficient on the basis of transactions required to effect exchange may be represented by graphs which are trees.

In certain examples, intermediaries were shown in the distribution structure which divide the structure into two components, for example, Figure 21C (page 119 above) and Figure 23D (page 127 above). The graph of such a structure is said to consist of a forest of trees, which is discussed in greater detail in the next chapter. A distribution structure with greater than one component illustrates a possible effect on the distribution structure of economic distance. Each intermediary will attempt to add customers and locate suppliers until, because of increasing economic distance, the next firm added would increase marginal cost more than marginal revenue gained by increasing average order size. For each customer or supplier in the network, there
is one intermediary who can serve it more economically than the other intermediaries in the network, depending upon economic distance. For this reason, it is possible for components to evolve in the distribution network with different numbers of suppliers and customers, for example, the distribution structure in Figure 24. This, of course, includes the case where one intermediary may serve all suppliers and customers in the network most economically.

The assumption was made in this analysis that the distribution structure is defined for a homogeneous product. It should be noted that the same analysis will apply to a multiple product system. For example, the graph in

**FIGURE 24**

**DISTRIBUTION STRUCTURE WITH DISSIMILAR COMPONENTS**
Figure 14B (page 89 above) represents a network in which five products are produced, each of which is demanded by five customers, and transaction costs are assumed constant. Therefore, one intermediary evolves. If economies of scale are recognized, or if not all customers require all products, it is feasible for additional components to evolve which will be trees, based upon the assumptions of this chapter.
CHAPTER VII

EXTENSIONS OF THE THEORY OF CONTACT ADVANTAGE

The Index of Connectivity

Introduction

To restate conclusions of the previous chapter, the index of contact advantage, as a measure of extant distribution channel structure, has three major disadvantages:

1. The index is constructed by comparing two idealized structures—a complete graph and a tree graph. As such, it is not suited to an analysis of nonidealized, extant distribution channel structures.

2. The index has no upper bound, since the index magnitude is a function of the number of vertices in the system. Therefore, the ratio is not directly comparable between extant distribution channel structures.

3. The index allows only one intermediary in the network.

The index of contact advantage \( \frac{s(s - 1)/2}{s} \) was shown in the previous chapter to be equivalent, in graph
When it is stated in these graph theory terms, the index of contact advantage may be modified to overcome these deficiencies.

First, the number of edges may be allowed to vary by replacing \( v - 1 \), which is a measure of edges in a tree, by \( e \), the actual number of edges in the network. In any extant distribution structure, it is possible for the number of edges to vary from zero (the degenerate case in which no trading takes place) to the number of edges in a complete graph, \( \frac{v(v - 1)}{2} \). Of course the number of edges in a minimally connected graph (tree), \( v - 1 \), is the practical lower limit in any effectively functioning system.

Second, if the modified equation is inverted, the index is bounded by zero at the lower limit and one at the upper limit. The lower limit represents a degenerate graph (completely disconnected), and the upper limit represents a completely connected graph. Therefore, the index
measures the degree to which the graph is connected, and may be stated:  

\[
\text{Index of Connectivity} = \frac{e}{v(v-1)/2}
\]

\[0 \leq e \leq \frac{v(v-1)}{2}\]

**Interpretation of the Index of Connectivity**

**Analysis.** For convenience, the distribution structures illustrated by the graphs of Figure 14 and 15 are reproduced in Figure 25. The indices of transactional efficiency, contact advantage, and connectivity are computed for the graphs, and the results listed in Table 7.

The indices of transactional efficiency and contact advantage may be applied only to the specific distribution structures for which they were derived. In addition, application of the index of transactional efficiency to

---

1 This index is a non-planar equivalent to the gamma index \((\gamma)\), proposed by W. L. Garrison and D. G. Marble, "The Structure of Transportation Networks" (Northwestern University, Evanston, Illinois, 1961), p. 39 (mimeographed). It is also the inverse of a measure called the degree of connectivity by Z. Prihar, "Topological Properties of Tele-Communication Networks," Proceedings: Institute of Radio Engineers, XLIV (1956), 929-933.
FIGURE 25
FOUR GRAPHS REPRESENTING CENTRALIZED AND DECENTRALIZED EXCHANGE IN DISTRIBUTION STRUCTURES
### Table 7

**INDEX VALUES FOR SELECTED DISTRIBUTION STRUCTURES**

<table>
<thead>
<tr>
<th>Graph</th>
<th>Index of Transactional Efficiency</th>
<th>Index of Contact Advantage</th>
<th>Index of Connectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>25A</td>
<td>2.50</td>
<td>Not Applicable</td>
<td>0.56</td>
</tr>
<tr>
<td>25B</td>
<td>2.50</td>
<td>Not Applicable</td>
<td>0.18</td>
</tr>
<tr>
<td>25C</td>
<td>Not Applicable</td>
<td>2</td>
<td>1.00</td>
</tr>
<tr>
<td>25D</td>
<td>Not Applicable</td>
<td>2</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Index of Transactional Efficiency = \( \frac{CS}{C + S} \)

Index of Contact Advantage = \( \frac{S(S - 1)/2}{S} = \frac{(S - 1)}{2} \)

Index of Connectivity = \( \frac{e}{v(v - 1)/2} \)

where \( 0 \leq e \leq v(v - 1)/2 \)

- \( C \) = Number of retailers
- \( S \) = Number of producers
- \( v \) = Number of vertices
- \( e \) = Number of edges
Figure 25A results in the same value as when applied to Figure 25B, and the same is true when the index of contact advantage is applied to Figures 25C and 25D. On the other hand, application of the index of connectivity results in a value for each structure which depends upon the degree the structure is connected. For this reason, the index of connectivity may be applied unambiguously for comparison of the four distribution structures, and also for comparison of extant distribution structures.

Relation of distribution structures. The index of connectivity may be applied to any of the four distribution structures graphed in Figure 25. The first two structures are simply a special case of the second two. In Figures 25A and 25B, \( v_1 \) through \( v_5 \) are specified to be producers (suppliers) and \( v_6 \) through \( v_7 \) are specified to be retailers (customers). In Figures 25C and 25D, each producer (supplier) is also a customer of every other producer, but this is not explicitly represented in the graph. In Figure 25C, there are \( \frac{s(s-1)}{2} \) connections, or in terms of vertices, \( \frac{v(v-1)}{2} \) connections. In Figure 25A, there are \( s \times c \)
connections among suppliers and customers. There is also the possibility for the following connections:\footnote{2}

Each supplier connected to every other supplier:

\[
\frac{S(S - 1)}{2} \text{ connections}
\]

Each customer connected to every other customer:

\[
\frac{C(C - 1)}{2} \text{ connections}
\]

The total possible connections in Figure 25A, therefore, are:

\[
SC + \frac{S(S - 1)}{2} + \frac{C(C - 1)}{2} = \frac{2SC + S(S - 1) + C(C - 1)}{2} =
\]

\[
\frac{S^2 - S + C^2 - C + 2SC}{2} = \frac{(S + C)(S + C - 1)}{2}
\]

But in Figure 25A, \(S + C = v\), therefore

\[
\frac{(S + C)(S + C - 1)}{2} = \frac{v(v - 1)}{2}
\]

which is the formula for the number of edges in Figure 25C.

\footnote{2}{F. E. Balderston, "Communications Networks in Intermediate Markets," Management Science, January, 1958, p. 156, deals specifically with the distribution structures graphed in Graph A and B of Figure 14. In his analysis, he does, however, recognize the possibility of these other connections.}
Figure 25A, therefore, is a subgraph of the type of graph shown in Graph C, but with some of the possible connections missing. In other words, Figure 25C might represent a distributive network, from which subgraphs which are enterprise channel groups, such as Figure 25A, could be selected. For example, an enterprise channel group including \( v_1 \) and \( v_2 \) and their customers, selected from Figure 25C, is illustrated in Figure 26. The similarity of this structure to that of Figure 25A is evident.

![Graph](image)

**FIGURE 26**

AN ENTERPRISE CHANNEL GROUP

The index of connectivity, therefore, may be calculated unambiguously for any of the distribution structures graphed in Figure 25, and each distribution structure may be directly compared to each other distribution structure by means of the index.
Interpretation. The index of connectivity relates the actual number of edges in a graph to the possible number of edges. If the index is converted to a percentage, that is, multiplied by 100, it may be interpreted as the percent the graph is connected. For example, Figure 25A is 56 percent connected and Figure 25B is 18 percent connected, as compared to maximum connectivity (completeness). Figure 25C is of course 100 percent connected, because it is a complete graph.

The index of connectivity might be interpreted, in relation to distribution structures, such that small values are preferable to large values. A large value could indicate that the system is highly connected, thus with considerable room for improvement by adding intermediaries and reducing required transactions. On the other hand, a large value could indicate too many intermediaries in the network which also results in unnecessary transactions.

The desirability of low values for the index of connectivity would not, however, hold true as the index approaches zero. Figure 25B is a tree, having v - 1 edges, and with an index of connectivity value of 0.18. Removal of any edge reduces the index value, but obviously does not
improve the efficiency of the system, since removal of an edge from a tree disconnects the graph by isolating a vertex. Such an edge removal may therefore be interpreted as the isolation of a producer or retailer from the system.

It would be desirable, for this reason, to limit the range of the index to values for connected graphs, that is, $(v - 1) \leq e \leq \frac{v(v - 1)}{2}$. However, if the lower limit on the index were changed from 0 to $(v - 1)$, which is the number of edges in a minimally connected graph (tree), the index would no longer have a common lower bound for all networks. For example, Figures 25B and 25D are trees, and might be viewed equivalently as representing, on the basis of required transactions, most efficient distribution structures, yet they have index values of 0.18 and 0.33, respectively.

A more useful index would be one which utilizes the proposition that efficient distribution structures may be represented by graphs which are trees. Such an index should ideally have common upper and lower bounds to allow direct comparison of extant distribution structures.
Redundancy in Distribution Structures

Trees and Fundamental Circuits

To restate, a graph which is a tree may represent an efficient distribution structure, because every pair of vertices is joined by one and only one chain. Additional chords added to the tree represent, therefore, redundant chains, or paths, through the distribution structure. These redundant paths may be thought of as representing unnecessary, i.e., redundant, transactions.

If one chord is added to a tree, a unique single circuit called a fundamental circuit is formed. Figure 27 shows a graph which is not a tree, formed from Figure 25D (page 139 above), which is a tree, by joining $v_2$ and $v_3$ with a chord. A distribution channel interpretation of the structure graphed in Figure 27 might be a producer $v_2$ who sells to an intermediary $v_6$, but who also sells directly to $v_3$, in competition with $v_6$.

A fundamental circuit is formed by the edge progression including the branch joining $v_6$ and $v_2$, the chord joining $v_2$ and $v_3$, and the branch joining $v_3$ and $v_6$, which completes the circuit. Other fundamental circuits may be
formed from Figure 25D, for example by chords joining $v_3$ and $v_4$, or $v_4$ and $v_5$, etc.

A graph contains a set $E$ which has as elements $e$ edges. A tree subgraph $T$ in a graph has been shown to include exactly $v - 1$ edges. It is a theorem of graph theory that the complement of the edges in $T$ with respect to $G$ contains, therefore, $e - (v - 1)$ or $e - v + 1$ edges (chords).

The Zeroth Betti Number and First Betti Number (Cyclomatic Number)

The relationship between branches and chords may be stated more generally. Figure 28A is a disconnected graph,
Vertices \( v \) = 8, Edges \( e \) = 10, Components \( p \) = 2

A. A DISCONNECTED GRAPH \( G \)

B. TREES OF THE COMPONENTS OF \( G \)--A FOREST OF 2 TREES

Branches = \( v-p \) = 8-2 = 6
Chords = \( e-v+p \) = 10-8+2 = 4

FIGURE 28

A DISCONNECTED GRAPH AND ITS TREES
since no chains join $v_1, v_2, v_6, v_7$ or $v_8$ to $v_3, v_4$ or $v_5$.

A disconnected graph contains two or more parts which are not connected to each other; in this case one part contains $v_1, v_2, v_6, v_7$ and $v_8$, and adjacent edges, and a second part contains $v_3, v_4$ and $v_5$, and adjacent edges. These parts of the graph are connected subgraphs which are not contained in any larger connected subgraphs of the graph, and are called components of the graph. A count of the components of a graph constitutes one of the simplest indices for describing its structure, and is called $p$, the Zeroth Betti number.

A tree may be defined for each of the components of the graph, as in Figure 28B. If a graph consists of $p$ components, the $p$ trees thus specified are known as a forest of $p$ trees of the graph. There are $v - p$ edges (branches) in the forest, and $e - v + p$ edges (chords) not in the forest. These chords form $e - v + p$ fundamental circuits with the edges in the forest, and each chord represents a redundant path. The number of fundamental circuits in a graph is known as nullity, the first Betti number, or the cyclomatic number $\mu$. Thus, the cyclomatic number $\mu = e - v + p$ may be interpreted as an index of network structure which measures redundancy in the system.
The Zeroth Betti number and first Betti number (cyclomatic number) have as an important property that they are invariant under isomorphic transformation of a graph. A graph $G$ is isomorphic with a graph $G_1$ if the vertices and edges of $G$ can be placed in one-to-one correspondence with those of $G_1$ in such a way that incidences are maintained. Therefore, two graphs may be structurally identical even though they appear to be different. Figure 29 shows two graphs which are isomorphic.

![Graph G and Graph G_1](image)

**FIGURE 29**
**TWO ISOMORPHIC GRAPHS**

By comparing vertices, edges, and incidences, they can be seen to be structurally equivalent: $e_1 \sim v_1$ & $v_2$, $e_2 \sim v_2$ & $v_3$, etc. The first Betti numbers for $G$ and $G_1$ are both 3, which illustrates that if $G$ were isomorphically
transformed into $G_1$, the index would remain invariant and provide a consistent measure of structure.

The first Betti number, or cyclomatic number, for Figure 25B is $\mu = 10 - 11 + 1 = 0$, and for Figure 25D is $\mu = 5 - 6 + 1 = 0$. Both of these graphs are trees, which as nonredundant structures have cyclomatic numbers of zero. Figure 28B has a cyclomatic number of $\mu = 6 - 8 + 2 = 0$, since a forest is also a nonredundant structure. As the number of redundant edges (chords) added to a tree increases, the cyclomatic number also increases. The upper limit is reached, which will be different for each graph, when the graph is complete, since all possible chords are included in the graph, representing maximal redundancy.

The magnitude of the cyclomatic number for a complete graph is a function of the number of vertices in the graph. The cyclomatic number, therefore, has a lower limit of zero but no upper limit, which makes interpretation of the index difficult. It is perhaps desirable to transform this index in a manner which would yield common upper and lower limits for all networks, and which would maintain the property of isomorphic invariance. The cyclomatic number $\mu$, which is the actual number of fundamental circuits in a network, may be transformed to a ratio measure by normalizing $\mu$ in
relation to the maximum number of fundamental circuits in a complete graph. This maximum number is of course the number of edges in a complete graph \(v(v - 1)/2\) minus the number of edges in a tree \((v - 1)\). The number of edges

\[
[v(v - 1)/2] - (v - 1)
\]

is therefore the number of chords in a complete graph, or the maximum number of redundant edges. The ratio formed in this manner is:

\[
\frac{\mu}{[v(v - 1)/2] - (v - 1)} = \frac{e - v + p}{[v(v - 1)/2] - (v - 1)}
\]

The number of edges in a graph to which this ratio applies is restricted to \((v - p) \leq e \leq v(v - 1)/2\). The ratio has a range from 0 to 1.

**Redundancy Due to Nonessential Intermediaries**

The above ratio measures the degree of redundancy in a graph on the basis of redundant edges, or chords. In graphs of distribution structures, chords may therefore be interpreted as redundant transactions, i.e., transactions in addition to the minimum number required to effect exchange. A distribution structure represented by a graph which is a tree (i.e., no chords) on this basis has no redundant transactions. An additional type of redundancy is possible in distribution structures, however,
that which results from inclusion of nonessential intermediaries in a tree structure.

For example, in the distribution structure represented by Figure 30A, one producer sells to one retailer via a wholesaler. Inclusion of the wholesaler results in a redundant transaction (i.e., edge), since one transaction between the producer and retailer would suffice. Such a redundant edge, which is a branch of a tree, is called a $k$-redundant edge to differentiate it from the type of redundant edge (chord) discussed previously. It was proposed that distribution channel structures which are most efficient on the basis of transactions required to effect exchange may be represented by graphs which are trees. Trees containing $k$-redundant edges are not, however, most efficient structures, which necessitated the corollary that not all trees represent most efficient distribution structures (page 114).

Balderston has theorized that for some very simple distribution structures no intermediary can be justified. Generally, in this view, a minimum of two suppliers and two customers is required before one intermediary can be introduced. This minimum distribution structure is shown in

\[ \text{Ibid.}, \text{ p. 153.} \]
FIGURE 30
DISTRIBUTION STRUCTURES ILLUSTRATING K-REDUNDANCY
Figure 30E. Therefore, in this interpretation, any distribution structure with less than five participants, including an intermediary, is to some degree redundant. However, in this inquiry, it is possible for an intermediary to be justified for a network including as few as one supplier and two customers, or two suppliers and one customer. For example, in Figure 30C, $S_1$ and $S_2$ sell to $W_1$, who performs a collecting activity. The intermediary $W_1$ may make up large quantities of a good from smaller quantities supplied from $S_1$ and $S_2$, and then ship the large quantity to $C_1$. This recognizes that movement in large quantities is generally more economical than movement in smaller quantities, consistent with the previous assumption concerning the relation of costs to order size (see page 117 above).

It should be noted that Figure 30B, which is isomorphic with Figure 30A, is not redundant, because it does not include an intermediary. The interpretation of $k$-redundancy depends, therefore, upon the nature of the business units associated with the vertices. For this reason, it is necessary to determine the existence of $k$-redundant edges by inspection of the graph, and determination of the business units associated with the vertices. Graph theory does not differentiate between the various vertices in a
A method is developed in the next section to identify only those edges in the graph which have a probability of being k-redundant.

In Figure 30A, one of the edges is k-redundant. Only one chord is possible in Figure 30A, between S\textsubscript{1} and C\textsubscript{1}. Figure 30A contains one of two possible redundant edges and is, therefore, 50 percent redundant. Figure 30D contains intermediary W\textsubscript{2}, which is nonessential. Using the ratio developed in the previous section, Figure 30D has

\[
\frac{v(v - 1)/2 - (v - 1)}{6} = 6
\]

possible chords and one k-redundant edge. It is therefore 

\[\frac{k}{6 + k} = \frac{1}{7} = .14\] or 14 percent redundant, where k is the number of k-redundant edges. Figures 30B, 30C, and 30E are not redundant, nor is Figure 30F, in which W\textsubscript{2} may accumulate goods from S\textsubscript{1} and S\textsubscript{2}, transfer a large quantity of goods to W\textsubscript{2}, who breaks bulk to C\textsubscript{1} and C\textsubscript{2}.

The ratio proposed in the previous section may be adapted to reflect intermediary or k-redundancy. This is accomplished by adding k to the numerator and denominator of the ratio, as was done for Figure 30D above, which preserves the 0 to 1 range of the index.
The Index of Structural Redundancy

The index thus formed may be termed the index of structural redundancy, and stated:

\[
\text{Index of Structural Redundancy} = \frac{\mu + k}{\left[v(v - 1)/2\right] - (v - 1) + k}
\]

or

\[
\frac{e - v + p + k}{\left[v(v - 1)/2\right] - (v - 1) + k}
\]

where

\[(v - p) \leq e \leq v(v - 1)/2\]

\[k = \text{number of } k\text{-redundant edges (by inspection)}\]

It may be applied to distribution structures to measure the degree of redundancy, and has a range from 0 to 1. If the index is multiplied by 100, it may be interpreted as percent redundant. The index of structural redundancy might also be viewed as a measure of the degree of inefficiency in a distribution structure, on the basis of transactions required to effect exchange, since it is a measure of nonessential

\[4\text{This index is a non-planar equivalent to the alpha index (a), proposed by Garrison and Marble, loc. cit.}\]
paths in the structure (redundant edges, or chords, and k-
redundant edges). Thus a structure which is 40 percent re-
dundant, might on this basis also be interpreted to be
40 percent inefficient.

The Index of Structural Efficiency

It is perhaps more useful to utilize the complement
of the index of structural redundancy, or the percent effi-
cient of a structure. Therefore, a structure which is 40
percent inefficient (redundant) could be interpreted as
60 percent efficient, on the basis of transactions required
to effect exchange. The complement of the index of struc-
tural redundancy, or one minus the index of structural re-
dundancy, is termed here the index of structural efficiency,
and stated:

\[
\text{Index of Structural Efficiency} = 1 - \left[ \frac{e - v + p + k}{[v(v - 1)/2] - (v - 1) + k} \right]
\]

It has the same restrictions as the index of structural
redundancy.

The cyclomatic number, index of structural re-
dundancy, and index of structural efficiency are computed for
each distribution structure graphed in Figure 25 (page 139) and listed in Table 8.

**TABLE 8**

**CYCLOMATIC NUMBER $\mu$, INDEX OF STRUCTURAL REDUNDANCY, AND INDEX OF STRUCTURAL EFFICIENCY FOR FOUR DISTRIBUTION STRUCTURES**

<table>
<thead>
<tr>
<th>Graph</th>
<th>$\mu$</th>
<th>Index of Structural Redundancy</th>
<th>Index of Structural Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>25A</td>
<td>16</td>
<td>0.44</td>
<td>0.56</td>
</tr>
<tr>
<td>25B</td>
<td>0</td>
<td>0</td>
<td>1.00</td>
</tr>
<tr>
<td>25C</td>
<td>6</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>25D</td>
<td>0</td>
<td>0</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\[ \mu = e - v + p \]

Index of Structural Redundancy = \[ \frac{\mu + k}{[v(v - 1)/2] - (v - 1) + k} \]

Index of Structural Efficiency =

\[ 1 - \text{Index of Structural Redundancy} \]

All with restrictions:

\[ (v - p) \leq e \leq v(v - 1)/2 \]

\[ k = \text{number of k-redundant edges (by inspection)} \]
Figures 25B and 25D are trees, which are proposed to be most efficient distribution structures, on the basis of required transactions. This is reflected by the indices which indicate such structures are 0 percent redundant or 100 percent efficient. Figure 25C, which is completely connected, represents a distribution structure with maximum redundancy, since it contains all possible redundant edges, and thus is 100 percent redundant or 0 percent efficient on the basis of transactions required. Figure 25A is 44 percent redundant, or 56 percent efficient on the basis of transactions required.

Recapitulation

The Theory of Contact Advantage, as developed by several scholars over a period of years, has been interpreted in the previous chapters by means of graph theory, analyzed, and several resultant implications and extensions discussed. An implication of the theory used to develop the index of contact advantage and index of transactional efficiency is that distribution structures which are efficient on the basis of transactions required to effect exchange may generally be represented by graphs which are trees. Trees
are minimally connected graphs, in which every pair of vertices is connected by one and only one chain, and therefore represent distribution structures with the minimum number of transactions. It was proposed that distribution structures which have tree patterns may evolve because of economies of scale resulting from increased order sizes and reduction in aspects of economic distance.

The index of contact advantage and index of transaction efficiency, analysis has shown, apply only to the specific distribution structure for which they were developed. Moreover, each index compares a certain structure which includes one intermediary to that same structure without an intermediary. Each index applies to a network with not more than one intermediary and has a value which has no upper limit. For these reasons, the indices are not useful for comparison of extant distribution structures.

The index of connectivity was developed from the index of contact advantage, modified to overcome the above deficiencies. This index has a lower limit of 0 and an upper limit of 1, corresponding respectively to a completely unconnected (disconnected) graph and a completely connected graph. It provides an invariant measure of structure which may be applied to any distribution structure. Since a
distribution structure which may be represented by a tree might be viewed as a most efficient structure, the edges in a graph in addition to those in a tree (chords) may represent redundant transactions. Therefore, low values for the index might be viewed as desirable. However, the range of the index includes disconnected graphs, which have fewer edges than do trees, representing distribution structures in which some firms are not served by the channel system. Such distribution structures are therefore not efficient, even though their graphs will have low index values. It was desirable, therefore, to develop an index which makes use of the proposition that distribution structures efficient on the basis of transactions required may be represented by graphs which are trees.

The index of structural redundancy, and its complement, the index of structural efficiency, were developed for this purpose. Distribution structures represented by graphs which are trees have values of 0 percent redundant or 100 percent efficient, and those represented by complete graphs have values of 100 percent redundant or 0 percent efficient. In addition to counting edges which represent transactions which are redundant because they are chords, the indices count edges which represent inclusion of
nonessential intermediaries in a tree, or k-redundant edges. These developed indices have the following advantages over the indices of contact advantage and transactional efficiency:

1. The indices have upper and lower bounds, and provide invariant measures of distribution structure.

2. Values of the indices may be readily interpreted.

3. The indices may be applied to any distribution structures, including those with several levels of intermediaries, and those which vary in some manner from ideal structures.

Graph theory has provided a useful tool with which to analyze the theory of contact advantage, and with which to theorize about the nature of distribution structure. The developed indices are measures which reflect the theoretical concepts of efficiency resulting from the theory of contact advantage. The ultimate test of such theory, however, depends upon its eventual usefulness for analysis of extant distribution structure.
CHAPTER VIII

APPLICATION OF
THE INDEX OF STRUCTURAL EFFICIENCY TO
EXTANT DISTRIBUTION STRUCTURES

Introduction

In addition to providing a possible conceptual insight into the nature of distribution structure, the index of structural efficiency might be put to practical use in analyzing extant distribution structures. There are many possibilities for redundancies in extant distribution structures, that is, in those extant distribution structures which vary to some degree from the most efficient tree pattern. It is potentially useful for the channel analyst to have a means to identify these redundancies, and to have a measure of structural efficiency for comparing one structure to other structures in the economy.

It is not uncommon for a producer to be unaware of
what markets his product is reaching, and consequently, through which channels it has been transferred. Such a situation makes the probability of redundancies very great, since it implies lack of control over channels. Some producers, in an effort to learn where, when, how, and by whom their products have been sold, use the device of enclosing warranty registration questionnaires with their products.¹ The warranty registration must be completed and mailed in by the purchaser to put the product warranty in force, which enables the producer to gather market information.

In other cases, redundancies can result from channel shift, that is, from horizontal channel modifications. For example, baby food was at one time marketed primarily through drug retailers, via drug wholesalers. However, baby food began to appear on the shelves in supermarkets. Due to the demands of the customer group, who desired to purchase baby food with their other food purchases, the supermarkets had purchased stocks from the drug wholesalers, setting up the redundant distribution structure graphed in Figure 31.

An additional example of redundancy in distribution channels is found in the aerospace industry. Prime contractors purchase many thousands of parts, which are stocked in inventory and supplied as needed to test and launch sites. Generally, parts requirements are determined by listing parts numbers or names from drawings of components and sub-systems, which are primarily designed and built by subcontractors. It is not uncommon for identical parts to be
purchased from several different suppliers, stocked separately, and supplied as unique parts to the usage site. This in part results because identical parts often have several different part numbers or may have been given different names by different manufacturers.

In simple distribution structures, it is perhaps relatively easy to determine the degree of redundancy by visual inspection of the structure. This is not, however, the case with more complex distribution structures which are made up of many producers, intermediaries, and customer groups. It would perhaps be very difficult to visually graph a complex distribution structure, that is, to represent the structure by actually drawing the graph. Such a graph would cover a large area of paper, and would perhaps not be particularly useful due to its complexity. Fortunately, the characteristics of graph theory allow more efficient mathematical representations of graphs which represent distribution structures. For example, in Table 5 (page 80 above) a graph was expressed by enumerating edges and vertices and their incidences. The fundamental characteristics of a graph may be stated with a visual graph, with a listing such as in Table 5, and in a perhaps more useful manner, with a matrix.
The Adjacency Matrix

One such matrix\(^2\) is the adjacency matrix \(A\). In a graph \(G\) with \(v\) vertices, the adjacency matrix is a \(v \times v\) matrix where each row and each column corresponds to a specific vertex in the graph. The elements of the matrix are given values of one or zero, in the boolean arithmetic sense, depending upon whether an edge joins the vertex associated with the \(i^{th}\) row to that associated with the \(j^{th}\) column. In other words, the element of the matrix \(a_{ij} = 1\) if there is an edge incident to vertex \(i\) and vertex \(j\), that is, joining \(v_i\) and \(v_j\). The elements of the principal diagonal, or \(a_{ij}\) where \(i = j\), is specified to be one or zero, depending upon the characteristics of the particular problem under investigation. The elements of the principal diagonal for present purposes are specified to be zero, because elements of one are not possible under the axiom system which governs simple graphs.

The distribution structures represented by Figure 30D and 30F, and one which is a redundant variation of

Figure 30 F, are reproduced in Figure 32 along with their associated adjacency matrices. In Figure 32D, for example, $v_3$ is joined with $v_1$, $v_2$, and $v_4$; therefore, the row labeled $v_3$ has elements of one in the columns labeled $v_1$, $v_2$, and $v_4$, and elements of zero in remaining columns.

**Determining the Degree of the Vertices and Edges in the Graph**

It is possible to determine the degree of each vertex directly from the adjacency matrix, by summing the row or column associated with that vertex. For example, the sum of the elements across the row $v_3$ in Figure 32D is three, which is the degree of $v$. The degrees of the vertices of each graph in Figure 32 are listed in Table 9 and summed to indicate the degrees of all vertices in each graph. As was stated previously, it is a theorem of graph theory that the number of edges in a graph is equal to one-half of the sum of the degrees of all vertices in the graph. Therefore, the number of edges in each graph may be calculated directly from the adjacency matrix, and are listed in Table 9.
FIGURE 32
THREE DISTRIBUTION STRUCTURES AND THEIR ASSOCIATED ADJACENCY MATRICES
TABLE 9

DEGREES OF THE VERTICES AND VARIABLES DETERMINED FROM THE ADJACENCY MATRIX

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Degree $\delta$ of the Vertices</th>
<th>Graph 32D</th>
<th>Graph 32F</th>
<th>Graph 32G</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1$</td>
<td></td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$v_2$</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td>$v_3$</td>
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<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$v_4$</td>
<td></td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$v_5$</td>
<td></td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$v_6$</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\Sigma \delta$ in $G$</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Edges in $G$</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p$</td>
<td>1</td>
<td>1</td>
<td>1</td>
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</tbody>
</table>
Determining the Number of k-Redundant Edges

Other relationships are directly attainable from the adjacency matrix. For example, analysis of structures which may have a k-redundant edge (e.g., Figure 32D), results in the knowledge that in order for a k-redundant edge to exist, there must be at least one vertex with a degree of two. For example, in Figure 32D, \( v_4 \) has degree of two, and is the endpoint of a k-redundant edge. All vertices of degree two, however, are not necessarily endpoints of k-redundant edges. For this reason, it is necessary to determine the type of business enterprise associated with vertices directly joined by an edge with the vertex that has degree two. It should be noted, however, that the number of edges which must be examined for k-redundancy are greatly reduced by first examining the degree of vertices. In Figure 32D, \( v_3 \) and \( v_4 \) are both wholesalers, and no intermediate sort is performed by \( v_4 \), which verifies that the edge joining \( v_3 \) and \( v_4 \) is k-redundant. On the other hand, in Figure 32G, \( v_1 \) and \( v_5 \) both have a degree of two, but there are no k-redundant edges as these vertices are joined to a vertex where an intermediate sort may occur. The edge \( v_1 \) to \( v_5 \) is, however, a chord, which is a redundant edge, but not k-redundant, and will
therefore be counted by \( \mu \) in the formula for the index of structural efficiency. The number of \( k \)-redundant edges for the graphs are listed in Table 9.

**Determining the Number of Components in the Graph**

**Test for the Zeroth Betti Number \( p > 1 \)**

It is possible to test by means of the \( A \) matrix whether the graph represented by the matrix has a Zeroth Betti number \( p > 1 \), that is, whether it consists of more than one component. This may be accomplished by computing \( A^{v-1} \), that is, raising the matrix \( A \) to the \( v - 1 \) power (powering the matrix). If \( p = 1 \), all off-diagonal elements \( a_{ij} \) in the \( A^{v-1} \) matrix will be non-zero, while if \( p > 1 \), some elements will be zero. Powering the adjacency matrix manually is a very laborious process for even a relatively small graph; however, a computer can be programmed to accomplish this task. Powering the matrix has other important implications, which are briefly discussed in the following chapter. Since in many analyses powering \( A \) is desirable, it thereby provides as a by-product the test to determine whether \( G \) has \( p > 1 \).
The Value of the Zeroth Betti Number \( p \)

If \( p > 1 \), it is still necessary to determine its value. The following matrix manipulation determines the value of \( p \). It may be applied directly if it is not desirable to test for \( p > 1 \) by powering the adjacency matrix, otherwise only if the test indicates that \( p > 1 \).

An elementary matrix is one obtained by performing elementary operations on the unit (identity) matrix \( I \). For example, for a \( 3 \times 3 \) unit matrix, the following elementary matrices result from interchanging rows.

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
\end{bmatrix}
\]

Generally, there are \( i = p! \) elementary matrices, where \( p \) is

the rank of the unit matrix, e.g., $3! = 6$ elementary matrices in the example above.

The inner products of the rows are zero and the length of the rows are unity, therefore, each elementary matrix is orthogonal. Calling the second elementary matrix above $\Pi_2$, the result of $\Pi_2 \Pi_2^T$ should therefore be $I$ (where superscript $T$ means transpose, i.e., the rows and columns of the matrix are interchanged).

$$\Pi_2 \Pi_2^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

The elementary matrix $\Pi_2$ is formed by interchanging rows two and three. If $A$ is premultiplied by $\Pi_2$, the resultant matrix is $A$ with rows two and three interchanged. If $A$ is postmultiplied by $\Pi_2^T$, columns two and three are interchanged.

Therefore, $\Pi_i A \Pi_i^T$ results in interchanging rows and columns of $A$, and $\Pi_i A \Pi_i^T = AI$. The transformation of the graph represented by $\Pi_i A \Pi_i^T$ maintains, therefore, an isomorphic relation with $A$. The other elementary matrices operate similarly.
Consider the distribution structure which is graphed and stated by the adjacency matrix in Figure 33.

If $A$ is premultiplied by an elementary matrix $T$ such that the first row and fifth row are interchanged, and the product postmultiplied by $T^T$ such that the first and fifth
column are interchanged, the following matrix results:

\[
\Pi A \Pi^T = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0
\end{bmatrix}
\]

The objective of this operation was to bring down the zeros, if possible, into the lower left hand corner of the matrix. Then \( \Pi A \Pi^T \) is partitioned such that \( A_1 \) and \( A_4 \) are square matrices along the diagonal, while \( A_3 \) and \( A_4 \) are not necessarily square:

\[
\Pi A \Pi^T = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0
\end{bmatrix} = \begin{bmatrix}
A_1 & A_2 \\
A_3 & A_4
\end{bmatrix}
\]

Note that \( A_3 \) has all zero elements. If it is possible to find a \( \Pi \) that will transform a matrix \( A \) into this
form, $A$ is said to be decomposable. If no such $T$ makes the transformation, $A$ is of course indecomposable. Note that $A_1$ and $A$ are indecomposable. Since $A$ is decomposable into two components, $p = 2$. The partition of the graph corresponds to the two components of the graph in Figure 33. Indecomposable graphs have one component, i.e., the graphs of Figure 32, each of which has $p = 1$.

Figure 34 illustrates a distribution structure with three components, and its associated $A$ matrix partitioned to indicate a submatrix which is also decomposable, i.e., $p = 3$. Note that labeling the vertices to group components in this example eliminated the necessity of manipulation. The matrix manipulation developed in this section may become laborious by hand, but computers may be programmed to perform the manipulation efficiently.

---

4 Taro Yamane, *Mathematics for Economists* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962), p. 336. The partitioning method is, generally, to form submatrices in the all corner of the matrix, beginning with a $1 \times 1$ submatrix, and to increase the rank until either the condition is satisfied or the submatrix exhausts the matrix.
THREE-COMPONENT DISTRIBUTION STRUCTURE AND PARTITIONED ADJACENCY MATRICES
Potential for Distribution Structure Improvement

By constructing and manipulating the adjacency matrix $A$, a channel analyst is able to generate the values of variables needed to calculate the index of structural efficiency:

$$
\text{Index of Structural Efficiency} = 1 - \frac{e - v + p + k}{[v(v - 1)/2] - (v - 1) + k}
$$

where

- $e$ = edges in graph
- $v$ = vertices in graph
- $p$ = components of graph
- $k$ = number of $k$-redundant edges, by inspection of 2 $\delta$ vertices

and

$$(v - p) \leq e \leq v(v - 1)/2$$

The values for the variables are listed in Table 9 (page 171) for the distribution structures graphed in Figure 32 (page 170).

The adjacency matrix is constructed by determining connections between vertices, which in the present analysis are enterprises in an enterprise channel group. It is
therefore possible for a distribution channel analyst to construct $A$ from a knowledge of immediate customers of each channel group member, without the necessity of first drawing the graph. An important assumption is that the members of the enterprise channel group will cooperate by disclosing the identity of their customers. Possible prerequisites for such cooperation are not examined here, but are well-documented elsewhere. It is sufficient for present purposes to assume that the information is obtainable because of factors such as a channel member having sufficient power to require disclosure, mutual expectations by the channel members that cooperation can increase channel group performance, or confidential treatment of such information by a party such as a trade association.

Assume that Manufacturers $M_1$ and $M_2$ in Figure 7 (page 55, reproduced in Figure 35) are typical producers who have concerned themselves only with the links in the channels directly adjacent to them, and that the assumptions

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6 McVey, op. cit., p. 29.
FIGURE 35

AN ENTERPRISE CHANNEL GROUP WITH ORDER SIZE
AS LENGTH OF THE EDGES.
on pages 116-117 above are in effect. The vertical sequences in the distribution channels below the first level quite probably "just grew like Topsy, without direction or intent . . . ," \(^7\) because \(M_1\) and \(M_2\) were concerned only with selling to the first level intermediaries, and not with the effectiveness of the enterprise channel group as a system. However, \(M_1\) and \(M_2\) learn that there may be opportunities for system improvement through channel leadership, and arrange for a distribution channel analysis.

The channel analyst gathers information about the members of the channel group and their customers, and the order sizes associated with each transaction (connection) between channel group members. In this manner, the \(m \times m\) adjacency matrix shown in Figure 36 is formed, where the \(a_{ij}\) entries are the order sizes associated with each edge in the network. It is not necessary to visually graph, or chart, the enterprise channel group in order to specify its adjacency matrix. In fact, the adjacency matrix can be stored in computer memory. However, for comparison, the enterprise channel group is shown in Figure 35. In addition, the order size associated with each transaction (edge) \(^7\) *Ibid.*, p. 28.
### FIGURE 36

**ADJACENCY MATRIX FOR AN ENTERPRISE CHANNEL GROUP**

<table>
<thead>
<tr>
<th></th>
<th>M₁</th>
<th>M₂</th>
<th>W₁</th>
<th>W₂</th>
<th>W₃</th>
<th>W₄</th>
<th>R₁</th>
<th>R₂</th>
<th>R₃</th>
<th>R₄</th>
<th>R₅</th>
<th>R₆</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
<th>C₆</th>
<th>C₇</th>
<th>C₈</th>
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<td>M₂</td>
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*Degree 6*  
\[ \sum a_{ij} = 144 \]

\[ \sum \delta = 64 \]
is shown by numbers on the edges, i.e., the lengths of the edges. The market supply and demand conditions represented by this distribution structure are shown in Table 10.

TABLE 10

MARKET CHARACTERISTICS AND CALCULATED VARIABLES
FOR AN ENTERPRISE CHANNEL GROUP

<table>
<thead>
<tr>
<th>Market Characteristics</th>
<th>Supply</th>
<th>Demand</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_1 = 12$ units</td>
<td>$C_1 = 2$ units</td>
<td>Vertices $v = 20$</td>
</tr>
<tr>
<td></td>
<td>$M_2 = 14$ units</td>
<td>$C_2 = 4$ units</td>
<td>Edge $e = 32$</td>
</tr>
<tr>
<td></td>
<td>$C_3 = 4$ units</td>
<td></td>
<td>Zeroth Betti Number $p = 1$</td>
</tr>
<tr>
<td></td>
<td>$C_4 = 4$ units</td>
<td></td>
<td>$k$-Redundant Edges $k = 0$</td>
</tr>
<tr>
<td></td>
<td>$C_5 = 4$ units</td>
<td></td>
<td>$\sum \sum$ Order Sizes $= 144$ Units</td>
</tr>
<tr>
<td></td>
<td>$C_6 = 4$ units</td>
<td></td>
<td>Average Order Size $= 144/32 = 4.5$ Units/Transaction</td>
</tr>
<tr>
<td></td>
<td>$C_7 = 2$ units</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_8 = 2$ units</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All of the variables needed to compute the index of structural efficiency are directly obtainable, or may be computed from, the adjacency matrix. The number of vertices in the graph is directly obtainable from the adjacency.
matrix, since \( v = m \). Similarly to previous examples, the degree of each vertex may be computed by counting the number of non-zero entries in each row of the adjacency matrix, from which the number of edges may be computed:

\[
e = \sum \delta/2 = 64/2 = 32 \text{ edges.}
\]

In addition, the order sizes associated with each edge are summed across rows to determine the sum of orders \( \sum a_{ij} \) associated with each vertex. The sum of the column formed in this manner is the sum of all orders in the enterprise channel group, \( \sum \sum a_{ij} \), from which the average order size is computed:

\[
\frac{\sum \sum a_{ij}}{e} = \frac{144 \text{ Units}}{32 \text{ Edges}} = 4.5 \text{ Units/Edge (i.e., Transaction)}
\]

Vertices \( C_2, C_4, \) and \( C_8 \) each have \( \delta = 2 \), as shown in Figure 36, so these vertices must be examined to determine if they are endpoints for \( k \)-redundant edges. For example, \( C_2 \) is joined to \( R_1 \) and \( R_2 \), and therefore the edges joining \( C_2 \) to \( R_1 \) and \( R_2 \) do not exhibit \( k \)-redundancy. No elementary matrix can be found which will allow a partition of the matrix into the form

\[
\begin{bmatrix}
A_1 & A_2 \\
\hline
A_3 & A_4
\end{bmatrix}
\]
where \( A_1 \) and \( A_4 \) are square matrices and \( A_3 \) has all zero entries, therefore \( A \) is indecomposable and the Zeroth Betti number \( p = 1 \). The values of the variables computed from the adjacency matrix are listed in Table 10, and provide the necessary information for calculating the index of structural efficiency:

\[
\text{Index of Structural Efficiency} = 1 - \frac{\alpha - \upsilon + p + k}{\left[\frac{\upsilon(\upsilon - 1)}{2}\right] - (\upsilon - 1) + k} = \frac{32 - 20 + 1}{\left[\frac{20(19)}{2}\right] - 19} = 0.924 \times 100 = 92.4 \text{ percent efficient on the basis of transactions required}
\]

The Manufacturers \( M_1 \) and \( M_2 \) decide that they should attempt to improve the efficiency of the enterprise channel group through which their products are marketed. They agree that their present customers should all be served, and that each manufacturer will retain his present market share. The channel group improvement is to be accomplished by reallocating goods among intermediaries. They also have learned that a distribution structure which may be represented by a graph which is a tree represents the most efficient distribution structure, i.e., a structure that is
100 percent efficient, on the basis of transactions required to effect exchange. There are three tree patterns, shown in Figure 37, which may be formed to satisfy supply and demand requirements of the system. A comparison of the average order sizes for these tree pattern structures to that of Figure 35 is shown in Table 11, along with the number of transactions required to effect exchange.

**TABLE 11**

**COMPARISON OF DISTRIBUTION STRUCTURE VALUES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Required Transactions</th>
<th>Average Order Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>37A</td>
<td>12</td>
<td>6.5 Units/Transaction</td>
</tr>
<tr>
<td>37B</td>
<td>14</td>
<td>5.6 Units/Transaction</td>
</tr>
<tr>
<td>37C</td>
<td>14</td>
<td>5.6 Units/Transaction</td>
</tr>
<tr>
<td>35</td>
<td>32</td>
<td>4.5 Units/Transaction</td>
</tr>
</tbody>
</table>

Average order size, 37A =

\[
\frac{12 + 14 + 14 + 12 + 2 + 4 + 4 + 4 + 4 + 4 + 2 + 2}{12} = \frac{78}{12} = 6.5 \text{ Units/Transaction}
\]

Figures 37B, 37C, and 35 similarly computed
FIGURE 37

DISTRIBUTION STRUCTURES REPRESENTED BY TREES DERIVED FROM AN ENTERPRISE CHANNEL GROUP.
GRAPH C

FIGURE 37 (continued)
All of the distribution structures, which are represented by graphs which are trees, are more efficient on the basis of required transactions, than the original distribution structure in Figure 35. A distribution structure represented by the tree in Figure 37 that provides the number of intermediaries necessary to minimize total transaction costs due to scale economies, as shown in Figure 22 (page 123 above), would, on this basis, be selected for the improved distribution structure.

Recapitulation

It is unnecessary to draw the graph of a complex distribution structure, since the graph may be represented directly in matrix form. One such matrix is the adjacency matrix $A$, formed by matrix entries of one for each adjacency relationship between vertices, and zero otherwise. All of the values of variables necessary to compute the index of structural efficiency are obtainable directly from $A$, or from manipulating $A$: number of vertices, number of edges, number of components, and number of $k$-redundant edges in a graph.
Because of the manipulative advantages of matrix algebra, large extant distribution structures may be analyzed to determine their degree of structural efficiency, which is based upon the fewness of transactions required to effect exchange in the distribution structure. If the index value for the structure is low, indicating a large number of redundant transactions, it may be desirable to analyze the distribution structure for possible improvement, in conjunction with other criteria of efficiency.
CHAPTER IX

SUMMARY AND IMPLICATIONS

Summary

This dissertation has explored how the distributive network might be decoupled into segments called enterprise channel groups for analysis, and has explored the extent to which graph theory might provide an effectual means to conceptually analyze aspects of distribution channel structure contained in the Theory of Contact Advantage. The Theory of Contact Advantage is based primarily upon the proposition that inclusion of intermediaries in a distribution structure reduces the number of transactions required to effect exchange, and that a distribution structure with the least number of transactions represents, on this basis, an efficient distribution structure. Scholars have proposed the index of transactional efficiency and the index of contact advantage as measures of this aspect of efficiency in
distribution channel structures. Each index is derived, however, for a specific distribution structure, allows only one intermediary in the structure, and has no upper limit. For these reasons, the indices do not provide measures which are useful for analysis of extant distribution channel structure, although they are useful to illustrate the underlying theory.

The basic proposition contained in the Theory of Contact Advantage has been analyzed and interpreted in graph theory terms to determine its implications for analysis of distribution channel structure. Basically, in graph theory terms, a distribution structure with the least number of transactions required to effect exchange, i.e., on this basis, a most efficient structure, may be represented by a graph which is a tree. Based upon this interpretation, an index of structural efficiency has been developed, which measures the degree to which distribution structures conform to this most efficient tree form. A distribution structure which may be represented by a graph which is a tree is 100 percent efficient on the basis of transactions required to effect exchange, while a distribution structure represented by a graph with all possible edges, interpreted as
having the maximum number of redundant transactions, is on this basis 0 percent efficient.

In order for graph theory to be useful for analysis of extant distribution structure, it must be capable of dealing with the complexities of real world distribution systems. This dissertation has explored how the close relation of matrix algebra to graph theory might be utilized, so that the manipulative advantages of matrix algebra might become attendant to analysis of distribution channel structure. An adjacency matrix A, which is mathematically equivalent to a graph, may be constructed directly from empirical observation of the distribution structure to be analyzed, without the intermediate step of representing the structure visually as a graph. In fact, the adjacency matrix may be contained completely in computer memory; thus, it is unnecessary to represent visually the adjacency matrix itself. The values of all variables contained in the index of structural efficiency are directly obtainable from the adjacency matrix or from manipulation of the adjacency matrix. The adjacency matrix has been constructed for an enterprise channel group, and its use to provide the information for analysis of distribution structure illustrated.
Graph theory has provided a useful tool with which to analyze conceptually aspects of distribution channel structure, and, because of its relation to other branches of mathematics, graph theory may provide a means to deal with complex extant distribution structures comprising many business enterprises and their interconnections. The scope of this inquiry, however, was by necessity limited in order to provide an area researchable within time and energy constraints of the dissertation. As elaborated in the limitations delineated in Chapter II, conclusions of this inquiry must therefore be viewed as hypothetical, pending empirical testing. In addition, it should be reiterated that it was necessary to modify the index, stated in graph theory terms, to accommodate an aspect of structure which may be unique to analysis of distribution structures. That aspect was the possibility of k-redundant edges in a graph which is a tree, as elaborated in Chapter VII. Since graph theory does not differentiate between the identities of the enterprise associated with vertices in a graph, it is necessary to examine vertices of degree two (all k-redundant edges join two vertices, at least one of which must have degree two) to determine if an adjacent edge is k-redundant. Although it was possible in this inquiry to incorporate this aspect
of distribution channel structure into the index of structural efficiency in a manner that preserves the upper and lower limits of the index, it may indicate a possible limitation of graph theory for analysis of other aspects of distribution structure, or at least that caution should be used in its application.

The index of structural efficiency may be applied unambiguously to all distribution structures, and has a determinate range which readily enables direct comparison of structures. The index is, therefore, more general in its application than the indices formulated previously, and provides a potentially useful measure for analysis of extant distribution structure. In addition, graph theory has been useful in formulating an explanation for evolution of intermediaries in distribution structures which are nonredundant, as a means for validating, on a molar basis, the logic of the proposition pertaining to fewness of transactions in distribution structures. To this extent, graph theory has resulted in making concepts contained in the Theory of Contact Advantage more explicit, and more potentially useful for empirical analysis of extant distribution structures. This inquiry may provide a conceptual groundwork for such empirical analysis, since it is susceptible to cumulative
refinement. Through research and debate, the concepts and methods begun here might be sufficiently developed to provide a means for future empirical study of distribution channel structure.

**Implications for Future Research**

This section is provided to lend perspective to this inquiry by relating it to areas of possible future research. The concepts developed here are capable of cumulative refinement, or fit into a hierarchy of complexity. The dissertation research falls within the area of static structure analysis. Since it is a partial analysis, limited to a specific theory, a necessary first step will be to expand this beginning into a more comprehensive analysis. In order to develop quantitative explanations of system behavior, it is first necessary to understand structural implications of the system.

**Structural Implications**

There are certain graph theory concepts contained in this dissertation that have structural implications which may merit further analysis. For example, the degree of a point is to some extent a measure of the importance of an enterprise, represented by the point, to an enterprise
channel group. Thus, a wholesaler represented by a point with a high degree, representing many contacts with other enterprises in the system, may be more important to the system than a wholesaler represented by a point with a lower degree. Similarly, the concept of distance may be useful in assessing the importance of an enterprise to an enterprise channel group. An enterprise which is connected to the other enterprises in a group by chains containing the fewest number of edges might be considered the most accessible in the system. Specific indices have been developed to measure the accessibility of points to the system. An enterprise which is most accessible to the other enterprises in the network, or the central place, may be in the most favorable position to assume the role of the centralizing organization, or channel leader, in the network.


Davidson, Stern, and Phillips have discussed the nature of the centralizing organization. The accessibility of an enterprise to the other enterprises in an enterprise channel group may be determined by powering the adjacency matrix associated with the enterprise channel group, in the manner discussed by Garrison and Marble.

The concepts of degree and distance are examples of the many graph theory concepts which might be examined for application to analysis of distribution channel structure. These concepts may provide useful insights for understanding structural implications in distribution channel systems. In addition to graph theory concepts introduced in this dissertation, there are of course many other concepts which may be receptive to future research.

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7 W. L. Garrison and D. F. Marble, "The Structure of Transportation Networks" (Northwestern University, Evanston, Illinois, 1961), pp. 30-34. (Mimeographed.)
Static/Dynamic Analysis

Once the system has been more completely defined, static/dynamic analysis may become possible. In this level of analysis, units of measurement could be defined for the flows in the system. Attention may then be focused on the flows through the system, where previously analysis was limited to relationships, or the network itself. The characteristics of a distribution structure might be defined in terms of its ability to transmit these flows efficiently. Analysis, however, would remain close to the static level, as motive or driving forces are not specifically included in the model at this stage. Flow analysis requires specification of the direction of the edges which represent flows, which is not possible under the axiom system for ordinary graphs used in this inquiry. Graphs in which edges are directed, or digraphs, were introduced briefly in Chapter IV and are governed by the axiom system developed in the branch of graph theory called digraph theory.

An assumption was made in this inquiry that a transaction simultaneously effects the title, logistics, and payment flows. This simplification allows analysis of an enterprise channel group in terms of a single graph, as
relationships between the enterprises can be viewed in terms of the connections which transmit the three flows. In extant distribution structures, this assumption may be valid only in certain cases; it is quite likely that title, logistics and payment flows will occur at different times and will involve different sets of enterprises. For example, in many cases not all enterprises who hold title to the product will physically handle the product. For this reason, it may be necessary to decouple the enterprise channel group into separate flow networks, one for each of the three flows, as elaborated in Chapter III. Structural implications of these flow networks may then be analyzed in terms of the specific flow through that flow network. The criteria which must be chosen for evaluating each flow network will of course depend upon the specific characteristics for that network.

The axioms governing simple or ordinary graphs, which have been used in this inquiry, permit at most one edge between a given pair of points. This feature of ordinary graphs imposes limitations on the ability of the distribution channel analyst to represent the three flows on one graph, unless, as was done in this inquiry, the simplifying assumption is made that the flows occur simultaneously. The separate evaluation of each of the flow networks may be
conceptually valuable; however, at some time it may become possible to consolidate the various flow networks into one representation of the enterprise channel group by allowing more than one edge between a pair of points. A beginning has been made in graph theory to deal with these types of problems, called coloring problems, although little theory has yet been developed.\(^8\)

**Empirical Analysis**

In a third level of development, an analogue or simulation model might be constructed. Various levels of flow could be conducted through the system, and the performance of the system measured. This stage would tend to determine the internal consistency of the system, and the model might become capable of predictive use, in that the actual performance of the elements of the system could be compared with expected results determined from a prior knowledge of the inputs. The model, however, would remain essentially abstract, since inputs at this stage would be arbitrarily selected.

The validated model might then be tested by means of empirical research. This empirical research depends, in part, upon refinement of necessary data gathering techniques which primarily involve distribution cost analysis. These techniques of distribution cost analysis are based upon the premise that all marketing expenses are incurred to insure performance of the marketing functions. Functions, in turn, may be related to a specific flow through a network, for example, the logistics flow is accomplished due to the performance of the transportation and storage functions. The cost of performing the functions attendant to a flow may be in this manner related to the volume of that flow through the network. Thus, a cost per unit of throughput may be determined for a specific distribution structure, for

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10 Longman and Schiff, op. cit., p. 106.
example, the cost per unit of product due to performance of transportation and storage functions in the logistics flow network. In this manner, propositions about efficiency of distribution structures, such as the ones contained in this dissertation, may be tested for validity. If the outputs of the model are verifiable by costing studies and observation, it may be useful as a predictive model.

The model developed at this point would primarily describe the more mechanistic aspects of extant channel performance. The reasons for changes in behavior, or changes in expectations, would not be explicitly considered. However, what is learned by behavioral study of distribution channels might eventually be combined with this more mechanistic structural analysis. For example, graph theory has been applied to the study of role structures in organizations.\textsuperscript{11} It has been proposed that role and role theory may be important to an understanding of the behavior of firms in distribution channels,\textsuperscript{12} and graph theory might therefore


be useful in this analysis. If behavioral and structural analyses can be coordinated, it may be possible to formulate a dynamic multi-equation model in terms of behavioral as well as structural equations.
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