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AN ANALYSIS OF THE OBJECTIVES OF A FIRST YEAR CALCULUS SEQUENCE, A TEST FOR THE ACHIEVEMENT OF THESE OBJECTIVES, AND AN ANALYSIS OF RESULTS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * * *

The Ohio State University
1967

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CHAPTER I

INTRODUCTION

Background for the problem

Throughout the past twenty years a great deal of concern has been expressed about the need for changes in mathematics education. One result of this interest has been the creation of new programs and new texts designed to introduce into the curriculum those revisions considered most necessary by various authorities. While this emphasis on new material has been invaluable in attempting to produce an immediate remedy for the problems of mathematics education, it appears to this investigator that not enough attention has been paid to the question of objectives at the classroom level. Vaguely stated and broadly conceived objectives do not appear to offer the classroom teacher any assistance in planning experiences by which the objective may be achieved or in constructing tests to evaluate the students' achievement. Such statements of objectives seem to have produced a lack of understanding and awareness on the part of the classroom teacher concerning the importance of objectives in placing the proper perspective on new content within a course and within the total curriculum.
The lack of explicitly defined sets of objectives extends to the college level. Most college texts and college bulletins restrict themselves to an outline of content to be covered, or to a general statement of objectives which is of little value to the college teacher in the preparation of learning experiences and tests.

Since the students' backgrounds are varied, and contacts between teacher and student relatively infrequent, the student is generally evaluated solely on the basis of test results; this is particularly true in mathematics because of the widespread belief that the material is of an objective nature. However, if the tests are to have validity, they must measure the extent to which the objectives of the course have been achieved. The college teacher of mathematics, concerned with assigning a grade, usually constructs an instrument to test for the acquisition of the facts and techniques of the course, rather than first defining a set of objectives and then constructing a test to measure the achievement of these objectives. As a result, a large part of the students' progress is not evaluated and many worthwhile objectives of a course are ignored by the teacher.

Unfortunately it is extremely difficult to prevent mechanical skills and rote responses from dominating any examination. The difficulty of formulating questions testing understanding and creative ability is magnified by the uncontrollable tendency of teachers to "teach for the exams"(33;20).\footnote{The first set of digits represents the Bibliography entry; the second represents the page number.}
This emphasis on tests for the purpose of assigning a grade may also affect the way in which the student perceives the objectives of a course. While it is generally agreed that an awareness of the objectives on the part of the student makes for a better learning situation, the student in college mathematics courses must often discover the objectives for himself, through the instructor's teaching or his tests. If the student is evaluated solely on the basis of test results, he soon learns that the "important" objectives are those which the tests measure.

**Definition of terms**

Many educators disagree on the way in which educational objectives should be defined. Among those who believe that educational objectives should be stated in terms of student behavior are Dressel, Furst, and Mager. Dressel defines objectives as "explicit statements descriptive of the competencies and the traits which a program intends to develop in those who engage in it" (10;28). Later he states:

> The objectives of instruction must point directly to the students, to the changes which the instructor hopes will occur in them. . . . Objectives should indicate the student behavior which is desired and do so in such a way that it can be identified as it gradually develops (10;35,36).

According to Furst:

> an educational objective may be defined as a desired change in behavior. . . . It is not something the instructor does; it is not the same as course content; and it is not a fundamental life value (16;30).
Mager believes:

1. An instructional objective is a statement that describes an intended outcome of instruction.

2. An objective is meaningful to the extent it communicates an instructional intent to the reader and does so to the degree that it describes or defines the terminal behavior expected of the learner (25;43).

An educational objective will be defined in this study in the sense in which it is defined above, i.e., an educational objective is a statement of a desired change in the student as a result of exposure to the educational process, expressed in terms of the behaviors characteristic of its achievement.

The taxonomy or the taxonomy of educational objectives will mean the taxonomy of educational objectives developed by Bloom (cognitive domain) and Krathwohl (affective domain).

The cognitive domain is defined as that part of the taxonomy which includes "those objectives which deal with the recall or recognition of knowledge and the development of intellectual abilities and skills" (2;7).

The affective domain is defined as that part of the taxonomy which deals with "objectives which describe changes in interest, attitudes, and values, and the development of appreciations and adequate adjustment" (2;7).

The definitions given above will be used throughout the remainder of this work.
Statement of the problem

The first year of the calculus sequence at The Ohio State University involves the study of the fundamental portions of differential and integral calculus, the investigation of special functions, and the development of integration techniques. This material is developed in a two quarter sequence designated as Math 440 and Math 441. During the second year a treatment of polar coordinates, vectors, three dimensional analytical geometry, and linear algebra is presented together with the study of partial derivatives, multiple integrals, and infinite series. This material is developed in a two quarter sequence designated as Math 542 and Math 543. At that point the required program in the calculus terminates and the student may take any one of a number of mathematics courses which builds up his knowledge of the calculus, e.g., differential equations or advanced calculus.

It seems logical to assume that there are objectives peculiar to the calculus sequence. It also seems logical to assume that there are objectives common to the calculus and other branches of mathematics. If these objectives can be identified, the question naturally arises as to whether or not a testing device exists or can be constructed which effectively measures the achievement of these objectives.

The purpose of this study is to identify a set of objectives for the first year calculus sequence at The Ohio
State University, to construct an instrument to test for the achievement of these objectives, and to compare and contrast student rating of objectives with faculty rating.

Description of research procedure

A collection of proposed objectives was created using the taxonomy and various other sources. Eight members of the mathematics faculty at The Ohio State University were asked to rate these statements. The results of this rating were the basis for selecting several objectives for which test items were constructed. A number of these items, accepted by a panel of faculty members and colleagues, were used to create a preliminary test form which was administered to fifty-seven students at the Mansfield and Lima branches of The Ohio State University during the spring quarter of 1967. An analysis of the test results was conducted and this analysis used as a basis for revisions. The revised test form was administered to a group of one hundred ninety-seven students on the main campus of The Ohio State University during the final week of the spring quarter of 1967. This group is the population on which the conclusions of this study will be based.

Limitations and delimitations

Among the limitations and delimitations of the study, the most restrictive are:

1. The small number of faculty members used to rate
the proposed objectives. It should be remembered, however, these are all the members of the mathematics faculty involved in teaching or planning the calculus sequence.

2. The sample population may not have been representative of the total population. While no attempt was made to select the sample at random, the students were assigned to course sections in an arbitrary manner and the sections used for the study were chosen on the basis of availability.

3. The test was administered in the last week of the quarter before the final examination and after a regularly scheduled examination. As a result, the students may not have been highly motivated to participate in the study. The students were told, in an effort to improve their motivation, that a good score on the test would be taken into consideration in borderline grade situations; a poor score on the test would not be taken into consideration in assigning course grades.

4. The scoring of the second half of the test was less objective than the first because of its problem-solving nature. To make the scoring more reliable each problem was assigned a value of four or less and three scorers were employed rather than one. The method of assigning a score for each problem will be discussed in Chapter III.

5. The conclusion of the first year calculus sequence was chosen as the most likely place in the sequence to administer the test for the achievement of the first year
objectives for two reasons. First, the students would have been exposed by that time to most of the fundamental ideas and techniques of elementary calculus. Second, the student population of the second year sequence, which is composed primarily of mathematics and engineering majors, is more select than the population of the first year sequence. This selectivity in the sample population might have exaggerated the degree of achievement of the objectives.

6. The entire range of objectives relating to teaching methods and pedagogy were omitted from consideration on the assumption that their inclusion would make the scope of this study too broad and unwieldy.

7. The inclusion of content objectives, e.g., the student should know and be able to use the chain rule for differentiation, was rejected because most elementary calculus textbooks agree strongly on content (although not necessarily on arrangement) of the fundamental portions of differential and integral calculus; for a typical sample of content, see Fisher and Ziebur (15) or Thomas (41).

8. It became apparent in attempting to construct a set of objectives for the first year calculus sequence at The Ohio State University that the heterogeneity of the student population might create a problem. The same sequence of courses (Math 440 and Math 441) is taken by students planning to major in mathematics, mathematics education, the sciences and engineering, and the liberal arts. While there
are many objectives which would apply to all of these groups, there are others which are uniquely suited to only one group or whose importance might vary from group to group. Because of the limitations of time and of the scope of this study, it was decided to construct a set of objectives, each of which is important to all the subgroups of the population. Different degrees of emphasis might then be placed on certain objectives in the set for each of the subgroups.

9. Finally, the objectives for which the test items were constructed are only the more important objectives which the faculty selected. These objectives were selected on the basis of their importance at The Ohio State University and may not be applicable at other colleges and universities.

Overview of the study

The remainder of the study is organized as follows: Chapter II will be devoted to a review of the literature; in Chapter III the research procedure will be presented and discussed; Chapter IV will consist of an analysis of the data collected; finally, in Chapter V conclusions based on the analysis of the data will be presented and suggestions for further study and related research will be made.
CHAPTER II

REVIEW OF THE LITERATURE

The literature on evaluation, item construction, and test construction is quite extensive and any review brief enough to be accommodated within this study would be virtually useless. The following books proved to be most helpful:

1. Furst, Constructing Evaluation Instruments
2. Lindquist, Educational Measurement

The plan of this chapter is to review the literature related to the taxonomy, and its use to generate and classify objectives and test items. Attention will be concentrated on Handbook I (cognitive domain) of the taxonomy because its earlier publication has permitted it to be more thoroughly studied than Handbook II (affective domain). An extensive, annotated bibliography for Handbook I may be obtained from Dr. Richard Cox, Assistant Professor of Education, University of Pittsburgh.

A number of taxonomies or classification systems have been proposed for the domain of education and research on learning and teaching. Zinn (42) proposes a two dimensional model to express the relationship among these systems.
The first dimension concerns the purpose or use of the system and may be grouped into the purposes of (a) communication among educators, students, and parents; (b) specification of instructional activity as might be required in the writing of a textbook or examination; and (c) classification and organization of training procedures as well as research results. The second dimension concerns the referents for the language of the taxonomy and may be grouped into the referents of (a) common English; (b) technical English; and (c) symbolism or diagramatic codes. A particular taxonomy may be located in this two dimensional matrix by asking the question, "What does who want to achieve with what material?" For example, Gagne (17) has a classification system in which he refers to stimulus processing. If a researcher wanted to use this system to organize the literature on learning in the social sciences, this system would be located in class (c) of the first dimension and class (b) of the second dimension of Zinn's matrix. The taxonomy of Bloom (cognitive domain) and Krathwohl (affective domain), for the purposes of this study, would be located in class (b) of the first dimension and class (a) of the second dimension.

Bloom et al., have attempted a classification of educational objectives into three broad areas, cognitive, affective, and psychomotor. The cognitive domain, to which those objectives which deal with the recall or recognition of knowledge and the development of intellectual abilities and
skills (2;7) are related, is subdivided as follows: level 1--knowledge, level 2--comprehension, level 3--application, level 4--analysis, level 5--synthesis, level 6--evaluation. The affective domain, to which those objectives which deal with interests, attitudes, appreciations, values, and emotional sets or biases (21;7) are related, is subdivided as follows: level 1--receiving, level 2--responding, level 3--valuing, level 4--organization, and level 5--characterization by a value complex. Condensed descriptions of the analyses of the cognitive and affective domains are found in Appendix A. The psychomotor domain, to which those objectives relating to the manipulative or motor skill areas (2;7), are related, has not yet been published by the authors of the taxonomy.

The taxonomy is designed to classify objectives stated in terms of intended student behaviors because the authors believe that the changes occurring as a result of educational experiences "can be represented by a relatively small number of classes of behavior." It does not attempt to classify "the instructional methods used by teachers or the ways in which teachers relate themselves to the students or the instructional material" or "the particular subject matter or content being studied. . ." (2;7).

The analysis of the cognitive domain is apparently based on the assumption that the arrangement of levels is hierarchal from simple to complex and from concrete to
abstract, and that the heirarchy of levels is cumulative, i.e., any given category consists of the processes stipulated by all lower-level categories and, in addition, a process which is unique to it from the standpoint of lower order categories. For consistency, an objective should be placed in the most complex class which is appropriate and relevant to the behavior implied by that objective. Studies conducted by Stoker (38) and by Stoker and Kropp (39) support the hierarchal hypothesis. Studies conducted by Zinn (43) and Smith and Paterson (38) suggest that it is impossible to substantiate a simple-to-complex continuum. Stoker and Kropp (39) present data which support the cumulative hypothesis.

The analysis of the affective domain is structured along a continuum according to the process of internalization which "represents a continuous modification of behavior from the individual's being aware of a phenomena to a pervasive outlook on life that influences all his actions" (21;33). No research was found testing this method of structuring.

The unity of the taxonomy as a single classification system is emphasized by the following parallels drawn between the analyses of the cognitive and the affective domain:

1. knowledge--receiving
2. comprehension--responding
3. application--valuing
4. analysis and synthesis--conceptualization
5. evaluation--organization and characterization (21;49)
Several difficulties arising from the use or attempted use of Handbook I (cognitive domain) have been reported. The placement of an objective, as noted by the authors (2;9), may not be unique and the more complex behaviors tend to include the simpler ones, making test results at the higher levels difficult to evaluate. Cox (6) classified three hundred seventy-nine multiple choice items using the taxonomy categories and found that the one hundred most discriminating items were not distributed among the taxonomy levels in the same proportion that the original three hundred seventy-nine items were. He concludes that statistical item selection has a biasing effect in favor of items at the lower levels. Kropp, Stoker, and Banshaw (22) point out the problems in interpretation which result from measuring the "product response" in place of the "process response," i.e., the correct response in place of the correct process of arriving at that response. They were not able to locate any empirical studies which would clarify the relationship between these two response measures. Romberg and Kilpatrick (35) report difficulty in classifying mathematical test items which can be missed at several stages. They suggest that a problem having several parts at different levels of the taxonomy should, perhaps, have each part tested separately but they indicate that the problem of devising items to test these various parts is not an easy one.
The lack of comprehensiveness of Handbook I (cognitive domain) is indicated by the number of modifications reported. McGuire (27) revises the original categories into the categories

1.0 knowledge
2.0 generalization
3.0 problem-solving of a familiar type
4.0 problem-solving of an unfamiliar type
5.0 evaluation
6.0 synthesis.

Romberg (34) lists the following seven levels of intellectual activity in mathematics adapted from Handbook I (cognitive domain).

1. knowing: terminology, facts, properties, reasons, principles, structure.
2. manipulating: carrying out algorithms.
3. translating: changing from one language to another.
4. applying: making comparisons, selecting appropriate facts and techniques.
5. analyzing: analyzing data, recognizing relevant and irrelevant information, seeing patterns, isomorphisms, symmetry.
6. synthesizing: specializing and generalizing, formulating problems, constructing a proof or a problem.
7. evaluating: validating answers, judging reasonableness of answers, validating the solution process, criticizing the solution process.

Several studies have been reported which use the taxonomy to generate or classify objectives or test items.
Mau (26) used Handbook I (cognitive domain) to construct an instrument with fifty objectives for undergraduate home management. A total of three hundred ninety-seven undergraduate home management professors rated the objectives as either essential, desirable, little or no importance, or cannot classify. Cognitive objectives for home management courses were formulated by combining revised statements of the highest ranking objectives that represented all taxonomy classes, with specific content components of home management. Allen (1) used Handbook I (cognitive domain) and Handbook II (affective domain) to classify the objectives of social studies. These objectives were used to develop test items and separate tests for knowledge of terminology, knowledge of generalization, comprehension, and reasoning were developed. Milholland (28) sent a questionnaire listing sixteen objectives of the first course in psychology (nine taken from Handbook I) to three hundred colleges. Test items from several sources were then assigned to relevant objectives. Satisfactory items were located or produced for only the objectives from Handbook I. Hunkins (20) submitted a collection of fifty-nine multiple choice questions to two judges and used those items receiving 100% agreement among the judges and the investigator to construct a test of forty-two items to be administered in a forty-five minute period. The investigator reported difficulty in constructing items at the levels of comprehension, application, analysis, and synthesis.
Romberg and Kilpatrick (35), in a study on evaluation in mathematics education, identified the basic mathematical topics that students are expected to master at each grade level, K-12, classified these topics into a system of behavior skills based on the cognitive domain, and then constructed sample test items to illustrate the various topics in the curriculum. The investigator restricted themselves to cognitive behavior because it had been described and investigated in enough detail for its components to be discussed, and to test items with written responses because these seemed the most practical.

The following chapter will present and discuss the research procedures used in generating a set of objectives for the first year calculus and constructing a test to evaluate the achievement of these objectives.
CHAPTER III

RESEARCH PROCEDURE

This chapter describes the method of procedure for the study including the generation of statements of objectives, the construction and selection of test items, the construction and administration of a preliminary test form and the administration of a revised test form.

Statement of objectives

The construction of a statement to be used as an educational objective involves the consideration of several principles. Esbensen proposes that an objective should specify performance, conditions, and extent, i.e., an objective should specify what is being done, how it is to be done, and how much should be done (12). This seems to be consistent with Mager's criteria for writing objectives, which are:

1. Identify the terminal behavior by name.

2. Try to further define the desired behavior by describing the important conditions under which the behavior will be expected.

3. Specify the criteria of acceptable performance by describing how well the learner must perform to be considered acceptable (25;12).

The specificity of an educational objective is partly determined by the scope of the objective, e.g., an objective
for a daily lesson may be much more specific in language and implied behaviors than an objective for a ten week course. In general, "objectives should be made explicit but they must also be implicit in the activities demanded of students" (10;36). It was decided that the purposes of this study would be served best by statements of objectives which were specific with respect to the behaviors expected of the students but not restrictive with respect to the activities by which these behaviors might be achieved. No attempt would be made to spell out the daily activities which the student should perform in the classroom, but each statement would be constructed to include only one student behavior.

A survey was made of twenty-seven calculus textbooks published during the past eighteen years in an attempt to obtain a set of objectives for the first year calculus sequence. A list of these texts is found in Appendix B. The following statements are typical of the statements of objectives found in the preface of most of them:

1. To present the fundamental concepts of analytics and the calculus in such a manner as to give the student . . . a clear idea of both the methods and the uses of these branches of mathematics (9).

2. To introduce undergraduate students to the elementary concepts of mathematical analysis from the point of view of contemporary mathematics (14).

3. To present the ideas which lie at the heart of the calculus (30).

4. To give the student a sound understanding of the fundamental concepts of calculus and a thorough appreciation of its many applications (32).
5. Edification of the student in the nature of mathematics as an edifice of logic (40).

6. To provide for a thorough understanding of the limit of a sum process, the limit of the difference quotient, and the possible practical applications of the calculus (36).

These statements appeared to be of little value for this study since none were stated in behavioral terms. A similar investigation of college catalogues and bulletins was equally fruitless since most of these presented only an outline of topics to be covered.

An investigation of the literature of mathematics education was only slightly more rewarding. The objectives of the calculus stated by Butler and Wren (5;585) are very similar to the statements of objectives in calculus texts. Dubish (11;11,13,14,15) states an objective for each of the subgroups of the student population, i.e., mathematics majors, mathematics education majors, science and engineering majors, and liberal arts majors, but these are still a long way from the behavioral statements suggested by Mager and Esbensen. The Twenty-sixth Yearbook of the National Council of Teachers of Mathematics (29;72) contains a fairly well written set of objectives but these are also not stated in behavioral terms and are so general that they might be applied to any mathematics course.

A more definitive approach was found in the publications of the Committee on the Undergraduate Program in Mathematics. The committee decided on the following objectives
for the pre-graduate preparation of research mathematicians:

The student should be introduced to the language of mathematics, both in its rigorous and idiomatic forms. He should be able to give clear explanations of the meaning of certain fundamental concepts, statements, and notations. He should acquire a degree of facility with selected mathematical techniques, know proofs of a collection of basic theorems, and have experience with the construction of proofs. He should be ready to read appropriate mathematical literature with understanding and enjoyment. He should learn from illustration and experience to cultivate curiosity, and the habit of experimentation, to look beyond immediate objectives, and to make and test conjectures.

The student should by these means be led to seek an understanding of the place of mathematics in our culture, in particular, to appreciate the interplay between mathematics and the sciences (8;5).

These statements come much closer to the criteria for objectives stated in behavioral terms, but they are not written specifically for the calculus. The purpose of the introductory calculus course, the Committee states, is "to introduce the ideas of derivatives and integrals with their principal interpretations and interrelations and to develop the simpler techniques of differentiation and integration for the elementary functions. . ." (7;31). This last statement seemed no more useful than statements found in calculus textbooks.

It was decided, at this point, to construct a set of objectives using the taxonomy of educational objectives. A list of possible objectives was constructed for each of the four categories of calculus students which Dubish cites. These objectives were constructed from source statements in calculus textbooks and the literature of mathematics education.
by analyzing the statements for the behaviors which they implied at each level of the cognitive and affective domains. To cite an example, Dubish's objective for mathematics education majors—to have students see the bearing that advanced mathematics can have on their high school teaching—gave rise to the following objectives at level 1 (recall) of the cognitive domain:

1. Knowledge of the specific facts and terminology of the first year calculus sequence, e.g., the meaning of $D_{xy}$, $y'$, \( \int_a^b f(x) \, dx \), \( \lim_{x \to a} f(x) \).

2. Knowledge of the historical development of calculus in terms of major concepts, i.e., the development of integration by the Greeks to calculate the area of regular polygons, and the development of differentiation in the seventeenth century to treat rates of change.

3. Knowledge of the power and limitations of the calculus in unifying and explaining high school topics.

4. Knowledge of the historical and the sophisticated approaches to important problems in order to guide high school students in making discoveries.

5. Knowledge of the important theorems and generalizations of the first year calculus sequence, e.g., differentiation and integration theorems.

Dubish's objective was then analyzed at level 2 (comprehension) and several objectives, based on the behaviors implied at this level, were constructed. The process was repeated for each of the remaining levels of the cognitive domain and for all of the levels of the affective domain.

For such lists of objectives, certain ones were selected which seemed to be applicable to all the students.
After revising the objectives to make the required behaviors specific and to make the language of the objectives uniform, the following list of proposed objectives for the first year calculus sequence was obtained. The order of the objectives in this list was determined by a table of random numbers to prevent any bias in the rating process, which will be discussed in the following section. The letter (C--cognitive; A--affective) and the numeral following each objective refer to the domain and level of the domain at which the objective is written. It should be remembered that the levels and sublevels of the taxonomy are not necessarily mutually exclusive, and, therefore, the placement of an objective is not unique.

The student should:

1. Be willing to expand mathematical knowledge by independent reading.  
   A 2.2

2. Be aware of the potential use of calculus in areas where it is not now being used.  
   A 1.1

3. Define intuitively the technical terms of first year calculus, e.g., the limit of a function, continuity, derived function, critical point, definite integral.  
   C 2.20

   C 5.20

5. Demonstrate a knowledge of the historical development of the major concepts of calculus, i.e., development of integration by the Greeks to calculate area under a curve; development of differentiation in the seventeenth century to treat rates of change.  
   C 1.22

6. Recognize several different applications of a particular principle.  
   C 1.23
1. Experience enjoyment as a result of comprehending the calculus used in a practical application.  
2. Be alert to possible solutions and methods of attack on a non-traditional nature.  
3. Develop study habits which are orderly and precise.  
4. Isolate the logical structure underlying a proof.  
5. Be willing to try calculus as a tool in physical problems.  
6. Be aware of the historical contributions of calculus to our civilization.  
8. Attempt to conceptualize the deductive nature of mathematics.  
9. Be aware of the role of calculus in modern science, business, industry, and the humanities.  
10. Indicate the logical fallacies in a proposed proof.  
11. Read mathematics texts or periodicals independently with some degree of comprehension (provided the content is a reasonable extension of the student's range of knowledge).  
12. Give examples of the way in which calculus unifies and explains certain topics of secondary school mathematics, e.g., use of limits to evaluate the trigonometric functions, use of integration to derive formulas for area and volume.  
13. Apply calculus to physical problems which are traditionally solved by calculus.  
14. Make use of various methods of proof, e.g., deduction, and proof by contradiction.  
15. Recognize the historical or classical approach to certain problems.
22. Recognize the limitations of calculus in solving physical problems. C 6.20

23. Construct proofs of original statements with some degree of rigor (provided the content is a reasonable extension of the student's mathematical knowledge). C 5.10

24. Identify all the symbols used in first year calculus, e.g., \( \frac{dy}{dx} \), \( f' \), \( \int f \), \( \int_a^b f(x) \, dx \). C 1.11

25. Distinguish between heuristic arguments and formal proofs. C 3.00

26. Be aware of the interrelationship of calculus and other broad areas of mathematics. C 1.23

27. Attempt to conceptualize the logical structure of calculus. C 5.30

28. Compute the derivative and definite integral of common functions, e.g., polynomials, trigonometric functions, logarithmic functions, exponential functions. C 3.00

29. Be characterized by a desire to understand and create mathematics. A 5.2

30. Identify the geometric interpretations of the derivative and the definite integral, e.g., the derivative as the slope of the tangent line and the definite integral as the area under a curve. C 2.00

31. Make inferences from given statements. C 2.30

32. Analyze problems into fundamental relationships. C 4.10

33. Be curious about the extent to which calculus is used in twentieth century civilization. A 1.2

34. Recognize the importance of the fundamental theorem of calculus in relating differentiation and integration. C 2.20

35. Apply the geometric interpretations of the derivative and the definite integral. C 3.00
Rating of proposed objectives
by faculty members

The proposed set of objectives above along with some examples of test items was submitted to eight members of the mathematics department at The Ohio State University. These were all the faculty members involved in teaching and planning the calculus sequence or who were familiar with the sequence. The following letter of instruction was included with the proposed objectives:

The following is a list of statements which might be used as objectives for the first year calculus sequence (440, 441) here at Ohio State University, including some examples of test items. The proposed objectives are stated in terms of student behavior and each could be preceded by the phrase "the student should" or "the student must." The test items are included to clarify the meaning of arbitrarily chosen statements and to suggest a method of testing for the achievement of particular objectives.

I would like you to indicate those statements which are appropriate as objectives by assigning to each the numeral 0, 1, 2, or 3 in the following manner:

0. If you think the statement is not an objective of the sequence.

1. If you think that the statement represents an objective of minor importance.

2. If you think that the statement represents an objective of moderate importance.

3. If you think that the statement represents an objective of major importance.

Make your decisions on the basis of whether or not you think the statement is an appropriate objective for all students taking the sequence, e.g., mathematics majors, engineering majors, mathematics education majors, and liberal arts majors.

If you think a statement represents an important objective, mark it so, even though it may seem difficult to test for achievement of the objective.
or the test item may seem crude. Please add any objectives which you feel are important but do not appear on the list and alter any statement (or test item) in order to make it more meaningful or useful.

Thank you for your cooperation.

Each faculty member was interviewed after he had rated the proposed objectives and many helpful suggestions and clarifications resulted.

The mean of the scores for each of the proposed objectives and the number of faculty members rating each of the proposed objectives is given in Table 1.

It was arbitrarily decided to use only those objectives with a mean rate of 2.00 or more in constructing the test, in order to keep the scope of the study within reasonable bounds. Table 2 contains a list of those objectives with a mean rate of 2.00 or more and the level of the taxonomy at which the objective was placed.

**Item construction**

A number of test items were constructed for each objective in Table 2; a complete set of these items, grouped according to the related objective, is found in Appendix C. Each item in the set was constructed in an effort to make its solution depend upon the behavior suggested by the objective. Items at a higher level of the taxonomy might include behaviors from a lower level because of the hierarchial nature of the taxonomy. Every effort was made to avoid constructing or including items whose solution required a higher level


<table>
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<tr>
<th>Objective</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>7</td>
<td>6</td>
<td>6</td>
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Note: The number of raters is sometimes less than eight because some faculty members did not respond to every proposed objective.
<table>
<thead>
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<th>Level</th>
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</thead>
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<td>I. Compute the derivative and the definite integral of common functions, e.g., polynomials, trigonometric functions, logarithmic functions and exponential functions.</td>
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<td>II. Identify the geometric interpretations of the derivative and the definite integrals, e.g., the derivative as the slope of the tangent line and the definite integral as the area under a curve.</td>
<td>3.00</td>
<td>2.00C</td>
</tr>
<tr>
<td>III. Apply the geometric interpretations of the derivative and the definite integral.</td>
<td>3.00</td>
<td>3.00C</td>
</tr>
<tr>
<td>IV. Identify all the symbols used in first year calculus, e.g., $dy$, $f'$, $\int f$, $\int_a^b f(x)dx$</td>
<td>3.00</td>
<td>1.11C</td>
</tr>
<tr>
<td>V. Define intuitively the technical terms of the first year calculus, e.g., the limit of a function, continuity, derived function, critical point, definite integral.</td>
<td>3.00</td>
<td>2.20C</td>
</tr>
<tr>
<td>VI. Recognize the importance of the Fundamental Theorem of calculus in relating differentiation and integration.</td>
<td>2.87</td>
<td>2.20C</td>
</tr>
<tr>
<td>VII. Recognize several different applications of a particular principle.</td>
<td>2.71</td>
<td>1.23C</td>
</tr>
<tr>
<td>VIII. Apply calculus to physical problems which are traditionally solved by calculus.</td>
<td>2.66</td>
<td>3.00C</td>
</tr>
<tr>
<td>IX. Outline a method of solution to a problem.</td>
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<td>5.20C</td>
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<tr>
<td>X. Be willing to try calculus as a tool in physical problems</td>
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<th>Mean Rate</th>
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<td>XI. Distinguish between heuristic arguments and formal proofs.</td>
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<td>3.00C</td>
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<tr>
<td>XII. Analyze problems into fundamental relationships</td>
<td>2.16</td>
<td>4.10C</td>
</tr>
<tr>
<td>XIII. Make inferences from given statements.</td>
<td>2.14</td>
<td>2.30C</td>
</tr>
<tr>
<td>XIV. Be alert to possible solutions and methods of attack of a non-traditional nature.</td>
<td>2.00</td>
<td>1.1A</td>
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<tr>
<td>XV. Be aware of the interrelationship of calculus and other broad areas of mathematics</td>
<td>2.00</td>
<td>1.23C</td>
</tr>
<tr>
<td>XVI. Experience enjoyment as a result of comprehending the calculus used in a practical application</td>
<td>2.00</td>
<td>2.3A</td>
</tr>
</tbody>
</table>

behavior or a trick. The ease or difficulty with which the investigator was able to construct test items was not directly related to the level of the taxonomy at which the objective was stated. The objective, "the student should apply the geometric interpretation of the derivative and the definite integral," which is at level three (application) of the cognitive domain, gave rise to a large number of items. The objective "the student should be aware of the interrelationship of calculus and other broad areas of mathematics," which is at level one (recall) of the taxonomy, did not lend itself easily to the construction of test items and considerable effort was required to develop acceptable items.
The test items in Appendix C were presented to a panel of faculty members and colleagues who were asked to judge the appropriateness of each item for an objective. Their criticism led to the discarding of some items, the revision of others, and the inclusion of new items. As a result of this procedure, acceptable items could not be produced for three of the objectives in Table 2.

Objective X pertains to the students' willingness to try calculus as a tool in physical problems. The most profitable approach seemed to be a series of problems which could be solved with equal difficulty by calculus or by other methods, defining achievement of the objective as choosing the method using calculus. It was pointed out, however, by several faculty members that this might be more of a test of recognition of type problems than an indication of willingness to try calculus.

Objective XI concerns the ability of the student to distinguish between heuristic arguments and formal proofs. The difficulty here centered on the meaning of "formal proofs," and the meaning of proof itself. Depending on the mathematical maturity of the student, any one of a number of arguments might serve as a "formal proof," and that which constitutes a proof for one student might conceivably not be a proof for another.

Objective XII relates to the ability of the student to analyze problems into fundamental relationships. It was
concluded after several unsuccessful attempts at constructing test items that objective IX (outline a method of solution to a problem) is sufficient for objective XII but objective IX is not necessary for objective XII. That is, the student who outlines a method of solution to a problem must be aware of the fundamental relationships involved in the problem in order to construct a meaningful outline but it is possible for a student to pick out several key relationships in a problem without being able to organize these into an outline for a solution of the problem.

There were no restrictions placed on the type of item to be used for a particular objective or for the test as a whole. However, to insure uniformity of scoring, "paper and pencil" items were used rather than interview or observation techniques. Occasionally, the statement of an objective suggested a particular type item; for example, objective IV, "identify all the symbols used in first year calculus," seemed very easy to test with a matching item, using symbols in one column and their meanings in the other. In general, four types of items were tried for each objective--multiple choice, matching, classification or key-list, and written response.

Multiple choice type items consist of a stem with several choices as endings. Every effort was made to have each choice grammatically correct for a given stem; each item was constructed with only one correct ending; and ambiguously worded stems or endings were rewritten. The matching type
item is fundamentally a group of multiple choice items, all of which employ the same responses (19;147). The stems are arranged in one column and the common responses in another. There is only one correct response for each stem; each response might be used for more than one stem. The classification or key-list type is another variation of the multiple choice. It consists of a list of statements or other items which are to be classified according to a common set of categories (16;263). The written response type item requires the student to write the solution to a problem, to write an equation, to write an outline of a method of solution, or to draw a graph.

**Test construction**

Those items which the panel found acceptable were used to construct a preliminary test form. The items selected for inclusion were studied as a group to insure that no information in one item might lead the student to the proper solution in another item. The number of items related to each objective was roughly proportional to the importance of the objective as determined by the faculty rating, although in some instances, there were not enough acceptable items for a particular objective and too many for another. It was decided to retain as many acceptable items as possible without strongly overloading the test on any one objective.

The decision to administer the test during a regular forty-eight minute class period was prompted by the
availability of the students at those times. Because of the large number of objectives and related test items, the test was divided into halves on the basis of type of item, i.e., the multiple choice and related items were arranged into a test given during one period, and the written response items formed a second test given during a separate period. A copy of the preliminary test materials is found in Appendix D.

The first half of the test, which had a forty-five minute time limit, required the students to make forty-nine responses. The items were grouped according to type, i.e., multiple choice, matching, or key-list, and arranged into a booklet consisting of three sections. An answer sheet, similar to commercial ones which may be machined scored, was provided and a page of directions was attached to the front of the test booklet. The directions indicated that the final score would be the total number of correct responses with no penalty for guessing.

The second half of the test, which also had a forty-five minute time limit, consisted of a single page on which the eight written response questions were grouped into two sections. An answer booklet was provided with space for the student to show the work done in arriving at each answer.

Each item on the first half of the test, with the exception of number 16, had exactly one correct response and could be scored correct or incorrect. Number 16, a classification item, was an attitude inventory to test the students'
enjoyment as a result of comprehending the calculus used in a practical application.

The second half of the test was scored independently by three graders using the same scoring key. The scoring sheet was constructed so that each grader could not see the scores of the other graders, in an effort to make the scores independent. The mean of these scores was computed for each item and the item was scored correct if the mean was strictly greater than half of the point value of the problem. The item was scored incorrect if the mean was less than or equal to half the point value of the problem.

A test criticism form to be used with the preliminary test form was also constructed; a copy of this form is found in Appendix D. The form asked the student to evaluate from very good to very poor the following properties of the test:

1. Clarity of directions
2. Clarity of wording of items
3. Balance of difficulty of items
4. Time limits

The criticism form also asked the student for his general comments on the test.

Method of analysis of results

A general analysis performed on the total scores of the first half of the test would supply information on the mean and median scores, the standard deviation, the reliability,
and the standard error. An item analysis performed on the responses to the items on the first half of the test would supply information on the difficulty and the discrimination of each item, and on the pattern of students' responses. These analyses would be performed on an IBM 7094 using the OSU Item Analysis Program.

The question of how to report the results of the test in order to determine how well the group had performed on individual items, on each objective, and at each level of the taxonomy was a difficult one to answer. If the student chose the correct response, he may have known the correct answer or he may have guessed. Therefore, the number of students who did not achieve the desired behavior was greater than or equal to the number of failures recorded and we can consider the number of failures recorded for each item a lower bound of failure.

It was decided to establish a percent of failure for each item by dividing the lower bound of failure by the number of students taking the test. This percent would then be used as a measure of the performance of the group on each item.

A measure of the performance of the group on an objective would be established by computing the mean of the percents of failure for all the items related to that objective.

A measure of the performance of the group at a level of the taxonomy would be established by computing the mean of the percents of failure for all the items related to that level.
The means established in this way would then permit a comparison among the performances on the objectives and among the performances at the taxonomy levels.

**Administration of the preliminary test form**

The preliminary test form was given to a total of fifty-seven students at the Mansfield and Lima Branches of The Ohio State University. The investigator was present each time the test was given and made observations which will be discussed in the next section. The first and second halves of the test were administered in succeeding class periods with a ten minute rest period between halves because of conflicts with scheduled classes. The materials for the first half of the test, consisting of the test booklet with attached directions, an answer sheet, and scratch paper, were given to the student at the beginning of the first period. The student was asked to read the directions and was asked if he had any questions. The student was reminded of the time limits and told to begin. The student was told to stop writing after forty-five minutes and the answer sheets were collected. The student was then given a copy of the test criticism form and was asked to spend a few minutes filling it out. Both the test booklet and form were then collected.

Following a ten minute rest period the student was given the materials for the second half of the test which consisted of the test and an answer booklet. The student was
told that the same general directions applied to the second half of the test and was told to begin. The student was told to stop writing after forty-five minutes and the answer booklets were collected. The student was then given a second copy of the test criticism form and asked to spend a few minutes completing it. After he had completed the form, both test and form were collected.

Revision of the preliminary test form

The summary statistics of the general analysis of the total scores on the first half of the preliminary test form are given in Table 3. The item difficulty and the phi coefficient of item discrimination for each item are given in Table 4.

Several factors were considered in evaluating the data in Tables 3 and 4 and using them to revise the test. First, the small sample population necessitated the comparison between the groups of students in the upper and lower halves of the test rather than the groups in the upper 27.5% and lower 27.5% respectively as is more customary. This information would decrease the confidence placed in the item difficulty and the item discrimination since there is no way of telling how the students are distributed within the upper and lower halves. For example, if the students in the upper half were concentrated below 72.5%, then the difficulty would decrease and the discrimination would decrease. Second, the reliability
**TABLE 3**

**SUMMARY STATISTICS OF THE TOTAL SCORES ON THE FIRST HALF OF THE PRELIMINARY TEST FORM**

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TABLE 4

ITEM DIFFICULTY AND ITEM DISCRIMINATION FOR THE ITEMS ON THE FIRST HALF OF THE PRELIMINARY TEST FORM AS COMPUTED BY THE ITEM ANALYSIS

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<th>Diff</th>
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<td>-.01</td>
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<td>.19</td>
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<td>18.6</td>
<td>.74</td>
<td>.31</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Item number 16 does not appear in this list because it evaluated a student opinion and did not require a specific answer.
was computed by Kuder-Richardson formula #20, Kuder-Richardson formula #21, and an odd-even split. The high reliability value reported by all of these methods seemed to indicate that confidence could be placed in the reliability of the test. Third, the similarity of the mean and the median suggested that the sample population was a good representation of the total population. Finally, the data reported in Table 4 indicated that the item discrimination using the phi coefficient was low for most of the items on the test. This coefficient of discrimination is related to the item difficulty and tends to be low for items which are difficult or easy (10;453). The item difficulty coefficient is the percent of the sample population responding correctly, i.e., if a large number of students got the item right, the item would have a high item difficulty coefficient.

The second half of the test was scored by three graders using the same scoring key. No computations were performed on these scores because the results were too low to be of any value. Some possible reasons for this will be discussed in the following paragraphs.

Two conditions of the test situation were immediately obvious to the investigator. First, the forty-five minute time limit was too generous for the first half of the test; most of the students had completed the test after thirty-five minutes. Second, the students were not prepared mentally to take a test in two succeeding test periods; this was
evidenced by the fact that most of the students stopped trying to solve the problems on the second half after twenty minutes. One possible reason for this is that the students were not motivated to take the test.

The test criticism form confirmed the above observations. In general, the students thought the first half of the test was easy, with too much time provided, and the second half of the test was hard. The second half of the test might have appeared harder in view of the opinion that the first half was easy or the students may have been accustomed to the multiple choice type items and were not able to change to a written response type. The test criticism form also yielded important information on items which the students found difficult and items which they thought could have been more clearly worded.

The foregoing discussion prompted the following revisions. A number of items with high item difficulty coefficient and low item discrimination coefficient were replaced with items requiring the same behavior, but more knowledge of the content in order to make the first half of the test more challenging. Items of moderate difficulty were added to the first half of the test to make the forty-five minute time limit more meaningful. Questions and directions which the students thought were ambiguously worded or confusing were rewritten. A question was added to the answer sheet asking the student to indicate the amount of calculus he had studied
before entering the Math 440-441 sequence. The student had to indicate that he had had (1) no calculus, (2) 0 to 6 weeks of calculus, (3) 6 to 12 weeks, or (4) 12 weeks to a half year of calculus before entering the sequence. The directions and one problem in the first section of the second half of the test were rewritten. The remaining three items were replaced with items which were considered better tests of the behavior being tested. The format of the answer booklet was also changed to eliminate some confusion regarding the directions.

A copy of the revised test and answer keys are found in Appendix E, along with the scoring instructions and score sheet for the second half of the test. The distribution of the items on the revised test according to type is found in Table 5. The distribution of the items on the revised test among the objectives and the levels of the taxonomy is found in Table 6.

An inspection of Table 5 seems to indicate an over-abundance of multiple choice items, but this is misleading since the matching and key-list items each contain several parts. Table 6 reflects the faculty's choice of objectives at the lower levels of the cognitive domain and its preference for cognitive rather than affective objectives.
<table>
<thead>
<tr>
<th>Objective</th>
<th>Rate</th>
<th>Multiple Choice</th>
<th>Match</th>
<th>Key List</th>
<th>Written</th>
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<tbody>
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<td>1, 2, 3, 4, 5</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>3.00</td>
<td>6, 8</td>
<td></td>
<td>22</td>
<td></td>
</tr>
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<td>III</td>
<td>3.00</td>
<td>10, 11, 14, 16</td>
<td></td>
<td>7</td>
<td></td>
</tr>
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<td>23, 24</td>
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<td>6, 8</td>
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<td>1, 3</td>
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<td>2.28</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>2.16</td>
<td></td>
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<td>2.00</td>
<td>13, 15</td>
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<td></td>
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<tr>
<td>XVI</td>
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<td>20</td>
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</tr>
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</table>
TABLE 6

DISTRIBUTION OF OBJECTIVES IN TABLE 2 AND RELATED TEST ITEMS FROM THE REVISED TEST FORM ACCORDING TO LEVELS OF THE COGNITIVE AND AFFECTIVE DOMAINS

<table>
<thead>
<tr>
<th>Level</th>
<th>Objective</th>
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<th>Second Half</th>
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<tbody>
<tr>
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<td></td>
<td>Cognitive Domain</td>
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</tr>
<tr>
<td></td>
<td>VII</td>
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<td>13,15</td>
<td></td>
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<td>6,8</td>
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<td></td>
<td>V</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VI</td>
<td>17,19</td>
<td>6,8</td>
</tr>
<tr>
<td></td>
<td>XIII</td>
<td>9,12</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>I</td>
<td>1,2,3,4,5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>10,11,14,16</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>VIII</td>
<td>18</td>
<td>1A,3A,4</td>
</tr>
<tr>
<td></td>
<td>XI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>XII</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>IX</td>
<td></td>
<td>1B,3B</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Affective Domain</td>
</tr>
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<td>XIV</td>
<td></td>
<td>2,5</td>
</tr>
<tr>
<td>2</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>XVI</td>
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</table>

Administration of revised test form

The revised test form was administered to one hundred ninety-three students registered in Math 441 at The Ohio State University in the spring quarter of 1967. The first half of the test was administered to all students taking part in the experiment on Monday, June 5, and the second half
administered on Tuesday, June 6. In order to motivate the students, they were informed of the dates on which the test would be given two weeks prior to the test, of the general format of each day's test, and of the content included in the test. The students were told that preparation for this test would be a good preparation for the final examination in the course. The students were also told that the results of the test would be used to help determine grades in borderline cases. The test was administered during regular class periods and the investigator consulted with each of the other three instructors who administered the test in an effort to insure uniformity of procedure. The students were not given test criticism forms at the end of each half of the test as they were in the preliminary administration of the test. Instead, an objective rating sheet was distributed on the first day of the test with instructions to complete the rating and return the sheet the following day. A copy of the objective rating sheet is found in Appendix E. The data obtained from this administration of the test are the data on which the conclusions of this study are based. These data will be analyzed in the following chapter.
CHAPTER IV

ANALYSIS OF RESULTS

A general analysis of the test results will be presented in this chapter as well as a discussion of the achievement of the objectives related to the cognitive and affective domains, and a comparison of the faculty and student ratings of the objectives.

General analysis of the test results

The first half of the revised test was administered to one hundred ninety-three students at the end of the first year calculus sequence. Sixteen of these students did not respond to the question concerning previous experience with calculus and were eliminated from the sample. Another sixteen students indicated that they had studied calculus for more than six weeks before entering the Math 440-Math 441 sequence at The Ohio State University. These students were eliminated from the sample on the assumption that they had acquired a certain familiarity with the content of the first year calculus sequence. This familiarity might have influenced positively the results of the test. The total number of students in the sample population for the first half of the test was, therefore,
one hundred sixty-one. One hundred fifty-four of these students took the second half of the test.

A general analysis was performed on the total scores on the first half of the test as described in Chapter III; the summary statistics of this analysis are given in Table 7.

**TABLE 7**

**SUMMARY STATISTICS OF THE TOTAL SCORES ON THE FIRST HALF OF THE REVISED TEST FORM**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Students Taking the Test</td>
<td>161</td>
</tr>
<tr>
<td>Upper 27.50 percent</td>
<td>44</td>
</tr>
<tr>
<td>Lower 27.50 percent</td>
<td>44</td>
</tr>
<tr>
<td>Number of Items in Test</td>
<td>57</td>
</tr>
<tr>
<td>Mean Score, Total</td>
<td>22.48</td>
</tr>
<tr>
<td>Mean Score, Upper</td>
<td>31.93</td>
</tr>
<tr>
<td>Mean Score, Lower</td>
<td>13.64</td>
</tr>
<tr>
<td>Median Score</td>
<td>22.00</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>7.43</td>
</tr>
<tr>
<td>Reliability Calculated by Kuder-Richardson Formula #20</td>
<td>0.81</td>
</tr>
<tr>
<td>Standard Error</td>
<td>3.23</td>
</tr>
<tr>
<td>Reliability Calculated by Kuder-Richardson Formula #21</td>
<td>0.77</td>
</tr>
<tr>
<td>Standard Error</td>
<td>3.59</td>
</tr>
<tr>
<td>Reliability Calculated by Odd-Even Split</td>
<td>0.87</td>
</tr>
<tr>
<td>Standard Error</td>
<td>2.65</td>
</tr>
</tbody>
</table>
The items on the first half of the test were grouped into three subtests. The first subtest contained the items related to level one (recall) of the cognitive domain and the second and third subtests contained the items related to the second (comprehension) and third (application) levels of the cognitive domains, respectively. Analyses were also performed on the total scores on these subtests as described in Chapter III; the summary statistics of these analyses are given in Table 8. An investigation of the data in these tables reveals that the reliability of the entire test and the reliabilities of the subtests related to levels one and two of the cognitive domain were high. The reliability of the subtest related to level three was low because of the small number of items. There were insufficient items at level five (synthesis) of the cognitive domain and at levels one (receiving) and two (responding) of the affective domain to perform similar analyses which would be meaningful.

The mean score and the median score for the entire test were similar and the mean score and median score for each of the subtests were similar. This suggests that the sample population was a good representation of the total population.

A correlation among the scores of the three graders of the second half of the test was computed in order to check on the adequacy of the scoring instructions. The correlation between the first and second graders was .83; the correlation
TABLE 8

SUMMARY STATISTICS OF THE TOTAL SCORES ON THE SUBTESTS RELATED TO LEVELS 1, 2, AND 3 OF THE COGNITIVE DOMAIN

<table>
<thead>
<tr>
<th>Summary for Level 1 (Recall)</th>
<th></th>
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<tbody>
<tr>
<td>Total Number of Students Taking the Test</td>
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<tr>
<td>Upper 27.50 percent</td>
<td>44</td>
</tr>
<tr>
<td>Lower 27.50 percent</td>
<td>44</td>
</tr>
<tr>
<td>Number of Items in Test</td>
<td>23</td>
</tr>
<tr>
<td>Mean Score, Total</td>
<td>11.00</td>
</tr>
<tr>
<td>Mean Score, Upper</td>
<td>16.61</td>
</tr>
<tr>
<td>Mean Score, Lower</td>
<td>5.30</td>
</tr>
<tr>
<td>Median Score</td>
<td>11.54</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.63</td>
</tr>
<tr>
<td>Reliability Calculated by Kuder-Richardson Formula #20</td>
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</tr>
<tr>
<td>Standard Error</td>
<td>1.99</td>
</tr>
<tr>
<td>Reliability Calculated by Kuder-Richardson Formula #21</td>
<td>0.77</td>
</tr>
<tr>
<td>Standard Error</td>
<td>2.24</td>
</tr>
<tr>
<td>Reliability Calculated by Odd-Even Split</td>
<td>0.88</td>
</tr>
<tr>
<td>Standard Error</td>
<td>1.57</td>
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</table>

<table>
<thead>
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</thead>
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<td>Total Number of Students Taking the Test</td>
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<tr>
<td>Upper 27.50 percent</td>
<td>44</td>
</tr>
<tr>
<td>Lower 27.50 percent</td>
<td>44</td>
</tr>
<tr>
<td>Number of Items in Test</td>
<td>22</td>
</tr>
<tr>
<td>Mean Score, Total</td>
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<tr>
<td>Mean Score, Upper</td>
<td>10.64</td>
</tr>
<tr>
<td>Mean Score, Lower</td>
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</tr>
<tr>
<td>Median Score</td>
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</tr>
<tr>
<td>Standard Deviation</td>
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<tr>
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</tr>
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</tr>
<tr>
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</tr>
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<td>Standard Error</td>
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### TABLE 8 (Contd.)

**Summary of Level 3 (Application)**

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</tr>
<tr>
<td>Upper 27.50 percent</td>
<td>44</td>
</tr>
<tr>
<td>Lower 27.50 percent</td>
<td>44</td>
</tr>
<tr>
<td>Number of Items in Test</td>
<td>10</td>
</tr>
<tr>
<td>Mean Score, Total</td>
<td>4.10</td>
</tr>
<tr>
<td>Mean Score, Upper</td>
<td>6.16</td>
</tr>
<tr>
<td>Mean Score, Lower</td>
<td>2.14</td>
</tr>
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<td>Median Score</td>
<td>4.05</td>
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<tr>
<td>Standard Deviation</td>
<td>1.69</td>
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</tr>
<tr>
<td>Standard Error</td>
<td>1.33</td>
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</tbody>
</table>

between the second and third graders was .89; the correlation between the first and third graders was .78.

Table 9 contains the number and percent of incorrect responses for each item on the first half of the test. An investigation of the number of omissions for each item indicated that the students may not have had sufficient time to complete the first half of the test. This inference was strengthened by the way in which the omissions were grouped; i.e., many students made no response to any item after a certain point in the test. The greatest number of omissions occurred on the items in section C. It appeared that counting the omissions as incorrect responses would grossly over-rate the percent of failure. A more meaningful estimate of the percent of failure would, perhaps, be obtained by dividing the
<table>
<thead>
<tr>
<th>Item</th>
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<th>Responses</th>
<th>Percent Incorrect Responses</th>
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</thead>
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<td>56</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
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TABLE 9 (Contd.)

<table>
<thead>
<tr>
<th>Item</th>
<th>Failures</th>
<th>Responses</th>
<th>Percent Incorrect Responses</th>
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<td>25.4</td>
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</tr>
<tr>
<td>25.6</td>
<td>23</td>
<td>55</td>
<td>42</td>
</tr>
</tbody>
</table>

Note: Items 20.1, 20.2, 20.3 and 20.4 do not appear on this list because they evaluate a student opinion. The results on these items will be discussed in the section on the achievement of objectives related to the affective domain.

The number of incorrect responses by the total number of responses, multiplying by 100, and rounding off to the nearer integer. This would produce a lower bound of failure for each item which might be much less distorted than the one obtained by dividing the sum of the number of failures and the number of omissions by the total number of students. In the
first case, we are assuming that the students who omitted the item would respond in a manner similar to those who did respond. In the second, we are assuming that all the students who omitted the item would get it wrong. The second assumption seems less reasonable in view of the lack of time, the fact that the items in the section with the greatest number of omissions are related to low level cognitive objectives, and the fact that most of the students who did respond to these items responded correctly.

Table 10 contains the number and percent of incorrect responses for each item on the second half of the test. There seemed to be sufficient time for this half of the test so omissions were regarded as failures. The percent failure for each item is the number of failures divided by the number of students who took the test.

Analysis of the results relating to the cognitive domain

A summary of the results of the test for level one (recall) of the cognitive domain is found in table 11. Those items with a percent of failure greater than or equal to forty will be discussed.

Item 23.2 requires the student to identify the symbol, y'. The correct response is (a) the instantaneous rate of change in y with respect to x. Many of the students chose (b) the differential of y. The similarity of the words derivative and differential may be the cause for this error.
TABLE 10

NUMBER AND PERCENT OF INCORRECT RESPONSES FOR EACH ITEM ON SECOND HALF OF THE TEST

<table>
<thead>
<tr>
<th>Item</th>
<th>Number of Failures</th>
<th>Percent of Failures</th>
</tr>
</thead>
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</tr>
<tr>
<td>1B</td>
<td>139</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>86</td>
<td>55</td>
</tr>
<tr>
<td>3A</td>
<td>127</td>
<td>82</td>
</tr>
<tr>
<td>3B</td>
<td>146</td>
<td>94</td>
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<td>4</td>
<td>141</td>
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</tr>
<tr>
<td>8</td>
<td>133</td>
<td>86</td>
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</tbody>
</table>

Note: The total number of students who took the second half of the test was 154.
### TABLE 11

Mean percent of failure for each of the items related to the objectives at level one of the cognitive domain, for the objectives at level one, and for level one as a whole

<table>
<thead>
<tr>
<th>Level</th>
<th>Objective</th>
<th>Item</th>
<th>Percent Failure</th>
<th>Mean Percent for Objective</th>
<th>Mean Percent for Level</th>
</tr>
</thead>
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<td>39</td>
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<td>23.2</td>
<td>71</td>
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<td>23.2</td>
<td>15</td>
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<td>23.6</td>
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<tr>
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<td>24.2</td>
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<td>24.6</td>
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</tr>
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<td>21.9</td>
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<td>21.10</td>
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<tr>
<td>XV</td>
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<td>55</td>
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<td></td>
<td></td>
<td>15</td>
<td>59</td>
<td></td>
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</tbody>
</table>

Item 23.4 requires the student to identify the symbol, \( \frac{\Delta y}{\Delta x} \). The correct response is (f), the slope of the chord line. Many of the students chose (a), the instantaneous rate of change of y with respect to x. This seems to imply some confusion in the minds of the students concerning the difference between the quotient function and its limit.
Item 24.1 requires the student to identify the symbol, \( \lim_{x \to a} f(x) \). The correct response is (d), the limit of the function \( f(x) \) as \( x \) approaches \( a \). Many of the students chose (a), the limit of the function \( f(x) \) as \( x \) approaches \( a \) from the right.

Item 24.4 requires the student to identify the symbol, \( f(x) \to L \) as \( x \to a \). The correct response is (d), the limit of a function \( f(x) \) as \( x \) approaches \( a \). There is no clearly defined pattern of incorrect responses, but (a) and (b), the limit of the function \( f(x) \) as \( x \) approaches \( a \) from the left, were chosen by many students. The results on this item and on item 24.1 seem to indicate that the students would benefit from a greater exposure to the concept and symbolism of one-sided limits.

Items 21.1, 21.4, 21.6, 21.9, and 21.10 require the student to indicate whether the given problem was usually solved by differentiation, integration, or neither. The correct response for 21.1 is (b), differentiation; for 21.6 (c), neither; for 21.9 (a), integration; for 21.10 (b), differentiation.

Item 21.1 contains a common maximum-minimum type problem. Many of the students thought the problem was usually solved by "integration." A few students thought it could be solved by "neither."

Item 21.4 contains an integration problem of the type associated with volumes which are not solids of revolution.
The problem may be found on page 262 of the second edition of Calculus and Analytic Geometry by Fisher and Ziebur (15). "Differentiation" was slightly favored over "neither" among the incorrect responses to this item.

Item 21.6 requires the student to identify the method of solution of the problem, "Compute the time required for an automobile to travel a given distance at a constant rate." Nearly as many students chose "differentiation" as the correct response, "neither," and relatively few chose "integration."

Item 21.9 contains an integration problem related to the exponential function. More students chose "differentiation" than the correct response "integration"; the number of students who chose "neither" was nearly as great as the number who chose the correct response.

Item 21.10 contains a maximum-minimum type problem whose solution involves a trigonometric function. More students chose "neither" than the correct response, "differentiation"; the number of students who chose "integration" was nearly as great as the number who chose the correct response.

The students did not recognize two of the three integration problems and two of the five differentiation problems in this section. Assuming that the problems are of a type with which the student should have been familiar, the results for these items seem to imply that the students would benefit
from exposure to a greater variety of integration problems.

Item 13 is as follows:

The circumference of a unit circle can be proved to be $2\pi$ units by

a. measuring the circumference.
b. multiplying the length of the diameter by $\pi$.
c. computing the arc length for the circle.
d. evaluating $\int_0^2 \pi \, dt$.
e. means of formulas beyond the scope of Math 440 and Math 441.

The correct response is (c); the incorrect responses (d) and (e) were chosen by a large number of students. Choice (a) and choice (b) might have been eliminated by the students as being too obvious.

Item 15 is as follows:

The most accurate approximation of a root of a fifth degree polynomial is found by

a. graphing the polynomial.
b. applying the fundamental theorem of calculus.
c. substituting values into the polynomial.
d. using Newton's method.
e. using L'Hopital's rule.

The correct response is (d); the incorrect response chosen most often was (b). There does not seem to be any reason for choosing (b) most often except as a guess by the student at a connection between the expression "fundamental theorem of calculus" and the correct response.

Items 13 and 15 are intended to test the student's awareness of the interrelationship of calculus and other broad areas of mathematics. The results on these items indicate
that more attention should be paid to making the students
aware of this interrelationship.

The objective which seems to have been achieved most
successfully at level one (recall) is objective IV, "the
student should identify all the symbols used in the first
year calculus, e.g., \( \frac{dy}{dx}, f', \int f, \int_{a}^{b} f(x) \, dx \)." The mean
percent of failure for objective XI (the student should be
aware of the interrelationship of calculus and other broad
areas of mathematics) is misleading because of the small
number of items related to this objective. A larger number
of items would help validate the mean percent of failure
reported.

The mean percent of failure for all the items related
to level one (recall) is 35.

A summary of the results of the test for level two
(comprehension) of the cognitive domain is found in Table 12.
Those items with a percent of failure greater than or equal
to forty will be discussed.

Item 6 asks the student for the usual geometric inter-
pretation of the symbol \( f'(x_0) \). The correct response is (a),
the slope of the function at \( x_0 \). A large number of students
chose (b), the slope of the line tangent to \( f(x) \) at any
point. The choice (b) seems to imply a confusion between
\( f'(x) \), the slope of the function at any point, and \( f'(x_0) \),
the slope of the function at the point \( x_0 \).
TABLE 12

MEAN PERCENT OF FAILURE FOR EACH OF THE ITEMS RELATED TO THE OBJECTIVE AT LEVEL TWO OF THE COGNITIVE DOMAIN, FOR EACH OF THE OBJECTIVE AT LEVEL TWO, AND FOR LEVEL TWO AS A WHOLE

<table>
<thead>
<tr>
<th>Level</th>
<th>Objective</th>
<th>Item</th>
<th>Percent Failure</th>
<th>Mean Percent for Objective</th>
<th>Mean Percent for Level</th>
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<td>8(Sec Half)</td>
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<td>51</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Item 22.2, 22.3, 22.4, and 22.5 are related to the following diagram and instructions.
(22) Let \( y = f(x) \) be defined by the graph below and let the letters \( a, b, c, d, e, \) and \( h \) represent numbers in the domain of \( f \). Select the letter for which each statement becomes true; if none of the letters makes a statement true, select \( n \). Each response may be used more than once, but there is only one correct answer for each statement.

Item 22.2 requires the student to select the letter representing a number in the domain of the function \( f \) at which \( f'(x) \) is defined and \( f(x) \) is not continuous. The correct response is \( (n) \). There is no obvious pattern of incorrect responses although responses \( (a) \) and \( (d) \) were chosen by many students.

Item 22.3 requires the student to select the letter at which \( f'(x) = 0 \) and \( f(x) \) is neither a maximum nor a minimum. The correct response is \( (e) \). Several students chose \( (c) \) and \( (h) \), implying an awareness of the meaning of \( f'(x) = 0 \). These students failed to apply the second condition of the statement.

Item 22.4 requires the student to select the point at which \( f'(x) \) is undefined and \( f(x) \) is a relative but not an absolute minimum. The correct response is \( (a) \). A large number of students chose responses \( (d) \) and \( (n) \). Response \( (d) \)
satisfies the condition that \( f'(x) \) is undefined but \( f(d) \) is an absolute minimum. The students choosing \((n)\) probably forgot that the endpoints of an interval may be maximum or minimum values.

Item 22.5 requires the student to select the point at which \( f'(x) = 0 \) and \( f(x) \) is an absolute minimum. The correct response is \((n)\). Response \((d)\) was chosen by several students; response \((h)\) was chosen more often than any other response, including the correct response. The students choosing \((d)\) omitted the condition that \( f'(x) = 0 \) and those choosing \((h)\) omitted the condition that \( f(x) \) be an absolute minimum.

There seemed to be a tendency on the part of the students responding incorrectly to items 22.2, 22.3, 22.4, and 22.5 to focus on one property of the desired value and to ignore any other properties.

Item 25.3 requires the student to select the statement which explains or defines the statement "The graph of \( f(x) \) is an unbroken curve on the interval \((a,b)\)." The correct response is \((b)\), "\( f(x) \) is continuous on the interval \((a,b)\)." There is no pattern among the incorrect responses, and the time would imply that the percent failure might actually be lower than reported.

Item 25.6 requires the student to select the statement which explains or defines the statement, "The graph of \( f(x) \) has no sharp corners when \( a < x < b \)." The correct response is \((a)\) "\( f'(x) \) exists on the interval \((a,b)\)." There is no
pattern among the incorrect responses for this item and the comment made for item 25.3 is applicable here also.

Item 17 is as follows:

If \( \int_0^x (3t^2 + 4t - 1) \, dt = 2 \), then the smallest solution \( x \) is

a. less than -1  

b. -1  

c. 0  

d. 1  

e. greater than 1

The correct response is (a). The incorrect responses chosen most often were (b) and (d). This may be due to the fact that 1 and -1 are both solutions of the equation.

Items 19.1, 19.2, 19.3, 19.4, and 19.5 require the student to answer questions about the function,

\[ F(x) = \int_0^x |\sin t| \, dt \quad \text{for} \quad x \in [-\pi, 2\pi]. \]

Item 19.1 asks the student for the interval on which \( F(x) \) is positive. The correct response is (b), \( x \in [0, 2\pi] \) only. The fact that most students believed \( F(x) \) to be \(|\sin x|\) is indicated by the large number of students choosing response (a), all \( x \).

Item 19.2 asks the student for the interval in which \( F(x) \) is increasing. The correct response is (e) everywhere. A large number of students chose response (b) the interval \([-\pi/2, \pi/2]\), and response (d) the intervals \([-2\pi, -3\pi/2]\), \([-\pi, -\pi/2]\), \([0, \pi/2]\), and \([\pi, 3\pi/2]\). Response (d) is the set of intervals on which \(|\sin x|\) is increasing.

Item 19.3 asks the student for a point at which \( F(x) \) has a horizontal tangent. The correct response is (d), \( \pi \).
A large number of students chose response (b), $\frac{\pi}{2}$, which is a point at which both $\sin x$ and $|\sin x|$ have a horizontal tangent.

Item 19.4 asks the student for a point in the interval $(-\pi, \pi)$ at which $F(x)$ has a maximum value. The correct response is (a) no point. Responses (c) 0 and $\pi$, and (d) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, were each chosen by many students. The function $F(x)$ has horizontal tangents at 0 and $\pi$ but $F(x)$ is not maximum at either, and $\pi$ is not in the given interval. The function $|\sin x|$ has maximum values at $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ which may be the reason why (d) was chosen so often.

Item 19.5 is as follows:

$F(x)$ is differentiable because

a. $|\sin t|$ is continuous
b. $F(x)$ is continuous
c. $\lim_{x \to c} (F(x+ h)-F(x))$ exists for all $c$ in $[-2\pi, 2\pi]$
d. $|\sin t|$ is positive for all $t$
e. $\lim_{\Delta t \to 0} \frac{|\sin t|}{\Delta t} = 0$

The correct response is (b). The most popular incorrect response was (c). This response looks like the definition of the derivative of a function which is probably the reason for its being chosen so often.

The responses to items 19.1 through 19.5 seem to indicate that the concept of a function defined in terms of the definite integral is not well understood by the students. The use of $|\sin x|$ for $\int_0^X |\sin t| \, dt$ by the students seems to imply a confusion between $F(x)$ and $F'(x) = |\sin x|$. 
Item 6 (second half) is as follows:

If \( f(t) \) is continuous then \( F(x) = \int_a^x f(t)\,dt \) defines a continuous function. Write a mathematical expression for the slope of the line tangent to \( F \) at a point with \( x \) coordinate \( x_0 \).

Most students did not respond to this question. Some gave the response "\( m = F'(x_0) \)." The fact that the slope of the tangent line is the value of the derived function was evidently known by the students, but they seemed unable to apply this knowledge to a function defined in terms of an integral.

Item 8 (second half) is as follows:

The processes of differentiation and of finding the definite integral involve two different concepts. Write a mathematical identity showing how differentiation and the definite integral are related for a function which is continuous in the interval \((a,b)\).

Very few students gave the proper response \( \int_a^b f(x)\,dx = F(b) - F(a) \) where \( F'(x) = f(x) \). Many students were able to give both the formal definition and the geometric interpretation of the derivative and the definite integral but were unable to relate the two using the fundamental theorem of calculus. Some students gave the response "\( \int dy = y + c \)" without going any farther.

Item 9 is as follows:

Which of the following is a valid inference for the hypotheses listed?

Hypotheses: 1. \( f(x) \) is a quadratic function
   2. the derived function \( f'(x) \) has a negative slope everywhere.
a. \( f(x) \) is concave up
b. \( f(x) \) is increasing everywhere
c. \( f(x) \) has a point of inflection
d. \( f(x) \) rises and then falls
e. \( f(x) \) is decreasing everywhere

The correct response is (d). Approximately half the students chose (e) implying that the phrase "the derived function \( f'(x) \)" was misread as "the derived function of \( f(x) \)."

Item 12 is as follows:

Which of the following is a valid inference for the hypotheses listed below?

Hypotheses: 1. \( f(x) \) is a quadratic function.
               2. \( g(x) \) is a linear function.
               3. \( x=a \) and \( x=b \) are the \( x \) coordinates of the points of intersection of \( f(x) \) and \( g(x) \) with \( a<b \).
               4. For \( a<x<b \), \( f(x) - g(x) \) is positive.

a. \( f(x) \) is increasing everywhere.
b. \( f(x) \) is decreasing everywhere.
c. \( f(x) \) is concave down everywhere.
d. \( f(x) \) is concave up everywhere
e. \( f(x) \) is positive everywhere.

The correct response is (c). The incorrect responses are nearly evenly distributed among (a), (d), and (e). There is no apparent reason for the popularity of any of these choices.

The objective which appears to have been achieved most successfully at level 2 (comprehension) is objective V, "The student should define intuitively the technical terms of the first year calculus, e.g., the limit of a function, continuity, derived function, critical point, definite integral." Additional test items would have to be administered before confidence could be placed in the mean percent of failure for objective XIII, "The student should make
inference from given statements." The mean percent of failure for objective VI, "The student should recognize the importance of the Fundamental Theorem of calculus in relating differentiation and integration," and the percent failure on the items related to this objective seem to imply that there is a lack of understanding by the students of the way in which the derivative and the definite integral are related and of the role of the fundamental theorem of calculus in this relationship.

The mean percent of failure for all items related to level 2 (comprehension) is 57.

A summary of the results of the test for level 3 (application) of the cognitive domain is found in Table 13. Those items with a percent of failure greater than or equal to forty will be discussed.

Item 1 is as follows:

\[
\frac{d}{dx} \left( \ln \left( \frac{x}{x + 1} \right) \right) = \]

a. \( \frac{1}{x + 1} \)  

b. \( \ln x - \ln (x + 1) \)  

c. \( \frac{x}{(x + 1)^3} \)  

d. 1  

e. \( (x^2 + x)^{-1} \)

The correct response is (e). The incorrect response chosen most often was (a) which omits the use of the chain rule in differentiating the function \( \ln u \).
TABLE 13

MEAN PERCENT OF FAILURE FOR EACH OF THE ITEMS RELATED TO THE OBJECTIVES AT LEVEL 3 OF THE COGNITIVE DOMAIN, FOR EACH OF THE OBJECTIVES AT LEVEL THREE, AND FOR LEVEL THREE AS A WHOLE

<table>
<thead>
<tr>
<th>Level</th>
<th>Objective</th>
<th>Item</th>
<th>Percent Failure</th>
<th>Mean Percent for Objective</th>
<th>Mean Percent for Level</th>
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</thead>
<tbody>
<tr>
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<td>56</td>
<td>48</td>
<td>61</td>
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<td></td>
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<td>III</td>
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<td>7(Sec Half)</td>
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<tr>
<td></td>
<td>1A(Sec Half)</td>
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<td></td>
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<td></td>
<td>4 (Sec Half)</td>
<td>91</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Item 3 is as follows:

\[ \frac{d}{dx} (x^3 e^{2x^2}) = \]

- a. \( e^{2x^2} [2x^4 + 3x^2] \)
- b. \( 4x^3 e^{2x^2} + 3x^2 e^{2x^2} \)
- c. \( x^2 [xe^{4x} + 3e^{2x^2}] \)
- d. \( x^3 e^{2x^2} + 3x^2 e^{2x^2} \)
- e. \( e^{2x^2} [4x^4 + 3x^2] \)

The correct response is (e). The most popular incorrect response was (d) which uses the product rule for differentiation correctly but assumes that \( \frac{d}{dx} (e^u) = e^u \).
Item 4 is as follows:
\[ \int \tan 2x \sec^3 2x \, dx \]
\[
\begin{align*}
\text{a. } & \frac{\tan^2 2x}{4} + c \\
\text{b. } & \frac{\sec^3 2x}{6} + c \\
\text{c. } & \frac{\sec^3 2x}{3} + c \\
\text{d. } & \frac{\tan^2 2x}{2} + c \\
\text{e. } & \frac{\sec^4 2x}{8} + c
\end{align*}
\]

The correct response is (b). The incorrect responses are nearly evenly distributed among the other choices with choice (c) receiving slightly more than the rest. This choice is an improper use of the chain rule.

Item 5 is as follows:
\[ \int_0^1 3x \, dx = \]
\[
\begin{align*}
\text{a. } & e^{\ln^3} \\
\text{b. } & \frac{9}{2} - 3 \\
\text{c. } & \ln \left( \frac{1}{3} \right)^2 \\
\text{d. } & \frac{e^{\ln^3}}{\ln^3} \\
\text{e. } & 2(\ln 3)^{-1}
\end{align*}
\]

The correct response is (e). The incorrect responses were fairly evenly distributed among choices (a), (b), and (d). There is no apparent reason for the popularity of these responses.

Item 10 is as follows:

Suppose that \( f(x) \) is defined by the graph

\[ \int_a^c f(x) \, dx \]

Which of the following is the best approximation to

\[ \int_a^c f(x) \, dx? \]
\[ a. \frac{1}{2} \frac{(f(c) - f(a))}{(c - a)} \quad \text{d.} \quad \frac{1}{2} (f(a) - f(c)) \]
\[ b. \ (f(a) - f(c)) (a - c) \quad \text{e.} \quad \frac{1}{2} (a - c) (f(c) - f(a)) \]
\[ c. \frac{f(c) - f(a)}{c - a} \]

The correct response is (e). Response (c), which represents the slope of the line connecting the points (a, f(a)) and (c, f(c)), was chosen much more often than the correct response.

Item 11 asks the student for the numerical value of \( f'(\pi) \) if \( f(x) \) is tangent to \( y = |x| \) at \( x = \pi \). The correct response is (c), 1. Response (d), 0, was chosen by many students and may be a result of interpreting the symbol "\( f'(\pi) \)" as "\( \frac{d}{dx} (f(\pi)) \)."

Item 14 is as follows:

Let \( f(x) \) be defined by the following graph: and let
\[ m = \int_a^b f(x) \, dx, \quad n = \int_b^c f(x) \, dx, \quad p = \int_a^c f(x) \, dx, \quad q = \int_b^a f(x) \, dx \]

If the numbers \( m, n, p, q \), are arranged in increasing magnitude, the correct order is
\[ a. \quad q < n < p < m \quad \text{b.} \quad n < p < m = q \quad \text{c.} \quad m < p < n < q \]
\[ d. \quad q = m < n < p \quad \text{e.} \quad q < p < n < m \]

The correct response is (e). The most popular incorrect responses were (b) and (c). Response (b) assumes that
\[ \int_{a}^{b} f(x) \, dx = \int_{b}^{a} f(x) \, dx \] and response (c) reverses all the inequality signs in the correct response. The reason for choosing (c) may be confusion regarding the inequality symbol.

Item 16 requires the student to choose the closest approximation to \[ \int_{0}^{1/2} \frac{1}{1 + x^6} \, dx \] from among the five responses. The correct response is (c), .5. Choice (a), .005 was the most popular incorrect response. When the percent failure on items 10 and 16 are compared with the percent failure on item 8, it appears as if the student is able to identify the geometric interpretation of the definite integral, but unable to apply this interpretation to a concrete situation.

Item 18 is as follows:

If a particle moves vertically so that \( t \in [0,b] \), \( (b > 0) \), the velocity \( v \) is greater than zero and the acceleration \( a \) is less than zero, then the particle

a. is speeding up.
b. slowing down.
c. will change directions.
d. is falling.
e. is in equilibrium.

The correct response is (b); the incorrect response chosen most often was (d).

Item 1A (second half) is as follows:

A kite is 40 feet high, with 50 feet of string out. If the kite moves horizontally at 5 feet/second away from the boy flying it, how fast is the string being let out?

The correct answer is 3 feet/second. Most of the students responding to this item drew a triangle and labeled its sides.
Many used the Pythagorean relationship to find the length of one side, but this is as far as they were able to proceed. They did not realize that it was necessary for them to differentiate in order to complete the solution.

Item 3A (second half) is as follows:

A ball is rolled over a level lawn with initial velocity of 24 feet/second. Due to friction the velocity decreases at a rate of 6 feet/second\(^2\), i.e., \( \frac{dv}{dt} = -6 \text{ feet/second}^2 \). How far will the ball roll?

The correct answer is 48 feet. Many students who responded incorrectly to this item realized that \( \int v = s + c \) if \( s \) is the distance function and \( c \) is a constant but did not realize that they were given the acceleration, \( a = \frac{dv}{dt} \). Some were able to find an antiderivative of \( \frac{dv}{dt} \) but failed to use the initial conditions to solve for the constant of integration.

Item 4 (second half) is as follows:

Find the volume of the solid obtained by rotating the region bounded by the coordinate axes, the line \( x = 1 \), and the function \( y = e^{-x^2} \) about the y axis.

A large number of students did not respond to this item. Many of those who did were not able to go beyond the recognition of this item as one whose solution involved integration. This was obvious from the integral sign with limits of integration 0 and 1, and nothing else in the response area. Very few students were able to set up the general formula for finding the volume of a solid of revolution using either the method known as the "disc" method or the method known as the
method of "hollow cylinders." Both of these methods received a great deal of attention in the study of applications of integration in Math 441.

The objective which seems to have been achieved most successfully at level three (application) of the cognitive domain is objective I, "The student should compute the derivative and definite integral of common functions, e.g., polynomials, trigonometric functions, logarithmic functions, and exponential functions." The high mean percent of failure for objective VIII, "The student should apply calculus to physical problems which are traditionally solved by calculus," is heavily influenced by the inclusion of items from the second half of the test. The students were much less successful on the written response items than on the multiple choice items. There seems to be a need for more exposure to and work with physical applications of both differentiation and integration.

There are two items related to objective IX, "The student should outline a method of solution to a problem," which is the only objective at level 5 (synthesis) of the cognitive domain.

Item 1B (second half) and item 3B (second half) require the students to outline their method of solution to items 1A and 3A respectively. Very few of the students who responded correctly in 1A or 3A were able to give an acceptable outline of their method of solution. Similarly,
very few of the students who responded incorrectly to 1A or 3A were able to give an acceptable outline of their method of solution.

Item 1A is a common related rate type problem studied as an application of differentiation. The method of solution of this type problem was evidently not remembered from the first part of the sequence.

Item 3A involves the solution of a differential equation using initial conditions. This was one of the first topics taken up by the students in the study of integral calculus so the method of solution of the problem should have been familiar.

Although a larger number of items would be necessary to validate the results on objective IX, the very high percent of failure seems to imply that a great deal of work needs to be done in making the student aware of the structure of a solution of a problem. The results on the written response items might have been much better if the student had been able to outline a method of solution and then follow that outline to complete the solution.

Using the mean percent of failure as a measure of the achievement of a particular level of the taxonomy, we can compare the achievement at the first three levels of the cognitive domain. The greatest amount of achievement seems to have occurred at level 1 which involves those behaviors and test situations which emphasize the remembering, either
by recognition or recall, of ideas, material, or phenomena (2;62). The students were most successful at this level in identifying symbolism which is a behavior associated with the knowledge of specifics. They were also very successful in recognizing several different applications of a particular principle, a behavior which may be associated with the knowledge of the universals and abstractions in a field.

There seems to have been much less achievement at level 2, which involves those behaviors necessary to know, when confronted with a communication, what is being communicated, and to be able to make some use of the material or ideas contained in it (2;89). The students were most successful, at this level, in defining intuitively the technical terms of the first year sequence; this behavior may be associated with the interpretation of a communication. The students were very unsuccessful in recognizing the importance of the fundamental theorem of calculus in relating differentiation and integration which may be associated with the behaviors relating to extrapolation, i.e., going beyond the recognition of the formula \( \int_a^b f(x) \, dx = F(b) - F(a) \) as a means of evaluating the definite integral to an understanding of the way in which it unites differential and integral calculus.

The achievement at level 3, which involves the application of an abstraction, method, theory, or principle, seems to be comparable to the achievement at level 2. The
students were most successful, at this level, in computing the derivative and integral of several common functions. Their success on this objective, however, was not nearly as great as their success on the best achieved objectives at levels 1 and 2.

The achievement at level 5 (synthesis) cannot properly be compared with the achievement at the other levels because of the small number of items at that level.

Analysis of the results relating to the affective domain

There are two items related to objective XIV at level 1 (receiving) of the affective domain; objective XIV states, "The student should be alert to possible solutions and methods of attack of a non-traditional nature"; item 2 (second half) requires the student to find the positive number which exceeds its square by the greatest amount; item 5 (second half) requires the student to compute the derivative of $y = 4 \sin^2 x \cos^2 x + \cos^2 2x$. Both of these items may be solved using knowledge from the first year calculus sequence. Item 2, however, may be solved easily by a finite substitution process and item 5 becomes a trivial differentiation if two trigonometric identities are first applied to $y$. Most of the students responding to this question blindly applied a method from the calculus without first attempting to find a simplified method, even though the directions specified that they did not have to use calculus. The results on these items
seem to imply that the students do not analyze each problem for its unique features, but rather, classify it as a particular type and then proceed to apply a standard method of solution.

The mean percent of failure for objective XIV as computed from Table 10 is 60.

There are four items related to objective XVI at level 2 (responding) of the affective domain; objective XVI states, "The student should experience enjoyment as a result of comprehending the calculus used in a practical application." Items 20.1 20.2, 20.3, 20.4 are grouped under the following instructions:

Assuming that you get the correct result to a problem involving an application of calculus, rate each of the statements below (a) through (e) as follows:

a. if you strongly disagree
b. if you disagree
c. if you are undecided
d. if you agree
e. if you strongly agree

Item 20.1 is the statement, "Getting the correct response without understanding the method of solution is upsetting."
Item 20.2 is the statement, "Getting the correct answer is enjoyable even without understanding the method of solution."
Item 20.3 is the statement, "Getting the correct answer when the method of solution is understood is generally neither enjoyable nor unenjoyable."
Item 20.4 is the statement, "Getting the correct answer is enjoyable only when the method of solution is clear."
Table 14 contains the number and percent (to the nearer integer) of the responding students who chose each response. Eighty percent of the students who responded to item 20.3 either disagreed or strongly disagreed with the statement. Seventy-eight percent of the students disagreed or strongly disagreed with the statement in item 20.2. Seventy-seven percent of the students agreed or strongly agreed with the statement in 20.1. Seventy percent of the students responding to item 20.4 agreed or strongly agreed with the statement. There appears to be a discernible trend in these responses, i.e., the students claim they experience some kind of emotion when they arrive at the correct response (20.3), they claim they consider arriving at the correct response unenjoyable (20.2) and even upsetting (20.1) if the understanding of the method of solution is lacking, and they claim they experience enjoyment only when the method of solution is clear (20.4). This seems to imply that objective XVI has been achieved. The extent to which the objective has been achieved would appear to be considerable if the number of students responding at the extremes of the response scale was any indication of success, and if there was no "halo" effect, i.e., if the students responded as they actually felt rather than as they thought they were expected to respond.
TABLE 14

NUMBER AND PERCENT OF STUDENTS RESPONDING TO EACH CHOICE
IN ITEMS 20.1, 20.2, 20.3, AND 20.4

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<tr>
<th>20.1</th>
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<th>12</th>
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<th>61</th>
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<tr>
<td></td>
<td>Percent</td>
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<td>9</td>
<td>28</td>
<td>42</td>
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</tbody>
</table>

Faculty-student rating
of objectives

The students in the sample population were asked to express the extent to which they felt the statements on pages 23 through 25 were objectives of the Math 440-Math 441 calculus sequence. The rating scale they used was the same one the faculty had used, i.e.,

0. If they felt the statement did not represent an objective.
1. If they felt the statement represented an objective which received minor emphasis.
2. If they felt the statement represented an objective which received moderate emphasis.
3. If they felt the statement represented an objective which received major emphasis.

One hundred thirty-five students returned the rating sheets and on the basis of these ratings a mean rate was computed for
each of the thirty-five statements. Table 15 contains a summary of the ratings and the corresponding mean value on each of the proposed objectives for both the faculty and the students. The means were then arranged in descending order, i.e., for the students, statement 28 with a mean rate of 2.82 was ranked 1, statement 10 with a mean rate of 2.79 was ranked 2, and so forth. The faculty ratings were ranked in the same manner. Table 16 contains the faculty mean and rank, and the student mean and rank for each statement. The difference in rank was computed by subtracting the student rank from the faculty rank. The rank difference correlation coefficient, $\rho$, between these rankings was computed as .74 (18;372). For significance at the .01 level, a coefficient of .43 is necessary (18;201). As a check on the rank-difference coefficient $\rho$, the product-moment coefficient of correlation, $r$, was computed using the means as raw scores (18;143). The value of $r$ was found to be .77. Hence, there is no significant difference in the way in which the faculty and the students ranked the set of proposed objectives of the first year calculus sequence. There were instances, however, in which the faculty and student ranks differed considerably on individual statements. Some of these instances will now be discussed.

Statement 1--the student should be willing to expand mathematical knowledge by independent reading--was ranked 18 by the faculty and 30.5 by the students. Statement 17--the
### TABLE 15
RATING AND MEAN RATING FOR EACH OF THE PROPOSED OBJECTIVES

<table>
<thead>
<tr>
<th></th>
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TABLE 16
MEAN, RANK, AND RANK DIFFERENCE FOR EACH OF THE PROPOSED OBJECTIVES

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Note: In the case of tied means, the mean rank was assigned to all the tied values, e.g., statements 3, 24, 28, 30, and 35 were rated 3.00 by the faculty and therefore all were ranked 3.
student should read mathematics texts or periodicals independently with some degree of comprehension (provided the content is a reasonable extension of the student's range of knowledge)—was ranked 19.5 by the faculty and 30.5 by the students. Hence, the students seem to consider independent reading less important than did the faculty.

Statement 10— the student should isolate the logical structure underlying a proof—was ranked 17 by the faculty and 2 by the students. Statement 14— the student should attempt to conceptualize the deductive nature of mathematics—was ranked 31.5 by the faculty and 19 by the students. Statement 27— the student should attempt to conceptualize the logical structure of calculus—was ranked 30 by the faculty and 12 by the students. The ranking of these three statements appears to imply that the students envisioned a much more abstract foundation for the calculus sequence than the faculty. The students seemed to feel that deduction and logic played a more dominant role than the faculty considered necessary or important. It is highly probable, however, that the students' interpretation of deduction and logic is different from that of the faculty, for example, the students may have felt that a rigorous proof of a particular theorem was an attempt at conceptualizing the logical structure of calculus while the faculty had in mind a development of the calculus starting with the properties of the real number system.
Statement 25—the student should distinguish between heuristic arguments and formal proofs—was ranked 11 by the faculty and 32 by the students. There is good reason to suspect that the students did not know the meaning of the word heuristic and, as a result, rated the objective lower than the faculty.

The following chapter contains a summary of the study, a statement of the major conclusions of the study, and some suggestions for related research.
CHAPTER V

SUMMARY AND CONCLUSIONS

Summary

A collection of proposed objectives for the first year calculus sequence at The Ohio State University was generated using the taxonomy of educational objectives developed by Bloom (cognitive domain) and Krathwohl (affective domain), and statements from the literature of mathematics education. These objectives were rated on the basis of importance by members of the mathematics faculty. Several statements were chosen on the basis of the rating, and test items were constructed for these statements. A selection process involving faculty members and colleagues resulted in a number of acceptable test items. A preliminary test form constructed from these items was administered to a group of fifty-seven calculus students at two of the University branches; analyses of the results on this preliminary test were used to construct a revised test. The revised test was administered to a group of one hundred ninety-three students concluding the first year calculus sequence at the main campus of The Ohio State University. The students also rated the proposed objectives using the same rating scale employed by the faculty.
The results on the revised test and the rating of the proposed objectives provided the data on which the following conclusions are based.

Conclusions

Assuming that the items of the test are valid tests of the behaviors implied by the objectives and assuming that the test is reliable, the following conclusions seem to be warranted by the analysis of the test results:

1. The group of objectives associated with level 1 (recall) of the cognitive domain was achieved more satisfactorily than the group of objectives associated with level 2 (comprehension) or level 3 (application).

2. The achievement of the group of objectives at level 2 was comparable to the achievement of the group of objectives at level 3.

3. The objective achieved most successfully among the three objectives related to level one was "The student should identify all the symbols used in first year calculus, e.g., \( \frac{dy}{dx}, f', \int f, \int_a^b f(x) \, dx \)" (objective IV).

4. The objective achieved most successfully among the four objectives related to level two was "The student should define intuitively the technical terms of the first year calculus, e.g., the limit of a function, continuity, derived function, critical point, definite integral" (objective V).
5. The objective achieved most successfully among the three objectives related to level 3 was "The student should compute the derivative and the definite integral of common functions, e.g., polynomials, trigonometric functions, logarithmic functions, and exponential functions" (objective I).

An investigation of the rating of the proposed objectives by the faculty and the students implies the following conclusions:

1. There was no significant difference in the overall way in which the faculty and the students ranked the set of proposed objectives of the first year calculus sequence.

2. The faculty ranked objectives related to independent reading much higher than the students did.

3. The faculty ranked objectives related to deduction and logic much lower than the students did.

Suggestions for further study and related research

This study raised many questions in the mind of the investigator. Some of these questions were answered in the course of the research and the reporting of the research. Many more remain unanswered and it is in the hope of pointing others in the direction of solutions to these questions that the following suggestions for further study and related research are made.
Repeated administration of the test with increased time limits would be both useful and rewarding. The lack of certainty concerning the lower bounds of failure for the items in the later part of the first half of the test would be reduced and more appropriate time limits for the test could be established.

A validation of the relationship between the test items and the objectives they are supposed to be testing could be accomplished by having several judges classify the items, first by taxonomy levels and then by objectives within the levels. The classification of the judges could be compared and used to set up a reference classification. The extent to which the author's classification agrees with the reference classification could be taken as a measure of the appropriateness of individual items in testing each objective.

The statement of additional objectives and the construction of additional test items at the higher levels of the cognitive domain are essential if the achievement of more complex cognitive behaviors is to be studied and measured.

It became apparent in the attempt to construct acceptable test items that "paper and pencil" tests were an ineffective method of evaluating affective behaviors. Check-list and observation techniques seem more effective but the difficulty in interpreting and scoring introduced by these methods appears to make their use in the classroom impractical.

Measures of affective behaviors which are more adequate, more
refined, and more objectively scored and evaluated should be developed, and more emphasis placed on the observation and development of "interests, appreciations, attitudes, values, and emotional sets or biases" (21:7) in the classroom.

Attempts, such as Romberg's (34) to adapt the taxonomy to mathematical behaviors might result in valuable insight into the problem of the way in which mathematics is learned.

A complete list of objectives for a course or a sequence of courses might be very long. Even a partial list (as evidenced by this study) may be too long to be tested in a short period of time. Studies similar to this one, focusing on only a few of the objectives, might permit in-depth testing of particular behaviors. Another possibility might be the use of several tests, each concentrating on a relatively few objectives; each of these tests could be administered to a part of the student population and the results extrapolated to the entire group.

The construction of banks of items related to particular objectives is under way in many areas. These projects are attempts at carefully selecting and classifying items with respect to objectives. After submitting his objectives to the bank, the teacher receives several test items appropriate for each objective; these items may be used directly or as type items to test for the desired behavior. Further studies into the classification of objectives for the calculus,
and the construction and classification of good test items would promote the development of such item banks.

Studies relating the importance that a student attaches to an objective to his performance on items related to that objective might shed light on the way in which attitudes in mathematics affect the learning of mathematics. Studies comparing student achievement and attitude to the degree of faculty-student agreement on objectives might be equally valuable.

Finally, the procedures used in this study might be extended to construct objectives and related tests for:

1. each of the subpopulations of students taking the calculus sequence, e.g., mathematics majors, science majors, education majors.

2. the teaching behaviors necessary in college mathematics classrooms.

3. the entire calculus sequence.

4. any college mathematics course.

It is hoped that this study will not be viewed as an end product but rather the raw material from which many useful products may be distilled.
APPENDIX A

CONDENSED VERSIONS OF THE COGNITIVE
AND AFFECTIVE DOMAINS
COGNITIVE DOMAIN

KNOWLEDGE

1.00 KNOWLEDGE

Knowledge, as defined here, involves the recall of specifics and universals, the recall of methods and processes, or the recall of a pattern, structure, or setting. For measurement purposes, the recall situation involves little more than bringing to mind the appropriate material. Although some alteration of the material may be required, this is a relatively minor part of the task. The knowledge objectives emphasize most the psychological processes of remembering. The process of relating is also involved in that a knowledge test situation requires the organization and reorganization of a problem such that it will furnish the appropriate signals and cues for the information and knowledge the individual possesses. To use an analogy, if one thinks of the mind as a file, the problem in a knowledge test situation is that of finding in the problem or task the appropriate signals, cues, and clues which will most effectively bring out whatever knowledge is filed or stored.

1.10 KNOWLEDGE OF SPECIFICS

The recall of specific and isola ble bits of information. The emphasis is on symbols with concrete referents. This material, which is at a very low level of abstraction, may be thought of as the elements from which more complex and abstract forms of knowledge are built.

1.11 KNOWLEDGE OF TERMINOLOGY

Knowledge of the referents for specific symbols (verbal and non-verbal). This may include knowledge of the most generally accepted symbol referent, knowledge of the variety of symbols which may be used for a single referent, or knowledge of the referent most appropriate to a given use of a symbol.

*To define technical terms by giving their attributes, properties, or relations.

*Familiarity with a large number of words in their common range of meanings.

*Illustrative educational objectives selected from the literature.
1.12 KNOWLEDGE OF SPECIFIC FACTS

Knowledge of dates, events, persons, places, etc. This may include very precise and specific information such as the specific date or exact magnitude of a phenomenon. It may also include approximate or relative information such as an approximate time period or the general order of magnitude of a phenomenon.

*The recall of major facts about particular cultures.

*The possession of a minimum knowledge about the organisms studied in the laboratory.

1.20 KNOWLEDGE OF WAYS AND MEANS OF DEALING WITH SPECIFICS

Knowledge of the ways of organizing, studying, judging, and criticizing. This includes the methods of inquiry, the chronological sequences, and the standards of judgment within a field as well as the patterns of organization through which the areas of the fields themselves are determined and internally organized. This knowledge is at an intermediate level of abstraction between specific knowledge on the one hand and knowledge of universals on the other. It does not so much demand the activity of the student in using the materials as it does a more passive awareness of their nature.

1.21. KNOWLEDGE OF CONVENTIONS

Knowledge of characteristic ways of treating and presenting ideas and phenomena. For purposes of communication and consistency, workers in a field employ usages, styles, practices, and forms which best suit their purposes and/or which appear to suit best the phenomena with which they deal. It should be recognized that although these forms and conventions are likely to be set up on arbitrary, accidental, or authoritative bases, they are retained because of the general agreement or concurrence of individuals concerned with the subject, phenomena, or problem.

*Familiarity with the forms and conventions of the major types of works, e.g., verse, plays, scientific pagers, etc.

*To make pupils conscious of correct form and usage in speech and writing.
1.22 KNOWLEDGE OF TRENDS AND SEQUENCES

Knowledge of the processes, directions, and movements of phenomena with respect to time.

*Understanding of the continuity and development of American culture as exemplified in American life.

*Knowledge of the basic trends underlying the development of public assistance programs.

1.23 KNOWLEDGE OF CLASSIFICATIONS AND CATEGORIES

Knowledge of the classes, sets, divisions, and arrangements which are regarded as fundamental for a given subject field, purpose, argument, or problem.

*To recognize the area encompassed by various kinds of problems or materials.

*Becoming familiar with a range of types of literature.

1.24 KNOWLEDGE OF CRITERIA

Knowledge of the criteria by which facts, principles, opinions, and conduct are tested or judged.

*Familiarity with criteria for judgment appropriate to the type of work and the purpose for which it is read.

*Knowledge of criteria for the evaluation of recreational activities.

1.25 KNOWLEDGE OF METHODOLOGY

Knowledge of the methods of inquiry, techniques, and procedures employed in a particular subject field as well as those employed in investigating particular problems and phenomena. The emphasis here is on the individual's knowledge of the method rather than his ability to use the method.

*Knowledge of scientific methods for evaluating health concepts.

*The student shall know the methods of attack relevant to the kinds of problems of concern to the social sciences.
1.30 **KNOWLEDGE OF THE UNIVERSALS AND ABSTRACTIONS IN A FIELD**

Knowledge of the major schemes and patterns by which phenomena and ideas are organized. These are the large structures, theories, and generalizations which dominate a subject field or which are quite generally used in studying phenomena or solving problems. These are at the highest levels of abstraction and complexity.

1.31 **KNOWLEDGE OF PRINCIPLES AND GENERALIZATIONS**

Knowledge of particular abstractions which summarize observations of phenomena. These are the abstractions which are of value in explaining, describing, predicting, or in determining the most appropriate and relevant action or direction to be taken.

*Knowledge of the important principles by which our experience with biological phenomena is summarized.

*The recall of major generalizations about particular cultures.

1.32 **KNOWLEDGE OF THEORIES AND STRUCTURES**

Knowledge of the body of principles and generalizations together with their interrelations which present a clear, rounded, and systematic view of a complex phenomenon, problem, or field. These are the most abstract formulations, and they can be used to show the interrelation and organization of a great range of specifics.

*The recall of major theories about particular cultures.

*Knowledge of a relatively complete formulation of the theory of evolution.

**INTELLECTUAL ABILITIES AND SKILLS**

Abilities and skills refer to organized modes of operation and generalized techniques for dealing with materials and problems. The materials and problems may be of such a nature that little or no specialized and technical information is required. Such information as is required can be assumed to be part of the individual's general fund of knowledge. Other problems may require specialized and technical information at a rather high level such that specific knowledge and skill in dealing with the problem and the materials are required. The abilities and skills objectives emphasize the mental processes of organizing
and reorganizing material to achieve a particular purpose. The materials may be given or remembered.

2.00 COMPREHENSION

This represents the lowest level of understanding. It refers to a type of understanding or apprehension such that the individual knows what is being communicated and can make use of the material or idea being communicated without necessarily relating it to other material or seeing its fullest implications.

2.10 TRANSLATION

Comprehension as evidenced by the care and accuracy with which the communication is paraphrased or rendered from one language or form of communication to another. Translation is judged on the basis of faithfulness and accuracy, that is, on the extent to which the material in the original communication is preserved although the form of the communication has been altered.

*The ability to understand non-literal statements (Metaphor, symbolism, irony, exaggeration).

*Skill in translating mathematical verbal material into symbolic statements and vice versa.

2.20 INTERPRETATION

The explanation or summarization of a communication. Whereas translation involves an objective part-for-part rendering of a communication, interpretation involves a reordering, rearrangement, or a new view of the material.

*The ability to grasp the thought of the work as a whole at any desired level of generality.

*The ability to interpret various types of social data.
2.30 EXTRAPOLATION

The extension of trends or tendencies beyond the given data to determine implications, consequences, corollaries, effects, etc., which are in accordance with the conditions described in the original communication.

*The ability to deal with the conclusions of a work in terms of the immediate inference made from the explicit statements.
*Skill in predicting continuation of trends.

3.00 APPLICATION

The use of abstractions in particular and concrete situations. The abstractions may be in the form of general ideas, rules of procedures, or generalized methods. The abstractions may also be technical principles, ideas, and theories which must be remembered and applied.

*Application to the phenomena discussed in one paper of the scientific terms or concepts used in other papers.

*The ability to predict the probable effect of a change in a factor on a biological situation previously at equilibrium.

4.00 ANALYSIS

The breakdown of a communication into its constituent elements or parts such that the relative hierarchy of ideas is made clear and/or the relations between the ideas expressed are made explicit. Such analyses are intended to clarify the communication, to indicate how the communication is organized, and the way in which it manages to convey its effects, as well as its basis and arrangement.

4.10 ANALYSIS OF ELEMENTS

Identification of the elements included in a communication.

*The ability to recognize unstated assumptions.
*Skill in distinguishing facts from hypotheses.
4.20 ANALYSES OF RELATIONSHIPS

The connections and interactions between elements and parts of a communication.

* Ability to check the consistency of hypotheses with given information and assumptions.

* Skill in comprehending the interrelationships among the ideas in a passage.

4.30 ANALYSIS OF ORGANIZATIONAL PRINCIPLES

The organization, systematic arrangement, and structure which hold the communication together. This includes the "explicit" as well as "implicit" structure. It includes the bases, necessary arrangement, and the mechanics which make the communication a unit.

* The ability to recognize form and pattern in literary or artistic works as a means of understanding their meaning.

* Ability to recognize the general techniques used in persuasive materials, such as advertising, propaganda, etc.

5.00 SYNTHESIS

The putting together of elements and parts so as to form a whole. This involves the process of working with pieces, parts, elements, etc., and arranging and combining them in such a way as to constitute a pattern or structure not clearly there before.

5.10 PRODUCTION OF A UNIQUE COMMUNICATION

The development of a communication in which the writer or speaker attempts to convey ideas, feelings, and/or experiences to others.

* Skill in writing, using an excellent organization of ideas and statements.

* Ability to tell a personal experience effectively.
5.20 PRODUCTION OF A PLAN, OR PROPOSED SET OF OPERATIONS

The development of a plan of work or the proposal of a plan of operations. The plan should satisfy requirements of the task which may be given to the student or which he may develop for himself.

*Ability to propose ways of testing hypotheses.

*Ability to plan a unit of instruction for a particular teaching situation.

5.30 DERIVATION OF A SET OF ABSTRACT RELATIONS

The development of a set of abstract relations either to classify or explain particular data or phenomena, or the deduction of propositions and relations from a set of basic propositions or symbolic representations.

*Ability to formulate appropriate hypotheses based upon an analysis of factors involved, and to modify such hypotheses in the light of new factors and considerations.

*Ability to make mathematical discoveries and generalizations.

6.00 EVALUATION

Judgments about the value of material and methods for given purposes. Quantitative and qualitative judgments about the extent to which material and methods satisfy criteria. Use of a standard of appraisal. The criteria may be those determined by the student or those which are given to him.

6.10 JUDGMENTS IN TERMS OF INTERNAL EVIDENCE

Evaluation of the accuracy of a communication from such evidence as logical accuracy, consistency, and other internal criteria.

*Judging by internal standards, the ability to assess general probability of accuracy in reporting facts from the care given to exactness of statement, documentation, proof, etc.

*The ability to indicate logical fallacies in arguments.
6.20 JUDGMENTS IN TERMS OF EXTERNAL CRITERIA

Evaluation of material with reference to selected or remembered criteria.

*The comparison of major theories, generalizations, and facts about particular cultures.

*Judging by external standards, the ability to compare a work with the highest known standards in its field—especially with other works of recognized excellence.
1.0 RECEIVING (ATTENDING)

At this level we are concerned that the learner be sensitized to the existence of certain phenomena and stimuli; that is, that he be willing to receive or to attend to them. This is clearly the first and crucial step if the learner is to be properly oriented to learn what the teacher intends that he will. To indicate that this is the bottom rung of the ladder, however, is not at all to imply that the teacher is starting de novo. Because of previous experience (formal or informal), the student brings to each situation a point of view or set which may facilitate or hinder his recognition of the phenomena to which the teacher is trying to sensitize him.

The category of Receiving has been divided into three sub-categories to indicate three different levels of attending to phenomena. While the division points between the sub-categories are arbitrary, the subcategories do represent a continuum. From an extremely passive position or role on the part of the learner, where the sole responsibility for the evocation of the behavior rests with the teacher—that is, the responsibility rests with him for "capturing" the student's attention—the continuum extends to a point at which the learner directs his attention, at least at a semiconscious level, toward the preferred stimuli.

1.1 AWARENESS

Awareness is almost a cognitive behavior. But unlike Knowledge, the lowest level of the cognitive domain, we are not so much concerned with a memory of, or ability to recall, an item or fact as we are that, given appropriate opportunity, the learner will merely be conscious of something—that he take into account a situation, phenomenon, object, or stage of affairs. Like Knowledge it does not imply an assessment of the qualities or nature of the stimulus, but unlike Knowledge it does not necessarily imply attention. There can be simple awareness without specific discrimination or recognition of the objective characteristics of the object, even though these characteristics must be deemed to have an effect. The individual may not be able to verbalize the aspects of the stimulus which cause the awareness.

Develops awareness of aesthetic factors in dress, furnishings, architecture, city design, good art, and the like.
Develops some consciousness of color, form, arrangement, and design in the objects and structures around him and in descriptive or symbolic representations of people, things, and situations.

1.2 WILLINGNESS TO RECEIVE

In this category we have come a step up the ladder but are still dealing with what appears to be cognitive behavior. At a minimum level, we are here describing the behavior of being willing to tolerate a given stimulus, not to avoid it. Like Awareness, it involves a neutrality of suspended judgment toward the stimulus. At this level of the continuum the teacher is not concerned that the student seek it out, nor even, perhaps, that in an environment crowded with many other stimuli the learner will necessarily attend to the stimulus. Rather, at worst, given the opportunity to attend in a field with relatively few competing stimuli, the learner is not actively seeking to avoid it. At best, he is willing to take notice of the phenomenon and give it his attention.

Attends (carefully) when others speak—in direct conversation, on the telephone, in audiences.

Appreciation (tolerance) of cultural patterns exhibited by individuals from other groups—religious, social, political, economic, national, etc.

Increase in sensitivity to human need and pressing social problems.

1.3 CONTROLLED OR SELECTED ATTENTION

At a somewhat higher level we are concerned with a new phenomenon, the differentiation of a given stimulus into figure and ground at a conscious or perhaps semiconscious level—the differentiation of aspects of a stimulus which is perceived as clearly marked off from adjacent impressions. The perception is still without tension or assessment, and the student may not know the technical terms or symbols with which to describe it correctly or precisely to others. In some instances it may refer not so much to the selectivity of attention as to the control of attention, so that when certain stimuli are present they will be attended to. There is an element of the learner's controlling the attention here, so that the favored stimulus is selected and attended to despite competing and distracting stimuli.

Illustrative objectives selected from the literature follow the description of each subcategory.
Listens to music with some discrimination as to its mood and meaning and with some recognition of the contributions of various musical elements and instruments to the total effect.

Alertness toward human values and judgments on life as they are recorded in literature.

2.0 RESPONDING

At this level we are concerned with responses which go beyond merely attending to the phenomenon. The student is sufficiently motivated that he is not just 1.2 Willing to attend, but perhaps it is correct to say that he is actively attending. As a first stage in a "learning by doing" process the student is committing himself in some small measure to the phenomena involved. This is a very low level of commitment, and we would not say at this level that this was "a value of his" or that he had "such and such an attitude." These terms belong to the next higher level that we describe. But we could say that he is doing something with or about the phenomenon besides merely perceiving it, as would be true at the next level below this of 1.3 Controlled or selected attention.

This is the category that many teachers will find best describes their "interest" objectives. Most commonly we use the term to indicate the desire that a child become sufficiently involved in or committed to a subject, phenomenon, or activity that he will seek it out and gain satisfaction from working with it or engaging in it.

2.1 ACQUIESCENCE IN RESPONDING

We might use the word "obedience" or "compliance" to describe this behavior. As both of these terms indicate, there is a passiveness so far as the initiation of the behavior is concerned, and the stimulus calling for this behavior is not subtle. Compliance is perhaps a better term than obedience, since there is more of the element of reaction to a suggestion and less of the implication of resistance or yielding unwillingly. The student makes the response, but he has not fully accepted the necessity for doing so.

Willingness to comply with health regulations.
Obeys the playground regulations.

2.2 WILLINGNESS TO RESPOND

The key to this level is in the term "willingness," with its implication of capacity for voluntary activity. There is the implication that the learner is sufficiently committed to exhibiting the behavior that he does so not just because of a fear of punishment, but "on his own" or voluntarily. It
may help to note that the element of resistance or of yielding unwillingly, which is possibly present at the previous level, is here replaced with consent or proceeding from one's own choice.

Acquaints himself with significant current issues in international, political, social, and economic affairs through voluntary reading and discussion.

Acceptance of responsibility for his own health and for the protection of the health of others.

2.3 SATISFACTION IN RESPONSE

The additional element in the step beyond the Willingness to respond level, the consent, the assent to responding, or the voluntary response, is that the behavior is accompanied by a feeling of satisfaction, an emotional response, generally of pleasure, zest, or enjoyment. The location of this category in the hierarchy has given us a great deal of difficulty. Just where in the process of internalization the attachment of an emotional response, kick, or thrill to a behavior occurs has been hard to determine. For that matter there is some uncertainty as to whether the level of internalization at which it occurs may not depend on the particular behavior. We have even questioned whether it should be a category. If our structure is to be a hierarchy, then each category should include the behavior in the next level below it. The emotional component appears gradually through the range of internalization categories. The attempt to specify a given position in the hierarchy as the one at which the emotional component is added is doomed to failure.

The category is arbitrarily placed at this point in the hierarchy where it seems to appear most frequently and where it is cited as or appears to be an important component of the objectives at this level on the continuum. The category's inclusion at this point serves the pragmatic purpose of reminding us of the presence of the emotional component and its value in the building of affective behaviors. But it should not be thought of as appearing and occurring at this one point in the continuum and thus destroying the hierarchy which we are attempting to build.

Enjoyment of self-expression in music and in arts and crafts as another means of personal enrichment.

Finds pleasure in reading for recreation.

Takes pleasure in conversing with many different kinds of people.
3.0 VALUING

This is the only category headed by a term which is in common use in the expression of objectives by teachers. Further, it is employed in its usual sense: that a thing, phenomenon, or behavior has worth. This abstract concept of worth is in part a result of the individual's own valuing or assessment, but it is much more a social product that has been slowly internalized or accepted and has come to be used by the student as his own criterion of worth.

Behavior categorized at this level is sufficiently consistent and stable to have taken on the characteristics of a belief or an attitude. The learner displays this behavior with sufficient consistency in appropriate situations that he comes to be perceived as holding a value. At this level, we are not concerned with the relationships among values but rather with the internalization of a set of specified, ideal, values. Viewed from another standpoint, the objectives classified here are the prime stuff from which the conscience of the individual is developed into active control of behavior.

This category will be found appropriate for many objectives that use the term "attitude" (as well as, of course, "value").

An important element of behavior characterized by Valuing is that it is motivated, not by the desire to comply or obey, but by the individual's commitment to the underlying value guiding the behavior.

3.1 ACCEPTANCE OF A VALUE

At this level we are concerned with the ascribing of worth to a phenomenon, behavior, object, etc. The term, "belief," which is defined as "the emotional acceptance of a proposition or doctrine upon what one implicitly considers adequate ground" (English and English, 1958, p. 64), describes quite well what may be thought of as the dominant characteristic here. Beliefs have varying degrees of certitude. At this lowest level of Valuing we are concerned with the lowest levels of certainty; that is, there is more of a readiness to re-evaluate one's position than at the higher levels. It is a position that is somewhat tentative.

One of the distinguishing characteristics of this behavior is consistency of response to the class of objects, phenomena, etc. with which the belief or attitude is identified. It is consistent enough so that the person is perceived by others as holding the belief or value. At the level we are describing here, he is both sufficiently consistent that others can identify the value, and sufficiently committed that he is willing to be so identified.
Continuing desire to develop the ability to speak and write effectively.
Growing in his sense of kinship with human beings of all nations.

3.2 PREFERENCE FOR A VALUE

The provision for this subdivision arose out of a feeling that there were objectives that expressed a level of internationalization between the mere acceptance of a value and commitment or conviction in the usual connotation of deep involvement in an area. Behavior at this level implies not just the acceptance of a value to the point of being willing to be identified with it, but the individual is sufficiently committed to the value to pursue it, to seek it out, to want it.

- Assumes responsibility for drawing reticent members of a group into conversation.
- Deliberately examines a variety of viewpoints on controversial issues with a view to forming opinions about them.
- Actively participates in arranging for the showing of contemporary artistic efforts.

3.3 COMMITMENT

Belief at this level involves a high degree of certainty. The ideas of "conviction" and "certainty beyond a shadow of a doubt" help to convey further the level of behavior intended. In some instances this may border on faith, in the sense of it being a firm emotional acceptance of a belief upon admittedly nonrational grounds. Loyalty to a position, group, or cause would also be classified here.

The person who displays behavior at this level is clearly perceived as holding the value. He acts to further the thing valued in some way, to extend the possibility of his developing it, to deepen his involvement with it and with the things representing it. He tries to convince others and seeks converts to his cause. There is a tension here which needs to be satisfied; action is the result of an aroused need or drive. There is a real motivation to act out the behavior.

- Devotion to those ideas and ideals which are the foundations of democracy.
- Faith in the power of reason and in methods of experiment and discussion.
4.0 ORGANIZATION

As the learner successively internalizes values, he encounters situations for which more than one value is relevant. Thus necessity arises for (a) the organization of the values into a system, (b) the determination of the interrelationships among them, and (c) the establishment of the dominant and pervasive ones. Such a system is built gradually, subject to change as new values are incorporated. This category is intended as the proper classification for objectives which describe the beginnings of the building of a value system. It is subdivided into two levels, since a prerequisite to interrelating is the conceptualization of the value in a form which permits organization. Conceptualization forms the first subdivision in the organization process, Organization of a value system the second.

While the order of the two subcategories, seems appropriate enough with reference to one another, it is not so certain that 4.1 Conceptualization of a value is properly placed as the next level above 3.3 Commitment. Conceptualization undoubtedly begins at an earlier level for some objectives. Like 2.3 Satisfaction in response, it is doubtful that a single completely satisfactory location for this category can be found. Positioning it before 4.2 Organization of a value system appropriately indicates a prerequisite of such a system. It also calls attention to a component of effective growth that occurs at least by this point on the continuum but may begin earlier.

4.1 CONCEPTUALIZATION OF A VALUE

In the previous category, 3.0 Valuing, we noted that consistency and stability are integral characteristics of the particular value or belief. At this level (4.1) the quality of abstraction or conceptualization is added. This permits the individual to see how the value relates to those that he already holds or to new ones that he is coming to hold.

Conceptualization will be abstract, and in this sense it will be symbolic. But the symbols need not be verbal symbols. Whether conceptualization first appears at this point on the affective continuum is a moot point, as noted above.

Attempts to identify the characteristics of an art object which he admires.

Forms judgments as to the responsibility of society for conserving human and material resources.
Objectives properly classified here are those which require the learner to bring together a complex of values, possibly disparate values, and to bring these into an ordered relationship with one another. Ideally, the ordered relationship will be one which is harmonious and internally consistent. This is, of course, the goal of such objectives, which seek to have the student formulate a philosophy of life. In actuality, the integration may be something less than entirely harmonious. More likely the relationship is better described as a kind of dynamic equilibrium which is, in part, dependent upon those portions of the environment which are salient at any point in time. In many instances the organization of values may result in their synthesis into a new value or value complex of a higher order.

Weighs alternative social policies and practices against the standards of the public welfare rather than the advantage of specialized and narrow interest groups.

Develops a plan for regulating his rest in accordance with the demands of his activities.

5.0 CHARACTERIZATION BY A VALUE OR VALUE COMPLEX

At this level of internalization the values already have a place in the individual's value hierarchy, are organized into some kind of internally consistent system, have controlled the behavior of the individual for a sufficient time that he has adapted to behaving this way; and an evocation of the behavior no longer arouses emotion or affect except when the individual is threatened or challenged.

The individual acts consistently in accordance with the values he has internalized at this level, and our concern is to indicate two things: (a) the generalization of this control to so much of the individual's behavior that he is described and characterized as a person by these pervasive controlling tendencies, and (b) the integration of these beliefs, ideas, and attitudes into a total philosophy or world view. These two aspects constitute the subcategories.

5.1 GENERALIZED SET

The generalized set is that which gives an internal consistency to the system of attitudes and values at any particular moment. It is selective responding at a very high level. It is sometimes spoken of as a determining tendency, an orientation toward phenomena, or a predisposition to act in a certain way. The generalized set is a response to highly generalized phenomena. It is a persistent and consistent
response to a family of related situations or objects. It may often be an unconscious set which guides action without conscious forethought. The generalized set may be thought of as closely related to the idea of an attitude cluster, where the commonality is based on behavioral characteristics rather than the subject or object of the attitude. A generalized set is a basic orientation which enables the individual to reduce and order the complex world about him and to act consistently and effectively in it.

Readiness to revise judgments and to change behavior in the light of evidence.
Judges problems and issues in terms of situations, issues, purposes, and consequences involved rather than in terms of fixed, dogmatic precepts or emotionally wishful thinking.

5.2 CHARACTERIZATION

This, the peak of the internalization process, includes those objectives which are broadest with respect both to the phenomena covered and to the range of behavior which they comprise. Thus, here are found those objectives which concern one's view of the universe, one's philosophy of life, one's Weltanschauung—a value system having as its object the whole of what is known or knowable.

Objectives categorized here are more than generalized sets in the sense that they involve a greater inclusiveness and, within the group of attitudes, behaviors, beliefs, or ideas, an emphasis on internal consistency. Though this internal consistency may not always be exhibited behaviorally by the students toward whom the objective is directed, since we are categorizing teachers' objectives, this consistency feature will always be a component of Characterization objectives.

As the title of the category implies, these objectives are so encompassing that they tend to characterize the individual almost completely.

Develops for regulation of one's personal and civic life a code of behavior based on ethical principles consistent with democratic ideals.
Develops a consistent philosophy of life.
APPENDIX B

LIST OF CALCULUS TEXTBOOKS


APPENDIX C

OBJECTIVES FROM TABLE 2 AND RELATED TEST ITEMS
OBJECTIVE: Identify all the symbols used in first year calculus, e.g., $\frac{dy}{dx}$, $f'$, $\int f$, $\int_a^b f(x)dx$

Test Items:

1) Place one letter from Column B in each blank in Column A; each item of Column B may be used more than once.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{dy}{dx}$</td>
<td>a) the instantaneous rate of change in $y$ with respect to $x$.</td>
</tr>
<tr>
<td>$y'$</td>
<td>b) the differential of $y$.</td>
</tr>
<tr>
<td>$\int_a^b f(x)dx$</td>
<td>c) the limit of the function $f(x)$ as $x$ approaches $a$.</td>
</tr>
<tr>
<td>$\frac{\Delta y}{\Delta x}$</td>
<td>d) the sum of the values of $f(x)$ at $1, 2, 3, \ldots, n$.</td>
</tr>
<tr>
<td>$\sum_{k=1}^{n} f(x)$</td>
<td>e) the definite integral.</td>
</tr>
<tr>
<td>$dy$</td>
<td>f) the slope of the chord line.</td>
</tr>
<tr>
<td>$b$</td>
<td>g) the indefinite integral.</td>
</tr>
</tbody>
</table>
2) Place one letter for Column B in each blank in Column A; each item of Column B may be used more than once.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lim_{x \to a} f(x) = L )</td>
<td>a) The limit of the function ( f(x) ) as ( x ) approaches ( a ) from the right.</td>
</tr>
<tr>
<td>( \int f(x) , dx )</td>
<td>b) the limit of the function ( f(x) ) as ( x ) approaches ( a ) from the left.</td>
</tr>
<tr>
<td>([a, b])</td>
<td>c) the point with ( x ) coordinate ( a ) and ( y ) coordinate ( b ).</td>
</tr>
<tr>
<td>( \lim_{x \to a^-} f(x) )</td>
<td>d) the limit of the function ( f(x) ) as ( x ) approaches ( a ).</td>
</tr>
<tr>
<td>( f(x) \to L ) as ( x \to a )</td>
<td>e) the set of all real numbers greater than or equal to ( a ) and less than or equal to ( b ).</td>
</tr>
<tr>
<td>( y^{[n]} )</td>
<td>f) the antiderivative of ( f(x) ).</td>
</tr>
<tr>
<td>( \frac{d}{dx} ) f(x)</td>
<td>g) the ( n )th derivative of a function.</td>
</tr>
</tbody>
</table>

3) \( \frac{d}{dx} f(x) \) is another way of writing

a) \( \lim_{\Delta x \to 0} f(x) \).

b) \( \frac{\Delta y}{\Delta x} \).

c) \( f'(x) \).

d) \( \lim_{x \to 0} f(x) \).

e) none of these.
OBJECTIVE: Be willing to try calculus as a tool in physical problems

Test Items:

1) Assuming that a physical problem has two methods of solution of equal difficulty, one using calculus and the other using geometry, which one would you use and why?

2) Suppose that two towns, c miles apart, are on the same side of a straight river, one at a distance A miles from the river and the other at a distance B miles. If a pumping station is located on the river between the towns, what is the shortest pipe line connecting the two towns to the station? (Hint: Reflect the position of one town about the straight river bank.)

3) Prove the rectangle of maximum area which can be inscribed in a circle is a square.

4) The sum of two positive integers is thirty-six. Determine the integers if their product is to be a maximum.
OBJECTIVE: Identify the geometric interpretation of the derivative and the definite integral, e.g., the derivative as the slope of the tangent line and the definite integral as the area under a curve

Test items:

1) If \( f(x) \) is continuous on \([a, b]\), then \( \int_a^b f(x)\,dx \) may be interpreted geometrically as

a) the area under \( f(x) \) between \( a \) and \( b \) provided \( f(x) > 0 \).

b) an approximation to the area under \( f(x) \) between \( a \) and \( b \), provided \( a \) and \( b \) are both positive.

c) the definite integral from \( a \) to \( b \) for any integers \( a \) and \( b \).

d) a rectangle whose base has length \( b - a \) and whose height is \( f(c) \) where \( c \) is in \([a, b]\).

e) none of these.

2) Suppose that \( f(x) \) is defined by the graph:

![Graph](image)

Which of the following is the best approximation to \( \int_a^c f(x)\,dx \)?

a) \( \frac{1}{2}(c-a) (f(a) - f(c)) \)

b) \( f(c) - f(a) \)

c) \( \frac{f(c) - f(a)}{c - a} \)

d) \( \frac{1}{2}(f(c) - f(a)) \)

e) \( \frac{1}{2} \frac{f(c) - f(a)}{c - a} \)
3) Each of the following diagrams represents a fundamental idea of the calculus. Place the proper letter in the blank below each diagram.

\[ 
\begin{array}{cccc}
\text{a) } & \text{b) } & \text{c) } & \text{d) }
\end{array}
\]

\[ 
\begin{array}{cccc}
\text{a) } \int_{a}^{b} f(x) \, dx & \text{b) } f(x) \text{ is continuous at } c & \text{c) } f'(c) \text{ does not exist} & \text{d) } \lim_{x \to c} f(x) \text{ exists}
\end{array}
\]

\[ 
\begin{array}{cccc}
\text{e) } (c, f(c)) \text{ is a relative minimum}
\end{array}
\]

4) If a function \( f(x) \) is tangent to \( y = |x| \) at \( x = \pi \) then \( f'(\pi) \) has the numerical value \( \frac{1}{\pi} \).

5) Suppose that \( f'(x) = 4 \) at \((1,3)\) and that \( f(x) \) is continuous. Find an approximation for \( f(1.25) \).

6) Let \( f(x) \) be given by the following graph:

Order the numbers below in ascending magnitude:

\[ 
\begin{array}{cccc}
\text{a) } \int_{a}^{b} f(x) \, dx & \text{b) } \int_{b}^{c} f(x) \, dx & \text{c) } \int_{a}^{c} f(x) \, dx & \text{d) } \int_{b}^{a} f(x) \, dx
\end{array}
\]

\[ 
\begin{array}{cccc}
\text{d) } & \text{b) } & \text{c)} & \text{a) }
\end{array}
\]
7) If \( f'(x_0) \) exists, it may be interpreted geometrically as

a) the value of the derivative at \( x_0 \).

b) the value of the function at \( x_0 \).

c) the value of the slope of the function at \( x_0 \).

d) the value of \( \frac{dy}{dx} \) at \( x_0 \).

e) none of these.
OBJECTIVE: Compute the derivative and the definite integral of common functions, e.g., polynomials, trigonometric functions, logarithmic functions, exponential functions

Test items:

1) \( \frac{d}{dx} (\sin \sqrt{x^2 + 1}) = \)
   a) \( \cos \sqrt{x^2 + 1} \)
   b) \( \frac{x}{\sqrt{x^2 + 1}} (\sin \sqrt{x^2 + 1}) \)
   c) \( \left(\frac{x}{\sqrt{x^2 + 1}}\right) (\cos \sqrt{x^2 + 1}) \)
   d) \( \sin \left(\frac{x}{\sqrt{x^2 + 1}}\right) \)
   e) \( \cos \left(\frac{x}{\sqrt{x^2 + 1}}\right) \)

2) \( \frac{d}{dx} [\ln (3x^2 - 3x + 6)] = \)
   a) \( \frac{2x - 1}{x^2 - x + 2} \)
   b) \( \ln (6x - 3) \)
   c) \( \frac{1}{3x^2 - 3x + 6} \)
   d) \( \frac{6x - 3}{\ln(3x^2 - 3x + 6)} \)
   e) \( \frac{6x - 3}{\ln(3x^2 - 3x + 2)} \)

3) \( \frac{d}{dx} (e^{4 - 3x^2}) = \)
   a) \( e^{4 - 3x^2} \)
   b) \( -6x \)
   c) \( (4 - 3x^2)e^{4 - 3x^2} - 1 \)
   d) \( (4 - 3x^2)e^{-6x} \)
   e) \( (-6x)e^{4 - 3x^2} \)

4) \( \int e^{\sin x} \cos x \, dx = \)
   a) \( e^{\sin x} \sin x \)
   b) \( e^{\sin x} \)
   c) \( e^{-\cos x} \sin x \)
   d) \( e^{-\cos x} \cos x \)
   e) \( e^{-\cos x} \cos x \)
5) \[ \int x \sqrt{3 - 2x^2} \, dx = \]
\begin{align*}
a) & \quad -\frac{1}{4}(3 - 2x^2)^{\frac{3}{2}} \\
(d) & \quad -\frac{1}{6}(3 - 2x^2)^{\frac{3}{2}} \\
& \quad \text{b) } x^2(3 - 2x^2)^{\frac{3}{2}} \\
& \quad \text{c) } (3 - 2x^2)^{\frac{3}{2}}
\end{align*}

6) \[ \int_2^4 (2x - 3x^2) \, dx = \]
\begin{align*}
(a) & \quad -44 \\
(b) & \quad -48 \\
(c) & \quad -52 \\
(d) & \quad 44 \\
(e) & \quad 52
\end{align*}

7) \[ D_x(3x^2 + 4)^5 = \]
\begin{align*}
a) & \quad 5(3x^2 + 4)^4 \\
& \quad \text{b) } 5(3x^2 + 4)(6x) \\
& \quad \text{c) } 5(6x + 0)^4 \\
(d) & \quad 5(3x^2 + 4)^4(6x) \\
& \quad \text{e) } 5(6x + 0)\]
OBJECTIVE: Define intuitively the technical terms of first year calculus, e.g., the limit of a function, continuity, derived function, critical point, definite integral

Test Items:

1) Express in your own words the meaning of each of the following ideas; do not use such technical terms as limit, derivative, infinite series, antiderivative.
   a) The limit of a function at a point exists.
   b) A function is continuous at a point.
   c) The point (c,f(c)) is a critical point of the function f(x).
   d) The definite integral of a non-negative function exists for a given interval.
   e) The indefinite integral of a function exists.

2) Each statement in Column B described a mathematical idea in rather loose non-technical terms. Match one item of Column A with each item of Column B; the items of Column A may be used more than once.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $f'(x)$ exists on the interval $(a,b)$.</td>
<td>4) $f(x)$ is neither rising nor falling.</td>
</tr>
<tr>
<td>(2) $f(x)$ is continuous on the interval $(a,b)$.</td>
<td>1) $f(x)$ has a tangent line at every point between a and b.</td>
</tr>
<tr>
<td>$\int_{a}^{b} f(x)dx$ exists.</td>
<td>3) The area under the graph of $f(x)$ can be found if $f(x) &gt; 0$ for $a &lt; x &lt; b$.</td>
</tr>
<tr>
<td>(3)</td>
<td>2) $f(x)$ is an unbroken curve.</td>
</tr>
<tr>
<td>(4) $f'(c) = 0$.</td>
<td>5) $f(x)$ settles down as $x$ gets close to $c$.</td>
</tr>
<tr>
<td>(5) $\lim_{x \to c} f(x) = L.$</td>
<td>2) The graph of $f(x)$ may be traced without lifting the pencil.</td>
</tr>
<tr>
<td></td>
<td>1) $f(x)$ has no sharp corners.</td>
</tr>
</tbody>
</table>
OBJECTIVE: Outline a method of solution to a problem

Test Items:

1) Write a short specific statement or paragraph outlining the way in which you would solve this problem. (Do not compute the solution!)

   The radius of the front wheel of a child's tricycle is 10 inches and the rear wheels each have a radius of 6 inches. The pedals are fastened to the hub of the front wheel by arms that are 7 inches long. How far does a pedal travel when the rear wheels make one revolution as the rider follows a straight line?

2) Write a short specific statement or paragraph outlining the way in which you should solve this problem. (Do not compute the solution!)

   A missile moves along a vertical path according to the equation $S = 145t - 16t^2$. How high does it go?

3) Arrange the statements below in the order that represents the steps which you would use to solve this problem.

   A trapezoidal plate is submerged vertically in water with its upper edge horizontal and 5 feet below the surface and its lower edge 12 feet below the surface. If the upper and lower edges are 6 and 8 feet long respectively, find the total force on one side of the plate.

   (a) draw a diagram
   (b) evaluate the integral
   (c) determine what's being sought
   (d) set up an element of force

   Step 1 $\Box(a)$; Step 2 $\Box(c)$; Step 3 $\Box$; Step 4 $\Box$. 
OBJECTIVE: Apply the geometric interpretation of the derivative and the integral

1) Draw the graph of a continuous function \( f(x) \) defined on \([0,2]\) such that \( f'(x) > 0 \) when \( x \neq 1 \) and

a) \( f'(x) \) is continuous with \( f'(1) = 0 \).

b) \( f'(1) \) is undefined.

c) \( f'(x) \) is continuous and \( f'(1) \rightarrow +1 \) as \( x \rightarrow 1 \).

[Note: Draw a separate graph for each of parts a, b, and c.]

2) Let \( a < b < c < d \) be elements of the domain of a continuous function \( f \). Sketch the graph of the function if all of the following conditions hold:

a) \( \int_{b}^{c} f(x) \, dx + \int_{c}^{d} f(x) \, dx > 0 \)

b) \( \int_{c}^{d} f(x) \, dx < 0. \)

c) \( \int_{a}^{b} f(x) \, dx = \int_{b}^{c} f(x) \, dx \)

d) \( \int_{a}^{c} f(x) \, dx > 0 \)
3) Let \( y = f(x) \) be given by the graph below and let \( a, b, c, d, e, g, h \) be elements of the domain of \( f(x) \). Fill in the blanks below with the element which makes the phrase a true description. Each element may be used only once; if none of the elements make the phrase a true description, put N.A. in the blank.

\[
\begin{array}{c}
f'(x) \text{ is undefined and } f(x) \text{ is defined.} \\
f'(x) = 0 \text{ and } f(x) \text{ is a relative maximum.} \\
f'(x) = 0 \text{ and } f(x) \text{ is neither a maximum nor a minimum.} \\
f'(x) \text{ does not exist and } f(x) \text{ is a relative minimum.} \\
\text{N.A.} \ f'(x) = 0 \text{ and } f(x) \text{ is an absolute minimum.} \\
f'(x) \text{ is undefined and } f(x) \text{ is not continuous.}
\end{array}
\]
OBJECTIVE: Recognize several different applications of a particular principle

Test Items:

1) Which of the following type problems cannot be solved by integration methods?
   a) Work
   b) Volume
   c) Hydrostatic Pressure
   d) Acceleration
   e) Force

2) List below four different applications of the process of differentiation (integration).
   a) ___________________________ c) ___________________________
   b) ___________________________ d) ___________________________

3) The problem of finding the center of gravity of an object is usually solved by one of the fundamental processes of calculus. Write below the name of that process and as many different applications as you think of other than the one above.

   ___________________________ ___________________________
   ___________________________ ___________________________

4) In each blank place one of the following:
   D if the statement represents an application of differentiation;
   I if the statement represents an application of integration;
   N if the statement represents an application of neither.

   D The maximum volume of a sphere under certain restraints.
   N The compound interest which a given amount of money acquires in a given number of years.
   D The time at which a projectile reaches its greatest height.
   I The work done in pumping all of the water out of a cylindrical tank.
The biggest rectangle which can be inscribed in an ellipse.

The time required for an automobile to travel a given distance at a fixed rate.

5) The concepts of differentiation, integration, and continuity all involve the concept of
   a) slope.
   b) area.
   c) limit.
   d) summation.
   e) increment.

6) Which of the following, if any, is not an application of the derivative?
   a) the slope of a tangent line to a curve
   b) velocity
   c) maximum-minimum problems
   d) L'Hospital's rule
   e) all are applications
OBJECTIVE: Be aware of the interrelationship of calculus and other broad areas of mathematics

Test Items:

1) Calculus cannot be used to prove theorems about
   a) geometry.
   b) algebra.
   c) mathematical logic.
   d) statistics.
   e) topology.

2) Which of the following areas of mathematics makes the most use of applications of calculus?
   a) topology
   b) statistics
   c) mathematical logic
   d) abstract algebra
   e) game theory

3) List three broad areas of mathematics, other than calculus, which make use of applications of calculus.
   a) __________________________
   b) __________________________
   c) __________________________
OBJECTIVE: Experience enjoyment as a result of comprehending the calculus used in a practical application

Test Items:

1) Assuming that you get the correct result to a problem involving an application of calculus, put one of the numerals 1 through 5 in each blank below. Use:

(1) If you strongly disagree
(2) If you disagree
(3) If you are undecided
(4) If you agree
(5) If you strongly agree

___ Getting the correct answer without understanding the method of solution is upsetting.
___ Getting the correct answer is enjoyable even without understanding the method of solution.
___ Getting the correct answer when the method of solution is understood is generally neither enjoyable nor unenjoyable.
___ Getting the correct answer is enjoyable only when the method of solution is clear.

2) In each pair of items below, underline the one which is more enjoyable to you.

(a) Getting the correct answer to a problem only when the method of solution is understood.
(b) Getting the correct answer to a problem whether or not the method of solution is understood.
(c) Trying the problem whether or not a correct solution is obtained.
(d) Understanding the principle behind a solution, whether or not the problems involving the principle can be solved.

(a,b)  (b,c)  (a,d)
(a,c)  (b,d)  (c,d)
OBJECTIVE: Distinguish between heuristic arguments and formal proofs

Test Items:

1) Which of the following statements is the least rigorous proof of the fact that \( \lim_{x \to 1} f(x) = 3 \) when \( f(x) = 2x + 1 \)?

   a) Given \( \varepsilon > 0 \) then \( \delta = \frac{\varepsilon}{2} \) satisfies the condition that
      \( |f(x) - 3| < \varepsilon \) when \( 0 < |x - 1| < \delta \).

   b) In order for \( f(x) \) to be in \( (3 - \varepsilon, 3 + \varepsilon) \), \( x \) must be in
      \( (1 - \frac{\varepsilon}{2}, 1 + \frac{\varepsilon}{2}) \) for every positive number.

   c) If \( x \to 1 \) then \( f(x) \to 3 \).

   d) For every positive number \( \varepsilon \), \( f(x) \) must be in an \( \varepsilon \) neighborhood of 3 when \( x \) is in an \( \frac{\varepsilon}{2} \) neighborhood of 1.

   e) \( f(x) = 3 \) when \( x = 1 \).

2) Which of the following statements is the most rigorous proof of the fact that \( D_x x^n = n x^{n-1} \) where \( n \) is an integer?

   a) \( D_x x^5 = 5x^4 \).

   b) \( \lim_{x \to a} \frac{x^n - a^n}{x - a} = nx^{n-1} \).

   c) The slope of the function \( f(x) = x^n \) at \( (a, a^n) \) is \( na^{n-1} \).

   d) \( \int nx^{n-1} \, dx = x^n \).

   e) For every \( \varepsilon > 0 \) there is a \( \delta > 0 \) such that \( |x^n| < \varepsilon \)
      whenever \( 0 < |x - a| < \delta \).
3) Which of the following is the most rigorous proof of the fact that the area of a circle of radius \( r \) is \( \pi r^2 \)?

a) The limit of the sum of the areas of rectangles inscribed inside a semi-circle is \( \frac{\pi r^2}{2} \).

b) \( \int_0^r 2\pi t \, dt = \pi r^2 \).

c) The area of a circle of radius \( r \) is defined as \( \pi r^2 \).

d) \( \lim_{n \to \infty} l(n) = \lim_{n \to \infty} C(n) = \pi r^2 \) where \( l(n) \) and \( C(n) \) are the areas of regular inscribed and circumscribed polygons of \( n \) sides.

e) The area of a sector is \( \frac{1}{2} r \times \) length of arc in radians so the area of the circle is \( \frac{1}{2} r \times \) circumference, i.e., \( \frac{1}{2} r \times (2\pi r) = \pi r^2 \).
OBJECTIVE: Make inferences from given statements

Test Items:

1) Which of the following is not a valid inference for the hypotheses listed?

Hypotheses: 1) \( f(x) \) is a quadratic function.

2) The derived function, \( f'(x) \), has a negative slope everywhere.

a) \( f(x) \) rises and then falls.

b) \( f(x) \) crosses the x axis.

c) \( f(x) \) is concave down.

d) \( f(x) \) has a maximum point.

e) \( f''(x) \) is negative.

2) Which of the following is not a valid inference for the hypotheses listed?

Hypotheses: 1) \( f(x) \) is a continuous function.

2) \( g(x) \) is a continuous function.

\( \frac{1}{f(x)} \) is a continuous function.

b) \( f(x) \cdot g(x) \) is a continuous function.

c) \( f(x) + g(x) \) is a continuous function.

d) \( f(g(x)) \) is a continuous function.

e) \( cf(x) \) is a continuous function if \( c \) is a constant.

3) Which of the following is a valid inference for the hypotheses listed below?

Hypotheses: 1) \( f(x) \) is a quadratic function.

2) \( g(x) \) is a linear function.

3) \( x = a \) and \( x = b \) are the x coordinates of the points of intersection of \( f(x) \) and \( g(x) \) with \( a < b \).

4) For \( a < x < b \), \( f(x) - g(x) \) is positive.
a) $f(x)$ is concave up everywhere.

b) $f(x)$ is concave down everywhere.

c) $f(x)$ is increasing everywhere.

d) $f(x)$ is decreasing everywhere.

e) $f(x)$ is positive everywhere.
OBJECTIVE: Recognize the importance of the fundamental theorem of calculus in relating differentiation and integration

Test Items:

1) The fundamental theorem of calculus states
\[ \int_a^b f(x)dx = F(b) - F(a). \]

The importance of this theorem lies in the fact that it relates the basic concepts of

a) function and integration.
b) limit and integration.
c) continuity and integration.
d) differentiation and integration.
e) slope and integration.

2) The processes of differentiation and integration involve two different concepts. Write a mathematical identity showing how differentiation and integration are related.

3) Differentiation involves the limit of a quotient function and integration involves the limit of an infinite sum. Write a mathematical identity showing how these two different concepts are related.

4) When you first learned how to integrate, you found the value of the integral by computing the sum of the area of rectangles inscribed and circumscribed about the function in some interval and then you took the limit as the length of the base of the rectangles decreased. Later you learned the short cut method of evaluating the definite integral
\[ \int_a^b f(x)dx = F(b) - F(a). \]

What important relationship permitted you to make that short cut?
5) Choose from the list below the statement which best relates the concept of differentiation to integration.

a) \[ \int_a^b f(x) \, dx = \lim_{\Delta x \to 0} \sum f(x^*) \Delta x. \]

b) \[ \int f(x) \, dx = F(x) + c \text{ where } F'(x) = f(x). \]

c) \[ D_x \left( \int f(x) \, dx \right) = f(x). \]

d) \[ \int_a^b f(x) \, dx = F(b) - F(a) \text{ where } F'(x) = f(x). \]

e) There is no relationship between differentiation and integration.
OBJECTIVE: Analyze problems into fundamental relationships

Test Items:

1) The solution of the following problem is dependent upon a small number of important relationships. After the problem, list those relationships.

Show that the volume of the largest right circular cylinder that can be inscribed in a given right circular cone is 4/9 the volume of the cone.

2) The following is a list of important mathematical concepts:

(a) Pythagorean Theorem
(b) Continuity
(c) Differentiation
(d) Integration
(e) ab=0 iff a = 0 or b = 0
(f) x^2 > 0 for every real number except zero
(g) Similar triangles

In the blank before each problem, put the letter of those concepts involved in the solution of the problem. If none of the concepts is involved in the solution, put an N in the blank.

1. Point A and B move along the x and y axes respectively in such a way that the perpendicular distance, r inches, from the origin to AB is constant. How fast is OA changing when \( OB = 2r \) and B is moving toward 0 at a rate of 0.3r in/sec.?
OBJECTIVE: Be alert to possible solutions and methods of attack of a non-traditional nature

Test Item:

Solve the following problems in any correct manner you choose:

1) What number exceeds its square by the greatest amount?

2) Compute the derivative of $y = 4 \sin^2 x \cos^2 x + \cos^2 2x$
OBJECTIVE: Apply calculus to physical problems which are traditionally solved by calculus

Test Items:

1) Find the volume of the solid obtained by rotating the region bounded by the coordinate axes, the line \( x = 1 \) and the function \( y = e^x \) about the x axis.

2) How much work is required to empty a tankful of water by pumping it over the rim if the tank is a hemisphere with a radius 2 feet?

3) A rectangle is to have one side along the x axis and two vertices on the circle with radius 5 and center at the origin. What is the largest perimeter that such a rectangle can have?
APPENDIX D

PRELIMINARY TEST MATERIALS AND TEST CRITICISM FORM
A TEST OF SELECTED CONCEPTS  
FROM THE FIRST YEAR CALCULUS SEQUENCE

Directions:

This examination is divided into two parts, each of which has a 45 minute time limit. Do not spend too much time on any one item. There is no penalty for guessing so you may answer an item even though you are not certain the answer is correct. You should, however, avoid wild guesses since they will distort the examination results. Your score will be the number of correct responses.

Each part of the examination will have a separate answer sheet. Mark all answers clearly on the answer sheet; do not make any marks on the examination itself. Example:

1. \( \frac{d}{dx} f(x) \) is another way of writing

a) \( \lim_{x \to 0} f(x) \)  
b) \( \frac{\Delta y}{\Delta x} \)  
c) \( f'(x) \)  
d) \( \lim_{x \to 0} f(x) \)  
e) \( \frac{f(x)}{x} \)

Answer Sheet: 1. a b c d e

Use the examination booklet for any scratch work. It is suggested that you use pencil rather than pen since pencil will not smear and it is easier to erase.

Some of the items may pertain to material which you have not yet covered so don't become discouraged if you can't answer all the items.

The results of this examination can be very useful to you and to your teachers if you work to the best of your ability. It will tell you how well you have mastered some of the basic concepts and operations of the beginning calculus courses and it will help your teachers to evaluate and to revise, if necessary, those beginning courses. If you do not make a sincere effort, the results will be misleading and the time spent in taking the examination will be wasted. Look upon this examination as a chance to show what you are really capable of doing and you will enjoy taking it.
Section A—Each of the multiple choice questions in this section has five responses but only one correct answer. For each question, blacken the space corresponding to the response which is the correct answer.

1) \( D_x(3x^2 + 4)^5 = \)
   a) \( 5(3x^2 + 4)^4 \)  
   b) \( 5(3x^2 + 4)(6x) \)  
   c) \( 5(6x + 0)^4 \)  
   d) \( 5(6x + 0) \)  
   e) \( 5(3x^2 + 4)^4(6x) \)

2) \( \frac{d}{dx}(\sin \sqrt{x^2 + 1}) = \)
   a) \( \cos \sqrt{x^2 + 1} \)  
   b) \( \left(\frac{x}{\sqrt{x^2 + 1}}\right)\left(\sin \sqrt{x^2 + 1}\right) \)  
   c) \( \left(\frac{x}{\sqrt{x^2 + 1}}\right)\left(\cos \sqrt{x^2 + 1}\right) \)  
   d) \( \sin \left(\frac{x}{\sqrt{x^2 + 1}}\right) \)  
   e) \( \cos \left(\frac{x}{\sqrt{x^2 + 1}}\right) \)

3) \( \frac{d}{dx}(e^{(4 - 3x^2)}) = \)
   a) \( e^{(4 - 3x^2)} \)  
   b) \( e^{-6x} \)  
   c) \( (4 - 3x^2)e^{(4 - 3x^2)} - 1 \)  
   d) \( (4 - 3x^2)e^{-6x} \)  
   e) \( (-6x)e^{(4 - 3x^2)} \)

4) \( \int x \sqrt{3 - 2x^2} \, dx = \)
   a) \( -\frac{1}{4}(3 - 2x^2)^{\frac{3}{2}} + c \)  
   b) \( -\frac{1}{6}(3 - 2x^2)^{\frac{3}{2}} + c \)  
   c) \( (3 - 2x^2)^{\frac{3}{2}} + c \)  
   d) \( x^2(3 - 2x^2)^{\frac{3}{2}} + c \)  
   e) \( -\frac{3}{8}(3 - 2x^2)^{\frac{3}{2}} + c \)

5) \( \int_2^4 (2x - 3x^2) \, dx = \)
   a) 52  
   b) 44  
   c) -52  
   d) -48  
   e) -44
6) If \( f'(x_0) \) exists, it is usually interpreted geometrically as

a) the slope of the function at \( x_0 \).

b) the value of the function at \( x_0 \).

c) the derivative at \( x_0 \).

d) the value of \( \frac{dy}{dx} \) at \( x_0 \).

e) none of these.

7) The concepts of differentiation, integration, and continuity all involve the concept of

a) slope.  
c) area.  
e) increment.

b) limit.  
d) summation.

8) If \( f(x) \) is continuous on \([a,b]\), then \( \int_a^b f(x)\,dx \) is usually interpreted geometrically as

a) the area under \( f(x) \) between \( a \) and \( b \) provided \( f(x) > 0 \).

b) an approximation to the area under \( f(x) \) between \( a \) and \( b \), provided \( a \) and \( b \) are both positive.

c) the indefinite integral from \( a \) to \( b \) for any integers \( a \) and \( b \).

d) a rectangle whose base has length \( b - a \) and whose height is \( f(c) \) where \( c \) is in \([a,b]\).

e) none of these.

9) Which of the following is not a valid inference for the hypotheses listed?

Hypotheses: 1) \( f(x) \) is a quadratic function.
2) The derived function, \( f'(x) \) has a negative slope everywhere.

a) \( f(x) \) rises and then falls.

b) \( f(x) \) has a maximum point.

c) \( f(x) \) is concave down.

d) \( f(x) \) crosses the \( x \) axis.

e) \( f''(x) \) is negative.
10) Suppose that $f(x)$ is defined by the graph:

Which of the following is the best approximation to $\int_a^c f(x)\,dx$?

a) $\frac{1}{2} \frac{(f(c) - f(a))}{(c - a)}$

b) $f(a) - f(c)$

c) $\frac{f(c) - f(a)}{c - a}$

d) $\frac{1}{2} (f(a) - f(c))$

e) $\frac{1}{2} (c-a) (f(a) - f(c))$

11) If a function $f(x)$ is tangent to $y = |x|$ at $x = \pi$ then $f'(\pi)$ has the numerical value

a) $-\pi$. b) $-1$. c) 1. d) 0. e) $\pi$.

12) Which of the following is a valid inference for the hypotheses listed below?

Hypotheses: 1) $f(x)$ is a quadratic function.
2) $g(x)$ is a linear function.
3) $x = a$ and $x = b$ are the $x$ coordinates of the points of intersection of $f(x)$ and $g(x)$ with $a < b$.
4) For $a < x < b$, $f(x) - g(x)$ is positive.

a) $f(x)$ is increasing everywhere.
b) $f(x)$ is decreasing everywhere.
c) $f(x)$ is concave down everywhere.
d) $f(x)$ is concave up everywhere.
e) $f(x)$ is positive everywhere.
13) The circumference of a unit circle can be proved to be 2 \( \pi \) units by

a) measuring the circumference.
b) multiplying the length of the diameter by \( \pi \).
c) computing the arc length for the circle.
d) evaluating \( \int_0^2 \pi \, dt \).
e) means of formulas beyond the scope of Math 440 and 441.

14) Let \( f(x) \) be defined by the following graph: and let

\[
m = \int_a^b f(x) \, dx, \quad n = \int_b^c f(x) \, dx,
\]

\[
p = \int_a^c f(x) \, dx, \quad q = \int_b^a f(x) \, dx.
\]

If the numbers \( m, n, p, q \) are arranged in increasing magnitude, the correct order is

a) \( q < n < m < p \). 
b) \( n < p < m = q \). 
c) \( m < p < n < q \). 
d) \( q = m < n < p \). 
e) \( q < n < p < m \).

15) The most accurate approximation of a root of a 5th degree polynomial is found by

a) graphing the polynomial.
b) applying the fundamental theorem of calculus.
c) substituting values into the polynomial.
d) using Newton's method.
e) using L'Hôpital's Rule.

SECTION B

16) Assuming that you get the correct result to a problem involving an application of calculus, rate each of the statements below a through e as follows:

a) if you strongly disagree
b) if you disagree
c) if you are undecided
d) if you agree
e) if you strongly agree

16.1) Getting the correct answer without understanding the method of solution is upsetting.
16.2) Getting the correct answer is enjoyable even without understanding the method of solution.
16.3) Getting the correct answer when the method of solution is understood is generally neither enjoyable nor unenjoyable.
16.4) Getting the correct answer is enjoyable only when the method of solution is clear.

17) For each problem below, select
(a) if it is usually solved by integration,
(b) if it is usually solved by differentiation, or
(c) if it is usually solved by neither integration nor differentiation.

17.1) Find the maximum volume of a cone inscribed in a sphere of fixed radius.
17.2) A 100 ft. cable weighing 5 lb/ft supports a 500 lb. safe. Find the work done in winding 80 ft. of the cable on a drum.
17.3) Find the acceleration of a projectile which moves vertically along the path \( s = 16t^2 + 156t + 84 \) at the time at which it reaches its greatest height.
17.4) The position of a point \( t \) is given as \( x = \frac{1}{2}t^2 \), \( y = \frac{1}{9}(6t + 9)^{3/2} \); find the distance the point travels from \( t = 0 \) to \( t = 4 \).
17.5) Find the dimensions of the largest rectangular plot that can be enclosed with a fixed amount of fencing.
17.6) Compute the time required for an automobile to travel a given distance at a constant rate.

18) Let \( y = f(x) \) be defined by the graph below and let the letters \( a, b, c, d, e, \) and \( h \) represent numbers in the domain of \( f \). Select the letter for which each statement becomes true; if none of the letters makes a statement true, select \( n \). Each response may be used more than once.

18.1) \( f'(x) \) does not exist, \( f(x) \) is defined and \( x \) is in \( (a,b) \).
18.2) \( f'(x) \) is defined and \( f(x) \) is not continuous.
18.3) \( f'(x) = 0 \) and \( f(x) \) is neither a maximum nor a minimum.
18.4) \( f'(x) \) is undefined and \( f(x) \) is a relative but not an absolute minimum.
18.5) \( f'(x) = 0 \) and \( f(x) \) is an absolute minimum.
18.6) \( f'(x) = 0 \) and \( f(x) \) is a relative maximum.
Section C—In the matching exercises below, blacken the answer space corresponding to the statement in Column B which defines or explains each symbol or statement in Column A. Each statement in Column B may be used more than once.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.1) ( \frac{dy}{dx} )</td>
<td>a) the instantaneous rate of change in y with respect to x.</td>
</tr>
<tr>
<td>19.2) ( y' )</td>
<td>b) the differential of y.</td>
</tr>
<tr>
<td>19.3) ( \int_{a}^{b} f(x) , dx )</td>
<td>c) the limit of the function ( f(x) ) as ( x ) approaches a</td>
</tr>
<tr>
<td>19.4) ( \frac{\Delta y}{\Delta x} )</td>
<td>d) the sum of the values of ( f(x) ) at ( 1, 2, 3, \ldots, n ).</td>
</tr>
<tr>
<td>19.5) ( \sum_{x=1}^{n} f(x) )</td>
<td>e) the definite integral.</td>
</tr>
<tr>
<td>19.6) ( dy )</td>
<td>f) the slope of the chord line.</td>
</tr>
<tr>
<td></td>
<td>g) the indefinite integral</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.1) ( \lim_{x \to a} f(x) )</td>
<td>a) the limit of the function ( f(x) ) as ( x ) approaches a from the right</td>
</tr>
<tr>
<td>20.2) ( \int f(x) , dx )</td>
<td>b) the limit of the function ( f(x) ) as ( x ) approaches a from the left</td>
</tr>
<tr>
<td>20.3) ( [a, b] )</td>
<td>c) the point with ( x ) coordinate ( a ) and ( y ) coordinate ( b )</td>
</tr>
<tr>
<td>20.4) ( \lim_{x \to a^-} f(x) )</td>
<td>d) the limit of the function ( f(x) ) as ( x ) approaches a</td>
</tr>
<tr>
<td>20.5) ( f(x) \to L ) as ( x \to a )</td>
<td>e) the set of all real numbers greater than or equal to ( a ) and less than or equal to ( b )</td>
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<td>20.6) ( y[n] )</td>
<td>f) the antiderivative of ( f(x) )</td>
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<td>g) the ( n )th derivative of a function</td>
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21) **Column A**

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<td>21.1) ( f(x) ) is neither rising nor falling when ( x ) is</td>
<td>a) ( f'(x) ) exists on the interval ( (a,b) ).</td>
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<td>between ( a ) and ( b ).</td>
<td>b) ( f(x) ) is continuous on the interval ( (a,b) ).</td>
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<td>21.2) The area under the graph of a continuous function ( f(x) &gt; 0 )</td>
<td>c) ( \int_a^b f(x) , dx ) exists.</td>
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<td>can be found for ( a &lt; x &lt; b ).</td>
<td>d) ( f'(x) = 0 ) for all ( x ) in the interval ( (a,b) ).</td>
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<td>21.3) The graph of ( f(x) ) is an unbroken curve on the interval ( a,</td>
<td>e) \lim_{x \to a} f(x) = L.</td>
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<td>b ).</td>
<td>f) ( f(x) ) is not continuous on the interval ( (a,b) ).</td>
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<td>21.4) ( f(x) ) settles down as ( x ) gets close to ( a ).</td>
<td>g) ( f'(x) &gt; 0 ) for all ( x ) in the interval ( (a,b) ).</td>
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<td>21.5) The graph of ( f(x) ) may be traced without lifting the pencil.</td>
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<td>21.6) The graph of ( f(x) ) has no sharp corners when ( a &lt; x &lt; b ).</td>
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A Test of Selected Concepts from the First Year Calculus Sequence
(PART I)

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A TEST OF SELECTED CONCEPTS
FROM THE FIRST YEAR CALCULUS SEQUENCE

PART II

Section A: Solve the word problems 1 thru 4 in any correct manner you choose (you do not necessarily have to use Calculus). Arrange your solution neatly and show all work; do not perform arithmetic computations. In #1 and #3, write a short outline of your method of solution.

1) A kite, 150 feet above the ground, is moving horizontally at a rate of 20 ft/sec. away from a boy holding the string. How fast is the string being let out when the kite is 250 feet from the boy?

2) The sum of the length and width of a rectangle is 36 inches. Determine the dimensions if the area of the rectangle is to be a maximum.

3) A weight of 50 lbs. stretches a spring 2 inches beyond its natural length. How much work is required to stretch it those 2 inches?

4) Find the volume of the solid obtained by rotating the region bounded by the coordinate axes, the line $x = 1$ and the function $y = e^x$ about the $x$ axis.

Section B: The questions in this section require you to draw a graph, write an identity, or compute a derivative. Be sure that all symbols are legible and show your work when possible.

5) Compute the derivative of $y = 4 \sin^2 x \cos^2 x + \cos^2 2x$.

6) If $f(t)$ is continuous then $F(x) = \int_a^x f(t)dt$ defines a continuous function. Write a mathematical expression for the slope of the line tangent to $F$ at a point with $x$ coordinate $x_0$. 
7) Let \(a < b < c < d\) be numbers in the domain of a continuous function \(f\). Sketch a graph of the function if all of the following conditions hold:

a) \(\int_b^c f(x)\,dx + \int_c^d f(x)\,dx > 0\)  
b) \(\int_c^d f(x)\,dx < 0\)  
c) \(\int_a^b f(x)\,dx = \int_b^c f(x)\,dx\)  
d) \(\int_a^c f(x)\,dx > 0\)

8) The processes of differentiation and of finding the definite integral involve two different concepts. Write a mathematical identity showing how differentiation and the definite integral are related for a function which is continuous in the interval \((a,b)\).
A Test of Selected Concepts from the First Year Calculus Sequence
(PART II)

SECTION A

1)

2)

3)
Test Criticism Form

To the Student: Your comments and suggestions will be a great help in improving this test. Evaluate each of the following test qualities on a five point scale as follows:

1 - Very poor
2 - Poor
3 - Adequate
4 - Good
5 - Very Good

A. Clarity of directions: Indicate any directions which were confusing or not clear.

B. Clarity of wording of items: Indicate (by number, if possible) those items which were ambiguously worded.

C. Balance of difficulty of items: Indicate whether the test was too easy or too hard, and why.

D. Time limits: Indicate whether too little or too much time was provided.

General Comments:
APPENDIX E

REVISED TEST MATERIALS AND OBJECTIVE RATING SHEET
A TEST OF SELECTED CONCEPTS  
FROM THE FIRST YEAR CALCULUS SEQUENCE 

Directions:

Do not spend too much time on any one item. There is no penalty for guessing so you may answer an item even though you are not certain the answer is correct. You should, however, avoid wild guesses since they will distort the examination results. Your score will be the number of correct responses.

Mark all answers clearly on the answer sheet; do not make any marks on the examination itself. Example:

1. \( \frac{d}{dx}(f(x)) \) is another way of writing

a) \( \lim_{\Delta x \to 0} \frac{f(x)}{\Delta x} \)  
b) \( \frac{dy}{dx} \)  
c) \( f'(x) \)  
d) \( \lim_{x \to 0} f(x) \)  
e) \( \frac{f(x)}{x} \)

Answer sheet: 1. a b c d e

Use the examination booklet for any scratch work. It is suggested that you use pencil rather than pen since pencil will not smear and it is easier to erase.

The results of this examination can be very useful to you and to your teachers if you work to the best of your ability. It will tell you how well you have mastered some of the basic concepts and operations of the beginning calculus courses and it will help your teachers to evaluate and to revise, if necessary, those beginning courses. If you do not make a sincere effort, the results will be misleading and the time spent in taking the examination will be wasted. Look upon this examination as a chance to show what you are really capable of doing and you will enjoy taking it.
Section A—Each of the multiple choice questions in this section has five responses but only one correct answer. For each question, blacken the space corresponding to the response which is the correct answer.

1) \[ \frac{d}{dx} \left( \ln \left( \frac{x}{x + 1} \right) \right) = \]
   a) \( \frac{1}{x} \)
   b) \( \ln x - \ln(x + 1) \)
   c) \( \frac{x}{(x + 1)^3} \)
   d) 1
   e) \( (x^2 + x)^{-1} \)

2) \[ \frac{d}{dx} (\sin \sqrt{x^2 + 1}) = \]
   a) \( \cos \sqrt{x^2 + 1} \)
   b) \( \left( \frac{x}{\sqrt{x^2 + 1}} \right) \left( \sin \sqrt{x^2 + 1} \right) \)
   c) \( \left( \frac{x}{\sqrt{x^2 + 1}} \right) \left( \cos \sqrt{x^2 + 1} \right) \)
   d) \( \sin \left( \frac{x}{\sqrt{x^2 + 1}} \right) \)
   e) \( \cos \left( \frac{x}{\sqrt{x^2 + 1}} \right) \)

3) \[ \frac{d}{dx} (x^3 e^{2x^2}) = \]
   a) \( e^{2x^2} [2x^4 + 3x^2] \)
   b) \( 4x^3 e^{2x^2} + 3x^2 e^{2x^2} \)
   c) \( x^2 [xe^{4x} + 3e^{2x^2}] \)
   d) \( x^3 e^{2x^2} + 3x^2 e^{2x^2} \)
   e) \( e^{2x^2} [4x^4 + 3x^2] \)

4) \[ \int \tan 2x \sec^3 2x \, dx = \]
   a) \( \frac{\tan^2 2x}{4} + c \)
   b) \( \frac{\sec^3 2x}{6} + c \)
   c) \( \frac{\sec^3 2x}{3} + c \)
   d) \( \frac{\tan 2x}{2} + c \)
   e) \( \frac{\sec^4 2x}{8} + c \)
5) \( \int_0^1 3^x \, dx = \)
   a) \( e^{\ln^3} \)
   b) \( \frac{9}{2} - 3 \)
   c) \( \ln\left(\frac{1}{3}\right)^2 \)
   d) \( \frac{e^{\ln^3}}{\ln^3} \)
   e) \( 2(\ln 3)^{-1} \)

6) If \( f'(x_0) \) exists, it is usually interpreted \textit{geometrically} as
   a) the slope of the function at \( x_0 \).
   b) the slope of the line tangent to \( f(x) \) at any point.
   c) the derivative.
   d) the value of the function at \( x_0 \).
   e) the slope of the chord line at \( x_0 \).

7) The concepts of differentiation, integration, and continuity all involve the concept of
   a) slope.
   b) limit.
   c) area.
   d) summation.
   e) increment.

8) If \( f(x) \) is continuous on \([a, b]\), then \( \int_a^b f(x) \, dx \) is usually interpreted \textit{geometrically} as
   a) the area under \( f(x) \) between \( a \) and \( b \) provided \( f(x) > 0 \).
   b) an approximation to the area under \( f(x) \) between \( a \) and \( b \), provided \( a \) and \( b \) are both positive.
   c) the indefinite integral from \( a \) to \( b \) for any integers \( a \) and \( b \).
   d) a rectangle whose base has length \( b - a \) and whose height is \( f(c) \) where \( c \) is any number in \([a, b]\).
   e) the sum of the areas of inscribed rectangles with width \( \Delta x \).

9) Which of the following is a valid inference for the hypotheses listed:
    Hypotheses: 1) \( f(x) \) is a quadratic function.
    2) The derived function \( f'(x) \) has a negative slope everywhere.
   a) \( f(x) \) is concave up.
   b) \( f(x) \) is increasing everywhere.
   c) \( f(x) \) has a point of inflection.
   d) \( f(x) \) rises and then falls.
   e) \( f(x) \) is decreasing everywhere.
10) Suppose that \( f(x) \) is defined by the graph:

Which of the following is the best approximation to \( \int_a^c f(x) \, dx \)?

- a) \( \frac{1}{2} \left( \frac{f(c) - f(a)}{c - a} \right) \)
- b) \( (f(a) - f(c))(a - c) \)
- c) \( \frac{f(c) - f(a)}{c - a} \)
- d) \( \frac{1}{2} (f(a) - f(c)) \)
- e) \( \frac{1}{2} (a - c)(f(c) - f(a)) \)

11) If a function \( f(x) \) is tangent to \( y = |x| \) at \( x = \pi \) then \( f'(\pi) \) has the numerical value

- a) \( -|\pi| \)
- b) \( -1 \)
- c) 1
- d) 0
- e) \( \pi \)

12) Which of the following is a valid inference for the hypotheses listed below?

Hypotheses: 1) \( f(x) \) is a quadratic function.
   2) \( g(x) \) is a linear function.
   3) \( x = a \) and \( x = b \) are the x coordinates of the points of intersection of \( f(x) \) and \( g(x) \) with \( a < b \).
   4) For \( a < x < b \), \( f(x) - g(x) \) is positive.

- a) \( f(x) \) is increasing everywhere.
- b) \( f(x) \) is decreasing everywhere.
- c) \( f(x) \) is concave down everywhere.
- d) \( f(x) \) is concave up everywhere.
- e) \( f(x) \) is positive everywhere.

13) The circumference of a unit circle can be proved to be \( 2\pi \) units by

- a) measuring the circumference.
- b) multiplying the length of the diameter by \( \pi \).
- c) computing the arc length for the circle.
- d) evaluating \( \int_0^{2\pi} \pi \, dt \).
- e) means of formulas beyond the scope of Math 440 and 441.
14) Let \( f(x) \) be defined by the following graph, and let

\[
m = \int_a^b f(x) \, dx, \quad n = \int_b^c f(x) \, dx,
\]

\[
p = \int_a^c f(x) \, dx, \quad q = \int_b^a f(x) \, dx.
\]

If the numbers \( m, n, p, q \) are arranged in increasing magnitude, the correct order is

a) \( q < n < p < m \).

b) \( n < p < m = q \).

c) \( m < p < n < q \).

d) \( q = m < n < p \).

e) \( q < n < p < m \).

15) The most accurate approximation of a root of a 5th degree polynomial is found by

a) graphing the polynomial.

b) applying the fundamental theorem of calculus.

c) substituting values into the polynomial.

d) using Newton's method.

e) using L'Hopital's rule.

16) Which of the following numbers most closely approximates

\[
\int_0^{1/2} \frac{1}{1+x^6} \, dx?
\]

a) .005  b) .05  c) .5  (d) 5  e) 50

17) If \( \int_0^x (3t^2 + 4t - 1) \, dt = 2 \) then the smallest solution \( x \) is

a) less than -1  b) -1  c) 0  d) 1  e) greater than 1

18) If a particle moves vertically so that for \( t \in [0, b] \) (\( b > 0 \)), the velocity \( v \) is greater than zero and the acceleration \( a \) is less than zero, then the particle

a) is speeding up.

b) slowing down.

c) will change directions.

d) is falling.

e) is in equilibrium.
19) Define $F(x) = \int_{0}^{x} |\sin t| \, dt$ for $x \in [-2\pi, 2\pi]$.

19.1) $F(x)$ is positive for
   a) all $x$. 
   b) $x \in [0, 2\pi]$ only.
   c) $x \in [0, \pi]$ only. 
   d) $x \in [0, \frac{\pi}{2}]$ only. 
   e) no $x$.

19.2) $F(x)$ is increasing
   a) nowhere. 
   b) on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
   c) on the interval $[0, 2\pi]$. 
   d) on the intervals $[-2\pi, -\frac{3\pi}{2}]$, $[-\pi, -\frac{\pi}{2}]$.
   e) everywhere.

19.3) $F(x)$ has a horizontal tangent at
   a) $\frac{\pi}{4}$. 
   b) $\frac{\pi}{2}$. 
   c) $\frac{2\pi}{3}$. 
   d) $\pi$. 
   e) $\frac{3\pi}{2}$.

19.4) $F(x)$ has a maximum value in $(-\pi, \pi)$ at
   a) no point. 
   b) $\pi$ only.
   c) $0$ and $\pi$. 
   d) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.
   e) $\frac{\pi}{2}$ only.

19.5) $F(x)$ is differentiable because
   a) $|\sin t|$ is continuous.
   b) $F(x)$ is continuous.
   c) $\lim_{x \to c} (F(x + \Delta x) - F(x))$ exists for all $c$ in $[-2\pi, 2\pi]$.
   d) $|\sin t|$ is positive for all $t$.
   e) $\lim_{\Delta t \to 0} |\sin t| = 0$. 

SECTION B

20) Assuming that you get the correct result to a problem involving an application of calculus, rate each of the statements below, a through e, as follows:
   a) if you strongly disagree
   b) if you disagree
   c) if you are undecided
   d) if you agree
   e) if you strongly agree

20.1) Getting the correct answer without understanding the method of solution is upsetting.
20.2) Getting the correct answer is enjoyable even without understanding the method of solution.
20.3) Getting the correct answer when the method of solution is understood is generally neither enjoyable nor unenjoyable.
20.4) Getting the correct answer is enjoyable only when the method of solution is clear.

21) For each problem below, select
   (a) if it is usually solved by integration,
   (b) if it is usually solved by differentiation, or
   (c) if it is usually solved by neither integration nor differentiation.

21.1) Find the maximum volume of a cone inscribed in a sphere of fixed radius.
21.2) A 100 ft. cable weighing 4 lb./ft. supports a 500 lb. safe. Find the work done in winding 80 ft. of the cable on a drum.
21.3) Find the acceleration of a projectile which moves vertically along the path \( s = -16t^2 + 156t + 84 \) at the time at which it reaches its greatest height.
21.4) When two pieces of 1 inch quarter round are cut so as to fit together in the corner of a room, how much material is cut away from each piece?
21.5) Show that the rectangle with fixed perimeter which encloses the most area is a square.
21.6) Compute the time required for an automobile to travel a given distance at a constant rate.
21.7) A wire 2 ft. long is to be used to form a circle and a square. How much wire should be used to form the circle if the sum of the area enclosed by the square and the circle is to be a maximum?
21.8) What is the largest angle of a right triangle with hypotenuse 13 inches and one leg 12 inches long.
21.9) The rate at which radium decomposes is proportional to the amount present. If it takes 1 gram of radium 1700 years to decrease to half a gram, how much of the original 1 gram is present after 2000 years?
21.10) A picture 4 feet high is hung on a wall so that the bottom edge is 3 feet above eye level. Find the distance from the wall at which you would get the "best view."

22) Let \( y = f(x) \) be defined by the graph below and let the letters \( a, b, c, d, e, \) and \( h \) represent numbers in the domain of \( f \). Select the letter for which each statement becomes true; if none of the letters makes a statement true, select \( n \). Each response may be used more than once, but there is only one correct answer for each statement.

![Graph](image)

22.1) \( f'(x) \) does not exist, \( f(x) \) is defined, and \( x \) is in \( (a, b) \).
22.2) \( f'(x) \) is defined and \( f(x) \) is not continuous.
22.3) \( f'(x) = 0 \) and \( f(x) \) is neither a maximum nor a minimum.
22.4) \( f'(x) \) is undefined and \( f(x) \) is a relative but not an absolute minimum.
22.5) \( f'(x) = 0 \) and \( f(x) \) is an absolute minimum.
22.6) \( f'(x) = 0 \) and \( f(x) \) is a relative maximum.
Section C—In the matching exercises below, blacken the answer space corresponding to the statement in Column B which defines or explains each symbol or statement in Column A. Each statement in Column B may be used more than once, but there is only one correct answer for each item.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
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<tbody>
<tr>
<td>23.1) ( \frac{dy}{dx} )</td>
<td>a) the instantaneous rate of change in y with respect to x</td>
</tr>
<tr>
<td>23.2) ( y' )</td>
<td>b) the differential of y</td>
</tr>
<tr>
<td>23.3) ( \int_{a}^{b} f(x) , dx )</td>
<td>c) the limit of the function ( f(x) ) as ( x ) approaches ( a )</td>
</tr>
<tr>
<td>23.4) ( \frac{\Delta y}{\Delta x} )</td>
<td>d) the sum of the values of ( f(x) ) at ( 1, 2, 3, \ldots, n )</td>
</tr>
<tr>
<td>23.5) ( \sum_{x=1}^{n} f(x) )</td>
<td>e) the definite integral</td>
</tr>
<tr>
<td>23.6) dy</td>
<td>f) the slope of the chord line</td>
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<td>g) the indefinite integral</td>
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<tr>
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<tbody>
<tr>
<td>24.1) ( \lim_{x \to a} f(x) )</td>
<td>a) the limit of the function ( f(x) ) as ( x ) approaches ( a ) from the right</td>
</tr>
<tr>
<td>24.2) ( \int f(x) , dx )</td>
<td>b) the limit of the function ( f(x) ) as ( x ) approaches ( a ) from the left</td>
</tr>
<tr>
<td>24.3) ([a,b])</td>
<td>c) the point with ( x ) coordinate ( a ) and ( y ) coordinate ( b )</td>
</tr>
<tr>
<td>24.4) ( \lim_{x \to a^-} f(x) )</td>
<td>d) the limit of the function ( f(x) ) as ( x ) approaches ( a )</td>
</tr>
<tr>
<td>24.5) ( f(x) \to L ) as ( x \to a )</td>
<td>e) the set of all real numbers greater than or equal to ( a ) and less than or equal to ( b )</td>
</tr>
<tr>
<td>24.6) ( y[n] )</td>
<td>f) the antiderivative of ( f(x) )</td>
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<td></td>
<td>g) the ( n )th derivative of a function</td>
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<tr>
<td>Column A</td>
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<tr>
<td>25.1) f(x) is neither rising nor falling when x is between a and b.</td>
<td>a) $f'(x)$ exists on the interval $(a, b)$.</td>
</tr>
<tr>
<td>25.2) The area under the graph of a continuous function $f(x) &gt; 0$ can be found for $a &lt; x &lt; b$.</td>
<td>b) $f(x)$ is continuous on the interval $(a, b)$.</td>
</tr>
<tr>
<td>25.3) The graph of $f(x)$ is an unbroken curve on the interval $(a, b)$.</td>
<td>c) $\int_a^b f(x) , dx$ exists.</td>
</tr>
<tr>
<td>25.4) $f(x)$ gets close to $L$ as $x$ gets close to $a$.</td>
<td>d) $f'(x) = 0$ for all $x$ in the interval $(a, b)$.</td>
</tr>
<tr>
<td>25.5) The graph of $f(x)$ may be traced without lifting the pencil.</td>
<td>e) $\lim_{{x \to a}} f(x) = L$.</td>
</tr>
<tr>
<td>25.6) The graph of $f(x)$ has no sharp corners when $a &lt; x &lt; b$.</td>
<td>f) $f(x)$ is not continuous on the interval $(a, b)$.</td>
</tr>
<tr>
<td></td>
<td>g) $f'(x) &gt; 0$ for all $x$ in the interval $(a, b)$.</td>
</tr>
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A Test of Selected Concepts from the First Year Calculus Sequence
(PART I)

Name ___________________________ Date ____________

Last First Middle

Amount of calculus in high school or another college prior to Math 440 and 441; circle one:

a) none  b) 0 to 6 weeks  c) 6 to 12 weeks  d) 12 weeks to 1/2 year

SECTION A

1. __________________________
2. __________________________
3. __________________________
4. __________________________
5. __________________________
6. __________________________
7. __________________________
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13. __________________________
14. __________________________
15. __________________________
16. __________________________
17. __________________________
18. __________________________

SECTION B

19.1 __________________________
19.2 __________________________
19.3 __________________________
19.4 __________________________
19.5 __________________________

20.1 __________________________
20.2 __________________________
20.3 __________________________
20.4 __________________________

21.1 __________________________
21.2 __________________________
21.3 __________________________
21.4 __________________________
21.5 __________________________
21.6 __________________________
21.7 __________________________
21.8 __________________________
21.9 __________________________
21.10 __________________________
A TEST OF SELECTED CONCEPTS
FROM THE FIRST YEAR CALCULUS SEQUENCE

PART II

Section A: Solve the word problems 1 thru 4 in any correct manner you choose (you do not necessarily have to use Calculus). Arrange your solutions neatly and show all work when possible. In addition to a solution, write a short outline of your method of solution to #1 and #3.

1) A kite is 40 ft. high, with 50 ft. of string out. If the kite moves horizontally at 5 ft./sec. away from the boy flying it, how fast is the string being let out?

2) What positive number exceeds its square by the greatest amount?

3) A ball is rolled over a level lawn with initial velocity 24 ft./sec. Due to friction the velocity decreases at a rate of 6 ft./sec^2, i.e., \( \frac{dv}{dt} = -6 \text{ ft./sec}^2 \). How far will the ball roll?

4) Find the volume of the solid obtained by rotating the region bounded by the coordinate axes, the line \( x = 1 \), and the function \( y = e^{-x^2} \) about the y axis.

Section B: The questions in this section require you to draw a graph, write an identity, or compute a derivative. Be sure that all symbols are legible and show your work when possible.

5) Compute the derivative of \( y = 4 \sin^2 x \cos^2 x + \cos^2 2x \).

6) If \( f(t) \) is continuous then \( F(x) = \int_a^x f(t)dt \) defines a continuous function. Write a mathematical expression for the slope of the line tangent to \( F \) at a point with \( x \) coordinate \( x_0 \).
7) Let \( a < b < c < d \) be numbers in the domain of a continuous function \( f \). Sketch a graph of the function if all of the following conditions hold:

\[
\begin{align*}
\text{a)} & \quad \int_{b}^{c} f(x) \, dx + \int_{c}^{d} f(x) \, dx > 0 \\
\text{b)} & \quad \int_{c}^{d} f(x) \, dx < 0 \\
\text{c)} & \quad \int_{a}^{b} f(x) \, dx = \int_{b}^{c} f(x) \, dx \\
\text{d)} & \quad \int_{a}^{c} f(x) \, dx > 0 
\end{align*}
\]

8) The processes of differentiation and of finding the definite integral involve two different concepts. Write a mathematical identity showing how differentiation and the definite integral are related for a function which is continuous in the interval \((a,b)\).
A Test of Selected Concepts from the First Year Calculus Sequence (PART II)

Name ___________________________ Last _______________ First _______________ Middle _______________ Date _______________

SECTION A

1) \[ S^2 = G^2 + (40)^2 \]
\[ \frac{dG}{dt} = 5 \text{ ft./sec.} \]
\[ 2S \frac{dS}{dt} = 2G \frac{dG}{dt} \]
\[ \text{at } t_0, G = \sqrt{S^2 - (40)^2} = \frac{30}{30} \text{ ft.} \]
\[ \frac{dS}{dt} = ? \]

Hence

At \( t_0 \), \( S = 50 \text{ ft.} \)

\[ 100 \frac{dS}{dt} = 2(30)(5) \text{ or } \frac{dS}{dt} = 3 \text{ ft./sec.} \]

Outline

1) Sketch Picture
2) Label information; determine what's being sought
3) Pythagorean Relationship
4) Differentiate with respect to time
5) Evaluate \( G \)
6) Substitute

2) Using Differentiation:

Let \( x \) be any positive real number. Then the amount \( A \) by which \( x \) exceeds its square is \( A = x - x^2 \).

\[ A' = 1 - 2x = 0 \Rightarrow x = \frac{1}{2} \]

3)

\[ \frac{dv}{dt} = -6 \]
\[ v = -6t + C \]
\[ v_0 = 24 \Rightarrow v = -6t + 24 \]
\[ S = -3t^2 + 24t + c \]
\[ S_0 = 0 \Rightarrow S = -3t^2 + 24t \]
\[ v = 0 \text{ when } t = 4 \]

\[ \therefore S = -3(4)^2 + 24(4) = 48 \]

1) Find equation for velocity
2) Use initial velocity
3) Find equation for distance
4) Evaluate constant
5) Set \( v = 0 \) to get time \( t \), when ball stops
6) Substitute \( t \) into distance equation
4) 
\[ V = \int_0^1 2\pi x f(x) \, dx \]
\[ = \int_0^1 \pi e^{-x^2} 2x \, dx \]
\[ = -\pi \left[ e^{-x^2} \right]_0^1 \]
\[ = -\pi \left[ \frac{1}{e} - 1 \right] \]

SECTION B

5) Without Simplifying:
\[ y = 4 \sin^2 x \cos^2 x + \cos^2 2x \]
\[ y' = 4[\sin^2 x(2\cos x)(-\sin x) + \cos^2 2x(-2\sin 2x)] \]
\[ = 4\sin^2 x \cos^2 x - \sin^2 2x + \cos^2 2x \]
\[ = \sin^2 2x + \cos^2 2x \]
\[ = 1 \]
\[ \therefore y' = 0 \]

6) 
\[ F(x) = \int_0^x f(t) \, dt \Rightarrow F'(x) = f(x) \]
Slope of the tangent line to \( F(x) \) at \( x_0 \) is \( F'(x_0) = f(x_0) \).
7) A Possible Solution

Graph Not Unique

\[ \int_a^b f(x) \, dx = F(b) - F(a) \text{ where } F'(x) = f(x). \]
A Test of Selected Concepts from the First Year Calculus Sequence
(PART II)

Name  Scoring Directions  Date

Last  First  Middle

Section A

1)

1A.

0 - Nothing
1 - Correct labels
2 - Pythagorean Theorem
3 - Differentiation
4 - Substitution

1B.

0 - Nothing
1 - Partial Outline
2 - Complete Outline

2)

0 - Nothing
1 - Attempting by differentiation
2 - Correct solution by differentiation
4 - Correct solution by intuition, geometry, substitution

3)

3A

0 - Nothing
1 - Correct Velocity Formula
2 - Correct Distance Formula
3 - Correct Value for Time
4 - Substitution

3B

0 - Nothing
1 - Partial Outline
2 - Complete Outline
4)

0 - Nothing
1 - Correct limits
2 - $\int_a^b 2\pi xy \, dx \text{ or } \int_a^b x^2 \, dy$
3 - $\int_0^1 2 \pi x e^{-x^2} \, dx$
4 - Correct evaluation

SECTION B

5)

0 - Nothing
1 - Attempting to differentiate without simplifying
2 - Correct derivative without simplifying
3 - Simplifying before differentiating
4 - Correct derivative after simplifying

6)

0 - Nothing
2 - $F'(x) = f(x)$
4 - $F'(x_0) = f(x_0)$
7)

0 - Nothing
1 - Any part of the graph correct
2 - Graph entirely correct

8)

0 - Nothing
2 - $\int_a^b f(x)dx = F(b) - F(a)$
4 - $\int_a^b f(x)dx = F(b) - F(a)$ where $F'(x) = f(x)$
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<th>1st</th>
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<td>0-4</td>
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<td>1A</td>
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Express the extent to which you feel the following statements were objectives of the 440, 441 calculus sequence. Assign to each statement one of the numerals 0 thru 3 as follows:

0 if you feel the statement does not represent an objective.
1 if you feel the statement represents an objective which received minor emphasis.
2 if you feel the statement represents an objective which received moderate emphasis.
3 if you feel the statement represents an objective which received major emphasis.

Having completed the 440, 441 calculus sequence, the student should:

1. Be willing to expand mathematical knowledge by independent reading.
2. Be aware of the potential use of calculus in areas where it is not now being used.
3. Define intuitively the technical terms of first year calculus, e.g., the limit of a function, continuity, derived function, critical point, definite integral.
5. Demonstrate a knowledge of the historical development of the major concepts of calculus, i.e., development of integration by Greeks to calculate area under a curve; development of differentiation in seventeenth century to treat rates of change.
6. Recognize several different applications of a particular principle.
7. Experience enjoyment as a result of comprehending the calculus used in a practical application.
8. Be alert to possible solutions and methods of attack of a non-traditional nature.
9. Develop study habits which are orderly and precise.
10. Isolate the logical structure underlying a proof.
11. Be willing to try calculus as a tool in physical problems.
12. Be aware of the historical contributions of calculus to our civilization.
14. Attempt to conceptualize the deductive nature of mathematics.
15. Be aware of the role of calculus in modern science, business, industry, and the humanities.
16. Indicate the logical fallacies in a proposed proof.
17. Read mathematics texts or periodicals independently with some degree of comprehension (provided the content is a reasonable extension of the student's range of knowledge).
18. Give examples of the way in which calculus unifies and explains certain topics of secondary school mathematics, e.g., use of limits to evaluate the trigonometric functions, use of integration to derive formulas for area and volume.
19. Apply calculus to physical problems which are traditionally solved by calculus.
20. Make use of various methods of proof, e.g., deduction, and proof by contradiction.
21. Recognize the historical or classical approach to certain problems.
22. Recognize the limitations of calculus in physical problems.
23. Construct proofs of original statements with some degree of rigor (provided the content is a reasonable extension of the student's mathematical knowledge).
24. Identify all the symbols used in first year calculus, e.g., \( \frac{dy}{dx}, f', \int f, \int_a^b f(x)dx \).
25. Distinguish between heuristic arguments and formal proofs.

26. Be aware of the interrelationship of calculus and other broad areas of mathematics.

27. Attempt to conceptualize the logical structure of calculus.

28. Compute the derivative and the definite integral of common functions, e.g., polynomials, trigonometric functions, logarithmic functions, exponential functions.

29. Be characterized by a desire to understand and create mathematics.

30. Identify the geometric interpretations of the derivative and the definite integral, e.g., the derivative as the slope of the tangent line and the definite integral as the area under a curve.

31. Make inferences from given statements.

32. Analyze problems into fundamental relationships.

33. Be curious about the extent to which calculus is used in twentieth century civilization.

34. Recognize the importance of the fundamental theorem of calculus in relating differentiation and integration.

35. Apply the geometric interpretations of the derivative and the definite integral.


