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DISSEMINATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By
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* * * * * *

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1967

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CHAPTER I

INTRODUCTION

As early as 1961 Heer\(^1\) suggested that the degeneracy of the traveling wave modes in a resonant electromagnetic system would be removed by rotation and a beat frequency of \(2m \Omega\) between the clockwise and counterclockwise modes would be observed. Initial considerations indicated that the frequency separation was proportional to the angular momentum of the photon and the rotation rate of the structure; i.e., \(\frac{\Delta \nu}{\nu} = \frac{\Omega \cdot J}{h \nu}\). Subsequent analyses\(^2,3\) indicated that the quantity of importance was the moment of energy flux rather than the angular momentum, whereas the distinction is only readily apparent in the presence of macroscopic matter.

After the development of high gain CW gaseous optical


masers,\(^4\) experimental investigations were made more readily possible, and, verification of the theory was provided by Macek and Davis and Cheo and Heer\(^5\) utilizing the 1.153 micron and 3.39 micron transitions respectively of He-Ne masers in rectangular Fabry-Perot cavities. In neither instance was verification of the theory for electrodynamics of accelerating macroscopic media provided.

This dissertation contains (1) a review of the theory of the electrodynamics for dispersive media in accelerated frames of reference; (2) a review of the basic properties of a gaseous optical maser especially those related to the

\[
\begin{align*}
2P^5s[1/2]_1 & \rightarrow 2P^5p[3/2]_2 \\
2P^5s[1/2]_1 & \rightarrow 2P^5p[3/2]_2
\end{align*}
\]

transitions (Racah notation) of Neon; (3) a detailed description of the triangular resonant cavity, recombine, rotating platform, detector, and associated equipment used in


the experimental investigation; (4) the experimental procedure necessary for aligning the cavity, attaining the optimum active maser medium and accumulating the data; and (5) conclusions and discussion on the beat frequency of a rotating closed path optical maser in the presence of dispersive media, bias beating, power sensitive frequency pulling due to the presence of the "Lamb dip" in the doppler broadened gain curve, and, methods to attain optimum stabilization of the system.

Review of Historical Development

Many experimental and theoretical investigations were conducted during the first half of this century to study the effect of rotation on light. The most notable of the early investigations was that of Michelson and Gale in which the effect of the earth's rotation on light was observed by noting the displacement of fringes produced by two interfering light beams traveling clockwise and counterclockwise paths around an enclosed area of approximately one-tenth square mile. The theory of this effect had already been developed on the basis of

---

6 G. Sagnac, Compt. Rend. 157 708 and 1410 (1913) and B. Pogany, Ann. Physik 85 244 (1928).

a fixed aether by Michelson as early as 1904 and was later extended to the theory of relativity by Siberstein in 1921.

Motivated by these experiments, Heer suggested in 1961 that rotation with respect to the fixed stars could be detected by measuring the beat frequency resulting from the splitting of a degenerate mode in a rotating resonant cavity. Subsequently, at the Third International Conference on Quantum Electronics, Heer presented a more detailed analysis using the equations of electrodynamics in a covariant form. The frequency separation was attributed to the modification of the electromagnetic field upon the transformation from an instantaneous rest frame to the rotating system. These developments prompted an ever continuing presentation of experimental and theoretical investigations on the subject. The experimental work to date has been incomplete inasmuch as the electrodynamics for macroscopic matter or dispersive media were not considered, with the

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8A. A. Michelson, Phil. Mag. 8 716 (1904).
result that the distinction between the angular momentum and the moment of energy flux for the resonant photon was not evident.

**Purpose of the Dissertation**

Khromykh\(^{13}\) and Heer\(^{14}\) have shown that the beat frequency for a rotating resonant cavity in the presence of dispersive media or with dispersive media moving with respect to the cavity can be expressed to a first order approximation by

\[
\frac{\Delta \nu}{\nu} = 2c^{-1}\left\{2\Omega \cdot d\Sigma + \int (\mu^2 - 1) \mu v d\mu/dv | \mu \cdot \hat{n} d\sigma \right\} / \int (\mu(\nu_0) + \nu_0 d\mu/dv) d\sigma
\]

where
- \(\Omega\) = the rotation rate of the cavity
- \(\nu\) = the index of refraction
- \(\nu_0\) = the oscillation frequency

and
- \(\mu\) = the velocity of the medium with respect to the cavity.

Heer\(^{15}\) has further shown that this is entirely consistent with the expression

\[
\frac{\Delta \nu}{\nu} = 4c^{-1} \Omega \cdot \int d\nu \frac{\tau \times (E \times H)}{d\nu} + O(\Omega^2)
\]

\[
/ \int d\nu (E \cdot D + B \cdot H)
\]

---


\(^{14}\) C. V. Heer, J. A. Little, and J. R. Bupp (to be published).

\(^{15}\) Ibid.
obtained from a consideration of Minkowski's energy-momentum tensor for an almost plane wave expansion of a beam of finite cross section. It is therefore the aim of this dissertation to verify the above formulations by examining the effect of macroscopic matter on the beat frequency of a rotating resonant cavity.
CHAPTER II

THEORY OF THE ELECTRODYNAMICS FOR AN ACCELERATED SYSTEM OF REFERENCE

The purpose of the following chapter is to briefly summarize the theoretical development necessary for an understanding of the electrodynamics of macroscopic matter in an accelerated frame of reference. The theory has been thoroughly formulated by Heer in a series of papers presented over the past several years. The summary is most satisfactorily divided into three basic areas of development. The first section deals with Minkowski's form of the energy-momentum tensor; the second outlines the analysis of a convenient plane wave expansion leading to the development of the index of refraction measured by an observer in an accelerated system; and the third includes the effect of the optical maser medium.

Minkowski's Energy-Momentum Tensor

For macroscopic matter, Maxwell's field equations of electrodynamics can be written in a covariant form as

\[ \text{Minkowski's Energy-Momentum Tensor} \]

\[ \text{For macroscopic matter, Maxwell's field equations of electrodynamics can be written in a covariant form as} \]

\[ \text{See for example, Chapter I references 1, 2, 3, and 14.} \]
\[
\frac{\partial F_{\alpha \beta}}{\partial x^\gamma} + \frac{\partial F_{\beta \gamma}}{\partial x^\alpha} + \frac{\partial F_{\gamma \alpha}}{\partial x^\beta} = 0
\]

and

\[
(-g)^{-1/2} \frac{\partial}{\partial x^\beta} \left[ (-g)^{-1/2} H^{a \beta} \right] = j^\alpha
\]

If we consider the four vector \( F_{\alpha \beta} j^\beta \) which is the analogue of the four force density we can write with the aid of the covariant field equations:

\[
F_{\alpha \beta} j^\beta = -\frac{3}{2} \frac{\partial}{\partial x^\beta} (F_{\alpha \beta} H^{\mu \beta}) + 1/4 \frac{\partial}{\partial x^\alpha} (F_{\beta \mu} H^{\beta \mu}) + 1/4 \left[ \frac{\partial F_{\mu \beta}}{\partial x^\alpha} H^{\beta \mu} - F_{\beta \mu} \delta H^{\beta \mu} / \partial x^\alpha \right]
\]

This leads immediately to the equation

\[
F_{\alpha \beta} j^\beta + 1/4 \left[ F_{\mu \beta} \delta H^{\mu \beta} / \partial x^\alpha - F_{\mu \beta} \delta H^{\mu \beta} / \partial x^\alpha \right] = -\frac{\partial S_{\alpha \beta}}{\partial x^\beta}
\]

The quantity \( S_{\alpha \beta} \) is defined to be Minkowski's form of the energy-momentum tensor

\[
S_{\alpha \beta} = F_{\alpha \nu} H^{\beta \nu} - 1/4 \delta_{\alpha \beta} F_{\mu \nu} H^{\mu \nu}
\]

the covariant divergence of which is equal to the four force \( f_\alpha \).

The components of this tensor expressed in non-covariant vectors can be written as

\[
S_{t x} = -c^{-1} \left( E \times B \right)_x
\]

\[
S_{x t} = c \left( D \times B \right)_x
\]

and

\[
S_{t t} = -1/2 \left( E \cdot D + B \cdot H \right)
\]

These components \( S_{t x}, S_{x t}, \) and \( S_{t t} \) are recognized as being of the usual form for the energy flux, the

---

momentum, and the energy density respectively in an instantaneous rest frame. If upper case indices are used to denote an instantaneous rest frame and lower case indices to denote the accelerated frame of observation, the transformation of the energy-momentum tensor from the instantaneous rest frame to the system of observation can be written as:

\[ S^v_\phi = A^a_\phi A^v_\beta S_{a\beta} \]

The relationship between the energy density in the two reference frames can be expressed as:

\[ S^T_T = S^t_t + (g_{ta}/g_{tt})S^a_t \quad (2-4) \]

For a stationary metric the \( g_{\alpha\beta} \) are independent of the local time variable \( t \), and, for short periods of time where losses are negligible, time independent complex normal mode solutions to Maxwell's equations can be obtained.\(^3\)

Minkowski's total energy-momentum tensor can be decomposed into an electromagnetic and a mechanical part, in which development a transparent medium is defined as a system which does not even locally exchange energy with the electromagnetic field. For the case \( f_t = 0 \) it can be shown that \( \frac{\partial S_{\tau\beta}}{\partial x^\beta} = 0 + O(\Omega^2) \).

For nondissipative walls \( E \times H \) vanishes at the walls and \( \int d\gamma S^t_t = 0 \) i.e., the integral of the

\(^3\text{Heer, Phys. Rev. 134, op. cit.}\)
energy density over the volume of the cavity is a conserved quantity.

The frequency \( \nu = \frac{1}{\hbar} \frac{d\nu}{dV} S_{t}^{\top} \) of a resonant photon in the cavity modes can be written from expression 2-4 as:

\[
\nu = \frac{1}{\hbar} \frac{d\nu}{dV} \left[ S_{T}^{T} - \left( \frac{g_{ta}}{g_{tt}} \right) S_{t}^{a} \right] \tag{2-5}
\]

This system is degenerate in that \( \pm \) values of \( S_{t}^{a} \) are permitted. If the cavity contains media which is not magnetically optically active, the integral \( \int dV S_{t}^{T} \) is the same for both the + direction traveling wave modes and the frequency separation between resonant photons in the two modes can be written as:

\[
\frac{\nu^{+} - \nu^{-}}{\nu} = \left( \frac{\hbar \nu}{2} \right)^{-1} 2 \int dV \left( \frac{g_{ta}}{g_{tt}} \right) S_{t}^{a}
\]

\[
= \left[ \int dV S_{t}^{T} \right]^{-1} 2 \int dV \left( \frac{g_{ta}}{g_{tt}} \right) S_{t}^{a}
\]

and from equation 2-3

\[
\frac{\Delta \nu}{\nu} = \frac{4c^{-1} \mu \cdot \int dV \cdot (E \times H) + O(2)}{\int dV \left( E \cdot D + B \cdot H \right)} \tag{2-6}
\]

This expression is correct to first order in \( \mu \) hence the frequency separation can be determined to the same order of accuracy using the form of \( E, D, B, \) and \( H \) in the instantaneous rest frame. From equation 2-6 the dependence of the frequency separation on the moment of the energy flux of the photon is readily apparent.
Plane Wave Analysis

Although equation 2-6 is a detailed formulation for the frequency separation between the clockwise and counterclockwise traveling waves, it is not in a form that can be readily evaluated for dispersive media or with media moving with respect to the rotating resonant cavity. This type of problem is more satisfactorily treated if a plane wave expansion is utilized to describe the electromagnetic field in the cavity.

In this short wavelength approximation where an electromagnetic wave can be described in terms of its amplitude and phase, the propagation of waves in a dispersive media which is moving with respect to an observer in an accelerated frame of reference may be analyzed in terms of three basic invariants. If these three quantities are determined in one reference frame they are known in any other frame. These invariants can be described as being the phase $d\phi = k_\alpha dx^\alpha$, an invariant $k_\alpha k^\alpha$ which provides physical information, and the transformation of frequencies between reference frames by $k_\alpha U^{\alpha}$. The index $\alpha$ is used to designate the covariant or contravariant quantities in the general accelerated frame of reference A of the observer; the index $\phi$ will be used to denote a reference frame I instantaneously at rest with respect to A and the index
\( \phi' \) to denote a reference frame \( \Gamma' \) instantaneously at rest with respect to the dispersive medium.

In the reference frame at rest with respect to the moving medium the square of the propagation vector

\[
k_{\phi'} = \left( \omega_{\Gamma'}, \mu \hat{n}; i \omega_{\Gamma'} \right) \frac{\hat{n}}{c}
\]

can be written as

\[
k_{\phi'} k_{\phi'} = \omega_{\Gamma'}^2 \left( \frac{1}{u_{\Gamma'}^2} - \frac{1}{c^2} \right) = k_{\Gamma'}^2 \left[ u^2 (k_{\Gamma'})^2 - 1 \right] = k_a k_a
\]

where \( u(k_{\Gamma'}) \) is the index of refraction and \( \omega_{\Gamma'} = c k_{\Gamma'} \) expresses the frequency dependence. Since the four velocity has a single component, namely \( u_{\Gamma'} = c \), the transformation of frequencies can be written as

\[
k_{\phi'} u_{\phi'} = k_a u^a = k_{\Gamma'} u_{\Gamma'} = k_{\Gamma'} c = \omega'
\]

The contravariant components of the four velocity can be written as

\[
u^a = \frac{dx^a}{d\tau} = (\Gamma, u^a, \Gamma c)
\]

where \( u^a = \frac{dx^a}{d\tau} \) is the velocity of the moving medium with respect to the accelerated system. \( \Gamma \) is the analogue of the Lorentz factor and to a first order approximation is unity.

\[
\Gamma = [ \left( 1 - (g_{ta}/g_{tt} 1/2) u^a/c \right)^2 - u^2/c^2]^{-1/2}
\]

\[
= 1 + O(2)
\]
The spatial distance between two points can be written as
\[ d\sigma^2 = \gamma_{ab} \, dx^a \, dx^b \]
where
\[ \gamma_{ab} = g_{ab} + \gamma_a \gamma_b \]
\[ \gamma_a = g_{ta} / \sqrt{-g_{tt}} \]

In the case of a stationary metric, that is, one that does not depend on \( x^4 \), \( \gamma_{ab} \) is equal to the spatial portion of the four-dimensional metric \( g_{ab} \) and a frequency \( \omega \) may be introduced
\[ \omega = c k_t = c (-g_{tt})^{1/2} k_T = (-g_{tt})^{1/2} \omega_0 \] (2-9)

The spatial components of the wave vector are given by the expression
\[ k_a = (k_a k^a - k_t / g_{tt})^{1/2} \gamma_{ab} n^b + g_{ta} k_t \] (2-10)
where \( n^b = dx^b / d\sigma \) is the contravariant projection along \( \sigma \) the path direction. The relationship between \( k_T \) and \( k_t \) may now be determined by utilizing equations 2-7, 2-8, 2-9, and 2-10.

\[ k_T = c^{-1} k_a U^a \]
\[ = c^{-1} (k_a k^a - k_t^2 / g_{tt})^{1/2} u^a n^b \delta_{ab} + k_t \]
where
\[ k_a k^a - k_t^2 / g_{tt} = k_T^2 [\mu^2 (k_T) - 1] + k_t^2 \]
\[ = k_t^2 [k_T^2 (\mu^2 (k_T) - 1) + 1] / k_t^2 \]
so that

\[ k_T = k_t \left( \frac{1 + k_{T'}^2}{k_t^2} \right)^{1/2} \delta_{ab} \frac{u_{anb}}{c} + k_t \]

solve by an iteration to find \( k_T \)

\[ k_T = k_t \left( 1 + \mu(k_T') \frac{u_{anb}}{c} \delta_{ab} \right) \quad (2-11) \]

where \( \mu(k_T') \) has the form

\[ \mu(k_T') = \mu(k_t) + \mu \nu c^{-1} \frac{d\nu}{dv} \delta_{ab} u_{anb} \]  

(2-12)

Noting that the phase \( d\phi = k_a dx^a \) can be written

\[ d\phi = (k_a dx^a / d\sigma) d\sigma - cdt \]

we are now in a position to solve for the phase using equations 2-10, 2-11, and 2-12. Since \( r = 1+O(2) \) and \( \gamma_{ab} = \delta_{ab} + O(2) \) the space is Euclidean through first order and the following expressions are correct to the same order of accuracy

\[ \gamma_{ab} u_{anb} = u \cdot \hat{n} \]

(2-13)

\[ g_{tt} = 1 \]

\[ g_{ta} n^a = c^{-1} (\vec{\psi} \times \tau) \cdot \hat{n} \]

The phase can be written from equations 2-10, 2-11, as

\[ d\phi = [(k_a k^a - k_t^2 / g_{tt}) \gamma_{ab} u_{anb} - g_{ta} k_t n^a] d\sigma - cdt \]

and utilizing equations 2-12 and 2-13

\[ d\phi = k_t \left[ (\mu(k_T') + (\mu^2 - 1) \delta_{ab} u_{anb} / c + g_{ta} n^a) d\sigma \right] - cdt \]

\[ - 2\pi \nu \left[ (\mu(\nu) + c^{-1} [(\mu^2 - 1) + \nu \frac{d\nu}{dv}] \right] u \cdot \hat{n} \]

(2-14)

\[ + c^{-1} (\vec{\psi} \times \tau) \cdot \hat{n} d\sigma - cdt \]
The quantity in the curly brackets {} may be interpreted as the index of refraction \( \mu_a \) as measured by an observer in the accelerated system.

In the short wavelength approximation the total phase is invariant and stationary to a variation. The classical path

\[
\delta \mu_a \, d\sigma = 0 \quad (2-15)
\]

follows from Fermat's principle \( \delta/dt = 0 \) between two stationary points. Since the phase was determined to first order the phase may be computed to the same accuracy along the path for \( \Omega = 0 \). Using the resonant condition for a closed path \( c^{-1} \int \mu_a \, d\sigma = q \), where \( q \) is a large integer, the frequency separation between the clockwise and counterclockwise traveling waves can be written as

\[
\frac{\nu_{CW} - \nu_{CCW}}{\nu_0} = 2c^{-1} \left\{ \int [(\Omega \times \tau) \cdot \hat{n}] \, d\sigma + \int [(\mu^2 - 1) + \nu \, d\mu] u \cdot \hat{n} \, d\sigma \right\} \frac{d\sigma}{d\nu} \frac{\nu_0 \, d\mu}{d\nu} \]

where \( \nu_0 \) is evaluated at \( \Omega = 0 \). For a closed path the line integral \( \int [(\Omega \times \tau) \cdot \hat{n}] \, d\sigma \) transforms to the surface integral \( 2\Omega \cdot dS \), consequently the final form for the beat frequency can be written

\[
\frac{\Delta \nu}{\nu_0} = 2c^{-1} \left\{ 2\Omega \cdot dS + \int [\mu^2 - 1 + \nu \, d\mu / d\nu] u \cdot \hat{n} \, d\sigma \right\} \frac{d\sigma}{d\nu} \frac{\nu_0 \, d\mu}{d\nu} \]

\[
\left\{ \int \left[ \mu (\nu_0) + \nu_0 \, d\mu / d\nu \right] d\sigma \right\} \]

\( (2-17) \)
This expression is in complete agreement with equation 2-6 for an almost plane wave expansion of a beam of finite cross section. Modification of this expression for the specific experimental geometry and conditions will be made in Chapter VII to properly analyze the data.

Effect of the Optical Maser Medium

In a recent paper, Lamb\textsuperscript{4} presented a self consistent analysis to describe the operation of a gaseous optical maser. The treatment was one in which he assumed that an electromagnetic field (standing wave) in a cavity polarized the moving atoms in the active maser medium. This macroscopic polarization was then used as a source term in Maxwell's equations to determine conditions on threshold, output power as a function of cavity tuning, frequency pulling, combination tones, and frequency locking. Heer and Graft\textsuperscript{5} have extended this method to obtain the electric polarization for atoms with angular momentum interacting with traveling electromagnetic waves and with a static magnetic field. A plane wave expansion of the form

\[ \Phi(r,t) = \sum_{n,q} \{ a_n \phi_{nq} \exp i (\phi_{nq} - \omega_{nq} t) \} \]

\[ + \sum_{n,q} \{ b_n \phi_{nq} \exp i (\phi_{nq} + \omega_{nq} t) \} \] + C.C.


was utilized to describe the field in the cavity. In this expansion $A_{\sigma q}$ and $B_{\sigma q}$ are the amplitudes of the waves in the $+\sigma$ and $-\sigma$ directions respectively, and, are slowly varying functions of time or $\sigma$. $\sigma$ is along the actual optical path and $\omega_{\sigma q A}$ is determined by the resonant condition for a closed path

\begin{equation}
\phi_{\sigma q A} = c^{-1} \omega_{\sigma q A} \int_{0}^{\sigma} \omega_{A} d\sigma
\end{equation}

The subscript $\sigma$ denotes the two possible canonical states of polarization for the photon. Standing wave modes of the type presented by Fox and Li\(^6\) and extended to closed path resonators by Collins\(^7\) and Clark\(^8\) are appropriate to describe the field in the cavity, but for the problem at hand, a plane wave expansion has the same general character and is more convenient.

Heer and Graft have shown (equations 29 (a) and 29 (b) of reference 5) that the polarization is related to $E$ and $E^3$ by second and fourth order tensors respectively. For the specific case of having the active medium contained in a Brewster's angle discharge tube, the electric vector is linearly polarized throughout the

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\(^6\)A. G. Fox and T. Li, Bell System Tech. J \textbf{40} 453 (1961).


maser medium. Additional conditions such as single mode operation, large doppler broadening, and a transverse magnetic field permit considerable simplification of the relationship which can be expressed:

\[-\varepsilon_0^{-1} P(\omega_A) = A[a(\omega_A) - b(\omega_A)|A|^2 - c(\omega_A, \omega_B)|B|^2]\]

and

\[-\varepsilon_0^{-1} P(\omega_B) = B[a^*(\omega_B) - b^*(\omega_B)|B|^2 - c^*(\omega_B, \omega_A)|A|^2]\]

The coefficients are of the form \(a = a' + ia''\) and are given by

\[a(\omega) = \text{const } Z \left( \omega - \omega_{ab}, \Gamma_{ab}, D \right)\]  \hspace{1cm} (2-19)

\[b(\omega) = \text{const } \exp - \frac{(\omega_{ab} - \omega)^2}{D^2} \left[ 2\Gamma_{ab} \right]^{-1}\]

\[c(\omega) = \text{const } \exp - \frac{(\omega_{ab} - \omega)^2}{D^2} \left[ i(2\omega_{ab} - \omega_A - \omega_B) + 2\Gamma_{ab} \right]^{-1}\]

Where \(Z\) is the integral for doppler broadening, \(\omega_{ab}\) is the natural maser frequency and \(D\) is the doppler width. \(\omega_A\) and \(\omega_B\) are the frequencies in the A and B directions respectively.

An index of refraction can be introduced by use of the relationship

\[\varepsilon_0^{-1} P = (\mu^2 - 1)E\]

or

\[\mu = 1 + 1/2\varepsilon_0^{-1} P/E\]

(2-20)

From equation 2-19 the approximate index of refraction can be written

\[\mu(\omega_A) = 1/2 \left[ -a^*(\omega_A) + b^*(\omega_A)|A|^2 + c^*(\omega_A, \omega_B)|B|^2 \right]\]

(2-21)

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A similar expression applies for $\mu (\omega_B)$. Equation 2-21 will be analyzed in Chapter VI in a discussion of bias beating and power sensitive frequency pulling.
CHAPTER III

THE HELIUM-NEON OPTICAL MASER

Javan, Bennett, and Herriott\(^1\) successfully operated the first gaseous optical maser in 1961 utilizing the \(4s[1/2]^1_1 - 3p[3/2]_2\) (Racah notation) transition of Neon. In the half decade since, literally thousands of maser transitions\(^2\) in gases have been observed including the exceptionally high gain \(5s[1/2]^0_1 - 4p[3/2]_2\) line in Neon. Since the He-Ne optical maser is the most integral component in the experimental apparatus for this investigation, it is deemed necessary at this juncture to include a chapter in this dissertation to consider the basic aspects of such a system. Special emphasis will be placed on the \(4s[1/2]^0_1 - 3p[3/2]_2\) (1.153 micron) and \(5s[1/2]^0_1 - 4p[3/2]_2\) (3.39 microns) transitions in Neon.

An optical maser may be considered to be analogous to a common electronic oscillator, consisting of a resonant optical cavity with cavity losses compensated

\(^1\)Javan, Bennett, and Herriott, op. cit.

for by means of some feedback mechanism. This mechanism is that of emission of radiation due to a stimulated transition from an overpopulated upper energy level to a lesser populated level in a given active medium. The necessary topics for a basic understanding of a maser system will be included under the following sections:

2. Atomic energy levels and transition probabilities.
3. General theory.
4. Excitation mechanisms.
5. Optical resonant cavities.
6. Frequency characteristics.

Condition for Maser Action

Given an optical resonant cavity it is essential to introduce a mechanism to replace energy losses for the purpose of maintaining oscillation. This is achieved via the phenomenon of stimulated emission. Atoms in the active medium in an excited state, optically coupled to a lower state, are stimulated by radiation at the appropriate frequency to emit further radiation thereby reinforcing the stimulating emission. The physics of the process becomes exceedingly complex when the interaction of the radiation field with the active medium is analyzed. This point will be considered further in the
section on general theory. The important consideration for maser action, therefore, is the establishment of population inversion. This terminology means that the upper level population must be considerably greater than that of the lower level and hence inverted with respect to the normal Boltzmann distribution.

Atomic Energy Levels and Transition Probabilities

Many of the articles concerning the He-Ne optical maser represent the Neon energy levels by means of the obsolete and essentially arbitrary Paschen notation. Unfortunately this has probably arisen due to the fact that L-S coupling does not adequately explain much of energy level systems. A much more suitable labeling scheme for the He-Ne energy levels is that arising from Racah's $j$-$l$ coupling. This coupling technique produces reasonably good wave functions for cases where the electrostatic interaction is much smaller than the spin orbital term for the parent ion with the opposite holding true for the external electron.

Racah's $j$-$l$ coupling.—In this coupling method, the angular momentum $l$ of the running electron is coupled to the angular momentum $j$ of the parent to yield an intermediate angular momentum $K$.

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K is then coupled to the spin angular momentum \( \hat{s} \) to yield the total angular momentum \( \hat{J} \) for the configuration 
\[
\hat{J} = \hat{K} + \hat{s}
\]

The energy levels are labeled in this scheme by denoting the parent configuration followed by the \( K \) and \( J \) values.

\( e.g., \) Parent \([K]_J\)

A superscript \( o \) may appear indicating an odd parity otherwise the parity is even. The general structure of a configuration in this coupling is that of pairs having the general characteristics of doublets with \( K \) assuming half integral values and \( J \) assuming integral values.

Appropriate final wave functions may be constructed using the values of \( j, K, \) and \( J \) and the corresponding Clebsch-Gordon coefficients in the following representation:

\[
|J, J_z, K, S\rangle = (-1)^{S-K-J_Z} \frac{1}{(2J + 1)^{1/2}} \cdot \sum_{m_K, m_S} \cdot C(K, S, J, m_K m_S - J_Z) |K m_K S m_S\rangle
\]

where \( m_K \) and \( m_S \) are the projections of \( K \) and \( S \) along \( z \) and \( |K, m_K, S m_S\rangle \) represents the product wave function. These final wave functions may then be used to evaluate the matrix elements of the electric dipole
operator between any two states, leading to the transition probabilities in the form:

\[ \left| \frac{J_{Z_1} J_{Z_2} | \langle K_1 S J J Z_1 | T_{pq} (\ell) | K_2 S J Z_2 \rangle}{\sqrt{\frac{(2J_1+1)(2J_2+1)(2K_1+1)(2K_2+1)}{g(2J_1+1)}}} \right|^2 \]

\[ = \frac{1}{g(2J_1+1)} \times \left\{ \frac{J_{1J} \frac{J_{21}}{2}}{K_{1K} \frac{1}{2}} \right\}^2 \times \left| \frac{\ell_1 | T_p (\ell) | \ell_2}{} \right|^2 \]

where \( \left\{ \frac{J_{1J} \frac{J_{21}}{2}}{K_{1K} \frac{1}{2}} \right\} \) indicates a Racah 6-j coefficient and \( T_{pq} \) is the electric dipole tensor operator.

Koster and Statz\(^4\) used this representation to evaluate the transition probabilities for all the 4s-3p transitions in Neon. A tabulation of the relative line strengths illustrates that the strongest transitions are those where \( \Delta K = \Delta J = \pm \Delta \ell \). The only strict selection rules for these transitions are that \( \Delta J = 0, \pm 1 \) with 0→0 transitions forbidden.

**General Theory**

The most appropriate theory for the operation of a multimode gaseous optical maser is that presented by Lamb\(^5\) and extended to more general cases by Heer and Graft.\(^6\) The end results of the theory are quite


\(^5\)Lamb, Jr., op. cit.

\(^6\)Heer and Graft, op. cit.
ponderous but the general groundwork is relatively straightforward and will be outlined in the succeeding section.

The maser action is assumed to arise from the establishment of a population inversion between two excited states a and b lying far above the ground level in the active medium. A perturbation V(t) with matrix elements between levels a and b induces an atomic polarization. In general this perturbation may include both states of polarization as well as radiation traveling in the + and - z directions. It is demonstrated in both analyses that a density matrix description for the ensemble of atoms is a handy model for treating cases having possible degeneracies in both the radiation field and the atomic levels.

If the Hamiltonian for an atom is H then the equation of motion for the density matrix can be written as

$$i\hbar \dot{\rho} = H\rho - \rho H$$

In the general two level problem where a and b are connected by the perturbation V(t) it is convenient to truncate the density matrix to include only these levels.

$$\rho = \begin{pmatrix} \rho_{aa} & \rho_{ab} \\ \rho_{ba} & \rho_{bb} \end{pmatrix}$$

Heer and Graft\textsuperscript{7} have shown that this differential equation

\textsuperscript{7}Ibid.
Fig. 1.—Helium-Neon Energy Level Diagram
for $\rho$ has an exact integral solution of the form

$$\rho(t) = \lambda \Gamma^{-1} + (i\hbar)^{-1} \int_0^\infty ds \, T^+(s) \{ [V(t-s) \rho(t-s) - 
\rho(t-s) V(t-s)] T(s) \}
$$

where $T(s) = \exp \left[-(1/2 \Gamma - i\hbar^{-1} H_0)s\right]$. For the two level problem this integral may be solved directly to yield a series expansion for the density matrix.

$$\rho = \rho_0(n) \rho(n) = \lambda \Gamma^{-1}
$$

$$\rho(n)(t) = \hbar^{-1} \int_0^\infty ds T^+(s) \{ [V\rho(n-1) - \rho(n-1)V] \}(t-s) T(s)
$$

In most cases of interest the electric dipole interaction is the perturbation of primary importance i.e., $V(t) = \Phi \cdot \mathcal{E}$ where $\Phi$ is the electric dipole operator.

The macroscopic electrical polarization, for the given interaction, can be determined from the trace of $\rho \Phi$

$$\mathcal{P}(\mathcal{r},t) = N<\text{tr} \rho(\mathcal{r},t) \Phi> \quad \text{and} \quad N<\text{tr} \rho(\mathcal{r},t) \Phi>
$$

where the averaging is performed over the atomic velocity distribution. This polarization can now be used as a source term in Maxwell's equation:

$$-\nabla \cdot \nabla \mathcal{E} - c^{-2} \frac{\partial^2 \mathcal{E}}{\partial t^2} = c^{-2} \varepsilon_0^{-1} \frac{\partial^2 \mathcal{P}}{\partial t^2}
$$

to obtain conditions on threshold, output power as a function of tuning, frequency pulling, stability, etc.
**Excitation Mechanisms**

Javan has pointed out that in a system of two gases where one gas has a metastable state lying close in energy to an excited state in the second gas, a large cross section for inelastic collisions can be expected, resulting in a transfer of excitation from an atom in the metastable state to the excited state of an atom in the second gas. This excitation process is known as a collision of the second kind, and is the primary means of producing the population inversion necessary for maser action in the He-Ne system. In this case a collision of the second kind occurs between the long lived $2^3S$ and $2^1S$ Helium metastables and the $2p^54s$ and $2p^55s$ excited levels of the Neon configuration.

$$\text{He}(2^3S) + \text{Ne} \rightarrow \text{He} + \text{Ne}(2p^54s)$$

$$\text{He}(2^1S) + \text{Ne} \rightarrow \text{He} + \text{Ne}(2p^55s)$$

The energy spread of the levels in the $4s$ and $5s$ configurations is only of the order of $kT$ and they become selectively populated with respect to the $2p^53p$ configuration which is too far removed from the Helium metastables to be populated in a similar manner. The lifetimes of these $4s$ and $5s$ levels are an order of magnitude longer than those of the levels in the $2p^53p$ configuration which decay rapidly to the $2p^53s$ levels. This lower configuration ($2p^53s$) consists of metastable
levels and other levels which become effectively metastable due to resonant trapping. A problem does exist here in that the $2p^53p$ levels may be populated by electron impact with the $2p^53s$ levels thereby destroying the necessary population inversion. To eliminate this possible excitation mechanism the electron density must be suppressed and the $2p^53s$ metastables destroyed. This is normally accomplished by operating at low pressures and by making the cross section of the maser tube sufficiently small so that the metastables may diffuse to the walls.

It should be noted that even some lines with zero transition probabilities oscillate. This is due to the fact that the collision of the second kind process is so powerful that extremely high degrees of population inversion can occur allowing weak lines to oscillate apparently violating selection rules.

**Optical Resonant Cavities**

**Fabry-Perot interferometer.**—The most common resonator used to establish a high radiation density for the production of maser action is the Fabry-Perot interferometer. This interferometer consists of two flat parallel reflecting plates of radius $a$, separated by a distance $b$. Such a resonator was used by Schawlow and
Townes\textsuperscript{9} and later by Javan et al\textsuperscript{10} in obtaining maser action in the optical region using solid and gaseous active mediums respectively.

The classical view of the interferometer indicates that resonances at all angles are possible. The approximate treatment by Schawlow and Townes\textsuperscript{11} used the assumptions that the amplitude of the field in the resonator is zero at the edges of the plates and that there is no phase variation across the surface of the plates. Fox and Li\textsuperscript{12} have illustrated clearly that neither of these assumptions is valid. In their treatment of the problem, the field distribution was calculated by considering a uniform plane wave originating from one of the plates and to compute, using an integral equation from Huygen's Principle, the distribution at the opposite plate. The reflected portion is then used as a starting distribution for a second similar determination. After a large number of successive reflections at the plates, the distribution remained unchanged after each transit except for an overall reduction due to diffraction losses. This steady


\textsuperscript{10}Javan, Bennett, and Herriott, \textit{op. cit.}

\textsuperscript{11}Schawlow and Townes, \textit{op. cit.}

\textsuperscript{12}Fox and Li, \textit{op. cit.}
state distribution is assumed to represent a normal mode for the resonator. The exact amplitude distribution, phase distribution and diffraction loss per transit is found to depend on the geometry of the system and particularly the quantity $a^2/b\lambda$ where $\lambda$ is the free space wavelength.

These normal modes are specified as in standard waveguide practice by a $\text{TEM}_{nmq}$ designation where $n$ and $m$ specify the axial and radial symmetry of the mode and $q$ is determined by the resonance condition $q\lambda = 2b$. In all cases considered, the loss per transit for the antisymmetrical modes was found to exceed that of the lowest-order symmetrical mode. For a case where the interferometer contains an active medium, oscillation of a given mode occurs only when the gain exceeds the loss presented by the mode.

Confocal resonators.—Other resonators frequently utilized for optical cavities are those configurations using concave mirrors or combinations of concave mirrors with optical flats. The most common of these arrangements is the confocal system\textsuperscript{13} using two mirrors of equal radii of curvature, situated on a common axis, separated by a distance equal to twice the focal length. The confocal system is preferred over other arrangements

\textsuperscript{13}P. Connes, Revue d'Optique \textbf{35} 37 (1956).
due to the fact that it is easier to align and exhibits smaller diffraction losses.

**Frequency Characteristics**

For most optical maser transitions the spectral lines are inhomogeneously broadened as the result of three basic mechanisms: (1) natural broadening due to the finite lifetime of the excited state; (2) Doppler broadening due to thermal motion of the atoms; and (3) pressure broadening due to atomic or ion-electron collisions. The result is that the spectral line has a finite width $\Delta \nu$ centered about the transition frequency $\nu_0$.

The natural width of a transition can be estimated by considering the uncertainty principle $\Delta E \Delta t \sim h$. This implies that the energy spread $\Delta E = h\Delta \nu$ is of the order of $h/\Delta t$, where $\Delta t$ is the lifetime of the excited state, or that $\Delta \nu_n \sim 1/\Delta t$. Measurements$^{14}$ of $\Delta t$ for the 1.153 $\mu$ transition in Neon would indicate, by this argument, a natural line width of the order of $10^7$ CPS.

Since the pressures used in He-Ne optical masers are extremely low, broadening due to the Doppler shift dominates over the collision (pressure broadening) effects. In this case, the fractional gain per unit

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length is a Gaussian function given by

$$G(v) = G_0 \exp - \frac{[v-v_0]^2}{0.6 \Delta v_D}$$

where $\Delta v_D$ is the width of the line at half maximum. $\Delta v_D$ can be written as

$$\Delta v_D = 2v_0 \left( \frac{2kT \ln 2}{Mc^2} \right)^{1/2}$$

where $k$ is Boltzmann's constant, $T$ is the absolute temperature and $M$ is the atomic mass.

Calculations for the 3.39 micron and 1.153 micron lines in Neon indicate that the Doppler widths are ~270 Mc and ~800 Mc respectively. For most cavities the longitudinal mode spacing ($\Delta v = c/2b$) is of the order of ~100 Mc so that several longitudinal modes may oscillate under the large Doppler curve.

Lamb dip.—The theory of Lamb predicts that the Gaussian Doppler curve may exhibit a Lorentzian shaped dip, having a width of the same order as the natural width, at line center. This dip has been observed experimentally and is assumed in Chapter VI to contribute appreciably to power sensitive frequency pulling.

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16Lamb, Jr., op. cit.

CHAPTER IV

THE EXPERIMENTAL APPARATUS

The experimental apparatus was designed for the purpose of investigating the effect of macroscopic media on the beat frequency of a rotating closed path optical maser. To this end it was necessary to construct an exceedingly stable triangular oscillator with a rotational drive and accessory equipment capable of determining effects as small as .25 percent of the empty cavity beat frequency.

The overall basic apparatus consists of a triangular resonant cavity and a He-Ne discharge tube mounted on a platform with a variable drive. A recombiner for the cw and ccw traveling waves, is provided external to the cavity along with a detector suitable for measuring the beat frequency between the slightly degenerate signals. An oscilloscope and a frequency-period counter are used as associated equipment to continuously monitor the output of the detector.

Description of the Resonant Cavity

The resonant cavity consists of two plane dielectric mirrors and one spherical concave dielectric mirror mounted
at the vertices of an 87.5 cm. equilateral triangle. The plane mirrors are formed by depositing multiple layers of suitable dielectric materials on circular quartz substrates 1 inch diameter and 3/8 inch thick with front surfaces flat to better than 1/20 wavelength. The spherical mirror is formed in the same manner using a 1 inch diameter, 3/8 inch thick quartz substrate with the front surface having a 10 meter radius of curvature. The dielectric layers are of the proper thickness and number\(^1\) to provide a maximum reflectivity of 99.7 percent of 1.153 micron light incident at an angle of 30°. (Light polarized in a plane perpendicular to the plane of incidence experiences a reflectivity of 99.7 percent whereas light polarized in the plane of incidence experiences a slightly lower reflectivity.) The material and the number of the dielectric layers is held as proprietary information by the manufacturer, Perkin-Elmer Corporation, Norwalk, Connecticut; but, Born and Wolf\(^2\) indicate that ordinary high reflection dielectric mirrors are formed by depositing a periodic succession of alternate quarter wave films of dielectrics having exceedingly high and low indices of refraction respectively, with a

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\(^2\)Ibid., p. 64.
low index film adjacent to the substrate. Commonly used dielectrics are zinc sulfide \((n = 2.3)\) or titanium dioxide \((n = 2.45)\) for the high index films, and cryolite \((n = 1.35)\) or magnesium fluoride \((n = 1.38)\) for the low index films.

A cavity in this configuration could appropriately be designated as a triangular "pseudo confocal" Fabry-Perot cavity. The stability condition \(0<\xi<\xi_f, \xi_y\) as stated by Collins\(^3\) \((\xi_f = (R \cos \theta_i)/2 \text{ and } \xi_y = (R \sec \theta_i)/2)\) is easily satisfied for a triangular cavity of length 262.5 cm utilizing a spherical mirror of radius 10 meters.

The corner reflectors are mounted semi-rigidly in dual gimbaled holders of the variety supplied by the Lansing Corporation, Ithaca, New York. (See Illustration 1.) Adjustments as small as \(10^{-4}\) radians about a vertical or horizontal axis can be made by means of micrometer screws magnetically coupled to the inner and outer gimbals respectively. The outer gimbal is made of stainless steel to provide proper spring loading for the adjustment. The magnet contact surface for the inner gimbal is ground spherical to a radius of curvature equal to the lever arm for inner gimbal adjustment. This feature provides independent control of the outer

\(^3\)Collins, op. cit.
gimbal by preventing the inner gimbal micrometer screw from dragging the magnet. The holder for the spherical mirror is slightly modified by lowering the inner gimbal micrometer screw and magnet to the bottom of the mount. This modification is necessary to prevent the clockwise output signal, taken through this mirror, from striking the micrometer barrel.

Recombination of the clockwise and counterclockwise signals is provided external to the cavity by means of a totally aluminized mirror and a beam splitter (50 percent transmission at 1.0 micron) mounted in similar adjustable holders at the vertices of a 6 cm equilateral triangle behind the spherical mirror (See Figure 2). In this manner the CW and CCW signals are directed within a solid angle of $10^{-6}$ steradians to a lead sulfide infrared photo detector.

The alignment procedure for both the cavity and the recombiner will be outlined in detail in Chapter V.

**Description of the Maser Medium and Discharge Tube**

A 5:1 mixture of He and isotopic Ne (90% Ne$^{20}$ and 9% Ne$^{22}$) at a pressure of approximately 1 torr was used as the active medium for the initial investigations. Cheo$^4$ has indicated that this mixture and pressure

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$^4$P. K. Cheo, private communication.
Fig. 2.—Triangular "Pseudoconfocal" Cavity and Recombiner

L = Discharge Tube
M = Cavity Mirrors
M' = Recombiner Mirrors
F = Test Medium
D = Detector
produces the maximum gain for oscillation on the 1.153 micron transition of Ne when contained in a discharge tube of the dimensions used in this particular experiment. Subsequent investigations in the laboratory have substantiated this contention, although it was found that the gain curve was essentially flat over a reasonable range of mixtures and pressures (4:1 to 6:1 and .7 torr to 1.3 torr).

The maser medium is contained in a 30 inch quartz tube with an 8 mm I.D. and 10 mm O.D. 3 mm thick quartz windows with both surfaces flat to 1/20 wave are epoxied to the ends of the tube which are cut at anti-parallel Brewster's angles (55° 20' for quartz). (See Figure 3.) The discharge tube, as a unit, is supplied by Laboratory Optics, Plainfield, New Jersey.

Later, more successful, investigations (See Chapter V) were carried out using a 10:1:1 mixture of He, Ne20 and Ne22 at a pressure of 1.4 torr contained in a 6 mm I.D. tube of the type described previously.

A quartz-pyrex seal is provided at a right angle to one end of the tube to allow filling the tube with the proper mixture. To this seal a barium getter can be attached. The getter which is provided by the O.S.U. Communications Lab has a barium coated filament which can be fired by passing a current of approximately
Fig. 3.—Brewster's Angle Discharge Tube
7 amperes through it (the actual current varies with each getter). This procedure provides purification of the maser medium by the removal of water vapor and other residual gases. Previous cleaning of the mixture has been provided by passing the gas over a silica gel trap immersed in a liquid nitrogen bath. The overall procedure normally produces a mixture suitable for 300-400 hours operation.

The tube is mounted in the cavity in such a manner that the normal to the windows lies in a plane perpendicular to the platform. In this arrangement, light transmitted through the Brewster's angle windows is polarized in the plane that experiences maximum reflection at the corner reflectors.

**Excitation Source**

Excitation of the maser medium is provided by a radio frequency transmitter inductively coupled to seven copper air-gap electrodes around the discharge tube. The transmitter is a Collins 32 V 2 capable of providing 150 watts of rf power. Although 14 MHz was selected as the excitation frequency for maser operation, the Collins transmitter allows continuous band switching from 3.5 MHz to 29.7 MHz. The output network is capable of matching unbalanced resistive loads from 26 to 200 ohms through proper adjustment of the final tuning and
antenna loading controls. Matching of the transmitter output with the copper electrodes is effected by means of the following circuit.

![Circuit Diagram](image)

The primary coil is fabricated by winding four turns of 1/8 inch copper tubing at 1-1/2 inch diameter. The 1 inch diameter secondary coil of 10 turns of 1/8 inch copper tubing is wired in parallel with a 0-100 µf variable capacitor. Proper resonance of the circuit at 14 MHz is ensured by monitoring with a grid dip meter.

**Holding Device for Transparent Material**

The transparent medium is introduced into the cavity in the form of 1 inch diameter quartz (fused silica) flats 1/2 inch thick, both surfaces flat to better than 1/20 wave and having less than 0.2 seconds wedge. (Supplied by Laboratory Optics, Plainfield, New Jersey.) In order to ensure the lowest possible loss at each surface, the flats are arranged in pairs at anti-parallel Brewster's angles. (The surfaces are parallel to the windows of the discharge tube.) To accomplish this, an aluminum rack was fabricated by the Physics Department Shop in which anti-parallel pairs of 1/2 inch slots were cut at angles of 55° 20' (by means of a sine bar) with
respect to a horizontal surface. The rack can accommodate from 2-12 flats in pairs and can be adjusted in the vertical and horizontal planes by means of knurled adjustment screws. (See Illustration 2.)

Rotating Platform and Drive Mechanism

The rotating platform and variable drive mechanism are essentially the same as that constructed for previous investigations by Cheo.

The top of the platform is a 42" x 42" aluminum plate, 1-3/4 inches thick, on which the cavity and discharge tube are mounted. An L-shaped aluminum piece 1-3/4 inch thick is bolted to one corner of the main plate so that adequate space is provided for the recombiner mirrors and the detector. The main plate is bowed onto a 20 inch diameter ball bearing by means of 3/8 inch diameter brass rods with two 2' x 2' x 6" limestone slabs (approximately 600 pounds) sandwiched between. The entire unit is resting on a reinforced concrete slab 32" x 34" x 4" with rubber isomode pads between. The ball bearing is made of two circular steel plates with a circular V-groove cut at an 18 inch diameter to accommodate 76 precision carbon chrome alloy balls (3/4 inch diameter, spherical to 0.000025 inches). Before aligning

Illustration 2.—Holder for Quartz Windows
the cavity (see alignment procedure, Chapter V), it was necessary to level the platform to within a maximum deviation of \(0.0001\) inch per foot using the precision level from the student shop.

The variable rotation mechanism for the platform consists of a \(1/4\) h.p. G.E. capacitative A.C. motor with a shaft rotation rate of 1725 RPM, a reversible variable speed control (Zero-Max model 10E400R), a 29 to 1 single worm speed reducer (Abart Gear and Machine Co.), a 2 inch diameter sprocket, and a double steel roller chain.

The drive system is capable of producing clockwise and counterclockwise platform rotation rates from zero to 9 degree/second by means of the screw control on the Zero-Max. The input and output shafts of the Zero-Max are connected by means of flexible couplings to the A.C. motor and the speed reducer respectively. The double steel roller chain is wrapped around the 2 inch sprocket on the output shaft of the speed reducer and pinned by means of two 8-32 brass screws to the upper plate of the 20 inch bearing. The bottom half of the roller chain is allowed to be in contact with the lower (stationary) housing of the 20 inch bearing, providing a friction drag thereby reducing backlash. It will be noted in Chapter V that the method for accumulating data permits
a reasonable uncertainty in the rotation rate but is quite sensitive to backlash.

Associated Equipment

A lead sulfide (Kodak Ektron N-2) detector (mounted as shown in Figure 4) is used to detect the frequency separation between the CW and the CCW modes. The detector has excellent sensitivity at 1.153 microns having a \( D^* \) of \( 10^{11} \) and a spectral sensitivity \( S_i \) of the order of \( 10^3 \). These characteristics normally used for evaluating photodetectors are defined as follows:

\[ D^* (\text{Cm/}\text{watt sec}^{1/2}) = \frac{V (\Delta f)^{1/2}}{J \text{NEP}^{1/2}} \]

\[ = \frac{A\Delta f}{\text{NEP}} \]

where

\[ V = \text{rms signal in volts} \]
\[ \Delta f = \text{bandwidth in cycles per second} \]
\[ J = \text{rms value of radiant energy flux density in watts per cm}^2 \]
\[ N = \text{rms noise in volts} \]
Fig. 4.—Detector and Detector Mount
A = sensitive area in cm\(^2\)

NEP = noise equivalent power; the rms value
of minimum radiant energy flux in
watts necessary to give a signal to
noise ratio of unity.

The spectral sensitivity \(S_i\) is numerically equal to
the signal in microvolts produced by a photodetector
having 1 volt bias applied across the detector and load
resistor when irradiated by a flux density of 1 micro-
\(\text{watt/cm}\(^2\) at a chopping frequency of 90 CPS.

\[
S_i = \frac{V}{JE} \frac{(R_c + R_l)^2}{4R_cR_l}
\]

where

\(V\) = rms value of signal in volts
\(J\) = rms value of radiant energy flux
density in \(\text{watts/cm}\(^2\)
\(R_c\) = detector dark resistance
\(R_l\) = load resistance
\(E\) = bias in volts across detector
and load resistor.

Although the response time of an N-2 detector is
reportedly poor (500-1000 Msec.) no appreciable diminu-
tion of the output amplitude is noted until an input
of 5000-6000 cps is exceeded.

A bias voltage is applied to the detector from a
circuit enclosed in a 2" x 2" x 3" aluminum box mounted
directly to the detector holder by means of a coaxial T.

The bias circuit is as follows:

```
  Detector ← 100 x Preamp
```

The output of the detector is fed through an AC coupled 100 x preamplifier (Tektronix type 123) with a bandpass of 3cps to 25 KC/sec to a Tektronix type 561 oscilloscope and a Computer Measurements Co. Model 201C Frequency-Period counter. The oscilloscope utilizes as plug-in units a type 63 differential amplifier and a type 67 time base. The counter is triggered externally by means of brushes mounted on two adjacent corners of the rotating platform and a third brush rigidly mounted on the wall to function as the switch in the following circuit:

```
  10 M
  67.5V
  .01 μf

  Pin 8
  Pin 2
```

This circuit supplies a negative pulse sufficient to activate the flip flop circuit in the counter so that the counting process may be started and stopped at specific points during platform rotation.

A complete diagram is shown in Figure 5.
Fig. 5.—Block Diagram of Complete Experimental Arrangement
CHAPTER V

EXPERIMENTAL PROCEDURE

In order to investigate the effect of transparent media on the beat frequency of a closed path optical maser, it is necessary to have a stable oscillator with reasonably high gain, transparent materials with optical surfaces that present high transmission characteristics, minimal wave front distortion and exceedingly low feedback of the CW signal into the CCW\(^1\) (and vice versa), a detection and recording system that presents sensitive and accurate accumulation of data and a table rotation drive that allows minimal backlash. Toward this end, the procedure outlined in this chapter has been implemented over a matter of several years. The sections will be ordered to include the successful methods followed by the investigative procedures leading toward the optimum system. These areas could be categorized as follows:

1. Successful investigation with Homosil quartz at 1.153 microns in a triangular

\(^1\)See Chapter VI for discussion of feedback.
Fabry-Perot cavity.

2. Modifications to eliminate the effect of the highly dispersive character of the active medium.

3. Initial investigations with KBr at 3.39 microns in a square Fabry-Perot cavity.

4. Investigations with borosilicate glass, carbon tetrachloride and glacial acetic acid at 1.153 microns in a triangular Fabry-Perot cavity.

Successful Investigation with Homosil Quartz at 1.153 Microns

In view of the requirements that the system possess good stability, relative ease of alignment, and wavelength of operation such that inexpensive test media is readily available, a 1.153 micron maser mounted in a triangular cavity was utilized for the final investigations. The test medium was introduced into the cavity in the form of 1.26 cm. thick quartz flats arranged in pairs at anti-parallel Brewster's angles. (See Chapter IV for complete description of the holding device.) The flats used were Homosil quartz (Laboratory Optics Co., Plainfield, New Jersey) with both surfaces flat to better than 1/20 wavelength and having no measurable wedge. This arrangement was found to introduce relatively low
loss and feedback into the cavity. With as many as 12 flats, the dc level of oscillation was reduced by only 30 percent with no apparent increase in the level of bias beating.

As will be discussed in succeeding sections, this experimental arrangement was selected after several arduous investigations with other cavity configurations, wavelengths, and media.

Once this system was adopted as being the most satisfactory, data was taken to verify the theoretical formulation for the beat frequency in the presence of macroscopic media. The data was accumulated by recording on the CMC frequency-period counter, the number of beats through a given angle \((90^\circ \pm \Omega_{\text{earth}} T)\). \(\Omega_{\text{earth}}\) is the angular rotation rate in \(^\circ/\text{sec}\) and \(T\) is the period of rotation for the platform. The counter was triggered by means of the external circuit described in Chapter IV while at the same time the beat waveform was continuously monitored on the Tektronix oscilloscope to assure that the beats remained stable through a given rotation. The number of beats was recorded as a function of table rotation rate, number of flats in the cavity and the dc power level of the maser. As will be noted in Chapter VI, frequency-pulling effects caused by the change in maser dc power level caused some difficulty. Reasonably good success in eliminating this
problem was found in using an attenuating flat in the cavity to control the dc level while maintaining the transmitter power transfer constant. Previously the dc level had to be maintained constant by means of altering the transmitter loading as each additional pair of Homosil flats was placed in the cavity. The attenuating flat (borosilicate glass flat to $\frac{1}{20}$ wavelength with no measurable wedge) was mounted nearly perpendicularly to the cavity axis in a Lansing type mirror holder. Angular adjustments as small as $10^{-4}$ radians are sufficient to maintain a constant dc level. Utilizing this technique, the data conformed reasonably well to the predicted values. (See Chapter VII.)

**Modifications to Eliminate the Highly Dispersive Character of the Active Medium**

As will be emphasized in Chapter VI the mechanism for the power sensitive frequency pulling is that of dispersion in the active medium when the mode of oscillation lies near line center. Because of this it was decided to take advantage of the isotopic broadening presented by a 10:1:1 He-Ne$^{20}$-Ne$^{22}$ active medium. With the resultant line broadened from $\sim 800$ MHz to $\sim 1500$ MHz, the region of highest power lies approximately 350 MHz from either isotopic resonance. The dispersion in this region can be shown to be lower by a factor of almost 100.
Such a mixture at a pressure of 1.4 torr contained in a 6 mm i.d. Brewster's angle discharge tube was utilized in the final phase of the investigation. The data taken with this much improved mixture (See Chapter VII) was far superior to any previous data in that frequency pulling no longer needed to be considered and a reduction in the residual bias was observed.

Initial Investigations with KBr at 3.39 Microns

The initial investigations were for the purpose of examining the effect of potassium bromide (K Br) on the beat frequency of a square cavity optical maser operating on the 3.39 micron transition of neon. The active medium was a 5:1 mixture of helium and isotopic neon at a pressure of 0.8 torr contained in an r.f. excited 8 mm i.d. discharge tube with Brewster's angle windows. Alignment of the cavity for this high gain line (gain $10^4$/meter) was a relatively simple task. The cavity consisted of three 1-1/2 inch diameter x 1/4 inch thick quartz flats (front surface figure of 1/20 wavelength), two of which were totally aluminized and the other aluminized for 2 percent transmission at 1 micron (Evaporated Coating, Inc., Huntington Valley, Pennsylvania) and a 1-1/2 inch diameter x 1/2 inch thick 10 meter radius spherical quartz substrate (front surface figure of 1/4 wavelength) aluminized for total reflection. The mirrors
were mounted in Lansing Corporation adjustable mirror mounts at the corners of a square 92 cm x 92 cm. Since the aluminized surfaces exhibit nearly total reflection of visible light it was a straightforward procedure to trace a collimated light beam around the cavity ensuring that it closed back upon itself. This requirement alone is normally adequate to align a resonant cavity for the 3.39 micron transition. Additional refinement for optimum dc level can be made while observing the maser oscillation.

The K Br was introduced into the cavity in the form of two 3/4 inch wide x 3/4 inch high x 6 inch long rods, the end surfaces of which were polished flat to 1/20 wavelength with less than 10 seconds wedge between the ends. The ends were additionally coated with dielectric layers, the nature of which is held as proprietary information by Perkin-Elmer Corporation, Norwalk, Connecticut, for less than 1/4 percent reflection at 3.39 microns. The rods were held in aluminum troughs which permitted adjustment only in the horizontal plane. (Adjustment in the vertical plane could be made by means of shims under the mounts.)

Oscillation was obtained with both rods in the cavity but the resulting data was considerably different than anticipated; a point which is discussed in detail in Chapter VI. Despite the high transmission
characteristics of K Br, the introduction of both rods into the cavity reduced the dc level of oscillation from 4 volts to 10 millivolts, a fact, which as will be noted later, produces large frequency pulling effects. Considerable deterioration of the anti-reflection coatings prevented further investigations with K Br.

At this juncture it was decided to alter the cavity to provide oscillation on the 1.153 micron line of He-Ne since more, less expensive, materials are available with high transmission characteristics at the shorter wavelength.

Cavity tubing to enhance dc power levels.--During these initial investigations it was noted that the power level of oscillation of the 3.39 micron He-Ne infrared maser depended on the bore of the tubing used in the maser cavity.² This effect was regarded as enhancement of the cavity Q for the proper tube bore. The maser configuration, which is shown in Figure 6, is a 92 x 92 cm Fabry-Perot square cavity in which is mounted a 6 mm i.d. He-Ne discharge tube with Brewster angle windows. Upon insertion of a 75 cm length of 6 mm bore Pyrex tube into one leg of the cavity, a 75 percent increase in the power level of oscillation was observed. Insertion of additional lengths of Pyrex tubing into the remainder

Fig. 6.—Placement of Cavity Tubing

M = Cavity Mirrors
T\(_1,2,3\) = Cavity Tubing
D = Discharge Tube
of the cavity indicated that the increase is proportional
to the length of the inserted tubing. An increase of
approximately 1 percent per cm of tubing was obtained
or the power level increased by a factor of three upon
the insertion of 200 cm of 6 mm bore tubing into the
cavity. The effect diminished toward the original power
level for larger bore tubing. Above 9.5 mm i.d. no
effect was observed. Decreasing the tubing bore below
4.5 mm caused an attenuation of the power level. Enhance­
ment of the power level has also been produced using
cavity tubing of Inconel, stainless, or quartz, but the
results were not as marked. The optimum enhancement of
the power level upon insertion of tubing of this latter
type was of the order of 0.3 percent per cm as compared
to 1 percent per cm for Pyrex. The enhancement was
greatest for a 6 mm bore. No difference in the results
was noted when the cavity was altered from a system
utilizing four optical flats to a "pseudoconfocal"

        An explanation for the observed phenomena may be
given in terms of the coefficient of reflection for
almost grazing incidence. At grazing angles the reflec­
tion coefficient is very nearly unity. The angular
divergence in this system was of the order of $10^{-3}$
radians, so that even a medium as lossy as Pyrex for
infrared radiation may become an excellent reflector.
Utilizing this additional internal reflection, the Pyrex tubing served as an efficient guide for the infrared radiation.

Investigations with Borosilicate Glass, Carbon Tetrachloride and Glacial Acetic Acid at 1.153 Microns

Alignment of the square cavity for oscillation of the 1.153 micron transition was not as simple a task as experienced on the higher gain line (3.39 micron gain = $10^4$/meter; 1.153 micron gain = 10%/meter). Initially, attempts were made to substitute dielectric mirrors (reflectivity of 99.7 percent for 1.153 microns at a 45° angle of incidence) one at a time for the aluminized mirrors, allowing the maser to still oscillate at 3.39 microns. (Despite the high loss presented by the 1.153 micron dielectric mirrors to 3.39 micron light, the maser continued to oscillate on the 3.39 micron line with two dielectric and two aluminized mirrors comprising the cavity.) Unfortunately, alignment adequate for oscillation on the low gain line could not be attained by this technique. Additional alignment procedures (e.g., tracing a collimated beam around the cavity and sighting around the cavity with a telescope) were entirely unsatisfactory. Because of these difficulties, it was decided to change the cavity configuration to that of an equilateral triangle; a configuration which should...
present some ease of alignment for the following reasons:

1. Three points determine a plane, and
2. Rays in the horizontal plane are basically self closing in a triangle.

A triangular cavity was formed by first scribing the largest diameter circle (diameter of 98.5 cm) that would still allow sufficient space for the mirror holders on the table. The circle was then divided into six equal arcs using a beam compass. Alternate points were then used as the vertices of an equilateral triangle 87.5 cm on a side. Dielectric mirrors (99.7% reflectivity for 1.153 micron light incident at an angle of 30°) were mounted at the vertices of the triangle in adjustable holders of the Lansing Corporation variety. Two mirrors are flats (surface figure of 1/20 wavelength) while the third (output mirror) is a spherical mirror (surface figure of 1/4 wavelength) with a 10 meter radius. A recombiner for the CW and CCW signals was constructed using the extension of the two sides of the cavity triangle which intersect at the spherical mirror. ( Provision is made for the displacement of the signal passing through the 3/8 inch mirror.) These two lines, behind the spherical mirror, are used to construct another equilateral triangle 6 inches on a side at the vertices of which are mounted a mirror aluminized for total reflection at 1.0 micron and a mirror aluminized
for 50 percent reflection at 1.0 micron. These five reflectors comprise the triangular "pseudo-confocal" Fabry-Perot cavity and external recombiner as illustrated in Figure 2.

Alignment of the new cavity was still no easy task for the low gain line (1.153 microns). Many schemes were tested unsuccessfully; namely, tracing a collimated light beam along the cavity axis, sighting around the cavity with a telescope, and, attempting to send the signal from a visual autocollimator (K and E model 71-4210) around the cavity. All methods failed due to the low reflectivity of the cavity mirrors for visible light. One unsuccessful procedure exhibited some promise. This entailed coupling the output of a linear confocal 1.153 micron maser into the cavity through one cavity mirror. This technique failed for one reason; the cavity had to be reasonably well aligned prior to implementing the coupling of energy into it.

Finally, it was decided to utilize the fact that rays are basically self closing in the horizontal plane of a triangle.\(^3\) This implies that the mirrors should be relatively insensitive to horizontal adjustments, whereas, the vertical adjustments could be ultrasensitive. (This fact was later verified.) Because of this, the vertical tilt of the mirrors was precisely

\(^{3}\text{J. E. Kilpatrick, private communication.}\)
established through the following steps:

1. The table was leveled to 0.001 inch per foot utilizing the precision level from the Student Shop.

2. A DoAll black granite gauge block (having precisely a 90° angle established between the bottom and front surfaces) was placed on top of a DoAll black granite surface plate (flat to .0001 inch across the surface) in the center of the triangle, meanwhile ensuring that the table and surface plate remained level to .0001 inch per foot.

3. A fully aluminized optical flat (both surfaces flat to 1/40 wavelength with no measurable wedge) was attached to the front surface of the gauge block and used to establish a vertical reference for the K and E visual autocollimator. The three cavity mirrors were then established to within 0.2 seconds of this vertical plane using the autocollimator on the front surfaces.

A less precise technique was necessary to provide a triangle in the established plane. A surveyor's
transit was used in conjunction with a crosshair of #40 AWG wire positioned on the desired cavity axis to determine the horizontal adjustments by sighting both directions around the cavity. This procedure with the transit was repeated with the discharge tube mounted in the cavity to properly position the tube. After this entire scheme was completed, only a very minute \(10^{-4}\) radians adjustment on the vertical micrometer of one mirror was necessary to refine the cavity alignment sufficiently for maser oscillation at 1.153 microns.

Alignment of the recombiner was accomplished quite simply by placing an infrared image converter at a distance of 4 to 5 meters from the front surface of the beam splitter. The infrared spots arriving at the image converter from the beam splitter and the fully aluminized mirror were overlapped visually. This procedure normally produces alignment of the CW and CCW signals to within \(10^{-4}\) radians which is adequate for the observation of beats between the two slightly degenerate signals. Final alignment of the cavity and recombiner was completed as the cavity rotated, optimizing the beats for stability and amplitude.

A 3/4" x 3/4" x 10" rod of Schott BK-7 borosilicate glass was placed in the cavity, in a mount adjustable in the vertical and horizontal planes. The rod
had both ends polished flat to 1/20 wavelength coated with dielectric layers for less than 1/4 percent reflection at 1.153 microns and having less than 1 minute nonparallelism between the ends. The rod was aligned by first establishing a feedback mirror to use as a reference external to the cavity. When the feedback mirror is properly adjusted, entrainment with the cavity mode can be accomplished. This mirror was then used to establish a normal reference plane with which the visual autocollimator was utilized to determine the borosilicate rod placement to better than 1 second. Usage of the BK-7 rod was unsuccessful in that oscillation was not allowed with the rod in the cavity.

Liquids such as CCL₄ and glacial acetic acid with high transmission at 1.153 microns were also attempted. (The acid was filtered by multiple passage through a 0.5 micron Millipore filter to remove solid impurities.) In several attempts the liquids were contained in tubes with borosilicate windows held both rigidly and semi-rigidly at the tube ends. The windows had a surface figure for both surfaces of better than 1/20 wavelength, with the outer surface coated (Perkin-Elmer Corporation) for less than 1/4 percent reflection at 1.153 micron. The mismatch between the borosilicate and the liquid at the inside surface produced a reflection of only
0.036 percent. In no case was oscillation allowed with the liquids in the cavity.

Two observations for failure were readily apparent:

1. Small variations in the index of refraction due to fluctuations as small as $10^{-2}$° K in the liquid temperature are sufficient to produce excessive wave-front distortion.

2. The anti-reflection coatings supplied by Perkin-Elmer Corporation are unsuitable for usage in a maser cavity (a fact which was hesitantly confirmed by Perkin-Elmer Corporation).

Oscillation is barely allowed with coated flats singly in the cavity. At no time was oscillation allowed with more than one anti-reflecting window in the cavity; whereas, two uncoated borosilicate flats (producing 16 percent total reflection loss) reduced the dc level of oscillation to about 50 percent of the empty cavity level.
CHAPTER VI

DISPERSION OF THE ACTIVE MEDIUM

During the course of the initial investigations with an active medium of Helium and natural neon, an extremely undesirable feature, detrimental to the measurement of absolute rotation, was noted. The beat frequency was observed to be strongly power dependent. This fact was first suspected when the observed shift in the beat frequency on insertion of a slightly lossy medium was considerably larger than anticipated from equation 2-17. Subsequent rotation data (See Figure 7) recorded as the level of excitation was varied showed in fact that the beat frequency could be pulled by as much as ten percent. On the supposition that the effect is due to dispersion in the active medium it is deemed necessary at this point to carefully analyze the one isotope model for a closed-path oscillator with an eye toward determining methods for eliminating this detrimental feature.

One Isotope Model

Since a plane wave expansion is a good approximation for the electromagnetic field in the cavity (See Chapter
Fig. 7.—Dependence of the Beat Frequency on the Excitation Level
II), Maxwell's equation yields two almost orthogonal solutions at $e^{-i\omega A t}$ and $e^{+i\omega B t}$

$$[\omega^2 A - \omega^2] A - 2i\omega A \dot{A} = \varepsilon_{0}^{-1} \omega^2 P(\omega A)$$

and

$$[\omega^2 B - \omega^2] B + 2i\omega B \dot{B} = \varepsilon_{0}^{-1} \omega^2 P(\omega B)$$

where the following assumptions have been made: (1) the A and B terms are slowly varying functions of space and time so that second derivatives may be neglected and (2) the field has been polarized by Brewster's angle windows so that the y-components can be set equal to zero.

If expression 2-18 for the polarization is included as the source term in equation 6-1, the equations of motion for the time dependent amplitudes can be written as

$$2i\dot{A} = \omega A [\frac{-g\Omega - iQ^{-1} + 2(\omega - \omega A) + A(\omega A) - b(\omega A)}{\omega A} |A|^2 - c(\omega A \omega B) |B|^2]$$

and

$$2i\dot{B} = \omega B [\frac{g\Omega + iQ^{-1} + 2(\omega - \omega B) + A(\omega B) - b(\omega B)}{\omega B} |B|^2 - c(\omega A \omega B) |A|^2]$$

where the term $g\Omega$ has been introduced to account for rotation of the cavity.\(^1\) The coefficients $a(\omega), b(\omega),$ and $c(\omega)$ have been presented previously (see equation 2-19) as

$$a(\omega) = \text{const} \ Z (\omega - \omega_{ab}, \Gamma_{ab}, D)$$

\(^1\)Heer and Graft, op. cit.
\[
\begin{align*}
    & b(\omega) = \text{const} \times \left[ \exp \left( (\omega_{ab} - \omega_0)^2 \right) \right] \times (2\Gamma_{ab})^{-1} \\
    & c(\omega) = \text{const} \times \left[ \exp \left( (\omega_{ab} - \omega_0)^2 \right) \right] \frac{i(2\omega_{ab} - \omega_A - \omega_B + 2\Gamma_{ab})}{D^2}^{-1}
\end{align*}
\]

For a steady state condition, i.e., \( \dot{A} = \dot{B} = 0 \). A solution for the output power would be of the form
\[
|A|^2 = |B|^2 = a'(\omega)/[b'(\omega) + c'(\omega)]
\]

\[\text{(6-3)}\]

\[\frac{e_0-1}{P} = \frac{1}{\mu^2 - 1} \text{E as } \mu = 1 + \frac{1}{2} \epsilon_0^{-1} P/E \text{ and from 2-18}
\]

\[
\begin{align*}
    & \mu_A = \frac{1}{2} (a'(\omega_A) - b'(\omega_A)) |A|^2 - c' |B|^2) \\
    & \mu_B = \frac{1}{2} (a'(\omega_B) - b'(\omega_B)) |B|^2 - c' |A|^2)
\end{align*}
\]

For a broad gaussian the \( a'(\omega) \) and \( b'(\omega) \) terms are essentially frequency independent in the region of \( \omega_{ab} \) so that the dispersive effect must arise from variations

\[\text{Lamb, op. cit.}\]
Fig. 8.--Typical One-isotope Power Curve
in the $c'$ term, i.e.,

$$\frac{du}{dv} = \frac{dc'}{dv}$$

\begin{equation}
= -2\left(2\omega_{ab} - \omega_A - \omega_B\right) 2\Gamma_{ab} \frac{\left(2\omega_{ab} - \omega_A - \omega_B\right)^2 + 4\Gamma_{ab}^2}{\left(2\omega_{ab} - \omega_A - \omega_B\right)^2}^2
\end{equation}

This term is plotted on an expanded scale in the vicinity of $\omega_{ab}$ in Figure 9. It can be noted that the active medium can contribute a dispersive term $vd\mu/dv$ as large as 0.3 to the denominator of equation 2-17:

$$\frac{\Delta v}{\nu_0} = 2c^{-1}\left\{2/\Omega \cdot dS\right\}\left\{\mu(\nu_0) + v \frac{d\mu}{dv} d\sigma\right\}^{-1}$$

with a resultant change in the beat frequency as large as ten percent (assuming the active medium fills one-third of the cavity). This is in good agreement with experimental observations. Measurement of the beat frequency with a KBR rod in the cavity indicated a shift $\sim 10\%$ larger than anticipated. Observations taken soon after with a thin lossy flat showed a shift in the beat frequency $\sim 9\%$ (see data, Figure 10). These observations are consistent with the theory proposed here.

Three methods are immediately suggested to eliminate this problem: (1) operate sufficiently away from the power dip; (2) operate at a fixed power level or (3) utilize a two isotope active medium to take advantage of the resultant broadened doppler curve.
Fig. 9.—Dispersion Near Line Center
Fig. 10. $\Delta \nu_{\text{beat}}$ vs $\Omega$ for KBr and Lossy Quartz
The first of these suggestions is unrealizable since the longitudinal mode spacing has been measured to be 112 MHz; one mode will normally be in the vicinity of line center. The second method was actually implemented with reasonably good success by utilizing an attenuation flat to control cavity losses. (See discussion Chapter VII.)

The third technique, i.e., using a two isotope active medium, is the one that warrants prime consideration. This method was put into effect with a 10:1:1 mixture of He:Ne\textsuperscript{20}:Ne\textsuperscript{22} producing exceedingly gratifying results. The dependence of the beat frequency on the power level is essentially eliminated (illustrated in Figure 7) with the added advantage that the residual bias was lessened (see Chapter VII). It is highly probable that the reduction in bias was primarily due to the utilization of a new discharge tube, but at this point, in addition to an analysis of the two isotope model, a discussion of the mechanism for bias beating will be included to find justification for the improvement.

**Two Isotope Model**

If two isotopes are present in a 1:1 ratio in the active medium the electric polarization can be calculated
by noting that
\[ P = N \text{tr} \rho \Phi + \sum_a N_a \text{tr} \rho_a \Phi \]
or
\[ P = N/2 \left( \text{tr} \rho_1 \Phi + \text{tr} \rho_2 \Phi \right) \]

Using the results of Heer and Graft\(^3\) this can be written as

\[
P(\omega_A) = -\frac{\epsilon_A}{2} \left[ a_1(\omega_A) + a_2(\omega_A) - (b_1(\omega_A) + b_2(\omega_A)) |A|^2 - (c_1(\omega_A, \omega_B) + c_2(\omega_A, \omega_B)) |B|^2 \right]
\]

with a similar expression for \(P(\omega_B)\) where the subscripts 1 and 2 correspond to the two isotopes.

Following the same pattern as the one isotope model with equations of motion for the amplitudes can be written from Maxwell's equation as:

\[
2iA = \omega_A \left\{ -g_A - iQ^{-1} + \frac{1}{2} \left( \frac{\omega_0 - \omega_A}{\omega_A} \right) + \frac{1}{2} \left[ a_1(\omega_A) + a_2(\omega_A) - (b_1(\omega_A) + b_2(\omega_A)) |A|^2 - (c_1(\omega_A, \omega_B) + c_2(\omega_A, \omega_B)) |B|^2 \right] \}
\]

and

\[
2iB = \omega_B \left\{ g_B + iQ^{-1} + \frac{1}{2} \left( \frac{\omega_0 - \omega_B}{\omega_B} \right) + \frac{1}{2} \left[ a_1^*(\omega_B) + a_2^*(\omega_B) - (b_1^*(\omega_B) + b_2^*(\omega_B)) |B|^2 - (c_1^*(\omega_A, \omega_B) + c_2^*(\omega_A, \omega_B)) |A|^2 \right] \}
\]

Under the steady state requirement that \(\dot{A} = \dot{B} = 0\) the power output can be shown to be of the form:

\[
|A|^2 = |B|^2 \left\{ e^{-\left(\omega_A b_1 - \omega_0\right)^2/D_1^2} + e^{-\left(\omega_A b_2 - \omega_0\right)^2/D_2^2} \right\} b_1 - b_2 + c_1 - c_2
\]

or

\[ (6-7) \]

\(^3\)Heer and Graft, op. cit.
\[ |A|^2 = |B|^2 \times \{ \Gamma_{ab} [\{ (2\omega_{ab1} - \omega_A - \omega_B)^2 + 4\Gamma_{ab}^2 \}] \times \{ \{ (2\omega_{ab1} - \omega_A - \omega_B)^2 + 4\Gamma_{ab}^2 \} \{ (2\omega_{ab2} - \omega_A - \omega_B)^2 + 4\Gamma_{ab}^2 \} + \Gamma_{ab}^2 \times \{ (2\omega_{ab1} - \omega_A - \omega_B)^2 + 4\Gamma_{ab}^2 \} + \{ (2\omega_{ab2} - \omega_A - \omega_B)^2 + 4\Gamma_{ab}^2 \} \}^{-1} \]

A typical power curve is given in Figure 11 for the case where \( |\omega_{ab1} - \omega_{ab2}| < D_1, D_2 \) and \( \Gamma_{ab1} = \Gamma_{ab2} \) (typical for case of Ne\(^{20}\) and Ne\(^{22}\)). It will immediately be noted that the region of highest power is near neither of the power dips at \( \omega_{ab1} \) and \( \omega_{ab2} \) but is a broad flat region between. In this region the cavity frequencies cannot be appreciably pulled with changing power.

Following the pattern of the previous section, the index of refraction can be shown to be

\[
\mu_A = \frac{1}{4} [a_1^\prime(\omega_A) + a_2^\prime(\omega_A) - (b_1^\prime(\omega_A) + b_2^\prime(\omega_A))] |A|^2
\]

\[ -(c_1^\prime(\omega_A, \omega_B) + c_2^\prime(\omega_A, \omega_B)) |B|^2 \]

Again, the dispersion can be due only to the \( c^\prime(\omega_A, \omega_B) \) terms.

\[
v \frac{d\nu}{dv} = [\frac{dc_1^\prime}{dv} + \frac{dc_2^\prime}{dv}] \]

In the region of interest \( c_1^\prime \) and \( c_2^\prime \) are slowly varying functions of \( \omega \) and the dispersive term \( v d\nu/dv = 10^{-3} \).

This term will produce negligible effect on the measurement of the beat frequency.
Fig. 11.—Typical Two-isotope Power Curve
Recently Hutchings et al and Lee and Atwood\(^4\) have observed experimentally that the beat frequency could be pulled by as much as ten percent in a single isotope mixture. In both experiments the cause was attributed as being due to a nonreciprocity in the index of refraction contributing to bias beating. In the case of the experiment carried out by Lee and Atwood the evidence is irrefutable, but, in the present observations, dispersion near \(\omega_{ab}\) is the culprit as will be obvious after the succeeding paragraph on bias beating.

**Bias Beating**

The phenomenon of bias beating results from non-reciprocal scatter of part of one of the traveling waves into the other.\(^5\) The net result for \(|A|^2 \neq |B|^2\) is that \(\mu_A \neq \mu_B\) producing a difference in optical path lengths in the two directions of propagation.

Using the results of the preceding sections the nonreciprocity or difference in \(\mu_A\) and \(\mu_B\) can be shown to be

\[
\delta \mu = \mu_A - \mu_B \approx 1/2 [c^-(\omega_A, \omega_B) - b^-] [|A|^2 - |B|^2].
\]


It is obvious that this term can be varied in the vicinity of $\omega_{ab}$ by the $c'(\omega_A,\omega_B)$ term producing an effective non-reciprocal dispersion. This is obviously the source for the measurements made by Lee and Atwood\textsuperscript{6} since the beat frequency could be adjusted in both directions about the unpulled value.

Despite the fact that the nonreciprocity is strongly power dependent near line center due to the rapid variation in the $c'$ term it should be noted that this is a very weak source of error in this experiment. If the nonreciprocity were sufficiently large the data points of the type displayed in Figure 7 for the CW and CCW senses of direction would either converge or diverge as the power level was varied. This would be due to the fact that $\Delta \omega$ bias would add to $\Delta \nu_{\text{beat}}$ in one sense of direction and subtract in the other. Such is not the case since the CW and CCW data points maintain a fairly uniform separation.

A small residual bias is present in the system as will be noted in Chapter VII but not of the magnitude as noted in previous experiments.\textsuperscript{7} This is probably due to

\textsuperscript{6}Lee and Atwood, \textit{op. cit.}

\textsuperscript{7}See for example Hutchings et al, \textit{op. cit.} and Lee and Atwood, \textit{op. cit.}
the fact that all optical components were laboriously cleansed prior to each data run, maintaining a very low level of backscatter.

For the two isotope case, the nonreciprocity in the index of refraction (using equation 6-8) can be shown to be

$$\delta \nu = \frac{1}{4} [c_1' (\omega_A, \omega_B) + c_2' (\omega_A, \omega_B) - b_1' - b_2'] [|A|^2 - |B|^2]$$

and from the resonant condition for the cavity

$$\Delta \omega_{BIAS} = \frac{\omega_0}{4} [c_1' + c_2' - b_1' - b_2'] [ |A|^2 - |B|^2 ]$$

In the region of operation, i.e., several hundred megahertz from either atomic line, the $c'$ terms are several orders of magnitude smaller than the $b'$ terms so that we can write

$$\Delta \omega_{BIAS} = \frac{\omega_0}{4} [b_1' + b_2'] [ |A|^2 - |B|^2 ]$$

It is obvious here that bias cannot be appreciably decreased using a two isotope mixture but its power dependence can be almost entirely removed.
CHAPTER VII

EXPERIMENTAL RESULTS AND ANALYSIS

The analysis of the experimental results presented in the succeeding sections can be more easily performed if equation 2-17

\[
\frac{\Delta \nu}{\nu_0} = \frac{2c^{-1} \left( 2\mathbb{S} \cdot d\mathbb{S} + \int \left[ (\mu^2 - 1) + \mu v d\mu/d\nu \right] y \cdot nd\sigma \right)}{\int \left[ \mu (\nu_0) + \nu_0 d\mu/d\nu \right] d\sigma}
\]

is reduced to a less formidable form using the specific geometry and conditions maintained throughout the experimentation.

Air currents have been minimized during the actual observations so that the motion of the air in the cavity is due primarily to rotation of the table (i.e., \( \mathbf{v} = \hat{\mathbf{S}} \times \mathbf{r} \)). The quantities \( \mu^2 - 1 \) and \( \mu v d\mu/d\nu \) are of the order of \( 3 \times 10^{-4} \) or less consequently the integral over \( d\sigma \) in the numerator may be neglected. Under this condition, equation 2-17 for the specific cavity geometry of an equilateral triangle of side length \( D \) may now be written as:

\[
\frac{\Delta \nu}{\nu} = \frac{\sqrt{3} D \omega}{3c} \left( \int [\mu (\nu_0) + \nu_0 d\mu/d\nu] d\sigma \right)
\]  

(7-1)

The actual data was taken not by observing the beat frequency but by recording the total count through
a fixed angle at a nearly uniform rotation rate. This total count for an empty cavity (i.e., \( \nu = 1.00 \)) is given by

\[
N_0 = \int_0^T \Delta v dt = \frac{\sqrt{3D}}{3\lambda}
\]  

(7-2)

where \( \theta = \theta_{\text{lab}} + \Omega_{\text{earth}} T \cos \gamma \).

This allows us to rewrite \( N_0 \) as

\[
N_0 = \frac{\sqrt{3D}}{3\lambda} \theta_{\text{lab}} + \Delta \nu_{\text{earth}} T
\]

(7-3)

where \( \theta_{\text{lab}} \) is the fixed angle as measured in the laboratory frame, \( \Delta \nu_{\text{earth}} \) is the contribution to the beat frequency due to the component of Earth's rotation normal to the platform and \( T \) is the actual period of rotation.

The modification on \( \theta \) is necessary due to the fact that the system indicates \( \Delta \nu \) with respect to the fixed stars and not with respect to the laboratory frame. The (+) and (-) signs indicate table rotations with and counter to Earth's rotation respectively. For equal times of rotation in the clockwise and counterclockwise directions, the average count experimentally should be identical to \( N \) calculated using the value of \( \theta \) measured in the laboratory. For \( D = .875 \) meters, \( \lambda = 1.153 \) microns, and \( \theta_{\text{lab}} = 1.562 \) radians, the calculated value for \( N_{\text{ave}} \) is

\[
N_{\text{ave}} = \frac{\sqrt{3}}{3} \frac{D}{\lambda} \theta_{\text{lab}} = 684,000 \text{ counts}
\]

(7-4)
This value is reduced slightly to 683,000 if the thickness \( (t = 3 \times 10^{-3} \text{ meters}) \) of the Brewster's angle windows on the discharge tube is taken into account.

As will be noted in succeeding sections, this value is almost exactly realized experimentally in the case where an isotopic mixture of Neon is used in the active medium.

Equations 7-3 and 7-4 clearly illustrate why this method of accumulating data is far superior to recording beat frequencies. In measuring \( N \), small non-uniformities in rotation rate during a small incremental part \( \delta t \) of the total period \( T \) will contribute negligibly only to the \( \Delta v_{\text{earth}} T \) term and not to \( N_{\text{ave}} \). Whereas if \( \Delta v_{\text{beat}} \) were measured during one of these "jerks" an entirely erroneous value would be recorded.

Introduction of Transparent Media

When transparent media is introduced into the cavity it is necessary to integrate the denominator of equation 7-1 along the actual path. For the specific case of having \( m \) windows of thickness, \( t = .0126 \text{ meters} \) positioned at Brewster's angle, the expression for \( N_{\text{ave}} \) can be written in terms of the original path and the new optical path as:

\[
N_{\text{ave}} = 683,000 \left[ 1 + \sum_{\text{m}} \frac{.0126(\mu' - \cos \theta)}{3D \cos \theta} \right]^{-1}.
\]
The change in optical path length per window is $0.0126 \left( \frac{\mu' - \cos \theta}{\cos \beta} \right)$, where $\mu'$ includes both the index of refraction and the dispersive term. The angles $\theta$ and $\beta$ for placement at Brewster's angle are illustrated in Figure 12.

For fused silica, the index of refraction and dispersion are taken to be $\mu = 1.449$ and $\nu = 0.0141$ for which values $N_{ave}$ can be calculated as

$$N_{ave} = 683,000 \left[ 1 + 0.2443 \mu \right]^{-1} \quad (7-6)$$

**Experimental Results**

The experimental results that conform most nearly to the theory are those that were recorded under conditions of minimal dispersion in the active medium. The two techniques utilized to minimize or eliminate this dispersive effect (see Chapter VI) were: (1) maintaining constant cavity losses by utilizing an attenuating flat and (2) taking advantage of the isotopic broadening presented by a 10:1:1, He-Ne$^{20}$Ne$^{22}$ active medium. Representative data accumulated with these two methods will be presented and analyzed in detail in the following paragraphs.

---

Fig. 12.—Placement of Quartz Flats
(Diagram of Optical Path)
Attenuation method. In this method a constant cavity Q was maintained by utilizing a borosilicate flat as an attenuator in the cavity. As each additional pair of fused silica windows was positioned in the cavity, the attenuator was rotated slightly ($10^{-4}$ radians) to maintain constant cavity losses. The result was that the fractional shift in the beat frequency was essentially unaffected by the active medium. The data accumulated by this technique (see Figure 13) is not entirely satisfactory in that the empty cavity value of $N_{\text{ave}}$ falls slightly lower than that predicted by equation 7-4. The experimental value of 678,000 falls approximately 3,000 counts lower than the value recalculated to take into account the 1.27 cm thickness of the attenuating flat. This difference can be accounted for if we note that the dc level of oscillation for this data was 25 mv. The information presented in Chapter VI (Figure 7) indicates that we were probably experiencing a constant, although reduced, shift due to the dispersion in the active medium.

Despite this slight shortcoming in the data, one encouraging note is sounded by the fact that the slope of the data curve is very nearly that predicted by equation 7-6. If the empty cavity value of 678,000 is used, equation 7-6 predicts a count of 653,000 for twelve (1.26 cm) fused silica flats. The experimental
Fig. 13.—N vs Number of Quartz flats
(Attenuation Method)
value of 652,500 falls well within the range of the experimental scatter.

Isotopic broadening.—Because of the separation in the Ne\textsuperscript{20} and Ne\textsuperscript{22} lines due to the difference in isotopic mass, the Doppler width can be broadened from 800 MHz to 1500 MHz by utilizing a 50-50 isotopic active medium. The resultant Doppler curve (see Figure 11) has a broad low dispersion region of approximately 700 MHz width near line center. Using a 10:1:1 mixture of He-Ne\textsuperscript{20}-Ne\textsuperscript{22} the dispersion term is reduced by a factor of 100 and the necessity of maintaining a constant cavity Q is eliminated. Using such an isotopic mixture, the data (see Figure 14) conforms extremely well with the theory. With twelve windows of thickness 1.26 cm in the cavity, equation 7-6 predicts a shift in \( N_{\text{ave}} \) from 683,000 to 659,000 counts. A value of 658,000 counts is observed experimentally. The difference of 1000 counts is of the same order as the experimental scatter.

Residual bias.—The phenomenon of bias beating has been considered previously in Chapter VI and has been attributed to the mechanism of non-reciprocal back-scatter. Experimentation with a feedback mirror external to the cavity indicates that this is indeed the case. Changing the intensity of either the CW or CCW signals by as little as 10\textsuperscript{-7} is sufficient to
Fig. 14.—N vs Number of Quartz Flats
(Isotopic Broadening)
produce bias beats with frequency in the kilocycle range. Because of this, all surfaces in the cavity must be absolutely clean to minimize backscatter.

The effect of even a small residual bias can be easily observed in both Figures 13 and 14. For example in Figure 13 where \( T_{CW} = 121 \) seconds and \( T_{CCW} = 127 \) seconds, equation 7-3 predicts (for \( \Delta v_{\text{earth}} = 20.5 \) cps) a separation between the clockwise and the counterclockwise data points of 5100 counts. A separation of some 14,000 counts is observed suggesting a constant residual bias of \( \approx 8900 \) counts or \( \Delta v_{\text{bias}} = 36 \) cps.

The data accumulated with the isotopic mixture illustrates a reduced residual bias over these previous investigations. For example, looking at Figure 14, where \( T_{CW} = 136 \) seconds and \( T_{CCW} = 148 \) seconds, a separation between the clockwise and counterclockwise points of 5800 counts is predicted. The observed separation of \( \approx 10,000 \) suggests that the residual bias contributes \( \approx 4,000 \) counts or \( \Delta v_{\text{bias}} = 15 \) cps. Since the analysis of Chapter VI suggests that the residual bias should be substantially the same for both cases, it is probable that this reduction in the residual bias is the result only of the utilization of a new, clean, discharge tube.
Conclusions

The following conclusions can be drawn from this research:

1. The excellent agreement between the experimental data and equation 2-17 indicates that, at least to first order in $\Omega$, a covariant formalism is adequate for analyzing problems in an accelerated frame of reference.

2. Since equations 2-6 and 2-17 are consistent for a plane wave expansion of a beam of finite cross section, it can be concluded that the frequency separation is proportional to the moment of the energy flux and, only in the case of vacuum, proportional to the moment of the angular momentum. This is in agreement with Minkowski's formalism in that transparent media do not even locally exchange energy with the electromagnetic field.

3. Isotopic broadening of the Doppler line is a convenient method to minimize the dispersive nature of the active medium. Some reduction in residual bias was noted while using an isotopic mixture, but this could easily be due to the fact that a new, clean discharge tube was employed.

4. Cavity tubing of the proper I.D. can be utilized to reduce losses, enhancing the maser dc power level.
5. A mechanism such as non-reciprocal backscatter that tends to make $|A|^2$ and $|B|^2$ differ by as little as $10^{-7}$ is sufficient to produce bias beats in the kilocycle range.

6. Some residual bias is inherent in the system due to the difficulty in maintaining a completely dust-free configuration.

7. Accumulating data by recording the total count in both directions is far preferable to observing frequencies in that the effects of bias and Earth's rotation are minimized.

8. A triangular cavity is considerably easier to align than a square cavity due primarily to the property of rays being essentially self-closing in the horizontal plane.
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†Pages 11-16 are based on a set of unpublished notes by C. V. Heer.