This dissertation has been microfilmed exactly as received 67-16,285

HANLON, James Towers, 1938-
NOISE INDUCED GAIN SATURATION IN THE ULTRA-
SONIC TRAVELING WAVE AMPLIFIER.

The Ohio State University, Ph.D., 1967
Engineering, electrical

University Microfilms, Inc., Ann Arbor, Michigan
NOISE INDUCED GAIN SATURATION IN THE
ULTRASONIC TRAVELING WAVE AMPLIFIER

DISSERTATION
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
James Towers Hanlon, B.E.E., M.S.

The Ohio State University
1967

Approved by

E.M. Boone
Adviser
Department of Electrical Engineering
ACKNOWLEDGEMENTS

This project was begun at Bell Telephone Laboratories and completed at The Ohio State University under contract with the Laboratories. Without the consultation of Dr. D. L. White, co-discoverer of the ultrasonic amplifier, and the encouragement of two able supervisors, Mr. John Rowen and Dr. J. E. May, Jr., this work would not have been supported or completed. It is a pleasure to acknowledge their assistance.

The author is also indebted to many of his Bell Laboratories colleagues whose work has complemented this effort. The patience and skill of Mr. S. S. Bearder and Mr. C. Schmidt are evident in the mechanical perfection of the experimental units. Dr. N. F. Foster has contributed his efficient uhf transducers. Mrs. E. T. Handleman's efforts prefected the cadmium sulfide material used. Dr. F. G. Eggers' careful lessons in measurement technique have also left their mark on this work.

The author is also deeply indebted to his adviser, Prof. E. M. Boone. Without Prof. Boone's continued friendship and assistance, he would not have been able to return to graduate work and to complete either the doctorate studies or this paper.

Most importantly, the author must thank his own wife, Kathleen, and his children who have sacrificed material benefits and have put up with a busy, preoccupied husband and father. Without them, it would not have been worth the effort.
VITA

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 10, 1938</td>
<td>Born - Cincinnati, Ohio</td>
</tr>
<tr>
<td>1961</td>
<td>B.E.E., The Ohio State University, Columbus, Ohio</td>
</tr>
<tr>
<td>1962</td>
<td>M.S., The Ohio State University, Columbus, Ohio</td>
</tr>
<tr>
<td>1962-1965</td>
<td>Member of Technical Staff, Bell Telephone Laboratories, Murray Hill, New Jersey</td>
</tr>
<tr>
<td>1965-1967</td>
<td>Instructor, Department of Electrical Engineering, The Ohio State University, Columbus, Ohio</td>
</tr>
</tbody>
</table>

PUBLICATIONS


FIELDS OF STUDY

Major Field: Electrical Engineering

Studies in Applied Mathematics,
Professor H. D. Colson

Studies in Solid State Electron Devices,
Professor M. O. Thurston

Studies in Vacuum Electron Devices,
Professor D. T. Davis

Studies in Circuit Theory of Electron Devices,
Professor E. M. Boone

Studies in Quantum Mechanics,
Professor R. L. Mills
Professor L. C. Brown
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>VITA</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>viii</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. SMALL SIGNAL THEORY</td>
<td>4</td>
</tr>
<tr>
<td>Propagation Equations</td>
<td></td>
</tr>
<tr>
<td>Gain, Loss, and Bunching</td>
<td></td>
</tr>
<tr>
<td>III. DEPARTURES FROM SMALL SIGNAL THEORY</td>
<td>16</td>
</tr>
<tr>
<td>Current State-of-the-Art</td>
<td></td>
</tr>
<tr>
<td>Experimentally Observed Gain Saturation</td>
<td></td>
</tr>
<tr>
<td>IV. NOISE SATURATION</td>
<td>24</td>
</tr>
<tr>
<td>Evidence Supporting Noise Saturation</td>
<td></td>
</tr>
<tr>
<td>Experimental Evaluation of Acoustic Flux</td>
<td></td>
</tr>
<tr>
<td>Saturation Level</td>
<td></td>
</tr>
<tr>
<td>Saturation Level Dependence on Carrier</td>
<td></td>
</tr>
<tr>
<td>Concentration</td>
<td></td>
</tr>
<tr>
<td>A Physical Interpretation of Noise Saturation</td>
<td></td>
</tr>
<tr>
<td>Signal Saturation in the Presence of</td>
<td></td>
</tr>
<tr>
<td>Noise Flux</td>
<td></td>
</tr>
<tr>
<td>V. DEVICE IMPLICATIONS</td>
<td>54</td>
</tr>
<tr>
<td>Significance of Noise and Signal Saturation</td>
<td></td>
</tr>
<tr>
<td>Power Levels to the Design of a Device</td>
<td></td>
</tr>
<tr>
<td>VI. CONCLUSIONS</td>
<td>62</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>63</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Small Signal Prediction of Frequency Dependent Shear Mode Ultrasonic Gain</td>
<td>12</td>
</tr>
<tr>
<td>2.</td>
<td>Small Signal Prediction of Voltage Dependent Shear Mode Ultrasonic Gain and Observed Data Points for Sample #280-2 at 220 MHz</td>
<td>14</td>
</tr>
<tr>
<td>3.</td>
<td>Small Signal Prediction of Voltage Dependent Shear Mode Ultrasonic Gain and Observed Data Points for Sample #280-2 at 350 MHz</td>
<td>20</td>
</tr>
<tr>
<td>4.</td>
<td>Small Signal Prediction of Shear Mode Gain and Measured Points for Sample #280-2 at 550 MHz. Observed Gain Saturates for This Operating Condition</td>
<td>21</td>
</tr>
<tr>
<td>5.</td>
<td>Small Signal Prediction of Shear Mode Gain and Measured Points for Sample #280-2 at 550 MHz. Observed Gain Saturates Well Below the Theoretical Maximum for This Operating Condition</td>
<td>22</td>
</tr>
<tr>
<td>6.</td>
<td>550 MHz Signal Output, Noise Output, and Current-Voltage Characteristics for Sample #280-2. Drift Potential is 140 volts and Loop Gain is Less than 1.0.</td>
<td>25</td>
</tr>
<tr>
<td>7.</td>
<td>550 MHz Signal Output, Noise Output, and Current-Voltage Characteristics for Sample #280-2. Drift Potential is 150 volts, Loop Gain Exceeds 1.0, and Amplifier Saturates After Several Transits</td>
<td>25</td>
</tr>
<tr>
<td>8.</td>
<td>550 MHz Signal Output, Noise Output, and Current-Voltage Characteristics for Sample #280-2. Drift Potential is 200 volts and Amplifier Saturates During First Transit</td>
<td>25</td>
</tr>
<tr>
<td>9.</td>
<td>A Sketch of the Signal Portion of Figure 7 Showing Pertinent Features of Figures 6, 7, and 8</td>
<td>26</td>
</tr>
<tr>
<td>10.</td>
<td>A Sketch of the Experimental Ultrasonic Amplifier Showing Transducers, Buffer Rods, and Drift Field Connections</td>
<td>32</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>11. Small Signal Gain and Measured Noise Figure for Sample #280-2 as a Function of Frequency</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>12. Measured Noise Saturation Power as a Function of Carrier Concentration</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>13. An Idealized Ultrasonic Amplifier Showing Gain Angle and Transducer Angle and the Noise Power-Gain Product Incident upon the Output Transducer</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>14. Experimentally Observed Signal Power Saturation at 480 MHz</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>15. Experimentally Observed Signal Saturation and Bunching Parameter, 140 volt Potential</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>16. Experimentally Observed Signal Saturation and Bunching Parameter, 150 volt Potential</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>17. Experimentally Observed Signal Saturation and Bunching Parameter, 160 volt Potential</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>18. Experimentally Observed Signal Saturation and Bunching Parameter, 165 volt Potential</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>19. Experimentally Observed Signal Saturation and Bunching Parameter, 170 volt Potential</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>20. Experimentally Observed 480 MHz Fundamental and 960 MHz Second Harmonic Outputs for the Operating Conditions of Figure 17</td>
<td>54</td>
<td></td>
</tr>
<tr>
<td>21. The Variation of Output Plane Noise Power as a Function of Drift Potential and the Corresponding Small Signal Gain Variation Combine as Shown to Delineate the Dynamic Range for Sample #280-2 at 740 MHz</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>22. Theoretical Ultrasonic Gain and Dynamic Range for the Operating Condition of Figure 21</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>23. Maximum Ultrasonic Gain as a Function of Frequency for a Noiseless Shear Mode Amplifier Terminated in 290⁰K</td>
<td>61</td>
<td></td>
</tr>
<tr>
<td>Table</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>1. Amount of Saturation at Unity Bunching for the Drift Potentials of Figures 15 through 19</td>
<td>52</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

To the average engineer, the word "ultrasonics" is perhaps associated with a twenty kilocycle cleaning bath or a vibrating soldering iron or welding tool in his laboratory. But though he may not realize it, the uses and the spectrum of ultrasonic energy extend far beyond these familiar items. Ultrasonic delay lines are made which operate at frequencies up to or beyond a hundred megacycles whose dispersion characteristics can be tailored to virtually any requirement (1). High frequency ultrasonic waves are finding increasing use in broad-banded light modulators and in light deflection systems (2). Ultrasonic pulse-echo techniques have great utility in nondestructive material testing (3), with ultrasonic holography offering the possibility of a three dimensional picture of the interior of solids. Many basic physical phenomena involving phonon interactions can be observed or stimulated by ultrasonic techniques (4).

The traveling wave ultrasonic amplifier (5) which is the subject of this investigation may be a valuable component in many higher frequency ultrasonic systems. Its basic mechanism is rather simply understood. In a piezoelectric material, a traveling ultrasonic wave is accompanied by a traveling,
space-alternating, electric field. If the material is in addition a semiconductor, this electric field will produce a coupling and an energy interchange between the ultrasonic wave and any mobile carriers. In particular, those carriers which are drifting with a velocity close to the ultrasonic wave velocity will experience a strong, cumulative interaction with the wave, extracting momentum from it and attenuating it if they are slower and delivering momentum to it and amplifying it if they are faster.

This basic mechanism is potentially useful, as it can produce the ultrasonic analog of many transmission devices common to the microwave art. Its most obvious utility is as an amplifier, where it is capable of producing nearly 80 decibels of signal gain. With an appropriate feedback path, not at all a problem in ultrasonic media where end reflections are often bothersome, it will oscillate in a stable manner with milliwatts of output. Because sonic velocity in the coupled ultrasonic-electron system is a function of electron velocity, it can be used as a variable phase shifter. Appropriate material orientation so that only specific ultrasonic components are piezoelectrically coupled to the electrons will allow it to be used as an effective mode filter. Its non reciprocal forward gain and reverse loss make it ideal as an ultrasonic isolator.

Such possibilities have been recognized for several years, and a thorough engineering evaluation of the phenomenon used as an ultrasonic signal amplifier has been carried out from a
theoretical basis (6). The work to be described here was undertaken to fill in several unknown quantities in this engineering evaluation which could not be obtained from purely theoretical considerations as they are understood at this time.

Chapter 2 attempts to provide a brief review of the basic, small signal theory for the reader who is unfamiliar with the area. Chapter 3 discusses experimentally observed departures from small signal gain predictions, and Chapter 4 presents evidence linking these departures to high level noise saturation. The measured ultrasonic noise figure is a highly useful byproduct of this portion of the study. Chapter 4 also describes the rudiments of a physical theory describing the generation of noise within the amplifier, and in addition presents the experimentally observed signal saturation characteristics in the presence of high noise flux levels. Chapter 5 discusses the significance of the noise and signal saturation power limits to the design of a device.
CHAPTER II
SMALL SIGNAL THEORY

The analysis reported here is essentially that proposed by White (5). It is the first of a number of such analyses (7-9) all of which predict essentially the same small signal behavior for the frequencies of interest, and it is at the same time the most straightforward of the lot. Because of the materials in which ultrasonic amplification is actually observed, the analysis will deal only with plane waves in an n-type semiconductor. Rationalized MKS units will be presumed throughout.

Propagation Equations

Consider an acoustic wave propagating in the x-direction of a piezoelectric semiconducting medium. Define a strain, \( S \), a stress, \( T \), and a displacement, \( u \), such that:

\[
S = \frac{\partial u}{\partial x} \quad \text{and} \quad \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \quad (1a, b)
\]

where \( \rho \) is the mass density of the medium. Further, assume that the medium is characterized by a piezoelectric constant, \( e \), whose units are coulombs per square meter, such that \( S \) produces an electric field in the x-direction. Then the equations of state corresponding to this one-dimensional problem are:
where $E$ is the electric field, $D$ the electric displacement, $c$ the elastic constant at constant electric field, and $\varepsilon$ the dielectric permittivity at constant strain. Normally, the constant coefficients of equation (2) are tensors; but the case in question is one in which there is only a single, shear mode ultrasonic wave traveling in the x-direction coupled to an electric field in the x-direction. In such a case, the constants become scalars.

Using equation (2a) for the stress, the wave equation in an elastic medium becomes:

$$\varrho \frac{\partial^2 u}{\partial t^2} = \frac{\partial T}{\partial x} = c \frac{\partial^2 u}{\partial x^2} - \varepsilon \frac{\partial E}{\partial x}.$$  (3)

With a zero piezoelectric constant, this equation degenerates to a simple elastic wave equation. The presence of an electric field in a piezoelectric medium modifies the propagation, and in the solution of the wave equation $E$ must be found in terms of the material displacement, $u$.

Two additional relations which are helpful in this development are Poisson's equation and the continuity equation. Poisson's equation in the present, one-dimensional case is most conveniently expressed as:

$$\frac{\partial D}{\partial x} = Q$$  (4)

where $Q$ is space charge density, and electric displacement is specified by equation (2b). The equation of continuity is:
where \( J \) is the electrical current density.

The space charge density may in turn be written as:

\[
Q = -q n_s \tag{6}
\]

where \( q \) is the magnitude of the electron charge and \( n_s \) is electron density required to produce charge density \( Q \).

When the number of electron collisions per second is large compared to the frequency of the wave motion, the current in an n-type semiconductor is described by the familiar relation:

\[
J = q \mu n_c E + q D_n \frac{\partial n_c}{\partial x} \tag{7}
\]

where the first term above is due to drift and the second to diffusion. In this equation, \( \mu \) is the electron mobility, \( n_c \) the conduction band electron density, and \( D_n \) the electron diffusion coefficient.

As discussed in reference 5, the density of conduction band electrons will be written as:

\[
n_c = n_o + f n_s \tag{8}
\]

where \( n_o \) is the equilibrium carrier concentration, not a function of time or position, and where the deviation from equilibrium space charge density in the presence of an ultrasonic wave is \( q n_s \). As a portion of this space charge may be held in traps, the fraction \( f q n_s \) appears in the conduction band. With no trapping,
all the space charge is due to conduction electrons and \( f = 1 \).

With trapping whose relaxation time is of the order of an ultrasonic period, \( f \) becomes complex to describe the phase shift introduced by such trapping between the \( n_s \) population and the ultrasonic wave (6).

Equations (4) through (8) are combined to eliminate \( J, n_s, \) and \( n_c \). The resulting expression in the electric displacement \( D \) is:

\[
- \frac{\partial^2 D}{\partial x \partial t} = \mu \frac{\partial}{\partial x} \left[ \left( q n_o - f \frac{\partial D}{\partial x} \right) E \right] - f D n \frac{\partial^3 D}{\partial x^3}.
\]  

(9)

In a small signal case, the electric field may be written as:

\[
E = E_0 + E_1 e^{j(kx - \omega t)}
\]  

(10)

where \( E_0 \) is the "drift field" due to an applied dc voltage and \( E_1 \) is the phasor representation of the ac field accompanying an ultrasonic strain wave traveling in the medium with propagation constant \( k \). This propagation constant is generally a complex quantity:

\[
k = \frac{\omega}{v_s} + j \alpha
\]  

(11)

where \( \alpha \) is the attenuation constant in nepers/m, \( \omega \) is the radian frequency of the field, and \( v_s \) is the phase velocity of the ultrasonic wave.

The material displacement accompanying the ultrasonic wave can be represented in like manner as:

\[
u(x,t) = u_1 e^{j(kx - \omega t)}.
\]  

(12)
Equations (1a) and (2b) are then used to eliminate D in equation (9). If the second order terms are omitted, that is the dc and harmonic terms which result when $e^{j(kx - \omega t)}$ is squared, $E_1$ then becomes:

$$E_1 = \frac{-jkeu_1}{\epsilon} \frac{1}{1 + \frac{j\sigma}{\epsilon \omega} \left[ 1 + f \mu \frac{k}{\omega} E_0 + j D_n \omega f \left( \frac{|k|}{\omega} \right)^2 \right]^{-1}}$$  \hspace{1cm} (13)$$

where $\sigma = \mu q n_0$, the material conductivity.

The next step is to use this expression for $E_1$ in the wave equation (3) to solve for the propagation constant, $k$. Inserting $E$ from (10) and $u$ from (12) into (3) and cancelling the common $e^{j(kx - \omega t)}$ term, there results:

$$-\omega^2 \rho u = -c k^2 u - jekE_1.$$  \hspace{1cm} (14)$$

If it is desired to demonstrate the modification of the elastic constant due to the piezoelectric coupling, (14) above should be written as:

$$-\omega^2 \rho u = -c' k^2 u$$  \hspace{1cm} (15)$$

where $c'$, the modified elastic constant, is then seen from (14) using (13) for $E_1$ to be:

$$c' = c \left[ 1 + \frac{e^2}{\epsilon c} \frac{1}{1 + \frac{j\sigma}{\epsilon \omega} \left[ 1 + f \mu \frac{k}{\omega} E_0 + j D_n \omega f \left( \frac{|k|}{\omega} \right)^2 \right]^{-1}} \right].$$  \hspace{1cm} (16)$$

The solution for the propagation constant $k = jk + \omega/v_s$ in equation (15) is a fourth order equation with complex coefficients. The expression may be greatly simplified, however,
since $|k| \ll \omega/v_s$, by setting $k = \omega/v_s$ in (16) for $c'$. It is also customary to use the dielectric relaxation frequency, $\omega_c = \sigma/\epsilon$, a quantity which White has christened the "diffusion frequency," $\omega_D = v_s^2/f_D n$, and to define a coefficient $\gamma = 1 + (f\mu e/\epsilon v_s)$. This $\gamma$ is a measure of the ratio of the electron drift velocity to the velocity of sound. In the case of no trapping where $f = 1$, $\gamma$ is a positive quantity when the electrons are slower than the sound wave and a negative quantity when they are faster. The quantity $e^2/\epsilon c$ is also represented by $K^2$.

Using this notation and these approximations, the complex elastic coefficient, the ac field, the ac carrier concentration, and the ac current density are found. It is seen directly from equation (16) that:

$$c' = c \left[ 1 + K^2 \left( \frac{1}{1 + j \frac{\omega_c}{\omega} \left( \frac{1}{\gamma + j \frac{\omega}{\omega_D}} \right)^{-1}} \right) \right].$$

(17)

In a similar manner, from (13) and using $S = \frac{3u}{\partial x} = j k u_1 e^{j(kx-\omega t)}$,

$$S_1 e^{j(kx-\omega t)};$$

$$E_1 = -\frac{e}{\epsilon} \frac{\gamma + j \frac{\omega}{\omega_D}}{\gamma + j \frac{\omega_c}{\omega} + \frac{\omega}{\omega_D}} S_1.$$  (18)

From (2b), (4), and (6), and using $S$ as immediately above and $E$ from (10), the carrier concentration is found to be:

$$n_s = e \frac{\omega_b}{q v_s} \frac{S_1}{\gamma + j \left( \frac{\omega_c}{\omega} + \frac{\omega}{\omega_D} \right)} = -\frac{\sigma}{q v_s} \frac{E_1}{\gamma + j \frac{\omega}{\omega_D}}.$$  (19)
where $n_s$ here is also presumed to be the complex amplitude of the term which propagates as $e^{i(kx - \omega t)}$. From (7), (8), and (10), and using $J = J_0 + J_1 e^{i(kx - \omega t)}$, the current density is:

$$J_1 = \frac{\sigma E_1}{\nu \gamma + j \frac{\omega}{\omega_D}} = -\frac{\sigma e}{\gamma + j \left(\frac{\omega_c}{\omega} + \frac{\omega}{\omega_D}\right)} S_1.$$  \hspace{1cm} (20)

Using the complex elastic coefficient from (17) in (15) and neglecting small terms commensurate with $K^2 \ll 1$ and $\omega \ll \omega/v_s$, approximate solutions for $\omega$ and $v_s$ are obtained from the imaginary and real portions of the equation respectively.

$$\omega = K^2 \frac{\omega_c}{v_s} \frac{1}{\nu \gamma + \frac{\omega_c}{\omega_D} \left(1 + \frac{\omega_D}{\omega_c \omega}\right)^2}$$  \hspace{1cm} (21)

$$v_s = \left(\frac{\omega}{\nu}\right)^\gamma \left[1 + \frac{K^2}{2} \frac{\omega_c}{\omega_D} \frac{1 + \frac{\omega_D}{\omega_c \omega} + \frac{\omega^2}{\omega_D \gamma^2}}{\nu \gamma + \frac{\omega_c}{\omega_D} \left(1 + \frac{\omega^2}{\omega_c \omega_D}\right)^2} \right]$$  \hspace{1cm} (22)

When $\gamma$ is negative, $\omega$ is negative and there is gain within the medium. When there is no trapping, $f = 1$, a negative $\gamma$ occurs when $E_0$ exceeds $-v_s/\mu$ or when the electrons, which are of course negatively charged carriers, drift in the same direction and at a somewhat greater velocity than the ultrasonic wave.
Gain, Loss, and Bunching

For an amplifier of length \( L \), the amplitude of a traveling ultrasonic strain wave is changed by the factor \( e^{-\omega L} \). Since power is proportional to the square of strain, the power gain of this amplifier is then:

\[
G = e^{-2\omega L} .
\]  

(23)

As can be seen from expression (21) for \( \chi \), the gain is a function of frequency through \( \omega \) and applied drift field through \( \gamma \).

The frequency for which \( G \) and also \( \alpha \) are maximum is independent of \( \gamma \) and is found from (20) to be:

\[
\omega_{\text{max}} = \sqrt{\omega_c \omega_D} .
\]  

(24)

In practice, since \( \omega_D \) is virtually constant, and since \( \omega_c = \sigma/\epsilon \) can be adjusted by chemical doping or, as in the case of photo-sensitive cadmium sulfide, by external illumination, this equation is used to find the proper conductivity for the desired frequency of maximum gain.

At a given frequency, the value of \( \gamma \) which makes the gain maximum is found in turn to be:

\[
\gamma_{\text{max}} = \pm \left( \frac{\omega_c}{\omega} + \frac{\omega}{\omega_D} \right) .
\]  

(25)

The positive value corresponds to maximum loss, the negative value to maximum gain.

Figure 1 shows a typical plot of ultrasonic gain predicted
Figure 1. Small Signal Prediction of Frequency Dependent Shear Mode Ultrasonic Gain for Sample #280-2, \( L = 1.65 \text{ mm} \), \( \mu = 272 \text{ cm}^2/\text{v sec}, \nu_a = 1.78 \times 10^5 \text{ cm/sec}, \omega_c = 0.4846 \times 10^9 \text{ sec}^{-1}, f_{\text{max}} = 239 \text{ MHz}, \gamma_{\text{max}} = -0.644, \omega_D = 0.4658 \times 10^{10} \text{ sec}^{-1} \).
by (23) and (21) with frequency variable for several values of $\gamma$. Note that the gain mechanism is quite broad-banded and thus potentially interesting for device purposes.

Figure 2 shows a plot of gain with drift potential the variable at a frequency near $\omega_{\text{max}}$ given by (24). The gain is zero at $\gamma = 0$, the "crossover point." The $\Theta$ marks are observed data points and are discussed in detail in a later section.

As Figure 2 demonstrates, this amplifier is a non reciprocal device. The gain for a wave traveling in the forward direction is not generally the same as the loss a similar wave experiences traveling in the reverse direction. Thus, depending on the chosen operating parameters, this device may operate as an unconditionally stable amplifier, as an isolator with near zero forward gain and high reverse loss, or even as an ultrasonic oscillator.

Besides the gain equation, an additional relation from the small signal theory significant to the current investigation is the ratio of the alternating component of carrier concentration, $n_a$ from (19), to the equilibrium concentration, $n_o$. Noting that $e\omega_c/qv_s$ in (19) can be written $e\mu n_o/\epsilon v_s$, this "bunching parameter" becomes:

$$\frac{n_a}{n_o} = \frac{e\mu}{\epsilon v_s} \frac{S_1}{\gamma + j \left( \frac{\omega_c}{\omega} + \frac{\omega}{\omega_d} \right)}.$$
Figure 2. Small Signal Theory Prediction of Shear Mode Ultrasonic Gain and Measured Data Points for Sample #280-2 at 220 MHz. Resistivity is 2585 ohm cm, Mobility is 272 cm²/volt sec. Length, Sonic Velocity, and \( \omega_0 \) for This and Subsequent Figures are as Noted on Figure 1.
Obviously, there will be an upper bound on the small signal theory as this bunching parameter approaches unity. The details of this signal saturation process have been investigated experimentally in this project.

There are many other device parameters which might be discussed at this point. Suffice it to say that May (6) has made a thorough engineering evaluation of the amplifier and has outlined a procedure for obtaining a "best" design. The project to be described has verified several points assumed by May which could not be predicted from the existing theory. In addition, a further limitation on the maximum obtainable ultrasonic gain not envisioned by May or White has been discovered and characterized.
CHAPTER III
DEPARTURES FROM SMALL SIGNAL THEORY

**Current State of the Art**

Amplification of ultrasonic waves in a piezoelectric semiconductor was first reported by Hutson, McFee, and White in 1961 (10). The simple, macroscopic theory describing the interaction of plane elastic waves with the free carriers in a piezoelectric semiconductor presented in the previous chapter was outlined shortly thereafter by Hutson and White (5, 11). The main result of this theory, the expression for ultrasonic gain as a function of drift field given by equations (21) and (23), was subsequently verified by a number of other theoretical approaches (7-9).

Following these original experiments, many investigators entered the field. Although the experimental acoustic gains which they observed were often impressive, nearly all results reported seemed to fall short of the theoretical predictions (12-16). A notable exception to this problem has been reported recently by White, Handleman, and Hanlon (17) who have conducted a series of experiments on carefully controlled CdS and have produced amplifiers which do exhibit theoretical characteristics from at least 50 MHz to 1000 MHz.
With theoretical material becoming available, it is reasonable to propose the design of an ultrasonic amplifier based on the Hutson and White model. As already mentioned, May (6) has made a rather extensive analysis along these lines using in addition the results of a number of theoretical and experimental investigations on transducers, noise, and electron bunching. He has developed techniques by which an optimum amplifier may be designed for a given set of operating specifications.

The dynamic range of the amplifier predicted by this model is limited on the low signal end by noise and on the high signal end by amplifier saturation on signal power. This saturation is presumed to occur when all of the electrons available for interaction are bunched, or when the bunching parameter of equation (26) is unity. This assumption has been experimentally verified by Hanlon (16) and by Ishiguro et. al. (14) for amplification in inhomogeneous material.

Further investigations have demonstrated, however, that saturation on signal power is not necessarily the mechanism which will place an upper bound on the dynamic range. Especially in cases which are designed to produce a large maximum ultrasonic gain at some frequency, the signal gain, even in "theoretical" material, is found to saturate at an intermediate level independent of the signal level. Evidence indicates that this saturation can be attributed to the buildup of ultrasonic noise flux within the amplifier on the initial ultrasonic pass. Further evidence demonstrates a definite relationship between the free
carrier concentration and the noise flux power level at which this saturation occurs.

**Experimentally Observed Gain Saturation**

In the past, it has often been difficult to determine the mechanism responsible for deviations between experimentally observed and theoretically predicted gains of ultrasonic amplifiers. Several investigators (9) including the author have found nonuniformities in the gain along the signal path of a supposedly homogeneous sample. These nonuniformities have customarily been ascribed to non-ohmic drift field contacts and to material inhomogeneities. In general with this kind of sample, the observed gain is usually far from that theoretically predicted with respect to both its absolute value and the shape of its observed variation with drift field.

Ishiguro, Uchida, and Suzuki have postulated the existence of a trapping effect with a relaxation time on the order of an ultrasonic period, which according to their model produces deviations from the simple theory similar to those which they had observed and to those mentioned above.

Suzuki's comment at the 1964 IEEE International Convention where he first presented his trapping paper pretty well summed up the state of the ultrasonic amplifier art to that date. One had to find a piece of "good-natured cadmium sulfide" in order to produce a successful amplifier, and that kind of material was very seldom found and then only by luck.
A major step forward in this material problem was realized when Handleman, White, and Hanlon (16) announced that they had been able to prepare a number of cadmium sulfide amplifiers which behaved according to the small signal theory from at least 50 to 1000 MHz. Figures 2 and 3 illustrate the close agreement between predicted and observed behaviors of one such amplifier made available to the author. Other samples of this same material have produced even better agreement.

But the remarkably good fit between theory and observation in this "improved" material illustrated by these figures does not occur for all possible operating conditions. A sharp and well defined break-away from the small signal theory is observed for conditions which produce relatively large amounts of gain, as is illustrated in Figures 4 and 5. These figures compare the predicted and observed behaviors of the same amplifier which produced the "good" curves of Figures 2 and 3. The only significant change in the amplifier between these two operating conditions is resistivity, lower in the later cases in order to shift the frequency of maximum gain upward as specified by equation (24). As is apparent, even in this "theoretical" material, some phenomenon is occurring which produces a saturation in the observed, first pass, ultrasonic gain (18) when the amplifier drift field exceeds a certain critical value.

The sharp definition with which this phenomenon occurs in material whose small signal characteristics can actually be described by the existing, explicit equations has allowed the
Figure 3. Small Signal Theory Prediction of Shear Mode Ultrasonic Gain and Measured Data Points for Sample #280-2 at 220 MHz. Resistivity is 1815 ohm cm, Mobility is 281 cm$^2$/volt sec.
Figure 4. Small Signal Theory Prediction of Shear Mode Ultrasonic Gain and Measured Data Points for Sample #280-2 at 550 MHz. Resistivity is 641 ohm cm, Mobility is 275 cm²/volt sec. The Observed Gain Saturates Below the Theoretical Maximum Value for this Operating Condition.
Figure 5. Small Signal Theory Prediction of Shear Mode Ultrasonic Gain and Measured Data Points for Sample #280-2 at 550 MHz. Resistivity is 459 ohm cm, Mobility is 282 cm²/volt sec. The observed Gain Saturates Below the Theoretical Maximum Value for this Operating Condition.
quantitative evaluation of the mechanism which produces it. As the following sections will show, it is possible to predict the drift field at which saturation will occur for any amplifier given several experimentally determinable parameters. Thus the gain limitation imposed by this heretofore unevaluated saturation phenomenon can be quantitatively analyzed by the designer of an ultrasonic amplifier.
Evidence for Noise Saturation

It is experimentally observed that the drift field at which measured gain departs from theoretical gain is also the field at which the sample's dc current saturates. Figures 6, 7, and 8 illustrate the onset of this saturation behavior in time, showing drift voltage-current characteristics and sonic output for three different drift fields. Figure 6 shows a field such that saturation does not occur, Figure 7 a field such that the sample begins to saturate after several ultrasonic transit times, and Figure 8 a field such that saturation occurs before the completion of the first ultrasonic transit of the amplifying material.

Figures 6, 7, and 8 each show two photographs of a triple channel oscilloscope trace displaying voltage, current, and sonic output. To facilitate their interpretation, Figure 9 has been prepared as a line drawing showing the pertinent details of the upper of the two oscilloscope traces of Figure 7. This upper trace shows the time variation of amplifier voltage, current, and sonic output with sonic signal input. The lower trace of Figures 6, 7, and 8 shows the same voltage and current curves, but the sonic output trace is now a narrow band, 10 megahertz sample of...
Plate 1. 550 MHz Signal Output with Current-Voltage Characteristics (above) and Noise Output with Current-Voltage Characteristics (below) for Amplifier Sample #280-2 with a Resistivity of 459 ohm cm and a Mobility of 282 cm²/volt sec.

Figure 6. Drift Potential is 140 volts and Amplifier Loop Gain is Less Than 1.0.

Figure 7. Drift Potential is 150 volts and Amplifier Loop Gain Exceeds 1.0. Amplifier Saturates After Several Transits.

Figure 8. Drift Potential is 200 volts. Amplifier Saturates During First Transit.
Figure 9. A Sketch of the Signal Portion of Figure 7 Showing the Following Pertinent Features of the Figures of Plate I:

1. Waveform of Applied Drift Voltage.
2. Waveform of Current. Scale is Adjusted so that Initial Voltage and Current Magnitudes Coincide.
3. Envelope of Ultrasonic Signal Output.
4. Receiver Noise Output Before Amplification Turn-on.
5. Drift Pulse Turn-on.
6. Ultrasonic Output Turn-on, Delayed from Drift Pulse Turn-on by the Ultrasonic Transit Time of the Sample and the Output Buffer Rod.
7. Sample Saturation Begins as Current Falls Below Voltage.
the noise generated by the amplifier with no signal input.

The sonic output traces are a composite, wave envelope pattern obtained by overlapping a succession of amplified signal pulses. Each signal pulse is slightly shorter than one amplifier round trip time; and the individual pulses which make up the total envelope have been successively delayed after drift field turn on and photographically overlapped to make up the composite envelope shown. A gating circuit has been used in the receiver following the sonic output amplifier port to remove the extraneous noise and echoes which follow the trailing edge of each main pulse in the composite. Sonic output lags the drift field pulse by the transit time of the sample and the output buffer rod.

The lower oscilloscope traces show the ungated noise output of the amplifier with no signal input for each operating condition. Detector gain for the noise trace is 46 decibels more than for the signal trace in Figure 6, 26 decibels more in Figure 7, and 13 decibels more in Figure 8.

Similar non-ohmic behavior has been observed in inhomogeneous material (19,20) and has been explained (21,22) on the basis of acoustoelectric current buildup in a region of high acoustic flux intensity which opposes the drift current at this point. The result is that a very high field develops across this region, and the field distributed across the ohmic remainder of the sample is adjusted so that this section delivers enough flux to the saturated region to maintain saturation. Additional field applied to the sample after saturation is largely absorbed by the
saturated region.

If this model is applied to the homogeneous amplifier, it becomes apparent that saturation would occur near the output plane. The noise wavefront leaving the input plane as the drift field is turned on grows exponentially as it proceeds down the sample, and it finally attains the saturation level at some particular distance away from the input. The high field region then forms between this point and the output plane, and departure from small signal theory occurs.

This type of saturation can occur for any bias condition producing forward gain, regardless of whether there is round trip gain for flux reflected from the amplifier output plane. Because it occurs when flux is amplified to the saturation level, it fixes an upper bound on the amount of gain which can be obtained from an ultrasonic amplifier.

If the remaining ohmic material still produces net round trip gain at some frequencies, echoes of the initial wavefront will add to the net noise power in the earlier portions of the amplifier and decrease the distance down the sample at which saturation occurs. This decrease will occur in a stepwise fashion, and it will continue until the amplifying region shrinks and its field drops so that the loop gain is insignificant. This is the case illustrated by Figure 8.

Figure 7 illustrates a case with lower forward gain, but with net loop gain. Saturation sets in, but only after several transits. Figure 6 illustrates a case with still lower forward
gain and net loop loss. No saturation occurs, as the sample is too short for it to develop. The sample remains ohmic in the steady state, and gain continues to behave according to theory.

The traces of noise output also behave as expected. The noise output of Figure 6 remains at a low level and shows no particular irregularities. The noise of Figure 7, observed with 20 decibels less receiver gain than in Figure 6, builds to a much higher level and begins to saturate as the sample becomes non-ohmic. The noise of Figure 8, observed with 33 decibels less receiver gain than in Figure 6, shows definite steps in its output level and seems to reach a final, high level, saturated output.

Thus it seems reasonable to conclude that the phenomenon responsible for the gain saturation observed in this amplifier is based on the buildup of acoustic noise flux to some critical level. The idea of an abrupt saturation level is probably an oversimplification, but certainly a convenient one leading to useful design information. Observation shows that saturation sets in very quickly. A 1.65 mm long sample goes from first pass ohmic to saturated operation over a drift potential change of about three volts. Thus the single, critical level approximation seems a reasonable simplification.

Using material which behaves in the non-saturated state according to theory, it has been possible to measure a quantity associated with the total acoustic power level just at the onset of saturation. Further, it has been found that this power level
is related to the concentration of carriers in the material. The following sections will describe these measurements.

Experimental Evaluation of Acoustic Flux Saturation Level

For material which behaves according to the existing, small signal theory, it becomes a reasonable task to evaluate the total net acoustic flux sensed at the amplifier output plane in the modes which activate the output transducer. Essentially, one must sum the output noise power produced in a particular operating condition over all frequencies. If this is done for the operating condition which just produces saturation on the first ultrasonic pass, the acoustic flux level associated with the onset of saturation can then be evaluated.

Such an evaluation is most readily accomplished through the use of the noise figure model of an amplifier. All of the noise generated throughout the length of the amplifier is represented in this model by an equivalent noise generator at the input of a noiseless amplifier of the same gain.

The noise figure, $F$, of an amplifier is defined \(^{(23,24)}\) as the ratio of the total noise power per unit bandwidth (at a corresponding output frequency) available at the output to that portion of this power engendered at the input frequency by the input terminated at the standard noise temperature of $290\degree K$. The total noise flux, $N$, seen by the output transducer can then be obtained by evaluating the following integral:
where \( G(f) \) is the frequency-dependent gain predicted by small signal theory in this case and corrected for lattice loss as may be necessary, \( F(f) \) is the frequency dependent noise figure which must be experimentally determined, \( k \) is Boltzmann's constant, and \( T_0 \) is the standard 290\(^\circ\)K noise temperature. Numerical evaluation of the above integral using observed values of \( F \) shows that the frequency limits of integration can be collapsed to the 20 decibel down points of the gain relation while retaining nearly all contributions to \( N \) and to the 10 decibel down points if accuracy to two significant figures is sufficient.

The experimental basis for estimation of the saturation value of acoustic flux in material whose \( G(f) \) can be described theoretically then rests upon a measurement of amplifier noise figure, \( F \), as a function of frequency throughout the range in which gain is no less than 20 decibels below the peak gain generated. Thus it is well to examine in some detail the experiment by which this measurement has been made.

Previous reports (16,17) have been published of measured values of the acoustic spot noise figure of a cadmium sulfide ultrasonic amplifier. The same technique reported in the first of these references has been used in this work.

An experimental amplifier investigated is shown in Figure 10. It consists of a CdS shear mode sample, 1.65 mm long and 3.20 mm by 4.88 mm in cross section, bonded with sodium silicate to two,
Figure 10. A Sketch of the Experimental Ultrasonic Amplifier
Showing Vapor Plated CdS Transducers, Quartz Buffer Rods, and dc Drift Field Connections.
1.27 cm long buffer rods of BC cut quartz. Each buffer rod has an evaporated, shear mode, CdS transducer on its outer end (25). Ohmic contact is made by the dc field leads to an indium doped skin, and resistivity is adjusted by controlling the level of tungsten light which illuminates the sample.

Amplifier noise figure has been measured using a pulsed adaptation of the CW signal generator technique (24). These measurements have been standardized by comparing them with and correcting them to standard noise source measurements of the associated UHF receiver. The noise output of the amplifier is sampled only during the first sonic pass after drift field has been applied, thus excluding from this measurement any round trip buildup effects of acoustic noise flux. The rise time of the bias pulse is adjusted so that no significant harmonics fall into the frequency range of significant amplifier gain.

The system described can be considered to be composed of three networks in cascade, an ultrasonic amplifier whose noise figure is sought imbedded in the middle of two, lossy transducer structures. The overall noise figure, \( F \), of such a system is given by (24) as:

\[
F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}
\]

(28)

where \( F_1 \) is the noise figure of the input transducer, \( G_1 \) is the gain of the input transducer, \( F_2 \) is the noise figure of the ultrasonic amplifier and \( G_2 \) is its gain, and \( F_3 \) is the noise figure of the output transducer. In order to solve for the
desired quantity $F_2$, it is necessary to know $F_1$ and $G_1$ in every case and often to know $F_3$ as well when the combined $G_1 G_2$ product is not sufficient to make the third term above negligible (26).

As is well known, the noise figure of a matched attenuator held at the standard noise temperature of $290^\circ K$ is just equal to its loss. Thus, for the $290^\circ K$ amplifier, $F_1$ and $1/G_1$ are the same number. In like manner, the noise figure $F_3$ of the output transducer also equals its matched insertion loss. Actual measurements of the "dark" insertion loss of the entire package taken when there are no carriers present in the cadmium sulfide to interact with the transmitted sonic wave yields a loss equal to that of the combined loss of the two transducers alone (27). Thus:

$$L_{\text{Dark}} = L_1 \times L_3$$

or in decibels

$$L_{\text{Dark}} = L_1 + L_3$$  \hspace{1cm} (29)

To evaluate $L_1$ and $L_3$ separately, it is necessary to find another measurable relationship involving them independent of equation (29). For this purpose, it will be assumed that the acoustic noise figure of the amplifier when it is biased for forward gain in one direction is the same as its acoustic noise figure when it is biased for forward gain in the opposite direction and when the roles of the input and output transducers have been interchanged. This assumption is not too drastic in view of the self-consistency of the resulting figures and in view
of the virtually identical gain curves which are observed under such conditions.

If the ultrasonic amplifier's gain, $G_2$, is large enough so that the last term of equation (28) can be neglected, then the noise figure measured from section 1 to section 3 is:

$$F_{13} = F_1 + \frac{F_2 - 1}{G_1}$$

Taking into account that $1/G_1 = L_1 = F_1$, this becomes:

$$F_{13} = F_1 + (F_2 - 1) L_1 = F_1 F_2 = L_1 F_2$$

or in decibels:

$$F_{13} = L_1 + F_2$$

(30)

In like manner, the noise figure measured in the opposite direction is in decibel notation:

$$F_{31} = L_2 + F_2$$

(31)

Equations (30) and (31) combine to give the second necessary equation, which in decibel notation reads:

$$L_1 - L_3 = F_{13} - F_{31}$$

(32)

Equation (32) may then be combined with equation (29) to yield the loss of the individual transducers:

$$L_1 = \frac{L_{\text{Dark}} + (F_{13} - F_{31})}{2}, \quad L_3 = \frac{L_{\text{Dark}} - (F_{13} - F_{31})}{2}$$

(33)
Once the losses of the individual transducers have been evaluated, equation (28) can then be solved for the desired acoustic noise figure, \( F_2 \).

Figure 11 shows an observed curve of spot noise figure as a function of frequency along with the theoretically calculated gain over the same frequency range for this operating condition. While this particular operating condition does not produce the lowest noise figure observed, it does typify the generally observed frequency behavior of noise figure for many operating conditions. Generally, noise figure is observed to decrease moderately for frequencies lower than that of maximum gain and to increase moderately for higher frequencies as is shown. The character of this variation is such as to make reasonable the approximation \( F(f) = F(f_{\text{max gain}}) \) in the integral of equation (27), especially when one realizes that the major contribution to this integral is made by the portion of the frequency spectrum whose gain is within 10 decibels of the maximum.

Thus by combining measured values of \( F \) with theoretical values of gain, equation (27) can be made to yield the magnitude of the noise flux sensed by the amplifier output transducer. In particular, if the values appropriate to the onset of first-pass, non-ohmic behavior are used, the saturation level of the acoustic flux can be evaluated.
The Major Contribution to the Total Output Noise Flux Comes from the Portion of the Spectrum within 10 dB of Maximum Gain.

Figure 11. Small Signal Theoretical Gain and Experimentally Measured Noise Figures for Sample #280-2 Plotted as a Function of Frequency. Resistivity is 2460 ohm cm, Mobility is 266 cm²/volt sec, dc Drift Potential is 180 volts.
Saturation Level Dependence on Carrier Concentration

The phenomenon of amplifier saturation on signal power has been treated by White(5) and by May(6). It is postulated that signal power saturation occurs when the local bunching of carriers reaches some fraction, possibly as great as unity, of the total carrier concentration. The saturation of an amplifier on noise flux is more difficult to describe mathematically than is signal saturation. One might begin by supposing that saturation occurs when local electric fields due to noise flux are sufficient to produce sonic velocity in a significant number of carriers. Such an approach for signal flux would lead to the signal saturation described by White. Thus, as signal saturation is dependent upon carrier concentration, one is also lead to look for a dependence of the noise flux saturation level on carrier concentration.

Carrier concentration for a given operating condition has been determined as suggested by White, Handleman, and Hanlon(17) by measuring the zero drift field sonic attenuation and the drift mobility of the sample in a given operating condition. Equation (21) for attenuation is then fitted to these two points by adjusting the dielectric relaxation frequency which equals the material conductivity divided by its dielectric constant. Conductivities calculated in this manner are found to agree with ohmmeter measurements to within 20 percent for higher resistance operating conditions and to much better precision for lower resistivities.
Figure 12. Measured Noise Saturation Power Evaluated According to Equation (27) as a Function of the Measured Carrier Concentration.
Figure 12 shows the result of plotting saturation level noise flux calculated by evaluation of equation (27) against carrier concentration. From left to right, the points on this plot correspond to operating conditions which produce maximum gain at 320, 380, 480, 560, and 740 MHz. It is evident from this figure that a relationship exists between the noise flux saturation level and the carrier concentration. Extrapolating from the curve, it is found that:

\[ P_{ns} = 7.85 \times 10^{-51} n^{2.6} \text{ mW} \]  

(34)

where \( P_{ns} \) is the saturation level of acoustic flux in milliwatts and \( n \) is the density of carriers per cubic meter.

To summarize, equation (34) represents an experimentally observed relationship between the carrier concentration and the maximum amount of noise flux which this sample can support while remaining ohmic. The implications of this relationship for device design will be considered.

A Physical Interpretation of Noise Saturation

The observed values of noise saturation power level, \( P_{ns} \), are useful by themselves because of their immediate application in the device design area. The purpose of this section, however, is to make an order of magnitude estimate of the physically more basic phenomenon involved in noise saturation. In the process, several facts significant to the geometry of device design will become apparent.
Let Figure 13a represent an ultrasonic amplifier with an output transducer of diameter $D$, biased so that gain exists within a cone $\Theta_g$. Let the transducer diameter be much greater than the wavelength of sound, $D \gg \lambda$, so that it is reasonable to suppose that the transducer responds to all signals incident upon it within the angle $\Theta_{tr} = \lambda/D$ to the perpendicular.

The total noise power within the sample is the sum of the powers contained in each individual noise mode which the sample can support. Since the number of these modes is a function of the sample dimensions, this total noise power also depends on sample dimensions. It may be expressed as:

$$P_I = \sum_{\text{all modes}} kT B$$

where $P_I$ is the total internal noise power within the sample, $k$ is Boltzmann's constant, $T$ is the effective temperature for a given mode which may be a function of gain and of direction, and $B$ is the equivalent noise bandwidth.

It is useful to consider a sketch of the expected dependence of $P_N(\Theta)G(\Theta)$, the noise power, gain product incident upon a unit area of the amplifier output plane as a function of the angle from the normal, $\Theta$. Such a picture is shown in Figure 10b. The power per unit area is expected to fall off for angles different from zero because the gain in these directions decreases. The gain decreases because of the decreasing electric field component in these directions and also because of decreasing piezoelectric
Figure 13: (a) Idealized Ultrasonic Amplifier Showing Gain Angle, $\theta_g$, and Output Transducer angle, $\theta_{tr}$.

(b) The Resulting Noise Power-Gain Product Incident Upon the Transducer as a Function of Angle $\theta$ from Vertical Incidence.
constant. For directions near the normal, however, neither of these effects are very important, and so the curve should show a rather flat peak around $\Theta = 0$.

The amount of power delivered to an external matched load by a lossless transducer of diameter $D$ would be:

$$P_{Ntr} = A_{tr} \int_{\Theta=0}^{\Theta_{tr}} \int_{f=0}^{f_{\text{max}}} P_N(\Theta, f) G(\Theta, f) d\Theta df$$

where $P_{Ntr}$ is the power output of the transducer which is taken to be lossless, $A_{tr}$ is the transducer area, $\pi D^2/4$, $f_{\text{max}}$ is the maximum frequency of the acoustic phonon branch, $\Theta_{tr} = \lambda/D = c/fD$ is the "transducer angle," $f$ is frequency, $c$ is the velocity of sound, and the product $P_N G$ which is a function of angle $\Theta$ and frequency $f$ is as illustrated in Figure 13b.

This is a rather difficult integral to evaluate because of the frequency and angular dependence of its integrand. If, however, the approximation is made that $\Theta_{tr} \ll \Theta_o$ and therefore that $P_N(\Theta) \approx P_{\text{max}}$ over the entire transducer lobe; and if further the gain function $G(f)$ has a pronounced maximum about a particular frequency $f_o = c/\lambda_o$, then:

$$P_{Ntr} \approx A_{tr} P_{\text{max}} \Theta_{tr}^2 G_{o B}$$

where $\Theta_{tr}^2$ results from the $\Theta$ integration and $G_{o B}$ is the noise gain - bandwidth product of the amplifier within the transducer gain lobe resulting from the frequency integration. Using $A_{tr} = \pi D^2/4$ and $\Theta_{tr} = \lambda_o/D$, equation (37) becomes:
As the area of a transducer increases, then, its angle decreases in such a way as to cancel the effect of the changing area as shown in equation (38) and thus to make the output noise power, $P_{Ntr}$, independent of transducer area. If $P_{Ntr}$ is independent of transducer area, the amplifier noise figure will also be independent of area as can be seen by a consideration of equation (27).

**Signal Saturation in the Presence of Noise Flux**

Since both signal and noise saturation mechanisms depend on carrier concentration, it is important for the device designer to know whether high levels of noise flux in the output plane of the amplifier will reduce the level at which signal power saturation occurs.

The experimental amplifier previously described was operated at fields from well below to somewhat above that producing first pass noise saturation. Figure 14 shows its electrical power input and output characteristics with resistivity adjusted to produce maximum gain at the observation frequency, 480 MHz. First pass saturation is observed to occur at a 165 volt drift potential.

This figure shows clearly that the onset of noise saturation
Figure 14. Experimentally Observed Signal Power Saturation of Sample #280-2 at 480 MHz. Resistivity is 640 ohm cm, Mobility is 282 cm²/volt sec, dc Drift Potential is a Parameter. The Power Input and Output Are Measured at the Electrical Terminals and Include Transducer Loss.
is not observed to reduce the signal saturation level. Thus, for an amplifier operating near noise saturation with a near maximum signal output, an increase in drift potential would not increase the signal output beyond the previous signal power saturation limit. This presumably detrimental saturation effect might be used to relax power supply regulation requirements if operation on a steep portion of the gain curve is proposed.

By the technique described in the previous section, the losses of the individual transducers in this experiment can be evaluated, and the absolute, acoustic, signal power levels at the amplifier input and output can then be calculated. Using equation (26), values of the bunching parameter corresponding to observed acoustic outputs can also be calculated. These are displayed along with the acoustic input and output powers in Figures 15 through 19. The carrier bunching, previously called n_\text{c}, has been denoted by n_1 in these figures. These cases correspond to the five cases shown in Figure 14. Note that in all cases, the power output is virtually a linear function of the power input for values of the bunching parameter below 0.5, and that it seems to saturate for a bunching parameter somewhat less than 2.0. As shown in Table I, the amount of saturation evaluated at the intersection of the n_1/n_0 = 1 line and the projected, low level, dynamic response decreases as drift field increases. In no case is it observed to be worse than 1.75 decibels.

While these figures present saturation data at only one
Figure 15. Experimentally Observed Signal Power Saturation of Sample #280-2 at 480 MHz. Bunching Parameter $n_1/n_0$ is Plotted Vertically. Drift Potential is 140 volts, Other Parameters Are as in Figure 14.
Figure 16. Experimentally Observed Signal Power Saturation of Sample #280-2 at 480 MHz Showing Bunching Parameter. Drift Potential is 150 volts, Other Parameters Are as in Figure 14.
Figure 17. Experimentally Observed Signal Power Saturation of Sample #280-2 at 480 MHz Showing Bunching Parameter. Drift Potential is 160 volts, Other Parameters Are as in Figure 14.
Figure 18. Experimentally Observed Signal Power Saturation of Sample #280-2 at 480 MHz Showing Bunching Parameter. Drift Potential is 165 volts, Other Parameters Are as in Figure 14.
Figure 19. Experimentally Observed Signal Power Saturation of Sample #280-2 at 480 MHz Showing Bunching Parameter. Drift Potential is 170 volts, Other Parameters Are as in Figure 14.
Table I. Amount of Saturation at Unity Bunching for the Drift Potentials of Figures 15 through 19.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Drift Potential</th>
<th>Amount of Saturation (^\ast)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>140 volts</td>
<td>1.3 decibels</td>
</tr>
<tr>
<td>16</td>
<td>150 volts</td>
<td>1.75 decibels</td>
</tr>
<tr>
<td>17</td>
<td>160 volts</td>
<td>1.3 decibels</td>
</tr>
<tr>
<td>18</td>
<td>165 volts</td>
<td>0.9 decibels</td>
</tr>
<tr>
<td>19</td>
<td>170 volts</td>
<td>0.5 decibels</td>
</tr>
</tbody>
</table>

*Measured at the intersection of the \(n_1/n_0 = 1\) level with the projected low level response.
frequency and resistivity, they are typical of observations made for conductivities producing maximum gains at frequencies from 200 MHz to 740 MHz.

Figure 20 repeats the 480 MHz fundamental frequency saturation curve of Figure 17 and shows in addition the measured acoustic level of the 960 MHz second harmonic. Note that at the point where the fundamental signal bunching parameter is 1.0, second harmonic output is 21.5 decibels below fundamental output and that it drops further below the fundamental output for lower bunching and output levels. This should represent a tolerable amount of second harmonic distortion for most device purposes.

Third and higher harmonics could in principle also be measured, but equipment limitations prevented it in this case. They should not represent a material increase in the total harmonic distortion.
Figure 20. Experimentally Observed 480 MHz Fundamental and 960 MHz Second Harmonic Output for the Operating Condition Illustrated in Figure 17. Electrical Input is Restricted to 480 MHz.
CHAPTER V

DEVICE IMPLICATIONS

Significance of Noise and Signal Saturation Power Levels to the Design of a Device

With a knowledge of the noise figure and carrier concentration of a projected amplifying device, it is possible to predict the region of drift fields in which this device will follow the small signal theory and also to examine its expected dynamic range. Essentially, dynamic range is limited on the low signal end by amplifier noise and on the high signal end by signal power saturation. Noise saturation is important because it limits the maximum signal gain which can be obtained from a device with a given noise figure. Because of this phenomenon, it is not necessarily possible to design an ultrasonic amplifier of any desired gain; but rather practical amplifiers are limited to a maximum gain on the order of 80 decibels.

Figure 21 shows for one particular operating condition how the power limitations change as a function of bias voltage. The effective input noise power curve is calculated from the measured amplifier noise figure and has been drawn for a one megahertz noise bandwidth. This corresponds roughly to a one megahertz signal bandwidth between half power points. Should the design
Figure 21. The Variation of Output Plane Noise Power as a Function of Drift Potential and the Corresponding Small Signal Gain Variation Combine as Shown to Delineate the Dynamic Range for Sample 280-2 at 740 MHz. Resistivity is 275 ohm cm, Mobility is 277 cm²/volt sec.
bandwidth be different, as it probably will be, this curve will move up or down by a factor of 3 decibels each time the bandwidth is accordingly doubled or halved.

The total noise power curve is a plot of the total noise power incident upon the amplifier output plane as predicted by equation (27) evaluated for the various values of bias voltage. The only actual point of significance on this curve from the amplifier design standpoint is the point at which it intersects the noise saturation power level predicted by equation (34). This point marks the upper boundary of the region in which small signal theory predicts the ultrasonic gain. The total noise power curve is shown dashed above this point, as these higher noise power levels, although predicted by equation (27), do not occur.

The curve of signal saturation power is found from the strain level found from equation (26) when the bunching parameter $n_s/n_0$ is unity. Note that this strain experiences a slight increase with bias voltage. The input power level which just produces this signal saturation level in the output plane is then calculated by subtracting from the saturation power the gain predicted by small signal theory, all operations being performed of course in decibel notation. This accordingly is plotted as the "signal saturation input power level" and also is dashed in the region where small signal theory does not apply.

The difference between this maximum input signal which produces saturation in the output and the effective noise power
input then represents the dynamic range of the amplifier. Figure 22 shows this dynamic range along with the small signal gain as a function of the bias voltage for the case of Figure 21, as noted before for a one megahertz effective noise bandwidth. As this curve demonstrates, the dynamic range of this amplifier deteriorates considerably as it is operated at bias voltages which produce higher ultrasonic gains.

The curves of Figures 21 and 22 have been prepared from data measured and calculated for a sample with a 50 mil transducer. If the transducer area is changed, the aperture angle also changes inversely as the area. Thus, for a transducer in a nearly isotropic noise field, the available noise power output which is the product of aperture, area, and flux is not changed. The signal saturation level, however, is a direct function of transducer area. Providing other factors such as electrical matching and power dissipation will allow it, the dynamic range can be improved by increasing the transducer area. Such a change would move the signal saturation power level of Figure 21 upward, but would not affect the noise power input level.

The curves of Figures 21 and 22 also pertain to a sample 1.65 mm long. For an amplifier of any length, noise saturation will occur when the noise power at the output plane as calculated using equation (27) reaches the level specified by equation (34). The only term in equation (27) sensitive to a change in length is the gain, \( G(f) \). A longer sample, for example, would require a smaller net gain per unit length to reach noise saturation at
Figure 22. Theoretical Ultrasonic Gain and Dynamic Range for the Operating Condition of Figure 21.
its output, and thus the applied drift field would be smaller. Since to a first order approximation the variation of gain with field is one only of relative magnitude, the basic contour of G(f) as a function of frequency does not change as can be seen in Figure 1. In other words, the maximum gain obtainable is not a strong function of sample length. Thus for gain levels near the maximum, it would be possible to balance length and drift field against each other to obtain the most efficient or otherwise desirable operating point; but it would not be possible to obtain a substantially greater gain by an increase in length.

The maximum obtainable ultrasonic gain for the 1.65 mm sample is shown as a function of frequency in Figure 23. For this figure it has been assumed that the amplifier itself is noiseless, that is that its noise figure is 0 decibels, and that its input termination is at 290°K. For an amplifier whose noise figure is greater than zero decibels, the maximum obtainable gain at a particular frequency will be the number for the noiseless amplifier less a number of decibels equal to the amplifier noise figure. This, of course, just takes into account the additional noise generated within the amplifier.

Practically speaking, then, Figure 23 represents a curve of maximum obtainable ultrasonic gain for a noiseless, shear mode cadmium sulfide amplifier of any length. While it is plotted only from 520 to 740 MHz, the range over which experimental evidence has been collected, it can easily be extended by evaluating the respective equations at the appropriate frequency.
Figure 23. Maximum Ultrasonic Gain as a Function of Frequency for a Noiseless, $F = 0$ db, Shear Mode Amplifier, 1.65 mm long, whose input termination is at $290^\circ$K.
CHAPTER VI
CONCLUSIONS

The acoustic flux level which occurs at noise saturation in a shear mode, cadmium sulfide ultrasonic amplifier made of "theoretical" material has been sampled for a number of different operating points. This level has been found to be dependent upon the concentration of carriers in the region in which noise saturation occurs. The importance of this phenomenon, and the limitations it imposes upon the use of the ultrasonic amplification phenomenon for device purposes have been indicated, and several other important parameters such as carrier bunching and harmonic distortion have been evaluated at noise levels approaching saturation.

Further evaluation of this phenomenon in other materials proposed for ultrasonic amplifying devices should be done. It might also be useful to carry out the theoretical calculation of the total saturation noise level suggested by equation (35) to obtain an estimate of the total acoustic flux which it would be possible to generate using this noise saturation phenomenon.
REFERENCES


12. See, for example, (10) above for reasonably close agreement and (9) and (13) for disagreement with theory.


18. "First-pass gain" is the gain observed during the time of the first ultrasonic transit after the drift field has been turned on. Thus, saturation effects due to buildup of sonic noise flux over several round trips are excluded from these observations.


26. A fourth term describing the noise contribution of the laboratory signal detector is also required. Its form is 

\[ \frac{F_4 - 1}{G_1 G_2 G_3} \]

Since \( F_4 \), the detector noise figure, can be measured separately and since the product \( G_1 G_2 G_3 \) is merely the overall insertion gain of the experiment, readily determined and usually large enough so that the entire term can be neglected, this term will be omitted for simplicity. In any event, it is easily measured for all operating conditions.

27. Cadmium sulfide lattice attenuation must be subtracted from the "dark loss" to obtain the "combined transducer loss" at higher frequencies where it becomes significant.