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EARTH GRAVITY MODELS BASED ON DISTINCT
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EARTH GRAVITY MODELS BASED ON
DISTINCT 5° by 5° VALUES OF
TOPOGRAPHY AND MOHO

Dissertation

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By

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PREFACE

The solution of many problems in geodesy requires a thorough and detailed knowledge of the gravity field of the earth. Methods for prediction of the field have been developed in order to aid and complement the physical measurements. This is done by filling-in the gaps in the observed data, or by estimating their effects on the total field.

Earth models, which account for the effect of the topography and its compensation, have been suggested by various authors. These models are theoretical in more than one way, because they are based on some hypothesis of equilibrium or equimass, and utilize standard densities.

The scope of this study was to suggest, establish and analyze unique models of the earth, based on the mean topography and distinct mean values of crustal thickness or depth to the Moho, and the densities. Computer programs for the models were prepared and checked. Such models and programs could become very handy with increased data about the actual Moho and density distribution.

The author is deeply indebted to many individuals and institutions for their help in preparing this study. Outstanding among them is the adviser, Dr. U. A. Uotila, for his guidance, advice, encouragement, and data furnished. Drs. I. I. Mueller and R. H. Rapp supplied valuable comments and suggestions.
Special thanks go to The Ohio State University and the Department of Geodetic Science for the high level of instruction maintained and for the excellent facilities made available. Developing the model would not have been feasible if it were not for the OSU Computer Center, its staff, and the IBM 7094 Computer.

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ABSTRACT

The problem of predicting gravity anomalies is presented and earth-gravity models, whose anomaly field could closely approximate the free-air anomaly field are introduced. These models are based on distinct 5° x 5° mean values of the elevation and the crustal thickness.

Mean crustal thickness values were estimated for all 5° x 5° surface elements.

A study of the elevations and crustal thicknesses, together with comparisons to an Airy-Heiskanen model and mass-balance considerations, yielded a mass-model for the earth.

Four gravity models were developed, which handle the above data and consider the densities as constant, distinct, or constant and distinct for each 5° x 5° compartment. The densities could also be read-in.

Working formulas and procedures for computation of model anomalies in the various models were chosen, modified, or derived, and checked out.

Computer programs were designed for the above formulas, procedures, and models, and were checked out. The programs compute the total model anomaly either at specified points, independently, or for the entire earth as a whole.

The computation is done at the mean coordinate of integer 5° x 5° compartments or at the center of gravity of same. The computed effect can be limited to a specified angular distance from the point in question if the effect of the neglected zones is not desired or negligible.
Two partial results of the model anomaly computations are recorded; namely, the accumulated effect of all 5° x 5° compartments outside the computation compartment and the accumulated effect of all surface elements, excluding a spherical cap contained within the computation compartment. These partial results could be useful for prediction purposes and for comparisons to similar computational systems.

The various programs were test-run, and execution times were obtained.

Possible comparisons between model anomalies and observed anomalies or other authors' model anomalies were discussed and demonstrated. Related statistical analysis techniques were reviewed and incorporated.

Test runs with the existing data indicated that the model with constant density did not approximate the free-air anomaly field.

Those models which use distinct densities yield a favorable comparison to the free-air anomaly field, which indicates their possible and existing superiority over zero-anomaly assumption.

The model which uses distinct densities below a crustal layer of constant density shows the greatest versatility. Its results indicate prediction accuracy approaching ± 10 mgals.

Error analysis of the precision and accuracy of the model anomalies and of the prediction accuracy was undertaken. Possible systematic effects on model anomalies were analyzed and related to crustal-thickness data close to the computation point.
1. INTRODUCTION

1.1 General problems of geodesy

Geodesy, as the name implies, is concerned with the size and shape of the earth. Its practical application is to furnish maps and coordinates of points on or close to the surface of the earth [78]. The coordinates can be in any system as long as they all refer to the same system, which should be well defined. Furnishing coordinates is the general problem of geodesy.

Through the ages the demands for accuracy have increased along with the increase in areas to be mapped and connected. Thus classical geodesy has developed from local, independent surveys to the use of various mathematical models for the earth, onto which surface points are tied and their coordinates fixed.

There is also a purely geometric solution to the problem, which is becoming more feasible with the advent and improvement of long-distance measuring techniques. This is by means of direct observations of spatial angles and distances between points [25], [3, p. 185], [49], or by observations of satellites [68], [74], both on a world wide basis. These solutions will not be treated here.

The model for the figure of the earth is a reference surface like a sphere or an ellipsoid of revolution, which closely approximates the figure of
the earth. How this surface is selected is a separate problem. It would suffice here to mention, for instance, that the earth ellipsoid [27, p. 68] has the rotation axis of the earth for minor axis, and all the masses outside it are distributed inside it in such a way that the ellipsoid is an equipotential surface.

1.2 Gravimetry

The reference surface used for the coordinate system is regular, although not necessarily yielding simple expressions for computations on it. The physical surface is highly irregular, and the fact that local surveys are based on the direction of the local gravity complicates the matter.

For the transformation between the physical surface and the reference one, physical geodesy [88], [14], [17] introduced the concepts of center of gravity, rotation axis, undulations of the geoid (or height anomalies), deflections of the vertical, and related terms. The values of these, together with the directly observable geodetic data between points on the surface, allow the transformation which yields coordinates of physical points in the reference system. The solution could be for example through Stokes' and Vening Meinesz's approaches [17, p. 265, 258], [75, p. 10], or with those of Hirvonen [20], Molodensky [52], [54], [56], [78], or Bjerhammar [2]. These approaches may or may not require gravity reductions which involve additional problems introduced by mass transportations and lack of knowledge of density distribution.
Whatever the approach, however, one thing is common to all of them: Complete gravimetric coverage of the earth is essential for best results. Since such coverage is an endless, expensive physical endeavour, one must resort to predictions in order to fill in the gaps between points where gravimetric data are available. The same procedure could be applied to error analysis of the effect of the unsurveyed areas on specific gravimetric quantities. Both prediction and error analysis are very important tasks required of modern geodesy.

1.3 Newton's law of attraction

The gravitational force of attraction $F$ between two mass elements $m$ and $M$ separated by a distance $D$ between the centers of gravity is expressed in Newton's law

$$F = k \frac{mM}{D^r}, \quad (1)$$

with the constant of gravitation $k$. The force is directed on the line connecting the two centers of gravity.

This law came about by Kepler's Laws of motion and is the very basis of all gravity computations. Since it is empirical there is a definite inaccuracy associated with its terms. The power of $r$ is very close to 2, as best obtainable today, and the force is apparently independent of relative motions.

The constant $k$ has been shown, [70], to be independent of material to one part in 200,000 (Eötvös). It is similarly independent of radioactivity.
and chemical effects and of temperatures up to 1000°C (Shaw) to at least one
millionth (while it could perhaps vary with time); yet it is known only to four
significant figures. It is very interesting to note the fact that in so many
areas the very fundamental quantities are not known to more than 5 to 7 figures.
Geodesy, for instance, uses the speed of light for many computations—and it
is known to six figures only in vacuum, and even with less accuracy in the
atmosphere, due to refraction. Yet the situation with k, or rather with kM
which is customarily considered the relevant constant, is a little better since
a change in it will only change the scale and will not have differential effects.
Physical geodesy deals mainly with differences in gravity rather than absolute
values. Besides, kM is known with much better accuracy than k itself, to 6 digits.

The value of k as obtained from direct measurements is 6.673±0.003 x
10^-8 cm^3 gr^-1 sec^-2 [17, p. 156]. The value of kM as obtained from moon
probes is 3.986009±0.000006 x 10^30 cm^3 sec^-2 [66, p. 10].

Geodesy is mainly concerned with the attraction T of the body M (the
earth) at the mass point m, which is obtained from equation (1) for m = 1 and
r = D and is called the acceleration (gravitation) T:

$$ T = k \int \frac{dM}{r^2}. $$

Similarly the attraction in any direction can be expressed through the poten-
tial V of the same mass M at the point m,

$$ V = k \int \frac{dM}{r}. $$
when the directional derivative is taken; namely,

\[ T_i = \frac{\partial V}{\partial i} \quad (4) \]

1.4 Gravity and normal gravity

On the surface of the earth (or for points rigidly connected to it), the term gravity \( g \) at a point implies the vectorial resultant of the acceleration (gravitation) due to the attraction of the earth's mass as per equation (4), together with the deceleration due to the centrifugal effect of the earth's rotation around its axis, both computed at that point. Attraction and tidal effects of heavenly bodies can be neglected for most practical applications \([59, p. 2]\), as well as the attraction of the atmosphere \([18, p. 63]\).

The inclusion of the centrifugal effect could be made simply by changing the potential \( V \) in equation (3) to the total potential \( W \),

\[ W = k \int \frac{dM}{M} - \frac{\rho^2 \omega^2}{2} \quad (5) \]

with \( \rho \) the point's distance from the rotation axis, and \( \omega \) the rotation speed in rad./sec. The centrifugal effect can be omitted in cases when attraction comparisons are made between points with the same distance \( \rho \). It is actually canceled out in computations of gravity anomalies.

Some remarks about \( V \) are worthwhile. The scalar \( V \) as well as the vector \( g = \nabla V \) are both unique, finite, continuous and have second derivatives - or they are harmonic. But what about the second derivatives of \( V \)? Since \( g \) is \( (\text{grad. } V) \), and since \( (\text{grad.}) \) field does not have curl,
\[ \nabla \times \mathbf{g} = 0, \quad (6) \]

it must have divergence

\[ \nabla \cdot \mathbf{g} \neq 0. \quad (7) \]

Therefore,

\[ \nabla^2 V \neq 0, \quad (8) \]

or the second derivative of the potential is not continuous. The discontinuity occurs at points of mass density change, and is indicative of a difference in the radius of curvature of the \( V \) surface at these points.

The possibility of a discontinuity in the first derivative of \( g \) should be kept in mind in connection with problems of upward and downward continuation of the gravity field and related matters.

The unit of \( g \) in the c.g.s. system is called gal, for Galileo Galilei. The practical unit in physical geodesy is the milligal, mgal.

Knowledge of the absolute value of \( g \) is quite important for many branches of science. The definition of the dyne—the unit of force in the c.g.s. system—is based on gravity. Consequently, all units involving dynes, grams and their derivations will depend on \( g \). In geodesy, the absolute knowledge is not so critical, (except when comparing results from satellites and gravimetry [31, p. 521], or generally from two different gravimetric systems or models). This is because most of the computations are done with anomalies, or differences between two numbers in the same system and not with the number
itself. Thus not much harm is done even if the equatorial value of normal gravity is not right, or the Potsdam or the local system should be shifted by several mgals, so long as all gravity values, observed and theoretical, refer to the same system [16, p. 201], [85, p. 11], [82].

In order to avoid dealing with big numbers when matters of gravity are concerned, the concept of normal gravity has been introduced. Normal gravity is the gravity of the normal earth [57, p. 72]. It describes the force field of the spherop. The surface of the spherop is represented by a rotational ellipsoid, if terms smaller than the square of the flattening are neglected. The ellipsoid, in turn, is chosen so that it is close to the figure of the mean sea level. The difference between an ellipsoid and a spherop is that the latter is an equipotential surface, like the mean sea level, while a homogeneous ellipsoid with the mass and rotational speed of the earth is not. The potential on the spherop is $U$, and $\gamma$ is just $\nabla U$.

Normal gravity for a specific spherop can be computed at any point on or outside it to any degree of accuracy [20, p. 29].

1.5 The geoid

Mean sea level is a natural reference surface for actual surveys, and for counting potential differences from, and it is an equipotential surface which is called the geoid. Now the problem of classical geodesy is to give the coordinates of all points as projected on the ellipsoid, which is a relatively easy surface to compute on. Choosing an ellipsoid suitable for geodesy,
or best fitting, etc., could actually be immaterial to the solution of the coordinates, so long as the ellipsoid is kept close enough to the geoid, to allow for neglecting high-order terms in series development. However, for the ease of computations, one should recall that McLaren's theorem states, in fact [45, p. 61], that the attraction at an exterior point to an ellipsoid of revolution is the same as that of a homogeneous ellipsoid with the same total mass, so long as the former has homogeneous confocal layers. Since the attraction of a rotating ellipsoid (homogenous, or having homogeneous confocal layers) is well established, too, it would be tempting to regard the earth as such an ellipsoid. This, by way of accounting for mass deviations from a homogeneous ellipsoid, including all the masses outside the ellipsoid [40]. The ellipsoid should also have the total mass of the earth, which restricts the selection, and the axis of rotation and centers of gravity should coincide.

The well-known Stokes' approach follows the procedure above and allows to determine the geoid if there are no masses outside it, and if a complete gravity coverage exists on the surface of the geoid [75, p. 10].

The separation between the (earth) spherop and the geoid is called the geoid undulation, N; positive value means that the geoid is above the spherop. It should be noted, though, that Stokes' solution, utilizing gravity anomalies, does not actually specify the size of the reference ellipsoid; it rather refers to a certain shape. Solution of the size (or the equatorial radius a) and of the zero-order undulation N₀ necessitates the use of the
constant $kM$ (from lunar probes), the flattening $f$ (from close-orbit satellite analysis) and the rotational velocity of the earth $\omega$, in addition to the gravity material [66].

In this fashion the absolute undulations can be found, with expected accuracy for $N_0$ of $\pm 10$ meters. The individual undulations depend on the reference system in use. In the systems most commonly used today the undulations will have a range of probably less than $\pm 100$ m from the value of $N_0$.

The discussion above relates to the classical concepts of geodesy. Quite a few "modern" or new approaches have been suggested [20], [50], [51], [52], [54], but they seem to yield practically the same results for the coordinates [52], [71], [77], and there are still arguments in favour of keeping the geoid "alive" [42].

2. GRAVITY ANOMALIES

2.1 Gravity anomalies in general

A gravity anomaly $\Delta g$ is the discrepancy between the value of the observed gravity $g$, reduced or corrected in any or many ways, and the corresponding value of the normal gravity $\gamma$. A cautious, even though hazy, definition like that could hold in any gravimetric system. A definition of the gravity anomaly field as a function of a disturbing potential, $T = W - U$, is feasible [76, p. 6]. Its form is
\[
\Delta g = - \frac{\partial^2 T}{\partial \rho^2} - \frac{\partial^2 U}{\partial \rho^2} N,
\]

with \( \rho \) the direction of the normal, \( U \) the spheropotential and \( N \) the undulation.

The former definition seems to the author to be more demonstrative, however, and is closer related to his studies than the latter one. The emphasis in the definition is on the clause "reduced or corrected in any or many ways" and on the adjective "corresponding".

In classical geodesy "corresponding value" means the value at the corresponding ellipsoidal point (which is actually on the spherop). The point is where the normal to the ellipsoid passes through the point on the geoid, reached by following the curved plumb line from the surface point. The angle, at the geoid, between the two lines here mentioned is called the deflection of the vertical \( \theta \). Usually its components are used; \( \xi \) along the meridian and \( \eta \) along the prime vertical. The deflection can be ignored for purposes of reducing the point from the geoid to the ellipsoid [89, p.22].

Moreover, in classical geodesy the above quoted clause will actually reduce the observed gravity to the geoid and will determine the specific meaning and the name assigned to the gravity anomaly obtained. Thus, for instance, reducing the value of \( g \) with a free-air reduction will yield a free-air anomaly. Similarly, further application of Bouguer and isostatic corrections will result in isostatic anomaly, etc.

A discussion of the different systems of anomalies is found in so many places, that the author feels obliged not to try it here. Only a short survey
Gravity anomalies are the outcome of comparison between theoretical and observed gravity values and constitute the essential material for computations of related quantities like undulations, deflections, etc. Here, again the literature is too extensive for the author to write about. Reference is made to [3], [11], [17], [41], [52], and [85].

One important remark, though, is that no matter what the anomaly system used, the final results for the physical value of the undulations and the deflections will be the same. This remark should be qualified by saying that the conditions necessary for the successful application of Stokes' method are fulfilled or accounted for by the indirect effect, etc. [23, p. 37], [41, p. 149], [3, p. 418], and [76, p. 5]. To recapitulate, the conditions are:

1) There are no masses outside the geoid, yet — in case masses are theoretically transferred —

2) There is no change in the total mass.

3) There is no change in the location of the center of gravity, and

4) There is no surface deformation of the geoid.

2.2 Free-air correction

Gravity on the surface of the earth has a considerable vertical gradient (about -0.3 mgal/m). The free-air correction \( \delta g \) accounts for the elevation
of the station above the reference surface in the sense that it gives the effect of this elevation on the gravity value. It does not yield the actual change in gravity, had it been physically measured at the reference surface, unless the route between the points is short and in vacuum.

The transfer of the observation point to the reference surface is in "free air." This is part of the Stokes' requirements; namely, that there are no masses above the geoid. Therefore, the free-air correction is utilized in all the "classical" anomaly systems, as well as in some "modern" approaches (Hirvonon [20], Molodenski [46]).

The free-air correction should be taken in the field of the remaining masses within the geoid \( M^* \) [79, p. 10]. Moreover, it should be integrated along the plumb-line \( H \) in the form

\[
\delta g_{r,*,*} = \int \frac{\partial^2 g}{\partial H^2} \, dH, \tag{10}
\]

but this is seldom feasible [13]. The practical approximation is to use a constant gradient, and a mean one at that, or at least a regional value rather than the observed gradient [59, p. 188]. Some consideration should, nevertheless, be given to the local curvature of the geops [79, p. 11], [17, p. 149].

The free-air correction is usually taken as the derivative of the normal gravity,

\[
\delta g_r = \frac{\partial g}{\partial H}. \tag{11}
\]

This can be represented as a series or otherwise [6]. For a specific reference ellipsoid, for instance, the International Ellipsoid [20, p. 30],
\[
\delta g = -0.3085507H - 0.0002270H \cos 2\varphi_0 + 0.0000005H \cos^2 2\varphi_0 \\
+ 0.00007254H^2 - 0.00000011H^2 \cos 2\varphi_0 + \ldots, 
\]  
(12)

with \(\varphi_0\) the latitude of the point and \(\delta g\) and \(H\) in mgal and meter (or gal and km), respectively.

The first term is the main one, and is sufficient for most computations. It can be easily verified by the data for a spherical earth, where the change in gravity is

\[
\delta g_S = \frac{\partial g_S}{\partial R} \, dR. 
\]

The gradient is

\[
\frac{\partial g_S}{\partial R} = \frac{-2g_0}{R_0} \left(1 - \frac{3H}{2R_0} + \ldots\right), 
\]

(13)

\(dR\) is \(H\), and \(g_0\) and \(R_0\) are the attraction and the radius of the sphere at sea level (about 981 gal and 6371 km).

2.3 Some gravity anomaly systems

Gravity anomaly systems — whether proposed or in actual use — are numerous, but none of them directly satisfies all of Stokes' conditions, p. 11.

Of the few that come close to it in some respect, or are extensively used, four will be briefly surveyed below. These are the systems of free-air, Bouguer, isostasy and Rudzki anomalies. The reduction inherent in each is discussed, and its effects. The addition of the observed gravity and the subtraction of the normal gravity to obtain the specific anomaly are common to all systems. The reference surface is usually the geoid.
1. The free-air reduction \( \delta g \) combines the free-air correction with the terrain effect. It actually removes the topography above the reference surface, transfers the point (gravity) at the terrain to the reference surface, then replaces the mass of the topography as an infinitely thin layer at that surface. In practice, therefore, the resulting anomaly does not consider the effects of the topography and its compensation, or any permanent or varying effect of mass discrepancies within the earth.

2. The Bouguer reduction removes the topography above the reference surface to infinity, then reduces the point (gravity) to the reference surface. The removal of topography is usually done by accounting for the terrain effect, the Bouguer plate, and the curvature of the earth. The mass of the topography is never replaced back. The resulting anomaly considers the effects of the topography, therefore, but not its compensation, or other mass discrepancies.

3. In the isostatic reduction the mass deviations above or below the reference surface (topography or ocean) are replaced within that surface, where their assumed compensation takes place. The point (gravity) at the terrain is transferred to the reference surface. The resulting anomaly constitutes the effects of the mass discrepancies from the assumed isostatic system.

4. The Rudzki reduction replaces the topographic masses outside the reference surface within it in such a way that the potential on it remains unchanged. In this process the masses are changed, however. The point (gravity) is then transferred to the reference surface.
The first three systems above can be regarded as variants of the isostatic system, with regard to mass transfers. Namely, the Bouguer system is an isostatic system with infinite depth of compensation; the isostatic system is one with a finite depth of compensation, and the free-air system is one with zero depth of compensation. Since some sort of isostatic equilibrium does prevail [15, p. 19], the isostatic anomalies will generally be the smoothest and the free-air ones the roughest of the three.

The free-air anomalies are locally correlated to the elevation of the observation point, or to the relief of the area. The Bouguer anomalies are usually less correlated to the elevations, and mainly reflect the subterranean mass irregularities (including compensation effects); thus, they are suitable for geophysical exploration. The isostatic anomalies are the least dependent on elevations, and indicate effects of mass discrepancies unaccounted for by the compensation system.

Stokes' conditions are met as follows:

1. No masses outside the geoid — satisfied by all four systems.
2. No change in the total mass of the earth — satisfied by the free-air and isostatic systems only. Rudzki's system changes the mass a little, while the Bouguer one changes it considerably.
3. No change in center of gravity — closest agreement in the isostatic and Rudzki's systems, worse in free-air, worst in Bouguer.
4. No surface deformation on the geoid — satisfied by Rudzki's system only. The free-air, isostatic, and Bouguer systems bring about deformations on the order of a few meters, scores or hundreds of meters, respectively.
All the effects above mentioned should be accounted for before the correct values of the undulations and the deflections are found.

2.4 The free-air anomaly system

The expression for the free-air anomaly $\Delta g$ is:

$$\Delta g = (g_B + \delta g_{AB}) - \gamma_A^*,$$

with the observed gravity $g_B$, the normal gravity (at the spheroid) $\gamma_A^*$, and the free-air reduction from B to A, $\delta g_{AB}$.

The free-air anomaly system is outstanding among the many existing anomaly systems:

1. Computation of the free-air correction is relatively simple and easy.
2. The system closely satisfies Stokes' conditions; and
3. The system is relevant to the author's studies. It can be shown that certain "model earth" anomaly fields can approximate the free-air anomaly field, and, therefore, be used for prediction purposes. This point, which is the essence of the study undertaken, is further pursued on the next chapter, p. 23 on.

2.5 Prediction of gravity anomalies

2.51 Predictions in general

In order to fill-in gaps in the observed gravity field of the earth, geodetic science resorts to prediction procedures. The prediction of
gravimetric data, or (what it usually amounts to) the prediction of gravity anomalies, can be executed in several ways: Statistically or geophysically or combined. In the general survey that follows no distinction is made between prediction of point- or mean-value [67], or between the various anomaly systems, unless so specified. A few of the suggested methods are as follows:

1. Using zero isostatic anomalies or their equivalent (outdated).
2. Using line or surface-fitting techniques, or development in Fourier's series or spherical harmonics of the existing gravity anomaly field.
3. Using correlations with the size of the prediction compartment, or with elevations, heat-flow data, or autocorrelation.
4. Using spherical harmonics derived from the motion of satellites.
5. Using spherical harmonics derived from the elevations (and squares of elevations).
6. Using elevations and geophysical data about the distribution of densities and depth of the Moho.

A good review of some of the prediction methods can be found in [17], [31], [37], [61].

2.52 Predictions using gravity anomalies alone

The free-air gravity anomaly field changes very rapidly on the surface of the earth, as it is critically related and affected by close topographic
features and hidden mass distributions. Straightforward interpolation and extrapolation could yield doubtful results unless careful consideration to local effects is given, which is not inherent in the method. Even when the interpolation is based on neighboring squares, the expected accuracy of prediction is on the order of $\frac{1}{2}$ the standard deviation of the mean anomaly [12, p. 8]. The problem is less critical when prediction is made of the mean value for a large area, which then leads to the trivial case of the prediction zero for the earth as a whole.

Predictions based on available data can be aided by development into spherical harmonics.

In a development into spherical harmonics there is an optimal number for the degree of harmonics developed as a function of the number, location and grouping of the data points or the size of the square (if mean values are used). Usually developments were kept well below the 50th degree and include only a few of the zonals above degree eight. It would be worthwhile to mention that the higher harmonics of the gravity field do not necessarily decrease in magnitude and that some scientists doubt the total reliability of spherical harmonics development for prediction purposes. There is also the problem of inadmissible terms in the solution [81], [24], [39], problems of convergence on the surface of the earth, and more.
2.53 Predictions using correlations

Correlation between any two variables can be expressed through covariances and correlation coefficients [65, p. 4], see p. 114.

Correlation of mean free-air anomalies with the size of the compartment was studied in [21]. The results indicated that there is a critical size of the prediction square above which it is better to use a zero anomaly rather than a point anomaly as a representative. This size is about 2.6 degrees for the side, where the error of representation (E) exceeds the mean anomaly (G). However, it is better, then, to mean the point and zero anomaly. If there is not even one point anomaly in the square the function G will indicate the standard error for zero assumption. This function decreases to zero for the earth as a whole, which is reasonable yet trivial.

Correlation with the distance between the data points has been studied among others in [64], [67]; correlation with heat-flow in [32], and correlation with the elevation of the point in [53], [55], [65], [79, p. 22], [85, p. 20], with quite consistent results for free-air and Bouguer anomalies. The latter correlation method will not work, of course, for anomaly system which is independent of the elvation — like an ideal isostatic system might be. Even in imperfect systems it can be misleading in many areas where specific correlation patterns exist, like around islands, trenches, coasts, etc. [37, p. 30].

A good correlation-prediction technique should incorporate more than just elevations or distances: Perhaps a combination of them together with
Moho depths and density data will be the best; but then this is exactly the data unknown.

2.54 Predictions using spherical harmonics from satellite observations

Earth satellites are held in orbit by a balance between the gravity pull of the earth and the centrifugal force created by their movement. Perturbations in the orbit will be caused mainly by discrepancies between the theoretical field in which the satellite is supposed to be moving (spherical earth) and the actual gravity field of the earth. Gravity anomalies in a specific anomaly system (allowing for a specific reference surface) are caused by mass discrepancies from the model earth, which have also some perturbing effects on the orbit. Thus, it follows that spherical harmonics of the gravity anomalies can be related to the spherical harmonics derived from the motion of satellites (through some related assumptions) [31, p. 533].

Only general and broad outlines of the anomalies can be detected in this method, which is to say that only the lower harmonics will be reliable [32, p. 13], [75, p. 11]. Published coefficients do not usually exceed the 15th degree and agree reasonable well among publications [34, p. 476], [33, p. 5185], [57, p. 124]. Recently gravimetric and satellite data were compared in [35]. Kaula's findings admit variations in the quality between different solutions from satellite data and show that the arithmetic mean of the independent solutions is better than any of them.
Computations based on gravimetric data can not yet solve even for the lower degree spherical harmonics, however, due to unfavorable distribution of gravity stations on the globe. The lower degree terms from satellite orbits could be incorporated, therefore, in the analysis of the gravity field. One could either adopt them and find the higher degree terms from gravity material [32, p. 1], [84, p. 11], or one could establish a model based on them together with a model based on topography [83, p. 2], etc.

In this connection it is obvious that a combination of gravimetric, astrogeodetic and satellite data will yield a better solution than just the latter. This has been done among others in [30], [31].

2.55 Predictions using spherical harmonics from elevations

The known elevations of the surface of the earth (and their squares) can be developed into spherical harmonics \([H]_n\), [63], \([H^2]_n\), [26]. Then a model is established, based on these terms and on Airy's Isostatic System. This model allows computations of the coefficients for a spherical harmonics development of gravity anomalies and related quantities [26], [28], [69], [83], [86], [87].

This model is interesting since it is almost equivalent to an actual isostatic model, assuming that the main part in the anomalies is due to the topography and its compensation — yet it predicts anomalies without being
based on gravity material. Its results can be checked at any point or area where gravity material exists, without previously utilizing this data. Of course, problems related to the use of a spherical harmonics development still exist; namely, to what degree is the development reliable? Does it converge on the surface of the earth?

2.56 **Predictions using elevations and geophysical data**

The more data about the mass distribution in the earth's crust incorporated in the model, the more reliable and closer to the actual earth it becomes.

Topography and its compensation account for a big part in the gravity anomalies, so that these effects should be removed from the measured gravity field in order to obtain the net gravity anomaly field. The effect of topography and its compensation could, conversely, closely represent the free-air anomaly field [37, p. 28], [36, p. 5].

A much better situation would exist if — rather than just assuming a certain isostatic model — a model could be developed which uses actual depths of the Moho (where the density discontinuity in the crust occurs). Such a model is the subject of this study.

Even after accounting for the actual depth of the Moho, the problem of the density distribution itself is still open. More and more geological
factors are needed for a thorough and good analysis or prediction of gravity material [8, p. 130], [9, p. 85], [10, p. 2], [91].

3. EARTH GRAVITY MODEL

3.1 The general model

The gravity anomalies represent the discrepancies between the gravity field of the "model earth" (theoretical gravity) used and the (reduced) gravity field of the actual earth. These discrepancies are the unaccounted-for factors in the anomaly system. In a free-air anomaly system, neither the topography and its compensation, nor the hidden mass discrepancies in the crust are actually accounted for (since a condensation of the topography at the geoid practically restores its effect). Other factors are neglected too; such as daily mass transportations on or below the surface of the earth and attraction and tidal effects of heavenly bodies. The latter have a tertiary, negligible effect only, while the mass discrepancies have a secondary and the topography and its compensation have a primary effect.

 Examination of the relative contributions of the effects above to the gravity anomalies leads to a twofold practical conclusion:

1. If the topography and its compensation would be correctly accounted for, there will be left only the effect of the hidden masses which could then be investigated.
2. The free-air anomaly field of the earth could be closely approximated by computing the effect of the topography and its compensation in a suitable reference system.

Theoretical and practical formulas and devices which try to account for the topography and its compensation are numerous. Reference is again made to [17], [37].

The author studied a model which evolved from the second conclusion above and incorporated it into a completely computerized program. In principle, the model is based on mean elevations, mean depths of Moho, and mean densities for a certain size of compartment all over the globe. The program computes the corresponding mean values of the total effect of the topography $T$ and its compensation $C$.

The computed effect includes the attraction (disturbance) and Bruns' term caused by mass discrepancies from the theoretical "model earth" chosen, which is spherical, nonrotating, and has specific depths and densities for the mean crust, the subcrust, and the ocean-water. Neglecting the rotation of the earth and its bulge is permitted, since their effects are effectively canceled out in the observation and computation process yielding the (free-air) anomaly. The effects of the mass discrepancies deduced from the topography and Moho and their densities, superimposed on the model earth, will thus describe a "model anomaly" field. Model anomalies are not strictly gravity anomalies (as used in the classical geodesy), even though both disturbance and Bruns' term are incorporated into them, because no comparison is
made between an observed (reduced) and theoretical gravity. The term should be regarded, therefore, as descriptive only (or between quotation marks) throughout the text. Similar approach should be taken toward other model-derived quantities, such as "model gravity," "normal gravity of model sphere," in the proceeding subheadings.

If the model chosen to represent the mean earth is suitable, the model anomalies would closely represent the free-air anomalies of the actual earth (depending also on the gravity formula used).

Using reliable Moho values, or crustal thicknesses, has an advantage over any isostatic system since this is exactly (part of) the information missing in the latter. When using isostasy, the depth of the Moho is computed through the equilibrium theory, which could not, beforehand, account for under- or over-compensation or regional compensation as the actual Moho indicates.

The problem of the density is the same in both approaches.

A somewhat similar study has been suggested in [87, p. 4].

3.2  Constant-density model A

3.21  Introduction of the model

The first model to be tried was one with constant density of the crust. The practical approach to the model is illustrated in Figure 1. All continuous surfaces are represented by distinct area means of 5° x 5° (or other size).
Earth model A; Constant density

The data includes mean elevations above the geoid (AB), and mean crustal thickness values (EB) or depth of the Moho (EA). Reference name of this model is A.

The model earth consists of a sphere (OC), whose concentric homogeneous layers increase in density from a model $\delta_{sub}$ towards the center, and of a spherical-shell crust (CA) with a constant density $\delta_{crust}$. The topography (or ocean, AB) projects above (or below) the geoid which
envelopes the crust and has its specific density discrepancy from $\delta_{\text{crust}}$. Similarly the compensating layer between the model subcrust and the actual Moho (CE) has its density discrepancy from $\delta_{\text{sub}}$.

With the data above, the model attraction due to the topography, $T$, and due to the compensation layer, $C$, can be computed at any point $B$, for every value of crust (AC). The effect $T$ is computed with reference to the geoid (considered a sphere in the model), and the densities of the crust and water. The effect $C$ is similarly computed with reference to the model subcrust (a sphere). This leaves out the effect of the crustal shell (AC) and the inner sphere (OC), which amounts to the bigger part of the total attraction at point $B$. This practice is justified in the next subheading.

Fixing the depth of the model CRUST (AC) is somewhat arbitrary, but it would make sense to put it where the mean Moho falls as shown in the next subheading. It should be remembered, though, that a change (CD) of $\pm 1$ km in the depth of CRUST, at a depth of 30 km produces a spherical-shell effect of about $\pm 50$ mgals at the surface point $B$ (depending on the densities used). It could vary by at most 0.5 mgal at various surface points, due to their elevation. The effect should be considered in the actual model if the layer (CD) exists; namely, when the model subcrust does not coincide with the mean Moho of the earth. See p. 88.
3.22 The model and free-air anomalies

From expression (14), p. 16, for the free-air anomaly \( \Delta g \), the value of the observed gravity at \( B \), \( g_B \), can be written as

\[
g_B = (\gamma_A - \delta g_{AB}) + \Delta g. \tag{15}\]

Here \( \gamma_A \) is the value at \( A^* \) (the spheroid) of the normal gravity for the sphere \( ODA^* \) (the middle letter designates the location of the subcrust in the system, Fig. 1). The free air reduction \( \delta g_{AB} \) can be substituted by the free-air correction in flatland, or at any case when strictly \( 5^\circ \times 5^\circ \) mean elevations are used, since the terrain effect is small. Its notation is in the sense

\[
\delta g_{AB} = g_B - g_A, \text{ in free air.}
\]

Another quantity can be found, \( \bar{g}_B \), which is the total attraction, or model gravity at \( B \):

\[
\bar{g}_B = \gamma_B + cd \bar{F}_B + (T + C)_B. \tag{16}\]

Here \( \gamma_B \) is the value at \( B \) of the normal gravity for the model sphere \( OCA \) (with its CRUST), and \( cd \bar{F}_B \) indicates the effect of the shell CD at \( B \). The term \( (T + C)_B \) is the attraction effect of the topography and model compensation as discussed in the previous subheading.

Expanding (16) one obtains

\[
\bar{g}_B = (\gamma_A - \delta \bar{g}_{AB}) + cd \bar{F}_B + (T + C)_B. \]
Note that the $\gamma_A$ is at the geoid point A. The term $\delta g_{BA}$ is the free-air reduction between B and A for the model sphere. This reduction can be replaced by the regular free-air correction, and also the effect of the shell CD at B approximated by that one at A (which is allowed with accuracy better than $\pm 0.5$ mgal for $\pm 1$ km in CD, p. 27).

When these steps are taken one obtains

$$\overline{g_B} = (\gamma_A - \delta g_{BA}) + c_0 F_A + (T + C)_B.$$  \hspace{1cm} (17)

But $\gamma_A = \gamma_A* - BR$; namely, the value at the spheroid differs from the value at the geoid by Bruns' term, $-BR = \delta \gamma_A*$. Inserting the last relation in (17) yields

$$\overline{g_B} = (\gamma_A* - \delta g_{BA}) + c_0 F_B + (T + C)_B - BR.$$  \hspace{1cm} (17')

Comparing this with equation (15) one finds

$$(T + C)_B - BR = \Delta g + (\overline{g} - g)_B - c_0 F_A,$$ or

$$\Delta \overline{g} = \Delta g + (\overline{g} - g)_B - c_0 F_A.$$  \hspace{1cm} (18)

Thus the relation between model anomaly $\Delta \overline{g}$ and free-air anomaly $\Delta g$ is established.

The model anomaly is composed of the disturbance and Bruns' term,

$$\Delta \overline{g} = (T + C - BR).$$  \hspace{1cm} (19)

The term $(\overline{g} - g)_B$ in equation (18) is the difference between the model gravity and the actual gravity, which is the effect at B of the unaccounted factors,
secondary and tertiary in nature (as on p. 23). It can be neglected for most practical purposes. If one uses a standard system for the model, instead of distinct Moho values and model subcrust, one obtains zero for the effect of the spherical shell (since it does not exist),

\[ c_0 F_A = 0. \]

In that case also equation (18) yields

\[ (\varepsilon_0 - g_0) = -\Delta g_\infty. \]  \hspace{1cm} (20)

This result is in agreement with [92, p. 103].

The term \( F \) in itself is constant for every point (or zero if the model crust coincides with the mean Moho), and can be easily accounted for (or eliminated). The correction term \( BR \) is Bruns' term for the model, and amounts to the total differential of \( \gamma \) with respect to the elevation difference \( A^*A = N \). It can be replaced by the free-air correction for the mean earth, 0.3086 \( N \), if the value of the model (geoid) undulation is known. Otherwise, and generally, it can be computed from the disturbing potential (of the mass discrepancies) \( \Delta V \) as

\[ BR = 2\Delta V/R, \]  \hspace{1cm} (21)

with \( R \) the mean radius of the earth. This Bruns' term [85, p. 32] converts the model disturbance \( (T + C)_0 \) into the model anomaly \( (T + C - BR)_0 \), as in (19).
Model anomalies for this constant-density model A, using the available Moho data, did not seem to represent the free-air anomalies too well. Some results are demonstrated in 4.331, p. 124.

3.3 Distinct-densities model B

3.3.1 Introduction of the model

After the constant-density model A, another model was introduced, which uses distinct densities. The practical approach to this model is illustrated in Figure 2. Mean elevation and Moho for all $5^\circ \times 5^\circ$ are represented, together with distinct values of densities. How to obtain the latter, if not previously known, is discussed in 3.85, p. 90. Reference name of this model is B.

Similar to the previous model, the model earth consists of a sphere (OF), whose concentric homogeneous layers increase in density from model $\delta_{ub}$ towards the center, and of a spherical-shell crust (FA) with a constant density $\delta_{rust}$. The topography (AB) projects above (or below) the geoid which envelopes the crust, and has its specific (distinct) density discrepancy from $\delta_{rust}$. Similarly, the compensating layer between the model subcrust and the actual Moho (FE) has its density discrepancy from $\delta_{rust}$ [for F deeper than the actual Moho—as in Figure 2]. But, unlike model A before, there is another density discrepancy to be considered; namely, between the model $\delta_{rust}$ and the distinct $\delta_1$ along (EA).
Figure 2

Earth model B; Distinct densities
With the above data one can compute the model attraction effects of
the shaded layers in Fig. 2: The crustal layer $T_1$ above sea level (AB) and
$T_2$ below sea level (AE), or—at sea—that of the water and the crustal layer.
The model attraction effects of the subcrust $S$ (EF) above a certain constant
depth $F$ is also included. The level $F$ should be where the masses (or pres-
sures) are in equilibrium, or deeper—at any rate below the deepest Moho
value, if densities are computed from some equimass or equipressure rela-
tions. Instead of computing total attraction effects of the shaded layers, dis-
crepancies from a simple mean model can be considered only. In this case
the sphere $F$ will not always be below $E$, but will be identical to the sphere
$D$ in Figs. 1 or 2. The entire procedure is justified in the next subheading.

3.32 The model and free-air anomalies

The procedure is similar to this for the constant-density model $A$, p. 28. The analysis is first made with the assumption that all distinct
densities have the same value. Accounting for the differences comes later.

The normal gravity $\gamma_A^*$ at $A^*$ (the spheroid) can be separated into
two components: $\bar{\gamma}_A$, the normal gravity of the inner sphere $OF$ at $A^*$ and
$\rho_A \bar{\gamma}_A^*$, the attraction effect of the shell $FA$ at $A^*$. [A third component, due
to the spherical shell of thickness $CD$ between the model subcrust ($F$) and
the mean Moho ($E$) is set to zero beforehand, since the model subcrust and
mean Moho do coincide. This is similar to the procedure with model $A$,
p. 30.]. With these components, equation (15), p. 28, becomes
Further expansion of the free-air correction yields

$$g_B = o_B A_f + f_A G_A + f_A G_B - f_A G_{AB} + \Delta g.$$  \hspace{1cm} (22)

Here the new free-air corrections \(o_B G\), \(f_A G\) are the partial contributions of the sphere (OF) and the shell (FA), respectively.

The model gravity \(\bar{g}_B\) at B can be expressed as

$$\bar{g}_B = o_B G_B + f_A G_B + (T_1 + T_2 + S)_B,$$  \hspace{1cm} (23)

with \(o_B G\) the normal model gravity of the sphere (OF) at B, and  

\(f_A G_B\) the normal model gravity of the shell (FA) at B; and  

\((T_1 + T_2 + S)_B\) the model disturbance due to the shaded areas in Fig. 2 (after comparison to the model).

The term \(f_A G_B\) can be separated into its components;

$$f_A G_B = f_A G_A - f_A G_{AB} \quad \text{, and further to}$$

$$f_A G_B = f_A G_A* - f_A G_{A*} - f_A G_{AB}.$$  \hspace{1cm} (24)

Here \(f_A G_A*\) is the normal model gravity of the shell (FA) at \(A^*\), where

\(f_A G_{AB}\) is its corresponding free air correction from B to A, and  

\(f_A G_{A*} A^*\) is Bruns' term for the shell (FA) between A and \(A^*\).
Similar to (23), the term \( \overline{\gamma}_B \) can be separated into its components:

\[
\overline{\gamma}_B = \overline{\gamma}_A^* - \overline{\delta \gamma}_A^{*A} - \overline{\delta \gamma}_{AB} ,
\]

(25)

with \( \overline{\gamma}_A^* \) the normal model gravity of the sphere (OF) at \( A^* \),

\( \overline{\delta \gamma}_{AB} \) its corresponding free-air correction between \( B \) and \( A \), and

\( \overline{\delta \gamma}_A^{*A} \) Bruns' term for the sphere OF between \( A \) and \( A^* \).

When relations (24) and (25) are inserted in (23), one obtains

\[
\overline{g}_B = \overline{\gamma}_A^* - \overline{\delta \gamma}_{AB} + \delta A G_A^* - \delta A \overline{\delta \gamma}_{AB} + (T_1 + T_2 + S)_B - BR,
\]

(26)

with the notation for the complete Bruns' term

\[
BR = \overline{\delta \gamma}_A^{*A} + f_A \delta \gamma_A^* ,
\]

which amounts to

\[
BR = \delta \gamma_A^{*A} .
\]

Here, as before, Bruns' term converts the model disturbance \((T_1 + T_2 + S)_B\) into the model anomaly

\[
\Delta g = (T_1 + T_2 + S - BR)_B .
\]

(27)

Comparing equation (26) to (22), one finds

\[
\Delta \overline{g} = \Delta g + \overline{g} - g_B .
\]

(28)
Here, as before in model A, the term \((g - g)_b\) is the effect at B of the unaccounted factors.

Equation (28) provides the relation between the model anomalies and the free-air gravity anomalies (but see also p. 23).

Now the general case for this model should be considered — the existence of distinct crustal densities \(\delta_1, \delta_2\) for the column (BE), Fig. 2. In that case computation of the effects \((T_1, T_2\) and \(S)\) for each compartment will utilize the corresponding density discrepancies between the distinct \(5^\circ \times 5^\circ\) mean density and the model densities; namely, \((\delta_1, \delta_1 - \delta_{\text{crust}}\) and \(\delta_{\text{sub}} - \delta_{\text{crust}}\), respectively.

Results for model B anomalies are analyzed in 4.332, p. 127.

3.4 Models with two densities

The actual density of the crust probably increases with depth. So it would be a better model which considers more than one distinct density for each \(5^\circ \times 5^\circ\) element. Multiple-layered models could be numerous, and rather complicated to work with. Two modified two-layered models were studied as feasible models, and will be mentioned below. Consider Figure 3.
The two models differ from the previous ones by that they have a part of the column with constant density $\delta_0$. In model C this part includes positive elevations only (AB), while in model D the constant density refers to a specific fraction of the crustal thickness (A'B). The distinct densities $\delta_1$ will not—as a rule—be the same in both cases. These models will be discussed later in 4.333, p. 130, 134. Results of model D anomalies are also analyzed in 4.5, p. 142.
3.5 General

The more complicated models C and D, p. 37, are only an expansion of the models A and B, previously discussed. Computation is more laborious, but no theoretical difficulties are added.

Equation (18), p. 29, or (28), p. 35, or a similar one is the theoretical basis for the specific model used. It means mathematically that the effect of the topography and its compensation, together with Bruns' term, computed from the model at surface points B, can closely approximate the free-air anomaly field (after constant—or slightly varying—terms $F_b$ are considered).

The comparison between model anomalies and mean observed free-air anomalies should be made with caution, and might involve some difficulties, though, since the two anomaly sets (generally) arise from different systems. The difference can be expressed between the normal gravity for the earth (used for the free-air anomaly) and the model normal gravity (used for the model anomaly). Even if the model utilized does represent the mean earth, and therefore both anomalies seemingly refer to the same system — the model anomaly refers to the mean earth, while the gravity anomaly incorporates a theoretical gravity. The latter would change with the specific reference ellipsoid used.

All these difficulties can be resolved, however, when the residuals $V'$ between the two sets of anomalies are made to average zero on each latitude zone. This procedure eliminates all constant deviations, while the
effects of uncertainties in the (equatorial normal gravity $\gamma_e$, or the flattening $f$ of the ellipsoid which comprise the uncertainty in the) normal gravity $\gamma$, are indeed constant on each latitude zone, see p. 109. When such comparison is impossible, as the usual case of prediction is, the resulting model anomalies might have to be modified in some fashion, to correspond to — and represent — free-air anomalies for a specific gravity formula.

A significant meaning could be associated with the models A-D, regarding recent suggestions [5], [56], to compute the gravity anomalies at the surface point, which amount to correcting $\gamma$ by $\delta g$.

Remark: The computation point should always be at the surface. At sea, therefore, it is at mean sea level, and not at the top of the submerged crust.

All the models used throughout the text are grouped together on p. 91.

3.6 Introduction of a computer to the model

3.61 General use of the computer

The computation of model anomaly at each surface point $B$ should be extended over the entire surface of the earth. This task renders itself best to computations in compartments, or "squares" between meridians and parallels. And, most of all, it can be adopted for handling in a computer.

The advantages of the computerized "squares" system are the great speed of operation and the fact that all the quantities necessary for the
computations need to be set (or estimated) only once, for each square on the
globe; they do not vary with the computation point, as the case in other systems
is. The programming, checking, "debugging" and test-run processes
associated with the computerized system are tedious and unnerving, but once
the program is running, it has a fantastic power in itself, and can rather
easily be modified to fit varying situations or data. In other words,
establishing a model and setting up a workable program could in many cases
be considered as valuable as obtaining reasonable model — or confirmed —
results.

In general what the computer does in the following studies is to store
the data provided of mean elevation and crustal thickness, together with
some constants. Densities are fed-in, or computed according to the specific
model. Then it computes and stores the partial contributions of each square
to the model disturbance T and C at each point, together with Bruns' term.
Then it prints the results.

This is a very schematic picture and will be clarified throughout the
text. Note that the computation is done simultaneously for partial effects of
all squares, and not for the total at individual squares. This procedure has
a great meaning and value, expressed in computer-time. Several programs
for total point-by-point computation are described, too.
3.62 Basic computational techniques

3.621 Rigorous formulas for attraction

The computation point $P_0$ in Figure 4 is situated at the elevation $H_o$ above the sphere (radius $R_o$). The radius-vector to the point $P_0$ is

$$r = R_o + H_o,$$

and the radius-vector to any compartment at $P$ is

$$\rho = r - h,$$

with $h$ the elevation difference. The direction of the arrows for $\rho$ and $h$ indicates their positive reckoning, similar to [38], but not quite the same.

Figure 4

Configuration for attraction on a sphere
The mass element at $P$ in the spherical system is

$$dm = \delta \rho^2 \cos \varphi \phi d\phi d\lambda$$  \hspace{1cm} (29)$$

with $\delta$ the density, and $(\varphi, \lambda)$ the (lat., long.) of $P$.

The attraction $dF_0$ in the direction of $D$ of the mass $dm$ at $P_0$, according to equation (1), is

$$dF_0 = k \frac{\delta \rho^2 \cos \varphi \phi d\phi d\lambda}{D^2}.$$ \hspace{1cm} (30)$$

The attraction in the vertical direction $dF_r$ can be found from Figure 4, as

$$dF_r = dF_0 \frac{Z}{D}.$$ \hspace{1cm} (31)$$

One substitutes now

$$Z = (r - \rho \cos \psi),$$
$$D = (r^2 - 2r\rho \cos \psi + \rho^2)^{\frac{1}{2}},$$
$$\cos \psi = \sin \varphi \sin \varphi_0 + \cos \varphi \cos \varphi_0 \cos (\lambda_0 - \lambda),$$

with $(\lambda_0 - \lambda)$ the longitude difference and $\psi$ the angular distance between $P_0$ and $P$. Putting in the integral form with the proper borders of the attracting compartment $\varphi_1$ to $\varphi_2$ in latitude and $\lambda_1$ to $\lambda_2$ in longitude, one obtains the vertical attraction $\Delta F_r$ at $P_0$ as

$$\Delta F_r = k\delta \int_{\rho_1}^{\rho_2} \int_{\phi_1}^{\phi_2} \int_{\lambda_1}^{\lambda_2} \rho^2 \cos \varphi \left[ \frac{r - \rho [\sin \varphi \sin \varphi_0 + \cos \varphi \cos \varphi_0 \cos (\lambda_0 - \lambda)]]}{[r^2 - 2r\rho \sin \varphi \sin \varphi_0 + \cos \varphi \cos \varphi_0 \cos (\lambda_0 - \lambda)] + \rho^2} \right]^{\frac{3}{2}} d\lambda d\phi d\rho.$$ \hspace{1cm} (32)$$

\hspace{1cm} d\lambda d\phi d\rho.
The integration and summation of $\Delta F_r$ for all the squares will comprise the total vertical attraction $F_r$ at the point $P_0$. Depending on the elevation and density of the square, the term $F_r$ will be either T or C (or $T_1$, $T_2$ or S), at $P_0$. The same procedure should be repeated for all the points $P_0$ on the surface of the earth. Note that $\rho$, $h$, $\varphi$ and $\psi$ vary from point $P$ to another; $h$ and $\psi$ vary with each computation point $P_0$ also, but the size ($\Delta \varphi \times \Delta \lambda$) of the compartment at $P$ could be kept constant (in degrees). Keeping the size constant might render some computations easier to perform and is the usual practice in global studies; likewise in the model. Varying the size of the compartment ($\Delta \varphi \times \Delta \lambda$) at different latitude zones might have the advantage that effects of the compartments could be made to approach the same order of magnitude. It has been used, for example, in [23].

The explicit solution of the integrals from equation (32) is very complex and probably altogether impossible [17, p. 182].

### 3.622 Mass-line formulas for attraction

Integration of the attraction of a compartment bordered by latitudes, longitudes and radius-vectors does not produce explicit mathematical expressions. For practical reasons, therefore, the volume of the compartment above is replaced by the volume defined by the solid angle $d\sigma$ at O which is equivalent to $(\Delta \varphi \times \Delta \lambda)$ and by the same boundary radius-vectors $\rho_1$ and $\rho_2$. Figure 4,
Then the integration is performed on the mass line from $\rho_1$ to $\rho_2$. This procedure, which follows [38], is justified as long as the effects of the size, the shape, and the height of the square are negligible. Idealizing and extending the power of the computer, one can theoretically assume that the compartments can be subdivided in such a way that the condition is met. Otherwise the size-and-shape effects should be investigated and accounted for, of course. Some practical problems on this line are resolved in the text.

Another approximation could be to compute the effects of the mass as a column in a rectangular coordinate system [22], but this will be rather complicated on the sphere.

In the mass-line system, the element of mass is

$$dm = \delta \rho^2 \, d\rho \, d\sigma.$$  \hspace{1cm} (33)

Note that the integration is made with respect to $\rho$ only. Following (30) and (31) one obtains

$$\Delta F_r = k\delta \, d\sigma \int_{\rho=\rho_1}^{\rho=\rho_2} \frac{\rho^2 \left( r - \rho \cos \psi \right)}{(r^2 - 2rp \cos \psi + \rho^2)^{3/2}} \, d\rho.$$  \hspace{1cm} (34)

The two integral forms needed for the solution of (34) — both with the same denominator, but different numerators — can be solved. The results follow.
\[\Delta F_r = k_0 d|\sigma [\frac{-3r^2 \cos \psi + 6r_0 \cos^2 \psi - r_0 - \rho^2 \cos \psi}{(r^2 - 2r_0 \cos \psi + \rho^2)^2} + \rho_2 \ln \{(r^2 - 2r_0 \cos \psi + \rho^2)^{1/2} + \rho - r \cos \psi\}]\] 

Using \(y_{1,2} = -h_{1,2}/r\)

and \(X = \sin \frac{\psi}{2}\), expression (35) can be converted into

\[\Delta F_r = k_0 d|\sigma \left[ \frac{(1 - 16X^2 + 24X^4) + (3 - 20X^3 + 24X^4)y + (-1 + 2X^2)y^2}{(4X^2 + 4X^2y + y^2)^{1/2}} y_2 \right.\]

\[- 2 \left(1 - 6X^2 + 6X^4\right) \ln \{(4X^2 + 4X^2y + y^2)^{1/2} + y + 2X^2\} \}

Inserting the limits, with

\(T_{1,2} = 4X^2 + 4X^2y_{1,2} + y_{1,2}^2\)

and \(i = (T_2/T_1)^{1/2}\) and \(j = y_1/y_2\) one obtains:

\[\Delta F_r = k_0 d|\sigma \left[ \left(1 - 16X^2 + 24X^4\right) (1 - i) \right.\]

\[+ (3 - 20X^3 + 24X^4) (1 - ij)y_2 \]

\[+ (-1 + 2X^2) (1 - ij^2)y_2^2 \] \left[ \frac{1}{\sqrt{T_2}} \right.\]

\[- 2 \left(1 - 6X^2 + 6X^4\right) \ln \frac{\sqrt{T_2} + y_2 + 2X^2}{\sqrt{T_1} + y_1 + 2X^2} \].

The formulas (35), (36), (37) correspond to Kukkamäki's (4), (6), (7) [38, p. 6].

3.623 Mass-line formulas for potential

The potential of the mass element dm at \(P_0\), Figure 4, p. 41, is given as

45
 Integration of equation (38) will yield the partial contribution $\Delta V$ of
the mass element $dm$ to the disturbing potential $V$. In order to obtain
expressions similar to those of the attraction (35), (36), (37), the mass
line procedure is introduced again. Substituting the proper terms in expression
(38) one obtains

$$
\Delta V = k \delta \, d\sigma \, \int_{\rho=\rho_1}^{\rho_2} \frac{\rho^2}{(r^2 - 2 \rho \cos \varphi + \rho^2)\sqrt{2} \, \rho^2} \, d\varphi.
$$

Equation (39) corresponds to Kukkamäki's (8) [38, p. 7], and its solution
gives

$$
\Delta V = k \delta \, d\sigma \left[ \frac{1}{2} (\rho + 3 \cos \psi) (r^2 - 2 \rho \cos \psi + \rho^2) \right]^{\rho_2}_{\rho_1}
+ \frac{r^2}{2} (3 \cos^2 \psi - 1) \ln \left\{ (r^2 - 2 \rho \cos \psi + \rho^2) \sqrt{2} + \rho - r \cos \psi \right\}.
$$

Using $y_1, z = - h_{1,2}/r$

and $X = \sin \psi/2$, as on p. 44, one obtains

$$
\Delta V = k \delta \, d\sigma \, r^2 \left[ (2 + y/2 - 3 X^2) (4 X^2 + 4 X^2 y + y^2) \sqrt{2} \right]^{y_2}_{y_1}
+ (1 - 6 X^2 + 6 X^4) \ln \left\{ (4 X^2 + 4 X^2 y + y^2) \sqrt{2} + y + 2 X^2 \right\}.
$$

Expressions (40) and (41) for the potential, are very similar to
expressions (35) and (36) for the attraction. They could easily be incor-
porated together, which will be required for the model anomaly, as in
equations (19),(21), pp. 29-30.
3.624 Working formulas

In formula (37), the terms $T$ are always positive and (37) holds even when $X = 0$ with negative $y_1$ and $y_2$. This can be verified through L'Hopital's procedure of differentiations [1], or by checking the resulting formulas for the attraction of a spherical shell. Formula (37) has not been used in this study, however, due to a flaw inherent in the form of the term $j$. The elevation difference $h_2$ will be zero for compartments that have the same elevation as the computation point (when the attraction of the topography is concerned); thus yielding $y_2 = 0$. The computer will then be required to determine the value for $j = y_1/0$. The O.S.U. IBM 7094 fails to give the right answer; it substitutes instead $j = y_1$.

For computation of the model disturbance either (35) or (36) could be used. Some preliminary investigation and programming has been done on (37); and, therefore, the final working formula was (36), which is quite similar to (37) in construction. Formula (35) has been used for checking purposes later and proved to be a little inferior to (36).

For the potential computation (or model anomaly), equation (41) which corresponds to (36) was used.
3.63 Related computational techniques

3.631 Size of squares and borderlines

The size, shape and possibly the height of the compartment should be accounted for, once they affect the anomaly computed, or else the working procedures should be changed to achieve independence.

When the distance between the squares becomes small (as in the case of neighboring squares on the same parallel, especially close to the poles, or when computing the effect of the square on itself), the above-mentioned factors cannot be ignored.

One approach to the problem will be to have smaller and smaller squares, with decreasing distance. Test areas and theoretical test runs have suggested that from an angular distance of about 4° inward, 5° x 5° squares should be cut into 2.5° x 2.5°, or smaller squares. To clarify the problem, consider Figure 5. Consideration is made of attraction only, since it constitutes the main part in the anomaly.

The computation point A and the compartments T₁, C₁ are on the same parallel close to the equator E, or close to the pole, P. The compartments have the same (Δφ° x Δλ°). (The compartments have been projected to show their varying sides on the parallel S₁ and angular distance ψ₁. The other side of the compartment, on the meridian, is constant). The attraction vector will not be the same for cases E and P, the latter being bigger. The reason for that is that the volume (approximately) decreases with cos φᵣ, and its
distance $D_p$ also decreases (approximately) with $\cos \varphi_p$; therefore, the total effect from equation (1), p. 3, will be an increase of (approximately) $1/\cos \varphi_p$. The vertical attraction will, however, be affected by the direction of the vector also. This effect is very critical especially close to the pole. There the almost horizontal direction of the attraction from $T_p$ will cancel only a small part of the attraction of $C_p$. The same holds for the subdivision of the square: Differences will express themselves much more in the effect of $C_p$ than in that of $T_p$. The absolute value through $5^\circ \times 5^\circ$ computation is always too big, compared to results from a smaller size of compartments.

The same arguments hold when computing the attraction of the square at the computation point inside it. The regular working formulas for a mass-line may yield results, but these might not be too realistic since they all do
not account for the distribution of the mass, or the shape of the square.

When dealing with the square itself a chopping procedure can be used. This will cover the area down to a specific angle $\theta$ from the computation point $P_0$, represented as the distance $d_\theta$ on the sphere, as in Figure 6. The inner zone is referred to as a "spherical cap" and computed accordingly. A good approximation of the circle, as well as the solution to the problem mentioned at the outset of this section will be achieved when the size of the secondary squares is small, compared with $d_\theta$.

Another procedure could be to represent the entire square by the equivalent spherical cap $D_\theta$, which has the same area. This would probably yield results similar to those of the mass line procedure.

Figure 6

A compartment on the sphere
Another total approach to the problem at the pole could be to try and keep the area of the compartment reasonably big, if not constant all over the globe. Trying the latter will involve difficulties, related to the nature of the sphere. Using compartments bigger than $5^\circ \times 5^\circ$ could be feasible [23, p.45], but will involve re-estimation of the data and modifications of programs. Arguments in favour of this approach are that the data itself at the poles is unreliable, and that the total effect of these areas on the rest of the world is small.

The very same argument can be mentioned in favour of the equal-$\Delta \lambda$ technique. Of course, one is getting in trouble at the poles in either way. Considering the unreliable data at the poles, and its small effect on zones close to the equator, one is lead to the practical solutions as follows:

1. Not to compute the effects of the polar compartments on each other within the polar zone, or on themselves, but rather to replace all of them with a spherical cap. The cap will have the mean elevation of the compartments in question, and the result of the anomaly caused by it will represent the individual values at the squares. This result, with the degree of approximation inherent will probably be better than

2. To take the subdivision procedure straight forward. Just the amount of test-runs required in order to determine the proper borderlines is so big, that this procedure was dropped.

Instead of endlessly subdividing the square, the main program in this
study was modified to compute at one stage the model anomaly effects of each
and every $5^\circ \times 5^\circ$ on all the others, except the reciprocal relations at the polar
zones. This allows a later combination of a cap effect, or $5^\circ \times 5^\circ$, or a finer
procedure or combinations to one's liking. The entire computation of the effect
of the polar zones on themselves can be omitted, if concern is not of what is
happening at the poles.

3.632 Size of the spherical cap

The attraction and potential of the spherical cap can be computed as a
function of $\theta$, the central angle of the cap, Figure 6, p. 50. The value assigned
to this angle is irrelevant to the solution, except for two important practical
considerations, which do restrict the choice:

1. It would be very interesting to assign to $\theta$ the constant value of the border
   of zone O in the Hayford system [17, p. 161]. Then the model could
   compute at one stage the total attraction of the outer zones (18-1). Since
   there exist maps of the latter in the Airy-Heiskanen system, $T=30$ [29],
a valuable comparison is feasible. The values obtained in this fashion
   could be very useful for prediction purposes, as [29] is. But at the same
time,

2. The entire cap should be contained in one compartment, otherwise problems
   of varying elevations and borderlines arise.

Both conditions can be met, using $5^\circ \times 5^\circ$ squares, as long as $|\varphi_2| \leq 55^\circ$.

52
Thus, it has been decided to use a constant $\theta^{\text{rad}} = 166.700/R_0^\text{m}$, for that zone, which will allow the comparison mentioned in number one. For $|\phi| > 55^\circ$ the $\theta$ varies with each particular $\phi$, and its value is set as

$$\theta^\circ = 2.4 \cos \phi,$$

which is just contained within the compartment of $5^\circ \times 5^\circ$ at that latitude, and no comparison to [29] is possible.

### 3.633 Working formulas for spherical cap

A cone $\theta$ intersects a spherical shell with radii $\rho_1$ to $\rho_2$, Figure 7. The notations $\rho_1$, $\nu$ indicate radius vector and angle from the origin $O$. The mass element $dm$ is on the ring $PP$ with its radius vector $\rho$ and angle $\nu$. The vertical attraction $dF_0$ at $P_0$ of the mass element is sought first, and then the total attraction $\Delta F$ there (which is vertical). [Later on, the potential is developed also, and the attraction from it].

![Figure 7](image-url)

**Figure 7**

Spherical cap
The mass element in Figure 7 is a ring with

\[ dm = 2\pi \delta \rho^2 \sin \nu \, d\rho \, d\nu \quad (42) \]

and the attraction, similar to equation (1), p. 3, is

\[ dF_D = \frac{k \cdot 2\pi \delta \rho^2 \sin \nu \, d\rho \, d\nu}{D^2} \quad (43) \]

Using relation (31), p. 42, one substitutes \( Z = (\rho_0 - \rho \cos \nu) \),

\[ D = (\rho_0^2 + \rho^2 - 2 \rho_0 \rho \cos \nu)^{1/2} \]

and putting in the integral form one obtains

\[ \Delta F = 2\pi k \delta \int_0^\theta \int_{\rho_0 - \rho_0 \rho \cos \nu}^{\rho_0 + \rho_0 \rho \cos \nu} \frac{\rho^2 \sin \nu}{(\rho_0^2 + \rho^2 - 2 \rho_0 \rho \cos \nu)^{3/2}} \, d\rho \, d\nu \quad (44) \]

Solution to the inner integral with respect to \( \rho \) is not available in the literature, and it is probably unsolvable. Consideration of the smallness of \( \Delta \rho / \rho \) allows a good and easy approximation. It is based on the mathematical properties of the integral as follows:

Suppose \( f(\rho) \) is a function of \( \rho \). If \( \Delta \rho \) is small enough, one can use the mean radius \( \rho_{12} = (\rho_1 + \rho_2)/2 \) and substitute

\[ \int_{\rho_1}^{\rho_1 + \Delta \rho} f(\rho) \, d\rho \approx f(\rho_{12}) \Delta \rho \quad (45) \]

which amounts to taking the area measured by the integral to be rectangular.
Using this approximation one obtains

\[ \Delta F = 2\pi k \delta \rho_2^2 \Delta \rho \int_{\nu=0}^{\theta} \frac{(\rho_0 - \rho_{12} \cos \nu)}{(\rho_0^2 + \rho_{12}^2 - 2\rho_0 \rho_{12} \cos \nu)^{3/2}} \sin \nu \, d\nu , \]  

(46)

which is solved immediately as

\[ \Delta F = 4\pi k \delta \rho_{12}^2 \Delta \rho \left[ \frac{1}{a (a \cos \nu + b)} \right] \left\{ \rho_0 + \rho_{12} \left( \cos \nu + \frac{2b}{a} \right) \right\}^{\theta} \nu=0 . \]  

(47)

This solution, with \( a = -2\rho_0 \rho_{12} \) and \( b = \rho_0^2 + \rho_{12}^2 \) is very nicely suited for computer uses. Note that subtraction between the brackets must be performed after the limits are substituted.

Formula (47) could be manipulated to a more explicit form,

\[ \Delta F = \frac{2\pi k \delta \rho_2^2 \Delta \rho}{\rho_0^2} \left[ 1 - \frac{\rho_2 \cos \theta - \rho_{12}}{(\rho_0^2 + \rho_{12}^2 - 2\rho_0 \rho_{12} \cos \theta)^{3/2}} \right] . \]  

(48)

The latter is close to the Bouguer effect of the plane (a case when \( \rho_0 \to \infty \)).

A graphical approximation can be obtained from Figure 7, p. 53, as

\[ \Delta F \approx (2\pi k \delta \Delta \rho) \left( 1 + \frac{X}{D} \right) . \]  

(49)

A special case of equation (48) appears in [26, p. 75], when \( \rho_2 = \rho_0 \), or the spherical cap reaches to the computation point. The equation given there reads (effectively):

\[ \cdots \]
It uses, however, the further approximation of \( \rho_2 = \rho_1 \), and is therefore less accurate. The derivation (48) given here by the author is more general, and more detailed.

In case the approximation (45) is expected to be too far off, numerical integration of (44) will be necessary. This will be done after the integration with respect to \( \nu \) was performed; namely, on

\[
\Delta F = \frac{2\pi k \delta A^2}{\rho_0} \left[ \frac{1}{(\rho_0^2 + \rho^2 - 2\rho_1 \rho \cos \theta)^{3/2}} \right],
\]

with

\[
A = -2\rho_0 \rho
\]

and

\[
B = \rho_0^2 + \rho^2.
\]

This form is again suitable for computer use.

The expression for the potential \( \Delta V \) of the spherical cap in Figure 7, p.53, is found readily using equation (38), p.46, It gives

\[
\Delta V = 2\pi k \delta \int_{\nu=0}^{\nu=\rho} \frac{\rho^2 \sin \nu}{(\rho_0^2 + \rho^2 - 2\rho_0 \rho \cos \nu)^{3/2}} \, d\nu \, d\psi.
\]

Solution of the inner integral with respect to \( \rho \) is possible [4, p.70], but the next integration seems hopeless then. Thus one must start integration with respect to \( \nu \) [4, p.61], then integrate with respect to \( \phi \). The solution is

\[
\Delta V = \frac{2\pi k \delta}{\rho_0} \left[ \left( \frac{\rho_2}{3} \rho_0 \cos \theta \right) + \frac{\rho_2}{3} \rho_0 \cos^2 \theta \right] \left( \frac{1}{3} - \frac{\cos^2 \theta}{2} \right) + \frac{\rho_2}{3} \rho_0 \cos \theta \left( \frac{1}{3} - \frac{\cos^2 \theta}{2} \right)
\]

with \( D_1 = (\rho_2^2 + \rho_1^2 - 2\rho_1 \rho \cos \theta)^{3/2} \).

At this stage it is clear that a rigorous, closed form for the attraction \( \Delta F \) is possible. It could be arrived at from (53) by differentiating the potential; namely,

\[
\Delta F = \frac{3}{\rho_0} \left( \frac{\Delta V}{\rho_0} \right).
\]

The differentiation is done with respect to the vertical direction at \( \rho_0 \), or \( \rho_1 \). The result is

\[
\Delta F = \frac{2\pi k \delta}{\rho_0} \left[ \left( \frac{\rho_2}{3} \rho_0 \cos \theta \right) + \frac{\rho_2}{3} \rho_0 \cos^2 \theta \right] \left( \frac{1}{3} - \frac{\cos^2 \theta}{2} \right) + \frac{\rho_2}{3} \rho_0 \cos \theta \left( \frac{1}{3} - \frac{\cos^2 \theta}{2} \right)
\]

with \( D_1 \) as before.
Equation (55) is a closed expression for the attraction of the spherical cap, believed to be original. It might be too cumbersome for actual computations, though, and could be replaced by an approximation like (47), or numerical integration like (51). A demonstration of the above is given on p. 97.

3.634 Computation of the distance

The angular distance $\phi$ between two points 1 and 2 on a sphere is given through the cosine law as

$$\cos \phi = \sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos (\lambda_2 - \lambda_1)$$  \hspace{1cm} (56)

when $\varphi_1$ is the latitude and $\lambda_i$ is the longitude of the point.

The accuracy of this formula decreases considerably when $\phi$ approaches zero. In cases like these another solution [72] gives better results:

$$\sin^2 \phi = [\sin (\lambda_2 - \lambda_1) \cos \varphi_2]^2 + [\sin \varphi_2 \cos \varphi_1 - \sin \varphi_1 \cos \varphi_2 \cos (\lambda_2 - \lambda_1)]^2.$$  \hspace{1cm} (57)

The quantity $X = \sin \frac{\phi}{2}$ used for equations (36), (37), p. 45, will be found from (56) as:

$$X = \sqrt{\frac{1 - \cos \phi}{2}},$$  \hspace{1cm} (58)

or, from (57) as

$$X = \sin \left[ \frac{1}{2} \arctan \sqrt{\frac{\sin^2 \phi}{1 - \sin^2 \phi}} \right].$$  \hspace{1cm} (59)
This is due to functions lacking in the computer.

Actual tests have been performed to determine whether the loss of accuracy is significant, in case the cosine formula is used. It seems that five significant figures can be obtained accurately, even at 1° separation. The effect of the 5° x 5° squares is computed using this formula. For the inner squares the sine formula is used. Double precision computations will give higher accuracy, but this might not be necessary for prediction purposes, and thus unadvisable to use.

3.635 Area of compartment

For computation of the attraction and the potential and for weighting purposes, the area of the compartment or square in question is needed. The area should be expressed in steradians.

The element $d\sigma$ of the area on a sphere, Figure 6, p. 50, is

$$d\sigma = \int_{\varphi_1}^{\varphi_1+\Delta\varphi} \int_{\lambda_1}^{\lambda_1+\Delta\lambda} \cos \varphi \, d\lambda \, d\varphi .$$  \hspace{1cm} (60)

For very small squares one can take

$$d\sigma \approx \cos \varphi \, \Delta\lambda \, \Delta\varphi ,$$  \hspace{1cm} (61)

with $\varphi$ somewhere between $\varphi_1$ and $\varphi_2$; but the exact expression is:

$$d\sigma = (\sin \varphi_2 - \sin \varphi_1) \, \Delta\lambda .$$  \hspace{1cm} (62)
3.636 Center of gravity of compartment

The computation of anomaly can be performed at any point, but if the mass-line method is to be used then the mass-line might be located along the center of gravity (C. G.) of the attracting compartment (but see p. 95). Since the computations are reciprocal, one might as well also compute at the center of gravity. For that purpose the knowledge of the C. G. is needed. The actual deviations in the anomaly, if the C. G. is replaced by the mean value of \( \varphi \) and \( \lambda \), might be negligible.
In Figure 8 the letters on the right indicate co-latitudes, on the left—latitudes. The flattened compartment will approximate the three-dimensional spherical segment for small $\Delta \varphi$ and $\Delta \lambda$. The co-latitude of the center of gravity will be computed first, then the latitude, from it.

The location of the center of gravity $C$ of a general figure, on an $X$ axis, can be found as the quotient

$$C_x = \frac{\int x \, d\sigma}{\int d\sigma},$$

with $x$ the coordinate of the (center of gravity of the) area element $d\sigma$ [4, p. 55].

For the computation below, the $X$ axis is the meridian $\lambda$. The figure whose center of gravity is sought is the shaded segment bordered by the co-latitudes $\beta_1$ and $\beta_2$, and by the longitudes $(\lambda - \Delta \lambda/2)$ and $(\lambda + \Delta \lambda/2)$. Distances, areas and moments are expressed in their proper angular units. The area element is located with $\beta$—the distance from the pole $O$, and $\alpha$—the angle from the $X$ axis at the pole.

The area element $d\sigma$ equals $\beta \, d\lambda \, d\beta$. The $X$ coordinate of $d\sigma$ equals $\beta \cos \alpha$.

The numerator in equation (63), the first moment, will therefore be

$$2 \int_{\beta=\beta_1}^{\beta_2} \int_{\alpha=0}^{\Delta \lambda/2} \beta \cos \alpha \, d\alpha \, d\beta = \frac{2}{3} \sin \frac{\Delta \lambda}{2} (\beta_2^3 - \beta_1^3), \quad (64)$$

and the denominator, the area, is
So the value for the co-latitude of C.G., from (63), is

\[
\beta_{c.g.} = \frac{2}{3} \frac{\Delta \lambda}{\sin^2 \frac{\Delta \lambda}{2}} \left( \frac{\beta^2_2 - \beta^2_1}{\beta_2^2 - \beta_1^2} \right).
\]  

(66)

The right hand side in (66) can be still rearranged a little. Noting that in case one uses constant \(\Delta \varphi\), with \(K\) the number of the parallel zone; \(\beta_1 = K \Delta \varphi\), and \(\beta_2 = (K + 1) \Delta \varphi\). Plugging these in (66) one obtains

\[
\beta_{c.g.} = \frac{4}{3} \sin \frac{\Delta \varphi}{\Delta \lambda} \frac{\Delta \varphi}{3K} [3K (K + 1) + 1]/(2K + 1).
\]  

(67)

Note that the location of the center of gravity for the figure (\(\Delta \varphi \times \Delta \lambda\)) is independent of \(\Delta \lambda\) for small \(\Delta \lambda\). This is how it should be, indeed. In the reverse case; namely, when the location varies with \(\Delta \lambda\), the solution will be wrong. This is obvious, however, since the solution made some approximations related to the size of \(\Delta \lambda\), in flattening the spherical figure, and could not handle a bigger \(\Delta \lambda\).

Accounting for the smallness of \(\Delta \lambda\), and also setting \(\Delta \varphi = \Delta \lambda\), one obtains from (67)

\[
\beta_{c.g.} = \frac{2}{3} \Delta \varphi [3K (K + 1) + 1]/(2K + 1),
\]  

(68)
and then

$$\varphi_{c.o.} = \frac{\pi}{2} - \beta_{c.o.}$$

Equation (68) gives the co-latitude of the center of gravity for the small compartment \((\Delta \varphi \times \Delta \lambda)\). The factor \(K\) is the co-latitude zone number of the compartment in the coordinate system used. It is usually an integer, but could be a mixed fraction if the specific location of the compartment does not fit the \((\Delta \varphi \times \Delta \lambda)\) system. The units of \(\beta\) and \(\Delta \varphi\) should match, but they could both be degrees, grads or radians. The proper value for \(\pi/2\) should be chosen accordingly.

The procedure for computation of the center of gravity is compact, easy, neat, and computer-oriented. A demonstration of results obtainable from the use of equation (68) is given on p. 93.

3.64 Summary of the procedure

The practical procedure for the computations, at a specific point, of the attraction and the potential of the many compartments on the globe is suggested as follows:

1. The inner zone of the compartment around the point is regarded as a spherical cap, Figure 6, p. 50. The limit of the cap is a distance of \(166.700/R_o\) \(\text{km}\) (if \(|\varphi_2| < 55^\circ\)),

or \(2.4^\circ \cos \varphi\) (if \(|\varphi_2| > 55^\circ\)).

2. The rest of the square is subdivided into secondary squares. A small enough division seems to be \(0.25^\circ \times 0.25^\circ\).
3. The effect of the 5° x 5° squares is computed reciprocally, all over the world (or up to a specific distance between squares), except

4. At the chosen polar latitude zones, where only effects of other zones are computed reciprocally and the inter-effects are omitted. Afterwards the latter can be added through 5° x 5°, 1° x 1°, or cap, etc. Otherwise complete omission of inter-effects at selected polar region can be achieved.

5. Some 5° x 5° squares, close to the computation compartment, might be subdivided (into 1° x 1°, for example). The limit for this operation could be set constant, or vary with the latitude.

Working formulas have been developed for the two cases involved (square or cap), and for the parameters utilized (latitude of C.G., area, and the distance).

This résumé is schematic-theoretic only, and does not necessarily represent the order of operations in the computer. For example, a certain relation between two points, distance-wise, might be investigated for pairs of points all over the globe — rather than the total effect at particular points with varying distances between the pairs. Also the problem of the densities was not mentioned here.
3.7 Computer programs

3.71 The distance

The computation of the distance comes before the computation of the model anomaly. In a straightforward approach to the problem, the tendency will be to compute the total anomaly at individual points. This is easy to visualize. One sets up a computation point, computes a distance to a compartment, finds the anomaly and adds it to the effect of the rest of them. When the summation of the partial effects covers the whole earth, the computation moves to the next point.

Such an operation has one advantage, that each point is computed independently of the others and can be so checked. By its nature it is the proper procedure for prediction. But a big disadvantage is that the number of distances computed will be very large. Distance computations involve taking trigonometric functions and a square root, which is time consuming. Even pre-tabulation and the use of $x^d$ in (36), p. 45, does not solve the problem completely.

The total number of $5^\circ \times 5^\circ$ compartments on a sphere is 2592. From each of these there are 2591 physical distances, to the rest of the squares, which totals 6,715,872. Of course, due to symmetry one can immediately realize that only a quarter of these are necessary (Figure 9). But even that is too much. The real number of different distances on a sphere (for $5^\circ \times 5^\circ$) is only 12,636.
To realize the number 12,636 one must remember that the pattern of 4 distances in Figure 9 can be rotated around the NS axis without creating any new distances. Thus, from the zone ±(90 - 85) there are 1331 distances to all the squares and the number decreases by $2 \times 37$ for each consecutive latitude zone until it reaches 73 at the zone ±(5 - 0). The total number in this series is 

$$\frac{18}{2} \times (36 + 2) \times 37 - 18 = 12,636.$$ 

On the same parallel zone there are 36 distances and between parallel zones there are 37.

The great advantage of the simultaneous-computations system over the point-computation system is the saving of time. The point method in a certain test run took about 0.067 minute per point for the total attraction excluding the
square itself. On two latitude zones it amounts to about 9.6 minutes, while the same zones took only 3.5 minutes to compute in the other method. The savings in time are considerable, indeed. More detailed time estimates are to be found in 4.13, p. 103. Disadvantages are the fact that its programming is much more complicated, and that it computes the total effect at all the points simultaneously, so to speak, and results are available at the end only. What the computer actually does is to set up 2 parallel zones and pick up a symbolic pair of squares on them (so that the increasing \( \lambda \) between them will go up to 180°, no more). Then it computes the distance and with this particular distance in memory, it searches around to locate the pairs of squares which can use this distance. Then it computes the attraction and potential reciprocally, between the pair, stores, and runs along. When the distance has been exhausted from complete usage whenever it occurs, it is discarded and a new distance is used.

3.72 Elevation and density

Ideally, if the density distribution were known, each square could have its own \( \delta \) value, for the topography and for the compensation layer (as well as for any mass deviation known of, or perhaps several density values, as well). Due to lack of data in that area the first model which was in use, the constant density model A, p. 25, used three standard densities, for the crust, the subcrust and the water. Actually in use were differences between densities — (water to crust) \( \delta_1 \), and (subcrust to crust) \( \delta_2 \), and the density of the crust, \( \delta_c \).
The program could be extended to handle distinct densities as in model B, as well as multilayered models C, D, which was the actual case here. For demonstration purposes only the model A above will be analyzed, as to the correct match of density with vertical boundaries.

From the data of elevation and crustal thickness (here indicated as MOHO) for any two compartments, the computer is supposed to come up with the terms \(\delta\) and \(h\), Figure 4, p. 41; and using \(X\) to compute \(\Delta F\) and \(\Delta V\) from the corresponding equations, then assign it to the right compartment as \(T\) or \(C\), Figure 1, p. 26.

In order not to deal with total attraction and potential, but with differences from a model only, a standard CRUST is introduced, Figure 10, (same as in Figure 1, p. 26). Its theoretical meaning is quite different from that of the normal crust in an isostatic system, since one already knows the depth of the Moho. The CRUST is not the theoretical basis for an isostatic system, whereupon depths of Moho are computed, but rather a practical convenience for the

![Diagram](image-url)

**Figure 10**

Elevation, MOHO, and CRUST
computation of the effect of the distinct (measured) Moho. It could be set as
the depth of the mean Moho. In its practical application, however, p. 88, the
CRUST is similar to the normal crust.

A good anomaly field should average zero on the whole earth. In this
model one could change the depth of the CRUST by CD to average the model
anomalies to zero, or to minimize the r.m.s. attraction, etc. All these
changes amount to just subtracting a constant number from each value, as a
change in the C contribution, p. 27.

At any rate, the values of the elevations and the Moho at the two
arbitrary compartments, together with the CRUST should determine the
specific δ and h terms to be used in (36). Reference is made to Figures 10
and 4, p. 68, 41, and to formula (36), p. 45. The two compartments can
interchange their roles as attracting element and computation point. The com-
putation point (subindex 0) can be either on land (positive elevation) or at sea
(negative elevation), in which case the computational elevation is taken zero.
The attracting compartment (subindex i) can likewise have + elevation, and its
Moho may be below or above the lower edge of the model CRUST, since one
deals with distinct values of Moho, not theoretical. All the possible cases can
be summarized in tabular form, Table 1.
Table 1

Determination of $h$ and $\delta$

<table>
<thead>
<tr>
<th>For $H_1$</th>
<th>For $H_0$</th>
<th>$T \geq 0$</th>
<th>$T &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_2 = H_o - H_1$</td>
<td>$-H_1$</td>
<td>$&gt; CRUST + H_1$</td>
<td>$CRUST + H_1$</td>
</tr>
<tr>
<td>$h_1 = H_o$</td>
<td>$0$</td>
<td>$H_o + MOHO$</td>
<td>$MOHO$</td>
</tr>
<tr>
<td>$\delta = \delta_o$</td>
<td>$\delta_o$</td>
<td>$\delta = \delta_2$</td>
<td>$\delta_2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For $H_0$</th>
<th>$&gt; 0$</th>
<th>$&lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_2 = H_o + CRUST$</td>
<td>$CRUST$</td>
<td>$MOHO$</td>
</tr>
<tr>
<td>$h_1 = H_o$</td>
<td>$&lt; CRUST + H_1$</td>
<td>$CRUST$</td>
</tr>
<tr>
<td>$\delta = \delta_1$</td>
<td>$\delta_1$</td>
<td>$\delta = -\delta_2$</td>
</tr>
</tbody>
</table>

1 $H_1$ is the elevation of the attracting compartment.
2 $H_o$ is the elevation of the computation point.
3 $h_i$ is the upper ($i = 2$) or lower ($i = 1$) border of attracting mass,
   $\delta$ is the corresponding density discrepancy.

When computation is done in the attracting compartment itself, $H_1$
should be replaced by $H_o$, the elevation of the computation point.

Picking up the correct $\delta$ and $h_i$ terms for the more complicated models
will not be discussed here. The procedure can be checked from the statement
listing for the specific model, in the appendixes.

3.73 The main program

The main program computes model anomalies at all $5^\circ \times 5^\circ$ surface
elements, simultaneously — so to speak. Partial attraction and potential
effects between pairs of compartments are computed, and stored in their
proper locations. The total model anomaly at a specific point is generally unknown until the entire operation has been completed. The program produces world maps of the model anomalies, rather than individual point values. The name for this program is program M.

3.731 Schematic flow chart

To clarify the logic of the program consider Figure 11, the generalized flow chart. It is for the case when the effect of the polar zones on themselves is included.

3.732 General order of operations

The division into three parts in the flow-chart, Figure 11, follows the logic of operations as in 3.64, p. 63. It also reproduces the partial effects of attraction and potential from outside the $5^\circ \times 5^\circ$ compartment and from outside Hayford's zone O, as well as the total effect at the point. The two outside effects (on anomalies) could be very valuable for detailed interpolation, between computation points. Especially suited for this will be the first one, giving the effect outside the $5^\circ \times 5^\circ$ itself. This is because its values would change more smoothly between points than that of the second set, and also because it has regular, straight border-lines of $5^\circ \times 5^\circ$.

Part I of the program computes the effect (on the model anomaly) of all $5^\circ \times 5^\circ$ surface elements, on each other. It therefore excludes the effect of
Figure 11

Schematic flow chart,
Main program

1. Preparations
2. Initial latitude
3. Lower latitude
4. Polar zone?
5. Polar zone
6. Printout
7. Longitude difference
8. Distance
9. Close?
10. Subdivide
11. Initial longitude
12. Symm. to equator
13. Symm. to meridian
14. Anomaly
15. All?
16. Printout
17. Compartment
18. Limit θ
19. Distances
20. Anomaly
21. All?
22. Printout
23. Compartment
24. Limit θ
25. Anomaly
26. All?
27. Printout
28. Mean zero
29. Reduce
End

PART I

PART II

PART III
each compartment on itself, which has usually a big — if not the biggest —
contribution to the total anomaly. It is considered in Parts II and III.

Part III computes the effect of the spherical cap contained in the com-
partment, and Part II computes the effect of the remainder, within the com-
partment, through a subdivision process.

The order of specific operations is indicated in the frames drawn. Some
general comments are in place here. Reference is to the number in the flow-
chart.

1. The preparation stage comprises the read-in of data and constants,
and the computations of the variables and constants used throughout
the program (center of gravity and area of squares, densities, if
necessary, etc.).

2. The two compartments, between which reciprocal (attraction and
potential) effects are sought, are generally on two separate latitude
zones. The initial latitude zone in the 5° x 5° system varies from 0
(at the pole) to 17 (at the equator). The southern hemisphere (zones
18-35), is covered in step (12.).

3. The second latitude zone is always to the south (for the southern
hemisphere — to the north) of the initial one. It does not reach lower
than the symmetrical latitude to the initial one, because cases like
that have already been covered by the symmetry to the equator (12.)
for a higher initial zone (2.).
4., 5., and 6. are side steps only, designed to obtain a value for the effect at the polar zones. They may or may not be used at all for later evaluation. Their inclusion, though, assures the possibility of correctly accounting for the effect of the polar zones on themselves, if needed. This would be done as follows: From the results of the total effect of all 5° x 5° (16.) at the polar zones subtract the effect of the 5° x 5° in the zones on themselves. Then apply any (improved) method, if any, to obtain better results for the zones.

The rest of the squares are not seriously affected by this operation, even though it may reflect in (28.). Conversely, the reciprocal computation could be limited to some specific latitude zones, which might exclude the polar zones altogether.

7. A longitude difference is set up between the two parallel zones. This allows

8. The computation and storage of a distance, not yet attached to physical points on the earth. This distance can usually be utilized in four places, as seen from Figure 9, p. 66; namely, in cases symmetrical to the initial case with respect to the equator or to the meridian.

11. The initial point is set up in longitude also. Now the distance can be used for

14. The actual computation of the attraction and potential between the pair of compartments, reciprocally.

9. and 10. subdivide the 5° x 5° compartment into 1° x 1° subsquares, in case the distance is shorter than a specified control limit.
12. The symmetrical case to the equator is computed, except when the two latitude zones in question are already symmetric.

13. Before the initial point is moved on the same latitude zone (keeping the same distance), the symmetrical case to the meridian is computed. Exception to this is when the two compartments are on the same great circle.

12., 13., and 14. are repeated as the initial point moves on its parallel. In that connection it should be mentioned that while the point moves through $72 \times 5^\circ$ on the parallel, the longitude difference for computing the distance only reaches $36 \times 5^\circ$ (symm. to meridian). Note also that when the latter is indeed $180^\circ$, the point moves through $36 \times 5^\circ$ only.

16. When the effects of all $5^\circ \times 5^\circ$ have been considered (15.) and summed in their proper locations, a regular printout and punching process takes over.

In parts II and III operation goes from compartment to compartment independently, since the auto-effect is sought, (but see (19.).)

17. A compartment is picked up and

18. The limiting $\theta$ for the inner cap is set up, or computed.

19. The pattern of distances from the computation point to the subsquares is figured and stored for later use on the same parallel (or the symmetrical one).
20. Computation of the partial effects of the subsquares takes place, and these are added to the total. This is done, of course, only in case the distance is bigger than the outer limit of the cap.

22. After all the squares have been computed in this fashion (21.) a printout list will allow the comparison of model disturbance to $|\varphi| \leq 55^\circ$.

23., 24. are similar to 17., 18.

25. Computation of the cap is performed, the result stored.

27. The total effect is printed out, after all compartments have been considered (26.).

At this stage the computations of the model have been completed and additional corrections or modifications could be performed. For example, the model anomaly field might be desired to mean zero for the entire earth.

28. Averaging the effects to zero over the whole world means subtracting the (weighted) mean value from each of the values.

29. The subtraction gives the reduced values for the model, on which further studies and investigations can be based.

28. and 29 could be omitted, of course, or others taken in their stead.

The statement listing for the main program M, which computes model anomalies at all $5^\circ \times 5^\circ$ elements is presented in appendix A. Explanations are in appendix B. Appendix C gives modifications from program M to program M1, which can handle the two-layered model D, with varying upper crust. Relevant comments are given in appendix D.
The programs mentioned above are tabulated, with regard to their applications, on p. 91.

3.74 The point program

The point program was designed in order to obtain and/or check point-by-point values from the main program. It has been later used to a much wider degree, due to the limiting factor of machine-time required for a complete run of the main program. Reference name for this program is point program P.

The point program P is in a sense a modification and simplification of the main one, M. It is very straightforward, and yields relatively quick results for specific points. It would be the only procedure applicable to computations at odd locations of latitude and longitude. In this case, however, modifications to the program P will be needed, to account for longitude difference of non-integer multiples of 5°. Also the computation inside the square should be changed, or omitted altogether, because there might not be place for a spherical cap around the computation point within the compartment.

Better and more detailed data could likewise be employed inside and outside the square, like 1° x 1° means, etc. Its use for computations over the entire earth might be inefficient, as has been pointed before, p. 66. These problems will be evaluated later on.

The general order of operations follows that of the main program M. Part I computes the effect of all 5° x 5° outside the compartment in question.
Part II computes the effect of the subsquares inside the compartment, and Part III computes the effect of the inner cap. The main difference is that each point is considered by itself, and no distances are precomputed and stored for later use (which could be done, however, if there are repeated latitudes in the list of points). The result is one number for the attraction and the potential at the point, rather than an array for the entire earth, obtained through reciprocal computations, as in program M.

The statement listing for program P is in appendix E, comments in appendix F. Appendix G gives modifications from program P to program P1, which can handle the two-layered model D, with varying upper crust. Relevant comments are in appendix H.

All the programs mentioned above are tabulated, with regard to their applications, on p. 91.

3.8 Input data

3.81 Topography

The input 5° x 5° mean elevations used for the model were taken from the files of the Department of Geodetic Science, The Ohio State University. These values are slightly different from the 5° x 5° values estimated and listed in [37], appreciably different from those listed in [43].

Area weighted mean for the earth is +630 m for land compartments (28.7%), and -3604 for sea compartments (71.3%).
Regular area weighted mean for the earth, considering elevations only is therefore - 2387 m.

The mean when masses are considered is $\overline{H} = -2274$ m (or -2287 m) [in water], and

The mean when the water is condensed is $\overline{H^*} = -1400$ m (or -1475 m) [in air]. Standard densities used here are 2.67/1.027 (or 2.89/1.027), respectively.

3.82 **Crustal thickness**

3.821 **General data**

Values for the mean crustal thickness in $5^\circ \times 5^\circ$ compartments have been estimated by the author from a small scale map, [73]. They are listed in Appendix I. Coordinates (LAT, LON) refer to N-W corner of the compartment.

The work above was mostly carried on under Air Force Contract No. AF23(601)-4132, OSURF Project No. 1891. The contract was supervised by Dr. Uotila, and administered by Aeronautical Chart and Information Center, St. Louis, Missouri 63118. The values published under the contract [87], were preliminary.

Regular area weighted mean of crustal thickness is 19.856 km.

The mean with condensation of water is 20.844 km (20.768 km).
The depth to the bottom of the mean crust, using data from p. 79, is 22.243 km (or again 22.243 km), with densities 2.67/1.027 (or 2.89/1.027), respectively. Some graphical representation can be found in Figure 12, p. 83.

The standard error of the estimated mean based on the map is probably about ±2 km. The absolute accuracy of the contours of the map is unknown to the author, could probably reach ±3 km, or might be systematically wrong at places.

It is doubtful that man will ever be able to measure the depth of the Moho with direct methods, all over the earth (even though a project was started, in order to actually drill a hole right to the subcrust [13]). Indirect methods of deduction are geophysical, by interpretation of seismic data [91], or through theoretical correlation to Bouguer anomalies [5], [46].

The model in this study is based on distinct values of the crustal thickness, and its results are quite sensitive to changes in the latter. Such is the case with most of the existing anomaly systems; and perhaps even worse, since no observed data is incorporated. In these theories the discrepancies are well hidden, usually, behind the assumed relationship between the elevation and the Moho. This point will be discussed again later on.

3.822 Comparisons to short profiles

In order to obtain some idea about the reliability of the source map [73], values listed in [48] were checked. These values refer to relatively short seismic refraction profiles, reversed or unreversed.
The recognition of the Moho is not easy [92, p. 106], as evidenced by varying results obtained even for the same profile (up to 20km difference),[48]. The comparison was based on area means for closely grouped profiles. The standard deviation for 146 groups compared to the map was about ±5 km. The agreement was somewhat better at sea than on land. The result is indicative only, due to uneven distribution of the profiles, and the fact that they actually represent point values.

A similar comparison to the source map [73] was made with 5° x 5° mean crustal thicknesses listed in [44]. The comparison was made at 171 squares (omitted several squares where the deviation between the two values compared was bigger than ±20 km, with only one observation listed in [44]). The standard deviation was ±6.2 km.

The map in use gives mostly 10 km contour lines, for which a standard deviation of ±3 km might be anticipated as a measure of precision. The accuracy would be worse. The results from the short seismic profiles do not really contradict the source map listing. They rather indicate that its accuracy could be on the order of ±3 km, or worse.

3.823 Comparisons to an isostatic system

More preliminary work was undertaken, using the distinct mean elevations and crustal thicknesses. Values for the normal crust $T$ in the Airy-Heiskanen isostatic system have been computed. The conversion formulas were derived
from the ones in [17, p. 136], and appear in [87, p. 4].

It is widely believed that a likely value for the normal thickness of the crust is 30 km. The weighted mean from the data above, with standard densities 3.27/2.67/1.027 yielded 28.49 km.

Using the Airy-Heiskanen system above, theoretical values of the crustal thickness were derived and computed from elevations, for all 5° x 5° surface elements. This theoretical set (referred to as SET II) produced an area-weighted mean for the crustal thickness of 21.348 km. With condensation of water the mean is 22.336 km, as compared to 21.77 km, found in [37, p. 39]. [The latter is misprinted as 22.40 km].

SET II above was compared to SET I (the estimated values of the crustal thickness of the earth, appendix I). Graphical representation is in Figure 12. The comparison gave a standard deviation of about ±8 km for the 5° x 5° mean crustal thickness, for the average-sized compartment (0.00484 steradians). The estimated thicknesses have a corresponding standard deviation of probably less than ±5 km. The two measures above suggest that this particular Airy-Heiskanen system (SET II) could perhaps only approximate the available crustal data. It indicates, therefore, that the standard densities utilized for the isostatic model would have to be changed, or even distinct 5° x 5° densities substituted. All this might have to be done, if the available crustal thickness data (SET I) is to be used in the model.

The Airy-Heiskanen system could, nevertheless, provide rather reasonable
model anomalies, as judged by their comparison to mean observed free-air gravity anomalies, p. 123.

3.824 Correlation with elevation

Valuable information can be obtained from correlation of Moho depth, $M$, to elevation, $H$. If an isostatic system with specific $(\Gamma, \delta_{\text{crust}}, \delta_{\text{sub}}, \delta_{\text{water}})$ prevails, one would expect to find a correlation approximating a broken line of two branches, of the type
\[ M = T + \frac{\delta_{\text{crust}}}{\delta_{\text{sub}} - \delta_{\text{crust}}} (+H) \]  

(70)

for land, and

\[ M = T + \frac{\delta_{\text{crust}} - \delta_{\text{water}}}{\delta_{\text{sub}} - \delta_{\text{crust}}} (-H) \]  

(71)

for sea.

If, however, the sea water is condensed into land density, the resulting artificial elevations \( H^* \) will have a straight line representation,

\[ M = T + \frac{\delta_{\text{crust}}}{\delta_{\text{sub}} - \delta_{\text{crust}}} (\pm H^*) \]  

(72)

Consequently, a straight line fit of the form

\[ M = A + B H^* \]  

(73)

was tried for Moho depths derived from the crustal thickness data of SET 1 [73] and the elevations. It should be noted, beforehand, that \( A = T \) if an isostatic system prevails, but that \( A \) is not the mean crustal thickness. This is so because the area-weighted mean elevation over the earth (with condensation) is not zero but \( H^* \), p. 83. For the same reason it might be more illustrative in some cases to count elevations from other reference level than mean sea level, m.s.l.

Adjustment to fit equation (73) was performed, including all \( 5^\circ \times 5^\circ \) compartments, for several pairs of densities. Results are in Table 2.

The results listed have all about the same standard deviation, and vary but within the limits of rejection around the values

\[ A = 34.1 \pm 0.14 \text{ km} \quad \text{and} \quad B = 8.0 \pm 0.07 , \]  

with
### Table 2

Correlation coefficients for Moho and elevation

<table>
<thead>
<tr>
<th>Density</th>
<th>Rock</th>
<th>Water</th>
<th>A*</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.75</td>
<td>1.030</td>
<td>33.968</td>
<td>8.230</td>
</tr>
<tr>
<td></td>
<td>2.75</td>
<td>1.027</td>
<td>33.975</td>
<td>8.218</td>
</tr>
<tr>
<td></td>
<td>2.80</td>
<td>1.027</td>
<td>34.016</td>
<td>8.149</td>
</tr>
<tr>
<td></td>
<td>2.85</td>
<td>1.027</td>
<td>34.055</td>
<td>8.084</td>
</tr>
<tr>
<td></td>
<td>2.90</td>
<td>1.027</td>
<td>34.092</td>
<td>8.021</td>
</tr>
<tr>
<td></td>
<td>2.95</td>
<td>1.027</td>
<td>34.127</td>
<td>7.962</td>
</tr>
<tr>
<td></td>
<td>3.00</td>
<td>1.027</td>
<td>34.160</td>
<td>7.905</td>
</tr>
<tr>
<td></td>
<td>3.05</td>
<td>1.027</td>
<td>34.191</td>
<td>7.851</td>
</tr>
<tr>
<td></td>
<td>3.10</td>
<td>1.027</td>
<td>34.222</td>
<td>7.799</td>
</tr>
</tbody>
</table>

\( m_\circ = \pm 0.375 \text{ km} \) (weights in steradian). These results indicate a standard deviation of Moho for the average-sized 5° x 5° compartment of \pm 2 km (or \pm 3.7 km), when its elevation is 0 m (or 2000 m), respectively.

A check for the particular pair of \((A, B)\), and the densities is provided by substituting the mean (condensed) elevation \(\overline{H^*}\) in equation (73). The resulting \(M\) should then be the depth of the mean Moho (condensed), all in the same density system.

The first solution in Table 2 used the same densities as [92, p. 105], and could therefore be compared to its results. These were \(A = 33.2 \text{ km}\) and \(B = 7.5\), with \(m_\circ = \pm 11 \text{ km}\). \[Assuming the word "uncertainty" means standard deviation.\]

The comparison is not decisive, since results derived from many (2592) area means are compared to those obtained from (about 315) point values. It
seems to indicate, though, that the mathematical structure (straight line fit) might not be significant for the data in [92]; but it is significant for the data used in the model [73]. This demonstrates the degree of isostatic equilibrium which prevails for the data used.

3.825 Mass balance

The preceding subheading does not consider mass balance in the model, which is discussed below.

A model which is introduced to represent the earth must keep the total mass unchanged. Therefore the model computation level defined by CRUST, (Figure 1, p. 26 ) and the model densities must be balanced. This balance should consider the effect of all $5^\circ \times 5^\circ$ compartments, using their elevations and crustal thicknesses.

Several solutions for a pair of $\delta_{\text{sub}}/d\delta$ for different CRUST values, which are in mass balance, are given in Table 3. The term $d\delta$ stands for $(\delta_{\text{sub}} - \delta_{\text{crust}})$. The listing is for $d\delta$ which satisfies the balance.

The solutions were arrived at by listing the total mass discrepancy for the whole earth, as computed from various combinations of the data (CRUST, $\delta_{\text{crust}}$, $d\delta$). A solution is where the total mass discrepancy is zero.

Mass considerations, together with the results from the previous chapter, prescribe the procedure for establishing the earth model. It is as follows:

Start with a chosen $\delta_{\text{crust}}$ for the model, and a density for water. Using the
### Table 3

**CRUST and densities in mass balance**

<table>
<thead>
<tr>
<th>CRUST#2</th>
<th>d(\delta)</th>
<th>2.5</th>
<th>2.6</th>
<th>2.7</th>
<th>2.8</th>
<th>2.9</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>0.62</td>
<td>0.67</td>
<td>0.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>0.50</td>
<td>0.53</td>
<td>0.57</td>
<td>0.61</td>
<td>0.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.43</td>
<td>0.47</td>
<td>0.50</td>
<td>0.53</td>
<td>0.56</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>0.38</td>
<td>0.41</td>
<td>0.44</td>
<td>0.47</td>
<td>0.50</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>0.35</td>
<td>0.37</td>
<td>0.39</td>
<td>0.42</td>
<td>0.45</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>0.31</td>
<td>0.33</td>
<td>0.36</td>
<td>0.38</td>
<td>0.40</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>0.29</td>
<td>0.31</td>
<td>0.33</td>
<td>0.35</td>
<td>0.37</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>0.26</td>
<td>0.28</td>
<td>0.30</td>
<td>0.32</td>
<td>0.34</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>0.24</td>
<td>0.26</td>
<td>0.28</td>
<td>0.30</td>
<td>0.31</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>0.23</td>
<td>0.24</td>
<td>0.26</td>
<td>0.28</td>
<td>0.29</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>0.21</td>
<td>0.22</td>
<td>0.24</td>
<td>0.26</td>
<td>0.28</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>0.20</td>
<td>0.21</td>
<td>0.23</td>
<td>0.24</td>
<td>0.26</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.19</td>
<td>0.20</td>
<td>0.21</td>
<td>0.23</td>
<td>0.24</td>
<td>0.26</td>
<td></td>
</tr>
</tbody>
</table>

5° x 5° mean elevation and Moho values, solve for (A, B) in equation (73), p. 84. Comparison of (72) and (73) will yield d\(\delta\) (from B). The term A will be CRUST.

The model should be checked to see if it is in balance, the operation repeated until the conditions are reasonably satisfied.

#### 3.826 Conclusions

Considering all the evidence above, it does not seem likely that the mean Moho (for the average 5° x 5° compartment) is accurate to the nearest km. In that state of affairs the use of constant densities would bring about a standard deviation for the model anomalies of ± 50 mgals depending on the
density (i.e. - 50 mgals for 1 km at 30 km depth with $2.67/3.27$; or - 30 mgals for same with $2.90/3.20$, etc.). Some other devices should be employed in order to reduce big discrepancies to model anomalies. This has been mentioned in 3.2 and on p. 21, and is shown on p. 124.

At the same time some insight into proper data for a mass model has been gained, as shown in the preceding subheading, which will be utilized in the proceeding one.

### 3.83 Mass model

The mass model chosen used

$$
\delta_{\text{crust}} = 2.89, \text{ and } \delta_{\text{water}} = 1.027.
$$

Using the described procedure, p. 84, it yielded

$$
A = 34.084 \text{ km, and}
$$

$$
B = 8.0338
$$

which in turn gave

$$
d\delta = 0.36, \text{ or } \delta_{\text{sub}} = 3.25.
$$

This system is extremely close to mass equilibrium, as evidenced by computing the weighted mean mass discrepancy for the entire earth from the mass model and the $5^\circ \times 5^\circ$ mean elevation and Moho data. The result is

$$
2.58 \times 10^{-7} \text{ km gr/cm}^3.
$$

Graphical representation of the mass model is in Figure 12, p. 83, in the shaded column.
The crustal density of this model, 2.89, is close to the densities suggested in [92, p. 106], all being rather higher than customarily thought of. Standard density of 2.67 is the basis of an Airy-Heiskanen system, etc. It should be noted, though, that this density might be too close to the densities of surface rocks as to yield a good model, since the density at the Moho is probably on the order of 3.3. An increase in the mean density (from 2.67) is indicated, as the case in this model is (to 2.89). A multi-layered model might be better, but it should be noted that under consideration is the mass model only. From it total mass, and then densities, could be derived. In many cases the total mass to a certain depth is of concern, not the actual layer structure.

3.84 Constants

The constants read-in into the machine are listed as comments in the statement listing, appendixes A and E, and are explained in the corresponding appendix B. A few additional remarks are proper at this point.

The set of 3 standard densities \([D, D(1), D(2)]\) read-in refer to the mass model as discussed above. So is the term CRUST, the model mean Moho.

The terms CME and CMM which are the Condensed Mean Elevation (depth) and Condensed Mean Moho are those found in Figure 12, p. 83. They must be precomputed from the \(5^\circ \times 5^\circ\) mean elevations and Moho data. Note that CME is set positive for mean negative elevation.
3.85 Distinct densities

The distinct $5^\circ \times 5^\circ$ densities might be read-in, which is why this subheading is under Input Data.

No matter what kind of computational model is used — A, B, C, or D, p. 91 — the mass model (p. 88) stays the same one. The distinct densities, if computed for a certain gravity model, will be deduced from the mass model by use of the rest of the data, including distinct elevations and crustal thicknesses.

Comment: In a few cases the computed distinct density $\delta_{\text{crust}}$ might be bigger than $\delta_{\text{sub}}$. It is, however, only a mathematical device and does not have too much of a physical meaning. It should not be forced to be smaller than $\delta_{\text{sub}}$, because this change will throw the gravity model off in anomalies and mass balance, p. 130.

3.9 Résumé

A short résumé of the various models and programs in the text might prove useful for better understanding and for quick reference.

Table 4 represents the models, mentions how the $5^\circ \times 5^\circ$ densities are set (or computed). It gives also the reference pages for each model and the program which can handle it.

The same mass-model is used for all models below. It has the mean elevation and mean Moho of the earth and corresponding densities of crust and subcrust.
Table 4
Models and programs

<table>
<thead>
<tr>
<th>Model</th>
<th>$5^\circ \times 5^\circ$ densities</th>
<th>Page</th>
<th>Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Constant</td>
<td>25</td>
<td>M; P</td>
</tr>
<tr>
<td>B</td>
<td>Distinct</td>
<td>31</td>
<td>M; P</td>
</tr>
<tr>
<td>C</td>
<td>Distinct (below m.s.l.) + Constant (above m.s.l.)</td>
<td>36</td>
<td>M; P</td>
</tr>
<tr>
<td>D</td>
<td>Distinct (lower crust) + Constant (upper crust)</td>
<td>36</td>
<td>M1; P1</td>
</tr>
</tbody>
</table>

All densities can be read-in, if available. Computed densities are obtained from equi-mass considerations (compared to mass-model). In model D the upper crust can be at most 40% of the total crust. Constant densities for models C and D are pre-set.

The main program M computes model anomalies for all $5^\circ \times 5^\circ$ elements. Point program P computes model anomalies at individual locations. Both programs handle model A, B, and C, or read-in densities. Model D is handled by programs M1 and P1 only.

The printout listing includes nine sets of numbers (or maps) — one each for model disturbance, Bruns' term and model anomaly; for the accumulated effect from outside the square, from outside the spherical cap, and total. Most of these sets are separated, in addition, to effects of the topography (or ocean) and of its compensation.
**4. RESULTS**

4.1 Preliminaries

Quite a few preliminary and preparatory measures should be taken before a major program can even be test-run. This study was no exception in this respect. The detailed investigations led to the determination of the size and boundaries of the subsquares, and gave a general impression of the logic, the organization and accuracy of the final program.

All major programs, and all the models utilized, are summarized on p. 91. It would, therefore, be a proper reference for all of them.

4.11 Geometry on a sphere

Programs in this group checked the geometry on a sphere with regard to the area and center of gravity of the squares, and also for the distance generated between pairs of points on the surface for either the center of gravity of the compartments or mean latitudes. The distances finally yielded a table of distances, arranged in the order of their appearance in the main program. The table is for values of either the mean latitude or the center of gravity of the square, expressed in degrees. It is not represented in this text, though.

The center of gravity of the compartment is computed using equations (68) and (69), p. 63. The procedure is original and needed some additional checking. To demonstrate its workability, Table 5 lists results obtained for
Table 5

Center of gravity of compartments

<table>
<thead>
<tr>
<th>$\Delta \phi^o$</th>
<th>$\phi^o$</th>
<th>$^1K$</th>
<th>C. G.$^o$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Colat.</td>
</tr>
<tr>
<td>5</td>
<td>90-85</td>
<td>0</td>
<td>3.333</td>
</tr>
<tr>
<td></td>
<td>45-40</td>
<td>9</td>
<td>47.544</td>
</tr>
<tr>
<td></td>
<td>5- 0</td>
<td>17</td>
<td>87.524</td>
</tr>
<tr>
<td>2</td>
<td>90-88</td>
<td>0</td>
<td>1.333</td>
</tr>
<tr>
<td></td>
<td>45-43</td>
<td>22.5</td>
<td>46.007</td>
</tr>
<tr>
<td></td>
<td>2- 0</td>
<td>44</td>
<td>89.004</td>
</tr>
</tbody>
</table>

$^1$ Latitude zone number, from the pole.

A small sample of compartments in two ($\Delta \phi \times \Delta \lambda$) systems; namely, 5° x 5° and 2° x 2°.

The results above show the effect of the meridian convergence and the size of the compartment on its center of gravity. It also demonstrates the use of non-integer zone number K.

The 5° x 5° areas were easily checked by their summation to 4π.

This was followed by some smaller programs, designed to check on the accuracy of the cosine formula for distances on a sphere. The conclusion was that double precision computation with the cosine formula outside the computation compartment will assure 10 significant digits, while inside the compartment double precision computation using the sine formula is preferable. [Further investigation proved that using double precision techniques would be superfluous for most prediction purposes. Thus the final programs use single precision].
4.12 Attraction on a sphere

The first program in this group was one that computed attraction between two compartments on a sphere, having specified area, elevation, Moho and angular distance between them. The results were checked by hand computations, and found exact.

Then the problem of the subdivision of squares was tackled. The effect of the three neighboring squares 1, 2, 3 (E, S.E. and S. to the computation compartment, respectively) was found for varied data. This has been checked with the results of a subdivision of the neighboring squares into $1^\circ \times 1'$ and $0.5^\circ \times 0.5'$, and smaller. Table 6 gives a small demonstration of the computation procedure. Investigated in this instance is an area with mean elevation of +2000 m, mean crust of 41,500 m, and densities 2.67/3.27. The model CRUST is 30,000 m. Location of the computation point is latitude zone 5, 65° - 60° N. The computation is between the centers of gravity of the compartments. Listed are the effects from the topography (T) and from the crust (C), and their total. Quite a few tests were performed, out of which it was concluded that in most cases a subdivision of the neighboring compartments is advisable, especially at the polar zones, on the parallel.

Another problem relates to the center of gravity (C.G.) of the compartment, p. 60. The question is whether the mass-line should be located at the C.G., as suggested by geometric considerations?
Table 6
Effect of neighboring squares

<table>
<thead>
<tr>
<th>Square</th>
<th>Attraction in mgals.</th>
<th>From 5° x 5°</th>
<th>From 1° x 1°</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T</td>
<td>C</td>
<td>Total T</td>
<td>C</td>
</tr>
<tr>
<td>1 (E)</td>
<td>1.25</td>
<td>-12.42</td>
<td>-11.17</td>
<td>1.13</td>
</tr>
<tr>
<td>2 (SE)</td>
<td>0.80</td>
<td>-2.15</td>
<td>-1.35</td>
<td>0.84</td>
</tr>
<tr>
<td>3 (S)</td>
<td>0.72</td>
<td>-1.70</td>
<td>-0.87</td>
<td>0.74</td>
</tr>
</tbody>
</table>

The geometric significance of the C.G. might be misleading, when matters of attraction and potential are concerned. This is due to the non-linear nature of the effects integrated over the compartment. Representation of the latter by a mass-line situated at the C.G. might not always be the best, or necessarily better than mean-latitude position. This was verified through some test runs at various locations on the globe. The comparison was made between attraction values derived from a mean- or C.G.-located mass-line, for 5° x 5°, and attraction values obtained from a numerical integration process, with subsquares of 0.1° x 0.1°. The differences vanish, of course, when the compartment is far from the computation point. A conclusion from all the above was that a subdivision of the 5° x 5° elements close to the computation point might be inevitable, at any location. A possible subdivision into 1° x 1° was incorporated into all programs, either with a constant limit for the process (for example 5° 2), or a varying one (for example 10. 2° cos ϕ), or a combination of both.
In establishing the constant-density model $A$, p. 27, some effects of the shell CD had to be evaluated. The derivations follow below.

The mass of the shell, Figure 1, p. 26, is

$$M_{c0} = \frac{3}{4} \pi \delta \left[ (R - \text{CRUST})^3 - (R - \text{CRUST} - \text{CD})^3 \right] \times 10^6 \text{gr/km}.$$  

The attraction $F_A$ at $A$, using equation (1), p. 3, is

$$F_A = k \frac{M_{c0}}{R^2} \text{gal/km},$$

or

$$F_A \approx 4\pi k \delta \left\{ 1 - 2 \frac{\text{CRUST} - \text{CD}}{R} \right\} \text{CD} \times 10^7 \text{mgal/km}. \quad (74)$$

For $\delta = 0.6$, $k = 6.67 \times 10^{-8}$, $R = 6371 \text{km}$, with CRUST $= 30 \text{km}$ and CD $= 1 \text{km}$, one obtains

$$F_A \approx 50.29 \times 0.9909 \times 1 = 49.83 \text{mgals}.$$  

The change in $F$ with the elevation $H$ of the point $B$ is approximately

$$\frac{dF_A}{dR} \text{dR}, \text{ with } \text{dR} = H.$$ The derivative should not be taken from equation (74) but from the preceding expression, in which only the $R$ in the denominator is variable. Thus,

$$dF_A \approx -2 \frac{F_A}{R} H. \quad (75)$$
For the same data as above, and \( H = \frac{1}{2} 5 \text{ km}, \) the effect is

\[
dF_{ab} = \pm \frac{49.83 \times 2 \times 5}{6371} \approx \pm 0.08 \text{ mgal.}
\]

This was all for 1 km of shell CD. Thicker shell will produce linearly bigger effect. However, the shell in the model utilized is very thin — or indeed zero — and the differential effects of elevations are negligible.

The next program gave the effect of the inner cap, and checked the cases discussed in 3.633, p. 53. To recapitulate, the problem was to evaluate some approximate solutions (47), (51), (50) to an unsolvable expression (44) for the attraction. The comparison between the solutions above was extended to include the correct effect, obtainable from the closed relation (55).

Jung's approximation (50), p. 56, evaluates the integral with the lower limit of the radius-vector \( \rho_1 \). The author's approximation (47) uses the mean radius \( \rho_{12} \), and should yield better results than the former. In order to check both solutions, a numerical integration (51) was added. It uses a Gaussian integration procedure, available at the O.S.U. Computer Center as the library subroutine GAUSS. Jung's values were found by hand computation. An example is presented in Table 7. The data set used was \( \text{CRUST} = 30 \text{ km, and densities 3.20/2.70}. \) The point location is given by the (lat/long) of the N-W corner of the compartment. The results are designated JUNG, APPROX., and NUM. INT., for equations (50), (47), and (51), respectively.
Table 7

Effect of cap, A

<table>
<thead>
<tr>
<th>Pt.</th>
<th>¹Jung</th>
<th>²Approx.</th>
<th>³Num. Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>45° N/30° E</td>
<td>- 77.17</td>
<td>- 77.87</td>
<td>- 77.87</td>
</tr>
<tr>
<td>70° E</td>
<td>- 151.55</td>
<td>-175.77</td>
<td>-175.97</td>
</tr>
<tr>
<td>75° E</td>
<td>- 250.00</td>
<td>-302.87</td>
<td>-303.58</td>
</tr>
<tr>
<td>135° E</td>
<td>+ 11.77</td>
<td>+ 1.55</td>
<td>+ 1.59</td>
</tr>
<tr>
<td>45° N/210° E</td>
<td>- 12.55</td>
<td>- 39.12</td>
<td>- 38.99</td>
</tr>
</tbody>
</table>

¹ Equation (50), p. 56 [27, p. 75].
² Equation (47), p. 55.
³ Equation (51), p. 56.

Results in mgal.

These results emphasize what had been stated on p. 56. Attraction of a cap, derived with Jung's equation, might be off by as much as 16%, or what is more serious — they could be off by 50 mgals. The values in the last two columns agree very well with each other, so that either one could be used. The numerical integration might as well be used since its time requirement is rather small (even though it is bigger than the time required for the approximate solution: 0.006 min. compared to 0.001 min, per point).

Comparisons between Jung's approximation (50), the author's numerical integration (51), and the author's closed expression for the attraction (55) were then undertaken. An example is given in Table 8. The data set is
\[ \rho_o = 6371.2 \text{ and } \rho_2 = 6341.2 \text{ km, for} \]

\[ \theta = 0.03 \text{ rad, with} \]

\[ 2\pi k\delta = 1000 \text{ E.} \]

The results are designated JUNG, NUM. INT., and CLOSED, for equations (50), (51), and (55), respectively.

Table 8

<table>
<thead>
<tr>
<th>( \rho_1 \text{ km} )</th>
<th>( \text{Jung} )</th>
<th>( \text{Num. Int.} )</th>
<th>( \text{Closed} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6335.2</td>
<td>0.3973</td>
<td>0.5010</td>
<td>0.5013</td>
</tr>
<tr>
<td>6338.1</td>
<td>0.2032</td>
<td>0.2529</td>
<td>0.2530</td>
</tr>
</tbody>
</table>

1 Lower limit of cap

2 Equation (50), p. 56 [27, p. 75].

3 Equation (51), p. 56.

4 Equation (55), p. 57.

Results in gals.

This table shows again that the numerical integration yields results very close to the correct ones, for the attraction of the spherical cap.

Next came the question of the "steps" between the cap and the rest of the square, Figure 6, p. 50. To evaluate the effect of the gaps, the total attraction effect of the 5° x 5° element was computed using the model (cap and 0.25° x 0.25° subsquares) and using triple numerical integration (\( \Delta \phi \Delta \lambda \Delta h \)),

99
either with \((0.25^\circ \times 0.25^\circ \times 150\text{m})\) or \((0.1^\circ \times 0.1^\circ \times 100\text{m})\) increments. The results are denoted \((\text{Model}), (\text{Int}_{0.25}), \text{and (Int}_{0.10})\), respectively. Several tests at various points confirmed that the effect of the border zones between the cap and the subsquares inside the \(5^\circ \times 5^\circ\) is negligible. Moreover — considering the fact that the integrations here are \(5 - 50\) times slower than the model computation \((0.25 - 2.40\text{ min. compared to 0.05 min., per point})\) — it is evident that they should not be employed. An example: For the data \(\text{CRUST} = 30\text{ km},\) and densities \(2.70/3.20,\) at the compartment \(40^\circ \text{N}/30^\circ \text{E},\) the results for the attraction effect of \(C\) (compensation) were: \(-61.57\) (Model); \(-61.52\) (Int. \(0.25\)); \(-61.60\) (Int. \(0.10\)) mgal.

The investigation of the proper subdivision of the computation compartment itself is closely related to the above. Programs checked and compared the results obtained from subsquares of \(0.125\times 0.125, 0.25\times 0.25,\) and \(0.1\times 0.1.\) Subdivision into \(0.25\times 0.25\) beyond the cap was found to be satisfactory. The area of these subsquares together with the area of the cap also summed up the total area of the compartment to \(3 - 4\) significant figures, correctly.

At this point all the results of these programs could be modified and incorporated into one main program, program \(M,\) which computes model (attraction and) anomalies at all \(5^\circ \times 5^\circ.\)

The point program \(P,\) which computes model anomalies at specific points only, was designed and checked, too. Two working formulas for attraction were used, for comparison's sake, (35) and (36), p. 45. Both formulas yielded the same results indeed. The final working programs were based on (36).
Several trials were made to limit the computation of (attraction and) model anomaly to a certain distance $\psi$ from the computation point, p.64. It stands to reason that the effect of far-away compartments might be negligible, and/or not desirable to be considered. The immediate outcome of limiting the computation distance will be great savings in time. The question is to find what the closest limit of $\psi$ could be. Table 9 will demonstrate the total model anomaly, in mgals, of the $5^\circ \times 5^\circ$ compartments outside the computation square, related to the limiting angular distance $\psi$. The data set is CRUST = 34,084 km, densities 2.89/3.25, using distinct-densities model B, p.31.

Table 9

Limiting the computation distance

<table>
<thead>
<tr>
<th>Point</th>
<th>$\psi$</th>
<th>180</th>
<th>120</th>
<th>90</th>
<th>60</th>
<th>45</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$45^\circ$ N $0^\circ$ E</td>
<td>15.54</td>
<td>4.97</td>
<td>4.18</td>
<td>4.05</td>
<td>3.90</td>
<td>3.61</td>
<td></td>
</tr>
<tr>
<td>$70^\circ$</td>
<td>8.62</td>
<td>8.12</td>
<td>7.44</td>
<td>6.72</td>
<td>6.38</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>$140^\circ$</td>
<td>5.10</td>
<td>4.69</td>
<td>4.12</td>
<td>3.53</td>
<td>3.24</td>
<td>5.78</td>
<td></td>
</tr>
<tr>
<td>$210^\circ$</td>
<td>-4.90</td>
<td>-5.35</td>
<td>-5.63</td>
<td>-6.26</td>
<td>-6.78</td>
<td>-3.11</td>
<td></td>
</tr>
<tr>
<td>$45^\circ$ N $280^\circ$ E</td>
<td>6.87</td>
<td>6.37</td>
<td>5.98</td>
<td>5.47</td>
<td>4.95</td>
<td>3.83</td>
<td></td>
</tr>
<tr>
<td>$5^\circ$ N $150^\circ$ E</td>
<td>-3.88</td>
<td>-4.19</td>
<td>-4.51</td>
<td>-5.14</td>
<td>-5.66</td>
<td>-2.76</td>
<td></td>
</tr>
</tbody>
</table>

\(^1\)Model B anomalies, mgals.

The circle around the computation point, within which the attraction and potential effects are considered should be selected in such a way, that outside effects are negligible. It is evident that the loss of accuracy becomes significant —
above 1 mgal — when the computation distance is limited below $\psi = 120^\circ$.

The topographic-isostatic attraction effects from Hayford's zones 10 to 1, for various isostatic systems [60] (especially for the Airy-Heiskanen system $T = 30$ km), indicate attraction effects on the order of a few mgals. The corresponding indirect effect (Burns' term), listed in the files of the Department of Geodetic Science at O. S. U., is almost constant everywhere — about -0.8 mgal. Zone 10 starts at $7^\circ 51'30''$ from the computation point. Limiting the computation distance to about $10^\circ$ will inevitably cause errors on the order of several mgals. Table 9 confirms this conclusion, but it also seems to indicate that the discrepancies do not decrease quite rapidly with the computation limit.

The lowest limit for computation of model anomalies could, therefore, be set at $\psi = 120^\circ$, instead of the complete $180^\circ$. The effects of the neglected zones will amount to less than ±1 mgal, while the time savings will be considerable. Lower limits of computation could be set, in case lower accuracy is tolerable. One should note, however, that the effects of the neglected zones are mainly systematic for an area, rather than random. Controls have been set in all the programs to limit the computation distance to any of the angles in Table 9. See appendix B.

Remark: Computation of model disturbance (attraction) can be limited to about $\psi = 30^\circ$, since the attraction effects of the zones beyond is on the order of ±0.5 mgals.
All the programs use single precision techniques. The accuracy should easily suffice for prediction purposes. Modifications to allow double precision procedures could be made, for special cases. The time requirements will be much increased. Time estimates follow in the next subheading.

4.13 Execution times

Execution times vary considerably, with the model and program used, with the degree of precision utilized and with the limiting border for the entire computation and for the subdivision of neighboring squares.

The time estimates are based mainly on the point program P, which computes model anomalies at individual points. Actual execution times, and time estimates, are available for double and single precision operations, p. 93, for some models. As a rule, single precision computations cut down the time requirements, and savings of about 30% are the average.

The computer is the IBM 7094, the language is SCATRAN. Execution times to 0.001 min. are readily available, by calling library subroutines.

Execution times with double-precision operations are available for some attraction computations only. A few of them will be quoted below, and compared with results for single precision operations.

Mean execution times for the distinct-densities model B, p. 31, are given in Table 10, for both precisions. The time is in minutes, for one point attraction computation and for the whole earth (2592 points). The parts are as on p. 71; the limiting angular distance as on p. 101. All neighboring
squares, to a distance of 5.2°, were subdivided into 1° x 1°, p. 95 (except where specified otherwise).

Table 10

Execution times, model B

<table>
<thead>
<tr>
<th>Precision</th>
<th>Part I</th>
<th>Part II</th>
<th>Part III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>180</td>
<td>120</td>
<td>90</td>
</tr>
<tr>
<td>Double</td>
<td>1 pt.</td>
<td>0.66</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>1711</td>
<td>1348</td>
</tr>
<tr>
<td>Single</td>
<td>1 pt.</td>
<td>0.25</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>648</td>
<td>466</td>
</tr>
</tbody>
</table>

Mean times in minutes, for point program P.
1 Neighboring squares, to a distance of 5.2°, are subdivided into 1° x 1°.
2 Neighboring squares not subdivided.
3 Equation (51), p. 56.
4 Equation (47), p. 55.
5 Must use double precision for integration.
6 Sequence in computation of attraction, p. 71.

Execution times for the two-layered model C, which uses the same program P as above, are the same. The constant-density model A uses a little less time. The two-layered model D requires an increase of about 30% for Part I, because it uses program P1. In it the attraction is computed four times, instead of three times in program P, as is shown in appendix D, p. 193. [All the models are briefly summarized on p. 91].
The use of program P for all 2592 (5° x 5°) will require, therefore,
from 30 to 4 hours for double precision, and
from 12 to 2 hours for single precision
(depending on the limit of computation ψ, being
from 180° to 30°, respectively).

For ψ = 120° the times are about 24 hours (D.P.) and 9 hours (S.P.).

Time estimates for the main program M are rather difficult to come
by. They should be less than the ones for program P, as explained in 3.71,
p.65. Execution times for latitude zones 17 and 18 (5°N - 5°S) might aid the
estimates. Using the same model B as for Table 10, some execution times
are listed in Table 11.

Table 11

Execution times, various programs

<table>
<thead>
<tr>
<th>Program</th>
<th>P</th>
<th>P1</th>
<th>M</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Precision</td>
<td>D.</td>
<td>S.</td>
<td>D.</td>
<td>D.</td>
</tr>
<tr>
<td>Part</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Subdivision</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>I, ψ = 30°</td>
<td>8.6</td>
<td>4.5</td>
<td>11.5</td>
<td>3.6</td>
</tr>
<tr>
<td>II</td>
<td>5.1</td>
<td>2.2</td>
<td>6.7</td>
<td>5.1</td>
</tr>
<tr>
<td>III</td>
<td>1.3</td>
<td>0.9</td>
<td>1.7</td>
<td>1.3</td>
</tr>
</tbody>
</table>

1Double or single precision.
2Sequence in the computation of attraction, p.71.
3Subdivision of neighboring squares into 5°x5°, to a distance of 5.2°.
4Same as corresponding main programs.
Times in minutes for lat. zone (5°N-5°S), 144 pts.
The savings in time, between using the main program M or the point program P, are up to 50% of the time for Part I. Execution times for Parts II and III would have stayed virtually the same. This would indicate a total time requirement, for attraction computations with program M,

from 15 to 3 hours for double precision, and

from 7 to 2 hours for single precision (depending on the limit of computation $\psi$, being

from $180^\circ$ to $30^\circ$, respectively).

For $\psi = 120^\circ$ the times are about 13 hours (D.P.) and 4 hours (S.P.).

The final programs for model anomaly computations were all single precision. The range of execution time depends on the limit of computation $\psi$, and on the limit of subdivision into $1^\circ \times 1^\circ$, p. 95. The suggested procedure allows a variable limit of $10.2^\circ \cos \varphi$ for latitudes below $60^\circ$, and a constant limit of $5.2^\prime$ above $60^\circ$. Limiting the subdivision of neighboring squares to a constant of $5.2^\prime$ will result in a big increase in execution time at the poles, but it might be needed for best accuracy. At the polar zones the computation could be skipped altogether, or replaced by another procedure, p. 64. Some time estimates can be gained from Table 12. The programs in use were point programs P and P1. Listing is for location at $45^\circ$ and at the pole.

A considerable saving in time is possible by using the main program M (for the whole world) rather than program P (point by point). Execution times for the two parallel zones between $5^\circ$N-$5^\circ$S are given in Table 13.
### Table 12
Execution times, various models

<table>
<thead>
<tr>
<th>Model</th>
<th>Loc</th>
<th>Subdiv.</th>
<th>$\psi^0$</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
<th>120°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>45°</td>
<td>10.2$\cos \varphi$</td>
<td>1 pt.</td>
<td>0.13</td>
<td>0.08</td>
<td>0.06</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>All</td>
<td>337</td>
<td>212</td>
<td>161</td>
<td>143</td>
<td>44</td>
</tr>
<tr>
<td>B</td>
<td>45°</td>
<td>5.2°</td>
<td>1 pt.</td>
<td>0.18</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>All</td>
<td>454</td>
<td>325</td>
<td>235</td>
<td>205</td>
<td>44</td>
</tr>
<tr>
<td>B</td>
<td>90°</td>
<td>5.2°</td>
<td>1 pt.</td>
<td>0.18</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>All</td>
<td>454</td>
<td>325</td>
<td>235</td>
<td>205</td>
<td>44</td>
</tr>
<tr>
<td>D</td>
<td>45°</td>
<td>5.2°</td>
<td>1 pt.</td>
<td>0.25</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>All</td>
<td>640</td>
<td>571</td>
<td>411</td>
<td>337</td>
<td>117</td>
</tr>
<tr>
<td>D</td>
<td>90°</td>
<td>5.2°</td>
<td>1 pt.</td>
<td>0.22</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>All</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part I</th>
<th>Part II</th>
<th>Part III</th>
</tr>
</thead>
<tbody>
<tr>
<td>45°</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>60°</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>90°</td>
<td>0.13</td>
<td>0.06</td>
</tr>
<tr>
<td>120°</td>
<td>0.18</td>
<td>0.06</td>
</tr>
<tr>
<td>180°</td>
<td>0.25</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Time in minutes for point programs P and P1.

1. Location of computation point.
2. Limit of distance for subdivision into $1^\circ \times 1^\circ$.

A rough time estimate of model anomaly computations for the whole world will be on the order of 10 hours. This is for program M, using the limit of the computation $\psi = 120^\circ$, and subdividing neighboring squares into $1^\circ \times 1^\circ$ within a distance of $10.2\cos \varphi$ (for $\varphi \leq 60^\circ$) or $5.2^\circ$ (for $\varphi > 60^\circ$). The computation is skipped at the polar zone (above $80^\circ$ or $85^\circ$). More detailed or accurate time estimates are not available.
Table 13

Execution times, various programs

<table>
<thead>
<tr>
<th>Program</th>
<th>P</th>
<th>P1</th>
<th>M</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>I, $\psi=30^\circ$</td>
<td>5.2</td>
<td>5.76</td>
<td>6.48</td>
<td>3.35</td>
</tr>
<tr>
<td>$10.2^\circ \cos \phi$</td>
<td>7.92</td>
<td>9.36</td>
<td>6.12</td>
<td>6.80</td>
</tr>
<tr>
<td>$\psi=120^\circ$</td>
<td>25.22</td>
<td>35.56</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>II</td>
<td>2.52</td>
<td>3.02</td>
<td>2.35</td>
<td>2.65</td>
</tr>
<tr>
<td>III</td>
<td>0.86</td>
<td>1.15</td>
<td>1.21</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Time in minutes for lat. zones 5° N-5° S.

1Sequence in the computation of anomaly, p. 71.
2Limit of distance for subdivision into 1° x 1°.

4.2 Analysis of results

Once the programs are operational, one can start with test-runs. Comparisons are made between the model anomalies and observed free-air gravity anomalies (or another author's model anomalies). Statistical methods and procedures should be applied to the analysis [90, p. 336], [80]. A review of the statistical analysis of the results, in the form adopted for this specific study, follows below. The remarks and procedures are typical of the same.

Comparison between the model anomalies $\Delta g$ and the observed anomaly $\Delta g$ is made on each parallel zone, and yields the original residuals,

$$V'_1 = \Delta g_1 - \Delta g_{1}.$$  

(76)
The residuals \( V' \) do not seem to be random, having rather distinct waves.

Besides, their average,

\[
\bar{V}' = \frac{\left[ V' \right]}{n},
\]

is not zero. This bias should be accounted for, if it has a physical meaning [as when changes in the normal gravity formula can affect the residuals, see p. 39]. Reducing the residuals to have a mean zero on each parallel zone is done by subtracting the bias; namely,

\[
V_i = V'_i - \bar{V}'.
\]

The new variance \( [VV] \) will yield the standard deviation of the reduced system, \( v_v \), as

\[
v_v = \pm \sqrt{\frac{[VV]}{n-1}}.
\]

In case the original variance \( [V'V'] \) is precomputed (or if it is required for some statistical analysis), the relation between the two variances can be used in either direction, instead of following (79) explicitly. This relation is

\[
[VV] = [V'V'] - \bar{V}'[V'] .
\]

For investigation of a subgroup (on the parallel zone), with its population \( j \), a relation similar to the one above would be

\[
[V_j V_j] = [V'_j V'_j] - 2\bar{V}'[V'_j] + j\bar{V}'^2 \quad (j = 1, 2, \ldots, j).
\]
Note that here the reduced variance \([ V_j V_j ]\) of the subgroup could be smaller or bigger than the old one \([ V_j' V_j' ]\). This is so because the mean deviation \(\overline{V}'\) (to be subtracted from each \(V_j'\)) does not equal the mean deviation of the subset, \(\overline{V}_j'\).

Equation (81) is a generalization of (80). Both might be utilized, especially when the computations are done by hand.

The standard deviation of the reduced residuals, \(m_v\), is smaller than that of the original ones, \(m_v'\). The reduction brings about an improvement. However, the statistical significance of the reduction should be analyzed also.

The hypothesis \(H_1\), relating the observed anomalies \(\Delta g\) to the model anomalies \(\Delta \overline{g}\), is taken as

\[
H_1 : \Delta g = a + \Delta \overline{g} + \epsilon ,
\]

with the bias \(a\), and the error \(\epsilon\) [47, p.172]. The null-hypothesis \(H_0\) is

\[
H_0 : a = 0 .
\]

The solution of the bias is immediate, \(a = \overline{V}'\).

The test statistic for the examination of the hypothesis \(H_1\) is

\[
F = \frac{(\text{Mean Square})_{\text{num}}}{(\text{Mean Square})_{\text{den}}} ,
\]

as in Table 14. The number of observations is \(n\).

The tabulated value of \(F\), with \((1, n-1)\) degrees of freedom, for a specific level of significance \(x\) (for example 5%), can be found in [47, p.394].

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Table 14

Variance analysis, H1

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Parameters</th>
<th>Deg. of Freedom</th>
<th>Residuals</th>
<th>Sum Square</th>
<th>Mean Square</th>
<th>F_{1,n-1}</th>
</tr>
</thead>
<tbody>
<tr>
<td>H0</td>
<td>0</td>
<td>n</td>
<td>[V'V']</td>
<td>MS_v = [V'V']/n</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H1</td>
<td>1</td>
<td>n-1</td>
<td>[VV]</td>
<td>MS_r = [VV]/(n-1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff.</td>
<td>(-) 1</td>
<td>1</td>
<td>SS_0 = [V'V'] - [VV]</td>
<td>MS_o = SS_0/1</td>
<td>F = MS_o/MS_r</td>
<td></td>
</tr>
</tbody>
</table>

The hypothesis is significant if the computed value is bigger than the tabulated one. In most cases it was found (for the given data) that the bias was not statistically significant.

The bias can be similarly investigated [7, p. 126] by analysis of hypothesis

\[ H2 : \Delta g = \Delta \bar{g} \]

or; namely, the actual bias is zero.

The test statistic used is

\[ t = \frac{\bar{V}'}{n/m_v} \] (86)

The tabulated value of \( t \), with \((n-1)\) degrees of freedom and the same level of significance \( x \) as above, can be found in [7, p. 384] (but enter the table with \((1 - x/2)\) because it is cumulative). The hypothesis is significant if the absolute value of the computed \( t \) is smaller than the tabulated one. Most cases confirmed that the bias had no statistical significance, for the available data.

The original residuals \( V' \), as well as the reduced ones \( V \), indicate that
systematic effects are probably involved. One such effect might be coming from wrong crustal thicknesses used, p. 132. Assuming that the correlation is mainly with the crustal thickness of the square in question (crust), a linear hypothesis \( H_3 \) could be tried,

\[
H_3 : \Delta g = a + b \cdot \text{crust} + \Delta g + \epsilon. \tag{87}
\]

Rearranging, this hypothesis is

\[
H_3 : V' = a + b \cdot \text{crust} + \epsilon, \tag{88}
\]

and the null-hypothesis is therefore

\[
H_0 : a = b = 0. \tag{89}
\]

The least-squares solution for \( a \) and \( b \) can be had from

\[
b = \frac{[V' \cdot \text{crust}] - [V'][\text{crust}]/n}{[\text{crust}^2] - [\text{crust}]/n}, \tag{90}
\]

and

\[
a = \frac{[V'] - b [\text{crust}]/n}{n}. \tag{91}
\]

The new residuals \( d \) can be computed as

\[
d_i = V'_i - (a + b \cdot \text{crust}_i). \tag{92}
\]

This relation yields zero for \( d \), when the mean values \( \bar{V} \) and \( \bar{\text{crust}} \) are inserted.

The new standard deviation \( m_d \) is slightly smaller than \( m_v \). The question
of how significant is the line-fit is checked in Table 15, with the F test.

Table 15

Variance analysis, H3

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Parameters</th>
<th>Deg. of Freedom</th>
<th>Residuals</th>
<th>Mean Square</th>
<th>F&lt;sub&gt;3, n-3&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>H0</td>
<td>0</td>
<td>n</td>
<td>$[V'V']$</td>
<td>$MS_v = [V'V']/n$</td>
<td></td>
</tr>
<tr>
<td>H3</td>
<td>2</td>
<td>n-2</td>
<td>$[dd]$</td>
<td>$MS_d = [dd]/(n-2)$</td>
<td></td>
</tr>
<tr>
<td>Diff.</td>
<td>(-) 2</td>
<td>2</td>
<td>$SS_D = [V'V']-[dd]$</td>
<td>$MS_D = SS_D/2$</td>
<td>$F = \frac{MS_D}{MS_v}$</td>
</tr>
</tbody>
</table>

The hypothesis H3 was found statistically significant in most cases, for the same level of significance $x$ as before. This means that the residuals have some degree of correlation to the individual crustal thickness.

The standard deviations $m$, from samples on various latitude zones, could give some idea about the general accuracy $\sigma$ of the prediction with the model. They could also indicate whether the samples belong to the same population. The test statistic is

$$\left(\frac{X^2}{n}\right) = \frac{m^2}{\sigma^2}$$

with the degrees of freedom $n$ [7, p. 105]. For the same level of significance $x$, the tables [7, p. 386] yield two numbers: $A = \frac{X^2}{n} \cdot m^2$, and $B = \frac{X^2}{n} \cdot \sigma^2$.

Expression (93) will limit the value of $\sigma$, therefore, between

$$\frac{m}{\sqrt{B}} < \sigma < \frac{m}{\sqrt{A}}$$

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Another indication about the general population can be found from a different F test [7, p.107]. The test statistic is

\[ F_{n_i, n_j} = \frac{m_i}{m_j}, \]  

(95)

with \( i \) and \( j \) indicating the two subgroups to be compared (two parallel zones). With the same level of significance \( x \), and with the two degrees of freedom \( n_i \) and \( n_j \), the tables [7, p.390] give two numbers: C for \( (x/2) \) and D for \( (1-x/2) \). The samples belong to the same population if the computed \( F \) is between the tabulated values \( C \) and \( D \).

Still another tool can be used for the investigation of both residuals and model anomalies. This is the correlation coefficient. The correlation coefficient \( \rho \) between two variables \( x \) and \( y \) is defined as

\[ \rho = \frac{COV(x,y)}{\sigma_x \sigma_y}, \]  

(96)

with the covariance \( COV \), and the standard deviation \( \sigma \). The coefficient can be estimated from a population of \( x \) and \( y \) with mean values \( \bar{x} \) and \( \bar{y} \), by

\[ r = \frac{[(x_i - \bar{x})(y_i - \bar{y})]}{\sqrt{[(x_i - \bar{x})^2][(y_i - \bar{y})^2]}}. \]  

(97)

This can be expanded, for computational reasons, to

\[ r = \frac{[xy] - [x][y]/n}{\sqrt{[x^2] - [x]^2/n}([y^2] - [y]^2/n/n)}. \]  

(98)

The correlation coefficient can vary between \( \pm 1 \). The higher the (absolute
value of the coefficient — the more correlated the variables are. This does not always imply that the standard deviation decreases with increased correlation coefficient, since the latter can not detect scale errors between the compared variables. For prediction purposes the coefficient correlating model anomalies and free-air gravity anomalies should approach +1, of course. In the text that follows, the correlation coefficient might be given as a number, or literally, on a scale based on the grades "low", "medium" and "high".

Remark: Most of the statistical tests require or imply normal distribution of the residuals. (Some information can be gained, though, even if this is not the case; likewise when systematic errors might be affecting the results).

4.3 Test runs

After the preliminary stages, there were still quite exhaustive and rather numerous test-runs to be performed, for the various models and programs involved.

4.31 Program tests

The program tests checked the programs as a whole for results, and also for mode of operation, especially in the main programs M and M1.

4.311 Point program P

The ultimate test for a program would be to compare results obtained through its use to the known or precomputed values. Such values could easily be
established, using artificial data. Tests of this kind were used to check the point
program P, and the discrepancies indicated a standard deviation for the total model
anomaly of about ±1 mgal. This was anticipated, p. 102. For example, consider a
case in which all the distinct 5° x 5° values for the elevation, the crustal thickness
and the densities were set up as constant 1000 m, 31 000 m and 3,1/3,6, respect-
tively. Using R = 6000 km, k = 7 x 10^{-3} and CRUST = 30 km and the distinct-
densities model B, the point program established the two shells with density discri-
pancies of -0.1 and 3.0 and border radii of (5970 to 6000) and (6000 to 6001) km,
respectively. The total attraction of these two shells should be 1.4 mgal everywher
on the surface of the model. The program yielded 2.1 and 2.5 mgal, at latitude 4
and 5°, respectively. Other data yielded similar, and better results.

4.312 Main program M

Using internal time controls, the main program M was first checked out
with regard to the order of operations; namely, which cells were filled? How, an
when? As the order indicated proper operation, the program was modified in a
way that allowed computation of total model anomaly starting from the equator out
to the poles (NOCOMP control, appendix B). This allows comparison of total resu
from the main program M to ones obtained from the point program P, in locations
not at the polar zones.

The comparison showed that the programs were yielding exactly the same
results, at every point.
4.32 Model comparisons

The model test-runs were used to obtain total attraction and anomaly values in sample areas, in order to study and evaluate the various models and deduce the best of them. Comparisons to models by other authors were also included.

The test samples might be too small for positive conclusions, but they should be at least indicative of the prevailing situation.

All computations were done at the center of gravity of the compartment, and were carried out to a distance of \( \psi = 120^\circ \), unless specified otherwise. The limit for subdivision of neighboring squares is \( 10.2^\circ \cos \phi \).

4.321 Comparison to an Airy-Heiskanen system

The effect of the Hayford zones 18-1 in the Airy-Heiskanen system \( T=30 \), and densities \( 3.27/2.67/1.027 \) is given in [29]. One model A with constant densities, p.25, can handle this system, if the proper (Airy) crustal thickness data is precomputed and read-in. This was done, and computations on latitude zone 9 (45°-40°N) carried out for the effect of the Hayford zones 18-1. The results are listed in Table 16, p.118, as \( A_{\alpha \beta} \). The corresponding values from [29] appear as \( K \). The sign of the latter was reversed to represent the effect and not the reduction. The listed longitude is that of the N-W corner of the compartment. The rest of the table concerns the following subheadings.

The r.m.s. deviation of the effect from zones 18-1 was \( m_k = \pm 27.4 \text{ mgal} \).
Table 16

Model anomalies, various authors

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1 (5° x 5°) mean observed free-air anomaly [84], lat. 45°-40° N.
2 Attraction of Hayford's zones 18-1, Airy Heiskanen model, T = 30 [29].
3 Corresponding model A disturbances.
4 Uotila's model anomalies, Airy-Heiskanen system, T = 30 [86].
5 Corresponding model A anomalies.
6 Kivioja's model anomalies, Airy-Heiskanen system, T = 30 [37 p. 89].

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### Table 16 (Continued)

**Model anomalies, various authors**

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</table>
while the standard deviation of the residual between the model $A_{out}$ and $K$ was $m_{v'} = \pm 4.7$ mgals. This measure points out that the constant-density model $A$ would indeed represent the effect of the Hayford zones 18-1 with an accuracy approaching that of [29] (and considerably better than the assumption of zero disturbance, in case the maps above are not available). The standard deviation of $\pm 4.7$ mgals between the models is rather good agreement, even though the mean residual is $\bar{V}' = 1.35$ mgal, and there could possibly have been some systematic effects on one or both models. These could have come from differences in elevation data and/or data evaluation in the models, especially close to the point of computation.

The statistical analysis, following subheading 4.2, p. 108, is listed in Table 17. It shows that the bias is significant (both $F_{v'}$ and $\tau_{v'}$), and that the straight-line fit between the residuals and the crustal thickness is even more significant. The adjustments bring about some improvement in the standard deviation, especially noted for $m_d$. The correlation coefficient between neighboring residuals is high, also suggesting some systematic effects. Even so, the prediction correlation (of $A_{out}$ with $K$) is very high (0.997).

The indirect effect for the same Airy system, and for the same zones 18-1, is listed in the files of the Department of Geodetic Science at O.S.U. These effects are everywhere negative, ranging in magnitude from $-0.7$ to $-1.0$ mgal. The corresponding Bruns' term from model $A_{out}$ was about $-0.7$ mgal, everywhere.
Table 17

Analysis of various authors' models

<table>
<thead>
<tr>
<th>Author</th>
<th>( ^1 \text{Kirkki} )</th>
<th>( ^2 \text{Uotila} )</th>
<th>( ^3 \text{Kivioja} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{V} )</td>
<td>1.35</td>
<td>-0.79</td>
<td>1.29</td>
</tr>
<tr>
<td>( ^5 F_{\tilde{V}} )</td>
<td>6.85*</td>
<td>0.97</td>
<td>1.42</td>
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<tr>
<td>( ^4 )</td>
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<td>(3.99)</td>
<td>(3.99)</td>
</tr>
<tr>
<td>( ^5 t_{\tilde{V}} )</td>
<td>2.62*</td>
<td>-0.99</td>
<td>1.19</td>
</tr>
<tr>
<td>( ^7 F_4 )</td>
<td>(2.00)</td>
<td>(2.00)</td>
<td>(2.00)</td>
</tr>
<tr>
<td>( ^{10} R )</td>
<td>109.80**</td>
<td>4.86*</td>
<td>0.93</td>
</tr>
<tr>
<td>( ^{10} )</td>
<td>(3.14)</td>
<td>(3.14)</td>
<td>(3.14)</td>
</tr>
<tr>
<td>( ^5 \pm m_{\tilde{V}} )</td>
<td>4.57</td>
<td>6.86</td>
<td>9.29</td>
</tr>
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<td>( ^6 \pm m_{\tilde{V}} )</td>
<td>4.37</td>
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<td>9.19</td>
</tr>
<tr>
<td>( ^7 \pm m_4 )</td>
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</tr>
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<td>( ^8 \text{Resid. Corr.} )</td>
<td>High</td>
<td>Med. Low</td>
<td>Med.</td>
</tr>
<tr>
<td>( ^9 \text{Pred. Corr.} )</td>
<td>0.997</td>
<td>0.774</td>
<td>0.734</td>
</tr>
</tbody>
</table>

Lat. 45°–40°N. \( N = 72 \) squares. Model A with Airy crustal data.

1 Comparison between attraction of Hayford's zones 18-1 [29] and model A disturbances.

2 Comparison between Uotila's model anomalies [86] and model A anomalies.

3 Comparison between Kivioja's model anomalies [37, p. 89] and model A anomalies.

4 Computed value and (tabulated); level of significance 5%.

Asterisk * indicates significance.

5 Original residual.

6 After reduction of mean.

7 After straight-line fit.

8 Correlation between neighboring residuals.

9 Correlation between effects analyzed.
4.522 Comparison to Kivioja's model

L. Kivioja computed the total effect of the topography and its compensation in the Airy-Heiskanen system, $T = 30$, with densities $3.27/2.67/1.027$. The computation included attraction effects only, but the results were treated and adjusted as model anomalies [37, p. 88]. The constant-density model $A$ was used again, with the proper crustal thickness values derived from the elevations in this Airy system, to compute the model anomalies. The computation was done on the 9th parallel zone ($45^\circ - 40^\circ$N), as in the preceding subheading which was a part of it. The results are listed in Table '6, p. 118, under $A$. The corresponding values from [37] are listed under $L$.

The results gave $m_L = \pm 12.9$ mgals, and $m_V = \pm 9.3$ mgals. The latter measure is rather high, but it should be noted that the $L$ values have been altered or adjusted, so as to better represent the free-air anomaly field of the earth. In other words, the comparison to the $L$ values is not indicative of either how well the effects of the topography and its compensation were considered in the author's model $A$, or how closely they can represent the actual free-air anomalies. In addition, there could have been some effects arising from the slightly different sets of $5^\circ \times 5^\circ$ mean elevations used.

The statistical analysis, following subheading 4.2, p. 108, is listed in Table 17, p. 121. The bias of 1.29 mgals is not statistically significant; nor is the straight-line fit. Thus there is no marked improvement in the standard deviations for $V$ and $d$. Correlation coefficients between neighboring residuals are medium in size. The correlation coefficient between the two models is
medium-high (0.734).

For future reference, a comparison between the model A anomalies for the Airy system above and the observed free-air anomalies was made. In Table 16 these appear as \( A \) and \( \Delta g \), respectively (p. 118); and in Table 26 as \( A_{\text{re}} \) and \( \Delta g \) (p. 151). Results give \( m_{\Delta g} = \pm 15.1 \) and \( \pm 19.1 \) mgals, and \( m_{\gamma} = \pm 13.3 \) and \( \pm 15.1 \) mgals, for all the squares, and for the ones that have (16-25) observed \( 1^\circ \times 1^\circ \), respectively. This means that the specific Airy-Heiskanen system utilized here, and computed through the constant-density model A, gives slightly better results for the prediction than assuming zero anomaly.

4.323 Comparison to Uotila's model

Uotila [86] computed model free-air anomalies for the same Airy-Heiskanen system mentioned before (\( T=30 \) km and densities \( 3.27/2.67/1.027 \)), based on the elevations and squares of elevations with Jung’s derivations [26]. These model-anomalies could be compared to the author’s A values in Table 16, since they refer to the same system, p. 118. Uotila’s model anomalies are listed under U.

The results give \( m_{\Delta g} = \pm 10.8 \) mgals, while \( m_{\gamma} = \pm 6.9 \) mgals. The agreement between the two models is not exceptionally good.

The statistical analysis, following subheading 4.2, p. 108, is listed in Table 17, p. 121. The bias of -0.79 mgal is not statistically significant by itself, while the straight-line fit is significant (or so is the relation of residuals to crustal thicknesses). The improvement in the standard deviation value is not big. The
correlation coefficients between neighboring residuals are medium-low, and the coefficient between the two models is medium-high (0.774).

4.33 Model test-runs

All model computations were performed at the center of gravity of the compartment. The limit of computation is $\psi = 120^\circ$, unless otherwise specified; and the limit for subdivision of neighboring squares is $10.2^\circ \cos \varphi$.

Analysis of results is done in two groups, one for compartments containing $(16-25) \, 1^\circ \times 1^\circ$ anomalies, and the other for all the compartments on the parallel zone. A similar procedure was introduced in [87].

A short reference for model classification is on p. 91.

4.331 Constant-density model A

This model assumes constant densities, as described in 3.2, p. 25. It was the first model to be tried. Results did not agree very well with the observed mean free-air anomalies, with standard deviations between them on the order of $\pm 50$ mgals. Obviously then, the models investigated were far from representing the anomaly field of the earth. Thus evaluation and trial-and-error procedures with various densities seemed hopeless. For demonstration, Table 18 lists results on the 9th latitude zone $(45^\circ \text{-}40^\circ \text{N})$. The data set is: CRUST = 34.084 km, and the densities $3.25/2.89/1.027$. The model anomalies are coded $A$ in the table. The mean observed $5^\circ \times 5^\circ$ anomalies from [84] are listed under $\Delta g$, together with
Table 18

Various model anomalies, I

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<tr>
<th>Long(^\circ)E</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
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<td>36</td>
<td>36</td>
<td>35</td>
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<td>43</td>
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<td></td>
<td>44</td>
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<td>47</td>
<td>52</td>
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<td>63(\frac{1}{2})</td>
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<tr>
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<td>12</td>
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<td>9</td>
<td>14</td>
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<tr>
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<td>20</td>
<td>5</td>
<td>29</td>
<td>15</td>
<td>11</td>
<td>4</td>
<td>4</td>
<td>26</td>
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<th>150</th>
<th>160</th>
<th>170</th>
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<tbody>
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<td>-8</td>
<td>5</td>
<td>7</td>
<td>-6</td>
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<tr>
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<td>-10</td>
<td>5</td>
<td>7</td>
<td>6</td>
<td>12</td>
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<td>0</td>
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<td>-13</td>
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<td>-10</td>
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<td>9</td>
<td>21</td>
<td>-1</td>
<td>3</td>
</tr>
<tr>
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<td>-9</td>
<td>4</td>
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<td>20</td>
<td>-5</td>
<td>-9</td>
</tr>
</tbody>
</table>

1 (5° x 5°) mean observed free-air anomaly [84], lat. 45°-40°N.
2 Model A anomaly. Constant densities 3.25/2.89/1.027.
3 Model B anomaly. Mass-model: CRUST=34.084, densities above.
4 Model D anomaly. Mass-model above. Const. density 2.60, 0.4 of crust.
Computation limit: \(\phi = 30°\) or 120°.
5 Residuals for D\(120\), rounded. V' - original, V - reduced, d - line fitting.

125
Table 18 (Continued)

Various model anomalies, I

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<th>Long(^\circ)E</th>
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<td>6(2)</td>
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<td>16</td>
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<tr>
<td>((1^\circ \times 1^\circ))</td>
<td>2 (3)</td>
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<td>- 9</td>
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<td>- 6</td>
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</tr>
<tr>
<td>((1^\circ \times 1^\circ))</td>
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<td>25</td>
<td>25</td>
<td>25</td>
<td>22</td>
<td>14</td>
<td>8</td>
<td>12</td>
<td>7</td>
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<tr>
<td>(\Delta g^{**\text{cal}})</td>
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<td>- 7</td>
<td>- 11</td>
<td>7</td>
<td>10</td>
<td>- 16</td>
<td>- 21</td>
<td>- 1</td>
<td>14</td>
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<td>- 6</td>
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<td>V(^{'})</td>
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<td>- 18</td>
<td>- 23</td>
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<td>9</td>
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</table>
the number of $1^\circ \times 1^\circ$ in each. The model uses the mean densities and Moho used for the mass-model, thus eliminating the spherical shell effect $\bar{F}$, p. 30. Likewise, the mean density discrepancy for the model is $0.00008 \text{ km gr/cm}^3$, which is rather low, p. 88.

The comparison between model A anomalies and the mean free-air gravity anomalies is done in two groups, for squares with $(16-25) \ 1^\circ \times 1^\circ$ anomalies, and with $(0-25)$. The statistical analysis follows 4.2, p. 108, and is summarized in Table 19. The bias is statistically insignificant, while the line-fit is significant for the whole group (but not for the $(16-25) \ 1^\circ \times 1^\circ$ subgroup). The standard deviations are high, and improve only a little by the adjustment. The correlation coefficient between model and observed anomalies is medium; however, the prediction accuracy is so much worse than the standard deviation of the observed mean anomalies, that this model must be rejected. The use of other data sets usually made the picture look even worse, with existing crustal thicknesses used.

4.332 Distinct-densities model B

This model computes the distinct $5^\circ \times 5^\circ$ densities in the manner described in subheading 3.85, p. 90. Results for the mass-model with CRUST = 34.084 km and densities 3.25/2.89/1.027 for the latitude zone $(45^\circ-40^\circ N)$ are given in Table 18, p. 125, under B. Comparison can again be made to the observed free-air anomalies from [84]. These are listed as $\Delta g$, together with the number of $1^\circ \times 1^\circ$ mean anomalies used for the $5^\circ \times 5^\circ$ mean.
Table 19

Analysis of various model anomalies

<table>
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<th>$^2B$</th>
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<td>(4.28)</td>
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<td>$^8\pm m_{\Delta g}$</td>
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<td>15.1</td>
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<tr>
<td>$^9$Resid. Corr.</td>
<td>Low</td>
<td>Low</td>
<td>M. High</td>
<td>Low</td>
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<tr>
<td>$^{10}$Pred. Corr.</td>
<td>0.495</td>
<td>0.638</td>
<td>0.255</td>
<td>0.436</td>
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</table>

Lat. 45°-40°N. Comparison between model and observed anomalies [84]. N-61/24.

1 Model A anomalies. Constant densities 3.25/2.89/1.027.
2 Model B anomalies. Mass-model: CRUST=34.084, densities above.
3 Model D anomalies. Mass-model above. Const. density 2.60, 0.4 of crust.
4 Computed value and (tabulated); level of significance 5%.
Asterisk * indicates significance.
5 Original residuals.
6 After reduction of mean.
7 After straight-line fit.
8 Observed anomalies.
9 Correlation between neighboring residuals.
10 Correlation between anomalies analyzed.
The comparison between model B anomalies and the mean free-air gravity anomalies follows 4.2, p.108, and is summarized in Table 19, p.128. The bias is statistically insignificant, while the straight-line fit is significant, indeed. The standard deviations of the residuals after adjustment is smaller than the standard deviation of the observed anomalies (or even before adjustment, for the (16-25) group). This means that the model anomalies would represent the mean free-air anomalies better than zero anomalies. The difference between the two standard deviations above is not very big, however, and there is evidence to suspect some systematic effects. Neighboring residuals show waves, and are moderately correlated. Other tests; namely, Abbe-Helmert, sign change, and distribution criteria also suggest systematic effects.

The grouping of residuals by the number of $1^\circ \times 1^\circ$ mean free-air anomalies in the square seems to have improved and clarified the prospect for the prediction accuracy. Implication from this might raise some doubt about the accuracy of the mean observed anomalies, especially in the low-$(1^\circ \times 1^\circ)$ category—which is quite reasonable. A grouping into land and sea compartments was not found meaningful enough to be practiced here.

Comment 1: Kaula [35] gives recent $10^\circ \times 10^\circ$ mean free-air anomalies deduced from satellite observations. In order to gain some appreciation of the $5^\circ \times 5^\circ$ mean free-air anomalies in [84], a comparison was made between the two sets. A system of $10^\circ \times 10^\circ$ means can not directly be compared to one with $5^\circ \times 5^\circ$ means. Therefore the latter were averaged into $10^\circ \times 10^\circ$ means, wherever there
appeared four $5^\circ \times 5^\circ$ values (at 228 out of the total of 648 squares). The mean deviation was $\bar{\gamma}' = -0.5$ mgal, and the standard deviation $m_{\gamma'} = 16.4$ mgals. This means that Kaula's anomalies do not, in effect, contradict Uotila's.

Comment 2: The model B computes distinct model densities for each compartment. At some locations the computed density was even higher than the model density for the subcrust. The former should not be corrected, or forced to be smaller than the subcrust density, since it is a model density only. In fact, forcing or changing the computed density distorts the mass balance and the model anomalies (to the worse).

4.333 Two-layered model C

This model computes the distinct $5^\circ \times 5^\circ$ densities, after assuming a constant density for the topography above mean sea level [19, p. 6], see p. 37. Results for the mass-model data CRUST = 34.084 km, and densities 3.25/2.89/1.027 for 8 points on latitude zone 11 ($35^\circ$-$30^\circ$N) are demonstrated in Table 20. The model is listed under C, and uses various (constant) densities above m.s.l.; namely, 2.60, 2.70, 2.80 and 2.89. For comparisons, the mean observed free-air anomaly and the result from the model B with the same mass-model are listed also.

It is evident that the model C anomalies can not differ very much from model B anomalies, just by the changing of the constant density above mean sea level. It stands to reason, too, since the effect on the topography which could be changed by it is not prominent, compared to the effects below sea level. At points
Table 20

Various model anomalies, II

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<th>Long.°E</th>
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<th>70</th>
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<th>²Fra.</th>
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<tr>
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<tr>
<td>D</td>
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<td>-5.56</td>
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</table>

Lat. 35°-30° N.

¹ Constant density used for topography (C) or upper crust (D).
² Upper crust as fraction of total crust.
³ Model D with fraction zero becomes model B.
in a sea area the effect is negligible.

Altogether there is not much advantage in using model C over B. Even their mean mass discrepancies for the whole earth, with the mass-model data 34.084 - 3.25/2.89/1.027, are almost the same: 2.63 and 2.57 x 10^-7 km gr/cm^3 respectively. These are both extremely small.

Note that in Table 20, p.131, model (C, 2.89) does not always give the same anomalies as model B. This is how it should be, since the latter does not - as a rule - use the standard density 2.89 for the topography.

4.334 Changes in Moho

The results for model B in Table 18, p.125, indicate possible systematic effects, in that the deviation V between the reduced model anomaly and the free-air gravity anomaly goes through a wave motion. In other words, the deviation has the same sign and comparable size for big areas. Some correlation was found to the crustal thickness of the individual compartment; but the main source for such waves could be systematically wrong Moho values, area-wise. A little investigation of that possibility was undertaken, and an example given in Table 21.

Two areas were chosen: A - at land, where the residuals were negative, and B - at sea, where they were positive. The areas are also designated by their latitudes and longitudes, in the table. By comparing the distinct crust thickness of each compartment to the corresponding thickness (derived from the elevation) in an Airy-Heiskanen system, an indication as to the direction of the Moho change.
Table 21

Effect of Moho changes

A. Land area

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<th>2In</th>
<th>3V</th>
<th>Ch. V</th>
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B. Sea area

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<table>
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<th>320</th>
<th>325</th>
<th>330</th>
<th>335</th>
</tr>
</thead>
</table>

Model B anomalies. Mass model: CRUST = 34.084, densities 3.25/2.89/1.027.
1 Model anomaly effect from outside the compartment.
2 Model anomaly effect from within the compartment.
3 Deviation between total (1 + 2) model B anomaly and free-air anomaly [84].
4 Change in the Moho, km.
5 Before change.
6 After change affected.

133
can be obtained. It so happened, that this test suggested to decrease the crustal thickness in A, and to increase it in B. The change was affected as a smooth, regional one as shown by the encircled numbers in Table 21. It did by no means amount to the total discrepancy between the crustal thicknesses above. Yet the residuals V prior to the change, were affected by it in the right direction, to become (absolutely) smaller. The effect was more pronounced at the land area than at sea.

The local-regional change of Moho changed the model anomalies in — and close to — the location of the change only.

Most of the effect in the change comes from the compartment itself (IN), and less from outside it (OUT), of course.

Altogether it has been demonstrated that systematically wrong Moho (or crustal thickness) values, in an area, constant or waves, could bring about systematically wrong anomalies, constant or waves.

4.335 Two-layered model D

The fact that the two-layered model C can not, probably, differ much in results from model B opens up a possibility for another model, D, which could be more versatile. Why not use a model which utilizes two densities: One constant, for the upper crust, and the other variable with the compartment, for the lower crust? This would perhaps be closer to the actual phenomenon of increasing density with depth, and has been suggested for example in [62, p.513].
It would also mainly amount to change in the effect of the deeper crust, if the upper part were kept lighter. This would actually be similar in results to change in the Moho, as in the preceding subheading.

Contrary to the previous model C, changes between model D anomalies and model B anomalies can be expected for sea compartments also, since there always exists a crustal thickness which would be divided into two layers as described above (and on p. 37), thus making model D different from B.

Some short test-runs for model D are listed in Table 20, p. 131. The two numbers after the model name stand for the constant density used for the upper crust, and the fraction of the total crust which is considered upper crust. The case when the fraction is zero is actually model B (for any constant density specified); and so do the results witness.

The versatility of this model has been demonstrated, yet finding a good data set for its purposes might prove difficult. Two examples are listed in Table 18, p. 125. The same data as for the model B was employed; namely, CRUST = 34.084 km and densities 3.25/2.89/1.027. The model has an upper crust with density 2.60, extending to 0.4 of the total crust. The statistical summary in Table 19, p. 128, shows the improvement in this model from model B.

The comparison between model D anomalies and the mean free-air gravity anomalies follows 4.2, p. 108. For demonstration purposes, the same model D is used twice, with the limit of computation extended to $\psi = 30^\circ$ and to $\psi = 120^\circ$. As was found before, p. 101, the latter should yield the total anomalies to within 1 mgal or less, while the effects of the neglected zones beyond $30^\circ$ in the former
might be on the order of several mgals. The results were different, of course, and — surprising enough — seemingly worse for the case with $\psi = 120^\circ$. This only points out to the fact that systematic (or other) effects should be accounted for before comparison and analysis take place, and that a good agreement between model anomalies and observed anomalies does not necessarily signify a correct model.

The statistical summary for $\psi = 120^\circ$ in Table 19, p. 128, shows the bias to be statistically insignificant, while the straight-line fit to crustal thickness is significant, and yields smaller $m_4$ than $m_V$. All standard deviations for residuals are smaller than standard deviations for the observed anomalies, thus model anomalies useful for prediction of free-air gravity anomalies.

The improvement from the distinct-densities model $B$ to the two-layered model $D$ is real, as demonstrated by smaller $m$ values, as well as by higher correlation coefficients for the prediction, and generally lower coefficients for the correlation between neighboring residuals. Just the same, there still are possible systematic effects in model $D$, as suggested by the correlation coefficient, the long waves of residuals, and by the criteria of sign change, distribution and Abbe-Helmert. The three sets of residuals; namely, $V'$, $V$ and $d$ were also presented in Table 18, p. 125, in order to show their distribution.

The grouping into (0-25) and (16-25) $1^\circ \times 1^\circ$ in the square is meaningful, and yields better results for the prediction in the high-( $1^\circ \times 1^\circ$) bracket.

Establishing the best model $D$ might prove a tedious job. An adjustment procedure could perhaps be employed, to solve for the best fraction of the crust.
and its constant density. More than two layers could be visualized, not just of constant densities, etc. The possibilities are unlimited, except by the notion that the model chosen should be simple and represent the general state of affairs, rather than be a unique solution for so many (limited) fitting points.

4.34 Review of model evolution

Chronologically, here is the case history of the various models and procedures discussed, in review. Consider also Table 4, p. 91.

1. From the investigation of the input data, the apparently best mass-model to represent the mean earth was found.

2. The gravity model with standard densities, $A$, was established and working formulas developed for the main program $M$. Test-runs for a sphere with artificial data were performed. Numerical integration techniques were checked for some parts of the program.

3. The point program $P$ was developed for the same model, and results checked to the ones obtained from $M$.

4. Residuals between the free-air anomalies and model $A$ anomalies were too big to allow predictions with model $A$, even for successive changes of CRUST and densities in the mass-model.

5. The model with distinct densities, $B$, was established, and all programs modified to its use.

6. Model $B$ anomalies approximated the free-air anomaly field of the earth to a much better degree than before. In most cases they were better than assuming
zero anomaly.

7. Two-layered model C with constant density of the topography was introduced and programs modified accordingly.

8. The improvement with model C over B was not marked, due to the limitations on the effect of topography.

9. The two-layered model D, similar to C, but in which the constant density extends to a portion of the crust, was introduced and programs modified again.

10. The versatility of model D was demonstrated and encouraging results obtained for the possible prediction accuracy.

4.4 Prediction accuracy

Some of the possible sources affecting the precision of the model anomalies are listed below. These relate to computation formulas, techniques and procedures, as discussed in the preceding subheadings. They are specific to the model used and in that respect—-independent of the data. Some absolute errors which could arise from the data, and from the applications of model anomalies to prediction purposes are mentioned later. The (±) indicates root-mean-square deviation.

1. One source of error could be inaccuracies in the computations of the spherical caps, p. 56. The use of numerical integration with small enough increments should, in effect, eliminate this source.

2. The border areas between the cap and the subsquares, Figure 6, p. 50, could be another source of error. This effect was shown to be about ±0.1 mgal.
3. The computation for the subsquares inside the $5^\circ \times 5^\circ$ compartment shows another source of about $\pm 0.1$ mgal, p. 100.

4. Outside the square the inaccuracy due to taking the mass-line formula for $5^\circ \times 5^\circ$ compartment might bring about $\pm 0.5$ mgal. For neighboring squares the effect is mainly systematic, and could be much bigger than $\pm 0.5$. A subdivision of the neighboring squares (into $1^\circ \times 1^\circ$, for example) will reduce this error to the former limit. [This is closely related to the problem presented in (7.) below].

5. Limiting the computation to a specific angular distance should not bring about errors of more than $\pm 0.3$ mgal, even for $\psi = 120^\circ$, p. 101.

6. In deriving the model anomalies, certain approximations were made, which relate to the attraction of a spherical shell, p. 27. The inaccuracies introduced are zero, in most cases. In other cases they could be accounted for, or neglected if the error contributed is small enough.

   Altogether these errors will probably amount to less than $\pm 1$ mgal. The model was set up to predict free-air gravity anomalies which, for $5^\circ \times 5^\circ$, have a standard deviation of about $\pm 15$ mgals. Thus it can be said that the intrinsic model precision should suffice for this purpose.

   The absolute accuracy of the model anomalies is a completely different matter. The earth gravity model is based on some data; namely, elevation and crustal thickness for all $5^\circ \times 5^\circ$ elements. Errors in this data will be reflected to some extent, in any model utilized. The problem might end up in selecting a model whose anomalies best fit the actual anomalies, or better than the others, and trying to reason why it is so and what it implies. Some more effects arise
from the fact that the model anomalies are used for prediction of free-air gravity anomalies, and from the employ of mean values for the data.

7. One of the main sources of absolute errors is the very use of $5' \times 5'$ means, especially near to or at the computation point. It is hard to estimate what this effect will be. Tests indicated that it could probably reach $\pm 5$ mgals and more, for specific points, and is mainly systematic for areas. A solution for this problem is only by using more detailed data close to the computation point, perhaps down to $30' \times 30'$ means, or even $5' \times 5'$, etc. This is closely related to the general procedure of prediction when the effect outside some border-line is reasonably well known, and detailed investigation is necessary within the borders. This might have been investigated, even though the model is strictly based on $5' \times 5'$ means.

8. The second, and main source of possible errors is the crustal thickness data (and/or density data). It was pointed out that errors to the order of $150$ mgals can be expected from it, when a constant density is used together with the data of crustal thickness and elevation. Better models, which utilize distinct densities, could bring this down to between $\pm 10$ and $\pm 15$ mgals. Evidence is, however, that the crustal thickness data used is systematically wrong, areawise; which induces big systematic errors, again areawise.

9. The data on elevations could be considered accurate enough, compared with the preceding one.

10. Model anomalies should represent free-air gravity anomalies as close as possible. The comparison between model and observed anomalies reveals one more discrepancy, or absolute error. It stems from inadequate gravity formula,
p. 39, but could be considered here. The effect is constant for each parallel zone, and could be deduced statistically or eliminated by correcting the gravity formula.

11. Beyond that, there is a theoretical flaw in the prediction. Strictly speaking, model anomalies differ from free-air gravity anomalies by the effects of the hidden mass discrepancies, p. 24. These effects are unknown, could be on the order of a few mgals, for 5° x 5°.

12. Another problem is related to the mean value predicted, and is two-fold. On the one side: The model anomaly is computed at one point, which is supposed to represent the mean effect. Better result could be had by computing a mean value from several points within the compartment. On the other side: When comparing the model anomaly to the mean observed anomaly, account should be made of the number of 1° x 1° means incorporated in the latter. Comparative analysis of the prediction accuracy should be based on well-established 5° x 5° means only.

The biggest effect on the prediction accuracy is, by far, that of the Moho depths (and/or densities). Unless better data is utilized, the prediction of free-air anomalies with model anomalies might not yield results better than ±10 mgals. This standard deviation is smaller than the root-mean-square value for 5° x 5° mean free-air anomalies (±15 mgals); yet a considerable part of the individual deviation might be systematic.
4.5 Two-layered model D anomalies

Quite a few model test-runs were performed, in order to find a model which will best represent free-air anomalies. The two-layered model D seems to be the most versatile of all the models studied (indeed it includes all as special cases). The most promising model D so far, was one based on the mass-model:

\[ \text{CRUST} = 34.084, \text{ and densities } 3.25/2.89/1.027 \text{ (mean earth).} \]

It has a constant density of 2.60 for the upper crust, extending to 40% of the total crust. Some runs of this model, for various latitude zones, are listed in Table 22, under D. The limit of computation is \( \psi = 120^\circ \), and the limit for subdivision of neighboring squares is \( 10.2^\circ\cos\varphi \). Listed are also the mean observed free-air anomalies, and the number of \( 1^\circ \times 1^\circ \) means utilized for it. Analysis of the results is in the next subheading.

4.51 Analysis of model D anomalies

The comparison between model D anomalies listed in Table 22, and the mean free-air anomalies follows 4.2, p.108. The results are summarized in one form in Table 23, p.147.

The bias on each parallel zone is statistically insignificant; yet in most cases the line fitting to crustal thickness is significant. The standard deviations for the residuals are smaller than the r.m.s. value of the free-air anomalies, so the former can be used to predict the latter. There are, however, systematic effects on the residuals, as evidenced by their distribution, by the correlation coefficient between neighboring residuals, and as suggested by the distribution,
Table 22

Model D anomalies

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1 (5° x 5°) mean observed free-air anomalies [84].
2 Model D anomalies. Mass-model: CRUST = 34.084, densities 3.25/2.89/1.027;
   constant density 2.60, 0.4 of crust.
## Table 22 (Continued)

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<td>Crust</td>
<td>25</td>
<td>24</td>
<td>23</td>
<td>22</td>
<td>8</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>(1° x 1°)</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>12</td>
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<td>12</td>
<td>16</td>
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<tr>
<td></td>
<td>Δg</td>
<td>-13</td>
<td>-6</td>
<td>-13</td>
<td>-12</td>
<td>-6</td>
<td>4</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>5</td>
<td>Crust</td>
<td>25</td>
<td>24</td>
<td>23</td>
<td>22</td>
<td>8</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>(1° x 1°)</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>12</td>
<td>15</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Δg</td>
<td>-13</td>
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<td>-13</td>
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<td>-6</td>
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<td>6</td>
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</tr>
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<td>D</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>-5</td>
</tr>
</tbody>
</table>
### Table 23

Analysis of model D anomalies

<table>
<thead>
<tr>
<th>(1° x 1°)</th>
<th>(0 - 25)</th>
<th>(16 - 25)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lat. N</strong></td>
<td>50 45 40 5</td>
<td>50 45 40 5</td>
</tr>
<tr>
<td><strong>No.</strong></td>
<td>58 61 60 43</td>
<td>30 24 22 14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(0 - 25)</th>
<th></th>
<th>(16 - 25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V'$</td>
<td>2.16 3.18 1.93 2.00</td>
<td></td>
<td>-0.30 1.29 -0.00 9.75</td>
</tr>
<tr>
<td>$F_{V'}$</td>
<td>2.55 3.41 1.00 1.53</td>
<td></td>
<td>0.02 0.16 0.00 2.70</td>
</tr>
<tr>
<td>$t_{V'}$</td>
<td>(4.01) (4.00) (4.00) (4.08)</td>
<td></td>
<td>(4.18) (4.28) (4.32) (10.13)</td>
</tr>
<tr>
<td>$F_d$</td>
<td>1.60 1.85 1.00 1.24</td>
<td></td>
<td>-0.13 0.41 -0.00 1.64</td>
</tr>
<tr>
<td>($\pm m_v$)</td>
<td>(2.00) (2.00) (2.00) (2.02)</td>
<td></td>
<td>(2.04) (2.07) (2.08) (3.18)</td>
</tr>
<tr>
<td>(3.17)</td>
<td>(3.15) (3.16) (3.23)</td>
<td></td>
<td>(3.34) (3.44) (3.48) (19.00)</td>
</tr>
<tr>
<td>($\pm m_{\Delta g}$)</td>
<td>10.5 13.8 15.1 10.8</td>
<td></td>
<td>12.3 15.7 14.0 16.4</td>
</tr>
<tr>
<td>($\pm m_v$)</td>
<td>10.3 13.5 15.0 10.6</td>
<td></td>
<td>12.3 15.6 14.0 11.9</td>
</tr>
<tr>
<td>($\pm m_d$)</td>
<td>9.6 12.7 13.8 10.5</td>
<td></td>
<td>11.9 13.6 13.1 5.5</td>
</tr>
<tr>
<td>($\pm m_{\Delta g}$)</td>
<td>11.0 15.1 15.2 10.4</td>
<td></td>
<td>13.7 19.1 15.0 18.6</td>
</tr>
<tr>
<td>Pred. Corr.</td>
<td>0.322 0.403 0.339 0.163</td>
<td></td>
<td>0.431 0.504 0.412 0.979</td>
</tr>
</tbody>
</table>

Comparison between model D and observed anomalies [84].

1 Number of squares on parallel zone, in each class.
2 Computed value and (tabulated); level of significance 5%. Asterisk * indicates significance.
3 Original residuals.
4 After reduction of mean.
5 After straight-line fit.
6 Observed free-air gravity anomalies [84].
7 Correlation between neighboring residuals.
8 Correlation between anomalies analyzed.
9 Sample too small.
Abbe-Helmert and sign change criteria.

The grouping into (0-25) and (16-25) $1^\circ \times 1^\circ$ is significant, and yields better results for the prediction in the high $1^\circ \times 1^\circ$ subgroup. This would again emphasize that the mean free-air anomalies are not errorless, and should somehow be weighted by their reliability, for comparisons' sake.

The four samples listed in Table 22, p. 143, yield differing results for the distribution of the residuals and their statistical implications. The limits of the prediction accuracy, and some visual evidence about the homogeneity of the results can be gained from the $x^2$ test, p. 113. A statistical analysis of this form is given in Table 24, p. 149. The limits of $\sigma$, the prediction accuracy, range from below $\pm 10$ to over $\pm 20$ mgals, and rather overlap each other.

The problem whether or not the samples belong to one population can be checked with the aid of an $F$ test, p. 114. A statistical analysis of this form, for all the combinations between the samples, is given in Table 25, p. 150. It is striking to see that all samples are in the same population, in the (16-25) $1^\circ \times 1^\circ$ class; but not quite so, in the (0-25) class. This stresses again the significance of the grouping into $1^\circ \times 1^\circ$ classes, as a measure of reliability for the mean anomalies.

Remark: The effectiveness of the statistical analysis above might be — in some cases — limited by the presence of systematic errors in the residuals. Nevertheless, the general outline of the situation indicated is probably valid.
Table 24

Model D predictions

<table>
<thead>
<tr>
<th>Lat. °N</th>
<th>Resid.</th>
<th>No.</th>
<th>$\frac{x^2}{n}$</th>
<th>$^4\pm m$</th>
<th>$^5\pm \sigma$</th>
<th>Pred. Corr. No.</th>
<th>$\frac{x^2}{n}$</th>
<th>$^4m$</th>
<th>$^5\sigma$</th>
<th>Pred. Corr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>V'</td>
<td>58</td>
<td>1.41;0.667</td>
<td>10.5</td>
<td>8.8-12.8</td>
<td></td>
<td>30</td>
<td>1.58;0.543</td>
<td>12.3</td>
<td>9.7-16.7</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td></td>
<td>10.3</td>
<td>8.6-12.6</td>
<td></td>
<td></td>
<td>9.6</td>
<td>8.1-11.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>d</td>
<td></td>
<td>10.5</td>
<td>8.1-11.8</td>
<td>0.322</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>V'</td>
<td>61</td>
<td>1.39;0.675</td>
<td>13.8</td>
<td>11.7-16.8</td>
<td></td>
<td>24</td>
<td>1.65;0.508</td>
<td>15.6</td>
<td>12.2-21.9</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td></td>
<td>13.5</td>
<td>11.4-16.4</td>
<td>0.403</td>
<td></td>
<td>12.7</td>
<td>10.7-15.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>d</td>
<td></td>
<td>10.7-15.5</td>
<td></td>
<td></td>
<td>0.339</td>
<td>13.6</td>
<td>10.6-19.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>V'</td>
<td>60</td>
<td>1.39;0.673</td>
<td>15.1</td>
<td>12.8-18.4</td>
<td></td>
<td>22</td>
<td>1.69;0.490</td>
<td>14.0</td>
<td>10.8-20.0</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td></td>
<td>15.0</td>
<td>12.7-18.3</td>
<td>0.339</td>
<td></td>
<td>13.8</td>
<td>11.7-16.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>d</td>
<td></td>
<td>12.8-18.4</td>
<td></td>
<td></td>
<td>0.339</td>
<td>13.1</td>
<td>10.1-18.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>V'</td>
<td>43</td>
<td>1.47;0.619</td>
<td>10.8</td>
<td>8.9-13.7</td>
<td></td>
<td>7</td>
<td>3.12;0.072</td>
<td>11.9</td>
<td>6.7-44.0</td>
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<tr>
<td></td>
<td>V</td>
<td></td>
<td>10.6</td>
<td>8.8-13.5</td>
<td>0.163</td>
<td></td>
<td>10.5</td>
<td>8.7-13.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>d</td>
<td></td>
<td>10.5</td>
<td>8.7-13.4</td>
<td></td>
<td>0.163</td>
<td>5.5</td>
<td>3.1-20.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Model D anomalies - Mass model: CRUST = 34.084, densities 3.25/2.89/1.027.

Constant density 2.60, 0.4 of crust.

1 Residuals; original, after reduction of mean, after straight-line fit, respectively.
2 Number of squares on parallel zone, in each class.
3 From tables; level of significance 5%.
4 R.m.s. deviation of sample, between model D and observed anomalies [84].
5 Limits of prediction accuracy.
6 Correlation between anomalies analyzed.
7 Sample too small.
### Table 25

**Model D population**

<table>
<thead>
<tr>
<th>$1^\circ \times 1^\circ$</th>
<th>(0-25)</th>
<th>$\frac{3(m_i/m_j)}{\text{depm}}$</th>
<th>(16-25)</th>
<th>$\frac{(m_i/m_j)^n}{\text{depm}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Zones</strong></td>
<td>$i/j$</td>
<td>$F_{i-1,j-1}$</td>
<td>$\bar{V}$</td>
<td>$V$</td>
</tr>
<tr>
<td>$50^\circ/45^\circ$</td>
<td>58/61</td>
<td>0.59-1.68</td>
<td>0.58</td>
<td>0.57</td>
</tr>
<tr>
<td>$50^\circ/40^\circ$</td>
<td>58/60</td>
<td>0.59-1.68</td>
<td>0.48</td>
<td>0.49</td>
</tr>
<tr>
<td>$50^\circ/5^\circ$</td>
<td>58/43</td>
<td>0.57-1.80</td>
<td>0.95*</td>
<td>0.85*</td>
</tr>
<tr>
<td>$45^\circ/40^\circ$</td>
<td>61/60</td>
<td>0.60-1.67</td>
<td>0.84*</td>
<td>0.85*</td>
</tr>
<tr>
<td>$45^\circ/5^\circ$</td>
<td>61/43</td>
<td>0.57-1.79</td>
<td>1.64*</td>
<td>1.45*</td>
</tr>
<tr>
<td>$40^\circ/5^\circ$</td>
<td>60/43</td>
<td>0.57-1.79</td>
<td>1.97</td>
<td>1.73*</td>
</tr>
</tbody>
</table>

Model D anomalies. Mass model: CRUST=34.084, densities 3.25/2.89/1.027. Constant density 2.60, 0.4 of crust.

1. Number of squares on each parallel zone.
2. From tables; level of significance 5%.
3. Asterisk * indicates significance (same population).
4. Residuals; original, after reduction of mean, after straight-line fit, respectively.

#### 4.52 Prediction comparison for various authors' models

A prediction procedure can be tested by comparing its results to known values. The measures of accuracy are the standard deviation of the residuals, p.109, and the correlation coefficient between the two systems, p.114. Results of such a comparison are demonstrated in Table 26, p.151.

The models used, with their code names were:

1. (Kivioja) Kivioja's model anomalies [37, p. 89]. These are actually attraction effects of the topography and its compensation for the Airy-Heiskanen system T=30, adjusted in some way to fit the free-air anomalies area-wise.
Table 26

Prediction, various authors

<table>
<thead>
<tr>
<th>Model</th>
<th>1°×1°</th>
<th>(0–25)</th>
<th>(16–25)</th>
<th>Δg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kivioja</td>
<td>Uotila</td>
<td>D_{120}</td>
<td>A_{eq},</td>
</tr>
<tr>
<td>$\bar{V}'$</td>
<td>2.38</td>
<td>5.18</td>
<td>3.18</td>
<td>4.75</td>
</tr>
<tr>
<td>$\gamma_{Fv}'$</td>
<td>2.90</td>
<td>8.41*</td>
<td>3.41</td>
<td>7.79*</td>
</tr>
<tr>
<td>$\gamma_{tv}'$</td>
<td>(4.00)</td>
<td>(4.00)</td>
<td>(4.00)</td>
<td>(4.00)</td>
</tr>
<tr>
<td>$\gamma_{F_d}$</td>
<td>1.70</td>
<td>2.90*</td>
<td>1.85</td>
<td>2.79*</td>
</tr>
<tr>
<td>$\gamma_{F_d}$</td>
<td>(2.00)</td>
<td>(2.00)</td>
<td>(2.00)</td>
<td>(2.00)</td>
</tr>
<tr>
<td>$\gamma_{F_d}$</td>
<td>7.82*</td>
<td>6.35*</td>
<td>5.54*</td>
<td>8.74*</td>
</tr>
<tr>
<td>$\gamma_{F_d}$</td>
<td>(3.15)</td>
<td>(3.15)</td>
<td>(3.15)</td>
<td>(3.15)</td>
</tr>
</tbody>
</table>

Lat. 45°–40° N. Comparison between model and observed anomalies [84].

N=61/24 squares.

1 Kivioja's model anomalies [37, p. 89].

2 Uotila's model anomalies [86].

3 Model $D_{120}$ anomalies. Mass-model: CRUST = 34.084, densities 3.25/2.89/1.027.

Constant density 2.60, 0.4 of crust.

4 Model A anomalies for Airy-Heiskanen system, T=30.

5 Observed free-air gravity anomaly [84].

6 Computed value and (tabulated); level of significance 5%. Asterisk * indicates significance.

7 Original residuals.

8 After reduction of mean.

9 After straight-line fit.

10 Correlation between neighboring residuals.

11 Correlation between anomalies analyzed.

151
2. (Uotila) Uotila's model anomalies [86, p. 6]. These are based on the elevations and squares of elevations, and Jung's developments of spherical harmonics for the same Airy system [26].

3. (D123) The author's model D123 anomalies, with constant density 2.60 for 0.4 of the crust, as in Table 22, p. 143. They are based on the topography and on the crustal thickness data from [73].

4. (A_{h_u}) The author's model A anomalies, as in Table 16, p. 118. They are based on the topography and on the crustal data derived from it with the Airy-Heiskanen system $T = 30$ km.

The comparison was with the mean observed gravity anomalies ($\Delta g$), on the 9th parallel zone ($45^\circ - 40^\circ N$) [84].

For the total population (0-25), all models are significantly correlated to the local crustal thickness. Kivioja's model is outstanding among the four, yielding (statistically) the best prediction model, as evidenced by its prediction accuracy and correlation to the free-air anomalies, and by the correlation of neighboring residuals. It should be remembered, though, that Kivioja's anomalies have been obtained after some fit of model disturbances to the gravity anomalies utilized in his studies, thus making his results dependent on the latter. The other models have about the same quality, with the Airy model being slightly better than Uotila's model and the author's.

For the higher $1^\circ \times 1^\circ$ class (16-25) the picture is consistently different. The models—except for the author's model—are not significantly correlated to
The prediction accuracy for all models is smaller than the r.m.s. free-air anomaly. The results are much improved for the higher (l^2 \times l^0) class, which emphasizes the necessity of having a reliable observed mean anomaly.

It is very interesting to note the relation between Uotila's anomalies and model A_{m+1}. anomalies. Both are based on the same Airy-Heiskanen model, and show the same characteristics, indeed (test statistics F_1, t_1, and F_1). The author's model D_{120} in the (l6-25) class is still statistically correlated to crustal thickness, while the rest of the models are not. This points to somewhat inferior crustal data utilized for the author's model, p. 134.

Kivioja's prediction accuracy for this sample is a little better than the anticipated one of m \sim \pm 12 mgal (for combined ocean and land and coast areas) [37, p. 96]. The prediction accuracy, as well as the high correlation between Kivioja's model and the free-air anomalies, are probably outcomes of his adjustment process, which makes for some fit of model disturbances to observed anomalies in broad areas. In addition to that there are some minor differences in the elevation data used between this model and the others.

The spherical harmonics development of the elevations (and squares of elevations) used in Uotila's model does not seem to appreciably alter its results from the original model A_{m+1}.

The author's model, which is based on another data set shows its dependence on it. Nevertheless, it gives results similar to those of Uotila's and Airy's models.
5. CONCLUSIONS

A few concluding remarks are in place here. Consider also Table 4, p. 91.

1. The mass-model utilized for most of the gravity models seems to be a pretty good approximation for the mean earth. It is based on the available elevations and crustal thicknesses. Its data includes a model CRUST of 34.084 km, and densities 3.25/2.89/1.027. Establishing a mass-model is not an easy task, as can be judged from the procedures in subheading 3.83, p. 88.

2. The gravity model A, with the standard density, p. 25, does not seem to yield values good enough to represent free-air anomalies (with data above).

3. The models B and C which essentially use distinct density values, p. 31 and 36, yield standard deviations of the model anomalies from the observed ones on the order of the r.m.s. deviation for the latter or smaller. This indicates that these model anomalies would be better to use than zero anomalies.

4. The model D, which uses distinct densities and a constant density together, p. 36, is very versatile and yields the best results for the available data. The prediction accuracy approaches ±10 mgals, and is smaller than the r.m.s. deviation of the free-air anomalies.

5. Of great value are the various gravity models established and their related computer programs. The two program sets, P and M, carry computations from point to point or for the earth as a whole, respectively. The P programs will be especially suited for point predictions while the M ones will be better for complete world maps.
6. The prediction procedure should preferably exclude the effect of the 5° x 5° compartment itself (or even some neighboring squares), which should be evaluated using more detailed data for elevations, crustal thicknesses and densities. The values of the effect from outside the compartment could be pretabulated in maps, obtainable from main programs M, or easily computed for the neighboring compartments through point programs P. It could conversely be computed directly for the point inside the 5° x 5°, not necessarily at its center of gravity or mean coordinates, by a modified point program. A prediction procedure based on the outside effect of 5° x 5° compartments has advantages, in that the borders of the compartments are always integer degrees and that the effect itself will in most cases run smoother—from point to point—than the effect for the Hayford zones 18-1, for example.

7. Limiting the distance of computation is in most cases feasible, with considerable time-savings.

8. A broad statistical analysis of results is essential for model evaluation.

9. Comparisons of model anomalies to 5° x 5° mean observed anomalies is more reliable with increased number of 1° x 1° utilized for the 5° x 5° mean. The deviations seem to emphasize that a 5° x 5° mean observed value is doubtful when only a small number of 1° x 1° is used, rather than imply more about the model anomalies, or put them in question.

Some problems are eminent even when comparison is made to a reliable set of observed mean gravity anomalies. For example: How well does the
prediction point represent the observed mean? What is the effect of wrong
gravity formula used? On top of it all a big effect could arise unless detailed
data is substituted for $5^\circ \times 5^\circ$ means of elevation and crustal thickness,
especially close to the computation point.

10. Comparisons of model anomalies to model anomalies by other
authors, even if based on similar models and data, might not be easy to perform.
Extreme caution should be taken in order to account for all the deviations,
approximations and adjustments incorporated in the various models.

11. Further investigation could be carried on, regarding possible sources
of systematic effects on the models. It is already evident that better Moho values
and densities are needed, which would yield model anomalies less systematically
correlated to them. The models discussed in this study are based on the assumed
elevation, Moho and density values, and are, therefore, restricted by their
accuracy. This restriction is inherent in any anomaly system, however
carefully the difficulty may be hidden.

In this connection it might be worthwhile to recall that the familiar Airy-
Heiskanen system yields model free-air anomalies which are rather close to the
author's model anomalies. For example, Table 26, p. 151, indicates on latitude
zone $45^\circ -40^\circ$ N a standard deviation between the Airy model anomalies and the observed
free-air anomalies of $\sigma_m = \pm 13.3$ mgals. The author's model D gives $\sigma_{m_v} = \pm 13.5$
mgals. The r.m.s. deviation for the same sample observed anomalies is $m_{\Delta g} = 
\pm 15.1$ mgals. So it is evident that using the Airy system could be better than using
zero anomalies, and could approximate the author's models with the current data.
12. Some interesting minor problems which are related to the study may
deserve further investigation. Among them could be:

A. Rigorous solutions and/or detailed practical procedures for the
   computation of attraction and potential on a sphere, including border-
   lines, location of mass-line (if used), formulas, etc.

B. Computation of attraction and potential on a spherical cap above the
   computation point, and of spherical cap situated asymmetrically to
   the computation point.

C. Procedures at the pole or close to the computation point.
APPENDIXES
APPENDIX A

Statement listing, main program M

SHARMA D.  JOAN BEA350  50  150  1000  117  5/66
SHARMA D.  JOAN BEA350  50  150  1000  117  5/66

* PLEASE PRESS SWITCH 6 IF TIME EXCEEDS.* BUT LET PRINT.

*** RUN: STATEMENT LISTING, PUNCH SYMBOLIC BINARY, SCATRAN

START READ INPUT *FORM1* ((ELEV(1)I=0.1*1.L.2592)) - 0115
READ INPUT *FORM1* ((MOHO(1)I=0.1*1.L.2592)) - 0116
F FORM1 (1215) - 0117
READ INPUT *FORM5* (CME.CMM) -
F FORMS (2F10.5) - 0119
READ INPUT *FORM2* (MU.NU.NUR.KGRA.Crust.D(11)D(2)) - 0120
TETA*PRINT* - 0121
F FORM2 (311*12* F20.15* F10.0* F10.0* F5.0*3* F6.3* F3.1*11) - 0122
WRITE OUTPUT *FORM4* (MU.NU.NUR.KGRA.Crust.D(11)D(2)) - 0123
2*tETA* - 0124
F FORM4 (3HOM*11*3X* JHU=*11*3X* 4HNU=*12*3X* 4HRO=*1) - 0125
F10.7*3X*2HR*= F9.1*6HOKGRA*= F8.6*3X*6HCRUST= F6.3*3X*
6HDENS=*3* F6.3*3HDT0*= F9.1* - 0126
READ INPUT *FORM1* (NOCOMP) - 0128
INTEGER (ELEV) - 0129
FLOATING (KGRA) - 0130
C
DATA
ELEVATION IN METERS; CRUSTAL THICKNESS IN 10 METERS; 0131
CME = MEAN DEPTH (WATER COND.); CIMP = MEAN CRUST; 0133
KM = UNITS AND CONSTANTS; 0134
M = 1; COMPUTES AT MEAN LAT. 1; AT CME 0135
MU = 1; CONST. DENSITY; READS IN 2* COMP.(A); 3* COMP. 0136
BNU = 0; COMP. TO 180 DEG.; 1* 120; 2* 90; 3* 60; 4* 45 DE 0137
G = NUR = 72; NUMBER OF 5 DEGREES ON ONE PARALLEL; 0138
C
R = RHO DEGREES; 0139
R = MEAN RADIUS OF EARTH; IN METERS; 0140
KGRA = CONST. OF GRAVITY X 100000; 0141
CRUST = T OR ABSE. DEPTH OF COMPUTATION, IN METERS; 0142
D = DENSITY OF (MODEL) CRUST; 0143
D(1) = DENSITY OF WATER - D; 0144
D(2) = D - DENSITY OF (MODEL) SUBCRUST; 0145
TETA = D; THE INNER BORDER OF SPHERE CAP; 0146
PRINT = 0; PRINTOUT CONTROL; 0147
NOCOMP = NO. OF LAT. ZONES; FROM POL. NOT COMPUTED; 0148
DIMENSION (ELEV(2592*NUR)MUHO(2592*NUR)LI(3)F1(36)*L(16) 0149
CO(36)COLONG(37)AREA(36)*T(2592*NUR)C(2592*NUR)II(12) 0150
TETA(2)*A01(2)*F101(20)*5101(20)*CO01(20)*DI(200)*PM(18)F 0151
25921) - 0152
UPPER COMMON (PUT(2592*NUR)I*IPUT(12)*ITP(12)) - 0153
DO THROUGH (PRD)I=0.1*1.L.2592- 0154

159
SUMELE = SUMEL + SUM / 72.0 - 0.253
WMOHO = WMOHO + NMO * AREA(1) / 72.0 - 0.254
SUMO = SUMO + NMO / 72.0 - 0.255
MWELEV = MWELEV / TAREA + 0.5 - 0.256
PROVIDED (MWELEV < 0.0) ; MWELEV = MWELEV - 1 - 0.257
MMWMOH0 = WMOHO / TAREA + 0.5 - 0.258
MELEV = MELEV / NCAP + 0.5 - 0.259
PROVIDED (MELEV < 0.0) ; MELEV = MELEV - 1 - 0.260
MWELEV = SUMW / NCAP + 0.5 - 0.261
CALL SUBROUTINE (TCAP1,TCAP2,CAP1,CAP2,PTWC1,PTWC2,PCWC) = CAP - 0.262
\( \theta \) (T) ; CAPOLE, MELEV, MWELEV, D(0), IPK, CRUST, SIGWAT, SIGMA - 0.263
SIGSUB(MUSSTADEN) - 0.264
CALL SUBROUTINE (TCAP1,TCAP2,CCAP,PC1,PC2,PCCC) = CAP - 0.265
(0) ; CAPOLE, MELEV, MWELEV, D(0), IPK, CRUST, SIGWAT, SIGMA - 0.266
MUSSTADEN) - 0.267
TRANSFER TO (E6) PROVIDED (L.E.1) - 0.268
WRITE OUTPUT * FIG1 - 0.269
(F1H = 1T1H = 16*4, MEAN & 9X = 6Hw, MEAN & Q ON NORTH POLE CAP) - 0.270
NEG. ELEV. REFS TO WATER DENSITY (**) - 0.271
TRANSFER TO (E7) - 0.272
WRITE OUTPUT * FIG2 - 0.273
(F1H = 1T1H = 1Q *. SOUTH POLE CAP (*)) - 0.274
WRITE OUTPUT * FIG3 (MELEV, MWELEV, MMOHO, MWMHO, TCAP1, TCAP2, E) - 0.275
TWCAP1 + TWCAP2 + CCAP + CWCAP + TCAP1 + TCAP2 + CCAP + TWCA - 0.276
P * PTCA1 + PTCA2 + PTWC1 + PTWC2 + PCWC + PTCA1 + PTCA2 + TWCA - 0.277
CWC + PTCA1 + PTCA2 + TCAP1 + TCAP2 + PTWC1 + PTWC2 + PCWC + CAP - 0.278
P * PCWC + CWCAP + PTCA1 + PTCA2 + PC + CWCAP + TCAP1 + TCAP2 + CCAP + PTWC1 + PTWC2 + PCWC + TWCA - 0.279
TWCAP1 + TWCAP2 + CWCAP) - 0.280
TRANSFER TO (E6) PROVIDED (L.E.1) - 0.281
WRITE OUTPUT * FIG3 - 0.282
(F1H = 1T1H = 16*4 MEAN & 9X = 6Hw, MEAN & Q ON NORTH POLE CAP) - 0.283
TRANSFER TO (E6) PROVIDED (L.E.1) - 0.284
WRITE OUTPUT * FIG2 - 0.285
(F1H = 1T1H = 1Q *. SOUTH POLE CAP (*)) - 0.286
WRITE OUTPUT * FIG3 (MELEV, MWELEV, MMOHO, MWMHO, TCAP1, TCAP2, E) - 0.287
TWCAP1 + TWCAP2 + CCAP + CWCAP + TCAP1 + TCAP2 + CCAP + TWCA - 0.288
P * PTCA1 + PTCA2 + PTWC1 + PTWC2 + PCWC + PTCA1 + PTCA2 + TWCA - 0.289
CWC + PTCA1 + PTCA2 + TCAP1 + TCAP2 + PTWC1 + PTWC2 + PCWC + CAP - 0.290
P * PCWC + CWCAP + PTCA1 + PTCA2 + PC + CWCAP + TCAP1 + TCAP2 + CCAP + PTWC1 + PTWC2 + PCWC + TWCA - 0.291
TWCAP1 + TWCAP2 + CWCAP) - 0.292
MIN1 CONTINUE - 0.293
E3 CONTINUE - 0.294
MIN CALL SUBROUTINE (ITIME) = (CLOCK (-)) - 0.295
DO THROUGH (DSI) * I = 0 * I * L * 18 - 0.296
COSLIM = COSE * (2*1*RO5 * CO(1)) - 0.297
TRANSFER TO (PRINT) IF SWITCH (6) - 0.298
DO THROUGH (DSI) * I = 0 * I * L * (36 - 2*1) - 0.299
TRANSFER TO (PRINT) IF Switch (6) - 0.300
MIN1 = 1 + 11 - 0.301
TRANSFER TO (POLE) PROVIDED (NOCOMP; E; O; AND; NCAP; L; O; AND; I) - 0.302
I.E.1.-
TRANSFER TO (DS11) PROVIDED (NCAP.G.NUCOMP. AND (1.E.1.NCAP.
OR (1.E.NUCOMP. AND 1.E.GE.(36-NOCOMP))))-
TRANSFER TO (DS11) PROVIDED (NCAP.E.NUCOMP. AND (1.E.NOC.
OR (1.E.NOCOMP. AND 1.E.GE.(36-NOCOMP))))-
NORECI=0-
PROVIDED (1.E.NOCOMP. NORECI=1-
DO THROUGH (DSJJ) JJJ=0.1. JJJ=37-
TRANSFER TO (PRINT5) IF SWITCH (6)-
TRANSFER TO (DS5) PROVIDED (JJ.*E.0)-
TRANSFER TO (DS6) PROVIDED (JJ.*E.36)-
COSINE=SIN(I.*E.111)+C0(I.)*CO(I.)*COLUM(JJ)-
XSO=(1.-COSINE)/2.*-
TRANSFER TO (DS3)-
DS5 TRANSFER TO (DS7) PROVIDED (1.E.*0)-
A=*ABS*(F1(I.)*F1(111))-
TRANSFER TO (DS8)-
DS7 A=0.-
TRANSFER TO (DS8)-
DS6 A=180.96*R0.*ABS*(F1(I.)*F1(111))-
DS8 X=SIN*(A/2.4)-
XSO=X*X-
COSINE=COS*(A)-
DS3 TRANSFER TO (PIPO1.PIP1.PIP2) PROVIDED (NU-1)-
PIPO1 TRANSFER TO (DSJJ) PROVIDED (COSINE*L.E.-0.6)-
TRANSFER TO (PIPO0)-
PIPO2 TRANSFER TO (PIPO3.PIP4.PIP5) PROVIDED (NU-3)-
PIPO3 TRANSFER TO (DSJJ) PROVIDED (COSINE*L.E.0.1)-
TRANSFER TO (PIPO)-
PIPO4 TRANSFER TO (DSJJ) PROVIDED (COSINE*L.E.0.5)-
PIPO5 TRANSFER TO (PIPO6) PROVIDED (NU.G.04)-
TRANSFER TO (DSJJ) PROVIDED (COSINE*L.E.07010676)-
TRANSFER TO (PIPO)-
PIPO6 TRANSFER TO (DSJJ) PROVIDED (COSINE*L.E.08602531)-
PIPO9 X=1.*16.*XSO+24.*XSO*XSO-
X2=X+1.2.-4.*XSO-
X3=1.2.*XSO-
X4=1.*6.*XSO+6.*XSO*XSO-
X5=2.-3.*XSO-
DS4 DO THROUGH (DSJ) J=0.1. J=72-
TRANSFER TO (PRINT5) IF SWITCH (6)-
DO THROUGH (DSL) L=0.1. L=32-
TRANSFER TO (PRINT5) IF SWITCH (6)-
TRANSFER TO (DS9) PROVIDED (L.*E.0)-
K=35-1-
K=K+35-111-
TRANSFER TO (DS10)-
DS9 K=1-
**DS10**

TRANSFER TO (DS11) PROVIDED (3*E-0) - 0.351

TRANSFER TO (DS12) PROVIDED (J*E+00*K*J*E+36) - 0.352

DO THROUGH (DSM) IM=-1.2*IM*LE*2 - 0.353

TRANSFER TO (PRINT5) IF SWITCH (6) - 0.354

JJJ=J+IM*JJ - 0.355

TRANSFER TO (DS13) PROVIDED (JJJ=L*00*K*J*G*71) - 0.356

TRANSFER TO (ATT1) - 0.357

**DS13**

JJJ=AABS*(72.EDABS*JJJ) - 0.358

**ATT1**

KJ=72*K+J - 0.359

KKKJJJ=72*KKK+JJJ - 0.360

TRANSFER TO (SH11) PROVIDED (3.0*E-5*AND*COSINE*L*E-LIM) - 0.361

**R**(1*E-5*AND*COSINE*L*E-996) - 0.362

CALL SUBROUTINE (TK1*TK2*TKK1*TKK2*CK*CKKK*PK1*PK2*PKKK) - 0.363

PKKK2*PK3*PKK3=ATT1*(ELEV(KJ)+ELEV(KKKJJJ)+MUHU(KJ)+MUHU(KKKJJJ)+KGRA+CRUSTSIGMA.MU.0REC1.COMO+MU+CHW1) - 0.364

GMA=SIGSUB+MU.NOREC1.STADEN1*COMO+MU.NOREC1*SIGMA.0REC1.0STADEN - 0.365

POT(KJ)=POT(KJ)+PK1+PK2+PK3 - 0.366

POT(KKKJJJ)=POT(KKKJJJ)+PKK1+PKK2+PKK3 - 0.367

T(KJ)=T(KJ)+T1+T2 - 0.368

T(KKKJJJ)=T(KKKJJJ)+T1+T2 - 0.369

C(KJ)=C(KJ)+CK - 0.370

C(KKKJJJ)=C(KKKJJJ)+CKKK - 0.371

DSM CONTINUE - 0.372

**DS14**

TRANSFER TO (DS14) PROVIDED (J*E+00*K*J*E+36) - 0.373

TRANSFER TO (DSL) - 0.374

**DS17**

L=2 - 0.375

TRANSFER TO (DSL) - 0.376

**DS11**

TRANSFER TO (DSUJ) PROVIDED (J*E+00*K*J*E+36) - 0.377

**DS12**

TRANSFER TO (DS15) PROVIDED (J*E+36*AND*11*E+00*AND*J*G*71) - 0.378

JJJ=J+JG - 0.379

TRANSFER TO (DS18) PROVIDED (JJJ=L*00*K*JJJ*G*71) - 0.380

TRANSFER TO (ATT2) - 0.381

**DS18**

JJJ=AABS*(72.EDABS*JJJ) - 0.382

**ATT2**

KJ=72*K+J - 0.383

KKKJJJ=72*KKK+JJJ - 0.384

CONTINUE - 0.385

DSM CONTINUE - 0.386
TRANSFER TO (SH12) PROVIDED ((1<5\ AN) C O S I N E = L (G05LIM) = 0 0400
\* (1<LE=5 AND COSINE<0.996)) - 0401
CALL SUBROUTINE (TK1, TK2, TK1, T(KKKI), T(KKK2), CK, CKKK, PK2, PK2, PKK2) = 0402
PKK2, PK2, PKK3 = ATT (1, ELEV(KJ), ELEV(KKJ), MOHOU(KJ), MOHOU(K) = 0403
KKK1, J, ELEV(KKJ), X, KKK1, KKK2, KKK3, KKKJ = 0404
GMA, SIGSUB, MU, NOREC1, SIGA, SIGSUH, MU, NORECI, STADEN) = 0405
POT(KJ) = POT(KJ) + PK1 + PK2 + PK3 - 0406
POT(KKKJ) = POT(KKKJ) + PKK2 + PKK3 - 0407
T(K) = T(K) + TK1 + TK2 - 0408
T(KKKJ) = T(KKKJ) + T(KKK1) + T(KKK2) - 0409
C(K) = C(K) + CK - 0410
C(KKKJ) = C(KKKJ) + CKKK - 0411
TRANSFER TO (CONT) - 0412

SH12 CONTINUE -
CALL SUBROUTINE (TK1, TK2, TK1, T(KKKI), T(KKK2), CK, CKKK, PK2, PK2, PKK2) = 0413
PKK2, PK2, PKK3 = ATT (1, ELEV(KJ), ELEV(KKJ), MOHOU(KJ), MOHOU(K) = 0414
KKK1, J, ELEV(KKJ), AREA(K), AREA(KKK), E0(K), X, X1, X2, X3, X4, X5 - 0415
GMA, SIGSUB, MU, NOREC1, STADEN) = 0416
POT(KJ) = POT(KJ) + PK1 + PK2 + PK3 - 0417
POT(KKKJ) = POT(KKKJ) + PKK2 + PKK3 - 0418
T(K) = T(K) + TK1 + TK2 - 0419
T(KKKJ) = T(KKKJ) + T(KKK1) + T(KKK2) - 0420
C(K) = C(K) + CK - 0421
C(KKKJ) = C(KKKJ) + CKKK - 0422
CONT CONTINUE -
TRANSFER TO (DS17) - 0423

DS15 J = 72 -
TRANSFER TO (DS17) - 0426

DSL CONTINUE - 0427

DSJ CONTINUE - 0428

DSJJ CONTINUE - 0429

DSL CONTINUE - 0430

POLE WRITE NO HEADING *FPO0 - 0431

F FPO0 (1H1) -
DO THROUGH (POLO), IPOLA=0, 1, IPOLA=L, 2 - 0432
IK=1 -
PROVIDED (IPOLA*E, 1, IK=35-1 - 0433
WRITE NO HEADING \*FPO1 IK, (IK, IPOLA)+C(IPOLA, 1, IPOLA = 0, 1, IPOLA=L, 2) - 0434

F FPO1 (90) EFFECT OF POLAR ZONES ON SQUARES IN ZONE 112+0* WITHO U T T HEIR OWN EFFECT ORDER - EAST FROM ZERO */1H0/1H0/20+0* - 0441
MODEL ATTRACTION* MGAL*/(6(1PE20*8)) - 0442

POLO CONTINUE - 0443
DO THROUGH (POLO), IPOLA=0, 1, IPOLA=L, 2 - 0444
IK=1 -
PROVIDED (IPOLA*E, 1, IK=35-1 - 0445
WRITE NO HEADING \*FPO2 (IK, IPOLA)+C(IPOLA, 1, IPOLA = 0, 1, IPOLA=L, 2) - 0446
F FOP02 (1H1/1H0/1H0/1H0/20X*Q*RUNS TERM* MGAL*+(6(1P20+E))  0449
- DO THROUGH (POLO2)*IPOLA=0*1*IPOLA=L*2-
  IK=1-
  PROVIDED (IPOLA+E*1)*IK=35-1-
POLO2 WRITE NO HEADING *FOP03* ((POT(IK*IPUL)+T(IK*IPUL)+C(IK*1)
POLO1*IPOLA=0*1*IPOLA=L*72)-  0455
F FOP03 (1H1/1H0/1H0/1H0/20X*Q*MODEL ANOMALY* MGAL*+(6(1P20+E))  0457
- TRANSFER TO (DS19)-
DS18A CONTINUE -  0459
PRINT WRITE NO HEADING *FSPR1-
F FSPR1 (Q*1OUTSIDE THE SQUARE* MGALS* COORDINATE OF NEW CORNER*)  0462
- PUNCH CARDS *FSPU1-
F FSPU1 (Q* ------  5X5 OUTSIDE SQUARE*)  0465
- TRANSFER TO (PRINT) PROVIDED (NCAP*G*NOC)-
  WRITE OUTPUT *FSMN2* (5*NCAP)-  0467
F FSMN2 (Q*EFFECTS INSIDE POLAR CAPS OF *12*0* DEGREES ANL SUMS  0469
PRINT WRITE OUTPUT *PING1-
F PING1 (1H0/1H0/1H0/30X*Q*MODEL ATTRACTION*) -  0472
- PUNCH CARDS *PONG1-
F PONG1 (Q* ------  0474
- DO THROUGH (PPR2)*I3=0*1*13*L*6-
  WRITE NO HEADING *FPN2-  0476
F FPR2 (1H1/1H0/1H0/1H0/1H0)-  0478
  DO THROUGH (PPR2)*I2=0*1*12*L*36-
  DO THROUGH (PPR1)*I1=0*1*11*L*12-
  NI=72*12+12*13+11-
  IT(I1)=T(NI)+C(NI)+0*5-
  PROVIDED (IT(I1)*L*0);IT(1)=IT(11)-1-
PPR1 CONTINUE -  0481
  WRITE OUTPUT *FPR3* ((IT(15)*15=0*1*15*L*12)*90-5*12*6*13)-  0483
F FPR3 (25X*I12*5*4*X214)-  0485
  PUNCH CARDS *FPU2* ((IT(15)*15=0*1*15*L*12)*90-5*12*6*13)-  0487
F FPu2 (7X*12*15*13*12)-  0489
PPR2 CONTINUE -  0491
  WRITE NO HEADING *PING2-
F PING2 (1H1/1H0/1H0/30X*Q*RUNS TERM*)-  0493
  PUNCH CARDS *PONG2-
F PONG2 (Q* ------  HRUNS TERM*)  0494
  DO THROUGH (PING22)*I3=0*1*13*L*6-
  WRITE NO HEADING *FPR2-  0495
  DO THROUGH (PING22)*I2=0*1*12*L*36-
0497
DO THROUGH (PING21) I1=0,1,11,12= 0498
NI=72*12*13+11= 0499
IPOT(11)=POT(NI)+0.5= 0500
PROVIDED (IPOT(11)*L=0,15=0115,15=12) = 0501
PING21 CONTINUE = 0502
WRITE OUTPUT .FPR3 (((IPOT(15+0115,L=12) = 01*12*6,01*13)= 0503
3)= 0504
PUNCH CARDS .FPU2 (((IPOT(15+0115,L=12) = 01*12*6,01*13)= 0505
PING22 CONTINUE = 0506
WRITE NO HEADING *PING3- 0507
F PING3 (11=1110+/1J+10140//10XQ*MODEL ANOMALY*) - 0508
F PONG3 (Q* ------- MODEL ANOMALY*) - 0509
DO THROUGH (PING32) I1=0,1,13,16= 0510
WRITE NO HEADING *FPR2- 0511
DO THROUGH (PING32) I1=0,1,12,16= 0512
DO THROUGH (PING31) I1=0,1,11,12= 0513
NI=72*12*13+11= 0514
ITP(11)=T(NI)+C(NI)*POT(NI)+0.5= 0515
PROVIDED (ITP(11)*L=0,15=0115,15=12) = 0516
PING31 CONTINUE = 0517
WRITE OUTPUT .FPR3 (((ITP(15)+0115,L=12) = 01*12*6,01*13)= 0518
= 0519
PUNCH CARDS .FPU2 (((ITP(15)+0115,L=12) = 01*12*6,01*13)= 0520
PING32 CONTINUE = 0521
IPRINT 1=IPRINT 1- 0522
TRANSFER TO (END) IF SWITCH (6)- 0523
TRANSFER TO (PU1*PU2*PU3) PROVIDED (IPRINT =) 0524
PU1 CONTINUE = 0525
CALL SUBROUTINE (ITIME)=RCLOCK()= 0526
WRITE OUTPUT .FOT1 (ITIME)= 0527
F FOT1 (Q*ITIME FOR COMP. OF 5X5 OUTSIDE SQUARE = **-3P13 031)- 0528
CALL SUBROUTINE (ITIME)=ZCLOCK()= 0529
DO THROUGH (DPO) I1=NOC,16,18= 0530
TRANSFER TO (PRINTO) IF SWITCH (6)- 0531
TETA(1)=166700/2= 0532
PROVIDED (11=7,11=7) 0533
DO THROUGH (DP1)LATI=0,1,18= 0534
LATI=20*LATI- 0535
A01(LATI)=RO1* (COS*RO1*LATI)-COS* (RO1*(1LATI+1))- 0536
TRANSFER TO (DP4) PROVIDED (MES=1)- 0537
F011(LATI)=RO9-R01*ILATI- 0538
TRANSFER TO (DP5)- 0539
DP4 F011(LATI)=RO9-R01*(3*ILATI*(ILATI+1)+1)/(3*ILATI+1)- 0540
DP5 S101(LATI)=SIN* (F011(LATI))= 0541
C01(LATI)=COS* (F011(LATI))= 0542
DO THROUGH (DP1) ILONG=0,1,11,10= 0543
DRO1=R001*(ILONG+0.5)= 0544
COLO01 = \cos(DRO01) - 0.547
SIL001 = \sin(DRO01) - 0.548
ILALO = 10 * LATI + LONG
DP1
CALL SUBROUTINE (DIS(ILALO)) = DIST*(SI(1)*CO(1)*COS(LATI) - 0.549
COO1(LATI) * SIL001 * COLO01) - 0.550
DO THROUGH (PS1) * L = 0 . . L * 2 - 0.551
K = 1 - 0.552
PROVIDED (L * E = 1) * K = 35 - 1 - 0.553
DO THROUGH (PS1) * J = 0 . . J * 72 - 0.554
KJ = 72 * K + J - 0.555
DO THROUGH (PS1) * N = 0 . . N * L * 200 - 0.556
TRANSFER TO (PS1) PROVIDED (DIS(N) * L * LATA(1)) - 0.557
XT = \sin(0.5 * DIS(N)) - 0.558
CALL SUBROUTINE (T(1) + T(1) + T(1) + T(1) + T(1) + T(1) + T(1)) = LATPA * LLEV - 0.559
(KJ) * MOHO(KJ) = AOI(N) * EC0(KJ) * KT * KGRA * R * CRUST * SIGWAT * SIGMA - 0.560
SIGSUB * MU * STADEN - 0.561
PT(KJ) = PT(KJ) + PC1 + PC2 + PC3 - 0.562
T(KJ) = T(KJ) + T(1) + T(1) + T(1) - 0.563
C(KJ) = C(KJ) + CK - 0.564
PS1 CONTINUE - 0.565
DP0 CONTINUE - 0.566
PRINT0 WRITE NO HEADING * FOPRI - 0.567
F FOPRI (Q*1 OUTSIDE INNER CAP, MGALS * CORDS OF NEW INNER CAP) - 0.568
PUNCH CARDS * FOPU1 - 0.569
F FOPU1 (Q* OUTSIDE CAP) - 0.570
TRANSFER TO (PRINT) - 0.571
PU2 CONTINUE - 0.572
CALL SUBROUTINE (T1IME) = R*CL (1) - 0.573
WRITE OUTPUT * FOT2 (T1IME) - 0.574
F FOT2 (Q* TIME FOR COM P OUTSIDE INNER CAP = *) - 0.575
CALL SUBROUTINE (T1IME) = Z*CL (1) - 0.576
DO THROUGH (PAD1) * L = 0 . . L * 2 - 0.577
K = 1 - 0.578
PROVIDED (L * E = 1) * K = 35 - 1 - 0.579
DO THROUGH (PAD1) * J = 0 . . J * 72 - 0.580
KJ = 72 * K + J - 0.581
CALL SUBROUTINE (T1 * T2 * CC * PC1 * PC2 * PC3) = CAP * (TITA(1) + TITA(1) + ELEV * KJ * MOHO(KJ) + PC1 + PC2 + PC3 - 0.582
T(KJ) = T(KJ) + T1 * T2 - 0.583
C(KJ) = C(KJ) + CC - 0.584
PAD1 CONTINUE - 0.585
PADO CONTINUE -
PRINTC WRITE NO HEADING *FCPR1-
F FCPR1 (Q*TOTAL EFFECT*MGALS* COORD. OF NEW CORNER*)-
PUNCH CARDS *FCPU1-
F FCPU1 (Q* TOTAL*UNREduced-----)
TRANSFER TO (PRINT) -
FPU3 TRANSFER TO (PU4) PROVIDED (IPRINT*G*3)-
CALL SUBROUTINE (TIME)=RCLK.-
WRITE OUTPUT *FOT3**TIME**-
F FOT3 (Q*TIME FOR COMP. OF CAP = *, -1PH8.3) -
TRANSFER TO (GI) PROVIDED (NOCOMP*G*)-
PV=0.
PVV=0.
DO THROUGH (POP). I=0..I..L.36-
PVV=0.
DO THROUGH (POP2). J=0..J..L.72-
POP0 PVL=PVV+T(1. J)+C(1. J)+TOT(1. J)-
POP PV=PV+PVL*AREA(1)-
DG=PV*RO/720.-
INC=20.*DG+O.5-
DO THROUGH (POP1). I=0..I..L.2597-
POP1 C(I)=C(I)+DG-
DO THROUGH (POP4). I=0..I..L.36-
PVV=0.
DO THROUGH (POP3). J=0..J..L.72-
POP20 PVVL=PVV+T(I. J)+C(I. J)+TOT(I. J)*P.2-
POP2 PVV=PVV+PVL*AREA(1)-
PMO=SQRT**(PVV/2591.)-
DO THROUGH (POP3A). I=0..I..L.18-
POP3A PM(I)=PMO/SQRT**(AREA(1))-}

F FPOP (Q*TOTAL EFFECT REDUCED TO AVERAGE ZERO, MGALS COORD. OF NEW CORNER*1H0/1H0/G*0WEIGHTED MEAN EFFECT MGALS) BEF. ORE REDUCTION **F10.2/Q** CORR. INCREASE (M) IN CRUST **110/1H0/G*O*STAND. DEVE. OF EFFECT AFTER RED. IN LATE ZONES*/
SHOZONE.10X4HS.0.1H0/(14.10X.F6.2)) -
PUNCH CARDS *FPUP-
F FPUP (Q* TOTAL EFFECT--------- REDUCED*) -
WRITE OUTPUT*FDI1*(PMM)-
F FDI1 (Q*ST. DEVE. FOR MEAN AREA OF 5X5 SQUARE = **F10.2) -
TRANSFER TO (PRINT)-
PUL4 CONTINUE -
F GI WRITE OUTPUT *FP0PL-
F FPOP (Q* EFFECT OF CRUST ONLY, TOTAL MGALS* COORD. OF NEW CORNER*)-
PUNCH CARDS *FPUP1-
F FPUP1 (Q*--------- EFFECT OF CRUST--------- *) -
PUNCH CARDS *PONG1-
WRITE OUTPUT *PING1-
DO THROUGH (POP4) i3=0..13*L*6-
WRITE NO HEADING *FPR2-
DO THROUGH (POP4) i2=0..12*L*36-
DO THROUGH (POP3) i1=0..11*L*12-
NI=72*12+12*13+11-
IT(11)=T(NI)+0.5-
PROVIDED (IT(11)*L*G), IT(11)=IT(11)-1-
POP3 CONTINUE -
WRITE OUTPUT *FPR3* ((IT(15)+13=0..11*L*12)*00-5*16*0*13) 0/34
- PUNCH CARDS *FPR2* ((IT(15)+15=0..15*L*12)*00-5*16*0*13) 0/36
POP4 CONTINUE -
PUP1 CONTINUE -
DAN1 CONTINUE -
END CALL SUBROUTINE ()=ENDJOB()-
END PROGRAM (START)-
--- STATEMENT LISTING; PUNCH SYMBOLIC BINARY; SCATRAN
--- SUBROUTINE (DIS)=DIST(S:SI:COI:SI:CO01:SI:COL01:CO01)- 0/16
SIFI2=(SI:CO01:CO01)*P.2+(SI:CO01:SI:CO01:CO01:CO01):DI-
SIFI=SORT(SIFI2)- 0/18
DIS=ATAN(SIFI/SORT(1.-SIFI2))- 0/19
NORMAL EXIT -
END SUBPROG-
--- SUBROUTINE (TKK1,TKK2,CKK,PKK1,PKK2,PKK3)=ATTINAE(LV,MOH)- 0/23
•A01=EK*XT*KGRA*R*CRUST*SIGWAT*SIGMA*SIGSUB*MU*STADEN)- 0/24
DIMENSION ();(4))- 0/26
INTEGERS (ELEV)-
FLOATING (KGRA)- 0/27
XSQ=XT*XT- 0/28
X1=1.-16.*XSQ+24.*XSQ*XSO-
X2=X1+2.-4.*XSQ-
X3=-1.+2.*XSQ-
X4=1.-6.*XSQ+6.*XSQ*XSO-
X5=2.-3.*XSQ-
HC0=ELEV-
AM0=MOHO-ELEV-
D(0)=EK-
PROVIDED (MU:E:*3)*D(0)=STADEN-
D(1)=SIGWAT-EK-
D(2)=EK-SIGSUB-
D(3)=EK-SIGMA-
PROVIDED (ELEV*L:O)*HC0=0-
RPHC0=R+HC0-
DO THROUGH (DAN)*J=0..11*L*3-
TRANSFER TO (DA30,DA31,DA32) PROVIDED (J-1)- 0/31
DA30 TRANSFER TO (DA4) PROVIDED (FLEV*L:O)- 0/34
DA=DA(0)-
TRANSFER TO (DA45)-
0/46
0/47
0/48
0/49
0/50
0/51
0/52
0/53
0/54
0/55
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0/57
0/58
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DA4
DA=O(1)-
DA45
H2=O.N-
H1=ABS*ELFV-
TRANSFER TO (DA5)-
DA31
TRANSFER TO (DA6) PROVIDED (AMO+L.CRUST)=
DA=O(2)-
H2=HCO+CRUST-
H1=HCO+AMO-
TRANSFER TO (DA5)-
DA6
DA=O(2)-
H2=HCO+AMO-
H1=HCO+CRUST-
TRANSFER TO (DA5)-
DA32
DA=O(3)-
H2=HCO-
H1=HCO+CRUST-
TRANSFER TO (DA7) PROVIDED (1*ENL+O)+
TRANSFER TO (PIL) PROVIDED (J*F+2)-
CKK=O.N-
PKK3=O.N-
TRANSFER TO (DAN)-
PIL
TKK2=O.N-
PKK2=O.N-
TRANSFER TO (DAN)-
DA7
Y1=-H1/RPHCO-
Y2=-H2/RPHCO-
SORT1=SORT*(4*Y5*S0*(1*Y1)+Y1*Y1)-
SORT2=SORT*(4*Y5*S0*(1*Y2)+Y2*Y2)-
XLN=X4*XLN-((SORT2+Y2*Y2*X5*Y1)/((SORT2+Y1+Y2*X5*Y1))- 
HiACK=(X1+X2*Y2+X3*Y2*Y2)/SORT2-(X1+X2*Y1+X3*Y1*Y1)/SORT2+1-1*
*XLN-
HiACK=(X5*Y2*2)+SORT2-(X5+Y1*2)*SORT1+XLN-
CONST=KGR*DA01*RPHCO*2-
F=CONST*HiACK-
PF=-2*CONST*RPHCO*HiACK/R-
TRANSFER TO (KENO, KEN1, KEN2) PROVIDED (J-1)-
KENO
TKK1=F-
PKK1=PF-
TRANSFER TO (DAN)-
KEN1
CKK=F-
PKK3=PF-
TRANSFER TO (DAN)-
KEN2
TKK2=F-
PKK2=PF-
DAN
CONTINUE -
NORMAL EXIT -
END SUBPROGRAM -
SUBROUTINE (TC1, TC2, CC, PK1, PK2, PK3)=CAP (T0, T1, ELF, MOHO- 
K=TPK+CRUST, SIGWAT, SIGMA, SIGSU, SUB, MUS, STADIN)-
INTEGERS (ELEV)- 0796
DIMENSION (D(4),D1(3),PRA(3),RA(3),RAS(3))- 0797
PRECISION (2*RO*R1*R2*SOGER*FUN*FIN*TATB*COST*DCOS*)- 0798
TA=T1- 0799
TR=T0- 0800
COST=DCOS*(TA)- 0801
RO=R+ELEV- 0802
PROVIDED (ELFV*L0) , R0=R- 0803
COSQ=COST*COST- 0804
RCOS=RO*COST- 0805
ROS=RO*RO- 0806
D(0)=EK- 0807
PROVIDED (MU•E•3) , D(0)=STADEN- 0808
D(1)=SIGWAT-EK- 0809
D(2)=EK-SIGS23- 0810
D(3)=EK-SIGMA- 0811
DO THROUGH (DAN) = L=0•1•L0•3- 0812
TRANSFER TO (D10* D11* D12) PROVIDED (L-1)- 0813
D10 TRANSFER TO (D2) PROVIDED (ELEV*L0)- 0814
R1=R- 0815
DR=ELEV- 0816
DEN=D(0)- 0817
TRANSFER TO (D3)- 0818
D2 R1=R+ELEV- 0819
DR=ELEV- 0820
DEN=D(1)- 0821
TRANSFER TO (D3)- 0822
D11 DR=MOHO-ELEV-CRUST- 0823
DEN=D(2)- 0824
TRANSFER TO (D4) PROVIDED (MOHO•ELEV+CRUST)- 0825
DR=DR- 0826
DEN=DEN- 0827
R1=R-CRUST- 0828
TRANSFER TO (D3)- 0829
D4 R1=R+ELEV-MOHO- 0830
TRANSFER TO (D3)- 0831
D12 DR=CRUST- 0832
DEN=D(3)- 0833
R1=R-CRUST- 0834
TRANSFER TO (D7) PROVIDED (DEN•NE•0•)- 0835
TRANSFER TO (PIL) PROVIDED (L•E•2)- 0836
CC=0•- 0837
PK3=0•- 0838
TRANSFER TO (DAN)- 0839
PIL TC2=0•- 0840
PK2=0•- 0841
TRANSFER TO (DAN)- 0842
D7 R2=R1+DR- 0843
CALL SUBROUTINE (FIN)*GAUS* (R1* R2* SOGLR)- 0844

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RA(1)=R1-
RA(2)=R2-
RAS(1)=R1*R1-
RAS(2)=R2*R2-

DO THROUGH (PILA),I=1,1,1,3-

D(I)=SQRTR(RO+RAS(I)-2*RA(I)*RCOS)/RO-

PILA

PRA(I)=D(I)*(RAS(I)^2-RCOS*RA(I)/6+RO^3/3-RCOS*RCOS^3/5+*)

RAS(I)*RA(I)/(3*NO)-RAS(I)/2*O*RO**RO*RCOS*(1-RCOS)*KJN(RA(I)

1/RO-COST*D(I(I))-*

PI=2*TPK*DEN*(PRA(2)-PRA(1))/R-
BRA=TPK*DEN*FIN-

TRANSFER TO (KENO),KEN1,KE21 PK(OVIPR) (1,-1)=

KENO
TC1=BRA-
PK1=PI-

TRANSFER TO (DAN)=

KEN1
CC=BRA-
PK3=PI-

TRANSFER TO (DAN)=

KEN2
TC2=BRA-
PK2=PI-

TRANSFER TO (DAN)=

FUNCTION (FUN)=SQRTR (RADIUS)-

EXTERNAL (RO,COST)-

PRECISION (2*FUN*AR*BR*COST+USQRT*RADIUS)-

AR=2*RO*RADIUS-

BR=RO+RADIUS*RADIUS-

FUN=2*RADIUS*RADIUS*(RO+RADIUS*COST+HR/AR)/2*USQRT*AR-

NORMAL EXIT -

END SUBPROGRAM -

DAN

CONTINUE -

NORMAL EXIT -

END SUBPROGRAM -

SUBROUTINE (TK1,TMK2,TKK1,TMM,TKK2,CK,CMM,PK1,PK,PPK1,PPK2)

JAKGRA,R*CRUST,SIGWAT,SIGMA,SIGSUH,MU,NOR1,STADEN,1,(0,R

O*M*RO9*RF1)-

DIMENSION (D(4))-,

INTERS (ELK,ELKKK)-

FLOATING (KGRA)-

PK1=0-

PK3=0-

PK2=0-

PKKK1=0-

PKKK3=0-

PKKK2=0-

TK1=0-

CK=0-

TK2=0-
TKKK1=0.
CKKK=0.
TKKK2=0.

DO THROUGH (P); I=NOREC I+1; I*L*2-
TRANSFER TO (P1) PROVIDED (I*G*0)-

HCO=ELK-
HA=ELKKK-
IT=KKKA-
IH=KA-
JT=JJJA-
JH=JA-
AMO=MOKKK-ELKKK-
D(0)=EKKK-
Provided (MUE.3), D(0)=STADEN-
D(1)=SIGWAT-EKKK-
D(2)=EKKK-SIGSUB-
D(3)=EKKK-SIGMA-
TRANSFER TO (P2)-
P1

HCO=ELKKK-
HA=ELK-
IT=KA-
IH=KKKA-
JT=JA-
JH=JJJA-
AMO=MOKKK-ELKKK-
D(0)=EK-
Provided (MUE.3), D(0)=STADEN-
D(1)=SIGWAT-EK-
D(2)=EK-SIGSUB-
D(3)=EK-SIGMA-

P2

CONTINUE-
Provided (HCO*L.0*), HCO=0.*
RPHCO=R+HCO-

DO THROUGH (P); K1=5*IT; I*K1*L*5*(I+1)-
A=(COS*(K1/RO)-COS*(((K1+1)/RO)))/RO-
TRANSFER TO (S4) PROVIDED (MUE.1)-
F11=(89.5-K1)/RO-
TRANSFER TO (S5)-

S4

F11=RO9-RR1*(3*K1*(K1+1)+1)/(2*K1+1)-
S5

DO THROUGH (P); J1=5*JT; I*J1*L*5*(JT+1)-
D10=(J1-5*JH-2*)/RO-
COS=51(IH)*SIN(F11)+CO(IH)*COS*(F11)*COS*(D10)-
X50=(1.-COS.)/2.-
X1=1.-6.*X50*(4.-6.*X50)-
X2=X1+2.-4.*X50-
X3=-1.+2.*X50-
X4=1.-6.*X50*(1.-X50)-
X5=2.-3.*X50-
DO THROUGH (P); J=0+1; J*L.*3-
TRANSFER TO (P30,P31,P32) PROVIDED (J-1)-
TRANSFER TO (P4) PROVIDED (HA*L*O*)-

P30
DA=D(0)-
H2=HCO-MA-
H1=HCO-
TRANSFER TO (P5)-

P31
DA=D(1)-
H2=HCO-
H1=HCO-MA-
TRANSFER TO (P5)-
TRANSFER TO (P6) PROVIDED (AMO*L*CRUST)-

P4
DA=D(0)-
H2=HCO-MA-
H1=HCO-
TRANSFER TO (P5)-

P6
DA=D(2)-
H2=HCO+AMO-
H1=HCO+CRUST-
TRANSFER TO (P5)-

P32
DA=D(3)-
H2=HCO-
H1=HCO+CRUST-
TRANSFER TO (P) PROVIDED (DA*E*O*)-

P8
Y1=-H1/RPHCO-
Y2=-H2/RPHCO-
SORT1=SORT1*(4*XSQ*(1+Y1)+Y1*Y1)-
SORT2=SORT1*(4*XSQ*(1+Y2)+Y2*Y2)-
XLN=X4*LN1/((SORT2+Y2+2*XSQ)/(SORT1+Y1+2*XSQ))-1
BRACK=(X1+X2*Y2+X3*Y2*Y2)/SORT2-(X1+X2*Y1+X3*Y1*Y1)/SORT1-2
XLN=
PRACK=(X5+Y2/2)*SORT2-(X5+Y1/2)*SORT1+XLN-
CONST=KGRA*DA*A*RPHCO-
F=CONST*BRACK-
PF=-2*CONST*NPHCO*PRACK/R-
TRANSFER TO (P7) PROVIDED (I*G*O*)-
TRANSFER TO (KENO*KEN1*KEN2) PROVIDED (J-1)-

KEN0
TK1=TK1+F-
PK1=PK1+PF-
TRANSFER TO (P)-

KEN1
CK=CK+F-
PK3=PK3+PF-
TRANSFER TO (P)-

KEN2
TK2=TK2+F-
PK2=PK2+PF-
TRANSFER TO (P)-

P7
TRANSFER TO (LO0*LO1*LO2) PROVIDED (J-1)-

LO0
TKKK1=TKKK1+F-
PKKK1=PKKK1+PF-
TRANSFER TO (P)-
L01  CKKK=CKKK+F-
P3KK3=PKKK3+PF-
TRANSFER TO (P)-
L02  TKKK2=TKKK2+F-
P3KK2=PKKK2+PF-
P CONTINUE-
NORMAL EXIT-
END SUBPROGRAM-
SUBROUTINE (TK1,TK2,TKKK1,TKKK2,C=,CKKK,PK1,PK2,PKKK1,PKKK2,PK)
PK3,PKKK3)=ATT(ELK,ELKKK,MOK,MOKK,AKK,AKKK,KK,ELKKK,X1,X2)
X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19,X20)
1000  DIMENSION (D(4))-
1001  INTEGERS (ELK,ELKKK)-
1002  FLOATING (KGRA)-
1003  PK1=0.*-
1004  PK3=0.*-
1005  PK2=0.*-
1006  TK1=0.*-
1007  CK=0.*-
1008  TK2=0.*-
1009  DO THROUGH (P)+1=NOREC1,i+1,L*2-
1010  TRANSFER TO (P1) PROVIDED (1*,,-,,-)-
1011  HCO=ELK-
1012  HA=ELKKK-
1013  A=AKKK-
1014  AMO=MOKKK=ELKKK-
1015  D(0)=EKKK-
1016  PROVIDED (MU*E3),D(0)=STADLN-
1017  D(1)=SIGWAT-EKKK-
1018  D(2)=EKKK-SIGSUB-
1019  D(3)=EKKK-SIGMA-
1020  TRANSFER TO (P2)-
1021  P1  HCO=ELKKK-
1022  HA=ELK-
1023  A=AKK-
1024  AMO=MOK-ELK-
1025  D(0)=EK-
1026  PROVIDED (MU*E3),D(0)=STADFN-
1027  D(1)=SIGWAT-EK-
1028  D(2)=EK-SIGSUB-
1029  D(3)=EK-SIGMA-
1030  P2  CONTINUE-
1031  PROVIDED (HCO*L*O*),HCO=0.*-
1032  RPHCO=R+HCO-
1033  DO THROUGH (P)+J=0*1,J*L*3-
1034  TRANSFER TO (P37,P31,P32) PROVIDED (J-1)-
1035  P30  TRANSFER TO (P4) PROVIDED (HA*L*O*)-
1036  DA=D(0)
H2=HCO-HA -
H1=HCO -
TRANSFER TO (P5) -
P4 DA=D1(1) -
H2=HCO -
H1=HCO-HA -
TRANSFER TO (P5) -
TRANSFER TO (P6) PROVIDED (AMO*CRUST) -
DA=D1(2) -
H2=HCO+CRUST -
H1=HCO+AMO -
TRANSFER TO (P5) -
P6 DA=D1(3) -
H2=HCO -
H1=HCO+CRUST -
TRANSFER TO (P8) PROVIDED (DA*NE*O*) -
TRANSFER TO (PIL1) PROVIDED (1*1) -
TRANSFER TO (PIL2) PROVIDED (J*F*? ) -
CK=O*-
PK3=O* -
TRANSFER TO (P) -
PIL2 TK2=O* -
PK2=O* -
TRANSFER TO (P) -
PIL1 TRANSFER TO (PIL12) PROVIDED (J*F*? ) -
CKKK=O* -
PKKK3=O* -
TRANSFER TO (P) -
PIL12 TKKK2=O* -
PKKK2=O* -
TRANSFER TO (P) -
P8 Y1=H1/RPHCO -
Y2=H2/RPHCO -
SORT1=SQRT(4*X5Q*(1+Y1)+Y1*Y1 ) -
SORT2=SQRT(4*X5Q*(1+Y2)+Y2*Y2 ) -
XLN=X4*LN((SORT2+Y2+2*X5Q)/(SORT1+Y1+2*X5Q)) -
BRACK=(X1+X2*Y2+X3*Y2*Y2)/SORT2 - (X1+X2*Y1+X3*Y1*Y1 )/SORT1 -
XLN -
PRACK=(X5+Y2/2)*SORT2-(X5+Y1/2)*SORT1+XLN -
CONST=KGRA*DA*A*RPHCO -
F=CONST*BRACK -
PF=2.*CONST*RPHCO*PRACK/R -
TRANSFER TO (P7) PROVIDED (1*0*0 ) -
TRANSFER TO (KEN0*KEN1*KEN2) PROVIDED (J-1 ) -
KENJ TK1=F -

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APPENDIX B

Explanations, main program M

The explanations below relate to the statement listing in appendix A.

The computer is the IBM 7094. The language is SCATTRAN, as indicated.

The main program M computes model anomalies for the whole world.

Many of the statements are clear and serve an obvious purpose; they will be regarded briefly only. Some statements have indeed a deeper meaning and implications. Outstanding among these will be discussed here. Reference is to the card-or line-number, printed at the right (or to the outcome of its execution).
reads input, elevation and crustal thickness (referred to as MOHO) of the compartments. The input is in the integer mode; elevations in meters, crustal thickness in 10 meters. In (155) the latter is converted into meters.

reads the mean depth and the mean crust (water condensed). These should be precomputed, see 3.84, p. 89. Other constants and controls are read in (120-121) and (128). M indicates computation at the mean latitude (if 0) or at the center of gravity of the compartment (if 1). MU tells whether there is a set of densities available to be read-in (if 1), or that these will be considered as constant (if 0), or computed through a distinct-densities model B, p. 31 (if 2), or a two-layered model C, p. 36 (if 3), or D, p. 36 (if 4). The latter model is not used in this program, however — it is operational in the modified program M1 only. See p. 91, and appendix C. NU limits the computation outside the square to a distance of 180° (all compartments considered, if 0), 120° (if 1), 90° (if 2), 60° (if 3), 45° (if 4), or 30° (if 5); see 4.12, p. 101. NUR is the dimension vector of ELEV, MOHO, T, C, and POT, which could aid in the identification and the subscripting of variables, as well as in the printout process. RO is ρ'. R is the mean radius of the sphere representing the earth. KGRA is k, the constant of gravity, multiplied by 100 000 (for use with units of mgal and meter). CRUST is the term CRUST, the model mean Moho depth, p. 26. D, D(1), D(2) are the density differences (land-air, seawater-land, crust-subcrust). TETA is zero, the inner limit of the spherical cap, and IPRINT is a counter, set to zero, used
for the printout. (128) reads-in the number of latitude zones from the pole, for which no model anomaly is required, NOCOMP.

(160-187) are general preliminary operations, which precompute the constants necessary. (170) is needed for computation of distances. (169-184) store the values of $\phi$, $\sin(\phi)$ and $\cos(\phi)$, for either mean value of latitude or C.G. of the compartment, as determined by (171). The area of the $5' \times 5'$ squares in different latitude zones is computed in (183-184). (185-187) stores the constant model densities.

(188-222) obtain a set of density values. These are either read-in (189), or set constant (192-193), or computed with a distinct-densities model B (196-201), or a two-layered model C (208-221). The latter uses the constant density of the topography, STADEN (208).

This is the end of the preparatory stage. At this point actual computations could start. A sample computation has been inserted here (228-293), in order to demonstrate how the effect of the polar zones on themselves could be taken care of. In this example it is done by computing the mean of the elevation and the Moho, and assigning these values to one cap.

The number of latitude zones in the polar caps, p. 64, is set as NCAP, (223). (232) repeats the operation for the South Pole cap. The mean elevation and crustal thickness are obtained through (233 - 261). Weighted mean and regular mean are computed, to show the significantly different results obtainable. In use, later, are the results from the weighted system (289 - 291).
The fraction 0.5 and the unit in (256-261) are needed for rounding-off purposes. Actual computations for the caps are done by calling a subroutine (262-267).

In order to obtain execution times for certain stages, statements like (294) and (528) have been inserted at some places.

(295) through (432) perform the work of Part 1, computation of effects from outside the square, p. 71. (461-524) print and punch the results. All do-loops (except for IM) terminate at (428).

(295) sets the initial latitude zone, zero at the pole (90°-85°). (297) and alike are controls, put in for check-runs, and guarantee a printout in case the job is forced off the computer when execution time exceeds the estimate.

(298) sets the latitude interval, and assures that the other latitude zone (300) will not be below the symmetrical latitude with respect to the equator.

(301-302) transfer out for a while to (434-459), if a computation at the polar zones is performed. It prints out the accumulated effect of the polar zones on themselves, coming each from outside the square. The return is by (459).

The criterion for the print is interesting: It is when the lower latitude reaches the border of the polar zone, that one must print (302). At a later stage the computed values will include effects of other zones as well.

In the polar caps there are no 5° x 5° computations (303-306). There is no reciprocal computation in the zone for which no model anomalies are required (308), only its effect on other zones is considered.

(309) sets the longitude interval. There are special cases that arise.
In (311) the two squares in question will be on the same meridian, and in (312) they are on opposite ones. In (316) they are actually the same square. All the cases above have trivial results for the angular distance between the points. The general case follows, and needs to be computed through the cosine formula (313). The computations could be limited to a specific angular distance between the compartments through (325-336), if resulting effects from beyond that limit are negligible, or omitted.

The quantity $X_{SQ} = \sin^2 \psi/2$, and related terms are then computed (323), (337-341).

The actual fix for longitude for the points in question is made in (342), but this will immediately be developed into the pattern in Figure 9, p. 66.

(347-348) establish the symmetrical case with respect to the equator. Now, again, there are special cases to be considered. If the two compartments are really the same one (352 and 391), then the computation is skipped, since the square itself is to be considered in Parts II and III, p. 73. If the points are just on the same latitude, there is a case where they are on the opposite meridian (392-393). In that case the pair of points should be moved around on the parallel through 180° only, not 360°. Thus the variable governing this do-loop must be changed to stop it (426). The case when the points are on a great circle (353) is similarly checked.

The general case follows. The symmetry with respect to the meridian is established in (354) and (356). This is done by either adding or subtracting
the longitude difference to or from the initial longitude. Due to this algebraic operation, longitude indicators bigger than 71 (355° - 0°) or smaller than 0 (0° - 5°) can arise (357). These are reduced in (359).

Points on the same parallel, or on the same great circle again require special consideration, since in their cases no repetition symmetrical to the meridian is done (394).

The limit for subdivision of neighboring squares into 1° x 1' is set as (10.2° cos φ) for squares below 60° latitude (296), and approximately 5.2' for squares above 60° (400-401).

The final outcome of these procedures is to compute the model anomaly, including disturbance and Bruns' term, through the subroutine ATT. (370-379) or (414-417). The neighboring squares which are subdivided, are considered in subroutine ATT1. (364-367) or (402-405). The partial effects of the squares are added into their proper locations in (368-373), etc.

The printout and punching process (461-523) is repeated several times in the program, with different titles of course. A counter (524) distinguishes between the cases.

Part II, the effects from outside the cap within the square, p. 73, uses (531-578). The angle θ is set to the outer limit of Hayford's zone O (534), unless |φ| > 55° (535). The square is then divided into 400 (0.25° x 0.25') subsquares; but only half of them are considered, due to symmetry (545).

The distance to each subsquare is found through a subroutine (550-551). The
entire pattern of distances is stored, to be used later around the parallel and its southern reflection (554).

Computation of the accumulating effect of the subsquares is through the subroutine ATTRA. (560-562). Only cases which are beyond the limit \( \theta \) are considered (558).

Part III, the effect of the cap, p. 73, starts like Part II (579-588). Then the computation is performed by reference to the subroutine CAP. (589-591), which has already been used at (262-267).

After the printout (597-605), the operation has been completed. A few more statements were added here, so as to obtain the reduced system that does average zero over the whole earth. This can be done only if the computations yielded model anomalies everywhere on the earth (606). The existing average effect is found (614), and all values reduced accordingly (616-617). The corresponding increment in the value of the CRUST is approximated by (615). R. m. s. deviation of the model anomalies in latitude zones is computed (624-625), and also for a square of mean area (626). A printout follows (627-637).

Since the reduction above affects — in a sense — the numbers associated with \( C \) only, an additional printout is done for \( T \) alone (638-657). This is the end of the program (660).

Subroutine DIST. (716-721) computes the distance between two points, using the sine formula. This is for the subsquares outside the cap, within the computation compartment.
Subroutine ATTRA. (722-793) computes the effects of the subsquare. It is a special case of ATT, to be discussed below.

Subroutine CAP. (794-877) computes the effects of the spherical cap. The procedure is repeated three times (812): First for the effect of the topography or seawater, $T_1$, then for the compensation, $C$, and then for the crust between sea level and the model CRUST, $T_2$. $R_i$ is the radius-vector to the computation point; $R_i$ - the same for the lower boundary of the spherical cap, and $DR$ - its thickness, always positive. The computation is done by numerical integration (844), (866-874), p.56, performed in double precision (798), (868), etc.

Subroutine ATT, (1000-1109) is the one that computes the effects of the topography and its compensation reciprocally between two squares. One distinction is between the computation of $T_1$, $T_2$, or $C$ (1037). Then the procedure modifies Table 1, p.70, and determines the correct values of $HA (h)$ and $DA (\sigma)$ (1038-1059). $HCO$ is $H$, $A$ is $d\sigma$. The computation might be not reciprocal also (1013).

Subroutine ATT1. (878-999) is a modification of ATT. Changes reflect the fact that the square in question must be subdivided into 25 ($1^\circ\times1^\circ$) subsquares, and partial effects added together, to constitute the total effect. Instead of $(K,J)$ in the $5^\circ\times5^\circ$ system for (lat,long), one has $(K1,J1)$ in the $1^\circ\times1^\circ$ system (927), (933). The longitude difference reflects the use of the two systems (934).
APPENDIX C

Changes listing, main program M1

SHARNI. D. 
JOHN REA350  50  150  1000  11/5/66
SHARNI. D. 
JOHN BEA350  50  150  1000  11/5/66

* PLEASE PRESS SWITCH 6 IF TIME EXCEEDS, HUT LET PRINT.

*** RUN, STATEMENT LISTING, PUNCH SYMBOLIC BINARY, SCATRAN

M        INSERT (13)-
C        MU  IS  DISREGARDED IN THIS PROGRAM. DENSITIES COMPUTED (C)

M        REPLACE (51,72)-
C        READ INPUT (FORM5, (STADEN,FRA)-

C        STADEN = STANDARD DENSITY OF (FRA).-
C        FRA = FRACTION OF CRUST, LESS THAN 0.4. -

M        DELETE (75,75)-
M        REPLACE (79,83)-

M        EMASS = EMASS2 + DAM*SIGSUR*MOHO(1)*FRA*STADEN/10000. -

M        TRANSFER TO (EK8)-

E7        EMASS = EMASS2 + DAM*SIGSUR*MOHO(1)*FRA*STADEN-ELFV11)*SIGWAT) /

E7        /10000. -

E7        E0(1) = 100000*EMASS/(MOHO(1)*(1.*FRA))-

M        REPLACE (125,126)-

M        CALL SUBROUTINE (TCAP1,TCAP2,TCAP4,CWCAP,PTWC1,PTWC2,PTWC4,PCWC) -

M        E7        CALL SUBROUTINE (TCAP1,TCAP2,TCAP4,CAP,TE(0),CAPOLE,MWELEV,MMMOHO,D(0),TPKR,CRUST

M        CALL SUBROUTINE (TCAP1,TCAP2,TCAP4,CAP,TE(0),CAPOLE,MWELEV,MMMOHO,D(0),TPKR,CRUST

M        SIGWAT,SIGMASUR,FRA,STADEN)-

M        E7        GMA, SIGSUB, FRA, STADEN)-

M        TOTM = TCAPI + TCAP2 + TCAP4 -

M        E7        TIP = TWCAP1 + TWCAP2 + TWCAP4 -

M        POTM = TPC1 + TPC2 + TPC4 -

M        PIP = TWCAP1 + TWCAP2 + TWCAP4 -

M        REPLACE (133,136)-

E7        WRITE OUTPUT (FIG3, (MELEV, MWELEV, MMMOHO, MWWMOHO, TOTM, TIP, CAPI

E7        CAPI, CWCAP, TOTM, IP, CAPI, PCAP, POTM, PCAP, CCAP, PCAP, PCAP, PP2

E7        CAPI, CWCAP, TOTM, IP, CAPI, PCAP, CCAP, PCAP, POTM, PCAP, CAPI,

M        REPLACE (211,215)-

F        POT(KJ) = POT(KJ) + PK1 + PK2 + PK3 + PK4. -

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POT(KKKJJJ) = POT(KKKJJJ) + PK1 + PK2 + PK3 + PK4 -
T(KJ) = T(KJ) + TK1 + TK2 + TK4 -
T(KKKJJJ) = T(KKKJJJ) + TKK1 + TKK2 + TKK4 -

REPLACE (220, 224) -
CALL SUBROUTINE (TK1, TK2, TK4, TKK1, TKK2, TKK4, CK, CKKK, PK1 + PK2, PK4, PKKK1, PKKK2, PKKK4, PK3, PKKK3) = ATT (ELEV(KJ), MOHO(KJ), AREA(K), AREA(KKKJJJ), E0(KJ), E0(KKKJJJ), T1, X1, X2, X3, X4, X5, SIGR, SIGMA, SIGSUB, FRA, NITRE, STADEN) -
POT(KJ) = POT(KJ) + PK1 + PK2 + PK3 + PK4 -
T(KKKJJJ) = T(KKKJJJ) + TKK1 + TKK2 + TKK4 -

REPLACE (241, 245) -
CALL SUBROUTINE (TK1, TK2, TK4, TKK1, TKK2, TKK4, CK, CKKK, PK1 + PK2, PK4, PKKK1, PKKK2, PKKK4, PK3, PKKK3) = ATT (ELEV(KJ), MOHO(KJ), AREA(K), AREA(KKKJJJ), E0(KJ), E0(KKKJJJ), T1, X1, X2, X3, X4, X5, SIGR, SIGMA, SIGSUB, FRA, NITRE, STADEN) -
POT(KJ) = POT(KJ) + PK1 + PK2 + PK3 + PK4 -
T(KKKJJJ) = T(KKKJJJ) + TKK1 + TKK2 + TKK4 -

REPLACE (250, 254) -
CALL SUBROUTINE (TK1, TK2, TK4, TKK1, TKK2, TKK4, CK, CKKK, PK1 + PK2, PK4, PKKK1, PKKK2, PKKK4, PK3, PKKK3) = ATT (ELEV(KJ), MOHO(KJ), AREA(K), AREA(KKKJJJ), E0(KJ), E0(KKKJJJ), T1, X1, X2, X3, X4, X5, SIGR, SIGMA, SIGSUB, FRA, NITRE, STADEN) -
POT(KJ) = POT(KJ) + PK1 + PK2 + PK3 + PK4 -
T(KKKJJJ) = T(KKKJJJ) + TKK1 + TKK2 + TKK4 -

REPLACE (377, 379) -
CALL SUBROUTINE (TK1, TK2, TK4, TKK1, TKK2, TKK4, CK, CKKK, PK1 + PK2, PK4, PKKK1, PKKK2, PKKK4, PK3, PKKK3) = ATT (ELEV(KJ), MOHO(KJ), AOI (K), E0(KJ), T1, X1, X2, X3, X4, X5, SIGR, SIGMA, SIGSUB, FRA, NITRE, STADEN) -
POT(KJ) = POT(KJ) + PK1 + PK2 + PK3 + PK4 -
T(KKKJJJ) = T(KKKJJJ) + TKK1 + TKK2 + TKK4 -

REPLACE (402, 404) -
CALL SUBROUTINE (TC1, TC2, TC4, CC, PC1, PC2, PC3, PC4, PC3) = CAP (T1, T0, TETA, ELEV(KJ), MOHO(KJ), E0(KJ), T1, X1, X2, X3, X4, X5, SIGR, SIGMA, SIGSUB, FRA, NITRE, STADEN) -
POT(KJ) = POT(KJ) + PC1 + PC2 + PC3 + PC4 -
T(KKKJJJ) = T(KKKJJJ) + T1, T0, TETA, ELEV(KJ), MOHO(KJ), E0(KJ), T1, X1, X2, X3, X4, X5, SIGR, SIGMA, SIGSUB, FRA, NITRE, STADEN) -
POT(KJ) = POT(KJ) + PK1 + PK2 + PK3 + PK4 -
T(KKKJJJ) = T(KKKJJJ) + TKK1 + TKK2 + TKK4 -

REPLACE (446, 448) -
CONTINUE -
FIND CHANGES -
READ SYMBOLIC BINARY -
**STATEMENT LISTING: PUNCH SYMBOLIC HINARY: SCATRAN**

M REPLACE (7,7) -
   SUBROUTINE (TKK1, TKK2, TKK4, CKK, PKK1, PKK2, PKK4, PKK3) = ATTRA,
   (ELEV, MOHO, AN1, EK, KT, KGRA, CRUST, SIGWAT, SIGMA, SIGSUB, FRA, STADEN)

M REPLACE (19,23) -
   ELMO=FRA*MOHO-ELEV-
   D(0)=EK-SIGMA-
   D(1)=SIGWAT-SIGMA-
   D(2)=EK-SIGSUB-
   D(3)=STADEN-SIGMA-

M REPLACE (26,26) -
   DO THROUGH (DAN) * J=0*1*J*L*4 -

M REPLACE (29,29) -
   DA=STADEN-

M INSERT (33) -
   PROVIDED (ELMO*L*0) * H1=HCO+ELMO -

M REPLACE (40,40) -

M REPLACE (44,49) -

DA32 TRANSFER TO (DA33) PROVIDED (J*F*3) -
   TRANSFER TO (DA32A) PROVIDED (ELMO*L*0) -
   DA=D(1) -
   H2=HCO-
   PROVIDED (ELEV*L*0) * H2=HCO-ELEV-
   H1=HCO+CRUST-
   TRANSFER TO (DA5) -

DA32A DA=EK-
   H2=HCO+ELMO-
   H1=HCO-
   TRANSFER TO (DA5) -

DA33 DA=D(0) -
   H2=HCO+ELMO-
   PROVIDED (ELMO*L*0) * H2=HCO-
   H1=HCO+CRUST-
   PROVIDED (AMO*L*CRUST) * H1=HCO+AMO -

DA5 TRANSFER TO (DA7) PROVIDED (DA*NE*0) -
   TRANSFER TO (PIL1*PIL2*PIL3) PROVIDED (J-2) -

PIL1 CKK=0. -

M REPLACE (52,52) -

PIL2 TKK2=0. -

M INSERT (54) -

PIL3 TKK4=0. -
   PKK4=0. -
   TRANSFER TO (DAN) -

M REPLACE (72,73) -

KEN2 TRANSFER TO (KFN3) PROVIDED (J*F*3) -
   TKK2=F -
   PKK2=PF -
TRANSFER TO (DAN)-

KEN3

TKK4=F-

PKK4=PF-

M

REPLACE (77*77)-

SUBROUTINE (TC1+TC2+TC4+CC*PK1+PK2+PK4+PK7)=CAP*(T0+T1*ELF

V+MOHO+EK*TPK+R.CRUST*SIGWAT+SIGMA*SIGSUB*FRA*STADEN)-

M

REPLACE (89*94)-

D(0)=EK-SIGMA-

D(1)=SIGWAT-SIGMA-

D(2)=EK-SIGSUB-

D(3)=STADEN-SIGMA-

AMO=MOHO-ELFV-

ELMO=FRA*MOHO-ELFV-

DO THROUGH (DAN)*L=0*1*L*4-

M

INSERT (96)-

TRANSFER TO (D10A) PROVIDED (ELMO*E*O*)-

M

REPLACE (99*100)-

DEN=STADEN-

TRANSFER TO (D3)-

D10A

R1=R-ELMO-

DR=ELEV+ELMO-

DEN=STADEN-

TRANSFER TO (D3)-

M

REPLACE (105*105)-

D11

DR=AMO-CRUST-

M

REPLACE (109*109)-

DEN=SIGSUB-SIGMA-

M

REPLACE (112*114)-

D12

R1=R-AMO-

TRANSFER TO (D3)-

D12

TRANSFER TO (D13) PROVIDED (L.E*1)-

TRANSFER TO (D12A) PROVIDED (ELMO*L*O*)-

DR=ELMO-

PROVIDED (ELFV*L*O) OR=ELMO+ELEV-

M

REPLACE (117*119)-

TRANSFER TO (D3)-

D12A

DR=ELMO-

DEN=EK-

R1=R-

TRANSFER TO (D3)-

D13

DEN=D(0)-

TRANSFER TO (D14) PROVIDED (AMO*L*CRUST)-

DR=CRUST-ELMO-

PROVIDED (ELMO*L*O*) OR=CRUST-

R1=R-CRUST-

TRANSFER TO (D3)-

D14

DR=AMO-ELMO-

PROVIDED (ELMO*L*O*) OR=AMO-

R1=R-AMO-
D3 TRANSFER TO (D7) PROVIDED (DFN*NF*0*)-
TRANSFER TO (PIL1*PIL2*PIL3) PROVIDED (L-2)-

PIL1 CC=0*-
M REPLACE (122*122)-
PIL2 TC2=0*-
M INSERT (124*)-
PIL3 TC4=0*-
PK4=0*-
TRANSFER TO (DAN)-
M REPLACE (143*144)-
KEN2 TRANSFER TO (KEN3) PROVIDED (L.E.3)-
TC2=BRA-
PK2=PI-
TRANSFER TO (DAN)-
KEN3 TC4=BRA-
PK4=PI-
M REPLACE (157*157)-
SUBROUTINE (TK1,TK2,TK4,TKK1,TKK2,TKK4,CK,CKKK,PK1,PK2,
P<4,PKKK1,PKKK2,PKKK4,PK3,PKKK3) ATT1*(FLK*ELK*K*,MOK,MOKKK*,F
K*EKKK,*KA*J,KKA*J,JJJA*KR*CRUST*SIGWAT*SIGMA*SIGSUB*FRA*
RECP*STADEN*SIGCO*RO*MR*RO*RR*1)-
M INSERT (163*)-
PK4=0*-
M INSERT (166*)-
PKKK4=0*-
M INSERT (169*)-
TK4=0*-
M INSERT (172*)-
TKKK4=0*-
M REPLACE (182*186)-
FLMO=FRA*MOKKK*ELKKK-*
D(0)*EKKK-SIGMA-
D(1)=SIGWAT-SIGMA-
D(2)*EKKK-SIGSUB-
D(3)=STADEN-SIGMA-
M REPLACE (195*199)-
ELMO=FRA*MOK*ELK-*
D(0)*EK-SIGMA-
D(1)=SIGWAT-SIGMA-
D(2)*EK-SIGSUB-
D(3)=STADEN-SIGMA-
M REPLACE (218*218)-
DO THROUGH (P)*J=0*1*J*L*4-
M REPLACE (221*221)-
DA=STADEN-
M INSERT (223*)-
PROVIDED (FLMO*L*0*)+H1=HCO+FLMO-
M RFPLACE (234*234)-
P6 DA=SIGSUB-SIGMA-
M  REPLACE (238,240) -

P32  TRANSFER TO (P33) PROVIDED (J*F*3) -
     TRANSFER TO (P32A) PROVIDED (ELMO*L*O*) -
     DA=D(0) -
     H2=HCO -
     PROVIDED (H*O) H2=HCO-HA -
     H1=HCO+ELMO -
     TRANSFER TO (P5) -

P32A  DA=D(0)+SIGMA -
     H2=HCO+ELMO -
     H1=HCO -
     TRANSFER TO (P5) -

P33  DA=D(0) -
     H2=HCO+ELMO -
     PROVIDED (FLMO*L*O*) H2=HCO -
     H1=HCO+CRUST -
     PROVIDED (AMO*L*CRUST) H1=HCO+AMO -

M  REPLACE (260,260) -

KEN2  TRANSFER TO (KEN3) PROVIDED (J*F*3) -
     TK2=TK2+F -

M  INSERT (262) -

KEN3  TK4=TK4+F -
     PK4=PK4+PF -
     TRANSFER TO (P) -

M  REPLACE (270,271) -

LO2  TRANSFER TO (LO3) PROVIDED (J*F*3) -
     TKKK2=TKKK2+F -
     PKKK2=PKKK2+PF -
     TRANSFER TO (P) -

LO3  TKKK4=TKKK4+F -
     PKKK4=PKKK4+PF -

M  REPLACE (275,275) -

SUBROUTINE (TK1*TK2*TK4*TKKK1*TKKK2*TKKK4*CK*CKKK*PK1*PK2*PK4*PKKK1*PKKK2*PKKK4*PK3*PKKK3) ATT (ELK*ELKK*MOK*MOKK*AK +AKKK*EK*EKKK*X1*X2*X3*X4*X5*XSQ*KGRA*K*CRUST*SIGWAT*SIGMA*SIGSUB*FRA*NOREC1*STADEN) -

M  INSERT (281) -
     PK4=0 -

M  INSERT (284) -
     TK4=0 -

M  REPLACE (291,295) -

FLMO=FRA*MOKKK-ELKKK -
     D(0)=FKKK-SIGMA -
     D(1)=SIGWAT-SIGMA -
     D(2)=FKKK-SIGSUB -
     D(3)=STADEN-SIGMA -

M  REPLACE (301,305) -

ELMO=FRA*MOK-ELK -
     D(0)=EK-SIGMA -
M \( n(\square) = \text{SIGWAT-SIGMA-} \)
M \( D(2) = \text{EK-SIGSUB-} \)
M \( D(3) = \text{STADEN-SIGMA-} \)
M REPLACE (309*309) -
M DO THROUGH (P) * J=0+1 * J=L+4 -
M REPLACE (312*312) -
M DA=STADEN-
M INSERT (314) -
M PROVIDED (FLMO*L*0*) * H1=HCO+FLMO-
M REPLACE (325*325) -
P6 DA=SIGSUB-SIGMA-
M REPLACE (329*331) -
P32 TRANSFER TO (P31) PROVIDED (J*E*3) -
P32A TRANSFER TO (P32A) PROVIDED (FLMO*L*0*) -
M DA=D(3)-
M H2=HCO-
M PROVIDED (HA*L*0*) * H2=HCO-HA-
M H1=HCO+FLMO-
M TRANSFER TO (P5) -
P32A DA=D(0)+SIGMA-
P33 DA=D(0)-
M H2=HCO+FLMO-
M PROVIDED (FLMO*L*0*) * H2=HCO-
M H1=HCO+ELMO-
M PROVIDED (AMO*L*CRUST) * H1=HCO+AMO-
M REPLACE (334*335) -
M TRANSFER TO (P31,P31,P31) PROVIDED (J-2) -
PIL01 CK=0,-
M REPLACE (341*342) -
PIL03 TK4=0,-
PIL03 PK4=0,-
M TRANSFER TO (P) -
PIL1 TRANSFER TO (P1L1,P1L2,P1L3) PROVIDED (J-P) -
PIL11 CKKK=0,-
M INSERT (347) -
PIL13 TKKK4=0,-
PIL13 PKKK4=0,-
M TRANSFER TO (P) -
M REPLACE (366*366) -
M KFN2 TRANSFER TO (KFN3) PROVIDED (J*E*3) -
M TK2=F-
M INSERT (368) -
M KFN3 TK4=F-
M PK4=PF-
M TRANSFER TO (P) -
M REPLACE (376*377) -
APPENDIX D

Comments, main program M1

The main program M, appendix A, can not handle the two-layered model D, p. 36. Modifications into program M1 are needed. The changes listing in appendix C, when submitted together with the main program M, will result in the program M1. A complete statement listing might not give such a clear picture of modifications, as well as being voluminous, having most of the statements already appear in program M.

The card number at the right-hand side is the card (line) number in the main program M, which is referred to by the modification.

The effects of the topography and its compensation must be computed in four parts here, for 4 vertical density zones within each compartment (742 and alike). The zones yield the effect of topography or water (T1), and the rest of the upper crust (T3), which has a constant density. The lower crust above the model CRUST gives (T3), and (C) is the compensation effect. The
corresponding density discrepancies are found in a slightly different manner than in program M (735), since the density stored is that of the lower crust (216) and not that of the total crust.

The fraction of the upper crust with constant density is limited to 0.4 (188), due to physical considerations, as well as model difficulties. A problem arises when this fraction happens to be above sea level. Special caution is then needed, to decide upon the proper densities and their vertical border lines, as evidenced by the numerous conditional statements (749 and alike).

APPENDIX E

Statement listing, point program P

SHARN1. D* JOBN BEA350 50 150 1000 11/5/66
SHARN1. D* JOBN BEA350 50 150 1000 11/5/66
*** RUN STATEMENT LISTING PUNCH SYMBOLIC BINARY SCATHAN
START READ INPUT *FORM1* (FLEV(1)^1=0:1:1.1E+2592))-
       READ INPUT *FORM1* (MQHO(1)^1=0:1:1.1E+2592))-
       F FORM1 (1215)-
       READ INPUT *FORM5* (CME*CMM)-
       F FORM5 (2F10.5)-
       READ INPUT *FORM2* (M*MU*NU*UR*RO*KGRA*CRUST.D/D1*D(2)-
       *TETA)-
       F FORM2 (311.12* F20.15* F10.0* F10.8* F5.0*3 F6.3* F3*1)-
       WRITE OUTPUT *FORM4* (M*MU*NU*UR*RO*KGRA*CRUST.D/D1*D(2)-
       2*TETA)-
       F FORM4 (3HOM*11.3X*3HNU*11.3X*3HNU*11.3X* 4HNU*12.3X*3HRG*-
       F10.7*3X*2HR* F9.1*6HOKGRA* F8.6*3X*6HCRUST* F7.1*3X*-
       6HDENS*3* F6.0*3X*3HT0* F3*1)-
       INTEGERS (ELEV*FLINP*ELKKK)-
       FLOATING (KGRA)*-
       DATA-
       ELEVATION IN METERS CRUSTAL THICKNESS IN 10XMETERS-
       CME = MEAN DEPTH (WATER CONDENSED) CMM = MEAN CRUST K-
       M CONTROLS AND CONSTANTS

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C
NP = NUMBER OF COMP* PTS* MAXIMUM 100* 0068
NLAT = LAT. OF COMP* PT* IN 5X5 DEG.; 0 FOR 40-85 N. 0069
NLONG = LONG. OF SAME* 0 FOR 0-5 E. 0070
SIGSUB=D(0)-D(2)- 0071
SIGWAT=D(0)+D(1)- 0072
TRANSFER TO (EL0,EL1,EL2) PROVIDED (MUI-1)- 0073
EL1 READ INPUT *FORM3*((E0(1)*I=01*1*L.2592))- 0074
F FORM3 (12F5.3)- 0075
   DO THROUGH (EL4);I=01*1*L.2592-
   E1(1)=SIGWAT-E0(1)- 0077
   E2(1)=F0(1)-SIGSUB- 0078
EL4 E3(1)=E0(1)-D(0)- 0079
   TRANSFER TO (EL5)- 0080
EL0 DO THROUGH (EL3);I=01*1*L.2592-
   E0(1)=D(0)- 0082
   E1(1)=D(1)- 0083
   E2(1)=D(2)- 0084
EL3 E3(1)=0.- 0085
   TRANSFER TO (EL5)- 0086
EL2 TRANSFER TO (EL6) PROVIDED (MUI.G.2)- 0087
EMASS2=CMM*D(0)- 0088
   DO THROUGH (EK5);I=01*1*L.2592-
   AM=(MOHO(I)-ELEV(I))/1000.- 0090
   DAM=AM-CMM-CME-
   TRANSFER TO (EK1) PROVIDED (ELV(I)*L.0)- 0092
EMASS=EMASS2+DAM*SIGSUB-
   TRANSFER TO (EK2)- 0094
EK1 EMASS=EMASS2+DAM*SIGSUB+ELV(I)*SIGWAT/1000.- 0095
EK2 E0(1)=1000.*EMASS/MOHO(I)- 0096
   CONTINUE-
   E1(1)=SIGWAT-F0(1)- 0098
   E2(1)=E0(1)-SIGSUB- 0099
   E3(1)=F0(1)-D(0)- 0100
EK5 CONTINUE-
   TRANSFER TO (EL5)- 0102
EL6 READ INPUT *FDEN,*STADEN,- 0103
F FDEN (F10.3)- 0104
G STADEN = STANDARD DENSITY; ABOVE SEA LFVFL ONLY. 0105
EMASS2=CMM*D(0)- 0106
   DO THROUGH (EK9);I=01*1*L.2592-
   AM=(MOHO(I)-ELEV(I))/1000.- 0108
   DAM=AM-CMM-CME-
   E0(1)=STADEN-
   TRANSFER TO (EK7) PROVIDED (ELV(I)*L.0)- 0111
EMASS=EMASS2+DAM*SIGSUB-STADEN*ELEV(I)/1000.- 0112
   SIG2=EMASS/AM-
   TRANSFER TO (EK8)- 0114
EK7 EMASS=EMASS2+DAM*SIGSUB+SIGWAT/ELEV(I)/1000.- 0115
   SIG2=1000.*EMASS/MOHO(I)- 0116
FK8
CONTINUE -
E1(I)=SIGWAT-SIG2-
E2(I)=SIG2-SIGSUR-
E3(I)=SIG2-D(0)-

FK9
CONTINUE -

EL5
EMM=0.-
DO THROUGH (EL7)*I=0*1*I*L*2592-
IA=1/72-
AM=MOHO(I)-ELEV(I)-
PROVIDED (ELEV(I)*CFL*0.)*EMMLA=E(I(I)-ELEV(I)-
PROVIDED (ELEV(I)*L*0.)*EMMLA=-E(I(I)-ELEV(I)-

EL7
EMM=EMM+EMMLA*AREA(IA)-
EMM=EMM*RO(CRUST*720*1)-
WRITE OUTPUT *FIGK * (EMM)-

F FIGK
(Q*MEAN DEN. DISCREPANCY =*1PE13*6) -
DO THROUGH (PAM7)*INP=0*1*INP*L*INP-
INPLAT=NLAT(INP)-
KAK=72*INPLAT+NLONG(INP)-
ELINP=ELEV(KAK)-
MOINP=MOHO(KAK)-
HCO=ELINP-
PROVIDED (HCO*L*0.)*HCO=0.-
RPHCO=R+HCO-
FS(0)=E3(KAK)-
FS(1)=E1(KAK)-
FS(2)=E2(KAK)-
FS(3)=E3(KAK)-
CALL SUBROUTINE (ITIME)=ZCLOCK(*)-
T=0.-
C=0.-
PT=0.-
PC=0.-

C
RESULTS IN MGALS-:
DO THROUGH (PAM3)*J=0*1*J*L*36-
DO THROUGH (PAM3)*J=0*1*J*L*72-
KIK=72*J+-
ELKKK=ELEV(KIK)-
MOKKK=MOHO(KIK)-
FS(0)=D0(KIK)-
FS(1)=E1(KIK)-
FS(2)=E2(KIK)-
FS(3)=E3(KIK)-
JJ=ABS(J-NLONG(INP))-1
PROVIDED (JJ*G*36)+JJ=72-JJ-
TRANSFER TO (PAM5) PROVIDED (JJ*E*0)-
TRANSFER TO (PAM6) PROVIDED (JJ*E*36)-
COSINE=SI(1)*SIN(INPLAT)+CO(1)*CO(INPLAT)*COLUMG(JJ)-
TRANSFER TO (PAM4)-
PAM5  TRANSFER TO (PAM3) PROVIDED (1*F1*INPLAT)- 0166
      A=A*ABS(F1(1)-F1(INPLAT))-
      TRANSFER TO (PAM8)- 0167
PAM6  A=180*RO-ABS*(F1(1)+F1(INPLAT))-
PAM8  COSINE=COS(6)- 0170
PAM4  TRANSFER TO (PIPO,PIP1,PIP2) PROVIDED (NU-1)- 0171
PIP1  TRANSFER TO (PAM3) PROVIDED (COSINF*L-O*5)- 0172
PIP2  TRANSFER TO (PAM3) PROVIDED (COSINF*L-O*4)- 0173
PIP3  TRANSFER TO (PAM3) PROVIDED (COSINF*L-O*3)- 0174
PIP4  TRANSFER TO (PAM3) PROVIDED (COSINF*L-O*2)- 0175
PIP5  TRANSFER TO (PAM3) PROVIDED (NU*0*4)- 0176
PIP6  TRANSFER TO (PAM3) PROVIDED (COSINF*L-O*6)- 0177
      DO THROUGH (SHA1)- 0178
      A=COS((18/RO)-COS(((18+1)/RO))/RO)- 0179
      TRANSFER TO (SHA2) PROVIDED (M*F1)- 0180
      F11=(89.5-18)/RO- 0181
      TRANSFER TO (SHA3)- 0182
SHA2  F11=RO9-RR1*(3*IB*(18+1)+1)/(2*18+1)- 0183
      DO THROUGH (SHA3)- 0184
      OLD=(JB-5*NLUNG(INP)-2)/RO- 0185
      COSINE=SIN(F11)*SIN(INPLAT)+COS(F11)*COS(INPLAT)*COS(OLD)- 0186
      CALL SUBROUTINE (TK1,TK2,CK+PK1+PK2=PK3)=ATTRAC.(FLKKK+FS)- 0187
      MOKKK(A1+COSINE)- 0188
      T=T+TK1+TK2- 0189
      C=C+CK- 0190
      PC=PC+PK3- 0191
      PTC=PT+PC- 0192
      SHA1  CONTINUE- 0193
      TRANSFER TO (PAM3)- 0194
PIPO  CALL SUBROUTINE (TK1,TK2,CK+PK1+PK2=PK3)=ATTRAC.(FLKKK+FS)- 0195
      MOKKK(A1+COSINE)- 0196
      T=T+TK1+TK2- 0197
      C=C+CK- 0198
      PC=PC+PK3- 0199
      PTC=PT+PC- 0200
      SHA3  CONTINUE- 0201
      TRANSFER TO (PAM3)- 0202
PIPO  CALL SUBROUTINE (TK1,TK2,CK+PK1+PK2=PK3)=ATTRAC.(FLKKK+FS)- 0203
      MOKKK(A1+COSINE)- 0204
      T=T+TK1+TK2- 0205
      C=C+CK- 0206
      PC=PC+PK3- 0207
      PTC=PT+PC- 0208
      SHA3  CONTINUE- 0209
      WRITE OUTPUT FORME- 0210
      M=INP+FS(O)- 0211
      PAM3  CONTINUE- 0212
FORME

(1H/1H0/Q*0*COMPUTATION *=12*5X0*POINT LOCATION *=13*1H* 0215
13*9X0*ELEVATION *=15*5X0*CRUST THICKNESS *=15*5X* 0216
Q*0*DENSITY *=16*6X3/1H0/1H0/28XQ*0*ACUMULATED EFFECT (MGALs) 0217
*20XQ*0*TIME (MIN.)*1H0/1H0/1H0/1H0/1H0/3/H0/1H0 0218
- CALL SUBROUTINE (ITIME)=RCLOCK*()- 0219
WRITE OUTPUT *FOT1*(ITIME+TC+TC+PT+PT+C+PC+TC+PT 0221
C)- 0222

F FOT1

(0*COUTSIDE SQUARE*59X-3PB8*3/1H0/
Q*0*DISTURBANCE*9X3(F9*3/11X)/Q*0*RUNING TEWF*3X(F9*3/11X) 0224
11X) Q*0*ANOMALY*7X3(F9*3/11X)/1H0)- 0225
CALL SUBROUTINE (ITIME)=ZCLOCK*()- 0226
TETA(1)168700/R- 0227
PROVIDED (INPLAT=7) TETA(1)=(24/1H0)*SCU(INPLAT)- 0228
DO THROUGH (DP1) ILAT=0*1+ILAT1*20- 0229
ILAT=20*INPLAT+ILAT1- 0230
AO1(ILAT)=RO01*(COS*(RO01*(ILAT1+1)))- 0231
TRANSFER TO (DP4) PROVIDED (MEF)- 0232
F101(ILAT1)=RO01*(RO01*(ILAT1)-CO01*(ILAT1+1)))- 0233
TRANSFER TO (DP5)- 0234

DP4

F101(ILAT1)=RO01*(RO01*(ILAT1+1)+1)/(2*ILAT1+1)- 0235
S101(ILAT)=SIN*(F101(ILAT1))- 0236
CO01(ILAT)=COS*(F101(ILAT1))- 0237
DO THROUGH (DP1) * ILONG=0*1+ILONG1*10- 0238
DRO01=RO01*(ILONG+0*5)- 0239
COLO01=COS*(DRO01)- 0240
S1001=SIN*(RO01)- 0241
ILALO=10*ILAT1+ILONG- 0242

DP5

CALL SUBROUTINE (DIS(ILALO))=TETA*(1)*(INPLAT)*CO(INPLAT)- 0243
O1(ILAT1)=CO01(ILAT1)*S1001*C001)- 0244
DO THROUGH (PS1) N=0*1+N+S*200- 0245
TRANSFER TO (PS1) PROVIDED (DIS(N)*L*TETA(1)) 0246
XT=SIN*(0*5*DIS(N))- 0247
CALL SUBROUTINE (TKK1,TKK2,CKK,PKK1,PKK2,PKK3)=ATTR+ILLIN- 0248
P*FS*MOINP*AN1(N/10),XT)- 0249
T=T+TKK1+TKK2- 0250
C=C+CKK- 0251
TC=T+C- 0252
PT=PT+PKK1+PKK2- 0253
PC=PC+PKK3- 0254
PTC=PT+PC- 0255

PS1

CONTINUE- 0256
CALL SUBROUTINE (ITIME)=RCLOCK*()- 0257
WRITE OUTPUT *FOT2*(ITIMEF+TC+TC+PT+PT+C+PC+TC+PT 0258
C)- 0259

F FOT2

(0*COUTSIDE CAP*6PB6*3PB8*3/1H0/
Q*0*DISTURBANCE*3X3(F9*3/11X)/Q*0*RUNING TEWF*3X(F9*3/11X) 0261
11X) Q*0*ANOMALY*7X3(F9*3/11X)/1H0)- 0262
CALL SUBROUTINE (ITIME)=ZCLOCK*()- 0263
CALL SUBROUTINE (TC1,TC2,CC,PC1,PC2,PC3)=CAP*(TET0(0),TETA 0.264
(1)+ELNP*FS*MOINP)-
T=T+TC1+TC2-
C=C+CC-
TC=T+C-
PT=PT+PC1+PC2-
PC=PC+PC3-
PTC=PT+PC-
CALL SUBROUTINE (ITIME)=RCLOCK(*)-
WRITE OUTPUT *FOT3*(ITIME,T,CT,TC:1,PC,PTC,T+PT,C+PC,TC+PT
C)-
F FOT3
(Q*INSIDE CAP*63X*-3PBB*3/IHO/
Q* DISTURBANCE*3X*3(F9.3*11X)/Q*ARUNS TERM*7X*3(F9.3*11X1/IHO)-
TRANSFER TO (DAP)-
SUBROUTINE (TK1,TK2,CK,PK1,PK2,PK3)=ATTRAC*(FLKKK,DK,MOKKK,
A+KK=COS)-
UNIVERSAL (KGRA,RP,HPCH CO,HCOCRUST)-
XSO=(1-COS)/2-
X1=1.0-1.0*XSO+2.0*XSO*XSO-
X2=X1+2.0-4.0*XSO-
X3=-1.0+2.0*XSO-
X4=1.0-6.0*XSO+6.0*XSO*XSO-
X5=2.0-3.0*XSO-
DIMENSION (D(4))-1
INTEGER (FLKKK)-
HA=ELKKK-
A=AKKK-
AMO=MOKKK-ELKKK-
DO THROUGH (P): IP=0*1*1*3-
TRANSFER TO (P30,P31,P32) PROVIDED (IP=1)-
P30 TRANSFER TO (P4) PROVIDED (FLKKK,L=0)-
DA=D(0)-
H2=HCO-HA-
H1=HCO-
TRANSFER TO (P5)-
P4 DA=D(1)-
H2=HCO-
H1=HCO-HA-
TRANSFER TO (P5)-
P31 TRANSFER TO (P6) PROVIDED (AMO,L*,CRUST)-
DA=D(2)-
H2=HCO+CRUST-
H1=HCO+AMO-
TRANSFER TO (P5)-
P6 DA=D(2)-
H2=HCO+AMO-
H1=HCO+CRUST-
TRANSFER TO (P5)-

200
P32  DA=D(3)-
    H2=HCO-
    H1=HCO+CRUST-
  P5  TRANSFER TO (P7) PROVIDED (DA*NE=0.)-
    TRANSFER TO (PIL) PROVIDED (IP*E=2)-
    CK=0.*-
    PK3=0.*-
    TRANSFER TO (P)-
PIL  TK2=0.*-
    PK2=0.*-
    TRANSFER TO (P)-
P7  Y1=-H1/RPHCO-
    Y2=-H2/RPHCO-
    SQR1=SQR((4.*X5Q*(1.+Y1)+Y1*Y1)-
    SQR2=SQR((4.*X5Q*(1.+Y2)+Y2*Y2)-
    XLN=X4*LN((SQR2+Y2+2.*X5Q)/(SQR1+Y1+2.*X5Q))-;
    BRACK=(X1+X2+X3*Y2+Y2)/(SQR2-.1+X2*Y1+X3*Y1*Y1)/SQR1-2
    *XLN-
    PRACK=(X5+Y2/2.)*SQR2-(X5+Y1/2.)*SQR1+XLN-
    CONST=KGRA*DA*A/RPHCO-
    F=CONST*BRACK-
    PK=-2.*CONST*RPHCO*PRACK/R-
    TRANSFER TO (KEN0*KEN1*KFN2) PROVIDED (IP-1)-
KEN0  TK1=F-
    PK1=PF-
    TRANSFER TO (P)-
KEN1  CK=F-
    PK3=PF-
    TRANSFER TO (P)-
KEN2  TK2=F-
    PK2=PF-
P  CONTINUE -
    NORMAL EXIT -
    FND SUBPROGRAM -
    SUBROUTINE (DIS)=DIST(S11*CO1*S101*CO01*SIL001*COL001)-
    S1F12=(SIL001*CO01)*P.2+(S101*CO1-S11*CO01*COL001)*P.2-
    S1F1=SQR((S1F12)-
    DIS=ATAN((S1F1/SQR((1.-S1F12))-)
    NORMAL EXIT -
    END SUBPROGRAM -
    SUBROUTINE (TK1,TK2,CK,PK1,PK2,PK3)=ATTRA (ELEV,DOMOHO,A01
    *XT)-
    UNIVERSAL (KGRA*R*RPHCO/HCO+CRUST)-
    DIMENSION (N(4))- 
    INTEGERS (ELEV)-
    X5Q=XT**XT-
    X1=1.+16.*X5Q+24.*X5Q*X5Q-
    X2=X1+2.*4.*X5Q-
    X3=-1.+2.*X5Q-
TRANSFER TO (DAN) -

TK2=F-
PK2=PF-

DAN
CONTINUE -
NORMAL EXIT -
END SUBPROGRAM -
SUBROUTINE (TC1,TC2,CC,PK1,PK2,PK3)=CAPA(TC11,LL,LL2,MOHO)

- INTEGERS (ELEV) -
DIMENSION (D14),D1(3),PRA(7),RA(7),RA(7) -
PRECISION (2,R0,R1,R2,SUGER,RADIUS,FUN(lm)TA=T1) -
TA=T1 -
TB=TO -
PRECISION (2,DCOST(COST) -
COST=DCOST(TA) -
UNIVERSAL (TPK,CRUST,R) -
R0=R+ELEV -
PROVIDED (FLEV,L1,RO=RO -
COST=COST,COST -
RCOST=R0,COST -
R0=R0,R0 -
DO THROUGH (DAN) -L=0*1*L,L,L7 -
TRANSFER TO (D10,D11,D12,PROVIDED (L-1) -

D10
TRANSFER TO (D2) PROVIDED (FLEV,L7) -
R1=R -
DR=ELEV -
DEN=D(11) -
TRANSFER TO (D3) -

D2
R1=R+ELEV -
DR=ELEV -
DEN=D(3) -
TRANSFER TO (D3) -

D11
DR=MOHO-ELEV-CRUST -
DEN=D(1) -
TRANSFER TO (D4) PROVIDED (MOHO,L1+ELEV,CRUST) -
DR=DR -
DEN=DEN -
R1=R-CRUST -
TRANSFER TO (D3) -

D4
R1=R+ELEV-MOHO -
TRANSFER TO (D3) -

D12
DR=CRUST -
DEN=D(3) -
R1=R-CRUST -
TRANSFER TO (D7) PROVIDED (IFNE,NE,0) -
TRANSFER TO (PIL) PROVIDED (LF,2) -
CC=CC -
PK3=PK3 -
TRANSFER TO (DAN) -
PL IL T  C2*0*- " > P P M
PK2*0*- '>*•
TRANSFER TO (DAN)-

PK2*PI- TRANSFER TO (DAN)-

D7

R2=R1+DR-
CALL SUBROUTINE (FIN)=GAUSS,(R1,R2*SOGER,)-
RA(1)=R1-
RA(2)=R2-
RAS(1)=R1*#R1-
RAS(2)=R2*#R2-
DO THROUGH (PILA)i=1;1;1;L;3-
DI(1)=SORT,(ROS+RAS(1)-2*RA(1)*RCOS),/3-
PILA
PRD(1)=DI(1)*(RAS(1)/3-#COS*RA(1)/6++(13)-#COS*HCS**-2++)-
RAS(1)*RA(1)*/3**#R0, RAS(1)*2**#R0*HCS*5**-1**#COS*#D1**3**-
D1/RO-COST+DI(1))-
PI=2*TPK*DEN*(PRD(2)-PRD(1))/R-
BRA=TPK*DEN*FIN-
TRANSFER TO (KEINO+KEN1+KEN2) PROVIDED (L-1)-

KEN0
TC1=BRA-
PK1=PI-
TRANSFER TO (DAN)-

KEN1
CC=BRA-
PK3=PI-
TRANSFER TO (DAN)-

KEN2
TC2=BRA-
PK2=PI-
TRANSFER TO (DAN)-
FUNCTION (FUN)=SOFER,(RADIUS)-
EXTERNAL (RO,COST)-,
PRECISION (2*FUN*AR*BR*COST+DSRQR,*(RADIUS))-,
AR=2*#RO+RADIUS-
BR=RO+RADIUS+RADIUS-
FUN=2*RADIUS*RADIUS*((RO+RADIUS*(COST+2*#BR/AR))/DSRQR,+(2,2,5**AR)-
#COST+BR)-(RO+RADIUS*(1+2*#BR/AR))/DSRQR,*(AR+#A)/AR-
NORMAL EXIT -
END SUBPROGRAM -

DAN CONTINUE -
NORMAL EXIT -
END SUBPROGRAM -

DAP CONTINUE -

END CALL SUBROUTINE ()=ENDJOB,()-
END PROGRAM (START)-

*** DATA
PIL
TC2=0*
PK2=0*
TRANSFER TO (DAN)

D7
R2=R1+OR
CALL SUBROUTINE (FIN) =GAUSS (R1,R2,SOGER*)
RA(1)=R1-
RA(2)=R2-
RAS(1)=R1*R1-
RAS(2)=R2*R2-
DO THROUGH (PILA)*I=1*1+1*L+3-
DI(1)=SQRT(ROS+RAS(1)-2*KRA(1)*RCOS),/R0-
PRA(1)=DI(1)*((RAS(1)/3.0-RCOS*RA(1)/6.0+ROS/3.0-RCOS*RCOS/2.0)+
RAS(1)*RA(1)/(3.0*R0)-RAS(1)/2.0+0.5*R0*RCOS/(1.0-COSQ)*LN(10*RA(1))-

PILA
R0=COST+DI(1))-
P1=-2.0*TPK*DEN*(PRA(1)-PRA(1))/R-
BRA=TPK*DEN*FIN-
TRANSFER TO (KEN0,KEN1,KEN2) PROVIDED (L-1)-

KEN0
TC1=BRA-
PK1=P1-
TRANSFER TO (DAN)-

KEN1
CC=BRA-
PK3=P1-
TRANSFER TO (DAN)-

KEN2
TC2=BRA-
PK2=P1-
TRANSFER TO (DAN)-
FUNCTION (FUN)=SOGER*(RADIUS)-
EXTERNAL (RO+COST)-
PRECISION (2.0*AR*R0,COST,DSQRT,*RADIUS)-
AR=2.0*R0*RADIUS-
BR=R0*R0+RADIUS*RADIUS-
FUN=2.0*RADIUS*RADIUS*((RO+RADIUS*(COST+2.0*BR/AR))/DSQRT*(AR+
*COST+BR)-((RO+RADIUS*(1.0+2.0*BR/AR))/DSQRT*(AR+BR)))/AR-
NORMAL EXIT-
END SUBPROGRAM-

DAN
CONTINUE-
NORMAL EXIT-
END SUBPROGRAM-

DAP
CONTINUE-
END SUBPROGRAM-

PAM7
CONTINUE-
END CALL SUBROUTINE ()=ENDJOB();-
END PROGRAM (START)-
*** DATA
APPENDIX F

Comments, point program P

The comments below relate to the statement listing in appendix E. Reference is to the card number, printed at the right, in the statement listing.

The reader is assumed familiar with the main program M already. A few remarks will suffice here, since this program P evaluates the same models. A big simplification is that program P computes model anomalies at individually specified points only; it is therefore quite straightforward. The distance between two compartments is not stored, p. 65. The entire cumbersome system which evaluates where this distance can be re-used is eliminated.

In the input there are several variables missing from the main program M, for obvious reasons. NP is the number of points to be computed, and NLAT(i), NLONG(i) - their coordinates in the 5° x 5° system (63-66). I and J (151-152) are the coordinates of the attracting element. I varies from 0 at (90° -85° N) to 35 at (85° -90° S); J varies from 0 at (0° -5° E) to 71 at (355° -0° E).

Program P has more storage space than the main program M, can store four sets of density discrepancies (34), instead of one set of densities only.

The limit for subdivision of neighboring squares is 5.2°. A change to another constant limit will reflect in (183) alone. A limit varying with the latitude must also be declared somewhere between (134) and (183), not inclusive.
APPENDIX G

Changes listing, point program Pl

**SHARNI* D.**  JOBN BEA350  50  150  1000  11/ 5/66
**SHARNI* D.**  JOBN BEA350  50  150  1000  11/ 5/66

*** RUN, STATEMENT LISTING, PUNCH SYMBOLIC BINARY, SCATRAN

M  INSERT (12)-  0022
C  MU IS DISREGARDED IN THIS PROGRAM, DENSITIES COMPUTED (C).

M  REPLACE (15,15)-  0034
C  DIMENSION (E0(2592),E2(2592),MAM(2592),ELMO(2592),FS(4),FS (4))-

M  REPLACE (51,110)-  0073
C  SIGMA=D(10)-

EL24 READ INPUT, FORMS,(STADEN,FRA)-
C  STADEN = STANDARD DENSITY OF (FRA).
C  FRA = FRACTION OF CRUST, LESS THAN 0.4.-
C  EMASS2=CMM*D(0)*1000,-
C  DO THROUGH (EK15), I=0,1,10,2592-
C  MAM(I)=MOHO(I)-ELEV(I)-
C  ELMO(I)=MOHO(I)-FRA-ELEV(I)-
C  DAM=MAM(I)-(CMM+CME)*1000,-
C  TRANSFER TO (EK11) PROVIDED (ELEV(I)*L0)-
C  EMASS=EMASS2+DAM*SIGSUB-MOHO(I)*FRA*STADEN-
C  TRANSFER TO (EK12)-

EK11 EMASS=EMASS2+DAM*SIGSUB-MOHO(I)*FRA*STADEN+ELEV(I)*SIGWAT-
EK12 SIG3=EMASS/(MOHO(I)**(1,-FRA))- E0(I)=SIG3-D(0)-
EK15 CONTINUE -

M  INSERT (115)-  0137
C  ELMOK=ELMO(KAK)-
C  MAMK=MAM(KAK)-

M  REPLACE (120,122)-  0142
C  FS(1)=D(1)-
C  FS(2)=E2(KAK)-
C  FS(3)=STADEN-D(0)-

M  INSERT (133)-  0155
C  ELMOKK=ELMO(KIK)-
C  MAMKK=MAM(KIK)-

M  REPLACE (135,137)-  0157
C  ES(1)=D(1)-
C  ES(2)=E2(KIK)-
C  ES(3)=STADEN-D(0)-

M  REPLACE (171,172)-  0194
C  CALL SUBROUTINE (TK1,TK2,TK4,CK,PK1,PK2,PK4,PK3)=ATTRAC.-
C  (ELKKK,ES,MOKKK,ELMKK,MAMKKK,A1,COSINE)-

206
T = T + TK1 + TK2 + TK4 -
M REPLACE (175, 175) - 0199
PT = PT + PK1 + PK2 + PK4 -
M REPLACE (180, 181) - 0204

CALL SUBROUTINE (TK1, TK2, TK4, CK, PK1, PK2, PK4, PK3) = ATTRA(
ELKKK, ES, MOKKK, ELMOKK, MAMKKK, AREA(1), COSINE) -
T = T + TK1 + TK2 + TK4 -
M REPLACE (184, 184) - 0209
PT = PT + PK1 + PK2 + PK4 -
M REPLACE (214, 215) - 0248
CALL SUBROUTINE (TK1, TK2, TK4, CK, PK1, PK2, PK4, PK3) = ATTRA(
ELINP, FS, MOINP, ELMOK, MAMK, AD1(N/10), XT) -
T = T + TK1 + TK2 + TK4 -
M REPLACE (218, 218) - 0253
PT = PT + PK1 + PK2 + PK4 -
M REPLACE (226, 227) - 0264
CALL SUBROUTINE (TC1, TC2, TC4, CC, PC1, PC2, PC4, PC3) = CAP(
TETA(0), TETA(1), ELINP, FS, MOINP, ELMOK, MAMK) -
T = T + TC1 + TC2 + TC4 -
M REPLACE (230, 230) - 0269
PT = PT + PC1 + PC2 + PC4 -
M REPLACE (237, 238) - 0279
SUBROUTINE (TK1, TK2, TK4, CK, PK1, PK2, PK4, PK3) = ATTRA(
ELKKK, D, MOKKK, ELMOK, MAMK, AKKK, COS) -
UNIVERSAL (KGRA, R, RPHCO, HCO, CRUST, STADEN, SIGMA, SIGSUR) -
M REPLACE (249, 250) - 0292
DOT THROUGH (P) = IP = 0, 1, 1 P.L. 4 -
M REPLACE (253, 253) - 0296
DA = STADEN -
M INSERT (255) - 0298
PROVIDED (ELMO.L.O.), H1 = HCO + ELMO -
M REPLACE (261, 261) - 0304
TRANSFER TO (P6) PROVIDED (MAM.L.CRUST) -
M REPLACE (264, 267) - 0307
H1 = HCO + MAM -
TRANSFER TO (P5) -
P6 DA = SIGSUB - SIGMA -
H2 = HCO + MAM -
M REPLACE (270, 280) - 0313
TRANSFER TO (P33) PROVIDED (IP.E.3) -
TRANSFER TO (P32A) PROVIDED (ELMO.L.O.) -
DA = D(3) -
H2 = HCO -
PROVIDED (HA.L.O.), H2 = HCO - HA -
H1 = HCO + ELMO -
TRANSFER TO (P5) -
P32A DA = D(0) + SIGMA -
H2 = HCO + ELMO -
H1 = HCO -
TRANSFER TO (P5)-

P33 DA=D(0)-
H2=HCO+ELMO-
PROVIDED (ELMO,L.O.), H2=HCO-
H1=HCO+CRUST-
PROVIDED (MAM,L.CRUST), H1=HCO+MAM-

P5 TRANSFER TO (P7) PROVIDED (DA,NE,0*)-
TRANSFER TO (PIL1,PIL2,PIL3) PROVIDED (IP=21)-

PIL1 CK=O*-
PK3=O*-
TRANSFER TO (P)-

PIL2 TK2=O*-
PK2=O*-
TRANSFER TO (P)-

PIL3 TK4=O*-
PK4=O*-
TRANSFER TO (P)-
REPLACE (298,299)-
KEN2 TRANSFER TO (KEN3) PROVIDED (IP,E,3)-

TK2=F-
PK2=PF-
TRANSFER TO (P)-

KEN3 TK4=F-
PK4=PF-
M REPLACEM (309,310)-
SUBROUTINE (TK1,TK2,TK4,CK,PK1,PK2,PK4,PK3)-ATTRA (ELEV,
D*MOHO,ELMO,MAM,A01,XT)-
UNIVERSAL (KGRA,R,RP,CO,HC,CRUST,STADEN,SIGMA,SIGSUB)-

M REPLACEM (319,320)-
DO AT THROUGH (DAN)*J=0*4,J,L,4-

M REPLACEM (326,329)-
PROVIDED (ELMO,L.O.), H1=HCO+ELMO-
TRANSFER TO (DA5)-

DA31 TRANSFER TO (DA6) PROVIDED (MAM,L.CRUST)-

M REPLACEM (332,335)-
H1=HCO+MAM-
TRANSFER TO (DA5)-

DA6 DA=SIGSUB-SIGMA-

M REPLACEM (338,340)-
DA32 TRANSFER TO (DA33) PROVIDED (J,E,3)-
TRANSER TO (DA32A) PROVIDED (ELMO,L.O*)-

DA=D(3)-
H2=HCO-
PROVIDED (ELEV,L.O.), H2=HCO=ELEV-
H1=HCO+ELMO-
TRANSFER TO (DA5)-

DA32A DA=D(0)+SIGMA-
H2=HCO+ELMO-

208
H1=HCO-
TRANSFER TO (DA5)-
DA33 DA=D(0)-
H2=HCO+ELMO-
PROVIDED (ELMO.L.O.S1L.E)-H2=HCO-
M
H1=HCO+CRUST-
M
PROVIDED (MAM.L.CRUST)-H1=HCO+MAM-
M
REPLACE (342,348)-
TRANSFER TO (P1L1,P1L2,P1L3) PROVIDED (J-2)-
PIL1 CK=0.-
PK3=0.-
TRANSFER TO (DAN)-
PIL2 TK2=0.-
PK2=0.-
TRANSFER TO (DAN)-
PIL3 TK4=0.-
PK4=0.-
TRANSFER TO (DAN)-
M
REPLACE (366,367)-
KEN2 TRANSFER TO (KEN3) PROVIDED (J.E.3)-
TK2=F-
PK2=PF-
TRANSFER TO (DAN)-
KEN3 TK4=F-
PK4=PF-
M
REPLACE (372,372)-
SUBROUTINE (TC1,TC2,TC4,CC,PK1,PK2,PK4,PK3)=CAP*(T0,T1,
ELEV,D,MOMO,ELMO,MAM)-
M
REPLACE (379,379)-
UNIVERSAL (TPK,CRUST,R,STADEN,SIGMA,SIGSUB)-
M
REPLACE (385,385)-
DOTHROUGH (DAN)*L=0.1*L*L.4-
M
INSERT (387)-
TRANSFER TO (D10A) PROVIDED (ELMO,L.O.3)-
M
REPLACE (390,391)-
DEN=STADEN-
TRANSFER TO (D3)-
D10A R1=R=ELMO-
DR=ELEV+ELMO-
DEN=STADEN-
TRANSFER TO (D3)-
M
REPLACE (396,396)-
D11 DR=MAM=CRUST-
M
REPLACE (400,400)-
DEN=SIGSUB-SIGMA-
M
REPLACE (403,407)-
D4 R1=R=MAM-
TRANSFER TO (D3)-
D12 TRANSFER TO (D13) PROVIDED (L.E.3)-
TRANSFER TO (D12A) PROVIDED (ELMO,L,0*) -
DR=ELMO-
PROVIDED (ELEV,L,0*) , DR=ELMO+ELEV-
DEN=D(3)-
R1=R-ELMO-
TRANSFER TO (D3)-

D12A DR=ELMO-
DEN=D(0)+SIGMA-
R1=R-
TRANSFER TO (D3)-

D13 DEN=D(0)-
TRANSFER TO (D14) PROVIDED (MAM,L,CRUST)-
DR=CRUST-ELMO-
PROVIDED (ELMO,L,0*) , DR=CRUST-
R1=R-CRUST-
TRANSFER TO (D3)-

D14 DR=MAM-ELMO-
PROVIDED (ELMO,L,0*) , DR=MAM-
R1=R-MAM-

M REPLACE (409,415)-
TRANSFER TO (PIL1,PIL2,PIL3) PROVIDED (L-2)-
PIL1 CC=0*- 
PK3=0*- 
TRANSFER TO (DAN)-
PIL2 TC2=0*- 
PK2=0*- 
TRANSFER TO (DAN)-
PIL3 TC4=0*- 
PK4=0*- 
TRANSFER TO (DAN)-
M REPLACE (434,436)-
KEN2 TRANSFER TO (KEN3) PROVIDED (L,E,3)-
TC2=BRA- 
PK2=PI- 
TRANSFER TO (DAN)-
KEN3 TC4=BRA- 
PK4=PI- 
TRANSFER TO (DAN)-
M END CHANGES-
READ SYMBOLIC BINARY-

### DATA
APPENDIX H

Comments, changes listing P1

The point program P, appendix E, can not handle the two-layered model D, p. 36. Modifications into program P1 are given in appendix G. When they are submitted together with program P, the modified program P1 will result.

A few comments will suffice, since the modifications are similar to those in appendix D. Reference is to the old card number, appearing in the changes.

Full sets of Moho depth (MAM) and depth to the bottom of the upper crust (ELMO) can be stored. In addition to that, two sets of density discrepancies are stored: (E₀) between the lower crust and the model crust, and (Eₐ) between the lower crust and the model subcrust (73).

APPENDIX I

Crustal thickness, 5° x 5° means

The 5° x 5° mean crustal thicknesses were estimated by the author from a small scale map [73]. They are listed below in units of 10 meters. The precision of the mean is on the order of ±2 km. The accuracy of the map contours is unknown to the author, could probably reach ±3 km, and be systematically wrong in areas.
CRUSTAL THICKNESS, SX5 DEGREE MEANS

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**CRUSTAL THICKNESS**: 5x5 DEGREE MEANS

**UNITS**: 10 m.
CRUSTAL THICKNESS, 5X5 DEGREE MEANS

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99 Tab.8 ... ...

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Tab.8 ...

(add column 5)