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AN INVESTIGATION OF AERODYNAMIC CONTROLS
AT HYPersonic MACH NUMBERS

DISSERTATION
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
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1966

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The subject of this dissertation is the investigation of aerodynamic controls at hypersonic Mach numbers in the presence of separated and expansion type flow fields over the wing surfaces. The surfaces over which the various types of flow fields occur are a function of the combination of angle of attack and control surface deflection.

The purpose of this study was to determine aerodynamic control effectiveness and control surface hinge moments as a function of control surface location; the experimental results were compared with computations based on inviscid theory. In addition, the effects of coupling of the flow fields over both wing surfaces resulting from cross-flow through the control surface gap was presented and analyzed.

The plan of the research program was to measure, by experimental techniques, the surface pressure distributions over a wing with both leading and trailing edge controls at hypersonic speeds. The pressure distributions were then integrated to obtain two-dimensional force and moment coefficients, lift-to-drag ratio, and center of pressure location for both control locations. In addition, the experimental results were compared with inviscid theory based on a modified blast wave equation for the surface pressure distribution.
ACKNOWLEDGMENTS

The author is indebted to Professor John D. Lee of The Ohio State University for his criticism, helpful guidance, and encouragement during the course of the research. In addition, the author wishes to thank Professor Rudolph Edse of The Ohio State University for his interest in this research effort. Personal thanks and appreciation are extended to Col. Andrew Boreski of the Aerospace Research Laboratories' Staff and to Dr. Robert Korkegi of the Hypersonic Research Laboratory who made this work possible. Acknowledgment must be made of the support received from personnel of the Systems Research Laboratory. My appreciation is extended to Messrs. J. Randle Greenup and William D. Humphries for their assistance during the experimental phase of the program. A special debt of gratitude is extended to Mr. Robert Linhart for his contributions to the preparation of the computer program used to reduce the data. Thanks is extended to Messrs. David Jones and Jeffrey Pitsinger for their assistance in preparing the tables and curves. Finally, a special thanks to Miss Karen Hill and Miss Betty Dilts of the Hypersonic Research Laboratory of the Aerospace Research Laboratories and Mrs. Evelyn Shaw for typing the manuscript.

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Studies in Mathematics, Professors Louis Brand, Louis Doty, and Henry D. Colson

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Studies in Supersonic Aerodynamics, Professor Louis Doty and Garvin L. Von Eschen

Studies in Hypersonic Aerodynamics and Boundary Layer Theory, Professors Lan Wong, Garvin L. Von Eschen, and John D. Lee

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SYMBOLS

\( \textbf{Kn}_d \)  
Knudsen number based on the pressure lead inside diameter

\( p \)  
pressure in m.m. of Hg. abs.

\( T \)  
temperature in °R

\( d \)  
inside diameter of the orifice in inches

\( \lambda \)  
mean-free-path in inches

\( a_1, a_2, a_3, a_4, a_5, b_3, b_1, b_2, b_3, \text{ and } b_4 \)  
power series coefficients as defined by equation (A-5)

\( f \)  
longitudinal Mach number gradient correction factor

\( \gamma \)  
ratio of specific heats

\( X \)  
longitudinal distance along the model in inches

\( M \)  
Mach number

\( \Delta M/\Delta X \)  
longitudinal Mach number gradient

\( \alpha \)  
Angle of attack in degrees [or radians in equation (1)]

\( \delta \)  
Control surface deflection in degrees (nose up is positive)

\( P_0' \)  
total pressure behind the bow shock in m.m. of Hg. abs.

\( S \)  
model surface distance in inches

\( D \)  
leading edge wing thickness in inches
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<td>$P_2/P_1$</td>
<td>pressure ratio across local shock wave at control surface hinge line</td>
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<td>$R_e D$</td>
<td>free-stream Reynolds number referred to the wing thickness</td>
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<td>$f_1(\gamma), f_2(\gamma), f_3(\gamma)$</td>
<td>the Kholyavko blast wave theory functions given by equations (2), (3), and (4), respectively</td>
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<td>$E_1$</td>
<td>Lee's function given in equation (6)</td>
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<td>$\phi_{P_m}/P_o$</td>
<td>the vibrational degree of freedom correction for the Rayleigh equation (taken from Reference 1)</td>
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<td>$\beta$</td>
<td>constant as given by equation (45) and (46)</td>
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<td>$(p/p_t)_v$</td>
<td>pressure ratio across local expansion fan at control surface hinge line</td>
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<td>$p(\theta)$</td>
<td>the surface pressure over a semi-infinite cylinder</td>
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<td>$\theta$</td>
<td>the angle between the longitudinal axis of the wing and the pressure vector in degrees</td>
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<td>$A_n$</td>
<td>Gregorek-Korkan values of the Fourier coefficients</td>
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<td>$C_l$</td>
<td>integrated two-dimensional lift coefficient</td>
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<td>$q_\infty$</td>
<td>free-stream dynamic pressure in m.m. of Hg. abs.</td>
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<td>$C = C_w + C_f$</td>
<td>total wing chord length in inches</td>
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<td>wing chord in inches</td>
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<tr>
<td>$C_f$</td>
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<td>$C_D$</td>
<td>integrated two-dimensional pressure drag coefficient</td>
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<tr>
<td>$C_{m_{1/2}}$</td>
<td>integrated two-dimensional pitching moment coefficient about the mid-chord point</td>
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<tr>
<td>$h$</td>
<td>distance from wing trailing edge to the control surface hinge line in inches</td>
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<tr>
<td>$C_H$</td>
<td>integrated two-dimensional control surface hinge moments</td>
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<td>$\Delta p$</td>
<td>local pressure difference between the compression and expansion surfaces in m.m. of Hg. abs.</td>
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<td>$\eta$</td>
<td>the blast wave term given by equation (8)</td>
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### SYMBOLS
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#### Subscripts
- \( m \) or \( tc \): pressure corrected for thermal creep
- \( R \): uncorrected pressure and temperature at the cold end of the pressure lead
- \( W \): wall conditions
- \( e \): empirical correction factor
- \( H \): the Hall correction factor and the hinge line condition
- \( \infty \): free-stream conditions
- \( \Delta \): refers to the wedge conditions
- \( ns \): normal shock wave conditions
- \( sh \): shoulder conditions
- \( u \): upper wing surface
- \( l \): lower wing surface

#### Constants
- \( K_1 \): \( 0.5063 \times 10^{-7} \) in - mm of Hg abs/°R
- \( \gamma \): 1.400
I. INTRODUCTION

In recent years serious consideration has been given to manned aerodynamically controlled re-entry vehicles. The primary advantage of these vehicles is their capability of maneuvering, thereby enabling the pilot to choose a desired flight trajectory and landing area, rather than a pre-determined trajectory and impact point as in the case of a ballistic vehicle. Assuming these new vehicles are equipped with conventional aerodynamic control surfaces, certain serious control surface problems present themselves. At hypersonic speeds these problems manifest themselves in the form of high local heat transfer rates and possible loss of control effectiveness due to flow separation when the control surface is deflected. Flow separation is not peculiar to the hypersonic flight regime; however, its effects are more pronounced at these flight conditions as compared to flight at low speed or supersonic speeds. Although some experimental and theoretical data with reference to control surfaces and flow separation can be found in the literature they are usually limited to a sharp leading edge flat plate at zero degrees angle of attack followed by a wedge afterbody. More important, only the compression side of the configuration has been studied with no reference to the pressure distribution on the expansion surface.
In this study a more realistic case was investigated, the configuration was a wing with a semi-cylindrical leading edge equipped with a conventional aerodynamic control surface having a slight gap between the wing and the control surface to permit freedom of movement. The aerodynamic complexities encountered by this particular configuration are: (1) the presence of a separated and reattached flow field over one wing surface and a local expansion fan at the hinge line over the other wing surface, both resulting from control surface deflection; (2) vorticity due to bow shock curvature; (3) increased boundary layer thickness caused by the presence of a hot wall \(0.50 \leq T_w/T_o \leq 0.72\); and (4) the existence of cross-flow through an open gap in the vicinity of the control surface hinge line.

The effects of a separated flow field and the local expansion fan at the hinge line is shown schematically in Figures 17 and 18 for two typical cases where \(\alpha \geq 0^\circ\) at \(\delta > 0^\circ\) and \(\alpha > 0^\circ\) at \(\delta < 0^\circ\), respectively. This effect is discussed further in Section IV.

The effects of vorticity due to bow shock curvature on the local pressure distribution are discussed by Hayes and Probstein\(^{24}\) and Oguchi\(^{25}\) for the two-dimensional flat plate with both sharp and blunt leading edge. Based on the results of these investigations, it is believed that the effect of vorticity on surface pressures is negligible.

The presence of a hot wall probably will produce some difference in the surface pressure distribution as compared
with the cold wall case. The higher wall temperatures cause a thickening of the boundary layer in such a manner as to produce an "effective body" which is blunter and whose "effective" surfaces are "wedge-like" in nature rather than flat as compared with the actual physical configuration.

The effect of cross-flow through the open gap between the wing and the control surface does present some difficulties at hypersonic speeds since it couples the flow fields over both wing surfaces. This particular problem is discussed in some detail in Section IV.

Because of the complexities of the combined physical flow fields described above and the difficulties involved in putting them into mathematical form, no theory is presently available which takes into account the above mentioned phenomena simultaneously; therefore, the aeronautical engineer is forced to depend greatly on experimental measurements. The primary purpose of this research program was to determine the gross effects of the combined physical phenomena previously described on the integrated force and moment coefficients as a function of angle of attack, control surface deflection, and control location with respect to the wing. Although some analysis with reference to the local surface pressures and the effect of cross-flow through an open control gap on these surface pressures are presented, it was not the intent of this investigation to study the details of highly localized flow fields. For such a study a new and highly specialized model would be required; in addition, the
experimental program would be carried out in a facility equipped with a contoured nozzle rather than a conical nozzle in order to avoid tunnel effects on the separated region over the model resulting from undesirable conical nozzle pressure gradients. The configuration investigated is shown in Figure 5 and the location of the pressure orifices and temperature thermocouples used to measure surface pressures and temperatures, respectively, are given in Table 1. Tests were conducted at a Mach number of 13.87 and a Reynolds number, based on leading edge thickness, of approximately 3935.0. The ratio of wall temperature to stagnation temperature varied from 0.72 near the leading edge to approximately 0.50 at the trailing edge. Experimental pressure distributions were obtained for various combinations of angle of attack \((-12^\circ \leq \alpha \leq +12^\circ)\) and control surface deflections \((-18^\circ \leq \delta \leq +18^\circ)\); these pressure distributions were then integrated to obtain the corresponding force and moment coefficients, lift-to-drag ratio, and center of pressure location. These data are compared with inviscid theory based on a modified blast wave equation for the surface pressure distribution.

Included is a correlation of the experimentally obtained pressure distribution and related force and moment coefficients with the blast wave theory and a modified blast wave equation for attached flows at various angles of attack with a non-deflected control surface. Also presented is an analysis of the effects of cross-flow through the open control
surface gap. Finally, the force and moment coefficients, lift-to-drag ratio, the center of pressure location, control effectiveness, and hinge moments are analyzed as a function of control deflection and control surface location. All these experimental results are also compared with inviscid theory.

The experimental data were corrected for the excitation of the vibrational degree of freedom of the air molecules, thermal creep effects within the pressure-lines, and for the longitudinal Mach number gradient in the conical nozzle resulting from source flow. The details of these corrections are given in the Appendix. Included in the Appendix is an error analysis and a short analysis to determine by how much the flow over the model departs from ideal two-dimensional conditions. In addition, a discussion of liquefaction on the free-stream variables and the pressure distribution over the wing is also included.
II. DESCRIPTION OF TEST EQUIPMENT

A. The Wind Tunnel

The experimental data reported was obtained in the 4-inch hypersonic wind tunnel of the Aerospace Research Laboratories. The wind tunnel is mounted vertically and because of the low mass flow compared to the available air supply, it can be considered a continuous flow facility. High pressure air is supplied to the wind tunnel from an air supply system which stores approximately 1800 cu. ft. of pre-dried air at atmospheric temperature and at a pressure of 3000 psia. The diffuser back-pressure is decreased to approximately two millimeters of mercury through the use of a three stage vacuum pump system.

The primary components of the 4-inch wind tunnel are the heater, a nozzle, the open jet test cabin, a diffuser, and the related tunnel control system. The heater is an electrical resistance type heater with a maximum power input of 37 kilowatts.

Attached to the heater is a conical nozzle designed for a Mach number of 14.0, it was made up of two sections bolted together. The throat section has a throat diameter of 0.05 inch and is made of copper with back side water cooling. The nozzle flow expands from the throat to outlet through
a stainless steel conical section with a half angle of 7.5°.

The nozzle extends into a test cabin which is a simple box, one foot on each side, and is equipped with two 6-inch ports which can be used for flow visualization. The nozzle location within the test cabin and the 6-inch ports can be seen in both Figures 1 and 4. One side of the test cabin has a removable plate for purposes of model installation, instrumentation, and support mechanisms.

The test cabin is attached to a converging-constant area-diverging diffuser. The upstream portion of the diffuser is back-side water cooled. It is an axi-symmetric design with a physical inlet diameter of 4.40 inches. The constant area second throat section is 3.41 inches in diameter and 21.58 inches in length. The divergent section has a half angle of 3.5° and is 4.9; inches in length.

An external view of the 4-inch hypersonic wind tunnel is shown in Figure 1.

B. The Test Instrumentation

The instrumentation used in this program consisted of a pressure probe and its related transducer for calibrating the nozzle, and a rotary valve combined with a 1/10 psia variable reluctance transducer for purposes of measuring the wing surface pressures.

The total head probe and its transducer was designed to traverse the complete jet and was used to measure the
total head pressure in the free-stream. Knowing the tunnel stagnation conditions and the measured total head pressure, the lateral Mach number distribution at various longitudinal positions was calculated using the Rayleigh pitot equation; the results are shown in Figure 73.

The rotary valve and a 1/10 psia variable reluctance transducer were manifoldd together and the rotary valve was remotely controlled. The wing pressure leads were connected directly to the rotary valve. A time lag check under actual testing condition, revealed the pressures required a maximum of 20 seconds to stabilize. Based on these results, the rotary valve was cycled only when the 20-second time period was reached. In addition, pressure stabilization was verified at each point by visually monitoring the digital voltmeter, from which the surface pressures were obtained.

C. **The Wing Model**

The configuration investigated is shown in Figure 3; it consists of a flat plate wing with a semi-cylindrical leading and trailing edge and an aerodynamic control surface. The wing thickness, which was governed by blockage tests at angle of attack, was 0.125 inch while the total chord was 2.250 inches and its span was 4.50 inches. That portion of the wing which passed through the jet shear layer had a sharp leading edge, the purpose was to avoid tunnel flow breakdown at angle of attack. The wing's control surface had a chord length of 0.700 inch and a span of 2.700 inches, its hinge
line was 0.600 inch ahead of the control trailing edge as shown in Figure 5. In order to insure freedom of movement of the control surface about its hinge line under thermal conditions, a gap of 0.005 inch existed between the wing and the control surface. Since the wing was very thin, it was instrumented only on one side and the data was obtained for both surfaces by pitching the model in two directions. A total of 17 pressure orifices, whose inside diameter was 0.047 inch, were placed on the model centerline and six thermocouples were offset from the centerline by 0.25 inch. These thermocouples were used to measure the wing wall temperature. Five pressure orifices and three thermocouples were on the control surface; the remaining orifices and thermocouples were on the wing. The location of these pressure orifices and thermocouples are shown in Figure 5 and their coordinates are tabulated in Table 1. The thermocouples were chromel, chromel-alumel and were housed in an insulated stainless steel sheath whose outside diameter was 0.025 inch. The model was mounted at one end of its span to a model support strut which was then mounted in the support system housing as shown in Figure 2. The model and the model support strut rotated within the support system housing for purposes of setting the angle of attack. A second drive shaft, which was attached to the control surface, was contained within the model support strut and was used to set the control surface deflection with reference to the wing chord line. Both the pressure leads and the thermocouple wires were of sufficient length to be taken from the model through the
model support strut and external to the wind tunnel as a single unit where they were connected to the proper instrumentation. Because of the great number of instrumentation leads, it became necessary to bring these leads out of both ends of the model wing span. Those leads which were at the free end of the wing were bent around and outside the test jet as shown in Figure 3. The model and the related equipment were designed so it could be either injected or retracted from the test core manually, enabling starting the tunnel with an empty test section. A view of the model in the test cabin is given in Figure 4 which shows the relative position of these pressure leads when the wing was injected in its test position.

The thermocouples were connected to a Brown indicator and the wing wall temperatures were recorded manually. The pressure leads were connected to a rotary valve and a 1/10 psia variable reluctance transducer. The transducer output and the corresponding pressure orifice location were inputs to a X-Y plotter where the data was plotted as local pressure in volts versus the distance along the wing. In addition, all pressures were read directly from a digital voltmeter connected in parallel with the X-Y plotter.
III. PRESSURE DISTRIBUTIONS AND RELATED FORCE AND MOMENT COEFFICIENTS FOR ATTACHED FLOW FIELDS

A. Comparison of Typical Pressure Distribution with Inviscid Theory

In aeronautics, knowledge of the pressure distribution over a flight vehicle is important. Once the pressure distribution is known, either by theoretical or experimental techniques, it can then be integrated to obtain the force and moment coefficients acting on the vehicle. At hypersonic speeds, the available theories for predicting the pressure distributions are limited because of the complex flow field involved. The blunt flat plate wing is a good example, the only available theory for such a configuration is Sedov's blast wave theory for inviscid flows at zero angle of attack, and its extension to angle of attack by Kholyavko. At present no theory is known to exist for a blunt flat plate wing with a laminar boundary layer. In this particular investigation the experimental data was obtained at sufficiently high Reynolds number so the flow field can be considered to be inviscid, in view of this fact, the data are compared with inviscid theory.

In this section, consideration will be given to the case where the flow is attached to the wing within the angle of attack range of \(-12^\circ \leq \alpha \leq +12^\circ\) at \(\delta = 0^\circ\). Before discussing
the integrated force and moment coefficients, it will be necessary to first consider the surface pressure distributions since the proper integration of these pressure distributions result in the corresponding coefficients. For purposes of comparing with the data, the inviscid surface pressure distribution shall be obtained in accordance with the methods of Kholyavko and Lee. The surface pressure distribution based on Kholyavko's blast wave theory yields the expression

\[ \frac{P}{P_0} = \gamma M^2 \left( \frac{P_0}{P_0} \right) \left\{ f_1(\gamma) \left[ \frac{C_D N}{\frac{X}{D}} \right]^{2/3} + f_2(\gamma) \left[ \frac{C_D N}{\frac{X}{D}} \right]^{1/3} \right. \]

\[ \left. + f_3(\gamma) a^2 \right\} \]  

(1)

where

\[ f_1(\gamma) = \left[ \frac{(\gamma + 1)(\gamma - 1)}{36 \left(3\gamma - 1\right)^2} \right]^{1/3} \]  

(2)

\[ f_2(\gamma) = \frac{5}{3} \left( \frac{3\gamma - 2}{7\gamma - 1} \right) \left[ \frac{(\gamma + 1)^2 (\gamma - 1)}{6 \left(3\gamma - 1\right)^2} \right]^{1/3} \]  

(3)

and

\[ f_3(\gamma) = \frac{(\gamma + 1)(3\gamma - 2)^2}{2 \left(7\gamma - 1\right)^2} . \]  

In the second term of equation (1), the plus sign is taken for the compression surface while the minus sign is taken for the expansion side of the wing.

The second equation to be considered is Lee's modified blast wave equation. This semi-empirical relationship is simply the linear superpositioning of the first term in
equation (1) (Sedov's blast wave theory for zero degree angle of attack) and the wedge pressure term; it is written as

\[ \frac{P}{P_0} = \frac{E_1}{(S/D)^{2/3}} + \frac{(P_\infty^A)}{P_0} \frac{(P_\infty)}{P_0^ns} \]  

(5)

where \( E_1 \) is a function of angle of attack and Mach number. Its evaluation was based on the pressure at the upstream shoulder of the wing and is

\[ E_1 = (\frac{S}{D}^{2/3})_{sh} \left[ \frac{(P_r)}{P_0 sh} - \frac{(P_\infty^A)}{P_\infty} \frac{(P_\infty)}{P_0^ns} \right]. \]  

(6)

Equation (5) is the only semi-empirical equation for a configuration similar to the one under investigation which has been shown to have widespread agreement with experimental data over a great range of Mach numbers \( (1.5 \leq M \leq 14.0) \), angle of attack, and angle of sweep back at Reynolds numbers which are sufficiently high so the flow can be considered to be inviscid in nature (see Reference 26). Any departures from this form are probably due to viscous interaction, interference effects as a result of control surface deflections, or possible errors in the setting of the angle of attack of the wing during the experimental phase of the investigation.

Since the present interest involves the integrated force and moment coefficients, it is best to investigate the difference in the pressures over the upper and lower wing
surfaces rather than the individual pressure distribution over each wing surface. This form of presentation is justified by the fact that the difference in surface pressures appears as the integrand in the integrals of the force coefficients. Although only the surface pressure differences will be analyzed it must be recalled that in the case of the pitching moment and the hinge moment coefficients the integrand becomes the moment of the difference in the surface pressures about the pitch center and the hinge line, respectively. The difference in the surface pressures based on the blast wave theory is obtained from equation (1) and can be shown to be

$$\frac{\Delta p}{p_0} = \frac{1}{P_0} \left[ p_k(x) - p_u(x) \right] = n \alpha x^{1/3}$$  \hspace{1cm} (7)

where

$$n = 2\gamma f_2(\gamma) \left( \frac{p_0}{P_0} \right)^{\frac{2}{n_2}} M^2 (C_{D, N})^{1/3}$$  \hspace{1cm} (8)

The semi-empirical equation of Lee yields an equivalent expression obtained from equations (5) and (6) which states that

$$\frac{\Delta p}{p_0} = \frac{\Delta E_1}{(S_D)^{1/3}} + \zeta$$  \hspace{1cm} (9)

where $\zeta$ is a function of the free-stream Mach number and the difference in the wedge pressure ratio for the two wing surfaces and is
\[ \zeta = \left( \frac{P_{\infty}}{P_0} \right)_{ns} \left[ \left( \frac{P_\Delta}{P_\infty} \right)_{u} - \left( \frac{P_\Delta}{P_\infty} \right)_{u} \right] \]

the constant \( \Delta E_1 \) is simply

\[ \Delta E_1 = (E_{1u} - E_{1u}) = \left( \frac{C}{\beta} \right)^{2/3} \left\{ \left[ \left( \frac{P_{\infty}}{P_0} \right)_{sh} - \left( \frac{P_{\infty}}{P_0} \right)_{sh} \right] - \zeta \right\}. \]

It is important to notice the difference in the forms of equation (7) and equation (9). As the surface distance approaches infinity, the blast wave theory in the form of equation (7) states that the surface pressure difference tends to zero as an asymptote; this does not agree with the actual physical phenomenon. In Lee's semi-empirical equation as the surface distance approaches infinity, the surface pressure difference approaches the wedge pressure difference as an asymptote which is in agreement with the actual observations of such phenomenon. For this reason equation (9) should give slightly better results than equation (7) when the Reynolds number is high enough so the flow can be considered to be inviscid.

In order to make use of equations (5) and (9), some knowledge is required of the wedge pressure ratio and the shoulder pressure ratio. The wedge pressure ratio for the compression surface can be obtained from the free-stream Mach number and the angle of attack along with the oblique shock wave relationship. For the expansion surface of the
wing, the wedge pressure ratio can be obtained from the Prandtl-Meyer relation provided the free-stream Mach number and the angle of attack are known. The shoulder pressure ratio as a function of the angle of attack was obtained from the equation given in Reference 20 for the surface pressures over a two-dimensional cylinder. This particular expression is discussed in greater detail in the next section. Re-written in a more useable form it was found to be

$$\left(\frac{P_{sh}}{P_0}\right) = 0.320 - 0.455 \sin \alpha - 0.195 \cos 2 \alpha$$

$$+ 0.035 \sin 3 \alpha - 0.005 \cos 4 \alpha .$$

(12)

In order to insure that equation (12) is valid when a two-dimensional semi-cylindrical cap is followed by a flat plate it is most desirable to compare it with experimental data. In this particular investigation, the shoulder pressure could not be measured directly since the size of the model did not permit the placement of pressure orifices in the vicinity of the shoulder. Because of this fact, it became necessary to obtain the shoulder pressures by extrapolation of the downstream pressure distribution back to the shoulder. This was carried out on a computer by the method of least squares. A similar process was applied to the data given in References 19 and 28. The results of these calculations are shown in Figure 6 where
they are compared with equation (12). The agreement between the extrapolated data points and equation (12) is considered to be reasonably good.

Because of the complex trigometric form of equation (12) it is desirable to have a somewhat simplified form of this expression for purposes of performing "hand" calculations. As a first approximation, the shoulder pressure ratio can be assumed to vary linearly with angle of attack such that

\[
\left( \frac{P}{P_0} \right)_{sh} = 0.00523 \alpha + 0.1249 .
\]  \hspace{1cm} (13)

From Figure 6 it can be seen that equation (13) is a good approximation of the shoulder pressure ratio in the angle of attack range \(-16^\circ \leq \alpha \leq +8^\circ\), however as the angle of attack is increased beyond this range the deviation between equations (12) and (13) become significant and equation (12) should be used exclusively.

Some typical surface pressure differences for the non-deflected control surface case are shown in Figures 7 through 10. They are at angles of attack of \(+8^\circ\), \(+4^\circ\), \(-4^\circ\), and \(-8^\circ\), respectively, and are for a Mach number of 13.93 at a Reynolds number, based on the leading edge thickness, of approximately 3750. Also superimposed on these figures are the blast wave theory given by equation (7) and Lee's semi-empirical equation in the form of equation (9). For each angle of attack, equation (9) agrees very well with the
experimental data in the range of surface distance of $1.78 \leq \frac{S}{D} \leq 5.88$. In the same region of the surface distance, the blast wave theory predicts a slightly lower value of the difference in the surface pressure ($\Delta p/p_\infty$) at $\alpha > 0^\circ$ and it predicts a somewhat higher value of $\Delta p/p_\infty$ than the experimental data when $\alpha < 0^\circ$.

In general, it can be stated that for values of $S/D > 5.88$ equation (9) agrees reasonably well with the data at the higher angles of attack while equation (7) appears to agree best at the lower angles of attack. This fact is not too serious a problem since the integrated force and moment coefficients are not greatly affected as shall be shown in the sections to follow. The general conclusion to be drawn from the data presented in Figures 7 through 10 is that when the angle of attack is positive, the values of $\Delta p/p_\infty$ predicted by equation (9) are generally higher than the experimental points while that predicted by equation (7) are lower. For the case where the angles of attack are negative, the trends are reversed, i.e., the blast wave theory predicts a value of $\Delta p/p_\infty$ which is higher than the experimental data while Lee's semi-empirical equation predicts values which are slightly lower.
B. Comparison of Force and Moment Coefficients with Those Obtained from the Blast Wave Theory and a Modified Blast Wave Equation

1. The Equations Used in Integrating the Pressure Distributions. - The purpose of this section is to outline the method used to obtain the two-dimensional coefficients of lift, pressure drag, pitching moment about the mid-chord point, control surface hinge moment, lift-to-drag ratio, and the non-dimensionalized location of the center of pressure. In the resulting equations, both the drag coefficient and the lift-to-drag ratio are based on pressure drag alone.

The two basic assumptions made in the derivation are:

1) The force and moment coefficients were based on the pressure distribution over a configuration which consisted of three parts. They are the semi-cylindrical leading edge, the wing section, and the control surface. (2) A discontinuity occurs in the surface pressures at the upstream shoulder. This assumption has been verified from experimental data of the pressure distribution over a semi-cylindrical leading edge followed by a flat plate afterbody. The integrations were carried out with these assumptions in mind.

Since the model could not be equipped with pressure orifices in the semi-cylindrical leading edge, it became necessary to obtain the surface pressures in the region $0 \leq S/D \leq 0.7854$ by other means. A literature survey indicated
that the Gregorek-Korkan\textsuperscript{20} equation could be used in the nose region with greater accuracy than any other method for a cylindrical section. As an example, if Newtonian theory is used, the integrated pressure drag coefficient at zero degrees angle of attack is approximately 7\% higher than the corresponding coefficient based on the integration of the Gregorek-Korkan equation. Reference 20 indicates the pressure distribution over such a configuration is independent of Mach number and Reynolds number in the Mach number range of 8 to 15 and the Reynolds number range $2000 \leq R_{eD} \leq 75,000$. In addition, Gregorek and Korkan noticed the experimental pressure distribution could be described by a Fourier series of the cosine form. The resulting expression was used in the form

$$p(\theta) = p_0^c \sum_{n=0}^{4} A_n \cos n\theta \quad 0 \leq \theta \leq \pi$$

(14)

where $\theta$ is the angle between the free-stream velocity vector and the local pressure vector for zero degrees angle of attack. The value of the Fourier coefficients as determined by comparison with experimental data are tabulated below.

<table>
<thead>
<tr>
<th>$n$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_n$</td>
<td>0.320</td>
<td>0.455</td>
<td>0.195</td>
<td>0.035</td>
<td>-0.005</td>
</tr>
</tbody>
</table>

Two sets of general equations were derived depending upon the location of the control surface with respect to the
wing. In each case, the equations for force and moment coefficients are dependent on the specific geometry of the configuration under investigation. Because of the lengthy mathematical manipulations, only the final results are presented.

a. The Wing with a Trailing Edge Control

The integrated two-dimensional coefficients for the lift, pressure drag, pitching moment, and hinge moment, respectively, for the wing with the trailing edge control were found to be of the form

\[ C_L = \frac{1}{q_\infty c} \left\{ \int \frac{\cos \alpha}{\cos \delta} \right\} \]

\[ + \cos \alpha \int_{D/2}^{C_w} \Delta p \, dx + \cos (\alpha + \delta) \int_{C_w}^{C_w + C_f \cos \delta} \Delta p \, dx \]  

(15)

\[ C_D = \frac{1}{q_\infty c} \left\{ \int \frac{\sin (\alpha + \delta)}{\cos \delta} \right\} \]

\[ + \sin \alpha \int_{D/2}^{C_w} \Delta p \, dx + \sin (\alpha + \delta) \int_{C_w}^{C_w + C_f \cos \delta} \Delta p \, dx \]  

(16)


\[ C_{m_{12}} = \frac{1}{q_{\infty} C_{z}} \left\{ \frac{D}{2} \sum_{n=0}^{4} \frac{d_{\alpha} \sin \alpha}{\alpha} + \left[ \frac{C}{2} \right] + \left\{ C_{w} \tan \delta \right\} \right\} \]

\[ + (C_{f-h}) \sin \delta \left\{ \tan \delta \right\} \int_{C_{w}}^{C_{w}+C_{f} \cos \delta} \Delta p \, dx \]

\[ + \int_{D \Delta p}^{\frac{C}{2} \cos \delta} \Delta p \left( \frac{C_{w}-x}{\tan \delta} \right) \int_{C_{w}}^{C_{w}+C_{f} \cos \delta} \Delta p \, dx \]

(17)

\[ C_{H} = \frac{1}{q_{\infty} C_{f} \cos \delta} \int_{C_{w}}^{C_{w}+C_{f} \cos \delta} \Delta p \left[ (C_{f-h}) + \frac{1}{\cos \delta} (C_{w}-x) \right] \, dx \]

(18)

In equation (15) and equation (16), the first two terms on the right side of both equations are Fourier series in the sine and cosine of the angle of attack and are the contribution of the semi-cylindrical leading edge of the wing to the lift and drag coefficient, respectively. These terms result from the application of equation (14) to a wing at angle of attack. The third term on the right side of both equations is the contribution of the wing \((0.785 \leq S/D \leq 12.384)\) to the respective lift and drag coefficients while the fourth term is the contribution of the control surface to these same coefficients.
In equation (17), the Fourier series is the contribution of the semi-cylindrical leading edge section from \(0 \leq S/D \leq 0.785\) to the pitching moment coefficient. The second and fourth terms are the contribution of the control surface while the third term is the contribution of the wing in the range \(0.785 \leq S/D \leq 12.384\) to this same pitching moment coefficient.

In the above expressions

\[
\Delta p = p_\Delta (x) - p_u(x) = f(x) \quad ;
\]

the new Fourier coefficients are

\[
d_{1n} = A_n \int_0^{\pi/2} \sin n \xi \sin \xi \, d\xi
\]

and

\[
f_{1n} + A_n \int_0^{\pi/2} \cos n \xi \cos \xi \, d\xi
\]

These coefficients come about where equation (14) is applied to the wing at angle of attack. The new Fourier coefficients expressed in terms of the Gregorek-Korkan Fourier coefficients are tabulated below.

<table>
<thead>
<tr>
<th>(n\rightarrow)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_{1n})</td>
<td>0</td>
<td>(\pi/4 A_1)</td>
<td>(2/3 A_2)</td>
<td>0</td>
<td>(-4/15 A_4)</td>
</tr>
<tr>
<td>(f_{1n})</td>
<td>(A_0)</td>
<td>(\pi/4 A_1)</td>
<td>(1/3 A_2)</td>
<td>0</td>
<td>(-4/15 A_4)</td>
</tr>
</tbody>
</table>
The dynamic pressure \( q_\infty \) used in the above equations was based on the free-stream condition at the nose of the model and was corrected for the excitation of the vibrational degree of freedom of the test medium. The correction was applied in accordance with the method described in Reference 1.

At this point, it is most desirable to return to the equation for the pressure drag coefficient and consider the special case where \( \alpha = \delta = 0^\circ \). Under these conditions, equation (16) reduces to

\[
C_D = \frac{P_0 \cdot D}{q_\infty C} \sum_{n=0}^{4} f_{1n}
\]

where the drag coefficient is referred to the wing chord. Re-written, referred to the wing thickness \( D \) and replacing the dynamic pressure by its Mach number equivalent, the above function becomes

\[
C_{DN} = \frac{2}{\gamma} \left( \frac{P_\infty}{P_0} \right) (T_0) \frac{M^2}{(\gamma+1) M^2} \left( \frac{P_\infty}{P_0} \right)_{ns} \sum_{n=0}^{4} f_{1n}
\]

where

\[
\left( \frac{P_\infty}{P_0} \right)_{ns} = M^2 \left[ \frac{2}{(\gamma+1) M^2} \right]^{\gamma / \gamma - 1} \left[ \frac{2 \gamma}{\gamma+1} M^2 - \frac{\gamma-1}{\gamma+1} \right]^{1 / \gamma - 1}
\]

(23)
At hypersonic Mach numbers \((2\gamma/(\gamma+1)) M^2 \gg \gamma - 1/\gamma + 1\), therefore, the term \(\gamma - 1/\gamma + 1\) in equation (23) can be neglected. When the resulting form of equation (23) is substituted into equation (22) it becomes

\[
C_{DN} = \frac{2 \sum_{n=0}^{4} f_{1n}}{\frac{\phi_{p_0}}{p_0}} \left[ \frac{1}{\gamma} \left( \frac{\gamma+1}{2} \right)^{1/\gamma-1} \right]. \tag{24}
\]

Equation (24) states the leading edge pressure drag coefficient at \(\alpha = \delta = 0^\circ\) is independent of Mach number and is only slightly a function of the state of the gas. Assuming that the vibrational degree of freedom is not excited, then \(\phi_{p_0}/p_0(T_0)\) is unity and equation (24) gives a drag coefficient of 1.349 as compared with the Newtonian value of 1.445 for a two-dimensional semi-cylindrical section and values as determined by Gregorek and Korkan\(^{20}\) for a two-dimensional cylinder of 1.318 for a Mach number of 13.87 and 1.385 for the limiting Mach number condition for a cylindrical cap. Equation (24) was used for the evaluation of the surface pressure difference as determined by Kholyavko's blast wave theory in the form of equation (7).

b. The Wing with a Leading Edge Control

For the control surface treated as a leading edge control, the integrated two-dimensional force and moment coefficients become
\[
C_L = \frac{1}{q_\infty C} \left\{ p_0 D \left[ \cos (\alpha + \delta) \sum_{n = 0}^{4} d_{ln} \sin n (\alpha + \delta) \right. \right. \\
\left. \left. - \sin (\alpha + \delta) \sum_{n = 0}^{4} f_{ln} \cos n (\alpha + \delta) \right] + \cos (\alpha + \delta) \int_{D/2}^{C_F} \frac{C_f}{\Delta p} \, dx \right\} \\
+ \frac{\cos \alpha}{\cos \delta} \int_{\Delta p}^{C_F+C_W \cos \delta} \frac{C_f}{\Delta p} \, dx
\]

\[
C_D = \frac{1}{q_\infty C} \left\{ p_0 D \left[ \cos (\alpha + \delta) \sum_{n = 0}^{4} f_{ln} \cos n (\alpha + \delta) \right. \right. \\
\left. \left. + \sin (\alpha + \delta) \sum_{n = 0}^{4} d_{ln} \sin n (\alpha + \delta) \right] + \sin (\alpha + \delta) \int_{D/2}^{C_F} \frac{C_f}{\Delta p} \, dx \right\} \\
+ \frac{\sin \alpha}{\cos \delta} \int_{\Delta p}^{C_F+C_W \cos \delta} \frac{C_f}{\Delta p} \, dx
\]
\[
C_{m1/2} = \frac{1}{q_\infty C^2} \left\{ p_0 D \left\{ \left( C_w - \frac{C_f}{2} \right) \cos \delta + C_f \frac{D}{2} \right\} \sum_{n=0}^{4} d_{1n} \sin n (\alpha + \delta) \right. \\
- \left\{ \left( C_w - \frac{C_f}{2} \right) \sin \delta + (C_f - h) \tan \delta \right\} \sum_{n=0}^{4} f_{1n} \cos n (\alpha + \delta) \right\} \\
+ \int_{\Delta p} \left[ \left( C_w - \frac{C_f}{2} \right) \cos \delta + C_f - x \right] dx + \frac{1}{C_f + C_w} \sin \delta \\
\left. + \frac{1}{C_f} (C_f - x) \right] dx \right\} \\
\text{and } C_H = \frac{1}{q_\infty C^2} \left\{ p_0 D (h - \frac{D}{2}) \sum_{n=0}^{4} d_{1n} \sin n (\alpha + \delta) + \int_{\Delta p} (h - x) dx \right\}
\] (27)

In equation (25) and equation (26), the first two terms on the right side of each equation are the contribution of the semi-cylindrical leading edge to the lift and drag coefficient, respectively. In both cases, the third term is the contribution of the forward control surface to these same force coefficients while the fourth term in both equations is the contribution of the wing to both the lift and drag coefficients.

In the case of the pitching moment coefficient, the term breakdown can be seen in equation (27). The contribution
of the leading edge to the pitching moment coefficient is the two Fourier series terms in equation (27). The third term on the right side is the contribution of the leading edge control surface to this same moment coefficient while the fourth term is the contribution of the wing to the pitching moment coefficient.

Equation (28) is the hinge moment coefficient for the wing with a leading edge control surface. It is made up of two terms; the first term is a Fourier series in the effective angle of attack \( \alpha_{\text{eff}} = \alpha + \delta \) which is the contribution of the leading edge to the hinge moment coefficient while the second term is the contribution of the remaining control surface to this same hinge moment coefficient.

In each case, the lift-to-drag ratio \((L/D)\) was obtained by dividing the coefficient of lift by the pressure drag coefficient. The non-dimensionalized location of the center of pressure \((X_{cp}/c)\) was obtained by taking the ratio of the pitching moment about the mid-chord point to the product of the wing chord and the lift coefficient. The use of the lift coefficient in place of the normal force coefficient in computing the center of pressure location is justified by the fact that all the data was taken at relatively small angles of attack.

2. The Equations Obtained from the Blast Wave Theory

The force and moment coefficients resulting from the blast wave theory were obtained on the basis of equation (1).
Upon substitution of equation (7) into equations (15) through (18) for \( \delta = 0^\circ \), the force and moment coefficients based on the blast wave theory are

\[
C_L = \frac{1}{q_\infty C} \left\{ \frac{p_0 D}{q_\infty C^2} \left[ \cos \alpha \sum \frac{d_{1n}}{n!} \sin n \alpha - \sin \alpha \sum \frac{f_{1n}}{n!} \cos n \alpha \right] \right. \\
\left. + \frac{3}{2n} \left[ C^{2/3} - (\frac{D}{2})^{2/3} \right] \alpha \cos \alpha \right\} 
\]

(29)

\[
C_D = \frac{1}{q_\infty C} \left\{ \frac{p_0 D}{q_\infty C^2} \left[ \cos \alpha \sum \frac{f_{1n}}{n!} \cos n \alpha + \sin \alpha \sum \frac{d_{1n}}{n!} \sin n \alpha \right] \right. \\
\left. + \frac{3}{2n} \left[ C^{2/3} - (\frac{D}{2})^{2/3} \right] \alpha \sin \alpha \right\} 
\]

(30)

\[
C_{My} = \frac{1}{q_\infty C^2} \left\{ \frac{p_0 D}{q_\infty C^2} D(C-D) \sum \frac{d_{1n}}{n!} \sin n \alpha + 3n \left( \frac{C}{4} \left[ C^{2/3} - (\frac{D}{2})^{2/3} \right] \\
- \frac{1}{5} \left[ C^{5/3} - (\frac{D}{2})^{5/3} \right] \right) \alpha \right\} 
\]

(31)

\[
C_H (\text{rear}) = \frac{3n}{q_\infty C^2 f} \left\{ \frac{1}{2} (C-h) \left( C^{2/3} - C_w^{2/3} \right) - \frac{1}{5} \left( C^{5/3} - C_w^{5/3} \right) \right\} \alpha . 
\]

(32)

The hinge moment coefficient for the leading edge control surface is obtained by taking \( \delta = 0^\circ \) and introducing equation (7) into equation (28) yielding
The Equations Obtained from a Modified Blast Wave Equation

The force and moment coefficients based on Lee's semi-empirical equation for the surface pressure distribution as given by equation (5) are obtained by introducing equation (9) into equations (15) through (18) for \( \delta = 0^\circ \). This leads to the following expressions

\[
C_T = \frac{1}{\rho_\infty C_f} \left\{ \sum_{n=0}^{\infty} d_{1n} \sin n \alpha + 3n \left[ \frac{h}{2} \left( \frac{C_f^2}{(D/2)^2} \right) \right] \right\}
\]

(33)

3. The Equations Obtained from a Modified Blast Wave Equation

The force and moment coefficients based on Lee's semi-empirical equation for the surface pressure distribution as given by equation (5) are obtained by introducing equation (9) into equations (15) through (18) for \( \delta = 0^\circ \). This leads to the following expressions

\[
C_L = \frac{D_0^2 \rho \cdot D}{\rho_\infty C} \left\{ \cos \alpha \left[ g_1 (AE_1) + g_2 \zeta + \sum_{n=0}^{\infty} d_{1n} \sin n \alpha \right] - \sin \alpha \sum_{n=0}^{\infty} f_{1n} \cos n \alpha \right\}
\]

(34)

where \( \zeta \) and \( AE_1 \) are functions of the angle of attack and Mach number and are given by equations (10) and (11). The geometric constants in equation (34) are

\[
g_1 = 3 \left( \sqrt[3]{\frac{C}{D}} - \sqrt[3]{\frac{1}{2}} \right)
\]

(35)

\[
g_2 = \left( \frac{C}{D} - \frac{1}{2} \right)
\]
The pressure drag coefficient becomes

$$C_D = \frac{D^2 \rho}{q \infty C} \left\{ \cos \alpha \sum_{n=0}^{4} \ln \cos n \alpha + \sin \alpha \left[ g_1 (\Delta E_1) + g_2 \right] + \sum_{n=0}^{4} d_{1n} \sin n \alpha \right\}$$

(36)

and the pitching moment coefficient about the mid-chord point was

$$C_{ml/2} = \frac{D^2 \rho}{q \infty C^2} \left\{ h_1 (\Delta E_1) + h_2 \xi + \frac{1}{2} (C-D) \sum_{n=0}^{4} d_{1n} \sin n \alpha \right\}$$

(37)

where the new geometric constants are

$$h_1 = \frac{1}{2} C g_1 - \frac{3}{4} D \left( \sqrt[4]{\frac{C}{D}} - \sqrt[4]{\frac{1}{2}} \right)$$

$$h_2 = \frac{1}{2} C g_2 - \frac{1}{2} D \left[ \left( \frac{C}{D} \right)^2 - \left( \frac{1}{2} \right)^2 \right]$$

(38)

The hinge moment coefficient for the trailing edge control location is

$$C_H (\text{rear}) = \frac{D^2 \rho}{q \infty C_F} \left\{ F_1 (\Delta E_1) + F_2 \xi \right\}$$

(39)

with the geometric constants given as
\[ \kappa_1 = 3 (C-h) \left( \sqrt[3]{\frac{C}{D}} - \sqrt[3]{\frac{C}{W}} \right) - \frac{3D}{4} \left[ \sqrt[4]{\frac{C}{D}} - \sqrt[4]{\frac{C}{W}} \right]^4 \]

\[ \kappa_2 = (C-h) \left( \frac{C}{D} - \frac{C}{W} \right) - \frac{1}{2}D \left[ \left( \frac{C}{D} \right)^2 - \left( \frac{C}{W} \right)^2 \right] \]

The hinge moment coefficient for the control surface at the leading edge was obtained by letting \( \delta = 0^\circ \) and substituting equation (5) into equation (28) which becomes

\[ C_H(\text{forward}) = \frac{P\delta D}{q\omega C_f} \left\{ m_1 (\Delta E_1) + m_2 \zeta + \left(h-\frac{1}{2}D\right) \sum_{n=0}^{4} d_1 n \sin n \alpha \right\} \]

where the geometric constants are

\[ m_1 = 3h \left( \sqrt[3]{\frac{C_f}{D}} - \sqrt[3]{\frac{1}{2}} \right) - \frac{3D}{4} \left( \sqrt[4]{\frac{C_f}{D}} - \sqrt[4]{\frac{1}{2}} \right)^4 \]

\[ m_2 = h \left( \frac{C_f}{D} - \frac{1}{2} \right) - \frac{1}{2}D \left[ \left( \frac{C_f}{D} \right)^2 - \left( \frac{1}{2} \right)^2 \right] \]

With the above equations formulated for a wing whose flow field is fully attached \( (\delta = 0^\circ) \), it is now possible to compare the integrated force and moment coefficients obtained experimentally with those obtained by integration of the pressure distributions given by the blast wave theory and Lee's semi-empirical relationship for the pressure distribution. The experimental pressure distributions obtained for the wing with both a trailing edge and a leading edge control surfaces were integrated to obtain the related
force and moment coefficients. The equations used were equation (15) through (18) for the trailing edge control surface and equation (25) through (28) for the leading edge control surface data. The experimental force and moment coefficients were then compared with those obtained by integration of Kholyavko's blast wave theory for the pressure distribution as given by equation (1) and Lee's semi-empirical equation for the pressure distribution based on the shoulder conditions given by equation (5). This comparison is presented in the following sections.

4. The Lift Coefficient. - The lift coefficient as a function of angle of attack for $\delta = 0^\circ$ with the control surface location as a parameter is presented in Figure 11. The data is for a Mach number of 13.86 and a Reynolds number of approximately 3909. Superimposed is the lift coefficient obtained from the blast wave theory and Lee's semi-empirical equation in the form of equation (34).

The data indicates the lift coefficient is independent of the location of the control surface with respect to the wing. It also indicates the blast wave theory predicts a lower lift coefficient than the experimental data. Equation (34), on the other hand, agrees very well with the experimental data. This results from the fact that equation (9) agrees reasonably well with the experimental surface pressure differences as was previously shown.
In general, it can be stated that equation (34) and the experimentally obtained lift coefficient at these test conditions agree reasonably well.

5. The Drag Coefficient. - The drag coefficient as a function of angle of attack and control surface location is shown in Figure 12 for $M = 13.86$, $Re_D = 3900$, and $\delta = 0^\circ$. The experimental data indicates that there is a negligible difference in the data with reference to control surface location. The leading edge control surface having the slightly higher drag throughout the angle of attack range of the investigation. The difference in these experimental data varies from 2% to a maximum of approximately 6%. At $\alpha = \delta = 0^\circ$, both configurations should yield the same value for the pressure drag coefficient. As can be seen from Figure 12 this is not the case. The drag coefficient for the leading edge control is about 2% greater than that for the trailing edge location. Although these differences are within the experimental accuracy of the tests, it is also conceivable that they may be a result of cross-flow through the control surface gap which is located in a region of high pressure gradient for the leading edge control and in a region of low pressure gradient for the trailing edge control.

The interesting point is that equation (36) agrees reasonably well with both sets of experimental data while equation (30) yields a lower value of the drag coefficient
than either set of experimental data over the range of angle of attack of $-12^\circ \leq \alpha \leq 12^\circ$.

6. The Lift-To-Drag Ratio. - Figure 13 is a plot of L/D versus the angle of attack with control surface location as a parameter. Also shown are the results of blast wave theory calculations based on equations (29) and (30) and Lee's semi-empirical calculations resulting from the use of equations (34) and (36). The value of L/D is independent of control surface location; in addition, the calculations based on the semi-empirical equation appear to agree much better with the experimental data than the blast wave theory. The calculations based on equations (29) and (30) yield lift-to-drag ratios which are somewhat lower than both the semi-empirical calculations and the experimental data.

7. The Pitching Moment Coefficient. - The pitching moment coefficient is given in Figure 14. As can be seen, there is a very slight difference in the pitching moment coefficient as a function of control surface location in the range of angle of attack given as $-12^\circ \leq \alpha \leq 12^\circ$. The experimental data over this range of angle of attack exhibits a slight non-linearity. The blast wave theory does not predict this non-linearity even though it does predict a reasonable order of magnitude for the pitching moment in the range $-6^\circ \leq \alpha \leq 6^\circ$. The semi-empirical equation given by
equation (37) not only agrees very well with the experimental data but it also appears to predict the slight non-linearity at the higher angles of attack.

8. **Location of the Center of Pressure.** - Shown in Figure 15 is a plot of the location of the center of pressure for $\delta = 0^\circ$, $M = 13.86$, and $Re_D = 3909$. The experimental data shows that the center of pressure location is relatively independent of control surface location and angle of attack. Its average value is approximately 7% of the chord length ahead of the mid-chord point of the wing. It should be noted that the blast wave theory as given by equations (29) and (31) agrees reasonably well with the experimental data while the semi-empirical equation calculated on the basis of equations (34) and (37) yields a lower value of the center of pressure location of approximately 2.5% of the chord length ahead of the mid-chord point. The difference between the blast wave theory and the semi-empirical method yields a slightly lower pitching moment and a higher lift coefficient than the blast wave theory.

9. **The Hinge Moment Coefficient.** - The hinge moment coefficient for $\delta = 0^\circ$ is presented in Figure 16 as a function of angle of attack with the control surface location as a parameter. The experimental data indicates that the leading edge control has a positive slope of $\partial C_H/\partial \alpha = 0.0845$ per degree while the trailing edge control yields a negative
slope of $\frac{\partial C_H}{\partial \alpha} = -0.00363$ per degree. Furthermore, at any given angle of attack the leading edge control yields a higher hinge moment coefficient than the trailing edge control. This is attributed to the fact that the leading edge control is located in the region where blast wave effect is predominant. In this region, the pressure difference between the compression and expansion surface is greater than that for the trailing edge control location. Because of this, the leading edge control has the higher hinge moment.

Superimposed on these curves are the hinge moment coefficients calculated from the blast wave theory as given by equations (32) and (33) and the semi-empirical method given by equations (39) and (41), respectively. For both the trailing edge and the leading edge control surface, the blast wave theory is slightly lower than the experimental data. The semi-empirical technique agrees very well with the experimental data.

The data for attached flow ($\delta = 0^\circ$) is summarized in Table 2. It is concluded that the force and moment coefficients as calculated from the blast wave theory predicts values which are low compared to the experimental data. The only exception is the location of the center of pressure. The blast wave theory yields a center of pressure location somewhat greater than the experimental data. The coefficients calculated based on the semi-empirical pressure distribution given by equation (5) result in a lift coefficient and a
lift-to-drag ratio slightly higher than the experimental data. All other force and moment coefficients based on equation (5) agree very well with experiment. Finally, the semi-empirical method predicts a center of pressure location lower than that indicated by experiment.
IV. COUPLING OF FLOW FIELDS BY THE CROSS-FLOW EFFECT

A. A General Description of the Flow Field for the Trailing Edge Control

When designing aerodynamic control surfaces, the standard procedure is to incorporate a gap between the wing and the control surface in order to allow freedom of movement of the control. The presence of an open gap produces problems at hypersonic speeds, the problems resulting from such a gap are of great importance. It is the purpose of this section to indicate the changes which take place in the surface pressures when a cross-flow exists through such a gap.

The flow field "model" described below is based on surface pressure distributions obtained during this investigation with a wing equipped with a control surface which had an open gap.

The surface pressure data obtained in these tests can be categorized into two cases; (1) Case I - when $\alpha \geq 0^\circ$ at $\delta > 0^\circ$ and (2) Case II - when $\alpha \geq 0^\circ$ at $\delta < 0^\circ$. This is permissible since the flow fields for $\alpha \leq 0^\circ$ at $\delta < 0^\circ$ are similar in nature to Case I and those for $\alpha \leq 0^\circ$ at $\delta > 0^\circ$ are similar to Case II. These two cases are shown schematically in Figures 17 and 18,
respectively. The flow field within the laminar boundary layer in the presence of separation and reattachment is indicated in these diagrams but it shall not be discussed in detail since it has been adequately described by Lee and Reeves,8 Neilson et al.,9 and Holden.10 The discussion will be restricted to the surface pressures and how they are related to cross-flow through the gap.

Figure 17 is a schematic representation of the case \( \alpha \geq 0^\circ \) at \( \delta > 0^\circ \). However, in order to better understand the effect of cross-flow, it is necessary to consider two other conditions first. They are the flow fields for
\( \alpha > 0^\circ \) at \( \delta = 0^\circ \) and \( \alpha \geq 0^\circ \) at \( \delta > 0^\circ \) when the gap is sealed. These conditions are superimposed on the schematic.

When \( \alpha > 0^\circ \) at \( \delta = 0^\circ \), both surfaces of the wing exhibit a pressure distribution which is "blast-like" in form, i.e., the surface pressures decrease with increasing surface distance approximately as the two-thirds power of the surface distance. The pressure distribution for \( \alpha \geq 0^\circ \) at \( \delta = 0^\circ \) is used as the reference condition in the normalization of the data. This process will be discussed later.

Assuming the wing and the control surface are both displaced in the positive direction so \( \alpha \geq 0^\circ \) at \( \delta > 0^\circ \) with a sealed control surface gap, the point to be made is that the flow fields over the two surfaces are not coupled and can be treated separately. The compression surface supports a separated flow field while the
expansion surface exhibits a normal or attached flow field. Under these conditions, the compression surface exhibits an attached flow field in the region $0 \leq S/D \leq (S/D)_{sep}$ and at $S/D = (S/D)_{sep}$ the flow separates from the wing. This manifests itself as a pressure rise in the surface pressure distribution, indicating a change in flow direction resulting in a "separation shock wave" at this point over the wing. Practically every case investigated shows the separation pressure rise to be masked by the blast wave effect. Following the separation pressure rise and in the range $(S/D)_{sep} \leq S/D \leq (S/D)_{Hinge}$, the surface pressure continues to decrease very slightly with increasing surface distance; however, the pressures are higher than those for $\alpha \geq 0^\circ$ at $\delta = 0^\circ$. This region is sometimes referred to as the "dead air" region of the separated flow field and contains the circulation vortex. At approximately $S/D = (S/D)_{Hinge}$, the surface pressures exhibit a second pressure rise. Once again the flow direction is changed resulting in a second shock wave. This shock wave is usually called the "reattachment" shock since the flow reattaches itself to the wing in the general vicinity slightly up-stream of the base of this shock wave.

In view of the fact that the flow fields are uncoupled, the expansion surface pressure distribution can be considered separately. From $0 \leq S/D \leq (S/D)_{Hinge}$, the pressure decreases with increasing surface distance similar to the predictions of the blast wave theory. Slightly
up-stream of the control surface hinge line the flow field undergoes a further expansion due to control surface deflection resulting in a local expansion fan over the area of the hinge line. This region is shown in the surface pressure distribution as a sudden decrease in the pressures (a very favorable pressure gradient). Then as S/D is further increased, the surface pressures either continue to decrease slightly or become relatively constant as the trailing edge is approached.

In order to see this effect more clearly, the data were normalized with respect to the pressure distribution for $\alpha \geq 0^\circ$ at $\delta = 0^\circ$. It was normalized in such a manner so the local flow fields would appear as a percent change over and above the pressure distribution for $\alpha \geq 0^\circ$ at $\delta = 0^\circ$ (provided the angle of attack is the same for both pressure distributions). Mathematically, the normalization process can be written as

$$
\frac{\Delta \left(\frac{P_r}{P_0}\right)_{\delta = 0}}{\left(\frac{P_r}{P_0}\right)_{\delta = 0}} = \frac{\left(\frac{P_r}{P_0}\right)_{\delta \neq 0^\circ}}{\left(\frac{P_r}{P_0}\right)_{\delta = 0^\circ}} - 1. \quad (43)
$$

When this normalization process is applied to the compression side of the wing, the pressure distributions have the appearance of those typical of separated and reattached flow fields. When the same method is applied to the expansion surface, the flow field appears as a local
expansion fan in the vicinity of the control surface hinge line. These normalized curves are also shown schematically in Figure 17.

At this point, the actual configuration investigated can be considered, i.e., $\alpha > 0^\circ$ at $\delta > 0^\circ$ with an open control surface gap. Under these conditions, the flow fields over both surfaces of the wing are coupled by the cross-flow through the gap. Furthermore, the cross-flow direction is always from the compression side through the gap into the expansion side of the wing. This particular condition produces some interesting results with reference to the flow field over both wing surfaces.

In general, the cross-flow through the gap seems to alleviate the pressure levels over the compression surface as indicated in Figure 17. It is believed that this process moves the separation point downstream while the reattachment point moves upstream when compared with the sealed gap condition for the same angle of attack and control deflection.

The more interesting problem occurs over the expansion surface. Since these two flow fields are coupled by the so-called "cross-flow" effect, the expansion surface no longer exhibits a "pure" local expansion fan in the vicinity of the hinge line. The new flow field is more complex. In general, the experimental pressure distributions indicates that in the region from the leading edge to a point about five wing thicknesses downstream of the leading edge the pressures are similar to those when $\alpha > 0^\circ$ at $\delta = 0^\circ$.
Beyond this point, the local pressures are higher than those at \( \alpha \geq 0^\circ \) at \( \delta = 0^\circ \) in the same region of the wing. This higher pressure level extends to a location slightly upstream of the gap position at which point the pressure distribution decreases because of the local expansion fan due to control deflection. Following this region, the pressures over the control surface either gradually decrease or become relatively constant until the trailing edge is reached. This condition is shown in Figure 17 both as a standard pressure plot and in its normalized form. The second method of presentation shows more clearly the cross-flow effect over the expansion surface. The explanation of the flow field over the expansion side due to this coupling cannot be too definitive since there is an insufficient amount of information with regard to the physical phenomenon taking place. Flow visualization techniques with silicone oils, temperature sensitive paints, and graphite tuff revealed no significant or enlightening information. Based on the pressure distributions and technical judgment, it can only be postulated that one of two possible phenomena must result. Either the cross-flow through the gap acts as a perturbation on the laminar boundary layer over the expansion surface causing it to grow at a rate other than the surface distance to the one-half power, or if the cross-flow is sufficiently strong it can conceivably produce a local region of either
isentropic compression or separation ahead of the gap location. It should be mentioned that in all probability the flow is not separated in the region of the control surface as evidenced by the fact that the pressure gradient in this region is a favorable one.

As an example, consider Figure 19 where the curves drawn through the experimental data are for a Mach number of 13.93 at a Reynolds number of 3684 and the angle of attack is positive at four degrees. Superimposed are the cases where the control surface is not deflected and where $\delta = +10^\circ$. When the control surface is deflected to $\delta = +10^\circ$, the compression surface undergoes separation and reattachment; however, the separation point is not detectable in this particular type of presentation. The reattachment pressure rise can be readily seen in the vicinity of the gap location. The corresponding normalized surface pressure distribution is given in Figure 20. In this case, the separation point and the separation pressure rise is still not well defined; however, it is believed to be at approximately $S/D = 7.0$.

The pressure distribution for $\alpha = +4^\circ$ at $\delta = +10^\circ$ over the expansion surface is shown in Figure 19. The effect of cross-flow can best be observed when this pressure distribution is compared with the distribution for $\alpha = 4^\circ$ at $\delta = 0^\circ$. The pressure distribution over the region $0 \leq S/D \leq 2.8$ is approximately similar for both
cases. Starting at S/D = 2.8, the pressures are higher for \( \alpha = +4^\circ \) at \( \delta = +10^\circ \) when compared to the condition \( \alpha = +4^\circ \) at \( \delta = 0^\circ \). The higher pressure level over the expansion surface in this region is attributed to the cross-flow effect. It can be seen better in the normalized plot of this same data as given in Figure 21. In the vicinity of S/D = 11.8, the pressure starts to decrease very rapidly indicating the presence of an expansion fan located near the gap at S/D = 12.8 after which the surface pressures continually decrease over the control surface until the trailing edge is reached.

It should be noted that the surface pressure over the compression surface is approximately 1.6 times that over the expansion side of the wing at the gap location. It is this pressure difference which produces the above-mentioned cross-flow.

Figures 24 and 25 represent typical normalized pressure distribution where \( \alpha \geq 0^\circ \) at \( \delta > 0^\circ \). Figure 24 shows the separated and reattached flow field on the compression side for positive angle of attack at various positive control surface deflections. Figure 25 shows the corresponding expansion surface pressure distributions as altered by the cross-flow effect when compared to the pressure distribution for \( \alpha = +4^\circ \) at \( \delta = 0^\circ \).

The second case to be considered is for \( \alpha \geq 0^\circ \) at \( \delta < 0^\circ \) which is shown schematically in Figure 18.
In general, it can be stated that the direction of the cross-flow depends upon the order of magnitude of the surface pressures in the vicinity of the control surface gap with relation to each other. In most of the cases where the angle of attack was positive and control surface deflections negative, it was found that the surface pressures at the gap location were greater over the lower side of the wing than on the upper surface. Under these conditions, separation and reattachment occurs over the upper surface and is due to both deflected control surface and cross-flow through the control surface gap. The effect of cross-flow under these conditions is manifested by movement of the separation point upstream and the reattachment point downstream as compared to the case where \( \alpha \geq 0^\circ \) at \( \delta < 0^\circ \) with a sealed gap. In addition, the separation point is masked even more than it was when \( \alpha \geq 0^\circ \) at \( \delta > 0^\circ \). This can be seen in Figure 20 and again in Figure 26 where the angle of attack is +4° for various negative control surface deflection angles. It is important to note that the separation pressure rise is completely void under this condition of cross-flow. Although there is no conclusive evidence, it is conceivable that separation under this particular condition of cross-flow may occur as far forward as the upstream shoulder of the wing; however, this is not believed to be the case.
It is of interest to notice from Figure 18 that the lower surface of the wing has a higher surface pressure resulting from the flow undergoing a compression as the streamline adjacent to the wing passes through the bow wave. This region is followed by a decrease in pressure near the hinge line due to the presence of a local expansion fan resulting from the control surface deflection. Since the flow is attached over this surface, the pressure distribution near the hinge line in the normalized form appears as a sudden decrease in local surface pressures and no upstream effects appear as in the case for $\alpha > 0^\circ$ at $\delta > 0^\circ$. The lack of this upstream effect is also seen in Figures 21 and 27. In both cases, the effect of the expansion fan does not make its appearance until $S/D = 11.8$ for each control surface deflection in the range $-12^\circ \leq \delta \leq -2^\circ$ at $\alpha = +4^\circ$.

In the previous discussions of Cases I and II with reference to cross-flow effects through an open gap the conditions chosen were for positive angles of attack since the cross-flow effect is greatest under these circumstances. In addition, all the previous discussions show the combined effects of angle of attack and control surface deflection on cross-flow and their related surface pressure distributions. In order to see the effect of control surface deflection alone on cross-flow, the case $\alpha = 0^\circ$ at various control surface deflections must be considered. These data are shown in Figures 22 and 23 for $\alpha = 0^\circ$ at
control surface deflections varying from $\delta = -2^\circ$ to $\delta = -12^\circ$ in two degree increments for $M = 13.79$ and $Re_D = 3808$. (The data for positive control surface deflection at zero degrees angle of attack is not presented since it is exactly the same as that at $\alpha = 0^\circ$ and $\delta < 0^\circ$ with the exception that the compression and expansion flow fields occur over the opposite wing surfaces.)

Figure 22 shows the normalized surface pressure distribution on the compression or upper surface of the wing. It indicates that the flow separates from the wing upstream of the hinge line and then reattaches at some point on the control surface. In addition as the control surface deflection is increased, the local pressure level also increases. For example, at $S/D = 14.0$ for $\delta = -2^\circ$ the local pressure is approximately 12% greater than the pressure at the same surface location for the condition $\alpha = \delta = 0^\circ$ while at $\delta = -12^\circ$ and $S/D = 14.0$ the local pressure is about 35% higher than the pressure at the same position when the wing is at $\alpha = \delta = 0^\circ$.

For $\alpha = 0^\circ$ at $\delta < 0^\circ$ the pressures on the upper surface of the wing at the gap location are greater than those over the lower surface at the same location; for this reason a cross-flow exists from the upper surface through the open gap into the lower surface. This cross-flow causes an upstream effect over the lower surface similar to that described previously, it can be seen in Figure 23.
where the normalized surface pressures are plotted versus the non-dimensionalized surface distance for $\alpha = 0^\circ$, $M = 13.79$, and $R_{eD} = 3808$ with the control surface deflection as a parameter. As shown in Figure 23 the effect of cross-flow due to control surface deflection alone can be felt as far upstream as $S/D = 2.8$ for all control surface deflections in the range $-2^\circ \leq \delta \leq -12^\circ$. The maximum difference in the local surface pressures due to cross-flow resulting from control surface deflection alone occurs at $S/D = 11.8$. For $\delta = -2^\circ$ the local pressure at this point is about 15% higher than the local pressure when $\alpha = \delta = 0^\circ$; for $\delta = -12^\circ$ the local pressure is approximately 25% greater than the local pressure when $\alpha = \delta = 0^\circ$. Based on these facts it can be concluded that the cross-flow effect due to control surface deflection alone can be significant.

B. A General Description of the Flow Field for the Leading Edge Control

A detailed description of the individual flow fields and their coupling due to cross-flow will not be carried out since the basic concept is similar to that given previously for the trailing edge control. However, it should be mentioned that the effects of cross-flow for the leading edge control based on the surface pressures at the location of the gap indicate that in most cases the separated flow
field is due to a combination of control surface deflection and cross-flow. Figures 28 through 31 show the normalized surface pressure as a function of surface distance for $M = 13.87$ and $Re_D = 4086$ at $a = +4^\circ$ for the control deflection range $-12^\circ \leq \delta \leq +12^\circ$. For $\delta = +12^\circ$ the lower surface pressure is about five times as great as the upper surface pressure at the gap location and it decreases with decreasing control surface deflection to a value of 2.5 times the value of the upper surface pressures at $\delta = 0^\circ$. It is this pressure difference which produces the previously mentioned cross-flow. Figures 28 and 30 show the normalized surface pressures for the separated surfaces for $\delta > 0^\circ$ and $\delta < 0^\circ$, respectively at $a = +4^\circ$. In both cases, it is significant to notice that the flow separates from the wing in the vicinity of the gap location and does not reattach. In other words, the wing behind the hinge (or gap) location is completely separated. It is also interesting to notice that separation is stronger for the negative control surface deflections since the flow direction is turned through a greater angle as compared to the case when the control is deflected in the positive direction. From Figure 30, it can be seen that this fact appears as a greater pressure rise for the same absolute value of the control deflection angle as compared with those in Figure 28.

The corresponding normalized expansion flow fields are presented in Figures 29 and 31 for $\delta > 0^\circ$ and $\delta < 0^\circ$, 
respectively. These curves simply indicate that at the higher control surface deflection the expansion fan located in the vicinity of the gap location is felt as far forward as the front shoulder. The effect downstream of the gap location is approximately the same for both \( \delta > 0^\circ \) and \( \delta < 0^\circ \), i.e., the percent decrease for a given absolute value of the control deflection as compared to the condition \( \alpha = +4^\circ \) at \( \delta = 0^\circ \) is about the same.

Summarizing the effect of cross-flow, it can be stated that

(1) For a wing with a trailing edge control at conditions of \( \alpha \geq 0^\circ \) at \( \delta > 0^\circ \), the cross-flow produces a small effect over the separated wing surface, the major effect is over the wing surface which undergoes an expansion type flow field. In particular, the pressure distribution upstream of the expansion fan is affected indicating either a boundary layer growth other than the familiar laminar boundary layer relationship of the surface distance to the one-half power or possibly some type of unexplained local separation.

(2) For a wing with a trailing edge control under the conditions \( \alpha \geq 0^\circ \) at \( \delta < 0^\circ \), the cross-flow effects combined with control deflection increases the degree of separation over one wing surface. The other wing surface undergoes a local expansion in the vicinity of the hinge
line. Furthermore, the separation point is completely masked by the blast wave plus cross-flow effects.

(3) When the wing is equipped with a leading edge control at conditions of $\alpha > 0^\circ$ at both $\delta > 0^\circ$ and $\delta < 0^\circ$, the cross-flow is such that it combines with control surface deflection to produce separation without re-attachment over one surface while the other surface undergoes an expansion type of flow.
V. THE EFFECT OF CONTROL SURFACE LOCATION ON THE FORCE AND MOMENT COEFFICIENTS

The purpose of this series of investigations was to determine the variation of the integrated force and moment coefficients with control surface deflection and control location. Of primary importance is the variation of the control effectiveness parameter ($\frac{\partial C_L}{\partial \delta}$), the pitching moment, the center of pressure location, and the control hinge moment with physical location of the control surface on the wing.

Since no theory exists which takes into account simultaneously such physical phenomena as cross-flow through an open control gap, separated and reattached local flow fields over one wing surface with local expansion flow fields in the vicinity of the hinge line over the other surface, the experimental data presented in this section are compared with inviscid theory based on equation (5).

A. Comparison of Typical Pressure Distribution with Inviscid Theory

Typical pressure distributions over a wing equipped with a trailing edge control surface for the case when the control surface is deflected are presented in Figures 32 through 35. Similar curves for the wing with a leading
edge control are presented in Figures 36 through 42.
As was done previously for the surface pressure distributions for the non-deflected control surface, these curves are presented in their non-dimensional form of the difference in the surface pressures of the upper and lower surfaces as a function of the normalized surface distance.

In the cases where the wing was investigated with a trailing edge control surface equation (5) was used to calculate the inviscid value of $\Delta p/p_0$ in the region $0.7854 \leq S/D \leq (S/D)_{Hinge}$. For the region which extends from $(S/D)_{Hinge} \leq S/D \leq 18.571$, a revised form of equation (5) must be used; namely,

$$\frac{\Delta p}{p_0} = \frac{\beta}{(S/D)^{2/3}} + \zeta \quad (44)$$

where the value of $\beta$ changes depending on whether the control surface is deflected in the positive or negative direction. For positive deflections of the control surface ($\delta > 0^\circ$), the value of $\beta$ becomes

$$\beta = \left(\frac{S}{D}\right)^{2/3} \left[\left\{\frac{E_{1/2}}{(S/D)^{2/3}} + \left(\frac{p_A}{p_\infty}\right)_{ns} \left(\frac{p_2}{p_1}\right) - \left(\frac{p_A}{p_t}\right)_{ns} \right\} \right] \quad (45)$$

and for negative control surface deflections ($\delta < 0^\circ$), it is
The pressure ratio \( \frac{p_2}{p_1} \) in both equations (45) and (46) is the pressure rise across the local shock wave due to control surface deflection and is calculated based on the local inviscid Mach number upstream of the hinge line and the control surface deflection angle plus the oblique shock relationship. The pressure ratio \( \frac{p}{p_{t_v}} \) in these same equations is calculated based on the pressure ratio upstream of the hinge line, the control surface deflection angle and the Prandtl-Meyer relationship since a local expansion fan occurs in the vicinity of the hinge line on the opposite wing surface from which a local shock wave appears.

Figure 32 presents the experimental data for the trailing edge control in the form \( \frac{\Delta p}{p_0} \) versus the normalized surface distance for a Mach number of 13.93 and a Reynolds number of 3698.0. The wing angle of attack is \( +4^\circ \) and its control surface deflection angle is \( +8^\circ \). Superimposed on the data is the inviscid theory computed from equations (5) and (44). In the range of surface distance \( 0.785 \leq S/D \leq 5.88 \), the experimental data and equation (5) agree reasonably well; however, as the surface distance is increased from \( S/D = 5.88 \), there is a
departure between the experimental points and the inviscid theory. This departure is attributed to the fact that the theory does not take into account the actual physical phenomena of combined flow separation and local expansion fans previously mentioned. In the region over the control surface the inviscid theory predicts a higher value of \( \Delta p/p_0 \) than the experimental data. This higher pressure difference is reflected in the integrated force and moment coefficients in that the inviscid theory gives force coefficients which are slightly higher than the experimental data. In addition, the inviscid theory yields a pitching moment which is greater negatively than the experimental data. For the same reason, the hinge moment based on equation (44) is also slightly greater negatively than that obtained experimentally.

Figure 33 is a similar curve as described above with the exception that the control surface deflection is +4° rather than +8° as was the case in Figure 32. As can be seen, the results are somewhat similar in that the inviscid theory and the experimental data agree reasonably well over the up-stream section of the wing; however, once the interactions of the separated flow field sets in, the usual departure between the data and the inviscid theory is observed. The greatest departure between the theory and the experimental data again takes place over the control surface.
In addition to comparing the experimental data with inviscid theory an attempt was made to compare a few points based on the Hankey-Cross method\textsuperscript{29} with both the experimental data and the inviscid calculations. In making this determination the pressure distribution over the compression surface was computed by the Hankey-Cross method\textsuperscript{29} which is a simplified version of the Lees-Reeves technique.\textsuperscript{8} The surface pressure distribution for the expansion surface was calculated by equation (9) over the wing surface and equation (44) for the control surface. The corresponding plots of $\Delta p/p_0$ versus surface distance for these two cases are shown in Figures 32 and 33, respectively. In both cases the normalized surface pressure difference agrees with the inviscid calculations in the range of surface distance $0.7854 \leq S/D \leq 10.78$. In the region of the hinge line the value of $\Delta p/p_0$ based on this method gives a different result when compared with the inviscid theory, in that the Hankey-Cross method gives a continuous function typical of viscous effects while the inviscid theory predicts a discontinuity at the hinge line. Over the control surface the value of $\Delta p/p_0$ as calculated by the Hankey-Cross method approaches the inviscid value as an asymptotic solution; in addition, it yields values of $\Delta p/p_0$ higher than the experimental data. It can be concluded from these two comparisons that the integrated force and moment coefficients based on this method should yield
results very near those obtained when integrating the
inviscid values of Δp/ρ₀ over the surface distance.
The fact that this is the case is shown in Figures 45, 
49, 52, 56, 59, 63, and 69 where the integrated lift, 
control effectiveness parameter, drag, lift-to-drag ratio, 
pitching moment, center of pressure location, and hinge 
moment coefficients are presented for the trailing edge 
control when α = +4° at δ = +4°, +8°, and +12°, respectively 
(shown as solid square points). The integrated coefficients 
resulting from the Hankey-Cross method (for the compression 
surface) agree very well with the coefficients obtained by 
integration of the inviscid theory and they also agree 
reasonably well with the experimental data. These results 
are considered to be fairly good in light of the fact that 
the Hankey-Cross method is a simplification of the Lees-
Reeves theory which in itself is not an exact solution.
It should be noted that neither of these two techniques 
take into account either the cross-flow effect or the local 
pressure gradients. Because of these differences more 
theoretical and experimental research is required in order 
to better understand the effects of control surface deflec-
tion on a separated flow field in the presence of a pressure 
gradient and cross-flow through an open control surface gap.

Figure 34 is a plot of Δp/ρ₀ versus the normalized 
surface distance for α = +4° and δ = 0° at a Mach number 
of 13.79 and a Reynolds number of 3815.0. In the range
0.7854 \leq S/D \leq 5.88, the experimental data are about 7.5% higher than the value of $\Delta p/p_0$ based on inviscid theory; however, at values of the normalized surface distance where $S/D > 5.88$, the experimental data falls below the inviscid theory. As an example at $S/D = 15.8$, the experimental point is about 14% lower than the theory predicts.

The case when the angle of attack of the wing is positive and its control surface is deflected in the negative direction is given in Figure 35. Here, the Mach number is 13.77 and the Reynolds number is 3829, while the angle of attack is $+4^\circ$ and the control surface is deflected negatively to $-4^\circ$. The trend previously noted is present once again. Over the leading edge of the wing, the inviscid theory and the experimental data agree reasonably well until the net effects of separated flows and local expansion fans, due to control surface deflection, become effective. This point seems to be at approximately $S/D \geq 6.00$; as the surface distance increases beyond $S/D = 6.0$, the deviation between the data and the inviscid theory also increases. At $S/D = 16.8$, the experimental data is approximately four times greater than the calculated inviscid theory at the same location.

For the wing with a leading edge control, the inviscid surface pressure differences were computed using equation (5) for the control surface ($0.7854 \leq S/D \leq 5.89$). In this
particular case, the value $E_1$, given by equation (6), must be evaluated at the "effective" angle of attack of the control surface which is the algebraic sum of the wing angle of attack and the control surface deflection angle. In computing the inviscid value of $\Delta p/p_0'$ over the wing surface ($5.89 \leq S/C \leq 16.781$), the equation used was equation (44) where the value of $\beta$ is obtained from equation (46) for all cases when the control surface deflection is positive ($\delta > 0^\circ$) and equation (45) was used when the control surface is deflected in the negative direction ($\delta < 0^\circ$). It should be noted that the use of equations (45) and (46), with respect to the direction of control surface deflection for the leading edge control, is reversed to that when the control surface is placed at the trailing edge of the wing. Calculations for the leading edge control location were carried out for a Mach number of 13.87 and a Reynolds number of approximately 4090.0 for a wing angle of attack of $+4^\circ$ at various control surface deflections. These results are compared with the experimental data in Figures 36 through 42 where the value $\Delta p/p_0'$ is plotted versus the normalized surface distance. Figures 36, 37, and 38 are for positive control surface deflection of $+12^\circ$, $+8^\circ$, and $+4^\circ$, respectively. The general trend is similar in each case. The inviscid theory agrees very well with the experimental data in the range of $0.7854 \leq S/D \leq 5.89$ at which point the interactions
due to control surface deflection become evident, then in the vicinity of the hinge line, there is a sudden decrease in $\Delta p/p_0$. In each case as the surface distance is increased from $S/D = 5.89$ to the trailing edge of the wing, both the theory and the data show a monotonic decrease in the value $\Delta p/p_0$; however, the experimental data is slightly greater in magnitude when compared with the inviscid theory.

The cases when the control surface is deflected in the negative direction are shown in Figures 40, 41, and 42 for control surface deflections of $-4^\circ$, $-8^\circ$, and $-12^\circ$, respectively. The Mach number, Reynolds number, and angle of attack are similar to those listed previously. These figures show some interesting trends, for example, when the wing angle of attack is $+4^\circ$ and the control surface is deflected to $-4^\circ$; the effective angle of attack of the control surface is zero. This fact is reflected in Figure 40 in that both the inviscid theory and the experimental data yield values of $\Delta p/p_0 = 0$ in the region from $S/D = 0.785$ to approximately $S/D = 3.8$ at which point the experimental value of $\Delta p/p_0$ increase with increasing surface distance until it becomes relatively constant over the wing surface. The data over the wing surface is much higher than the inviscid theory predicts.

Figures 41 and 42 indicate the variation of $\Delta p/p_0$ along the surface for $\delta = -8^\circ$ and $\delta = -12^\circ$, respectively.
In both cases the general trend of the theory and the experimental data agrees reasonably well, however, the experimental values of $\Delta p/p_0$ over the control surface are much higher than the inviscid theory. One possible explanation for this deviation is the method used to compute the inviscid curve. It may be possible that the shoulder conditions for the "effective" angle of attack of the control surface is not exactly correct thereby leading to an error in the calculation of the inviscid curve. The agreement between the theory and the data over the wing surface in the range $5.89 \leq S/D \leq 16.78$ is better for $\delta = -12^\circ$ than for $\delta = -8^\circ$. In both cases the experimental data are slightly higher than the values predicted by the inviscid theory.

In the sections that follow the coefficients obtained by integration of the experimental pressure distributions are presented and superimposed are the inviscid theory calculations. The theoretical values for the trailing edge control were obtained by using equation (9) in the integral involving the wing surface and equation (44) in the integrals for the trailing edge control surface provided the proper value of $\beta$ as given in equations (45) and (46) are used in equation (44). These expressions were substituted into equations (15) through (18) in order to arrive at the inviscid theory for the trailing edge control location. The inviscid theory for the leading
edge control was obtained in a somewhat similar manner, here equation (9) and equation (44), in their proper form, were introduced into equations (25) through (28).

B. The Lift Coefficient and Control Effectiveness

Figures 43, 44, and 45 are plots of the two-dimensional lift coefficient as a function of control deflection with control surface location as a parameter for \( \alpha = 0^\circ, +2^\circ, \) and \( +4^\circ, \) respectively and superimposed on these curves are the corresponding inviscid theory. At each angle of attack, the lift coefficient for the wing with the trailing edge control increases linearly with increasing control deflection angle. For the leading edge control location, the lift increases in a very slightly non-linear manner (even though it was drawn linear) with increasing control surface deflection. This non-linearity will be noticed later in the discussion of the control effectiveness parameter. At every angle of attack investigated, the wing with the leading edge control produces a greater lift than the wing equipped with a trailing edge control surface. For example, if one considers the case where the control surface deflection is held constant at \( \delta = +10^\circ, \) then for \( \alpha = 0^\circ \) the wing with the leading edge control has a lift coefficient which is about three times as great as that for the wing with the trailing edge control. At
\( \alpha = +2^\circ \) and \( \alpha = +4^\circ \) for \( \delta = +10^\circ \), the lift of the wing with the leading edge control is about twice as great as the wing with the trailing edge control.

At each angle of attack and control surface location the inviscid theory yields a lift coefficient slightly greater than the experimental data. As was shown previously, this can be attributed to the fact that the inviscid surface pressure difference \((\Delta p/p_0)\) as given by equations (5) and (44) usually predict a slightly higher value than that obtained experimentally.

The variation of angle of attack with control surface deflection for the zero lift condition is shown in Figure 46. This variation is a linear function; the angle of attack decreased with increasing control surface deflection.

\[
\left( \frac{\partial \alpha}{\partial \delta} \right)_{C_L = 0} = -0.432 \text{ for the wing with the leading edge control as compared to a theoretical value of} \]

\[
\left( \frac{\partial \alpha}{\partial \delta} \right)_{C_L = 0} = -0.512 \text{ and} \left( \frac{\partial \alpha}{\partial \delta} \right)_{C_L = 0} = -0.125 \text{ for a wing with a trailing edge control as compared to the inviscid theory which yields} \left( \frac{\partial \alpha}{\partial \delta} \right)_{C_L = 0} = -0.203.
\]

The most significant parameter in this series is the plot of \( \partial C_L / \partial \delta \) as a function of the control deflection angle for various angles of attack and control location. It is the so-called "control effectiveness parameter;" the greater it is, the more effective the surface is as an aerodynamic control. Figures 47, 48, and 49 are plots of the control effectiveness parameter for \( \alpha = 0^\circ, +2^\circ \), and \( +4^\circ \),
respectively. From these figures it becomes obvious that the wing with the leading edge control is more effective aerodynamically than that with a trailing edge control. This trend is valid for all the angles of attack investigated. In each case, the wing with the forward control is about five to six times more effective than the wing with the rear control location.

The control effectiveness parameter for the wing with the trailing edge control appears to be independent of both angle of attack and control deflection angle. Its average experimental value is approximately $\frac{\alpha C_L}{\alpha \delta} = 0.00126$ as compared with an average inviscid calculation of $\frac{\alpha C_L}{\alpha \delta} = 0.00222$.

From Reference 27, if it is assumed that a wing has an NACA 0009 airfoil section with a control surface located at the wing's trailing edge which has a chord length of 27% of the total wing chord, it has a subsonic control effectiveness of approximately $\frac{\alpha C_L}{\alpha \delta} = 0.0405$. Although the airfoil section is different than the configuration under investigation it can be used to indicate the difference in control effectiveness at subsonic speeds as compared to hypersonic flight conditions. As can be seen the subsonic value of control effectiveness is about thirty times as great as the hypersonic value. The control effectiveness parameter for the wing with the leading edge control is slightly non-linear with control surface deflection.
When the control surface is deflected at \( \delta = +10^\circ \), the control effectiveness has a value of \( 3C_L/3\delta = 0.0053 \) for \( \alpha = 0^\circ \) and \( +2^\circ \) as compared to a theoretical value of \( 3C_L/3\delta = 0.0075 \) at \( \alpha = 0^\circ \) and \( 3C_L/3\delta = 0.0079 \) at \( \alpha = +2^\circ \). At \( \alpha = +4^\circ \) for \( \delta = +10^\circ \), it increases to \( 3C_L/3\delta = 0.0082 \) while the inviscid theory yields a value of \( 3C_L/3\delta = 0.0113 \) for the same conditions.

C. The Pressure Drag Coefficient

The pressure drag coefficient as a function of control surface deflection and control location are shown in Figures 50 through 52 for \( \alpha = 0^\circ, +2^\circ, \) and \( +4^\circ \). The data exhibits the expected trend, i.e., the drag increases with increased control surface deflection and angle of attack. In each case the wing with the leading edge control has a higher drag than the wing with the trailing edge control. In Figure 50, it should be noted that at \( \alpha = \delta = 0^\circ \) both configurations should yield the same value for the drag coefficient. Although they differ by only about 2-1/2%, it is believed that this effect may be a result of cross-flow through the wing gap. In one case, the gap is located in the region of high pressure gradient while in the other the gradient is much smaller.

The effect of changing flow fields resulting from control deflection can best be observed in the drag coefficient data for the wing with the trailing edge control.
As the angle of attack is increased, the experimental drag curve becomes very flat for negative control surface deflections. The onset of this leveling off of the drag coefficient is rapid and is almost complete at as low an angle of attack as $\alpha = +4^\circ$.

In every case investigated the inviscid theory and the experimental drag data agree reasonably well. For the trailing edge control location the agreement between theory and data is very good at small control deflections; at the higher control surface deflections the theory yields drag coefficients which are at most about 7% higher than the experimental drag. When the wing was equipped with a leading edge control surface this trend appears to be reversed. Better agreement between the inviscid theory and the experimental data occurs at the higher control surface deflections while at the lower values the theory predicts drag coefficients about 3% lower than the experimental data.

The variation of angle of attack with control deflection angle for the minimum drag coefficient is given in Figure 53. In the range of angles shown, the minimum drag coefficient varies from 0.0768 to 0.0798 for the trailing edge control and from 0.0780 to 0.0820 for the leading edge control. For both configurations, the angle of attack decreases non-linearly with increasing control
deflection. The agreement between theory and experimental data is very good.

D. The Lift-to-Drag Ratio

The lift-to-drag ratio is an important parameter since it governs the range flown by an aerodynamic vehicle. The greater the value of L/D, the farther an aircraft will fly for a given set of flight conditions. For this reason, the lift-to-drag ratio as a function of control surface deflection and control location with respect to the wing is presented. The experimental data is presented in Figures 54 through 56 for \( \alpha = 0^\circ, +2^\circ, \) and \( +4^\circ \), respectively. In general, it can be stated that the wing with a leading edge control has a higher lift-to-drag ratio than the one with the control located at the trailing edge. This trend prevails for all combinations of angles of attack and control surface deflection investigated. Assuming the control surface deflection is held constant at \( \delta = +14^\circ \), it is noticed that when \( \alpha = 0^\circ \) the lift-to-drag ratio for the leading edge control location is approximately three times as great as that for the rear control location. For the same value of the control surface deflection, i. e., \( \delta = +14^\circ \) at \( \alpha = +2^\circ \) and \( +4^\circ \), Figures 55 and 56 reveal that the lift-to-drag ratio for the leading edge control is a little over twice as great as that for the wing with a trailing edge control.
In general, both the theory and the experimental data yield a linear function of L/D with control surface deflection; in addition, for all angles of attack and control surface deflection the inviscid theory yields values of L/D only slightly greater than the experimental data.

E. The Pitching Moment Coefficient

The behavior of the pitching moment coefficient about the mid-chord point is shown in Figures 57 through 59 for three angles of attack; namely, \( \alpha = 0^\circ, +2^\circ, \) and \( 4^\circ, \) respectively. All the data is for \( M = 13.85 \) and \( R_e = 3935. \) Within the range of flap deflection of \(-18^\circ \leq \delta \leq +18^\circ,\) both the theory and the experimental data of the pitching moment varies linearly with control deflection. This applies to both control surface locations. The primary difference is that the leading edge control case has a positive slope while the trailing edge control exhibits a negative slope. The slopes based on the data are summarized in Table 4 and are relatively independent of both angle of attack and control surface deflections. The leading edge control has a value of \( \frac{\partial C_{m_{1/2}}}{\partial \delta} = 0.0014 \) per degree and for the trailing edge control it is \( \frac{\partial C_{m_{1/2}}}{\partial \delta} = -0.00044 \) per degree. The slope of the pitching moment curve for the leading edge control location is about
3.5 times as great as that when the control is located at the trailing edge.

At $\alpha = 0^\circ$ the inviscid theory agrees very well with the data for both control surface location. As the angle of attack is increased to $\alpha = +2^\circ$ and $\alpha = +4^\circ$ the inviscid theory predicts a slightly higher pitching moment than the experimental data for the leading edge control location while for the trailing edge control location the theory yields lower values of the pitching moment than those obtained experimentally.

If cross-plots of the pitching moment versus the lift coefficient are made, one can investigate the longitudinal static stability derivative ($\partial C_{m|\frac{1}{2}} / \partial C_L$) as a function of control location. This was carried out for the experimental data and the results are given in Table 4. It appears that the static stability derivative for each control location is relatively independent of both angle of attack and control surface deflection. The average values of the experimental static stability derivatives are $\partial C_{m_{\frac{1}{2}}} / \partial C_L = -0.35$ and 0.26 for the trailing edge and leading edge control location, respectively. This implies that the wing with the trailing edge control is statically stable while the one with the leading edge control is unstable.
The variation of the angle of attack with control deflection required to maintain a trimmed flight condition \( (C_{\frac{m}{2}} = 0) \) is shown in Figure 60. The wing with a trailing edge control requires a higher control deflection for trimming out than the wing equipped with a leading edge control provided it occurs at the same angle of attack. In addition, the trim condition varies linearly for the leading edge control and non-linearly for the trailing edge location. As can be seen the inviscid theory agrees reasonably well with the data for the control surface located at the trailing edge; however in the case of the leading edge control the inviscid theory predicts lower values of the angle of attack for a given control deflection in order to maintain trim.

F. The Center of Pressure Location

Figures 61 and 64 are plots of the center of pressure location at \( \alpha = 0^\circ \) for the trailing edge and leading edge control surface location, respectively. The location of the center of pressure is independent of control surface deflection for both physical locations of the control surface with respect to the wing. When the control surface is at the leading edge of the wing, the center of pressure is about 12% of the total wing chord length ahead of the mid-chord of the wing and agrees very well with the theory. When the wing is equipped with a trailing edge control, the
center of pressure is approximately 15% of the total chord length behind the mid-chord point which also agrees with the inviscid theory. For the wing with the control located at the trailing edge, Figures 62 and 63 are plots of the center of pressure movement as a function of control deflection for $\alpha = +2^\circ$ and $+4^\circ$, respectively. Shown on these curves are the calculations based on inviscid theory. The general trend in both cases is similar, and both the theory and the data agree reasonably well. At $\alpha = +2^\circ$ the center of pressure moves from a point about 8% of a chord length behind the mid-chord point at $\alpha = +18^\circ$ to about 20% of the chord length ahead of the mid-chord point at $\alpha = -8^\circ$. The experimental vertical asymptote ($x_{cp}/C = \infty$) was found to occur at $\alpha = -17^\circ$ while the inviscid theory predicts an asymptote at $\alpha = -11.8^\circ$. Both the theory and the experimental data indicates a center of pressure movement which is "hyperbolic" in nature and is similar to the moment of the center of pressure in subsonic speeds. At $\alpha = +4^\circ$ for the trailing edge control location the general trend of both the theory and experimental data are in agreement with each other and are similar to those observed at the lower angle of attack of $\alpha = +2^\circ$. The primary difference is the location of the vertical asymptote ($x_{cp}/C = \infty$). As the angle of attack is increased to $\alpha = +4^\circ$ the theoretical
asymptote occurs at a higher negative control surface
deflection of $\delta = -18.4^\circ$ as compared to the case where
$\alpha = +2^\circ$.

The movement of the center of pressure as a function
of control surface deflection at $\alpha = +2^\circ$ and $4^\circ$ for the
leading edge control location is given in Figures 65 and
66, respectively. Included in these figures is the
horizontal asymptote ($x_{cp}/C = \infty$) appears to agree much better for this case as
compared to the trailing edge control location. At
$\alpha = +2^\circ$ the experimental asymptote occurs at $\delta = -4.8^\circ$
while the theory predicts it to occur at $\delta = -3.8^\circ$, and
at $\alpha = +4^\circ$ the experimental vertical asymptote was found
to occur at $\delta = -9.7^\circ$ while the theory indicates an asymptote
to occur at $\delta = -7.6^\circ$. For both angles of attack the center
of pressure moves from some point about 15% of a chord
length ahead of the mid-chord point at high positive control
deflections and goes off the trailing edge to infinity at
$\delta = -4.8^\circ$ for $\alpha = +2^\circ$ and at $\delta = -9.7^\circ$ for $\alpha = +4^\circ$. It
then returns from infinity on the leading edge of the wing
and moves rearward once again. At both $\alpha = +2^\circ$ and $\alpha = +4^\circ$
it approaches a point about 15% of a chord length ahead of
the mid-chord point at $\delta = -20^\circ$ as its horizontal asymptote.
It should be noted that agreement between the theory and
data is very good in the range of control surface deflections from $\delta_{x_{cp}/c} = \infty \leq \delta \leq +20^\circ$ while in the region $-20^\circ \leq \delta \leq \delta_{x_{cp}/c} = \infty$ the inviscid theory yields a center of pressure location much farther to the rear than the experimental data.

The important points to be brought out are, (1) as the angle of attack is increased, the vertical asymptotes occur at higher negative control surface deflections for both control locations while the horizontal asymptotes do not vary significantly with increasing angle of attack, and (2) the movement of the center of pressure at hypersonic speeds is remarkably similar to that at subsonic speeds. Both can be expressed mathematically as a hyperbolic function of $X_{cp}/c$ versus the control surface deflection angle.

G. The Hinge Moment Coefficient

The magnitude of the control hinge moment and its linearity is very important to the aircraft designer. It is desirable to keep the hinge moments small in order to permit low stick forces. It is also desirable to have the hinge moment vary linearly with both angle of attack and control deflection since it simplifies calculations. It was found from experiments carried out at various constant values of the control deflection angle that the hinge moment did vary linearly with angle of attack. The results
are given in Table 3 where $\frac{\partial C_H}{\partial \alpha} = -0.00469 \text{ per degree}$ for the trailing edge control and $\frac{\partial C_H}{\partial \alpha} = 0.00843 \text{ per degree}$ for the control in front of the wing. In both cases, these values are practically independent of control surface deflection.

Figures 67 through 69 show both the experimental data and inviscid calculations of the variation of the control hinge moment as a function of control deflection and control location. These data are for $\alpha = 0^\circ$, $+2^\circ$, and $+4^\circ$, respectively. For each constant value of the angle of attack, the hinge moment coefficient is smaller for the wing with the trailing edge control. In addition, both the theory and the data indicate that the hinge moment varies linearly in all cases investigated in the range $-18^\circ \leq \delta \leq +18^\circ$. The primary difference is that the leading edge control surface has a positive slope while the wing with the trailing edge control has a negative slope. This information is tabulated in Table 4. As can be seen the experimental values are approximately $\frac{\partial C_H}{\partial \delta} = -0.00168 \text{ per degree}$ and $\frac{\partial C_H}{\partial \delta} = 0.00796 \text{ per degree}$ for the trailing edge and leading edge control surface location, respectively.

The fact that the wing with the leading edge control has hinge moments coefficients which are higher than that with a trailing edge control is attributed to the fact that in the former case the control surface is at a higher
"effective" angle of attack than the latter. In addition, the leading edge control is operating in a relatively undisturbed flow field while the trailing edge control is behind the complex flow field of the wing.

A final point of interest with respect to the control hinge moment is the relationship between the angle of attack and control deflection for the stick-free condition \( (C_H = 0) \) which is shown in Figure 70. For both control surface location, both the data and the theory indicates that the angle of attack must be linearly decreased with increasing control surface deflection in order for stick-free flight conditions to be maintained.

In summarizing the results of all the data and theory presented in these sections, it was surprising to find that most of the force and moment coefficients for both control surface location varied linearly with control surface deflection. This is particularly true when it is recalled that the flow field is complicated by the combined presence of such non-linear effects as vorticity due to bow shock curvature, separated and expansion type flow fields coupled together by the cross-flow effect. It can only be concluded that the combined onset of these effects result in the linear character of the data presented.
VI. CONCLUSIONS

An experimental and theoretical investigation was conducted on a flat plate wing with a semi-cylindrical leading edge equipped with both leading edge and trailing edge control surfaces. The data was taken at a Mach number of 13.87 and an approximate Reynolds number of 3935 over combined angle of attack (-12° ≤ α ≤ +12°) and control surface deflections (-18° ≤ δ ≤ +18°). Pressure distribution and the related force and moment coefficients were compared with the blast wave theory and a modified blast wave equation. The results of this research are summarized below:

(1) Most of the theoretical and the experimental force and moment coefficients for both control surface location exhibit a linear variation with control surface deflection. This could not be anticipated since the flow field is complicated by such non-linear effects as separation and local expansions coupled by cross-flow through the gap. It can only be concluded that these combined physical effects result in the near linear variation of the data.
(2) In general, it was found both theoretically and experimentally that the control effectiveness was about five times as great for the leading edge control as compared with that for the trailing edge control. In addition the control effectiveness for a trailing edge control is about thirty times as great at subsonic speeds as it is in the hypersonic regime.

(3) The wing with the leading edge control has force and moment coefficients which are greater than the trailing edge control location throughout the range of angle of attack and control deflections of this investigation.

(4) The wing with the leading edge control was statically unstable while the one with a trailing edge control was stable in the longitudinal mode.

(5) The location of the center of pressure at zero degrees angle of attack is relatively independent of control deflection and agrees very well with inviscid theory. For the leading edge control location, the center of pressure was about 12% of the chord length ahead of the mid-chord point while the trailing edge location had a center of pressure at approximately 15% of the chord length behind the mid-chord of the wing. As the angle of attack is increased, the movement of the center of pressure for both control surface locations became highly non-linear. Its movement is very similar to the movement
of the center of pressure at subsonic speeds and can be represented as a hyperbolic function of $X_{cp}/c$ versus the control surface deflection angle.

(6) The hinge moment coefficient for both control surface location varies linearly with both angle of attack and control deflections within the range of angles investigated and in both cases the agreement with theory is good. The wing with the trailing edge control exhibited lower hinge moments than the leading edge control configuration in the range $-18^\circ \leq \delta \leq +18^\circ$.

(7) A few points were calculated based on a simplified Lees-Reeves method; it was found that the surface pressure distributions agreed reasonably well with both the inviscid theory and the experimental data. The integrated force and moment coefficients as a result of this method are about the same as those obtained from inviscid theory and agree reasonably well with the data also. Since this technique does not account for either pressure gradients or the effects of cross-flow through an open control surface gap it is recommended that more theoretical research should be conducted in an attempt to explain these phenomena.

(8) For a deflected trailing edge control surface, the separated flow field over one of the wing surfaces probably reattach in the vicinity of the hinge line. In the case of the wing with the leading edge control surface,
separation due to control deflection probably occurred without reattachment. These phenomena are limited to combinations of angle of attack in the range \(-12^\circ \leq \alpha \leq +12^\circ\) and control deflections of \(-18^\circ \leq \delta \leq +18^\circ\).

(9) The effects of cross-flow through an open gap between the wing and the control surface may produce some complex flow field coupling. The effect is most pronounced when the control is at the trailing edge for conditions given as \(\alpha > 0^\circ\) at \(\delta > 0^\circ\). With an open gap, the flow fields over the two wing surfaces cannot be treated independently of each other since they are coupled by the cross-flow through the gap. In addition, for small angles of attack and control surface deflection the effects of cross-flow on the integrated force and moment coefficients is negligible. It is felt that at very high angles of attack and control surface deflection the effect of cross-flow on the integrated force and moment coefficients may become important. Based on the finds of the present investigation it is obvious that more detailed research is required in order to better understand the cross-flow phenomenon.
A. Error Analysis

During any experimental research program, the possibility of errors in testing and measuring techniques exist. Needless to say, it is most desirable to keep these errors to a minimum if the data is to be considered reliable. Assuming all systems are leak-proof, possible sources of errors could result from

(a) The improper setting of the tunnel stagnation conditions for a given Mach number.

(b) In the setting of the model angle of attack and control surface deflection angle.

(c) The variation of free-stream Mach number and local pressures over the model due to changing nozzle and model wall temperatures, respectively.

(d) Variation in pressures due to changes in transducer calibration.

Let us consider the possible error in the free-stream Mach number due to a change in the stagnation conditions. For a nominal Mach number of 13.87 at a stagnation pressure and temperature of 1000 psia and 2200°R, respectively, the pressure could be set within ±5 psia while the temperature
was held by the automatic control within \( \pm 10^\circ R \). This corresponds to a \( \pm 1/2\% \) change in the stagnation pressure and a \( \pm 0.46\% \) change in the stagnation temperature. The combined effect is reflected as an approximate change in the free-stream Mach number of \( \pm 0.30\% \) which was considered to be as good as could be expected.

The second source of possible error was in the setting of the wing angle of attack and the control surface deflection. During the initial stages of the research program, these angles were set and locked manually. It was found that this technique was not sufficiently accurate. The angle of attack and the control surface deflection could only be set within \( \pm 0.10 \) degrees. In addition, the locking mechanism on the control surface was not positive. These combined effects contributed to some minor difficulties in obtaining good repeatability of the data. This was overcome by the use of a micro-readout electrical pot system for both the angle of attack and the control surface deflection angle. In addition, a two-lock system was used in locking the control surface at the desired angle. This new system permitted setting both angles within \( \pm 2 \) minutes of the desired value which corresponds to a percentage error on these angles of about \( \pm 0.17\% \).

The variation of the free-stream Mach number and the local pressure over the model because of changing nozzle and model boundary layer thickness due to wall
temperature variations was noted very early in the research program. This problem was circumvented by heating the nozzle and the model until both reached a state of thermal equilibrium. The nozzle was allowed to heat up for approximately thirty minutes at which point the model was injected and allowed to heat up for another 10 minute period. When both the free-stream total pressure and the model wall temperatures became invariant with time (thermal equilibrium) then, and only then, was the data taken.

Finally, another source of error could be attributed to the non-repeatability of the calibration of the transducers used to measure the free-stream total head pressures and the model surface pressures. The calibration constant for the transducer used to measure the total head pressure in the free-stream \( (p_\infty) \) varied over a period of one year from a maximum value of 21.067416 m.m./volt to a minimum value of 20.588235 m.m./volt. The percent change in the measured total pressure based on the R.M.S. value of the calibration constant is approximately 2.3%. Over the same period of time, a second transducer was used to measure the model pressures. The calibration constant for this transducer varied from a maximum of 3.494158 m.m./volt to a minimum of 3.411324 m.m./volt. The change in model pressures due to a change in this transducer calibration constant based on its R.M.S. value is about 2.4%.
B. Corrections Applied to the Data

In reducing the data, three corrections were applied; they are (a) a correction for the excitation of the vibrational degree of freedom of the test medium, (b) a correction for thermal creep, and (c) a correction for the longitudinal Mach number gradient. An analysis of each of these corrections and their application is given below.

1. Vibrational Degree of Freedom Correction

The correction for the vibration degree of freedom of the test medium was applied to all the data. These corrections were taken from Reference 1 where they are derived in detail based on the assumption of a simple harmonic oscillator.

2. Thermal Creep Correction

The thermal creep correction was required since the flow through the various pressure leads experiences some type of low density phenomenon resulting from a longitudinal temperature gradient which produces an axial pressure gradient within these pressure leads. Because of this, the measured pressure at the cold end of the pressure lead may be different than the actual surface pressure which is usually at a much higher wall temperature. For this reason, a thermal creep correction was applied to
all the data. The particular correction used was a modified form of Howard's equation given in Reference 2 as

\[
\frac{\Delta P/P_{avg.}}{\Delta T/T_{avg.}} = \frac{1}{2 \left[ 1 + \frac{2.46(K_{nd} + 3.15)^2}{K_{nd}(K_{nd} + 24.6)} \right]}. \tag{A-1}
\]

Prior to applying equation (A-1), it was necessary to prove its validity over the combined range of surface pressure and wall temperatures encountered on the wing during an actual test. This verification was carried out in a specially designed piece of equipment shown schematically in Figure 7. The apparatus consists of a sealed bell jar connected to a line leading to a vacuum pump and related shut-off valve, also connected to this line was a surge tank and a bleed-in valve. The bell jar pressure \(p_R\) or \(p_{corr}\) was measured by means of an orifice placed in the vacuum line near the bell jar. The temperature \(T_R\) was measured with an iron-constantine thermocouple at approximately the same location. Placed within the bell jar was a pressure lead of the same type used in the model tested. It was a stainless steel tube with an inside diameter of 0.047 inch. The "test end" of this pressure lead was equipped with an electrical resistance heater with an external power supply. The heater was insulated to prevent heat being transferred to the air within the bell
A chromel, chromel-alumel thermocouple was attached to the hot end of the pressure tube in order to set the proper "wall temperature" ($T_w$). At the other end of the pressure lead, a transducer was used to measure the pressure ($p_m$). All pressures were manually recorded from a digital voltmeter and the temperatures were recorded on a Brown recorder.

The pressure in the bell jar ($p_R$ or $p_{corr}$) was set by the proper manipulation of the vacuum pump and the atmospheric bleed-in valve. The temperature ($T_w$) was obtained by the application of power to the heater until the proper wall temperature was read on the Brown recorder. At this point, the pressure $p_m$ and the temperature $T_R$ was recorded, from which the following was calculated:

$$\frac{\Delta P}{P_{avg.}} = \frac{(P_m - P_R)}{(T_w - T_R)} \left( \frac{1}{2} \right) \left( \frac{P_m + P_R}{T_w + T_R} \right)$$  \hspace{1cm} (A-2)$$

and

$$K_{nd} = \frac{2K_1}{d} \left( \frac{T_w + R_R}{P_m + P_R} \right)$$ \hspace{1cm} (A-3)$$

These quantities were obtained as a function of the de-gasing time, the length of the pressure lead, and the wall temperature. The results of this test are shown in Figure 72.

The data agree very well with the data obtained by Howard.
and Arney and Bailey, and equation (A-1). This is particularly true in the range of Knudsen numbers from \(0 \leq K_{nd} \leq 1.5\). Above \(K_{nd} = 1.5\), the scatter in the data is within the scatter of all the data available. This scatter is due to the difficulty in reproducing the lower pressure with any degree of accuracy. Based on this test, it was concluded that equation (A-1) was valid in the range of interest of this research program.

In order to make equation (A-1) more practical for use in a computer program, it was necessary to change its form slightly. In Reference 2, the Knudsen number based on the tube inside diameter was given as \(K_{nd} = 2\lambda/d\) where the mean-free-path was taken as \(\lambda = K_1 T_{avg}/P_{avg}\). Since the value of \(p_m\) and \(p_R\) are not too different, it is assumed for reasons of simplification, that \(\lambda = K_1 T_{avg}/p_R\). This leads to the expression

\[
p_m = \sum_{i=1}^{5} a_i p_R^i / \sum_{i=0}^{4} b_i p_R^i \quad \text{(A-4)}
\]

where the power series coefficients are a function of the wall temperature, the temperature at the cold end of the pressure lead, and the inside diameter of the tube. These coefficients are
\[ a_1 = \left(3T_w + T_R\right) \left(\frac{K_1}{d}\right)^4 \left(T_w + T_R\right)^3 \]
\[ a_2 = \left(157.44 T_w + 59.04 T_R\right) \left(\frac{K_1}{d}\right)^3 \left(T_w + T_R\right)^2 \]
\[ a_3 = \left(2100.64 T_w + 890.03 T_R\right) \left(\frac{K_1}{d}\right) \left(T_w + T_R\right) \]
\[ a_4 = 838.75 \left(\frac{K_1}{d}\right) \left(T_w + T_R\right) \]
\[ a_5 = 120.09 \]
\[ b_0 = \left(T_w + 3T_R\right) \left(\frac{K_1}{d}\right)^4 \left(T_w + T_R\right)^3 \]
\[ b_1 = \left(59.04 T_w + 157.44 T_R\right) \left(\frac{K_1}{d}\right)^3 \left(T_w + T_R\right)^2 \]
\[ b_2 = \left(890.32 T_w + 2100.64 T_R\right) \left(\frac{K_1}{d}\right)^2 \left(T_w + T_R\right) \]
\[ b_3 = b_4 \]
\[ b_4 = a_5 \]

An example of the order of magnitude of the thermal creep correction can be observed in Figure 76. The percentage thermal creep correction is given as a function of the normalized surface distance for zero degrees angle of attack and flap deflection at \( M = 13.80 \) and \( R_{e_D} = 3520 \). It is noted that the correction is less than 1% at the leading edge of the wing where the pressures and temperatures are both high. The correction increases monotonically with increasing surface distance to a maximum value of approximately 10% at the wing trailing edge. This particular case is considered to be an average representation of the order-of-magnitude of this correction.
The same curve is plotted in a somewhat different manner in Figure 78. At angle of attack, the correction for thermal creep will be smaller on the compression surface than the case discussed because of the higher pressures. They will be greater for the expansion surface since the pressures are less than the previously discussed case. This is particularly true since the wall temperatures are almost invariant with angle of attack because the model was allowed to come to thermal equilibrium before data was taken. An example of the percent thermal creep correction at angle of attack is given in Figure 79. In the presence of separated flow fields, this correction is even smaller since the local pressures are larger for approximately the same wall temperature.

3. **Longitudinal Mach Number Gradient Correction**

Experimental research conducted in a conical nozzle presents two additional problems with reference to corrections which must be applied to the data. One of these corrections is to the angle of attack because of flow inclination. The second correction is to be applied to the surface pressures since the wing is in a flow which has a longitudinal Mach number gradient. The angle of attack correction due to source flow was negligible (at $\alpha = 12^\circ$, the maximum correction was $\Delta\alpha = 0.03^\circ$), therefore, the present data was presented without such a
correction. The longitudinal Mach number gradient correction was a significant correction and warrants a closer investigation.

From Reference 5, it becomes obvious that the Mach number gradient correction is a function of model configurations, model attitude, bow wave shape and strength, plus local flow fields interaction. In the present investigation, most of these effects are present, thereby rendering a theoretically derived correction for the longitudinal Mach number gradient correction extremely difficult. For this reason, it was necessary to find an empirical correction which would be independent of the above mentioned variables.

Before describing the details of this correction, it may be noteworthy to point out the order-of-magnitude of the longitudinal Mach number gradient. Figure 73 gives both the lateral and longitudinal Mach number distribution for a conical nozzle at a stagnation temperature of 2200°R for a stagnation pressure of 1000 psia. The longitudinal Mach number gradient is 0.314 per inch. This corresponds to a change in Mach number over the wing chord of approximately 0.70.

An insight to an empirical correction was obtained from Vas and Bogdonoff's work. Tests were conducted on a sharp flat plate (D = 0.019 inch) in helium at a Mach number of 11.6. In order to determine the effect of
conical flow fields over the flat plate, it was tested in both a contoured nozzle and a conical nozzle. This data is presented in Figure 74. In an attempt to apply a longitudinal Mach number gradient correction to the data obtained in a conical nozzle, Vas and Bogdonoff normalized the surface pressures with respect to the free-stream pressure at the nose of the flat plate. Figure 74 indicates the procedure resulted in data that was lower than the data obtained in a contoured nozzle. When the same data was normalized with respect to the free-stream pressure at the orifice location without the flat plate in the flow, it resulted in data which was too high with respect to the contoured nozzle data.

A close inspection of these results indicates that if the data is normalized with respect to the average value of the free-stream pressure at the leading edge and the local free-stream at the orifice location for the tunnel empty case, it should fall on the contoured nozzle data. This was applied to the Vas and Bogdonoff data and as can be seen in Figure 74, it agrees very well with the contoured nozzle data. This leads to an empirical correction of the form

\[ P = f e^{P_{tc}} \]  

(A-6)
where the correction factor is

$$f_e = \frac{2}{1 + \frac{\frac{d}{dx}(x)}{\frac{d}{dx}(0)}}.$$  \hspace{1cm} (A-7)

Next, it was necessary to verify the correction factor given in equation (A-7) with data taken in air as a test medium. In addition, the variation of the correction over the wing chord should have the same form as predicted by existing simplified theories. This was done by comparing the results of equation (A-7) with the longitudinal Mach number gradient correction derived by Hall for a blunt flat plate at zero degrees angle of attack. Hall's forms were given as

$$p = f_H p_{tc}$$  \hspace{1cm} (A-8)

where the correction factor was

$$f_H = \frac{1}{1 - \frac{4\gamma}{5(\gamma-1)(1+\gamma)} \left( \frac{x}{M} \right) \left( \frac{\Delta M}{\Delta x} \right)}.$$  \hspace{1cm} (A-9)

The Hall correction factor increases with increasing distance along the chord for a given Mach number and Mach number gradient. It is important to notice that for conical nozzles, the empirical correction factor given by equation (A-7) varies in the same manner since the free-stream static pressure decreases with increasing longitudinal distance.
We can conclude, therefore, that the form of equation (A-7) is correct. The percent difference between the two correction factors with reference to the Hall correction factor for this particular case becomes

$$\frac{\Delta f}{f_H} = \frac{f_e - f_H}{f_H} = 2 \left[ \frac{1 - \frac{43}{5} (Y-1) (14+Y)}{M} \left( \frac{\Delta M}{\Delta x} \right) \right] \frac{P_e(x)}{P_e(0)} - 1 \quad (A-10)$$

Equation (A-10) has been calculated for the blunt flat plate wing at a Mach number of 13.80 and zero degrees angle of attack. This data is presented in Figure 77 for a longitudinal Mach number gradient of 0.314 per inch. The percent difference increases monotonically with increasing longitudinal distance. The empirical factor given by equation (A-7) overcorrects as compared to the Hall factor by a maximum of five percent at the trailing edge. Since the variation is not too great and within experimental accuracy of the data, equation (A-7) appears to be a reasonable correction. Finally, both correction factors were compared for surface pressure data obtained with air as a test medium. The corrected conical nozzle data was compared with experimental data obtained in a contoured nozzle by Lee\textsuperscript{19} for the same Reynolds number per foot of 0.0325 x $10^6$ and free-stream Mach number of approximately 14.2 at zero degrees angle of attack. Both the Hall correction and the empirical correction as given by
equation (A-7) were applied to the conical nozzle data over and above the thermal creep correction. The results are shown in Figure 75 and the agreement is excellent. The percentage correction of this particular correction factor for these same conditions is shown in Figure 76. The percent correction including both thermal creep and longitudinal Mach number gradient increases with increasing surface distance. The maximum total correction is about 25% at the trailing edge, of which approximately 10% is due to thermal creep and 15% is a result of longitudinal Mach number gradient. In order to insure that equation (A-7) was equally valid for the compression and expansion surfaces, a second comparison was made at an angle of attack of five degrees. This comparison is given in Figure 79. It indicates that equation (A-7) is a valid correction for the longitudinal Mach number gradient correction for both surfaces.

Based on these findings, all the data was corrected for the longitudinal Mach number gradient using the empirical expression given by equation (A-7).

C. Two-Dimensionality of the Data

During the present investigation, another problem which caused great concern was the two-dimensionality of the data taken in an open jet wind tunnel equipped with a conical nozzle. In order to minimize the conical nozzle
effects on the data, it was felt that the orifices on the model should be aligned on the tunnel centerline. This was considered necessary in view of the fact that in conical nozzles there is no flow inclination in this region. In order to verify this effect, both oil flow and paint studies were made. The stream-lines at the orifice centerline location indicated the flow was two-dimensional in the range of angle of attack $-12^\circ \leq \alpha \leq +12^\circ$. Because of this, all the data obtained in this investigation were taken with the line of orifices on the tunnel centerline.

The second point with reference to the two-dimensionality of the data was the effect of spanwise flow over the model resulting from a pressure differential between the model centerline location and the cabin pressure. In order to check this effect, several tests were conducted to determine the two-dimensionality of the flow when the model was equipped with and without "end plates."
The model without end plates was designed to completely span the open jet. The results of only one such test are presented. Figure 80 shows the tunnel empty lateral Mach number distribution at the location of the leading edge of the wing. The data indicates a test core extending from $-1.0'' \leq y \leq +1.0''$ and external to this core is the jet shear layer. The positioning of the end plates were symmetrical with respect to the orifice centerline location and were placed in the jet shear layer as shown in Figure 80.
The pressure distribution over the wing as a function of surface distance is shown in Figure 81. The parameter in Figure 81 is the endplate location taken as \( y_p = \pm 1.3985", \pm 1.6485", \) and \( \pm 1.7735". \) Superimposed is the data taken without end plates and is represented as \( y_p = 0. \) It is important to recall that the data without end plate compared very well with the data obtained by Lee\(^{19}\) in a contoured nozzle with a shroud around the model, thus eliminating any possible spanwise flow or end effects.

As shown in Figure 81, the data taken with end plates at \( y_p = \pm 1.7735" \) agree reasonably well with the data obtained without end plates. This agreement can be attributed to the fact that either the end plate is in the subsonic portion of the jet shear layer and no end plate shock wave is present; or the end plate is in the supersonic portion of the shear layer, but the conditions are such that the end plate bow shock strikes the model boundary layer so far from the orifice location that the spanwise effects of separation are dissipated and no appreciable effect is noted at the orifice location. As the end plates are moved inward to \( y_p = \pm 1.6485" \), there is a slight change in the pressure distribution as compared with the data obtained without end plate. Then as the end plates are moved further toward the line of orifices so that \( y_p = \pm 1.3985" \), the pressure distribution is typical of
one in which the boundary layer is separated and reattached resulting from shock impingement from the bow wave off the end plates.

The effects of end plate location on the data can be better observed if a cross plot is made of normalized surface pressure versus end-plate location with normalized surface distance as a parameter. The values of S/D were chosen as S/D = 8.79 and 10.79 in the separation plateau and S/D = 14.78 and 16.78 were in the reattachment plateau region for $y_p = +1.3985$ inches. The results are shown in Figure 82. For all values of S/D, the surface pressure decreased with increasing end-plate distance and appear to approach the value of surface pressures obtained without end plates as an asymptote. Recalling that the data without end plate agreed very well with that obtained in Reference 19 with a shroud, it was felt that the closest one could come to two-dimensional flow under the circumstances was with the wing spanning the open jet. It is believed that under these conditions the jet shear layer adjusts itself to the spanwise pressure gradient in such a way as to have reasonably good two-dimensional flow on the tunnel centerline up to $-12^\circ \leq \alpha \leq +12^\circ$. Based on this test, all the data obtained in the research program was conducted with the wing spanning the open jet.
D. Liquefaction Effects

Theoretical calculation based on the Clasius-Clapeyron vapor pressure equation indicates that for a Mach number of 13.86 and a stagnation pressure of 1000 psia the stagnation temperature required to avoid liquefaction of the components of air should be about 2750°R. Although the 4-inch hypersonic wind tunnels will operate at these elevated stagnation temperatures, the heater life is greatly reduced. In order to increase the longevity of the heater, it was necessary to conduct the research program at a stagnation temperature of 2200°R. Under these conditions, the test medium is slightly supersaturated. In order to insure that this condition of supersaturation did not affect the aerodynamic results, a liquefaction check was made. This consisted of two types of tests; (1) a check of the free-stream conditions and how they varied with increasing supersaturation, and (2) a check of the pressure distribution over the model at α = -δ = 0° as a function of increasing amounts of supersaturation. Both tests were conducted at the same Mach number (M = 13.86) and stagnation pressure (p₀ = 1000 psia) of the research program. In both tests, the degree of free-stream supersaturation was increased by a decrease in the stagnation temperature.
Figure 83 shows the free-stream conditions in percentage change over those obtained at $p_0 = 1000$ psia and $T_0 = 2200^\circ R$ (at approximately $M = 13.86$) versus the stagnation temperature of the tunnel. The total head pressure data was the actual experimental data taken while the other free-stream parameters were calculated on the basis of the total head pressure measurements and the tunnel stagnation pressure. This technique is not exactly valid since the ratio of specific heats and the universal gas constant is not known once supersaturation is encountered. Figure 83 indicates that with decreasing stagnation temperature from $T_0 = 2200^\circ R$ to approximately $T_0 = 1650^\circ R$ the Mach number remains relatively constant; however, the free-stream static pressure, free-stream dynamic pressure, and the total head pressure behind a normal shock decreases by a few percent of their respective value at $T_0 = 2200^\circ R$. In the range $1400^\circ \leq T_0 \leq 1650^\circ$, there is a very slight increase in these parameters with decreasing stagnation temperature. Finally, at $T_0 \leq 1400^\circ R$ the effect of liquefaction sets in very rapidly. In order to be on the conservative side, Daum21 chose $T_0 = 1650^\circ R$ as the approximate onset of liquefaction rather than $T_0 = 1400^\circ R$.

The second liquefaction test was made for the purpose of verification of the experimental limit for the onset of liquefaction obtained in the previous test. This test was
made with a blunt flat plate wing at $\alpha = \delta = 0^\circ$ and $p_0 = 1000$ psia for a liquefaction free Mach number of approximately 13.86.

The surface pressures of the wing were taken as a function of increasing supersaturation by decreasing the stagnation temperature. These data are presented in Figure 84 in percentage difference based on the surface pressures obtained at the highest stagnation temperature of $2200^\circ R$. The data are given as a function of the stagnation temperature with the normalized surface distance as a parameter. In the range $1650^\circ R \leq T_0 \leq 2200^\circ$, the surface pressures are independent of the stagnation temperature and for all values of $S/D$ they are about 97.5% of the surface pressures for $T_0 = 2200^\circ R$. In this range, it appears that some liquefaction may be present; however, it is insignificant. At approximately $T_0 = 1600^\circ R$, there is a sudden change in the surface pressure indicating that the onset of liquefaction has begun to show its effects on the data. As the stagnation temperature is decreased more and more beyond $T_0 = 1600^\circ R$, the surface pressures become very unpredictable because of increased liquefaction effects.

The two tests seem to agree that a safe experimental limit for these test conditions is about $T_0 = 1650^\circ R$. In order to minimize the effect of supersaturation during
the present investigation, the data were obtained at a
stagnation temperature of 2200°R for $p_0 = 1000$ psia at
$M = 13.86$. 


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27. Tamburello, Vito; Smith, Bernard J.; and Silvers, H. Norman, "Wind Tunnel Investigation of Control-Surface Characteristics of Plain and Balanced Flaps on a NACA 0069 Elliptical Semispan Wing," NACA ARR L5118, February 1946.


C = 2.250 INCHES  h = 0.600 INCHES  
C_w = 1.545 INCHES  g = 0.005 INCHES  
C_f = 0.700 INCHES  S/D = 0.7854 (FRONT SHOULDER)  
D = 0.125 INCHES  S/D = 17.7854 (REAR SHOULDER)  

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<th>THERMOCOUPLE N.</th>
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<th>FLAP FORWARD (S/D)_{HINGE} = 5.8854</th>
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**TABLE 1** CO-ÖRDINATES FOR THE PRESS ORIFICIES AND THERMOCOUPLE LOCATION.
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<th>α</th>
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<th>C&lt;sub&gt;D&lt;/sub&gt;</th>
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<th>C&lt;sub&gt;m&lt;/sub&gt;</th>
<th>X&lt;sub&gt;cp&lt;/sub&gt;</th>
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**Table 2** Comparision of test data force coefficients, lift to drag ratio, and the center of pressure location with those calculated by the blast wave theory and a semi-empirical equation for M = 13.87, Re<sub>D</sub> = 3935, and θ = 0°.
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<th>FLAP LOCATION</th>
<th>M</th>
<th>$Re_D$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$\frac{\partial C_H}{\partial \alpha}$</th>
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</tbody>
</table>

**TABLE 3** TABULATION OF $\frac{\partial C_H}{\partial \alpha}$ AS A FUNCTION OF FLAP LOCATION, ANGLE OF ATTACK, AND FLAP DEFLECTION
### Table 4: Tabulation of Some Aerodynamic Derivatives

As a function of Flap Location, Angle of Attack, and Flap Deflection.

<table>
<thead>
<tr>
<th>FLAP LOCATION</th>
<th>M</th>
<th>( \text{Re}_D )</th>
<th>( \alpha )</th>
<th>( \delta )</th>
<th>( \partial C_L / \partial \delta )</th>
<th>( \partial C_{m_{1/2}} / \partial \delta )</th>
<th>( \partial C_{m_{1/2}} / \partial \alpha )</th>
<th>( \partial C_L / \partial \alpha )</th>
<th>( \partial C_H / \partial \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>REAR</td>
<td>13.86</td>
<td>3770.0</td>
<td>0</td>
<td>-18 to +18</td>
<td>0.00129</td>
<td>-0.00045</td>
<td>-0.35090</td>
<td>-0.00171</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+2</td>
<td>0.00140</td>
<td>-0.00045</td>
<td>-0.31764</td>
<td>-0.00173</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+4</td>
<td>0.00111</td>
<td>-0.00043</td>
<td>-0.36930</td>
<td>-0.00164</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+6</td>
<td>0.00124</td>
<td>-0.00044</td>
<td>-0.35757</td>
<td>-0.00165</td>
<td></td>
</tr>
<tr>
<td>FORWARD</td>
<td>13.87</td>
<td>4057.0</td>
<td>0</td>
<td>-18 to +18</td>
<td>(variable)</td>
<td>0.00148</td>
<td>0.29496</td>
<td>0.00808</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+2</td>
<td>0.00146</td>
<td>0.29709</td>
<td>0.00798</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+4</td>
<td>0.00140</td>
<td>0.25320</td>
<td>0.00800</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+6</td>
<td>0.00126</td>
<td>0.18649</td>
<td>0.00778</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIG. 1 EXTERNAL VIEW OF THE 4" HYPERSONIC WIND TUNNEL, THE MODEL AND RELATED INSTRUMENTATION.
FIG. 2 A VIEW OF THE MODEL AND THE MODEL SUPPORT SYSTEM.
FIG. 3  CLOSE-UP VIEW OF WING PLUS FLAP.
FIG. 4 VIEW OF MODEL IN THE TEST CABIN.
FIG. 5 FULL SCALE SCHEMATIC DRAWING OF THE WING MODEL UNDER INVESTIGATION.
SHOULDER PRESSURE RATIO

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>M</th>
<th>Re&lt;sub&gt;D&lt;/sub&gt;</th>
<th>SYM</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARL</td>
<td>13.87</td>
<td>3,935</td>
<td>O</td>
</tr>
<tr>
<td>ref.(28)</td>
<td>14.10</td>
<td>35,000</td>
<td>□</td>
</tr>
<tr>
<td>ref.(19)</td>
<td>14.28</td>
<td>16,000</td>
<td>◐</td>
</tr>
<tr>
<td></td>
<td>14.20</td>
<td>12,700</td>
<td>△</td>
</tr>
<tr>
<td></td>
<td>12.28</td>
<td>21,300</td>
<td>△</td>
</tr>
<tr>
<td></td>
<td>12.20</td>
<td>13,700</td>
<td>△</td>
</tr>
<tr>
<td></td>
<td>9.95</td>
<td>15,000</td>
<td>△</td>
</tr>
<tr>
<td></td>
<td>7.01</td>
<td>20,600</td>
<td>△</td>
</tr>
<tr>
<td></td>
<td>6.96</td>
<td>7,500</td>
<td>△</td>
</tr>
</tbody>
</table>

EQN.(12)

EQN.(13)

FIG 6 SHOULDER PRESSURE RATIO VERSUS ANGLE OF ATTACK

α = ANGLE OF ATTACK IN DEGREES

EXPANSION SIDE → COMPRESSION SIDE
FIG. 7 $\Delta P/P_0^*$ VERSUS SURFACE DISTANCE WITH FLAP REAR AT $\alpha = +8^\circ, \delta = 0^\circ, M=13.93; Re_D=3703$.

FIG. 8 $\Delta P/P_0^*$ VERSUS SURFACE DISTANCE WITH FLAP REAR AT $\alpha = +4^\circ, \delta = 0^\circ, M=13.93; Re_D=3703$.

FIG. 9 $\Delta P/P_0^*$ VERSUS SURFACE DISTANCE WITH FLAP REAR AT $\alpha = -4^\circ, \delta = 0^\circ, M=13.76; Re_D=3821$.

FIG. 10 $\Delta P/P_0^*$ VERSUS SURFACE DISTANCE WITH FLAP REAR AT $\alpha = -8^\circ, \delta = 0^\circ, M=1376; Re_D=3840$. 
FIG. 11 LIFT COEFFICIENT VERSUS THE ANGLE OF ATTACK WITH THE
FLAP LOCATION AS A PARAMETER AT \( \beta = 0^\circ \).

FIG. 12 DRAG COEFFICIENT VERSUS ANGLE OF ATTACK WITH THE
FLAP LOCATION AS A PARAMETER AT \( \beta = 0^\circ \).

FIG. 13 LIFT TO DRAG RATIO VERSUS ANGLE OF ATTACK WITH THE
FLAP LOCATION AS A PARAMETER AT \( \beta = 0^\circ \).

FIG. 14 PITCHING MOMENT COEFF VERSUS ANGLE OF ATTACK WITH THE
FLAP LOCATION AS A PARAMETER AT \( \beta = 0^\circ \).
FIG. 15 CENTER OF PRESSURE LOCATION VERSUS THE ANGLE OF ATTACK WITH FLAP LOCATION AS A PARAMETER AT $\beta = 0^\circ$.

FIG. 16 HINGE MOMENT COEFFICIENT VERSUS THE ANGLE OF ATTACK WITH FLAP LOCATION AS A PARAMETER AT $\beta = 0^\circ$. 

<table>
<thead>
<tr>
<th>SYM</th>
<th>FLAP</th>
<th>$M$</th>
<th>$Rd$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>13.87</td>
<td>4057</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.86</td>
<td>3770</td>
</tr>
</tbody>
</table>
FIG. 17 SCHEMATIC DIAGRAM OF THE FLOW FIELD OVER A BLUNT FLAT PLATE AT $\alpha \geq 0$ AND $\delta \geq 0$. (NOT TO SCALE)
FIG. 18 SCHEMATIC DIAGRAM OF THE FLOW FIELD OVER A BLUNT FLAT PLATE AT $\alpha \geq 0$ AND $\delta \leq 0$. (NOT TO SCALE)
FIG. 19 LOCAL PRESSURE RATIO VERSUS SURFACE DISTANCE
WITH FLAP REAR AT $M=13.95$ AND $Re_{D}=3684$.

FIG. 20 $\Delta (P/P_0)_{S/D}$ VERSUS NORMALIZED SURFACE DISTANCE FOR
Flap Rear At $a=4^\circ$ $M=13.95$ and $Re_{D}=3684$.

FIG. 21 $\Delta (P/P_0)_{S/D}$ VERSUS NORMALIZED SURFACE DISTANCE FOR
Flap Rear At $a=4^\circ$ $M=13.95$ and $Re_{D}=3684$. 
Fig. 22 \( \frac{\Delta (p/p_0)}{(p/p_0)_{\delta=0}} \) versus normalized surface distance for Flap Rear at \( \alpha = 0 \), \( M = 13.79 \) and \( Re_D = 3808 \).

Fig. 23 \( \frac{\Delta (p/p_0)}{(p/p_0)_{\delta=0}} \) versus normalized surface distance for Flap Rear at \( \alpha = 0 \), \( M = 13.79 \) and \( Re_D = 3808 \).
FIG. 24 $\frac{\Delta(p\rho)_{\text{dyne} \cdot \text{cm}^{-2}}}{\Delta(y)}_{\text{dyne} \cdot \text{cm}^{-2}}$ VERSUS NORMALIZED SURFACE DISTANCE FOR Flap REAR At $\alpha = 4$. $M = 13.93$ and $Re_y = 3698$.

FIG. 25 $\frac{\Delta(p\rho)_{\text{dyne} \cdot \text{cm}^{-2}}}{\Delta(y)}_{\text{dyne} \cdot \text{cm}^{-2}}$ VERSUS NORMALIZED SURFACE DISTANCE FOR Flap REAR At $\alpha = 4$. $M = 13.93$ and $Re_y = 3698$.

FIG. 26 $\frac{\Delta(p\rho)_{\text{dyne} \cdot \text{cm}^{-2}}}{\Delta(y)}_{\text{dyne} \cdot \text{cm}^{-2}}$ VERSUS NORMALIZED SURFACE DISTANCE FOR Flap REAR At $\alpha = 4$. $M = 13.77$ and $Re_y = 3832$.

FIG. 27 $\frac{\Delta(p\rho)_{\text{dyne} \cdot \text{cm}^{-2}}}{\Delta(y)}_{\text{dyne} \cdot \text{cm}^{-2}}$ VERSUS NORMALIZED SURFACE DISTANCE FOR Flap REAR At $\alpha = 4$. $M = 13.77$ and $Re_y = 3832$. 
FIG. 28 \( \frac{\Delta (P/P_0)}{\Delta (P/P_0)_{b,,}} \) VERSUS NORMALIZED SURFACE DISTANCE FOR Flap FRONT At \( \alpha = 4^\circ \), \( M = 13.87 \) and \( Re_\theta = 4087 \).

FIG. 29 \( \frac{\Delta (P/P_0)}{\Delta (P/P_0)_{b,,}} \) VERSUS NORMALIZED SURFACE DISTANCE FOR Flap FRONT At \( \alpha = 4^\circ \), \( M = 13.87 \) and \( Re_\theta = 4087 \).

FIG. 30 \( \frac{\Delta (P/P_0)}{\Delta (P/P_0)_{b,,}} \) VERSUS NORMALIZED SURFACE DISTANCE FOR Flap FRONT At \( \alpha = 4^\circ \), \( M = 13.87 \) and \( Re_\theta = 4087 \).

FIG. 31 \( \frac{\Delta (P/P_0)}{\Delta (P/P_0)_{b,,}} \) VERSUS NORMALIZED SURFACE DISTANCE FOR Flap FRONT At \( \alpha = 4^\circ \), \( M = 13.87 \) and \( Re_\theta = 4087 \).
FIG. 32 \( \Delta P/P_i \) VERSUS SURFACE DISTANCE WITH FLAP REAR AT 
\( \alpha = +4^\circ; \beta = +8^\circ; M = 13.93; Re_D = 3698. \)

FIG. 33 \( \Delta P/P_i \) VERSUS SURFACE DISTANCE WITH FLAP REAR AT 
\( \alpha = +4^\circ; \beta = +4^\circ; M = 13.95; Re_D = 3698. \)

FIG. 34 \( \Delta P/P_i \) VERSUS SURFACE DISTANCE WITH FLAP REAR AT 
\( \alpha = +4^\circ; \beta = 0^\circ; M = 13.79; Re_D = 3815. \)

FIG. 35 \( \Delta P/P_i \) VERSUS SURFACE DISTANCE WITH FLAP REAR AT 
\( \alpha = +4^\circ; \beta = -4^\circ; M = 13.77; Re_D = 3629. \)

INVISCID THEORY

HANKEY-CROSS METHOD

HINGE LINE

\( S/D = \text{NORMALIZED SURFACE DISTANCE} \)
FIG. 36 $\Delta p/p^*$ VERSUS SURFACE DISTANCE WITH FLAP FRONT AT $\alpha = +4^\circ$; $\beta = +12^\circ$; $M = 13.87$; $Re_\alpha = 4086$.

FIG. 37 $\Delta p/p^*$ VERSUS SURFACE DISTANCE WITH FLAP FRONT AT $\alpha = +4^\circ$; $\beta = +9^\circ$; $M = 13.87$; $Re_\alpha = 4086$.

FIG. 38 $\Delta p/p^*$ VERSUS SURFACE DISTANCE WITH FLAP FRONT AT $\alpha = +4^\circ$; $\beta = +4^\circ$; $M = 13.87$; $Re_\alpha = 4087$.

FIG. 39 $\Delta p/p^*$ VERSUS SURFACE DISTANCE WITH FLAP FRONT AT $\alpha = +4^\circ$; $\beta = 0^\circ$; $M = 13.87$; $Re_\alpha = 4087$. 

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FIG. 40 \( \Delta P/P_i \) VERSUS SURFACE DISTANCE WITH FLAP FRONT AT 
\( \alpha = +4^\circ; \beta = -4^\circ; M = 13.87; Re_D = 4062 \)

FIG. 41 \( \Delta P/P_i \) VERSUS SURFACE DISTANCE WITH FLAP FRONT AT 
\( \alpha = +4^\circ; \beta = -8^\circ; M = 13.82; Re_D = 4124 \)

FIG. 42 \( \Delta P/P_i \) VERSUS SURFACE DISTANCE WITH FLAP FRONT AT 
\( \alpha = +4^\circ; \beta = -12^\circ; M = 3.84; Re_D = 4106 \).
FIG. 43 LIFT COEFFICIENT VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = 0^\circ$.

FIG. 44 LIFT COEFFICIENT VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = +2^\circ$.

FIG. 45 LIFT COEFFICIENT VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = +4^\circ$.

FIG. 46 ANGLE OF ATTACK VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER FOR $C_L = 0$. 

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FIG. 47 CONTROL EFFECTIVENESS PARAMETER VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = 0^\circ$

FIG. 48 CONTROL EFFECTIVENESS PARAMETER VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = 2^\circ$

FIG. 49 CONTROL EFFECTIVENESS PARAMETER VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = 4^\circ$
FIG. 50 DRAG COEFFICIENT VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = 0^\circ$.

FIG. 51 DRAG COEFFICIENT VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = +2^\circ$.

FIG. 52 DRAG COEFFICIENT VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = +4^\circ$.

FIG. 53 ANGLE OF ATTACK VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER FOR $C_{D_{\text{MIN}}}$ CASE.
FIG. 54 LIFT TO DRAG RATIO VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = 0^\circ$.

FIG. 55 LIFT TO DRAG RATIO VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = +2^\circ$.

FIG. 56 LIFT TO DRAG RATIO VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = +4^\circ$. 
FIG. 57 PITCHING MOMENT COEFF. VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = 0^\circ$.

FIG. 58 PITCHING MOMENT COEFF. VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = +2^\circ$.

FIG. 59 PITCHING MOMENT COEFF. VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER AT $\alpha = +4^\circ$.

FIG. 60 ANGLE OF ATTACK VERSUS FLAP DEFLECTION WITH THE FLAP LOCATION AS A PARAMETER FOR TRIM ($C_{m_{\alpha}} = 0$).
FIG. 61 CENTER OF PRESSURE LOCATION VERSUS FLAP DEFLECTION WITH THE FLAP AT THE REAR AT $\phi = 0^\circ$.

FIG. 62 CENTER OF PRESSURE LOCATION VERSUS FLAP DEFLECTION WITH THE FLAP AT THE REAR AT $\phi = +2^\circ$.

FIG. 63 CENTER OF PRESSURE LOCATION VERSUS FLAP DEFLECTION WITH THE FLAP AT THE REAR AT $\phi = +4^\circ$. 
Fig. 64 Center of Pressure Location Versus Flap Deflection with the Flap Forward at $\alpha = 0^\circ$.

Fig. 65 Center of Pressure Location Versus Flap Deflection with the Flap Forward at $\beta = 2^\circ$.

Fig. 66 Center of Pressure Location Versus Flap Deflection with the Flap Forward at $\alpha = 4^\circ$. 
FIG. 71 SCHEMATIC OF EXPERIMENTAL EQUIPMENT USED TO VERIFY THE KNUDSEN EQUATION.

FIG. 72 $\frac{[\Delta P/\text{Pavg.}]}{[\Delta T/\text{Tavg.}]}$ VERSUS THE KNUDSEN NUMBER BASED ON THE TUBE INSIDE DIAMETER.

$K_{nd}$ = KNUDSEN NUMBER BASED ON TUBE I.D.

\[
\frac{\Delta P/\text{Pavg.}}{\Delta T/\text{Tavg.}} = \frac{2.46(K_{nd}+3.15)}{1+K_{nd}(K_{nd}+24.6)}
\]
Fig. 73 Mach number versus longitudinal and lateral distance in inches for $P_e = 1000$ psf at $T_e = 2200^\circ$R.

Fig. 74 $P/P_e$ versus surface distance for a sharp flat plate tested in helium at $M = 1.16$ with $D = 0.019$ in. (Ref. 4).

Fig. 75 Surface pressure versus surface distance for $\alpha = 8 = 0^\circ$ at $M = 13.80$; $P_e = 715$ psia; and $T_e = 1910^\circ$R.

Fig. 76 Percent correction versus surface distance for $\alpha = 8 = 0^\circ$ at $M = 13.80$; $P_e = 715$ psia; and $T_e = 1910^\circ$R.
FIG. 77 PERCENT DIFFERENCE IN LONGITUDINAL MACH NUMBER GRADIENT CORRECTIONS VERSUS SURFACE DISTANCE.

FIG. 78 LOCAL PRESSURE VERSUS SURFACE DISTANCE.

FIG. 79 LOCAL PRESSURE VERSUS SURFACE DISTANCE.
FIG. 80 MACH NUMBER VERSUS LATERAL DISTANCE AT A STATION 1/4" DOWNSTREAM OF NOZZLE OUTLET.

FIG. 81 NORMALIZED SURFACE PRESSURE VERSUS SURFACE DISTANCE FOR $\alpha = 0^\circ$, $M = 13.78$, AND $Re_D = 3818$ WITH $Y_p$ AS A PARAMETER.

FIG. 82 SURFACE PRESSURE RATIO VERSUS END PLATE POSITION FOR $\alpha = 0^\circ$, $M = 13.78$ AND $Re_D = 3818$ WITH S/D AS A PARAMETER.
FIG. 83 PERCENT CHANGE OF VALUE AT $T_o = 2200^\circ R$ VERSUS THE STAGNATION TEMPERATURE FOR $P_o = 1000$ psia

FIG. 84 PERCENT CHANGE OF MODEL PRESS. VERSUS THE STAGNATION TEMPERATURE FOR $P_o = 1000$ psia AND $\alpha = \delta = 0$. 

$T_o =$ STAGNATION TEMPERATURE IN DEGREES RANKINE

APPROXIMATE ONSET OF LIQUEFACTION