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DISSERTATION

Presented in Partial Fulfillment of the Requirements for the
Degree Doctor of Philosophy in the Graduate School of
The Ohio State University

By

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* * * * * * *

The Ohio State University
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ABSTRACT

The prime objective of this study was to develop photogrammetric procedures and methodology to obtain accurate spacing and velocity data on platoons of vehicles as they progress along the roadway. These data, continuous in time and space, are required to effectively study traffic flow characteristics and develop and validate theories of traffic flow. It is impossible to obtain these same data with comparable accuracy through practical measurement techniques on the ground.

Seven photographic flights were made using either Infrared Ektachrome or Plus-X panchromatic film. Photographs of the traffic surrounding a test car were taken at one-second intervals with a Maurer P-2, 70-mm. aerial reconnaissance camera mounted in a helicopter. The photographic scale approximated 1:12,000 in each flight. Thus, each photograph covered a little less than one-half mile of roadway.

A sequence of 101 photographs from one flight was selected for detailed analysis. This sequence provided a good example of platoon interactions -- the traffic characteristics changing from free-flowing to congested and then back to free-flowing within the 100-second time period.

Photo-identifiable ground control points, such as lamp post bases
and manhole covers, were selected at an average spacing of 186 feet. Forty-one points were required to control the 7500 ft. of roadway included in the study section. A third-order ground survey was made to determine the ground coordinates of these points.

The photo coordinates of the center of the photograph, each ground control point, and the front-center of each vehicle imaged on each of the 101 photographs were read and recorded at the OML-Bendix AP/C Analytical Stereoplotter. An average of 38 vehicles and eight to ten ground control points appeared on each photograph.

A computer program was written to process these data on the Ohio State University 7094 computer. The program provided for the conversion of the vehicle photo coordinates to ground coordinates through suitable transformations based on the ground control points. These data were then used to determine the accumulative distance traveled along the roadway by each vehicle in each photograph. The spacings between vehicles on the same photograph and the velocity of each vehicle whenever it appeared on consecutive photographs were computed from the accumulative distances. A total of approximately 3700 spacings and velocities were computed.

Time-distance diagrams were drawn for each lane to show the vehicle trajectories and graphically illustrate the traffic flow characteristics.

The traffic flow data obtained are in error because of inaccuracies in the data collection and reduction equipment and techniques and due to approximations made in the computation procedures. An error analysis indicated that the average errors in the spacing and velocity
determinations were 0.75 ft. and 0.4 mph., respectively. The error analysis includes discussion of the effect of the focal plane shutter, tilt of the camera axis, vertical alignment of the roadway, horizontal alignment of the roadway, vehicle height above the roadway, accuracy of the photo coordinate determinations, lens and film distortions, and accuracy of the time interval between exposures.

The photogrammetric techniques formulated and tested in this study can provide a type of data not heretofore available to those studying traffic flow phenomena — accurate traffic flow data continuous in both time and space. Accurate vehicle trajectories, with corresponding spacing and velocity data, were obtained. The major bottleneck was, and still is, the digitization of the data available on the photographs. This factor alone may well determine the economic feasibility of the aerial survey technique for future traffic studies.
CHAPTER I

INTRODUCTION

The phenomena of traffic flow have been and are being investigated in the research studies of various disciplines, and numerous theories have been advanced to describe and predict the movement of vehicles on roads and streets.

Two general approaches are taken by investigators of traffic flow—macroscopic and microscopic. The macroscopic approach considers the principles governing the simultaneous movement of a large number of vehicles. The microscopic approach considers traffic movement on the basis of the behavior of the driver-vehicle unit.

At present there is no generally accepted or validated theory or model of traffic flow which deals with the propagation of disturbances in highway traffic, the amplification and attenuation of such disturbances, and their effect on traffic volumes and safety. The lack of knowledge concerning the interactions of vehicles as they progress along the roadway—involving both the macroscopic and microscopic aspects of traffic flow—is at least partially due to the absence of satisfactory techniques to collect the essential data. Better definition and understanding of the basic traffic flow parameters are required if theoretical investigations are to lead to improved traffic flow through modification of these parameters.
In order to study traffic flow characteristics, it is necessary to obtain spacing and velocity data, at relatively short time intervals, on a group of vehicles as it progresses along the roadway. Data collected in the past have been limited in the number of vehicles studied (car-following models) or in the number of spacing and velocity determinations for each vehicle. Aerial photographs with standard overlap generally do not provide an image of a given vehicle on more than two or three of the photographs, thereby permitting only two or three spacing determinations and one or two velocity determinations. Wires or pneumatic tubes on the roadway, or observers, have been used to collect data on many vehicles. These methods, however, permit only a few spacing or velocity determinations on a given vehicle, and the data collection is limited to a pre-selected location.

**Objectives**

The prime objectives of this study were to develop a photogrammetric procedure and methodology for the collection of accurate traffic flow data and to illustrate the feasibility and limitations of the techniques advocated. Fulfillment of these objectives necessitated an extensive error analysis to establish the accuracy of the resultant data.

**Method of Investigation**

A test car, identifiable from the air, is introduced into the traffic stream and is followed by a helicopter, which maintains a position directly above the test car as it progresses along the highway. Vertical photographs of the traffic and roadway for approximately
one quarter of a mile ahead of and behind the test car are taken with a Maurer P-2, 70-mm. reconnaissance camera at a known time interval. Essentially the same group of vehicles is imaged on each photograph since the driver of the test car has been instructed to "float" with the traffic.

The photographic negatives are mounted on glass plates and the photo coordinates of each vehicle and several photo-identifiable natural ground control points (such as lamp post bases and manhole covers) are read at the QMI-Bendix AP/C Analytical Plotter.

The photo coordinate data are transferred to punched cards for processing in an IBM 7094 computer. The vehicle photo coordinates are converted to ground coordinates through suitable transformations based on the ground control points. The accumulative distance (termed D-distance in this study) along the roadway from an arbitrary point at the beginning of the test section to each vehicle on each photograph is computed from the vehicle ground coordinates.

Spacings are determined by subtracting the D-distance of the following vehicle from that of the lead vehicle. The velocity of a vehicle is determined by dividing the change in its D-distance on two successive photographs by the known time interval between exposures.

The D-distances of each vehicle are plotted as ordinates against time as abscissa to obtain vehicle trajectories on a time-distance diagram. This diagram graphically illustrates variations in volume, density, spacings, and velocities as a group of vehicles progresses along the roadway.
Accomplishments

Accurate vehicle trajectories, with concurrent spacing and velocity data, for a group of vehicles as it progresses in time and space can be derived through the use of the techniques developed in this study. The techniques are essentially photogrammetric although a non-photogrammetric camera was used. The resultant data may be used in studies of traffic volume-speed-density relationships, speed distributions, lane changing, car following maneuvers, and platoon characteristics.

A supporting error analysis demonstrates the accuracy of the data obtained and includes discussion of the effect of the focal plane shutter of the camera, tilt of the camera axis, vertical and horizontal alignment of the roadway, vehicle height above the roadway, accuracy of the photo coordinate determinations, lens and film distortions, and the accuracy of the time interval between exposures.
CHAPTER II

PHOTOGRAPHIC METHODS IN TRAFFIC ENGINEERING

An extensive literature survey was conducted at the outset of this study to determine the state-of-the-art with regard to traffic flow data acquisition through photographic techniques. This initial survey indicated that a technique to collect accurate data on the interactions of groups of vehicles as they progress along the roadway was not available.

The literature survey was not restricted to traffic flow studies by aerial photography alone. Articles and papers dealing with all types of photographic traffic studies were reviewed. These reports were of significant value in that they pointed out the difficulties encountered and attendant solutions, if such were found, by other researchers. Many common problems exist in photographic traffic studies, particularly with regard to equipment and data reduction techniques.

The literature reviewed as a part of this study was categorized into five sub-groups which are discussed separately in the remainder of this chapter.

These sub-groups are:

Traffic Flow -- Aerial Photography
Traffic Flow -- Terrestrial Photography
Traffic Problems

8
Transportation Studies

Miscellaneous Studies Using Photography

Traffic Flow—Aerial Photography

The first reported traffic study utilizing photography was made in 1927 by the Maryland Aerial Traffic Survey (1). (Since aerial photography and the profession of Traffic Engineering were both in their infancy at that time, there were probably no significant earlier studies.) A large-format camera was used in the study to obtain aerial photographs for a traffic volume count on the highway between Baltimore, Maryland, and Washington, D.C.

In 1947 Greenshields presented a paper outlining the potential of aerial photographs in traffic analysis (2). Attempts were made to obtain volume and density data using a movie camera and helicopter; and later, a military aircraft. The resultant data were unsatisfactory in both cases. In concluding the paper, Greenshields advised using a larger format camera to provide greater accuracy and coverage.

At the 1952 Highway Research Board Meeting, Forbes and Reiss described a study on driver behavior by airphotos (3). Forbes had previously attempted a similar study using a 16-mm. movie camera from a blimp, with unsatisfactory results. In the later study, a modified 35-mm. movie camera and a light airplane were employed.

Forbes and Reiss' paper describes the data collection equipment problems in great detail and some of the advantages and disadvantages of various types of cameras and aircraft. The authors designed a special intervalometer but were unable to build it due to manpower and time limitations. A self-orienting mount was also conceived, but not built.
Two rolls of pictures were obtained (1200 exposures per roll). The photographs were of good quality (sharp enough to permit magnification of 40 diameters), but only relatively short stretches of highway were included in the picture continuously due to aiming difficulties. The intervalometer used was not entirely satisfactory since the intervals were not accurate enough in the minimum time range (2 and 3 second).

Forbes and Reiss concluded the method was practicable for the collection of information on driver behavior that would have been very difficult to obtain in any other way.

Martin Wohl reported a study using a Sonne Stereo Continuous Strip Camera in 1959 (4). The development of the basic formulas for determining vehicle speeds, volumes, and headways are included in his article. In a companion article (5), Mr. Sickle reported some of the results obtained from flights over one length of highway in a light plane. The authors reported 20 to 30 vehicle speed measurements could be made in an hour, and pointed out the accuracy of the data obtained depends on the accuracy of the determination of the ground speed of the aircraft. (Use of the Sonne Camera permits the determination of the speed of a vehicle only once per flight, but the total length of highway is imaged on one stereo strip.)

A paper by F. A. Wagner and Adolf May, Jr. in 1963 describes the procedures used in making traffic density studies by aerial photography (6). Wagner and May felt aerial photography would be useful in their freeway traffic operation studies as the problem locations were not well-defined or isolated.

Density is the traffic parameter measured in that study, because
of its suitability as a single measure of the level of freeway operation. It is particularly advantageous to use aerial photography to measure density, as this quantity can be measured directly on the photograph in vehicles per lane-mile. (The alternative ground method requires independent determination of the number of vehicles and their corresponding speeds at a fixed point and then computation of density from these data. The resultant "computed density" does not necessarily exist on any portion of the roadway.)

Three types of cameras were used in preliminary studies—16 mm., 70 mm., and 5 x 5 in. (127 mm.) formats. All provided usable results, but the 5 x 5 in. format camera, (a K -24 camera), was deemed best for the intended purpose. Two types of aircraft were used -- a helicopter and light plane. The light plane was selected for extensive use due to its greater speed and stability.

In the operational studies, photographs were taken with 50 percent overlap and the freeway section under study was flown at 5- to 10-minute intervals. The film scale was 1:10,000, providing approximately 4250 feet of ground coverage per photograph.

Traffic density data was obtained from the photographs. Traffic densities in vehicles per lane-mile for sections roughly one-quarter to one-half mile in length were computed by dividing the total vehicle count by the number of lanes and then dividing that result by the section length. Results are presented as traffic-density contour maps -- an original concept of the authors. These "maps" illustrate variations in traffic flow in distance and time. The source, duration, and extent of congestion can be determined from the "maps."
William T. McCasland published a paper in 1963 in which he described a comparison study of two techniques of aerial photography — strip photography and time-lapse photography — as applied to freeway traffic operations studies (7). The objectives of the study, conducted by the Texas Transportation Institute at Texas A & M University, were to determine operational characteristics of freeway traffic, factors affecting the level of service, and to compare the two aerial photography techniques.

The time-lapse camera used had a 24-in. focal length and photographs were taken at time intervals of three to four seconds at a scale of 1:1250. The continuous-strip camera had a 4-in. focal length and photographs were taken at a scale of 1:3000. The time lag between exposures for the strip camera was 0.5 second. The photographs were taken over a six-mile section of the Gulf Freeway in Houston, Texas.

Ground counts of the traffic were made and control vehicles equipped with speed recording equipment made travel time runs during the photographic flights. The data obtained included a time reference, vehicle position, location by lane, direction of travel, and type of vehicle. Very little ground control was established and vehicle positions were determined by stationing from construction plans.

Vehicle positions were measured to the nearest one-hundredth of an inch on the strip photography. The airplane speed, which affects vehicle speed determinations, assumed constant over each flight, was computed from the total time required to traverse the test section. (An error in airplane speed results in the same percentage error in vehicle speed.) Since no "true" velocities were known, comparisons
were limited to reproducibility. An aerial survey firm, utilizing photogrammetric equipment, compiled velocities of a single sample of 50 vehicles on two separate occasions. The mean speed determined in the second set of measurements differed from the mean speed determined in the first set by 0.03 mph. The standard deviation of the differences in paired velocity measurements was 2.33 mph. A member of the research staff at Texas Transportation Institute produced comparative values of 0.85 mph and 4.78 mph by direct scale measurements on the photograph. The difference in mean speed as determined by the aerial survey company and the project observer (using less elaborate equipment), was 1.7 mph and the standard deviation of the differences in paired velocity measurements was 5.0 mph. The average speed of the 50 vehicles was 34.7 mph.

Vehicle positions were determined to the nearest one-hundredth of an inch on the time-lapse photography, and the time interval between photographs had an accuracy of ± 0.1 second. The maximum error in vehicle speeds due to the inaccuracy of these two measurements would be ± 3 mph at 70 mph and ± 1 mph at 15 mph. (This assumes no error in the scale determination and transformation of photo positions to ground positions.)

The reported results show that the cost of determining vehicle velocities by continuous-strip photography was somewhat less than by time-lapse photography.

Acceleration-deceleration data could not be obtained from the continuous-strip photography as only one velocity determination could be made for each vehicle. Several vehicles were imaged five times or more.
on the time-lapse photography, permitting multiple velocity determinations and therefore a limited number of acceleration-deceleration values. The number of times a vehicle appeared in the photographs depended on the airplane speed (120-140 mph.) relative to the vehicle speed and the overlap between adjacent photographs (60 percent in that study).

Summary

None of the studies discussed in this section resulted in means to obtain traffic flow data continuous in both time and space. All of these studies were directed toward studying the characteristics of traffic on a pre-specified section of roadway rather than the continuous interactions of a group of vehicles. Most of the earlier studies were hampered by inadequate equipment.

The basic formulas for obtaining velocity and spacing data with a Sonne Stereo Continuous Strip Camera were developed and reported by Wohl and Sickle. This camera was also used in McCasland's study. The principal advantage of using this camera is that the entire section of highway being studied is imaged on two continuous photographs. However, only one velocity value per vehicle can be determined for each flight.

Wagner and May concentrated on obtaining density data, and also relied on multiple flights to obtain data continuous in time. Only approximate vehicle positions were required as data reduction consisted of counting the number of vehicles between stations along the roadway.

Some phases of McCasland's study were somewhat similar to the present study in that multiple velocity data were obtained for some vehicles. However, the velocity determinations were separated by several
seconds and only three or four velocity values per vehicle were obtained.

In general, those who attempted to acquire metric data used large format cameras to optimize accuracy and coverage. The interval between exposures was determined by overlap considerations or the running speed of the camera in the case of movie cameras. No attempts were made to stay with one group of vehicles.

The spacing and velocity data obtained in the present study are more accurate than any similar data reported in other aerial photographic studies.

**Traffic Flow -- Terrestrial Photography**

In a paper presented at the 1933 Highway Research Board Meeting, Dr. Bruce D. Greenshields described a method devised to study traffic behavior by terrestrial photogrammetry (8). This material for this paper was taken from his dissertation at the University of Michigan.

Photographs were taken with a 16-mm. movie camera driven by an electric motor to assure a constant time interval between exposures. The time interval could be varied from one-half second to two seconds. The timing was checked by including the image of a photographic timer with a sweep hand in each picture. The camera, stationed on a tripod 300 feet from the roadway, provided photographs with a roadway coverage of 125 feet.

The pictures were subsequently projected on a screen previously ruled with parallel markings. The projector was then adjusted so that these rulings represented five or ten feet. The distances traveled and spacings between cars were estimated and recorded.

About 6000 photographs were studied and data on volume, speed, and
spacing obtained. Greenshields developed a spacing-velocity relationship based on the results of this study.

In 1959, Donald O. Covault at Georgia Institute of Technology studied traffic flow characteristics using a motor-driven movie camera mounted on a stable tripod (9). Pictures were taken at a rate of 100 frames per minute, or a 0.6-second interval. Shutter speed was one-thirtieth of a second. Color film was used to aid in vehicle identification.

The pavement at the study location was pre-marked with three stripes of white adhesive tape placed perpendicular to the roadway centerline at 50-ft. intervals. The pictures were then taken at an oblique angle from a distance of approximately 200 feet.

For analysis, the film was projected through a time-and-motion study projector. This provided a rigid set up, so that lines corresponding to the tape on the highway and reference marks, such as light poles, could be drawn on the screen, and then the projected image matched with these marks. Lines representing 25-ft. intervals were also drawn on the screen.

Traffic volumes, vehicle speeds, and classification data were determined in five-minute increments by counting and visual position interpolations.

Since velocity determinations required considerable effort, a sample of 15 to 25 vehicles per 5-minute time period was used.

Covault concluded that time-lapse photography was feasible for the analysis of traffic flow and pointed out that non-technical people could extract the data.
Donald G. Capelle and Charles Pinnell, at the Texas Transportation Institute, chose the motion picture method of data collection for their study of capacity at signalized diamond interchanges in 1960 (10). This method was chosen over the 20-pen recorder and manual counting methods because it required a minimum of field personnel and allowed observance and recreation of all traffic events.

Pictures were taken at the rate of 10 frames per second with a 16-mm. motion picture camera mounted on a hydraulic platform truck. The platform could be extended to a height of 35 feet. Reference lines were placed perpendicular to traffic lanes at study locations.

A special counter, attached to the projector, indicated the frame number at all times during the evaluation. Thus, the operator could determine the number of frames between any two events. By utilizing the constant camera speed, elapsed time between events could be determined.

Data on the following operational characteristics were obtained on an individual lane basis: (a) traffic volumes by composition; (b) starting delays after signal change to green; and (c) time-headways between successive vehicles entering the intersection area, by composition.

Seven studies, made at two diamond interchanges, provided data on the operational characteristics of over 4000 vehicles. These data were used in the development of a method for determining the capacity and design of conventional diamond interchanges.

In 1962 R. O. Worrall, University of Wales, published an article based on research work done at Northwestern University (11).

Five cameras operated simultaneously at different positions
provided data for a study of traffic behavior on the approaches to a left-hand exit on the Congress Expressway in Chicago. Information obtained included volumes, densities, speeds and speed changes, weaving movements, travel paths and hazardous maneuvers.

Worrall used a 16-mm. movie camera modified to run at a frame interval of one-half to one second. Vehicle positions were determined in relation to a grid premarked at the edge of the highway. The photographs shown in the article are high-obliques and scale variation is severe. Vehicle positions could be estimated to an accuracy of ±10 ft. up to a distance of 400 feet from the camera. Evaluations of travel times, requiring two separate time measurements, could be made to an accuracy of one-half the time interval between exposures. (Considering the aforementioned position accuracy, it is apparent that speeds must be calculated over a considerable distance, and consequently time, to insure acceptable results.)

Summary

All four of the studies reviewed in this section employed a stationary movie camera to obtain data on traffic movements over a relatively short section of highway or at intersections.

Photogrammetric cameras or data reduction equipment were not used in any of these studies. Spacings and velocities could only be approximated, as interpolation of distance between pavement markings and/or interpolation of time between photographic frames were required.

The stationary movie camera technique is well-suited for obtaining data on traffic volumes, vehicle classifications, and various maneuvers (such as left turns) at locations where problems are known to exist and
when a complete and permanent record of the total situation is desired. However, data collection is confined to specific, relatively small areas, and it is not possible to obtain multiple spacing and velocity data for individual vehicles.

The techniques described in this section could not be used to meet the objectives of the present study, which require vehicle position measurements as the vehicle progresses along the roadway.

**Traffic Problems**

The use of aerial photography in the study of various specific traffic problems constitutes yet another application receiving considerable attention by traffic engineers. In general, the following studies were concerned with obtaining data on traffic phenomena other than flow characteristics.

Joseph F. Rice presented a paper at the 42nd Annual Meeting of the Highway Research Board, in 1963, in which he described the use of the aerial survey method by the Washington, D.C. Department of Highways and Traffic (12). He points out that most traffic problems occur during the rush hours, and aerial survey techniques are particularly useful in that they permit the gathering of traffic data in sufficient scope and detail during those critical periods. The photographers can be dispatched to problem locations and capture the critical traffic events on film for future study or reference.

Traffic engineers shared the Police Department surveillance helicopter with that department. Thus, recurring problem areas were subjected to joint enforcement and engineering inspection.

Aerial photographs were taken from the helicopter at flight heights
ranging from 200 to 2000 ft. The traffic engineers obtained photographs for operational studies with a hand-held 35-mm. camera. Time-lapse photography was also obtained with a K-24 aerial camera.

In one case, traffic congestion prevented cars from entering the traffic stream from off-street parking lots. Study of aerial photographs taken at the site revealed that the location of bus loading operations was one of the major elements contributing to the congestion. Subsequent shifting of bus loading areas has resulted in substantial improvements.

Another problem area was a complex intersection which tended to become a bottleneck during rush hours. The only way to obtain a perspective of the total problem was to view it from the air.

Photographs, taken at 5-second intervals, aided in operational research on the traffic signal control systems at two intersections. Pedestrian crossing problems were also studied with the aid of these photographs.

At this same meeting, Thomas N. Tamburri discussed California's Aerial photography inventory of freeways in the Los Angeles and San Francisco areas (13). A total of 348 miles of existing freeways was photographed at a scale of 1:2400. This provided a ground coverage width of 1000 ft. Where ramp terminals were located more than 700 to 800 ft. from the flight line, additional coverage along the crossroads was obtained. Total cost of the photography, excluding engineering by the California Division of Highways, was $7,130.40 or $20.49 per mile of coverage.

Traffic engineers use the photographs to study accident prone
locations, vehicle paths in relation to geometric features, and signing problems. The photographs permit parking prohibition studies and studies concerned with striping, median barriers, illumination, and pavement markings with a minimum amount of field work.

A pilot study on the use of these photographs in pinpointing individual accident locations was reported. Multilith prints of the photographs were furnished to the California Highway Patrol after prominent landmarks had been identified. The investigating officer indicates the location and type of accident on one of these prints (costing roughly three cents) and attaches the print to each accident report.

Project Sky Count of the Port of New York Authority was described by Thomas D. Jordan at this same Highway Research Board Meeting (14). Transportation operations analysis is the reported specialty of this project.

Photographs taken as a part of Project Sky Count were taken from a light airplane at a scale of 600 to 900 ft. per inch. Jordan reported that modification of a helicopter to permit photographic flights was being considered. (This modification has subsequently been accomplished.)

The first traffic study using aerial photographs was carried out over the western approaches to the Lincoln Tunnel. Photographs were taken at 5-second intervals on each of nine flights over the study area. The flights, made at seven minute intervals, lasted about one minute each. These photographs provided data on vehicular speeds (based on movement during a 10-second period), traffic densities, traffic volumes, and en route travel times over various route segments.

The second study was carried out at the New York International
Airport to determine the characteristics of highway traffic leaving the airport during a two hour period. Data similar to that at the Lincoln Tunnel Study was obtained.

The system was employed to study traffic on the center span of the George Washington Bridge, using techniques and data analyses similar to those employed in the two previous studies. Jordan reported a variance of less than one percent between traffic volume as determined by the Sky Count technique and bridge toll transactions for a three hour study period.

A follow-up article on Project Sky Count appeared in the December, 1964, issue of Traffic Engineering, authored by Justin H. Dickins (15). This article describes some of the studies conducted using the helicopter envisioned in Mr. Jordan's earlier paper. The helicopter, equipped with a K-17 aerial camera, was used to obtain photographs to study adequacy of traffic light signals, and to derive frequency distributions of headways and spacings in the traffic stream as related to selected speeds or densities.

Dickins described the use of the Sky Count technique to study traffic behavior on the Holland Tunnel Rotary, where side-swiping accidents occur frequently. The objective was to determine the utilization of rotary exists and selection of local street destinations in terms of vehicle class and lane of origin at the tunnel exit portal. (This task would have been almost impossible by ground techniques, as it involved the simultaneous tracking of several vehicles.) Inter-lane weaving patterns were also studied and compared to the side-swipe accident locations. Traffic Engineers then proposed changes in the existing
traffic control system to reduce or eliminate the critical weaving conditions. After the proposed alterations have been made, the Sky Count technique will be used to determine their effectiveness.

Traffic flow on the busiest ramps of the George Washington Bridge was improved through signing and lane channelization designed on the basis of a study of Sky Count photographs.

Traffic engineers also studied parking lot usage, parking meter areas, and taxi feeder lines. A pavement marking inventory was also conducted by aerial photography.

In an unpublished paper, Walter P. Smith, Jr., California Division of Highways, discussed the use of aerial photography to study traffic delay due to a lane closure (16). Sixteen passes over the study area, made at approximately nine minute intervals, provided a total of 114 photographs for this study. The scale was 1 inch to 400 ft. The analysis consisted of making vehicle counts at various sections. Knowing the elapsed time between exposures, vehicle-minutes spent within the study area were computed. This value was compared to normal conditions in the same area to determine delay due to lane closure.

Summary

The studies discussed in this section had objectives which differed from the objectives of the present study. However, review of these studies was helpful because many of the same problems (particularly with regard to equipment and data reduction) were encountered.

Transportation Studies

Photogrammetry has proved to be a particularly useful tool in
transportation and urban planning, as these studies require up-to-date physical data and land use inventories.

In 1955, the American City magazine carried an article on the use of aerial photography in the transportation planning studies in New Orleans (17). Individual photographs showed detailed portions of the city and, when assembled into a mosaic, gave a quick view of the entire metropolitan New Orleans area.

In 1962, transportation and urban planning received added impetus from the 1962 Federal Aid Highway Act which required a master plan in urban areas with a population over 50,000, if these areas were to receive federal funds for highways. Also, under the Housing Act of 1954, federal funds are available to help cities with a population of less than 50,000 prepare master plans.

In an article published in Photogrammetric Engineering, January, 1964, Dr. Robert D. Turpin outlined the applications of photogrammetry and photo-interpretation in transportation engineering (18). Turpin pointed out that aerial photographs would be useful in the assessment of existing and prior conditions; to determine geographic features, land use and value, parking facilities and traffic patterns (including congestion and flow, accident patterns, and traffic concentrations); and in the evaluation of economic, industrial, and sociological conditions.

Turpin states that the complexity of modern planning requires a method of data collection and evaluation which will be comprehensive rather than provide masses of uncorrelated studies. It is essential to see the whole "picture" at one time. Photogrammetry meets this need
better than ground-inventory methods, which are hampered by the problem of correlation and the impossibility of re-creation of conditions in existence at the time of data collection. Turpin also suggests that aerial photographs are particularly effective in public relations as a means of convincing various groups that conditions actually exist as reported, that the data collected are valid, and that the projections are reasonable.

In the previously mentioned article by Justin H. Dickins, aerial traffic reconnaissance is mentioned as one of the functions of Project Sky Count (15). Traffic reconnaissance is undertaken to provide comprehensive traffic information for use in transportation planning and determination of general traffic conditions in large areas. Areas up to 600 square miles have been covered. Flights are conducted at high altitudes and have been as long as four hours. From an examination of the resultant photography, traffic engineers determine the locations which warrant further detailed study.

Bruce E. Hewlett, Puget Sound Regional Transportation Study, in a published article (19) discussed the use of aerial photograph comparisons as a means of determining changes in the location and quantity of urban land use over a period of years. Hewlett concludes that aerial photographic interpretation, measurement, and analysis is unexcelled for the stated purposes. Aerial photographs provide a firm basis for making land use forecasts and for checking their accuracy through periodic aerial resurveys.

Robert R. Wagner describes a method of aerial photograph comparison analysis to interpret and measure changes in the land use in areas
around highway interchanges (20). The study was conducted by the Agricultural Research Service, U.S. Department of Agriculture, and concluded that reasonably accurate estimates of land use change can be obtained at minimum expense.

**Miscellaneous Studies Using Photography**

Some of the studies described in the following paragraphs can be thought of as component parts of the previous categories. Others are significantly unique and do not fall under any of the previous classifications. Louis L. Clearwater described an inventory of traffic controls conducted by aerial photography (21) and Maurice C. Olson discussed the use of aerial photography in traffic-signal planning (22) in papers presented at the 1959 California Street and Highway Conference held at the University of California.

Henry A. Barnes, Commissioner of Traffic of New York City, found the use of aerial photographs advantageous in the traffic signal system modernization currently being carried out in New York City (23). Aerial photography was used to supplement the available maps for the basic intersection drawings. More than 7000 individual drawings were needed to implement the plans and specifications for installation of signals at approximately 2700 intersections.

An article appears in the June, 1959, *Traffic Engineering* magazine in which Sven Tynelius describes a parking survey made in Sweden through the use of aerial photographs (24). Stereo photographs were taken so that vehicles which were moving could be easily distinguished from those that were parked. The article points out some of the advantages
and disadvantages of aerial photographic methods as compared to ground-survey techniques.

The use of terrestrial photographs in accident investigations was reported to the International Society of Photogrammetry by the English Delegation at the Ninth Annual Congress on Photogrammetry, London, England, 1960. The Road Research Laboratory uses photographs in on-the-spot accident investigations to make a permanent record of the accident site showing essential road features such as curbs, markings, light poles, trees, road geometry, and the position of the vehicles concerned. A stereometric camera is used which can be elevated to heights of up to 45 feet by a pneumatically operated telescopic pole.

Robert D. Turpin and Clyde E. Lee report the determination of statistical information concerning the actual height of drivers' eyes above the roadway surface (25). An aerial camera was converted for use on the ground at relatively close range with the camera axis horizontal. Driver eye heights were measured directly on photographs taken from a camera station near the edge of a highway as vehicles passed by. The principal advantage of the method is that the data were collected under realistic conditions—in this case, with the car in motion.

Although photography is not involved, it seems appropriate to mention aerial surveillance—i.e., the use of an airborne observer in a helicopter or light plane to reduce delay time to motorists caused by some specific incident by transmitting information through a local radio station. A thorough discussion of this subject was presented by Morton I. Weinberg and John R. Sharpe at the 34th Annual Meeting of the Institute of Traffic Engineers in Miami, Florida (26). Weinberg and
Sharpe concluded that an airborne observer system in a metropolitan area can provide benefits to the public (such as savings in vehicle operating costs due to reduction of delay time by avoiding traffic tie-ups) much in excess of its cost.
CHAPTER III

DATA COLLECTION

Planning

Before data collection could begin, the criteria for the data required to study the desired traffic flow phenomena had to be established. Specifications were needed for the time interval between velocity and spacing determinations, the accuracy of the spacing and velocity determinations, the minimum length of roadway coverage, and the minimum duration of the data collection.

A 20-minute test run in an instrumented vehicle provided a continuous trace of velocity versus time. No photographs were taken during this test run. The test was conducted during the afternoon peak flow on the Olentangy Freeway and Interstate 71 in Columbus, Ohio. The vehicle was also subjected to an erratic acceleration - deceleration pattern on a side street to produce more severe velocity changes.

Evaluation of the resultant velocity-time trace showed that significant changes in the velocity pattern could go undetected if the time interval between "points" on the graph exceeded one second. On the other hand, no significant improvement in the definition of the velocity trace could be gained by obtaining velocity values at a shorter time interval. Since the data reduction task for the same study duration increases as the number of photographs increases, the maximum acceptable
time interval of one second between velocity determinations was
specified for this study.

Review of the literature and discussions with Dr. Joseph Treiterer,
Research Supervisor and Assistant Professor at The Ohio State University
led to the conclusion that velocities must be determined within ± 1.0
mph. (1.5 ft./sec.) to be satisfactory for detailed investigations of
traffic flow characteristics. Hence, travel distances were needed
within ± 1.5 ft., as the time interval between exposures had already been
specified as one second.

For the procedures used in this study,

distance travelled = final position - initial position.
The variance of the distance, then, is equal to the sum of the variances
of the two position determinations. Therefore,

\[(1.5)^2 = \sigma_p^2 + \sigma_p^2 = 2 \sigma_p^2\]

\[\sigma_p = \frac{1.5}{\sqrt{2}} \approx 1.0 \text{ ft.}\]

where \(\sigma_p^2\) is the variance of the position determination.
Thus positions had to be determined to within ± 1.0 ft. (The actual
teach distance computation is somewhat more complicated, as is shown
later in the error analysis section, but the above simplification may
be made for planning purposes.)

At the time of the initial planning of this study, the procedure
for obtaining photo coordinates had not yet been determined. Automatic
photo scanning was being considered and it was reported these systems
could determine photo coordinates to the nearest 0.001 inch. The
maximum scale would be limited to 0.001 in. = 1.0 ft. in this case, or 1:12,000. Since the Maurer P-2 camera had a focal length of 3 in., the maximum flying height would be 3000 ft. above the roadway.

Consideration was also being given to the use of first-order photogrammetric plotters or comparators, for which the least reading was 0.01 mm. If a scale of 0.01 mm. = 1.0 ft. is specified, the scale ratio is 1:30,500 and the corresponding maximum flying height is 7600 ft. above the roadway.

A test flight was made in April, 1964 to check the camera, mount, and intervalometer and to obtain traffic photography to aid in the determination of reasonable values for the minimum roadway coverage and minimum data collection duration for the traffic flow study. Photographs were taken for one minute at an altitude of 7,200 ft. above the roadway; one minute at 1,800 ft.; and two minutes at 3,600 feet. Scales of this photography are nominally 1:24,000; 1:6,000; and 1:12,000, respectively. This test flight also pointed out the need for communication between the test car and the helicopter. Otherwise, the whole flight program must be planned in advance and it will not be possible to alter the program to take advantage of special situations which may develop and be spotted by the observer in the helicopter.

Preliminary evaluation of the photographs indicated that they were quite sharp and considerable magnification could be used in studying them without detrimental blurring. This alleviated fears that the vibrations in the helicopter might affect the quality of the photographs, particularly at low air speeds.

Vehicles could be identified with ease on the 1:24,000 scale
photography, but there were few photo-identifiable points along the roadway which could be used as ground control points. The lens and film distortions in the camera used are relatively large in magnitude and are erratic. Preliminary investigations, included in the error analysis, had shown that closely spaced ground control points, near the roadway, would be required to compensate for the distortions.

One of the prime advantages of the aerial survey technique is that it may be used whenever an interesting traffic pattern develops. The study location need not be specifically determined until after the helicopter is in the air and the observer has had an opportunity to scan a considerable length of roadway to either see or anticipate those patterns of interest. If ground control points must be pre-marked, this advantage is markedly reduced.

Light poles, manhole covers, guard rail posts and ends, paved ramp noses, and other roadside features were positively identifiable on the 1:12,000 scale photography. The roadway coverage at this scale is 2250 ft. Thus, vehicles within about 1100 ft. of the test vehicle were imaged on these photographs. Examination of the traffic platoon movements indicated this coverage to be sufficient. Therefore, all subsequent flights were flown at a scale of approximately 1:12,000.

It was also apparent from this test flight that a study duration of four minutes, which is the maximum for the Maurer P-2 camera equipped with a 50-ft. film magazine at one photograph per second, would be sufficient to record platoon formation and dissipation and/or the movement of the test vehicle into and out of most traffic jams.

It was concluded that the equipment and procedures used in the test
flight, with slight modifications, would be capable of obtaining the required data if the scale of the photographs approximated 1:12,000.

Equipment

Camera

A Maurer P-2, 70-mm. reconnaissance camera was used in this study. It was obtained, on loan, from the U.S. Air Force. This camera is equipped with a Kodak Ektar Lens, f/2.8, with a focal length of 3 inches. The camera has a focal plane shutter and may be set at shutter speeds of 1/500, 1/1000, or 1/2000 of a second. The camera is shown in Figure 1.

The Maurer P-2 camera will recycle at 6 frames per second on runaway or it may be pulse-operated. In this study, the camera was run at one frame per second. The electrical requirements are a supply of 28 volts D.C., 4.3 amperes. A single frame will be exposed if the pulse length is between 90 and 120 milliseconds.

Film magazines with capacities of 50-ft. and 100-ft. of film are available for this camera. In this study, two 50-ft. capacity magazines were used so that two films could be taken during the same flight. The negative size is 2-1/4 x 2-1/4 inches. The maximum number of photographs that can be taken without reloading is 247.

A glass plate with an inscribed grid is located between the focal plane shutter and the film. Thus, every photograph contains an image of this grid. There are no other fiducial markings. The camera was designed for reconnaissance or strike damage assessment and there is no vacuum back or pressure plate to insure film flatness. The film has perforations along both edges and is transported through a slot.
Fig. 1.—Camera, Mount, Intervalometer, and Frame Counter
formed by the grid-inscribed glass plate and the front plate of the magazine by sprocket gears in the magazine. Film tension is maintained by a clutch system on the film spool holders. As previously mentioned, lens and film distortions are relatively large and erratic.

Camera mount

A special camera mount was designed and built for this study. This mount is shown with the camera in Figure 1 and again in Figure 2, where it is installed in the helicopter. This Bell 47J2 helicopter is owned by the Ohio Department of Highways.

The frame holding the camera is mounted on a board which replaces a section of the helicopter floor when the camera is installed. The frame itself consists of a cardanic link which permits rotation of the optical axis in any direction. A bulls-eye type level bubble and a control stick approximately two feet long are attached to the frame. The camera operator can maintain the camera optical axis vertical by keeping the level bubble centered through movement of the control stick. The level bubble is fairly sensitive and tilt of the optical axis can be held to less than two or three degrees if the camera operator is able to devote full attention to this task.

Special care was taken to campen the transfer of vibrations from the helicopter to the camera. Rubber pads of varying stiffnesses were used between the mounting board and frame and between the frame and camera. Rubber sleeves and grommets were used for all the bolts at these connections.
Fig. 2.—Camera and Auxiliary Equipment Installed in Helicopter
Intervalometer

The intervalometer used in this study is also shown in Figures 1 and 2. This is an Intervalometer Camera Control -- Type CP-3, manufactured by the Abrams Instrument Corporation, and was supplied to the project by the U.S. Air Force. The intervalometer can be set for intervals between 0.2 seconds and 6.0 seconds in steps of 0.1 second. The mean interval between exposures for this study was 0.9982 seconds, with a standard deviation of ± 0.0042 seconds. The setting was 1.0 second.

The intervalometer is enclosed in a control box which also contains a switch for supplying current to the camera, and another switch for changing the f-stop during flight to compensate for changing light conditions. The camera may be activated by a push-button to take single photographs when the intervalometer is not being used.

A frame counter is connected to the intervalometer. This counter is not imaged on the photograph, but records the number of photographs taken so that the operator may determine how much film has been used at any time. This frame counter is in a separate housing and is shown in Figures 1 and 2.

Traffic Photography

After the test flight, six additional 50-ft. rolls of film were used to obtain photography of traffic flow in the Columbus, Ohio area. Five of these rolls were taken over Interstate 71 between Fifth Avenue and Morse Road during the afternoon peak flow and one roll was taken over the Olentangy Freeway just prior to an Ohio State University football game. Three of the rolls, including the one over the
Olentangy Freeway, were Kodak Ektachrome Infrared Aero Film, and the other three were Kodak Plus-X Panchromatic Film.

The Ektachrome Infrared film, which is a "false-color" film (sometimes called camouflage detection film), exhibits decided advantages in sharpness and removes the shadow problem, which is bothersome in the black-and-white photography, as it is sometimes difficult to differentiate between the image of a black car and its shadow. (For higher altitude photography, the haze-penetrating properties of the infrared film would provide an additional advantage.) Also, the color, although false, is of some aid in vehicle and control point identification. The panchromatic film is acceptable, however, and easier and cheaper to obtain and process.

Each of the rolls of film was examined for photographic quality and traffic flow patterns. A sequence of 101 photographs from Flight No. 3, on which panchromatic film was used, was selected for detailed analysis. (An enlargement of one of these photographs is shown as Figure 3.) This film was selected primarily because it contained an interesting traffic pattern. Within the 100-second time interval, the traffic changed from a free-flowing state with high speeds and low densities to a relatively congested state with low speeds (less than 5 mph.) and high density, and then back to the free-flowing state.

The photographs in this group were very sharp, but the flying height was somewhat lower than designed. The resultant scale was approximately 1:9400 and the roadway coverage about 1750 feet on each photograph. The total roadway coverage, and length of section included in this study, was 7350 feet. The average photographic overlap was
Fig. 3——Typical Traffic Study Photograph (Enlarged)
approximately 97 percent and the average helicopter speed was 40 mph.

Ground control points along the roadway were selected from inspection of the photographs. The majority of the points selected were light poles, but manhole covers, guardrail ends and posts, breaks in curb lines, and ramp noses were also used. The average spacing between ground control points was 186 feet, with a maximum spacing of 260 feet. Eight to ten ground control points appear on each photograph. However, due to the large overlap, a total of only 41 ground control points was required for the 101 photographs. The ground coordinates of these points were established through a standard third-order ground survey.
CHAPTER IV

DATA REDUCTION

Extraction of the data available on the photographs is a time consuming and relatively expensive process. Automatic vehicle and ground control point coordinate determination by film scanners was investigated in the study. However, the use of this type of equipment was not practicable for the detailed data extraction required. In general, available equipment can not determine vehicle positions accurately enough and/or requires considerable (and expensive) programming.

The coordinates of the vehicles and ground control points on a few photographs were determined using the drawing table coordinatograph attached to the Wild A7 Autograph at The Ohio State University. The least count on the coordinatograph is 0.01 mm., or 10 microns. The microscope in the coordinatograph provides 6X magnification. This method of data reduction was very slow because it is difficult to reach the guide-crank for positioning the measuring mark while looking through the microscope and because the coordinates had to be read from the dials and then recorded by hand. The Wild A7 Autograph itself could not be made available to this project due to a heavy work load.

The Nistri TA3 Stereocomparator at The Ohio State University was also used to extract coordinate data from a few photographs. However,
this equipment was not functioning properly and was consequently inefficient. The automatic read-out was not operable and coordinates had to be recorded by hand. Also, some of the read-out indicator lights were not working properly, and considerable care had to be exercised to assure that correct readings were being obtained. The least count on this instrument is 0.002 mm., or 2 microns.

The OML-Bendix AP/C Analytical Stereoplotter at the Ohio Department of Highways was used for the data reduction of the 101 photographs selected for detailed analysis. This equipment was made available to the project during off-hours. (Any high-order comparator, monocular or stereo, can be used with equal efficiency to obtain the data required in this study if it provides automatic coordinate print-out and it is possible to include pre-set identification numbers in the print-out with the coordinates.)

The AP/C Analytical Stereoplotter

The AP/C Analytical Stereoplotter, shown in Figure 4, was developed by Ottico Meccanica Italiana and the Bendix Corporation. The basic concepts were developed by U. Helava of the National Research Council of Canada. The AP/C is a commercial version of the AP/2, which was developed by OML-Bendix for the U.S. Air Force.

The AP/C has three fully-integrated components. The principal unit is a high precision stereocomparator. This comparator accepts glass plate diapositives or film transparencies (such as used in this study) in all formats up to 9.5 x 9.5 inches. Optical magnification of 10 times the photo scale was used in this study, but 6x and 14x magnifications
Fig. 4.--The OMI-Bendix AP/C Analytical Stereoplotter
are also possible. The least-count in the read-out is 0.001 mm., or one micron.

The second component consists of a general purpose electronic digital computer. This computer may be used "on-line" with the stereocomparator to process data obtained from the stereocomparator, or the computer may be used "off-line" to independently perform engineering and statistical computations.

The third component is an X-Y plotter, which can be driven by the computer directly. Coordinate values provided to the computer may be plotted with or without further corrections.

Thus, the AP/C, as a system, can extract data from a stereomodel or a single photograph, digitize these data, process the data in the computer and correct for any known systematic errors, and then provide results in a digital or analogue mode.

Data from the stereomodel or photograph may be obtained through a bank of Nixie-type tubes for visual digital read-out, a punched paper tape and an electric typewriter for digital recording, as well as the X-Y coordinatograph for analogue tracings.

**Data Extraction Procedure and Format**

The photographs on each roll of film were numbered in sequence from 1 to 247 (approximately). Photographs 90 through 190 on roll 3 were the 101 photographs selected for detailed analysis. A grease pencil was used to number the photographs.

The 101 photographs were then mounted, with cellophane tape, on standard 9 x 9 inch photographic glass plates. No special alignment of the grid lines was necessary as the coordinate system for a given
photograph is arbitrary and may be different for each photograph. The photographs were mounted with the emulsion side out so that proper focus would be obtained in the AP/C.

Each glass plate, containing nine photographs, was then put in the left plate carrier of the AP/C in turn, and photo coordinates on all nine photographs read using a single origin for the entire plate. The photo coordinates of the point under the measuring mark, with accompanying identification numbers, were printed out on the electric typewriter each time the proper console switch was pressed. These data can also be simultaneously punched on a paper tape when the AP/C is functioning properly. However, at the time of the data reduction for this project, the punch was not operating properly.

The photo-coordinate data for a typical photograph is shown in Figure 5. The first line on the left side of the data sheet gives the data for the center of the photograph, as defined by the grid. The first number is an identification number which is set on a bank of dials on the stereoplotter console. The (0000001) indicates this point is the center of the photograph. The second number (-0080027) is the x-coordinate of the point under the measuring mark in microns. The third number (-0075730) is the y-coordinate of the point in microns. The fourth number (0030119) is a second identification number -- the first three digits, 003, are the film roll number, and the last four digits, 0119, are the photograph number.

Immediately following this line, on the left side of the data sheet, are the data for the ground control points. The identification number consists of four zeros and a three digit number which identifies the
Fig. 5.—Photo Coordinate Data Format
specific ground control point. (Ground control points 210 through 219 appear on this photograph.) The other three numbers in the line represent the same items as for the photograph center point.

The data for the vehicles in the inside lane follow the ground control point data. The first digit, (1), of the identification number (1818819) identifies the lane in which the vehicle whose coordinates are being measured is travelling. The next three digits (818) are the identification number of the vehicle whose coordinates are shown in the second and third numbers, and the last three digits (819) are the identification number of the vehicle immediately in front of the vehicle whose coordinates are being measured. This identification technique permits the tracking of each individual vehicle even if it changes lanes. The next two numbers in the line are the x and y photo coordinates of the front-center of the vehicle, and the last number repeats the film roll and photograph identification.

Thus, vehicle 818 is in the inside lane and is following vehicle 819. The x and y photo coordinates of vehicle 818 are (-0105237, -0079151) in photograph 119 of film roll 3. The coordinate origin is arbitrary, but all coordinates on this photograph are on the same coordinate system.

The numbers on the right side of the data sheet are the corresponding data for the vehicles in the outside lane (lane 2).

The coordinates of the vehicles were obtained, in order, from the rear to the front in the direction of travel. A negative sign in front of the lane number indicates the vehicle was under a grade-separation structure, and the true photo coordinates could not be obtained. In this
case, the x and y photo coordinates are disregarded in the data processing. The last three digits of the first number in the line are set at 999 when the photo coordinates of the front vehicle on the photograph are being measured.

A data sheet similar to the one shown in Figure 5 was obtained for each of the 101 photographs. The total data reduction time was 40 hours for the 101 photographs, for an average of 24 minutes per photograph. An average of 38 vehicles and 9 ground control points appear on each photograph.

The data on these data sheets was then transferred to punched cards for processing on the electronic computer.
CHAPTER V

DATA PROCESSING AND EVALUATION

Computer Programming

A computer program was developed to calculate the spacing between successive vehicles on each photograph and the velocity of each vehicle that appeared on two successive photographs from the photo coordinate data. The program was written in SCATRAN (similar to FORTRAN) computer language for the IBM-7094 computer at The Ohio State University.

Ground coordinates of vehicles

The photo coordinates of the vehicles on each photograph were transformed to ground coordinates through the standard transformation equations:

\[
\begin{align*}
X_G &= A X_p + B Y_p + E \\
Y_G &= A Y_p - B X_p + F
\end{align*}
\]

where \(X_G\) and \(Y_G\) are ground coordinates, \(X_p\) and \(Y_p\) are the photo coordinates and \(A, B, E,\) and \(F\) are transformation coefficients. This transformation provides for a rotation, translation, and scale-change between the two coordinate systems. Thus, the azimuth orientation, origin position, and scale of the photo coordinate system may be completely arbitrary.
The coefficients were determined through the ground control points data. Transformation coefficients were calculated for each interval between successive ground control points on each photograph by substituting the measured photo coordinates and known ground coordinates of the two ground control points in the above transformation equations and then solving the resultant simultaneous equations. A total of approximately 800 sets of transformation equations were required to transform all the vehicle photo coordinates to ground coordinates, as there are eight to ten ground control points (and therefore seven to nine intervals) on each of the 101 photographs.

The first ground control point in the direction of travel of the vehicles was assigned the coordinates (1000.00 ft., 1000.00 ft.) so that negative ground coordinates for the vehicles would not be obtained. The x-axis of each of the photograph coordinate systems and the X-axis of the ground coordinate system were established such that they were nearly parallel to the roadway, with increasing x and X in the direction of travel.

The x photo coordinate of each photograph on each photograph was compared with the x photo coordinates of the ground coordinates on the same photograph to determine in which ground control point interval the vehicle was located. Then the corresponding transformation coefficients were used to compute the ground coordinates of the vehicle. If the x photo coordinate of the vehicle was less than the x photo coordinate of the first ground control point on the photograph, the transformation from photo coordinates to ground coordinates for the vehicle was made using the coefficients derived for the first interval
Similarly, if the vehicle had passed the last ground control point, the coefficients based on the last two ground control points on the photograph were used.

**Accumulative distance computations**

The spacing between vehicles or the distance travelled by a vehicle in the time interval between exposures can be determined by computing the difference in ground position based on the ground coordinates. However, these quantities may also be readily determined if the accumulative distance along the roadway to each vehicle from some "starting point" is known. This accumulative distance, or "D-distance", identifies the position of the vehicle with respect to the roadway centerline. A change in D-distance for a given vehicle over some time period represents the true distance travelled by the vehicle, whereas the distance between the initial and final coordinates is not the travel distance if there are any curves in the roadway. The accumulative travel distance is also used in the construction of time-distance diagrams, which are of particular interest in traffic flow analysis.

The computer program was designed to compute the D-distance of each vehicle on each photograph, based on the vehicle ground coordinates. The curved roadway was approximated by a series of 17 straight-line segments, defined by 18 "D-points". These 18 points were specific vehicles in the inside lane on specific photographs, and since their ground positions had already been determined through the transformation procedure described in the previous subsection, 18 points along the centerline of the inside lane of the roadway were obtained. (The
accumulative distance along the outside lane was assumed to be equal to that along the inside lane because the two curves included in the section of roadway involved in this study have large radii and they form a reverse curve.) In the curved section of the roadway, these D-points were selected at short intervals to assure that the distance along the chord between the D-points provided a close approximation to the actual roadway distance along the curve.

The distance between D-points was calculated from their ground coordinates, established in the manner previously described; i.e.,

\[ D = \left[ (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 \right]^{\frac{1}{2}} \]

where \((x_1, y_1)\) and \((x_{i+1}, y_{i+1})\) are the ground coordinates of adjacent D-points.

The first D-point was assigned an arbitrary D-distance sufficiently large to assure that all of the vehicles would have positive D-distances. To find the D-distance to a given vehicle, it was necessary to first determine the D-distance to the last D-point the vehicle had passed and then add to this the distance to the vehicle from this last D-point. Thus:

\[ D_v = A (x_v - x_1) + D_1 \]

where:

- \(D_v\) = accumulative distance, or D-distance, to the vehicle
- \(A = (D_{i+1} - D_1) / (x_{i+1} - x_1)\), where \(D_{i+1}\) is the D-distance to the D-point immediately ahead of the vehicle, \(D_1\) is
the D-distance to the last D-point passed by the vehicle and \( X_{i+1} \) and \( X_i \) are the ground X-coordinates of the corresponding D-points. Thus, \( A \) expresses the ratio between changes in D-distance to changes in X-coordinates for any point between D-points (i) and (i+1).

The D-distance to any D-point is equal to the D-distance to the previous D-point plus the distance from the last D-point to the one under consideration.

D-distances to the vehicles beyond the last D-point were computed using the \( D_v \) equation used for the last D-point interval, as the roadway continued in a straight line as defined by the last two D-points. The D-distance to vehicles preceding the first D-point were computed using the \( D_v \) equation for the first D-point interval. In this case, however, \( D_1 \) was equal to the D-distance of the first D-point and the quantity \( (X_v - X_1) \) was negative (but never larger than \( D_1 \) in magnitude).

**Spacing and velocity calculations**

The spacing or distance headway, between two adjacent vehicles is equal to the difference in their D-distances. This spacing distance is measured from the front-center of the following vehicle to the front-center of the lead vehicle. The clear distance between vehicles is equal to the calculated spacing minus the length of the lead vehicle.

The computer program was designed to calculate the spacing between vehicles whenever possible. No spacing value can be obtained for the lead vehicle on each photograph or when either the lead or following vehicle is under a grade-separation structure.

The average velocity of a vehicle during the time interval between
exposures can be determined by dividing the distance travelled during this time by the time interval. The travel distance is equal to the D-distance for the vehicle on the second photograph minus the D-distance of the same vehicle on the first photograph. The time interval used in this study was one second.

The velocity was computed for each vehicle each time it appeared on two successive photographs. The velocity could not be computed for the one-second interval preceding the photograph on which the vehicle appeared for the first time, nor could the velocity be determined for the time intervals in which the vehicle was under a grade-separation structure during or at either end of the time interval.

The scale ratio, or ratio of the distance on the photograph between a given pair of ground control points to the ground distance between the same two points, was computed for each pair of ground control points on each photograph. Examination of these ratios provides a quick check for gross errors in the photo-coordinate determinations for the ground control points. The flying height of the helicopter above the roadway can also be determined from these scale ratios, as the focal length of the camera is known. The variations in these scale ratios were also used to estimate the tilt of the photograph about an axis perpendicular to the roadway, as explained in Chapter VII.

**Flow charts**

The flow chart shown in Figure 6 presents an outline of the procedure followed in processing the raw data—photo and ground coordinates of ground control points and photo coordinates of vehicles—to obtain vehicle spacings and velocities for traffic flow analysis. This flow
Enter Data:
- Photo Coordinates (x and y) for each photo
- Photograph number
- Center of photograph
- Control Points
- Vehicles (with identification numbers)
- Ground Coordinates (X and Y)
- Control Points
- Data for D Calculations
  - Photo number
  - Vehicle to be used (with x and y)
  - Control point before and after vehicle (with x and y)
- Parameters
  - Number of photographs
  - Number of control points
  - Range of vehicle identification numbers
  - Necessary constants

Find Ground Coordinates of D Points
Equations: X = Ax + By + E
          Y = -Bx + Ay + F
Use ground coordinates and photo coordinates of
control points adjacent to each D point to
determine A, B, E, and F.
Solve equations for ground coordinates of D points
using determined coefficients and photo coordinates.

Find Accumulative Distance (D) Along Road
Assign first point an arbitrary value D1
Then D_{i+1} = D_i + \sqrt{(X_i + 1 - X_1)^2 + (Y_i + 1 - Y_1)^2}

Determine Relationship Between D and X
Equation: D = AX + B
Use values of D and X for successive pairs of
D points to determine A and B.
Result will be a set of equations for D in terms
of X for the range of X covered in the
photographs.

Convert Photo Coordinates on Photo m to Ground Coordinates
Equations: X = A_p x + B_p y + E_p
          Y = -B_p x + A_p y + F_p
Use ground coordinates and photo coordinates
of adjacent ground control points. Determine
A_p, B_p, E_p, F_p. Result will be n - 1 sets of equations
where n is the number of ground control points
on the photograph.
Solve appropriate equations for ground coordinates of
vehicles using determined coefficients and
photo coordinates.
Solve appropriate equation for ground coordinates of
photo center.

Fig. 6.—Outline of Data Processing Procedure
Determine Accumulative D for Each Vehicle
Use set of equations previously derived:
\[ D = AX + B \]
Use X determined above to select proper equation and then compute D's.

Compute Ground Coordinates and Accumulative D for Vehicles on Photograph \( m + 1 \)
Use same methods as for photograph \( m \).

Compute Spacings and Velocities
Headways will be difference in D's for successive vehicles on same photograph.
Velocities will be difference in D's for same vehicle on successive photographs divided by time factor.

Compute X, Y, D, Spacing, and Velocity for all Vehicles on all Photographs
Use methods outlined above.

Output:
- Print X, Y of all D points used in step 2
- Print D, A, and B for all equations derived in step 4
- For each photograph:
  - Print scale \( (A^2 + B^2) \) for each set of ground control points
  - Print X, Y of photo center
  - Print X, Y, D, S, and V for each vehicle
    (V will be indeterminate in first photograph)

Fig. 6.—Continued
chart summarizes the procedures described in the previous three sub-
sections.

The flow chart shown in Figure 7 details the actual steps used in
the data processing. A program in a computer language other than
SCATRAN can be written from this flow chart if desired. A listing of
the variables used in the program follows the detailed flow chart.

The program, as written, processed the data in successive pairs
of photographs. This procedure was adopted to minimize the storage
requirements, as it required the storage of the photo-coordinate data
and computed quantities for no more than two photographs at any given
time.

Traffic Flow Data

A copy of a typical computer output page is shown in Figure 8.
This sheet presents the data computed for the vehicles in the outside
lane on photograph 119.

For any given line, the number in the first column is the identifi-
cation number of the vehicle to which the subsequent data on the same
line applies. The second column, headed FOLLOWING, shows the identifi-
cation number of the vehicle immediately in front of the vehicle identi-
fied in the first column, except for the last line, where the number is
999. This indicates that vehicle 541 was the lead vehicle on this photo-
graph, and thus the vehicle ahead of it can not be positively identified.

The third and fourth columns, headed GROUND X and GROUND Y, give the
ground coordinates, in feet, of the front-center of the vehicle identified
in the first column. The asterisks in these two columns for vehicles 526
Fig. 7.—Computer Program Flow Chart
Fig. 7.--Continued
Fig. 7—Continued
LIST OF VARIABLES USED IN COMPUTER PROGRAM

NG  Number of ground control points.
NPI  Integer equal to first control point number minus one.
NXD  Number of points for "D" calculation.
SCAF  Scale conversion factor.
NPT  Total number of photographs.
TIMPA  Average time between photographs.
XG(I)  X ground coordinate of ground control point.
YG(I)  Y ground coordinate of ground control points.
XD(I)  X photo coordinate of D-points.
YD(I)  Y photo coordinate of D-points.
NPI(I)  Identification number of ground control point preceding D-point.
XP1(I)  X photo coordinate of control point preceding "D"-point. Later as X ground coordinate of D-point.
YP1(I)  Y photo coordinate of control point preceding D-point. Later as Y ground coordinate of D-point.
XP2(I)  X photo coordinate of control point following D-point.
YP2(I)  Y photo coordinate of control point following D-point.
DEN  Temporary storage for denominator of coefficients.
A  Transformation coefficient to coordinates of D-points
B  Transformation coefficient to ground coordinates.
C  Transformation coefficient
D  Transformation coefficient
NPTC  Counter for number of photographs.
DMAG  Distance between two successive D-points.
AD(I)  Transformation coefficient
Transformation of ground coordinate X to "D" for two successive D points.

BP(I)  Transformation coefficient

XPC  X photo coordinate of principal point on photograph being processed. Later used as X ground coordinate of same point.

YPC  Y photo coordinate of principal point on photograph being processed. Later used as Y ground coordinate of same point.

NRUN  Flight number.

NP  Photograph number.

LN  Lane number plus vehicle identification number (Sign plus first four digits of identification number). Later used as lane number only. (Sign plus first digit).

NPP(I)  Control point number minus NPI.

XPP(I)  X photo coordinate of control point on photograph being processed.

YPP(I)  Y photo coordinate of control point on photograph being processed.

NNP  Number of control points on photograph being processed.

SDEN  Temporary storage for denominator of transformation coefficients.

AP(I)  Transformation Coefficient
Transformation of photo coordinates to ground coordinates for any point lying between 2 successive control points on photograph being processed.

BP(I)  Transformation Coefficient

CP(I)  Transformation Coefficient

DP(I)  Transformation Coefficient

TEMP  Temporary storage cell.

SCAFA(I)  Scale factor for distance between two successive control points.

NPN  Control point number.

NPNP  Number of next control point after control point number NPN.

NCL(I)  Vehicle identification number for vehicle in lane 1.

NFL(I)  Identification number of vehicle ahead of vehicle NCL(I).

XCL(I)  X photo coordinate of vehicle NCL(I). Later used as X ground coordinate of same vehicle.
YCl(I) Y photo coordinate of vehicle NC1(I). Later used as Y ground coordinate of same vehicle.

DOT 1 Temporary "D" value of first vehicle in lane one of first photograph.

NC2(I) Vehicle identification number for vehicle in lane two.

NF2(I) Identification number of vehicle ahead of vehicle NC2(I).

XC2(I) X photo coordinate of vehicle NC2(I). Later used as x ground coordinate of same vehicle.

YC2(I) Y photo coordinate of vehicle NC2(I). Later used as x ground coordinate of same vehicle.

NNCl1 Number of vehicles in lane one for photograph being processed.

DOT 2 Temporary "D" value of first vehicle in lane two on first photograph.

DOT Smaller of DOT 1 or DOT 2.

DN2(I) "D" value of vehicle NC2(I) on photograph being processed.

DN1(I) "D" value of vehicle NC1(I) on photograph being processed.

NNC22 Number of vehicles in lane two for the photograph being processed.

JJ Counter to determine number of "D" intervals vehicle being processed has passed.

DSUM Accumulative "D" value along line connecting D-points.

KKKK Check counter.

NTNC1(I) Temporary storage for NC1(I).

TDN1(I) Temporary storage for DN1(I).

NTNC2(I) Temporary storage for NC2(I).

TDN2(I) Temporary storage for DN2(I).

NNNC11 Temporary storage for NNCl1.

NNNC22 Temporary storage for NNC22.

Vl(I) Velocity of vehicle in lane one.
KKKKK  Check counter.
V2(I)  Velocity of vehicle in lane two.
H1(I)  Headway of vehicle in lane one.
H2(I)  Headway of vehicle in lane two.
<table>
<thead>
<tr>
<th>VEHICLE</th>
<th>FOLLOWING</th>
<th>GROUND X</th>
<th>GROUND Y</th>
<th>DISTANCE(FTI)</th>
<th>HEADWAY(FTI)</th>
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</tbody>
</table>

Fig. 8.—Computer Output Format
and 532 indicate that these two vehicles were under the grade-separation structure at the time photograph 119 was taken.

The D-distance, or distance along the roadway from an arbitrary starting point, is shown in the fifth column, headed DISTANCE (FT). Asterisks appear in this column for vehicles 526 and 532 as the ground coordinates, which are required to compute the D-distance, are not available for these two vehicles on this photograph.

The sixth column, headed HEADWAY (FT), gives the distance from the front-center of the vehicle identified in the first column to the front-center of the vehicle identified in the second column. This quantity is equal to the difference in D-distance for the two vehicles. Considering vehicles 517 and 518, for example

\[ 1848.11 - 1770.25 = 77.86 \]

The asterisks for vehicles 534, 526, 532, and 541 indicate the D-distance for either the lead or following vehicle (or both) is missing. For vehicle 534, 526, and 532 this is due to vehicles 526 and 532 being under a grade-separation structure. In the case of vehicle 541, the preceding vehicle is not on the photograph.

The last column, headed VELOCITY (MPH), gives the average velocity during past one second of the vehicle identified in the first column. The values given for this photograph were obtained by subtracting the D-distance for a given vehicle on photograph 118 from the D-distance for the same vehicle on photograph 119; dividing this difference by the time interval of one second; and then converting the resultant velocity in feet per second to miles per hour. Asterisks appear on the lines for vehicles 526 and 532 because the D-distances on photograph 119 (and
possibly on photograph 118 also) are not available. The D-distance for vehicle 527 is shown in the data for photograph 119, but this vehicle was under the grade-separation structure on photograph 118, and consequently the D-distance is not available on that photograph and the velocity cannot be determined.

Data similar to that shown in Figure 8 were computed for each of the vehicles on each of the 101 photographs. Thus, spacing, or distance headway, and velocity data are available for a group of vehicles at one-second intervals for the duration of the study. In all, a total of approximately 3700 spacing and velocity determinations were made.

**Time-Distance Diagrams**

Time-distance diagrams for each lane were constructed from the computer output data. These diagrams are shown in Figures 9 and 10. For each photograph, the accumulative distance along the roadway from the zero of the "Distance" axis (the D-distance) to each vehicle was plotted in a vertical axis at the corresponding time. The vehicle trajectories were drawn by connecting the consecutive D-points for each vehicle. The velocity of the vehicle at any time is equal to the slope of its trajectory at the corresponding time.

A vertical section through the time-distance diagram represents the traffic situation at a given instant. The density of vehicles on the roadway at any time may be found directly by counting the number of vehicle trajectories which cross the corresponding time line within a selected length of roadway.

A horizontal line through the time-distance diagram represents the traffic situation at a given point along the road. The traffic flow
Fig. 9.—Vehicle Trajectories (Inside Lane)
Fig. 10.—Vehicle Trajectories (Outside Lane)
volume at any point along the roadway may be determined by counting the number of vehicle trajectories that cross the horizontal line within the selected time interval.

Analysis of the time-distance diagrams indicates that the helicopter velocity, which is equal to the slope of the band of trajectories, exceeded that of the group of vehicles—particularly those vehicles in the outside lane. Better "tracking" of a platoon of vehicles is possible if an observer accompanies the pilot and camera operator in the helicopter, and if radio communication between the helicopter and test vehicle is provided.

A total of 71 vehicles appeared on at least one of the photographs, but the average number of vehicles per photograph was 38. Only two of the vehicles, numbers 830 and 831, appeared on all 101 photographs. However, about 70 percent of the vehicles appeared on more than 50 photographs.

Trajectories which start and stop in the middle of the band of trajectories represent vehicles that switched lanes during the duration of the study. For example, vehicle 833 first appeared in the inside lane following vehicle 834 at time equals 16 seconds; then switched to the outside lane following vehicle 555 at time equals 62 seconds; and then returned to the inside lane behind vehicle 836 at time equals 77 seconds.

A sharp drop in vehicle velocities is evident at a distance value of about 3700 ft. on the time-distance diagram for the outside lane—particularly for those vehicles arriving after time equals 50 seconds. It can be observed that successive vehicles must decelerate at earlier
distance values. Thus, a "shock-wave" of rapid deceleration is progress­
gress back along the roadway in the direction opposite to the direc­tion of travel. The helicopter, and consequently the photo-coverage, continued on down the roadway, however, and the resultant high-density traffic state and the dissipation of this wave phenomenon were not captured on the photographs.

Platoon characteristics, flow density and volume variations, passing manuevers and lane changing (including gap acceptance) are all apparent on these time-distance diagrams, which provide a graphical display of the interaction of several vehicles as they progress along the roadway in time and space. Accurate numerical data for detailed analyses of these traffic characteristics or for use in the testing and validation of traffic flow theories are available in the accompanying computer output.

**Man Power, Equipment, and Costs**

If the developmental work in obtaining the final data of this study is disregarded, the following breakdown of man power and equipment time results. Times similar to these can be expected on a "production" basis.

**Man Power:**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photography and attendant operations</td>
<td>10</td>
</tr>
<tr>
<td>Selection of photos for detailed study</td>
<td>4</td>
</tr>
<tr>
<td>Mounting of photos on glass plates</td>
<td>4</td>
</tr>
<tr>
<td>Reading coordinates</td>
<td>80</td>
</tr>
<tr>
<td>Card punching</td>
<td>18</td>
</tr>
<tr>
<td>Ground control</td>
<td>32</td>
</tr>
<tr>
<td>Plotting time-distance diagrams</td>
<td>16</td>
</tr>
</tbody>
</table>

\[
\text{Total: } 164 \text{ man hours}
\]
Equipment:

- Helicopter: 1 hour
- AP/C Analytical Stereoplotter: 40 hours
- IBM Keypunch: 18 hours
- IBM 7094 Computer: 3 minutes

These time estimates are based on obtaining complete data from 101 photographs, covering a data collection period of less than two minutes, with an average of 38 vehicles per photograph.

Man power and equipment cost rates will vary with the organization performing the work, and total cost can be estimated by applying the appropriate rates.

Most of the work can be done by technicians on a production basis, and an overall average wage rate of $3.00 per hour seems reasonable. Thus, the total man power cost to obtain the final data in the present study on a production basis may be estimated at $492. The Ohio Department of Highways estimates the cost of the helicopter at $80.00 per hour, and a reasonable charge for the rental of the AP/C at $12.00 per hour. The rental for a key punch is $0.35 per hour, and the total computer charge is estimated at $45.00. Thus, the total equipment cost may be estimated at $611. Approximately $20.00 should be added to this figure for the cost of the film and processing. Since 3700 vehicle spacings and velocities were determined in the present study, the average cost per vehicle "data point" (both spacing and velocity) would have been $1123/3700 = $0.30 at production rates. If a smaller comparator had been used, the rental rate would have been reduced to $5.00 per hour. This would have resulted in an overall cost of $843, and a cost
per vehicle data point of $0.23. It is not possible to compare these costs with ground methods, as this type of data has never been collected on the ground.
CHAPTER VI

ALTERNATIVE EQUIPMENT AND PROCEDURES

Before and during the present study equipment and procedures other than those adopted were investigated. The specific techniques used were selected to fit the project objectives within rather narrow time and cost limitations. In some cases, alternative equipment or procedures appeared to be better suited for the task, but these could not be used primarily because time consuming and/or expensive developmental work was required. If a project of larger scope is envisioned, these alternatives should be re-examined.

Data Collection

Other types of cameras

The Maurer P-2, 70-mm. reconnaissance camera used in this study was loaned to The Ohio State University at no cost by the U.S. Air Force. Other types of cameras may be more suitable, but purchase of an expensive camera was not feasible.

The J. A. Maurer Company has developed a Model 222, 70-mm. reconnaissance camera which is similar to the Model P-2, but the shutter speed may be increased to 1/4000 second and an automatic exposure control system has been added. This camera, with intervalometer, is priced at approximately $9,000. A vacuum back magazine, to insure film flatness, is available at additional cost.
The Maurer Model 500 Aerial Reconnaissance Camera can use either five-inch wide film, resulting in a 4½ x 4½ inch photograph format useful in intersection studies, or 70-mm. film resulting in a 4½ x 2½ inch format for maximum economy in roadway studies. This camera is equipped with a 250 ft. magazine which will permit the taking of approximately 650 photographs without reloading. At one frame per second, this would lengthen the possible study duration to over ten minutes while providing twice the roadway coverage at the same photographic scale used in this study. A clock or frame counter image may be recorded on the photograph. Vacuum film flattening and image motion compensation are provided. The larger format, increased magazine capacity, and film flattening are significant advantages, but the list price of this camera is $23,000.

The Aerojet Delft TA-7M 70-mm. aerial camera can be recycled at rates from 1 to 15 frames per second, but cannot be pulse-operated. The minimum exposure interval is 1/600 second, but high speed lenses up to f/1.1 are available. A double stage disc focal plane shutter is used and automatic exposure control is provided. The price is unknown.

Cameras with formats other than 70-mm. were also considered. The minimum photographic scale which will supply the necessary detail and assure that natural ground control points in sufficient density can be found is approximately 1:10,000. Therefore, the use of 35-mm. film will reduce the roadway coverage to a maximum of about 1000 ft. Similarly, 16-mm. film will provide a coverage of only 500 ft. Roadway coverages as short as these will not provide the desired data.
Standard mapping cameras, with a 9 x 9 in. format, have a minimum recycling time of approximately three seconds. This is too slow to satisfy the objectives of the present study. In other studies, where an interval between exposures of three seconds can be tolerated, standard mapping cameras will provide the advantage of more coverage at the same scale, the same coverage at a larger scale, or some combination of these. Also, lens and film distortions will be smaller and better defined.

The E-24 camera has a 5 x 5 inch format and can recycle at rates up to three frames per second. However, the maximum film capacity is 56 ft., which provides only 125 exposures. In general, this is not enough photographs to encompass the traffic phenomena to be studied.

It is believed that automatic coordinate extraction will be possible only if the interval between exposures is reduced to about 1/5 second, to permit vehicle identification from photograph to photograph by anticipating position changes. A large capacity film magazine will be needed in this case, and this factor should be considered in the selection of an alternative camera.

The camera for any future study should be selected to best fit that project's objectives. The most versatile camera encountered in the equipment investigations of this study is the Maurer Model 500, which seems to be suitable for any photogrammetric traffic study.

Infrared scanners

Mr. Jack D. Frank, in his paper entitled "Traffic Data Acquisition from Aerial Infrared Imagery" (27), discussed the possibility of using
infrared imagery in the collection of data on total vehicle counts, traffic density, vehicle velocities, and spacings between vehicles.

An airborne infrared line scan system makes continuous sweeps across the earth's surface in a direction transverse to the line of flight, using a detector with a small instantaneous field of view. The "sweep" is carried forward by the forward motion of the aircraft. The detector produces an electrical signal which is processed electronically and utilized to intensity modulate a recording lamp or cathode ray tube. These, in turn, expose the photographic film.

This equipment is not being used for traffic studies at present and would have to be modified to provide data on traffic platoon movements. The line scan system as described, produces a continuous image as the aircraft moves forward and does not "retake" the same ground area over and over again. A double scan will provide two images that may be used for a single velocity determination for each vehicle.

However, the system appears to have a very significant characteristic for those types of traffic studies which can utilize a single or double scan in that the image of each "ground spot" is individually defined and processed electronically as a "bit". Thus, if vehicles can be identified by their "bit" characteristics, their position at a specific time would be identifiable and automatic coordinate extraction could be obtained by processing this information. As mentioned before, coordinate extraction is the major bottleneck of the present methods. Another advantage is that this system would operate equally well during the day or night, as it does not depend on a visual image.
Economic considerations may limit this technique, if developed, to projects of larger scope than the present study.

Data Reduction

Comparators

The CMI-Bendix AP/C Analytical Stereoplotter was used to obtain photo coordinates in this study, but less expensive equipment should be satisfactory. The AP/C was used essentially as a precise monocular comparator. Monocular comparators providing satisfactory accuracy can be purchased for approximately $30,000, whereas the price of the AP/C is about $135,000. The output from most of the less expensive instruments can be entered on punch cards directly, thus eliminating the keypunching operation of this study.

Projection-type coordinate reading instruments are also available. In this type, the photographic transparencies are projected onto a ground glass screen, with magnification of up to 20 to 30 times. The operator views the projected image directly rather than through eyepieces as is the case with comparators, and positions a cross-hair or other measuring mark on the desired point through the use of handwheels or a joystick control. The cost of sufficiently accurate equipment of this type is approximately $30,000 to $50,000.

A stereocomparator may be desirable for some traffic studies. The Zeiss PSK Precision Stereocomparator, with typewriter and card punch output, will provide maximum versatility. It is possible to supplement the coordinate data with identification numbers. The instrument provides 8, 12, or 16 times magnification, can handle formats up to $9\frac{1}{2} \times 9\frac{1}{2}$
inches, and coordinate readings are to one micron. The price is approximately $50,000.

**IBM scanning system**

A system developed by the IBM Corporation digitizes a photograph (or selected portions of a photograph), transforms the digitized image to vehicle positions, and then processes these data to obtain any desired output. This scanner converts each 0.001 inch area of the photograph into a "gray scale," digitized 0 through 7. Vehicles are detected by analyzing the digital output for characteristic patterns. Correlation of vehicles between successive photographs are also accomplished through the gray scale patterns, which vary from vehicle to vehicle.

This system is considered technically feasible by the IBM Corporation, but further research and development are required for an economic operating system. At present, the major problems are that the system cannot satisfactorily handle curved highways, more and better programs are needed for the pattern analyses, position determination accuracy for a scale of 1:10,000 is too low, and data processing costs are relatively high as a very large computer memory is required and extensive programming is necessary to extract the significant data.

**Philco scanning system**

The Philco Corporation has also developed a film scanner and has been supplied sample photographs by this project for testing purposes. To date, no meaningful results have been obtained. The system is reportedly faster than the one described by the IBM Corporation, but at
present the output is limited to the total number of vehicles on the photograph and the positions of these vehicles are not provided.

**Programmable film reader**

The Programmable Film Reader developed by Information International, Inc. provides a means of automatically reading and digitizing photographic data. The principal advantage of this equipment over other film scanners is that the film reading process is selective, being controlled by a stored computer program which is designed to locate and track only the data of interest on the film. A separate computer program is designed for each application.

The programmable light source has a raster containing 16,384 x 16,384 programmable light points. If the raster is set to cover the area of interest on 70-mm film, a "spot size" of 19.6 microns, or about 8 inches at ground scale on the traffic photography used in this study, is obtained. The effective spot size can be reduced by scanning the photograph in parts and then correlating the parts. The instrument determines the gray scale density (which may be divided into up to 64 levels) of each "spot".

Prices of the Programmable Film Reading Systems range from $240,000 to approximately $350,000. However, an earlier model than the one just described is available for service; i.e., films can be sent to Information International, Inc. for processing. It is estimated this system could do in an hour the work accomplished by two men at a monocular comparator for a week.

The Programmable Film Reader available for service utilizes a 1024 x 1024 raster, and thereby produces resolution only 1/16 that of the
instrument previously described. Thus, the spot size would be quite large at ground scale if an attempt were made to scan the whole photograph with one raster, and it will be necessary to read the photograph in parts to obtain the desired definition. This procedure can be programmed with no significant difficulty. However, a large number of similar, positively-identifiable ground control points will likely be required. This will necessitate the pre-setting of ground control points and thus limit the study area to that marked.

It is estimated that the cost of developing a suitable program to obtain traffic flow data similar to that obtained in the present study would approximate $7500. In order to make automatic tracking of vehicles from one photograph to the next by position estimation techniques possible, photographs would have to be taken at a significantly faster rate—probably four or five photographs per second. This presents no real problem if a camera with a film magazine of sufficient capacity to provide a suitable study duration is used, as the overall cost of data reduction would still be significantly reduced, even though the quantity would be increased.

Comments on desirable future developments

There are no instruments presently available which can automatically determine photo coordinates of vehicles on a roadway without further development or programming. The Programmable Film Reader can probably be adapted to the task, but only through considerable programming and only if the data collection procedure is changed, as discussed previously.
Data reduction equipment suitable for traffic studies would have the following characteristics:

a. Be able to process complete rolls of film automatically. One of the problems which arises here is that only the portion of each photograph containing the roadway should be processed. Since some tilt and rotation of the camera about a vertical axis is unavoidable and since roadway geometry varies, the "strip" to be examined will occupy a different position and orientation on each photograph. A possible solution might be to outline the area to be scanned on each photograph with a grease pencil or similar marking device. This would require cursory examination of each photograph, which is desirable anyway to check for camera malfunctions or extraordinary events.

b. Provide ground coordinates of each vehicle on each photograph to the nearest six inches for scales of 1:10,000 or larger. This accuracy requirement will necessitate study of the optimum "spot size" if a scanning process is employed. The density of ground control points required to provide suitable transformations will be dependent on the camera and film characteristics.

c. Provide tracking of individual vehicles from one photograph to the next. Tracking may be accomplished through either "position estimation" techniques or vehicle "signature" comparisons. Position estimation techniques utilize the positions of a vehicle on preceding photographs to estimate the range of possible locations on the photograph being examined. This projected area is then scanned to find the exact location of the vehicle. This procedure requires that photographs be taken at a relatively rapid rate (approximately 4 or 5 frames per second).
so that the projected area will be small and discrete. Vehicle signatures have been used by the IBM Corporation to track vehicles on successive photographs. Each vehicle is represented in the computer by several hundred "gray scale" digits. Each vehicle will produce a unique pattern, or signature, which can be compared to the patterns in the preceding photograph for identification. Shadows, re-orientation of the vehicles from one photograph to the next, the possibility of two similar vehicles being in the same area, and computer capacity and time will present problems, but the interval between exposures can be extended to at least one second, and probably more.

d. Be versatile with respect to data input. The data reduction equipment should be capable of handling all scales of 1:20,000 or larger. (The accuracy of output, of course, will vary with the photographic scale.) The equipment should be designed to process a wide range of formats (16-mm to 9 x 9 inches) and various film types (panchromatic, color, Ektachrome Infrared). Various roadway alignments, overhead structures, roadside developments, and types of vehicles (automobiles of varying sizes, trucks, busses, trailers, motorcycles, etc.) will be encountered and the reduction equipment must be capable of handling these variations with little or no additional instruction.

e. Provide the desired data at minimum cost and time. It is unlikely that a single organization engaged in traffic research or operations will be able to justify purchase of the data reduction equipment envisioned. Thus, it is desirable that an instrument with these capabilities be made available on a service basis. This service should provide an identification number, ground coordinates, velocity, and spacing for each vehicle on each photograph. Individual
organizations could then make any additional computations they might desire. Processing time and cost of this service should compare favorably with the manual methods used in the present study.
CHAPTER VII

DISCUSSION OF ERRORS

The computed positions of the vehicles are in error due to inaccuracies in the data collection and coordinate reading equipment and techniques, and due to assumptions made in the computation methods. The effect of the various sources of error on vehicle positions and attendant general considerations are discussed initially. The effect of these errors on the spacing between vehicles is discussed next. The spacing involves two vehicles on one photograph. Velocity determination errors are then considered—involving one vehicle on two photographs.

Effect of Tilt

Tilt is defined as the angle between the optical axis of the camera and a vertical line. It is also the angle which the image plane (photograph) makes with a horizontal plane. Tilt is ordinarily resolved into two components. One component is about a line parallel to the flight line—termed the $\omega$-tilt or $x$-tilt. The other component is taken about an axis normal to the $x$-axis and is termed $\phi$-tilt or $y$-tilt.

Although an effort was made to keep the camera axis vertical during the photographic flight by hand-orientation using a bulls-eye level, a certain amount of tilt was inevitable. The effect of this tilt on
the calculated ground position of a vehicle is shown in Figure 11.

Points A and B are ground control stations which are imaged at a and b on the photograph. \( v_t \) is the image of a vehicle \( V_t \) on the roadway between ground points A and B. In the transformation of photo coordinates to ground coordinates used in this study, the assumption was made that the ground position of any vehicle between points A and B could be found by multiplying the photo distance between the vehicle and the image of one control point by the scale factor determined by the ratio \( ab/AB \). This is not quite correct, as is illustrated in Figure 11. If \( k \) is the ratio of \( (v_t \ b)/AB \), then the position of the vehicle will be computed, with respect to B as \( k (AB) \), which is the position indicated by point \( V_c \). This discrepancy between the true position of the vehicle, \( V_t \), and the computed position, \( V_c \), is indicated as \( E \)--the error. Note that the error due to the assumptions in the computation procedure will become zero as the vehicle approaches either point A or point B.

In deriving an expression for \( E \), the following quantities (illustrated in Figure 11) are required.

- \( f \)--focal length of camera
- \( H \)--flying height above roadway
- \( t \)--component of tilt angle about an axis normal to the roadway alignment. (The other component will not effect \( E \).)
- \( p \)--the center of the photograph (also has point where the optical axis pierces the image plane).
- \( \alpha \)--the angle from the optical axis to the ray from the lens to ground control point A.
- \( \beta \)--the angle from the optical axis to the ray from the lens to ground control point B.
- \( \theta_t \)--the angle from the optical axis to the ray from the lens to the true position of the vehicle in question.
- \( \gamma_c \)--the angle from the optical axis to the ray from the lens to the calculated position of the vehicle.
Fig. 11.—Error in Computed Vehicle Position Due to Camera Tilt
N— the nadir point, a point vertically below the camera station at the moment of exposure.

n— image of nadir point

i— the isocenter; a point halfway (in terms of the angles subtended at L) between the nadir point and p, the principal point.

In Figure 11, the horizontal line intersecting the photograph at i represents a vertical photograph with the same focal length and from the same flying height. The primed letters a', b', n', v_t', v_c' represent the images that points A, B, N, V_t, V_c would have had on a vertical photograph. These points are then rotated about the isocenter to their corresponding positions on the tilted photograph, shown by the double primed letters — a'', b'', v_t'', and v_c''.

The displacement due to tilt, then, is the distance from the actual image to the double primed position, i.e., aa'', bb'', v_t v_t'', and v_c v_c''. This displacement is given by:

\[ aa'' = ia - ia'' = ia - ia' = (pa - ip) - (n'a' + n'i) \]

Where:

\[ pa = x_a = (f) \cdot \tan \alpha \] (x is distance from p to given point)

\[ ip = x_i = (f) \cdot \tan(\alpha/2) \]

\[ n'a' = (f) \cdot \tan(\alpha-t) \]

\[ n'i = (f) \cdot \tan(\alpha/2) \]

Therefore:

\[ aa'' = (f) \cdot \tan \alpha - (f) \cdot \tan(\alpha/2) - (f) \cdot \tan(\alpha-t) - (f) \cdot \tan(\alpha/2) \]

\[ = f [\tan \alpha - 2 \tan(\alpha/2) - \tan (\alpha-t)] \]

(The above basic derivation is given in "Photogrammetry", by Francis H. Moffitt (28).)
Similarly:

\[ bb'' = f \left[ \tan \beta - 2 \tan(t/2) - \tan(\alpha-t) \right] \]

\[ v_t v_t'' = f \left[ \tan \gamma_t - 2 \tan(t/2) - \tan(\nu_t-t) \right] \]

\[ v_c v_c'' = f \left[ \tan \gamma_c - 2 \tan(t/2) - \tan(\nu_c-t) \right] \]

Then:

\[ pa'' = pa - aa'' = f \left[ 2 \tan(t/2) + \tan(\alpha-t) \right] \]

\[ pb'' = pb - bb'' = f \left[ 2 \tan(t/2) + \tan(\beta-t) \right] \]

\[ pv_t'' = pv_t - v_t v_t'' = f \left[ 2 \tan(t/2) + \tan(\nu_t-t) \right] \]

\[ pv_c'' = pv_c - v_c v_c'' = f \left[ 2 \tan(t/2) + \tan(\nu_c-t) \right] \]

Distances on the vertical photograph are:

\[ a'b' = a''b'' = pa'' - pb'' = f \left[ \tan(\alpha-t) - \tan(\beta-t) \right] \]

\[ v_t'b' = v_t''b'' = pv_t'' - pb'' = f \left[ \tan(\nu_t-t) - \tan(\nu_c-t) \right] = k(a''b'') \]

\[ v_c'b' = v_c''b'' = pv_c'' = f \left[ \tan(\nu_c-t) - \tan(\beta-t) \right] \]

The "error" on the vertical photograph is:

\[ e = v_t v_c'' = v_t''b'' - v_c''b'' = f \left[ \tan(\nu_t-t) - \tan(\nu_c-t) \right] \]

Ground distances are:

\[ AB = NA - NB = H \left[ \tan(\alpha-t) - \tan(\beta-t) \right] \]

\[ V_C B = NV_C - NB = H \left[ \tan(\nu_C-t) - \tan(\nu_c-t) \right] = k(AB) \]

\[ V_T B = NV_T - NB = H \left[ \tan(\nu_t-t) - \tan(\beta-t) \right] \]

The ground error is:

\[ E = V_T B - V_C B = H \left[ \tan(\nu_t-t) - \tan(\nu_c-t) \right] \]

Substituting:

\[ E = V_T B - V_C B = H \left[ \tan(\nu_t-t) - \tan(\beta-t) - (k)\tan(\alpha-t) \right] + (k)\tan(\beta-t) \]
Or:

\[ E = \frac{H}{f}e = \frac{H}{f} \left[ (k^2)\tan(\alpha-t) - (k^2)\tan(\phi-t) - (f)\tan(\mu^c-t) + (f)\tan(\phi-t) \right] \]

Solving for \( \tan(\mu^c-t) \):

\[ \tan(\mu^c-t) = (2k)\tan(\alpha-t) - (2k)\tan(\phi-t) + (2)\tan(\phi-t) - \tan(\mu^c-t) \]

\[ = 2\left[ (k)\tan(\alpha-t) + (1-k)\tan(\phi-t) - \frac{k}{2}\tan(\mu^c-t) \right] \]

Substituting this value in the previous formula for ground error gives:

\[ E = H \left[ \tan(\mu^c-t) - (2k)\tan(\alpha-t) - 2(1-k)\tan(\phi-t) + \tan(\mu^c-t) \right] \]

\[ = 2H \left[ \tan(\mu^c-t) - (k)\tan(\alpha-t) + (k-1)\tan(\phi-t) \right] \]

Since \( \mu^c \) is a function of \( k, \alpha, \) and \( \phi \), it must be expressed in that form before differentiation is possible.

\[ k = \frac{x_v-t - x_b}{x_a - x_b} = \frac{(f)\tan(\mu^c_t) - (f)\tan(\phi)}{\tan(\alpha) - (f)\tan(\phi)} = \frac{\tan(\mu^c_t) - \tan(\phi)}{\tan(\alpha) - \tan(\phi)} \]

Hence:

\[ \tan(\mu^c_t) = k(\tan(\alpha) - \tan(\phi)) - \tan(\phi) \]

\[ \tan(\mu^c-t) = \frac{\tan(\mu^c_t) - \tan(t)}{1 + \tan(\mu^c_t)\tan(t)} \]

With appropriate substitutions and combining terms:

\[ E = 2H \left[ k(\tan(\alpha) - \tan(\phi)) + \tan(\phi) - \tan(t) \right] \]

\[ + (k-1) \frac{\tan(\phi) - \tan(t)}{1 + \tan(\phi)\tan(t)} \]

\[ - k \frac{\tan(\alpha) - \tan(t)}{1 + \tan(\alpha)\tan(t)} \]
This equation, for the error in determining a vehicle's position due to disregarding any tilt of the photograph in the transformation procedure, contains only factors which can be obtained by direct measurements on the tilted photograph and knowledge of the focal length of the taking camera. Thus, the error can be determined if the tilt is known.

The magnitude of the error, \( E \), obviously gets larger as the flying height increases or the distance between ground control points gets larger (reflected in the quantity \( (\tan \alpha \cdot \tan \phi) \)). The error also increases with an increase in tilt.

The value of \( k \) which maximizes \( E \) may be determined by differentiating the error equation with respect to \( k \), setting the derivative to zero, and solving for \( k \). This results in the following relationship:

\[
k = \frac{c - \sqrt{c}}{c - 1}
\]

where:

\[
c = \frac{(\tan \phi)(\tan t) + 1}{(\tan \alpha)(\tan t) + 1}
\]

Analysis of the 101 photographs revealed that the maximum tilt (component along roadway) was seven degrees; the actual range being from 7° in one direction to 5° in the other. (The method used to determine the tilt is described in the next sub-section.)

Further examination of the project data revealed the maximum spacing of ground control points to be 260 ft. The flying height was nearly constant, varying less than five percent, at 2350 feet above ground level. For ground control point placement relative to the photograph, the critical case occurs when one of the control points is
located at the lower end of the photograph. This corresponds to an \( \alpha \) value of +20°30' for the photography used in this study. (The tangent of this angle is determined by dividing the distance from the center of the photograph to the outermost control point by the focal length of the camera.) The distance from the nadir point to this outermost point for a tilt of 7° is:

\[
\text{Distance} = H \left[ \tan(\alpha - t) \right] = 2350(\tan 13°30') = 564 \text{ ft.}
\]

The critical value of \( \alpha \) is such that:

\[
H \left[ \tan(\alpha - t) \right] = 564 - 260 = 304 \text{ ft.}
\]

Solution of this equation gives a value of +14°23' for \( \alpha \).

Thus, if the maximum interval between ground points occurs at the worst possible position on the photograph with the maximum tilt, and a vehicle is imaged so as to obtain the worst possible value of \( k \), the error in the vehicle's calculated position, due to tilt, will be 1.79 ft., where \( k = 0.498 \).

An error of this magnitude did not occur in this study, as the combination of events described above did not occur. The error varies if any one of the factors is not at the "worst" value. This variation is shown in Table 1. The various combinations represent the following conditions.

Case I. The worst combination of events possible, using the worst values that occurred for the various factors.

Case II. Tilt was reduced to its median value of two degrees—the other factors remaining the same. (only five of the 101 photographs were tilted five degrees or more. Eighty-five percent were tilted less than three degrees.) This change will affect the critical value of \( \alpha \) as shown previously.
### TABLE 1

**POSITION ERROR DUE TO TILT FOR VARIOUS CASES**

<table>
<thead>
<tr>
<th>Case</th>
<th>$t$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$k$</th>
<th>Error (ft.)</th>
<th>Reduction from Case I (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$7^\circ$</td>
<td>$+20^\circ30'$</td>
<td>$+14^\circ23'$</td>
<td>0.4983</td>
<td>1.79</td>
<td>--</td>
</tr>
<tr>
<td>II</td>
<td>$2^\circ$</td>
<td>$+20^\circ30'$</td>
<td>$+14^\circ37'$</td>
<td>0.4996</td>
<td>0.46</td>
<td>74</td>
</tr>
<tr>
<td>III</td>
<td>$7^\circ$</td>
<td>$+3^\circ07'$</td>
<td>$-3^\circ07'$</td>
<td>0.4983</td>
<td>1.74</td>
<td>3</td>
</tr>
<tr>
<td>IV</td>
<td>$7^\circ$</td>
<td>$+20^\circ30'$</td>
<td>$+16^\circ09'$</td>
<td>0.4988</td>
<td>0.93</td>
<td>48</td>
</tr>
<tr>
<td>V</td>
<td>$7^\circ$</td>
<td>$+20^\circ30'$</td>
<td>$+14^\circ23'$</td>
<td>0.2500</td>
<td>1.36</td>
<td>24</td>
</tr>
<tr>
<td>VI</td>
<td>$2^\circ$</td>
<td>$+2^\circ15'$</td>
<td>$-2^\circ15'$</td>
<td>0.2500</td>
<td>0.19</td>
<td>89</td>
</tr>
</tbody>
</table>
Case III. The angle $\alpha$ was changed so that half the photograph will have a more critical value of $\alpha$ and half will have a less critical value. This reflects the effect of a vehicle's relative position on the photograph on the calculated error. All other factors were held at their most critical value.

Case IV. The spacing between ground control points was reduced from 260 ft. to 186 ft. The tilt and $k$ were given their most critical values.

Case V. The factor $k$ was given a value of 0.25. This factor expresses the position of a vehicle in relation to the ground control points. On the average, half the vehicles should be within one-quarter of a ground control spacing of a ground control point. The other factors remained at their critical values.

Case VI. Median values for all the factors were used.

The magnitude of the error decreases almost linearly with a decrease in the tilt angle. Comparing Case I with Case II, it is seen that reducing the tilt from $7^\circ$ to $2^\circ$ (a reduction of 71%) decreases the maximum possible vehicle position error on the photograph by 74 percent.

Comparing Case III with Case I shows that the position of the vehicle image relative to the center of the photograph has little effect on the position error. A reduction of only 3 percent is obtained if the vehicle image is assumed at the center of the photograph. The error decreases only by 6.7 percent as the vehicle travels from the "low" side of the photograph to the "high" side. (The previous statement is true if the vehicle remains at its critical $k$ value relative to the ground control points and the ground control point spacing remains at the maximum value of 260 feet.) Thus, for the tilts to be expected and the range of $\alpha$ values possible for photography with the camera used, the position of the vehicle image relative to the photograph center has little effect on the magnitude of the position error.
Case IV illustrates the effect of reducing the ground control point spacing. A reduction of 28 percent in spacing reduces the error by 48 percent.

Comparing Case V with Case I illustrates the effect of varying the vehicle's position relative to the ground control points. If the vehicle is at the quarter-point instead of the mid-point, the error will be 24 percent smaller. (The error is zero, of course, when $k = 0$ or 1.0.)

Case VI represents the error when none of the factors is at its critical value, but all are at a value which represents a "median" condition for the factor--i.e., about half the time it will be worse and half the time it will be better. This reduces the error by 89 percent—to two inches on the ground.

The effect of the tilt on vehicle spacing and velocity determinations will be discussed later. This error is systematic; i.e., it can be determined and compensated for if desired. All the required information is available: focal length of the camera; photo coordinates of the vehicles and ground control points; and the ground coordinates of the control points. All other parameters used in determining the error can be derived from these quantities.

The tilt can probably be held to three degrees or less in subsequent flights. There was only one person, other than the pilot, in the helicopter during the flight used in this study. This person had to watch the traffic and instruct the pilot as well as keep the level bubble on the camera mount centered during flight. The larger tilts in this flight occur in "groups", covering five or ten seconds, and all tilts in the group are in the same direction. In future flights, it is
advisable to have an observer so that the camera operator can concentrate on maintaining the camera optical axis vertical.

**Determination of tilt angle**

Referring back to Figure 11, the scale at any "point" on the line representing the photograph is defined as the ratio of the length $L_a$ (from the lens to the photograph) to the length $L_A$ (from the lens to the ground point).

Thus:

$$S_p = \frac{L_a}{L_A} = \text{scale at a "point"}$$

$$L_a = \frac{x_a}{\sin \alpha}$$

$$L_A = \frac{H}{\cos(\alpha - t)}$$

$$S_p = \frac{L_a}{L_A} = \frac{x_a}{H} \left( \frac{\cos(\alpha - t)}{\sin \alpha} \right)$$

Using the trigonometric substitution for the difference of two angles:

$$S_p = \frac{x_a}{H} \left[ \frac{(\cos \alpha)(\cos t) + (\sin \alpha)(\sin t)}{\sin \alpha} \right] = \frac{x_a}{H} \left[ \sin t + \frac{\cos t}{\tan \alpha} \right]$$

$$\tan \alpha = \frac{x_a}{f}$$

Therefore:

$$S_p = \frac{x_a}{H} \left[ \sin t + \frac{(f)(\cos t)}{x_a} \right] = \frac{(x_a)(\sin t)}{H} + \frac{(f)(\cos t)}{H}$$

Differentiating $S_p$ with respect to $x_a$:

$$\frac{dS_p}{dx_a} = \frac{\sin t}{H}$$
If \( x_a \) is measured positive to the right, as is ordinarily done, the formula becomes:

\[
\frac{dS}{dx_a} = - \frac{\sin t}{H} = - \frac{\sin t}{(f)(S,F_*)}
\]

Where \((S,F_*)\) is the scale factor = \(\frac{1}{S}\).

Thus, the change in scale due to tilt, with respect to a distance measured along the photograph, is linear.

An examination of the scale factor data showed that photograph 93 was tilted more than any other. The scale for nine intervals across the photograph was available as the photo contained the images of ten ground control points. It was assumed the scale at a point half-way between pairs of ground control points was equal to the scale over the interval. Scale (equal to the inverse of the scale factor) was plotted against the x-coordinate of mid-points of the corresponding pair of ground control point images on the photograph, as is shown in Figure 12.

A line of best fit was drawn and the slope of this line determined to be \(-18.22 \times 10^{-8}\) per mm. Hence:

\[
\sin t = + 0.000001822(8954)(75.82) = 0.1237
\]

\[
t = + 7^\circ
\]

where 8954 is the scale factor at the center of photograph 93 and 75.82 mm. is the focal length of the camera.

In the preceding derivation, it was assumed the ground was flat. On the section of freeway used in this study, the maximum grade was 2%. This corresponds to a ground "tilt" of 1°. The section covered by
Fig. 12.—Scale Variation on Photograph No. 93
photograph 93 is essentially level, so that the ground tilt does not enter the problem. The method used to determine tilt gives the tilt of the photograph relative to the ground, and this is the tilt required in the error equation. If true tilt, with respect to a plumb line is required, the ground "tilt" must be added or subtracted. If the section of roadway being studied does not have a constant grade, the tilt determination will be adversely affected. Solution would be possible if the ground profile were known.

Effect of Vertical Alignment

The section of roadway covered in this study is relatively level—the maximum grade being 2%. Table 2 summarizes the vertical alignment as shown on construction plans.

The length of roadway traversed by a vehicle differs from the calculated length (assuming the alignment was horizontal) by only 0.47 feet in a total distance of approximately 7350 feet. The length differentials for the vertical curves were approximated by assuming straight-line sections from the end of the vertical curve to the center point. The discrepancy at the maximum grade of two percent is only 0.02 feet per 100 feet. This is about equal to the accuracy of the ground survey, and represents a scaling error of 1:5000.

Therefore, for the grades encountered in this study, no significant error in determining vehicle travel distances is introduced by assuming the roadway is level.

Effect of Horizontal Alignment

The horizontal alignment of the section of roadway studied is
### TABLE 2

**VERTICAL ALIGNMENT OF ROADWAY**

<table>
<thead>
<tr>
<th>Approximate Station</th>
<th>Grade (%)</th>
<th>Length of Section (ft.)</th>
<th>Length of Differential (ft.) a</th>
</tr>
</thead>
<tbody>
<tr>
<td>162+00</td>
<td>V.C.</td>
<td>230</td>
<td>0.01</td>
</tr>
<tr>
<td>164+30</td>
<td>+2.00</td>
<td>560</td>
<td>0.11</td>
</tr>
<tr>
<td>169+90</td>
<td>V.C.</td>
<td>1160</td>
<td>0.06</td>
</tr>
<tr>
<td>181+50</td>
<td>-2.00</td>
<td>190</td>
<td>0.04</td>
</tr>
<tr>
<td>183+40</td>
<td>V.C.</td>
<td>600</td>
<td>0.03</td>
</tr>
<tr>
<td>189+40</td>
<td>+2.00</td>
<td>830</td>
<td>0.18</td>
</tr>
<tr>
<td>197+70</td>
<td>V.C.</td>
<td>600</td>
<td>0.01</td>
</tr>
<tr>
<td>203+70</td>
<td>+0.32</td>
<td>1780b</td>
<td>0.02</td>
</tr>
<tr>
<td>218+00</td>
<td>V.C.</td>
<td>400</td>
<td>0.00</td>
</tr>
<tr>
<td>222+00</td>
<td>-0.32</td>
<td>700</td>
<td>0.01</td>
</tr>
<tr>
<td>229+00</td>
<td>V.C.</td>
<td>300</td>
<td>0.00</td>
</tr>
<tr>
<td>232+00</td>
<td></td>
<td>7350</td>
<td>0.47</td>
</tr>
</tbody>
</table>

a (Distance along roadway) minus (Survey, or horizontal, distance)

b Includes station equation of approximately 350 ft.
defined in Table 3. There are only two curves—a 1°15' curve of 2400 feet and a 1° curve of 1600 feet.

In determining the relationship between $D$, distance traveled along the road from some arbitrary point, and the ground-coordinate system, it was necessary to approximate the roadway centerline curves by a series of straight lines. Specific vehicles on specific photographs were chosen to be the points at the ends of these straight line sections. Care was taken to select vehicles which were in the center of the lane. Another criterion for selecting the vehicles used to define the centerline was that a straight line between adjacent "control vehicles" was to be within the lane throughout its length.

The centerline for the section of highway under study was defined by 17 straight-line sections—defined by 18 vehicle-points. Ten lines were used to approximate the 2400-foot 1°15' curve. The longest line was 329.40 feet. The angle subtended by a chord of this length may be determined from the following formula:

$$L.C. = 329.40 = 2R\sin\left(\frac{\Delta}{2}\right)$$

where $R = \text{radius of 1°15' curve}$

$$\Delta = 4.1184^\circ$$

The corresponding length of circular arc is:

$$L = (100)\frac{\Delta}{D} = \frac{411.84}{1.25} = 329.47 \text{ ft.}$$

Thus, the true distance traveled was 0.07 feet longer than the calculated, or straight-line, distance. Similar corrections were computed for the other nine lines used to define this curve. The difference in length
<table>
<thead>
<tr>
<th>Approximate Station</th>
<th>Length of Section (ft.)</th>
<th>Degree of Curvature</th>
<th>Length Differential (ft.)&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>162+00</td>
<td>500</td>
<td>tangent</td>
<td>0</td>
</tr>
<tr>
<td>167+00</td>
<td>2400</td>
<td>1°15'</td>
<td>0.42</td>
</tr>
<tr>
<td>191+00</td>
<td>400</td>
<td>tangent</td>
<td>0</td>
</tr>
<tr>
<td>195+00</td>
<td>1600</td>
<td>1°</td>
<td>0.17</td>
</tr>
<tr>
<td>211+00</td>
<td>2450&lt;sup&gt;b&lt;/sup&gt;</td>
<td>tangent</td>
<td>0</td>
</tr>
<tr>
<td>232+00</td>
<td>7350</td>
<td></td>
<td>0.59</td>
</tr>
</tbody>
</table>

<sup>a</sup> (Arc length) minus (Sum of straight-line lengths)

<sup>b</sup> Including station equation of approximately 350 ft.
arc minus sum of straight-lines) is 0.42 feet over a total arc length of 2400 feet.

Similar calculations for the 1° curve, which was approximated by six straight lines, resulted in a total difference in length of 0.17 feet, with the greatest single difference being 0.07 feet for a 380 ft. chord.

Thus, the error due to approximating the actual curved centerline by a series of straight lines totals 0.59 feet over the total roadway length of about 7350 feet. The maximum rate of "shortening" was:

\[
\frac{0.07}{329.47} = \frac{1}{4700}
\]

This error is also a scaling error and, like the error created by assuming a level profile, is a systematic error which could be compensated for if desired. However, the magnitude is such that no compensation is warranted.

**Effect of Vehicle Relief**

In determining the photo coordinates of the vehicles, the center of the front of each vehicle is used as the target. Unfortunately, this target point is not at the roadway elevation whereas the ground control points are. This vehicle relief with respect to the roadway creates an error in the vehicle position determination due to the perspective nature of a photograph. (See Figure 13.)

The front of vehicle A in Figure 13 has the same ground X-coordinate as point Q. Both the front of vehicle A and point Q will be imaged at the center of the photograph at point a, or q. (Assuming no tilt.) Therefore, there will be no error in the calculated position.
Fig. 13.—Vehicle Relief Displacement
Vehicle B, on the other hand, will be measured at b on the photograph even though the front of the vehicle is at the same ground X-coordinate as point R which will be imaged at r -- different from b. The effect is to give vehicle B a calculated ground position of R'. This error, RR', will have a negative value and will vary with the angle \( \alpha \) and the height of the vehicle's front bumper. The maximum possible value of \( \alpha \) for the photography used in this study is \( 20^\circ 30' \) and the average vehicle bumper height is approximately 1.5 feet. Thus, the maximum position error for vehicles on the left side of the photograph is:

\[
\text{Error} = -(1.5)(\tan 20^\circ 30') = -0.56 \text{ ft.}
\]

This error decreases in absolute magnitude as the vehicle approaches the center of the photograph.

Similarly, the position of vehicle C is affected because the front of the hood is used for a target when a vehicle is on the right half of the photograph. Thus, its computed X-ground coordinate will be S' whereas its true X-coordinate is S. The error will be positive in this case. The hood of a car approximately 3.0 feet above the roadway, and thus the maximum possible error will be:

\[
\text{Error} = (3.0)(\tan 20^\circ 30') = +1.12 \text{ ft.}
\]

Vertical photography was assumed in the above discussion. The effect of tilt on vehicle relief displacement is illustrated graphically in Figure 14. If \( \phi \) is the angle from the plumb line to the edge of the photograph, no vehicle hood in front of vehicle C will be imaged on the photograph and the maximum possible error is as cited above. If the
Fig. 14. — Effect of Tilt on Vehicle Relief Displacement
photograph is tilted an angle $t$, vehicle $D$ will appear on the photograph and the error due to its relief will be:

$$E = H \left[ \tan(\alpha + t) \right]$$

The maximum tilt experienced in this study was $7^\circ$. Thus the maximum possible error for any vehicle on any photograph is:

$$E = 2350(\tan 27^\circ30') = 1.56 \text{ ft.}$$

This tilt will reduce the length of roadway to the left of the plumb line which will be covered on the photograph. Thus, the maximum error on the left side becomes:

$$E = -H \tan(\alpha - t) = -2350(\tan 13^\circ30') = -0.36 \text{ ft.}$$

The errors due to vehicle relief at the left side, center, and right side of the photograph for various tilt angles are given in Table 4.

As in the case of the errors due to tilt, these errors due to vehicle relief are systematic and can be computed for any vehicle if the tilt of the photograph, the focal length of the taking camera, and the photo coordinates of the vehicle are known. The effect of these errors on headways and velocities is relatively small, as will be shown later.

**Accuracy of Photo Coordinate Determinations**

The photo coordinates of all the ground control points and vehicles appearing on one photograph (photograph number 113) were read twice to obtain data on the accuracy of the coordinate measurements.
TABLE 4

EFFECT OF TILT ON ERRORS DUE TO VEHICLE RELIEF

<table>
<thead>
<tr>
<th>Tilt</th>
<th>Left Side of Photograph</th>
<th>Center of Photograph</th>
<th>Right Side of Photograph</th>
</tr>
</thead>
<tbody>
<tr>
<td>+7°</td>
<td>-0.36</td>
<td>+0.37</td>
<td>+1.56</td>
</tr>
<tr>
<td>+2°</td>
<td>-0.50</td>
<td>+0.10</td>
<td>+1.24</td>
</tr>
<tr>
<td>0°</td>
<td>-0.56</td>
<td>0</td>
<td>+1.12</td>
</tr>
<tr>
<td>-2°</td>
<td>-0.62</td>
<td>-0.05</td>
<td>+1.00</td>
</tr>
<tr>
<td>-7°</td>
<td>-0.78</td>
<td>-0.18</td>
<td>+0.72</td>
</tr>
</tbody>
</table>
A total of ten ground control points and 41 vehicle points were read and then repeated.

The Nistri AP/C Analytical Plotter was used to obtain the photo coordinates. The least-count on the print-out from this instrument was 0.001 mm., or 1 micron, at the image scale.

The values shown as $\Delta x$'s in Table 5 were obtained by subtracting the first x reading on a given point from the second x reading. The $\Delta y$'s were obtained in a similar manner. The average value of $\Delta x$ is $-4$ microns and the average value of $\Delta y$ is also $-4$ microns. If the reading errors are truly random, these values should be zero. Hence, it was assumed there was a coordinate shift between the time of the first and second readings. (This is probable, as some orientation elements on the Nistri Plotter were changed and then reset between readings.) Four microns were added to each of the $\Delta x$ and $\Delta y$ values and the adjusted differences listed as $\Delta x_a$ and $\Delta y_a$. These values, then, represent the differences in the two readings of the 51 points when both readings are referred to the same coordinate system.

An estimate of $s^2$, the variance of the coordinate measurements, may be obtained by taking the sum of the squares of the differences between the duplicate measurements and dividing by twice the number of points, or, what is equivalent in this case, by the total number of observations. Thus:

$$s^2 = \frac{\sum d^2}{2k}$$

### TABLE 5

**Differences of Paired Coordinate Measurements (microns)**

<table>
<thead>
<tr>
<th>Control Points</th>
<th>( \Delta x )</th>
<th>( \Delta x_a )</th>
<th>( \Delta y )</th>
<th>( \Delta y_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>208</td>
<td>-7</td>
<td>-3</td>
<td>-3</td>
<td>+1</td>
</tr>
<tr>
<td>209</td>
<td>-11</td>
<td>-7</td>
<td>-7</td>
<td>-3</td>
</tr>
<tr>
<td>210</td>
<td>-5</td>
<td>-1</td>
<td>+5</td>
<td>+9</td>
</tr>
<tr>
<td>211</td>
<td>-9</td>
<td>-5</td>
<td>-1</td>
<td>+3</td>
</tr>
<tr>
<td>212</td>
<td>-5</td>
<td>-1</td>
<td>-7</td>
<td>-3</td>
</tr>
<tr>
<td>213</td>
<td>+1</td>
<td>+5</td>
<td>-1</td>
<td>+3</td>
</tr>
<tr>
<td>214</td>
<td>+3</td>
<td>+7</td>
<td>-7</td>
<td>-3</td>
</tr>
<tr>
<td>215</td>
<td>-5</td>
<td>-1</td>
<td>-3</td>
<td>+1</td>
</tr>
<tr>
<td>216</td>
<td>-19</td>
<td>-15</td>
<td>-3</td>
<td>+1</td>
</tr>
<tr>
<td>217</td>
<td>+9</td>
<td>+13</td>
<td>-7</td>
<td>-3</td>
</tr>
</tbody>
</table>

**Vehicles**

<p>| 818            | -6             | -2             | -12          | -8           |
| 819            | +4             | +8             | -2           | +2           |
| 820            | 0              | +4             | -10          | -6           |
| 821            | +2             | +6             | -4           | 0            |
| 822            | +8             | +12            | -12          | -8           |
| 823            | +2             | +2             | -8           | +4           |
| 824            | +4             | +8             | +4           | +2           |
| 825            | +4             | +8             | +4           | +2           |
| 827            | -8             | -4             | +2           | +4           |
| 828            | -2             | +2             | +2           | +4           |
| 533            | -12            | -8             | -8           | -4           |
| 829            | +4             | +8             | -2           | -2           |
| 830            | -12            | -8             | -8           | -4           |
| 831            | -8             | -4             | -8           | -4           |
| 832            | -2             | +2             | -8           | -4           |
| 833            | -4             | 0              | 0            | +4           |
| 834            | -12            | -8             | +4           | +8           |
| 835            | -10            | -6             | 0            | 0            |
| 517            | -5             | -1             | -14          | -10          |
| 518            | -11            | -7             | -2           | +2           |
| 519            | -3             | +1             | 0            | +4           |
| 520            | -5             | -1             | +2           | +6           |
| 539            | -5             | -1             | 0            | +4           |
| 521            | -15            | -11            | -2           | +2           |
| 522            | -5             | -1             | -4           | 0            |
| 523            | -9             | -5             | +2           | 0            |
| 524            | -5             | -1             | -4           | 0            |
| 536            | -6             | -2             | -8           | 0            |
| 525            | -3             | +1             | -4           | 0            |
| 826            | -5             | -1             | -4           | 0            |
| 534            | -1             | +3             | -2           | +2           |</p>
<table>
<thead>
<tr>
<th>Vehicles</th>
<th>$\Delta x$</th>
<th>$\Delta x_a$</th>
<th>$\Delta y$</th>
<th>$\Delta y_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>526</td>
<td>-3</td>
<td>+1</td>
<td>-8</td>
<td>-4</td>
</tr>
<tr>
<td>532</td>
<td>-5</td>
<td>-1</td>
<td>-6</td>
<td>-2</td>
</tr>
<tr>
<td>527</td>
<td>-1</td>
<td>+3</td>
<td>-6</td>
<td>-2</td>
</tr>
<tr>
<td>528</td>
<td>-1</td>
<td>+3</td>
<td>0</td>
<td>+4</td>
</tr>
<tr>
<td>531</td>
<td>-5</td>
<td>-1</td>
<td>-8</td>
<td>-4</td>
</tr>
<tr>
<td>530</td>
<td>+5</td>
<td>+9</td>
<td>+2</td>
<td>+6</td>
</tr>
<tr>
<td>535</td>
<td>-1</td>
<td>+3</td>
<td>0</td>
<td>+4</td>
</tr>
<tr>
<td>537</td>
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<td>-13</td>
<td>+2</td>
<td>+6</td>
</tr>
<tr>
<td>538</td>
<td>+5</td>
<td>+9</td>
<td>-8</td>
<td>-4</td>
</tr>
<tr>
<td>540</td>
<td>+1</td>
<td>+5</td>
<td>-2</td>
<td>+2</td>
</tr>
</tbody>
</table>
For the control points, the variance and standard deviation of the x and y measurements are estimated as:

\[
\begin{align*}
\sigma_x^2 &= \frac{\sum (x_k - \bar{x})^2}{2k} = \frac{554}{2(10)} = 27.70 \\
\therefore s_x &= \pm 5.27 \text{ microns} \\

\sigma_y^2 &= \frac{\sum (y_k - \bar{y})^2}{2k} = \frac{138}{2(10)} = 6.90 \\
\therefore s_y &= \pm 2.63 \text{ microns}
\end{align*}
\]

The standard deviation of the position determination is:

\[
\sigma_p = \sqrt{\sigma_x^2 + \sigma_y^2} = \pm 5.88 \text{ microns}
\]

Similar values for the vehicle position determinations are:

\[
\begin{align*}
\sigma_x^2 &= \frac{\sum (x_k - \bar{x})^2}{2k} = \frac{1330}{2(41)} = 16.22 \\
\therefore s_x &= \pm 4.03 \text{ microns} \\

\sigma_y^2 &= \frac{\sum (y_k - \bar{y})^2}{2k} = \frac{992}{2(41)} = 12.10 \\
\therefore s_y &= \pm 3.48 \text{ microns}
\end{align*}
\]

\[
\sigma_p = \pm 5.32 \text{ microns}
\]

The fact that the standard deviation in control point position determination is larger than that for the vehicle position determination may indicate there is some difficulty in identifying these control points and that the points used—natural features, primarily light poles—will not be acceptable if a significantly smaller scale photography is used in the future. However, a larger sample of comparison coordinate determinations, particularly in the case of control points, are required to validate this conclusion as the inherent error of the coordinate
reading instrument is in the order of ±4 microns, which approximates the standard error of the position determinations.

It is interesting to note the operators could reproduce their readings for the center of the front of the vehicle, indicated by $s_y$, better than the front of the vehicle itself, indicated by $s_x$.

The values obtained for the standard deviations of position determinations at the image may be expanded to ground values through the scale factor.

$$ S_{PG} = \frac{H}{f} s_p = \frac{(2350 \text{ ft.})(40.0058 \text{ mm.})}{75.82 \text{ mm.}} $$

$$ = \pm 0.182 \text{ ft.} \quad \text{(Ground control points)} $$

$$ S_{PV} = \frac{H}{f} s_p = \frac{(2350 \text{ ft.})(40.00532 \text{ mm.})}{75.82 \text{ mm.}} $$

$$ = \pm 0.165 \text{ ft.} \quad \text{(Vehicles)} $$

The standard deviation in determining the distance between two ground control points is:

$$ S_{LG} = \sqrt{2} S_{PG} = \sqrt{2}(\pm 0.182 \text{ ft.}) = \pm 0.257 \text{ ft.} $$

These measurement deviations are random and no compensation is possible. The effect of the measurement accuracy on the accuracy of headway and velocity determinations will be discussed later.

**Lens and Film Distortions**

Lens and film distortions are discussed together as it is not feasible to separate the two in this case. The camera used, a Maurer P-2,
is basically a reconnaissance type camera and was not designed for photogrammetric measurements.

Significant differences in this camera as compared with photogrammetric cameras are:

a. No vacuum system to assure film flatness. The film is pulled into position from the front to the rear of the camera by a pair of sprockets at the rear of the platen. During exposure the film lies in a slot formed by the platen to the rear and a glass plate with an inscribed grid in front. Since this slot must be wide enough to permit the film to be drawn through and the film is drawn from one end, film flatness cannot be assured. The roadway is imaged as a line running from the front of the camera to the rear. Due to the "pivoting" effect of the lens, vehicles that are in front in the direction of travel are imaged at the rear of the camera.

b. No distortion curves are available for the lens. A check with the U.S. Bureau of Standards indicated they had no calibration information on lenses or cameras of this type.

c. A calibrated focal length was not available. The nominal focal length is three inches.

d. Perpendicularity of the optical axis with respect to the image plane is not assured, as the camera was not designed for metric photogrammetry.

e. There is no assurance the optical axis intersects the image plane at the center of the grid.

A test was designed and run to determine the magnitude of the combined lens and film distortions. Targets, four-inches square in size, were set up in a straight line at 25.00-feet intervals on level ground, and photographs were taken from a station 400 feet from the center target on a line perpendicular to the row of targets. Care was taken to point the optical axis, as defined by the center of the grid at the center target. It was not possible, however, to check the alignment after the film magazine was replaced and some tilting did occur.

Ten of the photographs were taken with the camera oriented such
that the targets defined a line perpendicular to the long axis of the film. The other eight were taken with the camera rotated 90° about the optical axis. In this latter position the row of targets was parallel to the long axis of the roll of film, which is similar to the orientation of the roadway on the photographs taken from the helicopter. Two photographs of each type were selected for detailed analysis.

In general, the tangential component of the lens distortions is small, and only radial distortions are investigated. The standard field technique for doing this consists of computing the angle from the optical axis to various targets; multiplying the tangents of these angles by the focal length of the lens; and comparing these radial distances with the measured radial distances from the principal point of the photograph to the target images.

In this study the optical axis could not be pointed precisely at the center target. Thus, the line of targets was not parallel to the image plane and the angles from the optical axis to the various targets could not be computed directly.

a. "Moving" the optical axis to the target line. This introduces errors of a secondary magnitude because the computed distortions radiate along a line which misses the optical axis by a small amount—about three millimeters on the four photographs studied.

b. The line of ground control points is "tilted" so that it is perpendicular to the optical axis.

c. The distance from the "moved" optical axis to each of the targets on the "tilted" line was computed, and converted to a theoretical radial distance through use of an estimated focal length. The focal length was estimated at 75.82 mm. by scaling several lengths near the center of the photographs and comparing them with known lengths on the ground. The camera position was also known.
d. "Radial" distances from the moved optical axis to each of the target images were computed from the measured coordinates. (Coordinate measurements for the lens and film distortion study were made at the Nistri TA3 Stereocomparator at The Ohio State University.)

e. Distortions were determined by comparing the radial distances found in (c) with those in (d).

The method used to transfer the optical axis to the target line is illustrated in Figure 15.

Points A and B are the two targets nearest to the principal point of the photograph, point C. It is desired to find the coordinates of point D, which is the point on the target line nearest point C. In Figure 15:

\[ \mathbf{r}_D = \mathbf{r}_A + \frac{k}{|\mathbf{r}_B - \mathbf{r}_A|} (\mathbf{r}_B - \mathbf{r}_A) \]

where \( k \) is the length AD and \( \mathbf{r}_i \) is the vector from \((0,0)\) to any point \( i \).

\[ (\mathbf{r}_C - \mathbf{r}_A) \cdot (\mathbf{r}_B - \mathbf{r}_A) = |\mathbf{r}_C - \mathbf{r}_A| |\mathbf{r}_B - \mathbf{r}_A| \cos \theta \]

\[ k = |\mathbf{r}_C - \mathbf{r}_A| \cos \theta \]

Therefore:

\[ k = \frac{(\mathbf{r}_C - \mathbf{r}_A) \cdot (\mathbf{r}_B - \mathbf{r}_A)}{|\mathbf{r}_B - \mathbf{r}_A|} \]

The expression for \( \mathbf{r}_D \) may be written as:

\[ \mathbf{r}_D = \mathbf{r}_A + \frac{(\mathbf{r}_C - \mathbf{r}_A) \cdot (\mathbf{r}_B - \mathbf{r}_A)}{|\mathbf{r}_B - \mathbf{r}_A|^2} (\mathbf{r}_B - \mathbf{r}_A) = \mathbf{r}_A + \beta (\mathbf{r}_B - \mathbf{r}_A) \]

\( \mathbf{r}_D \) may be resolved into components:

\[ X_D = X_A + \beta (X_B - X_A) \]

\[ X_D = X_A + \beta (X_B - X_A) \]
Fig. 15.—Transfer of Optical Axis to Target Line
\[
Y_D = Y_A + \beta (Y_B - Y_A)
\]

With proper substitutions:
\[
\beta = \frac{(X_C - X_A)(X_B - X_A) + (Y_C - Y_A)(Y_B - Y_A)}{(X_B - X_A)^2 + (Y_B - Y_A)^2}
\]

These latter equations may be combined to give:
\[
X_D = X_A + \frac{(X_C - X_A)(X_B - X_A)^2 + (Y_C - Y_A)(Y_B - Y_A)(X_B - X_A)}{(X_B - X_A)^2 + (Y_B - Y_A)^2}
\]
\[
Y_D = Y_A + \frac{(X_C - X_A)(X_B - X_A)(Y_B - Y_A) + (Y_C - Y_A)(Y_B - Y_A)^2}{(X_B - X_A)^2 + (Y_B - Y_A)^2}
\]

In Figure 16, the solid line BAG represents the target line and 
0 represents the camera station, as seen from above. The desired 
direction of the optical axis is OA, but the actual direction is OD'. 
Thus, the image plane is not parallel to line BAG as is desired, but is 
parallel to an imaginary target line rotated through the angle \( t \) from 
the true line. The images of targets \( B', A', G' \) would occupy the same 
position on the photograph as target \( B, A, \) and \( G \).

If distortion is assumed negligible at the center of the photograph, 
the angle \( t \) may be found as follows:
\[
\tan t = \frac{(ad)}{f}
\]

where \( (ad) \) is measured on the photograph.
Fig. 16. — Tilt of Target Line
The angle $\theta$ is known from the original target and camera station layout, i.e.:

$$\tan \theta = \frac{AG}{QA} = \frac{\text{target spacing}}{\text{distance from camera to center target}}$$

The theoretical (undistorted) position of target $G$ is at $g$, distance $(dg)$ from the moved center of the photograph (point $D'$). If $t$ and $\theta$ are measured positive clockwise from the line $QA$, the distance $(dg)$ is:

$$dg = r_g = f \left[ \tan(\theta - t) \right] = f \left[ \frac{\tan \theta - \tan t}{1 + \tan \theta \tan t} \right]$$

This distortion at point $g$ may be determined by comparing the distance $r_g$ to the distance from the transferred center of the photograph to the target image as measured on the photograph. (Actually, the component of the radial distortion parallel to the imaged target line is obtained.)

The procedure and necessary computations for determining the combined lens and film distortion in target photograph number 17 are indicated in Table 6. Columns (2) and (3) give the average of two readings for the $x$-coordinate and $y$-coordinate of each target. (The photo coordinate system has been rotated $90^\circ$ to approximately parallel the ground coordinate system.) The coordinates of the transferred optical axis are shown as $C'$. Columns (4) and (5) show the ground coordinates of the targets.

Column (6) gives the tangent of the angle at the camera station from the central target to each of the other targets. This tangent is
TABLE 6
DETERMINATION OF LENS AND FILM DISTORTIONS

<table>
<thead>
<tr>
<th>Pt.</th>
<th>x (mm.)</th>
<th>y (mm.)</th>
<th>X (ft.)</th>
<th>Y (ft.)</th>
<th>Tan Θ</th>
<th>K</th>
<th>r_t (mm.)</th>
<th>r_o (mm.)</th>
<th>d (mm.)</th>
</tr>
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<td>0.38289</td>
<td>29.031</td>
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<td>0.32009</td>
<td>24.269</td>
<td>24.177</td>
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<td>0.25724</td>
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<td>0.19465</td>
<td>14.758</td>
<td>14.725</td>
<td>-0.033</td>
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<td>0</td>
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<td>9.990</td>
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<td>5.264</td>
<td>5.264</td>
<td>0.000</td>
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<td>0</td>
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<td>0.00690</td>
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<td>0.523</td>
<td>0.000</td>
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<td>0.05558</td>
<td>4.214</td>
<td>4.209</td>
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<td>+0.18750</td>
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<td>13.676</td>
<td>13.683</td>
<td>+0.007</td>
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<td>428.064</td>
<td>207.810</td>
<td>+100.00</td>
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<td>+0.25000</td>
<td>0.24268</td>
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<td>18.402</td>
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<td>+0.37500</td>
<td>0.36715</td>
<td>27.837</td>
<td>27.821</td>
<td>-0.016</td>
</tr>
<tr>
<td>C</td>
<td>409.672</td>
<td>205.546</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C'</td>
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<td>207.606</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
equal to the ground X-coordinate divided by 400 ft. The value of $K$ shown in Column (8) is equal to:

$$K = \left| \frac{\tan \theta - \tan t}{1 + (\tan \theta)(\tan t)} \right|$$

For photograph 17, the tilt was $0^\circ24'$. The theoretically correct (non-distorted) radial distance from the transferred optical axis to each target is shown in Column (8) and is equal to $K(f)$; where $f$ is the focal length, 75.82 mm. Column (9) gives the distance from transferred optical axis at $C'$ to each target image, as computed from the measured coordinates.

The distortions, based on the assumed focal length, are indicated in Column (10), and are equal to Column (9) minus Column (8). Thus, a positive distortion indicates the image was displaced away from the theoretical position.

Figures 17 and 18 show the distortion as a function of the position of the image on the film. Photographs 4 and 17, shown in Figure 17, were oriented such that the targets formed a line perpendicular to the long axis of the film. Photographs 8 and 14, shown in Figure 18, were oriented such that the targets formed a line parallel to the long axis of the film. The distortion curves were oriented to be similar to the target orientations for more realistic comparisons.

Examination of the distortion curves shows that the combined lens and film distortions are fairly large and erratic. For example, the value of distortion at a radius of -20 mm. in Figure 18 for the two photographs is +16 microns and -52 microns—a difference of 68 microns. Thus, if the two curves were averaged and a correction made based on
Fig. 17.—Distortions Perpendicular to Long Axis of Film
Fig. 18.—Distortions Parallel to Long Axis of Film
this average curve, the residual distortion after correction would still be 34 microns, which amounts to about 1.1 feet at the scale of the traffic photography. If more photographs had been examined, it is almost certain that even larger variations would have been found. It was decided some other procedure to compensate for the lens and film distortions would be necessary to obtain the accuracy required in the traffic study.

The curves in Figure 18, which show the distortions in the portion of the photograph in which the road appears, also indicate the previously computed average value of 75.32 mm. for the focal length to be approximately correct, and no adjustment based on the distortion curves was deemed justified.

A procedure utilizing the ground control established for the traffic study was devised to determine the effect of the lens and film distortions or position determinations. (Actually, the effect of all errors are reflected in the results to be discussed. However, the photographs used for this distortion study were all nearly vertical—all tilted less than 2°—so that tilt errors were minimal. Horizontal and vertical alignment was previously shown to be negligible. The ground control points used were manhole covers so that the pointing accuracy is probably considerably better than that reported previously, which was based on lamp posts for ground control. The manhole covers make excellent targets as they are circular in shape and, at image scale, have a diameter about twice that of the measuring mark. Vehicle relief errors do not apply, as only ground control points are used.)
The procedure devised is as follows: (See Table 7.)

a. The ground coordinates of point 221 were computed based on the photo coordinates of points 220, 221, and 222 and the ground coordinates of points 220 and 222. A rotation-translation transformation was used; i.e.

\[ X = x \cos \theta + y \sin \theta + A \]
\[ Y = -x \sin \theta + y \cos \theta + B \]

A, B, and \( \theta \) were determined from points 220 and 222. The resultant equations were used to find the coordinates of point 221. These coordinates are shown in Columns (2) and (3).

b. The ground coordinates of point 221 found in part (a) were compared to the true ground coordinates of point 221 and the differences indicated as errors in Columns (4) and (5).

c. The \( X \) and \( Y \) errors were combined to determine the position error, \( P \), shown in Column (6).

d. Similar computations were carried out using points 219 and 223 as known ground control points. This increases the spacing between "known" ground control points from approximately 500 feet to approximately 900 feet. The results are shown in Column (7). (The distance from 220 to 222 is 499.8 ft. and point 221 is 1 foot from the center. The distance from 219 to 223 is 908.4 ft. and point 221 is 16 feet from the center.)

The position errors are shown in Figure 19 for both ground control spacings as a function of the distance to point 221 from the center of each photograph. The errors decrease significantly when the ground control spacing is decreased from 900 ft. to 500 ft. The "error" really amounts to differential distortion, as the two "known" ground control points are also distorted in position. However, their absolute distortion is of no consequence as they are "tied" to their true ground positions through the ground coordinates.

Figure 20 is a plot of average and maximum differential distortion for a point midway between the two known control points versus control point spacing, based on photographs 132 through 146. Obviously,
### TABLE 7

**POSITION ERRORS OF POINT 221 AS COMPUTED FROM OTHER GROUND CONTROL POINTS**

<table>
<thead>
<tr>
<th>Photograph</th>
<th>Ground X</th>
<th>Ground Y</th>
<th>X Error (ft.)</th>
<th>Y Error (ft.)</th>
<th>P Error (ft.) 500' Spacing</th>
<th>P Error (ft.) 900' Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
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<tr>
<td>129</td>
<td>4671.43</td>
<td>839.60</td>
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<td>-0.18</td>
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<td>130</td>
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<tr>
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<tr>
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Fig. 19.—Differential Lens and Film Distortions
Fig. 20.—Effect of Control Point Spacing on Differential Distortions
the differential distortion is zero if the ground control spacing is zero. Thus, three points are known. Smooth curves were drawn through the sets of three points. At the average ground control spacing of 186 feet used in the traffic photography, the average differential distortion may be estimated at 0.10 ft., and the maximum at 0.25 ft. At the maximum spacing of 260 feet, the corresponding values are 0.17 ft. and 0.45 ft. These values apply to points within approximately 14 mm. of the center of the photograph. Examination of Figure 19 indicates the differential distortions will be somewhat larger at the extreme edges of the photographs, particularly at the left edge. (Points within a ground distance of 250 ft. of the edge of the photograph were studied, however.) Doubling the above values will provide error estimates on the safe side.

Figure 19 shows the differential distortions to be minimal at the center of the photograph and to increase as the distance from the center increases—particularly to the left. Thus, the combined lens and film distortions appear to be at least partially systematic. However, partial compensation can be obtained with considerable difficulty and uncertainty at best. The effect of these errors on headway and velocity computations will be discussed in a later section.

**Accuracy of Time Interval Between Exposures**

The interval between exposures was controlled by an Intervalometer Camera Control-Type CF-3, which was borrowed from the U.S. Air Force at Wright-Patterson Field in Dayton, Ohio. It is possible to set this intervalometer for intervals from 0.2 second to 6.0 seconds in 0.1 second
intervals. For the traffic study, an interval of 1.0 second was used.

Since the accuracy of the velocity determinations depends on the accuracy of the time interval as well as the position accuracy, a test was devised and run to determine the interval accuracy.

The intervalometer could have been tested using an electronic counter. However, past experience indicated the camera might react in a slightly irregular fashion even if the pulse interval were accurate. Thus, a procedure was designed which would test the camera-intervalometer system as a whole.

The test apparatus is shown in Figure 21. The circumference of the rotating wheel was divided into 100 equal parts which are marked by a vertical line and numbered from 1 to 100. The wheel was mounted on a vertical axis and driven at the rate of one revolution per second by means of a 60-cycle synchronous motor. Thus, if a reference line is set up in front of the wheel, the marks on the wheel will pass the reference line at 0.01-second intervals.

The camera was set up in front of the reference marker and an auxiliary lens added to permit taking photographs at a short range. The camera itself is focused at infinity. Photographs were taken of the rotating wheel at an f-stop of 2.8 and an exposure time of 1/1000 second. The camera was run from the intervalometer which in turn was connected to a 28-volt D.C. source in the Ohio State University Electrical Engineering Laboratory. During the test, a strobotac was used to check that the velocity of the wheel did not vary.

The photographs obtained were of good quality and it was easy to estimate the "wheel reading" to the nearest 1/10 of a division, or
Fig. 21.—Time Interval Testing Apparatus

1 - Testing Device
2 - Camera and Mount
3 - Frame Counter
4 - Intervalometer
5 - Strobotac
6 - Oscilloscope
0.001 second. The time intervals between exposures were obtained by subtracting each reading from the previous one.

A total of 157 photographs were taken, which provided 156 intervals between exposures. The average interval between exposures was 0.9982 seconds and the standard deviation was ±0.0042 seconds, which is roughly 0.4 percent of the interval. The minimum time interval between exposures was 0.986 seconds and the maximum was 1.007 seconds. Thus the extreme values were within roughly one percent of the mean.

The effect of the accuracy of the time interval between exposures for the camera-intervalometer system on velocity computations will be discussed in the appropriate section.

**Second-Order Errors**

The effect of each error is discussed separately in this section. It is realized that second-order errors resulting from combinations of the individual errors are also present. For example, the position error due to tilt for a vehicle exactly opposite a control point is zero if the vehicle "target" is at the same elevation as the roadway. However, if the vehicle and control point are near the edge of the photograph, the vehicle appears to be either ahead of or behind the control point due to vehicle relief, and the computed position error due to tilt will have some small value—0.04 ft. for the worst possible combination of events.

The second-order errors are not discussed in detail as it is believed that they would not materially affect the spacing and velocity determinations. A full investigation of the second-order errors could constitute a dissertation in itself.
Effect of Errors on Spacing Determinations

The various errors discussed in the preceding pages of this chapter affect the spacings between vehicles as computed and shown in the computer print-out. The focal plane shutter introduces an additional error. The magnitudes of the position errors have already been described, but the resultant spacing error will depend on the differential position error of the two vehicles involved and not the absolute position error. Some errors tend to compensate whereas others do not.

Focal plane shutter

The Maurer P-2 camera used for this study has a focal plane shutter. This type of shutter consists of a curtain with a narrow slit in it. When the camera receives an actuating pulse, the slit slides across the space just in front of the film, thereby exposing each part of the film for a time determined by the width of the slit and the speed of the shutter. A given "point" on the film will be exposed from the time the front edge of the slit passes it until the rear edge passes the point. The speed of the shutter remains constant and various exposure times are obtained by varying the width of the slit.

The focal plane shutter permits very short exposure times—down to 1/2000 second for the Maurer P-2 camera—but may introduce distortions in the resultant photograph because different portions of the photograph are exposed at different times. If the camera station and objects being photographed are at rest these distortions will not occur. However, in the traffic study the vehicles and the camera station (helicopter) are moving. Thus, vehicles occupy different positions on the photograph.
than they would if the whole photograph were exposed instantaneously. If one vehicle is "caught" on the photograph just as the slit starts its trip across the film and a second is "caught" just as the slit finishes its trip, the second vehicle will have moved from the position it occupied when the first vehicle was photographed and the relative position, and thus the spacing, will be in error. (The first vehicle has moved from its "photographed" position by the time the second vehicle is photographed.)

The Maurer P-2 camera was mounted in the helicopter in such a way that the slit travelled in a direction from the front of the helicopter to the rear. Due to the pivoting effect of the lens, this means the ground was photographed as a "sweep" in the direction of travel of the vehicles, which move in the same direction as the helicopter.

The time-space relationship in which the photograph is obtained and effect on measured spacing between vehicles is shown in the distance-time diagram of Figure 22, where:

D—vector representing lead vehicle. Tan θ_D is the velocity of vehicle D.
C—vector representing the following vehicle. Tan θ_C is the velocity of vehicle C.
H—vector representing the helicopter or camera-station. Tan θ_H is the velocity of the helicopter.
PP'—vector representing the photograph. This vector represents the sweep of the focal plane shutter over the terrain. Tan θ_P is the velocity of the sweep and is equal to the velocity of
Fig. 22.—Effect of Focal Plane Shutter on Spacing Determinations
the helicopter plus the velocity of the shutter multiplied
by the scale factor, or \( V_H + \frac{H}{T} V_S \).

\( D_1 \) - position of vehicle C at time \( t_1 \), when it is registered on
the film.

\( D_2 \) - position of vehicle D at time \( t_1 \), when vehicle C is registered
on the film.

\( D_3 \) - position of vehicle D at time \( t_2 \), when it is registered on the
film.

The true spacing between vehicles C and D is \( (D_2 - D_1) \), whereas the
spacing measured on the film is \( (D_3 - D_2) \).

\[ E = \text{error} = D_3 - D_2 = V_D(t_2 - t_1) \]

\[ S + E = \text{true spacing} + \text{error} = (D_2 - D_1) + (D_3 - D_2) = V_p(t_2 - t_1) \]

\[ \frac{E}{S + E} = \frac{V_D(t_2 - t_1)}{V_p(t_2 - t_1)} = \frac{V_D}{V_p} \]

or,

\[ E = \frac{V_D}{V_p} (S + E) = \frac{V_D}{V_p} (D_3 - D_1) = \frac{V_D}{V_p} \text{ (measured spacing)} \]

True spacing = measured spacing - \( E \)

Therefore:

True spacing = \((\text{measured spacing})(1 - \frac{V_D}{V_p})\)

= \((\text{measured spacing}) \left[ \frac{V_H + \frac{H}{T} V_S - V_D}{V_H + \frac{H}{T} V_S} \right] \)
If the lead vehicle is stopped, \( V_D = 0 \); no correction will be necessary. The magnitude of the correction obviously increases as the velocity of the lead vehicle increases. The magnitude of the error depends on the helicopter velocity also; decreasing as the helicopter velocity increases. The error is not dependent on the velocity of the following vehicle, but is directly proportional to the spacing between the vehicles.

In the traffic photography, the maximum velocity observed was roughly 60 mph. and the minimum helicopter velocity was about 20 mph. The maximum measured spacing was 376 feet. The speed of the shutter is 38 in./sec., the flying height 2350 feet, and the focal length of the camera was previously determined to be 2.985 in. Thus, \( \frac{H}{F}(V_g) = 20,350 \) mph.

If all the variables occurred in the worst combination, the error would be:

\[
E = \frac{V_D}{V_p} (\text{measured spacing}) = \frac{60}{20 + 20,350(376)} = 1.11 \text{ ft.}
\]

For the average spacing of 95 ft. and average vehicle (and helicopter) velocity of 34 mph., the error would be 0.16 ft. It is estimated the error in the measured spacing due to the effect of the focal plane shutter will be less than 0.5 feet in 95% of the determinations. The error is always positive, i.e., the measured spacing is always somewhat longer than the true spacing.

This error is systematic and approximate corrections may be computed. The corrections will not be exact as the velocities used in the correction formula are not known precisely. The corrections will
be affected very little by the inaccurate velocities however.

**Tilt**

It was shown previously that the maximum possible position error due to tilt would be 1.79 ft. This assumes the worst possible combination of the range of variables experienced in the traffic photography. Since it is possible that one vehicle would be in the "worst" situation and another might be adjacent to a ground control point and thus have zero position error, the maximum spacing error is also 1.79 feet. Since the tilt can be in either direction and the second vehicle could be at a preceding or following ground control point, the spacing error could be plus or minus.

For a given photograph, the position error has the same sign, or is zero, for all vehicles. Thus, if the two vehicles are both between the same two ground control points, the position errors will have the same sign and the spacing error will be partially compensated.

If median values are used for the variables in the determination of the position error due to tilt of the photograph, the error is 0.19 ft. Since partial compensation is obtained in the spacing computation in all cases except when one of the vehicles is at a ground control point, it is conservative to estimate the effect of tilt on spacing has a median value near 0.12 ft.

This error may be plus or minus, but is systematic and corrections can be computed if desired. (In future flights, tilt can be held to less than 3° with relatively little difficulty and the resultant maximum error would be 0.69 ft. rather than 1.79.)
Vertical alignment

The maximum rate of position error due to assuming the roadway level was 0.02 ft. per 100 ft. Thus, for a maximum spacing of 376 ft., the spacing error would be 0.07 ft. For an average spacing of 95 feet the error would be 0.02 ft., if the vehicles were on the maximum grade. When the grade is less than maximum the error will be correspondingly less. The error is always plus as grade lengths are longer than the assumed level distance. The error is systematic and corrections can be computed.

Horizontal alignment

The maximum rate of position error due to approximating the curved roadway by a series of straight lines was 0.022 ft. per 100 ft. This is the same magnitude as the error due to vertical curvature and is insignificant in the determination of vehicle spacings for traffic studies. However, the error is always plus, and corrections may be computed.

Vehicle relief

In a vertical photograph, the maximum possible position error due to vehicle relief is +1.12 ft., as indicated in Table 4. If another vehicle were following the one with maximum position error, it would be $1750/2 - 376 = 500$ ft. from the nadir point of the photograph (or farther, if the spacing was less than the maximum of 376 ft.). The position error for this trailing vehicle would be:

$$\frac{e}{3 \text{ ft.}} = \frac{500 \text{ ft.}}{2250 \text{ ft.}}$$

(See Figure 14)

$$e = +0.64'$$
The two position errors have the same sign and would compensate in the spacing determination. Thus, the maximum spacing error due to vehicle relief for a vertical photograph would be $(1.12 - 0.64) = +0.48'$. If the average spacing of 95 ft. is used instead of the maximum, the resultant spacing error would be $+0.12$ ft.

Tilting the photograph will not increase these spacing errors as the change in position error for the two vehicles is a function of the change in distance from the vehicles to the nadir point—equal to the spacing—not the absolute position error.

While position errors of vehicles on the trailing half of the photograph have negative values, the spacing error will be positive as the following vehicle has the larger magnitude error on this half. The spacing errors for the vehicles on the left side of the photograph have smaller magnitudes as the position errors have smaller magnitudes, as indicated in Table 4.

These errors may also be corrected for as they are systematic. In all cases the error is positive and the computed spacings are longer than the actual spacings.

**Pointing accuracy**

The spacing between vehicles is determined by subtracting the D-distance, or accumulative distance travelled by a vehicle from some arbitrary starting point, of the following vehicle from the D-distance of the lead vehicle.

If both vehicles are in the same D-point interval, only the errors in the vehicle positions are involved. However, if the lead vehicle is
in one D-interval and the following vehicle in another, the errors in
determining the positions of the D-points are involved. At worst, the
spacing will completely straddle one D-interval; i.e., the following
vehicle will be in one D-interval and the lead vehicle will be in the
second D-interval ahead.

If the accumulative distance to the beginning of the D-interval
containing the following vehicle is designated $D_1$ and the accumulative
distance to the beginning of the D-interval containing the lead vehicle
is designated $D_3$, the $D$ values for the two vehicles may be expressed
as:

$$D_L = D_3 + A_3 X_L$$

$$D_F = D_1 + A_1 X_F$$

where $A_1$ and $A_3$ are coefficients to transform ground $X$-coordinates to
distance from the preceding D-point. The spacing is equal to:

$$S = D_L - D_F = (D_3 - D_1) + A_3 X_L - A_1 X_F = a_1 + a_2 + a_3$$

The variance of a linear combination of observations is equal to the
sum of the variance of the observations if the observations are inde­
dependent. ("Statistical Analysis in Chemistry and the Chemical Industry," by C. A. Bennett and N. L. Franklin, John Wiley & Sons, Inc., 1954, page 49.) Therefore:

$$\sigma_s^2 = \sigma_{a_1}^2 + \sigma_{a_2}^2 + \sigma_{a_3}^2$$
The ground coordinates of various vehicles are necessary to determine $a_1$, $a_2$, and $a_3$. The variance of vehicle ground coordinates is required if the variances of these factors are to be determined. The ground coordinates of vehicles are determined by:

\[
X = Ax + By + C \\
Y = Bx + Ay + D
\]

where $x, y$ are photo coordinates and $A, B, C, D$ are transformation coefficients which were determined by solving the same equations using two ground control points. To determine the variance in the ground coordinates of vehicles, it will be necessary to first determine the variance in the coefficients. If the two ground control points used are designated 1 and 2, the following equations for the coefficients may be derived:

\[
A = \frac{(y_2 - y_1)(y_2 - y_1) + (x_2 - x_1)(x_2 - x_1)}{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
B = \frac{(x_2 - x_1)(y_2 - y_1) + (y_2 - y_1)(x_1 - x_2)}{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
C = x_1 - Ax_1 - By_1 \\
D = y_1 - Ay_1 + Bx_1
\]

It was assumed the ground control was errorless. This is not exactly true, but the error in ground control coordinate determination is small compared to the pointing accuracy. The precision standard for
third order surveying is 1/5000 for distance determinations. This permits an error of only 0.02 ft. over an average vehicle spacing of 95 ft. and may be considered insignificant. If the coordinate system is translated so that point 1 has coordinates (0,0), C and D become equal to X and Y, respectively. If the X, Y coordinates are errorless, then the variances of C and D will also be zero. C and D are translation coefficients and, as such, are not dependent on photo-coordinate determinations.

The standard deviations in measuring photo coordinates of ground control points were determined as \( s_{x_1} = \pm 5.27 \) microns and \( s_{y_1} = \pm 2.63 \) microns. (See earlier section of this chapter entitled Accuracy of Photo Coordinate Determinations). These are estimates of \( \sigma_{x_1} \) and \( \sigma_{y_1} \), the true standard deviations of the measurements.

Define:

\[
(X_2 - X_1) = x
\]

\[
(Y_2 - Y_1) = y
\]

\[
(x_2 - x_1) = x
\]

\[
(y_2 - y_1) = y
\]

The variances of X and Y—ground control coordinate differences—were assumed equal to zero, as mentioned previously.

The variances of x and y may be estimated as follows:

\[
\sigma_x^2 = \sigma_{x_2}^2 + \sigma_{x_1}^2 = 2\sigma_{x_1}^2
\]

\[
\sigma_y^2 = \sigma_{y_2}^2 + \sigma_{y_1}^2 = 2\sigma_{y_1}^2
\]
\[ \sigma_{x_1}^2 = \sigma_{x_2}^2 = \sigma_{x_1}^2 \] is the standard deviation of the x-photo coordinate determination for ground control points. The best estimate of this standard deviation is \( s_x \), determined from a sample of 10 ground control points. Similar statements hold true for the y standard deviations. Thus:

\[ \sigma_x^2 = 2(5.27)^2 = 55.40 \text{ microns}^2 \]
\[ \sigma_y^2 = 2(2.63)^2 = 13.80 \text{ microns}^2 \]

The variance of an arbitrary function, \( z = f(x_1, x_2, \ldots, x_n) \), where \( x_1, x_2, \ldots, x_n \) are independent random variables, may be found as:

\[ \sigma_z^2 = \left( \frac{\partial z}{\partial x_1} \right)^2 \sigma_{x_1}^2 + \left( \frac{\partial z}{\partial x_2} \right)^2 \sigma_{x_2}^2 + \ldots + \left( \frac{\partial z}{\partial x_n} \right)^2 \sigma_{x_n}^2 \]


With previously defined substitutions,

\[ A = \frac{Y + X}{x^2 + y^2} \]

The variance of \( A \) is determined as:

\[ \sigma_A^2 = \left( \frac{\partial A}{\partial x} \right)^2 \sigma_x^2 + \left( \frac{\partial A}{\partial y} \right)^2 \sigma_y^2 \]

\[ \sigma_A^2 = \left[ \frac{y^2 x + 2xy - x^2 y}{(x^2 + y^2)} \right]^2 \sigma_x^2 + \left[ \frac{x^2 y - 2xy - y^2 x}{(x^2 + y^2)} \right]^2 \sigma_y^2 \]

This equation can be evaluated for each set of two adjacent ground control points on each photograph. Ground control points 222 and 223
were selected as typical as they are 184.02 feet apart, which is near
the average distance between ground control points of 186 feet. The
pertinent data for these points are tabulated below:

<table>
<thead>
<tr>
<th></th>
<th>Point 222</th>
<th>Point 223</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-ground coordinate</td>
<td>4916.60</td>
<td>5097.69</td>
</tr>
<tr>
<td>Y-ground coordinate</td>
<td>791.21</td>
<td>758.48</td>
</tr>
<tr>
<td>x-photo coordinate (Photo 141)</td>
<td>-79.804</td>
<td>-73.916</td>
</tr>
<tr>
<td>y-photo coordinate (Photo 141)</td>
<td>0.764</td>
<td>0.590</td>
</tr>
</tbody>
</table>

\[
X = X_{223} - X_{222} = 181.09 \text{ ft.} = 55196 \text{ mm.}
\]

\[
Y = Y_{223} - Y_{222} = -32.73 \text{ ft.} = -9976 \text{ mm.}
\]

\[
x = x_{223} - x_{222} = 5.888 \text{ mm.}
\]

\[
y = y_{223} - y_{222} = -0.174 \text{ mm.}
\]

Substituting these values in the above equation:

\[
\sigma_A^2 = \frac{(0.174)^2(5.888) - 2(5.888)(-0.174)(-32.73) - (5.888)^2(181.09)^2}{(5.888)^2 + (-0.174)^2} \times 0.0000554
\]

\[
+ \frac{(5.888)^2(-0.174) - 2(5.888)(-0.174)(55.196) - (-0.174)^2(-9976)^2}{(5.888)^2 + (-0.174)^2} \times 0.0000138
\]

Solving:

\[
\sigma_A^2 = 142.94 + 0.12 = 143.06
\]

The equation for B can be written as:

\[
B = \frac{Xy - Yx}{x^2 + y^2}
\]
The variance of $B$ is:

$$\sigma_B^2 = \left(\frac{\partial B}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial B}{\partial y}\right)^2 \sigma_y^2$$

$$\sigma_B^2 = \left[\frac{x^2 - 2xy - y^2}{(x^2 + y^2)^2}\right] \sigma_x^2 + \left[\frac{x^2 + 2xy - y^2}{(x^2 + y^2)^2}\right] \sigma_y^2$$

Substituting and solving:

$$\sigma_B^2 = 2.065 + 35.547 = 37.612$$

It was pointed out in the previous discussion that $\sigma_C^2$ and $\sigma_D^2$ are both equal to zero (assuming the ground control is errorless).

The values of $A$ and $B$ are determined by substituting the appropriate values in the previous equations for these quantities:

$$A = \frac{(-9.976)(-0.174) + (55.196)(5.888)}{(5.888)^2 + (-0.174)^2} = 271.37$$

$$B = \frac{(55.196)(-0.174) - (-9.976)(5.888)}{(5.888)^2 + (-0.174)^2} = 40.809$$

The ground distance from ground control point 222 to any vehicle between control points 222 and 223 may be determined by:

$$X_v = Ax_v + By_v$$

$$Y_v = -Bx_v + Ay_v$$

where $X_v$ is the $x$-component of the distance, $Y_v$ is the $y$-component, $x_v$ equals the $x$-photo coordinate of the vehicle minus $x_{222}$, $y_v$ equals the $y$-photo coordinate of the vehicle minus $y_{222}$, and $A$ and $B$ are 271.37 and 40.809 respectively.
The variance in \( x_v \) is:

\[
\sigma_{x_v}^2 = \left( \frac{\partial x_v}{\partial A} \right)^2 \sigma_A^2 + \left( \frac{\partial x_v}{\partial x_v} \right)^2 \sigma_{x_v}^2 + \left( \frac{\partial x_v}{\partial B} \right)^2 \sigma_B^2 + \left( \frac{\partial x_v}{\partial y_v} \right)^2 \sigma_{y_v}^2
\]

\( \sigma_{x_v}^2 \) and \( \sigma_{y_v}^2 \) are the variances in determining the photo distance between a ground control point and a vehicle. Thus:

\[
\sigma_{x_v}^2 = \text{variance in } x\text{-coordinate for ground control point plus variance in } x\text{-coordinate for vehicle}
\]

\[
= 27.70 + 16.22 = 43.92 \text{ microns}
\]

\[
= 0.00004392 \text{ mm.}
\]

Similarly:

\[
\sigma_{y_v}^2 = 0.00001900 \text{ mm.}
\]

To determine \( \sigma_{x_v}^2 \) for vehicle 543 in photo 141:

\[
x_v = -74.733 - (-79.804) = 5.071
\]

\[
y_v = -0.198 - 0.590 = -0.788
\]

\[
\sigma_{x_v}^2 = x_v^2 \sigma_A^2 + y_v^2 \sigma_B^2 + x_v^2 \sigma_{x_v}^2 + y_v^2 \sigma_{y_v}^2
\]

\[
= (5.071)^2(143.06) + (271.37)^2(0.00004392) + (-0.788)^2(37.612) + (40.809)^2(0.00001900)
\]

\[
= 3678.8 + 3.2 + 23.4 + 0.0 = 3705.4
\]

Therefore \( \sigma_{x_v}^2 = (3705.4)^{\frac{1}{2}} = 60.87 \text{ mm.} \approx 0.1997 \text{ ft.} \)

Similarly:

\[
\sigma_{y_v}^2 = x_v^2 \sigma_A^2 + y_v^2 \sigma_B^2 + x_v^2 \sigma_{x_v}^2 + y_v^2 \sigma_{y_v}^2
\]

\[
= 967.12 + 0.07 + 88.83 + 1.40 = 1057.42
\]
Therefore \( \sigma^2_{y_v} = (1057.42)^2 \) = 32.52 mm. = 0.1067 ft.

Since the variances in the \( X_v \) and \( Y_v \) coordinates are partially due to variances in the transformation coefficients, the variance in vehicle ground coordinates will vary with the position of the car with respect to the ground control interval used to determine the transformation coefficients. The transformation coefficients \( A \) and \( B \) represent a rotation and scaling transformation. Thus, as the distance to the vehicle under consideration from the center of rotation increases, the variance in its position will increase. The maximum distance the vehicle can be from the initial ground control point is limited to the spacing between ground control points which averages 186 feet on the ground or approximately 6.0 mm. on the photo.

Thus, the variance in the ground coordinates of each vehicle on each photograph will be different and may be computed for each individual case. For further analysis, however, the values derived for vehicle 543 on photograph 141 will be used as representative. These values approximate the maximum variances for vehicles between ground control points of average spacing.

The first term in the spacing equation is \( a_1 = (D_3 - D_1) \) where \( (D_3 - D_1) \) is the distance from D-point 1 to D-point 3. This distance is equal to \( (D_3 - D_2) + (D_2 - D_1) \), or:

\[
a_1 = (D_3 - D_2) + (D_2 - D_1) = \left[ \left( X_{D_3} - X_{D_2} \right)^2 + \left( Y_{D_3} - Y_{D_2} \right)^2 \right]^\frac{1}{2} + \left[ \left( X_{D_2} - X_{D_1} \right)^2 + \left( Y_{D_2} - Y_{D_1} \right)^2 \right]^\frac{1}{2}
\]
The D-points are vehicles and the variances will be assumed equal to that of vehicle 543 on photograph 141; i.e.:

\[ \sigma_{X_{D_3}}^2 = \sigma_{X_{D_2}}^2 = \sigma_{X_{D_1}}^2 = (0.1997)^2 = 0.03988 \text{ ft.}^2 \]

\[ \sigma_{Y_{D_3}}^2 = \sigma_{Y_{D_2}}^2 = \sigma_{Y_{D_1}}^2 = (0.1067)^2 = 0.01138 \text{ ft.}^2 \]

Since the variance of each of the \(X_D\) coordinates is 0.03988 \(\text{ft.}^2\), the variance of the difference in two \(X_D\) coordinates will be

\[ \sqrt{0.03988 \text{ ft.}^2} = 0.05639 \text{ ft.}^2 \].

Similarly the variance of the difference in two \(Y_D\) coordinates is 0.01609 \(\text{ft.}^2\). As mentioned before, the worst situation that can occur, with a maximum vehicle spacing of 376 ft., is that the lead vehicle be in the second D-interval ahead of the D-interval of the following vehicle. The pertinent data for a situation of this type that occurred in the traffic photography are given below.

<table>
<thead>
<tr>
<th></th>
<th>(D_1)</th>
<th>(D_2)</th>
<th>(D_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X)</td>
<td>2690.14</td>
<td>2939.35</td>
<td>3149.95</td>
</tr>
<tr>
<td>(Y)</td>
<td>1190.31</td>
<td>1181.52</td>
<td>1163.08</td>
</tr>
</tbody>
</table>

Then

\[ (X_{D_3} - X_{D_2}) = x_2 = 210.60 \]

\[ (X_{D_2} - X_{D_1}) = x_1 = 249.21 \]

\[ (Y_{D_3} - Y_{D_2}) = y_2 = -18.44 \]

\[ (Y_{D_2} - Y_{D_1}) = y_1 = -8.79 \]

With the above substitutions:

\[ a_1 = (D_3 - D_2) + (D_2 - D_1) = (x_2^2 + y_2^2)^{\frac{1}{2}} + (x_1^2 + y_1^2)^{\frac{1}{2}} \]
The variance of \( a_1 \) is:

\[
\sigma_{a_1}^2 = (\frac{\partial a_1}{\partial x_2})^2 \sigma_{x_2}^2 + (\frac{\partial a_1}{\partial y_2})^2 \sigma_{y_2}^2 + (\frac{\partial a_1}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial a_1}{\partial y_1})^2 \sigma_{y_1}^2
\]

\[
= \frac{x_2^2 \sigma_{x_2}^2 + y_2^2 \sigma_{y_2}^2}{x_2^2 + y_2^2} + \frac{x_1^2 \sigma_{x_1}^2 + y_1^2 \sigma_{y_1}^2}{x_1^2 + y_1^2}
\]

\[
= \frac{(210.60)^2(0.05639) + (-18.44)^2(0.01609)}{(210.60)^2 + (-18.44)^2} + \frac{(249.21)^2(0.05639) + (-8.79)^2(0.01609)}{(249.21)^2 + (-8.79)^2}
\]

\[
= 0.05608 + 0.05634
\]

\[
= 0.11242
\]

\[
\sigma_{a_1} = 0.3353 \text{ ft.}
\]

Thus, the standard deviation in the determination of the distance from \( D_1 \) to \( D_3 \) (a total of 460.76 feet) due to coordinate reading accuracy is approximately 0.33 ft. or 4 in. (subject to the assumptions made in this analysis). For comparison the allowable error in a third-order survey would be 0.09 feet.

The second term in the spacing equation is \( a_2 = A_3 x_L \) where \( A_3 \) is a coefficient to transform ground \( X \)-coordinates to \( D \)-distances and \( x_L \) is equal to the \( X \)-coordinate of the lead vehicle minus the \( X \)-coordinate of point \( D_3 \), or \((X_V - X_3)\). \( A_3 \) may be written:

\[
A_3 = \frac{D_4 - D_3}{x_{D_4} - x_{D_3}} = \left[ \frac{(X_{D_4} - X_{D_3})^2 + (Y_{D_4} - Y_{D_3})^2}{X_{D_4} - X_{D_3}} \right]^{\frac{1}{2}}
\]
The case that will give a maximum value of $\sigma_s^2$ has been described before and is illustrated below:

Therefore, the lead vehicle for this error study should be approximately

\[(376 - 211) = 165 \text{ ft. in front of point } D_3.\] Vehicle 536 in photo 122 satisfies this condition. The pertinent data are given below:

<table>
<thead>
<tr>
<th></th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>Vehicle 536</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>3149.95</td>
<td>3362.23</td>
<td>3312.43</td>
</tr>
<tr>
<td>$Y$</td>
<td>1163.08</td>
<td>1133.79</td>
<td>1154.87</td>
</tr>
</tbody>
</table>

Then:

\[
(X_{D_4} - X_{D_3}) = X_D = 212.28
\]

\[
(Y_{D_4} - Y_{D_3}) = Y_D = -29.29
\]

\[
(X_V - X_{D_3}) = X_L = 157.48
\]

With the above substitutions:

\[
a_2 = \frac{\left(\frac{\partial a_2}{\partial X_D}\right)^2 \sigma_{X D}^2 + \left(\frac{\partial a_2}{\partial Y_D}\right)^2 \sigma_{Y D}^2 + \left(\frac{\partial a_2}{\partial X_L}\right)^2 \sigma_{X L}^2}{X_D}
\]

The variance of $a_2$ is:

\[
\sigma_{a_2}^2 = \left(\frac{\partial a_2}{\partial X_D}\right)^2 \sigma_{X D}^2 + \left(\frac{\partial a_2}{\partial Y_D}\right)^2 \sigma_{Y D}^2 + \left(\frac{\partial a_2}{\partial X_L}\right)^2 \sigma_{X L}^2
\]

where $\sigma_{X D}^2 = 0.05639 \text{ ft.}^2$, which is the variance in the difference in $X$-coordinates of two vehicles, and $\sigma_{X L}^2 = 0.01609 \text{ ft.}^2$, which is the variance in the difference in $Y$-coordinates.
The third term in the spacing equation is $a_3 = -A_1 x_p$, which is similar in form to $a_2$. Coordinates of $D_1$, $D_2$, and the following vehicle are involved rather than $D_3$, $D_4$, and the lead vehicle. The negative sign becomes positive when the terms are squared.

Vehicle 52? in photo 103 is used as the following vehicle and computations similar to those for $a_2$ result in:

\[ X_D = 249.21 \]
\[ Y_D = -8.79 \]
\[ X_F = 249.03 \]

and \( \sigma_{a_3}^2 = 0.00000 + 0.00002 + 0.05646 = 0.05648 \text{ ft}^2 \)

The variance in the spacing between vehicles was previously stated as:

\[ \sigma_s^2 = \sigma_{a_1}^2 + \sigma_{a_2}^2 + \sigma_{a_3}^2 \]
Substituting the computed values of the three terms on the right:

\[
\sigma_s^2 = 0.11242 + 0.05764 + 0.05648 = 0.22654 \text{ ft}^2
\]

\[
\sigma_s = 0.476 \text{ ft.}
\]

Thus, the standard deviation in the determination of the spacing between two vehicles, due to the variance in reading photo coordinates, for a specific set of conditions will be 0.48 ft. Since each spacing determination involves some change in the D-point spacings, the position of the D-points relative to the ground control, and/or the position of the vehicle relative to the D-points, and since all these factors are influenced by the variance in the photo coordinate determination, the variance in each spacing determination will be different. The error in spacing due to variance in photo coordinate determination is a combination of random and systematic errors; i.e., the variance of any one spacing may be determined, but the variance for each spacing will be different.

The combination of "events" used in calculating the variance 0.22654 ft.\(^2\) is considered to be one of the worst possible combinations and is not the variance for an actual spacing. For instance, only a very few spacings involve three D-intervals, because the spacing exceeds the minimum D-point interval of 211 ft. in only a few cases, and the majority of the large spacings occur on the long straight-away where the D-interval is large. If only two D-intervals are involved, \(\sigma_{a_1}^2\) will be approximately halved. If only one D-interval is involved, which occurs more than half the time, \(\sigma_{a_1}^2\) will be zero. (However, it is possible for \(\sigma_{a_2}^2\) and \(\sigma_{a_3}^2\) to be larger in this case.)
In summary, the standard error of the spacing due to inaccuracies in the coordinate determinations will rarely, if ever, exceed 0.50 feet for the traffic photography. On the other hand, the variance in determining the distance between D-points is equal to approximately 0.0562 ft.\(^2\) (See previous calculation for variance of quantities \((D_3 - D_2)\) and \((D_2 - D_1)\).) This corresponds to a standard deviation of 0.237 ft. Since the spacing depends on finding the distance to the two vehicles from one or more D-points in two separate determinations, the minimum spacing standard deviation is \(\sqrt{2} (0.237 \text{ ft.}) = 0.335 \text{ ft.}\). Hence, even though the standard deviation in the spacing between specific vehicles due to the inaccuracies in determining photo coordinates is not easy to determine, it is within a fairly narrow range and is small in magnitude.

**Lens and film distortions**

The effect of the combined lens and film distortions on spacing determinations is difficult to assess due to the irregularity of these distortions for a given photograph and because these distortions vary so much from one photograph to another. These variations were illustrated in Figures 17 and 18. The more ground control that is available, the less effect these distortions will have on vehicle positions because each ground control point "ties" the photograph to the ground and only differential distortions between ground control points have any effect. The absolute distortions then have no bearing on spacing or velocity determinations. With this in mind, ground control points were established at relatively short intervals—averaging 186 feet with a maximum of 260 feet. Thus, an attempt was made to reduce the effect of the
distortions to a magnitude such that they could safely be disregarded.

Figure 20 shows the differential distortion at the point midway between two ground control points for various ground control spacings, as determined from the traffic photography data. At the maximum spacing of 260 ft., the average distortion for a vehicle half-way between these two points would be 0.17 ft. at ground scale. The maximum value would be 0.45 ft. However, due to the nature of the derivation of the data for Figure 20, these curves are valid for the middle three-quarters of the photograph only. This is indicated in Figure 19, where the data points extend to a radial distance of 21 mm. only, whereas the distance to the edge of the photograph is 28 mm. If the differential distortion curve in Figure 19 for the 500 ft. spacing is extended to a radial distance of 28 mm., the maximum value would approximate 2.6 ft. rather than the 1.97 ft. shown. This represents an increase of 30 percent. If the maximum error for a 260 ft. spacing is increased by a similar percentage, a value of 0.59 ft. is obtained.

Figures 17 and 18 indicate the errors at two closely spaced points due to differential distortion have the same sign as the edge of the photograph is approached, which is the critical case. Thus, the worst situation occurs when the following vehicle is near the edge of the photograph—thereby having a maximum differential distortion of 0.59 ft.—and the lead vehicle is next to a ground control point and therefore is not affected by differential distortion. This situation will result in spacing error of 0.59 ft. If the lead vehicle is either ahead of or behind the control point, an error due to differential distortion will exist, but this error will have the same sign as the error in the
following vehicle's position, and therefore will tend to compensate
for the error in the following vehicle's position rather than increase
it.

Since the position errors due to differential distortions have the
same sign for adjacent ground control intervals, except near the center
of the photograph where the distortions are very small, some compensa-
tion will almost always exist. As mentioned before, and as shown on
Figure 20, the average differential distortion for an average ground
control spacing amounts to 0.10 ft. at ground scale. The error in
spacing determinations due to the differential distortions will increase
as the edge of the photo is approached and as the ground control spacing
increases. In most cases, there will be some compensation of the differ-
tential distortion errors resulting from the positions of the lead and
following vehicles with respect to the ground control points.

Summary

The effect of each of the various errors discussed in this section
on the spacing determinations depends on many factors. Thus, although
the resultant spacing error for a given case—i.e., two specific vehicles
on a specific photograph—may be determined with fair accuracy (and con-
siderable computation), a meaningful "average" error, "maximum" error
or standard deviation is not easily obtained.

The "maximum" and "average" spacing error, as discussed in this
section are tabulated in Table 8. It is important to remember the qual-
ifications in the techniques used to derive these quantities. For
instance, the maximum possible spacing error due to tilt may be found
by assuming the worst combination of the factors involved, i.e., one
TABLE 8
EFFECT OF VARIOUS ERRORS ON SPACING DETERMINATIONS

<table>
<thead>
<tr>
<th>Item</th>
<th>Max. Error (ft.)</th>
<th>Avg. Error (ft.)</th>
<th>Type of Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Plane Shutter</td>
<td>+1.11</td>
<td>+0.16</td>
<td>Systematic</td>
</tr>
<tr>
<td>Tilt</td>
<td>±1.79</td>
<td>±0.12</td>
<td>Systematic</td>
</tr>
<tr>
<td>Vertical Alignment</td>
<td>+0.07</td>
<td>___</td>
<td>Systematic</td>
</tr>
<tr>
<td>Horizontal Alignment</td>
<td>+0.07</td>
<td>___</td>
<td>Systematic</td>
</tr>
<tr>
<td>Vehicle Relief</td>
<td>+0.48</td>
<td>+0.12</td>
<td>Systematic</td>
</tr>
<tr>
<td>Pointing Accuracy</td>
<td>±0.50</td>
<td>±0.40</td>
<td>Random &amp; Systematic</td>
</tr>
<tr>
<td>Lens &amp; Film Distortions</td>
<td>±0.59</td>
<td>±0.10</td>
<td>Random &amp; Systematic</td>
</tr>
</tbody>
</table>
vehicle is at the worst possible k value between the two ground control points with the maximum spacing, at the edge of the photograph with the maximum tilt, and the other vehicle is at a ground control point. This will result in a spacing error of 1.79 ft., as shown previously. However, this combination of events did not occur in the traffic photography—the ground control points with maximum spacing were not even on the photograph with maximum tilt.

The errors in Table 8 are not summed because the totals have no meaning. Some of the conditions required to obtain the maximum spacing error due to one type of error contradict the conditions required to obtain the maximum spacing error due to another type of error. For example, to obtain the maximum possible value of the spacing error due to tilt, the vehicles must be separated by one-half ground control point interval, whereas the vehicles must be separated by the maximum spacing of 376 ft. to obtain the maximum spacing error due to the focal plane shutter or vehicle-relief.

An examination of Table 8 reveals that efforts should be made to keep the tilt to 2° or less (this will reduce the maximum error to 0.46 ft.); investigate the use of a camera with better defined lens and film distortions, or reduce the ground control spacing (note the average error is quite small); and possibly correct the spacing for the effect of the focal plane shutter and vehicle-relief (although the average error is quite small). Also, examination of the average errors shows that it is undesirable to obtain the photo coordinates with any less accuracy. This requires that a high-order comparator be used (unless larger spacing errors are permissible).
It seems safe to assume the average error in the determination of the spacing between two vehicles is near 0.75 ft., and probably tends to be positive; i.e. the computed spacing is generally too large. The maximum error is estimated to be less than 3.5 ft.

**Effect of Errors on Velocity Determinations**

The various errors in the photogrammetric techniques used in this study affect the vehicle velocities as computed. All of the errors previously mentioned contribute to errors in the distance traveled by the vehicle between exposures. These distance errors combine with timing errors and produce velocity errors. Most of the distance errors are similar to the spacing determination errors. The essential differences are that the distances traveled are generally shorter, and two photographs are involved for each vehicle. travel distance. The focal plane shutter introduces another error.

**Focal plane shutter**

The time-distance relationships between the vehicle, helicopter, and the two photographs are illustrated in Figure 23, where:

- $C_t$—vector representing the true path of the vehicle being studied.
- $\tan \theta_{C_t}$ is the true velocity of the vehicle.
- $C_c$—vector representing the calculated path of the vehicle being studied. $\tan \theta_{C_c}$ is the calculated velocity of the vehicle.
- $H$—vector representing the path of the helicopter, which parallels the road. $\tan \theta_H$ is the velocity of the helicopter.
- $P_1$—vector representing the first photograph. This vector represents the sweep of the focal plane shutter slit over the ground
Fig. 23.—Effect of Focal Plane Shutter on Velocity Determinations
as it exposes the terrain. \( \tan \theta_p \) is the velocity of the sweep and is equal to the velocity of the helicopter plus the shutter velocity in the camera multiplied by the scale factor, or \( V_H + \frac{H}{f} V_s \).

\( P_2 \) -- vector representing the second photograph. The slope of this line is the same as for \( P_1 \) since the helicopter velocity is assumed constant over the interval between exposures.

A--ground control point
B--ground control point
1--position of vehicle when it is exposed on first photograph.
2--position of vehicle when it is exposed on second photograph.
3--position of vehicle one second (or other set time interval) after the vehicle was at position 1; i.e. \( t_3 - t_1 \) is the time interval between exposures--0.9982 seconds for the traffic photography.

4--\( D_4 \) is the point on the road that was exposed one "time interval" after the vehicle was at position 1. \( D_4 \) is the point exposed on photograph 2 at time \( t_2 \). Note that 1--4 is parallel to \( H \).

The true velocity of the car is:

\[
V_{C_t} = \tan \theta_{C_t} = \frac{D_3 - D_1}{t_3 - t_1} = \frac{D_2 - D_1}{t_2 - t_1}
\]

The calculated velocity is obtained by dividing the measured distance traveled, \( D_2 - D_1 \), by the time interval between exposures, or:

\[
V_{C_c} = \tan \theta_{C_c} = \frac{D_2 - D_1}{t_3 - t_1}
\]
The error in the velocity determination is:

\[ E = V_{C_c} - V_{C_t} = \frac{D_2 - D_3}{t_3 - t_1} - \frac{D_2 - D_1}{t_2 - t_1} \]

\[ E = (D_2 - D_1) \frac{(t_2 - t_1) - (t_3 - t_1)}{(t_3 - t_1) \cdot (t_2 - t_1)} = \frac{(D_2 - D_1)}{(t_3 - t_1)} \frac{(t_2 - t_3)}{(t_2 - t_1)} = \frac{(t_2 - t_3)}{V_{C_c} (t_2 - t_1)} \]

From Figure 23:

\[ (t_2 - t_3) = \frac{(D_2 - D_4)}{V_P} \]

\[ (t_2 - t_1) = \frac{(D_2 - D_1)}{V_{C_t}} \]

\[ (D_2 - D_1) = (D_2 - D_4) + (D_4 - D_1) \]

Combining the previous four equations:

\[ E = \frac{V_{C_c}}{V_P} \frac{V_{C_t}}{V_P} \left[ 1 - \frac{(D_4 - D_1)}{(D_2 - D_1)} \right] \]

Also from Figure 23:

\[ (D_4 - D_1) = V_H (t_3 - t_1) \]

\[ (D_2 - D_1) = V_{C_c} (t_3 - t_1) \]

Therefore:

\[ E = \frac{V_{C_c}}{V_P} \frac{V_{C_t}}{V_P} \left[ 1 - \frac{V_H}{V_{C_c}} \right] = \frac{V_{C_t}}{V_P} \cdot (V_{C_c} - V_H) \]
Substituting $V_{C_t} = V_{C_c} - E$ and $V_P = V_H + \frac{H}{f}V_S$;

$$E = \frac{V_{C_c} - E}{V_H + \frac{H}{f}V_S} (V_{C_c} - V_H)$$

Solving for $E$:

$$E = \frac{V_C - V_H}{\frac{H}{f}V_{C_c} + 1}$$

The true velocity may be obtained from:

$$V_{C_t} = V_{C_c} - E = V_{C_c} \left[ V_H + \frac{H}{f}V_S \right]$$

The focal length is 75.82 mm., the flying height averages 2350 ft., and the shutter speed was determined to be 38 in/sec. Hence, the factor $\frac{H}{f}V_S = 20,350$ mph. when the appropriate unit conversions are made.

The expression for the error may be rewritten as:

$$E = \frac{V_C - V_H}{20350 + 1}$$

It is apparent that the error is zero when the calculated velocity (also true velocity since error is zero) equals the helicopter velocity. In this case, the time between exposures of the vehicle will be equal to the time interval between the beginnings of the two photos; i.e., the time interval will be equal to that assumed.
The maximum error occurs when the calculated velocity of the vehicle is a maximum (roughly 60 mph.) and the velocity of the helicopter is a minimum (approximately 20 mph.). In this case:

\[ E = \frac{60 - 20}{\frac{20350}{60} + 1} = \frac{40}{340} = +0.12 \text{ mph.} \]

If the helicopter velocity exceeds that of the vehicle, the numerator of the error expression, and thus the error, will be negative. The worst combination for this case is a helicopter velocity of 60 mph. and a vehicle velocity of 30 mph. This results in an error of -0.04 mph. Thus, the error ranges from -0.04 mph. to +0.12 mph. A negative error indicates the true velocity was greater than the calculated velocity. Note that as the vehicle velocity approaches zero, the denominator of the error expression approaches infinity and the error approaches zero. This error is systematic and corrections can be computed. However, the helicopter velocity is not known precisely, and the corrections would not be exact. For the conditions experienced in the traffic photography, the errors due to this source are of such magnitude that they do not warrant compensation.

**Tilt**

Since two photographs are involved in the velocity determinations, the error in a velocity determination will be a function of the differential position errors for the vehicle on the two photographs. The worst situation that can occur is for the vehicle to be next to a ground control point on the first photograph, and then 88 ft. from this ground control point on the second. (The maximum distance a vehicle can
travel is the one second interval between exposures is 88 ft., as the maximum velocity is 60 mph.)

The maximum position errors occur on the photograph with the maximum tilt of 7°. If this condition is assumed for the second photograph, then the tilt of the first photograph is of no concern, as the position error will be zero when the vehicle is next to a ground control point regardless of the tilt. (It is true that the position error of the vehicle on one photograph could have the opposite sign from that of the same vehicle on the next photograph. However, examination of the data shows that the maximum change in tilt between successive photographs was 3°. Thus, the magnitudes of the position errors would be very small, as the tilts would have to be small.)

If the vehicle is not at a ground control point on the first photograph a position error will exist which will have the same sign as the position error in the second photograph and compensation will decrease the differential position error. The maximum rate of change of the position error with \( k \) occurs as \( k \) approaches 0.00 or 1.00. The maximum position error occurs for the maximum spacing of ground control points, which is 260 ft. Thus, the \( k \) value of the vehicle on the second photograph will be \( 88/260 = 0.34 \). If the maximum ground control point spacing is assumed at the edge of the photograph with the worst tilt, and \( k \) is 0.34, the position error will be 1.60 ft. (Derivation of position error formula is given in the discussion of tilt at the beginning of this chapter.) The error may be positive or negative as the tilt may have either sign. By the same reasoning set forth in the discussion of the effect of tilt on vehicle spacing determinations, it is conservative to
estimate the travel distance error has a median value near 0.12 ft. Compensation may be larger or smaller than in the case of the spacing determinations because the tilt of the two photographs may be different. On the other hand, in the velocity determinations the vehicle moves an average of only 50 ft. whereas the spacing between vehicles averages 95 ft.

If the vehicle velocity is zero, the vehicle will not change position with respect to the ground control points and there will be apparent movement due to the change in position error between the two photographs. If the first photograph is tilted 1°, the maximum position error will be 1.02 ft. If the next photograph is tilted 7°, the error for the same vehicle will be about 1.79 ft. Thus, the vehicle will apparently move 1.79 - 1.02 = 0.77 ft. even though it was standing still. This corresponds to a velocity error of 0.51 mph. This is an extreme value, and requires the worst combination of several factors.

The maximum velocity error caused by the error in the computed travel distance due to tilt will be:

\[
\frac{1.60}{(0.9982)(1.47)} = 1.09 \text{ mph.}
\]

where 0.9982 is the time interval between exposures in seconds and 1.47 is a conversion factor from fps. to mph. Similarly, the median velocity error due to tilt may be estimated at 0.08 mph.

This velocity error may be positive or negative and corrections may be computed. If the tilt is held to less than 3° in future flights, compensation will not be necessary as velocity errors due to tilt will all be less than 0.43 mph, and the average will be even smaller than 0.08 mph.
**Vertical alignment**

The maximum travel distance error due to assuming the roadway is level is \((0.02)(88) = 0.018\) ft. This corresponds to a velocity error of 0.01 mph., and does not warrant further consideration.

**Horizontal alignment**

The velocity error due to approximating the curved roadway with a series of straight lines is also about 0.01 mph., and warrants no further discussion.

**Vehicle relief**

In a vertical photograph, the maximum possible position error due to vehicle relief is \(+1.12\) ft. (See Table 4) If the preceding photograph were also vertical, the vehicle would be a maximum of \((88 - 30) = 58\) ft. from the edge of the photograph, or at least \(1750/2 - 58 = 817\) ft. from the nadir point. (The 30 ft. must be subtracted because the helicopter has traveled at least that far.) The position error for the vehicle on that photograph would be:

\[
\frac{e}{3\text{ ft.}} = \frac{817\text{ ft.}}{2350\text{ ft.}} \quad (\text{See Figure 14})
\]

\[
e = 1.04\text{ ft.}
\]

The two position errors have the same sign and would compensate. Thus, the maximum error in the travel distance would be \(1.12 - 1.04 = 0.08\) ft. At an average velocity of 34 mph., the corresponding travel distance error would be 0.04 ft. Corresponding velocity errors are 0.05 mph. and 0.03 mph. respectively.

If the second photograph were tilted \(7^\circ\), the maximum position error
has been shown to be 1.56 ft. The vehicle will be \( H(\tan 27^\circ 30') = 2350(0.52057) = 1223 \) ft. from the nadir point for the second photograph. However, the maximum change in distance for the vehicle from the nadir point for two successive photographs remains 58 ft., as the maximum velocity of the vehicle relative to the nadir point (helicopter) is 40 mph. Hence, the difference in position errors remains 0.08 ft. Thus, the range of velocity determination errors due to vehicle relief is 0.00 (for vehicles with the same velocity as the helicopter) to +0.05 mph.

The position errors of the vehicles on the trailing half of the photographs are negative, but the travel distance error will be positive as the position error on the first photograph will be larger in magnitude than on the second photograph. Thus, the computed travel distance will be larger than the true travel distance and the velocity error will be positive.

This error is systematic and the appropriate reduction in calculated velocity could be applied if desired.

**Pointing accuracy**

The travel distance is determined by subtracting the D-distance, or accumulative distance travelled by a vehicle from some arbitrary starting point, of the vehicle in one photograph from the D-distance of the same vehicle in the next photograph.

The error in the travel distance is of the same nature as the error in spacing discussed previously. However, the maximum travel distance is only 88 ft. whereas the spacings were as large as 377 ft. Thus, the worst case for travel distance error involves two D-intervals at most,
instead of three D-intervals as in the spacing determination. The worst case is illustrated below:

Let $D_L$ be the D-distance of the vehicle in the second photograph and $D_F$ be the D-distance of the same vehicle in the first photograph. Then:

$$D_L = D_2 + A_2 X_L$$

$$D_F = D_1 + A_1 X_F$$

where the terms have the same meanings as in the discussion of pointing accuracy in relation to spacing determination errors. The travel distance is:

$$T = D_L - D_F = (D_2 - D_1) + A_2 X_L - A_1 X_F$$

The variance of the terms $(D_2 - D_1)$ was determined to be 0.05634 ft.$^2$ in the discussion of spacing determination errors. The variance of the third term, $-A_1 X_F$, was found to be 0.05648 ft.$^2$ in that section. The variance for the second term, $A_2 X_L$, must be computed as a vehicle approximately 88 ft. in front of control point $D_2$ must be used. The formula used and computation methods are the same as for finding the variance of $a_2$ in the previously mentioned section. The resultant variance of the second term is 0.05697 ft.$^2$ (Vehicle 825 on photograph 110 was used for the lead vehicle.)
The variance of the travel distance is equal to the sum of the variances of the three terms in the travel distance expressions; i.e.

\[ \sigma_T^2 = 0.05634 + 0.05697 + 0.05648 = 0.16979 \]

\[ \sigma_T = (0.16979)^{1/2} = 0.412 \text{ ft.} \]

As pointed out previously, the variance for a particular travel distance will depend on the D-point spacings of the vehicle relative to the ground control points, and the position of the D-points in relation to the ground control points. The minimum standard deviation to be expected is 0.335 ft. Thus, the range between minimum and maximum is quite small. The velocity error corresponding to a travel distance error of 0.412 ft. is 0.28 mph.

**Lens and film distortion**

Vehicle velocity determinations involve one vehicle on two successive photographs. The travel distance error due to lens and film distortion is determined by the difference in differential distortion for the vehicle on the two photographs.

The maximum position error to be expected was shown to be 0.59 ft. in the discussion of the effect of distortions on the spacing determination. This occurs when the vehicle is mid-way between the two ground control points with maximum spacing and is imaged near the edge of the photograph. It is possible that the vehicle might still be mid-way between the ground control points on the succeeding photograph as vehicle velocities do approach zero on the traffic photography. Even though the differential distortion is erratic, it is unlikely that the
value on the succeeding photograph would be greatly different, as the vehicle must occupy nearly the same position (the only change being due to the movement of the helicopter and differential tilt).

Table 7 shows the maximum change in ground X-coordinate error between successive photographs to be 0.84 ft.; between photographs 131 and 132. The values given in column (4) are the X-error for the same point (ground control point 221) in each photograph and thus the situation is similar to that of a vehicle with zero velocity. However, these errors are for a 500 ft. spacing, and the maximum spacing between ground control points in the traffic photography is 260 ft. Figure 20 indicates that reducing the spacing from 500 ft. to 260 ft. reduces the maximum differential distortion to 30%. If this same percentage is applied to the differential error between successive photographs, a value of $(0.84)(0.30) = 0.25$ ft. is obtained.

At an average ground control spacing of 186 ft., it is possible for a vehicle travelling 60 mph. to cover nearly one-half of the ground control point interval in one second. Thus, the vehicle could have the maximum differential distortion for a spacing of 186 ft. -- which is 0.25 ft. -- on one photograph and then have zero differential distortion on the next photograph by being adjacent to a ground control point.

Both of the cases discussed above result in a travel distance error of 0.25 ft. and they represent extreme examples. Thus, the travel distance error due to the combined lens and film distortions varies in magnitude from zero to 0.25 ft. and may be either positive or negative. The velocity error corresponding to a travel distance error of 0.25 ft. is 0.17 mph.
Accuracy of time interval between exposures

The average interval between exposures was found to be 0.9982 seconds, with a standard deviation of \( \pm 0.0042 \) seconds. The extreme values in a sample of 136 interval determinations were 0.986 seconds and 1.007 seconds. The standard deviation is 0.42\% of the time interval, and the velocity determinations will be affected by this amount.

"Maximum error" is frequently defined as 2 standard deviations, or 0.84\% in this case. The velocity error, in mph., for various velocities is shown below:

<table>
<thead>
<tr>
<th>Velocity (mph)</th>
<th>Error (mph.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>0.08</td>
</tr>
<tr>
<td>40</td>
<td>0.17</td>
</tr>
<tr>
<td>60</td>
<td>0.25</td>
</tr>
</tbody>
</table>

For the range of velocities experienced in the traffic study, the magnitude of the velocity determination error ranges from zero to 0.50 mph. and may be either positive or negative. For the average velocity of 34 mph., the standard deviation is \( \pm 0.14 \) mph.

Summary

Like the spacing determination error, the net velocity determination error is dependent on many factors, some of which can be determined with confidence and corrected for if desired (such as the effect of the focal plane shutter and vehicle relief), and others which are random and/or poorly defined (such as the errors due to the lens and
film distortions). In some cases the errors compensate, in others they are additive.

"Maximum" and "average" values, subject to the qualifications made in deriving or estimating these quantities, are shown in Table 9. Some of the values, particularly the "average" values, are estimates based on study of the data and results, and can not be derived precisely.

Table 9 indicates that tilt is the factor which may influence the velocity determination accuracy the most. However, the maximum value—1.09 mph.—requires a specific combination of the worst values of several variables, which is unlikely to occur. (Note that the average error is quite small.) Also, this value can be reduced to 0.51 mph. by maintaining the tilt at 3° or less. This error is systematic and can be computed if desired.

The lens and film distortion is poorly defined and may change considerably from photograph to photograph. The values shown were estimated from data derived from the central 75% of the photographs. Since lens distortions, and probably film distortions for this camera, increase as the edge of the photograph is approached, larger errors may occur there.

The values in Table 9 are not summed because the errors are not additive. To obtain the maximum error due to a given factor, certain conditions are necessary which may be contrary to those required to obtain the maximum value due to another type of error.

Examination of the error values in Table 9 indicates that efforts should be made to minimize tilt and that the accuracies of the coordinate
<table>
<thead>
<tr>
<th>Item</th>
<th>Max. Error (mph.)</th>
<th>Avg. Error (mph.)</th>
<th>Type of Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Plane Shutter</td>
<td>+0.12</td>
<td>+0.04</td>
<td>Systematic</td>
</tr>
<tr>
<td>Tilt</td>
<td>±1.09</td>
<td>±0.08</td>
<td>Systematic</td>
</tr>
<tr>
<td>Vertical Alignment</td>
<td>+0.01</td>
<td>---</td>
<td>Systematic</td>
</tr>
<tr>
<td>Horizontal Alignment</td>
<td>+0.01</td>
<td>---</td>
<td>Systematic</td>
</tr>
<tr>
<td>Vehicle Relief</td>
<td>+0.05</td>
<td>+0.02</td>
<td>Systematic</td>
</tr>
<tr>
<td>Pointing Accuracy</td>
<td>±0.28</td>
<td>±0.25</td>
<td>Random &amp; Systematic</td>
</tr>
<tr>
<td>Lens &amp; Film Distortions</td>
<td>±0.17</td>
<td>±0.09</td>
<td>Random &amp; Systematic</td>
</tr>
<tr>
<td>Time Interval</td>
<td>±0.50</td>
<td>±0.14</td>
<td>Random</td>
</tr>
</tbody>
</table>
reading device used and the time interval between exposures are limiting factors in gaining further velocity determination accuracy.

The effect of vehicle relief is not as great in velocity determinations as in spacing determinations because the distances involved in velocity determinations are much smaller. For the range of vehicle and helicopter velocities encountered in this study, the fact that the camera employed focal plane shutter was not a significant disadvantage.

The average error in the determination of vehicle velocities for the traffic photography used in this study is estimated at approximately 0.4 mph., with a maximum error near 1.0 mph.
CHAPTER VIII

CONCLUSIONS

General

The photogrammetric techniques formulated and tested in this study can provide a type of data not heretofore available in the study of traffic flow phenomena—accurate traffic flow data continuous in both time and space. Specifically, accurate vehicle trajectories, with corresponding vehicle spacing and velocity data, were obtained. Data of this type are required to effectively study traffic platoon characteristics and may be used in many other traffic studies, such as validation of traffic flow theories and investigations of traffic volume-speed-density relationships, speed distributions, lane changing, passing and car-following maneuvers.

It is practically impossible to obtain these same data with comparable accuracy through ground data collection techniques. An important advantage of the aerial survey method is that it does not influence the traffic characteristics being measured, as the drivers are not aware that measurements of their driving actions are being made. Data collected on the ground through the use of pneumatic tubes, wires, other visible devices, or observers near the roadway, are generally suspect from this standpoint, as the influence of the presence of data collection apparatus on the traffic characteristics can not be evaluated.
Another significant advantage of the aerial survey methods over ground data collection techniques in the study of traffic flow phenomena is that the specific location of the data collection or the specific group of vehicles to be studied does not have to be selected until the outset of the actual data collection. The normal traffic situation can be observed from the helicopter hovering over the general area of interest, and then the helicopter can move to a specific location and photography initiated when a traffic pattern of particular interest develops. This procedure results in highly efficient data collection, as it is possible to secure data on the most informative patterns that occur on a considerable length of roadway. Ground data collection apparatus, on the other hand, must be set up in advance and it can only be hoped that the desired traffic patterns will occur at the pre-selected data collection site. It is also possible to alter the flight plan and record unusual and unpredictable traffic events with aerial photography.

Still another advantage of the aerial survey technique is that a permanent and complete record of the total situation is available on the photographs. Thus, if analysis of the initial data indicates the need for additional information, it can be obtained.

It is evident from the discussion of costs at the end of Chapter V that the major bottleneck in the photogrammetric technique for obtaining traffic flow is in the digitization of the data available on the photographs. This factor alone may well determine the economic feasibility of the aerial survey method for a given traffic study. Approximately 70 percent of the cost of obtaining the final data, as outlined
in Chapter V, is directly attributable to the data reduction process.

Other Specific Conclusions

In addition to the general conclusions regarding adequacy of the photogrammetric techniques to obtain traffic flow data, advantages of the aerial survey method, and the discussion of costs, the following specific conclusions can be drawn as a result of this study:

1. The Maurer P-2, 70mm. camera, the project-designed camera mount, and the Abrams Type CP-3 Intervalometer performed in a satisfactory manner.

2. A camera with a focal plane shutter can be used for this type of study. A photogrammetric camera, however, would be advantageous over the Maurer P-2 camera.

3. A study duration of four minutes is usually sufficient to record platoon formation and dissipation or the movement of a test vehicle into and out of a traffic jam.

4. A time interval between exposures of one second is satisfactory to define traffic characteristics—a longer interval will not provide sufficiently definitive data and a shorter interval will increase the data reduction task without appreciable increase in definition.

5. Photo coverage of approximately 2000 feet of roadway is sufficient, but a longer length is desirable.

6. The scale of the photography should be at least 1:10,000, so that spacing and velocities can be determined with sufficient
accuracy and to insure that photo-identifiable ground control points near the roadway will be available in sufficient density.

7. Vehicle shadows are sometimes difficult to differentiate from the vehicles themselves, but this problem did not exist when Ektachrome Infrared film was used. This latter type of film also facilitated vehicle identification from one photograph to another. However, panchromatic film is less expensive to purchase and process.

8. An observer should accompany the pilot and camera operator in the helicopter to help select the traffic situation to be photographed, aid the pilot in maintaining the helicopter position above the test vehicle, and allow the camera operator to concentrate on maintaining the camera optical axis vertical so that excessive tilts are not obtained.

9. An observer should accompany the driver of the test vehicle, and be in radio contact with the helicopter to aid the pilot maintain the desired position.

10. At times, a traffic situation which would provide data better suited to the project objectives will occur ahead of or behind the test vehicle, and the flight plan should be altered by the observer to take advantage of this situation at the expense of losing the test vehicle. In this case, the observer should select an alternate vehicle to follow.

11. The OMI-Bendix AP/C Analytical Stereoplotter is suitable for
the task of determining the necessary photo coordinates. A less expensive precise comparator would serve the purpose, however, if automatic read-out, with identification numbers, is possible.

12. The procedure and computer program for obtaining vehicle ground coordinates, D-distances, vehicle spacings, and vehicle velocities are satisfactory. Minor modifications will be required if more than two lanes of traffic are involved.

13. The time-distance diagram is a clear and meaningful way to present the data obtained.

14. The average and maximum error in the spacing determinations are estimated at 0.75 ft. and 3.5 ft. respectively. Major sources of these errors are: tilt of the camera axis; accuracy of the photo coordinate measurements; lens and film distortions; and effect of the focal plane shutter.

15. The average and maximum error in the velocity determinations are estimated at 0.4 mph. and 1.0 mph. respectively. Major sources of these errors are: accuracy of the time interval between exposures; accuracy of the photo coordinate measurements; tilt of the camera axis; and lens and film distortions.

16. The objective of this study was met—i.e., to obtain accurate traffic flow data, continuous in both time and space, through photogrammetric techniques.
CHAPTER IX

RECOMMENDATIONS FOR FUTURE STUDY

The techniques and methodology to obtain accurate data on spacing between vehicles and vehicle speeds for a group of vehicles as they progress along the roadway have been developed in this study. This new capability will be particularly useful in studies directed toward the verification of a present theory of traffic flow, or the development of a new theory.

Traffic engineering problems which could not previously be accurately defined numerically can be studied with new insight. These studies may include the investigation of flow characteristics on approaches to intersections, on urban arterials, and on express highways. Slightly different specifications with respect to data accuracy, roadway coverage, interval between exposures—or simply more data—may be desirable in such investigations.

Some advantage and improvement over the techniques and methodology developed in the present study may be obtained as a result of further research in the following areas:

a. Data reduction. This matter, which proved to be the major bottleneck of the present procedure, should receive prime attention. Investigations during the present study indicated automatic coordinate extraction was not practicable for the
project objectives and scope, but may be feasible in the near future or for studies of larger scope. If manual methods are employed the use of less expensive comparators than the AP/C should be investigated. Coordinated measuring devices utilizing projection of photographic transparencies onto a glass measuring table should increase operator efficiency.

b. The use of other cameras may be advantageous and should be investigated. For instance, stereophotogrammetric techniques, using a Wild RC8 camera, could provide a roadway coverage of 4500 ft. and an accuracy of ±0.15 ft. (ground scale) in coordinate determinations. Errors due to focal plane shutter, tilt, vehicle relief, and lens and film distortions would be practically eliminated. Disadvantages of larger format cameras include possible sacrifice in minimum recycling time (three seconds for a Wild RC8 camera), greater film cost, possible difficulty in obtaining and processing film, and problems associated with mounting the camera in the helicopter.

c. The use of sensors, such as infrared scanners and radar, for traffic data acquisition should be considered. These types of equipment offer the possibility of relatively simple automatic data reduction. It is likely that projects of larger scope than the present study will be required in order for these techniques to be economically feasible.

d. Panchromatic Plus-X and Ektachrome Infrared films were used in the present study. The use of other film types—particularly
standard color films and fast panchromatic films—may be advantageous in other traffic investigations. Relatively fast films will usually be required, as most critical traffic situations occur either early or late in the day.

e. The computer program could be refined to include compensation for some of the systematic errors, such as the effect of the focal plane shutter, tilt of the camera axis, horizontal and vertical alignment of the roadway, and vehicle relief. Corrections for lens and film distortions could also be included if another camera is used for which distortion curves can be obtained. The program could also be expanded to handle more than two lanes of traffic. The desirability and necessity of these changes will depend on the proposed study.
REFERENCES


