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DISSENTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Earl James Zwick, B. S., M. Ed.

* * * * * *

The Ohio State University
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Approved by

[Signature]  
Adviser
Department of Education
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CHAPTER I

THE PROBLEM

The high degree of diversity in the mathematical backgrounds of entering freshmen in American universities and colleges has made placement in mathematics a matter of concern for the institutions. The problem has been further complicated and the range in backgrounds has been further broadened by recent curricular changes in secondary mathematics.

Any solution to the problem has been made more difficult by large enrollment increases in many institutions in recent years. In 1957 Rickover wrote:

Within the next fifteen years, six million youngsters will clamor for admission to our institutions of higher learning. Colleges and universities are now being exhorted to prepare for the flood, and they are criticized severely when they refuse to expand in order to become mammoth high schools where overgrown children must be taught . . . where many are mathematically illiterate yet confidently expect to become engineers because this is a profession where money is to be made.1

While many people would dispute some of Rickover's statements on education, the fact remains that placement problems have multiplied in recent years with the growth

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in enrollments in colleges and universities. This problem of overcrowding is expected to become worse as future enrollments increase according to projected figures.

At The Ohio State University placement examinations indicate that approximately thirty per cent of the entering freshmen do not have the mathematical background to compete successfully in the traditional college mathematics courses. The solution to this problem at Ohio State has been to offer remedial mathematics courses for these students.

Placement in mathematics at Ohio State is presently based on a series of mathematics achievement tests. Those students scoring below a pre-assigned level on the initial test, test A, are required to take a second test, test B. Students scoring below a pre-assigned level on this test are required to take a remedial course in mathematics. Those students who passed test A are also given a second test, test D, at a more advanced level to determine their placement level in the mathematics sequence of courses. This sequence includes advanced placement and an honors sequence for the most gifted students who intend to take one or more years of mathematics.

The remedial students are assigned to Mathematics 400 if they do not intend to complete a calculus sequence or to Mathematics 401 if they intend to complete such a sequence. This study is concerned with the remedial aspects of the placement program.
The large number of entering freshmen determined by the placement tests to be deficient in mathematical background would seem to make the effectiveness of the remedial program important to the University. It is the purpose of this investigation to determine the degree of effectiveness of this program. The problem under investigation includes the following aspects:

1. The determination of the predictive efficiency of the present system of selection for remedial students.

2. The determination of the extent to which the remedial mathematics courses remedy the weaknesses identified by the placement examinations.

Included in the study is an analysis of the errors made by the students on placement test B with their resulting implications for remedial mathematics and for secondary mathematics. Also included is an analysis of the high school preparation of one of the groups of remedial students.

The general hypotheses tested in this study are these:

1. The mathematics placement tests significantly distinguish the students who are likely to pass their initial college-level mathematics course with a grade of "C" or better from those who are unlikely to do so.

2. Mathematics 400 and 401 increase the mathematical skills of the students taking the courses to the degree
that they are able to enjoy success in subsequent mathematics courses equal to that of comparable students whose backgrounds are such that these students do not require remedial work in mathematics.

3. Students who have completed two units of high school algebra perform significantly better in Mathematics 400 than students who have completed one unit of high school algebra.

The mathematics program at Ohio State for remedial students includes two options for the students. For the students who want or need to take a sequence of calculus courses in their programs, Mathematics 401 is followed by Mathematics 439, which is a course in college algebra and trigonometry. For the students who do not need a sequence of calculus courses, Mathematics 400 is followed by Mathematics 416. For elementary education students, Mathematics 400 is followed by Mathematics 410.

The courses with which this study is primarily concerned are described as follows in the general catalogue of the university:

Mathematics 400: Arithmetic and Elementary Algebra
This course consists of a review of arithmetic combined with topics from elementary algebra and geometry.

Mathematics 401: Intermediate Algebra and Trigonometry
A review of material which is usually contained in a second high school algebra course and in one semester of high school trigonometry.
Mathematics 416: First Year College Mathematics

The sequence 416, 417 is designed as a terminal sequence of courses in mathematics and to prepare students to enter a calculus sequence. 416 treats sets, functions, algebra, graphs, and vector spaces.

Mathematics 439: Algebra and Trigonometry

Inequalities, functions, graphs, exponential and logarithmic and trigonometric functions and their graphs, complex numbers, inverse functions.2

The study was limited by the fact that it basically covered only the fall and winter quarters of 1963-4 and did not pursue the students' progress in mathematics beyond their first college-level mathematics courses. A small group of engineering students were studied beyond this stage in their mathematical training. Data on this group covered the period from their entrance as freshmen in the fall quarter of 1961 through the summer quarter of 1963.

Chapter II contains a report on the related literature. The following chapter is devoted to the methods and procedures used in the study. In Chapter IV the results are presented in terms of the hypotheses tested in the study, and the last chapter includes a summary of the study,

the conclusions, and the recommendations made on the basis of the results.
CHAPTER II

SURVEY OF THE RELATED LITERATURE

Much has been written in recent years on the subject of college remedial work. A search of the literature revealed many studies and articles concerned with the philosophy behind the offering of remedial work, the advantages and disadvantages of offering it in colleges and universities, the prevalence of remedial courses in American higher education institutions, the various proposals for solving the problems of diversity in the backgrounds of the entering freshmen, and the prediction of success and point averages and grades in college courses.

In order to avoid being overly repetitive, this survey included the references which provided a crossection of the literature dealing with the major topics which relate to college remedial courses in general and to college remedial mathematics in particular.

Many of the references in the literature relating to the subject of college remedial work were focused upon the philosophy behind the offering of such work in higher education institutions.

On one side were those who claimed that remedial course work should not be offered in college. One of the arguments to support such a stand pointed to the amount of
the college faculty's time which was devoted to the needs of entering freshmen who, "read badly, write poorly, and figure inaccurately if at all...." The writer added, "...from one-tenth to one-third of the freshman class are not equipped with the basic skills expected of a qualified student."\(^3\)

It was further claimed that such courses were the product of "misplaced pedagogical emphasis" and that this aggravated the problem of overpopulation in the colleges. The misplaced emphasis referred to that on the needs of poorly prepared students. The alternative which was often offered was selective admissions under which such courses would not be needed since deficient students would not be admitted.\(^4\)

With respect to mathematics in particular, the arguments against remedial courses were aptly stated by Stone.

Clearly the college can hardly undertake a richer and more advanced program in mathematics unless it is willing to eliminate a considerable amount of deadwood from the traditional college curriculum. There are a good many obvious candidates for elimination, subjects which should either be taught in high school or not be taught at all. Let us cite college algebra, solid geometry, most of numerical trigonometry, descriptive

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geometry, and some topics in the calculus. Since a great many — perhaps a majority — of our colleges are still teaching these subjects, the work of reform has to begin by banishing them from the college curriculum. Only when this has been accomplished can the college mathematics program be given its proper scope and be brought to the proper level of quality.\(^5\)

The problem is not so easily solved in the cases of most state universities and colleges. Many states, Ohio included, have two basic laws which are relevant to the question. The first law states that only one year of mathematics is required for graduation from high school. This requirement can be satisfied by anything that might be taught by a particular mathematics department, including general mathematics, shop mathematics, or business arithmetic. The second law states that any graduate of a state-approved high school may enter the University. The mathematical preparation of the entering freshmen may vary from virtually no preparation to a thorough preparation through analytic geometry and even topics from calculus. The laws themselves prevent us from simplifying the problem by establishing entrance requirements.\(^6\)

There were some further arguments favoring the


offering of college remedial courses. One of these was stated by Hughes:

A number with low entrance qualifications achieve university success and vice versa; this applies whether the entrance measure is in terms of intelligence quotient, objective-achievement, traditional examinations, or success at the secondary level. ...universities and colleges have in fact an obligation to those whom they admit. ...the university and the college (should) accept the task of developing to the maximum all the intellectual talents and abilities of which they are the guardians. If this is accepted by these bodies as a correct attitude, then offerings which are 'appropriate to higher education' and which 'do belong in college and university curriculums' can be presented with a diversity of approach and with such necessary aids as to obtain the maximum effect in a diversity of personalities.⁷

A second such argument, although it was written in 1940, did not seem to have lost its timeliness.

Quite a number of young men do not make up their minds that they want to enter engineering or some other field which requires professional training in mathematics until they actually reach the university. Most of these have taken only the customary two years of high school mathematics. It therefore seems incumbent upon the university to provide some means of making a quick review of elementary algebra, a course corresponding to the second year of algebra, and a course in trigonometry available to these students.⁸

The question now arose concerning the prevalence of remedial courses in American higher education institutions.


⁸H. M. Bacon, "High School Mathematics in the University," The Mathematics Teacher, XXXIII (January, 1940),
A 1961 publication of the Committee on the Undergraduate Program in Mathematics, a committee of the Mathematical Association of America, stated the results of a survey of two hundred eighty-six colleges and junior colleges with respect to the course offerings in mathematics of the institutions. The surveyors estimated that twenty-five per cent of the students in four-year colleges and forty-one per cent of the students in junior colleges had available a course in remedial mathematics. The committee defined remedial mathematics as a "review of arithmetic and/or review of elementary algebra and plane geometry." ⁹

A survey by Lindquist of 877 institutions revealed that 24.9 per cent of the entering freshmen began their mathematical training at a level lower than college algebra and trigonometry. Further at public colleges the survey report indicated that approximately one-third of the students began at the lower level. ¹⁰

By number of institutions the survey report showed that about 57 per cent of the schools questioned offered, as their lowest level course, one which was below the college algebra and trigonometry level. Approximately 38 per cent

⁹Committee on the Undergraduate Program in Mathematics, A Catalogue Survey of College Mathematics Courses, Mathematical Association of America, 1961.

of the institutions offered college algebra and trigonometry as their lowest level course, and the remaining 5 per cent offered analytic geometry and calculus or above as the initial course in mathematics. ¹¹

Various solutions have been offered to the problem of diversity in the backgrounds of entering college freshmen. As stated previously, the most popular solution seemed to be selective admissions. The case for selective admissions was stated by McEwan, "There appears to me to be no obligation on the part of the state to provide equal education to all its citizens though I believe we should provide equal educational opportunity for all. The distinction seems to me related rather to the size of the college or university than to the nature of its government and support." ¹²

Jorgensen added, "Equal opportunity does not mean equal or identical education for all individuals. It means rather, that education at all levels shall be available to every qualified person." ¹³


¹³A. N. Jorgenson, "College for Everyone?," College Admissions, College Entrance Examination Board, 1957, p. xi.
With respect to mathematics in particular Rosenbloom favored the idea of selective admissions.

The first general aim of all these programs is to make a course in calculus, supplemented by analytic geometry, the standard beginning course for college freshmen. This course will ordinarily require a foundation of four years of mathematics in high school.

The first consequence is that within a few years the colleges will be able to reduce drastically their courses in college algebra, trigonometry, and remedial high school mathematics.... Colleges must set time limits on offering these courses for credit and a further time limit beyond which they will not offer these courses at all.\(^\text{14}\)

This same idea was echoed by a report of a conference on engineering education. The report stated:

The calibre and continuity of the high school mathematics preparation are basic to any discussion of the mathematics program for engineering students. The conferees were not willing to accept a policy of lowered admission standards, with remedial work as part of the normal program. Rather, it was believed that engineering colleges should insist that those students who wish to complete their training in the normal time must come prepared to start a course in calculus and analytic geometry at the beginning of their freshman year.\(^\text{15}\)

Williams recommended remedial classes as "the best temporary solution. The best final solution is to cover


less material in high school mathematics but cover it more thoroughly." 16

Placing the burden of solution of the problem of remedial work on the high schools was advocated by others as the best method of solution. One such advocate was Hartley who stated:

...first the administration should establish for each freshman course rigid prerequisites in terms of present ability. The administration should assign its best teachers to the classes populated by the poorly prepared students.

...pressures can be brought by the accrediting agencies upon the secondary schools so that a high school diploma will be a symbol of competence in the subjects pursued, and a guarantee that the minimum essentials of each course have been mastered .... Students are not at fault. The system in which they are entangled is to blame. 17

Hildebrandt advocated more study of the problem before any definite conclusions could be made.

What is needed is a definite testing program, which would determine the capabilities and weaknesses of the individual student in mathematics and in areas that could be related to success or failure in the study of mathematics. Such tests given during the first weeks of the year should not only show the skill or lack of skill in handling specific basic concepts of arithmetic and algebra, but should also supply information on such factors as the ability to make correct observations, to handle data on information correctly, to draw correct conclusions, and to reason properly. Certain standards in mathematics for


entrance into our freshman courses should also be agreed upon on a national basis.\textsuperscript{18}

Remedial mathematics as a permanent part of the college program for the foreseeable future was the solution advocated by some educators. A panel discussion of a sectional meeting of the Mathematical Association of American reported the following results.

There is a preponderence of evidence that the average college freshman is poorly prepared in mathematics. This does not necessarily mean that the student is dull or has a low I. Q. He may only be the victim of circumstances. Secondary education has reached a new high as to confusion of objectives. In many cases the high school diploma is little more than a belated birth certificate or perhaps a certification of a more or less satisfactory record of attendance at school for twelve years. The colleges have little choice in the matter of accepting the high school product and must start the students where they are and proceed. It is recommended that tests of mathematical competence be administered to incoming freshmen and the course be adjusted accordingly. At first there should be a very careful presentation of elementary topics from a new viewpoint, if possible. The beginning freshman needs review, he needs encouragement, and he surely needs to establish confidence in his ability to master mathematics. ...many students may need to spend as much as sixteen to twenty quarter hours on pre-calculus mathematics.\textsuperscript{19}

\textsuperscript{18}E. H. C. Hildebrandt, "For A Better Mathematics Program in the College," The Mathematics Teacher, XLIX (February, 1956), p. 92.

These same sentiments were expressed by Cameron in the *American Mathematical Monthly*.

State universities have their own peculiar problems. Many of them are due to a heterogeneous student body, representing an incredibly wide range of ability, preparation, and interest. ...in state universities with relatively unselected freshman classes, differentiated programs of study appear to be the only feasible way of providing the type of education appropriate to the various levels of ability.20

A doctoral dissertation on remedial mathematics in state universities and colleges drew conclusions to the effect that the institutions who reported years of experience and research in giving remedial mathematics were the ones who expressed the most satisfaction with the results of such courses. The author further added, "...that pre-freshman mathematics courses represent valuable instructional experiences is evident from the large number of institutions which express satisfaction with their results in offering such courses and from the high percentage of students who pass college mathematics after taking pre-freshman mathematics."21

More closely related to the present study was the literature pertaining to experiences in placement in mathematics in various institutions. Since the mathematics


program at this level had changed significantly in recent years both with respect to the college freshman program and with respect to the high school program, it was decided that a study made more than ten years ago would not apply to the present needs of students. Therefore, only studies made since 1954 are included in this review.

The amount of change in the curriculum was partially due to the new secondary mathematics programs. A report at Oklahoma State University indicated that approximately twelve per cent of the incoming freshmen in the fall of 1962 had been taught some mathematics under the School Mathematics Study Group program.\(^{22}\)

With respect also to high school preparation, Johnson reported that students who had taken intermediate algebra in high school performed significantly better in their initial mathematics course in college than the students with one year of algebra. The difference in performance was found to be independent of the difference in aptitude test scores between the two groups of students.\(^{23}\)

The prediction of college grades

At the University of Houston, "Of the seventeen


predictor-measures based on test scores, four 'achievement-type' tests yielded the greater number of significant relationships with the twenty-eight criterion-measures based on twenty-seven courses and QPA (quality point average)."\(^{24}\)

At the University of Maryland, using the American College Test scores, the Maryland Algebra Aptitude Test score, and the Reading Comprehension Test score, Marches found that

Even when intervals of achievement are extended to form broad dichotomies such as success and failure in mathematics versus upper-half and lower-half percentile scores, only one of the five entrance test measurements shows promise of being a useful instrument in predicting success or failure in mathematics course work. In this test relationship, only the Maryland Algebra Aptitude Test has a sufficiently high chi-square value that is statistically significant beyond the conventionally accepted \(0.05\) probability level.\(^{25}\)

Coddington at the University of California studied 7000 freshmen in 1963, using the Scholastic Aptitude Tests. Correlations were so low that the institution no longer requires entrance examinations. In predicting grade point averages, the correlations were \(0.18, 0.32, 0.29,\) and \(0.40\) respectively for the SAT Verbal, the SAT Mathematics, high


Graybeal's study at the University of North Carolina yielded the following results:

1. The best single predictor of either achievement or success in college algebra was the measure of incidental or residual knowledge of fundamental algebraic processes as indicated by scores on a diagnostic or pre-test in the subject.

2. Past record in mathematics courses played a leading role in predicting success, as indicated by instructor's marks in college algebra course work.

3. Intelligence test scores and results of reasoning ability tests played a much more influential role in predicting achievement, as measured by standardized tests in algebra, than in predicting success.

4. Rank in high school graduating class was a predictor of secondary importance for both criteria.

5. Personal interests played non-existent roles in predicting achievement, but rather influential supporting roles in predicting success.

6. Vocational interests seemed to play minor roles

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in predicting achievement and non-existent roles in predicting success.27

At the University of Oklahoma, where the Ohio State Psychological Examination, the Iowa High School Content Examination, and a mathematics placement examination were used, a study revealed the following findings:

1. The probably error of prediction of college grade points was approximately two-thirds of a point, based on regression equations in which the independent variables were the placement test raw score, the decile rank on the mathematics section of the Iowa test, the decile rank on the Ohio State Psychological Exam total score, and high school mathematics grade points. The error of prediction increased only slightly when only the first two variables were used.

2. From a practical standpoint, the students could be grouped using only the mathematics achievement tests.

3. The placement test served better as an instrument for separation of students into sections for intermediate algebra and a first course in mathematical analysis than for separation of students into sections for remedial mathematics and intermediate algebra.

4. Measures of general ability showed too much

overlap between mathematics groups to serve as a basis for separation into ability groups in mathematics.28

A study at the University of Houston indicated:

1. That a mathematics screening test developed locally was the best single predictor of success in a student's initial mathematics course.

2. That a course which was partially remedial - in that two-fifths of the time was devoted to remedial work - significantly improved the background of the students so that they could compete successfully with the more adequately prepared students.

3. That a completely remedial course for students with no high school background in mathematics, or for students scoring low on the screening test, did not increase the skills of students sufficiently to enable them to compete successfully in the more difficult mathematics courses.29

Hoerres and O'Dea, studying the predictive value of the American Council on Education (ACE) examinations, emphasized the fact that each institution must establish its own set of validities with respect to the placement


instruments used. They stated, "Unless the relationships of the American Council on Education scores with specific course marks in any given institution using the instrument are known, it is not possible to predict the outcome of courses with any likelihood of accuracy."³⁰

In summary, there seemed to be general agreement among educators that the problem of variation in background of entering freshmen existed, but that there was little agreement with respect to the best means of remedying the situation. There were educators and others who would exclude all remedial work from higher education institutions, usually on the basis that such work is the obligation of the high schools. Others saw differentiated instruction, including remedial work, as the only solution to the problem of diversity in the backgrounds of entering freshmen. Reports of studies indicated that many institutions were satisfied with the results of remedial instruction in mathematics. Some reporters indicated doubt about the use of predictive instruments to select the students who need remedial instruction, usually citing poor correlations between the predictors and course grades. However, the majority of the investigators concerned with predicting

success rather than grades expressed satisfaction with the results of the predictions.

Among the studies on predicting grades or success in freshman college mathematics courses, the majority of the results indicated that achievement-type tests in mathematics were the best criteria for predicting success or for grouping students into ability levels. The variability in the reliability of the tests indicated in the various studies showed the need for establishing local norms for a particular institution and the need for continuous revision of the norms to solve the problem of the constantly changing student body.
CHAPTER III

METHODS AND PROCEDURES

All students who enter Ohio State University are administered a series of guidance or placement examinations by the testing center of the university. Included in the battery of examinations are the Ohio State Psychological Examination, the American College Testing Program series, and the Ohio State University Mathematics Placement Tests. The subjects in this study were selected from all of the students who entered Ohio State University during the fall quarter of 1963. In addition, one sample consisted of students who entered as first quarter freshmen during the fall quarter of 1961.

Selection of subjects

In the fall of 1963, the Orientation and Testing Center provided the information which was available on all students who entered the university for the first time during the fall quarter of the 1963-64 academic year. From this group it was possible, using the data available, to select the students who were first quarter freshmen during the fall quarter of 1963 and who completed one of the courses with which this study is concerned during the fall quarter of 1963 or the winter quarter of 1964. All students who entered the university with prior college credits were
excluded from the study, even though they might otherwise have met all of the requisite conditions.

A second group of students, entering freshmen in the college of engineering during the fall quarter of 1961, were selected on the basis of being exactly one point below the passing score on the placement test which separated the remedial students from the non-remedial students.

The following information was available for the students who were entering freshmen, who were full-time students, and who were enrolled for a mathematics course during the fall quarter of 1963:

1. Student's identification number
2. Student's name
3. College of enrollment
4. Sex
5. Ohio State Psychological Examination total raw score
6. Mathematics Placement Test A raw score
7. Form of second mathematics placement test (B or D)
8. Mathematics Placement Test B or D raw score
9. American College Testing Program standard scores
10. Class status, as first quarter freshman
11. Mathematics placement level
12. Mathematics courses completed during the fall quarter of 1963 and the winter quarter of 1964
The data on the students who fulfilled the requirements necessary for inclusion in the study were transferred from the testing center's cards and from the final grade sheets of the mathematics department to International Business Machine cards to permit rapid statistical analysis.

The following groups of students were included in the various phases of the study:

1. All students who completed Mathematics 416 during the fall quarter of 1963.
2. All students who completed Mathematics 400 during the fall quarter of 1963 and mathematics 416 during the winter quarter of 1964.
3. All students who were enrolled for mathematics 410 during the winter quarter of 1964.
4. All students who completed Mathematics 439 during the fall quarter of 1963.
5. All students who completed Mathematics 401 during the fall quarter of 1963 and Mathematics 439 during the winter quarter of 1964.
6. All students who were entering freshmen in the college of engineering during the fall quarter of 1961 and who scored exactly eighteen on mathematics placement test 3.

Definition of terms

Some of the terms used in the study were assigned operational definitions in order to delineate their scope.
more clearly. These terms and their operational definitions are presented below:

**Success:** The term "success" or "successful" will be restricted to those students who received a final grade of "A", "B", or "C" for the mathematics course under consideration.

**Unsuccessful:** The term "unsuccessful" will be restricted to those students who received a final grade of "D" or "F" for the mathematics course under consideration.

**Pass:** The term "Pass" when applied to a placement examination score will mean that the score was above a preassigned cut-off score on that examination.

**Fail:** The term "fail" or "failure" when applied to a placement examination score will mean that the score was less than or equal to a preassigned cut-off score on that examination.

The mathematics screening test battery consisted of three tests which divided the students into four levels. Students scoring less than 24 on test A, which was required of all entering students, were required to take test B while the students who passed test A were required to take test D. Students scoring less than 19 on test B were assigned level IV status, while those students who scored more than 18 but less than 30 on the test were assigned level III status and the remainder were designated as level II students. On test D the cut-off score was 14, and students failing the test were assigned as level II while the passing students were assigned as level I.
Briefly, the mathematics placement program followed the following schedule.

Test A Scores

0-13 - level IV
0-23 - Test B 19-29 - level III
30-40 - level II

Test D 0-14 - level II
15-25 - level I

Mathematics course placement based on the four levels was made according to the following schedule.

**Placement level I (Advanced Placement):** A student wanting or needing calculus enrolled in Mathematics 440 (Analytic Geometry and Calculus). If he took 440, he received credit for Mathematics 439. Although the department recommended that students in level I take 440, a student who was certain that he did not want to take calculus could enroll for Mathematics 416.

**Placement level II:** A student wanting or needing calculus enrolled in Mathematics 439. Other students enrolled for Mathematics 416.

**Placement level III:** A student wanting or needing calculus enrolled in Mathematics 401. Other students enrolled for Mathematics 416.

**Placement level IV:** Every student in this level was required to take a remedial course. Students in the calculus sequence enrolled for Mathematics 401. Otherwise, they enrolled in Mathematics 400.

The remedial students then were the level IV, non-calculus students and the levels III and IV, calculus
sequence students. Briefly stated, the placement levels assigned students according to the following schedule:

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<th>Level</th>
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<td>I</td>
<td>Mathematics 440</td>
<td>Mathematics 416</td>
</tr>
<tr>
<td>II</td>
<td>Mathematics 439</td>
<td>Mathematics 416</td>
</tr>
<tr>
<td>III</td>
<td>Mathematics 401</td>
<td>Mathematics 416</td>
</tr>
<tr>
<td>IV</td>
<td>Mathematics 401</td>
<td>Mathematics 416</td>
</tr>
</tbody>
</table>

Description of the instruments used in the study

The instruments utilized in this study were the Ohio State Psychological Examination (abbreviated OSEP), the American College Testing Program series (abbreviated ACT), and the mathematics placement series which was developed at Ohio State University.

Ohio State Psychological Examination. This test provided a scale of mental ability and scholastic aptitude. Only the total raw scores were used in the study to determine the predictive efficiency of the test.

American College Testing Program. The basic ACT test battery consisted of four tests averaging forty-five minutes in length. This study used the scores from the following areas.

1. English: An eighty-item, fifty minute test measuring the student's understanding and use of the basic elements in correct and effective writing.

2. Mathematics: A forty-item, fifty minute test measuring the student's general reasoning ability. The test
emphasized the solution of practical quantitative problems such as are encountered in almost any field of college instruction, and included a sampling of formal mathematical techniques developed in secondary mathematics.

The scores on the tests were converted to standard scores, and a composite score was provided. This score was the arithmetic mean of the standard scores on the above two tests, the Social Studies Test, and the Natural Sciences Test. The standard scores on the ACT English and Mathematics tests and the ACT composite score were evaluated as predictors of mathematical success in the study.

The fundamental idea behind the ACT battery was that the best way to predict success in college was to measure as directly as possible the abilities that the student would have occasion to employ in his college work. The tests placed primary emphasis on what the student could do with what he had learned rather than on what he had learned in the sense of detailed and specific descriptive information. They attempted to identify the students who had developed their abilities most fully during their preceding school years rather than to identify the brightest students or those who learned most easily.

In some cases which were considered in this study, Scholastic Aptitude Test (SAT) scores were available instead of ACT scores. In these cases, the testing center supplied
a conversion scale from which it was possible to assign ACT scores.

Ohio State University Mathematics Placement Tests.
This series of tests was developed at Ohio State by the mathematics department in conjunction with the testing center for the purpose of placing students in their initial mathematics course.

1. Test A: A thirty-item general screening test which was required of all students. The test tentatively separated the students into two groups. The passing group consisted of students who are regarded as tentative advanced placement students. The test was essentially an achievement test designed to cover the basic concepts of secondary mathematics.

2. Test B: At a more elementary level than Test A, this forty-item test was designed to identify those students who needed a remedial course in mathematics before taking either Mathematics 416 or Mathematics 439. It was an achievement test which attempted to determine competence in arithmetic skills and reasoning ability as well as the ability to handle the basic concepts of secondary mathematics.

3. Test D: At a more advanced level than Test A, this test contained twenty-five items designed to identify the advanced placement students. It was an achievement test which covered the basic concepts found in a college algebra and trigonometry course.

There had been no comprehensive study which dealt
with the reliability and the validity of the mathematics series, but exploratory attempts to confirm the validity of the tests had been completed by the testing center and the mathematics department. Results of these studies tended to support the a priori assumption that the validity was satisfactory. One of the purposes in this study was to determine the reliability of the series. Informal studies, completed while refining the tests, indicated satisfactory reliability with respect to the test items.
CHAPTER IV

RESULTS

The results of the study are presented in terms of the hypotheses listed in Chapter I. The third hypothesis relating to the effect of a second unit of high school algebra on performance in Mathematics 400 is discussed first because of its secondary importance and to discuss it after the main issues in the study would be anticlimactic. The first and second hypotheses were tested with respect to each of the remedial courses, and to avoid unnecessary repetition and confusion both of the hypotheses were tested with respect to Mathematics 400-416 and Mathematics 401-439 in the same sections. The results, therefore, are presented in the following order: the effect of a second unit of high school algebra on performance in Mathematics 400, testing the hypotheses with respect to the Mathematics 400-416 sequence, and testing the hypotheses with respect to the Mathematics 401-439 sequence.

The effect of a second unit of high school algebra on performance in mathematics 400 (hypothesis 3)

A sample of 361 students was selected on the basis of enrollment for Mathematics 400 during the same hour in the fall quarter of 1963. The group was picked at random from three such groups enrolled during the quarter. A question-
naire, a sample of which appears in Appendix A, was circulated to the students in the sample to collect data on their high school preparation in mathematics. The remedial course was selected for this study to test the effect of a second unit of high school algebra on performance in a college-oriented situation for the case of the students who, according to the results of the mathematics placement tests, were the products of instruction in algebra which for some reason had not "taken." The idea was to discover whether the "exposure" to the second year of algebra made a significant difference in the students' remedial mathematics grades. The mean grade in second year algebra for the portion of the sample having two years of algebra was 1.23, which is considerably below average based on a four-point system. The mean grades for the two subgroups of students were 1.87 and 1.79 respectively for the "two-year" and the "one-year" groups. The distributions of grades in high school algebra and in Mathematics 400 for the groups are shown in Appendix B in Tables 12-16.

Analysis of the difference in means of the samples showed that the group having two years of high school algebra had a mean grade of 2.37 in Mathematics 400 with a standard deviation of .967 and a standard error of estimate of .1209. The corresponding figures for the group having one year of high school algebra were 2.26, 1.06, and .0620 respectively for the mean grade, standard deviation, and
standard error of estimate. The mean difference of .61 yielded a T-score of 4.49, which indicated that the group of students having two years of high school algebra performed significantly better than the other group in the course. The difference was found to be significant at the .01 level of confidence.

On the basis of these results, the third hypothesis was established. Computation of the T-score, as well as the other such computations made in the study, were made following the procedure outlined in Garrett's book on educational statistics.\textsuperscript{31}

The computations were made using an International Business Machines 1620 computer. The computer programs used in the study are presented in Appendix C, and they are written in AFIT FORTRAN machine language.

Results pertaining to the non-calculus sequence of courses

To aid in the analysis of the effectiveness of Mathematics 400, an item count was made on placement test B. The results of this count are shown in Table 1. Shown in the table are the per cents of a group of remedial students who answered the particular items correctly as compared to the per cents of a group of non-remedial students who answered

<table>
<thead>
<tr>
<th>Item</th>
<th>Per Cent Correct by Non-remedial Group</th>
<th>Per Cent Correct by Remedial Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Placement Test</td>
<td>Final Exam.</td>
</tr>
<tr>
<td>1</td>
<td>95.3</td>
<td>77.4</td>
</tr>
<tr>
<td>2</td>
<td>79.7</td>
<td>57.1</td>
</tr>
<tr>
<td>3</td>
<td>85.6</td>
<td>61.0</td>
</tr>
<tr>
<td>4</td>
<td>70.3</td>
<td>26.0</td>
</tr>
<tr>
<td>5</td>
<td>75.4</td>
<td>43.3</td>
</tr>
<tr>
<td>6</td>
<td>63.3</td>
<td>15.6</td>
</tr>
<tr>
<td>7</td>
<td>98.0</td>
<td>36.1</td>
</tr>
<tr>
<td>8</td>
<td>71.1</td>
<td>40.3</td>
</tr>
<tr>
<td>9</td>
<td>88.0</td>
<td>47.6</td>
</tr>
<tr>
<td>10</td>
<td>67.1</td>
<td>22.0</td>
</tr>
<tr>
<td>11</td>
<td>88.1</td>
<td>58.9</td>
</tr>
<tr>
<td>12</td>
<td>91.4</td>
<td>61.7</td>
</tr>
<tr>
<td>13</td>
<td>27.4</td>
<td>25.4</td>
</tr>
<tr>
<td>14</td>
<td>70.3</td>
<td>46.3</td>
</tr>
<tr>
<td>15</td>
<td>22.3</td>
<td>3.7</td>
</tr>
<tr>
<td>16</td>
<td>74.3</td>
<td>39.0</td>
</tr>
<tr>
<td>17</td>
<td>73.0</td>
<td>33.3</td>
</tr>
<tr>
<td>18</td>
<td>26.1</td>
<td>4.0</td>
</tr>
<tr>
<td>19</td>
<td>77.1</td>
<td>27.9</td>
</tr>
<tr>
<td>20</td>
<td>52.9</td>
<td>11.4</td>
</tr>
</tbody>
</table>
TABLE 1—Continued

<table>
<thead>
<tr>
<th>Item</th>
<th>Per Cent Correct by Non-remedial Group</th>
<th>Per Cent Correct by Remedial Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Placement Test</td>
<td>Final Exam.</td>
</tr>
<tr>
<td>21</td>
<td>22.7</td>
<td>11.4</td>
</tr>
<tr>
<td>22</td>
<td>44.1</td>
<td>14.9</td>
</tr>
<tr>
<td>23</td>
<td>88.7</td>
<td>45.6</td>
</tr>
<tr>
<td>24</td>
<td>81.7</td>
<td>45.7</td>
</tr>
<tr>
<td>25</td>
<td>76.1</td>
<td>31.3</td>
</tr>
<tr>
<td>26</td>
<td>46.0</td>
<td>12.3</td>
</tr>
<tr>
<td>27</td>
<td>17.0</td>
<td>8.3</td>
</tr>
<tr>
<td>28</td>
<td>76.7</td>
<td>41.7</td>
</tr>
<tr>
<td>29</td>
<td>87.7</td>
<td>45.1</td>
</tr>
<tr>
<td>30</td>
<td>22.8</td>
<td>11.4</td>
</tr>
<tr>
<td>31</td>
<td>82.4</td>
<td>44.7</td>
</tr>
<tr>
<td>32</td>
<td>61.0</td>
<td>32.3</td>
</tr>
<tr>
<td>33</td>
<td>86.1</td>
<td>46.0</td>
</tr>
<tr>
<td>34</td>
<td>30.1</td>
<td>14.3</td>
</tr>
<tr>
<td>35</td>
<td>43.9</td>
<td>29.4</td>
</tr>
<tr>
<td>36</td>
<td>30.0</td>
<td>22.7</td>
</tr>
<tr>
<td>37</td>
<td>12.5</td>
<td>11.0</td>
</tr>
<tr>
<td>38</td>
<td>83.1</td>
<td>49.6</td>
</tr>
<tr>
<td>39</td>
<td>47.4</td>
<td>22.8</td>
</tr>
<tr>
<td>40</td>
<td>59.3</td>
<td>27.6</td>
</tr>
</tbody>
</table>
the items correctly. Each sample contained 700 students selected by choosing every seventh test sheet from the tests available in the testing center. Also included in the table are the per cents correct for 29 of the items which were included as part of the final examination for 473 remedial students who were selected, for convenience, on the basis of enrollment for Mathematics 400 at the same hour during the fall quarter of 1963.

Analysis of the errors made by the entering freshmen on placement test B produced some interesting results. It should be pointed out that the students who took test B had already failed test A, but they had placed in the upper two-thirds of their high school graduating classes. Even among the group of students who failed test B, approximately 95 per cent had completed at least one year of high school algebra, and about 20 per cent had completed two years of algebra. The following facts stood out as being significant:

1. 20 per cent of the non-remedial group and 43 per cent of the remedial group could not find one half per cent of a whole number successfully.

2. 30 per cent of the non-remedial group and 74 per cent of the remedial group could not solve an equation of the form \( \frac{x}{a} + \frac{1}{b} = \frac{1}{c} \), where \( a, b, \) and \( c \) were integers with an obvious common multiple.

3. 25 per cent of the non-remedial group and 52 per cent of the remedial group could not divide \( \frac{34}{5} \) by \( \frac{8}{15} \).
4. 84 per cent of the remedial group could not simplify the fraction \((\frac{1}{2} - \frac{2}{3})/ (\frac{2}{4} + \frac{4}{5})\).

5. 29 and 60 per cent respectively of the non-remedial and remedial groups either did not know that the speed is equal to the distance divided by the time or could not divide 360 by \(8\frac{1}{4}\).

6. More than half of the remedial students could not solve a linear equation with integral coefficients which required only adding 3 to both sides of the equation and dividing by -1.

7. 33 and 88 per cent respectively of the non-remedial and remedial students could not successfully solve a pair of simultaneous linear equations of a relatively elementary nature.

8. 41 per cent of the remedial students could not find the altitude of a triangle given the formula for the area, the area, and the base.

9. 38 per cent of the remedial students could not change \(13/35\) to a decimal correct to two places.

10. More than half of the remedial students could not find the number of revolutions a wheel, with the circumference given in inches, makes in a mile. The fact that 5280 ft. = 1 mile was given in the problem.

11. Virtually no one in either group could make a simple application of the definition of a negative exponent.
12. 61 per cent of the remedial students could not match the statement of the distributive law for positive integers in numerical form with its statement in symbolic form.

13. Although approximately 31 per cent of the remedial students had completed a course in high school plane geometry, only 38 per cent of the group could make a numerical application of the similar triangle ratios.

14. Virtually no one in either group could successfully find the product of two complex numbers. The most common answer given was the algebraic sum of the two numbers.

15. Only 28 per cent of the remedial students could evaluate an expression of the form $ax^2 + bx + c$, where $a$, $b$, and $c$ were integers and $x$ was given as $-2$.

16. Approximately half of the non-remedial students and virtually none of the remedial students could solve a quadratic equation which was easily factored.

17. Almost half of the non-remedial students and virtually none of the remedial students could combine $\sqrt{3}$ and $\sqrt{32}$ successfully.

18. More than half of the remedial students could not solve a proportion of the form $\frac{x + a}{b} = \frac{c}{d}$ where $a$, $b$, $c$, and $d$ were integers.

19. Almost 70 per cent of the remedial students could not find a value for a centigrade temperature, given the formula, $F = \frac{9}{5}C + 32$, and a value for $F$. 
20. More than half of the non-remedial students and almost ninety per cent of the remedial students could not successfully subtract a pair of relatively simple algebraic fractions.

21. More than half of the remedial students could not solve a linear equation with literal coefficients.

22. Virtually no one in either group could determine the point at which a given linear equation cut the Y-axis.

23. More than half of the remedial group could not solve the problem of finding two numbers given their sum and their difference.

24. Approximately 65 per cent of the remedial group and about 40 per cent of the other group did not understand the meaning of the scientific notation for a number.

25. More than half of the remedial students could not successfully multiply algebraically a trinomial by a monomial.

26. Approximately half of the remedial students could not determine the nature of the third angle of a triangle with respect to being acute, obtuse, or right, given the measure of the other two angles.

These statements seem to indicate a need for some type of program to improve the mathematical backgrounds of the remedial students. In fact, they seem to cast some doubt concerning the adequacy of the preparation of at least
a segment of the students who passed Test 3. It would be
doubtful whether a student who failed the test would have
any chance whatever were he placed in a college level mathe­
matics course.

It is interesting, in comparing the per cents
correct for the non-remedial students on the placement test
with the per cents correct for the remedial students after
they had completed Mathematics 400, that the latter per
cents were significantly higher (a difference of ten per cent
or more) on eight of the items and significantly lower on two
of the items. This would seem to indicate that if test 3
were successful in identifying the skills and understandings
which were necessary for success in Mathematics 416, then
Mathematics 400 improved the backgrounds of the remedial
students to the extent that they should have been able to
compete successfully in that course. It should be pointed out,
however, that factors such as time lapse between learning and
testing, motivation provided by the necessity of attaining
a given point average, and differences in learning situations
between the college classroom and the high school classroom
could not be considered in analyzing the per cents.

The effectiveness of the various placement tests in
predicting grades and success in mathematics courses was
selected as the best criteria to judge the effectiveness of
the selection program in mathematics. This phase of the
study included three sections, computation of correlation
coefficients between the scores on the placement tests and Mathematics 416 grades, a regression analysis of the scores on the tests versus the grades in Mathematics 416, and computation of biserial correlations between the test scores and pass-fail status in Mathematics 416.

For computing the correlation coefficients between the grades and the test scores, a computer program, which appears in Appendix C, was used. Tables 2, 3, and 4 show the results of the computations respectively for level IV students, for level III students, and for the groups combined. The samples included all students who completed Mathematics 416 during the fall quarter of 1963 and had level III status on the placement tests and all level IV students who completed Mathematics 400 and 416 during the fall and winter quarters.

Although the correlations were not extremely high, the scores on the Ohio State mathematics tests proved to be the most closely correlated with course grades. The correlations for the level IV students showed that it was very difficult to exhibit much more than a casual relationship between placement tests and course grades in Mathematics 416 for remedial students. This was undoubtedly due to differences caused by the fact that unequal gains were made by the students who took remedial mathematics, and the predictive capacity of the test scores was decreased by this variability in improvement of mathematical backgrounds.
TABLE 2
CORRELATION BETWEEN THE PLACEMENT EXAMINATION SCORES AND GRADES IN MATHEMATICS 416 FOR LEVEL III STUDENTS

\[ N = 475 \]

<table>
<thead>
<tr>
<th>Test</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ohio State Psychological</td>
<td>.068</td>
</tr>
<tr>
<td>Ohio State Math. Test A</td>
<td>.286</td>
</tr>
<tr>
<td>Ohio State Math. Test B</td>
<td>.260</td>
</tr>
<tr>
<td>ACT English</td>
<td>.111</td>
</tr>
<tr>
<td>ACT Mathematics</td>
<td>.157</td>
</tr>
<tr>
<td>ACT Composite Score</td>
<td>.109</td>
</tr>
</tbody>
</table>

TABLE 3
CORRELATION BETWEEN THE PLACEMENT EXAMINATION SCORES AND GRADES IN MATHEMATICS 416 FOR LEVEL IV STUDENTS

\[ N = 157 \]

<table>
<thead>
<tr>
<th>Test</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ohio State Psychological</td>
<td>.125</td>
</tr>
<tr>
<td>Ohio State Math. Test A</td>
<td>.133</td>
</tr>
<tr>
<td>Ohio State Math. Test B</td>
<td>.126</td>
</tr>
<tr>
<td>ACT English</td>
<td>.116</td>
</tr>
<tr>
<td>ACT Mathematics</td>
<td>.134</td>
</tr>
<tr>
<td>ACT Composite Score</td>
<td>.130</td>
</tr>
</tbody>
</table>
TABLE 4
CORRELATION BETWEEN THE PLACEMENT EXAMINATION
SCORES AND GRADES IN MATHEMATICS 416
FOR LEVELS III AND IV STUDENTS

N = 632

<table>
<thead>
<tr>
<th>Test</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ohio State Psychological</td>
<td>.098</td>
</tr>
<tr>
<td>Ohio State Math. Test A</td>
<td>.241</td>
</tr>
<tr>
<td>Ohio State Math. Test B</td>
<td>.197</td>
</tr>
<tr>
<td>ACT English</td>
<td>.141</td>
</tr>
<tr>
<td>ACT Mathematics</td>
<td>.147</td>
</tr>
<tr>
<td>ACT Composite Score</td>
<td>.134</td>
</tr>
</tbody>
</table>

The regression analysis of the grades in mathematics versus the placement scores was also completed using a computer program. The sample from the preceding section was also used for this analysis. The results were computed using the Mathematics 416 grade as the dependent variable and the various test scores as independent variables. A prediction of the grade of a particular student (Y), using a regression equation, would be found by evaluating

\[ Y = a_0 + a_1X_1 + a_2X_2 + \ldots + a_nX_n \]

where \( n \) varies from 1 to 6 depending on how many test scores were used in the particular equation. All of the 63 possible combinations of variables were computed to determine the best combination to use. The multiple correlation, which is the correlation between the
grades received by the students and the grades which would be computed from the regression equation, was used to determine the best combination of variables for predictive purposes. Table 5 shows the data for the best choice for a single variable and for two variables. The entire list of possibilities appears in Table 17 in Appendix B. The addition of a third or of any number of variables to the regression equations did not materially increase the multiple correlation, the correlation for all six variables being .2632 as opposed to the correlation for the best two variables of .2617.

**Table 5**

**REGRESSION EQUATIONS FOR THE PREDICTION OF MATHEMATICS 416 GRADES**

\( N = 632 \)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Correlation</th>
<th>Regression Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_2 ) (Test A)</td>
<td>.2445</td>
<td>[ Y = .0556x_2 + 1.428 ]</td>
</tr>
<tr>
<td>( x_2, x_3 ) (Test B)</td>
<td>.2617</td>
<td>[ Y = .0511x_2 + .0517x_3 + 1.359 ]</td>
</tr>
<tr>
<td>( x_2, x_3, x_5 ) (ACT Math.)</td>
<td>.2617</td>
<td>......</td>
</tr>
</tbody>
</table>

Test A proved to be the best single predictor of Mathematics 416 grades, and Test A combined with Test B proved to be the best combination of predictors, although the correlations were not very high.

Evaluating the ability of the placement tests to
predict student success in Mathematics 416, as defined in Chapter III, was accomplished using the biserial correlation technique. The significance of the correlations was tested using the appropriate table in Garrett's book.\textsuperscript{32}

The results of this evaluation are presented in Table 6.

\begin{table}
\centering
\caption{Biserial Correlations Between Placement Scores and Pass-Fail Status in Mathematics 416}
\label{tab:table6}
\begin{tabular}{|l|c|c|c|}
\hline
\textbf{Test} & \textbf{r}_{\text{bis}} & \textbf{Predictive Efficiency} & \textbf{Best Cut-off Score} \\
\hline
OSPE & .128 & 52.2\% & 87 \\
Test A & .263 & 60.0\% & 19 \\
Test B & .267 & 65.4\% & 23 \\
ACT Eng. & .163 & 50.9\% & 19 \\
ACT Math. & .193 & 59.7\% & 22 \\
ACT Comp. & .218 & 57.9\% & 21 \\
\hline
\end{tabular}
\end{table}

The table also includes a measure called the "predictive efficiency" which is the per cent of correct decisions which would have been made with respect to success in Mathematics 416 if the best cut-off score had been used. All of the biserial correlations were found to be significant at the .01 level of confidence.

\textsuperscript{32}Ibid, p. 201.
Test B of the placement series was determined to be the best predictor of success when the correlation or the predictive efficiency was used as the measure of the test's predictive ability.

On the basis of these studies, the first hypothesis with respect to Mathematics 416 was given a qualified acceptance, the qualification being that predicting grades in the course on the basis of the test scores proved to be unsatisfactory, but predicting success in the course proved to be adequate using the scores.

The hypothesis that Mathematics 400 improves the backgrounds of the remedial students sufficiently so that they are able to compete successfully in Mathematics 416 was tested by an analysis of the grades received by matched groups of students, one group consisting of students who completed Mathematics 400 and 416 during the fall and winter quarters of 1963-64 and the other group consisting of students who completed Mathematics 416 during the fall quarter of 1963. The groups were matched identically on the dichotomous variables of sex and enrollment status, identically on college of enrollment, and as closely as possible on OSPE scores, ACT English scores, and ACT Composite scores. No attempt was made to match the mathematics placement scores since the level IV group would necessarily have lower scores on these tests than the
other group. The means and standard deviations of the test scores for the 114 matched pairs are shown in Table 7.

TABLE 7
MEANS AND STANDARD DEVIATIONS OF THE MATCHED MATHEMATICS 416 GROUPS FOR THE CONTROL VARIABLES

\( n = 114 \) Pairs

<table>
<thead>
<tr>
<th>Variable</th>
<th>Remedial Group</th>
<th>Non-remedial Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S. D.</td>
</tr>
<tr>
<td>OSPE</td>
<td>72.03</td>
<td>21.32</td>
</tr>
<tr>
<td>ACT Eng.</td>
<td>18.33</td>
<td>3.94</td>
</tr>
<tr>
<td>ACT Comp.</td>
<td>19.50</td>
<td>2.89</td>
</tr>
</tbody>
</table>

The differences in group means were tested by computing a T-score for each difference. The resulting T-scores of .0185, .5195, and .6429 respectively for the OSPE, ACT English, and ACT Composite scores showed that no cumulative bias occurred by matching each pair as closely as possible on these variables.

Analysis of the grades attained by the two groups revealed that the remedial group received an arithmetic mean grade of 2.2105 with a standard deviation of 1.1881, while the non-remedial group received a mean grade of 2.0964 with a standard deviation of .9730. A T-score of .7928 for the .1141 difference in the means showed that although the remedial group mean was higher, the difference was not statistically significant at the .01 level of confidence.
A comparison of the grades of groups selected on the basis of being within three points and four points above or below the cut-off score on test B of the placement series yielded similar results. In both cases the remedial group received slightly better grades, but the differences in means were not statistically significant at the .01 level of confidence. The statistics for these groups appear in Tables 18 and 19 in Appendix B.

The fact that the remedial students performed as well as, and in this case slightly better than, the non-remedial students supports the validity of the second hypothesis with respect to Mathematics 400.

A further attempt to evaluate the effectiveness of Mathematics 400 was made by circulating the questionnaire, a sample of which appears in Appendix A, to the Mathematics 410 classes during the last week of the winter quarter of 1964. Analysis of the replies received yielded little in the form of definite conclusions. A summary of the replies is shown in Table 20 in Appendix B. It is interesting that the students gave the Mathematics 400 work in the arithmetic processes and in number bases a vote of confidence with respect to being helpful in Mathematics 410, and, despite the fact that all of the respondents had completed Mathematics 400, more than half of them regarded their preparation for Mathematics 410 as merely adequate or less than adequate.
Results pertaining to the calculus sequence of courses

Testing the effectiveness of the placement procedure for the calculus-sequence students was accomplished by computing the correlations between the scores on the placement tests and the Mathematics 439 grades, by making a regression analysis of the test scores versus the grades, and by computing biserial correlations between the test scores and pass-fail status in Mathematics 439.

Table 6 shows the results of computing the correlations between the placement scores and the course grades.

<table>
<thead>
<tr>
<th>Test</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ohio State Psychological Exam.</td>
<td>.295</td>
</tr>
<tr>
<td>Ohio State Mathematics Test A</td>
<td>.421</td>
</tr>
<tr>
<td>Ohio State Mathematics Test B*</td>
<td>.339</td>
</tr>
<tr>
<td>Ohio State Mathematics Test D*</td>
<td>.419</td>
</tr>
<tr>
<td>ACT English</td>
<td>.221</td>
</tr>
<tr>
<td>ACT Mathematics</td>
<td>.392</td>
</tr>
<tr>
<td>ACT Composite Score</td>
<td>.310</td>
</tr>
</tbody>
</table>

Tests B and D samples consisted of that part of the 976 students who took that particular test.
The sample for this phase of the study consisted of all students who completed Mathematics 439 during the fall quarter of 1963 and all students who completed Mathematics 401 and 439 during the fall and winter quarters of 1963-64. The greatest correlation occurred between test A and the course grades. The coefficients for tests B and D are somewhat deceiving since the sample for test B, all of the students who failed test A, consisted of the students in the lower end of the group, and the sample for test D, all students in the group who passed test A, consisted of the students at the higher end of the group. It had been shown in the other analyses that the correlations became much lower when only the students at the lower end were used. All of the correlations were found to be statistically significant at the .01 level of confidence.

The regression analysis was made on the same sample as the simple linear correlations. As in the previous regression analysis, a computer program was used, and the best predictive measures were selected in terms of the best single predictor, the best pair of predictors, and the best combination of three predictors. The addition of a fourth variable added negligibly to the multiple correlation. The data on the best predictors appear in Table 9. Table 21 in Appendix B lists the multiple correlations for all of the combinations of variables.
TABLE 9

REGRESSION EQUATIONS FOR THE PREDICTION OF MATHEMATICS 439 GRADES

\( N = 976 \)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Correlation</th>
<th>Regression Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_2 ) (Test A)</td>
<td>.3923</td>
<td>( Y = .310X_2 - 4.627 )</td>
</tr>
<tr>
<td>( X_2, X_3 ) (Test D)</td>
<td>.4942</td>
<td>( Y = .221X_2 + .152X_3 - 4.627 )</td>
</tr>
<tr>
<td>( X_2, X_3, X_5 ) (ACT Math.)</td>
<td>.5187</td>
<td>( Y = .171X_2 + .134X_3 + .063X_5 - 4.971 )</td>
</tr>
</tbody>
</table>

Again test A emerged as the best single predictor, the combination of tests A and D the best pair of predictors, and tests A and D combined with the ACT Mathematics Test the best trio of predictors. All of the multiple correlations were found to be significant at the .01 level of confidence.

The same sample was again used in the evaluation of the ability of the placement tests to predict student success in Mathematics 439, and the procedure was identical to that used in the analysis of the test scores' ability to predict success in Mathematics 416. The biserial correlations and the predictive efficiencies of the tests were computed and recorded along with the best cut-off score for each of the predictors. This data appears in Table 10. All of the correlations were found to be significant at the .01 level of confidence.
### TABLE 10

**BISERIAL CORRELATIONS BETWEEN PLACEMENT SCORES AND PASS-FAIL STATUS IN MATHEMATICS 439**

*N = 976*

<table>
<thead>
<tr>
<th>Test</th>
<th>r_bis</th>
<th>Predictive Efficiency</th>
<th>Best Cut-off Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSP2</td>
<td>.347</td>
<td>60.2%</td>
<td>37</td>
</tr>
<tr>
<td>Test A</td>
<td>.501</td>
<td>70.6%</td>
<td>26</td>
</tr>
<tr>
<td>Test B*</td>
<td>.362</td>
<td>60.7%</td>
<td>24</td>
</tr>
<tr>
<td>Test D*</td>
<td>.571</td>
<td>71.4%</td>
<td>9</td>
</tr>
<tr>
<td>ACT Eng.</td>
<td>.205</td>
<td>55.5%</td>
<td>20</td>
</tr>
<tr>
<td>ACT Math.</td>
<td>.423</td>
<td>63.2%</td>
<td>26</td>
</tr>
<tr>
<td>ACT Comp.</td>
<td>.382</td>
<td>60.6%</td>
<td>24</td>
</tr>
</tbody>
</table>

*Tests B and D results were computed on that part of the sample which included these tests.*

A cursory study of Table 10 would indicate that Test D emerged as the best predictor of success; however, the sample for test D did not include the students who failed test A, while the sample for test B included only these students. When a biserial correlation was computed for test A using only the sample for test D, the results showed test A to be the better predictor. The predictive efficiency for test A when applied to this same sample rose to 78.12 per cent.

On the basis of the preceding sections, the present methods of selection for Mathematics 401 and 439 were
accepted as the most efficient, and the first hypothesis with respect to the calculus sequence was accepted.

Testing of the hypothesis with respect to the effectiveness of the calculus-sequence remedial course, Mathematics 401, was completed in two phases. In the first phase two groups of students, the remedial group having completed Mathematics 401 and 439 during the fall and winter quarters of 1963-64 and the non-remedial group having completed Mathematics 439 during the fall quarter of 1963, were matched identically on the dichotomous variables of sex and enrollment status, identically on college of enrollment, and as closely as possible on OSPE, ACT English, and ACT Composite scores. The groups could not be matched on the mathematics placement scores since these scores were used initially to divide the students into groups of remedial and non-remedial students. The means and standard deviations for the variables are shown in Table 11.

| TABLE 11 |
| MEANS AND STANDARD DEVIATIONS OF THE MATCHED MATHEMATICS 439 GROUPS FOR THE CONTROL VARIABLES |
| N = 262 pairs |

<table>
<thead>
<tr>
<th>Variable</th>
<th>Remedial Group</th>
<th>Non-remedial Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S. D.</td>
</tr>
<tr>
<td>OSPE</td>
<td>72.60</td>
<td>18.31</td>
</tr>
<tr>
<td>ACT Eng.</td>
<td>12.46</td>
<td>4.00</td>
</tr>
<tr>
<td>ACT Comp.</td>
<td>21.11</td>
<td>2.97</td>
</tr>
</tbody>
</table>
Computation of T-scores for each of the differences between the group means revealed no significant difference for the OSPE and ACT English. However, the difference between the means on the ACT composite score was revealed to be statistically significant. Since this difference was in favor of the non-remedial group, and since it was unlikely that it could be lowered because the ACT composite score is partially dependent on the ACT Mathematics score, it was decided that the analysis would be continued on the groups with the qualification that if the non-remedial group achieved higher grades in Mathematics 439 than the remedial group, then the results would have to be considered as inconclusive.

Analysis of the grades attained by the groups in Mathematics 439 revealed that the remedial group achieved a mean grade of 2.5374 with a standard deviation of 1.0515, while the non-remedial group achieved a mean grade of 1.9863 with a standard deviation of 1.0561. The T-score of 4.4826 for the .5511 mean difference was found to be significant at the .01 level of confidence. The remedial group, therefore, performed significantly better than the non-remedial group in Mathematics 439.

A comparison of the grades of two groups of students selected on the basis of being within three points of the cut-off score on Test A yielded similar results. The remedial group again performed significantly better than the
The second phase of testing the effectiveness of Mathematics 401 attempted to provide a measure of breadth to the study. A group of students, all freshmen in the college of engineering during the fall quarter of 1961, was selected on the basis of being exactly one point below the cut-off score on placement test B. The registrar's office provided the grades received by these students in their mathematics courses through the summer of 1963. The group was subdivided into remedial and non-remedial subgroups by being allowed to choose their first mathematics course as either Mathematics 401 or 439.

Analysis of the subsequent success of these groups, although based on small numbers, yielded some useful results. The remedial group performed significantly better than the non-remedial group in Mathematics 439 and the following first course in analytic geometry and calculus. Subsequent courses in the sequence did not have sufficient numbers of the group enrolled to yield significant results because of drop-outs and transfers to other areas of instruction, but when all mathematics courses except 401 taken by the group as a whole were considered, the data showed that the remedial group, averaging 2.70 for all courses, performed significantly better than the non-remedial group which
averaged 1.81 for all mathematics courses. The statistics for these groups are shown in Table 23 in Appendix 3. The survival rate for the remedial group calculated as of the fall quarter of 1963 was 36 per cent, while the rate for the non-remedial group was 24 per cent.

On the basis of the preceding two sections, the second hypothesis with respect to Mathematics 401 was accepted.

Hypothesis 2: Mathematics 400 and 401 increase the mathematical skills of the students taking the course to the degree that they are able to enjoy success in subsequent mathematics courses equal to that of comparable students whose backgrounds are such that these students do not require remedial work in mathematics.
CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Restatement of the problem

Increasing enrollment in higher education institutions has made the problem of course placement a matter of concern to educators in order that valuable time and space not be wasted on students who are enrolled in courses for which they are not prepared. Reports of previous studies indicate that mathematical aptitude tests and mathematical achievement tests are the most effective instruments in differentiating mathematically deficient students from students with adequate mathematical backgrounds.

This investigation was concerned primarily with determining the effectiveness of the mathematics placement program at The Ohio State University in selecting the students with deficient mathematical backgrounds and with the effectiveness of the remedial program in remedying the deficiencies.

Selection of the subjects was based on the student's being a first quarter freshman in the fall quarter of 1963 and taking one of the courses involved in the study during the fall quarter of 1963 or the winter quarter of 1964. One small sample consisted of freshmen in the fall quarter of 1961 in the College of Engineering.
The criterion measures used in the study were the grades in the mathematics courses, on the basis of 4 points for an "A", 3 points for a "B", 2 points for a "C", 1 point for a "D", and no points for an "E", and success in the mathematics courses (a grade of A, B, or C).

**Testing the hypotheses**

The results of the study were stated in terms of the hypotheses. Testing the effectiveness of the remedial mathematics courses was done in the case of Mathematics 400 by matching 114 pairs of remedial and non-remedial students on all of the pertinent variables except scores on the mathematics placement tests, and analyzing their performance in Mathematics 416. The results showed that the remedial group, averaging 2.21, performed slightly but not significantly better than the non-remedial group which averaged 2.09 in the course.

Similar analyses made on two groups of students, one selected on the basis of being within three points and the other selected on the basis of being within four points of the cut-off score on the placement tests, yielded like results. In both cases the remedial part of the group performed slightly but not significantly better than the non-remedial part of the group in Mathematics 416.

An attempt to evaluate the effectiveness of Mathematics 400 by circulating a questionnaire to students who had completed the course during the fall quarter of 1963
and Mathematics 410 during the winter quarter of 1964 yielded inconclusive results.

In the case of Mathematics 401, the results were more conclusive. Analysis of the performance of 282 matched pairs in Mathematics 439 showed that the group which had taken the remedial course performed significantly better than the group which had not taken the remedial course, the mean grades being 2.54 and 1.99 respectively for the remedial and non-remedial groups. Similar results were obtained when the grades of the group of 51 students who scored three points or less below the cut-off score on the placement test was compared with the grades of the group of 75 students who were within three points of the cut-off score on the passing side. The mean grades were 2.92 and 2.00 respectively for the remedial and non-remedial groups. The mean difference of .92 yielded a T-score of 5.126 which is significant at the .01 level of confidence.

The results of the study confirm the first hypothesis.

Hypothesis 1: Mathematics 400 and 401 increase the mathematical skills of the students taking the courses to the degree that they are able to enjoy success in subsequent mathematics courses equal to that of comparable students whose backgrounds are such that these students do not require remedial work in mathematics.

Hypothesis 2: The mathematics placement tests significantly distinguish the students who are likely to
pass their initial college-level mathematics course with a "C" or better from those who are unlikely to do so.

The second hypothesis was tested in three phases: computing correlations between the mathematics course grades and placement scores, a regression analysis of course grades versus placement test scores, and computing biserial correlations between the placement test scores and pass-fail status in the courses. Also computed for each test was a measure called the predictive efficiency, which was the per cent of correct decisions which would have been made had the best cut-off score for that particular test been used.

For Mathematics 416 poor correlations were obtained between course grades and test scores, but nevertheless mathematics placement tests A and B, which are the measures presently used for course placement, emerged as the best criteria for placement. The multiple correlation, using a regression equation with tests A and B as the independent variables, was .26. Addition of other variables added little to the correlation. Prediction of success rather than grades in Mathematics 416 on the basis of biserial correlations and predictive efficiencies of the tests was more fruitful. Test E of the placement series emerged as the best predictive measure with a correlation of .27 and a predictive efficiency of 65.4 per cent.

For Mathematics 439 better correlations between the course grades and the placement scores were obtained than
for Mathematics 416. Tests A and D of the placement series emerged as the best predictors of both grades and success. The multiple correlation, using a regression equation and tests A and D as predictors, was .49, and addition of other variables added little to the correlation. Prediction of success rather than grades on the basis of biserial correlations and predictive efficiencies showed that test A of the placement series with a biserial correlation of .50 and a predictive efficiency of 71.4 per cent was the best predictive measure.

The results showed that prediction of grades in Mathematics 416 was unsatisfactory, that prediction of grades in Mathematics 439 was somewhat effective, but that prediction of success was quite satisfactory for both courses. Since it was the main goal of the placement program to predict success rather than grades, the results confirmed the second hypothesis.

Hypothesis 3: Students who have completed two units of high school algebra perform significantly better in Mathematics 400 than students who have completed one unit of high school algebra.

Information was obtained on the amount of algebraic preparation of a sample of 361 students who studied Mathematics 400. Analysis of the course grades showed that the group which had completed one year of high school algebra averaged 2.26 in Mathematics 400, while the group which had
completed two years of high school algebra averaged 2.87. Analysis of the mean difference yielded a T-score of 4.49. A T-score of 4.49 is significant at the .01 level of confidence. The third hypothesis was accepted on the basis of the results.

Conclusions

The results of the study give the placement and remedial programs of The Ohio State University a vote of confidence. The results are overwhelming for the Mathematics 401-439 sequence of courses where the remedial course did an excellent job of preparing students with low-level preparation in mathematics to compete successfully in college algebra and trigonometry. The present placement program, with minor revisions of the cut-off scores, proved to be the best possible program which could be devised using the data available.

In the Mathematics 400-416 sequence of courses, the results showed more modest gains by the students taking the remedial course, but the level IV students, after taking Mathematics 400, were able to compete on an equal basis with the level III students who had passed placement test B. The present placement program for the students in this sequence, also with minor revisions of the cut-off scores, provided the best grouping of students for differentiated instruction.

For the high school student, the results of the study support the conclusion that a second unit of high
school algebra considerably enhances the student's chances of performing successfully in a remedial course in mathematics even when the student was relatively unsuccessful in the second course in high school algebra.

**Recommendations for action to be taken**

It was concluded on the basis of the results of the study that the present remedial program was relatively effective in accomplishing its purpose of preparing students who are deficient in mathematical background to compete in college-level mathematics courses. With maximum efficiency in mind, the following recommendations are respectfully made:

1. It is recommended that the present placement procedure be retained with the cut-off score for level IV changed from 18 to 23 to provide for maximum efficiency in the placement of students. It is further recommended that the cut-off score on Test E which differentiates level III students from level II students be further examined in the light of the results of this study. The "best cut-off score" obtained in this study was influenced by the fact that the group of students between the present cut-off score, 26, and the score obtained in the study, 23, had completed Mathematics 401. The students in this group performed significantly better in Mathematics 439 than their counterparts on the passing side of the cut-off score. It is suggested that groups of students in this range be
placed in each of levels II and III and their performances in Mathematics 439 be studied to find the cut-off score which results in the maximum efficiency.

2. It is recommended that the teachers for the remedial courses be carefully selected as instructors who have the methodology training to teach in terms of basic understandings rather than in terms of a "bag of tricks" to solve meaningless problems.

The statistics shown in the item analysis of test B of the placement series indicate that the instructors in the remedial courses should assume little in the way of mathematical skills on the part of the students. The analysis showed a uniform lack of understanding of the basic mathematical principles of secondary mathematics rather than merely a lack of manipulative skills. When a problem appeared on test B as a simple application of an algebraic manipulative skill or an arithmetic skill, and when the problem appeared in standard notation, the per cent of correct responses increased greatly over the correct response to a problem which was an application of a basic mathematical concept where the understanding of a mathematical principle was necessary.

The dependence of the remedial students on notation was shown by placing a pair of simultaneous equations on the final examination in Mathematics 400 in the conventional x and y form and the same pair of equations on the test using \( x_1 \) and \( x_2 \) as the variables. The per cent correct for
the conventional notation was 80.5, while the per cent correct for the other notation was 47.2.

Other examples on both the placement test and the final examination in the remedial course also showed the need for teaching for understanding of basic principles rather than for manipulative skills.

3. It is further recommended that the high schools evaluate their mathematics programs in light of the above recommendation so that the source of the problem of remedial students might be found. They should search for reasons why approximately thirty per cent of the incoming freshmen at Ohio State cannot pass a set of placement examinations which cover only the basic essentials of high school mathematics. They should try to find out what per cent of their graduates were in the group of remedial students who could not solve a pair of simultaneous equations of a very elementary nature, had no idea what a minus sign before an exponent meant, could not subtract a pair of relatively simple algebraic expressions, could not make a simple application of the similar triangle ratios, could not combine $\sqrt{8}$ and $\sqrt{32}$ successfully, and could not successfully substitute -2 into a quadratic expression with integral coefficients. This was despite the fact that over 95 per cent of the students had completed one year of high school algebra, despite the fact that more than 80 per cent of the students had completed a course in plane geometry, despite
the fact that about 20 per cent of the students had completed two years of high school algebra, and despite the fact that the entire group of students had placed in the upper two-thirds of high school graduating classes.

It would seem that this thirty per cent of the entering freshman class, along with a considerable number of students who managed to pass the tests but still had a background of a questionable nature, had completely missed the whole idea of mathematics, having few manipulative skills and fewer mathematical understandings.

That some of these students can be "saved" has been shown by this study. The fact that a student scoring 5 correct out of 40 items on test B could be successful in both Mathematics 400 and 416 testified to this fact.

Recommendations for further study

1. It is recommended that a continuing study of the remedial program be conducted to meet the changing needs of the entering freshman population. The high school mathematics program has been undergoing revolutionary changes in recent years, and the students who have received instruction under the new programs will enter the university in increasing numbers in future years. Continuous study to revise the test norms and to revise the tests themselves will be necessary to insure that the placement program does not become outdated.
2. A study in depth should be made to test the effect of the remedial courses on the students' performances in the total mathematics program. Such a study would provide the answer to the question as to whether the remedial program merely helps the student to prolong his stay in college an extra quarter or two rather than fulfills its purpose of helping the student to acquire certain mathematical skills which for some reason he did not acquire in his high school training.

3. The fact that Mathematics 401 more effectively prepared students for Mathematics 439 than Mathematics 400 did for Mathematics 416 suggests a need to study the content of Mathematics 400 with the purpose of coordinating it more closely with the content of Mathematics 416.

4. It is recommended that the secondary schools, by means of a testing program similar to the placement examinations, re-evaluate their mathematics curricula in the light of what has been shown with respect to a large group of their graduates. Such a study would help them to determine the areas in which their students are receiving inadequate instruction or for some other reasons are not learning the basic skills and understandings which should be expected of a competent high school graduate. The ideal end of such a study would be that successful completion of a high school mathematics course would imply that the student completing the course possessed certain mathematical skills and understandings which he could apply in appropriate situations.
APPENDIX A

Questionnaires
<table>
<thead>
<tr>
<th>Course Title</th>
<th>No. of Units*</th>
<th>Grade</th>
<th>Year Taken</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Math.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Year Algebra</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plane Geometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advanced Algebra</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid Geometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trigonometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Others (please name)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*1 unit is 1 year's study (for semester courses use ½).

If you listed others such as Senior Mathematics, please describe the material covered briefly and list, if possible the title and author(s) of the text used.

**PREVIOUS COLLEGE MATHEMATICS (IF ANY)**

<table>
<thead>
<tr>
<th>At OSU</th>
<th>At other colleges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course No.</td>
<td>Grade</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
QUESTIONNAIRE - MATHEMATICS 410

Name______________________________ Instructor____________________

A. To what degree do you feel that your preparation for Mathematics 410 was adequate? (Circle one number.)

Very Adequate 5 4 3 2 1 Inadequate

B. If you took Mathematics 400 at OSU please complete the following questions.

1. To what degree do you feel that Math. 400 helped you in Math. 410? (Circle one number.)

Very Much 5 4 3 2 1 Very Little

2. Do you feel that the material covered in Math. 400 was: (Circle one number)

Too Difficult 5 4 3 2 1 Too Easy

3. Rate the following areas with respect to being helpful preparation for Math. 410. (Circle one number in each line.)

Arithmetic processes Very Helpful 5 4 3 2 1 Not Helpful
Number Bases 5 4 3 2 1
Algebraic equations 5 4 3 2 1
Complex numbers 5 4 3 2 1
Algebraic fractions 5 4 3 2 1
APPENDIX B

Supplementary Tables
### TABLE 12

DISTRIBUTION OF SECOND YEAR ALGEBRA GRADES FOR THE MATHEMATICS 400 GROUP HAVING TWO UNITS OF HIGH SCHOOL ALGEBRA

<table>
<thead>
<tr>
<th>Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1</td>
<td>3</td>
<td>20</td>
<td>32</td>
<td>12</td>
</tr>
</tbody>
</table>

### TABLE 13

DISTRIBUTION OF FIRST YEAR ALGEBRA GRADES FOR THE MATHEMATICS 400 GROUP HAVING TWO UNITS OF HIGH SCHOOL ALGEBRA

<table>
<thead>
<tr>
<th>Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2</td>
<td>10</td>
<td>33</td>
<td>23</td>
<td>0</td>
</tr>
</tbody>
</table>

### TABLE 14

DISTRIBUTION OF FIRST YEAR ALGEBRA GRADES FOR THE MATHEMATICS 400 GROUP HAVING ONE UNIT OF HIGH SCHOOL ALGEBRA

<table>
<thead>
<tr>
<th>Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>8</td>
<td>34</td>
<td>139</td>
<td>112</td>
<td>0</td>
</tr>
</tbody>
</table>
### TABLE 15

**DISTRIBUTION OF MATHEMATICS 400 GRADES FOR THE GROUP HAVING TWO UNITS OF HIGH SCHOOL ALGEBRA**

\[N = 68\]

<table>
<thead>
<tr>
<th>Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>20</td>
<td>24</td>
<td>19</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

### TABLE 16

**DISTRIBUTION OF MATHEMATICS 400 GRADES FOR THE GROUP HAVING ONE UNIT OF HIGH SCHOOL ALGEBRA**

\[N = 293\]

<table>
<thead>
<tr>
<th>Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>40</td>
<td>79</td>
<td>106</td>
<td>54</td>
<td>14</td>
</tr>
</tbody>
</table>
TABLE 17
MULTIPLE CORRELATIONS BETWEEN MATHEMATICS 416 GRADES AND PLACEMENT TEST SCORES

N = 632

<table>
<thead>
<tr>
<th>Variable(s)</th>
<th>Correlation</th>
<th>Variable(s)</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
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### Table 18

Data for the Mathematics 416 Groups with Within Three Points of the Passing Score on Test B

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<tr>
<th>Group</th>
<th>Mean Grade</th>
<th>Standard Deviation</th>
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T-score = 1.4280

### Table 19

Data for the Mathematics 416 Groups with Within Four Points of the Passing Score on Test B

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T-score = 1.3938
TABLE 20

RESULTS OF THE QUESTIONNAIRE GIVEN TO
THE MATHEMATICS 410 STUDENTS

\( N = 69 \)

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Table 21

Multiple Correlations Between Mathematics Grades and Placement Test Scores

N = 976

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### TABLE 22
DATA FOR THE MATHEMATICS 439 GROUPS WITHIN THREE POINTS OF THE PASSING SCORE ON TEST A

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<th>Standard Deviation</th>
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T-score = 5.1265
### TABLE 23
DATA ON THE 1961 ENGINEERING GROUP

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<th>Non-remedial Group</th>
<th>T-score</th>
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APPENDIX C

Computer Programs
COMPUTER PROGRAM FOR T-SCORES

100 FORMAT(//40HT0 READ IN GRADES OF GP A, PRESS START)
101 FORMAT(7XF3.0)
102 FORMAT(//40HT0 READ IN GRADES OF GP B, PRESS START)
103 FORMAT(//28HARITHMETIC MEAN OF GROUP A =F9.4)
104 FORMAT(//28HARITHMETIC MEAN OF GROUP B =F9.4)
105 FORMAT(//31HSTANDARD DEVIATION OF GROUP A =F10.4)
106 FORMAT(//31HSTANDARD DEVIATION OF GROUP B =F10.4)
107 FORMAT(//39HST. ERROR OF THE MEAN OF GP. A =F10.4)
108 FORMAT(//39HST. ERROR OF THE MEAN OF GP. B =F10.4)
109 FORMAT(//52HST. ERROR OF THE DIFFERENCE BETWEEN)
110 FORMAT(22HTHE MEANS =F10.4)
111 FORMAT(//20X,9HT SCORE =F10.6)
112 FORMAT(///20X,19H---------------------)
113 FORMAT(//43HADJUST PAPER FOR PRINT OUT, THEN PRESS START)
130 FORMAT(//29HREAD IN NUMBER OF CARDS-J2,I2)
       SUMXA=0.0
       SMXA2=0.0
       SUMXB=0.0
       SMXB2=0.0
       PAUSE
       PRINT130
       READ,J2,I2
       AJ2=J2
       BI2=I2
       PRINT100
       PAUSE
       DO 10 J=1,J2
       READ101,XA
       XA2=XA*XA
       SUMXA=SUMXA+XA
       10 SMXA2=SMXA2+XA2
       PRINT102
       PAUSE
       DO 20 I=1,I2
       READ101,XB
       XB2=XB*XB
       SUMXB=SUMXB+XB
       20 SMXB2=SMXB2+XB2
       AMEAN=SUMXA/AJ2
       BMEAN=SUMXB/BI2
       STDVA=SQRT(AJ2*SMXA2-(SUMXA*SUMXA))/AJ2
       STDVB=SQRT(BI2*SMXB2-(SUMXB*SUMXB))/BI2
       SIGMA=STDVA/SQRT(AJ2)
       SIGMB=STDVB/SQRT(BI2)
       SIGDF=SQRT((SIGMA*SIGMA)+(SIGMB*SIGMB))
       T=ABS(AMEAN-BMEAN)/SIGDF
       PRINT113
       PAUSE
PRINT103,AMean
PRINT104,BMean
PRINT105,STV
PRINT106,Sigma
PRINT108,SigmB
PRINT109
PRINT110,SigDF
PRINT111,T
PRINT112
STOP
END
COMPUTER PROGRAM FOR SIMPLE CORRELATIONS

34 FORMAT(//12X,26H SIMPLE LINEAR CORRELATION)
37 FORMAT(//19H EXPLAINED VARIANCE,11X,F15.2)
39 FORMAT(//15H TOTAL VARIANCE,15X,F15.2)
40 FORMAT(//27H COEFFICIENT OF CORRELATION,2X,F15.3)
41 FORMAT(//27H STANDARD ERROR OF ESTIMATE,3X,F15.2)
42 FORMAT(//4H A =.2X,F10.5)
43 FORMAT(//4H B =.2X,F10.5)
44 FORMAT(//21H UNEXPLAINED VARIANCE ,8X,F15.2)
45 FORMAT(7X12,1X14)
46 FORMAT(7X12,6X13)
47 FORMAT(7X12,10X13)
48 FORMAT(7X12,14X13)
49 FORMAT(7X12,18X13)
50 FORMAT(7X12,22X13)
51 FORMAT(//10X,25H ALL CORRELATIONS IN ORDER)
52 FORMAT(//10X,F18.6)
53 FORMAT(//10X,40HCORR BET GRADE AND OSPE SCORE)
54 FORMAT(//10X,42HCORR BET GRADE AND EXAM A SCORE)
55 FORMAT(//10X,42HCORR BET GRADE AND EXAM B SCORE)
56 FORMAT(//10X,50HCORR BET GRADE AND ACT ENG SCORE)
57 FORMAT(//10X,47HCORR BET GRADE AND ACT MATH SCORE)
58 FORMAT(//10X,48HCORR BET GRADE AND ACT COMP SCORE)
59 FORMAT(//10X,42HCORR BET SUCCESS AND OSPE SCORE)
60 FORMAT(//10X,44HCORR BET SUCCESS AND EXAM A SCORE)
61 FORMAT(//10X,44HCORR BET SUCCESS AND EXAM B SCORE)
62 FORMAT(//10X,49HCORR BET SUCCESS AND ACT ENG SCORE)
63 FORMAT(//10X,49HCORR BET SUCCESS AND ACT MATH SCORE)
64 FORMAT(//10X,50HCORR BET SUCCESS AND ACT COMP SCORE)
65 FORMAT(10X14,21X12)
66 FORMAT(15X13,17X12)
67 FORMAT(19X13,13X12)
68 FORMAT(23X12,9X12)
69 FORMAT(28X13,5X12)
70 FORMAT(31X13,1X12)

DIMENSION COFV(12)

PRINT34
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J=0
M=0
N=0
L SUMY=0.0
SUMX=0.0
SUMXY=0.0
SUMXS=0.0
SUMYS=0.0
L NUM=0.0
GO TO(4,5,6,7,8,85,86,87,88,89,90),1
64 PRINT53
GO TO 19
77 PRINT54
GO TO 19
78 PRINT55
   GO TO 19
91 PRINT56
   GO TO 19
92 PRINT57
   GO TO 19
93 PRINT58
   GO TO 19
94 PRINT66
   GO TO 19
95 PRINT67
   GO TO 19
96 PRINT68
   GO TO 19
97 PRINT69
   GO TO 19
98 PRINT70
   GO TO 19
99 PRINT71
   GO TO 19
4 READ45,IX,IY
   GO TO 10
5 READ46,IX,IY
   GO TO 10
6 READ47,IX,IY
   GO TO 10
7 READ48,IX,IY
   GO TO 10
8 READ49,IX,IY
   GO TO 10
9 READ50,IX,IY
   GO TO 10
35 READ79,IX,IY
   GO TO 10
86 READ80,IY,IX
   GO TO 10
87 READ81,IY,IX
   GO TO 10
88 READ82,IY,IX
   GO TO 10
89 READ83,IY,IX
   GO TO 10
90 READ84,IY,IX
10 X=IX
    Y=IY
3 UNUM=UNUM+1.0
   XY=X*Y
   XSQ=X*X
   YSQ=Y*Y
14 SUMX=SUMX+X
SUMY = SUMI + Y
SUMXY = SUMXY + XY
SUMXS = SUMXS + XSQ
SUMYS = SUMYS + YSQ
IF (SENSE SWITCH 9) 17, 26
26 GO TO(4, 5, 6, 7, 8, 9, 85, 86, 87, 88, 89, 90), I
17 ZMY = SUMY / NUM
A = (SUMXS * SUMY - SUMX * SUMYS) / (NUM * SUMXS - (SUMX * SUMX))
B = (NUM * SUMXY - SUMX * SUMY) / (NUM * SUMXS - (SUMX * SUMX))
EXVAR = (A * SUM + B * SUMXY - ZMY * SUMY) / NUM
UNEXV = (SUMYS - A * SUMY - B * SUMXY) / NUM
TOTV = (SUMYS - ZMY * SUMY) / NUM
R = SQRT(1.0 - UNEXV / TOTV)
M = M + 1
COFV(M) = R
SEOF = SQRT(UNEXV)
25 I = I + 1
N = N + 1
GO TO(64, 77, 78, 91, 92, 93, 94, 95, 96, 97, 98, 99), N
19 PRINT 42, A
PRINT 43, B
PRINT 37, EXVAR
PRINT 44, UNEXV
PRINT 39, TOTV
PRINT 40, R
PRINT 41, SEOF
J = J + 1
GO TO(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 23), J
22 PRINT 40, R
J = J + 1
GO TO(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 23), J
23 PRINT 51
DO 24 M = 1, 12
PRINT 52, COFV(M)
24 CONTINUE
STOP
END
COMPUTER PROGRAM FOR REGRESSION ANALYSIS

100 FORMAT(7X,714)
101 FORMAT(11X,14,13,13,13,13,13,12)
102 FORMAT(/12HN0 SAMPLES=15,13H, RAW S OF S=E11.4)
103 FORMAT(/10X,15,1X,15,1X,E15.8)
104 FORMAT(/4OHNVAR R-SQ, RES S OF S VARIABLE LIST)
105 FORMAT(/13,F8.4,E12.4,16,613)
106 FORMAT(/28HREAD IN SORTED VARIABLE LIST)
107 FORMAT(/13HRID S OF S=E11.4,12H, MULT CORR=F7.4)
108 FORMAT(/9H VAR NO 13,7H COEF=E11.4)
109 FORMAT(/15HERROR F63-48-47)
998 NN=1
999 DO 1 I=1,9
 1 M(I)=0
    KKK=1
    DO 60 J=1,9
      IX(J)=0.0
      DO 60 J=1,121
    60 X(J)=0.0
 2 A(J)=0.0
    DO 61 J=1,8
    61 B(J,I)=0.0
    X(I)=1.0
    INC=36
 3 N=0
 45 GO TO(4,5),NN
 C READ IN SAMPLES AND BUILD UP TRIANGULAR MATRIX
 C DEPENDENT VARIABLE GRADE
 4 READ100,IX(8),IX(2),IX(3),IX(4),IX(5),IX(6),IX(7)
    GO TO 8
 C DEPENDENT VARIABLE SUCCESS
 5 READ101,IX(2),IX(3),IX(4),IX(5),IX(6),IX(7),IX(8)
 8 DO 90 LM=2,8
 90 X(LM)=IX(LM)
    K=0
    DO 9 I=1,8
    9 DO 9 J=1,8
      K=K+1
      A(K)=A(K)+X(I)*X(J)
    N=N+1
 10 IF (SENSE SWITCH 9)10,85
 11 N=0
 12 IF (SENSE SWITCH 1)11,13
 13 PRINT102,N,A(36)
    IF (SENSE SWITCH 1)11,13
 C PUNCH MATRIX OF SUMS OF PRODUCTS
 11 K=0
    DO 12 I=0,7
 12 DO 12 J=1,7
      K=K+1

12 PRINT103,I,J,A(K)
13 T=A(INC)
   S=1./T
   K1=1
   KP=1
   II=2
   KS=8
   IF (SENSE SWITCH 2)14,16
14 PRINT104
16 K2=8-KP
   L1=KP-2
   DO 41 K=K1,K2
      INC=INC-KS
      KS=KS-1
      I2=I1+KS-KP
      J1=I2+1
      P=1./A(I1-1)
      DO 18 1=II,K
         Q=P*A(I)
         L=J1-I
         J2=L+I2
         DO 17 J=J1,K
            JK=J-L
            JN=J+INC
            A(JN)=A(J)-Q*A(JK)
      17 A(JN)=A(J)-Q*A(JK)
      J1=J2+1
      IF (K+L1)44,50,51
      50 T=A(JN)
         S=1./T
      51 P=A(JN)
         Q=(T-P)*S
         N=K-1
         IF (SENSE SWITCH 2)19,20
19 PRINT105,N,Q,P,M(1),M(2),M(3),M(4),M(5),M(6),M(7)
20 PUNCH105,N,Q,P,M(1),M(2),M(3),M(4),M(5),M(6),M(7)
37 III=III+1
38 IF (III-64)41,21,40
40 IF (III-128)41,21,44
41 II=I2+INC+2
      M(K2)=0
      K1=K2-1
      IF (K1-1)44,39,42
      KP=M(K1)+3-K1
      KS=KS+2
      INC=INC+2*KS-1
      I1=J1-2*INC+KP*(2*KS+3-KP)/2
      GO TO 16
21 PRINT106
C READ IN COMBINATIONS
22 READ,N,Q,P,M(1),M(2),M(3),M(4),M(5),M(6),M(7)
23 LL=N+1
   M(LL+1)=7
   QQ=SQRT(Q)
   PRINT107,P,QQ
C SELECT MATRIX ELEMENTS
   KK=1
   III=1
   DO 29 I=0,6
      IF (M(III)-I)44,25,24
24   KK=KK+8-I
      GO TO 29
25   JJL=III
   DO 28 J=I,7
      IF (M(JJL)-J)44,27,28
27   B(III,JJL)=A(KK)
      JJL=JJL+1
28   KK=KK+1
      III=III+1
29 CONTINUE
C SOLVE FOR REGRESSION COEFFICIENTS AND OUTPUT
   DO 36 KK=1,LL
      P=1./B(KK,KK)
      JJL=KK+1
      DO 30 JJ=JJL,III
         B(JJ,KK)=B(KK,JJ)
30   B(KK,JJ)=B(KK,JJ)*P
   DO 36 I=1,LL
      IF (I-KK)32,34,31
31   JJL=I
32   P=B(I,KK)
   DO 33 J=JJL,III
33   B(I,J)=B(I,J)-B(KK,J)*P
   IF (LL*KK)44,35,36
34   PRINT108,M(I),B(I,III)
35 CONTINUE
      IF (SENSE SWITCH 3)39,52
36   GO TO 22
39   NN=2
      GO TO 999
44   PRINT109
      PAUSE
      GO TO 998
END
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AUTOBIOGRAPHY

I, Earl Zwick, was born in Canton, Ohio, on May 20, 1931. I received my secondary education in the schools of Canton, Ohio. My undergraduate work was done at John Carroll University and Kent State University, the latter school granting me the Bachelor of Science degree in 1953. After a period of military service, I returned to Kent State University where I was granted the Master of Education degree in 1957. I was a public school teacher in the Akron, Ohio, school system from 1955 until 1961. In September, 1961, I came to The Ohio State University where I attended a National Science Foundation Academic Year Institute during the 1961-62 academic year. At the end of that year, I was appointed Instructor at The Ohio State University, where I specialized in the Department of Mathematics. In 1963, I accepted a position as Assistant Professor of Mathematics at Indiana State College, and I now hold this position.