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TWO-PHASE RADIO FREQUENCY HEATING OF A PLASMA
CONFINED IN A MAGNETIC MIRROR SYSTEM

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * * *

The Ohio State University
1964

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INTRODUCTION

For the nuclei of the hydrogen isotopes, the mean binding energy per nucleon is less than it is for nuclei of slightly higher mass number. As a result, nuclear energy can be obtained by fusion of the hydrogen isotopes. The fusion reactions of interest and the amount of energy given to each particle are:

\[
\begin{align*}
    D + D &\rightarrow \text{He}^3 \ (0.82 \text{ Mev}) + n \ (2.45 \text{ Mev}) \\
    D + D &\rightarrow \text{T} \ (1.01 \text{ Mev}) + p \ (3.02 \text{ Mev}) \\
    D + \text{T} &\rightarrow \text{He}^4 \ (3.5 \text{ Mev}) + n \ (14.1 \text{ Mev}) \\
    D + \text{He}^3 &\rightarrow \text{He}^4 \ (3.6 \text{ Mev}) + p \ (14.7 \text{ Mev}) .
\end{align*}
\]

The tritium and helium-3 required for the last two reactions, are produced in small quantities in the first two reactions.

The reaction rate for each of the above reactions can be expressed as

\[
R = n_1 n_2 \bar{\sigma} \bar{v} \text{ interactions } / (\text{cm}^3 \cdot \text{sec}),
\]

where

- \(n_1\) = the density of the first type of nuclei,
- \(n_2\) = the density of the second type of nuclei,
- \(\bar{\sigma} \bar{v}\) = the average of the product of the reaction cross section and the relative velocity for the proper velocity distribution.

Note: For the D-D reactions, \(n_1 = n_2\).

Consider for example, the D-D reaction rate for a kinetic temperature \((\frac{3}{2}m\nu^2)\) of 10 kev (assuming a Maxwellian distribution of velocities)
and an ion density of $10^9 \text{ions/cm}^3$ is 1 reaction per second. It has been shown that $\bar{v}$ for each of the mentioned reactions increases as the kinetic temperature increases. Therefore, to obtain an appreciable reaction rate, high ion densities and kinetic temperatures are necessary. The confinement and production of such ion densities and ion temperatures are the subjects of many research projects in plasma physics.

Several different magnetic field configurations have been used to confine a plasma. Two of the more important ones are the "mirror" and cusped configurations. A typical "mirror" configuration is represented by Figure 1. The loss of particles out the ends of the system is greatly reduced by the presence of the stronger fields (or mirrors) at each end. In theory, however, the magnetic mirror system is unstable, because in the confining region the magnetic field lines are concave toward the plasma.

A typical cusped configuration is represented by Figure 2. The particle loss from a cusped system is expected to be quite high. The advantage of a cusped system is that the magnetic field lines are convex toward the plasma and hence stable.

There are, also, several different methods of obtaining high ion temperatures. This paper will be concerned only with radio frequency heating of a plasma confined by a combination of the above field configurations. The applied radio frequency fields will be designed primarily for the heating of ions.
Figure 1. Variation of Axial Magnetic Field
Strength in a Typical Magnetic Mirror
Configuration.
Figure 2. Typical Cusped Magnetic Field.
CHAPTER I
MOTION OF AN ION IN A ROTATING ELECTRIC
FIELD AND REVOLVING MAGNETIC FIELD

General theory

The non-relativistic motion of an ion in a rotating electric field can easily be solved for the following configuration:

\[ E_x = E_0 \sin \omega t \]  \hspace{1cm} (1.1)
\[ E_y = E_0 \cos \omega t \]  \hspace{1cm} (1.2)
\[ E_z = 0 \]  \hspace{1cm} (1.3)

Since the electric field is time dependent, there must also be a magnetic field present. Expansion of Maxwell's equation, curl \( \mathbf{B} = (1/c^2)\mathbf{E} \), gives the following equations:

\[ \left( \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) = \left( \frac{1}{c^2} \right) \omega E_0 \cos \omega t \]  \hspace{1cm} (1.4)
\[ \left( \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} \right) = \left( \frac{1}{c^2} \right) \omega E_0 \sin \omega t \]  \hspace{1cm} (1.5)
\[ \left( \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} \right) = 0 \]  \hspace{1cm} (1.6)

If \( B_x = B_y = 0 \), equations (1.4-6) reduce to

\[ \left( \frac{\partial B_z}{\partial y} \right) = \left( \omega/c^2 \right) E_0 \cos \omega t \]  \hspace{1cm} (1.7)

and

\[ \left( \frac{\partial B_z}{\partial x} \right) = \left( \omega/c^2 \right) E_0 \sin \omega t \]  \hspace{1cm} (1.8)

The solution to equations 1.7 and 1.8 is

\[ B_z = \left( \omega E_0 / c^2 \right) (y \cos \omega t + x \sin \omega t) + C \]  \hspace{1cm} (1.9)
Since we are also interested in the motion of an ion in the presence of a strong constant axial magnetic field \( B_{z0} \), one should set \( C \) equal to \( B_{z0} \). The time dependent portion of equation 1.9 (which will be referred to as \( \Delta B_z \)) is represented in Figure 3. This portion of \( B_z \) can be properly called a revolving magnetic field. Figure 4 contains a more general representation of this revolving field.

The problem to be solved then is that of an ion moving in a rotating electric field and revolving magnetic field, which is superimposed on a constant axial magnetic field. For the fields defined by equations 1.1, 1.2, and 1.9, the non-relativistic equation of motion

\[
\mathbf{m} \ddot{\mathbf{r}} = \mathbf{qE} + \mathbf{q(}\hat{x} \times \mathbf{B}) \tag{1.10}
\]

becomes

\[
\dot{x} = \left(\frac{q}{m}\right)E_0 \sin \omega t + \left(\frac{qE_0 \omega}{mc^2}\right)y (\cos \omega t + x \sin \omega t) + \left(\frac{qB_{z0}}{m}\right) \tag{1.11}
\]

\[
\dot{y} = \left(\frac{q}{m}\right)E_0 \cos \omega t - \left(\frac{qE_0 \omega}{mc^2}\right)x (\cos \omega t + x \sin \omega t) - \left(\frac{x(qB_{z0})}{m}\right) \tag{1.12}
\]

\[
\dot{z} = 0. \tag{1.13}
\]

By setting \( \alpha = y + ix \), the system of equations, 1.11 and 1.12, can be written

\[
\ddot{\alpha} = \dot{\alpha} + i\alpha = \left(\frac{qE_0}{m}\right)(\cos \omega t + i \sin \omega t) + \left(\frac{qB_{z0}}{mc^2}\right)(x \sin \omega t + y \cos \omega t) + \frac{(qE_0)}{m} \tag{1.14}
\]

Equation 1.14 can be further reduced to

\[
\ddot{\alpha} - K_1 e^{i\omega t} - iK_2 \dot{\alpha} - i\omega \alpha K_3 (\alpha e^{i\omega t} + \alpha e^{-i\omega t}) = 0 \tag{1.15}
\]

where

\[
K_1 = \left(\frac{qE_0}{m}\right), \quad K_2 = \left(\frac{qB_{z0}}{m}\right), \quad \text{and} \quad K_3 = \left(\frac{qE_0}{mc^2}\right).
\]
Figure 3. A Plot of $\Delta B_z = \frac{\omega E_0}{c^2} (ycos\omega t + x sin\omega t)$. 
Figure 4. A General Representation of a Revolving Magnetic Field.
The exact solution of the above differential equation is not obvious, because of the non-linearity introduced by the last term. The last term contains the information about the revolving magnetic field. If it is assumed that the magnitude of the revolving field is much less than the constant axial magnetic field, this last term can be neglected. The problem is now simply that of an ion moving in a rotating electric field and a constant axial magnetic field.

Motion of an ion in a rotating electric field

Let us consider the case when the frequency of the rotating electric field is equal to the cyclotron frequency of the deuteron,

\[ \omega = \frac{qB_0}{m} = K_2. \]  \hspace{1cm} (1.16)

Neglecting the last term in equation 1.15, one obtains the following differential equation

\[ \ddot{\alpha} - i\omega \dot{\alpha} = K_1 e^{i\omega t}. \]  \hspace{1cm} (1.17)

The form of the solution to equation 1.17 is

\[ \alpha = A_1 + A_2 e^{i\omega t} + K_1 t e^{i\omega t}. \]  \hspace{1cm} (1.18)

Where \( A_1 \) and \( A_2 \) can be determined from the initial conditions:

\[ \alpha_0 = A_1 + A_2 = y_0 + ix_0 \]  \hspace{1cm} (1.19)

and

\[ \dot{\alpha}_0 = i\omega A_2 + K_1 = i\dot{x}_0 + \dot{y}_0. \]  \hspace{1cm} (1.20)

The solution of equations 1.19 and 1.20 for \( A_1 \) and \( A_2 \) gives

\[ A_1 = y_0 - (x_0/\omega) + i(x_0 + (y_0 - K_1)/\omega) \]  \hspace{1cm} (1.21)

and

\[ A_2 = (\dot{x}_0/\omega) - i(\dot{y}_0 - K_1)/\omega. \]  \hspace{1cm} (1.22)
The form of $x$ and $y$ can be obtained from equation 1.18.

$$
x = x_0 + \left( \frac{y_0 - K_1}{\omega} \right) (1 - \cos \omega t) + \left( \frac{x_0}{\omega} \right) \sin \omega t + K_1 t \sin \omega t \tag{1.23}
$$

$$
y = y_0 + \left( \frac{y_0 - K_1}{\omega} \right) (\cos \omega t - 1) + \left( \frac{x_0}{\omega} \right) \sin \omega t + K_1 t \cos \omega t \tag{1.24}
$$

Equations 1.23 and 1.24 show that an ion moving in a constant rotating electric field and a strong axial magnetic field will not remain within a limited distance of the $z$-axis for an indefinite period of time. The trajectory of the ion in the $xy$-plane is a spiral.

**Change in energy and in radius of an ion**

The change in energy of an ion moving in a rotating field can be calculated under certain conditions. Let us assume that the ion is moving in a circular orbit, the radius of which is

$$r_g = \frac{m \nu_{\perp}}{q B_{z_0}} = \frac{\sqrt{2mW_{1}}}{q B_{z_0}} \tag{1.25}$$

where

- $m$ = mass of ion,
- $v_{\perp}$ = velocity of ion $\perp$ to $z$-axis,
- $q$ = charge of ion,

and that the ion is in phase with the rotating electric field (in phase means that $\mathbf{v}_{\perp}$ and $\mathbf{E}$ are parallel). Figure 5 shows the geometry of the calculation. The change in energy per cycle ($\Delta W_{1}$) in the $xy$-plane is given by

$$\Delta W_{1} = \oint q \mathbf{E} \cdot d\mathbf{s} \tag{1.26}$$

where $d\mathbf{s}$ is an incremental path length on the circle in Figure 5.

Since $d\mathbf{s}$ and $\mathbf{E}$ are always parallel, equation 1.26 becomes

$$\Delta W_{1} = r_g q E \int_{0}^{2\pi} d\phi \tag{1.27}$$

hence

$$\Delta W_{1} = 2\pi r_g q E \tag{1.28}$$
Figure 5. Cyclotron Rotation of an Ion.
The change in radius per cycle can be obtained by differentiating equation 1.25.

\[ \Delta r_g = \left( \frac{\sqrt{2m}}{qB_{z0}} \right) \frac{1}{2} \omega \Delta \tilde{W} \]  \hspace{1cm} (1.29)

Substituting equation 1.28 into 1.29 gives

\[ \Delta r_g = \left( \frac{2\pi mE}{qB_{z0}^2} \right) \]  \hspace{1cm} (1.30)

which is a constant. These calculations have shown that the change in energy per cycle increases as the radius of the orbit increases and that the change in radius per cycle is a constant.

Motion of an ion in a revolving magnetic field

The exact motion of an ion in a revolving magnetic field is quite complicated (see equation 1.15). The drift motion of the ion, however, can easily be approximated. Let us consider an ion in the field configuration described in Figure 4. If the ion's orbit does not contain the axis about which the field is revolving, the ion will spend half of its orbit in a \( B_{z0} + \Delta B_z \) field and the other half in a \( B_{z0} - \Delta B_z \) field. The resultant motion is shown in Figure 6. The drift of the orbital center \( (r_c) \) per cycle is

\[ \Delta r_c = r_{g2} - r_{g1}. \]  \hspace{1cm} (1.31)

Substituting the correct expressions for \( r_{g1} \) and \( r_{g2} \) into equation 1.31 gives

\[ \Delta r_c = \left( \frac{\sqrt{2m\tilde{W}}}{q} \right) \left[ \frac{1}{\left( B_{z0} + \Delta B_z \right)} - \frac{1}{\left( B_{z0} - \Delta B_z \right)} \right] \]  \hspace{1cm} (1.32)

which is approximately equal to

\[ \Delta r_c = \left( \frac{2\sqrt{2m\tilde{W}}}{qB_{z0}^2} \right) \Delta B_z. \]  \hspace{1cm} (1.33)
Figure 6. The Drift Motion of an ion in a Revolving Magnetic Field.
Balance of drift motion and radius expansion

When an ion is moving in both the revolving magnetic field and the rotating electric field described by equations 1.1, 1.2, and 1.9, the magnetic field causes the ion in phase with the electric field to drift toward the z-axis. Figure 7 indicates the geometry under consideration. In this case the $\Delta r_g$ of equation 1.30 and the $\Delta r_c$ of equation 1.33 are in opposite directions. Figure 8 represents the different possible balance conditions. Since $\Delta r_g$ is a constant and $\Delta r_c$ is proportional to $W^2$, it is possible as the energy varies to observe all of the cases represented in Figure 8 for a given ion. In other words as the energy increases, the balance condition changes from case 1 to case 3.
Figure 7. Motion of an Ion in a Rotating Electric Field and Revolving Magnetic Field.
Case 1
\[ \Delta r_c < \Delta r_g \]

Case 2
\[ \Delta r_c = \Delta r_g \]

Case 3
\[ \Delta r_c > \Delta r_g \]

Figure 8. Balance of Drift Motion and Radius Expansion.
CHAPTER II

CHANGE OF PHASE DUE TO NON-UNIFORM MAGNETIC FIELDS

Change of phase due to radial variation in the axial magnetic field \( B_{z0} \)

In Chapter I it was assumed that the axial magnetic field \( B_{z0} \) was constant at every point in the xy-plane. In the typical experimental system, however, there will be a radial variation in the axial magnetic field. Let us assume the axial field can be represented by the following Taylor's expansion about the z-axis.

\[
B_z = B_{z0} + r\left(\frac{dB_z}{dr}\right)_{r=0} + \frac{1}{2}r^2\left(\frac{d^2B_z}{dr^2}\right)_{r=0} + \ldots \quad (2.1)
\]

In order to simplify this calculation, let us further assume that the ion is rotating about the z-axis (this means that in Figure 7, \( r_c = 0 \)). Hence, the magnetic field in which the ion is moving is

\[
B_z = B_{z0} + r_c\left(\frac{dB_z}{dr}\right)_{r=0} + \frac{1}{2}r_c^2\left(\frac{d^2B_z}{dr^2}\right)_{r=0} + \ldots \quad (2.2)
\]

If the rotating frequency of the electric field is held fixed at the ion cyclotron frequency given by equation 1.16, there will develop a phase difference between the rotating electric vector and the instantaneous direction of the motion. Figure 9 displays this phase relationship. This phase angle can be represented as

\[
\phi = \phi - 2\pi n \quad (2.3)
\]

\( n \) = the number of cycles through which the \( \mathbf{E} \) vector has rotated.

The change in the phase per cycle can be calculated from equation 2.3.

\[
\frac{d\phi}{dn} = \frac{d\phi}{dn} - 2\pi \quad (2.4)
\]
Figure 9. The Phase Relationship Between the Rotating Electric Field and the Instantaneous Ion Direction.
The change in the ion's angle of rotation ($\phi$) per cycle is given by
\[ (d\phi/dn) = 2\pi f_c T \] (2.5)
where
\[ f_c = (qB_z/2\pi m) = \text{(the cyclotron frequency of the ion)}, \]
\[ T = (2\pi m/qB_{zo}) = \text{(the period of the rotating electric field)} , \]
As a first approximation, let
\[ B_z = B_{zo} + r_g (dB_z/dr)_{r=0} \] (2.6)
hence
\[ (d\phi/dn) = (2\pi/B_{zo}) (B_{zo} + r_g (dB_z/dr)_{r=0}). \] (2.7)
Substituting equation 2.7 into equation 2.4, one obtains
\[ (d\phi/dn) = (2\pi r_g/B_{zo}) (dB_z/dr)_{r=0}. \] (2.8)
If equation 2.8 is differentiated once, the following differential equation is obtained
\[ (d^2\phi/dn^2) = (2\pi/B_{zo}) (dB_z/dr)_{r=0} (dr_g/dn). \] (2.9)
The change in radius per cycle is no longer a constant since the phase between the rotating $\vec{E}$ vector and the ion velocity is constantly changing. The change in energy per cycle, equation 1.28, becomes
\[ (\Delta W_i/\Delta n) = 2\pi r_g qE \cos \Theta. \] (2.10)
Substituting equation 2.10 into equation 1.29, one finds that
\[ (\Delta r_g/\Delta n) = (2\pi mE/qB_{zo}^2) \cos \Theta. \] (2.11)
As a result, the following differential equation must be solved for the change in phase,
\[ (d^2\Theta/dn^2) = a \cos \Theta \] (2.12)
where
\[ a = (4\pi^2 mE/qB_{zo}^3) (dB_z/dr)_{r=0}. \]
By letting \( \frac{d\Theta}{dn} = p \), one can show that the number of cycles required for the phase to change from \( \Theta_0 \) to \( \Theta_1 \) is

\[
n = \left( \frac{\Theta_1 - \Theta_0}{(2a \sin t + c)^{\frac{1}{2}}} \right) \quad (2.13)
\]

where

\[
c = (8 \pi r_{go}^2 / B_{zo}) (dB_z/dr)_{r=0}^2
\]

and

\[
r_{go} = \text{radius when } \Theta = \Theta_0.
\]

The exact solution of equation 2.13 is not obvious, however it can be approximated by a series expansion. However, for the present discussion the following approximation will be used:

\[
\frac{d\Theta}{dn} = \left(2\pi <r_g>/B_{zo}\right)(dB_z/dr)_{r=0} = \text{constant} \quad (2.14)
\]

where

\[
<r_g> = \text{the mean value of } r_g \text{ in the region where the phase change occurred.}
\]

Therefore, the number of cycles required for the phase to change from \( \Theta_0 \) to \( \Theta_1 \) is

\[
n = (\Theta_1 - \Theta_0) \left[ B_{zo} / \left(2\pi <r_g>(dB_z/dr)_{r=0}\right) \right]. \quad (2.15)
\]

Equation 2.15 clearly indicates that the larger \( (dB_z/dr)_{r=0} \) for a given \( B_{zo} \) the fewer cycles required for an ion to get out of phase (\( \vec{E} \) perpendicular to \( \vec{v}_L \)) with the rotating electric field. Consequently, the radial uniformity of the axial magnetic field must be carefully regulated when one wants to heat ions by the ion cyclotron resonance principle. The revolving magnetic field discussed in Chapter I will also contribute to the radial non-uniformity of the axial magnetic
field. This contribution to the change in phase was not included in this calculation because of the difficulty involved in estimating its exact effect. In the experimental system to be described in Chapter IV, the radial non-uniformity in the revolving magnetic field is less than the radial non-uniformity in the axial magnetic field.
CHAPTER III
MOTION OF AN ION IN A ROTATING MAGNETIC FIELD

Motion in a rotating magnetic field
generated by four parallel currents

Let us consider the problem of an ion moving in the fields induced by the current configuration described in Figure 10. The magnetic field induced by each wire can be calculated from

\[ \vec{B} = \text{curl} \, \vec{A} \quad (3.1) \]

For a single wire of finite length, Figure 11, the vector potential \( \vec{A} \) can be expressed as

\[ A_z(\varphi, z, t) = (\mu_0 I/4\pi) \left( \int_0^l \frac{dz'}{(\varphi^2 + (z - z')^2)^{1/2}} \right) \quad (3.2) \]

The only non-zero component of the magnetic field for a single wire is

\[ B_\varphi = (\mu_0 I/4\pi) \left( \frac{l-z}{(\varphi^2 + (l-z)^2)^{1/2}} + \frac{z}{(\varphi^2 + z^2)^{1/2}} \right). \quad (3.3) \]

The total magnetic field induced by the current configuration described in Figure 10 can be calculated by superimposing four terms of the form expressed in equation 3.3. The resulting expression would be too complicated to be used in an exact solution of the equations of motion. However, the resulting magnetic field rotates about the \( z \)-axis. This rotation is demonstrated in Figure 12.

This configuration of current wires generates a rotating cusped magnetic field in addition to a rotating magnetic field. The normal cusped magnetic field configuration\(^5\) used to enhance the stability of a plasma is described in Figure 2. The normal cusped field therefore
Figure 10. Geometry for Four Parallel Current-Carrying Wires.
Figure 11. Geometry for Expressing the Vector Potential for a Straight Wire.
Figure 12. Rotating Magnetic Field Generated by the Four Parallel Wires in Figure 10.
is a stationary quadrapole, but the cusped field now being discussed is a rotating dipole. This rotating dipole should also increase the stability of the plasma, since the magnetic field lines are convex towards the plasma over a considerable part of the cross section. In addition, the rotation of this configuration at a high frequency should reduce the possibility of an instability occurring at the points where the magnetic field lines are not convex toward the plasma.

Motion of an ion in a uniform rotating magnetic field

The non-relativistic motion of a charged particle in a rotating magnetic field can be solved for the following configuration:

\[ B_x = B_0 \cos \omega t \]  \tag{3.4}  
\[ B_y = B_0 \sin \omega t \]  \tag{3.5}  
\[ B_z = \text{constant} \]  \tag{3.6}  

Since the magnetic field is time dependent, there must also be an electric field present. Expansion of Maxwell's equation, \( \text{curl } \vec{E} = -\vec{B} \), gives the following equations:

\[ \left( \frac{\partial E_z}{\partial y} \right) - \left( \frac{\partial E_y}{\partial z} \right) = \omega B_0 \sin \omega t \]  \tag{3.7}  
\[ \left( \frac{\partial E_z}{\partial x} \right) - \left( \frac{\partial E_x}{\partial z} \right) = \omega B_0 \cos \omega t \]  \tag{3.8}  
\[ \left( \frac{\partial E_y}{\partial x} \right) - \left( \frac{\partial E_x}{\partial y} \right) = 0. \]  \tag{3.9}  

If \( E_x = E_y = 0 \), equations 3.7, 3.8, and 3.9 reduce to

\[ \left( \frac{\partial E_z}{\partial y} \right) = \omega B_0 \sin \omega t \]  \tag{3.10}  
\[ \left( \frac{\partial E_z}{\partial x} \right) = \omega B_0 \cos \omega t. \]  \tag{3.11}  

A solution to equations 3.10 and 3.11 is

\[ E_z = \omega B_0 (x \cos \omega t + y \sin \omega t). \]  \tag{3.12}  

For the fields defined above, the non-relativistic equations of motion

\[ m\ddot{x} = q\dot{E} + q(\dot{\mathbf{r}} \times \dot{\mathbf{B}}) \quad (3.13) \]

becomes

\[ \dot{m}\dot{x} = q(yB_0 - zB_0 \sin \omega t) \quad (3.14) \]
\[ m\dot{y} = q(zB_0 \cos \omega t - xB_0) \quad (3.15) \]
\[ m\ddot{z} = qB_0 (\omega x \cos \omega t + \omega y \sin \omega t + \dot{x} \sin \omega t - \dot{y} \cos \omega t). \quad (3.16) \]

Equation 3.16 can also be written

\[ m(d\dot{z}/dt) = qB_0 (d/dt)(x \sin \omega t - y \cos \omega t). \quad (3.17) \]

After integration, equation 3.17 becomes

\[ \dot{z} = q(B_0/m)(x \sin \omega t - y \cos \omega t) + C. \quad (3.18) \]

Substituting equation 3.18 into equations 3.14 and 3.15, one obtains

\[ \dot{x} = (qB_0/m)y - (q^2B_0^2/m^2)[(x \sin \omega t - y \cos \omega t) \sin \omega t] - (qB_0/m)C \sin \omega t \quad (3.19) \]
\[ \dot{y} = (-qB_0/m)x + (q^2B_0^2/m^2)[(x \sin \omega t - y \cos \omega t) \cos \omega t] + (qB_0/m)C \cos \omega t \quad (3.20) \]

Let \( \alpha = x + iy. \) (3.21)

The above system of equations can then be written

\[ \partial = \dot{x} + iy = (qB_0/m)(\dot{y} - i\dot{x}) - (qB_0/m)C(\sin \omega t - i \cos \omega t) \]
\[ - (q^2B_0^2/m^2)(x \sin^2 \omega t + y \cos^2 \omega t) - (y + ix) \sin \omega t \cos \omega t). \quad (3.22) \]

Making use of the following identities

\[ \sin 2\omega t = 2 \cos \omega t \sin \omega t \]
\[ \cos 2\omega t = 1 - 2 \sin^2 \omega t = 2 \cos^2 \omega t - 1 \]
\[ e^{i\omega t} = \cos \omega t + i \sin \omega t, \]

one can rewrite equation 3.22 as
\[ 
\dot{\alpha} = -i(qB_0/m)(\dot{\chi} + i\dot{\psi}) + i(qB/m)Ce^{i\omega t} 
- (qB_0^2/m^2)[\frac{1}{2}(x + iy) + \frac{1}{2}(iy-x)(\cos2\omega t + isin2\omega t)]. 
\]  
(3.23)

Equation 3.23 can be further reduced to
\[ 
\dot{\alpha} + iK_1\dot{\alpha} + \frac{iK_2^2}{2}\dot{\alpha} - \frac{iK_2^2}{2}\alpha e^{i2\omega t} = iK_2Ce^{i\omega t} 
\]  
(3.24)

where
\[ 
K_1 = (qB_0/m) 
\]
and
\[ 
K_2 = (qB_0/m). 
\]

Substituting \( \alpha = fe^{i\omega t} \) into equation 3.24, one obtains a differential equation which has constant coefficients.
\[ 
\ddot{f} + i(2\omega + K_1)f + (\frac{1}{2}K_2^2 - K_1\omega - \omega^2)f - \frac{iK_2^2}{2}f^* = iK_2C 
\]  
(3.25)

A general solution to equation 3.25 is
\[ 
f = Ae^{i\gamma t}\phi + De^{-i\gamma t}\phi + iE. 
\]  
(3.26)

Substituting equation 3.26 into 3.25, one obtains the following equation:
\[ 
iK_2C = [-A\gamma^2 - A\gamma(2\omega + K_1) + (\frac{1}{2}K_2^2 - K_1\omega - \omega^2)A - \frac{1}{2}iAK_2^2]e^{i\gamma t} + \phi 
+ [-D\gamma^2 + D\gamma(2\omega + K_1) + (\frac{1}{2}K_2^2 - K_1\omega - \omega^2)D - \frac{1}{2}iAK_2^2]e^{-i\gamma t} + \phi 
+ i[K_2^2 - K_1\omega - \omega^2]E. 
\]  
(3.27)

Equation 3.27 is satisfied if the following three conditions are valid:
\[ 
-A\gamma^2 - A\gamma(2\omega + K_1) + (\frac{1}{2}K_2^2 - K_1\omega - \omega^2)A = \frac{1}{2}DK_2^2 
\]  
(3.28)
\[ 
-D\gamma^2 + D\gamma(2\omega + K_1) + (\frac{1}{2}K_2^2 - K_1\omega - \omega^2)D = \frac{1}{2}iAK_2^2 
\]  
(3.29)
\[ 
E = K_2C/(K_2^2 - K_1\omega - \omega^2). 
\]  
(3.30)

Equations 3.28 and 3.29 can be combined to obtain
\[(A/D) = \frac{1}{2} K_2^2 / (-\gamma^2 - \gamma(2\omega + K_1) + \frac{1}{2} K_2^2 - K_1\omega - \omega^2)\]
\[= (-\gamma^2 + \gamma(2\omega + K_1) + \frac{1}{2} K_2^2 - K_1\omega - \omega^2) / \frac{1}{2} K_2^2 \quad (3.31)\]

The last equality in equation 3.31 leads to an equation which can be solved for \(Y\).

\[\gamma^4 - \gamma^2 (2\omega + K_1)^2 + (\frac{1}{2} K_2^2 - K_1\omega - \omega^2)^2 = 2 \gamma^2 (\frac{1}{2} K_2^2 - K_1\omega - \omega^2) = (\frac{1}{2} K_2^2)^2 \quad (3.32)\]

Therefore

\[2 \gamma^2 = K_2^2 + 2 K_1\omega + 2 \omega^2 + K_1^2 \]
\[\pm [(K_2^2 + K_1^2 + 2 K_1 \omega + 2 \omega^2)^2 - 4(K_1 \omega + \omega^2)(K_1 \omega + \omega^2 - K_2^2)]^{1/2} \quad (3.33)\]

or

\[\gamma_{1,2} = \sqrt{\frac{1}{2}} (K_2^2 + K_1^2 + 2 K_1 \omega + 2 \omega^2 \pm \sqrt{\beta})^{1/2} \quad (3.34)\]

where

\[\beta = (K_1^2 + K_2^2 + 2 K_1 \omega + 2 \omega^2)^2 - 4(K_1 \omega + \omega^2)(K_1 \omega + \omega^2 - K_2^2). \quad (3.35)\]

One can now write down the final form for \(\alpha\).

\[\alpha = e^{i \omega t} (A_1 e^{i \gamma_1 t} + \phi_1 + D_1 e^{-i \gamma_1 t} + \phi_1 + A_2 e^{i \gamma_2 t} + \phi_2 + D_2 e^{-i \gamma_2 t} + \phi_2 + e^{i \gamma_3 t} + \phi_3 + e^{-i \gamma_3 t} + \phi_3) \quad (3.36)\]

The form of \(x\) and \(y\) obtained from equation 3.36 will be periodic only if \(\gamma_{1,2}\) are real numbers. The necessary conditions for \(\gamma_{1,2}\) to be real can be obtained from equation 3.34 and can be stated as follows:

\[K_1^2 + K_2^2 + 2 K_1 \omega + 2 \omega^2 > \sqrt{\beta} \quad \text{and} \quad \beta > 0. \quad (3.37)\]

Hence

\[(K_1^2 + K_2^2 + 2 K_1 \omega + 2 \omega^2)^2 > 4(K_1 \omega + \omega^2)(K_1 \omega + \omega^2 - K_2^2) > 0. \quad (3.38)\]
The left inequality is always satisfied. The right inequality requires that either
\[ K_1 \omega > -\omega^2 + K_2^2 \]
or
\[ K_1 \omega < -\omega^2. \]  (3.39)
Substituting the definitions of \( K_1 \) and \( K_2 \) into equation 3.39 leads to either
\[ (qBz/m \omega) < -1 \]
or
\[ (qBz/m \omega) > -1 + (qBz/m \omega)^2. \]  (3.40)

If a deuteron is to move within a limited distance from the z-axis equation 3.40 must be satisfied. Let us consider the case where the frequency of the rotating magnetic field is equal to the ion cyclotron frequency of the deuteron.
\[ \omega = (qBz/m) = K_1 \]  (3.41)
Equation 3.40 will be satisfied providing
\[ B_0 < \sqrt{2} B_{zo}. \]  (3.42)
Equation 3.42 implies that no periodic solutions are possible if the axial magnetic field is removed. Therefore, a uniform rotating magnetic field alone does not enhance the confinement of the plasma.

If the frequency of the rotating magnetic field equals the ion cyclotron frequency, which is much less than the electron cyclotron frequency, the electrons tend to be tied to the lines of force where the ions are not. As a result, large circulating currents are
induced. These currents cause ohmic heating of the plasma. Although a uniform rotating magnetic field does not enhance confinement, it does increase the heating of the plasma.
CHAPTER IV
EXPERIMENTAL SYSTEM

Introduction

In Chapter I, II, and III the motion of an ion in certain magnetic and electric field configurations was developed. It was shown that an ion moving in phase with a rotating electric field gains a definite amount of energy each cycle of the electric field. As a result, a rotating electric field could effectively be used to heat a plasma confined in a magnetic mirror system. It was, also, shown that an ion moving in a revolving magnetic field has a resultant drift motion. However, it is possible to phase the rotating electric field and revolving magnetic field, so that an ion in phase with the rotating electric field will drift toward the axis about which the electric field is rotating. This inward drift motion will enhance the confinement of the plasma.

In Chapter III, it was shown that a configuration of four parallel wires could be used to generate a rotating magnetic field. In addition this rotating magnetic field appears like the cusped field induced by a rotating current dipole. The fact that the magnetic field lines are convex toward the plasma and that the configuration is rotating at a high frequency should enhance the stability of the plasma. The rotating magnetic field, also induces circulating currents in the plasma, and these currents cause ohmic heating of the plasma.
All of the theory developed in the preceding chapters was for electric and magnetic field configurations superimposed upon a constant axial magnetic field. Such a constant axial magnetic field is a desired characteristic of the central region of a magnetic mirror system. Chapter II contained an approximation of the phase change introduced by a radial variation in this axial magnetic field. Such a phase change is obviously detrimental to the heating capabilities of a rotating electric field. Also, if the field lines of the mirror system are not parallel to the z-axis, the charged particles present in the plasma will be forced to move in the axial direction.

It would be desirable, therefore, to design an inductive coil configuration which induces electric and magnetic fields similar to those described in the preceding chapters. In other words, there should be a region which is characterized by a rotating electric field and revolving magnetic field. This region would be the primary heating region of the system. A second region which is characterized by a rotating cusped magnetic field would allow secondary heating of the plasma in what should be a more stable geometry. In order not to introduce impurities with a high atomic number into the plasma, this coil configuration should be mounted outside of the plasma.\(^9\) This restriction reduces considerably the number of desirable coil configurations.

**Geometry of the radio frequency coils**

It has been shown that an ion in cyclotron resonance with a single phase alternating current will drift toward the single phase current.\(^{10}\) As a result, if the current flowing in two wires was phased
90 degrees apart; the ion would drift first toward one wire and then toward the other. This implies that an ion would drift toward the intersection of two perpendicular wires carrying currents phased 90 degrees apart. It would, therefore, be desirable to construct a coil configuration composed of radial wires which intersect at right angles at the z-axis. Since it is not possible to pass wires through the plasma, an external coil configuration must be used to achieve the above effect.

The fields induced by a straight wire, Figure 11, carrying a single phase alternating current can be derived from the vector potential expressed in equation (3.2). The only non-zero component of the magnetic field induced by a straight wire is

\[
E_\Phi = (\mu_0 I / 4\pi \rho) \left[ \frac{(l-z)}{\sqrt{\rho^2 + (l-z)^2}} + \frac{z}{(\rho^2 + z^2)^{3/2}} \right]. \tag{4.1}
\]

The only non-zero component of the induced electric field is

\[
E_z = -(\mu_0 I / 4\pi \rho) \ln \left[ \frac{l-z + \sqrt{\rho^2 + (l-z)^2}}{l-z + \sqrt{\rho^2 + z^2}} \right]. \tag{4.2}
\]

Figure 13, shows the direction of the electric and magnetic fields expressed by equations (4.1) and (4.2). It is apparent from Figure 13 that the direction of the fields in the region below the wire is the same as in the region beside the wire. Therefore, the fields in the region below the current carrying wire appear as if they were induced by an imaginary wire passing through that region. Consequently, the intersection of radial currents can be approximated by the current configuration contained in Figure 14.

In order to induce the currents described in Figure 14, a closed circuit loop must exist and this loop must contain a power source.
Figure 13. Direction of the Magnetic and Electric Field Induced by a Straight Wire.
Figure 14. A Current Configuration that Approximates Perpendicular Radial Currents.
The current path of this closed loop or coil is indicated in Figure 15. The variable capacitor shown in Figure 15 is used to adjust the resonant frequency of the load circuit to the fixed frequency of the power source. In the plane perpendicular to the plane defined by the coil 1 in Figure 15, there exists an identical coil 2. The current in coil 2 is 90 degrees out of phase with that of coil 1, Figure 16. In the plane perpendicular to the z-axis at point (a) of Figure 15, a current configuration similar to that of Figure 14 is obtained. In this plane there is induced a revolving magnetic field, Figure 17; and a rotating electric field, Figure 18. The phasing of the revolving magnetic field is the same as that discussed in Chapter 1. As a result, the drift motion of an ion in phase with the rotating electric field will be directed toward the z-axis. Figure 19 indicates how the coils were constructed and attached inside the magnetic mirror coils. Figure 20 shows the final form of the coil for each phase. Due to limited space the variable capacitors were mounted outside of the magnet coils, Figure 21.

The revolving magnetic field configuration shown in Figure 17 was obtained from the proper form of equation (4.1) for the coils shown in Figure 19. The revolving property of the field was experimentally verified by passing the proper DC currents through each of the coils and measuring the magnetic field with a Model 120 Bell Gaussmeter. The results obtained were in agreement with Figure 17. The average magnitude of this revolving magnetic field (ΔBz) is .004I0 gauss. Where I0 is the amplitude of the RF current passing through the coils. The average magnitude of the rotating electric field is .05I0 volts/cm.
Figure 15. Current Path for one Phase of the Radio Frequency Coil Configuration.
Figure 16. Two-Phase Alternating Currents.
Figure 17. Revolving Magnetic Field Induced by Perpendicular Currents with a Phase Difference of 90 Degrees.
Figure 18, Rotating Electric Field Induced by Perpendicular Currents with a Phase Difference of 90 Degrees.
Figure 19. The Mounting of the RF Coils.
Figure 20. Final Form of the RF Coil for Each Phase.
Figure 21. The Mounting of the Variable Capacitors.
In the radio frequency coil configuration there are four regions that are characterized by a revolving magnetic field and rotating electric field. These are the regions directly below the vertical currents in Figure 15. In these regions the plasma is heated by the ion cyclotron resonance effect. It also should be noted that adjacent regions are 180 degrees out of phase. The regions directly below the horizontal currents in Figure 15 are characterized by a rotating magnetic field. This rotating magnetic field is the same as that developed in Chapter 3. Consequently, ohmic heating of the plasma occurs in these regions.

Radio frequency power supplies

Figure 22 is a block diagram of radio frequency power supplies and load circuits. The power output of the last amplifier stage is 20 kw into each phase at 10 kv. Each phase of the load circuit is tuned by a variable capacitor to the frequency of the crystal oscillator. The phasing of the coils is monitored by a Tektronix 555 dual beam oscilloscope. The load coils are water cooled to minimize temperature effects. RMS currents of 500 amperes should be obtained in each phase with the power supplies described above.

Magnetic mirror system

The "mirrors" of the magnetic bottle are produced by two strong end solenoids. Additional coils are placed in the central region in order to obtain a more uniform axial magnetic field in the central volume, Figure 23. The variation in the magnetic mirror field along the z-axis is indicated in Figure 24. The magnetic mirror ratio for
Figure 22. RF Power Supplies and Load Circuits.
Figure 23. The Magnetic Mirror System.
Figure 24. Magnetic Mirror Field.

mirror ratio $= \frac{B_{zm}}{B_{zo}} = 1.87$
this system is 1.57. The resonant field strength is approximately 5000 gauss and occurs at a magnet current of 1000 amperes. The axial magnetic field in the central region was measured with a Model 240 Bell Incremental Gaussmeter and these results were used to determine the final positioning of the central magnet coils. The variation in axial magnetic field of the final system was less than 1% in the central volume defined by $0 \leq \rho \leq 2.75''$ and $4'' \leq z \leq 4''$, Figure 25. These measurements indicated that

$$<x_g>(dE_z/dr)_r = 0 \left(1/B_{z0}\right) = (8/5000).$$

(4.3)

Using equation 2.15, one finds that the approximate number of cycles required for $\Theta_1 - \Theta_0 = (\pi/2)$ is 156. At 4 mc, this corresponds to approximately 40 microseconds. With the present power supplies this time is sufficient to heat an ion to the maximum deuteron (30 kev) energy that can be confined inside the pyrex vacuum chamber.

The charged particles in the plasma are tied to the magnetic flux lines, which penetrate the plasma. Consequently, the pyrex vacuum chamber was blown to fit the contour of the magnetic flux lines, Figure 26; in order to minimize the number of hot ions hitting the glass walls. In other words, the glass wall never crosses a magnetic flux line which is contained in the central region. The inside diameter of the pyrex vacuum chamber is 5.5 inches in the central region. This diameter allows the confinement of 30 kev deuterons in a 5000 gauss axial magnetic field.
$B_z = 63$ gauss in positive ($z$) direction.

Indicated values are deviations from 63 gauss.

Figure 25. Incremental Gauss Plot.
Figure 26. Pyrex Bottle and Magnetic Flux Lines.
CHAPTER V
EXPERIMENTAL RESULTS

Introduction

Electrical coupling of the two phases through the shield on the inside of the central magnet coils and the magnet coils themselves had to be overcome in order to maintain a 90 degree phase difference between each coil. This coupling (2) is represented by the circuit diagram in Figure 27. With the power supply connected across the entire RF coil, Figure 27, it was possible to maintain a phase difference of 90 degrees between each phase. Under these operating conditions the impedance seen by the power supply is much higher than the impedance required for proper matching. Consequently, the maximum power available for each phase is about 3 kw at 9 kv.

Considerably more power could be supplied to each coil if it were not necessary to maintain a phase difference of 90 degrees. However, experimental results show that proper phasing of the RF coils (proper phasing means that the electric and magnetic fields rotate as described in Chapter IV) makes a considerable difference in the properties of the plasma generated. The light emitted from the ends of the pyrex bottle and the electron density at the center of the bottle were measured for the currents phased as in Figure 16 and the opposite phasing (-90 degrees). These measurements were made when the axial
Figure 27. Coupling between Phase 1 and Phase 2.
magnetic field was at the calculated resonant field strength of 5000 gauss. At this field strength and when the currents are phased as in Figure 16, the ions and the induced electric field will rotate in the same direction at the same frequency. When the phasing is reversed the induced electric field and the ions will rotate in opposite directions. Figure 28 contains the observed results. The electron density was 4 times greater for the proper phasing. Also, the light emitted was 40% greater.

Change in loading of rf power supplies

In an attempt to observe ion cyclotron resonance the change in loading of RF power supplies was observed. The plate current meter for the 6424 amplifier stage was nulled at a point where the plate current was a minimum. The change in plate current was then read on a more sensitive scale. Since the plate voltage on the 6424 tubes remains constant, the change in plate current is a measure of the loading of the RF power supplies. The zero indicated in Figure 29 corresponds to the point where the plate current was nulled. Since the tuning of the power supplies remained constant, the power output increased as the plate current increased. Consequently, in Figure 29, ion cyclotron resonance was not indicated by a change in the loading of the RF power supplies; since the calculated ion cyclotron resonance occurs at $B_z = 5000$ gauss.

Electron density

The electron density at the center of the magnetic mirror system was measured by means of a microwave interferometer, Figure 30. The
### Power = 2.5 kw/Ø, $B_{zo} = 5000$ gauss

<table>
<thead>
<tr>
<th>Phasing</th>
<th>Light Emitted</th>
<th>Electron Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>+90</td>
<td>2.5 ft. candles</td>
<td>$4 \times 10^{10}$ electrons/cm³</td>
</tr>
<tr>
<td>+90</td>
<td>2.5 ft. candles</td>
<td>$5 \times 10^{10}$</td>
</tr>
<tr>
<td>+90</td>
<td>2.5 ft. candles</td>
<td>$3 \times 10^{10}$</td>
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<td>-90</td>
<td>1.5 ft. candles</td>
<td>$.5 \times 10^{10}$</td>
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<td>-90</td>
<td>1.8 ft. candles</td>
<td>$1.6 \times 10^{10}$</td>
</tr>
<tr>
<td>-90</td>
<td>1.9 ft. candles</td>
<td>$.8 \times 10^{10}$</td>
</tr>
</tbody>
</table>

Figure 28. Light Emitted and Electron Density at the Resonance Condition when the Phase Difference is ± 90 Degrees.
Figure 29. Change in RF Power versus Axial Magnetic Field for Deuterium at 14 Micons.

Change in RF Power to 6424 Tubes (kilowatts)

Deuterium - 14 microns
Power - 2.5 kw/Φ
Plate volts - 78 kv
Plate current - .9 amps.
(before null)
Figure 30. Schematic Representation of the Microwave Interferometer.
klystron had a frequency range of 27-30 kilo-megacycles. The maximum densities were observed at a deuterium gas pressure of 14 microns, Figure 31. The peak in electron density at 4500 gauss is an indication of ion cyclotron resonance. At a deuterium gas pressure of 1 micron, Figure 32, the electron density is 3 times greater than the background at 4700 gauss. The half width of this resonance peak is about 1000 gauss, Figure 32.

The increase in electron density at low axial magnetic field strengths is due to increased heating of the free electrons. Since the electrons rotate in the opposite direction of the ions in an axial magnet field, the rotating electric field (10 v/cm) and the electrons rotate in opposite directions. Also, the radius of the electron orbit is inversely proportional to the axial magnetic field strength, equation 1.25. Consequently, at high magnetic field strengths the electrons rotate in very tight orbits about the magnetic flux lines and absorb very little energy. However at zero axial magnetic field the mean free path for an electron in deuterium gas at 14 microns is about 4 cm. Therefore in a half cycle of the rotating electric field an electron can gain 40 ev of energy. This energy is sufficient to excite and ionize a deuterium atom.

**Light emitted from ends of pyrex bottle**

A W-7 Photometer made by Weston Electrical Instrument Corp. was used to measure the light emitted from the ends of the pyrex bottle. The light emitted increased rapidly as the axial magnetic field strength approached zero, Figure 33. This increase results from the
Figure 31. Electron Density versus Axial Magnetic Field for Deuterium at 14 Microns.

Deuterium - 14 microns
Power - 2.5 kW/Φ
Figure 32. Electron Density versus Axial Magnetic Field for Deuterium at 1 Micron.
Figure 33. Light Emitted versus Axial Magnetic Field for Deuterium at 14 Microns.
exciting of the deuterium atoms caused by increased heating of the free electrons.

While making these measurements, the location of the source of this light changed as the deuterium gas pressure was changed. At a gas pressure of 14 microns, the central region of the bottle cross section was dark in comparison with the surrounding bright region. At a deuterium gas pressure of 1 micron, the central region of the bottle cross section was bright in comparison with the surrounding region. Since the induced electric field is weaker in the central region of the bottle cross section than in the surrounding region, the amount of ionization should be less in the central region. Since the mean free path increases as the gas pressure decreases, an ion exists for a longer period of time in the rotating electric field and revolving magnetic field. Therefore, the brightness of the central region at the lower gas pressure supports the theory developed in Chapter I concerning the inward drift motion of ions in a properly phased revolving magnetic field and rotating electric field.
CONCLUSIONS

In theory, an ion moving in phase with a rotating electric field gains a definite amount of energy each cycle of the electric field. As a result, a rotating electric field can effectively be used to heat a plasma confined in a magnetic mirror system. Also, in a properly phased revolving magnetic field and rotating electric field, an ion in phase with the rotating electric field will drift toward the axis about which the electric field is rotating. The brightness of the central region at a gas pressure of 1 micron supports this theory concerning the inward drift motion of ions moving in a properly phased revolving magnetic field and rotating electric field.

Mathematically, it was shown that a uniform rotating magnetic field alone does not enhance the confinement of the plasma. Also, a radial non-uniformity in the axial magnetic field will introduce a phase difference between the ion motion and the rotating electric field. Such a phase difference reduces the heating capabilities of a rotating electric field.

A radio frequency coil configuration which incorporated the properties of a rotating electric field, revolving magnetic field, and rotating magnetic field was constructed. This coil configuration was used in conjunction with a magnetic mirror system capable of confining 30 kev deuterons. The electron density was 4 times greater and the light emitted 40% higher for the proper phasing of this two

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phase system in comparison with the 180 degree reversal of this phasing.

At a deuterium gas pressure of 14 microns a peak in electron density at 4500 gauss indicated ion cyclotron resonance. At a deuterium gas pressure of 1 micron, the electron density was 3 times greater than the background at 4700 gauss. The half-width of this resonance peak was about 1000 gauss. Ion cyclotron resonance was not indicated by a change in the loading of the RF power supplies at a deuterium gas pressure of 1 micron or at 14 microns. For a deuterium gas pressure of 14 microns, the electron density and the light emitted increased as the axial magnetic field was decreased because of increased heating of the free electrons.
REFERENCES


