INELASTIC SCATTERING OF 6.2 MEV PROTONS BY MAGNESIUM

DISSERTATION

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By

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INTRODUCTION

The earliest example of a direct nuclear reaction is the Oppenheimer Phillips\(^1\) deuteron stripping reaction. It was observed that when a beam of deuterons was incident upon a nucleus, many protons were liberated, although the coulomb barrier was well above the incident energy of the deuterons. This process is pictured as one in which the proton in the deuteron is held away from the nucleus by the coulomb force while the neutron enters the nucleus and is captured. Although this picture of the reaction is not actually in the class of direct reactions to be discussed below, it is in sharp contrast to the compound nucleus theory which pictures the whole deuteron as entering the nucleus, with the ejection of one of the protons at some later time.

Deuteron stripping at high energies (190 Mev) was considered by Serber\(^2\), who computed the angular distribution of the emitted protons and neutrons by assuming that the nucleus was merely a geometrical barrier to a nucleon. When one of the constituents of the deuteron struck the nucleus, it was absorbed, and its partner continued on with approximately the velocity of the incident deuteron, thus giving an angular distribution sharply peaked in the forward direction.

The first calculation on the stripping process which included the properties of the nucleus was performed by Butler.\(^3\) The wave function describing the reaction was divided into two regions and was forced to match at the boundary; namely, the surface of

\(^3\) Butler, G. H., Phys. Rev. 58, 449 (1940).
the nucleus. The wave function for \( \psi_n \big| \psi_0 \) (\( \psi_n \) is the position of the neutron and \( \psi_0 \) is the nuclear radius) consisted of an incident plane wave of deuterons and the exterior wave functions for a neutron bound in a potential well, with the associated protons being free. For \( \psi_n \big| \psi_0 \), the wave function consisted of out-going protons and a neutron captured into a state of well-defined angular momentum. This calculation yielded the result that the angular distribution of emitted protons is proportional to \( j_l(Qr_o) \), where \( j_l \) is the spherical Bessel function of order \( l \), \( Q \) is the magnitude of the momentum transfer, and \( r_o \) is, again, the nuclear radius. The function \( j_l \) has a principal maximum at zero for \( l = 0 \), and this maximum moves to larger angles as \( l \) increases. Thus if one observes an angular distribution conforming to some value of \( l \), one can determine the parity and limit the spin of the state in which the residual nucleus was left. This has since become a valuable tool of nuclear spectroscopy.

A similar angular distribution for the stripping process has been computed by Bhatia, Huang, Newns and Huby by using a Born approximation approach. The cross section is proportional to the square of a matrix element between two states, one of which describes the incident deuterons and the target nucleus, the other describing out-going protons and the residual nucleus, with the captured neutron in a well-defined orbit. In this calculation the integration is carried out over all space except
that for which \( n \leq N \). Recently Butler has proposed a theory of direct reactions which includes stripping as a special type of a more general class of reactions. Direct reactions are those in which an incident particle interacts strongly with one of the nucleons at the surface of the nucleus, exchanging momentum and energy with it. Butler's theory then includes stripping, pick-up, inelastic scattering, \((p,n)\) and \((p,\alpha)\) reactions. It is shown that the angular distributions of the emitted particles are similar to those of the stripping reactions. This theory is outlined in the next section.
THEORY OF DIRECT REACTIONS

According to Butler, nuclear reactions can be described as occurring in two ways. The first process is the formation and subsequent decay of the compound nucleus. In such a reaction the incident particle, which we take to be a proton, enters the nucleus where it quickly exchanges energy and momentum with the other members of this compound nucleus. The compound nucleus then remains in this highly excited state until one of the nucleons is given enough energy to escape from the nucleus, thus leaving the residual nucleus of the reaction. Because of the relatively long existence of the compound nucleus, it retains no "memory" of how it was formed, so its decay is determined solely by the properties of the compound system, i.e., spin, parity, and energy. If only one level of the compound nucleus is excited, the angular distribution of the emitted protons will be symmetric about a plane perpendicular to the beam direction. This symmetry exists because the incident proton can carry into the nucleus only angular momentum perpendicular to the beam direction. If more than one level of the compound nucleus is involved, there will be interference terms from levels of different parity, leading to asymmetry about 90 degrees. If, however, a large number of levels are excited, these interference terms are expected to cancel out because of their random phases, thus yielding again, an angular distribution which is symmetric about 90 degrees.
The second process is the direct reaction, in which the incident proton interacts with one of the nucleons at the surface of the target nucleus, and the emitted particle immediately leaves the scene of the collision. We shall consider the case in which the emitted particle is the incident proton. It is shown below that the angular distribution of the emitted proton depends on the angular momentum given to the residual nucleus during the reaction, and in general, this angular distribution is not symmetric with respect to the plane perpendicular to the beam direction.

We make the assumption that the first approximation to the wave function describing any nuclear reaction is one which describes the formation and decay of the compound nucleus. At great distances from the nucleus this wave function $\Psi$ consists of a plane wave of incident protons and an out-going spherical wave of all the particles that can be emitted in the reaction. Close to the nucleus, the proton wave is distorted by the forces acting on it. Because of the short range of nuclear forces (we shall neglect the coulomb force here) we may say that the proton wave is slightly distorted near the nucleus, while inside the nucleus, the wave function is a complicated function of all the nucleons involved. We now make the assumption that the energy of the incident proton is sufficient to excite many levels of the residual nucleus. In this case the compound nucleus will most often decay to one of the highly excited levels of the residual nucleus, since the density
of such levels is large. Thus the function $\psi$ describes a process in which the first few excited states of the residual nucleus are little represented. On the other hand, the direct reaction, which is essentially a collision between two nucleons, is most likely to involve small interchange of energy, and thus leave the residual nucleus in one of its lower states. The description of these reactions may then be thought of as a perturbation on the original wave function $\psi$, and the angular distribution of the associated protons may be calculated from perturbation theory. It is important to remember that the interior of the nucleus may be eliminated from the calculation since, if the proton wave is strong inside the nucleus, it leads to formation of the compound nucleus which does not decay to the low lying states in which we are interested.

We use the Born approximation to compute the angular distribution $f(\theta)$, which is given by the expression:

$$
\begin{align*}
    f(\theta) &= A \left| \int (K^i_r, K^f_r) \right|^2 \\
    &= A \left| \int \phi^*_f \nabla \phi_i \right|^2 \\
    &= (1.1)
\end{align*}
$$

where $\phi_f$ is the final state, $\nabla$ is the appropriate interaction potential, and $\phi_i$ is the initial state. The integration extends over all configuration space except for $R < R_e$. The initial state is taken to be $\psi_0(\xi) = k_r, \vec{r}_p$, where $k_p$ is the propagation vector of the incident proton whose coordinate is $\vec{r}_p$, and $\psi_0(\xi)$ is the wave function describing the target nucleus, the coordinates of which are lumped in the symbol $\xi$. Similarly, the final
state shall be taken as \( U^e(\xi, \lambda) \), where \( U^e(\xi) \) describes the nucleus in the excited state \( \xi \) and \( \lambda_p \) is the propagation vector for a proton traveling in the direction \( (\phi, \phi') \). For simplicity we shall ignore the spins of the particles involved (if the proton does not flip its spin this will make no essential difference) and take the potential \( V \) to be \( V_o \delta(\lambda_p - \lambda_n) \), where \( V_o \) is constant and \( \delta(\lambda_p - \lambda_n) \) is the Dirac delta function. The matrix element thus has the form

\[
I = \int U^e(\xi, \lambda) e^{-i \vec{k}_p \cdot \vec{\lambda}_p} V_o \delta(\vec{\lambda}_p - \vec{\lambda}_n) U_o \mu e^{i \vec{k}_p \cdot \vec{\lambda}_n} d\gamma.
\]  

(1.2)

We now make the expansions

\[
U^e(\xi, \lambda_n) = \sum_j V_j(\xi) F_j(\lambda_n)
\]

(1.3)

\[
U_o(\xi, \lambda_n) = \sum_k W_k(\xi) G_k(\lambda_n),
\]

where \( \xi \) represents the coordinates of all the nucleons in the nucleus resulting from the extraction of the \( n \)th neutron. We can now integrate over the coordinates \( \xi \), leaving the matrix element in the form

\[
I = \sum_s \int F^*_s(\lambda_n) G_s(\lambda_n) V_o \delta(\lambda_n - \lambda_p) e^{i (\vec{k}_p - \vec{k}_p') \cdot \vec{\lambda}_n} d\lambda_n \cdot d\lambda_p.
\]  

(1.4)
Integrating over $\Omega_f$, we obtain

$$I = \sum_s \int F_s^*(\vec{r}_n') G_s(\vec{r}_n) V_0 \ e^{i \vec{Q} \cdot \vec{r}_n} d\Omega_n,$$  \hspace{1cm} (1.5)

where $\vec{Q} = \vec{k}_f - \vec{k}_f'$. We now consider some one term in the sum over $s$ and therefore drop subscripts. We expand the functions $F$ and $G$ in terms of spherical harmonics. Thus

$$F(\vec{r}_n) = \sum_{l,m} f(l_n, l, m) Y_{l,m}(\theta_n, \phi_n)$$

$$G_l (\vec{r}_n) = \sum_{l', m'} g(l_n, l', m') Y_{l', m'}(\theta_n, \phi_n),$$  \hspace{1cm} (1.6)

where $Y_{l,m}$ is the spherical harmonic and $f$ and $g$ are the associated radial functions. Upon substitution of (1.6) into (1.5) the matrix element takes the form

$$I = \int \sum_{l,m} f \ g \ Y_{l,m}(\theta, \phi) Y_{l', m'}(\theta_n, \phi_n) \ e^{i \vec{Q} \cdot \vec{r}_n} d\Omega_n.$$

Each of the products of spherical harmonics may again be expanded in a sum of spherical harmonics, so the matrix element becomes

$$I = \int \sum_{l,m} f \ g \ Y_{l,m}(\theta, \phi) \ e^{i \vec{Q} \cdot \vec{r}_n} d\Omega_n,$$  \hspace{1cm} (1.8)
Similarly we may expand the exponential in terms of Legendre polynomials which are just spherical harmonics for which $m = 0$.

Thus

$$i \mathbf{Q} \cdot \mathbf{r} = \sum_{\ell} A_{\ell} j_{\ell}(\mathbf{Q} \cdot \mathbf{r}) \gamma_{\ell}^0(\theta, \phi),$$

(1.9)

where $j_{\ell}(\mathbf{Q} \cdot \mathbf{r})$ is the spherical Bessel function of order $\ell$. In this expansion we have chosen the $z$ axis to lie along the vector $\mathbf{Q}$, which is in the direction of momentum transfer. We now integrate over $\theta$ and $\phi$, making use of the orthogonality properties of the $\gamma_{\ell}^m$. The matrix element thus takes the form

$$I = \sum_{\ell} J_{\ell}^2 \int dr \, A_{\ell} j_{\ell}(\mathbf{Q} \cdot \mathbf{r}) \gamma_{\ell}^0(\theta, \phi) \cdot \gamma_{\ell}^0(\theta, \phi).$$

(1.10)

From the expansions (1.6) we see that the functions $f_{\ell}$ and $g_{\ell}$ are the radial wave functions for a particle bound in a potential well, and for $\gamma_{\ell}^m$, they must be the exponentially decaying solutions of this problem. Thus we may write

$$f_{\ell} (\mathbf{r}, l, m) = C_{\ell} h_{\ell}(i \beta \mathbf{r}),$$

$$g_{\ell} (\mathbf{r}, l', m') = D_{\ell} h_{\ell}(i \beta \mathbf{r}),$$

(1.11)

where the $h_{\ell}(i \beta \mathbf{r})$ are the spherical Hankel functions of argument $i \beta \mathbf{r}$ and the $\beta$ are related to the corresponding binding energies by

$$\frac{\beta^2 \hbar^2}{2m} = E.$$ 

(1.12)
The matrix element may now be written as a sum of integrals of

\[ \sum_{n=1}^{\infty} a_n^2 h_i(x) h_o(x') j_l(Q n) \Delta n. \]  

The only term that involves the angle of scattering is the \( j_l(Q n) \) in which the scattering angle enters in \( Q \), the magnitude of the momentum transfer. We shall now determine the limits on \( L \) in the sum.

In the original matrix element the term

\[ U^*(J_F) U(J_i) \]  

occurred, where \( U \) is the nuclear wave function with total angular momentum \( J \). We can expand this product in terms of angular momentum eigenfunctions of the nucleus, obtaining

\[ U^*(J_F) U(J_i) = \sum_{J} A(J) U(J), \]  

where

\[ A(J) = \sum_{J=|J_i-J_F|}^{J} \]  

We have chosen to expand this product in the form

\[ U^*(J_F) U(J_i) = \sum_{J} V_s(J_J) \sum_{R} \sum_{L} Y_{m}^{mm} \sum_{r} V_{r}^{*}(J_J) Y_{r}^{*}. \]  

or

\[ U^*(J_F) U(J_i) = \sum_{J} V_s(J_J) V_{r}^{*}(J_J) \sum_{L} \]  

(1.16)
The double sum over $S$ and $R$ may again be expressed as a sum of angular momentum eigenfunctions. Consider the term of that sum for which $J$, the total angular momentum, is zero. Then, since the sum in (1.17) must correspond to the sum in (1.15) the values of $L$ are limited to

$$J_i + J_F \geq L \geq |J_i - J_F|.$$  \hspace{1cm} (1.18)

For any other term of that sum, the value of $L$ will be limited to a smaller range included in (1.18). Therefore the overall allowed values of $L$ are given by (1.18). This allows us to make the interpretation that $L$ is the angular momentum carried into the nucleus during the collision. The angular distribution will then be given by a sum of terms of the form (1.13) subject to the selection rules (1.18). In most cases the selection rules will limit the allowable values of $L$ to a small number.

In order to understand how the integral (1.13) depends on $\varphi$, consider that term for which $J_i = 0$ and $J_F = L$. This corresponds to a shell model picture of the nucleus in which the uncoupled particle has zero angular momentum in the initial nucleus and angular momentum $L$ in the final nucleus. The integral (1.13) then becomes

$$\int_{J_i}^{\infty} h_0(i\rho) h_L(i\rho') \hat{p}_L(q') d\rho', \hspace{1cm} (1.19)$$
The Hankel functions have the form
\[ h_l(i\rho \lambda) = e^{-\rho \lambda / \rho} \]
\[ h_l(i\rho' \lambda) = e^{-\rho' \lambda / \rho'} \left[ \frac{\Lambda}{\rho + \rho'} + \frac{B}{(\rho + \rho')^2} + \cdots + N_{L-1} \right]. \]  
(1.20)

If \( \rho' \lambda \) is large enough so that only the first term contributes significantly, we may use the approximation
\[ h_o(i\rho \lambda) h_l(i\rho' \lambda) \approx \frac{\rho + \rho'}{\rho' \lambda} h_l(i\rho + \rho' \lambda). \]  
(1.21)

Since the \( \lambda \) varies slowly in comparison to the other functions in the integrand, we replace it by its value at \( \lambda \), so the matrix element becomes (neglecting multiplicative constants)
\[ I = \int_{\lambda_0}^{\infty} \frac{1}{\lambda} h_l(i\rho + \rho' \lambda) \int_{\lambda}^{\lambda'} \mathcal{A} \ d\lambda. \]  
(1.22)

The wave equations satisfied by \( h_l \) and \( j_l \) are
\[ \frac{1}{\lambda} \frac{d}{d\lambda} \left( \lambda^2 \frac{d h_l}{d\lambda} \right) + \left[ \mathcal{Q}^2 - \frac{l(l+1)}{\lambda^2} \right] h_l = 0 \]  
(1.23)

and
\[ \frac{1}{\lambda} \frac{d}{d\lambda} \left( \lambda^2 \frac{d j_l}{d\lambda} \right) + \left[ \mathcal{Q}^2 - \frac{l(l+1)}{\lambda^2} \right] j_l = 0. \]  
(1.24)
Multiplying (1.32) by \( j_L(\alpha \lambda) \) and (1.24) by \( h_L(i\beta) \) and subtracting (1.23) from (1.24) we obtain

\[
\mathcal{N}^2 (Q^i - g^i) h_L j_L = \mathcal{N}^2 \left( \frac{\partial^2}{\partial \lambda^2} \right) j_L - \mathcal{N}^2 \left( \frac{\partial^2}{\partial \lambda^2} \right) h_L, \tag{1.25}
\]

or

\[
\mathcal{N}^2 (Q^i - g^i) h_L j_L = \mathcal{N}^2 \left( \frac{\partial^2}{\partial \lambda^2} \right) j_L - \mathcal{N}^2 \left( \frac{\partial^2}{\partial \lambda^2} \right) h_L + 2 \lambda \left[ h_L \frac{\partial^2}{\partial \lambda^2} j_L \right]. \tag{1.26}
\]

Now, the Wronskian of \( j_L \) and \( h_L \) is defined to be

\[
W(j_L, h_L) = j_L \frac{\partial h_L}{\partial \lambda} - h_L \frac{\partial j_L}{\partial \lambda}, \tag{1.27}
\]

so

\[
\frac{\partial W}{\partial \lambda} = j_L \frac{\partial^2 h_L}{\partial \lambda^2} - h_L \frac{\partial^2 j_L}{\partial \lambda^2}. \tag{1.28}
\]

Thus we see that (1.26) becomes

\[
\mathcal{N}^2 (Q^i - g^i) h_L j_L = -\mathcal{N}^2 \frac{\partial W}{\partial \lambda} - 2 \lambda W = -\frac{\partial}{\partial \lambda} (\mathcal{N}^2 W), \tag{1.29}
\]

and the matrix element may be written

\[
\mathcal{I} (\mathcal{K}_p, \mathcal{K}_{p'}) = W \left[ \left( \frac{\partial}{\partial \lambda} (\mathcal{N}^2) \right), h_L \left( \frac{\partial^2}{\partial \lambda^2} \right) \right]_{\lambda = \lambda_0}. \tag{1.30}
\]
The angular distribution for this direct reaction is proportional to

\[ f(\phi) \sim \mathcal{W} \left[ J_L, h_L \right]_{\lambda = \lambda'} \]  

(1.31)

In the general case, the integrals (1.13) must be calculated numerically, assuming that one is using some model of the nucleus which permits assignment of values for \( \lambda \) and \( \lambda' \).

In the derivation above, it was assumed that the incident proton did not enter far into the nucleus before suffering a collision. This gave rise to a well-defined value of \( \lambda_e \) which appears in the final expression for the angular distribution. In the event that such penetration does occur, one would expect any sharp peaks to be smeared out, thus making it impossible to tell by the angular distributions whether a direct reaction had occurred. However, there is still a fundamental difference between the direct reaction and the compound nucleus reaction such that the two can be distinguished experimentally.

The compound nucleus is produced by adding to the target nucleus a proton with momentum in the beam direction. This direction then acts as a symmetry axis for emission of the proton and the later gamma ray. Since the proton carries into the nucleus no angular momentum along this axis, there is no preferred direction on it, and therefore the gamma radiation will be symmetric with respect to the plane perpendicular to the beam direction.

The direct reaction transfers momentum to the nucleus in the recoil direction. Angular momentum transferred to the nucleus
is therefore in a plane perpendicular to this direction. This can also be seen in the derivation of the angular distribution where the term $\gamma_L^M$ with $M=0$ was picked out to describe the excited neutron. Thus, with the recoil direction as an axis of quantization, we may say that the excited nucleus has gained in the $J$ quantum number, but not in the $M$ number. In the special case that $J_i = 0$ and $J_f = 2$ as in $^{24}\text{Mg}$, we may say that the excited nucleus is in the state $J = 2$ and $M = 0$, where $J$ is the total angular momentum and $\ell$ is the angular momentum carried in by the proton. Thus the $^{24}\text{Mg}$ nucleus, after a direct reaction leaving it in its first excited state will emit gamma radiation in coincidence with the protons leaving that state such that the distribution of gamma radiation, $Z_{J\ell\ell'}$, is given by

$$Z_{J\ell\ell'} = \frac{1}{2} \left( 1 - \frac{\ell(\ell+1)}{\ell L + 1} \right) \left| \gamma_L^{\ell+1} \right|^2 + \frac{1}{2} \left( 1 - \frac{\ell(\ell-1)}{\ell L + 1} \right) \left| \gamma_L^{\ell-1} \right|^2$$

(1.32)

In the case at hand $\ell = 2$ and $m = 0$. Thus

$$Z_{J\ell\ell'} = \frac{1}{2} \left| \gamma_L^2 \right|^2 + \frac{1}{2} \left| \gamma_L^{-2} \right|^2.$$  

(1.33)

In these relations the $\gamma_L^{\pm m}$ are the usual spherical harmonics and, in particular,

$$\gamma_L^{\pm 1} = \left[ \frac{\pm 1}{2\sqrt{2\pi}} \right] \sqrt{2} \sin \theta \cos \theta \gamma_L^0 = \gamma_L^{-1}.$$  

(1.34)

The spatial distribution will be given by the square of the
expression (1.34) which becomes, upon using the usual trigonometric identities

\[ \mathcal{Z}_{l,0} = \frac{4 \sin^2 \theta}{\sin \phi} \]  

(1.35)

where it should be noted that \( \phi \) is now measured from the direction of nucleus recoil which is now the axis of quantization.

From the remarks made at the beginning of this section, one would expect direct reactions to be most readily observed when the energy of the incident beam is high enough to excite many levels of the residual nucleus. It is of interest to inquire to what extent these reactions occur at low energies where there is competition between direct and compound nucleus reactions. In an attempt to partially answer this question, the experiments described in the following pages have been performed.
MEASUREMENT OF ANGULAR DISTRIBUTIONS

In order to determine experimentally the angular distribution to be compared with \( f(\phi) \), one must be able to answer the question, "For a given number of incident protons, how many with energy \( E \) are scattered into the element of solid angle \( d\alpha \) making an angle \( \theta \) with the original beam?" For this, one needs a beam of protons, reasonable well-defined both in energy and direction, a properly situated target, a means of detecting and energy analysing the protons scattered into \( d\alpha \) at \( \theta \), and a method of determining when a given number of protons has struck the target. This section describes the overall scheme for satisfying the above requirements.

The source of protons is the Ohio State University cyclotron which produces a 6.2 Mev proton beam. This beam is divergent upon leaving the cyclotron. In order to increase the intensity of the beam at large distances from the cyclotron, a pair of quadrupole focussing magnets are placed around the evacuated tube carrying the beam. The beam then passes through a 15 degree sector type deflecting magnet which focusses the beam in the horizontal plane such that the focal point is at the target. This magnet deflects protons of different energies through different angles. Thus, if one places a wall with a narrow slit just in front of the target, only those protons within a narrow range of energy will be allowed to strike the target, this energy range being determined by the slit width and the magnet parameters. The direction of the beam is then determined geometrically by this
slit and another slit or aperture placed just after the deflecting magnet.

The target is positioned in the geometrical center of the scattering chamber by a combination mechanical and optical system described in the next section. The target is made from ordinary magnesium ribbon available from any chemical house. The main requirement on the target is that it be so thin that a proton which traverses the entire thickness of the target, reacts with a nucleus at the far surface, and traverses the entire thickness again by back scattering, will lose a small amount of energy in comparison with the energy resolution of the detecting apparatus. The preparation of such targets is described in the next section.

The proton detector is a Cesium Iodide crystal which is free to move on a circle in the plane of the beam. The target is at the center of this circle. The crystal converts a fraction of the energy lost by the proton in the crystal into visible light. Over the range of energies used, this fraction is constant. If the crystal is thick enough to completely stop the protons, the intensity of the resulting light pulses will be proportional to the energy of the scattered protons. In the case of Mg$^{24}$, there are two groups of protons observable with the energy of the beam available; those which are elastically scattered and therefore have the original energy of the beam, (neglecting the energy of recoil of the nucleus) and those which leave the nucleus in its first excited state at 1.37 Mev, which have a correspondingly lower energy. The light pulses from the crystal travel through
a lucite light pipe to a photomultiplier tube which converts them into electrical pulses whose heights are proportional to the intensity of the light pulses. At this point the energy spectrum of the scattered protons is converted into a spectrum of electrical pulses. These pulses are then amplified by a linear amplifier to a height of about 60 volts and then introduced into a pulse height discriminator. The discriminator produces an output pulse of constant amplitude whenever a pulse is put into it, providing that the input pulse is greater than some value $V$ and less than some value $V+w$, where $w$ is the window width. The output pulses are then introduced into a scaler which records the total number of output pulses in some time $T$, $T$ being determined by the incident beam intensity. If the incident proton beam were constant in intensity, one could record the reading of the scaler for some time $t$ and some setting of $V$, repeat this for a series of values $V$ and thus obtain an energy spectrum of protons striking the crystal at some angle $\theta$. One would then see two groups of pulses sized, corresponding to the two proton groups of $\text{Mg}^{2+}$ mentioned above. The total number of pulses in each group would then be equal to the total number of protons scattered at that angle corresponding to that energy. By repeating this observation for some set of angles $\theta$, one obtains the angular distribution desired.

In practice, one can not obtain a proton beam of constant intensity, and therefore the practice of counting for a set time would lead to incorrect and inconsistent results. Further,
in general, the target will not be uniform, and as the beam wanders over the allowed region of the target, it will sometimes strike areas of greater thickness and at other times less thickness. This will be equivalent to allowing the total number of target nuclei to vary with time, so that two successive measurements will, in general, not agree. This difficulty is over come by placing another proton detector at some fixed angle with respect to the beam direction, and shutting off the moveable detector when the fixed detector has recorded a predetermined number of scattered protons. For a given value of \( N_pN_n \), where \( N_p \) is the number of incident protons and \( N_n \) is the number of nuclei in the target, a certain number of protons will be scattered into the fixed detector. Conversely, when the fixed detector has received a certain number of protons, the product \( N_pN_n \) is determined. This satisfies the last of the necessary conditions stated at the beginning of this section. A detailed description of the apparatus used will be found in the next section.
A schematic diagram of the scattering system is shown in Figure 1. Protons from the cyclotron enter a brass pipe of 2 inch inside diameter and pass through the fields of the quadrupole focussing magnets. The first magnet is placed 25 1/2 inches from the cyclotron vacuum tank and the second is 8 1/2 inches further. The beam then passes through the field of the deflecting magnet which is 62 inches from the last focusing magnet. The deflected beam is then focused at the center of the scattering chamber, which is 84 inches from the deflecting magnet.

The deflecting magnet serves to both focus a parallel beam of protons and to provide a means of energy discrimination of the incident beam. Consider the schematic drawing of Figure 2. The pole face of the magnet is a sector of a circle of radius $R$ whose center is at the point $A$. A proton entering the field along the path (a) moves on a circular path whose center is at $A$, and is therefore deflected through an angle $\alpha$. A proton entering along the path (b) will also move on a circular path of radius $R$, but whose center is now at point $B$. From the diagram, one sees that it will be deflected through the angle $\beta$. We have the relations

$$R \sin \phi = l \sin \alpha$$

$$\beta = \alpha + \phi.$$  

(2.1)
For small angles we may replace \( \sin x \) by \( x \) and then the relations (2.1) yield

\[
\beta = \alpha + \frac{\ell}{R} \alpha. \tag{2.2}
\]

We see that the additional deflection is proportional to the displacement of the incident path from the center of the magnet. This is just the condition that gives focussing at point \( C \).

Consider now the diagram of Figure 3. As before, a proton entering the center of the magnet with energy \( E \) will be deflected through the angle \( \alpha \). A proton with an energy which differs from \( E \) by \( dE \) will move on a circle with center \( dR \) from \( A \). As before, we may write

\[
\beta = \frac{dR}{R} \alpha + \alpha. \tag{2.3}
\]

The shift in focal point, \( dD \), is then given by

\[
dD = C_1 \frac{dR}{R} \alpha. \tag{2.4}
\]

Now, the radius of curvature, \( R \), is proportional to the square root of the energy, \( E \), so we may write

\[
\frac{dR}{R} = \frac{k^{\frac{1}{2}}}{k E^{\frac{1}{2}}} \frac{dE}{E} = \frac{dE}{2 E}. \tag{2.5}
\]
From (2.5) and (2.4) we have

\[ \frac{\delta E}{\delta d} = \frac{2}{G \alpha} \frac{E}{c} \]  \hspace{1cm} (2.6)

Using the values \( E = 6 \text{ Mev}, \ G = 84 \text{ inches} \) and \( \alpha = 15 \text{ degrees} \), we obtain the value of 0.55 Mev per inch.

This magnet requires a current of approximately 15 amperes, which is provided by a generator whose field current is regulated so as to maintain a constant magnetic field. The magnet current passes through a low resistance, thus providing a reference voltage which is proportional to the magnet current. This voltage is bucked against a battery through a galvanometer. A beam of light is reflected from the mirror of the galvanometer onto a phototube which controls the power supply for the generator field. Thus any change in magnet current is fed back to the generator in such a way as to cancel itself. The effectiveness of the regulation is displayed by another galvanometer which measures the voltage drop across a second series resistor. This system indicates that the average current is held constant to approximately 1 part in 500.

The scattering chamber is shown schematically in Figure 4. The wall of the chamber is made of a piece of pipe of inside diameter 10 inches and height 6 inches. The wall thickness is 1/8 inch. A brass flange 1/2 inches thick is soldered to the pipe to form the bottom of the chamber. Through the center of this bottom flange passes a 1/2 inch diameter brass rod which supports the
DIAGRAM OF SCATTERING SYSTEM

FIGURE 1
FIGURE 2

BEAM FOCUSING
FIGURE 3

BEAM DEFLECTION
FIGURE 4
target. An O-ring in the flange provides the vacuum seal. The target may be rotated through 360 degrees. To the top of the pipe is soldered another flange which is provided with two grooves for ball bearings and an O-ring. The lid of the chamber rests on the ball bearings which are at the proper height to provide enough compression of the O-ring for a vacuum seal, but not so much as to prevent rotation of the lid by hand when it is under vacuum. Through the lid passes a lucite pipe of diameter 0.870 inches. A cesium iodide crystal is cemented to the lucite with paraffin. The back of the lucite rod has been cut away at an angle of 45 degrees so that light from the crystal will be reflected up the rod. This flat surface has been covered with a layer of aluminum by evaporation so as to increase the reflecting properties.

The crystal was cut with a razor blade until its thickness was approximately 0.017 inches and its face was a square whose sides measure approximately 3/16 inch. It is desirable that the crystal have as small a volume as possible in order to decrease its total cross section for gamma rays and neutrons. With this size crystal the background from the cyclotron produced no noticeable effect.

Surrounding the lucite rod is a collimator made of copper tubing. It is held in position by a flange screwed to the underside of the lid. (this flange is not shown in Figure 4) The degree of collimation of the scattered protons is determined by
a hole drilled in a lucite rod inserted in the horizontal tube of the collimator. Just in front of the crystal this hole has diameter 0.120 inches, and at a distance of 1 11/16 inches, the diameter is 9/32 inches. This allows the crystal to intercept only those protons which have passes through a circle of about 1/2 inch diameter at the center of the chamber.

The chamber has four ports of 3 inch diameter spaced at 90 degrees. The proton beam enters through the collimating system placed in one of these ports (shown on the right in Figure 4) and after passing through the target impinges upon a faraday cup which is lined with tungsten. Since tungsten has a high atomic number, the protons can not overcome the coulomb barrier and interact with the nucleus. The use of such high atomic number materials where the beam strikes prevents production of troublesome gamma radiation. The faraday cup is bolted to an insulating stand and the beam current is read on a microammeter connected to the faraday cup through a Stupakoff seal.

The third port is connected through a right angle pipe of 3 inch diameter to a Consolidated Vacuum Corp. type MC-275 diffusion pump backed by a Cenco Megavac mechanical pump. Between the pump and the chamber is a water cooled baffle to prevent oil vapors from the pump from entering the system. The vacuum system is capable of producing a pressure of $10^{-5}$ mm of mercury in the scattering chamber. It has been found that the system works equally well when the diffusion pump is not used, and therefore,
in order to minimize oil contamination of the target, only the mechanical pump is turned on during a run.

The fourth port is covered with a flange which is equipped with a 2 inch diameter hole through which passes an RCA 5819 photomultiplier tube. The hole contains an O-ring which fits snugly around the phototube so as to provide vacuum seal. A cesium iodide crystal is cemented to the face of the tube with paraffin, thus forming the elements of the fixed detector. The crystal is about 0.020 inches thick and its face is a square of sides approximately 3/8 inch. The size has been chosen to give a convenient counting rate. A mu metal shield fits snugly into the port and thus shields the tube from stray magnetic fields. An aluminum disk is bolted to the front of the shield. This disk has a 5/8 inch hole in the center, and into this hole has been pressed a copper tube which allows the crystal to see only the center of the chamber and therefore only the protons scattered from the target.

The proton beam passes through a 1/4 inch long brass pipe of 1/2 inch diameter. The beam direction is defined by a circular aperture in this tube and the width of the target itself. The aperture is 0.200 inches in diameter and is placed 15 inches from the target, which has width 0.050 inches. The half angle spread in beam direction is then calculated to be 0.48 degrees. This collimating system is shown in Figure 5. The aperture is cut from 0.010 thick tantalum metal sheet which has been spot welded to a steel insert which fits snugly into the brass tube.
FIGURE 5

BEAM COLLIMATOR TUBE
The inside of the tube has been lined with lead sheet to prevent production of gamma radiation. Of particular interest is the anti-scattering system inserted into the tube just in front of the target. Since protons are scattered from any surface which is struck by the beam, it is necessary to prevent the moveable detector from seeing the collimating system. This is accomplished by the steel insert placed in the front of the tube. The last place from which protons can be scattered is the edge of the tantalum sheet shown in the diagram of Figure 5. Protons scattered from this edge can not be found behind the dotted line because of the 1/2 inch long wall on the other side of the tube. If the moveable detector is kept behind this line, it can receive protons only from the target. The width of the aperture between the tantalum edge and the protecting wall is approximately 0.100 inches. This allows the moveable detector to be brought to an angle of 13 degrees with respect to the incident beam before it can see the beam collimator.

The target is made from a piece of magnesium ribbon which originally is 0.005 inches thick and 1/8 inches wide. It is first reduced in thickness by scraping it with a razor blade while it is lying upon a glass plate. When it is about 0.001 inches thick it is waved gently in a weak solution of sulfuric acid until it is observed to bend with the waving motion. It is then rinsed in distilled water and allowed to dry. This method produces a target of thickness about 0.0003 inches. After drying, it is cut with ordinary scissors and a steady hand
to a width of about 0.05 inches. The target should be as narrow as possible since motion of the beam across the target results in a change in the solid angle intercepted by the detector. The solid angle $\Omega$ is given by

$$\Omega = \frac{A}{R^2},$$

where $A$ is the area of the detector and $R$ is the distance from the target. By differentiation we have

$$\frac{\partial \Omega}{\partial R} = 2 \frac{\partial R}{R}.$$

For a target with a width of 0.05 inches and distance $R$ of 3.75 inches this results in an uncertainty of 2.6 percent.

It is also necessary that the target be accurately positioned on the axis of rotation of the lid so that the solid angle intercepted by the detector is independent of its position. When the lid was machined, the center was marked, and this mark forms the main reference point. The target must be mounted on a line which goes through this mark and is perpendicular to the underside surface of the lid. Such a line is obtained by placing an accurate right angle ruler on the lid when it is lying upside down, and causing one edge of the ruler to touch the center mark. A small rod is screwed into the lid about 3/4 inch from the center mark. This rod is equipped with an adjustable pointer such that the tip of the pointer can be made to touch the edge of the ruler at the altitude of the detector crystal. When locked in place, the tip of the pointer is at the center of the chamber. When the lid is placed on the chamber, one can insure that the beam goes through the center of the chamber by sighting down the
beam collimator. By lining up the two beam collimator apertures, the pointer and the center of the detector crystal, one can calibrate the angle markings on the lid. Using this method, one can determine the angle of the detector with respect to the beam direction to within approximately 1/3 degrees. In the same way, one can center the target by raising the pointer so that it is just above the target and then sighting through two ports spaced at 90 degrees. Target alignment is accurate to within about 0.01 inches.

Both of the RCA 5819 phototubes are supplied with the correct voltages by a Haumer model N401 high voltage regulated power supply. This supply provides a potential of -800 volts to a resistive voltage dropping circuit which is attached to the tube. The signals from both the moveable and fixed detectors pass through long co-axial cables from the cyclotron room to an adjacent room containing the counting equipment. Each signal is passed through a linear amplifier and pulse height discriminator designed by Dr. P. Jastram of The Ohio State University. The output of each discriminator is then sent to an Atomic model 1040 scaler. The discriminator for the fixed detector is set such that it counts all pulses above a certain value, this value being such that both the elastic and inelastic peaks are counted. When the scaler has recorded 100,000 counts, it turns off both itself and the scaler for the moveable detector. The discriminator for the moveable detector is then set at a certain window width and a pulse height spectrum is then taken. A typical spectrum of Mg$^{24}$ is shown in Figure 6. The large peak on the right is
PULSE HEIGHT SPECTRUM
OF SCATTERED PROTONS

$\theta = 90^\circ$

**Figure 6**
caused by elastically scattered protons, and the smaller peak just to the left is caused by protons which leave the nucleus in the first excited state at 1.37 Mev. The resolution, which is defined as the width of the peak divided by the pulse height, is about 6 percent.

It is observed that there is a drift in the electronic system such that the position of these peaks oscillates during the time necessary to observe the spectrum. Since the cross section is proportional to the area under the peak, a small drift which occurs while the window is being moved over the peak will considerably distort the peak, causing a large uncertainty in the angular distribution. These peaks, however, have the gaussian form, so that any fraction of the area under the peak will be proportional to the cross section. This fraction is taken by opening the window to a width of about 7 volts, and moving the window over just the peak under observation. Usually four points are taken, and the highest value taken to be proportional to the cross section. This is repeated several times at each angle until the operator feels that the results are sufficiently consistent. Often, one can observe a drift taking place during the measurement and delete that particular run. After the value of the peak is obtained in this manner, the window is moved to a position just below the peak, and a measurement of the background is made. The difference between the peak value and the background is taken to be the quantity of interest. This is usually reproducible.
to within about 4 percent.

In the forward direction the elastic peak increases very much in size due to the large cross section for coulomb scattering in the forward direction. This results in high counting rates, which cause the background to become quite large. For this reason, it is felt that observations at angles less than 40 degrees are unreliable. In taking the angular distributions, the moveable detector is first set at 40 degrees and then moved by increments of 10 degrees until 160 degrees is reached. Some or all of the points between are then taken to insure that the angular distribution obtained is not, in reality, a time distribution.

The angular distribution for the inelastic peak was taken for four energies by inserting different thicknesses of aluminum foil into the path of the beam just before the final beam collimator. The variation of cross section was observed as a function of energy by replacing the fixed detector with a current integrator connected to the faraday cup and mounting the target on the collimating tube so that it hung in front of the beam. The front collimator was replaced with a simple circular aperture of diameter 0.080 inches and the target was widened to that all the beam had to pass through the target. A circular wall was then
attached to what was the target holder shaft, and in this wall were drilled eight holes, each covered with a different thickness of aluminum. By rotation of the shaft, the beam energy could be changed by increments corresponding to the energy loss in 0.01 mm of aluminum. During these measurements the detector angle was set at 70 degrees.

The results of these measurements are displayed and discussed in the next section.
RESULTS OF ANGULAR DISTRIBUTION MEASUREMENTS

Curves of the variation of cross section with energy for a detector angle of 70 degrees are shown in Figure 7. A foil thickness of $10^{-3}$ cm decreases the energy of the protons by about 150 keV. The curve for the elastic peak, shown as a solid line, increases with decreasing energy, as would be expected for pure coulomb scattering. The local peak at a thickness of about $2 \times 10^{-3}$ cm could indicate an increase in the cross section for compound nucleus formation, or more generally, interference between coulomb and compound nucleus scattering. The dotted curve for the inelastic peak has a similar local maximum (actually a flattening out) at the same foil thickness, and shows another peak at a foil thickness of $6 \times 10^{-3}$ cm. It was decided that the angular distributions of most interest would be those corresponding to the peaks and valleys of the yield curves of Figure 7. Accordingly, four distributions were taken, corresponding to foil thicknesses of 0, 2, 3.8, and 6 milicentimeters. These angular distributions are shown in Figures 8 through 11. The distribution expected from pure direct reaction is shown in Figure 12. The experimental data have been corrected as to correspond to the distribution in the center of mass. The direct reaction distribution is seen to have one peak centered at about 70 degrees. A similar peak appears in Figure 8 and Figure 9 where the energy degrading foils are of thicknesses 0 and $2 \times 10^{-3}$ cm respectively. In both cases these curves rise in the backward direction, whereas the theoretical curve shows a pronounced drop in the backward direction. As the
VARIATION OF CROSS SECTION WITH ENERGY

THICKNESS OF ABSORBING FOIL, CM $\times 10^3$

Figure 7
ANGULAR DISTRIBUTION OF PROTONS
SCATTERED BY Mg\textsuperscript{24} (Q = -1.37 MEV)
MAXIMUM BEAM ENERGY: 6.2 MEV

Figure 8
ANGULAR DISTRIBUTION OF PROTONS
SCATTERED BY Mg$^{24}$ (Q = -1.37 MEV)
BEAM ENERGY DEGRADED BY .02 mm AL FOIL

Figure 9
ANGULAR DISTRIBUTION OF PROTONS SCATTERED BY Mg$^{24}$ ($Q = -1.37$ MEV)
BEAM ENERGY DEGRADED BY .038 mm AL FOIL

Figure 10
ANGULAR DISTRIBUTION OF PROTONS SCATTERED BY Mg\textsuperscript{24} (Q = -1.37 MEV)

BEAM ENERGY DEGRADED BY .06 mm AL FOIL
THEORETICAL ANGULAR DISTRIBUTION FOR DIRECT REACTION

\( E = 6.2 \text{ MEV (LAB)} \)

\( r_o = 5.5 \times 10^{-13} \text{ CM} \)

**Figure 12**
foil thickness becomes $3.8 \times 10^{-3}$ cm, this peak disappears completely, the distribution showing only a steady rise in the backward direction. Finally, for a foil thickness of $6 \times 10^{-3}$ cm, we again have a peak which is now very broad and centered at about 110 degrees. From the rapid variation of the angular distribution with energy, it appears that no clear comparison between the Butler curve and the experimental curves can be made.

Measurements of angular distributions at higher energies have often supported the direct reaction theory. In particular, there is good agreement between the theory and experiment in the inelastic scattering of 31.5 Mev alpha particles by Mg, leading to the 1.37 Mev level. The value of $r_0$ chosen to fit these data was $5.5 \times 10^{-13}$ cm. On the other hand, the data at 18 Mev for inelastic scattering of protons from the same level does not agree with the theoretical curve nearly as well. However, the results of angular correlation measurements for this reaction agree very well with the theoretical $\sin^2(2\theta)$ curve. This has been interpreted as a penetration of the incident proton into the nucleus before undergoing the direct reaction. Greenlees, Haywood, Kuo and Petravic have presented the angular distributions for a series of energies around 9 Mev. These distributions were found to vary rapidly with energy as did the distributions in the present work. This rapid variation was assumed to be due to interference between compound nucleus formation and direct reactions. Using this assumption, we can attempt to make a rough estimate of the
relative strengths of direct reaction and compound nucleus reaction.

We write the angular dependence of the wave function for outgoing protons in the form

\[ f(\theta) + g(\theta), \quad (5.1) \]

where \( f(\theta) \) is the amplitude for direct reaction and \( g(\theta) \) is the amplitude for compound nucleus formation. The angular distribution is then given by the absolute value of the square of this expression,

\[ \sigma(\theta) = \left| f'(\theta) + g'(\theta) + 2 f(\theta) g(\theta) \right|^2. \quad (5.2) \]

Since at backward angles the expression \( f'(\theta) \) is small, the observed angular distributions in the backward angle must be just \( g'(\theta) \), and for some particular angle we will call this just \( g'. \) Consider the angle \( 180^\circ - \theta \). The value of the distribution function is given by \( (5.2) \) but because of the assumed symmetry of \( g(\theta) \) about 90 degrees we may replace \( f(\theta) \) by \( g - g' \). The value of the distribution function at this angle is some fraction \( A \) of the value for the corresponding backward angle. Thus we may write

\[ f^2 + g^2 + 2 fg = A g'. \quad (5.3) \]
Solving this equation for \( f \) we obtain

\[
    f = g \left[ -1 \pm \sqrt{A} \right].
\]  

(5.4)

Since \( f \) must become zero if the angular distribution is symmetric about 90 degrees \((A = 1)\), we choose the upper sign. Consider now the data from the distribution of Figure 9. Using the angles 140 degrees and 40 degrees, we obtain a value of \( A \) which is very close to unity. On the other hand, the distribution is obviously not symmetric about 90 degrees, indicating that the above analysis is in error.

The analysis above depends upon the smallness of the theoretical curve in the backward direction. This curve, however, was derived without taking into account the effects of coulomb forces and any distortion of the incident waves by nuclear forces. These effects would tend to smear out the peak and raise the curve for large angles. Thus, even if the correct curve were available, it would be impossible to carry out an analysis of this type. It appears that an experimental test of the presence of direct reactions at these energies must utilize the angular correlation technique.
At the end of the theory section was given the angular distribution of gamma rays emitted from a $^\text{Mg}^{24}$ nucleus which had been raised to its first excited state (1.37 MeV) by a direct reaction with a proton which transferred linear momentum $Q$ to the nucleus during the collision. This distribution was shown to have the form $\sin^2(2\theta)$, where $\theta$ is the angle between the vector $Q$ and the propagation vector $K$ of the gamma ray. The target will emit a gamma for every proton which is inelastically scattered, regardless of the direction of the scattered proton. We are interested only in those gammas associated with a proton scattered with propagation vector $K$. In order to make this distinction, the electronic apparatus is arranged so as to count only those gammas which are in time coincidence with a proton traveling with propagation vector $K$. The proton detector of the previous experiment serves again as the proton detector, and the moveable proton detector is replaced by a gamma detector. The angular distributions may now be measured exactly as in the previous experiment.

Any detected gamma which is not in time coincidence with a proton could not be of the type in which we are interested, and is, quite properly, ignored by the electronic apparatus. If the gamma is in time coincidence, it still may not be the gamma associated with the proton because the electronic apparatus has a finite time resolution which is of the order of $10^{-8}$ second. If the counting rate in the gamma counter is large enough, there
will be some accidental counts. The experiment must therefore be arranged in such a way that the ratio of true coincidences to accidental coincidences is large.

The detectors used in this experiment are capable of discriminating between protons and gammas of different energies, so we shall assume that the only protons and gammas present are those with the appropriate energies. Consider a target in which there are $N$ interesting collisions per second. An interesting collision is one in which the proton is scattered inelastically, leaving the nucleus in the 1.37 Mev excited state. The true coincidence counting rate is given by the expression

$$C_T = N \cdot \mathcal{A}_p \cdot \varepsilon_p \cdot \mathcal{A}_\gamma \cdot \varepsilon_\gamma,$$

where $\mathcal{A}$ is the fraction of the total solid angle intercepted by a detector, and $\varepsilon$ is the efficiency of that detector. The subscripts refer to either proton or gamma detector. The accidental coincidence rate is given by the product of the total time available for such a coincidence, and the counting rate in the other counter. Thus

$$C_A = \left[ N \cdot \mathcal{A}_p \cdot \varepsilon_p \right] \left[ N \cdot \mathcal{A}_\gamma \cdot \varepsilon_\gamma \right] \tau,$$

where $\tau$ is the resolving time of the apparatus. The figure of merit is the ratio of true to accidental coincidences, given by

$$C = \frac{1}{N \cdot \tau}.$$  

The value of $\tau$ in the circuit used is about $2 \times 10^{-3}$ seconds.
The value of $N$ can be estimated from the counting rates in the previous experiment. It was observed that the largest counting rate for protons in the moveable detector was about 1000 counts per second. From the dimensions of the crystal and its distance from the target, one can then calculate a value of $N$ which is $1.55 \times 10^7$ events per second. This gives a lowest estimate for the ratio of true to accidental counting rates of 3. Assuming that the minimum acceptable value of $C$ is 10, one may simply reduce the product of beam intensity and target volume by a factor of about three.

The circuitry and techniques of this measurement are given in the next section.
APPARATUS FOR COINCIDENCE MEASUREMENT

A block diagram of the system is shown in Figure 13. The fixed proton detector has been described previously. The gamma detector is a sodium iodide crystal whose diameter is one inch and whose length is two inches. The crystal is mounted on a lucite light pipe which passes through the lid of the scattering chamber described earlier. A photon passing through the crystal may lose energy by photoelectric absorption, Compton scattering and pair production. All of these processes give rise to fast electrons in the crystal which then gives off a light pulse proportional to the energy of these electrons. One may then perform a pulse height analysis of the signals from the attached photomultiplier tube to determine the energy spectrum of the gammas striking the crystal. Because of the three effects mentioned above, these spectra are much more complicated than a proton spectrum, and require careful analysis for interpretation of the data. The pulse height discriminator to which the gamma pulses pass was calibrated by placing a sample of Na\textsuperscript{22} near the chamber and observing the spectrum. Na\textsuperscript{22}, which decays by $\beta^+$ emission, gives off gamma rays of energy 1.28 Mev. The $\beta^+$, upon annihilation with an electron in the Na\textsuperscript{22} sample, produces two gammas of energy 0.51 Mev. The pulse height spectrum for Na\textsuperscript{22} is shown in Figure 14. The resolution is rather poor, but one can distinguish the photoelectric peak for the 1.28 Mev gamma at a pulse height of 57 volts, and the peak for the 0.51 Mev gamma at a pulse height of 20 volts. In the actual measurements of
Figure 13
PULSE HEIGHT SPECTRUM
OF Na$^{22}$

Figure 14
the angular distributions, the gamma counters were set to accept all pulses of height greater than 55 volts, which included the photoelectric peak for the 1.37 Mev gammas from the Mg target as well as part of their Compton distribution.

Since the volume of the NaI crystal is quite large, it was necessary to shield the chamber with lead to reduce the gamma flux from the cyclotron. Preliminary experiments indicated that the flux of neutrons in the cyclotron room produced as much background as the gammas. Slow neutrons are absorbed by the iodine nuclei to produce $^{128}$I, which decays by $\beta$ emission with a half life of 25 minutes. The crystal, which had been exposed to these neutrons, was found to be quite active even after it was removed from the cyclotron room. Measurements of the half life of the activity in the crystal yielded a value of about 25 minutes.

The chamber was therefore surrounded by a wall of lead of 4 inch thickness. The top and bottom were covered with a 2 inch thick layer of lead. This lead box was then surrounded by 2 inches of borax, since boron has a high capture cross section for slow neutrons. Finally, the borax was surrounded by a wall of paraffin of about 3 inches thickness to slow down any fast neutrons in the room. These efforts were successful in producing a tolerable background. The operation of the electronic system is as follows: Pulses from both the proton and gamma detectors are amplified by broad band amplifiers whose signals are introduced into a fast coincidence circuit. This circuit produces an output pulse if,
and only if it receives a signal from both detectors within a period of $2 \times 10^{-8}$ seconds. The signals from each detector also pass through a linear amplifier and pulse height analyser whose output signals are applied to the slow coincidence circuit. This circuit is such that it produces an output pulse if, and only if it receives a pulse from the fast coincidence circuit and both pulse height analysers within a period of about $10^{-6}$ seconds.

The output of the slow coincidence circuit is then applied to a scaler which records the coincidences. Thus, this scaler will record only those events which correspond to the simultaneous detection of a proton and gamma, both the appropriate energy.

The readings of the scaler are then plotted for each angle of the gamma detector, thus giving the desired angular distribution.

The determination of the fast coincidence or non-coincidence of signals from the two detectors is based upon the simple addition of two pulses of equal height. If they occur at different times, the overall signal consists of two separate pulses. If they occur at the same time, the signal is the sum, and therefore is twice as large as a single original pulse. This output signal is then used to trigger a one shot multivibrator which will fire only on the large pulse. The pulse lengths of the signals from the detectors are of the order of $10^{-6}$ seconds and are therefore too long to be used in the addition mentioned above. These pulses are converted into short uniform pulses in the following way: The pulse from a detector is introduced into a broad band
amplifier whose negative output is applied to the grid of a sharp cut-off pentode. If the amplification is sufficiently great, the signal will bias this pentode beyond cut-off very quickly, and hold it there for the duration of the signal. The voltage at the plate of this pentode will therefore have the form of a step function. Connected to the plate is a shorted length of coaxial cable. The step function generated at the plate travels down this cable, and is reflected back with opposite polarity and delayed by $6 \phi$, where a foo (denoted by $\phi$) is defined to be $10^{-9}$ seconds. The signal at the plate therefore consists of the sum of two step functions of opposite sign, with a delay of $6 \phi$. The result is a rectangular pulse of very short duration. In practice, the rise time of the step function limits the time resolution to about $20 \phi$. A block diagram of this circuit is shown in Figure 15. The sum of these short pulses appears at the common junction of the plates of the input tubes and passes through a diode which is biased by the voltage appearing across a potentiometer connected to its cathode. This potentiometer is adjusted so that the ratio of the coincidence pulse height to the single pulse height is a maximum. This signal then passes through an amplifier, and is applied to the grid of the univibrator.

For correct operation of the electronic system, it is necessary to properly adjust the bias on the discriminator diode in the fast coincidence circuit and the triggering level of the
univebrator. Furthur, in general, a simultaneous detection of a gamma and a proton will not lead to simultaneous pulses at the fast coincidence circuit because of differences in cable lengths, phototube parameters and crystal differences. It is therefore necessary to introduce a delay in one of the cables from the detectors such that the coincidence rate is maximized. Preliminary experiments utilizing a pulse generator yielded the results shown in Figure 16. A similar set of measurements using the annihilation gammas from Na$^{22}$ is shown in Figure 17. When the bias on the diode was set too low, none of the pulses were large enough to trigger the univibrator. As the bias was increased, a plateau was reached. At this point the double pulses from true coincidences would fire the univibrator, but not the single pulses. Finally, as the bias setting was increased, a point was reached where any pulse would fire the univibrator. The operating point is then chosen to be about in the middle of this plateau. The situation with the triggering level of the univibrator is obviously similar.

With the diode bias and triggering level set at the proper values, measurements were made of the coincidence rate with delay, using the actual signals from the proton and gamma detectors. These results are displayed in Figure 18. The half width of these curves indicates a resolution of about $2 \times 10^{-8}$ seconds. In the measurements of the angular distribution, the proton pulse height discriminator was set to straddle the inelastic peak, and the gamma
FAST COINCIDENCE CIRCUIT

Figure 15
COINCIDENCE DELAY CURVES FOR PULSER

DIODE = 40, DELAY = -8 μsec

COINC. DELAY CURVE
DIODE = 40, MULTI = 79

16.5 μsec

UNIVIBRATOR-BIAS CURVE
DIODE = 40, DELAY = -8 μsec

DIODE BIAS CURVE
MULTI = 79, DELAY = -8 μsec

COUNTS

10^4

10^3

10^2

10^1

10^0

COUNTS

10^4

10^3

10^2

10^1

10^0

DELAY, μsec (10^-9 sec)

20 30 40 50 60 70 80

DIODE BIAS, ARB. UNITS

Figure 16
COINCIDENCE DELAY CURVES FOR Na$^{22}$

COINC DELAY CURVE
DIODE = 40, MULTI = 79

DIODE BIAS CURVE
MULTI = 79, DELAY = -8 $\mu$s

DIODE BIAS, ARB. UNITS

COUNTS

10^{-6}  10^{-5}  10^{-4}  10^{-3}  10^{-2}  10^{-1}  10^0  10^1  10^2

DELAY, $\mu$s ($10^{-9}$ sec)

75  80

Figure 17
COINCIDENCE DELAY CURVE FOR Mg$^{24}$

Figure 18
discriminator was set to count all pulses above 55 volts, which included the photoelectric peak of the 1.37 Mev gammas from the target. The fixed proton counter was set to turn off all counting equipment when it had recorded $10^6$ pulses. A typical set of readings gives a corresponding time of 663 seconds and a total number of gamma counts of $3.18 \times 10^6$. The number of coincidences was 1326. Using a resolving time of $2 \times 10^{-8}$ seconds we obtain an accidental counting rate of 0.145 per second. The observed coincidence rate was 2 per second, thus yielding a ratio of true to accidental counting rate of 13.1.
RESULTS AND CONCLUSIONS

The data from the angular correlation measurement are shown in Figure 19. For each point, the accidental counts have been calculated and subtracted from the data, leaving only the true coincidence counts. On these points has been plotted the curve

\[ N = 8.6 + 6.0 \sin^2 (2\theta), \]

which agrees fairly well with the experimental points. In the expression above, \( \theta \) is the angle between the detector and the nuclear recoil direction, which is calculated to be 36 degrees. If it is assumed that the distribution of gammas from nuclei excited by the compound nucleus reaction is isotropic, one can estimate from the parameters of the curve above that the ratio of direct to compound nucleus reactions is about 0.7. In general, however, the distributions from the compound nucleus reactions are not isotropic, but just symmetric about 90 degrees, which implies at least the existence of direct reactions, if not a quantitative estimate of their strength.

We shall now consider in more detail the compound nucleus process. The compound nucleus formed by absorption of a proton by Mg\(^{24}\) (J=0) has some angular momentum \( J \) with projection \( m = 0 \) along the beam direction. This nucleus then decays by emission of a proton to a state in which \( J = 2 \) and \( m = 0, \pm 1 \) or \( \pm 2 \). From
ANGULAR CORRELATION OF GAMMAS
FROM THE REACTION $\text{Mg}^{24}(p, p')\text{Mg}^{24}$

**Figure 19**
equation (1.32) we have that the angular distribution of the gamma radiation will consist of the sum

\[ Z = A \sin^2 (2\theta) + B \left( 3 \cos^2 \theta \right) + C \sin^4 \theta. \]

All of these terms are symmetric about 90 degrees and each has a local peak at 90 degrees. Let us assume that we have the direct reaction peak at about 80 degrees and the local peak from compound nucleus at 90 degrees, both being of equal magnitude. The resulting curve would then have its peak at 85 degrees. The data, however, shows a peaking at about 80 degrees, so if the compound nucleus peak exists, it must be quite small. It appears then that the observed experimental peak must be mostly due to the direct reaction, and the original estimate based on isotropic background is valid.

We can now apply this to the observed angular distributions. Let us assume that the density of levels in the compound nucleus is such that each is well defined. The angular distribution of protons will be symmetric about 90 degrees. The observed distributions, however, are asymmetric, indicating that there is interference with the compound nucleus distribution and the direct reaction distribution. Assume now that the density is such that several levels overlap. The angular distribution will then be of the form
where the $f_i$ are the wave functions for out-going protons from the compound nucleus levels, and $g$ is the corresponding function for direct reaction protons. We have seen that $g$ is about as large as the contributions from all these levels, so it is likely that all the $f_i$ are small in comparison with $g$. Thus the interference term involving $g$ will be several times as large as that due to interference between the compound nucleus levels, and the main cause of asymmetry is again the direct reaction.

If the number of levels involved is large, the analysis is similar. It is assumed that the interference terms from the compound nucleus levels will cancel out because of their random phases, and one would expect also that the same would be true of the $g$ interference term. However, there are $\binom{n}{l}$ terms in the $f$ interference, thus allowing a large chance for canceling, while there are just $n$ terms in the $g$ interference sum. Further, the $g$ is much larger than any of the $f_i$, so that any interference is again due to the direct reaction. It appears then, that direct reactions make a significant contribution at these energies, and are the main cause of asymmetries in the angular distributions.
I, Henry Allyn Lackner, was born in Brooklyn, New York, on April 11, 1931. I received my secondary school education in the public school of Woodmere, New York, and my undergraduate training at Union College, Schenectady, New York. The Master of Science degree was awarded me by the University of Delaware, Newark, Delaware, in 1954. At the University of Delaware I held the positions of Teaching Assistant and Research Assistant. In October 1954, I was appointed Teaching Assistant in the Department of Physics and Astronomy at The Ohio State University. In June 1956 I was appointed Research Assistant under Dr. H.J. Hausman, supervisor of the Cyclotron Laboratory. I held that position for two years while completing the requirements for the Doctor of Philosophy degree.