A STUDY OF ROTARY
PNEUMATIC SERVOMECHANISMS

DISSERTATION

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ERNEST OTTO DOEBELIN, B. Sc., M. Sc.

The Ohio State University
1958

Approved by

[Signature]
Adviser
Department of Mechanical Engineering
The University assumes no responsibility for the accuracy or correctness of any of the statements or opinions expressed in this dissertation.
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STATEMENT OF THE PROBLEM

The object of the investigation was to develop theoretical and experimental methods for the analysis of the dynamic performance of pneumatic servomechanisms using rotary motors as the power element. The supply pressure for the system was taken to be the order of 100 psig. These systems are intended primarily as devices for positioning inertia and friction loads in response to electrical or mechanical commands.
SUMMARY OF RESULTS

Linearized analyses of vane-type motors and axial-piston type motors driven by conventional four-way closed-center servo valves were carried out. Open-loop and closed-loop transfer functions were obtained for feedback systems using electrical motion transducers, electronic amplifiers and electromagnetic torque motors.

Experiments were run on such a system using an axial-piston motor. These experiments were mainly step-function tests on the closed-loop system. The effects of size of step-function, amount of gain, added pneumatic damping and valve dither were investigated. In general, the performance was found to be better for large commands than for small, due to the presence of considerable coulomb friction. Velocity and acceleration saturation effects were noted for large signals. Increase in gain caused a decrease in stability, but this could be partially offset by added pneumatic damping which is easily introduced. Valve dither improved valve hysteresis but had little effect on overall system performance due to large coulomb friction in the motor.
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LIST OF SYMBOLS

\[ A_p = \text{area of piston, inch}^2 \]
\[ A_v = \text{effective area of motor vanes, inch}^2 \]
\[ B = \text{equivalent viscous friction coefficient of motor and load, referred to motor shaft, inch-lb}_f\text{-sec/radian} \]
\[ D = \text{differential operator, l/sec} \]
\[ e_c = \text{feedback voltage, volts} \]
\[ e_e = \text{error voltage, volts} \]
\[ e_R = \text{reference or command voltage, volts} \]
\[ J = \text{equivalent moment of inertia of motor and load, referred to motor shaft, inch-lb}_f\text{-sec}^2 \]
\[ J_{\text{equiv}} = \text{equivalent inertia of axial-piston motor, inch-lb}_f\text{-sec}^2 \]
\[ J_L = \text{load inertia, inch-lb}_f\text{-sec}^2 \]
\[ J_{OS} = \text{inertia of output shaft, inch-lb}_f\text{-sec}^2 \]
\[ J_{WP} = \text{moment of inertia of wobble-plate about its own axis, inch-lb}_f\text{-sec}^2 \]
\[ K = \text{gain constant, radians/sec-inch} \]
\[ K_{\text{amp}} = \text{amplifier and torque motor sensitivity, inches/volt} \]
\[ K_{dv} = \text{discharge valve area constant, inch}^2/\text{inch} \]
\[ K_{iv} = \text{inlet valve area constant, inch}^2/\text{inch} \]
\[ K_{ml} = \text{motor leakage constant, inch}^5/\text{lb}_f\text{-sec} \]
\[ K_{mt} = \text{motor torque constant, inch-lb}_f/\text{lb}_f/\text{inch}^2 = \text{inch}^3 \]
\[ K_{TR} = \text{feedback transducer sensitivity, volts/radian} \]
\( M_{CR} \) = mass of connecting rod, lb\( f \)-sec\(^2\)/inch
\( M_P \) = mass of piston, lb\( f \)-sec\(^2\)/inch
\( M_{WP} \) = mass of wobble-plate, lb\( f \)-sec\(^2\)/inch
\( N \) = number of pistons
\( P_d \) = absolute pressure in discharge volume, lb\( f \)/inch\(^2\)
\( P_d^* \) = average absolute pressure in discharge volume, lb\( f \)/inch\(^2\)
\( P_i \) = absolute pressure in inlet volume, lb\( f \)/inch\(^2\)
\( P_i^* \) = average absolute pressure in inlet volume, lb\( f \)/inch\(^2\)
\( P_s \) = absolute supply pressure, lb\( f \)/inch\(^2\)
\( R \) = gas constant, inch\(^2\)/sec\(^2\)-°R
\( R_m \) = effective radius for pressure force on vanes, inch
\( R_{WP} \) = wobble-plate radius to connecting rod, inch
\( R_{WPCG} \) = distance of wobble-plate center of mass from output shaft axis, inch
\( T \) = absolute temperature, °R
\( t \) = time, sec
\( V_d \) = discharge volume of motor, inch\(^3\)
\( V_i \) = inlet volume of motor, inch\(^3\)
\( V_m \) = motor displacement per radian, inch\(^3\)/radian
\( V_s \) = sonic velocity, inch/second
\( \ell_v \) = valve stem travel, inch
\( \phi \) = wobble-plate tilt angle, radians
\( \zeta \) = damping ratio, dimensionless
\( \Theta_c \) = controlled position angle, radians
\[ \theta_m = \text{motor angle of rotation, radians} \]

\[ \rho_d = \text{mass density of gas in discharge volume, lb}_f^{-1} \text{sec}^2/\text{inch}^4 \]

\[ \rho_d^* = \text{average value of discharge density, lb}_f^{-1} \text{sec}^2/\text{inch}^4 \]

\[ \rho_i = \text{mass density of gas in inlet volume, lb}_f^{-1} \text{sec}^2/\text{inch}^4 \]

\[ \rho_i^* = \text{average value of inlet density, lb}_f^{-1} \text{sec}^2/\text{inch}^4 \]

\[ \rho_s = \text{mass density of supply air, lb}_f^{-1} \text{sec}^2/\text{inch}^4 \]

\[ \tau_1, \tau_2 = \text{time constants, sec} \]

\[ \omega_m = \text{motor angular velocity, radians/sec} \]

\[ \omega_n = \text{undamped natural frequency, radians/sec} \]
CHAPTER I
INTRODUCTION AND BACKGROUND

1.1. Application of Fluid-Power Control. Fluid-power control systems using both liquids and gases as operating media have been used for many years. To put subsequent discussions into proper perspective, it will be useful to review briefly the most important of these applications at this time.

1.1.1. Systems for Position Control. Many fluid-power control systems have the task of positioning some object in response to a command. These systems may be either open-loop or feedback control systems.

Typical of common open-loop control systems are the pneumatic and hydraulic clamping devices which position and hold parts while they are being machined. These are open-loop systems since the desired position is established by a fixed stop, and not by measurement of the actual position and correction of errors, as in a feedback system.

Another very common open-loop system, usually pneumatic, is the diaphragm valve widely used in the process industries. Here a valve stem is positioned in response to air pressure signals. The pressure force on one side of the diaphragm is balanced by a spring force on the
other side, so that valve stem position is proportional to air pressure.

Hydraulic servomechanisms have found wide applica-
tion in various types of contouring lathes and die-sinking machines. These are true feedback systems since the actual position of the cutting tool is measured and com-
pared to the desired position (usually embodied in a tem-
plate or three-dimensional model) and if error is present, corrective action is instituted. A relatively recent var-
iation of this application is the so-called numerical pos-
tioning control. Here the information as to the desired shape of the work-piece is stored in magnetic tape, punched cards. This part of the system is electronic or electro-
mechanical, but the cutting tool or workpiece must still be positioned and this is often best done with hydraulics.

Often the diaphragm valve mentioned earlier does not have sufficient speed and accuracy for a given process. Valve positioners, which are true feedback systems, have been developed to fill this need. These can be either hydraulically or pneumatically operated and are finding increasing application as processes become faster-acting and quality specifications more rigid.

Many processes involving plastics, rubber, paper, etc. are concerned with a moving web of material. This web must be controlled in its lateral position as it
travels through the process and winds up on the takeup roll. One common solution to this problem involves both pneumatics and hydraulics. A small air nozzle is placed at the edge of the moving web and perpendicular to it. As the web moves laterally, it uncovers the nozzle more or less, changing the back pressure of the nozzle. This error signal is fed to a hydraulic actuator which positions the takeup roll so as to maintain a straight edge on the roll as it winds up.

In electric-arc metal-melting furnaces, the electric power consumed is kept constant by varying the distance of the electrodes from the metal to be melted. The heavy electrodes are in general positioned by an electro-hydraulic control system.

In modern aircraft, both open and closed loop fluid power controls are used for many purposes. Simple on-off hydraulic and pneumatic systems open and close doors, lower landing wheels, position seats, etc. More complex hydraulic systems position the aircraft control surfaces in response to signals from the human pilot or an automatic pilot. Hydraulics are almost universally used in this application because of the high forces necessary and the premium on weight-saving. To get optimum performance from modern jet engines when they are used over wide ranges
of operating conditions, the following devices have been
developed.

1. Variable-geometry burners and afterburners
2. Variable-pitch compressor and turbine stator blades.
3. Variable ejectors
4. Variable inlet area and geometry
5. Thrust reversers

All of these involve positioning some object or objects at a certain place upon a command. Hydraulic and pneumatic systems are widely used in such applications.

The steering of large ships has been accomplished with the aid of hydraulic servomechanisms for many years. The first such applications were made about 1870 and this type of steering has been very popular ever since.

Water turbines of the Kaplan type for generating hydroelectric power are well suited for installations where the head may vary considerably. This is due to the fact that high efficiency is maintained by changing the pitch of the blades. The pitch-changing device is usually a hydraulic servomechanism.

Fluid-power controls have many military applications. Hydraulic systems are widely used for positioning of all kinds of guns on land and aboard ships. Drives for many radar antennas are hydraulically actuated.
In recent years, power steering for cars, trucks and buses has become much more popular. Many of these power steering devices are hydraulic servomechanisms. A pneumatic system for steering trucks and buses has also been developed. This has the advantage that these vehicles invariably have air brakes, so that a source of pneumatic power is already available.

1.1.3. Systems for Speed Control. In some applications, the controlled variable is not the position, but rather the speed of linear motion or rotation of some object.

Many machines require a variable-speed drive to give the best performance under different operating conditions. Variable speed of hydraulic and pneumatic cylinders and rotary motors is easily accomplished with valves or variable-displacement pumps. In simple systems where high accuracy is not required, open-loop operation is adequate. For more stringent requirements, feedback may be necessary.

In some cases, it is desired to hold the speed constant at some preset value in the face of varying loads or other disturbances. This is the general problem of speed governing and applies to all kinds of turbines and engines. Hydraulic governors are widely used because of their simplicity and reliability and the ease with which integral and derivative effects may be obtained.
Feeds and speeds on many machine tools are obtained hydraulically, since a stepless variation is possible from the lowest to the highest rates.

In the processing of continuous-strip materials, hydraulic speed controls are widely used to synchronize the various machines which are interconnected to give a continuous process.

1.1.3. Systems for Force or Torque Control. Certain control problems require the application of controlled force or torque rather than motion.

One application of this type is the air brake so widely used on large motor vehicles. Here the driver applies a certain amount of foot pressure against a spring-loaded treadle. This opens a valve, allowing air to flow from a reservoir to the brake chambers. As the pressure builds up, the valve gradually closes, due to a pressure force deflecting a valve spring. When the pressure reaches the value called for by the driver's foot pressure, the valve closes and the air pressure is held at a constant value. Thus the driver can control the amount of braking effort in a continuous manner from low values up to the maximum, which is limited by the reservoir pressure of the system (about 100 psig.).
In the processing of continuous-strip materials, it is often necessary to control the tension in the moving strip as it passes through various interconnected machines. The strength of the material may vary from point to point in the process, so it may be necessary to control tension at different levels to prevent breakage but still insure uniform tracking and windup. This type of control has been accomplished by regulating the pressure drop across hydraulic motors which drive the rolls carrying the strip material. Since the motor torque is proportional to pressure drop, this regulates the torque and thereby the tension in the material.

Another application of force control is found in the pneumatic force servos recently developed for actuating control surfaces in aircraft. The usual approach to this problem has been a position-control system rather than a force-control system. That is, the control-surface position is made proportional to the stick position. This means that the correcting aerodynamic forces will depend on speed, altitude, etc., since a given stick position will not give the same corrective effort at 30,000 feet as it would at sea level. At present, this non-uniformity is corrected by the use of air-data computers which vary the control parameters as the airplane changes speed,
altitude, etc., in order to give a uniform corrective effort. There is some feeling at present that a more direct approach to this problem is to control the force on the airplane control surface rather than its position. In this way, a given stick motion always results in the same corrective force on the airplane. This concept is presently being developed.

1.2. Survey of Previous Work in Fluid-Power Control.
Considerable analytical and experimental work intended to place the analysis and design of fluid-power control systems on a more rational basis has been carried out. A survey of the more significant published material is given in this section. It should be appreciated that much important work has been done on military projects which is not available to the author and the general public.

No textbooks devoted exclusively to feedback control systems using fluid power are known to the author. Standard works on servomechanisms devote at most a few pages to simplified treatments of some common hydraulic and pneumatic components. The majority of the more advanced work will thus be found in papers, articles and reports.

The most comprehensive general treatment of this area is given in a series of articles by R. Hadekel (1). This series is entitled "Hydraulic and Pneumatic Servos," but
the emphasis is definitely on hydraulics. The few pneumatic examples are of the low-power, low performance process control type. Hadekel's opinion of pneumatic servos is summed up in the following quotation. "As regards pneumatic servos, the situation is different in many respects. Their performance is rarely of a high order, and they are not suitable for high powers." This opinion is not entirely born out by recent developments. An interesting comparison of electric and hydraulic motors is given by Hadekel. He states that all forms of hydraulic motors are superior to their electrical counterparts with regard to performance. The basic parameter here is the torque-to-inertia ratio. The torque of an electric motor is limited by the tractive force between a magnet pole and an armature which does not in practice exceed 60 psi. The corresponding "stress" in a hydraulic motor is the working pressure, which may be as high as 5000 psi. Since the inertia of hydraulic motors is generally smaller than that of electric motors, the torque-to-inertia ratio is clearly superior. This series of articles discusses quite thoroughly pump control, valve control, power supplies, valve flow characteristics and compensation methods. A quite similar series of articles is authored by J. M. Nightingale (2).
A major method of fluid-power control is by means of valves, so considerable study of this component has been carried out. A series of two articles by Shearer and Lee (3) discusses the selection of control valves for both hydraulic and high-performance pneumatic systems. An interesting comparison of a complete hydraulic and a comparable pneumatic system, both operating at 800 psi is given. A numerical example is worked through and a general comparison of hydraulic and pneumatic systems is presented.

A paper by Johnson, Schmid and Warshawsky (4) gives an excellent review of problems involved in the application of electrohydraulic servo valves. Topics covered include types of valves, static and dynamic characteristics, system characteristics and reliability. A good description, including numerical values, is given of twelve commercially available valves.

Forces on valve spools due to flow through the valve have presented some problems in hydraulic systems and have been the subject of several papers (5,6,7). This problem is relatively less important in pneumatic systems, since the forces are dependent on fluid density, which is of course much less for air than for oil.

The design of valves for high performance pneumatic servos operating at about 1000 psi is discussed in a
paper by Lee and Shearer (8). A new type of valve construction is developed and results of tests on an electro-pneumatic servo are given.

A paper by Eckman, Taft and Schuman (9) describes the design, development and testing of an electrohydraulic servomechanism of ultrahigh (200 ops) frequency response. Linearised analyses and non-linear analog computer studies are compared with actual system performance.

Application of hydraulic pump control to high power industrial systems has been treated in several papers. Analysis of typical speed and position control systems for presses, shears and grinders is covered in a paper by Mitka (10). Speed and tension controls for processing of continuous webs are discussed by Malmquist (11). Davis and Mitka (12) report on laboratory performance measurements on speed control systems.

The earliest thorough study of high-performance pneumatic servos is to be found in a Sc. D. thesis by Shearer (13). This work is further amplified in two papers (14) by Shearer and one by Ezekiel and Shearer (15). These works treat quite exhaustively the analysis of linear motion pneumatic servos. Linearised and non-linear analyses and analog computer studies are carried out and experimental tests of components and complete systems.
are run. The emphasis is on high pressure (about 1000 psi) military applications. The analysis, design and testing of a small, high performance pneumatic servo is described in an interesting paper by Reethof (16). A simplified treatment of linear-motion pneumatics is given by Levenstein (17). He also discusses a simple method of stabilization by means of tanks and flow restrictions.

Haroum and Templin (18) discuss pneumatic force-servos for aircraft control surfaces. A paper by Williams (19) describes qualitatively the performance of a new type of rotary air motor especially developed for high-temperature aircraft applications. No analysis is given and feedback-type applications are not stressed. A very interesting comparison of air turbines and positive-displacement motors is presented.

A comprehensive analytical study of the design of vane-type rotary air motors is given in an article by Teichmann (20). Only non-reversible motors are studied, so the application to feedback systems is restricted.

1.3. Basis of the Current Interest in Pneumatic Control. Most of the work on high performance pneumatic servos mentioned in Section 1.2. was directly sponsored or indirectly influenced by military agencies. The pressing problem which motivated these studies is the problem of
temperature, as encountered in high speed aircraft and missiles. The conventional method of control in this area involves the use of hydraulic actuators as the power device. By necessity, these hydraulic systems must be placed in high temperature environments. As temperatures continue to rise, it has been found that new methods of actuation must be developed since hydraulics are severely limited as to temperature. Since there is little prospect of the temperature problem diminishing in the future, a number of investigations into pneumatic systems were begun, since these systems are much less sensitive to temperature.

1.4. Motivation for the Present Investigation. As can be seen from section 1.2, the majority of investigations in the field of fluid-power control have been in the area of hydraulics. Hydraulic servomechanisms have been found to be superior to electrical types in many cases where relatively high power is required and thus have found rather extensive application. The applications of pneumatic control prior to about 1945 were to be found almost entirely within two specific areas.

One type of application concerns the clamping and positioning of work-pieces against fixed stops for machining purposes using air cylinders with a pressure of about 100 psig. This is an open loop type of system
since the correct position is established by the fixed stops and not by measurement of the actual position, comparison to the desired position and correction in case of error. Design of systems of this type is relatively straightforward and few difficult problems have been encountered. Speed of application and magnitude of impact forces can be controlled by use of orifices in inlet and exhaust lines.

The other major field of application of pneumatic control has been in the area termed "industrial process control." This involves feedback control systems for controlling temperature, pressure, tank level, fluid flow and similar variables in industrial processes. The function of most of these systems is to maintain the controlled variable at or near a fixed desired value or "set point" in the face of disturbances. A wide variety of pneumatic devices have been developed to perform measuring, transmitting, controlling and actuating functions for such systems. These pneumatic devices have been used for many years and are noted for their simplicity, low cost and reliability. In refineries and other plants involving explosive fumes, all-pneumatic systems offer an additional safety advantage over electrical systems in which sparks may occur. Of course electrical systems may be designed
to be explosion proof, but this entails additional cost. The air pressure used for transmitting information and generating various control modes such as proportional, derivative and integral has been standardized as the range 3 to 15 psig for practically all commercial pneumatic controllers. At this low pressure level, the compressibility of air severely limits the dynamic performance of such systems. Also, forces available for accelerating moving parts are low since they are directly proportional to the pressure. The wide acceptance of such systems in industry must mean, however, that these dynamic limitations are of secondary importance. This is, as a matter of fact, the case. The reaction times of most industrial processes have been in the past, and are to a large extent today, measured in minutes or even hours. Thus, even though the dynamic performance of the pneumatic control system may seem very poor when compared to the split-second response times of hydraulic and electric systems, it is more than adequate for most industrial control problems. In fact, the pneumatic system is usually so much faster than the controlled process itself, that the dynamic response of the controller has been neglected in most analyses.

The recent awakening of interest in high-performance pneumatic servomechanisms has been due mainly to problems
associated with high temperatures in aircraft and missile systems as outlined in section 1.3. These studies have been concerned with linear-motion systems using very high pressures (1000 psig or more). The rather promising results of these investigations led to speculation as to the performance which might be achieved at intermediate pressures (about 100 psig) which are readily available in most factories. It was felt that pneumatic servomechanisms operating at these pressures might fill a gap between low-powered electric systems and high-powered hydraulic systems. It was also decided that rotary-motion systems rather than linear motion systems would be considered. This decision was based on two main premises:

1. No work on rotary motion pneumatic systems at any pressure was uncovered in the literature search.

2. Rotary motors have considerably less air volume under compression than do comparable cylinder-piston actuators, thus giving a "stiffer" system.
CHAPTER II
ANALYSIS OF ROTARY PNEUMATIC SERVOMECHANISMS

2.1. Basic System Configurations for Fluid-Power Control. Whether operating with liquid or gas or with rotary or linear-motion motors, certain basic arrangements of components are possible in fluid-power control systems. These will now be listed and discussed.

2.1.1. Pump Control. Two basically different methods of modulating the flow of fluid power to a fluid motor have been used. These are called pump control and valve control.

In the method called pump control the generator of fluid-power is a variable-displacement pump. This pump is driven at constant speed by an electric motor or other rotary power source, and its displacement may be varied continuously, so that any flow rate from maximum in one direction, through zero, to maximum in the other direction may be obtained. Thus the fixed-displacement fluid motor may be driven in either direction at any speed within the range of the pump or may be left stationary. Pump control has been widely used in hydraulic systems, especially when the amount of power involved is large, say the order of 20 HP. The reason for this is that although the equipment necessary for a pump control system is relatively complex and expensive,
the efficiency is considerably higher than alternative methods. Thus, at high power levels, and where weight and space requirements are not critical, pump control has been extensively applied in military and industrial servomechanisms.

When we consider the possibility of using pump control for pneumatic servomechanisms at 100 psig working pressure, the prospects are not encouraging. When used in hydraulic systems, the pump-motor combination forms an essentially closed system, since the suction of the pump is connected to the discharge of the motor and the discharge of the pump to the inlet of the motor. The hydraulic fluid thus circulates through the system and only small quantities need be added to make up leakage. This arrangement could not be used with pneumatic systems due to the compressibility of the working fluid. That is, with the system at rest with the pump delivery set at zero, if a command signal tells the pump to start to deliver air to the motor, the only source of air for the pump is that air trapped in the discharge side of the motor, the line connecting motor discharge to pump inlet, and the inlet of the pump. This small quantity of air could not be compressed to a high enough pressure to give good dynamic response, if, in fact, any response at all. The obvious
solution to this problem is to connect the pump inlet to atmosphere, not to the motor discharge, but this allows pumping in one direction only. To get reversible operation, additional valves would be necessary on the pump to connect the proper side to atmosphere at the proper time. It is seen that the system is getting very complicated, and since one of the main advantages hoped for in 100 psig pneumatic control is simplicity and low cost, pump control cannot be seriously considered. Also, no commercially available variable-displacement air compressors of the type needed here are known to the author.

3.1.2. Valve Control. In a valve control system the power supply is a reservoir of fluid maintained at essentially constant pressure by an independent pumping system which is not a part of the feedback control system. This reservoir of pressurized fluid is connected to the rotary or translatory fluid motor through a valve. The opening and closing of this valve controls the flow of fluid power to the motor. The advantages of valve control over pump control are simplicity and higher speed of response. The major disadvantage is poor efficiency, which is inherent in such systems since the valve dissipates a considerable portion of the fluid power at its metering orifices. The method of valve control seems well suited to
pneumatic servomechanisms and therefore this type of system will be studied in greater detail.

Many different configurations of valves have been developed to meet the requirements of various types of applications in the hydraulic control field. These same types of valves are equally applicable to pneumatic control even though certain details of design might profitably be varied. Since the various types of valves are adequately discussed in the literature, this material will not be repeated here. The valve configuration which seems most suitable for our purposes is that normally called the closed-center four-way valve.

2.1.3. Configuration of the Complete System. The system to be studied has the basic function of positioning a shaft in angular rotation in response to commands which might be electrical or mechanical. Angles greater than $360^\circ$ are of interest for two reasons. First, the driven load may be geared down and thus the motor may make several revolutions while the load turns only a fraction of a revolution. Second, the motor rotation may be used to drive a load in translation by use of a lead screw, and thus multiple revolutions are again of interest. Thus, both fractions and multiples of revolutions are of interest.

A block diagram of the system to be studied is given in Figure 1.
2.2. Rotary Pneumatic Motors for Servomechanism Applications. We are primarily interested in studying one component of the complete system shown in Figure 1. This is the power actuator or motor. The analysis will be confined to existing designs rather than to studies of possible new types of motors. Two basically different types of air motors are in common use in industry. Although these motors are not at present utilized in servomechanism type applications, it is possible to apply them to such systems. Since they were thus not specifically designed to meet the requirements of feedback control systems, their operating characteristics may not be the best possible when used in this way. The two types of motors studied are the vane-type and the axial-piston type. (It should be noted here that a short time before the completion of this dissertation several new types of air motors were introduced by two companies in the aircraft accessories field. These are especially designed for feedback type applications, and though they are used almost exclusively in military aircraft systems at present, they may find increasing use in industry. These new motors are briefly described in the Appendix. At the time of this writing, no experimental results on the performance of these motors in feedback systems was available.)
2.3.1. Analysis of Vane-Type Motor Driven by Closed-
Center Four-Way Valve. The vane-type motor is probably
the most widely used type of air motor because of its low
cost and simplicity. Figure 2 shows the essential fea-
tures of such a motor and the closed-center four-way valve
used to control it.

Such motors may be purchased in either a reversible
or non-reversible type of design. For servomechanism
applicators, reversibility is usually necessary and the
performance should be the same for either direction of
rotation. Figure 2 shows the reversible type of motor.
The only difference in a non-reversible motor is that
the annular space between housing and rotor can increase
in volume in going from inlet to discharge, thus allow­
ing the air to undergo a net expansion and perform more
useful work. Inspection of Figure 2 (reversible motor)
shows no net expansion, and thus reversible motors are
somewhat less efficient than non-reversible ones. The
reversible motor can not have a net expansion if the
performance is to be the same for both directions of
rotation. This efficiency penalty is actually not very
great, and in servomechanisms, efficiency is rarely a
major performance criterion.

In analyzing this motor-valve combination a simpli-
Figure: Block Diagram

Pneumatic Servomechanism

Command Angle \( \Theta_R \)

Error \( \Theta_E \)

Measuring Device

\[ \Theta_E = \Theta_R - \Theta_e \]

Valve

Motor and Load

Controlled Variable \( \Theta_c \)
fied model of the mode of operation of the motor will be used. The purpose of the analysis is to obtain a linear differential equation with constant coefficients relating the valve stem motion to the motor rotation. A description of the motor operation according to the model is as follows. Assume the motor at rest and the valve closed. If the valve is now opened, one side of the motor is connected to the high pressure supply and the other side is vented to atmosphere. Air will start to flow through the inlet port of the valve into the inlet volume, $V_1$, of the system. This inlet volume consists of the tubing connecting the valve to the motor, the inlet passage of the motor itself, and some portion of the motor displacement volume. The last-named component of the inlet volume actually varies somewhat as the motor rotates, but an average value equal to one half of the total annular space between rotor and housing will be used here. As air flows into the inlet volume $V_1$, a pressure $P_1$ builds up in this space. At the discharge side of the motor, a similar volume $V_d$ and pressure $P_d$ exist. In general, because of the symmetry required in a reversible motor, $V_1$ equals $V_d$, but of course $P_1$ need not equal $P_d$. Now if $P_1$ and $P_d$ are different at any instant of time, a torque will be exerted on the rotor. This torque will be assumed to be directly proportional to the
Fig. 2. Vane Motor with Four-Way Closed-Center Valve
pressure difference \( P_j - P_d \). The proportionality factor equals the area, \( A_v \), of the vanes exposed to the pressure difference times the mean radius \( R_m \), at which the pressure force acts. Although this varies somewhat as the motor rotates, the motor torque constant, \( K_{mt} \), will be assumed constant.

Based on the simplified model, the inlet volume has one inflow and two outflows. The inflow is the flow through the valve inlet port. We assume that the pressure drop across the inlet valve port is always large enough to give sonic velocity at the port. The mass flow rate of air through this port then depends only on the supply pressure \( P_s \) and the port area. We assume that \( P_s \) is constant and that the valve port area varies linearly with valve stem travel \( L_v \). One outflow from the inlet volume is the air trapped between adjacent vanes and carried from the inlet volume to the discharge volume. If the displacement of the motor per radian is \( V_m \) and the angular velocity of the rotor is \( \dot{\theta}_m \), the mass flow rate of air out of the inlet volume is the product of \( V_m, \dot{\theta}_m \) and \( \rho \), where \( \rho \) is the mass density of the air in the inlet volume. The other outflow from the inlet volume is a leakage flow. That is, all the air that enters the inlet volume is not moved to the discharge volume by being carried there by the vanes. Some of the air leaks
beyond clearances, and this mass flow rate is assumed proportional to the product of \( \rho \) and the pressure difference \( P_1 - P_d \).

In considering flows in and out of the discharge volume \( V_d \), we see that there are two inflows and one outflow. The two inflows are, of course, the same as the two outflows of the inlet volume. The outflow of the discharge volume is the flow through the valve discharge port. We again assume sufficient pressure drop across the valve port to give sonic velocity so that the outflow depends on the port area and the pressure \( P_d \) only.

In relating the pressures \( P_1 \) and \( P_d \) to the volumes in which they exist, and to the inflows and outflows of these volumes, a simple relationship employing the perfect gas law may be used. This involves considerable approximation, but is in line with the other simplifying assumptions already made. More refined assumptions result in complicated and highly non-linear equations, from which little information can be gleaned about the general effects of the various parameters. It is hoped that the present analysis eliminates the complicating details while retaining the essential features of the system.
The perfect gas law may be written as

\[ pV = mRT \]  \hspace{1cm} (2.1)

where

- \( p \) = Absolute pressure
- \( V \) = Volume
- \( m \) = Mass
- \( R \) = Gas constant
- \( T \) = Absolute temperature

If we assume \( V, R \) and \( T \) are constant, then we can write this law in differential form:

\[ dp = \frac{RT}{V} \frac{dm}{dt} \] \hspace{1cm} (2.2)

and upon dividing through by \( dt \), where \( t \) is time, we get

\[ \frac{dp}{dt} = \frac{RT}{V} \frac{dm}{dt} \] \hspace{1cm} (2.3)

This law then gives a relationship between the time rate of change of pressure and the time rate of change of mass for a fixed volume. This relation may be directly applied to the problem at hand if we assume the inlet volume \( V_1 \) and the discharge volume \( V_d \) to be constant.
Consider first the inlet volume \( V_i \). For this volume,
\[
\frac{dm}{dt} = \text{mass inflow rate} - \text{mass outflow rate} \quad (2.4)
\]
Thus,
\[
\frac{V_i}{RT} \frac{dP_i}{dt} = (K_{iv} x_v) \dot{V}_s \rho_s - V_m \dot{\Theta}_m P_i - C_{il} P_i (P_i - P_d) \quad (2.5)
\]
where the symbols have the following meanings:

- \( V_i \) = inlet volume, inch\(^3\)
- \( R \) = gas constant, inch\(^2\)/sec\(^2\)-O\(^R\)
- \( T \) = absolute temperature of gas, °O\(^R\). Assumed constant and uniform throughout the system
- \( P_i \) = absolute pressure in inlet volume, lbf/inch\(^2\)
- \( t \) = time, seconds
- \( K_{iv} \) = inlet valve area constant, inch\(^2\)/inch
- \( x_v \) = valve stem travel from neutral, inch
- \( \dot{V}_s \) = sonic velocity corresponding to temperature \( T \), inch/second
- \( \rho_s \) = mass density of supply air corresponding to supply pressure \( P_s \) and temperature \( T \), lbf/sec\(^2\)/inch\(^4\)
- \( V_m \) = motor displacement volume per radian, inch\(^3\)/radian
- \( = \frac{2 \pi R_m A_v}{2 \pi} = R_m A_v \)
\[ \dot{\theta}_m = \text{motor angular velocity, radians/second} \]
\[ \rho_i = \text{mass density corresponding to pressure } P_i \]
\[ \text{and temperature } T, \text{ lb}_f^{-\text{sec}}^{-2}/\text{inch}^4 \]
\[ K_{ml} = \text{motor leakage constant, inch}^5/\text{lb}_f^{-\text{sec}} \]
\[ \text{(an experimentally-determined average value)} \]
\[ P_d = \text{absolute pressure in discharge volume, } \text{lb}_f^{-2}/\text{inch}^2 \]

In examining equation 2.5, two non-linear terms remain to be linearized. The flow rate due to air trapped by the vanes, \( v_m \dot{\theta}_m \rho_i \), contains the product \( \dot{\theta}_m \rho_i \) of two dependent variables, and is therefore non-linear. This will be linearized by assuming that for any given process, an average value of the mass density \( \rho_i^* \) may be used. The other non-linear term is the product \( \rho_i (P_i - P_d) \). Here again an average value of mass density \( \rho_i^* \) will be used in order to achieve a linear expression. The linearized equation then becomes

\[
\frac{dP_i}{dt} + \left[ \frac{K_{ml} \rho_i^* RT}{V_i} \right] P_i - \left[ \frac{K_{ml} \rho_i^* RT}{V_i} \right] P_d \\
+ \left[ \frac{V_m \rho_i^* RT}{V_i} \right] \frac{d\theta_m}{dt} = \left[ \frac{K_{vl} V_s \rho_s RT}{V_i} \right] \chi_v \tag{2.8}
\]
For the discharge volume, a similar equation may be set up:

\[
\frac{dV_d}{dt} = \frac{V_m}{R} \rho_c \dot{m} + K_{dv} \rho_d^*(P_v - P_d) - (k_{dv} \chi_r) \dot{V}_v \rho_d
\]  

(2.7)

where

\[V_d = \text{discharge volume, inch}^3 \text{ (numerically equal to } V_i)\]

\[K_{dv} = \text{discharge valve area constant, inch}^2/\text{inch}\]

\[\rho_d = \text{mass density corresponding to pressure } P_d \text{ and temperature } T, \text{ lb-ft-seo}^2/\text{inch}^4.\]

In equation 2.7, the term \(k_{dv} \chi_r \dot{V}_v \rho_d\) must be linearized. If \(J_y\) is thought of as a known (forcing) function of time, then the term is not non-linear, but rather linear with variable coefficient. When the system is connected in a closed loop, however, \(J_y\) becomes a dependent rather than an independent variable, and then the term would be truly non-linear. The term will be linearized by assuming an average value \(\rho_d^*\) for \(\rho_d\). The equation then reads

\[
\frac{dP_d}{dt} + \left[\frac{K_{mv} \rho_d^* RT}{V_d}\right] P_d - \left[\frac{K_{mv} \rho_c^* RT}{V_d}\right] P_c
\]  

\[- \left[\frac{V_m \rho_c^* RT}{V_d}\right] \frac{d\Theta_m}{dt} = - \left[\frac{K_{dv} \dot{V}_v \rho_d^* RT}{V_d}\right] \chi_r. \]  

(2.8)
The final equation which can be written, relating the variables $P_i$, $P_d$, $\Theta_m$ and $I$, is the equation relating torque, inertia and acceleration for a rotating body.

From Newton's law we have
\[
\sum \text{Torques} = J \alpha \quad (2.9)
\]

In this case, pressure torque + friction torque + load torque = $J \alpha$. \quad (2.10)

We will assume that all friction in motor and load is viscous and that there is no external load torque acting.

Then
\[
K_m (P_i - P_d) - B \frac{d\Theta_m}{dt} = J \frac{d^2\Theta_m}{dt^2}, \quad (2.11)
\]

where
\[
K_m = \text{motor torque constant, inch}^3
\]
\[
= R_m A_v = V_m
\]
\[
B = \text{equivalent viscous friction coefficient of motor and load, referred to motor shaft, inch} - \text{lb}_{f} - \text{sec/radian}
\]
\[
J = \text{equivalent moment of inertia of motor and load, referred to motor shaft, inch} - \text{lb}_{f} - \text{sec}^2.
\]
It is appreciated that the foregoing equations are very crude and do not take into account a number of important phenomena such as

1. Temperature changes.

2. Valve flow relation change when velocity drops below sonic.

3. Valve flow relation change when valve stem is moved in opposite direction to that assumed in above equations.

4. Effects of varying densities which were assumed constant.

5. Effect of dry friction.

6. Effect of valve overlap or underlap, and many others.

This relatively crude approach is felt to be justified as an initial attempt at dynamic analysis of these motors, since more exact statements lead to hopeless complication.

Since we are interested in a relation between valve stem motion and resulting motor rotation, equations 2.6, 2.8 and 2.11 may be combined to eliminate \( P_1 \) and \( P_4 \). To facilitate this, the equations will be rewritten in operator form with the complicated coefficients replaced by single letters.
\[
[D + \mu_1] P_z - [\mu_1] P_d + [\mu_2 D] \Theta_{\mu_1} = [\mu_3] X_v \quad (2.12)(2.6)
\]

\[
- [\mu_1] P_z + [D + \mu_1] P_d - [\mu_2 D] \Theta_{\mu_1} = - [\mu_4] X_v \quad (2.13)(2.8)
\]

\[
[\mu_5] P_z - [\mu_5] P_d - [\mu_6 D^2 + \mu_7 D] \Theta_{\mu_1} = 0 \quad (2.14)(2.11)
\]

Here,

\[
D = \frac{d}{dt}
\]

\[
\mu_1 = \frac{K m \rho \mu^* R T}{V_z}
\]

\[
\mu_2 = \frac{V m \rho \mu^* R T}{V_z}
\]

\[
\mu_3 = \frac{K m v \bar V_3 \rho \mu^* R T}{V_z}
\]

\[
\mu_4 = \frac{K m v \bar V_3 \rho \mu^* R T}{V_d}
\]

\[
\mu_5 = K m t
\]

\[
\mu_6 = J
\]

\[
\mu_7 = B
\]
We can now eliminate $P_1$ and $P_d$ by the use of determinants:

\[
\begin{vmatrix}
D + C_1 & -C_1 & C_3 X_v \\
-C_1 & D + C_1 & -C_4 X_v \\
C_5 & -C_5 & 0
\end{vmatrix}
\]

\[\Theta_m = \frac{C_5 (C_3 + C_4) X_v}{C_6 D^4 + (2C_1 C_6 + C_1) D^3 + 2(C_2 C_5 + C_7) D^2} \] (2.15)

By expanding these determinants, we get

\[\Theta_m = \frac{C_5 (C_3 + C_4) X_v}{C_6 D^4 + (2C_1 C_6 + C_1) D^3 + 2(C_2 C_5 + C_7) D^2} \] (2.16)

\[\Theta_m = \frac{C_5 (C_3 + C_4) X_v}{C_6 D^3 + (2C_1 C_6 + C_1) D^2 + 2(C_2 C_5 + C_7) D} \] (2.17)

\[
C_6 \frac{d^3 \Theta_m}{dt^3} + (2C_1 C_6 + C_7) \frac{d^2 \Theta_m}{dt^2} + 2(C_2 C_5 + C_7) \frac{d \Theta_m}{dt} = C_5 (C_3 + C_4) X_v .
\] (2.18)
From equation 3.17, the open-loop transfer function is

\[ \frac{\Theta_m}{X_v} = \frac{C_5 (C_3 + C_4)}{D \left[ C_6 D^2 + (2C_1C_6 + C_7)D + 2(C_2C_5 + C_1C_7) \right]} \quad (2.19) \]

This transfer function is of the form

\[ \frac{\Theta_m}{X_v} = \frac{C}{D \left[ D^2 + 2 \wp \omega_n D + \omega_n^2 \right]} \quad , \quad (2.20) \]

where

\[ \wp \quad \text{damping ratio, dimensionless} \]

\[ \omega_n \quad \text{undamped natural frequency, radians/sec.} \]

By evaluating these quantities,

\[ \omega_n = \sqrt{2(C_2C_5 + C_1C_7) \frac{C_6}{C_6}} = \sqrt{\frac{2P^* \text{RT}}{V_n J} \left( K^2 + BK \right)} \quad (2.21) \]
\[
\gamma = \frac{C_i + \frac{C_1}{2C_0}}{\omega_n} = \frac{K_{ml} P_i^* RT}{V_i} + \frac{B}{2J} \sqrt{\frac{2 P_i^* RT}{V_i J} (K_{ml}^2 + BK_{ml})}.
\]

(2.22)

A large value of \( \omega_n \) indicates a high speed of response and a low value of \( \gamma \) is characteristic of a lightly damped highly oscillatory system. If we define \( P_i^* \) as an average pressure corresponding to \( P^* \), \( R \) and \( T \), equations 2.21 and 2.22 may be rewritten as follows:

\[
\omega_n = \sqrt{\frac{2 P_i^*}{V_i J} (K_{ml}^2 + BK_{ml})}.
\]

(2.23)

\[
\gamma = \frac{B + \frac{2 K_{ml} P_i^* J}{V_i}}{2 \sqrt{\frac{2 P_i^*}{V_i} (K_{ml}^2 + BK_{ml}) J}}.
\]

(2.24)
Another form of the transfer function \( \frac{\Theta_m}{X_v} \) which is sometimes useful is

\[
\frac{\Theta_m}{X_v} = \frac{K}{\tau_1 D (1 + \tau_2 D)(1 + \tau_2 D)} \tag{2.30}
\]

where

\[
K = \text{gain constant} = \frac{C_S (C_2 + C_4)}{2 (C_2 C_S + C_1 C_1)} \tag{2.31}
\]

\[
K = \frac{K_{mT} \bar{V} \left[ K_{iv} P_s + K_{iv} P_d \right]}{2 P_z^* \left[ K_{mT}^2 + B K_{mT} \right]} \tag{2.32}
\]

\( \tau_1, \tau_2 = \text{time constants}. \) These can be easily found by solving the quadratic equation in any given numerical case.

2.2.2. Analysis of Axial-Piston Motor Driven by Closed-Center Four-Way Valve. The axial-piston type of motor is quite common in the hydraulic field, but is not so widely used in pneumatic applications. Only one manufacturer of axial-piston air motors was found, so the analysis will be based on this design.
A simplified diagram of the axial-piston air motor and closed-center four-way valve is shown in Figure 3. Air from the inlet port of the valve flows through the distributor to the cylinders of the motor. The distributor is directly connected to the motor output shaft and rotates with it. The function of the distributor is to provide the proper valving action so that the cylinders are alternately connected to the supply and exhaust ports of the valve in proper relationship with the rotation of the motor shaft. Thus, each cylinder is caused to go through its cycle of intake, expansion and exhaust in synchronism with the motor rotation. This type of motor is reversed simply by shifting the four-way valve, just as in the vane-type motor. It should be noted, however, that due to the valving action of the distributor, the axial-piston motor can have a net expansion of the air and still be reversible. This was not possible with the vane-type motor.

The force on the pistons acts on the wobble plate tending to rotate it in its bearing. This torque reacts against the fixed bevel gear, thus giving a torque on the motor output shaft. Usually, four or more cylinders are used to eliminate dead spots and give a more uniform torque output.
The analysis of the axial-piston motor will be based on the same simplified model as was used for the vane motor. Expressions must thus be derived for the following quantities, since they are not found in the same way as for the vane motor:

1. Motor inertia
2. Motor torque constant
3. Motor displacement volume
4. System inlet and discharge volume

All other variables and parameters have the same meanings and are found in the same way as for the vane motor.

2.2.2.1. Determination of Motor Inertia Referred to Motor Output Shaft. In the vane motor, the inertia is essentially constant since the vanes are quite light and do not change their effective inertia much as the motor rotates. In the axial-piston motor, the motion of the various parts is quite complicated and the inertia varies with angle of rotation. An approximate constant equivalent inertia will be calculated based on equivalence of kinetic energy.

A kinematic analysis of the wobble-plate mechanism is necessary to determine the velocities used in computing the kinetic energy. No analysis for this particular form of the wobble-plate was found in the literature, so
Fig. 4. Skeleton Diagram of Wobble-Plate Mechanism
a partial analysis sufficient for the present purpose
was undertaken.

Figure 4 shows a skeleton diagram of the mechanism
without the piston and cylinder. The first step will be
to obtain equations for the motion of the center of one
of the ball socket joints at the wobble plate in terms of
the angle of rotation $\theta_m$ of the motor output shaft. If
the output shaft is rotated an angle $\theta_m$ from the $\theta_m = 0$
position, as shown in Figure 4, certain motions will
result. Exactly the same motion would result if-

1. The fixed bevel gear and the wobble plate are
considered as one piece ("welded" together) and the
fixed bevel gear is allowed freedom to rotate.

2. The output shaft is rotated an angle $\theta_m$.

3. The output shaft is fixed, wobble plate and
fixed bevel gear are considered free to move again, and
the fixed bevel gear is rotated through an angle $-\theta_m$.
The motion due to steps 1 and 2 is represented by

\[
Z_i = R_{wp} \sin \beta \tag{2.33}
\]

\[
X_i = -R_{wp} \cos \beta \sin \theta_m \tag{2.34}
\]

\[
Y_i = -R_{wp} \cos \beta \cos \theta_m . \tag{2.35}
\]
The motion due to step 3 is shown by

\[ Z_z = R_{wp} \sin \beta (\cos \Theta_m - 1) \]  
\[ (2.36) \]

\[ X_z = R_{wp} \left[ \sin \Theta_m \cos \Theta_m + \cos \beta \sin \Theta_m (1 - \cos \Theta_m) \right] \]  
\[ (2.37) \]

\[ Y_z = R_{wp} \left[ \cos \beta \cos \Theta_m (1 - \cos \Theta_m) - \sin^2 \Theta_m \right]. \]  
\[ (2.38) \]

The total resultant motion becomes thus,

\[ Z = R_{wp} \sin \beta \cos \Theta_m \]  
\[ (2.39) \]

\[ X = R_{wp} (1 - \cos \beta) \sin \Theta_m \cos \Theta_m \]  
\[ (2.40) \]

\[ Y = -R_{wp} (\sin^2 \Theta_m + \cos \beta \cos^2 \Theta_m). \]  
\[ (2.41) \]
At this point, certain approximations will be introduced to facilitate analysis:

1. The connecting rod (which has exceedingly complex motion) will be replaced by two equal concentrated masses, one at the piston and one at the ball-socket joint at the wobble plate.

2. The \( z \) motion of the piston will be assumed to be the same as the \( z \) motion of the ball-socket joint on the wobble plate.

We can now compute the velocities of the various parts. The velocity of the wobble-plate end of the connecting rod has three components:

\[
\frac{dz}{dt} = \frac{d\bar{z}}{d\theta_w} \frac{d\theta_w}{dt} = -R_{wp} \omega \sin \beta \sin \Theta_w
\]

\[
\frac{dx}{dt} = \frac{d\bar{x}}{d\theta_w} \frac{d\theta_w}{dt} = R_{wp} \omega \left( 1 - \cos \beta \right) \left[ \cos^2 \Theta_w - \sin^2 \Theta_w \right]
\]

\[
\frac{dy}{dt} = \frac{d\bar{y}}{d\theta_w} \frac{d\theta_w}{dt} = -2 R_{wp} \omega \left( 1 - \cos \beta \right) \sin \Theta_w \cos \Theta_w.
\]
The magnitude of the resultant velocity is

\[ |V| = \sqrt{\left( \frac{dz}{dt} \right)^2 + \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2}. \]  

(2.45)

Substituting from equations 2.42, 2.43 and 2.44 and simplifying, we get

\[ |V| = R_{wp} \omega_m \sqrt{\sin^2 \beta \sin^2 \Theta_m + (1 - \cos \beta)^2}. \]  

(2.46)

To compute the kinetic energy, we need the square of the velocity:

\[ |V|^2 = R_{wp}^2 \omega_m^2 \left[ (1 - \cos \beta)^2 + \sin^2 \beta \sin^2 \Theta_m \right]. \]  

(2.47)

We see that this varies with \( \Theta_m \). The average value of \( |V|^2 \) over one cycle is

\[ \left( \frac{|V|^2}{\text{Av.}} \right)_{\text{Av.}} = \frac{1}{2\pi} \int_0^{2\pi} R_{wp}^2 \omega_m^2 \left[ (1 - \cos \beta)^2 + \sin^2 \beta \sin^2 \Theta_m \right] d\Theta_m. \]  

(2.48)
Evaluating the integral, we get

\[
\left( |V| \right)^2_{\text{avg.}} = \frac{R^2 \omega_m^2}{2} \left[ 3 - 4 \cos^2 \beta + \cos^2 \beta \right].
\]  

(2.49)

Assuming the \( z \) motion of the piston to be the same as the \( z \) motion of the wobble-plate end of the connecting rod, we can compute the velocity of the piston:

\[
V_p = \frac{d^2 p}{d \Theta_m} \frac{d \Theta_m}{dt} = R_{wp} \omega_m^2 \sin \beta \sin \Theta_m.
\]  

(2.50)

The average value of \( V_p^2 \) over one cycle is

\[
\left( V_p \right)^2_{\text{avg.}} = \frac{R_{wp}^2 \omega_m^2 \sin^2 \beta}{2 \pi} \int_0^{2\pi} \sin^2 \Theta_m d \Theta_m.
\]  

(2.51)

\[
\left( V_p \right)^2_{\text{avg.}} = \frac{R_{wp}^2 \omega_m^2 \sin^2 \beta}{2}.
\]  

(2.52)

The total kinetic energy of the system is made up of several parts. Each of these will now be computed.

I. Output shaft. The output shaft itself, the air distributor and any other parts that rotate with the output shaft have an inertia \( J_{os} \). The kinetic energy of these parts is

\[
\text{K.E.} = \frac{J_{os} \omega_m^2}{2}. 
\]  

(2.53)
II. Wobble-plate. The kinetic energy of the wobble-plate due to rotation of its center of mass about the output shaft axis is

\[ K.E. = \frac{M_{wp} R_{wpca}^2 \omega_m^2}{2} \]  \hspace{1cm} (2.54)

In addition to this, the wobble-plate has kinetic energy due to its rotation about its own axis. Its angular velocity about its own axis is \( \omega_m \), so this kinetic energy is given by

\[ K.E. = \frac{J_{wp} \omega_m^2}{2} \]  \hspace{1cm} (2.55)

Thus the total kinetic energy of the wobble-plate becomes

\[ K.E. = \frac{\omega_m^2}{2} \left[ M_{wp} R_{wpca}^2 + J_{wp} \right] \]
where

\[ M_{wp} = \text{mass of wobble-plate} \]

\[ R_{wpzq} = \text{distance of wobble-plate center of mass from output shaft axis} \]

\[ J_{wp} = \text{moment of inertia of wobble-plate about its own axis}. \]

III. Piston and connecting rod. The kinetic energy of the piston and its half of the connecting rod is

\[
\text{K.E.} = \left( M_p + \frac{M_{CR}}{2} \right) \frac{R_{wp}^2 \omega_m^2 \sin^2 \beta}{4}, \text{PER CYLINDER.}
\]  

(2.57)

The kinetic energy of the wobble-plate end of the connecting rod is

\[
\text{K.E.} = \left( \frac{M_{CR}}{2} \right) \frac{R_{wp}^2 \omega_m^2}{4} \left[ 3 - 4 \cos \beta + \cos^2 \beta \right], \text{PER CYLINDER.}
\]

(2.58)

The total kinetic energy of piston and connecting rod is

\[
\text{K.E.} = \frac{R_{wp}^2 \omega_m^2}{8} \left[ \left( 2M_p + M_{CR} \right) \sin^2 \beta + M_{CR} \left( 3 - 4 \cos \beta + \cos^2 \beta \right) \right] \quad \text{(2.59)}
\]

\[
\text{K.E.} = \frac{R_{wp}^2 \omega_m^2}{8} \left[ M_{CR} + 2M_p \sin^2 \beta + M_{CR} \left( 3 - 4 \cos \beta \right) \right]
\]

\[
\text{K.E.} = \frac{R_{wp}^2 \omega_m^2}{4} \left[ 2M_{CR} \left( 1 - \cos \beta \right) + M_p \sin^2 \beta \right], \text{PER CYLINDER.}
\]
where

\[ M_{cr} = \text{mass of connecting rod} \]

\[ M_p = \text{mass of piston}. \]

The total kinetic energy of the system is the sum of the above components and is given by

\[
K.E. = \frac{\omega_m^2}{2} \left\{ \sum_j M_j \frac{R_j^2}{2} + \frac{N R_{wp}^2}{2} \left[ 2 M_{cr} (1 - \cos \beta) + M_p \sin^2 \beta \right] \right\}
\]

where

\[ N = \text{number of cylinders}. \]

If we define the equivalent lumped, constant inertia as an inertia which has the same kinetic energy as the total kinetic energy of the system, we get

\[
\frac{J_{equiv} \omega_m^2}{2} = \frac{\omega_m^2}{2} \left\{ \sum_j M_j \frac{R_j^2}{2} + \frac{N R_{wp}^2}{2} \left[ 2 M_{cr} (1 - \cos \beta) + M_p \sin^2 \beta \right] \right\}
\]

or

\[
J_{equiv} = \sum_j M_j \frac{R_j^2}{2} + \frac{N R_{wp}^2}{2} \left[ 2 M_{cr} (1 - \cos \beta) + M_p \sin^2 \beta \right].
\]
Fig. 5. Torque versus angle of rotation for a single cylinder with constant pressure.

Fig. 6. Resultant torque for a four cylinder motor with unequal inlet and discharge pressure.
If there is a load inertia which has a value $J_L$ when referred to the motor shaft, the total inertia of the system is

$$J = J_{\text{equiv}} + J_L.$$ \hspace{1cm} (2.63)

This $J$ is the same $J$ as used in the analysis of the vane motor.

2.2.2.2. Determination of Motor Torque Constant. As in the case of the vane motor, the torque due to air pressure is assumed proportional to the pressure drop across the motor. We will now derive an expression for the proportionality constant.

For constant inlet and discharge pressures, the variation of torque with angle of rotation for a single cylinder is shown in Figure 5. The shape of the curve is very complicated due to the complex kinematics of the wobble-plate, however, the curve is approximately sinusoidal and we will assume it so in all further calculations. Under this assumption, we need only find the amplitude to get a mathematical equation for it, since the period is clearly $2\pi$. The maximum value of the torque occurs at $\Theta_m = 90^\circ$ and its value is

$$T_{\text{max}} = (P \Delta P \sin \beta) R \omega_p,$$ \hspace{1cm} (2.64)
where \( A_p \) is the area of the piston and \( p \) is the pressure.

The equation describing the variation of the torque with angular rotation of the motor is

\[
T = p A_p R_w p \sin \beta \sin \Theta_m.
\]

(2.65)

Because of the valving action of the distributor, the inlet pressure acts on the piston for about one-half cycle (180°) and the discharge pressure for the other half cycle. Thus we have:

\[
T = P_i A_p R_w p \sin \beta \sin \Theta_m, \quad 0^\circ < \Theta_m < 180^\circ.
\]

(2.66)

\[
T = P_d A_p R_w p \sin \beta \sin \Theta_m, \quad 180^\circ < \Theta_m < 360^\circ.
\]

(2.67)

The fact that the torque is negative (resisting the motion) for the second half-cycle is taken care of by the negative sign of \( \sin \Theta_m \) when 180° \(< \Theta_m < 360°. Since there are \( N \) cylinders, there is a smoothing effect on the torque as shown in Figure 6 for the case of 4 cylinders. Assuming constant speed operation with \( P_i \) and \( P_d \) both constant,
an average value of torque may be calculated:

\[
\tau_{\text{avg.}} = N A \rho R_w p \sin \beta \left[ \frac{1}{2\pi} \int_0^{2\pi} (P_0 - P_d) \sin \theta \, d\theta \right]
\]

or

\[
\tau_{\text{avg.}} = \frac{N A \rho R_w p \sin \beta (P_0 - P_d)}{\pi}.
\]

If we now assume that this relationship between torque and pressure holds on an instantaneous as well as an average basis, our motor torque constant is defined as

\[
K_{\text{mt}} = \frac{N A \rho R_w p \sin \beta}{\pi}
\]

which is analogous to \( I_{\text{mt}} \) for the vane motor.

2.2.2.3. Motor Displacement Volume. The volume displaced per revolution, assuming 100 per cent volumetric efficiency, is given by:

\[
\text{Volume} = N A \rho (2 R_w p \sin \beta);
\]

and the displacement per radian is

\[
V_m = \frac{N A \rho R_w p \sin \beta}{\pi}.
\]
Fig. 7. Block Diagram of Electro-Pneumatic Positioning Servomechanism
This quantity is analogous to $V_m$ for the vane motor, and we see that, just as in the vane motor, it is equal numerically to $K_{ot}$.

### 2.3.3.4. System Inlet and Discharge Volume

As in the vane motor, the system inlet and discharge volumes are made up of the tubing connecting the valve to the motor, the flow passages in the motor itself, and some portion of the motor displacement volume. The last named component of this volume is the only one which presents any difficulty. Actually, the number of cylinders exposed to inlet pressure varies as the motor rotates and also depends on the number of cylinders. Furthermore, if say, two cylinders were both open to inlet pressure at some instant, they would not have the same volumes since the individual pistons would be at different phases of their stroke. We will get a conservative (too large) value for the inlet and discharge volumes if we assume that—

1. The inlet port angular spacing is $\frac{2\pi}{N}$ radians.
2. The inlet for a given cylinder lasts $\pi$ radians.
3. If a cylinder is open at all, its full-stroke volume is exposed.

Under these assumptions, the contribution of the motor displacement volume to the inlet or discharge volumes is

$$NR_w A_p \sin \beta$$

If this is added to the other com-
ponents of inlet or discharge volume, a $V_i$ or $V_d$ completely analogous to the similar quantities for the vane motor will be obtained.

2.3. The Complete Feedback Control System. The open-loop transfer function relating valve stem motion to motor rotation was derived for the vane motor in section 2.2.1. Exactly the same form of equation holds for the axial-piston motor. It is only necessary to substitute the equivalent parameters derived in section 2.2.2. We can now obtain the transfer functions for a complete system.

A block diagram for a complete system is shown in Figure 7. The command or reference value is in the form of a voltage $E_r$. The controlled variable $\Theta_c$ (same as $\Theta_m$) is measured with an electrical transducer, which has a transfer function $K_{TR}$ volts/radian. The command voltage and the voltage $E_c$ are compared in the error detector, and if not equal, an error voltage $E_\varepsilon$ is fed to the amplifier. This voltage is amplified and causes the torque motor to drive the valve stem to a position proportional to the error voltage. The amplifier and torque motor have a transfer function $K_{AMP}$ inches/volt.
We may now derive the system transfer functions.

\[(e_E)(K_{\text{AMP}}) = X_v\]  

\[
\Theta_c = \frac{K X_v}{D (1 + T_1 D) (1 + T_2 D)} \quad (2.74)
\]

\[
\Theta_c = \frac{K K_{\text{AMP}} K_{\text{TR}} X_E}{D (1 + T_1 D) (1 + T_2 D)} \quad (2.75)
\]

\[
\Theta_c = \frac{K K_{\text{AMP}} K_{\text{TR}} \Theta_E}{D (1 + T_1 D) (1 + T_2 D)} \quad (2.76)
\]

The open-loop transfer function becomes

\[
\frac{\Theta_c}{\Theta_E} = \frac{K K_{\text{AMP}} K_{\text{TR}}}{D (1 + T_1 D) (1 + T_2 D)} \quad (2.77)
\]

The closed-loop transfer function is

\[
\frac{\Theta_c}{\Theta_R} = \frac{1}{\frac{T_1 T_2}{K K_{\text{AMP}} K_{\text{TR}}} D^3 + \left(\frac{T_1 + T_2}{K K_{\text{AMP}} K_{\text{TR}}}\right) D^2 + \frac{1}{K K_{\text{AMP}} K_{\text{TR}}} D + 1} \quad (2.78)
\]
With these transfer functions available, the study of system performance and system compensation, if necessary, is a routine application of well-known analysis methods and will not be gone into here.
3.1. Scope and Purpose of the Experimental Study.

It should be clear to the reader that the analyses of chapter II involve assumptions that are very gross indeed. The main purpose of such a simplified treatment is to attempt to bring out the pertinent factors affecting the performance with a reasonable expenditure of analytical effort. It is not to be expected that numerical predictions of this theory will agree, except in order of magnitude, with the actual behavior. It is felt that such a method of attack is justified since the use of more accurate assumptions results in expressions of such complexity that their solution and interpretation appears rather hopeless at present. It is possible that more accurate assumptions of simplicity equal to those used in the present study might be discovered but the limits of time precluded extensive pursuit of this possibility.

Ideally, the experimental phase of a study such as the present one would concern itself with evaluation of the accuracy and ranges of validity of the various simplifying assumptions. In this particular case, due to the number, complexity and magnitude of the assumptions, such an approach was not deemed feasible in the light of limitations of time and available experimental facilities. It was thus
Fig. 8. Torque-Speed Curves for Air Motors with 90 Psig Supply
decided that the overall objective of the investigation would be best served by a limited study of a system assembled from commercially available components. The purpose of this test would be only to give a basis for deciding whether further study of such systems was warranted in light of their possible commercial application.

3.2. Description of the System Studied. The actual system studied consisted of:

1. A rotary air motor as the power device;
2. A ten-turn potentiometer excited with a DC voltage for measuring the angular position of the motor shaft;
3. An electronic servo-amplifier with a summing circuit for comparing voltages representing the actual and the desired position of the motor shaft. This amplifier provided a differential output current proportional to the error voltage;
4. A servo-valve driven by an electromagnetic torque motor. This device receives the differential current from the amplifier and positions the valve in proportion to it. The ports of the valve are connected to the air motor.

A block diagram of this system is shown in Figure 7. The purpose of the system is to angularly position the load shaft in response to commands $\xi_R$. 
3.2.1. Description of the Individual Components. A description of the individual components of the above system and some of the problems associated with their choice and application will now be presented.

3.2.1.1. The Air Motor. At the time this study was begun, there were no rotary air motors on the market specifically designed for servomechanism type applications. However, conventional vane and axial-piston types were available with the feature of reversibility necessary for servo-applications. It was decided to secure one motor of each type for study. (Note: It is interesting to observe that about the time of conclusion of this study, 3 new types of rotary air motors especially suited to servo-type applications were introduced by two aircraft accessory manufacturers. This development can be taken as an indication of the interest in devices of this type and the present availability of equipment undoubtedly superior to that tested in the present study. See Appendix for a brief description of these new motors.)

The vane type motor is of very simple design as shown in Figure 2. This particular motor has the steady-state performance curve shown in Figure 8. Being reversible, this motor has no net expansion of the working fluid and is thus less efficient than comparable non-reversible motors, which do have a net expansion.
The axial-piston motor has the steady state performance curve shown in Figure 8. The design of this motor is quite complex. It uses a wobble-plate and bevel gearing to transform the linear motion of the pistons into rotary motion of the output shaft. This mechanism was analyzed in section 2.2.3. The motor has four cylinders which are all valved by a single rotating air distributor which rotates with the output shaft and properly synchronizes the valve action with the piston motion. The rotation of the air distributor causes each cylinder, in turn, to go through the following cycle:

1. Intake. Cylinder open to supply. 100.7°
2. Expansion. Cylinder sealed. 36.3°
3. Exhaust through discharge port of servo valve. (A restricted exhaust.) 100.7°
4. Seal. Cylinder sealed. This seal is necessary to isolate event 3 from event 5. 36.3°
5. Exhaust to atmosphere. (This is a direct exhaust to atmosphere through a large port and serves to increase volumetric efficiency by removing low pressure air from the cylinder more effectively than could be done by event 3. 49.7°
6. Seal. Cylinder sealed. This seal is necessary to isolate event 6 from event 1. 36.3°

\[360.0°\]
(The angles given above are the nominal values for cylinder ports of zero width. Actually, since the ports have a width of 20° the above angles should each be increased by 20°.)

This motor is reversed simply by interchanging the inlet and discharge ports by shifting the four-way servo valve. The performance is identical for either direction of rotation. Also, because of the internal valving of the motor a net expansion is possible even though the motor is reversible. This is not possible with a vane type reversible motor. The main effect of the net expansion is an increase in efficiency.

Both of the above motors were selected on the basis of their being the smallest reversible motors of their type commercially available. This restriction on size was necessitated by the small air flow capacity of the servo valve. (See discussion in section 3.2.1.2.)

3.2.1.2. The Servovalve. In searching for a suitable valve, the same problem was encountered as for the motors. That is, since pneumatic actuators have not been used in such applications, valves have not been designed and manufactured for these requirements. The only metering-type valves of large flow capacity available are slow-acting process control valves. These valves would be so much slower than the air motor under study that the dynamic
effects of the valve would completely mask the behavior of the motor.

Since it was not desired to engage in the design and construction of valves, another solution had to be found. Now there are many hydraulic servovalves of varying design and capacity commercially available. These are not designed for low-pressure pneumatic use but might be suitable in this instance. The paper (8) by Lee and Shearer described a new type of valve which they developed especially for high-pressure pneumatic systems. This valve incorporates several features which make it especially suitable for such applications. It was discovered that this type of valve was actually in commercial production as a valve for both hydraulics and high pressure pneumatics. The only drawback involved in the use of this valve for our application is that the flow capacity of the valve would be quite small at low (100 psig) air pressures. Preliminary calculations showed that although the valve did not have sufficient flow capacity to drive the motors at their full power capacity, it might give some significant results. Since the calculations were of an approximate nature and this was the only valve available, it was decided to purchase such a valve.

The valve purchased had inlet and exhaust ports which each have an area of .0023 square inches when the valve is
wide open. The valve is wide open when the differential current supplied to the torque motor is 40 ma. The resonant frequency of the valve and torque motor is about 340 cycles per second, so its response speed will greatly exceed that of the pneumatic actuator.

3.3.1.3. The Servoamplifier. This amplifier is designed to be used with AC-excited transducers. It has its own 5000 cps oscillator to provide excitation for the transducer. The suppressed-carrier signal from the transducer is fed to an AC amplifier and phase inverter and then through a phase sensitive demodulator and a two-section R-C filter. This reduces the suppressed carrier signal to a D.C. signal of proper magnitude and polarity. This D.C. signal is fed to a summing circuit where it is combined with the command voltage signal to form an error signal. This error signal is fed through a conventional direct coupled DC amplifier with one stage of voltage amplification and a push-pull stage of power amplification. The output differential current is fed to the coils of the torque motor which positions the servovalve.

Since our system uses a potentiometer transducer with DC excitation, the oscillator and AC amplifier section of the amplifier were not used, and the amplifier was modified to accept the D.C. potentiometer signal directly. When
FIG. 9. OVER-ALL VIEW OF TEST SETUP
modified in this way, the amplifier has a gain adjustable from zero to 160 differential ma per volt of error signal.

An additional feature of this amplifier is the provision for adding a dither voltage at the summing circuit. This voltage is a sinusoidal voltage which is supplied by some outside source of relatively high frequency. Its purpose is to keep the static friction broken loose by superposition of a small oscillatory motion.

3.3.1.4. The Potentiometer. The potentiometer is the device which measures the angular position of the output shaft and generates a voltage signal proportional to this position. Since we wish to investigate large motions, a 10-turn potentiometer was selected. This allows a rotation of 10 revolutions before running into its stops. In selecting the total resistance of a potentiometer two conflicting considerations must be compromised. First, a high sensitivity (volts per degree of rotation) is in general desired so that high accuracy may be achieved without excessive amplifier gain. High sensitivity is obtained by exciting the potentiometer with a high voltage. This, however, is limited by the heat generated. The potentiometer which we intend to use has a thermal rating of 2 watts. This
means that the maximum excitation voltage is given by the following equation:

\[ E_{\text{max}} = \sqrt{2R} \]

where \( R \) is the total resistance of the potentiometer. It appears that any \( E_{\text{max}} \), and thus any sensitivity, could be achieved simply by choosing \( R \) large enough. This is not true, however, due to another consideration. This has to do with the linearity of the output signal of the potentiometer. The output voltage is a strictly linear function of angular position only if the output is open circuited. If the output is not open circuit but is connected to another circuit which draws current from the potentiometer, the linearity is distorted. The degree of this distortion depends on the ratio of the potentiometer resistance to the resistance of the attached circuit. If this ratio is very low, the distortion will be negligible. A compromise must be struck and for this application it resulted in a potentiometer resistance of 10,000 ohms. This means that the maximum excitation voltage based on heat limitations is 141 volts. The maximum sensitivity of this potentiometer would thus be 0.039 volts per degree. With the amplifier gain at its maximum value and the potentiometer excited with its maximum voltage, the angular error necessary to cause the valve to open wide is 6.4°. If this value
of system gain does not cause absolute instability, the steady-state error must be less than 6.4°. Since this 6.4° is measured at the motor shaft, the steady-state accuracy of a practical system would usually be considerably better than 6.4°. The reason for this is that when the motor is actually used to drive a load inertia of some sort, there is a considerable gear reduction from the motor to the load. For example, if the gear reduction is 20 to 1, the accuracy of load positioning would be one twentieth of 6.4°. (This assumes the motion is measured at the motor and that the gearing itself introduces no errors.) This accuracy of 0.32° is quite respectable, so the sensitivity of the system was judged adequate.

3.2.1.5. The Load. No mention has been made of the load which the motor drove in this study. Actually no external load was used. The only load driven by the motor (other than its own inertia) was the small inertia and friction of the measuring potentiometers.

3.2.2. Auxiliary Equipment. Several pieces of auxiliary equipment which do not enter into the analysis of the closed-loop system will now be described.

3.2.2.1. The Pneumatic Power Supply. The servovalve is supplied with compressed air from a pneumatic power supply and modulates the flow of this power to the air
Value Opening Indication Circuit

Motor Shaft Angle Measuring Circuit

Step Function Switch and Summing Circuit

Fig. 10. Auxiliary Circuitry
motor in response to commands from the control system.
The pneumatic power supply used in this study consisted of-
1. A compressed air line bringing compressed air from the university power house.
2. A pressure regulator.
3. A storage tank of volume approximately 10 ft$^3$
4. A filter which used both centrifugal effects and a porous bronze filter element to remove dirt.
5. A lubricator which added small quantities of finely divided oil to the air.
The arrangement of this equipment is shown in Figure 9.
The main difficulty encountered with this equipment was the erratic variation of the pressure supply from the power house. Also, the maximum pressure which could be reliably maintained even for a short time was 80 psig, which is somewhat lower than was desired. The purpose of the large storage tank was to provide a reservoir of essentially constant pressure from which short bursts of energy could be extracted by the control system. The lubricator helps to reduce sticking of the control valve which is especially critical in pneumatic systems. It also serves to reduce friction in the air motor.

3.3.2.2. Electrical Power Supplies. The servo amplifier requires a DC voltage regulated at $\pm$ 300 volts for the plate supply and also 6.3 volts AC or DC for
FIG. 11. VIEW OF AMPLIFIER, VALVE, FILTER
AND LUBRICATOR
the heaters. These voltages were obtained from a standard commercial electronic power supply of ample current capacity.

The DC voltage for exciting the potentiometer was obtained from dry batteries.

3.3 Instrumentation. The system was tested only for its response to commands which were step changes in displacement. Thus it was necessary only to record the time-variation of the motor shaft angular position. Since a potentiometer for measuring this position is actually part of the control system, it might seem that we could simply take our voltage proportional to position from this potentiometer and feed it to an oscillograph for recording purposes. This would be possible if the oscillograph had a very high input impedance, since it would then not draw any current from the control system amplifier input circuit. However, the impedance level at the servoamplifier input is the order of 1 megohm, while the input impedance of the oscillograph is about 400 ohms. Thus, connecting the input of the oscillograph directly across the potentiometer would effectively short-circuit the amplifier input. This problem could be solved if an isolating amplifier of considerably higher input impedance than the servoamplifier were available. It could be connected between the potentiometer and the oscil-
FIG. 12. VIEW OF MOTOR, FEEDBACK POTENTIOMETER
AND MEASURING POTENTIOMETER
lograph and thus not affect the action of the control system. Such an amplifier was not available, so another solution was necessary. A simple answer to the problem was to gear another potentiometer to the present one and take the motion measurement from it. Since this potentiometer is completely isolated electrically from the control system, the above-mentioned loading problem is not present. The total resistance of this new ten turn potentiometer was selected as 100 ohms.

The oscillograph used to record the data was a light-beam type of instrument but it does not require any darkroom work to develop the paper. The paper develops itself when exposed to ordinary fluorescent light for a few seconds, thus giving the record immediately. The galvanometers used had a flat frequency response from 0 to 60 cycles per second and a sensitivity of 8.54 millivolts per inch deflection when the circuit had the correct damping resistance. The total resistance of the circuit should be 425 ohms to give the optimum damping (64 per cent of critical damping). The full scale deflection of the galvanometers is 8 inches, thus allowing rather accurate measurements from the record. Paper speeds of 0.2, 1.5 and 25 inches per second are quickly available by means of a push-button transmission. These speeds are accurate to $\pm$ 5 per cent.
This accuracy was felt to be sufficient for the present study so the time measurements on the record were based on the record drive speed, rather than using timing lines from an oscillator of known frequency.

3.3.1. Auxiliary Circuitry. Several auxiliary circuits were developed to allow proper and convenient application of step functions and measurement of the resulting response.

Since step functions of various magnitudes were to be studied, it would be necessary to adjust the sensitivity of the measuring potentiometer so that full-scale traces would be obtained for all sizes of step functions. A circuit for accomplishing this is shown in Figure 10. The 25,000 ohm potentiometer is adjusted to back out the voltage of the measuring potentiometer when the system is at its zero position. When the measuring potentiometer moves, an unbalance voltage is fed to the oscillograph. The magnitude of this voltage can be adjusted by means of the 10 ohm potentiometer. Since the impedance looking to the left at terminals a-b is less than 10 ohms, the proper damping will be obtained for the galvanometer.

The circuit arrangement used for generating the step function commands is shown in Figure 10. With the switch open, the two potentiometers are adjusted so that the desired error voltage will appear at the amplifier input terminals when the switch is closed. With the circuit shown, the
amplifier input is never open circuit. Previous to develop-
ment of the circuit of Figure 10, a somewhat simpler cir-
cuit to accomplish the same purpose was tried. In this
circuit the amplifier input was open circuit when the switch
was open and it was found that the amplifier oscillated at
high frequency under these conditions, giving out an audi-
ble screech. It was noted that this did not occur if the
amplifier input was not open circuit, so the arrangement
of Figure 10 was developed and this operated properly.

To get accurate time measurements, the initiation of
the step-function command must be clearly marked on the
oscillograph record. This can be done by applying a pulse
to another galvanometer of the oscillograph when the step
change is made. (A total of 3 galvanometers were available
in the oscillograph.) The voltage pulse cannot be obtained
from the input circuit of the amplifier since the oscil-
lograph would act as a short circuit in this high impedance
input. However, the marker pulse can be measured at the
amplifier output rather easily, and since the frequency
response of the amplifier is much higher than that of the
complete control system, the time delay in going from the
input to the output of the amplifier will be negligible.
The circuit for accomplishing this measurement is shown in
Figure 10. Note that we are actually measuring the torque
motor current, and since the valve position is directly proportional to this current, the galvanometer trace will actually represent valve position as a function of time. There are two limitations to this statement:

1. The proportionality of current and valve position holds strictly only for static conditions. However, since the frequency response of the torque motor and valve is listed by the manufacturer as 370 cycles per second, the proportionality should be very good for our relatively slow system.

2. There are two limiting processes possible in the valve motion-torque motor current relationship. That is, the valve can strike its wide-open mechanical stop while the current is still increasing. Also, the amplifier will saturate at a constant current when excessive input signals are received. Since the saturation occurs at values of current greater than those necessary to drive the valve against its stops, the galvanometer can show an increasing current when the valve is actually at a constant wide-open position.

The above limitations must be observed in interpreting the trace representing valve opening on the oscillograph record. These limitations are not important since this trace was used for qualitative study only. The main purpose
was to mark the initiation of the step change and this was done quite accurately.

3.4. The Test Program. The following list describes the experimental tests that were performed. Details of the methods of running the tests will be found in section 3.4.1.

A. Preliminary tests

1. Calibration of air tank pressure gage
2. Dimensional measurements on air motor
3. Static friction tests on air motor
4. Static tests to determine whether valve is capable of driving motor
5. Effect of dither on valve performance

B. Tests on Complete Feedback Control System

1. Step-function response for 22.5°, 45°, 360° and 720° steps
2. Effect of gain on step function response
3. Effect of dither on step-function response
4. Effect of added pneumatic damping on step-function response

3.4.1. Methods of Performing the Tests

A. Preliminary tests

1. Calibration of air tank pressure gage. The gage was calibrated by using an ordinary dead weight gage tester.
2. The air distributor was removed from the axial-piston motor and measurements made with a micrometer and 6 inch scale to determine the valve cycle in terms of angle of rotation. (See section 3.2.1.1. for results of this measurement.)

3. Static friction tests on air motor. The torque necessary to slowly turn the motor over by hand was measured by using a spring scale and lever arm fastened to the output shaft. In another test, with no external torque on the output shaft, the motor was connected to the air supply through the pressure regulator and the pressure gradually increased from zero psig until the motor started to rotate. This starting pressure was recorded.

4. Static tests to determine whether valve is capable of driving motor. The air supply, servovalve, and air motor were connected in series, open loop. The servo-valve was opened wide and the response of the motor noted qualitatively.

5. Effect of dither on valve performance. With 18 psig of supply pressure, the servovalve was caused to travel from full open in one direction to full open in the other, by the application of stepwise increasing error voltage. The motion of the valve was measured with a dial indicator with very light spring loading. The above test
was then repeated with a dither voltage of 150 cycles per second added to the error voltage. The amplitude of the dither voltage was small and did not cause any perceptible motion of the dial indicator. Touching the valve plate with the hand, however, resulted in the sensation of definite "buzz".

B. Tests on Complete Feedback Control System.

1. Step-function response for 22½°, 45°, 360° and 720° steps. The procedure for obtaining the response to a step-function of any size was as follows. First, the measuring circuit (see Figure 10) was adjusted to give nearly a full-scale galvanometer deflection for the size step being-used. The air tank pressure and the DC voltage on the control system potentiometer were measured and recorded. The command voltage was adjusted so that the neutral point would be at mid-travel (5 turns) of the control system potentiometer. This then allows 5 turns of motor rotation before the stops are reached. Since the largest commands were 2 turns (720°), this provided ample safety margin for overshooting. To provide a zero reference for output angle on the oscillograph record, the amplifier gain was turned to its highest value and the motor output shaft rotated by hand until the valve was heard to be at its neutral position, the system being fully connected as a closed loop.
This neutral position can be detected very accurately by the valve sound, and with the gain turned up high, a very small rotation of the motor shaft can be detected. With the system in this neutral position, the oscillograph record drive switch is "jogged", causing a short line to be drawn on the record by the output-angle galvanometer. This is the zero reference mark for output angle. Now, the step function switch (Figure 10) is opened, deactivating the feedback system. The gain is set at the value desired and the motor shaft is turned a number of degrees equal to the size of the step change desired away from its neutral position by hand. The number of degrees turned through is found by counting the number of teeth on the potentiometer gear that pass a fixed reference. The deflections of the galvanometer light spot in going from the neutral position to the initial error position is measured on the calibrated screen of the oscillograph. This establishes the record scale factor in degrees rotation per inch of light-spot deflection. To perform the test, the oscillograph record drive is started, allowed to get up to speed and then the step-function switch is quickly closed. The system responds, and when the response is finished, the record drive is shut off. The record is torn off and developed immediately under the room fluorescent light.
Fig. 13. Effect of Dither on Valve Hysteresis
2. Effect of gain on step-function response. The procedure outlined in part "1" above was carried out for a range of amplifier gain settings.

3. Effect of dither on step-function response. A 150 cycle per second dither voltage was added to the error voltage and several runs were made to see the effect.

4. Effect of added pneumatic damping on step-function response. It was found that additional system damping could easily be obtained by blocking the free exhaust port of the air motor. This meant that all motor exhaust had to go through the servovalve discharge port. This port is quite small and the flow restriction gives a damping effect.
CHAPTER IV
RESULTS AND CONCLUSIONS

4.1. Preliminary Tests. The static friction tests on the air motors gave the following results. The static friction torque of the vane-type motor was about 1.7 inch-pounds and was quite constant and uniform as the motor was turned over slowly. The axial-piston motor gave quite an erratic friction torque, the average value of which was about 5 inch-pounds. The variation of the friction torque with angular position can undoubtedly be charged to the action of the wobble-plate mechanism. The air pressure necessary to just start the motors turning was about 6 psig for the vane-type motor and 20 psig for the axial-piston type.

With the servovalve connected to the vane-type motor, the supply pressure was raised to 80 psig and the servovalve opened wide. The motor barely turned over, showing that this valve did not have adequate flow capacity for the vane-type motor. This may be explained as follows. In the vane-type motor, the air enters through the supply port of the servovalve, goes through the motor and then out through the discharge port of the servovalve to the atmosphere. Thus the air flow path consists of the resistance of the valve supply port, the resistance of the motor, and the resistance of the valve discharge port all in series.
TABLE 1

EFFECT OF ADDED PNEUMATIC DAMPING ON 45° STEP-FUNCTION RESPONSE

<table>
<thead>
<tr>
<th>ITEM</th>
<th>TEST NO. 1</th>
<th>TEST NO. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Added Damping ?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Damping Ratio</td>
<td>.06</td>
<td>.4</td>
</tr>
<tr>
<td>Frequency, cycles/second</td>
<td>8.7</td>
<td>7.7</td>
</tr>
<tr>
<td>Rise Time, seconds</td>
<td>.088</td>
<td>.082</td>
</tr>
<tr>
<td>Settling Time, seconds</td>
<td>.31</td>
<td>.21</td>
</tr>
<tr>
<td>Maximum Overshoot, per cent</td>
<td>98%</td>
<td>87%</td>
</tr>
<tr>
<td>Steady-State Error, degrees</td>
<td>14</td>
<td>7.6</td>
</tr>
<tr>
<td>Maximum Velocity, RPM</td>
<td>340</td>
<td>370</td>
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</table>
The leakage of the vane-type motor from inlet to discharge is so great that the motor flow resistance is actually considerably less than the flow resistance of the servo valve ports. Thus it is impossible to develop a large pressure drop across the motor since all the pressure drop occurs across the valve ports. With essentially no pressure drop across the motor, little torque can be developed. This valve-motor combination was therefore unsuitable and no further tests were run on it. This does not mean that this type of motor is necessarily unsuitable for servo applications. With a proper size valve, good performance might be attained. This test was repeated with the axial-piston motor, and considerably better performance was noted because of two factors:

1. The axial-piston motor has less leakage than the vane type, especially when starting from rest. In a vane-type motor, part of the sealing force at the vane-housing contact is due to centrifugal force, which is zero when the motor is at rest. The axial-piston motor leakage is through the piston-cylinder fit, which is quite close and independent of speed.

2. As mentioned earlier, part of the cycle of the axial-piston motor consists of a free discharge directly
<table>
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<th>TEST NUMBER</th>
<th>5</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
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<tbody>
<tr>
<td>Gain, percent</td>
<td>5</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Overshoot, degrees</td>
<td>39</td>
<td>44</td>
<td>44</td>
<td>42</td>
<td>47</td>
<td>44</td>
<td>49</td>
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<td>Overshoot, percent</td>
<td>87</td>
<td>97</td>
<td>97</td>
<td>94</td>
<td>105</td>
<td>97</td>
<td>109</td>
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<tr>
<td>Settling time, seconds</td>
<td>.21</td>
<td>.22</td>
<td>.23</td>
<td>.28</td>
<td>.29</td>
<td>.29</td>
<td>.29</td>
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<tr>
<td>Rise time, seconds</td>
<td>.082</td>
<td>.078</td>
<td>.080</td>
<td>.082</td>
<td>.082</td>
<td>.080</td>
<td>.080</td>
</tr>
<tr>
<td>Steady-state error, degrees</td>
<td>7.6</td>
<td>0</td>
<td>1.8</td>
<td>11</td>
<td>8</td>
<td>2.2</td>
<td>7.9</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>.4</td>
<td>.07</td>
<td>.07</td>
<td>.06</td>
<td>.06</td>
<td>.07</td>
<td>.06</td>
</tr>
<tr>
<td>Frequency, cycles per second</td>
<td>7.7</td>
<td>10</td>
<td>10</td>
<td>10.4</td>
<td>10</td>
<td>8.7</td>
<td>9.1</td>
</tr>
<tr>
<td>Maximum velocity, RPM</td>
<td>370</td>
<td>370</td>
<td>380</td>
<td>370</td>
<td>380</td>
<td>390</td>
<td>380</td>
</tr>
</tbody>
</table>
to atmosphere, \(\textit{not}\) through the discharge valve port). This would reduce the back pressure and give a greater torque.

Although the valve did not have adequate capacity for driving the axial-piston motor at full speed, the performance was sufficiently good to warrant further tests on the complete system.

The tests of the effect of dither on servo valve performance showed a considerable decrease in hysteresis when dither was used. These results are shown in Figure 13.

4.3. Tests on Complete Feedback Control System. The results of tests on the complete feedback control system using the axial-piston air motor will now be presented.

Test No. 1 was a 45° step function. The supply pressure was 80 psig and the gain was 5 per cent valve opening per degree of angular error. The form of the record of motor shaft angle versus time is shown in Figure 14. The response is seen to be quite oscillatory and the rise time (time to the peak of the first overshoot) is 86 milliseconds. The settling time (time to reach new steady state) is 310 milliseconds. The maximum overshoot is 98 per cent. For purposes of rough comparison, the response may be considered to be that of a linear, second-order system. For such systems, the damping ratio,
zeta, can be obtained by measuring two successive over-
shoots and finding their ratio. If this is done for
this system, zeta is found to be 0.06, which represents
a very lightly damped and thus highly oscillatory system.
The frequency of oscillation of the system is measured
as 8.7 cycles per second. For the 45° step change the
system exhibits a 14° steady-state error. This is due
mainly to motor leakage and coulomb friction. With a
gain of 5 per cent valve opening per degree of error, a
steady-state error of 14° means that the system can come
to rest with the valve 70 per cent open. This again
emphasizes the inadequate flow capacity of the valve,
which was mentioned earlier. The maximum velocity at-
tained can be estimated by measuring the slope of the
curve. This was about 240 revolutions per minute. Since
the size of the step was 45° and the gain was 5 per cent
valve opening per degree, the valve operated in a non-
linear fashion since it was wide (100 per cent) open
part of the time. The way that the motion comes to an
abrupt halt on the third overshoot is further evidence
of strong non-linear behavior. A linear system with a
damping ratio of 0.06 would have had many more cycles of
oscillation before coming to rest. This "sticking" of
the system is due primarily to a relatively large amount
Fig. 17. Test No. 19

Motor Shaft Angle

Time
### TABLE 3

**EFFECT OF GAIN AND ADDED PNEUMATIC DAMPING ON 360° STEP-FUNCTION RESPONSE**

<table>
<thead>
<tr>
<th>TEST NUMBER</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Added Damping?</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Gain, Percent</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Overshoot, degrees</td>
<td>0</td>
<td>0</td>
<td>32</td>
<td>32</td>
<td>31</td>
<td>9</td>
</tr>
<tr>
<td>Overshoot, percent</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Settling time, seconds</td>
<td>.20</td>
<td>.19</td>
<td>.26</td>
<td>.39</td>
<td>.39</td>
<td>.33</td>
</tr>
<tr>
<td>Rise time, seconds</td>
<td>.20</td>
<td>.19</td>
<td>.20</td>
<td>.18</td>
<td>.19</td>
<td>.23</td>
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<tr>
<td>Steady-state error, degrees</td>
<td>16</td>
<td>7</td>
<td>14</td>
<td>9</td>
<td>13</td>
<td>12</td>
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<tr>
<td>Damping ratio</td>
<td>1</td>
<td>1</td>
<td>.24</td>
<td>.22</td>
<td>.18</td>
<td>.40</td>
</tr>
<tr>
<td>Frequency, cycles per second</td>
<td>0</td>
<td>0</td>
<td>7.0</td>
<td>7.2</td>
<td>7.7</td>
<td>8.0</td>
</tr>
<tr>
<td>Maximum velocity, RPM</td>
<td>480</td>
<td>490</td>
<td>490</td>
<td>540</td>
<td>510</td>
<td>410</td>
</tr>
</tbody>
</table>
MOTOR SHAFT ANGLE

TEST NO. 19

TIME

TEST NO. 21

TEST NO. 23

TEST NO. 24

Test No. 18. Tests No. 19, 21, 23 & 24
of coulomb friction inherent in the wobble-plate design. This is accentuated by the variability of the motor torque coefficient with angle of rotation due to changes in the mechanical advantage of the mechanism as it rotates. The leakage also varies with angle, and all these effects together give the motor a tendency to stick at some position as it slows down.

Test No. 2 was a repeat of test No. 1 and all measured characteristics agreed with 5 per cent.

Tests No. 3 and 4 were identical with 1 and 2 except that a dither voltage was applied to the valve. No noticeable improvement in performance was noted. This is probably due to the fact that the fault lies in the motor, not the valve. The dither voltage is sufficient to keep the valve friction broken loose, but the limited flow capacity of the valve does not allow sufficient oscillatory pressure to develop to keep the motor friction from causing stickings. As a matter of fact, there is some question as to the desirability of dithering such a motor, since the ball-socket joints of the wobble-plate would be subjected to considerable wear and possible fretting fatigue.

Test No. 5 was a repeat of No. 1 except for one significant difference. In test No. 5, the free exhaust port of the motor was plugged, forcing all the exhaust
gas to travel through the discharge port of the servo-valve. It was hoped that this restricted exhaust would increase the system damping. Figure 15 shows the results of test No. 5. The damping is clearly better and the speed of response has not been noticeably deteriorated. For purposes of comparison, the data are tabulated in Table 1.

Test No. 8 was a repeat of No. 5 and the numerical values of response characteristics were in good agreement for both tests.

Tests No. 7 and 8 were repeats of test No. 5 with dither added. No significant effect was noticed, so no further tests were run using dither.

Test No. 9 was a repeat of test No. 5 (45° step with added damping) but with a higher gain setting on the amplifier. For test No. 5 the gain was 5 per cent valve opening per degree of angular error, whereas in test No. 9 the gain was 8 per cent. Figure 16 shows the response for test No. 9. We see that the higher gain has overcome the added damping, making the system quite oscillatory again. Test No. 10 was a repeat of test No. 9 and agreed with it. Tests No. 11 and 12 were repeats of test No. 5 with still higher gain, in this case 10 per cent. Figure 16 shows the response for test No. 11. Tests No. 13 and 14 used the highest gain available, 14 per cent, and the
response for test No. 13 is shown in Figure 16. Table 2 on page 93 allows comparison of performance criteria for various values of gain, all other parameters being held constant.

Tests 9 and 10 showed the same effects of increased gain that would be expected from a linear system. That is, the damping ratio is less, the frequency of oscillation is higher, and the steady-state error is less. This trend is however, not continued for tests 11 and 12 which had still higher gain. The frequency of oscillation is only slightly higher (about 2 per cent) and the damping ratio slightly less. (It should be remembered that these terms are used very loosely and the method of determining the numerical values is rather crude.) This insensitivity of frequency and damping ratio to gain changes may be explained by the fact that the valve is thoroughly saturated for most of the first few cycles for all the values of gain used here, and since the frequency and damping ratio measurements were based on these early cycles, they show little change when the gain is varied. The most important departure from linearity, however, is exhibited by the steady-state error. An otherwise linear system with a small amount of coulomb friction will show a decrease in steady-state error with an increase in
<table>
<thead>
<tr>
<th>TEST NUMBER</th>
<th>27</th>
<th>28</th>
<th>29</th>
</tr>
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<tbody>
<tr>
<td>Added Damping?</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Gain, percent</td>
<td>10</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>Overshoot, degrees</td>
<td>35</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Overshoot, percent</td>
<td>5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Settling time, seconds</td>
<td>.56</td>
<td>.49</td>
<td>.41</td>
</tr>
<tr>
<td>Rise time, seconds</td>
<td>.31</td>
<td>.45</td>
<td>.41</td>
</tr>
<tr>
<td>Steady-state error, degrees</td>
<td>5</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>.15</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Frequency, cycles per second</td>
<td>6.8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maximum velocity, RPM</td>
<td>480</td>
<td>370</td>
<td>400</td>
</tr>
<tr>
<td>Velocity at first zero-crossing, RPM</td>
<td>480</td>
<td>250</td>
<td>280</td>
</tr>
</tbody>
</table>
Fig. 19. Tests No. 27, 28, 29

Motor shaft angle

Test No. 27

Test No. 28

Test No. 29

Time
gain. This is because the coulomb friction acts as a load torque in the steady-state, and errors due to load torque are easily shown to be decreased by increasing gain. The present system, however, shows radically different performance in tests 11 and 12. An increase in gain caused an increase in steady-state error of considerable magnitude. This can be explained in the following way. The steady-state error is determined by the position at which the motor finally "sticks" when it comes to rest. This in turn is determined by the momentum of the moving parts, the torque due to air pressure and the friction. For a given size of step, say 45°, it is entirely possible that these effects would combine in such a way that a larger gain would give more steady-state error than a smaller gain. For the same gain settings, a different size of step might reverse these results. Tests No. 13 and 14 also showed similar erratic behavior. Tests 11 and 14 show steady-state errors which indicate that the valve was wide open and the motor stationary in steady state. This behavior can be charged to the small flow capacity of the valve, and the variable friction, mechanical advantage and leakage of the motor.

Tests No. 15, 16, 17 and 18 were 45° step changes at 80 psig pressure, but with the free exhaust port open, rather than plugged. The respective gains were 8, 10, 13
and 14. All these responses were wildly oscillatory, but not absolutely unstable. In an attempt to cause sustained or divergent oscillations, the supply pressure was raised to 90 psig. (This could only be held for a short time and could not always be attained, so 80 psig was the standard pressure used for all the other tests.) Test No. 19 was run at this pressure with a gain of 14. Figure 17 shows the response, which is seen to be bordering on absolute instability. A slight further increase in gain or pressure would certainly cause sustained or divergent oscillations.

Tests 19 through 24 were 360° step functions with supply pressure of 80 psig and various gains. The effect of added pneumatic damping is also studied. These results are summarized in Table 3 and Figure 18. The response curves show that a velocity saturation occurs since the valve is wide open long enough for the motor to attain a steady speed. Plugging the free exhaust port reduces this steady speed only slightly. The damping is seen to be much better for a 360° step than for a 45° step and would be acceptable for many applications even without added pneumatic damping. Plugging the free exhaust port to give added damping makes a significant improvement with little loss in speed of response. Tests 19 and 20 seem to give good response even though the gain is quite low. Ordinar-
ily, such a low gain would result in large steady-state error, but such does not seem to be the case here. Actually, some 45° step-function tests were tried with these low gains, but the motor barely responded so no data was taken. It is felt that the gains of 2 and 3 are thus really not acceptable and that the seemingly good performance of tests 19 and 20 can be explained as follows. For a 360° step, a gain of 2 per cent valve opening per degree of error means the valve will be wide open long enough to get the motor up to a good speed (480 RPM). When the motor gets close to the desired position, the valve is shut down partially but the motor has considerable momentum to carry it on without the aid of torque due to air pressure. However, it does decelerate and comes to rest quite close to the desired value. Thus, the relatively small steady-state error is due to the magnitude of the command, and the performance for smaller steps or gradual variations would be poor, making these low gains unsuitable for practical applications. Comparing the 45° step response with the 380° step response, say tests 11 and 22, we see that the overshoot is less in both actual value and also in per cent of command for the larger size command. For a linear system, the per cent overshoot is the same for all sizes of steps, and thus the actual value of
the overshoot is directly proportional to the size of step. In the system under study, however, the velocity is limited at some saturation value, and thus commands larger than a certain value do not cause larger velocities. Also, there is a saturation of acceleration before the saturation of velocity. That is, once the maximum available pressure drop across the motor is established, the driving torque has reached a limit and larger command signals do not cause a larger acceleration. These deviations from linear behavior help to explain the trend of overshoot with increasing size of command for the system studied. They do not, however, entirely explain why a 360° step change could exhibit less overshoot than a 45° step change, as was noted in comparing tests 11 and 22. An additional factor which could explain this is the fact that the frictional torque resisting motion increases with speed. Thus, even though the 360° step function response has a greater maximum velocity (540 RPM) than the 45° (370 RPM) and might be expected to overshoot more because of its greater momentum, the larger frictional torque could conceivably decelerate the motor more quickly in the case of the larger command. That is, in the 45° step, when the motor is nearing the neutral position (valve closed) the driving torque is still greater than the resisting frictional torque and the motor
is still accelerating. For the 360° step, on the other hand, the torques are balanced (constant velocity) when the motor crosses the neutral position.

Tests No. 25 and 26 were 22½° step-function without added pneumatic damping and with supply pressure of 80 psig. Test No. 25 had a gain of 10 and Test No. 26 had a gain of 14. Both tests showed large overshoots and undershoots and strong coulomb friction effects ("sticking"). No further tests were run at this size of step since it was felt that the size of command was too close to the average steady-state error (about 10 degrees) exhibited in all the other tests to give reliable results.

Tests No. 27, 28 and 29 were 720° step-function tests with a supply pressure of 80 psig. Table 4 and Figure 19 summarize the results of these tests. The only new aspect of behavior noticed in these tests was the deceleration occurring before the first zero-crossing for tests 28 and 29, which had the free exhaust port plugged to give added damping. This characteristic was not present in the comparable 360° tests to any great extent. It appears then that with the free exhaust port plugged, the system does not come to its final velocity within one revolution or even two revolutions. The system accelerates to its maximum velocity in about one-fourth revolution and maintains
this velocity until the end of the first revolution. Then a deceleration takes place during the second revolution, reducing the speed by about 25 per cent. To get the true steady-state speed corresponding to 100 per cent valve opening, tests of 4 or 5 revolution size would be necessary.

Tests 27, 28 and 29 were the last tests run on this system.

4.3. General Conclusions. The tests show the effects of several important non-linearities in the system. The saturation due to the limiting of the valve flow is apparent for the larger signals. This type of behavior is not at all unusual for servomechanisms, since electric and hydraulic amplifiers and actuators all exhibit saturation-type behavior for large signals. To compare analytical performance predictions based on linear models with experimentally measured behavior it is necessary to restrict the study to small signals. In this system, however, the additional non-linearity due to large coulomb friction has such a strong effect, especially for small signals, that comparison to linear predictions is difficult.

It is clear that if systems such as that studied here are to become practical, suitable valves must be developed. Several problems are involved here. The main one, of course,
is simply that of flow capacity. Valves of sufficient flow capacity and dynamic response are necessary to fully realize the potential of rotary pneumatic actuators. The dynamic response need not be nearly as good, however, as that of the valve used in the present investigation. Since the motor has a frequency of oscillation of the order of 10 cycles per second, the valve need not have a dynamic response exceeding this.

The present investigation involved use of a valve of inadequate flow capacity and a supply pressure of 80 psig. If larger valves were available and the usual 100 psig pressure were used, faster response could be expected. However, proper system damping would then very likely become an even more serious problem. If the system were still partially electrical, compensating networks might profitably be employed for added stabilization. A simpler and more desirable solution would be the pneumatic stabilizing tanks discussed in reference 17.

In designing pneumatic motors for feedback applications, considerations of coulomb friction and leakage seem to be most critical. With a larger valve, the vane-type motor might give good performance, since its coulomb friction is low and uniform. The new air motors which have just recently been introduced by two manufacturers of aircraft accessories show considerable promise for feedback
applications. Not only has friction been reduced, but these motors have attached their own control valves, thus solving two of the biggest problems. No dynamic response measurements on these motors as used in feedback systems were available at the time of this writing, but their specifications (see Appendix) certainly lead one to expect characteristics better than those of the system studied here.

Whether rotary pneumatic actuators driven with 100 psig air are suitable for any given industrial application will depend on the specifications for that particular application. It is not within the scope of this study to make such specific recommendations but rather to give an indication of the character of the performance to be expected from such systems.
APPENDIX

RECENT DEVELOPMENTS IN
AIR MOTOR DESIGN
An indication of the increasing interest in rotary pneumatic servos is evidenced by the recent development of three new types of air motors. Development work has been going on for several years on these new designs, but information was released to the general public too late to include detailed analyses in this dissertation. Although no analyses or results of tests on feedback applications of these motors have been published, the author feels sure this work has been done and will be published in the near future by the companies responsible for the development of these motors. Since these motors have been specifically developed for servo applications, their performance is undoubtedly superior to that of the motor tested in this investigation. However, since these new motors were developed for high temperature aircraft applications, the prices may be somewhat out of line with industrial standards. For the sake of completeness, these motors are here briefly described.

One pneumatic power unit is a gear-type air motor and thus gives no expansion of the air as it travels through the motor. It is available in horsepower ratings of .6 to 30 HP and operates on gas pressure of from 10 to 300 psig. The motor is designed to run in 1000°F environment and on 1000°F inlet air and has its own integral four-way control
valve. The torque is proportional to the supply pressure and the leakage and friction are said to be unusually low. A high torque-to-inertia ratio and dynamic braking (due to compressor action when the valve is closed) allow quick starting and stopping.

Another new motor, the cam-piston motor, is completely novel in design. Three pistons whose faces are in the shape of cams are mounted in line on a single ball-bearing spline. Air pressure is admitted to the "cylinder" spaces in proper sequence by a rotating distributor. As the pistons move axially along the spline, they contact cam rollers fixed to the motor housing, and thereby cause the spline shaft to rotate. The valving arrangement and cam shape is such that the "cylinders" are double acting, thus giving the motor 18 power strokes per revolution. This gives a torque which is very uniform with rotation of the motor. This motor is available in horsepower ratings from 4.5 to 20 HP and can be used in temperatures from -65°F to 100°F. Supply air pressure may be from 10 to 200 psig.

Finally, the nutating-disk air motor is similar in operation to the common nutating disk water meter. Two nutating disks are used and bevel gearing is necessary to prevent rotation of the disks about their own axes. Spe-
cial internal valving is necessary to give the proper inlet-exhaust sequencing. By operating the two disks 180° out of phase, inertia unbalance is said to be eliminated. The construction of the motor is such that no clearance volume is necessary. Large inlet and exhaust ports allow the use of low supply pressures with good performance. In comparison to the cam-piston motor, the nutating-disk motor gives faster response and is suitable for higher speeds, but is limited in operating temperature to 500°F. This motor is available in horsepower ratings from 13 to 53 HP. Operating pressure is from 5 to 120 psig.
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I, Ernest Otto Doebelein, was born in Durlach, Germany, November 25, 1930. My parents and I came to the United States when I was three years old. We settled in Cleveland, Ohio, where I attended school through the fourth grade. We then moved to North Ridgeville, Ohio, where I finished my secondary school education. I attended Case Institute of Technology from 1948 to 1952, receiving my B. S. in Mechanical Engineering in June, 1952. I then attended the Ohio State University, financing my education by means of a half-time job as Research Assistant in Mechanical Engineering. I obtained my M. S. in 1954 and immediately started on my Ph. D. program. At the same time, I accepted a position as full-time instructor in Mechanical Engineering. I held this position for four years while completing requirements for my Ph. D. degree.