DHAKA, Vir Abhimanyu Singh, 1932—
THE EFFECT OF A MAGNETIC FIELD ON THE
FORWARD CHARACTERISTICS OF LONG GER­
MANIUM JUNCTION DIODES.

The Ohio State University, Ph.D., 1962
Engineering, electrical

University Microfilms, Inc., Ann Arbor, Michigan
THE EFFECT OF A MAGNETIC FIELD ON THE
FORWARD CHARACTERISTICS OF LONG
GERMANIUM JUNCTION DIODES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Vir Abhimanyu Singh Dhaka, B.Sc., M.Sc.(Phys.), M.Sc.(E.E.)

The Ohio State University
1962

Approved by

[Signature]
Advisor
Department of Electrical Engineering

PLEASE NOTE:
Figure pages are not original copy. They tend to "curl". Filmed in the best possible way.
University Microfilms, Inc.
ACKNOWLEDGEMENTS

The author is deeply grateful to his adviser, Professor M. O. Thurston, for his continued guidance and helpful suggestions. The author also wishes to express his gratitude to Professor E. M. Boone for his active interest, encouragement and understanding.

The author wishes to take this opportunity to thank his colleagues in the Electron Device Laboratory for their friendship and technical assistance. Special thanks are due to Mr. Henry Pagean for his willing cooperation and help in all the laboratory and machine work.

The financial support of Air Research and Development Command of the United States Air Force is gratefully acknowledged.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>11</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>ix</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Magnetoresistance</td>
<td>1</td>
</tr>
<tr>
<td>THEORETICAL WORK</td>
<td>9</td>
</tr>
<tr>
<td>Preliminary Considerations</td>
<td>9</td>
</tr>
<tr>
<td>Analysis of an Unsymmetric p-n Junction with Ohmic Contacts at both Ends in a Magnetic Field</td>
<td>16</td>
</tr>
<tr>
<td>Analysis of an Unsymmetric pnn Junction in a Magnetic Field</td>
<td>25</td>
</tr>
<tr>
<td>Discussion of Theoretical Evaluations and other Considerations</td>
<td>35</td>
</tr>
<tr>
<td>EXPERIMENTAL WORK</td>
<td>41</td>
</tr>
<tr>
<td>Diode Fabrication</td>
<td>41</td>
</tr>
<tr>
<td>Experimental Investigations of the Diodes in the Magnetic Field</td>
<td>46</td>
</tr>
<tr>
<td>Galvanomagnetic Amplifier Using a long P-n Junction as its Active Element</td>
<td>64</td>
</tr>
<tr>
<td>Input Power</td>
<td>68</td>
</tr>
<tr>
<td>Equivalence for the Output Circuit</td>
<td>70</td>
</tr>
<tr>
<td>Deduction of $\Delta R$ in Terms of $\Delta B$</td>
<td>72</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>--------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Measurements and Results of the Operation of the Galvanomagnetic Amplifier</td>
<td>75</td>
</tr>
<tr>
<td>Application of a Thin Rectangular Diode for Measuring Magnetic Fields</td>
<td>78</td>
</tr>
<tr>
<td>Discussion of Additional Applications</td>
<td>84</td>
</tr>
<tr>
<td>CONCLUSION</td>
<td>88</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>89</td>
</tr>
<tr>
<td>LIST OF REFERENCES</td>
<td>95</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>96</td>
</tr>
<tr>
<td>AUTOBIOGRAPHY</td>
<td>99</td>
</tr>
</tbody>
</table>
LIST OF TABLES

1. Characteristic Properties of the Tested Diodes 49
2. Effect of Magnetic Field on Diode Current 54
3. Dependence of the Diffusion Length on the Magnetic Field 58
4. Output Voltage of the Diode for Different Magnetic Fields 83
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Illustration</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Diagram showing the initial deflection of electrons in a magnetoresistance element by a magnetic field (perpendicular to the plane of the page) to create a transverse electrical field.</td>
<td>2</td>
</tr>
<tr>
<td>2. Diagram showing the transverse current produced by shorting the Hall voltage.</td>
<td>4</td>
</tr>
<tr>
<td>3. Schematic diagram of a rectangular p-n junction diode with ohmic base contact.</td>
<td>6</td>
</tr>
<tr>
<td>4. Injected minority carrier distribution in the base region of the diode with a ohmic base contact.</td>
<td>6</td>
</tr>
<tr>
<td>5. Schematic diagram of a rectangular p-n junction diode with a majority carrier base contact.</td>
<td>27</td>
</tr>
<tr>
<td>6. Injected minority carrier distribution in the base region of the diode with a majority carrier base contact.</td>
<td>27</td>
</tr>
<tr>
<td>7. The relative resistance change in the magnetic field due to different geometrical form. R_B is the resistance in the magnetic field and R_0 is the resistance at zero flux density.</td>
<td>38</td>
</tr>
<tr>
<td>8. The Jig used for Alloying.</td>
<td>42</td>
</tr>
<tr>
<td>9. The Setup Used for Alloying.</td>
<td>44</td>
</tr>
<tr>
<td>10. The Setup Used for Measuring the Forward Characteristics of the Diodes in the Magnetic Field.</td>
<td>47</td>
</tr>
</tbody>
</table>
11. The shaft used to hold the diode in the magnetic field.

12. Forward Characteristics of XR 30-1 Diode for Varying Values of the Magnetic Field Shown in Table 2.

13. Forward Characteristics of XR 30-2 Diode for Various Values of the Magnetic Field Shown in Table 2.

14. Forward Characteristics of XR 30-0 Diode for Various Values of the Magnetic Field Shown in Table 2.

15. Forward Characteristics of XR 20-0 Diode for Various Values of the Magnetic Field Shown in Table 2.

16. Forward Characteristics of XR 20-2 Diode for Various Values of the Magnetic Field Shown in Table 2.

17. Forward Characteristics of XR 20-1 Diode for Various Values of the Magnetic Field Shown in Table 2.

18. Forward Characteristics of XR 30-1 Diode for different Magnetic Field Strengths.

19. Forward Characteristics of XR 30-1 Diode for Different Field Strength with Current on Log Scale 1) 0 gauss 2) 500 gauss 3) 1000 gauss 4) 2000 gauss 5) 4000 gauss 6) 10,000 gauss.


22. Dependence of the Voltage Across the Diode XR 30-1 on the Magnetic Field Intensity for Different Currents.

24. Complete Assembly of the Galvanomagnetic Amplifier.  66
25. Disassembled Galvanomagnetic Amplifier.  67
26. Equivalent Output Circuit of the Galvanomagnetic Amplifier.  74
27. Equivalent Output Circuit of the Galvanomagnetic Amplifier.  74
28. Forward Characteristics of the Diode for Different Values of the Magnetic Field Normal to the Broad Face
   1) 0 gauss
   2) 500 gauss
   3) 1000 gauss
   4) 2000 gauss
   5) 3000 gauss
   6) 5000 gauss
   7) 7000 gauss.  80
29. Forward Characteristics of the same Diode as above for the same Field Strengths, but not the Field being Normal to the Narrow Face of the Diode.  80
30. The circuit for obtaining a.c. output by giving rocking motion to the diode in the magnetic field.  81
31. A typical a.c. output for 500 gauss magnetic field in the above arrangement.  81
32. The arrangement for the rocking motion of the diode.  82
33. Diagram showing the use of the diode as a pressure gauge.  85
34. The diode used as a fluxmeter.  85
35. The diode used as a regulator.  87
36. The diode used for automobile ignition timing.  87

viii
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
<td>Area of the central leg of the core</td>
</tr>
<tr>
<td>$B$</td>
<td>Magnetic Flux density</td>
</tr>
<tr>
<td>$B_c$</td>
<td>Amplitude of the alternating flux density in the core of the coil</td>
</tr>
<tr>
<td>$B_e$</td>
<td>Amplitude of the alternating flux density in the diode</td>
</tr>
<tr>
<td>$B_o$</td>
<td>Bias flux density in the diode</td>
</tr>
<tr>
<td>$D_p$</td>
<td>Diffusion constant of the holes in the n-type semiconductor</td>
</tr>
<tr>
<td>$C$</td>
<td>A factor in the exponent of high injection diode equation</td>
</tr>
<tr>
<td>$E, E_0$</td>
<td>Applied direct voltage</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Alternating voltage impressed on the magnetic coil (rms)</td>
</tr>
<tr>
<td>$G_m$</td>
<td>Maximum power gain</td>
</tr>
<tr>
<td>$H$</td>
<td>Magnetic Field strength</td>
</tr>
<tr>
<td>$I$</td>
<td>Alternating current through the diode (rms)</td>
</tr>
<tr>
<td>$I_c$</td>
<td>Alternating current through the coil (rms)</td>
</tr>
<tr>
<td>$I_{dc}$</td>
<td>Direct current through the diode</td>
</tr>
<tr>
<td>$J$</td>
<td>The current density through the diode</td>
</tr>
<tr>
<td>$J_n$</td>
<td>Electron current density through the diode</td>
</tr>
<tr>
<td>$J_p$</td>
<td>Hole current density through the diode</td>
</tr>
</tbody>
</table>
$J_{S1}$ Saturation current density through the diode when both contacts are ohmic

$J_{S2}$ Saturation current density through the pnn diode

$K$ Ratio $\frac{B_c}{B_e}$

$L$ Inductance of the magnetic circuit

$L_p$ Effective diffusion length of the holes in the n-type base region

$N$ Density of the donors

$P_o$ Output power

$P_{om}$ Maximum output power

$Q$ Figure of merit of the input circuit $\frac{L}{R_c}$

$R$ Resistance of the diode at the operating point

$R_B$ Resistance of the diode in the magnetic field

$R_c$ Alternating current resistance of the input circuit

$R_L$ Load resistance

$R_o$ Resistance of the diode at zero field

$S$ The slope of the $R$ versus $B$ characteristics for the diode

$T$ Temperature in degrees Kelvin

$V, V_e$ Total voltage applied across the diode

$V_o$ Voltage across the junction

$Z_l$ Input impedance of the magnetic circuit

$b$ Ratio of electron mobility to hole mobility

$d$ Length of the diode

$f$ Frequency
\( i_\text{e} \) Instantaneous current through the diode

\( i_x \) Instantaneous current in the input circuit

\( k \) Boltzmann constant

\( n \) Density of electrons in the conduction band

\( n \) Degenrate n-type semiconductor

\( p \) Density of holes in the valence band

\( p \) Density of holes in the valence band

\( p_n \) Equilibrium density of holes in the n-type base

\( q \) Electronic charge

\( p \) Mobility of holes

\( n \) Mobility of electrons

\( \rho \) Resistivity of semiconductor material

\( o \) Initial resistivity of the semiconductor

\( p \) Lifetime of the injected holes in the n-type base

\( \text{Instantaneous alternating flux in the core in Maxwells} \)

\( m \) Amplitude of the alternating flux in the core in Maxwells

Angular velocity
INTRODUCTION

The resistance of metals at low temperatures changes when the metal is subjected to a magnetic field. The effect is noticeable at room temperature in some metals, intermetallic compounds, and semiconductors and is strongest in materials with high charge carrier mobilities. This effect has been noticed in bismuth, germanium, silicon, lead sulphide, indium antimonide, indium arsenide, and gallium arsenide.

Magnetoresistance

This phenomenon of magnetoresistance is closely associated with the Hall effect discovered by E. H. Hall in 1879. When a conductor carrying an electric current is placed in a magnetic field perpendicular to the direction of the current, the current carriers are deflected towards the sides of the conductor, as shown in Figure 1. As a result of this deflection there is a piling up of electrical charge on the sides of the conductor. This creates a potential difference across the sample perpendicular to the plane of the magnetic field and the applied electric field. This potential difference is called the Hall voltage.
Fig. 1. Diagram showing the initial deflection of electrons in a magnetoresistive element by a magnetic field (perpendicular to the plane of the page) to create a transverse electric field.
Another result of this deflection of the charge carriers is an increase in the resistance of the conductor. This magnetoresistance results mainly from the fact that not all the current carriers have the same velocity. For a given magnetic field and its associated Hall field only those current carriers which move with the average velocity will move in a straight undeflected path along the conductor. The current carriers which have velocities larger or smaller than the average will be deflected and traverse a longer path. Thus their contribution to the conduction will be reduced and the resistance of the conductor will be increased.

The magnetoresistance effects described are characterized by three factors: (1) existence of a transverse (Hall) electric field, (2) absence of transverse current, and (3) no change in the number of current carriers. It is possible, however, to arrange conditions so that the Hall electric field is made zero or very small. This is shown in Figure 2, where Hall electrodes which are placed on the sides of the conductor are shorted. A transverse current flows and the existing Hall voltage is reduced to zero and thus the Hall field which was balancing the Lorentz force on the current carriers vanishes, increasing the extent to which the carriers are deflected and also the number of the carriers which are deflected. This is because there is
Fig. 2. Diagram showing the transverse current produced by shorting the Hall voltage.
little or no voltage preventing this deflection. However, the number of the current carriers involved in the conduction process does not change because the predominant conduction is by majority carriers.

However, if a conduction process based upon the minority carriers is subjected to a transverse magnetic field, it can be shown (see Appendix) that the number of the current carriers and their distribution depend upon the effective diffusion length of the minority carriers in the base region. This effective diffusion length is in turn dependent upon the mobility of the current carriers. So any change in the magnetic field results in a change in the number of the current carriers available for conduction and, thus, enhances the magnetoresistance effect. Also, at high injection levels, the Hall voltage is not sufficient to counterbalance the magnetic deflection of the current carriers and this results in further enhancement of the effect.

The conduction in a forward biased p-n junction is by the minority carriers injected from the p-type region into the n-type base. A schematic of such a diode is shown in Figure 3 and the distribution of the injected carriers is shown in Figure 4.

In the study undertaken, long germanium diodes were studied in magnetic fields at room temperature. The magneto-
Fig. 3. Schematic diagram of a rectangular p-n junction diode with ohmic base contact.

Fig. 4. Injected minority carrier distribution in the base region for the diode with ohmic base contact.
resistance in a rectangular p-n diode specimen was found to be comparable to the magnetoresistance effect in Corbino disk samples of indium antimonide (mobility $\approx 100,000 \text{ cm}^2/\text{volt sec}$) and this fact is of importance since a rectangular geometry is preferable to a disk design in many applications. Also the temperature sensitivity at room temperature is better for a germanium device than an InSb device. For several cases, sensitivity figures are also compared with InSb and GaAs Hall effect devices and are shown favorable. One additional advantage is that this is a two-terminal device in comparison to a four-terminal Hall effect device.

Several practical applications are considered. They can be classified according to the method of producing the magnetic field. One group utilizes a constant magnetic field usually obtained from a permanent magnet and acts as a transducer of mechanical motion into changes in electrical resistance. Another group of devices uses a variable magnetic field created by an electromagnet consisting of an excitation coil, a core structure, and a small air gap in which the magnetodiode is placed.

Typical applications are: transducers, magnetic flux measurements and control, solid state electronic controls, serve controls, contactless potentiometers, solid state d.c. to a.c. converters, magnetodiode amplifiers.
Theoretical expressions for the effect have been obtained in this study and devices made on germanium have been tested in the magnetic field. Two applications of this device were investigated: (1) as an active element in a galvanomagnetic amplifier and (2) as a fluxmeter for the measurement of both a.c. and d.c. magnetic flux.
THEORETICAL WORK

Preliminary Considerations

When a sharply unsymmetric p-n junction diode with ohmic contacts at both ends is forward biased and the level of injection is high, the current voltage characteristics have the form:

\[ J = J_{s1} \left( e^{\frac{qV}{kT}} - 1 \right) \]  

(1)

where \( q \) is the electronic charge; \( J \) is the current density through the diode; \( V \) is the total voltage across the diode; \( k \) is the Boltzmann constant; \( T \) is the absolute temperature in degrees Kelvin; and

\[ b = 2 \frac{Cosh \frac{d}{L_p}}{b + 1} \]  

(2)

\[ b = \frac{\mu_n}{\mu_p} \]  

is the ratio of the mobilities of electrons and holes; \( d \) is the length of the diode; \( L_p \) is the effective diffusion length. \( J_{s1} \) is the saturation current density which is a complex function of the parameters of the diode and of the material. For the case of an ohmic base contact, Rediker and Sawyer have shown that the value of \( J_{s1} \), when the base is n-type, is

\[ J_{s1} = \frac{g_{np}D_p}{L_p} \cosh \frac{d}{L_p} \]  

(3)
where $L_p$ is the diffusion length of the injected holes in the n-type base, $D_p = \frac{kT \mu_p}{q}$, and $p_n$ is the equilibrium concentration of minority carriers in n-type base. $J_{S1}$ can, therefore, be written as

$$J_{S1} = \frac{kT \mu_p p_n}{L_p} \cosh \frac{d}{L_p}$$

(4)

The current-voltage relationship given by equation (1) can be simplified for two limiting cases

1) When $\frac{d}{L_p} \ll 1$.

This condition is true in the case of most industrial diodes. For this case $\cosh \frac{d}{L_p} \approx 1$ and $C = 2$, and equation (1) reduces to

$$J = J_{S1} \left( e^{\frac{qV}{2kT}} - 1 \right) .$$

In this case the distribution of injected carriers as a function of distance from the junction and consequently the forward current of the diode does not depend on the diffusion length. Any change in the forward characteristics of such a diode because of an applied magnetic field is because of the change in $J_{S1}$. This was observed by Kaufman$^3$ for the case of industrial germanium diodes. In these diodes $0.1\% \left( \frac{4J}{J} \right) < 0.6\%$ in a magnetic field of 2800 gauss.
2) $e^{\frac{d}{L_P}} \gg 1$

In this case the coefficient $C$ in the exponent of equation (1) can be written as

$$C = 2 \left( b + \frac{\frac{1}{2}e}{L_P} \right) \frac{d}{b+1} = \frac{e^{\frac{d}{L_P}}}{b+1}$$

and in the current-voltage characteristics $C$ becomes strongly dependent on the effective diffusion length of the injected carriers. Since $C$ itself occurs in the exponent, small changes in the diffusion length lead to a very large change in the forward current.

In diodes with a large $\frac{d}{L_P}$ ratio the distribution of injected carriers and consequently the resistance of the base of the diode is determined by the diffusion length of the injected carriers. The forward current in these diodes depends basically not on the resistance of the p-n junction but on the resistance of the base region. If the diffusion length of the injected carriers becomes smaller, a decrease in the concentration of non-equilibrium carriers takes place, which in turn leads to an increase in the base resistance. This then necessitates a change in the distribution of the voltage across the diode, and if a constant voltage were applied across the diode then the voltage across the p-n junction available for injection decreases. This causes a decrease in the injected current which leads to a complementary increase in the resistance of the semiconductor requiring
further adjustment of the voltages. Because of this mutually constructive phenomenon, forward current is very sensitive to changes in diffusion length. A small change in the diffusion length of the injected carriers, therefore, affects the forward characteristics in a significant manner. Now the diffusion length

\[ L_p = \frac{\sqrt{kT}}{q} \mu_p \tau_p \]

and thus \( L_p \) depends on the mobility \( \mu_p \) and on the lifetime \( \tau_p \) of the injected carriers.

One of the ways in which mobility and, therefore, the effective diffusion length can be modulated is by applying a magnetic field transverse to the direction of the current flow. Moving holes and electrons experience a deflection in a magnetic field. As a consequence their mobility decreases which brings about a corresponding decrease in \( L_p \).

Effect of magnetic field on the current-voltage characteristics of a forward biased sharply unsymmetric p-n junction diode with ohmic contacts at both ends has been considered by Karakushan and Stafeev for the condition of a constant voltage across the junction. In what follows, this treatment has been extended to cover the case when a constant current is maintained through the diode as the magnetic field is applied. Also, an analysis of the behavior of a more likely case of an unsymmetric p-n junction with a majority carrier base contact (p nm) is made.
The only volume parameter of semiconductors that is affected by a magnetic field is the mobility of the carriers. When the applied magnetic field is perpendicular to the direction of the carrier flow, they are deflected and this reduces their mean free path in the direction of the electric field. This results in a decrease of the mobility and causes an increase of the semiconductor's resistivity in the magnetic field.

For a single crystal of semiconductor

\[ \rho = \frac{1}{q (\mu_p^p + \mu_n^p)} \]  

where \( \rho \) is the resistivity in ohm cm and \( p \) and \( n \) are the equilibrium concentrations of holes and electrons in the crystal. The above equation can also be written as

\[ \rho = \frac{1}{q \mu_p^p \left( \frac{1}{p} + \frac{1}{bn} \right)} \]  

The variation in the resistivity with change in the magnetic field, assuming that mobility is the only volume parameter changing and that \( b = \frac{\mu_n}{\mu_p} \) does not change with the applied magnetic field, can be written as

\[ \frac{d\rho}{dH} = -\frac{1}{q \left( \frac{1}{p} + \frac{1}{bn} \right)} \frac{1}{\mu_p^p} \frac{d\mu_p}{dH} \]

\[ = -\frac{1}{q \mu_p^p \left( \frac{1}{p} + \frac{1}{bn} \right)} \frac{1}{\mu_p^p} \frac{d\mu_p}{dH} \]

\[ \therefore \frac{1}{\rho^p} \frac{d\rho}{dH} = -\frac{1}{\mu_p^p} \frac{d\mu_p}{dH} \]
Here the change in the resistivity is solely because of the change in the mobility. The number of the equilibrium carriers involved in the conduction process is not affected to any appreciable extent. Any change in the diffusion length because of the change in the mobility does not enter into the picture for conduction in a semiconductor without minority injection. But this change becomes very significant when minority carrier injection is responsible for the conduction process as is the case for a forward biased p-n junction. The change in the diffusion length with applied magnetic field can be derived by using Einstein's relation which for holes is

$$\mu_p = \frac{q}{kT} D_p$$

(9)

where $D_p$ is the diffusion constant of holes in the n-type base region and

$$D_p \tau_p = L_p^2$$

(10)

$\tau_p$ = hole lifetime.

Substituting for $D_p$ from (10) and into (9) we obtain

$$L_p = \sqrt{\frac{kT}{q} \mu_p \tau_p}$$

(11)

Differentiating with respect to $H$, one obtains

$$\frac{dL_p}{dH} = \frac{1}{\hbar} \sqrt{\frac{kT}{q} \mu_p \tau_p} \frac{1}{\mu_p} \frac{d\mu_p}{dH}$$

which can be written as
Combining equations (8) and (12) yields

\[
\frac{1}{L_p} \frac{dL_p}{dH} = \frac{1}{\mu_p} \frac{d\mu_p}{dH}
\]

(12)

The distribution of the injected carriers depends upon the diffusion length and is given by the relation

\[
p - p_n = p_n (e^{\frac{qV_o}{KT}} - 1)
\]

(14)

where \(V_o\) is the voltage across the junction. The derivation for this distribution is shown in the Appendix. The relation shows that the injected carrier distribution is dependent on the diffusion length which in turn depends upon the magnetic field. Thus, a change in the magnetic field in the case of a forward biased diode decreases not only the mobility, but also the distribution and concentration of the injected carriers. This results in a much larger change in the forward resistance of the diode than would be expected for the semiconductor material alone.

Unsymmetric p-n diodes with both contacts ohmic and pnn diodes have been analyzed for the conditions of constant current and constant voltage for the two limiting cases of low and high injection.
Figure 3 shows a schematic diagram of a long p-n diode with ohmic contacts at both ends; d is the length of the base of the diode. The ohmic contact at x = d is an infinite recombination velocity contact. The carrier distribution for such a situation is shown in Figure 4. The boundary conditions require that

\[ P_{x=d} = P_n. \]

Expressions giving the current-voltage characteristics for the two limiting cases of low injection and high injection have been derived.¹

a) For the case of low injection levels in the forward direction the current-voltage characteristics are expressed by the relation

\[ J = J_{Sl} \left( e^{\frac{q(V - JR_T)}{kT}} - 1 \right) \]

where

\[ JR_T = J p0 d \left[ 1 + (b-1) \frac{L_p}{d} \tanh \frac{d}{L_p} \right] \]

is the voltage drop across the base of the diode. J is the current density and its dimensions are amp/cm² and R_T has the dimensions of ohm cm².

and

\[ J_{Sl} = \frac{kT \mu p P_n}{L_p} \ln \frac{d}{L_p} \]

(17)
V is the voltage applied across the diode.

We proceed with the differentiation of equation (15) with respect to H and evaluate the value of \( \frac{1}{J} \frac{dJ}{dH} \). A comparison of this value with the change of resistivity of the semiconductor with the magnetic field, i.e. \( \frac{1}{\rho} \frac{d\rho}{dH} \) will directly show the enhancement of the apparent magneto-resistance effect in the diode when operated under some specific conditions.

Differentiating equation (15) w.r.t. H and keeping the voltage constant so that \( \frac{dV}{dH} = 0 \), we get

\[
\frac{dJ}{dH} = \frac{dJ_S}{dH} \left( e^{\frac{q(V - J\rho_H)}{kT}} - 1 \right) + J_S \left( -J \frac{d\rho_H}{dH} - \rho_H \frac{dJ}{dH} \right) \frac{q}{kT} .
\]

Transposing and substituting \( J + J_S = J_S e^{\frac{q(V - J\rho_H)}{kT}} \), we get

\[
\frac{dJ}{dH} \left[ 1 + (J + J_S) \frac{q}{kT} \rho_H \right] = \frac{J}{J_S} \frac{dJ_S}{dH} - \frac{q}{kT} J (J + J_S).
\]

\[
\frac{d\rho_H}{dH} (18)
\]

\[
\frac{dJ_S}{dH} \text{ and } \frac{d\rho}{dH} \text{ can be evaluated from equations (16) and (17)}
\]
\[ J_{S1} = \frac{k_T \lambda_{ppn}}{I_p} \cosh \frac{d}{I_p} \]

\[ \frac{dJ_{S1}}{dH} = k_T \frac{\lambda_{ppn}}{I_p} \left( \frac{1}{I_p^2} \frac{d\lambda_{ppn}}{dH} - \frac{\lambda_{ppn}}{I_p} \frac{dI_p}{dH} \cosh \frac{d}{I_p} \right) + \frac{\lambda_{ppn}}{I_p} \frac{dL_p}{dH} \frac{dL_p}{dH} \]

\[ \text{Csch}^2 \frac{d}{I_p} = \frac{k_T \lambda_{ppn}}{I_p} \mu \frac{1}{I_p^2} \left( \frac{1}{I_p} \frac{d\mu}{dH} - \frac{1}{I_p} \frac{dI_p}{dH} \right) \cosh \frac{d}{I_p} \]

Substituting from equation (13) for

\[ \frac{1}{I_p} \frac{d\mu}{dH} \quad \text{and} \quad \frac{1}{I_p} \frac{dI_p}{dH} \]

\[ \frac{dJ_{S1}}{dH} = -\frac{1}{2} \frac{1}{\rho_o} \frac{d\rho_o}{dH} \left[ k_T \frac{\lambda_{ppn}}{I_p} \cosh \frac{d}{I_p} + k_T \frac{\lambda_{ppn}}{I_p} \frac{dL_p}{dH} \frac{1}{\sinh^2 \frac{d}{I_p}} \right] \]

\[ = -\frac{1}{2} \frac{1}{\rho_o} \frac{d\rho_o}{dH} \left[ J_s + J_{Sd} \frac{1}{\cosh \frac{d}{I_p} \sinh \frac{d}{I_p}} \right] \]

or \[ \frac{dJ_{S1}}{dH} = -\frac{1}{2} \frac{1}{\rho_o} \frac{d\rho_o}{dH} \left[ 1 + \frac{2d}{L_p \sinh \frac{2d}{L_p}} \right] J_s \]

(19)

And since \[ R_T = \rho_d \left[ 1 + (b - 1) \frac{L_p}{d} \tanh \frac{d}{I_p} \right] \]

assuming \[ b \leq 1 \]

\[ R_T = \rho_d \]
and \( \frac{dR_T}{dH} = \frac{d}{dH} \frac{\rho \rho}{\rho} = \frac{R_T}{\rho} \frac{d\rho}{dH} \) \hspace{1cm} (20)

Substituting from equations (19) and (20) in equation (18) we obtain

\[
\frac{dJ}{dH} \left[ 1 + (J + J_{S1}) \frac{R_T}{kT} \right] = -\frac{1}{2} \frac{d\rho}{dH} \left[ 1 + \frac{2g}{kT} \right]
\]

\[
(J + J_{S1}) \frac{R_T}{L_p \text{Sinh} d \frac{1}{L_p}} \]

For the case when \( \frac{d}{L_p} < 1 \), \( \text{Sinh} \frac{2d}{L_p} \approx \frac{2d}{L_p} \) and the above expression reduces to

\[
\frac{1}{J} \frac{dJ}{dH} \approx -\frac{1}{2} \frac{2g}{kT} \frac{(J + J_{S1}) R_T}{1 + (J + J_{S1}) \frac{R_T}{kT}} \frac{1}{\rho} \frac{d\rho}{dH}
\]

Thus, at low injection in diodes with \( \frac{d}{L_p} < 1 \), the change in the forward characteristics is not any different from the change in the resistivity of the semiconductor material itself.
For $\frac{d}{L_p} > 2$, $\text{Sinh} \frac{2d}{L_p} \gg \frac{2d}{L_p}$ and we can write

$$\frac{1}{J} \frac{dJ}{dH} = -\frac{1}{2} \left[ 1 + \frac{\frac{q}{kT} \left( J + J_{S1} \right) R_T}{1 + \frac{q}{kT} \left( J + J_{S1} \right) R_T} \right] \frac{1}{\rho_0} \frac{d\rho_0}{dH}$$

which can be approximated as

$$\frac{1}{J} \frac{dJ}{dH} \approx -\frac{1}{\rho_0} \frac{d\rho_0}{dH} \quad (22)$$

Thus, at low injection in long diodes the dependence of the diode current on the applied magnetic field is the same as the resistivity of the single crystal itself.

For the case of constant current flow through the diode junction at low injection one can arrive at the following expression,

$$\frac{1}{V} \frac{dV}{dH} = \frac{kT}{q} \frac{1}{\ln \frac{J}{J_{S1}} + J_{RT}} \left\{ \frac{1/2 \left( 1 + \frac{2d}{L_p} \text{Sinh} \frac{2d}{L_p} \right) J}{\left( J + J_{S1} \right) \frac{q}{kT}} \right\} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \quad (23)$$

In the case of the diodes with $\frac{d}{L_p} < 1$

$$\frac{1}{V} \frac{dV}{dH} = \frac{kT}{q} \frac{1}{\ln \frac{J}{J_{S1}} + J_{RT}} \left\{ \frac{J}{\left( J + J_{S1} \right) \frac{q}{kT}} + J_{RT} \right\} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \quad (24)$$

which indicates a lesser sensitivity to the applied magnetic field than the resistivity of the single crystal itself.
For diodes with $\frac{d}{L_p} > 2$,

$$\frac{1}{V} \frac{dV}{dH} = \frac{1}{q} \ln \frac{J}{J_{S1}} + JR_T \left[ \frac{2(J + J_{S1}) a}{KT} + JR_T \right] \frac{1}{\rho_0} \frac{d\rho_0}{dH}$$

and the sensitivity is not very appreciably increased.

b) When the injection level is high it has been shown that

$$J = J_{S1} e^{\frac{qV}{2KT}}$$

This expression considers the effect of the field in the base and takes into account the fact that when the injection level is greater than the equilibrium carrier density, a portion of the applied voltage modifies the majority carrier distribution to maintain charge neutrality and is not available to cause minority carrier injection.

$$C = 2 + \frac{b + \cosh \frac{d}{L_p}}{b+1}$$

when $\frac{d}{L_p} < 1$, as is the case in most industrial diodes, the value of $C$ reduces to 2 and we obtain the well known relation

$$J = J_{S1} e^{\frac{qV}{2KT}}$$

and for the case of constant applied voltage we obtain

$$\frac{dJ}{dH} = \frac{dJ_{S1}}{dH} e^{\frac{qV}{2KT}}$$
which on substitution from equation (19) becomes

\[
\frac{dJ}{dH} = -\frac{1}{2} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \left[ 1 + \frac{2d}{L_p \ sinh \frac{2d}{L_p}} \right] J_{S_1} e^{\frac{qV}{2kT}}
\]

And since \( \frac{d}{L_p} < 1 \), this becomes

\[
\frac{1}{J} \frac{dJ}{dH} = -\frac{1}{\rho_0} \frac{d\rho_0}{dH}
\]

(27)

which is the same as for a single crystal and, therefore, no advantage comes into the picture. An analysis for the case when constant current is maintained through the diode shows no definite superiority over the single crystal sensitivity.

However, when a high injection is maintained for a diode in which \( \frac{d}{L_p} > 2 \), we have the relation

\[
J = J_{S_1} e^{\frac{qV}{kT}}
\]

and for the case of constant voltage across the diode we get

\[
\frac{dJ}{dH} = \frac{dJ_{S_1}}{dH} e^{\frac{qV}{kT}} + J_{S_1} e^{\frac{qV}{kT}} \left( -\frac{1}{C^2} \frac{dC}{dH} \right).
\]

Substituting for \( \frac{dJ_{S_1}}{dH} \) from equation (19) we get

\[
\frac{dJ}{dH} = -\frac{1}{2} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \left[ 1 + \frac{2d}{L_p \ sinh \frac{2d}{L_p}} \right] J_{S_1} e^{\frac{qV}{kT}} + J_{S_1} e^{\frac{qV}{kT}} \left( -\frac{1}{C^2} \frac{dC}{dH} \right), \ or
\]

\[
e^{\frac{qV}{kT}} \left( -\frac{1}{C^2} \frac{dC}{dH} \right), \ or
\]
$$\frac{1}{J} \frac{dJ}{dH} = -\frac{1}{2} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \left[ 1 + \frac{2d}{L_p \sinh \frac{2d}{L_p}} \right] + \frac{qV}{\sigma kT} \left( -\frac{1}{c} \frac{dc}{dH} \right)$$

when \( \frac{d}{L_p} \) is large

$$\frac{1}{J} \frac{dJ}{dH} = -\frac{1}{2} \frac{1}{\rho_0} \frac{d\rho_0}{dH} + \frac{qV}{\sigma kT} \left( -\frac{1}{c} \frac{dc}{dH} \right) \quad (28)$$

Now

$$b = \cosh \frac{d}{L_p}$$

$$c = 2 \frac{b + 1}{b + 1}$$

and

$$\frac{dc}{dH} = 2 \left( -\frac{dL_p}{dH} \frac{\sinh \frac{d}{L_p}} {b + 1} \right)$$

and

$$\frac{1}{c} \frac{dc}{dH} = \frac{1}{2} \frac{d}{L_p} \frac{\sinh \frac{d}{L_p}} {b + \cosh \frac{d}{L_p}} - \frac{1}{\rho_0} \frac{d\rho_0}{dH}$$

when \( \frac{d}{L_p} \) is large, \( \frac{\sinh \frac{d}{L_p}} {b + \cosh \frac{d}{L_p}} \approx \tanh \frac{d}{L_p} = 1 \)

so that

$$\frac{1}{c} \frac{dc}{dH} = \frac{1}{2L_p} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \quad . \quad (29)$$

Substituting equation (29) into (28) we obtain

$$\frac{1}{J} \frac{dJ}{dH} = -\frac{d}{2L_p} \left[ \frac{qV}{\sigma kT} + \frac{L_p}{d} \right] \frac{1}{\rho_0} \frac{d\rho_0}{dH}$$

and at sufficiently high current density \( \frac{qV}{\sigma kT} > \frac{L_p}{d} \)

so that we may write
\[ \frac{1}{J} \frac{dJ}{dH} = - \frac{d}{2L} \frac{qV}{\sigma kT} \frac{1}{\rho_o} \frac{d\rho_o}{dH} \]

which can be written as

\[ \frac{1}{J} \frac{dJ}{dH} = - \frac{d}{2L} \ln \frac{J}{J_{S1}} \frac{1}{\rho_o} \frac{d\rho_o}{dH} \]  \hspace{1cm} (30)

which shows that at high injection in diodes with large values of \( \frac{d}{L} \) the current changes to a much larger extent with the magnetic field than does the resistivity of the single crystal under the same conditions. But for very long diodes (ratio \( \frac{d}{L} \) ) it is not possible to satisfy the condition of high injection. The values of \( \frac{d}{L} \) in the range from 6 to 10 were considered suitable for these investigations.

If, on the other hand, the current through the diode is kept constant, the analysis shows that the sensitivity of the voltage is not as great but is still quite significant in comparison to the sensitivity of the resistivity of a single crystal. For this analysis we again start from the voltage-current characteristics valid at high injection levels

\[ J = J_{S1} e^{\frac{qV}{\sigma kT}} \]

and since \( J = \text{constant} \)

\[ \frac{dJ}{dH} = 0 = \frac{dJ_{S1}}{dH} e^{\frac{qV}{\sigma kT}} + J_{S1} e^{\frac{qV}{\sigma kT}} \frac{qV}{\sigma kT} \left( \frac{1}{C} \frac{dV}{dH} - \frac{V}{C^2} \frac{dC}{dH} \right) \]

\[ = - \frac{1}{2} \frac{1}{\rho_o} \frac{d\rho_o}{dH} J_{S1} e^{\frac{qV}{\sigma kT}} + J_{S1} e^{\frac{qV}{\sigma kT}} \frac{qV}{\sigma kT} \frac{qV}{\sigma kT} \left( \frac{dV}{dH} - \frac{V}{C} \frac{dC}{dH} \right). \]
Rearranging we get

\[ \frac{q}{CkT} \frac{dV}{dh} = \frac{1}{2} \frac{1}{\rho_0} \frac{d\rho_0}{dh} + \frac{qV}{CkT} \frac{d}{2L_p} \frac{1}{\rho_0} \frac{d\rho_0}{dh} \]

\[ \frac{q}{CkT} \frac{dV}{dh} = \frac{1}{2} \left( 1 + \frac{qV}{CkT} \frac{d}{L_p} \right) \frac{1}{\rho_0} \frac{d\rho_0}{dh} \]

and

\[ \frac{1}{V} \frac{dV}{dh} = \frac{1}{2} \left( \frac{CkT}{Vq} + \frac{d}{L_p} \right) \frac{1}{\rho_0} \frac{d\rho_0}{dh} \]

assuming that the condition of high injection is satisfied, we can write

\[ \frac{1}{V} \frac{dV}{dh} = \frac{d}{2L_p} \frac{1}{\rho_0} \frac{d\rho_0}{dh}. \]  

(31)

The restriction on equation (31) is that \( \frac{d}{L_p} \) cannot be increased indefinitely because then it is not possible to maintain a high level of injection. The appropriate values of ratio \( \frac{d}{L_p} \) lie in the range of 6 to 10 for optimum sensitivity.

Analysis of an Unsymmetric pnn Junction in a Magnetic Field

The majority carrier contact \( n^+ \) is essentially a highly doped alloy contact. A space charge region is formed at the \( nn^+ \) junction. Under the conditions of operation shown in Figure 5, electrons are injected from \( n^+ \) region into n-type base. Condition of charge neutrality throughout requires accumulation of holes in the vicinity of this junction. Thus, the distribution of holes in the base region assumes the form
shown in Figure 6.

Here again we proceed to investigate theoretically the effect of the magnetic field on the current-voltage characteristics of such a diode structure for various physical dimensions and for different conditions of operation. Two limiting cases considered are for low and high injection.

a) For low injection level the current voltage characteristics are given by the relationship

\[ J = J_{S2} \left( e^{\frac{q(V - IR_T)}{kT}} - 1 \right) \]  \hspace{1cm} (32)

where \( J_{S2} \) is shown by Hall5 to be given by

\[ J_{S2} = \frac{kT \mu_p n_0}{L_p} \text{Tanh} \frac{d}{L_p} \] \hspace{1cm} (33)

In order to evaluate the dependence of the current-voltage characteristics we proceed to differentiate equation (32) w.r.t. the magnetic field \( H \).

For the case when the voltage across the diode is kept constant, we obtain

\[
\frac{dJ}{dH} = \frac{dJ_{S2}}{dH} \left( e^{\frac{q(V - JR_T)}{kT}} - 1 \right) + (J_{S2} e^{\frac{q(V - JR_T)}{kT}} \frac{q}{kT})
\]

\[
\left( - J \frac{dR_T}{dH} - R_T \frac{dJ}{dH} \right).
\]

Substituting \( J + J_{S2} = J_{S2} e^{\frac{q(V - JR_T)}{kT}} \) we obtain
Fig. 5. Schematic diagram of a rectangular p-n junction diode with a majority carrier base contact.

Fig. 6. Injected minority carrier distribution in the base region of the diode with a majority carrier base contact.
\[
\left[ 1 + (J + J_{S2}) \frac{a}{kT} \right] \frac{R_T}{\frac{R_T}{J_{S2}}} \frac{dJ}{dH} = \frac{J}{J_{S2}} \frac{dJ_{S2}}{dH} - J \left( J + J_{S2} \right) \frac{a}{kT} \frac{dR_T}{dH}
\]

Now \( J_{S2} = \frac{kT\beta_p n}{L_p} \) \( \text{Tanh} \frac{d}{L_p} \) \( (34) \)

\[
\frac{dJ_{S2}}{dH} = kT\beta_p \left[ \frac{\frac{dL_p}{dH}}{L_p} - \frac{\frac{dL_p}{dH}}{L_p} \right] \text{Tanh} \frac{d}{L_p} - \frac{\frac{dL_p}{dH}}{L_p} \frac{dL_p}{dH} \text{Tanh} \frac{d}{L_p}
\]

\[
\text{Sech}^2 \frac{d}{L_p} = \frac{kT\beta_p n}{L_p} \left[ \left( \frac{1}{L_p} \frac{dL_p}{dH} - \frac{1}{L_p} \frac{dL_p}{dH} \right) \text{Tanh} \frac{d}{L_p} - \frac{\frac{dL_p}{dH}}{L_p} \frac{dL_p}{dH} \right]
\]

Substituting for \( \frac{1}{\mu_p} \frac{dL_p}{dH} \) and \( \frac{1}{L_p} \frac{dL_p}{dH} \) in terms of \( \frac{1}{\rho_0} \frac{d\rho_0}{dH} \) from equation (13) we obtain

\[
\frac{dJ_{S2}}{dH} = \frac{kT\beta_p n}{L_p} \left[ \frac{1}{2} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \text{Tanh} \frac{d}{L_p} - \frac{\frac{dL_p}{dH}}{L_p} \frac{dL_p}{dH} \right]
\]

\[
\frac{\text{Tanh} \frac{d}{L_p}}{\text{Sinh} \frac{d}{L_p} \text{Cosh} \frac{d}{L_p}} = \frac{1}{2} \left( 1 - \frac{d}{L_p} \text{Sinh} \frac{d}{L_p} \text{Cosh} \frac{d}{L_p} \right) \frac{J_{S2}}{\rho_0} \frac{d\rho_0}{dH}
\]

or

\[
\frac{dJ_{S2}}{dH} = -\frac{1}{2} J_{S2} \left( 1 - \frac{2d}{L_p} \text{Sinh} \frac{2d}{L_p} \right) \frac{1}{\rho_0} \frac{d\rho_0}{dH} \tag{35}
\]

which for \( \frac{d}{L_p} > 2 \), becomes very small and may be neglected, and
from equation (20) we have

\[
\frac{dR_T}{dH} = \frac{R_T}{\rho_o} \frac{d\rho_o}{dH}
\]

Substituting equations (35) and (20) into equation (34) we get

\[
\left[ 1 + (J + J_{S2}) \frac{a}{kT} R_T \right] \frac{dJ}{dH} = - \frac{1}{2} \frac{1}{\rho_o} \frac{d\rho_o}{dH} J_{S2} \]

\[
- \frac{q(V - JRT)}{kT} \left( e^\frac{q(V - JRT)}{kT} - 1 \right) \left( 1 - \frac{2d}{L_p \sinh \frac{2d}{L_p}} \right) (J + J_{S2}) \]

\[
\cdot \frac{a}{kT} \frac{JR_T}{\rho_o} \frac{d\rho_o}{dH} .
\]

(36)

Simplifying and transposing

\[
\frac{1}{J} \frac{dJ}{dH} = - \frac{1}{2} \frac{1}{\rho_o} \frac{d\rho_o}{dH} \left[ \frac{1}{L_p \sinh \frac{2d}{L_p}} + \frac{2a}{kT} (J + J_{S2}) R_T \right]
\]

\[
1 + \frac{a}{kT} (J + J_{S2}) R_T
\]

(37)

For those diodes in which \( \frac{d}{L_p} < 1 \), the above expression can be approximated to

\[
\frac{1}{J} \frac{dJ}{dH} \approx - \frac{1}{2} \frac{1}{\rho_o} \frac{d\rho_o}{dH} \frac{2a}{kT} (J + J_{S2}) R_T \]

\[
1 + \frac{a}{kT} (J + J_{S2}) R_T
\]

(38)

which gives a much smaller dependence than in the case of the resistivity of single crystal material. This dependence is also smaller than in a p-n diode with ohmic contacts at both ends. When \( \frac{d}{L_p} > 2 \), equation (37) becomes
\[ \frac{1}{J} \frac{dJ}{dH} = - \frac{1}{2} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \left( \frac{2a (J + JS_2) RT + 1}{kT} \right) \]  
\[ \frac{1}{J} \frac{dJ}{dH} = - \frac{1}{2} \left[ \frac{\frac{a}{kT} (J + JS_2) RT}{1 + \frac{a}{kT} (J + JS_2) RT} \right] \frac{1}{\rho_0} \frac{d\rho_0}{dH} \]

which is the same as in the case of the p-n diode with ohmic contacts (eq. 22) and the same conclusions apply here also.

**Constant Current Operation**

Starting from the relation

\[ J = JS_2 \left( e^{\frac{q(V - JR_T)}{kT}} - 1 \right) \]

the differentiation w.r.t. \( H \) yields

\[ \frac{dJ}{dH} = 0 = \frac{dJS_2}{dH} \left( e^{\frac{q(V - JR_T)}{kT}} - 1 \right) + JS_2 \left( e^{\frac{q(V - JR_T)}{kT}} - 1 \right) \]

\[ \frac{q}{kT} \left( \frac{dV}{dH} - J \frac{dR_T}{dH} \right) \]

Substituting for \( \frac{dJS_2}{dH} \) and \( \frac{dR_T}{dH} \) and making use of the relation in equation (40) we get

\[ 0 = - \frac{1}{2} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \left( 1 - \frac{2d}{L_p \text{Sinh} \frac{2d}{L_p}} \right) J + (J + JS_2) \frac{a}{kT} \left( \frac{dV}{dH} - \frac{JR_T}{\rho_0} \frac{d\rho_0}{dH} \right) \]
Rearranging and transposing we get

\[
(J + J_{S2}) \frac{q}{kT} \frac{dV}{dH} = \frac{1}{2} \frac{J}{\rho_0} \frac{d\rho_0}{dH} \left( 1 - \frac{2d}{L_p \sinh \frac{2d}{L_p}} + \frac{2q}{kT} \right) (J + J_{S2}) R_T
\]

or

\[
\frac{dV}{dH} = \frac{1}{2} \frac{J}{\rho_0} \frac{d\rho_0}{dH} \left( \frac{1 - \frac{2d}{L_p \sinh \frac{2d}{L_p}} + \frac{2q}{kT}}{(J + J_{S2}) \frac{q}{kT}} \right) (J + J_{S2}) R_T
\]

which for comparison purposes can be rewritten as

\[
\frac{1}{V} \frac{dV}{dH} = \frac{1}{2} \frac{J}{\rho_0} \frac{d\rho_0}{dH} \left( \frac{1 - \frac{2d}{L_p \sinh \frac{2d}{L_p}} + \frac{2q}{kT}}{(J + J_{S2}) \frac{q}{kT}} \right) (J + J_{S2}) R_T
\]

when \( \frac{d}{L_p} < 1 \), this above relation reduces to

\[
\frac{1}{V} \frac{dV}{dH} = \frac{J}{V} \frac{1}{2} \frac{d\rho_0}{dH} \left( 2 R_T \right)
\]

\[
\frac{1}{V} \frac{dV}{dH} = \frac{JR_T}{V} \frac{1}{\rho_0} \frac{d\rho_0}{dH}
\]

In long diodes where \( \frac{d}{L_p} > 2 \), equation (42) becomes

\[
\frac{1}{V} \frac{dV}{dH} = \frac{J}{V} \left( 1 + \frac{2q}{kT} (J + J_{S2}) R_T \right) \frac{1}{(J + J_{S2}) \frac{q}{kT}} \frac{1}{\rho_0} \frac{d\rho_0}{dH}
\]

b) High Injection Level

The current voltage characteristics of a p-n-p diode at high injection can be represented by

\[
J = J_{S2} e^{\frac{qV}{2kT}}
\]
where $J_{S2}$ and $C$ have been previously defined. For $\frac{d}{L_p} < 1$, $C = 2$ and the above relation reduces to

$$ J = J_{S2} e^{\frac{qV}{2kT}} $$

and in the case of constant voltage operation

$$ \frac{dJ}{dH} = \frac{dJ_{S2}}{dH} = e^{\frac{qV}{2kT}} $$

Substituting for $\frac{dJ_{S2}}{dH}$ from equation (35) we obtain

$$ \frac{dJ}{dH} = \frac{1}{2} \frac{1}{\rho_o} \frac{d\rho_o}{dH} \left( 1 - \frac{2d}{L_p \sinh \frac{2d}{L_p}} \right) J_{S2} e^{\frac{qV}{2kT}} $$

and can be rewritten as

$$ \frac{1}{J} \frac{dJ}{dH} = - \frac{1}{2} \frac{1}{\rho_o} \frac{d\rho_o}{dH} \left( 1 - \frac{2d}{L_p \sinh \frac{2d}{L_p}} \right) \quad (45) $$

which for $\frac{d}{L_p} < 1$, gives an extremely small value of $\frac{1}{J} \frac{dJ}{dH}$

and makes a thin diode under high injection almost independent of the influence of the applied magnetic field.

In the case of constant current operation the magnetic field dependent relation becomes

$$ \frac{dJ}{dH} = 0 = \frac{dJ_{S2}}{dH} e^{\frac{qV}{2kT}} + J_{S2} \frac{qV}{2kT} \frac{dV}{dH} $$

On substituting for $\frac{dJ_{S2}}{dH}$ and transposing and simplifying we obtain
which for small \( \frac{d}{L_p} \) values \( \left( \frac{d}{L_p} < 1 \right) \) gives \( \frac{1}{V} \frac{dv}{dH} \approx 0 \), almost independent of the applied magnetic field.

The above derivation shows that for the pnn\(^+\) diode structure, when \( \frac{d}{L_p} \) is small the forward characteristics of the diode are much less sensitive to the applied magnetic field than is the resistivity of the crystal itself.

For the case when high injection is maintained in long pnn\(^+\) diodes \( \left( \frac{d}{L_p} > 2 \right) \). The following treatment shows the dependence of the characteristics on the magnetic field.

\[ J = J_{S_2} e^{\frac{qV}{eKT}} \]

for the constant voltage operation

\[ \frac{dJ}{dH} = \frac{dJ_{S_2}}{dH} e^{\frac{qV}{eKT}} J_{S_2} e^{\frac{qV}{eKT}} (-\frac{1}{c} \frac{dc}{dH}) \cdot \]

Substituting for \( \frac{dJ_{S_2}}{dH} \) from equation (35) and for \( \frac{dc}{dH} \) from equation (29) we obtain on simplification

\[ \frac{dJ}{dH} = -\frac{1}{2} \frac{1}{\rho^o} \frac{d\rho^o}{dH} \left( 1 - \frac{2d}{L_p \sinh \frac{2d}{L_p}} \right) - \frac{J_{S_2}Q}{ekT} \frac{d}{dH} \frac{1}{\rho^o} \frac{d\rho^o}{dH} \]

or

\[ \frac{1}{J} \frac{dJ}{dH} = -\frac{1}{2} \frac{1}{\rho^o} \frac{d\rho^o}{dH} \left[ 1 - \frac{2d}{L_p \sinh \frac{2d}{L_p}} + \frac{qV}{ekT} \frac{1}{\rho^o} \frac{d\rho^o}{dH} \right] \quad (47) \]
and for $\frac{d}{L_p} \gg 2$, the above relation reduces to

$$\frac{1}{j} \frac{dJ}{dH} = - \frac{d}{2L_p} \frac{qV}{\rho_0} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \quad (48)$$

which is the same as the expression obtained for a long p-n junction with ohmic contacts. Thus, in long diodes with high injection the sensitivity of the change in the forward characteristics with the magnetic field is of the same order of magnitude regardless of the type of the base contact.

For the case of constant current through the diode we proceed by differentiating the equation

$$J = J_{S2} e^{\frac{qV}{\rho_0} \frac{1}{\rho_0}}$$

$$\frac{dJ}{dH} = 0 = \frac{dJ_{S2}}{dH} e + J_{S2} \frac{qV}{\rho_0} \frac{1}{\rho_0} \left( \frac{1}{c} \frac{dV}{dH} - \frac{V}{c^2} \frac{dc}{dH} \right)$$

Since for diodes with $\frac{d}{L_p} \gg 2$, $\frac{dJ_{S2}}{dH}$ is negligibly small we can write

$$\frac{1}{V} \frac{dV}{dH} = \frac{1}{c} \frac{dc}{dH}$$

which from equation (29) can be written as

$$\frac{1}{V} \frac{dV}{dH} = \frac{d}{2L_p} \frac{1}{\rho_0} \frac{d\rho_0}{dH} \quad (49)$$

From equations (48) and (49) it can be seen that for diodes with $\frac{d}{L_p} \gg 2$, the dependence of the current flow on the magnetic field for pnp diode is of the same order as in the case of a p-n diode with ohmic contacts.
Discussion of Theoretical Evaluations and Other Considerations

It was shown that for the case of a long forward biased p-n junction operating at high injection level the dependence of the current on the magnetic field is given by the expression

$$\frac{1}{J} \frac{dJ}{dH} \bigg|_{V_c=0} = -\frac{d}{2L} \ln \frac{J}{J_S} \cdot \frac{1}{\rho_0} \frac{d\rho_0}{dH}$$

In the case of a germanium diode with $\frac{d}{L} \neq q$ and for the forward current such that $\frac{J}{J_S} \approx 10^2$, $\frac{1}{J} \frac{dJ}{dH} = -23 \frac{1}{\rho_0} \frac{d\rho_0}{dH}$ or that the sensitivity of the forward current to the magnetic field is 23 times the sensitivity of the resistivity of the single crystal semiconductor.

The theoretical treatment so far concerns only the body properties of the material and does not take into account the surface conditions and their effect upon the sensitivity of the diode to the magnetic field. Effect of other parameters is also important and this is discussed below.

1. The magnetoresistance phenomenon is more pronounced in materials with large mobility values. The basic parameter that changes on the application of the magnetic field is the mobility. In general, the higher the initial value of the mobility the greater will be its sensitivity to the magnetic field. Also in this case the material should be such that good p-n junctions can be made into it. Good unsymmetric junctions can be made in germanium by the
alloying process and the mobility values are also high.

2. The diffusion length of the injected holes is proportional to the square root of the lifetime. Larger is the initial value of the diffusion length, more susceptible it will be to the variation in the magnetic field. Thus, long lifetime is desireable in the base region of the diode to obtain as large a value of the diffusion length \( L_p \).

3. From equation (8) we have

\[
\frac{d \rho_0}{dH} = -\rho_0 \frac{1}{\mu_p} \frac{d \mu_p}{dH}
\]

so that in order that \( \frac{d \rho_0}{dH} \) is large, initial resistivity of the material should be large. Also the injection level at which conductivity modulation takes place will be lower for a more intrinsic material. For this reason the starting material should be high resistivity material.

4. In this case of the ordinary magnetoresistance phenomenon in an extrinsic semiconductor, the geometry of the sample is an important factor influencing the sensitivity. This is so because in an extrinsic semiconductor the conductance is predominantly due to the majority carriers. On the application of the magnetic field, these carriers are deflected to a side and set up an electric field. The voltage thus developed, called Hall voltage exerts a force on the carriers to counterbalance the Lorentz force with the result that those carriers which move with the average velocity are no longer deflected. If this voltage is shorted out the magnetoresistance effect increases because now there is no voltage to oppose the deflection of the carriers. If
the length-to-width ratio is small, some of the shorting of the Hall voltage can take place via the contacts and this results in an increased magnetoresistance effect. Quantitative calculations of this effect have been made by a number of workers.\textsuperscript{6} Samples with small length-to-width ratio or with Corbino\textsuperscript{7} disk geometry have the optimum values and it can be seen for magnetoresistance effects in InSb studied by Weiss and Welker\textsuperscript{8} and shown in Figure 7.

For an intrinsic semiconductor both carriers are present in comparable numbers in the base region. Also, for the case of high injection their number is comparable in the base region. When a current flows, electrons and holes move in opposite directions and a magnetic field will tend to deflect the oppositely moving and oppositely charged carriers in the same direction. A counteracting electric field, which is set up in the presence of a magnetic field by an initial movement of charge, cannot simultaneously balance the magnetic deflection of both carriers in view of the opposite forces exerted by the electric field on the two carriers. Consequently, in the steady state holes and electrons will flow in equal numbers in the direction of magnetic deflection. The equality of the two currents is required in order that the net transverse current be zero in accordance with the principle of charge conservations. The Hall field is then that electric field which insures
Fig. 7. The relative resistance change in the magnetic field due to different geometrical form. $R_B$ is the resistance in the magnetic field and $R_o$ is the resistance at zero flux density.
the necessary equality of transverse hole and electron flows.

But the currents for the individual carriers perpendicular to the longitudinal direction of current flow do not vanish; the carriers themselves are not conserved. In fact, hole electron pairs must be continuously generated in the region or on the surfaces from which the transverse current flows and must recombine in the region or on the surface to which it flows. If the processes for recombination and generation are slow and result in large deviations of the carrier concentrations from their steady state concentration, holes and electrons tend to accumulate on that side of the sample towards which they are deflected. Since they are deflected in equal number, no space charge will result from their accumulation. When a gradient in carrier concentration is set up across the sample, further accumulation is opposed by the resulting diffusion currents. These diffusion currents constitute another means by which the effects of magnetic deflection may be balanced under steady-state condition.

If, on the other hand, the processes of recombination and generation are fast, no such large accumulation of the carriers will take place on the surface; and, consequently, no balancing of the magnetic deflection will take place by diffusion currents.
Thus, it is possible to nullify the forces opposing the magnetic deflection of the carriers by (1) operating at a high level of injection so that \( p = n \) and (2) by having a large surface recombination velocity. If these two conditions are satisfied the geometrical shape will not be of importance unless some physical dimensions become comparable to the diffusion length. Thus, one can use rectangular samples and still not have the Hall field oppose the deflection of the carriers by the magnetic field.

In the experiments performed, rectangular samples of 20 ohm cm and 30 ohm cm germanium were used. The lifetime in the material itself was 170 \( \mu \)secs.
EXPERIMENTAL WORK

Diode Fabrication

P-n junctions used in these investigations were made here in the Electron Device Laboratory. The junctions were made by alloying indium pellets on to low dislocation density and high lifetime 30 ohm cm n-type germanium. The alloying process was carried out in a Marshal furnace at 600°C under low pressure after flushing the alloying tube with nitrogen. A jig was fabricated for the purpose of alloying and is shown in Figure 8. This jig provides a weight on the indium dot and thus helps wetting. The weight was made of stainless steel and the remainder of the jig was made of high quality graphite. Oxidation of the steel weight in a wet hydrogen atmosphere was required to prevent the wetting of the weight itself. For best results, alloying was carried out in vacuum after flushing the alloying tube with nitrogen.

Rectangular samples of different physical dimensions, as well as circular samples in the shape of Corbino disk, were processed to achieve clean, single crystal wafers with flat parallel faces.

The condition of the surface of the samples was
Fig. 8. Jig used for Alloying
found to have marked influence on the quality of the resultant junctions. Experiments indicated that the surface damage by lapping as well as polishing penetrated as much as two mils into the wafer. In order to obtain satisfactory junctions it is important to remove this damaged surface. This was accomplished by using the following chemical etch.

- 70% Nitric Acid 3 parts by volume
- 48% Hydrofluoric Acid 1 part by volume
- 99.7% Glacial Acetic Acid 3 parts by volume

It was necessary to agitate the etch in order to obtain a regular plane surface.

Indium, which is an acceptor dopant, was used for alloying the junction. Indium was selected for its softness as well as its convenient alloying temperature. Pure indium pellets made by Accurate Specialties Co., Woodside 77, New York, were used. Indium disks were taken through a complete degreasing cycle before attaching them to germanium under weight. The junctions were formed at the center of the end face of the rectangular samples and at the center of the Corbino disks. The alloying process was carried out in an evacuated tube. The set-up used is shown in Figure 9. The temperature was raised linearly to 600°C and held there for ten minutes to establish equilibrium conditions. The plateau at the peak temperature is desirable from the control as well as reproduction point of view. Slow cooling is very impor-
Fig. 9. Setup used for Alloying
tant for preserving lifetime in germanium and for a small value of the reverse bias saturation current. The cooling was done at 20°C per minute for the first 250°C after peak temperature.

After alloying and before any contacts were made, the diodes were etched to remove oxides and high conductivity layers across the junction. Contact to the indium alloyed side was made by soldering 2 mil diameter gallium doped silver wire. Silver wire was used to eliminate the possibility of contamination during subsequent etching. The ohmic contact was made by moderate sandblasting of the other end and plating nickel on to it by the Electroless process. A mechanically sound contact with good ohmic properties could then be soldered to it. In the case of the diode with n⁺ contact, antimony doped tin was used as a dopant and was carried through the alloying cycle when indium was being alloyed. Subsequent to making the contacts, the junction area was again etched in the solution described earlier.

The junction area of the diodes fabricated was 2 x 10⁻² sq cm. The lifetime of the injected minority carriers was measured by the method suggested by Lederhandler and Giacoletto and was found to range from 10 microsecond to 25 microsecond. In general, diodes with good current voltage characteristics were obtained. Diodes having more than 40 micro amperes current at 20 volts reverse bias were rejected.
Experimental Investigations of the Diodes in Magnetic Fields

P-n diodes having different lengths were investigated in the experimental set-up shown in Figure 10. The equipment consisted of (1) a Varian magnet; (2) a Tektronix diode tester; (3) an oscilloscope Polaroid Camera; (4) an adjustable shaft to mount the diode; and (5) a kerosene bath for efficient heat transfer.

The mounting arrangement of the diode is shown in Figure 11. It is made of a lucite \( \frac{1}{4} \)" diameter rod. The diode can be positioned as desired under a pressure spring. The whole shaft can be clamped and held rigidly. The orientation of the diode with respect to the magnetic field can be accomplished by turning the knob on the shaft. During measurements the sample was placed in a distilled kerosene bath to allow for better heat transfer.

These diodes were examined in magnetic fields up to 15,800 gauss. The applied magnetic field was transverse to the direction of current flow. The forward characteristics of these diodes were recorded for different values of the magnetic field. For increasing magnetic fields, the forward resistance of the diode increased and, therefore, adjustment of applied voltage so as not to heat up the sample, became important. Table 1 gives the physical properties and dimensions of the tested diodes. Diodes were
Fig. 10. Setup used for Measuring the Forward Characteristics of the Diodes in the Magnetic Field
Fig. 11. The shaft used to hold the diode in the magnetic field.
TABLE 1
CHARACTERISTIC PROPERTIES OF THE TESTED DIODES

<table>
<thead>
<tr>
<th>Sample</th>
<th>Base Material</th>
<th>n, carriers/cm$^3$ at 300°K</th>
<th>Dimensions, mils</th>
</tr>
</thead>
<tbody>
<tr>
<td>XR 30-1</td>
<td>30 ohm cm n-type Ge</td>
<td>5.78 x 10^{13}</td>
<td>30 x 60 x 240</td>
</tr>
<tr>
<td>XR 30-2</td>
<td>30 ohm cm n-type Ge</td>
<td>5.78 x 10^{13}</td>
<td>30 x 60 x 160</td>
</tr>
<tr>
<td>XR 30-0</td>
<td>30 ohm cm n-type Ge</td>
<td>5.78 x 10^{13}</td>
<td>270 (diameter) 30 (thickness)</td>
</tr>
<tr>
<td>XR 20-1</td>
<td>20 ohm cm n-type Ge</td>
<td>8.66 x 10^{13}</td>
<td>20 x 60 x 240</td>
</tr>
<tr>
<td>XR 20-0</td>
<td>20 ohm cm n-type Ge</td>
<td>8.66 x 10^{13}</td>
<td>270 (diameter) 30 (thickness)</td>
</tr>
</tbody>
</table>
made in two geometries; cylindrical and planer rectangular.

The graphs showing the variation of the current with the magnetic field are shown in Figures 12 to 17 for the six diodes tested. Table 2 summarizes the results of these figures. Table 2 shows that the change in the magnitude of the current for a small change in the magnetic fields is quite large. The change in the forward resistance is large and compares quite favorably with the magnetoresistance effect in InSb.

From these experiments, it was observed that the XR30-1 diode has the best sensitivity to the magnetic field. The effect of the geometrical shape on the sensitivity is only the result of the change in the ratio \( \frac{d}{L_p} \). When the injection is high, Hall voltage becomes negligible. This becomes further clear by comparing the change in the characteristics of the Corbino disk sample Figure 14, and the rectangular sample Figure 12. Since there is no definite advantage to the use of circular Corbino disk geometry and making an ohmic contact to its periphery was difficult, only rectangular samples were used in further investigations.

By comparing the characteristics for XR30-1 and XR20-1 in Figures 12 and 17, one can see that the extent of the change is larger for the higher resistivity material. Experimentally, it was not possible to establish a sharp optimum value for the ratio \( \frac{d}{L_p} \) for the sensitivity. This
Fig. 12. Forward Characteristics of XR 30-1 Diode for Varying Values of the Magnetic Field Shown in Table 2

Fig. 13. Forward Characteristics of XR 30-2 Diode for Various Values of the Magnetic Field Shown in Table 2
Fig. 14  Forward Characteristics of XR 30-0 Diode for Various Values of the Magnetic Field Shown in Table 2

Fig. 15  Forward Characteristics of XR 20-0 Diode for Various Values of the Magnetic Field Shown in Table 2
Fig. 16 Forward Characteristics of XR 20-2 Diode for Various Values of the Magnetic Field

Fig. 17 Forward Characteristics of XR 20-1 Diode for Various Values of the Magnetic Field
<table>
<thead>
<tr>
<th>Magnet Field Gauss</th>
<th>XR 30-1 V=16 volts</th>
<th>XR 30-2 V=9 volts</th>
<th>XR 30-0 V=10 volts</th>
<th>XR 20-0 V=12 volts</th>
<th>XR 20-2 V=11/4 volts</th>
<th>XR 20-1 V=11/4 volts</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>175</td>
<td>170</td>
<td>380</td>
<td>460</td>
<td>176</td>
<td>178</td>
</tr>
<tr>
<td>1470</td>
<td>105</td>
<td>80</td>
<td>219</td>
<td>370</td>
<td>155</td>
<td>158</td>
</tr>
<tr>
<td>3400</td>
<td>59</td>
<td>30</td>
<td>200</td>
<td>235</td>
<td>92</td>
<td>114</td>
</tr>
<tr>
<td>6500</td>
<td>32</td>
<td>16</td>
<td>120</td>
<td>114</td>
<td>48</td>
<td>64</td>
</tr>
<tr>
<td>9500</td>
<td>22</td>
<td>15</td>
<td>85</td>
<td>105</td>
<td>33</td>
<td>44</td>
</tr>
<tr>
<td>12000</td>
<td></td>
<td>67</td>
<td>82</td>
<td>27</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>13700</td>
<td>13</td>
<td>14</td>
<td>57</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14800</td>
<td></td>
<td>65</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15800</td>
<td>8</td>
<td>50</td>
<td>60</td>
<td>19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
is further discussed in the next section. It was, however, observed that for diodes with $\frac{d}{L_p} > 4$, the difference in the characteristics of the diodes with a $n^+$ base contact or ohmic contact was not noticeable. The characteristics of another XR30-1 diode are shown in Figure 18. The diode base length was 210 mils. The ratio $\frac{d}{L_p}$ for this diode was 7.48.

The characteristics of Figure 18 are plotted again in Figure 19 on a semi-log graph. The change in the slope of the straight portion of this graph with the applied magnetic field indicates a change in the value of $C$ in the current-voltage characteristics and since

$$\frac{qV}{kT}$$

$$J = J_s e^{-C}$$

where

$$C = 2 \frac{b + \cosh d}{b + 1} \frac{L_p}{b + 1}.$$

We assume that $b$, the ratio of electron and hole mobility does not change significantly with the magnetic field. Thus, any change in the value of $C$ is caused by a change in the value of $L_p$, the diffusion length.

Typical calculations of the effective diffusion length of holes in $n$-type base were made for different magnetic field strengths. As expected, the effective diffusion length was found to decrease with increasing
Fig. 18. **Forward Characteristics of XR30-1 diode for different magnetic field strengths.**
Fig. 19. Forward characteristics of XR 30-1 diode for different field strength with current on log scale. 1) 0 gauss, 2) 500 gauss, 3) 1000 gauss, 4) 2,000 gauss, 5) 4,000 gauss, 6) 10,000 gauss
magnetic field. These values are shown in the table below.

**TABLE 3**

DEPENDENCE OF THE DIFFUSION LENGTH ON THE MAGNETIC FIELD

<table>
<thead>
<tr>
<th>Magnetic Field Strength (in gauss)</th>
<th>( \frac{d}{L_p} )</th>
<th>Effective Diffusion Length, ( L_p ) (in mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.48</td>
<td>.68</td>
</tr>
<tr>
<td>2</td>
<td>7.55</td>
<td>.673</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2000</td>
<td>7.97</td>
</tr>
<tr>
<td>5</td>
<td>4000</td>
<td>8.28</td>
</tr>
<tr>
<td>6</td>
<td>10,000</td>
<td>8.47</td>
</tr>
</tbody>
</table>

Figure 20 shows the change in the forward resistance of the diode. The incremental resistance at \( V = 25 \) volts forward bias changed by a factor of 15.6 for a change in the magnetic field from 0 to 10,000 gauss. This variation in resistance compares well with the magnetic resistance change in InSb at room temperature shown earlier in Figure 7.

Figure 21 shows the dependence of the current in the diode on the magnetic fields for different forward voltages. It is seen that for higher current the sensitivity is higher. This increase is in accordance with the theory. The maximum sensitivity of the diode tested was

\[
\frac{\Delta J}{\Delta H} = 40 \, \mu a/\text{gauss}
\]
Magnetic field intensity in thousands of gauss

Fig. 20. Change of forward resistance of the XR-30-1 diode for different values of the magnetic field

1) Forward d.c. resistance
2) Forward incremental resistance at Forward bias
   \[ V = 25 \text{ volts} \]
Fig. 21. Dependence of the current in diode XR30-1 on the magnetic field intensity for different forward voltages.
Further increase in the injection level is limited by heating of the sample.

Figure 22 shows the dependence of the voltage across the diode on the magnetic field for different values of current flowing through it. The voltage across the diode increases with the applied magnetic field. The maximum sensitivity measured was

$$\frac{\Delta V}{\Delta H} = \gamma \text{mV/ gauss}$$

Discussion of the Results Obtained

In the germanium material itself from which the diodes were made, the magnetoresistance at room temperature for a magnetic field of 5,000 gauss gave a value $\frac{\Delta p}{p_0}$7%. The sample had the same physical dimensions as the XR30-1 diode. From Figure 18 it can be seen that for XR30-1 diode $\frac{\Delta R}{R_0}$ = 500% for the same applied field. This shows that in the case of the diode the sensitivity increases by a factor of 71 which is about three times as much as predicted by the theoretical considerations.

The probable reason for this deviation is the fact that the only parameter whose change with magnetic field was taken into account was the diffusion length. No account has been taken of the change in the diffusion length as a function of the injection level. With increasing magnetic field
Fig. 22. Dependence of the voltage across the diode XR30-1 on the magnetic field intensity for different currents.
the injection level decreases and, therefore, the change in injection level also begins to influence the diffusion length.

But probably a still more significant factor is the absence of the counterbalancing Hall voltage in the diode since for this physical shape of single crystal germanium sample where the length is more than four times the width the Hall voltage appreciably reduces the magnetoresistance effect and, therefore, $\frac{\Delta \rho}{\rho_0}$ is not as large as it would be without the Hall voltage.

No surface effects have been taken into account. As long as the dimensions of the sample are large in comparison to the diffusion length, their effect can be ignored, but when they become comparable increased scattering of carriers by surface defects and higher surface recombination velocity interferes with the magnetic deflection and, consequently the degree of the effect is reduced.

However, the effect of the magnetic field is large enough to consider possibilities of devices using such diodes as control or sensing elements. A two terminal device with sensitivity to the magnetic field at room temperature comparable to InSb magnetoresistance devices can very effectively replace a large number of Hall devices with increased efficiency and added reliability because they have only two rather than four contacts.
Two such devices analyzed are:

(1) An amplifier using such a diode as its active element, and

(2) A fluxmeter which acts as its own chopper and gives an a.c. output.

Galvanomagnetic Amplifier
Using a Long p-n Germanium Junction as Its Active Element

Since the forward resistance of the long diodes changes quite markedly with the applied magnetic field, possibilities of achieving amplification were investigated. The schematic diagram of the circuit used is shown in Figure 23. The complete assembly of this circuit is shown in Figure 24.

The input circuit was made from a ferrite core assembly available from Ferroxcube Corporation of America, Saugerties, New York. This is not necessarily a configuration of optimum design, but was selected because of availability. The core assembly was made of a manganese-zinc ferrite. This material, Ferroxcube B4, is very suited for low frequencies and has extremely small hysteresis loss and high permeability. The pot core assembly has an outside diameter of 1.77 inches and the overall height was 1.39 inches. The diameter of the leg is 0.65 inch and its length is 1.06 inch. The core assembly is composed of five separate pieces, two disks (top and bottom), two rings, and one central leg and are shown in Figure 25. An air gap was lapped on to the central leg to accommodate the diode.
Fig. 23. Schematic diagram of the galvanomagnetic amplifier studied.
Fig. 24 Complete Assembly of the Galvanomagnetic Amplifier
Fig. 25. Disassembled Galvanomagnetic Amplifier
The design of the suitable magnetic coil was carried out from the normal induction curve and incremental permeability data available from the Ferroxcube Corporation of America. 2500 turns of AWG-30 enamel wire formed the coil. An additional 300 turns were used for d-c magnetic field biasing of the diode. All measurements were carried out at room temperature.

The performance of the diode was evaluated theoretically. In what follows an expression for the power gain has been developed.

Input Power

For the input magnetic coil the following relationship holds

\[ L \frac{di}{dt} = N_1 \frac{d\phi}{dt} \times 10^{-8} \text{ volts} \]  

(50)

where \( L \) is the inductance of the magnetic circuit in henries, \( i_x \) is the instantaneous input current in amperes, \( N_1 \) is number of turns in the input coil and \( \phi \) is instantaneous flux in the central core in Maxwells.

For sinusoidal variations

\[ \phi = \phi_m \sin \omega t \]  

(51)

where \( \phi_m \) is the peak value of the flux and is equal to

\[ A_c B_c \]  

where

\[ A_c = \text{Area of the central leg of the core in cm}^2 \]

\[ B_c = \text{Peak a.c. flux density in the central leg of the core} \]
\[
\begin{align*}
\frac{di_x}{dt} &= \frac{N_1 A_c B_c \omega \cos \omega t}{L} \times 10^{-3} \quad (52) \\
and \quad i_x &= \frac{N_1 A_c B_c \sin \omega t}{L} \times 10^{-8} \text{ amps} \quad (53) \\
and \quad i_{\text{rms}} &= I_c = \frac{N_1 A_c B_c \sin t}{\sqrt{2} L} \times 10^{-8} \text{ amps} \quad (54)
\end{align*}
\]

Because of leakage of the flux and difference in the areas of the central leg and the diode, the flux density \(B_c\) may not equal the flux density \(B_e\) in the sample, so that we define

\[
K = \frac{B_c}{B_e}
\]

Thus,

\[
I_c = \frac{N_1 A_c k B_e}{\sqrt{2} L} \times 10^{-18} \text{ amps} \quad (55)
\]

And if \(\omega L \gg R_c\) where \(R_c\) is the a.c. resistance of the input circuit the input voltage is

\[
E_c = \omega L I_c
\]

or

\[
E_c = \frac{N_1 A_c k B_e \omega}{\sqrt{2}} \times 10^{-8} \text{ volts} \quad (56)
\]

and the input power is given by \(I_c^2 R_c\) and is

\[
P_1 = \frac{(N_1 A_c k B_e)^2 R_c}{2L} \frac{R_c}{L} \times 10^{-16} \text{ watts}
\]

\[
= \frac{(N_1 A_c k B_e)^2 \omega}{2L} \frac{\omega}{Q} \times 10^{-16} \text{ watts} \quad (57)
\]
Equivalence for the Output Circuit

An equivalence for the output circuit of the galvanomagnetic amplifier shown in Figure 23 can be obtained as below. The resistance $R$ is subjected to a periodic or a non-periodic variation $\Delta R$ and the output is obtained across the load resistance $R_L$.

For small signal analysis, it can be assumed that the resistance of the long diode element can be considered constant at any particular value of the magnetic flux density. To the first approximation the region of operation is linear. The slope of the forward biased current-voltage characteristics depends upon the applied magnetic field.

The current through the diode at high injection can be written as

$$i_e = f (V_e, R)$$

so that for the small signal case one can write

$$\Delta i_e = \left. \frac{\partial i_e}{\partial V_e} \right|_R \Delta V_e + \left. \frac{\partial i_e}{\partial R} \right|_{V_e} \Delta R$$  \hspace{1cm} (58)

and when $R$ is constant, $\Delta R = 0$ and we get

$$\frac{\Delta i_e}{\Delta V_e} = \left. \frac{\partial i_e}{\partial V_e} \right|_R = \frac{1}{R}$$  \hspace{1cm} (59)

When $V_e$ is constant, $\Delta V_e = 0$ and we get

$$\frac{\Delta i_e}{\Delta R} = \left. \frac{\partial i_e}{\partial R} \right|_{V_e}$$
But \( i_e = \frac{v_e}{R} \)
and: \( \frac{\partial i_e}{\partial R} = \frac{-v_e}{R^2} \) \( \text{ (60)} \)

From equation (58), (59) and (60) we get
\[
\Delta i_e = \frac{1}{R} \Delta v_e - \frac{v_e}{R^2} \Delta R \quad \text{ (61)}
\]

Now
\[
\Delta v_e = - \Delta i_e R_L
\]
\[
\therefore \Delta i_e = - \frac{R_L}{R} \Delta i_e - \frac{v_e}{R^2} \Delta R \quad \text{ (62)}
\]

Transposing
\[
\Delta i_e (1 + \frac{R_L}{R}) = - \frac{v_e}{R^2} \Delta R
\]
and we get
\[
\Delta i_e = - \frac{v_e}{(R + R_L)} \frac{\Delta R}{R} \quad \text{ (63)}
\]

But
\[
i_e = \frac{E_o}{R + R_L}
\]
\[
v_e = i_e R = \frac{E_o R}{R + R_L}
\]
and
\[
\Delta i_e = \frac{-E_o \Delta R}{(R + R_L)^2} \quad \text{ (64)}
\]

In the above expression
\[
i = i_e + \Delta i_e
\]
and
\[
R = R_o + \Delta R
\]

From this it is possible to arrive at the Thevinin's equivalent for the a.c. part for the output circuit of the amplifier.

This is shown in Figure 26 which amounts to a voltage source...
of voltage,

\[ V = -I_o \Delta R \]

and of internal resistance \( R_o \). This equivalent circuit is valid even when \( R_o \) is not purely resistive, but is a complex quantity.

**Deduction of \( \Delta R \) in Terms of \( \Delta B \)**

The change in \( R, \Delta R \) is dependent upon the characteristics of the semiconductor material, the junction properties and the change in the magnetic flux density in the sample. The resistance \( R \) of the diode is equal to the zero field resistance \( R_o \), plus the increase in the resistance \( \Delta R \) caused by the increase in the field \( \Delta B \). For small changes, and assuming that the region of operation is linear

\[ R = R_o (1 + \frac{\Delta R}{R_o}) \]  \hspace{1cm} (65)

\[ \frac{\Delta R}{R_o} \text{ is a function of the flux density } B. \]

To a satisfactory approximation for small changes one can write

\[ \Delta R = \frac{dR}{dB} \cdot \Delta B \]  \hspace{1cm} (66)

We define the ratio \( \frac{dR}{dB} = S \), the variation in the resistance of the diode with the change in the flux density.

**Calculation of the Output Power**

Substituting equation (66) into (64) we obtain

\[ \Delta i_e = \frac{-E_o S \Delta B}{(R + R_L)^2} \]  \hspace{1cm} (67)
Here $\Delta i_e$ is the total change in $i_e$ and $\Delta B$ is the change in $B$.

The value of the resistance increases whether the magnetic flux is in one direction or another. Thus, if an alternating magnetic flux of frequency $f$ is applied, the output would be at twice the frequency. In order that the input frequency be preserved at the output, it is necessary that a biasing d.c. magnetic field greater than twice the amplitude of the alternating magnetic field be applied across the diode so that the resultant flux always remains in the same direction. In the above expressions $\Delta B$ is the total change in the magnetic flux density and, hence, is twice the amplitude of alternating magnetic flux density, therefore, $\Delta B = 2B_e$. Similarly, $\Delta i_e$ is the total change in the current and

$$\text{hence } \Delta i_e = \frac{2\sqrt{2}}{2} I$$

or

$$I = \frac{1}{\sqrt{2}} i_e$$

where $I$ is the rms value of the alternating current in the diode. Substituting this in equation (67), we get

$$I = - \frac{E S B_e}{\sqrt{2} (R + R_L)^2} \quad \text{(68)}$$

or

$$I = - \frac{I_{dc} S B_e}{\sqrt{2} (R + R_L)} \quad \text{(69)}$$

where $I_{dc}$ is the direct current through the diode. The Thevinin's equivalent circuit for this is shown in Figure 27.
Fig. 26. Equivalent output circuit of the galvanomagnetic amplifier.

Fig. 27. Equivalent output circuit of the galvanomagnetic amplifier.
The output power is \( I^2 R_L \) and can be written as

\[
P_o = \frac{(I_{dc} B_e)^2 R_L}{2 (R + R_L)^2} \text{ watts} \quad (70)
\]

For maximum output power \( R_L = R \) and

\[
P_{om} = \frac{(I_{dc} B_e)^2}{8R} \quad (71)
\]

The d.c. power dissipated in the element is

\[
P_d = I_{dc}^2 R \quad (72)
\]

\[
\therefore P_{om} = \frac{P_d}{8R^2} \quad S^2 B_e^2
\]

and maximum power gain is

\[
G_m = \frac{P_{om}}{P_i} = \frac{P_d}{8R^2} \quad S^2 B_e^2 \quad 2L \frac{Q}{(N_1 A_c k B_e)^2 \omega} \times 10^{16}
\]

\[
= \frac{P_d}{8R^2} \quad S^2 \quad \frac{L}{(N_1 A_c k)^2} \frac{Q}{\omega} \times 10^{16} \quad (73)
\]

Measurements and Results of the Operation of the Galvanomagnetic Amplifier

General Radio 650 A impedance bridge was used to measure the inductance \( L \) and the quality factor \( Q \) of the magnetic circuit. Both \( L \) and \( Q \) depend upon the air gap in the central core. The active element was the XR 30-1 diode whose thickness was 30 mils. An air gap 36 mils wide was lapped in
the central core to accommodate the sample. With this air gap and a very carefully machined magnetic circuit the inductance \( L \) was 3.1 henries and \( Q \) was found to be 18.3. This measurement was made at 60 cycles. Alternating current resistance of the magnetic circuit computed from the values of \( L \) and \( Q \) was

\[
R_{ac} = \frac{\omega L}{Q} = \frac{377 \times 3.1}{18.3} = 65 \text{ ohms}
\]

In the magnetic circuit the reluctance of the air gap containing the element is much greater than the reluctance of the rest of the magnetic circuit. Hence, a reduction in the magnetic permeability of the core material as a result of the bias field does not greatly alter the effective permeability of the magnetic path. The inductance, therefore, does not change greatly with the direct current flowing in a part of the winding.

The core material, Ferroxcube \( B_4 \) has low retentivity and, therefore, a very small hysteresis loss. The resistivity of the core material is \( 10^5 \) ohm cm and, therefore, the eddy current loss is extremely small. Because of this high resistivity, it was possible to glue the diode directly on to the central core. An aluminum clamping frame was used to hold the whole assembly firm and rigid. Typical results for the operating device were as follows:

At the input

\[
E_0 = 5.3 \text{ volts rms}
\]

\[
Z_1 = 1170 \text{ ohms}
\]

\[
R_{ac} = 65 \text{ ohms}
\]
At the output

\[ I = 25 \text{ m.a.} \]
\[ R_L = 680 \text{ ohms} \]
\[ V_0 = 0.8 \text{ volts peak a.c.} \]

From these values one can calculate the input power, output power, and power dissipation

\[ P_d = 0.25 \text{ watts} \]
\[ P_i = 1.31 \times 10^{-3} \text{ watts} \]
\[ P_o = 0.47 \times 10^{-3} \text{ watts}. \]

So that

\[ \frac{\text{output power}}{\text{Input power}} = 0.36. \]

This is a significant ratio when note is taken of the fact that the region of the magnetic field influencing the sample was only \( 72 \times 10^{-4} \) square inches. Though an effort was made to concentrate the field in a smaller area, yet the flat top of the core had a much bigger area and considerable leakage. The area of cross section of the central core was 0.35 square inch. So approximately only 1/50th of the field is being put to use. The magnetic circuit was not designed for optimum performance. Taking into account all these factors, it appears that an optimum design can give a power gain of about 15 or 16.

An attempt was made to use thin samples so that it is possible to work with a smaller air gap since it will result in less reluctance and, therefore, greater flux density. But, it was found that when the thickness of the sample became comparable to the diffusion length (\( L_p = 25 \text{ mils} \)) the
sensitivity of the diode for a magnetic field normal to the wider face decreased. This decrease in the sensitivity is because of the fact that after the injection of the minority carriers across the junction, the major force acting on the carriers is that of diffusion and a large number of them will go over to the surface and recombine there rather than be available for deflection by the magnetic field.

Analysis of the dependence of the direction of the magnetic field in the plane transverse to the direction of the current flow was made for a rectangular diode. This is discussed in the next section and it has been shown that this result can be used for measuring magnetic fields very accurately.

Application of a Thin Rectangular Diode for Measuring Magnetic Fields

A test on a diode whose dimensions were

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>240 mils</td>
</tr>
<tr>
<td>width</td>
<td>115 mils</td>
</tr>
<tr>
<td>thickness</td>
<td>20 mils</td>
</tr>
</tbody>
</table>

showed that the apparent magnetoresistance effect was strongly dependent on the direction of the magnetic field in the plane normal to the direction of the current in the diode. Current-voltage characteristics for different magnetic fields were obtained for the following two cases:

1. When the magnetic field is normal to the broad face and the carriers are deflected towards the narrow face.
2. When the magnetic field is normal to the narrow face and the carriers are deflected towards the broad face.

The characteristics for these two cases are shown in Figures 28 and 29. It is readily seen that the magnetoresistance effect is much more pronounced in the second case. For Case 1 the forward resistance of the diode on applying a magnetic field of 500 gauss changed from 260 ohms to 304 ohms only, while for Case 2 the change was from 260 ohms to 510 ohms for the same magnetic field.

This suggests the possibility that if this diode be turned with its axis of rotation perpendicular to the direction of the magnetic field and the circuit shown in Figure 30 is used, an alternating voltage will be produced across the resistance. This alternating voltage can be displayed on an oscilloscope and is shown in Figure 31.

The motion imparted to the diode was such that it turned through 90 degrees and then repeated the motion in the opposite direction. An arrangement like this can have soldered contacts. A continuous rotation required slip ring contacts which produced a large amount of contact noise.

The rocking motion of the diode was achieved by coupling it to a small motor so that when the shaft of the motor revolved, the diode executed only the rocking motion. The arrangement is shown in Figure 32. The angle through
Fig. 28. Forward characteristics of the diode for different values of the magnetic field normal to the broad face of the diode. 1) 0 gauss, 2) 500 gauss, 3) 1000 gauss, 4) 2000 gauss, 5) 3000 gauss, 6) 5000 gauss, 7) 7000 gauss.

Fig. 29. Forward characteristics of the same diode as above for the same magnetic field strengths but now the field being normal to the narrow face of the diode.
Fig. 30. The circuit for obtaining a.c. output by giving rocking motion to the diode in the magnetic field.

Fig. 31. A typical a.c. output for 500 gauss magnetic field in the above arrangement.
Fig. 32. The arrangement for the rocking motion of the diode.
which the diode turned could be controlled by adjusting the position of the pivot on the shaft of the motor. The output voltage recorded was as shown in Table 4.

**TABLE 4**  
Output Voltage for Different Magnetic Fields

<table>
<thead>
<tr>
<th>Field Strength</th>
<th>Peak-to-peak alternating voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 gauss</td>
<td>0 volts</td>
</tr>
<tr>
<td>500 gauss</td>
<td>0.33 volts</td>
</tr>
<tr>
<td>1000 gauss</td>
<td>0.66 volts</td>
</tr>
<tr>
<td>2000 gauss</td>
<td>1.40 volts</td>
</tr>
<tr>
<td>3000 gauss</td>
<td>2.5 volts</td>
</tr>
<tr>
<td>5000 gauss</td>
<td>4.50 volts</td>
</tr>
<tr>
<td>7000 gauss</td>
<td>5.7 volts</td>
</tr>
</tbody>
</table>

The speed of the motor was 30 rpm. A change of speed from 20 rpm to 60 rpm did not have any effect on the amplitude of the output and higher speeds of turning can be used. The output is alternating and can, therefore, be amplified and the sensitivity of measurements can be increased.

For the purpose of comparison, the d.c. voltage developed for 7 K. gauss across a GaAs Hall probe marketed by Bell Inc., Columbus, Ohio, measured with \( R_L = 10 \) K. ohm was only 70 millivolts.
Discussion of Additional Applications

The effect of the magnetic field on the forward characteristics of thick based germanium p-n junction diodes is so marked that they can be made use of in a large number of applications. Some of them are:

1. A potentiometer without moving contacts.
2. As pressure gauges, microphones, accelerometers, etc., where a longitudinal displacement can easily be converted into changes in electric current. An arrangement is shown in Figure 33.
3. Flux density, measurement and control. Since forward resistance depends upon the magnetic field, flux may be measured directly. Either the current change or the voltage drop can be used for feedback control purposes. The bridge circuit of Figure 34 can be used for the measurement of the flux.
4. Voltage and current regulators. These diodes can be used as control elements in either voltage or current regulators. A possible circuit is shown in Figure 35.
5. Automobile ignition timing. The distributor-breaker contacts have always been a source of trouble in automobiles. Through use of a series of permanent magnets, such as shown
Fig. 33. Diagram showing the use of the diode as a pressure gauge.

Fig. 34. Diagram showing the use of the diode as a flux meter.
in Figure 36, a magnetic flux field can be generated which will pass through a diode which holds an electronic timing transistor switch in the normal position. Coincident with the flux field, a transistor energizes the spark coil to produce conventional ignition, etc.
Fig. 35. The diode used as a regulator element.

Fig. 36. The diode used for automobile ignition timing.
CONCLUSION

The effect of the magnetic field on long germanium diodes has been found to be quite significant. In principle, any electronic device based upon the Hall effect can be replaced by a similar device based upon the effect discussed here. These devices should have much higher efficiency and sensitivity than those of the Hall effect devices. Besides these advantages, it is a two-terminal device which should result in significant reliability improvement.
APPENDIX

Analysis of the Distribution of the Injected Carriers for an Unsymmetric p-n Junction

For the junctions studied the p-n junction was made by alloying indium on to 30 ohm cm germanium. The density of the holes in the indium alloyed p-type is $\approx 10^{21} \text{cm}^{-3}$ and the density of electrons in the n-type germanium is $5.78 \times 10^{13} \text{cm}^{-2}$ and the junction is unsymmetric. The current flowing through the p-n junction can be considered purely as hole current.

The equations which govern the behavior of the carriers in the n-type region of the diode described above are

\begin{align*}
J_p &= q \left( \mu_p p^E - qD_p \frac{\partial p}{\partial x} \right) \quad (A-1) \\
J_n &= q \left( \mu_n n^E + qD_n \frac{\partial n}{\partial x} \right) \quad (A-2) \\
\frac{\partial p}{\partial t} &= - \frac{p - D_n}{\tau_p} - \frac{1}{q} \frac{\partial J_p}{\partial x} = 0 \quad (A-3)
\end{align*}

where $J_p$ and $J_n$ are the hole and electron current densities respectively and

\[ J = J_p + J_n \quad (A-4) \]

where $J$ is the total current density, $p$ and $n$ are the densities of holes and electrons; $D_n$ is the equilibrium density.
of the holes in n-type region; $q$ is the electronic charge; $D_p$ and $D_n$ are the coefficients of diffusion of holes and electrons.

$$D_p = \frac{kT}{q} \mu_p$$

$$D_n = \frac{kT}{q} \mu_n$$

$\tau_p$ is the lifetime of minority carriers in the n-type base.

$E$ is the intensity of the electric field in the base region.

Assuming that charge neutrality is maintained throughout

$$n = N + p$$

where $N$ is the density of donors.

Adding equations (A-1) and (A-2) we obtain

$$J_p + J_n = J = q \mu_p (p + bn) E - q (D_p \frac{\partial p}{\partial x} - D_n \frac{\partial n}{\partial x})$$

where $b = \frac{\mu_n}{\mu_p}$, the ratio of electron and hole mobilities.

Substituting $\frac{\partial p}{\partial x} = \frac{\partial n}{\partial x}$ in (A-5) and assuming the variation in $x$ direction only we get

$$E = E = \frac{J}{q \mu_p (b+1) p + bN} - \frac{kT}{q} (b-1) \frac{dp}{dx}$$

Substituting (A-6) in (A-1) we obtain

$$J_p = q \mu_p \left( \frac{J}{q \mu_p (b+1) p + bN} - \frac{kT}{q} (b-1) \frac{dp}{dx} \right)$$

$$- kT \mu_p \frac{dp}{dx}$$
\[
J_p = \frac{J_p}{(b+1) p+bN} - D_p q(b-1) \frac{p}{(b+1)p+bN} \frac{dp}{dx} - D_p q \cdot \frac{dp}{dx} \tag{A-7}
\]

Differentiating this equation with respect to \(x\) we obtain
\[
\frac{dJ_p}{dx} = J \left[ \frac{1}{(b+1) p+bN} \frac{dp}{dx} - \frac{(b+1)p}{(b+1)p+bN} \frac{dp}{dx} \right] - D_p q(b-1) \left[ \frac{1}{(b+1)p+bN} \right] \frac{dp}{dx} - D_p q \left[ \frac{(b-1)p}{(b+1)p+bN} \right] \frac{d^2p}{dx^2}
\]

\[
\frac{dJ_p}{dx} = J \left[ \frac{1}{(b+1) p+bN} \frac{(b+1)p}{(b+1)p+bN} \frac{dp}{dx} \right] - D_p q(b-1) \left[ \frac{1}{(b+1)p+bN} \right] \frac{dp}{dx} - D_p q \left[ \frac{(b-1)p}{(b+1)p+bN} \right] \frac{d^2p}{dx^2}
\]

Simplifying
\[
\frac{dJ_p}{dx} = J \left[ \frac{1}{(b+1) p+bN} \frac{(b+1)p}{(b+1)p+bN} \frac{dp}{dx} \right] \frac{dp}{dx} - D_p q(b-1) \left[ \frac{1}{(b+1)p+bN} \right] \frac{dp}{dx} - D_p q \left[ \frac{(b-1)p}{(b+1)p+bN} \right] \frac{d^2p}{dx^2}
\]

or
\[
\frac{dJ_p}{dx} = J \left[ \frac{1}{(b+1) p+bN} \frac{(b+1)p}{(b+1)p+bN} \frac{dp}{dx} \right] \frac{dp}{dx} - \frac{D_p q(b-1)}{(b+1)p+bN} \frac{d^2p}{dx^2}
\]

or
\[
\frac{dJ_p}{dx} = J \left[ \frac{1}{(b+1) p+bN} \frac{(b+1)p}{(b+1)p+bN} \frac{dp}{dx} \right] \frac{dp}{dx} - \frac{D_p q(b-1) bN}{(b+1) p+bN} \left( \frac{dp}{dx} \right)^2
\]

or
\[
\frac{dJ_p}{dx} = \frac{J bN}{(b+1) p+bN} \left( \frac{dp}{dx} \right)^2 - \frac{D_p q(b-1) bN}{(b+1) p+bN} \left( \frac{dp}{dx} \right)^2
\]

or
\[
\frac{dJ_p}{dx} = \frac{J bN}{(b+1) p+bN} \left( \frac{dp}{dx} \right)^2 - \frac{D_p q(b-1) bN}{(b+1) p+bN} \left( \frac{dp}{dx} \right)^2
\]

or
\[
\frac{dJ_p}{dx} = \frac{D_p q(b-1) bN}{(b+1) p+bN} \left( \frac{dp}{dx} \right)^2
\]

\[
\frac{dJ_p}{dx} = \frac{D_p q(2p+N)}{(b+1) p+bN} \frac{d^2p}{dx^2} \tag{A-9}
\]
For steady state

\[ q \frac{P - P_n}{\tau_P} = - \frac{\partial J_p}{\partial x} \]

Substituting for \( \frac{\partial J_p}{\partial x} \) in (A-9) transposing and simplifying we obtain

\[ \frac{d^2p}{dx^2} + \frac{N (b-1)}{(2p+N)(b+1)p} \left( \frac{dp}{dx} \right)^2 - \frac{JN}{qD_p(2p+N)(b+1)pN} \frac{dp}{dx} = 0 \] (A-10)

Mathematical solution of this equation is difficult and complex. But a relative importance of the various terms involved can be estimated as below.

1. If \( p \ll N \), the second and third terms in (A-10) approach \( \frac{P}{N} \) and become negligible because \( \frac{p}{N} \) is very small.

2. If \( p \gg N \), the second and third terms in (A-10) approach \( \frac{N}{p} \) and can again be neglected because \( \frac{N}{p} \) becomes very small.

Thus, second and third terms can be neglected both in n-type and p-type regions. With this approximation (A-10) can be written as

\[ \frac{d^2p}{dx^2} - \frac{P - P_n}{D_p \tau_p} \frac{(b+1)p + bN}{(2p+N)b} = 0 \] (A-11)

and a solution of this equation can be written as

\[ p = P_n + Ae^{\frac{-x}{L_p}} + Be^{\frac{x}{L_p}} \] (A-12)

where \( L_p \) is the diffusion length of the injected carriers.
and $A$ and $B$ are constants which depend on the boundary conditions. Assuming that Boltzmann distribution is valid so that when

$$x = 0 \quad \frac{qV_0}{kT} = p = p_n^e$$

where $V_0$ is the voltage drop across the space charge region of the p-n junction.

The second contact is ohmic. This contact has infinite rate of recombination, so that when

$$x = d \quad p = p_n$$

Applying these boundary conditions (A-12) becomes at $x = 0$

$$p_n^e = p_n + A + B$$

or

$$A + B = p_n \left( e^{\frac{qV_0}{kT}} - 1 \right) \quad (A-13)$$

at $x = d$

$$P_n = p_n + A e^{\frac{-d}{L_p}} + B e^{\frac{d}{L_p}}$$

or

$$A = -Be^{\frac{2d}{L_p}} \quad (A-14)$$

Substituting (A-14) in (A-13)

$$\frac{d}{BeL_p} \left( e^{\frac{-d}{L_p}} - e^{\frac{d}{L_p}} \right) = p_n \left( e^{\frac{qV_0}{kT}} - 1 \right)$$
or
\[ B = -p_n \left( e^{\frac{qV_o}{kT}} - 1 \right) \frac{d}{2 \sinh \frac{d}{L_p}} \]  \hspace{1cm} (A-15)

and
\[ A = p_n \left( e^{\frac{qV_o}{kT}} - 1 \right) \frac{d}{2 \sinh \frac{d}{L_p}} \]  \hspace{1cm} (A-16)

Substituting these values for \( A \) and \( B \) in (A-12) we obtain
\[ p = p_n + p_n \left( e^{\frac{qV_o}{kT}} - 1 \right) \frac{\sinh \frac{d-x}{L_p}}{\sinh \frac{d}{L_p}} \]  \hspace{1cm} (A-17)

(A-17) shows that the distribution of \( p \) depends upon the diffusion length \( L_p \).
LIST OF REFERENCES


BIBLIOGRAPHY


AUTobiography

I, Vir Abhimanyu Singh Dhaka, was born at Malout Mandi, India, on December 24, 1932. I had my high school education from Bharatpur High School, Bharatpur, and obtained the Bachelor's degree from St. John's College, Agra. From 1953 to 1955 I attended Ratputana University and graduated with Master's degree in Physics. I taught Physics at Jain College, Ferozepore for two years before joining the Graduate School of the Ohio State University. In 1959, I obtained the Master's degree in Electrical Engineering. While a graduate student at the Ohio State University, I held the position of Graduate Assistant, Research Assistant and Research Associate in the Department of Electrical Engineering.