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INVESTIGATION OF THE EFFECTS OF ELECTROSTATIC FIELDS ON HEAT TRANSFER AND BOUNDARY LAYERS

Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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The Ohio State University
1962

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Table 1  Power Requirements for Corona Wind 169
INTRODUCTION

The influence exerted by an electric field on gases and liquids has been known for many years. Primary attention was given to electrolytic phenomena and to gas discharges. Relatively little work, however, was done on the effects of electric fields acting alone on the flow of fluids. Such field effects tend to be rather subtle in contrast to the relatively large fluid body force generated by the interaction of an electric field and a transverse magnetic field. This latter fluid-electromagnetic interaction gives rise to the whole field of magnetohydrodynamics (MHD). The interest in MHD has received wide attention in recent years because of potential applications in power generation, controlled fusion, and space propulsion. The subtle electric field effects, however, merit consideration apart from the mass of the work done on magnetohydrodynamics. The fluid-electric interactions are generally distinct from those in the MHD field.

An electric field can influence not only fluids containing charged particles and conducting fluids, but neutral non-conducting fluids as well. This wide range of action may provide controllable body forces within the fluid without the high degree of ionization customarily used in magnetohydrodynamics. Changes in fluid properties, such as viscosity or conductivity, can also occur in the presence of a field, and thus the entire behavior of fluid flow and heat transfer may be affected.

It is the purpose of this research to explore the possible interactions between electric fields and fluids. Because of the wide range of possible influences, an extensive survey of possible electric field-fluid interactions was undertaken to provide a sound basis for the specific areas of research chosen. A summary of the interactions is presented in Appendix I. Included in this summary are the basic electrical
phenomena involved, descriptions of various experimentally observed interactions, and a discussion of the potential application of the phenomena.

Two specific electric-fluid interactions were selected for more detailed investigations, the effects of ion motion in gases, and the motion of neutral particles. In the study of the ion motions, a combined experimental and analytical program was undertaken of the effects of a corona discharge on the free convection from a heated plate. A Mach-Zehnder interferometer was used in these studies. The analytical approach was based upon the Navier Stokes, energy, Von Karman, and electric field equations. Correlations between the theoretical solutions obtained and the test data were accomplished. The motion of neutral particles as represented by dielectrophoresis was studied only in an exploratory fashion to verify the possible actions which could be obtained through the application of an electric field. An explanation of the corona wind and dielectrophoresis is included in Appendix I.
EXPERIMENTAL INVESTIGATION OF THE EFFECT
OF CORONA WIND ON FREE CONVECTION

This section covers the experimental work done in studying one possible interaction between an electrical field and a fluid. The experiment chosen, as discussed briefly in the previous section, is that of a corona discharge impinging on a heated flat plate. The section will describe the experimental phases of the work and a description of the phenomena observed.

Aims of the Corona Wind-Heat Transfer Tests

In order to study the possible effects which could be achieved by electric fields acting on ions in gases, it was necessary to choose a phenomenon which could provide a suitable source of ions. Many possible sources exist including combustion flames, ultra-violet radiation of the gas, and others. A very controllable source, and one which is simple to obtain, is that of the ions in a corona discharge (1)*.

The phenomenon of corona wind is discussed in Appendix I; however, the basic action will be reviewed briefly here. If a fine needle point electrode is located a short distance away from another blunt electrode, it is observed that a stream of air moves away from the point when high voltages are applied between the point and the other electrode. This is illustrated in Figure 1.

![Diagram of corona wind](image)

*Numbers in parentheses in text refer to References given at the end of the dissertation.
Around the fine corona point an intense electrical field exists. Ions, existing in air naturally or created by this intense field, are accelerated by the field. These ions then collide with the neutral molecules of the air and near the point further ionization takes place. Away from the region of the point the collisions are elastic and a streaming of the air results. The net effect is the corona wind moving away from the charged point. The velocities obtainable from corona wind in air are very small, of the order of a few feet per second. However, if this slight streaming were made to impinge upon a heated plate, a change in the flow field and temperature distribution should result. It was also recognized that the free convection occurring from the heated plate in still air would provide a very sensitive phenomenon to explore with the corona wind or with any other electrostatic influence which might exist. Consequently a combined free convection-electrostatic test was planned.

To actually conduct such a test, a Mach Zehnder interferometer was utilized (2). Using the interference patterns of light, the interferometer can record very small changes in the temperature field around the test specimen. Any changes induced by the corona wind or electrostatic field should thus be recorded by the interferometer as changes in the interference pattern. Thus the experiment actually conducted is based upon studying a very sensitive phenomena (free-convection) with an extremely sensitive instrument (the interferometer) to detect any potentially small influences of the applied electrostatic field. The data obtained from the interferometer, although of considerable value, have the limitation that only temperature distributions are obtained. No information can be obtained about the velocity distribution in the vicinity of the plate. To overcome this difficulty, it was necessary to study pressure effects separately.
Many electrode configurations were conceived to provide suitable corona discharge as well as the field shaping necessary to control the phenomena. In addition, electrodes were made which would provide fields without a corona discharge. These electrode configurations were used to provide base line data with no ions, and to explore any possible effects which might occur not directly associated with ion motion. Because the corona wind tests indicated some very interesting results, most of the work was concentrated upon the corona discharge. The work with the other electrode configurations was exploratory only.

Description of Experimental Apparatus

Heat Transfer Equipment

A Mach-Zehnder interferometer was used to study the effects on the heat transfer. The theory of operation of an interferometer is covered in numerous publications and will not be discussed in detail (2, 3). Both infinite fringe and finite fringe patterns were used in the study. Because the infinite fringe patterns gave graphic and readily usable results most of the runs were made with this setting. The phenomena were observed both with monochromatic light of 5641 A and with white light. All data were taken with the monochromatic light source, although the white light was effective in visually studying the phenomena.

Figure 2 shows the over-all test apparatus with the interferometer located in the background. Depicted are the high voltage supply at the left, the working section of the interferometer in the center, and the free-convection heater power supply on the table. Figure 3 indicates the Polaroid Camera located at the rear of the interferometer, which was used to record the fringe patterns obtained. The controls at the right are used to adjust the interferometer to the fringe pattern desired.
Figure 2
Overall view of interferometer installation.
Figure 3

Close up of camera mounting on interferometer.
The interferometer was located in a small building isolated from outside traffic and disturbances. All windows were covered over and taped to keep out extraneous light. The influence of random convective currents in the room caused occasional trouble with the use of the interferometer. It became necessary to keep all heat turned off in the room for several hours prior to testing and throughout the tests. Likewise, even though the room had been apparently sealed, changes in outside conditions of strong winds, bright sunlight, or rapidly changing temperature caused fluctuations in the action of the interferometer. Consequently testing was accomplished only on relatively calm days with all heat turned off.

The specimen used to study free convection was an electrically heated flat aluminum plate 6 inches by 10 inches in size. The test plate had been used by personnel of the Mechanical Engineering Department of the Air Force Institute of Technology for a series of heat transfer tests. It had been found to have a very uniform temperature distribution over a range of temperatures. The plate was heated by electrical resistance heaters located in the rear of the plate which were well insulated electrically from the metal plate itself. Iron-constantan thermocouples were located in the plate in the positions shown on Figure 4. The thermocouple junctions were located in holes drilled in the plate which had been filled in so that the front of the plate was smooth. During the course of the tests, four reference temperatures on the plate were read whenever data were taken with the interferometer. These temperatures were observed to remain within one degree Fahrenheit of each other in almost all of the runs. The greatest difference was slightly over 1°F.

The plate is shown mounted vertically in the interferometer in Figure 5. Shown also in the figure are a typical electrode and the traversing mechanisms used.
Figure 4
Locations of thermocouples on heated plate.
Figure 5

Typical interferometer installation of heated plate and electrode.
Temperatures were read by means of a Rubicon Instruments potentiometer using an ice-water bath reference temperature for the thermocouples.

A.C. current was used to power the heaters in the plate in the early phases of the test. The power dissipated in the heaters was controlled by means of voltage regulators. The variation of temperature over the plate could be controlled by adjusting the applied voltage to specific heaters, and the mean plate temperature could be varied by a master voltage control. Although this power supply provided very good temperature regulation for the free convection tests without applied fields, it was found to cause difficulty with the microammeter used to measure corona currents. During initial studies of the combined corona-heat transfer tests, it was noted that the microammeter read very erratically whenever the A.C. heater supply was turned on. Because of this, the A.C. power supply was replaced by automotive storage batteries to provide a ripple-free D.C. power source. Variations in voltages required for temperature control were secured by a suitable rheostat. With this arrangement, the difficulty with the microammeter was eliminated and the instrument was steady through all its ranges. Plate temperatures were adjusted to uniform values and they remained within the $1^\circ$F tolerance indicated previously. Figure 6 shows the heater power supply. The D.C. supply controls are located at the front of the table, while the A.C. Variacs are shown at the rear of the table. The batteries used are located below the table, and the voltmeters and ammeters used to monitor heater input power are located in the center of the table. The thermocouple potentiometer is in the rear of the table and a microammeter used to measure corona current is at the right.
Figure 6
Heater power supply
Figure 6
Electric Apparatus

A 0-50,000 volt power supply was used to provide the necessary electrical field for the corona discharge. It had a current capacity of up to 2 milliamperes which was more than adequate for the tests accomplished. The design of the power supply allowed only a negative ground. Because of the arrangement of the test equipment it permitted only the study of a positive point corona. Current was measured through the use of a Greibach microammeter. This instrument had ranges from 0-1 microamperes up to 0-1000 microamperes.

Voltage measurements were taken through the use of three separate instruments. One voltmeter was incorporated as part of the power supply. This instrument was read during all heat transfer runs. The voltmeter had a 0-50,000 volt range. The second voltmeter used was an RCA Voltomyst utilizing a high voltage probe. This instrument drew undesirable currents in its operation, which, however, could be accounted for and calibrations set up to correct for them. The third voltmeter used was a Sensitive Research Instrument Corporation electrostatic voltmeter with a range of 0-40,000 volts. It was used during precise voltage, current, and pressure measurements of various corona discharges. Independent of which voltmeter was used in the tests, it was found that the current was the most sensitive parameter to control and as a consequence all settings were based upon current. It was often noted that large variations in current occurred even though the changes in voltage were barely discernable.

For the tests with dielectrophoresis an A.C. power supply was used. The supply consisted of a neon transformer connected to a Variac to provide a voltage range of zero to 20,000 volts. Several milliamperes current could be drawn from this source.
Electrodes

Several electrodes were fabricated and tested. Typical ones are shown in Figure 7. The electrode at the left consists of a series of No. 7 needles soldered precisely at 1/2 inch intervals along a 1/8 inch brass rod 11 inches long. The alignment of the tips of the needles was done carefully in a jig in order to obtain minimum distortion in the interferometer due to inaccuracies of needle arrangement. The needles in this case served as the corona discharge points. The electrode located at the front of the picture is a non-discharging one made of 1/8 inch brass rod, shaped in the form of an oval 12 inches long. The electrode in the rear of the picture is a grid of fine wires 0.010 inch in diameter. The wires are spaced at one inch intervals and are 12 inches long. The wires act as the corona discharge source. The electrode at the right is a flat aluminum plate 6 inches by 10 inches in size. It is not heated. It was used as a non-discharging electrode to provide an approximately parallel field.

The installation of an electrode is shown in Figure No. 5, in which the 6 x 10 heated aluminum plate is mounted in the interferometer. A single wire electrode can be seen mounted at a 2 centimeter distance in front of the plate. This single wire electrode is typical of the type used throughout the tests. It is a single stainless steel wire put in tension to keep it straight, electrically insulated from the mounting attachments. A small stranded wire lead is brought from the corona wire out to the attachment of the high voltage lead. The electrode is mounted in a traversing mechanism which allows movement in three perpendicular coordinate directions. Thus the corona wire may be moved up and down or its distance from the plate varied as required. The wire shown in the figure is 12 inches long of 0.004 inch diameter.

Both the plate and the electrode were electrically insulated from the remainder of the equipment. The plate was maintained at ground potential
Figure 7
Electrodes used for interferometer tests.
by the ground return lead to the power supply. All the sharp edges of the plate or electrodes were rounded over or covered with cement or corona dope to minimize localized undesirable corona discharge. The surface of the plate itself was burnished to form a clean polished surface free from sharp scratches. In spite of the precautions taken, however, at very high values of the applied voltage, some localized discharge was found to occur both from spots on the plate, as well as from distinct points on the various electrodes. Fortunately these localized discharges appeared to occur only at voltages and currents which were considered beyond the range of interest in the study.

**Pressure Survey Equipment**

In order to interpret the phenomena observed with the interferometer, it became necessary to study the pressure distribution between various electrodes and the flat plate. Review of past data obtained by Chattock (4), Harney (5), Stuetzer (6), and Robinson (7), revealed that a sharp pressure peak was to be expected directly beneath the fine wire during a corona discharge. None of these investigators, however, had published any data for the case of fine wires placed parallel to a flat plate. To determine such pressure distribution, tests were run by using the various electrodes and a flat plate provided with a series of pressure taps. The arrangement of the electrodes and pressure plate is similar in appearance to the arrangement of the electrodes shown in Figure No. 5. The heated plate was replaced by the pressure plate and pressures were recorded during the corona discharge. No interferometer or heat transfer data were taken during the pressure surveys. Two separate tests of pressure variation were set up. In the first sequence an inclined manometer board was tilted to a 10° angle. By means of suitable flood lights sharp definition of the meniscus was established for all the tubes. Figure 8 shows the manometer board with the flood lights. To measure the
pressures, a suitable surface was fabricated from a 7 inch by 12 inch aluminum plate. A series of holes were drilled at 0.2 inch spacing with a No. 54 drill. The holes were drilled along the horizontal and vertical centerlines of the plate. One hole was drilled at the center of the plate and the remaining holes were spaced outwardly up to two inches away from the center of the plate. The back of the plate was counter drilled so that 1/16 inch metal tubing could be inserted. All tubes were sealed by a Glyptol varnish as they were pressed into the plate. The pressure measuring plate can be seen located on top of the manometer board in Figure 8. Plastic tubing was used to connect the pressure taps to the manometer board. All leads were made as short as possible, and all connections were made as tight as possible to minimize the possibility of leakage. The entire surface of the plate was polished. All nicks, scratches, and rough edges were smoothed out, and corona dope was applied at all positions of potential local discharge away from the center of the plate.

In order to obtain useful data on the very small pressure differences, it was necessary to employ special procedures in the use of the manometer board. A Polaroid camera was mounted above the manometer board, perpendicular to the board to minimize distortion. The Polaroid picture was then read by means of a Gaertner Comparator. Before every test sequence reference pictures were taken, and after each sequence another reference picture was taken. Room temperature changes had to be watched carefully to minimize drift effects. By comparing the position of the meniscus on each data run with the corresponding reference values, it was possible to read values of pressure to less than 0.001 inches of water. Repeatability was good. This technique provided simultaneous readings of pressure across the face of the plate and thus gave a good indication of the form of the pressure distribution.
Figure 8
Overall view of inclined manometer showing spot light installations and 7 x 12 plate.
The second technique of measuring pressure utilized the same electrodes and pressure plate as described before. The pressures generated were read, however, by means of a Chattock micromanometer. This pressure test is shown in Figure 9. The high voltage power supply, the electrostatic voltmeter, ammeter, as well as the pressure plate and manometer can all be seen. Figure 10 shows the Chattock micromanometer. This manometer provides only one pressure reading at a time. Thus for a pressure survey it is necessary to change the pressure taps in sequence along the rear of the plate. The plastic tubing lead was made as short as possible to minimize the lag in the system. Readings were made to determine the lag time before stable readings were achieved. After the tubing was shortened and the lines made as clear as possible it was found that a stable value was achieved well within one minute.

The Chattock manometer used was a two-fluid “U” tube. The fluids used in this case were kerosene and distilled water. A Gaertner Comparator was used to read the level of the maniscus. It provided an increase in sensitivity of the instrument as well as improving ease of reading. The Chattock manometer required considerable adjustment and care in its operation if consistent data were to be obtained. During the operation of the entire pressure measurement apparatus, it became evident that considerable caution would have to be exercised during tests. Room temperature variations, outside winds, and gusts readily affected not only the individual readings but zero shifts as well. Variations in humidity affected the corona discharge. Consequently pressure tests were run only on quiet days with the room heat turned off, and on days when the relative humidity was not high. Figure 11 illustrates a typical set-up with the single wire in place a short distance in front of the pressure plate.
Figure 9
Overall view of pressure survey arrangement using Chattock micromanometer.
Figure 10
Close-up view of Chattock micromanometer and Gaertner comparator.
Figure 11
Corona wind pressure plate set up
showing electrode in place.
Experimental Procedures

Because the electrostatic phenomena produce forces which are small in magnitude, it was necessary to watch for all extraneous disturbances that could occur in the test environment as well as the accuracy of the instrumentation. The difficulties with the temperature variations, gusty weather, and humidity have been indicated previously. To alleviate the presence of small random air motions in the vicinity of the test section of the interferometer, an enclosure was placed about the test section. It was located in such a fashion so as to not materially affect the free convection flow of air near the plate and the flow of air from the plate and the flow of air from the corona discharge. Use of the enclosure resulted in more stable test patterns.

An interferometer provides data on the isothermal lines in the fluid around a heated body. It does this by sensing the average density changes in the working section. A free convection test of a flat plate in still air will reveal a series of fringes located in front of the plate. By measuring the spacing of the fringes and through calculations of the temperatures which each fringe represents it is possible to determine accurately the temperature profile through the thermal boundary layer as well as the local heat transfer coefficients. A typical interferometer picture is shown in Figure 12. An end view of the test specimen is seen. The flat heated surface is seen at the right. The fringes are evident in front of the plate. In this particular case the plate temperature is 92°F, and the room temperature is 64.1°F. The "T" shaped rod located at the bottom provides a reference length for data reduction. The end view of the electrode support rod can be seen at the left side. The fine corona wire is located at the end of the rod but cannot be discerned in the picture. No voltage is applied in this interferogram. When a voltage is applied between the fine wire and the plate and changes to the thermal
Figure 12
Typical picture as seen with the interferometer.
No applied field.
boundary layer occur, these are indicated by distortions in the fringe patterns. In general, if the fringes are brought closer together causing a thinning of the thermal boundary layer, then an increase in the heat transfer results. If the boundary layer is pulled outwardly, then this would indicate a decrease in the heat transfer rate. Thus when viewing the data presented in subsequent photographs, these basic points should be borne in mind as an aid in interpreting the phenomena.

The accuracy of the data taken depends both on the environmental conditions as well as on the instrumentation used. Temperatures measured with the thermocouples and the thermometers could be read to within 0.1°F. The interferometer fringe patterns contain errors due to the variation in density in the horizontal direction along the plate. This error can be compensated for and a compensation is included in the data reduction method. A second error is introduced by the end effects of the finite width plate. The largest inaccuracy comes in the actual measurements of the spacings of the fringes. Although the Gaertner comparator used to measure the spacing can provide readings of .0001", the actual determination of the centerline of the fringes introduces far greater uncertainties. When the fringes are spread far apart and are diffuse in character a sizeable uncertainty can result. Conversely, when the fringes are compacted one upon the other, the individual lines are difficult to locate and an uncertainty is introduced.

The voltage and current data are also subject to inaccuracies. As mentioned previously the voltage used in corona discharge phenomena is a relatively insensitive parameter in the region of the initial flow of current. In order to initiate the corona discharge, the voltage control knob of the power supply was turned slowly until the initial change in current could be observed. The voltage reading was of little assistance in determining the starting point. Many calibrations were made with the
three voltmeters described previously. The precision electrostatic voltmeter was used as a standard to calibrate the other instruments. The electrostatic voltmeter was available for use during only a portion of the work and as a consequence recourse was made to the less accurate instruments.

Current measurements made with the Greibach microammeter are considered to be accurate representations of the currents flowing. Unfortunately corona discharge currents are very erratic. The fundamental nature of the discharge currents with a positive point consists of a series of random current pulses accompanying electron avalanches toward the positive point. Slightly varying atmospheric conditions in the vicinity of the wire also could cause variations in the corona current. Consequently, the pointer of the microammeter moved continually and erratically during all readings. These same variations have been noted by other investigators (5, 7). Several readings were usually taken, but because of drifting which would occur to either the high side or low side, the averages were not truly representative. Efforts were made in all tests to take readings at the most stable current value in the range under consideration. Because of this erratic behavior it is difficult to assign a value for the accuracy of the current data even though it is a most sensitive parameter in this investigation. From study of many data points it is believed that the current values are within 5% in the small current ranges (less than 10 μa) and considerably better than that for higher currents.

The pressure data contain several possible sources of inaccuracies. With both manometers the possibility existed of the meniscus sticking to the wall of the tubes and thus introducing errors. If continuous direct light were directed on either manometer, all readings became meaningless. Both volumetric changes as well as distortions of the apparatus occurred whenever direct light was used for more than an instant.
Consequently, when working with the inclined manometer it was necessary to flash on the lights momentarily while taking a picture of the tubes. The Chattock manometer was particularly sensitive to random external disturbances. As indicated before, changes in temperature, drafts, outside winds, and aircraft all could cause erratic readings. However, even doors being closed, cabinets opened, other tests being conducted in remotely located areas could cause severe transients in the pressure readings. An ideal location to perform these tests would have been an air conditioned room with controlled humidity, with extremely rugged walls, and with heavy thermal insulation.

With the pressure equipment set up in the same room as the interferometer, the disturbances due to the movement of people was substantially eliminated. Testing was accomplished on what appeared to be very quiet and calm days. Even under the best conditions a troublesome drift of the reference zero occurred. Many runs were obtained under good test conditions in which the zero shift was within .0001 inches of water, and these data show a consistent pattern with little shifting and distortion. Unfortunately some of the runs consumed several hours of time and small atmospheric changes may have occurred which were not readily detectable and zero shifts occurred. Data obtained under these conditions were erratic and did not correlate well. In spite of the difficulties it is believed that reliable data were taken. It will be presented in detail in the next section on test results.

By far the most troublesome problem in the whole experiment was the difficulty of being able to repeat the tests to obtain the same data with a high degree of precision (within one percent). The factors causing this which have been mentioned before are of considerable importance, but fundamental to this problem is the erratic character of the corona discharge itself. Variations of the corona due to small changes in humidity,
local ionization, random convective currents are very difficult to control. The average mobility of the ions is considerably affected by the presence of moisture in the air. Since pressure rise varies approximately inversely as ion mobility, pressure rise due to corona discharge can be expected to vary with humidity. Clear cut corrections for the influence of humidity on pressure rise are apparently not available, and as a consequence the pressure rise data were not corrected for the humidity effects. It should be noted, however, that the basic interactions between corona discharge and free-convection heat transfer occurred for all ranges of humidity from very dry cold days, to foggy days where the humidity was close to 100%.

Results of the Experimental Investigation

Method of Data Interpretation

The heat transfer coefficients were obtained from the interferograms by measuring the spacing of the fringes and through the use of the calculation procedures indicated in Appendix II. Readings were taken at ten separate locations along each plate. In some cases more positions were included. The data is presented in the plots of heat transfer coefficients along the surface of the plate. The effects of varying current and wire spacing are shown. Data are also presented to illustrate the usual characteristics of corona discharge in the form of a voltage-current graph.

Interferometer Data

The results of the combined electric field-convection tests are shown in the accompanying photographs and the graphs which correspond to them. The most significant results are presented and interpreted. The first sequence of tests incorporates the heated plate in a vertical position. A single stainless steel wire .004 inch in diameter located 2 cm away from the plate is used as an electrode. This configuration provides an
intense electrical field near the wire, and a highly non-uniform electrical field in the region in front of the plate. The wire is horizontal, and is located at the centerline of the plate. The potential applied between the wire and the plate is raised from zero slowly up to values sufficient to give currents of approximately 150 µa.

Plates I and II present the picture as seen with the interferometer as the voltage is increased. Picture A is the reference with no applied field. The plate temperature is 119°F and the room temperature is 69°F. In this exploratory run, no attempt was made to hold the plate temperature constant at high corona current values. At the high values of current it was noted that the plate temperature dropped to approximately 100°F with a constant heater current. Picture A shows a typical free convection pattern. The rods holding the fine wires are visible at the left. The "T" located at the bottom of the picture provides a reference length. The fringe pattern shown in this picture remains unchanged as the potential is applied as long as the current remains essentially zero. As soon as a slight current begins to flow some changes in the pattern would occur. This behavior is characteristic of all tests conducted with the corona discharge.

Picture B illustrates the initial disturbance of the thermal boundary layer. It occurs just at the initial onset of corona current. Subsequent pictures viewed in sequence show the marked distortion of the isothermal lines as the current is increased. The patterns shown are fairly steady for a given stabilized current except for pictures D and E which are slightly unsteady. The isothermal lines are extremely steady for currents in excess of 10 µa. Study of the pictures reveals that depending upon the value of current, regions of increased or decreased heat transfer can be noted. The data on heat transfer coefficients $h_x$ in this sequence are considered quite accurate for values of current less than
Plate 1

Interferometer pictures of the effects of corona discharge on heat transfer. Vertical heated plate. Plate temperature approximately 119°F. Room temperature 69°F. Wire diameter 0.004 inch. Wire 2 cm. in front of plate at the centerline.
A. No electric field 0 KV

B. 5.41 KV 0.70 microamps

C. 5.8 KV 1.85 microamps

D. 6.1 KV 3.10 microamps

Plate I
Plate II

Interferometer pictures of the effects of corona discharge on heat transfer. Continuation of sequence shown in Plate I.
Plate II

E. 6.4 KV    5.9 microamps  F. 7.5 KV    22.8 microamps

G. 10.0 KV    74.2 microamps  H. 13.4 KV    193 microamps
10 μa. Above this value sufficient plate cooling occurred to introduce some error in $h_x$. Consequently no $h_x$ data are presented for currents greater than 10 μa. The values of the heat transfer coefficient corresponding to the first five pictures is shown in Figure 13. In this figure the heat transfer coefficient is plotted at various locations along the plate for several values of corona discharge current. The abscissa, $x$, represents the distance in the vertical direction along the plate surface. The origin is at the centerline of the plate, three inches from the bottom edge of the metal plate as seen in picture A. The abscissa in Figure 13 indicates that distances to the right of $x = 0$ are for the lower part of the plate, and distances to the left of $x = 0$ are for the upper part of the plate. It is quite evident from a study of the data in Figure 13 that a marked change in the magnitude and distribution of $h_x$ results as a consequence of the applied electrical field. For small currents the variation along the plate is extremely large, while at higher currents the values of $h_x$ tend to become more symmetric about the centerline.

Plate III presents the results with the same configuration as before except that the temperature difference between plate and room air is maintained constant throughout the test sequence. Stabilized test points were obtained and multiple records made of the phenomena. Accurate data were obtained up to high values of current. Picture A shows the reference pattern with no applied field. The changes in the fringes are evident as the current is increased. At the higher values of current, the thermal boundary layer becomes extremely thin and it can be expected that the heat transfer rate has increased greatly. Unfortunately this extreme thinness of the boundary layer makes it difficult if not impossible to determine the local values of heat transfer coefficient. The heat transfer coefficients which correspond to this sequence are shown in Figure 14.
Figure 13
Heat transfer coefficients for vertical plate, 0.004 inch diameter wire located 2 cm distance at centerline.
Figure 13

$z$-distance (inches)

top of plate  bottom of plate

$h_x$

- 1.9 $\mu\alpha$
- 0.8 $\mu\alpha$
- 0.0 $\mu\alpha$

$h_x$

- 6.0 $\mu\alpha$
- 3.1 $\mu\alpha$
- 0.0 $\mu\alpha$
Plate III

Effects of corona discharge on heat transfer. Vertical heated plate. Room temperature 64°F. ΔT held constant at 27°F. 0.004 inch wire located 2 cm. from plate at centerline.
Figure 14

Heat transfer coefficients for vertical plate, 0.004 inch diameter wire located 2 cm. distance at centerline. Second sequence.
The reference curve with no field applied can be obtained with considerable accuracy. The values of heat transfer coefficient for the higher values of current indicate a larger degree of scatter, and in the center of the plate, could not be determined.

The next series of these tests was conducted with the 0.004 inch wire moved to a distance of 5 cm away from the plate. The temperature difference was maintained at 34°F. Plate IV shows the results. The thermal boundary layer is squeezed down as the field is increased. The main difference between this case and the previous one for corresponding high currents is that the thinning of the thermal layer is more uniform across the plate. The values of heat transfer coefficient which correspond to these figures are shown in Figure 15. The difficulties encountered in reducing the data are illustrated again at the higher current values.

Plate V shows a test sequence with the 0.004 inch wire moved out to 10 cm. The higher currents were not reached in this case because the applied voltage required became excessively high. At high voltages, local points of corona discharge occurred on the plate and on discreet points of the wire. Such intense local discharges tended to give erroneous impressions of the value of the current which actually drove the corona wind being studied. Heat transfer coefficients are shown in Figure 16.

In the next sequence of tests, the heated plate was placed so that the flat heated surface faced downward. The 0.004 inch wire was placed 2 cm away from the plate at the centerline. The temperature difference was held constant at 27°F. Plate VI illustrates the influences on the isotherms as the current begins to flow. Only the range from 0 to 4.55 µa is indicated in this sequence. The gradual squeezing of the boundary layer is clearly evident. Carefully controlled experiments indicated that the pattern shown is very symmetrical about the centerline for all values
Plate IV

Effects of corona discharge on heat transfer. Vertical heated plate. Room temperature 61°F. ΔT held constant at 34°F. 0.004 inch wire located 5 cm. from plate at centerline.
A. 0 KV 0 microamps
B. 8.3 KV 4.8 microamps
C. 11.5 KV 19.9 microamps
D. 23.3 KV 158 microamps
Plate IV
Heat transfer coefficients for vertical plate, 0.004 inch diameter wire located 5 cm. distance at centerline.
Plate V

Effects of corona discharge on heat transfer. Vertical heated plate. Room temperature 53°F. ΔT held constant at 34°F. 0.004 inch wire located 10 cm. from plate at centerline.
Data difficult to read at high currents and at center of plate.

Figure 16

Heat transfer coefficients for vertical plate, 0.004 inch diameter wire located 10 cm. distance at centerline.
Plate VI

Effects of corona discharge on heat transfer. Horizontal heated plate. Room temperature 60°F. ΔT held constant at 27°F. 0.004 inch wire located 2 cm. from plate at centerline.
A. 0 KV   0 microamps  
B. 5.0 KV   1.05 microamps  
C. 5.2 KV   2.54 microamps  
D. 5.7 KV   4.55 microamps  
Plate VI
Figure 17
Heat transfer coefficients for horizontal plate, 0.004 inch diameter wire located 2 cm. distance at centerline.
Plate VII

Effects of corona discharge on heat transfer. Horizontal heated plate.
Room temperature 61°F. ΔT held constant at 34°F. 0.004 inch wire
located 2 cm. from plate at centerline.
Plate VII
Figure 18
Heat transfer coefficients for horizontal plate, 0.004 inch diameter wire located 2 cm. distance at centerline. Second sequence.
Plate VIII

Effects of corona discharge on heat transfer. Horizontal heated plate. Room temperature 61°F. ΔT held constant at 34°F. 0.004 wire located 5 cm. from plate at centerline.
Plate VIII

A. 0 KV 0 microamps  
B. 8.7 KV 4.45 microamps  
C. 11.8 KV 21.2 microamps  
D. 23.3 KV 158 microamps
Figure 19

Heat transfer coefficients for horizontal plate, 0.004 inch diameter wire located 5 cm distance at centerline.
of current. The heat transfer coefficients which correspond to these pictures are shown in Figure 17. The method of presentation of data for the horizontal plate is similar to that used for the vertical plate. Since the configuration was symmetrical about the centerline of the plate, only the data for one half of the plate are presented, from the centerline out to the edge where $x = 3$ inches. The gradual change in magnitude and distribution is clearly evident. The variation of $h_x$ becomes bell shaped as the current is applied. At the end of the plate where $x = 3$, the end effects at the edge of the plate cause a localized increase. This characteristic increase can be seen from the bottom curve of free convection with no field applied. It is interesting to notice that for currents of a few microamperes the local values of $h_x$ can be changed several fold.

Plate VII presents test data for another set of runs with the same configuration as used in the previous sequence. The temperature difference was maintained at $27^\circ\text{F}$. The 0.004 inch diameter wire is placed 2 cm below at the centerline. The current range in this sequence covers 0 $\mu\text{A}$ to 159 $\mu\text{A}$. The thinning of the thermal boundary layer is quite evident. At the high values of current, the patterns resemble those found with the vertical plate. The heat transfer coefficients are shown in Figure 18. The values at high currents, and the values near the center tend to be scattered for the reason indicated before.

In the next sequence of tests, the same configuration of horizontal plate is used, but the wire is moved to 5 cm away. The temperature difference is maintained at $34^\circ\text{F}$. Plate VIII indicates the results of the tests. The same thinning down of the boundary layer is evident. The shape is somewhat flatter than with the 2 cm case. The heat transfer coefficients are shown in Figure 19. The change in magnitude and distribution is evident. Once again, at higher current values the data becomes
scattered due to the difficulty of reading the interferograms. Large changes in the heat transfer coefficient again result from a small current flow.

Pressure Survey Data

Tests were conducted of the pressure rise obtained with a corona wind to provide both data on the magnitudes of the pressures as well as the variation of the pressure along the plate. The test configuration consisted of a single wire located parallel to the plate at the centerline. Three wire diameters were tested, .0005, 0.004, and 0.010 inches. The wire was placed at the centerline at various distances in front of the plate. The wire was oriented so that it ran parallel to the 7” dimension of the pressure plate. The pressure rise obtained at the centerline of the plate is shown in Figures 20, 21, and 22. The data in all figures indicate an approximately linear pressure-current characteristic, which is in accordance with the theories of corona wind. Some zero shift occurred, but the data are fairly orderly. A typical zero shift can be seen in the 2 cm case of Figure 22. A small influence of spacing of the wire can be noted, however any trends due to wire spacing are not consistent as the wire size is changed. The magnitudes of the pressure for a given current likewise do not seem to be markedly influenced by wire size. No clear cut trends of the effects of wire size or wire spacing can be determined from the data. All data presented in these three figures were taken with the Chattock micromanometer.

The next sequence presents the results of the studies of pressure distribution. Data were taken with the Chattock micromanometer. A .004 inch wire was used for all these tests. Figures 23, 24, and 25 present the results for three wire spacings, 2 cm, 4 cm, and 6 cm. The magnitudes of the centerline pressures at a given current do not vary greatly.
Figure 20

Stagnation pressure at the centerline of 7'' x 12'' flat plate for various wire spacings. Wire diameter 0.0005 inches.
Figure 21

Stagnation pressure at centerline of the 7" x 12" flat plate for various wire spacings. Wire diameter 0.004 inches.
Figure 22

Stagnation pressure at centerline of the 7" x 12" flat plate for various wire spacings. Wire diameter 0.010 inches.
Figure 23

Corona wind pressure distribution over the surface of the 7" x 12" plate. 0.004 wire at centerline along the 7" dimension, 2 cm. away from plate.
Figure 24

Corona wind pressure distribution over the surface of the 7" x 12" plate. 0.004 wire at centerline along the 7" dimension, 4 cm. away from plate.
Figure 25

Corona wind pressure distribution over the surface of the 7" x 12" plate. 0.004 wire at centerline along 7" dimension, 6 cm. away from plate.
This was indicated in the previous test data. The shape of the curves at all spacings is similar. Considerable testing had shown that the pressure distributions were symmetrical, and consequently one-half of the distribution is shown. Because the corona discharge does not necessarily occur in a line perpendicular from the wire to the plane, some shift in the peak pressure position can be expected. Study of the curves reveals that the centerline of the corona wind actually did move away from the geometric centerline of the plate in some cases.

Figure 26 presents the results of another series of tests at 2 cm spacing. The data taken at the lower three currents were strongly affected by random zero shifting. Although the curves seem to be of reasonable proportion, it will be seen in a subsequent curve, that these data are unusable. These curves are included solely to indicate the characteristic variability of the corona phenomenon.

Figure 27 presents typical pressure distributions obtained with the inclined manometer. The data were obtained to get the general shape of the pressure distribution. The absolute values of these data are not as good as those obtained with the Chattock manometer. As can be seen, the same bell shaped appearance of the data occurs as was found with the previous data.

**Current-Voltage Relations**

Many sets of data on the variation of current with applied voltage were taken during the course of the testing. Since it proved difficult to relate the voltage to either test technique or data interpretation, only one representative voltage curve will be presented. Figure 28 presents the results for a corona discharge with a 0.004 inch wire located at the centerline of a 7" × 12" flat plate. The wire ran in the direction of the 7" dimension. Data are presented for various wire spacings. It is believed that the results are typical of those found with corona discharge.
Figure 26

Corona wind pressure distribution over the surface of the 7" x 12" plate. 0.004 wire at centerline along 7" dimension, 2 cm. away from plate. Data for low currents very erratic.
Figure 27

Typical pressure data from inclined manometer, \( a = 2 \text{ cm} \).
Figure 28
Typical current-voltage curves for a corona discharge with 0.004 wire at centerline along 7" dimension of a 7" x 12" flat plate, 4 cm. away.
Figure 28

![Graph showing the relationship between voltage (KV) and current (µA) for different distances (cm).](image)
Discussion of Test Results

The purpose of this section is to describe in some detail the phenomena which were observed in the foregoing sequence of tests on the interferometer. Reference to Plate I reveals an unusual movement of the thermal boundary layer. At the initial flow of current the isotherms are "pulled-out" from the heated plate. At somewhat greater currents this "pull-out" becomes larger. This phenomenon occurred in all runs with the vertical plate, and with several electrode configurations. As the current was raised to even higher values, the "pull-out" disappeared as indicated by the pictures in Plate I. The cause of the "pull-out" was investigated in considerable detail. Pressure distribution measurements at low currents failed to provide any accurate data on the pressure field in the region of pull-out. The "pull-out" shape appeared to move opposite to the direction expected from corona wind action. Any dipole forces acting in the air should have acted in a direction opposite to that appearing in the "pull-out" also. (Discussion of dipole forces may be found in Appendix I.) A possible explanation of the "pull-out" phenomenon lay in the action of two impinging fluid streams. With a heated vertical plate, a convective stream moves upwards along the plate. If the corona wind strikes the plate, it can be expected that the stream will split and try to move upwards and downwards. The corona stream moving downwards will buck the upward streaming of the thermal boundary layer. It appeared possible therefore that as the two fluid streams met, a change in the temperature field would occur which could have an unanticipated form. In order to explore the possibility that the "pull-out" phenomenon was due to an interaction of the corona wind and the thermally heated stream, the sequence of tests previously shown in Plates VI, VII, and VIII were conducted. The corona wire was placed exactly at the center
of an accurately leveled horizontal plate at a short distance below the plate. Upon application of the field, the center of the corona wind stream would be located at the point of zero longitudinal velocity of the thermal boundary layer. Thus any corona streaming after striking the plate would split into two streams, and each stream would move in the same direction as the streaming in the thermal boundary layer. If no "pull-out" occurred during such a test, then the "pull-out" phenomenon must have been associated with the fluid flow impingement. Conversely if the "pull-out" did occur, then the phenomenon would require some other explanation, and conceivably could be a new type of interaction. The tests with the horizontal plate were set-up, both with the hot side up and the hot side down. The convective cells involved in free convection with the upper face hot made it impossible to interpret the results. The tests with the bottom heated and the corona wire at several spacings were most successful. The results are shown in Plates VI and VII. As can be seen from the pictures, as the field is applied and the current flows, no "pull-out" is evident with either wire spacing. This result was repeated several times for verification. It was necessary, however, to conduct the experiments extremely carefully so that extraneous disturbances did not confuse the results. Since the "pull-out" did not occur, it was felt that the "pull-out" phenomenon observed with the vertical plate tests was due to the impingement of fluid streams, and not an unexplained electrostatic effect.
ANALYTICAL STUDY OF THE EFFECT OF CORONA DISCHARGE ON FREE CONVECTION HEAT TRANSFER

An analysis of the influence of the applied electric field on the heat transfer of the heated flat plate was undertaken. It was done to try to learn possible trends which might occur as well as to aid in obtaining a better understanding of the phenomena. The analytical work is broken down into phases including electrical field effects, heat transfer theory, the combined problem, and an approximate solution to the horizontal plate case.

Electric Field-Corona Discharge Relations

In analyzing corona discharge phenomena it is essential to obtain relationships covering the field strength, current distribution, current-voltage relationships and the induced pressure rise. The theory is based upon the recognition that a definite space charge, \( \rho_c \), exists. Two simple illustrations of solutions will be presented; a one dimensional case, and a cylindrical electrode configuration.

The electric field equations from Appendix I are:

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \quad (1) \]

\[ \vec{J} = \sigma_c \vec{E} + \rho \vec{v} \quad (2) \]

\[ \vec{F} = \rho \vec{E} \quad (3) \]

Pressure Rise with Space Charge

In a gas where the total current carriers are associated with the ions, \( \vec{J} = \rho_c \vec{v}_t \), where \( \vec{v}_t \) is the total velocity of the moving charge. It is assumed that ions of only one sign exist away from the immediate vicinity of the corona point, and the mobility of these ions is approximately constant. (A discussion of the concept of ion mobility may be
found in Appendix I.) Then since the total velocity of the charge (ions) is equal to the sum of the gas motion and ion motion the following relationship exists.

\[ \vec{v}_t = \text{velocity of ions relative to the gas} + \text{velocity of the gas.} \]

The velocity of the ions relative to the gas is equal to \( KE \).

\[ \vec{v}_c = KE + \vec{v} \]

where \( \vec{v} \) is gas velocity. The current density becomes

\[ J = \rho_c \vec{v}_c = \rho_c (KE + \vec{v}) \]

using

\[ \rho_c = \epsilon (\nabla \cdot \vec{E}) \]

\[ J = \epsilon (\nabla \cdot \vec{E})(KE + \vec{v}) \]

Considering the relative magnitudes of \( KE \) and \( \vec{v} \) for a typical case of corona wind

\[ KE \approx 100 \text{ m/sec} \]

The usual air velocities in a corona discharge are below 5 m/sec. Thus \( \vec{v} \) is considerably smaller than \( KE \) in this case and it will be neglected. If air is blown past the corona point at higher velocities, then \( \vec{v} \) could become significant (5).

\[ J \approx \rho_c KE = KE (\nabla \cdot \vec{E}) \vec{E} \]
In the stagnation region under the impinging corona wind stream, the velocities will be very small. The boundary layer thickness in this region will likewise be very small, and the pressure across the boundary layer will be approximately constant. Consequently the pressure rise occurring due to the electrical field will occur in the region of the air where essentially non-viscous potential flow occurs. Consequently in the Navier Stokes equations (8,9)

\[ \rho \frac{\partial u_i}{\partial t} = \mathbf{X}_i - \frac{\partial P}{\partial x_i} + \nu \nabla^2 u_i + \frac{\kappa}{\Delta} \frac{\partial \Delta}{\partial x_i} \]

\[ \mathbf{X} = \mathbf{F}, \quad \Delta = \frac{\partial u_i}{\partial x_i}, \]

where \( \mathbf{F} \) the electrical body force is equal to \( \mathbf{X} \); the body force exerted on the fluid. The pressure gradient and the electric field body forces are of considerably greater magnitude than the viscous or inertia forces, and consequently the equations reduce to

\[ \nabla P = \rho \mathbf{E} \]

and

\[ \nabla P = \mathbf{J} / \kappa \]

follows. This indicates that the pressure gradient is proportional to current density and inversely proportional to the ion mobility. In a one dimensional case, the equation becomes

\[ \frac{\partial P}{\partial y} = \mathbf{J} / \kappa \]

\[ P = \int_{y_0}^{y_1} (\mathbf{J} / \kappa) dy \]

In a one dimensional model of cross-section \( A \), the current density would be related to the current \( i \) by:

\[ \mathbf{J} = i / A \]

\[ \rho = \frac{i}{kA} \int_{y_0}^{y_1} dy = (i / kA) (y_1 - y_0) \]
This predicts a linear rise in pressure as either the current or the electrode spacing is increased. The distance \( y_o \) is a reference distance located a very short distance away from the region of intense corona discharge.

If the electrode geometry is cylindrical, then for a constant current,
\[
\bar{J} = i/A
\]
and \( A \) is given as a function of radius by
\[
A = l r \theta_c
\]
where \( l \) is the length of the fine wire electrode, \( r \theta_c \) is the arc length at any particular radius \( r \), and \( \theta_c \) is the angle subtended by the outer electrode.

\[
\begin{align*}
p &= \int_0^r \frac{i}{k} \frac{dr}{r} \\
&= \left( \frac{i}{k l \theta_c} \right) \ln \frac{r}{r_0} \\
&= \left( \frac{i}{k l \theta_c} \right) \ln \frac{R}{r_a}
\end{align*}
\]

If the outer electrode radius is \( R \), and it is assumed that the reference radius at the edge of the intense corona is \( r_a \), then the pressure at \( R \) is
\[
p = \left( \frac{i}{k l \theta_c} \right) \ln \frac{R}{r_a}
\]

This relationship would indicate that the pressure is once again proportional to the current, but varies as the logarithm of the radii ratio as the spacing is changed, or wire size is changed.

If relationships between field strength and current are desired, then it can be shown (1, 7) for the linear case:
\[
E^2 - E_0^2 = \frac{2i}{ekA} (y - y_o)
\]
Where

\[ E = E_0 \ @ y = y_0 \]

For the cylindrical case the relationship can be shown to be

\[ E^2 = \left( \frac{2i}{\varepsilon \kappa} \right) + \frac{c_n}{r^2} \]

\[ c_n = (r_a E_a)^2 - 2i r_a^2 / \varepsilon \kappa \]

Where \( E_a \) is the field strength at \( r_a \) (1).

A complete listing of the various relationships of current, voltage and pressure for simple geometries is given by Steutzer in reference 6.

Cobine (1) derives the relationship between corona current and applied voltage for a cylindrical electrode configuration in the form

\[ i = \frac{2kV(V-V_c)}{R^2 \ln R/r_a} \]

where \( V_c \) is voltage at which corona starts. He points out that the general relationship between current and voltage for corona discharge is of the form

\[ i = CV(V-V_c) \]

\( C \) is a constant which depends primarily upon the geometry of the system. This form has been checked by many investigators and has been found to hold quite well at low current values.

In order to determine the pressure rise occurring between the fine wire and flat plate configuration used in the experimental phase, attempts were made to derive the space charge limited current, voltage and pressure relationships. The boundary conditions in the problem of a round
cylinder at one side and a flat surface at the other side proved to be difficult to handle mathematically. Since it was desirable to have explicit equations for the field and current distribution, numerical solutions and representations were not attempted. Approximations based upon a cylindrical configuration were tried but when these were matched with the form of the pressure curves obtained from test data, the correlation was very poor.

Pressure Rise with Low Charge Density

Since small currents had produced very significant results in the experimental phase, it was decided to investigate the possible use of the space-charge-free field equations. Without space charge, the problem of determining the field became an elementary one. The one case of a long wire located above an infinite conducting plane is covered in many sources (10, 11, 12). For close spacing and small wire size, it was assumed that the test configuration might be represented adequately by this theoretical model.

The equations for the two dimensional field based upon the method of images as as adapted from reference (10) are:

\[ E_x = \frac{\lambda}{2\pi \epsilon} \left[ \frac{y}{R_1^2} - \frac{y}{R_2^2} \right] \]

\[ E_y = \frac{\lambda}{2\pi \epsilon} \left[ \frac{y+a}{R_1^2} - \frac{y-a}{R_2^2} \right] \]

where the various distances are represented in Figure 29. \( \lambda \) represents charge per unit length.

At the surface of the plate \( y = 0 \), and \( R_1 = R_2 = R \) This leads to the following relations

\[ E_x = 0 \] at the plane surface
Figure 29
Field of a wire parallel to a plane.
The equation for voltage is
\[ V = \frac{\lambda}{2\pi\varepsilon} \ln \frac{2a}{r_a} \]
which is valid for large spacings relative to the wire size.

Letting \( Z = x/a \)
\[ E_y = \frac{Z(V_a)}{\ln (2ay/a)} \left[ \frac{a^2}{x^2 + a^2} \right] \]

The relationship for the pressure in a dielectric due to the applied electrostatic field is given by (13, 14)
\[ \Delta p = \frac{1}{2} \varepsilon E^2 \]
Strictly speaking this relationship holds only for the pressure rise in a dielectric between two parallel plates. It does not hold precisely for the case of a corona discharge. However, Stuetzer (6) indicates that relationships of the form
\[ \Delta p = \frac{1}{2} \varepsilon E_{max}^2 \cdot (G) \]
hold quite well for corona discharges. \( G \) is a constant which depends on geometry. The maximum value for \( E \) will occur near the plate where \( E_x \) is small. Consequently as an approximation
\[ \Delta p = \frac{1}{2} \varepsilon E_y^2 (G) \]
with the constant \( G \) left undetermined for the moment.
Using this relationship (26) and equation 25, the expression for pressure can be written as

\[
\Delta \rho = 2 \varepsilon \left[ \frac{\left( \frac{V}{a} \right)}{\ln \left( \frac{2a}{x} \right)} \right]^2 \left[ \frac{1}{1 + z^2} \right] (G) 
\]

When \( x = 0, z = 0 \), then at the centerline

\[
\Delta \rho_c = 2 \varepsilon \left[ \frac{\left( \frac{V}{a} \right)}{\ln \left( \frac{2a}{x} \right)} \right]^2 (G) 
\]

where \( p_c \) is centerline pressure. If the reference atmospheric pressure is assumed to be zero for convenience, then \( \Delta \rho_c = p_c \)

\[
\frac{p}{p_c} = \left[ \frac{1}{1 + z^2} \right]^2 
\]

This expression provides a pressure distribution over the flat plate under the assumptions of no space charge effects, that \( \rho = \frac{1}{2} \varepsilon E(x) \) and that the influence of \( E_x \) is relatively small. Equation 29 is plotted in Figure 30. To check whether this expression would provide a reasonable representation of the actual phenomena, comparison was made with experimental data. The comparison is shown in Figures 31, 32, and 33. These figures present the same pressure distribution data as discussed previously in Figures 23, 24 and 25, except that the data have been normalized. The magnitudes of all pressures have been normalized by dividing by the centerline peak pressure. The distances along the plate were transformed by the introduction of the new variable \( Z \), where \( Z = x/a \), and \( a \) is the distance from the wire to the plate. These plots show that the form of the pressure distribution for a given wire spacing remains approximately the same as the current...
Figure 30
Normalized theoretical pressure distribution assuming $p_c = 0$. 

\[ \frac{P}{P_c} = \frac{1}{(1 + z^2)^2} \]
Figure 31

Normalized pressure distribution over the surface of the 7" x 12" plate. 0.004 wire at the centerline along the 7" dimension, 2 cm away from the plate.
Figure 32

Normalized pressure distribution over the surface of the 7" x 12" plate. 0.004 wire at the centerline along the 7" dimension, 4 cm. away from plate.
Figure 33

Normalized pressure distribution over the surface of the 7" x 12" plate. 0.004 wire at the centerline along the 7" dimension, 6 cm. away from the plate.
is varied widely. In addition the shape of the distribution remains essentially the same as the spacing is changed. It can be seen that the normalized distribution is more peaked in the 6 cm. case, but the differences are not large. Figure 34 presents the normalized data of Figure 26. It is included solely to illustrate the large changes which can arise from small zero shifts in the micromanometer. The data obtained at the lower three values of current in this figure were all found to have small random shifts in the zero reference of the manometer. As can be seen from Figure 34 these data indicate a wide variance and were not used further.

As can be noted from Figures 31, 32 and 33 the correlation between the shape of the pressure curve for the test data and the theory is quite good for the smaller values of "a." At the larger values of "a" the correlation becomes poorer. This may possibly be attributed to the fact that the assumed infinite plate, two dimensional model becomes a poor approximation as the spacing becomes large. Based upon the comparison with the test data, the variation \[ \frac{\rho}{\rho_c} = \left[ \frac{1}{1 + z^2} \right]^2 \]
is considered to be a reasonable representation so long as "a" is considered small. It provides information on the shape of the pressure distribution only.

In attempting to calculate \( p_c \) from the form \( p_c = k e^{\frac{V a}{\ln (2\gamma a)}} \), it was found to be difficult to determine the geometric constant \( G \) because of the boundary conditions mentioned in the space charge case. Consequently in calculating center pressures, the data obtained during the pressure tests were used. The relationship used between center-line pressures and current was in accord with the equations for the one dimensional case and the cylindrical configuration.
Figure 34

Normalized pressure distribution over the surface of the 7" x 12" plate. 0.004 wire at centerline along 7" dimension, 2 cm. away from plate. Data for low currents illustrate erratic behavior due to zero shifts.

Note -
Data for 5, 10, 25 μA currents are erratic and should not be used.
The test data shown in Figures 20, 21 and 22 verify the linear relationship of pressure and current. Using $p = C_1 i$ the constant $C_1$ becomes essentially a geometric shape factor for a given ion mobility. $C_1$ then is determined from Figures 20, 21, or 22 for the specific wire size and wire spacing chosen.

Thus in establishing the mathematical model of the pressure produced, an experimental constant has been introduced. The resulting expression, however, has the advantage of being quite simple in form.

**Convective Heat Transfer without Applied Electric Fields**

To provide a background for the work done to obtain a solution, a brief summary of relevant heat transfer problems will be given. Flat plate forced convection, vertical plate free convection, and stagnation point heat transfer will be covered. The starting point in all these cases are the following equations (8).

**Equation of Continuity**

\[
\frac{\partial u_i}{\partial x_i} = 0
\]

**Navier Stokes Equations**

\[
\rho \frac{d u_i}{d t} = \chi_i - \frac{\partial p}{\partial x_i} + \mu \nabla^2 u_i + \frac{\mu}{3} \frac{\partial A}{\partial x_i}
\]

**Energy Equation**

\[
\rho \frac{d C_p T}{d t} = \frac{1}{\rho m c_p} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i} (k \frac{\partial T}{\partial x_i}) + \frac{\mu}{\rho} \nabla \cdot \nabla T - \frac{1}{\rho m} \mu X_i + q_x + q_y
\]
Where
\[ \Phi = \int \left\{ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} \right)^2 \right\} \]

\[ + \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 - \Delta \}\]

where
- \( q_r \) = radiation energy
- \( q_g \) = internal heat generated
- \( X_1 \) is a body force
- \( \tau \) is time; \( \Delta = \frac{\partial u_i}{\partial x_i} \)

**Forced Convection-Laminar Flow**

With the usual assumptions of Blasius flow (3, 15), the equations for the case of heat transfer to a flat plate in laminar flow become

**Continuity:**
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

**Navier Stokes:**
\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \]

**Energy Equation:**
\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{\mu_c} \left( \frac{\partial u}{\partial y} \right)^2 \]

Use of the stream function
\[ \psi = (\nu \chi u)^{1/2} f(y) \]

and the transformation \( \eta = y \left( \frac{\mu_\omega}{\nu} \right)^{1/2} \) allows the Navier Stokes equations to be transformed to an ordinary differential equation.
whose solution is tabulated in many sources (3, 9). The solution to the energy equation is handled through the same transformation. The energy equation becomes

\[
\frac{d^2 T}{d \eta^2} + \frac{f}{2 \eta} \frac{d}{d \eta} \left( \frac{d T}{d \eta} \right) = \left( \frac{-u_\infty^2}{J_m \rho} \right) N_{Pr} f''
\]

The solution to this equation is given by

\[
h = -k \left( \frac{u_\infty}{V_k} \right)^{1/2} \left[ \left( \frac{f''}{N_{Pr}} \right) \int_0^{\eta} \left( \frac{f''}{N_{Pr}} \right)^{1/2} d\eta \right]_{\eta=0}
\]

A reasonable approximation to this solution is given by (16)

\[
h = k \left( \frac{u_\infty}{V_k} \right)^{1/2} \left( 0.332 N_{Pr}^{-1/3} \right)
\]

\(N_{Pr}\) is the Prandtl Number

**Free Convection Vertical Plate**

In the free convection from a vertical plate, a body force arises due to the effect of gravity. The equations in this case become (8)

**Navier Stokes**

\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \rho - \frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}
\]

**Energy**

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_P} \frac{\partial^2 T}{\partial y^2}
\]
The transformation of the equations in this case uses

$$
\bar{y} = 4 V C x^{\frac{3}{4}} F(\eta)
$$

$$
\eta = c y / x^{\frac{1}{4}}
$$

$$
C = \sqrt{\frac{g (T_w - T_o)}{4 V^2 T_o}}
$$

where

- $T_o$ is free stream temperature
- $T_w$ is wall temperature
- $\theta = T - T_o / T_w - T_o$

The equations become

- Navier Stokes
  $$
  F''' + 3 F F'' - \overline{F'}^2 + \theta = 0
  $$

- Energy
  $$
  \theta'' + 3 N_o F \theta' = 0
  $$

Solutions for this set of equations are listed in references 3 and 9.

**Stagnation Point Heat Transfer**

The stagnation point problem arises when a fluid stream impinges upon a surface and a zone of zero velocity is formed under the stream. The case of an infinite stream striking a cylinder or flat plate is well covered in the literature (8, 15). Both theoretical and experimental evidence support the assertion used in the solutions that the velocity of the flow outside of the boundary layer increases linearly from the stagnation point. This variation holds only in the vicinity of the stagnation point, but it is most useful.
Using the relationship \( U_\infty = \beta_1 x \) the pressure gradient term in the Navier Stokes equations becomes

\[-\frac{1}{\rho} \frac{\partial p}{\partial x} = \beta_1^2 x\]

and the Navier Stokes equations become

\[u \frac{\partial u}{\partial y} + v \frac{\partial u}{\partial y} = \beta_1^2 x + v \frac{\partial^2 u}{\partial y^2}\]

Using

\[\psi = (\nu \beta_1)^{1/2} x F(\eta)\]

\[\eta = (\beta_1 / \nu)^{1/2} y\]

the equation is transformed to the ordinary equation

\[F'''' + F F'' - (F')^2 + 1 = 0\]

The energy equation for this case is given by

\[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}\]

which with the transformation becomes

\[\theta'' + N_p F' \theta' = 0\]

where

\[\theta = \frac{T - T_w}{T_0 - T_w}\]

The solution of these equations is given in reference (15).

The foregoing brief sketch of three of the fundamental problems of convective heat transfer are included solely as background material. Some of the efforts made in tackling the problem of heat transfer with corona discharge were based upon the concepts involved in these solutions. Their inclusion provides for ready reference.
Combined Corona Wind-Convection Problem

In order to formulate a suitable mathematical model, it is necessary to set down as completely as possible the physical system which is to be described. In the present work, two separate physical cases will be considered. The corona wind impinging on a horizontal plate with the bottom heated will constitute one case, and the corona wind impinging on a heated vertical plate will constitute the second case. These are shown in Figures 35 and 36. In the first case the buoyancy force is symmetrical about the center of the plate. Any streaming which occurs due to thermal buoyancy or corona wind is additive and no regions of stagnation should occur.

The buoyancy force in the case of the horizontal plate acts somewhat differently than in the vertical plate case. For a vertical plate, the buoyancy force acts along the plate, and causes the convective streaming to move along the surface. For a horizontal plate the buoyancy force acts perpendicular to the plate, and any streaming which results occurs because of the upward movement of the heated air at the edge of the plate. Consequently, in the center of the plate, the buoyancy force which acts perpendicular to the small streaming taking place will be neglected in the analysis of the horizontal plate.

In the second case, the vertical plate presents an interesting but a formidable problem. The same complex addition of corona effects and thermal effects occur as in the first case, but in addition, at certain values of current, the magnitude of corona wind will approach that of the free convection streaming. Thus at least two separate problems present themselves. In the region above the centerline of the impinging jet of air, the two streams will be additive and a combined free-convection and forced convection problem results. The thermal body force in this case is in the same direction as the external corona wind stream. In the
Figure 35
End view of horizontal plate with bottom heated subjected to corona discharge in air.

Figure 36
End view of vertical plate with front heated subjected to corona discharge in air.
region below the centerline the situation is changed. Here the corona wind streams in the opposite direction to the thermal body force. At some value of current, the magnitude of the corona wind will be approximately the same as that of the upward streaming heated air. Although it is doubtful that the distribution of velocity of the two streams as they impinge will be the same, the net result is that some type of stagnation region should result. This complex problem is illustrated graphically by the interferometer pictures of the vertical heated plate shown in Plate I of a previous section.

The approaches used in attempting to solve these problems included consideration of solutions of the Navier Stokes and energy equations with the concept of an impinging two dimensional finite jet, and the use of the Von Karman integral equations.

Two Dimensional Jet Approach

Study of all the data and photographs of the corona wind indicated that the impinging jet of air appeared to be very narrow in cross section before it was deflected by the plate. The horizontal plate pictures as well as the vertical plate pictures at high currents indicated that the streaming after impingement was approximately symmetrical. A possible model then was conceived to be a thin two dimensional jet of air impinging upon a flat surface. The case of the horizontal plate with the thermal body force included was considered from this viewpoint first. If this approach would turn out to be promising it was felt that the determination of the width of the jet from the test data could then be undertaken.

The case of the horizontal flat plate with an impinging jet was considered in the following manner. As a first approximation it was assumed that the viscosity and boundary layer would not markedly influence the flow field and pressure distribution of the jet of air. This assumption
allowed the use of potential flow theory in the region away from the surface of the plate. One possible technique for handling this case is through the use of the hodograph transformation \(17, 18\). The problem is shown in Figure 37.

Using the transformation \( \mathcal{F} = (\mathcal{U} - i\mathcal{V})/\mathcal{U} \)

the jet is mapped onto the upper half of the unit circle. \( t = -\mathcal{F} \left( \mathcal{F} + \frac{1}{\mathcal{F}} \right) \)

maps upper half of the unit circle onto the upper half of the \( t \) plane.

The flow in the \( t \) plane is equivalent to a source flow of strength \( \frac{\mathcal{U}b}{\pi} \).

The complex potential \( \mathcal{W}(z) = \mathcal{U} + i\mathcal{V} \) becomes for this case,

\[
\mathcal{W}(z) = \frac{\mathcal{U}b}{\pi} \ln \left( \frac{z^2 + 1}{z^2 - 1} \right)
\]

using \( \frac{dw}{dz} = \frac{dw}{ds} \frac{ds}{dz} \) it can be shown that

\[
dz = \frac{zb}{\pi} \left( \frac{1}{z^2 - 1} - \frac{1}{z^2 + 1} \right)
\]

From which

\[
z/b = \left( z/\pi \right) \left( \tan^{-1} \mathcal{F} + \tanh^{-1} \mathcal{F} \right)
\]

At the surface of the plate \( v = 0 \) and this equation becomes

\[
x/b = \left( z/\pi \right) \left( \tan^{-1} \mathcal{U}/\mathcal{V} + \tanh^{-1} \mathcal{U}/\mathcal{V} \right)
\]

This is an implicit equation. Expressing the terms in series and reducing the equation,

\[
x/b \cong \frac{4}{\pi} \left[ \mathcal{U}/\mathcal{V} + \frac{1}{3} \left( \mathcal{U}/\mathcal{V} \right)^3 + \frac{1}{5} \left( \mathcal{U}/\mathcal{V} \right)^5 + \ldots \right]
\]
Figure 37
Transformation of the impinging two dimensional jet shown in the Z plane.
Using a reversion of series technique

\[ \frac{u}{U} = \frac{\pi x}{4 b} \left[ 1 - \frac{1}{5} \left( \frac{x}{b} \right)^2 + \cdots \right] \]

The form of the equations for velocity and pressure calculated on an incompressible basis is shown in Figure 38.

The most revealing aspect of this figure is that the linear term of the expression for \( \frac{u}{U} \) holds quite well in the vicinity of the centerline. If the pressure and velocity relationships of the corona wind jet would be similar to these curves, then potentially a very simple calculation for boundary layer profile and heat transfer could be made. In the central region where \( \chi < \frac{4u}{\pi U} b \), the stagnation solution discussed previously would apply, and in the outer region where \( \chi > \frac{4u}{\pi U} b \) the flat plate laminar flow solution would apply.

A comparison between the theoretical thin jet pressure distribution and a typical pressure distribution obtained from test is shown in Figure 39. Several attempts were made to adjust the relative shapes of the curve by choosing various values of \( b \), the theoretical jet half width. Unfortunately the form of the pressure curves did not agree well with the test curves. It could be expected that correlation with heat transfer coefficients based upon this method might not prove to be very good.

Navier Stokes Approach

Since both test data and the thin jet theory indicated that the region of large pressure variation would be confined to a region close to the centerline, it was felt that the combined thermal body force-corona wind problem could be tackled in the outer regions of the plate assuming
Figure 38
Pressure and velocity distribution at the surface of a plate under narrow ideal jet.
Figure 38

**Pressure Variation**

\[ \frac{\Delta P}{\frac{1}{2} \rho U^2} \]

\[ \frac{X}{b} = \frac{2}{\pi} (\tan^{-1} \frac{U}{b} + \tanh^{-1} \frac{U}{b}) \]

\[ \frac{U}{b} = \frac{\pi}{4} \frac{X}{b} \]

**Velocity Variation**

\[ \frac{U}{U} \]

\[ \frac{X}{b} = \frac{2}{\pi} (\tan^{-1} \frac{U}{b} + \tanh^{-1} \frac{U}{b}) \]
Figure 39
Comparison of the narrow ideal jet pressure distribution with a curve based upon data of corona discharge from 0.004" wire with 2 cm. spacing.
the wind velocity in this region to be approximately constant. The physical model which was considered next was that of the second case as represented by Figure 36.

This figure illustrates the two regions of flow, above the centerline and below the centerline. If the free convection effects are large, and the upper half of the plate is considered, then the body force acts in the direction of external flow.

\[ \rho g \left( \frac{T_w - T_o}{T_o} \right) \] is the body force

In the region away from the centerline, the equations become

Navier Stokes

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g \left( \frac{T_w - T_o}{T_o} \right) \Theta + \gamma \frac{\partial^2 u}{\partial y^2} \]... 56

Energy

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} \]... 57

Where the term \( \left( \frac{\nu}{\rho m C_p} \right) \left( \frac{\partial u}{\partial y} \right)^2 \) is dropped from the energy equation because the velocities are extremely small. This set of equations is, of course, exactly the same set as used for the free convection vertical plate case reviewed previously. The significant change in the problem comes with the different boundary conditions. As the corona wind streams over the surface it will have a velocity distribution which varies from zero at the surface to some maximum away from the surface and then it will decay back to zero farther out from the plate. The shape of this total distribution is complex. If one considers the problem of the spreading of a free jet, even this is difficult to handle. In the case of corona discharge, it is difficult to know the form of the highly localized velocity
distribution, and consequently the shape of the wake is in doubt. This shape would form the outer region of the corona wind after it turned the corner. On the other hand the width of the boundary layer growing beneath the corona wind would be very small for some distance away from the center stagnation region. It was then assumed consequently that the thickness of the corona wind would be large relative to the boundary layer thickness, and that the velocity at the edge of the boundary layer would be the value given by the peak of the corona wind. These assumptions are represented in Figure 40 where schematic representations of temperature and velocity profiles are shown. \( U_\infty \), the impinging velocity, is the velocity of the corona discharge.

Using the free convection transformations the equations are the same as before

\[
F'''' + 3 F F''' - 2 (F')^2 + \theta = 0
\]

\[
\theta'' + 3 N_p F \theta' = 0
\]

The boundary conditions are

\[
y = 0, \quad y = 0, \quad u = 0, \quad v = 0, \quad T = T_w
\]

\[
y = \delta, \quad \eta = \infty, \quad u = u_f, \quad v = 0, \quad T = T_0
\]

which lead to

\[
F = 0, \quad F' = 0, \quad \theta = 1 \quad @ \quad y = 0
\]

\[
F'' = 0, \quad F' = \frac{u}{4 \sqrt{C^2 \sqrt{2} \nu}} \quad @ \quad y \rightarrow \delta
\]

These imply that the boundary conditions change at every position up the wall. If this approach is used, solutions for the equations would have to be made in discreet steps for selected values of \( x \).
Figure 40

Schematic representations of flow, velocity profiles, and temperature profiles illustrating boundary conditions.
For the case with the body force acting in the direction opposite to the direction of flow (19), the sign of the body force term is changed and the equations become
\[
F'' + 3FF' - z(F)' - \Theta = 0 \tag{60}
\]
\[
\Theta'' + 3N\frac{F}{F'} F\Theta' = 0 \tag{61}
\]
The boundary conditions are the same as for the previous case. Consequently a solution proceeding in this fashion would have to be done in discreet steps also.

Different forms for the transformations were tried, but no clear-cut approach to handle the combined free convection-forced convection was evident. Review of other combined convection work did not reveal anything directly applicable, and it was decided to follow the integral equation approach.

**Von Karman Integral Equation Approach**

The integral equations of boundary layer and heat transfer theory for steady flow are given as (9).

**Momentum Equation:**
\[
\frac{\partial}{\partial x} \int_0^{\delta} \rho u^2 dy - u_0 \frac{\partial}{\partial x} \int_0^{\delta} \rho u dy = - \delta \frac{\partial \Phi}{\partial x} - g \int_0^{\delta} \rho \frac{T - T_0}{\rho} dy - \mu \frac{\partial u}{\partial y} \bigg|_{y=0} \tag{62}
\]

**Energy Equation:**
\[
T_m \frac{\partial}{\partial x} \int_0^{\delta_T} \rho u T dy - T_0 \frac{\partial}{\partial x} \int_0^{\delta_T} \rho u dy = - \int_0^{\delta_T} \frac{\partial \Phi}{\partial y} dy + \int_0^{\delta_T} \frac{\partial T}{\partial y} \bigg|_{y=0} \frac{\partial u}{\partial y} \bigg|_{y=0} \tag{63}
\]
where \( \delta_T \) is the thickness of the thermal boundary layer. Because the velocities involved in the phenomenon are so small, the viscous dissipation term was dropped in the energy equation.
The procedure used in solving the integral equations is to assume reasonable forms for the velocity and temperature distributions in the boundary layer which meet certain prescribed boundary conditions. Although the technique is not precise, it does offer insight into the problem as well as providing solutions useful in engineering.

**Analytical Solution of the Horizontal Plate Case**

In the application of the Von Karman integral equations to the case of heat transfer from the bottom of a heated flat plate subject to corona wind, it was assumed that the thermal body force in this case was small and could be neglected. The integral equations become

\[
\frac{\partial}{\partial x} \int_0^\delta \rho u^2 dy - u_0 \frac{\partial}{\partial x} \int_0^\delta p dy = -\delta \frac{\partial p}{\partial x} - \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}
\]

\[
J_m C_p \left[ \frac{\partial}{\partial x} \int_0^\delta \rho u T dy - T_0 \frac{\partial}{\partial x} \int_0^\delta p dy \right] - \int_0^\delta \frac{\partial p}{\partial x} = -J_m k \left( \frac{\partial T}{\partial y} \right)_{y=0}
\]

In this case, it was decided to attempt to determine what variation in boundary layer thickness and heat transfer coefficient could be obtained throughout the entire range, from the centerline outwardly. The body force effects due to the applied electrical field were included through the use of the pressure rise distribution previously derived under the section on electric field-corona discharge relations. Thus the analysis consisted primarily of determining the variations of boundary layer thickness and heat transfer coefficients under the influence of a prescribed pressure distribution.

**Pressure-Velocity Relation**

In considering the fluid flow in the region of the impinging jet of corona wind, it is necessary to examine the nature of the relationship between velocity and pressure. For incompressible flow, Bernoulli’s
equation holds. If, however, a body force is introduced, the relationship is not as simple, even if the viscous effects are considered small. From thermodynamic reasoning it can be shown that if work is done on the fluid, for example by the body force, then the constant of integration will change across streamlines, and the condition of irrotationality will not hold even for incompressible flow. Because of this, one cannot assume the relationship \( p_c = p + \frac{1}{2} \rho u^2 \) will hold throughout the fluid stream above the boundary layer without more careful consideration.

In the region outside the boundary layer the viscous effects can be considered to be small and consequently the Navier Stokes equations become

\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \chi - \frac{\partial p}{\partial x} \tag{66}
\]

\[
\rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \gamma - \frac{\partial p}{\partial y} \tag{67}
\]

If \( X_i = \rho_c E_i \)

Using \( E_i = \frac{\partial V}{\partial x_i} \) where \( V \) is the potential (voltage)

\[
\frac{\partial \chi}{\partial x_i} = -\rho_c \frac{\partial V}{\partial x_i} \tag{68}
\]

For very small currents, if \( \rho_c \) is assumed to be a scalar constant other than zero,

\[
X_i = -\frac{\partial \rho_c}{\partial x_i} V \tag{69}
\]

and the equations become for incompressible flow,

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial}{\partial x} (\rho + \rho_c V) \tag{70}
\]

\[
u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial}{\partial y} (\rho + \rho_c V)
\]
then

\[
(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) dx = - \frac{1}{\rho} d(p + \rho V) \tag{71}
\]

\[
(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}) dy = - \frac{1}{\rho} d(p + \rho V) \tag{72}
\]

\[
(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) dx - (u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}) dy = 0
\]

Letting

\[
M = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}
\]

\[
N = -(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y})
\]

\[
M dx + N dy = 0 \tag{73}
\]

Using Green's theorem for the plane (20)

\[
\iint_A \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) dx dy = -\oint_c (M dx + N dy) \tag{74}
\]

and

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{30}
\]

it can be shown that

\[
u \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) = v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{75}
\]
This condition holds for all values of \( u \) and \( v \) if \( \nabla u = 0, \nabla v = 0 \), and implies that if \( \rho_c \) can be considered a scalar constant, then irrotationality will follow. The foregoing is a demonstration that the use of the condition of irrotationality is a reasonable one for the case of very small space charge.

\[
\int_0^u \left( u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} \right) dx = -\frac{1}{\rho} \partial \frac{(\rho + \rho_c V)}{
}
\]

\[
\frac{p}{2} (u^2 + v^2) = -p - \rho_c V + p_0
\]

\[
p + \frac{p}{2} (u^2 + v^2) = p_0 - \rho_c V
\]

where \( p_0 \) is the constant of integration. Near the flat plate surface, \( v = 0 \), and \( V \) approaches zero. Consequently, Bernoulli's equation holds approximately near the wall,

\[
p + \frac{p}{2} u^2 = p_0
\]

When \( x = 0, u = 0, p = p_c \), it follows that \( p_0 = p_c \), where \( p_c \) is the pressure at the centerline of the corona discharge.
Using the relation previously derived, for the pressure variation under a corona discharge,

\[ \rho = \rho_c \left[ \frac{1}{1 + z^2} \right]^2 \]

the Bernoulli equation becomes:

\[ \frac{\rho_u^2}{2} = \rho_c \left[ 1 - \left( \frac{1}{1 + z^2} \right)^2 \right] \]

With this relation, the solution of the integral equations can be undertaken. Three forms of velocity and temperature profiles are assumed, linear, parabolic, and cubic. Because the parabolic distribution solution provides the clearest picture of the techniques involved, it will be carried through completely. The linear and cubic solutions will be indicated only.

**Parabolic Solution**

The parabolic profile assumed is:

\[ u = u_x (\frac{2y}{\delta} - \frac{y^2}{\delta^2}) \]

which meets the boundary conditions of

\[ y = 0, \quad u = 0 \]
\[ y = \delta, \quad u = u_x, \quad \frac{\partial u}{\partial y} = 0 \]
\[ \Theta = T - T_w \; ; \quad \Theta = \Theta_0 \left( \frac{2y}{\delta} - \frac{y^2}{\delta^2} \right) \]

which meets the boundary conditions of

\[ y = 0, \quad T = T_w, \quad \Theta = 0 \]
\[ y = \delta_T, \quad T = T_0, \quad \Theta = \Theta_0, \quad \frac{\partial \Theta}{\partial y} = 0 \]
Substitution of the velocity profile into the momentum equation
\[
\frac{2}{\partial x_0} \int \rho \frac{d^2 y}{\partial x} u_x \frac{d^2 y}{\partial x} \int \rho u dy = -\delta \frac{\partial p}{\partial x} - \mu \frac{d^2 u}{\partial y^2}
\]
leads to
\[
\frac{\delta}{\partial x} \rho \frac{d^2 (u_x^2)}{\partial x} - \frac{2}{3} \rho u_x \frac{d^2 (u_x^2)}{\partial x} + \delta \frac{\partial p}{\partial x} = -\frac{2}{\delta} \mu \frac{d u_x}{\partial x}
\]
which upon further manipulation becomes
\[
\frac{d \delta^2}{dx} - \left[ \frac{\frac{3}{u_x^2}}{\frac{d}{dx}} \frac{d}{dx} \frac{d^2 u_x + 5p}{\partial x} \right] \delta^2 = \frac{\delta}{u_x} \frac{301}{\delta}
\]
Considering the bracketed term and the Bernoulli relation
\[
p + \rho \frac{u_x^2}{2} = \rho_c
\]
\[
\frac{u_x^2}{2} = \frac{\rho c}{\rho} - \frac{2p}{\rho}
\]
\[
\frac{3}{u_x^2} \frac{d}{dx} \frac{d^2 u_x + 5p}{\partial x} = \frac{3}{u_x^2} \frac{d}{dx} \frac{2p}{\rho} + \frac{3p}{\rho}
\]
Since \( \rho_c \) is a constant with respect to \( x \)
\[
\frac{3}{u_x^2} \frac{d}{dx} \frac{2p}{\rho} + \frac{3p}{\rho} = \frac{q}{2} \frac{1}{(\rho u_x^2/2)} \frac{dp}{dx}
\]
But
\[
\frac{dp}{dx} = -\frac{d}{dx} \rho \frac{u_x^2}{2}
\]
and
\[
-\frac{q}{2} \frac{1}{(\rho u_x^2/2)} \frac{d}{dx} \left( \frac{\rho u_x^2}{2} \right) = -\frac{q}{2} \frac{d}{dx} \left( \ln \frac{\rho u_x^2}{2} \right)
\]
The momentum equation becomes

\[ \frac{d\delta^2}{dx} + \frac{g}{2} \left[ \frac{d}{dx} \left( \ln \frac{\rho u_x^2}{2} \right) \right] \delta^2 = \frac{30V}{u_x} \]

The solution to this equation is

\[ \delta^2 = e^{-\int \frac{dx}{\alpha} \left( \ln \frac{\rho u_x^2}{2} \right)} \int e^{\int \frac{dx}{\alpha} \left( \ln \frac{\rho u_x^2}{2} \right)} \cdot \frac{30V}{u_x} \, dx \]

\[ + c e^{-\int \frac{dx}{\alpha} \left( \ln \frac{\rho u_x^2}{2} \right)} \]

\[ \delta^2 = \frac{30V}{\left( \frac{\rho u_x^2}{2} \right)^{\frac{3}{2}}} \int \left( \frac{\rho u_x^2}{2} \right)^{\frac{3}{2}} \, dx + \frac{c}{\left( \frac{\rho u_x^2}{2} \right)^{\frac{3}{2}}} \]

For the boundary layer thickness to remain finite at \( x = 0, u_x = 0 \), \( c \) must be zero.

\[ \delta^2 = \frac{30V \sqrt{\frac{\rho}{2}}}{\left( \frac{\rho u_x^2}{2} \right)^{\frac{3}{2}}} \int_0^x \left( \frac{\rho u_x^2}{2} \right)^{\frac{3}{2}} \, dx \]

From

\[ \frac{\rho u_x^2}{2} \equiv \frac{\rho u^2}{2} = \rho_c \left[ 1 - \left( \frac{i}{1 + e^2} \right)^2 \right] \]

where \( u = u_x \), the velocity outside the boundary layer,

\[ \delta^2 = 15u_a \sqrt{\frac{2}{\rho \rho_c}} \frac{1}{\gamma^\frac{3}{2}} \int_0^\delta \frac{z^2}{\gamma^2} \, dz \]
where
\[
\gamma = \left[ 1 - \left( \frac{1}{1+z^2} \right)^2 \right]
\]
\[z = x/a\]

\[a = \text{spacing of corona wire from plate.}\]

Using a somewhat tedious process, the integral for this parabolic case can be integrated exactly. \(\gamma^4\) is expanded and integrated term by term to give:

\[
\begin{align*}
F(z) &\equiv \int_0^z \gamma^4 \, dz = z - 0.899023 \left( \frac{z}{1+z^2} + \tan^{-1} z \right) \\
&\quad + 0.7333984 \frac{z}{(1+z^2)^2} + 0.5867188 \frac{z}{(1+z^2)^3} \\
&\quad - 0.3542411 \frac{z}{(1+z^2)^4} - 0.3148810 \frac{z}{(1+z^2)^5} \\
&\quad + 0.0773810 \frac{z}{(1+z^2)^6} + 0.0714236 \frac{z}{(1+z^2)^7}
\end{align*}
\]

Thus an explicit solution for the square of the thickness of the boundary layer is obtained. The behavior of the expression for \(\delta^2\) in the neighborhood of the origin must be investigated closely, because the expression \(\gamma^8/\delta\) approaches zero at the origin. Applying L'Hopital's rule,

Let

\[
\frac{f(z)}{g(z)} = \frac{\int \gamma^4 \, dz}{\gamma^8/\delta}
\]

Then

\[
\begin{align*}
\frac{f(z)}{g(z)} &= \frac{\gamma^4}{\gamma^8/\delta} d\gamma/\delta \\
&= \frac{2}{9} \frac{\gamma^{12} d\gamma}{d\gamma} \\
&= \frac{2}{9} \frac{\gamma^{12}}{4z(1+z^2)^3} = \frac{1}{18} \frac{(2+z^2)^{1/2}}{(1+z^2)^2}
\end{align*}
\]

\[
\lim_{z \to 0} \frac{f(z)}{g(z)} = \sqrt{\frac{2}{18}}
\]
At the origin then

\[ \delta^2 = \left( \frac{5}{3} \mu a \right) \frac{z}{\rho p_c} \]

\[ \delta = \left( \frac{5}{3} \mu a \right)^{1/2} \left( \frac{1}{\rho p_c} \right)^{1/4} \]

For larger values of \( z \)

\[ \delta^2 = 15 \mu a \sqrt{z} \frac{\int_0^z y^4 \, dz}{y^{1/2}} \]

The shape of the boundary layer is given by

\[ \left( \frac{\delta}{15 \mu a \sqrt{z/p_c}} \right)^2 = \left[ \frac{\int_0^z y^4 \, dz}{y^{1/2}} \right]^2 \]

This function is plotted in Figure 41. The function shows the finite value at the origin, and the subsequent increase as \( z \) gets larger.

The energy equation will be considered next.

\[ \text{Im} Q \left[ \frac{\partial}{\partial x} \left( \rho u T dy - T_0 \frac{\partial}{\partial x} \int_0^x \rho u dy \right) \right] - \int_0^x \frac{\partial}{\partial x} \left( p \frac{\partial}{\partial y} \right) dy = -\text{Im} k \left( \frac{\partial T}{\partial y} \right) \bigg|_{y=0} \]

Substituting \( \Theta = T - Tw \)

\[ \Theta = \Theta_0 \left( 2y \delta_T - y^2 \delta_T^2 \right) \]

\[ \frac{\partial}{\partial x} \left[ \int_0^x (u \Theta - u \Theta_0) dy \right] - \frac{1}{\rho \text{Im} p} \frac{\partial}{\partial x} \int_0^x d y \frac{\partial}{\partial y} = -\lambda \left( \frac{\partial \Theta}{\partial y} \right) \bigg|_{y=0} \]

\[ -\Theta_0 \frac{d x}{\delta} \left[ 1 - \frac{\delta x}{\delta} \right] \frac{d u \delta_T}{\delta x} - \frac{\delta x}{\delta} \frac{U x \delta_T}{\rho \text{Im} p} \left[ 1 - \frac{\delta x}{\delta} \right] = -2 \lambda \Theta_0 \frac{d x}{\delta_T} \]
Figure 41

Variation of the theoretical velocity boundary layer thickness for a parabolic velocity profile.
Where \( \delta_\ast \equiv \delta T / \delta \) and is assumed not to vary appreciably with \( x \).

\[
\frac{\partial \delta_\ast^2}{\partial x} + \left\{ \frac{1}{u_{h}^{2}} \frac{\partial u_{h}^{2}}{\partial x} - 20 \frac{\delta_\ast - 3}{\delta \theta} \frac{\partial \delta_{s}/\partial x}{\rho m \rho \theta_0} \right\} \delta_{s} = -\frac{120 \chi}{\delta_\ast (\delta_\ast - 5) U_k}
\]

whose solution is:

\[
\delta_{s}^2 = e^{-\int f_1(x) dx} \left[ \int e^{f_2(x)} dx + C e^{-\int f_1(x) dx} \right]
\]

where

\[
f_1(x) dx \equiv \left\{ \frac{1}{u_{h}^{2}} \frac{\partial u_{h}^{2}}{\partial x} - 20 \frac{\delta_\ast - 3}{\delta \theta} \frac{\partial \delta_{s}/\partial x}{\rho m \rho \theta_0} \right\}
\]

\[
f_2(x) dx \equiv -\frac{120 \chi}{\delta_\ast (\delta_\ast - 5) U_k}
\]

\[
\int e^{-f_1(x)} dx = \int u_{h}^{2} e^{-\left( \frac{\delta_\ast - 3}{\delta \theta} \frac{20 \rho}{m \rho \theta_0} \right) x} dx = \int u_{h}^{2} e^{-\left( \frac{\delta_\ast - 3}{\delta \theta} \frac{20 \rho}{m \rho \theta_0} \right) \frac{120 \chi}{\delta_\ast (\delta_\ast - 5) U_k}} dx
\]

Substituting

\[
P = \rho C \left[ \frac{1}{1 + z^2} \right]^2
\]

\[
\alpha_1 \equiv \left( \frac{\delta_\ast - 3}{\delta \theta} \frac{20}{\rho m \rho \theta_0} \right)
\]

\[
u_k^2 = \frac{z \rho c \gamma}{\rho}
\]
and noting that for the solution to remain finite at the origin, \( c = 0 \),

\[
\delta_T^2 = \frac{-120 \alpha a \xi \rho c (\xi + \xi^2)^2}{\delta \xi (\delta \xi - 5) \sqrt{2 \rho c \gamma}} \int_0^\infty \frac{e^{-\xi \rho c (\xi + \xi^2)^2}}{\xi^{3/2}} d\xi
\]  

Using

\[
e^\xi = 1 + \xi + \frac{\xi^2}{2!} + \frac{\xi^3}{3!} + \ldots\]

\[
e^{-\xi \rho c (\xi + \xi^2)^2} \approx 1 - \xi \rho c (\xi + \xi^2)^2
\]

using the first two terms. The expression for \( \delta_T^2 \) becomes

\[
\delta_T^2 = \frac{-120 \alpha a \xi \rho c (\xi - \xi^2)^2}{\delta \xi (\delta \xi - 5) \sqrt{2 \rho c \gamma}} \int_0^\infty \frac{e^{-\xi \rho c (\xi - \xi^2)^2}}{\xi^{3/2}} d\xi
\]

This is a complex expression for which no exact solution was found.

The expression \( \alpha_1 p_c \) will be examined further. At the centerline

\[
p_c = \frac{p}{2} \bar{U}^2, \text{ where } \bar{U} \text{ is the peak impinging velocity.}
\]

\[
\alpha_1 p_c = \frac{\delta \xi - 3}{\delta \xi - 5} \cdot \frac{20 \rho c}{\rho T_m C_p \theta_0}
\]

\[
= \frac{\delta \xi - 3}{\delta \xi - 5} \cdot \frac{10 \bar{U}^2}{T_m C_p \theta_0} = \frac{\delta \xi - 3}{\delta \xi - 5} \cdot 10 N e_k
\]

where \( N e_k \equiv \frac{\bar{U}^2}{T_m C_p \theta_0} \)

To determine the order of magnitude of \( \alpha_1 p_c \), consider typical values of the various terms.
Typically
\[ T_0 = 20^\circ F \]
\[ U = 10 \text{ ft/sec} \]
\[ c_p = 0.24 \text{ BTU/lb} \text{ °F} \]
\[ N_{Ek} = 8.32 \times 10^{-4} \]

For values of \( \delta^* \) from 0 to 3, the expression \[ \frac{\delta^* - 3}{\delta^* - 5} \] lies between 3/5 and 0. Assuming a value of 1/2 inch which occurs when \( \delta^* = 1 \)

\[ \alpha \rho c = 4.16 \times 10^{-3} \]

Since the pressure relation
\[ \frac{\rho}{\rho_c} = \left[ \frac{1}{1 + z^2} \right]^2 \]
varies between 1 and 0 as \( z \) varies from 0 to infinity,

\[ \frac{\alpha \rho c}{\rho_c} \left( \frac{1}{1 + z^2} \right)^2 \]

will always be smaller than \( 4.16 \times 10^{-3} \) for \( \delta^* = 1 \).

Values of \( \delta^* \) above 2 are unlikely. Evaluation of \( \frac{\delta_T}{\delta_0} \) at specific points verified this. If \( \delta^* \) becomes greater than 3 the analysis would become questionable, certainly in the region \( \delta^* = 5 \).

\[ 0 \leq \frac{\alpha \rho}{\rho_c} < 4.16 \times 10^{-3} \quad \text{for all values of } z. \]

\[ \lesssim 1.004 \]

For the small pressures involved in a corona discharge, then,

\[ e^{\alpha c p} \approx 1 \]

\[ \delta_T^2 = \frac{-120 \alpha a}{8 \cdot \delta^*(\delta^* - 5) \sqrt{2p}} \int \sqrt{y^2 + z^2} \]
The value of $\delta_T^{-2}$ as $z \to 0$ is indeterminate. Applying L'Hopital's rule

$$\frac{f(z)}{g(z)} = \frac{1}{y} \frac{d}{dz} \sqrt{\frac{y}{\gamma}}$$

$$\frac{f'(z)}{g'(z)} = \frac{\frac{d}{dz} \sqrt{\frac{y}{\gamma}}}{\frac{d}{dz} \sqrt{\frac{1}{\gamma}}} = \frac{\sqrt{\frac{y}{\gamma}}}{4z(1+z^2)^{-3}} = \frac{(2+z)^{3/2}}{4}$$

$$\lim_{z \to 0} \frac{f(z)}{g(z)} = \frac{\sqrt{z}}{4}$$

At the origin

$$\delta_T^2 = \frac{-30 \alpha \delta_s}{\delta_x (\delta_x - 5)} \sqrt{\frac{\rho_c}{\rho}}$$

By definition $\delta_* = \delta_T^2$. The value of $\delta_*$ at the origin can be determined as follows using the values of $\delta_T^{-2}$ and $\delta^2$ at the origin.

$$\delta_0^2 = \frac{\delta_T^2}{\delta^2} = \frac{-30 \alpha \delta_s \sqrt{\rho_c}}{(\delta_T^2) a \mu \sqrt{\frac{\rho_c}{\rho}}}$$

$$= \frac{-18 N_{Pr}^{-1}}{\delta_x (\delta_x - 5)}$$

For air, with $N_{Pr} = 0.71$ a graphical solution yields $\delta_* = 2.048$.

For values of $z > 0$, the ratio $\frac{\delta_T^2}{\delta_x}$ varies in a more complex fashion. For small pressures this ratio becomes,

$$\frac{\delta_*^2}{\delta^2} = \frac{\delta_T^2}{\delta_x} = \frac{-4 N_{Pr}^{-1}}{\delta_x (\delta_x - 5)} \left\{ \frac{\int \gamma^{1/2} d\gamma/\gamma}{\int \gamma^{3/2} d\gamma/\gamma^{1/2}} \right\}$$

As $z$ approaches infinity, the bracketed term approaches $\int dz/\gamma$ and the ratio approaches 1.
At very high $z$,

$$\delta_\infty^2 = -4 \frac{N_p}{\delta_\infty^2} \frac{1}{(\delta_\infty - \delta)}$$

For a Prandtl number of unity, $\delta_\infty = 1$. For air, $\delta_\infty = 1.05$. Thus a variation in $\delta_\infty$ does occur with a change in $z$. This is contrary of course to the assumption originally made in solving the integral equations. The effect of the change in $\delta_\infty$ will be considered in evaluating the overall results.

The heat transfer coefficients can be calculated from

$$h_x = -\frac{k}{\Theta} \frac{\partial T}{\partial y} \bigg|_{y=0}$$

Using $\Theta = T - T_w$ and $\Theta = \Theta_0 \left( \frac{y}{\delta_r} - \frac{y^2}{\delta_r^2} \right)$

$$h_x = \frac{2k}{\delta_r}$$

Expressing this in terms of the velocity boundary layer,

$$h_x = \frac{2k}{\delta_\infty} \left( \frac{2 \rho c}{(30a\mu)^{1/2}} \right)^{1/4} \frac{1}{\left[ F(2) / 8^{1/2} \right]^{1/2}}$$

Using the pressure-current relationship found in the previous section,

$$\rho_c = \zeta c_i$$

$$h_x = \frac{2k}{\delta_\infty} \left( \frac{2 \rho c_i}{(30a\mu)^{1/2}} \right)^{1/4} \frac{1}{\left[ F(2) / 8^{1/2} \right]^{1/2}}$$

Near the origin $\delta_\infty = 2.048$
At the origin
\[ h_x = \frac{2k}{2.048} \left( \frac{3}{5} \right)^{\frac{1}{2}} \left( \rho c_i \right)^{\frac{1}{4}} \left( \frac{a_i}{c_i} \right)^{\frac{1}{2}} \]
where \( c_1 \) is an experimental and geometric constant. This equation predicts that the heat transfer coefficient at any station should vary as the one-fourth power of the current.

Variations in the shape of the velocity and temperature profiles which are assumed may materially affect the magnitudes of the boundary layer thickness and heat transfer coefficient, but should have relatively small influence on the form of the variation with distance. In order to get an estimate of the changes caused by profile shape two other profiles were assumed, linear and cubic.

**Linear Solution**

For the linear solution the following curves were assumed:
\[ u = u_x \frac{y}{\delta} \]
\[ \theta = \theta_0 \frac{y}{\delta_T} ; \quad \theta = T - T_0 \]

with the boundary conditions of
\[ y = 0 , \quad T = T_w , \quad \theta = 0 , \quad u = 0 \]
\[ y = \delta , \quad u = u_x \]
\[ y = \delta_T , \quad T = T_0 , \quad \theta = \theta_0 \]

Substituting these into the integral equations, one obtains for the velocity boundary thickness,
\[ \delta^2 = 6a\mu \sqrt{\frac{2}{\rho p}} \int_0^\infty y^{\frac{1}{2}} dz \]
The thermal boundary layer is given by

\[
\delta_T^2 = \frac{12 \alpha a \left[ 1 - 3 N_{E_L} \left( \frac{1}{1 + z^2} \right)^2 \right]}{\delta_* \sqrt{2 \rho_c / \rho}} \int_0^z y^{1/2} \left[ 1 + 3 N_{E_L} \left( \frac{1}{1 + z^2} \right)^2 \right] dy
\]

At the origin

\[
\delta_* = \frac{3 \alpha \delta_*}{\delta_*} \sqrt{1 / \rho_c}
\]

For air \( \delta_* = 1.92 \)

For larger values of \( z \)

\[
\frac{\delta_T^2}{\delta^2} = \frac{12 \alpha a \sqrt{2 \rho_c / \rho} \left[ \frac{1}{60 \mu \sqrt{2}} \int_0^z y^{1/2} dy / \delta_* \right]}{\delta_{*} \sqrt{2 \rho_c / \rho}}
\]
The heat transfer coefficient is given by,

\[ h_x = -\frac{k}{\beta_o} \left( \frac{\partial T}{\partial y} \right) \theta \]

\[ h_x = \frac{k}{\delta_x} \left( \frac{2 \rho p c_c}{12 \alpha \mu} \right)^{1/4} \left[ \int \frac{z^{9/2} \partial z}{\gamma_5^{1/2}} \int d\gamma \right]^{1/2} \]

The values for \( z \) are given by the following approximation:

\[ \int \frac{z^{9/2} \partial z}{\gamma_5^{1/2}} = \frac{1}{\gamma_5} \left\{ z - 9.36973 \left[ \frac{1}{1+z^2} + \tan^{-1} z \right] \right. \]

\[ + \cdot 48233.6 \frac{z}{(1+z^2)^2} + 70021.1 \frac{z}{(1+z^2)^3} - 542473 \frac{z}{(1+z^2)^4} \]

\[ - 48233.6 \frac{z}{(1+z^2)^5} + 197886 \frac{z}{(1+z^2)^6} + 182664 \frac{z}{(1+z^2)^7} \]

\[ - 0.29514 \frac{z}{(1+z^2)^8} - 0.027777 \frac{z}{(1+z^2)^9} + - - - \}

Values between \( z = 0 \) and \( z = 1 \) can be obtained from the curve in Figure 42. The curve in this region was obtained by graphical integration. At the origin where \( \int \frac{z^{9/2} \partial z}{\gamma_5^{1/2}} = \frac{\sqrt{2}}{20} \)
Figure 42
Variation of the boundary layer parameters.
Cubic Solution

For the cubic solution the following curves were assumed:

\[ u = U_\infty \left[ \frac{3}{2} \left( \frac{y_\infty}{\delta} \right) - \frac{1}{2} \left( \frac{y_\infty}{\delta} \right)^3 \right] \]

\[ \theta = \Theta_0 \left[ \frac{3}{2} \left( \frac{y_\infty}{\delta_T} \right) - \frac{1}{2} \left( \frac{y_\infty}{\delta_T} \right)^3 \right] \]

with \( \Theta = T - T_w \) as before. These equations meet the boundary conditions of

\[ y = 0, \quad u = 0, \quad \frac{d^2 u}{dy^2} = 0, \quad \theta = 0, \quad \frac{d^2 \theta}{dy^2} = 0 \]

\[ y = \delta, \quad u = U_\infty, \quad \frac{du}{dy} = 0 \]

\[ y = \delta_T, \quad \theta = \Theta_0, \quad \frac{d\theta}{dy} = 0 \]

Substituting these into the integral equations, one obtains for the velocity boundary layer thickness

\[ \delta = \frac{140}{13} a \mu \left( \frac{z}{\rho_\infty} \right)^{1/2} \int \frac{\gamma^{10/13}}{\gamma^{1/3}} \frac{dz}{\gamma^{10/13}} \]

\[ z \to 0 \quad \int \gamma^{10/13} \frac{dz}{\gamma^{10/13}} = \frac{13}{24} \sqrt{2} \]

\[ \delta_0 = \frac{7c}{61} \frac{a \mu}{\rho_\infty} \left( \frac{1}{\rho_\infty} \right)^{1/2} \]

\[ \delta_0 = (\frac{7c}{61})^{1/2} (a \mu)^{1/2} \left( \frac{1}{\rho_\infty} \right)^{1/2} \]
The thermal boundary layer is given by

\[ \delta^2 = \left\{ \frac{540 a \alpha \left[ 1 - \frac{45}{2} \left( \frac{\delta_x - \delta_y}{2 \delta_x} \right)^2 \right]}{(27 \delta_y - 2 \delta_x^3) \left( 2 \delta_y \right)^{\frac{1}{2}}} \right\} \frac{1}{1 + \left( \frac{2 \delta_y}{\rho} \right)^2} \]

\[ \times \left\{ \int_0^z \left[ 1 + \frac{45}{2} \left( \frac{\delta_x - \delta_y}{2 \delta_x} \right)^2 \right] \frac{dz}{y^{\frac{1}{2}}} \right\} \]

\[ \delta_y = \frac{540 a \alpha}{(27 \delta_y - 2 \delta_x^3) \left( \frac{2 \delta_y}{\rho} \right)^{\frac{1}{2}}} \int_0^z y^{\frac{1}{2}} \frac{dz}{y} \]

For \( z = 0 \),

\[ \delta_y = \frac{135 a \alpha}{(27 \delta_y - 2 \delta_x^3) \left( \frac{2 \delta_y}{\rho} \right)^{\frac{1}{2}}} \]

The ratio of the boundary layer thicknesses at the origin is given by

\[ \frac{\delta_y}{\delta^2} = \frac{135 a \alpha \left( \frac{\delta_y}{\delta_x} \right)^{\frac{1}{2}} \left( \frac{61}{72} \right) \left( \frac{\delta_y}{\delta_x} \right)^{\frac{1}{2}}} {27 \delta_y - 2 \delta_x^3} \] \[ \times a \mu \]

\[ = \frac{135}{27 \delta_y - 2 \delta_x^3} \left( \frac{61}{72} \right) N_{Pr}^{-1} \]

\[ 16 \delta_x^3 - 216 \delta_y^3 = -915 N_{Pr}^{-1} \]

For air a graphical solution yields 2.088 for \( z = 0 \). As \( z \to \infty \), for a Prandtl number of one, \( \delta_x \) approaches 1.0. The equation is

\[ 27 \delta_x^3 - 2 \delta_y^3 = 2.51 N_{Pr}^{-1} \]

The heat transfer coefficient is given by

\[ h_x = -\frac{k}{\delta_x} \left( \frac{\partial T}{\partial y} \right)_{y=0} \]

\[ h_x = \frac{3}{2} \frac{k}{\delta_x} \left( \frac{13/280 a \alpha}{(2 \delta_y \delta_x)^{\frac{1}{2}}} \right)^{\frac{1}{2}} \]

\[ \left\{ \int_0^z \frac{dz}{y^{\frac{1}{2}}} \right\}^{\frac{1}{2}} \]

\[ h_x = \frac{3}{2} \frac{k}{\delta_x} \left( \frac{13/280 a \alpha}{(2 \delta_y \delta_x)^{\frac{1}{2}}} \right)^{\frac{1}{2}} \]

\[ \left\{ \int_0^z \frac{dz}{y^{\frac{1}{2}}} \right\}^{\frac{1}{2}} \]
The function \( \int_{\gamma}^{z} \frac{10^9}{2e} d\gamma \int \frac{\gamma}{\gamma} \) is shown in Figure 42. The values in the region \( z \approx 0 \) to \( z \approx 1 \), are obtained by graphical integration. For values of \( z \) greater than one, the following expression can be used,

\[
\int_{\gamma}^{z} \frac{10^9}{2e} d\gamma \int \frac{\gamma}{\gamma} = \frac{1}{\gamma} \left\{ \frac{z}{9141.61} \left( \frac{z}{1+z^2} + \tan^{-1}z \right) + 776.297 \frac{z^2}{(1+z^2)^2} + 630.396 \frac{z^3}{(1+z^2)^3} - 4264.8 \frac{z}{(1+z^2)^4} - 3792.8 \frac{z^5}{(1+z^2)^5} + 1237.29 \frac{z^6}{(1+z^2)^6} + 1142.11 \frac{z^7}{(1+z^2)^7} - 0.135 \frac{z^8}{(1+z^2)^8} - 0.01684 \frac{z^9}{(1+z^2)^9} + \cdots \right\}
\]

At the origin

\[
h_z = \frac{3}{2} \frac{k}{\gamma} \frac{Z}{\gamma} \frac{Z}{\gamma} \frac{(\gamma + \gamma')^{1/2}}{(\gamma + \gamma')^{1/2}}\]

Combined Thermal Body Force and Corona Wind

The case of the vertical plate subjected to the combined electric field and thermal body forces was considered from the integral equation approach also. The area where the corona wind velocity is approximately constant was considered in detail. The pressure gradient term was not included in the initial analysis. It was decided that the form of the equations should be investigated first in the region where the pressure was approximately constant. Based on the outcome of this initial analysis, the pressure term could be reconsidered.
It was assumed that the velocity and thermal boundary layer thicknesses were the same. The following velocity and temperature profiles were assumed.

\[ U = U_0 \left[ \frac{3}{2} \left( \frac{y_d}{\delta} \right) - \frac{1}{2} \left( \frac{y_d}{\delta} \right)^2 \right] + U_x \left( \frac{y_d}{\delta} \right)^2 \]

\[ \frac{T - T_0}{T_w - T_0} = \left[ 1 - \frac{y_d}{\delta} \right]^2 \]

where

- \( U_0 \) = velocity due to the corona wind
- \( U_x \) = velocity due to the free convection

These profiles were substituted into integral equations. The resulting equations obtained are:

**Momentum:**

\[
\frac{1}{105} \int \frac{d\bar{x}}{dx} \left( st. 625 U_0^2 + 9.5 U_0 U_x + U_x^2 \right) = \frac{U_0}{24} \int \frac{d\bar{x}}{dx} \left( 15 U_0 + 2 U_x \right)
\]

\[ = \frac{G}{3} \left( \frac{T_w - T_0}{T_0} \right) - \frac{2}{3} \left( \frac{3}{2} U_0 + U_x \right) \]

**Energy:**

\[ \int \frac{d\bar{x}}{dx} \left( 3 U_0 + U_x \right) = 6 U_x \]

An intricate coupling exists and no simple analytic solution was found for this set of equations. It would be possible to determine a solution for specific cases through numerical methods, but because of the limitations of time no further work was expended on this approach. Because of the difficulty experienced even with a constant pressure distribution, the variation of pressure \( \frac{P}{P_c} = \left[ \frac{1}{1 + \epsilon^2} \right]^2 \) was not introduced into the analysis.
Summary of Analysis

The analysis of the horizontal plate based upon the Von Karman integral approach appears to have been a fruitful course to follow. The magnitude of the heat transfer coefficient is predicted to vary as $l^{1/3}$ and explicit relationships are available for the variation of $h_x$. Because of the need for assuming velocity and temperature profiles in the integral method, it cannot be expected that the absolute magnitude of the heat transfer coefficient will be precise. The trends, however, should be representative of the actual variations of $h_x$ if the physical model used is a reasonable approximation to the actual test model. The validity of the assumption of very low space charge distortion of the field equations can only be ascertained after comparison with the test data.

The analysis of the horizontal plate by means of the thin jet approach appears to provide an approximate technique for estimating the effects of the corona discharge on heat transfer. The difficulty of estimating the proper jet width limits the utility of this method.

In the next section a comparison will be made between the analytical predictions and the actual test data for the horizontal plate case.
CORRELATION OF TEST DATA WITH THE
ANALYTICAL SOLUTION

This section will be devoted to the comparison of the analytical solutions and the test data obtained from the interferometer. The comparison made is limited to the case of the horizontal plate with the bottom side of the plate heated. The configuration considered is the 0.004 inch diameter wire located at 2 cm and at 5 cm below the plate. Both the integral equation solutions and the two dimensional thin jet solution will be compared.

In order to compare the theory with the test data the constants are evaluated at the mean temperature of the air in front of the plate. The values of the thermal conductivity, density, and viscosity of air at various temperatures were taken from Jakob and Hawkins (16).

The constant in the equation for pressure \( p = C_i \) is obtained from Figure 20 in which the data for the 0.004 inch wire is plotted for various wire spacings.

For \( A = 2 \text{ cm} \)

\[ C_i = 3.54 \times 10^{-4} \text{ p.s.f./}\mu\text{a} \]

For \( A = 5 \text{ cm} \)

\[ C_i = 3.80 \times 10^{-4} \text{ p.s.f./}\mu\text{a} \]

These values of \( C_i \) are derived from the test data on the pressure plate with the wire located vertically. The plate dimension vertically is 7 inches. However, the corresponding dimension of the heated plate is 10 inches. The current flowing to the plate for any given pressure will be proportional to the length of the wire and the plate. A test run was made
to verify this. The pressure plate was masked off with electrical insulating tape in a series of steps and pressure-current measurements taken. The results shown in Figure 43 indicate that the linear relation holds. Correcting $c_1$ for the difference in plate size:

\[d = 2\, \text{cm}\]

\[c_1 = 2.48 \times 10^{-4} \, \text{p.s.f.} / \mu \text{a}\]

\[d = 5\, \text{cm}\]

\[c_1 = 2.66 \times 10^{-4} \, \text{p.s.f.} / \mu \text{a}\]

For a typical case

\[T_{\text{mean}} = 79^\circ \text{F}\]

\[V = 1.686 \times 10^{-4} \, \text{sq ft/sec}\]

\[k = 0.0151 \, \text{BTU/hr-ft-}^\circ \text{F}\]

\[\rho = 0.0736 \, \text{lbs/cu ft}\]

**Integral Equation Solution**

Substituting these values into the equations for heat transfer coefficient,

(a) equation 123 for the linear profile, assuming $\delta$ variation to be small becomes

For $a = 2\, \text{cm}$

\[h_\chi = 0.466 \, i^{\frac{1}{4}} \left( \frac{\int_0^\infty \frac{\delta}{y^5} \, dz}{y^s} \right)^{-\frac{1}{2}}\]

For $a = 5\, \text{cm}$

\[h_\chi = 0.300 \, i^{\frac{1}{4}} \left( \frac{\int_0^\infty \frac{\delta}{y^5} \, dz}{y^s} \right)\]
Figure 43
Variation in currents for various pressure plate widths.
(b) equation 140 for the parabolic profile, assuming $\delta^*$ variation to be small becomes

For $a = 2 \text{ cm}$

$$h_x = 0.553 i^{1/2} \left( \frac{\int \delta^* \mathrm{d}z}{\sqrt{a}} \right)^{-1/2}$$

For $a = 5 \text{ cm}$

$$h_x = 0.356 i^{1/2} \left( \frac{\int \delta^* \mathrm{d}z}{\sqrt{a}} \right)^{-1/2}$$

(c) equation 157 for the cubic profile, assuming $\delta^*$ variation to be small becomes

For $a = 2 \text{ cm}$

$$h_x = 0.479 i^{1/4} \left( \frac{\int \delta^* \mathrm{d}z}{\sqrt[3]{a}} \right)^{-1/2}$$

For $a = 5 \text{ cm}$

$$h_x = 0.309 i^{1/4} \left( \frac{\int \delta^* \mathrm{d}z}{\sqrt[3]{a}} \right)^{-1/2}$$

**Test Data Correlation**

Typical solutions of heat transfer coefficient for the $a = 2 \text{ cm}$ case are shown in Figures 44, 45, and 46. The test data for the corresponding cases are shown in Figures 47 and 48.

From a comparison of the test data and the theoretical solutions at $a = 2 \text{ cm}$, it can be seen that the trends of the solutions are good. The magnitude of $h_x$ differs depending upon the profile considered. This had been anticipated as was discussed in the previous section. To afford a more direct comparison, the theory and test data are shown together in Figure 49. This figure indicates that the trends are good. At the end of
Figure 44
Theoretical variation of heat transfer coefficient, parabolic profile; $a = 2$ cm.
Figure 45

Theoretical variation of heat transfer coefficient, linear profile; $a = 2 \text{ cm}$. 
Figure 46

Theoretical variation of heat transfer coefficient, cubic profile; $a = 2$ cm.
Heat transfer coefficients for horizontal flat heated plate at low currents, a = 2 cm. Wire diameter 0.004 inch.
Figure 48

Heat transfer coefficients for horizontal flat heated plate at high currents, a = 2 cm. Wire diameter 0.004 inch.
Figure 49

Direct comparison of theory and test data, $a = 2$ cm. Wire diameter 0.004 inch.
the plate the data and theory diverge. A large portion of this difference is believed to be due to the end effects of the plate. The comparison for a = 5 cm. is shown in Figure 50. It can be seen from both Figures 49 and 50 that the theory predicts a somewhat more rapid decrease of $h_x$ as $x$ increases than is shown by the test data. Since it appeared that the use of the fixed value of $\delta^*$ might introduce sizeable variations in $h_x$ as $z$ became large, a study was undertaken to determine the explicit variation of $\delta^*$ for all three velocity distributions.

The variation of $\delta^*$ with $x$ for the parabolic case is given by

$$\delta_x^2 = \frac{4 N_{r'}^{-1}}{\delta^* (S - \delta^*)} \left\{ \frac{\int_0^x \overline{\delta^* d\ell}}{\int_0^x y^{3/2} d\ell / y^{3/2}} \right\}$$

For the linear case

$$\delta_x^2 = \frac{N_{r'}}{\delta^*} \left\{ \frac{\int_0^x \overline{\delta^* d\ell}}{\int_0^x y^{3/2} d\ell / y^{3/2}} \right\}$$

For the cubic case

$$\delta_x^2 = \frac{25.05}{27 \delta^* - 2 d^2} N_{r'}^{-1} \left\{ \frac{\int_0^x \overline{\delta^* d\ell}}{\int_0^x y^{10/3} d\ell / y^{10/3}} \right\}$$

The values of the integrals were determined by graphical integration of the numerator. For specific values of $z$, the values of $\delta^*$ could be determined. The linear case provided an explicit solution

$$\delta_x = N_{r'}^{-1/3} \left\{ \frac{\int_0^x \overline{\delta^* d\ell}}{\int_0^x y^{3/2} d\ell / y^{3/2}} \right\}^{1/3}$$
Figure 50

Direct comparison of theory and test data. a = 5 cm. Wire diameter 0.004 inch.
Figure 50

Parabolic - 10.2 μa
Cubic - 10.2 μa
Linear - 10.2 μa

Parabolic - 10.2 μa
Parabolic - 2.6 μa

x-distance (inches)
which exhibits the familiar \( \mathrm{N_{Pr}}^{-1/3} \) form found in the forced convection theory. The values of \( \delta^* \) for the parabolic and cubic cases were obtained by a careful graphical solution of the 4th order and 5th order equations. The variation of \( \delta^* \) is shown in Figure 51.

The heat transfer coefficients modified by the inclusion of the variation of \( \delta^* \) are shown in Figures 52, 53, 54, 55, 56, and 57. The test data for \( a = 5 \text{ cm.} \) is shown in Figure 58. A better comparison of the modified theory and the test data can be found in Figure 59 where the \( a = 2 \text{ cm.} \) curves and test data are compared at two values of the current. The agreement in shape of the theoretical curves is now considered to be excellent. The end effects of the plate are clearly evident in the test data. Since the changes made in \( \delta^* \) had no effect at the origin, the small deviation between test and theory remain. This difference is reflected in the relatively uniform displacement of the theoretical curve from the test data. Since one of the most sensitive parameters to determine the entire investigation was the absolute magnitude of pressure, it is not altogether unexpected that differences in magnitude exist.

Figure 60 presents the comparison for \( a = 5 \text{ cm.} \) Similar results to those presented for the 2 cm. case are evident. The shape of the curves is in good agreement except where the end effects of the plate are evident. The absolute magnitudes of the heat transfer coefficients also differ somewhat from the test data.

The wide variation with corona wind of heat transfer coefficient over the plate implies a sizeable variation of the temperature profile in the boundary layer. This variation is presented for a typical case in Figure 61. The temperature profiles are plotted at several locations along the plate. It can be observed that the profiles do vary, and that the slopes of the curves increase greatly near the origin under the impinging jet of air.
Figure 51

Variation of $S_*$ for various profiles.
Figure 52

Modified heat transfer coefficients, parabolic profile; $a = 2 \text{ cm}$.
Figure 53

Modified heat transfer coefficients, linear profile; $a = 2$ cm.
Figure 54

Modified heat transfer coefficients, cubic profile, $a = 2$ cm.
Figure 55

Modified heat transfer coefficient, parabolic profile, $a = 5$ cm.
Figure 56

Modified heat transfer coefficients, linear profile, \( a = 5 \text{ cm} \).
Figure 57

Modified heat transfer coefficients, cubic profile; \( a = 5 \text{ cm} \).
Figure 58

Heat transfer coefficients for horizontal flat heated plate, $a = 5$ cm.

Wire diameter 0.004 inch.
Figure 59

Direct comparison of modified theory with test data for $a = 2$ cm.
Figure 59

- Parabolic - 5.6 μm
- Cubic - 5.6 μm
- Linear - 5.6 μm

- Parabolic - 19.1 μm
- Linear - 19.1 μm

h_x vs. x-distance (inches)
Figure 60

Direct comparison of modified theory with test data for $a = 5$ cm.
Figure 61

Variation of temperature in the thermal boundary layer on a flat horizontal plate under influence of corona discharge.

a = 2 cm. Current 5.6 microamperes.
The theory predicted that the heat transfer coefficient should vary as $i^{\frac{1}{3}}$ under the action of the corona wind. To verify that such a trend exists, the values of $h_x$ were plotted against $i^{\frac{1}{3}}$. The results are shown in Figure 62 and 63. The current varies from 0.32 $\mu$A to 180 $\mu$A for the 2 cm. case, and from 2.6 $\mu$A to 180 $\mu$A for the 5 cm. case. Data are plotted for several locations along the horizontal plate. The data at $x = 0$ for the a = 2 cm. case were very erratic at high currents and are not included. It can be seen that straight lines passed through the points correlate with the data quite well. Consequently the prediction that the heat transfer coefficient should vary as $i^{\frac{1}{3}}$ is substantiated, Figure 64 presents data on the variation of the mean heat transfer coefficient with current. The ratio of the average heat transfer coefficient over the surface of the plate with applied field to the average free convection heat transfer coefficient is plotted against $i^{\frac{1}{3}}$. It can be seen that a linear relationship also would hold well for this case.

It is apparent from a review of the theory and the experimental work, that large changes in heat transfer over the free-convection case can be achieved through the use of the corona wind. Substantial increases occur for very small currents. The power required for the action is low. Some typical values of the power expended are shown in Table I. The values are those for some of the curves shown in Figures 47, 48, and 58.

**Thin Jet Solution**

The last correlation to be made is between the thin jet and the test data. The equations used were

Stagnation region (9)

$$k < \frac{d u}{\pi U} b$$

$$h_x = 0.570 N_{Pr}^{\frac{2}{3}} k \left( \frac{\pi u}{b} \right)^{\frac{1}{2}}$$

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Figure 62

Variation of local heat transfer coefficient with corona current
a = 2 cm. Wire diameter 0.004 inch.
Figure 63

Variation of local heat transfer coefficient with corona current,
a = 5 cm. Wire diameter 0.004 inch.
Figure 64

Variation of mean heat transfer coefficient for horizontal plate.

0.004 inch wire.
TABLE I

Power Requirements for Corona Wind Horizontal Plate, 0.004 inch wire at two spacings.

<table>
<thead>
<tr>
<th>Wire Spacing cm.</th>
<th>Current</th>
<th>Voltage KV</th>
<th>Power watts</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>.32</td>
<td>5.00</td>
<td>.0016</td>
</tr>
<tr>
<td>2</td>
<td>1.05</td>
<td>5.00</td>
<td>.0053</td>
</tr>
<tr>
<td>2</td>
<td>2.54</td>
<td>5.17</td>
<td>.0131</td>
</tr>
<tr>
<td>2</td>
<td>4.55</td>
<td>5.67</td>
<td>.0258</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>6.0</td>
<td>.0336</td>
</tr>
<tr>
<td>2</td>
<td>5.6</td>
<td>6.34</td>
<td>.0648</td>
</tr>
<tr>
<td>2</td>
<td>10.2</td>
<td>6.67</td>
<td>.127</td>
</tr>
<tr>
<td>2</td>
<td>19.1</td>
<td>8.00</td>
<td>.328</td>
</tr>
<tr>
<td>2</td>
<td>41.0</td>
<td>8.83</td>
<td>.703</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>8.33</td>
<td>.022</td>
</tr>
<tr>
<td>5</td>
<td>2.6</td>
<td>8.67</td>
<td>.039</td>
</tr>
<tr>
<td>5</td>
<td>4.55</td>
<td>10.00</td>
<td>.102</td>
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<tr>
<td>5</td>
<td>10.2</td>
<td>11.85</td>
<td>.251</td>
</tr>
<tr>
<td>5</td>
<td>21.2</td>
<td>14.19</td>
<td>.561</td>
</tr>
<tr>
<td>5</td>
<td>39.5</td>
<td>18.18</td>
<td>1.429</td>
</tr>
</tbody>
</table>

These data correspond to the data in Figures 47, 48 and 58.
Blasius region (16)

\[ h_x = \frac{k}{(\frac{v}{v_x})^{\frac{1}{2}}} \left( 0.332 \frac{N_{Pr}^{\frac{1}{2}}}{2} \right) \]

Study of the pressure distribution revealed that an approximate equivalent jet half width for \( a = 2 \text{ cm.} \) would be

\[ b = \frac{a}{2} \]

Using this value, and substituting

\[ \overline{U} = \left( \frac{2p_c}{\rho} \right)^{\frac{1}{2}} \]

\[ \rho_c = C, i = 2.478 \times 10^{-4} \]

The equation for the stagnation region becomes

\[ h_x = \left( \frac{\gamma}{4} \right)^{\frac{1}{8}} \left( 5.570 \frac{N_{Pr}^{o.2}}{\kappa} \right) \left( \frac{2\overline{U}}{\rho} \right)^{\frac{1}{4}} \left( \frac{2 \times 2.478 \overline{U}}{\rho} \right)^{\frac{1}{4}} \]

Typically for air at 79°F and \( a = 2 \text{ cm.} \)

\[ h_x = 1.98 i^{\frac{1}{4}} \]

This expression holds from \( x = 0, \) to \( x = 2a/\pi. \) Beyond \( x = 2a/\pi, \) the Blasius solution is used. Fitting the Blasius solution so that \( x = 2a/\pi, \) the heat transfer coefficients of the two regions are the same, results in the following expression,

\[ X > 2a/\pi \]

\[ h_x = 2.23 i^{\frac{1}{4}} X^{\frac{1}{4}} \]

where \( x \) is given in centimeters in this case. These relationships are plotted in Figure 65. Only the calculations for \( a = 2 \text{ cm.} \) are presented. Comparison of this curve with the test data of Figure 48 indicates a fair degree of correlation. The right order of magnitude is achieved and the shapes of the curves approximate the test data, except of course in the region of the ends of the plate. The primary problem in this method lies in the choice of a suitable jet half width, \( b. \) No accurate nor logical method presents itself by which to determine \( b. \) Since both magnitude, and to some extent shape are affected by the choice of \( b, \) this method can be considered only an approximation.
Figure 65
Heat transfer coefficients based upon thin jet solution $a = 2$ cm.
EXPLORATORY TESTS ON FREE CONVECTION
WITH OTHER ELECTRODES

In the overall investigation of the electrostatic effects on free convection heat transfer, many electrode configurations were used. At the outset of the testing, a Model "T" Ford automotive spark coil was used as a high voltage source. The voltage waveform was found to be very erratic, including both a D.C. component and random A.C. components. The spark coil was useful in providing some interesting pictures of effects of the corona discharge on the interferometer patterns. Plate IX illustrates tests made with a two dimensional row of pins and the spark coil. Picture A illustrates the effect on the isothermal lines an instant after the field is applied. A large swirl can be seen moving upwards at the top of the picture. Picture B shows the effect of the corona discharge under continuous application of the field. In both pictures the row of needles is represented by the gray line pointing upwards towards the center of the plate. The dual lines on the left side of the picture are the support rods for the needles.

Plate X indicates an initial test using a grid of 0.010 inch diameter wires placed in front of the heated plate. The wire grid is shown in the rear of Figure 7. The electric field is impressed between the fine wires and the heated plate. The voltage is applied by means of the automotive spark coil. Both pictures indicate a decided pullout of the thermal boundary layer in the vicinity of the wires. These pictures were taken in the initial exploratory phase of the work. The photography is very poor, but the pictures are included because the phenomenon they show had an important impact on subsequent testing. Many of the subsequent vertical plate tests with a single wire described in a previous section were aimed at understanding this pullout. The pullout phenomenon appears to indicate
A. Just after field applied
B. Steady state with field

Plate IX
Corona discharge to a heated plate. Using a row of needles and spark coil.
A. Approximately 1 cm spacing  B. Approximately 2 cm spacing

Plate X

Electrical field applied to a wire grid located in front of the vertical heated plate. Spark coil used.
an action counter to that of corona wind, and even counter to the action which would be produced by dielectric streaming. As indicated in Appendix I dielectric streaming would occur as the result of a body force produced by the non-uniform electric field. However, the density gradient and thus the dipole moment per unit volume is in the direction to cause the dielectric streaming to move away from the fine wires. As discussed in the section under testing, it is believed that the pullout occurs as an interaction between fluid streams, one stream of which is highly viscous, the boundary flow. Thus it appears that the first impression of the picture may be a misleading one.

Plate XI illustrates tests with the same wire grid in place. The high voltage applied was provided by the D.C. power supply. At the very low currents, between 1 μa and 5 μa the pullout phenomenon observed with the spark coil could be observed. Pictures in the low current range are not included. Picture A is a reference with no field applied. The remaining pictures indicate the effect of increased currents on the isothermal lines. It can be seen that the thermal boundary layer is made thinner by the application of the electric field. The entire flow field in this configuration is erratic and apparently very turbulent.

Plate XII presents the test results using a specially prepared surface for the heated plate. This surface consisted of a series of 0.004 inch wires attached to the surface of an insulating sheet of plastic. The wires were spaced one inch apart. The individual wires were parallel and ran in the horizontal direction along the plastic sheet. The plastic sheet was cemented to the front of the heated plate. The fine wires were located away from the plate surface facing the free room air. Dropping resistors were placed between each of the wires, and were so chosen that approximately a current of 10 μa per wire would flow for an applied voltage of 30 KV. It was believed that this configuration would provide
Plate XI

Interferometer pictures of the effects of corona discharge on heat transfer. Vertical heated plate. Room temperature 52°F. ΔT held constant at 34°F. 0.010 inch wire grid located horizontally at 2 cm. from plate. D. C. voltage applied.
Plate XI
Plate XII

Interferometer pictures of the effects of corona discharge on heat transfer. Boundary layer grid glued to vertical heated plate. Plate temperature approximately 100°F. Room temperature 54°F. Oval positive electrode 1 5/8″ below bottom edge of heated plate.
for local sources of ions within the boundary layer. By placing the other electrode at various locations and applying a voltage, the possible effects on the boundary layers could be observed with the interferometer. Picture A illustrates the reference case with no applied field. The wires are so small, they cannot be seen at the surface of the plate. The external electrode is positioned 1 5/8 inches below the bottom edge of the plate. This electrode is an oval rod shown in the front of Figure 7. It is not visible in the interferometer pictures. Picture B shows the effects when the field is applied. Even at high currents very little effect of the field is found. It is apparent that any attempt to work at the surface of a body will have to be done with a great deal of careful forethought.

Plate XIII illustrates the effect of a uniform electric field on the thermal boundary layer of the vertical heated plate. A flat 6 inch by 10 inch aluminum electrode, unheated, was placed 1.1 cm in front of the heated plate. The electrical field was applied. Picture A was taken without an applied field. In Picture B a strong field was applied. No corona discharge occurred and no measurable currents were observed. Changes to the fringe patterns are not apparent. Tests of this type were repeated for several electrode spacings and various applied voltages. Voltages up to and including that necessary for breakdown (sparking) were applied. No visible effects on the isothermal lines were found for applied fields below breakdown potentials. It thus appears that application of a uniform field in air with very low ionization produces no measurable changes on the free convection heat transfer.

No further discussion of the tests presented in this section will be given. The pictures are included primarily to indicate the coverage of work accomplished and to stimulate thought.
Plate XIII

Effect of uniform electric field on free convection heat transfer.
EXPLORATORY TESTS WITH DIELECTROPHORESIS

This section will cover very briefly a series of tests made to explore the body forces obtainable through the use of the dielectrophoresis body force. The phenomenon of dielectrophoresis is explained in detail in Appendix I. In summary, a force can act on a particle placed in a non-uniform electric field even if that body does not possess a net electrical charge. By virtue of the charge displacements occurring on bodies in electric fields an electrical body force arises. Experiments conducted by Gemant and Pohl (21, 22) have indicated that motion of the particles, either solid or liquid can be achieved. To verify these results, tests were conducted and are reported in this section. In addition some original work was done on the motion of droplets of water in a simulated “Zero-G” model.

Description of Test Apparatus

Figure 66 presents the entire test set up. At the right of the bench is a D.C. high voltage power supply, next to it is a neon transformer connected with a Variac used to provide a variable source of A.C. high voltage. The test model is located at the center of the picture. It can be seen more clearly in Figure 67. Various electrode shapes are shown at the left of the glass container. The actual test model is a glass container approximately 8 inches in diameter provided with a sealed stem at its right side through which one electrode lead extends. Various shaped electrodes can be placed in the bottom of the container. The other electrodes are mounted onto the electrode holder shown directly above the center of the container. The electrical field is applied between the upper electrode holder and the lower electrode. The fluids used in the experiments are a mixture of one half CCl₄ and silicone D.C. 200 fluid. The fluids were mixed and the attempt was made to obtain a fluid with the
Figure 66
Overall view of test set-up for dielectrophoresis experiments.
DIELECTROPHORESIS

DANGER
HIGH VOLTAGE
CAPACITOR
DISCHARGE
Figure 67

Close up of dielectrophoresis equipment showing electrodes.
same specific gravity as that of water. Although it had been anticipated that the fluids were completely miscible, in actual use, an interface between two phases of the fluid formed. At this interface it was possible to suspend droplets of water. The fluids were nonconducting.

Test Results

Figures 68 and 69 illustrate electric wind effects. A point is used as the central electrode. It is located just above the surface of the liquid. The other electrode can be seen beneath it. The lower electrode is in the fluid at the bottom of the container. With a field applied, waves are generated on the surface of the liquid as the corona wind strikes the surface. This is the same phenomenon as considered previously in the sections on heat transfer.

The next series of pictures indicate the effect of a nonuniform A.C. field on the fluid. The central electrode shown in Figure 70 is a round ball approximately 1/8 inch in diameter located 3/16 inch above the surface of the fluid. The same lower electrode is used as described before. Figure 71 shows that upon application of the A.C. voltage the fluid jumps up and clings to the surface of the ball. Figure 72 shows a ring central electrode located above the surface. Figure 73 indicates the fluid clinging to the wire under the action of the field. These experiments merely represent pictorial verification of Gemant's and Pohl's work.

The last pictures illustrate the action of the non-uniform field on suspended water droplets. Figure 74 shows the fluid model. The upper electrode is a small round surface electrically insulated by a very heavy coating of wax. The lower electrode is a sheet of aluminum foil placed below and outside of the glass container. The non-uniform field is established between these two electrodes. Two relatively large droplets of water can be seen suspended upon the interface existing within the fluid model. Figure 75 shows that after the field was applied the water droplets
Figure 68
Illustration of electric wind demonstration, no applied field.
Figure 69

Electric wind test with applied field showing surface waves.
Figure 70

Dielectric attraction test, no applied field. Sphere located just above liquid surface.
Figure 71
Dielectric attraction test with applied field.
Figure 72

Dielectric attraction test, no applied field. Ring electrode just above surface.
Figure 73

Dielectric attraction test with applied field.
Figure 74

Dielectric influence on water droplets suspended in a non-conducting liquid. No applied field.
Figure 75

Dielectric influence on water droplets suspended in a non-conducting liquid, after the application of the electric field.
moved slowly toward the central electrode, and upon reaching it coalesced to form a single droplet. This sequence portrays the actual existence of the non-uniform field body force. In order to observe the phenomenon, considerable care had to be exercised in conducting the tests. If a bare central electrode were used, when the droplet reached it, the droplet would be repelled and broken up.

The exploratory tests covered in this section are included solely to demonstrate that the non-uniform field effects actually exist. It was not possible in the time available for accurate instrumentation to be obtained and used. It was found that approximately 10,000 volts A.C. were required to move the droplets of water, but unfortunately currents were not measurable with the A.C. milliammeter available. The pictures, however, do indicate that some measure of fluid orientation and control are possible with the non-uniform electric fields.
Starting with the fundamental question, what are the possible effects of an electrostatic field on the boundary layer and heat transfer in a fluid, an entire field of investigation was opened. The review of literature and theory in several fields of endeavor revealed that a broad range of influences were possible to obtain. The question that followed was, were the phenomena controllable and useful?

Two series of experiments were then conducted to determine if the influences could be controlled, or made to provide useful results. The one series consisted of the corona wind effects on free convection heat transfer. An extensive experimental and analytical program was undertaken and accomplished. Large changes in heat transfer were noticed with the application of electric fields. The correlation achieved between the theory and the test data appeared to be very good. While the actual magnitudes of the heat transfer coefficients were not exactly predicted, the trends indicated were in excellent agreement.

The second series of experiments were purely exploratory in nature, and were accomplished to demonstrate some actions of the dielectrophoresis phenomenon.
CONCLUSIONS AND RECOMMENDATIONS

As a result of the overall survey conducted on the possible interactions between electric fields and fluids, it was concluded that many possible actions exist. The actions include

1. Ion drift phenomena due to an applied field
2. Charged particles motion in an A.C. field
3. Phoresis motions
4. Dielectrophoresis phenomena
5. Changes in fluid properties with applied fields.

Because of these actions, body forces are available of sufficient magnitude to provide useful functions. Prior experiments have indicated that both partially-ionized and neutral fluids can be influenced. It appears logical to utilize these body forces in carefully selected regions in the fluid, for example, in the boundary layer. In such a way, changes in heat transfer or drag can be obtained.

The specific experimental work conducted in the present study, clearly indicates that a sizeable change in heat transfer can be obtained through the use of the corona discharge. The application of the Von Karman integral equations and electric field theory lead to an analytical solution which correlates well with the test data for a typical test configuration. The demonstration tests of dielectrophoresis, likewise indicate that neutral fluid and particle motion is possible under non-uniform A.C. electric fields.

It is believed that the phenomena involved in electrofluidmechanics can have significant importance in a wide range of fluid mechanic and heat transfer problems. Much basic work must be done to understand the various phenomena, their limitations and their scope.
It is recommended that a continuing effort be expended in this area of work. Many possible avenues of research are evident, including effects on viscosity, thermal conductivity, transition, separation, drag and heat transfer. Potential areas of application may be large if the phenomena can be controlled at will.
# SYMBOLS

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
<th>UNITS</th>
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<tr>
<td>A</td>
<td>area</td>
<td>sq. ft.</td>
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<td>a</td>
<td>distance-corona wire to plate</td>
<td>cm.</td>
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<td>b</td>
<td>impinging jet half width</td>
<td>cm.</td>
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<td>specific heat at constant pressure</td>
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REFERENCES


OTHER BIBLIOGRAPHICAL MATERIAL


APPENDIX I

STUDY OF ELECTRIC FIELD-FLUID INTERACTIONS
APPENDIX I

INTERACTIONS OF ELECTROMAGNETIC FIELDS

FUNDAMENTAL INTERACTIONS

The fundamental forces available are electrical in nature. To properly orient later discussion, both electric and magnetic effects will be reviewed briefly.

Electrostatic Influence

A fundamental concept in electricity is that of Coulomb’s law of repulsion between two charged particles of like sign.

\[
F = \frac{q_1 q_2}{4 \pi \varepsilon r^2} \hat{a}
\]

where:
- \(F\) = force,
- \(q_1, q_2\) = charge,
- \(\varepsilon\) = permittivity (dielectric constant),
- \(r\) = distance between charges, and
- \(\hat{a}\) = unit vector.

This relationship leads to the concept of the electric field, or force per unit charge, \(F = qE\), where \(E\) is the electric field strength. The simplest action of an electric field is that of the force induced on a charged particle placed in that field. Particles that can be influenced in this manner are electrons, positive and negative ions, and a whole host of macroscopic particles. Typical macroscopic particles are charged dust particles, droplets of liquid, impurities in solutions, colloids, or any other aggregations that can sustain an electrical charge. Although the magnitude of the action for the various particles may be different, the basic motion is that of the drift of the charged particle as a result of the force induced by the electric field, as shown in Figure 1.

![Figure 1. Drift of ions in a uniform field](image)
In contrast to this simple Coulomb attraction is the subtle influence of the electric field on neutral molecules. The action of polarization in a dielectric illustrates this phenomenon. When an ideal nonconducting dielectric is placed between the plates of a charged condenser, the theory of dielectric action indicates that a change occurs in the electrical orientation of the molecules. A slight displacement of the positive and negative charge within the molecules is induced by the field existing between the plates of the condenser. This slight shift of charges is the basis of the induced polarization of the dielectric (Ref. 1). This action is not limited to condensers, but exists wherever there are bodies in an electric field, both macroscopic bodies and atomic particles. A polarized particle, because of the shift of charge, has an electrical moment equal to the product of the charge times the distance between charges. In addition to these induced dipoles, many molecules have permanent dipoles that are independent of an external field. Typical molecules having permanent dipoles are water and oxygen.

The significant feature of the electrical dipole, either induced or permanent, is that it affords a means of applying a controlled force directly to the individual neutral molecule. Two effects are possible. The polarized molecule can be oriented in the direction of the local field. In a nonuniform field, a direct force on the molecule exists to move the molecule in the field. The molecules will be oriented in both uniform and nonuniform fields, but a force will act on the polarized molecules only in a nonuniform field. It should be remembered that with macroscopic bodies of high conductivity, no large internal fields can exist and no internal charge separations can take place. External charge-induction effects can take place, however.

The last electrostatic influence to be considered is that of the mechanical stress induced in a dielectric due to the applied field. Under the influence of the field, a tension known as electrostriction is produced within the material. This tension is analogous to the phenomenon of magnetostriction and is related to the piezoelectric effect in crystals. The internal stress is the internal reaction to the forces exerted on the dielectric by the charged plates (i.e., the field between the plates).

Magnetic Influences

Let us next consider the effects of magnetism. Magnetic influences include paramagnetism, diamagnetism, and ferromagnetism. A bar of material which aligns itself parallel to an externally applied magnetic field is an example of paramagnetism. Paramagnetism occurs when the electron orbits, which act as small magnetic dipoles, become oriented with the applied field. Ferromagnetism is similar to paramagnetism but the effect is much, much larger. Ferromagnetism is best illustrated by the familiar iron magnet. A diamagnetic body tends to align itself perpendicular to the applied field because of the gyroscopic precession of the electron orbits within the molecules of the substance.

Both paramagnetic and diamagnetic influences are weak when compared to ferromagnetism, and diamagnetic effects are only approximately one-tenth to one-hundredth those of paramagnetism. Diamagnetism can be observed only in substances which have no paramagnetism, since even a small paramagnetic influence will mask out the diamagnetic effect. Certain inert gases such as N₂ exhibit diamagnetism, whereas O₂ exhibits paramagnetism (Ref. 1). Under the influence of a divergent magnetic field, paramagnetic substances are attracted toward the converging field lines while a diamagnetic substance would be repelled. Thus, small body forces are available within the fluids, even in non-ferromagnetic materials.
Another influence which can act upon a fluid is that of magnetic stress. A substance placed into an applied field will experience an internal stress which is proportional to the square of the field strength.

The last magnetic influence to be discussed is that of the $\mathbf{J} \times \mathbf{B}$ Lorentz interaction which occurs when an electrical current flows in a magnetic field. This force is large and is the driving action behind electric motors. Since the current can be a stream of charged particles in a fluid, a large body force can be produced in a fluid if suitable electrical and magnetic fields are superimposed. As mentioned previously, this interaction forms the basis for magnetohydrodynamics (Refs. 2, 3, and 4).

In summary, several basic field and material interactions are possible. There are the very strong Coulomb electrostatic repulsion, magnetic repulsion (ferromagnetic or induced fields), and the $\mathbf{J} \times \mathbf{B}$ Lorentz force. More subtle influences are those due to electrical dipole action and the diamagnetic and paramagnetic action. Although these effects are relatively small, they can provide a useful body force within the fluid. This discussion, however, will deal almost exclusively with the effects of applied electric fields acting alone.

OBSERVED INTERACTIONS OF ELECTRIC FIELDS

The previous discussion indicated that an applied electrical field could act on substances placed within its sphere of action. If this is so, then one should expect to find numerous examples where such influence has been observed (Refs. 5 and 6). The first observation of electric-fluid interaction was the motion of a candle flame between the charged plates of a condenser. When a high potential was applied, the flame displaced laterally and assumed a somewhat fan-shaped appearance, as shown in Figure 2. The high temperatures in a flame provide a copious supply of ions. These ions tend to drift laterally in the applied field and distort the shape of the flame.

---

**Figure 2. Distortion of a flame by an electric field**
Electrolytic action in liquids was first observed long ago, and played a key role in the early development of chemistry. Electrolytic action in a solution is due to the drift of the ions to the anode and cathode. More recently, gaseous conductors have played an important role in the development of electronics. Various gas tubes, discharge tubes, arcs, and similar uses of conducting gases have been studied extensively. All of these involve moving charges -- ions, electrons, or mixtures -- under the action of the field.

Let us consider next a large group of lesser known effects. Dust particles that carry a charge may be removed by an applied field. If a natural charge does not exist on the particles, suitable charges can be induced. This action forms the basis for the various electrostatic precipitators used in home and industry. The air close to a waterfall is charged (Ref. 7); the fine mist carried aloft is generally negatively charged and the spray nearer the waterfall has a positive charge. The charges are thought to be induced by the shattering of the water upon impact and the bubbling at the water surface. Similar electrification phenomena include charged sprays over ocean waves, spray out of nozzles, and steam exhausted through a nozzle (Refs. 8 and 9). Further discussion on the mechanism of charging in these cases will be taken up later.

Electric wind, because of its recent application to lifting devices, has received a great deal of attention. Electric wind is a current of air flowing from a highly charged point electrode in air (Refs. 5 and 6). It can be demonstrated by applying a high voltage between a needle point and a plane surface, as shown in Figure 3. The air stream so generated will blow out a candle flame. A corollary effect is that of electric wind (ion drag) pressure generation (Ref. 10). In this case, several highly charged points are placed in series, and the resultant electric wind builds up a pressure. Large numbers of ions are created at the point due to the intense local fields, and these ions drift in the electric field. Momentum is transferred between the ions and the neutral molecules, resulting in a streaming of the gas as a whole. The effect is considerably greater in liquids than in gases.

![Figure 3. Electric wind induced by a charged point](image-url)
The next group of effects are considered together. These effects all involve the movement of charged particles in a field, but not necessarily ions. Only a listing of the effects will be given at this point; details will be discussed later. The phenomena are: streaming potential or flow electrification, electrophoresis, dielectrophoresis, electro-endosmosis, and Dorn effect (Refs. 11, 12, and 13). Streaming potential is observed as the build-up of extremely high voltages when fuels are pumped through lines. Many cases indicate a high static charge was generated in the fueling of aircraft that has caused large sparks and fires. This phenomena has been the subject of considerable study to determine the causes and means of control. Electrophoresis is presently utilized extensively in biochemistry as a means for separating and identifying various substances, such as proteins, for example. Electrophoresis has also been used for plating or coating, since this technique provides a relatively high rate of build-up. Fluids have been pumped by using the electrophoretic effect. Dielectrophoresis is the drift of uncharged particles in a nonuniform electric field. It has been used to separate various materials suspended in solution and to pump or to change the orientation of dielectric fluids. This effect has a very significant influence on free-convection heat transfer in liquids. Electro-endosmosis has been observed in liquids as the ability of a field to elevate a portion of a fluid in a container. If a container is divided into two segments connected by a porous plug, applying a voltage across the fluids will change the elevation of the fluids in the two segments. The Dorn effect is similar to the foregoing. The Dorn effect is the difference in potential that is built up in a liquid as a suspensoid slowly settles downward in a container under the influence of gravity.

Another phenomenon is the Kerr cell. The Kerr cell acts as an electrical light shutter. An electric field applied to certain liquids will change the ability of the liquid to transmit light. By applying a rapidly oscillating potential to the liquid, a beam of light can be cut up into segments. This phenomenon is dependent upon the inter-relationship between the refractive index and the dielectric constant of a given material. This relationship was first deduced by Maxwell from his field equations (Refs. 1 and 14).

Electrostriction produces a change in the volume of a substance. Because of this, thermodynamic properties can possibly change. All thermodynamic quantities that depend upon volume can be influenced, and the influences would be a function of the square of the local field strength.

Electric propulsion uses electric field effects extensively. The thermal arc thrustor uses an extremely high current arc to heat the hydrogen propellant to very high temperatures and a nozzle to expand the gases and produce useful thrust. The ion engine accelerates the ions of the propellant (cesium, for example) to very high velocities. The cesium ions are ejected by the field to produce useful thrust at specific impulse values of 5000 to 20,000 seconds.

Power may be generated by the inverse electric wind effect (Refs. 15, 16, and 17). Analyses and preliminary tests have shown that the deceleration of charged particles (ions or droplets, for example) can be used for producing useful power. This action formed the basis of the Armstrong electrostatic generator, which was used late in the 19th Century.

The last effects to be mentioned in this section are not directly related to fluids. They may be of general interest, however, and do illustrate the influence of fields on materials. Field emission is used either to extract electrons from the surface of a metal or, with sufficient strength, to extract the ions from the crystal lattice. This action forms the basis for the electron-field and the ion-field microscopes. Field emission has also been used to provide a source of electrons for various electronic devices. Exploding wires demonstrate a combination of electrical effects when enormous currents are discharged through small
wires during a very short time interval. Static charging phenomena are probably the oldest form of electrification known. However, the basic mechanisms of charge separation in solid-solid contact is complex and apparently not clearly understood. Actions of contact potential, Volta potential, adsorbed layers of water, and physical transfer of electrons and ions under high local fields are all involved in various forms of static electrification. Tribo electrification or frictional electrification is of this type and occurs as solid-solid contact phenomena in dry surfaces (Ref. 7). While solid-solid electrification is not of primary concern here, the mechanism of charge generation by air flowing over a surface and combustion gas flowing out of a nozzle, for example, will directly influence the state of charge in flowing fluids.

Thus, many effects of electric fields on fluids have been observed. Some have been studied extensively, and others little or not at all. The wide range of effects, however, encourages study of electric-fluid interaction.

OBSERVED EFFECTS OF ELECTROMAGNETIC FIELDS

The actions of electromagnetic fields on fluids has led to the development of electromagnetic pumps and many magnetohydrodynamic applications. Many, many effects have been shown to take place, including apparent changes of fluid properties, velocity profiles, boundary layers, and heat transfer. These effects are thoroughly covered in the literature on magnetohydrodynamics. In this study of electric field effects, therefore, magnetic field effects will not be discussed further.
ACTION OF ELECTRIC FIELDS ON PARTICLES

ACTION ON CHARGED PARTICLES

To provide a firm basis for a more detailed discussion of the observed electric field effects, a review of the action of fields on particles will be covered next. Fundamental to this discussion are the basic equations of electricity and magnetism. Maxwell's relations for electromagnetism summarize the basic phenomena of electricity (Ref. 3). These relations are:

\[
\begin{align*}
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{E} &= \rho \\
\nabla \cdot \mathbf{B} &= 0 \\
\mathbf{J} &= \sigma (\mathbf{E} + \nabla \times \mathbf{B}) + q \mathbf{v} \\
\mathbf{F} &= q \mathbf{E} + \mathbf{J} \times \mathbf{B} \\
\mathbf{D} &= \epsilon \mathbf{E} = \epsilon_0 \mathbf{E} + \mathbf{P} \\
\mathbf{B} &= \mu \mathbf{H}
\end{align*}
\]

where:

\[
\begin{align*}
\mathbf{J} &= \text{current density}, \\
\rho &= \text{charge density}, \\
\mathbf{D} &= \text{electric flux density}, \\
\mathbf{E} &= \text{electric field strength}, \\
\epsilon &= \text{dielectric constant}, \\
\mathbf{P} &= \text{polarizability}, \\
\epsilon_0 &= \text{dielectric constant (vacuum)}, \\
q &= \text{charge}, \\
\mathbf{v} &= \text{velocity of charge}, \\
\mathbf{B} &= \text{magnetic flux density},
\end{align*}
\]
\( \mathbf{H} \) = magnetic field,
\( \mu \) = magnetic permeability, and
\( \sigma \) = electric conductivity.

For the cases involving no externally applied magnetic field, the number of applicable equations is reduced and simplified. To illustrate the use of the equations, consider the motion of a single ion in a uniform field formed by two flat plates. The applicable equations are:

\[
\nabla \cdot \mathbf{B} = \rho \\
\mathbf{B} = \mu \mathbf{E} \\
\mathbf{E} = -\nabla V
\]

where:

\( V \) = potential.

If \( E \) is a constant, then

\[
\nabla = \frac{\partial}{\partial x} \\
E_x = \text{constant} = -\frac{\partial V}{\partial x} \\
V = -E_x x + C_1 \\
C_1 = 0 \quad \Rightarrow \quad V = 0 \quad \text{at} \quad x = 0.
\]

Therefore,

\[
V = -E_x x. \quad (3)
\]

Consequently, voltage is a linear function of distance.

Under the influence of the field, \( \mathbf{F} = q \mathbf{E} \),

\[
m \ddot{x} = q E_x \\
\dot{x} = \frac{q}{m} E_x t + C_1,
\]

where:

\( m \) = the mass of the particle.

If at \( t = 0, \dot{x} = 0 \), then \( C_1 = 0 \), and:

\[
x = \frac{q}{2m} E_x t^2 + C_2.
\]
If at $t = 0$, $x = 0$, then $C_2 = 0$, and:

$$x = \frac{q}{2m} \varepsilon x t^2.$$  \hspace{1cm} (4)

This result, which is similar to the motion of a body falling in a uniform gravitational field, represents the motion of a charged particle in a uniform field. It could also represent the motion of an ion between collisions when moving in a neutral gas.

Motion of a charge in the field of another point charge will occur along a radial line. Because of spherical symmetry, Poisson's equation will reduce in complexity. Poisson's equation can be written as:

$$\nabla \cdot \mathbf{D} = \rho$$

or

$$\nabla^2 V = -\frac{\rho}{\varepsilon}.$$  

If the moving charge is small and is assumed to have negligible influence on the applied field, then

$$\nabla^2 V = 0$$

In spherical coordinates:

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}.$$  

Symmetry reduces this equation to:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) = 0,$$

and then:

$$r^2 \frac{\partial V}{\partial r} = C_1,$$

$$dV = \frac{C_1}{r^2} \, dr$$

$$V = -\frac{C_1}{r} + C_2.$$  

If $V = 0$ at $r \to \infty$, then $C_2 = 0$.

Therefore:

$$V = -\frac{C_1}{r}$$  \hspace{1cm} (5)
and

\[ E = -\frac{\partial V}{\partial r} = \frac{C_i}{r^2} . \]

Using this, the equation of motion is:

\[ \frac{d^2 r}{dt^2} = \frac{q}{m} \frac{C_i}{r^2} \cdot \frac{1}{r^2} . \]

Let \( v = dr/dt \) and \( dr = vdt \). Then:

\[ v \frac{dv}{dt} = \left( \frac{qC_i}{m} \right) \frac{dr}{r^2} \]

\[ \frac{v^2}{2} = -\frac{qC_i}{m} \int_{r_0}^{r} \frac{dr}{r^2} + C_2 . \]

If \( v = v_0 \) at \( r = r_0 \), then:

\[ (v^2 - v_0^2) = \frac{2qC_i}{m} \left( \frac{1}{r_0^2} - \frac{1}{r^2} \right) . \]  \( (6) \)

This is the equation for the velocity of motion of a charged particle in a spherically shaped electrical field.

The motion of a moving charged particle in a magnetic field will be of interest later, so we will derive the equation for this motion at this point. The force on a moving charge or on a current in a magnetic field is analogous to the forces and torques on the windings of an electric motor. It can be represented by:

\[ F = J \times B \]

where:

\[ J = \frac{q}{m} \frac{dv}{dt} . \]

Therefore:

\[ F = q \frac{dv}{dt} \times B . \]

If the charged particle enters a magnetic field moving with a velocity, \( v \), in a plane normal to the magnetic flux, then the equation reduces to a scalar equation:

\[ F = qvB . \]  \( (7) \)

The direction of this force is normal to both the magnetic field and the velocity of motion. Such a force causes the direction of motion to change but not the magnitude of the velocity. Consequently, the force is constant in magnitude and the direction is always perpendicular to the velocity. Such a force causes the particle to move in a circle. The acceleration of the particle and the radius of the circle can be readily determined by:

\[ F = ma = qvB \]

\[ \frac{a}{m} = \frac{qvB}{m} = \text{acceleration}. \]
In a circular path:

\[ a = \frac{v^2}{r} \]

Equating these two expressions:

\[ \frac{qvB}{m} = \frac{v^2}{r} \]

\[ r = \frac{mv}{qB} \]  \hspace{1cm} (8)

A charged particle moving without an external electric field will begin to rotate about the magnetic lines of force when entering a magnetic field. If, in addition, the particle has a velocity in the direction of the field, it will spiral along the flux line at a constant radius. This elementary introduction to the action of the moving charge in a magnetic field, we believe, will be sufficient for the subsequent discussion.

As a charged particle moves in a dense fluid under the influence of an electric field, its motion is retarded by an effective drag exerted by the fluid. This effect on the motion of an ion moving through a gas is included in the concept of ion mobility (Refs. 18 and 19). As the ion moves in a field, it accelerates quickly to a characteristic velocity along the field, called the ion drift velocity. In dense gases, this velocity is approximately proportional to the applied field strength, providing the field strengths are not so high as to cause breakdown (a spark for example).

The drift velocity, then, is given by:

\[ v = KE \]  \hspace{1cm} (9)

where: \( K \) = the ion mobility.

This motion, as can be noted from the following equations, differs from the motion of a single ion in free space where the field is uniform:

Single ion: \( v = \frac{(qE)}{m} \)

Ion in a gas: \( v = KE \).

Based upon the elementary kinetic theory of gases:

\[ K = \frac{eL}{m \bar{v}} \]  \hspace{1cm} (10)

where:

\( e \) = charge on the ion,

\( L \) = mean free path of gas,

\( \bar{v} \) = mean speed of molecules, and

\( m \) = mass of the ion.
This expression is based upon the assumption that the ion accelerates between impacts with neutral molecules after having been stopped at each collision. Derivations by Compton, Langmuir, and others have led to more refined expressions for mobility. The simple equation (Eq. 10), however, illustrates the general dependence of mobility on the basic characteristics of the gas and ion, namely, that mobility depends upon the mean free path, mass, and mean free speed of the molecules, as well as the magnitude of the charge on the ion. It has been observed during tests that the mobilities of negative and positive ions of the same molecules are different. Negative ions usually move more rapidly through a given field than positive ions. Because of this phenomenon, the distribution of ions in a uniform field will generally not be symmetric.

To illustrate the possible distribution of charged particles under the influence of a uniform field, two examples will be given. First, the distribution will be established assuming none of the charges are neutralized at the wall, i.e., no current flows.

If no mass motion of the fluid occurs and a stationary state is reached, then the pressure rise due to ion distribution in the field will be given by:

\[
\frac{dp}{dx} = -F_x
\]

where: \( F_x \) = the force acting on the ion gas due to the electric field.

\[
F_x = \rho E_x
\]

\[
\rho = n_i \cdot q_i
\]

\[
F_x = n_i \cdot q_i \cdot E_x
\]

where:

\( E_x \) = field in x direction,

\( n_i \) = number of ions/unit volume, and

\( q_i \) = charge per ion.

From kinetic theory:

\[
\rho = \frac{1}{3} \cdot \frac{\gamma}{q} \cdot \sigma^2 = \frac{\gamma RT}{qm}
\]

where:

\( \gamma \) = assumed constant, and

\[
\frac{\gamma}{q} = \text{ion gas density} = n_i \cdot m
\]

Substitute this into the equation for pressure rise:

\[
RT \cdot \frac{d n_i}{dx} = -q_i \cdot E_x \cdot n_i
\]
Therefore:

\[
\frac{q_i E_x x}{RT} = A e^{-n_i}
\]

Thus, the ion density distribution in this simple illustration is of the same exponential form as the variation in density of the air in an isothermal atmosphere.

Next, let us consider gaseous conduction between two parallel plates with an applied potential. As the voltage is raised, the charges migrate to the plates. Negative ions move toward the positive plate, and positive ions move more slowly toward the negative plate. As the voltage is increased, a large number of charged particles move close to the plate and effectively cancel the applied field in the vicinity of the electrode. This condition exists in front of both plates and results in a distribution of potential between the plates; a sharp drop in potential occurs near each electrode and a very small drop in the center as shown in Figure 4.

The congregation of charge near the plates is known as space charge. Space charge can also occur in the vicinity of a heated cathode emitter. As large numbers of electrons are emitted, the electrons in the vicinity of the cathode shield the cathode and create a space charge directly in front of the cathode. The amount of current that can flow is then limited.

![Figure 4. Illustration of space charge](image-url)
In a high density gas, the potential distribution for electrons (Ref. 19) is:

\[ V = \frac{2}{3} \left( \frac{8 \pi J}{K} \right) x^\frac{3}{2}, \tag{12} \]

and the current density is:

\[ J = \frac{q}{4} \frac{K V^2}{8 \pi x^3}. \tag{13} \]

In a high vacuum, the current distribution for electrons is given by Child's law:

\[ J = \left( \frac{2}{3} \frac{q}{m} \right) \frac{V^\frac{3}{2}}{a \pi x^2}. \tag{14} \]

It can be seen that the current distributions, and thus the charge distributions, can vary widely, even in the simple case of a uniform field between flat plate electrodes.

Knowledge of the distribution of ions is important because the forces exerted within a fluid can depend directly on the number of ions that exist in the fluid. In the previous discussion of ion motion in fields, the only factors taken into account were the electric field and the ion density. Effects of charge generation, recombination, and diffusion have not been considered. These influences can be large in specific cases, but the solution of the complete equations is complex. The complete equations for the simple one-dimensional case with both positive and negative ions (Ref. 19):

\[
\begin{align*}
q - a_i n_1 n_2 + D_1 \frac{d^2 n_1}{dx^2} - K_1 \frac{d}{dx} (E n_1) &= 0 \\
q - a_i n_1 n_2 + D_2 \frac{d^2 n_2}{dx^2} + K_2 \frac{d}{dx} (E n_2) &= 0 \\
\frac{d E}{dx} &= -(n_1 - n_2) e
\end{align*}
\]

where:

- \(a_i\) = coefficient of recombination,
- \(n_1\) = number of positive ions,
- \(n_2\) = number of negative ions,
- \(q\) = rate of charge generation,
- \(D_1\) = coefficient of diffusion,
- \(K_1\) = mobility of positive ions, and
- \(K_2\) = mobility of negative ions.
Directed motion of a charge normally takes place under the action of a steady-state electric field. If a time-varying, AC, field is applied, we might expect the charge to merely oscillate back and forth in the field. A charge in a uniform time-varying field will oscillate so. A charged particle in a nonuniform time-varying field, however, will act differently, depending on the phase between the velocity and the field. For illustration, consider a charge moving under the influence of a divergent field. If the charge moving under the influence of a divergent field. If the charge moves outward when repelled, it covers a definite distance from the center of the field. At this point in space, the magnitude of the field is decreased, but the particle has acquired a definite velocity. As the sign of the field is reversed, the particle is accelerated back toward the center of the field, but the accelerating force is smaller than the previous repulsion force. With a purely sinusoidal variation of field, the particle cannot regain its original position. As a consequence, the particle may acquire a mean motion away from the center of the field, as shown in Figure 5. The ion, therefore, acquires a net drift as the result of the non-steady applied AC field (Ref. 20).

Figure 5. Motion of a charged particle in an AC field
The equation of motion of a particle in a spherical field of the Coulomb type would be:

\[ m \ddot{r} = \frac{q_1 q_2}{4 \pi \varepsilon r^2} \sin \omega t \]  

(16)

where:

- \( m \) = mass of particle,
- \( q_1 \) = charge of particle,
- \( q_2 \) = charge at center of field,
- \( \varepsilon \) = dielectric constant, and
- \( \omega \) = frequency of field variation.

Letting

\[ a = \frac{q_1 q_2}{4 \pi \varepsilon m} \]

the equation can be written as:

\[ \ddot{r} = \frac{a}{r^2} \sin \omega t \]  

(17)

Since this equation is nonlinear, an elementary solution does not appear possible. It is somewhat similar to Hill's equation and Mathieu's equation which arise in the analysis of the forced oscillation of an inverted pendulum. This equation would have to be solved by perturbation or iterative methods. Attempts to study the stability by Poincare's method were not successful due to the form of the \( \frac{\sin \omega t}{r^2} \) term.

If a linear-force field gradient is assumed possible, then the equation of motion becomes:

\[ \ddot{r} = a r \sin \omega t \]  

(18)

This is also a Mathieu equation, and no elementary solution is possible.

Care must be used in interpreting any motion effect. The scope of this research study does not permit studying the behavior of such equations extensively. In any specific case, a step-by-step iterative solution can be utilized.

In summary, this section has outlined the factors that affect the motion of a charged particle in an externally applied field. The illustrations indicate how a particle drifts in the field and possible distributions of charge.

**DIELECTRIC ACTION ON NEUTRAL PARTICLES**

The next topic to be considered in electrofluid interactions is dielectric effects. The most common use of dielectric effects is in capacitors. When an ideal dielectric is placed between charged plates, the electrons are tightly bound and cannot move freely through the material to the plates. Since the charges cannot move freely to neutralize the applied field, an internal field will exist within the dielectric (Ref. 1).
Considering Coulomb's law:

\[ F = \frac{q_1 q_2}{4 \pi \epsilon_0 r^2} \]

it can be seen that as the value of the permittivity (dielectric constant) is increased, the force between the charges is reduced. Likewise, the electric field between the charges is reduced. The action of the dielectric stems from the formation and orientation of molecular electric dipoles within the dielectric. When an atom is placed in an electric field, the positively charged nucleus and the electron cloud separate slightly, as shown in Figure 6. This displacement of charge creates an electrical dipole moment: the positive nucleus tends to move toward the negatively charged plates and the electron cloud shifts toward the positively charged plates. This field-induced dipole will form with all atoms and molecules in varying degrees, and in uniform and nonuniform electric fields.

![Diagram of dipole displacement](image)

Figure 6. Formation of induced dipole-displacement of charge

Many molecules have dipoles as an inherent part of their structure due to the arrangement of the atoms in the molecule. A typical molecule with a dipole is water. The total dipole moment of any molecule is the sum of its permanent and field-induced dipoles. The formulation for the electrical dipole is illustrated by the expression of Debye (Refs. 1 and 21):

\[ \eta = \eta_0 + \frac{\mu^2}{3kT} \]  

(19)
where:
\[ \eta = \text{polarizability of a single molecule}, \]
\[ \mu = \text{permanent dipole of the molecule}, \]
\[ k = \text{Boltzmann's constant}, \]
\[ T = \text{absolute temperature}, \]
\[ \eta_0 = \text{constant not influenced by temperature}. \]

Since the dipole moment is given by:

\[ \rho = \eta E \]

where:
\[ \rho = \text{molecular dipole moment and} \]
\[ E = \text{field strength}, \]

then:

\[ \rho = E \left( \eta_0 + \frac{\mu^2}{3kT} \right). \]  \(20\)

An electric field applied to a dipole will cause the dipole to orient itself in the direction of the field. If the molecules of the dielectric have a large permanent dipole, the orientation effects can be strong. In studies of electro-fluid mechanics, any interactions with neutral molecules can be expected to be most pronounced in those fluids containing molecules of large dipole moments. Such, indeed, is the case, as will be illustrated later in discussions of electroviscosity and heat transfer. The mechanisms of interaction between the dipoles, ions, and the applied field, however, are of fundamental importance and, consequently, dipoles will be considered here in more detail (Ref. 3).

Let us examine the nature of the electrical dipole, as shown in the following sketch:
The dipole moment, given by \( p = q \ell \), is the product of the charge strength and the distance separating the two equal but opposite charges. If the distance between the charges were zero, the net charge would be zero and the resultant field would be zero. As the charges are separated, however, the dipole is formed. The system of the two charges is electrically neutral, but there is a net resultant field from the dipole.

The potential at A due to +q is:

\[
V_1 = \frac{+q}{4 \pi \varepsilon r_1}
\]

and due to -q is:

\[
V_2 = \frac{-q}{4 \pi \varepsilon r_2}
\]

Since \( \ell \neq \ell_2 \), the net potential at A for relatively large distances is

\[
V = \frac{q \ell \cos \theta}{4 \pi \varepsilon r^2} = \frac{p \cos \theta}{4 \pi \varepsilon r^2}
\]

(21)

Using the relation \( E = \nabla V \) to find the field strength, we find:

\[
E = -\hat{r} \frac{\partial V}{\partial r} - \hat{\theta} \frac{1}{r} \frac{\partial V}{\partial \theta}
\]

\[
= \hat{r} \frac{q \ell \cos \theta}{2 \pi \varepsilon r^3} + \hat{\theta} \frac{q \ell \sin \theta}{4 \pi \varepsilon r^3}
\]

(22)

where:

\( \hat{r} \) = a unit vector in \( r \) direction, and

\( \hat{\theta} \) = a unit vector in \( \theta \) direction.

The most important points to be noted from these equations are that a resultant field exists and that it varies inversely as the cube of the distance from the dipole.

Polarization of a dielectric is defined as the dipole moment per unit volume:

\[ P = \frac{q \ell}{V} = \text{polarization per unit volume.} \]

The flux density in a dielectric is related to the polarization by the equation:

\[ D = \varepsilon_0 E + P = (\varepsilon_0 \frac{P}{\varepsilon}) E \]

But

\[ D = \varepsilon E \]
Therefore:

\[ \epsilon = \epsilon_0 + \frac{P}{E}, \]

\[ P = E (\epsilon - \epsilon_0) \]  \hspace{1cm} (23)

Consider the action of an applied nonuniform field on a dipole located within the field. Assume that the dipole has oriented itself along the direction of the field, as shown in Figure 7.

![Figure 7: Action of nonuniform field on a dipole](image)

The summation of forces in the x direction per unit volume gives:

\[ \sum F_x = -qE + \left( qE + q \frac{\partial E}{\partial x} \delta x \right) = \left( q \delta x \right) \frac{\partial E}{\partial x}. \]

Since the distance traveled along the x direction between charges is \( l \), then \( \delta x = l \), and:

\[ \sum F_x = q l \cdot \frac{\partial E}{\partial x}. \]

The force per unit volume is:

\[ F_x = q l \cdot \frac{\partial E}{\partial x} = P \frac{\partial E}{\partial x}. \]

Substituting for \( P \), we obtain:

\[ F_x = (\epsilon - \epsilon_0) E \left( \frac{\partial E}{\partial x} \right) = \frac{\epsilon - \epsilon_0}{2} \frac{\partial E^2}{\partial x} \]  \hspace{1cm} (24)
This equation indicates that a body force exists within a dielectric whenever there is a gradient in the field strength. If \( \nabla \mathbf{E} \) is zero, then no body force exists. The magnitude of the body force depends upon the difference in permittivity between the dielectric and vacuum and the gradient of the square of the field strength. Since the force is a function of the square of the field strength, its direction remains the same regardless of the sign of the field. An alternating nonuniform field (time varying) would thus result in a force which is always directed toward the region of highest intensity, as shown in Figure 8. As an example, consider the field around the end of a needle point. The field lines converge to this point, and the field strength increases as the point is approached. In such a case, the dielectric body force would be directed toward the point of the needle, both for DC and AC fields.

From a macroscopic view of the interactions, this dielectric body force may be the most important interaction phenomenon in fluids. There are, however, other actions.

The electric stress generated by the attraction of the charges acts as a stress on the surface of a dielectric and as an internal pressure within the dielectric. The electric surface stress results from the lines of force emanating from charges distributed over a surface. The pull on a surface having a charge density, \( \sigma \), is given by:

\[
F_{\text{sur}} = \int_{0}^{\sigma} \mathbf{E} \, d\sigma = \frac{1}{\varepsilon_{0}} \int_{0}^{\sigma} \sigma \, d\sigma = \frac{\sigma^2}{2\varepsilon_{0}}.
\]

At the surface of the conductor with a surface charge density, \( \sigma' \):\n
\[
D_{\text{normal}} = \sigma ; \quad \varepsilon_{0} \mathbf{E} = \sigma.
\]

Therefore:

\[
F_{\text{sur}} = \frac{\sigma^2}{2\varepsilon_{0}} = \frac{1}{2} \varepsilon_{0} E^2.
\]

The pressure within the dielectric is given by:

\[
p = \frac{(\varepsilon - \varepsilon_0)}{2} \varepsilon E^2.
\]

This result is obtained by equating the dielectric body force to the resisting internal pressure on a unit volume of the dielectric (Ref. 22).

The final interaction to be considered in this section is that of the dipole interaction with charged particles (Ref. 23). In a fluid that contains both charged particles and molecules with dipoles, the charged particles may attract and influence the neutral dipole molecules. The charged particles (ions, for example) act as the point of a needle and thus provide an intense nonuniform electric field close to the particle. As a polar molecule enters this region, it is attracted in a manner analogous to that of the dielectric body in a nonuniform field. In the event the approaching molecule does not have a permanent dipole (nonpolar), then a dipole moment is induced by the field of the charge. Induced dipole attractions in nonpolar fluids, however, will be smaller than those in strong polar fluids. Although the significance of this dipole-charge interaction in macroscopic fluids is not clear, the interaction may lead to "clustering" effects because the neutral molecules would tend to cluster around the charged particle. If such effects occur to an appreciable extent, they could influence fluid properties (such as viscosity), and fluid behavior (velocity profiles, etc.). The various force relationships in the neutral-charged particle interactions are complex; further information is provided in References 18 and 24.
DIPOLE ACTION AND INDUCED FORCE BY NONUNIFORM FIELD WITH CENTER OF FIELD POSITIVE

DIPOLE ACTION AND INDUCED FORCE BY NONUNIFORM FIELD WITH CENTER OF FIELD NEGATIVE

Figure 8. Force on a dipole
The nature of dielectric influences thus tends to be somewhat more complex than the more simple drift effects of the charged particle motion. These influences are of primary importance in the study of fluids, however, because we can create body forces through dielectric action within a fluid that does not contain ions.

PHORESIS

Phoresis is derived from the Greek word meaning motion. As applied to problems in physical chemistry, it relates to the drifting or motion of particles in an electric field. Several phoretic effects have been shown to take place. Ion motion during electrolysis is not generally included in phoretic effects; these effects usually pertain only to particles that are larger than atoms or molecules. These particles may include colloids, bubbles, powders, suspensions, droplets, and similar submacroscopic particles. In most cases, the phoresis effects concern motion of particles in liquids, usually nonconducting liquids. These effects are identified as follows:

1. Cataphoresis
2. Electrophoresis
3. Electro-endosmosis
4. Streaming potential
5. Dorn effect
6. Dielectrophoresis

Each one of these phenomena will be explained. Before doing so, however, we will review the concept of the Helmholtz double layer as discussed in the work of Klinkenberg and van de Minne (Ref. 12) and Loeb (Ref. 7).

At the interface between two phases (liquid and solid or bubble and liquid, for example) the charge is generally unevenly distributed. At an electrode, ions of one sign may go into solution, or ions from the liquid may be adsorbed at the surface of the electrode. In either case, a charge is built up on the surface. Assume, for illustration, that the electrode surface becomes charged positively, as shown in Figure 9. Then an accumulation of negatively charged ions builds up adjacent to the surface. These negative ions are strongly attracted by the positive surface through Coulomb attraction and form a layer of "bound" or "fixed" charge. Further into the fluid, a positively charged diffuse layer has been induced by the "bound" negative layer. The net charge of this diffuse layer diminishes until it reaches a neutral state. The combined "bound" layer and diffuse layer constitute the electrical double layer, or Helmholtz double layer.

Various experiments have shown that the diffuse layer can be moved with a mass motion of the surrounding fluid. The fluid then acquires the net charge that existed within the diffuse layer. The "bound" charge remains on the surface so that the charge is opposite to that of the moving fluid. If the fluid is a very good conductor (i.e., aqueous solutions), then the net charge difference is quickly dissipated and little potential difference is generated. If the fluid has very low conductivity, however, the charges do not neutralize readily and a large charge separation can result. High voltages can thus be generated.
The charge per unit area, \( \sigma \), of the fixed and mobile layer is given by:

\[
\sigma = \epsilon \frac{V_d}{\delta}
\]

(27)

and \( \delta \) is given by:

\[
\delta = \sqrt{\frac{\epsilon k T}{2n Z e^2}}
\]

(28)

where:

- \( \sigma \) = charge per unit area,
- \( V_d \) = potential difference across double layer,
- \( \delta \) = thickness of the double layer,
- \( n \) = number of dissociated molecules per unit volume,
- \( Z \) = valency of ions, and
- \( e \) = unit charge.
Usually, the potential difference, $V_d$, is given as the electrokinetic potential, $\zeta$, or the Zeta potential. It is similar to $V_d$ but is described as:

$$\zeta = \frac{\sigma}{\epsilon} = \frac{4 \pi \delta \sigma}{\epsilon}$$  \hspace{1cm} (29)$$

Electrophoretic mobility, analogous to ion mobility in a gas, is given by:

$$\kappa = \frac{\zeta \epsilon}{4 \pi \mu}$$ \hspace{1cm} (30)$$

where:

$\mu = $ viscosity.

With the concept of the Helmholtz double layer in mind, we can now describe the various types of phoretic motion.

**Cataphoresis**

In cataphoresis, many small solid particles suspended in a liquid are involved. At the surface of each particle, an associated double layer will exist. Under the action of an imposed electric field, the outer portion (mobile layer) of the double layer will move toward one electrode and the solid particle with the bound layer will move in the other direction. Cataphoresis occurs with bubbles as well as solid particles. The forces involved are small and the effects on the large particles will be small.

**Electrophoresis**

Electrophoresis is essentially the same phenomenon as cataphoresis. It, however, can be construed to mean all effects of electric fields on phoresis phenomena (Ref. 25).

**Electro-endosmosis**

If a potential difference is deliberately set up along the axis of a narrow tube or across a plug, the liquid is pumped through the tube. If the container is a closed volume, a pressure is built up to a value sufficient to prevent further flow. The pressure generated is the electro-osmotic pressure. The action in this case is due to motion of the charged layers of the Helmholtz double layer.

**Streaming Potential**

Streaming potential or flow electrification is a very common phenomenon that occurs whenever a fluid flows through a tube. The fluid must have low conductivity to demonstrate the effect. As the fluid traverses the pipe, the mobile layers from the surfaces of the entrained particles are carried downstream, resulting in a large charge separation. Extremely large potential differences may result. Streaming potential effects have caused many disastrous petroleum-product fires.

**Dorn Effect**

The Dorn Effect is also known as the settling potential. When a large quantity of suspended particles settles in a nonconducting fluid, a very large potential difference
can be set up between the top and bottom of the container. The potential is caused by the shearing of the double layers as the particles with their layers drift slowly down through the fluid. A charge separation occurs, and because of the low conductivity of the fluid, a high potential difference is established. In time, of course, the charges drift and neutralize each other, but the rate of neutralization depends on the conductivity of the fluid. Very large potential differences have been observed in experiments conducted by the petroleum industry.

**Dielectrophoresis**

The dielectrophoresis phenomenon is not due to the shearing action on the Helmholtz double layer but, rather, to the action of a nonuniform field on a fluid dielectric or particles in a fluid (Refs. 13 and 26). The body force applied to the dielectric by the nonuniform field has been covered in the section on dielectric effects. Dielectrophoresis results from the motion of the dielectric fluid from the action of the field. In addition, the field can cause an induced charge distribution on any particles suspended in the fluid. This induced charge distribution results in a body force on the macroscopic particles such as was experienced by molecules. Polarization may also result from a field-induced displacement of the double layer, and thus provide a strong induced dipole. Dielectrophoresis motions tend to be relatively complex and any observations or conclusions concerning particle action must be approached with considerable caution. For example, the same field that causes a dielectrophoretic drift can also shear the double layer so that the two influences become intermixed. Dielectrophoresis can move materials in relation to each other if there is a difference in their dipole moments, either induced or natural. Action can be expected between the phases of a given fluid because the dipole moment (per unit volume) changes considerably between the liquid and vapor phases, for example.

In summary, phoresis effects generally occur from the shearing action on electrical double layers, which results in a charge separation. This charge separation can either generate a potential difference or cause the particles to drift under the influence of an external field. Dielectrophoresis results from the application of nonuniform fields and may occur without double-layer action.

**ELECTROFLUID INTERACTIONS**

We have reviewed the action of electric fields, dielectric action, and phoretic effects. It is now possible to go into further detail in the discussion of the more important interactions.

**ELECTRIC WIND AND ION DRIFT**

Electric wind is manifested by the flow of air from a highly charged needle point. The electric pinwheel utilizes the wind generated from two charged points at the ends of a pivoted arm to rotate the arm. In the vicinity of the points, a very intense electric field exists. In such a field, the surrounding gas is ionized to a high degree due, primarily, to the collisions of ions and electrons with the neutral molecules. Stray ions or electrons within the gas (due to natural cosmic ray ionization, for example) may gain enough velocity between collisions to ionize the neutral molecules. The new ions are likewise accelerated by the field and tend to move away from the intense field, to a point where
the field strength is lower. Here, elastic collisions occur with neutral molecules, which leads to a drifting motion of both the ions and the neutral molecules. On a macroscopic scale, this drifting is a local mass motion or wind.

When the ions and neutral molecules move away from the point of high charge, a change of momentum takes place and a reaction on the body results. This is the force that turns the electric pinwheel. This same phenomenon can produce a low velocity flow of a mass of gas by using sufficient points or a grid of fine wires, as shown in Figure 10. Such arrangements can induce fluid flow or pumping, or produce useable thrust; they form the basis for electric wind, ion drag pressure, and similar phenomena. Fundamental limitations to this type of device are the power required to achieve mass flow and the potential that can be applied before breakdown (sparking).

The action of a field on flames is distinct from the electric wind phenomena, since the ions exist in the flames and need not be generated by the field. As a consequence, the field effects on flames are the result of the motion of particles within the flame itself. Some portions of the flame contain a greater negative charge, and others contain a greater positive charge. This variation in the net charge leads to severe distortions of the flame as the field is applied (Figure 2). Tests on flames indicate a much higher degree of ionization exists in the reaction zone than is expected under equilibrium conditions (Ref. 17). Applied fields can lead not only to the distortion of the flame, but to mass motion of the air surrounding the flame. This action is a logical corollary to the electric wind. Experiments show that definite changes in heat transfer can also result (Refs. 27 and 28).

When ionization exists in a gas, the distribution of ions may be distorted even though the average charge density is zero (the mass of the fluid is electrically neutral). Since the mobility of the negative and positive ions is substantially different, an applied field
will cause unsymmetrical motion of the ions and would likewise tend to cause slightly
different mass motion of the fluid. Such action is limited by the amount of recombination
taking place and the ambipolar diffusion coefficient. Ambipolar diffusion occurs as a
result of the combined effects of the diffusion of a single specie and the attractive
Coulomb forces of unlike charges. As the positive and negative ions move in the applied
field, a charge separation occurs with a resultant attraction between the oppositely
charged areas in the fluid. In addition, as the ions are separated, natural diffusion acts
to return the charges to a more uniform distribution. The combined influence of diffusion
and charge separation is known as ambipolar diffusion. The net result of these effects is
to reduce the rate of drift and the amount of distortion produced by the field.

The most outstanding illustration of the potentially powerful influence of ion drift is
Clark's experimentation (Ref. 28). This work shows that flow streams can be diverted
through large angles, boundary layers and heat transfer can be influenced, and even sur­
face erosion can be inhibited (Figures 11, 12). These experimental data indicate effects
on flow quite out of proportion to results ordinarily expected by the actions of fields. They
point to the importance of conducting further detailed and systematic study of the electro­
fluid interactions.

A corollary to the ion drift effect is the possibility of using ion motion to generate elec­
trical power. This possibility is based upon the inverse process of electric wind (Refs. 15,
16, and 17). In inverse electric wind, the ions are not accelerated by a given potential dif­
ference, but are slowed down by the field and return their energy to an external circuit.
This purely electrical field effect does not involve external magnetic fields. References
15, 16, and 17 discuss ion-motion power generation thoroughly.

FLUID ELECTRIFICATION AND ELECTROVISCOITY

Since the phenomenon of spray electrification has its foundation in the action of the
Helmholtz double layer, the main features of this action were covered under phoresis.
The charge generated in the exhaust of steam through nozzles and the spray of liquids
from nozzles and from waterfalls is the result of shearing action on the double layer.
Bubbles that rise to the surface of a liquid and burst lead to strong charging phenomena
from the double-layer effect. Several experiments have been conducted in which fluids
have been sprayed out of nozzles into regions of intense electric fields (Refs. 7, 8, and
29). The level of charging was considerably higher than that obtained by double-layer
action alone. When the field was increased, the spray configuration was altered consider­
ably, since droplet size, distribution, and charge are influenced to a large extent by the
field.

The phenomenon of flow electrification was covered in considerable detail under
phoresis. The shearing of the double layer builds up a large charge. A critical factor is
the level of conductivity of the fluid. If the conductivity of the liquid exceeds 50 x 10^-12
mhos per centimeter, the charges dissipate themselves quickly within the fluid. The
hydrocarbon liquids in which charge separation takes place would be extremely poor
conductors of electricity if no impurities were present. If the liquid were very pure, no
charge is generated, but impurity concentrations of one part in a billion is sufficient to
cause electrification. On the other hand, small amounts of certain additives in various
petroleum products increase the conductivity enough to dissipate quickly any charges
developed (Ref. 12). Mixtures of hydrocarbons or additions of water increase charging
effects.
Figure 11. Schlieren photograph showing impingement of reducing oxy-hydrogen flame against metal plate charged with -10,000 volts relative to burner nozzle.

Figure 12. Schlieren photograph of 250°F air flow between electrically insulated metal plate and field shaping electrode with -10,000 volts to plate and 10,000 volts to field shaping electrode.
The importance of the impurities or additives on the electric-fluid interaction is also born out by the series of experiments of Andrade and Dodd, and Dobinski (Refs. 30 and 31). Dobinski observed that viscosity changes when an electric field is applied to a flowing liquid. He deduced that the change in viscosity varies as the square of the field strength. He deduced that the effect depended upon the amount of impurities in the liquid; as the impurities were removed, the field-induced viscosity was progressively reduced to that value for the liquid without a field. Viscosity was influenced only in polar fluids. Other experimenters verified these results. Andrade and Dodd in a series of very careful tests corroborated the earlier findings of Dobinski. Viscosity increased as the square of the field strength, and the effect was most pronounced on polar molecules. With very pure fluids, no inherent increase in viscosity was apparent with the application of a field (Ref. 32). Consequently, the impurities existing in a liquid are of primary importance. These impurities affect the amount of charge build-up in flow electrification, the rate of decay of the charge in a liquid, and the viscosity of the fluid. The degree of impurity required to show these influences is very small. Ordinary commercial grade fluids contain sufficient impurities for the effects to be evident. Water, for example, has a significant influence on various nonconducting fluids, and water is present in most practical fluids.

The influences on viscosity were explained by Andrade and Dodd as being due to the accumulation of charges in the liquid. The action of the charges on a polar liquid led to clustering and, subsequently, increasing the viscosity. Thus, the characteristic distribution of charge within a field may provide a similar variation in the viscosity in the fluid. The data of Andrade and Dodd indicate the increases in viscosity can be as high as two to one. This influence on viscosity is of great significance to the investigator of fluid behavior and heat transfer because viscosity is a primary factor in these phenomena.

DIELECTRIC EFFECTS ON FLOW AND HEAT TRANSFER

Several effects from the body force available on neutral nonconducting fluids through the use of dielectric action have been noted. Gemant reported that the surface of a liquid can be drawn upward towards a charged needle point (Ref. 24). In a series of experiments, Pohl demonstrated dielectric action by spraying liquids upward from the surface (Refs. 13 and 26). The primary purpose of Pohl's work was to use the action to separate various suspended particles from a nonconducting solution. A nonuniform field was established between an outer circular conducting wall and a fine internal wire. Applying a voltage varying from zero to 80 kilovolts across the electrodes separated the suspensoid readily. Polarization is achieved by the action of the field on the suspended particles. The charge on the particles is induced by the applied field to form a dipole. This dipole could result from either double-layer action or the movement of mobile charges in the suspensoid particle. The action requires a highly divergent field and high field strengths. The motion of the particles is independent of the direction of the field strength, as predicted by the theory of dielectric action. The action is strongest for large particles where large polar moments can be generated.

The separation depends on the difference in the dielectric constant between the solute and the solvent. For the solid particles to be moved toward and be separated at the central electrode, the solids must have a higher dielectric constant than the surrounding liquid. When the liquid has a higher dielectric constant than the suspensoid, the liquid is induced to move toward the central wire, which causes a mixing action within the fluid. In much of Pohl's work, both electrophoresis and dielectrophoresis effects occur; the results must be viewed with this factor in mind. Dielectric effects are significant and
provide an effective body force within the fluid, even though their absolute magnitude is small, as shown in Figure 13.

Figure 13. Electric and dielectric effects
Few experiments have been conducted on the influence of an electric field on heat transfer. The work usually has been directed at free convection in liquids, with a little exploratory work on boiling. In almost all cases, a nonuniform field was established by placing a fine central electrode in a cylindrical outer electrode. The wire was heated electrically to establish a field between the wire and the outer cylinder. Significant increases in heat transfer result; increases of over 100 percent of the heat transfer without a field have been recorded (Refs. 33, 34, 35, and 36). Senftleben studied the phenomena in gases extensively and attacked the problem on a thermodynamic basis. His analysis showed that the dielectric stress effect would be greater for a cold gas than for a hot gas, and consequently the cold gas would tend to stream toward the central electrode. This induced streaming would increase the rate of heat transfer. Although sound analyses seem to have been conducted, no electrophoretic and dielectrophoretic actions are considered. If the fluids were of very high purities, these effects could be neglected in the evaluation. Unfortunately, purity does not usually seem high enough to preclude the action of phoresis in convective heat transfer. Whether phoretic effects are present or not, strong influences on free convection appear possible. The final result may be a mixture of influences, but the net action may produce useful results. Further work in this area should logically consider the phoretic influences on heat transfer as well as other streaming phenomena induced by the field.

It is apparent that many important factors influence heat transfer and boundary layer phenomena. Although the details of all the action are not clear, sufficient knowledge exists to initiate definite explorations on heat transfer and boundary layers. Various combinations of ion drift, induced viscosity, phoretic effects, and stress-induced streaming may be used to achieve a specific result.

OTHER ELECTRIC EFFECTS

Static charging of solids is a complex phenomenon. Other aspects of the influence of fields on solids, however, will be mentioned. Mendel and Weinig found, during a series of tests, that the application of high fields to the surface of a magnesium oxide crystal produced dislocations in the crystal (Ref. 37). Two types of dislocation were found, a surface dislocation and a dislocation extending through the interior of the crystal. The dislocations were not explained, but the second type of dislocation implies strong internal field effects. More striking effects were found by Clark (Ref. 28). Surface erosion of a cartridge starter turbine was effectively inhibited by the application of a bucking voltage. Cross sections from turbines after operation showed evidence of subsurface pitting. The origin of the interior pits is not clear, but Clark hypothesized that this pitting may result from some field-solid interaction. Clark, in a further work (Ref. 38), indicated that appreciable electrical effects occurred during bubble motion and cavitation. Tests indicated that electromagnetic fields were established about the bubbles rising in a fluid. When the bubbles burst upon contact with the surface, local electromagnetic fields were induced. In the same work, fluid turbulence was also found to generate local electromagnetic fields.
APPLICATION OF ELECTRIC INTERACTIONS TO THE MECHANICS OF FLUIDS AND HEAT TRANSFER

The following section presents a broad outline of the possible ways in which the electro-fluid interactions can be utilized to influence fundamental fluid behavior.

FLUID PROPERTIES

Fundamental to any investigation of fluid behavior is the nature of the fluid properties. Important properties include viscosity, conductivity, diffusion, and compressibility. If any of these properties change, flow fields, drag, or heat transfer could change. We must therefore consider the fluid properties initially. Experimentation on viscosity of liquids by Andrade and Dodd, indicated that the viscosity of fluids of normal purity changes considerably (Ref. 31). The effects were most pronounced in polar liquids, in these tests which were limited to uniform fields. Tests by Leidenfrost and Schmidt (Ref. 36) revealed an influence on conductivity. It is expected that the influence on diffusion would be strong because of the ambipolar effects.

BOUNDARY LAYERS

Of fundamental importance to the motion of real fluids is the boundary layer existing between a moving fluid and its boundary. The boundary layer depends on viscosity, local flow velocity, surface condition, Reynolds number, and many other parameters. In ordinary applications, boundary layer is a relatively small portion of the total flow field. In the case of flow in pipes and in certain cases of external aerodynamics, however, the boundary layer can be extensive. Whenever the boundary layer is changed, the total flow field surrounding a body can be influenced significantly. As the boundary layer changes under the influence of the adverse pressure gradient in a diffuser, for example, a point is reached where the flow in the boundary layer detaches from the surface. This separation usually results in the entire flow stream separating in the diffuser. Diffuser efficiency and pressure recovery are markedly reduced by this separation. The divergence angles of conical diffusers for subsonic flow are limited to approximately 8 degrees to prevent such separation.

Boundary-layer flow is categorized as laminar and turbulent. At very low Reynolds numbers, laminar flow exists. In pipes of small diameter and low rates of flow, laminar flow exists over the entire length of the pipe. In external aerodynamics, the laminar flow exists on the leading edge of the body; as flow progresses over the body, however, a transition to turbulent flow will occur at a critical Reynolds number. Laminar flow is characterized by the smooth flow of layers of fluid, relatively low drag, and generally low heat transfer. Turbulent flow is characterized by extensive internal mixing of the various layers of the stream and generally higher drag and heat transfer rates.

All phenomena of fluid flow and heat transfer are intimately related to the boundary layer. The importance of the boundary layer has led to the use of suction in laminar flow control on aircraft and to the use of pressurized fluid injection to control the boundary layer. Such techniques have demonstrated very low net drag due to a stabilized laminar layer, or have delayed separation so as to increase stall angles and lift coefficients and reduce drag. Fluid injection into the boundary layer has been used to vary the heat transfer rates between the body of the fluid and the wall. It is a logical corollary, therefore, to
inquire as to what influence the electric field may have on the boundary layer. Since several electrical interactions are possible, this question would require a rather broad systematic program of investigation. Such a program must consider (1) the type of fluids, (2) the type and degree of ionization, and (3) the type fields.

Extreme care must be exercised so that the influence can be separated. Electrophoretic and ionic influences can become intermixed. Oscillating field effects on ions and dielectrophoresis can occur in a combination.

The condition of the fluid-wall interface is of vital importance to the resistance of a moving fluid. From a macroscopic viewpoint, the velocity of a fluid drops to zero at the wall because the fluid "sticks" to the wall. The velocity then increases from the wall out to the free stream flow. Such a macroscopic viewpoint gives no insight as to the nature of the fluid-wall contact. Viewed on a kinetic theory basis, the neutral molecules strike the wall and are momentarily adsorbed at the surface of the crystal lattice. Adsorption results from the electrical forces at the molecular level (Refs. 7 and 39). The kinetic energy of the impinging molecules is given up to the wall in the process. A short interval later, these molecules acquire sufficient energy from the surface to leave the wall. Although the approaching molecules may have had a directed component of velocity (mass velocity of the flow) in addition to random motion, they leave the surface with a random orientation. The net change in momentum normal to the surface leads to the fluid exerting pressure on the wall. The net change in the tangential velocity due to adsorption at the surface leads to a frictional drag at the surface, with subsequent fluid shear. Consequently, the nature of the impact of the incident molecule and the type of reflection will influence the average flow velocity near the boundary significantly. When the density of a gas is low enough, the phenomenon of slip flow occurs. In this case slipping apparently occurs at the wall and the velocity is not completely reduced to zero at the interface.

The importance of the fluid-wall interaction on velocity and boundary layer leads one to the question of the possible influences of the electro-fluid interactions at this interface. If small body forces could be created directly at the interface, the entire boundary layer and flow field could be influenced. If highly localized interactions could be induced, then some degree of slip flow in dense gases might be possible. Such microscopic influences will require special attention as to the condition of the surface, since local irregularities or crystal imperfections could lead to relatively intense fields. If suitable field shaping were possible on a microscopic scale, some influence on the surface flow might be possible.

TRANSITION FROM LAMINAR TO TURBULENT FLOW

The transition from laminar to turbulent flow is a sensitive parameter in aerodynamics or fluid mechanics. Surface roughness, fluid properties, degree of fluid turbulence, and pressure gradient all play an important role in the onset of turbulence. The apparent natural instabilities in the boundary layer lead to roll-up and then to turbulence. If sufficient damping exists, this local turbulence can decay. Because of the sensitivity of the onset of transition, removing small amounts of fluid by suction have been successful in stabilizing the laminar layer. The very sensitivity of this instability may be an excellent area for study of electrical interactions. Field influences on the basic mechanism of transition and the possible changes to the onset of turbulence can be studied. Increased local viscosity or variable viscosity could affect the local damping in the region of initial instability. Body forces derived from any of the many electrical effects could possibly be applied to accelerate portions of the boundary layer. From the results of past porous
suction tests, the magnitude of the body forces should be very small. Only careful experiments can indicate if any modification to transition can actually be achieved.

SEPARATION

Although separation is not as critical a parameter as transition, it should be studied, none the less, to determine whether any significant influences can be achieved through the action of electric fields. The approach used in considering separation must be similar to that for boundary layers. Possible effects of the field on the shape of the local velocity profile at separation and the influence on reattachment should be considered.

DRAG

External drag and internal pressure drop in a pipe result from the fluid shearing stresses at the wall. The surface shear stress is in direct proportion to the slope of the boundary-layer velocity profile at the wall. Any change to the boundary layer by an applied field will have a direct effect on the drag or the pressure drop. Any changes in boundary layer, transition, or separation can significantly affect the shearing stress, drag, and pressure drop.

HEAT TRANSFER

Forced convection depends upon the manner in which heat transfer is accomplished through the boundary layer. All studies of the boundary layer will therefore apply directly to heat transfer. Since a thermal boundary layer is established in conjunction with the velocity boundary layer, heat transfer must be examined specifically. A variation of conductivity or viscosity (Prandtl Number) of the fluid could markedly influence the temperature profile. The internal heat generated by ohmic heating within the fluid may have a significant influence.

Fluid mixing directly affects heat transfer. Mixing interchanges hot and cold slugs of fluid and greatly increases heat transfer. Such techniques as shaped fields, controlled ionization, or seeding could possibly be used to increase local mixing. It may be possible to reduce heat transfer at a wall by using similar techniques to inhibit mixing.

The work already accomplished on free convection has indicated a strong influence of divergent fields. Free convection is marked by the existence of a very small driving force. When the surface of a still fluid is heated, the heating causes the warmer, less dense fluid to rise. This local, induced convective flow adds to the pure conduction through the fluid to provide heat transfer. The driving force is the small buoyancy in the heated layer. An additional body force applied near the surface should significantly influence heat transfer; such effects have actually been observed (Refs. 34, 35, and 36).

Important areas to be studied are electro-fluid heat transfer under condition of zero gravity and multiphase fluids. Under zero gravity, no free convection can occur. Existing body forces are zero or have been cancelled by the orbital trajectory. Electric-fluid interactions could be very effective in both fluid orientation and heat transfer. The suitable interaction might move the fluid against or away from a given surface. We must have a sounder understanding of the basic interactions, however, before we can use them in actual Zero G environments.
Heat transfer in multiphase systems is complex. Boiling and condensing phenomena are usually approached on a semiempirical basis. Two-phase or multiphase flow processes also present considerable difficulty in heat transfer. The addition of electric fields to these phenomena may provide not only techniques for studying the flow and heat-transfer mechanisms, but means for directly influencing the phenomena. The key feature in phase change is the large change in volume. When a liquid changes to a vapor, many changes may occur, including: (1) a large change in the dipole moment per unit volume; (2) a change in ion drift velocity or in the damping of motions; (3) influences on bubbles from Helmholtz double-layer action; and (4) the tendency for droplets, when formed, to be ionized. All these factors provide possible means of directly influencing the mechanisms of boiling or condensing. Changes in bubble size, rate of formation and emission from the surface, as well as effects on the films and the droplets and their removal, should be studied. The most important factor in the two-phase fluid case is that a potentially large change in body force exists between the gaseous and liquid phase.

**SOURCES OF IONIZATION**

In the application of fields to gases, sufficient ions must be provided if ion drift interactions are to be utilized. Several possible sources of these ions exist. The combustion chambers of engines, including turbines, ramjets, rockets, and starters, provide exhaust gases with a relatively high degree of ionization. Frictional heating generates local skin temperatures on aircraft and missiles operating at high Mach numbers that are high enough to ionize a portion of the air. Since the trend for future vehicles seems to be in the direction of increasing combustion and surface temperatures, these sources may provide even greater supplies of ions in future applications. Moreover, it is in these two areas -- the combustion chamber and skin -- that electro-fluid interactions may be most significant and offer the greatest potential utility. Other possible techniques for obtaining ions include the use of ultraviolet radiation, X-rays, soft beta particles, soft alpha particles, and microwaves. Seeding the gas films with materials that are readily ionized and matched to the characteristics of the primary fluid can also be utilized with any of the basic techniques. Based on the work done so far, it is believed that the areas requiring ionization would be highly localized and relatively small.

**POWER AND VOLTAGE CONSIDERATIONS**

The exploration of electro-fluid interactions may require power ranging from a few volts to several kilovolts. It is anticipated that the power required will be very small. Unless an arc is formed in applying the field, the currents involved in the interactions will be very small. The tests conducted to date indicate that a few milliamperes of current may be sufficient to affect the fluid, but larger amounts may be required for conducting fluids. The power required will, of course, be directly related to the work done on the fluid and the efficiency of the process. If a mass of fluid is to be accelerated or a mass velocity is to be maintained, then power requirements may be very high, regardless of the efficiency of the process. It is expected, however, that the most important influences can be achieved by acting in small regions. We may be able to limit the electro-fluid effects to the boundary layer, to the transition zone, or to the region near the surface in convection, for example. The interactions can be used to trigger or stabilize natural flow phenomena. Control of the fluid can be achieved not by applying brute force, but rather by modifying the boundary layer, such as is done in various existing forms of boundary layer and flow control. The application of fields to such localized areas should require very small currents and, consequently, even with high potentials, the power required would be...
small. The nature of electrostatics usually involves very high potentials and relatively low currents.

These electrical requirements are distinct from the normal electrical circuit requirements. Electrostatic generators rather than electromagnetic generators are required. In electrostatics, it is relatively simple to produce potentials of thousands or even hundreds of thousands of volts but with very small currents. Small laboratory generators weighing just a few pounds readily produce potentials of this order, but it is doubtful that they can be used for practical applications. Other sources of potential may be inherent in the flow system. Charge was found to be generated in flow of fluids through pipes and nozzles and during bubbling and settling, for example. Charge exists in aircraft static charging and in the operation of a helicopter rotor. Recorded data indicate potentials up to hundreds of thousands of volts.

From such system-developed voltages two separate possibilities exist:

1. The natural charging phenomena may be used as a source of the desired potential. These high charges may be directed through suitable conductors to local areas of application. Considerable difficulty will undoubtedly be encountered in attempting to utilize these potentials, but we believe the concept is worthy of some consideration.

2. The converse possibility is also of fundamental importance. If large charges are generated by the over-all system (flow through a network of pipes, airflow over an aircraft, etc.) then very high local fields may exist in small areas within the system. Such fields may interact within the fluid system in an uncontrolled manner and may also affect the measurements of basic phenomena. A field-solid interaction may also take place. Intense fields would tend to localize in areas of discontinuity within the structure. It is of interest to note that the macroscopic phenomena of fretting, fatigue, and failure usually occur at such discontinuities. Although it is almost pure conjecture that fields could significantly influence these phenomena, the possibility must be considered in any over-all discussion of the electric field interactions.
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<tr>
<th>Symbol</th>
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<td>$\mathbf{a}$</td>
<td>unit vector</td>
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<td>$n_i$</td>
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<tr>
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<td>velocity</td>
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<td>$z$</td>
<td>valency of atom</td>
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LIST OF SYMBOLS (Continued)

\( c_t \) = coefficient of recombination
\( \gamma \) = specific weight
\( \delta \) = double-layer thickness
\( \varepsilon_0 \) = permittivity of free space
\( \varepsilon \) = permittivity of substance
\( \zeta \) = zeta potential
\( \eta \) = polarizability of a molecule
\( \mu \) = viscosity
\( \rho \) = charge density
\( \sigma \) = surface charge density
\( \omega \) = frequency of alternating field
APPENDIX I

LIST OF REFERENCES


LIST OF REFERENCES (Cont'd)


LIST OF REFERENCES (Cont'd)


APPENDIX I

OTHER BIBLIOGRAPHIC MATERIAL


APPENDIX II
CALCULATION OF HEAT TRANSFER COEFFICIENTS

The calculation of the heat transfer coefficients from the interferograms is a straight forward process (1). It is based upon the fundamental equations of heat transfer and the unique characteristics of the interferometer. Consider the basic heat transfer equations first. In convection heat transfer from a wall, it is assumed that the wall is at rest. It then follows that the heat must pass from the wall through the fluid layer by conduction, and the simple equation of heat conduction can be used,

\[ Q = -k A \frac{\partial T}{\partial y} \bigg|_{y=0} \]

Where \( A \) is the area through which the heat passes. The defining equation for the heat transfer coefficient is

\[ Q = h A (T_w - T_0) \]

Equating these two expressions, one finds

\[ h = -\frac{k}{T_w - T_0} \frac{\partial T}{\partial y} \bigg|_{y=0} \]

The value of \( k \) is determined at a temperature representative of the fluid layers adjacent to the wall. Following Sparrow (2), the reference temperature used in evaluating the fluid properties is

\[ T_e = T_w - 0.38 (T_w - T_0) \]

The values of the properties of air, including density, viscosity, and Prandtl number were taken from references (1, 3). \( \frac{\partial T}{\partial y} \bigg|_{y=0} \) is determined from the interferometer data. Eckert and Soehngen have derived an equation for the temperature difference between ambient
and any fringe in the boundary layer (4). A complete tabulation of values of this equation is given by Evalenko (1). These values are reproduced in Table I. To use the table, it is necessary to know the density, and the number of the fringe being considered. The fringe temperature is then given by

\[ T_{Fi} = T_c \sum_{ij} \left( \frac{\lambda_r e}{L c^2 \Delta a} \right)^i \]

In order to determine the temperature gradient at the wall, it is necessary to measure the spacing of the fringes precisely. The interferometer pictures were read by means of a Gaertner comparator. This instrument could read distances to within 0.0001 inch. Each picture taken shows a small "T" located at the bottom. This "T" provided a reference length of three inches from which a scale factor could readily be obtained for each photograph. Lines were carefully scribed on the photographs at selected stations along the heated plate. Distance readings were made for all fringes at a given station, and then corrected by applying the scale factor for that photograph. Knowing the temperature of each fringe and its spacing, it is then possible to plot the temperature profiles. From the temperature profiles, the slope at the wall was determined for several typical cases. For the same cases the slopes based upon the inner two fringes were determined and compared with the slopes based upon the plotted temperature profiles. Since the two slopes agreed quite well, the determination of the remaining temperature gradients was based upon the slopes determined by the temperatures and spacings of the inner two fringes.
Table I

\[
\sum_{ij} \left( \frac{\lambda_0 F_i}{LC \gamma_a} \right)^j
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\( F \) is the fringe number
\( \rho \) is the density of the air based upon \( T_R \)
Appendix II-References


APPENDIX III
LIST OF EQUIPMENT USED

The nomenclature for the equipment used in the experimental phase is listed in this section.

1. High voltage D. C. power supply.
   Type: D. C. power supply.
   Rating: 0-2 ma
          0-50 KV
   Make: Sterling Instruments Co., Detroit, Michigan
   Model: PS-50

2. Microammeter
   Type: Illuminated Spot microammeter
   Rating: 0-1-3-10-100-300-10000 μa
   Make: Greibach Instruments Corp., New Rochelle, N. Y.
   Model: Number 500.

3. High Voltage Voltmeter
   Type: Electrostatic voltmeter, double pivoted moving vane.
   Rating: 0-10-20-30-40 KV
   Make: Sensitive Research Instruments Corp., New Rochelle, N. Y.
   Model: ESH

4. High Voltage Voltmeter
   Type: Vacuum tube voltmeter plus high voltage probe
   Rating: 0-50 KV
   Make: Radio Corporation of America, Harrison, N. J.
   Model: WV-97A and WG-289 High Voltage Probe
5. Thermocouple Potentiometer
   Type: Portable precision potentiometer
   Make: Rubicon Instruments Div., Minneapolis-Honeywell Regulator Co., Minneapolis, Minn.

6. Inclined manometer
   Type: 40 tube band with reservoir
   Specific gravity of fluid .956

7. Micromanometer
   Type: Chattock two fluid micromanometer (Kerosene and water
   Make: Frank G. Wahlen, Chicago

8. Traversing microscope
   Type: micrometer traverse - Range 0-4” traverse
   Make: Gaertner Scientific Corp., Chicago, Ill.
   Model: Gaertner Comparator
I, Henry René Velkoff, was born in Cleveland, Ohio, May 14, 1921. I received my secondary education in the public schools of Fort Wayne, Indiana, and my undergraduate training at Purdue University, which granted me the Bachelor of Science in Mechanical Engineering in 1942. Following graduation I worked with the Flight Test Division of Lockheed Aircraft Corporation as an analyst. I then took part in the formation of a small research company developing a jet-driven helicopter. I was called into service with the Army and subsequently stationed at Wright Field, Ohio, where I worked as a project engineer in the development of helicopter rotors. Following separation from service in August 1947, I was employed by the Air Force at Wright Field, I then held several positions in a group responsible for the development of helicopter and VTO propulsion systems. During this period of time I undertook graduate studies at the Ohio State University Graduate Center at Wright Field and was awarded the Master of Science degree in March 1952. My work continued to be in the field of rotary wing development, and I subsequently became Chief of the group. In 1957 scope of the work of the group was expanded to include the transmissions and small gas turbines associated with helicopter and VTO systems. I continued academic studies at the Ohio State Graduate Center and in 1959 I was selected by the Air Force to undertake full-time graduate study. In September, 1959, I enrolled in the Graduate School, Ohio State, Columbus, Ohio, where I specialized in Mechanical Engineering. In August, 1960, I returned to the Wright-Patterson Air Force Base where I worked on problems in heat transfer, fluid mechanics, and recovery. During this period I completed the investigation reported herein.