This dissertation has been microfilmed exactly as received

GERHARD, Glen Carl, 1935—
ON THE CURRENT-VOLTAGE CHARACTERISTIC OF PIN AND PSN DIODES AT HIGH INJECTION LEVELS.

The Ohio State University, Ph.D., 1963
Engineering, electrical

University Microfilms, Inc., Ann Arbor, Michigan
ON THE CURRENT-VOLTAGE CHARACTERISTIC
OF PIN AND PSN DIODES AT
HIGH INJECTION LEVELS

DISSERTATION
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Glen Carl Gerhard, B.E.E., M.Sc.

The Ohio State University
1963

Approved by

[Signature]
Adviser
Department of Electrical Engineering
ACKNOWLEDGMENTS

The writer wishes to express his appreciation to Dr. Marlin O. Thurston, who has given him much time and advice during the course of this work. He is also indebted to Mr. John W. Gray for several stimulating discussions and to Mr. C. Henry Pagean for preparation of the samples used.
TABLE OF CONTENTS

ACKNOWLEDGEMENT .......................... ii

LIST OF ILLUSTRATIONS .................... iv

CHAPTER 1 Introduction .................... 1

  2 Long Base Diode Theory .................. 8

    2.1 Linear Theory for Heavily Conductivity-modulated Diodes ....... 8

    2.2 The Formulation of the General Problem ......................... 37

    2.3 The Theory of Lampert and Rose ............................. 54

3 Experimental Diodes ...................... 63

  3.1 Experimental Verification of Simple Theory .......... 63

  3.2 Probe Analysis of Germanium Structures ..................... 69

  3.3 An Alternative PSN Diode Theory ......................... 94

  3.4 Other Effects: Multiple Terminal PIN Devices ...... 107

4 Summary .................................. 116

APPENDIX A ................................ 118

LITERATURE CITED ......................... 123

AUTOBIOGRAPHY ............................ 128

iii
## LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The Variation of Equilibrium Free Carrier Densities with Distance for a PSN Diode with an n-type Base Region.</td>
<td>2</td>
</tr>
<tr>
<td>2.</td>
<td>Carrier Density Distributions in the Base Region of the Diode of Figure 1 for Low Injection Levels.</td>
<td>2</td>
</tr>
<tr>
<td>3.</td>
<td>Model of PSN Diode for the Simplified Analysis.</td>
<td>15</td>
</tr>
<tr>
<td>4.</td>
<td>The Ratio $p_0/n_0$ as a Function of the Ratio $L/L$.</td>
<td>19</td>
</tr>
<tr>
<td>5.</td>
<td>The Ratio $n_0/p_0$ as a Function of the Ratio $L/L$.</td>
<td>19</td>
</tr>
<tr>
<td>6.</td>
<td>$-p_0/p_0'$ $L$ as a Function of the Ratio $L/L$.</td>
<td>21</td>
</tr>
<tr>
<td>7.</td>
<td>The Parameter $C$ as a Function of $L/L$.</td>
<td>23</td>
</tr>
<tr>
<td>8.</td>
<td>The Parameter $D$ as a Function of $L/L$.</td>
<td>24</td>
</tr>
<tr>
<td>9.</td>
<td>The Carrier Density Distribution.</td>
<td>25</td>
</tr>
<tr>
<td>10.</td>
<td>The Potential Distribution.</td>
<td>28</td>
</tr>
<tr>
<td>11.</td>
<td>Base Voltage Drop as a Function of $L/L$.</td>
<td>29</td>
</tr>
<tr>
<td>12.</td>
<td>The Dependence of Mobility on Electric Field for Germanium.</td>
<td>42</td>
</tr>
<tr>
<td>13.</td>
<td>$(\Theta_0/\Theta_F)$ and $(\Theta_F/\Theta_0)^{1/2}$ as Functions of $E$.</td>
<td>52</td>
</tr>
<tr>
<td>14.</td>
<td>The Current-Voltage Characteristic Measuring Circuit.</td>
<td>64</td>
</tr>
<tr>
<td>15.</td>
<td>The Current-Voltage Characteristic for a Silicon PSN Diode.</td>
<td>68</td>
</tr>
<tr>
<td>16.</td>
<td>Determination of a Probe's Floating Potential.</td>
<td>71</td>
</tr>
<tr>
<td>17.</td>
<td>Conceptual Representation of a PSN Structure with Floating Potential Probes.</td>
<td>74</td>
</tr>
<tr>
<td>18.</td>
<td>Current-Voltage Characteristic of Diode A.</td>
<td>76</td>
</tr>
</tbody>
</table>
19. Null Circuit to Check Floating Probe Potentials 78
20. Potential Distributions for Diode A. 79
21. Current-Voltage Characteristics of Diode D. 84
22. Initial Potential Distributions of Diode D. 85
23. Potential Distributions of Diode D After Re-etching. 88
24. The Circuit for Carrier Drift Experiments. 91
25. Conceptual Distributions of Free Carrier Densities, Net Excess Carrier Density, Electric Field Intensity, and Potential in the n-type Base Region of a PSN Diode under High Injection Conditions. 97
26. Yu's Triode Chargistor. 108
27. Charger Output Characteristics Given by Yu. 108
28. Charger Characteristics for Experimental Triode Chargistor. 110
29. Chargistor Equivalent Circuit. 110
30. Gate Current-Voltage Characteristics for an Experimental Chargistor. 113
31. Circuit and Waveform for Relaxation Oscillations. 114
CHAPTER 1

Introduction

Recently, there has been a revival of interest in the general class of PIN junction devices. These devices, in their basic form, consist of a region heavily doped with acceptor impurities and a region heavily doped with donor impurities separated by a region of the original semiconductor material as shown in Figure 1. This last region may be either intrinsic or lightly doped with either acceptors or donors. In the first case, the term PIN applies; while, in the second case, PSN is sometimes used where the S refers to the "soft" or light doping present.

These diodes are different from ordinary PN junctions in that holes are injected from the p⁺ region at one end into the base region, while electrons are injected from the n⁺ region at the other end of the base region giving rise to a double injection current with two junction potential drops. There is, in addition, a voltage drop across the base layer since the holes and electrons must recombine after injection giving rise to carrier gradients and a resulting electric field. The situation can be better understood through the use of Figure 2. If the base region is very long compared with the characteristic diffusion length of either species of carrier, at low injection levels the carrier densities will decay with distance to
Figure 1. The Variation of Equilibrium Free Carrier Densities with Distance for a PSN Diode with an n-type Base Region

Figure 2. Carrier Density Distributions in the Base Region of the Diode of Figure 1 for Low Injection Levels
their thermal equilibrium values, and the resistance of the base region will be largely determined by the normal resistivity and physical dimensions of the base layer as well as the diffusion lengths themselves. Under such conditions the problem reduces to that of a pair of diodes placed back to back, and the necessary theoretical considerations may be found in a text such as Jonscher 1.

If the base region is very short, the carrier densities may remain quite high across its entire length and cause negligible voltage drop. If the base region is reasonably long but the applied bias is large enough such that the carrier densities are still reasonably larger than their thermal equilibrium values, a moderate voltage drop results. It is this latter case that is of concern in this paper.

The credit for the first analysis of the PIN diode goes to R. N. Hall 2 who, in 1952, described the current-voltage characteristic of a power rectifier using this structure. Very little additional theory appeared in the literature until 1955 when A. Herlet and E. Spenke published a series of three papers 3 dealing with this subject in some detail. In addition, Herlet published a separate paper 4 showing the effect of changing the width of the base layer. The following year, there appeared a number of papers by Kleinman 5, Kinman, et al 6, Rashba and Tolpygo 7, and Spenke 8. Kinman, et al, and Spenke treated the case of a junction and recombination surface diode which may be denoted as a PIR device. It is interesting to note that the result of either
treatment is that the diode forward current is proportional to $I = \frac{qV}{2kT}$, which agrees with Hall. Kleinman developed a theory for a PIN diode which yielded a $I = \frac{qV}{kT}$ characteristic which does not agree with experiment; Fletcher believes that this is due to Kleinman's failure to treat adequately the effect of recombination in the base region. In any case, Kleinman's analysis does not apply to long base diodes.

In addition to the papers already mentioned, there have been several valuable contributions by V. I. Stafeev. In one paper, he gives a critique of all the analyses up to that point; this is worth summarizing here. He points out that, at high current densities, one must consider the properties of the N-I junction as well as those of the P-I junction and that Hall did not take sufficient cognizance of this fact. Hall, while neglecting the voltage drop in the semiconductor, obtained a characteristic of the form $J = J_0 (\frac{qV}{2kT} - 1)$ where $J_0$ is not the simple saturation current density, $J_s$, of an ordinary diode but is more a complex quantity with $J_0 > J_s$. Herlet and Spenke also agree that the coefficient 2 appears in the denominator of the exponent for PIN and PSN diodes; Spenke, as previously mentioned, found the same result for PIR structures when the ratio, $L/L^*$, of the base layer thickness to the effective diffusion length was small. He did consider the voltage drop across the base for intrinsic material. However, Herlet and Spenke in all of their work cited here assumed that the hole and electron mobilities were equal, i.e. $\mu_p = \mu_n$. This assumption causes the hole and electron distributions to be symmetrical; hence the results of their
analysis cannot be applied to an actual diode. Rashba and Tolpygo determined the current-voltage characteristic of a PIN device by neglecting diffusion and considering the transport of carriers by the electric field only. However, as it will be shown in the next chapter, at high levels of injections the electric field appearing in the base region in the vicinity of the junctions causes the diffusion coefficient to increase, resulting in a negligible change in the distributions of excess (non-equilibrium) carriers in the base region and, hence, a negligible change in the voltage drop across the base. As Stafeev points out, the field present under such conditions does not draw many excess carriers into the sample and, therefore, Rashba and Tolpygo's current-voltage cannot be expected to apply to real diodes. Experimental evidence for this is presented by Stafeev 13.

Fletcher considered the case of extremely high injection levels for which the voltage drop across the p-n junction approaches that of the base region. However, such a condition presupposes infinite carrier densities at both sides of the junction, an assumption which is somewhat objectionable. This difficulty, coupled with the further assumption that \( \mu_p = \mu_n \), does not allow this theory to be useful in predicting real diode characteristics. Stafeev 12 makes an analysis of the current-voltage characteristics of PIR structures using typical values of conductivity and carrier mobilities. This analysis assumes that the injection is high enough so that the carrier densities are much greater than their thermal
equilibrium values. In fact, his analysis at this point is quite similar to the simplified theory to be presented in Chapter 2. However, the formulation and use of boundary conditions is not the same; and, hence, neither is the form of the final results.

In 1961, Lampert and Rose published an analysis of volume-controlled double-injection currents in trap-free semiconductors. A few of the assumptions in this paper were misleading and were corrected in subsequent papers by Lampert. The results of these analyses will be discussed in detail in a later section. Experimental studies along the lines of these papers have been made by Larrabee for the trap-free case and by Holynak and Holynak et al. for the case of large trapping center densities.

This dissertation contains two main sections. The first of these deals with theoretical analysis of PSN structures and begins with a simplified linear analysis. Following this, the problem is reformulated in more general terms resulting in a completely nonlinear differential equation which cannot be readily solved. The relation derived here is more general than that programmed on a computer by Lieb, Jackson, and Root which treated the case of injection levels intermediate to those satisfying the high and low injection theories. This section concludes with a review of and commentary on the newer theory of Lampert and Rose.

The second of the main sections deals with experimental studies. Actual PSN diode characteristics are compared with those
predicted by the simple theory. Experiments on very long PSN germanium structures are described, and the results of a "probe" analysis of the potential distributions of several diodes are presented. These are compared with data predicted by the Lampert and Rose theory.

In conjunction with this, some lifetime experiments with "hot" carriers are described; some modifications of existing theory are then proposed. Finally, a review of different observed effects is presented along with a description of a PSN triode structure.

It will be assumed that the reader is reasonably familiar with general semiconductor concepts and with ordinary p-n junction theory. The latter area is taken to include the fundamental formulations for carrier transport, continuity of current, charge neutrality, and physical conditions at a junction.

Rationalized mks units will be used throughout this paper, but the dimension of length in all constant and parameters will be expressed in terms of centimeters as is common practice in semiconductor literature. Any deviation from a commonly used symbol for a given parameter will be so noted by the use of a defining equation in the text.
CHAPTER 2

Long Base Diode Theory

2.1 Linear Theory for Heavily Conductivity-modulated Diodes

In order to better understand the principles of operation of a PIN or PSN diode, a simplified linear theory will be presented. This theory follows conventional lines in that it uses a linearized simple differential equation with well known exponential solutions which are then matched to derived boundary conditions. While some of the simplifying assumptions may not satisfy actual conditions in a diode over a large range of current density, they do make the problem mathematically tractable.

For a steady state situation, it may be assumed that the variation of the free hole and electron densities at any point in a diode model are zero with respect to time. Hence, the continuity equations may be set down as

\[ \frac{\partial p}{\partial t} = - \frac{1}{e} \overline{A} \cdot J_p - R_p = 0 \quad (2.1) \]

and

\[ \frac{\partial n}{\partial t} = \frac{1}{e} \overline{A} \cdot J_n - R_n = 0 \quad (2.2) \]

where \( R_p \) and \( R_n \) denote the recombination rates for holes and electrons respectively. For a one-dimensional geometry, equations (2.1) and (2.2) may be written as
The carrier transport equations are

\[ J_p = \sigma_p \bar{E} - eD_p \Delta p \]  \hspace{1cm} (2.5)

and

\[ J_n = \sigma_n \bar{E} + eD_n \Delta n \]  \hspace{1cm} (2.6)

Upon dividing equation (2.6) by the factor

\[ b = \mu_n/\mu_p, \]  \hspace{1cm} (2.7)

one obtains

\[ \frac{J_n}{b} = e \mu_p (n-p) E + eD_p \frac{dn}{dx} \]  \hspace{1cm} (2.8)

If (2.5) is subtracted from (2.8), there results

\[ \frac{J_n}{b} - J_p = e \mu_p (n-p) E + eD_p \left( \frac{dn}{dx} + \frac{dp}{dx} \right) \]  \hspace{1cm} (2.9)

This may be differentiated with respect to \( x \) to obtain

\[ \frac{1}{b} \frac{d}{dx} \left( \frac{J_n}{b} \right) = e \mu_p \frac{d}{dx} \left[ (n-p) E \right] + eD_p \left[ \frac{d^2n}{dx^2} + \frac{d^2p}{dx^2} \right] \]  \hspace{1cm} (2.10)

Rearranging (2.3) and (2.4) yields

\[ \frac{d}{dx} J_p = -e R(p) \]  \hspace{1cm} (2.11)

and

\[ \frac{d}{dx} J_n = e R(n) \]  \hspace{1cm} (2.12)
which, upon substitution into (2.10) yield

\[
\frac{e}{b} R(n) + e R(p) = e \mu_p \frac{d}{dx} \left[ (n-p) E \right] + eD_p \left[ \frac{d^2 n}{dx^2} + \frac{d^2 p}{dx^2} \right] \cdot (2.13)
\]

Dividing (2.13) by e and collecting terms, the following general result is obtained:

\[
0 = D_p \left[ \frac{d^2 n}{dx^2} + \frac{d^2 p}{dx^2} \right] + \mu_p \frac{d}{dx} \left[ (n-p) E \right] - \left[ \frac{R(n)}{b} + R(p) \right] \cdot (2.14)
\]

At this point it is necessary to spell out the various assumptions that are necessary to make the problem mathematically tractable without the use of a digital computer. The first assumption has already been used to obtain (2.14).

i) The geometry is one-dimensional by virtue of the use of a model diode with constant cross-sectional area, homogeneous material, and planar junctions, one at each end.

ii) The lifetime is constant and is the same for both holes and electrons. This will be discussed in detail later, but it is a good approximation for the high injection case. It will be assumed that

\[
R(n) \equiv \Delta n / \tau \quad (2.15)
\]

and that

\[
R(p) \equiv \Delta p / \tau \quad (2.16)
\]

iii) The injected carrier densities are quite large at each junction and remain large everywhere
in the base region with respect to the thermal equilibrium hole and electron densities, p_T and n_T. This may be expressed as n = p in view of the next assumption.

iv) Quasi-neutrality holds everywhere; that is \( \Delta p \approx \Delta n \).

v) The mobilities are constant with respect to carrier density and electric field intensity; this is not always a valid assumption, and it will be shown that a decrease in mobilities for high electric fields can explain apparent discrepancies in experimental results as compared to theoretical calculations. Without this assumption, the mathematics becomes quite formidable.

vi) The mobility ratio \( b = \mu_n/\mu_p \) is constant. This is at least as good as an assumption as (v) above and generally is a great deal better.

vii) The base region of the diode is homogeneous with respect to doping; and, hence, n_T and p_T are constant and independent of position for any given temperature.

viii) The injected excess carriers are all free and are not bound in traps or recombination centers; this is a reasonably good approximation for well-purified semiconductors.
ix) The junctions are abrupt and have emitter efficiencies of unity. The latter specification may become invalid at sufficiently high injection densities as will be mentioned later.

If (2.15) and (2.16) are substituted in (2.14), so that

\[ 0 = D_p \left( \frac{a^2 \Delta n}{dx^2} + \frac{\Delta n}{dx} \Delta p \right) + \mu_p \frac{d}{dx} \left[ (n-p) E \right] - \left( \frac{\Delta n}{b} + \frac{\Delta p}{c} \right) \frac{l}{c} , \tag{2.17} \]

it may be seen that, under the assumptions (i) through (viii), this expression reduces to

\[ 0 = 2D_p \frac{d^2 \Delta p}{dx^2} - \left( \frac{l}{b} + 1 \right) \frac{\Delta p}{c} \cdot \tag{2.18} \]

This may be multiplied by b and rearranged to give

\[ \frac{d^2 \Delta p}{dx^2} = \frac{(b + 1)}{2bD_p} \frac{\Delta p}{c} \cdot \tag{2.19} \]

If the ambipolar diffusion length \( L \) is defined as

\[ L^* \equiv \frac{2b}{\sqrt{b + 1}} \frac{L_p}{c} \tag{2.20} \]

where \( L_p \equiv \sqrt{D_p} \) is the usual diffusion length for holes, (2.19) becomes

\[ \frac{d^2 \Delta p}{dx^2} = \frac{1}{L^*^2} \Delta p \tag{2.21} \]

which is the well known diffusion equation. The field terms in (2.17) are neglected only because \((p-n) = 0\) under assumptions (iii) and (iv), not because the field itself is assumed to be zero. Since the total carrier densities, \( p \) and \( n \), may be separated into their
thermal equilibrium and excess densities as

\[ p = \Delta p + p_T \]  
(2.22)

and

\[ n = \Delta n + n_T, \]  
(2.23)

equation (2.17) may be rewritten as

\[ 0 = n_p \frac{d^2}{dx^2} (\Delta n + \Delta p) + \mu_p \frac{d}{dx} \left[ (\Delta n - \Delta p) E \right. \]
\[ \left. + (n_T - p_T)E \right] = \frac{b + 1}{b} \Delta p. \]  
(2.24)

Since \( D_p = \frac{kT}{e} \mu_p \) by Einstein's relation and since \((n_T - p_T)\)
is a constant for a given material, (2.24) may be rearranged as

\[ \frac{d}{dx} \left[ (\Delta n - \Delta p) E \right] = (p_T - n_T) \frac{dE}{dx} + \frac{kT}{e} \frac{d^2}{dx^2} (\Delta n + \Delta p) = \frac{(b+1)\Delta p}{b\mu p_T}. \]  
(2.25)

The transition from this equation to (2.21) may be explained in the following manner. Assumption (iv), that \( \Delta n = \Delta p \), allows the first term to be dropped. The validity of dropping the second term, however, may not be quite so apparent. Since the base region of the diode under consideration is basically of high resistivity, the quantity \((p_T - n_T)\) will rarely be greater than \(10^{14}\) \(\text{cm}^{-3}\) for either germanium or silicon. The divergence of the electric field will be that given by Poisson's equation,

\[ \frac{dE}{dx} = \frac{\varepsilon}{\varepsilon} (\Delta p - \Delta n); \]  
(2.26)

and, since \( \Delta p = \Delta n \), it may be reasonably expected that the second term will be negligible compared to the third term. This is
particularly so since the quantity $\Delta n + \Delta p$ is assumed to be several orders of magnitude greater than either $n_T$ or $p_T$. Thus, if the length of the base region is not so long as to invalidate assumption (iii), thereby allowing all the injected carriers to recombine in the base region, the third term will dominate; and, equation (2.21) results, giving the so-called diffusion solution.

The diffusion solution is based upon the previously derived equation (2.21) $d^2\Delta p/dx^2 = (L^*)^2\Delta p$ based, in turn, upon the assumptions already listed. Since quasi-neutrality was assumed, there is a corresponding equation for the excess electron density,

$$d^2\Delta n/dx^2 = (L^*)^2 \Delta n.$$  (2.27)

The one-dimensional model assumed here is shown in Figure 3. The center of the base region is at $x = 0$; the $p^+$ junction is located at $x = -L/2$ and the $n^+$ junction is located at $x = +L/2$.

The solution to both (2.21) and (2.27) has the form

$$\Delta n \equiv \Delta p = A \frac{x}{L^*} + B \frac{L^*}{x}$$  (2.28)

where $A$ and $B$ are arbitrary constants. The boundary conditions here are the values of $\Delta p = \Delta n$ and $d \Delta p/dx = d \Delta n/dx$ at the edges of the base region, $x = -L/2$ and $x = +L/2$. They may be listed, using (2.28) as

$$F_0 \equiv \Delta p \bigg|_{x = -L/2} = A \in \frac{L/2L^*}{L/2L^*} + B \in \frac{-L/2L^*}{-L/2L^*},$$  (2.29)

$$n_0 \equiv \Delta n \bigg|_{x = +L/2} = A \in \frac{-L/2L^*}{L/2L^*} + B \in \frac{L/2L^*}{L/2L^*},$$  (2.30)
Figure 3. Model of PSN Diode for the Simplified Analysis
\[ p_0' \equiv \frac{d\Delta p}{dx} \bigg|_{x = -L/2} = -\frac{A}{L^2} \epsilon + \frac{B}{L}, \quad (2.31) \]

and

\[ n_0' \equiv \frac{d\Delta n}{dx} \bigg|_{x = +L/2} = -\frac{A}{L^2} \epsilon + \frac{B}{L}, \quad (2.32) \]

Before these can be used to evaluate the constants \( A \) and \( B \) in terms of the total current density, \( J \), the conditions at the junctions must be discussed in more detail.

Starting with the \( p^+ - i \) junction at \( x = -L/2 \), if the assumption of unity emitter efficiency is valid, all of the injected current must be that due to holes and, using equation (2.6),

\[ J_n \bigg|_{x = -L/2} = 0 = (\sigma_n E + e D_n \frac{d\Delta n}{dx}) \bigg|_{x = -L/2} \quad (2.33) \]

Solving (2.33) for the electric field intensity at the junction,

\[ E \bigg|_{x = -L/2} = -\frac{e D_n}{\sigma_n} \frac{d\Delta n}{dx} \bigg|_{x = -L/2} \quad (2.34) \]

which, using the Einstein relation and the definition of \( \sigma_n \), becomes

\[ E \bigg|_{x = -L/2} = -\frac{kT}{\epsilon p_0} p_0' \quad (2.35) \]

This may be substituted into (2.5) giving

\[ J = J_p \bigg|_{x = -L/2} = -\frac{\mu p_0}{p_0} \frac{kT}{p_0} p_0' = -eD_n p_0' \quad (2.36) \]

or

\[ J = - (\mu p_0 kT + e D_n) p_0' \quad (2.37) \]
Again using the Einstein relation, (2.37) becomes

\[ J = -2 \mu_p kT p_0' \]  

(2.38)

Hence, the diffusion constant and mobility are effectively doubled at the junction as compared with the solution when the electric field intensity is assumed to be zero. (This latter case has been considered by many authors including vander Ziel 22).

Equation (2.38) may be used to solve for \( p_0' \) in terms of the total current density as

\[ p_0' = -\frac{J}{2\mu_p kT} \]  

(2.39)

This same procedure may be used to determine the relation between \( n_0' \) and \( J \) at the other junction. Starting with equation (2.5), the entire current density must be that due to injected electrons and

\[ J_p \bigg|_{x = +\frac{L}{2}} = 0 = (\sigma_p E - e D_p \frac{d\Delta p}{dx}) \bigg|_{x = +\frac{L}{2}} \]  

(2.40)

Again, solving for \( E \), there results

\[ E \bigg|_{x = \frac{L}{2}} = \frac{eD_p}{\sigma_p} \frac{d \Delta p}{dx} \]  

(2.41)

and, upon using the Einstein relation and the definition of \( \sigma_p \) again,

\[ E \bigg|_{x = \frac{L}{2}} = \frac{kT}{e n_0} n_0' \]  

(2.42)

is obtained. This may be substituted into (2.6) to obtain
\[ J = J_n \left| x = \frac{L}{2} \right. = \frac{\mu_n n_0}{n_0} \frac{kT}{n_0} n_0' + kT \mu_n n_0' \] (2.43)

which simplifies to

\[ J = 2 \mu_n kT n_0' \] (2.44)

with the same doubling of mobility as at the other junction. This may be solved for \( n_0' \) giving

\[ n_0' = \frac{J}{2 \mu_n kT} \] (2.45)

It may then be seen that, using (2.39) and (2.45)

\[ \frac{p_0'}{n_0} = \frac{-J}{2 \mu_p kT} \cdot \frac{2 \mu_n kT}{J} = -\frac{\mu_n}{\mu_p} \] (2.46)

which, with the use of the ratio of mobilities, \( b \), becomes

\[ p_0' = -b n_0' \] (2.47)

Equations (2.39), (2.45), and (2.47) may now be used to evaluate the constants \( A \) and \( B \) of (2.28) under the conditions set forth in (2.29) through (2.32). The algebra involved here is straightforward but tedious, and the reader is referred to appendix A for the details. The results are summarized below.

The ratio of injected carriers is

\[ \frac{p_0}{n_0} = \frac{b(2L/L^*) + 2}{(2L/L^* + 1) + 2b} \frac{L/L^*}{L^*} \] (2.48)

This ratio and its inverse are plotted as functions of the base width to ambipolar diffusion length ratio \( L/L^* \) in Figures 4 and 5.
Figure 4. The Ratio \( \frac{p_0}{n_0} \) as a Function of the Ratio \( \frac{L}{L^*} \)

Figure 5. The Ratio \( \frac{n_0}{p_0} \) as a Function of the Ratio \( \frac{L}{L^*} \)
respectively for $b = 2.1$ and $b = 3.0$. These values of $b$, for germanium and silicon respectively, were chosen on the basis of room temperature and low field values of mobilities; and, their use is at least as good as the assumptions regarding the mobilities themselves. It may be noted that, as $L/L^*$ approaches zero, $p_0/n_0$ in equation (2.48) approaches unity; while, as $L/L^*$ becomes large, $p_0/n_0$ approaches the mobility ratio, $b$, itself.

The constants $A$ and $B$ are

$$A = \frac{p_0 \left( \frac{L}{2L^*} - \frac{n_0}{p_0} \right)}{2 \sinh (L/L^*)} \quad (2.49)$$

and

$$B = \frac{p_0 \left( \frac{n_0}{p_0} \right)}{2 \sinh (L/L^*)} \quad (2.50)$$

The relation between $p_0$ and $p_0'$ is found to be

$$p_0 = -p_0' L^* \tanh (L/L^*). \quad (2.51)$$

This last relation is independent of the actual value of $p_0$ since $n_0/p_0$ is independent of $p_0$ as long as none of the original assumptions is violated. A plot of the ratio $-p_0/p_0'L^*$ as a function of $L/L^*$ is shown in Figure 6.

If (2.39) is substituted for $p_0'$ in (2.51), and (2.51) in turn is substituted for $p_0$ in (2.49) and (2.50), the latter equations become

$$A = \frac{JL^* \left( \frac{L}{2L^*} - \frac{n_0}{p_0} \right)}{4 \mu p kT \sinh (L/L^*) \tanh (L/L^*)} \quad (2.52)$$
Figure 6. \[-p_0/p_o' L^*\] as a Function of the Ratio $L/L^*$
These constants may be normalized by defining

$$C \equiv \frac{A \mu_p kT}{J L^*}$$  \hspace{1cm} (2.54)

and

$$D \equiv \frac{B \mu_p kT}{J L^*}$$  \hspace{1cm} (2.55)

These are shown in Figures 7 and 8 as functions of $L/L^*$. The evaluation of these quantities allows the carrier distribution of equation (2.28) to be calculated. Normalized distributions are plotted as functions of $x$ in the base region for different values of $L/L^*$ in Figure 9.

Once the carrier distribution is known, the electric field intensity, the potential drop, and the excess or non-neutralized space charge may be calculated as functions of $x$. If the transport equations (2.5) and (2.6) are added, the resulting equations may be solved for the field intensity to give

$$E = \frac{J - e D_p (b-1) \Delta p / dx}{\epsilon \mu_p (b+1) \Delta p}$$  \hspace{1cm} (2.56)

Using (2.28) and the definitions of the parameters $C$ and $D$, this becomes

$$E = \frac{J \mu_p kT - e D_p J (b-1) \frac{D}{L^*} \frac{C}{e} \frac{X/L^*}{x/L^*}}{\epsilon \mu_p (b+1) J L^* \left[ C \frac{x/L^*}{x/L^*} + D \frac{x/L^*}{x/L^*} \right]}$$  \hspace{1cm} (2.57)

which, using the Einstein relation, simplifies to
Figure 7. The Parameter C as a Function of $\frac{L}{L^*}$
Figure 8. The Parameter D as a Function of L/L∗.
Figure 9. The Carrier Density Distribution
This is independent of the current density, $J$, since the conductivity modulation of the base region keeps pace with the current density to maintain a constant voltage drop until the high injection assumptions break down.

Equation (2.58) may be integrated to obtain the potential drop and differentiated to obtain the net space charge. The integral for the former is

$$
V(x) = -\frac{kT}{eL^*(b+1)} \int_{-L/2}^{x} \left[ \frac{1+(b-1)(C \in -x/L^* D \in -x/L^*)}{C \in -x/L^* + D \in -x/L^*} \right] \right] \cdot (2.58)
$$

where $V(-L/2) = 0$. This may be integrated using equations (110) and (111) from Peirce 23 to yield

$$
V(x) = \frac{-kT}{eL^*(b+1)} \left[ \frac{L^*}{\sqrt{CD}} \tan^{-1} \left( \frac{x}{L^*} \right) \cdot \sqrt{\frac{D}{C}} \right] \tan^{-1} \left( \frac{2x/L^*}{L_D} \right) \cdot \ln \left( \frac{D+C \in -2x/L^*}{-L^*} \right) \right]_{-L/2}^{x} (2.60)
$$

This equation may be evaluated by taking the values of $C$ and $D$ from Figures 7 and 8 respectively. Inserting the limits gives
\[ v(x) = \frac{-kT}{e(b+1)} \left\{ \left[ \tan^{-1} \left( \frac{x/L^* - D}{C} \right) \right] - \tan^{-1} \left( \frac{-L/2L^* - D}{C} \right) \right\} \]

\[ -\frac{1}{2} \left[ \ln \left( \frac{C + D \in \frac{-L/L^*}{C + D \in 2x/L^*}}{C + D \in \frac{L/L^*}{D + C \in -2x/L^*}} \right) + \ln \left( \frac{D + C \in \frac{L/L^*}{D + C \in -2x/L^*}}{D + C \in \frac{L/L^*}{D + C \in -2x/L^*}} \right) \right] \]
Figure 10. The Potential Distribution
Figure 11. Base Voltage Drop as a Function of $L/L^*$
which shows the amount of unneutralized excess charge. Since holes are injected at \( x = -L/2 \), \((\Delta p - \Delta n)\) is positive there; and, since electrons are injected at \( x = +L/2 \), \((\Delta p - \Delta n)\) must be negative there. Obviously, there will be one value of \( x \), denoted as \( x_o \), for which \((\Delta p - \Delta n) = 0\) exactly. This may be found by setting (2.63) equal to zero. Since the denominators and constants are positive, they may be eliminated leaving

\[
0 = C \in \frac{-x_o/L^*}{D , x_o/L^*} - 2(b-1)(c^2 \in \frac{-2x_o/L^* + d \in 2x_o/L^*}{2x_o/L^*}) \tag{2.65}
\]

This is a transcendental equation which must be solved for a specific ratio of base length to diffusion length. However, for reasonably large values of \( L/L^* \), only the first two terms of (2.65) are significant. For such a case, (2.65) becomes

\[
C \in \frac{-x_o/L^*}{D , x_o/L^*} = 0 . \tag{2.66}
\]

Multiplying by \( \in x_o/L \),

\[
C - D \in 2x_o/L^* = 0 \tag{2.67}
\]

and

\[
\in 2x_o/L^* = c/D \tag{2.68}
\]
Taking the natural logarithm of both sides,

\[ \frac{2x_0}{L^*} = \ln \left( \frac{C}{D} \right) \]  

(2.69)

and

\[ x_0 = \frac{L^*}{2} \ln \left( \frac{C}{D} \right) \]  

(2.70)

The discussion of the diffusion solution is complete except for the current-voltage characteristic. In addition to the potential drop across the base region, \( V_B \), given by equation (2.62) and Figure 11 there will be two additional potential drops across the \( p^+ - i \) and \( n^+ - i \) junctions, denoted respectively as \( V_p \) and \( V_n \). Jonscher 21 and others have discussed this problem, and a short summary will suffice here. If the free hole and electron densities are denoted as \( p(a) \) and \( n(a) \), respectively, on one side of an abrupt junction, and similarly as \( p(b) \) and \( n(b) \) on the other side, then assuming that the carriers on both sides of the junction are in thermal equilibrium with one another, Boltzmann's equilibrium conditions give

\[ \frac{p(b)}{p(a)} = \frac{n(a)}{n(b)} = e^{-(V_D - V)/kT} \]  

(2.71)

\( V_D \) is the usual internal diffusion potential across the junction when side \( a \) contains a greater net acceptor density than side \( b \), and \( V \) is the externally applied junction bias. The first part of (2.71) gives the familiar product rule

\[ p(b) n(b) = p(a) n(a) \]  

(2.72)

which, for the case of intrinsic material in either region becomes

\[ p(b)n(b) = p(a)n(a) = n_i^2 e^{\frac{eV'}{kT}} \]  

(2.73)
since the electron and hole concentrations in purely intrinsic material, \( n_1 \) and \( p_1 \), are equal. When reasonably high injection levels are attained, the separation of the quasi Fermi levels, \( V' \), may differ from the applied junction bias.

The diffusion potential \( V_D \), across a p-n junction is given in

\[
\exp \left( -\frac{eV_D}{kT} \right) = \frac{p_n}{n_n} = \frac{p_P}{n_n} \quad (2.74)
\]

At the p+ -n junction this becomes

\[
\exp \left( -\frac{eV_D}{kT} \right) = \frac{p_T}{p_P} \quad (2.75)
\]

while (2.71) becomes

\[
\frac{p_0}{p_P'} = e^{\left( V_D - V \right)/kT} \quad (2.76)
\]

where \( p_P' \) is the non-equilibrium free hole density on the p+ side of the junction and \( p_0 \) is the density of holes injected into the base region as previously defined. If (2.76) is multiplied by the inverse of (2.75), there results

\[
\frac{p_0}{p_T} \frac{p_P}{p_P'} = e^{V/kT} \quad (2.77)
\]

Since the original assumptions require that the injection density \( p_0 \) not approach a magnitude requiring the majority carrier density in the p+ region to deviate from its thermal equilibrium value, it is proper to equate

\[
p_P = p_P' \quad (2.78)
\]
which allows \( \frac{eV}{kT} \)
\[ P_0 = P_T \left( \frac{eV}{kT} \right) \]
to be written. \( \text{(2.79)} \)

Likewise, it can be shown that the corresponding relation at the
\( n^+ - i \) junction is given by
\[ \frac{eV}{kT} \]
\[ n_o = n_T \left( \frac{eV}{kT} \right) \]
\( \text{(2.80)} \)

where \( n_o \) and \( n_T \) are as previously defined. Denoting the \( p^+ - i \) junction potential drop as \( V_p \) and the corresponding \( n^+ - i \) junction potential drop as \( V_n \), \( \text{(2.77)} \) and \( \text{(2.78)} \) may be solved to obtain
\[ V_p = \frac{kT}{e} \ln \left( \frac{P_0}{P_T} \right) \]
\( \text{(2.81)} \)

and
\[ V_n = \frac{kT}{e} \ln \left( \frac{n_o}{n_T} \right) \]
\( \text{(2.82)} \)

The quantities \( P_0 \) and \( n_o \) may be replaced by their equivalent expressions, \( \text{(2.29)} \) and \( \text{(2.30)} \) using \( \text{(2.54)} \) and \( \text{(2.55)} \) to evaluate the constants \( A \) and \( B \). In this manner one obtains
\[ P_0 = \frac{JL^*}{\mu^* \mu_T kT} \left( C \in \frac{L/2L^*}{-L/2L^*} + D \in \frac{L/2L^*}{-L/2L^*} \right) \]
\( \text{(2.83)} \)

and
\[ n_o = \frac{JL^*}{\mu^* \mu_T kT} \left( C \in \frac{-L/2L^*}{L/2L^*} + D \in \frac{L/2L^*}{-L/2L^*} \right) \]
\( \text{(2.84)} \)

Then \( \text{(2.81)} \) and \( \text{(2.82)} \) become
\[ V_p = \frac{kT}{e} \ln \left[ \frac{JL^*}{\mu^* \mu_T kT P_T} \right] \left( C \in \frac{L/2L^*}{-L/2L^*} + D \in \frac{-L/2L^*}{L/2L^*} \right) \]
\( \text{(2.85)} \)

and
\[ V_n = \frac{kT}{e} \ln \left[ \frac{JL^*}{\mu^* \mu_T kT n_T} \right] \left( C \in \frac{-L/2L^*}{L/2L^*} + D \in \frac{L/2L^*}{L/2L^*} \right) \]
\( \text{(2.86)} \)
Assuming negligible contact losses, the total applied voltage drop across the diode, \( V_T \), therefore, is the sum of equations (2.62), (2.85), and (2.86) or

\[
V_T = |V_B| + V_p + V_n \quad \text{(2.87)}
\]

Substituting (2.85) and (2.86) into (2.87) gives

\[
V_T = |V_B| + \frac{kT}{e} \ln \left[ \frac{J^* L^*}{\mu_p k T p T} \right] (C \in \frac{L/2L^*}{D \in -L/2L^*})
+ \frac{kT}{e} \ln \left[ \frac{J^* L^*}{\mu_p k T p T} \right] (C \in \frac{-L/2L^*}{D \in L/2L^*}) \quad \text{(2.88)}
\]

which upon defining \( J_1 = 1 \text{ amp/cm} \) as a dimension factor, may be rearranged to

\[
V_T = \left| V_B \right| + \frac{2kT}{e} \ln \left( J/J_1 \right) + \frac{2kT}{e} \ln \left( \frac{J_1 L^*}{\mu_p k T} \right) + \frac{kT}{e} \ln
\]

\[
\left[ \frac{1}{P_{\text{TrT}}} \left( C \in \frac{L/2L^*}{D \in -L/2L^*} \right) \left( C \in \frac{-L/2L^*}{D \in L/2L^*} \right) \right] \quad \text{(2.89)}
\]

which, upon taking the product in the last term and using (2.73), simplifies, in turn, to

\[
V_T = \left| V_B \right| + \frac{2kT}{e} + \frac{2kT}{e} \ln \left( \frac{J}{J_1} \right) + \frac{2kT}{e} \ln \left( \frac{J_1 L^*}{\mu_p k T} \right)
+ \frac{kT}{e} \ln \left[ \ln \left( C^2 + D^2 + 2CD \cosh \left( \frac{L}{L^*} \right) \right) \right] \quad \text{(2.91)}
\]

This factor contains all the geometrical and physical constants involved but neither the current density nor the applied total
voltage; these quantities have been purposely separated from the rest of the terms in (2.90). The magnitude of $V_B$ is independent of current density as long as the lifetime is not modulated by changing current through the diode. While the lifetime does change with current density in most diodes, for high injection conditions it will become constant as previously assumed. Using (2.91) equation (2.90) becomes

$$V_T = \frac{2kT}{e} \ln \left( \frac{J}{J_i} \right) + G. \quad (2.92)$$

This may be solved for $J$ to obtain

$$J = J_i e^{(V_T - G)/2kT}. \quad (2.93)$$

The quantity $(V_T - G)$ may be considered as "effective potential" appearing in the high injection equation. The factor $G$ may be readily evaluated for a given diode since curves for $C$, $D$, and $V_B$ have already been presented as figures 7, 8 and 11, respectively. It has, however, been determined by trial and error that the results are more accurate if equation (2.88) is used in actually calculating current-voltage characteristics. Equation (2.93) is of academic interest because it shows the factor of two in the denominator of the exponent. In view of this, the discussion of the current-voltage characteristics will be terminated at this point. However, a further word concerning emitter efficiency is in order. Since it has been assumed that the junctions are both strongly asymmetrically doped, the emitter efficiency is nearly unity for reasonably low values of injection; that is, the current is carried almost entirely
by carriers flowing into the high resistivity base region. If the forward bias is increased to the point where the density of carriers in the base region becomes comparable in magnitude with the carrier density in the heavily doped region, the emitter efficiency decreases. When this occurs, the original assumption (ix) of unity emitter efficiency is no longer valid; and, the analysis no longer holds.

Hence, the doping of the end regions should be at least an order or two of magnitude above the carrier densities in the base region at the junctions. Likewise, the assumption (iii) of substantial conductivity modulation breaks down whenever the carrier densities fall below a level one or two orders of magnitude greater than the majority carrier density at thermal equilibrium. A glance at Figure 9 shows that, for a diode with the ratio $L/L^* = 6$, the ratio of the minimum carrier density to the maximum is in the vicinity of $6 \times 10^{-2}$. The thermal equilibrium carrier density in the base should not, therefore, be much greater than two orders of magnitude below the minimum density at the $p^+$ junction, $p_0$. If, according to the preceding discussion, the emitter efficiency is to remain near unity, the doping of the $p^+$ region must be two orders of magnitude greater than $p_0$. Thus, this value of doping will be $1.67 \times 10^{15}$ greater than that in the intrinsic base region—a condition that, while not completely unrealizable in actuality, nevertheless poses a limit on the maximum value of $L/L^*$ for which this analysis applies. It is apparent that this limit is in the neighborhood of $L/L^* = 6$. 
2.2 The Formulation of the General Problem

The analysis of the preceding section was based on a large number of simplifying assumptions; these were necessary in order that the problem be mathematically tractable such that a straight-forward algebraic solution was possible. However, a more rigorous and general analysis may be set up, with some of the more restrictive assumptions omitted, to serve as a starting point for an investigation using a digital computer. Such a program would be ambitious, to say the least, since the programming would require much effort and time; but, actual solutions of a sufficiently general set of equations would be of great interest and value.

The analysis to be presented in this section will retain several of the less restrictive assumptions of the preceding section, namely (i), the specification of a one-dimensional geometry; (vii), the specification of a homogeneous base region; and, (ix), the assumption of unity emitter efficiencies at each junction. This last assumption is made in order to confine the study to effects in the base region rather than effects directly attributed to junctions conditions. In addition to these assumptions, it will also be assumed that the density of recombination centers, \( N_R \), will not be significantly greater than the carrier densities at thermal equilibrium, i.e. that the excess carriers are essentially free and are not bound in traps or recombination centers. The recombination centers may be lattice defects or ionized impurity sites, and these effects will be discussed in detail later.
The general problem may be set up in the same manner as the simplified one by reformulating the carrier recombination and taking notice of the electric field intensity dependence of such parameters as the carrier mobilities and lifetimes. The starting place here will be equation (2.11):

\[ 0 = \frac{d^2\Delta n}{dx^2} + \frac{d^2\Delta p}{dx^2} + \mu_p \frac{d}{dx} \left[ (\Delta n - \Delta p + n_T - p_T) E \right] \]

\[ = \left[ \frac{R(n)}{b} + R(p) \right]. \]

Ordinarily the Einstein linear relation between the diffusion constant and drift mobility would be used next. However, this relation is valid only when there is a linear relation between the electric field and the drift mobility. The mobilities do vary as functions of free carrier densities; 25 but, the diffusion constant varies in the same manner, and so the Einstein relation is valid.

For a material with given free carrier densities at constant temperature, the Einstein relation will not hold for large values of electric field intensity because the drift velocities become non-linear junctions of electric field indicating changing mobilities. Hence, the value of the diffusion constant, \( D_p \), will be dictated by the free carrier densities of the material under consideration.

It would be advisable to examine briefly the physical mechanisms involved under the application of an electric field. A detailed discussion of this subject is found in Gunn 26.
With zero applied electric field, the carriers have frequent collisions with the lattice while exhibiting random motion and, hence, zero net velocity in order to maintain the thermal equilibrium distributions of carrier energies and momenta. Statistically speaking, there exists a mean free path between collisions and a mean thermal velocity $\bar{v}_T$ given by 27

$$\bar{v}_T = \left( \frac{3kT_0}{m^*} \right)^{1/2}$$

(2.94)

where $m^*$ denotes the relevant effective mass, and $T_0$ is the temperature common to the lattice and carriers.

If a moderate electric field is applied, the carriers are accelerated during their free flight between collisions and, hence, gain kinetic energy. However, this energy is transferred to the lattice during successive collisions; when the rate of energy transferred to the lattice equals the rate of gain of carrier energy for the applied field, there occurs a steady state in which the original equilibrium energy and momenta distributions become disturbed. These new non-equilibrium distributions yield a net non-zero value of average drift velocity corresponding to the fact that the average energy per carrier is now slightly greater than the thermal energy, $3kT_0/2$, and is given by $K. E. = \frac{3kT_c}{2}$, where $T_c$ is defined as the equivalent carrier temperature. Since obviously $T_c > T_0$, the carriers are said to be "hotter" than the lattice.

As long as the drift velocity, $V_d$, is small in comparison with $\bar{v}_T$, the difference between the equilibrium and non-equilibrium
values of kinetic energy is quite small; and, the carrier temperature is only slightly greater than the lattice temperature. Under these conditions the drift velocity is proportional to the applied electric field as

$$V_d = \mu_0 E,$$  \hspace{1cm} (2.95)

and the constant of proportionality is the usual constant low field mobility $\mu_0$. As the electric field is increased, the carriers become "hotter", and the corresponding energy difference becomes greater. This causes modification of the collision processes, and equation (2.95) is no longer valid. The now non-linear relationship may be expressed as

$$V_d = \mu(E) \cdot E$$  \hspace{1cm} (2.96)

where the mobility $\mu(E)$ is no longer constant but is now field dependent. At first the departures from linearity may be described by

$$\mu = \mu_0 = k_1 E^2$$  \hspace{1cm} (2.97)

where $k_1$ is a constant. While this square-law dependence has been observed, its magnitude is relatively small. A great deal more serious departure from (2.95) takes place at fields of the order $10^3$ V/cm. in germanium at room temperature and at slightly higher fields in silicon. Theoretical studies predict a dependence of the form

$$\mu \propto E^{-1/2}.$$  \hspace{1cm} (2.98)

At higher fields, at least in n-type germanium, the drift velocity approaches a constant which turns out to be the thermal velocity $10^7$ cm/sec. Approximately $10^4$ volts/cm is necessary to achieve the onset of this velocity saturation. Gunn 29 has published curves
of drift velocity as a function of electric field intensity for holes and electrons in both germanium and silicon.

More recently, Conwell 30 has published curves of mobility as a function of electric field in germanium for both holes and electrons showing theoretically calculated curves and the experimental results of several other investigators. A first order algebraic approximation for the experimentally-derived curves as shown in Figure 12 may be obtained by curve-fitting. There are substantial regions where the curves are essentially linear; and, hence they may be approximated with less than 10% maximum error by the straight line segments as constructed on the curves. Since the abscissa is a logarithmic scale, it will be expedient to define

\[ y \equiv \log(\varepsilon). \quad (2.99) \]

Then, for n-type germanium where electrons are the majority carrier, the mobility ratio \( \mu/\mu_{n0} \) is given by

\[ \frac{\mu}{\mu_{n0}} = 1, \quad E < 5.2 \times 10^2 \text{ volt/cm.} \quad (2.100) \]
\[ \frac{\mu}{\mu_{n0}} = 1 - 0.62(y - 2.717), E \geq 5.2 \times 10^2 \text{ volt/cm.} \]

At about \( 10^4 \) V/cm of electric field this approximation becomes invalid; however, this should be adequate for most applications.

For holes in the p-type germanium the mobility ratio is given by

\[ \frac{\mu}{\mu_{p0}} = 1, \quad E < 6.78 \times 10^2 \text{ volt/cm.} \quad (2.101) \]
\[ \frac{\mu}{\mu_{p0}} = 1 - 0.58(y - 2.832), E \geq 6.78 \times 10^2 \text{ volt/cm.} \]
Figure 12. The Dependence of Mobility on Electric Field for Germanium
Alternatively, the corrected portions of (2.100) and (2.101) may be written in terms of $E$ respectively as

$$\frac{\mu_n}{\mu_{no}} = 2.68 - .62 \log (E) \quad (2.102)$$

and

$$\frac{\mu_p}{\mu_{po}} = 2.64 - .58 \log (E). \quad (2.103)$$

Similar relations for silicon can be determined from Gunn's curves.

Substituting (2.102) and (2.103) into (2.104), there results, for electric field intensities greater than $6.8 \times 10^3$ V/cm.,

$$0 = D_p \left[ \frac{d^2 \Delta n}{dx^2} + \frac{d^2 \Delta p}{dx^2} \right] + \mu_{po} (2.64 - .58 \log (E))$$

$$- \frac{1}{b} \left[ E \left( \frac{d \Delta n}{dx} - \frac{d \Delta p}{dx} \right) + (\Delta n - \Delta p) \frac{dE}{dx} + (n_T - p_T) \frac{dE}{dx} \right] - \frac{R(n)}{b} - R(p). \quad (2.104)$$

which, since $(n_T - p_T)$ is constant, may be rearranged to

$$0 = D_p \left[ \frac{d^2 \Delta n}{dx^2} + \frac{d^2 \Delta p}{dx^2} \right] + \mu_{po} (2.64 - .58 \log (E))$$

$$- \frac{1}{b} \left[ E \left( \frac{d \Delta n}{dx} - \frac{d \Delta p}{dx} \right) + (\Delta n - \Delta p) \frac{dE}{dx} + (n_T - p_T) \frac{dE}{dx} \right] - \frac{R(n)}{b} - R(p). \quad (2.105)$$

It is necessary to examine the mobility ratio, $b$, for this case.

Using (2.102) and (2.103) it may be seen that

$$b \equiv \frac{\mu_n}{\mu_p} = \frac{\mu_{no} (2.68 - .62 \log (E))}{\mu_{po} (2.64 - .58 \log (E))} = \frac{\mu_{no}}{\mu_{po}} \quad (2.106)$$

does not vary greatly with $E$ and, hence, can be considered constant.

There yet remains the most important modification of the simplified analysis—that of choosing the proper recombination
functions, \( R(n) \) and \( R(p) \), for the large ranges of carrier densities and electric field intensities involved. For semiconductors like germanium and silicon, recombination at low electric fields is due mainly to indirect transitions via recombination centers in the forbidden gap; such centers may be due to an ionized impurity or a lattice defect. If the amount of charge bound in traps \( \Delta n_t \) satisfies

\[
\Delta n_t \ll n_T, p_T
\]  

(2.107)

(which is a reasonable assumption for near-intrinsic highly purified semiconductors) then trapping effects may be ignored.

The statistics of the recombination process through a single set of imperfection levels have been considered by Hall 31 and by Shockley and Read 32. The probabilities of capture of holes and electrons by the centers are denoted by \( C_p \) and \( C_n \) respectively and are given by

\[
C_p = N_r \bar{V} \sigma_p
\]  

(2.108)

and

\[
C_n = N_r \bar{V} \sigma_n
\]  

(2.109)

where \( N_r \) is the density of centers, \( \bar{V} \) is the average velocity of the carriers, \( \sigma_p \) is the capture cross section for holes, and \( \sigma_n \) is the capture cross section for electrons.

It is also necessary to define

\[
n' = n \frac{e^{-E_T/kT}}{N_c}
\]  

(2.110)

and

\[
p' = p \frac{e^{-(E_G - E_T)/kT}}{N_v}
\]  

(2.111)
where $N_c$ is the density of states in the conduction band, $N_v$ is the density of states in the valence band, $E_g$ is the band-gap energy, and $E_J$ is the electron ionization energy of the recombination centers. It should be noted here that the energies are positive quantities, measured as the energy difference between the bottom of the conduction band and the level in question. Thus $n'$ is the density of free electrons when $E_J = E_F$, the Fermi level. Likewise, $p'$ is the density of free holes for the same condition.

The results of the Shockley-Read calculations will be given directly; the reader is referred to the literature previously cited for details. It is, however, assumed that

$$E_r < E_1 \frac{n}{2} + \frac{n^2}{2} \frac{kT}{2} \ln \left( \frac{N_v}{N_c} \right)$$

(2.112)

where $E_1$ is the energy corresponding to the Fermi level in an intrinsic sample. This requires that $n' > n_1 > p'$; for the reverse case where $E_r > E_1$, it is necessary only to interchange the roles of holes and electrons.

The results may be divided into three separate cases. For Case I the following assumptions are made:

1. The excess carrier density is small compared with the thermal equilibrium carrier densities and, hence, $\Delta n = \Delta p$.

2. The electron and hole lifetimes are equal. The lifetime is defined as

$$\tau = \frac{\Delta n}{U}$$

(2.113)
where \( U \) is defined as the rate of steady state hole-electron pair generation and hence the net rate of both electron and hole capture. \( U \) is given by

\[
U = \frac{C_n C_p (p_n - p'n')}{C_n (n + n') + C_p (p + p')} \quad (2.114)
\]

Substituting (2.114) into (2.113) and making use of the first assumption above gives

\[
\tau = \tau_o, \text{ the low level lifetime}
\]

\[
\tau_o = \tau_{po} \frac{n_T}{n_T + p_T} + \tau_{no} \frac{p_T}{n_T + p_T} \quad (2.115)
\]

where \( \tau_{po} \equiv C_p^{-1} \) is the lifetime for holes in highly n-type material and \( \tau_{no} \equiv C_n^{-1} \) is the lifetime for electrons in highly p-type material.

For Case II the following assumptions are necessary:

1. The excess carrier density is not small compared with the thermal equilibrium carrier densities.

2. The electron and hole lifetimes are equal.

For this case, (2.113) becomes

\[
\tau = \tau_o \frac{1 + a \Delta n}{1 + c \Delta n} \quad (2.116)
\]

where

\[
a = \frac{\tau_{po} + \tau_{no}}{\tau_{po} (n_T + n') + \tau_{no} (p_T + p')} \quad (2.117)
\]

and

\[
c = (n_T + p_T)^{-1} \quad (2.118)
\]

If \( a > c \), \( \tau \) increases with increasing excess electron density, \( \Delta n \); if \( a < c \), \( \tau \) decreases with \( \Delta n \). As very large values of \( \Delta n \) are
obtained, the high injection constant lifetime given by

\[ \tau_{\infty} = \tau_{p0} + \tau_{n0} \]  \hspace{1cm} (2.119)

is reached.

For Case III the situation is more complicated but of great interest. Here the following assumptions are made:

1. The excess carrier density is small enough compared to thermal equilibrium carrier densities so that only first-order terms in \( \Delta n \) and \( \Delta p \) need be considered.

2. The electron and hole lifetimes are different, and the density \( N_r \) of recombination centers is not negligible compared with \( n_T \) and \( p_T \).

Now it is necessary to define two steady state lifetimes:

\[ \tau_p = \frac{\Delta p}{U} \]  \hspace{1cm} (2.120)

and

\[ \tau_n = \frac{\Delta n}{U} \]  \hspace{1cm} (2.121)

The requirement now for electrical neutrality is that

\[ \Delta p - \Delta n = N_r \Delta f_R \]  \hspace{1cm} (2.122)

where \( \Delta f_R \) represents the fraction of filled centers in excess of those filled at thermal equilibrium. The lifetimes then become

\[ \tau_p = \frac{\tau_{n0} (p_T + p^{'}) + \tau_{p0} n_T + n^{'}}{n_T + p_T + N_r (1 + n_T/n^{'})^{-1}} \]  \hspace{1cm} (2.123)
and
\[ \tau_n = \frac{\tau_{po} (n_T + n') + \tau_{no} p_T + p' + N_r (1 + p_T/p')^{-1}}{n_T + p_T + N_r (1 + p_T/p')^{-1} (1 + p'/p_T)^{-1}}. \quad (2.124) \]

Since \( n_T/n' = p_T/p' \), because of the law of mass action, the denominators of (2.123) and (2.124) are equal. The occupation of the recombination centers may be expressed as a function of the Fermi level as
\[ n_r = N_r \left( 1 + \exp \left( \frac{E_F - E_r}{kT} \right) \right)^{-1} \quad (2.125) \]
where \( n_r \) is the density of occupied centers and \( n_{r0} \) is this density for thermal equilibrium conditions. Then the following relations may be determined:
\[ N_r (1 + n_T/n')^{-1} = n_r = n_{r0} \quad (2.126) \]
and
\[ N_r (1 + p_T/p')^{-1} = n_{r0} \quad (2.127) \]
Using these, and dividing (2.126) by (2.124), one arrives at the expression for the ratio of the electron lifetime to the hole lifetime,
\[ \frac{\tau_n}{\tau_p} = \frac{\sigma_n (n_T + n') + \sigma_p (p_T + p' + n_{r0})}{\sigma_p (p_T + p') + \sigma_n (n_T + n' + N_r - n_{r0})}. \quad (2.128) \]
It is interesting to note that, if \( N_r = 0 \), equations (2.123) and (2.124) reduce to (2.115) where the hole and electron lifetimes were assumed to be equal. If \( N_r = \), the lifetimes become
\[ \tau_p^{-1} = \tau_{po}^{-1} \left( n_r/N_r \right) \quad (2.129) \]
These may be interpreted by observing that $\tau_p^{-1}$ is $\tau_{po}^{-1}$ times the probability that a center is already occupied by an electron so that it is ready to capture a hole and that $\tau_n^{-1}$ is $\tau_{no}^{-1}$ times the probability that a center is empty so that it may capture an electron. Different values of $\tau_n$ and $\tau_p$ imply, of course, different capture cross sections for holes and electrons; and, indeed, this seems to be the case. Lampert has predicted negative resistance regions in the current voltage characteristics of double injection diodes doped with a suitable impurity for which $\frac{\sigma_p}{\sigma_n} > 1$; experimental verification of this has been published by Holynak et al. The mechanism of Case III does not play an important role for the low recombination center density assumed in the analysis in this paper. However, there is one further recombination mechanism study that is of interest here. At the higher values of electric field intensities when the carriers become sufficiently heated, the high injection level lifetime of Case II is no longer adequate to describe the situation. Recalling from (2.108), (2.109), and the definitions of $\tau_{po}$ and $\tau_{no}$ the facts that

$$\tau_{po} = (N_r \bar{V} \sigma_p)^{-1} \quad (2.131)$$

and

$$\tau_{no} = (N_r \bar{V} \sigma_n)^{-1}, \quad (2.132)$$

it may be seen that as the carriers become "hotter", $\bar{V}$ increases, and the lifetime $\tau_\infty = \tau_{po} + \tau_{no}$ decreases correspondingly. However, if either of the capture cross sections, $\sigma_p$ or $\sigma_n$, decreases as $\bar{V}$
increases, the lifetime may decrease less rapidly, remain constant, or even increase. This problem has been considered by a number of workers, most recently by Ridley and Watkins \(^{34}\) and by Zucker and Conwell \(^{35, 36}\). The latter pair have investigated the kinetics of recombination in high fields using the approach applied by Shockley and Read for low fields. They obtained a relation between the change of carrier concentration in a microwave field and the speed-dependence of the cross sections and applied it to the case of electron capture of negatively charged copper centers in germanium. Since copper tends to be the most electrically active impurity in germanium, a short discussion of their results is warranted.

If \(C_n^o\) is the probability of capture of an electron by a center for low values of electric field and \(C_n^E\) is the same probability for large values of field, then the ratio of the average of \(C_n^E\) to \(C_n^o\) may be fundamentally related to the ratio of high field to low field carrier distribution temperatures, \(\Phi_E / \Phi_o\). In particular if the assumption is made that the cross section is proportional to \((V)^j\), one obtains

\[
\frac{<C_n^E>}{C_n^o} = \frac{1}{T} \int_0^T (\frac{\Phi_E}{\Phi_o}) \left(\frac{j+1}{2}\right) dt
\]

\[= <(\Phi_E/\Phi_o) \left(\frac{j+1}{2}\right)> T. \quad (2.133)
\]

For hot electrons in germanium it has been found that the carrier velocity distribution is well approximated over a wide range of fields by a Maxwell-Boltzmann distribution \(^{37}\). \(\Phi_E\) is, therefore,
the effective temperature of this distribution for any value of electric field \( E \).

For \( j = 0 \), the capture cross section is independent of \( \bar{V} \), and the capture cross section decreases with increasing \( E \). This may be seen by observing the ratios \( \Theta_0/\Theta_E \) and \((\Theta_E/\Theta_0)^{1/2}\) for electrons in germanium as a function of d.c. field in Figure 13. If the cross section varied as \( \bar{V}^{-1} \) corresponding to \( j = -1 \), \( C_n^E / C_n^0 = 1 \), and there is no change in recombination rate with electric field. However, it has been found experimentally that the cross section for electron capture of copper centers with a single negative charge decreases with increasing speed of carriers. Using 2.85 MHz microwaves up to peak fields of \( 10^4 \) \( \text{v/cm} \), Zucker and Conwell \(^{35}\) found that, for a sample of germanium with \( 10^{11} \) \( \text{cm}^{-3} \) of copper centers and \( p_e = 2 \times 10^{15} \) \( \text{cm}^{-3} \), the capture cross-section for electrons decreases almost as rapidly as \( v^{-1} \). The amount of decrease of the cross-section was greater for fields of peak value less than \( 3 \times 10^3 \) \( \text{v/cm} \) for most samples, however, so that no complete conclusions can be drawn. Yet, it is apparent that the net effect is that the lifetime involved does tend to decrease as the electric field is increased; this point will be brought up again in the next chapter.

Returning to the previous analysis, it can be seen that Case I of the Shockley-Read mechanism can be used in equation (2.114) for the low-injection case where the Einstein relation holds and the hole mobility \( \mu_p \) is constant. Substituting (2.115) into (2.113) and observing that \( U = R(n) = R(p) \), (2.114) may be rewritten as
Figure 13. \((\theta_o/\theta_E)\) and \((\theta_E/\theta_o)^{1/2}\) as Functions of \(E\)
As the carrier concentrations are increased, the recombination term approaches Case II and the limiting value of constant high injection lifetimes of equation (2.119) is reached yielding

$$0 = D_p \left[ \frac{d^2 \Delta n}{dx^2} + \frac{d^2 \Delta p}{dx^2} \right] + \mu_p \frac{d}{dx} \left[ (\Delta n - \Delta p + \eta_T - \eta_T) E \right]$$

$$- \left[ \frac{\Delta n}{b} + \Delta p \right] \cdot \frac{1}{\tau_{\infty}} \cdot (2.134)$$

If $\tau_{\infty}$ does not vary rapidly with electric field, then for fields greater than $5.2$ to $6.8 \times 10^2$ v/cm, the recombination term of (2.135) may be substituted into (2.103):

$$0 = D_p \left[ \frac{d^2 \Delta n}{dx^2} + \frac{d^2 \Delta p}{dx^2} \right] + \mu_p \frac{d}{dx} \left[ (\Delta n - \Delta p + \eta_T - \eta_T) E \right]$$

$$- \left[ \frac{\Delta n}{b} + \Delta p \right] \cdot \frac{1}{\tau_{\infty}} \left[ \frac{\Delta n}{b} + \Delta p \right] \cdot (2.136)$$

If $\tau_{\infty}$ does vary with electric field intensity because of the velocity dependence of a capture cross section, the magnitude of dependence must be known before any further formulation can be useful.
The equations (2.134), (2.135), and (2.136) are all quite non-linear and cannot be solved with a direct algebraic approach. In the interest of completeness, it should be remembered that along with these equations, Poisson's equation (2.26), and the boundary conditions (2.39), (2.45), and (2.47) must be satisfied.

2.3 The Theory of Lampert and Rose

The Lampert and Rose theory of the injected plasma in semiconductors was, at the time of publication, the latest of a series of papers dealing with both single and double injection into insulators. Mott and Gurney showed in 1940 that, at an injecting metal-insulator contact, electrons may "boil" off the metal into the conduction band of the insulator. This situation is analogous to that in a vacuum diode having a thermionic cathode. In 1955, Rose showed that the space-charge-limited current flow in an insulator with an injecting contact is strongly affected by energy levels in the forbidden gap because of imperfections. Lampert then published a simplified theory of the single injection space-charge-limited current in an insulator with traps. The double injection problem was investigated in some detail by Parmenter and Ruppel; Lampert then published a simplified version of this theory. It was at this point that Lampert and Rose applied insulator theory to the case of a semiconductor double injection diode.

It was implied in this analysis that the double injection current in a semiconductor was also space-charge-limited; however, it will be apparent to the reader that recombination plays the dominant role in the analysis to follow and that the currents are
actually recombination-controlled. Lampert emphasized this in a succeeding paper 15 on double-injection currents in order to clear up any misunderstanding; a complete summary of his double injection analysis for both insulators and semiconductors may be found in an even more recent article 16.

Jonscher 43 has pointed out that, at a very high forward bias, a p-n junction may exhibit voltage-current characteristics of the form \( J = K (V - V_D)^2 \) where \( K \) is a constant and \( V_D \) is the diffusion potential. Likewise, diffusion solutions for a PIN structure under very high bias can yield a square-law characteristic; Lampert and Rose point out that their analysis, which also yields a square-law dependence of current density upon applied voltage, is in no way related to these other works.

The point at which Lampert and Rose's theory (hereafter designated as the L-R theory) differs from that already presented is in the interpretation of equation (2.25). They indicate that, although the diffusion term will dominate near the contacts, in a sample which is many diffusion lengths long the magnitude of this term drops off exponentially going away from a contact. The characteristic length for this exponential drop is a diffusion length. It may be remembered from section 2.1 that, for values of the ratio \( L/L^* \) much greater than 6, some of the assumptions of the diffusion theory were no longer valid because the base was no longer sufficiently conductivity-modulated to allow the thermal equilibrium carrier densities, \( p_T \) and \( n_T \), to be ignored. It is for such large values of \( L/L^* \) that the L-R analysis should be considered. In such
a long sample the field terms should be dominant over most of the base region; and, because of this, the L-R analysis neglects the diffusion term entirely.

If the carrier densities in most of the base region are simply the thermal equilibrium values, then the current-voltage characteristic should be determined for the most part by Ohm's law

\[ J = e (\mu_n n_T + \mu_p p_T) \frac{V}{L} \tag{2.137} \]

Here it has been assumed that the electric field intensity is simply given by the ratio \( E = \frac{V}{L} \) where \( V \) is the voltage drop across a base region of length \( L \). If the injection level is increased to the point where \( n = p n_T, p_T \) and \( \bar{n} \) is defined as the average value of \( n = p \) over the length of the base region, the current density given by

\[ J = e (b + 1) \mu_p \frac{V}{L} \tag{2.138} \]

where it has been assumed that \( |n-p| < \bar{n} \) as in the previous analysis.

As was previously stated, Lampert and Rose have made assumptions regarding equation (2.25)

\[
\frac{d}{dx} \left[ (\Delta n - \Delta p) E \right] = (p_T - n_T) \frac{dE}{dx} \\
+ \frac{kT}{e} \frac{d^2}{dx^2} \left( \Delta n + \Delta p \right) = \frac{(b + 1) \Delta p}{b \mu_p T}
\]

which vary from those made in section 2.1, the variation depending on the particular portion of the current-voltage characteristic being discussed. It should be noted that the equivalent form of
(2.25) in the L-R theory differs slightly from that given above in that the terms involving \((\Delta n - \Delta p)\) and \((p_T - n_T)\) are reversed. This is due to an unusual formulation of the continuity equations whereby the signs in front of the carrier recombination terms are just the opposite of those seen in most semiconductor literature. In order to be consistent, the L-R theory considers Poisson's equation to be

\[
\frac{dE}{dx} = \frac{e}{\varepsilon} (\Delta n - \Delta p) \tag{2.139}
\]

which is also of opposite polarity as compared to (2.26).

The reader may remember that the first and second terms of (2.25) were dropped in favor of the third term, leaving the simple diffusion equation. The L-R theory, on the other hand, points out that until a high enough injection level is reached such that \(\Delta n - \Delta p = n_T - p_T\), the second term will be dominant over the first. Since the L-R theory assumes negligible diffusion effects in the base region, the third term is also dropped leaving

\[
-(p_T - n_T) \frac{dE}{dx} = \frac{(b + 1) \Delta p}{b \mu_p C} \tag{2.140}
\]

Lampert and Rose now assume that the divergence of the electric field may, by simple dimensional analysis, be replaced by

\[
\frac{dE}{dx} = \frac{V}{L^2} \tag{2.141}
\]

and that the excess carrier density \(\Delta n\) may be replaced by the average value of electron density, \(\bar{n}\), to obtain
Using the fact that \( b \mu_p \equiv \mu_n \), this may be rearranged to give

\[
(b + 1) \frac{n}{b} = \frac{(b + 1) n}{b \mu_p}.
\]  (2.1h3)

Substitution of (2.1h3) into (2.138) gives the result

\[
j = e \mu_n \mu_p \frac{(n_T - p_T)}{L^3}, \quad n_T > p_T.
\]  (2.1h4)

This square-law regime of the current-voltage characteristic has been designated the "semiconductor regime" by Lampert because of the appearance of the thermal equilibrium free carrier densities in the relation above. The same result was obtained from a more rigorous derivation with the exception of a multiplying factor of 9/8. It is immediately apparent to the reader that the appearance of the quantity \( n_T - p_T \) in a current-voltage characteristic equation in this manner is somewhat unusual. However, Lampert and Rose defend this dependence of current on the free carrier density difference by pointing out that recombination kinetics require this to be so.

The voltage at which the current-voltage characteristic makes a transition from Ohm's law, equation (2.137), to the square law, equation (2.1h4), is determined from the intersection of the curves corresponding to these equations. The transition voltage is then given by

\[
-j \propto L^3, \quad n_T > p_T.
\]
If the base region is sufficiently "soft" so that \( p_T/n_T \ll 1 \) and \( \mu_p p_T/\mu_n n_T \ll 1 \), (2.145) may be rewritten as

\[
\tau = \tau_{tr, n-s} = \frac{L^2}{\mu_p V_{tr, n-s}} \frac{1 + \frac{\mu_p}{\mu_n} \frac{n_T}{n_T}}{1 - \frac{p_T}{p_T}} \quad (2.146)
\]

where \( \tau_{tr, n-s} \) is the hole transit time across the base region which is equal to the lifetime. Returning to equation (2.25), Lampert and Rose note that, at sufficiently high voltages, the first term dominates the second term since now \( |\Delta p - \Delta n| > |p_T - n_T| \) over most of the base, and the electric field intensity itself has increased. The quantity \( (\Delta n - \Delta p) \) in (2.25) may be replaced by its equivalent expression from Poisson's equation

\[
(\Delta n - \Delta p) = -\frac{\epsilon}{e} \frac{dE}{dx} \quad (2.147)
\]

whereby the governing equation becomes

\[
-\frac{\epsilon}{e} \frac{d}{dx} \left[ \frac{dE}{dx} E \right] = \frac{(b + 1) n}{\mu_n} \tau \quad (2.148)
\]

The L-R analysis uses a simple dimensional analysis here whereby the quantity

\[
-\frac{d}{dx} \left[ E \frac{dE}{dx} \right] = \frac{V^2}{L^4} \quad (2.149)
\]

Again \( \Delta n \) is replaced by its average value \( \bar{n} \), to give
\[(b + 1) \frac{v}{n} = \frac{\varepsilon \tau \mu_n}{e} \frac{V^2}{L^4} \quad (2.150)\]

which, upon substitution into (2.138) provides a new current-voltage characteristic

\[J = \varepsilon \tau \mu_n i_p \frac{V^3}{L^4} \quad (2.151)\]

This needs only to be multiplied by the numerical factor of 6.9% to agree exactly with the result of a rigorous analysis [41].

The transition voltage for this region is obtained by solving for the intersection of the curves corresponding to (2.151) and (2.141). This gives

\[V_{tr,s-I} = \frac{eL^2 (n_T - p_T)}{6.9% \varepsilon} \quad (2.152)\]

Again, if \(p_T \approx n_T\), the electron transit time is given by (2.153)

\[t_{n,s-I} = \frac{L^2}{\mu_n V_{tr,s-I}} \]

which differs from the dielectric relaxation time \(t_n \equiv \varepsilon/\varepsilon_n \mu_n\), by the factor, 6.9%.

There are a number of points in the L-R theory that require closer scrutiny. Foremost among these is the questionable assumption that the injecting contacts themselves play no important role in determining the current-voltage characteristic. As implied in the L-R analysis, the electric field intensity in the base region becomes reasonably large when high forward bias voltages are applied; a large electric field will tend to sweep injected carriers away from the
junctions into the base region where they recombine. If diffusion is ignored and the contacts are to impose no restraint on the flow of carriers, the presence of infinite carrier densities is implied. Lampert and Rose point this out but maintain that this does not appreciably alter the results since the diffusion that does take place affects only a small portion of the base region. Nevertheless in the semiconductor regime where, the author agrees, the current is quite recombination-limited, the junctions do play an important part. Since the assumption of an emitter efficiency of unity at the ultra-high injection levels specified here may be a poor one, the effective rate of recombination at the junction itself as well as the bulk lifetime may well determine the current-voltage characteristic.

It was pointed out in the analysis that the current-voltage characteristic would not be appreciably changed by a lifetime which was a function of position. As long as the defect density is less than the thermal equilibrium carrier densities, this is apparently true. However, when this specification is not fulfilled, the hole and electron lifetimes may no longer be equal; and, the current-voltage characteristic may take on quite a different form. Lampert's paper on this subject predicts negative resistance effects; this is mentioned here as such effects will be discussed in the chapter to follow.

The last point to be considered with respect to the L-R analysis is that Lampert and Rose assumed the lifetimes and
mobilities to be field independent. It will be shown in the next chapter that this assumption will not be satisfied for large values of electric field intensity and that this fact may be a major contributor to the discrepancy between measured and predicted results.
CHAPTER 3
Experimental Diodes

3.1 Experimental Verification of Simple Theory

In order to determine whether the simplified theory as developed in section 2.1 would check with measurements on actual FSN diode structures, the current-voltage characteristics of several diodes were carefully measured using d.c. pulse techniques to eliminate any temperature dependency. The measuring circuit used is shown in Figure 11. A 600 μsec. pulse duration was used to allow transients to die out, and a 2.5 c.p.s. repetition rate was used to insure a low value of duty cycle and, hence, a minimum amount of diode temperature rise. The lifetimes of the diodes' base regions were measured using a battery and mercury-wetted contact vibrating chopper with the method of Lederhandler and Giacoletto 45. Lifetimes measured in this manner are those determined by the base region immediately adjacent to the junction; if the lifetime in the bulk of the base region differs from this value, all calculations based on diffusion length will contain some error.

A typical calculation for a silicon double-diffused diode will be shown next and compared with the actual measured characteristics. This diode was made by diffusion of boron and phosphorous into opposite sides of a wafer of 90 ohm-centimeter
Figure 14. The Current-Voltage Characteristic Measuring Circuit
resistivity p-type silicon. The thickness of the base region was approximately 30 mils or 7.63 x 10^{-2} cm. The diode itself was ultrasonically cut from the wafer in the form of a cylinder of 100 mils diameter with an active area of approximately .05 cm^2. Nickel studs were soldered to the ends which had been twice plated in an electroless nickel bath; a heat treatment between the plating operations lowered the contact resistance and improved the mechanical strength of the bond. A polishing etch with deionized water rinse was used shortly before measurements were taken to insure a clean surface.

The measured lifetime was 25 μsec. which, using a hole mobility $\mu_p = 400 \text{ cm}^2/\text{volt-sec.}$, gives an ambipolar diffusion length, $L^*$, of $1.93 \times 10^{02} \text{ cm}$. The ratio, $L/L^*$, then is 3.96. Using this information, the voltage drop across the base, $V_B$, and the constants $C$ and $D$ may be determined from Figures 11, 7 and 8 respectively. The current-voltage characteristic may then be determined from equation (2.93) directly or by first calculating the injected carrier concentrations, $p_0$ and $n_0$, as a function of current density and using these in conjunction with (2.82), (2.83) and (2.88) to determine the total voltage drop. The latter method will be illustrated here since some of the intermediate details may be of interest. The following information has already been determined from theoretical curves or assumptions as previously mentioned:
Using these values in conjunction with the previously mentioned equations, the calculated data may be presented in Table 1. It may be seen that, for \( J = 1 \text{ amp/cm}^2 \), the majority carrier equilibrium density, \( p_T \), will begin to be comparable with the carrier densities in the middle of the base region; and, hence, the theory is not valid for values below that point. The calculated current-voltage characteristics compare reasonably favorably with the measured characteristics as can be seen in Figure 15. This was not the case for all diodes--some diodes with shorter lifetimes did not provide so good an agreement between theory and experiment. The fact that lifetime near a heavily diffused junction might be different from that in the bulk material can easily explain part of the discrepancy. In addition, the lifetime is actually a function of carrier density as discussed in section 2.3.
Calculated Volt-Ampere Characteristic of a Silicon Diode

Table 1

<table>
<thead>
<tr>
<th>I (ma.)</th>
<th>J (amp)</th>
<th>P_o (cm^{-3})</th>
<th>n_o (cm^{-3})</th>
<th>V_p (volts)</th>
<th>V_n (volts)</th>
<th>V_T (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.0</td>
<td>5.95 x10^{15}</td>
<td>1.47 x10^{15}</td>
<td>.0883</td>
<td>.525</td>
<td>.8133</td>
</tr>
<tr>
<td>80</td>
<td>1.6</td>
<td>9.52 x10^{15}</td>
<td>3.53 x10^{15}</td>
<td>.10</td>
<td>.547</td>
<td>.847</td>
</tr>
<tr>
<td>120</td>
<td>2.4</td>
<td>1.43 x10^{16}</td>
<td>5.3 x10^{15}</td>
<td>.110</td>
<td>.557</td>
<td>.867</td>
</tr>
<tr>
<td>160</td>
<td>3.2</td>
<td>1.91 x10^{16}</td>
<td>7.07 x10^{15}</td>
<td>.1175</td>
<td>.564</td>
<td>.881</td>
</tr>
<tr>
<td>320</td>
<td>6.4</td>
<td>3.82 x10^{16}</td>
<td>1.41 x10^{16}</td>
<td>.135</td>
<td>.582</td>
<td>.917</td>
</tr>
</tbody>
</table>
Figure 15. The Current-Voltage Characteristic for a Silicon PSN Diode
At higher bias voltages all of the diodes tested showed the same departure from the theoretical characteristics as in Figure 15. The greatest contribution to the divergence of the two curves is that resulting from decreasing emitter efficiency. The theoretically calculated hole densities, \( p_0 \), are on the order of \( 10^{17} \text{cm}^{-3} \) at the point where this error makes itself noticeable. The equilibrium majority carrier density of the heavily doped region near at least one of the diffused junctions probably is not much greater than that value, and one of the critical assumptions for the theory is no longer satisfied. This would, of course, invalidate the calculations under such conditions.

### 3.2 Probe Analysis of Germanium Structures

In order to determine whether Lampert and Rose's theory of double injection recombination-limited currents provides an adequate description of the conduction mechanism in long base PIN or PSN diodes, an experimental program was set up with particular emphasis on obtaining information concerning the potential distribution in such devices. The method of probing the surface of a device with a micro-manipulator equipped with a fine-pointed probe and a high impedance voltmeter would seem feasible at first, but this procedure can lead to sizeable errors. Because the base regions of the devices of interest here are inherently of high resistivities, the point-contact formed by the probe on the surface tends to be rectifying and, hence, to possess a high contact impedance. In addition to this, surface layers and channels can indicate a potential distribution
considerably different from that actually present in the bulk of the material. While "forming" the point contact by passing a current pulse through it reduces the contact impedance, the depth of penetration is slight; and, the problems, while reduced in magnitude, still exist. Also measurements made in this manner are usually no reproducible since the point has to be resharpened and reformed once it is removed from the surface.

In order to reduce these difficulties to the point where completely reproducible results could be obtained, a "floating junction" technique of probing was applied. The principles of this method are somewhat similar to those of the "floating barrier" procedure used by Moore and Webster 17 to measure the surface recombination of germanium if the area covered by their "barrier" is drastically reduced. The "floating barrier" consists of a film of material on the surface which will form a blocking or rectifying contact. Indium was used with n-type germanium in the experiments to be described. Since the resistivity of the p-layer under the indium was probably not greater than 0.001 ohm-cm., 18 it may be concluded that this layer, when compared to the high resistivity material underneath, acts as an equipotential.

Figure 16 shows a cross section of a floating "barrier" or "probe" consisting of an indium sphere alloyed to a high resistivity germanium bar. When an electric field is applied to the bar, the alloyed junction "sees" a potential variation along its length in the n-type material and will float at some intermediate potential which
Figure 16. Determination of a Probe's Floating Potential
will be denoted as $V_f$. Part of this junction is biased in the reverse direction, collecting thermally generated holes from the n-type bar; the rest of the junction is biased in the forward direction, injecting holes back into the bar. The net current, of course, remains zero. The p-region then acts as a translator of holes rather than as an area of low surface-recombination-velocity. Moore and Webster have called this translating action the "feed-in, feed-out" effect.

In order for such a junction to be used as a potential probe in a double-injection diode where both hole and electron currents are present, several conditions must be satisfied.

1) The hole translating effect of the probe should be minimized so as not to emphasize any surface effects such as conducting channels.

2) The presence of the probe should not disturb the carrier distributions present in the bulk of the material when the probe is absent.

3) The total range of voltage under the probe junction should be as small as possible as compared with the total voltage applied to the bar in order to obtain reasonable resolution when making potential distribution measurements.

4) The floating voltage, $V_f$, should not be appreciably different from that of the average potential, $1/2 (\Delta V)$, under the junction.

5) The probe should be alloyed to a depth sufficient to penetrate any surface inversion layer.
The first four conditions are met when the maximum longitudinal dimension, in this case the diameter, of the junction, d, is quite small when compared to the length of the bar. In addition, it has been shown that the floating potential given by

\[ V_f - V_a = \frac{kT}{e} \left[ \frac{e\Delta V/kT}{1 - e\Delta V/kT} \right] = \frac{a}{d} \Delta V \] (3.1)

is in good agreement with experiment. When typical values of \( \Delta V \) for small probe areas are substituted into (3.1), it may be seen that little error is introduced; for a potential difference measurement between two identical probes any error tends to cancel out. The last condition, necessitated by the problem of inversion layers present on the surfaces of high resistivity semiconductors, is least serious for n-type germanium where careful etching removes any such layer present. For p-type germanium and both n and p-type silicon, etchants can cause such layers; alloying probes through them eliminates measurement error.

For the potential probing experiments, the area was kept as small as possible by placing the indium probes on the surface in the form of spheres, 7 mils in diameter. The actual diameter of the circular alloyed region was somewhat smaller. A drawing of a typical experimental structure is shown in Figure 17. Since the germanium bars were nominally 30 mils wide, this did not appreciably affect the potential distribution. The n-type germanium bars used in these experiments had a resistivity of 30 ohm-cm, and possessed a lifetime of 175 \( \mu \)sec, before fabrication.
Figure 17. Conceptual Representation of a PSN Structure with Floating Potential Probes
Diode A was fabricated on a bar measuring 29 x 15.2 x 200 mils long after first cleaning the bar in absolute alcohol and trichlorethylene and giving it a light etch in a mixture of 10 parts HNO₃ and 1 part HF. The p⁺ contact was 95% indium and 5% gallium. The five probes spaced along the bar were indium, while the n⁺ contact was a pellet of 63% tin, 35% lead, and 2% antimony. Initial alloying was done at 450°C in a tank nitrogen atmosphere with a moderately slow cooling period to fix the contact and probes in place. The diode was then etched in a diluted 10-1 etch and thoroughly rinsed with deionized water. It was then realloyed in a vacuum at 550°C with very slow cooling. Following this, it was etched in a modified CP-4 mixture containing 3 parts HNO₃, 1 part HF, 4 parts deionized water, and 4 parts glacial acetic acid. It was then etched in straight HF and thoroughly rinsed in deionized water.

The current-voltage characteristics were determined using pulse techniques and the same circuit of Figure 14 with the exception that the unit pulse generator and amplifier were replaced with a more powerful pulsed klystron power supply having a fixed pulse duration of 250 μsec. and a repetition rate of 25 c.p.s. The characteristics so obtained are shown in Figure 18. The logarithmic plot of the low current characteristic is essentially linear; while for currents above 100 milliamperes, the characteristics pass briefly through a region where current is a function of V² and then to a region where the current is a function of V³ or of a greater power of V. The pulse generator had insufficient current capabilities to
Figure 18. Current-Voltage Characteristic of Diode A
allow any further increase in voltage. Since the calculated transit times for Lampert and Rose's theory indicate that the insulator regime cannot be attained with the biases used in this experiment, it must be concluded that the last part of the characteristic is not caused by the recombination-limited mechanisms of the L-R theory. Since the onset of this condition is at about 130 volts, one can solve for the corresponding trap density using the equation given by Lampert for the transition voltage,

\[ V_{TFL} = \frac{eN_t L^2}{\epsilon} \]  

which, for this diode, gives a trap density, \( N_t \), on the order of \( 1 \times 10^9 \text{ cm}^{-3} \). While this value seems low, it is at least consistent with annealed high resistivity material; and, hence, Lampert's theory for high trapping center densities cannot be used.

While making the probe analysis, it was necessary to determine whether the floating potential as measured with an oscilloscope equipped with a high impedance (50 megohm) input was the actual floating voltage, neglecting the small contact potential difference. For this purpose a pulse-nulling system was used; when the voltage applied to the probe is that corresponding to zero bias for the probe junction, the current through a series resistance should be zero. Experiments showed that if the probe impedance was high enough, it gave identical results with the null method. The circuit used is shown in Figure 19. The potential distributions so obtained are shown in Figure 20.
Figure 19. Null Circuit to Check Floating Probe Potentials
Figure 20. Potential Distributions for Diode A
It may be observed that, when the applied voltage approaches the apparent threshold of 130 volts, the recombination rate appears to be enhanced in the center of the diode. The electric field becomes sufficiently large there to produce "hot" carriers. As discussed in (2.2), the mobilities under this condition decrease with a correspondingly smaller drift velocity than would otherwise be expected. Hence, the carriers might tend to spread out without any large local concentrations of space charge; this would cause the potential distribution to be more uniform. If, however, the recombination rate should increase with an increase of an electric field, the presence of a region with field intensity higher than its surroundings would be a self-sustaining phenomenon. Since a point of constant electric field appears to fall within the high field area, it may be assumed that this is the point, \( x_0 \), discussed earlier where \((\Delta p - \Delta n) = 0 \) exactly; and, the variations of electric field are quite large at the ends of the regions surrounding that point. It may be observed that the rate of change of electric field with distance, \( dE/dx \), is positive on the p+ side of \( x_0 \) and is negative on the n+ side of \( x_0 \). This corresponds to
\[
(\Delta p - \Delta n) > 0, \quad -\frac{L}{2} \leq x < x_0 \quad (3.3)
\]
and
\[
(\Delta p - \Delta n) < 0, \quad x_0 < x \leq \frac{L}{2} \quad (3.4)
\]
which agrees with the theory developed in chapter 2. For large values of the ratio \( L/L \) it may be shown that equation (2.70) reduces to
\[
x_0 = \frac{L}{2} \frac{\ln b}{\ln b} \quad (3.5)
\]
While this diode probably does not satisfy the assumptions of chapter 2, it is interesting to note that the calculated value based on a hole mobility of 1700 cm²/volt-second and the original material bulk lifetime of 175 μsec gives a value that lies in the vicinity of the observed $x_0$. The calculated value is denoted by $x_0$ in Figure 20.

It is difficult to show whether or not Lampert and Rose's equation for the semiconductor regime, (2.14)

$$J = e \frac{\tau \mu_n \mu_p}{(n_T - p_T)} \frac{V^2}{L^2}$$

actually conforms to experimental data. As with Larrabee's experiments, using the lifetime of the bulk material before fabrication does not allow the theory a fair chance, as the lifetime after alloying is generally lower. In this case inserting the original lifetime into the equation and using a value of $L$ that is either the total diode length or the effective length over which most of the potential drop occurs always results in a value of current density that is several orders of magnitude higher than the observed value. Also, since reasonably high electric field intensities are involved, the quantities $\tau$, $\mu_n$, and $\mu_p$ are not constant as discussed in (2.3).

The voltage $V_{tr, \infty}$ at which the transition from Ohm's law to the square law takes place is given by (2.14). Again, putting the previously mentioned numbers into this equation, using the full length for $L$, one obtains $V_{tr, \infty} = 0.0289$ volts, which is obviously erroneous. Hence, the effective value of $\tau$ must be considerably lower than this. Using a value of $V_{tr, \infty} = 120$ volts, a value of
lifetime, \( \tau = 0.06 \, \mu \text{s} \), is obtained; this seems rather low considering the original starting material and relatively low alloying temperatures involved. Hence, one is led to believe that either Lampert's theory cannot be expected to agree quantitatively with experimental results or that another mechanism is entering into the experiment so that the theory does not apply anyway. As a further check, the voltage, \( V_{\text{tr}, \ s-I} \) at which the transition from the semiconductor regime to the insulator regime takes place may be calculated from (2.152). From this, \( V_{\text{tr}, \ s-I} = 6.9 \times 10^4 \) volts, and it may be deduced that the observed \( V^3 \) dependence of current at very high injection levels results from other considerations than those governing the "insulator" regime.

As a final comment on Lampert's theory, it may be observed that, although his assumption that \( \frac{dE}{dx} = \frac{V}{L^2} \) is dimensionally correct, it does not actually represent the physical situation since the fact that \( \frac{dE}{dx} \) is not constant, but rather changes sign, has been shown experimentally.

In the course of this investigation a number of diodes were fabricated; the results already shown for diode A are not atypical. Several etches were used to prepare diodes for measurements; the previously mentioned mixture of 3 parts \( \text{HNO}_3 \) and 1 part HF mixed in turn with equal volumes of glacial acetic acid and deionized distilled water appeared to leave the cleanest surfaces. While the potential distribution on diode A provided sufficient information to obtain a reasonable replica of the electric field variation, the accuracy
suffered because of the small number of probes. Another diode was fabricated with eight alternately staggered indium probes along the length of the base region to provide greater resolution.

Diode D was fabricated from a 30 x 5 x 200 mil bar of 30 ohm-cm n-type germanium; the thicknesses of many of the diodes fabricated after diode A were reduced because of the maximum current limitation imposed by the pulse generator. After a thorough cleaning an n-type contact of 63% tin, 35% lead, and 2% antimony and a p-type contact of indium were alloyed on opposite ends of the bar; the previously described indium probe arrangement was simultaneously alloyed. As before, the initial alloying took place in tank nitrogen and was followed by a heat treatment at 500°C in vacuum with very slow cooling. A pure HF etch with a deionized water rinse was used between the two procedures. A light HNO₃-HF cleanup etch completed the fabrication. The reverse breakdown voltage was approximately 580 volts. As for diode A, all current-voltage characteristics were taken using the d.c. pulse arrangement that has been described. The forward-bias characteristic is shown in Figure 21, and potential distributions for several values of current density appear in Figure 22. Here a new phenomenon occurs—that of a negative resistance! While much has been written on negative resistance in PIN devices during the current year, all experimental results were obtained for semiconductor materials which had been saturated with electrically active impurities such as iron, gold, and copper. Theory predicts negative resistance current-voltage characteristics when the density of trapping centers
Figure 21. Current-Voltage Characteristics of Diode D
Figure 22. Initial Potential Distributions of Diode D
for materials such as those just mentioned is large compared with the
injected carrier densities. However, for the voltage probing
experiments, every effort was made to avoid contamination by such
impurities and, hence, the appearance of negative resistance
characteristics cannot be explained solely on the basis of previous
theory.

Lampert 15, 16 considers the case in which an acceptor
recombination level lies below the Fermi level so that, at thermal
equilibrium, the levels are filled by electrons. He assumes that the
hole capture cross section, \( \sigma_p \), of these centers is considerably
larger than the electron capture cross section, \( \sigma_n \). Under conditions
of double injection most of the holes recombine because the ratio of
hole to electron diffusion length \( L_p/L_n \) is rather small; and, hence
the current is essentially carried completely by electrons. However,
if sufficient bias is placed across the base region such that the
hole transit time becomes equal to the low-level lifetime, the current
becomes a two-carrier current and increases through a negative
resistance region until the voltage attains a minimum value,

\[
V_M = \left( \frac{\sigma_n}{\sigma_p} \right) V_{th}
\]

(3.6)

where \( V_{th} \) is the critical value of voltage for which the hole life-
time and transit time are equal.

When \( V_{th} \) is reached, the holes can reach the electron in-
jecting end of the base region before recombining; this has the
effect of replacing the electrons in the recombination centers with
holes so that, for each arriving electron, one hole is annihilated
and the hole and electron lifetimes become equal. As higher biases are applied, the square-law and cube-law current-voltage characteristics previously discussed in conjunction with the L-R theory are predicted.

In order to determine whether the observed characteristics of diode D were in any way related to the model just presented, it was necessary to take all available precautions to make certain that junction and surface effects were minimized. Therefore, diode D was re-etched just prior to remeasurement; the etchant used was that previously mentioned as giving the best results. The reverse breakdown voltage was increased to some value over 1000 volts, well above the maximum range of the instrumentation. At the same time the scatter in the potential distribution data was decreased indicating a reduction of surface effects. The new current-voltage characteristic is plotted in Figure 21 along with the original one. The potential distributions are shown in Figure 23. As may be seen, for low values of current density the potential distribution across the base region is reasonably uniform. However, as high injection levels are attained, the electric field reaches a large maximum value near the center of the base region. At a certain critical voltage, the current density rapidly increases and the potential distribution is considerably altered in that the region of maximum electric field moves closer to the hole injecting contact. For current densities slightly lower than those corresponding to this critical voltage, some of the probe voltages appear to be unstable indicating a sort of "switching" or change in mechanism. For higher values of current
Figure 23. Potential Distributions of Diode D after Re-etching
density, the potential distribution becomes more uniform again no longer shows a localized large electric field intensity.

The electric field as determined from the slope of the potential distribution curve in Figure 23 showed a maximum value of $7 \times 10^3$ volts/cm$^2$. This is at a current of 100 ma, which corresponds to a current density of 100 amp./cm$^2$ using the diode dimensions as measured with a microscope equipped with a calibrated eyepiece. Thus, nearly all of the applied voltage appears across a central portion of the base region. Since the diffusion length can be roughly determined from the potential distribution of the diode for lower current densities and can be seen to be no greater than $L/8$ for this diode, it would seem that this diode $D$ meets the assumptions set forth in Lampert and Rose. However, this is only a very narrow range over which the current is proportional to $V^2$. If the average field in the different regions of the base is evaluated and the hole transit time, $t_p$, across the base region is determined from these fields and the corresponding drift velocities given by Gunn 29 it is seen that $t_p \geq 5 \times 10^{-7}$ sec, which can be safely assumed to be considerably less than the hole lifetime. It would not appear that the square-law regime is related so directly to the quality of hole transit time and lifetimes as reasoned by Lampert and Rose. However, the fact that the transit time is considerably less than the carrier lifetime in the base region is important as will be shown in the next section.

For the large electric field intensities inherent in the measured potential distributions the electron drift velocity is
nearly constant since the mobility varies with $E^{-1}$. The recombination rate may become field-dependent as discussed in chapter 2. Before providing a theoretical basis for the observed results, one additional experiment concerned with lifetime should be mentioned. Since the defect density and the velocity-dependence of the capture cross-sections were not known for the actual samples used in these experiments, an attempt was made to determine the dependence of the hole lifetime on electric field intensity using a pulse method which was described originally by Haynes and Shockley. The actual circuit used is shown in Figure 24. Small alloyed junctions were used instead of point contacts and a micromanipulator allowed contact to be made to any collector selected. The theory of such a measurement is quite simple; a sweeping field pulse is applied to ohmic contacts at the ends of an n-type bar with constant cross section. If, after all transients have passed, a pulse is applied to an emitter junction, holes will be injected and will drift toward the negative contact. If collector contacts are located as shown, a signal will appear across the resistor in series with a collector proportional to the remaining hole density. If, for an applied field, $E$, $t_1$ is the time interval separating the beginning of hole injection at the emitter and their arrival at the first collector and if $t_2$ is a corresponding time interval for a second collector located further down the bar as shown, then the average hole mobility is given by

$$\mu_p \equiv \frac{V_p}{E} = \frac{\Delta x}{(t_2-t_1)E} \quad (3.7)$$
Figure 24. The Circuit for Carrier Drift Experiments
where \( \bar{v}_p \) is the drift velocity for holes and \( \Delta x \) is the distance separating the two collectors. If the recombination is assumed to be the usual simple mechanism such that the excess hole density decreases exponentially with time as

\[
\Delta p(t) = \Delta p(0) e^{-t/\tau},
\]

(3.8)

then the ratio of hole densities at the two collectors is given by

\[
\frac{\Delta p(t_2)}{\Delta p(t_1)} = \frac{e^{-t_2/\tau}}{e^{-t_1/\tau}} = e^{-(t_2-t_1)/\tau}.
\]

(3.9)

The lifetime is therefore given by

\[
\tau = \frac{(t_2 - t_1)}{-\ln(\Delta p(t_2)/\Delta p(t_1))}.
\]

(3.10)

Since the signal appearing across the collector resistor should be proportional to \( \Delta p(t) \), the area observed under the waveform after correction for the electrostatic potential signal should permit \( \Delta p(t) \) to be computed; and, the ratio of two areas from two identical collectors should allow the lifetime to be determined from equation (3.10).

Since the waveform spreads out because of diffusion in the hole packet, it is important to use an emitter pulse which is quite sharply defined and has a sharp leading edge when compared to the pulse duration.

In the actual experiments, tin was alloyed to the ends of 30 and 40 ohm-cm n-type germanium bars; these contacts were then
sandblasted to provide a truly ohmic contact. Indium spheres were alloyed on one surface of the bar to form emitter and collector junctions. The sandblasted ends were masked with a trichlorethylene-soluble wax while the rest of the structure was etched in the previously described surface cleanup etching bath; cleaning in several trichlorethylene baths completed the operation. Two General Radio type 1217B unit pulse generators were used to provide the injection pulse. The first was synchronized to the beginning of the sweeping pulse and provided a delayed pulse, 150 µsec. later, to drive the second pulse generator which then provided the actual emitter pulse, 0.2 µsec. in duration. Unfortunately, the injected pulse was not sufficiently rectangular to permit accurate determination of the time differences, \( t_1 - t_2 \). Hence all lifetime measurements were relative rather than absolute; however, the results may be simply stated. For drift velocities, \( V \), corresponding to electric fields from approximately 700 volts/cm and up, the apparent hole lifetime decreases indicating that the capture-cross-sections involved are not linear in \( V^{-1} \) or a greater power of velocity. As previously stated, Conwell and Zucker have performed more elaborate and reliable experiments but their work was confined to germanium samples which had been deliberately contaminated with copper. Their results showed a dependency of the electron capture cross section proportional to \( V^j \) where \( j \) lay between 0 and -1 and, for one sample, was closer to -1. Hence, while no general conclusions may be reached about the exact relationships that will occur, it seems fairly safe to state that, at the present time,
there is no experimental evidence to suggest that \( j \) will be a positive quantity. Since the carriers spend the least time in the regions of high electric field intensity, the effects of a different lifetime in such a region are minimized.

3.3 An Alternative PSN Diode Theory

The Lampert-Rose Theory does not adequately describe the current-voltage characteristics of the diodes tested; indeed, it hardly appears to apply to very many diodes at all. However, a careful analysis of the potential distribution of a diode along with its current voltage characteristic can give additional insight into the physical mechanisms involved. While most of the discussion to follow centers about one experimental diode, similar results were observed for a number of diodes.

At low and medium injection levels, the carriers decay toward their thermal equilibrium levels because of recombination; the degree of conductivity modulation increases as the current density and the corresponding injection levels are raised. At moderately high injection levels the carrier distributions follow a combination of linear and exponential laws depending on the base region length among other things. If the carrier densities everywhere are sufficiently greater than their thermal equilibrium values, then the theory of section (2.1) gives at least a sufficient qualitative picture. As the applied forward bias is increased, the carrier densities become greater tending to reduce the electric field intensity and the field-driven current; the current density then becomes more diffusion-
controlled. However, the carriers can only be injected as fast as
they can recombine in the base region if reasonable emitter
efficiencies are assumed; the current is recombination-limited as
mentioned earlier. Thus, if the applied voltage and resulting
electric field intensity are to be increased still further, thus
increasing the carriers' velocities, the current can increase only
if the carriers can recombine sufficiently within a time corre-
sponding to the carrier transit time across the base region. The
constant lifetime model imposes a limit here. When the transit
time across the base region for an injected hole becomes equal to
the lifetime, any further increase across the base region will tend
to cause a pile-up of carriers in the base region with a resultant
departure from space charge neutrality. The carriers cannot continue
to pile up without affecting the recombination rate, however; and,
if the carriers approach a density of approximately $10^{18}$ cm$^{-3}$,
direct recombination lowers the effective lifetime and limits the
carrier density at any point to a reasonable value consistent with
the transit time and the direct recombination lifetime.

$$\tau_d = \frac{1}{v \sigma p} \quad (3.11)$$

where $v$ is the carrier velocity and $\sigma$ is the capture cross section
of holes for electrons; $\sigma$ is about $10^{-20}$ cm$^2$. It would not be
expected that departures from space charge neutrality would occur
near the injecting contacts themselves; any large field immediately
at the contact would cause carrier depletion which, in turn, would
cause the emitter efficiency to fall. If carrier depletion does not take place so that the junction conditions (2.33) and (2.40) are reasonably satisfied, there will be diffusion-controlled currents in addition and the rate of change of field, $\frac{dE}{dx}$, will not be large. As the carriers traverse the base region, they are recombining. If the transit time for example an injected hole, is longer than its average lifetime, then the chances of its recombining with an electron are good; and, the current, while recombination-controlled, is not recombination-limited or saturated. If a hole can traverse the base region in a shorter time than its average lifetime, then its chances for recombination are not so good, and it must either accumulate with other such holes at the base region or be collected by the electron injecting contact thus lowering that contact's emitter efficiency. A similar picture may be drawn for injected electrons. Because of the higher mobility of electrons, it may be surmised that they will have a greater tendency to accumulate than holes and, hence, any unusual departures from neutrality will probably be predominantly regions of excess electron density.

The net excess carrier density, free carrier density, potential, and electric field intensity distributions for a diode under moderately high injection conditions are shown in Figure 25. The point at which $\Delta n = \Delta p$ exactly, denoted by $x_0$, is shown to be between the center of the base region and the hole injecting contact. This is due to the greater electron mobility as was discussed in section (2.1). The governing equation,
Figure 25. Conceptual Distributions of Free Carrier Densities, Net Excess Carrier Density, Electric Field Intensity, and Potential in the n-type Base Region of a PSN Diode under High Injection Conditions.
\begin{equation}
0 = D_p \left[ \frac{d^2n}{dx^2} + \frac{d^2p}{dx^2} \right] + \mu_p \frac{d}{dx} \left[ (n-p) E \right],
\end{equation}

\begin{equation}
= \left[ \frac{R(n)}{b} + R(p) \right]
\end{equation}

(2.14)
determines these distributions; but, a simple solution cannot be given here as the problem becomes the general non-linear problem discussed in section (2.2). However, critical portions of the potential distribution can be qualitatively discussed.

The regions near the injecting contacts are quite heavily conductivity-modulated, and diffusion effects may be expected to play an important role. With the proper value of lifetime for the carrier densities involved, the diffusion equation used in the simplified theory

\begin{equation}
\frac{d^2\Delta p}{dx^2} = \frac{1}{(L^*)^2} \Delta p
\end{equation}

(3.12)
may be used with previously discussed boundary conditions to describe the carrier distribution. An exponential variation is expected; however, the ambipolar diffusion length may be a function of carrier density.

In the center of base region where high electric fields predominate, the diffusion terms of the governing equation (2.14) may be safely neglected leaving the equation from which Lampert and Rose formulated their theory,

\begin{equation}
\mu_p \frac{d}{dx} \left[ (n-p) E \right] = \frac{R(n)}{b} + R(p) \cdot
\end{equation}

(3.13)
If a constant lifetime model is assumed with equal hole and electron lifetimes, and (3.13) is also divided by \( \mu_p \), there remains

\[
\frac{d}{dx} \left[ (n-p) E \right] = \frac{\Delta n}{b \mu_p \tau} + \frac{\Delta p}{\mu_p \tau}.
\]  

(3.14)

This may be expanded and rearranged to give

\[
E \frac{d}{dx} (n-p) + (n-p) \frac{dE}{dx} = \frac{b \Delta p + \Delta n}{\mu_p b \tau}.
\]  

(3.15)

Since the region of interest is located in the immediate vicinity of the point \( x = x_0 \) where the electric field is a maximum and \( \Delta p = \Delta n \) exactly, \( \frac{dE}{dx} = 0 \) in this region and (3.16) becomes, after dividing by \( E \)

\[
\frac{d}{dx} (\Delta n - \Delta p) = \Delta n \quad \frac{b + 1}{\mu_p b \tau E}.
\]  

(3.17)

None of the quantities in the brackets on the right hand side of this equation varies rapidly in the very close vicinity of \( x = x_0 \). While \( \Delta n = \Delta p \) at \( x_0 \), the derivatives \( d\Delta p/dx \) and \( d\Delta n/dx \) are not necessarily equal as is usually assumed for simplified diode theories. The net space charge has a maximum on either side of the region under discussion and is positive for \( x < x_0 \) and negative for \( x > x_0 \). Therefore, if the electron density has a positive-valued derivative, the hole density derivative must be negative-valued. If it can be approximated that

\[
\frac{d \Delta p}{dx} = - \frac{d \Delta n}{dx},
\]  

(3.18)
then (3.17) becomes

$$2 \frac{d \Delta n}{dx} = \Delta n \left( \frac{b + 1}{\mu_p b \tau E} \right). \quad (3.19)$$

This last assumption is probably true within a factor no greater than the ratio of carrier mobilities, b. Equation (3.19) may be integrated to give

$$\ln (\Delta n) = \left( \frac{b + 1}{2b \mu_p \tau E} \right) (x - x_0) + \ln (\Delta n_0) \quad (3.20)$$

where $\Delta n_0$ is the excess electron density at $x = x_0$. This is equivalent to the expression

$$\Delta n = \Delta n_0 \left( \frac{b + 1}{2b \mu_p \tau E} \right) (x - x_0). \quad (3.21)$$

For values of $x$ where $E$ is quite large, the exponent is small and the carrier densities vary slowly. As the edge of the high field region is reached and $E$ begins rapidly to decrease, the exponent increases and the carrier densities tend to rise more rapidly. However, (3.21) also loses validity at this point as $E$ becomes a function of $x$ and $\frac{d \Delta p}{dx} = \frac{d \Delta n}{dx}$ again. A numerical example may be taken from the data shown in Figure 22; at a current density of 100 amp./cm$^2$ and a point of maximum electric field intensity where $E = 7 \times 10^3$ volts/cm., the value of the coefficient of the exponent is approximately .05.

If diffusion currents are negligible as assumed, (2.5) and (2.6) become

$$J_p = e \mu_p E \quad (3.22)$$
and

\[ J_n = e \mu_n E \quad (3.23) \]

respectively. Since the carrier mobilities are not constant for some of the fields in question, these equations may be replaced with their equivalent expressions using the actual carrier drift velocities, \( v_p \) and \( v_n \), as

\[ J_p = e v_p \quad (3.24) \]

and

\[ J_n = e v_n \quad (3.25) \]

The ratio of hole current density to electron current density is then given by

\[ \frac{J_p}{J_n} = \frac{v_p}{v_n} \quad (3.26) \]

If the field is low enough so that the ratio of the velocities is that of the low field mobilities, \( b \), the ratio \((3.26)\) has the value of \( b^{-1} \). The difference in carrier densities in a region of reasonably strict charge neutrality is that for thermal equilibrium as expressed in

\[ (p - n) = p_T - n_T \quad (3.27) \]

If \((3.24)\) and \((3.25)\) are added,

\[ J = J_p + J_n = e (p v_p + n v_n) \] results. \((3.28)\)

If these last two equations are solved simultaneously, it may be shown that, for a given electric field intensity, \( E \), and a current density, \( J \),

\[ n = \frac{J/e - v_n (p_T - n_T)}{v_p + v_n} \quad (3.29) \]
The carrier velocities, as determined from Gunn, for a
field intensity of $7 \times 10^3$ volts/cm. are $v_n \approx 6 \times 10^6$ cm./sec. and
$v_p \approx 2 \times 10^6$ cm./sec. Again, for diode D, for which $p_T = 1.3 \times 10^{13}$ cm.$^{-3}$ and $n_T = 5.5 \times 10^{13}$ cm.$^{-3}$, at a current density of $J = 100$
amp./cm$^2$, as before, it may be shown that $n \approx 1.5 \times 10^{11}$ cm.$^{-3}$ and
$p \approx 1.1 \times 10^{11}$ cm.$^{-3}$. Thus, the region of high electric field inten­
sity is not highly conductivity-modulated, and the conceptual
distribution of Figure 25 appear to be reasonably valid representations.

It has been pointed out that, as the bias voltage is
increased above that corresponding to medium high injection levels,
the carriers tend to "pile up" at the ends leaving a region of high
electric field near the center of the base region. As long as the
transit time for the minority carrier in the base region is longer
than its lifetime, the recombination rate controls the current
density but does not cause it to saturate. The main effect of the
higher injected carrier densities under such conditions is simply to
cause a high degree of conductivity modulation and the theory of
section (2.1) applies as well as any. If, however, the bias is
increased still further, the carriers cannot recombine as fast as
they are swept across the base region. Since few electrons can cross
the junction into the $p^+$ region if the junction has a reasonable
emitter efficiency, most must be accumulated near the junction. The
same may be said with respect to holes near the junction at the $n^+$
region. With such high carrier densities at the junctions, the field is constrained to an ever-narrower region in the base. Since carriers are swept rapidly through this latter region to the junction regions, the carrier densities there are near those at thermal equilibrium as has been shown. The current-voltage characteristic should no longer follow the exponential characteristic described earlier, but should tend to saturate; the increase in current density should now be much less per volt of bias increase as compared with the situation at lower injection levels. If one looks at the continuity equations (2.3) and (2.4)

\[
- \frac{1}{e} \frac{d J_p}{dx} = R(p) = 0
\]

and

\[
\frac{1}{e} \frac{d J_n}{dx} = R(n) = 0
\]

and, using the constant lifetime model, rearranges them to

\[
d J_p = - e \frac{\Delta p}{t} \ dx \quad (3.31)
\]

and

\[
d J_n = e \frac{\Delta n}{t} \ dx \quad (3.32)
\]

then these equations may be integrated and added to give the dependence of the current density on the lifetime and carrier densities as

\[
J = \frac{e}{t} \int_{-L/2}^{+L/2} (\Delta n - \Delta p) \ dx \quad (3.33)
\]

The applied bias, then, merely increases the areas under the curves representing \(\Delta p\) and \(\Delta n\); the difference between these areas is
represented by the integral. Each time a hole and electron re­combine, a new carrier of each species may be injected at the proper junction giving a double contribution to the current. Since the distribution of carriers is so highly non-linear, J does not vary as the maximum carrier density but depends on the net space charge distribution. It should be pointed out that the current density becomes inversely proportional to the lifetime when the transit time for the minority carrier is short enough such that a change in lifetime is not reflected as a change in the equivalent series resistance of the base region. The suggestion that the current density can be proportional to the inverse of the lifetime is also contrary to the theory of Lampert and Rose, who did also assume that the transit time was shorter than the lifetime. While the carrier densities and the net space charge density are certainly determined for a given bias by the effective recombination rate, it would not be expected that the parameters would be proportional to the square of the lifetime as the Lampert-Rose theory would require.

Obviously, the injection level cannot be increased indefinitely without causing a radical change in the emitter efficiency at, at least, one of the junctions. As the voltage is increased, the distributions of Δp and Δn are altered such that the integral on the right hand side of (3.33) is increased causing the current density to become larger. Eventually, the magnitude of the density of the carrier injected at one of the junctions will become comparable in magnitude to the majority carrier density on the heavily doped side of the junction. When this occurs, the junction
becomes, for all intents and purposes, transparent or non-existent for both species of carriers. Since the current density is somewhat saturated up to this point because the carriers cannot recombine quickly enough to avoid accumulating near the junctions allowing both species of carrier to flow across the junction eliminates much of the need for recombination in the base region. Thus, when this point is reached, the current increases very rapidly for a small range of increasing voltage. Additional recombination takes place at the ohmic contacts to the heavily doped regions so that the current flow becomes that of a plasma in which the holes and electrons flow in opposite directions with little carrier recombination and a minimum of space charge density. Since the saturation effects are no longer present, it should take less voltage to maintain any given current density than was previously required. This, of course, indicates that a region of negative resistance might be expected here. Again very little in the way of a simple current-voltage relation can be given analytically here as the problem is at least as non-linear and mathematically intractable here as it was at lower current densities. It will suffice to say that the potential distribution should be more linear than for the saturated condition.

If the current voltage characteristics of diode D as shown in Figure 21 are observed, it may be seen that there is a very definite and substantial region that appears to be limited by some saturation effect. The current density is initially given by

\[ J = \epsilon_0 u v, \quad 10 \leq v \leq 70 \text{ volts}, \quad (3.34) \]
which upon a further increase in applied voltage becomes

\[ J = 32 e^{-0.0027v}, \quad 140 \leq v \leq 280 \text{ volts.} \quad (3.35) \]

Thus, it may be seen that the change in the exponential dependence of current density upon voltage becomes quite pronounced. It may also be observed from the potential distribution curves in Figure 23 that the potential drop does become localized as the current density becomes saturated and causes the large electric field intensities mentioned previously. The maximum electric field observed was \( 7 \times 10^3 \) volts/cm which is sufficient to cause the electron drift velocity to saturate. Thus, any further increase in field would cause the electron injection level to increase more rapidly. This fact, would lead one to believe that the electron injecting junction would be the first to have its emitter efficiency drastically reduced. It may be noted that the potential distribution for the next higher measured value of current density shows the region of maximum electric field shifting toward the hole injecting contact indicating changes in the carrier distributions near the \( n^+ \) junction.

For still higher values of current density a region of negative resistance is encountered, and the potential distribution tends to become more linear as expected. The accuracy of the measurements taken in this region of negative resistance is not so good as that for the remainder of the characteristic because the internal impedance of the pulse generator used in series with the negative resistance of the diode caused the system to be unstable at times. In fact, the
voltage would rise and fall over a range of several hundred volts at the very low frequency of 0.3 c.p.s.

This concludes this discussion of the alternative PSN diode theory. While no compact expression for the current-voltage characteristic has been derived, it may be safely stated that the theory of recombination-saturated current is consistent with both current-voltage characteristic and potential distribution measurements for several PSN diode structures. This is, at least, more than can be said for the theory of Lampert and Rose.

3.4 Other Effects: Multiple Terminal PIN Devices

In the course of these investigations a number of interesting effects were observed during experiments with PIN structures possessing three or more contacts. While little in the way of analysis will be presented here, some of the experiments and devices to be described may prove interesting to other investigators.

Most of these effects were observed while making measurements on a class of experimental PIN devices denoted as "chargistors" by their originator, H. N. Yu 53. A chargistor consists of a long base PIN diode with one or more additional injecting contacts spaced between the main electrodes. A typical triode chargistor is shown conceptually in Figure 26. A supposedly typical device as described by Yu was fabricated from a bar of 45 ohm-cm resistivity n-type germanium; indium was used for the p-type gate and feeder electrode while a lead-tin-arsenic alloy was
Figure 26. Yu's Triode Chargistor

Figure 27. Charger Output Characteristics Given by Yu
used for the charger electrode. The charger output characteristics for this device are shown in Figure 27; no charger voltage scale was given. The dotted line shows the usually PIN diode characteristic obtained with the gate open circuited. The curve for $V_g = 0$ shows the effect of short circuiting the gate to the feeder. Yu claims that the gate acts as an electrostatic potential shield and that changes in the charger voltage do not appreciably affect the region between the gate and the feeder. Thus, when the gate is at zero potential very little current should flow as is shown. Raising the gate potential allows a certain amount of current to flow from the feeder to the charger. Since the gate junction is reverse biased for reasonably large values of $V_c$, there is some cause for skepticism at this point. If the junction is "clean" such that its reverse leakage or saturation current is reasonably small, it is not clear to the writer how it can act as a potential shield, particularly to the degree claimed. In order to investigate this, various chargistor structures were fabricated on 30 ohm-cm. resistivity n-type germanium bars using various lengths and contact spacings. None of these structures showed the gate to have the control suggested by Yu. The most successful of these devices was fabricated using the original dimensions of Yu's devices, 200 x 30 x 10 mils thick, and using indium and a 95% tin-57% antimony alloy for contact materials. The spacing between the gate and feeder was very narrow, being on the order of 0.16 mm. The current voltage characteristics were taken using previously described pulse techniques and are shown in Figure 28.
Figure 28. Charger Characteristics for Experimental Triode Chargistor

Figure 29. Chargistor Equivalent Circuit
These characteristics do not bear a great deal of resemblance to Yu's and are more suggestive of conductivity modulation. The characteristic curve for \( V_g = 0.4 \) volts shows a discontinuity in the neighborhood of \( V_c = 30 \). This was observed in many structures as a small hysteresis loop appearing at one point on the current-voltage characteristic. The location of such a loop could be regulated via the gate voltage.

The observation that the gates of most of the experimental structures appeared to conductivity modulate the diode rather than to act as an electrostatic shield was partially verified by observing the current-voltage characteristic of the gate of a chargistor for various values of charger current. The equivalent circuit for the device appeared as shown in Figure 29. The resistance, \( R_g \), or the region between the gate and feeder is much smaller than \( R_c \), the resistance between the gate and charger. When there is no applied charger voltage, the gate-feeder circuit should have the exponential current-voltage characteristic of a thin base PIN diode. As the charger voltage and current are increased, a higher value of gate voltage will be needed to cause the gate diode to conduct. When the gate injects carriers, the region between the gate and feeder becomes conductivity modulated, and \( R_g \) decreases. This causes a larger effective potential to appear across the gate diode since \( V_g \) also decreases. Thus, the current-voltage characteristic for the gate should have a negative resistance region which should become more prominent as the charger current increases. This is
born out by measurements of this characteristic as shown in Figure 30.

Another effect that has been noticed is one that is sometimes quite troublesome while making measurements. Several devices exhibited a tendency of oscillate under certain conditions. One of these was observed on a sample of 20 ohm-cm resistivity n-type germanium used in drift mobility experiments. This bar had ohmic contacts at each end and an injecting contact of indium near one end. The ohmic contacts were of a tin-lead-antimony alloy and were sandblasted. When the circuit shown in Figure 31 was used, voltage tunable oscillations appeared at the indium contact. Both relaxation and sinusoidal waveforms were observed. One possible explanation for these oscillations is that if holes are injected at the emitter, the conductivity of the region between the emitter and the grounded contact may be modulated to a large extent. This causes a negative resistance region to be formed which may give rise to the oscillations. This device is not unlike a filamentary transistor $h^+$.

Only one additional effect will be discussed here; this occurred only for a few devices. Some germanium diode units used in the chargistor studies had a very low saturation current in the forward direction with breakdown voltages up to 200 volts. Such diodes were photosensitive and could be switched to another stable state having a much higher saturation current. Holynak 18 has
Figure 30. Gate Current-Voltage Characteristics for an Experimental Chargistor
Figure 31. Circuit and Waveform for Relaxation Oscillations
also observed this effect. He has sectioned 20 to 30 different units made with a wide range of alloys and has not found any evidence to support a suggestion of R. N. Hall 55 that these diodes may be a form of a pnnpn switch. There are many questions as yet unanswered here.
CHAPTER I

Summary

In this dissertation, the theory of PIN and PSN diodes has been reviewed and extended. A simplified linear theory for the high injection case has been presented and compared with experiment. The theory for diodes to which the simplified theory does not apply has been critically examined with emphasis on application to experimental devices. It has been shown that the theory of Lampert and Rose is not sufficient to describe adequately the current voltage characteristics of long PIN diodes at high injection levels and that current saturation effects must be accounted for. A qualitative theory is presented which explains the experimentally observed current-voltage characteristics. A method for obtaining reproducible measurements of the potential distribution in n-type germanium diodes base regions has been described and the results of such measurements discussed. The general non-linear differential equation governing the carrier distributions and current flow has been discussed with particular attention to recombination and mobility variations. Finally, some unexplained experimental effects have been presented.

It is obvious to the reader at this point that there is much yet to be learned about the detailed operation of junction
devices. An investigation of the general problem at high injection levels using a digital computer would serve a useful purpose here; setting up the program might prove to be a formidable task, but the information gained thereby would be of great interest to a large number of workers in the semiconductor field. Some of the effects now considered as spurious or troublesome could, if understood, possibly lead to new applications for existing device structures. Since semiconductor device material are no longer limited to germanium and silicon, semiconductors such as gallium arsenide, indium antimonide, cadmium sulphide, and cadmium selenide may find increasing application in these devices as each of their individual technologies progresses.
Appendix A

The Determination of the Constants A and B

Equations (2.28) through (2.32) with (2.47) give the following conditions:

\[ \Delta p = \Delta n = A e^{-x/L^*} + B e^{x/L^*} \]  \hspace{1cm} (A-1)

\[ p_0 = A e^{L/2L^*} + B e^{-L/2L^*} \]  \hspace{1cm} (A-2)

\[ n_0 = A e^{-L/2L^*} + B e^{L/2L^*} \]  \hspace{1cm} (A-3)

\[ p_0' = \frac{-A}{L^*} e^{L/2L^*} + \frac{B}{L^*} e^{-L/2L^*} \]  \hspace{1cm} (A-4)

\[ n_0' = \frac{-A}{L^*} e^{-L/2L^*} + \frac{B}{L^*} e^{L/2L^*} \]  \hspace{1cm} (A-5)

\[ P_0' = -b n_0' \]  \hspace{1cm} (A-6)

If (A-4) and (A-5) are multiplied by L and substituted into (A-6),

\[ L/2L^* \quad -L/2L^* \quad -L/2L^* \quad L/2L^* \]

\[-A e^{L/2L^*} + B e^{-L/2L^*} = bA e^{-bB} \]  \hspace{1cm} (A-7)

is the result. This may be rearranged to
Solving for $B$, one obtains

$$B = A \left( \frac{b \in \mathbb{L}/2L^* + \epsilon \in L/2L^*}{b \in L/2L^* + \epsilon \in -L/2L^*} \right) \quad (A-9)$$

Dividing $(A-2)$ by $(A-3)$, one obtains

$$\frac{P_0}{n_0} = \frac{A \in \mathbb{L}/2L^* + B \in -L/2L^*}{A \in -L/2L^* + B \in L/2L^*} \quad (A-10)$$

which, upon the substitution of $(A-9)$ for $B$, becomes

$$\frac{P_0}{n_0} = A \in \mathbb{L}/2L^* + A \in -L/2L^* \left( \frac{b \in \mathbb{L}/2L^* + \epsilon \in L/2L^*}{b \in L/2L^* + \epsilon \in -L/2L^*} \right) \quad (A-11)$$

If both the numerator and the denominator of $(A-11)$ are multiplied by $\epsilon \in \mathbb{L}/2L^*$, and if the constant, $A$, is cancelled out, there results

$$\frac{P_0}{n_0} = \left( \frac{b \in \mathbb{L}/2L^* + \epsilon \in L/2L^*}{b \in L/2L^* + \epsilon \in L/2L^*} \right) \quad (A-12)$$

which may be simplified by collecting all the terms in both the numerator and denominator over a common denominator, giving
\[
\frac{P_0}{n_0} = \frac{\varepsilon}{(b \in L/2L^* + \varepsilon - L/2L^*) + (b \in L/2L^* + \varepsilon L/2L^*)}.
\]

which may be simplified to

\[
\frac{P_0}{n_0} = \frac{b(\varepsilon L/2L^* + \varepsilon - L/2L^*) + 2L/2L^*}{(b \in 3L/2L^* + \varepsilon - L/2L^*) + 2b \in L/2L^*}.
\]  (A-13)

Both the numerator and denominator of this may be multiplied by

\[
\varepsilon L/2L^*
\]

to obtain

\[
\frac{P_0}{n_0} = \frac{b(\varepsilon 2L/L^* + 1) + 2 \in L/L^*}{(b \in 2L/L^* + 1) + 2b \in L/L^*}.
\]  (A-15)

This ratio and its inverse will appear in most of the expressions relating \(A\) and \(B\) to \(P_0^*\).

If (A-2) is multiplied by \(\varepsilon L/L^*\) there results

\[
\varepsilon L/L^* P_0 = A \varepsilon - L/2L^* + B \varepsilon - 3L/2L^*.
\]  (A-16)

If (A-3) is subtracted from this

\[
\varepsilon L/L^* P_0 - n_0 = B(\varepsilon - 3L/2L^* - \varepsilon L/2L^*).
\]  (A-17)

is obtained, giving
\[ B = \frac{n_0 - \epsilon}{\epsilon \frac{L}{2L^*} - \epsilon \frac{-L}{2L^*}} \quad (A-18) \]

Multiplying (A-2) by \( \epsilon \frac{L}{L^*} \),

\[ \epsilon \frac{L}{L^*} \frac{P_0}{P_0} = A \epsilon \frac{3L}{2L^*} + B \epsilon \frac{L}{2L^*}, \quad (A-19) \]

and again subtracting (A-3) from this,

\[ \epsilon \frac{L}{L^*} \frac{P_0}{P_0} - n_0 = A \left( \epsilon \frac{3L}{2L^*} - \epsilon \frac{-L}{2L^*} \right) \quad (A-20) \]

is obtained, giving

\[ A = \frac{P_0 \epsilon \frac{L}{L^*} - n_0}{\epsilon \frac{3L}{2L^*} - \epsilon \frac{-L}{2L^*}}. \quad (A-21) \]

This may be simplified to

\[ A = \frac{P_0 \epsilon \frac{L}{2L^*} - n_0 \epsilon \frac{-L}{2L^*}}{\epsilon \frac{L}{L^*} - \epsilon \frac{-L}{L^*}} \quad (A-22) \]

and

\[ A = \frac{P_0 \left( \epsilon \frac{L}{2L^*} - n_0 / P_0 \epsilon \frac{-L}{2L^*} \right)}{2 \sinh (L/L^*)} \quad (A-23) \]

Likewise, (A-18) may be simplified to

\[ B = \frac{n_0 \epsilon \frac{L}{2L^*} - P_0 \epsilon \frac{-L}{2L^*}}{\epsilon \frac{L}{L^*} - \epsilon \frac{-L}{L^*}} \quad (A-24) \]
and

\[ B = \frac{P_0 \left( \frac{n_0}{P_0} \in L/2L^* - \in -L/2L^* \right)}{2 \sinh (L/L^*)} \]  \hspace{1cm} (A-25)

It is now desirable to find the relation between \( P_0' \) and \( P_0 \). If (A-24) is multiplied by \( L^* \),

\[ L^* P_0' = -A \in L/2L^* + B \in -L/2L^* \]  \hspace{1cm} (A-26)

is obtained. The results of (A-23) and (A-25) may be substituted for \( A \) and \( B \) respectively, giving

\[ L^* P_0' = \frac{L/2L^* -L/2L^* L/2L^*}{2 \sinh (L/L^*)} \]

\[ P_0(-\frac{n_0}{P_0} \in L/2L^* - \in -L/2L^*) \]

\[ + \frac{P_0(-\frac{n_0}{P_0} \in L/2L^* - \in -L/2L^*)}{2 \sinh (L/L^*)} \]  \hspace{1cm} (A-27)

which, upon division by \( P_0 \), simplifies to

\[ \frac{L^* P_0'}{P_0} = \frac{n_0/P_0 - \in -L/L^* - \in -n_0/P_0}{2 \sinh (L/L^*)} \]  \hspace{1cm} (A-28)

Combining the terms in the numerator, one obtains

\[ \frac{L^* P_0'}{P_0} = \frac{2 \cosh (L/L^*)}{2 \sinh (L/L^*)} = - \coth (L/L^*) \]  \hspace{1cm} (A-29)

and

\[ P_0 = -P_0' L^* \tanh (L/L^*) \]  \hspace{1cm} (A-30)
LITERATURE CITED


8. Spenke, E., "Durchla and Sperreieigenschaften eines p-i-metal


18. Holonyak, N., To be published


55. Hall, R. N., Private communication.
I, Glen Carl Gerhard, was born in Albion, New York, on March 1, 1935. I received my secondary-school education at the Bloomfield Central School, East Bloomfield, New York, and my undergraduate training at Syracuse University, which granted me the Bachelor of Electrical Engineering degree in 1956. From the Ohio State University, I received the Master of Science degree in 1958. In October, 1958, I was appointed Instructor in Electrical Engineering and Research Associate in the Electron Device Laboratory, both at the Ohio State University. I held these positions for four years while completing the requirements for the Doctor of Philosophy degree.

Since September, 1962, I have held the position of Engineer at the Electronics Laboratory of the General Electric Company in Syracuse, New York.