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APPLICATION OF THE FAST CYCLOTRON WAVE OF A MAGNETICALLY
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Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

DEAN TRAFFORD DAVIS, B.E.E., M.Sc.

*

The Ohio State University

1961

Approved by

E. M. Boone
Advisor
Department of Electrical Engineering
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APPLICATION OF THE FAST CYCLOTRON WAVE OF A MAGNETICALLY
FOCUSED ELECTRON BEAM TO FREQUENCY MULTIPLICATION

CHAPTER I
INTRODUCTION

Mushrooming communication requirements in the past two decades
have stimulated efforts to extend the usable portion of the radio
frequency spectrum to shorter and shorter wavelengths. The current
frontier might be said to be that portion of the spectrum extending
from one centimeter to one millimeter in wavelength or the frequency
range from 30 to 300 kHz. Although some sources are available in
this region, the majority of those on the current market are varia-
tions of devices and techniques that have been employed at lower fre-
quencies, and, in general, they leave much to be desired with respect
to efficiency and life.

Traditional Signal Sources

The majority of devices which may be utilized to generate sig-
nificant amounts of coherent microwave energy in the millimeter wave-
length region are electron tubes utilizing an electron beam interact-
ing with an electromagnetic field in such a way that a portion of the
kinetic energy of the electrons is delivered to the field and thus
converted to radio frequency energy. In order that this energy be
coherent, the electron beam must be bunched, or converted to an ac
beam. This process involves velocity modulation of the beam in such a manner that a "compression" space charge wave is produced in the electron beam, so that the current density at a given point in the device is a periodic function of time. This bunching process can occur in two general ways, as described in the following two paragraphs.

In devices of the klystron type, the electrons in the beam are velocity modulated during an initial transit through a short concentrated ac field, and then permitted to drift in a region free of high frequency field, during which time the faster electrons overtake slower electrons and form "bunches", or regions of high charge density. These bunches are then permitted to pass through a second concentrated ac field in such phase that the denser portions of the beam pass through a decelerating field and deliver a portion of their kinetic energy to the field. In the two-gap klystron, the drifting and bunching occur in a region of constant dc potential, while in the reflex klystron the drifting and bunching are accompanied by a deceleration in a retarding dc field and eventual reversal of direction, after which the bunches of electrons return through the same gap employed to produce the initial velocity modulation.

The second type of bunching differs from the first in that the interaction of the beam with the ac field is a continuous process, since the electromagnetic field in this case is produced by some sort of structure that will sustain a traveling wave. The beam in this case moves with an average velocity greater than the velocity of propagation of the wave on the structure, and the energy transfer is a gradual one. One might consider the bunching process to be due
primarily to interaction in the early portion of the structure, and the final energy interchange to occur predominantly in the latter portion of the structure. Since the electron velocity is greater than that of the wave on the structure, it is apparent that the structure must have a rather low phase velocity in order that relativistic electron velocities may be avoided. The reduction of propagation velocity is normally accomplished by means of a periodic structure or by means of a circuit such as a helix, in which the effective velocity of the field with respect to the beam is reduced by causing the wave to travel a considerably longer path between two points than that taken by the electrons.

This distributed type of interaction has been employed in such devices as traveling wave tubes, magnetrons, carcinotrons, backward wave oscillators, and crossed field devices.

In scaling oscillators employing these mechanisms into the low millimeter wavelength range, the inherent difficulties are practically independent of the particular type of device involved. As the dimensions of the associated circuitry are directly proportional to the wavelength of operation, the physical size of the device soon reaches practical limitations imposed by mechanical tolerances, heat dissipation, circuit losses, and cathode operating and starting current densities. (Since the circuit dimensions decrease linearly with wavelength, the required current density must increase at least as the square of the frequency. Increased circuit losses cause the required starting current density in the reflex klystron to increase roughly as the five-halves power of the frequency.)
In spite of the above limitations, electron tubes have been developed which produce energy in the wavelength range below three millimeters. However, the majority of these are inefficient devices having a rather short life. In order to generate energy at millimeter wavelengths and avoid the problem of cathode loading, the idea of utilizing the harmonic content of an electron beam bunched at a lower frequency to excite a circuit which will support an electromagnetic field at some integral multiple of the driving frequency appears attractive. Self-excited frequency multipliers of this type have been investigated by Cornetet\(^1\) and Thurston\(^2\) of the Electron Device Laboratory of The Ohio State University. Cornetet studied a self-excited frequency multiplier utilizing a floating-drift-tube klystron as the fundamental oscillator, while the tube investigated by Thurston utilized a retarding-field oscillator. Both utilized a beam velocity modulated and bunched to produce a fundamental wavelength of one centimeter or longer, but used to excite an appropriate resonator in the four to five millimeter range. Harmonic output powers of the order of tenths of milliwatts were obtained with fundamental output powers of the order of several milliwatts.

The self-excited frequency multiplier overcomes the problem of starting current to a very great extent and thus somewhat reduces the problem of cathode loading and the attendant short life, although the problems with respect to the circuit dimensions and construction at the high output frequencies remain unchanged.

A serious disadvantage of the klystron frequency multiplier which is not predicted by simple klystron theory is the poor conversion
efficiency from the driving frequency to a given harmonic. In a two-gap frequency multiplier of the conventional klystron type or of the similar types investigated by Cormet1 and Thurston2 (which are also essentially two gap multipliers), the actual harmonic power available is far less than that which would be indicated by a Fourier analysis of the current. The first order theory does predict a dependence on signal level and drift angle, but does not indicate the rather serious losses introduced by space-charge debunching. Debunching effects are not particularly serious in conventional klystron amplifiers, but in frequency multipliers the debunching forces produce a decided reduction in the harmonic components present in the beam current. This particular effect is accentuated by the fact that the debunching forces act continuously throughout the drift region, while the bunching force exists only in the interaction gap. The adverse effects of debunching forces in frequency multiplication may be overcome to a certain extent by use of a distributed output circuit. This has been done by Bates3 in a traveling wave frequency multiplier which utilized a low frequency helix for bunching at the fundamental frequency and a high frequency helix for interaction at the harmonic. These two helices are placed in cascade and are designed as rather high gain amplifier circuits. Since amplification occurs both at the fundamental and the harmonic frequencies, the conversion loss inherent in klystron frequency multipliers is avoided, and a conversion gain is realized instead.

One rather noticeable disadvantage for some applications is inherent in the use of broadband output circuitry such as a helix.
Unless the output circuit is highly dispersive so that undesired frequencies may be easily suppressed by adjustment of the beam voltage, several harmonics may be present in the output simultaneously at relatively low orders of frequency multiplication, since the separation between harmonics can quite easily become small compared to the pass-band of the structure for circuits of this type.

It appears that a desirable frequency multiplier should have the following characteristics. First of all, it should employ a mechanism which is not appreciably affected by the adverse effects of space charge debunching. Secondly, continuous interaction in the output circuit should be employed. The third desirable feature is an output circuit selective enough to reject unwanted multiples of the fundamental driving frequency.

**The Fast-Cyclotron-Wave Frequency Multiplier**

A device having (to some extent) all the desirable characteristics previously mentioned is a frequency multiplier utilizing the fast cyclotron wave on an electron beam which is focused by a magnetic field along its axis and subjected to transverse excitation at a frequency near the cyclotron resonance of the focusing field. The concept of space charge waves in electron streams is one generally known. Restoring forces generated in velocity modulation tubes by space charge fields and in transverse field tubes by the focusing field produce a pair of such waves, commonly called the slow and the
The phase velocities of these two waves are

\[ u_{1,2} = u_0 \frac{1}{1 + (\omega_c/\omega)} \]  \hspace{1cm} (1-1)

where \( u_0 \) is the axial velocity of the electron beam, \( \omega_c \) is the resonant frequency of the restoring system, such as the plasma frequency or the cyclotron frequency in the case of magnetic focusing, and \( \omega \) is the frequency of excitation. As \( \omega \) approaches \( \omega_c \), the two propagation velocities become widely separated, and coupling to only one mode becomes a simple matter. The case of interest for the device under consideration is the fast wave which appears in the form of transverse rotational electron motion of constant angular velocity corresponding to the electron cyclotron frequency of the magnetic focusing field. This wave is called the fast cyclotron wave. If the exciting field is tuned to the cyclotron frequency of the focusing field, the electrons will move in phase with the excitation voltage. An electron initially in an accelerating field will remain in an accelerating field, increasing its kinetic energy and thus its radius of rotation, since angular velocity must remain constant. Electrons initially in a decelerating phase of the field will continue to decelerate, giving up energy and decreasing their radii of rotation, eventually moving to the axis of rotation near the initial center of the beam, from which they can be moved only by an accelerating field.

Thus, the electron beam leaves the region of transverse excitation rotating about its original axis, with the kinetic energy of rotation representing the energy delivered to the beam by the exciting
high frequency field. If this beam is then passed through a region consisting of two parallel plates connected to a resonant circuit tuned to the cyclotron frequency of the focusing field and coupled to an external load, the rotating electron beam will induce a current in the plates, and the rotational energy of the beam will be converted to high frequency energy and delivered to the external load. No amplification occurs in such a device, but the energy can be transferred from the input to the output coupler with nearly one hundred percent efficiency if the operating conditions are properly adjusted.

If the two parallel plates representing the output circuit are replaced by a magnetron type resonator tuned to the kth harmonic of the cyclotron frequency and having 2k vane pairs or cavities, the rotating electron beam can induce a harmonic current in the resonator and can move continuously in a retarding phase of the harmonic field. Under these conditions the rotational energy of the electron beam can be converted into radio frequency energy at the harmonic, and for the ideal case of an infinitely thin beam, one hundred percent conversion efficiency could be realized. The actual efficiency obtainable is a function of beam diameter, maximum radius of rotation, order of the harmonic, and beam-to-circuit coupling.

A frequency multiplier of this type has been constructed at the Electron Device Laboratory of The Ohio State University and is currently under investigation. Figure 1(a) shows the arrangement of the electron gun and associated circuitry, while Figure 1(b) shows the envelope of the rotating electron beam. In Figure 1(a), K is the cathode of the electron gun, F is the focusing electrode, A is the
(a) Fast Cyclotron Wave Frequency Multiplier

(b) Envelope of The Rotating Electron Beam

Figure 1.
anode or accelerating electrode, and $C_1$ is the input coupler driven
by the source $S$, while $C_0$ represents the output coupler or harmonic
circuit, which is connected to a load represented by $R_L$; $C$ is the
collector or target which intercepts the electron beam after it has
delivered its energy of rotation to the output coupler, while the
arrow $B$ indicates the direction of the magnetic field.

In Figure 1(b) the changes in the path of the electron beam as
it interacts with the transverse high frequency fields are indicated.
As the electrons move from the electron gun anode to the entrance of
the first interaction space $C_{I1}$, the trajectories remain relatively
uniform, since this region is relatively free of high frequency field.
During the passage from $C_{I1}$ to $C_{I2}$, the electrons experience cyclon
tron acceleration due to interaction with the high frequency field,
and the high frequency energy delivered to the beam is stored as ro-
tational kinetic energy by the electrons. The increasing radius of
rotation during this passage through the input coupler is indicated
in the figure. During the passage from $C_{I2}$ to $C_{O1}$, the radius of
electron rotation remains constant, due to the absence of high fre-
quency field in this region. As the rotating beam passes through the
output coupler, the rotational motion of the electrons causes a cur-
rent to be induced in the output coupler which flows in the load $R_L$,
producing a retarding electric field which decelerates the electrons,
converting the rotational kinetic energy back to high-frequency energy
and delivering it to the load. This process is indicated in Fig-
ure 1(b) by a decrease in the radius of rotation as the beam passes
through the output coupler. After delivering their rotational energies to the output circuit, the electrons then strike the collector and are removed from the interaction space.

A complete analysis of the interaction mechanism is contained in the following chapters.
References


CHAPTER II
GENERATION OF FAST CYCLOTRON WAVES

Analysis of the Interaction Mechanism

The production of a fast cyclotron wave on a magnetically fo-cused electron beam involves principles which are fundamental in na-ture and quite familiar in certain applications. A variety of beam and circuit arrangements can be employed, but the fundamental inter-action mechanism is the same in every case.

One of the circuits most commonly employed for excitation of fast cyclotron waves on electron beams moving in axially directed magnetic fields is the Cuccia coupler. In its simplest form, this coupler may be represented by two parallel plates, driven by an ex-ternal high frequency source in such a way that a time-varying elec-tric field exists between them, normal to the axis of the electron beam. This arrangement is shown in Figure 2.

In order to determine the nature of the interaction, one might consider the equations of electron motion for this particular case. The electrons enter the input region with an axial velocity in the z-direction due to a constant accelerating potential $V_b$. In this re-gion, the beam is focused by a z-directed component of magnetic field of flux density $B$. In the figure shown, the two plates are oriented parallel to the y-z plane and are driven by a high frequency voltage of the form $v(t) = V_1 \cos \omega t$. Thus, an x-directed component of electric field exists between the two plates, and if one assumes the
Figure 2.

Cuccia Coupler
length and width of the plates to be large compared to the spacing d, this field is of uniform intensity $E_1$, where $E_1 = -\frac{V_1}{d}$.

Under the conditions stated above, the equations of electron motion may be formulated.

The equations of motion associated with each of the three axes are as follows.

$$\ddot{x} = -eB\sin\omega t - eB\dot{y} \quad (2-1)$$
$$\ddot{y} = eB\dot{x} \quad (2-2)$$
$$\ddot{z} = 0 \quad (2-3)$$

where $E_1$ is the amplitude of the time varying ac field, $B$ is the flux density of the magnetic focusing field, $e$ is the electronic charge, and $m$ is the electronic mass. For purposes of simplification, it may be assumed that the beam is driven at cyclotron resonance, which is the most favorable operating condition. This requires that

$\omega = \omega_c = eB/m$. Then Equations (2-1) and (2-2) can be combined to yield

$$\dddot{x} + \omega_c^2 \dot{x} = \left(\frac{e}{m}\right)\omega_c E_1 \sin\omega_c t \quad (2-4)$$

or letting $e/m = \eta$, the electronic charge to mass ratio,

$$\dddot{x} + \omega_c^2 \dot{x} = \eta\omega_c E_1 \sin\omega_c t \quad (2-5)$$

and similarly,

$$\dddot{y} + \omega_c^2 \dot{y} = -\eta\omega_c E_1 \cos\omega_c t \quad (2-6)$$

In order to obtain solutions of Equations (2-5) and (2-6) one may consider an electron entering the coupler at time $t = t_1$ with negli-
gible initial transverse accelerations and velocities. This imposes
the following boundary conditions:

\[ \begin{align*}
\text{At } t = t_1, \quad & i = 0, \quad \ddot{i} = 0 \\
& \dot{y} = 0, \quad \ddot{y} = 0.
\end{align*} \]

Solution of Equation (2-5) for \( \dot{x} \) as a function of \( t \) yields

\[ \dot{x} = -\frac{\gamma E_1}{\omega_c^2} \left[ \frac{\sin \omega_c (t - t_1)}{\omega_c} + (t - t_1) \cos \omega_c t \right]. \tag{2-7} \]

A similar solution may be obtained for \( \dot{y} \).

It may be noted that the second term in the brackets increases
linearly with increasing time. Therefore, only the second term is
significant and is to be retained for large transit angles through
the coupler (i.e., for \( \omega_c (t - t_1) \gg 1 \), which is the case for the ma-

majority of fast cyclotron wave devices).

The particular integral solutions for \( \dot{x} \) and \( \dot{y} \) may then be in-
tegrated to obtain expressions for \( x \) and \( y \) of the form

\[ \begin{align*}
\dot{x} &= -\frac{\gamma E_1}{2 \omega_c} (t - t_1) \sin \omega_c t \\
\dot{y} &= +\frac{\gamma E_1}{2 \omega_c} (t - t_1) \cos \omega_c t.
\end{align*} \tag{2-8} \tag{2-9} \]

The above equations indicate that the \( x \) and \( y \) coordinates de-
scribe harmonic motion of linearly increasing amplitude as the elec-
trons travel through the coupler.

Qualitatively, it may be stated that the electrons lie on the
line directrix of a cone, and as the electron beam leaves the coupler,
it has a rotational motion of angular velocity $\omega_c$, while the radius of rotation is dependent on the transit time $(t-t_1)$ and the electric field intensity $E_1$.

The maximum radius of rotation $r$ corresponds to the amplitude of either the $x$ or $y$ coordinate, or

$$r = \frac{N E_1}{2 \omega_c} (t-t_1). \quad (2-10)$$

A similar result may be obtained as a special case of interaction of a rotating electron beam with a tangential electric field at any given harmonic of the cyclotron frequency. This will be reserved for a later section.

The rotational kinetic energy of the electrons represents the energy delivered to the beam by the high frequency field. The high frequency driving power required may be determined by noting that the rate of energy storage is simply the product of the energy stored by a single electron and the number of electrons leaving the coupler per unit time.

The rotational kinetic energy stored by a single electron is given by

$$W_e = \frac{m \omega_c^2 r^2}{2}, \quad (2-11)$$

where $\omega_c$ is the cyclotron frequency, $r$ is the radius of rotation, and $m$ is the electronic mass.

The energy stored by $N$ electrons is simply

$$N W_e = \frac{N m \omega_c^2 r^2}{2}, \quad (2-12)$$
but $N$ is given by
\[ N = \frac{I_0}{e}, \quad (2-13) \]

where $I_0$ is the beam current in amperes and $e$ is the electronic charge in coulombs. The rate at which energy is stored by the beam, or the power delivered from the source to the beam is thus given by
\[ P_{\text{in}} = N \omega_e = \frac{I_0 m \omega_e^2 r^2}{2e} , \quad (2-14) \]

where all quantities are as previously defined. Thus, the input power to the beam is a function of beam current, operating frequency, and the radius of rotation of the beam as it leaves the interaction region of the input coupler. It may be noted that the energy storage capability of the beam can be related quite simply to the required field intensity $E_1$ and the transit time $\tau = (t-t_1)$. Squaring Equation (2-10), one obtains
\[ r^2 = \frac{n^2 E_1^2 \tau^2}{4 \omega_e^2} . \quad (2-15) \]

Substitution of the right-hand side of Equation (2-15) in Equation (2-14) yields
\[ P_{\text{in}} = \frac{I_0 \alpha E_1^2 \tau^2}{8m} . \quad (2-16) \]

Thus, in order to store energy at a given rate in the rotational motion of the electron beam, the product of the electric field intensity and the transit time must be a constant.

Since $E_1 = -V_1/d$, $P_{\text{in}}$ is also given by
\[ P_{\text{in}} = I_0 e V_1^2 \tau^2/8m d^2 . \quad (2-17) \]
The transverse electronic conductance of the beam may be defined by noting that

\[ P_{\text{in}} = \frac{V^2}{2} G_b / 2, \quad (2-18) \]

where \( G_b \) is the conductance presented by the beam to the high frequency field, and thus by equating the right-hand sides of Equations (2-17) and (2-18) one may obtain

\[ G_b = \frac{I_0 e r^2 / 4 m d^2}{2}. \quad (2-19) \]

Now if \( l_1 \) is the length of the interaction region of the input coupler, then

\[ \tau = \frac{l_1}{\sqrt{2(e/m)V_b}}, \quad (2-20) \]

where \( V_b \) is the accelerating potential of the electron gun.

Substitution of Equation (2-20) in Equation (2-19) yields an expression for the transverse conductance presented by the beam to the high frequency field in terms of the dc beam conductance and the dimensions of the coupler, or

\[ G_b = \frac{I_0 l_1^2}{8 V_b d^2} = \frac{G_0 l_1^2}{8 d^2}. \quad (2-21) \]

The appearance of \( l_1 \) and \( d \) in Equation (2-21) is physically reasonable, as one would expect the conductance to be greater for a longer interaction space, while the induced current would be reduced for larger values of \( d \).

If the input coupler is to be reasonably efficient, at least
one-half the high frequency energy delivered by the driving source should be stored on the beam. This requires that the beam conductance be at least equal to the circuit loss conductance when referred to the interaction region of the coupler. This condition may be approximated by proper design of the circuit and the electron gun, and can normally be realized with a reasonable amount of success in practical tube designs.

Effects of Finite Beam Diameter

In the ideal case, the electron beam passing through the input coupler would be infinitely thin, and the maximum radius of rotation of the electron beam would be \( d/2 \), or one-half the spacing between the plates. Thus, the saturation input power level, or the level beyond which the electrons would graze the coupler and be collected on the plates would be given by

\[
P_{\text{max}} = \frac{I_0 \omega_c^2 d^2}{8 \eta}
\]

(2-22)

In the practical case, the beam has a finite radius \( r_b \), and the electrons in the beam will graze the plates when the radius of rotation is \( (d/2 - r_b) \) as shown in Figure 3, and the saturation input power is given by

\[
P_{\text{max}} = \frac{I_0 \omega_c^2 (d - 2r_b)^2}{8 \eta}
\]

(2-23)

which may be obtained by substitution of the appropriate value for
Grazing Orbit of the Electron Beam for Saturation Input

Figure 3.
the radius of rotation in Equation (2-14). In both Equations (2-22) and (2-23), \( \eta \) is the electronic charge-to-mass ratio.

Thus, the maximum input power to the coupler is limited to a value less than that given by Equation (2-22). Examination of Equation (2-23) indicates that the maximum input power before interception of the beam occurs approaches zero as the beam diameter approaches the spacing between the plates, and thus the coupler becomes useless if the beam fills the interaction space entirely.

Another factor of significance is the signal level at which the input coupler is driven. It is customary to define a signal level in conventional electron tubes in terms of a normalized circuit voltage. It may be noted from Equation (2-10) that the radius of rotation of the electron beam is proportional to the magnitude of the applied high frequency field or ac circuit voltage. Thus, for purposes of convenience, the signal level may be expressed in terms of the radius of rotation of the beam as it leaves the coupler. This quantity may be normalized with respect to the maximum radius of rotation for the ideal case, which is simply one-half the spacing between the couplers. The signal level is then given by

\[
a = \frac{r_1}{d/2} = \frac{2r_1}{d},
\]

where \( r_1 \) is the radius of rotation of the beam as it leaves the coupler, and \( d \) is the coupler spacing. In order to achieve the maximum degree of coupling to the output circuit, the diameter of the aperture through which the beam passes in the output circuit must be equal to the spacing \( d \) between the plates in the input circuit. Thus,
if the radius of the output circuit aperture is \( r_a \), then \( d = 2r_a \) and \( a \) is given by

\[
\alpha = \frac{2r_1}{d} = \frac{r_1}{r_a} .
\]  

(2-24b)

Thus, the amount of power stored as rotational energy in the electron beam may be expressed in terms of the signal level \( a \), beam current \( I_0 \), angular frequency \( \omega_c \), and the dimensions of the circuit. One may rewrite Equation (2-14) in the form,

\[
P_{in} = \frac{a^2 \omega_c^2 d^2}{8 \eta} = \frac{a^2 I_0 \omega_c^2 r_a^2}{2 \eta} ,
\]  

(2-25)

where \( \eta \) is the electronic charge-to-mass ratio as before.

**Efficiency of the Input Coupler**

One of the primary difficulties encountered in storing high frequency energy on the beam is that of obtaining a favorable ratio between the circuit conductance and the electronic conductance of the beam as given by Equation (2-21). Since the driving source must deliver power to the circuit loss conductance as well as to the beam, the total high frequency power input is given by

\[
P_{in} = \frac{V^2 (G_b + G_c)}{2} ,
\]  

(2-26)

and the efficiency with which the high frequency energy is transmitted to the beam is given by

\[
\eta = \frac{G_b}{G_b + G_c} = \frac{1}{1 + (G_c/G_b)} .
\]  

(2-27)
For the type of coupler employed in the multiplier under consideration, $G_c$ may be calculated with reasonable accuracy as though the coupler were a short circuited parallel plate transmission line operating in the TEM mode. The coupler is shown in Figure 4.

The conductance presented to a transverse $x$-directed electric field at the center of the coupler is approximately given by\textsuperscript{2,3,4}

$$G_c = \frac{2l_1}{d (R_0) Q_{eff}}$$  \hspace{1cm} (2-28)

where $l_1$ is the length of the interaction region parallel to the axis of the electron beam, $d$ is the spacing between the plates, $R_0$ is the intrinsic resistance of free space, and $Q_{eff}$ is the loaded $Q$ of the circuit. (Ideally, $G_c$ would be lower by a factor of $\pi/4$, but fringing fields tend to increase it from the ideal figure.) As indicated in the figure, the width of the coupler is a half wavelength at the frequency of excitation, which corresponds to the cyclotron frequency of the magnetic focusing field.

If one substitutes the expressions for $G_c$ and $G_b$ given by Equations (2-28) and (2-21) into Equation (2-27), the resulting expression for $\nu_1$ is of the form

$$\nu_1 = \frac{1}{1 + \frac{16d}{Q_1 G_0 R_0 Q_{eff}}}$$  \hspace{1cm} (2-29)

For high efficiencies to be realized, the second term in the denominator must be small compared to unity, which is a condition often difficult to realize.
Figure 4.
As an example, one might consider the following case:

\[ P_{in} \text{ (power delivered to beam)} = 100 \text{ watts} \]
\[ a = 0.8 \]
\[ d = 2.5 \text{ mm} \]
\[ l = 10 \text{ mm} \]
\[ \omega_c = 2\pi \times 10^{10} \]
\[ I_0/v_b^{3/2} = 0.5 \times 10^{-6} \]
\[ r_e/r_b = 5 \]

For the above conditions, the required beam current is 8.97 milliamperes, which requires a beam voltage of 685 volts, and the beam conductance is thus

\[ G_0 = \frac{I_0}{v_b} = \frac{8.97 \times 10^{-3}}{685} = 1.315 \times 10^{-5} . \]

The effective \( Q \) that may normally be realized in this type of circuit is of the order of 100, while \( R_0 = 120 \, \text{n} \). If these values are substituted into Equation (2-30) a value of 0.11 is obtained for \( \sqrt{1} \). This indicates rather inefficient coupling, and a driving power of 910 watts would be required in this case to deliver 100 watts to the beam. The efficiency is somewhat improved at higher power levels if the beam current is adjusted to the optimum value in each case. This is due to the fact that the beam conductance increases with increasing beam current if the perveance of the electron gun is a constant. The efficiency of the coupler as a function of the RF power on the beam is plotted in Figure 5, while Figure 6 shows the power delivered to the beam as a function of driving power. Two cases are indicated in
Figure 5.

Coupler Efficiency vs. Beam Power

- $Q_i = 10 \text{ mm}$
- $d = 2.5 \text{ mm}$
- $\alpha = 0.8$

1. Permeance $= 5 \times 10^{-6}$
2. Permeance $= 1.0 \times 10^{-6}$

$\omega_c = 2\pi \times 10^{10}$

Power Delivered to Beam in Watts
### Beam Power vs Exciting Power

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<th>Exciting Power</th>
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#### Notes
- $L_1 = 10 \text{ mm}$
- $d = 2.5 \text{ mm}$
- $\alpha = 0.8$
- $P_{\text{perv}} = 0.5 \times 10^{-6}$
- $P_{\text{perv}} = 1.0 \times 10^{-6}$
- $\omega_c = 2\pi \times 10^{10}$
the plot. In both instances, the physical dimensions of the coupler are unchanged, but in one case, the microperveance of the electron gun is 0.5, and in the second case the microperveance has been increased to 1. The higher perveance gun results in an improvement in efficiency, since $G_0$ is greater at every power level. The most significant improvement might appear to be obtained from a change in the ratio $\ell_1/d$, but the length $\ell_1$ is normally limited to a value smaller than the width of the waveguide carrying the fundamental signal, which is 22.5 mm for X-band or 3 cm excitation. The value of $d$ should be at least 4 or 5 times the diameter of the electron beam employed. Since the electron gun employed in the experimental tube has an anode aperture of about 0.25 mm, the maximum beam diameter is probably about 0.5 mm, allowing for transverse oscillations of the electron trajectories under no-signal conditions. Thus, a minimum practical value for $d$ would be about 2 mm. The saturation signal level $\alpha$ for this particular case would be 0.75, due to the change in the ratio of $d$ to the beam diameter. The required beam current is 0.16 ma/watt of RF power on the beam. If one calculates the beam conductance as a function of RF power on the beam and substitutes the values obtained into Equation (2-29) with $\ell_1 = 22.5$ mm and $d = 2$ mm, the curves of coupler efficiency shown in Figure 7 will be obtained. Again, the characteristics were calculated for electron guns having microperveances of 0.5 and 1, which were the two cases shown in Figure 5. The longer interaction space definitely improves the interaction efficiency, although some difficulty may be experienced in the
Figure 7 - Coupler Efficiency vs. Beam Power

- $l_s = 225 \text{ mm}$
- $d = 2 \text{ mm}$
- $\alpha = 0.75$

1. Pervance $= 0.5 \times 10^{-4}$
2. Pervance $= 1.0 \times 10^{-4}$

$\omega_c = 2\pi \times 10^{10}$

Power Delivered to Electron Beam in Watts

Coupler Efficiency
mechanical design of the circuit when an attempt is made to realize this condition experimentally.

Since the magnetic fields required for electron cyclotron resonance in the centimeter region are of the order of a few kilogauss and must be maintained throughout the interaction space including the output circuit as well as the input coupler, it is desirable that the total length of the interaction region be as short as possible to reduce the size of the associated magnetic circuitry. Any practical design must be a compromise between the size of the required magnet or solenoid and the coupler efficiency as indicated in Figures 5 and 7.

In order to make an intelligent choice of coupler length, a plot of coupler efficiency as a function of interaction length $l$ is a useful aid. Figure 8 gives the coupler efficiency as a function of interaction length for $d = 2\, \text{mm}$, $a = 0.75$, $\omega_c = 2\pi \times 10^{10}$ and $P_{\text{in}} = 100\, \text{watts}$ (power on the beam). The curves shown are for micro-pervances of 0.5 and 1 as shown in the preceding calculations. A mechanically convenient length for the interaction region is of the order of 10 to 12 mm, so practical coupler efficiencies for the pervances utilized in the calculations are in the range from 15.6 to 25.2 percent. This means that the driving power required to produce 100 watts of high frequency power on the beam is between 640 and 400 watts. For experimental purposes this amount of power may be obtained by attenuating the output of a pulsed magnetron, although the performance of the multiplier should be quite similar for either cw or pulsed excitation. Some difficulty is to be expected with magnetron excita-
Figure 8. Coupler Efficiency vs Coupler Length

- **d = 2 mm**
- **l, variable**
- **α = 0.75**
- **P_in = 100 watts on beam**
- **ωC = 2π x 10^8**
- **Pervane = 0.5 x 10^-6**
- **Pervane = 1.0 x 10^-6**
tion due to the fact that the frequency stability of the source is a primary factor in determining the overall performance of the tube. However, the power handling capabilities of the tube should be the same unless the beam is pulsed as well as the drive. The power input to the beam, and thus the energy storage capability, is limited by the dissipation capability of the collector, since very little of the beam is intercepted by the circuit. For high power operation, the collector may be water cooled.

One other approach to improving the efficiency of the Cuccia coupler may be suggested by a careful examination of Equation (2-29). It is possible by rearrangement of this equation to obtain an expression for the dc beam conductance required to realize a given efficiency for a particular set of circuit dimensions. This expression has the form

$$G_0 = \frac{16 \, d}{L_1 \, R_0 \, Q_{eff}} \frac{\nu_1}{(1 - \nu_1)}.$$  \hfill (2-30)

For the dimensions used in the calculations for Figures 5 and 6, the coupler efficiency as a function of dc beam conductance is plotted in Figure 9. Theoretically it is possible to obtain a given efficiency at any power level by designing an electron gun of sufficiently high pervance to obtain the required beam current at a beam conductance corresponding to that associated with the given efficiency. However, a sample calculation will reveal the fact that reasonable efficiencies can be obtained only at medium to high power levels. The tube considered in the plot of Figure 9 has a current requirement of
Figure 9.

Coupler Efficiency vs. Beam Conductance

$\ell_1 = 10 \text{ mm}$

$d = 2.5 \text{ mm}$
0.16 mA/watt. The required beam current for a beam input power of 100 milliwatts would be 16 microamperes. If one attempted to realize a coupler efficiency of 50 percent at this power level, the required beam voltage would be 0.19 volt if one were to obtain the dc beam conductance of 85 micromhos indicated in the figure. A beam power of one watt would likewise require a beam voltage of 1.9 volts, which is still of the order of magnitude of the thermal emission velocities of the electrons. An input power of 10 watts would require a beam current of 1.6 milliamperes and a beam voltage of 19 volts, which is within the realm of possibility, even though the required microperveance of the electron gun would be 19.5. Higher power inputs can be handled with greater ease. At the 200 watt level, the electron gun would be required to deliver 32 mA at 380 volts, which corresponds to a microperveance of 4.35. Some difficulty may be experienced in realizing high values of perveance in a magnetically shielded electron gun, although the currents involved in the sample calculations are small enough that it is feasible to design guns with a rather low convergence and still avoid excessive cathode loading.

If an attempt is made to operate the tube at lower power levels by designing for a lower efficiency, some gain in operating range is realized, but it is marginal in nature. If an electron gun is designed to permit the coupler to operate at 10 percent efficiency, operation at the 100 milliwatt level would still not be feasible. The required dc beam conductance would be 8.9 micromhos, which would require a beam voltage of 1.7 volts for the required current of 16 microamperes. This voltage is again of the order of magnitude of the
emission velocities of the electrons, and does not represent a practical operating condition. An increase in the beam power level to one watt multiplies both quantities by 10, and the resulting requirement of 0.16 milliamperes at 17 volts is well within the realm of possibility. However, the drive requirement from the source is 10 watts, which represents a very slight advantage in operation over operation of the previous tube at a beam power level of 10 watts. It would seem reasonable to conclude that practical operation of the Cuccia coupler in the region corresponding to electron cyclotron frequencies of 10 kilomegacycles and higher must involve driving powers of the order of 10 watts or more. At higher excitation frequencies, the beam to circuit coupling problem becomes even more severe, so a pronounced decrease in realizable efficiencies may be expected at higher driving frequencies. It is unlikely that the Cuccia coupler in its simplest form would be satisfactory at frequencies higher than about 30 kilomegacycles.

Little attention has been given to the dc efficiency of the beam, but it may be noted in each numerical example that the high frequency power on the beam is in excess of the dc power on the beam, unlike conventional traveling wave devices. The only beam parameters appearing in Equation (2-25), which defines the power handling capability of the beam are the current $I_0$ and the signal level $\alpha$, which is limited by the beam diameter. In general, the interaction efficiency is improved at higher beam conductances as indicated by Figure 8, so it would appear that the ac and dc efficiency improve.
simultaneously. One exception to this rule occurs in certain types of fast cyclotron wave quadrupole amplifiers which employ a space varying dc pumping field rather than a time varying field, in which case the dc power on the beam must always exceed the high frequency power. However, this limitation does not appear in the case of the fast cyclotron wave frequency multiplier (see Appendix A).
References


CHAPTER III
THE HIGH FREQUENCY POTENTIAL DISTRIBUTION
IN THE INTERACTION SPACE

The harmonic resonator or output circuit in the Fast-Cyclotron-Wave Frequency Multiplier is quite similar to the multiple resonator structures employed in cylindrical magnetrons. One significant difference lies in the fact that the cathode at the center of the cylindrical magnetron forms a ground plane for the high frequency field of the circuit, and thus modifies the ac potential distribution in the interaction space. In addition, the nature of the high frequency field in the region remote from the circuit is not of such great importance in magnetrons, as most of the interaction between the electron stream and the ac field is confined to a region near the resonant structure itself. In the Fast-Cyclotron-Wave Frequency Multiplier, as the kinetic energy of electron rotation is converted to high frequency energy, the electrons move away from the circuit, and therefore the variation of the high frequency field as a function of radial position is of primary importance. The actual physical configuration of the individual resonators has little or no effect on the nature of the potential distribution or interaction mechanism. However, the circuit must be periodic in the angular direction, and of such periodicity that the rotating electron beam passes through k wavelengths of the kth harmonic field for each complete rotation of the electron beam at the fundamental or cyclotron frequency. Thus,
the angular field variation at the kth harmonic must be of the nature of \( \cos k\theta \) or \( \sin k\theta \), in order for k wavelengths to exist in one revolution around the structure. This will be approximately the case if the circuit consists of 2k uniformly spaced interaction gaps coupled to 2k corresponding resonators, which are tuned to a frequency \( \omega = k\omega_c \), where \( \omega_c \) is the electron cyclotron frequency, and k is an integer denoting the order of the frequency multiplication desired. A typical slot and vane resonator of this type is shown in Figure 10 for the case \( k = 4 \). In the figure, the radius of the aperture \( r_a \) is defined as the distance from the axis of the electron beam to a vane tip. Dimension b is approximately one-quarter wavelength at the desired output frequency. The dimension h is determined by geometrical limitations and is limited to a value

\[
h \leq \pi r_a/k. \tag{3-1}\]

The equality of course would hold only for the case of vanishingly thin vane tips.

In order to determine the potential distribution in the interaction space, it will be assumed that all quantities vary harmonically with time. If the time dependence is eliminated from the equation, the high-frequency potential distribution can be obtained by means of a quasi-static solution of La Place's equation in cylindrical coordinates. Thus, the quasi-static potential distribution, \( V(r, \theta) \) must satisfy

\[
\frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{1}{r^2} \frac{\partial^2 V}{\partial \theta^2} = 0. \tag{3-2}\]
FIGURE 10

FOURTH HARMONIC RESONATOR FOR THE FAST CYCLOTRON WAVE FREQUENCY MULTIPLIER
It is assumed that there is no potential variation along the z axis, which will be the case if the individual resonators act as shorted quarter-wave transmission lines operating in the TEM mode.

The high frequency potential at the origin must be zero, as there is an even number of vanes having equal potentials of opposing polarity at any given instant of time. If an instant of time is chosen when all circuit potentials are going through a maximum, and the line $\Theta = 0$ is chosen through the center of an interaction gap, $V(r, \Theta)$ must satisfy the following boundary conditions.

(a) $V(r, \Theta) = R(r)T(\Theta)$ (variables separable)
(b) $V(0, \Theta) = 0$
(c) $V(r_a, \Theta) = f(\Theta)$
(d) $V(r_a, 2mA/k) = 0 \quad m = 0, 1, 2, 3, \ldots$
(e) $V[r_a, (\lambda m + 1)/2k] = + V_{k/2} \quad m = 0, 1, 2, 3, \ldots$
(f) $V[r_a, (\lambda m - 1)/2k] = - V_{k/2} \quad m = 0, 1, 2, 3, \ldots$

Substitution of condition (a) in Equation (3-2) requires

$$R^nT + (1/r)R'T + (1/r^2)RT'' = 0 \quad (3-3)$$

or, dividing by RT,

$$(R^n/R) + (1/r)(R'/R) + (1/r^2)(T''/T) = 0 \quad (3-4)$$

Multiplying by $r^2$ and rearranging, one obtains

$$\frac{r^2R^n + rR'}{R} = - \frac{T''}{T} \quad (3-5)$$

Since the left-hand side of Equation (3-5) is a function of r
only, and the right-hand side is a function of $\theta$ only, both sides must be equal to some constant which will be denoted as $\mathcal{K}^2$.

Thus, from the right-hand side of Equation (3-5) one obtains

\[ T^* = -\mathcal{K}^2 T, \quad (3-6) \]

which has the solution

\[ T = A \sin \mathcal{K} \theta + B \cos \mathcal{K} \theta, \quad (3-7) \]

where $A$ and $B$ are constants determined by the boundary conditions.

Now condition (d) requires that $B = 0$, while (d) and (e) may be satisfied if $\mathcal{K} = k$. Therefore,

\[ T = A \sin k\theta, \quad (3-8) \]

where $A$ is yet to be determined.

If $\mathcal{K}^2 = k^2$, then the left-hand side of Equation (3-5) yields

\[ r^2R'' + rR' - k^2R = 0, \quad (3-9) \]

which has the solution

\[ R = C_1 r^k. \quad (3-10) \]

Then from (a),

\[ V(r, \theta) = C_1 r^k \sin k\theta, \quad (3-11) \]

and (d) and (e) are satisfied if $C_1A = V_k/2r_a^k$, so that

\[ V(r, \theta) = \frac{V_k}{2} \left(\frac{r}{r_a}\right)^k \sin k\theta. \quad (3-12) \]

This is the general form of the potential distribution in the inter-
action space which will be used to determine the general interaction relations in Chapter IV.

The primary interaction will be considered to be with the $\theta$-directed component of electric field, which is given by

$$E_{\theta} = -\frac{1}{r}(\partial V/\partial \theta),$$  \hspace{1cm} (3-13)

which is of the form

$$E_{\theta} = \frac{k V_k}{2 r_a} \left( \frac{r}{r_a} \right)^{k-1} \cos k \theta.$$  \hspace{1cm} (3-14)

Thus, the tangential force acting on the electrons decreases as $(r/r_a)^{k-1}$, which indicates that the coupling between the electron beam and the resonator is severely reduced at the higher harmonics, since the field associated with a given value of $r$ and thus a given power level will be considerably less.

Qualitatively, it might be expected that the interaction efficiency will be reduced at higher orders of multiplication. This effect is demonstrated in the general analysis of the interaction mechanism in the following chapter.
CHAPTER IV

INTERACTION BETWEEN THE ROTATING ELECTRON BEAM
AND THE HARMONIC CIRCUIT

General Interaction Mechanism

As discussed in previous sections, the output circuit utilized in the FCFM is similar to that employed in cylindrical magnetrons. In the general case the circuit for the kth harmonic will be some variation of a slot and vane structure and will have 2k cavities, or k vane pairs. Under operating conditions, the rotating electron beam moves in such a way that it interacts with the tangential electric field component of the n-mode of the circuit. If the resonators of the structure are tuned to the kth harmonic of the cyclotron frequency of the magnetic field, then synchronism will exist between the rotational motion of the electron beam and the electric field of the structure. This will allow the electron beam to move in a continuously retarding phase of the field as it will pass one resonator every half cycle, and will permit continuous conversion of the rotational kinetic energy of the beam to high frequency electromagnetic energy. As the rotational energy is delivered to the high frequency field, the radius of rotation decreases, and in the ideal case of an infinitely thin beam, all the rotational energy of the beam could be delivered to the circuit. In this case, one hundred percent conversion efficiency from the fundamental to the harmonic should be possible. In the practical case, the efficiency is a function of the
beam and circuit dimensions as well as the circuit losses and signal level, which will be shown in the following sections.

As stated previously, the rotational energy of the beam is converted to high frequency energy by continuous motion in a retarding phase of the high frequency field. This situation is illustrated in Figure 11. The four electrodes of alternating polarity represent the vanes of a second harmonic output circuit. The dotted arrows indicate the direction of the force on the electron beam corresponding to the indicated polarity of the field at a given instant of time. The cross-hatched circle in the first quadrant represents the electron beam in cross-section. The small curved arrow marked $\omega_e$ indicates the direction of rotation of the beam, which is against the retarding polarity of the field. One half cycle later, the position of the beam will have changed by ninety degrees, but all polarities and lines of force will have reversed at this time, so that the beam will still be moving in a retarding field, and thus, the rotational kinetic energy will continue to be delivered to the high frequency field and converted to high frequency energy.

In the general case, it is possible to make a first order calculation of the rate of energy conversion, and determine the energy delivered as a function of the transit angle through the harmonic resonator. The high frequency potential distribution in the interaction space may be expressed as a function of polar coordinates $r$ and $\theta$, and is of the form

$$V(r, \theta) = (V_N/2)(r/r_0^2) \cos k\omega t \sin k\theta, \quad (h-1)$$
Rotating Beam in a Decelerating Field

FIGURE II
where \( V_k \) is the maximum value of the time varying voltage existing between adjacent vanes, \( r_a \) is the aperture radius or distance from the center of the interaction space to a vane tip, and \( k \omega \) is the frequency of the time varying voltage induced in the resonators by the rotating electron beam.

An exact solution of the interaction between the electron beam and the time varying field of the harmonic circuit is rather involved, but a reasonable first order solution can be obtained by utilizing the energy relationships involved.

The rotational kinetic energy of an electron is given by

\[
W_e = \frac{m \omega_c^2 r^2}{2}, \tag{4-2}
\]

where \( m \) is the electronic mass, \( \omega_c \) is the angular frequency of the rotational motion or the cyclotron frequency, and \( r \) is the radius of rotation.

As the radius of rotation of the electron changes by an incremental amount \( dr \), the incremental energy change which occurs is of the form

\[
dW = m \omega_c^2 r \, dr. \tag{4-3}
\]

Since the change in rotational kinetic energy is due to interaction between the moving electron and the time varying field of the circuit, the incremental energy change may be expressed in terms of the work done on the electron by the field.

In this case, it will be assumed that the primary interaction
is with the tangential or $\theta$-directed component of electric field.

Then one may write

$$dW = F \cdot ds = -e \frac{1}{r} \frac{\partial V(r, \theta)}{\partial \theta} \cdot r \, d\theta$$

$$= -e \frac{\partial V}{\partial \theta} \cdot d\theta.$$  \hspace{1cm} (4-4)

Now

$$\frac{\partial V}{\partial \theta} = \frac{V_k}{2} \left( \frac{r}{r_a} \right)^k k \cos k\omega t \cos k\theta,$$ \hspace{1cm} (4-5)

and

$$m\omega_c^2 r \, dr = -e \frac{V_k}{2} \left( \frac{r}{r_a} \right)^k k \cos k\omega t \cos k\theta \, d\theta.$$ \hspace{1cm} (4-6)

The rate at which this energy interchange occurs is given by

$$m\omega_c^2 r \frac{dr}{dt} = -e \frac{V_k r_k}{2 r_a^k} k \cos k\omega t \cos k\theta \frac{d\theta}{dt}.$$ \hspace{1cm} (4-7)

Now $\theta = \omega_c t$, where $\omega_c$ is the cyclotron frequency of the magnetic field, and thus $k\theta = k\omega_c t$, while $d\theta/dt = \omega_c$, a constant.

Substitution of these expressions in Equation (4-7) yields

$$m\omega_c^2 r \frac{dr}{dt} = -e \frac{V_k r_k}{2 r_a^k} k \omega_c \cos k\omega_c t \cos k\omega_c t,$$ \hspace{1cm} (4-8)

which may be rearranged to obtain the relation

$$\frac{dr}{r^{k-1}} = -e \frac{V_k k}{2 r_a^k \omega_c} \cos k\omega t \cos k\omega_c t \, dt.$$ \hspace{1cm} (4-9)
Integration of Equation (4-9) yields

$$\int_{r_1}^{r} \frac{dr}{r^{k-1}} = -\frac{r V_k k}{2r_a k \omega_c} \int_{t_1}^{t} \cos k \omega t \cos k \omega_c t \, dt, \quad (4-10)$$

where \( r \) and \( r_1 \) correspond to the radii of rotation of the electron at times \( t \) and \( t_1 \), respectively.

For the case of synchronous interaction, \( \omega = \omega_c \), and Equation (4-10) has the form

$$\int_{r_1}^{r} \frac{dr}{r^{k-1}} = -\frac{r V_k k}{2r_a k \omega_c} \int_{t_1}^{t} \cos^2 k \omega_c t \, dt, \quad (4-11)$$

or

$$\int_{r_1}^{r} \frac{dr}{r^{k-1}} = -\frac{r V_k k}{2r_a k \omega_c} \int_{t_1}^{t} (1 + \cos 2k \omega_c t) \, dt, \quad (4-12)$$

and

$$\int_{r_1}^{r} \frac{dr}{r^{k-1}} = -\frac{r V_k k}{2r_a k \omega_c} \left[ t + \frac{\sin 2k \omega_c t}{2k \omega_c} \right]_{t_1}^{t}. \quad (4-13)$$

If \( t - t_1 \) involves an integral number of cycles so that the second term on the right-hand side contributes nothing, the integration yields

$$\int_{r_1}^{r} \frac{dr}{r^{k-1}} = -\frac{r V_k k}{2r_a k \omega_c} \left[ t - t_1 \right] \quad (4-14)$$
If one considers the fundamental or input resonator, \( k = 1 \), and Equation (4-14) reduces to the form

\[
\frac{r - r_1}{r_1} = -\frac{\gamma_2 v_1 [t - t_1]}{h r_a \omega_c}, \tag{4-15}
\]

which states that the radius of rotation for interaction in a two-pole coupler is a linear function of time, which was the result obtained in Chapter II by a different method. Equation (4-15) becomes Equation (2-10) if \( 2r_a = d \).

Another case of interest is the case for \( k = 2 \), where the interaction occurs in a four-vane resonator tuned to the second harmonic of the cyclotron frequency. In this case, the integration yields

\[
\ln \frac{r}{r_1} = -\frac{\gamma_2 v_2}{2r_a^2 \omega_c} (t - t_1),
\]

or

\[
\frac{r}{r_1} = \epsilon - \frac{\gamma_2 v_2 (t - t_1)}{2r_a^2 \omega_c}. \tag{4-16}
\]

Thus, for the case of interaction in the second harmonic resonator, the radius decreases exponentially with time. In the general case where \( k \) is \( \geq 3 \), the integration yields

\[
\frac{1}{2 - k} \left[ \frac{1}{r^{k-2}} - \frac{1}{r_1^{k-2}} \right] = -\frac{\gamma v_k k [t - t_1]}{h r_a^k \omega_c}. \tag{4-17}
\]

For purposes of simplification it becomes convenient to define \( r_1 \), the radius of rotation of the beam as it leaves the input coupler and
enters the harmonic resonator in terms of the signal level \( \alpha \) as given by Equation (2-24b). Then,

\[ r_1 = \alpha r_a, \quad (4-16) \]

and

\[ r_{k-2} = \alpha^{k-2} r_a^{k-2}. \quad (4-19) \]

Substitution of these expressions in Equation (4-17) and solving for \( r \) yields

\[ r_{k-2} = \frac{\text{h}\alpha^k r_a^k \omega_c^2}{\text{d} \gamma V_k k(k-2) \omega_c(t-t_1) + \text{h}\alpha^2 \omega_c^2 r_a^2}. \quad (4-20) \]

From this expression the radius of rotation may be calculated as a function of time for a given harmonic and gap voltage \( V_k \). This will be discussed further in conjunction with the problem of realizable interaction efficiency.

**Limitations on Interaction Efficiency**

If the beam were infinitely thin, all the rotational energy stored by the electrons would be available for interaction, and the conversion efficiency from the fundamental driving frequency to the harmonic would approach one hundred percent.

In any practical tube, however, the beam must be of finite thickness. This prevents realization of the theoretical efficiency.

If the beam has a radius \( r_b \), the maximum permissible radius of rotation in the output coupler is \( r_a - r_b \), since this is the radius at which the beam will graze the anode. Thus, the kinetic energy
that can be stored by the electrons in the beam is limited to a value

\[ W = \frac{m \omega_c^2 (r_a - r_b)^2}{2} \]  \hspace{1cm} (h-21)

The portion of this energy that can be delivered by the beam to a harmonic resonator is limited by beam loading effects. As the beam delivers rotational energy, its radius decreases to a point where the beam subtends a half wavelength on the circuit from a point on the axis of rotation.

When the radius decreases below this value, the beam remains in a given portion of the field for more than a half cycle and consequently begins to remove energy from the high frequency field. This minimum radius of rotation can be expressed as a function of beam radius and harmonic number in the form

\[ r_{\text{min}} = \frac{2 k r_b}{n} \]  \hspace{1cm} (h-22)

where \( k \) is the number of the harmonic and \( r_b \) is the radius of the electron beam.

The maximum conversion efficiency of the device at a given harmonic may then be expressed by

\[ \nu = \frac{m \omega_c^2 r_{\text{max}}^2 - m \omega_c^2 r_{\text{min}}^2}{m \omega_c^2 r_{\text{max}}^2} \]

\[ = 1 - \frac{r_{\text{min}}^2}{r_{\text{max}}^2} \]  \hspace{1cm} (h-23)

where \( \nu \) is the high frequency electronic conversion efficiency,
\( r_{\text{max}} \) is the grazing radius \((r_a - r_b)\), and \( r_{\text{min}} \) is given by Equation (4-22). If these expressions are substituted into Equation (4-23), one obtains

\[
\nu_k = \left[ 1 - \frac{4 k^2}{\pi^2 \left( \frac{r_a}{r_b} - 1 \right)^2} \right]. \tag{4-24}
\]

It becomes apparent that the ratio of the anode aperture radius to the beam radius is an extremely important parameter in the design of an efficient multiplier. Efficiency as a function of harmonic order for various values of \( r_a/r_b \) is plotted in Figure 12. The efficiency is improved as the ratio \( r_a/r_b \) increases. The general performance of the FCWM is improved with a thinner beam, not only because of the decreased value of \( r_{\text{min}} \), but because the beam can pass much closer to the circuit, and the beam-to-resonator coupling is improved.

As indicated in the figure, the efficiency is decreased at a given harmonic for lower values of \( r_a/r_b \). By setting the right-hand side of Equation (4-24) equal to zero, the minimum value of \( r_a/r_b \) for generation of a given harmonic can be determined. Thus,

\[
0 = \left[ 1 - \frac{4 k^2}{\pi^2 \left( \frac{r_a}{r_b} - 1 \right)^2} \right]
\]

or

\[
\left( \frac{r_a}{r_b} - 1 \right)^2 = \frac{4 k^2}{\pi^2} ,
\]

and

\[
\left( \frac{r_a}{r_b} \right) - 1 = \frac{2k}{\pi} . \tag{4-25}
\]
Theoretical Maximum Conversion Efficiency as a Function of Harmonic Order

Figure 12.
Thus, for harmonic output to exist at the kth harmonic, it is necessary that the ratio $r_a/r_b$ satisfy the relation

$$\frac{r_a}{r_b} > \frac{2k}{n} + 1. \quad (h-26)$$

Thus, the required aperture radius for a given beam diameter must increase linearly with the order of the desired harmonic, or if the aperture radius is to remain constant, the beam diameter must be decreased in approximately inverse proportion to the harmonic number.

**Optimum Transit Angle Through the Harmonic Resonator**

The minimum useful radius of rotation in the harmonic resonator was shown in the preceding section to be given by

$$r_{\text{min}} = \frac{2k r_b}{n}. \quad (h-22)$$

In order to realize the maximum efficiency at any given harmonic, the transit time of the electrons through the output resonator should be adjusted to a value which permits all the available portion of the rotational kinetic energy to be extracted from the beam. This requires that the electrons remain in the interaction space until the radius of rotation of the electron beam reaches the value given by Equation (h-22).

Equation (h-20) expresses the radius of rotation as a function of the transit time in the output resonator. The optimum transit time may be determined from this relation by setting $r = \frac{2k r_b}{n}$ and solving for the transit time ($t - t_1$).
For the special case of the second harmonic

\[
\frac{-\eta v_2(t-t_1)}{2 \omega_c r_1^2} = r - r_1 \in \frac{2 \omega_c r_1^2}{n},
\]

where \( r_1 \) is the radius of rotation at \( t = t_1 \) and is given by \( n r_a \), the product of the input signal level and anode radius. The optimum transit time could thus be obtained in the case of the second harmonic by letting \( r = hr_b/n, \), \( r_1 = or_a \), and rewriting Equation (4-27) in the form

\[
\frac{-\eta v_2(t-t_1)}{2 \omega_c r_1^2} = \frac{hr_b/n - or_a}{2 \omega_c r_1^2} \in \frac{h r_b/n}{n or_a}.
\]

Taking the logarithm of both sides and rearranging yields

\[
\ln \frac{hr_b/n}{n or_a} = - \frac{-\eta v_2(t-t_1)}{2 \omega_c r_1^2}.
\]

Solving Equation (4-29) for \( (t - t_1) \) yields the relation

\[
(t - t_1) = \frac{2 \omega_c or_a^2}{\eta v_2} \ln \frac{n or_a}{hr_b}.
\]

or \( \theta_0 \), the optimum transit angle in terms of the cyclotron frequency, is given by

\[
\theta_0 = \omega_c (t - t_1) = \frac{2 \omega_c or_a^2}{\eta v_2} \ln \frac{n or_a}{hr_b}.
\]

For the general case of the kth harmonic, the radius of rotation?
tion in the output resonator is given by Equation (4-20) and is of the form

\[ r^{k-2} = \frac{\hbar e^k r_a^k \omega_0^2}{\alpha^k \eta V_k k(k-2)\omega_0(t-t_1) + \hbar \omega_0^2 e^2 r_a^2} \]  \hspace{1cm} (4-20)

When \((t-t_1)\) is optimum, then \(r = 2kr_b/\pi\), and Equation (4-20) becomes

\[ \left[\frac{2kr_b}{\pi}\right]^{k-2} = \frac{\hbar e^k r_a^k \omega_0^2}{\alpha^k \eta V_k k(k-2)\omega_0(t-t_1) + \hbar \omega_0^2 e^2 r_a^2} \]  \hspace{1cm} (4-32)

Solving Equation (4-32) for \(Q_0\) yields

\[ Q_0 = \omega_0(t-t_1) - \frac{\hbar \omega_0^2 r_a^2}{\eta V_k k(k-2)} \left[\left(\frac{\pi r_a}{2kr_b}\right)^{k-2} - \frac{1}{\alpha^{k-2}}\right] \]  \hspace{1cm} (4-33)

In order to calculate the optimum transit angle for any given harmonic, it is necessary to know the input signal level \(\alpha\), and the harmonic circuit voltage \(V_k\).

In order to determine the latter, the interaction admittance of the harmonic circuit must be known. Then the circuit voltage, assuming optimum operating conditions, is given by

\[ \frac{V_k^2 G_k}{2} = \nu_k P_{in} \]  \hspace{1cm} (4-34)

where \(G_k\) is the circuit conductance referred to the interaction gap, \(\nu_k\) is the optimum conversion efficiency for the harmonic under consideration, and \(P_{in}\) is the RF power stored as rotational energy on the beam. If one assumes a simple circuit of the slot and vane type, the circuit conductance may be approximated to a fair degree of
accuracy by treating the slots as shorted quarter-wave parallel plate transmission lines, as in the case of the fundamental resonator.

The conductance of a single slot at resonance is given by \(^1,2,3\)

\[
G = \frac{l_2}{h R_0 Q_{\text{eff}}},
\]

(4-35)

where \(l_2\) is the thickness of the resonator, \(h\) is the slot height, \(R_0\) is the intrinsic resistance of free space, and \(Q_{\text{eff}}\) is the effective \(Q\) of the circuit. (Ideally, \(G\) would be \(\pi/4\) times this quantity, but fringing effects increase it somewhat.) The total circuit conductance is given by

\[
G_k = \frac{2k l_2}{h R_0 Q_{\text{eff}}},
\]

(4-36)

since there are \(2k\) slots around the periphery of the structure and all have RF voltage existing across them. Practical realization of this condition would require strapping of the resonators, but the error will probably be small in the unstrapped case, although a certain amount of difficulty is to be expected due to the lack of uniformity among the individual cavities caused by unavoidable tolerances in fabrication.

Another factor to be noted is the variation of the slot height \(h\) with the order of frequency multiplication required. The required \(2k\) slots for the \(k\)th harmonic limit \(h\) to a value

\[
h \leq \pi r_\alpha/k.
\]

(4-27)

The equality holds only for the case of vanishingly thin vane tips.
For this case, if one substitutes Equation (4-37) into Equation (4-36) and sets \( R_0 = 120 \times \), the expression for \( G_k \) becomes

\[
G_k = \frac{2 \frac{k^2 l_2}{120 \pi^2 \frac{r_a Q_{\text{eff}}}}}{r_a}
\]  

(4-38)

As a number of parameters actually determine the optimum transit angle in the harmonic resonator, it is necessary to select some typical values in order to determine the effects of the harmonic order on the transit angle.

As an example, a tube will be chosen with the following characteristics.

- **Fundamental wavelength** = 3 centimeters
- **Anode radius** \( r_a \) = 1.25 millimeters
- **RF power stored on the beam** = 100 watts
- **Beam radius** \( r_b \) = 0.125 millimeters
- **Cyclotron frequency** \( \omega_c \) = \( 2\pi \times 10^{10} \) radians/second

In order to determine the optimum transit angle, one must assume a signal level \( \alpha \), since the efficiency is a function of signal level. In the example, it will be assumed that the tube is being driven at saturation input, which corresponds to the value of \( \alpha \) given by

\[
\alpha = \frac{r_a - r_b}{r_a} = 0.9
\]  

(4-39)

Since \( \frac{r_a}{r_b} \) is known, one may employ Equation (4-24) to determine the efficiency as a function of harmonic order. Once the efficiency and input power are determined, Equation (4-38) can be employed to deter-
mine the circuit conductance, and \( V_k \) may then be calculated. Solution of Equation \((4-34)\) for \( V_k \) yields

\[
V_k = \sqrt{\frac{2V_k P_{\text{in}}}{\alpha_k}}. \tag{4-40}
\]

The values of \( r_a, a, r_b, V_k, \) and \( \omega_c \) may be inserted in Equations \((4-31)\) and \((4-33)\) to determine the optimum transit angle as a function of harmonic number. For the data given, the values of efficiency, power output, circuit conductance, and gap voltage are given in Table I.

**TABLE I**

Operating Parameters for Optimum Efficiency

<table>
<thead>
<tr>
<th>( k )</th>
<th>( V_k )</th>
<th>( P_{\text{out}} ) (watts)</th>
<th>( G_k )</th>
<th>( V_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.98</td>
<td>98</td>
<td>0.34 x 10^{-3}</td>
<td>760</td>
</tr>
<tr>
<td>3</td>
<td>0.95</td>
<td>95</td>
<td>0.765 x 10^{-3}</td>
<td>502</td>
</tr>
<tr>
<td>4</td>
<td>0.92</td>
<td>92</td>
<td>1.361 x 10^{-3}</td>
<td>368</td>
</tr>
<tr>
<td>5</td>
<td>0.875</td>
<td>87.5</td>
<td>2.13 x 10^{-3}</td>
<td>286</td>
</tr>
<tr>
<td>6</td>
<td>0.82</td>
<td>82</td>
<td>3.06 x 10^{-3}</td>
<td>231</td>
</tr>
<tr>
<td>7</td>
<td>0.755</td>
<td>75.5</td>
<td>4.17 x 10^{-3}</td>
<td>190</td>
</tr>
<tr>
<td>8</td>
<td>0.68</td>
<td>68</td>
<td>5.45 x 10^{-3}</td>
<td>158</td>
</tr>
</tbody>
</table>

In the above table, the conductance has been calculated by assuming a value for \( l_2 \) of 6 millimeters. The effective \( Q \) has been chosen as 100.
The optimum transit angles for this set of parameters and operating conditions have been calculated and plotted in Figure 13. For the case of the second harmonic, or \( k = 2 \), the calculation was made by use of Equation (4-31), while Equation (4-33) was employed for \( k \geq 3 \).

If one notes the magnitudes of the transit angles, it would appear that the choice of parameters for this example is rather poor, since the angle required for the sixth harmonic is 1140 radians, or approximately 182 cycles at the fundamental frequency. This is largely due to an unfortunate choice of \( r_a \), which can cause the beam to be poorly coupled at the higher harmonics, due to the rapid decrease in the fringing field. In addition, the choice of \( r_a \) determines the current required for a given amount of input power and thus determines the electronic admittance of the beam. A mismatch between beam and circuit can lead to inefficient operation.

A slight change in tube parameters can produce a radical change in the optimum transit angle.

If one reduces the value of \( r_a \) by a factor of two, maintaining the same beam diameter, input power, and fundamental frequency, the optimum transit angle will have the values plotted in Figure 14.

No values are given for harmonics higher than the sixth, since it cannot exist for the conditions chosen. However, the transit angles for those harmonics which can be generated are much more reasonable. The calculations in both figures are based on a resonator thickness of 6 millimeters. The required beam voltage for generation of the fourth harmonic under the conditions of Figure 13 is 0.91 volts,
Pin = 100 Watts
\( I_a = 1.25 \text{ mm} \)
\( I_b = 10 \)
\( I_e = 6 \text{ mm} \)
\( \omega_c = 2\pi \times 10^{10} \)

Optimum Transit Angle vs. Harmonic Number

Figure 13.
Optimum Transit Angle in Radians (Fundamental)

- \( P_{in} = 100 \text{ Watts} \)
- \( a_a = 6.25 \text{ mm} \)
- \( a_a/r_0 = 5 \)
- \( a_e = 6 \text{ mm} \)
- \( w_c = 2\pi \times 10^{10} \)

**Figure 14.**

Optimum Transit Angle vs. Harmonic Number

Harmonic Number \( K \)
which is somewhat impractical, as it approaches the order of magnitude of the thermal emission velocities of the electrons, while the value for fourth harmonic generation in the case of Figure 14 is 175 volts. The beam current required to store 100 watts in the first case is 6.87 milliamperes, while that required in the second case is 35.9 milliamperes. As indicated by the calculations, the conditions of operation where the dc power on the beam is of the same order of magnitude as the RF power lead to values of voltage and current more easily realizable in a practical electron gun, although it is not necessarily true that performance in terms of efficiency is improved as this condition is approached.

As a further matter of interest, the calculations for the second tube might be conducted again, with the input power reduced to 10 watts. This would require that the beam current be reduced by a factor of 10. The only change in the calculation of transit angle would be a reduction in the circuit voltage by a factor of $\sqrt{10}$, which would require that the transit angles be longer by the same factor. If one calculates the required beam voltage for each case, it will be noted that this quantity is also reduced by a factor of 10, which implies that the conversion efficiency in the output resonator is unchanged as long as the dc conductance of the electron beam remains unchanged.

This would lead one to believe that it should be possible to obtain an expression relating efficiency and beam conductance similar to that obtained for the fundamental resonator in Chapter II. This will be done in the following section.
With respect to the curves plotted in Figures 13 and 14, the qualitative results are quite similar, even though the transit angles associated with a given harmonic differ greatly for the two cases. As the order of the harmonic increases, the required transit angle increases, due to the reduction in gap voltage and beam-to-resonator coupling for higher values of \( k \). However, as the order of the harmonic increases beyond a certain point, the minimum useful radius of rotation increases rapidly, causing a drastic decrease in the realizable efficiency, and the available portion of the stored energy is removed from the beam in a shorter time.

Electronic Admittance of the Beam in the Output Resonator

In the second chapter, an expression was obtained for the transverse electronic conductance of the beam by considering the loading effects of the beam on the high frequency field of the input coupler. A similar expression for the electronic conductance of the beam at the harmonic frequency may be obtained by relating the output power to the induced voltage. The radius of rotation in the output region is given by

\[
\begin{align*}
\sigma^{k-2} &= \frac{k \rho \kappa \omega_c^2}{\frac{1}{\kappa} \int \frac{k(k-2)\omega_c(t-t_1) + \hbar \omega_c^2 a^2 r_a^2}{k \rho \kappa \omega_c(t-t_1) + \hbar \omega_c^2 a^2 r_a^2}}. \\
\end{align*}
\]

This may be rearranged to yield

\[
\begin{align*}
\frac{\alpha \omega_c^2 \omega_c(t)}{\kappa} &= \frac{\hbar \omega_c^2 a^2 r_a^2}{\left[\left(\frac{\alpha \omega_c^2}{\kappa}\right)^{k-2} - 1\right]}.
\end{align*}
\]
Squaring both sides, one obtains

\[ a^2 k \gamma^2 v_k^2 \omega_0^2 (t-t_1)^2 = 16 \alpha^4 \frac{1}{\omega_c} \left[ \left( \frac{\alpha x}{r_a} \right)^{k-2} - 1 \right]^2 \]

Now if \((t-t_1) = \tau_2\), the transit time through the output region, and \(r = r_2\), the final value of the radius of rotation, one obtains

\[ a^2 k \gamma^2 v_k^2 \omega_0^2 \tau_2^2 = 16 \alpha^4 \frac{1}{\omega_c} \left[ \left( \frac{\alpha x}{r_2} \right)^{k-2} - 1 \right]^2 . \]

Now

\[ \tau_2^2 = \frac{\ell_2^2}{2 \gamma v_b} , \]

where \(\ell_2\) is the length of the output interaction region, \(v_b\) is the dc beam voltage, and \(\gamma\) is the electronic charge-to-mass ratio.

If one substitutes Equation (4-44) in Equation (4-43) and divides both sides of the result by \(2\gamma a^2 r_a^2\) while multiplying by \(I_o\), one obtains

\[ a^2 k -2 \frac{v_k^2 \omega_0^2 \ell_2^2 I_o}{h v_b r_a^2} = \frac{16 \alpha^2 r_a^2 \omega_0^2 I_o}{2 \gamma} \left[ \left( \frac{\alpha x}{r_2} \right)^{k-2} - 1 \right]^2 , \]

or, rearranging,

\[ a^2 k -2 \frac{v_k^2 \omega_0^2 \ell_2^2 I_o}{6 \hbar v_b r_a^2} = P_{in} \left[ \left( \frac{\alpha x}{r_2} \right)^{k-2} - 1 \right]^2 , \]

where \(P_{in}\) is the RF power stored as rotational kinetic energy on the electron beam, and is given by

\[ P_{in} = \frac{a^2 I_o \omega_0^2 r_a^2}{2 \gamma} . \]
Now \( P_{\text{in}} = P_{\text{out}}/\nu_k \), where \( \nu_k \) is the conversion efficiency at the kth harmonic, so that

\[
\frac{a^{2k-2} \nu_k^2 k^2(k-2)^2 \ell_2^2 I_0}{6u \nu_b r_a^2} = \frac{P_{\text{out}}}{\nu_k} \left[ \left( \frac{a r_a}{r_2} \right)^{k-2} - 1 \right]^2. \quad (4-47)
\]

Now it is true that

\[
\frac{\nu_k^2 G_e}{2} = P_{\text{out}}, \quad (4-48)
\]

and if one solves Equation (4-47) for \( P_{\text{out}} \), one obtains

\[
\frac{\nu_k^2}{6u \nu_b r_a^2} \left[ \left( \frac{a r_a}{r_2} \right)^{k-2} - 1 \right]^2 G_0 = P_{\text{out}}, \quad (4-49)
\]

and equating coefficients with Equation (4-49), one obtains

\[
G_e = \frac{\nu_k^2 \ell_2^2 k^2(k-2)^2 \nu_k}{6u \nu_b r_a^2 \left[ \left( \frac{a r_a}{r_2} \right)^{k-2} - 1 \right]^2} G_0. \quad (4-50)
\]

Now the conversion efficiency is given by Equation (4-23), which may be solved for \( r_{\text{min}}/r_{\text{max}} \). This corresponds to \( r_2/\alpha r_a \) in the above expression, and it may be noted that

\[
r_2/\alpha r_a = (1 - \nu_k)^{1/2}. \quad (4-51)
\]
Substitution of Equation (4-51) in Equation (4-50) yields

\[ G_e = \frac{a^{2k-2} k^2(k-2)^2 \nu_k l_2^2}{\left(\left(\frac{1}{1-\nu_k}\right)^2 - 1\right)^2} \rho \rho_\alpha \right] \quad \text{(4-52)} \]

It may be noted that Equation (4-52) is somewhat similar in form to Equation (2-21), having the same dependence on geometrical parameters and dc beam conductance.

Noting that the electronic conductance actually developed by the beam must equal the circuit conductance, \( G_k \), one may state

\[ G_k = \frac{a^{2k-2} k^2(k-2)^2 \nu_k l_2^2}{\left(\left(\frac{1}{1-\nu_k}\right)^2 - 1\right)^2} \rho \rho_\alpha \right] \quad \text{(4-53)} \]

or

\[ \frac{G_o}{G_k} = \frac{\rho \rho_\alpha^2}{l_2^2} \left(\left(\frac{1}{1-\nu_k}\right)^2 - 1\right)^2 a^{2k-2} k^2(k-2)^2 \nu_k \right] \quad \text{(4-54)} \]

and one obtains an expression relating conversion efficiency, circuit conductance, and dc beam conductance.

Equation (4-54) thus defines a conversion efficiency parameter analogous to Pierce's gain parameter \( \phi \) for a traveling wave tube.\(^4\)

For given values of \( \rho_\alpha \), \( l_2 \) and input signal level \( a \), one may insert various values of conversion efficiency and obtain the required value of \( G_o/G_k \). Of course, the maximum values of efficiency realizable are
those determined by the dimensions of the beam and circuit as indicated in Figure 12. For purposes of convenience, an efficiency parameter $\gamma_k$ may be defined such that Equation (4-54) has the form

$$\frac{G_o}{G_k} = \frac{32 r_a^2}{L_2^2 a^{2k-2}} \gamma_k,$$

where

$$\gamma_k = \left[ \frac{1}{1 - \gamma_k} \right]^2 - 1,$$

and other quantities are as previously defined. The parameter $\gamma_k$ is plotted in Figure 15 for $k = 3, 4, 5,$ and $6$. It might be noted that for efficiencies below 0.5 or fifty percent, $\gamma_k$ is smaller for the higher harmonics, while for efficiencies greater than fifty percent, $\gamma_k$ increases rapidly for the high harmonics, and more slowly for the lower ones. Over much of the range, the values are nearly equal, as indicated by the fact that the curves are nearly coincident over a fairly wide range.

Of more interest than the value of the parameter $\gamma_k$ itself, however, is the actual ratio $G_o/G_k$ required to realise a given efficiency at a particular harmonic. In order to determine this relation, it is necessary to make a choice of certain design parameters. It will be assumed that the signal level $\alpha$ is 0.6 in all cases, which corresponds to saturation input when $r_a/r_b = 5$. The dimensions chosen will be $r_a = 1.25$ mm, and $L_2 = 6$ mm. Then $(r_a/L_2)^2 = 1/23,$
and Equation (4-55) reduces to

\[
\frac{\omega_0}{\omega_k} = \frac{1.4}{(0.8)^{2k-2}} \gamma_k .
\]  \hspace{1cm} (4-56)

Values of \( \gamma_k \) may be obtained from Figure 15 and inserted in Equation (4-56). The results are plotted in Figure 16, where the conversion efficiency is given as a function of the ratio \( \omega_0/\omega_k \) for the 2nd, 3rd, 4th, 5th, and 6th harmonics. No calculation has been made for harmonics higher than the 6th, as the minimum radius of rotation is normally the primary limiting factor in determining the realizable efficiency at higher harmonics.

As an example of the use of the curves in Figure 16, it will be assumed that \( r_a/r_b = 5 \), \( a = 0.8 \), \( L_2 = 6 \text{ mm} \), and \( r_a = 1.25 \text{ mm} \). In addition, it will be assumed that a magnetically shielded electron gun is employed with a perveance of \( 0.5 \times 10^{-6} \). If the high frequency power input to the beam is 100 watts, the required beam current as given by Equation (2-25) is 8.97 milliamperes, and the beam voltage will be 682 volts. This yields a value for \( G_0 \) of \( 1.315 \times 10^{-5} \); \( G_k \) may then be calculated from Equation (4-36) with an assumed value of \( h \) which is chosen so that the slot height and vane thickness are equal. In this case, \( h \) is one-half the value given by the equality in Equation (4-37), or

\[
h = \frac{n}{2k} r_a \quad \text{or} \quad h = \frac{n}{2k} r_a .
\]  \hspace{1cm} (4-57)

*The calculation for the special case \( k = 2 \) was obtained by means of a similar derivation from Equation (4-27).
Figure 15.
Conversion Efficiency $\eta_k$ vs. Normalized Beam Conductance $G_0/G_k$ for $\alpha = 0.8$ and $\rho_2/\rho_4 = 4.8$.

Figure 16.
In addition, it will be assumed that the effective Q of the slots is 100. The values of \( G_0/G_k \) may then be calculated for each value of \( k \), and the conversion efficiency may be obtained as the ordinate of the point where the vertical line corresponding to a particular value of \( G_0/G_k \) intersects the appropriate curve. For the case given, using a gun with a microperveance of 0.5, the efficiencies obtained are plotted in Figure 17 as a function of harmonic number. It may be noted in every case that the conversion efficiency is below the theoretical maximum, which is obtained from Figure 12.

If the electron gun is designed for a microperveance of 1.0, the beam conductance will be increased by a factor of 1.588, if all other quantities remain unchanged. The conversion efficiencies for this case are plotted in Figure 18, and again it may be noted that the efficiencies obtained are less than the theoretical maximum in every case, although some improvement is noted. Theoretically, it should be possible to design a tube to operate at the optimum efficiency, although the beam conductance required would be high enough to require beam voltages close to the level of the thermal emission velocities in the majority of cases. Qualitatively, the results for the output coupler are similar to those obtained for the input circuit. One is forced to operate under conditions corresponding to values of efficiency less than the theoretical maximum in order to obtain reasonable requirements for the electron gun. The curves in Figure 16 show a tendency to saturate around the region where \( G_0/G_k = 0.48 \), so improvements in efficiency obtained by increasing the dc beam conductance beyond this point are extremely marginal. As a
Maximum and Actual Conversion Efficiencies vs Harmonic Number

Figure 17.
Maximum and Actual Conversion Efficiencies vs Harmonic Number

Figure 18.
matter of interest, one might obtain the values of $G_o/G_k$ corresponding to optimum efficiency by the combined use of Figures 12 and 16. These values are given in the following table for the case $r_a/r_b = 5$.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$k$</th>
<th>$G_o/G_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.89</td>
<td>0.460</td>
</tr>
<tr>
<td>3</td>
<td>0.77</td>
<td>0.924</td>
</tr>
<tr>
<td>4</td>
<td>0.59</td>
<td>0.400</td>
</tr>
<tr>
<td>5</td>
<td>0.366</td>
<td>0.152</td>
</tr>
<tr>
<td>6</td>
<td>0.085</td>
<td>0.014</td>
</tr>
</tbody>
</table>

In practice, the optimum values of $G_o/G_k$ are quite difficult to realize, since the circuit conductance is limited by the geometry of the device, and the perveance of the electron gun cannot be extremely high, or transmission difficulties may be experienced, due to the presence of the required magnetic focusing field. It may be noted, however, that beam conductance does increase with beam current, if the perveance of the electron gun remains constant, so that the device does tend to function more efficiently at higher power levels for a given set of dimensions.

Assuming that the gun voltage and beam current may always be
adjusted to optimize the efficiency, the beam conductance may be calculated for various power levels with all other conditions as given in Figures 16 and 17, and a family of curves may be obtained giving the conversion efficiency as a function of input power for each of the various harmonics. (See Figure 19.) It may be noted that the conversion efficiency increases quite rapidly with input power up to about 100 watts, after which the curves tend to saturate.

This type of performance, which achieves its greatest efficiency at power levels of the order of 100 watts or higher, seems to be typical of conventional crossed-field devices as well as those belonging to the fast-cyclotron-wave category. This characteristic becomes more pronounced as the device is scaled to higher and higher frequencies unless the diameter of the electron beam can be scaled in direct proportion to the operating wavelength (Appendix C). However, since the electron beam is magnetically focused, it is desirable to avoid electron guns with an extremely high concentration ratio and the high transverse electron velocities normally associated with such guns. Thus, scaling of the beam may cause a severe increase in cathode loading. At higher power levels, the radius $r_a$ may be increased, and the beam radius $r_b$ may be larger for a given value of efficiency. In any case, the multiplier shows good possibilities of developing large amounts of power in the millimeter range with excitation at three centimeters. The conversion efficiencies indicated in Figure 19 are higher than those which will be obtained in actual practice, inasmuch as they represent purely electronic conversion ef-
\( r_a = 1.25 \, \text{mm} \)
\( r_x = 6 \, \text{mm} \)
\( x = 0.9 \)
\( \omega R = 2 \pi \times 10^{10} \)

Pervance = \( 5 \times 10^{-6} \)

**Conversion Efficiency vs RF Power Input to the Electron Beam**

Figure 19.
ficiencies and do not include the effects of input and output circuit losses, which are appreciable. However, even when these are included, the efficiencies should be high enough to be interesting. Another rather important conclusion to be drawn from Figures 16 through 19 is the fact that design of the electron gun to obtain a particular dc beam conductance is not nearly as important as it might seem at first glance, since optimum performance from a fixed perveance gun can be obtained at only one power level, whereas the performance of the fixed perveance gun employed in the calculation of Figure 19 is quite acceptable over a rather wide range of input powers. This is a significant factor in the design of the experimental multiplier, which will be discussed in the next chapter.
References


CHAPTER V

EXPERIMENTAL STUDIES

Design of the Experimental Tube

Numerous problems are encountered in the design of a practical fast-cyclotron-wave device for operation in the centimeter and millimeter wavelength ranges. One of the most severe problems in that of the design of a suitable electron gun, which is normally the major source of difficulty in any microwave tube, whether it is intended to be a low noise device or a power source. It was thought that a magnetically shielded Pierce gun having a microperveance of 0.5 to 1.0 would be suitable for feasibility studies, while design of an optimum electron gun could be attempted after the general characteristics of the multiplier had been demonstrated. The gun actually employed was scaled from one used in a floating drift tube klystron frequency multiplier by Cornetet. The original version was designed for a beam diameter of 0.050 inch. A 0.050 inch diameter beam was employed by Cuccia in a fast wave coupler designed to operate at 800 Mc, and direct scaling for x-band operation would have required a 0.005 inch diameter beam in the gun under consideration. It was felt that 0.010 inch was the smallest practical beam diameter for a physically realizable design, so this was the value chosen. The original gun had a nominal microperveance of 0.5. The measured microperveance of the scaled version was approximately 0.4. However, this figure is on the basis of cathode current alone. The actual beam microperveance
obtained was about 0.2 due to the fact that the transmission was rather poor in the low voltage range, which was the operating region of interest.

A cross section of the experimental gun is shown in Figure 20. The focusing electrode shown in the figure is operated at a negative potential with respect to the cathode and permits a measure of control of the permeance and transmission. It may be noted that the anode forms a portion of the magnetic shield. When the gun was scaled, it was necessary to reduce the anode thickness, and thus the shielding effect of the anode was reduced. Evidently, the shielding was inadequate for the magnetic fields of three kilogauss or more required for cyclotron resonance at x-band, since the focusing electrode seemed to have less control in the presence of full field than in the original model. It was realized that the inferior performance of the gun would cause an attendant deterioration in the performance of the multiplier, but it was deemed advisable to proceed with the tests.

Figure 21 shows the construction of the experimental tube. Because of the limitations of the external magnetic circuit employed, it was desirable to keep the air gap as small as possible. Thus, both the input and output couplers were placed within a distance equal to the long dimension of the fundamental wave guide, as shown in the diagram. This no doubt introduces additional losses due to the loading effects of the end spaces on the individual resonators, but was dictated by the operating limitations of the available solenoid. The air gap in the magnetic circuit is the distance between the anode of
Electron Gun

FIGURE 20

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GUN MOUNT &amp; SHIELD</td>
</tr>
<tr>
<td>2</td>
<td>ANODE</td>
</tr>
<tr>
<td>3</td>
<td>CATHODE</td>
</tr>
<tr>
<td>4</td>
<td>FOCUSING ELECTRODE</td>
</tr>
<tr>
<td>5</td>
<td>CATHODE MOUNT</td>
</tr>
<tr>
<td>6</td>
<td>CERAMIC SPACER</td>
</tr>
<tr>
<td>7</td>
<td>CATHODE INSULATOR</td>
</tr>
<tr>
<td>8</td>
<td>CERAMIC RETAINING RING</td>
</tr>
</tbody>
</table>
FIGURE 21

1 COLLECTOR
2 OUTPUT COUPLER
3 CERAMIC SPACER
4 OUTPUT WAVEGUIDE
5 WAVEGUIDE TRANSFORMER
6 INPUT COUPLER
7 WAVEGUIDE TRANSFORMER
8 INPUT WAVEGUIDE
9 GUN ANODE

Experimental Tube
the electron gun and the collector, both of which were made of Armco iron and copper plated.

As discussed in Chapters II and IV, the limiting efficiency of either a fundamental or harmonic coupler is dependent on the ratio of the coupler length to the diameter of the coupler aperture (or spacing for the case of the fundamental circuit). The type of circuitry employed in the experimental FCWFM determined the maximum length for both input and output couplers as indicated in the following discussion.

Figure 22 shows an exploded view of the essential portions of the circuitry. Both the input and output circuits are coupled to their respective wave guides by means of quarter wave transformers which are shown in the figure. The long dimension of either transformer is limited to the width of the waveguide employed at the desired frequency. If a resonator operating in a TEM mode is employed, the length of the resonator should be of the order of one-half guide width or less. (In addition, the attempt to achieve a minimum air gap is again a consideration.) The excitation supplied to the multiplier was to be at x-band. The inside dimensions of the fundamental waveguide (WR-90) employed were thus 0.900 inch by 0.400 inch. Since the length of the input resonator was to be approximately one-half the major dimension of the input guide, a value of 0.400 inch or approximately 10 millimeters was chosen for \( L_1 \). For the harmonic resonator, the value of \( L_2 \) depends on the output frequency.

Three output wave guides were mounted in the cartridge of the experimental tube in order to permit the utilization of output
Exploded View of Input and Output Circuits

Figure 22.
circuits at different harmonics of the input frequency. The guides chosen were WR-42, WR-28, and WR-12. The nominal driving frequency of the multiplier was chosen as 9375 Mc, since the only high power x-band driver available initially was a pulsed magnetron operating at this frequency. The second harmonic thus would be 18,750 Mc, which falls within the frequency range of the TE_{10} mode in the WR-42 guide, which has a major dimension of 0.1*20 inch. The value of $l_2$ for the second harmonic resonator was chosen to be 0.240 inch, which is slightly over half the major dimension of the associated wave guide. The third harmonic and fourth harmonics are 28,125 Mc and 37,500 Mc which fall within the 26.5 to 40 KMc range of the TE_{10} mode in the WR-28 guide. The value of $l_2$ chosen for both these resonators was 0.160 inch. Since it was desirable to make some provision for measurements at much higher harmonics, the WR-12 wave guide was made the third choice. The TE_{10} mode in this wave guide covers the range from 60 to 90 KMc, and includes the seventh, eighth, and ninth harmonics of the nominal driving frequency. The resonators for this frequency range are quite difficult to make, and have not yet been designed. However, the length of the couplers would be less than the 0.120 inch major dimension of the WR-12 guide. A probable choice would be 0.080 inch.

No provision was made for output at the fifth and sixth harmonics, but it was felt that the available output frequency ranges would permit adequate demonstration of the multiplier characteristics.

Since simple slot and vane resonators were employed, the slots in the harmonic resonator were made one quarter wave deep at the
required operating frequency. The fundamental resonator was treated as a resonant half-wave line operating in the TEM mode in accordance with the discussion in Chapter II. Figure 23 shows the essential dimensions of the fundamental and third harmonic resonators. The dimensions of the quarter-wave transformers employed to couple the resonators to the wave guide are given in Figure 24. The values given are scaled from a standard design in Collins.¹

Since $l_1$ and $l_2$ were somewhat fixed by the geometry of the design and $r_b$ was rather definitely determined by the minimum practical diameter of the electron beam, the only dimension remaining to be chosen was $r_a$, the radius of the coupler aperture, which is also one-half the input coupler plate spacing "d". An examination of Equation (2-29) will reveal the desirability of a high ratio of $l_1$ to $d$ for the purpose of obtaining good efficiency in the input coupler. A similar requirement is true for the case of the output coupler. From Equation (4-26) it may also be noted that

$$\frac{r_a}{r_b} \geq \frac{(2\pi/n) + 1}{k}$$

where $k$ is the desired harmonic number. Now $r_b$ would be 0.005 inch were it possible to maintain the ideal condition of Brillouin flow. It is not unreasonable to assume that the beam radius may fluctuate as much as two to one in the experimental model, although this can be minimized if careful attention is given to the conditions under which the electron beam enters the magnetic field. If the two-to-one fluctuation is assumed, the maximum value of $r_b$ becomes 0.010 inch. For generation of harmonics through the fourth, the requirement is that
(a) Fundamental Resonator

Resonator Details
Figure 23.

(b) Harmonic Resonator

6 Slots .020 high, .105 deep
Aperture .040 radius
(a) Fundamental Waveguide Transformer
Design Frequency 9375 Mc

(b) 3rd Harmonic Waveguide Transformer
Design Frequency 28,125 Mc

Figure 24.
\( r_a/r_b \) be greater than 3.54. Thus, \( r_a \) was chosen as four times the assumed maximum value of \( r_b \), or \( r_a = 0.040 \) inch. It was realized that redesign of the coupler would be required for generation of the higher harmonics, but for the initial tests, it was desirable to design for the highest possible input coupler efficiency. Some improvement in circuit efficiency could probably be obtained by the use of re-entrant resonators, but one of the problems in designing a multiplier of this type is to make the resonant frequency of the harmonic circuit an exact integral multiple of the fundamental, and sources of error in both design and fabrication are minimized by the use of the simple resonant slot.

**Theoretical Performance of the Experimental Tube**

With the design completed, it is possible to calculate the expected performance of the experimental tube.

It was noted previously that the actual microperveance of the beam obtained in the scaled experimental gun was 0.2 rather than the value of 0.5 obtained in the original version. The efficiency of both input and output couplers will be reduced due to the resultant decrease in the conductance of the electron beam. The values of the various parameters for the initial design are as follows.

\[
\omega_c = 2\pi (9375) \times 10^6 \quad \text{(fundamental frequency)}
\]

\[ l_1 = 0.400 \text{ inch} \]

\[ l_2 = 0.240 \text{ inch}, \ k = 2 \\
0.160 \text{ inch}, \ k = 3 \\
r_a = 0.040 \text{ inch} \]
\[ \alpha = 0.75 \quad (\text{saturation level for } r_a/r_b = 4) \]

\[ I_0/\sqrt{V_B} = 0.2 \times 10^{-6} \]

For the preceding set of values, the current requirement for the electron beam as determined from Equation (2-25) is 185 microamperes per watt of RF power at saturation input.

Figure 25 shows the required driving power for saturation input as a function of beam current, while Figure 26 shows the theoretical output power for saturation input as a function of beam current for the second, third and fourth harmonics. Figure 27 shows the power output as a function of the power input for the same three harmonics. These calculations are based on the expressions derived in Chapters II and IV.

The calculations were not carried beyond 5 milliamperes of beam current, as the driving power required at this current is 418 watts, which is somewhat difficult to obtain. However, the theoretical characteristics with the existing gun appeared to be adequate for a check of the simple theory, although the performance left much to be desired.
Figure 25:

Theoretical Driving Power vs Beam Current

\[ P_{th} = 2 \times 10^{-6} \]

\( \alpha = 0.75 \)

\( f_c = 9375 \text{ MHz} \)

\( d = 0.080 \text{ inch} \)

\( l = 0.400 \text{ inch} \)
Theoretical Output Power vs Beam Current

Output Power in Milliwatts

Beam Current in Milliamperes

Figure 26.
Theoretical Output Power vs Driving Power

Theoretical Output in Milliwatts

Driving Power in Watts

Figure 27.

$\ell_2 = 0.160 \text{ inch}$

$r_a = 0.040 \text{ inch}$

$f_c = 9375 \text{ Mc}$

$\alpha = 0.75$

$P_{Pervance} = 2 \times 10^{-6}$
Experimental Results

Initially, the only source available for high power excitation at x-band was a pulsed magnetron having a peak power output in the 50 kilowatt range. In order to obtain the power levels of the order of a few hundred watts which were necessary to drive the FCWM it was necessary to attenuate the output of the magnetron with an absorbing load and a 20 db directional coupler. This reduced the power level to the order of 500 watts peak. The output was then controlled by means of a calibrated flap attenuator having a range from zero to 20 db.

Two sets of tests were run with pulsed 9375 Mc excitation. The first test was conducted on a second harmonic multiplier. Some difficulties were noted with respect to testing of the second harmonic version at the power levels involved. One of these was due to the fact that the second harmonic of the driving magnetron was quite pronounced and fed through into the output circuit of the multiplier even when the multiplier beam was turned off. However, some cyclotron interaction did occur, as about a 3 db increase in second harmonic output was noted when the magnetic field was adjusted to cyclotron resonance with the beam turned on. The aperture in the shield between the fundamental and output resonators was evidently too large in this particular tube, or the problem of second harmonic feed-through would not have been quite so pronounced. For this reason, a design modification was made. The aperture in the shield plate was reduced to the minimum diameter necessary to permit the rotating beam to pass through at saturation input. In addition, the decision was
made to conduct further experimental work at the third and higher harmonics, to reduce the possibility of feed-through from the driver to an even greater degree.

Pulse tests on the third harmonic version proved to be somewhat unsatisfactory as well, although the problem of feed-through was eliminated. Some output was obtained, but it was highly unstable, and difficult to optimize adjustments. In addition, the pulse modulator itself produced some spurious signals in the detection circuitry. These were identified by the fact that none of the microwave circuit adjustments, either x-band or third harmonic, had any effect on their magnitude. Careful attention to grounding eliminated the unwanted signals, and a very small amount of fast cyclotron wave output was obtained at a frequency of 28,125 Mc.

Several reasons for the small output existed, but the most obvious one was the fact that the input circuit was resonant at the wrong frequency, and therefore much less efficient than the simple theory would predict. The resonant frequency of the input coupler was measured and found to be 9700 Mc, which differed by 3.5 percent from the design value of 9375 Mc. A similar discrepancy probably exists in the harmonic resonator, although this measurement was not performed. Since the tolerances in fabrication and assembly could account for errors of this magnitude, it became apparent that a tunable CW source would be essential for any significant tests.

A CW magnetron having a tuning range from 8,775 to 10,475 Mc was obtained, and the third harmonic multiplier was tested at an excitation frequency of 9,700 Mc. The arrangement of the equipment is
shown schematically in Figure 28. As shown in the figure, a 20 db
directional coupler and cross guide coupler were connected in cascade
to permit monitoring the input power to the multiplier, which was in
the 100-watt range and higher. It was expected that the output power
would be of the order of several milliwatts, so some difficulty was
experienced in finding a suitable detector for the output. A calo-
rimeter power meter was available, but the response was far too slow
to permit optimizing adjustments. In the initial tests, a bolometer
was used a few inches from the end of the output wave guide as no
suitable attenuator was available. However, some characteristics of
the multiplier were obtained with this method of measurement. The
relative output power as a function of beam current is shown in Fig-
ure 29. The maximum obtained corresponds to a few milliwatts or more.
The exact figure in this case was unknown due to the unknown amount
of attenuation in the air between the end of the wave guide and the
bolometer mount. Also, the output coupling was certainly less than
ideal under these circumstances, due to the mismatch at the end of
the output wave guide. However, the shape of the characteristic is
quite similar to those shown in Figure 26, which would indicate that
the simple theory is qualitatively correct. The output powers indi-
cated in Figure 26 represent the portion of the high frequency energy
on the electron beam which is converted to harmonic energy, and do
not include the effects of output circuit losses. These figures are
also based on the assumption that the resonant frequencies of the
fundamental and harmonic resonators have an integral relationship,
Schematic Diagram of Equipment Arranged for CW Tests

Figure 28.
Relative Output vs Beam Current

Excitation Frequency 9700 Mc
Output Frequency 29,100 Mc

Test No. 1
Test No. 2

Figure 29.
which is probably not the case, considering the results of the measurements on the fundamental resonator.

Tuning of the multiplier is quite critical. It is important to have good guide-to-resonator coupling for efficient power transfer at the fundamental. The quarter-wave transformers did not prove to be well matched at the resonant frequency, and the E-H tuner was required to reduce the standing wave ratio to a value near unity. Unfortunately, the cyclotron resonance loading as controlled by the magnetic field and beam current as well as the E-H tuner adjustment strongly affected the output frequency of the magnetron so that there was a pronounced interaction among the critical adjustments. An attempt was made to frequency modulate the magnetron to permit adjustments to be made with greater ease, but even a modulating voltage of 600 volts (anode potential 4.5 kv) did not vary the frequency more than 7 megacycles. The changes in output frequency due to variations in loading were greater than this, so although the adjustments were somewhat less critical, they were still difficult to make. The characteristics shown in Figure 29 were obtained with modulated output. The efficiency apparently does increase with beam current as predicted by the theory. It might be noted that the actual conversion efficiency of the fast-cyclotron-wave mechanism is not as bad as it would appear to be at first glance, inasmuch as the maximum power that could be delivered to the 4 milliamperes beam was 21.6 watts, based on the figure of 185 microamperes per watt at saturation input which was calculated for the experimental tube.

A second series of CW tests were run on the third harmonic
multiplier after a suitable attenuator had been obtained to permit direct measurement of the third harmonic output power.

The electron gun performed somewhat more favorably in these tests than in the previous ones. Two sets of curves were run. In Figure 30, the beam current was 4 milliamperes at 1000 volts and a power output of 17 milliwatts was obtained at a frequency of 29,100 Mc under these conditions. The driving power was in the 150-200 watt range and fluctuated somewhat, so it is not known accurately. The maximum RF power on the beam could only be of the order of 22 watts as in the previous case, so the actual harmonic conversion efficiency was 0.077 percent, which is over an order of magnitude smaller than the ideal figure. However, as mentioned previously, the theoretical figure of 500 milliwatts given in Figure 26 does not include circuit losses and assumes ideal tuning (i.e., harmonic cavity and fundamental cavity frequencies are integral multiples). The electron gun was re-adjusted, and it was found that 7.5 milliamperes of beam current could be obtained at 1000 volts. Figure 31 shows the output power as a function of beam current for this case. An output power of 28 milliwatts was obtained at 29,100 Mc with a beam current of 7.5 milliamperes. The maximum power on the beam for this case would be about 41 watts at saturation input. The conversion efficiency for this case would be 0.0684 percent which is about the same as that obtained previously. A slight saturation effect may be noted on this curve, which would indicate that the driving power was insufficient to produce a grazing orbit at the beam current obtained, due to the inefficient coupling. The actual output of the magnetron could not
Figure 30 - Power Output vs Beam Current

Excitation Frequency 8700 Mc
Output Frequency 29,100 Mc
Figure 31 - Power Output vs Beam Current

Excitation Frequency 9.700 Mc
Output Frequency 29.100 Mc

Output Power in Milliwatts

Beam Current in Milliamperes
be controlled very accurately, since no variable attenuators were available for such a high power level. The theoretical drive requirements have certainly been verified within a factor of 2 or 3, however. It was necessary to obtain excitation beyond the hundred watt level to obtain significant results, which is in accordance with the theoretical analysis.

Figures 32 and 33 show the variation of the output with magnetic field. This is in good agreement with the simple theory (Appendix B) which would predict a cyclotron mode width of 680 gauss for the input coupler with the dimensions given. The value actually measured was 825 gauss in one case and 900 gauss in the other, which correspond to differences of 21.4 percent and 32.4 percent, respectively. These figures are well within the limits of experimental error.
Relative Output vs Magnetic Field

- Magnetic Field in Gauss
- Relative Output Power

$V_b = 1000\,V$
$V_o = 4\,ma$

Figure 32.
Relative Output vs Magnetic Field

Magnetic Field in Gauss

$V_b = 1000 \, V$
$I_o = 7.5 \, ma$

Figure 33.
Summary and Conclusions

A theoretical analysis and some preliminary experimental results have been given, indicating that the fast-cyclotron-wave frequency multiplier can be operated in the centimeter wavelength range, and the output obtained at 29,100 Mc or 1.03 cm seems to indicate that good possibilities exist for utilizing the device further into the millimeter wavelength range, although it will not be as efficient as originally supposed. The primary problem to be solved is the development of a suitable electron gun, as the circuitry is not too critical aside from the coupling requirements.

Cuccia obtained 5 percent overall conversion efficiency from a similar multiplier operating at a frequency of 800 megacycles with the output at 3200 megacycles or the fourth harmonic. However, this was obtained with an electron gun which delivered a beam current of 45 milliamperes at 600 volts, whereas the experimental centimeter wavelength multiplier utilized a gun which produced a maximum of 7.5 ma. at 1000 volts, due to poor transmission. Inasmuch as the efficiencies of both the input and output coupler increase almost linearly with beam conductance in the low efficiency range, a scaled gun which would duplicate this performance should increase the output power for a given drive level by nearly two orders of magnitude, which would yield outputs of the order of watts. Design of an adequately shielded high permeance gun will require an extended development program and should be the main objective of research designed to extend the range of the multiplier to low millimeter wavelengths. An
immersed gun cannot be employed, due to cathode loading limitations at the required current densities.

If the associated problems can be solved, the FCWF shows good possibilities of becoming a highly competitive millimeter wavelength generator, inasmuch as it has excellent reliability characteristics. The 28 milliwatts output at 29,100 Mc with 150-200 watts excitation at the fundamental is not extremely good performance, but the proper electron gun should improve this by about two orders of magnitude. If this can be accomplished, generation of a few hundred milliwatts at 3 millimeters is well within the realm of possibility and will justify the author's original estimate of the capabilities of the device.
References


APPENDIX A

POWER HANDLING CAPABILITY OF THE ELECTRON BEAM

In Chapter II it is stated that the dc power input to the electron beam must exceed the high-frequency power on the beam in the case of a fast-cyclotron-wave amplifier employing a space-varying pumping field, rather than a time-varying field. This limitation does not apply to the fast cyclotron wave frequency multiplier or other devices which do not employ the spatially-varying pumping mechanism. However, the power which can be handled by a tube with a given geometry does have a fundamental limit that is imposed by the nature of the necessary magnetic focusing field. The maximum current density that can be permitted in a magnetically-focused electron beam is limited to a value determined by the condition for which the radial space-charge force in the beam equals the restraining force of the magnetic focusing field.

This condition is satisfied by the relation

$$\omega_c^2/2 = \omega_p^2,$$

where \(\omega_c\) is the electron cyclotron frequency, \(\eta B\), and \(\omega_p\) is the plasma frequency of the electron beam given by

$$\omega_p^2 = \eta p / \varepsilon_0,$$

where \(p\) is the charge density, \(\eta\) is the electronic charge-to-mass ratio, and \(\varepsilon_0\) is the permittivity of vacuum.
Now
\[ p = \frac{I_o}{(\sqrt{2} \pi V_b) n r_b^2}, \]
where \( V_b \) is the accelerating potential of the electron gun, \( I_o \) is the direct current in the electron beam, and \( r_b \) is the beam radius.

Thus
\[ \frac{\omega_c^2}{2} \frac{v_c}{e} \frac{n l_o}{\pi} \frac{\pi I_o}{\varepsilon_0 \sqrt{2 \pi V_b} n r_b^2}, \]
or
\[ I_o = \frac{\omega_c^2 \varepsilon_0 \sqrt{2 \pi V_b} n r_b^2}{2 \pi} \frac{\omega_c^2 \varepsilon_0 V_b n r_b^2}{\sqrt{2 \pi}}, \]

which is the normal expression for Brillouin current.

Now the high frequency power on the beam is given by
\[ P_{rf} = \frac{a^2 I_o \omega_c^2 r_a^2}{2 \pi}, \]
where \( a \) is the signal level as defined in Chapter II, and is given by
\[ a = (1 - r_b/r_a) \]
for saturation input.

Thus, the maximum RF power that can be stored on the beam is given by substituting Brillouin current into the power expression.

This yields
\[ P_{rf,\text{max}} = \frac{a^2 \omega_c^2 n r_a^2 r_b^2 \varepsilon_0 V_b}{(2 \pi)^{3/2}}, \]
or

\[ P_{\text{RF max}} = \left(1 - \frac{r_b}{r_a}\right)^2 \frac{\frac{1}{2} \pi n r_a^2 r_b^2 \varepsilon_0 \sqrt{V_b}}{(2\pi) \sqrt{m}^3} \]

It should be noted that for a given ratio of \( r_a / r_b \) that the power handling capability of a given basic design is independent of frequency as long as all dimensions including those of the beam are scaled linearly with wavelength, since the term \( \frac{1}{c} r_a r_b^2 \) will remain constant.

If one considers the dimensions employed in the experimental tube,

\[ r_a = 10^{-3} \]
\[ r_b = 0.25 \times 10^{-3} \]
\[ \omega_c = 2\pi \times 10^{10} \]
\[ \varepsilon_0 = (1/36\pi) \times 10^{-9} \]
\[ m = 1.76 \times 10^{11} \]

substitution of the given values into the maximum power equation will yield a value for \( P_{\text{RF max}} \) of \( 20\mu \sqrt{V_b} \).

Now for the dimensions given, the required beam current is approximately \( 0.16 \) milliamperes/watt of high frequency power. Thus, the beam current is given by

\[ I_0 = 0.16 \times 10^{-3} P_{\text{RF}} \],

where \( P_{\text{RF}} \) is the high frequency power on the beam in watts. Now the dc input to the beam is given by

\[ P_{\text{dc}} = I_0 V_b \],
but the minimum value of $V_b$ for the tube under consideration is given by

$$V_b = \frac{(P_{RF})^2}{(2\Omega)^2}.$$  

Substitution of the expressions for voltage and current in terms of the high frequency power into the expression for dc power yields

$$P_{dc} = 3.86 \times 10^{-9} (P_{RF})^3$$

Over a rather wide range, the required dc input power to the beam will be very much less than the high frequency power, due to the fact that the coefficient of $(P_{RF})^3$ on the right-hand side of the preceding equation is quite small. The power input for which the dc beam power must equal the high frequency power may be obtained from the above equation by setting $P_{dc} = P_{RF}$. For the case under consideration, this occurs at a power level of approximately 16 kilowatts, which would correspond to a beam current of 2.56 amperes and a beam voltage of 6.25 kilovolts. At lower power levels, the dc power required rapidly becomes a small percentage of the high frequency power, so that the dc beam efficiency is not a primary consideration. However, it will obviously be of rather pronounced importance in the kilowatt range. In addition, this limitation determines the maximum dc beam conductance that may be realized at a given power level, and therefore limits the maximum conversion efficiency that may be realized. In general, this will present no problem for tubes designed to operate in the range of a few hundred watts and slightly lower, as other factors normally place more severe limitations on the efficiency.
APPENDIX B
ELECTRONIC BANDWIDTH

In Chapter IV the general interaction mechanism was treated for the synchronous case only. The performance of the coupler for asynchronous operation is of interest with respect to its restrictions on bandwidth of operation.

From Equation (4-10)

\[
\int_{r_1}^{r} \frac{dr}{r^{k-1}} = -\frac{\eta v_0}{2\pi r_k \omega_c} \int_{t_1}^{t} \sin k\omega t \sin k\omega_c t \, dt
\]

\[
= -\frac{\eta v_0}{4 \pi r_k \omega_c} \left[ \frac{\sin k(\omega - \omega_c)t}{k(\omega - \omega_c)} - \frac{\sin k(\omega + \omega_c)t}{k(\omega + \omega_c)} \right]_{t_1}^t
\]

and if \( \frac{\omega - \omega_c}{\omega + \omega_c} \ll 1 \), the second term in the brackets becomes negligible, and one obtains

\[
\int_{r_1}^{r} \frac{dr}{r^{k-1}} = -\frac{\eta v_0}{4 \pi r_k \omega_c} \left[ \frac{\sin k(\omega - \omega_c)t}{k(\omega - \omega_c)} \right]_{t_1}^t
\]

\[
= -\frac{\eta v_0}{4 \pi r_k \omega_c} \frac{\sin k(\omega - \omega_c)t}{k(\omega - \omega_c)} - \frac{\sin k(\omega - \omega_c)t}{k(\omega - \omega_c)} \frac{t_1}{t_1}
\]

As \( (\omega - \omega_c) \to 0 \), \( \sin k(\omega - \omega_c)t \to k(\omega - \omega_c)t \)
and
\[
\int_{r_1}^{r} \frac{dr}{r^{k-1}} \frac{-\gamma v_0 k}{4\pi r k \omega_c} (t - t_1),
\]
which is the result obtained in Equation (4-11).

The extinction bandwidth of the coupler may be obtained by setting the integral on the left-hand side of Equation (4-10) equal to zero, which corresponds to zero net energy interchange between the electron beam and the high-frequency field.

This requires that
\[
\sin k(\omega - \omega_c)t = \sin k(\omega - \omega_c)t_1,
\]
which occurs for
\[
k(\omega - \omega_c)(t - t_1) = 0, \pm 2\pi m.
\]
The value 0 is a trivial case, and the value of m corresponding to the extinction bandwidth of the principle mode of the coupler is \(m = 1\).

Thus
\[
k(\omega - \omega_c)(t - t_1) = \pm 2\pi
\]
or
\[
\omega - \omega_c = \frac{\pm 2\pi}{k(t - t_1)}
\]
where \(\omega_c\) is the extinction frequency. In the input coupler, \(k = 1\), and the extinction bandwidth is defined by the values of \(\omega\) for which
\[
\omega - \omega_c = \frac{\pm 2\pi}{(t - t_1)}.
\]
The bandwidth to extinction points is thus of the form

\[ B W_e = \frac{2(\omega_e - \omega_c)}{2n} = 2(f_e - f_c) = \frac{2}{t - t_1}, \]

or, if \( t - t_1 = \tau \), the transit time through the input coupler,

\[ B W_e = \frac{2}{\tau}. \]

If \( \tau = N T_c \), where \( T_c = 1/f_c \), the extinction bandwidth thus becomes

\[ B W_e = \frac{2f_c}{N}, \]

where \( f_c \) is the cyclotron frequency, and \( N \) is the number of cyclotron orbits executed by an electron passing through the input coupler.

In general, the circuit \( Q \) will have a more pronounced effect in determining the operating bandwidth than the extinction frequencies defined by the preceding relation.

With respect to the output circuit, the frequency of the induced current will always be at or near a multiple of the cyclotron frequency of the rotating beam, so that the circuit bandwidth places a definite restriction on the adjustment of the magnetic field. With the circuitry employed, one should not expect practical bandwidths greater than one percent. Broadband operation would require use of different structures entirely.
APPENDIX C  
EFFECTS OF SCALING ON EFFICIENCY  

From Equation (4-55)

$$\frac{G_o}{G_k} = \frac{g^2 r_a^2 \gamma_k}{L_2^2 \alpha^{2k-2}} ,$$

where

$$\gamma_k = \frac{\left[ \left( \frac{1}{1 - \nu_k^2} \right)^{\frac{k-2}{2}} - 1 \right]^2}{k^2(k-2)^2 \nu_k} ,$$

and other quantities are as previously defined.

Now $\gamma_k$ is a function of $\nu_k$ and $k$ only, independent of the actual frequency of operation.

From Equation (4-38),

$$G_{k_{\min}} = \frac{2k^2 L_2}{120 n^2 r_a Q_{\text{eff}}} .$$

Substitution of this value for $G_k$ in Equation (4-55) yields

$$\gamma_k = \frac{15 n^2 G_o L_2}{8 k^2 r_a Q_{\text{eff}} \alpha^{2k-2}} .$$

Now

$$\alpha_{\text{max}} = \frac{r_a - r_b}{r_a} = 1 - \frac{r_b}{r_a} .$$
\[ \delta_{\text{max}} = \frac{15 \pi^2}{8k^2} g_0 Q_{\text{eff}} \left( 1 - \frac{r_b}{r_a} \right)^{2k-2} \frac{l^2}{r_a}, \]

and the conversion efficiency associated with a given value of \( \delta_k \) is independent of everything except the order of frequency multiplication if all dimensions are scaled linearly with excitation wavelength. It is to be noted that \( (1 - r_b / r_a) \) should be as close to unity as possible, while \( l^2 \) should be large compared to \( r_a \). In practice, \( r_b \) is determined by the minimum realizable beam diameter, so that the condition

\[ l^2 \gg r_a \gg r_b \]

is limited by the restrictions on the electron gun more than any other. Eventually, cathode loading becomes the primary limitation. Another factor which will tend to reduce efficiency at higher frequencies is the reduction in realizable values of \( Q_{\text{eff}} \) due to increased circuit losses.


I, Dean Trafford Davis, was born in Minerva, Ohio, May 14, 1927. I received my secondary school education at Gallia Academy High School in Gallipolis, Ohio. I enrolled in the College of Engineering of The Ohio State University in 1946 and obtained the degree Bachelor of Electrical Engineering in 1951. In 1953, I enrolled in the Graduate School and joined the staff of the Electron Device Laboratory, where I was employed as a Research Assistant while completing the requirements for the degree Master of Science which I received in 1955. In October 1955 I joined the teaching staff of the Department of Electrical Engineering as an Instructor and continued in this position while completing the requirements for the Doctor of Philosophy degree.