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NASH, Robert Thornton, 1929—
A MULTIREFLECTOR MERIDIAN-TRANSIT RADIO-TELESCOPE ANTENNA FOR THE OBSERVATION OF WAVES OF EXTRA-TERRESTRIAL ORIGIN.

The Ohio State University, Ph.D., 1961
Engineering, electrical
University Microfilms, Inc., Ann Arbor, Michigan
A MULTIREFLECTOR MERIDIAN-TRANSIT RADIO-TELESCOPE ANTENNA
FOR THE OBSERVATION OF WAVES OF EXTRATERRESTRIAL ORIGIN

DISSERTATION
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Robert Thornton Nash, B.S., M.S.

The Ohio State University
1961

Approved by

[Signature]
Adviser
Dept. of Electrical Engineering
ACKNOWLEDGEMENTS

It is indeed impossible to note all of those who have taken part in realizing the Ohio State-Ohio Wesleyan Radio Telescope. Dr. J. D. Kraus, who is Director of The Ohio State University Radio Observatory, originated the fundamental concept of the antenna and without his direction this instrument would not have come into existence. During the mechanical design and the construction Mr. David Lipphardt and Mr. Bertan Morrow provided valuable assistance. While the test program was being conducted the comments of Dr. H. C. Ko were most helpful, as was the assistance of Mr. Robert Townsend in obtaining measurements. Finally there are many past undergraduate students who aided in the construction for a large proportion of the assembly work was performed by students. Mr. Clayton Fletcher and William Ryan were helpful in the preparation of the drawings and photographs. My wife, Frances, was most diligent in her completion of the manuscript and is indeed deserving of my thanks. The construction of this instrument has been made possible by grants to The Ohio State University by the National Science Foundation.
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</table>
INTRODUCTION

The program in radio astronomy was begun at The Ohio State University in 1951 by Professor J. D. Kraus. At its inception the program was one in which the measurement of the magnitude of the background radiation was of prime interest. This work culminated in the publication of a map of the radio sky between $-35^\circ$ and $60^\circ$ declination at a frequency of 250 megacycles. It became apparent even before this work was completed, that there was a necessity for larger apertures, a resulting increase in sensitivity, and a greater frequency range.

In 1953 the concept of the antenna system described herein was brought forth by Professor Kraus as a possible answer to the question of obtaining feasible antenna systems of large aperture. A study was made and a scale model of the system was constructed and tested by the author in 1954 and 1955. These tests demonstrated the fundamental properties of the system, and on this basis construction was begun in 1956 of a prototype instrument. The antenna consists of a fixed paraboloid, a conducting ground and a tiltable flat sheet, on a north-south axis. The axis of the paraboloid is coincident with the ground and therefore an image is present. The flat sheet may be moved through 50 degrees, which tips the antenna beam through 100 degrees. The design and measured performance of one half of the prototype is the subject of this dissertation.
CHAPTER I
Fundamental Aspects of the Design

The Antenna System

The antenna in a radio telescope is that portion of the system which acts as the collector of the incoming radio noise present at the point of observation. The total power collected by the antenna may be given by the following expression (1).

\[ W = \frac{A_{em}}{2} \int \int B(\theta, \phi, f) D(\theta, \phi, f) \, d\Omega \, df \, (\text{watts}) \]  

where

- \( W \) = received power in watts
- \( A_{em} \) = effective aperture of the antenna, (meters\(^2\))
- \( B(\theta, \phi, f) \) = brightness of the radio sky in (watts m\(^{-2}\) cps\(^{-1}\) rad\(^{-2}\))
- \( D(\theta, \phi) \) = normalized antenna power pattern
- \( d\Omega = \sin \theta \, d\theta \, d\phi \)
- \( df \) = elemental bandwidth
- \( \theta \) = zenith angle
- \( \phi \) = azimuth angle
- \( f \) = frequency

If only the power received per unit bandwidth is considered we have

\[ P_f = \frac{A_{em}}{2} \int \int B(\theta, \phi) \, D(\theta, \phi) \, d\Omega \cdot (\text{watts cps}\(^{-1}\)) \]  

Now \( A_{em} \) may be given by

\[ A_{em} = A_p \sum_{k=1}^{N} G_k \, (\text{meters}^2) \]  

where

- \( A_p \) = physical aperture (meters\(^2\))
Fig. 1 Coordinates
$G_K$ = efficiency factors for the system. (0 ≤ $G_K$ ≤ 1)

The several $G_K$'s are due to spill over and illumination shape, leakage, phase errors, aperture blocking, feed efficiency and, in this case, the effect of the second reflector. The first two factors must be considered together; however, in the case of the other factors independence is assumed based on the premise that each is small and that it may be treated as a perturbation. $D_n(\theta, \phi)$ in general may be considered to be simply dependent on the relative field intensity across the aperture. These various factors will be considered in sequence and finally the several effects will be brought together to determine the effective aperture as a function of frequency.

The Illumination-Spillover Relationship

The aperture factor is a function of the proportion of energy intercepted by the reflector system and also the spatial amplitude distribution of this energy. Figure 1 depicts the antenna system. In this analysis a rectangular aperture with an inphase field located at the focal point illuminates the reflector and the electric field vector is in the vertical direction. This rectangular aperture is in reality the mouth of an electromagnetic horn, and therefore the field intensity across the reflector is readily calculable. In the dimensionless coordinates $u$ and $v$ the field amplitude distribution will be given by (2)

$$g_1(u) g_2(v) = \left[ \frac{\sin v}{v} \right] ^2 \left[ \cos u \left( 1 - \frac{\ln^2}{\pi^2} \right) \right]$$

(4)

where $u = na \lambda \sin \alpha$, $v = nb \lambda \sin \beta$

$$u_0 = \text{value of } u \text{ corresponding to the angle } \alpha_0 \text{ to the edge of}$$
reflector

\[ v_o = \text{value of } v \text{ corresponding to the angle } \beta_o \text{ to the top of} \]
reflector or

\[ u_o = na \sin \alpha \]
\[ v_o = nb \sin \beta_o \]

Therefore the proportion of the total power intercepted by the reflector is

\[
\frac{P_I}{P_T} = \frac{\int_{-\beta_o}^{\beta_o} \int_{-\alpha_o}^{\alpha_o} g_1^2(a) g_2^2(\beta) \sin(90-\beta) \, da \, d\beta}{2n \int_{-\beta_o}^{\beta_o} \int_{-\alpha_o}^{\alpha_o} g_1^2(a) g_2^2(\beta) \sin(90-\beta) \, da \, d\beta}
\]

(6)

where the integrals represent the proportions of power intercepted by the reflector in the horizontal and vertical directions. The horizontal proportion will be evaluated numerically while

\[
\int_{-\beta_o}^{\beta_o} g_2^2(\beta) \sin(90-\beta) \, d\beta = \frac{2}{\pi} \left[ \frac{\cos(2n\beta_0) - 1}{2n} \right] + Si(\frac{2n\beta_0}{F})
\]

(7)

Now the gain of the antenna is given by

\[
G = \frac{4n}{\lambda^2} \frac{P_I}{P_T} \left[ \int_{A_F} H(E, \eta) \, dE \, d\eta \right]^2 \left[ \int_{A_F} |H(E, \eta)|^2 \, dE \, d\eta \right]^2
\]

(8)
and the following coordinate transformations will be helpful.

\[
\frac{\xi}{F} = \tan \alpha \quad \frac{\zeta}{F} = \tan \beta \\
\frac{\xi}{F} = \sin \alpha \quad \frac{\zeta}{F} = \sin \beta
\]

which yields from combination with equation (5)

\[
u = \frac{\xi}{F} \quad \eta = \frac{\zeta}{F} \quad \text{nb}
\]

let \( \xi^* = \frac{\xi}{\xi^0} \quad \eta^* = \frac{\zeta}{\zeta^0} \)

then \( u = \frac{\xi^0}{F} \, \text{na} \, \xi^* \quad v = \frac{\zeta^0}{F} \, \text{nb} \, \eta^* \)

with \(-1 \leq \xi^* \leq 1 \quad -1 \leq \eta^* \leq 1 \)

\( H (\xi, \eta) \) the main aperture field intensity distribution may be

assumed to be parabolic and therefore

\[
H (\xi, \eta) = \left[ 1 - (1 - \xi^1 (u_0) \xi^* \xi^2 \right] \left[ 1 - (1 - \xi^2 (v_0) \eta^* \eta^2 \right]
\]

If we let \( K_1 = 1 - \xi^1 (u_0) \quad K_2 = 1 - \xi^2 (v_0) \).

then

\[
G = \frac{4 \pi P_F}{\lambda^2} \frac{P_T}{P_T} \frac{1}{\int_{A_P} (1-K_1 \frac{\xi^2}{\xi^0} (1-K_2 \frac{\eta^2}{\eta^0} \, d\xi d\eta)^2} \\
\int_{A_P} \left( 1-K_1 \frac{\xi^2}{\xi^0} (1-K_2 \frac{\eta^2}{\eta^0} \right)^2 \, d\xi d\eta
\]

(9)
Upon evaluation of the integrals

\[ G = \frac{l_p A_p P_T}{\lambda^2} \frac{\left| \int_{-1}^{1} (1-K_1 e^{z^2}) e^z dz \right|^2}{\int_{-1}^{1} (1-K_1 e^{z^2})^2 dz^2} \]

\[ \times \left| \int_{0}^{1} (1-K_2 d^z) dz \right|^2 \]

(10)

Upon evaluation of the integrals

\[ G = \frac{l_p A_p P_T}{\lambda^2} \frac{\left[ 2 + g_1 (u_o) \right]^2}{9 \left[ 1 - \frac{2}{3} (1-g_1) (u_o) + \frac{1}{3} (1-g_1) (u_o)^2 \right]} \]

\[ \times \frac{\left[ 2 + g_2 (v_o) \right]^2}{9 \left[ 1 - \frac{2}{3} (1-g_2) (v_o) + \frac{1}{3} (1-g_2) (v_o)^2 \right]} \]

(11)

where \( A_p \) = physical aperture of the reflector (m²)

Since \( C_o, J_o, \) and \( P \) are presumably known and

\[ u_o = \frac{C_o}{P} na \lambda \quad v_o = \frac{J_o}{P} nb \lambda \]

it is possible to maximize equation (11) by a proper choice of \( a \lambda \) and \( b \lambda \). This is shown in Figure 2 where it will be noted the \( G_1 \) is maximum, the product of curves IV and V; when \( g_1(u_o) \) and \( g_2(v_o) \) are .35 which indicates that the power level at the reflector edge should be about one tenth the value at the reflector center. The optimum
I. ILLUMINATION FACTOR
II. HORIZONTAL SPILLOVER
III. VERTICAL SPILLOVER
IV. PRODUCT OF I + II
V. PRODUCT OF I + III

Fig. 2 Optimum illumination
values of $g_1(u_0)$ and $g_2(v_0)$ in turn lead to $a^\lambda$ and $b^\lambda$.

Since a waveguide frequency band covers a range of 1.52:1 it may be divided into two parts with ranges of 1.23:1. This much frequency swing may be tolerated for a horn of constant physical aperture with a 1% gain change. The performance is tabulated in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Ideal Aperture Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1(u_0)$ or $g_2(v_0)$</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>$0.45$</td>
</tr>
<tr>
<td>$0.35$</td>
</tr>
<tr>
<td>$0.25$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>frequency</th>
<th>$a^\lambda(2\ell_o=180')$</th>
<th>$b^\lambda(\gamma_o=70')$</th>
<th>Side Spillover (%)</th>
<th>Top Spillover (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.9 f_o$</td>
<td>4.15</td>
<td>3.90</td>
<td>2.2</td>
<td>10.3</td>
</tr>
<tr>
<td>$f_o$</td>
<td>4.60</td>
<td>4.35</td>
<td>3.4</td>
<td>11.8</td>
</tr>
<tr>
<td>$1.11 f_o$</td>
<td>5.10</td>
<td>4.80</td>
<td>6.0</td>
<td>13.2</td>
</tr>
</tbody>
</table>

The gain has been computed on the basis of a perfectly conducting ground, which creates an electrical image and a resultant field intensity which is maximum at the ground plane. If the conducting ground is not present the gain ideally remains unchanged if $b^\lambda$ is doubled and the coordinate origin for $\gamma$ is shifted vertically to the
center of the aperture. The gain for a uniformly illuminated aperture is given by (5)

$$G_0 = \frac{\ln Ap}{\lambda^2}$$

(12)

and thus the factor in the fourth column of Table I may be regarded as an aperture efficiency factor where the basis for comparison is the gain expression in equation (11). For optimum conditions

$$G = \frac{\ln}{C^2} (.757 Ap) = \frac{\ln}{C^2} (A_{em})$$

(13)

and

$$A_{em} = .757 Ap$$

or

$$G_1 = .757$$

(14)

and $A_{em}$ will be referred to as the maximum effective aperture. In succeeding sections it will be shown that $A_{em}$ will be further reduced due to leakage, random errors, feed inefficiencies and the effect of the second reflector.

The Reflectivity of the Antenna Surface

The surface of a reflector need not be a continuous conducting sheet, in order to behave as a reflector of radio frequency energy. Consider a grid of cylindrical posts which are parallel to the plane of polarization of a plane wave which is normally incident upon the grating. The grid is present in an infinite medium of characteristic impedance $Z_0$ (Figure 3a). An equivalent circuit for this grating is shown in Figure 3b where the value of the normalized reactance is given by (6)

$$\frac{X_A}{Z_0} = \frac{c}{\lambda} \text{Im} \left( \frac{c}{\rho d} \right)$$

(15)
Fig. 3 Grating.
The value of $X_b$ is neglected for it may be assumed to be equal to zero for the situations to be considered.

Two cases will be considered in which the diameter $d$ is .208 cm. The first is one in which the spacing between posts is $2.5h$ cm. This is the case for the center 180° of the parabola and the entire flat reflector. The second situation is for the outer 90° on each side of the paraboloid where the spacing is 7.62 cm. The normalized reactances as a function of frequency are given by

$$c = 2.5h \text{ cm} \quad \frac{X_a}{Z_0} = \frac{3.43}{\lambda} \quad (16)$$

$$c = 7.62 \text{ cm} \quad \frac{X_a}{Z_0} = \frac{18.7}{\lambda} \quad (17)$$

with $\lambda$ expressed in centimeters. The normalized admittance presented at terminals $m-n$ will be represented by,

$$Y_{m-n} = 1 - j \frac{\lambda}{3.43} \quad \text{for } c = 2.5h \text{ cm.} \quad (18)$$

$$Y_{m-n} = 1 - j \frac{\lambda}{18.7} \quad \text{for } c = 7.62 \text{ cm.} \quad (19)$$

The general expression for the voltage transmission coefficient in equation (20) may now be evaluated.

$$\mathcal{Y} = \frac{2}{1 + Y_{m-n}} \quad (20)$$

Upon insertion of the two values for $Y_{m-n}$ expressions for $\mathcal{Y}$ as a function of wavelength result.

$$\mathcal{Y} = \frac{2}{2 - j \frac{\lambda}{3.43}} \quad \text{for } c = 2.5h \text{ cm} \quad (21)$$
The power transmitted through the grating is given by

\[ J = \frac{2}{2-j} \frac{\lambda}{18.7} \quad c = 7.62 \text{ cm} \quad (22) \]

\[ \frac{P_T}{P_o} = |J|^2 \quad (23) \]

In tabular form these expressions are evaluated for the two grating separations throughout the range of interest (Table II).
### TABLE II
Grating Transmission Factors
\( c = 2.54 \text{ cm.} \)

<table>
<thead>
<tr>
<th>( \lambda (\text{cm.}) )</th>
<th>( \frac{x}{Z_0} )</th>
<th>( J_1 )</th>
<th>( \frac{P_T}{P_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>.057</td>
<td>.113</td>
<td>.013</td>
</tr>
<tr>
<td>30</td>
<td>.114</td>
<td>.222</td>
<td>.048</td>
</tr>
<tr>
<td>20</td>
<td>.172</td>
<td>.314</td>
<td>.104</td>
</tr>
<tr>
<td>15</td>
<td>.229</td>
<td>.415</td>
<td>.172</td>
</tr>
<tr>
<td>12</td>
<td>.286</td>
<td>.497</td>
<td>.215</td>
</tr>
<tr>
<td>10</td>
<td>.343</td>
<td>.565</td>
<td>.310</td>
</tr>
</tbody>
</table>

\( c = 7.62 \text{ cm.} \)

<table>
<thead>
<tr>
<th>( \lambda (\text{cm.}) )</th>
<th>( \frac{x}{Z_0} )</th>
<th>( J_2 )</th>
<th>( \frac{P_T}{P_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>.062</td>
<td>.120</td>
<td>.014</td>
</tr>
<tr>
<td>150</td>
<td>.125</td>
<td>.242</td>
<td>.058</td>
</tr>
<tr>
<td>100</td>
<td>.187</td>
<td>.350</td>
<td>.115</td>
</tr>
<tr>
<td>75</td>
<td>.250</td>
<td>.417</td>
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<td>60</td>
<td>.312</td>
<td>.500</td>
<td>.270</td>
</tr>
<tr>
<td>50</td>
<td>.376</td>
<td>.600</td>
<td>.360</td>
</tr>
<tr>
<td>37</td>
<td>.500</td>
<td>.707</td>
<td>.500</td>
</tr>
<tr>
<td>30</td>
<td>.625</td>
<td>.780</td>
<td>.600</td>
</tr>
</tbody>
</table>

The reflection coefficient which may be used can be evaluated in terms of \( J \) from the following:
\[ p = \tau - 1 \]

The reflection coefficients being represented by,

\[ p_1 = \tau_1 - 1 \]

and

\[ p_2 = \tau_2 - 1 \]

Since there are two screens then

\[ G_2 = \left(1 - \frac{P_T}{P_0}\right)^2 \]

where \( G_2 \) is the proportion of power reflected and these values are given for \( c = 2.54 \text{ cm.} \) in Table III.

**TABLE III**

Grating Transmissions Factors for Two Reflections

<table>
<thead>
<tr>
<th>( \lambda \text{ (cm.)} )</th>
<th>( G_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>.971*</td>
</tr>
<tr>
<td>30</td>
<td>.907</td>
</tr>
<tr>
<td>20</td>
<td>.803</td>
</tr>
<tr>
<td>15</td>
<td>.686</td>
</tr>
<tr>
<td>12</td>
<td>.570</td>
</tr>
<tr>
<td>10</td>
<td>.475</td>
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</tbody>
</table>

These values will be employed later in the determination of the overall antenna gain. It will be noted that at low frequencies the equivalent inductive reactance is zero.

The Feed Efficiency

The aperture used to illuminate the paraboloidal reflector is assumed to be rectangular with dimensions \( a \) and \( b \) respective-
ly for width and height. Uniform phase across the aperture is implicit in the gain determination, and in addition only half the vertical dimension of the aperture was physically present.

A compound electromagnetic horn will produce the desired aperture condition if it is vertically polarized and has the appropriate values for $a_\lambda$ and $b_\lambda$. The field intensity will be uniform in the $y$ direction and vary as

$$E = \cos \frac{nx}{a}$$  \hspace{1cm} (28)

in the horizontal direction. Uniformity of phase across the horn mouth will produce a gain which was determined in the last section. Any deviation from a condition of phase uniformity across the aperture, or horn mouth will result in a gain diminution. This section will be devoted to a consideration of this question.

For any horn antenna with flare angles $\gamma_{xa}$ and $\gamma_{yb}$ the phase at any point in the $y$ plane, at the mouth will be given by

$$\text{phase} = \frac{L_\lambda}{\cos \gamma_y} - L_\lambda$$  \hspace{1cm} (29)

$$\text{phase} = L_\lambda (1 + \frac{y^2}{2}) - L_\lambda$$  \hspace{1cm} (30)

$$\text{phase} = L_\lambda \frac{y^2}{2L_\lambda} = \frac{y^2}{2L_\lambda}$$  \hspace{1cm} (31)

where $\gamma_y = \frac{y}{L_\lambda}$

$$y \leq \frac{b_\lambda}{2}$$

Thus at the top of the horn mouth
In the $x$ plane where $y = 0$

$$\text{phase error} = \frac{x^2}{2L^{-1/2}} x - \frac{a}{2}$$

(33)

and at the edge of the horn mouth

$$\text{max. phase error} = \frac{a^2}{8L^{-1/2}}$$

(34)

This phase error is quadratic and it is of interest to determine both the gain loss of the feed as a function of the error and also the resultant deviation from a spherical phase front of the radiation from the horn, which produces a further gain loss in the main aperture plane.

In the vertical plane, $x = 0$ the relative field intensity is given by

$$g_3(v) = \int_0^1 e^{jvy} e^{-jK_3y^2} dy$$

(35)

where

$$K_3 = \frac{n}{4L^{-1/2}} y^* = \frac{2y}{b^{-1/2}} 0 \leq y^* \leq 1$$

or

$$g(v) = \int_0^1 (\cos vy^* \cos K_3y^2 - j \cos vy^* \sin K_3y^2) dy^*$$

(36)

This may be evaluated by expansion into a power series and it is desirable to evaluate this expression in the range

$$0 \leq v \leq 3.0$$

and thus terms through $v^8$ must be retained. The normalized field

$$\text{max. phase error} = \frac{b^2}{8L^{-1/2}}$$

(32)
intensity after evaluating the integrals will be

$$g(v) = 1 - \frac{K_3^2}{10} + \frac{K_3^4}{215} - v^2 \left( \frac{1}{6} - \frac{K_3^2}{28} + \frac{K_3^4}{528} \right)$$

(37)

$$+ v^4 \left( \frac{1}{120} - \frac{K_3^2}{432} + \frac{K_3^4}{7180} \right)$$

$$- v^6 \left( \frac{1}{15,850} - \frac{K_3^2}{15} + \frac{K_3^4}{259,000} \right)$$

$$+ v^8 \left( \frac{1}{363,000} - \frac{K_3^2}{1,047,000} + \frac{K_3^4}{16,450,000} \right)$$

$$- \left\{ \frac{K_3^3}{3} - \frac{K_3^3}{12} + \frac{K_3^5}{1,220} \right\}$$

$$- v^2 \left( \frac{K_3^3}{10} - \frac{K_3^3}{108} + \frac{K_3^5}{3120} \right)$$

$$+ v^4 \left( \frac{K_3}{168} - \frac{K_3^3}{1560} + \frac{K_3^5}{43,200} \right)$$

$$- v^6 \left( \frac{K_3}{6480} - \frac{K_3}{56,200} + \frac{K_3}{1,470,000} \right)$$

$$+ v^8 \left( \frac{K_3}{41,300} - \frac{K_3^3}{3,620,000} + \frac{K_3^5}{91,900,000} \right) \right\}$$

This expression is evaluated for three values of $K_3$ and for

$0 \leq v \leq 2.5$ in Table IV.
TABLE IV
E Plane Field Intensity

<table>
<thead>
<tr>
<th>v</th>
<th>g_3(v)</th>
<th>g_3(v)</th>
<th>g_3(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K_3 = 0</td>
<td>K_3 = \pi/4 = \lambda/8</td>
<td>K_3 = \pi/2 = \lambda/4</td>
</tr>
<tr>
<td>0</td>
<td>1.000</td>
<td>.972</td>
<td>.896</td>
</tr>
<tr>
<td>.5</td>
<td>.960</td>
<td>.935</td>
<td>.863</td>
</tr>
<tr>
<td>1.0</td>
<td>.843</td>
<td>.823</td>
<td>.769</td>
</tr>
<tr>
<td>1.5</td>
<td>.662</td>
<td>.660</td>
<td>.630</td>
</tr>
<tr>
<td>2.0</td>
<td>.452</td>
<td>.416</td>
<td>.480</td>
</tr>
<tr>
<td>2.5</td>
<td>.238</td>
<td>.272</td>
<td>.346</td>
</tr>
</tbody>
</table>

The field intensity is less for small v and greater for large v than the values for the ideal case, i.e. (K_3 = 0). The phase is also seen to change as v increases which will produce a phase retardation across the main aperture. The relative vertical gain for the feed is approximately equal to

\[
\left( g_3 (v) \right)^2 = 1 - \frac{K_3^2}{12.7} \tag{38}
\]

where \( K_3 \) = maximum error in radians

The gain loss for the X plane (y = 0) will be calculated, but the phase for \( u > 0 \) will not be for the effect is negligible.

In the horizontal plane (y = 0) the relative field intensity is for \( u = 0 \)

\[
g_{14} (0) = \int_{-1}^{1} (1 - x^2) e^{-j K_{14} x^2} \, dx \tag{39}
\]

19
where \( K_4 = \frac{n a^2}{4 \lambda A} \)

Evaluation through a power series expansion yields

\[
\frac{g_{14}(0)}{1.365} = \left(1 - \frac{K_4^2}{10 + 216} - \frac{1}{2} \frac{K_4^4}{11 + 264} \right)
- j \left[ \frac{K_4^3}{3} - \frac{K_4^5}{1320} - \frac{K_4^3}{5} - \frac{K_4^5}{54 + 1560} \right]
\]

The factor 1.365 accounts for the power distribution across the aperture and equation (40) is evaluated in Table V.

**TABLE V**

**H Plane Field Intensity**

<table>
<thead>
<tr>
<th>( K_4 )</th>
<th>( g_{14}(u = 0) )</th>
<th>( \frac{g_{14}(0, K_4)^2}{g_{14}(0, 0)^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.91</td>
<td>1.000</td>
</tr>
<tr>
<td>( \pi/4 )</td>
<td>.902 ( \angle -2^\circ )</td>
<td>.980</td>
</tr>
<tr>
<td>( \pi/2 )</td>
<td>.863 ( \angle -17.2^\circ )</td>
<td>.895</td>
</tr>
</tbody>
</table>

The phase errors in the H plane of the horn mouth do not affect the pattern as much as the phase errors in the E plane, since the edge contribution is smaller in the H plane.

The additional effect of the phase change in the v coordinate on the overall gain will now be considered. From Table IV the phase change across the reflector in the v coordinate may be
found. This is very nearly equal to

\[ \text{phase} = \left| K_5 \gamma^*^3 \right| \]  \hspace{1cm} (4.1)

where the \( K_5 \)'s are given by

\[
\begin{array}{cc}
K_3(\text{rad}) & K_5(\text{rad}) \\
0 & 0 \\
\pi/4 & .32 \\
\pi/2 & .53 \\
\end{array}
\]

The \( K_5 \)'s are determined by taking the value of the phase error for \( v = 2.25 \) which is that value of \( v \), which was shown to optimize the gain. The normalized relative on axis field intensity is then

\[ g_1(0) = \int_0^1 (1 - .65 \gamma^*^2) e^{j K_5 \gamma^*^3} d\gamma^* \]  \hspace{1cm} (4.2)

\[ g_1(0) = \left[ 1 - \frac{.65}{3} + \frac{.65 K_5^2}{18} + \frac{K_5^4}{312} - \frac{.65 K_5^4}{360} \right] \]  \hspace{1cm} (4.3)

\[ -j K_5 \left[ \frac{1}{4} - \frac{K_5^2}{80} - \frac{.65}{6} + \frac{.65 K_5^2}{72} \right] \]

The value of \( g_1(0) \) for several values of \( K_5 \) is given in Table VI.
The gain reduction factors are approximately

\[ G_4 = 1 - \frac{K_3^2}{12} \quad (\phi \text{ plane}) \quad K_3 \text{ in radians} \quad (44) \]

\[ G_3 = 1 - \frac{K_4^2}{23} \quad (\theta \text{ plane}) \quad K_4 \text{ in radians} \quad (45) \]

The length may now be determined immediately for a horn which will have a given efficiency. Expressing the gain factors in terms of the horn parameters,

\[ G_4 = 1 - \frac{b^2}{18 L^2} \quad (\phi \text{ plane}) \quad (46) \]

\[ G_3 = 1 - \frac{a^2}{37 L^2} \quad (\theta \text{ plane}) \quad (47) \]

For optimum operation with \( \theta_0 = 70^\circ \) and \( 2 \theta_0 = 180^\circ \), \( b_\lambda = 4.35 \) and \( a_\lambda = 4.65 \). Thus if \( L_\lambda = 25 \) then

\[ G_4 = 1 - .032 = .968 \]

\[ G_3 = 1 - .020 = .980 \]

Thus the feed may be operated at approximately 95% efficiency for lengths of the order of 25 wavelengths.

For the feed used during the test program \( b_\lambda = 9.3 \), \( a_\lambda = 5 \) and \( L_\lambda = 43 \) thus

\[ G_4 = 1 - .21 = .79 \]

\[ G_3 = 1 - .01 = .99 \]

These figures will be referred to later in the determination of the overall antenna gain.
TABLE VI
Non Spherical Factor

<table>
<thead>
<tr>
<th>K₃(rad)</th>
<th>K₅(rad)</th>
<th>g₁(0)</th>
<th>g₁(0,K₅)²/g₁(0,0)²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>.783</td>
<td>1.000</td>
</tr>
<tr>
<td>n/4</td>
<td>.32</td>
<td>.781 /3.2°</td>
<td>.995</td>
</tr>
<tr>
<td>n/2</td>
<td>.53</td>
<td>.775 /5.0°</td>
<td>.98</td>
</tr>
</tbody>
</table>

An overall gain reduction factor for the feed may now be determined in terms of the horn phase error. This is summarized in Table VII.

TABLE VII
Feed Gain Reduction Factors

E Plane or $\gamma$ Plane

<table>
<thead>
<tr>
<th>K₃(rad)</th>
<th>g₃(0,K₃)²/g₃(0,0)²</th>
<th>K₅</th>
<th>g₅(0,K₅)²/g₅(0,0)²</th>
<th>g₃(0,K₃)² g₅(0,K₅)²/g₃(0,0)² g₅(0,0)²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
<td>0</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>n/4</td>
<td>.945</td>
<td>.32</td>
<td>.995</td>
<td>.94</td>
</tr>
<tr>
<td>n/2</td>
<td>.805</td>
<td>.53</td>
<td>.98</td>
<td>.79</td>
</tr>
</tbody>
</table>

H Plane or $\psi$ Plane

<table>
<thead>
<tr>
<th>K₄</th>
<th>g₄(0,K₄)²/g₄(0,0)²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>n/4</td>
<td>.980</td>
</tr>
<tr>
<td>n/2</td>
<td>.895</td>
</tr>
</tbody>
</table>
The Perturbation in the Antenna Near Field Caused by the Flat Reflector

It is not completely correct to determine the antenna gain simply by considering the paraboloid. The flat sheet, present in the Fresnel zone of the paraboloid, changes the plane of the phase front and in addition produces a slight gain loss due to phase variations across the sheet. At the edges, the phase variations are quite large and it will be shown that there is an optimum width for the flat sheet.

The field intensity at some point \((p,q)\) in the aperture plane of the flat sheet is given by

\[
\mathcal{E}_1(p,q) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(1-K_1^2)(1-K_2^2)}{z} e^{-2\pi j z d \phi} \mathcal{J} \tag{48}
\]

An approximation of \(z\) will be shown to be quite appropriate.

\[
z = z_0^2 + (f-p)^2 + (\phi-q)^2 \tag{49}
\]

The distance \(z\) may be given approximately by

\[
z = z_0 + \frac{(f-p)^2}{2z_0} + \frac{(\phi-q)^2}{2z_0} \tag{50}
\]

and the approximation is in error less than 1\% if

\[
(f-p) \text{ or } (\phi-q) \leq \frac{z_0}{5.35} \tag{51}
\]

The phase of the radiation arriving at point \(p\) will be retarded by

\(2\pi\) when

\[
(f-p)^2 = 2z_0 \tag{52}
\]

or 360\(\pi\) degrees when

\[
(f-p)^2 = 2\pi z_0 \tag{53}
\]
whence

\[(I - p) = \sqrt{2n} \sqrt{z_0} \]  \hspace{1cm} (54)

If expressions 51 and 54 are equated we may find the zone number \( n \) for which the 1% approximation is good.

\[2n z_0 = \frac{z_0^2}{5.35^2} \]  \hspace{1cm} (55)

\[n = \frac{z_0}{5.35} \]  \hspace{1cm} (56)

Since the field intensity at \((p, q)\) is nearly completely determined by the contributions from the first several full wave zones, these approximations will be found to be very reasonable. Equation (48) now may be written as

\[g_1(p, q) = \frac{e^{-2\pi j z_0}}{z_0} \int_{-\infty}^{\infty} e^{-2\pi j \frac{(I-p)^2}{2z_0}} d\xi \int_{-\infty}^{\infty} e^{-2\pi j \frac{(q-q)^2}{2z_0}} d\eta \]  \hspace{1cm} (57)

If \(K_1 = 0\), and \(K_2 = 0\) then this expression simply reduces to the Fresnel integral which might be evaluated from tables. However, in this case, for the \(K\)'s non zero, another technique must be resorted to in order to determine the field intensity in the \(p-q\) plane. This may be accomplished if an integration is performed over individual zones, and then a summation is carried out over all zones. In the first horizontal zone,

\[g_1(p) = \frac{e^{2\pi j z_0}}{z_0} \int_{p-\sqrt{2z_0}}^{p+\sqrt{2z_0}} (1-K_1\xi^2) e^{-2\pi j \frac{(I-p)^2}{2z_0}} d\xi \]  \hspace{1cm} (58)
\[ \begin{align*}
\frac{e^{2\pi nj_0}}{z_0} & \sum_{n=1/4}^{1} \int_{P+/2z_0}^{P+/2z_0(n-1/4)} (1-K_1^2) e^{-2\pi j \left( \frac{p}{2z_0} \right)^2} d\xi \\
\frac{e^{2\pi nj_0}}{z_0} & \sum_{n=1/4}^{1} \int_{P+/2z_0(n-1/4)}^{P+/2z_0(n-1/4)} (1-K_1^2) e^{-2\pi j \left( \frac{p}{2z_0} \right)^2} d\xi
\end{align*} \]

\[ n = 1/4, 1/2, 3/4, 1 \]

The first full wave zone is evaluated in \( \lambda/4 \) increments while succeeding zones are evaluated as whole zones or fractional zones. Thus for \( n \) an integer

\[ \mathcal{E}_1(p) = \frac{e^{2\pi nj_0}}{z_0} \sum_{n=2}^{n_0} \int_{P+/2z_0}^{P+/2z_0(n-1)} (1-K_1^2) e^{-j2\pi n \left( \frac{p}{2z_0} \right)^2} d\xi \\
+ \frac{e^{2\pi nj_0}}{z_0} \sum_{n=2}^{n_0} \int_{P+/2z_0(n-1)}^{P+/2z_0(n-1)} (1-K_1^2) e^{-j2\pi n \left( \frac{p}{2z_0} \right)^2} d\xi \]

where \( n_0 \) corresponds to the zone number at the edge of the aperture. It is assumed that the average value of \( f(\xi) \) may be used for a given zone and it will therefore be taken outside the integral. Thus we have

\[ \mathcal{E}_1(p) = \frac{e^{2\pi nj_0}}{z_0} \left\{ \sum_{n=2}^{n_0} (1-K_1^2) \int_{P+/2z_0(n-1)}^{P+/2z_0(n-1)} e^{-j2\pi n \left( \frac{p}{2z_0} \right)^2} d\xi \right\} \]
\[ + \sum_{n=2}^{n_0} \left( 1 - K_1 \xi^2 \right) \int \frac{e^{-j2\pi n(\xi-p)^2}}{p-\frac{1}{2z_0}} \, d\xi \] (61)

If the integrals can be evaluated then \( g_1(p) \) can be found by completing the summation. The value of these integrals is given in Table VIII.
### TABLE VIII
Quarter and Full Wave Zone Factors

<table>
<thead>
<tr>
<th>n</th>
<th>$\sqrt{2n} - \sqrt{(n-1/4)^2}$</th>
<th>$\sqrt{2n} - \sqrt{(n-1/4)}^2 \int_{p+2(n-1/4)}^{p+2n} \frac{e^{-2n\psi(p-c)^2}}{2z_0} d\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4</td>
<td>.707</td>
<td>.633 $/-29^\circ$</td>
</tr>
<tr>
<td>1/2</td>
<td>.293</td>
<td>.294 $/-133^\circ$</td>
</tr>
<tr>
<td>3/4</td>
<td>.225</td>
<td>.203 $/-224^\circ$</td>
</tr>
<tr>
<td>1.0</td>
<td>.189</td>
<td>.170 $/-45^\circ$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>n</th>
<th>$\sqrt{2n} - \sqrt{(n-1)}$</th>
<th>$\sqrt{2(n-1)} - \sqrt{(n-1-1)} \int_{p+2(n-1)}^{p+2n} \frac{e^{-2n\psi(p-c)^2}}{2z_0} d\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>.586</td>
<td>.032 $/-90^\circ$</td>
</tr>
<tr>
<td>3.0</td>
<td>.449</td>
<td>.015 $/-90^\circ$</td>
</tr>
<tr>
<td>4.0</td>
<td>.379</td>
<td>.009 $/-90^\circ$</td>
</tr>
<tr>
<td>5.0</td>
<td>.334</td>
<td>.006 $/-90^\circ$</td>
</tr>
<tr>
<td>6.0</td>
<td>.302</td>
<td>.004 $/-90^\circ$</td>
</tr>
<tr>
<td>7.0</td>
<td>.277</td>
<td>.003 $/-90^\circ$</td>
</tr>
<tr>
<td>8.0</td>
<td>.259</td>
<td>.003 $/-90^\circ$</td>
</tr>
<tr>
<td>9.0</td>
<td>.242</td>
<td>.003 $/-90^\circ$</td>
</tr>
<tr>
<td>10.0</td>
<td>.230</td>
<td>.002 $/-90^\circ$</td>
</tr>
</tbody>
</table>

Beyond the tenth zone, full zones need not be considered. However, partial zones do need to be included. Table IX gives half zone values for n less than ten.
### TABLE IX

**Half Wave Zone Factors**

\[ \begin{array}{cccc}
\text{n} & \int_{/2n-\pi(\text{n}-1/2)}^{p+\pi/2} e^{-2n_0^2 \frac{(x-p)^2}{\pi^2}} \, dx & \int_{/2n-\pi(\text{n}-1/2)}^{p+\pi/2} e^{-2n_0^2 \frac{(x-p)^2}{\pi^2}} \, dx \\
1.5 & .318 & .637 /-90^\circ & .202 /-90^\circ \\
2.0 & .268 & .170 /+90^\circ & .170 /+90^\circ \\
2.5 & .236 & .150 /-90^\circ & .150 /-90^\circ \\
3.0 & .213 & .135 /+90^\circ & .135 /+90^\circ \\
3.5 & .197 & .125 /-90^\circ & .125 /-90^\circ \\
4.0 & .182 & .116 /+90^\circ & .116 /+90^\circ \\
4.5 & .172 & .109 /-90^\circ & .109 /-90^\circ \\
5.0 & .162 & .103 /+90^\circ & .103 /+90^\circ \\
5.5 & .154 & .098 /-90^\circ & .098 /-90^\circ \\
6.0 & .148 & .094 /+90^\circ & .094 /+90^\circ \\
6.5 & .141 & .089 /-90^\circ & .089 /-90^\circ \\
7.0 & .136 & .086 /+90^\circ & .086 /+90^\circ \\
7.5 & .132 & .084 /-90^\circ & .084 /-90^\circ \\
8.0 & .127 & .081 /+90^\circ & .081 /+90^\circ \\
8.5 & .123 & .079 /-90^\circ & .079 /-90^\circ \\
9.0 & .119 & .076 /+90^\circ & .076 /+90^\circ \\
9.5 & .116 & .074 /-90^\circ & .074 /-90^\circ \\
10.0 & .114 & .072 /+90^\circ & .072 /+90^\circ \\
\end{array} \]

The full zone values in Table VIII are obtained by adding the half zone

29
values in Table IX. For quarter zones beyond \( n = 10 \) see Table X.

**TABLE X**

Large Quarter Zone Factors

\[
\begin{array}{cccc}
\hline
n & \sqrt{\frac{2n}{2(n-1/4)}} & \int_{\frac{p+2n}{p+2(n-1/4)}} e^{-j2\pi \frac{(c-p)^2}{2z_0}} \, dc \\
10.25 & .055 & .05 /-45^\circ \\
20.25 & .040 & .036 /-45^\circ \\
30.25 & .032 & .029 /-45^\circ \\
\hline
\end{array}
\]

Thus for \( n \) greater than ten, full zone contributions may be neglected but partial zone contribution should be included. This method may be applied to determine the near field pattern or far field pattern, of any aperture with any parabolic illumination taper. Table XI gives values for the system herein described.
TABLE XI
Frequency-Zone Relations

<table>
<thead>
<tr>
<th>$f$ (mc)</th>
<th>$z_0$ ($\lambda$)</th>
<th>$2z_0$ ($\lambda$)</th>
<th>$2\gamma_0$ ($\lambda$)</th>
<th>$\sqrt{2} z_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2050</td>
<td>1000</td>
<td>376</td>
<td>146</td>
<td>44.7</td>
</tr>
<tr>
<td>1380</td>
<td>675</td>
<td>253</td>
<td>98</td>
<td>36.8</td>
</tr>
<tr>
<td>900</td>
<td>440</td>
<td>165</td>
<td>64</td>
<td>29.7</td>
</tr>
<tr>
<td>594</td>
<td>290</td>
<td>109</td>
<td>42</td>
<td>24.1</td>
</tr>
<tr>
<td>390</td>
<td>190</td>
<td>71</td>
<td>28</td>
<td>19.5</td>
</tr>
<tr>
<td>255</td>
<td>125</td>
<td>47</td>
<td>18</td>
<td>15.8</td>
</tr>
</tbody>
</table>

The value of the field intensity is given for several situations in the succeeding table which were determined for $z_0 = 4\lambda$ or $f = 900$ m.c. and are generally applicable for the frequencies above 250 m.c. For frequencies below 250 mc. the reflector becomes less than one zone in height and these effects must be considered more carefully.
### TABLE XII

**Intensities at 900 Megacycles**

<table>
<thead>
<tr>
<th>p(Å)</th>
<th>Description</th>
<th>g1(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>center</td>
<td>.93 / -45°</td>
</tr>
<tr>
<td>23</td>
<td>center</td>
<td>.90 / -42°</td>
</tr>
<tr>
<td>40</td>
<td>center</td>
<td>.80 / -42.5°</td>
</tr>
<tr>
<td>61</td>
<td>center</td>
<td>.67 / -43°</td>
</tr>
<tr>
<td>67</td>
<td>edge</td>
<td>.59 / -35°</td>
</tr>
<tr>
<td>72</td>
<td>edge</td>
<td>.47 / -33.5°</td>
</tr>
<tr>
<td>82</td>
<td>edge</td>
<td>.20 / -49.4°</td>
</tr>
<tr>
<td>93</td>
<td>edge</td>
<td>.11 / -125°</td>
</tr>
<tr>
<td>98</td>
<td>edge</td>
<td>.086 / -169°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>q(Å)</th>
<th>Without Ground Plane</th>
<th>g1(q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>center</td>
<td>1.03 / -35°</td>
</tr>
<tr>
<td>38</td>
<td>edge</td>
<td>1.07 / -42.3°</td>
</tr>
<tr>
<td>44</td>
<td>edge</td>
<td>.902 / -42.5°</td>
</tr>
<tr>
<td>54</td>
<td>edge</td>
<td>.655 / -28°</td>
</tr>
<tr>
<td>64</td>
<td>edge</td>
<td>.332 / -48.6°</td>
</tr>
<tr>
<td>75</td>
<td>edge</td>
<td>.101 / -146°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>q(Å)</th>
<th>With Ground Plane</th>
<th>g1(q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>center</td>
<td>.935 / -41.6°</td>
</tr>
<tr>
<td>12</td>
<td>center</td>
<td>.912 / -42.1°</td>
</tr>
<tr>
<td>34</td>
<td>center</td>
<td>.760 / -41°</td>
</tr>
<tr>
<td>43</td>
<td>center</td>
<td>.735 / -41.5°</td>
</tr>
<tr>
<td>53</td>
<td>center</td>
<td>.495 / -33.4°</td>
</tr>
<tr>
<td>64</td>
<td>edge</td>
<td>.222 / -48.5°</td>
</tr>
<tr>
<td>74</td>
<td>edge</td>
<td>.105 / -127°</td>
</tr>
</tbody>
</table>
It will be noted that for values of \( p \) or \( q \) greater than \( \alpha \) or \( \beta \), the phase changes very rapidly indicating that \( p_0 \) should be equal to or less than \( \alpha \) and \( q_0 \) less than \( \beta \). The first condition can be satisfied but the second cannot. It will also be noted that \( q \)-plane phase variations are greater without the ground plane than with the ground plane. Graphic integration gives a total power loss, for \( p \) greater than \( \alpha \) and \( q \) greater than \( \beta \) of about 2\%. The average peak phase errors and corresponding gain losses are given below.

\[
\text{av. peak error} \quad \left[ g_1 (0) \right]^2
\]

<table>
<thead>
<tr>
<th>Plane Conditions</th>
<th>Peak Error</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )-plane</td>
<td>0.017</td>
<td>0.994</td>
</tr>
<tr>
<td>( q )-plane (with ground plane)</td>
<td>0.017</td>
<td>0.994</td>
</tr>
<tr>
<td>( q )-plane (without ground plane)</td>
<td>0.028</td>
<td>0.984</td>
</tr>
</tbody>
</table>

Thus the total gain factor for the flat sheet may be taken to be

\[
\begin{align*}
\text{with ground plane} & \quad G_5 = 0.968 \\
\text{without ground plane} & \quad G_5 = 0.958
\end{align*}
\]

If \( p_0 \) is greater than \( \alpha \) and \( q_0 \) greater than \( \beta \) an additional loss results. Thus equality between lengths and heights of the two reflectors yields the optimum performance.

**Effect of Random Errors**

Gain loss due to leakage and feed inefficiency have been considered previously. The additional losses due to random errors which occur in the reflectors remain to be considered. In this analysis it will be assumed that any random errors which are present will be less than one quarter wave length and that they may be resolved into a Fourier series representing the phase across the aperture. In this case
\[ \psi (\xi^*) = \frac{a_0}{2} + \sum_{K_8 = 1}^{\infty} a_{K_8} \cos K_8 \xi^* + \sum_{K_8 = 1}^{\infty} b_{K_8} \sin K_8 \xi^* \]  

(62)

where \( \psi (\xi^*) = \) phase function

\[-1 \leq \xi^* \leq 1 \text{ (dimensionless)}\]

The field intensity in the direction of the maximum will be

\[ e_1(0) = \frac{Ap}{2} \int_{-1}^{1} f(\xi^*) e^{-j\left(\sum_{K_8 = 1}^{\infty} a_{K_8} \cos K_8 \xi^* + \sum_{K_8 = 1}^{\infty} b_{K_8} \sin K_8 \xi^* \right) d\xi^*} \]

(63)

Since \( \frac{a_0}{2} \) does not affect the gain it has been omitted. The aperture amplitude function \( f(\xi^*) \) is for generality given by

\[ f(\xi^*) = (1 - K_1 \xi^* \ 2)\]

or

\[ e_1(0) = \frac{Ap}{2} \int_{-1}^{1} \left[1 - K_1 \xi^* \right]^2 \ e^{-j\left(\sum_{K_8 = 1}^{\infty} a_{K_8} \cos K_8 \xi^* + \sum_{K_8 = 1}^{\infty} b_{K_8} \sin K_8 \xi^* \right) d\xi^*} \]

(64)

The phase function upon conversion to an infinite produce gives

\[ e_1(0) = \frac{Ap}{2} \int_{-1}^{1} (1 - K_1 \xi^* \ 2 \ \prod_{K_8 = 1}^{\infty} \left[\cos(a_{K_8} \cos K_8 \xi^*) - j \sin(a_{K_8} \cos K_8 \xi^*) \right] \]

\[ \left[\cos(b_{K_8} \sin K_8 \xi^*) - j \sin(b_{K_8} \sin K_8 \xi^*) \right] d\xi^* \]

(65)

and since the two sine terms are small they may be dropped for
\[ a_{k8} \ll 1 \text{ and } b_{k8} \ll 1. \]

The field intensity function becomes then

\[ \mathcal{E}_1(0) = \frac{A \Phi}{2} \int_{-1}^{1} (1-K_1 \mathcal{L}^2) \prod_{K_8=1}^{\infty} \cos \left( a_{k8} \cos \theta_8 \right) \cos \left( b_{k8} \sin \theta_8 \right) \, d\mathcal{L}^* \]

\[ (66) \]

The infinite product can be expressed in terms of zero order Bessel functions and equation (66) will then become

\[ \mathcal{E}_1(0) = \frac{A \Phi}{2} \int_{-1}^{1} (1-K_8 \mathcal{L}^2) \prod_{K_8=1}^{\infty} J_0(a_{k8}) J_0(b_{k8}) \, d\mathcal{L}^* \]

\[ (67) \]

or

\[ \mathcal{E}_1(0) = \prod_{K_8=1}^{\infty} J_0(a_{k8}) J_0(b_{k8}) \frac{A \Phi}{2} \int_{-1}^{1} (1-K_1 \mathcal{L}^2) \, d\mathcal{L}^* \]

\[ (68) \]

From equation (68) it may be seen that the gain reduction is independent of the aperture amplitude distribution and an error amplitude factor may be defined as

\[ g_6 = \prod_{K_8=1}^{\infty} J_0(a_{k8}) J_0(b_{k8}) \]

\[ (69) \]

If equation (68) is expanded into a power series and only second order terms of \( a_{k8} \) and \( b_{k8} \) are retained \( g_6 \) becomes

\[ g_6 = 1 - \sum_{K_8=1}^{8} \left( \frac{a_{k8}}{2} \right)^2 + \left( \frac{b_{k8}}{2} \right)^2 \]

\[ (70) \]
If we let
\[
\left( \frac{c_1}{2} \right)^2 = \sum_{K=1}^{\infty} \left( \frac{aK}{2} \right)^2 + \left( \frac{bK}{2} \right)^2
\]
then
\[
\varepsilon_6 = 1 - \left( \frac{c_1}{2} \right)^2
\]  \hspace{1cm} (72)

or the gain factor is given by
\[
\varepsilon_6 = (\varepsilon_{g1})^2 = \left[ 1 - \left( \frac{c_1}{2} \right)^2 \right]^2 \approx 1 - 2 \left( \frac{c_1}{2} \right)^2
\]  \hspace{1cm} (73)

where \( c_1 \) is the peak horizontal error in radians. Since a similar expression exists for the \( \mathcal{Z} \) plane we have
\[
G_6 = G_{6\mathcal{Z}}, G_{6\mathcal{Z}} = 1 - 2 \left( \frac{c_1}{2} \right)^2
\]  \hspace{1cm} (74)

where \( \left( \frac{c}{2} \right)^2 = \left( \frac{c_1}{2} \right)^2 + \left( \frac{c_2}{2} \right)^2 \)

Table XIII gives \( G_6 \) for various values of \( c \).

<table>
<thead>
<tr>
<th>TABLE XIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Error Factors</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c (( \lambda ))</th>
<th>G_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>1.00</td>
</tr>
<tr>
<td>.05</td>
<td>.95</td>
</tr>
<tr>
<td>.10</td>
<td>.81</td>
</tr>
<tr>
<td>.15</td>
<td>.61</td>
</tr>
</tbody>
</table>

Two reflections produce gain losses which may be considered to be the produce of the gain factor for each reflection, and for equal losses in each reflector.
TABLE XIV
Factor for Two Reflections

<table>
<thead>
<tr>
<th>c (\lambda)</th>
<th>G6</th>
</tr>
</thead>
<tbody>
<tr>
<td>.000</td>
<td>1.000</td>
</tr>
<tr>
<td>.035</td>
<td>.950</td>
</tr>
<tr>
<td>.07</td>
<td>.810</td>
</tr>
<tr>
<td>.105</td>
<td>.610</td>
</tr>
</tbody>
</table>

For the antenna system herein described it has been estimated that the peak errors in the reflector are no more than one centimeter or four-tenths of an inch. With a normal distribution the most probable peak error would be half this, and since the phase error is twice the reflector error the gain factor G6 as a function of frequency is given in Table XV.

TABLE XV
Random Error Loss with Frequency

<table>
<thead>
<tr>
<th>f (megacycles)</th>
<th>G6</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>.990</td>
</tr>
<tr>
<td>1000</td>
<td>.955</td>
</tr>
<tr>
<td>1500</td>
<td>.905</td>
</tr>
<tr>
<td>2000</td>
<td>.825</td>
</tr>
<tr>
<td>2500</td>
<td>.760</td>
</tr>
<tr>
<td>3000</td>
<td>.625</td>
</tr>
</tbody>
</table>
For high efficiency the peak error must be kept to less than \( \frac{\lambda}{20} \) and \( \frac{\lambda}{10} \) would be considered as the limit for any useful operation for a double reflector system. The effect of the displacement of an entire section is considered later.

The Antenna Gain

The various factors which have been determined individually in the preceding sections may now be considered together to determine the overall gain of the antenna. The various factors are listed below.

\[
\begin{align*}
G_1 & \text{ due to taper and spillover} \\
G_{3-4} & \text{ due to feed efficiency} \\
G_5 & \text{ due to the effect of the flat sheet} \\
G_2 & \text{ due to leakage} \\
G_6 & \text{ due to random errors}
\end{align*}
\]

The first three are independent of frequency, however, the last two are not. These factors and the overall gain factor for a system with a ground plane are given in Table XVI for \( 2\gamma_o = 180^\circ \) and \( \gamma_o = 70^\circ \).

**TABLE XVI**

Gain Factor with Conducting Ground

<table>
<thead>
<tr>
<th>f(megacycles)</th>
<th>( G_1 )</th>
<th>( G_{3-4} )</th>
<th>( G_5 )</th>
<th>( G_2 )</th>
<th>( G_6 )</th>
<th>( G_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>.76</td>
<td>.95</td>
<td>.968</td>
<td>.974</td>
<td>.990</td>
<td>.672</td>
</tr>
<tr>
<td>1000</td>
<td>( n )</td>
<td>( n )</td>
<td>( n )</td>
<td>.907</td>
<td>.955</td>
<td>.605</td>
</tr>
<tr>
<td>1500</td>
<td>( n )</td>
<td>( n )</td>
<td>( n )</td>
<td>.802</td>
<td>.905</td>
<td>.506</td>
</tr>
<tr>
<td>2000</td>
<td>( n )</td>
<td>( n )</td>
<td>( n )</td>
<td>.686</td>
<td>.825</td>
<td>.395</td>
</tr>
<tr>
<td>2500</td>
<td>( n )</td>
<td>( n )</td>
<td>( n )</td>
<td>.570</td>
<td>.760</td>
<td>.304</td>
</tr>
</tbody>
</table>
The tests which were conducted took place with different feed efficiencies and a different flat sheet effect. The computed gain factors are shown in Table XVII. In both tables the surface error is assumed to be .2".

**TABLE XVII**

Gain Factors Without Conducting Ground

<table>
<thead>
<tr>
<th>f(mc)</th>
<th>$G_1$</th>
<th>$G_{3-4}$</th>
<th>$G_5$</th>
<th>$G_2$</th>
<th>$G_6$</th>
<th>$G_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>.76</td>
<td>.78</td>
<td>.958</td>
<td>.974</td>
<td>.990</td>
<td>.550</td>
</tr>
<tr>
<td>1000</td>
<td>&quot;</td>
<td>&quot;</td>
<td>.907</td>
<td>.955</td>
<td>.490</td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>&quot;</td>
<td>&quot;</td>
<td>.802</td>
<td>.905</td>
<td>.413</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>&quot;</td>
<td>&quot;</td>
<td>.686</td>
<td>.825</td>
<td>.322</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>&quot;</td>
<td>&quot;</td>
<td>.570</td>
<td>.760</td>
<td>.235</td>
<td></td>
</tr>
</tbody>
</table>

These tests were conducted at $= 58.5^\circ$ by observing drift curves of the Radio Star, Cassiopeia A, through the antenna beam. An additional gain reduction of 6 per cent was present due to additional spillover because $\delta > 40^\circ$. The effective aperture may be determined from the relationship

$$A_{em} = \frac{2K}{G} \frac{T_{ANT}}{G}$$

(75)

$K = $ Boltzmann constant $= 1.38 \times 10^{-23}$

(joules/degree K)

$T_{ANT} = $ antenna temperature (degree Kelvin)

$G = $ flux density of observed source (janskys)

$G$ was assumed to have the relationship (7-8)
Fig. 4 Effective aperture.
The physical aperture in this case is 970 m² and in Figure 4, the measured points and several theoretical curves are given. There is an uncertainty of ± 10 per cent in the measured values from the calibration used and the source spectrum, but it will be noted that the agreement is quite good between the measurement limits of 1200 to 2000 mc. for a surface error of .25". Since the feed was above ground, reflections would occur which will diminish the gain and enhance the side lobes. This effect would enter as a higher apparent surface error and thus the true surface tolerance should be less than .25".

The Pattern Shape

Whereas the gain or effective aperture of an antenna is dependent on the proportion of energy which is coupled into the forward direction, the shape of the antenna main lobe may be determined simply from the field amplitude distribution across the aperture. It was assumed in a previous section that the amplitude could be represented by equation (77).

$$H(\mathbf{E}, \mathbf{\gamma}) = (1-K_1 E^2) (1-K_2 \mathbf{\gamma}^2).$$

Further it was demonstrated that the antenna gain was optimized when $K_1 = K_2 = .65$ and the half power beam widths, side lobe levels and first zero widths may be determined from equation (77) (9). The gain is ideally the same with or without the ground plane, and it may be
found from the expression listed below.

\[
R.A._{HP} = \frac{59}{2 \theta_o \lambda}
\]  
(78)

\[
\delta_{HP} = \frac{59}{\theta_o \lambda} \delta \leq 40^\circ
\]  
(79)

\[
\delta_{HP} = \frac{59}{\theta_o \lambda /2 \sin(\frac{130^\circ - \delta}{2})} \delta \geq 40^\circ
\]  
(80)

\[
R.A._{n-n} = \frac{138}{2 \theta_o \lambda}
\]  
(81)

\[
\delta_{n-n} = \frac{138}{\theta_o \lambda} \delta \leq 40^\circ
\]  
(82)

\[
\delta_{n-n} = \frac{138}{\theta_o \lambda /2 \sin(\frac{130^\circ - \delta}{2})} \delta \geq 40^\circ
\]

The side lobes in either plane are approximately 17 db below the peak intensity or about 2 per cent of the main lobe. These parameters are tabulated over a range of frequencies in Table XVIII for \(2 \theta_o = 180^\circ\) and \(\theta_o = 70^\circ\) which is the reflector size used when the tests were performed.

**TABLE XVIII**

<table>
<thead>
<tr>
<th>(f) (mc)</th>
<th>(R.A._{HP})</th>
<th>(R.A._{n-n})</th>
<th>(\delta_{HP})</th>
<th>(\delta_{HP})</th>
<th>(\delta_{HP})</th>
<th>(\delta_{HP})</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>(61^\circ)</td>
<td>(150^\circ)</td>
<td>(165^\circ)</td>
<td>(1.81)</td>
<td>(2.03)</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>(32^\circ)</td>
<td>(75^\circ)</td>
<td>(83^\circ)</td>
<td>(91)</td>
<td>(1.03)</td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>(21^\circ)</td>
<td>(50^\circ)</td>
<td>(55^\circ)</td>
<td>(60)</td>
<td>(68)</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>(16^\circ)</td>
<td>(38^\circ)</td>
<td>(42^\circ)</td>
<td>(46)</td>
<td>(52)</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>(13^\circ)</td>
<td>(30^\circ)</td>
<td>(33^\circ)</td>
<td>(36)</td>
<td>(41)</td>
<td></td>
</tr>
</tbody>
</table>
During June 1960 R.A.$_{\text{HP}}$ and $\delta_{\text{HP}}$ were measured at $\delta = 58.5^\circ$ with $f = 9140 \text{ mc}$. The antenna gain was not determined for this series of observations because the feed efficiency was unknown. A full feed above ground was used and ground effects tend to deteriorate the pattern, producing higher side lobes, but the results are still in excellent agreement with the expected values. A drift curve obtained during that series is shown in Figure 5.

Measurements were made in the spring of 1961 to determine the performance of the antenna between 1000 and 2000 megacycles. Representative patterns are shown in Figure 6 through 8 and the measured performance is tabulated below.

**TABLE XIX**

Measured Beam widths

<table>
<thead>
<tr>
<th>Frequency (megacycles)</th>
<th>R.A.$_{\text{HP}}$ (degrees)</th>
<th>$\delta_{\text{HP}}$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9140</td>
<td>.35$^\circ$</td>
<td>1.10$^\circ$</td>
</tr>
<tr>
<td>1200</td>
<td>.26$^\circ$</td>
<td>---</td>
</tr>
<tr>
<td>1350</td>
<td>.24$^\circ$</td>
<td>---</td>
</tr>
<tr>
<td>1500</td>
<td>.23$^\circ$</td>
<td>---</td>
</tr>
<tr>
<td>1650</td>
<td>.20$^\circ$</td>
<td>---</td>
</tr>
<tr>
<td>1800</td>
<td>.19$^\circ$</td>
<td>---</td>
</tr>
<tr>
<td>1940</td>
<td>.19$^\circ$</td>
<td>.65$^\circ$</td>
</tr>
</tbody>
</table>

A measured vertical pattern obtained at 1940 megacycles per second is shown in Figure 9. The half power width is about 20 per cent
greater than expected and the side lobes are quite high. This is attributed to the ground reflections which are present, and it appears that this effect is also present in the horizontal pattern structure. Completion of the ground plane will remove this pattern deterioration.
JUNE 14, 1960
FREQUENCY = 945 mc

Fig. 5 Cassiopeia A
APRIL 6, 1961
FREQUENCY = 1200 mc
$T_A = 396°$

Fig. 6 Cassiopeia A
APRIL 4, 1961
FREQUENCY = 1500 mc
$T_A = 274^\circ$
APRIL 1, 1961
FREQUENCY = 1940 mc
$T_A = 156^\circ$

Fig. 8 Cassiopeia A
Fig. 9  Vertical pattern  $f = 1940$ mc
CHAPTER II
Mechanical Consideration

The overall antenna system consists of four integral parts which include the paraboloid, flat reflector, ground plane and receiver room. These portions of the system were designed and constructed over a period between 1956 and 1961 under the direction of Professor Kraus with direct supervision by the author. The basic problem connected with each portion and the approach used will be described. The fundamental mechanical design problem is to place a reflecting surface in a particular place and maintain it there under widely varying conditions. Little difficulty is encountered in restraining the surface after placement, but considerable effort is necessary to place reflecting surfaces which have physical areas of the order of 1000 meter$^2$ in particular places to an accuracy of 0.01 meters. Welded steel structures, which could provide the necessary stiffness were employed and these were constructed in large jigs. The reflectors were designed to withstand wind velocities of 100 mph with stresses in the members to be no more than 10,000 P.S.I.; and since mild steel construction was used throughout, service factors of three or more are present. The drag of the screen is 23 pounds per square meter or 2.1 pounds per square foot for a 100 mph wind velocity. The supporting structure, however, contributes a greater drag, which in general produces a total wind load of four to six pounds per square foot of total area.
The Paraboloid

The curved reflector was designed to be built in individual sections thirty feet wide and seventy feet high. Two triangular towers support each section and each complete 2100 square foot section was assembled before erection. Provision is included to allow for adjustment of the reflecting surface at points every ten feet in each direction on the surface. Thus any error may be corrected after the entire mirror is completed or at any future time if any deformation should occur. The surface is actually made up of straight line segments which connect adjusting points every ten feet. These straight line segments do not deviate from the true shape by more than $0.011 \lambda$ at a frequency of 1500 mc; or produce a phase error greater than $0.022 \lambda$, which has essentially no effect on the overall antenna performance. The reflecting surface itself consists of vertical copper clad steel wires .08" in diameter, held under tension by nine pound window sash weights, and restrained every ten feet vertically. Figure 10 depicts one nearly completed section of the paraboloid.

The Ground Plane

The area between the two reflectors was filled with compacted earth to bring the altitude to within .2 feet of the axis. Two and one-half inches of concrete poured in lanes ten feet wide, provide a solid level surface which has a slope of .1 per cent to provide drainage. The surface tolerance is ± 1/4 inch and the conducting surface is .005" thick aluminum sheet which is cemented to the concrete surface.
Fig. 11 Flat Sheet Section
The Flat Reflector

The flat sheet consists of rigid driven sections, hinged at the axis, twenty feet wide and one hundred feet high spaced every forty feet with the intervening space spanned by loose beams. No adjustment was built into the flat sheet. However, the design considerations are generally the same as those for the paraboloid. The reflector surface may be tilted from within seven degrees of the vertical to within 33° of the horizontal axis. The declination range is thus from -36° to +60° which includes 85 percent of the sky area observable from Delaware, Ohio. The reflector is driven by electric winches at a rate of 4.75 degrees declination per minute. The loose joints will permit up to ±1.8° movement in declination for any section with respect to the adjacent ones. A section of the flat sheet nearing completion is shown in Figure 11. A friction locking system prevents motion of the reflector after it has been positioned with the winch. The bands and shoes are aluminum which has a high coefficient of friction.

Positioning Error

In the gain determinations it has, of course, been assumed the flat sheet is perfectly aligned. Any error in alignment will result in some reduction in efficiency and it is of interest to know the magnitude of the misalignment effects. The positioning phase error is linear and if it is assumed that reflector sections have alternately directed errors, then it may be shown that

\[ G_7 = 1 - \frac{(\sigma f K_{mc})^2}{6} \]
where $\sigma$ = peak setting error at reflector top (inches). This may also be expressed in terms of the angular error and
\[ G_7 = 1 - 17 (\Delta \delta f_{Kmc})^2 \]  
where $\Delta \delta$ = declination setting error in degrees. Thus an error of $+0.05^\circ$ would produce a 17 per cent gain loss at 2000 megacycles per second. It has been found that it is possible to position the reflector within $+0.02^\circ$ declination and therefore even at 2000 megacycles the gain loss should be less than three per cent. The slow motion of the reflector is responsible for the ease with which settings can be made.

**Receiver Room**

The receiver room is located under the prime focus point and has an area of approximately 500 square feet. Openings in the roof permit the ingress of radio frequency transmission lines. Temperature stability of the order of $+1^\circ$ Fahrenheit for periods of many hours is a result of the underground construction with no special precautions being taken.
CHAPTER III
Operational Aspects

Introduction

The gain determinations which were made and the pattern shapes which were discussed in Chapter I assumed conditions of perfect focusing. The situation may arise where movement of the feed from the focal point either accidentally or purposefully occurs. This motion may be either axial or transverse and the behavior under these conditions will now be considered. In addition, the feed antenna impedance is of interest, as is the ambient antenna temperature. Finally it will be shown that if multi-channel operation is desired, there is an optimum ratio for \( \frac{F}{2a_0} \).

Defocusing

The effect of defocusing may be present in the system at times either through purposeful introduction or by incorrect positioning of the feed phase center at the focal point of the paraboloid. Defocusing occurs when there is a displacement of the feed along the axis of the reflector. Let

\[ \Delta F_A \] feed displacement (wavelengths)
\[ \alpha \] angle between the axis and a line to a point on the paraboloid
\[ 2a_0 \] aperture width
\[ c \] horizontal distance from the axis to a point on the paraboloid
\[ F_A \] focal length
Displacement of the feed by an amount $\Delta F_A$ produces a phase change given by

$$\Delta \phi = \Delta F_A \cos \alpha$$

(86)

or for

$$\alpha \leq \pi/4$$

$$\Delta \phi = \Delta F_A \left(1 - \frac{\alpha}{2}\right)^2$$

(87)

The first term in (87) is simply an overall advancement or retardation which in no way affects the behavior of the antenna system, but the second term produces a quadratic phase error in the aperture plane. Treating only the second term and replacing $\alpha$ with the aperture coordinate and the focal length, the error becomes

$$\Delta \phi_{\text{error}} = -\Delta F_A \frac{\varepsilon^2}{2} = -\Delta F_A \frac{\varepsilon^2}{F^2} \varepsilon^{*2}$$

(88)

Let $\alpha = \frac{\varepsilon^2}{F^2} - 1 \leq \varepsilon^{*2} \leq 1$

and

$$K_\theta = -\frac{\Delta F_A}{2} \frac{2^2}{F^2} \frac{\varepsilon^2}{\varepsilon^{*2}}$$

Equation (88) may now be written as

$$\Delta \phi_{\text{error}} = K_\theta \varepsilon^{*2} \text{ radians}$$

(89)

It will be noted that the phase error is very dependent on the f number of the reflector system. In the following analysis which determines the gain loss due to defocusing, equation (90) is considered to be less than $\pi/4$ or $\pi/2$. With this restriction the aperture field will be resolved into two components in phase quadrature with each other. The total normalized field in the aperture plane assuming a quadratic taper is
$$H(\xi) = \left[1-K_1\xi^2\right] \cos(K_9\xi^2) + j \sin(K_9\xi^2)$$  \hspace{1cm} (91)$$

where 0 \leq K_1 \leq 1 and represents the aperture field intensity at the edge with respect to the center. For maximum gain $K_1 = .65$ which corresponds to a 10 dB taper. The total on axis field $g_1(u)$ at a distant point is given by

$$g_1(0) = 2 \int_0^1 \left[1-K_1\xi^2\right] \cos(K_9\xi^2) d\xi + j \int_0^1 \left[1-K_1\xi^2\right] \sin(K_9\xi^2) d\xi$$  \hspace{1cm} (92)$$

Expanding the phase error function in a series and integrating the total field will be

$$\frac{g_1(0)}{2} = \left[1 - \frac{K_1}{3} - \frac{K_9^2}{10} + \frac{K_1 K_9^2}{14} + \frac{K_9^4}{216} - \frac{K_1 K_9^4}{268}\right] + j \left[\frac{K_9^3}{3} - \frac{K_9^3}{42} - \frac{K_1 K_9^3}{5} + \frac{K_1 K_9^3}{54}\right]$$  \hspace{1cm} (93)$$

Table XX shows the effect of various phase errors for an aperture with a 10 dB taper.

**TABLE XX**

<table>
<thead>
<tr>
<th>$K_9/2\pi$</th>
<th>$\frac{g_1(0)}{2}$</th>
<th>$\left[\frac{g_1(0;K_9)}{g_1(0,0)}\right]^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.783</td>
<td>1.00</td>
</tr>
<tr>
<td>$\lambda/16$</td>
<td>.775 + j.08</td>
<td>.99</td>
</tr>
<tr>
<td>$\lambda/8$</td>
<td>.750 + j.16</td>
<td>.96</td>
</tr>
<tr>
<td>$\lambda/4$</td>
<td>.663 + j.27</td>
<td>.838</td>
</tr>
</tbody>
</table>
For \( \frac{K_0}{2\pi} \) less the one quarter wavelength the antenna gain is very nearly given by

\[
g = 1 - 2.6 \left( \frac{K_0}{2\pi} \right)^2
\]  \hspace{1cm} (94)

This may be rewritten in terms of the fundamental parameters of the antenna system, in which case

\[
g = 1 - 0.65 \Delta F \left( \frac{\xi_0}{F} \right)^4
\]  \hspace{1cm} (95)

Systems with large \( f \) numbers are very tolerant to defocusing as is illustrated in Table XXI.

<table>
<thead>
<tr>
<th>( \Delta F )</th>
<th>( \left( \frac{\xi_0}{F} \right)^2 )</th>
<th>( f ) number</th>
<th>( \left[ \frac{\xi_1(0,K_0)}{\xi_1(0,0)} \right]^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>.5</td>
<td>2.00</td>
<td>.995</td>
</tr>
<tr>
<td>1.0</td>
<td>.75</td>
<td>1.33</td>
<td>.987</td>
</tr>
<tr>
<td>1.0</td>
<td>1</td>
<td>1.00</td>
<td>.959</td>
</tr>
<tr>
<td>1.0</td>
<td>1.5</td>
<td>.67</td>
<td>.794</td>
</tr>
</tbody>
</table>

This material will be referred to again when the arrangement of multiple feeds is considered.

The Measurement of Defocusing Gain Loss

Measurements were made with the antenna system at a wavelength of 16.6 cm for a horizontal \( f \) number of 2.32. The results are summarized in Table XXII.
## TABLE XXII
Defocusing Factors

<table>
<thead>
<tr>
<th>$\Delta F_A$</th>
<th>$g$ Calc.</th>
<th>$g$ Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.995</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>0.978</td>
<td>1.00</td>
</tr>
<tr>
<td>6</td>
<td>0.951</td>
<td>0.945</td>
</tr>
<tr>
<td>8</td>
<td>0.913</td>
<td>0.885</td>
</tr>
</tbody>
</table>

Since the horizontal and vertical phase centers are in different position there is always some defocusing. However, the vertical $f$ number is six and then horizontal conditions predominate.

**Beam Steering**

Observations may be made to the east or west of the meridian by displacing the feed to west or east of the paraboloid axis. Different methods may be employed to accomplish these off-axis observations. In one instance a feed may be moved at a constant rate so that a certain portion of the sky is under continual observations. Another possibility is that several feeds may be arrayed around the focal point to produce multiple observations using several channels, which might be at any frequency.

**Gain Reduction**

A gain diminution is present when a feed displacement is carried out. This gain loss is a function of the lateral movement of the feed and also the $f$ number of the antenna. Let
\[ \Delta X = \text{lateral displacement of the feed in feet} \]

\[ F = \text{focal length in feet} \]

\[ 2 \ell_c = \text{width of paraboloid in feet} \]

\[ \alpha = \text{angle between axis and some point on the paraboloid} \]

\[ \gamma = \frac{\Delta X}{F} \]

The phase change is then given by

\[ \text{phase change} = -\Delta X \lambda \sin (\alpha + \gamma) \quad (96) \]

or for

\[ \alpha + \gamma \leq \frac{\pi}{4} \]

\[ \text{phase change} = \Delta X \lambda (\gamma + \alpha - \frac{(\gamma + \alpha)^3}{6}) \]

\[ = \Delta X \lambda (\gamma + \alpha) - \frac{\Delta X \lambda}{6} (\gamma + \alpha)^3 \quad (97) \]

The first term in equation (97) represents a linear phase shift which produces a beam shift. The second term represents a phase error across the aperture of the antenna. If \( \ell^* \) represents the dimensionless distance from the axis to a point in the aperture plane then

\[ \text{phase error} = \frac{-\Delta X \lambda \ell_c^3}{6F^3} (\Delta \ell^* + \ell^*)^3 \quad (98) \]

in the aperture plane.

The inphase field intensity component in a direction normal to the phase plane is then given by

\[ g_1(u) = \int_{-1}^{1} (1 - K_6 \ell^* \epsilon^2) e^{i \ell_0 u \ell^* + j \ell_0 K_7 \ell^*^3} \, dx \quad (99) \]

where

\[ \frac{K_7}{2\pi} = \frac{\Delta X \lambda \ell_c^3}{6F^3} (\Delta \ell^* + \ell^*)^3 \]
The exponentials may be expanded into trigonometric function so that

\[ g_1(u) = \int_{-1}^{1} (1-K_6 \xi^2) (\cos \xi_0 u \xi^* + j \sin \xi_0 u \xi^*) e^{-jK_7 \xi^*} d\xi^* \quad (100) \]

In order to evaluate the integral \( \frac{K_7}{2\pi} \) will be expanded and only terms containing \( \xi^* \) will be retained.

\[
\frac{K_7}{2\pi} = \frac{\Delta x_A (\xi_0)^3 \xi^* - \Delta x_A (\xi_0)^2 (\Delta x_A) \xi^*}{2} - \frac{\Delta x_A^I (\xi_0)(\Delta x_A) \xi^*}{2} - K_8 \xi^* - K_9 \xi^* - K_{10} \xi^* \quad (101)
\]

or

\[
\frac{K_7}{2\pi} = -K_8 \xi^* - K_9 \xi^* - K_{10} \xi^* \]

Thus

\[ g_1(u) = \int_{-1}^{1} (1-K_6 \xi^2) (\cos \xi_0 u \xi^* + j \sin \xi_0 u \xi^*) e^{jK_8 \xi^*} e^{jK_9 \xi^*} e^{jK_{10} \xi^*} d\xi^* \quad (102) \]

In general \( K_9 \) and \( K_{10} \) may be neglected for \( \frac{F}{\xi_0} > 1 \) but they must be considered for \( \frac{F}{\xi_0} < 1 \). First, second and third order phase errors are seen to be present when the feed is displaced. For \( \frac{F}{\xi_0} < 1 \) the value for \( g_1(u) \) will be after evaluation by power series.

\[ g_1(u) = 1 - \frac{K_6}{3} - \frac{K_7^2}{18} + \frac{K_6 K_7^2}{312} - \frac{K_7^4}{360} \]

\[ + \xi_0 u \frac{K_7}{7} \left( \frac{1}{3} - \frac{K_6}{6} - \frac{K_7^2}{78} \right) \]

\[ - u^2 \xi_0^2 \left( \frac{1}{6} - \frac{K_6}{10} - \frac{K_7^2}{78} + \frac{K_6 K_7^2}{44} \right) \]
The peak value of $g_1(u)$ will occur for some value of $u$ greater than zero. If the first three terms of $g_1(u)$ are taken it may be written as

$$g_1(u) = c_0 u K_7 c_1 - u^2 c_2$$  \hspace{1cm} (104)$$

The value of $u$ which makes $g_1(u)$ maximum may be found by differentiation. Thus

$$
\mathcal{E}_0 u_m = \frac{K_7 c_1}{2 c_2} = \frac{K_7 \left( 1 - \frac{K_6}{7} - \frac{K_7^2}{36} + \frac{K_6 K_7^2}{144} \right)}{2 \left( 1 - \frac{K_6}{10} - \frac{K_7^2}{36} + \frac{K_6 K_7^2}{144} \right)} \hspace{1cm} (105)$$

Since $K_6 = .65$ then the maximum normalized values of $g_1(u)$ are given in Table XXIII.

**TABLE XXIII**

Cubic Error Effects for Small $f$ Number

<table>
<thead>
<tr>
<th>$K_7$</th>
<th>$u_m \mathcal{E}_0$</th>
<th>$\frac{g_1(K_7, u_m)}{g_1(0,0)}$</th>
<th>$\left[ \frac{g_1(K_7, u_m)}{g_1(0,0)} \right]^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$\pi/4$</td>
<td>0.435</td>
<td>0.993</td>
<td>0.986</td>
</tr>
<tr>
<td>$\pi/2$</td>
<td>1.020</td>
<td>0.973</td>
<td>0.945</td>
</tr>
</tbody>
</table>
For $K_6 = .65$ $u_m$ may be given by

$$u_m = K_7 \frac{(.107 - .007 K_7^2)}{(.202 - .026 K_7^2)}$$

(106)

Since the gain factor can be expressed approximately by

$$\left[ \frac{\bar{g}_1(K_7, u_m)}{\bar{g}_1(0,0)} \right]^2 = 1 - \frac{K_7^2}{45}$$

(107)

it becomes in terms of the parameters $\Delta X, \mathcal{F}_o$ and $F$.

$$\frac{\bar{g}_1(K_7, u_m)}{\bar{g}_1(0,0)^2} = 1 - \frac{\Delta X^2}{45} \left( \frac{\mathcal{F}_o}{F} \right)^6$$

(108)

Figure 12 depicts the displacements possible for various losses and various antenna lengths $2 \mathcal{E}_o$ in feet when $\frac{\mathcal{F}_o}{F} > 1/2$. For $\frac{\mathcal{F}_o}{F} < 1/2$ the situation is somewhat different and in that case if $K_9$ and $K_{10}$ are retained we have

$$\bar{g}_1(u) = 1 - \frac{K_6}{3} - \frac{K_8^2}{2} \left( \frac{1}{7} - \frac{K_6}{9} \right) - \frac{K_9^2}{2} \left( \frac{1}{5} - \frac{K_6}{7} \right) - \frac{K_{10}^2}{2} \left( \frac{1}{3} - \frac{K_1}{5} \right) - K_8 K_{10} \left( \frac{1}{5} - \frac{K_1}{7} \right)$$

$$- \mathcal{E}_o u K_{10} \left( \frac{1}{3} - \frac{K_1}{5} \right) - \mathcal{E}_o u K_8 \left( \frac{1}{5} - \frac{K_1}{7} \right)$$

$$- \mathcal{E}_o^2 u^2 \left( \frac{1}{3} - \frac{K_1}{5} \right)$$

$$+ j K_9 \left( \frac{1}{3} - \frac{K_1}{5} \right) - j \mathcal{E}_o u K_9 K_{10} \left( \frac{1}{5} - \frac{K_1}{7} \right)$$

$$- j K_8 K_9 K_{10} \left( \frac{1}{7} - \frac{K_1}{9} \right) - j \mathcal{E}_o u K_8 K_9 \left( \frac{1}{7} - \frac{K_1}{9} \right)$$

(109)
Fig. 12  Beam steering.
and it will be noted that the $g_1(u)$ is now complex. The value of $E_0u$ which makes $g_1(u)$ maximum is very nearly equal to

$$E_0 u_m = -\frac{K_{10} \left(\frac{1}{3} - \frac{K_1}{5}\right) + K_8 \left(\frac{1}{5} - \frac{K_1}{7}\right)}{\left(\frac{1}{3} - \frac{K_1}{5}\right)}$$

(110)

Since

$$K_7 = K_8 + K_9 + K_{10}$$

(111)

we have

$$K_7 \max = |K_8| + |K_9| + |K_{10}|

K_7 \min = |K_8| + |K_{10}| - |K_9|

which correspond to the errors at $+\varepsilon_0$ and $-\varepsilon_0$ respectively. For $\frac{E_0}{F} = .214$ we have

TABLE XXIV

Cubic Error Effects for Large f Number

<table>
<thead>
<tr>
<th>$K_7\max$</th>
<th>$u_m$</th>
<th>$g_1(K_7, u_m)$</th>
<th>$\frac{g_1(K_7, u_m)}{g_1(0,0)}$</th>
<th>$\left[\frac{g_1(K_7, u_m)}{g_1(0,0)}\right]^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>n/6</td>
<td>.195</td>
<td>.998/2.80</td>
<td>.996</td>
<td></td>
</tr>
<tr>
<td>n/2</td>
<td>.59</td>
<td>.975/8.40</td>
<td>.952</td>
<td></td>
</tr>
</tbody>
</table>

The gain factor may be given approximately as

$$\left[\frac{g_1(K_7, u_m)}{g(0,0)}\right]^2 = 1 - \frac{K_7^2}{50}$$

or

$$= 1 - \frac{\Delta x^2}{45} \left(\frac{\Delta x}{F} + \varepsilon_0\right)^6$$

(112)
In general greater displacements are achieved with larger \( f \) numbers and this will be referred to later.

**Beam Steering Time**

The beam is tipped through an angle \( \delta \) where

\[
\delta = 57.3 \cdot \frac{\Delta X}{F} \text{ (degrees)} \tag{113}
\]

Since the earth rotates approximately one degree in four minutes and \( F \) is 120 feet the time interval \( \Delta t \) between the observation and the time the region would appear on the meridian is

\[
\Delta t = \pm \frac{0.545 \Delta X}{\cos \delta} \tag{114}
\]

where \( \delta \) is the declination at which the observation is being made, and \( \Delta X \) is the lateral displacement in feet. The total time that a source may be observed is twice the value in equation (114) or

\[
2 \Delta t = \pm \frac{1.09 \Delta X}{\cos \delta} \text{ (minutes)} \tag{115}
\]

For a constant loss

\[
2 \Delta t = \pm \frac{1.07 \Delta X}{\cos \delta \cdot f_{Kmc}} \text{ (minutes)} \tag{116}
\]

where \( \Delta X \) is the displacement at 1000 megacycles, which produces the desired loss. Tracking times for a 1 percent gain loss are given in Table XXV.
TABLE XXV

Tracking Time for a 1 Percent Gain Loss at 1000 M.C. at $\delta = 0^\circ$

<table>
<thead>
<tr>
<th>$2 \xi_0$ (feet)</th>
<th>$2 \Delta t$ (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>32.0</td>
</tr>
<tr>
<td>200</td>
<td>26.8</td>
</tr>
<tr>
<td>250</td>
<td>18.4</td>
</tr>
<tr>
<td>300</td>
<td>12.5</td>
</tr>
<tr>
<td>400</td>
<td>6.2</td>
</tr>
<tr>
<td>500</td>
<td>3.4</td>
</tr>
<tr>
<td>600</td>
<td>1.8</td>
</tr>
<tr>
<td>700</td>
<td>1.2</td>
</tr>
</tbody>
</table>

For frequencies greater than 1000 megacycles the constant loss beam steering time decreases and for frequencies below 1000 megacycles the time increases.

Declination Change and Right Ascension Change

Horizontal displacement of the feed from the focal point produces a change in the observed declination. The altitude of the antenna beam is given by

$$\sin \text{alt} = \cos \delta \cos (\text{lat} - \delta_s) \quad (117)$$

while the altitude of the point in the sky is represented by

$$\sin \text{alt} = \sin \delta_0 \sin \text{lat} + \cos \delta_0 \cos \text{lat} \cos \text{H.A.} \quad (118)$$

where

$$\text{H.A.} = \frac{\delta}{\cos \delta_s}$$
\[ \delta_s \] = declination setting (degrees)
\[ \delta_o \] = observed declination (degrees)
lat = latitude on earth (degrees)
\( \delta \) = displacement angle (degrees)

If (117) and (118) are equated the observed declination in terms of the feed displacement angle \( \delta \) is found to be

\[ \delta_o - \delta_s = -0.00872 \delta^2 \tan \delta_s \text{ (degrees)} \]

\[ = -28.7 \tan \delta_s \left( \frac{\Delta X}{F} \right)^2 \text{ (degrees)} \] (119)

Thus the actual observed declination is less than that indicated when \( \delta > 0 \) and it is greater than the declination at which the antenna is set when \( \delta < 0 \). This effect is most pronounced at high declinations.

There will also be a small correction which must be applied to the hour angle. The maximum was seen to occur for \( u < 0 \); and thus there will be a small correction which is always negative. Now

\[ U_m \approx 0.53 K_f \text{ (rad.)} \] (120)

and for \( \frac{\epsilon_o}{F} > 0.5 \)

\[ U_m \approx \frac{\pi \Delta x \lambda}{3} \left( \frac{\epsilon_o}{F} \right)^3 \times 0.53 \] (121)

The full half power width in this coordinate is 3.2 rad. thus

\[ U_m = \frac{0.53 \pi \Delta x \lambda}{9.6} \left( \frac{\epsilon_o}{F} \right)^3 \text{ (H.P. B.W.)} \] (122)

Since

\[ \Delta X = \Delta X \text{ (ft.)} \epsilon_{km.c.} \text{ (wavelength)} \] (123)

and

\[ \text{HPBW} = \frac{58}{2 \epsilon_o \epsilon_{km.c.}} \text{ (degrees)} \] (124)
the change in the $\delta$ coordinate is

$$\Delta \delta = \frac{5 \Delta x (ft)}{c_0} \left( \frac{c_0}{F} \right)^3 \text{ (degrees).} \quad (125)$$

or

$$\cos \delta \Delta t = \frac{20 \Delta x (ft)}{c_0} \left( \frac{c_0}{F} \right)^3 \text{ (minutes time.)} \quad (126)$$

where the source appears earlier or later as the feed is displaced to the east or west. The total hour angle is then

$$\cos \delta \Delta t = (-.545 \times + 20 \frac{\Delta x}{c_0} \left( \frac{c_0}{F} \right)^3 \text{ (minutes) (127)}$$

$$\Delta t \cos \delta = \Delta x \left( -.545 + \frac{20}{c_0} \left( \frac{c_0}{F} \right)^3 \right) \text{ (minutes) (128)}$$

For the tests described herein $\frac{2c_o}{F} = .43$ and this effect is negligible. The table below shows this effect for various values of $2 \frac{c_o}{F}$ and these are valid for gain losses less than five percent.

**TABLE XXVI**

Tracking Time Corrections

<table>
<thead>
<tr>
<th>$2 \frac{c_o}{F}$</th>
<th>$\frac{\Delta t \cos \delta}{\Delta x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.86</td>
<td>$-.545 + \frac{1.6}{c_0}$</td>
</tr>
<tr>
<td>1.29</td>
<td>$-.545 + \frac{5.1}{c_0}$</td>
</tr>
<tr>
<td>1.76</td>
<td>$-.545 + \frac{13.8}{c_0}$</td>
</tr>
</tbody>
</table>
The angle is changed by
\[ \gamma = \Delta X (-0.137 + \frac{(\frac{f^2}{F})^3}{5\sqrt{F}}) \text{ (degrees)} \] (129)

Thus for \( f \) numbers less than unity the correction will become appreciable. For \( \frac{f^2}{F} < 1/2 \) the complete expression for \( K_7 \) should be used.

Beam Steering Tests

Tests were conducted at a frequency of 1800 megacycles for \( 2L_0 = 180^\circ \) on the source Cassiopeia A for which \( \delta = 58.32^\circ \). Table XXVII summarizes the results of the beam steering test

**TABLE XXVII**

<table>
<thead>
<tr>
<th>Beam Steering Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta ) (A)</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( G ) (calc.)</th>
<th>( G ) (obs.)</th>
<th>( \Delta \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1%</td>
<td>1.00</td>
<td>.01</td>
</tr>
<tr>
<td>.3%</td>
<td>1.00</td>
<td>.03</td>
</tr>
<tr>
<td>.6%</td>
<td>1.00</td>
<td>.07</td>
</tr>
</tbody>
</table>

The day to day variations in this case exceed the gain variations observed. This indicates that multiple feeds may be used successfully.
The Primary Feed Impedance

In the determination of the feed impedance aperture blocking may be disregarded. The feed represents only about 0.1 percent of the reflector area and this may be neglected. Only the limits of the standing wave ratio will be computed and this will be done for $a_\lambda = 4.6$, $b_\lambda = 4.3$ and $L_\lambda = 25$. The impedance of the horn feed is determined almost entirely by the $E$ plane flare. The reflection coefficient $\Gamma_H$ is

$$\Gamma_H = \frac{1 - Y_H}{1 + Y_H}$$  \hspace{1cm} (130)

where $Y_H$ is the overall normalized horn admittance. Since $\Gamma_H$ is small it may be written as

$$|\Gamma_H| = |\Gamma_1| + |\Gamma_2|$$  \hspace{1cm} (131)

where

- $\Gamma_1 =$ throat reflection coefficient
- $\Gamma_2 =$ mouth reflection coefficient

The throat admittance is approximately

$$Y_1 = 1 - j 0.05 \hspace{1cm} |\Gamma_1| = 0.025$$  \hspace{1cm} (132)

for any length. The mouth reflection coefficient is (10)

$$|\Gamma_2| = 0.05$$

Thus

$$|\Gamma_H| = |\Gamma_1| + |\Gamma_2|$$  \hspace{1cm} (133)

and

$$|\Gamma_H| \leq 0.075$$  \hspace{1cm} (134)
Fig. 13 Standing wave ratio.
In addition there is a certain reflection associated with the transition between the coaxial line and the waveguide. It is given by $|\eta_t|$ and may be taken to be

$$|\eta_t| \leq 0.09$$

(135)

Thus the total reflection coefficient $T$ is

$$|\eta_T| = |\eta_t| + |\eta_2| + |\eta_1|$$

(136)

or

$$0.015 \leq \eta_T \leq 0.165$$

(137)

which corresponds to

$$1.03 \leq V.S.W.R. \leq 1.39$$

(138)

Figure (13) shows measured values of the VSWR for a full horn above ground used in the tests for which $b_\lambda = 9.3$, $a_\lambda = 5\lambda$, and $L_\lambda = 43$ at 1500 mc. The graduate upward trend as the frequency decreases is attributed to the effect of the ground.

The Antenna Temperature

If extremely low noise receivers are employed it is necessary that the ambient antenna temperature be known. Using the values found earlier for screen leakage and spillover this noise input may be determined. It is assumed that the surroundings have an effective temperature of $300^\circ$K and that the sky temperature is zero. The data are tabulated in Table XXVIII.
**TABLE XXVIII**

Antenna Temperature with Ground Plane

<table>
<thead>
<tr>
<th>f(mc) (Paraboloid)</th>
<th>Leakage I</th>
<th>Leakage II (Flat Sheet)</th>
<th>Sides</th>
<th>Total Ambient Antenna Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>12°</td>
<td>12°</td>
<td>10°</td>
<td>34°</td>
</tr>
<tr>
<td>1500</td>
<td>26°</td>
<td>24°</td>
<td>10°</td>
<td>60°</td>
</tr>
<tr>
<td>2000</td>
<td>44°</td>
<td>37°</td>
<td>10°</td>
<td>91°</td>
</tr>
</tbody>
</table>

The factors are determined by computing the proportion of the antenna impedance which is coupled through the two screens and also that coupled to the sides of the reflectors. When the ground plane is present no contribution appears from vertical spillover. The same values are given in Table XXIX for a system without the ground plane.

**TABLE XXIX**

Antenna Temperature without Ground Plane

<table>
<thead>
<tr>
<th>f(mc) (mc)</th>
<th>Leakage I</th>
<th>Leakage II</th>
<th>Sides</th>
<th>Bottom</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>10°</td>
<td>9°</td>
<td>10°</td>
<td>44°</td>
<td>73°</td>
</tr>
<tr>
<td>1500</td>
<td>21°</td>
<td>19°</td>
<td>10°</td>
<td>44°</td>
<td>93°</td>
</tr>
<tr>
<td>2000</td>
<td>35°</td>
<td>29°</td>
<td>10°</td>
<td>44°</td>
<td>118°</td>
</tr>
</tbody>
</table>

In addition the feeders will contribute a component which is dependent on the feeder efficiency. Thus the transmission lines may easily contribute large noise temperatures unless the efficiencies are high. These components are given by

\[ T_F = 300 (1-K_{10}) O_K \]

(139)

75
where \( T_F \) = transmission line input (°K) 
\( K_{10} \) = transmission line efficiency

Typical values are given in Table XXX.

**TABLE XXX**

Cable Efficiencies

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>( T_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>0</td>
</tr>
<tr>
<td>95%</td>
<td>15°</td>
</tr>
<tr>
<td>90%</td>
<td>30°</td>
</tr>
<tr>
<td>85%</td>
<td>45°</td>
</tr>
<tr>
<td>80%</td>
<td>60°</td>
</tr>
</tbody>
</table>

If a maser amplifier is to be used the antenna temperature could be lowered to about 10°K by making the screen more opaque.

**Optimum Operating Conditions**

It is of interest to determine if there are any optimum relationships which exist between focal length, reflector spacing and reflector width. As long as the flat sheet is some convenient distance to the rear of the focal point, the operation will be independent of its exact location. However, it will be shown that a certain optimum relationship does exist between the focal length and the reflector width. It has already been shown that a relationship between antenna gain and lateral feed displacement

\[
G_B = 1 - \frac{\Delta \lambda}{40.5} \left( \frac{L_F}{F} \right)^6
\]

(140)
exists where $X$ is expressed in wavelengths. A meridian transit instrument may observe a particular part of the sky only once in a sidereal day and if more than one feed could be coupled into more than one receiver, then information could be acquired at a greater rate.

This may be accomplished if a number of feeds are distributed about the focal point on either side of the axis. In the following analysis it is assumed that receivers are operated over a broad range of frequencies. If the wavelength corresponding to the highest frequency is $\lambda_1$, the wavelengths for the other channels will be given by

$$\lambda_n = 1.52(n-1)\lambda_1$$

which corresponds to the change between waveguide bands. It is also assumed that the highest frequency feed is on axis and that the ones at successively lower frequencies are located farther out. Then the displacements $d_n$ are given by

$$d_1 = 0$$

$$d_2 = \frac{a\lambda_1}{2} + \frac{\lambda_1}{2} + \frac{\lambda_2}{2} + \frac{a\lambda_2}{2}$$

$$d_3 = \frac{a\lambda_1}{2} + \frac{\lambda_1}{2} + \frac{\lambda_3}{2} + \frac{a\lambda_3}{2}$$

$$d_4 = \frac{a\lambda_1}{2} + \frac{\lambda_1}{2} + \lambda_2 + a\lambda_2 + \frac{\lambda_4}{2} + \frac{a\lambda_4}{2}$$

$$d_5 = \frac{a\lambda_1}{2} + \frac{\lambda_1}{2} + \lambda_3 + a\lambda_3 + \frac{\lambda_5}{2} + \frac{a\lambda_5}{2}$$

(11.2)

where $a\lambda_n = $ feed aperture width at a wavelength $\lambda_n$ and $\frac{\lambda_n}{2}$ is a
buffer zone on each side of the nth feed. If the distance \( d_n \) is written in terms of the wavelength for the nth feed we have

\[
\begin{align*}
\text{d}_1 &= 0 \\
\text{d}_2 &= \frac{a_1}{3.04} + \frac{\lambda_1}{3.04} + \frac{\lambda_2}{2} + \frac{a_2}{2} \\
\text{d}_3 &= \frac{a_1}{4.6} + \frac{\lambda_1}{4.6} + \frac{\lambda_3}{2} + \frac{a_3}{2} \\
\text{d}_4 &= \frac{a_1}{7} + \frac{\lambda_1}{7} + \frac{\lambda_2}{2.3} + \frac{a_2}{2} + \frac{a_4}{2} \\
\text{d}_5 &= \frac{a_1}{10.6} + \frac{\lambda_1}{10.6} + \frac{\lambda_3}{2.3} + \frac{a_3}{2} + \frac{a_5}{2}
\end{align*}
\]

This distance determination could be carried out for an even greater number of feeds. Now \( a_n \) is a function of the reflector width and the focal length. Figure (114) gives gain loss as function of displacement for various reflector widths, and also the greatest feed displacement necessary for a given number of feeds. If a given number of channels and a given loss is specified the greatest possible reflector width may be found.

**TABLE XXXI**

<table>
<thead>
<tr>
<th>Channels</th>
<th>1% Width</th>
<th>3% Width</th>
<th>5% Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>670'</td>
<td>820'</td>
<td>900'</td>
</tr>
<tr>
<td>3</td>
<td>560'</td>
<td>670'</td>
<td>735'</td>
</tr>
<tr>
<td>7</td>
<td>480'</td>
<td>610'</td>
<td>670'</td>
</tr>
</tbody>
</table>
Fig. 14 Optimum relations.
This can, of course, be given more generally in terms of \( \frac{F}{2F_0} \) or the f number.

**TABLE XXXII**

Generalized Optimum Conditions

<table>
<thead>
<tr>
<th>Channels</th>
<th>( \frac{1F}{2F_0} )</th>
<th>( \frac{3F}{2F_0} )</th>
<th>( \frac{5F}{2F_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>.63</td>
<td>.51</td>
<td>.47</td>
</tr>
<tr>
<td>3</td>
<td>.75</td>
<td>.63</td>
<td>.57</td>
</tr>
<tr>
<td>7</td>
<td>.88</td>
<td>.69</td>
<td>.63</td>
</tr>
</tbody>
</table>

To prevent serious aberations the gain loss should be held to about one percent thus the f number should be between .6 and .9 or the width between 480 and 680 feet. The optimum situation would appear to occur with approximately six channels and an f number of about 1.0. In this case simultaneous observations could be made over a range of eight to one in frequency. It should be noted that the relationship between the number of channels and the f number is applicable to any reflector antenna. For f numbers greater than 1.5 additional channels may be added almost indefinitely. It should be noted that if a Dicke system is employed two receivers could be operated from one feed giving two identical records. Thus for three or more channels the width of the Ohio State-Ohio Wesleyan Radio Telescope should be less than 480 feet if single channel feeds are used.
APPENDIX I

Far Field Patterns

It is of interest to determine expressions which relate the far field pattern to the aperture distribution. Two situations will be considered. In one the amplitude distribution is assumed to be symmetrical and in the other it is not. The far field intensity in the dimensionless coordinate is given by

\[ g(u) = \int_{-1}^{1} f(x) e^{jux} \, dx \quad (1) \]

Let \( f(x) = 1 - Kx^2 \) then

\[ g(u) = \int_{-1}^{1} (1-Kx^2) e^{jux} \, dx \quad (2) \]

This yields upon evaluation the following expression:

\[ g(u) = (1-K) \frac{\sin u}{u} - \frac{2K}{u^2} \cos u + \frac{2K}{u^3} \sin u \quad (3) \]

Another possibility exists if the amplitude distribution is asymmetrical. Thus

\[ g(u) = \int_{0}^{1} (1-Kx^2) e^{jux} \, dx \quad (4) \]

\[ = (1-K) \frac{\sin u}{u} - \frac{2K}{u^2} \cos u + \frac{2K}{u^3} \sin u \]

\[ + j \left[ \frac{1}{u} - \frac{\cos u}{u} + \frac{2K}{u^3} + \frac{K \cos u}{u} \right. \]

\[ - \left. \frac{2K \sin u}{u^2} - \frac{2K \cos u}{u^3} \right] \quad (5) \]
The last expression is seen to contain a quadrature component which is characteristic of an asymmetrical distribution. However the maximum field intensity and thus the gain will be the same as in the symmetrical case. It may be shown that the half power width decreases and the side lobe level increases in the case of the unsymmetrical distribution.
APPENDIX II

Power Intercepted by Reflector

We are interested in determining the amount of power which is intercepted by the paraboloid. It may be expressed as

\[
\frac{P_I}{P_T} = \frac{\frac{1}{2z_0} \int_{-\phi_0}^{\phi_0} \int_{-\theta_0}^{\theta_0} |F_2(\theta, \phi)|^2 \sin(90-\theta) \, d\theta d\phi}{\frac{1}{2z_0} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} |F_1(xy)|^2 \, dx \, dy}
\]

(1)

The aperture distribution is

\[F_1(x,y) = \cos\left(\frac{\pi x}{a}\right)\]

(2)

While the radiation intensity is

\[F_2(\theta, \phi) = \frac{2ab}{\pi} \left[\frac{\sin(nb \sin \theta)}{(nb \sin \theta)}\right] \left[\frac{\cos(na \sin \phi)}{1 - \frac{4}{n^2} (na \sin \phi)^2}\right]\]

(3)

Equation (1) is then

\[
\frac{P_I}{P_T} = \frac{\frac{1}{2z_0} \int_{-\phi_0}^{\phi_0} \int_{-\theta_0}^{\theta_0} \left[\frac{\sin(nb \sin \theta)}{(nb \sin \theta)}\right]^2 \left[\frac{\cos(na \sin \phi)}{1 - \frac{4}{n^2} (na \sin \phi)^2}\right]^2 \sin(90-\theta) \, d\theta d\phi}{\frac{1}{2z_0} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \cos^2\left(\frac{\pi x}{a}\right) \, dx \, dy}
\]

(4)

The power ratio will be given by equation (5)
\[
\frac{P_I}{P_T} = \frac{8ab}{n^2} \int_{-\theta_o}^{\theta_o} \int_{-\phi_o}^{\phi_o} \left[ \frac{\sin(na \sin\theta)}{na \sin\theta} \right]^2 \left[ \frac{\cos(nb \sin\phi)}{1 - \frac{4}{n^2} (nb \sin\phi)^2} \right]^2 \sin(90-\phi) \, d\phi \, d\theta
\]

Evaluation of the integrals will yield the proportion of power intercepted by the reflector.
BIBLIOGRAPHY


5. Silver, S., op. cit., p. 177.


I, Robert Thornton Nash, was born in Columbus, Ohio, on September 20, 1929. My secondary education was received in the public schools of Columbus, Ohio, and in the University School. I enrolled at The Ohio State University in 1947 and received the degree, Bachelor of Science in Physics in 1952. From 1953 to 1955 I was a graduate assistant and a research assistant in the Department of Electrical Engineering at The Ohio State University while completing the requirements for the degree Master of Science, which I received in 1955. I was appointed research associate in 1956 and instructor in 1957 while continuing my graduate studies.