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AN INVESTIGATION OF THE VARIATION OF INTERNAL HYSSTERETIC DAMPING IN METALS AS A FUNCTION OF STRESS STATE AND SPATIAL STRESS DISTRIBUTION

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

TRUMAN GRAY FOSTER, B.M.E., M.Sc.

* * * * *

The Ohio State University

1961

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ACKNOWLEDGMENTS

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The University assumes no responsibility for
the accuracy or correctness of any of the statements
or opinions expressed in this dissertation.
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<td>a</td>
<td>Radii</td>
</tr>
<tr>
<td>C</td>
<td>Damping coefficient</td>
</tr>
<tr>
<td>c</td>
<td>Neutral axis, distance to extreme fiber from</td>
</tr>
<tr>
<td>γ</td>
<td>Weight per unit volume</td>
</tr>
<tr>
<td>δ</td>
<td>Strain, shear</td>
</tr>
<tr>
<td>D</td>
<td>Diameter</td>
</tr>
<tr>
<td>d</td>
<td>Displacement</td>
</tr>
<tr>
<td>d</td>
<td>Diameter</td>
</tr>
<tr>
<td>ξ</td>
<td>Logarithmic decrement</td>
</tr>
<tr>
<td>E</td>
<td>Modulus of elasticity (Young's Modulus)</td>
</tr>
<tr>
<td>Ed</td>
<td>Energy, dissipated</td>
</tr>
<tr>
<td>ε</td>
<td>Strain, normal</td>
</tr>
<tr>
<td>F</td>
<td>Force or load</td>
</tr>
<tr>
<td>f</td>
<td>Frequency</td>
</tr>
<tr>
<td>G</td>
<td>Modulus of elasticity in shear</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational constant</td>
</tr>
<tr>
<td>h</td>
<td>Height, depth, or thickness</td>
</tr>
<tr>
<td>I</td>
<td>Inertia, moment of (mass)</td>
</tr>
<tr>
<td>I</td>
<td>Inertia, moment of (area)</td>
</tr>
<tr>
<td>J</td>
<td>Inertia, polar moment of (area)</td>
</tr>
<tr>
<td>K</td>
<td>Spring constant (load per unit deflection)</td>
</tr>
<tr>
<td>k</td>
<td>Gyration, radius of</td>
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<thead>
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<th>Description</th>
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<tr>
<td>k</td>
<td>Damping coefficient</td>
</tr>
<tr>
<td>L</td>
<td>Length</td>
</tr>
<tr>
<td>ℓ</td>
<td>Length</td>
</tr>
<tr>
<td>M</td>
<td>Moment of force, including bending moment</td>
</tr>
<tr>
<td>m</td>
<td>Mass</td>
</tr>
<tr>
<td>m</td>
<td>Damping exponent</td>
</tr>
<tr>
<td>µ</td>
<td>Poisson's ratio</td>
</tr>
<tr>
<td>n</td>
<td>Damping exponent</td>
</tr>
<tr>
<td>n</td>
<td>Revolutions per unit time</td>
</tr>
<tr>
<td>ρ</td>
<td>Density, mass per unit volume ( ( \frac{\text{g}}{\text{g}} ))</td>
</tr>
<tr>
<td>P</td>
<td>Force or load</td>
</tr>
<tr>
<td>P</td>
<td>Power</td>
</tr>
<tr>
<td>Q</td>
<td>Force or load</td>
</tr>
<tr>
<td>R</td>
<td>Radius</td>
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<tr>
<td>r</td>
<td>Radius</td>
</tr>
<tr>
<td>S</td>
<td>Stress</td>
</tr>
<tr>
<td>s</td>
<td>Distance, arc length</td>
</tr>
<tr>
<td>σ</td>
<td>Stress, normal</td>
</tr>
<tr>
<td>T</td>
<td>Period</td>
</tr>
<tr>
<td>T</td>
<td>Torque</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
</tr>
<tr>
<td>t</td>
<td>Thickness</td>
</tr>
<tr>
<td>τ</td>
<td>Stress, shear</td>
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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$U$</td>
<td>Energy, total strain per unit volume</td>
</tr>
<tr>
<td>$C$</td>
<td>Amplitude, torsional vibration</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Angle, position coordinate</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume</td>
</tr>
<tr>
<td>$V$</td>
<td>Energy, distortion</td>
</tr>
<tr>
<td>$W$</td>
<td>Energy, total strain</td>
</tr>
<tr>
<td>$\Delta W$</td>
<td>Energy, dissipated specific</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Circular frequency (2 xf)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Strain, amplitude</td>
</tr>
<tr>
<td>$Z$</td>
<td>Section modulus</td>
</tr>
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CHAPTER I

INTRODUCTION

STATEMENT OF THE PROBLEM

The Need for Research in the Area of Hysteretic Damping

Considerable research in the area of hysteretic damping energy loss in metals has been done in the past 150 years. In this research a clear distinction was not made between material damping loss and component damping loss. It was not recognized that the configuration of a vibrating component and the stress distribution within it should be carefully accounted for in computing the damping energy loss. Recently, it became evident that future research should reveal the hysteretic damping energy loss in terms of specific parameters for the particular material. These basic data could then be introduced into a rational analytical method, accounting for both the shape of the mechanical part and the stress distribution in it, to give engineering estimates of the damping loss.

The Objectives of This Study

The objectives of this study were: (1) to establish a rational analytical method for the determination of the hysteretic damping loss which would take into account the shape and the stress distribution
of machine components, (2) to obtain fundamental damping energy loss
data on some important engineering materials under various states of
stress and special stress distributions, and (3) to compare the re-
sults of (1) and (2).

METHODS USED TO ACCOMPLISH
THE OBJECTIVES

The Basic Torsional Case

To accomplish the previously listed objectives, basic torsional-
pendulum tests, hereafter referred to as Case I, were developed. The
specimens employed in these tests were hollow 3/8 inch diameter rods
having a nominal wall thickness of 1/16 inch. The torsional loading
developed a stress distribution in the specimen which was constant
along the length, but varied with the radius in the transverse direc-
tion. A basic approximation was employed in connection with these
tests. This approximation was the replacement of the actual 33-1/3
percent radial stress gradient with an assumed constant radial stress
distribution. Specifically, this was done by computing the average
shear stress at the center of the wall and using this stress as the
assumed uniform stress across the wall. For the above rod dimensions,
the average shear stress was 83-1/3 percent of the shear stress at
the outer fiber. These tests were designed in this fashion so that
the two damping loss constants m and k, page 19, could be determined
under conditions approximating a uniform state-of-stress.

Three types of materials were investigated; (1) low carbon
steel 1010 to 1025, nominally 1018, (2) 304 stainless steel, and (3) 2024-T3 aluminum alloy. These particular materials were chosen because (1) they are widely used in the fabrication of machine components and (2) they represent a variety of engineering materials having different crystallographic structures.

The specimens were fabricated by cutting the hollow tubes to their correct lengths and welding small flanges to their ends. No heat treatment or complex machining was employed in their manufacture. Thus, "as-received" materials were used, and it was the intention of this investigator to obtain results of the same order of accuracy that a designer could expect when making estimates of hysteretic losses, using nominal properties of engineering materials.

The Other Cases Considered

Analytical methods were developed for two other cases where the stress distribution was more non-uniform than in Case I. Case II was a torsional stress situation which involved a solid cylindrical specimen. The torsional loading developed a stress distribution in the specimen which was constant along the length, but varied from zero at the center of the rod to a maximum at its outer fiber.

Case III was a bending pendulum which employed a solid cylindrical rod loaded essentially in pure bending. This loading resulted in a stress distribution which was constant along the length of the specimen but varied from zero at the neutral plane to a maximum at the outer fibers.
Experimental results were obtained to verify the analytical development. The types of materials used in the manufacture of the solid cylindrical specimens were (1) 1018 low carbon steel, (2) 304 stainless steel, and (3) 2024-T4 aluminum alloy. These specimens were also fabricated in the same fashion as was employed for the hollow specimens.
CHAPTER II

THE IMPORTANCE OF INTERNAL HYSTERETIC DAMPING IN METALS

A BRIEF OUTLINE OF THE IMPORTANT INTERNAL-DAMPING RESEARCH AREAS

Research Prior to 1950

As early as 1781, Coulomb reflected on the possible damping mechanisms which caused the vibrational amplitude of his torsional pendulum to decay. He even attempted to prove that the damping energy losses were associated with internal friction in the material and not with air friction.

Lasan estimated that by 1950 approximately 1000 papers had been published on the subject of structural damping. Most of these papers, though dealing with many significant parameters, neglected the effects on damping energy loss caused by the shape and the stress distribution within the machine part.

Research in the Past Decade

In the past decade Lasan and his associates have been engaged actively in research in the area of structural damping. They have

---

concentrated on the measurement of the specific damping energy loss under resonant vibrational conditions, and were among the first to recognize that the stress distribution in a machine part could seriously affect the results of any damping measurements.

Marin\(^2\) and his associates have been concerned with the important problem of devising methods of analytically predicting the fatigue strength of machine parts. In these methods, they took into account the effects of structural damping, and they were concerned primarily with parts that vibrate in a resonant manner.

Zener\(^3\) has investigated the internal friction properties of metals at low stress levels. In general, these stress levels have been below those considered important with respect to machine parts.

His relaxation spectra,\(^4\) that is, plots of the loss tangent (logarithmic decrement divided by \(n\)) versus increasing vibration frequency at constant temperature, or versus decreasing temperature at constant frequency, exhibit several peaks. He found that the location and magnitude of these peaks are different for each material. He also has attempted to explain their existence by various microstructural damping mechanisms.


\(^4\)Lazen, p. 21.
THE IMPORTANCE OF HYSTERETIC DAMPING LOSS TO DESIGNERS

The Importance of High Structural Damping Loss

Modern airliners have disintegrated in flight. This catastrophic event frequently was brought about by fatigue failure resulting from excessive flutter-type vibration or impact caused by air turbulence. However, the situation usually was aggravated by the weakening of an air-frame component by a prior history of cumulative fatigue damage due to normal flight, and random acoustical loading.

Thus a structural component tends to resonate at some fundamental frequency or some multiple of this frequency in response to the load spectrum created by flight conditions. In general it will vibrate at various frequencies in response to the load spectrum. This response can be reduced by the introduction of structural damping either of an internal or material type or of an external dry-friction type. Consequently, it is very important that engineers develop methods for designing these structural components which include sufficient damping to reduce the vibrational amplification to acceptable levels.

Sometimes it is not feasible to build in appreciable external damping; then materials having inherently high internal damping have to be utilized. A typical example is the almost classical problem of the turbine or compressor blade.
The Importance of Low Internal Damping Loss

The preceding discussion indicates that high internal damping loss is frequently of engineering importance. On the other hand, low internal damping is sometimes important, such as in reducing the frictional losses associated with nearly pure rolling motion of mechanical parts. Internal hysteretic work is involved in the stress-strain relationship at the interface of rolling elements, and according to the results of recent researches reported by Tabor\textsuperscript{5} in England and Drutowski\textsuperscript{6} from the General Motor Corporation in Detroit, this is considered to be the primary loss associated with pure rolling motion. Thus, if a general analytical method can be devised which will estimate the amount of hysteretic energy loss under periodic loading conditions, it could be applied to rolling elements to compute, for example, frictional torques.


CHAPTER III

THE GENERAL CONCEPT OF HYSTERETIC DAMPING LOSS AND THE IMPORTANT PARAMETERS ASSOCIATED WITH THIS PHENOMENON

THE CONCEPT OF HYSTERETIC DAMPING LOSS

The Hysteretic Diagram

If a metal part is subjected to a uniaxial load cycle, then the stress-strain relationship at a point is the familiar hysteretic diagram. In this diagram, Figure 1, the area is grossly exaggerated. Actually the area involved is quite small and represents the hysteretic energy loss per cycle for a completely reversed uniaxial stress cycle.

![Hysteretic diagram](Image)

Fig. 1. Hysteretic diagram.
Hysteretic loss is associated with the time phase relationship between stress and strain, i.e., the strain lags behind the stress, and as long as the stress level remains within the elastic range, the area of the inscribed loop is, for most metals, very small. However, this area represents a finite energy loss per cycle which is ultimately dissipated as heat to the surroundings.

The Basic Hypothesis of This Investigation

The actual mechanism by which some of the strain work is converted into heat is not well understood. One theory involves micro-slip which is assumed to act on the shear planes of minimum strength. This theory suggests that the specific hysteretic damping energy loss might be proportional to the specific distortion energy level. This hypothesis was first suggested by Robertson and Yorgiadis, and is used in this study to form the basis of a general analytical method to compute the hysteretic damping loss for various specimen and load configurations.

LIMITING THE SCOPE
OF THE PROBLEM

The Significant Parameters Associated with Hysteretic Damping Loss

Many physical parameters affect the amount of hysteretic energy lost per unit volume per cycle. Those that are known to be significant for a particular material are the following:

- Stress amplitude
- Stress distribution
- Configuration of the component
- Stress history
- Vibrational frequency
- Temperature
- Magnetic field
- Crystalline structure

Limiting the Number of Parameters Used in This Study

Hysteretic energy loss in metal is very sensitive to metallographic structural changes. Grain size and aging effects definitely affect it. It is believed to be a function of the various types of lattice imperfections, their number and distribution. That it is


\[9\] Ibid., pp. 350, 352.
temperature sensitive has been shown by Kimball\textsuperscript{10} and Zener\textsuperscript{11}. That it is frequency sensitive has been demonstrated by Bennawits and Rotger\textsuperscript{12}. A number of investigations have shown that magnetic fields have a pronounced effect on hysteretic damping loss for ferro-magnetic materials. The effect of stress history, i.e., the number and level of prior stress-strain cycles, has been dramatically demonstrated by Lasan\textsuperscript{13}. He has shown for most metals, for stress levels up to the neighborhood of the endurance limit, that the effect of stress history is negligible, but above a certain limiting stress level the effect is drastic. His curves, plots of a damping loss parameter versus stress level for various specimens, nominally of the same structure but having different stress histories, diverge at this limiting stress level. He has shown that this divergence is a function of the stress history of the specimen.

In this study the last five of the parameters listed on page 11 were held constant. The temperature was held nearly constant by using the ambient conditions of the laboratory, page 14. Frequency variation was eliminated by using approximately the same values for all tests, page 13. All testing was done below the endurance limit of the material; therefore, the stress history parameter was not con-

\textsuperscript{11}Zener, p. 151.
\textsuperscript{12}Kimball, p. A-2h.
\textsuperscript{13}Zener, p. 55.
\textsuperscript{13}Lasan, p. 29.
considered significant. However, some minor crystalline structure variation was present because the solid specimens and the hollow specimens, nominally of the same material, did not come from the same metallurgical heats.

Thus the first three parameters, stress amplitude, stress distribution, and configuration of the specimen, were considered significant. The effects of changing these parameters were carefully considered in both the analytical and the experimental parts of this investigation.
CHAPTER IV

THE DEVELOPMENT OF A RATIONAL ANALYTICAL METHOD TO
COMPUTE THE ENERGY LOST BY HYSRSTERIC DAMPING

THE DETERMINATION OF AN
INTEGRAL FUNCTION TO
COMPUTE THE ENERGY LOST
BY HYSRSTERIC DAMPING

The Damping Loss for a
Non-Uniform State-of-Stress

It has been indicated that the hysteretic damping loss is a
function both of the stress amplitude and the state of stress; con­
sequently, if the stress amplitude is different for each point in the
stressed region, i.e., if stress gradients exist, the damping loss
will vary from point to point within the significantly stressed region
of a structural component. Now, if the state of stress changes from
point to point in a stressed machine part, then the distortion energy
will also vary, and the question arises, how can the damping loss be
computed for the entire region which is significantly stressed?

An Integration Process
Is Indicated

Since there are an infinite number of points, an integration
process is indicated. Other experimenters have also used integration
methods, namely Mentel,14 Coehardt,15 Marin and Sharma.16 They have in some cases used a distortion energy hypothesis in setting up their integral equation. They apparently did this for each individual case. However, it would be difficult, expensive, and really impossible to investigate the many different stress distributions to determine appropriate damping parameters. This dilemma may be resolved by generalizing; in this case, resorting to a comparison between a hypothesized parameter in a multiaxial situation and the actual value of the same parameter in an uniaxial situation.

A Description of the Analytical Method of This Study

The damping loss for a non-uniform stress distribution can be determined by developing a general theory. First, an integral function, summing up the energy dissipation by damping, over the significantly stressed region of the machine part can be set up. Next, the hypothesis can be made that if the peak amplitude of the specific distortion energy in the non-uniform state be equal to the peak amplitude of the specific distortion energy in a uniform stress state then the same amount of damping loss can be expected. Finally, all that is needed to determine analytically the damping in a non-uniform case


16Loc. cit., p. 135.
is two damping constants, k and a, page 19. These damping constants can be determined experimentally by applying the uniform stress case to a particular material having a particular crystalline structure.

This investigation utilized a simple case, having an approximately uniform stress distribution, to obtain the fundamental data for the material. This information was then used to calculate the damping loss in a general multiaxial situation where the state of stress was a function of position, and the dynamic loading was either cyclic or approximately cyclic.

A BRIEF DESCRIPTION OF THE BASIC MECHANISM OF HYSYERETIC DAMPING

The Relationship of Distortion Energy to Hysteresic Energy Loss

When a part is loaded, for example, in reversed torsion or reversed bending, it is believed that slip occurs within the crystalline micro-structure. This slip occurs on certain preferred crystallographic planes and in certain preferred crystallographic directions. The preferred planes and directions are collectively referred to as preferred slip systems. In iron,\textsuperscript{17} for example, which has a body-centered-cubic lattice, the preferred slip system is diagonally from a corner atom toward the body-centered atom, i.e., in the $\langle 111 \rangle$ directions and on $\{ 110 \}$ diagonal planes.

It has been demonstrated\(^\text{16}\) that the plastic behavior of crystals depends only on the resolved shear stress on the preferred crystallographic planes and is independent of stresses normal to these planes. The onset of plastic flow occurs when the resolved shear stress reaches a certain critical value.

If the stress levels are below the usual endurance strengths, the critical resolved shear stress is not attained and plastic flow does not occur. However, it is believed that some slip does occur, and that it is a gross manifestation of the generation of dislocations and the mobility of these dislocations.

Consequently, it is hypothesized that the hysteretic damping energy loss is related to the distortion energy which in turn is related to the energy required to generate and mobilize dislocations.

---

**The Basic Mechanism of Hysteretic Damping Loss**

In summary, when a torsional load is first applied to a thin walled tubular specimen, elastic energy is stored. In the process of storing this energy, even at low stress levels, a certain amount of it will be lost because of micro-slip work, that is, energy required to generate and mobilize the dislocations. This irreversible mechanism is believed to be the significant internal damping mechanism operating at ordinary temperatures, low frequencies, and stress levels below the endurance limit of the material.

\(^\text{16}\) Ibid., p. 6.
The Effect of Grain Orientation

It is difficult to derive a simple law, such as Coulomb's dry-friction law, for the frictional forces associated with the micro-slip phenomenon, and then from it write a function expressing the amount of frictional work done. The primary reason for this difficulty is that the metallic grains are randomly oriented. Thus, if such a law could be developed analytically, it probably would require a statistical approach to account for the random orientation of the grains. However, a random law is needed which relates hysteretic damping loss to the peak amplitude of the distortion energy, and this law should consider the type, grain size, and hardness of the material.

Experimental Determination of the Internal Friction Law

An actual test, for example, a resonant torsion test may be used to determine a random law relating the specific energy lost per cubic inch per cycle to the stress level. Robertson and Yorgiadis\textsuperscript{19} have determined experimentally such a law by curve fitting their data. Their law states that

\[ \Delta \omega = k \tau^m \]

\[ (1) \]

\textsuperscript{19}Robertson and Yorgiadis, p. A-178.
where \[ \Delta W = \text{energy loss in inch-lbs per cubic inch per cycle}, \]
\[ \tau = \text{amplitude in psi of the shear stress at the outer fiber of a solid cylindrical specimen loaded in simple torsion}. \]

They carefully pointed out that it was not correct to apply this law directly to the non-uniform torsional shear stress situation, because all of the fibers of the specimen were not stressed the same amount.

However, if the torsional specimen were hollow, then the shear stress would be approximately uniform, and this law would more nearly fit the situation. This uniform shear stress situation was the basis of this investigation. This law was assumed to apply to the hollow cylindrical specimen loaded in simple torsion. In this relationship \( k \) and \( m \) are considered to be the primary specific material damping parameters, and are dependent upon the type of material, its grain size, and its hardness. They also may be dependent upon other basic mechanical properties related to the crystalline structure. If \( k \) and \( m \) are determined experimentally, the stress can be related to the distortion energy and the relationship of energy loss to distortion energy can be developed for non-uniform states of stress and various specimen-load configurations.

**Introduction of the Distortion Energy Hypothesis into the Analytical Method**

It will be assumed in the following that \( \Delta W \) is related to the amplitude of the distortion energy. The distortion energy can be expressed in terms of the principal stresses and the appropriate elastic
The basic hypothesis proposed here says that if the point value of the distortion energy in a uniform stress distribution is equal to the point value of the distortion energy in a non-uniform stress distribution, then the energy lost due to hysteretic damping per cubic inch per cycle is the same. Thus if

\[ V_{\text{uniform}} = V_{\text{non-uniform}}, \]

then

\[ \Delta V_{\text{uniform}} = \Delta V_{\text{non-uniform}}. \]

Here uniform was used to indicate that the stress distribution in the part is approximately uniform throughout the region that is significantly stressed, and non-uniform indicated that significantly

\[
v = \frac{1 + \mu}{E} (s_1^2 + s_2^2 + s_3^2 - s_1 s_2 - s_2 s_3 - s_3 s_1). \quad (2)
\]

Then, since for the case of simple torsion,

\[ s_1 = \tau, \quad s_2 = -\tau, \quad s_3 = 0, \quad (3) \]

if (3) is substituted in (2), the result would be

\[ \tau = \left[ \frac{E}{1 + \mu} \right]^{1/2}. \quad (4) \]

Finally, if (4) is substituted into (1) the following expression would result,

\[ \Delta V = k \left[ \frac{E}{1 + \mu} \right]^{m/2}. \quad (5) \]
stress gradients are present. Thus it is possible to write

$$\Delta w = k \left[ \frac{E}{1 + \mu} \right]^{m/2}$$

(5)

where \(m\) and \(k\) are independent of the stress amplitude, and thus also independent of the amplitude of the distortion energy, and \(V\) is a function of position and amplitude of the stress components at a particular position.

To obtain the total energy dissipated in a machine component, the specific loss has to be integrated over the volume of metal that is significantly stressed. Thus

$$E_d = \int_{\text{Vol}} \Delta w \, d(\text{vol})$$

(6)

or

$$E_d = \int_{\text{Vol}} k \left[ \frac{E}{1 + \mu} \right]^{m/2} \, d(\text{vol}),$$

(7)

and this expression in general can be applied to any uniform or non-uniform stress distribution and any component configuration.

**Establishment of the Analytical Method on a Logarithmic Decrement Basis Rather than on an Energy Loss Basis**

A parameter often used as a measure of the hysteretic damping loss is the logarithmic decrement, defined mathematically by

$$\delta = \log_a \left( \frac{y_m}{y_{m+1}} \right),$$

(8)
where $y_m$ and $y_{m+1}$ are successive values of the amplitude of a decaying vibration. To facilitate the analysis of the experimental data it is convenient to use $\delta$, rather than $E_d$, because $\delta$ can easily be measured experimentally, and it is well known that\textsuperscript{20}

$$2 \delta \approx \frac{E_d}{\bar{W}},$$

(9)

where $\bar{W}$ is the total strain energy in the significantly stressed region of the component. Thus substitution of (6) into (9) gives

$$\delta = (1/2) \frac{\int \Delta W \, d(\text{vol})}{\int U \, d(\text{vol})},$$

(10)

where $U$ is the specific strain energy, i.e., the total strain energy at a point within the significantly stressed region.

\textsuperscript{20}Zener, p. 63.
CHAPTER V

ANALYTICAL RESULTS FOR THE THREE TEST-SPECIMEN CONFIGURATIONS CONSIDERED IN THIS STUDY

THE DERIVATION OF ANALYTICAL EXPRESSIONS FOR THE LOGARITHMIC DECREMENT FOR THE THREE CASES CONSIDERED IN THIS STUDY

Case I. A Hollow Bar in Simple Torsion

If a hollow bar were loaded in simple torsion then the actual stress distribution would be

\[ \tau = \frac{G r}{r} , \]

and at any point \( r \) the energy lost per unit volume per cycle was assumed to be

\[ \Delta W = k (\tau)^n . \]  

(1)

Then, if equation (10), page 22, were used, i.e.,

\[ \delta_I = (1/2) \frac{\int \Delta W \, d(\text{vol})}{\int U \, d(\text{vol})} , \]

(10)

and

\[ U = \left( \frac{1 + \mu}{E} \right) (\tau)^2 , \]

\[ \Delta W = k (\tau)^n , \]

and

\[ \tau = \frac{G r}{\lambda} , \]
were substituted into it. The result would be

\[
\delta_I = \frac{\frac{1}{2} \int_0^l \int_0^{r_2} \int_0^{r_1} k \left( \frac{\sigma_0}{l} \right)^m r^{m+1} \, dr \, d\phi \, ds}{\int_0^l \int_0^{r_2} \int_0^{r_1} \left( \frac{1 + \mu}{E} \right) \left( \frac{\sigma_0}{l} \right)^2 r^3 \, dr \, d\phi \, ds}
\]

which when integrated would give

\[
\delta_I = \left( \frac{h k \sigma_0}{m + 2} \right) \left( \frac{r_2^m}{r_1} - \frac{r_1^m}{r_2} \right), \tag{11}
\]

and for a hollow cylindrical specimen with an O.D. of 3/8 inch and an I.D. of 1/4 inch the following relations are true,

\[
\tau_1 = 0.8 \, \tau_{\text{uniform}}
\]

and

\[
\tau_2 = 1.2 \, \tau_{\text{uniform}}, \tag{12}
\]

where \( \tau_{\text{uniform}} \) is the shear stress at the center of the wall of the hollow specimen. Substitution of equation (12) into (11) gave the following expression,

\[
\delta_I = \left( \frac{h k \sigma_0}{m + 2} \right) \left( 1.2^{m-2} - 0.8^{m-2} \right) \tau_{\text{uniform}} \tag{13}
\]

or

\[
\delta_I = c \, \tau_{\text{uniform}}. \tag{14}
\]
Case II. A Solid Bar in Simple Torsion

If a solid bar were loaded in simple torsion, the actual stress distribution would be

\[ \tau = \frac{G r}{l}, \]

and at any point the energy lost per unit volume per cycle was assumed to be

\[ \Delta W = k \tau^m. \]

Then, if equation (10), page 22, were used, i.e.,

\[ \int_\Pi \frac{1}{2} \int_{\text{vol}} \Delta W \, d(\text{vol}) \int_{\text{vol}} U \, d(\text{vol}) \]

and

\[ U = \frac{1 + \mu}{E} (\tau)^2, \]

\[ \Delta W = k \tau^m, \]

and

\[ \tau = \frac{G r}{l}, \]

were substituted into it, the result would be

\[ \delta_{\Pi} = \frac{\frac{1}{2} \int_{0}^{l} \int_{0}^{2\pi} \int_{0}^{s_2} k \left( \frac{\varepsilon}{\lambda} \right)^m r^{m+1} \, dr \, d\phi \, ds}{\int_{0}^{l} \int_{0}^{2\pi} \int_{0}^{s_2} \left( \frac{1 + \mu}{E} \right) \left( \frac{\varepsilon}{\lambda} \right)^2 r^3 \, dr \, d\phi \, ds}, \]

which when integrated would give

\[ \delta_{\Pi} = \left( \frac{h k g}{m+2} \right) (\tau_2)^{m-2}, \quad (15) \]

where

\[ \tau_2 = \tau (s_2). \]
This result, equation (15), is identical with Lazan's\textsuperscript{21} result for the same situation.

Case III. A Solid Bar in Pure Bending

Introduction.— If a solid round bar were loaded in pure bending, a non-uniform stress distribution would be set up in the bar. In order to apply equation (10), page 22, to this situation, it would be necessary to adopt a theory relating the energy loss per unit volume per cycle to some specific variable that changes from point-to-point within the significantly stressed region of the bar. One hypothesis, which could be used, states that the energy loss per unit volume per cycle can be related to the distortion energy per unit volume.

Distortion energy hypothesis.— If the distortion energy hypothesis were adopted, equation (10) could be written

\[
\delta_{\text{III}} = \frac{1}{2} \int_{\text{Vol}} k \left[ \left( \frac{1}{1 + \mu} \right) \frac{V}{\sqrt{1 + \nu}} \right]^{\gamma/2} d(\text{vol})
\]

and for the state-of-stress in a round bar in pure bending

\[
\nu = \left( \frac{1 + \mu}{2\mu} \right) (\sigma_b)^2,
\]

and

\[
U = \left( \frac{1}{2E} \right) (\sigma_b)^2,
\]

where
\[ \sigma_b = \left( \frac{M_0}{I_0} \right) r \sin \beta^* , \]

and
\[ I_0 = \frac{\pi a^4}{4} . \]

If these equations were substituted into (16) the following equation would result:

\[
\delta_{III} = \frac{1}{\frac{1}{2} k} \left[ \frac{1}{3} \right] \int_0^l \int_0^{2\pi} \int_0^{s_2} \left( \frac{M_0}{I_0} \right)^m (\sin^m \beta) r \, dr \, d\phi \, ds
\]

which when integrated would give

\[
\delta_{III} = \left[ \frac{8 \epsilon S}{\sqrt{n} (m+2)(3)^{m/2}} \right] \left[ \frac{n}{\frac{m}{2} \frac{n}{2} + 1} \right] (\sigma_2)^{m-2} , \tag{17}
\]

where \( \sigma_2 = \sigma (a_2) \).

This result, if multiplied by the factor \([1.02]\), would agree with Lasan's result\(^{22}\) for the same situation, assuming \( \sigma = \gamma_3 \tau \).

Maximum shear stress hypothesis.— If, instead of the distortion energy hypothesis, the hypothesis were adopted that the energy

\^\text{Using cylindrical coordinates.}

\(^{22}\)\textit{Ibid.}, p. 12.
loss per unit volume per cycle is related to the level of the maximum shear stress at a point, then equation (18) would result.

First, the principal normal stresses in the pure bending cases are

\[
\sigma_1 = \sigma_x \\
\sigma_2 = 0 \\
\sigma_3 = 0 .
\]

Second, the principal shear stresses in this case are

\[
\tau_3 = \pm \frac{1}{2} (\sigma_2 - \sigma_3) = 0 \\
\tau_2 = \pm \frac{1}{2} (\sigma_3 - \sigma_1) = \pm \sigma_b/2 \\
\tau_1 = \pm \frac{1}{2} (\sigma_1 - \sigma_2) = \pm \sigma_b/2 .
\]

Thus the maximum principal shear stress for pure bending would be

\[
\tau_{III} = \tau_1 = \pm \sigma_b/2 ;
\]

and the maximum shear stress hypothesis would say that if

\[
\tau_{III} = \tau_I
\]

then

\[
\Delta W_{III} = \Delta W_I .
\]

If

\[
\tau_I = \tau_{III} = 1/2 \sigma_b
\]
were substituted into equation (10), the result would be

\[ \delta_{\text{III}} = \frac{1}{2} \int_{\text{Vol}} k \left( \frac{\sigma_b}{2} \right)^m d(\text{vol}) \]

where

\[ \sigma_b = (M_d/I_o) r \sin \beta^* \]

and

\[ I_o = \pi \frac{a^4}{h} \]

and

\[ U = (1/2E) (\sigma_b)^2 \]

Since

\[ W = \int_{\text{Vol}} U d(\text{vol}) = \frac{1}{8E} \left( \frac{M_o a^2}{I_o} \right)^2 a^2 \pi l \]

for a round bar in pure bending, then equation (10) could be written

\[ \delta_{\text{III}} = \frac{\frac{8}{E} \int_0^l \int_0^{2\pi} \int_0^{a/2} \left( \frac{1}{2} \right)^m \left( \frac{M_o a^2}{I_o} \right)^m r^{m+1} (\sin^m \beta) \, dr \, d\beta \, ds}{\left( \frac{M_o a^2}{I_o} \right)^2 a^2 \pi l} \]

which when integrated would give

\[ \delta_{\text{III}} = \left[ \frac{8}{E \sqrt{n} (m+2)} \right] \frac{\Gamma\left( \frac{m}{2} + \frac{1}{2} \right)}{\Gamma\left( \frac{m}{2} + 1 \right)} (\sigma_2)^{m-2} \]  

(18)

where

\[ \sigma_2 = \sigma (a_2) \]

*Using cylindrical coordinates.
Now if
\[ \delta_{\text{III}}(s) = \delta \text{ for the maximum shear stress hypothesis} \]
and
\[ \delta_{\text{III}}(D) = \delta \text{ for the distortion energy hypothesis} \]
then
\[ \delta_{\text{III}}(s) = \left[ \frac{(3)^{1/2}}{(2)^{n/2}} \right] \delta_{\text{III}}(D), \]
and to obtain the curves for \( \delta_{\text{III}}(s) \) versus \( \sigma_2 \), all that would be required would be to multiply the corresponding ordinates for \( \delta_{\text{III}}(D) \) by the constant \( B \), where \( B \) would be defined by
\[ B = \left[ \frac{(3)^{1/2}}{(2)^{n/2}} \right]. \]

Exhibited in Table 1 are the values of \( B \) for the three materials used in this investigation.

### Table 1

<table>
<thead>
<tr>
<th>Material</th>
<th>( n )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1018 Steel</td>
<td>2.793</td>
<td>0.670</td>
</tr>
<tr>
<td>2 304 Stainless Steel</td>
<td>3.616</td>
<td>0.580</td>
</tr>
<tr>
<td>3 2024 Aluminum Alloy</td>
<td>4.66</td>
<td>0.516</td>
</tr>
</tbody>
</table>
SUMMARY OF THE
ANALYTICAL RESULTS

Case I. Essentially
Uniform Torsion

When equation (10) was applied to this case, it was found, page 24, that

\[ \delta = C \frac{p}{n_{\text{uniform}}} \]  \hspace{1cm} (14)

This equation states that the hysteretic energy loss, as measured by the logarithmic decrement, is a power function of the average shear stress across the wall of the hollow torsional specimen. If this power function is plotted on a log-log type graph, then \( C \) and \( n \), respectively, determine its location and slope, Figure 8.

A similar interpretation would apply to the equations for the other two cases considered in this investigation. In these equations \( k \) and \( m \) are used, rather than \( C \) and \( n \). The constants \( k \) and \( m \) are used because they are more correctly the basic damping constants for a particular material.

The above constants are related by the following equations,

\[ C = \left( \frac{k + 1}{m + 2} \right) \left[ \left( \frac{r_2}{r_{\text{ave}}} \right)^{m-2} - \left( \frac{r_1}{r_{\text{ave}}} \right)^{m-2} \right] \]  \hspace{1cm} (20)

and

\[ n = m - 2. \]  \hspace{1cm} (21)
Case II. Non-Uniform Torsion

For Case II equation (10) gave

\[ \delta_{II} = \left( \frac{k_0}{m+2} \right) (\tau_2)^{m-2} . \]  \hfill (15)

Case III. Non-Uniform Bending

If the distortion–energy hypothesis, that is, equation (5) were introduced into equation (10), the result would be

\[ \delta_{III} = \left[ \frac{8 k_0}{(m+2) (3)^{m/2}} \frac{\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{m}{2} + 1\right)} \right] (\tau_2)^{m-2} . \]  \hfill (17)

However, if instead of a distortion–energy hypothesis, a maximum–shear–stress hypothesis were adopted, the result would be,

\[ \delta_{III} = \left[ \frac{8 k_0}{(m+2) (2)^m} \frac{\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{m}{2} + 1\right)} \right] (\tau_2)^{m-2} . \]  \hfill (18)
CHAPTER VI
THE EXPERIMENTAL APPROACH USED TO MEASURE THE ENERGY DISSIPATED BY HystERetic DAMPING

THE DEVELOPMENT OF
A TESTING FACILITY

Features of the Experimental Set-Up

The testing facility was designed to include the following desirable features.

1. In order to minimize losses due to end effects, i.e., losses associated with stress gradients in those regions where the specimen is attached to other parts, it was considered very important to make the significantly stressed region of the specimen large. This was accomplished by making the specimen long and slender.

2. For the basic case, i.e., the uniformly stressed case, the specimen should be economical to manufacture. This feature was incorporated by designing the specimen so that it could be fabricated by cutting commercially available tubing to the correct length and welding a simple flange onto each end.

Minimising Frictional Losses

In the design of the testing facility, great care was exercised to minimize the external frictional losses. If such losses were large relative to the internal friction, large errors could be
introduced which would obscure the experimental results. Welded joints were used on the ends of the specimen in order to eliminate any Coulomb type of friction in this region. The specimens were mounted in torsional and bending types of pendulums, which were in turn suspended from the ceiling with hard, fine music wires. With these measures the unwanted losses were of four types: (1) hysteretic losses in the suspension wires, (2) aerodynamic or windage losses, (3) hysteretic losses in the welded flanges, and (4) strain gage lead-wire losses.

A Study to Determine Desirable Load Configurations and Specimen Shapes

A detailed study of all types of load configurations and specimen shapes was made. The results of this study showed that a hollow bar loaded in pure torsion is one of the simplest load-specimen configurations. This configuration has essentially a uniform state of stress, and is economical to manufacture, and the external losses are minimal.

The next least complicated cases were a solid bar loaded in pure torsion and a solid bar loaded in pure bending. These latter cases have non-uniform stress distribution; however, this was desirable to check the theory. This preliminary study gave rise to the three cases which were used throughout this investigation.
EXPERIMENTAL EQUIPMENT USED
TO TEST THE SPECIMENS

Release Fixture

A release fixture was designed and built in which a pendulum, either a torsional or a bending pendulum, could be inserted, loaded, and released. The release mechanism consisted of four pins which engaged four holes in the pendulum. These pins were rapidly retracted by four AC solenoids. Thus the pendulum was released and allowed to decay freely. This fixture worked well for the two torsional pendulums, but was difficult to operate for the bending pendulum. In the latter situation, unless the release fixture were rapidly removed from the vicinity of the bending pendulum, the pendulum would strike the fixture, and the high amplitude portion of the test would be ruined.

It was discovered that if relatively large masses were employed for both the torsional and the bending types of pendulums, they could be set in operation by hand. Thereafter, the release fixture was not used.

Experimental Apparatus and Strain Measuring Equipment

Figures 2 and 3 show the main features of the two types of pendulums used in this investigation. The vertical pendulum was used for both hollow and solid cylindrical specimens, i.e., Case I and II. Also illustrated in these pictures are the strain measuring and recording equipment used for all experimental tests. The strain gage
bridges, located on the specimens, were of the usual type having four active gages arranged to compensate for temperature and for bending in both torsional types of pendulums and for temperature and torsion in the bending pendulum. A typical specimen is shown in Figure 4.

Originally it was planned to operate the bending pendulum in a vertical position. However, when this was attempted, an unwanted extra mode of vibration was superimposed on the desired mode. This two-degree-of-freedom effect was practically removed by the horizontal set-up indicated in Figure 2.
Fig. 2. A view of the torsional pendulum used for both uniform torsion, Case I, and non-uniform torsion, Case II.
Fig. 3. A view of the bending pendulum used for non-uniform bending, Case III.
Fig. 4. A view of a typical specimen used for both the torsional and bending pendulums.
CHAPTER VII

EXPERIMENTAL RESULTS

THE TYPE OF MATERIAL, LABORATORY CONDITIONS, AND FREQUENCY USED FOR EACH TEST

Introduction

The significant parameters believed to affect hysteretic damping loss were listed in Chapter III, page 11. In this investigation the temperature and the magnetic fields were the ambient conditions of the laboratory. However, the temperature and the humidity were recorded, page 44. The effects of frequency change and material-structural change were not studied; consequently, the results can only be interpreted for the type of material and the frequency range actually used.

Type of Specimens, Material and Shape

The nominal physical properties of the materials used in the manufacture of each specimen are listed in Table 2. These materials were chosen so that a significant range of engineering metals would be included. Figure 5 shows the nine specimens used in this investigation.
<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Material</th>
<th>Treatment</th>
<th>Ultimate Strength x10^{-3}, psi</th>
<th>Yield Strength x10^{-3}, psi</th>
<th>Modulus of Elasticity x10^{-7}, psi</th>
<th>Modulus of Rigidity x10^{-6}, psi</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1018 C.R. Steel</td>
<td>As Rec'd.</td>
<td>64</td>
<td>45</td>
<td>3</td>
<td>11.5</td>
<td>0.30</td>
</tr>
<tr>
<td>2</td>
<td>2024-T3 Al. Alloy</td>
<td>As Rec'd.</td>
<td>70</td>
<td>48</td>
<td>1</td>
<td>3.85</td>
<td>0.33</td>
</tr>
<tr>
<td>3</td>
<td>304 Stainless Steel</td>
<td>As Rec'd.</td>
<td>85</td>
<td>40</td>
<td>2.8</td>
<td>12.5</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>1018 C.R. Steel</td>
<td>As Rec'd.</td>
<td>64</td>
<td>45</td>
<td>3</td>
<td>11.5</td>
<td>0.30</td>
</tr>
<tr>
<td>5</td>
<td>304 Stainless Steel</td>
<td>As Rec'd.</td>
<td>85</td>
<td>40</td>
<td>2.8</td>
<td>12.5</td>
<td>0.30</td>
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<tr>
<td>6</td>
<td>2024-T3 Al. Alloy</td>
<td>As Rec'd.</td>
<td>70</td>
<td>48</td>
<td>1</td>
<td>3.85</td>
<td>0.33</td>
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<td>As Rec'd.</td>
<td>70</td>
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<td>1</td>
<td>3.85</td>
<td>0.33</td>
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<tr>
<td>8</td>
<td>304 Stainless Steel</td>
<td>As Rec'd.</td>
<td>85</td>
<td>40</td>
<td>2.8</td>
<td>12.5</td>
<td>0.30</td>
</tr>
<tr>
<td>9</td>
<td>1018 C.R. Steel</td>
<td>As Rec'd.</td>
<td>64</td>
<td>45</td>
<td>3</td>
<td>11.5</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Fig. 5. View showing the nine specimens used in this study.
The Natural Frequency and the Ambient Conditions of the Laboratory for Each Test

The natural frequency of each test is listed in Table 3. In Table 4 the ambient conditions in the laboratory which existed at the time of the test are listed.

TABLE 3
The Natural Frequency of Each Test

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Test Number</th>
<th>Type of Test</th>
<th>Natural Frequency (cps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23</td>
<td>III</td>
<td>5.35</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>III</td>
<td>3.33</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>III</td>
<td>5.33</td>
</tr>
<tr>
<td>4</td>
<td>14C</td>
<td>II</td>
<td>3.12</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>II</td>
<td>3.00</td>
</tr>
<tr>
<td>6</td>
<td>15B</td>
<td>II</td>
<td>1.80</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>I</td>
<td>1.52</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>I</td>
<td>2.72</td>
</tr>
<tr>
<td>9</td>
<td>19</td>
<td>I</td>
<td>2.83</td>
</tr>
</tbody>
</table>
### TABLE 4

The Ambient Conditions for Each Test

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Temperature at Barometer (°F)</th>
<th>Barometric Pressure (in. Hg.)</th>
<th>Wet Bulb Temp. (°F)</th>
<th>Dry Bulb Temp. (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>88.0</td>
<td>29.49</td>
<td>77.0</td>
<td>85.0</td>
</tr>
<tr>
<td>24</td>
<td>88.0</td>
<td>29.49</td>
<td>77.0</td>
<td>85.0</td>
</tr>
<tr>
<td>25</td>
<td>88.0</td>
<td>29.41</td>
<td>75.0</td>
<td>83.0</td>
</tr>
<tr>
<td>14C</td>
<td>83.5</td>
<td>29.49</td>
<td>73.5</td>
<td>81.5</td>
</tr>
<tr>
<td>16</td>
<td>83.5</td>
<td>29.49</td>
<td>73.5</td>
<td>81.5</td>
</tr>
<tr>
<td>15B</td>
<td>83.5</td>
<td>29.49</td>
<td>73.5</td>
<td>81.5</td>
</tr>
<tr>
<td>20</td>
<td>87.0</td>
<td>29.43</td>
<td>75.0</td>
<td>85.0</td>
</tr>
<tr>
<td>18</td>
<td>87.0</td>
<td>29.43</td>
<td>75.0</td>
<td>85.0</td>
</tr>
<tr>
<td>19</td>
<td>87.0</td>
<td>29.43</td>
<td>75.0</td>
<td>85.0</td>
</tr>
</tbody>
</table>
DATA REDUCTION METHOD USED TO
YIELD LOGARITHMIC DECREMENT

The Strain Versus
Time Record

The strain-versus-time charts obtained from the recorder were
nearly flawless and exhibited the usual logarithmic decay character-
istic (Figure 6). Initial and final calibration signals were re-
corded by the application of a known precision resistor across one
leg of the strain-gage bridge. These calibration signals were used
not only to calculate the strain levels, but were used also to meas-
ure the sixty-cycle electrical noise present. All strain amplitude
measurements were corrected for noise. Alongside the strain-time
record, a one-cycle-per-second timing signal was recorded, and in
addition, the total elapsed time, as measured by a stop-watch, was
marked on the record.

Three Methods of
Data Reduction

In order to obtain $\delta$, the logarithmic decrement, from the
strain-time records, three methods were examined. These were:

(1) $\delta = \frac{y}{y}$,

(2) $\delta = (dy/dx)_1 \left( \frac{y_1}{y} \right)$,

and

(3) $\delta = \left( \frac{1}{n} \right) \log_e \left( \frac{y_1}{y_n} \right)$.
where

\[ T = \text{period, seconds}, \]
\[ n = \text{number of vibrations between } y_0 \text{ and } y_n. \]

The first method, if used, would give inaccurate results, because the magnitude of \( \Delta y \) was of the same order as the error in measuring it. The second and third methods were compared in detail, and they yielded nearly identical results. Since the third method was the most convenient to apply, it was adopted. Also, it was noted that if this method were judiciously applied, it would yield nearly instantaneous values of \( \dot{\varphi} \) versus instantaneous values of the stress level. This could be done by the proper choice of \( n \). If \( n \) were chosen so that the decrease in amplitude were essentially linear over \( n \), then the average value of \( \dot{\varphi} \) would be equal to the instantaneous value of \( \dot{\varphi} \) at mid-range of \( n \).

Also, if this instantaneous value of \( \dot{\varphi} \) were quoted at the average amplitude, then the average and the instantaneous amplitude would be the same at mid-range of \( n \), and the final result would provide an instantaneous value of \( \dot{\varphi} \) quoted at the correct instantaneous value of the stress level.
Fig. 6. View of a typical strain-time record.
Experimental Determination of
the Overall Unwanted Losses in
the Bending Pendulum Experiments

The overall unwanted losses included:

1. Hysteretic losses in the suspension wires.

2. Hysteretic losses in the end-welds used to attach the
   specimens to their fastened flanges.

3. Windage losses associated with the vibration of the end-
   bells of the bending pendulum.

4. Windage and hysteresis losses associated with the vibra-
   tion of the strain-gage lead wires.

To experimentally determine an overall loss decrement, one of
the end-bells was disconnected from the specimen in the bending pen-
dulum set-up. The end-bell was set in motion with an initial ampli-
tude of 10°. It was allowed to decay freely. The total time re-
quired for it to come to rest was forty minutes.

During the above test the natural frequency was measured
several times. The average of these measurements gave 24
vibrations
per minute. Therefore, the total number of oscillations of the end-
bell was 960 cycles. Thus,

\[ \xi_{\text{ave}} = \frac{1}{960} \approx 0.001 , \]

which would be an estimate of the logarithmic decrement at \( \theta = 5^\circ \).

This is illustrated in Figure 7.
Fig. 7. Estimation of the logarithmic decrement.

This estimate was made for one end-bell suspended by two wires. Since two end-bells and four wires were involved in the actual set-up, this loss decrement should be doubled, giving \( \delta_{\text{ave}} = 0.002 \).

This estimate of the unwanted loss decrement included only the effects of items 1 and 3. A rough estimate might allow an increase of the above figure by 50 percent to account for items 2 and 4. If this were done, the result would be \( \delta_{\text{ave}} = 0.003 \).

This correction factor was applied to both the 1018 steel and the 304 stainless steel bending pendulum test results. A similar correction factor was estimated for the aluminum bending pendulum results. Because the starting amplitude in this test was nearly double those used for the steel specimens, it was believed that the
loss decrement would be nearly twice the estimated amount for the steel specimens; therefore, \( \Delta_{\text{ave}} = 0.006 \).

This correction factor was applied to the 2024 aluminum alloy bending pendulum test results. This correction factor was of the same order of magnitude as the loss decrement actually measured. Therefore, these results are obscure and unreliable because the error in the estimation of the overall loss correction factor is unknown. The same, but not quite as serious, situation exists in the application of the correction factor to the results of the 304 stainless steel test. In this situation, the correction factor applied was nearly one-half the magnitude of the loss decrement measured, and again the results are obscure and unreliable.

**EXPERIMENTAL DETERMINATION OF THE OVERALL UNWANTED LOSSES IN THE TORSIONAL PENDULUM EXPERIMENTS**

The upper half of the torsional pendulum, supported by one wire, was set in motion. It took 6 hours and 50 minutes for it to come to rest. Several values of the period of vibration were determined and their average value was 26 seconds per vibration. Consequently, the total number of vibrations was

\[
\frac{6,600 \text{ sec}}{26 \text{ sec/vib}} = 1600 \text{ vibrations,}
\]

and

\[
\Delta_{\text{ave}} = \frac{1}{1600} = 0.0006.
\]
This measurement was for a large amplitude of nearly 180°; therefore, the decrement for an amplitude of approximately 10° crudely would be \( \delta_{\text{ave}} = 0.0006/9 = 0.00007 \). Since the preceding measurement was timed for one end-bell suspended by one wire, the computed loss decrement must be doubled because the actual torsional experiment used two end-bells and two suspension wires. This would give \( \delta_{\text{ave}} = 0.00014 \).

Because the strain-gage lead wires did not vibrate in the torsional experiment and the effect of the end-welds is believed to be quite small, no additional correction factor for these items need be included in the above estimate.
EXPERIMENTAL RESULTS

Correction of the Results for Windage and Wire Losses

After the values of $\delta$ versus stress level were determined by the method described on page 145, they were corrected for windage and suspension-wire losses, as outlined in the previous section starting on page 48.

Tabular Presentation of Results

Tables 5 through 13 show the experimentally determined values of the logarithmic decrement versus the corresponding stress level and indicate how the correction factors were applied for the nine tests conducted in this investigation.

Graphical Presentation of Results

Figures 8 through 16 are plots of the logarithmic decrement versus the peak stress in the cycle. It should be noted for Case I tests that $\delta$ is plotted versus the average or uniform shear stress across the wall of the hollow specimen. In Case II and III types, the value of $\delta$ is plotted versus the outer fiber stress, shear stress for Case II, and bending stress for Case III.
### TABLE 5

Results of Test 19, Uniform Torsion of a Hollow 1018 C.R. Steel Rod (Case I)

<table>
<thead>
<tr>
<th>( \tau_{\text{uniform}} )</th>
<th>( \delta_{\text{uncorr.}} )</th>
<th>( \delta_{\text{losses}} )</th>
<th>( \delta_{\text{corr.}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15,330</td>
<td>0.01061</td>
<td>0.00014</td>
<td>0.01050</td>
</tr>
<tr>
<td>12,000</td>
<td>0.00958</td>
<td>0.00014</td>
<td>0.00944</td>
</tr>
<tr>
<td>8,670</td>
<td>0.00745</td>
<td>0.00014</td>
<td>0.00731</td>
</tr>
<tr>
<td>6,670</td>
<td>0.00668</td>
<td>0.00014</td>
<td>0.00654</td>
</tr>
<tr>
<td>5,120</td>
<td>0.00543</td>
<td>0.00014</td>
<td>0.00529</td>
</tr>
<tr>
<td>4,020</td>
<td>0.00470</td>
<td>0.00014</td>
<td>0.00456</td>
</tr>
<tr>
<td>3,265</td>
<td>0.00407</td>
<td>0.00014</td>
<td>0.00393</td>
</tr>
<tr>
<td>2,670</td>
<td>0.00294</td>
<td>0.00014</td>
<td>0.00280</td>
</tr>
<tr>
<td>1,530</td>
<td>0.00239</td>
<td>0.00014</td>
<td>0.00225</td>
</tr>
<tr>
<td>h9h</td>
<td>0.00106</td>
<td>0.00014</td>
<td>0.00092</td>
</tr>
</tbody>
</table>

### TABLE 6

Results of Test 14C, Non-Uniform Torsion of a Solid 1018 C.R. Steel Rod (Case II)

<table>
<thead>
<tr>
<th>( \tau_2 )</th>
<th>( \delta_{\text{uncorr.}} )</th>
<th>( \delta_{\text{losses}} )</th>
<th>( \delta_{\text{corr.}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9,710</td>
<td>0.01930</td>
<td>0.00014</td>
<td>0.01920</td>
</tr>
<tr>
<td>5,720</td>
<td>0.01470</td>
<td>0.00014</td>
<td>0.01450</td>
</tr>
<tr>
<td>3,920</td>
<td>0.00960</td>
<td>0.00014</td>
<td>0.00950</td>
</tr>
<tr>
<td>3,020</td>
<td>0.00760</td>
<td>0.00014</td>
<td>0.00750</td>
</tr>
<tr>
<td>3,460</td>
<td>0.00600</td>
<td>0.00014</td>
<td>0.00580</td>
</tr>
<tr>
<td>2,090</td>
<td>0.00510</td>
<td>0.00014</td>
<td>0.00500</td>
</tr>
<tr>
<td>1,310</td>
<td>0.00350</td>
<td>0.00014</td>
<td>0.00340</td>
</tr>
<tr>
<td>715</td>
<td>0.00220</td>
<td>0.00014</td>
<td>0.00210</td>
</tr>
<tr>
<td>560</td>
<td>0.00190</td>
<td>0.00014</td>
<td>0.00180</td>
</tr>
</tbody>
</table>
TABLE 7

Results of Test 23, Non-Uniform Bending of a Solid 1018 Steel Rod (Case III)

<table>
<thead>
<tr>
<th>$\sigma_2$</th>
<th>$\delta_{uncorr}$</th>
<th>$\delta_{losses}$</th>
<th>$\delta_{corr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21,000</td>
<td>0.01730</td>
<td>0.00300</td>
<td>0.01430</td>
</tr>
<tr>
<td>17,200</td>
<td>0.01570</td>
<td>0.00300</td>
<td>0.01270</td>
</tr>
<tr>
<td>12,450</td>
<td>0.01700</td>
<td>0.00300</td>
<td>0.01400</td>
</tr>
<tr>
<td>9,200</td>
<td>0.01240</td>
<td>0.00300</td>
<td>0.00940</td>
</tr>
<tr>
<td>7,310</td>
<td>0.01060</td>
<td>0.00300</td>
<td>0.00760</td>
</tr>
<tr>
<td>6,000</td>
<td>0.00880</td>
<td>0.00300</td>
<td>0.00580</td>
</tr>
<tr>
<td>5,000</td>
<td>0.00980</td>
<td>0.00300</td>
<td>0.00680</td>
</tr>
<tr>
<td>2,980</td>
<td>0.00610</td>
<td>0.00300</td>
<td>0.00310</td>
</tr>
<tr>
<td>1,575</td>
<td>0.00490</td>
<td>0.00300</td>
<td>0.00190</td>
</tr>
<tr>
<td>1,130</td>
<td>0.00290</td>
<td>0.00300</td>
<td>---</td>
</tr>
</tbody>
</table>

TABLE 8

Results of Test 18, Uniform Torsion of a Hollow 304 Stainless Steel Rod (Case I)

<table>
<thead>
<tr>
<th>$\tau_{uniform}$</th>
<th>$\delta_{uncorr}$</th>
<th>$\delta_{losses}$</th>
<th>$\delta_{corr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15,700</td>
<td>0.006890</td>
<td>0.00014</td>
<td>0.00675</td>
</tr>
<tr>
<td>13,900</td>
<td>0.004930</td>
<td>0.00014</td>
<td>0.00485</td>
</tr>
<tr>
<td>12,370</td>
<td>0.003200</td>
<td>0.00014</td>
<td>0.00306</td>
</tr>
<tr>
<td>10,780</td>
<td>0.002180</td>
<td>0.00014</td>
<td>0.00204</td>
</tr>
<tr>
<td>9,050</td>
<td>0.001570</td>
<td>0.00014</td>
<td>0.00143</td>
</tr>
<tr>
<td>7,380</td>
<td>0.000995</td>
<td>0.00014</td>
<td>0.00086</td>
</tr>
<tr>
<td>5,970</td>
<td>0.000737</td>
<td>0.00014</td>
<td>0.00060</td>
</tr>
<tr>
<td>2,860</td>
<td>0.000333</td>
<td>0.00014</td>
<td>0.00019</td>
</tr>
<tr>
<td>2,210</td>
<td>0.000216</td>
<td>0.00014</td>
<td>0.00010</td>
</tr>
<tr>
<td>1,090</td>
<td>0.000164</td>
<td>0.00014</td>
<td>0.00002</td>
</tr>
</tbody>
</table>
### Table 9

Results of Test 16, Non-Uniform Torsion of a Solid 304 Stainless Steel Rod (Case II)

<table>
<thead>
<tr>
<th>$\tau_2$</th>
<th>$\delta_{uncorr.}$</th>
<th>$\delta_{losses}$</th>
<th>$\delta_{corr.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19,200</td>
<td>0.00256</td>
<td>0.00014</td>
<td>0.00262</td>
</tr>
<tr>
<td>12,800</td>
<td>0.00173</td>
<td>0.00014</td>
<td>0.00159</td>
</tr>
<tr>
<td>9,070</td>
<td>0.00117</td>
<td>0.00014</td>
<td>0.00103</td>
</tr>
<tr>
<td>6,220</td>
<td>0.00094</td>
<td>0.00014</td>
<td>0.00080</td>
</tr>
<tr>
<td>5,500</td>
<td>0.00080</td>
<td>0.00014</td>
<td>0.00076</td>
</tr>
<tr>
<td>4,620</td>
<td>0.00064</td>
<td>0.00014</td>
<td>0.00050</td>
</tr>
<tr>
<td>3,680</td>
<td>0.00067</td>
<td>0.00014</td>
<td>0.00053</td>
</tr>
<tr>
<td>2,820</td>
<td>0.00045</td>
<td>0.00014</td>
<td>0.00031</td>
</tr>
<tr>
<td>1,830</td>
<td>0.00038</td>
<td>0.00014</td>
<td>0.00024</td>
</tr>
<tr>
<td>1,060</td>
<td>0.00027</td>
<td>0.00014</td>
<td>0.00013</td>
</tr>
</tbody>
</table>

### Table 10

Results of Test 25, Non-Uniform Bending of a Solid 304 Stainless Steel Rod (Case III)

<table>
<thead>
<tr>
<th>$\sigma_2$</th>
<th>$\delta_{uncorr.}$</th>
<th>$\delta_{losses}$</th>
<th>$\delta_{corr.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25,600</td>
<td>0.00790</td>
<td>0.00300</td>
<td>0.00490</td>
</tr>
<tr>
<td>20,500</td>
<td>0.00630</td>
<td>0.00300</td>
<td>0.00330</td>
</tr>
<tr>
<td>17,050</td>
<td>0.00560</td>
<td>0.00300</td>
<td>0.00260</td>
</tr>
<tr>
<td>14,550</td>
<td>0.00500</td>
<td>0.00300</td>
<td>0.00200</td>
</tr>
<tr>
<td>12,700</td>
<td>0.00410</td>
<td>0.00300</td>
<td>0.00110</td>
</tr>
<tr>
<td>11,280</td>
<td>0.00391</td>
<td>0.00300</td>
<td>0.00091</td>
</tr>
<tr>
<td>10,050</td>
<td>0.00383</td>
<td>0.00300</td>
<td>0.00083</td>
</tr>
<tr>
<td>5,700</td>
<td>0.00354</td>
<td>0.00300</td>
<td>0.00055</td>
</tr>
<tr>
<td>3,040</td>
<td>0.00332</td>
<td>0.00300</td>
<td>0.00032</td>
</tr>
<tr>
<td>1,385</td>
<td>0.00225</td>
<td>0.00300</td>
<td>0.00013</td>
</tr>
</tbody>
</table>
### TABLE 11

Results of Test 15B, Uniform Torsion of a Hollow 2024S-T3 Aluminum Alloy Rod (Case I)

<table>
<thead>
<tr>
<th>$\tau_{\text{uniform}}$</th>
<th>$\delta_{\text{uncorr.}}$</th>
<th>$\delta_{\text{losses}}$</th>
<th>$\delta_{\text{corr.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11,030</td>
<td>0.002500</td>
<td>0.000114</td>
<td>0.00236</td>
</tr>
<tr>
<td>8,770</td>
<td>0.000920</td>
<td>0.000114</td>
<td>0.00078</td>
</tr>
<tr>
<td>7,580</td>
<td>0.000460</td>
<td>0.000114</td>
<td>0.00032</td>
</tr>
<tr>
<td>6,340</td>
<td>0.000336</td>
<td>0.000114</td>
<td>0.00019</td>
</tr>
<tr>
<td>5,200</td>
<td>0.000272</td>
<td>0.000114</td>
<td>0.00013</td>
</tr>
<tr>
<td>4,500</td>
<td>0.000252</td>
<td>0.000114</td>
<td>0.00011</td>
</tr>
<tr>
<td>3,740</td>
<td>0.000192</td>
<td>0.000114</td>
<td>0.00005</td>
</tr>
<tr>
<td>2,900</td>
<td>0.000173</td>
<td>0.000114</td>
<td>0.00003</td>
</tr>
<tr>
<td>2,120</td>
<td>0.000134</td>
<td>0.000114</td>
<td>—</td>
</tr>
<tr>
<td>1,680</td>
<td>0.000110</td>
<td>0.000114</td>
<td>—</td>
</tr>
<tr>
<td>1,315</td>
<td>0.000080</td>
<td>0.000114</td>
<td>—</td>
</tr>
</tbody>
</table>

### TABLE 12

Results of Test 15B, Non-Uniform Torsion of a Solid Aluminum Alloy Rod (Case II)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\delta_{\text{uncorr.}}$</th>
<th>$\delta_{\text{losses}}$</th>
<th>$\delta_{\text{corr.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7,630</td>
<td>0.001635</td>
<td>0.000114</td>
<td>0.00150</td>
</tr>
<tr>
<td>6,830</td>
<td>0.000966</td>
<td>0.000114</td>
<td>0.00083</td>
</tr>
<tr>
<td>6,250</td>
<td>0.000850</td>
<td>0.000114</td>
<td>0.00071</td>
</tr>
<tr>
<td>5,620</td>
<td>0.000655</td>
<td>0.000114</td>
<td>0.00051</td>
</tr>
<tr>
<td>4,840</td>
<td>0.000560</td>
<td>0.000114</td>
<td>0.00042</td>
</tr>
<tr>
<td>4,060</td>
<td>0.000466</td>
<td>0.000114</td>
<td>0.00032</td>
</tr>
<tr>
<td>2,820</td>
<td>0.000405</td>
<td>0.000114</td>
<td>0.00026</td>
</tr>
<tr>
<td>2,090</td>
<td>0.000311</td>
<td>0.000114</td>
<td>0.00020</td>
</tr>
<tr>
<td>1,635</td>
<td>0.000301</td>
<td>0.000114</td>
<td>0.00021</td>
</tr>
<tr>
<td>1,290</td>
<td>0.000301</td>
<td>0.000114</td>
<td>0.00016</td>
</tr>
<tr>
<td>995</td>
<td>0.000272</td>
<td>0.000114</td>
<td>0.00013</td>
</tr>
</tbody>
</table>
### TABLE 13

Results of Test 24, Non-Uniform Bending of a Solid 2024-T3 Aluminum Alloy Rod (Case III)

<table>
<thead>
<tr>
<th>$\sigma_2$</th>
<th>$\delta_{uncorr.}$</th>
<th>$\delta_{losses}$</th>
<th>$\delta_{corr.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18,750</td>
<td>0.00690</td>
<td>0.00600</td>
<td>0.0090</td>
</tr>
<tr>
<td>16,400</td>
<td>0.00640</td>
<td>0.00600</td>
<td>0.0040</td>
</tr>
<tr>
<td>14,000</td>
<td>0.00590</td>
<td>0.00600</td>
<td>---</td>
</tr>
<tr>
<td>11,700</td>
<td>0.00590</td>
<td>0.00600</td>
<td>---</td>
</tr>
<tr>
<td>9,200</td>
<td>0.00660</td>
<td>0.00600</td>
<td>0.0060</td>
</tr>
<tr>
<td>6,860</td>
<td>0.00640</td>
<td>0.00600</td>
<td>0.0040</td>
</tr>
<tr>
<td>4,260</td>
<td>0.00620</td>
<td>0.00600</td>
<td>0.0020</td>
</tr>
<tr>
<td>2,410</td>
<td>0.00480</td>
<td>0.00600</td>
<td>---</td>
</tr>
<tr>
<td>1,463</td>
<td>0.00520</td>
<td>0.00600</td>
<td>---</td>
</tr>
<tr>
<td>917</td>
<td>0.00380</td>
<td>0.00600</td>
<td>---</td>
</tr>
</tbody>
</table>
\[ \delta_1 = \left( \frac{4 \cdot \kappa \cdot G}{M+2} \right) (1.2 \cdot \tau^2 - 0.8 \cdot \tau^2) \tau^{M-2} \]

WHERE:
- \( \kappa = 5.66 \times 10^{-6} \)
- \( M = 2.793 \)
- \( \tau = 1.81 \times 10^{-12} \)
- \( G = 11.5 \times 10^6 \)

**Figure 8** - Experimental results for Case I, uniform torsion, hollow 1018 steel specimen, test no. 19.
\[ \varepsilon = \left( \frac{4K_2G}{M+2} \right) (\tau_2)^{M-2} \]

WHERE:
- \( M = 2.793 \)
- \( K_2 = 1.81 \times 10^{-12} \)
- \( G = 11.50 \times 10^6 \)

FROM CASE I

\( \tau \) - UNIFORM SHEAR STRESS AT OUTER FIBER, PSI

FIGURE 9 - EXPERIMENTAL AND ANALYTICAL RESULTS FOR CASE II, NON-UNIFORM TORSION, SOLID 1018 STEEL SPECIMEN, TEST NO. 14C.
ANALYTICAL EXPRESSION BASED ON A DISTORTION ENERGY HYPOTHESIS:

\[ \sigma_{III} = \frac{8AE}{\sqrt{m(m+2)(2m+1)}} \left( \frac{m}{m+1} \right)^{m-2} \]

WHERE:
- \( m = 2.793 \)
- \( k = 1.34 \times 10^{-12} \)
- \( E = 30.0 \times 10^6 \)

FROM CASE I

○ - EXPERIMENTAL POINTS, CORRECTED FOR WINDAGE AND WIRE LOSSES

ANALYTICAL EXPRESSION BASED ON A MAXIMUM SHEAR STRESS HYPOTHESIS:

\[ \sigma_{III} = \frac{8AE}{\sqrt{m(m+2)(2m+1)}} \left( \frac{m}{m+1} \right)^{m-2} \]

\( \sigma_2 \) - BENDING STRESS AT OUTER FIBER, PSI

FIGURE 10 - EXPERIMENTAL AND ANALYTICAL RESULTS FOR CASE III, NON-UNIFORM BENDING, SOLID 1018 STEEL SPECIMEN, TEST NO. 23.
\[ \Delta = \left( \frac{4\mu G}{M+2} \right) (1.2M^{-2} - 0.8M^{-2}) \]

WHERE:
\[ M = 3.816 \]
\[ \mu = 1.45 \times 10^{-17} \]
\[ G = 1.25 \times 10^{6} \]

FIGURE II - EXPERIMENTAL RESULTS FOR CASE I, UNIFORM TORSION, HOLLOW 304 STAINLESS STEEL SPECIMEN, TEST NO. 18.
\[ \delta'' = \left( \frac{4E}{(m+2)} \right) (\tau')^{m-2} \]

\[ \tau' = \frac{G}{2\pi R} \]

WHERE:

\[ m = 3.816 \]
\[ \kappa = 1.45 \times 10^{-17} \]
\[ G = 125 \times 10^6 \]

\( \tau' \) - SHEAR STRESS AT OUTER FIBER, PSI.

\( \delta'' \) - LOGARITHMIC DECREMENT

FIGURE 12 - EXPERIMENTAL AND ANALYTICAL RESULTS FOR CASE II, NON-UNIFORM TORSION, SOLID 304 STAINLESS STEEL SPECIMEN, TEST NO 16.
ANALYTICAL EXPRESSION BASED ON A
DISTORTION ENERGY HYPOTHESIS.

\[ \delta = \frac{8 \cdot E}{\sqrt{m+1} (m^2 + \frac{1}{2})} \cdot \frac{r}{(m^2 + 1)} \cdot \sigma_2^{m-2} \]

WHERE:
- \( m = 3.816 \)
- \( \varepsilon = 1.45 \times 10^{-17} \)
- \( E = 2.8 \times 10^6 \)

- EXPERIMENTAL POINTS, CORRECTED FOR WINDAGE AND WIRE LOSSES

ANALYTICAL EXPRESSION BASED ON A MAXIMUM SHEAR STRESS HYPOTHESIS.

\[ \delta = \frac{8 \cdot E}{\sqrt{m+2} \cdot 2^m} \cdot \frac{r}{(m^2 + 1)} \cdot \sigma_2^{m-2} \]

\( \sigma_2 \): BENDING STRESS AT OUTER FIBER, PSI.

FIGURE 13 - EXPERIMENTAL AND ANALYTICAL RESULTS FOR CASE III, NON-UNIFORM BENDING, SOLID 304 STAINLESS STEEL SPECIMEN, TEST NO. 25.
$\log_{10} T = \frac{4 \mu G}{m^2} (1.2 m^{-2} - 0.8 m^{-2}) \gamma m^{-2}$

Where:

$\mu = 466$

$G = 1.10 \times 10^{-20}$

$\gamma = 3.85 \times 10^6$

Figure 14 - Experimental results for Case I, uniform torsion, hollow 2024-T3 aluminum alloy specimen, test no. 20.
Figure 15: Experimental and Analytical Results for Case II, Non-Uniform Torsion, Solid 2024-T4 Aluminum Alloy Specimen, Test No. 15B.

\[ \gamma = 4.6 \times 10^{-2} \text{ from Case I} \]

Where:

- \( \gamma \) = shear strain
- \( \sigma \) = shear stress at outer fiber
- \( \sigma = 385 \times 10^6 \text{ psi} \)

\[ \sigma = \left( \frac{4K}{2^{\gamma}} \right) \left( \frac{L}{L_m} \right)^{m-2} \]
ANALYTICAL EXPRESSION BASED ON A DISTORTION ENERGY HYPOTHESIS:

\[ \sigma = \frac{8E}{\sqrt{\pi}(m+2)^{m+2}} \left( \frac{m}{m+2} \right)^{m+2} \frac{1}{r} \left( \frac{m}{m+2} + 1 \right) \]

WHERE:

\[ m = 4.56 \]
\[ E = 10.0 \times 10^6 \]

\[ E = 11.0 \times 10^6 \]

FROM CASE I

FIGURE 16 - EXPERIMENTAL AND ANALYTICAL RESULTS FOR CASE II, NON-UNIFORM BENDING, SOLID 2024S-T4 ALUMINUM ALLOY SPECIMEN, TEST NO. 24.
CHAPTER VIII
CONCLUSIONS AND RECOMMENDATIONS

COMPARISON OF THE EXPERIMENTAL
RESULTS WITH THE ANALYTICAL
RESULTS

Case I. Uniform Torsion of a
Hollow Cylindrical Specimen

A study of Figures 8, 11, and 14 shows that little difficulty
was experienced in fitting equation (13) to the experimental data
for the materials used in this investigation.

Case II. Non-Uniform Torsion of
a Solid Cylindrical Specimen

A study of Figures 9, 12, and 15 shows some agreement between
the experimental results and equation (15). The greatest deviation
exists for 304 stainless steel, Figure 12, where the slope of the
experimental results is quite different from the slope of the line
which represents equation (15). At the extremities of this plot,
where the deviations are the greatest, the predicted results would
be off by a factor of 2 to 1. In the center region of this plot the
difference between the theoretical expression and the experimental
results is small; therefore, reasonable predictions of the damping
loss could be made for shear stresses ranging from 3000 psi to
8000 psi. Figure 9, for 1018 steel shows that the damping constant m

67
is correct, but it also shows that the other damping constant $k$ is in error. Figure 15, for 2024-T3 aluminum alloy, shows fair agreement between theoretical and experimental results for stress levels above 4000 psi. Below this level the comparison deviates sharply.

Case III. Non-Uniform Bending of a Solid Cylindrical Specimen

A casual study of Figures 10, 13, and 16 shows some agreement between experimental results and the theoretical developed equation (17). However, a close study of this situation for Figures 13 and 16, i.e., 304 stainless steel and 2024-T3 aluminum alloy, reveals that the magnitude of the overall correction factor for windage, suspension wire, and strain-gage lead-wire losses is on the same order of magnitude as the value of $\delta$ being measured. Thus, the results are obscure, and it is perhaps remarkable that the trend of the points appears to justify the hypothesis. This situation does not exist in Figure 10, 1018 steel in bending. Here the magnitude of $\delta$ was an order of magnitude larger than the overall correction factor, and the trend of the points quite definitely justifies the hypothesis.
REASONS FOR DEVIATION BETWEEN
EXPERIMENTAL AND ANALYTICAL
RESULTS

A comparative study of Figures 11 and 12, also Figures 14 and
15, show for both stainless steel and aluminum alloy that the slopes
of the experimental plots for the hollow torsional case were quite
different from those obtained for the solid torsional specimens.
This deviation could be the result of different crystallographic
structures, i.e., the hollow specimen versus the solid specimen.
These two types of specimens, though nominally of the same material,
were brought to their final shapes by different wrought processes.

Figure 9, 1018 steel shows that the damping constant \( m \) is cor-
rect, but the other damping constant \( k \) is in error. The sources of
this deviation could be (1) experimental error in determining the
correction factor applied to \( \delta \), (2) the fundamental error involved
in assuming that the hollow torsional case represents a uniform shear
stress situation, and (3) the difference in the metallographic struc-
ture that actually exists between the solid 1018 specimens and the
hollow 1018 specimen used in this investigation. The error involved
in \( k \) is probably due to the first two factors because the metallo-
graphic structural difference would perhaps affect \( m \) as well as \( k \).

The large deviation between theory and experiment in Figure 15
at low stress levels, below 4000 psi, might be attributed to errors
in measurement. It should be noted that in this region the magni-
tudes of the logarithmic decrement were very small; therefore, the
plot is very sensitive to errors in measurement.
In Figure 10 for 1018 steel in bending it is interesting to note that the line representing equation (17) falls between the uncorrected results and the corrected results. Since the correction factor applied to \( \delta \) is quite crude, it could be in error on the high side, if it were, then the theoretical and experimental results actually are in better agreement than indicated.

Six of the nine plots, i.e., Figures 9, 10, 11, 13, 14, and 15, show an accelerated rate of hysteretic energy dissipation at high stress levels. This increased rate could be the result of regenerative multiplication of dislocations. It is generally considered that these dislocations processes are stress sensitive, and that they increase with an increased stress level.

**GENERAL CONCLUSIONS**

Two general conclusions were made:

1. The results substantiate the analytically developed equations for hollow and solid torsional pendulums, Cases I and II.

2. The results indicate only that the theoretical development and the distortion energy hypothesis may be correct for non-uniform bending, Case III.
RECOMMENDATIONS
FOR FUTURE STUDY

An important recommendation would be to devise a suspension scheme to reduce the unwanted damping in the bending set-up. This might be a difficult task because it is believed that a significant portion of this damping was due to Coulomb type damping either at the specimen end-flanges or in the suspension-wire connections.

Also, in the bending set-up the strain-gage lead wires vibrated and caused a wooden beam, which positioned these lead wires, to sympathetically vibrate. The amplitude of this vibration might be reduced by leading out these wires exactly at one of the nodal points and running them along one of the pendulum suspension wires.

Very few improvements would be necessary or warranted for the torsional pendulum. This apparatus was nearly free of unwanted friction. However, two changes would improve the general accuracy of the results obtained. First, the hollow specimens could be manufactured from thinner-walled tubing, thus approximating more closely the state of uniform stress. However, this change may not be as easy as it might appear, because some difficulty would be expected in welding the end-flanges to the thin-walled tubular specimens. Second, a smaller diameter wire might be used for the suspension wires. This would reduce the external hysteretic losses to a minimum. Also, this second measure would lower a very slight second-degree-of-freedom effect noted in this system. This low frequency mode was caused by a windup in the suspension wires, and its final effect was a slight scalloping of the strain versus time records.
Another important recommendation would be to investigate the effect of slight material structural differences on the magnitude of the damping exponent. This would perhaps justify or nullify the belief that the difference in slope between the hollow torsional case and the solid torsional case was due to this factor.
BIBLIOGRAPHY

Books


Bulletins


Periodicals


I, Truman Gray Foster, was born in Bedford, Indiana, February 3, 1920. I received my secondary school education in the public schools of Millersburg, Ohio, and my undergraduate training at The Ohio State University, which granted me the Bachelor of Science degree in Mechanical Engineering in 1948.

I immediately began my graduate studies and also accepted an Instructorship at The Ohio State University. In June, 1950, I received my Master of Science degree in Mechanical Engineering with a major in machine design.

From 1950 until 1954, I was employed at the University as a research associate and group leader on several projects in the field of fatigue of metals. Meanwhile, I began course work toward a doctorate and also taught part time in the machine design area. In 1954, I withdrew from the research projects so that I could teach full time and also devote more effort toward the completion of the Doctor of Philosophy degree.