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EVIDENCES FOR DIRECT INTERACTIONS AT LOW ENERGIES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Harry Fred Bewsher Jr., B.Sc., M.Sc.

The Ohio State University

1960

Approved by

[Signature]

Adviser

Department of Physics and Astronomy
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I. Introduction

Recently the results of many nuclear scattering reactions have indicated that for those energies of the incident beam which are able to leave the final target nucleus in any of many excited states, a significant portion of the reaction can often be described as occurring by means of a direct-interaction mechanism. In a very general way the analysis of direct reactions requires the consideration of only a small number of the total number of degrees of freedom in order to make predictions which are in good agreement with experiment. Whereas, the analysis of compound-nucleus reactions requires the consideration of many intermediate degrees of freedom. At present, there is considerable research effort to determine which degrees of freedom are important for the different reactions and what approximations are feasible in the description of the experimental results.

If a \((p,p')\) reaction proceeds by way of a compound-nucleus mechanism, the manner of decay of the compound state is independent of the method of formation, and depends only on the energy and upon the statistical weights and barrier penetration probabilities for the
different decay alternatives. If many of the levels of the compound nucleus are excited, the angular distribution of the emitted protons would be symmetric about the plane perpendicular to the incident beam direction. The interference between levels of opposite parity average out. The spectrum of energies of the emitted protons would be Maxwellian, except for the effects of the barrier penetration. All of these predictions have been found to be violated to a large degree in many experiments.

The angular distribution of the inelastically scattered protons, which leave the final target nucleus in one of its low-excited states, often approximates very closely the predictions of the direct reaction theory. In formulating the wave function which describes such collisions, it is convenient to use a function having a compound nucleus form plus a direct reaction part as a small perturbation. Several direct reaction theories are summarized in the next chapter. Each of these theories predicts that the angular distribution function of the inelastically scattered protons should have diffraction-like oscillations.

Another method of determining the reaction mechanism for \((p,p')\) scattering is to study the angular correlation between the inelastically
scattered protons and their associated decay gammas. If this correlation depends strongly on the angle of detection of the inelastically scattered protons, then a large portion of the reaction can be described by a direct reaction process. In the next chapter, the angular correlation theory is developed for the case in which the ground state of the target nucleus has spin 0 and the first excited state has spin 2.

The Ohio State University cyclotron accelerates protons to approximately 6.5 Mev. For light nuclei the density of excited levels is small in this energy region; therefore, the compound nucleus which is formed by the absorption of such incident protons by a light nucleus has available only a few channels of decay. Hence, it might be expected that the formation of one of the low-excited states of some nucleus by a \((p,p')\) reaction would proceed predominantly by a compound nucleus reaction. Lackner, Dell and Hausman measured the angular correlation between the gammas emitted from


the 1.38 Mev state of Mg which was excited by a \((p,p')\) reaction. The angular correlation strongly indicated the presence of some direct interaction.

(2) Seward measured the angular correlations between

protons leaving the target nuclei in their first excited state and their associated gammas. He used $^{24}\text{Mg}$ and $^{52}\text{Cr}$ targets. For both targets, there appeared to be a significant amount of direct interaction except for the cases in which the incident proton beam energy was less than the coulomb barrier energy.

It was decided to investigate the possible presence of direct $(p,p')$ reactions using various light-nuclei targets. The results of the angular correlation runs using the following targets; $^{6}\text{Li}$, $^{7}\text{Li}$, $^{32}\text{S}$, $^{28}\text{Si}$, $^{52}\text{Cr}$, $^{20}\text{Ne}$, $^{27}\text{Al}$ and $^{12}\text{C}$, are contained in this thesis. The energy of the incident proton beam was varied in the $^{28}\text{Si}$ experiments, to determine the effect of a possible resonance in the compound state formed by the absorption of an incident proton by a $^{28}\text{Si}$ nucleus.
II Theory

Introduction

Much recent evidence indicates that for \((p,p')\), \((p,d)\), \((\gamma,\gamma')\), \((n,p)\) etc. reactions, in which the final residual target nucleus is left in one of its low-lying excited states, the reaction often proceeds predominantly by a direct-interaction mechanism as opposed to a compound-nucleus mechanism. In many of the experiments described above, the differential cross section of the inelastically scattered particles was found not to be symmetric about 90°, but rather to be peaked in the forward direction and possessing diffraction-like oscillations. The energy distribution of the inelastically scattered particles was often found to contain many more high-energy particles than the Maxwellian energy spectrum. These are all evidences of a direct-interaction mechanism and contrary to the predictions of the compound-nucleus interaction theory.

Theory of Compound-Nucleus Reactions

In a compound-nucleus reaction, when an incident nuclear particle bombards a target nucleus the two join to form a compound nucleus which may or may not be in one of its excited states depending on the kinetic energy of the incident particle. In the
compound nucleus there are assumed to be strong interactions between all the nucleons; consequently, the incident particle loses its identity and the total energy and momentum of the compound nucleus is shared by all the nucleons in some complicated manner. This compound state retains no "memory" of how it was formed. Its decay depends only on the properties of the compound state, i.e., spin, parity, and energy.

The angular distribution of the emitted particles would be symmetrical about a plane perpendicular to the incident beam direction if only one level of the compound nucleus is excited. This can be seen in the following way. The parity of the wave function, $\psi$, for the reaction is definite and is equal to the parity of the single excited level of the compound nucleus. The parity of $\psi$ equals the parity of the residual nucleus wave function times the parity of the emitted particle wave function. Assuming the parity of the wave function which describes the residual nucleus is definite, the parity of the emitted particle wave function is also definite. Hence, the angular distribution of the emitted particles is symmetric about the plane perpendicular to the incident proton direction. If two or more levels
of a compound nucleus are excited, the distribution function of the emitted particle may be asymmetric about 90° (center of mass coordinates) because of the interference of terms of different parity. However, if a large number of compound nucleus levels are excited then the interference terms are expected to cancel out and again the distribution of the emitted particles would be symmetric about 90°.

**Theory of Direct Reactions**

In a direct reaction, the incident particle interacts strongly with only one (or a small number) of the target nucleons, and the emitted particle immediately leaves the residual target nucleus. Below is an outline of a direct interaction theory which was formulated by S.T. Butler. The theory will be applied to the (p,p') case since this is the interest of this thesis.

Let a nucleus \( \hat{X} \) be bombarded with protons of well defined energy and direction. Many of these protons may be captured into one or more levels of the compound nucleus which subsequently can decay in a variety of ways. We will set the first approximation, labelled \( \psi_r \), to the total wave function of the total system equal to that portion of the reaction:

which proceeds according to the compound-nucleus reaction theory. Let us now put $\psi$ in the form

$$\psi = \psi_{el} + \phi$$  \hspace{1cm} (2.1)$$

where $\psi_{el}$ is that portion of $\psi$ which describes the elastic scattering and $\phi$ represents the remainder of all scattering processes of $\psi$. Let us assume that there will always be a contribution from the $\phi$ portion, i.e., inelastic compound nucleus reactions. This will occur if the energy of the incident proton forms a level of excitation of the compound nucleus which has many modes of decay available. An important observation which can be made about the function $\phi$ is that if the energy of the incident protons is sufficiently high so that there exists a large spread in the energy of the protons which decay through the various allowed channels, then $\phi$ will be made up mainly of wave functions which describe outgoing protons of such energies that the final target nuclei will be left in their highly excited states. The reason for this is that the density of the levels of the final nucleus is in general very much greater for high excitations than for low; hence, there are many more channels of decay available to the compound nucleus which lead to a
highly excited final target nucleus than to one of the low excited final states. In particular, it would be expected for high energy incident protons that a compound nucleus reaction would contribute only slightly to the elastic scattering.

Direct reaction processes which do not proceed by way of a compound nucleus formation will form the second approximation, $\psi_2$, to the total wave function. As an example, an incident proton might interact strongly with a single surface nucleon of the initial target nucleus with the result that the incident proton will be either elastically or inelastically scattered. It is quite possible that in general the number of direct processes are small enough compared to the number of compound nucleus processes so that perturbation methods may be used in the calculation of the $\psi_2$. However, one important aspect of these processes should be noted. Direct reactions, such as described above, are very likely to involve little energy exchange and to leave the final nucleus in a low-lying state. Thus direct interactions are most likely to produce transitions to just those states which are little represented in $\psi_1$.

The differential cross section for this (p,p') reaction is
\[
\frac{d\sigma}{d\Omega}(k_p, k'_p) = \frac{M_p^2}{(2\pi\hbar^2)^2} \frac{k'_p}{k_p} \sum_{\mathcal{A}_V} |I|^2. \tag{2.2}
\]

\[\sum_{\mathcal{A}_V}\] implies a summation over final spins and an average over initial spins.

\(I\), the interaction matrix, is, according to perturbation theory, the following

\[
I(k_p, k'_p) = \int d\tau \psi_i(k_p; P) \mathbf{V}_{pP} \psi^*_{p,t}(k'_p; P) \tag{2.3}
\]

\(\mathbf{V}_{pP}\) is a sum of effective two-particle interactions between the incident proton and the nucleons of the target nucleus.

\(\psi_{el,t}\) is the function describing the elastic scattering of protons of wave-vector \(k'_p\) by the final nucleus in state \(t\).

\(\psi_{el,t}\) will be put into the form

\[
\psi_{el,t} = \mathcal{N}_t (\mathcal{S}) \psi_{p,t}(k'_p, \mathcal{S}_p) \tag{2.4}
\]

where \(\mathcal{N}_t\) is the wave function of the final nucleus in state \(t\).

\(\psi_{p,t}(k'_p, \mathcal{S}_p)\) is the wave function of the scattered proton having a wave vector of \(k'_p\).
Since the $\phi$ in $\Psi_i$ will be expected to contribute only slightly to the interaction matrix, we have

$$I(k_p, k'_p) = \int \mathcal{D}T \Psi_{el} \left[ \Psi_{eff}^{*} \right] \Psi_{el, c} =$$

$$\int \mathcal{D}T \mathcal{N}_0(T) \mathcal{N}^{*}_t(T) \left[ \Psi_{eff}^{*}(k_p, \nu_p) \Psi_{el, c}^{*}(k'_p, \beta_p) \right] \quad (2.5)$$

where $\Psi_{el}$ has been approximated by

$$\Psi_{el} = \mathcal{N}_0(T) \Psi_{p}(k_p, \nu_p), \quad (2.6)$$

where $\mathcal{N}_0(T)$ is the wave function of the ground state of the nucleus, and $\Psi_{p}(k_p', \nu_p)$ is the wave function of the incident proton having a wave vector of $k_p$.

In order to simplify the evaluation of the interaction integral, spin functions will not be introduced into the wave functions. The spins will be considered only in so far as they determine selection rules.

Let us now consider merely the direct interaction $\Psi_{eff}$ between the proton and one nucleon of the initial nucleus.

Thus,

$$I(k_p, k'_p) = \int \mathcal{D}T \mathcal{D} \nu_p \mathcal{D} \nu'_p \mathcal{N}_0(T) \mathcal{N}_t^{*}(T) \Psi_{eff}^{*}(k_p, \nu_p) \Psi_{el, c}^{*}(k'_p, \beta_p) \quad (2.7)$$

where $\mathcal{F}$ represents the coordinates of all the nucleons of the target nucleus except the coordinates of the nucleon interacting with the incident proton.

It is convenient to make use of the fact that the zero-range approximation for the proton-nucleon interaction
is known to be a very good approximation in direct
interactions for the energies usually employed
experimentally.

Then,
\[ I(k_p, k_{p'}) = \int \delta^3(q) \, dq \, N_v(\hat{q}) \, N_u(\hat{q}) \, \frac{2}{r^2} \sqrt{r^2(k_r, \alpha_n)} \times V_0(\alpha_n) \, |\psi_p(k_p, \alpha_n)|^2 \, |\psi_{p'}(k_{p'}, \alpha_n)|^2. \quad (2.8) \]

It is now convenient to make the following expansions:
\[ N_v(\hat{q}) = N_v(\hat{q}) = \sum_{s, s'} U_s(\hat{q}) \, F_s(\alpha_n), \]
\[ N_u(\hat{q}) = N_u(\hat{q}) = \sum_{s, s'} U_{s'}(\hat{q}) \, G_{s'}(\alpha_n). \quad (2.9) \]

The integration over \( \hat{q} \) yields
\[ I(k_p, k_{p'}) = \sum_{s, s'} \int \delta(\alpha_n) \, F_s(\alpha_n) \, G_{s'}^*(\alpha_n) \, V_0(\alpha_n) \times |\psi_p(k_p, \alpha_n)|^2 \, |\psi_{p'}(k_{p'}, \alpha_n)|^2. \quad (2.10) \]

Let us expand \( F_s \) and \( G_{s'} \) in spherical harmonics.

\[ F_s(\alpha_n) = \sum_{l, m} f_s \, Y_{l, m}(\Theta_n, \Phi_n), \]
\[ G_{s'}(\alpha_n) = \sum_{l', m'} g_{s'} \, Y_{l', m'}(\Theta_n, \Phi_n). \quad (2.11) \]

where \( l \) and \( M \) will be limited by selection rules.

Now let us assume that there is one predominating
term in the summation over \( S \). This is certainly
true for the pure shell model in which the nucleon
responsible for the scattering will merely have its
single-particle state changed, without affecting
the state of the core. Also we shall assume that the
summations over \( l \) and \( M \) are limited to one value
each, and in particular we shall set \( l = 0 \), thus \( M = l \)
(say) is limited by selection rule.

\[ \left| J_i + J_f \right| \leq l \leq J_i + J_f \quad (2.12) \]
$J$ represents the spin of a target nucleus. In addition, $l$ is even or odd depending on whether there is a parity change or not. Also, $m_l = m(\text{say})$ will be limited to one value depending on the initial and final spin projections.

Under these conditions

$$I(k_p, k'_p) = \int d\rho_n \, f_3(\rho_n) \, q_s(\rho_n, l, m) \, Y_{l,m}(\Theta_n, \Phi_n)$$

$$\times V_0(\rho_n) \, \psi_0(k_p, \rho_n) \, \psi_{l,m}^*(k'_p, \rho_n). \quad (2.13)$$

Due to the short mean free path for capture of the protons into compound nucleus states, the integral (2.13) will receive contributions only from values of $\rho_n > R_0$ out in the "tail" of the interacting nucleon density distribution.

We shall take

$$f_3(\rho_n) = A \, h_0(i \kappa \rho_n)$$

$$q_s(\rho_n, l, m) = B(l, m) \, h_1(i \kappa' \rho_n), \quad (2.14)$$

where $(\frac{\hbar^2}{2M_n}) \kappa^2$ is the binding energy of the interacting nucleon in the initial nucleus, and $(\frac{\hbar^2}{2m_f}) \kappa'^2$ is the binding energy in the final nucleus, and where

$h_1$ is the spherical Hankel function of the first kind. Equation (2.13) becomes

$$I(k_p, k'_p) = V_0 \, A \, B(l, m) \, \int d\rho_n \, h_0(i \kappa \rho_n) \, h_1(i \kappa' \rho_n)$$

$$\times Y_{l,m}(\Theta_n, \Phi_n) \, \psi_0(k_p, \rho_n) \, \psi_{l,m}^*(k'_p, \rho_n) \quad (2.15)$$

if $V_0(\rho_n)$ is assumed to be independent of $\rho_n$.

Since the portion of the total reaction which proceeds by direct interaction is very small for
Let \( n_c < R_o \), a lower limit of integration of \( n_c = R_o \) will be placed on the integral (2.15). The wave functions \( \psi_p \) and \( \psi_{p,t} \) will be assumed to be plane waves for \( n_c > R_o \).

We now obtain the result

\[
I(k_p, k'_p) = V_o A B(l, m) \int_{n_c \geq R_o} d \rho_c e^{i \Omega \rho_c} Y_{lm}(\Theta, \Phi) \tag{2.16}
\]

where \( \Omega = k_p - k'_p \)

and \( Y_{lm} \) is the spherical Bessel function. The Hankel functions have the form

\[
H_0(iKn_n) = e^{-Kn_n}, \quad H_1(iK'n_n) = e^{-Kn_n} \left[ \Phi + \frac{B_1}{(Kn_n)^2} + \frac{B_2}{(Kn_n)^4} + \frac{B_3}{(Kn_n)^6} + \cdots \right] \tag{2.18}
\]

Therefore, so long as \( K'R_o \) is sufficiently large we can make the following approximation

\[
H_p(iK'n_n) H_0(iK'n_n) \approx \frac{K + K'}{K K' R_n} \left[ i(K + K') \right] \hat{f} \left\{ i(K + K') \right\} \tag{2.19}
\]

If we now assume the contributions to the integral of equation (2.17) come from values of \( n_n \) sufficiently close to the lower limit, we can replace \( \frac{1}{n_n} \) by \( \frac{1}{R_o} \).

Then the integral becomes

\[
I(k_p, k'_p) = V_o A B(l, m) S_m \left\{ \frac{4\pi (2l+1)}{2} \right\} \frac{(K + K')}{K K' R_o} \int_{n_c \geq R_o} d \rho_c \hat{f} \left\{ i(K + K') \right\} j_{l}(Q \rho_c) \tag{2.20}
\]
The wave equations which \( h_x(iK\nu) \) and \( j_x(Q\nu) \) satisfy are

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d h_x(iK\nu)}{dr} \right) + \left[ -\frac{k^2 - \frac{l(l+1)}{r^2} \right] h_x(iK\nu) = 0 \tag{2.21}
\]

and

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d j_x(Q\nu)}{dr} \right) + \left[ Q^2 - \frac{l(l+1)}{r^2} \right] j_x(Q\nu) = 0 \tag{2.22}
\]

Multiplying (2.21) by \( \frac{d h_x(iK\nu)}{d r} \) and (2.22) by \( j_x(Q\nu) \) and subtracting (2.21) from (2.22), we obtain

\[
r^2 (Q^2 + k^2) h_x j_x = h_x \frac{d}{dr} \left( r^2 \frac{d j_x}{dr} \right) - j_x \frac{d}{dr} \left( r^2 \frac{d h_x}{dr} \right)
\]

\[
= r^2 h_x \frac{d^2 j_x}{dr^2} - r^2 j_x \frac{d^2 h_x}{dr^2} + 2r \left[ h_x \frac{d j_x}{dr} - j_x \frac{d h_x}{dr} \right] \tag{2.23}
\]

The Wronskian of \( h_x, j_x \) is defined as

\[
W(j_x, h_x) = j_x \frac{d h_x}{dr} - h_x \frac{d j_x}{dr} \tag{2.24}
\]

Therefore,

\[
\frac{d}{dr} \frac{d W}{dr} = j_x \frac{d^2 h_x}{dr^2} - h_x \frac{d^2 j_x}{dr^2} \tag{2.25}
\]

Thus

\[
r^2 (Q^2 + k^2) h_x j_x = -r^2 \frac{d}{dr} \left( r^2 W \right) - 2r r W
\]

\[
= - \frac{d}{dr} \left( r^2 W \right) \tag{2.26}
\]

Equation (2.20) becomes

\[
I(k, k') = V_0 \pi B(l, m) S_{m,0} \left\{ 4\pi (2l+1) \right\}^{1/2} k \left( K + K' \right) P_0 \left[ Q^2 + (K + K')^2 \right] \left[ K K' W \left[ j_x(Q\nu), h_x(iK\nu) \right] \right] \tag{2.27}
\]

It is seen immediately from equation (2.17) that if the interacting nucleon distribution "tail" is infinitely thin so that the nucleon radius is well defined, then \( I(k, k') \) will be proportional to \( j_x(Q\nu) \) and the differential cross section will be proportional to \( |j_x(Q\nu)|^2 \). The function \( j_x(x) \) has a principal maximum at \( x = 0 \) for \( l = 0 \) and this
maximum moves to larger \( \lambda \) as \( l \) increases. Thus from the angular distribution of the protons scattered from a target which has been excited to a definite level, one might be able to determine the parity and a limit for the spin of this target nucleus level.

It will later be shown from the data discussed in this thesis, that the above outlined theory is too crude. A more refined treatment of direct reactions \(^{(4)}\) has been made by Levinson and Banerjee \(^{(4)}\). In their calculations, they employed distorted wave functions to represent the incoming and scattered protons instead of making the plane wave approximation. They also considered the effects of a finite range potential. As is shown by equation (2.2) the differential cross section for the \((p,p')\) reaction is

\[
\frac{d\sigma}{d\Omega} (k_F, k_p') = \frac{M_p^2}{(2\pi\hbar^2)^2} \frac{k_{p'}}{k_F} \sum_{J^M} |I|^2. \tag{2.28}
\]

However, the interaction matrix will be put into the more general form

\[
I = \left< \psi_c^f (j_f T_f T_z) \chi (\sigma \gamma m_\gamma) I_f M_f \right| \psi_c (j_i T_i T_z) \chi (\sigma \gamma m_\gamma) I_i M_i \right>, \tag{2.29}
\]

\[
\times \int F_F (-k_F, \Delta_r) \frac{V_{in}}{2} (1 + P_{in}^X)(1 - P_{in}^T P_{in}^T) F_i (k_i \beta_n) d^3 \beta_n
\]

\[
\times \int \psi_c^f (j_f T_f T_z) \chi (\sigma \gamma m_\gamma) I_f M_f \right>,
in which \( P \) represents an exchange type operator. The interaction is assumed to be of spin independent Serber exchange type. The \( \psi \) wave functions represent the target nucleus, while the incoming or scattered proton is represented by \( F(h, m_n) \times (\sigma, \tau m) \).

\( J, T, \) and \( T_z \) are the spin, isotopic spin and component of the isotopic spin quantum numbers of the target nucleus. Also \( \sigma, \tau, \) and \( m_\tau \) are the spin, isotopic spin and component of the isotopic spin quantum numbers of the incident or scattered scattered proton.

In setting up the interaction matrix a channel-spin representation has been employed. The channel spin and channel spin component are defined in the following way

\[
I = J + \sigma
\]

\[
M = M_J + M_\tau.
\]

(2.30)

The \( F \) wave functions are expanded in terms of partial waves in the following manner

\[
F(h, m_n) = \sum_{\ell} 2V_{\ell} i^{\ell} f_{\ell}(h, m_n) Y_{\ell,0}(\Omega_n).
\]

(2.31)

Where \( f_{\ell} \) is the solution of the radial Schrödinger equation

\[
\left[ \frac{d}{d r^2} + \frac{l(l+1)}{r^2} - V(r) + \frac{\ell^2}{r^2} \right] r^2 f_{\ell}(kr) = 0.
\]

(2.32)

\( V(r) \) represents the net potential between the projectile and the nucleus. Asymptotically, \( F(kr) \) goes into an
incoming plane wave and an outgoing spherical wave.

The value of the integral of equation (2.29) is given in the Levinson and Banerjee article. In a later article by the same two authors, this theory is applied to the inelastic scattering of

protons on $^{12}$C. In order to find the radial form of the incident and scattered proton wave functions, a Woods and Saxon potential was assumed for the $V(r)$ in equation (2.32). This potential has the form

$$-V(r) = \frac{\sqrt{V} + iW}{1 + \exp\left(\frac{r-R}{\alpha}\right)}$$

where $\sqrt{V}$ is the depth of the real well, $W$ is the depth of the imaginary well, $\alpha$ is the surface thickness, and $R$ the radius parameter.

The values of the parameters in the Woods and Saxon potential were determined by the elastic scattering of electrons on $^{12}$C. The radial wave functions thus obtained were then inserted into the interaction matrix. The potential appearing in equation (2.29) which is responsible for the inelastic collision, is taken to be of the form of a Yukawa potential. The results thus obtained are in much better agreement with the $(p,p')$ scattering of $^{12}$C than those predicted by the
Butler Theory. Both theories predict that the angular distribution function of the inelastically scattered protons should have diffraction type oscillations. The Butler theory predicts that the peak occurring in the most forward direction should have the highest maximum and each succeeding peak should decrease in amplitude; whereas in the Levinson and Banerjee theory the amplitudes of the peaks may or may not decrease with increasing angle.

Chase, Willets and Edmonds treated the problem of determining the angular distribution of the scattered particles by employing a rotational-optical model. Since the Coulomb interaction between the rotating target nucleus and the incident particle was neglected, it was expected that the theory would be most appropriate if the incident particles were neutrons.

The Hamiltonian for the interacting system is taken to be

$$H = -\frac{\hbar^2}{2m} \nabla^2 + T_{\text{rot}} + V(r, \theta),$$

(2.34)

where $T_{\text{rot}}$ is the rotational energy operator for the target which is assumed to be of fixed, axially-symmetric shape. The coordinates of the incident particle relative to the nuclear principal axes
are \((r, \theta', \phi')\). The complex optical potential representing the interaction between the incident particle and the deformed target is \(V(r, \theta')\). Spin of the incident particle, and hence spin-orbit coupling, are omitted.

It is convenient to expand \(V(r, \theta')\) as

\[
V(r, \theta') = \sum_{\lambda} N_{\lambda}(r) P_{\lambda}(\cos \theta')
\]  

\[(2.35)\]

in which, for a plane-reflectionally invariant shape, \(\lambda\) takes on only even values.

We now write the wave function corresponding to an incoming partial wave of orbital angular momentum \(l\) with \(z\) projection \(m = 0\) as

\[
\psi^l = \sum_{l' I'} \frac{1}{\lambda_{l' I'}} U_{l' I'}^l(r) U_{l' I'}^{l' 0}(\theta, \phi, \Theta_1, \Theta_2)
\]

\[(2.36)\]

where \(U_{l' I'}^{l' 0}\) contains the entire angular dependence for both the incident particle and the target nucleus. \(U_{l' I'}^{l' 0}\) constitutes an eigenfunction of total angular momentum \(l\), with projection \(0\), composed of angular momentum \(I'\), the angular momentum of the target nucleus; and \(l'\), the orbital angular momentum of the incident particle. Hence

\[
U_{l' I'}^{l' 0} = \sum_{m} (l' I' m, -m | l' 0) Y_{l'm}^*(\Theta, \Phi) Y_{I' m}^{l' 0}(\Theta_1, \Theta_2)
\]

\[(2.37)\]
where \((\theta, \phi, \Theta_1, \Theta_2)\) are measured relative to the space fixed z axis which is the direction of the incident beam. The wave functions of the target nucleus are determined from the following Schrödinger equation:

\[
T_{\text{rot}} D_{mK}^{\pm}(\Theta_1, \Theta_2) = \frac{\hbar^2}{2I} [\mathcal{I}(\mathcal{I}+1) - K^2] D_{mK}^{\pm}(\Theta_1, \Theta_2) \tag{2.38}
\]

where \(I\) is the moment of inertia for the rotation. Here \(D_{mK}^{\pm}\) is the usual (unnormalized) symmetric-top wave function corresponding to a state of angular momentum \(I\) with projections \(m\) and \(K\), respectively, along the incident particle direction and the nuclear symmetry axis. In the development of this theory the value of \(K\) was restricted to zero with the initial target spin \(I = 0\) (i.e. to even-even targets). Other states of the target nucleus can have

\[
I = 0, 2, 4, 6, \ldots \tag{2.39}
\]

The normalized nuclear wave functions are

\[
\mathcal{Y}_{\pm m}(\Theta_1, \Theta_2) = \left[\frac{(2I+1)}{4I^2}\right]^{1/2} (-)^m D_{m0}^{\pm}(\Theta_1, \Theta_2). \tag{2.40}
\]

The Schrödinger equation

\[
H \psi^\ell = E \psi^\ell \tag{2.41}
\]

yields the following coupled set of differential equations for the \(U^\ell_{J', I}(\nu)\) functions

\[
\left\{ \frac{\hbar^2}{2m} \left[ -\frac{d^2}{dn^2} + \frac{l(l+1)}{n^2} \right] + \frac{\hbar^2}{2I} I'(I'+1) + N_0(\nu) \right\} \frac{d^2}{d\nu^2} U^\ell_{J', I}(\nu) \]

\[
- \frac{\hbar^2}{2I} \sum_{l} \langle l''; l''; l' | \mathcal{P}^2_{\text{red}}(\cos \Theta') l''; l' | l; l' \rangle \times U^\ell_{J, I}(\nu) = 0.
\]
For $\lambda$ greater than the maximum extent of the potential $V(\lambda, \Theta)$, the radial wave functions $U^{\ell}_{\lambda, \lambda^\prime}(\nu)$ have the form
\[ \frac{1}{\nu} U^{\ell}_{\lambda, \lambda^\prime}(\nu) = \eta_{\lambda^\prime}^{(1)}(k_0 \nu) + \eta_{\lambda^\prime}^{(2)}(k_0 \nu), \]
where the $\eta_{\lambda^\prime}^{(1)}$, $\eta_{\lambda^\prime}^{(2)}$ are the usual outgoing and incoming spherical Hankel functions, respectively.

The channel wave number $k_{\lambda^\prime}$ is defined as follows
\[ k_{\lambda^\prime} = \sqrt{\frac{2mE}{h^2} - \frac{m}{\ell} \ell(I + 1)}. \]

The differential cross section for scattering in which the target nucleus has been raised to the $I^+$ rotation level is given in terms of the $\eta_{\lambda^\prime}^{(2)}$ by
\[ d \sigma_{\lambda^\prime}^{(\text{rot})}(\Theta) = \frac{1}{4k_0^2} \sum_{\lambda^\prime = -I}^{+I} \sum_{L = 0}^{\infty} \sum_{m' = -L}^{+L} \sum_{m'' = -L}^{+L} (-1)^{m''} \tilde{A}_{\lambda^\prime m'}^{I^+} \tilde{A}_{\lambda^\prime m''}^{I^+} \times (1')^{oo} \times (1')^{m'}_{\lambda^\prime m''} \times P_L(\cos \Theta), \]

\[ \tilde{A}_{\lambda^\prime m'}^{I^+} = \sum_{\lambda = |\ell - I|}^{\ell + I} (2\ell + 1) (2\ell' + 1) (\ell' m', -m')_{\lambda^\prime m''} \times (S_{\lambda^\prime} S_{\ell', \ell''} - \delta_{\lambda' \lambda}), \]

This theory predicts that the distribution function of the inelastically scattered protons possesses diffraction type oscillations which, in general, are not symmetrical about the plane normal to the incident beam direction.

**Angular Correlation Theory**

The target nucleus which is excited by the inelastic...
scattering of protons may decay from the excited state, which will be referred to as state (f), to the ground state by emitting a gamma ray. In angular correlation theory, we are concerned with analyzing the angular distribution of the gamma rays relative to their associated inelastically scattered protons. The study of the gamma ray angular distribution can give additional information about the formation of the excited target nucleus state (f). The inelastic scattering of protons generally does not form the excited state (f) in a random way, rather the angular momentum \( J_f \) of this state will tend to lie along some preferred axis. The gamma ray angular distribution will have as its axis of symmetry the preferred axis of \( J_f \); and the form of this distribution will depend on the magnitude of \( J_f \). Hence angular correlation experiments can be employed as an aid in determining both the magnitude and direction of \( J_f \).

Let us consider what occurs in the case of a reaction which proceeds by the compound-nucleus mechanism. If the spin of the proton is neglected, the angular momentum which the incident proton brings into the target nucleus is in a plane perpendicular to the incident beam direction. This plane then acts as a symmetry plane for emission of the proton and the
gamma ray. In the event that a single level of the compound nucleus is excited, the wave function representing this compound nucleus has a definite parity. Hence the wave function describing the outgoing proton and the residual nuclear excited state has a definite parity. If the residual nucleus is left in a $2^+$ state, which is the case for most of the experiments described in this thesis, then the wave function describing the outgoing proton has a definite parity. The wave function describing the gamma, which is emitted when the $2^+$ excited nuclear state decays to the ground state, has a definite parity. In summary,

$$\psi(\text{compound nucleus}) = \psi(2^+ \text{ excited state of target nucleus}) \psi(\text{scattered proton})$$

$$= \psi(\text{ground state of target nucleus}) \psi(\text{scattered proton}) \psi(\text{Gamma ray})$$

Hence the angular distribution of the gammas which are detected in time coincidence with protons inelastically scattered into a solid angle, $d\Omega(\Theta, \Phi)$, will be symmetrical about the plane perpendicular to the incident proton direction.

If several overlapping levels of the compound nucleus are excited as a result of the absorption of the incident proton by the target nucleus, the
parity of the wave function which represents the excited compound nucleus might not have a definite parity. In this case the wave function of the proton emitted by the compound nucleus also might not have a definite parity. Hence the wave function representing the gamma, which is emitted when the excited target nucleus decays to the ground state, has no definite parity. Therefore, the angular distribution function of the gamma ray is not necessarily symmetrically about the plane perpendicular to the incident beam direction.

In the event a \((p,p')\) reaction proceeds by a direct interaction mechanism (and the plane-wave approximation for the incident and outgoing proton wave function is appropriate), the angular momentum transferred to the nucleus is in a plane perpendicular to the direction of the linear momentum transfer, \(Q\), where

\[ Q = K_i - K_f \]  

(2.48)

If the \(z\)-axis is chosen to lie along the \(Q\) direction, then the angular momentum, which is imparted to the nucleus by the inelastically scattered proton, is perpendicular to the \(z\)-axis. The even-even nuclei which
were used as targets for the angular correlation experiments described in this thesis have a 0+ ground state and a 2+ excited state. When the 2+ state decays to the 0+ state, a pure electric quadrupole gamma is emitted. The angular distribution function, $Z_{lm}(\Theta, \Phi)$, for pure electric radiation $(l, m)$ is

$$Z_{lm}(\Theta, \Phi) = \frac{1}{2} \left[ \frac{1 - m(m+1)}{l(l+1)} \right] Y_{l,m}$$

For the case under consideration, $l=2, m=0$ if the $z$-axis is chosen to be in the direction of the $\Theta$ direction. Then

$$Z_{20}(\Theta, \Phi) = \frac{1}{2} \left| Y_{2,0} \right|^2 + \frac{1}{2} \left| Y_{2,-1} \right|^2$$

where

$$\left| Y_{2,0} \right| = \left[ \frac{\sqrt{\frac{15}{8} \pi}}{2} \right]^{\frac{1}{2}} \sin \Theta \cos \Theta = \left| Y_{2,-1} \right|$$

The angular distribution of the emitted gammas is proportional to the square of $Z_{20}(\Theta, \Phi)$ which is a constant times $\sin^2 \Theta$ where $\Theta$ is measured with respect to the $\Theta$ direction.

Levinson and Banerjee also treat the angular

$$Z_{lm}(\Theta, \Phi)$$

References:

correlation problem by employing distorted waves for
the wave functions of the incoming and outgoing
proton states. For the case of a 2+ state decaying
by gamma emission to a 0+ state, the distorted wave
approach gives a gamma angular distribution of
\[ A + B \sin^2(\theta - \theta_0) \] where \( \theta_0 \) is measured from some axis
which only approximates the nuclear recoil direction.

The angular correlation of three nuclear radia-
tions is described by Satchler. A more comprehensive


treatment of angular correlations is given by

(10) Devons and Goldfarb.

(10) Devons, S. and Goldfarb, L.J.B., Encyclopedia
of Physics 2, 362 (1957).

(11) Austern has summarized the various direct reaction

(11) Austern, N., University of Pittsburgh,
Pittsburgh, Pennsylvania, Fast Neutron Physics-
Chapter V (Unpublished).

theories.
III Experimental Equipment

The Scattering System

Figure 1 is a block diagram of the scattering system. The Ohio State University cyclotron served as the source of the 7.0 Mev protons which were used in the experiments described in this thesis. The proton beam, extracted electrostatically from the cyclotron vacuum system, is diverging as it enters the 2\textsuperscript{in} o.d. brass pipe. In order to obtain a sufficiently intense beam current striking the target, a pair of quadrupole magnets were placed around the brass pipe. The magnetic field of these magnets forces the protons into paths parallel to the axis of the brass pipe. The focused beam is then bent through an angle of 15° by the field of the deflecting magnet. This magnet resolves the beam energy such that the protons which finally enter the scattering chamber have an energy spread less than ± 50 kev.

The beam enters the scattering chamber by passing through the .025" diameter hole of a tantalum collimator which is placed in the entrance

\begin{enumerate}
\item Bowsher, H.F., M.S. Thesis, Ohio State University, Dept. of Physics (1956).
\end{enumerate}
pert of the chamber.

The alignment of the collimator is such that it removes that portion of the proton beam which is not travelling toward the geometrical center of the chamber where the target is located.

Proton Detection

As a result of the interaction between the incident beam protons with the target, the outgoing protons will be scattered at various angles. A large fraction of the scattered protons are travelling in a direction making a sufficiently small angle with the incident beam direction that they enter the Faraday cage. These protons finally return to the electrical ground potential after passing through a one Megohm resistor. The beam current can be calculated by measuring the electrical potential of the Faraday cage relative to the ground potential. Since the cyclotron is pulsed at 11.1 Mc/sec, only an average potential is measured; therefore, only the time averaged beam current is known. The magnitude of the proton beam was of the order of $10^{-7}$ to $10^{-8}$ amperes.

A small fraction of the incident protons are scattered by the target nuclei into the small solid angle which contains the proton detector. The detector is a CsI (Tl) disk crystal.
approximately .020" in thickness and approximately 1/4" in diameter. Since paraffin is nearly transparent and has an index of refraction approximating those of lucite and CsI, it is used to connect the CsI crystal to the lucite light pipe. The light pipe in turn is connected to an EMI 9536B photo-multiplier tube. Apiezon vacuum grease is used to make good an optical contact between the light pipe and the photo tube. By employing different light pipes of various shapes, it is possible to have the proton detector crystal located at any one of the angles (37°, 60°, 90°, 120°). However, in order to change the detector angle it is necessary to return the system to atmospheric pressure, remove the lid to the scattering chamber and then exchange lucite light pipes. Much effort was spent on determining empirically the best shape for each of these light pipes so that maximum light would be transferred from the crystal to the phototube. The light pipes also were highly polished for the same reason.

The EMI photo tube is pressed into a 2" hole in one of the ports to the scattering chamber. This hole contains an O-ring which fits snugly around the photo tube so as to provide a vacuum seal. This proton detection method gave about 10% energy
resolution.

**Gamma Detection**

The interaction between an incident proton and a target nucleus may result in inelastic scattering of the proton and the excitation of the target nucleus. This target nucleus may then return to its former ground state by emitting a gamma ray. A cylindrical NaI crystal (2 in length and 1 in diameter), which is supported by a lucite light pipe, is used as the gamma detector. Good optical contact is obtained by applying Apiezon vacuum grease between the light pipe and crystal. As shown in Figure 2 the light pipe, which contains a 90° bend, projects through the lid of the scattering chamber and is attached to an EMI 9536 B photo tube. A thin layer of castor oil has been found to provide a good optical contact between the light pipe and photo tube.

The gamma detecting angle can be changed remotely since the lid to the scattering chamber, which supports the gamma detecting assembly, rests on ball bearings and can be rotated remotely through a selsyn system. The gamma detector is protected from protons by having a .020" tantalum foil glued to its front face. This gamma detecting system has an energy resolution
PHOTO-TUBE

NOT TO SCALE

O-RING GROOVE

BEARING GROOVE

NaI CRYSTAL

TARGET

COLLIMATOR

LUCITE LIGHT PIPE

SCATTERING CHAMBER

FIGURE 2
of no better than 20% mainly due to the light loss in the light pipe especially near the 90° bend. However, the nature of the experiments is such that better energy resolution is not required.

The Alignment of the Target

The gamma detector tracks a circular path which is concentric with the geometrical center of the scattering chamber; therefore, if the beam does not strike the target at this center point the solid angle which the gamma detector presents to the bombarded area of the target depends on the angle of gamma detection. This change of solid angle of detection adversely affects the angular correlation runs.

The following method was used in aligning the target and the path of the incident beam. The entrance port collimator containing the .025" hole was roughly positioned by applying the following optical method. A point light source was inserted along the axis of the 2" o.d. brass pipe at a point about 36" in front of the chamber. A brass plate containing a cylindrical hole was properly placed on the floor of the scattering chamber so that the axis of the hole fell along the axis of the scattering chamber. A pointer was then inserted into this brass plate.
hole. The length of the pointer was such that the tip was located at the geometrical center of the scattering system. The point light source and the pointer tip then established a line which was to become the path of the proton beam. The position of the entrance port collimator was adjusted so that while watching the pointer tip, it was possible to look through the collimator hole and see the point light source.

To further check the alignment of the beam path a paper target was placed at the center of the chamber and bombarded by the proton beam until a small hole appeared in the paper. The target was then rotated about the axis of the scattering chamber and was bombarded again. The collimator was then moved so as to reduce the distance separating the two holes. This process was repeated until the separation of the holes was less than .006". It was then assumed that the beam was directed sufficiently close to the geometrical center of the chamber.

If a target were misaligned such that it was struck by the beam either before or after the beam had reached the chamber center, the angular correlation experiments would be adversely affected. Since the
two previously discussed tests would fail to reveal such a target misalignment, the following test was also employed. An angular distribution of the 4.8 Mev gamma rays in time coincidence with protons inelastically scattered in a certain predetermined solid angle was measured using Li as the target. Rose has proved that for all excited nucleus levels having spin $1/2$, the gamma angular distribution is isotropic. The first excited state of Li is known to have a spin $1/2$. Figure 3 shows the results of measuring the gamma angular distribution and within statistics the curve is isotropic. The system was then assumed to be properly aligned.


Energy Degradation of the Incident Beam

The energy of the protons leaving the cyclotron is fixed at approximately 7.0 Mev; however it was desirable to test the effect of change in incident beam energy on the angular correlation. The incident beam protons were degraded in energy by placing a thin platinum foil in the path of the beam. This foil also increases
Figure 3: Angular correlation, Li7 target.
the energy spread of the beam. Table 1 lists the most probable energy and the energy spread of the protons after the beam has passed through foils of various thicknesses.

The value of 7.0 MeV, as the energy of the protons leaving the cyclotron, was determined by the following method. A platinum foil (0.0001" thick) was used as the target. No energy degrading foils were placed in the path of the beam. The energy of the protons, which are scattered off the platinum target at an angle of 90° relative to the incident beam direction, was measured with the 90° double focusing magnet spectrometer. The energy was found to be 6.57 MeV. However, the energy of the incident protons is higher since the particles lose energy in passing through the target. For protons of this energy range the loss of energy due to atomic collisions and ionization interactions in a 0.0001" platinum foil is approximately 250 keV. Kinetic energy is also given to any target nuclei which interact with the incident proton.

If it is assumed that a proton and target-nucleus interaction occurs at the point, one half of the

---

INCIDENT BEAM ENERGY
VS. FOIL THICKNESS

<table>
<thead>
<tr>
<th>FOIL</th>
<th>BEAM ENERGY</th>
<th>BEAM ENERGY SPREAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO FOIL</td>
<td>7.0 MeV</td>
<td>100 keV</td>
</tr>
<tr>
<td>0.0001&quot; PLATINUM</td>
<td>6.7 MeV</td>
<td>200 keV</td>
</tr>
<tr>
<td>0.0002&quot; PLATINUM</td>
<td>6.5 MeV</td>
<td>200 keV</td>
</tr>
<tr>
<td>0.0003&quot; PLATINUM</td>
<td>6.3 MeV</td>
<td>200 keV</td>
</tr>
<tr>
<td>0.0004&quot; PLATINUM</td>
<td>6.2 MeV</td>
<td>200 keV</td>
</tr>
<tr>
<td>0.0006&quot; PLATINUM</td>
<td>5.8 MeV</td>
<td>200 keV</td>
</tr>
</tbody>
</table>

TABLE 1.
distance through the target, then the energy of the proton just after the interaction is \((6.57 + 0.25 \times \frac{2}{2})\text{ MeV} = 6.69\text{ MeV}\). The energy before the interaction was 6.86 MeV, since

\[
E(\text{after collision}) = E(\text{before collision}) \frac{(M+m)}{(M-m)}
\]

(3.1)

where \(M\) is the mass of the target nucleus (platinum) and \(m\) is the mass of the proton. The energy of the proton before it entered the target was \((6.86 + 0.25)\text{ MeV} = 7.10\text{ MeV}\).

The energy resolution of the spectrometer magnet is about one percent which means that for 6.5 MeV, the energy resolution is about 65 kev. The energy spread of protons elastically scattered from a platinum target has been measured to be about 120 kev. An upper limit for the energy spread of the protons entering the scattering chamber is obtained from the equation

\[
(\text{spread of energy in incident beam})^2 + (\text{energy resolution of magnet})^2 \leq (120\text{ keV})^2
\]

(3.2)

The inequality exists because there is also a spreading of the proton energies due to the beam passing through the target. This effect has been omitted in equation (3.2). The spread in energy
of the beam entering the chamber is less than ± 50 kev.

In order to obtain the maximum information about the angular correlation experiments, all protons other than those scattered from the target should not be detected by the proton (CsI) detector. To reduce the number of undesired protons being detected, collimation was arranged as shown in Figure 4.

A fraction of the protons spray outward, away from the main beam path, anytime the beam passes through a collimator hole or penetrates an energy degrading foil. As can be seen from the diagram, the smallest number of scatterings any proton can receive and still arrive at the CsI crystal is one, and this only occurs when a proton of the main beam is scattered off the target into the crystal detector. Some protons, which are not scattered off the target, still arrive at the detector but they have been scattered at least twice (eg, path ABC in figure 4). Their number is small compared to those scattered directly off the target.

Reduction of Gamma Background

The following precaution were taken to reduce the creation of the gamma rays not associated with protons inelastically scattered off the target.
Figure 4. Arrangement of Collimation
1. The inside wall of the last 18" of the 2\textsuperscript{nd} o.d. brass pipe is lined with lead.
2. The Faraday cage is lined with tantalum.
3. The energy degrading foils are platinum.
4. The .025\textsuperscript{th} collimator in the entrance port is made of tantalum.
5. The collimators which restrict the protons arriving at the CsI proton detector are made of tantalum.
6. Four inches of lead surround the scattering chamber.

Since lead, tantalum, and platinum have a large atomic number, the incident protons are unlikely to penetrate the coulomb barrier and excite these nuclei with the result that few gamma-rays will be produced.

Target Preparation

Figure 5 shows the arrangement of the equipment used in the evaporation process for making various targets. Both an oil-diffusion pump and a mechanical pump were used to evacuate the 8.5\textsuperscript{th} glass bell jar. Any one of several filaments (depending on the material to be evaporated) could be attached to the water cooled electrodes which are electrically insulated from the steel base plate. A copper plate,
FIGURE 5. EVAPORATION EQUIPMENT.
attached to the rod which projects through the steel base plate, can be rotated externally so that the target holder is either exposed to or concealed from the evaporating material. The filaments were made of either carbon rod with a small hole in which powdered material to be evaporated could be placed or made of tungsten wire shaped as a spiral basket.

The filaments were heated electrically by a power supply capable of producing as much as a hundred amperes.

Carbon targets were prepared from a commercial hydrocarbon called Formvar. The powdered Formvar is dissolved in 1-2 dichloroethane to form a solution which, when dropped lightly on the surface of water, spreads over the surface and quickly dries into a clear film. A wire loop is then lowered under the film and raised slowly. The film sticks to the loop and is self supporting. Formvar is obtainable from Shawinigan Resins Corporation, Springfield, Mass.

Chromium targets were prepared by placing a small piece of chromium metal in a .040" tungsten wire filament and then passing 75-80 amperes current through the filament. Some of the evaporated chromium was deposited on a glass plate which had been placed several inches from the filament. The chromium was removed from the glass by floating it on water.
or very weak sodium hydroxide solution. The chromium was lifted from the water with a Formvar target.

Lithium targets were prepared by evaporating a piece of metallic lithium placed in a .030 inch tungsten filament. The filament was heated with a current of about 30 amperes.

Sulfur targets were prepared by evaporating lead sulfide onto a Formvar film. The film was placed approximately six inches from the carbon-red filament. Since the powder melts from the bottom, the evaporation must take place slowly or the lead sulfide will be blown out of the filament by the escaping gas vapor.

Another method of preparing the sulfur targets was to evaporate sulfur onto a Formvar film. However, it was discovered that sulfur on Formvar evaporated when struck by the 7 Mev proton beam. To keep the sulfur from evaporating, the target was gold plated to conduct the heat of bombardment away quickly.

Silicon targets were prepared by drawing out a quartz fiber to the desired diameter and then fastening it to a wire ring. The quartz fiber was then gold plated to conduct away the heat of bombardment.

An aluminum target was prepared by gluing a 0.0001" aluminum foil to a target holder.
A neon target was prepared by enclosing neon gas inside a brass tube 2-1/2" long and 3/4" in diameter. The ends of the tube were fitted with valves for charging the tube. Two 1/2" holes were cut opposite each other to allow the cyclotron beam to enter and leave the target. A third hole was cut at 90° with respect to the first two holes to allow the scattered beam to leave the target. The holes were covered with 0.00011" nickel foil secured with Gelva V-25 adhesive and a thin copper clamp cut with matching holes. The Gelva V-25 adhesive was obtained from the Shawinigan Resin Corporation, Springfield, Mass. A second neon target was prepared in a similar fashion except the third-hole which permits the scattered beam to leave the target was cut at 60° relative to the first two holes. The two targets were then evacuated and filled with neon gas at 20 lb/sq.in.

The Electronic Circuitry

Figure 6 is a block diagram of the coincidence circuit. When a gamma ray strikes the NaI crystal a light pulse, whose intensity is proportional to the energy of the gamma, is produced. The light pulse traverses the lucite light pipe and finally arrives at the cathode of the EMI 9536B photo-tube. A
Figure 6. Diagram of coincidence circuit.
voltage pulse whose amplitude is proportional to the energy of the detected gamma is produced by the photo-tube. This electrical pulse travels through 120 feet of RG 62-U cable, in going from the EMI tube to the electronics room, and is then fed into a fast amplifier. The amplified pulse has approximately a 10 Volt negative amplitude and a rise time of about 5 nano-seconds. This pulse is then fed into the #1 fast coincidence circuit. Meanwhile, protons striking the CsI crystal produce light pulses which are converted into voltage pulses by a second EMI photo-tube. These pulses travel through a variable delay in addition to the 120 feet of RG-U cable and are then amplified by a fast amplifier. The variable delay can be changed by units of 2 nano-seconds. This permits one to adjust the time of transit of the proton voltage pulse so that its time of arrival at the fast coincidence circuit is the same as for the voltage pulse which corresponds to the gamma associated with the detected proton. The fast coincidence circuit sends a pulse to the triple coincidence circuit whenever the separation in time of arrival of a proton pulse and a gamma pulse is less than the circuit's resolving time. The resolving time of the fast coincidence circuit was measured to be approximately 25 nano-seconds.
As shown in Figure 6 the EMI voltage pulses are also amplified by slow linear amplifiers and then are fed into single channel pulse height analyzers. An energy window is placed on each of the two analyzers. Whenever an analyzer receives a voltage pulse corresponding to the detection of a radiation having an energy which lies within the preset energy window, the analyzer sends a pulse to the triple coincidence circuit. The triple coincidence circuit triggers the total coincidence scaler whenever it receives pulses simultaneously from the \#1 fast coincidence circuit and each of the single channel pulse height analyzers. In review, a coincidence count is recorded whenever a proton and a gamma, each with the appropriate energy, have a separation in time of detection equal to or less than the resolving time of the \#1 fast coincidence circuit.

The proton and gamma ray, which are detected in time coincidence, may be related in the sense that the excited target nucleus, which emitted the detected gamma, originally received its excitation energy by interacting with the detected proton. It is also possible that a proton and a gamma ray be detected in time coincidence and not be related in the manner just described. When the proton and gamma are
related, a coincidence shall be referred to as a "true" coincidence while the detection of a proton in coincidence with a non related gamma ray shall be referred to as an "accidental" coincidence. The circuitry above does not distinguish between true coincidences and accidental coincidences.

An estimation of the fraction of the total coincidence counts which are true coincidences can be made in the following way.

The accidental coincidence counting rate is

$$N_p N_g \frac{2 \tau}{\gamma}.$$  

The true coincidence counting rate is

$$N e_p \Omega_p f_p(\theta) e_\gamma \Omega_\gamma f_\gamma(\theta),$$

where

- $N_p$ = cyclotron beam strength.
- $N_p$ = rate of detection of protons having an energy within the proton pulse height analyzer energy window.
- $N_g$ = rate of detection of gammas having an energy within the gamma pulse height analyzer energy window.
- $2 \tau$ = the resolving time of the fast-coincidence circuit.
- $f_p(\theta)$ = the differential inelastic scattering cross section of the protons.
- $f_\gamma(\theta)$ = the function describing the angular distribution of the emitted gammas.
- $e_p$ = efficiency of the CsI crystal for proton detection.
- $e_\gamma$ = efficiency of the NaI crystal for gamma detection.
- $\Omega_p$ = solid angle of detection of the CsI crystal.
- $\Omega_\gamma$ = solid angle of detection of the NaI crystal.

The above procedure for determining the fraction of total coincidence counts which are true coincidences
is feasible for the situation where the proton beam is of constant strength. However, this method is impractical to apply to a cyclotron beam. The O.S.U. cyclotron beam is pulsed at a frequency of 11.1 Mc/sec, and protons are emitted during less than half of a cycle; therefore, the instantaneous proton beam intensity varies greatly from the measured or time averaged beam. The number of accidental coincidences varies with the square of the beam intensity and the number of true coincidences varies linearly with beam intensity. Consequently serious errors would result if the value of the time averaged beam intensity is used instead of the instantaneous value of the beam intensity.

In order to eliminate this possible source of error, a second coincidence circuit, which will be referred to as the #2 coincidence circuit, was added to the detector system. See Figure 6. The purpose of this added circuit was to measure accidental coincidences. Pulses from the output of the fast amplifiers were sent to the input of the #2 fast-coincidence circuits as well as to the #1 fast-coincidence circuits. A delay, equivalent to the time between consecutive cyclotron pulses, was inserted in the line leading to the input of the
#2 fast coincidence circuit. The outputs of the 
#2 coincidence circuit as well as those of the 
proton and gamma single channel pulse height 
analyzers were fed into the #2 triple coincidence 
circuit. In the event a proton and a gamma of the 
appropriate energies are detected such that the 
separation in the time of detection of the two radiations 
is equal to the period of the pulsed cyclotron beam 
plus or minus the resolving time of the #2 fast 
coincidence circuit, then the #2 triple coincidence 
circuit will trigger the scaler which is connected 
to its output. The number of coincidence counts 
recorded by the #2 triple coincidence circuit, 
within statistics, is equal to the number of 
accidental coincidences recorded by the #1 triple 
coincidence circuit. The #2 coincidence circuit can 
record no true coincidences since protons of one cycle-
tron pulse are being matched with gamma rays related 
to protons of the next cyclotron pulse. The number 
of true coincidences occurring during a run is taken 
to be the difference between the counts of #1 and #2 
coincidence circuits.

The Alignment of the Coincidence Circuits

It is important that the efficiencies and 
resolving times of the two coincidence circuits be the 
same. For the initial alignment, a fast pulser
was used as the source of pulses. The extra delay, which is in the line leading to the input of the #2 fast coincidence circuit, was removed so that the two coincidence circuits were connected identically. The pulses from the fast pulser were fed simultaneously to both the proton and gamma ray signal lines. The two coincidence circuits were adjusted so that the coincidence counts recorded in the two circuits were the same. This gave a check of the efficiencies and resolving times of the two circuits.

The coincidences of the two circuits were again recorded as a function of delay #1. However, the pulses resulted from the detection of protons, which were inelastically scattered off a Mg target, and their associated gamma rays. The coincidence circuits were adjusted so that the counting rates for both circuits were the same for all values of delay #1. Delay #2 was again inserted. While #1 was fixed at the value which maximized the counting rate of the #1 circuit, the coincidence counting rate of the #2 circuit as a function of delay #2 was measured. The desired value of delay #2, which is equal to the period of the pulsed cyclotron beam, is determined by noting a secondary maximum in the coincidence counting rate of the #2 circuit as a function of
delay #2. The primary maximum in the counting rate is excluded since true coincidence counts are also being included.

A test which was made prior to each angular correlation run was to observe the true coincidence rate, #1 minus #2, as a function of the base of the energy window (fixed width) in the proton pulse height analyzer. Figure 7 shows the energy spectrum of the protons (solid line) scattered from a neon gas target at an angle of 90° with respect to the incident beam. Plotted as a dotted line on top of the energy spectrum are the net coincidence counts as a function of the base line of the energy window. There are two peaks in the coincidence counts which correspond to the first and second excited states of Ne at 1.63 Mev and 4.26 Mev. Although the coincidences of circuits #1 and #2 were highest when the proton energy window bracketed the elastic proton peak, the net coincidence counting rate was zero within statistical limits.

**Normalization**

The proton signals, which are fed into the slow amplifier and finally to the single channel pulse height analyzer, are also fed into a second amplifier and pulse height analyzer (not shown in
FIGURE 7  COINCIDENCES AND PROTON SPECTRUM - Ne²° TARGET
Figure (6). The energy window of this second analyzer is set so that the scaler, which is connected to the analyzer, counts both the elastically and inelastically scattered protons. When a preset number of protons has been recorded, this scaler automatically shuts off the coincidence circuits.
IV Experiments

Below is a description of the angular correlation experiments. These experiments consist of exciting target nuclei by a \((p,p')\) reaction. The excited nuclei then decay by emitting a gamma ray. The angular distribution of the gamma rays, which are associated with the protons inelastically scattered in a certain direction, is then measured. The following targets \(\text{Li}^6, \text{Cr}^{52}, \text{Al}^{27}, \text{C}^{12}, \text{Ne}^{20}, \text{S}^{32}, \text{Si}^{28}\) were used.

A target of \(\text{Li}^6\) was obtained in the following way. Since in the naturally occuring lithium, the \(\text{Li}^6\) isotope composes only 7.4% of the total amount, it is necessary to use an enriched sample. Lithium metal, which was enriched to 99.3% \(\text{Li}^6\), was obtained from Oak Ridge National Laboratory. The lithium was evaporated onto a thin Formvar target.

An energy spectrum of the protons scattered from the target at an angle of 90° with respect to the incident beam direction, is shown in Figure 8. Peaks "a" and "b" correspond to protons elastically scattered from carbon and lithium respectively, whereas peak "c" corresponds to the inelastically scattered protons leaving \(\text{Li}^6\) in its 2.7-Mev first
excited (virtual) state. In order to see the origin of peak "d", it is necessary to study the reaction more closely. The first excited state of Li may split into a deuteron and an alpha particle. The peak "d" corresponds to the detection of deuterons and alpha particles as well as the inelastically scattered protons which excited the carbon nuclei in the Formvar to their first excited state (4.4 Mev).

The virtual state of Li might also decay to the ground state by the emission of a gamma ray. Since the energy resolution of the gamma detector is poor and also because of the high rate of detecting background gamma rays, the gamma energy spectrum can not be used to investigate whether an appreciable fraction of the excited Li nuclei decay by gamma emission. The angular correlation experiment might aid in determining the method of decay of the excited Li nuclei in the following way. If the excited target nuclei predominantly experience particle break-up as opposed to gamma emission, then there will not be a large net coincidence counting rate in the angular correlation experiment. The results of this experiment were, that, within statistical limits, the coincidence counts recorded by the two coincidence circuits were the same. This indicates that particle
PROTON DETECTOR ANGLE = 90°
break-up predominates over the gamma emission in the decay of the 2.2 Mev level of Li.

A chromium target was chosen because of the magnitude of the charge it possesses. The coulomb barrier for protons on chromium is about 6 Mev. This is approximately equal to the incident proton beam energy of the Ohio State University cyclotron.

Before an incident proton can interact strongly with the nucleons of the target, it must overcome the large coulomb barrier. Of the incident protons which are successful in overcoming this barrier, it might be expected that a large fraction would be elastically scattered. Both the elastically and inelastically scattered protons must overcome the same coulomb barrier. However, the inelastically scattered protons have less kinetic energy which greatly reduces their chances of penetrating the barrier.

The spectrum of protons scattered from the chromium target at a detector angle of 90° is shown in Figure 9. Peaks "a" and "b" correspond to protons scattered elastically from Cr and C, whereas peak "c" corresponds to protons leaving Cr in its 1.44-Mev first excited state. Figure 9 shows that a large fraction of the protons scattered from Cr were
FIGURE 9. PROTON SPECTRUM - C\textsuperscript{52} TARGET
scattered elastically. The scattering from carbon occurs because the target was prepared by evaporating metallic chromium onto thin Formvar backings.

Strong interactions occur between the incident proton and the nucleons of the target nucleus; therefore, the incident proton quickly loses its identity once it is inside the target nucleus. Because of the large barrier the time elapsing before a proton is emitted might be sufficient for the nuclear system to reach statistical equilibrium. This is especially true for the inelastically scattered protons. Figure 10 shows the angular correlation between the inelastically scattered protons and their associated decay gammas. The proton detector was set at 90°. As can be seen, the angular correlation curve is isotropic which indicates that statistical equilibrium was attained by the nucleons of the compound system.

(16) Seward measured the \( (p',\gamma) \) angular correlations using magnesium and chromium targets at various incident proton beam energies. His experiments were consistent with the assumption that the \( (p,p') \) reaction mechanism is predominantly compound nucleus formation when proton energies are below the barrier, and
$C^{52}_n(p,p'\gamma) 1.45 \text{ MeV}$

Proton Detector Angle = 90°

**Figure 10. Angular correlation - $C^{52}_n$ target.**
direct interactions become appreciable as seen as the energy of the scattered proton is well above the barrier.

It was decided to examine aluminum Al because its proton number was only one removed from a closed shell. The complexity of the energy spectrum of protons scattered from aluminum along with our inability to resolve the various levels, added to the difficulties of this experiment. The spectrum of the protons scattered from the aluminum target at 90° is shown in Figure 11. The highest energy peak corresponds to the unresolved proton groups from the first and second excited states in aluminum as well as the elastic peak. For the angular correlation experiment, the base line of the proton analyzer was set to count all protons corresponding to the first and second excited states as well as the elastically scattered protons. Consequently, coincidences were recorded between protons from both the first and second excited states and their resultant decay gamma rays.

Figure 11 also shows the energies, spins and parities of the ground state, and the first and second excited states of Al. The emitted gamma rays
FIGURE II. PROTON SPECTRUM - $^{27}$Al TARGET.
can result from the following decay schemes:
1. The 0.842 MeV state (1/2 +) decays to the ground state (5/2 +). The angular distribution of these gamma rays is isotropic. See page 35.
2. The 1.013 MeV state (3/2 +) decays to the 0.842 MeV state (1/2 +). The energy of these gamma rays is (1.013-0.842 = 0.171) MeV which is below the base line of the energy window for the gamma analyzer. Therefore, these decay gammas did not affect the angular correlation experiment.
3. The 1.013 MeV state (3/2 +) decays to the ground state (5/2 +). This decay would be described predominantly by the emission of a M-1 or E-2 radiation. The (3/2 +) excited state was prepared by a (p,p') reaction in which the ground state (5/2 +) had no preferential orientation. Hence, there is no unique direction for the orientation of the (3/2 +) excited nuclei. Therefore, the decay pattern of the (3/2 +) state would be expected to be very complex.

Figure 12 shows the net coincidence counts as a function of the gamma detector angle. The proton detector angle was at 90° with respect to the incident beam direction. Within statistics the distribution is isotropic. Due to the various decay
$\text{Al}^{27}(p, p'\gamma)$

PROTON DETECTOR ANGLE = 90°

FIGURE 12. ANGULAR CORRELATION - Al$^{27}$ TARGET.
schemes, it is not surprising that the distribution function of the gamma rays appears isotropic.

A considerable number of experiments have been performed on the inelastic scattering of protons leaving $^{12}$C in its 4.4 MeV first excited state. These experiments range in energy from 6.5 to 96 MeV.

Even at energies as high as 10 MeV, the angular distributions of the inelastically scattered protons have been measured as being symmetric about 90° (center-of-mass coordinates). At higher energies the angular distributions tend to be asymmetric about 90° and are peaked in the forward direction.

Angular correlation experiments performed by Sherr at 16.6 MeV agreed quite well with the predictions of the simple direction-reaction theories.

For incident protons having a small energy, the number of channels available for decay of the compound nucleus is seriously limited due to the
high excitation energy of the first few excited states in $^{12}$C, 4.4 and 7.6 MeV, and because of the energy required for competing reactions such as $(p,d)$ $(p,n)$, $(p,\alpha)$. Consequently, the probability is high that the compound nucleus will decay to the first excited state at 4.4 MeV. Since the direct-reaction contribution to the reaction is predicted to be small, its contribution to the angular distribution may be unobservable because of the large compound nucleus contribution. At high bombarding energies the number of channels open for compound nucleus decay is large. The probability of compound nucleus decay to the first excited state is consequently decreased and direct processes to the first excited state may then become observable.

Figure 13 shows the spectrum of protons scattered from a thin Formvar target. Peaks "a" and "b" correspond, respectively, to the protons scattered elastically from $^{12}$C, and the scattered protons which left the target nucleus in the 4.4 MeV state. The proton detector angle is 90°.

Figure 14 shows the results of the two angular correlation experiments. The proton detector was set at laboratory angles of 90° and 120° with respect to the incident beam direction. At
PROTON DETECTOR ANGLE = 90°

FIGURE 13. PROTON SPECTRUM - C^{12} TARGET.
\[ C^{12}(p,p'\gamma) 4.4 \text{ MeV.} \]

\[ W(\theta) = (195 \pm 6) + (88 \pm 34) \sin^2 2(\theta - \{51^\circ \pm 2^\circ\}) \text{ L.S.} \]

\[ W(\theta) = 180 + 115 \sin^2 2(\theta - 51^\circ) \text{ F.S.A.} \]

Figure 14. Angular Correlation - \( C^{12} \) Target.
forward angles, the large elastic scattering peak of protons from the hydrogen in the target overlapped the protons inelastically scattered from C. Consequently, only two proton detector angles were used in the angular correlation experiments.

Figure 14 shows the experimental points as well as their statistical standard deviations. The smooth curve is a least-squares fit to the experimental data. The equations (designated L.S.) give the parameters to the curve. The probable errors of the parameters are also included. The equations (designated F.S.A.) have been corrected for the effects of the finite solid angle of the gamma detector. The Appendix contains an outline of the procedure used in analyzing the data. The point $\Theta_R$, represented on each curve, is the classical recoil direction.

From the curves of Figure 14, a significant portion of the $(p,p')$ reaction can be explained in terms of a direct reaction process. However, earlier in this section, it was stated that the angular distribution of the inelastically scattered protons off C is symmetrical about 90° for incident protons in the 7 Mev range. Auster has shown that direct reactions can give this forward-backward symmetrical
scattering as a result of reflection. When either the incident or outgoing particles see the nucleus as being sufficiently transparent (the optical well being composed mainly of a real potential), there exists the possibility that a large peak in the angular distribution at or near the forward direction may be "imaged" in the backward direction. This is the result of the fact that a wave splits into two parts upon striking a barrier between two different media. One portion penetrates the barrier while the other portion is reflected.

Angular distributions of protons inelastically scattered from Ne have been done by Freemantle et al., Gibson et al., and by Kondo and Yamazaki. Again, as in the case with C, the angular distributions of the inelastically scattered protons tend to be symmetric about 90° (C.M.) for energies below 10 Mev.

The angular correlation of inelastically scattered protons and their associated gamma rays was undertaken with the expectation that these experiments would be more sensitive to direct reactions than
The results of the angular correlation runs are shown in Figure 15. The experimental points are shown along with their statistical standard deviations. The least-squares equations (and standard deviations) are given. The equations which take into account the effect of the finite solid angle of detection of the gamma detector are also included. The curves correspond to laboratory proton detector angles of 60°, 90°, and 120° with respect to the incident beam direction. Calculated classical nuclear recoil directions are indicated on the curves.

The angular correlations indicate that a large portion of the \((p,p')\) reaction proceeded by means of a direct-reaction mechanism.

Sulfur targets were prepared by evaporating sulfur onto thin Formvar backings. Gold was then evaporated onto the sulfur. The purpose of the gold film was to conduct away the heat which is produced by the energy lost by the incident beam bombarding the target.

The spectrum of protons scattered from the sulfur target is shown in Figure 16. Peak \(a\) corresponds to protons elastically scattered from gold, sulfur, and Formvar (carbon). Peak \(b\) corresponds to
Figure 15: Angular Correlation - N^2o Target

\[ W(\theta) = (990 \pm 35) + (1160 \pm 220) \sin^2 (\theta - 15^\circ) \]

\[ W(\theta) = (780 \pm 7) + (1310 \pm 61) \sin^2 (\theta - 35^\circ) \]

\[ W(\theta) = (367 \pm 8) + (223 \pm 45) \sin^2 (\theta - 62^\circ) \]

\[ W(\theta) = (328 \pm 176) \sin^2 (\theta - 62^\circ) \]

\[ W(\theta) = 1436 \sin^2 (\theta - 56^\circ) \pm 5.5^\circ \]

\[ W(\theta) = 12^\circ \pm 6^\circ \]

\[ W(\theta) = 60^\circ \pm 5^\circ \]

\[ V = 0^\circ \pm 5^\circ \]

\[ N^{2o}(p,p,n) 1.63 \text{ MeV} \]
PROTON DETECTOR ANGLE = 90°

FIGURE 16. PROTON SPECTRUM - $S^{32}$ TARGET.
inelastically scattered protons leaving $^3$S$^2$ in its 2.25 Mev first excited state. The width of the peaks is a result of the energy loss in the thick target.

The results of the angular correlation runs are shown in Figure 17. The data for these curves have been analyzed in the same manner as the data for the C$^{12}$ and Ne$^{20}$ curves.

Angular distribution experiments of protons inelastically scattered from Si have been reported by Yamabe et al. from 4.8 Mev to 5.7 Mev bombarding energy, and by Greenlees et al. from 8.0 to 9.4 Mev bombarding energy. For these cases,


the angular distributions of protons leading to the 1.78 Mev first excited state in Si have not been symmetric about 90° (C.M.).

A silicon target was prepared by drawing a quartz fiber down to a diameter estimated as being less than 0.001". A thin film of gold was then evaporated onto the fiber surface.
Figure 17. Angular Correlation

\[ W(\theta) = 950 + 607 \sin^2 (\theta - 90) \]

\[ W(\theta) = (105 \pm 5)(1 + 5 \sin^2 (\theta - 90)) \]

Proton Detector Angle = 118°

\[ W(\theta) = 593 + 202 \sin^2 (\theta - 90) \]

\[ W(\theta) = (18 \pm 5)(1 + 5 \sin^2 (\theta - 90)) \]

Proton Detector Angle = 90°

\[ W(\theta) = 604 + 303 \sin^2 (\theta - 70) \]

\[ W(\theta) = (64 \pm 5)(1 + 32 \sin^2 (\theta - 70)) \]

Proton Detector Angle = 60°

\[ \frac{S_3^2}{p(p,l)Z.25 \text{ MeV}} \]

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The results of the angular correlation runs are shown in Figure 18. The energy of the incident protons was 7.0 Mev. The curves correspond to laboratory proton detector angles of 60°, 90°, 120°, and 140°. No attempt was made to fit the 140° correlation run to a $A + B \sin^2 2(\Theta - \Theta_0)$ curve.

The angular distribution of the gamma rays is symmetric about 90° (C.M.) when the proton detector angle is either 60°, 90°, or 120°. This is contrary to the predictions of a direct reaction. However, this symmetry of the gamma radiation about 90° (C.M.) would be expected if there exists a single strong resonance which is excited in the compound nucleus. See page 24.

One way of testing the validity of the single, strong, compound-nucleus resonance explanation of the above results, is to reduce the energy of the incident protons. However, before the angular correlations could be rerun at the lower energies, the thin silicon target broke. This target will be designated as target #1.

A second silicon target (called target #2) was prepared by the method used in preparing target #1. However, the diameters of the two quartz fibers may
Coincidence:
counts

Int. & f.
tion
cycle

Si$_{28}^{28}$(pp'γ) 1.78 Mev.

Proton detector angle = 60°

$W(\theta) = (595 \pm 5) + (360 \pm 24) \sin^2 2(\theta - (46.5 \pm 1^\circ))$ L.S.

$W(\theta) = 511 + 506 \sin^2 2(\theta - 46.5^\circ)$ F.S.A.

Proton detector angle = 90°

$W(\theta) = (358 \pm 3) + (145 \pm 24) \sin^2 2(\theta - (46.5 \pm 1^\circ))$ L.S.

$W(\theta) = 302 + 220 \sin^2 2(\theta - 46.5^\circ)$ F.S.A.

Proton detector angle = 120°

$W(\theta) = (680 \pm 6) + (190 \pm 45) \sin^2 2(\theta - (47^\circ \pm 2^\circ))$ L.S.

$W(\theta) = 657 + 262 \sin^2 2(\theta - 47^\circ)$ F.S.A.

Proton detector angle = 140°

No equation or curve fitted to data.

Gamma ray detector angle.

Figure 18. Angular correlation - Si$_{28}^{28}$ Target #1.
have differed. Also, there may have been different amounts of gold plated on the two quartz fibers. It was decided to first repeat the angular correlation experiments on the silicon target #2 before changing the beam energy. This would provide a check on the alignment of the scattering system and the electronic circuitry.

The results of these runs are shown in Figures 19, 20, 22, and 24. The incident beam energy was 7.0 Mev. The angular distribution function of the gamma rays is not symmetric about 90° (C.M.) for any of the four runs. Also, the symmetry axis of the gamma ray distribution function shifts in a regular manner with a change in the proton detector angle. These last four runs indicate the presence of a direct interaction occurring in the \((p,p')\) reaction. These results do not agree with those obtained from the target #1 experiments.

A possible explanation for this difference in the results is based on the assumption that the thicknesses of the two targets were not the same. As the protons pass through a target, they lose energy as a result of atomic interactions. Consequently, if the two targets had different thicknesses, the amount of energy lost by the protons, due to the atomic
FIGURE 19. ANGULAR CORRELATION

\[ W(\theta) = \frac{22 + 24 \cos^2(\theta - 72^\circ)^2}{8}\]

INCIDENT BEAM ENERGY = 6.7 MeV

\[ W(\theta) = \frac{56 + 116 \cos^2(\theta - 75^\circ)^2}{8}\]

INCIDENT BEAM ENERGY = 7.0 MeV

PROTON DETECTOR ANGLE = 37^\circ
Figure 21. Angular Correlation - Sg²⁸ Target #2, Proton Detector Angle = 60°
INCIDENT PROTON ENERGY
= 7.0 MeV.

\[ W(\theta) = (480 \pm 28) + (176 \pm 17) \sin^2 2(\theta - \{58\pm 1\}) \text{ L.S.} \]

\[ W(\theta) = 459 + 212 \sin^2 2(\theta - 58^\circ) \text{ F.S.A.} \]

INCIDENT PROTON ENERGY = 6.7 MeV.

\[ W(\theta) = (390 \pm 11) + (139 \pm 58) \sin^2 2(\theta - \{60 \pm 2\}) \text{ L.S.} \]

\[ W(\theta) = 370 + 181 \sin^2 2(\theta - 60^\circ) \text{ F.S.A.} \]

**Figure 22: Angular Correlation - Si^{28} Target #2, Proton Detector Angle = 90°**
**INCIDENT PROTON ENERGY = 7.0 MeV.**

\[ W(\theta) = (978 \pm 7) + (218 \pm 35) \sin^2 2(\theta - (60^\circ \pm 1^\circ)) \text{ L.S.} \]

\[ W(\theta) = 943 + 284 \sin^2 2(\theta - 60^\circ) \text{ F.S.A.} \]

**INCIDENT PROTON ENERGY = 6.7 MeV.**

\[ W(\theta) = (500 \pm 19) + (150 \pm 90) \sin^2 2(\theta - (57^\circ \pm 1^\circ)) \text{ L.S.} \]

\[ W(\theta) = 476 + 196 \sin^2 2(\theta - 57^\circ) \text{ F.S.A.} \]

**GAMMA DETECTOR ANGLE**

**FIGURE 24. ANGULAR CORRELATION -**

**Si^{28} TARGET #2, PROTON DETECTOR ANGLE = 120^\circ**
interactions, would not be the same for both targets. A test of this possible explanation is to see if, for a different incident beam energy, the target #2 angular correlation results would agree the above target #1 results.

The angular correlation runs were taken on silicon target #2 at several different beam energies. The beam energies were in the range: 5.8 Mev to 7.0 Mev. The results of the runs are shown in Figures 19, 20, 21, 22, 23 and 24. The proton detector angles were 37, 60, 90, and 120. At none of these energies, are the results similar to those obtained with target #1. Since it is impossible to increase the incident beam energy above 7.0 Mev, we were not able to check whether higher energy protons incident on target #2 would give the results of target #1. However, decreasing the thickness of the target has nearly the same effect as increasing the incident beam energy.

A third target (called target #3) was then prepared. This target was quite thin. Figure 25 shows the results of an angular correlation run made on target #3. The proton detector angle was 60° with respect to the incident beam direction. The results approximate quite closely those obtained from target #1.
$S.2^9$ THREE DETECTORS, PROTON DETECTOR ANGLE = 60°

FIGURE 25. ANGULAR CORRELATION

(AMMA DETECTOR ANGLE

\[ W(\theta) = 600 + 390 \sin^2 (\theta - 45°) \]

INCIDENT PROTON ENERGY = 7.0 MEV

COINCIDENCE COUNTS / INTEGRATION CYCLE

0 20 40 60 80 100 120 140

0 100 200 300 400 500 600 700 800 900 1000
A 0.0002" platinum foil was then inserted into the path of the incident beam. This reduced the energy of the incident protons to 6.5 Mev. An angular correlation run was then made on target #3. The results of this run are shown in Figure 26. They resemble the results obtained for the case in which target #2 was bombarded by 7.0 Mev protons. It now appears that the thicknesses of targets #1 and #3 were nearly the same, whereas target #2 was thicker.
Figure 26: Angular Correlation - Si28 Target #3

Proton Detector Angle = 60°, Incident Proton Energy = 6.5 MeV

C O I N C I D E N C E COU N T S/INTEGRATION CYCLE

CoMMA D E TECT O R ANGLE

0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140° 150°
IV Conclusions

Table 2 lists the level of excitation of the compound nucleus, which is formed when a nucleus of a Li, Cr, Al, C, Ne, S, or Si target absorbs a 7.0 Mev incident proton. The level of excitation, in all these cases, is such that the compound nucleus has only a small number of decay channels available. For this reason, it was not necessarily expected that a direct-reaction mechanism would be observable. The purpose of these experiments was to search for any evidences of (p,p') direct reactions at these marginal energies.

Since the spin of the first excited state of 7 Li is 1/2, the angular distribution of the decay gamma rays is isotropic. Hence, the angular correlation experiment gives no indication of the interaction mechanism involved in the formation of the excited Li state. However, this target was useful in checking the alignment of the scattering system.

Due to the many decay schemes available for the emission of gamma rays from the first and second excited states of 27 Al, no conclusions could be drawn as to the interaction mechanism involved in the (p,p') scattering off 27 Al.

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EXCITATION ENERGY OF THE COMPOUND NUCLEI FORMED BY ABSORBING 7.00 MeV PROTONS.

<table>
<thead>
<tr>
<th>TARGET NUCLEUS</th>
<th>COMPOUND NUCLEUS</th>
<th>LEVEL OF EXCITATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Li}^6$</td>
<td>$\text{Be}^7$</td>
<td>11.50 Mev.</td>
</tr>
<tr>
<td>$\text{Al}^{27}$</td>
<td>$\text{Si}^{28}$</td>
<td>17.35 Mev.</td>
</tr>
<tr>
<td>$\text{C}^{12}$</td>
<td>$\text{N}^{13}$</td>
<td>8.36 Mev.</td>
</tr>
<tr>
<td>$\text{Ne}^{20}$</td>
<td>$\text{Na}^{21}$</td>
<td>13.37 Mev.</td>
</tr>
<tr>
<td>$\text{S}^{32}$</td>
<td>$\text{Cl}^{33}$</td>
<td>9.48 Mev.</td>
</tr>
<tr>
<td>$\text{Si}^{28}$</td>
<td>$\text{P}^{29}$</td>
<td>9.64 Mev.</td>
</tr>
</tbody>
</table>

TABLE 2
The other targets (C, Ne, S, Si and Cr) have (0 +) ground states and (2 +) first excited states. If, in the formation of the 2 + state by the (p,p') reaction, the angular momentum imparted to the target nucleus is in a definite plane, the axis perpendicular to this plane serves as the symmetry direction of the gamma ray angular distribution function. According to the Butler theory, this axis is the direction of the nuclear classical recoil direction, \( Q = K_i - K_f \). None of the angular correlation runs had a gamma distribution function which is symmetric about this axis.

The Levinson and Bannerjee theory predicts that the gamma ray distribution function should have the form: \( W(\Theta) = A + B \sin^2 Z(\Theta - \Theta_c) \) where \( \Theta_c \) only approximates the classical recoil direction. A least-squares curve of this form was fitted to the correlation data for the even-even nuclei. In most cases, the least squares curve fit quite nicely to the experimental data.

A reaction which proceed by a compound-nucleus mechanism could give a gamma angular distribution of the form \( W(\Theta) = A + B \sin^2 Z(\Theta - \Theta_c) \). However, it is difficult to see how a compound nucleus theory could predict:
1. A regular shift of the symmetry axis, $\Theta_0$, with a change in the proton detector angle.

2. A regular shift of the symmetry axis, $\Theta_0$, with a change in the incident beam energy.

A shift in the symmetry axis, $\Theta_0$, with a change in the proton detector angle was observed in the angular correlation experiments performed on $^{12}$C, $^{20}$Ne, and $^{28}$Si (target #2). It was also seen that as the proton detector angle increases, the symmetry axis, $\Theta_0$, decreases. This agrees with the direct-interaction theories. The angular correlation experiment was performed on $^{28}$Si (target #2) at various incident beam energies. There was observed to be a shift in the symmetry axis, $\Theta_0$, with a change in the incident beam energy. As the beam energy was lowered, the symmetry axis, $\Theta_0$, decreased. This also agrees with the direct-reaction theories.

The angular correlation experiments performed on $^{52}$Cr and $^{28}$Si (targets #1 and #3) gave a gamma distribution which was symmetric about 90 independent of the proton detector angle. These results can be explained by assuming the formation of a compound nucleus as an intermediate state in the $(p,p')$ reaction.
The length of time given to the counting of net coincidences for a certain gamma angle was determined in the following way. The protons detected by the CsI crystal were counted. When a preset number of protons were recorded, the coincidence circuits were turned off. This method of normalization is not accurate since it is impossible to distinguish between the protons scattered off the the target and the protons scattered off the collimation system, the energy degrading foils, the target holder, or the walls of the scattering system. The number of protons in the latter group varies for different proton detector angles and for different incident beam energies. The uncertainty involved in this method of normalization may be as high as 20\%.

The number of coincidences which were observed for the Si targets varied considerably with the incident beam energy. For the runs in which the gamma ray distribution function was symmetric about 90°, the coincidence counting rate was relatively high. This is additional evidence of a strong resonance in the compound nucleus. At lower beam energies where the symmetry axis, Θ, was not 90°, the number of coincidences changed slowly with beam energy.
The $^{32}$S angular correlation experiments yielded a gamma ray angular distribution which was symmetric about 90° for proton detector angles of 90° and 120°. However, the symmetry axis of the gamma ray distribution function was not 90° for a proton detector angle of 60°. A possible explanation of these results is that the angular distribution of the protons, which are inelastically scattered by direct interaction with $^{32}$S, is strongly peaked in the vicinity of 60°. Hence, only at a proton detector angle of 60°, would the direct-reactions become appreciable in comparison to the compound-nucleus reactions.

In all the angular correlation runs, there is a tendency for the number of coincidence counts to decrease with an increasing proton detector angle. This is a result of the method of normalization. The number of protons scattered from a target increases in the forward angles mainly due to the elastic coulomb scattering. Hence, the fraction of inelastically scattered protons is much less for the forward angles than for the backward angles.
Appendix

**Least-Squares Method of Determining Best Curve**

A curve of the form

\[ W(\Theta) = A + B \sin^2 2(\Theta - \Theta_0) \]  

(A.1)

was fitted to the measured gamma angular distribution for the various angular correlation experiments. The criterion of least-squares was used in calculating the parameters \( A, B, \) and \( \Theta_0 \). In order to evaluate most easily these parameters, the function \( W(\Theta) \) was transformed by means of trigonometric identities into the linear relation

\[ W(\Theta) = A + B_2 - \frac{B}{2} \left\{ \cos 4 \Theta_0 \cos 4 \Theta + \sin 4 \Theta_0 \sin 4 \Theta \right\} \]  

(A.2)

The following substitutions,

\[ A_1 = A + B_2 \]
\[ A_2 = -\frac{B}{2} \cos 4 \Theta_0 \]
\[ A_3 = -\frac{B}{2} \sin 4 \Theta_0 \]  

(A.3)

reduce equation (A.2) to

\[ W(\Theta) = A_1 + A_2 \cos 4 \Theta + A_3 \sin 4 \Theta. \]  

(A.4)

Now define \( Z(\Theta) \) as

\[ Z(\Theta) = \sum_{i=1}^{n} \rho_i (y_i - W(\Theta))^2 \]  

(A.5)

Here \( y_i \) is equal to the measured number of true coincidences at the gamma detector angle \( \Theta_i \), \( \rho_i \) is a weight factor, and the summation is taken over the \( n \) detector positions.

The three equations needed to evaluate \( A_1, A_2, \)
and $A_3$ are determined by taking the partial derivatives of $Z(\theta)$ with respect to $A_1, A_2$ and $A_3$ and setting the results of each of the partial differentiations equal to zero. The three equations are

\[ \sum_n w_i N_i - \sum_n w_i A_1 - \sum_n w_i \cos 4\theta_i A_2 - \sum_n w_i \sin 4\theta_i A_3 = 0, \]

\[ \sum_n w_i N_i \cos 4\theta_i - \sum_n w_i \cos 4\theta_i A_1 - \sum_n w_i \cos^2 4\theta_i A_2 \]

\[ = \sum_n w_i \sin 4\theta_i \cos 4\theta_i A_3 = 0, \]

\[ \sum_n w_i N_i \sin 4\theta_i - \sum_n w_i \sin 4\theta_i A_1 - \sum_n w_i \sin 4\theta_i \cos 4\theta_i A_2 \]

\[ = \sum_n w_i \sin^2 4\theta_i A_3 = 0. \]

The constants $A, B$ and $\theta_0$ are related to $A_1, A_2$ and $A_3$ as follows:

\[ \theta_0 = \frac{1}{4} \tan^{-1} \frac{A_3}{A_2}, \]

\[ B = -\frac{2A}{\cos 4\theta_0}, \]

\[ A = A_1 - B/2. \]

Probable Errors of Parameters $A, B$ and $\theta_0$.

If we repeatedly perform the experiment of measuring the number of true coincidences for $\theta = \theta_1$ and for a fixed proton detector angle, the results of the various experiments will differ. This variation of $y_i$ is the result of an uncertainty or probable error associated with $y_i$ which is due
to the statistical nature of nuclear reactions. If it can be assumed that the measured values of many experiments of $n_i$ would be Gaussian distributed, then the following derivation of the probable errors of $A, B,$ and $\Theta$ is valid.

It is seen in equation (A.1) that

$$W(\Theta) = W(A_1, A_2, A_3; \Theta)$$

(A.8)

Let $A_1^*, A_2^*$, and $A_3^*$ be those values of $A_1, A_2,$ and $A_3$ which give the least-squares best fitting curve to the data. Now

$$Z(\Theta) = \sum_{i=1}^{n} \left( \frac{n_i - W(A_1, A_2, A_3; \Theta)}{\sigma_i^2} \right)^2$$

(A.9)

Let $A$ represent the parameters $A_1, A_2, A_3$, and $A^*$ the parameters $A_1^*, A_2^*$, and $A_3^*$. We now Taylor-expand $Z(A)$ about $A^*$

$$Z(A) = Z(A^*) + \sum_{i=1}^{n} \frac{\partial^2 Z}{\partial A_i} \bigg|_{A_i^*} (A_i - A_i^*)$$

$$- \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 Z}{\partial A_i \partial A_j} \bigg|_{A_i^*} (A_i - A_i^*) (A_j - A_j^*) + \cdots$$

(A.10)

The second term of the expansion vanishes because

$$\frac{\partial^2 Z}{\partial A_i} = 0 \text{ at } A_i = A_i^*.$$

The following substitutions

$$B_i = A_i - A_i^*$$

and

$$H_{ik} = - \frac{\partial^2 Z}{\partial A_i \partial A_k} \bigg|_{A_i^*}$$

(A.11)
reduce equation (A.10) to

\[ Z(A) = Z(A^0) + \frac{1}{2} \sum_j \sum_k H_{jk} \beta_j \beta_k. \quad (A.12) \]

Let \( U \) be the unitary matrix which diagonalizes the symmetric matrix \( H \):

\[ U \cdot H \cdot U^{-1} = \begin{pmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{pmatrix} \equiv H \quad \text{where} \quad \mathcal{U} = U. \quad (A.13) \]

Let

\[ \beta \equiv (\beta_1, \beta_2, \beta_3) \quad \text{and} \quad \chi \equiv \beta \cdot U^{-1}. \quad (A.14) \]

The element of probability in the \( \beta \) space is

\[ d^3 \beta = C \exp \left[ -\frac{1}{2} (\chi \cdot U) \cdot H (\chi \cdot U) \right] d^3 \chi. \quad (A.15) \]

Since \( |U|^2 = 1 \) is the Jacobian relating the volume elements \( d^3 \beta \) and \( d^3 \chi \), we have

\[ d^3 \beta = C \exp \left[ -\frac{1}{2} \sum \frac{h_i}{2} \chi_i^2 \right] d^3 \chi. \quad (A.16) \]

Now we have

\[ h_i \delta_{ij} = \chi_i \chi_j. \quad (A.17) \]

Then

\[ \beta_a \beta_b = \sum_{i,j} \chi_i \chi_j \cdot U_{ia} U_{jb} = \sum_{i} U_{ai}^{-1} h_i \chi_i \cdot U_{ib} \quad (A.18) \]

Finally

\[ (\beta_i - \beta_i^*) (\beta_j - \beta_j^*) = (H^{-1})_{ij}. \quad (A.19) \]

For the case where

\[ W(A; \chi) = \sum_{i=1}^{m} \beta_i f_i(\chi), \quad (A.20) \]

it can be easily shown that

\[ H_{ij} = \sum_{l=1}^{p} \frac{f_i(\chi_l) f_j(\chi_l)}{f_{ij}(\chi_l)}. \quad (A.21) \]
In order to find the probable errors of the parameters \( A_1, A_2, \) and \( A_3 \) of equation (A.4), we set
\[ f_1 = 1, \quad f_2 = \cos 4\theta, \]
and \( f_3 = \sin 4\theta. \) \hspace{1cm} (A.22)

The probable errors are
\[ (A_1 - A_1') = \sqrt{(H_{11})^{-1}} = \Delta A_1, \]
\[ (A_2 - A_2') = \sqrt{(H_{12})^{-1}} = \Delta A_2 \text{ and } \]
\[ (A_3 - A_3') = \sqrt{(H_{13})^{-1}} = \Delta A_3. \] \hspace{1cm} (A.23)

The probable errors of the constants \( A, B \) and \( \Theta \) are determined from the equations
\[ \Delta A_1 = \Delta A + \frac{\Delta B}{2}, \]
\[ \Delta A_2 = -\frac{\Delta B}{2} \cos 4\Theta_0 + \frac{B}{2} \sin 4\Theta_0 (4\Delta \Theta_0), \]
and \( \Delta A_3 = -\frac{\Delta B}{2} \sin 4\Theta_0 - \frac{B}{2} \cos 4\Theta_0 (4\Delta \Theta_0). \) \hspace{1cm} (A.24)

In a set of notes based on a series of lectures given by Jay Orear, various phases of statistics which are of interest to physicists are presented.


Correction for Finite Solid Angle of \( \gamma \) Detector

M.E. Rose describes the method of correcting the least-squares best fitting curve to take into account the effect of the finite solid angle of detection.

Below is outlined the method of applying this correction to the angular correlation curves.

The correlation function may be represented in the form

\[ W(\Theta - \Theta_0) = A + B \sin^2 2(\Theta - \Theta_0) = \sum_{l=0}^{2} \alpha_l P_l(\cos(\Theta - \Theta_0)). \]  

(A.25)

Let \( \delta = \Theta - \Theta_0 \), then

\[ W(\delta) = A + B \sin^2 2\delta = A + 4B \sin^2 \delta \cos^2 \delta \]

\[ = A + 4B \left( 1 - \cos^2 \delta \right) \cos^2 \delta \]

\[ = A + 4B \cos^2 \delta - 4B \cos^4 \delta \]

\[ = \alpha_0 + \alpha_2 \left( \frac{3}{2} \cos^2 \delta - \frac{1}{2} \right) \]

\[ + \alpha_4 \left( \frac{35}{8} \cos^4 \delta - \frac{15}{4} \cos^2 \delta + \frac{5}{8} \right), \]

(A.26)

where

\[ \frac{35}{8} \alpha_4 = 4B, \]

\[ \frac{3}{2} \alpha_2 = \frac{4}{7} B, \text{ and} \]

\[ \alpha_0 = \frac{56}{15} B + A. \]

(A.27)

The constants \( A, B, \alpha_0, \alpha_2, \alpha_4 \) give the best least-squares curve to the data. We now proceed to find the values \( A', B', \alpha_0', \alpha_2', \alpha_4' \) which take into account the finite size of the gamma detecting crystal.

The absorption of the gamma radiation in the NaI crystal is proportional to \( \left[1 - e^{-\tau \mathcal{N}(\delta)}\right] \), where \( \mathcal{N}(\delta) \) represents the distance traversed by the gamma radiation incident on the crystal at an angle \( \delta \) with
the axis. Figure 27 shows

\[ \mathbf{N}_1(\beta) = 2 \sec \beta \quad \text{for } \alpha = \beta = \beta' = 0.1658 \text{ radians}, \]

\[ \mathbf{N}_2(\beta) = \cos \beta - \sec \beta \quad \text{for } \beta' < \beta < \beta' = 0.4636 \text{ radians}. \]

The absorption coefficient, \( \gamma \), depends on the material of the crystal detector as well as on the excitation energy.

The corrected function is

\[ W'(\xi) = \frac{\int W(\xi)(1 - e^{-\xi}) \, d\Omega}{\int (1 - e^{-\xi}) \, d\Omega} \quad (A.29) \]

where \( \Omega \) is the solid angle of detection.

The equation relating the primed \( \alpha' \) to the unprimed \( \alpha \) is

\[ \alpha' = \frac{\int_0^\beta (\cos \theta)(1 - e^{-\xi}) \sin \beta d\beta + \int_0^\xi P_1(\cos \beta)(1 - e^{-\xi}) \sin \beta d\beta}{\int_0^\beta (1 - e^{-\xi}) \sin \beta d\beta + \int_0^\xi (1 - e^{-\xi}) \sin \beta d\beta} \quad (A.30) \]

Knowing the values of the \((\alpha')^2\), the corrected values \( \alpha' \) and \( \beta' \) are determined from the equations

\[ \alpha_1' = - \frac{32}{35} \beta', \]

\[ \alpha_2' = \frac{8}{21} \beta', \]

\[ \alpha_0' = \frac{56}{105} \beta' + \beta'. \quad (A.31) \]
FIGURE 27. ARRANGEMENT OF NaI DETECTOR
I, Harry Fred Bowsher Jr., was born in Lima, Ohio, on February 26, 1931. I received my secondary school education in the public schools of Dayton, Ohio and Lima, Ohio. I received my college training at The Ohio State University, which granted me the Bachelor of Science and the Master of Science degrees in 1956. In September 1956, I was appointed Teaching Assistant in the Department of Physics and Astronomy at The Ohio State University. In September 1957, I was appointed Research Assistant under Dr. Hausman, supervisor of the Cyclotron Laboratory. I held that position for three years while completing the requirements for the degree Doctor of Philosophy.