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ESSAYS IN AGRICULTURAL FINANCE AND RISK MANAGEMENT

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate School
of The Ohio State University

By

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*****

The Ohio State University
2001

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The dissertation consists of three essays in agricultural finance and risk management. The essays apply innovative numerical techniques to several important issues ranging from optimal behavior of food processors to the provision of crop insurance.

The first essay presents a dynamic game model that captures the interaction between the processing capacity and oligopsonistic behavior of food processors. Unlike the conventional econometric approaches to oligopsony, the model incorporates a structural relationship between capacity, pricing decisions, and investment policies and takes into account the dynamic nature of capital management. Market power is calculated directly by comparing the optimal prices arising in the duopsony game with those in perfectly competitive and monopsony models. The effect of supply uncertainty on capacity and prices is analyzed. Spatial aspects of input supply distribution are also discussed. Numerical methods (orthogonal collocation) are used to compute and simulate the solution to the dynamic game.

The second essay presents economic analysis of the Standard Reinsurance Agreement (SRA), the document which governs the relationship between the Federal Crop Insurance Corporation and private insurance companies that deliver crop insurance products to farmers. The essay describes crop insurance products available to farmers, discusses the history of SRA and briefly outlines its provisions. It then presents the SRA Simulator, a tool designed to assist crop insurers and policymakers in assessing
the economic impact of the Agreement. Finally, the simulator is used to analyze the effects of the SRA on the rates of return of private insurance companies as well as potential improvements in both the Agreement and the ways the companies allocate their books of business across different reinsurance funds.

The third essay discusses recent developments in catastrophe insurance products, securitization of correlated risks, and application of these innovations to provision of crop insurance. The essay presents an analytical framework for pricing index-based insurance contracts and addresses several issues arising in design of such contracts. The suggested methodology is then illustrated with a case study of Nicaraguan rainfall insurance contracts.
Dedicated to my parents
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ESSAY 1

CAPACITY AND OLIGOPSONISTIC BEHAVIOR IN AGRICULTURAL FOOD PROCESSING: A DYNAMIC GAME APPROACH

1.1 Introduction

Capacity is a major factor affecting the pricing and production decisions of agricultural input processors. The infamous price collapse in the U.S. hog market in December 1998, for example, is largely attributed to an unexpected increase in hog supply, which forced the processing industry to work above its slaughter capacity for several weeks.

The effects of processing capacity on economic behavior are more profound in markets for agricultural inputs, which are typically oligopsonistic by nature. Increasing industry concentration in all major food-processing sectors (Rogers and Sexton [50]) and especially the meat-packing industry (Hayenga [25]) presents producers/farmers with fewer marketing outlets and thus makes them more vulnerable to the capacity constraints of existing processing plants.

1Testimony of J. Patrick Boyle, President and CEO of American Meat Institute, before Senate Committee on Agriculture [56].
For a long time oligopsonistic behavior received very little attention from mainstream economists, who mostly concentrated on the output side of imperfect competition and dismissed oligopsony as a special case of the former. However, Rogers and Sexton [50] strongly argue that such distinctive structural characteristics of agricultural input markets as high shipping cost, specialization of farmers and processors, limited input substitution, and bargaining power of selling and buying agents make them fundamentally different from classical oligopoly markets and thus warrant special attention to their modeling.

The last decade produced several studies devoted to the application of traditional oligopoly methods to various aspects of input competition (Schroeter [54], Azzam and Pagoulatos [2], Rogers and Sexton [50], Durham and Sexton [15], Bergman and Brännlund [8], Azzam and Schroeter [3], Muth and Wohlgenant [45]). However these papers mostly address the issue of estimating oligopsony and/or monopsony power econometrically and usually employ a static framework. Capacity, investment, and capital management, which are fundamentally dynamic processes, are ignored. The recent work by Richards, Patterson, and Acharya [49] seems to recognize the dynamic nature of oligopsonistic markets, but does not go beyond simply postulating game-theoretic behavior and quickly reducing it to an econometric model.

The conventional approach to econometric modeling of oligopsony behavior is to include a conjectural variation term in the input price equation. While conjectural variations allow one to account to some degree for the distortions introduced by imperfect competition, the very name of this term suggests imposing an ad hoc relation between the price and model parameters. Haskel and Martin [24], for example, consider a model that attempts to capture the effect of capacity on input pricing and competition by introducing a conjectural variation term linearly related to the slope of the marginal cost curve.
An alternative approach to modeling both the dynamic nature of capacity management and the oligopsonistic nature of competition in agricultural input markets is to use dynamic games. The game structure allows one to account explicitly for the interaction between market participants, while the dynamic features take into account such phenomena as capacity expansion, capital investment, and long-term pricing strategies. The fundamental difficulty in using a dynamic game approach is that the Bellman equations, which describe the game equilibria, generally do not have closed form solutions. In order to circumvent this limitation, the literature has traditionally relied on the linear-quadratic (L-Q) framework (see, for example, Karp and Perloff [32], Reynolds [48]). L-Q dynamic game models are analytically convenient because the optimal policy function is known to be linear with parameters that satisfy a well-defined nonlinear equation. Moreover, in some cases L-Q models or L-Q approximations to nonlinear models may provide fairly accurate representation of oligopolistic behavior especially around a deterministic steady state.

However, L-Q dynamic game models possess some unappealing properties for the analysis of many oligopolistic and oligopsonistic markets (Vedenov and Miranda [58]). In particular, L-Q specifications require the reward function to be quadratic, the state transition equation to be linear, and the decision variables to be unconstrained (see Reynolds [48] for an example). To accurately capture the effects of capacity in a dynamic framework, however, involves bound constraints (nonnegativity of investment) and functions with kinks (production costs with regime switching at the capacity barrier), which effectively render the L-Q approach useless.

Nevertheless, the Bellman equations of general type can be successfully solved numerically using well-established orthogonal collocation techniques (Rui and Miranda [51], Vedenov and Miranda [58]). While this method does not yield a closed
form solution, it provides a highly accurate approximate solution that may be used to analyze qualitative relationships by solving and simulating the model under different parameters.

The purpose of this essay is to present a dynamic game model of duopsony competition for an agricultural input product that explicitly incorporates the effect of capacity on players' decisions. The model also takes into account the spatial nature of input markets by considering a distribution of producers along a unit line and assuming a nonzero cost of transportation. The Bellman equations describing the problem are solved numerically using orthogonal collocation methods. The numerical solutions to the model are used to (1) analyze the relationship between capacity and pricing decisions, (2) compare equilibrium prices under monopsony, duopsony, and perfect competition, (3) analyze the spatial aspects of oligopsonistic behavior by varying the transportation cost, (4) model the effect of players' entry/exit on total capacity and farm-level prices, and (5) consider the effect of uncertainty in input supply on equilibrium capacity and prices.

1.2 Model

Consider two processors competing for a single input \( x \) and producing a single output product \( y \). In the spirit of Rogers and Sexton [50], assume that the producers are distributed along a unit line with the density \( h(s), s \in [0, 1] \), \( \int_0^1 h(s)ds = 1 \), while the processing plants are located at the end-points of the interval\(^2\). As a specific example, one may think of two hog slaughtering plants located at the opposite sides of a heavy hog producing region in Iowa and competing for the supply of market hogs from local hog finishers.

\(^2\)Throughout the essay, the subscript 1 refers to the processor located at the left end-point \( (s_1 = 0) \) while subscript 2 refers to the processor at the right end-point \( (s_2 = 1) \).
Further assume that the transportation cost is linear, with $T$ being the cost of shipment per unit distance. If $W_1$ and $W_2$ are prices offered by the processors, then the effective (net) price received by a producer at point $s$ under free-on-board (FOB) pricing is $w_1 = W_1 - Ts$, if shipping to Processor 1, and $w_2 = W_2 - T(1 - s)$, if shipping to Processor 2.

Assume that all market agents operate under perfect information and that prices set by processors are observable to all producers at no cost. Each producer ships to the plant offering the highest net price and supplies a quantity $x(w)$ that is a constant elasticity function of the net price

$$x(w) = \begin{cases} A\eta w^{\eta-1}, & w > 0 \\ 0, & w \leq 0 \end{cases},$$

where $w = \max\{W_1 - Ts, W_2 - T(1 - s)\}$. This supply function is similar to the one used by Azzam and Schroeter [2] in modeling oligopsony in the beef packing industry. It is flexible enough to account for stylized facts such as farmers' profit-maximizing behavior, opportunity to exit, and the possibility to sell a part or all of the farm production at an outside market if prices offered by either of the processors are extremely low. The parameter $A$ is essentially a measure of productivity of individual producers and may be either deterministic or stochastic.

Under FOB pricing, each processor has an effective market radius $R_i$ within which it collects all the supply offered by farmers. The total supply available to each processor is then

---

3The cost of price discovery is an interesting issue by itself, but goes beyond the scope of this essay.
\[
X_i(W_1, W_2) = \int_0^{R_i} h(s) x(W_i - Ts) ds.
\]

Assuming a uniform distribution of farmers, \( h(s) = 1 \), and using (1.1), we obtain

\[
X_i(W_1, W_2) = \int_0^{R_i} A \eta(W_i - Ts)^{\eta-1} ds = \frac{A}{T} (W_i^\eta - (W_i - TR_i)^\eta). 
\] (1.2)

An interesting effect here is that the market radius of each plant is affected by the prices set by both processors. There are three nontrivial situations that may arise. First, both plants split the total available supply. In this case, the market radius of each plant extends to the indifference point \( s^* \) such that

\[
W_1 - Ts^* = w_1(s^*) = w_2(s^*) = W_2 - T(1 - s^*), \quad s^* \in (0, 1). 
\] (1.3)

Second, one processor may capture the entire supply by setting its price so high that the net price received by the farmer at the opposite end of the unit line exceeds the price offered by the other processor. Third, both processors may set prices too low compared to the transportation cost so that the net price becomes negative in the middle region \([R_1, 1 - R_2]\) from which neither processor receives any supply. More formally,

\[
R_1 = \begin{cases} 
1, & s^* \geq 1, \\
0, & s^* \leq 0, \\
\frac{w_1}{T}, & \frac{w_1}{T} \leq s^*, \quad 0 < s^* < 1, \\
s^*, & \text{otherwise,}
\end{cases}
\]

and

\[
R_2 = \begin{cases} 
0, & s^* \geq 1, \\
1, & s^* \leq 0, \\
\frac{w_2}{T}, & \frac{w_2}{T} \geq s^*, \quad 0 < s^* < 1, \\
1 - s^*, & \text{otherwise,}
\end{cases}
\] (1.4)

where \( s^* = (W_1 - W_2 + T)/2T \) is the indifference point determined from (1.3). Note that the conditions \( W_1/T \leq s^* \) and \( W_2/T \geq s^* \) in the third case are equivalent.
Each plant $i$ processes acquired input $X_i$ into final product $Y_i$ according to the constant elasticity production function $Y_i = f(X_i) = \alpha_i X_i^{\beta_i}$, $i = 1, 2$. The final product is sold in the competitive outside market at the price $p$, with both processors acting as price-taking profit maximizers$^4$.

Now let us consider the cost side of plant operations. First, each processor pays for the acquired input at the price it sets. Note that for simplicity of presentation we do not consider explicitly the costs of other input factors such as labor. This is not an unreasonable assumption, since the cost of raw input usually accounts for a major part of the total cost. MacDonald and Ollinger [36], for example, indicate that the cost of live animal and meat input accounts for 74 percent of the hog slaughter costs, while labor carries only an 11 percent factor share. In addition, the cost of labor can be implicitly included in the cost of processing itself, which we discuss next.

In order to model the effect of capacity on processors’ decisions, we assume that each plant $i$ has some level of capital $K_i$ that affects processing cost. More specifically, we assume that each plant operates at a constant marginal cost as long as it processes input below the capital level$^5$. When the level of input exceeds the capital, the plant incurs a penalty in the form of increasing marginal cost. Formally, we write the production cost as

$$q_i(X_i, K_i) = \theta_i X_i + \frac{c}{3} (\max\{0, X_i - K_i\})^3.$$ (1.5)

$^4$The assumption of price-taking in the output market is made only for simplicity of presentation and is not restrictive. The model can be easily adjusted for the case of both input and output competition, e.g. by letting the output price $p$ to be a function of both output levels $Y_i$, $i = 1, 2$.

$^5$This constant marginal cost may account for the input factors other than the raw input itself, such as labor, administrative overhead, etc.

$^6$The cubic term guarantees that the marginal cost is continuously differentiable at the point of regime switch.
The above cost function does not exhibit economies of size. Although this could be easily corrected by adding a constant fixed cost to \( q_i \), this constant term would disappear in the first-order conditions and thus would not fundamentally change the profit-optimizing decisions. Therefore we choose to drop the constant term for transparency of presentation, since the conclusions of the analysis hold the same for a cost function exhibiting economies of size.

While capital here is not acting as a hard capacity in the sense that the plant cannot process input beyond the capital level, the cost specification in (1.5) does reflect the stylized fact that production above some natural level results in increasingly higher additional cost, e.g. overtime wages, additional equipment rental, etc. Besides, increasing the value of parameter \( \zeta \) makes the overproduction penalty more severe, so that at the limit the capital \( K_i \) becomes a hard capacity as \( \zeta \to \infty \). For practical purposes, however, even relatively small values of \( \zeta \) provide interesting results and significantly affect processor's behavior. Therefore throughout the essay we treat the capital stock \( K \) as capacity and use the words capital and capacity interchangeably.

Note that one traditional application of capacity in the economic literature is that of entry deterrence (Tirole [55]). In this case, the capacity levels are usually higher than those optimal under no entry threat and therefore do not constrain plants production decisions. However, it is the constraining nature of capacity that affects existing competition and thus prices. Thus, we choose not to consider entry deterrence in our essay in order to concentrate on the capacity-price relationship.

Since capital management is an inherently dynamic problem, we assume that there are two factors affecting each plant's capital level over time. First, the capital depreciates at some natural rate \( \xi \) and second, each processor can invest in the capital. If \( K_i \) is the current period capital level at plant \( i \) and \( z_i \) is the investment in capital, then the next period capital is \( K_i' = (1 - \xi)K_i + z_i \). We also assume that the capital
investment is costly, with the cost of investment being a constant elasticity function of the investment level \( c_i(z_i) = \gamma_i z_i^\gamma / \kappa \). Finally, we do not allow selling of assets, so that the only way a plant can eliminate excess capacity is to let it depreciate at the natural rate \( \xi \).

Processor \( i \)'s one-period profit can be thus written as

\[
\pi_i(W_1, W_2, z_1, z_2, K_i, K_2) = \bar{p} Y_i - X_i W_i - q_i(X_i, K_i) - c_i(z_i),
\]

where output \( Y_i \), cost of production \( q_i \), and cost of investment \( c_i \) are specified above. Each processor chooses a pricing strategy \( W_i \) and capital investment program \( z_i \) as to maximize the present value of the flow of current and future profits discounted at some rate \( \delta \) taking the opponent's actions as given.

The Markov perfect equilibrium of the above game, if it exists, must satisfy (Fudenberg and Tirole [19]) the pair of Bellman equations

\[
V_i(K_1, K_2) = \max_{z_i, W_i} \{ \bar{p} Y_i - X_i W_i - q_i(X_i, K_i) - c_i(z_i) + \delta V_i(K'_1, K'_2) \},
\]

subject to \( K'_i = (1 - \xi) K_i + z_i, \quad z_i \geq 0, \quad i = 1, 2, \)

where \( X_i(W_1, W_2), \quad i = 1, 2, \) are determined by (1.2) and (1.4). Note, however, that the effect of investment is not realized until the next period. At the same time, pricing decisions in the current period do not affect the future capital level. Therefore, in each period, processors operate under the fixed capacities \( K_i, \quad i = 1, 2, \) and optimize their short-run profits while taking the other player's actions as given. In the long run, they can adjust their capacity level by either investing in the capital stock \( K_i \) or allowing it to depreciate at the natural rate. This observation helps us to simplify the system of equations (1.7), since the short-run profit optimization problem can be solved in advance for all combinations of states \( K_1 \) and \( K_2 \).
More specifically, let $\tilde{\pi}_i(K_1, K_2)$ be a solution to the short-run profit optimization problem

$$\max_{W_i} \tilde{p}Y_i - X_i(W_1, W_2)W_i - q_i(X_i, K_i), \quad K_i \text{ fixed}, \quad i = 1, 2, \quad (1.8)$$

with $X_i(W_1, W_2), \quad i = 1, 2,$ determined by (1.2) and (1.4). Then the system in (1.7) can be rewritten as

$$V_i(K_1, K_2) = \max_{z_i} \{\tilde{\pi}_i(K_1, K_2) - c_i(z_i) + \delta V_i(K'_1, K'_2)\}, \quad (1.9)$$

$$\text{s.t.} \quad K'_i = (1 - \xi)K_i + z_i, \quad z_i \geq 0, \quad i = 1, 2.$$

The latter together with (1.8) is the model describing the behavior of both processors.

It is worth to mention here that the above dynamic model presents a fundamental challenge to the conventional L-Q methodology. First of all, all functions in the short-term profit are inherently nonlinear. Second, the cost function specified in (1.5) has a kink at the point $X_i = K_i$ that cannot be replaced by a linear or quadratic function. Finally, the nonnegativity of investment becomes a binding constraint for some $K_i$ (see Section 1.5). However, by using numerical solution methods, we can circumvent the limitations of the L-Q technique and find a quite accurate solution to the model.

The equations in (1.9) are functional equations, i.e. their solutions are not vectors, but rather entire functions of the state variables. More specifically, the solution to the system consists of the value function $V_i(K_1, K_2)$ itself, the optimal pricing policy $W_i(K_1, K_2)$, and the optimal investment strategy $z_i(K_1, K_2)$ defined over an entire state space of capital levels $(K_1, K_2)$. Note that even though each processor optimizes with respect to its own decision variables, the optimal strategies depend on capacities of both plants due to the interaction of both players' decisions in determining the market radii in (1.4).
The following section presents Bellman equations and profit maximization problems for several modifications of the above basic model. These are used later in the result section.

1.3 Modifications of the Basic Model

1.3.1 Monopsony

Assume that the single processing plant is located at the left end-point of the unit line (s = 0). The derivations of the duopsony model hold here with some obvious changes. Equation (1.2) still holds. The marketing radius $R$ is now determined only by the condition that the net price at any point $s$, $0 < s < R$, is nonnegative, i.e. $R = \min\{W/T, 1\}$. The short-term profit optimization problem becomes

$$\max_{W} \{\bar{p}Y - WX - q(X, K)\}, \quad K \text{ fixed}, \quad (1.10)$$

and the Bellman equation describing the long-term investment policy becomes

$$V(K) = \max_{z} \{\bar{\pi}(K) - c(z) - \delta V(K')\} \quad (1.11)$$

$$\text{s.t. } K' = (1 - \xi)K + z, \quad z \geq 0,$$

where $\bar{\pi}(K)$ is the optimal value in (1.10).

Note that in this model we ignore the threat of entry and assume that capacity only affects the processing capabilities. The model can be easily modified in order to adapt entry deterrence as firm’s objective, but this topic goes beyond the scope of the present essay.

\(^{7}\)For the monopsony case, we use the same notation as before but drop the subscripts.
1.3.2 Perfect Competition

Let a large number of small identical firms be located at each end-point of the interval so that capacity is not an issue. Each firm is a price taker on both output and input sides, processes input at the constant marginal cost $\theta_i$, and operates at zero profit in the short run. Equations (1.2) and (1.4) still hold and the competitive prices at each end-point of the unit line are then determined from the condition

$$\pi_i(W_1, W_2) = \bar{p}Y_i - X_i(W_i + \theta_i) = 0. \quad (1.12)$$

1.3.3 Consolidation

In this case we still have two plants, but the prices are now set at one central office so as to maximize the total flow of discounted future profits from both plants. The relevant Bellman equation is

$$V(K_1, K_2) = \max_{z_1, z_2} \{\tilde{\pi}(K_1, K_2) - (c_1(z_1) + c_2(z_2)) + \delta V(K'_1, K'_2)\}, \quad (1.13)$$

s.t. $K'_i = (1 - \xi)K_i + z_i, \quad z_i \geq 0, \quad i = 1, 2,$

where $\tilde{\pi}(K_1, K_2)$ is a solution to the short-term profit optimization problem

$$\max_{W_1, W_2} \{\bar{p}(Y_1 + Y_2) - (X_1W_1 + X_2W_2) - (q_1(X_1, K_1) + q_2(X_2, K_2))\}, \quad K_1, K_2 \text{ fixed}, \quad (1.14)$$

and $X_i(K_1, K_2)$ are again determined from (1.2) and (1.4).

1.3.4 Supply Uncertainty

Let the productivity parameter $A$ in (1.1) follow a Markov chain with two possible states. Let $\varepsilon_1$ and $\varepsilon_2$ be multiplicative productivity shocks associated with these
states and let \( Pr_{lm} \) be the probabilities associated with the transitions from state \( \varepsilon_l \) to \( \varepsilon_m \), \( l, m = 1, 2 \). Since the productivity parameter \( A \) eventually determines the supply \( X_i \) acquired by each plant, the shocks propagate into \( X_i \) and the system of equations (1.9) becomes

\[
V_i(K_1, K_2, \varepsilon_l) = \max_{z_i} \{ \bar{\pi}_i(K_1, K_2, \varepsilon_l) - c_i(z_i) + \delta E_z V_i(K_1', K_2', \varepsilon_l) \}, \quad (1.15)
\]

subject to

\[
K_i' = (1 - \xi)K_i + z_i, \quad z_i \geq 0, \quad i = 1, 2, \quad l = 1, 2,
\]

\[
E_z V_i(\cdot, \cdot, \varepsilon_l) = \sum_{m=1}^{2} Pr_{lm} V_i(\cdot, \cdot, \varepsilon_m),
\]

where \( \bar{\pi}_i(K_1, K_2, \varepsilon_l) \) is the optimal value in

\[
\max_{w_i} \bar{p}Y_i - X_i(W_1, W_2, \varepsilon_l)W_i - q_i(X_i, K_i), \quad K_i \text{ fixed}, \quad i = 1, 2. \quad (1.16)
\]

Note that the L-Q methodology, while admitting a simple form of uncertainty in the state transition rule, cannot handle more complex stochastic shocks such as those described above. Once again, numerical methods can be successfully used to obtain an accurate solution in the situation where other approaches are either inapplicable or simply do not exist.

1.4 Solution Methodology

Any conventional optimization techniques, e.g. a modification of the Newton method or various gradient methods (Miranda and Fackler [44], Judd [31]), can be applied to the short-term profit maximization problem in (1.8) as well as (1.10) and (1.16). Equation (1.12) is essentially a root-finding problem and can be solved by either function iteration or the Newton method.
Solution of the Bellman equations is a more challenging task, but can also be accomplished by using a special numerical technique called (orthogonal) collocation. The collocation method for dynamic games is a natural combination of methods used to compute an equilibrium of a static game and those used to compute the solution of a Bellman equation in a single-agent dynamic optimization problem.\footnote{Obviously, this does not imply that a Markov Perfect Equilibrium of the game can be obtained as a combination of a solution to some static game and a solution to some single-agent dynamic programming model.}

It is instructive to briefly summarize these two simpler numerical methods before describing in detail how to solve a general dynamic game (see also Judd \cite{Judd29, Judd31}, Miranda and Fackler \cite{MirandaFackler44}, Vedenov and Miranda \cite{VedenovMiranda58}, Rui and Miranda \cite{RuiMiranda51}). In addition, the methodology for solution of a single-agent optimization model can be used to solve the Bellman equations for modifications of the basic model presented in Section 1.3 (cases of monopsony and consolidation).

First, consider a simple static $m$-player game, in which each player $i$ receives a reward $f_i(z_i, z_{-i})$ that depends on his action $z_i$ and the actions of others $z_{-i}$. If a vector of actions $z^*$ is a Nash equilibrium of such a game, it must solve

$$\max_z f_i(z, z^*_{-i}) \quad (1.17)$$

for every $i$. An iterative method that can often successfully compute a solution to (1.17) is as follows:

0. **Initial Step:** Make an educated guess as to what $z^*$ is.

1. **Update Step:** Given the current iterate for $z^*$, update $z^*_i$ for each player $i$ by solving player $i$'s optimization problem while keeping $z^*_{-i}$ fixed at its current value.
2. Convergence Check: If the change in \( z^* \) from the previous iteration is less than some prescribed tolerance, stop; otherwise, return to Step 1.

If convergence is achieved, then the convergent \( z^* \) value solves (1.17), and thus must be a Nash equilibrium of the static \( m \)-player game.

Now consider a single-agent dynamic optimization model (see, for example, Subsections 1.3.1 and 1.3.3). In any period, the agent observes the state of an economic process \( K \), takes an action \( z \) that belongs to some state-contingent action space \( Z(K) \), and earns a reward \( f(K, z) \) that depends on both the state of the process and the action taken\(^9\). In general, the state of the economic process may follow a controlled Markov probability law. Specifically, the state of the economic process in the following period \( K' \) depends on the state and action in the current period and possibly an exogenous random shock \( \varepsilon \) that is unknown in the current period:

\[
K' = g(K, z, \varepsilon).
\]

The agent seeks a policy (or strategy) \( z^* \) of state-contingent actions \( z = z^*(K) \) that will maximize the present value of current and expected future rewards discounted at a per-period factor \( \delta \).

The solution of the single-agent dynamic optimization problem is characterized by the Bellman equation

\[
V(K) = \max_{z \in Z(K)} \{ f(K, z) + \delta \mathbb{E}_\varepsilon V(g(K, z, \varepsilon)) \}, \quad K \in K,
\]

whose unknowns are functions defined on the state space \( K \). Except in very special cases, the Bellman functional equation lacks an analytic closed form solution and can

\(^9\)Both \( K \) and \( z \) may be vectors.
only be solved approximately using computational methods. A variety of methods are available for computing approximate solutions to Bellman equations, including L-Q approximation and space discretization. However, in most applications, particularly stochastic models with bounded decisions, these methods either provide unacceptably poor approximations or are computationally inefficient (Miranda [43]).

A method that can efficiently offer a highly accurate numerical solution to most Bellman equations is the collocation method. The latter is a solution strategy rather than a specific technique. First, one approximates the unknown value function \( V \) using a linear combination of known basis functions \( \phi_1, \phi_2, \ldots, \phi_n \) whose coefficients \( a_1, a_2, \ldots, a_n \) are to be determined:

\[
V(K) \approx \sum_{j=1}^{n} a_j \phi_j(K).
\]

Second, the basis function coefficients \( a_1, a_2, \ldots, a_n \) are fixed by requiring the approximant to satisfy the Bellman equation, not at all possible states, but rather at \( n \) states \( K_1, K_2, \ldots, K_n \), called the collocation nodes. Many collocation basis–node schemes are available to the analyst, including Chebychev orthogonal polynomials and spline function approximation. The best choice of the basis–node scheme is application-specific, and typically depends on the curvature properties of the value function.

The collocation strategy replaces the Bellman functional equation with a system of \( n \) nonlinear equations in \( n \) unknowns. Specifically, to compute the approximate solution to the Bellman equation, or more precisely, to compute the \( n \) basis coefficients \( a_1, a_2, \ldots, a_n \) in the representation of the value function approximant, one solves the equation system.
\[
\sum_j a_j \phi_j(K_l) = \max_{z \in Z(K_l)} \left\{ f(K_l, z) + \delta E \sum_{j=1}^n a_j \phi_j(g(K_l, z, \varepsilon)) \right\}
\]

for \( l = 1, 2, \ldots, n \), which may be compactly expressed in vector form as the \textit{collocation equation}

\[
\Phi a = v(a).
\]

Here, \( \Phi \), the \textit{collocation matrix}, is an \( n \times n \) matrix whose typical \( ij^{th} \) element is the \( j^{th} \) basis function evaluated at the \( i^{th} \) collocation node,

\[
\Phi_{ij} = \phi_j(K_i)
\]

and \( v \), the \textit{conditional value function}, is a function from \( \mathbb{R}^n \) to \( \mathbb{R}^n \) whose typical \( i^{th} \) element is

\[
v_i(a) = \max_{z \in Z(K_i)} \left\{ f(K_i, z) + \delta E \sum_{j=1}^n a_j \phi_j(g(K_i, z, \varepsilon)) \right\}.
\]

The conditional value function gives the maximum value obtained when solving the optimization problem embedded in Bellman's equation at each collocation node, given the value function approximation implied by the coefficient vector \( a \).

In principle, the collocation equation may be solved using any nonlinear equation solution method. For example, one may write the collocation equation in the equivalent fixed-point form \( a = \Phi^{-1} v(a) \) and use function iteration method, which employs the iterative update rule

\[
a \leftarrow \Phi^{-1} v(a).
\]

Alternatively, one may write the collocation equation as a root finding problem \( \Phi a - v(a) = 0 \) and solve for \( a \) using Newton's method, which employs the iterative update rule
Here, \( v'(a) \) is the \( n \times n \) Jacobian of the conditional value function \( v \) at \( a \). A typical element of \( v' \) may be computed by applying the envelope theorem to the optimization problem in (1.18), namely

\[
v'_j(a) = \frac{\partial v_l(a)}{\partial a_j} = \delta E e_j (g(K_l, z_l, \epsilon))
\]

where \( z_l \) is the optimal argument in (1.18).

Regardless of which nonlinear equation solution method is used, the conditional value \( v_l(a) \) must be computed for every \( l \), i.e. the optimization problem embedded in Bellman's equation must be solved at every collocation node \( K_l \), taking the current coefficient vector \( a \) as fixed. Even though the Newton method has the additional requirement of computing the Jacobian of \( v \), however, this comes at only a small additional cost because most of the effort required to compute the derivative comes from solving the optimization problem embedded in Bellman's equation.

Now consider an \( m \)-player dynamic game model with each player's state described by one state variable\(^{10}\). In any period, each player \( i \) observes the state of an economic process \( K = (K_1, K_2, \ldots, K_m) \), takes an action \( z_i \) that belongs to some state-contingent action space \( Z_i(K) \), and earns a reward \( f_i(K, z_i, z_{-i}) \) that depends on the state of the process, the action taken by the player \( z_i \), and the actions taken by other players \( z_{-i} \).

In general, the state of the economic process follows a controlled Markov probability law. Specifically, the state \( K' \) in the following period depends on the states and actions of all the players in the current period and possibly an exogenous random shock \( \epsilon \) that is unknown in the current period:

\(^{10}\)The methodology can be easily extended to the case of more than one state variable per player.
Each player $i$ seeks a policy $z_i^*$ of state-contingent actions $z_i = z_i^*(K)$ that will maximize the present value of his own current and expected future rewards discounted at a per-period factor $\delta$, given the policies that will be followed by the other players.

The above dynamic game setup is similar to those discussed in Fundenberg and Tirole [19] (Chapter 13). The Markov perfect equilibrium of such a game is characterized by a system of $m$ simultaneous Bellman equations

$$V_i(K) = \max_{z_i \in \mathcal{Z}_i(K)} \left\{ f_i(K, z_i, z_{-i}^*(K)) + \delta \mathbb{E}_\varepsilon V_i(g(K, z_i, z_{-i}(K), \varepsilon)) \right\}, \quad K \in \mathcal{K},$$

whose unknowns are the value functions $V_i(\cdot)$ and the optimal policies $z_i^*(\cdot)$ defined on the state space $\mathcal{K}$.

The collocation method may be applied to this system of Bellman equations much as to a single Bellman equation. First, one approximates the unknown value functions $V_i$ using a linear combination of known basis functions $\phi_1, \phi_2, \ldots, \phi_n$ whose coefficients $a_{i1}, a_{i2}, \ldots, a_{in}$ are to be determined:

$$V_i(K) \approx \sum_{j=1}^{n} a_{ij} \phi_j(K).$$

Second, the basis function coefficients $a_{i1}, a_{i2}, \ldots, a_{in}$ are fixed by requiring the approximants to satisfy the Bellman equations, not at all possible states, but rather at $n$ collocation nodes $K_1, K_2, \ldots, K_n$. 

$$K' = g(K, z, \varepsilon).$$
The collocation strategy replaces the Bellman functional equations with a system of \( mn \) nonlinear equations in \( mn \) unknowns. Specifically, to compute the approximate solution to the \( i^{th} \) Bellman equation, or more precisely, to compute the \( n \) basis coefficients \( a_{i1}, a_{i2}, \ldots, a_{in} \) in the basis representation of the \( i^{th} \) value function approximant, one solves the equation system

\[
\sum_j a_j \phi_j(K_i) = \max_{z \in Z_i(K_i)} \left\{ f_i(K_i, z, z_{-i}) + \beta \mathbf{E} \sum_{j=1}^n a_j \phi_j(g(K_i, z, z_{-i}, c)) \right\}
\]

for \( l = 1, 2, \ldots, n \), where \( z_{-i} \) are the optimal actions taken by other players when the state is \( K_i \).

This system of equations may be again compactly expressed in vector form

\[
\Phi a_i = u_i(a_i, z_{-i}),
\]

where \( \Phi \) is the collocation matrix, and \( u_i \), player \( i \)'s conditional value function, is a function from \( \mathbb{R}^n \) to \( \mathbb{R}^n \) whose typical \( l^{th} \) element is

\[
v_u(a) = \max_{z \in Z_i(K_i)} \left\{ f_i(K_i, z, z_{-i}) + \beta \mathbf{E} \sum_{j=1}^n a_j \phi_j(g(K_i, z, z_{-i}, c)) \right\}
\]

for \( l = 1, 2, \ldots, n \). Again, \( a_i \) is the vector of basis coefficients associated with the approximation of player \( i \)'s value function, and \( z_{-i} \) is the vector of the other players' actions at the collocation states.

Just as in case of single-agent dynamic optimization model, the collocation equations for an \( m \)-player game may be solved using nonlinear equation solution methods. However, one cannot solve the individual player collocation equations independently because one player's optimal action depends on the actions taken by others. Still, one can solve collocation equations for the \( m \)-player game by integrating the iterative
strategies used to solve static games and single-agent dynamic optimization problems. For example, one may write the collocation equations in the equivalent fixed-point form $a_i = \Phi^{-1}v(a_i, z_{-i})$ and use function iteration method:

0. **Initial Step:** Select the degree of approximation $n$, a set of basis functions $\phi_j$, and a set of collocation nodes $K_i$; for each player $i$, make a guess for the basis coefficient vector $a_i$ and action vector $z_i$ at the collocation nodes.

1. **Solution Step:** Holding the basis coefficient vectors $a_i$ and action vectors $z_i$ constant, solve the optimization problem in (1.19) and compute $v_i(a_i, z_{-i})$ for each player $i = 1, 2, \ldots, m$; let $z'_i$ denote the action vector that solves (1.19).

2. **Update Step:** For every player $i$, update the action vector by setting

$$z_i \leftarrow z'_i$$

and update the basis coefficient vector by setting

$$a_i \leftarrow \Phi^{-1}v_i(a_i, z_{-i}). \quad (1.20)$$

3. **Convergence Check:** If the change in the coefficient vectors from the previous iteration is less than some prescribed tolerance, stop; otherwise, return to Step 1.

Alternatively, one may write the collocation equation as a root finding problem $\Phi a_i - v_i(a_i, z_{-i}) = 0$ and solve for the $a_i$ and $z_i$ using a mixed Newton method. In this case, the steps of the algorithm are exactly the same as above, with the update rule (1.20) in Step 2 replaced by

$$a_i \leftarrow a_i - [\Phi - v'_i(a_i, z_{-i})]^{-1}[\Phi a_i - v_i(a_i, z_{-i})].$$

21
Here, $v'_i(a_i, z_{-i})$ is the $n \times n$ Jacobian of the conditional value function $v_i$ with respect to $a_i$. A typical $j^{th}$ element of $v'_i$ may be computed by applying the envelope theorem to the optimization problem in (1.19):

$$v'_{ij}(a) = \frac{\partial v_i}{\partial a_j}(a_i, z_{-i}) = \delta E_z \phi_j(g(K_i, z'_i, z_{-i}, \varepsilon))$$

where $z'_i$ is the optimal argument in the maximization problem (1.19). As a variant of this mixed Newton method, one could also employ a mixed quasi-Newton method, replacing the derivative in the update rule with a finite difference approximation.

The best choice of iteration method is not always clear. If a good initial guess of the solution is available, then Newton's method can be considerably faster than function iteration. (Asymptotically, Newton's method is guaranteed to converge at a quadratic rate, while function iteration only at a linear rate.) Unfortunately, Newton's method can be highly sensitive to initial condition. If a poor starting value is chosen, the derivative information used by Newton's method at early iterations can be highly misleading and cause the algorithm to diverge. In these instances, function iteration method, which is asymptotically slower but more robust to initial values, may be a better choice. In practice, a combination of the two methods often works well. Here, function iteration method is used initially, but replaced by Newton's method after a small number of iterations. It is this combined method that is used in solving the oligopsony model.

The accuracy delivered by the approximation method depends on a number of factors, most notably the number of basis functions and collocation nodes $n$. Typically, the greater the degree of approximation $n$, the more accurate the resulting approximant (although this cannot be proven analytically), but the more expensive is its computation. For this reason, choosing a good set of basis functions and collocation
nodes is critical for achieving computational efficiency. Approximation theory sug‐
gests that Chebychev polynomials basis functions and Chebychev collocation points
will often make superior choices, provided the solution to the functional equation
is relatively smooth. Otherwise, linear or cubic basic splines may perform better
(Miranda [44], Judd [31]).

Note that both the Chebychev polynomials and splines are defined as functions
from \( \mathbb{R} \) to \( \mathbb{R} \). In order to accommodate these for the two-dimensional state space in
our model, the following approach can be used. First, a set of nodes and a correspond‐
ing set of basis functions are chosen in each dimension. Then the two-dimensional
grid is created as a Cartesian product of one-dimensional nodes. In the same way,
the two-dimensional basis functions are specified as products of corresponding one‐
dimensional basis functions.

Formally, if \( K_{1p}, p = \overline{1,n_1}, \) and \( K_{1q}, q = \overline{1,n_2}, \) are the sets of nodes for each
state variable, then the collocation nodes \( K_l, l = \overline{1,n}, n = n_1n_2, \) are defined as all
possible pairs \( K_l = \{ K_{1p}, K_{1q} \}, p = \overline{1,n_1}, q = \overline{1,n_1}. \) In the same way, if \( \phi_p(K_1) \)
and \( \phi_q(K_2) \) are basis functions defined on the state spaces \( K_1 \) and \( K_2, \) respectively,
then the two-dimensional basis functions \( \phi_l(K), l = \overline{1,n}, n = n_1n_2, \) are defined as
\( \phi_l(K) = \phi_l(K_1, K_2) = \phi_p(K_1)\phi_q(K_2) \) for all possible combinations of \( p = \overline{1,n_1} \) and
\( q = \overline{1,n_1}. \)

For the purposes of simulation, we used Chebychev basis functions and collocation
nodes with \( n_1 = n_2 = 25 \) nodes for each state (a 625-node grid). All numerical compu‐
tations were performed in MATLAB programming environment using the CompEcon
toolbox. The source code and detailed discussion of specific routines can be found in
Miranda and Fackler [44].
Finally, one must give some thought to the choice of method used to solve the maximization problem embedded in Bellman's equation. Of course, many methods are available for solving constrained optimization problems such as solving the first-order condition using Newton's method (see Miranda and Fackler [44], Judd [31]) or using a simple derivative-free golden section search algorithm (Press et al. [47]). We found that the Newton method was superior to the golden section search in both speed and accuracy.

1.5 Simulation Results

Throughout this section, we assume that both plants utilize an identical technology. We select the following benchmark parameters: for the supply function, \( A = 2, T = 2, \) and \( \eta = 1.5; \) for the price of output product \( \bar{p} = 8; \) for the transformation function, \( \alpha_1 = \alpha_2 = 1 \) and \( \beta_1 = \beta_2 = 0.6; \) for the cost of production function, \( \theta_1 = \theta_2 = 1 \) (the constant part of the marginal cost) and \( \zeta_1 = \zeta_2 = 2 \) (severity of the over-capacity production penalty); for the cost of investment function, \( \gamma = 1.5 \) and \( \kappa_1 = \kappa_2 = 0.3. \) Finally, the depreciation rate is set to \( \xi = 0.07 \) and the discount factor is set to \( \delta = 0.95. \)

First we present simulation results for the base case of two identical plants competing with each other. We then compare this behavior with that of monopsonistic and perfectly competitive markets. We also illustrate the effect of transportation cost on prices offered under each regime and compute the market power exerted by the duopsonists. Next we consider some policy implications, such as the effect on price and overall capacity if (1) a new plant is built in addition to an existing one and (2) both plants merge and fall under the same management. Finally, we demonstrate the effect of supply uncertainty on the long-term average capacity maintained by each plant.
1.5.1 Base case

Figures 1.1-1.6 summarize the optimal behavior of processors in the duopoly game. The optimal short-run pricing strategy obtained by solving (1.8) and optimal acquired input under this strategy are shown for Processor 1 in Figs. 1.1 and 1.2, respectively.

When Processor 1's own capacity is limited (small $K_1$), both the price it offers and the input it acquires are relatively low. As the capacity increases, Processor 1 finds it profitable to process more input and thus offers a higher price to attract it. At some point, capacity reaches the level where it does not constrain production decisions and acquired input is determined from the short-term optimization problem (1.8) with the constant marginal cost. The offered price is then set at some constant level to attract the necessary supply within the market radius, which corresponds to the plateau on both figures.

The effect of the other plant's capacity is similar. When Processor 2's capacity is low, it cannot compete adequately and Processor 1 can attract enough supply by offering a lower price. As Processor 2's capacity increases, it becomes more competitive. Processor 1 now has to offer higher prices yet acquires lower input. Finally, when both processors have sufficiently high capacities, they split the available supply and set their prices just high enough to acquire all the input from their respective market areas.

Shown in Fig. 1.3 is the optimal investment policy for Processor 1. Note that under no uncertainty, both processors will eventually arrive at some optimal capacity which they then maintain in perpetuity. This optimal level of capacity is the steady state of the dynamic game. At the steady state, the optimal strategy is to invest exactly enough to offset depreciation of capital, i.e. $z_i(K_i^*) = \xi K_i^*$, where $K_i^*$ is the

Since both plants are identical, we present all results for only one of them.
<table>
<thead>
<tr>
<th>Number of nodes</th>
<th>Computation Time</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 × 5</td>
<td>0.9s</td>
<td>2.1 \times 10^{-4}</td>
</tr>
<tr>
<td>7 × 7</td>
<td>1.0s</td>
<td>8.9 \times 10^{-5}</td>
</tr>
<tr>
<td>10 × 10</td>
<td>1.2s</td>
<td>6.9 \times 10^{-5}</td>
</tr>
<tr>
<td>14 × 14</td>
<td>2.2s</td>
<td>4.9 \times 10^{-5}</td>
</tr>
<tr>
<td>20 × 20</td>
<td>19.5s</td>
<td>4.2 \times 10^{-5}</td>
</tr>
</tbody>
</table>

Table 1.1: Computation times and $L_{\infty}$ norms of residuals for different numbers of approximation nodes.

steady-state capacity. When the processor's own capacity is lower than the steady-state level, the optimal policy is to invest in capital to raise it to the optimal level. On the other hand, when the plant has excess capacity, it is optimal to invest zero and allow capital to depreciate down to the steady-state level. Note again that this type of solution is impossible to obtain using the L-Q techniques, which do not allow binding constraints.

Figure 1.4 shows the short-run optimal profit corresponding to the pricing strategy in Fig. 1.1. An increase in own capacity results in increasing profits due to higher processing volume, while an increase in opponent's capacity decreases profits due to stronger competition. The value function shown in Fig. 1.5 is essentially a smoothed version of short-run profit.

Finally, shown in Fig. 1.6 are residuals of the Bellman equation computed at points other than the collocation nodes and expressed as a fraction of the value function itself. The sine-shaped structure of the residuals is common for computations using Chebychev polynomials. The norm of the residuals is of the order of $10^{-5}$ which represents extremely high accuracy.
In order to illustrate the trade-off between the approximation accuracy and computation time, we also ran the model with different numbers of nodes. The computation times and accuracy computed as an $L_\infty$ norm of the residuals for Chebychev polynomials and Chebychev collocation nodes are presented in Table 1.1. As mentioned before, higher degree of approximation results in higher accuracy but also higher computational times. Note also that the improvement in accuracy, while exists, is not very dramatic. This is explained by the kinks in the optimal decision functions. The approximation error is especially large in the area where optimal investment policy and optimal pricing policy become flat (Fig. 1.6). Since the $L_\infty$ norm computes the maximum absolute value in the range, this error dominates the accuracy estimate even though approximation accuracy improves dramatically in other areas as the number of nodes increases.

1.5.2 Alternative Market Structures and Effect of Transportation Cost

The models of processors’ behavior in cases of monopsony and perfect competition are derived in Section 1.3 (equations (1.10)–(1.12)). The specific functional forms and parameters for these models are the same as in the base case.

Presented in Figs. 1.7–1.9 are optimal prices for the duopsony, monopsony, and competitive markets, respectively, as functions of transportation cost. As economic theory suggests, prices offered by a monopsonist (Fig. 1.8) are always lower than those offered by a duopsonist\(^\text{12}\), which, in turn, are lower than those in the perfectly competitive market (Figs. 1.7 and 1.9, respectively). However, the behavior of optimal prices is very different in each situation.

\(^\text{12}\)Note that the relevant price comparison here should be made between the monopsonist and the duopsonist who is located at the same point (left end-point of the unit line).
In case of duopsony, transportation cost below \( T = 1 \) results in tighter competition since the total costs of hauling to either plant do not differ much. Both plants are thus forced to offer higher prices to attract input. As transportation cost increases, competition and hence prices decrease, reaching a minimum where the unit line is split exactly in half between the plants \((T \approx 1\) in Fig. 1.7).

At the next stage, the transportation cost is high enough to dampen the competition but still low enough to significantly affect net prices and thus overall supply. The offered prices increase almost linearly as plants try to extract adequate input from the available market area. At some point, however, the transportation cost becomes too high \((T \approx 2\) in Fig. 1.7) and starts to affect profit margins of the plants resulting in smaller and smaller market radii. At the same time, both plants are forced to increase prices to provide supply even from the shrinking market areas.

In case of monopsony (Fig. 1.8), on the other hand, prices monotonically increase with the transportation cost mostly to offset the latter. The kink around \( T = 1.5 \) is the point where the market radius of the monopsonist becomes less than the whole unit line. Finally, prices in the perfectly competitive market (Fig. 1.9) steadily increase in an attempt to offset the increasing cost of shipment.

Note, however, that increases in prices in all three markets are not large enough to compensate the losses of farmers to increased shipment costs. Moreover, for high transportation costs, the market radii in duopsony and monopsony cases do not even cover the whole unit line meaning that some farmers effectively are offered zero or negative net prices.

Another practical implication of these results is that decreasing competition means higher losses to producers especially if the cost of transportation is relatively high. Thus, technologies and government policies that decrease shipment costs (e.g. investments in infrastructure) are highly valued by farmers, particularly in imperfectly
competitive markets, because they stimulate competition. Also, benefits from policies to decrease shipment costs will likely be nonlinear in the case of imperfect competition and may exhibit threshold effects.

In order to measure the degree of market power exerted by the processors in each case, we used an approach similar to that used in traditional oligopsony models, e.g. Muth and Wohlgenant [45]. The general idea is that the observed input price is the sum of a "would be" competitive price and an additional term that measures the degree of market power. Since we consider identical conditions for all three market types, we compute market power here simply as the ratio \((W_c - W_d)/(W_c - W_m)\), where \(W_c\), \(W_d\), and \(W_m\) are the optimal prices offered under perfect competition, duopsony, and monopsony, respectively. The corresponding graph is shown in Fig. 1.10. As the transportation cost increases, the level of competition between the duopsonists declines, since each of them essentially becomes a monopsonist within its own market radius. Therefore, the degree of market power steadily approaches one, which corresponds to the purely monopsonistic market.

1.5.3 Policy Implications

Now consider two scenarios of dynamic market adjustment. First, assume that only one plant exists at the left end-point of the interval (pure monopsonist) that operates at the steady-state capacity. At some point, the second plant is built at the opposite end with capacity corresponding to the steady-state of the duopsony market. Presented in Figs. 1.11 and 1.12 are the adjustment paths for the total capacity and optimal prices offered to farmers.

After the new plant becomes operational, the former monopsonist finds itself competing in a duopsony market. Its original steady-state capacity is now too high for the new market. Since selling of assets is prohibited in the model, the optimal investment
strategy is to allow the extra capacity to depreciate at the natural rate down to the new steady-state. As a result, the overall capacity first jumps to the new level, but then gradually declines reaching in our numerical example a level that is 20 percent below the peak industry capacity. In a similar, albeit less extreme fashion, the offered price first increases due to additional competition, but then slowly decreases as the original plant adjusts its capacity to the new steady-state level.

Real-life examples of this scenario include situations where a competing firm challenges an incumbent firm in a particular spatial market or a farmer cooperative builds additional processing capacity to ensure local demand. In either case, new capacity can quickly drive out a (potentially large) portion of the incumbent firm’s capacity via depreciation so that the long-run capacity boost from a challenging plant will not equal its original capacity.

Similarly, long run prices may slowly retreat after a noticeable initial jump, perhaps leading producers to levy charges of price collusion when, instead, such a price retrenchment is a natural outgrowth of an optimal, arms-length dynamic adjustment. In other words, the incumbent will not continue its operations in the same way, but rather will adjust its plant capacity and pricing behavior to be optimal under new conditions. Thus profits from the new plant or the positive effect of the additional capacity will be at least partially offset by the changing market structure.

For the second scenario, assume that two identical plants originally compete in a duopsonistic market as in the base case. At some point, the owner of one plant buys out the other thus bringing both plants under the same management. Numerous examples of such consolidation have been observed in the food-processing industry and are of great concern to regulators and farm groups.
The formal model describing the optimal investment and pricing strategies for two centrally-managed plants is presented in Section 1.3 (equations (1.13) and (1.14)). The adjustment paths for overall capacity and prices offered to farmers by each plant are shown in Figs. 1.13 and 1.14.

After the buyout, the plants do not have to compete with each other, but rather set their prices as to optimally extract the available supply. The steady-state capacity in the new market is less than that in the duopoly case, albeit not by a large margin. Nevertheless, the plants adjust their capacities accordingly, which immediately results in overall lower prices offered to farmers. Thus consolidation may negatively affect farmers' welfare by both decreasing the price offered and the volume of product purchased (recall that the input purchased is a monotonically increasing function of the net prices (1.1)). In a real-world situation, specific characteristics of the plant may result in a much larger effect.

Note also that in our example, the total value of both plants in the steady state is $V_d = 256.6$, while the value of a single plant operating as a monopsonist is $V_m = 138.75$. Therefore, if the salvage value of a plant is more than $V_d - V_m = 117.85$, it becomes more profitable to shut down the newly acquired plant after a short period of time. Such a pattern is also often observed in the real world and only increases the negative effect on producers' welfare.

1.5.4 Supply Uncertainty

Supply uncertainty is endemic to agricultural production and, while demand uncertainty has been examined as a driver of capacity decisions under oligopoly (e.g. $^{13}$Recall that the value represents the discounted flow of current and future profits from a plant operating according to the optimal strategies in (1.13) and (1.14)).
Baldursson [4], Deneckere and Peck [13], Gabszewicz and Poddar [20]), little atten-
tion has been directed toward supply uncertainty (see Maskin [38] for an exception)
or the capacity–uncertainty linkage under oligopsony. In order to model uncertainty
in supply, we assume that the productivity parameter $A$ in (1.1) is stochastic and
follows a two-stage Markov chain. This may represent, for example, the cyclicality
observed in supply of hogs or cattle, with the Markov states corresponding to the
“high” and “low” levels of supply. The appropriate modifications of the basic model
are presented in Section 1.3 (equations (1.15) and (1.16)).

The values of the shocks and transition probability matrix determine the specific
characteristics of the process. In order to compare the optimal behavior of processors
under supply uncertainty with the base case, let us choose the shocks and probabilities
so that the following conditions are satisfied. First, the system spends on average half
the time in each state. Second, the expectation of the productivity parameter $A$ is
equal to that in the base case. Third, the variance of the shocks has a specific value,
which we vary parametrically. Finally, the autocorrelation of the process has a specific
value.

For the purposes of simulation, let us assume the worst-case scenario where the
autocorrelation of the process is zero (random walk). If $\sigma^2$ is the process variance,
then the values of the multiplicative shocks $\varepsilon_1 = 1 - \sigma$ and $\varepsilon_2 = 1 + \sigma$ along with the
transition probability matrix

$$
\mathbf{Pr} = \begin{bmatrix}
0.5 & 0.5 \\
0.5 & 0.5
\end{bmatrix}
$$

satisfy the above conditions.
We compute the long-term average capacity for each plant by running multiple Monte-Carlo simulations of the capacity path. An interesting effect observed here is that on average each plant chooses to keep capacity above the deterministic steady-state. Since there is a penalty for processing input above the capacity level, the plants maintain higher capacity in order to accommodate unexpected surges in supply\textsuperscript{14}.

Shown in Fig. 1.15 is the long-term average capacity in excess of the deterministic steady-state (in percents) for different values of the demand shock standard deviation $\sigma$. As the uncertainty about the supply increases, so does the extra capacity, even though it is heavily underutilized half of the time\textsuperscript{15}. Symmetric effects emerge from studies that examine oligopolists' capacity choices under stochastic demand (Gabszewicz and Poddar [20], Deneckere and Peck [13], Baldursson [4]) and stochastic supply (Maskin [38]).

Hence, the recent trend toward processing firms locking farmers into long-term contracts (e.g., packer contracts in the hog sector) with strict limits on delivery amounts and delivery times may be an attempt to reduce the local stochastic variation in supply and, in the long-run, may enable the firms to decrease often unused excess processing capacity.

One consequence of this excess capacity phenomenon is that farmers on average receive higher prices than in the deterministic case, since higher capacity results in higher competition and thus higher prices (Fig. 1.1). However, if contracts successfully minimize supply volatility, it may drive down the aggregate capacity and, hence, farm-level prices.

\textsuperscript{14}Such motivations are particularly important to processing firms if the cost penalty for breaching capacity is high. A trend toward higher penalties for surpassing capacity has been noticed, for example, in the hog slaughter sector where slaughter plants are more likely to be double-shifted on a regular basis than they were a decade ago. Hence, it is a greater burden to exceed capacity if a plant is already running a double shift five days a week.

\textsuperscript{15}Maskin [38] points out that the excess capacity driven by stochastic elements also has the side effect of decreasing the potential for entry by another firm.
1.6 Conclusion

Processing capacity is an important factor that needs to be taken into account in modeling processors' behavior in oligopsony markets. A dynamic game framework allows us to explicitly incorporate capacity in the model and take into account such effects as investment and depreciation, which are usually omitted in traditional static models. Numerical techniques are successfully used to solve the Bellman equations that characterize the solutions to the dynamic game and thus derive relationships between optimal capacity, pricing, and investment. Collocation methods provide a powerful and flexible tool for solving dynamic game models. Collocation methods do not impose restrictions on functional forms and allow constraints to be explicitly considered. The ability of these methods to handle different forms of uncertainty allows models to incorporate demand shocks and other random factors.

Simulation results highlight important implications of the model. Higher capacity means higher competition and higher prices offered to farmers. Transportation cost is a significant factor affecting the degree of competition between processors. High transportation costs essentially transform the duopsonistic market into two separate monopsonistic markets.

Two dynamic scenarios demonstrate the effect of structural changes in the market. Construction of an additional plant initially increases competition and total available capacity, but these effects are offset by the subsequent capacity adjustment of the existing plant. Consolidation results in decreasing overall capacity and hence lower prices offered to farmers.
Uncertainty in supply forces processors to keep capacity in excess of the certainty-equivalent steady-state level in order to avoid penalty for processing over capacity in the periods of high supply. Decrease in input variability would reduce the excess capacity but also result in lower average prices received by farmers.

Extensions of the present work may include further analysis of capacity-price relationship (e.g. with different specifications of the cost of production and product transformation functions) as well as estimation of model parameters for a specific case. The dynamic monopsony model derived as a part of more general analysis can be used separately either for simulations or, again, as a basis of estimation. The scenario of the monopsonist using capacity as entry deterrence can also be studied by slightly modifying the model. Finally, the spatial aspect of the competition can be further investigated by considering nonuniform distribution of producers and/or nonlinear transportation costs.
Figure 1.2: Acquired input under optimal strategy, Processor 1.
Figure 1.4: Short-term profit function under optimal strategy, Processor 1.
Figure 1.6: Residuals of the Bellman equation, Processor 1.
Figure 1.7: Optimal price at the duopsony steady state vs. transportation cost $T$. 
Figure 1.8: Optimal price at the monopsony steady state vs. transportation cost $T$. 
Figure 1.9: Optimal price in the purely competitive market vs. transportation cost $T$. 
Figure 1.10: Duopsony market power vs. transportation cost $T$. 
Figure 1.11: Capital transition path, new plant scenario.
Figure 1.12: Price transition path, new plant scenario.
Figure 1.13: Capital transition path, consolidation scenario.
Figure 1.14: Price transition path, consolidation scenario.
Figure 1.15: Long-time average excess capacity.
ESSAY 2

ECONOMIC ANALYSIS OF THE STANDARD REINSURANCE AGREEMENT

2.1 Introduction

U.S. government involvement in the provision of crop insurance goes back to 1930s. Title V of the Agricultural Adjustment Act of 1938 established the Federal Crop Insurance Corporation (FCIC) whose purpose at the time was to provide wheat insurance for farmers. Several subsequent Acts of Congress, most notably the Federal Crop Insurance Act of 1980, the Federal Crop Insurance Reform Act of 1994, and the Agricultural Risk Protection Act of 2000 expanded the scope of the federal insurance program, introduced additional types of coverage, and permitted the U.S. government to subsidize premium payments. Currently, federal crop insurance programs are available for more than 100 different crops in more than 3,000 counties across the U.S., although not every crop is covered in every county. In 2000, over 200 million acres, more than 75% of all eligible acreage, were enrolled in the programs. Total liability of the programs was $34.1 billion, which is almost ten times higher than in 1980.

Government interest in crop insurance has derived from the inability of the private insurance industry to provide affordable crop insurance products. While private hail insurance has existed as far back as late 1800s, early attempts by private
companies to offer comprehensive crop loss coverage has resulted in spectacular failures (Kramer [33]). Even today, the viability of nonsubsidized crop insurance is questioned by the majority of researchers in the area. The main reasons quoted in the literature include asymmetric information, which results in moral hazard (Chambers [11]) and adverse selection (Skees and Reed [52]), and a high systemic component present in a portfolio of crop risks (Miranda and Glauber [41], Mason, Hayes, and Lence [39]). Government involvement reduces the cost of underwriting for private companies through direct subsidies and results in better diversification of aggregate risk portfolio than any single company may achieve.

Rather than providing insurance directly to farmers, the U.S. government uses private insurance companies as a delivery mechanism utilizing the industry’s extensive network of agents and adjusters. While the types of the products and their features are determined by the Risk Management Agency (RMA) of USDA, the actual policies are underwritten by private insurance companies throughout the country. However, in order to attract private insurers to underwrite crop contracts, the government has to provide additional incentives. These incentives come in two forms. First, FCIC pays companies a direct subsidy for administrative and operating expenses on behalf of producers. Second, the government provides discounted reinsurance, since private reinsurance would be either inaccessible or prohibitively costly.

The terms and conditions under which FCIC subsidizes and/or reinsures crop insurance contracts sold by the private insurance companies are regulated by the Standard Reinsurance Agreement (SRA). The SRA represents a cooperative risk sharing agreement between FCIC and the companies to deliver crop insurance under the authority of the Federal Crop Insurance Act. The SRA is periodically renegotiated between the Risk Management Agency and the crop insurance companies. The version of the Agreement currently in effect was approved by the RMA and private
insurance companies in July 1997 and was subsequently amended by the Agricultural Research, Extension, and Education Reform Act of 1998. In addition, the Agriculture Risk Protection Act of 2000 provided that the Federal Crop Insurance Corporation must renegotiate the SRA once during the 2001 through 2005 reinsurance years.

Over time, changes in the crop insurance program structure have led to changing rates of returns of the companies involved in delivery of crop insurance. In particular, addition of new products, such as CAT coverage and revenue insurance has greatly expanded the scope of the crop insurance program. Increasing premium discounts and subsidies have also caused producers to shift to higher coverage levels and revenue products thus increasing program participation both in terms of the acreage insured and the coverage selected. At the same time, the geographical distribution of companies' books of business has undergone significant changes over the last twenty years. Finally, the premium rates determined by the SRA has been revised several times since the inception of the Federal Crop Insurance program in 1980.

Until recently, the RMA have lacked an analytical model that would allow it to evaluate the effects of various provisions of the SRA on companies' rates of return. The SRA Simulator was developed at the Ohio State University as a tool to assist policymakers in assessing the economic impact of the Agreement. The simulator uses the historical data on yields, prices, and insurance losses for each district, crop, and insurance product in order to simulate a distribution of the book of business resulting from underwriting crop insurance either in aggregate or for a specific company.

The purpose of this essay is to (1) describe the SRA as well as major crop insurance products available to farmers, (2) present the modeling methodology behind the SRA Simulator, and (3) use the simulator to address some of the problems that are currently of interest to both policymakers and the private insurance companies underwriting crop insurance contracts.
2.2 Crop Insurance Products

Currently, there are more than 20 types of insurance products available for different crops, although some of the products are in the pilot stage and/or apply to specific crops in specific counties. Discussed below are the insurance plans that comprise the major portion of the FCIC portfolio in terms of total liability. These contracts are also explicitly modeled in the SRA Simulator. More detailed information about these and other products, as well as current and historical participation data, are available from the RMA website at http://www.rma.usda.gov.

2.2.1 Yield-Based Insurance Products

The most common class of crop insurance products are the Multiple Peril Crop Insurance (MPCI) contracts, whose indemnities are based on the realization of yield compared to the actual production history (APH). In order to participate in the program, farmers need to submit historical yield data for a period of four to ten years which is then averaged to compute the APH or “expected” yield. Farmers can select the proportion of the expected yield as well as a proportion of the predicted (base) price\(^\text{16}\) that they want to insure. MPCI contracts pay indemnities when the realized yield falls below the selected coverage level. If \(\bar{Y}\) is the APH yield, \(y\) is the realized yield, \(\eta\) is the selected yield coverage level, and \(\pi\) is the selected percentage of the predicted crop price \(\bar{p}\), then the liability and the payoff of an MPCI contract per insured acre can be expressed as

\[ L = \max(0, \eta - \pi \cdot \bar{Y} - y) \]

\[ P = \max(0, \eta - \pi \cdot \bar{Y} - y) \]

\(^{16}\)The base prices are established and published annually by the RMA in the beginning of a contract cycle.
\[ L_{MPCI} = \pi \tilde{p} \tilde{y} \quad (2.21) \]

and

\[ I_{MPCI} = \pi \tilde{p} \max\{0, \eta \tilde{y} - y\}, \quad (2.22) \]

respectively. Available yield coverage levels \( \eta \) range from 50\% to 75\% at 5\% intervals. Coverage up to 85\% is also available for selected crops in selected counties. The percent of the predicted price \( \pi \) can be chosen between 55\% and 100\% also at 5\% intervals. Recent increases in premium subsidies for higher coverage levels have made such contracts more attractive and have significantly increased program participation.

A special type of MPCI is the Catastrophic coverage (CAT) contract introduced by the Federal Crop Insurance Reform Act of 1994 to replace other Federal disaster assistance programs. Originally, CAT contracts were equivalent to an APH yield product\(^{17}\) with \( \eta = 0.5 \) and \( \pi = 0.6 \), but in 1998 the price election requirement was reduced to \( \pi = 0.55 \) (current level). The premium on CAT is paid by the federal government, although farmers are required to pay a sign-up fee for each crop insured in each county. Initially, this fee was set at \$50 per crop, but later increased to \$60 in 1999 and \$100 in 2001.

APH contracts were the first crop insurance products made available to farmers by the Act of 1980. At the time, yield coverage was available at 50\%, 65\% and 75\% levels. Later, additional coverage levels were introduced and the program was expanded to include more crops in more counties. Even though other types of products have been introduced since 1980, the APH yield contracts remain the most popular insurance plan. In 2000, more than 1.3 million APH contracts (including CAT) were sold with the combined liability of \$16.5 billion (48\% of the total liability for the year).

\(^{17}\)The terms APH yield products, APH buy up products, and APH contracts are synonymous to MPCI contracts and used interchangeably throughout the essay.
2.2.2 Revenue Insurance Products

Unlike yield-based products, revenue insurance contracts guarantee a certain level of farmer’s revenue and pay indemnities whenever the actual combination of yields and prices results in revenue below that level.

2.2.2.1 Crop Revenue Coverage

The most common and popular revenue product is the Crop Revenue Coverage (CRC) which is available in all counties where APH contracts are available. Under the CRC plan, farmers can insure a percentage of the projected revenue. The projected revenue is computed as the product of the APH yield and the base price established by the RMA prior to the insurance period. Available coverages range between 50% and 75% at 5% intervals. Coverage up to 85% is available for crops and counties where 85% APH contracts are available.

While the projected revenue is used to establish the revenue guarantee and premiums, the actual payoff of a CRC contract also depends on the harvest-time price realization so as to provide a higher indemnity if the harvest-time price exceeds the base price up to a prescribed maximum. Price differentials between the base and harvest prices are limited to $1.50 for corn and grain sorghum, $0.70 for cotton, $3.00 for soybeans, and $2.00 for wheat.

Both the base and harvest prices are determined as averages of daily settlement prices for specific commodity futures contracts during a specific period of time. The particular futures contracts and price discovery periods used for the CRC price determination are listed in the insurance contract (see also [57]).
If \( \bar{y} \) and \( y \) are the APH and realized yields, respectively, \( \eta \) is the coverage level selected by the farmer, and \( p_b \) and \( p_h \) are the base and harvest prices, respectively, then the liability and payoff of a CRC contract per insured acre are determined as

\[
L_{CRC} = \eta \max\{p_b, p_h\} \bar{y}
\]

and

\[
I_{CRC} = \max\{0, \max(p_b, p_h) \eta \bar{y} - p_h y\},
\]

respectively.

Even though CRC is a relatively new line of products, it has been gaining in popularity among farmers. The total number of CRC policies sold grew from 177,329 with combined liability of $4.4 billion in 1998 to 407,703 with combined liability of $10.3 billion (30.1% of the total liability) in 2000.

### 2.2.2.2 Income Protection and Revenue Assurance

Two other revenue insurance products, the Income Protection (IP) and Revenue Assurance (RA) plans, are very similar in structure. Like the CRC contracts, they both insure a specific level of a farmer's income but use different prices to determine the liability and final payoff. The amount of protection is calculated as a product of coverage level selected by the farmer, the APH projected yield, and the projected price. Both contracts pay indemnities if the realized yield valued at the harvest price is lower than the guaranteed level.

The coverage level chosen can range between 50% and 75% at 5% intervals for IP and between 65% and 75% for RA. The projected and harvest prices are based on the average daily settlement prices for specific futures contracts or indices depending on the crop insured. In addition, the RA contract allows for a Harvest Price Option.
(HPO) election in which case the fall harvest price is used to recompute the revenue guarantee, if it is higher than the projected price. Aside from this difference, the liabilities and payoffs of an IP or RA contract per insured acre can be determined as

\[ L_{IP} = L_{RA} = p_p \eta \bar{y} \]  

(2.25)

and

\[ I_{IP} = I_{AR} = \max\{0, p_p \eta \bar{y} - p_h y\} \]  

(2.26)

respectively, where again \( \bar{y} \) and \( y \) are the APH and actual yields, respectively, \( \eta \) is the selected coverage level, and \( p_p \) and \( p_h \) are the projected and harvest prices, respectively.

The IP and RA plans account for a smaller share of insurance policies, but the number of contracts sold exhibited steady growth over the last several years with the combined liability of both plans accounting for $0.95 billion or 2.76% of the total FCIC portfolio in 2000.

Two new types of revenue insurance products, the Adjusted Gross Revenue (AGR) and Group Revenue Insurance Policy (GRIP), have been approved by FCIC in 1999. However, these programs are still in the experimental stage\(^\text{18}\) and are neither considered here nor currently implemented in the SRA Simulator.

2.3 1999 Standard Reinsurance Agreement

In order to be eligible for the FCIC subsidies and reinsurance, companies are required to offer all types of contracts for all crops approved by FCIC in any state in which

\(^{18}\text{The combined liability for these two products in 2000 was only }$0.13\text{ billion, or about 0.4% of the total book of business.}\)
the companies underwrite crop insurance in a given reinsurance year\(^{19}\). In addition, companies must have sufficient experience and adequate organization structure, as well as a satisfactory performance record, to underwrite contracts. Companies are also prohibited from selling supplemental insurance contracts that may shift risk to crop insurance contracts sold or subsidized by FCIC.

FCIC provides reinsurance for the participating companies in exchange for portion of insurance premiums collected by those companies. The reinsurance comes in two forms: proportional and nonproportional. Under the former, the companies cede their liability for ultimate net losses in exchange for an equal percentage of the associated net premiums. In other words, they completely transfer a portion of their book of business to FCIC. The nonproportional reinsurance is then applied to the remaining or retained portion of companies' books of business. Nonproportional reinsurance is similar to traditional reinsurance in that FCIC shares losses with the companies in exchange for a portion of their underwriting gain\(^{20}\).

Under the proportional reinsurance, each company may allocate the contracts to one of the three funds: Assigned Risk Fund, Developmental Fund, and Commercial Fund. Funds differ in the required level of retention and also in the FCIC shares of gains and losses from retained business under the nonproportional insurance.

The Assigned Risk Fund is characterized by the lowest required retention rate (20%) which makes it the primary designation for the high-risk contracts. In order to avoid concentration of the whole book of business in the Assigned Risk Fund, the

\(^{19}\)A reinsurance year is defined as a period from July 1 of any year through June 30 of the following year and identified by reference to the year containing June.

\(^{20}\)Underwriting gain is the amount by which net premiums collected by a company exceed its losses or the total indemnities it had to pay.
maximum cession limits are established for each state ranging from 10% of the net book of business in Hawaii and Vermont to 75% in Alaska, Georgia, Maine, Montana, Nevada, Rhode Island, Texas, Utah, and West Virginia.

The Developmental Fund requires the companies to retain at least 35% of the net book premium as well as the associated liability. Within the Developmental Fund, contracts are further designated into CAT Fund, Revenue Insurance Fund, or All Other Plans Fund. The retention percentages for these three funds may differ across state, but cannot be lower than 35%. A company may elect to retain more than 35% of its premium and associated liabilities, in which case the retention level can be chosen in 5% increments up to 100%.

Finally, designation of a contract to the Commercial Fund requires a company to retain at least 50% of the liability and associated net premiums. Just like with the Developmental Fund, the Commercial Fund is further subdivided into CAT, Revenue Insurance, and All Other Plans funds. Similar to Developmental Fund, a company may select a retention rate higher than 50% in 5% increments. A company may also chose to entirely forgo the use of the Commercial Fund in a particular state, in which case all contracts not designated to the Assigned Risk Fund are automatically placed in the Developmental Fund.

In addition to the minimum retention levels required by funds, each company must maintain a minimum overall retention level. The latter is equal to 35% of the company net book of business, unless the company designates more than 50% of its book of business in the Assigned Risk Fund or forgoes the use of the Commercial Funds. In these cases, the required minimum overall retention level is set at 22.5% of the company net book of business.
The nonproportional reinsurance is applied to the portion of companies' books of business retained after the proportional cession. The responsibilities of the companies for the retained losses, as well as their share of the underwriting gain, are specified for each fund and state and depend on the loss ratio of each company in a given reinsurance year. The loss ratio is defined as a ratio of the total indemnities paid by the company to the total retained premiums. When the loss ratio exceeds 100%, the company suffers a net underwriting loss, and when the loss ratio falls below 100%, the company earns a net underwriting gain.

The schedules of liabilities and underwriting gains retained by companies under different realizations of the loss ratios are shown in Tables 2.1 and 2.2. For each loss ratio range, the percentages in the tables apply to the fraction of each company's loss (gain) within that range. For example, if in a given state for a given reinsurance year a company experienced the net underwriting loss of 180% of the total amount of retained premiums for the CAT plans in the Commercial Fund, then the company retains 50% of losses between 100% and 160% and 40% of the remaining losses between 160% and 180%.

As the loss ratio increases, FCIC assumes a larger fraction of company's losses, up to 100% of the portion of losses in excess of 500% of the total retained premiums. At the same time, FCIC claims a larger fraction of companies' underwriting gains as their loss ratios decrease. Note that the Assigned Risk Fund provides the highest level of protection against losses but also leaves the companies the smallest fraction of the gains. The Commercial Fund, on the other hand, gives companies the highest return in case of the underwriting gain, but also leaves the largest portion of the net losses on the companies' balance.
Note that for modeling purposes, an SRA can be completely described by the required retention rates for each fund, the breakpoints of the loss ratio ranges, and the shares of the underwriting losses or gains retained by the companies within each range.

2.4 SRA Simulator

The Standard Reinsurance Agreement Simulator is a user-friendly Windows 98/NT program developed at the Ohio State University with the financial and logistical support of the Risk Management Agency and the Economic Research Service, U.S. Department of Agriculture. The following sections present the assumptions, data, and methodology behind the simulator.

2.4.1 Simulation Strategy

The main financial indicator of any company's performance is its rate of return. In case of an insurance company, the rate of return is driven by the net underwriting gain that is the difference between the premiums collected and indemnities paid. Since losses depend on the occurrence or nonoccurrence of random events, the rate of return is a random variable. The purpose of reinsurance is to reduce the downside variability of this random variable and possibly increase its expected value.

Under the SRA, the rate of return is determined by a particular realization of the company's loss ratio and the SRA parameters, i.e. retention rates, breakpoints, and shares. Therefore, in order to analyze the effects of changes in the SRA on the rates of return, one needs to model the distribution of the loss ratios by state and fund for each company reinsured by FCIC.
The random variables that ultimately drive loss ratios are the farm-level yields and prices, since they determine the insurance loss for any given contract through relationships like (2.22) and thus the aggregate losses for any company. A naïve approach thus would be to model the distributions of the farm-level yields and then compute the corresponding distributions of companies’ rates of return by aggregating over each company’s book of business.

Such an approach, however, has been heavily criticized by the industry representatives because simulations based solely on individual contract specifications without actual participation data taken into account neither reflect the adverse selection present in the crop insurance portfolio, nor take into account the additional losses companies incur due to moral hazard. As a result, the losses computed in this way typically underestimate the actual underwriting losses experienced by the companies and need to be adjusted to reflect the composition of insurance companies’ portfolios.

Ideally, one would like to have a long series of historical data on companies’ premiums and indemnities which then can be used either to fit a parametric distribution or as a basis of an empirical distribution. However, there are several serious obstacles which do not permit this approach. First of all, the number of contract types available under the crop insurance program have increased dramatically since 1980, with a large portion of products introduced after 1994. Therefore, historical loss data simply are not available for many contracts prior to 1994. Second, program participation has significantly increased over the last two decades both in terms of the acreage insured and coverage levels selected by the producers. This in turn led to broader pool of insured risk and decreasing variation in indemnities. Third, participating companies over time have changed the composition and the geographical
distribution of contracts in their books of business as well as allocation across reinsurance funds. Finally, premium rates\textsuperscript{21} have also changed over time, thus affecting historical realizations of companies’ gains and losses.

In order to circumvent data limitations and derive a distribution of loss ratios that accurately reflects the historical changes in the crop insurance programs, the following strategy is implemented. First, it is assumed that historical loss costs, or ratios of indemnities to the total liabilities, accurately reflect the actual distribution of underwriting losses. Specifically, it is assumed that loss costs by district, crop, and product over the historical period were generated by stationary data-generating processes that are uniform across companies and reinsurance funds.

The aggregate historical loss costs are available for the period of 1981–1998 only for selected APH yield contracts and thus do not provide information about the distribution of loss costs for other products such as CAT and revenue products. The missing loss costs therefore are simulated in the following way. The distributions of individual yields and prices are derived\textsuperscript{22} based on historical data and then used to simulate the loss costs for all products by district, crop, and year for the period from 1981 through 1998. The simulated values are then rescaled so that to replicate the historically observed aggregate loss costs.

The derived distribution of loss costs for each district, crop, and product is then combined with the data on liabilities and premium rates for the base year and aggregated to compute the distribution of loss ratios for each company by state and reinsurance fund. The available liability data for insurance companies are not disaggregated by product. However, the total liability data are available by district, crop, crop.

\textsuperscript{21}The premium rate of a contract is a ratio of its premium to the liability associated with the contract.

\textsuperscript{22}The specific methodologies used at each modeling step are presented in more details in later sections when discussing implementation of the SRA Simulator.
and insurance product. Therefore, an implicit assumption is made that the product composition of the book of business is uniform across all companies for each crop within each district.

Finally, the derived distribution of the loss ratios can be used along with the SRA parameters to compute the expectations and coefficients of variation of the rates of return by company, state, and/or fund.

### 2.4.2 Modeling Methodology and Program Implementation

At the most general level, the SRA Simulator consists of three program modules: the simulation routine (SRA.SIM), the preprocessing routine (SRA.PRE), and the runtime routine (SRA.RUN). Each module uses the output of the previous module as its input along with additional external data. The last module is provided with a graphical user interface (GUI) that allows the user to interact with the simulator by entering SRA parameters and saving and/or printing the program output. The modules are implemented in Fortran 95 and the GUI is written in Visual Basic.

The simulator covers six major crops and fourteen crop insurance products. All simulations are calibrated to the 1998 (base year) premium rates and distribution of book of business across crops and regions. The incorporated crops are barley (BRL), corn (CRN), cotton (CTN), soybeans (SOY), grain sorghum (SRG), and winter wheat (WHT). The incorporated insurance products are CAT coverage, APH buy up at \{50, 55, \ldots, 75\}\% coverage\(^{23}\) (six products), CRC at \{50, 55, \ldots, 75\}\% coverage (six products), and other revenue coverage (IP and RA).

\(^{23}\)A 35\% APH yield contract, which was introduced for a short period of time in 1993–94 is used for some internal simulations, but is not present in the base year product lineup.
Together, this combination of crops and products encompass about 74% of the total FCIC liability for the base year (1998). Although the excluded products and crops seem to comprise a relatively large fraction of the total liability, the modeling of those represents a fundamental difficulty because of scarcity of available data, crop- and region-specific contracts, and otherwise complicated idiosyncratic contract structures. Due to these limitations, the SRA Simulator likely does not capture the full effects of the SRA in states where the portion of business covered by the major six crops is relatively low, e.g. California and Florida. Still, the crops and contracts incorporated in the simulator represent a major portion of the crop risk exposure and adequately reflect the effects of the SRA on insurance companies.

Figure 2.1 shows the tree diagram of the SRA Simulator. The root entries represent external data files, while the links between the modules represent the outputs of one module used as inputs by another. The specific entries and operations performed within each module are described below.

2.4.2.1 SRA SIM Module

This module implements stochastic simulation micro-models of individual farms and uses them to derive the distribution of loss costs for all products by district and crop. The critical assumption underlying each farm's micro-model is that the realized ratio of farm yields to district\(^2\) yield is independently and identically distributed over time, independent of district yields, and independent across farms. In other words, it is assumed that each individual farm's yield can be represented as

\[
\log y_f = \log y_d + \log \epsilon, \tag{2.27}
\]

\(^2\)A district is a statistical unit intermediate between a county and a state. Each state is typically split into nine or ten districts and each district typically includes eight to twelve counties.
where $y_f$ is the farm yield, $y_d$ is the district yield, and $\varepsilon$ is a random shock drawn from an unknown distribution. This approach implies that the individual yield elasticities are equal to 1 for all farms within the district, a mildly restrictive assumption. However, the shocks $\varepsilon$ are modeled independently for each farm in order to capture the individual yield variability.

The expected APH yield used in computing the payoffs of crop insurance contracts (2.22), (2.24), and (2.26) is then represented as

$$\bar{y}_{APH} = \mathbb{E} \varepsilon \bar{y}_{det},$$

(2.28)

where $\bar{y}_{det}$ is the expected detrended district-level yield. Note that both the expectation and variance of $\varepsilon$ vary across farms. In addition, $\mathbb{E} \varepsilon \neq 1$ in general, since individual farm yields may be consistently above or below district averages.

Note that while the random shocks $\varepsilon$ are independent of district yields, there may be a significant correlation between the shocks at nearby farms within the district. If we were to fit a parametric distribution for $\varepsilon$, we would have to estimate parameters of an $n$-variate distribution rather than $n$ separate distributions, where $n$ is the number of farms in the district. Even for a simple case of ten farms and normal distribution with a symmetric variance-covariance matrix, this translates into 60 parameters that need to be estimated. However, the individual data are available for no more than ten years, so that fitting a parametric distribution is not feasible. Instead, the SRA Simulator uses the nonparametric empirical distribution obtained by treating all available realizations of $\varepsilon$ as equally probable.
The data used at this stage include the NASS historical district-level yield series for 1981–1998 and the individual farm yield series for more than a million insured units\textsuperscript{25} provided by RMA. The detrended district yields were computed by Alan Ker, University of Arizona, from raw NASS data using a nonparametric LOESS detrending procedure [22].

For APH products, the distribution of yields is enough to calculate the loss costs. Indeed, the prices enter both the indemnity (2.22) and the liability (2.21) as multipliers, and thus cancel out in computing the loss cost. However, this is not the case for the revenue products, since prices enter indemnity under the maximum operator. Therefore, the loss costs for revenue products depend on the base and harvest-time prices used to settle the contracts. The base (or projected) price is determined at or before signing the insurance contract and therefore is a parameter rather than a random variable. Thus the only stochastic component in (2.23)–(2.26) is the harvest price.

While historical yield data are readily available at the county level and above for all the crops included in the simulator, the associated price series are much less complete. Even when the price series are available for the entire simulation period, e.g. CBOT corn price series, we cannot reliably assume that those are drawn from a stationary distribution due to changing farm policies and support programs, inflation, and other factors. Therefore, instead of deriving the harvest price distribution from historical data, the harvest price movement is modeled for each crop as

\[
\log p_h = \log p_b + \alpha(\log \bar{y}_{nat} - \log \bar{y}_{nat}) + z, \tag{2.29}
\]

\textsuperscript{25}For the crop insurance purposes, a single farm may either represent a whole unit or be divided into several insured units. However, from modeling standpoint each unit may be considered as a separate farm. Therefore, for the rest of this section we refer to units as farms even though this may not accurately reflect the actual farm structure.
where \( p_h \) is the harvest price, \( p_b \) is the base (projected) price, \( y_{nat} \) is the detrended national yield, \( \bar{y}_{nat} \) is the expected detrended national yield, \( \alpha \) is the elasticity parameter, and \( z \) is a random shock independent of \( y_{nat} \) and distributed normally with zero mean and some variance \( \sigma^2 \).

The data on the national yields are available from the NASS database. The estimates of the elasticity parameters \( \alpha \) and the variances \( \sigma^2 \) of the random shocks were provided by the RMA along with the projected prices for each crop for the base year.

The actual simulations in the module are performed in the following sequence (see also Fig. 2.1). First the input data are read in the following arrays:

- \( yieldsdisthist(i_d, i_c, i_y) \) — historical district yields by district, crop, and year;
- \( yieldsdistdet(i_d, i_c, i_y) \) — detrended district yields by district, crop, and year;
- \( yieldsind(i_d, i_c, i_y, k) \) — historical individual yields by district, crop, year, and farm;
- \( prices(i_s, i_c, i_y) \) — parameters of the price model (2.29) by state, crop, and year.

Here and below \( i_d = 1, n_d \), \( i_c = 1, n_c \), \( i_y = 1, n_y \), and \( i_s = 1, n_s \) are counters for districts, crops, years, and states, respectively. The total numbers of districts \( n_d \) and \( n_s \) are determined by the available data, the total number of crops is \( n_c = 6 \), and the number of years is \( n_y = 18 \) (1981 through 1998).

\(^{26}\text{In order to avoid accumulation of subscripts, for the rest of this section the indices of array elements are presented in parenthesis following the name of the array. Also, the ranges of indices are omitted for brevity sake.}\)
Let $n_r$ be the number of actual yield data available for a farm $k$ growing crop $i_c$ in the district $i_d$ for the years $i_1, \ldots, i_m$. The realizations of the shock $\varepsilon$ in (2.27) are then computed as $\varepsilon(i_c, i_r, k) = \frac{\text{yieldsind}(i_d, i_c, i_r, k)}{\text{yieldsdisthist}(i_d, i_c, i_r)}$ and each is assigned a probability $1/n_r$.

Next, the whole set of shock realizations is applied to each detrended district yield in the data series so that the APH average yield for the farm is computed as

$$\bar{y}_{APH}(i_c, k) = \frac{1}{n_r} \sum_{i_r=1}^{n_r} \sum_{i_y=1}^{n_y} \varepsilon(i_c, i_r, k) \times \text{yielddistdet}(i_d, i_c, i_y).$$

The distributions of the harvest prices are computed in a similar way. The normal random variable $z$ is replaced by the discreet shocks $z_j(i_c)$ with the weights $w_j(i_c)$ using five-point Gaussian quadratures [31] and realizations of the harvest price are then computed as

$$p_h(i_c, i_y, j) = \exp \left\{ \log p_0(i_c) + \alpha \left( \log y_{nat}(i_c, i_y) - \frac{1}{n_y} \sum_{i_y=1}^{n_y} \log y_{nat}(i_c, i_y) \right) + z_j \right\},$$

where $p_0(i_c)$ is the base year projected price, $y_{nat}(i_c, i_y)$, $i_y = \overline{1, n_y}$, are the detrended national yields, and $j = \overline{1, 5}$ indexes the discreet shocks.

The distribution of the loss costs from insuring crop $i_c$ at the farm $k$ can now be simulated for each product. For the CAT and APH buy up the realizations of the loss costs are computed in the following way

$$\text{losscosts}(k, i_c, i_p, i_y) = \frac{1}{n_r n_y \bar{y}_{APH}} \sum_{i_r=1}^{n_r} \max\{0, \eta(i_p)\bar{y}_{APH} - \varepsilon(i_c, i_r, k) \times \text{yielddistdet}(i_d, i_c, i_y)\}$$

where $i_p = 1, \ldots, 7$ are the APH products and $\eta(i_p)$ are the corresponding coverage levels.
For the CRC products the procedure is similar, if only slightly more complicated due to prices:

\[
\text{losscostsim}(k, i_c, i_p, i_y) = \frac{1}{5n_r} \sum_{i_r=1}^{n_r} \sum_{j=1}^{5} \left[ \max\{0, \max\{p_b(i_c, i_y), p_h(i_c, i_y, j)\}\} \eta(i_p) \bar{y}_{APH} \right.
\]

\[
- p_h(i_c, i_y, j) \epsilon(i_c, i_r, k) \times \text{yielddistdet}(i_d, i_c, i_y) \left] \times \frac{1}{\eta(i_p) \max\{p_b(i_c, i_y), p_h(i_c, i_y, j)\} \times \bar{y}_{APH}},
\]

where \(i_p = 8, \ldots, 13\) are the CRC products and \(\eta(i_p)\) are the corresponding coverage levels\(^{27}\).

In the same way, for the IP and RA, the loss cost realizations are computed as

\[
\text{losscostsim}(k, i_c, i_p, i_y) = \frac{1}{5n_r} \sum_{i_r=1}^{n_r} \sum_{j=1}^{5} \left[ \max\{0, p_p(i_c, i_y)\} \eta(i_p) \bar{y}_{APH} \right.
\]

\[
- p_h(i_c, i_y, j) \epsilon(i_c, i_r, k) \times \text{yielddistdet}(i_d, i_c, i_y) \left] \times \frac{1}{\eta(i_p) p_p(i_c, i_y) \bar{y}_{APH}},
\]

where \(i_p = 14\) is the "other revenue products" index, with the coverage level set to 65%.

Finally, the simulated loss costs for each farm are accumulated by districts in which the farms are located. At the end, the SRA.SIM module generates the array \(\text{losscostsim}(i_d, i_c, i_p, i_y)\) that contains the equiprobable realizations \(i_y = 1, \ldots, n_y\) of loss costs for each district \(i_d\), crop \(i_c\), and product \(i_p\). This array is then passed to the next module where it is used in subsequent computations.

\(^{27}\)Note that for brevity sake the loss cost formula for the CRC does not explicitly take into account the maximum limits on price movements (Section 2.2.2.1). However, the appropriate adjustments are made in the actual simulation routine.
2.4.2.2 SRA_PRE Module

This module performs computations that do not depend on the user’s input and generally needs to be executed only once. In particular, the module (i) calibrates the product-specific loss cost distributions simulated by the SRA.SIM to conform to the historical aggregate APH buy up loss costs; (ii) computes the base year liabilities by district, crop, product, company (or organization), and fund; and (iii) aggregates the loss costs, liabilities, and premium rates to state, organization, and fund levels (Fig. 2.1). The output of the module is then used by the runtime routine SRA_RUN.

The input data read by the module include

- \textit{losscostbu}(i_d, i_c, i_y) — historical aggregate lost cost data for selected APH buy up products, by district, crop, and year;
- \textit{losscostsim}(i_d, i_c, i_p, i_y) — simulated distribution of loss costs by district, crop, product, and year (output of SRA.SIM);
- \textit{liabilityhist}(i_d, i_c, i_p, i_y) — historical liability weights of APH buy up products in the aggregate by district, crop, product, and year;
- \textit{liabbaseprod}(i_d, i_c, i_p) — base year liabilities by district, crop, and product;
- \textit{liabbaseorg}(i_d, i_c, i_o, i_f) — base year liability by district, crop, organization, and fund;
- \textit{premratedist}(i_d, i_c, i_p) — base year premium rates by district, crop, and product.

Here \(i_o = 1, n_o\) and \(i_f = 1, n_f\) are counters for organizations (insurance companies) and reinsurance funds. The total number of companies in the data set is \(n_o = 17\), and the total number of funds \(n_f = 7\) is determined by the SRA.
The first step in the computations is to adjust the simulated loss costs to replicate the observed aggregate loss costs for the selected APH buy up products. The main assumption here is that the bias in the simulated values is multiplicative and consistent across all products, although it may be different from year to year. If $B \subseteq \{1, \ldots, n_p\}$ is the index subset of products included in the aggregate loss costs data, then the calibration procedure can be outlined as follows. For each district, crop, and year the adjustment factor is computed as

$$ factor(i_d, i_c, i_y) = \frac{\text{losscostbu}(i_d, i_c, i_y)}{\sum_{i_p \in B} \text{liabilityhist}(i_d, i_c, i_p, i_y) \times \text{losscostsim}(i_d, i_c, i_p, i_y)} $$

and then applied to all the available products to generate the distribution of loss costs by district, crop, and product:

$$ \text{losscostdist}(i_d, i_c, i_p, i_y) = factor(i_d, i_c, i_y) \times \text{losscostsim}(i_d, i_c, i_p, i_y). $$

At the second step, the module disaggregates the base year liabilities by district, crop, product, organization, and fund. Here, it is assumed that the product composition for each crop within each district is the same across all organizations and fund. Therefore, the data on total liabilities by district, crop, and product are first used to compute the product shares

$$ \text{liabshare}(i_d, i_c, i_p) = \frac{\text{liabbaseprod}(i_d, i_c, i_p)}{\sum_{i_p=1}^{n_p} \text{liabbaseprod}(i_d, i_c, i_p)} $$

which are then applied to the liabilities by district, crop, organization, and fund to compute the disaggregated liabilities

$$ \text{liabilitydist}(i_d, i_c, i_p, i_o, i_f) = \text{liabshare}(i_d, i_c, i_p) \times \text{liabbaseorg}(i_d, i_c, i_o, i_f) \quad \forall i_p. $$
Finally, the module constructs the aggregate coverage data by state, organization, and fund which are required by the runtime module SRA.Run. First the liabilities are aggregated across states, crops, and products

\[ \text{liability}(i_s, i_o, i_f) = \sum_{i_d \in i_s} \sum_{i_c = 1}^{n_c} \sum_{i_p = 1}^{n_p} \text{liabilitydist}(i_d, i_c, i_p, i_o, i_f), \]

where \( i_d \in i_s \) indicates summation over all districts in a particular state. Next the total losses and total premiums are computed by district, crop, product, organization, and fund implicitly assuming that the loss costs and premium rates are the same across all organizations and funds

\[ \text{loss}(i_d, i_c, i_p, i_o, i_f, i_y) = \text{liabilitydist}(i_d, i_c, i_p, i_o, i_f) \times \text{losscostdist}(i_d, i_c, i_p, i_y), \]

\[ \text{prem}(i_d, i_c, i_p, i_o, i_f) = \text{liabilitydist}(i_d, i_c, i_p, i_o, i_f) \times \text{premratedist}(i_d, i_c, i_p). \]

The aggregated loss costs and premium rates are then computed by first aggregating the total losses and premiums across states, crops, and products, and then dividing them by the aggregate liabilities

\[ \text{losscost}(i_s, i_o, i_f, i_y) = \frac{\sum_{i_d \in i_s} \sum_{i_c = 1}^{n_c} \sum_{i_p = 1}^{n_p} \text{loss}(i_d, i_c, i_p, i_o, i_f, i_y)}{\text{liability}(i_s, i_o, i_f)}, \]

\[ \text{premrate}(i_s, i_o, i_f) = \frac{\sum_{i_d \in i_s} \sum_{i_c = 1}^{n_c} \sum_{i_p = 1}^{n_p} \text{prem}(i_d, i_c, i_p, i_o, i_f)}{\text{liability}(i_s, i_o, i_f)}. \]

The output of the module consists of a set of loss cost, liability, and premium rate records for each unique combination of state, organization, and fund. This data is then passed on to the SRA.Run for final computations.
2.4.2.3 SRA_RUN Module

The runtime module is coupled with a graphical interface that allows a user to analyze companies' rates of return under different scenarios and/or combinations of the SRA parameters. Once the user provides the necessary information, the interface passes it along with the necessary data to the numerical engine. The inputs used at this stage include

- \( \text{losscost}(i_s, i_o, i_f, i_y) \) — equiprobable realizations (indexed by year) of the loss cost distribution by state, organization, and fund (output of SRA.PRE);

- \( \text{liability}(i_s, i_o, i_f) \) — base year liabilities by state, organization, and fund (output of SRA.PRE);

- \( \text{premrate}(i_s, i_o, i_f) \) — base year premium rates by state, organization, and fund (output of SRA.PRE);

- \( \text{pctretain}(i_s, i_o, i_f) \) — base year retention rates by state, organization, and fund (can be modified by user).

The engine performs three essential sets of tasks. First, it converts the total liability to the retained liability using the data on retention rates by state, organization, and fund. Second, it uses the aggregated loss costs and premium rates to compute the loss ratios by state, organization, fund, and year

\[
\text{lossratio}(i_s, i_o, i_f, i_y) = \frac{\text{losscost}(i_s, i_o, i_f, i_y)}{\text{premrate}(i_s, i_o, i_f)},
\]

and adjusts these to account for FCIC sharing of losses and gains under the current set of the SRA parameters as input by the user.
\[\text{lossratioadj}(i_s, i_o, i_f, i_y) = SRA(\text{lossratio}(i_s, i_o, i_f, i_y)).\]

Here the \textit{SRA} operator represents the adjustment according to the schedules in Tables 2.1 and 2.2.

Next, the engine uses the adjusted loss ratios to compute the corresponding rates of return by state, organization, and fund

\[\text{return}(i_s, i_o, i_f, i_y) = 1 - \text{lossratioadj}(i_s, i_o, i_f, i_y). \quad (2.30)\]

Note that (2.30) expresses the rates as a percent of the total retained premiums. The user may also select to compute the rates as a percent of the \textit{maximum possible underwriting loss} (MPUL), which is simply the maximum loss that can be experienced by the organization ex-post to sharing losses with FCIC under the SRA contract. In this case, the rates of returns in (2.30) are simply multiplied by \(\text{prem}(i_s, i_o, i_f)/\text{MPUL}(i_o)\).

Another possibility is to express the rates of return as percent of the maximum \textit{probable} underwriting loss, which may be calculated, for example, as the worst losses over the period of analysis (1981–1998). This option is not currently implemented in the simulator but can be easily added if needed for future research.

Finally, the engine computes the means and coefficients of variations of the rates of return

\[
\hat{\text{mean}}(i_s, i_o, i_f) = \frac{1}{n_y} \sum_{i_y=1}^{n_y} \text{return}(i_s, i_o, i_f, i_y)
\]

\[
\hat{\text{cv}}(i_s, i_o, i_f) = \sqrt{\frac{1}{n_y} \sum_{i_y=1}^{n_y} (\text{return}(i_s, i_o, i_f, i_y) - \text{mean}(i_s, i_o, i_f))^2},
\]

and passes them back to the interface.
The final task performed by the interface is to output the relevant rate of return statistics. Both the expectations and the coefficients of variation of the rates of return are presented in two formats: by state and fund and by organization and fund.

2.5 Standard Reinsurance Agreement: Efficiency Analysis

In this section we attempt to use the SRA Simulator in order to analyze the effects of various reinsurance provisions on the rates of return of private insurance companies. More specifically, we consider three scenarios. First, we compare the expected rates of return and their coefficients of variation in cases with and without SRA. This is a relatively simple exercise, but it clearly illustrates the risk reduction effect of the SRA.

Second, we suggest a simple heuristic rule the companies may use to allocate their books of business across the three reinsurance funds. We then compare the expectations and coefficients of variation of the rates of return under the heuristic rule with those under the base year allocations chosen by the companies. While this approach falls short of a full-scale optimization, the majority of the companies can still improve their performance using the suggested rule. In other words, while the rule does not necessarily reach the Pareto-optimal mean-variance frontier, it does represent a Pareto improvement for most of the companies in the sense that it provides higher expected and less variable rates of return.

Finally, we use the SRA Simulator in order to analyze possible venues for simplifying the structure of the SRA. More specifically, we replace the existing schedules (Tables 2.1 and 2.2) by a simpler structure and then consider the effects of this new structure on companies' rates of return.
\[2.5.1 \text{ "With versus Without SRA" Analysis}\]

In order to evaluate the effect of government reinsurance provisions on the performance of private insurance companies underwriting the crop insurance contracts, we run the SRA simulator with different selection of the input parameters. The case with no reinsurance is equivalent to replacing all entries in Tables 2.1 and 2.2 by 100\% regardless of funds and loss ratios and also setting the retention rates to 100\% for all funds. The expected rates of return and their coefficients of variation under this scenario are presented for each company\textsuperscript{28} in Table 2.3.

In the same way, the expected rates of return and corresponding coefficients of variation under the current SRA can be computed using the SRA parameters in Tables 2.1 and 2.2. These results are presented in Table 2.5. The performance characteristics of each company with and without the SRA are shown in Fig. 2.2 as points on the mean-variance plane. Movements down and to the right indicate improvement of companies’ performance.

Fifteen out of seventeen companies significantly benefit from the government reinsurance by both increasing the expectations and decreasing variation of the rates of return. In other words, the SRA results in a strong Pareto improvement for these companies. The remaining two companies also decrease the variability of their return but end up with lower expected rates. These two outliers may be explained by either good diversification of these companies’ books of business or concentration of their business in the regions that naturally exhibit higher rates of return on crop insurance contracts.

\textsuperscript{28}The actual names of the companies are withheld to preserve privacy.
Tables 2.4 and 2.6 show how companies' expected rates of return change by specific funds. An interesting phenomenon observed here is that the SRA affects rates of return for different types of products in very different ways. While expected rates of return on CAT and revenue contracts significantly increase after the SRA, the expected rates of return on "other" (mostly APH buy up) products drop for both Commercial and Developmental Funds. One possible explanation is that the APH products are on average more profitable and result in expected loss ratio less than one, while both the CAT and revenue contracts tend to have expected loss ratios more than one. The SRA then decreases the gains from APH buy up contracts while decreasing losses from CAT and revenue contracts. In a sense, the SRA subsidize losses from one product group by gains from the other. However, since the overall effect of the SRA results in increased expected rates of return, the government ultimately has to pay the cost of this increase.

2.5.2 Heuristic Allocation Rule

Since different reinsurance funds provide different levels of protection, companies may selectively allocate their books of business in order to reduce the overall retained risk and increase their rates of return. Ideally, an optimal allocation rule can be obtained as a solution to a multivariate optimization problem that would determine in what fund to allocate contracts underwritten in each specific district, for each specific crop, and for each insurance product, as well as recommend optimal retention rates for each reinsurance fund. However, implementing such an optimization problem poses a significant challenge due to the enormous number of decision variables and the nonlinearity of the objective function. Indeed, with a typical state including ten districts, each district growing four crops on average, and fourteen contracts available for each crop, the number of decision variables for this allocation problem alone is 560.
In addition, retention rates need to be chosen for the Developmental and Commercial Funds. Adding more states increases this number even further and makes the problem practically impossible to address.

In order to make the problem more tractable, we suggest a simple heuristic rule for allocation of the book of business and then compare the rates of return under this rule with those observed in the base year. More specifically, for each district and crop, the expected loss ratio without reinsurance is computed based on historical data as described in Section 2.4.2.2. If the expected ratio is higher than a prespecified value near 1.0, then for each company, all the business for that district and crop is placed in the Assigned Risk Fund. Otherwise, the business is allocated to the Commercial Fund.

The rationale behind this rule is that the loss ratio above 1.0 represents expected loss for the district, and therefore a company might want to obtain as much reinsurance protection for that district as possible. The loss ratio below 1.0, on the other hand implies expected underwriting gain, and therefore the company may want to retain as much of that gain as possible.

Several adjustments need to be made to the allocation obtained according to this heuristic rule in order to conform to the SRA requirements. First, the Agreement specifies the upper limits for cession to the Assigned Fund, which vary by state. Therefore, after the initial allocation of business is done by district and crop, the overall cession level is computed for a given company in each state. If the amount ceded exceeds the maximum allowable for that state, the excess business is transferred to the Developmental Fund in equal proportion for each district and crop with no sorting according to the expected loss ratios. Second, since the Assigned Risk Fund imposes a mandatory 20% retention rate, the suggested rule may result in lower levels of retained premiums, i.e. smaller books of business. In order to evaluate the effect
of the suggested rule in a comparable setting, the amount of business retained by each company using the rule should be the same or higher than that observed in the base year. In order to achieve this, the retention rates for the Commercial and Developmental Funds are set to levels that result in the retained premiums close to those observed in the base year.

Presented in Table 2.7 are the expectations and coefficients of variation of the rates of return the companies would achieve by following the suggested heuristic rule with the loss ratio cut-off parameter set to 1.0. The table also present the retention levels set for both the Commercial and Developmental Funds for each company. The overall amount of the retained premiums under the heuristic rule is $1,175 million which about 3% less that the actual retained premiums of $1,210 million observed in the base year.

Twelve out of seventeen companies would enjoy the same or higher amount of retained premiums under the heuristic rule than in the base year. The remaining five companies would lose anywhere between 4.4% and 20% of their respective books of business. However, all seventeen companies would at least lower the variability and eleven out of seventeen companies would also increase the expectations of their rates of return.

Figure 2.3 summarizes the effect of the heuristic rule on companies' rates of return by comparing it with the empirical observations for the base year. Two points are shown for each company on a mean-variance plane with movement down and to the right corresponding to Pareto-improvements.

---

For simplicity, the same rates are used for both funds. Recall also that the retention rates for the Commercial and Developmental Funds must be set in 5% increments and obviously cannot exceed 100%.
The analysis indicates that private companies do not fully utilize the reinsurance capacities offered by the existing SRA. Even a simple heuristic rule may significantly improve the performance of the companies by both decreasing the variability and increasing the expectation of their rates of return without reducing retained premiums. Note also that the suggested rule may be further adjusted to the needs of a specific company, e.g. by varying the loss ratio cut-off parameter or modifying the retention rates separately for the Commercial and Developmental Funds.

2.5.3 Simplified SRA

One possible reason why insurance companies do not take advantage of the reinsurance protection offered by the SRA may be the inherent complexity of the Agreement. Nonlinear reinsurance schedules presented in Tables 2.1 and 2.2 make it harder to evaluate the effects of allocating the book of business in one or another way. Therefore, it may be of interest to investigate if a simpler SRA structure would provide a level of protection similar to the one offered under the current agreement.

To make our point, we consider the following SRA structure. The government provides a stop loss protection when the realized loss ratio is above some prespecified level and shares both in losses and gains in the same proportion when the loss ratio is below this level. This structure is essentially a two-parametric family of reinsurance agreements, one parameter being the stop loss point and the other being the fraction of gains and/or losses retained by the company below that point.

Obviously, different level of protection may be generated by freely varying both of the parameters. However, the cost of the reinsurance for government under some combinations of these parameters may be too high. Therefore, we impose the condition that the overall expected rate of return remains the same under the simplified SRA as it was in the base year under the existing SRA.
Note that fixing the overall expected rate of return does not guarantee that expected rates of return for each company will also remain the same. Since the SRA applies in the same way to all participating companies, we simply cannot adjust individual rates of return. However, we can use the remaining degree of freedom in the choice of SRA parameters to minimize the deviation of companies' expected rates of return from their base year values. These two conditions uniquely determine the stop loss point 1.46 and the universal share rate 0.54. In other words, under the simplified SRA, the government covers all losses in excess of 146% of the net premiums, while companies share 54% of losses and gains when their loss ratios are below 146%.

Presented in Table 2.8 are the expectations and coefficients of variation for companies' rates of return under this "optimal" simplified SRA. Figure 2.4 compares the effect of the latter with that of the current SRA on the mean-variance plane. Once again, moving down and to the right is beneficial for the companies, since it increases the expectations of the rates of return and decreases their variability.

Under the suggested simplified SRA, nine out of seventeen companies would attain both higher expectations and lower variability of their rates of return. Seven out of seventeen companies effectively would have the same expected rates of return but lower variability. Only one company would end up with lower expected rate of return, albeit also with lower variability. Note, however, that this is the same company (Company 8) which benefited the least from the current SRA as compared to the situation with no reinsurance at all (see Fig. 2.2).

In practice, replacing the whole SRA by the simplified version would not be beneficial for producers, since it creates a strong incentive for companies to concentrate their business in those regions where the rates of return are the highest. However, the simplified structure may be applied, for example, to the Commercial Fund only so as
to preserve the high risk protection provided by the Assigned Fund. The overall conclusion is that the Risk Management Agency can potentially rework the SRA into a simpler structure preserving or even reducing the risk borne by individual companies without increasing the expected government cost of reinsurance provision.

2.6 Conclusion

The essay presents an economic analysis of the Standard Reinsurance Agreement between the Federal Crop Insurance Corporation and private insurance companies providing crop insurance for farmers. The SRA Simulator allows policymakers to analyze the effects of the agreement on the distributions of companies' rates of return. The simulator is a user-friendly program that models farm-level distributions of yields and prices and aggregates them to compute rates of return of insurance companies delivering crop insurance products.

The essay uses the simulator to analyze the effect of current SRA on companies' rates of return and illustrate that the SRA is highly effective in reducing the risk borne by insurers. Two counterfactual scenarios are also analyzed, one suggesting a simple heuristic rule to allocate companies' books of business across reinsurance funds and the other suggesting a simplified structure of the SRA. The results suggest that private insurance companies do not fully utilize the reinsurance provisions offered by the SRA and can significantly improve their performance. Also, the government may simplify the current SRA structure to provide the same or better reinsurance at the same cost.

Further research may involve analyzing more sophisticated heuristic rules for allocation of the book of business, such as presorting districts/crop combinations by the expected loss ratio and allocating only the worst districts to the Assigned Risk Fund
up to the maximum allowable cession limit. One may also attempt to set up a formal optimization problem albeit under some simplifying assumptions such as identical allocation patterns across all products for a given district. In terms of modifying the SRA structure, an interesting research question may be whether the RMA can decrease the cost of reinsurance program while maintaining the rates of return of the private insurance companies.
### Loss Ratio between 100% and 160%

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<th>Revenue Insurance Plans</th>
<th>All Other Plans</th>
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<tr>
<td>Developmental</td>
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<td>30.0%</td>
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<tr>
<td>Assigned Risk</td>
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### Loss Ratio between 160% and 220%

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### Loss Ratio between 220% and 500%

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<td>—</td>
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</tr>
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Table 2.1: Retained liabilities by fund for different loss ratios.

*Note:* FCIC assumes the ultimate net losses in excess of companies’ retained ultimate losses as determined in the table. In addition, FCIC assumes 100% of the amount by which companies’ retained losses in a given state and fund exceed 500% of the retained net book premium for a given reinsurance year.
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<td>94.0%</td>
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Table 2.2: Retained underwriting gain by fund for different loss ratios.
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Table 2.5: Expected rates of return and coefficients of variations by company. Base year (1998) book of business, current SRA.
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<th>Assigned</th>
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Table 2.6: Expected rates of return and coefficients of variations by fund. Base year (1998) book of business, current SRA.
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<th>Company</th>
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Table 2.7: Expected rates of return and coefficients of variations by company. Heuristic allocation of the base year book of business.

\*The proportion of total premiums retained by companies in both Developmental and Commercial Funds.
\*Companies reduce the variability of their rates of return.
\*Companies both reduce the variability and increase the expectations of their rates of return.
\*Companies lose a part of their books of business.
<table>
<thead>
<tr>
<th>Company</th>
<th>Retained Premiums, $ million</th>
<th>Rate of Return, percent</th>
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Table 2.8: Expected rates of return and coefficients of variations by company. Base year (1998) book of business, simplified SRA.

*The company reduces both the variability and expectation of its rate of return.
↑Companies reduce the variability of their rates of return.
*↑Companies both reduce the variability and increase the expectations of their rates of return.
Figure 2.1: SRA Simulator Tree Diagram.
Figure 2.2: Mean-variance diagram for companies' returns with and without SRA.
Figure 2.3: Mean-variance diagram for companies’ returns under empirical and heuristic allocation.
Figure 2.4: Mean-variance diagram for companies' returns under current and simplified SRA.
3.1 Introduction

Agricultural production has always been a risky endeavor. Farmers constantly have to deal with unfavorable weather conditions, variability in prices of inputs and outputs, livestock disease outbreaks, pests, etc. The uncertainty of future incomes complicates both short-term production decisions and long-term planning (e.g. expansion of production or capital investments in machinery and equipment). It also renders lending institutions less willing to provide loans to farmers, since the probability of default is relatively high. Although some forms of self-insurance may be available to farmers (e.g. crop diversification or intertemporal income transfers), these have certain limitations and ultimately reduce farm profits in the long term.

Of all the risk factors affecting agricultural production, especially crop production, weather is typically the most significant. Weather phenomena are hard to predict (at least in the beginning of growing season) and even harder to mitigate against. Moreover, since unfavorable weather conditions such as floods or droughts often affect
large areas, the risks faced by different producers are correlated. The latter presents a stumbling block to traditional insurance, which is designed to pool a large number of small unrelated risks rather than widespread simultaneous (catastrophic) losses.

Traditionally, unwillingness or inability of insurance markets to provide risk management mechanisms for agricultural crop production has prompted many governments to step in with various forms of support of agricultural producers (subsidized loans, price-support programs, tax breaks, etc.). Unfortunately, the government support programs are often inefficient and come at high social cost (see, for example, discussion in Skees, Hazell, and Miranda [53] and Hazell [26]).

An emerging trend in recent years has been to develop new financial instruments (catastrophe options, catastrophe bonds) which would allow insurers to securitize correlated risks and circumvent the limitations of traditional reinsurance market. These innovations were stimulated partly by enormous losses suffered by the insurance industry from natural disasters in the late 1980s and early 1990s (hurricanes Hugo and Andrew, Northridge earthquake, etc.). In application to agricultural insurance, the innovations include area-yield insurance program and various exchange-traded area-yield contracts. A characteristic feature of these instruments is that their payoff depends on values of a specially designed measure, or index, related to the risk being hedged against.

The main advantage of indices is that they can be measured objectively and do not depend on individual actions of market participants. Transparency of index-based contracts along with the fact that they are uncorrelated with traditional financial instruments (stocks, bonds, etc.) make them attractive to investors outside of insurance industry. Insurers benefit by getting access to additional funds that can be used to indemnify large simultaneous losses caused by natural disasters.
In the same way, using index-based instruments in agricultural risk management (e.g. rainfall insurance, heating degree-days contracts, area-yield contracts, etc.) allows to circumvent problems faced by traditional insurance and provide farmers with an efficient hedge of weather-related risks. However, a design of an index insurance contract or financial instrument requires answering very important questions, such as what variable to use as an index, how to structure the indemnity schedule, how to price a contract, how to sell it, and whom to sell it to.

The purpose of the present study is to (1) review recent literature on catastrophe insurance and securitization of correlated risks; (2) provide a summary of recent developments in catastrophe insurance products; (3) discuss issues arising in design of index-based instruments; (4) provide an analytical framework for pricing such instruments; and (5) illustrate the above technique for Nicaragua rainfall insurance contracts.

3.2 Weather Risks and Traditional Insurance

Weather affects all areas of human activities. Since weather cannot be reliably predicted for more than few days ahead, it constitutes a risk factor. Some weather phenomena have positive effect and may be necessary for some economic processes, just as proper amounts of sunshine and rainfall are necessary for successful crop production. However, even the normal (average) weather patterns may have negative impact on business operations. One and the same spring rain, for example, may benefit a corn producing farmer yet adversely affect visitation of a nearby golf course. By the same token, a hot summer day may mean a booming business for an ocean resort, but a capacity overload for a local power station.
Even though unfavorable weather conditions may cause substantial losses to businesses, these pale in comparison with devastating effects of large scale natural phenomena such as hurricanes, tornadoes, earthquakes, floods or droughts. In recent years, several natural disasters have resulted in losses previously unheard of in the insurance industry. The reported damages of Hurricane Hugo (1989) totaled $4.2 billion, but these were dwarfed three years later by Hurricane Andrew, which caused $15.5 billion in insured damages. In 1994, Northridge earthquake in California resulted in losses in excess of $12 billion (McCarty and Spudeck [40]).

Although droughts and floods may attract less media attention than hurricanes or earthquakes, they usually affect much larger areas, last for extended periods of time, and have severe negative impact on agricultural production. According to National Climatic Data Center [9], the severe drought of 1988 resulted in total economic loss of $56 billion, while the Midwest flooding of 1993 caused more than $23 billion in damage, each alone being more costly than most hurricane seasons of the last two decades.

While the catastrophes of the last decade were arguably the costliest on record, natural disasters have always been present. During its long history, the insurance industry has had to develop some ways to cope with catastrophic events as well as more regular weather phenomena. Therefore, before proceeding with recent innovations in weather risk management instruments, it may be instructive to discuss how traditional insurance handles weather-related risks, where it succeeds, and where and why it fails.

3.2.1 Insurance of Non-Catastrophic Weather Risks

When it comes to negative effects of isolated weather phenomena, the traditional insurance provides relatively efficient instruments for managing risks related to those
phenomena. For example, Good Weather Insurance Agency, a Massachusetts-based company, has specialized in providing coverage for weather-related risks for more than 25 years. Insurance is offered in four general areas: sales promotions, special events, winter weather and income stabilization. Among company clients are golf courses, department stores, municipalities, airports, stadiums, etc. While specific policy terms differ from case to case, the general idea is that an indemnity payment is triggered by the occurrence of a pre-specified weather condition. Some examples include

- a policy for a Mid-Atlantic airport which for a premium of $42,500 provided a November-through-March total-inch snow protection with a 50" deductible, with every extra inch of snow paying $25,000;

- a policy for a restaurant in Portland, ME, which for a premium of $9,240 provided December through April coverage for every Saturday paying $7,500 when more than 3.5 inches of snow fell in a 24 hour period;

- a policy for a soccer game in Chicago which for a premium of $16,500 guaranteed payment of $100,000 if there was rain during more than two hours of a game day in April.

Since 1998, Holidair Insurance Services in Canada has offered a “sunshine insurance” for Canadian tourists visiting Florida. It provides that Canadian vacationers in Florida be reimbursed the total cost of their vacation package if more than one-tenth of an inch of rain is recorded, per day, during the hours of 10am and 4pm, for half of their vacation, plus one day. Similar services are offered by other companies both in the United States (weather division of Northland Insurance Companies,  

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30Source: company website
Worldwide Weather Insurance Agencies, Inc., etc.) and abroad (for instance, Onlineweather.com, part of ATLAS Holdings, Ltd., provides a wide range of weather insurance for all parts of the United Kingdom).

A common feature of all the contracts mentioned above is that they deal with single events and resulting losses affect only one client at a time. The probability of each event and associated losses can be calculated using historical data\footnote{Good Weather Insurance Agency claims to have precipitation records for "every single hour for every single day for thousands of locations for sometimes 100 years".} and conventional actuarial techniques. Moreover, an insurance company can diversify its risk portfolio by insuring different clients in different regions against different weather phenomena, or by insuring clients whose losses are negatively correlated.

The advent of the Internet has facilitated access to historical weather data and client–company information exchange. Not surprisingly, many companies advertise weather-related risk management products on their web sites, or spin-off specialized online divisions to handle this particular line of business.

3.2.2 Challenges of Catastrophe Insurance

As emphasized above, traditional insurance is quite comfortable with small risks that are uncorrelated across time and space. Unfortunately, major natural disasters do not meet either of these conditions, since they affect large areas simultaneously and result in enormous losses.

Figure 3.1 summarizes U.S. disasters in 1980–1999 with losses exceeding $1 billion \cite{9}. More than forty such events occurred during the 20 year period, with a combined cost in excess of $230 billion (in 1999 dollars). The general trend is that disasters are becoming more frequent and more costly. Various projections suggest
that the insurance industry may easily face a $50 billion disaster within foreseeable future. The situation is often aggravated by shifting demographics and a sharp increase of property value in disaster-prone areas. In the last 25–30 years, the population of the Southeast Atlantic coast almost doubled. Population growth rates in Florida and California have been two to three times higher than the national average. At the same time, the value of insured coastal property rose by more than 70% since 1988.

For an insurance company, a catastrophic event translates into hundreds or thousands of claims that need to be paid within a relatively short period of time. Since insurance companies usually pay claims out of the current stream of premiums (dynamic premium strategy), such a practice imposes a tremendous financial burden forcing insurers to quickly raise large amount of capital after a catastrophic event. To make things worse, many insurance companies face this same situation simultaneously and must compete against each other for external funds.

Vulnerability of traditional insurance to the financial stress imposed by catastrophic events was clearly demonstrated in the early 1990s. In the wake of Hurricane Andrew, 20 property and casualty insurance companies that operated in Alabama, Florida, Georgia, Mississippi, and South Carolina as of December 31, 1989 failed before January 1, 1994 (see [1]). The remaining companies became less willing to offer hurricane insurance in the disaster-prone areas or offered it at much higher premiums. Average rates in the Miami area, for example, increased by 65% between 1992 and 1995 (Jaffee and Russell [28]).

Similarly, after Northridge earthquake of 1994, the majority of insurance companies operating in California either ceased writing homeowner insurance or imposed strict limits on policies offered. More than one hundred insurance agencies requested an increase in premium rates from state commissioners.

33Five states most affected by the disaster
3.2.3 Reinsurance and Financing of Catastrophe Risks

Theoretically, sharing of natural disaster risk does not seem completely impossible. If, for example, an insurance company knows that a $1 billion disaster occurs once in ten years in an area with one million insured households, then all it needs to do in order to be able to meet the disaster loss is to collect a $100 premium per household per year over ten years. However, collection of internal funds is subject to several institutional constraints the insurance industry must deal with. The following is a brief summary of some of those constraints (see Jaffee and Russell [28] for detailed discussion).

1. Accounting Requirements. Existing accounting practices do not allow an insurance corporation to set aside a part of capital surplus to pay for a future catastrophic loss. The rationale is that the insurance company financial report provides a more realistic snapshot of company’s financial situation if income and losses are accounted for one year at a time, rather than over a period of several years. Therefore, an insurance company cannot accumulate several years worth of premiums for the sole purpose of covering large losses in case of a disaster. The company, of course, may allocate the retained earnings to a capital surplus account, but this account cannot have a special designation.

2. Taxation. Under the current taxation system, any addition to the capital surplus as well as any interest on the accumulated capital are taxed as corporate income. Therefore, insurance companies have even less incentive to set aside some sort of a “disaster fund” in order to pay for a possible catastrophic loss.

3. Agency Cost of Excess Cash. Accumulation of excess cash reserves by a publicly traded corporation entails various negative consequences from the managerial standpoint. First, internal cash is usually underpriced by investors, which means
that the company stock suffers in the short run. Second, a company with large
cash holdings becomes a target of potential takeovers. Finally, if an insurance
company is a part of a larger corporation, the parent company may consider
cash surplus as an easy source of capital, allocating it to other parts of the
corporation or to stockholders as dividends.

Even in case of mutual companies, which are less vulnerable to takeovers and
have better opportunities to create and preserve contingency funds, excess cash
reserves are often seen by policyholders as a good reason to decrease rates or
otherwise redistribute the accumulated surplus.

4. Moral Hazard. Limited liability creates a potential for moral hazard. For ex-
ample, an insurance company may decide to use the collected catastrophic in-
surance premiums for dividend payments in a year without a disaster, and then
file for bankruptcy when an actual catastrophe occurs.

Note that even if an insurance company were able to collect premiums over an
extended period of time, it would only break even on average. If the one-in-ten year
disaster discussed above happened on the second year of the contract, the company
would still face the problem of raising extra capital, since 90% of its contingency
funds would be in the form of yet unpaid premiums.

Since using internal funds to deal with catastrophic losses is not an option for
the majority of insurance companies, they usually turn to various sources of external
funds either before or after the event. The traditional source of ex-ante financing of
catastrophe risks (and large risks in general) is reinsurance. Some insurance com-
panies sell reinsurance along with other lines of business, although most specialize
exclusively in reinsurance (e.g. Swiss Re., Munich Re., London Reinsurance Group,
etc.).
By buying a reinsurance contract, an insurance company can share its catastrophe risk exposure with the reinsurer. In the simplest example, if a company A anticipates a potential loss of up to $500 million, it may sell the upper layer of its exposure (e.g. $250 million and above) to another company B for a premium. In case of a disaster, company A would pay claims up to the first $250 million. If the total losses exceeded this amount, then the reinsurance contract would be triggered and company B would provide additional funds to cover the losses between $250 and $500 million.

At the first glance, reinsurance provides a natural way out for the insurance companies facing catastrophe risks. Insurance companies get access to external capital, while reinsurers can diversify their portfolio by taking risks in different geographical regions of the United States or around the world (after all, what are the odds of a hurricane in Florida and forest fires in Latin America happening at the same time).

In reality, however, the picture is far from perfect. Froot [16] suggests that only a small fraction of catastrophic exposure borne by insurance companies is covered by reinsurance. According to various estimates, a $50 to $100 billion catastrophic event has a rather high probability of occurring within the next twenty years. However, the insurance industry by itself is thought to be able to absorb no more than $25 to $35 billion in catastrophic losses (see, for example, McCarty and Spudeck [40]). Moreover, very little of reinsurance in place covers industry-wide losses above $5 billion. In other words, the upper 90% of a potential $50 billion disaster losses are essentially unprotected by reinsurance and would have to be paid by the insurance companies that retained the exposure.

Another problem with catastrophe reinsurance is unjustifiably high prices, especially in the aftermath of large natural disasters. Industry-wide price per unit of ceded exposure rose steadily over the last ten years, with dramatic increases following
Hurricanes Andrew and Hugo and the Northridge earthquake (Froot and O’Connell [18]). Even though reinsurance prices began to decline in 1995, they still remain well in excess of the actuarially-fair levels.

Froot [16] outlines eight different explanations to these observed facts. While no single explanation fully accounts for the scarcity of catastrophe risk sharing, each of them sheds some light on the problem. The following is a brief summary of Froot’s arguments.

1. Lack of Reinsurance Supply. For some reasons, the reinsurance industry does not have enough capacity to accommodate a major part of catastrophic exposure. It may be costly for reinsurers to raise a large amount of capital or carry it on their balance sheets. As a consequence, the demand for reinsurance is higher than the available supply, which leads to high rates and fewer contracts.

   This is supported by empirical facts, which indicate that prices for reinsurance go up in the aftermath of a large catastrophic event, while the number of contracts sold goes down. Higher prices may also be explained by the increasing demand for insurance after a disaster, but the latter would also result in a higher volume of transactions, which has not been observed. In the long term prices eventually fall back. However, the observed pattern is consistent with the shortage of reinsurance capital at least in the short run.

2. Market Power of Reinsurers. Since there are relatively few large reinsurance companies, they may exercise considerable market power. As a result, reinsurers may maintain high prices and have no incentive to raise extra capital.

   The empirical facts in favor of this explanation are general consolidation within the reinsurance industry over time, increase in capital and market share of large reinsurers, and secular growth of reinsurance prices even without catastrophic events.

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34See [16], pp. 1–22, for detailed discussion.
taken into account. However, consolidation per se does not necessarily mean increasing market power and may be a reflection of general growth in the industry. There is also a possibility of some sort of interaction between market power and capital shortage, which together result in the observed price/quantity patterns.

3. Inefficiency of Corporate Form for Reinsurance. The high cost of reinsurance capital may be explained by agency cost of capital. If a reinsurance company is a publicly traded corporation, then management is forced to maintain some level of return. Empirical evidence suggests that reinsurance capital is often considered as "equity like" and requires an appropriate return above risk-free assets, even though catastrophic events are largely uncorrelated with fluctuations of the major financial markets. Thus reinsurance capital is costly, reinsurance rates are high, and insurance companies do not want to buy coverage.

4. Transaction Cost. The price of a reinsurance contract includes a substantial frictional component. Nonliquidity of the contract, various intermediary costs involved in underwriting a contract, and other factors may possibly drive up reinsurance rates and hence decrease the amount of reinsurance bought by insurance companies. Empirical facts, however, suggest that the transaction cost alone cannot account for the observed high levels of prices.

5. Moral Hazard and Adverse Selection. Once an insurer transfers risk to a reinsurer, the behavior of the former may change in that the insurer may become more careless in underwriting contracts in disaster-prone areas, may not enforce preventive measures, or otherwise expose itself to unjustified risks. Therefore, high reinsurance rates serve as a precautionary measure in order to prevent this type of behavior and to force insurers to care more about their risk portfolios.

\footnote{See also Bohn and Hall [10] for more insight on this issue.}
In a similar way, if risk transfer is relatively cheap, then insurers with above-average exposure would actively buy reinsurance, while those with less riskier portfolios would not. As a result, the reinsurer would have to face adverse selection in its own risk portfolio, with potential losses much higher than the reinsurance premiums collected. While this argument can explain high prices and low levels of risk transfer, it still does not account for cyclical movements in prices.

6. *Rate Control.* Many states (including California and Florida) require insurance companies to keep insurance rates below prescribed levels, which leaves very little, if any, margin of profit. Therefore, companies are often forced to resort to cost-cutting measures, of which saving on reinsurance is one option. While this argument does not explain generally high reinsurance rates, it does provide a plausible explanation to the observed low levels of risk transfer.

7. *Third-party ex-post financing.* In case of any major disaster, both insurer and insured expect federal and local governments to step in and provide relief funds. Reliance on emergency funding has increased over time. After hurricane Andrew (1992) FEMA relief covered about $1.8 billion out of estimated $32.4 billion total losses, while after hurricane Georges (1998), FEMA’s contribution rose to $2.4 billion out of the estimated $5.9 billion total losses. Other natural disasters have received similar treatment.

The consequences of this large-scale government disaster assistance are two-fold. On the one hand, it eliminates any incentive for private insurance to create a market for natural disaster contracts (e.g. flood insurance). Even if the incentive exists, government usually subsidizes rates for insurance contracts offered by private companies, essentially providing “free” reinsurance.

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36Source: Top Ten Hurricane Disasters Ranked by FEMA Relief Costs [online], available at http://www.fema.gov/library/top10hu.htm

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On the other hand, government interventions create potential moral hazard by encouraging new construction in the disaster-prone areas with the implicit promise of free relief funds in case of a major catastrophic event. By the same token, owners of property in potential disaster areas have less incentive to buy disaster insurance, since they know that the government would step in anyway and provide the same relief for free. While this explanation accounts for low volumes of reinsurance contracts, it still does not answer the question why prices for the existing reinsurance contracts are high.

8. **Behavioral factors.** People tend to underestimate the probability of rare events, which are too difficult to perceive. Therefore, they discount low probability, high consequence events at a very high rate and thus under-purchase insurance. Another behavioral factor is that individuals react differently to risk and uncertainty. When outcomes and their probabilities are clear (risk) the risk premium is easy to perceive. On the other hand, when neither probability nor outcome are well-defined (will a hurricane hit this particular area, and if yes, will it destroy this particular house?), decision makers are more uncertain about the situation and charge more to carry the exposure. This explanation also accounts for low volumes of risk transfer, although is less clear about the reasons of high cost of reinsurance.

No matter which of the above explanations accounts for the observed cost and volume of reinsurance, the fact, once again, remains that the combined capacity of the insurance and reinsurance industry is barely enough to cover losses from a mid-force hurricane sweeping through a heavily populated area of Florida, an average earthquake with epicenter in downtown San-Francisco, or a major drought in the Midwest. The industry is not capable of carrying the burden of catastrophic events on its own and has to search for alternative ways of financing.
3.3 Market Securitization of Weather-Related Risks

In a perfect world, risk sharing would be complete and the cost of ceding would be close to the actuarially-fair rate. High levels of catastrophe risk retention by retail insurance and high reinsurance rates are evidence of incomplete or imperfect markets. Applied to crop insurance, market imperfection translates into unacceptably high rates or even complete lack of risk protection for farmers.

In some cases, the market for a particular risk may simply not exist. In developing countries, traditional insurance faces many obstacles, including lack of appropriate institutions and financial infrastructure, unwillingness of insurance companies to underwrite contracts under uncertain conditions and high risk of default, unfamiliarity of local population with financial instruments, corruption at local level, and so on. Securitization, or transfer of risks to financial markets by means of specially designed financial instruments appears to be a natural way to introduce market efficiency, reduce transaction cost, and attract additional capital to financing of weather-related risks.

3.3.1 Appeal of Capital Markets

The idea of using capital markets as a way to share risks has been widely discussed and developed in the last decade especially in application to catastrophic insurance (Froot [16], Lewis and Davis [35], Doherty [14]). However, the same approach can also be used to provide an efficient and affordable insurance against natural disasters to farmers both in the United States and around the world. The main arguments in favor of market securitization of catastrophe risks appearing in the literature are as follows.
• Weather-related risks are uncorrelated with capital market fluctuations. According to a study by Guy Carpenter & Co. [23], correlation between returns from underwriting catastrophic reinsurance and S&P500 over a period from 1970 to 1994 was $-0.13$. The same number for the U.S. Government bonds was $-0.07$. Therefore, securities whose pay-offs are tied to occurrences of particular natural events should be very attractive to investors, allowing for better portfolio diversification. Insurers, on the other hand, would get access to much cheaper source of disaster-contingent funds, since such securities should not require return much higher than the risk-free assets (e.g. Treasury Bills).

• Financial markets provide capital far in excess of what is available in the insurance industry. A potential loss of $50$ billion due to a natural disaster is well below market volatility on an average trading day. Thus insurers can access a larger pool of capital for both *ex-ante* and *ex-post* financing of catastrophic events. In the same way, capital may be attracted to provide insurance in the areas where traditional insurance either fails or does not exist (e.g. agricultural production).

• Attraction of private capital eliminates a need for government involvement, at least in the form of direct financing of risks by taxpayers’ money. The government, however, may still play regulatory role in establishing appropriate financial institutions, monitoring, and so on.

Until recently, the only way an investor was able to add a weather-related risk exposure into his/her portfolio was through investing in an insurance or reinsurance company active in the weather-risk market. However, in doing so, the investor exposed him/herself to the full combination of risks bundled in the (re)insurance company portfolio, as well as the potential financial risks of the company ownership. Market
securitization, on the other hand, should allow one to invest in a particular type of risk exposure and choose an amount of that exposure one is willing to undertake. Therefore, a typical security tied to a weather-related risk should possess the following properties.

- The contract should be clearly related to the occurrence of a specific event during a pre-specified period of time. In order to avoid moral hazard, the specific event must be easily observed and easily monitored.

- The contract should be divisible so that an investor may buy a portion of risk exposure in any desired increments.

- The risk inherent in the contract should be clearly specified and separated from the basic business risks of the issuer.

- There should exist a well-functioning market for the contract which would allow for efficient pricing.

Early attempts at securitization of weather-related (mainly catastrophic) risks yielded mixed results (Lewis and Davis [35]). *Contingent surplus notes* were considered by AIG and Merrill Lynch even before Hurricane Andrew. Nationwide Insurance company issued $400 million in such notes in 1995, and Hannover Re issued an $85 million tranche in the same year.

Under the contingent surplus notes arrangement, invested capital is put in a special trust fund that issues coupon bearing debt-securities. The capital is initially protected by Treasury notes, but the insurer has a right to replace those by surplus notes if a pre-specified event occurs during the contract life-time. Surplus notes carry higher interest rate than Treasury notes, but provide the issuer with a secured source of
contingent funding. However, due to regulatory constraints, payments on surplus notes must be made only out of the insurer's surplus. Therefore, investors are still subject to the general business risk of the insurance company.

Nationwide did get access to a source of funding unavailable through the traditional reinsurance, although high pricing of the issue (250–300 basis points above Treasury notes) did not make this route much cheaper.

Catastrophe bonds were another instrument successfully used by several companies since 1997. The general idea of catastrophe bonds is that an insurance company issues interest-bearing debt securities either through a specially selected reinsurer or specially created company. The company reserves a right to change the interest rate on the bond contingent on a particular level of company-specific or industry wide losses. United Services Automobile Association's (USAA) issue of catastrophe bonds in 1997 was the largest and highly publicized placement at the time (Froot [17], Lewis and Davis [35]).

Under the arrangement, a special company, Residential Reinsurance Limited, was created in the Cayman Islands with the single purpose of providing reinsurance for USAA. Residential Re then issued a total of $477 million ($164 million principal-protected and $313 million principal-unprotected) in debt securities. Investors would receive LIBOR plus 273 basis points on principal-protected and LIBOR plus 576 basis points on principal-unprotected securities. If a triggering event occurred within the coverage period (one year), investors would lose either the interest or both the interest and the principal. The triggering event was defined in the contract as a single category three through five hurricane that would result in USAA losses higher than $1 billion. Residential Re would then cover about 80% of the $500 million layer in excess of $1 billion.
The issue was over-subscribed and generated considerable trading in the secondary market. It also showed a substantial interest of institutional and private investors in catastrophic risk-sharing securities. Other companies (Swiss Re, Reliance, etc.) followed with similar issues differing in volume, triggering conditions, and interest. Some of these transactions are summarized in Table 3.1.

*Contingent equity securities* were designed to provide insurance companies with *ex-post* sources of capital. As discussed in Section 3.2.2, in the aftermath of a catastrophic event, an insurance company may find itself in a desperate need of additional capital in order to meet hundreds if not thousands of claims simultaneously. Raising external capital in such a situation may prove very expensive, with capital being in short supply and investors concerned about the financial stability of the company. Under the contingent equity arrangement, the company essentially buys a call option, which allows it to get access to a pre-committed capital at pre-specified rates contingent on occurrence of a pre-specified event.

In 1994–97, Aon arranged several issues of contingent equity securities, which it called Catastrophe Equity Puts (CatEPuts). Among the insured were Hawaii Hurricane Relief Fund, Florida Windstorm Underwriting Association, and State Auto Insurance. More details on these and other transactions are shown in Table 3.1.

3.3.2 Index-Based Derivatives — Advantages and Disadvantages

One of the problems with early attempts to securitize risk sharing was that the triggering event or condition was usually tied to the financial situation or losses experienced by the insurance company. This created a potential for moral hazard, since outside investors could not monitor directly the actual financial situation or underwriting practices of the company.
The recent approach to dealing with moral hazard is to design contracts based on an objectively measured variable, or index (see, for example, Doherty [14]). The index may be, for example, an aggregate insurance loss of companies in the region or the industry overall. In application to agricultural production, an index may be an average county yield or the amount of rainfall received by a particular area during the growing season. The general idea is that the index variable does not depend on individual actions, can be easily monitored, and is transparent. Index-based insurance contracts and financial derivatives do not depend on financial situation of the company issuing the security. In other words, they carry very little credit risk. The transparency of the underlying variable makes them attractive to investors, who can easily monitor the state of their portfolio. This also facilitates creation of a secondary market for the derivatives, which can be traded on common exchanges just as options or futures.

The major downside of index-based securities is that the index may not necessarily track the losses of a particular insurer, crop producers, etc. In other words, index derivatives introduce so-called basis risk. If, for example, a contract pay-off is based on the average county yield in a particular year, then the farmers whose yields are below county average do not receive enough indemnification (if any) for their losses. By the same token, even though industry-average losses from a hurricane may not be too large, a particular insurance company may have a major insured interest in the area hit most severely. Under the index-based contract, this company may receive little to no payments, even though its losses are disproportionally high.

On the other hand, since the index-based contract indemnity does not depend on individual actions, the insured retains an undistorted economic incentive to mitigate against losses. If, for example, an insurance company purchases a contract linked to the industry-wide losses, then by investing in risk prevention measures (e.g. enforcing
building code, monitoring the insured property, etc.) it can decrease its own risk exposure to the level below industry average and thus earn an additional premium from the contract. With the standard reinsurance, such an adjustment made little sense, since it would be offset by the corresponding change in reinsurance rates.

Overall, index-based contracts provide an attractive and innovative way to share risk. During the last decade, several types of such contracts emerged and successively traded on major exchanges and OTC market. The catastrophe options and futures introduced at CBOT in early 1990s is probably the most well-known example of contracts whose pay-off is explicitly related to a realization of an index. More specifically, a set of indices of insurance industry losses maintained by Property Claims Services (PCS) are used as a basis of cash settlement for the contracts. The indices cover the entire U.S. as well as several separate regions (Florida, Texas, California, East, Northeast, etc.) represent reported quarterly losses. Pay-off occurs when the index exceeds a pre-specified level (strike).

Another example of index contracts which are designed to mitigate weather-related risks are Heating Degree Days (HDD) and Cooling Degree Days (CDD) options and futures contracts traded at the Chicago Mercantile Exchange. An HDD (CDD) is determined as a number of degrees by which the average temperature at a particular day exceeds (falls below) 65°F. This measure is of special interest for electric power companies, since it is closely correlated with demand on electricity. The 65°F mark is considered the temperature at which consumers use neither air conditioning, nor heaters. Any deviation from this benchmark results in higher electricity consumption. Therefore, by using a contract whose pay-off depends on the number of HDDs or CDDs during a particular month, a utility company can offset additional costs it
incurs during the periods of peak electricity consumption. Currently, the contracts are traded for some major U.S. cities (Atlanta, Chicago, Dallas, New York, etc.), but CME plans to extend the area of contract coverage to other locations.

In the rest of this essay we attempt to develop a methodology which can be used to design index-based insurance contracts for agricultural production. We further illustrate the methodology by a case study of Nicaragua rainfall insurance.

3.4 General Approach to Design and Pricing of Index Contracts

Several important issues must be addressed in designing and pricing index contracts for hedging agricultural risks (e.g. rainfall insurance). First, the contract should have a relatively simple and transparent structure. An individual buying the contract should not be confused as to when and under what circumstances the contract will pay off.

Second, the index should be easily observable and measurable on a regular basis. Further, there should be enough historical observations of the index in order to derive its distribution and relation to the actual risk being insured.

Third, using an index to compute indemnities rather than actual losses inevitably introduces some basis risk. Therefore, careful consideration should be given as to what variable (or combination of variables) to use as an index in order to minimize the basis risk to the extent possible.

Fourth, given the traditional accounting practices of insurance companies, the contract parameters should be specified in a way that conforms to specific premium rate and risk exposure requirements of the company issuing the contract.
While an index contract may be structured in many different ways with different coverage layers and provisions, we will focus our attention on a particular class of elementary contracts. Specifically, an elementary contract pays an indemnity \( f(\bar{v}) \) conditional on realization of the index \( \bar{v} \) according to the following schedule (Fig. 3.2)

\[
f(\bar{v}) = \begin{cases} 
  x, & \text{if } \bar{v} \leq \lambda i^*; \\
  x \frac{i^* - \bar{v}}{(1 - \lambda)i^*}, & \text{if } \lambda i^* < \bar{v} \leq i^*; \\
  0, & \text{if } i^* < \bar{v},
\end{cases}
\]

In other words, the contract pays whenever the index \( \bar{v} \) falls below a specified trigger \( i^* \), with the indemnity proportional to the difference between the index and the trigger. The maximum indemnity \$x \) is paid whenever the index falls below a critical value \( \lambda i^* \), \( 0 < \lambda < 1 \).

Although developed independently, this setup is somewhat similar to that used by Martin, Barnett, and Cobble [37]. The main differences are that we emphasize the insurance aspect of the contract and specify the payment structure so that indemnities are paid for rain deficiency rather than excess rainfall. This approach seems to be more suitable for crop production where drought negatively affects the harvest-time yield.

Elementary contracts in (3.31) are convenient for analysis, yet offer enough flexibility to construct more complicated instruments. Combining elementary contracts with different triggers \( i^* \), limit parameters \( \lambda \), and maximum liabilities \( x \), one can recreate or otherwise approximate more complicated, multi-layered indemnification schedules that may provide efficient risk protection whenever expected losses are not linearly related to index. The elementary contract also contains the simple "all-or-nothing" contract as a special case. Specifically, when \( \lambda = 1 \), the contract pays the maximum indemnity if the index falls below the trigger level \( i^* \), and nothing otherwise.
In order to specify a particular contract, we need to impose three conditions on the contract parameters $i^*$, $\lambda$, and $x$. These conditions can be chosen so that the contract has pre-specified properties. For the purposes of our analysis, it will be convenient to further standardize the contracts under consideration by requiring them all to have an expected indemnity, or pure premium, of $1. From the buyer’s standpoint this is a convenient normalization, since it allows him to see readily how much protection he can buy for $1 in pure premium by inspecting the indemnity schedule. The normalization, however, is not restrictive, since one can achieve any coverage level desired simply by buy the necessary number of $1 contracts.

We shall refer to an elementary contract with a pure premium of $1 as a standard contract. For a standard contract, the parameters must be chosen such that

$$1 = E_\xi f(\xi; i^*, \lambda, x) = x \left( \int_0^{\lambda i^*} h(i)di + \int_{\lambda i^*}^{i^*} \frac{i^* - i}{(1 - \lambda)i^*} h(i)di \right).$$

(3.32)

where $h(i)$ is the probability density function\(^{37}\) of the index $\xi$.

Insurance companies usually design contracts based on a premium rate, which is the ratio between the premium and largest risk on line. Because a standard contract has a pure premium of $1, the premium rate is simply equal to

$$\pi = \frac{1}{x},$$

(3.33)

where $x$ is the maximum liability. In other words, fixing the premium rate of a standard contract immediately fixes its maximum liability.

\(^{37}\)The distribution may be estimated by either parametric or nonparametric methods. If historical data are not available, either subjective probabilities or a structural model may be used to construct the distribution.
Thus, a standard contract can be uniquely identified by its premium rate $\pi$ and limit parameter $\lambda$. Condition (3.33) fixes the maximum liability $x$ given the premium rate, and condition (3.32) implicitly fixes the trigger $i^*$ given the limit parameter $\lambda$.

One of our goals is to develop a systematic approach to designing standard index contracts that are optimal, in some sense, for potential buyers. Let us look at the standard index contract from a buyer's standpoint. Assume that the individual's income $\bar{r}$ subject to an index-related risk can be expressed as a function $g$ of the index $\bar{r}$ and an independent random shock $\varepsilon$

$$\bar{r} = g(\bar{r}; \varepsilon),$$

(3.34)

where $\varepsilon$ essentially represents the basis risk.

In case of agricultural insurance, we assume that the buyer is a farmer involved in agricultural production, i.e. growing crops. The farmer's income, or net revenue $\bar{r}$ can be calculated as $\bar{r} = \bar{p} \cdot \bar{y} - c$, where $\bar{p}$ is the harvest time price, $\bar{y}$ is the crop yield, and $c$ is the total production cost. Depending on the particular situation, the risk faced by the farmer may be caused by uncertainty about either yields or prices.

Further, assume that the individual has some target income level $r^*$ he wishes to protect. The target income may be some fraction (e.g. 75%) of the expected income $\bar{r} = E g(\bar{r}; \varepsilon)$ or some other income level (e.g. the level at which the individual breaks even). Income lower than the target is considered to be a loss

$$\bar{L} = \max\{0, r^* - g(\bar{r}; \varepsilon)\}.$$ 

If the individual buys $N$ standard contracts defined by the parameters $\pi$ and $\lambda$ and conditions (3.32) and (3.33), his total loss with contracts is then

$^{38}$Similar analysis can be applied to any type of risk exposure with obvious changes in the assumptions and arguments.
\[ \bar{L}_c = \max \{0, r^* - [g(\bar{t}; \varepsilon) + N f(\bar{t}; \pi, \lambda) - N]\}. \]

We assume that the individual tries to avoid the downside loss at all states of nature and therefore wants to minimize his total expected root-mean square (RMS) loss. Hence, the optimal number \( N^* \) of standard contracts as well as the optimal parameters \( \pi \) and \( \lambda \) can be determined as a solution to the optimization problem

\[
\min_{\{N, \pi, \lambda\}} E_\varepsilon \left( \int_0^\infty \left[ \max \{0, r^* - [g(i; \varepsilon) + N f(i; \pi, \lambda) - N] \} \right]^2 h(i) di \right)^{1/2}. \tag{3.35}
\]

Given the above considerations, the process of designing and pricing of a standard index contract for a representative buyer can be outlined as follows\(^{39}\).

First, given the distribution of the underlying index \( \bar{t} \) and relation between the index and buyer's risk exposure \( \bar{L} \) (determined by the function \( g \) and the target income level \( r^* \)), the optimal number \( N^* \) of standard contracts and the contract parameters are determined by solving (3.35). The buyer is then offered a composite contract consisting of \( N^* \) standard contracts with the specific parameters \( \pi^*, \lambda, \) and \( z \).

Before proceeding with an application of the suggested methodology, it is worth mentioning several important points related to specifics of its implementation.

The standard contract structure provides a convenient basis for comparing contracts with different premium rates and triggers. Along with specifying the risk an insurance company undertakes, it also gives the buyer an opportunity to determine which contract provides the best coverage for the same price. The RMS loss measure is a transparent selection criterion, which allows rank-ordering of various contracts available on the market.

\(^{39}\)Obviously, the process should start with choosing an index. However, the criteria used at this stage are often case specific and hard to outline in general terms. Some of the issues arising in selecting an appropriate index are illustrated by the practical example in Section 3.5.
The composite contract does not necessarily need to consist of \( N^* \) identical contracts. In principle, one can combine contracts that differ in structure. For instance, we might consider a combination of contracts with different triggers and limit parameters to obtain coverage over several risk layers simultaneously. However, finding an optimal combination of multiple contracts of different structures renders the optimization problem more difficult by increasing the number of variables to be chosen. Alternatively, a more complicated contract may be first constructed as a weighted average of several standard contracts of different structure (in order to preserve the fixed premium rate and unit price) and then the composite contract may be constructed according to (3.35).

Instead of finding an optimal number of contracts \( N^* \), the problem may be reformulated in terms of the amount of money the buyer is prepared to pay for insurance. In this case, the number of contracts is fixed, and the optimization problem in (3.35) becomes a condition only on the parameters \( \lambda \) and \( \pi \). While fixing the commitment of the buyer may not result in the best available risk hedging, such an analysis may be important if buyer operates under tight budget constraints and cannot afford the best available insurance coverage. However, the pre-specified number of contracts may not necessarily be optimal in the sense that the buyer sometimes can achieve a greater risk protection by actually reducing his commitment level (see discussion in Section 3.5).

The suggested methodology assumes that the index distribution \( h(i) \) and the relation between the index and income \( g(\tilde{r}; \varepsilon) \) are known. In practice, however, only particular realizations of \( \tilde{r} \) and \( \tilde{r} \) are often available. In this case, the distribution of the index may be estimated by using either nonparametric techniques (e.g. kernel-smoothing) or by fitting the observed series by one of the standard distributions (e.g. by the maximum likelihood method). The relation \( g \) also has to be estimated from the
available data, with the appropriate functional form chosen based on either agronomic or statistical considerations. Both estimations may present a challenge in a practical situation. In case of the rainfall insurance, for instance, the structural relation $g$ may not necessarily be monotonic, since both too much and too little rain may be equally devastating for the growing crops. However, these difficulties are independent of the suggested design and pricing methodology and must be addressed when using any other pricing approach.

No matter how accurately the relation in (3.34) is estimated, there are ultimately some risks that cannot be hedged against by using an index contract. In case of crop production, harvest at a particular farm depends on a variety of factors, such as the soil moisture at planting time, the amount of rainfall, the temperature patterns during the growing season, and the amount of fertilizers in the soil. While one index, e.g., amount of rainfall, may account for several risk factors, there is always some risk attributed to other influential factors that are not correlated with the index. In other words, unless the index exactly reflects the risk exposure of the buyer, there is always some basis risk present. The latter may have several components, one caused by a nondeterministic relation between the index and targeted variable (random shock $\varepsilon$ in (3.34)), as well as those caused by spatial, temporal, and other factors. All these issues should also be taken into account in designing a contract (or a set of contracts) in order to provide protection as close to optimal, as possible.

### 3.5 Case Study of Nicaragua Rainfall Insurance

#### 3.5.1 Background

In order to illustrate the suggested methodology for designing and pricing index-based contracts, we applied it to development of rainfall insurance for Nicaragua. Nicaragua
is a small Central American country which was plagued by civil war and unrest for more than a decade in the 1970s and 1980s. At the present time, the country still lacks a solid financial infrastructure, and financial institutions are in the developing stage. Private insurance did not exist in Nicaragua until 1996. Even though the situation has improved during 1990s, the country still faced serious economic problems that were made worse by the devastating Hurricane Mitch in 1998.

According to CIA's *The World Factbook* [59], more than 50% of Nicaragua population live below the poverty line. Agricultural production contributes 34% to the GDP (1999 data) and employs more than 50% of the population. While some crops, such as bananas and coffee are produced for export purposes, others (maize, beans, etc.) are produced mostly for domestic consumption and are either traded locally or consumed by the farmers themselves. Therefore, bad harvest may not only mean a loss of income, but also a poor food supply for the following year.

Local insurance companies mostly carry the traditional lines of insurance, such as life, health, auto, and property casualty insurance. Foreign companies may be interested in providing insurance or reinsurance for agriculture, but they are often reluctant to conduct business in the country due to the unstable political environment, unfamiliarity of the population with insurance products, and generally low levels of income.

The Nicaraguan government, on the other hand, is interested in bringing such insurance in the country, since this would help to further stabilize the economy. The World Bank has an ongoing project to start a pilot crop insurance program in several regions of Nicaragua, which, if successful, could be extended to other crop-producing regions of the country. The present case study has been partially performed for this project and uses data provided by the World Bank and the Government of Nicaragua.
The rest of the section discusses issues arising in design of index-based insurance contracts for one of the regions in Nicaragua, the Departamento de Nueva Segovia. The region is located in the mountain foothills in the north-western part of the country along the Honduras border, with altitudes varying from 400 to 1,000 meters (1,300ft to 3,300ft). Humid tropical climate is predominant, with rainfall exhibiting regular patterns without a clearly observed dry season. Average temperatures range from 22 to 24 °C (70F to 75F) and the annual rainfall ranges from 1,200mm to 1,600mm (47 to 63 inches). The main crops grown in the region are corn/maize (15,700mz), beans (6,000mz), and rice (2,700mz), with growing season spanning May through November.

3.5.2 Primary Analysis

The first step in designing an index-based insurance contract is to determine the risk one wishes to insure against. All three main crops in the region are mostly grown for home (food) consumption and therefore yield uncertainty is the main risk factor faced by producers. Thus, for the purpose of illustration, we can assume that the prices are nonrandom and fixed. Equation (3.34) then becomes

40 Geographical, climatic, and economic data presented below were provided by the World Bank, which compiled them from various sources within Nicaragua including but not limited to government publications.
41 Nicaragua uses a combination of metric and local units of measurement, which are preserved throughout the section in order to avoid confusion. US equivalents to the metric values are given in parenthesis. For local units, conversion rules are given in footnotes.
42 Manzana (mz) is a local unit of area, 1 mz = 1.742 acres
43 All numbers represent average planted areas in 1990–1999.
44 The analysis can be easily extended to the case where both prices and yields are random (and possibly correlated with each other).
\[ \tilde{r} = \bar{p} g_{\tilde{y}}(\bar{\epsilon}; \varepsilon) - c, \]

where \( \bar{p} \) is a local price for a given crop, \( g_{\tilde{y}} \) is a function which relates realizations of index variable \( \bar{\epsilon} \) to the yield \( \tilde{y} \), and \( c \) is production cost.

Now the next step is to select an appropriate index, which can reflect the uncertainty in yields, yet be measured objectively and independently. Given the climatic characteristics of the region, the level of rainfall during the growing season seems to be a natural candidate for this role. Rainfall data are available for more than thirty years, which provides a good basis for contract pricing. In addition, the amount of rainfall can be related to potential yields through well-developed agronomic models (see, for example, [27] and extensive references therein). Other possible variables may include temperature variations, soil quality, etc. However, these parameters are quite uniform throughout the region and provide less information about the yields. The effect of these variables does not have to be completely ignored, though. In particular, they may be implicitly incorporated in the agronomic model (function \( g_{\tilde{y}} \)).

Note that even if the amount of rainfall is selected as a single representative index variable, there are still choices as to how exactly to specify the index. The major issue here is the trade-off between transparency of the contract structure and amount of basis risk. More specifically, there are three types of basis risk we need to be concerned about — temporal, spatial, and crop-specific.

### 3.5.2.1 Temporal Component

The sensitivity of yield to water varies over the stages of growth. A typical growth cycle can be divided into four phenological periods, viz. germination, bloom, development, and maturity. Across the periods, rainfall has different effects on the
prospective yield. For instance, beans are most sensitive to rainfall during the development stage. Historical variations of rainfall are also different during each period. Thus there are potentially four different time periods to base a contract on.

As an extreme case, one can create four different contracts, each based on rainfall during one of the phenological periods, and thus reduce the temporal basis risk to minimum. However, with each period being only 15 to 25 days long, such an arrangement would involve added transaction costs for both sellers and buyers (marketing, monitoring, etc.). Clearly, creating more than one contract for the entire growing cycle may not be economical.

Alternatively, the total amount of rainfall during the growing season can be selected as an index. The transaction cost involved in marketing such a contract would be lower than for four distinct contracts. However, the basis risk embodied by the single contract would be larger, since the total amount of rainfall completely ignores period-by-period variations in the effect of rainfall on plant growth.

Finally, a compromise approach is to combine the measurements of precipitation during each period into one index, e.g. by creating a weighted average with specially chosen weights so that a greater weight is attributed to rainfall during the period when the plant is most affected by the soil moisture level. The weights may also be chosen so as to minimize some criterion reflecting the buyer's risk.

3.5.2.2 Spatial Component

Rainfall patterns differ across locations within the same region. Measurements at one station may track precipitation level at nearby farms very closely, but diverge considerably from observations at farms located farther away. Thus, a contract based on measurements at one station would bear very little basis risk for some farmers, but perform very poorly for others.
As before, an ideal solution to the problem would be to create a separate contract for each station in the region. However, in case of Nueva Segovia, this may result in dozens of distinct contracts marketed in a geographical area comparable with Rhode Island.

A more reasonable approach is either to choose a central (in some sense) station as a reference point for the entire region, or, again, create a weighted average of the observations at different stations. The latter may not necessarily be a better alternative, since creating an average would require coordination of data collection from several different locations. In addition, the beginning of the growing season may vary slightly from one part of the region to another. Therefore, the temporal component of the basis risk is also involved, which further complicates the matter.

3.5.2.3 Crop-Specific Component

Different crops vary in their sensitivity to rainfall, duration of the growing season, and planting times. For example, maize has a growing season of 90 to 110 days, with planting time spread from mid-May till early November, while beans have growing season from 60 to 75 days and are usually planted in early September to mid-October. Obviously, a single contract cannot provide an optimal protection for all crops. Therefore, either the contracts should be specifically tailored for each crop, or a series of contracts should be developed for the whole season providing different levels of protection for different periods within the season. Once again, the crop-specific component of the basis risk may interact with temporal and spatial components in a complex way.

A particular decision of how much contract simplicity can be sacrificed to reduce basis risk must ultimately be left to the financial institution issuing the contract. However, the present study provides a methodology to compare different alternatives.
in making such a decision. Note that the methodology is quite general and can be easily applied to either another region or even another type of risk provided that the contracts can be specified in a way similar to (3.31).

In the following subsections, we first demonstrate some basic properties of index contracts based on the rainfall measurement for the whole growing season at only one station, and for only one crop. At the next step, we compare several combinations of rainfall measurements for different periods of the growth season at the same station and for the same crop. Then we extend the analysis to several weather stations and compare alternative approaches to spatial aggregation. Due to lack of adequate data, we do not explicitly consider the crop-specific basis risk. However, this type of analysis can be easily performed in a similar fashion. All numerical results presented below have been obtained in MATLAB programming environment on a Pentium III 700MHz computer under Windows 98SE operating system.

3.5.3 Base Case

Here we concentrate on the basic properties of an insurance contract for bean production based on rainfall measurements at el Jicaro measuring station. The precipitation data are available for the station from 1963 to 1999 with several gaps during 1980s. The total amount of rainfall during the growing season (75 days) is chosen as an index. During the observation period, the index varied from 217.7mm to 800.3mm (8.5 in to 31.5 in) with the mean 505.14mm (19.9 in) and median 466.7mm (18.4 in). The average bean yield in the region is 10 qq/m$^2$, with maximum reaching 16 qq/m$^2$.

$^{45}$Quintal (qq) is a local unit of weight, 1 qq = 101.43 lb
In order to relate the selected index to the main risk factor to crop producers, the rainfall observations must be transformed into potential yields. This can be done in two stages. First, the amount of rainfall is converted into a special measure called water saturation index (WSI), and then the WSI is converted into expected yield.

As mentioned before, the growing season can be divided into four phenological periods corresponding to different stages of plant development. During each period, the WSI reflects the amount of water a plant absorbs relative to the optimal level necessary for the plant growth. The WSI can be then translated into a probability of "harvest survival" during each period, namely

$$\Pr_{	ext{surv}}^{(i)} = 1 - k_y^{(i)} \left(1 - \text{WSI}^{(i)}\right), \quad i = 1, 4, (3.36)$$

where $k_y^{(i)}$ is a parameter that reflects plant sensitivity to water saturation. This parameter is crop- and period-specific and is estimated from agronomical models [27]. For beans, the values of $k_y^{(i)}$ for the corresponding phenological periods are equal to 0.2, 1.1, 0.75, and 0.2.

In order to obtain the expected yield for the season, the maximum expected yield should be multiplied by the survival probabilities for each period

$$y = y^* \prod_{i=1}^{4} \Pr^{(i)}. \quad (3.37)$$

Based on historical data, the maximum yield $y^*$ is estimated at 16qq/mz, while the local price for beans is estimated at $p = $35/qq. This information allows us to compute the expected revenue, given the WSI data for each of the four phenological periods.

The specific conversion procedure from precipitation measurements to WSI is rather complex and relies on a sophisticated agronomic model [27]. In addition, this model uses period-by-period observations of rainfall amount. However, the historical
data series available for the station include both the amount of rainfall and the WSI. Therefore, in order to relate the aggregate index (the total amount of rainfall during the growing season) to the final outcome (expected revenue), we approximate the agronomic relation by an *ad hoc* statistical model. In other words, using the WSI data we first generate the expected yield (and thus revenue) series from (3.36) and (3.37), and then regress the revenues on observed total amount of rainfall using the econometric model

\[
\log \left(1 - \frac{r}{y^* p}\right) = g(i; \varepsilon) = b_0 + b_1 \sqrt{i} + \varepsilon.
\]

The model provides a reasonable fit with \( R^2 = 0.45 \) and \( F \)-statistics equal to 22.11 \((p\text{-value of 0.0001})\). The estimated values of coefficients are \( b_0 = -1.125 \) and \( b_1 = -1.432 \), with both coefficients significant at any level of confidence \((p\text{-values of 0.0000 in both cases})\). The scatter plot of the expected revenues against the rainfall index as well as the fitted relationship between the two are shown in Fig. 3.3.

The next step in designing a contract is to construct the probability density function of the index. Since the data set contains only 29 observations, non-parametric methods would provide too rough an estimate of the underlying distribution. Instead, parameters of the standard gamma-distribution

\[
f(x|\alpha, \beta) = \begin{cases} \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} & \text{for } x > 0, \\ 0 & \text{for } x \leq 0 \end{cases}
\]

(see, for example, DeGroot [12]) are fitted\(^{46}\) to the data using the maximum likelihood method implemented in MATLAB Statistical Toolbox. The choice of the

\(^{46}\)Here we implicitly assume that the observed realizations of the index are i.i.d. drawn from the same distribution.
gamma distribution seems to be appropriate, since data are inherently nonnegative and skewed to the left. In addition, it allows for potential realizations of the index higher than those observed historically, which would not be the case, for example, with beta-distribution (see also discussion in Martin, Barnett, and Coble [37]. The estimated parameters of the distribution are $\alpha = 8.812$ with 95%-confidence interval $[2.762, 14.862]$ and $\beta = 0.057$ with 95%-confidence interval $[0.016, 0.099]$. 

In order to illustrate the basic properties of the contract, we first select the parameters of the standard contract ad hoc. More specifically, the premium rate is fixed at 2%, 5%, and 10%, which are levels commonly used in the insurance industry. The structural parameter $\lambda$ is fixed at 0, 0.5, and 1. In what follows, contracts with these choices of $\lambda$ are referred to as "fully proportional", "partially proportional", and "all-or-none", respectively. For the purposes of analysis, the target revenue level $r^*$ is assumed to be equal 100% of the expected revenue. The latter is computed as a sample average of revenue series obtained from (3.36) and (3.37) and is equal to $490.78/mz$.

Shown in Fig. 3.4 are representative indemnity schedules for different types of contracts (different parameter $\lambda$) and the same premium rate (5%). As the observed value of index decreases, the fully proportional contract triggers first, followed by partially proportional and "all-or-none". While the maximum liability and particular trigger points vary with the premium rate, the basic structure of each contract remains the same. This is further illustrated by Fig. 3.5 which shows how parameters of a particular contract (in this case, partially proportional) change with different premium rates. The general tendency is that a higher premium rate implies a lower maximum liability, but also a more probable triggering event.

47These names roughly reflect the payment structure of each contract.
In order to find the optimal standard contract specification and the optimal number of standard contracts for a typical bean producer in el Jicaro, we need to solve the optimization problem in (3.35). In order to do so, we use the following variation of the grid search method. First, a grid is created with respect to \( \lambda \) and \( \pi \) with 26 nodes in one direction (from 0 to 1 with the step of 0.04) and 31 node in the other direction (from 2% to 62% with the step of 2%). Since the parameters of the contract do not depend on the number of contracts bought, we can compute the trigger parameter \( \lambda^* \) and maximum liability \( x \) from (3.32) and (3.33) in advance for each point of the grid. Then we can vary the number of contracts \( N \) (as percentage of the expected revenue) with small increments, compute the RMS loss for each point of the grid, and find the minimum attainable RMS loss for a given commitment level.

Shown in Fig. 3.6 is the minimum attainable RMS loss versus the number of contracts bought. The graph has a clearly specified minimum, i.e. the optimization problem in (3.35) has a solution. For the minimum point \( N^* = 12.27 \) (2.5% of the expected revenue), the RMS loss versus the premium rate \( \pi \) and limit parameter \( \lambda \) is shown in Fig. 3.7. The isoquants of this surface are also shown in Fig. 3.8. The optimal choice of parameters \( \lambda \) and \( \pi \) are 0.12 and 26%, respectively. The minimum attainable RMS loss is equal to $0.82/mz.

Figures 3.4–3.8 further illustrate the conclusions of Section 3.4. First, the suggested RMS loss criterion provides a uniform way to compare contracts with different parameters. Second, the criterion allows the buyer to optimally choose the number of contracts in order to achieve the best protection. Third, the same premium rate does not necessarily imply the same coverage. Finally, the premium rate does not measure the price the buyer pays for the insurance. In other words, even if the buyer commits the same amount of money to purchase the insurance, his level of coverage can vary greatly depending on the choices of premium rate and contract parameters.
The specific choice of the contract obviously depends on buyer’s goals and preferences. Although the buyer does not have a direct control over $\lambda$ embedded in a particular contract, he can choose the most appropriate one from the selection offered by an insurer. In a region where drought is an uncommon phenomena, the “all-or-none” contract may be a better solution, since it provides an inexpensive protection against a rather rare event. On the other hand, if a farmer grows a crop that is very sensitive to the amount of water in the soil (e.g. rice), then a variant of the proportional contract may be a better choice. Even though the cost of the insurance is higher in this case, the farmer is protected to some extent even from a slight deficiency in the amount of rainfall.

3.5.4 Temporal Factors

In order to measure the basis risk introduced by using the total amount of rainfall as an index, we compare the minimum RMS losses attainable for contracts based on several other combinations of rainfall measurements. As mentioned in Subsection 3.5.3, for the four phenological periods (germination, bloom, development, and maturity), the sensitivity coefficients $k_y^{(j)}$ in (3.36) for beans are equal to 0.2, 1.1, 0.75, and 0.2. The same coefficients for maize are 0.4, 1.5, 0.5, and 0.2. Therefore, it seems reasonable to assume that either an average of rainfall measurements during each phenological period weighted with coefficients $k_y^{(j)}$, or the amount of rainfall during the most influential period (bloom) may better reflect the potential yield.

For the purposes of comparison, we use the same methodology and compute the optimal values of premium rate and limit parameter $\lambda$ as well as the optimal number of contracts $N^\ast$. The minimum attainable RMS losses are then compared across the following three index specifications:
• the total amount of rainfall (base case);

• the weighted average computed as

\[ \overline{\tau} = \frac{\sum_{j=1}^{4} k^{(j)} \tau_{j}}{\sum_{j=1}^{4} k^{(j)}} , \]

where \( \tau_{j} \) are rainfall measurements during each growth period, and coefficients \( k^{(j)} \) are shown above;

• the amount of rainfall during the second growth period.

The analysis was performed for all crop-station combinations for which rainfall and WSI data were available. The results are summarized in Table 3.2. Surprisingly, the weighted average turned out to be a better option only in one out of 29 cases. In all other situations, either the total amount of rainfall or rainfall during the second period performed better. This may be explained by the fact that the weighted average does not adequately reflect the effect of each period rainfall. Other weighting schemes may prove to be a better alternative, although the only way to find this out is by trial and error. As for the other two indices, there is no clear pattern as to what index is uniformly the best, since the optimal choice varies wildly both across crops and stations. Apparently, the index selection must be performed for each station separately using the suggested methodology.

3.5.5 Spatial Factors

As mentioned in Subsection 3.5.2.2, creating several contracts based on index observations at different points reduces basis risk, but increases the complexity of the
system. On the other hand, reducing the number of contracts decreases transaction cost, but introduces additional basis risk to farmers located far away from the measuring station. In order to evaluate this effect and choose an optimal contract for more than one station, the following methodology was implemented.

First, we fix a crop and consider all stations which have adequate data for this crop. Then, using the results in Table 3.2, the optimal index is chosen for each station. Finally, for each index we compute the minimum attainable loss at each station, assuming that only a single contract based on this index is available. The best index is then determined as the one which minimizes the aggregate loss at all stations.

The results for some crops are summarized in Tables 3.3–3.5. The first part of each table reports the minimum attainable RMS loss, while the second indicates the optimal number of contracts (as percent of the expected revenue). The rows correspond to stations, and columns correspond to indices. For example, the number at the intersection of the second row and third column shows the minimum attainable RMS loss for a Jicaro farmer if he uses a contract based on Murra rainfall. The last line represents the aggregate loss at all five stations for each choice of the index.

Once again, there is no clear winner in either case. No index uniformly dominates other for any crops. Moreover, even the best index in the aggregate sense varies from crop to crop. The specific choice of the index obviously has to be made by the company issuing the insurance contracts. However, the suggested methodology again provides enough information for making an informed decision.

3.6 Conclusion

Index-based contracts have been gaining popularity during the last decade as an innovative risk-management instrument. The advantages of index contracts include
transparency of the underlying variable, elimination of moral hazard, and reduction in adverse selection. Applied to agricultural production, index contracts have a potential to introduce efficient risk-management mechanisms in the areas where traditional insurance either fails or requires heavy subsidies from the government.

The suggested methodology of design and pricing index contracts allows one to compare contracts with different parameters and structure within a unified framework and also provides guidelines as to an optimal number of contracts a buyer needs to purchase in order to obtain the best risk protection. The methodology is applied to design and pricing of rainfall insurance contracts for Nicaragua.

Several important issues which arise in practical application include the trade-off between basis risk and contract transparency, derivation of index distribution and estimation of relationship between the index and the risk being hedged against. The agronomic model data supplied by the World Bank provided somewhat unrealistic estimates of the yields. More specifically, the variation of yields generated from the model was less than that reported in other sources. While this is not a flaw of the suggested methodology per se, which is quite robust and does not depend on the specific index chosen, a better understanding of the relation between yields and indices is crucial for successful implementation of the latter.

Further research may include designing institutional framework to introduce index-based insurance contracts both in Nicaragua and other locations. A better understanding of yield/weather relationship for specific crops and/or regions would also be useful. Finally, indices other than rainfall may be considered as alternative basis for agricultural insurance contracts.
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<th>Cedent</th>
<th>Type of Deal</th>
<th>Amount (thousands)</th>
<th>Date</th>
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<td>Florida Windstorm Underwriting Association</td>
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*withdrawn

Table 3.1: Capital market deals on catastrophe risk.
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<th>Station</th>
<th>Crop</th>
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Table 3.2: Minimum attainable RMS loss for different choices of index. Temporal basis.

Notes: Numbers and abbreviations after the crop indicate the length of the growing season in days and the planting time (Primera or Postrera). Numbers in bold face indicate the best available choice.

141
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<th>RMS Loss</th>
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<th>Murra</th>
<th>Quilali</th>
<th>San Fernando</th>
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<td><strong>377.87</strong></td>
<td><strong>11.42</strong></td>
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<table>
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Table 3.3: Minimum attainable RMS loss for different choices of index. Spatial basis. Beans 60 days, Primera.
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<td>33.28</td>
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<th><strong>Optimal Number of Contracts</strong></th>
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<th>Jicaro</th>
<th>Murra</th>
<th>Quilali</th>
<th>San Fernando</th>
</tr>
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Table 3.4: Minimum attainable RMS loss for different choices of index. Spatial basis. Beans 75 days, Postera.
### RMS Loss

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<tr>
<th></th>
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<th>Quilali</th>
<th>San Fernando</th>
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### Optimal Number of Contracts

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<thead>
<tr>
<th></th>
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<th>Murra</th>
<th>Quilali</th>
<th>San Fernando</th>
</tr>
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<tbody>
<tr>
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<td>4.75</td>
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<tr>
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<td>1.25</td>
</tr>
<tr>
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<td>3.75</td>
<td>8.75</td>
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Table 3.5: Minimum attainable RMS loss for different choices of index. Spatial basis. Beans 75 days, Primera.
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<th>Year</th>
<th>Billion Dollar U.S. Weather Disasters Since 1980</th>
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<tr>
<td>1980</td>
<td>Drought/Heat Wave $44.8 - 18,000 deaths</td>
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<tr>
<td>1983</td>
<td>Hurricane Alicia Florida Freeze $54.6 108 deaths</td>
</tr>
<tr>
<td>1986</td>
<td>Hurricane Elena Florida Freeze $2.0 No deaths</td>
</tr>
<tr>
<td>1989</td>
<td>Hurricane Hugo Florida Freeze $12.5 94 deaths</td>
</tr>
<tr>
<td>1990</td>
<td>South Plains Flooding $1.3 13 deaths</td>
</tr>
<tr>
<td>1991</td>
<td>Hurricane Bob Oklahomai, CA Flooding $2.0 10 deaths</td>
</tr>
<tr>
<td>1992</td>
<td>Hurricane Andrew Florida Freeze $32.0 61 deaths</td>
</tr>
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<td>1993</td>
<td>Eastern Storm, Blizzards SE Drought/Heat Wave $5.0 276 deaths</td>
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<tr>
<td>1994</td>
<td>Southeast Ice Storm Tropical Storm Alberta $3.2 51 deaths</td>
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<td>1996</td>
<td>California Flooding SE-SW Severe Weather $5.3 37 deaths</td>
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<td>1999</td>
<td>Ash-Texas Tornadoes OK KS Tornadoes $1.3 17 deaths</td>
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</tbody>
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<thead>
<tr>
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<td>1999</td>
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**Figure 3.1:** Billion dollar U.S. weather disasters.
Figure 3.2: Indemnity payments of a standard contract.
Figure 3.3: Relation between expected revenue and amount of rainfall. El Jicaro beans.
Figure 3.4: Indemnity schedule for different contract types. Standard contract ($1 in premium). Premium rate 5%, El Jicaro beans.
Figure 3.5: Indemnity schedule for different premium rates. Standard contract ($1 in premium). Partial-proportional contract, El Jicaro beans.
Figure 3.6: The minimum attainable RMS loss as a function of the number of contracts bought. Target revenue level \$490.78/mz, El Jicaro beans.
Figure 3.7: The RMS loss as a function of the premium rate $\pi$ and limit parameter $\lambda$ for the optimal number of contracts bought. Target revenue level $\$490.78/mz$, El Jicaro beans.
Figure 3.8: Contour representation of the RMS loss as a function of the premium rate $\pi$ and limit parameter $\lambda$ for the optimal number of contracts bought. Target revenue level $490.78/mz$, El Jicaro beans.
BIBLIOGRAPHY


