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UMI
THREE-DIMENSIONAL THERMAL AND RADIOMETRIC MODELING OF LAND MINE SIGNATURES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By

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* * * * *

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2001

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ABSTRACT

Several aspects of the IR signatures of land mines are examined. Three-dimensional thermal and radiometric models are developed to predict the infrared (IR) signatures of buried mines. A finite element method (FEM) based thermal model is developed to study temporal variations, the spatial structure of the signature, and environmental effects. A reference solution is presented for the integral equation that governs the temperature distribution in the case of a time invariant convection coefficient and air temperature. The integral equation solution is compared with the FEM model to assess the effects of various assumptions in the latter approach. A detailed representation of the TM-62 AT mine is developed for the thermal FEM model, and automatic mesh generation and adaptive mesh refinement algorithms are employed to improve the model’s accuracy. A radiometric model, which addresses both the spatial and spectral characteristics of the environment, is also presented to predict the IR signatures of buried mines. The effect of surface roughness on the mine signatures is investigated for several surfaces. Polarimetric IR signatures of surface mines were studied. A model based on a second order small perturbation method/small slope approximation is developed to examine the effects of material composition, geometry, and statistical surface properties. From these numerical simulations, it is possible to explain phenomena observed in IR mine signatures and to suggest techniques for improving detection.
This work is dedicated to the land mine victims.
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CHAPTER 1

INTRODUCTION

With tens of millions of land mines buried in over 70 countries, mine detection during peacetime (humanitarian demining) requires a robust, effective, and expeditious mine detection technology. It has long been known that a host of sensor technologies are capable of detecting mines [1]. Among these sensors are ground penetrating radars [2, 3, 4] (GPRs), metal detectors [5], infrared (IR) sensors, chemical sensors [6], condensed phase techniques [7, 8], and acoustic sensors [9, 10, 11].

Infrared (IR) sensors have several unique and highly desirable properties for demining. Chief among these are their abilities to detect mines at long range and to rapidly scan large areas. In addition, since the IR signature is independent of the electrical properties of the buried target, it complements a metal detector or a ground penetrating radar (GPR) in a sensor fused system.

The IR signature of a buried mine involves both thermal and radiometric issues, which will be discussed later in this chapter. Detection of land mines via thermal IR imagery is affected by several factors, including time of day, cloud cover, vegetation, surface irregularities, emissivity variations, and past meteorological conditions. There is a critical need to understand the effects of these environmental factors to improve detection performance and to better utilize IR sensors.
This chapter provides background information about the IR signatures of land mines and thermal IR imagery. This chapter is organized as follows: In Sect. 1.1 we summarized previous experimental and modeling studies. In Sect. 1.2 the thermal and radiometric processes that produce IR signatures are briefly discussed and a mathematical description of the process is presented. In Sect. 1.3 some experimental studies are presented to familiarize the reader with IR mine signature and thermal clutter. The organization of this dissertation is given in Sect. 1.4.

1.1 Previous Studies

1.1.1 Experimental

For many years experimental studies have been conducted to assess the detection performance of thermal IR imagery and to better understand the underlying phenomena. Among these studies we note the work of LeSchack and Del Grande [12] who describe the effects of emissivity on target signatures in the context of a two-color IR system. Janssen et al. [13] reported a study of basic phenomenology in which several sensors were used to examine the time evolution of signatures for both buried and surface mines. A related study is that of Maksymomko et al. [14] who measured the temperatures of both live and surrogate mines through multiple diurnal cycles. Simard [15] found a linear relation between the apparent temperature of buried mines and the soil temperature gradient on gravel roads. Russel et al. [16] showed that this gradient could be predicted from remote soil temperature measurements, making it possible to assess the potential effectiveness of an IR sensor prior to use. Pregowski and Swiderski [17] presented a qualitative discussion of the effects of soil cover and water content. Another work, obliquely related to the effort proposed here, is that of
Del Grande et al. [18], who discussed the use of computed tomography and a finite difference thermal analysis code (TOPAZ3D) for imaging structural flaws in materials. Discussions of the physics of non-thermal hyper-spectral sensors are rare. Among these works we note a paper by DePersia et al. [19] in which the spectral signatures of various types of soil disturbances are discussed.

1.1.2 Modeling

In spite of the above-mentioned experimental works, the phenomena that produce mine signatures are still poorly understood. Although models for IR signatures of buried mines are not well developed, there has been progress in modeling related problems. Thermal IR remote sensing of soil provides useful information for terrestrial studies such as characterizing geological materials, monitoring effusive volcanism, detecting fractures, and hazard assessment. Thermal IR data can also provide valuable information about temporal and spatial variations of soil temperature and this problem has been widely studied in the literature using both analytical and numerical techniques. Those models are one-dimensional, but they describe heat transfer at the soil-air interface and various environmental processes, which are critical components of three-dimensional thermal signature models. We summarize these efforts below, and we will further refer to these works during the mathematical description of the thermal processes in Sect. 1.2. Watson [20] used a Fourier series formulation [21] for periodic heating of a surface to obtain an analytical solution for the temperature distribution in a one-dimensional model of homogeneous soil heated by diurnal solar insolation. That work included the effects of skylight, atmospheric absorption, thermal radiation, and thermal conduction in soil. England et al. [22] included the effects
of wind-driven convective heat transfer between soil and air. Kahle [23] obtained a solution of the one-dimensional heat flow equation, including sensible and latent heat transfer. Watson [24] determined the surface temperature over a homogeneous layer by using the Laplace transform to obtain a relation between surface flux and surface temperature. England [25] used a one-dimensional model to study the radiometric characteristics of diurnally heated freezing and thawing soils. Liou and England [26] developed diurnal and annual models for freezing and thawing moist soils subject to annual insolation, radiant heating and cooling, and sensible and latent heat exchanges with the atmosphere. Liou and England [27] used a one-dimensional coupled heat and moisture transport model for bare, unfrozen, moist soils. They later extended the model to freezing soils [28].

1.2 Physical Description of the Process

An IR sensor measures the total emitted and reflected power from a surface; contributions of these components are not easily separated. Depending on the application, either the emitted or reflected component may constitute clutter. Thermal emission from the soil depends on the surface temperature and emissivity. In detection of buried objects, the emitted component carries the target-related information and, therefore, the temperature of the soil surface is of primary interest. We discussed the descriptions of the thermal and radiometric models in Sects. 1.2.1 and 1.2.2, respectively.

1.2.1 Thermal Model

A buried mine disturbs the surface temperature when the solar-driven diurnal flow of thermal energy into and out of the soil is affected by the mine. The surface
temperature distribution can be found through the solution of the three-dimensional heat transfer equation in the soil and the mine. In this section, we discuss the heat transfer mechanisms involved and the boundary conditions that govern the problem.

Three heat transfer mechanisms affect the surface temperature distribution as shown in Fig. 1.1. Conduction occurs in the soil and at the soil-mine interface. At the soil-air interface convective and radiative heat transfer mechanisms are present. Convection involves the transfer of heat between the soil and air, and is greatly affected by wind. Radiative heat transfer involves direct solar radiation, scattered sunlight, and thermal radiation between the earth and atmosphere. All of these processes are time varying (and herein, assumed periodic) over the diurnal cycle. Mathematical descriptions of the phenomena are as follows:

The temperature distribution in the soil and mine is described by the three-dimensional heat flow equation

$$C(r)\frac{\partial T(r,t)}{\partial t} = \nabla \cdot (K(r) \nabla T(r,t)),$$  
(1.1)

where \( T(r,t) \) [K] is the temperature, \( K \) [W m\(^{-1}\) K\(^{-1}\)] is the thermal conductivity of the material, and \( C \) [J m\(^{-3}\) K\(^{-1}\)] is the volumetric heat capacity. Thermal diffusivity \( \kappa \) [m\(^2\) s\(^{-1}\)] is defined as the ratio of the thermal conductivity to the volumetric heat capacity, which can be written as

$$\kappa_i(r) = \frac{K_i(r)}{C_i(r)}, \quad i = s, m$$  
(1.2)

where the subscripts \( s \) and \( m \) refer to the soil and mine, respectively. For piecewise constant properties in the soil and mine regions, Eq. (1.1) can be rewritten as

$$\nabla^2 T(r,t) - \frac{1}{\kappa_i(r)} \frac{\partial T(r,t)}{\partial t} = 0, \quad i = s, m$$  
(1.3)

5
Figure 1.1: The heat transfer mechanisms in the soil, at the soil-mine interface, and at the soil-air interface.
To complete the thermal problem description, boundary condition at the soil-air interface must be specified. At this boundary, convective and radiative heat transfer mechanisms are present. Numerous models have been presented in the literature to represent the heat transfer at a soil-air interface. Those models have been extensively studied in one-dimensional thermal analyses of homogeneous soil over a diurnal cycle [20, 22, 23, 26, 27, 28], and validated with measurements. The net heat flux into the ground $F_{\text{net}}$ can be written as (for $z$ positive downward)

$$F_{\text{net}}(t) = -k \frac{\partial T(r, t)}{\partial z} \bigg|_{z=0} = F_{\text{sun}}(t) + F_{\text{sky}}(t) - F_{\text{sh}}(t) - F_{\text{gr}}(t), \quad (1.4)$$

where $F_{\text{sun}}$ is the incident solar radiation reduced by cloud extinction, atmospheric absorption, soil albedo and the cosine of the zenith angle; $F_{\text{sky}}$ is the sky brightness with a correction for cloud cover; $F_{\text{sh}}$ is the sensible heat transfer from land to atmosphere due to convection; and $F_{\text{gr}}$ is the gray body emission from the soil surface.\(^1\)

The incident solar radiation can be measured experimentally or obtained from sophisticated models (e.g., MODTRAN4 [29]). A simple closed form approximation is often sufficient for exploratory studies. Watson [20] used

$$F_{\text{sun}}(t) = S_0(1 - A)(1 - C)H(t) \quad (1.5)$$

to approximate the short-wavelength solar flux. In Eq. (1.5) $S_0 = 1353 \text{ [W/m}^2\text{]}$ is the solar constant, $A$ is the ground albedo, $C$ is a factor that accounts for the reduction in solar flux due to cloud cover, and $H(t)$ is the local insolation function. This model employs atmospheric transmittance data given by Allen [30], approximated in a functional form by Watson [20]. A convenient approximation for the local insolation

\(^1\)In formulating this expression we have ignored evaporation and other movement of soil moisture, as well as changes in the state in the medium.
\( \mathcal{H}(t) \) is [20, 31]

\[
H(t) = \begin{cases} 
M(Z(t)) \cos Z'(t) & -t_r < t < t_s \\
0 & t_s < t < t_r 
\end{cases}
\] (1.6)

where \( t_s \) and \( t_r \) [h] are the local sunset and sunrise times with respect to midnight, \( M(Z) \) is the atmospheric transmission as a function of zenith angle \( Z \), and \( Z' \) is the local zenith angle for the inclined surface. The function \( M \) is modeled as

\[
M(Z) = 1 - 0.2\sqrt{\sec Z}
\] (1.7)

and \( Z \) is determined from

\[
\cos Z = \cos \lambda \cos \delta \cos \omega t + \sin \lambda \sin \delta.
\] (1.8)

in which \( \lambda \) is the local latitude and \( \delta \) is the declination angle of the sun

\[
\delta = -23.433^\circ \cos[2\pi(\text{month})/12]
\] (1.9)

The local zenith angle \( Z' \) can be determined from

\[
\cos Z' = \cos d \cos Z - \sin d (\sin \psi \cos \delta \sin \omega t - \cos \psi \sin \delta \cos \lambda - \sin \delta \sin \lambda \cos \omega t)
\] (1.10)

where \( d \) is the surface slope angle measured downward from the horizontal, and \( \psi \) is the azimuth of the slope angle measured counterclockwise from north. For flat ground, assumed here, \( d = 0 \).

The long-wavelength radiation from the atmosphere, \( \mathcal{F}_{\text{sky}} \), is given as

\[
\mathcal{F}_{\text{sky}}(t) = \mathcal{E} \sigma T_{\text{sky}}^4(t),
\] (1.11)

where \( \sigma = 5.67 \times 10^{-8} \text{ [W m}^{-2} \text{ K}^{-4}] \) is the Stephen-Boltzmann constant and \( T_{\text{sky}} \) [K] is an effective sky radiance temperature. The heat loss due to ground radiation is given by Stefan’s Law

\[
\mathcal{F}_{\text{gr}}(t) = \mathcal{E} \sigma T^4(t, z = 0),
\] (1.12)
where \( E \) [unitless] is the mean emissivity of the surface and \( T(t, z = 0) \) is the soil temperature at the soil-air interface. The sensible heat transfer between the surface and atmosphere is approximated by

\[
F_{sh}(t) = h(t)(T_{air}(t) - T(t, z = 0)).
\]  \hspace{1cm} (1.13)

where \( h \) [W m\(^{-2}\) K\(^{-1}\)] is a convection coefficient. Kahle [23] offered the following model for this parameter

\[
h(t) = \rho_a c_{pa} C_d(W(t) + 2)
\]  \hspace{1cm} (1.14)

where \( \rho_a = 1.16 \) [kg/m\(^3\)] is the density of air, \( c_{pa} = 1007 \) [J kg\(^{-1}\) K\(^{-1}\)] is the specific heat of air, \( C_d \) [unitless] is the wind drag coefficient chosen to be 0.002, and \( W(t) \) [m/s] is the wind speed. Kahle [23], drawing on measurements by Kondratyev [32], formulated an empirical model for the air temperature, which was later modified by England [26] to yield the expression

\[
T_{air}(t) = T_{0,air} - T_{del} \cos(2\pi(t - 2)/24),
\]  \hspace{1cm} (1.15)

where \( t \) is the local time as above, \( T_{0,air} \) is the the average air temp, and \( T_{del} \) is the peak value of the deviations. \( T_{0,air} \) and \( T_{del} \) are estimated from local meteorological data. Kahle also proposed a model for the sky temperature, which was modified by England [26] using Brundt’s formula [23]

\[
T_{sky}(t) = T_{air}(t)(0.61 + 0.05\sqrt{w})^{0.25},
\]  \hspace{1cm} (1.16)

where \( w \) [mmHg] is the water vapor pressure.

Using Eqs. (1.5) and (1.11)-(1.16) in Eq. (1.4) results in the boundary condition at the soil-air boundary. This condition involves the nonlinear function of the surface temperature distribution \( T^4 \), but over the limited temperature range observed in
mine detection (on the order of 10 K), the result can be linearized using a technique described by Watson [20]. The linearized boundary condition can be written as

$$\frac{\partial T(r, t)}{\partial z} \approx T(r, t) \frac{1}{K_s} (h(t) + 4\varepsilon\sigma T_{sky}(t)) - \frac{1}{K_s} (\mathcal{F}_{sun}(t) + h(t)T_{air}(t) + 4\varepsilon\sigma T_{sky}^4(t)).$$

(1.17)

### 1.2.2 Radiometric Model

The radiative components seen by the IR camera include (1) thermal emission from the soil surface (a portion of which comprises the desired signal), (2) soil-reflected sunlight, (3) soil-reflected skylight (thermal emission from the atmosphere and sunlight scattered by particles and air molecules), and (4) a negligible amount of thermal emission from the atmosphere directly to the camera. These components are illustrated in Fig. 1.2, and we write the spectral radiance emitted from the scene at position r in direction \((\theta, \phi)\) as follows

$$L_{rec}(\theta, \phi, r, \lambda) = L_{surf}(\theta, \phi, r, \lambda) + \int_0^{\pi/2} d\theta' \sin \theta' \int_0^{2\pi} d\phi' \mathcal{R}(\theta, \phi; \theta', \phi') \cdot [L_{sun}(\theta', \phi', r, \lambda) + L_{sky}(\theta', \phi', r, \lambda)]$$

(1.18)

where \(\mathcal{R}(\theta, \phi; \theta', \phi')\) is the bidirectional reflectance distribution function (BRDF) of the soil. The BRDF relates incident irradiance in direction \((\theta', \phi')\) and reflected radiance in direction \((\theta, \phi)\), and it has units of sr\(^{-1}\). If the emissivity of a material is independent of the direction, it is referred to as a diffuse emitter. In this work we model the soil surface as a diffuse reflector, which permits us to write \(E(\theta, \phi, r, \lambda) = E(r, \lambda)\) and \(\mathcal{R}(\theta, \phi; \theta', \phi') = \rho/\pi\) in which \(\rho\) is the reflectivity of the surface.

Thermal emission from the soil surface depends on the surface temperature and emissivity. The surface temperature can be obtained by a numerical solution of the
Figure 1.2: Contributors to a thermal image of the ground include direct sunlight, aerosol-scattered sunlight, thermal emissions from the air reflected by the ground, and thermal emissions from the soil.

...
of its radiation in the visible region of the spectrum. Radiation from objects with
temperatures below 800 K, however, remain mainly in the infrared region of the
spectrum and can not be seen with a naked eye. The peak radiation from an object
with a temperature of 1000 K, e.g., a piece of heated metal, occurs approximately
at 3 µm. Although, most of its radiation is in the IR region of the spectrum, it
emits some visible radiation as red light. As the temperature of the object increases,
the contribution of the shorter wavelengths become more pronounced. For example,
a light bulb with temperature approximately 2900 K emits white light. Terrestrial
objects generally have temperatures around 300 K. At this temperature the maximum
radiation occurs near 10 µm, which is the LWIR region of the spectrum.

The radiation diffusely emitted from a point \( r \) on the soil surface at temperature
\( T(r) \) can be written as the product

\[
L_{surf}(\lambda, r) = \varepsilon(\lambda, r)L_{BB}(\lambda, T(r)),
\]

where \( \varepsilon(\lambda, r) \) is the spectral directional emissivity. In Eq. (1.20) polarization effects,
which are often directional and may be significant, have been ignored.

From the above expressions it is clear that emissivity has a direct effect on the
thermal signature. In addition, grass and other forms of ground cover have a strong
effect on the heat flow process and on the reflection and radiation of thermal energy.
The emissivity of natural materials has been studied by Salisbury and D’Aria [33, 34].
In those works the emissivity of soils, rocks and vegetation was described for both the
3-5 µm and 8-14 µm bands. The spectral response of these materials is complicated,
but in general over the 3-5 µm band rocks and soils exhibit reflectance values of 5%
to 30%, while vegetation has reflectance values of 2% to 15%. For the 8-14 µm band
they found rock reflectance values of 1% to 10%. Vegetation reflectance values were
Figure 1.3: Planck's radiation law for different temperatures.
found to have approximately the same range. The emissivity of these materials can be obtained from the reflectivity values by using Kirchoff’s law for an opaque body,
\[ \mathcal{R} = 1 - \mathcal{E}. \]

Using the diffuse reflector approximation for the soil, the \( \delta \) dependence of \( L_{\text{sun}} \), and the angle invariance for \( L_{\text{sky}} \) we find

\[
L_{\text{rec}}(\theta, \phi, r, \lambda) = L_{\text{surf}}(r, \lambda) + \rho(r, \lambda) [L_{\text{sun}}(r, \lambda) + L_{\text{sky}}(r, \lambda)]
\] (1.21)

Equation (1.21) permits us to calculate the power incident on an IR detector surface \( D \) from the radiating soil surface \( S \). The power incident on the detector can be found by

\[
\Phi = \int_{\lambda_1}^{\lambda_2} d\lambda \int_D dD \int_S dS L_{\text{rec}}(\lambda, r) \frac{\cos \theta_1 \cos \theta_2}{R^2},
\] (1.22)

where \( R \) is the distance between a point \( r \) on surface \( S \) and a point on the detector \( D \); \( \cos \theta_1 \) and \( \cos \theta_2 \) are projections of the normal vectors for surfaces \( D \) and \( S \), respectively, in the direction of the radiation; and the spectral band of interest is \( \lambda_1 \) to \( \lambda_2 \). IR image formation using Eq. (1.22) will be discussed in Chapter 5.

1.3 Experimental Studies

In this section, experimental studies performed at the OSU ESL surrogate mine field using MWIR (2.2-4.6 \( \mu \)m) and LWIR (8-12 \( \mu \)m) imaging systems are summarized. These results illustrate some phenomena observed in IR imagery. We provide sample images which demonstrate IR mine signatures and thermal clutter. The experimental studies demonstrated in this section have two goals. In the first set of experiments, given in Section 1.3.2, the contrast changes of insulating and conducting mine signatures are examined, and qualitative differences in the magnitude of these
mine signatures are noted. The second set of experiments, presented in Section 1.3.3, attempts to demonstrate the shapes of IR mine signatures and distortion of those signatures due to thermal clutter. More detailed information on these experimental studies can be found in previous works by Sendur and Baertlein [31, 35, 36, 37].

1.3.1 Sensor Descriptions

Characteristics of the IR cameras used in the Ohio State University (OSU) ElectroScience Laboratory (ESL) surrogate mine field experiment are presented in Tables 1.1 and 1.2. A description of the camera features is as follows: Spectral response is the frequency (or wavelength) band over which the detector is sensitive. Two significant atmospheric IR transmission windows are 8-12 \( \mu m \) and 3-5 \( \mu m \). Sensors that operate in the former window are referred to as long-wave IR (LWIR) sensors, whereas those that operate in the latter window are known as mid-wave IR (MWIR) sensors. Sensor type determines the semiconductor material used in the photon detector, which determines the spectral band. Array size gives the size of the detector array, i.e., the number of image pixels in the horizontal and vertical directions. Field of view (FOV) is the angular range covered by the infrared camera in the horizontal and vertical directions. Instantaneous field of view (IFOV) is the angular range covered by an individual image pixel. An important performance criteria for the IR camera is the internally generated noise, expressed as the noise equivalent temperature difference (NEAT) which defines the weakest signature that can be detected. Pixel depth indicates the number of bits used to define an individual image pixel.

An IRRIS 160ST camera was procured from Cincinnati Electronics, and its characteristics are shown in Table 1.1. This sensor has a variable dynamic range, and
Spectral response | MWIR (2.2-4.6 µm)  
Sensor type      | InSb PV array     
Array size       | 160 (h) by 120(v) pixels 
FOV             | 9.1° by 6.8°      
IFOV            | 1 mrad           
NEΔT            | 0.025 K (typ.), 0.040 (max)  
Pixel depth     | 12 bits          

Table 1.1: IRRIS 160ST camera characteristics.

it can be interfaced directly to a personal computer, with the computer performing camera control and image acquisition. The camera FOV and image alignment were checked in the course of our studies and found to agree with the manufacturer's data. A two-point calibration is used, which was suggested by the manufacturer.

An Agema 1000 LWIR camera, described in Table 1.2, was borrowed from the US ARMY Night Vision Laboratory for a period of three months. This sensor, now produced by FLIR Systems, is a second-generation scanning array. It supports both a narrow field of view for target interrogation, and a wide field of view for scanning. While the LWIR band enjoys some advantages in reduced clutter, the LWIR detector is somewhat noisier, which complicates the choice of MWIR versus LWIR sensors for demining.

1.3.2 Sand Box Experiments

In our first sequence of measurements eight mine surrogates were buried outdoors in a region of homogeneous sandy soil and imaged with our MWIR camera. The surrogates were laid out in a 2×4 array as shown in Figures 1.4 and 1.5. In Fig. 1.4 the height of the camera is 64 inches and $D = 147$ inches resulting in observation
Spectral response | LWIR (8.0-12.0\(\mu\)m)
---|---
Sensor type | HgCdTe
Array size | 590 (h) by 401 (v) pixels
NFOV | 5\(^\circ\) by 3\(^\circ\)
WFOV | 20\(^\circ\) by 13\(^\circ\)
IFOV | .15 mrad (NFOV), .6 mrad (WFOV)
NEAT | < 0.2 K
Pixel depth | 12 bits

Table 1.2: Agema 1000 camera characteristics.

angles ranging from 12.3\(^\circ\) to 19.1\(^\circ\). Both good thermal conductors and good thermal insulators were used as surrogates. The four elements on the top row of Figure 1.4 were cast iron metal disks of 4.75 inch diameter and 0.5 inch thickness. The disks are black in color and have a 1 inch diameter hole through their center. The four elements in the lower row of the figure were styrofoam disks of roughly the same size. Pairs of conducting-insulating surrogates were buried at depths of 0 cm (top surface flush to ground), 1 cm, 2 cm and 3 cm. Thin squares (roughly 1 \times 1 \times 5 cm in size) of black carbon-impregnated foam were used as fiducial markers. A view of the mine grid, which approximates that presented to the IR camera is shown in Figure 1.6. Data were collected under a variety of sky conditions. These qualitative data evidence a number of important physical phenomena, and they illustrate the variability inherent in IR mine signatures. These data underscore the dependence of IR imagery on cloud cover, degree of illumination, and the history of these quantities. Representative results are illustrated in Figure 1.7. While the sky is cloudy (image 1), the white insulating mine surrogate on the surface (on the right) reflects the cool sky, producing a darker image. Only 30 minutes later under intermittent clouds (image 3)
both the black conducting mine and the white insulating mine have the same apparent temperature. One hour later under full sun (image 7) the white surface mine reflects the incident sunlight and appears warmer than the soil. Finally, after the mine field is shadowed (image 11), the warm metal surrogates radiate strongly, while the white insulating surrogates reflect the cold sky. The thin layer of soil over the insulating surrogates cools more quickly than the surrounding soil. In the same figure thermal energy stored in the black metal surface mine is being released, making it appear warmer than the background. The presence of the buried insulators is evident from the cooler areas over those mines, but there is no evidence of the buried conductors. The experiments using conducting and the insulating mines indicate that signatures of buried conducting mines are weaker than those of insulating mines, therefore, they are more difficult to detect.
1.3.3 ESL Surrogate Mine Field Experiments

Facilities

To better understand the signatures of buried mines and to get a better appreciation for IR clutter, additional experiments were performed. A surrogate mine field was created in the backyard of the ESL using 40 mine-like and clutter-like targets as shown in Figure 1.8. The identities of the buried objects are given in Table 1.3.3. The objects in this field had been in situ for more than 18 months at the time these data were acquired. Surface vegetation on the mine field was completely removed. The surface of the mine field is planar to within an estimated variation of ±2 inches. For most of the images presented in this work the cameras were positioned on the roof of the ESL building, roughly 25 feet above the level of the mine field and 40 feet
Figure 1.6: The view of the surrogate minefield from the IR camera. The camera FOV is smaller than the region pictured here.
Figure 1.7: Representative results from the small surrogate mine grid data set. Under cloudy skies (image 1) the white insulating surrogate is a better reflector. Under intermittent sun (image 3) the conducting and insulating surface surrogates have comparable apparent temperatures. Under full sun (image 7) the white surrogate reflects and the dark surrogate absorbs the incident sunlight. Finally, shortly after the mines are shadowed (image 11) the surface metal surrogate radiates, and the insulating surrogates appear cooler.
away, which results in observation angles ranging from 25.5° to 38.5° with respect to horizon.

Figure 1.8: The layout of the ESL surrogate mine grid. The key marked near each object is interpreted as the Abbreviation/Depth in inches. (See Table 1.3.3)

The test area was not cleared or otherwise conditioned for IR imaging prior to emplacing the targets. As a result, there may exist buried objects or voids in this area. The four concrete blocks shown in the figure were concealed in a lush grass cover that was present when the test objects were emplaced. It was only months later after the grass was removed that these blocks were discovered.
Figure 1.9: The layout of the MURI project mine test grid, indicating the selected regions for the clutch experiments. The square boxes show the regions chosen for the experiment.
Local Region Experiments

The experiments were performed in two regions of the above-described surrogate mine field as shown in Figure 1.9. Each of these regions include two 7 inch nylon or teflon discs, which are readily detectable in IR imagery. In one of the regions the discs are buried at a 1 inch depth and in the second region the discs are buried at 2 inches. To act as fiducial markers we used 10 cm square, white, styrofoam slabs. These inexpensive materials are good diffuse reflectors of sunlight during daylight hours, and they also block the upward flow of heat during evening hours.

Sendur and Baertlein [36, 37] reported that direct illumination of the scene by sunlight is the dominant MWIR clutter source, and skylight (both aerosol-scattered
sunlight and thermal emission by the air) is also a significant clutter source. On the basis of these findings, the MWIR images were acquired after sunset. Image mapping to ground coordinates is performed using a perspective transformation described in Appendix A. Figure 1.10 shows the results of an experiment performed on 9 June 1998. These data were collected in region 1 after sunset to avoid surface-reflected light. Figure 1.10(a) shows the fiducial marker geometry and the mine positions. Figures 1.10(b)-(d) are the images taken in region 1 at 19:10, 20:25 and 21:55, respectively. In these figures the fiducial marker positions are obvious, and one can also identify the mine signatures. Operating at night does not remove all reflected clutter, since the night sky continues to produce thermal radiation in the infrared region, which is reflected from the ground. Surface roughness, emissivity variations and buried anomalies can each be sources of clutter.

Figure 1.11 shows the results of an experiment performed on 13 October 1998 with the LWIR sensor. A sketch of the imaged region is given in Fig. 1.11 (a) including the fiducial marker geometry, mine positions and the location of the concrete block. Figure 1.11 (b), (c), and (d) are LWIR imagery captured on 13 October 1998 at 18:00, 19:00, and 21:00, respectively. Although the imagery are somewhat cluttered, the mine signatures can be identified in these figures.

Surrogate Mine Field Experiments

Images of larger areas of the ESL mine field were acquired to understand the responses of a greater variety of targets and to better characterize the IR clutter. Since the FOVs are different for the MWIR and the LWIR cameras, the images were acquired with different camera configurations and geometries. Fiducial markers are essential in establishing the position of objects within imagery, and here they
Figure 1.10: MWIR imagery collected in region 1 on 9 June 1998. The data were collected at different times after sunset: (a) Sketch of the region of interest including the mine positions, fiducial markers and the concrete block. (b) Image captured at 19:10. (c) Image captured at 20:25. (d) Image captured at 21:55.
Figure 1.11: (a) Sketch of the region of interest including the mine positions, fiducial markers and the concrete block. (b) Image captured by the LWIR camera on 12 October 1998 at 18:00. (c) Image captured at 19:00. (d) Image captured at 21:00
perform an additional function: The camera FOVs and the imaging geometry do not permit the entire minefield to be viewed in one image, and so it is necessary to combine several images to provide a complete view of the area. The fiducial markers provide an image-to-image registration capability. Gross registration of the images is also possible with correlation-based techniques after perspective transformations. At least four fiducial markers are included in each image so that a limited amount of fine correction is also possible.

Figure 1.12 illustrates the fiducial markers and image boundaries associated with MWIR data captured on 12 July 1998 at 00:25. The location of the camera for all tests is given in Table 1.4. Ground coordinates are referred to an origin at the lower-right-most target in the minefield. The locations and descriptions of the fiducials are given in Table 1.5. The current optics of the IRRIS camera has a relatively narrow FOV, and to span the entire mine field seven images were required. In the figure, red lines indicate ground-projected image borders and blue squares indicate fiducial markers. In acquiring each image a marker in the center of the camera display was pointed towards a pre-determined fiducial marker. As a result, the location of the center of the image is exactly known, and a perspective transformation can be performed using the camera-minefield geometry and the camera FOV. The captured images are numbered from left to right. These numbers are shown in green near the fiducial marker that corresponds to the image center. The black circles in Figure 1.12 identify specific clutter sample points whereas red stars identify the most readily IR-detectable mine surrogates.\(^2\) Figures 1.13 and 1.14 (a)-(g) show the perspective-transformed imagery.

\(^2\)In this work a “readily detectable IR target” is a surrogate of diameter greater than six inches buried at a depth less than or equal to two inches.
Table 1.4: Camera position for all LWIR and MWIR tests.

<table>
<thead>
<tr>
<th>Marker #</th>
<th>X pos.</th>
<th>Y pos.</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4.5&quot;</td>
<td>4' 7&quot;</td>
<td>2' x 2'</td>
</tr>
<tr>
<td>2</td>
<td>-5' 5&quot;</td>
<td>4' 7&quot;</td>
<td>2' x 2'</td>
</tr>
<tr>
<td>3</td>
<td>-10' 5&quot;</td>
<td>4' 7&quot;</td>
<td>2' x 2'</td>
</tr>
<tr>
<td>4</td>
<td>-15' 6&quot;</td>
<td>4' 7&quot;</td>
<td>2' x 2'</td>
</tr>
<tr>
<td>5</td>
<td>-20' 6&quot;</td>
<td>4' 7&quot;</td>
<td>2' x 2'</td>
</tr>
<tr>
<td>6</td>
<td>-25' 7&quot;</td>
<td>4' 7&quot;</td>
<td>2' x 2'</td>
</tr>
<tr>
<td>7</td>
<td>1' 3&quot;</td>
<td>-1' 1&quot;</td>
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<td>10' 1&quot;</td>
<td>4' x 4'</td>
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Table 1.5: Fiducial marker positions for 12 July 1998 MWIR tests.
Figure 1.12: The sketch of the ESL mine field showing the borders of the images. Lines: Image borders. Circles: Clutter sample points. Stars: IR-detectable mine surrogates. Squares: Fiducial markers. Numbers: The fiducial marker that corresponds to the center of the ith image, where i is the image number from left to right.

In the 12 July data set, images 2 and 4 contain the most IR-detectable mine surrogates. These correspond to Figures 1.13 (b) and (d). The mine signatures are very clear and there is minimal clutter, especially in Figure 1.13 (d). In Figure 1.13 (b) the mine signatures can be seen, but there is somewhat more clutter.

For the Agema camera, three images were used to span the mine field. The camera's wide FOV setting was used in all tests. The test procedure was similar to that described above for the MWIR tests. Figure 1.15 illustrates the fiducial marker...
Figure 1.13: Images captured by the MWIR camera on 12 July 1998 at 00:05. Images are numbered from left to right (a) Image #1 (b) Image #2 (c) Image #3 (d) Image #4
Figure 1.14: (e) Image #5 (f) Image #6 (g) Image #7
geometry for images captured 12 October 1998 at 21:00. Marker positions are given in Tables 1.6. In Figs. 1.16 (a)-(c), although the imagery has some clutter, the mine signatures are visible. In Fig. 1.16 (c), the mine-like object buried at 1 inch depth is especially clear.

<table>
<thead>
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<th>Y pos. [in.]</th>
<th>Size [in.]</th>
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Table 1.6: Fiducial marker positions for 12 October 1998 LWIR tests.
Figure 1.15: A sketch of the ESL mine field. **Lines:** Image borders. **Circles:** Clutter sample points. **Stars:** IR-detectable mine surrogates. **Numbered squares:** Fiducial markers. **Other squares:** Fiducial markers used in the MWIR experiment. **Numbers:** The fiducial marker that corresponds to the center of the $i$th image, where $i$ is the image number from left to right.
Figure 1.16: Images captured by the LWIR camera on 12 October 1998 at 21:00. Images are numbered from left to right (a) Image #1 (b) Image #2 (c) Image #3
1.4 Organization of this Dissertation

In this work, three-dimensional thermal and radiometric models are developed to predict the IR signatures of a buried mine. The work is organized as follows: In Chapter 2 a reference solution is presented for the integral equation that governs the temperature distribution. Using a periodic boundary condition in time at the planar interface, the temperature distribution in the lower-half space is expanded in a Fourier series. A volume integral equation for the Fourier series coefficients is obtained via Green’s second identity. The Green’s function for this problem is derived and the integral equation is solved using the method of weighted residuals. In Chapter 3 a three-dimensional thermal model is developed to study the passive IR signature of a mine buried under a smooth soil surface. Using the previously described mathematical model, a finite element method (FEM) based thermal model is used to study temporal variations, the spatial structure of the signature, and environmental effects. The integral equation solution, which was developed in Chapter 2, is compared with the FEM model to assess the effects of various assumptions in the latter approach. In Chapter 4 we describe a detailed representation of a TM-62 AT mine used with the FEM thermal model. We employed automatic mesh generation and adaptive mesh refinement algorithms to better discretize the computational volume leading to more accurate results. A quantitative comparison of mines with metal and plastic cases is given in that chapter. In Chapter 5 a radiometric model is presented to predict IR mine signatures. The radiometric model addresses both the spatial and spectral characteristics of the environment. Examples of the temporal evolution of IR imagery for smooth and rough soil surfaces are given in that chapter. In Chapter 6 a model is described for the polarimetric signature of surface mines. A model based on second
order small perturbation method/small slope approximation is developed to study the effects of material composition, geometry, and statistical surface properties. A summary and concluding remarks are presented in Chapter 7.
Predicting the thermal signature of a buried land mine requires modeling the complicated inhomogeneous environment and the structurally complex mine. It is useful, both in checking such models and in making rough calculations of expected signatures, to have an accurate, easily computed reference solution for a relatively simple geometry.

In this chapter we present an integral equation solution for a homogeneous cylindrical body (the mine model) buried in an infinite homogeneous half-space with a planar interface (the soil model). The solution of the volume integral equation is obtained using the method of weighted residuals (MWR), a technique also known to researchers in electromagnetics as the “Method of Moments (MoM)”. In Chapter 3, the integral equation solution is compared with a finite element model to assess the effects of various assumptions in the latter approach. This chapter is organized as follows: Sect. 2.1 provides a description of the three-dimensional heat flow equation, including the convective and radiative boundary conditions at the soil-air interface. Using a periodic boundary condition in time at the planar interface, the temperature distribution in the lower-half space is expanded in a Fourier series. In Sect. 2.2 the
problem geometry is introduced and integral representations for the temperature distribution in the soil and mine are obtained. A volume integral equation for the Fourier series coefficients is obtained via Green's second identity, and the Green's function for the Fourier coefficients is derived. The Green's function for this problem is derived in Sect. 2.3 and subsequently simplified and converted into a computationally efficient form. The numerical solution procedure for the Fourier series coefficients of the temperature distribution is formulated in Sect. 2.4. Numerical results are presented in Sect. 2.5. Summary remarks appear in Sect. 2.6.

2.1 Fourier series coefficients of temperature distribution

The physical processes and the heat transfer mechanisms that produce IR signatures of buried land mines were briefly discussed in Chapter 1 and will not be repeated here. We now present an integral equation based solution to Eq. (1.3). The boundary condition at the soil-air interface given by Eq. (1.17) is time-varying. If the convection coefficient \( h(t) \) is approximated by its mean value \( \bar{h} \), then this boundary condition becomes a periodic function at the diurnal rate. This approximation suggests that the temperature distribution \( T(r, t) \) can be written as

\[
T(r, t) = \sum_{n=-\infty}^{\infty} T_n(r)e^{i\omega t} \tag{2.1}
\]

where \( \omega = 2\pi/(86400) \) [rad s\(^{-1}\)] is the radian frequency of a diurnal cycle and \( T_n(r) \) is the \( n \)th Fourier coefficient of the temperature. Since \( T(r, t) \) is a real quantity, the coefficients satisfy \( T_n(r) = T^*_n(r) \) where \( * \) denotes the complex conjugate operator. Substituting Eq. (2.1) in Eq. (1.3) and assuming piecewise constant material properties, the Fourier coefficients of the three-dimensional heat-flux equation can be
written as
\[ \nabla^2 T_n(r) - \frac{i\omega n}{\kappa_s(r)} T_n(r) = 0, \quad i = s, m \] (2.2)

Solar heating is the dominant heat source in this problem, and this fact permits additional approximations. Replacing \( T_{air}(t) \) with its mean value \( \overline{T}_{air} \), and simplifying \( T_{sky} \) in a similar manner yields
\[ \frac{\partial T(r,t)}{\partial z} \approx T(r,t) \frac{1}{\kappa_s} \left( \frac{\bar{h}}{\kappa_s} + 4\varepsilon\sigma T_{sky}^3 \right) - \frac{1}{\kappa_s} \left( F_{sun}(t) + \bar{h}T_{air} + 4\varepsilon\sigma T_{sky}^3 \right) \] (2.3)

By using a Fourier expansion of the solar insolation function
\[ F_{sun}(t) = \sum_{n=-\infty}^{\infty} \mathcal{F}_n e^{i\omega nt} \] (2.4)
the boundary condition for the \( n \)th Fourier coefficient can be written as
\[ \frac{\partial T_n(r)}{\partial z} = \alpha T_n(r) + \beta_n \] (2.5)
where
\[ \alpha = \left( \frac{\bar{h} + 4\varepsilon\sigma T_{sky}^3}{\kappa_s} \right) \] (2.6)
\[ \beta_n = -\left( \mathcal{F}_n + \bar{h}T_{air} + 4\varepsilon\sigma T_{sky}^3 \right) / \kappa_s \] (2.7)

2.2 Integral representation for the temperature distribution

In this section an integral representation for the Fourier coefficients of temperature in the lower half space will be obtained for a cylindrical mine with radius \( \rho_0 \) and thickness \( \tau \), buried at depth \( h \) under a smooth soil surface (see Fig. 2.1). Homogeneous soil with thermal diffusivity \( \kappa_s(r) = \kappa_s \) and thermal conductivity \( \kappa_s(r) = \kappa_s \) is assumed. The thermal properties of the mine need not be uniform in the following formulation, and the position \( (r) \) dependence on the mine's thermal properties will be retained.
2.2.1 Problem Formulation

For a mine that is a body of revolution, the problem has rotational symmetry. Therefore, Eq. (2.2) can be written as

\[ \nabla_{\rho z}^2 T_n^s(r) - k^2_n T_n^s(r) = 0 \quad r \in \text{soil} \]

\[ \nabla_{\rho z}^2 T_n^m(r) - \hat{k}_m^2(r) T_n^m(r) = 0 \quad r \in \text{mine} \]  

(2.8)

where

\[ \nabla_{\rho z}^2 = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{\partial^2}{\partial z^2} \]  

(2.9)

\[ k^2_n = \frac{i \omega n}{\kappa_s} \]  

(2.10)

\[ \hat{k}_m^2(r) = \frac{i \omega n}{\kappa_m(r)} \]  

(2.11)

In Eq. (2.8) \( T_n^m(r) \) and \( T_n^s(r) \) represent the Fourier coefficients in the mine and soil regions, respectively.

We employ a standard method [38] of solving the differential operator equation defined in Eq. (2.2). Forming the inner product of that equation and the Green's
function $G$ yields

$$
\langle \nabla_{p_2}^2 T_n(\rho', z'), G(\rho, z; \rho', z') \rangle = \langle \nabla_{p_2}^2 G(\rho, z; \rho', z'), T_n(\rho', z') \rangle + R(\rho, z)
$$

(2.12)

where $\langle \cdot, \cdot \rangle$ is the inner product, which comprises integration over the entire lower half space. The Green’s function satisfies the equation

$$(\nabla_{p_2}^2 - k_n^2)G(\rho, z; \rho', z') = -\frac{\delta(\rho - \rho')}{\rho} \delta(z - z')$$

(2.13)

with boundary conditions defined below. The conjunct $R$ is an integral over the problem boundaries [38, p. 198] and is also defined below. Using the above property of $G$ and the differential equation satisfied by $T_n$ we immediately obtain

$$
T_n(\rho, z) = \int_{z_1}^{z_2} dz' \int_0^{\rho_0} d\rho' \rho' c(\rho', z') T_n(\rho', z') G(\rho, z; \rho', z') + R
$$

(2.14)

In this result $z_1 = h, z_2 = h + r$, we have exchanged primed and unprimed coordinates, and we have defined

$$
c(r) = k_n^2(r) - k_n^2 = \frac{\kappa_s - \kappa_m(r)}{\kappa_s \kappa_m(r)}
$$

(2.15)

Integration by parts is used to determine $R$ from its definition in Eq. (2.12). In doing so, we exploit the thermal boundary conditions, namely that temperature is continuous

$$
T_n^m(r = r_{ms}) = T_n^s(r = r_{ms})
$$

(2.16)

and thermal flux is continuous

$$
K_m \nabla T_n^m(r) \bigg|_{r=r_{ms}} = K_s \nabla T_n^s(r) \bigg|_{r=r_{ms}}
$$

(2.17)

where $r_{ms}$ denotes any point on the mine-soil boundary. The following result is obtained

$$
R(\rho, z) = -\int_0^{\infty} d\rho' \rho' Z^s(\rho, z; \rho', 0) + \rho_0 \int_{z_1}^{z_2} dz' F^m(\rho, z; \rho_0, z') \left(1 - \frac{K_m(\rho_0, z')}{K_s}\right)
$$
\[ + \int_{0}^{\rho_{0}} d\rho' \rho' \left( \mathcal{S}^{m}(\rho, z; \rho', z_{2}) \left( 1 - \frac{K_{m}(\rho', z_{2})}{K_{s}} \right) \\ - \mathcal{S}^{m}(\rho, z; \rho', z_{1}) \left( 1 - \frac{K_{m}(\rho', z_{1})}{K_{s}} \right) \right) \]

in which

\[ Z_{n}^{i}(\rho, z; \rho', z') = S_{n}^{i}(\rho, z; \rho', z') - T_{n}^{i}(\rho', z') \frac{\partial G(\rho, z; \rho', z'')}{\partial z''} \bigg|_{z'' = z'} \quad (2.19) \]

\[ F_{n}^{i}(\rho, z; \rho', z') = G(\rho, z; \rho', z') \frac{\partial T_{n}^{i}(\rho', z'')}{\partial \rho'} \bigg|_{\rho' = \rho'} \quad (2.20) \]

\[ S_{n}^{i}(\rho, z; \rho', z') = G(\rho, z; \rho', z') \frac{\partial T_{n}^{i}(\rho', z'')}{\partial z''} \bigg|_{z'' = z'} \quad (2.21) \]

In Eqs. (2.19)-(2.21) the subscript \( i \) can be \( s \) or \( m \), which refer to the soil and mine, respectively. By selecting the following boundary condition for the Green's function at the soil-air interface

\[ \frac{\partial G(\rho, z; \rho', z')}{\partial z} = \alpha G(\rho, z; \rho', z') \quad (2.22) \]

the integral representation for the temperature distribution \( T_{n}(\rho, z) \) in the lower half space can be obtained as

\[ T_{n}(\rho, z) = \int_{z_{1}}^{z_{2}} dz' \int_{0}^{\rho_{0}} d\rho' \rho' c(\rho', z') T_{n}^{m}(\rho', z') G(\rho, z; \rho', z') \]

\[ - \beta_{n} \int_{0}^{\infty} d\rho' \rho' G(\rho, z; \rho', z' = 0) \]

\[ + \rho_{0} \int_{z_{1}}^{z_{2}} dz' F^{m}(\rho, z; \rho_{0}, z') \left( 1 - \frac{K_{m}(\rho, z')}{K_{s}} \right) \]

\[ + \int_{0}^{\rho_{0}} d\rho' \rho' \left( \mathcal{S}^{m}(\rho, z; \rho', z_{2}) \left( 1 - \frac{K_{m}(\rho', z_{2})}{K_{s}} \right) \right) \]

\[ - \mathcal{S}^{m}(\rho, z; \rho', z_{1}) \left( 1 - \frac{K_{m}(\rho', z_{1})}{K_{s}} \right) \) \quad (2.23) \]

Equation (2.23) is an integral relation from which one can determine the temperature distribution anywhere in the lower half space by integrating the temperature distribution over the mine. If \( (\rho, z) \) is a point within the mine, this relation is a Fredholm's
integral equation of the second kind for the unknown mine temperature distribution.

In the absence of the mine, i.e., for a homogeneous half space, the first, third, and last terms in Eq. (2.23) vanish and the temperature distribution can be found directly from

\[ T_n(\rho, z) = -\beta_n \int_0^\infty d\rho' \rho' G(\rho, z; \rho', z' = 0) \]  

(2.24)

The Green's function for Eq. (2.23) is derived in Sect. 2.3. The numerical solution procedure for Eq. (2.23) is presented in Sect. 2.4.

### 2.2.2 Alternative Interpretation

An alternative view of the problem offers a physical interpretation of the expressions derived in Sect. 2.2.1. The key mathematical tool in this work is the volume equivalence theorem for the heat transfer equation. The volume equivalence theorem is widely used in electromagnetics [39, 40, 41] to determine the scattered electromagnetic fields in the presence of a material body in a homogeneous environment.

The original problem comprises a buried mine with thermal properties \( \kappa_m(\mathbf{r}) \) and \( K_m(\mathbf{r}) \) in homogeneous soil with thermal properties \( \kappa_s(\mathbf{r}) = \kappa_s \) and \( K_s(\mathbf{r}) = K_s \). A new equivalent problem can be defined in which the mine is replaced by homogeneous soil and an equivalent heat source \( Q(\mathbf{r}) \) as shown in Fig. 2.2. The temperature anomaly due to the inhomogeneity can be viewed as being generated by a so-called "induced" source \( Q(\mathbf{r}) \), which is proportional to the temperature distribution in the inhomogeneous volume. The derivation is well known in electromagnetics, but somewhat involved. Using Eq. (2.1) in Eq. (1.1) and the properties of the divergence operator, we obtain

\[ \mathcal{K}(\mathbf{r}) \nabla^2 T_n(\mathbf{r}) + \nabla \mathcal{K}(\mathbf{r}) \cdot \nabla T_n(\mathbf{r}) - i\omega n C(\mathbf{r}) T_n(\mathbf{r}) = 0 \]  

(2.25)
Figure 2.2: Volume equivalence for heat transfer equation. (a) The original problem, in which a buried mine with thermal properties $\kappa_m(\mathbf{r})$ and $\mathcal{K}_m(\mathbf{r})$ is present in homogeneous soil with thermal properties $\kappa_s(\mathbf{r}) = \kappa_s$ and $\mathcal{K}_s(\mathbf{r}) = \mathcal{K}_s$. (b) The equivalent problem, in which the mine is replaced by homogeneous soil and an equivalent distributed heat source $\mathcal{Q}(\mathbf{r})$. 
Dividing both sides of Eq. (2.25) by $K(r)$ we get

\[ \nabla^2 T_n(r) + \frac{\nabla K(r)}{K(r)} \cdot \nabla T_n(r) - k^2(r)T_n(r) = 0 \quad (2.26) \]

where

\[ k^2(r) = i\omega_n C(r) = k_n^2 + (\tilde{k}_n^2(r) - k_n^2)U_{\text{mine}}(r) \quad (2.27) \]

where $k_n^2$ and $\tilde{k}_n^2(r)$ are defined in Sect. 2.2.1 and

\[ U_{\text{mine}}(r) = \begin{cases} 1 & r \in \text{mine} \\ 0 & r \in \text{soil} \end{cases} \quad (2.28) \]

Substituting Eq. (2.28) into Eq. (2.26) and rearranging the terms yields

\[ \nabla^2 T_n(r) - k_n^2 T_n(r) = Q(r) \quad (2.29) \]

where $Q(r)$ is the fictitious heat source. This is given by two terms

\[ Q(r) = Q_v(r) + Q_s(r) \quad (2.30) \]

where

\[ Q_v(r) = c(r)T_n(r)U_{\text{mine}}(r) \quad (2.31) \]
\[ Q_s(r) = -\frac{\nabla K(r)}{K(r)} \cdot \nabla T_n(r) \quad (2.32) \]

The quantity $c(r)$ in Eq. (2.31) has been defined in Sect. 2.2.1. Also note that

\[ \frac{\nabla K(r)}{K(r)} = \left( 1 - \frac{K_m(\rho, z_1)}{K_s} \right) \delta(z - z_1) \left( u(\rho) - u(\rho - \rho_0) \right) \dot{\bar{z}} \\
- \left( 1 - \frac{K_m(\rho, z_2)}{K_s} \right) \delta(z - z_2) \left( u(\rho) - u(\rho - \rho_0) \right) \dot{\bar{z}} \\
- \left( 1 - \frac{K_m(\rho_0, \bar{z})}{K_s} \right) \delta(\rho - \rho_0) \left( u(z - z_1) - u(z - z_2) \right) \dot{\bar{\rho}} \quad (2.33) \]

where $u$ is the unit step function. Eq. (2.29) can be manipulated to yield a volume integral equation, which can be solved for the unknown, induced heat source. Using
this induced heat source, the temperature distribution in the lower half space can be found by integration. This volume formulation can be conveniently used to treat any inhomogeneous material.

Applying Green's second identity to the functions \( T_n(\rho, z) \) and \( G(\rho, z; \rho', z') \) with a source distribution \( Q(\rho, z) \) in the mine gives

\[
T_n(\rho, z) = -\int_{z_1}^{z_2} dz' \int_0^{\rho_0} d\rho' \rho Q(\rho', z')G(\rho, z; \rho', z') + \int_{\Gamma} \left[ T_n(\rho, z) \nabla G(\rho, z; \rho', z') - G(\rho, z; \rho', z') \nabla T_n(\rho, z) \right] \cdot \hat{n} \, ds \tag{2.34}
\]

where \( \Gamma \) denotes the boundaries (at the soil-air interface and at the mine) and \( \hat{n} \) is the unit normal on those boundaries. Using the definitions of the boundaries, Eq. (2.34) can be written as

\[
T_n(\rho, z) = -\int_{z_1}^{z_2} dz' \int_0^{\rho_0} d\rho' \rho Q(\rho', z')G(\rho, z; \rho', z') - \int_0^{\infty} d\rho' \rho Z^*G(\rho, z; \rho', 0) \tag{2.35}
\]

Equation (2.35) can be transformed into Eq. (2.23) by selecting the boundary condition for \( G \) as given by Eq. (2.22).

### 2.3 Green’s Function

The Green's function \( G \) is the solution of the heat transfer equation for an internal point source of heat. When \( G \) is known, a solution for an arbitrary source can be obtained via Eq. (2.23). In this section a Green's function will be derived, which satisfies Eq. (2.13) with the boundary condition on the soil-air interface given by Eq. (2.22). In addition to this boundary condition, the Green's function must have a finite value for \( G(\rho = 0, z; \rho', z') \) and must vanish as \( \rho \to \infty \) and \( z \to \infty \). In Sect. 2.3.1 the derivation is presented. In Sect. 2.3.2 the Green's function is simplified and transformed into a more appropriate form for numerical evaluation.
2.3.1 Derivation of the Green's Function

The Green's function is easily derived in the spectral domain. Taking the Hankel transform \[ f(k_p) = \int_0^\infty x J_0(k_p x) f(x) dx \] (2.36) of both sides of the Eq. (2.13) we obtain

\[ \left( \frac{\partial^2}{\partial z^2} - (k_p^2 + k_n^2) \right) \tilde{G}(k_p, z; \rho', z') = -J_0(k_p \rho') \delta(z - z') \] (2.37)

where \( \tilde{G}(k_p, z; \rho', z') \) is the Hankel transform of the Green's function \( G(\rho, z; \rho', z') \). In obtaining Eq. (2.37) the differential equation for the zeroth order Bessel function was used. The solution of the ordinary differential equation given by Eq. (2.37) with the boundary conditions at the interface can be obtained through well known methods.

We can show

\[ \frac{\tilde{G}(k_p, z; \rho', z')}{J_0(k_p \rho')} = \frac{1}{2\sqrt{k_n^2 + k_p^2}} \exp \left( -\sqrt{k_n^2 + k_p^2} z_\geq \right) \left[ \exp \left( \sqrt{k_n^2 + k_p^2} z_\leq \right) + r_0(k_n, k_p, \alpha) \exp \left( -\sqrt{k_n^2 + k_p^2} z_\leq \right) \right] \] (2.38)

where

\[ r_0(k_n, k_p, \alpha) = \frac{\sqrt{k_n^2 + k_p^2} - \alpha}{\sqrt{k_n^2 + k_p^2} + \alpha} \] (2.39)

The inverse Hankel transform, given by

\[ f(x) = \int_0^\infty k_p J_0(k_p x) \hat{f}(k_p) dk_p \] (2.40)

yields the desired result

\[ G(\rho, z; \rho', z') = \int_0^\infty dk_p J_0(k_p \rho') J_0(k_p \rho) \frac{k_p}{2\sqrt{k_n^2 + k_p^2}} \exp \left( -\sqrt{k_n^2 + k_p^2} z_\geq \right) \times \left[ \exp \left( \sqrt{k_n^2 + k_p^2} z_\leq \right) + r_0(k_n, k_p, \alpha) \exp \left( -\sqrt{k_n^2 + k_p^2} z_\leq \right) \right] \] (2.41)

where \( z_\leq \) and \( z_\geq \) stand for \( \min(z, z') \) and \( \max(z, z') \), respectively.
2.3.2 Simplification of the Green’s Function

In general, numerical evaluation of the Green’s function given by Eq. (2.41) is computationally challenging. Nonetheless, efficient, accurate calculation of \( G \) is crucial to a numerical solution of the integral equation. The integral given by Eq. (2.41) is similar to the Sommerfeld integrals \[42\], which have been studied extensively in the physics and electromagnetics literature. The major difficulties in the computation of these integrals can be summarized as follows:

- For large \( \rho \) and \( \rho' \) values, the functions \( J_0(k_p \rho) \) and \( J_0(k_p \rho') \) oscillate rapidly, leading to slow convergence.
- For \( |z - z'| \ll 1 \) or \( (z + z') \ll 1 \) the exponential terms in the integrals decay slowly, again producing slow convergence.

Straightforward manipulations ameliorate some of these problems. The quantity \( r_0(k_n, k_p, \alpha) \) can be expressed as

\[
r_0(k_n, k_p, \alpha) = 1 - \frac{2\alpha}{\sqrt{k_n^2 + k_p^2 + \alpha}}
\]

and, hence, the Green’s function can be expressed as

\[
G(\rho, z; \rho', z') = G_1(\rho, z; \rho', z') + G_2(\rho, z; \rho', z') + G_3(\rho, z; \rho', z')
\]

where

\[
G_1(\rho, z; \rho', z') = \int_0^\infty dk_p J_0(k_p \rho') J_0(k_p \rho) \frac{k_p}{2\sqrt{k_n^2 + k_p^2}} \exp \left( -\sqrt{k_n^2 + k_p^2}(z - z') \right)
\]

\[
G_2(\rho, z; \rho', z') = \int_0^\infty dk_p J_0(k_p \rho') J_0(k_p \rho) \frac{k_p}{2\sqrt{k_n^2 + k_p^2}} \exp \left( -\sqrt{k_n^2 + k_p^2}(z + z') \right)
\]

\[
G_3(\rho, z; \rho', z') = -\alpha \int_0^\infty dk_p J_0(k_p \rho') J_0(k_p \rho) \frac{k_p}{\sqrt{k_n^2 + k_p^2}(\sqrt{k_n^2 + k_p^2 + \alpha})} \cdot \exp \left( -\sqrt{k_n^2 + k_p^2}(z + z') \right)
\]
Employing the multiplication identity for the zeroth order Bessel functions used by Johnson [43]

\[ J_0(k_p \rho')J_0(k_p \rho) = \frac{1}{2\pi} \int_0^{2\pi} J_0(k_p \rho \phi) d\phi \]  

(2.47)

where

\[ \rho_t = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos \phi} \]  

(2.48)

and changing the order of integration we can rewrite Eqs. (2.44) and (2.45) as

\[ G_1(\rho, z; \rho', z') = \frac{1}{4\pi} \int_0^{2\pi} d\phi' \int_0^{\infty} dk_p J_0(k_p \rho_t) \frac{k_p}{\sqrt{k_n^2 + k_p^2}} \cdot \exp \left(-\sqrt{k_n^2 + k_p^2}(z - z_<)\right) \]  

(2.49)

\[ G_2(\rho, z; \rho', z') = \frac{1}{4\pi} \int_0^{2\pi} d\phi' \int_0^{\infty} dk_p J_0(k_p \rho_t) \frac{k_p}{\sqrt{k_n^2 + k_p^2}} \cdot \exp \left(-\sqrt{k_n^2 + k_p^2}(z + z')\right) \]  

(2.50)

Using following identity [38]

\[ \frac{\exp(-k\sqrt{r^2 + z^2})}{\sqrt{r^2 + z^2}} = \int_0^{\infty} \gamma J_0(\gamma r) \exp\left(-|z|\sqrt{\gamma^2 + k^2}\right) d\gamma \]  

(2.51)

permits Eqs. (2.49) and (2.50) to be simplified to

\[ G_1(\rho, z; \rho', z') = \int_0^{2\pi} d\phi' \frac{\exp(ik_n R_1)}{4\pi R_1} \]  

(2.52)

\[ G_2(\rho, z; \rho', z') = \int_0^{2\pi} d\phi' \frac{\exp(ik_n R_2)}{4\pi R_2} \]  

(2.53)

where

\[ R_1 = \sqrt{\rho'^2 + \rho^2 - 2\rho\rho' \cos \phi' + (z - z')^2} \]  

(2.54)

\[ R_2 = \sqrt{\rho'^2 + \rho^2 - 2\rho\rho' \cos \phi' + (z + z')^2} \]  

(2.55)

Similarly, substituting Eq. (2.47) into Eqs. (2.56) and changing the order of integration we obtain

\[ G_3(\rho, z; \rho', z') = -\frac{\alpha}{2\pi} \int_0^{2\pi} d\phi I_{G_3} \]  

(2.56)
where

\[ I_G = \int_0^\infty dk_\rho J_0(k_\rho \rho_1) \frac{k_\rho}{\sqrt{k_n^2 + k_\rho^2(\sqrt{k_n^2 + k_\rho^2} + \alpha)}} \exp\left(-\sqrt{k_n^2 + k_\rho^2}(z + z')\right) \quad (2.57) \]

We can express this in another form as suggested by Kuo and Mei [44]. Multiplying both sides of Eq. (2.57) by \(\exp(-\alpha(z + z'))\) and differentiating both sides with respect to \(z + z'\) yields

\[
\frac{\partial(I_G \exp(-\alpha(z + z')))}{\partial(z + z')} = -\exp(-\alpha(z + z')) \\
\times \int_0^\infty dk_\rho J_0(k_\rho \rho_1) \frac{k_\rho}{\sqrt{k_n^2 + k_\rho^2}} \exp\left(-\sqrt{k_n^2 + k_\rho^2}(z + z')\right) \quad (2.58)
\]

The integral in Eq. (2.58) can be evaluated using Eq. (2.51). After integrating both sides with respect to \(z\) with appropriate limits, \(I_G\) can be written as

\[
I_G = \exp(-\alpha(z + z')) \int_{z + z'}^\infty \frac{\exp(ik_nR_3)}{R_3} \exp(-\alpha\zeta) d\zeta \quad (2.59)
\]

where

\[
R_3 = \sqrt{\rho'^2 + \rho^2 - 2\rho' \rho \cos \phi' + \zeta^2} \quad (2.60)
\]

Using Eqs. (2.52), (2.53), (2.56), and (2.59) in conjunction with Eq. (2.43) we obtain the final form of the Green's function as

\[
G(\rho, z; \rho', z') = \frac{1}{4\pi R_1} \int_0^{2\pi} d\phi' \exp\left(ik_nR_1\right) + \frac{1}{4\pi R_2} \int_0^{2\pi} d\phi' \exp\left(ik_nR_2\right) \\
- \frac{\alpha}{2\pi} \int_0^{2\pi} d\phi \exp(-\alpha(z + z')) \int_{z + z'}^\infty d\zeta \frac{\exp(ik_nR_3)}{R_3} \exp(-\alpha\zeta) \quad (2.61)
\]

The last integral in Eq. (2.61) has infinite integration limits, but the integrand is a rapidly decaying function for typical values of \(\alpha\), and it is well approximated by an integral over a finite domain. Green's function given by Eq. (2.61) is evaluated and plotted for different parameters in Fig. 2.3. For the evaluation of the Green's
function an adaptive Gaussian quadrature is used. When the source and observation points approach one another, we have $R_1 \to 0$, and the Green's function has a well-known integrable singularity. The treatment of this case is discussed in Sect. 2.4.2 and Appendix B.

2.4 Numerical solution procedure

In this section we describe a numerical solution procedure for the integral equation in Eq. (2.23) to obtain the Fourier coefficients $T_n^m(r)$. Our approach is to solve for the temperature $T_n^m$ within the mine, which can then be used to evaluate Eq. (2.23) for the Fourier coefficients at the surface. The time history of the temperature is found by evaluating the Fourier series.

We employ the MWR in this work. As noted above, the electromagnetics community has developed an extensive body of knowledge on MWR solutions of integral equations in the temporal frequency domain under the guise of the so-called "method of moments" (MoM). The MoM discretizes the integral equation into a matrix equation, the solution of which is obtained using standard methods. Following the pioneering works of Richmond [45] and Harrington [46], an extensive literature has developed on this procedure, and good summary references are also available [47, 48, 49].

2.4.1 Matrix formulation

The solution of the integral equation using the MWR begins with a representation of the unknown temperature distribution over the buried mine using specified expansion functions and unknown coefficients. The problem at hand imposes constraints on these functions. The integral representation of the temperature distribution given in Eq. (2.23) requires both the temperature and its derivatives with respect to $\rho$ and
Figure 2.3: Green's function versus various parameters with $\alpha = 5.1273$, $\rho = 0$, and $z = 0$: (a) $n = 1$, $\rho' = 0.1$, and $z' \in [0,0.5]$; (b) $n = 1$, $\rho' = 0.2$, and $z' \in [0,0.5]$; (c) $n = 1$, $\rho' \in [0,0.5]$, and $z' = 0.05$; (d) $n = 1$, $\rho' \in [0,0.5]$, and $z' = 0.1$; (e) $n \in [1,10]$, $\rho' = 0.2$, and $z' = 0.05$; and (f) $n \in [1,10]$, $\rho' = 0.2$, and $z' = 0.2$. 

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Figure 2.4: An example discretization of the buried mine with 4 subdivisions in the \( \hat{\rho} \)-direction and 3 subdivisions in the \( \hat{z} \)-direction.

z. We employ expansion functions \( \Lambda(\rho, z) \) that are linear in \( \rho \) and \( z \), viz:

\[
T_{m}^{n}(\rho, z) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{m,n} \Lambda_{nm}(\rho, z)
\]  

(2.62)

where \( M \) and \( N \) are the number of divisions in the \( z \) and \( \rho \) directions, respectively. This representation yields a continuous, piecewise-linear representation of \( T_{m}^{n} \) and a piecewise constant form for its derivatives.

A sample discretization of the buried mine with \( N = 4 \) subdivisions in the \( \hat{\rho} \)-direction and \( M = 3 \) subdivisions in the \( \hat{z} \)-direction is illustrated in Fig. 2.4. The thermal properties over each subsectional basis function are assumed constant and taken into account in the constant \( c_{m,n} \), which can be defined as

\[
c_{m,n} = c(((\rho_n + \rho_{n-1})/2, (z_m + z_{m-1})/2)
\]  

(2.63)

where \( c \) was previously defined in Eq. (2.15) and the indexed values \( \rho_n \) and \( z_m \) are the subsection boundaries. By substituting the approximate temperature distribution of
Eq. (2.62) into the integral equation Eq. (2.23), we obtain

\[
\beta_n \int_0^\infty d\rho' \rho' G(\rho, z; \rho', z') = - \sum_{m=1}^M \sum_{n=1}^N A_{mn} \left[ \Lambda_{mn}(\rho, z) 
\right.
\]
\[
+ c_{mn} \int dz' d\rho' \rho' G(\rho, z; \rho', z') \Lambda_{mn}(\rho', z')
\]
\[
+ \rho_0 \sum_{m=1}^M A_{mn} \int dz' G(\rho, z; \rho' = \rho_0, z') \cdot 
\]
\[
\frac{d}{d\rho'} \Lambda_{nm}(\rho', z') \left( 1 - \frac{K_m(\rho, z')}{K_s} \right)
\]
\[
+ \sum_{n=1}^N A_{Mn} \int d\rho' \rho' G(\rho, z; \rho', z' = z_2) \cdot 
\]
\[
\frac{d}{dz'} \Lambda_{nm}(\rho', z') \left( 1 - \frac{K_m(\rho', z_2)}{K_s} \right)
\]
\[
- \sum_{n=1}^N A_{In} \int d\rho' \rho' G(\rho, z; \rho', z' = z_1) \cdot 
\]
\[
\frac{d}{dz'} \Lambda_{nm}(\rho', z') \left( 1 - \frac{K_m(\rho', z_1)}{K_s} \right)
\]
\]  

(2.64)

To complete the formulation, we express the left-hand side of Eq. 2.64 using the summation in Eq. (2.62), multiply both sides of Eq. (2.64) by testing functions \(w_{n'm'}(\rho, z)\) and integrate over the mine. Various testing functions have been used in the literature. Galerkin methods are known to be optimum in the least-square sense, but for integral equations they are also computationally expensive in terms of matrix-fill time. To reduce the computational cost, we have employed a point-matching technique. The result can be expressed as a linear system of equations

\[
\overline{Z}A = V \tag{2.65}
\]

for the constants \(A_{mn}\), which we represent by the matrix \(A\). The kernel of the integral equation appears in \(\overline{Z}\) which we refer to as the “impedance” matrix. Solving Eq. (2.65) using an appropriate technique yields the unknown coefficient vector \(A\), which can be used to compute the temperature distribution over the buried mine via
Eq. (2.62). If desired, the temperature distribution everywhere in the lower half-space can be computed using Eq. (2.23).

2.4.2 Impedance matrix, source vector, and singularity extraction

In this section evaluation of the matrix $Z$ is discussed. Rows of this matrix have indices derived from the $(\rho, z)$ expansion functions indexed by $m$ and $n$, with a similar relation for the columns involving $\rho'$ and $z'$. To make the notation more concise, we use the index $p$ to refer to a particular point $(\rho, z)$, which we denote $(\rho_n, z_m)$, with a similar convention for $\rho'$ and $z'$ using $q$. In general, $Z_{pq}$ involves integrals of the form

$$\int_{z_{mq}-1}^{z_{mq}} dz' \int_{\rho_{pq}-1}^{\rho_{pq}} d\rho' G(\rho_n, z_m; \rho', z') f(\rho', z') \tag{2.66}$$

where $f(\rho, z)$ is a continuous (at most linear) function of the arguments. Consider first the off-diagonal matrix elements $Z_{pq}$ with $p \neq q$. For this case, the testing point $(\rho_n, z_m)$ is not in the integration domain and the Green's function is a smooth function. In this case $Z_{pq}$ can be directly evaluated by numerical methods.

For the diagonal matrix elements $Z_{pp}$, however, the testing point $(\rho_n, z_m)$ lies in the integration domain. This results in $R_1 \rightarrow 0$ which causes a singularity as $R_1^{-1}$. Consequently, an integrand regularization technique (e.g., singularity extraction) is necessary in numerical evaluation of the integrals. Singularity extraction techniques are widely used in MoM solutions in electromagnetics for both surface and volume formulations [50, 51, 52, 53]. Techniques for bodies of revolution have also been studied [54, 55], which are now described.

In brief, calculation of the diagonal elements requires that the singular parts of the integrand are evaluated separately by analytical methods. A careful examination
suggests that the singular part of the Green's function $G(\rho, z; \rho', z')$ is due to function $G_1(\rho, z; \rho', z')$, which leads us to write

$$G_1(\rho, z; \rho', z') = G_1^R(\rho, z; \rho', z') + \int_0^{2\pi} d\phi' \frac{1}{4\pi R_1} f(\rho, z) \quad (2.67)$$

where $f(\rho, z)$ is a smooth function and

$$G_1^R(\rho, z; \rho', z') = \int_0^{2\pi} d\phi' \frac{\exp(ik_n R_1) - 1}{4\pi R_1} \quad (2.68)$$

The function $G_1^R(\rho, z; \rho', z')$ can be integrated numerically. For small elements a Taylor series expansion of the integrand yields

$$\lim_{R_1 \to 0} G_1^R(\rho, z; \rho', z') = \frac{k_n}{2} \quad (2.69)$$

The remaining singular integral in Eq. (2.67) has one of the following forms

$$\begin{bmatrix}
I_e(\rho_1, \rho_u; \phi_1, \phi_u; z_l, z_u) \\
I_\rho(\rho_1, \rho_u; \phi_1, \phi_u; z_l, z_u) \\
I_z(\rho_1, \rho_u; \phi_1, \phi_u; z_l, z_u)
\end{bmatrix} = \int_{\rho_1}^{\rho_u} d\rho' \int_{z_l}^{z_u} dz' \int_{\phi_1}^{\phi_u} d\phi' \cdot \frac{1}{\sqrt{\rho^2 + \rho'^2 - 2\rho \rho' \cos \phi' + (z - z')^2}} \begin{bmatrix}
1 \\
\rho' \\
z'
\end{bmatrix} \quad (2.70)$$

Evaluation of these functions is discussed in Appendix B.

Singularity extraction is not necessary in evaluating the source vector elements $V_p$, since $G(\rho, z; \rho', z')$ is evaluated at the testing point $z_{m_p} > 0$ while $z' = 0$.

### 2.4.3 Surface Temperature Distribution

The numerical procedure described in Sect. 2.4.1 yields the Fourier coefficients of the temperature distribution within the mine. The temperature distribution over the soil surface $z = 0$ is found by using Eqs. (2.1) and (2.64). The procedure offers some challenges. When $z = z' = 0$, the exponential factors in Eq. (2.61) are eliminated,
leading to slow convergence. It is shown in Appendix C that an approximate but accurate and efficient form for the Green's function is

\[
G(\rho, z = 0; \rho', z' = 0) = \frac{1}{2\pi} \int_0^{2\pi} d\phi' \frac{\exp(ik_n\rho_t)}{\rho_t} \left( -\alpha K_0(-ik_n\rho_<) \right) - \frac{\alpha^2}{2\pi} \int_0^{2\pi} d\phi' \frac{\exp(ik_n\rho_t)}{ik_n} \left( -\alpha^3 \frac{\rho_t}{4\pi} \int_0^{2\pi} d\phi' \frac{K_{-1}(-ik_n\rho_t)}{ik_n} \right) \tag{2.71}
\]

The Green's function representation given by Eq. (2.71) is used to evaluate the third term in Eq. (2.64) when the observation point is on the soil surface.

2.5 Results

In this section numerical results obtained with the integral equation solution are presented. Additional results comparing the integral equation and FEM solutions are given in a Chapter 3, where we consider additional cases. In this section, consider the simple case of circular cylinder of diameter 20 cm, height 7.5 cm, and depth 4.5 cm filled with a good thermal insulator (styrofoam).

An important parameter of this problem is the required number of harmonics in the Fourier expansion given by Eq. (2.1). The forcing function of this integral equation is the solar insolation function \( F_{\text{sun}}(t) \), the Fourier series of which is dominated by its first term. Therefore, we expect the temperature coefficients \( T_n \) to be rapidly convergent. Our numerical simulations support this expectation. An example Fourier harmonic distribution is given in Table 2.5. Based on our numerical experiments, we kept the first five Fourier harmonics in the expansion given by Eq. (2.1).

Figure 3.2 shows the surface temperature over the center of the mine as a function of time. In the second set of results, we investigated spatial variations in the surface temperature, by plotting the temperature along a cut through the center of the mine.
Table 2.1: Fourier harmonics of the temperature distribution at a point over the mine

| Harmonic # | $T_n$ | $|T_n|/|T_1|$ |
|------------|-------|-------------|
| 1          | $2.5351 + j3.8669$ | 1            |
| 2          | $0.3582 + j1.3392$ | 0.2998       |
| 3          | $0.0042 + j0.0945$ | 0.0205       |
| 4          | $0.0140 - j0.1289$ | 0.0280       |
| 5          | $0.0059 - j0.0248$ | 0.0055       |
| 6          | $-0.0058 + j0.0120$ | 0.0029       |
| 7          | $-0.0043 + j0.0094$ | 0.0022       |

We present the temperature at times that correspond to maximum signature contrast. Figure 3.3 illustrates the spatial distribution results. The integral equation solution took about 2 hours on a 300 MHz Pentium II machine and requires less than 20 MB of memory. Most of this time is used to fill and factor the $\mathbf{Z}$ matrix.

2.6 Summary

In this chapter, an integral equation has been formulated for the temperature over a buried land mine. Using the approximations of a time invariant convection coefficient and air temperature, and linearizing the radiation boundary condition about its average value, the case of periodic (diurnal) solar heating leads to a Fourier decomposition of the time variation. The integral equation for each Fourier component of the temperature distribution was solved using the method of weighted residuals. The resulting model provides a reference solution for a relatively simple geometry, which can be used to check more sophisticated FEM-based models. In Chapter 3,
Figure 2.5: The surface temperature distribution over the center of a cylindrical mine using the integral equation solution.

The integral equation solution is compared with a finite element model to assess the effects of various assumptions in the latter approach.
Figure 2.6: The spatial dependence of the surface temperature distribution: (a) When the mine signature has a maximum negative contrast, (b) When the mine signature has a maximum positive contrast.
CHAPTER 3

THREE-DIMENSIONAL FINITE ELEMENT THERMAL MODELING OF LAND MINE SIGNATURES

In this chapter a three-dimensional thermal model is developed to study the passive IR signature of mines buried under a smooth soil surface. Using the mathematical model described in Chapter 1, a FEM model is used to study temporal variations, the spatial structure of the signature, and environmental effects.

This chapter is organized as follows: The FEM formulation of the problem is given in Sect. 3.1. The accuracy and efficiency of the model developed herein are demonstrated in Sect. 3.2 by comparing FEM results with results of other techniques. Simulation results for several land mines are also presented in Sect. 3.2. Summary remarks appear in Sect. 3.3.

3.1 FEM Formulation

The FEM is a well-known, efficient computational technique widely used for the solution of heat transfer problems [56, 57]. The FEM permits modeling of arbitrarily shapes, which allows one to incorporate a rough soil surface and realistically shaped mines and anomalies. Furthermore, different thermal parameter values may be used over each spatial element, which permits modeling of inhomogeneous soil and mines.
The Galerkin formulation used here yields the best approximation in the variational sense. A spatial FEM formulation can be combined with a time-stepping scheme to obtain a solution for time-dependent heat transfer problems. In this section, the FEM formulation of the three-dimensional heat transfer equation is given, including the spatial and temporal discretizations of the problem.

An important part of any FEM implementation is the spatial discretization of the computational domain. In what follows we denote the computational volume by $\Omega$ and its boundary by $\Gamma$. We take this volume to have a rectangular parallelepiped (box) shape. The boundary planes in $x$ and $y$ are denoted $\Gamma_x$ and $\Gamma_y$ respectively. The lower boundary plane is denoted $\Gamma_z$. The volume and boundary of element $e$ are represented by $\Omega_e$ and $\Gamma_e$, respectively. The element boundaries on the soil-air interface are represented by $\Gamma_{e,\text{air}}$.

We require the solution of Eq. (1.3) with the boundary conditions in Eq. (1.17) when driven by a periodic source. As noted previously, the temperature ranges of interest are relatively small, and over this limited range the thermal properties of soil and mine are assumed to be independent of temperature and of time. Hence, phenomena such as drying and soil water movement are neglected in the current model. We also assume that the temporal and spatial dependencies of $T(r, t)$ are separable within an element.

Over each element $\Omega^e$ we approximate the temperature distribution as

$$T^e(r, t) = \sum_{i=1}^{N_e} T_i^e(t) \phi^e_i(r), \quad (3.1)$$

where $\phi^e_i(r)$ are specified interpolation functions, $N_e$ denotes the number of nodes over the subdivided finite element, and $T_i^e(t)$ are the nodal temperatures, which comprise the unknown coefficients in our representation. Using this expression in Eq. (1.1) and
using a Galerkin formulation to enforce weak equality we obtain

$$\int_{\Omega} \phi_j^e(\mathbf{r}) \left[ C(\mathbf{r}) \frac{\partial T^e(\mathbf{r}, t)}{\partial t} - \nabla \cdot (\mathbf{K}(\mathbf{r}) \nabla T^e(\mathbf{r}, t)) \right] d\mathbf{r} = 0 \quad \forall \Omega^e \in \Omega. \quad (3.2)$$

Continuity in temperature and flux are enforced on the surface $\Gamma^e$ shared by adjacent elements.

The boundary conditions imposed on external boundaries $\Gamma$ are determined from physical considerations. The boundary condition at the soil-air interface has been defined in Eq. (1.17). For the transverse coordinates $x$ and $y$, we assume the presence of an infinite rectilinear array of mines (e.g., a mine field), one of which is centered in our computational domain. As a result of symmetry arguments, we have

$$\frac{T(\mathbf{r}, t)}{\partial x} \bigg|_{\Gamma_x} = 0 \quad \text{and} \quad \frac{T(\mathbf{r}, t)}{\partial y} \bigg|_{\Gamma_y} = 0 \quad (3.3)$$

in $x$ and $y$, respectively. One can show numerically that, although an infinite array of mines is being analyzed, for a sufficiently large computational volume, there is negligible interaction between mines, and the results are a good approximation to the response of an isolated mine. For the boundary condition on $\Gamma_z$ (at the bottom face of the computational volume) we observe that at sufficiently large depths (a few tens of centimeters) the temperature is independent of time over a diurnal cycle. The required depth $h$ is on the order of the “diurnal depth”, which is given by $D = \sqrt{\kappa L_p / \pi}$, where $L_p = 24$ hours, can be predicted from one-dimensional models (see Appendix D) of soil [58] using thermal characteristics found in the literature [59, 60]. Furthermore, at sufficiently large depths the spatial derivative of the temperature is zero. On the basis of these findings the boundary condition at $\Gamma_z$ can be taken to be either

$$T(\mathbf{r}, t) \bigg|_{\Gamma_z} = \text{const.} \quad (3.4)$$
The constant in Eq. (3.4) can be derived from a one-dimensional model, noting that at sufficiently large depths the steady state solution exists. In this work we use the boundary condition given by Eq. (3.5), since the zero flux constraint is more natural for FEM formulations. Over the course of a year ($L_p = 365$ days), variations in the temperature distribution may extend to tens of meters [23], and if necessary, this lower boundary condition can be further modified by a small constant flux representing the seasonal trend or geothermal flux [20, 23].

After substituting Eq. (3.1) in Eq. (3.2) and incorporating the boundary conditions given by Eqs. (3.3), (3.4), and (1.17) the unknown nodal coefficients $T_i(t)$ can be expressed as a matrix equation

$$\mathbf{M} \dot{T} + \mathbf{K} T = \mathbf{F}, \quad (3.6)$$

in which boldface letters represent vectors, over-lined boldface letters represent matrices and the superposed dot on $T$ denotes the derivative with respect to time. The elements of these matrices and vectors are given by

$$M_{ij} = \int_{\Omega_e} C(r) \phi_i^e(r) \phi_j^e(r) dr, \quad (3.7)$$

$$K_{ij} = \int_{\Omega_e} \mathcal{K}(r) \left[ \frac{\partial \phi_i^e(r)}{\partial x} \frac{\partial \phi_j^e(r)}{\partial x} + \frac{\partial \phi_i^e(r)}{\partial y} \frac{\partial \phi_j^e(r)}{\partial y} + \frac{\partial \phi_i^e(r)}{\partial z} \frac{\partial \phi_j^e(r)}{\partial z} \right] dr$$

$$+ \int_{\Gamma_{e,air}} \left[ 4\mathcal{E} \sigma T^3_{sky}(t) + \sigma h(t) \right] \phi_i^e(r) \phi_j^e(r) dS, \quad (3.8)$$

and

$$F_i = \int_{\Gamma_{e,air}} \left[ 4\mathcal{E} \sigma T^4_{sky}(t) + \sigma h(t) T_{air}(t) + \mathcal{F}_{sun}(t) \right] \phi_i^e(r) dS. \quad (3.9)$$
The spatial discretization was accomplished using pentahedral ("prism"-shaped) elements. Linear interpolation functions $\phi_i(x)$ were chosen.

Temporal discretization of Eq. (3.6) leads to a finite difference formulation, which is solved using the Crank-Nicholson scheme. We have

$$\left(2\bar{M} + \Delta t \bar{K}_{t+\Delta t}\right)\mathbf{T}_{t+\Delta t} = \Delta t \left(\mathbf{F}_t + \mathbf{F}_{t+\Delta t}\right) + \left(2\bar{M} - \Delta t \bar{K}_t\right)\mathbf{T}_t. \quad (3.10)$$

Eq. (3.10) is solved at each time step to evaluate the nodal temperatures $\mathbf{T}_{t+\Delta t}$. Employing a dense LU decomposition technique for this solution is inefficient, because the matrices $\bar{M}$ and $\bar{K}$ are sparse. Banded matrix storage provides a partial remedy for this problem, but limitations in node numbering usually results in inefficient memory use. An efficient sparse solver is employed in our work. The sparse matrices are stored as vectors with non-zero elements, so that inefficient memory usage is avoided. For very large problems, however, an iterative technique will be more efficient.

An initial condition at $t = 0$ is also required. Although the solution procedure described here will converge to a steady state value using $T(x, t = 0) = \text{const}$ as the initial condition, the computation time can be reduced if the average depth-dependent temperature of homogeneous soil is used as a starting value. A suitable estimate is derived from a one-dimensional steady state model for homogeneous soil

$$T(x, t = 0) = \left(\frac{1}{L_p} \int_{L_p} F_{\text{sun}}(t)dt + 4\mathcal{E}\sigma T_{\text{sky}}^4 + hT_{\text{air}}\right)/\left(4\mathcal{E}\sigma T_{\text{sky}}^3 + h\right) \quad (3.11)$$

in which overbar indicates a time averaged value and $L_p$ denotes the 24 hour period. Equation (3.11) is derived by using a Fourier series expansion of the temperature with the boundary condition given by Eq. (1.17) and solving only for the dc term of the series. A similar expression neglecting the convection term was derived by Watson [20].
As mentioned above, the FEM is a widely used computational technique and a number of public domain FEM solvers exist. The code reported here is based on building blocks described by Akin [56] and modified to include time variation, sparse matrices, pentahedral elements, and problem-specific code for the boundary conditions, the matrix elements, and the source vector elements. In addition, a separate mesh generator was created using MATLAB for buried mine geometries.

3.2 Results

In this section numerical simulation results are presented to demonstrate the temporal and spatial dependence of IR mine signatures. The results are organized in three groups. In the first group the temperature of homogeneous soil is computed using three different techniques, namely, an analytical model, an integral-equation based body-of-revolution model, and the above-described FEM model. This 1-D solution provides a check on the temporal behavior. The analytical solution appears in Appendix D. The integral equation solution was described in Chapter 2. In the second group of results, the three-dimensional modeling capabilities of the FEM and integral-equation solutions are compared for a simple homogeneous target. In the third group the temporal and spatial temperature distributions of several land mines are studied and suggestions are presented for mine detection.

3.2.1 Thermal modeling of homogeneous soil

We used an analytical model developed by Watson [20] as a benchmark solution to evaluate the integral-equation and FEM solutions for a homogeneous half-space comprising damp soil. The predicted soil surface temperature is given in Fig. 3.1, which shows good agreement among the three solutions. The thermal properties of
the soil and of other materials used in the mine models are given in Table 3.2.1 [59, 60]. Here and throughout this work we use the model parameters \( S_0 = 1353 \, [W/m^2] \), \( \lambda = 35^\circ \), \( \delta = 0^\circ \), \( A = 0.3 \), \( Cl = 0.2 \), \( d = 0^\circ \), \( \psi = 0^\circ \), \( \epsilon = 1 \), \( t_R = 6:00 \, [h] \), \( t_S = 18:00 \, [h] \), \( w = 0.8 \, [mmHg] \), \( T_{sky} = 260 \, K \), and \( T_{air} = 289 \, K \). These values are appropriate for an experiment performed on the vernal equinox, on which the celestial equator intersects the ecliptic, resulting in zero declination of the sun. The duration of day and night are equal, which is used to select the sunrise and the sunset times. An average air temperature of 289 K is assumed, which is a typical value for Columbus, OH in spring. Figure 3.1 suggests that soil surface temperature values are hotter during the day. Minimum and maximum surface temperatures are observed around 06:00 and 14:00, respectively. In addition, there is a lag between the incident radiation and the temperature of the soil.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \kappa , [m^2s^{-1}] )</th>
<th>( \mathcal{K} , [Wm^{-1}K^{-1}] )</th>
<th>( D , [m] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil</td>
<td>( 5.0 \times 10^{-7} )</td>
<td>2.6</td>
<td>0.117</td>
</tr>
<tr>
<td>TNT</td>
<td>( 9.25 \times 10^{-8} )</td>
<td>0.234</td>
<td>0.0504</td>
</tr>
<tr>
<td>Air</td>
<td>0.0225</td>
<td>26.3</td>
<td>24.87</td>
</tr>
<tr>
<td>Plastic</td>
<td>( 1.4 \times 10^{-7} )</td>
<td>0.24</td>
<td>0.0621</td>
</tr>
</tbody>
</table>

Table 3.1: Thermal diffusivity (\( \kappa \)), conductivity (\( \mathcal{K} \)) and diurnal depth (\( D \)) for materials used in modeling mines
3.2.2 IR signature of a cylindrically shaped homogeneous mine

In this section a circular anti-tank mine simulant buried 4.5 cm under a perfectly smooth soil surface is studied. The simulant mine has a diameter of 20 cm and a height of 7.5 cm, and it is filled with trinitrotoluene (TNT), an explosive used in most land mines. The thermal properties of the materials used in the mine models are given in Table 3.2.1. It is noteworthy that TNT is a better thermal insulator than soil, and as we describe below, this fact has a significant effect on the signature. Figure 3.2 shows the surface temperature distribution at the center of the mine as a function of time. The results show good agreement between the codes. The difference between the surface temperature distributions are less than 0.5 K. A possible cause
of the difference between these solutions is the different spatial discretizations used in them. In the second set of results (Fig. 3.3) we investigated spatial variations in the surface temperature distribution. For this purpose we selected a cut through the center of the mine, and we present the temperature distribution as a function of radial distance from the center of the mine at different times. Figure 3.3 (a) and (b) illustrate the results at times when the mine signatures have maximum negative and positive contrast, respectively. Again, the FEM and integral equation models show good agreement. The integral equation solution took about 2 hours on a 300 MHz Pentium II machine and requires less than 20 MB of memory. Most of this time is used to fill and factor the $\mathbf{Z}$ matrix. The FEM solution took about 1 hour 40 minutes in the same machine and requires around 250 MB of memory. No significant attempts have been made to optimize either code.

![Figure 3.2: The surface temperature distribution over the center of a cylindrical mine using the integral equation solution and a finite element solution.](image-url)
3.2.3 IR signatures of anti-tank and anti-personal land mines

In this section we present the temperature distributions of several land mines using the FEM model. Numerical results are presented for a rectangular anti-personal (AP) mine, a rectangular anti-tank (AT) mine, and a circular AT mine, all of which are buried under a smooth soil surface. The temporal evolution of the temperature distribution is presented both at the surface and as a function of depth.

First, a square AT mine buried 3.74 cm under a smooth soil surface was considered. The computational volume, which had dimensions $1.4798 \times 1.4798 \times 0.3572$ m, was discretized by pentahedral elements formed on a rectangular grid of spacing 3.02 cm, 3.02 cm, and 1.88 cm in the $x$, $y$, and $z$ directions, respectively. The mine dimensions are similar to those of an M-19 mine, which was modeled as a rectangular parallelepiped with edge sizes $33.2 \times 33.2 \times 9.4$ cm. The mine was assumed to be composed of homogeneous TNT as shown in Fig. 3.4 (a). The overall computational domain was discretized using 91238 finite elements. This discretization results in 50000 spatial nodes, which also determines the size of the resulting matrix equation. The memory requirement of the FEM code for this discretization is 249 MB. A time increment of 360 seconds was chosen and the FEM solution was observed to reach steady after only two simulated diurnal cycles. The Crank-Nicholson scheme is unconditionally stable, which permits substantial freedom in choosing the time increment. The time step size was selected to give good resolution over a 24 hour period. The simulation takes about 4.5 hours on a 300 MHz Pentium II machine.

Figure 3.5 shows the surface temperature distribution over the rectangular AT mine. The results are presented as a sequence of images evaluated at three hour intervals. 
Figure 3.3: The spatial dependence of the surface temperature distribution: (a) when the mine signature has a maximum negative contrast, (b) when the mine signature has a maximum positive contrast.
Figure 3.4: Models for the surrogate mines used in the simulations (a) Rectangular anti-tank mine. (b) Rectangular anti-personal mine. (c) Cylindrical anti-tank mine.
time increments starting from sunrise. As is commonly observed in experimental im-
agery, contrast changes occur twice daily at the thermal "cross-over" times, and these
events appear in our simulations. The results suggest that the physical temperature
differences during the day peak at roughly 4.6 K. The temperature difference has a
maximum at the center of the mine and diminishes as we move away from the center.
The temperature distribution as a function of depth is presented in Fig. 3.6. The
location of the mine is indicated with a rectangle. The temperature distribution is
again given every three hours starting at sunrise. The figures present the evolution
of heat flow into the ground and the influence of the mine on this heat flow. The
surface temperature over the mine is cooler at dawn, and it warms as time proceeds.
As a result of its insulating properties (relative to soil) the mine tends to block the
flow of heat into the soil, causing the overlying soil to become hotter during the day.
During the night, the mine blocks the upward flow of energy in the soil, permitting
the layer of soil above the mine to cool more rapidly.

In the second set of simulations, a rectangular AP mine buried at 1 cm depth is
investigated. AP mines are typically placed close to the surface to permit triggering by
the small weight of a human. The size of the AP mine was chosen to be comparable to
that of an M-14 mine (a circular mine). The number of spatial elements used was the
same as the foregoing AT mine simulation, but the element dimensions were reduced
to 0.5 × 0.5 × 0.5 cm³. The AP mine is a smaller mine and we used a more detailed
model for it, as shown in Fig. 3.4 (b). The outer case of the AP mine is modeled
by a 0.5 cm thick plastic. The interior is modeled by TNT and air. This air is
intended to represent the gap over the explosive occupied (in part) by the triggering
mechanism. Figure 3.7 shows the surface temperature distribution over this mine,
Figure 3.5: Simulation of the soil surface temperature difference over an M-19 mine buried 3.74 cm under a smooth soil surface. The temperature distribution is evaluated (a) at dawn, (b) 3 hours after sunrise, (c) at noon, (d) 3 hours after noon, (e) at sunset, and (f) 3 hours after sunset.
Figure 3.6: Simulation of the temperature distribution at depth taken through the middle of the M-19 mine buried 3.74 cm under a smooth soil surface. The temperature distribution is evaluated (a) at dawn, (b) 3 hours after sunrise, (c) at noon, (d) 3 hours after noon, (e) at sunset, and (f) 3 hours after sunset.
which suggests that temperature differences up to roughly 2 K will be observed. The temperature distribution as a function of depth is presented in Fig. 3.8 at three hour time increments. The AP mine signature demonstrates characteristics similar to those of the AT mine. Our previous observations regarding thermal cross-over times and obstruction of heat flow by the mine are also applicable here. Two additional observations can be made regarding AP mines. First, although the amplitude of the AP mine signature is smaller than that of an AT mine, the AP mine still gives a detectable signature. Second, the air gap over the explosive has a significant effect on the heat flow as shown in Fig. 3.8.

We next consider the signature of a simulant circular AT mine [61] buried 6.64 cm. The simulant mine has a diameter of 25 cm and a height of 8.33 cm. The mine is assumed to be composed of TNT as shown in Fig. 3.4 (c). The computational volume has the dimensions 0.9984 \times 0.9984 \times 0.3154 m^3 and was discretized with elements of size 2.08 \times 2.08 \times 1.66 cm^3 in the x, y, and z directions, respectively. This discretization results in 87552 finite elements and 48020 nodes. The surface temperature distribution and the temperature distribution as a function of depth are given in Figs. 3.9 and 3.10, respectively. Figure 3.9 suggests a temperature difference of at most 1.73 K during the day.

In the previous three simulations, the mine has a rectangular shape, whereas in this simulation the mine is circular. We observe, however, that surface signatures of rectangular mines are not rectangular, but rather have somewhat rounded shapes. In Fig. 3.11, we quantify this blurring of the mine shape by showing isothermal contours for the circular and rectangular AT mines at several times. The results

\[\text{The noise-equivalent temperature difference in modern IR cameras is less than 0.1 K}\]
Figure 3.7: Simulation of the soil surface temperature difference over an M-14 mine buried 1 cm under a smooth soil surface. The temperature distribution is evaluated (a) at dawn, (b) 3 hours after sunrise, (c) at noon, (d) 3 hours after noon, (e) at sunset, and (f) 3 hours after sunset.
Figure 3.8: Simulation of the temperature distribution at depth taken through the middle of the M-14 mine buried 1 cm under a smooth soil surface. The presence of the air gap is evidenced by the different temperatures in the top and bottom halves of the mine. The temperature distribution is evaluated (a) at dawn, (b) 3 hours after sunrise, (c) at noon, (d) 3 hours after noon, (e) at sunset, and (f) 3 hours after sunset.
Figure 3.9: Simulation of the soil surface temperature difference over a cylindrical anti-tank mine buried 6.66 cm under a smooth soil surface. The temperature distribution is evaluated (a) at dawn, (b) 3 hours after sunrise, (c) at noon, (d) 3 hours after noon, (e) at sunset, and (f) 3 hours after sunset.
Figure 3.10: Simulation of the temperature distribution at depth taken through the middle of the cylindrical anti-tank mine surrogate buried 6.66 cm under a smooth soil surface. The temperature distribution is evaluated (a) at dawn, (b) 3 hours after sunrise, (c) at noon, (d) 3 hours after noon, (e) at sunset, and (f) 3 hours after sunset.
in Fig. 3.11 are for 16:00, which corresponds to the median between two thermal cross-over times. Figures 3.11 (a) and (b) present isothermal contours on the top surface of the buried mines, where the isotherms have the shape as the mine. The small distortion in Fig. 3.11 (a) from a perfect cylinder is due to approximation of a cylinder by pentahedral elements. Figures 3.11 (c) and (d) present the isothermal contours at a depth of half the mine burial depth. As we can see, isotherms for the cylindrical mine preserve their circular shape, but isotherms for the rectangular mine are rounded. In Figs. 3.11 (e) and (f) the isotherms are plotted on the soil surface, where additional rounding of the rectangular mine signature is observed. The soil overlying the land mine has the effect of a spatial low-pass filter on the IR image. The high spatial frequency components attenuate faster with increasing depth than the low spatial frequency components.

Finally, we investigate the dependence of the signatures on burial depth and time. For this purpose, we utilized results from the cylindrical AT mine shown in Fig. 3.4 (c). In the simulations, the mine is assumed to be buried at depths of 1.66 cm, 3.32 cm, 4.98 cm, and 6.64 cm. The signatures are plotted as a function of distance from the center. Figure 3.12 (a) shows signatures for mines buried at different depths at 15:00. The signatures have a Gaussian-like spatial dependence, which is little affected by different burial depths, although the amplitudes of the mine signatures are smaller for deeper mines. Figure 3.12 (b) shows the signatures at 21:00, and the results are in accordance with our foregoing observations. For the mine buried at 6.66 cm the mine signature is almost completely lost, but for other burial depths the signatures are still detectable. Figures 3.12 (c) and (d) illustrates that (except for a time dependent amplitude) the spatial dependence of the signature is largely independent of time.
Figure 3.11: Isothermal contours for cylindrical and rectangular anti-tank mines at 16:00 plotted at different depths (a) On the cylindrical mine surface. (b) On the rectangular mine surface. (c) At a depth of half of the mine burial depth for cylindrical mine. (d) At a depth of half of the mine burial depth for rectangular mine. (e) On the soil surface over the cylindrical mine. (f) On the soil surface over the rectangular mine.
Figure 3.12: The mine signatures as a function of distance from the center (a) Dependence of mine burial depth at 15:00. (b) Dependence of mine burial depth at 21:00. (c) Dependence of time of day for a mine buried at 3.32 cm. (d) Dependence of time of day for a mine buried at 6.64 cm.
3.3 Summary

A FEM-based three-dimensional thermal model has been developed to investigate the IR signatures of buried land mines. The behavior of this FEM model was compared with other formulations, including an analytical model of 1-D geometries and an integral equation solution (presented in Chapter 2). Good agreement is observed in these comparisons. Numerical simulations using different land mines and a variety of depths were presented.

Our work with this model supports the following observations:

- TNT, the explosive material used in most land mines, is a better insulator than soil. It obstructs the heat flow into and out of the soil. As a result, soil above the mine is hotter than the background soil during the day, but cooler during the night.

- The simulated peak contrast is consistent with experimental observations. For all observation times the mine-generated contrast has a peak at the center of the mine and it diminishes as we move away from the mine center.

- Two thermal “cross-over” times occur during a day, at which times contrast changes occur in the imagery. The mine signatures are completely lost at thermal cross-over times, and data acquisition should be avoided near those times, but, unfortunately, the time of these events depends on mine composition and burial depth.

- The spatial dependence of a mine signature is (with the exception of an amplitude scaling) largely independent of image acquisition time and burial depth.
• The soil overlying the land mine has the effect of a spatial low-pass filter on the IR image. Signatures of square mines buried several inches will appear similar to those of circular land mines.
CHAPTER 4

DETAILED MODELING OF THERMAL SIGNATURES

In this chapter we present a more detailed approach to FEM thermal modeling using the TM-62 AT mine as an example model. We employ automatic mesh generation and adaptive mesh refinement algorithms to discretize the computational volume in an effort to obtain more accurate results and to study the effects of small but significant structural features including the case of the mine.

This chapter is organized as follows: In Sect. 4.1 a detailed model for the TM-62 mine is presented. The components of that mine and its working principles are given in that section. Also, the thermal implications of a more complex mine structure are discussed therein. In Sect. 4.2 automatic mesh generation and adaptive mesh refinement algorithms are discussed. A decision metric, which determines if a finite element requires further subdivision during adaptive mesh refinement, is introduced in that section. Thermal modeling results for metal-cased TM-62M and plastic-cased TM-62P AT mines are presented in Sect. 4.3. The significance of the adaptive mesh refinement and the convergence of the results are discussed therein. Summary remarks appear in Sect. 4.4.
4.1 A detailed model of TM-62 anti-tank mines

Detailed information about 700 types of mines currently being used in different parts of the world have been compiled by the US Department of Defense [62]. In that archive, the mines are classified with respect to their target, case, size, shape, effect, and countries of manufacture and use.

Two members of the TM-62 AT mine family are investigated in this chapter. In the aforementioned land mine archive, it is reported that the TM-62 series of conventional AT blast mines are manufactured in the Russian Federation and its associated states. We focus on the metal-case (TM-62M) and the plastic-case (TM-62P) versions. The TM-62M and TM-62P are cylindrical AT mines. A TM-62M AT land mine is shown in Fig. 4.1. The TM-62M model that we developed is shown in Fig. 4.2. That model is a body of revolution.

The principal components of the TM-62 mine are the main explosive charge and ignition system. The main charge in an AT mine is significantly larger than that of an anti-personnel mine, because the main purpose of an AT mine is to damage vehicles. In a TM-62 mine the main charge is a large annulus of explosive initiated by the ignition system (see Fig. 4.1), which is located in the center of the mine. The ignition system is composed of secondary (initiating) explosives, a timing system, and a fuze assembly. The operation of the mine can be summarized as follows: When pressure is applied to the top of the mine, the timing system is activated. Timing is used to delay detonation until the vehicle's center travels to the mine's location. Next, the fuze assembly triggers the detonator, which is the first member of the secondary explosive train. The detonator is a very sensitive explosive, which can be initiated easily. The detonator ignites a larger explosive body called the booster. The
Trigger and timing assembly

Main charge

Detonator charge

Booster charge

Figure 4.1: (a) Photograph of a TM-62M AT land mine. (b) Construction of a TM-62M mine.
Figure 4.2: (a) The model developed for TM-62M mine. (b) The cross section of the TM-62 model.
explosive used in the booster is less sensitive than the detonator, and it initiates the main charge.

Relatively small structural features in a mine can have significant effects on thermal behavior and, as a result, more detailed modeling of the mine is required. Some of the important issues are as follows:

The thin outer cover of a land mine can be either plastic or metal. Plastics are typically thermal insulators, which block heat transfer, while metals are good thermal conductors, which facilitate heat transfer. An accurate model of the case is required to account for its role in the thermal signature.

As previously mentioned, the ignition system of a mine is complex, being composed of several metal and plastic parts with air gaps and secondary explosives. Prior experimental work suggests that small internal air gaps in the ignition system have a significant effect on the thermal signature. Such gaps are common in pressure activated mines, which require a void through which an internal structure is displaced. Modeling of these voids requires a greater level of detail.

4.2 Mesh generation

We noted above that a detailed model of the mine geometry and composition is necessary for a complete thermal analysis. For a more precise and robust discretization of the computational volume and for a more accurate solution of this detailed geometry, two major improvements are essential in the FEM solver, namely, automatic mesh generation and adaptive mesh refinement.

In a recent study, Lee [63] discussed advances in several aspects of the FEM for solving the three-dimensional time-harmonic Maxwell equations. He sketched the
research and development activities in six major technical areas of commercial field simulation software for analysing and designing RF/microwave devices. These fields are: automatic mesh generation, $H(\text{curl})$ conforming vector finite elements, fast and efficient matrix solution techniques, accurate mesh truncation methods, a posteriori error estimate and adaptive mesh refinement, and fast frequency sweep algorithms. Even though that work deals with applications in microwave engineering, the issues regarding automatic mesh generation and adaptive mesh refinement are immediately applicable to the solution of the heat transfer equation.

4.2.1 Automatic mesh generation

Discretization of the computational volume is an essential part of an FEM solution, and mesh generation has a significant effect on the accuracy and convergence of the solution, especially for complex geometries that involve both small and large volume features.

In our previous thermal modeling studies, we used meshes composed of pentahedral elements defined over a structured grid. This discretization permitted us to model arbitrarily shaped soil-air interfaces and canonical mine geometries including rectangular boxes and cylinders. For more complex geometries, tetrahedral elements have advantages, since they can approximate any arbitrarily shaped volume. Tetrahedral elements are also superior to the pentahedral elements because they are simplices in three dimensions. Using tetrahedral elements also allows us to represent a rough soil-air interface as a surface triangulations. In addition, Lee [63] reported that unstructured grids offer many advantages over structured grids in the discretization of a computational volume. To obtain these benefits, the previously described,
FEM-based, thermal model was modified to employ tetrahedral elements over a unstructured grid formed by automatic mesh generation.

Delaunay tessellation is an efficient and robust way to discretize arbitrarily shaped volumes using tetrahedral elements. Among the available Delaunay tessellation algorithms, Lee reported that the advancing front technique [64] offers some advantages over the Bower-Watson algorithm [65]. The advancing front technique starts from the boundaries of the computational domain and advances a front throughout the domain by constructing one tetrahedron at a time. Unfortunately, bad elements can be formed during automatic Delaunay tessellation. To overcome this problem, after mesh generation is completed, two algorithms, namely, smoothing and swapping can be employed to improve the mesh quality. In this study, an automatic mesh generation program is employed, which uses the advancing front technique in conjunction with smoothing and swapping operations.

4.2.2 Adaptive mesh refinement

The nature of a specific mesh or of the underlying the heat transfer solution may require a more accurate estimate of temperature in specific regions. This need can be addressed by using a refined (more detailed) mesh in those regions. Some reasons for mesh refinement include the following:

- There may be significant differences in the sizes of adjacent tetrahedra. The thermal energy stored in a large tetrahedron may be much larger than that in a smaller one. Further discretization of large elements may be required.

- The thermal properties of soil and mine constituents are different. For example the conductivity of a metal is much larger than the other materials used in the
mine. These differences in thermal properties may result in significant changes in temperature over short distances.

- Tetrahedra around temperature anomalies may require refinement to obtain an accurate representation of the temperature distribution. For the buried mine problem, large thermal gradients exist at the soil-air interface, which require finer discretization.

To identify mesh regions that require refinement, it is necessary to have a metric that takes all of the aforementioned factors into account. The metric used here is the maximum instantaneous deviation of the stored energy in a tetrahedron from the average stored energy throughout a day. The average value of the temperature distribution over the computational volume throughout a day is given as

$$T_0 = \frac{\int_{L_p} dt \int_{V} dV T(r, t)}{L_p \times V}$$

(4.1)

where $T(r, t)$ [K] is the temperature at point $r$ and time $t$. $L_p = 24$ [h] is the diurnal period and $V$ [m$^3$] is the computational volume of interest. Note that the results from only one diurnal cycle are used. Those results are obtained only after running the model to steady state. Let $\mathbf{r}_{c,n}$ represent the centroid of the $n$th tetrahedron. The deviation from the average temperature $T_0$ is given as

$$\Delta T_n(t) = T(\mathbf{r}_{c,n}, t) - T_0$$

(4.2)

The instantaneous deviation in stored energy is given as

$$\Delta E_n(t) = C(\mathbf{r}_{c,n}) \Delta T_n(t) V_n$$

(4.3)

where $C(\mathbf{r}_{c,n})$ [J m$^{-3}$ K$^{-1}$] is the volumetric heat capacity for the $n$th tetrahedron, and $V_n$ [m$^3$] is its volume. Using Eq. (4.3), the decision metric for the $n$th tetrahedron
can be defined as

$$\mathcal{M}_n = \max_{t \in (0, T_p)} \Delta E_n(t)$$  \hspace{1cm} (4.4)$$

This metric is used to decide whether tetrahedron $n$ requires further refinement. The automatic mesh generation and adaptive mesh refinement programs are developed by Prof. Jin-Fa Lee [65].

4.3 Results

In this section we present thermal modeling results for TM-62M and TM-62P AT mines over a diurnal cycle using the mine geometry and composition presented in Sect. 4.1. The process begins with a coarse mesh of the mine and soil obtained from the automatic mesh generation program. For a body of revolution, this program requires the description of the lines forming the boundaries of the regions in the $\hat{\rho}$-$\hat{z}$ cross section. The initial mesh involves 40989 tetrahedral elements. For this coarse discretization, we solved the heat transfer equation. Using the solution obtained with the coarse mesh, the decision metric for each tetrahedral element was obtained from Eq. (4.4), and the mesh was then refined at the appropriate tetrahedra. This refined mesh was used with the FEM based thermal model to obtain a new temperature distribution, and the solve-refine process was repeated. The number of tetrahedra and nodal points used in each iteration for TM-62M and TM-62P mines are given in Tables 4.1 and 4.2, respectively.

Figure 4.3 shows the final iteration of the surface temperature distribution over a TM-62M mine buried 5 cm under a smooth soil surface. Bilinear interpolation was used to transform the unstructured FEM grid to a uniform grid of image pixels. The results are presented as a sequence of images evaluated at three hour time increments.
starting from sunrise. The thermal cross-over times for the TM-62M mine occur around 12:00 and 21:00. At those times the image contrast is low, which causes the results to be noise-like as a result of the quasi-random mesh refinement process. The maximum negative contrast (-0.54 K) occurs around 08:00, and the maximum positive contrast (0.58 K) is observed around 17:00.

To quantify the effect of adaptive mesh refinement, we present in Fig. 4.4 the surface temperature difference between the first and last iterations. We observe a
significant decrease (about 0.5 K) in the temperature over the mine due to refinement. A temperature decrease is expected because the metal-case facilitates heat transfer, which causes the thermal energy to penetrate the mine thereby reducing its effect.

To understand the effect of metal versus plastic mine cases, we computed the surface temperature distribution over a TM-62P mine buried 5 cm under a smooth surface. The numerical simulations suggest about a one hour shift in the thermal cross-over times. For the TM-62P AT mine, thermal cross-overs occur around 11:00 and 20:00. A similar shift is observed in the times of maximum contrast. For the TM-62P, the maximum negative contrast (-1.03 K) occurs around 07:00, and the maximum positive contrast (1.53 K) is observed around 16:00. These results suggest that mines with plastic cases produce stronger thermal signatures than mines with metal cases. A similar result was observed in Chapter 1. Further discussion of this difference appears later in this section.

To show the effect of adaptive mesh refinement on the TM-62P signature, the temperature difference between the first and last iterations are plotted in Figure 4.6. We observe that the differences are not all of the same sign, as they were for the metal-cased mine. This is expected because the plastic case blocks the flow of heat flux deeper into the mine, which tends to enhance contrast (producing both positive and negative temperature shifts).

Next we focus on the convergence of the temperature results. For this purpose, we define the error for the nth iteration as

$$
\epsilon = \frac{1}{L_p} \int_{L_p} dt \left[ \frac{1}{S_a} \int_{S_a} dx \left( T_n(r, t) - T_{n-1}(r, t) \right) \right]
$$

(4.5)

where $S_a$ is the surface defined by the soil-air interface, and $T_n(r, t)$ is the temperature distribution at point $r$. Equation (4.5) is the time and space average absolute error
over the scene. Errors for TM-62M and TM-62P mines are plotted in Figs. 4.7 (a) and (b), respectively. For both cases, the results show rapid initial convergence, with a better final result for the TM-62P. This is expected, because the metal case permits more thermal flux to reach to deeper elements, which requires finer discretization over a larger volume. Conversely, the plastic case offers more uniform thermal properties, which can be modeled accurately with fewer elements. For the metal-cased mine the error levels are higher than the plastic-cased mine. However, the computational resources didn't permit the simulations with a larger number of tetrahedra.

In the next set of results we compare the thermal signatures of mines with metal and plastic cases. For this purpose, we plotted in Fig. 4.8 the surface temperature along a line passing through the origin. We present results at the following times: 07:00, 08:00, 16:00, and 17:00. These times correspond to signature contrast maxima for TM-62M and TM-62P mines. As expected, the signature of the plastic-cased mine is much stronger than that of the metal-cased mine.

4.4 Summary

In this chapter we described techniques for accurate thermal modeling of mines with small but significant structures. A detailed representation of the TM-62 AT mine geometry and composition was developed. This model was discretized using an automatic mesh generation program. The mesh was then further refined using an adaptive mesh refinement technique. Numerical simulations were done for mines buried at 5 cm, which is a typical depth for the TM-62 AT mine family. We observed that the signatures of plastic-cased mines show higher contrast than those of metal-cased mines. Furthermore, we observed a difference in the thermal cross-over times
Figure 4.3: Simulation of the soil surface temperature over an TM-62M mine buried 5 cm under a smooth soil surface. The temperature distribution is evaluated (a) 6:00, (b) 9:00, (b) 12:00, (b) 15:00, (b) 18:00, and (f) 21:00.
Figure 4.4: Soil surface temperature difference between first and last iterations for the TM-62M mine (a) 6:00, (b) 9:00, (b) 12:00, (b) 15:00, (b) 18:00, and (f) 21:00.
Figure 4.5: Simulation of the soil surface temperature over an TM-62P mine buried 5 cm under a smooth soil surface. The temperature distribution is evaluated (a) 6:00, (b) 9:00, (b) 12:00, (b) 15:00, (b) 18:00, and (f) 21:00.
Figure 4.6: Soil surface temperature difference between first and last iterations for the TM-62P mine (a) 6:00, (b) 9:00, (b) 12:00, (b) 15:00, (b) 18:00, and (f) 21:00.
Figure 4.7: Convergence behavior of (a) metallic-cased mines and (b) plastic-cased mines.
Figure 4.8: Comparison of the signatures of the TM-62M and TM-62P mines (a) 07:00 (maximum negative contrast for the plastic mine), (b) 08:00 (maximum negative contrast for the metallic mine), (c) 16:00 (maximum positive contrast for the plastic mine), (d) 17:00 (maximum positive contrast for the metallic mine).
for these mines. The cross-over times for the TM-62P occurs around 11:00 and 20:00, while for the TM-62M mine these times are 12:00 and 21:00. The highest contrast for the TM-62P and TM-62M mine signatures were observed around 16:00 and 17:00, respectively.
CHAPTER 5

THREE-DIMENSIONAL RADIOMETRIC MODEL FOR LAND MINES BURIED UNDER AN ARBITRARILY SHAPED ROUGH SURFACE

In addition to the mine's thermal signature, an IR image of the surface will also contain undesirable reflected light from natural sources. The latter comprises a form of clutter, which has a detrimental effect on mine detection performance. Natural sources are also wavelength dependent, which requires attention to the spectral content of those reflected components. Therefore, to better understand the IR signatures of buried land mines and clutter, a spectral radiometric model is required. There has been little work in radiometric modeling of mine signatures. Flynn et al. [66] developed a surface mine simulation tool for the UV-VNIR regime, but no prior work on radiometric modeling for buried mines has appeared.

In this chapter we describe a radiometric model capable of predicting the IR signatures of buried land mines. The model addresses both the spatial and spectral characteristics of the environment. This chapter is organized as follows: In Sect. 5.1 we summarize some enhancements to our FEM thermal model which permit more realistic modeling of the natural environment. The radiometric model is presented in Sect. 5.2. Sunlight and skylight models are given in Sect. 5.2.1. The simulation of
IR imagery is discussed in Sect. 5.2.2. Numerical results for a cylindrical anti-tank mine are presented in Sect. 5.3. Examples of the temporal evolution of IR imagery for smooth and rough soil surfaces are given therein. Summary remarks appear in Sect. 5.4.

5.1 Thermal model

As noted in previous chapters, a buried mine affects the flow of heat flux into and out of the soil, since its thermal characteristics are different than those of soil. This alteration of the heat flow results in a disturbance of the surface temperature, which is the primary source of the IR signatures of buried land mines\(^4\). As improvements to the thermal model, in this study we suggest two major modifications: (1) a rough soil-air boundary and (2) a more sophisticated model of the solar insolation.

Prior experimental work suggests that a rough soil-air interface has a strong influence on the clutter signature of soil. This roughness complicates the thermal modeling as a result of spatial variations in the absorbed downwelling solar radiance (via the variable local surface normal). Surface roughness also affects the calculation of the reflected sunlight and skylight, which arises in the radiometric model discussed next. A rough surface is easily incorporated in our thermal model by varying the tilt in the triangular surface facets. A sample discretization of a volume is given in Fig. 5.1.

The second modification to our previous model is the incorporation of a more realistic solar insolation function. In previous chapters we used a relatively simple

\(^4\)Other phenomena that may contribute to buried mine signatures include the difference in soil moisture content (and the concomitant change in soil emissivity and thermal properties) due to the moisture barrier formed by the mine. This barrier also affects the vitality of any overlying vegetation.
Figure 5.1: Sample spatial discretization of the computational domain. The volume is subdivided into pentahedral elements resulting in a triangular mesh for the soil surface, which is emphasized with thicker lines.
analytic function that addresses the time of day, cloud extinction, atmospheric absorption, and soil albedo. In this chapter the more sophisticated capability provided by MODTRAN [29] has been incorporated. MODTRAN provides several benefits including more accurate calculation of solar position for a given latitude and longitude and atmospheric modeling that includes the effects of absorption bands and weather conditions.

5.2 Radiometric model

As noted above, the IR signature of a buried mine includes various soil reflected components in addition to the thermal components, and the inclusion of those components is the principal contribution of this chapter. In Sect. 5.2.1 we review our models for the additional components. IR image formation is discussed in Sect. 5.2.2.

5.2.1 Sunlight and skylight models

Sunlight and skylight contribute to both the thermal and radiometric models. The short wavelength solar heating of the soil surface comprises the dominant source for the thermal model, and long wavelength reflected radiance is received by the IR sensor.

The radiance $L_{SUN}$ can be modeled with reasonable accuracy by a blackbody radiator. The solar radiance incident on the earth's atmosphere is essentially collimated and can be written in the form

$$L_{\text{sun}}(\theta, \phi, r, \lambda) = \pi L_{BB}(T, \lambda) \delta(\sin \theta - \sin \theta_0) \delta(\phi - \phi_0) / \sin \theta_0$$  \hspace{1cm} (5.1)

where $L_{BB}(T, \lambda)$ is the radiance of a blackbody at temperature $T$ and $(\theta_0, \phi_0)$ are the solar zenith and azimuth angles. Neckel and Labs [67] show that a blackbody
source at $\approx 5800$ [K] represents a good numerical fit to exoatmospheric measured radiance data in the visible and near IR bands. Thekaekara and Drummond [68] summarized data regarding components of the total radiation and their spectral distribution, and they proposed standard values for them. Measured solar radiation at different sites and models for meteorological and climatic factors are presented in a summary by Dogniaux[69] prepared for the European Community Programme on Solar Energy. The measurement techniques and instruments are described in Coulson [70]. Figure 5.2 (a) illustrates the predicted solar radiance at the outer surface of the atmosphere using a blackbody emitter at 5800 [K].

The atmosphere attenuates both the downwelling solar radiance and skylight and the reflected and emitted radiance. In many demining applications the distance between the IR camera and the soil surface is small and, hence, the attenuation of the down-welling solar radiance is of greatest concern. For airborne mine detection, attenuation of the emitted and reflected signals must also be considered.

Sophisticated modeling tools have been developed to deal with the complexities of the solar radiance and atmospheric transmission, absorption, and scattering phenomena. These models predict transmittance and radiance for sensor systems under varying atmospheric conditions. Among these computer codes, LOWTRAN (Low spectral resolution transmission), MODTRAN (Moderate spectral resolution transmission), and FASCODE (Fast atmospheric signature code) are commonly used. LOWTRAN is a one-parameter, band-model code that predicts atmospheric absorption and scattering for systems with low spectral resolution. It is computationally efficient, and it has an atmospheric database that can model six reference atmospheres with various
constituents. MODTRAN is a two-parameter, band-model code with a higher resolution than LOWTRAN (2 cm$^{-1}$ for MODTRAN versus 20 cm$^{-1}$ for LOWTRAN). FASCODE is a high-resolution code, which is usually reserved for studies involving very narrow bandwidth.

To model the atmosphere we have selected the 1976 US standard atmosphere. Temperature, pressure and water-vapor altitude profiles, and altitude profiles of relevant gases (ozone, methane, nitrous oxide, carbon monoxide, and others) were set to values of the 1976 US standard atmosphere, but we set the CO$_2$ mixing ratio to 365 ppmv, which is consistent with more recent data. The weather is assumed to be clear and sunny. To model boundary-layer aerosols, rural extinction was selected with a visibility to 23 km. (Visibility is related to aerosol extinction at 550 μm.) Figure 5.2 presents the solar spectral radiance at Columbus, OH on April 3. Figures 5.2 (b)-(f) show the solar radiance predictions with atmospheric effects at 10AM, 12PM, 2PM, 4PM, and 6PM, respectively.

The quantity $L_{SKY}$ depends on the composition of the local atmosphere and is more difficult to measure. It comprises wide-angle Rayleigh scattering by molecular constituents (very weak at IR wavelengths), small-angle Mie scattering by aerosols, and thermal radiation from the warm atmosphere. Although there has been extensive work in developing models for $L_{SKY}$, local changes in atmospheric particulates and water vapor content can greatly affect model accuracy. In addition, a number of molecular species (H$_2$O, CO$_2$, CO, N$_2$O, O$_3$, and CH$_4$) have absorption bands in the infrared, which affect the incident solar radiation [58, §3.5.2]. The thermal contribution is most important at long wavelengths, and in this work we approximated $L_{SKY}$ by a blackbody radiator at the local air temperature.
Figure 5.2: Predicted spectral irradiance at Columbus, OH on April 3. (a) Solar spectral irradiance with no atmospheric effects. (b) Prediction by MODTRAN at 10 AM. (c) 12 PM (d) 2 PM (d) 4 PM (d) 6 PM
5.2.2 IR image formation

The power incident on the detector can be found by Eq. (1.22). In this equation the detector area is typically small, and, the flux incident on each detector pixel is essentially constant. As a result, integration over the detector area is reduced to the product of the detector area and the flux falling on it.

The integration over the spectral band of interest $[\lambda_1, \lambda_2]$ is one-dimensional and presents no challenges for a narrow-band sensor. This integral is evaluated using a Gaussian quadrature rule. For a wider spectral domain with molecular absorption bands, evaluation of this integral is more challenging due to possible rapid variations in the integrand. To avoid possible numerical inaccuracy for larger spectral domains, we used adaptive Gaussian quadrature.

In terms of the computational time and memory requirements, the most challenging aspect of Eq. (1.22) is evaluation of the integral over the surface $S$, which is the area of the rough surface seen by an individual image pixel. The projected surface area can be large for some combinations of camera height, location, and observation angle. In addition, the surface may be self shadowing. Two types of shadowing are taken into account in the radiometric model, and these are illustrated in Fig. 5.3. Some parts of the soil surface can be invisible due to blocking as illustrated in Fig. 5.3 (a). To permit a general study of rough surfaces our radiometric model has been designed to deal with this form of shadowing, but practical sensor systems are often constrained to avoid these conditions, since they can lead to missed detections. Solar heating of the soil surface is also affected by shadowing as shown in Fig. 5.3 (b), and our model is also capable of handling this effect.
Figure 5.3: (a) Surface roughness can produce obscuration. (b) Heating of the soil surface is affected by shadowing.
Integration over the projected surface areas is done using numerical quadrature techniques and is illustrated in Fig. 5.4 (a). As noted above, the soil surface is described by triangular facets with varying normal vectors. Every image pixel is divided into two triangles, and the integration domains comprise the projection of these triangles (or abscissas for numerical integration) onto the rough surfaces. The details of the selection of the integration abscissas and the integration rule are discussed below.

The major difficulty at this point is the projection of the abscissas onto the surface facets. The challenge in this process is twofold: (1) deciding which surface facets contain the projections of the abscissas (shown Fig. 5.4 (a)), and (2) determining which of these facets will contribute to the integral at the specified integration point (shown in Fig. 5.4 (b)). This is a computationally challenging process due to the large number of facets and abscissas. For example, the relatively modest numerical simulations presented below in Sect. 5.3 involve $48 \times 48 \times 2 = 4608$ surface triangles and $120 \times 160 \times 6 = 57600$ abscissa values. Visible surface determination is a computationally expensive process for this large number of points. Techniques offered by computer graphics are essential for computational efficiency. Specifically, a z-buffer algorithm [71] is utilized to determine the facet associated with an integration point. Once the projection of the integration point over a facet is determined, evaluation of the integrand at that integration point is required. The physical temperature, emissivity and surface normal direction are known at the integration points and, therefore, the integrand in Eq. (1.22) can be obtained with the aid of Eq. (1.18).

The accuracy of the integration over these triangular domains affects the fidelity of the simulation, and a number of tests were done to verify this accuracy. Different adaptive Gaussian quadrature rules were used over a simplex coordinate system. For
Figure 5.4: (a) Projection of the integration abscissas onto the facets forming the rough surface. (b) Visible surface determination is required because of shadowing.
our application the number of surface facets is smaller than the number of image pixels, which implies that a facet is in the field of view of several pixels. Furthermore, the functional variations over these facets are smooth. Ultimately, we concluded that a non-adaptive three point quadrature rule produced acceptable results.

5.3 Results

The thermal and radiometric models described above were used to simulate the temporal and spatial signatures of mines buried under smooth and rough surfaces. In all simulations a steady wind speed of $W(t) = 2 \text{[m s}^{-1}]$ and an average air temperature of $T_{\text{air}} = 289 \text{[K]}$ were used. The thermal diffusivity ($\kappa$) and conductivity ($\mathcal{K}$) of soil were taken to be $5.0 \times 10^{-7} \text{[m}^2\text{ s}^{-1}]$ and $2.6 \text{[W m}^{-1}\text{ K}^{-1}]$, respectively. The mine was modeled as a homogeneous cylinder of TNT, for which we used $\kappa = 9.25 \times 10^{-8} \text{[m}^2\text{ s}^{-1}]$ and $\mathcal{K} = 0.234 \text{[W m}^{-1}\text{ K}^{-1}]$. Mine dimensions vary significantly. As a simple but representative target, we selected a simulant anti-tank mine [61] developed by the US Army. The simulant mine has a diameter of 25 cm, a height of 8.33 cm, and was buried 6.64 cm.

The radiometric model requires as input the triangular rough surface representation and the output of the thermal model. Using these data and the geometry given in Fig. 5.5, we constructed a virtual IR camera, the specification of which are given in Table 5.3. The IR camera is aimed at point $O$ (see Fig. 5.5), which is also the global coordinate origin for both the thermal and radiometric models. To cover the surface above the mine with sufficient resolution, an appropriate camera location
and height must be identified. For this work we selected a camera height of 5.7 meters, and a horizontal standoff (camera center to point O) of 1 meter. The resulting ground-projected field of view encompasses most of the computational volume.

<table>
<thead>
<tr>
<th>Spectral range</th>
<th>MWIR (4.4-5 μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Array size</td>
<td>160 (h) by 120 (v) pixels</td>
</tr>
<tr>
<td>FOV</td>
<td>9.1° (h) by 6.8° (v)</td>
</tr>
<tr>
<td>IFOV</td>
<td>1 mrad</td>
</tr>
</tbody>
</table>

Table 5.1: Specifications of the virtual camera used in the radiometric model.

5.3.1 Mines under smooth surfaces

Figure 5.6 shows the simulation results for the surrogate mine buried under a smooth surface. The results are presented as a sequence of images evaluated at three hour time increments starting from sunrise. These results show phenomena identified previously, namely, the so-called thermal “cross-over” times at 11 AM and 9 PM. The IR signature of the land mine is clearly seen. Clutter-like variations, which are ubiquitous in experimental studies do not appear in the simulations.

5.3.2 Random rough surface modeling using experimental data

Rough surfaces were constructed by specifying elevations for points in a rectilinear grid defined by the top surface of our FEM computational volume. The surface was then represented by triangles fitted to the grid points as shown in Fig. 5.1. The same representation was used by both the thermal and radiometric models.
Figure 5.5: Sketch of the region of interest including the IR camera. Point O is both the center of the field of view and the origin of coordinates. The image plane, the ground transformed image, the surface boundaries of the computational volume, and the global axis are illustrated.
Figure 5.6: Simulation of the IR signature of the SIM-25 mine buried 6.66 cm under a smooth soil surface. The radiometric model is used to predict the response at different times of the day: (a) At dawn. (b) 3 hours after sunrise. (c) At noon. (d) 3 hours after noon. (e) At sunset. (f) 3 hours after sunset.
To generate a random rough surface we require a description of the surface height statistics. In this work we used experimental data acquired by Salvati et al. [72] in support of a satellite remote sensing study. Those authors used a microtopographic laser scanner to measure surface heights on a grid of spacing 0.15 cm. The region sampled in their work was formerly cultivated farmland, which contained pronounced furrows. To simulate uncultivated soil surfaces we used data in the “down-furrow” direction and extended it isotropically. We approximated the resulting surface height spectrum using the following analytical expression

$$W(k_x, k_y) = \exp \left( -L_1(k_x^2 + k_y^2)^{1/2} - L_2(k_x^2 + k_y^2)^{1/4} \right)$$  \hspace{1cm} (5.2)$$

where $k_x$ and $k_y$ are spectral frequencies in the $x$ and $y$ directions, respectively, and we estimated $L_1 = 0.025$ and $L_2 = 0.25$ from the data. The experimental and theoretical surface height spectra are plotted in Fig. 5.7. In addition, we compared the histogram of the data with a Gaussian distribution using the Kolmogorov-Smirnov (KS) test. The KS levels of significance deviate from unity by less than $4 \times 10^{-3}$, which strongly imply a Gaussian distribution.

The problem of generating a rough surface with a zero-mean, Gaussian distributed, height profile $z = f(x, y)$ can be addressed by using the surface height spectrum $W(k_x, k_y)$ given by Eq. (5.2). Each spectral component is multiplied by a complex random number of unit magnitude and uniformly distributed phase $\psi_{m,n}$. We form the quantity

$$P(m, n) = e^{j\psi_{m,n}} \frac{2\pi N^2}{L} \sqrt{W \left( \frac{2m\pi}{LN}, \frac{2n\pi}{LN} \right)}$$  \hspace{1cm} (5.3)$$
Figure 5.7: Comparison of the experimental and model PSDs.
where \( N^2 \) is the number of points in the discretization of the surface profile and \( L \) is the edge-length of the surface. An example random surface generated in this manner is shown in Fig. 5.8.

Figure 5.9 shows the temporal evolution of a mine signature for the rough surface plotted in Fig. 5.8. Again, the results are presented as a sequence of images evaluated at three hour time increments starting from sunrise. In these figures it is harder to identify the mine signature, because the circular shape is lost. In addition, the imagery shows significant clutter, which could easily be mis-detected.

### 5.3.3 Random surface emissivity modeling

As noted in Sect. 5.2.2, the radiation emitted from a surface is a function of surface emissivity. In this section we investigate the effects of a random surface emissivity profile. Using studies by Salisbury and D’Aria [33, 34] as a guide, we assumed soil emissivity values ranging from 0.8 to 0.9. To define the spatial distribution of the emissivity, we assumed a Gaussian distribution and a Gaussian spectrum given by

\[
W(k_x, k_y) = \frac{L_x L_y e_s^2}{4\pi} \exp \left( -\frac{L_x^2 k_x^2}{4} - \frac{L_y^2 k_y^2}{4} \right),
\]

(5.4)

where \( L_x \) and \( L_y \) are correlation lengths in the \( x \) and \( y \) directions respectively, and \( e_s \) is the surface rms emissivity. In what follows we assume an isotropic surface with \( L_x = L_y = L \). Equation (5.4) is used with Eq. (5.3) to obtain a realization of the surface emissivity. The resulting values are scaled to the range [0.8, 0.9] to obtain the desired distribution. In Fig. 5.10 we have plotted an emissivity realization with correlation length \( L = 5 \) cm. Figure 5.11 shows the temporal evolution of the IR mine signatures for this emissivity. The emissivity variations break up the mine signature and the imagery shows significant clutter.
Figure 5.8: An example of a rough surface realized using a Gaussian spectrum with correlation length $L = 20$ cm.
Figure 5.9: Simulation of the IR camera response of the SIM-25 mine buried 6.66 cm under a rough soil surface with peak to peak height variations of 5 cm. The radiometric model is used to predict the IR camera response at different times of the day (a) At dawn. (b) 3 hours after sunrise. (c) At noon. (d) 3 hours after noon. (e) At sunset. (f) 3 hours after sunset.
5.3.4 Model Validation

Efforts have been made to validate the combined thermal-radiometric model using experimental data. We used 8-12 μm calibrated images of AT mines acquired by TRW at times near 10:00 AM and 12:00 PM. The images were acquired at the US Army Fort A.P. Hill, Site 71A mine lanes #13 (dirt) and #14 (gravel). Collateral information was collected including soil temperature at multiple depths (0.5", 2", 4", 8"), wind speed and direction, soil moisture content (one depth), air temperature, barometric pressure, and down-welling and up-welling radiance in the 0.3-3 μm and 3-50 μm spectral bands.

To predict the measured data, it was necessary to make assumptions about several unknown environmental parameters. Specifically, we assumed soil thermal conductivity=1.8 [W/mK] and diffusivity=10^{-6} [m^2/s] for the given moisture content in a clay-loam soil. The mine (an “EM-12” mine surrogate) is known to have a 30 cm diameter and 5.2 cm height. The mine's internal contents are not known, and because it is part of an active test site it was not possible to excavate the mine and examine it (to avoid disturbing the target for subsequent tests). Collateral information suggested a filling of styrofoam pellets and carnuba wax with an overlying void, which we modeled as a good thermal insulator. The thermal model's FEM mesh was constructed of pentahedral elements with cell dimensions of 1.27 cm (height) by 2.56 cm (base). Other environmental parameters were derived from the available data and models. A surface emissivity of 0.98 was assumed. MODTRAN was used to estimate the incident radiance as a function of time, and it was found to replicate measured values to an accuracy of 10%.
The measured data appear in Fig. 5.12 (a). To reduce small-scale image clutter related to surface roughness and emissivity variations we applied a low-pass filter to the original data, which produced the imagery shown in Fig. 5.12 (b). The model results appear in Fig. 5.12 (c), the shape of which is seen to compare well with Fig. 5.12 (b). A one-dimensional cut through the data is shown in Fig. 5.12 (d), which helps to quantify the agreement. We see that the signature shape and temperature contrast are fairly well predicted, but the model temperature is approximately 2K below the measured data. There are several possible sources for this error including incorrect parameter estimates and our use of periodic thermal boundary conditions in time. Warmer conditions the previous night can produce an offset that is not easily addressed without prior temperature data. The agreement in Fig. 5.12 is encouraging, but additional work in validation is clearly called for.

5.4 Conclusions

In this chapter, a radiometric model has been presented for the spatial and spectral IR signatures of buried land mines. Atmospheric effects and the spectral content of natural sources are taken into account via the MODTRAN model. Both the thermal and radiometric models used in this study incorporate a rough surface for the soil-air interface, which has implications for both thermal heating and for reflected radiometric components. The temperature distribution computed using the thermal model is combined with surface-reflected radiometric components to produce the image seen at an IR sensor. Simulations of mines under rough surfaces were performed, and it was found that surface roughness can add appreciable clutter. In addition, random variations in the surface emissivity profile were found to be another potential source
of clutter. An effort was made to validate the model using experimental data. It was found that the signature shape and temperature contrast are fairly well predicted.
Figure 5.10: An example surface emissivity realized using a Gaussian spectrum with correlation length $L = 5$ cm.
Figure 5.11: Simulation of the IR signature of the SIM-25 mine buried 6.66 cm under a smooth soil surface with a Gaussian surface emissivity profile. The radiometric model is used to predict the response at different times of the day: (a) At dawn. (b) 3 hours after sunrise. (c) At noon. (d) 3 hours after noon. (e) At sunset. (f) 3 hours after sunset.
Figure 5.12: (a) Raw experimental data. (b) Measured data after low-pass filtering to remove surface clutter. (c) Model prediction. (d) Comparison of model and experiment.
CHAPTER 6

ANALYSIS OF POLARIMETRIC IR PHENOMENA FOR DETECTION OF SURFACE MINES

In this chapter we address the problem of surface mine detection. It has long been recognized that man-made objects (including surface-laid land mines) tend to have different polarization characteristics than natural materials. Man-made objects such as mines have smooth surfaces and both reflect and emit radiation depending on their electrical and thermal properties and the imaging geometry. Conversely, natural objects such as soil, have rougher surfaces, and the reflected and emitted radiation depend on the statistical surface characteristics. As a result, natural backgrounds tend to have a different polarization signature than man-made objects and, hence, polarization can be a good discriminator of man-made and natural objects.

Although many experimental and empirical studies of polarimetric sensors have been reported in the literature, a better understanding of the theoretical basis of mine detection via polarimetric IR imagery is necessary. In this chapter, to address the aforementioned problem, a model based on the second order small perturbation method/small slope approximation (SPM/SSA) is developed to study the effects of material composition, geometry, and statistical surface properties on the polarimetric
IR signatures. This chapter starts with review of prior work on polarimetric sensors in Sect. 6.1. A description of the problem and radiometric quantities appear in Sect. 6.2. The polarimetric emissivity vector and the unpolarized emissivity are introduced in Sect. 6.3. Evaluation of the emissivity vector via the reciprocal active scattering problem using bistatic scattering coefficients is also discussed in that section. In Sect. 6.4 we introduce the polarized and unpolarized emissivity of a perfectly flat surface, which will be used in modeling the land mines. A second order SPM/SSA solution of scattering from a rough surface is presented in Sect. 6.5. Using this solution, the polarimetric emissivity vector and the unpolarized emissivity are formulated. Modeling results for polarimetric signatures of mines and soil surfaces are discussed in Sect. 6.6. A chapter summary is presented in Sect. 6.7.

6.1 Prior work with polarimetric IR sensors

Several groups have previously explored polarization for mine detection. DiMarzio et al. [73] investigated the use of polarimetric measurements with IR spectral imagery. They found polarimetric signatures in the 8-12 μm band were promising and complementary to spectral imaging for flush-buried land mines. Furthermore, they confirmed the earlier findings of Johnson et al. [74, 75] that disturbed soil tended to show a polarimetric signal. They suggested that this emissivity variation may not detect mines alone, but it could be used in conjunction with other sensors to reduce the false alarm rate. Larive et al. [76] made an experimental study of mine detection using polarimetric imagers. Their findings suggest that polarimetric IR imagery increases the contrast between the mines and the background soil, and that high spatial resolution offers additional benefits. In addition, they reported that the 8-12 μm
band is preferable to the 3-5 $\mu$m band for polarimetric imagery. In a more recent study, Larive et al. [77] presented an automatic detection algorithm for surface-laid and flush-buried mines in polarimetric imagery. The algorithm depends on the extraction of two features, namely axis tilt and ellipticity. They reported success in locating surface mines.

Barbour et al. [78] experimentally explored the effectiveness of polarimetric IR imagery for mine detection. They reported a benefit for polarimetric IR imagery, especially under heavy background clutter. They also pointed out the high signal-to-clutter ratio and the natural dual mode capability of thermography and polarization in the same image frame. Barnes et al. [79] described a 3-5 $\mu$m polarimetric system for detection of surface mines. The proposed system uses polarimetric features such as degree of polarization (DoP), degree of linear polarization (DoLP), degree of circular polarization, ellipticity and orientation of major axis. Their experimental studies suggest that DoP and DoLP tend to highlight man-made objects, ellipticity tends to highlight all the objects within the scene equally, and the polarization vector orientation contains information regarding the relative orientation of surfaces within the scene.

The detection performance of a 3-5 $\mu$m camera and a visible (VIS) camera with and without a polarization filter was investigated by de Jong et al. [80]. They reported higher detection rates for classification based on polarization features than based on intensity only. They further state that use of a polarization filter enhances IR performance, especially when there is low thermal contrast. Based on their experimental studies in homogeneous sand, the VIS camera performed better than the MWIR sensor. Conversely, in a forest background where there is extensive natural
clutter, the performance of the MWIR was reported to be superior. They reported further improvement when intensity and polarimetric results were fused.

Active sensors can be used for polarimetric detection independent of the diurnal cycle. Previously, active polarimetric systems have been used for various purposes. Miles et al. [81] reported a polarization-based active/passive sensor system which has been developed for the U.S. Army's STandoff MInefield Detection System (STAMIDS). They used two channels of near IR polarization reflectance information in addition to conventional thermal information. Although they reported good detection capability for both night and day operation, the detection rates were low for buried mines. Based on their studies, they concluded that the supplementary active polarization and reflectance data were superior to thermal data alone.

6.2 Problem description

The spectral radiance of a blackbody is given by Planck's radiation law (see Eq. (1.19)). Real materials emit less than a blackbody and the emitted spectral radiance generally depends on the transmitted and received directions and polarizations in addition to the temperature of the object and the wavelength. The IR camera receives spectral radiance $L_\beta$ emitted by the object, where $\beta$ denotes the polarization. This can be expressed as

$$L_\beta(\lambda, T, \theta, \phi) = \mathcal{E}_\beta(\lambda, \theta, \phi)L_{BB}(\lambda, T) \tag{6.1}$$

Furthermore, the total spectral radiance received from an opaque object also involves the reflected radiance. The reflectance for an opaque object can be determined via Kirchoff's law. Taking the reflected radiation into account, the total spectral radiance
from soil and mine can be expressed as

\[ L_{\text{mine}}(\lambda, T_{\text{mine}}, \theta, \phi) = \varepsilon_{\text{mine}}(\lambda, \theta, \phi)L_{BB}(\lambda, T_{\text{mine}}) \]
\[ + (1 - \varepsilon_{\text{mine}}(\lambda, \theta, \phi))(L_{\text{sky}}(\lambda) + L_{\text{sun}}(\lambda)) \quad (6.2) \]

\[ L_{\text{soil}}(\lambda, T_{\text{soil}}, \theta, \phi) = \varepsilon_{\text{soil}}(\lambda, \theta, \phi)L_{BB}(\lambda, T_{\text{soil}}) \]
\[ + (1 - \varepsilon_{\text{soil}}(\lambda, \theta, \phi))(L_{\text{sky}}(\lambda) + L_{\text{sun}}(\lambda)) \quad (6.3) \]

The components in Eqs. (6.2) and (6.3) are illustrated in Fig. 6.1. In Eqs. (6.2) and

Figure 6.1: The reflected and emitted radiance components from the rough soil surface and the smooth land mine surface.

(6.3), the emissivity expressions depend on the composition (optical properties) of
the soil and mine, statistical description of the surface, problem geometry, wavelength and polarization. Generally the soil and mine have different roughness and optical properties. Throughout this work the mine surface is assumed to be smooth, while the soil is assumed rough. The surface roughness depends on the size of the soil particles and any disturbances of the surface. This dissimilarity in material properties and surface roughness are the main sources of polarimetric signatures.

6.3 Emissivity for polarized and unpolarized sensors

In this section we derive the emissivity seen by polarized sensors, and reduce it to that seen by an unpolarized sensor. The polarization states of the electric field are characterized by four quantities known as Stokes parameters.

\[
I = \begin{bmatrix}
L_h \\
L_v \\
U \\
V
\end{bmatrix} = \begin{bmatrix}
\langle E_h E_h^* \rangle \\
\langle E_v E_v^* \rangle \\
2\text{Re}\langle E_h E_v^* \rangle \\
2\text{Im}\langle E_h E_v^* \rangle
\end{bmatrix}
\]

where \( E_h \) and \( E_v \) are the horizontal and vertical polarizations of the fields sensed by the receiver. Yueh and Kwok [82] state that the emission vector \( \mathcal{E} \) can be related to the Stokes parameters as

\[
\mathcal{E} = \begin{bmatrix}
\mathcal{E}_h \\
\mathcal{E}_v \\
\mathcal{E}_U \\
\mathcal{E}_V
\end{bmatrix} = c' \begin{bmatrix}
\langle E_h E_h^* \rangle \\
\langle E_v E_v^* \rangle \\
2\text{Re}\langle E_h E_v^* \rangle \\
2\text{Im}\langle E_h E_v^* \rangle
\end{bmatrix}
\]

where \( c' \) is a constant for a fixed frequency.

Kirchhoff's law relates the emissivity of a body to its absorptivity. For an object in thermal equilibrium, the amount of energy emitted by the object is equal to the energy absorbed by the object. Therefore, the emissivity of the object can be obtained by solving for the absorptivity of the object using the reciprocal active scattering
problem [83]. Consider a plane wave $E^i$ incident on a medium with area $A$. The power intercepted by the surface area $A$ is

$$P_i = \frac{|E^i|^2 A \cos \theta_i}{2\mu}$$  \hspace{1cm} (6.6)

where $\theta_i$ is the incidence angle and $\mu$ is the free space impedance. The total scattered power at range $R$ can be written as

$$P_s = \sum_{\beta=v,h} \int_0^{\pi/2} d\theta_s \int_0^{2\pi} d\phi_s R^2 \sin \theta_s \frac{|E^\beta|^2}{2\mu}$$  \hspace{1cm} (6.7)

where $\theta_s$ and $\phi_s$ describe the scattering directions. Using Eqs. (6.6) and (6.7) we can write the absorptivity as

$$a_\alpha = \frac{P_i - P_s}{P_i} = 1 - \frac{1}{4\pi} \sum_{\beta=v,h} \int_0^{\pi/2} d\theta_s \sin \theta_s \int_0^{2\pi} d\phi_s \gamma_{\beta\alpha}(\theta_s, \phi_s; \theta_i, \phi_i)$$  \hspace{1cm} (6.8)

where $\gamma_{\beta\alpha}(\theta_s, \phi_s; \theta_i, \phi_i)$ is the bistatic scattering coefficient defined as

$$\gamma_{\beta\alpha}(\theta_s, \phi_s; \theta_i, \phi_i) = \frac{4\pi R^2 |E^{\beta}_s|^2}{|E^i|^2 A \cos \theta_i}.$$  \hspace{1cm} (6.9)
Kirchoff’s law and the well known relation between emissivity and reflectivity $R$ for an opaque body

\[ \mathcal{E} = 1 - R \quad (6.10) \]

allow us to write

\[ \mathcal{E}_h = 1 - R_h = 1 - \frac{1}{4\pi} \sum_{\beta=v,h} \int_0^{\pi/2} d\theta_s \sin \theta_s \int_0^{2\pi} d\phi \gamma_{\beta h}(\theta_s, \phi_s; \theta_i, \phi_i) \quad (6.11) \]

\[ \mathcal{E}_v = 1 - R_v = 1 - \frac{1}{4\pi} \sum_{\beta=v,h} \int_0^{\pi/2} d\theta_s \sin \theta_s \int_0^{2\pi} d\phi \gamma_{\beta v}(\theta_s, \phi_s; \theta_i, \phi_i) \quad (6.12) \]

in which $R_h$ and $R_v$ are horizontal and vertical reflectivities, which are obtained by integrating the bistatic scattering coefficient over the upper hemisphere.

A fully polarimetric version of this equation can be given in terms of the Stokes vectors. Consider a plane wave

\[ \mathbf{E} = (\hat{\mathbf{v}}_i E_{v_i} + \hat{\mathbf{h}}_i E_{h_i})e^{i\mathbf{k} \cdot \mathbf{r}} \quad (6.13) \]

Using a scattering function matrix, the scattered field can be written as

\[ \begin{bmatrix} E_{h_s} \\ E_{v_s} \end{bmatrix} = \frac{e^{ikR}}{R} \begin{bmatrix} f_{hh} & f_{vh} \\ f_{hv} & f_{vv} \end{bmatrix} \cdot \begin{bmatrix} E_{h_i} \\ E_{v_i} \end{bmatrix} \quad (6.14) \]

where the first subscript denotes the incident polarization and second denotes the scattered polarization. Using Eqs. (6.14) the Stokes vector of the total reflected radiation can be written in terms of the incident field Stokes vector as

\[ \mathbf{I}_r = \frac{1}{4\pi} \int_0^{\pi/2} d\theta_s \int_0^{2\pi} d\phi \begin{bmatrix} \gamma_{hhhh} & \gamma_{uhh} & \text{Re}(\gamma_{hvh}) & -\text{Im}(\gamma_{hvh}) \\ \gamma_{huh} & \gamma_{vvh} & \text{Re}(\gamma_{huv}) & -\text{Im}(\gamma_{huv}) \\ 2\text{Re}(\gamma_{hhv}) & 2\text{Re}(\gamma_{uvh}) & \text{Re}(\gamma_{hhv} + \gamma_{uvv}) & -\text{Im}(\gamma_{hhv} + \gamma_{uvv}) \\ 2\text{Im}(\gamma_{hhv}) & 2\text{Im}(\gamma_{uvh}) & \text{Im}(\gamma_{hhv} + \gamma_{uvv}) & \text{Re}(\gamma_{hhv} + \gamma_{uvv}) \end{bmatrix} \cdot \mathbf{I}_i \quad (6.15) \]

where

\[ \gamma_{abcd} = \frac{4\pi f_{ab} f_{cd}^*}{A \cos \theta_i} \quad (6.16) \]
By considering an unpolarized incoming wave with

\[
I_i = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}
\]  

(6.17)

we can find the polarimetric emissivity as

\[
\begin{bmatrix} \mathcal{E}_h \\ \mathcal{E}_v \\ \mathcal{E}_U \\ \mathcal{E}_V \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{4\pi} \int_0^{\pi/2} d\theta \int_0^{2\pi} d\phi \begin{bmatrix} \gamma_{hhh} + \gamma_{vvh} & \gamma_{huh} + \gamma_{vuv} \\ \gamma_{vhu} + \gamma_{uvu} & 2\text{Re}(\gamma_{hhh} + \gamma_{vvh}) \\ 2\text{Im}(\gamma_{hhh} + \gamma_{vvh}) \end{bmatrix}
\]  

(6.18)

The first two rows are the same as Eqs. (6.11) and (6.12). An unpolarized sensor is sensitive to the intensity of the incoming signal. Therefore, if we choose an unpolarized incoming signal of amplitude 1 in the reciprocal active scattering problem and calculate the intensity, an unpolarized sensor will measure

\[
\epsilon_{up} = \frac{\mathcal{E}_h + \mathcal{E}_v}{2}
\]  

(6.19)

### 6.4 Emissivity of perfectly flat surfaces

In this section we investigate the emissivity seen by both polarized (vertical and horizontal) and unpolarized sensors. The IR detector views a perfectly flat, dielectric surface as shown in Fig. 6.3. For a perfectly flat surface the reflectivities \( R_h \) and \( R_v \) are determined from the Fresnel reflection coefficients as

\[
R_h = |R_h|^2
\]  

(6.20)

\[
R_v = |R_v|^2
\]  

(6.21)

where the reflection coefficients for horizontal and vertical polarizations are defined by

\[
R_h = \frac{k_{zi} - k_{Lzi}}{k_{zi} + k_{Lzi}}
\]  

(6.22)
in which $\epsilon_0$ and $\epsilon_1$ are the permittivities of the air and soil regions, respectively. In Eq. (6.23) $k_{zi}$ and $k_{1zi}$ represent the $z$ component of the wave vectors in air and soil regions, respectively. Using these equations together with Eqs. (6.11), (6.12), and (6.19) the polarized and unpolarized emissivities can be obtained.

In this work, we consider mines with plastic surfaces. To model plastic mines, we used values of 1.4 to 1.6 for the refractive indices of various plastics taken from the literature [84, 85, 86, 87]. We first studied polarized and unpolarized emissivity variations as a function of observation angle. The state of polarization for the reflected radiation is defined as

$$p = \frac{R_h - R_v}{R_h + R_v}$$  \hfill (6.24)
In Fig. 6.4 the horizontal and vertical polarized emissivities, unpolarized emissivity, and the state of polarization of the reflected radiation are presented for dielectric surfaces with refractive indices of $n = 1.4$, $1.5$, and $1.6$. In these results $\theta = 0^\circ$ corresponds to nadir viewing. The horizontally polarized emissivity is almost constant until $\theta = 40^\circ$ and then starts to decrease. This decrease accelerates beyond $\theta = 70^\circ$, and by $\theta = 90^\circ$ there is no horizontally polarized emission. The vertically polarized emissivity starts from the same value as the horizontally polarized emissivity, since the Fresnel coefficients are equal at $\theta = 0$. As we increase the observation angle, the vertically polarized emissivity increases smoothly until it reaches unity. This point corresponds to the Brewster angle, at which there is no reflection for the vertical component. After the Brewster angle the vertically polarized emissivity decreases sharply to zero. We can see from the graphs that if the relative permittivity of the surface increases, the emissivity values decreases. Figure 6.4 (c) presents emissivity variations seen by an unpolarized sensor. This emissivity is almost constant until $\theta = 60^\circ$, where it sharply decreases to zero. We noted above that for angles smaller than the Brewster angle, there is little variation in the unpolarized emissivity. This observation suggests that the diffuse surface approximation can be used if the observation angle is smaller than the Brewster angle. In Fig. 6.4 (d) reflected polarization is plotted as a function of observation angle. This value is zero when the horizontal and vertical components are equal (nadir viewing) and unity when the vertical reflected component is zero (the Brewster angle).
Figure 6.4: Emissivity and received polarization as a function of observation angle for plastic surfaces having different permittivity values. (a) Horizontally polarized emissivity. (b) Vertically polarized emissivity. (c) Emissivity variations seen by an unpolarized sensor.(d) Polarization of the reflected radiation.
6.5 The effects of surface roughness on IR emission

The emissivities seen by polarized and unpolarized sensors are now investigated for the rough surface case. The rough surface scattering problem will be solved to obtain the bistatic scattering coefficients, which can be used to determine the emissivity. Several approximate theories can be used to solve the rough surface scattering problem. In this work we present a second order SPM/SSA solution.

The SPM/SSA is essentially an iterative technique for rough surfaces with small slopes. A second-order SPM formulation decomposes the fields into coherent and incoherent parts. Zeroth-order SPM solutions are the reflected and transmitted fields of a perfectly smooth surface. These fields are characterized by the Fresnel reflection coefficients. A first-order SPM solution gives the lowest-order incoherent fields. A second-order solution gives the lowest-order correction to the coherent fields. SPM assumes that the surface variations are much smaller than the incident wavelength, but recent studies have shown that SPM can be used for emissivity calculations even when the surface height is large, if the small slope condition is satisfied [88].

Yueh et al. [89] presented the second-order reflectivity matrix in a compact form. Using the formulations in Section 3 and Appendices 2 and 3 of the aforementioned paper, the emissivity calculations can be done, but there are several ambiguities in the formulation, and the approach to numerical evaluation of the integrals is unclear. Johnson and Zhang [88], referred to hereafter as JZ, reported that separate evaluation of the coherent and incoherent terms requires very accurate evaluation of these integrals and may cause numerical problems due to the cancellation of large numbers. JZ addressed those problems by combining the coherent and incoherent terms in a compact form for the emissivity.
The emissivity expressions of JZ for a second order SPM/SAA are

\[
\begin{bmatrix}
\mathcal{E}_h \\
\mathcal{E}_v \\
\mathcal{E}_U \\
\mathcal{E}_V \\
\end{bmatrix} =
\begin{bmatrix}
1 - |R_{hh}^0|^2 & 0 & 0 & 0 \\
0 & 1 - |R_{vv}^0|^2 & 0 & 0 \\
\int_0^\infty dk'_p k'_p W(k'_p, \phi') & \int_0^\infty dk'_p k'_p W(k'_p, \phi') & \int_0^\infty dk'_p k'_p W(k'_p, \phi') & \int_0^\infty dk'_p k'_p W(k'_p, \phi') \\
\end{bmatrix}
\begin{bmatrix}
g_h(f, \theta_i, \phi_i, \epsilon, k'_p, \phi') \\
g_v(f, \theta_i, \phi_i, \epsilon, k'_p, \phi') \\
g_U(f, \theta_i, \phi_i, \epsilon, k'_p, \phi') \\
g_V(f, \theta_i, \phi_i, \epsilon, k'_p, \phi') \\
\end{bmatrix}
\]  

(6.25)

where $R_{hh}^0$ and $R_{vv}^0$ are the horizontally and vertically polarized flat surface Fresnel reflection coefficients, respectively, and $W(k'_p, \phi')$ is the surface spectrum. The quantities $g_h$, $g_v$, $g_U$, and $g_V$ are weighting functions given by

\[
g_h(f, \theta_i, \phi_i, \epsilon, k'_p, \phi') = 2 \text{Re}\{R_{hh}^{(0)*} f_{hh}^{(2)}\} + \frac{k_z}{k_z} \left[ |f_{hh}^{(1)}|^2 + |f_{hv}^{(1)}|^2 \right] F 
\]  

(6.26)

\[
g_v(f, \theta_i, \phi_i, \epsilon, k'_p, \phi') = 2 \text{Re}\{R_{vv}^{(0)*} f_{vv}^{(2)}\} + \frac{k_z}{k_z} \left[ |f_{vv}^{(1)}|^2 + |f_{hv}^{(1)}|^2 \right] F 
\]  

(6.27)

\[
g_U(f, \theta_i, \phi_i, \epsilon, k'_p, \phi') = 2 \text{Re}\{(R_{hh}^{(0)*} - R_{vv}^{(0)*}) f_{hh}^{(2)}\} 
+ \frac{2k_z}{k_z} \text{Re}\{f_{hh}^{(1)} f_{hh}^{(1)*} + f_{hv}^{(1)} f_{hv}^{(1)*}\} F 
\]  

(6.28)

\[
g_V(f, \theta_i, \phi_i, \epsilon, k'_p, \phi') = 2 \text{Im}\{(R_{hh}^{(0)*} + R_{vv}^{(0)*}) f_{hh}^{(2)}\} 
+ \frac{2k_z}{k_z} \text{Im}\{f_{hh}^{(1)} f_{hh}^{(1)*} + f_{hv}^{(1)} f_{hv}^{(1)*}\} F 
\]  

(6.29)

The $f_{ab}^{(1)}$ and $f_{ab}^{(2)}$ functions are given in Appendices 2 and 3 of Yueh et al.[89] with modifications proposed by JZ. The function $F$ was proposed by JZ to limit incoherent contributions to the upper hemisphere.

As noted by JZ, these results may be simplified extensively. In the absence of cultural or agricultural features (e.g., crop furrows), it is reasonable to assume that the soil surface spectrum is isotropic. JZ has shown that as a result of symmetry, $\mathcal{E}_U$ and $\mathcal{E}_V$ vanish in that case. The soil surface height variation are much larger than the sensor wavelengths, which permits a further approximation that leads to the result

\[
\begin{bmatrix}
\mathcal{E}_h \\
\mathcal{E}_v \\
\end{bmatrix} =
\begin{bmatrix}
1 - |R_{hh}^{(0)}|^2 & 0 \\
1 - |R_{vv}^{(0)}|^2 & 0 \\
\end{bmatrix}
\begin{bmatrix}
S^2 h_{hh}^{(0)}(\theta_i, \epsilon) \\
S^2 h_{vv}^{(0)}(\theta_i, \epsilon) \\
\end{bmatrix}
\]  

(6.30)
where $S^2$ is the slope variance and $h^{(0)}$ are shape functions defined as an asymptotic limit of $g$ functions (see JZ). Eq. (6.30) suggests that the emissivity vector for a rough soil surface is a function of observation angle, soil permittivity, and surface slope variance.

### 6.6 Results

It is a straightforward matter to use Eq. (6.30) to compute soil emissivity. For this work we required the complex permittivity of soil in the IR spectral regime. The composition of soil and the thermal and optical properties of its components are important in many applications [90, 91, 92, 93, 94], but surprisingly little data is available in the IR regime. Refractive index values in the visible and near-IR portions of the spectrum are available [95, 96] and will be used here. Near 0.616 μm the refractive index of soil with a mean particle radius of 10 μm is approximated as $1.6+0.002j$. Using this value the shape functions in Eq. (6.30) can be calculated, and they appear in Figure 6.5. From these data we compute the emissivities and state of polarization for a rough surface. The results are illustrated in Fig. 6.6 for different surface slope variances.

Predictions can be made from these data regarding the performance of polarimetric sensors of surface mines. A key issue is the polarization metric being used for detection. As noted in Section 6.1, different metrics have been used. The state of polarization $p$ for the reflected radiation defined in Eq. (6.24) can be used for this purpose, and it is plotted as a function of observation angle for mine-like surfaces and soil in Figs. 6.4 (d) and 6.6 (d), respectively. The results suggest that for the very smooth soil case, the mine and plastic surfaces produce similar $p$ values. As
the soil roughens, however, the peak value of $p$ moves to smaller observation angles. Operating the sensor at the angle of optimum $p$ may lead to improved detection.

Another widely used metric for mine detection is the DoLP, which is defined as

$$\text{DoLP} = \frac{L_h(\lambda, T, \theta, \phi) - L_v(\lambda, T, \theta, \phi)}{L_h(\lambda, T, \theta, \phi) + L_v(\lambda, T, \theta, \phi)}$$ \hspace{1cm} (6.31)

When the physical temperature of the soil surface and the mine surface are equal, i.e., when $T_{\text{mine}} = T_{\text{soil}}$, then DoLP is related to differences in the polarimetric emissivity terms $\mathcal{E}_\beta(\lambda, \theta, \phi)$. In this case Eq. (6.31) can be written as

$$\text{DoLP} \approx \frac{(\mathcal{E}_h - \mathcal{E}_v)}{(\mathcal{E}_h + \mathcal{E}_v)}$$ \hspace{1cm} (6.32)

In the visible and near-IR regime we have $(L_{\text{sun}}(\lambda) + L_{\text{sky}}(\lambda)) \gg L_{BB}(\lambda, T_{\text{mine}})$, and Eq. (6.32) can be approximated by

$$\text{DoLP} \approx \frac{(\mathcal{E}_h - \mathcal{E}_v)}{(\mathcal{E}_h + \mathcal{E}_v) - 2} \quad \text{visible and near-IR} \hspace{1cm} (6.33)$$
Figure 6.6: The emissivity and radiation polarization variations functions of observation angle for the rough soil surface for different slope variance values (a) Horizontal polarized emissivity variations for different permittivity values. (b) Vertical polarized emissivity variations for different permittivity values. (c) Emissivity variations as seen by an unpolarized sensor for different permittivity values. (d) Polarization of reflected radiation for different observation angles.
Conversely, for the LWIR regime we have \( (L_{\text{sun}}(\lambda) + L_{\text{sky}}(\lambda)) \ll L_{BB}(\lambda, T_{\text{mine}}) \), which leads to

\[
\text{DoLP} \approx \frac{(\varepsilon_h - \varepsilon_v)}{(\varepsilon_h + \varepsilon_v)} = p \quad \text{LWIR}
\]  

(6.34)

We plot the DoLP for a mine-like plastic surface and soil. To simulate the plastic cover of the land mine, we have chosen Nylon-66 (Polyhexamethylene-adipamide) which has a refractive index of 1.53 \[87\]. To model the rough soil surface we use a refractive index of \(1.6 + 0.002j\) with a range of surface slope variances. The DoLP results are presented in Fig. 6.7. For the smooth soil surface the DoLP values for Nylon-66 are similar to those of soil, but as the soil gets rougher, the DoLP values of Nylon-66 and soil are quite different. These results confirms the earlier experimental studies. In Fig. 6.8 the contrast in DoLP values between the mine and soil surfaces are plotted as a function of observation angle for different slope variances. The results suggest that to get a strong polarimetric signature, the data should be taken at observation angles beyond 35°.

6.7 Summary

The theoretical basis for passive polarimetric IR signatures was investigated for use in land mine detection. A model was developed for emission from mines and natural surfaces. Using a second order SPM/SSA solution of the reciprocal active scattering problem, the emissivity seen by both polarized and unpolarized sensors was studied for smooth and rough surfaces. To model non-metallic landmines, the smooth mine surfaces were represented by various plastics. The rough soil surface was modeled by a Gaussian surface spectrum with a complex refractive index. Results for the polarimetric mine signatures were presented. We confirmed the earlier
Figure 6.7: The DoLP as a function of observation angle for Nylon-66 and rough soil surfaces with different slope variance values

Figure 6.8: The DoLP contrast between mine and soil surface as a function of observation angle for different slope variance values
experimental findings that as the soil roughness increases the polarimetric signature contrast increases.
CHAPTER 7

SUMMARY AND CONCLUSION

7.1 Summary

Three-dimensional thermal and radiometric models have been developed to predict the IR signatures of land mines. First, the physical processes and heat transfer mechanisms that produce thermal IR signatures were briefly discussed, and a mathematical description of the processes was presented. Example experimental imagery were presented to familiarize the reader with IR mine signatures and thermal clutter. Experiments using conducting and insulating mines indicated that the signatures of buried conducting mines are weaker than those of insulating mines, and signature of flush buried mines are strongly affected by the downwelling radiance. In addition, images acquired on a surrogate mine field demonstrated the challenge of detecting mines in realistic clutter.

A significant contribution of this work is the development and validation of a combined thermal-radiometric model for mine signatures. Two thermal simulation codes were developed. A reference solution was presented for the integral equation that governs the surface temperature distribution for a buried body of revolution under a planar interface. That work is limited to the case of constant air temp and convection coefficient. Using a periodic boundary condition in time at the interface,
the temperature distribution in the lower-half space was expanded in a Fourier series. A volume integral equation for the Fourier coefficients was obtained via Green's second identity. The Green's function for this problem was derived and the integral equation was solved using the method of weighted residuals.

A FEM-based three-dimensional thermal model was also developed. The behavior of this FEM model was compared with other formulations, including an analytical model of 1-D geometries and the integral equation solution. Good agreement was observed in these comparisons. Numerical simulations were presented for a square AP mine, a square AT mine, and a circular AT mine buried under a smooth soil surface. The temporal evolution of the temperature distribution was presented both at the surface and as a function of depth. It was found that the thermal signatures of mines under smooth surfaces has a circular characteristic. In addition, it is observed that the thermal signatures are mostly independent of the time of day and the mine burial depth.

Numerical techniques were developed for analysis of mines with complex structures. A detailed representation of the TM-62 AT mine was developed. This model was discretized using an automatic mesh generation program, and further refined during the calculation using an adaptive mesh generation program. Using this detailed geometry and accurate discretization, a quantitative comparison of mines with metal and plastic cases was presented. It was found that the plastic-cased mines produce stronger signatures than metal-cased mines. Also a shift in the thermal cross-over times is observed for the mines with plastic and metallic cases.

A three-dimensional radiometric model with both spectral and spatial capabilities was presented. Atmospheric effects and the spectral content of natural sources were
taken into account via the MODTRAN model. Both the thermal and radiometric models used in this study incorporated a rough surface for the soil-air interface, which has implications for both thermal heating and for reflected radiometric terms. The temperature distribution computed using the thermal model was combined with surface-reflected radiometric components to produce the image seen at a virtual IR sensor. The model was validated using experimental data. It was found that the signature shape and temperature contrast were fairly well predicted.

A second significant contribution of this work is the creation of a model for passive polarimetric signatures. The theoretical basis for polarimetric IR phenomena was investigated for use in surface mine detection. A model was developed for emission from mines and natural surfaces under the assumption that there was no thermal contrast and that the incident radiance was unpolarized. Using a second order SPM/SSA solution of the reciprocal active scattering problem, the emissivity seen by both polarized and unpolarized sensors was studied for smooth and rough surfaces. To model non-metallic landmines, the mine surfaces were represented by various smooth plastics. The rough soil surface was modeled by a Gaussian surface spectrum with a complex refractive index. Results for the polarimetric mine signatures were presented. It was found that stronger polarimetric signatures are achieved for rougher soils, and that polarimetric signatures are stronger for larger observation angles.

7.2 Concluding remarks

Our work with the thermal and radiometric models support the following conclusions for IR mine detection:
• TNT, the explosive material used in most land mines, is a better insulator than soil. It obstructs the heat flow into and out of the soil. As a result, soil above the land mine is hotter than the background soil during the day, but cooler during the night.

• Trends in the simulated peak contrast are consistent with experimental observations. (1) For all observation times the mine-generated contrast has a peak at the center of the mine and it diminishes as we move away from the mine center. (2) Two thermal "cross-over" times occur during a day, at which times contrast changes occur in the imagery. The mine signatures are completely lost at thermal cross-over times and data acquisition should be avoided near those times. The time of these events depends on mine composition and burial depth.

• The spatial dependence of a mine signature is (with the exception of scaling) largely independent of image acquisition time and burial depth.

• The soil overlying the land mine has the effect of a spatial low-pass filter on the IR image. The surface thermal signatures of rectangular land mines buried several inches will appear similar to those of circular land mines.

• Plastic-cased mines produce stronger signatures than metal-cased mines. Plastics are good thermal insulators, which block the natural heat transfer, while the metal mine case facilitates that heat transfer.

• A shift in the thermal cross-over times is observed for the metallic-cased and plastic-cased mines. The cross over times for the plastic-case TM-62P occurs
around 11:00 and 20:00. For the metal-case TM-62M mine these times occur approximately one hour later.

- The contrast observed in a polarimetric sensor will tend to increase as the soil roughness increases. Mines have smooth surfaces, and their polarimetric signatures are similar to the smooth soil surfaces because of the similarity of their permittivity values. A polarimetric mine signature exists for rough soil surfaces, because the rough surface tends to produce unpolarized scattering. For strong polarimetric signatures, the imagery should be acquired at zenith angles of 35° or more.

7.3 Suggestions for future research

Mine detection using IR imagery depends on many uncontrolled factors which are determined by the environmental conditions in the mine-detection site. Vegetation is one of the factors which have been ignored in our models. Low-lying vegetation such as grass can enhance detection of surface mines, since sites with little surface vegetation may exhibit poor contrast for surface mines at all times. Conversely, vegetation hampers the detection of buried mines, since it reduces soil heating and complicates the surface signature. Incorporating a realistic vegetation model into the thermal and radiometric models is very desirable.

Another environmental factor that may contribute to buried mine signatures is the variation in soil moisture content (and the concomitant change in soil emissivity and thermal properties) due to the moisture barrier formed by the mine. We ignored the soil moisture movement in the thermal-radiometric model. The effects of soil moisture movement on the mine signatures need a detailed investigation.
In this work we focussed on the IR signatures of buried mines. Signatures of surface mines are also of great interest for humanitarian demining. Surface mine signatures are dramatically affected by the sudden changes in illumination (e.g., blocking of sunlight by clouds). In addition, modeling surface mine signatures requires more detailed mine geometries and compositions. A detailed thermal-radiometric model for surface mines should be one of the goals of the future research.

In Chapter 6 polarimetric signatures of surface mines are studied, but the incident radiation was assumed to be unpolarized. An important extension of that work would be to include the effects of polarized sources (both natural and artificial), because of its improved accuracy and its relevance to modern mine detection systems.
PERSPECTIVE TRANSFORMATIONS

A perspective transformation is used to map images captured by the IR cameras to ground coordinates using the camera-scene geometry and camera field-of-view information. Figure A.1 illustrates the camera position, image center point, and some parameters used in the transformation. The axes are chosen so that the ground lies in the $z = 0$ plane. The camera is indicated by C and its ground coordinates are given by $(X_c, Y_c, Z_c)$. The point on the ground that corresponds to the center of the image is indicated by O and its ground coordinates are $(X_o, Y_o, 0)$. The camera field of view is defined by the known angles $O_r$ and $O_m$. The projection of the point C onto the ground coordinates is shown by B. The distance between B and O can be found by

$$BO = \sqrt{(X_o - X_c)^2 + (Y_o - Y_c)^2} \quad (A.1)$$

Figure A.2 is the $z$-$\hat{z}$ plane view of Figure A.1. The camera depression angle $\theta$ and ground azimuth angle $\phi$ can be expressed in terms of the distances as

$$\theta = \arctan \frac{Z_c}{BO} \quad (A.2)$$

$$\phi = \arctan \frac{Y_o - Y_c}{X_o - X_c} \quad (A.3)$$
Figure A.1: The image plane and the ground transformed image. The camera location and its projection onto the ground plane are shown by points C and B, respectively. Point O is the center of the field of view.

Figure A.2: The $\hat{z}$-$\hat{s}$ plane view of Figure A.1
The distance between the camera and closest pixel in the ground transformed image can be written as
\[ CD = \frac{Z_c}{\cos(\pi/2 - \theta - O_r/2)} \]  
(A.4)
and the distance between the camera and the image plane at the closest pixel is given by
\[ CP = CD \cos(O_r/2) \]  
(A.5)
The digital image viewed by the camera is assumed to have row and column dimensions \(n_{row}\) and \(n_{col}\). This image can be projected onto the plane passing through point P. The row dimension \(P_r\) and column dimension \(P_c\) of a pixel in this plane are given by
\[ P_r = \frac{CP \tan(O_r/2)}{n_{row}/2} \]  
(A.6)
\[ P_c = \frac{CP \tan(O_c/2)}{n_{col}/2} \]  
(A.7)
The image pixel value at any ground coordinate \((x, y)\) is found by first converting ground coordinates to the \((s, t)\) coordinate system as follows:
\[ \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ \sin \phi & -\cos \phi \end{bmatrix} \begin{bmatrix} x - X_o \\ y - Y_o \end{bmatrix} + \begin{bmatrix} BO \\ 0 \end{bmatrix} \]  
(A.8)
Defining
\[ w = s/Z_c \]  
(A.9)
the image pixel coordinates \((x', y')\) corresponding to \((s, t)\) are found from
\[ \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \text{CP}/P_r & \text{CP}/P_c \end{bmatrix} \begin{bmatrix} w \tan \theta - 1 \\ 0 \\ w + \tan \theta \\ t/s \end{bmatrix} + \begin{bmatrix} \frac{n_{row}}{2} + 1 \\ \frac{n_{col}}{2} + 1 \end{bmatrix} \]  
(A.10)
In general \((x', y')\) will not correspond to integer pixel indices and some form of interpolation is required.
APPENDIX B

ANALYTICAL TREATMENT OF THE SINGULAR INTEGRAL

In this appendix we evaluate the functions $I_c(\rho_1, \rho_u; \phi_1, \phi_u; z_l, z_u)$, $I_p(\rho_1, \rho_u; \phi_1, \phi_u; z_l, z_u)$, and $I_z(\rho_1, \rho_u; \phi_1, \phi_u; z_l, z_u)$, which were introduced in Eq. (2.70). Singularity extraction is required in the evaluation of these functions if the testing point lies in the integration domain. The boundaries of this integral and the location of the singularity are illustrated in Fig. B.1.

Consider a small volume around the singularity. Such a volume can be defined by the cylindrical coordinates $\rho \in (\rho - \epsilon_\rho, \rho + \epsilon_\rho)$, $\phi \in (\phi - \epsilon_\phi, \phi + \epsilon_\phi)$, and $z \in (z - \epsilon_z, z + \epsilon_z)$. Figure B.2 demonstrates the top and side views of this volume. The functions can be written as

$$
\begin{bmatrix}
I_c(\rho_1, \rho_2; 0, 2\pi; z_l, z_u) \\
I_p(\rho_1, \rho_2; 0, 2\pi; z_l, z_u) \\
I_z(\rho_1, \rho_2; 0, 2\pi; z_l, z_u)
\end{bmatrix}
= 
\begin{bmatrix}
\left[I_c(\rho_1, \rho_2; 0, 2\pi; z_l, z - \epsilon_z)\right] \\
\left[I_p(\rho_1, \rho_2; 0, 2\pi; z_l, z - \epsilon_z)\right] \\
\left[I_z(\rho_1, \rho_2; 0, 2\pi; z_l, z - \epsilon_z)\right] \\
\left[I_c(\rho_1, \rho_2; 0, 2\pi; z_l, z + \epsilon_z)\right] \\
\left[I_p(\rho_1, \rho_2; 0, 2\pi; z_l, z + \epsilon_z)\right] \\
\left[I_z(\rho_1, \rho_2; 0, 2\pi; z_l, z + \epsilon_z)\right]
\end{bmatrix}
+ 
\begin{bmatrix}
\left[I_c(\rho_1, \rho - \epsilon_\rho; 0, 2\pi; z - \epsilon_z, z + \epsilon_z)\right] \\
\left[I_p(\rho_1, \rho - \epsilon_\rho; 0, 2\pi; z - \epsilon_z, z + \epsilon_z)\right] \\
\left[I_z(\rho_1, \rho - \epsilon_\rho; 0, 2\pi; z - \epsilon_z, z + \epsilon_z)\right] \\
\left[I_c(\rho + \epsilon_\rho, \rho_2; 0, 2\pi; z - \epsilon_z, z + \epsilon_z)\right] \\
\left[I_p(\rho + \epsilon_\rho, \rho_2; 0, 2\pi; z - \epsilon_z, z + \epsilon_z)\right] \\
\left[I_z(\rho + \epsilon_\rho, \rho_2; 0, 2\pi; z - \epsilon_z, z + \epsilon_z)\right]
\end{bmatrix}
$$

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Figure B.1: The singularity point and the boundaries of the singular integral.
Figure B.2: The small volume around the singularity (a) Top view. (b) Side view.
\[
I_c(\rho - \epsilon_p, \rho + \epsilon_p; \epsilon, \phi, 2\pi - \epsilon; z - \epsilon_z, z + \epsilon_z) \\
I_p(\rho - \epsilon_p, \rho + \epsilon_p; \epsilon, \phi, 2\pi - \epsilon; z - \epsilon_z, z + \epsilon_z) \\
I_z(\rho - \epsilon_p, \rho + \epsilon_p; \epsilon, \phi, 2\pi - \epsilon; z - \epsilon_z, z + \epsilon_z) \\
+ \begin{bmatrix}
I_c^{\text{sing}} \\
I_p^{\text{sing}} \\
I_z^{\text{sing}}
\end{bmatrix}
\]

(B.1)

where \( I_c^{\text{sing}} \), \( I_p^{\text{sing}} \), and \( I_z^{\text{sing}} \) are integrals of the singular kernels in the small volume. The first five terms on the right-hand side involve no singularities and can be calculated numerically. The limits of the integrals in Eq. (B.1) can be better visualized with the aid of Fig. B.2. The volume of the singular region is calculated as

\[
V_{\text{sing}} = \int_{z - \epsilon_z}^{z + \epsilon_z} dz' \int_{\rho - \epsilon_p}^{\rho + \epsilon_p} d\rho' \int_{\phi - \epsilon}^{\phi + \epsilon} d\phi' = 8\rho_0\epsilon_\rho\epsilon_\phi\epsilon_z
\]

(B.2)

The singular region can be approximated by a small sphere centered at the singularity with a radius of

\[
r_{\text{sing}} = \left(\frac{6\rho_0\epsilon_\rho\epsilon_\phi\epsilon_z}{\pi}\right)^{\frac{1}{3}}
\]

(B.3)

which provides an equal volume. Assuming \( \epsilon = \epsilon_\rho = \rho_0\epsilon_\phi = \epsilon_z \), the integral functions \( I_c^{\text{sing}} \), \( I_p^{\text{sing}} \), and \( I_z^{\text{sing}} \) can be calculated as

\[
I_c^{\text{sing}} \approx \int_0^{r_{\text{sing}}} r' dr' \int_0^\pi d\theta \sin \theta \int_0^{2\pi} d\phi' \frac{1}{r'} = 2\left(6\sqrt{\pi}\right)^\frac{3}{2} \epsilon^2
\]

(B.4)

\[
I_p^{\text{sing}} \approx \int_0^{r_{\text{sing}}} r' dr' \int_0^\pi d\theta \sin \theta \int_0^{2\pi} d\phi' \frac{\rho + r'\sin \theta}{r'} = 2\epsilon^2 \left(\rho_0(6\sqrt{\pi})^\frac{3}{2} + \pi \epsilon\right)
\]

(B.5)

\[
I_z^{\text{sing}} \approx \int_0^{r_{\text{sing}}} r' dr' \int_0^\pi d\theta \sin \theta \int_0^{2\pi} d\phi' \frac{r'\cos \theta}{r'} = 2\pi \epsilon^3
\]

(B.6)
When the source and observation points are on the soil surface, i.e., $z = 0$ and $z' = 0$, the Green's function given by Eq. (2.41) can be simplified as

$$G(\rho, z = 0; \rho', z' = 0) = \int_0^\infty dk_{\rho} J_0(k_{\rho}\rho') J_0(k_{\rho}\rho) \frac{k_{\rho}}{\sqrt{k_n^2 + k_{\rho}^2 + \alpha}} \quad (C.1)$$

Since $J_0(u)$ decays with $u^{-1/2}$ as $u \to \infty$, this integral converges slowly. The following approximation is valid for small $\alpha$

$$\frac{1}{\sqrt{k_n^2 + k_{\rho}^2 + \alpha}} \approx \frac{1}{\sqrt{k_n^2 + k_{\rho}^2}} \left(1 - \frac{\alpha}{\sqrt{k_n^2 + k_{\rho}^2}} + \frac{\alpha^2}{2(k_n^2 + k_{\rho}^2)} - \frac{\alpha^3}{3(k_n^2 + k_{\rho}^2)^{3/2}} \right) \quad (C.2)$$

Substituting Eq. (C.2) into Eq. (C.1), employing the multiplication identity for the zeroth order Bessel functions given by Eq. (2.47), and changing the orders of $\phi$ and $k_{\rho}$ integrals we obtain

$$G(\rho, z = 0; \rho', z' = 0) \approx \frac{1}{2\pi} \int_0^{2\pi} d\phi' \int_0^\infty dk_{\rho} J_0(k_{\rho}\rho')\frac{k_{\rho}}{\sqrt{k_n^2 + k_{\rho}^2}}$$

$$- \alpha \int_0^\infty dk_{\rho} J_0(k_{\rho}\rho') J_0(k_{\rho}\rho) \frac{k_{\rho}}{k_n^2 + k_{\rho}^2}$$

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The \( k_\rho \) integrals in Eq. (C.3) can be evaluated in closed form \([97, 98, 99]\), leading to

\[
G(p, z = 0; p', z' = 0) \approx \frac{1}{2\pi} \int_0^{2\pi} d\phi' \frac{\exp(ik_n\rho_1')}{\rho_1'} - \alpha K_0(-ik_n\rho_1)I_0(-ik_n\rho_1') - \frac{\alpha^2}{2\pi} \int_0^{2\pi} d\phi' \frac{\exp(ik_n\rho_1')}{ik_n} - \frac{\alpha^3}{4\pi} \int_0^{2\pi} d\phi' \frac{\rho_1 K_{-1}(-ik_n\rho_1)}{ik_n} \tag{C.4}
\]

where \( K_0, K_{-1}, \) and \( I_0 \) are modified Bessel functions. The remaining integrals are readily evaluated using quadrature.
APPENDIX D

ONE-DIMENSIONAL ANALYTICAL THERMAL MODELS

In this appendix, one-dimensional analytical thermal models for natural solar heating of soil and mines are developed. The analysis comprises extensions of work presented by Watson [20], who was able to obtain an analytical solution for the temperature of homogeneous soil heated by the diurnal cycle.

This work is organized in two parts. In Sect. D.1 we derive closed form expressions for the temperature distribution. In Sect. D.2 we present example results that illustrate these expressions.

D.1 Analysis

In Sect. D.1.1 we describe Watson’s model for periodic heating of homogeneous earth. In Section D.1.2 we address the presence of the mine via a straightforward extension of this model to plane stratified layers.

D.1.1 Homogeneous Ground

Consider first the heating of soil without a mine as shown in Fig. D.1. For a homogeneous medium the heat flow equation can be written as

\[ \kappa \frac{\partial^2 V(x, t)}{\partial x^2} = \frac{\partial V(x, t)}{\partial t} \]  

(D.1)
where \( V(x, t) \) [K] is the temperature distribution and \( \kappa \) [m\(^2\)/s] is the thermal diffusivity of the soil. Thermal diffusivity is the ratio of the thermal conductivity to the volumetric heat capacity. The boundary condition at the ground-air interface involves losses due to convection and radiation. These issues have been discussed in Chapter 2, where we show that \( V(x, t) \) must satisfy

\[
-K \frac{\partial V(x, t)}{\partial x} = -\varepsilon \sigma V^4(x, t) + \varepsilon \sigma T_{sky}^4(t) + h(T_{air}^4(t) - V(x = 0, t)) + S_0(1 - A)(1 - Cl)H(t)
\]

where the terms have been described in Chapter 2. For reference, a compilation of the thermal characteristics of some materials appears in Table D.1.1. The values given here are typical, but large variations are possible for most natural materials. (See for example, the properties for soil.) Using the linearization described in Eq. (1.17), this can ultimately be written as

\[
\frac{\partial V(x, t)}{\partial x} \approx V(x, t) \frac{1}{K_s} (h(t) + 4\varepsilon \sigma T_{sky}^3(t))
\]

Figure D.1: A homogeneous soil half space illuminated by sunlight.
<table>
<thead>
<tr>
<th>Material</th>
<th>$\kappa$ [m$^2$s$^{-1}$] $\times 10^3$</th>
<th>$\mathcal{K}$ [W m$^{-1}$ K$^{-1}$]</th>
<th>$D$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil (dry)</td>
<td>3.1</td>
<td>0.35</td>
<td>0.093</td>
</tr>
<tr>
<td>Soil (damp)</td>
<td>5.0</td>
<td>2.6</td>
<td>0.12</td>
</tr>
<tr>
<td>Soil (sandy)</td>
<td>3</td>
<td>0.39</td>
<td>0.091</td>
</tr>
<tr>
<td>Soil (moist clay)</td>
<td>5</td>
<td>1.26</td>
<td>0.12</td>
</tr>
<tr>
<td>Soil (coarse gravelly)</td>
<td>1.4</td>
<td>0.52</td>
<td>0.062</td>
</tr>
<tr>
<td>Clay</td>
<td>10.1</td>
<td>1.28</td>
<td>0.17</td>
</tr>
<tr>
<td>Gravel</td>
<td>8</td>
<td>1.26</td>
<td>0.15</td>
</tr>
<tr>
<td>Sandy gravel</td>
<td>14</td>
<td>2.51</td>
<td>0.20</td>
</tr>
<tr>
<td>Granite</td>
<td>12.7</td>
<td>1.9</td>
<td>0.19</td>
</tr>
<tr>
<td>Limestone</td>
<td>8.1</td>
<td>0.7</td>
<td>0.15</td>
</tr>
<tr>
<td>Sandstone</td>
<td>11.6</td>
<td>1.70</td>
<td>0.18</td>
</tr>
<tr>
<td>Concrete</td>
<td>4.8</td>
<td>0.92</td>
<td>0.12</td>
</tr>
<tr>
<td>Brick</td>
<td>3.1</td>
<td>0.45</td>
<td>0.092</td>
</tr>
<tr>
<td>Wood</td>
<td>1.2</td>
<td>0.17</td>
<td>0.057</td>
</tr>
<tr>
<td>Styrofoam</td>
<td>16</td>
<td>0.029</td>
<td>0.21</td>
</tr>
<tr>
<td>Bakelite</td>
<td>1.1</td>
<td>0.23</td>
<td>0.056</td>
</tr>
<tr>
<td>Lucite</td>
<td>0.9</td>
<td>0.15</td>
<td>0.050</td>
</tr>
<tr>
<td>Nylon</td>
<td>1.4</td>
<td>0.24</td>
<td>0.062</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>0.8</td>
<td>0.12</td>
<td>0.047</td>
</tr>
<tr>
<td>Teflon</td>
<td>1.2</td>
<td>0.26</td>
<td>0.057</td>
</tr>
<tr>
<td>Cast iron</td>
<td>121</td>
<td>57.0</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table D.1: Thermal characteristics of some materials.

\[- \frac{1}{\mathcal{K}_s} (S_0(1 - A)(1 - Cl)H(t) + h(t)T_{air}(t) + 4\epsilon\sigma T_{sky}^4(t)). \quad (D.3)\]

We begin the solution process by using a spectral expansion in time. The Fourier series coefficients of $V(x, t) - T_{sky}$ are given by

\[ \hat{V}_n(x) = \frac{1}{T} \int_0^T [V(x, t) - T_{sky}] e^{-i\omega n} \quad (D.4) \]
where the period $T = 24(60)^2$ [s] is one full day. Equation (D.1) can then be written in terms of the Fourier coefficients as

$$\kappa \frac{d^2 \hat{V}_n(x)}{dx^2} = -i\omega n \hat{V}_n(x)$$  \hspace{1cm} (D.5)

The solution of equation (D.5) is given by

$$\hat{V}_n(x) = C_{1,n}e^{-\beta_n x} + C_{2,n}e^{\beta_n x}$$  \hspace{1cm} (D.6)

where

$$\beta_n = \sqrt{-\frac{i\omega n}{\kappa}} = (1 - i)\sqrt{\frac{\omega n}{2\kappa}} = (1 - i)\sqrt{n/D}$$  \hspace{1cm} (D.7)

in which the characteristic depth $D$ is given by

$$D = \sqrt{\frac{2\kappa}{\omega}}$$  \hspace{1cm} (D.8)

Values of $D$ are given in Table D.1.1. We find that for typical soils $D$ is on the order of 10 cm. As $x \to \infty$, $\hat{V}_n(x, t)$ should be bounded, and imposing this condition equation (D.6) becomes

$$\hat{V}_n(x) = C_{1,n}e^{-(1-i)\sqrt{\frac{\omega n}{2\kappa}} x} = C_{1,n}e^{-(1-i)\sqrt{n/D} x/D}$$  \hspace{1cm} (D.9)

The quantity $H(t) + h(t)(T_{air} - T_{sky})/(S_0(1 - A)(1 - Cl))$ can also be expressed in a Fourier series with the following coefficients:

$$A_n e^{i\kappa n} = \frac{1}{T} \int_0^T dt [H(t) + h(t)(T_{air} - T_{sky})/(S_0(1 - A)(1 - Cl))] e^{-i\omega n t}$$  \hspace{1cm} (D.10)

The Fourier representation of the boundary condition in equation (D.2) is then

$$-\kappa \frac{\partial \hat{V}_n(x)}{\partial x} = -[4\kappa \sigma T_{sky}^3 + h]\hat{V}_n(x) + (1 - A)S_0 C A_n e^{i\kappa n}$$  \hspace{1cm} (D.11)

Using equation (D.9) in equation (D.11) and solving the resulting equation for $C_{1,n}$, we get

$$C_{1,n} = \frac{(1 - A)S_0 C A_n e^{i\kappa n}}{4\kappa \sigma T_{sky}^3 + h + \kappa \beta}$$  \hspace{1cm} (D.12)
It will be convenient to write this as a magnitude and phase

\[ C_{1,n} = C_n e^{+i\phi_n} \]  

where

\[ C_n = \frac{(1 - A)S_0 CA_n}{h + 4\varepsilon\sigma T_{sky}^3} \frac{1}{|1 + \gamma(1 - i)\sqrt{n}|} \]  

\[ \phi_n = \epsilon_n + \tan^{-1}\left( \frac{\gamma\sqrt{n}}{1 + \gamma\sqrt{n}} \right) \]  

\[ \gamma = \frac{K}{(h + 4\varepsilon\sigma T_{sky}^3)D} \]

We find that \( V(x, t) \) is given by

\[ V(x, t) = T_{sky} + \sum_{n=-\infty}^{\infty} C_n e^{-x\sqrt{\frac{2}{\kappa}}} e^{(n\epsilon t + \phi_n)} \]  

from which we can obtain the surface temperature \( V(0, t) \). Since \( \phi_n > \epsilon_n \) we see that the soil temperature always lags the solar insolation.

### D.1.2 Layered Ground

Now consider a one-dimensional mine that lies between depths \( L_1 \) and \( L_2 \) as shown in Fig. D.2. The thickness of the mine is given by

\[ \Delta = L_2 - L_1 \]

The solution of the heat flow equation in this case is expressed as \( V_1(x, t) \), \( V_2(x, t) \), and \( V_3(x, t) \) in regions 1, 2, and 3, respectively. These functions satisfy the heat flow equations in their region, namely:

\[ \kappa_j \frac{\partial^2 V_j(x, t)}{\partial x^2} = \frac{\partial V_j(x, t)}{\partial t} \quad j = 1, 2, 3 \]

where \( \kappa_1, \kappa_2 \) and \( \kappa_3 \) are respectively the thermal diffusivity of the upper soil layer, the mine-like material, and the lower soil layer. We will ultimately require that the
Figure D.2: A one-dimensional model of a buried mine. Soil resides in the upper and lower layers. The mine-like material occupies the layer $L_1 < X < L_2$.

The upper and lower media are the same, i.e., $\kappa_1 = \kappa_3$. At each soil-material interface the temperature is continuous\(^5\)

$$V_j(L_j, t) = V_{j+1}(L_j, t) \quad j = 1, 2$$  \hspace{1cm} (D.20)

and the derivatives must satisfy

$$\kappa_j \frac{\partial V_j(x, t)}{\partial x} \bigg|_{x=L_j} = \kappa_{j+1} \frac{\partial V_{j+1}(x, t)}{\partial x} \bigg|_{x=L_j}$$  \hspace{1cm} (D.21)

At the soil-air interface we have

$$- \left. \frac{\partial V_1(x, t)}{\partial x} \right|_{x=0} \approx -\alpha (V_1(0, t) - T_{sky}) + \frac{I_s(t) + h(t)(T_{air} - T_{sky})}{\kappa_1}$$  \hspace{1cm} (D.22)

\(^5\)This requirement implies that the mine and soil are in perfect thermal contact. In general there will be some finite contact resistance present at such a boundary, which we have ignored in this work.
where the constant \( \alpha \) is given by

\[
\alpha = \frac{h + 4\sigma \epsilon T_{sky}^3}{K_1} \quad [m^{-1}]
\]  

(D.23)

Proceeding as above, the functions \( V_1(x,t), V_2(x,t), \) and \( V_3(x,t) \) are expanded in Fourier series with coefficients

\[
\hat{V}_{nj}(x) = \frac{1}{T} \int_0^T [V_j(x,t) - T_{sky}]e^{-i\omega t} \quad j = 1,2,3
\]  

(D.24)

Substituting these Fourier series into equation (D.19) for \( j = 1,2,3 \) we obtain the ordinary differential equations

\[
\kappa_j \frac{d^2\hat{V}_{nj}(x)}{dx^2} = -i\omega n\hat{V}_{nj}(x) \quad j = 1,2,3
\]  

(D.25)

for which the solutions are

\[
\begin{align*}
\hat{V}_{n1}(x) &= C_{1,n}e^{-\beta_1 x} + C_{2,n}e^{+\beta_1 x} \quad 0 \leq x \leq L_1 \\
\hat{V}_{n2}(x) &= D_{1,n}e^{-\beta_2 x} + D_{2,n}e^{+\beta_2 x} \quad L_1 \leq x \leq L_2 \\
\hat{V}_{n3}(x) &= E_{1,n}e^{-\beta_3 x} \quad x > L_2
\end{align*}
\]  

(D.26) (D.27) (D.28)

where

\[
\beta_j = (1 - i)\sqrt{\frac{\omega n}{2\kappa_j}} \quad j = 1,2,3
\]  

(D.29)

The unknown coefficients \( C_{1,n}, C_{2,n}, D_{1,n}, D_{2,n}, \) and \( E_{1,n} \) are determined from the boundary conditions in a straightforward manner.

The problem can be treated via an analogy with transmission line theory, in which \( \beta_j \) is the propagation factor. We define a thermal “admittance”

\[
y_j = \kappa_j \beta_j \quad [W K^{-1} m^{-2}]
\]  

(D.30)
Using this definition, the equivalent admittance at \( x = 0 \) is \( \alpha K_1 \) which leads to the "reflection" and "transmission" coefficients at the surface

\[
\begin{align*}
 r_0 &= \frac{\beta_1 - \alpha}{\beta_1 + \alpha} \\
t_0 &= 1 + r_0 = \frac{2\beta_1}{\beta_1 + \alpha}
\end{align*}
\] (D.31, D.32)

For the remaining interfaces we define

\[
\begin{align*}
 r_j &= \frac{y_j - y_{j+1}}{y_j + y_{j+1}} \quad j = 1, 2 \\
t_j &= 1 + r_j = \frac{2y_j}{y_j + y_{j+1}} \quad j = 1, 2 \\
\frac{y_j}{y_{j+1}} &= \frac{1 + r_j}{1 - r_j} \quad j = 1, 2
\end{align*}
\] (D.33, D.34, D.35)

It is interesting to note that (like their electromagnetic analogs) these quantities are constants, i.e., independent of \( \beta \) for the media considered here. We find

\[
\hat{V}_n(x) = \frac{t_0(1 - A) S_0 C A_n e^{i\alpha}}{2y_1(1 - r_0 \hat{r}_1 e^{-2\beta_1 L_1})} \left\{ \begin{array}{ll}
e^{-\beta_1 x} + \hat{r}_1 e^{-\beta_1 L_1} e^{+\beta_1 (x - L_1)} & x' \leq x \leq L_1 \\
\frac{\frac{A_1 e^{-\beta_1 L_1}}{t_1 e^{-\beta_1 L_1}} \left[ e^{-\beta_2 (x - L_1)} + r_2 e^{-2\beta_2 \Delta} e^{+\beta_2 x} \right]}{1 + r_1 r_2 e^{-2\beta_2 \Delta}} & L_1 \leq x \leq L_2 \\
\frac{\frac{A_1 e^{-\beta_1 L_1}}{t_1 e^{-\beta_1 L_1}} \left[ e^{-\beta_2 (x - L_1)} + r_2 e^{-2\beta_2 \Delta} e^{+\beta_2 x} \right]}{1 + r_1 r_2 e^{-2\beta_2 \Delta}} & L_2 \leq x
\end{array} \right.
\] (D.36)

where

\[
\hat{r}_1 = \frac{r_1 + r_2 e^{-2\beta_2 \Delta}}{1 + r_1 r_2 e^{-2\beta_2 \Delta}}
\] (D.37)

To obtain the temperature distribution we simply evaluate the Fourier series in each region and add \( T_{sky} \).

The case of one layer over a homogeneous half space is given by \( \Delta \to \infty \), for which we find \( e^{-2\beta_2 \Delta} \to 0 \) and

\[
\hat{V}_n(x) = \frac{t_0(1 - A) S_0 C A_n e^{i\alpha}}{2y_1(1 - r_0 \hat{r}_1 e^{-2\beta_1 L_1})} \left\{ \begin{array}{ll}
e^{-\beta_1 x} + r_1 e^{-2\beta_1 L_1} e^{+\beta_1 x} & x' \leq x \leq L_1 \\
\hat{t}_1 e^{-\beta_1 L_1} e^{-\beta_2 (x - L_1)} & x > L_1
\end{array} \right.
\] (D.38)
D.2 Example Results

In this section, sample calculations based on the models developed above are provided. Where available, experimental results are compared with the calculation.

D.2.1 Homogeneous Earth

Plots of the temperature distribution in homogeneous soil appear in Figures D.3 and D.4. Here and throughout this section we use the model parameters $\lambda = 35^\circ$, $\delta = 0^\circ$, $A = 0.3$, $C = 0.8$, $d = 0$, $\psi = 0$, $\epsilon = 1$, $t_R = 600$, $t_S = 1800$ and $T_{air} = 289 \text{ K}$.

In addition, we have ignored convection ($h=0$). These values represent approximate values for an experiment performed on March 21. This date is the vernal equinox on which the celestial equator intersects the ecliptic, resulting in zero solar declination. The day and night are of equal length and this is used to select the sunrise and the sunset times. As previously mentioned, a smooth surface was selected, therefore, the surface normal angle is zero. An average air temperature of 289 K is assumed, which is a typical value for Columbus in spring. Soil thermal properties are those of damp soil (see Table D.1.1).

In Figure D.3 we show the predicted soil surface temperature as a function of time. This curve is compared with thermistor data collected during the DARPA Backgrounds experiment [100]. The absolute temperature readings depend on air temperature, ground cover, and the history of the solar insolation. To minimize the effects of this dependence we have normalized both the experimental and model results. Good agreement is observed. We have also plotted in Figure D.3 the insolation

\text{For reference, Washington D.C. is located at } 38^\circ53' \text{ latitude, } -77^\circ2' \text{ longitude.}
function $H(t)$. The results clearly show the lag $\phi_n - \epsilon_n$ between the incident radiation and the temperature of the soil.

Figure D.4 shows the soil temperature variation as functions of time and depth during one diurnal cycle, and evidences the exponential behavior noted for this model. Temperature variations are normalized to the range $[0, 1]$ to permit easy comparison with the figures that follow. Below $D \approx 10$ cm there is relatively little temperature variation, which indicates that mines will be essentially undetectable below this depth.

Figure D.3: Normalized surface temperature as a function of time using the model and DARPA Backgrounds measurements. We consider the limiting cases in which the mine is a good thermal conductor (iron) and a good thermal insulator (styrofoam)
Figure D.4: Temperature distribution in homogeneous soil as a function of time and depth for natural diurnal heating.

D.2.2 Layered Earth

Figure D.5 illustrates the surface temperature for the three-layer case in which the mine lies between $L_1 = 5$ cm and $L_2 = 10$ cm. We consider the limiting cases in which the mine is a good thermal conductor (iron) and a good thermal insulator (styrofoam). Figure D.6 illustrates the temperature difference between the layered earth cases and the homogeneous soil case. Comparing the conducting mine case with homogeneous ground we observe temperature differences in the range 0-2 [K] range. For the insulating mine the range increases to 5 K. The homogeneous soil curve intersects the half-space curves twice during the day. These intersection points are the well-known thermal "cross-over" times at which the mine signatures are lost.
In Figures D.7 and D.8 we plot the surface temperature distribution for burial depths $L_1 = 1, 3, 5, \text{ and } 10 \text{ cm with mine thickness } L_2 - L_1 = 5 \text{ cm. The intensity of the signature depends strongly on the burial depth. At depths near 10 cm the signature has decreased to much less than 1 K. This observation indicates that mines buried below this depth will have weak contrast in this soil.}

In Figures D.9 and D.10 we study the effect of soil conductivity on the signatures of both conducting and insulating mines. In both cases increasing soil conductivity leads to decreased surface temperature contrast. This finding is consistent with an electrical analog, which predicts that greater thermal resistivity between the soil surface and the mine will diminish the influence of the mine. In Figures D.11 and D.12 we present a similar study for soil diffusivity. In this case we find that increasing diffusivity tends to improve detection. The characteristic depth $D$ is proportional to $\sqrt{\kappa}$ and, hence, increasing diffusivity increases the interaction between the mine and the surface.

Figures D.13 and D.14 illustrate the temperature distribution in time and depth for the three-layer case involving conduction and insulating mines respectively. The mine depth is 5 cm and the mine thickness is 5 cm also. We find that the thermally conductive mine in Figure D.13 has relatively little effect on the temperature distribution at depth. Again using an electrical analog, the mine layer “shorts” the two soil regions, leading to little difference in temperature across the conductor. In contrast, the insulating mine case shown in Figure D.14 effectively blocks the flow of heat into and out of soil below the mine. The insulating mine is therefore, much easier to detect.
Figure D.5: Surface temperature for the three-layered ground.

Figure D.6: Difference in surface temperature for styrofoam and iron mine layers compared with homogeneous soil.
Finally, in Figure D.15 we study the effects of abrupt changes in time on the mine signatures. The figure shows the temperature with respect to homogeneous soil for an illumination of the form

\[
H(t) = \begin{cases} 
M(Z(t)) \cos Z'(t) & -t_r < t < t_c \\
0 & t_c < t < t_r 
\end{cases}
\]  
(D.39)

where \( t_c = 0.5T \) (mid day) is the cutoff-time for solar illumination. This model attempts to predict the response of the soil if the sun were to suddenly pass behind a cloud or be otherwise obscured. Figure D.15 illustrates the surface temperatures for a styrofoam or iron layer of thickness 5 cm when the illumination is as given in equation D.39. It is seen that when the illumination is obstructed, the temperature decreases rapidly. The decrease for a conducting mine is somewhat more abrupt than for an insulating mine as shown in Figure D.16. Temperature differences for the insulating mine are in the 0-5 K range. For the conducting mine temperature differences are in the 0-2 K range.
Figure D.7: Difference in surface temperature for the three-layered ground and iron mine layer. The depth of the mine is varied from 1 cm to 10 cm.
Figure D.8: Difference in surface temperature for the three-layered ground and styrofoam mine layer. The depth of the mine is varied from 1 cm to 10 cm.
Figure D.9: Difference in surface temperature for the three-layered ground and iron mine layer. The conductivity of the soil is varied from 1 to 5.
Figure D.10: Difference in surface temperature for the three-layered ground and sty­
rofoam mine layer. The conductivity of the soil is varied from 1 to 5.

Figure D.11: Difference in surface temperature for the three-layered ground and iron
mine layer. The soil diffusivity is varied from $1 \times 10^{-7}$ to $1 \times 10^{-6}$.  

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Figure D.12: Difference in surface temperature for the three-layered ground and styrofoam mine layer. The soil diffusivity is varied from $1 \times 10^{-7}$ to $1 \times 10^{-6}$. 
Figure D.13: Normalized temperature distribution for a three-layer earth as a function of time and depth. The middle layer has the thermal properties of iron.
Figure D.14: Normalized temperature distribution for a three-layer earth as a function of time and depth. The middle layer has the thermal properties of styrofoam.
Figure D.15: Surface temperature for the three-layered ground. The solar illumination is abruptly terminated at mid day.
Figure D.16: Difference in surface temperature for the three-layered ground. The solar illumination is abruptly terminated at mid day.


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