MACH BANDS AND RETINAL INTERACTION

DISSERTATION

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By

WERNER JOSEPH KOPPITZ, DIPLOM-PSYCHOLOGE, M.A.

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Approved by:

Philburn Ratliff
Adviser
Department of Psychology
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Introduction

Experimenting with rotating discs, supplied with black and white sectors, I have observed a phenomenon accidentally, the pursuit of which has directed me to a more general law of psychological optics (41, p. 303, translated from the German).

With these words Ernst Mach introduced the phenomenon today known in optics as Mach bands or Mach rings in the papers of the Vienna Akademie der Wissenschaften for 1866.

Table I (Appendix, p. 57) shows some of these discs, reproduced from the original etchings (Figures 1a, 1b, 2a and 2b). The impression which the observer receives from the rotating discs is rendered very skillfully in Figures 1c and 2c. That the pattern of the sectors does not influence the impression is readily seen. Sectors 1a and 1b do not appear different in rotation. Nor do sectors 2a and 2b. The rotation effect is determined solely by the ratio of black and white per circumference with the perplexing exception at β and γ. At these places, where the borders of the black sectors change their direction abruptly, and thus, the ratio between black and white alters discontinuously, rings appear, distinctly set off from the surrounding. A transition from light to dark evokes bright bands (Figure 1c); the reverse change leads to dark rings (Figure 2c).
It did not take Mach long to find out that the phenomenon is not restricted to rotating discs. A stationary field with a light distribution similar to the rotating discs yields the same contrast effects at the borders, where a sudden change in the light distribution takes place. If the field is round rings will appear; bands will be seen in a rectangular field. The form of the contrast phenomenon follows the outline of the border of the two adjacent fields of different illumination.

Since photographs of such fields show the contrast phenomenon also, the conclusion nearest at hand makes some properties of the physical light distribution responsible for the phenomena. Mach found an ingenious method to disprove such conclusion. Today it is much simpler to demonstrate it with the help of a photometric device. If we scan the field with a search tube of a photovoltmeter the indicator climbs steadily from dark to light. No increase or decrease in the reading occurs in the region of the bands. This indicates that the Mach bands are not generated by the physical distribution of the light, but must originate in the visual pathways of the observer.

From the beginning Mach was convinced that these contrast phenomena depend upon retinal interaction-

It seems to me that the discussed phenomena can be explained only by a reciprocal effect of neighbouring spots on the retina (41, p.317).
In Mach's second paper we find a still sharper formulation of this idea—

The brightness of a point on the retina depends... upon the illumination of a large retinal area that surrounds this point. Naturally the points closest to the original one have the most influence... (42, p.140).

The deviation of a sensation from the mean of the adjacent sensations is a central topic in Mach's Analyse der Empfindungen (45). With respect to light sensation Mach writes as follows.

Various experiments... led me to the conclusion that the illumination of a spot on the retina is felt in proportion to its deviation from the mean of the illumination of the adjacent positions. The value of the retinal positions in determining this mean is to be conceived as rapidly decreasing with their distance from the position under consideration, a fact which of course can only be explained as depending on an organic reciprocal action of the retinal elements on one another (45, p.178).

The deviation from the mean illumination of the neighboring elements can be expressed mathematically by the second partial derivatives of the intensity of illumination with respect to a two-coordinate system. A more penetrating treatment of Mach's mathematical formulations will be given in a later chapter comparing several mathematical treatments of the subject.

Without knowledge of Mach's previous work the astronomer H. Seelinger detected the same phenomena thirty years later while observing the shadow of the earth during an
eclipse of the moon (53). In the first volume of the Proceedings of the Physiological Society W. McDougall mentioned Mach bands for the first time in an American journal (48). August Kühl, a student of H. Seelinger, published a series of articles about the phenomena from 1921 to 1951 (32, 33, 34, 35). The only English psychologist who wrote about these border contrasts was R. H. Thouless (54). With the exception of August Kühl, the work of whom will be reported in connection with the discussion of the mathematical analysis and the practical application of the problem, these studies do not go beyond Mach's treatment of the phenomena.

Kurt Koffka and M. R. Harrower approach the problem in a completely different way. The contrast rings are regarded as being not in the same phenomenal plane as the rest of the field, but are considered detached figures on a ground, and, they are therefore subject to Gestalt rules. Conditions that favor Gestalt organization should favor Mach's bands also and vice versa. Color gradients are used in addition to brightness gradients to demonstrate these factors (30).

In the last 10 years increased interest was aroused in the problem of Mach bands probably because it was recognized that the phenomena are important in many applied fields, e.g., photography and television.
In 1948 Glenn A. Fry (17) published an article that describes not only an excellent experimental arrangement set-up for the projection of these rings but also a very interesting theory of inhibition due to changes in the electrical potential caused by stimulation.

Three very interesting studies appeared from the Instituto Nationale Di Ottica during the last two years. Adriana Fiorentini, the chief spirit behind these investigations, projected small areas of light upon an adapting field with Mach bands. The subject had the task of adjusting the intensity of the areas till they vanished. The intensity of the dots at the vanishing point was used to evaluate the brightness of the Mach band. One result of this study was the confirmation that the Mach effect depends not only upon the illumination of the field but also upon the steepness of the gradient of illumination (14).

In a second study emphasis was placed upon the time factor, and it was found that below an exposure time of 0.5 seconds the perception of Mach bands deteriorates rapidly (15) as exposure time decreases.

The influence of foveal and peripheral regions of the retina on the perception of the Mach bands was investigated in a third paper (16). Fiorentini observed differences between central and peripheral vision. These differences
are attributes to differences in the interaction mechanism of the rods and the cones. Observations of a hemeralopic subject supported the view that the vision of Mach bands is related to cone vision. The effect of the peripheral regions is to enhance the brightness of the graded field.

Not the brightness but the width of the bands is the subject of a study by McCollough (47). One series of measurements was taken with the gradient of illumination constant but at different levels of illumination; another series tested the effect of a change in the steepness of the gradient. The results show that there is a regular decrease of apparent width of a bright Mach band with increase of the luminance of the field (with the steepness of the gradient constant). A decrease in the width of the band is also observed if the gradient of the field changes from shallow to steep. McCollough indicates that, from a certain steepness of the gradient on, the Mach band vanishes, and a sharp contour is seen between the two fields of different brightness.

All investigators, whether they express it explicitly or not, locate the Mach effect in the periphery of the visual pathway. Some (17, 48) consider the contribution of higher stages past the retina in the process although most researchers follow Mach's original theory and locate the process in the retina. But nobody has designed an
experiment to demonstrate that higher centers are not
decisive for the process.

We attempted to investigate this question in a pre-
liminary study by presenting a graded field to one eye and
a uniform field to the other in such a way that the borders
of the two fields were coincident in stereoscopic view.
Such a presentation of the field does not have the effect
of evoking a Mach band at the border of the two half
fields. These results indicate that the band phenomenon
occurs at the retinal level.

The present experiment was planned to test the summa-
tion effect of two superimposed bands. The Mach effect
is apparently a characteristic of the retina in processing
visual stimuli of graded light distribution. One way of
gaining insight into these processes is observation of the
changes that the phenomenon undergoes when the illumina-
tion of the field is changed. This is the approach used
in most studies.

However, it was felt that this approach is lacking in
an essential feature. The Mach effect is phenomenological;
if we find ways to demonstrate the influences of these
phenomena upon each other, we might obtain additional infor-
modation and a better understanding of the underlying proc-
esses. The most direct way seemed to be the generation
of two Mach bands under the same conditions of illumination and superposing one upon the other.

There remained still the problem of expressing the results of such observations quantitatively. A measure suitable for this purpose is brightness discrimination on the background of the bands. In order to evaluate the contribution of regions distant from the Mach band in the entire process, the brightness measurements could not be limited to the band alone. Several points in equal distance were selected along the graded field and the apparent brightness measured in the same manner as for the band.
Method

Apparatus.

Two Bausch & Lomb Clason Acuity Meters were used as projectors. One of the acuity meters with a projection lamp type PH/500 T12F for 500 Watts at 120 Volts served to generate the adaptation field on a white cardboard screen at a distance of 450 cm. from the projector. The field had the size of 25 x 25 cm., including 4.8 of visual angle at a distance of 3 m. The Mach bands were produced by inserting a shield of \( \frac{1}{2} \) in. thick pressed wood into the light beam at a distance of 232 cm. from the projector. This distance was chosen because the Mach effect was optimal under the present conditions for this distance. The vertical Mach band appeared exactly in the middle of the adaptation field, separating it into a uniform highly illuminated right half and a left half, of which the shield cut off so much of the light that the illumination decreased gradually from light to dark. Or, expressed differently, the illumination of the left half was characterized by an extensive half shadow generated by the shield. In the following this light distribution is called the single band condition.

The insertion at the same distance of a second shield opposite the first one cut down part of the highly illuminated right half of the field and we obtained a graded
illumination there also. Again a bright Mach band appeared at the inner border. When the experimenter moved the second shield in the provided groove toward the first one, a position was easily reached where both bands were superimposed upon each other. In this case the middle vertical of the adaptation field showed the effect of two superimposed bands and from the middle, the region of maximal brightness, the light decreased to the right as well as to the left in intensity, reaching its minimum approximately 6 cm. from the midline. We refer to this situation in the following as the double band condition.

Levels of illumination of the adaptation field were controlled by Wratten neutral filters of density 0.5, 1.0, 1.5 and 2.0 inserted into the optics of the projector.

The second projector had the purpose of casting a vertical bar of light on different places of the adaptation field. This light bar served as stimulus for the brightness discrimination of the subjects. Several changes were necessary in order to equip the acuity meter for this purpose.

The discrimination stimulus was generated by a slit mounted close to the condensor of the projector and movable in horizontal direction very precisely by a micrometer screw. Thus it was possible to displace the test bar horizontally over a range of 10 cm. and to repeat any
position precisely as often as required. A photographic shutter controlled the exposure time. In order to obtain a luminance greater than the one exhibited by the bands a projection lamp type PH 750 T12P/ for 120 Volts and 750 Watts had to be provided for the projector. The heat generated by such a lamp made fan cooling of the light house mandatory.

Wratten neutral filters regulated the luminance of the brightness discrimination stimulus. Ten of the filters ranging from 0.01 to 0.9 densities in steps of 0.1 were arranged around the periphery of a rotating disc 33 cm. in diameter. Spring stops helped in changing the filters quickly and in locating them accurately. An additional filter of 1.0 density, mounted on a swing-arm, enabled the experimenter to change the test stimulus between 0.01 and 1.9 densities, a range far wider than required by the variability of the subjects.

In placing the projectors the experimenter had to take care of the difficulty of possible interference of the two light beams, one from the adaptation field projector, the other from the discrimination stimulus projector. Therefore both acuity meters were placed on a common platform at slight angles to each other. The deviation from the normal were 6.75° for the test bar projector and 9.2° for the field projector.
A black curtain separated the apparatus and the experimenter from the subject. Thus protection against stray light was guaranteed while the communication with the subject was not complicated. Table II (Appendix, p. 58) shows the experimental arrangement in form of a diagram.

Procedure.

The subject took his seat at a distance of 3 m. from the screen with the adaptation field and placed his chin in an adjustable head rest. For monocular view of the field he used the eye of his preference, the other eye being blindfolded. Two of the subjects preferred the left eye, the third subject used the right eye. The experimental space did not permit a seating arrangement in the normal of the screen. The deviation from the normal was, however, only approximately 10°.

The room was completely darkened and the subject for ten minutes adapted to the luminance of the field. The experimenter then introduced the graded field and explained the phenomenon to the subject. A demonstration of the double band situation followed. The subjects were asked to compare the brightness of the Mach bands in both situations.

After the subjects had given their reports, measurements of brightness discrimination began. For the single band condition the test spot appeared on the midline and
1, 2, 3 and 4 cm. to the left of the midline along the graded half field. This distance was sufficient to cover most of the range of the graded half field from maximum to minimum luminance. Seventy discriminations were recorded in each position. For the double band condition the midline was tested again and points 1, 2, 3 and 4 cm. to the right on the graded half field connected with the second Mach band.

The test stimulus extended vertically 58 minutes of visual angle, horizontally 5 minutes.

Two conditions of eye fixation were employed: 1) constant fixation of the midline, 2) fixation on the point where the test stimulus appeared. For condition one the fixation spot, a black dot 10 mm. in diameter (12 minutes of visual angle at a distance of 3 m.), was kept always on the midline and in the middle of the test bar; for condition two the dot was moved according to the position of the test light. While for condition two the test stimulus is always in the line of regard the following visual angles correspond to the four positions under condition one:

- 1 cm. to the right or left - 11.5 minutes,
- 2 cm. to the right or left - 23 minutes,
- 3 cm. to the right or left - 34.5 minutes,
- 4 cm. to the right or left - 46 minutes.
The sequence of presenting the discrimination stimulus followed a regular pattern. The subject kept his eye on the fixation point on the adaptation field. The experimenter announced "ready" and approximately one second later released the cable opening the shutter for the present time, thus projecting the test stimulus on the screen. Depending on the response of the subject -- on whether he had seen the stimulus or not -- the intensity of the test light was changed accordingly by turning the disc with the filters. Then the next presentation was announced. Though the timing of the presentation was not controlled, a regular rhythm of presentation established itself of approximately three responses per minute.

Testing technique.

A psychophysical technique, known as the "up and down" method and derived by Dixon and Mood (13), helped very much in cutting down the number of required presentations of the test stimulus. Nevertheless nearly 22,000 responses were recorded for evaluation. The characteristic feature of the technique is the use of the subject's last response for the determination of the next stimulus presentation. This not only keeps the number of presentations at a minimum, but gives the experimenter also constant control of the experimental session and helps him in detecting errors immediately.
Two prerequisites must be fulfilled in order to employ the technique:

1) The experimenter must have an approximate knowledge of the mean, otherwise the danger exists that too many trials are wasted in reaching the region of the mean, and the number of presentations is not sufficient for statistics based on large sample. In our case a few initial test trials made a determination of the approximate mean easy.

2) The steps in testing should not be bigger than 0.5 to 2.0°. Since it became apparent that the mean of the sigmas lies close to 0.09 log unit, the selected interval of one tenth of a log unit fell well inside the required range.

Subjects.

Three psychology graduate students ranging in age from 23 to 28 years served as subjects. Two had normal vision and did not wear any corrective lenses; the third subject used weak bifocal corrections, but not during the experiment.

Measurement of physical light distribution.

Since the test points were fairly close together, the Macbeth Illuminometer could not be used to distinguish the single points. Therefore it was used only in recording the highest and the lowest level of illumination of the
adaptation field. For a measurement of the single test spots a photovolt photometer type 520-M was employed.

First the luminance of the uniform adaptation field was measured with the Macbeth Illuminometer. Then the screen was removed and at its place, exactly in the same distance from the projector, the search unit of the photovolt meter was erected and another measurement of the same conditions of illumination was made. Thus the measurements of the photovolt photometer were standardized with the Macbeth Illuminometer.

After these initial measurements had established the standard, the single band condition was projected again. The search unit of the photovolt meter was now moved into the exact positions of the test stimuli and series of measurements for each point were taken. The same procedure was followed for the double band condition at all four levels of illumination.

Table III (Appendix, p. 59) shows the physical light distribution in logmillilamberts.

If one compares the midline values (M, Table III) it becomes apparent that in spite of the constant illumination of the field, the luminance under double band condition is consistently lower than the values under single band condition.
In order to render the results comparable, we have to make an allowance for the loss in illumination. The following transposition, equating the values obtained in the single band condition with the lower luminance of the double band condition, is an appropriate way to achieve this.

From the definition of transmission ($T$) as ratio of the intensity of transmitted light ($I_T$) to incident light ($I$) follows that

$$I_T = T \times I \quad (1)$$

Density ($D$) is defined as the logarithm of the reciprocal of the transmission to the base 10

$$D = \log_{10} \frac{1}{T} = -\log_{10} T$$

thus

$$-D = \log_{10} T$$

Writing (1) in logarithmic form and replacing $\log_{10} T$ by $-D$ we obtain

$$\log I_T = -D + \log I \quad (2)$$

Since we know $I$ from the photometric measurements of the field and $D$ is known from the recordings of the filter densities inserted into the light beam of the test stimulus, we obtain $\log I_T$ from (2). $\log I_T$ actually is $\log \Delta I$ and a plot of $\log \Delta I$ against $\log I$ over the whole range of illumination enables us to interpolate $\Delta I$ values for any illumination inside the measured range. In this manner the single band values were read from the individual
graph of each subject which corresponded to the illumination level present under double band conditions.

The coordination of the data collected in the single band situation to the level of the double band condition is connected with a certain loss of data. We cannot extrapolate the $\Delta I$ curve for intensity values below or above the measured range in the single band situation. Therefore no $\Delta I$ can be found for intensities below the lower limit of the intensity range of the single band condition.
Results

Before turning to the quantitative results of the experiment, we shall first mention the phenomenological reports of the subjects.

All subjects saw the Mach effect under both conditions very distinctly at the two higher levels of illumination, not so clearly at the third level of illumination. When the subjects compared the Mach bands in the single and in the double band condition, the band in the double band situation appeared brighter than that in the single band situation. The small difference in gradients, though readily detected by photometric measurements of the right and left half fields in the double band situation, could not be distinguished by the subjects.
The numerical results of the apparent brightness increments above the actual adaptation field are given in Charts 1, 2, 3 (Appendix, pp. 60, 61, 62) for each subject. Every entry represents the mean of a series of seventy presentations of the stimulus in logmillilamberts.

Tables IV, V, VI (Appendix, pp. 63, 64, 65) are a graphical representation of the results for the situation requiring constant fixation on the Mach band. A comparison of these graphs with the photometric measurement of the actual light distribution (Table III) shows that the same characteristics that appear on Table III cannot be found in the graphs representing the ΔI for the subjects.

The light distribution is characterized by the unequal steps between the different levels of luminance due to irregularities in the filters. Between levels 0.5 and 1.0 there is a difference of 0.2 log millilamberts, between 1.0 and 1.5, a step of 0.35 log millilamberts, while the difference between 1.5 and 2.0 amounts to approximately 0.1 log millilamberts. Only the results of subject KC (Table V) in the single band situation come close to the actual light distribution; the other subjects have nearly equal steps between the levels of luminance.

If we compare the graphs, we can distinguish two groups. Subject BR and subject KC reacted similarly to the task, subject JS (Table VI) not only has a considerably
lower differential threshold but also smaller differences between the single and the double band conditions.

The most astonishing result is the trend, especially clearly expressed in the discriminations of subject BR (Table IV), that a higher \( \Delta I \) is necessary to arouse brightness discrimination in the double band situation than in the single band situation. It will be remembered (Table III) that the photometric data showed slightly lower intensities for the double band situation. According to this loss in luminance we expect a lower \( \Delta I \) for such a situation. For the midline such a result, though in contrast to our expectations, is in agreement with the verbal report of the subjects as mentioned in the beginning of the chapter.

In order to test whether the differences between the single band and the double band conditions are due only to sampling errors or represent statistically significant differences, t-tests for the sample means were computed (12,p.124).

To bring both conditions to a comparable level, the transformation, mentioned above (p. 17), was carried out. It enabled us to compute 24 t-ratios, 19 of these are significant at the 1% level of significance, two at the 5% level of significance and three are situated within the region of acceptance.
Another check of the results was provided by the data obtained from the situation requiring fixation on the discrimination stimulus. After transformation nineteen t-ratios could be computed, eighteen of which are significant at the 1% level, one at the 5% level of significance.

The change of the Weber-ratio $\frac{\Delta I}{I}$ within the range of the measured luminances can be followed in Table VII (Appendix, p. 66). The ratios obtained with the background of a single Mach band are close to the ratios obtained from the measurements with the background of a uniform field without gradients of illumination. The ratios from the double band conditions lie well above the single band ratios. There are not points enough tested to determine the exact form of the curve for the Weber-ratio, it seems that between two and five millilamberts the ratio remains constant, for luminances lower than two millilamberts the ratio increases fast.

The effect of time exposure is shown on Table VIII (Appendix, p. 67). Since the exposure times of 0.04 sec. and 0.02 sec., used in the experiment, are above the critical duration (55, p.373), the actual effect of the different exposure times should not be very marked. Contrary to this expectation we find considerable differences between the $\Delta I$ values collected under stimulus exposure
of 0.04 and 0.02 sec. The $\Delta I$ ratios for subject JS range from 1.6 to 2.9, the mean ratio for subject JS is 1.8, for subject BR 2.0. These results fit well into results obtained in experiments with stimulus exposure below the critical duration subject to the Bunsen-Roscoe law.
Discussion.

As mentioned above (p. 7) all evidence indicates that the Mach effect is based upon peripheral processes. The most promising hypothesis, first advanced by Ernst Mach, considers the mutual influence of illuminated retinal elements responsible for the effect. Since Mach's time numerous experiments concerned with retinal interaction have increased our knowledge of these processes considerably. We will, therefore, attempt to relate the results of our experiment with the latest findings in the field of retinal interaction. However, before we can do so, it will be necessary to clarify and define the central concept appearing in investigations of retinal interaction, that is the concept of inhibition. Depending upon use by physiologically or psychologically inclined investigators the term designates quite different empirical observations. Moreover, the investigators in the same field do not always agree.

The concept of inhibition has a venerable tradition in psychology. It plays an important role in Johann Friedrich Herbart's psychology already. Herbart conceives inhibition (Hemmung) as a metaphysical force weakening the ideas of a person till they vanish below the "limen of
consciousness" (Schwelle des Bewusstseins), a metaphor apparently borrowed from the sensory physiology of his time.

At the end of the nineteenth century, G. Heyman (29) made the term inhibition the central concept of a general psychophysical law, which he formulated probably under the influence of Fechner's idea. The "law of inhibition" (Hemmungsgesetz) and its general application for all sense modalities do not concern us here; it will be sufficient to show what Heyman understood by inhibition.

A minimum threshold can be reached only under optimal conditions, that is, the complete absence of inhibitory stimuli. As soon as other stimuli are introduced into the field, of the same sense modality or of others, the threshold will rise.

Here we meet the first time the concept of inhibition as psychologists employ it in different forms. Inhibition has an influence upon the threshold and can be measured by the increase in threshold under the influence of inhibitory stimuli.

Physiologists, studying retinal interaction, arrived at different operational definitions.

The classical studies in the field of retinal interaction are the experiments of E. D. Adrian and R. Matthews (1, 2, 3) with excised eyes of conger eels. They measured
the latency and the rhythm of discharge in the optic nerve of the isolated preparation as a function of the light distribution and the intensity of the light used to excite the retina. When stimulating several spots on the retina simultaneously they found, with certain qualifications depending upon the distance and the size of the spots, a decrease in latency of the optic nerve response. The experimental conditions are, in principle, similar to the conditions of which Heymann's experiments are a prototype. A restricted area of the retina is stimulated and the reaction to this stimulation is measured. These data are compared with measurements of the same area with neighbouring regions stimulated simultaneously. However, depending upon the measures employed in the experiments, the conclusions drawn from the results are quite different in both cases. The rise in threshold, observed in the first group of experiments, leads to the assumption of an inhibitory mechanism, while Adrian and Matthews postulate a summation effect according to the decrease in measured latency of the response of the optic nerve.

In tracing the proper location of these processes Adrian and Matthews discovered that the effect of strychnine on the retina is most revealing (3). A strychnine solution applied to the retinal preparation will shorten the latency of optic nerve discharge, however only if several
spots are stimulated simultaneously. It has no effect when the light is cast upon one spot only.

This indicates that the photochemical processes of the receptors cannot be involved in the effect because we are not successful in single spot stimulation. Strychnine must have its effect upon the synaptic layers and facilitate their interconnections. The summation process of the four spot experiment, therefore, has to be located in the nervous layers of the retina, not in the receptors themselves.

The histology of the retina certainly supports this view, for the elaborate branching processes of the retinal neurons and those of the amacrine and horizontal cells would give an ample structural basis for communication between the different regions. When a restricted area of the retina is illuminated, there must be a widespread activation of the synaptic layers. Anything like an exclusive connection between a given group of receptors and a given ganglion cell must be out of the question (3,P286).

The methods that were so successful in animal experiments could not be employed in studies of the human retina. Fortunately in the flicker fusion method we possess a method which allows us to study retinal processes directly in the living human being. Why are we justified to accept the off as an index of retinal events? According to Graham (21,P333 ff.) there are three ways of measuring off:

1) In psychophysical studies -- the number of flashes at
the point at which the subject ceases to see flicker and perceives steady light.

2) In retinal action potential records — fusion is determined by the point at which the intermittent ripples of the potential disappear.

3) In optic nerve discharge records — the fusion point is the number of flashes required to produce a continuous discharge of impulses.

Essentially the cff is equivalent in all three cases to the effect produced by a steady light. While the third measurement is restricted to retinal preparations the first two measurements can be taken from human subjects. The results in all three different methods parallel one another quite well; therefore we can accept the fusion frequency method as an index of retinal events.

Granit and different collaborators used cff as a measure of retinal interaction in six "Comparative studies on the peripheral and central retina" (23). Their array of interaction phenomena is considerably wider than those observed by any investigators before them. Not only were they able to demonstrate spatial summation in the retina but temporal summation effects with supra- and subliminal stimuli also. Inhibition is defined in the last of theses papers (23) by the following experiment:
The cff of a test area (a semicircle of 1° of visual angle) was recorded. Then a second semicircle, illuminated by a steady light, appeared at the screen, and the cff for the test patch was determined again. When both areas were held at the same luminance level, a higher cff was measured in the presence of the inducing patch. A reduction of the luminance of the test patch to two thirds or a half of the luminance of the inducing field had a different effect. The cff did not increase at all but actually decreased slightly compared to the presentation of the test patch alone. "It seems necessary to classify the phenomenon as inhibition since the total effect of the interaction on the darker (semicircular testpatch) is removed...." (23, p. 669).

According to Granit inhibition seems to be chiefly a foveal property, at least at higher intensities of illumination. Experiments with peripheral regions of the retina did not bring the same clear cut results as the foveal areas. The localization of the inhibitory processes is assumed to be situated in the amacrine cells with their abundance of interconnections.

From Granit's experiments we can conclude that inhibition and summation measured by the cff are not different processes but the same, and the differences in the results
are due to the non-linear transformation of the stimuli by the retina.

Geldard (20), testing the effect of an inducing field of different luminance upon the foveal threshold, could not find inhibitory effects so long as the two fields were separated from each other. The higher the luminance of the inducing field the lower was the threshold of the test area, with one exception. When the inducing field was moved adjacent to the test field the threshold increased. Geldard admitted that this could be interpreted as inhibition, but he preferred another interpretation. The rise in threshold seemed to him an artifact partly due to the imperfection of light sources that they scatter light, partly due to refraction of the cornea, lens and other ocular media. Inducing field and test field have readily observed halos which overlapped when the two lights became contiguous. In Geldard's opinion the sensitivity of the test area is substantially lowered through fatigue by this halo effect. The correctness of such an explanation is indicated by a smaller increase of the threshold at lower luminances of the inducing field. Two antagonistic factors will be active in such a situation: 1) a facilitation effect resulting in induced heightening of the sensitivity of the fovea and 2) the fatigue effect from scattered light.
The facilitation effect could be attributed to the nervous layers of the retina, while the inhibition effect should be attributed to the flaws in the optical system not directly connected with the retina.

Beitel (5) was able to demonstrate that Geldard's results are valid only for the special case of approximately equal luminance in both the inducing and the test field. A facilitatory effect or as Beitel prefers to call it a summation effect, was observed with luminances from one to ten times the intensity of the light source and up to 17 minutes of separation between the test field and the inducing field. A rise of the inducing field to 100 times the intensity of the test patch resulted in an increase of the threshold. An inhibitory effect appears which becomes more marked with decreasing separation of the inducing and the test field.

Summation and inhibition effects could also be found for peripheral areas of the retina. With a separation of the fields up to 130 minutes of visual angle measurable summation was registered. The intensity of the inducing field had to be increased considerably to about 1000 times the intensity of the test field before an inhibitory effect appeared. A decrease in separation of the fields increased the inhibition effect, also a further rise in the intensity of the inducing field up to 10,000 times.
There is probably nobody who has done more to clarify the phenomena of retinal interaction than Hartline and his collaborators. From his laboratory come the best quantifiable results obtained mainly from preparations of the eye of the horse-shoe crab Limulus. The comparatively simple structure of the eye of these crustaceans offers valuable advantages for the study of retinal processes. Though the single ommatidia are connected by a plexus, no internuclear neurons could be detected in the plexus. Thus the recorded action potentials of any single nerve fiber represent the direct end-organ discharge. The only interaction effect observable in the eye of Limulus is inhibitory. However, Hartline remarks:

In the vertebrate retina interaction has even more complex effects, for it comprises both excitatory and inhibitory influence. Indeed, the diversity of the responses of various optic nerve fibers in the vertebrate eye is probably the result of a complex interplay of excitatory and inhibitory components of interaction in the retina. The inhibitory component of the interaction in the vertebrate retina is similar, in many respects, to the purely inhibitory interaction that we have found in the eye of Limulus ... (26, 670).

The action potentials arising from an isolated ommatidium furnish the measure of retinal interaction. They slow down when another ommatidium is stimulated. The effect is stronger with increasing intensity of stimulation of neighboring retinal units; it depends also upon the distance of the interfering ommatidia from the tested one.
The excellent control of the retinal process on the Limulus eye enables Hartline to limit the number of possible mechanisms for the processes by exemption of mechanisms that are not compatible with the observed data.

The inhibitory process cannot be based on a photochemical reaction, for the slowing-down of the discharge begins suddenly only a few tenths of a second after the onset of the stimulation of an ommatidium several millimeters from the tested receptor unit. Furthermore the latency of reaction to inhibition does not change markedly with change of distance between tested and interfering ommatidium.

Another possible explanation can be found in the electric currents generated by the retinal action potentials of the receptors in flowing through the volume conducting mass of the eye tissue. They might affect the activity of nearby retinal elements. It is, however, improbable that these currents influence retinal activity. They are small and transient, while the inhibition remains after an initial peak active for many seconds. Lorente de No's discussion of action currents of nerve in volume conductors (38, 38ff) mentions detailed experiments that support such conclusions, the time given for the action potential at the point of entrance being of the order of two milliseconds. Another argument against electric
currents can be found in the observation that the inhibitory effect vanishes as soon as the nerve strand of the tested ommatidium is completely isolated by dissection of the other strands. Such a dissection could hardly have altered the flow in the volume conducting medium.

This leaves only one conclusion. The inhibitory effect apparently depends on the integrity of the neural interconnections in the plexus of fibers just behind the layer of ommatidia. Here the process of inhibition, defined as a decrease in action potentials, must take place.

The relatively simple relationship between illumination and inhibition that Hartline found in the Limulus eye becomes more complicated if vertebrates are used for the experiments. Recording the action potentials from the optic nerve of the frog, Hartline (25) obtained three different forms of discharge: 1) "on" discharge -- during stimulation, 2) "off" discharge -- after cessation of stimulation, 3) "on-off" discharge -- a combination of 1) and 2) confined mainly to the onset and cessation of the stimulus.

Kuffler's study (37) of the discharge patterns after retinal stimulation demonstrates the intricacy of retinal interaction in vertebrates. He recorded the discharge patterns from the ganglion cells behind the unopened eye
of the cat. By exploration with small pencils of light, receptive fields (25) were mapped. These are minimal areas of the retina that must be illuminated to cause a discharge in the ganglion cell. Such receptive fields were usually found to be one to two millimeters in diameter dependent upon the light intensity. They shrink under high intensity and expand under low intensity. Kuffler reports the same three forms of discharge found by Hartline in the frog's retina. The discharge pattern from individual ganglion cells is not fixed. On, off, on-off discharges could be recorded from one ganglion cell according to the zones within its receptive field which were stimulated by a small spot of light. During dark-adaptation with dim background illumination spontaneous activity is regularly registered. It is reduced by illumination and by certain anesthetics.

There exists a general functional organization of the receptive fields. The central region responds to the least amount of stimulation, its discharge pattern is the opposite to the discharge pattern of the periphery. For instance the center may yield off-discharges, the periphery will produce on-discharges and vice versa. If the light is focused upon an intermediate region between periphery and center on-off discharges will appear. Thus it is possible to classify the receptive units into two
classes the on-center units and the off-center units dependent upon the center discharge of the fields. A conversion of one type into the other type by changing the conditions of illumination did not succeed.

Interaction of two light stimuli within a receptive field results in a modification of both discharges. If, however, one of the lights is much stronger than the other, the discharge is not markedly affected. Frequently a ganglion cell, while yielding an off-discharge, was unresponsive to a stimulation of the on-area of its receptive field during the period of off-discharge. Observing the interaction of two receptive fields with on-discharges Kuffler found that often a lack of addition of the excitatory influences developed. These instances tend to explain "some"anomalous" observations, such as lengthening of latent periods with stronger stimuli, or increased discharge frequencies with weaker ones" (37, p.67).

Kuffler distinguishes two different modes of inhibition in a paper especially dedicated to the problems of retinal interaction (36). Two small stimuli A and B are projected into the center of a receptive field with an "on-center discharge", A has an intensity below the threshold. Superimposed into any region with the exception of the center, A + B reinforce each other; projected into adjacent
regions in the center A does not add to B, but on the contrary reduces the combined response. This is one mode of inhibition encountered in the experiments.

The other inhibition process is connected with the off-discharge. After the stimulus is withdrawn from an off-region the connected ganglion cell reacts with a discharge. A simultaneous stimulation of the ganglion cell during the off-discharge via another pathway through an on-channel is either ineffective or greatly reduced.

The first inhibition process gives evidence for the existence of exclusively inhibitory pathways to a ganglion cell. The second, observed when two excitatory pathways converge on a ganglion cell, demonstrates that stimuli may evidently be either excitatory or inhibitory depending upon the background activity reaching the ganglion cell.

This survey of some of the main investigations concerned with the problem of retinal interaction shows the discrepancies in the use of a concept like 'inhibition' by different authors. The experimental situation is in principle the same in all these studies. An area of the retina is stimulated, and the reaction of the retina to this stimulation is recorded by different means. When a small area in the vicinity of the initial tested one is stimulated simultaneously, another record of the same area is taken. The effect which this additional stimulation
exercises upon the reaction of the subject serves as basis for the inference of a retinal interaction process. Psychologists using thresholds as measures infer inhibition if the subject has a higher threshold in the second situation. Physiologists study action potentials either in form of an electroretinogram, ganglion discharge or optic nerve discharge. If they find a delay, a slowing down or a decrease in activity in the second situation, they call the effect of the additional stimulated area inhibitory.

As a general rule higher intensities of the stimulus impinging upon the retina are followed by an increased discharge activity (21). Therefore a reduction in discharge activity is not always necessary to infer inhibition. To speak of inhibition in a physiological formulation it is sufficient that no increase in discharge activity occurs when an additional area is stimulated.

Higher intensities of the stimulus, on the other hand, are correlated with a higher threshold of detection of the stimulus, and inhibitory processes defined by psychologists are characterized by an increase in threshold. Consequently a rise in threshold should be correlated with a reduction or at least an unchanged activity of discharge. Apparently there is a discrepancy between both concepts of inhibition, since we have just seen that discharge
activity increases with increasing intensity of the stimulus.

In both cases the concept 'inhibition' is based upon different stages in the visual process. Action potentials are measures closest to the changes evoked by stimulation of the retina, but they cannot furnish any information about the process of seeing the stimulus. In psycho-physical experiments we must rely upon measurements several stages remote from the source of excitation and distorted by modes of reaction of the organism to excitation, e.g., the discrimination of the light stimulus. We cannot measure the inhibitory effect upon the retina, we measure the increased stimuli necessary to overcome the effects which we accordingly infer as inhibitory. Using such remote measures we have no way of saying anything about the activity of the nerve fibers, but it is reasonable to suppose that the inhibitory decline in discharge activity in order to keep the organism's response above threshold level. Thus it is not too difficult to bring both concepts of inhibition to an agreement.
The results of the present experiment show explicitly a less intense ΔI, necessary to arouse brightness discrimination in the single band situation, compared to the ΔI, necessary for discrimination in the double band situation in spite of the fact that the luminance was lower in the double band situation.

There are a number of studies in vision with the same results. As early as 1923 Martin (46) found that "a considerable reduction of the contrast limen can be obtained by stimulating to an approximately equal extent the parts of the retina surrounding the portion on which the contrast is to be observed (46, p.483)." Geldard's (20) and Beitel's (5) investigations contain concordant results. Geldard used off, Beitel the absolute threshold as measure. Both obtained lower threshold values when in addition to the test area a neighboring retinal area was illuminated. Beitel's results, however, are pertinent only for the midline. The gradient of illumination in the graded half field declines sharply and 30 mm. from the midline the intensity of the stimulus dwindles to a thousandth of the midline intensity. For such a difference between test field and inducing field Beitel found an increase of threshold intensity instead of the decrease observed at approximately equal intensity levels of both fields. In opposition to these findings our data do not change
significantly, \( \Delta I \) remains lower for all tested points along the luminance gradient. This discrepancy, however, can be explained as due to different conditions. In Beitel's experiment a completely dark stripe separated test field and inducing field, while we have a gradual change of luminance.

Crawford (9) demonstrated the effect of short exposures of the inducing field (1/2 sec.) upon the differential threshold in the fovea. A much higher \( \Delta I \) was necessary for a discrimination when the test stimulus coincided with the onset of the inducing stimulus. One tenth of a second after the onset of the inducing stimulus \( \Delta I \) decreases to nearly half the initial value. A slight increase of \( \Delta I \) occurred again toward the end of the exposure of the inducing field and shortly after cessation. The most striking result of these experiments in the increase of \( \Delta I \) before the onset of the inducing stimulus. The decrease of \( \Delta I \) after the inducing stimulus was effective for a while, indicates a similar effect to the one we were able to observe. The increase of \( \Delta I \) shortly before onset and cessation of the inducing field demonstrates that the process of interaction of both fields must be quite complicated.

Baker (4) measured the threshold intensity increment during the course of light adaptation. Since data
collected by the recovery method fit well into a photo-
chemical theory of the adaption process it could be ex-
pected the threshold increment $\Delta I$ also could be traced as
a dependent variable of $I$ present at the time of the
test. Thus a negatively accelerated monotonically increas-
ing function for $\Delta I$ should be the result. This was not
the case. Contrary to expectation the $\Delta I$ at the initial
low intensities was a maximum, declined sharply to a mini-
mum, increased again slightly, and after approximately
200 sec. the $\log \Delta I$ curve levelled off well below the
initial value. Disregarding the time factor the results
are in principle the same.

The evidence sampled above seems to be sufficient to
establish the correctness of our results. It remains the
task to find an explanation for them.

Graham, Brown and Mote (22) emphasized the spatial
characteristics of the visual excitatory process. An
illuminated retinal area does not exhibit a uniform degree
of excitation in all of its associated nerve fibers. The
main effect is supposed to be in the center, the smallest
degree of excitation occurs in the outermost zones. A
gradient of effect connects the center and the periphery
of the retinal region. This principle of explanation
cannot be brought into agreement with our results. If we
follow its lead the interaction effect should be largest
for the midline and practically zero at the outer border of the field. Our results do not show such a gradient, the effect is nearly equal over the whole tested area.

Baker explains his results in terms of the ability of the individual sense cell to respond to a stimulus.

The initial high rate of response to $I$ stresses the response limit of the cell and larger values of $\Delta I$ are required to produce additional effects until the rate of formation of photochemical products slows to a point where the rate becomes a limiting factor (21, p. 178).

This is a possible explanation during a process of light adaptation for which the field luminance is constantly changing, but it is very improbable that such characteristics of the sense cell could account for the results obtained under the conditions of our experiment. The luminance during any one series of measurements does not change; hence the rate of photochemical decay and regeneration must be constant after the initial adaptation and no additional effects are required. In addition to this argument we might add Hartline's discussion (see p. 31) eliminating a photochemical effect as cause for the interaction.

In order to formulate our results in terms of neural interaction we shall follow the hypotheses developed by Ratoosh and Graham in a brightness discrimination study.
(51). Two nearby points of a stimulated foveal region are considered, and it is assumed that

a small increase in retinal illumination at one point has two effects: it increases the activity at the point illuminated, and decreases the activity at the nearby point, this decrease becoming smaller as the distance between the points increases (51, p.374).

Two assumptions are made about the retinal activity, A.

1) For a threshold (absolute or difference) discrimination, the difference between the activity, A, in that part of the retina stimulated by the test field before ΔI is presented and the activity in the same region when ΔI is presented is constant. 2) The activity, A, in a retinal region is a negatively accelerated, monotonically increasing function of the illumination of that region.

One conclusion drawn from the two assumptions of retinal activity makes ΔI a monotonically increasing function of I. Assumption 2) and the initial assumption about the effect of an increase in illumination upon the activity A allow statements about the activity of the region where the discrimination stimulus is projected. It is determined not only by the intensity I of the illumination, but also by the surrounding illumination. A high luminance of the surround will increase the activity on the test area while low luminances of the surround will affect activity of the test region to a far smaller degree.
If we call the activity of the test area before $\Delta I$ is projected $A_I$ and the activity of the same area after $\Delta I$ is projected $A(I + \Delta I)$, then according to the first assumption

$$A(I + \Delta I) - A_I = \text{constant}$$

To satisfy this equation $A(I + \Delta I)$ will be smaller for small $A_I$ and larger for large $A_I$. Since, taking assumption two into account, $A$ is a monotonically increasing function of $I$, we can now make a statement about $\Delta I$, the brightness discrimination increment. It varies accordingly depending on activity, that is, for intense activity on the test region a large $\Delta I$ is needed; a restricted activity of the test area will provoke discrimination with a smaller $\Delta I$.

In the single band situation we have a strong illuminated half field to the right of the test region. We must expect a severe decrease of the activity on the midline as consequence of the increased activity of that area. Since the activity $A_I$ is reduced, $A(I + \Delta I)$ will be smaller also and so will $\Delta I$ according to the derivation above.

In the double band situation, on the other hand, the highly illuminated half field is replaced by a graded field that cannot exert much influence upon the activity of the midline test region. Since the reduction of $A_I$ is
much less a larger \( A_{(I + \Delta I)} \) will be necessary to keep the difference constant. Thus our results are in agreement with the evidence accumulated by previous investigations.

These interaction effects are not characteristic of vision alone. Quite similar processes are encountered in audition as described by Galambos and Davis (19). The spontaneous discharge of nerve impulses of single auditory nerve fibers in cats can sometimes be decreased or stopped by certain tones or noises. It is not necessary to go into details, but it may be mentioned that one of the possible explanations for the phenomenon is based upon a similar hypothesis of interaction employed in our case for the retina.

There seem to exist direct neural connections between different parts of the cochlea. Poljak found "spiral fibers" which enter a "plexus" under the organ of Corti and eventually innervate a group of hair cells at some distance from their point of entry. They are stated to "... give rise to fine twigs which lie imbedded between the hair cells and are in intimate contact with them." (49, p.301). The parallel to Hartline's findings concerning the importance of the retinal plexus to inhibition in the retina of Limulus (see, p. 32) is obvious and increases the probability that we will find the interaction effects mediated by the plexi interconnecting sensory elements.
Is there any other possibility to account for the results?

Boynton & Riggs (6) suppose that at higher levels of excitation stray light within the eyeball becomes a factor of increasing importance in visual responses. They were able to demonstrate this hypothesis by recording action potentials characteristic of scotopic vision from the fovea and even the blind spot. Applied to the present experiment stray light can only be effective in the single band situation at least to an appreciative degree. The effect can only be active in such a way that by multiple reflection and diffraction within the ocular chamber light is distributed equally over the whole retina. This may affect the darkest positions of the retinal field somewhat and therefore a small effect upon the steepness of the light gradient might result from stray light, but it cannot have marked effect upon the band region, where we are well in the middle range of luminances. Moreover, stray light effects are usually reported only in connection with glare sources (18) and since inducing field and test field are well in the middle range of luminances—between \(-1\) and \(1\) log millilamberts—a stray light effect is not very probable.

In this connection another question should be answered: Why was the natural pupil used for the experiments?
From Reeves (52) data we know that the pupil changes rapidly only for the high luminances of the field. The visual field in our experiment did not extend beyond 5° of visual angle, so that it should not be influenced by the pupil opening. A slight contraction in the single band situation would mean a reduction in the light reaching the retina under equivalent circumstances. Consequently \( \Delta I \) should increase in this situation. Since we found a decrease in \( \Delta I \) for the single band situation it makes our argument only stronger.

The field conditions in which Mach bands appear can be interpreted as a more general case of simultaneous contrast observed as early as Aristotle's time. Since the classic study of Hess and Pretori (28) the topic found a large number of investigators (11, 17, 20, 17). All studies agree -- some with qualifications (27) -- that apparent brightness of the test field decreases with an increase of the luminance of the inducing field. The depressing effect upon the apparent brightness of the test field is directly proportional to the luminance of the inducing field according to Diamond (11). These findings support our results. In discussing the results of the contrast experiment Diamond and Heinemann feel confident to exclude the possibility of stray light and prefer retinal interaction as an explanatory principle.
Quantitative Evaluation of Mach Bands.

Mach, in his first report, based the quantitative evaluation of the observed effect upon three facts derived from experiment (41:p313): 1) The rings remain unchanged in appearance when the intensity function of the illumination changes from \( i = f(x) \) to \( i = qf(x) \), where \( q \) is a constant; 2) The rings vanish when \( i = f(x) \) changes to \( i = p + f(x) \); where \( p \) is a large, positive constant; 3) The clearness of the effect is strengthened by increasing the deviation from the gradual change in illumination of the adjacent region. Mathematically expressed this means that the clearness of the rings depends on the second derivative of \( i \) with respect to \( x \), \( \frac{d^2i}{dx^2} \). Using Fechner's formula for the sensation of light Mach developed a formula which encompassed the experimental facts encountered in his experiments.

Many years later Mach (45) gave a final formulation to the problem. The core of the argument centers around the sentence: "...the illumination of a position on the retina is felt in proportion to its deviation from the mean of the illumination of the adjacent positions." A coordinate system is laid upon the retina and the intensity of illumination of a particular spot on the retina in respect to this coordinate system is given by \( i = f(x,y) \). Then the mean value determining the sensation can be
represented approximately by

\[ 1 + \frac{m}{2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \]

where \( m \) is a constant. The sensation becomes brighter if the second derivative becomes positive, darker if the second derivative becomes negative provided that the derivative does not become infinite. In this case the expression given above is useless.

Geometrically the second derivative can be interpreted as the change in the curve \( i = f(x,y) \) when it changes its direction. Applied to the conditions of our experiment the left part of the curve \( i = f(x,y) \) remains the same in both situations. The right branch of the curve runs parallel to the \( x \)-axis in the single band situation, while in the double band situation the right part is approximately the mirror image of the left part of the illumination curve. It is apparent immediately that the change is more acute in the double band situation than in the single band situation. According to Mach's expression for the sensation the double band situation must yield the stronger sensation than the single band situation. This agrees with the phenomenological reports of the subjects, but it is in contradiction with the results obtained in the brightness discrimination experiments.
Kühl, in his quantitative solution of the Mach effect (32), arrives at a more complex solution for the ultimate stimulation ("letzte" Erregung) on the retina

\[ e = i \sum_{n} \frac{\varphi(r)}{\sum_{n} \varphi(r)} \, dv, \]

where \( i = f(x,y) \) is the intensity function, and \( \varphi(r) \) the factor determining the contribution of every element ev situated at the distance \( r \) from the test-spot.

The only assumption underlying this derivation is not different from Mach's assumption that the retinal elements act and react on one another and that the contribution diminishes with the distance from the element under observation. Mathematically expressed \( \varphi(r) \) decreases rapidly starting its maximum at \( r = 0 \) and probably approaching zero asymptotically very soon.

If the secondary influence of the asymptotic branch is neglected, \( \varphi(r) \) may be replaced by \( \psi(r) \) which quickly becomes zero with increasing \( r \). In approximation, then,

\[ i = i \sum_{n} \frac{\psi(r)}{\sum_{n} \psi(r)} \, dv. \]

In order to carry out the summation it is necessary to develop \( i = f(x,y) \) for a neighboring spot

\[ i = f(x+h, y+k). \sum_{n} \psi(r) \, dv \]

can now be expressed by a sum of double integrals where each integral expands over a
neighboring spot in a quadrant of the xy-plane. Since \( \psi(r) \) vanishes for larger values of \( r \) the integration can set the limits from 0 to \( \infty \). By that the double integrals become constant factors, and we may write

\[
\int \int \psi(r^2 + k^2) \, dh \, dk = m
\]

\[
\int \int \psi(r^2 + k^2) h^2 \, dh \, dk = \int \int \psi(r^2 + k^2) k^2 \, dh \, dk = n
\]

For \( e \) we thus obtain

\[
e = 1 + \frac{4m1}{4m1 + 2n(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})}
\]

and replacing \( \frac{2n}{4m} \) by \( k \) and \( \Delta i \) for \( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) we can finally write

\[
e = \frac{1}{1 + \frac{k}{2} \Delta i}
\]

Applying the formula to the results of our experiment we note that the only variable in the formula is \( \Delta i \) if we compare the single with the double band situation. The intensity \( i \) remains the same in both situations though it is the discrete value of two different functions; \( k \) the contribution constant of the retinal elements to the final stimulation has been separated from the intensity of illumination and therefore is also the same under both conditions. Only the second derivative here called \( \Delta i \), is different in the single and the double band situation. We can use the same argument mentioned above (see p.48) and state that \( \Delta i \) in the double band situation is larger than
in the single band situation. According to Kühl's formula e must then be smaller in the double band situation and larger in the single band situation. Again we encounter the same contrast compared with the results of the experiment which we met comparing the results with Mach's quantitative formulation. A solution without the additional employment of retinal interaction theories is not possible.

G. A. Fry (17) developed a quantitative solution for the Mach effect in terms of electrical potentials between points on the retina. The total potential difference of a point P on the retina produced by all elements of area surrounding the point is given by

\[ E = K \int_{-\infty}^{\infty} I(x_1y) e^{\exp\left(-\frac{1}{2} \frac{x^2}{\sigma^2}\right)} e^{\exp\left(-\frac{1}{2} \frac{y^2}{\sigma^2}\right)} dx dy \]

The constant \( \sigma \) expresses the extent of the region which contributes to E at P. Thus E is the sum of all the contributing elements of the retina. Each element contributes in direct proportion to its intensity of illumination and reciprocal to its distance from the tested area. Since in our case the conditions are arranged in such a way that the intensity I remains constant for all values of y for a given x value, we can simplify Fry's formula and write:

\[ E = K \int_{-\infty}^{\infty} I(x_1y) e^{\exp\left(-\frac{1}{2} \frac{x^2}{\sigma^2}\right)} dx \]
Again we run into the same problem as before. The integral is bound to be larger in the single band situation on account of the higher total illumination, but our results yield the smaller differential threshold for this condition.

The mathematically most elegant solution of the differences between original and perceived light distributions that we find in the Mach effect does not come from psychology nor optics but from communication theory. Kovácsznay and Joseph (31) are actually not concerned about the Mach effect at all but about the processing of black and white pictures. They search for operators which transform the original function \( f(x,y) \) into the resulting function of the picture \( F(x,y) \). Of the two operators which determine the picture the first one called "contour enhancement" contains the conditions of the Mach effect. It is shown that a linear operator involving the Laplace operator of the picture function \( f(x;y) \) accomplishes the transformation of \( f(x,y) \) into \( F(x,y) \). From the characteristics of vision two more properties of the operator can be derived. The resulting picture remains the same when we shift it and when we turn it except for the shifting and turning. Therefore, the operator must be homogeneous and isotropic. If, in addition, the resulting picture \( F \) depends only on the behavior of \( f \) in the infinitesimal
neighborhood of the point \( P(x,y) \) we can restrict our attention to differential operators.

The most general linear isotropic operator is given by:

\[
F(x,y) = Af(x,y) + B\nabla^2 f(x,y) + C\nabla^4 f(x,y) + \ldots
\]

where

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}
\]

If only the first two terms are kept, we have the simplest nontrivial isotropic linear differential operator. With a convenient choice for \( A \) and \( B \), we obtain

\[
F(x,y) = f(x,y) - \gamma^2 \left[ \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right]
\]

and that is with the exception of the sign Mach's original formula.
APPENDIX
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Table VI

Single Band vs. Double Band

Subject J.S.

Comparison at: Yen
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AUTOBIOGRAPHY

Born November 5, 1915, in Berlin, Germany, I, Werner Joseph Koppitz, was educated in Germany. Having finished secondary education at the Kaiserin Augusta Gymnasium in Berlin in 1935 I was refused entrance to a university under the regulations of the Hitler regime. I became a mechanic and it was not before 1946 that I was able to start studies at the University of Berlin. In 1951 I obtained the degree of Diplom Psychologe from that institution. The difficult working conditions in Germany determined me to accept the opportunity to come to the United States. During the first year after my arrival, in 1952, I worked for the Department of Aviation Psychology at Ohio State University. At the beginning of the summer quarter of 1953 I entered Graduate School and received the M.A. degree in 1955 from Ohio State University. While completing the requirements for the degree Doctor of Philosophy I was assistant to Professor S. Renshaw and Professor Ph. Batoosh.