INVESTIGATION OF A SELF-EXCITED
DRIFT-TUBE KLYSTRON FREQUENCY MULTIPLIER
FOR USE IN GENERATING MILLIMETER WAVES

DISSERTATION

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By

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**TABLE OF CONTENTS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Review of Recent Frequency Multiplier Developments</td>
<td>3</td>
</tr>
<tr>
<td>SOME THEORETICAL CONSIDERATIONS OF THE SELF-EXCITED DRIFT-TUBE KLYSTRON FREQUENCY MULTIPLIER</td>
<td>12</td>
</tr>
<tr>
<td>Debunching Effects</td>
<td>37</td>
</tr>
<tr>
<td>THEORETICAL EFFICIENCY OF THE DTKFM</td>
<td>44</td>
</tr>
<tr>
<td>FREQUENCY LIMITATIONS  A Comparison between Frequency Multipliers, Reflex Klystrons, and Two-Gap Klystrons</td>
<td>45</td>
</tr>
<tr>
<td>DESIGN AND CONSTRUCTION OF THE SELF-EXCITED DTKFM</td>
<td>58</td>
</tr>
<tr>
<td>DTKFM Design Procedure</td>
<td>70</td>
</tr>
<tr>
<td>EXPERIMENTAL RESULTS OBTAINED WITH THE SELF-EXCITED DRIFT-TUBE KLYSTRON FREQUENCY MULTIPLIER</td>
<td>72</td>
</tr>
<tr>
<td>ONE CENTIMETER TO FIVE MILLIMETER DRIFT-TUBE KLYSTRON FREQUENCY MULTIPLIERS</td>
<td>91</td>
</tr>
<tr>
<td>SUMMARY AND CONCLUSIONS</td>
<td>96</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>101</td>
</tr>
</tbody>
</table>
## LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Self-excited frequency multiplier employing a retarding-field oscillator.</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>A cross-section of the self-excited frequency multiplier resonators. The harmonic waveguide also supports the drift tube.</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>Idealized schematic of the resonator gaps.</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>Relative power and current of each harmonic as a percentage of the fundamental.</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>Plot of Equation one for design purposes.</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>Harmonic gap position for optimum output relative to oscillator gap positions. Distance between oscillator gaps has been taken as unity.</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>Effect of gap transit angle on fundamental current.</td>
<td>38</td>
</tr>
<tr>
<td>8</td>
<td>Effect of gap transit angle on third harmonic current.</td>
<td>39</td>
</tr>
<tr>
<td>9</td>
<td>Effect of gap transit angle on fourth harmonic current.</td>
<td>40</td>
</tr>
<tr>
<td>10</td>
<td>Effect of gap transit angle on sixth harmonic current.</td>
<td>40</td>
</tr>
<tr>
<td>11</td>
<td>Plot of average beam coupling coefficient.</td>
<td>55</td>
</tr>
<tr>
<td>12</td>
<td>Approximate field configuration.</td>
<td>60</td>
</tr>
<tr>
<td>13</td>
<td>The electron gun for supplying a 0.050 inch beam.</td>
<td>64</td>
</tr>
<tr>
<td>14</td>
<td>Drift-tube klystron multiplier assembly.</td>
<td>65</td>
</tr>
<tr>
<td>15</td>
<td>Calibration of the magnetic field used in the DTKFM.</td>
<td>66</td>
</tr>
</tbody>
</table>
### LIST OF ILLUSTRATIONS (cont.)

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>Dimensions applicable in calculation of resonant wavelength.</td>
<td>68</td>
</tr>
<tr>
<td>17</td>
<td>Sketch indicating that the actual resonator appears to be two singly reentrant resonators back-to-back.</td>
<td>75</td>
</tr>
<tr>
<td>18</td>
<td>Cross-section of the drift-tube showing the harmonic resonator and its coupling system. It will be noted that the drift-tube is made in two pieces and the gap is beyond the center indicated by the dotted line.</td>
<td>80</td>
</tr>
<tr>
<td>19</td>
<td>Drift-tube and harmonic resonator design for location of the harmonic gap close to either the buncher or catcher gap.</td>
<td>86</td>
</tr>
<tr>
<td>20</td>
<td>Design of the one centimeter to five millimeter DTKFM.</td>
<td>92</td>
</tr>
<tr>
<td>21</td>
<td>Comparison of three types of multipliers.</td>
<td>99</td>
</tr>
</tbody>
</table>
INVESTIGATION OF A SELF-EXCITED DRIFT-TUBE
KLYSTRON FREQUENCY MULTIPLIER
FOR USE IN GENERATING MILLIMETER WAVES

Introduction

This investigation concerns a new device for generation of rf power in the millimeter wavelength range. The problem of generation of more than microwatts of rf power in the region below a wavelength of four millimeters is a difficult one if coherent continuous wave output is desired. Pulsed rf output in this region is available by various means such as the Columbia Radiation Laboratories 3.3 MM Magnetron and cw output has been obtained by backward wave oscillators such as developed by Karp.1 There are deficiencies in all these devices so that any new means of MM waves generation is of interest.

The idea of a self-excited frequency multiplier is a relatively new one.2 Its main advantage appears to be in extending the frequency range of transit time types of oscillators over that to which the oscillator can be directly scaled. In direct scaling, such problems as gun

design for good electronic interaction and heat dissipation limit the maximum frequency of oscillation to a value less than theoretically possible. The self-excited frequency multiplier utilizes an oscillator operating efficiently at some relatively high frequency. A harmonic resonator is positioned so that it can interact with the bunched beam of this fundamental oscillator. Since the bunched beam of transit time oscillators is rich in harmonic content, power can be delivered to the harmonic resonator in the same manner as power is delivered to the second resonator of a klystron amplifier. If the oscillator could be directly scaled to the harmonic frequency of the self-excited multiplier, the current necessary for the start of oscillation would be high. The multiplier does not have this problem since it is being driven by a beam that is already bunched. The main problem in this latter case is to couple the harmonic of the beam to the harmonic resonator efficiently.

The device to be discussed is a self-excited frequency multiplier in which the fundamental oscillator is an integral cavity two-gap klystron and the harmonic resonator is placed at an appropriate position in the drift tube between the two gaps. The device will be called a drift-tube klystron frequency multiplier and abbreviated as DTKFM. The design to be discussed is not the only one possible and quite probably not the best. The simplest design that could be used with existing equipment, and one which would lead to a fairly
rapid evaluation of the device characteristics was chosen for experimentation. As is true in the case of any new device, time is necessary to evaluate completely its merits and characteristics. It is not proposed that this paper presents a complete evaluation of the DTKFM but rather explores some of the capabilities and limitations of it.

For many years the easiest and most successful means of generating higher frequencies has been by the use of externally driven frequency multipliers. The most common one has been the crystal multiplier but velocity modulation multipliers have been used at wavelengths as short as about one centimeter. The crystal multiplier has been useful at shorter wavelengths but has the problem of burnout with excessive driving power and relatively low output power. With the advent of better crystal materials, these multipliers are being improved and they are relatively simple devices. There are many applications where they are useful and means of improvement of their operation are worthy of investigation. However, methods of generating higher output power must be considered.

A few articles have been presented in the literature in recent years concerning frequency multipliers. Some interesting developments have been presented and a brief review of these will be discussed.

**Review of Recent Frequency Multiplier Developments:**

Investigation of the so-called standard type of velocity modulation
frequency multiplier to determine a correct design procedure for particular models was made by V. J. Norris. This type of multiplier consists of a low frequency resonator which is driven externally and a harmonic resonator separated from it by a field free drift space. The beam is velocity modulated by the driven low frequency resonator; becomes bunched in the drift space; and delivers power to the harmonic resonator.

Norris' frequency multiplier delivered about 100 milliwatts of rf power at a frequency of 9375 megacycles with a driving frequency of 937.5 mc. He found that the simple theory of klystron bunching ignoring space charge effects predicts accurately the optimum bunching parameter required for his particular tube. However, it is not possible to predict the level of rf current in the beam for a multiplier using a high harmonic because of the space charge effects. With high values of modulating or driving voltage, the obtainable value of rf current is not reduced greatly below that predicted by theoretical considerations.

An improvement in operation of the velocity modulation frequency multiplier is presented by Matsuo. He proposes that several electron

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beams be used rather than one. This provides an increased total beam current at the harmonic resonator gap without serious de-bunching effects that would be present in a single beam of the same resultant current density. This tube has a single input and output cavity and these two cavities are common to all beams. The electron beams have different dc velocities. They are velocity modulated in common by the input cavity and the energy of the harmonics generated in each beam are picked up by the output cavity.

Matsuo's experimental tube used two electron beams and the wavelength of the driving source was 22.5 centimeters. Experiments were performed with frequency multiplications as large as seven but results were only presented for multiplications of 3 and 4. At the third harmonic of the driving frequency, it was found possible to obtain approximately twice the output power over that obtainable with both beams at the same velocity when the velocity difference between the beams was adjusted correctly. Thus the use of two beams should increase the efficiency of this type of frequency multiplier over the standard design but the complexity of the device is increased and the possibility of scaling to higher frequencies is more complicated.

Melchor and associates report that ferrites are usable as efficient frequency doublers. Using an input frequency of nine kilo-

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megacycles at a peak power level of 32 kilowatts they have obtained 8 kw at a frequency of 18 kmc. These results show that ferrites are superior to crystal doublers both as to efficiency and power handling capabilities but at present, higher multiplication ratios are not possible.

Coleman and Sirkis\textsuperscript{6} report a megavolt type of frequency multiplier called a Harmodotron. This device consists of a 20 kilovolt electron gun followed by a prebunching resonator and then a 10.8 cm. cavity accelerator which produces a one MEV bunched beam. This bunched beam then passes through a cylindrical resonator operating in the TM\textsubscript{018} mode to give an output wavelength of 8.31 mm. A peak power of one watt is obtained using a 5 milliampere pulsed beam. It should be theoretically possible to obtain a peak power of 0.4 watt at a wavelength of one mm using this same beam. This device shows promise in producing large output power in the millimeter wavelength region but considerable associated equipment is necessary to provide the beam energies necessary.

The usual frequency multiplier is not tunable over a very wide frequency range. It is thus interesting to note that Bates\textsuperscript{7} has


investigated a velocity modulated frequency multiplier which is tunable
over a wide frequency range. This device utilizes traveling wave cir-
cuits in the form of two forward wave single helices in cascade. A
dispersive helix was chosen for the output or harmonic section to per-
mit selective amplification of a particular harmonic of the input fre-
quency and a broadband helix was used for the input or driven section.

Bates shows that an electron beam modulated by a traveling
electric field has a high harmonic current content in the same manner
as is the case for klystron modulation. His experimental traveling
wave frequency multiplier has an input section operating from 0.5 to
1.0 kmc, an output section voltage tunable over the range of 2.0 to
4.0 kmc, and an output power level in the range of 20 to 100 mw with
a beam input power of from 2 to 4 watts. It was found that multiplica-
tion ratios of 10 or 15 gave substantial gain, and harmonic output of
tens of milliwatts has been obtained at harmonics as high as the
fortieth. The possibility of scaling this device to operate in the milli-
meter wavelength region looks formidable but possibly with the dis-
covery of a new circuit of some kind, this scaling could be accom-
plished.

A new type of high power frequency multiplier is reported by

\footnote{Uenohara, Michizo, Uenohara, Michiyuki, Masutani, T., and
vol. 45, pp 1419-1420, October, 1957.}

Uenohara and associates. This operates on the principal that a micro-
wave discharge in a controlled atmosphere delivers output frequencies which are integral multiples of the applied field frequency. Their experimental device consisted of a discharge gap composed of two cylindrical posts in an atmosphere at a pressure between 0.4 and 4.0 mm of mercury. About 1000 volts at 50 cps is applied across the discharge gap to help initiate the microwave discharge. The gap of approximately 3 mm in length is then driven at a frequency of 3000 megacycles at a power level of about 12 watts. The output power is picked up by the use of a high pass filter. With this device, the second through the fourth harmonic of the driving frequency were measured. The second harmonic had an output power of 60 mw and the fourth harmonic had an output power of 0.6 mw. The investigators indicate that they believe the mechanism by which the device operates provides a possibility for extending the generation of microwave power into the millimeter and submillimeter wavelength region but they discuss this no further.

The only self-excited frequency multiplier known to this writer is one investigated by Thurston. This device consists of a reflex type of oscillator called a retarding-field oscillator in which a density modulated beam not only drives a fundamental resonator at the end of its transit but also drives a harmonic resonator as well. A sketch of

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9Thurston, M. O., Op. Cit.
this device is shown in Fig. 1. The electron beam enters the fundamental resonator through a small aperture and becomes velocity modulated. The repeller is at a negative potential so that the beam is turned around and refocused on the anode. During this portion of the transit it may be assumed that the beam becomes density modulated. As the beam approaches or returns to the anode aperture it gives up energy to the field and thus supports oscillations. It is assumed that the various potentials have been adjusted to their proper values. The beam as collected on the anode aperture contains harmonic components so that by placing the gap of a harmonic resonator near the anode aperture, this resonator is also excited.

A practical frequency multiplier of this design has been made to operate at harmonics up to the seventh. The shortest wavelength detected to date has been at 3.3 mm. The output power at this wavelength is a fraction of a milliwatt, but the device has extended the frequency range of the oscillator and presents another means for generating energy at millimeter wavelengths. The conversion loss is found to be less for this self-excited frequency multiplier than that in the case of a crystal multiplier.

The harmonic output power of the retarding-field oscillator type as self-excited multiplier is relatively low but then the output power of the fundamental oscillator is also fairly low. If the fundamental oscillator were capable of delivering more power, then a larger
Fig. 1. Self-excited frequency multiplier employing a retarding-field oscillator.
amount of harmonic output should also be expected. This observation led to the idea of using the drift-tube klystron type of self-excited frequency multiplier. A larger amount of fundamental output power can be obtained so that the harmonic resonator will be driven harder and more harmonic power will be possible. The remainder of this paper discusses this type of a device.
SOME THEORETICAL CONSIDERATIONS
OF THE SELF-EXCITED
DRIFT-TUBE KLYSTRON FREQUENCY MULTIPLIER

A self-excited DTKFM is made up of a harmonic resonator in the drift-tube of a two-gap klystron oscillator. By placing the harmonic resonator in the drift-tube, it should be possible to minimize debunching effects in the velocity modulated beam. The ac power driving this resonator can be as large as it is possible to obtain in an oscillator without the necessity of an external rf driving source. There is also the advantage for some applications of having a device with two harmonically related frequencies in its output.

Since this frequency multiplier operates by extracting energy from the harmonics of a velocity modulated beam of electrons, some theory concerning this energy conversion will now be reviewed. ¹⁰

Figure 2 shows a cross section of the actual device and Fig. 3 shows an idealized schematic representation of the self-excited frequency multiplier. The feedback circuit between the output and input gaps is not shown because at present attention is focused only on the conditions in the beam. Figure 3 essentially represents two velocity modulation devices operating on the same electron beam with a common bunching gap. In the input or velocity modulation gap, it is assumed that there is a high frequency voltage impressed, the

Fig. 2. A cross-section of the self-excited frequency multiplier resonators. The harmonic waveguide also supports the drift-tube.
Fig. 3. Idealized schematic of the resonator gaps.
instantaneous value of which can be written as $V_1 \sin \omega t$. A dc beam current $I_0$, in which the electrons have been accelerated to a velocity $u_0$ due to a beam voltage $V_0$, passes through this region causing the electrons to have velocities greater and less than the value $u_0$. According to the usual assumption of the first order theory of bunching, $V_1/2V_0$ is less than one and the transit time across the first gap is small with respect to the period of one cycle of the rf oscillation. It is further assumed that the current density of the electron stream leaving this region is very nearly constant in time. Using the convention that the $rf$ voltage is positive when an electron is accelerated across the gap, each such electron gains an amount of energy equal to $e\beta V_1 \sin \omega t$, where $\beta$ is the beam coupling coefficient and is a measure of the effectiveness of the beam interaction with the gap voltage. The quantity $\beta$ is less than one and is important in the case of the DTKFM. It will be discussed more fully later. After the beam passes through this first gap it has a velocity $u$ given by $\mu u^2/2 = eV_0 + e\beta V_1 \sin \omega t$. This can be written as

$$u = u_0 \sqrt{1 + (\beta V_1/V_0) \sin \omega t} = u_0 \left[ 1 + (\beta V_1/2V_0) \sin \omega t + \ldots \right], \quad (1)$$

since $\mu u_0^2/2 = eV_0$ and $\beta V_1/V_0 \ll 1$.

After passing through the gap, the beam with a velocity as given in Eq. (1) is allowed to drift in a region free of dc and rf fields. Space charge effects are neglected due to kinematic effects, the faster electrons tend to overtake the slower ones that are ahead of them and
the result is the breaking up of the beam into groups or bunches.

The procedure for determining the instantaneous current at any particular distance $s$ along the beam from the first gap is to express the arrival time of the electrons at this distance in terms of the velocity given by Eq. (1). This leads to

$$\omega t_s = \omega t_1 + \theta_0 - X_0 \sin \omega t_1,$$

(2)

where $\omega$ is the radian frequency of interest,

$t_s$ is the time of arrival at point $s$,

$t_1$ is departure time from the first gap,

$\theta_0 = \omega_s/u_0$ is the dc transit time, and

$X_0 = \beta V_1 \theta_0/2V_0$ is the bunching parameter.

By using the principle of conservation of charge, one may state that electrons arriving at distance $s$ in the time interval $\Delta t_s$ are made up of one or more groups of electrons that have left the input gap during the intervals $\Delta t_1 = \left| dt_1/dt_s \right| \Delta t_s$. The total charge carried by the electrons arriving during the time $\Delta t_s$ is

$$I_0 \sum \Delta t_1 = I_0 \Delta t_s \sum \left| dt_1/dt_s \right|,$$

(3)

where the summation covers all times of departure $t_1$ that correspond to the same interval of time $t_s$. The total charge is also $i \Delta t_s$

where $i$ is the instantaneous current through a gap at distance $s$. Thus

$$i(t_s) = I_0 \sum \left| dt_1/dt_s \right|.$$

(4)
The inverse derivatives obtained from Eq. (2) may be used in Eq. (4) and the results plotted. It is found that for $X_0 = 1$ it is theoretically possible to have infinite peaks in the current pulses. It is the possibility of the presence of these infinite peaks of current that has made this type of device appealing as a frequency multiplier. The infinite peaks of current indicate an abundance of harmonic content. For values of $X_0 < 1$ the current no longer has infinite peak values but has harmonic content of considerable magnitude as may be seen if a harmonic analysis is made of the resultant current waveform.

A Fourier analysis of the current predicted by Eq. (4) shows that the instantaneous current components are given by

$$i_m = 2I_0 e^{-jm(\theta_0 - \pi/2)} J_m(mX_0),$$

where $m$ is the harmonic number and $J_m$ is the Bessel function of the first kind and order $m$. The magnitude of the harmonic current decreases in the same manner that the Bessel function decreases with increasing order. If the optimum bunching parameter is assumed for each harmonic, the ratio $|i_m/2I_0|$ decreases as shown in Fig. 4. The optimum bunching parameter is the value of $X_0$ giving maximum $\left|\frac{i_m}{2I_0}\right|$ and its value for each of the harmonics is given in Table I. This shows that the optimum bunching parameter approaches $X_0 = 1$ for increasing $m$.

\[\text{\textsuperscript{11}Ibid. p. 26}\]
### Table of Relative Power & Current of Each Harmonic as a Percentage of Fundamental

<table>
<thead>
<tr>
<th>m</th>
<th>Power</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>72.2</td>
<td>83.5</td>
</tr>
<tr>
<td>3</td>
<td>55.4</td>
<td>74.9</td>
</tr>
<tr>
<td>4</td>
<td>47.2</td>
<td>69.1</td>
</tr>
<tr>
<td>5</td>
<td>41.3</td>
<td>64.9</td>
</tr>
<tr>
<td>6</td>
<td>36.6</td>
<td>61.2</td>
</tr>
<tr>
<td>7</td>
<td>33.6</td>
<td>58.4</td>
</tr>
<tr>
<td>8</td>
<td>31.0</td>
<td>56.0</td>
</tr>
<tr>
<td>9</td>
<td>28.9</td>
<td>54.1</td>
</tr>
<tr>
<td>10</td>
<td>27.2</td>
<td>52.3</td>
</tr>
</tbody>
</table>

Fig. 4. Table of relative power & current of each harmonic as a percentage of fundamental.
TABLE I

Optimum Values of the Bunching Parameter

<table>
<thead>
<tr>
<th>m</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_{opt}</td>
<td>1.84</td>
<td>1.40</td>
<td>1.28</td>
<td>1.23</td>
<td>1.21</td>
<td>1.19</td>
<td>1.52</td>
<td>1.33</td>
<td>1.25</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Fig. 4 also shows the relative power that could be delivered to a resonator if $\beta$ were unity. Both these curves decrease slowly with increasing harmonic number. Physical limitations usually place the limit on the harmonic number that can be used in a frequency multiplier.

In the standard type of frequency multiplier it is rather easy to control the bunching parameter by controlling the driving source. In the DTKFM the problem is quite different in that the bunching parameter is determined by the design and operation of the oscillator.

The next consideration then should be in reference to the fundamental oscillator. The general approach is; (a) to design the fundamental oscillator; (b) determine from its operating characteristics the bunching parameter; (c) from b determine the position of the harmonic resonator gap for optimum output power; and (d) calculate the expected harmonic output power.

The fundamental oscillator is similar in design to that of the Floating-Drift-Tube Klystron with the major difference being that in the case of the experimental DTKFM tubes to be reported, the drift
tube voltage is kept at anode potential rather than being controllable. This is done because of the mechanical difficulties of isolating the drift tube electrically from the remainder of the circuit at the high frequencies that are to be considered. The dimensions become relatively small and thus the dc isolation of the drift tube would be very difficult.

The theoretical design equations for the Floating-Drift-Tube Klystron have been developed by Chodorow and Fan. The theory that is to be presented here will be similar to that presented by Chodorow and Fan but will be modified for the case of the DTKFM. Where differences between these theoretical treatments occur they will be pointed out.

The DTKFM fundamental oscillator analysis is similar to that for reflex tubes. One defines an electronic admittance determined by the velocity modulation and bunching process and a circuit admittance that depends only on the cavity. To obtain the electronic admittance, the current at the output gap is calculated in terms of the voltage at the input gap as was done in obtaining Eq. (5). Since the circuit admittance of importance is the admittance across the output gap, the electronic admittance must be written using the output gap

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voltage. In the experimental models of the DTKFM the input and output gaps are identical and symmetrically located in the resonator so that the voltage at the output gap is the same as the input gap voltage. With this in mind, the circuit admittance and the electronic admittance for the output gap are equated for steady state oscillations, and from the resultant complex equation two real equations are obtained,

\[ Y_e (2J_1(X_0)/x_0) \cos \phi = \omega C \left( \frac{1}{Q_0} + \frac{G_L}{Q_{ext}} \right) \]

\[ \frac{1}{Q_0} + \frac{G_L}{Q_{ext}} \tan \phi = \frac{2}{\delta} + \frac{B_L}{Q_{ext}} \]

where (7)

\[ \delta = \theta_0 - \pi/2 \]

\[ Y_e = \frac{2}{\theta_0} G_0/2 = \text{small signal electronic conductance}, \]

\[ G_0 = I_0/V_0, \]

\[ I_0, V_0, \] are direct beam current and voltage, respectively, \( Y_C^1 = \omega C \left( \frac{1}{Q_0^1} + j2\delta \right) = \frac{G^1_c}{1 + j2Q_0^1 \delta} = \text{equivalent circuit admittance at the second gap (including beam loading)}, \]

\[ Y_L^1 = \omega C \left( \frac{G_L}{Q_{ext}} + jB_L/Q_{ext} \right) = \frac{G_L^1}{G^1_L + jB_L^1} = \text{equivalent load admittance at the second gap}. \]

Chodorow and Fan\(^{13}\) introduce another parameter \( H = \frac{\rho_2 V_2}{\rho_1 V_1} \) that enters into the small signal electronic conductance and this is a significant parameter in determining the behavior of the Floating-Drift-Tube Klystron. This parameter takes into account the possibility of different input and output gaps due to different gap spacings.

\( ^{13}\)Ibid. p. 26.
The rf power delivered to the second gap is

\[ \frac{I_1 V_1}{2} = P_2 = \frac{\beta^2 I_0^2}{\gamma_e} X_0 J_1 (X_0) \cos \phi. \quad (8) \]

The power output to the load is

\[ P_L = \frac{\beta^2 I_0^2}{\gamma_e} \left[ X_0 J_1 (X_0) \cos \phi \frac{X_0^2 \omega C}{2 \gamma_e Q_0} \right]. \quad (9) \]

The load for optimum power is found by differentiating Eq. (9) with respect to \( X_0 \). Then, \( X_0 \) for optimum is given by \( \omega C/\gamma_e Q_0 = J_0(X_{00}) \) where \( X_{00} \) is the optimum \( X_0 \) with \( G_L = J_2(X_{00}) \), and the optimum power output is

\[ P_{L \text{ opt}} = V_0 I_0 \frac{\beta^2 G_0 X_{00}^2}{2 \gamma_e} J_2 (X_{00}) \quad (10) \]

and the corresponding efficiency is

\[ n_{L \text{ opt}} = \frac{X_{00}^2 J_2 (X_{00})}{\theta_0}. \quad (11) \]

In Chodorow's equation for optimum efficiency \(^{14}\) there is a multiplying factor of \( H \) which essentially determines the effective bunching angle defined as \( \theta_0/H \).

The electron transit time in the drift tube can be changed slightly by varying the anode voltage and this results in frequency modulation. If the drift tube is isolated, a larger amount of frequency modulation would be possible. In Eq. (6) \( \phi \) is related to the

\(^{14}\text{Ibid.} \text{ p. } 27.\)
anode voltage, while $\delta = \Delta f/f_0$ is linearly proportional to the operating frequency. By differentiation and substitution, the modulation sensitivity is found to be

$$\frac{df}{dV_0} = \frac{f_0\theta_0}{4V_0} \frac{y_e}{\omega C} \frac{2J_1(X_0)}{X_0} \sec \phi. \quad (12)$$

The half-power bandwidth can be found from the fact that the power output is equal to $P_L = \frac{V_2^2}{2} G_L^t / 2$ and $V_2 = V_1$. At the center of the mode with $X_0 = X_{0c}$,

$$P_L = \frac{1}{2} \left[ \frac{G_0 f_0 X_{0c}}{y_e} \right]^2 G_L^t. \quad (13)$$

If the load does not change with frequency, $X_0$ at the half-power point is $X_{0c}/2$. Substituting this value in equation (6) and solving for $2\delta$, one obtains the bandwidth

$$2\delta = \frac{G_0 f_0}{\omega C} \frac{\rho Z}{X_{0c}} \left[ 2J_1^2(\frac{X_{0c}}{2}) - J_1^2(X_{0c}) \right]^{1/2}. \quad (14)$$

Eq. (14) holds true with any load condition using the appropriate $X$.

A useful equation can be derived from Eq. (12) and Eq. (14). This is

$$\frac{2\delta}{df/dV_0} = \frac{4V_0}{\theta_0 J_1(X_{0c})} \left[ 2J_1^2(\frac{X_{0c}}{2}) - J_1^2(X_{0c}) \right]^{1/2}. \quad (15)$$

The operating parameter $X_{0c}$ can be calculated from the direct measurable quantities in the left hand side of Eq. (15) and known
quantities, \( V_0 \) and \( \theta_0 \). The function \( \frac{\theta_0}{4V_0} \frac{2\delta}{df_0/dV_0} \) is plotted in Fig. 5. By measuring the \( df/dV_0 \) characteristics of the DTKFM fundamental oscillator and using the other known quantities, Fig. 5 can be used to determine the operating parameter \( X_{0c} \). This is then usable in determining the harmonic gap position and other quantities related to the harmonic.

If both drift tube voltage and gap capacitance can be controlled, a more efficient fundamental oscillator is possible. By correct design of the gap capacitances a certain amount of improvement in efficiency should be expected over the case where both gaps are identical. This has not been done in the experimental tubes to be reported.

In an oscillator of the type used in the DTKFM it may be possible that the gap voltages will not be exactly the same even though the gaps are geometrically the same. If the output coupling iris is on the side of the resonator near the output gap, the coupling to a load will have some effect on this second gap voltage, which in turn, will affect the first gap voltage.

In the klystron amplifier or usual externally driven frequency multiplier it is possible to control the value of the bunching parameter by varying the driving voltage. This presents a unique problem in the case of the self-excited DTKFM since the driving
Fig. 5. Plot of equation one for design purposes.

\[ \frac{6}{4 V_o} \frac{26}{d f} \frac{d}{d V_o} \]
voltage at the bunching gap is determined by the operation of the oscillator and the degree of feedback coupling. The value of the effective harmonic bunching parameter in the DTKFM is obtained by placing the harmonic gap in a particular position and to some extent by the loading of the oscillator which varies $V_1$. The design of the oscillator determines the fundamental operating bunching parameter that can be found by experimental measurement and the use of Fig. 5. It is necessary to know this parameter so that $V_1$ can be determined and an optimum position for the harmonic gap located. The effect of loading on $V_1$ will depend on the degree to which the resonator is coupled to the load.

As is the case for two gap klystron oscillators, an electron that passed the buncher gap while the field was zero and changing from retarding to accelerating forms the center of a bunch and, for optimum output, must pass the catcher gap when the maximum retarding field occurs. If the phase angle between the buncher voltage and the catcher voltage is $\phi$, this is expressed as

$$\theta_0 + \phi + \frac{\pi}{2} = 2 \pi n$$

(16)

where $\theta_0 = 2 \pi f \tau$ is the transit angle and $n$ is an integer. The phase difference $\phi$ depends on the feedback coupling. The buncher voltage can be either in phase with the catcher voltage or $180^\circ$ out of phase in an integral cavity two gap oscillator; i.e., the angle $\phi$ has
two values. One value is approximately zero and the other value is approximately $\pi$ if the length of the feedback line is zero as is the case for the DTKFM. Eq. (16) can be rewritten as

$$\theta_0 = 2 \pi (n + 1/4) .$$  \hspace{1cm} (17)

Reducing the accelerating voltage increases the value of $n$ in Eq. (17) since the transit time is increased. As a matter of clarification, the significance of $n$ should be pointed out. The value of $n$ must be an integer if oscillation is to occur. A more useful value is given by

$$N = \theta_0 / 2\pi = fT = fs/u_0 ,$$  \hspace{1cm} (18)

where $s$ is the drift distance and $u_0$ the average velocity of the electrons. The value of $N$ should not be confused with the number of times the beam is bunched in the drift space. An oscillator normally operates at some point near the first maximum of the Bessel function curve since the buncher voltage will be decreased by reducing the acceleration voltage or overbunching, and the bunching parameter will remain essentially the same. This means that the beam is bunched only once in the drift space of an oscillator although exceptions to this are possible.

For the harmonic frequency, the bunching parameter may be written in the following ways:

\[ ^{15} \text{Harrison, A. E., Klystron Technical Manual, Sperry Gyroscope Company} \]
\[ x_{0m} = \omega_m S_{gm} V_{1/2m\mu_0 V_0} \]  

(19a)

\[ = \pi N_1 V_1 / V_0 \]  

(19b)

\[ = \pi N_2 V_1 / m V_0 \]  

(19c)

where \( \omega_m \) is the radian frequency of the harmonic catcher resonator and \( m \) is the harmonic number. \( N_1 \) and \( N_2 \) are the number of cycles which occur during transit from buncher to harmonic catcher. \( N_1 \) refers to the buncher frequency and \( N_2 \) refers to the catcher frequency. The beam coupling coefficient has been omitted in the above equations.

From the oscillating conditions \( \theta_0 \) will be known and from Fig. 5 the value of \( V_1 / V_0 \) may be calculated. Knowing this value of \( V_1 / V_0 \), one may use Eq. (19a) to determine the harmonic gap position \( S_{gm} \) using the optimum value of \( X_{0m} \) for the particular harmonic from Table I. This position will lie in the vicinity of the catcher gap but within the drift tube. This calculation does not take into account space charge debunching effects. The optimum position of the harmonic gap may be shifted due to these effects but probably the best way to determine the position is by experiment.

Referring to Eq. (19a), it is possible to plot the position of the harmonic gap relative to the two fundamental oscillator gaps. The ratio of \( V_1 / V_0 \) has been determined and will have some fixed value for all harmonics. The ratio \( \omega_m / m \) will be constant also and \( u_0 \) is
determined by oscillating conditions. Thus Eq. (19a) becomes

\[ S_{gm} = K X_{0m}, \]  

(20)

where \( K = 2\mu_0 V_0/\omega_m V_1 \) a constant for a particular oscillator design. Then for optimum harmonic output it is only necessary to plot Eq. (20) using optimum values of \( X_{0m} \) for each harmonic and the fundamental. The distance between the gaps of the oscillator has been taken as unity in plotting the harmonic gap position as a function of harmonic number, Fig. 6. Measurements can be made between the gap centers. As will be noted later in the experimental section and as can be seen from Fig. 6, the gap position for good output power is not critical. The dotted line of Fig. 6 shows the limiting position for optimum output at infinite harmonic number.

It is now interesting to consider second order effects in the bunching theory and note whether or not any significant changes in the harmonic content of the bunched beam is predicted.

To consider second order effects, Eq. (2) will be modified so as to include more terms of the Taylor series expansion. This then as the form

\[ \omega t_s = \omega t_1 + \theta_0 - X_0 \sin \omega t_1 + a_2 X_0^2 \sin^2 \omega t_1 + a_3 X_0^3 \sin^3 \omega t_1 \ldots \]  

(21)

Hamilton\textsuperscript{16} proceeds as follows to determine a relationship for \( X_0 \), \( a_2 \), and \( a_3 \). The properties of the drift space may be described by

\textsuperscript{16}Hamilton, Knipp, and Kuper, \textit{op. cit.}
Fig. 6. HARMONIC GAP POSITION FOR OPTIMUM OUTPUT RELATIVE TO OSCILLATOR GAP POSITIONS. DISTANCE BETWEEN OSCILLATOR GAPS HAS BEEN TAKEN AS UNITY.
expressing the electron transit angle through the drift space as a function, \( \theta (V_e) \), of the electron energy \( V_e \). The time of arrival at a distance \( s \) from the first gap is then given by

\[
\omega t_s = \omega t_1 + \theta_0 + \left( \frac{d\theta}{dV_e} \right)_0 (V_e - V_0) + (1/2) \left( \frac{d^2\theta}{dV_e^2} \right)_0 (V_e - V_0)^2 + \ldots \quad (22)
\]

where the subscript zero signifies evaluation for \( V_e = V_0 \).

If space charge effects are neglected and it is assumed that \( \Theta = 1 \), the electron energy may be considered sinusoidally modulated. Then the electron energy \( V_e \) is given by

\[
V_e = V_0 \left[ 1 + (\Theta V_1/V_0) \sin \omega t_1 \right] \quad (23)
\]

The coefficients in Eq. (21) become

\[
X_0 = -\Theta V_1 \left( \frac{d\theta}{dV_e} \right)_0 = a_1 X_0, \quad X_0 = +\Theta V_1/2 V_0 \theta_0 \quad (24a)
\]

\[
a_1 = -(2V_0/\theta_0) \left( \frac{d\theta}{dV_e} \right)_0, \quad (24b)
\]

\[
a_2 = \left( \frac{d^2\theta}{dV_e^2} \right)_0 /2 \left( \frac{d\theta}{dV_e} \right)_0^2, \quad (24c)
\]

\[
a_3 = -\left( \frac{d^3\theta}{dV_e^3} \right)_0 /6 \left( \frac{d\theta}{dV_e} \right)_0^3. \quad (24d)
\]

Hamilton then carries out the analysis for determining the instantaneous harmonic current in the same manner as in the first order approximation. This leads to the relation

\[
i_m = 2I_0 e^{-jm\theta_0} (1 + ja_2 X_0^2 \frac{d^2}{dx_0^2} + a_3 X_0^3 \frac{d^3}{dx_0^3} - a_4 X_0^4 \frac{d^4}{dx_0^4} + \ldots \ldots ) J_m (m X_0) \quad (25)
\]
A comparison of Eq. (25) with Eq. (5) will show that the first term in \( i_m \) is unaffected by finite gap voltage and arbitrary dc fields in the drift space. The only change is in the generalization of the bunching parameter as given by Eq. (24a).

Where \( a_3 \ll a_2 \ll a_1 \) the second term in Eq. (25) is the first order and next most important term. Since it is small and in quadrature to the first term it may be considered as producing a phase shift. The third and fourth terms are in phase with the first and lead to a change in amplitude.

To determine the effect of these generalizations on the harmonic current, Eq. (25) is rewritten as

\[
\frac{i_m}{2I_0} = e^{-jm(\theta_0 + \Delta_v \theta)} (1 + \Delta_v i_m/i_m) J_m(mX_0)
\]

where

\[
\Delta_v \theta = -\frac{a_2 X_0^2}{m} \frac{J_m'''(mX_0)}{J_m(mX_0)}
\]

\[
\Delta_v \frac{i_m}{i_m} = \frac{a_3 X_0^3}{m^2} \frac{J_m'''(mX_0)}{2m^2} - \frac{a_2 X_0^4}{m^2} \frac{J_m^4v(mX_0)}{J_m(mX_0)}
\]

For a field-free drift space, the coefficients \( a_1 \), \( a_2 \), and \( a_3 \) are: \( a_1 = +1 \), \( a_2 = 3/2\theta_0 \), \( a_3 = -5/2\theta_0^2 \). Thus Eq. (26) becomes

\[
\left| \frac{i_m}{2I_0} \right| = J_m(mX_0) - \left[ \frac{5}{2\theta_0^2 m^2} X_0^3 J_m'''(mX_0) \right. \\
+ \left. \frac{9}{8\theta_0^2 m^2} X_0^4 J_m^4v(mX_0) \right].
\]
The third and fourth derivatives of \( J_m(mX_0) \) are given by

\[
J_m^{\text{III}}(mX_0) = \frac{m^3}{8} \left[ J_m - 3(mX_0) - 3J_{m-1}(mX_0) \\
+ 3J_{m+1}(mX_0) - J_{m+3}(mX_0) \right]
\]

\[
J_m^{\text{IV}}(mX_0) = \frac{m^4}{16} \left[ J_m - 4(mX_0) - 4J_{m-2}(mX_0) + 4J_m(mX_0) \\
- 4J_{m+2}(mX_0) + J_{m+4}(mX_0) \right].
\]

For an assumed value of \( \theta_0 = 17.25 \), the effect of the more accurate approximation to the harmonic current given by Eq. (25) is shown in Table II.

### TABLE II

**Second Order Effects on Harmonic Current**

<table>
<thead>
<tr>
<th>Harmonic No.</th>
<th>First Approx. ( i_{m/2I_0} )</th>
<th>Second Approx. ( i_{m/2I_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.582</td>
<td>0.569</td>
</tr>
<tr>
<td>2</td>
<td>0.486</td>
<td>0.485</td>
</tr>
<tr>
<td>3</td>
<td>0.434</td>
<td>0.423</td>
</tr>
<tr>
<td>4</td>
<td>0.400</td>
<td>0.396</td>
</tr>
<tr>
<td>5</td>
<td>0.374</td>
<td>0.367</td>
</tr>
<tr>
<td>6</td>
<td>0.354</td>
<td>0.349</td>
</tr>
<tr>
<td>7</td>
<td>0.338</td>
<td>0.335</td>
</tr>
<tr>
<td>8</td>
<td>0.324</td>
<td>0.303</td>
</tr>
</tbody>
</table>

It will be noted that the second or more accurate approximation to \( i_{m/2I_0} \) is changed only slightly and there is no definite trend to the change. It thus appears that corrections to the first order theory of harmonic current content in a bunched beam will be of little importance in frequency multipliers. The value of the
bunching parameter used in determining Table II are those which optimize \( \left| \frac{i_m}{2I_0} \right| \) in the first order approximation.

Hamilton further shows that if beam loading and finite transit angle in the first gap are considered, the transit time to the gap in question is modified so that Eq. (2) becomes

\[
\omega t_s = \omega t_1 + \frac{\theta_1}{2} + \theta_0 - \frac{\psi X_0}{2} - \left[ X_0 + \frac{\Theta V_1 \theta_1}{4V_0} \right] \left[ \sin(\omega t_1 - \theta) - \theta \sin^2(\omega t_1 - \psi) \right] \tag{32}
\]

with

\[
\psi = V_1 N \theta_0^2 / 24 V_0 X_0 = -N \theta_0^2 / 12 \theta a_1 \theta_0
\]

\[
\gamma = V_1 P \theta_1 / 24 V_0 \theta = -P X_0 \theta_1 / 12 \theta^2 a_1 \theta_0 . \tag{33}
\]

\( P \) and \( N \) are the beam loading parameters given approximately by

\[
P = \frac{6}{\theta_1^3} (\theta_1 - \sin \theta_1)
\]

\[
N = \frac{24}{\theta_1^3} \left[ \sin \left( \frac{\theta_1}{2} \right) - \frac{\theta_1}{2} \cos \left( \frac{\theta_1}{2} \right) \right] . \tag{34}
\]

\( \theta_1 \) is defined as the first gap transit angle and it will be noted that \( \psi \) is independent of rf gap voltage.

The term \( X_0 + \Theta V_1 \theta_1 / 4V_0 \) is the contribution to the bunching parameter due to extending the drift space back to the center of the bunching gap; thus the additional term \( \Theta V_1 \theta_1 / 4V_0 \) can normally be neglected.

\[17\text{Ibid. p. 224.}\]
The second harmonic term, \( \sin (2 \omega t_1 - \psi) \), has more of an effect. This might be explained by saying that velocity modulation with a sinusoidal voltage across a finite gap is equivalent to velocity modulation in an infinitesimal gap by a fundamental and a second harmonic voltage as shown by Eq. (32). Since the fundamental and second harmonic voltages have the same symmetry about \( \omega t_1 - \psi = 0 \), the presence of the second harmonic does not change the phase of the bunch but does change the shape and thus the rf components of the bunch. It then appears to be worthwhile to investigate this effect on the harmonic content of the beam.

The transit time to a gap at distance \( s \) can be represented by the simplified relation

\[
\omega t_s = \omega t_1 - X_0 (\sin \omega t_1 - \psi \sin 2\omega t_1).
\]  

(35)

As in the usual harmonic analysis of the bunched beam

\[
i_m = \frac{I_0}{\pi} \int_{-\pi}^{\pi} e^{-jm(\omega t_s)} d(\omega t_1).
\]  

(36)

If Eq. (35) is substituted into Eq. (36), the following is to be evaluated:

\[
i_m = \frac{I_0}{\pi} \int_{-\pi}^{\pi} e^{-jm(\theta - X_0 \sin \theta)} \cdot j_m \psi X_0 \sin 2\theta d\theta.
\]  

(37)
where θ has replaced ωt₁. According to Gray, Mathews, and McRoberts,

\[ e^{jy \sin \theta} = \sum_{n=-\infty}^{\infty} J_n(y) e^{jn \alpha} \]  

Then

\[ i_m = \sum_{n=-\infty}^{\infty} J_n(m \gamma X_0) \int_{-\pi}^{\pi} e^{-j [(m^2 - 2n^2)\theta - mX_0 \sin \theta]} d\theta. \]  

By oddness of the argument,

\[ I_{m,n}(X_0) = \int_{-\pi}^{\pi} e^{-j [(m-2n)\theta - mX_0 \sin \theta]} d\theta \]  

becomes

\[ I_{m,n}(X_0) = 2 \int_{0}^{\pi} \cos [(m-2n)\theta - mX_0 \sin \theta] d\theta. \]  

The Bessel function relation gives

\[ J_p (X_0) = \frac{1}{\pi} \int_{0}^{\pi} \cos (pX_0 - X_0 \sin \theta) d\theta, \]  

Eq. (41) becomes

\[ I_{m,n}(X_0) = 2\pi J_{m-2n}(X_0), \]  

and

\[ i_m = 2I_0 \sum_{n=-\infty}^{\infty} J_n(m \gamma X_0) J_{m-2n}(mX_0). \]
Figs. 7, 8, 9, and 10 show a plot of $|i_m/z_0|$ as a function of $\phi$ for values as predicted by the first order theory. In all cases it appears possible to increase the harmonic content of the beam by controlling the first gap transit angle. However it is nearly impossible to obtain a $\phi$ large enough to achieve this possibility. For $\theta_1 = 5$, $P/\phi_1 = 0.05 \times |V_1/V_0 = 0.4$ and $\theta_1 = \pi$, $\phi = 0.05$. This is in the range where very little if any increase in fundamental or harmonic component is available. If the gap transit angle of the buncher becomes quite long, say near $2\pi$, the modulation coefficient becomes very low and it is difficult to obtain a decent bunching parameter $X_0$ in a reasonable length of drift space. Then the output of a practical multiplier would fall off seriously. At any rate it appears that a relatively large gap transit angle will not have too much of an adverse effect on the fundamental or harmonic content of the beam. This is of importance when operating at high frequencies where it may not be possible to keep the gap transit angle small.

**Debunching Effects**

The harmonic content of the beam is highly dependent on the degree to which the beam is bunched; the greater the degree of bunching, the better the harmonic content in the beam. The phenomenon of space charge debunching tends to thwart the ability to obtain a tightly bunched group of electrons.
Fig. 7. Effect of gap transit angle on fundamental current.
Fig. 8. Effect of gap transit angle on third harmonic current.
Fig. 9. Effect of gap transit angle on fourth harmonic current.

Fig. 10. Effect of gap transit angle on sixth harmonic current.
There is no theory that completely predicts the effects of this space charge debunching. This is because there are too many conditions which in general must be fulfilled. There are theories which give an indication of debunching effects and it is believed worthwhile to present some of the results of these.

The problem is broken into two parts which are considered separately. There are various assumptions made that cannot be maintained in practice but these make it possible to obtain a solution. Transverse and then longitudinal debunching is considered.

From simple considerations of spreading of a dc beam due to the space charge, a parameter $h/2\pi$ called the debunching wave number arises. $h$ is defined from

$$\left(\frac{h}{a}\right)^2 = \frac{60I_0c}{\mu_0V_0},$$

(45)

where $a$ is the beam radius and $c$ the velocity of light. The parameter $h$ also occurs in the transverse and longitudinal debunching theory.

Concerning transverse debunching, the following assumptions are made:

1. Drift-tube-wall effects are neglected.
2. Bunching parameter $X_0 \ll 1$.
3. Separation between bunch centers is large compared with drift tube radius.
If circumstances are such that the part of the beam that expands beyond the original diameter is wasted, Hamilton\textsuperscript{19} shows that the rf current is diminished by a factor.

\begin{equation}
1 - \frac{x_0}{12} (hs)^2.
\end{equation}

(46)

The effect of the drift tube wall is to increase this factor due to the positive image charge that is induced in it. For the case of the self-excited DTKFM this transverse debunching will be minimized by use of a magnetic field.

One assumption made in determining the effect of longitudinal debunching is in opposition to that used in the case of transverse debunching. The assumptions here are that;

1. Drift-tube-wall effects are negligible.

2. Beam diameter is much greater than the separation between bunch centers.

The method of attack for this case is to assume an unknown functional relation between bunching parameter and distance, and then find out what this functional relation must be in order to satisfy the physical laws governing space charge flow. These laws are (1) Poisson's equation relating space charge and the field, (2) the continuity equation relating current and time rate of change of charge density, and (3) Newton's law of motion relating force and

\textsuperscript{19}Hamilton, Knipp, and Kuper, op. cit.
acceleration. Carrying this process through, Hamilton shows the bunching parameter to be modified so that

\[ X(s) = X_0(s) \frac{\sin hs}{hs} \quad (47) \]

where

\[ X_0(s) = \frac{\omega s V_1}{u_0^2 V_0} \quad (48) \]

A restriction is placed on \( X(s) \) so that it must be much less than one. Under the conditions of this assumption the harmonic content of the beam is considerably reduced so that the effect on harmonic content is not predicted to any extent. However, it is further stated that \( \frac{u_0/\omega s}{\geq 6,6V_1/V_0} \) is a condition for drastic diminution in higher content.

A calculation has also been made for the case of a magnetically focused beam which is small in diameter with respect to the drift tube\(^{20}\). This leads to rather complicated expressions from which Beck makes the following conclusions. These are (1) that the debunching merely means that the effective value of \( \theta_0 \) is less than its geometric value and (2) that the conversion efficiency is decreased somewhat, but the bunch still occurs at the phase \( (\omega t_1-\theta_0) \).

All the theories developed tend to overestimate the effect of debunching. For longitudinal debunching, one reason may be

\[^{20}\text{Beck, A. H. W., Thermionic Valves, Cambridge University Press, pp 86-87, 1953.}\]
that the image effect of drift tube walls is neglected. This would have the effect of slightly enhancing the bunching. However, the results of these theories are used to approximate space charge effects.

THEORETICAL EFFICIENCY OF THE DTKFM

The theoretical efficiency of the self-excited DTKFM can be calculated in a manner similar to that for an amplifier. Assume the voltage developed across the gap of the harmonic resonator to be

\[ V_m = V_1 \sin m(\omega t - \delta) \]

Using Ramo's theorem the harmonic power extracted from the beam is the negative value of the time average of the product \( i_m V_m \). This power becomes, from Eq. (5),

\[ P_{bm} = -\frac{2I_0}{2\pi} \int_0^{2\pi} \rho_m V_1 \sin m(\omega t + \delta) J_m(mx_0) \cos m(\omega t - \theta_0) d(\omega t) \]

\[ = -1 \rho_m V_1 I_0 J_m(mx_0) \sin m(\theta_0 + \delta) \]

(50)

As in an amplifier, the harmonic resonator voltage adjusts its phase to extract maximum power from the beam so \( \theta_0 + \delta = (4n+3)\pi/2 \), \((n=0,1,2,...)\). Therefore \( \sin m(\theta_0 + \delta) = -1 \). The efficiency is \( P_{bm}/I_0V_0 \) and becomes

\[ \eta_m = \rho_m \Lambda J_m(mx_0) \]

(51)

where \( \eta_m \) is the efficiency at the \( m^{th} \) harmonic,

\( \rho_m \) is the beam coupling coefficient at the \( m^{th} \) harmonic,

and \( \Lambda = V_1/V_0 \). This equation applied only to the power delivered

\[ ^{21}\text{Ramo, S., Currents Induced by Electron Motion,} \]

to the resonator and is not in direct relation to the output power.

In an experimental tube to be discussed later, practical values of $\beta_m = 0.4, \Delta = 0.348$, and $X_0 = 2.4$ have been determined. Applying these values in Eq. (51) an efficiency on the order of 3% is indicated. Even though this appears to be low, if the fundamental output is in excess of 10 watts in the millimeter range, a considerable amount of harmonic output power can be expected.

Frequency Multipliers are inherently low efficiency devices but have advantages as has been pointed out.

**FREQUENCY LIMITATIONS**

A comparison between Frequency Multipliers, Reflex Klystrons and Two-Gap Klystrons.

This section presents a comparison between resonator type frequency multipliers, reflex klystron oscillators, and two-gap klystron oscillators concerning factors limiting the ultimate frequency at which the devices can operate.

If an electron beam passes through the gap of a resonator, the noise power delivered to the resonator is nearly the full shot noise of the beam modified by the beam coupling coefficient. Thus the noise power generated in the resonator is

$$N = 2eI_0BR_{sh} \beta^2$$

where $e$ is the charge of the electron.

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\( I_0 \) is the beam current, 
\( B \) is the frequency bandwidth, 
\( R_{sh} \) is the shunt resistance of the resonator, and 
\( \varphi \) is the beam coupling coefficient.

A simple reentrant resonator will be assumed and the noise power developed as a function of frequency will be determined. Assume the beam current to be constant. This will not be possible as frequency increases and will be taken into consideration later. For a specific cavity shape and mode of excitation, the \( Q \) varies as the square root of the resonant wavelength as does the shunt resistance. Then it is found that

\[
Q = \frac{f}{\Delta f} = \frac{f}{B} \quad \text{or}
\]

\[
B = \frac{f}{Q} = k_f \sqrt{f} = k_1 f^{3/2} \quad \text{and}
\]

\[
R_{sh} = k_2 / \sqrt{f},
\]

where \( k_1 \) and \( k_2 \) are proportionality constants. Assuming \( \varphi \) to be constant with scaling

\[
N = 2k_1 k_2 \left( f^{3/2} / f^{1/2} \right) \varphi^2
= 2k_1 k_2 f \varphi^2,
\]

where \( K = k_1 k_2 \).

Thus the noise power generated in a reentrant resonator is directly proportional to the frequency as the resonator is scaled. For detectable output of an amplifier or frequency multiplier, the beam
must be able to furnish power at the desired frequency in excess of this noise power.

The amount of current obtainable for use at extremely high frequencies places a further limitation on the amount of shot noise power that will be developed. A practical limit to electron beam diameter for a cylindrical beam at the present is approximately 0.010 inch. It will be assumed that no grids will be used in this comparison of the frequency limitations. The ultimate goal is to determine which device, i.e., oscillator or resonator type of frequency multiplier, has a better possibility of generating extremely high frequencies. The frequency multiplier and two gap klystron oscillator will be the devices which determine the amount of current $I_0$ that will be obtainable. This is due to the fact that the beam must pass through a tunnel in these devices. There the amount of current which can be passed through a tunnel of 0.010 inch in diameter and having an appropriate length must be determined.

Referring to the SERL Laboratories 8 millimeter klystron as a criterion of practical tunnel length, a value of 0.100 inch will be used. Thus we want to determine the amount of current which can pass through a tunnel 0.010 inch in diameter and 0.100 inch long.

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Spangenberg shows that when a beam is introduced into a cylinder with optimum entrance conditions, the maximum value of current that can be transmitted is given by

$$I_{\text{max}} = 1230 \ (V_{KV})^{3/2} \ (d/L)^2 \ \text{ma.} \quad (55)$$

where $V_{KV}$ is the beam potential in kilovolts,

d is the cylinder diameter, and

$L$ is the cylinder length.

Even with a strong axial magnetic field to prevent spreading, there is a maximum current that can be transmitted. As current is increased, the potential at the beam center drops below the value at the beam edge and finally decreases to such an extent that the beam is blocked by space charge action to a value of

$$I_{\text{max}} = 1.025 \ (V_{KV})^{3/2} \ \text{amperes.} \quad (56)$$

This is for a beam completely filling the tube and it will be noted that Eq. (56) is independent of the tube dimensions. If some neutralization is present because of positive ions, then it is possible to increase this limit by as much as six times.

The maximum current as predicted by Eqs. (55) and (56) will be found by assuming the beam voltage to be limited to 5000 volts. If the beam voltage is increased above this value, heat dissipation becomes a more difficult problem with the usual values of current neces-

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24 Spangenberg, K. R., op. cit.
Cathode materials available at the present can furnish in the order of 10 amperes per square centimeter of cathode area so that this is not the limitation that it was a few years ago.

Eq. (55) predicts a maximum current of 138 milliamperes and Eq. (56) predicts a maximum of 11.5 amperes. These values are theoretical and will be reduced in a practical tube. The reduction is due to such problems as heat dissipation and gun design. The amount of current that can be transmitted through the prescribed tunnel does not appear to be a limiting factor.

Referring again to the SERL 8 mm klystron oscillator, a current on the order of 0.10 ampere is possible with axial magnetic focusing and a simple gun design. It is necessary to dissipate 500 watts of power in the structure with this current. In a two gap klystron oscillator this is not a serious problem since most of the power can be dissipated in a massive externally cooled collector. A reflex klystron oscillator will have to operate at a power level considerably below 500 watts.

We shall now assume that it is possible to construct the two types of oscillators and the frequency multiplier at a wavelength of 4 mm. A reflex klystron operating at this wavelength has been reported in the literature, but to the author's knowledge, two gap klystrons have not been made this small. We further assume that it is possible
to maintain values of $Q$ and $R_{sh}$ as reported for the SERL Klystron. Values such as $Q_0 = 1500$, $Q_L = 500$, and $R_{sh} = 10^5$ ohms are reported by SERL. Assuming an intermediate value of $Q$, say $Q = 750$, the bandwidth $B$ becomes approximately $10^8$ cps. Eq. (54) then predicts a noise power of approximately 0.32 microwatt delivered to the resonator if the beam coupling coefficient is unity. Thus it will be necessary to have an rms beam current in excess of approximately $1.8 \times 10^{-3}$ ampere to develop detectable output power in a frequency multiplier.

According to Eq. (5), the rms harmonic current in a bunched beam is given by

$$I_m = \sqrt{2} I_0 J_m (mX_0), \quad (57)$$

where the nomenclature is as previously used. Assuming the optimum value for $mX_0$ in Eq. (57), it is theoretically possible to operate at the one hundredth harmonic of an oscillator at a wavelength of 40 cm and still have an rms harmonic current of approximately $19 \times 10^{-3}$ amp. This is well over the noise current but presents quite an optimistic viewpoint since debunching, reduced coupling coefficient, etc., have not been considered. This same viewpoint will be taken in the case of the oscillators.

For a reflex Klystron, it has been shown that the starting current for the onset of oscillations is given by

\[25\]

\[\text{Ibid. p. 590.}\]
\[ I_{\text{min}} = 2V_0G_{sh}/\theta_0 \beta^2. \quad (58) \]

This neglects space charge and considers only first order effects. Even with these assumptions, Eq. (58) presents a fairly accurate value for the starting current.

According to Eq. (58), the starting current decreases with increasing dc transit angle. However, this transit angle cannot be increased indefinitely since the beam trajectory and electronic interaction deteriorate rapidly. This will be especially true at very short wavelengths. Let it be assumed that it is possible to operate in the third electronic mode or with a transit angle of \(11\pi/2\) radians.

Assuming \(\beta\) to be unity and \(G_{sh} = 10^{-5}\) mho, one may predict from Eq. (58) that \(I_{\text{min}} \approx 5.8 \times 10^{-3}\) ampere. This is approximately three times greater than the shot noise current calculated earlier.

Actually the starting current would be considerably higher since it is inversely proportional to \(\beta^2\) and this quantity becomes small. This will also be true in the case of the two gap klystron oscillator as will be shown. In the frequency multiplier, coupling of the harmonic current to the resonator is directly proportional to \(\beta\) and this presents an advantage over the klystrons in this respect.

The starting current for the onset of oscillations in a two-gap klystron is given by\(^{26}\)

\(^{26}\)Ibid. p. 615.
where \( k \) is the coefficient of coupling of the resonators, \( Q_a \) and \( Q_b \) are the quality factors of the input and output resonators, respectively, and \( M \) is the assumed mutual inductance between the resonators.

The other parameters are as previously defined. With critical coupling assumed between the input and output resonators, Eq. (59) can be simplified by using the following relationships:

\[
k = k_c = \frac{1}{\sqrt{Q_a Q_b}}, \quad \text{and}
\]

\[
\omega M = \sqrt{R_a R_b},
\]

where \( R_a \) and \( R_b \) are the equivalent series resistances of the input and output resonators respectively. Eq. (59) then becomes

\[
I_{\min} = \frac{2}{e^{\phi_0}} \frac{1 + k^2 Q_a Q_b}{\omega M Q_a Q_b} V_0
\]  

(59)

By assuming the resonators to be identical and using the fact that for high \( Q \) circuits \( R_{sh} = Q^2 R \) series, one obtains

\[
I_{\min} = 4V_0 G_{sh}/e^{\phi_0}
\]  

(63)

where \( G_{sh} \) has replaced \( 1/R_{sh} \). This is twice the value of Eq. (58) but only orders of magnitude are of interest. If the same value of the dc transit angle as assumed for the case of the reflex klystron,
namely $\theta_0 = 11\pi/2$, is used, the starting current according to Eq. (63) would be approximately $11.6 \times 10^{-3}$ ampere. Thus the frequency multiplier needs less rf current for detectable output power than do either of the oscillators discussed. It is now possible to consider what will occur as frequency increases assuming oscillators and resonator multipliers can be constructed.

It was noted in Eq. (54) that the shot noise power developed in a resonator is directly proportional to the frequency and it is necessary to develop a signal greater than this value for detectable output. Since the power developed in a resonator is proportional to $I_m^2R_{sh}$, the necessary component of ac beam current to overcome the shot noise is proportional to the square root of the frequency. If it is assumed that $\theta$ and $\theta_0$ can be held constant, the starting current for oscillation of the klystrons is proportional to $G_{sh}$ and this in turn is proportional to the square root of the frequency. Thus the three devices are comparable in this respect. There remain then the problems of beam coupling coefficient variations and heat dissipation to be discussed.

When operating at values of beam power of the order of 500 watts, grids are not used. They cannot be made in small sizes to dissipate large amounts of heat without melting. This means that the interaction gap of the devices under discussion will be similar to
that shown by Fig. 11a. This interaction gap consists of two blunt edge cylinders of inner radius \( r \) and spaced a distance \( d \) apart. Variations such as cones rather than cylinders are used in these interaction gaps but the beam coupling coefficient is essentially the same as that in Fig. 11a.

This coupling coefficient is defined as the ratio of the peak energy actually gained by an electron as it passes through a gap to the energy which would be gained in a very quick transit at the time of maximum voltage. Pierce\(^{27}\) calculates the effective, \( \rho_s \), average \( \rho_a \), and axis \( \rho_0 \) values of the beam coupling coefficient for the geometry of Fig. 11a as a function of the radius \( r \) and the frequency. These values of \( \rho \) are modified by another function which takes into account the finite gap width \( d \). Fig. 11b indicates the variation of the average coefficient of coupling as a function of the radius \( r \) where \( \gamma d \) is a parameter. \( \gamma = \omega / u_0 \) where \( \omega \) is the radian frequency and \( u_0 \) is the dc beam velocity.

Whether or not the average or the rms value of the beam coupling coefficient is applied to a particular case depends upon the application. There is little difference in their numerical value but the average value \( \rho_a \) is in general slightly less; \( \rho_a \) has been selected for Fig. 11b since it presents a less optimistic viewpoint.

Fig. 11b. Plot of average beam coupling coefficient.
The value $\rho_a$ assumes for the hypothetical 4 mm oscillator or multiplier will now be examined. The assumption here is that the three devices have the same interaction gap. The beam and cylinder radius is 0.005 inch and it is further assumed that the gap spacing is 0.002 inch. For a beam voltage of 5000 volts, $y = 112$, $\gamma_r = 1.42$ and $\gamma_d = 0.57$ so that $\rho_a = 0.8$ from Fig. 11b. This indicates that the noise power delivered to the resonator of the frequency multiplier is decreased by 36\% according to Eq. (51). At the same time an increase of 36\% in the rms ac component of the harmonic beam power would be necessary to deliver the same detectable output that was available when $\rho_a$ was unity. For the two oscillators according to Eqs. (58) and (63), the direct starting current is increased by approximately 150\%. In general as frequency is increased, the value of $\rho_a$ becomes smaller for practical reasons so that the starting currents for the two oscillators become difficult to achieve. To the limit of feasible fabrication, it should be possible to obtain output from a resonator type of frequency multiplier whereas it may not be possible for oscillations to occur in the klystrons.

Concerning the problem of heat dissipation, the literature places a limit of about 3 mm wavelength on the reflex type of oscillator.

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\(\text{28 Carter, C.J., "Limitations on the Maximum Frequency of the Retarding-Field Oscillator," Technical Report, June 1953; Prepared under contract W33-038-ac-15162 with the Wright Air Development Command, Wright Air Development Center, Wright Patterson Air Force Base, Ohio.}\)
In this type of device the total electron beam is refocused on a cone that becomes very thin with increasing frequency. Even though this may be made of a good conducting material, it can only dissipate a small amount of heat and this limits the usable current. Thus the higher starting current and the lower operating current are tending toward the same limit. This is less of a problem in the two gap klystrons and the self-excited DTKFM since an axial magnetic field can be used to prevent the beam from hitting surfaces that are not capable of dissipating a large amount of power. A rugged collector which can be cooled dissipates most of the power.

The practical problem of constructing a two gap klystron to operate at a wavelength of 3 mm. or less appears to be difficult. An oscillator of this type can be built at a wavelength of 8 mm. as has been shown by the members of SERL. It may be possible to extend this design to shorter wavelength but the practical limit is fairly well defined.

There is also a practical limit to the construction of a resonator type self-excited frequency multiplier. From the preceding discussion this limit appears to be at shorter wavelengths than will be possible for either of the two oscillators. Another reason is that the self-excited frequency multiplier makes use of an oscillator operating at a wavelength where construction problems are not
difficult. The electron gun remains fairly simple and the component parts of a reasonable size. The harmonic resonator is the main component that becomes small.

Experiments to be discussed show that it is possible to operate the harmonic resonator in higher order cavity modes. If this can be done satisfactorily, the frequency limit can be extended slightly farther without decreasing the size of the harmonic resonator. Operation of oscillators in higher order cavity modes has been possible \(^{29}\) but adjustments are extremely critical and the output power is very small.

Johnson or thermal agitation noise in the resonator has been neglected in the preceding analysis since it is small with respect to the shot noise of the beam.

**DESIGN AND CONSTRUCTION OF THE SELF-EXCITED DTKFM**

The major portion of the design used for experimentation on the self-excited DTKFM were developed before the theory as presented earlier had been completed. This theory allows for a logical procedure in designing a DTKFM and will be outlined in detail later. At present, the general reasons for and explanations of the designs that

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have been tested will be discussed.

It was considered desirable to keep the design of the fundamental oscillator as simple as possible. As has been pointed out earlier, a harmonic resonator is to be placed in the drift region of a two gap klystron oscillator. A harmonic resonator in the drift tube places limitations on the diameter of the electron beam and the diameter of the drift tube. To minimize debunching effects, a short drift tube length was selected. Mechanical tuning of the oscillator leads to easier resonator design since it is difficult to design for exact frequencies. Many of the designs tested have not had any means of mechanical tuning so that it is perhaps fortuitous that good results have been obtained in these cases. The above factors and others indicated the use of a two gap single resonator oscillator similar to the floating drift tube klystron except that for simplicity the drift tube is maintained at anode potential. A sketch of this resonator construction along with the possible E field configurations is shown in Fig. 12. This construction has the advantage that it is only necessary to tune one resonator rather than two as is the usual case for two gap klystron oscillator if mechanical tuning is employed. Tuning can be accomplished by changing either of the gap spacings or placing a rotating paddle or disc in the resonator.
Fig. 12a.

Fig. 12b. Approximate field configuration.
The resonator of Fig. 12 can be excited in two ways. The gap voltages can be either in phase as shown in Fig. 12a or $180^\circ$ out of phase as shown in Fig. 12b. When the resonance is as in Fig. 12a, the dc transit angle between the gaps is given by $\theta_0 = 2\pi (n + 1/4)$ where $n$ is any integer. When the resonance is as in Fig. 12b, the dc transit angle is given by $\theta_0 = 2\pi (n + 3/4)$. The frequency of oscillation of these two modes will not be the same. With careful design it should be possible to separate these two modes sufficiently so that they will not interfere with each other. If conditions are correct, both modes can occur at the same time as has been found experimentally.

The dimension $G$ in Fig. 12 indicates the position in which an iris has been placed for coupling to the resonator. It is found experimentally that only one of the illustrated modes will couple through this iris. This is explained by noting the $E$ field configurations. The coupling from this iris is by $E$ field fringing through it into a waveguide. In Fig. 12b it will be seen that there is a component of $E$ field which will couple out the iris whereas in Fig. 12a the $E$ field becomes nearly zero in the plane of the iris. To couple to the mode of Fig. 12a it would be necessary to use a loop or possibly place a window or iris at the top of the resonator.

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It was decided that the supporting structure for the drift tube could also be used to couple the harmonic resonator to a load. A short piece of waveguide was used for this purpose resulting in a structure as shown by Fig. 2. This is a much larger supporting rod than would normally be used. It has not been noted that this causes any adverse effects but it does increase the difficulty of resonator design slightly for a specific frequency.

Available equipment limited the design value for the anode potential to about 3000 volts. This potential should furnish a gap transit time small enough that grids are not necessary and the rf output power should be of a magnitude so that measurement is not difficult. For the first tests, a wavelength of between 4 and 5 centimeters was used for the fundamental oscillator. Construction at these wavelengths is fairly easy and measuring apparatus was available. Due to the relatively small size of the harmonic resonator, it was necessary to select a beam diameter so that the beam coupling coefficient for the harmonic resonator would be as large as possible. A beam diameter of 0.050 inch appeared to be satisfactory. An electron gun developed at Services Electronics Research Laboratory was used as a basis for the design to furnish an electron beam of this diameter. This is a relatively simple low perveance Pierce gun.

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having a voltage controlled focus electrode near the cathode rather than the usual geometrically formed beam focusing electrode. Tolerances are less severe in this type of gun making it very desirable for experimental purposes. It was necessary to scale the gun developed at SERL since it had a beam diameter of 0.025 inch rather than 0.050 inch. The resultant structure and dimensions are as shown in Fig. 13.

The beam diameter is small and the distance that the electrons will have to travel is relatively large so that an axial magnetic field is needed. A mounting system was devised so that an available electromagnet could be used. The pole pieces in this mount were made so that the axial magnetic field increases in the direction of electron flow. This decreases the interception of the beam on the drift tube when the oscillator is operating. A bunched beam has greater space charge forces tending to spread the beam than does a dc beam of the same original diameter. With increasing magnetic field this extra spreading is minimized. The resultant experimental design is sketched in Fig. 14. A calibration of the magnetic field strength at the bottom and top of the waveguide between the pole pieces is shown in Fig. 15 as a function of the current through the magnet solenoid. The increasing magnetic field is easily seen in this figure.
Fig. 13. The electron gun for supplying a 0.050 inch beam.
Fig. 14. DRIFT-TUBE KLYSTRON MULTIPLIER ASSEMBLY
CALIBRATION OF THE MAGNETIC FIELD USED IN THE DTKFM

![Graph showing calibration of the magnetic field used in the DTKFM. The graph plots field strength in Gauss against magnet current in amperes. Two curves are shown: one for the top of the guide and one for the bottom of the guide. The x-axis represents magnet current (amperes) and the y-axis represents field strength (gauss).](image)
The electron gun of Fig. 13 under typical operating conditions furnishes a cathode current of 0.150 ampere at an anode potential of 3000 volts. Approximately 86% of this cathode current can be transmitted through a distance of approximately one inch when the focus electrode voltage is 400 volts and the axial magnetic field formed by the structure of Fig. 14 is used. The solenoid current was 2 amperes so the field varied between 1600 and 2650 gauss. When the oscillator is operating, it is not always desirable to have this high percentage of beam transmission. It appears that better beam coupling results if there is a certain amount of beam interception.

The dimensions of the resonators can be determined by the use of graphs or charts as presented by Hamilton, Knipp, and Kuper but it has been found that surprisingly accurate results can be obtained using an equation developed by Slater. This equation is developed for a doubly reentrant resonator as shown in Fig. 16a. The resulting equation is

$$\lambda_0 = \pi r_1 \sqrt{2L/d} \ln \frac{r_2}{r_1}$$  \hspace{1cm} (64)$$

where $\lambda_0$ is the resonant wavelength,

$r_1$ is the radius of the reentrant post,

$L$ is the cavity height,

\[32\] Hamilton, Knipp, and Kuper, op. cit.

Dimensions applicable in calculation of resonant wavelength.
\[ d \] is the gap width, and
\[ r_2 \] is the outer radius of the resonator.

The corresponding dimensions to be used in calculating the resonant wavelength of the oscillator of the DTKFM are shown in Fig. 16b. It is only necessary to use the dimensions at one gap and the total length of the resonator. When it is realized that the operation of this oscillator is similar to that of a reflex klystron, it is evident that this method for calculating the resonant wavelength is satisfactory. This equation appears to be accurate only for the \((n + 3/4)\) mode. If Eq. (64) is to be used to calculate the resonant wavelength of a singly reentrant cavity, the radii and the cavity height dimensions are used as before but the gap width \(d\) is reduced by one-half. By using Eq. (64) in this manner it will be found that the results are accurate to within less than 10\%. This will be demonstrated later in the experimental section.

The harmonic resonator is formed by making the oscillator drift tube in two or three pieces depending on design. The cavity dimensions can be machined or hobbed into three pieces and the components brazed together. The harmonic waveguide can be brazed into position in this same operation. It is not necessary to use a coupling waveguide directly into this resonator in all cases. It is possible to excite an output waveguide by direct radiation across
the fundamental resonator. This will be demonstrated in the discussion of the DTKFM having a one centimeter fundamental oscillator.

To complete this section, let us now outline a logical procedure for designing a complete self-excited DTKFM.

**DTKFM Design Procedure**

The design of the self-excited DTKFM can be divided into two parts. The first part consists of designing the fundamental oscillator and determining its operating characteristics. The second part consists of the selection of the harmonic number, design of the harmonic resonator, and from the operating characteristics of the fundamental oscillator, determining the location of the harmonic resonator in the drift tube.

In designing the fundamental oscillator, the selection of the operating frequency and the approximate anode potential is made from known conditions of the problem to be solved. The approximate cavity height will be determined by the spacing between the centers of the gaps. The gap spacing is determined from the conditions for oscillation, namely

\[ \theta_0 = \omega_0 t = \frac{\omega_0 s}{u_0} = 2\pi (n + 3/4), \quad (n = 0, 1, \ldots) \]

if the gap voltages are to be 180° out of phase. The number \( n \) is chosen so that the drift tube length is not so long that debunching
effects become large. The gaps should be designed for short transit angle and good beam coupling coefficient. The beam coupling coefficient can be checked by curves presented in Pierce's article \(^{34}\) or by Fig. 11b if grids are not to be used. The final dimensions of the resonator are then determined by Eq. (64) or use of the charts and graphs of Hamilton, Knipp, and Kuper \(^{35}\). Coupling the resonator to an output waveguide or coaxial line can be accomplished by either iris or loop. Loop coupling may be more satisfactory since this will couple to either or the possible cavity modes whereas iris coupling may not.

The operating characteristics of this design should now be determined. The bandwidth \(2\delta\) and the frequency sensitivity \(df/dV_0\) are determined by measurements. Since \(\theta_0\) and \(V_0\) are known, the operating bunching parameter \(X_{0c}\) can be determined from Fig. 5. This operating bunching parameter will be used to locate the harmonic resonator.

The desired harmonic number selected will probably have to be less than ten for ease of design and good output power. The same

\(^{34}\) Bell, and Hillier, op. cit.

procedure for resonator design as presented above can be used to
determine the dimensions for the harmonic cavity. To locate this
resonator in the drift tube, ratio $V_1/V_0$ is calculated from the oper-
ting bunching parameter and then used in Eq. (19a) where $X_{0m}$ is the
optimum value for the selected harmonic. As a further check, Fig. 6
may also be referred to.

EXPERIMENTAL RESULTS OBTAINED
WITH THE SELF-EXCITED DRIFT-TUBE KLYSTRON
FREQUENCY MULTIPLIER

It has been shown theoretically and quasi-theoretically that
the self-excited drift-tube klystron frequency multiplier has possi-
bilities of furnishing rf energy at short wavelengths. Before extreme-
ly short wavelengths can be investigated with acumen, it is necessary
to gain knowledge as to characteristics of the device at some conven-
ient longer wavelength. Actually, once the characteristics have been
determined at this lower frequency, it should only be a matter of
scaling and taking the usual precautions necessary for operation of
any microwave tube at extremely high frequencies. The precautions
are demanding, however, so that any investigation in the range of
short wavelengths is necessarily a long range project. For this rea-
son the majority of the following investigations are performed at
relatively long wavelengths. A brief investigation into operation at
very short wavelengths will be presented and some of the refine-
ments for future development will be discussed.

In general, the objective will be to determine conditions of operation for development of maximum harmonic output power. Whereas efficiency of performance is of interest in all electron devices, this factor loses its importance to some extent when the device involves generation of millimeter waves. This is because of present conditions that sources of generation in this wavelength range are few and difficult to produce so that any method possible for millimeter-wave generation, no matter how inefficient, is of interest. It may be that in a few years this will no longer be true but as of the present, it is true.

Several processes for improving the harmonic output power of the self-excited drift-tube klystron frequency multiplier are possible beyond that which is to be reported. An attempt to point out these processes will be made throughout the discussion. Many of the tests performed were for the purpose of determining the optimum position of the harmonic gap along the drift tube for best harmonic output. These tests were quite time consuming for the relative amount of information gained from them. This happens to be characteristic of much of electron vacuum tube research and a great deal of patience is necessary.

With the gun design and component mounting system completed as discussed in the previous sections, the next problem was to design
resonators for the fundamental and harmonic. The original design for these resonators was calculated from the graphs and tables of Hamilton, Knipp, and Kuper\textsuperscript{35}. These graphs and tables are for singly reentrant resonators and not for the design used in the oscillator of the drift-tube klystron. The drift tube klystron resonator looks like two singly reentrant resonators back-to-back as may be seen in Fig. 17 where the dotted line is through the center perpendicular to the axis. Thus the design was attempted from this observation.

Having decided on an operating voltage of 3000 volts as discussed in the previous section, a wavelength of 6 cm was selected for the oscillator. A transit angle of 11 radians was found to give a convenient distance of 0.449 inch between gap centers. To determine a possible gap width for the oscillator, the simple relation for beam coupling coefficient

\[ \rho = \frac{\sin \theta / 2}{\theta / 2} \]

was employed. This is generally used for gaps having grids and thus could lead only to an approximation of \( \rho \) for the drift-tube klystron oscillator. However, it was believed to be accurate enough for a first approximation. A gap width of 0.075 inch gave a beam coupling coefficient of

\textsuperscript{35} Hamilton, Knipp, and Kuper, op. cit.
Fig. 17. Sketch indicating that the actual resonator appears to be two singly reentrant resonators back-to-back. The dotted line shows the dividing point.
approximately 0.9 and was considered satisfactory.

Half the distance between the gap centers plus half a gap width, or approximately 0.26 inch was used as the height of a singly reentrant cavity and the remaining dimensions were determined from Hamilton, Knipp, and Kuper. This gave a center post diameter of 0.400 inch and an outside diameter of 0.800 inch. The total resonator of the drift tube klystron was then found by placing two of these singly reentrant resonators back-to-back with the center post forming the drift-tube. This design was believed to have too much gap capacitance for good operation so it was changed as in Fig.16b. This change was accomplished by increasing the resonator volume approximately the same proportional amount as the gap area or capacitance was decreased. This should not change the resonant frequency. The harmonic resonator was scaled from the resultant oscillator resonator by taking one reentrant section using direct scaling to form a singly-reentrant resonator operating at approximately

Rectangular irises were milled into each resonator for coupling to their respective waveguides. The fundamental resonator iris was formed in one-half the resonator as shown in Fig. 2 and the iris walls were made very thin. Heavy coupling to the harmonic resonator was desired so a large portion of this resonator was
actually removed and its output waveguide placed almost to the re-entrant center post. Very few design changes of these irises have been investigated. It is possible that improvement in the iris coupling of the resonators could lead to a definite improvement in operation of the device.

At the beginning of this investigation, a method was used for determining the optimum position of the harmonic gap that has now been replaced by the more accurate one previously described. The original method assumed an approximate depth of ac modulation of the beam \( V_1/2V_0 \). From the optimum bunching parameter \( X_{0m} \) for each harmonic, a harmonic transit angle \( \theta_{0m} \) could then be calculated and the gap position determined from the necessary transit time for this transit angle. There are too many approximations in this type of calculation but it was used to design the first few tubes. The method outlined in the previous section would be used in any further designs of the device.

In testing this design, it was found to oscillate at a wavelength of approximately 4 cm. Fortunately, the harmonic resonator was scaled from the oscillator resonator and harmonic output was obtained. Actually output at two slightly different wavelengths occurred for both the oscillator and harmonic. A wavelength of 1.38 cm or the third harmonic of the fundamental 4.14 cm and a
wavelength of 1.355 cm corresponding to a fundamental of 4.065 cm were measured. Similar results in the following modifications will be noted. At this stage of the investigation the reason for this phenomenon was not known. It was believed to be due to the large waveguide structure supporting the drift tube. Of course as will be substantiated later this is due to the possibility of exciting the two oscillator gaps either in phase of 180° out of phase. Nonuniformities make it possible for the frequencies to be slightly different. It is always found difficult to measure the output of one of these fundamental wavelengths because of the coupling arrangement.

Cold test measurements were performed on both resonators by using external oscillators which were frequency modulated and noting the patterns reflected from the resonators on a cathode ray oscilloscope. A pronounced dip in the pattern is observed when the external oscillator is tuned to the resonator frequency and in this manner the resonant frequency can be determined. The only values which could be measured were 4.14-cm wavelength for the fundamental and 1.37-cm for the harmonic. The method of coupling evidently prevented measurement of the other two wavelengths.

Two different tests were made on this oscillator. These tests differed in the point of location of the harmonic gap along the length of the drift tube. This change in location was simple to
accomplish since the drift tube was made symmetrical and could be
turned end for end, while the harmonic resonator was placed slightly
off center. Figure 18 demonstrates this and shows that the resona-
tor is formed by making the drift tube in two pieces. Also shown is
a portion of the supporting and coupling waveguide so that the method
of coupling to the resonator is easily seen.

With the drift tube in one position, the harmonic gap center
was 0.210 inch from the center of the buncher gap and upon reversal
this dimension became 0.255 inch. These particular dimensions a
arose from the original method of calculation for optimum harmonic
gap position. The drift tube was clamped into position by tightly
screwing together the outer two halves of the DTK Fundamental
oscillator resonator and in this way it was a relatively simple mat-
ter to change the gap position without constructing all new parts. This
fact undoubtedly leads to low Q for the fundamental resonator and
results in less efficient operation than would be possible if the com-
ponents were brazed together. However, this procedure was followed
throughout in all the tubes to be discussed so that small changes would
be easy to make.

With the harmonic gap in the closer position with respect to
the buncher gap, a third harmonic output power of approximately 100
milliwatts was measured at an anode voltage of 3000 volts and
Fig. 18. Cross section of the drift-tube showing the harmonic resonator and its coupling system. It will be noted that the drift tube is made in two pieces and the gap is beyond the center indicated by the dotted line.
collector current of 120 ma. Reversing the drift tube, 50 milliwatts of harmonic power was measured with an anode voltage of 2750 volts and a collector current of 110 ma. The difference in harmonic output power cannot be related to the gap position since the oscillator was not operating the same in both cases. A maximum fundamental output power of 13 watts was measurable in this second case but at an anode potential of 2525 volts and collector current of 90 ma. The outputs of both the fundamental oscillator and of the harmonic should occur at nearly the same anode potential. This discrepancy was later explained by careful wavelength measurement. It was found that the harmonic output was greatest at the wavelength corresponding to the fundamental wavelength which did not couple out of the resonator and vice versa. Thus, unfortunately, the best harmonic output corresponded to the in-phase mode of oscillation while the best measurable fundamental oscillator output corresponded to the out-of-phase mode of oscillation. This led to considerable confusion in many of the tests. Thus correlation with theory is difficult with these data.

It should be noted that the harmonic output was a function of both the magnetic field and the gas pressure. It is natural that the magnetic field would change the harmonic output since this can control the beam coupling coefficient. Increasing gas pressure would tend to cause excessive ionization and thus neutralize the beam bunching
to some extent.

The DTK oscillator was next modified slightly by decreasing the length of the gaps. This was accomplished by lengthening the drift tube so that the gap lengths would be 0.057 inch rather than the original 0.075 inch. To maintain nearly the same oscillator frequency, the area of the gaps was also decreased. This modification was made so as to have shorter gap transit angles and possibly larger gap voltage and better bunching. Cold test measurements gave a resonant wavelength of 4.24 cm which was considered satisfactory. Using this design in the self-excited oscillator, it was again found that oscillations occurred at two frequencies quite close together. This particular modification did not remove this difficulty. It was also found that approximately the same fundamental output power was obtained as before so that no improvement was realized here. This tends to agree with the calculations made considering second order bunching theory. The change made in gap transit angles should not change the bunching characteristic to any noticeable extent. However, this design was employed in the remainder of the tests.

To simplify the harmonic resonator fabrication and assembly, a simple rectangular parallelopiped to be used as the harmonic resonator was tested. The cavity height was made equal to the gap width
0.0125 inch and the cavity was designed to resonate at the fifth harmonic of the fundamental. A power output of 5 milliwatts at 8.48 mm wavelength was measured, indicating that this design does not couple with the beam satisfactorily and probably has too low a shunt resistance. For this reason the design was rejected and all further resonators were of the reentrant type.

When the drift-tube was modified to shorten the oscillator gaps, the supporting waveguide was also changed slightly so that harmonic output at the fourth harmonic of the fundamental was obtained. This new coupling slightly reduced the volume of the harmonic resonator.

Upon testing this modified tube, two wavelengths were detectable from both the harmonic and fundamental resonators. These were $\lambda_1 = 4.32$ cm, $\lambda_4 = 1.08$ cm, and $\lambda_1 = 4.8$ cm, $\lambda_4 = 1.2$ cm respectively where $\lambda_1$ is the fundamental wavelength and $\lambda_4$ is the harmonic wavelength. The maximum fundamental output power was about 15 watts at $\lambda_1 = 4.32$ cm and the harmonic output was approximately 50 milliwatts at $\lambda_4 = 1.2$ cm. Thus the same phenomenon as noted in the first model is again present. The harmonic gap was 0.210 inch from the first oscillator gap. The given output powers occurred under different voltage conditions. To obtain the maximum fundamental output, it was necessary to have an anode potential of 2800
volts at which voltage the harmonic output was 15 mw. To get maximum harmonic output, the anode voltage necessary was 2900 volts and the fundamental output was one watt. This is due of course to the fact that it takes different voltages to peak the fundamental oscillations.

It is interesting to note that higher frequency output was detectable from the harmonic resonator. By introducing a short section of RG-98/U waveguide having a cutoff wavelength of 7.52 mm at the output of the tube, a strong signal was detected at the sixth harmonic or approximately 7 mm. The mode shape was the same as that of the fundamental and the output power was estimated to be in the order of one milliwatt. Replacing the RG-98/U waveguide by RG-99/U which has a cutoff wavelength of 6.2 mw, a detectable signal at the eighth harmonic or approximately 5.4 mm was noted. This signal was quite weak.

By reversing the drift tube so that the harmonic gap is 0.255 inch from the first oscillator gap, a maximum output of 65 milliwatts was obtained fro the harmonic at \( \lambda_4 = 1.2 \text{ cm} \) and with an anode voltage of 3100 volts. No fundamental output power was measurable at this voltage. Reducing the anode potential to 2800 volts reduced the harmonic output to 10 mw but gave a fundamental power of 18 watts. Using the sections of small waveguide as before it was still possible
to detect the sixth and eighth harmonics in about the same magnitude as before. The results tend to agree with the theoretical considerations that the harmonic gap should be closer to the output gap of the oscillator for best harmonic output.

One final test has been made concerning variation of harmonic gap position. A design was constructed in which the gap could be placed extremely close to either the buncher or catcher gap. This is shown by Fig. 19. To obtain these extreme conditions it will be noted that a quarter wave coaxial line was necessary between the actual gap and the resonator due to space considerations. It was realized that this would lead to poor Q for the resonator but deemed desirable for placing the gap in these locations. An attempt was made to keep the resonator dimensions such that operation would be at the fourth harmonic. This was not successful since it operated at the fifth harmonic. For some reason, the fundamental oscillator itself did not operate well with this drift tube. Output at 9.4 mm was detected in both positions of the harmonic gap. It is of interest that harmonic output was obtained at all, with the gap at about a distance of 0.05 wavelength (fundamental) from the center of the bunched gap.

It can be seen from these results that the position of the harmonic resonator gap along the drift tube is not extremely critical
Fig. 19. Drift-tube and harmonic resonator design for location of the harmonic gap close to either the buncher or catcher gap.
with quite satisfactory results being obtained with the gap near the center of the drift tube. The tubes tested have had no means of tuning and since it is difficult to maintain exact harmonic relationship between resonators, it is indeed fortunate that results have been possible. By adding tuning to the oscillator so that an exact harmonic relationship between resonators would be possible, brazing all parts together, optimizing coupling, and improving $Q$, it would be possible to increase the efficiency considerably for both the fundamental oscillator and the harmonic.

To demonstrate that the formula developed by Slater and modified as previously discussed is accurate for resonant wavelength calculations for this tube, the following calculation will be made. The equation is repeated here with the discussed modification, for simplicity. It is

$$\lambda = 2\pi r_1 \sqrt{\frac{L}{d} \ln \frac{r_2}{r_1}}$$  \hspace{1cm} (65)$$

where the dimensions are as in Fig. 16b. For the fundamental oscillator, we use the dimensions of one gap and total resonator length. The resonator dimensions are then:

$$d = 0.057 \text{ inch} = 0.145 \text{ cm}$$

$$r_1 = 0.065 \text{ inch} = 0.165 \text{ cm}$$
From Eq. (65)

\[ \lambda_1 = 2\pi (1.165) \sqrt{\frac{1.52}{0.145}} \ln \frac{1.47}{0.065} = 4.72 \text{ cm} \]

as compared with an actual measured value of 4.76 cm. For the fourth harmonic resonator the dimensions are

\begin{align*}
&d = 0.012 \text{ inch} = 0.0305 \text{ cm} \\
r_1 = 0.040 \text{ inch} = 0.102 \text{ cm} \\
r_2 = 0.095 \text{ inch} = 0.241 \text{ cm} \\
L = 0.050 \text{ inch} = 0.127 \text{ cm}.
\end{align*}

Then Eq. (65)

\[ \lambda_4 = 2\pi (0.102) \sqrt{\frac{0.127}{0.0305}} \ln \frac{0.241}{0.102} = 1.218 \text{ cm} \]

as compared with the actual measured value of 1.20 cm. This calculation as far as the oscillator is concerned is only valid for the case where the gap voltages are 180° out of phase.

It is now of interest to check where the optimum harmonic gap position should be by the theory reviewed earlier. To make this calculation, it is necessary to measure the bandwidth \(2\delta\), the frequency sensitivity \(df/dV_0\), anode voltage \(V_0\), and frequency.

These measurements were made on the above oscillator having a fundamental wavelength = 4.31 cm and the following values determined.

For this oscillator, \(df/dV_0\) was measured to be 11.1 kc/volt and the
bandwidth $2\delta = 8.6$ Mc. The value of $\theta_0$ was determined from 
\[ \omega_0 S/u_0 \] where $\omega_0$ is angular resonant frequency, $s$ is the distance between gap centers, and $u_0$ is the beam velocity. For this tube 
\[ \omega_0 = \frac{5}{4} \pi \times 10^{10}, \quad s = 0.467 \text{ inch or } 1.19 \text{ cm}, \quad \text{and the tube operated at} \] 
\[ V_0 = 2800 \text{ volts so that } u_0 = 3.14 \times 10^9 \text{ cm/sec}. \] This gives a value of 16.6 for $\theta_0$ and should correspond to the $n + 3/4$ mode so that $\theta_0$ was considered to be $17.25 = 2\pi (n + 3/4)$ where $n = 2$. For these values,

\[ \frac{\theta_0}{4V_0} \frac{2}{df/dV_0} = 1.192 \] and from Fig. 5 the operating bunching parameter $X_{0c}$ is found to be 2.4. To calculate the operating-to-dc signal level ratio $V_1/V_0$, one needs to know the oscillator beam coupling coefficient $\rho$. This can be determined from the preceding data and tables from Pierce or Fig. 11b. It is found to be approximately 0.8. Thus from $X_{0c} = \frac{\rho V_1 \theta_0}{2V_0}$, and $X_{0c} = 2.4$, $V_1/V_0 = 0.348$. By solving Eq. (19a) for $S_{gm}$ and determining the gap position for the fourth harmonic we find

\[ S_{gm} = \frac{(X_{04}^2 u_0)}{\omega_m V/V_0} = 0.612 \text{ cm} \] (66)
or 0.241 inch. In this equation $X_{04}$ is the optimum bunching parameter for the fourth harmonic. It will be noted that the value $S_{gm} = 0.241$ inch is close to the 0.255 inch position as actually used in
some of the above tests and where the best results were obtained at the fourth harmonic.

The focusing magnetic field in the preceding oscillators was found to be 1000 gauss at the bottom of the waveguide and 1550 gauss at the top. This field gave sufficient focusing and beam interaction. The output power was a function of this field as mentioned previously.

Experimental verification has been obtained of the fact that the two oscillator wavelengths detected were due to exciting the gaps in phase or 180° out of phase. The center voltage for \( \lambda_1 = 4.23 \text{ cm} \) or \( \omega_0 = 4.45 \times 10^{10} \) radians per second was measured to be 2700 volts corresponding to a beam velocity \( u_0 = 3.07 \times 10^9 \) cm/sec. With the distance between gaps of 0.467 inch as above, \( \theta_0 = \omega_0 s/u_0 = 17.25 \) which agrees exactly with \( \theta_0 = 2\pi (n+3/4) \) with \( n = 2 \) and with the gap voltages out of phase. The center voltage for \( \lambda = 4.76 \text{ cm} \) or \( \omega_0 = 3.96 \times 10^{10} \) rad/sec was found to be 3100 volts corresponding to a beam velocity \( u_0 = 3.32 \times 10^9 \) cm/sec. Thus \( \omega_0 s/u_0 \) is 14.2 and agrees exactly with \( \theta_0 = 2\pi (n + 1/4) \) with \( n = 2 \) and with the gap voltages in phase.

An attempt has been made to tune the oscillator. Due to its construction, any method of tuning externally is difficult. At any rate, a small metal rotating paddle was inserted in the main volume
of the resonator. Its length was such that in one position the paddle extended nearly the total height of the resonator and by a rotation of 90°, it was perpendicular to the axis and in the center of the resonator. The paddle was inserted close to the harmonic waveguide so that its effect was masked considerably. Some tuning was possible but not enough to improve operation. It would be interesting to test the device if the oscillator gaps were adjustable for tuning.

ONE CENTIMETER TO FIVE MILLIMETER
DRIFT-TUBE KLYSTRON FREQUENCY MULTIPLIERS

Figure 20 shows a design cross section of the only tube tested in the millimeter range. This is drawn to a 4:1 scale so that an indication of size can be seen. Some of the intricacies can be noted by the make up of the harmonic resonator and drift tube. To from this resonator, the drift-tube had to be made of three pieces with the overall assembled length being only 0.104 inch. These three pieces must be kept in alignment and brazed together. For convenience, the two oscillator resonator halves were clamped together holding the drift tube in position. This again does not lead to high Q resonators.

The basis for the oscillator design was the SERL 8 mm klystron (see Ref. 23). This oscillator with special control over surface losses, close accuracy of assembly and in a sealed-off tube has been
Fig. 20. Design of the one centimeter to five millimeter DTKFM.
capable of delivering 25 watts of rf output power. If it were possible to reproduce this operation and have a harmonic resonator in the drift tube, large harmonic power could be expected at mm wavelengths.

Due to the size, a waveguide to the harmonic resonator is not possible. Coupling is accomplished by an iris into the harmonic resonator and direct radiation across the fundamental oscillator cavity to the output waveguide. The merits of this type of coupling have not been evaluated. The fundamental oscillator is iris coupled to its waveguide in a manner similar to the longer wavelength models.

The tunnel diameter for this tube is 0.025 inch and the focusing magnetic field necessary is increased. In the tests made, the field necessary for good operation varied between 2500 gauss at the bottom of the waveguide and 4000 gauss at the top. The electron gun was directly scaled from the one of Fig. 13 and operation was similar to that described previously.

An attempt was made to design the oscillator for operation at 8 mm wavelength and the harmonic resonator at the third harmonic or 2.67 mm. Due to discrepancies in fabrication and assembly, operation was obtained at $\lambda_1 = 1$ cm and $\lambda_2 = 5$ mm. Fundamental oscillator output power of 200 milliwatts was obtained with an anode voltage of 3200 volts, cathode current of 184 ma, collector or
transmitted current of 150 ma, and beam forming electrode voltage of 600. The second harmonic was fairly strong but the detection equipment used was inefficient. It was estimated that the 5 mm wavelength output was of the order of 0.1 mw but it could have been more. The coupling to both resonators is probably quite low because thick walled irises were used. A better coupling system could improve the output of both resonators.

The harmonic resonator had to be placed at the center of the drift tube for physical reasons of fabrication. It would be possible to vary this position slightly but not without difficulty. With the harmonic gap in this position, it would not be at an optimum point for best harmonic operation.

Using the actual dimensions of the harmonic resonator, the resonant wavelength was calculated by a method suggested by Ward. This calculation involves a field plot but is self-consistent and gives quite accurate results. The resonant wavelength was found to be 5.32 mm. Again tuning arrangement would be desirable to relate more closely and harmonically the fundamental and the

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harmonic resonators.

One further attempt has been made to increase the harmonic frequency. This was done by fabricating a new harmonic resonator having a longer gap and less capacitance. For reasons which have not yet been determined, the fundamental oscillator does not operate as well with this drift tube but the harmonic output is still detected at 5 mm.

Two interesting features were noted with this latter tube. First, the tube would not oscillate when the supposedly symmetrical drift tube was in one position, but if the drift tube were reversed, it would oscillate. The only other change that this entails is the output coupling to the waveguide. For oscillation to occur, the iris of the harmonic resonator was aligned with the fundamental waveguide. This then introduces the second interesting feature. With the iris so aligned the harmonic waveguide is coupled to the fundamental oscillator resonator only and yet harmonic output is obtained. In fact, harmonic output of about the same intensity is found in both the fundamental and harmonic waveguide.

If the second harmonic could be suppressed by using waveguide having a cutoff wavelength of less than 5 mm, it should be possible to obtain shorter wavelengths as was the case of the 4 cm tube. Actually, to get good results it will probably be necessary to
do the following:

1. Carefully process the materials for very smooth surfaces.

2. Observe an oxidizing and reducing procedure for obtaining high Q resonators.

3. Accurately align and braze all components into position.


5. Use sealed-off tubes rather than continuously-pumped models.

Most of these procedures are necessary for any millimeter-wave tube production and investigation.

SUMMARY AND CONCLUSIONS

The drift-tube klystron self-excited frequency multiplier has been described and shown to be a possible source of rf energy at millimeter wavelengths. The theory of operation has been developed and a design procedure outlined. This general procedure is to design the fundamental oscillator by usual known methods; determine the operating bunching parameter $X_{0c}$ from measured characteristics and the bandwidth relation

$$\frac{2S\theta_0}{4V_0 df/dV_0} = \frac{1}{J_1(X_{0c})} \left[ 2J_1^2 \left( \frac{X_{0c}}{2} \right) - J_1 \left( \frac{X_{0c}}{2} \right) \right]^{1/2}.$$
Calculate the beam coupling coefficient and thus determine the signal level $V_1/V_0$; and finally calculate the harmonic gap position from Eq. (19a) or (66).

$$S_{gm} = \frac{X_0}{\omega_m} \frac{2m^u_0}{V_1/V_0};$$

and the necessary value of $X_0$ for optimum $J_m(mX_0)$. This position has been shown to be at a point slightly beyond the center of the drift tube toward the catcher gap.

The effect of a long transit angle in the oscillator bunching gap has been shown to be of little importance as far as harmonic content of the beam is concerned. Effects of debunching have been considered but experimental results offer no definite conclusions concerning this phenomenon.

Frequency limitations concerning the DTKFM, reflex klystron oscillators, and two-gap klystron oscillators have been considered from the standpoint of beam shot noise and necessary beam current for onset of oscillations. In general, the conclusion is reached that to the limit of feasible fabrication it should be possible to obtain output from a resonator-type frequency multiplier whereas it may not be possible for oscillations to occur in the klystrons.

The experimental results in general tend to agree with the theoretical considerations. The effect of increasing the buncher gap
transit angle caused no noticeable change in harmonic output. The best harmonic output has been obtained with the harmonic resonator gap at a point beyond the center of the drift tube. Operation at millimeter wavelengths is possible as has been shown while at the same time it is becoming increasingly difficult to effect a two-gap klystron oscillator.

Figure 21 compares theoretical with experimental results obtained from the DTKFM, the self-excited retarding-field oscillator multiplier, and experimental results obtained with crystal multipliers. It can be seen that the self-excited retarding-field oscillator offers considerably better harmonic output power than does the crystal multiplier and the DTKFM offers better harmonic output power than either of these. The latter is true because of the possibility of higher power operation of the fundamental device. It appears that the DTKFM harmonic output is decreasing with harmonic number at about the rate theoretically predicted.

It is believed that continuation of the investigation of this device is desirable to optimize the design, increase the power output and to ascertain the upper frequency limitations and further characteristics. Investigation of the harmonic content of the beam after it leaves the second or catcher gap of the oscillator may prove to be informative. This information would be useful in determining
Fig. 21. Comparison of three types of multipliers.
the effective bunching and working of the beam by the oscillator.

At the same time, it may be possible to place harmonic resonators
so as to make use of this harmonic content either to improve the
present harmonic output or to add further harmonics.
BIBLIOGRAPHY


Thurston, M. O. "A Self-Excited Frequency Multiplier for the Millimeter Wavelength Range," Engineering Report No. 6 prepared under contract AF 33(616)-2225 with the Wright Air Development Command, Wright Air Development Center, Wright Patterson Air Force Base, Ohio (June, 1955).

AUTOBIOGRAPHY

I, Wendell Hillis Cornetet, Jr., was born in Huntington, W. Va., October 20, 1923. I received my secondary school education in the public schools of Huntington, W. Va., and my undergraduate training at Marshall College, Lafayette College, and West Virginia University, which granted me the Bachelor of Science in Electrical Engineering degree in 1948. From West Virginia University, I received the Master of Science in Electrical Engineering degree in 1951. While in residence there, I was an instructor in the Electrical Engineering Department during the years 1948-1951. In June 1951, I was appointed Research Assistant in the Electron Tube Laboratory under Professor E. M. Boone at The Ohio State University, where I have been specializing in electron tube research. In January, 1953, I was appointed as Research Associate at this same laboratory and in 1956, I was appointed as part time instructor in the Electrical Engineering Department. I am still employed in these two capacities at the present time.