THREE ESSAYS IN MONETARY ECONOMICS
WHAT DO WE LEARN FROM MONETARY ECONOMICS FOR THE LOST DECADE OF JAPAN?

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By

Ryo Kato, B.A., M.A.

* * * * *

The Ohio State University

2002

Dissertation Committee:
Paul Evans, Adviser
James Peck
Masao Ogaki

Approved by
Adviser
Department of Economics
This dissertation is comprised of three essays in monetary economics motivated by the *lost decade* of Japan, a long stagnation of the Japanese economy in the 90s.

The first essay (Chapter 2) provides useful tools in characterizing value functions of a certain class of dynamic optimization problem, which are widely applicable to analyzing economic agent’s decision making. I apply the tools developed in the essay to characterize the optimal monetary policy when nominal interest rate is bounded by zero or near liquidity traps. I obtain three predictions, namely, that the optimal monetary policy near liquidity traps will be (i) more expansionary, (ii) more aggressive than the Taylor rule and (iii) asymmetric. Those predictions are empirically consistent with Japanese data in mid-90s.

The second essay (Chapter 3) develops a dynamic general equilibrium model in which the interaction of corporate demand for liquidity and macroeconomic fluctuation can be analyzed. I extended a capital market model with asymmetric information introduced in Holmström and Tirole (1998) to an infinite horizon environment. My model is capable of replicating several empirical facts of business cycles, which have remained unexplained by preceding DGE studies, namely, deficiencies of the RBC model and other agency cost models.
The third essay (Chapter 4) explores empirical facts of the recent macroeconomic fluctuation of the Japanese economy, extending the model developed in Chapter 3. First, I show some evidence that the Japanese capital market is imperfect, which means that the economy can at most achieve the second best allocation of capital/credit. Based on the finding, I detect that re-allocation of wealth stifled economic growth in the early 90s. Further, I test the prediction of the Holmström and Tirole’s (1998) model to conclude that the risk sharing in short-term lending market was less effective in the 90 than in the 80s, which implies the violation of the second best allocation.
Dedicated to my wife, Tomoko
ACKNOWLEDGMENTS

First and foremost, I am deeply grateful to my adviser Professor Paul Evans without whom this dissertation could not have been completed. His technical advice based on his broad knowledge in economic theories as well as empirical studies always kept my research in the right direction. Moreover, I learned innumerable lessons and insights from him that are necessary for academic research in general.

My thanks also go to the members on my dissertation committee, Professor James Peck and Professor Masao Ogaki. I have been often inspired by Professor Peck’s advice and comments from game theoretical viewpoints. Professor Ogaki’s technical advices on econometrics were essential to conduct the empirical analysis. I would like to thank Professor George Alessandria, who is on my candidacy committee. He has devoted much time and efforts to discussing with me, from which I could derive deeper insights to my own DGE model.

The friendship of Shinichi Nishiyama, Takayuki Tsuruga, Lianfa Li and YongSe Kim is much appreciated and has led to many interesting and good-spirited discussions relating my research. I am indebted to my colleagues, Raphael Solomon and Ji Tao who always find the time for listening to my little problems that I encountered during my research. I am also indebted to Takuji Kawamoto at University of Michigan and Ichiro Fukunaga at London School of Economics for helpful discussions over midnight phone calls. My great thanks also go to my informants at Bank of Japan,
Kenji Nishizaki, Naoko Hara, and Makiko Nakamuro for collecting Japanese data. I know that I had troubled them helping me, to the extent which I am not even certain. Furthermore, I would like to express my gratitude to Professor Takashi Ui at Yokohama National University for his support and encouragement. He has been a “shadow supervisor” of my graduate study in the US.

Last, but not least, I would like to thank my wife Tomoko for her understanding and love during the past few years. Her devotion to me throughout my graduate studies was in the end what made this dissertation possible.
VITA

November 3, 1973 ......................... Born - Mie, JAPAN

1996 ........................................ B.A. Economics, University of Tokyo

1999 ........................................ M.A. Economics Ohio State University


PUBLICATIONS

Research Publications


FIELDS OF STUDY

Major Field: Economics

Studies in Macroeconomics and Monetary Economics: Professor Paul Evans
Studies in Applied Game Theory: Professor James Peck
Studies in Econometrics: Professor Masao Ogaki
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iv</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>v</td>
</tr>
<tr>
<td>Vita</td>
<td>vii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>xii</td>
</tr>
<tr>
<td>Chapters:</td>
<td></td>
</tr>
<tr>
<td>1. Preface</td>
<td>1</td>
</tr>
<tr>
<td>2. On the Constrained Dynamic Optimization with a Quadratic Reward</td>
<td>6</td>
</tr>
<tr>
<td>Function; An Application to the Optimal Monetary Policy near Liquidity</td>
<td></td>
</tr>
<tr>
<td>Traps</td>
<td></td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Theorems</td>
<td>8</td>
</tr>
<tr>
<td>2.2.1 Problem statement</td>
<td>8</td>
</tr>
<tr>
<td>2.2.2 Proofs</td>
<td>10</td>
</tr>
<tr>
<td>2.2.3 Remark: the role of uncertainty</td>
<td>19</td>
</tr>
<tr>
<td>2.3 Application to Monetary Policy with a Zero Bound on Nominal</td>
<td></td>
</tr>
<tr>
<td>Interest Rates</td>
<td>21</td>
</tr>
<tr>
<td>2.3.1 The model</td>
<td>22</td>
</tr>
<tr>
<td>2.3.2 Proposition 1: Expansionary policy</td>
<td>24</td>
</tr>
<tr>
<td>2.3.3 Proposition 2: Aggressive policy</td>
<td>26</td>
</tr>
</tbody>
</table>

viii
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3.4</td>
<td>Proposition 3: <em>Asymmetric</em> policy (decreasing aggressiveness)</td>
<td>28</td>
</tr>
<tr>
<td>2.4</td>
<td>Empirical Evidence</td>
<td>30</td>
</tr>
<tr>
<td>2.5</td>
<td>Concluding Remarks</td>
<td>34</td>
</tr>
<tr>
<td>3.</td>
<td>Liquidity, Infinite Horizons and Macroeconomic Fluctuations</td>
<td>38</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>38</td>
</tr>
<tr>
<td>3.2</td>
<td>Corporate Demand for Liquidity and Firms’ Investment: Holmström and Tirole (1998)</td>
<td>44</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Financial contract with liquidity demand</td>
<td>45</td>
</tr>
<tr>
<td>3.2.2</td>
<td>The role of the financial intermediary</td>
<td>50</td>
</tr>
<tr>
<td>3.3</td>
<td>The Dynamic General Equilibrium Model</td>
<td>52</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Optimization set-ups</td>
<td>53</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Recursive competitive equilibrium</td>
<td>55</td>
</tr>
<tr>
<td>3.4</td>
<td>Simulation</td>
<td>57</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Calibration</td>
<td>57</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Simulation results</td>
<td>59</td>
</tr>
<tr>
<td>3.5</td>
<td>Discussions</td>
<td>63</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Some empirical facts</td>
<td>63</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Relation to the <em>Lending View</em> and other studies</td>
<td>65</td>
</tr>
<tr>
<td>3.6</td>
<td>Concluding Remarks</td>
<td>68</td>
</tr>
<tr>
<td>4.</td>
<td>Does Financial Sector Distress Affect Business Cycles? Evidence from Japan</td>
<td>78</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>78</td>
</tr>
<tr>
<td>4.2</td>
<td>Theoretical background; Agency cost models</td>
<td>84</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Investment function in DGE models with imperfect capital market</td>
<td>85</td>
</tr>
<tr>
<td>4.2.2</td>
<td>The model for corporate liquidity demand; Holmström and Tirole (1998)</td>
<td>89</td>
</tr>
<tr>
<td>4.3</td>
<td>Evidence from aggregate data: Comparing RBC with agency cost models</td>
<td>95</td>
</tr>
<tr>
<td>4.4</td>
<td>Evidence from synthetic panel data</td>
<td>101</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Data source: The FSSCI</td>
<td>101</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Descriptive statistics</td>
<td>101</td>
</tr>
<tr>
<td>4.4.3</td>
<td>Estimation results</td>
<td>103</td>
</tr>
<tr>
<td>4.5</td>
<td>Conclusion</td>
<td>105</td>
</tr>
</tbody>
</table>

Bibliography                                                                                       110

Appendices:
A. Monte-Carlo Simulation using Kiyotaki-Moore model .......................... 115
   A.1 Model overview; Kiyotaki and Moore (1997) .................................. 115
   A.2 Monte-Carlo simulation .................................................................. 116

B. Structural VAR Estimation with a Long-Run Restriction ................... 118
   B.1 Blanchard-Quah technique ............................................................ 118
   B.2 Data ............................................................................................ 119
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 MLE result and Log-Likelihood ratio test</td>
<td>33</td>
</tr>
<tr>
<td>4.1 Monte-Carlo simulation results</td>
<td>89</td>
</tr>
<tr>
<td>4.2 Descriptive statistics</td>
<td>102</td>
</tr>
<tr>
<td>4.3 SUR estimation results</td>
<td>107</td>
</tr>
<tr>
<td>4.4 Covariance matrices for SURs</td>
<td>109</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>3-Concavity</td>
<td>17</td>
</tr>
<tr>
<td>2.2</td>
<td>Estimated monetary policy reaction function</td>
<td>37</td>
</tr>
<tr>
<td>3.1</td>
<td>Time schedule of events</td>
<td>46</td>
</tr>
<tr>
<td>3.2</td>
<td>Structure of moral hazard</td>
<td>48</td>
</tr>
<tr>
<td>3.3</td>
<td>Impulse response to productivity shock 1</td>
<td>70</td>
</tr>
<tr>
<td>3.4</td>
<td>Impulse response to productivity shock 2</td>
<td>71</td>
</tr>
<tr>
<td>3.5</td>
<td>Impulse response to productivity shock 3</td>
<td>72</td>
</tr>
<tr>
<td>3.6</td>
<td>Impulse response to wealth shock 1</td>
<td>73</td>
</tr>
<tr>
<td>3.7</td>
<td>Impulse response to wealth shock 2</td>
<td>74</td>
</tr>
<tr>
<td>3.8</td>
<td>Sensitivity analysis</td>
<td>75</td>
</tr>
<tr>
<td>3.9</td>
<td>Liquidity demand and degree of liquidity dependence: Japan</td>
<td>76</td>
</tr>
<tr>
<td>3.10</td>
<td>Liquidity demand and degree of liquidity dependence: US</td>
<td>77</td>
</tr>
<tr>
<td>4.1</td>
<td>A simulated path by the Kiyotaki and Moore model</td>
<td>88</td>
</tr>
<tr>
<td>4.2</td>
<td>Loans for working capital expenses/total loans</td>
<td>90</td>
</tr>
<tr>
<td>4.3</td>
<td>Auto-correlation function of output growth</td>
<td>97</td>
</tr>
</tbody>
</table>
4.4 Impulse response of the structural VAR to a transitory shock . . . . . . 98
4.5 Net wealth of corporate sector and household sector . . . . . . . . . 99
4.6 Corporate investment and output . . . . . . . . . . . . . . . . . . . . . . . 100
4.7 DLD-ROA plots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 106
4.8 Average DLD-ROA plots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 108
CHAPTER 1

PREFACE

The *Lost Decade*, which was formerly the word describing Latin American economy in the 80s, now reminds most people of the long stagnation of the Japanese economy in the 90s. My graduate studies are mostly motivated by this historic event in Japan, and the three essays in my dissertations are, to some extent, related to it in the sense that theoretical predictions are applicable to the Japanese experience in the 90s. Since each essay is self-contained and seemingly independent, in this preface here, I introduce the interrelated implications of the three essays from the viewpoint of application to the Japanese economy, which is the substantial motivation of the research.

In the first essay (Chapter 2), which presents a unique technique for characterizing value functions, an application of those tools are introduced in an attempt to provide a policy implication for the past of the decade. In the late 90s, we observe in the Japanese economy that the short-term nominal interest rate hit its lower bound, a well-known phenomenon, (but not that commonly observed all over the world in the latter half of the 20th century) a liquidity trap. In the application sections in Chapter 2, based on a widely used IS/AS model and the standard dynamic optimization theory,
I derive a suggestion on what could have been done for the Japanese economy before the decade was lost. Note that the policy implication is likely to be valid as long as inflation is low in that country, which is one of the most evident observations in the Japanese 90s.

In spite of the practical importance and outstanding impact of the liquidity trap, I take it as an outcome induced by underlying problems of the Japanese economy, rather than the cause creating the lost decade. Based on this view, the second essay (Chapter 3) seeks to provide a clue for characterizing the present state of the Japanese economy. Another key observation about the Japanese economy in the 90s is the extreme deterioration of corporate firms’ balance sheets after the dramatic decline of the stock/real estate market at the beginning of the decade. (Needless to say, deterioration in firms’ balance sheet is closely related to one of the most seriously discussed issues in current Japanese economy, the “non-performing loan problem” for commercial banks.) For this purpose, I develop a formal dynamic general equilibrium (DGE) model where heterogeneous agents, lenders (consumers) and borrowers (entrepreneurs) are playing a game in capital market with asymmetric information. Since asymmetric information prevents market mechanism from achieving the first-best allocation of economic resource, such as capital or liquidity, the model might carry an useful insight to understand the stagnation of the Japanese economy. The DGE model reveals a notable feature such that re-distribution of net wealth among heterogeneous economic agents creates a non-neutral effect on output as well as corporate investment. Actually in the model, one-time transfer of net wealth from firms (entrepreneurs) to households (consumers) causes a non-negligible downturn, given the presence of a certain magnitude of asymmetric information in capital market.
The economic intuition behind this prediction is simple. Suppose in the standard RBC economy, financial intermediation is, by construction, a costless activity, which is nearly a definition of the first-best economy. On the other hand, in the second-best economy, ex-ante distribution of net wealth does matter, since it alters the extent to which the economy would depend on financial intermediation. Essentially, an economy where the net wealth is distributed intensively to an un-productive sector (i.e., household) could be forced to endure lower economic performance via increased agency cost.

Now in the third essay (Chapter 4) I directly inspect the actual Japanese data from multiple angles in comparison with the theoretical predictions derived from my DGE model introduced in Chapter 3. First, I demonstrate that the stylized fact of the Japanese economy is inconsistent with the standard RBC model’s predictions to conclude that the economy can at most achieve the second-best allocation of capital/credit. Having confirmed the presence of imperfect capital market, (which could be created either by asymmetric information or other types of agency problems) I present evidence of some re-allocation of net wealth from corporate/productive sector to household sector at the beginning of the 90s. Recall my DGE model’s prediction, such that the re-allocation of net wealth would result in a downturn of output/corporate investment via increased agency costs, which is, as shown in the essay, evidently consistent with what the Japanese economy experienced in the early 90s. Furthermore, another question that one might ask next is, whether the Japanese economy achieves the second-best allocation in capital market in the 90s. Is there any misallocation of credit/capital or malfunctioning of the financial intermediation? It should be noted that my argument so far is perfectly confined within the second-best
economy. To examine the validity of the second-best allocation for the Japanese capital market, I test the prediction of a variant of the imperfect capital market model introduced in Holmström and Triole (1998), (note that it is a core feature of my DGE model constructed in the second essay) using Japanese data of the recent decades. The results of statistical testing reveal that the efficient risk sharing in the short-term lending market was less effective in the 90s than in the 80s, which implies violation of the second-best allocation of credit/capital.

Basically, I propose two types of inefficiency, one of which is within the second-best economy, and the other is outside of it. “Type 1” inefficiency is nearly self-evident even by a casual observation. On the other hand, I do not provide any good economic explanation for the mechanism behind the “type 2” inefficiency, violation of the second best allocation. It could be modeled as a non-Pareto optimal Nash equilibrium, but this is out of the scope of my study here. Another potentially missing viewpoint from this study is inefficiency stemming from final goods market. Mostly I focus the inefficiency that is created via agency problems in imperfect capital market, since it seems more reasonable to detect such misallocation of capital, reflecting the deteriorated balance sheets of the Japanese corporate sector. It is not necessarily correct to limit the source of inefficiency in factor markets and thus, further research focused on final goods market in a similar spirit could carry another insight for the lost decade of Japan.

Based on this analysis, I can propose policy implication for the future of the Japanese economy which might come after the decade. I do not know whether the promotion of productivity growth would increase economic welfare in the absence of the first-best economy. However, I can claim that any policy or reform which directly
ameliorates the malfunctioning financial intermediation is very likely to work, given
it would let the allocation of capital/credit get closer to the one achieved by the
second-best economy.
CHAPTER 2

ON THE CONSTRAINED DYNAMIC OPTIMIZATION WITH A QUADRATIC REWARD FUNCTION; AN APPLICATION TO THE OPTIMAL MONETARY POLICY NEAR LIQUIDITY TRAPS

2.1 Introduction

This essay provides convenient tools for characterizing the value functions of the infinite time dynamic optimization problem with a quadratic reward function and occasionally binding constraints on the control variable. Analytical inspection of the value function (or policy function) of the dynamic optimization problem with this type of constraints is scarce among the preceding studies. Although many numerical studies are available in the literature, (Christiano and Fischer(2000), Deaton(1989), and Orphanides and Wieland(2000) for example) analytical studies on the problem is not provided so far except for Carroll and Kimball (2001). They characterize the value function of the finite time horizon problem with the same type constraints as the one considered here using backward induction. On the other hand, the tools to be introduced here in this paper are convenient when the optimization horizon is infinite. Infinite horizon problems are potentially difficult since the popular technique for finite horizon problems, backward induction is not applicable in the case. I can resolve this
difficulty by using the techniques in this paper, which allow us to inspect the shape of the converged value function directly. In addition, the backward induction usually requires messy calculation, while the tools in this essay is less algebra-intensive. This practical convenience is another advantage of our new tools. However, it should be noted that my technique is not a substitute of the standard backward induction, but a complement of it, since each two technique should be applied to the problems with different time horizons.

Carrol and Kimball (1996) demonstrates that the third derivative of the value function is closely related to the concavity of the policy function (the consumption function in their case) in case of the unconstrained problem. Similarly, also in the constrained problem, inspecting the shape of the value function is necessary to characterize the policy function which is usually the interest of economists. However, in the constrained problem, higher order differentiability of the value function is not assured especially when the reward function is quadratic. The approach introduced in the essay does not depend on higher order differentiability for characterizing the value function and thus it can be widely applicable for this type of problem.

Further, I will demonstrate an application of the theorems to the monetary policy reaction function which is, I suppose, one of the most relevant and interesting examples of this kind of problem. Needless to say, since central banks can be regarded as living forever, their optimization problem inherently has an infinite horizon. The point is, when inflation is low, monetary policy is potentially constrained by the zero lower bound of nominal interest rate, which might bind in the future. It is well-known that the optimal monetary policy reaction function without the zero bound is linear in inflation and output due to the linear-quadratic nature of the problem. This is
regarded as a theoretical justification for the Taylor rule advocated in the cerebrated work by Taylor(1993). However, when the zero bound of the nominal interest rate is incorporated into the problem, a theorem provided in this essay implies that the optimal monetary policy reaction function is concave in inflation and output gap. This conclusion is consistent with some numerical simulations in previous studies. It should be noted that the concavity of the monetary policy reaction function implies asymmetry in the response. Intuitively, at any incremental change in state of the economy, monetary expansion is always greater than contraction when inflation is low. This kind of behavior of central bank is observed in a typical low inflation country, such as Japan.

This chapter is organized as follows. Section 2.2 introduces the problem statement and proves the main theorems. Section 2.3 argues the application of the theorems to monetary policy and derives several policy implications. Section 2.4 presents some empirical evidence which is consistent with the implications of the model. I will conclude the essay in section 2.5.

2.2 Theorems

2.2.1 Problem statement

The class of problems, denoted as (P) hereafter, which this essay is about to consider is,

\[
V(s_t) = \min_{x_{t+j} \leq k_{t+j}} \left\{ \sum_{j=0}^{\infty} \beta^j \frac{1}{2} (\lambda s_{t+j}^2 + x_{t+j}^2) \right\}
\]

\[
s.t. \quad s_{t+j+1} = \gamma s_{t+j} + \alpha x_{t+j}
\]

where \(s_{t+j} \in \mathbb{R}^1\) is a state variable, \(x_{t+j} \in \mathbb{R}^1\) is a control variable, and \(\alpha, \lambda, \) and \(\gamma\) are positive and constant parameters. \(k_{t+j} \in \mathbb{R}^{1+}\) is an exogenous constraint.
on the control variable.\footnote{This exogeneity condition is necessary for only theorem 1. As shown later, both theorem 2 and 3 will be proved without depending on this exogeneity condition.} Note that my interest is only in controllable initial states, namely $\{s_{t+j}\}_{j=0}^{\infty}$ is bounded such that there exists a converged value function $V(s_t)$.\footnote{In other words, I am assuming the regularity conditions on the discount rate so that the contraction mapping theorem could hold. For more rigorous argument as for this point, see Stokey and Lucas (1989).} Hereafter, I assume the $V(\cdot)$ is only once differentiable so that we could utilize the marginal value function $V'(\cdot)$. Let $S$ be the set of such initial states whose following paths are bounded.\footnote{The nature of $S$ is beyond the scope of this paper. The focus of the essay is not to characterize $S$, but rather to characterize the value function and policy function when $s_t \in S$, whatever $S$ looks like. See Chmielewski and Manousiouthakis (1996) for detailed argument about this “feasible set” $S$.}

For notational purpose let us state the unconstrained problem, $(P^u)$ as follows,

$$
V_u(s_t) = \min_{x_{t+j}} \left\{ \sum_{j=0}^{\infty} \beta^j \frac{1}{2} (\lambda s_{t+j}^2 + x_{t+j}^2) \right\}
$$

$$
s.t. \quad s_{t+j+1} = \gamma s_{t+j} + \alpha x_{t+j}
$$

(2.2)

It is well known that this problem admits the unique solution, $x_t = qs_t$ where $q = (1 - \theta_1)/\alpha \theta_1$ and $\theta_1$ is the larger root of the characteristic equation $\gamma z^2 - (1 + \beta \gamma^2 + \alpha^2 \beta \lambda) z + \beta \gamma = 0$.\footnote{See Sargent (1987) for example.} Note this problem has the certainty equivalence property due to its linear quadratic (LQ) regulator nature.

A good example of this class of problem is the standard consumption problem with a quadratic utility. It is a maximization version of $(P^u)$ such that

$$
V(s_t) = \max_{x_{t+j}} \left\{ \sum_{j=0}^{\infty} \beta^j \left( ax_{t+j} - \frac{b}{2} c_{t+j}^2 \right) \right\}
$$

$$
s.t. \quad w_{t+j+1} = R \left( w_{t+j} - c_{t+j} \right)
$$
where \( c_t \) is consumption, \( w_t \) is net wealth, \( R \) is a gross real interest rate on net wealth, and \( a \) and \( b \) are positive and constant parameters. When the consumer is confronted with a borrowing constrained, i.e. \( k_t \geq c_t \) this can be regarded as a maximization version of (P). As long as \( b \) is defined positive, essentially this problem has the same property as (P). Therefore most of arguments on (P) in this essay are applicable to this type of maximization problems, although I will focus minimization problems, (P) and (P") hereafter to avoid unnecessary confusion.

2.2.2 Proofs

**Theorem 1** Define a once differentiable function, \( h : \mathbb{R} \to \mathbb{R}, h(s_t) = V(s_t) - V_u(s_t) \), where \( s_t \) is the initial value of the state variable. If \( \gamma \geq 1 \), then, \( h'(s_t) \leq 0 \) for \( s_t \in S \).

Theorem 1 is proved with the help of Lagrange multiplier \( \psi_t \) on the constraint. Rewrite (P) in the following form, (P').

\[
\tilde{V}(s_t) = \min_{\lambda, \beta} \left\{ \frac{1}{2} \left( \lambda s_t^2 + x_t^2 \right) + \beta \tilde{V} (\gamma_s + \alpha x_t) + \psi_t (x_t - k_t) \right\}
\] (2.3)

Note that this value function (\( \tilde{V}(s_t) \)) is different from that of the original problem, \( V(s_t) \), since eqn (2.3) has the Lagrange multiplier in the right hand side. Careful attention should be paid on the sign of the Lagrange multiplier. As this is a minimization problem with the constraint \( x_t \leq k_t \), the sign in front of the multiplier must be set so that the multiplier has positive value when the constraint is strictly binding; that is, positive to keep \( x_t \leq k_t \).

Now here is the first order condition of (P').

\[
x_t + \alpha \beta \tilde{V}'(s_{t-1}) + \psi_t = 0.
\] (2.4)
Benveniste-Scheinkman formula can be applied to this so that,

\[ \tilde{V}'(s_t) = \lambda s_t + \beta \gamma \tilde{V}'(s_{t+1}). \]  

(2.5)

Eliminating \( \tilde{V}'(\cdot) \) from these two,

\[ \beta \gamma x_{t+1} - x_t - \alpha \beta \lambda s_{t+1} = \psi_t - \beta \gamma \psi_{t+1} \]  

(2.6)

Combining this equation with the transition equation yields the following second order difference equation.\(^5\)

\[ \beta \gamma s_{t+2} - \left(1 + \beta \gamma^2 + \alpha^2 \beta \lambda \right) s_{t+1} + \gamma s_t = \alpha \Psi_t \]

(2.7)

where \( \Psi_t \equiv \psi_t - \beta \gamma \psi_{t+1} \). Let \( \theta_{1,2} \) be the roots of the characteristic equation, \( \gamma z^2 - (1 + \beta \gamma^2 + \alpha^2 \beta \lambda) z + \beta \gamma = 0 \). As \( 1 > \beta > 0, \lambda > 0 \) and \( \gamma > 0 \) one root is in the unit circle and the other out of it so that \( \theta_1 \geq 1, 0 < \theta_2 < 1 \). Then I can derive the unique solution to this difference equation,

\[ \theta_1 s_{t+1} = s_t - \alpha \sum_{i=0}^{\infty} \theta_2^i \Psi_{t+i} \]  

(2.8)

Combining this with the transition equation leaves the optimal policy,

\[ x^*_t = \left(1 - \frac{\gamma \theta_1}{\alpha \theta_1} \right) s_t - \frac{1}{\theta_1} \sum_{i=0}^{\infty} \theta_2^i \Psi_{t+i}. \]

(2.9)

\( x^*_t = x^*(s_t) \) is the policy function of (P). Let \( x^{u*}_t = x^{u*}(s_t) \) be the policy function of \( (P^u) \). Then it is straightforward to show that \( x^{u*}_t = ((1 - \gamma \theta_1) / \alpha \theta_1) s_t \). That is, simply eliminating the second term in the right hand side of eqn(2.9) yields \( x^{u*}_t \).

Here, let us go back to the original problem set-up, namely, \( V(\cdot) \) instead of \( \tilde{V}(\cdot) \).

The first order conditions of (P) and \( (P^u) \) are respectively,

\[ x^*_t + \alpha \beta V' \left( s^C_{t+1} \right) = 0 \]

\[ x^{u*}_t + \alpha \beta V'_u \left( s^U_{t+1} \right) = 0 \]

\(^5\)Actually, eqn(1) is a nonlinear difference equation, since the term \( \Psi \) is also a function of \( s_t \).
Note that $s_{t+1}^C$ (constrained case) and $s_{t+1}^U$ (unconstrained case) are different. But the difference does not matter for my purpose due to the following relations. Again, applying Benveniste-Scheinkman formula yields,

\[
\begin{align*}
  x_t^* + \frac{\alpha}{\gamma} \left\{ V'(s_t) - \lambda s_t \right\} &= 0 \\
  x_t^{u*} + \frac{\alpha}{\gamma} \left\{ V'_u(s_t) - \lambda s_t \right\} &= 0.
\end{align*}
\]

Therefore, by combining these two with eqn(2.9), the key equations are derived as follows.

\[
\begin{align*}
  x_t^* - x_t^{u*} &= -\frac{\alpha}{\gamma} \left\{ V'(s_t) - V'_u(s_t) \right\} \\
  &= -\theta_1^{-1} \sum_{i=0}^{\infty} \theta_2^i \Psi_{t+i}
\end{align*}
\]

so that $h'(s_t) = (\gamma/\alpha) \theta_1^{-1} \sum_{i=0}^{\infty} \theta_2^i \Psi_{t+i}$, since $h'(s_t)$ is defined to be $V'(s_t) - V'_u(s_t)$.

Now, since all parameters are defined positive and $\theta_1 \geq 1$, it suffices to show

\[
\sum_{i=0}^{\infty} \theta_2^i \Psi_{t+i} \leq 0. \tag{2.10}
\]

Expanding the terms yields the following expression.

\[
\begin{align*}
  \sum_{i=0}^{\infty} \theta_2^i \Psi_{t+i} &= (\psi_t - \beta \gamma \psi_{t+1}) \\
  &\quad + \theta_2 (\psi_{t+1} - \beta \gamma \psi_{t+2}) \\
  &\quad + \theta_2^2 (\psi_{t+2} - \beta \gamma \psi_{t+3}) \\
  &\quad + \cdots \\
  &\quad + \theta_2^i (\psi_{t+i} - \beta \gamma \psi_{t+i+1}) \\
  &\quad + \cdots \\
  &= \psi_t + (\theta_2 - \beta \gamma) \sum_{i=0}^{\infty} \theta_2^i \psi_{t+1+i} \\
  &= \psi_t + (1 - \theta_1 \gamma) \sum_{i=0}^{\infty} \theta_2^i \psi_{t+i} \tag{2.11}
\end{align*}
\]
Recall that $\theta_1$ and $\theta_2$ are the roots of $\gamma z^2 - (1 + \beta \gamma^2 + \alpha^2 \beta \lambda) z + \beta \gamma = 0$ so that $\theta_1 \theta_2 = \beta$. The last line of eqn(2.11) is obtained by using this fact. My interest is the case when the constraint is not binding at the current period $t$, so that $\psi_t = 0$. Otherwise the optimal policy is trivially equal to $k_t$. Since the Lagrange multipliers are all non-negative and $\gamma \geq 1 \geq 1/\theta_1$ assures that $\sum_{i=0}^{\infty} \theta_i^2 \Psi_{t+i} \leq 0$. This proves the theorem 1.

As shown above, the condition $\gamma \geq 1$ is not necessary, but a sufficient condition for eqn(2.11) being negative. To obtain $\sum_{i=0}^{\infty} \theta_i^2 \Psi_{t+i}$ being negative, I need only $\gamma \geq 1/\theta_1$ instead of $\gamma \geq 1$, which implies $\gamma$ can be lower than one under a large value of $\theta_1 \geq 1$.

The theorem 2 is a restatement of Chmielewski and Manousiouthakis (1996) which is slightly modified here so that I could apply it more easily to the example later. Intuitively, theorem 2 is the investigation on the second derivative of the value functions. However, the value function of (P), namely $V(\cdot)$, is not necessarily twice differentiable as shown in Carroll and Kimball (2001).\textsuperscript{6} Intuitively, for a certain domain where the value function is twice differentiable, the theorem 2 implies the second derivative of the value function of (P) is at least as large as that of (P\textsuperscript{n}). Actually, as I will discuss later, when uncertainty is incorporated in the problem, value functions are most likely smooth everywhere. Hence this intuition may indeed be true in such a stochastic environment. Let me briefly introduce this theorem of Chmielewski and Manousiouthakis (1996) for illustrative purpose, since it is a natural complement for theorems 1 and 3, each of which investigates the first and third derivatives of the value function.

\textsuperscript{6}They show that the marginal value function of their problem has a kink. So it is not differentiable.
Theorem 2 (Chimielowski and Manousiouthakis (1996)) $h'(s_t)$ is monotone increasing in $s_t$ for $s_t \in S$.

Under the condition that the $V(s_t)$ is once differentiable, it suffices to show that $h(s_t) = V(s_t) - V_u(s_t)$ is a convex function.

Let $\{\tilde{x}_{t+j}^C\}_{j=0}^{\infty}, \{\tilde{x}_{t+j}^U\}_{j=0}^{\infty}$ be the optimal path of control variable for constrained and unconstrained problem respectively given initial states $\tilde{s}_t$ and $\bar{s}_t$. Let $\{\tilde{s}_{t+j}^C\}_{j=0}^{\infty}, \{\tilde{s}_{t+j}^U\}_{j=0}^{\infty}$ be the controlled optimal path of the state variable for constrained and unconstrained problem respectively, given initial states $\tilde{s}_t$ and $\bar{s}_t$. Now define $\pi_{t+j} = a\tilde{s}_{t+j} + (1 - a)\tilde{s}_{t+j}$ and $\bar{\pi}_{t+j} = a\tilde{x}_{t+j} + (1 - a)\tilde{x}_{t+j}$ where $0 \leq a \leq 1$. Linearity of the transition equation implies that the pair of sequences $\{\pi_{t+j}^C\}_{j=0}^{\infty}$, and $\{\pi_{t+j}^U\}_{j=0}^{\infty}$ under the constrained problem is feasible, but not necessary equal to the optimal paths given the initial state $\pi_t$. Also, it can be shown that the pair of sequences $\{\pi_{t+j}^C\}_{j=0}^{\infty}$, and $\{\pi_{t+j}^U\}_{j=0}^{\infty}$ under the unconstrained problem is feasible and optimal given the initial state $\pi_t$. Therefore, the following inequality holds.

$$V(\pi_t) - V_u(\pi_t) \leq \sum_{j=0}^{\infty} \beta^j \left( \lambda \left( \tilde{s}_{t+j}^C \right)^2 + \left( \tilde{x}_{t+j}^C \right)^2 \right) - \sum_{j=0}^{\infty} \beta^j \left( \lambda \left( \tilde{s}_{t+j}^U \right)^2 + \left( \tilde{x}_{t+j}^U \right)^2 \right).$$

(2.12)

Then it suffices to show that

$$\sum_{j=0}^{\infty} \beta^j \left[ \lambda \left( \tilde{s}_{t+j}^C \right)^2 + \left( \tilde{x}_{t+j}^C \right)^2 \right] - \lambda \left( \tilde{s}_{t+j}^U \right)^2 + \left( \tilde{x}_{t+j}^U \right)^2 \right]$$

$$\leq a \sum_{j=0}^{\infty} \beta^j \left[ \lambda \left( \tilde{s}_{t+j}^C \right)^2 + \left( \tilde{x}_{t+j}^C \right)^2 \right] - \lambda \left( \tilde{s}_{t+j}^U \right)^2 + \left( \tilde{x}_{t+j}^U \right)^2 \right]$$

$$+ (1 - a) \sum_{j=0}^{\infty} \beta^j \left[ \lambda \left( \tilde{s}_{t+j}^C \right)^2 + \left( \tilde{x}_{t+j}^C \right)^2 \right] - \lambda \left( \tilde{s}_{t+j}^U \right)^2 + \left( \tilde{x}_{t+j}^U \right)^2 \right]$$

which is equivalent in showing that

$$a(a - 1) \sum_{j=0}^{\infty} \beta^j \left[ \lambda \left( \tilde{s}_{t+j}^C - \tilde{s}_{t+j}^C \right)^2 + \left( \tilde{x}_{t+j}^C - \tilde{x}_{t+j}^C \right)^2 \right]$$

14
-\left( \lambda \left( \hat{s}_{t+j}^t - \bar{s}_{t+j}^t \right)^2 + \left( \hat{x}_{t+j}^t - \bar{x}_{t+j}^t \right)^2 \right) \\
\leq 0.

Now define $\bar{s}_{t+j} \equiv \hat{s}_{t+j} - \bar{s}_{t+j}$. Then again, it can be shown that the sequence $\{\bar{s}_{t+j}^C\}_{j=0}^{\infty}$ under the constrained problem is feasible, but not necessary optimal given the initial state $\bar{s}_t$. Similarly it can be shown that the sequence $\{\bar{s}_{t+j}^U\}_{j=0}^{\infty}$ under the unconstrained problem is feasible and optimal given the initial state $\bar{s}_t$. Therefore the following is true.

\[ V(\bar{s}_t) \leq \sum_{j=0}^{\infty} \beta^j \frac{1}{2} \left[ \lambda \left( \bar{s}_{t+j}^C \right)^2 + \left( \bar{s}_{t+j}^C \right)^2 \right] \]

\[ V_u(\bar{s}_t) = \sum_{j=0}^{\infty} \beta^j \frac{1}{2} \left[ \lambda \left( \bar{s}_{t+j}^U \right)^2 + \left( \bar{s}_{t+j}^U \right)^2 \right]. \]

But since the cost of constrained problem is higher or equal to the unconstrained problem for any initial state, it then follows that $V(\bar{s}_t) \geq V_u(\bar{s}_t)$, which in turn implies that

\[ \sum_{j=0}^{\infty} \beta^j \left[ \lambda \left( \bar{s}_{t+j}^C \right)^2 + \left( \bar{s}_{t+j}^C \right)^2 \right] \geq \sum_{j=0}^{\infty} \beta^j \left[ \lambda \left( \bar{s}_{t+j}^U \right)^2 + \left( \bar{s}_{t+j}^U \right)^2 \right]. \quad (2.13) \]

This proves the theorem 2.

The following corollary is also from Chmielewski and Manousiouthakis (1996).

**Corollary 1** $V'(s_t)$ is monotone increasing in $s_t$ for $s_t \in S$.

Due to the same reason as theorem 2, it is not appropriate to rely on the third order differentiability of $V(\cdot)$ for further analysis. However, it is still possible to characterize the value function further without relying on higher order differentiability. As long as the value function is once differentiable, the 3-concavity of the value function implies
the concavity of its marginal value function even if the marginal value function is kinked somewhere in it.

I first define the concept of the 3-concavity.7

**Definition 1 (3-Concavity)** A function \( f : [a, b] \rightarrow \mathbb{R} \) where \( b \leq 0 \) is said to be 3-concave on \([a, b]\) if for \( \forall z_1, z_2 \in [a, b] \) such that \( z_1 \neq z_2 \)

\[
\left[ \frac{1}{2} f(z_1) + \frac{1}{2} f(z_2) \right] - f \left( \frac{z_1 + z_2}{2} \right) \geq \left[ \frac{1}{2} f \left( \frac{z_1 + z_2}{2} \right) + \frac{1}{2} f \left( \frac{3z_2 - z_1}{2} \right) \right] - f(z_2)
\]

(2.14)

The inequality (2.14) is a special case of Levinson’s inequality (1964), which can be regarded as a higher-order Jensen’s inequality. As Jensen’s inequality is closely related to the concept of concavity, so is Levinson’s inequality to 3-concavity. The intuition of the inequality (2.14) can be captured by figure 2.1. Left-hand side of the inequality (i.e. denoted as \( A \)) represents the difference between the value of function evaluated at the mid-point of \( z_1 \) and \( z_2 \) to the mid-point of the cord from \( z_1 \) and \( z_2 \). It is possible to interpret \( A \) as the magnitude of convexity of a function in the domain \([z_1, z_2] \). Right-hand side of the inequality (i.e. denoted as \( B \)) can be interpreted in the similar fashion with a difference whose domain is now \([\frac{z_1+z_2}{2}, \frac{3z_2-z_1}{2}] \). Thus, intuitively, the function will be 3-concave if convexity decreases as \( z \) increases8.

Next, I state the lemma that links 3-concavity of the function to the concavity of the marginal function. The following lemma is a special case of the more general theorem that links \( n \)-concavity of a function to concavity of \((n - m)\)th derivative of the function. Here is the lemma.

---

73-concavity can be defined in more general form. But here I define it in slightly narrower fashion, since it is not only sufficient for the proof of theorem 3, but is more comprehensive for the application.

8Or, in the continuous analogy, \( f''(x) \) is decreasing in \( z \)
Lemma 1 If a function $f : \mathbb{R} \to \mathbb{R}$ is 3-concave on $[a, b]$, then the first derivative $f' : \mathbb{R} \to \mathbb{R}$ exists and is concave on $[a, b]$.

Proof. See Pečarić et al. (1992, pp.16).

I am now in the position to state the main theorem of this paper.

Theorem 3 $V'(s_t)$ is concave for $s_t \in S$
Proof is divided into two parts, namely, for (i) \( s_t \geq 0 \) and (ii) \( s_t < 0 \).

As for the case (i) it can be shown that \( V(s_t) = V_u(s_t) \). Since it is known that \( V_u(s_t) \) is quadratic in \( s_t \), if \( V(s_t) = V_u(s_t) \) for \( s_t \geq 0 \), then simply \( V'(s_t) \) is linear in \( s_t \) in that domain. The equivalence of \( V(s_t) \) and \( V_u(s_t) \) for \( s_t \geq 0 \) can be shown by the help of theorem 1. Since \( V(s_t) \geq V_u(s_t) \) for any \( s_t \in S \), the theorem 1 assures that \( V(s_t) \) approaches to \( V_u(s_t) \) from above as \( s_t \) increases. Also notice that for \( s_t = 0 \), \( V(0) = V_u(0) = 0 \), since \( k_{t+1} \geq 0 \). By theorem 1 and these two property directly means \( V(s_t) = V_u(s_t) \) for \( s_t \geq 0 \).

For the case (ii) \( s_t < 0 \), by lemma 1 it suffices to show that the following inequality, which is an alternative expression of eqn(2.14), holds for arbitrary two initial states which satisfy \( \hat{s}_t < \tilde{s}_t < 0 \).

\[
V(\hat{s}_t) + 3V(\tilde{s}_t) - 3V\left(\frac{\hat{s}_t + \tilde{s}_t}{2}\right) - V\left(\frac{3\tilde{s}_t - \hat{s}_t}{2}\right) \geq 0 \tag{2.15}
\]

Let \( \{\tilde{x}_{t+j}\}_{j=0}^{\infty} \) and \( \{x_{t+j}\}_{j=0}^{\infty} \) be the optimal path of control variable given initial states \( \hat{s}_t \) and \( \tilde{s}_t \). Let \( \{\hat{s}_{t+j}\}_{j=0}^{\infty} \) and \( \{\tilde{s}_{t+j}\}_{j=0}^{\infty} \) be the controlled optimal path of the state variable, given initial states \( \hat{s}_t \) and \( \tilde{s}_t \). Now define \( \tilde{x}_{t+j} = (\hat{x}_{t+j} + \tilde{x}_{t+j})/2 \), \( \tilde{\tilde{x}}_{t+j} = (3\hat{x}_{t+j} - \tilde{x}_{t+j})/2 \), \( \tilde{x}_{t+j} = (\hat{x}_{t+j} + \tilde{x}_{t+j})/2 \) and \( \tilde{\tilde{x}}_{t+j} = (3\hat{x}_{t+j} - \tilde{x}_{t+j})/2 \). Linearity of the transition equation implies that the pair of sequences \( \{\tilde{x}_{t+j}\}_{j=0}^{\infty} \), \( \{\tilde{x}_{t+j}\}_{j=0}^{\infty} \) and \( \{\tilde{\tilde{x}}_{t+j}\}_{j=0}^{\infty} \), \( \{\tilde{\tilde{x}}_{t+j}\}_{j=0}^{\infty} \) are feasible, but not necessary equal to the optimal path given the initial state \( \pi_t \) and \( \tilde{\pi}_t \). Therefore the following inequalities hold.

\[
V\left(\frac{\hat{s}_t + \tilde{s}_t}{2}\right) \leq \sum_{j=0}^{\infty} \beta^j \frac{1}{2} (\lambda \hat{s}_{t+j}^2 + \tilde{s}_{t+j}^2) = \sum_{j=0}^{\infty} \beta^j \frac{1}{2} \left[ \lambda \left(\frac{\hat{s}_{t+j} + \hat{s}_{t+j}}{2}\right)^2 + \left(\frac{\tilde{x}_{t+j} + \tilde{x}_{t+j}}{2}\right)^2 \right]
\]

\[
V\left(\frac{3\hat{s}_t - \tilde{s}_t}{2}\right) \leq \sum_{j=0}^{\infty} \beta^j \frac{1}{2} (\lambda \hat{s}_{t+j}^2 + \tilde{s}_{t+j}^2) = \sum_{j=0}^{\infty} \beta^j \frac{1}{2} \left[ \lambda \left(\frac{3\hat{s}_{t+j} - \tilde{s}_{t+j}}{2}\right)^2 + \left(\frac{3\tilde{x}_{t+j} - \tilde{x}_{t+j}}{2}\right)^2 \right]
\]

Consequently,

\[
V(\hat{s}_t) + 3V(\tilde{s}_t) - 3V\left(\frac{\hat{s}_t + \tilde{s}_t}{2}\right) - V\left(\frac{3\tilde{s}_t - \hat{s}_t}{2}\right)
\]

18
\begin{align*}
\geq V(\tilde{s}_t) + 3V(\tilde{s}_t) - 3 \sum_{j=0}^{\infty} \beta^j \frac{1}{2} \left[ \lambda \left( \frac{\tilde{s}_{t+j} + \tilde{s}_{t+j}}{2} \right)^2 + \left( \frac{\tilde{x}_{t+j} + \tilde{x}_{t+j}}{2} \right)^2 \right] \\
- \sum_{j=0}^{\infty} \beta^j \frac{1}{2} \left[ \lambda \left( 3\tilde{s}_{t+j} - \tilde{s}_{t+j} \right)^2 + \left( 3\tilde{x}_{t+j} - \tilde{x}_{t+j} \right)^2 \right] \\
= V(\tilde{s}_t) + 3V(\tilde{s}_t) - \sum_{j=0}^{\infty} \beta^j \frac{1}{2} \left[ 3 \left( \lambda \tilde{s}_{t+j}^2 + \tilde{x}_{t+j}^2 \right) + \left( \lambda \tilde{s}_{t+j}^2 + \tilde{x}_{t+j}^2 \right) \right] \\
= 0
\end{align*}

(2.16)

Here at this point I know that $V'(s_t)$ is concave for $s_t < 0$ and $V'(s_t)$ is linear for $s_t \geq 0$. To show the global concavity, (for any $s_t \in S$) I will demonstrate the following inequality. That is, for any arbitrary points such that $\hat{s}_t \leq 0$ and $\tilde{s}_t > 0$,

$$\frac{V'(\hat{s}_t) - V'(\tilde{s}_t)}{\hat{s}_t - \tilde{s}_t} \geq V''(\hat{s}_t) = c. \quad (2.17)$$

Note that for $s_t > 0$, $V''(s_t)$ exists and it is equal to some positive constant, ($=c$) since $V(\cdot)$ is quadratic as shown above. Now suppose that the eqn(2.17) is not true. Then there exists a pair of points, $\hat{s}_t \leq 0$ and $\tilde{s}_t > 0$, which satisfies,

$$\frac{V'(\hat{s}_t) - V'(\tilde{s}_t)}{\hat{s}_t - \tilde{s}_t} < V''(\hat{s}_t) = c. \quad (2.18)$$

Recall the implication of the theorem 2 here. Theorem 2 implies that the slope of any chord of $V'(s_t)$ (i.e. the left-hand-side of eqn(2.18)) is at least as steep as the slope of $V''(s_t)$ (i.e. $V''(s_t)$, which is equal to $c$) for any $s_t \in S$. Therefore, a contradiction. The theorem is proved.

2.2.3 Remark: the role of uncertainty

So far the argument is in the deterministic environment. Now in this subsection, I discuss the role of uncertainty. Basic idea provided here is that incorporating uncertainty into the problem makes the value/policy function smoother so that they are
more likely to be differentiable with higher order. Note that here I do not present a
rigourous theorem on the role of uncertainty, but rather focus the intuitive argument
how uncertainty creats more smoothness of value function.

I incorporate uncertainty into the problem in the following fashion. Let us think
of the following modi…ed problem, denoted as (PS), with uncertainty.

\[
\begin{align*}
J(s_t) &= \min_{x_{t+j} \leq k_{t+j}} E_t \left\{ \sum_{j=0}^{\infty} \beta^j \frac{1}{2} (\lambda s_{t+j}^2 + x_{t+j}^2) \right\} \\
\text{s.t. } s_{t+j+1} &= s_{t+j} + \alpha x_{t+j} + \varepsilon_{t+j+1}
\end{align*}
\]

where \( E \) is an expectation operator and \( \varepsilon_{t+j} \) is a stochastic disturbance with mean
zero and finite second moment. Rewriting the (PS) in the following form would be
informative.

\[
\begin{align*}
J(s_t) &= E_t \left[ \min_{x_{t+j} \leq k_{t+j}} \left\{ \frac{1}{2} (\lambda s_t^2 + x_t^2) + \frac{1}{2} \sum_{j=0}^{\infty} \beta^j \left( \lambda s_{t+j}^2 + x_{t+j}^2 \right) \right\} \right] \\
&= E_t \hat{V}(s_t)
\end{align*}
\]

where

\[
\hat{V}(s_t) = \min_{x_{t+j} \leq k_{t+j}} \left\{ \sum_{j=0}^{\infty} \beta^j \frac{1}{2} (\lambda s_{t+j}^2 + x_{t+j}^2) \right\} \\
\text{s.t. } s_{t+j+1} = s_{t+j} + \alpha x_{t+j} + \hat{\varepsilon}_{t+j+1}
\]

\( \{\hat{\varepsilon}_{t+j}\}_{j=0}^{\infty} \) is one realization of \( \{\varepsilon_{t+j}\}_{j=0}^{\infty} \). Therefore it is deterministic. With such
a deterministic and exogenous term \( \hat{\varepsilon}_{t+j} \), the nature of \( \hat{V}(s_t) \) is essentially identical to
\( V(s_t) \). Now since the expectation operator works as a summation with certain positive
weights, \( J(s_t) \) is a weighted average of multiple \( \hat{V}(s_t) \)'s with different deterministic
path of \( \hat{\varepsilon}_{t+j} \)'s. This is why \( J(s_t) \) can be thought to be much smoother than \( V(s_t) \).
Moreover, if the distribution of $\varepsilon_t$ is continuous, the degree of smoothness of $J(s_t)$ is expected to be even much higher. 

Consider the first order condition in this case is

$$x_t + \alpha \beta E_t J'(s_{t+1}) = 0.$$  \hspace{1cm} (2.20)

Because of the expected smoothness of $J(\cdot)$ as discussed above, the policy function $x_t = x(s_t)$ is likely to be differentiable to a higher order in such a stochastic environment.

### 2.3 Application to Monetary Policy with a Zero Bound on Nominal Interest Rates

In this section, I present an example of the constrained dynamic optimization problem of the kind. Perhaps the most familiar application is the standard consumption problem with a borrowing constraint considered in this paper.\(^{10}\) (the problem is actually a maximization problem rather than a minimization.) However, the application which I present here is the monetary policy reaction function with a zero lower bound on the nominal interest rate. I am motivated by two basic considerations. One is its infinite horizon nature – central banks live forever. The other is that in this problem possible domain of initial states lies over zero, which requires full portion of the theorems above.\(^{11}\)

Note that the concave nature of $V'(\cdot)$ (or 3-concavity of $V(\cdot)$ ) is preserved in $J'(\cdot)$, since the weights (probability measure) on $V'(\cdot)$s are all positive.

\(^{10}\)See Carroll and Kimball (2001).

\(^{11}\)Recall the proof of theorem 3 needs the careful treatment when $S$ spreads over both sides of zero. In the consumption problem, the state variable (=net wealth) cannot be negative inherently, which makes the nature of the problem more simple than the case of monetary policy presented here.
The model which I present here is standard in the literature of the inflation targeting such as Ball(1997) or Svensson (1997a) except for one aspect. That is, there is an additional constraint such that nominal interest rate cannot go below zero. This non-negativity constraint is becoming a serious concern for several central banks in developed countries, since many developed countries are successful in reducing inflation in this decade. Low inflation raises the probability that the non-negativity constraint would bind in the future, or in other words, the central bank would be caught in a “liquidity trap.” Once the economy is caught in a liquidity trap, the central bank can no longer stimulate the economy. In this sense, it is natural to think that the central bank is trying to avoid the zero lower bound to some extent. This is the intuition for why the zero bound affects the monetary policy reaction function. In the following subsection, I present three propositions as the characterization of monetary policy reaction function with such zero lower bound on nominal interest rate.

### 2.3.1 The model

The set-up of the model which I present in this section is based on Ball (1997) except for the non-negativity constraint (zero lower bound) on nominal interest rate. I consider a central bank which lives forever and minimizes the weighted sum of variances of output gap and inflation. First, assume that the central bank’s period-by-period loss function is given as

\[
L_t = \frac{1}{2} \left( y_t^2 + \lambda (\pi_t - \pi^*)^2 \right), \tag{2.21}
\]

where \(\pi\) and \(y\) denote the inflation rate and output gap, respectively, and \(\pi^*\) is the target inflation rate of the central bank. \(\lambda\) is a positive weight which represents the
preference of the central bank. The economy is described by the following conventional IS and AS type formulation,

\[ y_{t+1} = \rho y_t - \delta (i_t - E_t \pi_{t+1}) + \nu_{t+1} \]  
\[ \pi_{t+1} = \pi_t + \alpha y_t + \varepsilon_{t+1} \]  

(2.22)

(2.23)

Eqn (2.22) and eqn (2.23) stand for aggregate demand and supply function, respectively, where \( \nu \) and \( \varepsilon \) are random disturbances. Although eqn (2.22) contains a forward-looking variable, this can be substituted out by, \( E_t \pi_{t+1} = \pi_t + \alpha y_t \). Also by letting \( \hat{\rho} = \rho + \alpha \delta \), the central bank’s problem is now formulated as an intertemporal minimization problem with the objective,

\[
\min_{i_{t+j}} \quad E_t \sum_{j=0}^{\infty} \beta^j L_{t+j} \\
\text{s.t. } i_t \geq 0 \\
y_{t+1} = \hat{\rho} y_t - \delta i_t + \alpha \delta \pi_t + \nu_{t+1} \\
\pi_{t+1} = \pi_t + \alpha y_t + \varepsilon_{t+1}
\]  

(2.24)

(2.25)

Several remarks are in order. Without the zero bound on nominal interest rate, this problem takes the form of the well-known linear-quadratic (LQ) regulator problem. Therefore the policy function of the problem is linear in the state variable, i.e. inflation and output gap in this case. It is because of this that the Taylor rule is rationalized as a class of optimal reaction functions as proved in Ball (1997) and Svensson (1997a).\(^{12}\)

It should be noted that in the model adopted here, the role of households is not captured vividly. As discussed in McCallum (1996) IS equation can be regarded as

\(^{12}\)Notice that such Ball-Svensson models correspond to the unconstrained problem (\( P^u \)) in section 2.2.
the Euler equation of optimizing households if it contains expectation of future output gap in the right-hand side. My model here may be regarded as an approximation of the economy in which such forward-looking behavior of consumers is so small that it can be ignored.\textsuperscript{13} Also, it is convenient to regard the objective function as a second order approximation of social loss function in a sticky price environment.\textsuperscript{14}

The following subsections investigates the economic implications of each three theorem introduced in the previous section. In this application of monetary policy problem, my theorems respectively lead to the following three propositions, i.e., monetary policy with a zero bound on nominal interest rate will be (i) more expansionary and (ii) more aggressive than the Taylor rule, and exhibit (iii) an asymmetric response characterized by decreasing aggressiveness as the economy expands.

\textbf{2.3.2 Proposition 1: Expansionary policy}

Proposition 1 and 2 make sense in comparison with the unconstrained problem. So let me point out again, for convenience, that the policy function of the unconstrained problem (without eqn (2.25)) is linear in inflation and the output gap. Let us call this optimal linear rule "the Taylor rule" hereafter.

Before denoting proposition 1, I need to re-define both $\theta_1$ and $\theta_2$ here as follows.

$$
\theta_1 = \frac{\alpha^2 \beta \lambda + \beta + 1 + \sqrt{(\alpha^2 \beta \lambda + \beta + 1)^2 - 4 \beta}}{2}, \quad \theta_1 \geq 1
$$

$$
\theta_2 = \frac{\alpha^2 \beta \lambda + \beta + 1 - \sqrt{(\alpha^2 \beta \lambda + \beta + 1)^2 - 4 \beta}}{2}, \quad 1 > \theta_2 \geq 0
$$

\textsuperscript{13}See Svensson (1997b) and Woodford (1999) as for the issue of forward-looking consumers. Basically, if the model takes such behavior explicitly, the solution would not be unique and it depends on if the commitment by the central bank is possible or not. As long as focusing on "discrete" solution, namely an environment where such commitment is not possible, my argument goes through even in those "forward-looking" models.

Note that this is a special case of the original definition of $\theta_i$ where $\gamma = 1$. Now the first proposition is here as an application of theorem 1.\footnote{Notice the similarity in proposition 1 to the well-known notion of “precautionary saving” in consumption literature.}

**Proposition 1 (Expansionary policy)** Let $i^{Taylor} (\pi_t, y_t)$ be the optimal monetary policy reaction function when there is no zero bound on the nominal interest rate. Let $i^*(\pi_t, y_t)$ be the optimal monetary policy reaction function in the presence of the zero bound on the nominal interest rate. If $\rho \leq \bar{\rho} \equiv \theta_1^{-1} + \alpha \delta / (\theta_1 - \theta_2)$, then for any state $(\pi_t, y_t)$ where $i^*$ is strictly greater than zero, the monetary policy $i^*$ will be at least as expansionary as the monetary policy $i^{Taylor}$, i.e., $i^* \leq i^{Taylor}$.

Let us note the Taylor rule as the solution of the unconstrained problem from Svensson(1997a) as follows.

$$i^{Taylor} = \pi_t + \left( \alpha + \frac{\rho \theta_1 + \theta_1 - 1}{\delta \theta_1} \right) y_t + \left( \frac{\theta_1 - 1}{\alpha \theta_1 \delta} \right) (\pi_t - \pi^*) . \quad (2.26)$$

Although theorem 1 is not applicable directly here since I have two state variables in this case. However, it can be shown that a very similar procedure to the proof of theorem 1 will end up with the following result,

$$i^* = i^{Taylor} + \left( \frac{1}{\delta \theta_1} \right) \sum_{i=0}^{\infty} \theta_2^i E_t \Psi_{t+i} \quad (2.27)$$

where $\Psi_{t+i} = (\rho \beta \psi_{t+2+i} - (1 + \rho + \alpha \delta) \psi_{t+1+i} + \beta^{-1} \psi_{t+i}) \delta^{-1}$ (here are abusing notations slightly). Note that this eqn(2.27) is a variant of eqn(2.9). Then by this eqn(2.27) and by $\delta, \theta_1 > 0$, it suffices to show,

$$\sum_{i=0}^{\infty} \theta_2^i E_t \Psi_{t+i} \leq 0 . \quad (2.28)$$
Expanding the terms and multiplying $\delta$ on both side yields the followings.

$$
\delta \sum_{i=0}^{\infty} \theta^i E_t \Psi_{t+i} = \sum_{i=0}^{\infty} \theta^i (\beta^{-1} \psi_t - (1 + \rho + \alpha \delta) E_t \psi_{t+1} + \rho \beta E_t \psi_{t+2})
$$

$$
= \frac{1}{\beta} \psi_t - \left[ \rho + \alpha \delta + (1 - \frac{1}{\theta_1}) \right] E_t \psi_{t+1}
$$

$$
- \left[ \theta_2 \left( \theta_1 - \theta_2 \right) \left( \frac{1}{\theta_1} - \rho \right) + \alpha \delta \theta_2 \right] E_t \sum_{j=0}^{\infty} \theta^j \psi_{t+2+j}
$$

$$
= \frac{1}{\beta} \psi_t - \left[ \rho + \alpha \delta + (1 - \frac{1}{\theta_1}) \right] E_t \psi_{t+1}
$$

$$
- \theta_2 \left( \theta_1 - \theta_2 \right) \left( \rho - \rho \right) E_t \sum_{j=0}^{\infty} \theta^j \psi_{t+2+j}
$$

$$
\leq 0 \quad (2.29)
$$

where $\psi_t$ is assumed to be zero, since otherwise the optimal policy is trivially equal to zero at the current period. This proves the proposition.

\[2.3.3 \text{ Proposition 2: Aggressive policy}\]

**Proposition 2 (Aggressive policy)** For any state $(\pi_t, y_t)$ where $i^*$ is strictly greater than zero, the monetary policy $i^*$ will be at least as aggressive as the monetary policy $i^{Taylor}$, i.e., $\partial i^*/\partial \pi_t \geq \partial i^{Taylor}/\partial \pi_t$ and $\partial i^*/\partial y_t \geq \partial i^{Taylor}/\partial y_t$.

Although this applied problem seems to have two state variables, (inflation and output gap) it can be boiled down to a basic one-state/one-control case. Since technically it is equivalent to choose $i_t$ and $E_t y_{t+1}$, the value function can be written in the following form with one-state($s_t = \pi_t + \alpha y_t - \pi^*$) and one-control ($x_t = E_t y_{t+1}$) variable.

$$
J^C(s_t) = \min_{E_t y_{t+1} \leq k_t} E_t \left[ \frac{1}{2} \left( y^2_{t+1} + \lambda (\pi_{t+1} - \pi^*)^2 \right) + \beta J^C(s_{t+1}) \right]
$$
\[
+ \frac{1}{2} \left\{ y_t^2 + \lambda (\pi_t - \pi^*)^2 \right\} \\
= \min_{x_t \leq k_t} \left[ \frac{1}{2} \left( x_t^2 + \lambda s_t^2 \right) + \beta E_t J^C (s_{t+1}) \right] \\
+ \frac{1}{2} \left\{ y_t^2 + \lambda (\pi_t - \pi^*)^2 \right\} \\
\text{s.t. } s_{t+1} = s_t + \alpha x_t + \varepsilon_{t+1}
\]

where \( k_t = (\rho + \alpha \delta) y_t + \delta \pi_t \). Note that the last term in right hand side of the value function is fixed at period \( t \), hence it is irrelevant to the optimization procedure. As the result, the first order condition is also rewritten in the following equation,\(^{16}\)

\[
E_t y_{t+1} + \alpha \beta E_t J^{C^t} (s_{t+1}) = 0. \quad (2.30)
\]

Combining this first order condition with the IS equation so that I can obtain another closed form expression of the optimal reaction function (eqn(2.27)).

\[
i^* = \left( \alpha + \frac{\rho}{\delta} \right) y_t + \pi_t + \frac{1}{\delta} \alpha \beta E_t J^{C^t} (s_{t+1}) \quad (2.31)
\]

where \( s_{t+1} = \pi_{t+1} + \alpha y_{t+1} = (\pi_t + \alpha y_t + \varepsilon_{t+1}) + \alpha ((\rho + \alpha \delta) y_t + \delta \pi_t - \delta i_t^* + \nu_{t+1}) \). Thus this eqn(2.31) is a closed form expression of the optimal reaction function whose arguments are only current state variables. On the other hand, let the value function of the unconstrained problem be \( J^U (\cdot) \). When there is no zero bound, the third term of eqn(2.31) is replaced by \( \delta^{-1} \alpha \beta E_t J^{U^t} (s_{t+1}) \) for \( i^{Taylor} \). Now consider that \( i^* - i^{Taylor} \) can be rewritten in the following expression,

\[
i^* - i^{Taylor} = \delta^{-1} \alpha \beta E_t \left[ J^{C^t} (s_{t+1}) - J^{U^t} (s_{t+1}) \right]. \quad (2.32)
\]

\(^{16}\)One can verify that this first order condition is equivalent to the one that I derived in Section 2.2. See Svensson(1997) for this short-cut notation.
Recall that \( s_{t+1} \) inside each value function \( J^C(\cdot) \) and \( J^U(\cdot) \) is also a function of each \( i_t \).

Hence I can apply the implicit function theorem to this so that,

\[
\frac{\partial i^*}{\partial y_t} - \frac{\partial i^{Taylor}}{\partial y_t} = \alpha \left( \frac{\partial i^*}{\partial \pi_t} - \frac{\partial i^{Taylor}}{\partial \pi_t} \right) = \frac{\alpha^2 \beta}{\delta} \left[ \frac{E_t J_{tt}(s_{t+1})}{1 + \alpha^2 \beta E_t J_{tt}(s_{t+1})} - \frac{E_t J_{tt}(s_{t+1})}{1 + \alpha^2 \beta E_t J_{tt}(s_{t+1})} \right]. \tag{2.33}
\]

Here, note that Benveniste-Scheinkman formula gives \( J^0(s_t) = \lambda s_t + \beta E_t J^0(s_{t+1}) \).

Differentiating this with respect to \( s_t \) again leaves,

\[
J^{n}(s_t) = \lambda + \frac{\alpha \beta E_t J^{n}(s_{t+1})}{1 + \alpha^2 \beta E_t J^{n}(s_{t+1})}. \tag{2.34}
\]

Hence, to determine the sign of eqn(2.33) it suffices to show that,

\[
J^{C0}(s_t) \geq J^{U0}(s_t). \tag{2.35}
\]

Now I can apply theorem 2 under the condition that \( J(\cdot) \) is twice differentiable.

As I discussed in section 2.2, recall this differentiability condition is most likely to hold in the stochastic environment.

### 2.3.4 Proposition 3: Asymmetric policy (decreasing aggressiveness)

**Proposition 3 (Asymmetric policy)** For any state \((\pi_t, y_t)\) where \( i^* \) is strictly greater than zero, aggressiveness of the monetary policy \( i^* \) will be decreasing in each inflation and output gap, i.e., \( \partial^2 i^*/\partial \pi_t^2 \leq 0 \) and \( \partial^2 i^*/\partial y_t^2 \leq 0 \).

Let us rewrite \( E_t J^C(s_{t+1}) = \mathcal{X}(s_{t+1}) \) and the optimal choice of \( E_t y_{t+1} = \zeta(s_t) \) for notational purpose. Taking derivative of the first order condition eqn(2.30) with respect to \( s_t \) and apply implicit function theorem to acquire,

\[
\zeta'(s_t) = -\frac{\alpha \beta \mathcal{Y}''(s_{t+1})}{1 + \alpha^2 \beta \mathcal{Y}''(s_{t+1})}. \tag{2.36}
\]
Notice that $\zeta'(s_t) \leq 0$ by corollary 1. Repeat the same procedure to examine the second derivative of $\zeta(\cdot)$.

$$
\zeta''(s_t) = \frac{(1 + \alpha f')(1 + \alpha^2 \beta \tilde{\zeta}''') \alpha \beta \tilde{\zeta}''' - (1 + \alpha f')\alpha^3 \beta \tilde{\zeta}''''}{(1 + \alpha \tilde{\zeta}''')^2} \quad (2.37)
$$

$$
= -\frac{\alpha \beta \tilde{\zeta}''''(s_{t+1})}{(1 + \alpha^2 \beta \tilde{\zeta}'''(s_{t+1}))^3}. \quad (2.38)
$$

Applying Benveniste-Scheinkman formula leaves,

$$
J^C_{mn}(s_t) = \frac{\alpha \beta \tilde{\zeta}''''(s_{t+1})}{(1 + \alpha^2 \beta \tilde{\zeta}'''(s_{t+1}))^3}
$$

$$
= -\zeta''(s_t).
$$

As discussed in the previous section, I have good reason to think that $\tilde{\zeta}(\cdot)$ (and $J^C(\cdot)$) is very nice and smooth in stochastic models. Then here I am assuming that $\tilde{\zeta}(\cdot)$ (and $J^C(\cdot)$) is three times differentiable. Then, I can directly apply theorem 3 under the condition of three times differentiability to conclude $\zeta''(s_t) \geq 0$, since $J^C_{mn}(s_t) \leq 0$. Finally, recall that $\zeta(s_t) = \rho y_t + \alpha \delta \pi_t - \delta i^*$. Then,

$$
\frac{\partial^2 i^*}{\partial y_t^2} = -\frac{\alpha^2 \zeta''}{\delta} \leq 0
$$

$$
\frac{\partial^2 i^*}{\partial \pi_t^2} = -\frac{\zeta''}{\delta} \leq 0.
$$

If $J^C(\cdot)$ is not three times differentiable, what I can say is $\zeta(s_t)$ is concave in inflation and output gap. This concave reaction function (either in smooth or non-smooth cases) has an interesting implication. Suppose there happens a discrete change (not incremental) in inflation or output gap. Then monetary policy will be more aggressive during expansion than during contraction, since the aggressiveness itself is a decreasing function of inflation and output gap. In this sense, monetary policy with the zero bound is, to some extent, asymmetric.
2.4 Empirical Evidence

In this section, I test the implications of the propositions by estimating the monetary policy reaction function. I investigate Japanese data (from 1989 to 2000\(^{17}\)) since Japan is a typical (and maybe the most controversial) country where inflation is low. Several issues need to be addressed in estimating the policy reaction function. If the closed form expression of the policy reaction function is known, it is ideal to specify the regression form accordingly. Unfortunately, however, since the closed form expression of the policy reaction function does not exist as shown in the previous section, I run quadratic regressions to capture the increasing and (locally) concave nature of the policy reaction function. Since the objective of this empirical research is to detect the qualitative nature of the policy reaction function, this formulation is deemed sufficient for the purpose. In the regression, I expect that the coefficients on the quadratic terms \(\pi_t^2\) and/or \(y_t^2\) to be significantly different from zero, indeed negative, if the Bank of Japan’s monetary policy was consistent with the implication of the model.

The benchmark specification is in the spirit of the Taylor rule.

**Benchmark Specification:**

\[
i_t^# = c_0 + c_1 \pi_t + c_2 y_t + e_t \tag{2.39}
\]

\(^{17}\)This sample period needs some explanation. First of all, since I am interested in estimating the policy reaction function when the nominal interest rate is near zero, there is no point of including the samples when the nominal interest rate was high, such as in the 70's and early 80's. The second reason is due to the possibility of the structural break in the inflation target, \(\pi^\ast\). During the 70' and the 80's, when the inflation rate was high, it is likely that BOJ’s inflation target had been set higher than that of the 90's. Due to this possible (but unfortunately unobservable) revision of the inflation target, it is also likely that the policy reaction function was altered. Pooling the time series data from the 70's to the 90's in order to estimate the policy reaction function will likely yield inconsistent estimates.
where $e_t$ is assumed to be independently, identically and normally\textsuperscript{18} distributed with variance $\sigma^2$.

For the alternative specifications, I consider the following polynomial form.

**Alternative Specification:**

$$i_t^\# = c_0 + c_1 \pi_t + c_2 y_t + c_3 \pi_t^2 + c_4 y_t^2 + c_5 \pi_t y_t + e_t$$ (2.40)

where $e_t$ is, again, assumed to be independently and identically normally distributed with variance $\sigma^2$. The alternative specification – which can be interpreted as a second order Taylor approximation of the policy reaction function – is capable of capturing the concavity of the policy function with respect to $\pi$ and $y$ due to the presence of square-terms. Some caution must be exerted in estimating the above specifications. Due to some zero (or near zero) observations on the nominal interest rate data\textsuperscript{19}, if we simply conduct least squares estimation, such as OLS, on the two specifications above, the coefficient estimates will likely be biased. See, for instance, Cheung and Goldberger (1984). In order to obtain unbiased estimates, I therefore conduct a Tobit analysis.

Following the standard procedure in the Tobit analysis, I assume that “latent” interest rate $i_t^\#$ is observed only if it is greater than zero. Otherwise, it is left-censored

\textsuperscript{18}The assumption of normal distribution is purely auxiliary. However, this assumption is necessary in laying the ground work for the following Tobit analysis.

\textsuperscript{19}The Bank of Japan announced the zero-interest policy in February 1999. Since then, the Call Rate - the policy instrument of the Bank of Japan – was nil. However, it should be noted that, in practice, perfect fine-tuning of the Call Rate is not feasible since the BOJ’s open market intervention can only be implemented in a discrete manner. Indeed, the quarterly-average of the Call Rate after February 1999 was always slightly above 0 (0.03% to be specific). Nevertheless, taking the BOJ’s intention into account, during the period, I have counted that the zero-bound constraint as binding for the Call Rate during this period.
at zero. Expressing formally,

\[
i_t = \begin{cases} 
0 & \text{if } i_t^\# \leq 0 \\
& \text{if } i_t^\# > 0)
\end{cases}.
\]  

(2.41)

Thus, under this Tobit model, the probability of observed nominal interest rate to be zero is given as \( \Pr(i_t = 0) = 1 - \Phi \left( \frac{c'x_t}{\sigma} \right) \) and the likelihood of an observed positive nominal interest rate is given as \( f(i_t) = \phi \left( \frac{(i_t - c'x_t)}{\sigma} \right) / \sigma \), where \( c \) stands for the vector of coefficients, \( x_t \) is the vector of regressors, \( \Phi (\cdot) \) is the standard normal cdf and \( \phi (\cdot) \) is the standard normal pdf. Therefore, the log-likelihood function of the observed interest rates \((i_1, ..., i_T)'\) can be expressed as

\[
\ln L = \sum_{t=1}^{T} \left\{ 1 \left( i_t > 0 \right) \left[ - \ln(2\pi)/2 - \ln \sigma - \frac{(i_t - c'x_t)^2}{\sigma^2} \right] + 1 \left( i_t = 0 \right) \ln \left[ 1 - \Phi \left( \frac{c'x_t}{\sigma} \right) \right] \right\}
\]

(2.42)

where \( 1(\cdot) \) is an indicator function which takes a value of 1 if the condition inside the parenthesis is true and 0 otherwise. The results of Maximum Likelihood (ML) estimation are reported in Table 2.1.

The top portion of Table 2.1 reports the estimates of coefficient vector \( c \) and \( \sigma \) for four specifications. The benchmark specification is in the spirit of the Taylor rule. Alternative 1 allows for the concavity both in \( \pi \) and \( y \). Alternative 2 allows for the concavity in \( \pi \), but not in \( y \). Finally, Alternative 3 allows for the concavity in \( y \), but not in \( \pi \). Except for the case of Alternative 3, the coefficient estimates on quadratic terms were significantly negative. These results are consistent with the implications of the propositions discussed in the previous section.\(^ {20} \)

\(^ {20} \)The sign of coefficient on the cross term, \( \pi y \) might need explanation, although it is less important in economic sense. Theorem 3 predicts that the coefficient on the cross term, \( \pi y \) is also negative. However, table 1 shows the opposite sign. This is because of the strong multi-collinearity between \( \pi \) and \( y \). (Actually its correlation over the estimation period is 0.677.) Since inflation and output gap are highly positively correlated, observations do not spread well on the \( \pi - y \) field. This yields the
<table>
<thead>
<tr>
<th>Regressor</th>
<th>Benchmark</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
<th>Alternative 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>const.</td>
<td>1.610</td>
<td>3.873**</td>
<td>0.618</td>
<td>1.598</td>
</tr>
<tr>
<td></td>
<td>(0.415)</td>
<td>(0.387)</td>
<td>(0.393)</td>
<td>(0.301)</td>
</tr>
<tr>
<td>π</td>
<td>1.251</td>
<td>5.139**</td>
<td>2.770</td>
<td>3.946**</td>
</tr>
<tr>
<td></td>
<td>(0.243)</td>
<td>(0.702)</td>
<td>(0.743)</td>
<td>(0.190)</td>
</tr>
<tr>
<td>y</td>
<td>0.452</td>
<td>4.592**</td>
<td>-0.290</td>
<td>-1.450</td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td>(0.200)</td>
<td>(0.105)</td>
<td>(0.203)</td>
</tr>
<tr>
<td>π²</td>
<td>-</td>
<td>-</td>
<td>-0.568</td>
<td>-2.594**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.219)</td>
<td>(0.236)</td>
</tr>
<tr>
<td>y²</td>
<td>-</td>
<td>-</td>
<td>-0.056</td>
<td>-1.961*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.029)</td>
<td></td>
</tr>
<tr>
<td>πy</td>
<td>-</td>
<td>-</td>
<td>0.639</td>
<td>5.066**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.126)</td>
<td>(0.123)</td>
</tr>
<tr>
<td>σ</td>
<td>1.292</td>
<td>-</td>
<td>0.862</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.203)</td>
<td>(0.128)</td>
<td>(0.136)</td>
<td></td>
</tr>
</tbody>
</table>

Log-Likelihood Ratio Test

<table>
<thead>
<tr>
<th>Log-LR statistic</th>
<th>H₀: c₃=c₄=c₅=0</th>
<th>H₀: c₃=c₅=0</th>
<th>H₀: c₄=c₅=0</th>
</tr>
</thead>
<tbody>
<tr>
<td>log-LR statistic</td>
<td>31.183**</td>
<td>28.213**</td>
<td>20.542**</td>
</tr>
</tbody>
</table>

Note: Data source for the Call Rate is from BOJ’s Financial and Economics Statistics Monthly. I computed the quarterly average of the call rate to construct the data set for i. The numbers in the parenthesis denotes the standard error. In estimating the asymptotic variance of ML estimator, I used the algorithm proposed by Berndt, Hall and Hausman (1974). Rejection of the null hypothesis at the significance level of 5% and 1% is indicated by (*) and (**), respectively. Note that under the null of coefficient equal to zero, z-statistic is asymptotically distributed as the standard normal. Also note that log-LR statistic, under the null, is asymptotically \( \chi^2 \) distributed with degree of freedom corresponding to the number of restrictions imposed.

Table 2.1: MLE result and Log-Likelihood ratio test
In addition, in order to test the overall performance of the benchmark specifications against the alternative specification, I conducted a log-likelihood ratio test. The null hypothesis is that, for all the quadratic terms in each alternative specification, the coefficients are jointly equal to zero. By restricting the coefficient of second order terms as such, each alternative specification reduces to the benchmark specification. Therefore, under the null hypothesis, the test can be motivated as a performance test of the benchmark specification, which is a proxy for the Taylor rule, against the alternative specification, which allows for the policy reaction function to be (locally) concave. The result of the log-likelihood ratio test is reported on the bottom portion of Table 2.1. As can be seen, the test overwhelmingly rejects the null hypothesis for all cases – evidence against the benchmark specification. Figure 2.2 displays the shape of the estimated reaction function. It reveals that the function is indeed increasing and (locally) concave on the relevant domain.

2.5 Concluding Remarks

The tools provided in this essay are widely applicable for a class of dynamic optimization problems. Some other applications are, probably, consumption and investment functions with borrowing constraints. Especially, when an infinitely-lived firm has a quadratic cost function, which is assumed in some preceding empirical positive coefficient on the cross term. As a remedy for the multi-colinearity problem, I extract the first principle of \( \pi_t \) and \( y_t \), (denote this new data \( s_t \), since this \( s_t \) can be regarded as the consolidated state \( s_t = \pi_t + \alpha y_t \) in section 3) and ran a regression \( s_t \) on \( i_t \) as follows.

\[
i_t^* = c_0 + c_1 s_t + c_2 \left( \frac{1}{s_t} \right)
\]

Note this formulation can capture the asymptotic linear nature as well as the concavity. By this regression I confirmed that the sign of \( c_2 \) is significantly positive, which is consistent with the prediction of the model.
studies, and she is confronted with a borrowing constraint, the tools in this essay are applicable for characterizing the investment function. The key implication of the theorems for borrowing constraints is, even if the constraint is not binding in the current period, it can still affect the optimal choice of today’s investment/consumption. Numerical and analytical studies in consumption literature have found this property, but it seems less focused so far in the corporate investment literature. Due to the infinite horizon in the problem, the tools introduced in the paper can enrich the empirical testing methodology for borrowing constrained firms’ investment function. For the consumption problem with a borrowing constraint, the result would be much richer, if similar tools are available for the case with non-quadratic, such as CRRA, utility functions. This extension is one possible direction of further research.

As for the application to the monetary policy reaction function, perhaps the most serious shortcomings of the model is that it assumes no forward-looking behavior on the part of households. As I mentioned in section 2.3, if such forward-looking behavior is explicitly incorporated into the model, the optimal policy is no longer unique. Another concern is that the model ignores many important aspects of the real economy. When the nominal interest rate hits zero, the central bank in this model is essentially impotent in stimulating the economy. To derive any policy implication for an economy in such situation, namely in a liquidity trap, the simple IS/AS model is no longer sufficient for the purpose, and thus we need a more formal model in which I can analyze the detailed mechanism how and why the economy stays in the liquidity trap. Naturally, such model would capture the optimizing behaviors of not only policy makers, but also of households, firms and, perhaps, financial intermediary in a general equilibrium environment. Of course, it is far beyond the scope of this
paper. The point is that the policy implications provided in this paper are not for an economy which has already been caught in a liquidity trap, but for an economy which is about to go there.
(1) Inflation held at the average (=1.37%)

(2) Output gap held at the average (-0.83=\%)

Figure 2.2: Estimated monetary policy reaction function
CHAPTER 3

LIQUIDITY, INFINITE HORIZONS AND MACROECONOMIC FLUCTUATIONS

3.1 Introduction

This essay develops a computable dynamic general equilibrium model in which the role of liquidity over the business cycle can be analyzed. My focus is especially the corporate demand for liquidity and its influence on business cycles via firms’ investment decision rule. It is an empirical fact that corporations rely heavily on short-term debt for working capital expenses in the United States as well as in Japan.\(^{21}\) In the virtual economy with perfect information so that nothing prevents the classic Modigliani-Miller theorem (MM theorem, hereafter) from holding, whatever short-term financing is chosen, either by privately or publicly issued liquid asset, such decision is a trivial matter from the viewpoint of efficiency. Nonetheless, even if we ignore the complicated aspects of the financing contracts, corporate finance and the role of the banking sector is still of interest to business cycle researchers. In this line of literature, the real business cycle (RBC) framework with financial

\(^{21}\)See Einarson and Marquis (2001).
intermediary developed by Fuerst (1992)\textsuperscript{22} is a pioneering work. This framework was later intensively studied by Christiano (1991), Christiano and Eichenbaum (1993), Marquis and Einarson (2001), among others. The role of the financial intermediary in those models is to provide firms with financing for wage that must be paid to employees in advance of their sales of output. Overall, these models are well capable of explaining empirical business cycle facts, including bank loans and other financial variables. However, they are potentially flawed, because they fail to replicate the actual auto-correlation patterns of output and investment.\textsuperscript{23} One advantage of the model introduced in this essay is its superior performance to the Fuerst-Christiano style of RBC models in mimicking the actual auto-correlation patterns.

Another stream of studies on the interaction of corporate finance and the business cycle extends \textit{agency cost} models, which were originally developed in microeconomic contract theory, to macroeconomics. Roughly speaking, the difference between the value of the firm in what would be an ideal contracting situation and what is viable through negotiation is referred to as \textit{agency costs}.\textsuperscript{24} In agency cost models, the net present value (NPV) of an investment project is not maximized, simply because lenders and borrowers (entrepreneurs) have divergent incentives, so that for each agent NPV maximization could be suboptimal. The financial contract between lender and borrower is characterized by the nature of the concessions necessary to achieve at least a second best solution. It should be noted that for models with this agency cost, MM theorem is by construction violated, and thus the financial contract

\textsuperscript{22}Theoretical foundation of Fuerst-Christiano framework is based on the preceding study by Lucas(1990).

\textsuperscript{23}This is pointed out by Cogley and Nason (1995) and Gilchrist and Williams (2000).

\textsuperscript{24}Amaro de Matos (2001).
structure plays a non-trivial role in firms’ investment decisions. An early study of this type of financial contract is Townsend (1979). This Townsend’s study is well-known as the *optimal contract theory* with costly state verification. When the outcome of a project is private information to the entrepreneur, this asymmetric information creates a moral hazard problem. The entrepreneur may have an incentive to misreport the true outcome. The agency cost in this case is that a certain portion of the profit is lost for costly monitoring. More general framework of financial contract with agency cost is developed by Grossman and Hart (1986) and Hart and Moore (1990). In general, suppose a situation where the financial contract is constrained by the existence of unverifiable future variables, which are usually assumed to be unobservable ex-post by outsiders. They are not enforceable, in the sense that contracts cannot be written to condition payoffs on these unverifiable variables. In this context, the financial contract is herein understood as an instrument for reducing agency costs by creating incentives for entrepreneurs to “behave.” Hart and Moore’s (1994) framework is an example in which agency problems do not necessarily stem from asymmetric information. Even though the information is complete, where the entrepreneur cannot be replaced by others\(^{25}\) (or such replacement is highly costly) and renegotiation is difficult, some profitable projects are not financed. This is mainly because some portion of the profit must be paid to prevent entrepreneurs from threatening to repudiate.

In the 80s and 90s, these various types of financial structure were gradually taken into dynamic general equilibrium models (DGE models, hereafter) to investigate their outcome on business cycle dynamics. Williamson (1987) and Bernanke and Gertler’s

\(^{25}\)The notion of *human capital* can be brought in here. Unique human capital of the entrepreneur can play very similar role to that of private information in generating agency cost.
model (1989) reflect earlier attempts to construct DGE models with a Townsend-type financial contract based on costly state verification. Especially Bernanke and Gertler constructed an OLG model in which a financial market imperfection induces temporary shocks to firms’ net worth to be amplified and to persist. This mechanism is known as the financial accelerator. Similar modeling strategies can be found in Carlstrom and Fuerst (1997) and Kiyotaki and Moore (1997), although the former is based on Townsend’s financial contract, while in the latter Hart/Moore’s costly renegotiation contract is adopted as the central feature of the model. Both models consider infinitely-lived agents so that business cycle dynamics are easier to analyze due to the models’ tractability. These DGE models are mostly successful in replicating empirically reasonable business cycle dynamics, such as amplification, persistence, hump-shaped impulse response, and oscillations. Why are these DGE models with agency costs so successful in mimicking actual business cycle dynamics? In standard RBC models, a firm’s investment is merely a mirror of consumers’ savings. Recall that agency cost is only relative to a first best world leakage in process of transferring consumers’ savings to firms’ production inputs. By allowing agency cost to fluctuate endogenously, the tight link between savings and investment is softened, so that a firm’s investment decision can influence output dynamics independently of the consumer’s decision rule. Roughly speaking, the reason those DGE models can exhibit more empirically reasonable dynamics is because the trough in agency cost (usually at the peak of firm’s net worth) is delayed by one or two periods later behind the initial

\[\text{footnote}{\text{Greenwald and Stiglitz (1993) and Bernanke, Gertler and Gilchrist (2000) studied similar interest.}}\]

\[\text{footnote}{\text{An interesting recent study by Gilchrist and Williams (2000) presents a DGE model which generates hump-shaped dynamics of output. The key of their model is putty-clay technology of firm’s investment. This is another example that shows a certain device in firm’s investment decision rule leading to a realistic output dynamics, even though information is complete.}}\]

41
shock. Net worth is a state variable and thus limps behind. This is a commonly observed mechanism which drives the dynamics of most DGE models with agency cost.

This study is in the line of DGE models with agency cost. The core of our model, a unique financial contract structure, is taken from Holmstrom and Tirole (1998) (let us denote HT, hereafter) instead of other common financial contracts. The agency cost considered here arises from a standard moral hazard, which requires a certain portion of profit given to entrepreneurs in order to keep them diligent. This is simply because their effort is private information and therefore not enforceable. Although the style of moral hazard is quite standard and even traditional, HT model has an outstanding feature, such that corporate liquidity demand is motivated by the moral hazard. When a certain amount of profit is lost (must be given to the entrepreneur in my case as similarly handled in most moral hazard models) a firm’s value is strictly less than its maximum NPV. This wedge between a firm’s full value and its value for external investors entails the rationale on the firm’s needs for advance financing, namely, corporate demand for liquidity. Without this wedge, the firm can raise sufficient funds by issuing claims or obtaining a credit line from investors in advance, and that is enough to defray all the firm’s working capital expenses, even if such liquidity needs are stochastic. However, where a firm’s value for external investors is less than its full value, some of its projects may be terminated midstream, since it cannot finance working capital expenses by raising enough credit line or additional loans from the investors. To protect itself from such risks of liquidity shortage, a firm may want to hold liquidity reserves. This is the essential argument that HT makes in their paper.
This essay extends the HT model to an infinite horizon environment using a modeling strategy similar to that of Carlstrom and Fuerst (1997) and analyzes the business cycle dynamics that result from such liquidity-dependent corporate financing. The first notable result of the paper is that my model generates a hump-shaped impulse response very similar to that in Carlstrom and Fuerst, which is reported as an empirical fact in preceding business cycle studies.28 The result of this study enhances the view that hump-shaped output dynamics or similar persistence is common and robust outcome of various types of agency cost models.

Further, my DGE model in this essay provides several insights into other aspects of corporate liquidity demand and business cycles. The empirical fact is that corporate firms’ working capital expenses are pro-cyclical, while the degree to which firms rely on bank loans to finance their working capital expenses, measured as the volume in commercial and industrial loans relative to output, is counter-cyclical.29 My DGE model has an advantage over others in replicating this corporate financing structure over business cycles, in the sense that it successfully generates pro-cyclical demand for liquidity, while the degree of liquidity-dependence (measured as liquidity demand divided by investment expenditure) is counter-cyclical. Moreover, interestingly, this outcome exhibits clear similarity to existing empirical studies in the line of lending view literatures, such that corporate firms become highly dependent on bank loans in recessions.

28Empirics of hump-shaped dynamics of output is studied by Cogley and Nason (1995).
This chapter is organized as follows. Section 3.2 introduces a version of the moral hazard model of corporate liquidity demand presented in Holmström and Tirole (1998). Section 3.3 develops the DGE model in which the infinite horizon version of the HT-type corporate liquidity demand is embedded. Section 3.4 presents calibration and simulation results. Section 3.5 discusses some interpretations of empirical facts and how they are related to the lending view studies. I will conclude the paper in section 3.6.

### 3.2 Corporate Demand for Liquidity and Firms’ Investment: Holmström and Tirole (1998)

In this section I introduce the model of corporate liquidity demand presented by Holmström and Tirole (1998) in a slightly modified fashion. The HT model generates a unique investment function and corporate liquidity demand function. Later in section 3.3 they will be embedded in an otherwise standard dynamic general equilibrium model to analyze their influence on business cycle dynamics. Since the financial contract is only one period in length, I can consider the financial contract and investment behavior separately from the rest of the dynamic general equilibrium. In the following subsections, capital price $q$ and firms’ net worth $n$ are regarded as constant parameters which will be determined outside of the financial contract.

In the HT model, corporate liquidity demand is motivated by moral hazard. Holmström and Tirole’s (1998) arguments are summarized as follows. In a moral hazard model, the entrepreneur must be given a minimum share of profit in order to be motivated. Because of this, the value of external claims on the firm is strictly less than the full value of the firm. The wedge between the full value of the firm
and the external value of the firm prevents it from financing all projects that have positive net present value. This implies that liquidity shocks could force the firm to terminate a project midstream, even though the project has a positive continuation value. To avoid such risks, the firm wants to hold liquid reserves in the form of marketable assets that can be readily sold or credit lines provided by the financial intermediaries.

The HT model also demonstrates that in the absence of aggregate uncertainty, the financial intermediary can achieve production efficiency in the sense that private-issued liquid assets (including credit lines) are sufficient for insurance.\textsuperscript{30} I will rely on this result later in extending the model to an infinite horizon environment.

### 3.2.1 Financial contract with liquidity demand

The model shown here is essentially identical to that of HT, except for two minor modifications. One is that everything is going on within a period, while the original HT model is a three-period model. I can reconcile the difference by implicitly regarding each period is segmented into three sub-periods.\textsuperscript{31} The other is that capital goods are distinguished from consumption goods. The end-of-period capital price is $q$ in terms of consumption goods. Consideration of the financial contract can be separated from the rest of the general equilibrium, since the contract is only one-period in length. The financial contract is negotiated in the beginning of each period.

\textsuperscript{30}See proposition 2 in HT. Interestingly, the proposition reveals that full insurance cannot be achieved by financial market trading. Only financial intermediation can provide risk sharing by pooling idiosyncratic risks. Therefore, it is important to incorporate the financial intermediary as the third agent in my DGE model later.

\textsuperscript{31}This three sub-periods segmentation is purely for convenience. Sequence of events is described in Figure 3.1.
and is resolved by the end of that same period. General equilibrium issues affect the contract through the level of firms’ net worth, $n > 0$ and price of capital, $q > 0$.

Here is a two-goods economy, with consumption goods and capital goods. There are two types of agents, firms (entrepreneurs) and investors (consumers). Both are assumed to be risk neutral.\[^{32}\] A firm has access to a stochastic constant-returns-to-scale technology to convert an amount $i$ of consumption goods into $R_i$ of capital. In the midst of a period, an additional uncertain amount $\omega i$ of funds (as measured by capital goods) is necessary to cover working capital expenses and other cash needs. The liquidity shock $\omega$ is distributed according to the cumulative distribution function $\Phi(\omega)$ with a density $\phi(\omega)$. If $\omega i$ is paid, the project continues and a final pay-off is realized in sub-period 2. If $\omega i$ is not paid, the project terminates and yields nothing. Timing of events is described in Figure 3.1.

Figure 3.1: Time schedule of events

\[^{32}\]In the next section, risk averse consumers will be brought in as the source of outside fund supplier. However, in terms of the financial contract, they will be effectively risk neutral. Carlstrom and Fuerst (1997) denote two conditions which are sufficient for the risk neutrality as follows. Namely, (1) there is no aggregate uncertainty over the duration of the contract, and (2) the financial intermediary can take advantage of the law of large numbers to eliminate idiosyncratic risks. These two properties allow the financial intermediary to assure deterministic return to consumers. I will refer to this issue again later.
Investment is subject to moral hazard in that a firm (entrepreneur) privately chooses the probability $\pi$ that the project succeeds. The firm can either “behave” or “shirk.” If the firm behaves, the probability of success is $\pi_H$ (high) if it shirks, the probability of success is $\pi_L$ (low), where $\pi_H - \pi_L \equiv \Delta \pi > 0$. If the firm shirks, it enjoys a private benefit, $Bi > 0$, proportional to the level of initial investment $i$. The firm makes the decision on $\pi_H$ or $\pi_L$ after the continuation decision.

The net present value of the investment is maximized by continuing the project if and only if $\omega \leq \omega_1 \equiv \pi_H R$, that is, whenever the expected return $\pi_H R$ from continuation exceeds the cost $\omega$. HT refers to this $\omega_1$ as the first-best cutoff.$^{33}$ The firm has an endowment of net worth, $n > 0$ in the beginning of the period and can raise additional funds from outside investors. A contract with outside investors specifies the amount that the investors will contribute $i - n$, the initial scale of the project $i$, the contingencies in which the project is continued at the emergence of the liquidity shock (the cut-off level of the liquidity shock $\bar{\omega}$), and the distribution of the profit from the investment. Let $R_f i$ be the amount which the firm is paid when the project succeed. Generally $R_f$ can be contingent on $\omega$, but the second best contract is achieved by a contract such that the incentive compatible constraint, $\pi_H R_f \geq \pi_L R_f + B$, is binding. Namely, the entrepreneur’s share of profit is bounded by the minimum level which prevents it from shirking. With the binding incentive compatibility constraint, $R_f = B/\Delta \pi$, outside investors’ expected cash flow excluding

$^{33}$They assume

$$\max \{\pi_H R - \omega, 0\} \phi(\omega) \, d\omega - 1 > 0 \quad (3.1)$$

$$\max \{\pi_L R + B - \omega, 0\} \phi(\omega) \, d\omega - 1 \quad (3.2)$$

so that they could concentrate on contracts that implement the effort $\pi_H$. 

47
the liquidity shock, is $\pi_H (R - R_f) \equiv \omega_0$. $\omega_0$ is called pledgeable unit return from investment. The structure of moral hazard is illustrated in Figure 3.2.

![Figure 3.2: Structure of moral hazard](image)

Now I am ready to set up the optimal financial contract problem to choose $\{ i, \bar{\omega}, R_f(\omega) \}$.

\[
\text{max} \quad q i \pi_H R_f \int_{0}^{\bar{\omega}} \phi(\omega) \, d\omega \quad (3.4)
\]
\[
= q i \pi_H R_f \Phi(\bar{\omega}) \quad (3.5)
\]
\[
\text{s.t.} \quad i - n \leq q \left[ i \int_{0}^{\bar{\omega}} (\pi_H (R - R_f) - \omega) \phi(\omega) \, d\omega \right] \quad (3.7)
\]
\[
= q i \left[ \omega_0 \Phi(\bar{\omega}) - \int_{0}^{\bar{\omega}} \omega \phi(\omega) \, d\omega \right] \quad (3.8)
\]
\[
\equiv q i h(\bar{\omega}) \quad (3.9)
\]
\[
R_f \geq R_f^{\text{low}} = \frac{B}{\pi_H - \pi_L} \quad (3.11)
\]
This problem is to maximize a firm’s share of profit subject to investors’ break-even constraint eqn (3.9) and the firm’s incentive compatible constraint eqn (3.11). With the incentive compatible constraint binding, the remaining choice is \( i \) and \( \bar{\varphi} \). Since everything is linear in the problem, the break-even condition must hold with equality. This yields the following relation.

\[
i = \left( \frac{1}{1 - qh(\bar{\varphi})} \right) n
\]  

(3.12)

Substituting this into the objective function leaves an unconstrained problem with respect to \( \bar{\varphi} \).

\[
\max_{\bar{\varphi}} \left( \frac{q\Phi(\bar{\varphi})}{1 - qh(\bar{\varphi})} \right) \pi_H R_f n
\]

(3.13)

Here let us rewrite \( h(\bar{\varphi}) \),

\[
h(\bar{\varphi}) = \omega_0 \Phi(\bar{\varphi}) - \int_0^{\bar{\varphi}} \omega \phi(\omega) d\omega
\]

(3.14)

\[
= (\omega_0 - \bar{\varphi}) \Phi(\bar{\varphi}) + \int_0^{\bar{\varphi}} \Phi(\omega) d\omega
\]

(3.15)

Paying attention to the derivative \( h'(\bar{\varphi}) = (\omega_0 - \bar{\varphi}) \phi(\bar{\varphi}) \), the foc of (3.13) is

\[
q \int_0^{\bar{\varphi}} \Phi(\omega) d\omega = 1.
\]

(3.16)

Based on the closed form, I can define an implicit function of the optimal cutoff level of liquidity shock, \( \bar{\varphi} = \psi(q) \). Let us call this optimal cutoff induced by the second best financial contract “the degree of liquidity dependence” hereafter. Plugging this into eqn(3.12),

\[
i = \left( \frac{1}{1 - qh(\psi(q))} \right) n
\]

(3.17)

\[
\equiv k(q) n
\]

(3.18)
Thus investment is linear in $n$ with a factor of proportionality of $k(q)$, which exceeds one. HT calls this $k(\cdot)$ equity multiplier. Let me point out that very similar multipliers to this $k(\cdot)$ can be found in many other models with agency cost. In fact, it is a peculiar and common feature for those imperfect information models that investment requires a down payment. It can be shown that the investment function, eqn(3.18) is upward sloping in the capital price $q$ just as the investment function of the adjustment cost model is increasing in the shadow price of capital. A significant difference is that eqn(3.18) is not only a function of $q$, but also of the firm’s net worth $n$, which will be the key feature in generating unique dynamics in infinite horizon extension.

Finally, I introduce aggregate corporate liquidity demand $D$ and the “degree of liquidity dependence” $x$ which is defined as aggregate liquidity demand divided by aggregate investment $I$ for a later purpose in the empirical discussion.

$$D = q I \int_0^{\bar{\omega}} \omega \phi(\omega) d\omega \quad (3.19)$$

$$x = \frac{D}{q I} = \frac{\int_0^{\bar{\omega}} \omega \phi(\omega) d\omega}{\Phi(\bar{\omega}) \omega_1} \quad (3.20)$$

where aggregate investment $I = i \omega_1 \Phi(\bar{\omega})^{35}$ instead of $\omega_1 i$, since the fraction of investment projects whose liquidity shocks are larger than $\bar{\omega}$ are abandoned.

### 3.2.2 The role of the financial intermediary

One of HT’s fundamental questions is whether a privately issued liquid asset would be sufficient for achieving the (second) best outcome described above.  

---


35I am slightly abusing notations here. In the next section for dynamic general equilibrium analysis, I assume that fraction $\eta$ of the population of the economy is entrepreneurs. Hence to define the aggregate firm’s liquidity demand, $D$ and investment, $I$ they should be multiplied by the population weight $\eta$, that is, $D = \eta q I \int_0^{\bar{\omega}} \omega \phi(\omega) d\omega$ and $I = \eta \omega_1 \Phi(\bar{\omega})$ instead of those without $\eta$ shown here.
In the absence of aggregate uncertainty (idiosyncratic risk for each entrepreneur is independent), the answer is yes. I assume there is a continuum of entrepreneurs with unit mass. Thanks to the constant-returns-to-scale technology, there is no loss in assuming that entrepreneurs have identical net worth; the representative entrepreneur is endowed with \( n \) units of net worth at the beginning of the period. Then additional liquidity needs demanded by whole productive sector is

\[
D = qi \int_{0}^{\bar{\omega}} \omega \phi (\omega) d\omega
\]  

(3.21)

where \( i \) here is the representative firm’s investment. Note that by taking advantage of the law of large numbers, this amount \( D \) is a deterministic number. On the other hand, the maximum amount of the claims for the existing firms is equal to \( qi \omega_0 \Phi (\bar{\omega}) \equiv V \). Since \( qi \omega_0 \Phi (\bar{\omega}) - D = i - n > 0 \), in this economy without aggregate uncertainty, there can be sufficient amount of private-issued claims to meet firms’ additional liquidity demands. Now I herein incorporate the third agent, the financial intermediary.\(^{36}\) The financial intermediary collects all of the consumption goods and offers credit lines up to \( q \bar{\omega} i \) for each entrepreneur so that the second best financial contract described above can be implemented. The law of large numbers allows the intermediary to grant \( D \) of funds to entrepreneurs in total. As a result of each entrepreneur’s production with the credit line up to the second best cut-off, the

\(^{36}\)Carlstrom and Fuerst (1997) refer to this financial intermediary as a “capital mutual fund” (CMF). This notion was first incorporated by Williamson (1986). It should be emphasized that the notion of “mutual fund” referred to in the HT model is a different concept. The HT’s “mutual fund” has no risk pooling function. Actually, HT admits that their “mutual fund” cannot achieve production efficiency, and thus it is not classified as a kind of intermediary. Essentially, what distinguishes the intermediary here from other financial institutions is whether they have the ability to offer credit lines to entrepreneurs. Note that the value of marketable claims held by a firm cannot be made contingent on that firm’s idiosyncratic shock. Risk pooling can be implemented only via credit lines.
entrepreneurial sector as a whole can produce $V$ of capital at the end of the period. Note that again, the law of large number makes $V$ deterministic.

Essentially, the role of financial intermediary in this economy is *risk pooling* to achieve insurance. HT demonstrates that there can be many variants of this kind of financial intermediaries in the real world. Any variants that can offer credit lines to entrepreneurs are sufficient for the purpose.

### 3.3 The Dynamic General Equilibrium Model

In this section the investment function and liquidity demand function derived in the previous section are embedded into an otherwise standard DGE model.

Again there are two types of agents, firms (entrepreneurs) and consumers (investors) in the economy. The fraction $\eta$ of the population is entrepreneurs and the rest is consumers. Capital is produced from consumption goods using constant-return-to-scale technology which is specific to the entrepreneurs. This capital producing process takes place under moral hazard as described in the previous section. At the beginning of each period, entrepreneurs receive $i - n$ consumption goods from the financial intermediary as a part of the financial contract, and use them as inputs to produce capital. At the end of the period, newly produced capital is ready for use, if the entrepreneur’s project is not abandoned due to a liquidity shock. It is a substantial modification from the standard RBC in which there is an *ex-post* one-to-one technology for transforming consumption goods into capital. In the standard RBC model, a unit input of consumption goods is transformed into one unit of capital at
the end of the period. In this environment, it does not make sense to distinguish capital from consumption goods, and the price of capital is always equal to one. Hence, it is virtually regarded as a one-good economy.

In the economy assumed here, entrepreneurs receive external funds and credit lines via the financial intermediary. As shown in the previous section, the role of the intermediary here is to assure a certain return to consumers by providing entrepreneurs with collected consumption goods and necessary credit lines. Consumers who sold \( q \) units of consumption goods at the beginning of the period will receive one unit of capital from the intermediary in the end of the period. Because of this deterministic return over the duration of financial contract, consumers are regarded as effectively risk neutral in terms of the financial contract.

The economy is also populated with many firms producing a single consumption good. (I call them retailers to distinguish them from entrepreneurs.) Retailers are assumed to be free from moral hazard, and so I do not have to specify their financing contract. Instead, they are mechanically producing consumption goods at the level at which price equals marginal cost.

### 3.3.1 Optimization set-ups

- **Consumers’ optimization**

Consumers’ problem is standard. They are maximizing an infinite sum of discounted utility from consumption \((c_t)\) and labor supply \((l_t)\).

\[
\max : U^c = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{3.22}
\]

where \(\beta\) is a discount rate. At the beginning of each period, they rent previously accumulated capital at a rental rate \(r_t\), and purchase consumption goods at a price
of unity. (The consumption good is numeraire.) At the end of each period they purchase newly produced capital with the help of the financial intermediary. Also they supply their labor force at a wage rate \( w_t \). Consumers’ optimal conditions are summarized as follows.

\[
E_t \beta \left( \frac{q_{t+1} (1 - \delta) + r_{t+1}}{q_t} \right) \frac{u_{L,t}}{u_{c,t}} = w_t
\]

where \( \delta \) is the depreciation rate of capital.

- **Entrepreneurs’ optimization**

Entrepreneurs are risk neutral. They are maximizing an infinite sum of discounted consumption \( c_t^e \). Because of the moral hazard discussed before, the return on internal funds is higher than that on external funds. This higher return on internal funds induces entrepreneurs to postpone their consumption forever and thus the economy never reaches a steady state. To avoid oversavings of entrepreneurs, their discount factor \( \gamma \beta \) is assumed to be smaller than that of consumers: \( \gamma \beta < \beta \).\(^{37}\) Entrepreneurs’ optimization is written as follows.

\[
\max \quad U^e = E_0 \sum_{t=0}^{\infty} (\gamma \beta)^t c_t^e
\]

\[
s.t. \quad q_t K_{t+1}^e = (1 + \rho_t) n_t - c_t^e
\]

\[
where, \quad n_t = (1 - \delta) q_t K_t^e + r_t K_{t+1}^e + w_t^e
\]

\[
1 + \rho_t = \frac{q_t \pi_H R_{t} \Phi (\tilde{\omega}_t)}{1 - q_t \tilde{h} (\tilde{\omega}_t)}
\]

\(^{37}\) Another technique to avoid negative consumption of entrepreneurs is to assume simply that they consume a certain amount of wealth in each period. I can interpret this as that certain fraction of entrepreneur is dying in each period. This type of assumption is taken in Kiyotaki and Moore (1997) for example.
where $K^e_t$, $n_t$, and $\rho_t$ denote an entrepreneur’s capital, net worth and net return rate respectively. $w^e_t$ is a wage rate for entrepreneurs’ labor supply, which is fixed at one. This labor income assures positive net worth of entrepreneurs. Entrepreneurs invest whole $n_t$ at the beginning of a period to receive its return $q_t \pi H R_f \Phi (\tilde{\omega}_t)$ at the end of the period. Since $i_t = n_t / \{1 - q_t h (\tilde{\omega}_t)\}$ as shown in eqn(3.12), the return rate $\rho_t$ is defined as in eqn(3.26). Using this notation, entrepreneurs’ Euler equation can be written as $q_t = E_t \beta^\gamma (q_{t+1} (1 - \delta) + r_{t+1}) (1 + \rho_{t+1})$. It should be noted that net worth does not appear in this equation, so that this condition holds for any level of net worth.

- Retailers’ optimization

Retailers’ problem is again standard. Their production function is constant return to scale, such that $Y_t = v_t F (K_t, L_t, H_t)$, where $K_t, L_t$ and $H_t$ denote aggregate capital, labor input from consumers and labor input from entrepreneurs, respectively. $v_t$ is a random productivity shock which is normalized at one in steady state. Since retailers are free from moral hazard, their production always takes place at the efficient level of input, so that $r_t = v_t F_K (K_t, L_t, H_t)$, $w_t = v_t F_L (K_t, L_t, H_t)$ and $w^e_t = v_t F_H (K_t, L_t, H_t)$.

3.3.2 Recursive competitive equilibrium

The equilibrium of the economy is defined as the set of $K_{t+1}$, $K^e_{t+1}$, $H_t$, $L_t$, $n_t$, $i_t$, $c^e_t$, $c_t$, $q_t$, $x_t$, $D_t$, $w_t$, $r_t$, and $\tilde{\omega}_t$ which satisfies the following consumer’s decision rule (eqn(3.27)-(3.28)), entrepreneur’s decision rule and optimal financial contract (eqn(3.30)-(3.31)), retailer’s decision rule (eqn(3.38)-(3.39)), resource constraints (eqn(3.40)-(3.43)) and exogenous state transition (eqn(3.45)).
• Efficiency conditions and optimal financial contract

\[ c_t^{-\gamma} = E_t \beta \left( q_{t+1} (1 - \delta) + r_{t+1} \right) c_{t+1}^{-\gamma} \]  
(3.27)

\[ w_t = \frac{U_{L,t}}{U_{c,t}} \]  
(3.28)

\[ q_t = E_t \beta \gamma (q_{t+1} (1 - \delta) + r_{t+1}) (1 + \rho_{t+1}) \]  
(3.30)

\[ x_t = \frac{D_t}{q_t I_t} \]  
(3.31)

where, \( 1 + \rho_t = \frac{q_t \pi_H R_f \Phi(\bar{\omega}_t)}{1 - q_t h(\bar{\omega}_t)} \)  
(3.32)

\[ i_t = \left( \frac{1}{1 - q_t h(\bar{\omega}_t)} \right) n_t \]  
(3.33)

\[ D_t = \eta q_i \int_{0}^{\bar{\omega}_t} \omega \Phi(\omega) d\omega \]  
(3.34)

\[ I_t = \eta \Phi(\bar{\omega}_t) \omega_i i_t \]  
(3.35)

\[ \bar{\omega}_t = \psi(q_t) \]  
(3.36)

\[ r_t = v_t F_K(K_t, L_t, H_t) \]  
(3.38)

\[ w_t = v_t F_L(K_t, L_t, H_t) \]  
(3.39)

• Resource constraint and exogenous state transition

\[ Y_t = (1 - \eta) c_t + \eta i_t + \eta c_t^e \]  
(3.40)

where, \( Y_t = v_t F(K_t, L_t, H_t) \)  
(3.41)

\[ K_{t+1} = (1 - \delta) K_t + I_t \]  
(3.42)

\[ K_t^e = \frac{1}{q_t} \{ (1 + \rho_t) n_t - c_t^e \} \]  
(3.43)

where, \( n_t = \omega_t^e + \frac{1}{\eta} K_t^e q_t (1 - \delta) + r_t \)  
(3.44)

\[ v_{t+1} = \sigma v_t + (1 - \sigma) v^* \]  
(3.45)
Some remarks are in order. Eqn(3.40) denotes consumption goods market clear condition. Eqn(3.43) describes entrepreneur’s capital accumulation. Since investment projects with a liquidity shock larger than \( \bar{\omega}_t \) are abandoned as discussed in the previous section, newly produced capital \( I_t \) in eqn(3.35) is not equal to \( \omega_1 i_t \), but to \( \Phi(\bar{\omega}_t) \omega_1 i_t \) multiplied by the population weight \( \eta \). Productivity shock is specified as AR(1) as shown in eqn(3.45), where \( v^* \) denotes the normalized steady state level of productivity.

3.4 Simulation

3.4.1 Calibration

Let us start with the entrepreneurs’ technology. I assume a uniform distribution \([0, 2]\) for liquidity shocks as a benchmark. This distribution implies that the initial unit of investment requires the same amount of working capital expenses as the mean.\(^{38}\) Following a standard RBC environment, consumption goods are converted into capital via one-to-one transformation technology. That is, total expected return from unit investment \( = \omega_1 \Phi(\omega_1) \) is set at unity so that the technology of the entrepreneurial sector as a whole is one-to-one transformation in the presence of perfect information. Note that even in the case with symmetric information, fraction \( 1 - \Phi(\omega_1) \) of investment is abandoned, because of liquidity shocks. Given the uniform distribution, this one-to-one technology gives us \( \omega_1 \) set equal to 1.414. With the imperfect information in the economy, this technology does not assure the full return, since a certain portion of the return disappears due to the agency cost. In this sense, the unpledgeable part of the profit, which is given to the entrepreneur as the

\(^{38}\) As for this mean (or the upper bound) of the uniform distribution, I will examine alternative values in the following subsection for sensitivity analysis.
minimum share by incentive compatible constraint, is the most important parameter to calibrate. As Holmström and Tirole (1998) argue in their paper, where there is no moral hazard in the capital production process \((B = 0 \text{ or } \omega_1 = \omega_0, \text{ equivalently})\), entrepreneurs do not demand any liquidity, since the moral hazard is the essential motivation for advance financing. It should be emphasized that in a special case of my DGE model, where \(\omega_1\) is set equal to \(\omega_0\) (the entrepreneur’s share of the profit is zero), my DGE model collapses to the standard RBC model.\(^{39}\) Here, as a tentative value, I set \(\omega_0/\omega_1 = 0.75\), which implies 25% of the profit is given to the entrepreneur on average. This value of 25% is purely ad-hoc, so I will examine how the model is sensitive to this value later. Given these parameter settings, the ex-post capital production out of unit input \((= \omega_1 \Phi(\varphi))\) is 0.987 in the steady state, which implies about 1.2% of resource is lost during the capital production process as the result of agency cost.

For most of the other parameters and functional forms, I followed Carlstrom and Fuerst (1998). The consumers’ utility is additively separable in consumption and labor, such that \(U(c_t, l_t) = \left( c_t^{1-\theta} / 1 - \theta \right) + \mu \left( 1 - l_t \right) \), where \(\theta\) is set at 1.5. As for \(\mu\), it is chosen so that steady state labor supply is 0.3. Consumption goods production is Cobb-Douglas, such that \(F(K_t, L_t, H_t) = v_t K_t^{\alpha} L_t^{1-\alpha - \alpha'} H_t^{\alpha'}\), where \(\alpha\) is 0.3 and \(\alpha'\) is 0.01. Discount rates are \(\beta = 0.99\) and \(\gamma = 0.95\).

I am left with only two parameters: \(\eta\) which represents the proportion of entrepreneurs population; and \(\sigma\), the AR(1) coefficient for productivity shock. \(\eta\) as a

\(^{39}\)This is also a very similar point to Carlstrom and Fuerst (1997). In their model, if monitoring cost is set at zero, which implies no agency cost, their model collapses to the standard RBC model either.
tentative value is set at 0.3, which implies that 30% of the population is entrepreneurs
\( \sigma \) is chosen at 0.9.

3.4.2 Simulation results

Impulse response to a productivity shock

The solution technique that I utilize here is the standard undetermined
coeicient method. (the eigen decomposition method) Simulations are based on the
log-linearized model around the steady state of the non-stochastic part of the system.

I report simulation results in Figure 3.3-3.5. The figures show the response of my
“liquidity model” (denoted “LI” model hereafter) to a positive productivity shock.

For comparative purposes, the impulse response of the standard RBC model\(^{40}\) is presented in each panel. The notable feature of the LI model is a hump-shaped
response of output as shown in the upper panel in Figure 3.3. This is a sharp contrast
to the familiar RBC dynamics, that is, investment and output jump up on impact
and begin to decrease immediately. Cogley and Nason (1993, 1995) demonstrate the
dynamics of output (also labor hours and investment) of the RBC model inherited
from the auto-correlation structure of the productivity shock. Since the productivity
shock is assumed to be AR(1), the output dynamics of the RBC model appears
with sharp rise followed by monotone decline accordingly. The key variable which is
generating this contrast between two models is the behavior of the entrepreneur’s net
worth. Recall that the entrepreneur’s net worth is a combination of the entrepreneur’s
capital and wage/rental income. Although her income can jump to its stable path,
her capital cannot, since it is a state variable. Because of this, the initial response

\(^{40}\)Recall that the RBC model is a special case of the LI model where parameters are chosen such
that \( \omega_1 = \omega_0 \). This is a perfect information environment.
of net worth is limited. Similarly, the entrepreneur’s investment is a function of capital price and net worth, which directly means that investment is driven to jump by initial increase in capital price, while such jump is limited, since net worth is nearly fixed on impact. Cogley and Nason (1995) and Carlstrom and Fuerst (1997) report that output dynamics observed in actual U.S. time series data is consistent with this hump-shaped behavior. Economic intuition behind this hump-shape is as follows.

At an initial shock of increased productivity, the firm finds that her investment is now more profitable, but actual investment expenditure does not rise much until she can accumulate a certain amount of cash-flow. Without sufficient internal funds the required return rate on externally raised funds still remains high, which limits the increased profitability of investment. Essentially this is because investment needs a down payment as shown in section 3.2 due to the imperfect information that is given. Especially, let us compare the initial responses of investment and the capital price. The lower two panels in Figure 3.3 show that from periods 0 to 2, investment starts to boom gradually according to the increase in her net worth (or accumulated profit), while capital price has already started to fall. Since capital price is the proxy of profitability, my simulation results suggest that profitability and investment can move in opposite directions. This implication is consistent with many empirical studies reporting the poor explanatory power of Tobin’s Q in estimating investment function, since naive Q theory predicts a one-to-one relationship between profitability and investment. Among many empirical studies on firms’ investment, a recent work by Lamont (2000) provides an insightful empirical fact on the relationship between actual investment and profitability. According to the paper, U.S. data shows negative

\footnote{Among them, see Fazzari, Hubbard, and Petersen (1988) or Hoshi, Kasyhap, and Sharfstein (1991) for example.}
contemporaneous correlation between investment and current stock return, mainly because of investment lag. Lamont (2000) presents the empirical evidence that while actual investment expenditure responds to stock return with lags, their investment plan (based on survey data) reveals its positive contemporaneous correlation with stock return. I cannot specify the reason behind this lagged actual investment. However, potentially, the empirical evidence presented by Lamont (2000) seems to support my simulation results.

Another interest is the dynamic behavior of corporate demand for liquidity. The lower two panels in Figure 3.4 present the impulse response of corporate liquidity demand and the degree of liquidity-dependence. On impact of positive productivity shock, the capital price sharply rises as similarly observed in Tobin’s Q theory. Higher capital price implies that the entrepreneur’s investment is more profitable. This profitable environment reduces marginal benefit for holding liquidity, and thus the degree of liquidity-dependence (liquidity demand divided by investment) falls. The reason here should be emphasized. Firms tend to find more profitable investment projects in booms, and thus they do not need to rely on credit lines from a financial intermediary to withstand liquidity shocks. The lowest panel in Figure 3.4 reflects this lower marginal benefit for holding liquidity. On the other hand, liquidity demand (degree of liquidity-dependence multiplied by investment volume) itself shows the net effect of the fall in degree of liquidity dependence and increase in investment. Under my calibrated parameter settings, corporate liquidity demand stays almost still on impact, since these two opposite effects approximately cancel each other out. One quarter later, it starts to go up, since the latter effect, increase in investment, dominates the former effect.
Impulse response to a wealth shock

Wealth shock introduced here is a one time transfer of unit wealth from consumers to entrepreneurs. This experiment is insightful in understanding the nature of agency cost models. Recall that in standard RBC models, where information is complete and thus corporate finance is a trivial matter for business cycles, a transfer of wealth from one agent to another does not cause any real effect in the economy. On the other hand, in agency cost models, transfer of wealth among different agents induces a non-trivial real effect on a firm’s investment. This is because quantity of the internal funds of entrepreneurs (note that this is the firm’s net worth in my model here) plays a significant role in agency cost models. Figure 3.6-3.7 shows the result of this wealth shock simulation.

As can be seen in the figure, one time transfer of wealth from entrepreneurs to consumers causes a down turn of output, investment, and aggregate consumption in spite of the temporary increase in the household’s consumption. This implies that even if the aggregate net worth endowed in the economy as a whole stays constant, a re-distribution of wealth from firms to households can cause a recession, where agency cost is a non-trivial issue in that economy. In fact, this outcome is neither new nor surprising in the literature. Similar results of this wealth re-distribution are observed in other agency cost models such as Carlstrom and Fuerst (1997) and Kiyotaki and Moore (1997).

Sensitivity analysis

The most controversial parameters here would be the mean of the distribution of liquidity shock in terms of investment volume and the entrepreneur’s share
of profit. As for the mean of the liquidity shock, the amount of working capital expenses that are demanded on average for a unit of investment project should be empirically investigated. However, as shown in the upper panel in Figure 3.8, the output dynamics is pretty robust for various means of liquidity shocks. Essentially, this is because whatever distribution is expected, the optimal cut-off level of liquidity shock is mainly determined by the entrepreneur’s technology itself. In this sense, the profit share of the entrepreneur \( (1 - \omega_0/\omega_1) \) dramatically changes output dynamics. For instance, as mentioned in the previous subsection, with \( \omega_0/\omega_1 = 1 \), the dynamics of my model is the same as that of standard RBC, namely, the hump-shaped response of output and investment disappears. As can be seen in the lower panel of Figure 3.8, the hump-shaped dynamics requires a certain magnitude of agency cost to be generated. It seems that 10% for the entrepreneur’s share of profit \( (\omega_0/\omega_1 = 0.9) \) is sufficient for significant hump-shaped dynamics.

3.5 Discussions

3.5.1 Some empirical facts

As I have discussed in the previous section, the hump-shaped dynamics of output and investment are empirically verified by preceding studies. Here in this section, let us consider the empirical validity of the cyclical pattern of corporate demand for liquidity and the degree of liquidity dependence predicted by my model.

Figure 3.9 shows the Japanese data. In the upper panel of Figure 3.9, the solid line depicts bank loans for working capital expenses.\(^{42}\)

\(^{42}\) All the data series (except for bank loans for working capital expenses/investment for Japanese data) are detrended by using the HP filter with smoothing parameter =1000. As for bank loans for working capital divided by investment (for Japanese data), it is not detrended, since it is stationary.
The data can be a reasonable proxy for the liquidity provided via the financial intermediary. The dashed line is aggregate output. My model’s prediction on the relationship of these two variables is moderate positive correlation. This is because bank loan volume is the result of the net effect of the counter-cyclical degree of liquidity dependence and pro-cyclical investment expenditure. Actual correlation calculated by the data turns out to be 0.2. The lower panel reveals more clear cyclical patterns. As can be seen on the panel, the degree of liquidity dependence, which is measured as bank loans for working capital expenses divided by investment here, is apparently negatively correlated with output. Actual correlation within the sample period is -0.6.

Let us take a look at the U.S. data. The upper panel on Figure 3.10 shows commercial bank loans\(^{43}\) (the solid line) and output. (the dashed line)

Actual correlation of these two variables are 0.41 during the sample period. Similarly, in the lower panel of the same figure, the solid line depicts a proxy for the degree of liquidity dependence, measured as commercial bank loans divided by investment. Again, the panel reveals a clear counter-cyclical pattern of the degree of liquidity dependence. Actual correlation for U.S. data turns out to be -0.5. It should be noted that I am not claiming that most of the business cycle dynamics is driven by productivity shocks. Nonetheless, these casual observations on actual correlations seem to support my simulation results presented in the previous section.

\(^{43}\)As for U.S. data, I could not find the exact data for bank loans for working capital expenses. Instead, I show here total bank loans, which contain both the loans for fixed business investment and other kinds of long-term financing.
3.5.2 Relation to the *Lending View* and other studies

One of the major predictions of *Lending View* theory is that firms will be more bank loan-dependent in recessions. According to the lending view, this cyclical pattern of firms’ financing structure can be explained as follows. During recessions, investment projects are not so profitable on average as in booms; firms cut some of their projects whose returns do not exceed their financing cost. Since direct financing, such as equity finance, are usually more costly than loans from an intermediary, the result of cutting those unprofitable investment projects is to increase the ratio of bank loan financing in total financing. The mechanism which governs the cyclical fluctuations of financial structure in my model is slightly different from the standard lending view, but as I have already seen in the previous subsection, both the lending view and our model yield very similar predictions. The similarity is that in my model, firms become more liquidity-dependent in recessions, because lower profitability of their investment projects raises the marginal benefit of holding liquidity, while firms in the lending view demand more bank loan financing, because the marginal cost of obtaining it is lower. The difference is that the lending view considers asset substitution such as from bank loan to equity financing. This is in contrast to my model where the firm’s choice is whether to demand credit lines from banks or not. Let us see this point more precisely. Differentiating the first order condition (eqn(3.16)) in the financial contract to obtain,

\[
\frac{\partial \bar{\omega}}{\partial q} = -\frac{\int_0^\infty \Phi(\omega) d\omega}{q\Phi(\bar{\omega})} \leq 0
\]

which directly implies that the optimal credit line offered by the financial intermediary will decrease when investment projects become more profitable. This relation in
eqn(3.46) is the source of the negative correlation between capital price (investment profitability) and the firm’s degree of liquidity-dependence $x$ in eqn(3.20) as discussed so far. This can be verified by the following relation,

$$
\frac{\partial x}{\partial q} = \frac{\partial \omega}{\partial q} \frac{\partial x}{\partial \omega} = \frac{\partial \omega}{\partial q} \frac{\partial}{\partial \omega} \left( \int_{\omega_0}^{\omega_1} \omega \phi(\omega) d\omega \right) = \frac{\partial \omega}{\partial q} \frac{\omega_1 \phi(\omega) \int_{\omega_0}^{\omega_1} (\omega - \omega) \phi(\omega) d\omega}{(\Phi(\omega) \omega_1)^2} \leq 0.
$$

Basically, raising credit lines from the intermediary has a trade-off. A higher credit line is beneficial in withstanding larger liquidity shocks, while it reduces the investment profitability and thus the volume of investment. When investment projects are highly profitable on average, both firms and consumers are willing to cut credit lines, since they have larger investment volumes. Because of this mechanism, I find that the degree of liquidity dependence tends to be counter-cyclical in the simulation results, which is consistent with the actual data observation as shown in Figure 3.9 and 3.10.

Note that it is this point that my DGE model has an advantage over others with a different financial structure such as costly state verification. It is well known that some DGE models with agency costs tend to show anomalies regarding the cyclical pattern of financial aspects of the economy in spite of their superior performance in replicating the dynamics of the real variables such as investment and output.\(^{44}\) By adapting HT type financial contract instead of the costly state verification, my DGE model yields both a theoretically and empirically reasonable cyclical pattern.

\(^{44}\)For example, one anomaly observed in Carlstrom and Fuerst (1997) model is pro-cyclical risk premium, which does not appear in my model.
of a firm's financing structure, maintaining the auto-correlation dynamics of the real variables.

Another prediction of the lending view is that smaller firms, which are usually considered to be confronted with higher financing costs, are more bank loan-dependent. The HT model presents some similarity as for this point either. Eqn(3.16) implies that credit line given to a firm does not depend on either \( \omega_0 \) or \( \omega_1 \), but solely on \( q \). Consequently, the optimal credit line in terms of the firm’s NPV, \( \bar{\omega}/\omega_1 \) (or in terms of pledgeable value, \( \bar{\omega}/\omega_0 \)) is higher for a firm with lower \( \omega_1 \) (or \( \omega_0 \)). In other words, a firm with larger NPV tends to demand less liquidity than smaller firms in terms of their NPVs. This is a direct result from insurance provided by the financial intermediary. In addition, changes in \( \bar{\omega} \) with respect to profitability, that is, \( \partial \bar{\omega} / \partial q (\leq 0) \) is again constant over \( \omega_1 \), which can be interpreted as that a firm’s (maximum) liquidity demand, \( \bar{\omega}i \) divided by her NPV, \( \omega_1 i \) tends to be more sensitive to the profitability for smaller firms. Namely,

\[
\frac{d (\bar{\omega}/\omega_1^B)}{dq} > \frac{d (\bar{\omega}/\omega_1^A)}{dq}, \quad \text{for} \quad \omega_1^A > \omega_1^B. \tag{3.47}
\]

This relation is consistent with a common observation that smaller firms tend to fall in liquidity shortage during recessions (interpreted as periods when \( q \) is lower), in the sense that credit lines should be intensively allocated to smaller firms when the economy is in a recession.\(^{45}\)

\(^{45}\)However, general consequence of heterogeneity in firms’ NPV is not examined in my DGE model. My DGE model allows heterogeneity only for levels of net worth.
3.6 Concluding Remarks

Although the HT model is highly stylized, it requires a much less specific environment than it appears. Recall the calibration in section 3.4. I need to specify only two parameters and one distribution for entrepreneurial technology, namely $\omega_1$, $\omega_0$ and $\Phi$. Actually, as long as we are sticking to one-to-one transformation technology in capital production, $\omega_1 \Phi (\omega_1)$ must be set at one, and therefore only one parameter and one distribution need to be calibrated. This implies that the hump-shaped dynamics of output is robust to a broad class of models in which the investment process is characterized by a leakage due to moral hazard or imperfect information. My guess is that any reasonable theory which yields eqn (3.18), $i = k(q)n$ type investment function is consistent with hump-shaped dynamics of output. However, of course, this must await further research on this issue to be verified.

Another robust result of the model is that corporate demand for liquidity is procyclical, while the degree of liquidity-dependence is counter-cyclical. These predictions are consistent with empirical evidence presented in lending view literatures.

A potential flaw of my model in this essay is that it is lacking in the ability to analyze the role of public-supplied liquidity. A version of the HT model that provides a rationale for government-supplied liquidity has aggregate uncertainty present in the economy. Intuitively, the role of the government is to eliminate the aggregate uncertainty to achieve at least the second best outcome on the production side of the economy. However, our model is constructed on the assumption of no aggregate uncertainty in the capital production process.\textsuperscript{46} I need this assumption to

\textsuperscript{46}Note that productivity shock is not an aggregate uncertainty in this sense here. Since productivity shock is realized before the financial contract is implemented, and thus the current level
maintain modeling consistency. Especially in the presence of aggregate uncertainty, I cannot separate intra-period financial contract and the rest of the general equilibrium any more. Nonetheless, incorporating aggregate uncertainty and hence the role of government-supplied liquidity are potentially interesting, because of the following two advantages. One is that such a model can provide much richer implications on economic welfare. The other is that it allows us to analyze the business cycle patterns of liquidity premia on government-supplied securities, such as T-bills. For these purposes, we must await further research with this extension.

of productivity shock is in everyone’s information set when the financial contract is about to be implemented.
Figure 3.3: Impulse response to productivity shock 1
Figure 3.4: Impulse response to productivity shock 2
Figure 3.5: Impulse response to productivity shock 3
Figure 3.6: Impulse response to wealth shock 1
Figure 3.7: Impulse response to wealth shock 2
Figure 3.8: Sensitivity analysis
Figure 3.9: Liquidity demand and degree of liquidity dependence: Japan
Figure 3.10: Liquidity demand and degree of liquidity dependence: US
CHAPTER 4

DOES FINANCIAL SECTOR DISTRESS AFFECT BUSINESS CYCLES? EVIDENCE FROM JAPAN

4.1 Introduction

The recent stagnation of the Japanese economy and the East Asian crisis of the 90s have attracted renewed attention to the problems associated with financial sector distress and its consequences on economic growth. The events that led to the financial crisis have inspired many empirical researchers and theorists to develop both theoretical and empirical techniques to analyze the crisis. However, according to previous empirical studies of the Japanese economy, the extent to which such disruption of financial intermediation has contributed to the low economic growth of the Japanese economy in the 90s has not yet been established conclusively. This issue has been controversial among both academics and policy makers. Before reviewing the current state of the controversy, it will be instructive to explain the motivation of this essay and to classify the preceding studies according to their methodologies. The first group applies (1) the time series econometric technique based on aggregate data. Most of them employ vector auto-regression (VAR) or vector error correction (VEC) models. The second group exploits (2) the standard econometric approach based
on micro/panel data to test certain hypotheses derived from economic theory. The third group applies (3) the numerical simulation technique to calibrated dynamic general equilibrium (DGE) models. These three methodologies have advantages and disadvantages that will be discussed later in this essay. This essay takes a complementary approach that combines the techniques identified in 2 and 3 above to provide empirical evidence of the causality in which a distressed financial sector played a non-trivial role in creating the Lost Decade of Japan from the early stage of the 90s. In other words, the disruption of financial intermediation is not an unimportant consequence of exogenous factors such as the slowdown of productivity growth or a liquidity trap due to the zero lower bound on nominal interest rates.

In fact, the substantial body of the empirical studies on this issue admit, at least partially, a non-trivial role of the imperfection of the financial/capital markets in explaining the poor performance of the Japanese economy. Using the time-series (VAR/VEC) approach (1), Brunner and Kamin (1995), Bayoumi (1999), Bayoumi and Morsink (1999), Kwon (1999) and Motonishi and Yoshikawa (2000) share the view that financial sector distress significantly affected the economic growth of the Japanese economy, although their estimates vary both in length and magnitude. Motonishi and Yoshikawa (2000) argue that the financial distress-induced recession was relevant only after 1997, finding little evidence of it earlier in the 90s. By contrast, Bayoumi (1999) concludes from his historical decomposition of time series data that the decline of asset/real estate prices contributed significantly to the stagnation all through the 90s. Similarly, most of the studies in group 2 provide empirical evidence for a credit crunch in the 90s, but some of them claim that the statistical evidence for a credit crunch is strongest in the late 90s and that it seems to have had less effect in the early
Hayashi and Prescott (2002) offer the most unusual view of the studies from groups 2 and 3. They conclude that most of the downturn of the Japanese economy can be attributed to a slowdown in exogenous productivity growth, and thus that financial sector distress is irrelevant in explaining the stagnation.

That the views derived from VAR/VEC analysis are so different is not surprising, since it is well known that VAR/VEC models with many variables are highly sensitive to how they are specified, such as what lag length is chosen, what the order of the variables is and how the “structural” errors are handled. In particular, those VAR models must be identified by making assumptions, which may be debatable. As for the studies in group 2, there is a clear tendency for evidence to be found for a significant role of financial distress. The reason is that they test specific hypothesis about binding borrowing constraints\(^4^8\) such that contraction in bank lending pushes marginal investment projects abandoned via binding borrowing constraints. This view of the role of borrowing constraints gives rise to two hypotheses. The first hypothesis is that a capital crunch\(^4^9\) occurred. As discussed in Peek and Rosengren (1992), the capital crunch hypothesis focuses on the lenders’ balance sheet position as a factor which induces the contraction of bank lending. On the other hand, a credit crunch hypothesis focuses on borrowers’ net worth as a key factor influencing the fluctuation of bank lending. The capital crunch hypothesis is quite specific and most likely to be rejected by a substantial body of preceding studies.

\(^4^7\)See Gibson (1995) and Woo (1999) for examples.

\(^4^8\)See Woo (1999) and Hayashi and Prescott (2002) for those who reject the existence of major credit/capital crunch in the 90s. On the other hand, Kato, Ui and Watanabe (1999) and Kang and Stulz (1999) find evidence of negative effect from financial sector distress. Neither is based on the credit/capital crunch hypothesis specifically.

\(^4^9\)Some early works do not distinguish between a capital crunch and a credit crunch. To avoid confusion, I use the two terms for different phenomena according to the description in the text.
I claim that the credit crunch hypothesis is still too narrow to conclude that the disruption of financial intermediation did not affect the corporate investment and output fluctuation in the 90s. I propose a new view on this issue which can be supported both empirically and theoretically. First, from the theoretical point of view, it is easy to claim that rejection of the credit constraint hypothesis does not necessarily imply that financial sector distress is irrelevant to fluctuation in output and investment. Holmström and Tirole (1998) demonstrated that to achieve production efficiency, an efficient allocation of liquidity is necessary. In turn, excess lending/liquidity can be just as easily a source of inefficiency as a liquidity shortage. Essentially, given the presence of asymmetric information in the capital market, financial intermediaries must achieve risk sharing in the short-term lending market by efficiently allocating liquidity (credit lines, more specifically). Here emerges a pitfall of preceding empirical studies on the borrowing constraint. Suppose, a credit crunch occurs in some part of an economy, while excess lending occurs simultaneously in another part. By construction, then, there occurs an inefficient allocation of liquidity. Regression results based on the data of this economy would not necessarily show, overall, any evidence of a credit crunch due to the existence of the part with excess lending. In spite of the empirical suggestion, as Holmström and Tirole (1998) demonstrated, we cannot conclude from the standpoint of efficient risk sharing that in this economy the financial sector is doing perfectly. Further, the model of “forbearance lending” presented in Berglof and Roland (1997)\(^{50}\) is another example of a mechanism by which excess lending, the inverse of a credit crunch, could also increase the inefficiency of the economy.

\(^{50}\)A simplified application of their model to bank lending is provided by Kobayashi and Sekine (2002).
by bailing out non-performing corporate firms. The intuition of the model is straightforward. When the liquidation value of firms is low, (due to the decline in real estate prices, for instance) a Nash equilibrium could exist allowing inefficient continuation of the firm. The reason is that a lower liquidation value magnifies agency cost. The important insight of the model is that such an inefficient continuation problem, say _forbearance lending_, is most likely to be resolved by an optimally designed contract when firms’ liquidation value is sufficiently high.

This excess/forbearance lending view is relatively new and has not been much explored empirically so far. A pioneering work by Peek and Rosengren (2002) observes empirical evidence based on micro data from 1994 to 98 that the Japanese corporate firms with lower ROAs have attracted more bank lending and suggests the existence of inefficient forbearance lending. Additionally, it is easily recognized by casual observation that commercial bank lending to the non-manufacturing sector was subsequently increasing until 97, while that to the manufacturing sector was decreasing. Much empirical research points out that productivity of the non-manufacturing sector is relatively lower in Japan and indeed, it is non-manufacturing sector that has been experiencing more severe stagnation than the others all through the 90s. This observation seems consistent with the forbearance lending view as the result of inefficient financial intermediation.

Generally, one cannot tell much about whether the bank lending /liquidity is allocated optimally by looking at the correlation of lending with investment.\textsuperscript{51} My

\textsuperscript{51}In fact, it is easy to show that even in an explicitly credit-constrained economy, investment and lending are not necessarily highly correlated. I will demonstrate this point in section 4.3 based on a variant of the Credit Cycles model.
standpoint is as follows. All through the chapter, I pay much attention to the allocation of financial resources such as liquidity and net worth from the viewpoint of general equilibrium analysis. First, the fact is that Japanese macroeconomic data is more consistent with DGE models with agency cost than with standard RBC models; otherwise, it does not make any sense to focus on financial intermediation because of the classic Modigliani-Miller theorem. Hence it is at most the second-best efficiency that the economy could achieve in the presence of the agency cost. Given this observation, I detect the negative effect of financial sector distress, to some extent, on real output/firms’ investment in the 90s via increased agency costs. Note that this type of inefficiency comes up within the second-best economy, which means that second-best allocation is maintained by efficient risk sharing in the short-term bank lending market. Further, in this essay I test the side-condition implied by efficient financial intermediation based on synthetic panel data of the Japanese corporate sector. Based on the regression result, I argue that risk sharing was less effective in the 90s than in the 80s. Basically, I propose two types of inefficiency, namely, one in the second-best economy and the other outside of it. Note again that any increase in agency cost falls in the first criteria, while failure in achieving risk sharing is classified in the latter group. I detect that both of these types of inefficiency are relevant to the stagnation of the Japanese economy in the 90s. One might wonder that the implication of my finding is slightly shaky. In fact, I do not explicitly insist that financial sector distress has caused the slump of the Japanese economy. Rather, what I present here is the evidence of capital market imperfection and increased inefficiency in the financial intermediation in the 90s. Generally, standard economic theory does not support such a view that the slowdown in productivity growth (or zero bound
on nominal interest rates) would result in increased inefficiency in financial interme-
diation. Given this, let us ask a question: does any economic policy which promotes
productivity growth ameliorate the functioning of disrupted financial intermediation?
Maybe yes, maybe no. However, clearly any economic reform/policy which directly
fixes the disruption of the financial intermediation would raise the economic welfare
via increased efficiency in private firms’ investment opportunity.

This chapter is organized as follows. The next section introduces the theoretical
background of my argument, which is mostly taken from Holmström and Tirole
(1998). After reviewing potential flaws in preceding empirical studies, I will revisit
the central issue from a general equilibrium perspective. In section 4.3, I verify that
asymmetric information or other kinds of financial market imperfections are consist-
tent with Japanese macroeconomic data by comparing simulation results generated
by RBC and other DGE models with agency cost. I will provide evidence that agency
cost (recall that it is a “type 1” inefficiency) increased soon after the end of the bubble
period. Section 4.4 presents the empirical evidence of the violation of risk sharing,
\textit{i.e.}, increased inefficiency of “type 2.” I will conclude the essay in section 4.5.

4.2 Theoretical background; Agency cost models

A typical regression to examine the credit crunch hypothesis is as follows,$^{52}$
\[ \Delta I_t = c_0 + c_1 \Delta B_t \]  \hspace{1cm} (4.1)
where $I_t$ and $B_t$ denote a firm’s investment and bank lending respectively. Investment
can be substituted by output in some cases. This type of regression was originally

$^{52}$See Hayashi and Prescott (2002) for example.
developed in the *lending view* literature to test for the cash-flow sensitivity. Typically, the authors in this literature divide sample observations in two groups according to the degree of bank-dependence or size of the firms. Then if a significant difference is found between coefficients on cash-flow, they regard it as an evidence of the credit channel of monetary policy. Eqn (4.1) type credit crunch regression is in similar spirit. Those empirical researchers claim that if the credit crunch is a dominant phenomenon, changes in bank lending to the firm will be highly correlated with the firms’ investment due to the binding borrowing constraint. Nonetheless, the specification of eqn (4.1) is not necessarily consistent with predictions based on the rigorous model of agency problems between lenders and borrowers. I will discuss the validity of this type of regression from a general equilibrium perspective, as a testing method for the effect of financial intermediation’s distress on firms’ investment.

### 4.2.1 Investment function in DGE models with imperfect capital market

First, I review the mechanism of DGE models with an imperfect capital market and demonstrate that in those models, investment and bank lending are not necessarily highly correlated. In a typical DGE model with asymmetric information in the capital market, we find the following type of the break-even condition of the capital market.\(^{53}\)

\[
i_t - n_t = h(q_t) i_t
\]

which can be rewritten as,

\[
i_t = \frac{1}{1 - h(q_t)} n_t
\]

\(^{53}\)Rigorously speaking, eqn (4.3) is not an investment function, but it determines the input of consumption goods to produce capital.
where \( i, n, \) and \( q \) denote initial input of consumption goods, net worth of the firm and price of capital,\(^{54}\) respectively. The left hand side of eqn (4.3) represents initial input from outside investors (households, in most cases). The right hand side of the equation shows the return of the project where \( h(q_t) < 1 \). I do not specify what lies behind the \( h(q_t) \), since several different types of agency problems result in the similar function. Essentially, the condition shows the equivalence between input and its return, a break-even condition of the capital market where the opportunity cost is unity. Basically, asymmetric information creates agency cost, which prevents firms from financing their expenses up to the maximum NPV of the project. This is the principal reason for \( h(q_t) < 1 \). In this type of capital market, I do not observe any capital crunch, since the lenders’ net worth does not play any role here. Further, note that at any given level of borrower’s net worth, the market breaks even, which implies the loan demand is not rationed in any sense.

Another type of DGE model with agency cost stems from inalienable human capital as discussed in Hart and Moore (1993). A well-known example of such a DGE model is the “Credit Cycle” model presented in Kiyotaki and Moore (1997). That model has the following capital accumulation function, which is interpreted as a variant of investment function,

\[
K_{t+1} = \frac{1}{q_{t+1} - q_{t+2}/R} n_t
\]

where \( K_t \) and \( R \) denote capital stock and interest rate on the firm’s debt, respectively. Notice the similarity among equations (4.3) and (4.4). In both types of the model, the common feature is a downpayment for investment, which means a unit increase

\(^{54}\)Note that it is not Tobin’s Q, which represents the current and future profitability of the firm. Rather, the \( q \) here reflects the information of only current profitability.
in borrowers’ net worth is amplified by the equity multiplier which is the coefficient term on net worth both in eqn (4.3) and eqn (4.4). The key factor in understanding those imperfect capital market models is not the borrowing constraints per se, but the role of borrowers’ net worth. As can be verified by eqn (4.3) and (4.4), a firm’s investment depends on the level of its net worth as well as profitability of the project. This is in sharp contrast with the standard RBC model or Tobin’s $Q$ theory. Hence, a straightforward testing methodology should be on the role of borrowers’ net worth, rather than the binding borrowing constraints.

Why are the borrowing constraints of eqn (4.1) less important? Consider an explicit borrowing constraint embedded in Kiyotaki and Moore’s (1997) model. That is,

$$R \cdot B_t \leq q_{t+1}K_t$$  \hspace{1cm} (4.5)

where $B_t$ and $R$ denote debt and the gross interest rate on the debt, respectively. We can regard the debt here as bank lending for convenience. The theory predicts that bank lending is relevant to the borrower’s net worth,\footnote{In the Kiyotaki-Moore model, $q_{t+1}K_t$ is closely related to the borrower’s net worth.} which does not necessarily yield high correlation with investment and bank lending. Figure 4.1 shows a simulated fluctuation of firm’s investment and debt using the Kiyotaki-Moore model, where a borrowing constraint of the form eqn (4.5), is always binding.

Clearly, the two variables are not perfectly correlated, and peaks and troughs of the two do not coincide everywhere. I conducted simulations 100 times with 1,000 periods for each run. Table 4.1 reports the average correlation and $t$-statistics of the OLS regression based on the Monte-Carlo simulation.\footnote{See Appendix A for the details of the Monte-Carlo simulation. Matlab code for the simulation is available from the author on request.}
First column in Table 4.1 indicates the AR(1) coefficient (parameter indicating persistence) of the stochastic disturbance. Again, recall that the borrowing constraint eqn (4.5), is always binding in the model which generates the simulated data in figure 4.1. Nonetheless, a regression of the form eqn (4.1) does not detect the significant effect of bank lending on a firm’s investment due to simultaneity and other econometric problems. The misspecification problem is not negligible, since the agency cost model’s key prediction is the close linkage of borrowers’ net worth (note that it is a stock/state variable) to a firm’s investment. The bottom line is that one cannot rely on regressions of the form eqn (4.1) to test the general prediction of imperfect capital market models, but it is only valid to know whether the specific capital crunch hypothesis is relevant or not. Generally, it is not easy to detect the
<table>
<thead>
<tr>
<th>$t$-statistics on $c_1$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.9$</td>
<td>0.140</td>
</tr>
<tr>
<td>$\sigma = 0.6$</td>
<td>0.140</td>
</tr>
<tr>
<td>$\sigma = 0.2$</td>
<td>0.141</td>
</tr>
</tbody>
</table>

Table 4.1: Monte-Carlo simulation results

overall influence of financial market imperfections on real economic activity, such as output and investment, by running a single equation regression. Instead, we researchers need more careful analysis from multiple aspects.

4.2.2 The model for corporate liquidity demand; Holmström and Tirole (1998)

Another critique of credit crunch regressions is that they ignore the role of short-term lending, which is not intended to defray fixed investment expenditure, but rather to finance working capital. Before discussing the theoretical issues, let us take a brief look at the actual data. Figure 4.2 shows the aggregate data of the bank lending for working capital expenses over total bank loans.

On average, lending for working capital amounts to twice as much as for fixed investment. Hence, it is not surprising that total bank lending is not highly correlated with investment, since a substantial portion of the lending is carried out to finance working capital, not fixed investment expenditure. Essentially, it is fairly misleading to discuss the relation between bank lending and a firm’s behavior without understanding the role of short-term lending for working capital, namely, the firm’s demand for liquidity.
Figure 4.2: Loans for working capital expenses/total loans

This section introduces the theory for firms’ liquidity demand. My argument in this section and estimation in section 4.4 are mainly based on a variant of Holmström and Tirole’s (1998) model presented in Chapter 3.\textsuperscript{57} In the Holmström-Tirole (HT hereafter) model, corporate liquidity demand is motivated by moral hazard. Given a moral hazard, an entrepreneur must be given a minimum share of profit in order to be motivated. Because of this, the value of external claims on the firm is strictly less than the full value of the firm. The wedge between the full value of the firm and the external value of the firm prevents it from financing all projects that have positive net present value. This implies that liquidity shocks could force the firm to terminate a project midstream, even though the project has a positive continuation value. To

\textsuperscript{57}However, as I will discuss later, the empirical evidence from macro data is not only consistent with the specific HT model, but many other DGE models with agency cost/imperfect capital market.
avoid such risks, the firm demands credit lines or flexible short-term lending provided by financial intermediaries.

Here I present a brief review of a variant of the HT model introduced in Chapter 3 to derive testable predictions of the model. Suppose an economy where there exists a finite number (= \( m \)) of industries. In each industry, I assume that there exists a continuum of firms with a unit mass. First, let \( i \) be a firm’s initial input to acquire newly produced capital. In the absence of liquidity shocks, this project will yield \( \omega^0_i \) of capital to outside investors. However, in the model here, at some point during the investment project, an additional uncertain amount \( \omega_i \) (in terms of capital goods) of funds is necessary to cover working capital expenses and other cash needs. The liquidity shock \( \omega \) is distributed according to the cumulative distribution function \( \Phi (\omega) \) with a density \( \phi (\omega) \). If \( \omega_i \) is paid, the project continues and a newly produced capital goods, \((\omega^0 - \omega) i\), are sold at the end of the period. We allow the possibility that the distribution of liquidity shock is different from industry to industry, but each firm in the same industry is assumed to be confronted by the same distribution of liquidity shock. For notational purposes, let subscript \( j \) denote the \( j \)th industry and subscript \( t \) denote time index. For any firm in the \( j \)th industry, the following break-even condition for outside investors holds,

\[
i_{j,t} - n_{j,t} = q_{j,t} i_{j,t} \left[ \omega^0_j - \int_{0}^{\omega_{j,t}} \omega \phi_j (\omega) d\omega \right]
\]  

(4.6)

where \( n_{j,t} \) and \( \omega_{j,t} \) stand for internal fund/net worth of the firm and optimal cut-off level of liquidity shock. Note that a firm can withstand a liquidity shock as large as \( \omega_{j,t} i_{j,t} \), which implies the bank is willing to offer short-term lending up to this amount. Given this optimal choice of \( \omega_{j,t} \) (it is determined as a part of the optimal contract between firm and bank), the right hand side of eqn (4.6) shows the expected
return from the initial investment \( i_{j,t} - n_{j,t} \) on the left hand side. \( q_{j,t} \) denotes the profitability\(^{58}\) of the investment in \( j \)th industry at period \( t \). If we assume that there is a continuum of firms with a unit mass in each industry, the newly produced capital goods of the \( j \)th industry \( I_{j,t} \) will be equal to \( I_{j,t} = q_{j,t} \omega_{j,t} \Phi_j (\tilde{w}_{j,t}) \) owing to the law of large number.

On the other hand, the first order condition for the optimal credit lines \( (= \tilde{w}_{j,t} i_{j,t}) \) from the bank is characterized by the following optimal condition, which is a part of the optimal contract between bank and firm.

\[
\int_0^{\tilde{w}_{j,t}} \Phi_j (\omega_t) \, d\omega_t = \frac{1}{q_{j,t}} \tag{4.7}
\]

Note that the optimal choice of \( \tilde{w}_{j,t} \) does not depend on any firm-specific parameter, which enables us to ignore a firm’s index from \( \tilde{w}_{j,t} \). Based on this relation, we can define an implicit function \( \tilde{w}_{j,t} = \psi_j (q_{j,t}) \). Note that \( \psi' = - \left( \int_0^{\tilde{w}_{j,t}} \Phi_j (\omega_t) \, d\omega_t \right) / q_{j,t} \Phi (\tilde{w}_{j,t}) = - (q_{j,t}^2 \Phi (\tilde{w}_{j,t}))^{-1} \leq 0 \) by the implicit function theorem. We can plug this function into \( \int_0^{\tilde{w}_{j,t}} \omega_t \phi_j (\omega_t) \, d\omega_t \) to define the conditional mean of \( \omega_t \), \( \theta_{j,t} = \theta(q_{j,t}) \), which is defined as follows:

\[
\theta_{j,t} = \int_0^{\psi_j(q_{j,t})} \omega_t \phi_j (\omega_t) \, d\omega_t \tag{4.8}
\]

Let us examine the nature of \( \theta_{j,t} \) for later purpose here.

\[
\frac{\partial \theta_{j,t}}{\partial q_{j,t}} = \frac{\partial \theta_{j,t}}{\partial \tilde{w}_{j,t}} \frac{\partial \tilde{w}_{j,t}}{\partial q_{j,t}} = \frac{\partial \theta_{j,t}}{\partial \tilde{w}_{j,t}} \psi_j' \tag{4.9}
\]

\[
= \tilde{w}_{j,t} \phi (\tilde{w}_{j,t}) \left( \frac{-1}{q_{j,t}^2 \Phi (\tilde{w}_{j,t})} \right) \tag{4.10}
\]

\[
\leq 0 \tag{4.11}
\]

\(^{58}\)Some additional interpretation for this \( q \) is possible, such as price of capital or opportunity cost for the investment. We are implicitly assuming the working capital expense is more costly when \( q \) is high, since realized liquidity shocks is measured in capital.
This implies that the conditional mean of the liquidity shock is negatively correlated with profitability. The economic intuition behind this relation is that a firm is willing to hold liquidity when her investment projects are not so profitable that marginal benefit for holding liquidity is higher. Inversely, when her projects are more profitable, she does not have to hold much liquidity so that the project could survive. Consequently, investment profitability $q_{j,t}$ and marginal benefit for holding liquidity are negatively correlated, which generates the key relation, $\partial \bar{\omega}_{j,t}/\partial q_{j,t} = \psi' \leq 0$.

Now the first estimation is to test the optimal choice of the short-term lending offered to the industry. If the optimal credit line is implemented within the industry, aggregate bank lending for working capital in the $j$th industry, $D_{j,t}$ is,

$$D_{j,t} = q_{j,t} i_{j,t} \int_{0}^{\bar{\omega}_{j,t}} \omega_{j,t} \phi_{j} (\omega_{j,t}) d\omega_{j,t} \tag{4.12}$$

$$= q_{j,t} i_{j,t} \theta_{j,t} \tag{4.13}$$

Divide this $D_{j,t}$ by aggregate investment of the industry to obtain the “degree of liquidity dependence” $d_{j,t}$. The theory predicts that this degree of liquidity dependence is solely determined by $q_{j,t}$ as shown here

$$d_{j,t} = \frac{D_{j,t}}{I_{j,t}}$$

$$= \frac{\int_{0}^{\bar{\omega}_{j,t}} \omega_{j,t} \phi_{j} (\omega_{j,t}) d\omega_{j,t}}{\omega_{j,t}^{0} \Phi_{j} (\bar{\omega}_{j,t})}$$

$$= \frac{\theta_{j,t}}{\omega_{j,t}^{0} \Phi_{j} (\bar{\omega}_{j,t})} \tag{4.14}$$

where $\bar{\omega}_{j,t} = \psi_{j} (q_{j,t}), \theta_{j,t} = \int_{0}^{\bar{\omega}_{j,t}} \omega_{t} \phi_{j} (\omega_{t}) d\omega_{t}$ and $I_{j,t} = q_{j,t} \omega_{j,t}^{0} i_{j,t} \Phi_{j} (\bar{\omega}_{j,t})$. As for the notation for investment, I must avoid a confusion. Note that I observe newly installed capital, $I_{j,t}$ for actual “investment” data, instead of $i_{t,j}$, which stands for
initial input to yield capital. In other words, observed investment \((I_{j,t})\) consists of only the projects that survived, whose liquidity shocks are smaller than \(\tilde{\omega}_{j,t}i_{j,t}\).

According to the model introduced in chapter 3, the degree of liquidity dependence \(d_{j,t}\) will be negatively correlated with \(q_{j,t}\), i.e., \(\partial d_{j,t}/\partial q_{j,t} \leq 0\). I can verify the relation here.

\[
\frac{\partial d_{j,t}}{\partial q_{j,t}} = \frac{\partial d_{j,t}}{\partial \tilde{\omega}_{j,t}} \frac{\partial \tilde{\omega}_{j,t}}{\partial q_{j,t}}
\]

\[
= \left( \frac{\partial \theta_j}{\partial \tilde{\omega}_{j,t}} \right) \omega_j^0 \Phi_j (\tilde{\omega}_{j,t}) - \omega_j^0 \phi_j (\tilde{\omega}_{j,t}) \theta_j \psi'
\]

\[
= \left( \tilde{\omega}_{j,t} \Phi_j - \int_0^{\tilde{\omega}_{j,t}} \omega_t \phi_j (\omega_t) d\omega_t \right) \frac{\phi_j (\tilde{\omega}_{j,t})}{\omega_j^0 (\Phi_j (\tilde{\omega}_{j,t}))^2} \theta_j \psi'
\]

\[
= - (q\Phi)^{-3} \frac{\phi}{\omega^0}
\]

\[
\leq 0.
\]

Since the degree of liquidity dependence \(d_{j,t}\) is observable, I can conduct a statistical testing for the prediction of the theory here, namely, the negative sign of the derivative \(\partial d_{j,t}/\partial q_{j,t}\).

Here, I have to be aware that theoretical predictions are both on the conditional mean of the realized liquidity shocks, namely \(\theta_{j,t}\), and I cannot make any prediction on each individual \(\tilde{\omega}_{j,t}\) of each firm, since generally it is not observable. I test statistically whether the financial intermediation is functioning efficiently or not. When financial intermediation is efficiently conducted, the conditional mean of the observed liquidity shock, which implies they are met with short-term lending provided by banks, must show a systematically negative correlation with \(q_{j,t}\). In other words, I cannot observe for which individual firms the liquidity shock is hitting its upper bound, but I can
examine the firms’ conditional mean by creating industry-level aggregate data, which would be a function of $\theta_{j,t}$. 

Since I cannot specify the distribution of liquidity shock $\Phi(\cdot)$, here I run the following linear regression (first-order approximation), using industry level cohort data to test the condition predicted by the theory, namely to detect $\partial d_{j,t} / \partial q_{j,t} \equiv a_j^1 \leq 0$ for each $j$th industry. Consider the following cohort panel regression. Let $d_t = (d_{1,t} d_{2,t} \cdots d_{j,t} \cdots d_{m,t})'$, $x_t = (x_{1,t} x_{2,t} \cdots x_{j,t} \cdots x_{m,t})'$ and $q_t = (q_{1,t} q_{2,t} \cdots q_{j,t} \cdots q_{m,t})'$. Then I have,

$$d_t = A_0 + A_1 q_t + \varepsilon_t$$  \hspace{1cm} (4.21)$$

where $\varepsilon_t$ is a $m \times 1$ vector of error terms. $A_i$'s are $m \times m$ diagonal matrices where $\text{diag}(A_i) = (a_1^i a_2^i \cdots a_j^i \cdots a_m^i)$ are coefficient vectors to be estimated. Following the theory, I allow the possibility of a “macro shock,” which is common to all industries. This implies that the non-diagonal elements in the covariance matrix, $E(\varepsilon_t' \varepsilon_t)$ can be non-zero so that it should be estimated by a seemingly unrelated regression (SUR).

4.3 Evidence from aggregate data: Comparing RBC with agency cost models

In this section I compare the performance of the standard RBC model and DGE models with agency cost in replicating the stylized facts of the Japanese business cycles to detect the significant role of an imperfect financial market. As for the U.S., performance of RBC models has been intensively examined by earlier works, $^{59}$If I could assume an uniform distribution for $\Phi(\omega)$, the resulting regression will be a log-linear form such that $\log d_{j,t} = -0.5 \log q_{j,t} + \text{const.}$
which report several anomalies of the standard RBC models in comparison with the actual data. Cogley and Nason (1995) demonstrated that the standard RBC model lacks the ability to mimic the auto-correlation pattern and hump-shaped impulse response dynamics of output and investment, which are found in the actual U.S. data. Further, Carlstrom and Fuerst (1997) and Chapter 3 of this thesis have shown that DGE models with agency cost (asymmetric information in capital market) are well capable of replicating the auto-correlation patterns, which RBC models fail to explain. There are very few preceding works applying standard RBC models to the Japanese economy. Among those, Hayashi and Prescott’s (2002) study is a well-known (and perhaps the only) attempt. However, they do not mention Cogley and Nason’s critiques of RBC models. Here let us examine the property of actual time series data of the Japanese economy to see whether Cogley and Nason’s critique is valid to Japan as well as the United States.

Figure 4.3 shows the auto-correlation function (ACF) of the actual Japanese data (1979-2001), simulated data of the RBC model and an agency cost model. As can be seen in the panel, actual output growth in Japan exhibits at least the AR(2) auto-correlation pattern. Apparently, the RBC model is inferior to the agency cost model in replicating the ACF of the actual Japanese data, since it has no auto-correlation. Further, Figure 4.4 presents the impulse response function of estimated structural VAR using Japanese data. As for the VAR, I used the Blanchard-Quah technique following Cogley and Nason (1995).\textsuperscript{60} Evidently, the figure reveals that the hump-shaped impulse response function is true to the Japanese data as well as

\textsuperscript{60}The VAR model here is the same specification of that in Cogley and Nason (1995), a bivariate VAR with GDP and labor hours with orthogonalized (structural) errors. For more details, see Appendix B.
the U.S. data. Note that this type of hump-shaped dynamics is known as an anomaly of the standard RBC models, while agency cost models are well capable of replicating it.\(^{61}\)

These facts support agency cost models rather than RBC models. Now, given these facts, Figure 4.5 shows the net worth of each household sector and the productive sector (corporate firms excluding financial intermediaries).

As can be seen, until the end of the 80s, the time trends of both two sectors are quite similar, monotonically increasing. However, at the beginning of the 90s, these two trends begin to diverge. While the net worth of households (excluding

\(^{61}\)For the impulse response function of RBC and agency cost model, see Figure 3.3.
corporations) stays substantially constant over the 90s, the net worth of corporations declines significantly in the early 90s. This fact implies that the dramatic decline in the stock market does not affect the two sectors equally, but created re-distribution effect of wealth from the corporate sector to the non-corporate sector. It should be emphasized that such a re-distribution of wealth plays no role in the standard RBC economy, but it does matter in those DGE models with agency costs. It does not require much argument to understand that in an RBC economy with a perfect capital market, investment opportunities are completely exploited regardless of the distribution of wealth. Since financial intermediation is perfect, I can ignore any financial transaction, and what matters in the RBC environment is only marginal productivity, namely real factors. On the other hand, in the broad class of models
with agency costs, the ex-ante distribution of net worth does play a non-trivial role in characterizing the equilibrium of the economy. Recall a simple financial market model with asymmetric information. Agency cost is created via moral hazard or adverse selection, since lenders are information-inferior to borrowers. Essentially, financial intermediation is a costly activity because of the agency cost. Therefore, an economy where net worth is distributed intensively to the productive sector ex-ante can enjoy higher investment and output, since less financial intermediation is necessary. This economic intuition is verified by many DGE models with agency costs, such as Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997) and the model introduced in Chapter 3 of this thesis. These three studies present very
similar simulation results of a one-time transfer of wealth between households and the productive sector. Maintaining the aggregate net worth of the economy constant, redistribution of wealth from the productive sector to households can create a downturn in the economy. Note that the result presented by those agency cost models is indeed consistent with the observation on the Japanese economy in the early 90s.

Figure 4.6: Corporate investment and output
4.4 Evidence from synthetic panel data

4.4.1 Data source: The FSSCI

I constructed synthetic panel data based on the Financial Statements Statistics of Corporations by Industries (FSSCI) issued by the Japanese Ministry of Finance. In the FSSCI, sample observations are available quarterly for 23 consecutive years (from Winter/79 to Spring/2002). The FSSCI is based on a comprehensive survey run by the Ministry of Finance every quarter, which interviews randomly sampled going-concerns whose net worths are greater than 10 million yen. During the interviews, a number of questions are asked about firms’ quarterly estimates of such variables as profit, cost, assets, and debt. There are 20 cohorts of industries in the FSSCI, and those cohorts are classified according to Japan Standard Industrial Classification. Financial intermediaries and insurance companies are not contained in the FSSCI. As the theory suggests, I need synthetic panel (pseudo-panel) data instead of micro/individual data, since the hypothesis that I am to test is a prediction on the conditional mean of the individual observations.

4.4.2 Descriptive statistics

Before presenting the estimation results, here I report the descriptive statistics of the data in Table 4.2. In the first row, DLD stands for the degree of liquidity dependence (short-term debt divided by investment). The columns below the “adf” in the second row report the results of the augmented Dicky-Fuller test to examine the stationarity of the data series. The ADF test rejected the unit root hypothesis for

\footnote{See Table 4.2. More details on the classification are provided at the website of Statistics Bureau & Statistics Center. (http://www.stat.go.jp/english/info/seido/6.htm#1)}
<table>
<thead>
<tr>
<th>Industry/Variables</th>
<th>DLD</th>
<th>ROA(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std</td>
</tr>
<tr>
<td>1. Construction</td>
<td>41.45</td>
<td>9.66</td>
</tr>
<tr>
<td>2. Food</td>
<td>12.76</td>
<td>2.82</td>
</tr>
<tr>
<td>3. Textile</td>
<td>20.84</td>
<td>6.81</td>
</tr>
<tr>
<td>4. Paper &amp; Pulp</td>
<td>12.84</td>
<td>3.56</td>
</tr>
<tr>
<td>5. Chemical</td>
<td>10.12</td>
<td>2.06</td>
</tr>
<tr>
<td>6. Petrorium &amp; Coal</td>
<td>38.76</td>
<td>23.27</td>
</tr>
<tr>
<td>7. Ceramic</td>
<td>11.80</td>
<td>3.31</td>
</tr>
<tr>
<td>8. Iron &amp; Steel</td>
<td>13.18</td>
<td>4.45</td>
</tr>
<tr>
<td>9. Non-Ferrous Metal</td>
<td>15.56</td>
<td>5.57</td>
</tr>
<tr>
<td>10. Fablicated Metal</td>
<td>11.87</td>
<td>3.53</td>
</tr>
<tr>
<td>11. Machinery General</td>
<td>15.83</td>
<td>4.75</td>
</tr>
<tr>
<td>12. Electrical Machinery</td>
<td>5.97</td>
<td>1.46</td>
</tr>
<tr>
<td>13. Transport. Equipments</td>
<td>4.93</td>
<td>1.34</td>
</tr>
<tr>
<td>14. Precision Machinery</td>
<td>9.21</td>
<td>3.49</td>
</tr>
<tr>
<td>15. Electric &amp; Gas</td>
<td>2.80</td>
<td>0.55</td>
</tr>
<tr>
<td>16. Trans. &amp; Telecom</td>
<td>4.93</td>
<td>1.47</td>
</tr>
<tr>
<td>17. Wholesale</td>
<td>63.00</td>
<td>11.57</td>
</tr>
<tr>
<td>18. Retail</td>
<td>24.93</td>
<td>5.22</td>
</tr>
<tr>
<td>19. Real Estate</td>
<td>41.08</td>
<td>13.38</td>
</tr>
<tr>
<td>20. Service</td>
<td>8.98</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Table 4.2: Descriptive statistics

all industries, which is shown by zeros in the table. Further, Figure 4.7 presents the scatter plots of ROA against DLD for 20 industries for casual observation. Evidently, most of them exhibit negative correlations as the theory predicts.


4.4.3 Estimation results

Table 4.3 shows the estimates for diagonal elements in $A_1$ matrix in eqn (4.21) by the SUR.\textsuperscript{64} In the first column, which indicates the full sample (1979:4-2002:1) estimation results, the coefficients of 16 industries out of 20 show negative sign and 14 of them are significant at the 5% level. Note that significantly negative coefficients only imply that the actual data do not contradict the theoretical prediction. On the other hand, for industries whose coefficients are estimated significantly positive, namely, Iron & Steel and Electric & Gas, one can conclude that efficient allocations of liquidity are not achieved in those industries. This point needs to be explained more carefully. The possible null hypothesis for the efficiency of the $j$th industry is that the $a_{1j}$ is equal to zero. I test the null against the alternative that the $a_{1j}$ is strictly positive, which means the violation of the efficiency condition. Note that the statistical testing carried out here is one-sided. Therefore, if the regression detects positive sign at a certain significant level, that is strong evidence of inefficiency in the industry. Further, for industries whose estimates are insignificant, I could say that their efficiency is “suspicious” in the sense that they do not show the systematic negative correlation implied by the efficiency condition. Hence, it is weak evidence of inefficiency.\textsuperscript{65}

Now, having confirmed the interpretation of the estimation results, I conduct sub-sample estimates to examine whether there is a significant change before/after the

\textsuperscript{63}For some industries the result of the ADF test varies depending on their specifications, such as adding deterministic time trend or drift term. But for all the industries, there exists a specification of the ADF test which rejects the unit root hypothesis.

\textsuperscript{64}Covariance matrices of the error terms are reported in Table 4.4.

\textsuperscript{65}Note again that even if a negative coefficient is estimated at 5% level of significance, yet one cannot conclude the industry to be efficient. Therefore, the rejection of the null to detect inefficiency is a fairly strict testing requirement to extract truly inefficient industries.
so-called “bubble period” in the 80s. I divided the sample period in two at the end of 1991, which is generally considered the end of the bubble period.\textsuperscript{66} As a result, I observe that four industries, namely Food, Petroleum & Coal, Iron & Steel and Real Estate are inefficient in the 80s, while in the 90s, six industries are revealed to be inefficient. Those six industries whose efficiency are rejected are Chemical, Iron & Steel, Fabricated Metal, Machinery General, Electric & Gas and Transportation & Telecommunication. Further, four more industries are “suspected” to be inefficient in the 90s than the 80s by the insignificant estimates for $a_j^{1}$'s.

Further, Figure 5 shows the scatter plot of the average DLD and ROA over the sample period. As the theory predicts, we can observe negative correlation between them. This observation is consistent with the evidence on aggregate data introduced in Chapter 3, which implies the efficient management of liquidity allocation for the aggregate level. Further, Figure 5 presents an interesting property for the distribution of the industries whose coefficients are estimated significantly positive or insignificant. Clearly, those industries in which efficient allocations of liquidity are considered “suspicious” by the estimation for eqn (4.21) distribute within the oval near the origin in the figure 5, except for Chemical. One plausible interpretation for this observation is that the inefficiency results in the lower ROAs. But of course this is a shaky argument and we need further scrutiny to verify this conjecture.

\textsuperscript{66}See ESRI’s provisional determination of business cycle dates. The ESRI is the Japanese organization which corresponds to NBER in the US.
4.5 Conclusion

This study provides several critiques of preceding empirical studies on the Japanese economy, especially for those which focus on the credit/capital crunch hypothesis. The credit crunch hypothesis lacks the general equilibrium perspective, so that it could ignore the critical aspects of more general agency cost models in a DGE context. I do not claim that the lost decade of Japan is induced solely by financial sector distress, but suggest a more moderate view, namely, that the financial factors are not negligible when I take the indirect/general equilibrium effect of agency costs into account. It seems that some part of the literature confuses the credit crunch hypothesis with more general models with agency costs. It should be emphasized that rejection of the credit crunch hypothesis does not necessarily imply that financial sector distress is irrelevant. In fact, RBC models presume a quite specific environment, such that information is perfect in the financial market. However, it is most likely that financial market in real world is, to some extent, imperfect. Given the imperfect financial market, re-allocation of a borrower’s net worth has non-trivial effect on real economic performance, regardless of credit rationing. Hence, “type 1” inefficiency is indeed difficult to reject thoroughly by empirical analysis. However, the “type 2” inefficiency is moot. My estimation results depend on the prediction of a specific moral hazard model. There are other theories which result in inefficient allocation of credit/liquidity, such as forbearance lending as introduced in the literature of soft budget constraints. Further empirical analysis based on other theories on an inefficient financial market would be await to understand the interaction of financial factors and real economic activity of the Japanese economy in the 90s.
Figure 4.7: DLD-ROA plots
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Construction</td>
<td>-11.95</td>
<td>-4.93</td>
<td>-3.92</td>
</tr>
<tr>
<td></td>
<td>(1.48)</td>
<td>(2.17)</td>
<td>(1.39)</td>
</tr>
<tr>
<td>2. Food</td>
<td>-1.48</td>
<td>0.80**</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.47)</td>
<td>(0.49)</td>
<td>(0.63)</td>
</tr>
<tr>
<td>3. Textile</td>
<td>-5.70</td>
<td>-2.45</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>(1.29)</td>
<td>(1.13)</td>
<td>(2.28)</td>
</tr>
<tr>
<td>4. Paper&amp;Pulp</td>
<td>-4.11</td>
<td>-2.27</td>
<td>-3.50</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
<td>(0.55)</td>
<td>(0.77)</td>
</tr>
<tr>
<td>5. Chemical</td>
<td>0.16</td>
<td>0.11</td>
<td>2.26**</td>
</tr>
<tr>
<td></td>
<td>(0.28)</td>
<td>(0.29)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>6. Petrolium&amp;Coal</td>
<td>2.10</td>
<td>6.72**</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>(1.98)</td>
<td>(1.86)</td>
<td>(2.17)</td>
</tr>
<tr>
<td>7. Ceramic</td>
<td>-2.28</td>
<td>-2.32</td>
<td>-0.59</td>
</tr>
<tr>
<td></td>
<td>(0.34)</td>
<td>(0.33)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>8. Iron&amp;Steel</td>
<td>3.06**</td>
<td>3.02**</td>
<td>2.17**</td>
</tr>
<tr>
<td></td>
<td>(0.39)</td>
<td>(0.37)</td>
<td>(0.57)</td>
</tr>
<tr>
<td>9. Non-Ferrous Metal</td>
<td>-0.18</td>
<td>-0.89</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
<td>(0.49)</td>
<td>(0.57)</td>
</tr>
<tr>
<td>10. Fablicated Metal</td>
<td>-1.86</td>
<td>-1.63</td>
<td>1.48**</td>
</tr>
<tr>
<td></td>
<td>(0.38)</td>
<td>(0.49)</td>
<td>(0.60)</td>
</tr>
<tr>
<td>11. Machinery General</td>
<td>-2.04</td>
<td>-1.94</td>
<td>1.52**</td>
</tr>
<tr>
<td></td>
<td>(0.40)</td>
<td>(0.41)</td>
<td>(0.83)</td>
</tr>
<tr>
<td>12. Electrical Machinery</td>
<td>-1.41</td>
<td>-1.14</td>
<td>-0.75</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.17)</td>
<td>(0.18)</td>
</tr>
<tr>
<td>13. Transport. Equipments</td>
<td>-0.57</td>
<td>0.04</td>
<td>-2.29</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.10)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>14. Precision Machinery</td>
<td>-3.02</td>
<td>-1.08</td>
<td>-2.16</td>
</tr>
<tr>
<td></td>
<td>(0.34)</td>
<td>(0.38)</td>
<td>(0.42)</td>
</tr>
<tr>
<td>15. Electric&amp;Gas</td>
<td>0.14**</td>
<td>0.05</td>
<td>0.43**</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>16. Trans.&amp;Telecom</td>
<td>-0.67</td>
<td>-0.19</td>
<td>0.51**</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.32)</td>
<td>(0.29)</td>
</tr>
<tr>
<td>17. Whole Sale</td>
<td>-17.00</td>
<td>-8.41</td>
<td>-12.76</td>
</tr>
<tr>
<td></td>
<td>(4.46)</td>
<td>(4.71)</td>
<td>(6.47)</td>
</tr>
<tr>
<td>18. Retail</td>
<td>-5.60</td>
<td>-1.57</td>
<td>-15.86</td>
</tr>
<tr>
<td></td>
<td>(1.18)</td>
<td>(1.32)</td>
<td>(1.93)</td>
</tr>
<tr>
<td>19. Real Estate</td>
<td>-3.64</td>
<td>9.94**</td>
<td>6.17</td>
</tr>
<tr>
<td></td>
<td>(3.10)</td>
<td>(1.91)</td>
<td>(7.01)</td>
</tr>
<tr>
<td>20. Service</td>
<td>-1.23</td>
<td>-0.34</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(0.40)</td>
<td>(0.37)</td>
<td>(0.52)</td>
</tr>
</tbody>
</table>

Note: Numbers in () denote standard errors. (***) denotes significance at 5% for the one-sided test.

Table 4.3: SUR estimation results
Figure 4.8: Average DLD-ROA plots
<table>
<thead>
<tr>
<th>Year</th>
<th>Real Estate</th>
<th>Trans.&amp;Telecom</th>
<th>Electric&amp;Gas</th>
<th>Precison Machinery</th>
<th>Machinery General</th>
<th>Fablicated Metal</th>
<th>Service</th>
<th>Textile</th>
<th>Paper&amp;pulp</th>
<th>Chemical</th>
<th>Petroturn&amp;Coal</th>
<th>Iron&amp;Steel</th>
<th>Ceramic</th>
<th>Iron&amp;Steel</th>
<th>Retail</th>
<th>Whole Sale</th>
<th>Electric&amp;Gas</th>
<th>Fabricated Metal</th>
<th>Precision Machinery</th>
<th>Transport &amp; Telecommunications</th>
<th>Service</th>
<th>Retail</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>90.9</td>
<td>13.3</td>
<td>20.4</td>
<td>8.3</td>
<td>9.9</td>
<td>38.1</td>
<td>17.7</td>
<td>7.6</td>
<td>8.5</td>
<td>9.2</td>
<td>32.5</td>
<td>15.2</td>
<td>39.1</td>
<td>8.0</td>
<td>-0.2</td>
<td>16.1</td>
<td>0.9</td>
<td>18.0</td>
<td>3.2</td>
<td>1.4</td>
<td>7.8</td>
<td>28.9</td>
</tr>
<tr>
<td>1991</td>
<td>15.9</td>
<td>15.6</td>
<td>13.5</td>
<td>1.9</td>
<td>3.8</td>
<td>263.3</td>
<td>15.7</td>
<td>15.9</td>
<td>38.7</td>
<td>15.6</td>
<td>7.6</td>
<td>12.9</td>
<td>45.6</td>
<td>1.9</td>
<td>0.1</td>
<td>16.8</td>
<td>3.8</td>
<td>-2.9</td>
<td>18.8</td>
<td>6.9</td>
<td>6.3</td>
<td></td>
</tr>
<tr>
<td>1992</td>
<td>6.2</td>
<td>6.7</td>
<td>16.3</td>
<td>1.8</td>
<td>1.8</td>
<td>19.3</td>
<td>15.7</td>
<td>16.3</td>
<td>15.9</td>
<td>1.8</td>
<td>2.4</td>
<td>2.4</td>
<td>0.3</td>
<td>1.4</td>
<td>0.3</td>
<td>1.7</td>
<td>0.7</td>
<td>1.4</td>
<td>18.8</td>
<td>6.9</td>
<td>6.3</td>
<td></td>
</tr>
<tr>
<td>1993</td>
<td>0.4</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.9</td>
<td>14.4</td>
<td>1.7</td>
<td>1.6</td>
<td>1.7</td>
<td>1.6</td>
<td>1.2</td>
<td>1.2</td>
<td>0.1</td>
<td>0.7</td>
<td>0.7</td>
<td>1.2</td>
<td>0.7</td>
<td>0.2</td>
<td>18.8</td>
<td>6.9</td>
<td>6.3</td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>12.6</td>
<td>5.7</td>
<td>20.4</td>
<td>5.5</td>
<td>2.0</td>
<td>4.2</td>
<td>16.7</td>
<td>5.5</td>
<td>9.3</td>
<td>3.5</td>
<td>4.2</td>
<td>12.6</td>
<td>31.5</td>
<td>20.1</td>
<td>4.2</td>
<td>3.5</td>
<td>4.2</td>
<td>5.5</td>
<td>8.2</td>
<td>1.2</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>12.5</td>
<td>5.7</td>
<td>20.5</td>
<td>5.5</td>
<td>2.0</td>
<td>4.2</td>
<td>16.7</td>
<td>5.5</td>
<td>9.3</td>
<td>3.5</td>
<td>4.2</td>
<td>12.6</td>
<td>31.5</td>
<td>20.1</td>
<td>4.2</td>
<td>3.5</td>
<td>4.2</td>
<td>5.5</td>
<td>8.2</td>
<td>1.2</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>1996</td>
<td>12.5</td>
<td>5.7</td>
<td>20.5</td>
<td>5.5</td>
<td>2.0</td>
<td>4.2</td>
<td>16.7</td>
<td>5.5</td>
<td>9.3</td>
<td>3.5</td>
<td>4.2</td>
<td>12.6</td>
<td>31.5</td>
<td>20.1</td>
<td>4.2</td>
<td>3.5</td>
<td>4.2</td>
<td>5.5</td>
<td>8.2</td>
<td>1.2</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>12.5</td>
<td>5.7</td>
<td>20.5</td>
<td>5.5</td>
<td>2.0</td>
<td>4.2</td>
<td>16.7</td>
<td>5.5</td>
<td>9.3</td>
<td>3.5</td>
<td>4.2</td>
<td>12.6</td>
<td>31.5</td>
<td>20.1</td>
<td>4.2</td>
<td>3.5</td>
<td>4.2</td>
<td>5.5</td>
<td>8.2</td>
<td>1.2</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>12.5</td>
<td>5.7</td>
<td>20.5</td>
<td>5.5</td>
<td>2.0</td>
<td>4.2</td>
<td>16.7</td>
<td>5.5</td>
<td>9.3</td>
<td>3.5</td>
<td>4.2</td>
<td>12.6</td>
<td>31.5</td>
<td>20.1</td>
<td>4.2</td>
<td>3.5</td>
<td>4.2</td>
<td>5.5</td>
<td>8.2</td>
<td>1.2</td>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4: Covariance matrices for SURs
BIBLIOGRAPHY


APPENDIX A

MONTE-CARLO SIMULATION USING KIYOTAKI-MOORE MODEL

A.1 Model overview; Kiyotaki and Moore (1997)

This appendix briefly reviews the “full model” presented in Kiyotaki and Moore (1997). For more details of the model, see Kiyotaki and Moore (1997). First, they start with a linear production technology for entrepreneurs.

\[ y_t = (z_t + c) k_{t-1} \]

where \( y_t \) and \( k_t \) denote output and capital. A fraction \( ck_{t-1} \) of the produced goods is untradable so that the entrepreneur must consume for their own. \( z_t \) is a stochastic productivity shock. A borrowing constraint with the following specification is brought in here,

\[ R * b_t = q_{t+1} k_t \]

where \( R \), \( q_t \) and \( b_t \) stand for gross interest rate, price of capital and debt respectively.

Basic expression for the flow of fund,

\[ q_t (k_t - k_{t-1}) + \phi (k_t - \lambda k_{t-1}) + Rb_{t-1} + x_t - ck_{t-1} = z_t k_{t-1} + b_t \]
where $\phi (k_t - \lambda k_{t-1})$ denotes an input for reproduction of capital. For a fraction $\pi$ of the population can invest (entrepreneurs), while $(1 - \pi)$ (households) cannot. For a firm, investment is strictly better than consumption so that $x_t \geq ck_t$ and $R*bt = qt+1k_t$ are binding. Combining these binding constraints with eqn (A.3) yields,

$$\left(q_t + \phi - \frac{qt+1}{R}\right)k_t = (z_t + \lambda \phi + q_t)k_{t-1} - Rb_{t-1}. \quad (A.4)$$

On the other hand, households’ capital is just depreciating, namely, $k_t' = \lambda k_{t-1}'$. Combining these two to get the law of motion for aggregate capital,

$$K_{t+1} = (1 - \pi) \lambda K_{t-1} + \left(\frac{\pi}{q_t + \phi - qt+1/R}\right)\left[(z_t + \lambda \phi + q_t)K_{t-1} - Rb_{t-1}\right] \quad (A.5)$$

Further, aggregate debt follows,

$$B_t = RB_{t-1} + q_t(K_t - K_{t-1}) + \phi(K_t - \lambda K_{t-1}) - z_tK_{t-1} \quad (A.6)$$

Finally, the Euler equation of consumption or asset price path,

$$G'(K - K_t) + \frac{qt+1}{qt} = R \quad (A.7)$$

or in an alternative form $\psi(K_t) = qt - qt+1/R$, where $\psi(K_t) = G'(K - K_t) / R$.

**A.2 Monte-Carlo simulation**

From eqn(A.1), (A.2) and (A.3) the system will be the three equations as follows,

$$K_t = \left(\frac{\pi}{q_t + \phi - qt+1/R}\right)[(a + \lambda \phi + q_t)K_{t-1} - RB_{t-1}] + (1 - \pi) \lambda K_{t-1}\quad (A.8)$$

$$B_t = RB_{t-1} + q_t(K_t - K_{t-1}) + \phi(K_t - \lambda K_{t-1}) - aK_{t-1} \quad (A.9)$$

$$\psi(K_t) = qt - \frac{qt+1}{R} \quad (A.10)$$

In addition, I assume the productivity shock follows an AR(1) process with a random disturbance $e_t$ such that,

$$z_t = \sigma z_{t-1} + (1 - \sigma)z^* + e_t. \quad (A.11)$$
Let $\psi(K_t) = K_t - v$, $v = 2$, $\phi = 0.9$, $z^* = 1$, $\lambda = 0.975$, $\pi = 0.1$, $\sigma = 0.9$ and $R = 1.01$. These parameter values are taken from Kiyotaki and Moore (1997) except for $\sigma$ which does not appear in their original paper. Note that the system has three stable roots and one unstable root so that the Blanchard-Kahn theorem should be satisfied. Standard undetermined coefficient method is used for solving the model.

For the OLS regression, let investment be $I_t = q_tK_t - q_{t-1}K_{t-1}$. Then using artificially generated data, I run OLS regressions with the following specification,

$$I_t = c_0 + c_1B_t + v_t.$$  \hspace{1cm} (A.12)

where $c_0$ and $c_1$ are coefficients to be estimated and $v_t$ is an error term. I conducted the regression for 100 times with 1,000 periods for each run. Based on the regression results, I report average $t$-statistics and $R^2$ in section 4.2.1.
APPENDIX B

STRUCTURAL VAR ESTIMATION WITH A LONG-RUN RESTRICTION

B.1 Blanchard-Quah technique

The structural VAR estimation presented in chapter 4 is based on Blanchard-Quah technique, which Cogley and Nason (1995) uses to analyze the stylized facts of the US business cycles. For more rigorous argument of the structural VAR estimation, see Ogaki and Jang (2001) for example.

Consider a structural VMA expression of the bivariate system,

\[ X_t = A(0) \varepsilon_t + A(1) \varepsilon_{t-1} + \cdots \]  \hspace{1cm} (B.1)

\[ = \sum_{j=0}^{\infty} A(j) \varepsilon_{t-j} \]  \hspace{1cm} (B.2)

where \( \varepsilon \)'s are structural disturbances. By assumption, the two disturbances are orthogonal each other. In other words, \( Var(\varepsilon) = I \). On the other hand, consider the following reduced form of the system,

\[ X_t = e_t + C(1) e_{t-1} + \cdots = \sum_{j=0}^{\infty} C(j) e_{t-j} \]  \hspace{1cm} (B.3)
where e’s are reduced form errors. Let \( \text{Var}(e) = \Omega \) here. Notice the relation between structural errors and estimated errors such that,

\[
\varepsilon_t = A(0)^{-1}e_t
\]

(B.4)

Hence I need to know \( A(0) \) matrix to recover structural disturbances from estimated errors. To identify the \( A \) matrix, I can rely on two conditions, namely, 
\( e'e = A(0)A(0)' = \Omega \) and the upper left-hand-side entry in \( \sum_{j=0}^{\infty} A(j) = \left( \sum_{j=0}^{\infty} C(j) \right) A(0) \)

is equal to zero. Note that the second condition is derived from a long-run restriction such that the transitory shocks are neutral in the long-run. These two conditions provide four restrictions for \( A \) matrix, which are sufficient for identification.

To obtain \( \sum_{j=0}^{\infty} C(j) \) matrix, I estimated an unrestricted bivariate VAR(n),

\[
X_t = e_t + B(1)X_{t-1} + \cdots + B(n)X_{t-n}.
\]

(B.5)

Since eqn (B.3) is an alternative expression of eqn(B.5), the impulse response function of the two systems are identical. Then I can exploit the following relation to calculate \( \sum_{j=0}^{\infty} C(j) \),

\[
\sum_{k=0}^{\infty} \Gamma(k) = \sum_{j=0}^{\infty} C(j)
\]

(B.6)

where \( \Gamma(k) \) is the impulse response function derived from the estimated VAR.

**B.2 Data**

I use quarterly data of output and labor hours. Estimation sample period is 1980Q3-2002Q1 and lag length are chosen according to BIC. As discussed in Blanchard and Quah (1989), if output is \( I(1) \) and labor hour is \( I(0) \) so that they satisfy the conditions which B-Q methodology requires. However, labor hours data of Japan shows an apparent downward trend in the 80s. This has a clear reason.
From 1982, many firms eventually abandoned one day-off per week system and replace it by two day-off per week system. This reform was legally enforced by the newly constructed law in early 80s. Since this one-time reform had nothing to do with economic fluctuation, the data for this period is linearly detrended. For the rest of the details, I follow the original B-Q estimation.