ASTROPHYSICS FROM BINARY-LENS MICROLENSING

DISSERTATION

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By

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ABSTRACT

Microlensing events, especially ones due to a lens composed of a binary system can provide new channels to approach some of old questions in astronomy. Here, by modeling lightcurves of three binary-lens microlensing events observed by PLANET, I illustrate specific applications of binary-lens microlensing to real astrophysical problems.

The lightcurve of a prototypical caustics-crossing binary-lens microlensing event OGLE-1999-BUL-23, which has been especially densely covered by PLANET during its second caustic crossing, enables me to measure the linear limb-darkening coefficients of the source star in $I$ and $V$ bands. The results are more or less consistent with theoretical predictions based on stellar atmosphere models, although the nonlinearity of the actual stellar surface brightness profile may have complicated the interpretation, especially for $I$ band.

Next, I find that the model for the lightcurve of EROS BLG-2000-5, another caustics-crossing binary-lens microlensing event, but exhibiting an unusually long second caustic crossing as well as a very prominent third peak due to a close
approach to a cusp, requires incorporation of the microlens parallax and the binary orbital motion. Its projected Einstein radius is derived from the measured microlens parallax, and its angular Einstein radius is inferred from the finite source effect on the lightcurve, combined with an estimate of the source angular size given by the source position on the color-magnitude diagram. The lens mass is found by combining the above two quantities. This event marks the first case that the parallax effects are detected for a caustic-crossing event and the first time that the lens mass degeneracy has been broken.

The analysis of the lightcurve of MACHO 99-BLG-47, an almost normal event with a well-covered short-duration anomaly near the peak, shows that it is caused by an extreme-separation binary-lens. Although lightcurve anomalies that are similar to what was observed for MACHO 99-BLG-47 may result from either a planet around the lens or an extreme-separation binary, this result demonstrates that the two classes of events can be distinguished in practice.
Dedicated to everyone who reads this ...
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**FIELDS OF STUDY**

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al. (2000) and Fig 2.6] Also drawn are the contours of zero difference
(dotted line) and 5% difference (solid line). While the actual source
trajectory (y = 0) naturally traces well the zero-difference line, the
flux difference would also be extremely small for other trajectories
through the region, except very near the caustic.
Chapter 1

Introduction

While the redux of gravitational microlensing in the astrophysics was initiated by the suggestion that it can be used to probe the mass distribution of subluminous objects in the Galactic Halo (Paczyński 1986), the study during the last one and a half decades has revealed that microlensing provides unique opportunities to tackle a few of other outstanding questions in astronomy.

The most common and simplest form of microlensing is modeled as a pointlike light source being lensed by a single point of mass (hereafter PSPL) moving in a uniform linear fashion relative to the line of sight to the source. While its lightcurve, the record of the time-series observations of the apparent brightness change of the source during the course of the lensing event, is straightforward to model, it bears only a limited amount of information of physical interest. On the other hand, anomalous lightcurves, which are lightcurves that cannot be fit to the
standard form because the complexity of the lens/source system induces noticeable
signature upon it, can yield potentially a wealth of information of astrophysical
significance. In particular, if the microlensing is caused by a binary system, the
resulting “binary-lens” microlensing event turns out to be quite useful once it is
properly modeled, although that can be a formidable task.

If two point masses are separated by of order an Einstein radius (typically
several AU for Galactic bulge microlensing), gravitational “interference” between
them induces a unique magnification pattern that is usually much more complex
than a simple superposition of the patterns due to two independent mass points.
By observing the lightcurve of such an event, one can sample a one-dimensional
slice of these magnification maps, and in principle recover the configuration of the
lens system and the relative geometry between the lens system and the source
trajectory. Often these binary microlensing events are accompanied by caustic
crossings. Caustics are imaginary curves on the sky behind which the source is
infinitely magnified. By nature, the caustics are mathematical entities rather than
physical ones, so that the caustics are precisely defined and infinitely sharp. These
characteristics of caustics provide us with a method to probe very fine angular
structures of the object being lensed by observing the lightcurve while the caustic
moves over the source. In the lowest order, the effect is seen by the finite angular
extent of the source star temporally broadening the lightcurve over the crossing.
The complete modeling of these lightcurves usually yields a measurement of the angular radius of the source relative to the Einstein ring. For better observed caustic crossings, one may detect with reasonable accuracy, the effect of the intensity variation over the disk of the source star by analyzing the lightcurve.

Another notable application of binary-lens microlensing is a detection of extrasolar planets (Mao & Paczyński 1991; Gould & Loeb 1992). As with binaries, a planetary companion to the lens, which is essentially an extreme mass-ratio binary companion, induces a distortion in the observed microlensing lightcurve. Hence, one can infer the presence of a planetary companion by observing the lightcurve which is fittable only by a planetary system. Compared to other methods of detecting extrasolar planets, the microlensing technique is completely independent of the visual detection of the parent star (i.e., the lens), implying that it is in principle applicable to detect planets around stars of distance scales of kpc.

In the subsequent chapters, I model lightcurves of three binary-lens microlensing events observed by the Probing Lensing Anomalies NETwork (PLANET). Each of them provides a unique opportunity to demonstrate a specific application of binary-lens microlensing to astrophysical problems. In Chapter 2, the model of the lightcurve of a prototypical caustic-crossing binary-lens event, OGLE-1999-BUL-23, is used to derive the surface brightness profile of the source star, and the comparison of the result to theories is also discussed. One of the most
spectacular binary-lens events observed to date, EROS BLG-2000-5, is the main focus of the next chapter (Chapter 3). Modeling of this event requires a careful consideration of various effects in unprecedented detail. While this complexity has caused a considerable difficulty in finding an acceptable model, the final result contains enough information to determine the mass of the lensing object for the first time and to put a reasonable constraint on the kinematics of the lens/source system as well as on the location of the lens. In the last chapter (Chapter 4), the microlensing lightcurve of MACHO 99-BLG-47, which exhibits a short timescale deviation on top of an otherwise normal-looking lightcurve, thought to be a characteristic of planetary microlensing, is analyzed. I show that the lightcurve can be unambiguously identified not to be caused by a planetary deviation, but due to an extreme-separation roughly-comparable-mass binary. Despite the fact that lightcurves of extreme-separation binary-lenses and planetary deviations could look similar, this result implies that the lightcurve of a planetary system, if observed, can be distinguished from one due to a usual binary by a well-structured modeling.
Chapter 2

Source Limb-Darkening Measurement: OGLE-1999-BUL-23

2.1. Introduction

In PSPL microlensing events, the lightcurve yields only one physically interesting parameter $t_E$, the characteristic timescale of the event,

$$ t_E \equiv \frac{\theta_E}{\mu} ; \quad \theta_E \equiv \sqrt{\frac{2R_S}{D}}. \quad (2.1) $$

Here $D = AU/\pi_{rel}$, $\pi_{rel} \equiv \pi_L - \pi_S$ is the lens-source relative trigonometric parallax, $\mu \equiv \mu_S - \mu_L$ is the relative proper motion, $\theta_E$ is the angular Einstein radius, and $R_S \equiv 2GMc^{-2}$ is the Schwarzschild radius of the lens mass. However, many varieties of anomalous events have been observed in reality, and using their deviations from the standard PSPL lightcurve, one can deduce more information.
about the lens and the source. One example of information that can be extracted from anomalous events is the surface brightness profile of the source star (Witt 1995). If the source passes near or across the caustic, which is the region of singularity of the lens mapping and therefore at which the magnification for a point source is formally infinite, drastic changes in magnification near it can reveal the finite size of the source (Gould 1994; Nemiroff & Wickramasinghe 1994; Witt & Mao 1994; Alcock et al. 1997), and one can even extract its surface-brightness profile (Bogdanov & Cherepashchuk 1996; Gould & Welch 1996; Sasselov 1997; Valls-Gabaud 1998).

The falloff of the surface brightness near the edge of the stellar disk with respect to its center, known as limb darkening, has been extensively observed in the Sun. Theories of stellar atmospheres predict limb darkening as a general phenomenon and give models for different types of stars. Therefore, measurement of limb darkening in distant stars other than the Sun would provide important observational constraints on the study of stellar atmospheres. However, such measurements are very challenging with traditional techniques and have usually been restricted to relatively nearby stars or extremely large supergiants. As a result, only a few attempts have been made to measure limb darkening to date. The classical method of tracing the stellar surface brightness profile is the analysis of the lightcurves of eclipsing binaries (Wilson & Devinney 1971; Twigg & Rafert
1980). However, the current practice in eclipsing-binary studies usually takes the opposite approach to limb darkening (Claret 1998a), i.e., constructing models of lightcurves using theoretical predictions of limb darkening. This came to dominate after Popper (1984) demonstrated that the uncertainty of limb-darkening measurements from eclipsing binaries is substantially larger than the theoretical uncertainty. Since the limb-darkening parameter is highly correlated with other parameters of the eclipsing binary, fitting for limb darkening could seriously degrade the measurement of these other parameters. Multiaperture interferometry and lunar occultation, which began as measurements of the angular sizes of stars, have also been used to resolve the surface structures of stars (Hofmann & Scholz 1998). In particular, a large wavelength dependence of the interferometric size of a stellar disk has been attributed to limb darkening, and higher order corrections to account for limb darkening have been widely adopted in the interferometric angular size measurement of stars. Several recent investigations using optical interferometry extending beyond the first null of the visibility function have indeed confirmed that the observed patterns of the visibility function contradict a uniform stellar disk model and favor a limb-darkened disk (Quirrenbach et al. 1996; Hajian et al. 1998) although these investigations have used a model prediction of limb darkening inferred from the surface temperature rather than deriving the limb darkening from the observations. However, in at least one case, Burns et al. (1997) used interferometric imaging to measure the stellar surface brightness profile with
coefficients beyond the simple linear model. In addition, developments of high resolution direct imaging in the last decade using space telescopes (Gilliland & Dupree 1996) or speckle imaging (Kluckers et al. 1997) have provided a more straightforward way of detecting stellar surface irregularities. However, most studies of this kind are still limited to a few extremely large supergiants, such as α Orionis. Furthermore, they seem to be more sensitive to asymmetric surface structures such as spotting than to limb darkening.

By contrast, microlensing can produce limb-darkening measurements for distant stars with reasonable accuracy. To date, limb darkening (more precisely, a set of coefficients of a parametrized limb-darkened profile) has been measured for source stars in three events, two K giants in the Galactic bulge and an A dwarf in the Small Magellanic Cloud (SMC). MACHO 97-BLG-28 was a cusp-crossing event of a K giant source with extremely good data, permitting Albrow et al. (1999b) to make a two-coefficient (linear and square-root) measurement of limb darkening. Afonso et al. (2000) used data from five microlensing collaborations to measure linear limb darkening coefficients in five filter bandpasses for MACHO 98-SMC-1, a metal-poor A star in the SMC. Although the data for this event were also excellent, the measurement did not yield a two-parameter determination because the caustic crossing was a fold-caustic rather than a cusp, and these are less sensitive to the form of the stellar surface brightness profile. Albrow et al. (2000a)
measured a linear limb-darkening coefficient for MACHO 97-BLG-41, a complex rotating-binary event with both a cusp crossing and a fold-caustic crossing. In principle, such an event could give very detailed information about the surface brightness profile. However, neither the cusp nor the fold-caustic crossing was densely sampled, so only a linear parameter could be extracted.

In this chapter, I measure limb-darkening a star in the Galactic bulge by a fold-caustic crossing event, OGLE-1999-BUL-23, based on the photometric monitoring of the Probing Lensing Anomalies NETwork (PLANET; Albrow et al. 1998), and also develop a thorough error analysis for the measurement.

2.2. Limb Darkening Measurement from Caustic-Crossing Microlensing

Traditionally, the surface brightness profile of a star is parameterized by the axis-symmetric form of either

\[ S_\lambda(\vartheta) = S_\lambda(0) \left\{ 1 - \sum_m c_{m,\lambda} [1 - (\cos \vartheta)^m] \right\} , \quad (2.2a) \]

or

\[ S_\lambda(\vartheta) = S_\lambda(0) \left[ 1 - \sum_m c'_{m,\lambda} (1 - \cos \vartheta)^m \right] , \quad (2.2b) \]
where $\{c_{m,\lambda}^{(l)}\}$ is a set of limb-darkening coefficients which specifies the amount of limb darkening for any given star. Here $\vartheta$ is the angle between the normal to the stellar surface and the line of sight, i.e., $\sin \vartheta = \theta/\theta_*$, $\theta$ is the angular distance to the center of the star, and $\theta_*$ is the angular radius of the star. Detailed theoretical modelings of stellar atmospheres have shown that, for a given same finite number of coefficients, the form of equation (2.2a) is generally superior to that of equation (2.2b) for tracing the real surface brightness profile closely.

Since the observable consequence of microlensing is the change of the overall brightness of the source star and so does the effect of the limb darkening of the source on microlensing, it is convenient to normalize the surface brightness profile for the total flux $F_{s,\lambda} = (2\pi\theta_0^2) \int_0^1 S_\lambda(\vartheta) \sin \vartheta \, d(\sin \vartheta)$, instead of the central intensity $S_\lambda(0)$ as in equations (2.2). That is, there is no net flux associated with the limb-darkening coefficients. Hence, we adopt the modified surface brightness profile of the form,

$$S_\lambda(\vartheta) = \bar{S}_\lambda \left\{ 1 - \sum_m \Gamma_{m,\lambda} \left[ 1 - \frac{(m+2)\cos m \vartheta}{2} \right] \right\}, \quad (2.3)$$

which is a variant of equation (2.2a). Here $\bar{S}_\lambda \equiv F_{s,\lambda}/(\pi\theta_*^2)$ is the mean surface brightness of the source. In particular, corresponding to the two of the simplest parameterizations among the form of equation (2.2a)

$$S_\lambda(\vartheta) = S_\lambda(0) \left[ 1 - c_\lambda(1 - \cos \vartheta) \right], \quad (2.4a)$$
and
\[ S_\lambda(\vartheta) = S_\lambda(0) \left[ 1 - c_\lambda(1 - \cos \vartheta) - d_\lambda(1 - \sqrt{\cos \vartheta}) \right], \quad (2.4b) \]

we have the linear limb-darkening law
\[ S_\lambda(\vartheta) = \bar{S}_\lambda \left[ (1 - \Gamma_\lambda) + \frac{3\Gamma_\lambda}{2} \cos \vartheta \right], \quad (2.5a) \]

and the square-root limb-darkening law
\[ S_\lambda(\vartheta) = \bar{S}_\lambda \left[ (1 - \Gamma_\lambda - \Lambda_\lambda) + \frac{3\Gamma_\lambda}{2} \cos \vartheta + \frac{5\Lambda_\lambda}{4} \cos^{1/2} \vartheta \right], \quad (2.5b) \]

respectively. The transformations of the coefficients in equations (2.5) to the usual coefficients used in equations (2.4) are given by
\[ c_\lambda = \frac{3\Gamma_\lambda}{2 + \Gamma_\lambda}, \quad (2.6a) \]
\[ c_\lambda = \frac{6\Gamma_\lambda}{4 + 2\Gamma_\lambda + \Lambda_\lambda}; \quad d_\lambda = \frac{5\Lambda_\lambda}{4 + 2\Gamma_\lambda + \Lambda_\lambda}, \quad (2.6b) \]

The magnification of the limb-darkened source is found by the intensity-weighted integral of the microlensing magnification over the disk of the source. In particular, the magnification for the disk parameterized by equation (2.3) near the linear fold caustic behaving as one-sided square-root singularity is the convolution
\[ A = F_{s,\lambda}^{-1} \int_D d^2\theta A_r S_\lambda \]

\[ = \left( \frac{u_r}{\rho_*} \right)^{1/2} \left[ G_0 \left( -\frac{\Delta u_\perp}{\rho_*} \right) + \sum_m \Gamma_{m,\lambda} H_{m/2} \left( -\frac{\Delta u_\perp}{\rho_*} \right) \right], \quad (2.7a) \]

\[ G_n(\eta) \equiv \pi^{-1/2} \frac{(n + 1)!}{(n + 1/2)!} \int_{\max(n,-1)}^1 dx \frac{(1 - x^2)^{n+1/2}}{(x - \eta)^{1/2}} \Theta(1 - \eta), \quad (2.7b) \]

\[ H_n(\eta) \equiv G_n(\eta) - G_0(\eta). \quad (2.7c) \]

Here \( A_r \) is the point-source magnification near the fold caustic, \( \Delta u_\perp \) is the distance from the source center to the linear caustic in units of the Einstein ring radius, \( u_r \) is the characteristic lengthscale of magnification change associated with the caustic in units of the Einstein ring radius, \( \Theta(x) \) is the Heaviside step function, and \( \rho_* = \theta_*/\theta_E \) is the relative source radius with respect to the Einstein ring radius. Therefore, by decomposing the observed curve of microlensing magnification changes of the caustic-crossing source into a set of characteristic functions \( \{ G_0, H_{m/2} \} \), one can measure the limb-darkening coefficients \( \{ \Gamma_{m,\lambda} \} \) to describe the surface brightness profile of the source.

### 2.3. OGLE-1999-BUL-23

OGLE-1999-BUL-23 was originally discovered towards the Galactic bulge by the Optical Gravitational Lensing Experiment (OGLE; Udalski et al. 1992; Udalski,
Kubiak, & Szymański 1997). The PLANET collaboration observed the event as a part of its routine monitoring program after the initial alert, and detected a sudden increase in brightness on 1999 June 12. Following this anomalous behavior, PLANET began dense (typically one observation per hour) photometric sampling of the event. Since the source lies close to the (northern) winter solstice (RA = 18h07m45s14, Dec = −27°33′15″14; l = 3°64, b = −3°52), while the caustic crossing (1999 June 19) occurred nearly at the summer solstice, and since good weather at all four of PLANET sites prevailed throughout, they were able to obtain nearly continuous coverage of the second caustic crossing without any significant gaps. Visual inspection and initial analysis of the lightcurve revealed that the second crossing was due to a simple fold-caustic crossing (see § 2.4.1).

### 2.3.1. Data

PLANET observed OGLE-1999-BUL-23 with $I$- and $V$-band filters at four participant telescopes: the Elizabeth 1 m at South African Astronomical Observatory (SAAO), Sutherland, South Africa; the Perth/Lowell 0.6 m telescope at Perth Observatory, Bickley, Western Australia, Australia; the Canopus 1 m near Hobart, Tasmania, Australia; and the Yale/AURA/Lisbon/OSU 1 m at Cerro Tololo Interamerican Observatory (CTIO), La Serena, Chile. From

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2 http://www.astro.rug.nl/~planet/OB99023cc.html

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1999 June to August (2451338 < HJD < 2451405), PLANET obtained almost 600 images of the field of OGLE-1999-BUL-23. The additional observations of OGLE-1999-BUL-23 were also made at SAAO (HJD ≃ 2451440) and Perth (HJD ≃ 2451450; HJD ≃ 2451470). Here HJD is Heliocentric Julian Date at the center of exposure. The data reduction and photometric measurements of the event were performed relative to nonvariable stars in the same field using DoPHOT (Schechter, Mateo, & Saha 1993). After several rereductions, we recovered the photometric measurements from a total of 475 frames.

We assumed independent photometric systems for different observatories and thus explicitly included the determination of independent (unlensed) source and background fluxes for each different telescope and filter band in the analysis. This provides both determinations of the photometric offsets between different systems and independent estimates of the blending factors. The final results demonstrate satisfactory alignment among the data sets (see § 2.4.2), and we therefore believe that we have reasonable relative calibrations. Our previous studies have shown that the background flux (or blending factors) may correlate with the size of seeing disks in some cases (Albrow et al. 2000a,b). To check this, we introduced linear seeing corrections in addition to constant backgrounds.

From previous experience, it is expected that the formal errors reported by DoPHOT underestimate the actual errors (Albrow et al. 1998). and consequently
that $\chi^2$ is overestimated. Hence, we renormalize photometric errors to force the final reduced $\chi^2$/dof = 1 for our best fit model. Here, dof is the number of degrees of freedom (the number of data points minus the number of parameters). We determine independent rescaling factors for the photometric uncertainties from the different observatories and filters. The process involves two steps: the elimination of bad data points and the determination of error normalization factors. In this as in all previous events that have been analyzed, there are outliers discrepant by many $\sigma$ that cannot be attributed to any specific cause even after we eliminate some points whose source of discrepancy is identifiable. Although, in principle, whether particular data points are faulty or not should be determined without any reference to models, we find that the lightcurves of various models that yield reasonably good fits to the data are very similar to one another, and furthermore, there is no indication of temporal clumping of highly discrepant points. We therefore identify outlier points with respect to our best model and exclude them from the final analysis.

For the determination of outliers, we follow an iterative approach using both steps of error normalization. First we calculate the individual $\chi^2$’s of data sets from different observatories and filter bands with reference to our best model without any rejection or error scaling. Then, the initial normalization factors are determined independently for each data set using those individual $\chi^2$’s and the
number of data points in each set. If the deviation of the most discrepant outlier is larger than what is predicted based on the number of points and the assumption of a normal distribution, we classify the point as bad and calculate the new $\chi^2$'s and the normalization factors again. We repeat this procedure until the largest outlier is comparable with the prediction of a normal distribution. Although the procedure appears somewhat arbitrary, the actual result indicates that there exist rather large decreases of $\sigma$ between the last rejected and included data points. After rejection of bad points, 428 points remain (see Table 2.1 and Fig. 2.1).

2.4. Lightcurve Analysis and Results

To specify the lightcurve of a static binary-lens microlensing event with a rectilinear source trajectory requires seven geometric parameters: $d$, the projected binary separation in units of $\theta_E$; $q$, the binary mass ratio; $t_E$, the Einstein timescale (the time required for the source to transit the Einstein radius); $\alpha$, the angle of the source-lens relative motion with respect to the binary axis; $u_0$, the minimum angular separation between the source and the binary center – either the geometric center or the center of mass – in units of $\theta_E$; $t_0$, the time at this minimum; $\rho_s$, the source size in units of $\theta_E$. In addition, limb-darkening parameters for each wave band of observations, and the source flux $F_s$ and background flux $F_b$ for each telescope and wavelength band are also required to transform the lightcurve to a
Table 2.1: PLANET photometry of OGLE-1999-BUL-23. The predicted flux of magnified source is \( F = F_s A + F_{b,0} + \eta \theta_s = F_s [A + b_m + \hat{\eta} (\theta_s - \theta_{s,m})] \), where \( A \) is magnification and \( \theta_s \) is the FWHM of the seeing disk in arcsec. The values of \( b_m \) and \( \hat{\eta} \) are evaluated for the best model (wide w/ LD), in Table 2.2.

\[ \sigma_{\text{normalized}} = (\text{normalization}) \times \sigma_{\text{DoPHOT}} \]

\[ b_m \equiv (F_{b,0} + \eta \theta_{s,m})/F_s \]

\[ \hat{\eta} \equiv \eta/F_s \]

\[ \theta_{s,m} \text{ (arcsec)} \]

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<th>filter</th>
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<th>normalization(^a)</th>
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<th>( \hat{\eta} )^(c)</th>
<th>( \theta_{s,m} )^(d)</th>
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<td>41.14%</td>
<td>0.1794</td>
<td>1.715</td>
</tr>
</tbody>
</table>

\(^a\)\(\sigma_{\text{normalized}} = (\text{normalization}) \times \sigma_{\text{DoPHOT}}\)

\(^b\)Blending fraction at median seeing, \( b_m \equiv (F_{b,0} + \eta \theta_{s,m})/F_s \)

\(^c\)Scaled seeing correction coefficient, \( \hat{\eta} \equiv \eta/F_s \)

\(^d\)Median seeing disk size in FWHM
Fig. 2.1.— Whole data set excluding later-time baseline points (2451338 < HJD < 2451405), in $I$ (top) and $V$ (bottom) bands. Only the zero points of the different instrumental magnitude systems have been aligned using the result of the caustic crossing fit (§ 2.4.1); no attempt has been made to account for either different amounts of blended light or seeing corrections.
specific photometric system. However, Albrow et al. (1999c) advocated the use of a different set of parameters \((d, q, t_E, t_{cc}, \Delta t, Q, \ell; F_{cc}, F_{base})\) for the caustic-crossing binary event. Here, \(t_{cc}\) is the time of one of the caustic crossings (the time when the center of the source crosses the caustic), \(\Delta t = \theta_*/(\mu \sin \phi) = t_E \rho_\star \csc \phi\) is the timescale for the same caustic crossing (the half-time required for the caustic to move over the source), \(\phi\) is the angle that the source crosses the caustic, \(\ell\) is the position of caustic crossing parameterized by the path length along the caustic, and \(Q = u_r F_s^2 t_E \csc \phi\) is basically the relevant reexpressed form of the caustic strength \(u_r\). Two flux are the observed flux at the caustic crossing accounting only for the images not associated with the caustic singularity, \(F_{cc} = F_s A_{cc} + F_b\) and at the baseline \(F_{base} = F_s + F_b\). They are chosen to utilize the the caustics-crossing part of the lightcurve can be described generically and with a reasonable precision without knowing the full underlying geometry of the system and that the specification of the caustic crossing reduces the volume of parameter space to be searched for the solution. We adopt this latter parameterization for the analysis of OGLE-1999-BUL-23 although we eventually transform into and report the result in the standard parameterization.
2.4.1. Searching for $\chi^2$ Minima

In addition, we also use the method of Albrow et al. (1999c), which was specifically devised to fit the lightcurve of fold-caustics-crossing binary-lens events, to analyze the lightcurve of this event and find an appropriate binary-lens solution. This method consists of three steps: (1) fitting of caustic-crossing data using an analytic approximation of the magnification, (2) searching for $\chi^2$ minima over the whole parameter space using the point-source approximation and restricted to the non-caustic-crossing data, and (3) $\chi^2$ minimization using all data and the full binary-lens equation in the neighborhood of the minima found in the second step.

For the first step, we fit the $I$-band caustic-crossing data $(2451348.5 \leq \text{HJD} \leq 2451350)$ to the six-parameter analytic curve that characterizes the shape of the second caustic crossing of the source with linear limb darkening (eq. [2.5a]) (Albrow et al. 1999c; Afonso et al. 2000),

$$F(t; \Delta t, t_{cc}, Q, F_{cc}, \tilde{\omega}, \Gamma) = \left( \frac{Q}{\Delta t} \right)^{1/2} \left[ G_0 \left( \frac{t - t_{cc}}{\Delta t} \right) + \Gamma H_{1/2} \left( \frac{t - t_{cc}}{\Delta t} \right) \right] + F_{cc} + \tilde{\omega} \left( t - t_{cc} \right). \quad (2.8)$$

Note that this is the transformation of equation (2.7) from position-magnification space into time-flux space assuming the linear caustic and the rectilinear motion of the source relative to the lens. Figure 2.2 shows the best-fit curve and the data points used for the fit. This caustic-crossing fit essentially constrains the
search for a full solution to a four-dimensional hypersurface instead of the whole
nine-dimensional parameter space (Albrow et al. 1999c).

We then construct a grid of point-source lightcurves with model parameters
spanning a large subset of the hypersurface and calculate $\chi^2$ for each model using
the I-band non-caustic-crossing data. After an extensive search for $\chi^2$-minima over
the four-dimensional hypersurface, we find positions of two apparent local minima,
each in a local valley of the $\chi^2$-surface. The smaller $\chi^2$ of the two is found at $(d, q, \alpha) \simeq (2.4, 0.4, 75^\circ)$. The other local minimum is $(d, q, \alpha) \simeq (0.55, 0.55, 260^\circ)$. The results appear to suggest a rough symmetry of $d \leftrightarrow d^{-1}$ and $(\alpha < \pi) \leftrightarrow (\alpha > \pi)$, as was found for MACHO 98-SMC-1 (Albrow et al. 1999c; Afonso et al. 2000).

In addition to these two local minima, there are several isolated $(d, q)$-grid points
at which $\chi^2$ is smaller than at neighboring grid points. However, on a finer grid
they appear to be connected with one of the two local minima specified above. We
include the two local minima and some of the apparently isolated minimum points
as well as points in the local valley around the minima as starting points for the
refined search of $\chi^2$-minimization in the next step.

2.4.2. Solutions: $\chi^2$ Minimization

Starting from the local minima found in § 2.4.1 and the points in the local
valleys around them, we perform a refined search for the $\chi^2$ minimum. The $\chi^2$
Fig. 2.2.— Fit of the caustic-crossing data to the six-parameter analytic curve given by eq. (2.8). The time of second caustic crossing ($t_{cc}$) and the time scale of caustic crossing ($\Delta t$) are indicated by vertical lines. The instrumental SAAO $I$-band flux, $F_{18.4}$, is given in units of the zero point $I = 18.4$. 

\[
\begin{align*}
Q &= (15.3316 \pm 0.2291) F_{18.4} \text{ days} \\
\tilde{\omega} &= (-0.7490 \pm 0.0443) F_{18.4} \text{ day}^{-1} \\
t_{cc} &= (1349.1066 \pm 0.0006) \text{ days} \\
\Delta t &= (0.1707 \pm 0.0009) \text{ days} \\
F_{cc} &= (3.5671 \pm 0.0264) F_{18.4} \\
\Gamma &= (0.5192 \pm 0.0427)
\end{align*}
\]
minimization includes all the $I$ and $V$ data points for successive fitting to the full expression for magnification, accounting for effects of a finite source size and limb darkening.

As described in Albrow et al. (1999c), the third step makes use of a variant of equation (2.8) to evaluate the magnified flux in the neighborhood of the caustic crossing. Albrow et al. (1999c) found that, for MACHO 98-SMC-1, this analytic expression was an extremely good approximation to the results of numerical integration and assumed that the same would be the case for any fold crossing. Unfortunately, we find that, for OGLE-1999-BUL-23, this approximation deviates from the true magnification as determined using the method of Gould & Gaucherel (1997) as much as 4%, which is larger than our typical photometric uncertainty in the region of caustic crossing. To maintain the computational efficiency of Albrow et al. (1999c), we continue to use the analytic formula (2.8), but correct it by pretabulated amounts given by the fractional difference (evaluated close to the best solution) between this approximation and the values found by numerical integration. We find that this correction works quite well even at the local minimum for the other (close-binary) solution; the error is smaller than 1%, and in particular, the calculations agree within 0.2% for the region of primary interest. The typical (median) photometric uncertainties for the same region are 0.015 mag (Canopus after the error normalization) and 0.020 mag (Perth). In addition, we
test the correction by running the fitting program with the exact calculation at the
minimum found using the corrected approximation, and find that the measured
parameters change less than the precision of the measurement. In particular,
the limb-darkening coefficients change by an order of magnitude less than the
measurement uncertainty due to the photometric errors.

The results of the refined $\chi^2$ minimization are listed in Table 2.2 for three
discrete “solutions” and in Table 2.3 for grid points neighboring the best-fit solution
whose $\Delta \chi^2$ is less than one. The first seven columns describe the seven standard
geometric parameters of the binary-lens model. The eighth column is the time of
the second caustic crossing. The linear limb-darkening coefficients for $I$ and $V$
bands, $\Gamma_I$ and $\Gamma_V$, are shown in the next two columns (see eq. [2.5a]). The final
column is

$$\Delta \chi^2 \equiv \frac{\chi^2 - \chi^2_{\text{best}}}{\chi^2_{\text{best}}/\text{dof}},$$

(2.9)
defined as in Albrow et al. (1999c). The lightcurve (in magnification) of the best-fit
model is shown in Figure 2.3 together with all the data points used in the analysis.
<table>
<thead>
<tr>
<th>$d$</th>
<th>$q$</th>
<th>$\alpha^a$</th>
<th>$u_0^b$</th>
<th>$\rho_*$</th>
<th>$t_E$</th>
<th>$t_0^b$</th>
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<td>0.</td>
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Table 2.2: PLANET solutions for OGLE-1999-BUL-23

$^a$The lens system is on the righthand side of the moving source.

$^b$the closest approach to the midpoint of the lens system

$^c$LD $\equiv$ Limb Darkening
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<td>(×10⁻³)</td>
<td>(days)</td>
<td>(HJD)</td>
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Table 2.3: Models in the neighborhood of the best-fit solution
For typical binary-lens microlensing events, more than one solution often fits the observations reasonably well. In particular, Dominik (1999) predicted a degeneracy between close and wide binary lenses resulting from a symmetry in the lens equation itself, and such a degeneracy was found empirically for MACHO 98-SMC-1 (Albrow et al. 1999c; Afonso et al. 2000).

We also find two distinct local $\chi^2$ minima (§ 2.4.1) that appear to be closely related to such degeneracies. However, in contrast to the case of MACHO 98-SMC-1, our close-binary model for OGLE-1999-BUL-23 has substantially higher $\chi^2$ than the wide-binary model ($\Delta \chi^2 = 127.86$). Figure 2.4 shows the predicted lightcurves in SAAO instrumental $I$ band. The overall geometries of these two models are shown in Figures 2.5 and 2.6. The similar morphologies of the caustics with respect to the path of the source is responsible for the degenerate lightcurves near the caustic crossing (Fig. 2.6). However, the close-binary model requires a higher blending fraction and lower baseline flux than the wide-binary solution because the former displays a higher peak magnification ($A_{\text{max}} \sim 50$ vs. $A_{\text{max}} \sim 30$). Consequently, a precise determination of the baseline can significantly contribute to discrimination between the two models, and in fact, the actual data did constrain the baseline well enough to produce a large difference in $\chi^2$.

A fair number of preevent baseline measurements are available via OGLE, and those data can further help discriminate between these two “degenerate” models.
Fig. 2.3.— Magnification curve of the best-fit model taken from Table 2.2 for $(d,q) = (2.42, 0.39)$. Data included are SAAO (circles), Perth (inverted triangles), Canopus (triangles), and CTIO (diamonds). Filled symbols are for $I$ band, and open symbols are for $V$ band. Bottom panel shows a close-up of the time interval surrounding the second caustic crossing.
Fig. 2.4.— $I$-band lightcurves of the two “degenerate” models in SAAO instrumental $I$ magnitude. The solid line shows the best-fit model of a wide binary lens, $(d,q)=(2.42,0.39)$, and the dotted line shows the close binary-lens model, $(d,q)=(0.56,0.56)$. The filled circles show SAAO data points. Both models are taken from Table 2.2 and use the estimates of blending factors and baselines. The top panel is for the whole lightcurve covered by the data, and the bottom panel is for the caustic-crossing part only.
Fig. 2.5.— Lens geometries of the two “degenerate” models. The origin of the coordinate system is the geometric center of the binary lens, and the cross marks the center of the mass of the lens system. One unit of length corresponds to $\theta_E$. Closed curves are the caustics, and the positions of the binary lens components are represented by circles, with the filled circle being the more massive component. The trajectory of the source relative to the lens system is shown by arrows, the lengths of which are $2\theta_E$. 

$$(d,q)=(2.42,0.39)$$

$$(d,q)=(0.56,0.56)$$
Fig. 2.6.— Caustics of the two “degenerate” models with respect to the source path shown as a horizontal line. The similarity of the lightcurves seen in Fig. 2.4 is due to the similar morphology of the caustics shown here.
We fit OGLE measurements to the two models with all the model parameters being fixed and allowing only the baseline and the blending fraction as free parameters. We find that the PLANET wide-binary model produces $\chi^2 = 306.83$ for 169 OGLE points ($\chi^2$/dof = 1.83, c.f., Table 2.1) while $\chi^2 = 608.22$ for the close-binary model for the same 169 points (Fig. 2.7). That is, $\Delta \chi^2 = 164.04$, so that the addition of OGLE data by itself discriminates between the two models approximately as well as all the PLANET data combined. The largest contribution to this large $\Delta \chi^2$ appears to come from the period about a month before the first caustic crossing which is well covered by the OGLE data but not by the PLANET data. In particular, the close-binary model predicts a bump in the lightcurve around HJD $\approx 2451290$ due to a triangular caustic (see Fig. 2.5), but the data do not show any abnormal feature in the same region, although it is possible that binary orbital motion moved the caustic far from the source trajectory (e.g., Afonso et al. 2000). In brief, the OGLE data strongly favor the wide-binary model.

2.5. Limb Darkening Coefficients Measurement

Among our six data sets, data from SAAO did not contain points that were affected by limb darkening, i.e., caustic crossing points. Since the filters used at different PLANET observatories do not differ significantly from one another, we use the same limb-darkening coefficient for the three remaining $I$-band data sets.
Fig. 2.7.— Solid line shows the lightcurve of the best-fit PLANET model (wide w/LD), and the dotted line shows the PLANET close-binary model (close w/LD). The models are determined by fitting PLANET data only, but the agreement between the PLANET model (wide w/LD) and OGLE data is quite good. On the other hand, OGLE data discriminate between the two “degenerate” PLANET models so that the wide-binary model is very much favored, in particular, by the observations in (2451250 < HJD < 2451330). The baseline of the PLANET model (wide w/LD) is, $I_{OGLE} = 17.852 \pm 0.003$, which is consistent with the value reported by OGLE, $I_{OGLE} = 17.850 \pm 0.024$. 


diagram
The $V$-band coefficient is determined only from Canopus data, so that a single coefficient is used automatically.

For the best-fit lens geometry, the measured values of linear limb-darkening coefficients are $\Gamma_I = 0.534 \pm 0.020$ and $\Gamma_V = 0.711 \pm 0.089$, where the errors include only uncertainties in the linear fit due to the photometric uncertainties at fixed binary-lens model parameters. However, these errors underestimate the actual uncertainties of the measurements because the measurements are correlated with the determination of the seven lens parameters shown in Tables 2.2 and 2.3. Incorporating these additional uncertainties in the measurement (see the next section for a detailed discussion of the error determination), our final estimates are

$$
\Gamma_I = 0.534^{+0.050}_{-0.040} ; \quad \Gamma_V = 0.711^{+0.098}_{-0.095}, \quad (2.10a)
$$

$$
c_I = 0.632^{+0.047}_{-0.037} ; \quad c_V = 0.786^{+0.080}_{-0.078}. \quad (2.10b)
$$

This is consistent with the result of the caustic-crossing fit of § 2.4.1 ($\Gamma_I = 0.519 \pm 0.043$). Our result suggests that the source is more limb-darkened in $V$ than in $I$, which is generally expected by theories. Figure 2.8 shows the $I$-band residuals (in mag) at the second caustic crossing from our best-fit models for a linearly limb-darkened and a uniform disk model. It is clear that the uniform disk model exhibits larger systematic residuals near the peak than the
linearly limb-darkened disk. From the residual patterns, the uniform disk model
produces a shallower slope for the most of the falling side of the second caustic
crossing than the data requires; one can infer that the source should be more
centrally concentrated than the model predicts, and consequently the presence of
limb darkening. The linearly limb-darkened disk reduces the systematic residuals
by a factor of \(~ 5\). Formally, the difference of \(\chi^2\) between the two models is 172.8
with two additional parameters for the limb-darkened disk model, i.e., the data
favor a limb-darkened disk over a uniform disk at very high confidence.

Because of the multiparameter character of the fit, a measurement of any
parameter is correlated with other parameters of the model. The limb-darkening
coefficients obtained with the different model parameters shown in Table 2.3
exhibit a considerable scatter, and in particular, for the \(I\)-band measurement,
the scatter is larger than the uncertainties due to the photometric errors. This
indicates that, in the measurement of the limb-darkening coefficients, we need to
examine errors that correlate with the lens model parameters in addition to the
uncertainties resulting from the photometric uncertainties at fixed lens parameters.
This conclusion is reinforced by the fact that the error in the estimate of \(\Gamma\) from
the caustic-crossing fit (see Fig. 2.2), which includes the correlation with the
parameters of the caustic-crossing, is substantially larger than the error in the
linear fit, which does not.
Fig. 2.8.— Residuals from PLANET models of OGLE-1999-BUL-23 around the second caustic crossing. Top panel shows the residual for a model incorporating linear limb darkening (wide w/LD) and lower panel shows the same for a uniform disk model (wide no-LD). Both models are taken from Table 2.2. Symbols are the same as in Fig. 2.3. The residuals from the uniform disk are consistent with the prediction that the source is limb-darkened while the remaining departures from the limb-darkened model, which are marginally significant, may be due to nonlinearity in the surface brightness profile of the source star.
Since limb darkening manifests itself mainly around the caustic crossing, its measurement is most strongly correlated with $\Delta t$ and $t_{cc}$. To estimate the effects of these correlations, we fit the data to models with $\Delta t$ or $t_{cc}$ fixed at several values near the best fit – the global geometry of the best fit, i.e., $d$ and $q$ being held fixed as well. The resulting distributions of $\Delta \chi^2$ have parabolic shapes as a function of the fit values of the limb-darkening coefficient and are centered at the measurement of the best fit. (Both, $\Delta t$ fixed and $t_{cc}$ fixed, produce essentially the same parabola, and therefore we believe that the uncertainty related to each correlation with either $\Delta t$ or $t_{cc}$ is, in fact, same in nature.) We interpret the half-width of the parabola at $\Delta \chi^2 = 1$ ($\delta \Gamma_I = 0.031$, $\delta \Gamma_V = 0.032$) as the uncertainty due to the correlation with the caustic-crossing parameters at a given global lens geometry of a fixed $d$ and $q$.

Although the global lens geometry should not directly affect the limb darkening measurement, the overall correlation between local and global parameters can contribute an additional uncertainty to the measurement. This turns out to be the dominant source of the scatter found in Table 2.3. To incorporate this into our final determination of errors, we examine the varying range of the measured coefficients over $\Delta \chi^2 \leq 1$. The result is apparently asymmetric between the directions of increasing or decreasing the amounts of limb darkening. We believe that this is real, and thus we report asymmetric error bars for the limb-darkening measurements.
The final errors of the measurements reported in equations (2.10) are determined by adding these two sources of error to the photometric uncertainty in quadrature. The dominant source of errors in the $I$-band coefficient measurement is the correlation between the global geometry and the local parameters, whereas the photometric uncertainty is the largest contribution to the uncertainties in the $V$-band coefficient measurement.

Although the measurements of $V$- and $I$-band limb darkening at fixed model parameters are independent, the final estimates of two coefficients are not actually independent for the same reason discussed above. (The correlation between $V$ and $I$ limb-darkening coefficients is clearly demonstrated in Table 2.3.) Hence, the complete description of the uncertainty requires a covariance matrix.

$$
\mathbf{C} = \mathbf{C}_{\text{phot}} + \tilde{\mathbf{C}}_{\text{cc}}^{1/2} \begin{pmatrix} 1 & \xi \\ \xi & 1 \end{pmatrix} \tilde{\mathbf{C}}_{\text{cc}}^{1/2} + \tilde{\mathbf{C}}_{\text{geom}}^{1/2} \begin{pmatrix} 1 & \xi \\ \xi & 1 \end{pmatrix} \tilde{\mathbf{C}}_{\text{geom}}^{1/2},
$$

(2.11a)

$$
\mathbf{C}_{\text{phot}} \equiv \begin{pmatrix} \sigma_{V,\text{phot}}^2 & 0 \\ 0 & \sigma_{I,\text{phot}}^2 \end{pmatrix},
$$

(2.11b)

$$
\tilde{\mathbf{C}}_{\text{cc}}^{1/2} \equiv \begin{pmatrix} \sigma_{V,\text{cc}} & 0 \\ 0 & \sigma_{I,\text{cc}} \end{pmatrix},
$$

(2.11c)
\[ \tilde{\mathbf{c}}_{\text{geom}}^{1/2} = \begin{pmatrix} \tilde{\sigma}_{V,\text{geom}} & 0 \\ 0 & \tilde{\sigma}_{I,\text{geom}} \end{pmatrix}, \]  

(2.11d)

where the subscript \( \text{phot} \) denotes the uncertainties due to the photometric errors, \( \text{cc} \) denotes the correlation with \( \Delta t \) and \( t_{\text{cc}} \) at a fixed \( d \) and \( q \), \( \text{geom} \) denotes the correlation with the global geometry, and \( \xi \) is the correlation coefficient between \( \Gamma_V \) and \( \Gamma_I \) measurement. We derive the correlation coefficient using each measurement of \( \Gamma_V \) and \( \Gamma_I \), and the result indicates that two measurements are almost perfectly correlated (\( \xi = 0.995 \)). We accommodate asymmetry of the errors by making the error ellipse off-centered with respect to the best estimate. (See § 2.7 for more discussion on the error ellipses.)

### 2.6. Physical Properties of the Source Star

Figure 2.9 shows color-magnitude diagrams (CMDs) derived from a \( 2' \times 2' \) SAAO field and a \( 4' \times 4' \) Canopus field centered on OGLE-1999-BUL-23 with positions marked for the unmagnified source (S), the baseline (B), blended light (BL) at median seeing, and the center of red clump giants (RC). The source position in these CMDs is consistent with a late G or early K subgiant in the Galactic bulge (see below). Using the color and magnitude of red clump giants in the Galactic bulge reported by Paczyński et al. (1999) \([I_{\text{RC}} = 14.37 \pm 0.02, (V-I)_{\text{RC}} = 1.114 \pm 0.003]\), we measure the reddening-corrected color and magnitude.
of the source in the Johnson-Cousins system from the relative position of the source with respect to the center of red clump in our CMDs, and obtain

\[(V-I)_{s,0} = 1.021 \pm 0.044,\]  
(2.12a)

\[V_{s,0} = 18.00 \pm 0.06,\]  
(2.12b)

where the errors include the difference of the source positions in the two CMDs, but may still be somewhat underestimated because the uncertainty in the selection of red clump giants in our CMDs has not been quantified exactly.

From this information, we derive the surface temperature of the source; \(T_{\text{eff}} = 4830 \pm 100\) K, using the color calibration in Bessell, Castelli, & Plez (1998) and assuming \(\log g = 3.5\) and the solar abundance. This estimate of temperature is only weakly dependent on the assumed surface gravity and different stellar atmospheric models. To determine the angular size of the source, we use equation (4) of Albrow et al. (2000a), which is derived from the surface brightness-color relation of van Belle (1999). We first convert \((V-I)_{s,0}\) into \((V-K)_{s,0} = 2.298 \pm 0.113\) using the same color calibration of Bessell et al. (1998) and then obtain the angular radius of the source of

\[\theta_* = 1.86 \pm 0.13 \mu\text{as}\]

\[= 0.401 \pm 0.027 \, R_{\odot} \, \text{kpc}^{-1}.\]  
(2.13)
Fig. 2.9.— Color-magnitude diagram (CMD) of the field centered on OGLE-1999-BUL-23. Upper CMD is derived from $2' \times 2'$ SAAO images, and lower CMD is from $4' \times 4'$ Canopus images. The positions of the unlensed source (S), the baseline (B), blended light (BL) at median seeing, and the center of red clump giants (RC), are also shown. The extinction inferred from the (reddened) OGLE magnitude of the source in the $I$ band is, $A_I = 1.18$, which implies $E(V - I) = 0.792$ assuming the extinction law $A_I = 1.49E(V - I)$. 

41
If the source is at the Galactocentric distance (8 kpc), this implies that the radius of the source is roughly $3.2 \, R_{\odot}$, which is consistent with the size of a $\sim 1 \, M_{\odot}$ subgiant ($\log g = 3.4$).

Combining this result with the parameters of the best-fit model yields

$$\theta_E = \frac{\theta_s}{\rho_s} = 0.634 \pm 0.043 \text{ mas},$$

$$\mu = \frac{\theta_E}{t_E} = 13.2 \pm 0.9 \, \mu\text{as day}^{-1} = 22.8 \pm 1.5 \, \text{km s}^{-1} \, \text{kpc}^{-1}. \quad (2.14)$$

This corresponds to a projected relative velocity of $182 \pm 12 \, \text{km s}^{-1}$ at the Galactocentric distance, which is generally consistent with what is expected in typical bulge/bulge or bulge/disk (source/lens) events, but inconsistent with disk/disk lensing. Hence we conclude that the source is in the bulge.

As for properties of the lens, the projected separation of the binary lens is $1.53 \pm 0.10 \, \text{AU kpc}^{-1}$, and the combined mass of the lens is given by

$$M = \frac{c^2}{4G} \theta_E^2 D = 0.395 \pm 0.053 \, M_{\odot} \left( \frac{D}{8 \, \text{kpc}} \right), \quad (2.16)$$

where $D^{-1} = D_L^{-1} - D_S^{-1}$, $D_L$ is the distance to the lens, and $D_S$ is the distance to the source.
2.7. Limb Darkening of the Source

We compare our determination of the linear limb-darkening coefficients to model calculations by Claret, Díaz-Cordovés, & Giménez (1995) and Díaz-Cordovés, Claret, & Giménez (1995). For an effective temperature of $T_{\text{eff}} = 4830 \pm 100$ K and a surface gravity of $\log g = 3.5$, the interpolation of the $V$-band linear limb-darkening coefficients, $c_V$, of Díaz-Cordovés et al. (1995) predicts a value $c_V = 0.790 \pm 0.012$, very consistent with our measurement. However, for the $I$-band coefficient, the prediction of Claret et al. (1995), $c_I = 0.578 \pm 0.008$, is only marginally consistent with our measurement, at the 1.46-$\sigma$ level. Adopting a slightly different gravity does not qualitatively change this general result. Since we believe that the uncertainty in the color of the source is larger than in the limb-darkening coefficients, we also examine the opposite approach to the theoretical calculations – using the measured values of limb-darkening coefficients to derive the effective temperature of the source. If the source is a subgiant ($\log g \simeq 3.5$) as our CMDs suggest, the measured values of the limb-darkening coefficients are expected to be observed in stars of the effective temperature, $T_{\text{eff}} = 4850^{+650}_{-670}$ K for $c_V$ or $T_{\text{eff}} = 4200^{+390}_{-490}$ K for $c_I$. As before, the estimate from the $V$-band measurement shows a better agreement with the measured color than the estimate from the $I$-band. Considering that the data quality of $I$ band is better than $V$ band (the estimated uncertainty is smaller in $I$ than in $V$), this result needs to be explained.
Fig. 2.10.— Comparison of linear limb-darkening coefficients. The measured value from the best model is represented by a cross. One (solid line) and two (dotted line) $\sigma$ error ellipses are also shown. are displayed by dashed lines ($\log g = 3.5$). Model A is taken from Díaz-Cordovés et al. (1995) and Claret et al. (1995), B is from van Hamme (1993), and C is from Claret (1998b). In particular, the predicted values in the temperature range that is consistent with our color measurements ($T_{\text{eff}} = 4820 \pm 110 \text{ K for } \log g = 3.0; T_{\text{eff}} = 4830 \pm 100 \text{ K for } \log g = 3.5; \text{ and } T_{\text{eff}} = 4850 \pm 100 \text{ K for } \log g = 4.0$) are emphasized by thick solid lines. Model C’ is by Claret (1998b) for stars of $T_{\text{eff}} = 4850 \pm 100 \text{ K for } \log g = 4.0$. Although the measured value of the limb-darkening coefficients alone favors this model, the model is inconsistent with our estimation of the proper motion.
In Figure 2.10, we plot theoretical calculations of \((c_I, c_V)\) together with our measured values. In addition to Díaz-Cordovés et al. (1995) and Claret et al. (1995) (A), we also include the calculations of linear limb-darkening coefficients by van Hamme (1993) (B) and Claret (1998b) (C). For all three calculations, the \(V\)-band linear coefficients are generally consistent with the measured coefficients and the color, although van Hamme (1993) predicts a slightly smaller amount of limb darkening than the others. On the other hand, the calculations of the \(I\)-band linear coefficients are somewhat smaller than the measurement except for Claret (1998b) with \(\log g = 4.0\). (However, to be consistent with a higher surface gravity while maintaining its color, the source star should be in the disk, which is inconsistent with our inferred proper motion.) Since \(c_V\) and \(c_I\) are not independent (in both the theories and in our measurement), it is more reasonable to compare the \(I\)- and \(V\)-band measurements to the theories simultaneously. Using the covariance matrix of the measurement of \(\Gamma_I\) and \(\Gamma_V\) (see § sec:3), we derive error ellipses for our measurements in the \((c_I, c_V)\) plane and plot them in Figure 2.10. Formally, at the 1-\(\sigma\) level, the calculations of the linear limb-darkening coefficients in any of these models are not consistent with our measurements. In principle, one could also constrain the most likely stellar types that are consistent with the measured coefficients, independent of a priori information on the temperature and the gravity, with a reference to a model. If we do this, the result suggests either that the surface temperature is cooler than our previous estimate from the color or that
the source is a low-mass main sequence ($\log g \geq 4.0$) star. However, the resulting constraints are not strong enough to place firm limits on the stellar type even if we assume any of these models to be “correct.”

One possible explanation of our general result – the measured $V$-band coefficients are nearly in perfect agreement with theories while the $I$-band coefficients are only marginally consistent – is nonlinearity of stellar limb darkening. Many authors have pointed out the inadequacy of the linear limb darkening in producing a reasonably high accuracy approximation of the real stellar surface brightness profile (Wade & Rucinski 1985; Díaz-Cordovés & Giménez 1992; van Hamme 1993; Claret 1998b). Indeed, Albrow et al. (1999b) measured the two-coefficient square-root limb darkening for a cusp-crossing microlensing event and found that the single-coefficient model gives a marginally poorer fit to the data. The quality of the linear parameterization has been investigated for most theoretical limb-darkening calculations, and the results seem to support this explanation. Van Hamme (1993) defined the quality factors ($Q$ in his paper) for his calculations of limb-darkening coefficients, and for $4000 \, \text{K} \leq T_{\text{eff}} \leq 5000 \, \text{K}$ and $3.0 \leq \log g \leq 4.0$, his results indicate that the linear parameterization is a better approximation for $V$ band than for $I$ band. Similarly, Claret (1998b) provided plots of summed residuals ($\sigma$ in his paper) for his fits used to derive limb-darkening coefficients showing that the $V$-band linear limb-darkening has lower $\sigma$ than $I$-band.
and is as good as the $V$-band square-root limb-darkening near the temperature range of our estimate for the source of OGLE-1999-BUL-23. In fact, Díaz-Cordovés et al. (1995) reported that the $V$-band limb darkening is closest to the linear law in the temperature range $T_{\text{eff}} = 4500 \sim 4750$ K. In summary, the source happens to be very close to the temperature at which the linear limb darkening is a very good approximation in $V$, but is less good in $I$.

The actual value of the coefficient in the linear parameterization of a nonlinear profile may vary depending on the method of calculation and sampling. In order to determine the linear coefficients, models (A) and (C) used a least squares fit to the theoretical (nonparametric) profile by sampling uniformly over $\cos \vartheta$ (see eq. [2.5a]), while model (B) utilized the principle of total flux conservation between parametric and nonparametric profiles. On the other hand, a fold-caustic-crossing event samples the stellar surface brightness by convolving it with a rather complicated magnification pattern (Gaudi & Gould 1999). Therefore, it is very likely that neither of the above samplings and calculations is entirely suitable for the representation of the limb-darkening measurement by microlensing unless the real intensity profile of the star is actually the same as the assumed parametric form (the linear parameterization, in this case). In fact, the most appropriate way to compare the measurement to the stellar atmospheric models would be a direct fit to the (nonparametric) theoretical profile after convolution with the magnification
patterns near the caustics. In the present study, this has not been done, but we hope to make such a direct comparison in the future.
Chapter 3


3.1. Introduction

Although an initial objective of microlensing experiments was to probe the mass scale of compact subluminous objects in the Galactic halo (Paczyński 1986), the determination of the individual lens masses was in general believed not to be possible because the only generically measurable quantity is a degenerate combination of the mass with lens-source relative parallax $\pi_{\text{rel}}$ and proper motion $\mu$ (eq. [2.1]). Rather, it would be necessary to interpret the results statistically to infer an average mass scale of the lenses from the mean timescale of the observed events together with statistical estimates of $\pi_{\text{rel}}$ and $\mu$. However, Gould (1992) showed that there are additional observables that in principle can be used to break
the degeneracy between physical parameters and so yield a measurement of the lens mass $M$. If one independently measures the projected Einstein radius $\tilde{r}_E$

$$\tilde{r}_E \equiv \sqrt{2RS_D}, \quad (3.1)$$

as well as the angular Einstein radius $\theta_E$, then the lens mass $M$ is obtained by

$$M = \frac{c^2}{4G} \tilde{r}_E \theta_E
= 0.1227 \, M_\odot \left( \frac{\tilde{r}_E}{\text{AU}} \right) \left( \frac{\theta_E}{\text{mas}} \right). \quad (3.2)$$

Measurement of $\theta_E$ requires that the Einstein ring be compared with some “angular ruler” on the plane of the sky, while to measure $\tilde{r}_E$, one must compare the Einstein ring with some “physical ruler” in the observer plane.

The only angular ruler to be used for this purpose to date is the angular radius of the source $\theta_*$, which can be estimated from its dereddened color and apparent magnitude and the empirical color/surface-brightness relation (e.g., van Belle 1999). For the subset of microlensing events in which the source passes very close to or directly over a caustic, the finite angular extent of the source affects the lightcurve. The source radius in units of the Einstein radius $\rho_*$ can then be determined through analysis of the finite-source affected lightcurve, and thus, one can infer $\theta_E (= \theta_*/\rho_*)$ as well. While this idea was originally proposed for point-mass lenses, which have pointlike caustics (Gould 1994; Nemiroff & Wickramasinghe 1994; Witt & Mao

50
in practice it has been mainly used for binary lenses, which have linelike caustics and hence much larger cross sections (Alcock et al. 1997, 2000; Albrow et al. 1999a, 2000a; Afonso et al. 2000). In particular, one example of measurements from the caustic-crossing binary-lens event is found in Chapter 2.

All ideas proposed to measure $\tilde{r}_E$ are based on the detection of parallax effects (Refsdal 1966; Grieger, Kayser, & Refsdal 1986; Gould 1992, 1995; Gould, Miralda-Escudé, & Bahcall 1994; Hardy & Walker 1995; Holz & Wald 1996; Honma 1999): either measuring the difference in the event as observed simultaneously from two or more locations, or observing the source from a frame that accelerates substantially during the course of the event. Typically $\tilde{r}_E$ is of the order of several AU, and so the natural scale of the parallax baseline must be $\sim 1$ AU. This is the case for the two methods proposed to measure $\tilde{r}_E$: observing the event from a satellite in solar orbit and simultaneously on Earth (Refsdal 1966; Gould 1995); and detecting the distortion in the lightcurve due to the annual parallax effect (Gould 1992). The latter method utilizes that the reflex of the Earth’s orbital motion (annual parallax) induces a wobbling of the source’s passage through the Einstein ring, which in turn perturbs the lightcurve and $\tilde{r}_E$ can be determined from this perturbation. For this case, it is convenient to reexpress $\tilde{r}_E$ as the microlens parallax,

$$
\pi_E \equiv \frac{\pi_{\text{rel}}}{\theta_E} = \frac{\text{AU}}{\tilde{r}_E}.
$$

(3.3)
To date, this latter method is the only one by which $\tilde{r}_E$ has been measured (Alcock et al. 1995; Mao 1999; Soszyński, et al. 2001; Bond et al. 2001; Smith et al. 2001; Mao et al. 2002; Bennett et al. 2001). Unfortunately, this method requires that the event be rather long, $t_E \gtrsim 90$ days. To the extent that the Earth’s motion can be approximated as rectilinear, it is not possible even in principle to detect this effect. Over the relatively short timescales of typical events $t_E \sim 20$ days, on the other hand, the Earth’s motion can be approximated as uniform acceleration, which can lead to a potentially detectable asymmetry in the lightcurve (Gould et al. 1994).

However, this would yield a measurement of only a “projection” of the microlens parallax, $\pi_E \cos \alpha'$, where $\alpha'$ is the angle between the direction of the lens-source relative proper motion and that of the Earth’s acceleration vector at the peak of the event. Hence, even if this asymmetry were detected, it would give only an upper limit on $\tilde{r}_E$. To unambiguously measure $\pi_E$ requires that the event last long enough for the Earth’s acceleration vector to change substantially while the source lies within the Einstein ring.

However, it is possible to make parallax measurements using much shorter baselines. During a caustic crossing of a binary lens, the flux can change by several magnitudes as the caustic transits the star. The source radius projected onto the observer plane $\theta_\ast D$ can be of the order of $\sim 100 \ R_\odot$ for a main sequence star. Hence, observers on two continents could see fluxes differing by a few percent. This
would again yield only one projection of the microlens parallax but by repeating this procedure for the two caustic crossings of a single event, or by observing one crossing from three continents, one could obtain a full measurement of parallax (Hardy & Walker 1995; Gould & Andronov 1999). An important feature of this approach is that it also allows to measure $\theta_E$, which can be obtained from any caustic crossing. Hence, if this method were ever applied, it would yield the mass through equation (3.2). Unfortunately, the method requires extraordinary cooperation from the event itself, from the observatory directors, and from the weather. Consequently, it has not to date been successfully carried out.

It is expected that the use of the high-precision interferometric astrometry, most notably the *Space Interferometry Mission* (SIM) will be the best way to measure the individual lens mass (Paczyński 1998; Boden, Shao, & Van Buren 1998). With 10-µas level astrometry of SIM, one can directly measure $\pi_{\text{rel}}$. Furthermore, by monitoring the displacements in the light centroid caused by microlensing, $\theta_E$ can be also directly measured (Høg, Novikov, & Polanarev 1995; Miyamoto & Yoshii 1995; Walker 1995). Unlike most techniques mentioned earlier, neither of these measurements requires the event to be of a special class such as long timescale or caustic-crossing events. That is, individual lens masses can be measured for common PSPL events.
Here I propose a new method to measure parallax for a subclass of caustic crossing binaries and apply it to a binary-lens microlensing event EROS BLG-2000-5. The method requires the event to have at least three separate well-defined photometric peaks so that the source position can be precisely located relative to the lens geometry at three distinct times. Two of the peaks should be caused by caustic crossings: an entrance and an exit. It would be best for the third peak to be a cuspy-caustic crossing, but the chance of a two fold-caustic crossing accompanied by an additional cusp crossing is low. In the following, we investigate events where the third peak is due either to a cusp crossing or to a generic cusp approach.

3.2. Overview

If the event has two well-observed caustic crossings, plus a well-observed “cusp approach”, these features then provide nine “empirical parameters” – parameters that can be measured directly from the lightcurve without appealing to a model that depends upon a global lens geometry. They are the maximum flux $F_{\text{max}}$, the half-duration $\Delta t$, and the time at the maximum $t_{\text{max}}$ of each of the three peaks. The uncertainties in the measurements of these quantities are approximately

$$\frac{\sigma(F_{\text{max}})}{F_{\text{max}}} \sim \frac{\sigma(t_{\text{max}})}{\Delta t} \sim \frac{\sigma(\Delta t)}{\Delta t} \sim \frac{\sigma_{\text{ph}}}{\sqrt{N}} = 10^{-3} \left( \frac{N}{100} \right)^{-1/2} \sigma_{\text{ph}} 1\%,$$  (3.4)
where $\sigma_{ph}$ is the mean fractional error of each of the $N$ photometric measurements taken over the bump. Since $\Delta t$ is generally of the order of a day, the errors in $\Delta t$ and $t_{\text{max}}$ measurements can easily be of the order of a minute or smaller. The fact that six independent times can be measured with such precision, typically four or five orders of magnitude smaller than the characteristic timescale of the event $t_E$, is what makes the parallax measurement feasible.

In principle, any localizable feature found in the lightcurve may provide similar empirical parameters. What then makes peaks more significant as information posts than other features such as minima or points of inflection? The answer to this question is that the high precision measurements attainable for peaks can rarely be achieved for empirical parameters associated with the other localizable features, so that their merit as signature beacons is much weaker than for peaks. For instance, minima in microlensing lightcurves typically have widths that are ten times greater than peaks have (and by definition less flux as well). From equation (3.4), $\sigma(t) \propto (\Delta t/F)^{1/2}$, so that it is substantially more difficult to localize minima than peaks. Here, we assume Poisson noise ($\sigma_{ph} = \sqrt{F/F}$), and the same sampling frequency (i.e., $N \propto \Delta t$). Points of inflection are even more difficult to localize than minima.

Ordinarily, under the approximation that the lens-source relative motion is rectilinear, to specify a binary-lens lightcurve requires a set of seven global (or
geometric) parameters \((d, q, t_E, t_0, \alpha, u_0, \rho_\ast)\). However, since the actual motion is not rectilinear, these seven parameters will not be adequate to describe the event with very high precision, and in particular, subtle inconsistencies will be introduced among the nine precisely measured quantities mentioned earlier. We now show how these inconsistencies can lead to a parallax measurement.

### 3.3. Measurement of Parallax from Triple-Peak Events

In practice, the parallax will be measured by multidimensional fitting and subsequent \(\chi^2\) minimization. However, it is instructive for two reasons to identify in a systematic way the features of the event that permit \(\pi_E\) to be measured. First, this enables one to predict when an event will have a measurable \(\pi_E\). Second, there are technical difficulties associated with \(\chi^2\) minimization, and these can be ameliorated if the model parameterization is modified to reflect the underlying physics (Albrow et al. 1999c).

To understand how \(\pi_E\) is measured, we first show how some of the nine empirical parameters are related to one another in the absence of parallax, i.e., \(\pi_E = 0\). We will initially assume that the lens geometry \((d, q)\) and the fluxes \(F_s\) and \(F_b\) are known a priori. Then, at the peak of the cusp approach \(t_{\text{max, ca}}\), the source
position within the Einstein ring $u_{ca}$ can be determined very precisely from the precise measurements of $t_{\text{max,ca}}$ and $F_{\text{max,ca}}$. That is, at this time $t_{\text{max,ca}}$, the source must be somewhere along the cusp-approach ridge line, and its position on that line is determined from the inferred magnification $A = (F_{\text{max,ca}} - F_b)/F_s$.

Now consider the angles at which the source crosses each caustic line, $\phi_1$ and $\phi_2$. The half-duration of a caustic crossing is given by $\Delta t_{cc} = t_E \rho_s \csc \phi$. If this motion is assumed to be rectilinear, then $t_E$ becomes a well-defined quantity over the whole duration of the event, and thus,

$$\frac{\csc \phi_2}{\csc \phi_1} = \frac{\Delta t_{cc,2}}{\Delta t_{cc,1}}.$$  \hspace{1cm} (3.5)

The source trajectory is then the straight line that passes through $u_{ca}$ and has caustic-crossing angles that satisfy equation (3.5). From that equation, this angle can be determined with a precision of

$$\sigma(\alpha) = \frac{1}{|\cot \phi_2 - \cot \phi_1|} \left\{ \frac{\sigma(\Delta t_{cc,2})}{\Delta t_{cc,2}} \right\}^2 + \left[ \frac{\sigma(\Delta t_{cc,1})}{\Delta t_{cc,1}} \right]^2 \right\}^{1/2},$$  \hspace{1cm} (3.6)

which can be estimated using equation (3.4).

Next, we turn to the times of the two caustic crossings $t_{0,cc,1}$ and $t_{0,cc,2}$, at which the center of the source lies on the caustic lines. These are not the same as the times of peak flux $t_{\text{max,cc,1}}$ and $t_{\text{max,cc,2}}$, but they can be determined to the same precision (see eq. [3.4]) and are more convenient to work with. Let $u_{cc,1}$ and $u_{cc,2}$
be the positions in the Einstein ring of the two caustic crossings. For rectilinear motion, these satisfy the vector equation,

$$\frac{u_{cc,2} - u_{cc,1}}{t_{0,cc,2} - t_{0,cc,1}} = \frac{u_{ca} - u_{cc,1}}{t_{max,ca} - t_{0,cc,1}} = \mu_E.$$  \hspace{1cm} (3.7)

Here $\mu_E \equiv \mu/\theta_E$ so that the magnitude of the one side of the equation is the same as $t_E^{-1}$. However, if the acceleration due to the Earth’s orbital motion is not negligible, equation (3.7) will not in general be satisfied. This can be quantified in terms of $\delta t_2$, the difference between the measured value of $t_{0,cc,2}$ and the one that would be predicted from the other measured parameters on the basis of equation (3.7),

$$\delta t_2 \equiv (t_{0,cc,2} - t_{0,cc,1}) - \frac{|u_{cc,2} - u_{cc,1}|}{|u_{ca} - u_{cc,1}|}(t_{max,ca} - t_{0,cc,1}).$$  \hspace{1cm} (3.8)

To evaluate the relation between $\delta t_2$ and $\pi_E$, we make the approximation that the Earth’s acceleration vector projected on the sky $a_{\oplus,\perp}$ is constant for the duration of the caustic crossings and cusp approach. We first note that the magnitude of this acceleration is related to the parallax by

$$|a_{\oplus,\perp}| = \pi_E |\sin \psi| \left( \frac{\Omega}{r_{\oplus}/\text{AU}} \right)^2,$$  \hspace{1cm} (3.9)

where $\psi$ is the angle between the lines of sight toward the Sun and the event from the Earth, $r_{\oplus}$ is the distance between the Sun and the Earth, during the caustic
crossings, and \( \Omega = 2\pi \, \text{yr}^{-1} \). Then, after some algebra, we find

\[
\frac{\delta t_2}{t_3 - t_2} = \frac{|\sin \psi| t_E}{2} \left( \frac{\Omega}{r_\oplus / \text{AU}} \right)^2 \\
\times \left\{ -\pi_{E,\parallel} (t_2 - t_1) + \pi_{E,\perp} \left[ (t_3 - t_2) \cot \phi_2 - (t_3 - t_1) \cot \phi_1 \right] \right\},
\]

(3.10)

where we have used the simplified notations, \( t_{0,cc,i} \rightarrow t_i \) and \( t_{\text{max,ca}} \rightarrow t_3 \) and introduce a two-dimensional vector \( \boldsymbol{\pi}_E \) whose magnitude is \( \pi_E \) and whose direction is that of \( \boldsymbol{\mu} \). Here \( \pi_{E,\parallel} \) and \( \pi_{E,\perp} \) are the components of \( \boldsymbol{\pi}_E \) parallel and perpendicular to \( \mathbf{a}_{\oplus,\perp} \). Hence, by measuring \( \delta t_2 \) one can determine a particular projection of \( \boldsymbol{\pi}_E \) whose components are given by equation (3.10).

However, since \( \boldsymbol{\pi}_E \) is a two-dimensional vector, measurements of two independent components are required for the complete determination of \( \pi_E \). A second constraint is available from the width of the cusp approach \( \Delta t_{ca} \). We define \( \delta \ln \Delta t_{ca} \) in analogy to \( \delta t_2 \) and after some more algebra we find

\[
\delta \ln \Delta t_{ca} = \frac{|\sin \psi| t_E}{\cot \phi_2 - \cot \phi_1} \left( \frac{\Omega}{r_\oplus / \text{AU}} \right)^2 \\
\times \left\{ \pi_{E,\parallel} \left[ (t_3 - t_2) \cot \phi_1 + (t_2 - t_1) \cot \phi_3 - (t_3 - t_1) \cot \phi_2 \right] \\
+ \pi_{E,\perp} \left[ (t_3 - t_1) \cot \phi_3 \cot \phi_1 - (t_3 - t_2) \cot \phi_3 \cot \phi_2 - (t_2 - t_1) \cot \phi_2 \cot \phi_1 \right] \right\}.
\]

(3.11)

Measurement of \( \delta \ln \Delta t_{ca} \) therefore gives another projection of \( \boldsymbol{\pi}_E \). Furthermore, it is possible to obtain a third constraint on \( \boldsymbol{\pi}_E \) from the behavior of the lightcurve.
after the source has left the caustic, but while it still remains within the Einstein ring. For rectilinear motion, the Einstein time scale is well determined (see eq. [3.7]). If the latetime lightcurve drops off faster or slower than indicated by this timescale, it implies that the Earth’s acceleration vector has a component aligned with the direction of lens-source relative motion. The effect is similar to the one identified by Gould et al. (1994) but can be measured more easily because both $t_E$ and $t_0$ are determined very precisely from the lightcurve around the caustic-crossing region.

The relative orientation of these three constraints depends on the details of the lens geometry. In principle, they could all be roughly parallel, but this is unlikely; in general, it should be possible to combine the three projections to measure both components of $\pi_E$. If the event is sufficiently long (typically $t_E \gtrsim 60$ days) that the Earth moves $\gtrsim 1$ radian during an Einstein timescale, then the latetime behavior of the lightcurve will give information about both components of $\pi_E$. In this case there would be the fourth constraint.

The exact expression for the errors in the components of $\pi_E$ obtained from the first two constraints can be derived from equations (3.10) and (3.11), but these are extremely complicated and, for that reason not very interesting. However, reasonable estimates of these errors can be made as follows. First, we note that the error from applying the $\delta t_2$ constraint is dominated by the problem of determining
the change in $\alpha$ from the parallax to the nonparallax case. The error in the measurement of $\alpha$ is given by equation (3.6). On the other hand, the change in $\alpha$ (chosen for definiteness to be the angle at the time of the cusp approach) is given by

$$
(cot \phi_2 - cot \phi_1)\delta \alpha = \frac{2\delta t_2}{t_3 - t_2}.
$$

(3.12)

Hence, the fractional error in $\pi_{E,\delta t_2}$, the projection of $\pi_E$ measured by this constraint, is of the order of

$$
\frac{\sigma(\pi_{E,\delta t_2})}{\pi_{E,\delta t_2}} \sim \frac{\sigma(\alpha)}{\delta \alpha} \sim \frac{N^{-1/2}\sigma_{ph}}{|\sin \psi|\Omega^2(t_2 - t_1)t_E\pi_E},
$$

(3.13)

where we have made use of equations (3.4) and (3.6). For typical bulge parameters, $|\sin \psi| \sim 0.2$ and $\pi_E \sim 0.1$, and with good photometric coverage of the caustic crossings, $N^{-1/2}\sigma_{ph} \sim 10^{-3}$, the fractional error is $\sim 0.17 \times (t_E/50 \text{ days})^{-1} \times [(t_2 - t_1)/20 \text{ days}]^{-1}$ and hence $\pi_{E,\delta t_2}$ should plausibly be measurable. For the other constraint one finds similar expressions.

We now relax our assumption that $d, q, F_s,$ and $F_b$ are known a priori. Actually, if $(d, q)$ are known, $F_s$ and $F_b$ can be easily determined from, for example, the baseline flux $F_s + F_b$, and the minimum flux observed inside the caustic $A_{min}F_s + F_b$. Here $A_{min}$ is the magnification at minimum that, for a fixed $(d, q)$, is virtually independent of the minor adjustments to the trajectory due to parallax.
However, it is still necessary to relax the assumption that \((d,q)\) are known. In fact, these must be determined simultaneously with the parallax because changes in \((d,q)\) can have effects on the relative times of the caustic and cusp crossing and on the crossing angles, just as parallax can. Nevertheless, there are numerous other constraints on \((d,q)\) coming from the overall lightcurve, and so while \((d,q)\) and \(\pi_E\) can be expected to be correlated, they should not be completely degenerate. Hence, even allowing for degradation of the signal-to-noise ratio (S/N) for the \(\pi_E\) measurement due to the correlations between the projections of \(\pi_E\) and \((d,q)\), it should be possible to measure \(\pi_E\) with reasonable precision for events with two well-covered caustic crossings and a well-covered cusp approach.

3.4. Effect of Binary Orbital Motion

The measurement of the parallax discussed in the previous sections essentially relies upon the failure of the rectilinear approximation of the lens-source relative motion when the trajectory is overconstrained by available observations. For a relatively short timescale, what is actually measured from this is an instantaneous acceleration on the source trajectory, and this acceleration may contain significant contributions by other effects, a notable example of which is the binary orbital motion. The orbital motion of the binary lens projected onto the plane of the sky is observable in terms of a contraction or expansion of the binary separation \(\dot{d}\)
and the lateral rotation of the binary axis with respect to the fixed direction on the sky $\omega$. The apparent observable result of the latter effect may heuristically be understood as a centripetal acceleration $\sim u_c \omega^2$ on the source motion relative to the (static) lens system, where $u_c$ is the angular extent of the caustic in units of the Einstein ring. For a face-on circular binary orbit observed at the ecliptic pole, the ratio of the Earth’s acceleration to the projected acceleration due to the binary orbital motion is then

$$\frac{a_{\oplus}}{a_{\text{bin}}} = \frac{\pi E}{u_c \left( \frac{P}{\text{yr}} \right)^2}.$$ \hspace{1cm} (3.14)

Thus, depending on these parameters, either parallax or binary orbital motion could dominate. In addition for the general case, both the Earth and binary acceleration would be reduced by possibly very different projection factors (while the projection factor for the binary orbital motion is from the orbital inclination of the binary, the parallactic projection is due to the angle between the direction to the ecliptic pole and the line of sight to the event). Although the effect of $\dot{d}$ is, in general, not expressible analytically, one can expect it to be of a similar order of magnitude to that of $\omega$.

An unambiguous determination of the parallax therefore requires the measurement of the acceleration at least at two different times or not less than four independent constraints on the projection of the acceleration at different times.
We argued above that there are generically at least three constraints for the types of events under discussion and that there is a fourth constraint for sufficiently long events. In this latter case, parallax can be unambiguously discriminated from projected orbital motion. However, even when the event is short, so that there are only three constraints, it may still be feasible to measure the parallax. We first note that the orbital motion cannot significantly affect the latetime lightcurve – i.e., \( t_E \) is not influenced by the orbital motion – for it does not affect separations between the source and the binary center of mass. Hence, if one component of \( \pi_E \) is measured from the latetime lightcurve, it may be possible using equation (3.14) and Kepler’s Third Law to show that parallax is more important than projected orbital motion, in which case the caustic crossings can be used to determine the full parallax \( \pi_E \).

In brief, there is good reason to hope that events with three well-defined bumps can yield parallax measurements and, hence, mass measurements from the combination with the measurement of the angular Einstein radius. Whether this will be possible for any particular such event can only be determined by detailed modeling.
3.5. EROS BLG-2000-5

So far, we have argued that $\pi_E$ is measurable for a caustic-crossing binary event exhibiting a well-observed peak caused by a cusp approach in addition to the two usual caustic crossings. Here we provide the application of the method to the actual triple-peak caustic-crossing binary microlensing event EROS BLG-2000-5. In addition, the event has a relatively long time scale ($t_E \sim 100$ days), which is generally favorable for the measurement of $\pi_E$.

In fact, EROS BLG-2000-5 features many unique characteristics not only in the intrinsic nature of the event but also in the observations of it. These include a moderately well-covered first caustic crossing (entrance), a timely prediction – of not only the timing but also the duration – of the second caustic crossing (exit), the unprecedented four-day length of the second crossing, and two time-series of spectral observations of the source during the second crossing (Albrow et al. 2001b; Castro et al. 2001). To fully understand and utilize this wealth of information, however, requires detailed quantitative modeling of the event. In the rest of this chapter, we focus on the modeling of the geometry of the event based on the photometric observations made by the PLANET collaboration and find that both parallax and projected binary orbital motion are required to successfully model the lightcurve. Furthermore, as is usual for caustic-crossing events, we also measure the angular Einstein radius from the finite source effect during caustic crossings.
and the source angular size derived from the source position in the CMD. Hence, this is the first event for which both $\tilde{r}_E$ and $\theta_E$ are measured simultaneously and so for which the lens mass is unambiguously measured.

### 3.5.1. Data

On 2000 May 5, the Expérience pour la Recherche d’Objets Sombres (EROS)\(^1\) collaboration issued an alert that EROS BLG-2000-5 was a probable microlensing event ($\text{RA} = 17^h 53^m 11.5^s$, $\text{Dec} = -30^\circ 55' 35''$; $l = 359^\circ 14'$, $b = -2^\circ 43'$). On 2000 June 8, the Microlensing Planet Search (MPS)\(^2\) collaboration issued an anomaly alert, saying that the source had brightened by 0.5 mag from the previous night and had continued to brighten by 0.1 mag in 40 minutes. PLANET intensified its observations immediately, and has been continuing to monitor the event up to the 2001 season. Observations for PLANET were made from four telescopes: the Canopus 1 m near Hobart, Tasmania, Australia; the Perth/Lowell 0.6 m at Perth Observatory, Bickley, Western Australia, Australia; the Elizabeth 1 m at SAAO, Sutherland, South Africa; and the YALO 1 m at CTIO, La Serena, Chile. Data were taken in $V$ (except Perth), $I$ (all four sites), and $H$ (SAAO and YALO) bands. For the present study, we make use primarily of the $I$ band data. That is, we fit for the model using only $I$ band data, while the $V$ band data are used (together with

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\(^1\) [http://www-dapnia.cea.fr/Spp/Experiences/EROS/alertes.html](http://www-dapnia.cea.fr/Spp/Experiences/EROS/alertes.html)

\(^2\) [http://bustard.phys.nd.edu/MPS/](http://bustard.phys.nd.edu/MPS/)
the \( I \) data) only to determine the position of the source on CMD (see § 3.8). The lightcurve (Fig. 3.1) exhibits two peaks that have the characteristic forms of an entrance (A) and exit (B) caustic crossing (Schneider, Ehlers, & Falco 1992, also see fig. 1 of Gould & Andronov 1999) immediately followed by a third peak (C) which is caused by the passage of the source close to a cusp.

The data have been reduced in a usual way and the photometry on them has been performed by point spread function (PSF) fitting using DoPHOT (Schechter et al. 1993). The relative photometric scaling between the different telescopes is determined as part of the fit to the lightcurve which includes the independent determinations of the source and the background fluxes at each telescope. We find, as was the case for several previous events, that due to the crowdedness of the field, the moderate seeing conditions, and possibly some other unidentified systematics, the amount of blended light entering the PSF is affected by seeing, and that the formal errors reported by DoPHOT tend to underestimate the actual photometric uncertainties. We tackle these problems by incorporating a linear seeing correction for the background flux and rescaling the error bars to force the reduced \( \chi^2 \) of our best model to unity. For details of these procedures, refer Chapter 2 as well as Albrow et al. (2000a,b) and Gaudi et al. (2002).

We also analyze the data by difference imaging, mostly using ISIS (Alard & Lupton 1998; Alard 2000). We compare the scatter of the photometry on
Fig. 3.1.— PLANET $I$-band lightcurve of EROS BLG-2000-5 (the 2000 season only). Only the data points used for the analysis (“cleaned high-quality” subset; see § 3.5.1) are plotted. Data shown are from SAAO (circles), Canopus (triangles), YALO (squares), and Perth (inverted triangles). All data points have been deblended using the fit result – also accounting for the seeing correction – and transformed to the standard $I$ magnitude; $I = I_s - 2.5 \log[(F(t) - F_b)/F_s]$. The calibrated source magnitude ($I_s = 16.602$) is shown as a dotted line. The three bumps in the lightcurve and the corresponding positions relative to the microlens geometry are also indicated.
the difference images to that of the direct PSF fit photometry by deriving the normalized-summed squares of the S/Ns:

\[ Q \equiv \sum_i \left( \frac{F_s A_i}{\tilde{\sigma}_i} \right)^2 = \frac{N}{\chi^2} \sum_i \left( \frac{F_s A_i}{\sigma_i} \right)^2, \]

(3.15)

where \( F_s \) is the source flux derived from the model, \( A_i \) is the magnification predicted by the model for the data point, \( \sigma_i \) and \( \tilde{\sigma}_i \) are the photometric uncertainty (in flux) for individual data before and after rescaling the error bars [i.e., \( \tilde{\sigma}_i = \sigma_i (\chi^2/N)^{1/2} \)], and \( N \) is the number of the data points. Strictly speaking, \( Q \) defined as in equation (3.15) is model-dependent, but if the data sets to be compared do not differ with one another in systematic ways (and the chosen model is close enough to the real one), \( Q \) can be used as a proxy for the relative statistical weight given by the data set without notable biases. We find that difference imaging significantly improves the stability of the photometry for the data from Canopus and Perth, but it somewhat worsens the photometry for the data from SAAO and YALO. We suspect that the result is related to the overall seeing condition for the specific site, but a definite conclusion will require more detailed study and would be beyond the scope of the current research. We hope to be able to further improve the photometry on difference images in the future, but for the current analysis, we choose to use the data set with the better \( Q \) so that only for Canopus and Perth data sets, we replace the result of the direct PSF photometry with the difference imaging analysis result.
For the final analysis and the results reported here, we have used only a “high quality” subset of the data. Prior to any attempt to model the event, we first exclude various faulty frames and problematic photometry reported by the reduction/photometry software. In addition, data points exhibiting large (formal) errors and/or poor seeing compared to the rest of the data from the same site are eliminated prior to the analysis. In particular, the thresholds for the seeing cuts are chosen at the point where the behavior of “seeing-dependent background” becomes noticeably nonlinear with the seeing variation. The criterion of the seeing and error cut for each data set is reported in Table 3.1 together with other photometric information. In conjunction with the proper determination of the error rescaling factors, we also remove isolated outliers as in the case of OGLE-1999-BUL-23 presented in Chapter 2. Following these steps, the “cleaned high-quality” $I$-band data set consists of 1286 (= 403 SAAO + 333 Canopus + 389 YALO + 161 Perth) measurements made during the 2000 season (between May 11 and November 12) plus 60 additional observations (= 25 SAAO + 35 YALO) made in the 2001 season. Finally, we exclude 49 data points (= 19 SAAO + 20 Canopus + 10 Perth) that are very close to the cusp approach $[2451736.8 < \text{HJD} < 2451737.6]$ while we fit the lightcurve. We find that the limited numerical resolution of the source, which in turn is dictated by computational considerations, introduces errors in the evaluation of $\chi^2$ in this region of the order of a few, and in a way that does not smoothly depend on the parameters. These would prevent us from finding the
true minimum, or properly evaluating the statistical errors. However, for the final model, we evaluate the predicted fluxes and residuals for these points. As we show in § 3.7, these residuals do not differ qualitatively from other residuals to the fit.

3.6. PARAMETERIZATION

While to specify the lightcurve of a static binary event with a rectilinear source trajectory requires a set of seven geometric parameters ($d$, $q$, $t_E$, $t_0$, $\alpha$, $u_0$, $\rho_*$), to generally incorporate the annual parallax and the projection of binary orbital motion into the model, one needs four additional parameters. However, their inclusion, especially of the parallax parameters, is not a trivial procedure, since the natural coordinate basis for the description of the parallax is the ecliptic system while the binary magnification pattern possesses its own preferred direction, i.e., the binary axis. In the following, we establish a consistent system to describe the complete set of the eleven geometric parameters.
3.6.1. Description of Geometry

First, we focus on the description of parallax. On the plane of the sky, the angular positions of the lens and the source (seen from the center of the Earth) are expressed generally by,

\[ \varphi_S(t) = \varphi_{S,c} + (t - t_c)\mu_S + \pi_S \varsigma(t), \quad (3.16a) \]

\[ \varphi_L(t) = \varphi_{L,c} + (t - t_c)\mu_L + \pi_L \varsigma(t). \quad (3.16b) \]

Here \( \varphi_{S,c} \) and \( \varphi_{L,c} \) are the positions of the lens and the source at some reference time, \( t = t_c \), as they would be observed from the Sun, \( \mu_S \) and \( \mu_L \) are the (heliocentric) proper motion of the source and the lens, \( \pi_S \) and \( \pi_L \) are the annual trigonometric parallax, and \( \varsigma(t) \) is the Sun’s position vector with respect to the Earth, projected onto the plane of the sky and normalized by an astronomical unit (see Appendix B). At any given time, \( t \), \( \varsigma(t) \) is completely determined with respect to an ecliptic coordinate basis, once the event’s (ecliptic) coordinates are known. For example, in the case of EROS BLG-2000-5 (\( \lambda = 268.53, \beta = -7.50 \)), \( \varsigma = (0, r_\oplus \sin \beta) \) at approximately 2000 June 19, where \( r_\oplus \) is the distance between the Sun and the Earth at this time in astronomical units (\( r_\oplus = 1.016 \)). Then, the angular separation vector between the source and the lens in units of \( \theta_E \) becomes

\[ u(t) \equiv \frac{\varphi_S - \varphi_L}{\theta_E} = \nu + (t - t_c)\mu_E - \pi_E \varsigma(t), \quad (3.17) \]
where $\mathbf{v} \equiv (\varphi_{S,c} - \varphi_{L,c})/\theta_E$. Although equation (3.17) is the most natural form of expression for the parallax-affected trajectory, it is convenient to reexpress equation (3.17) as the sum of the (geocentric) rectilinear motion at the reference time and the parallactic deviations. In order to do this, we evaluate $u$ and $\dot{u}$ at $t = t_c$,

$$u_{t_c} \equiv u(t_c) = \mathbf{v} - \pi_E \varsigma_{t_c},$$  \hspace{1cm} (3.18a)

$$\dot{u}_{t_c} \equiv \dot{u}(t_c) = \mu_E - \pi_E \dot{\varsigma}_{t_c},$$  \hspace{1cm} (3.18b)

where $\varsigma_{t_c} \equiv \varsigma(t_c)$ and $\dot{\varsigma}_{t_c} \equiv \dot{\varsigma}(t_c)$. Solving equations (3.18) for $\mu_E$ and $\mathbf{v}$ and substituting them into equation (3.17), one obtains

$$u(t) = u_{t_c} + (t - t_c) \dot{u}_{t_c} - \pi_E \mathbf{D}_P,$$  \hspace{1cm} (3.19)

where $\mathbf{D}_P \equiv \varsigma(t) - \varsigma_{t_c} - (t - t_c) \dot{\varsigma}_{t_c}$ is the parallactic deviation. Note that $\mathbf{D}_P \simeq (\dot{\varsigma}_{t_c}/2)(t - t_c)^2$ for $t \sim t_c$, i.e., on relatively short time scales, the effect of the parallax is equivalent to a uniform acceleration of $-\pi_E \dot{\varsigma}_{t_c}$. Equation (3.19) is true in general for any microlensing event including PSPL events.

Next, we introduce the binary lens system. Whereas the parallax-affected trajectory (eq. [3.19]) is most naturally described in the ecliptic coordinate system, the magnification pattern of the binary lens is specified with respect to the binary
axis. Hence, to construct a lightcurve, one must transform the trajectory from
ecliptic coordinates to the binary coordinates. If the origins of both coordinates are
chosen to coincide at the binary center of mass, this transformation becomes purely
rotational. Thus, this basically adds one parameter to the problem: the orientation
of the binary axis in ecliptic coordinates. In accordance with the parameterization
of the projected binary orbital motion, one may express this orientation using the
binary separation vector, $d$, whose magnitude is $d$ and whose direction is that of
the binary axis (to be definite, pointing from the less massive to the more massive
component). With this parameterization, the projection of the binary orbital
motion around its center of mass is readily facilitated via the time variation of $d$.
If the time scale of the event is relatively short compared to the orbital period
of the binary, then rectilinear relative lens motion, $d = d_{t_c} + \dot{d}_{t_c}(t - t_c)$, will be
an adequate representation of the actual variation for most applications (see e.g.,
Albrow et al. 2000a). Then, the lightcurve of a rotating binary event with parallax
is completely specified by a set of eleven independent parameters ($d_{t_c}, \dot{d}_{t_c}, u_{t_c}, \dot{u}_{t_c},$
$q, \rho_*, \pi_E$). However, one generally chooses to make $t_c$ an independent parameter,
such as the time when $u_{t_c} \cdot \dot{u}_{t_c} = 0$. In that case, the set of eleven parameters
become ($d_{t_c}, \dot{d}_{t_c}, u_{t_c}, \dot{u}_{t_c}, q, \rho_*, \pi_E, t_c$).

Although the parameterization described so far is physically motivated, and
mathematically both complete and straightforward, in practice it is somewhat
cumbersome to implement into the actual fit. Therefore, we reformulate the above parameterization for computational purposes. For the analysis of EROS BLG-2000-5, we first choose the reference time $t_c$ as the time of the closest approach of the source to the cusp, and rotate the coordinate system so that the whole geometry is expressed relative to the direction of $d_{tc}$, i.e., the binary axis at time $t_c$ (see Fig. 3.2). We define the impact parameter for the cusp approach, $u_c$ ($\equiv |u_{tc} - u_{cusp}|$), and set $u_c > 0$ when the cusp is on the righthand side of the moving source. Then, $\dot{u}_{tc}$ is specified by $t'_E$ ($\equiv |u_{tc}|^{-1}$), the instantaneous Einstein timescale at time $t_c$, and by $\alpha'$, the orientation angle of $\dot{u}_{tc}$ with respect to $d_{tc}$.

In addition, we express $\dot{d}_{tc}$ in a polar-coordinate form and use the approximation that both the radial component, $\dot{d}$, and tangential component, $\omega$, are constant.

Under this parameterization, $\dot{d}$ corresponds to the rate of expansion ($\dot{d} > 0$) or contraction ($\dot{d} < 0$) of the projected binary separation while $\omega$ is the angular velocity of the projected binary-axis rotation on the plane of the sky. Finally, we define the microlens parallax vector, $\pi_E$, whose magnitude is $\pi_E$ and whose direction is toward ecliptic west (decreasing ecliptic longitude). In the actual fit, $\pi_{E,||}$ and $\pi_{E,\perp}$, the two projections of $\pi_E$ along and normal to $d_{tc}$, are used as independent parameters. Table 3.2 summarizes the transformation from the set of fit parameters $(d_{tc}, q, \alpha', u_c, t'_E, t_c, \rho_e, \pi_{E,||}, \pi_{E,\perp}, \dot{d}, \omega)$ to the set of the physical parameters $(d_{tc}, \dot{d}_{tc}, u_{tc}, \dot{u}_{tc}, q, \rho_e, \pi_E, t_c)$.
Fig. 3.2.— Geometry used for deriving the transformation shown in Table 3.2. The direction of $\mathbf{d}_c$ is chosen to be the $x$-axis while $\pi_E$ lies parallel to the direction of decreasing ecliptic longitude; $\hat{e}_w$. The reference time, $t_c$ is the time of the closet approach to the cusp, $\mathbf{u}_{cusp}$. 
3.6.2. The Choice of Fit Parameters

Judged by the number of fitting parameters alone, EROS BLG-2000-5 is by far the most complex event ever analyzed: compared to the runner-up, MACHO 99-BLG-47 (Bennett et al. 1999; Albrow et al. 2000a), it has two more geometric parameters and one more limb-darkening parameter. As a direct result, the path toward choosing a modeling procedure was substantially more tortuous than usual. We therefore believe that it is important to document this path, at least in outline, in order to aid in the modeling of future events.

As stated in § 3.6, the seven standard parameters for binary events are \((d, q, t_E, t_0, \alpha, u_0, \rho_*)\). Immediately following the first caustic crossing, we fit this crossing to five empirical parameters, including \(t_{cc,1}\) and \(\Delta t_1\), the time and half-duration of the first crossing (see § 2.4.1). We then changed our choice of parameters \((t_0, \rho_*) \rightarrow (t_{cc,1}, \Delta t_1)\) according to the prescription of Albrow et al. (1999c), effectively cutting the search space down from seven to five dimensions, and speeding up the search accordingly. This permitted us to accurately predict in real time not only the time, but also the (4-day) duration of the second crossing which in turn allowed two groups to obtain large-telescope spectra of the crossing (Albrow et al. 2001b; Castro et al. 2001). This was the first prediction of the duration of a caustic crossing.
Why is the substitution \((t_0, \rho_*) \rightarrow (t_{cc,1}, \Delta t_1)\) critical? Both \(t_{cc,1}\) and \(\Delta t_1\) are determined from the data with a precision \(\sim 10^{-3}\) day \(\sim 10^{-5}\) \(t_E\). Hence, if any of the parameters, \(\alpha, u_0, t_0, \rho_*,\) are changed individually by \(10^{-4}\) (subsequent changes of \(t_{cc,1}\) and \(\Delta t_1\) by \(10^{-4}\) \(t_E\)), this will lead to an increase \(\Delta \chi^2 \sim 100\). As a result, even very modest movements in parameter space must be carefully choreographed to find a downhill direction on the \(\chi^2\) hypersurface. By making \((t_{cc,1}, \Delta t_1)\) two of the parameters and constraining them to very small steps consistent with their statistical errors, one in effect automatically enforces this choreography.

In all the work reported here, we searched for \(\chi^2\) minima at fixed \((d,q)\), and repeated this procedure over a \((d,q)\) grid. We found for this event (as we have found for others) that regardless of what minimization technique we apply, if we search \((d,q)\) space simultaneously with the other parameters, then either we do not find the true \(\chi^2\) minimum or the search requires prohibitive amounts of time.

Following the second crossing, we added a linear limb-darkening (LD) parameter, but otherwise continued with the same parameterization. We found that the fitting process was then enormously slowed down because small changes \(\alpha, u_0, \) or \(t_E\) led to large changes in \(t_{cc,2}\) and \(\Delta t_2\) (the second crossing time and half-duration), whereas these quantities were directly fixed by the data. We therefore changed parameters \((\alpha, u_0, t_E) \rightarrow (t_{cc,2}, \Delta t_2, t_{axis})\), where \(t_{axis}\) is the time the source crossed the cusp axis. Hence, all five of the non-(\(d,q)\) parameters were
fixed more-or-less directly by the data, which greatly improved the speed of our parameter search. We thus quickly found the $\chi^2$ minimum for this (7+1)-parameter – seven geometry plus one limb-darkening – space. (Note that the LD parameter, like the source flux, the background flux, and the seeing correlation, is determined by linear fitting after each set of other parameters is chosen. Hence, it exacts essentially zero computational cost. We therefore track it separately.)

Since the lightcurve showed systematic residuals of several percent (compared to daily-averaged photometry errors $\ll 1\%$), we were compelled to introduce more parameters. We first added two parallax parameters; the magnitude $\pi_E$ and its relative orientation with respect to the binary, yielding a (9+1)-parameter fit. Since five of the nine geometrical parameters remained empirical, this procedure also converged quickly. However, while $\chi^2$ had fallen by several hundred (indicating a very significant detection of parallax), the problem of systematic residuals was not qualitatively ameliorated. This created something of a crisis. We realized that further improvements would be possible if we allowed for binary orbital motion. Lacking apparent alternatives, we went ahead and introduced projected binary orbital motion. This led to radical changes in our now (11+1)-parameterization. Allowing two dimensions of binary motion meant that both the orientation and the size of the binary separation could change. The latter induced changes in both the size and shape of the caustic, and so made it essentially impossible to define
the time parameters $\Delta t_{cc}$ in such a way that was at the same time mathematically consistent and calculable in a reasonable amount of time. We therefore went back to something very like the original geometric parameterization but with four additional parameters ($d$, $q$, $t_E$, $t_0$, $\alpha$, $u_0$, $\rho_*$, $\pi_E$, $\dot{d}$, $\omega$). Here the direction of $\pi_E$ contains the information on the direction of the binary axis relative to the line of ecliptic latitude at time $t_0$, when the source is closest to the binary center of mass in the Sun's frame of the reference. The binary orbital motion is incorporated via $\dot{d}$ and $\omega$. The quantities, $\alpha$, $u_0$, and $t_E$ are all given in the frame of the reference of the Sun as well.

However, while this parameterization has the advantage of mathematical simplicity, it would have introduced severe instabilities into the fitting procedure. At bottom, the problem is the same as the one that led to the substitution $(t_0, \rho_*) \rightarrow (t_{cc,1}, \Delta t_1)$ described above, but substantially more damaging. This is because the microlens parallax is a relatively poorly constrained quantity. Yet, within the framework of this parameterization, a change of the trial value of $\pi_E$ of only 1% (by itself) would shift $t_{cc,1}$ by $\sim 0.3$ day, and so induce $\Delta \chi^2 \sim 1000$. Thus, to avoid such huge $\chi^2$ jumps, even more careful choreography would have been required.

Instead, we made the following changes to the parameter scheme. First, we made the reference time, $t_c$, be the time of the closest approach of the source to the
cusp, rather than the closest approach to the center of mass. Second, we adopted, for the frame of reference, the frame of the Earth at \( t_c \) rather than the frame of the Sun. Our final choice of parameters is \((d_c, q, t'_E, t_c, \alpha', u_c, \rho_e, \pi_E, \dot{d}, \omega)\), where \( u_c \) is the impact parameter relative to the cusp and \( \alpha' \) and \( t'_E \) are evaluated at \( t_c \) and in the frame of reference of the Earth at the time. The change of reference frame is responsible for the form of the parallax deviation given in equation (3.19), but from a practical point of view it is very helpful so that the non-parallax parameters do not change very much when the parallax is changed: in particular they are similar to the solution without parallax (i.e., \( \pi_E \equiv 0 \)). The particular choice of the reference time \( t_c \) is useful because most of the “action” of the event happens close to this time, either during the cusp approach itself, or during the second caustic crossing a week previously. Hence, both \( t_c \) and \( u_c \) are relatively well fixed by the data, while the angle \( \alpha \) is also relatively well fixed since it is strongly constrained by the cusp approach and second crossing. As a consequence, we are able to find relatively robust minima for each \((d, q)\) grid point in about a single day of computer time, which is quite adequate to reach a global minimum.

Unfortunately, the \((11+1)\)-parameterization failed to qualitatively lessen the problem of systematic residuals. We then recognized that more LD freedom was required, and so added a square-root LD parameter in addition to the linear one. This reduced the systematic residuals to < 1%. As a result, we fit limb
darkening with a two-parameter form so that the final fit we adopted is an (11+2)-parameterization fit.

### 3.6.3. Terrestrial Baseline Parallax

In general, the Earth’s spin adds a tiny daily wobble of order $\sim R_\oplus / \tilde{r}_E$ (eq. [C.3]), where $R_\oplus$ is the Earth’s radius, to the source’s relative position seen from the center of the Earth as expressed in equation (3.17). Since $R_\oplus = 4.26 \times 10^{-5}$ AU, this effect is negligible except when the spatial gradient over the magnification map is very large, e.g., caustic crossings or extreme cusp approaches. Even for those cases, only the instantaneous offsets are usually what matters because the source crosses over the region of extreme gradient with a time scale typically smaller than a day. Hence, unless the coverage of the crossing from two widely separated observers significantly overlaps, the effect has been in general ignored when one models microlensing lightcurves.

However, in case of EROS BLG-2000-5, the second caustic crossing lasted four days, and therefore *daily modulations* of magnifications due to the Earth’s rotation, offset according to the geographic position of each observatory, may become important, depending on the actual magnitude of the effect (See also Honma 1999 for a similar discussion on the short time scale magnification modulation observed from an Earth-orbiting satellite). Hardy & Walker (1995) and Gould & Andronov
(1999) investigated effects of the terrestrial baseline parallax, for fold-caustic crossing microlensing events mainly focused on the instantaneous offsets due to the separation between observers. They argued that the timing difference of the trailing limb crossing for observations made from two different continents could be of the order of tens of seconds to a minute (Hardy & Walker 1995) and the magnifications near the end of exit-type caustic crossings could differ by as much as a few percent (Gould & Andronov 1999). Suppose that $\phi_2$ is the angle at which the source crosses the caustic, $A_{\text{max}}$ is the magnification at the peak of the crossing, and $A_{cc}$ is the magnification right after the end of the crossing. Then, for the second caustic crossing of EROS BLG-2000-5, since $t'_E \csc \phi_2 (R_\oplus / \tilde{r}_E) \simeq 9 (0.25 / \sin \phi_2)$ min and $\rho_s^{-1} (A_{\text{max}} / A_{cc}) (R_\oplus / \tilde{r}_E) \simeq 2 \times 10^{-2}$, the time for the end of the second caustic crossing may differ from one observatory to another by as much as ten minutes and the magnification difference between them at the end of the crossing can be larger than one percent, depending on the relative orientation of observatories with respect to the event at the time of the observations. Based on a model of the event, we calculate the effect and find that it causes the magnification modulation of an amplitude as large as one percent (Fig. 3.3). In particular, night portions of the observatory-specific lightcurves exhibit steeper falls of flux than would be the case if the event were observed from the Earth’s center. That is, the source appears to move faster during the night because the reflex of the Earth’s rotation is added to the source motion. This would induce a systematic bias in parameter
measurements if it were not taken into account in the modeling. We thus include the (daily) terrestrial baseline parallax in our model to reproduce the observed lightcurve of EROS BLG-2000-5. Here, we emphasize that this inclusion requires no new free parameter for the fit once geographic coordinates of the observatory is specified and $R_\oplus$ in units of AU is assumed to be known (see Appendix C).

In addition, we note the possibility of a simple test of the terrestrial baseline parallax. Figure 3.4 shows that the end of the crossing observed from SAAO is supposed to be earlier than in the geocentric model – and earlier than seen from South American observatories. Unfortunately, near the end of the second caustic crossing ($\text{HJD} \sim 2451733.66$), PLANET data were obtained only from SAAO near the very end of the night – the actual trailing limb crossing is likely to have occurred right after the end of the night at SAAO, while YALO was clouded out due to the bad weather at CTIO. (The event was inaccessible from telescopes on Australian sites at the time of trailing limb crossing.) However, it is still possible to compare the exact timing for the end of the second crossing derived by other observations from South American sites with our model prediction and/or the observation from SAAO. In particular, the EROS collaboration has published a subset of their observations from the Marly 1 m at the European Southern Observatory (ESO), La Silla, Chile, for the second crossing of EROS BLG-2000-5.
Fig. 3.3.— Prediction of deviations of lightcurves for SAAO and YALO from the geocentric lightcurve for a chosen model. The solid curve is the magnitude difference between the SAAO lightcurve and the geocentric one, and the dotted curve is the same for YALO. Nominal night portions (between 6 pm and 6 am local time) of the lightcurve are highlighted by overlaid dots.
Fig. 3.4.— Close-up of model lightcurves for the end of the second caustic crossing. The solid curve is modeled for SAAO observations, the dotted curve is for YALO, and the dashed curve is the geocentric lightcurve. The timing of the end of the second crossing for SAAO is earlier than for YALO by 11 minutes. For comparison purposes, all the lightcurve are calculated assuming no blend.
(Afonso et al. 2001). Comparison between their data and our model/observations may serve as a confirmation of terrestrial parallax effects.

### 3.7. Measurement of the Projected Einstein Radius

Table 3.3 gives the parameters describing the best-fit microlens model (see Appendix D for the discussion on the error determination) for the PLANET \textit{I}-band observations of EROS BLG-2000-5. We also transform the fit parameters to the set of parameters introduced in § 3.6.1. In Table 3.4, we provide the result of limb-darkening coefficient measurements. Because of the unprecedentedly long time scale of the second caustic crossing and the extremely close approach to the cusp, as well as the high quality of the data we adopt a two-parameter limb-darkening law (eq. [2.5b]). The measurements of two coefficients, \( \Gamma_I \) and \( \Lambda_I \) are highly anticorrelated so that the uncertainty along the major axis of error ellipse is more than 50 times larger than that along its minor axis. While this implies that the constraint on the surface brightness profile derived by the caustic crossing lightcurve is essentially one dimensional, its natural basis is neither the linear nor the square-root parameterized form. We plot, in Figure 3.5, the surface brightness profile indicated by the fit and compare these with theoretical calculations taken from Claret (2000). The figure shows that allowing the profile parameters to vary by 2-\( \sigma \) does not have much effect on the central slope, but cause a large change in
the behavior near the limb. We speculate that this may be related to the specific form of the time sampling over the stellar disk, but more detailed analysis regarding limb darkening is beyond the scope of the present study.

Figures 3.6 and 3.7 show “magnitude residuals”, \((2.5/\ln 10)[\Delta F/(AF_s)]\), from our best model. Note in particular that the residuals for the points near the cusp approach (HJD = 2451736.944) that were excluded from the fit do not differ qualitatively from other residuals. It is true that the mean residual for Perth on this night (beginning 4.0 hrs after \(t_c\) and lasting 2.3 hrs) was about 2% high. However, the Canopus data, which span the whole cusp approach from 1.2 hrs before until 7.7 hrs after \(t_c\), agree with the model to within 0.5%. Moreover, the neighboring SAAO and YALO points also show excellent agreement. See also Fig. 3.8 and especially Fig. 3.9.

From the measured microlens parallax \((\pi_E = 0.277 \pm 0.008)\), the projected Einstein radius is (eq. [3.3]);

\[
\tilde{r}_E = \frac{AU}{\pi_E} = 3.61 \pm 0.11 \text{ AU}.
\]  

We also derive \(t_E\), the heliocentric Einstein timescale;

\[
\mu_E = \dot{u}_{t_c} + \pi_E \ddot{s}_{t_c},
\]  

\[
t_E \equiv \mu_E^{-1} = 76.8 \pm 2.1 \text{ days},
\]
Fig. 3.5.— Surface brightness profile of the source star. The thick solid curve is the prediction indicated by the best fit model (see Table 3.4). In addition, the variation of profiles with the parameters allowed to deviate by 2-σ along the direction of the principal conjugate is indicated by a shaded region. For comparison, also shown are theoretical profiles taken from Claret (2000). The stellar atmospheric model parameters for them are \( \log g = 1.0 \), \([\text{Fe/H}] = -0.3\), and \( T_{\text{eff}} = 3500 \) K (dotted curve), 4000 K (short dashed curve), 4500 K (long dashed curve). Note that the effective temperature of the source is reported to be \( 4500 \pm 250 \) K by Albrow et al. (2001b).
Fig. 3.6.— “Magnitude” residuals from PLANET model of EROS BLG-2000-5. Symbols are the same as in Fig. 3.1. The top panel shows the lightcurve corresponding to the time of observations, the middle panel shows the averaged residuals from 15-sequential observations, and the bottom panel shows the scatters of individual residual points.
Fig. 3.7.— Same as Fig. 3.6 but focuses mainly on the “anomalous” part of the lightcurve. The middle panel now shows the daily averages of residuals.
and the direction of $\mu_E$ is 36°3 from ecliptic west to south. Since, toward the direction of the event, the Galactic disk runs along 60°2 from ecliptic west to south, $\mu_E$ points to 23°9 north of the Galactic plane. The overall geometry of the model is shown in Figure 3.8, and Figure 3.9 shows a close-up of the geometry in the vicinity of the cusp approach ($t \sim t_c$). Next, the projected velocity $\tilde{v}$ on the observer plane is

$$\tilde{v} = \mu_E \tilde{r}_E = (-74.5 \pm 3.1, 33.0 \pm 11.0) \text{ km s}^{-1}.$$  \hspace{1cm} (3.22)

Here the vector is expressed in Galactic coordinates. We note that the positive $x$-direction is the direction of the Galactic rotation – i.e., the apparent motion of the Galactic center seen from the Local Standard of Rest (LSR) is in the negative $x$ direction – while the positive $y$-direction is toward the north Galactic pole so that the coordinate basis is lefthanded.

### 3.8. Measurement of the Angular Einstein Radius

The angular radius of the source, $\theta_*$, is determined by placing the source on an instrumental CMD and finding its offset relative to the center of the red giant clump, whose dereddened and calibrated position toward the Galactic bulge are known from the literature. For this purpose, we use the data obtained from YALO. We find the instrumental $I$ magnitude of the (deblended) source $I_s$ from the fit
Fig. 3.8.— Geometry of the event projected on the sky. Left is Galactic east, up is Galactic north. The origin is the center of mass of the binary lens. The trajectory of the source relative to the lens is shown as a thick solid curve while the short-dashed line shows the relative proper motion of the source seen from the Sun. The lengths of these trajectories correspond to the movement over six months between HJD = 2451670 and HJD = 2451850. The circle drawn with long dashes indicates the Einstein ring, and the curves within the circle are the caustics at two different times. The solid curve is at $t = t_c$ while the dotted curve is at the time of the first crossing. The corresponding locations of the two lens components are indicated by filled ($t = t_c$) and open (the first crossing) dots. The lower dots represent the more massive component of the binary. The ecliptic coordinate basis is also overlayed with the elliptical trajectory of $\pi_E \varsigma$ over the year.
Fig. 3.9.— Close-up of Fig. 3.8 around the cusp approach. The source at the closest approach \((t = t_c)\) is shown as a circle. The solid curve is the caustic at \(t = t_c\) while the dotted curve is the caustic at the time of the first crossing. The positions of the source center at the time of each of the observations are also shown by symbols (same as in Figs. 3.1, 3.6, 3.7) that indicate the observatory. For the close-up panel, only those points that were excluded from the fit because of numerical problems in the magnification calculation (see §§ 3.5.1 and 3.7) are shown. Note that the residuals for all points (included these excluded ones) are shown in Figs. 3.6 and 3.7.
to the lightcurve. To determine the color, we first note that, except when the source is near (and so partially resolved by) a caustic, microlensing is achromatic. That is, the source is equally magnified in $V$ and $I$: $F_V = F_{V,b} + AF_{V,s}$ and $F_I = F_{I,b} + AF_{I,s}$. Hence, we assemble pairs of $V$ and $I$ points observed within 30 minutes (and excluding all caustic-crossing and cusp-approach data) and fit these to the form $F_V = a_1 + a_2 F_I$. We then find $(V - I)_{s, \text{inst}} = -2.5 \log a_2$. We also find the magnitude of the blend $I_b$ from the overall fit, and solve for the color of the blend $(V - I)_{b, \text{inst}} = -2.5 \log [a_2 + (a_1/F_{I,b})]$.

We then locate the source on the instrumental CMD and measure its offset from the center of the red giant clump; $\Delta(V - I) = 0.276 \pm 0.010$ and $\Delta I = 0.33 \pm 0.02$. Here, the quoted uncertainty includes terms for both the clump center and the source position. Finally, using the calibrated and dereddened position of the red clump, $[ (V - I)_{\text{RC}}, I_{\text{RC}} ] = [ 1.114 \pm 0.003, 14.37 \pm 0.02 ]$ (Paczyński et al. 1999), the source is at $(V - I)_{s,0} = 1.390 \pm 0.010$, $I_{s,0} = 14.70 \pm 0.03$. The procedure does not assume any specific reddening law for determining the dereddened source color and magnitude, but may be subject to a systematic error due to differential reddening across the field.

We also perform a photometric calibration for observations made at SAAO. The calibration was derived from images obtained on three different nights by observing Landolt (1992) standards in the 2000 season, and the resulting
transformation equation were confirmed with a further night’s observations in 2001. Applying the photometric transformation on the fit result for the source flux, we obtain the – unmagnified – source magnitude of $I_s = 16.602$ in the standard (Johnson/Cousins) system, while the standard source color is measured to be $(V - I)_s = 2.69$. By adopting the photometric offset between SAAO and YALO instrumental system determined by the fit, we are able to derive a “calibrated” YALO CMD for the field around EROS BLG-2000-5 (Fig. 3.10). The positions of the source (S), blend (B), and clump center (C) are also overlaid on the CMD. The CMD also implies that the reddening toward the field is $A_I = 1.90$ and $E(V - I) = 1.30$, which yields a reddening law, $R_{VI} = A_V/E(V - I) = 2.46$, nominally consistent with the generally accepted $R_{VI} = 2.49 \pm 0.02$ (Stanek 1996). If we adopt a power-law extinction model $A_\lambda \propto \lambda^{-p}$, then $R_{VI} = 2.46$ corresponds to $p = 1.37$. For this index, $E(B - V)/E(V - I) = 0.838$, which predicts a lower extinction than the spectroscopically determined reddening of $E(B - V) = 1.30 \pm 0.05$ by Albrow et al. (2001b).

Both the source and the blend lie redward of the main population of stars in the CMD. One possible explanation is that the line of sight to the source is more heavily reddened than average due to differential reddening across the field. Inspection of the images does indicate significant differential reddening, although from the CMD itself it is clear that only a small minority of stars in the field could
Fig. 3.10.— Calibrated CMD for the field around EROS BLG-2000-5, observed from YALO. The arrow shows the reddening vector. The positions of the source (S), blend (B), and the center of the red giant clump (C) are denoted by capital letters.
suffer the additional $\Delta A_V \sim 0.7$ that would be necessary to bring the source to the center of the clump. There is yet another indication that the source has average extinction for the field – i.e., the same or similar extinction as the clump center. The dereddened (intrinsic) color derived on this assumption, $(V-I)_{s,0} = 1.39$ is typical of a K3III star (Bessell & Brett 1988), which is in good agreement with the spectral type determined by Albrow et al. (2001b). On the other hand, if the source were a reddened clump giant with $(V-I)_{s,0} \sim 1.11$, then it should be a K1 or K2 star.

We then apply the procedure of Albrow et al. (2000a) to derive the angular source radius $\theta_*$ from its dereddened color and magnitude: first we use the color-color relations of Bessell & Brett (1988) to convert from $(V-I)_{s,0}$ to $(V-K)_{s,0}$, and then we use the empirical relation between color and surface brightness to obtain $\theta_*$ (van Belle 1999). From this, we find that

$$\theta_* = 6.62 \pm 0.58 \text{ \mu as}$$

$$= 1.42 \pm 0.12 \text{ R}_\odot \text{ kpc}^{-1},$$

where the error is dominated by the 8.7% intrinsic scatter in the relation of van Belle (1999). Alternatively, we use the different calibration derived for K giants by Beuermann, Baraffe, & Hauschildt (1999) and obtain $\theta_* = 6.47 \text{ \mu as}$, which is consistent with equation (3.23). Finally, we note that if the source were actually
a clump giant that suffered $\Delta A_V = 0.7$ extra extinction, its angular size would be 7.08 $\mu$as. Since we consider this scenario unlikely, and since in any event the shift is smaller than the statistical error, we ignore this possibility.

From this determination of $\theta_*$ and the value, $\rho_* = (4.80 \pm 0.04) \times 10^{-3}$, determined from the fit to the lightcurve, we finally obtain,

$$\theta_E = \frac{\theta_*}{\rho_*} = 1.38 \pm 0.12 \text{ mas}, \quad (3.24)$$

where the error is again dominated by the intrinsic scatter in the relation of van Belle (1999).

3.9. The Lens Mass and the Lens-Source Relative Proper Motion

By combining the results obtained in §§ 3.7 and 3.8, one can derive several key physical parameters, including the lens mass and the lens-source relative parallax and proper motion;

$$M = \frac{c^2}{4G} \tilde{r}_E \theta_E = 0.612 \pm 0.057 \, M_\odot, \quad (3.25)$$

$$D = \frac{\tilde{r}_E}{\theta_E} = 2.62 \pm 0.24 \text{ kpc}; \quad (3.26a)$$

$$\pi_{\text{rel}} = \frac{\text{AU}}{D} = 0.382 \pm 0.035 \text{ mas}, \quad (3.26b)$$
\[
\mu = \mu_E \theta_E = 18.0 \pm 1.7 \text{ \mu as day}^{-1}
= 31.1 \pm 2.9 \text{ km s}^{-1} \text{ kpc}^{-1}.
\] (3.27)

For the binary mass ratio of the best-fit model, \( q = 0.749 \), the masses of the individual components are 0.350 \( M_\odot \) and 0.262 \( M_\odot \), both of which are consistent with the mass of typical mid-M dwarfs in the Galactic disk. Using the mass-luminosity relation of Henry & McCarthy (1993), and adopting \( M_V = 2.89 + 3.37(V - I) \), the binary then has a combined color and magnitude \( M_I = 8.2 \pm 0.2 \) and \( (V - I) = 2.54 \pm 0.08 \). Since \( D^{-1} = D_L^{-1} - D_S^{-1} \), \( D \) is an upper limit for \( D_L \), i.e., the lens is located in the Galactic disk within 2.6 kpc of the Sun. Furthermore, an argument based on the kinematics (see § 3.10) suggests that \( D_S \gtrsim 8 \text{ kpc} \) so that it is reasonable to conclude that \( D_L \sim 2 \text{ kpc} \) \((m - M \simeq 11.5)\). Hence, even if the binary lens lay in front of all the extinction along the line of sight, it would be \( \sim 2 \text{ mag} \) fainter than the blend (B), and so cannot contribute significantly to the blended light. However, the proximity of the lens to the observer is responsible for the event’s long time scale and quite large parallax effect.
3.10. **The Kinematic Constraints on the Source Distance**

The projected velocity \( \tilde{v} \) (eq. [3.22]) is related to the kinematic parameters of the event by

\[
\tilde{v} = \mu D = \left( \frac{D}{D_s} v_S - \frac{D}{D_L} v_L + v_\odot \right)_\perp,
\]

where the final subscript, \( \perp \), serves to remind us that all velocities are projected onto the plane of the sky. Writing \( v_\odot = v_{\text{rot}} + v_{\odot, p} \) and \( v_L = v_{\text{rot}} + v_{L, p} \) as the sum of the Galactic rotation and the peculiar velocities and eliminating \( D_L \) in favor of \( D_S \), equation (3.28a) can be expressed as

\[
\tilde{v} = \left[ \frac{D}{D_S} (v_S - v_{\text{rot}}) - \left( 1 + \frac{D}{D_S} \right) v_{L, p} + v_{\odot, p} \right]_\perp.
\]

(3.28b)

For a fixed \( D_S \) and with a known Galactic kinematic model, the distribution function of the expected value for \( \tilde{v} \) can be derived from equation (3.28b). The measured value of \( \tilde{v} \) can therefore be translated into the relative likelihood, \( \mathcal{L} \) for a given \( D_S \) and the assumed kinematic model,

\[
-2 \ln \mathcal{L} = (\tilde{v} - \langle \tilde{v} \rangle) \cdot C^{-1} \cdot (\tilde{v} - \langle \tilde{v} \rangle) + \ln |C| + \text{constant},
\]

(3.29a)
\[
\langle \tilde{v} \rangle = x_S (\langle v_{S,\perp} \rangle - v_{\text{rot}}) - (1 + x_S) \langle v_{L,p,\perp} \rangle + v_{\odot,p,\perp} , 
\]
(3.29b)

\[
C = x_S^2 C[v_{S,\perp}] + (1 + x_S)^2 C[v_{L,\perp}] + C[\tilde{v}] ,
\]
(3.29c)

where \( C[v_{S,\perp}] \) and \( C[v_{L,\perp}] \) are the dispersion tensors for the source and the lens transverse velocity, \( C[\tilde{v}] \) is the covariance tensor for the measurement of \( \tilde{v} \), and \( x_S = D/D_S \).

We evaluate \( \mathcal{L} \) as a function of \( D_S \) assuming three different distributions for \( v_{S,\perp} \) which respectively correspond to source being in the near disk \( (\langle v_{S,\perp} \rangle = v_{\text{rot}} + \langle v_{S,p,\perp} \rangle) \), bulge \( (\langle v_{S,\perp} \rangle = \langle v_{S,p,\perp} \rangle) \), and the far disk \( (\langle v_{S,\perp} \rangle = -v_{\text{rot}} - \langle v_{S,p,\perp} \rangle) \). Adopting the Galactic rotational velocity, \( v_{\text{rot}} = (220, 0) \) km s\(^{-1}\), the solar motion, \( v_{\odot,p,\perp} = (5.2, 7.2) \) km s\(^{-1}\) (Binney & Merrifield 1998), and the kinematic characteristics of the lens and the source given in Table 3.5, we derive the relative likelihood as a function of source position (Fig. 3.11), and find that the measured value of \( \tilde{v} \) (eq. [3.22]) mildly favors the far disk over the bulge as the location of the source by a factor of \( \sim 2.3 \). The near disk location is quite strongly disfavored (by a factor of \( \sim 10.6 \) compared to the far disk, and by a factor of \( \sim 4.7 \) compared to the bulge).

The complete representation of the likelihood for the source location requires one to consider all the available constraints relevant to the source distance. In particular, these include the radial (line-of-sight) velocity measurement and the
Fig. 3.11.— Kinematic likelihood for $\vec{v}$ as a function of $D_S$. The three curves are for different distributions of the source velocity: near-disk-like; $v_S = v_{\text{rot}} + v_{S,p}$ (short-dashed line), bulge-like; $v_S = v_{S,p}$ (solid line), and far-disk-like; $v_S = -v_{\text{rot}} - v_{S,p}$ (long-dashed line). The top panel shows the likelihood derived using only the two-dimensional projected velocity information while in the bottom panel, the likelihood also includes the radial velocity information derived from the high resolution spectra of Castro et al. (2001).
number density of stars constrained by the measured color and brightness (or luminosity) along the line of sight. Although one may naively expect that a bulge location is favored by the high density of stars in the bulge, which follows from the fact that the line of sight passes within 400 pc of the Galactic center, it is not immediately obvious how the disk contribution compares to the bulge stars for the specific location of the source on the CMD (Fig. 3.10). Even with no significant differential reddening over the field, the particular source position, which is both fainter and redder than the center of the clump, can be occupied by relatively metal-rich (compared to the bulge average) first ascent giants in either the bulge or the far disk. Because first ascent giants become redder with increasing luminosity, the source must have higher metallicity if it lies in the bulge than in the more distant disk. Although there exists an estimate of the source metal abundance: \([\text{Fe/H}] = -0.3 \pm 0.1\) by Albrow et al. (2001b) based on VLT FORS1 spectra, to incorporate these measurement into a likelihood estimate would require a more thorough understanding of all the sources of error as well as a detailed model of the metallicity distribution of the bulge and the disk. For this reason, we defer the proper calculation of the likelihood in the absence of any definitive way to incorporate the specific density function, and restrict ourselves to the kinematic likelihood.
We determine the radial velocity of the source from Keck HIRES spectra of Castro et al. (2001), kindly provided by R. M. Rich. We find the line-of-sight velocity to be $\sim -100$ km s$^{-1}$ (blueshift; heliocentric), which translates to $\sim -90$ km s$^{-1}$ Galactocentric radial velocity accounting for the solar motion of 10 km s$^{-1}$. The derived radial velocity strongly favors bulge membership of the source since it is three times larger than the line-of-sight velocity dispersion for disk K3 giants, but is consistent with the motions of typical bulge stars (c.f., Table 3.5). Because the correlation between the radial and the transverse velocity for K3 giants either in the disk or in the bulge is very small, the likelihood for the radial velocity can be estimated independently from the likelihood for the transverse velocity, and the final kinematic likelihood is a simple product of two components. We find that the final kinematic likelihood including the radial velocity information indicates that the source star belongs to the bulge (Fig. 3.11): the likelihood for the bulge membership is about six times larger than that for the far disk membership.

Finally, we note that, if the source lay in the far disk, it should have an additional retrograde proper motion of $\sim 4$ mas yr$^{-1} \times (10$ kpc/$D_S$) with respect to the bulge stars, which should be measurable using adaptive optics or the Hubble Space Telescope.
3.11. Consistency between the Lens Mass and the Binary Orbital Motion

For the derived parameters, we find a projected binary lens separation

\[ r_\perp = dD_L \theta_E = 5.52 \text{ AU (5.25 AU)} \]

and transverse orbital speed

\[ v_\perp = [d^2 + (\omega d)^2]^{1/2} D_L \theta_E = 2.76 \text{ km s}^{-1} (2.62 \text{ km s}^{-1}) \text{ for } D_S = 10 \text{ kpc} \]

(8 kpc). We can now define the transverse kinetic and potential energies

\[ K_\perp = \frac{q}{(1 + q)^2} M v_\perp^2/2 \]

and \[ T_\perp = -\frac{q}{(1 + q)^2} GM^2/r_\perp, \]

and evaluate their ratio (here and throughout this section, we assume that \( D_S = 10 \text{ kpc} \)),

\[ \rho_\perp \equiv \left| \frac{T_\perp}{K_\perp} \right| = \frac{2GM}{r_\perp v_\perp^2} = \frac{c^2 \tilde{r}_E \theta_E [\tilde{r}_E^{-1} + (D_S \theta_E)^{-1}]^3}{d[d^2 + (\omega d)^2]} = 25.9 \pm 5.1. \quad (3.30) \]

Here, the error is dominated by the uncertainty in the measurement of \( \omega \).

Since \( r_\perp \leq r \) and \( v_\perp \leq v \), it follows that \( |T_\perp| \geq |T| \) and \( K_\perp \leq K \), and hence

\[ |T_\perp/K_\perp| \geq |T/K|, \]

where \( |T/K| \) is the ratio of the true three-dimensional potential and kinetic energies. The latter must be greater than unity for a gravitationally bound binary. This constraint is certainly satisfied by EROS BLG-2000-5. Indeed, perhaps it is satisfied too well. That is, what is the probability of detecting such a large ratio of projected energies? To address this question, we numerically integrate over binary orbital parameters (viewing angles, the orbital orientation and phase, and the semimajor axis) subject to the constraint that \( r_\perp = 5.5 \text{ AU} \) \( (D_S = 10 \text{ kpc}), \)

and at several fixed values of the eccentricity, \( e \). We assume a random ensemble
of viewing angles and orbital phases. The results shown in Figure 3.12 assume a Duquennoy & Mayor (1991) period distribution, but are almost exactly the same if we adopt a flat period distribution. All of the eccentricities shown in Figure 3.12 are reasonably consistent with the observed ratio, although higher eccentricities are favored.

One can also show that high eccentricities are favored using another line of argument. First, note that $|\mathbf{r}_\perp \times \mathbf{v}_\perp| = \omega r_\perp^2$ is the same as the projection of the specific angular momentum of the binary to the line of sight, $|(\mathbf{r} \times \mathbf{v}) \cdot \hat{n}| = 2\pi a^2(1 - e^2)|\cos i|/P$. Here $a$ and $P$ are the semimajor axis and the orbital period of the binary, $\hat{n}$ is the line-of-sight unit vector, and $i$ is the inclination angle of the binary. Combining this result with Kepler’s Third Law, $4\pi^2a^3 = GM P^2$, one obtains;

$$a(1 - e^2)^2 \cos^2 i = \left(\frac{\omega r_\perp^2}{GM}\right)^2 = \frac{4}{c^2} \frac{\omega^2 d^4}{r_E \theta_E [r_E^{-1} + (D_s \theta_E)^{-1}]^4} = 0.0013 \pm 0.0341 \text{ AU}.$$ (3.31)

From the energy conservation, $(v^2/2) - (GM/r) = -GM/(2a)$, one may derive a lower limit for $a$;

$$a = \frac{r}{2} \left(1 - \frac{rv^2}{2GM}\right)^{-1} \geq \frac{r_\perp}{2(1 - \varrho_\perp^{-1})} = 2.87 \pm 0.09 \text{ AU}.$$ (3.32)
Fig. 3.12.— Distributions of the ratio of transverse potential energy, \( |T_\perp| = \frac{q}{(1 + q)^2} GM^2/r_\perp \) to transverse kinetic energy, \( K_\perp = \frac{q}{(1 + q)^2} Mv_\perp^2/2 \), for binaries seen at random times and random orientations, for three different eccentricities, \( e = 0, 0.5, 0.9 \). Also shown is the 1 \( \sigma \) allowed range for EROS BLG-2000-5. Noncircular orbits are favored.
The corresponding lower limit for the binary period is $P \geq 6.22$ yr for $D_S = 10$ kpc.

Now, we can derive a constraint on the allowed eccentricity and inclination by dividing equation (3.31) by equation (3.32):

$$
(1 - e^2) |\cos i| \leq 0.021 \pm 0.281
$$

The constraint here essentially arises from the fact that the fit barely detects projected angular motion $\omega$, while the formal precision of its measurement corresponds to $\sim 80$ yr orbital period in 1-σ level. For the observed projected separation and the derived binary mass, this apparent lack of the binary angular motion therefore naturally leads us to conclude that the binary orbit is either very close to edge-on or highly eccentric (or both).

### 3.12. Another Look at MACHO 97-BLG-41

We have found that both parallax and binary orbital motion are required to explain the deviations from rectilinear motion exhibited by the lightcurve of EROS BLG-2000-5. In Albrow et al. (2000a), PLANET ascribed all deviations from rectilinear motion for MACHO 97-BLG-41 to a single cause: projected binary orbital motion. Could both effects have also been significant in that event as well? Only detailed modeling can give a full answer to this question. However, we can
give a rough estimate of the size of the projected Einstein ring $\bar{r}_E$ that would be
required to explain the departures from linear motion seen in that event.

Basically what we found in the case of MACHO 97-BLG-41 was that the
lightcurve in the neighborhood of the central caustic (near HJD $\sim 2450654$) fixed
the lens geometry at that time, and so predicted both the position of the outlying
causics and the instantaneous velocity of the source relative to the Einstein ring.
If this instantaneous relative velocity were maintained, then the source would have
missed this outlying caustic by about $\Delta u_{\text{obs}} \sim 0.4$ (Albrow et al. 2000a, fig. 3). On
the other hand, the predicted displacement of a caustic due to parallax is

$$\Delta u_{\text{pred}} = -\pi_E \mathcal{D}_P ; \quad (3.34a)$$

$$\mathcal{D}_P = \varsigma_{t_{\text{cc}},1} - \varsigma_{t_{\text{cc}},2} - (t_{\text{cc},1} - t_{\text{cc},2})\dot{\varsigma}_{t_{\text{cc}},2} . \quad (3.34b)$$

We find $|\mathcal{D}_P| = 0.072$, and hence,

$$\pi_E = 5.6 \frac{\Delta u_{\text{pred}}}{\Delta u_{\text{obs}}} . \quad (3.35)$$

That is, to explain by parallax the order of the effect seen requires $\bar{r}_E \sim 0.18$ AU,
which (using the measured $\theta_E = 0.7$ mas) would in turn imply a lens mass
$M \sim 0.015 M_{\odot}$, a lens distance of $D_L \lesssim 250$ pc, and the projected lens-source
relative transverse velocity on the observer plane (at time $t_{\text{cc},2}$) of only 13 km s$^{-1}$.  

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While these values cannot be strictly ruled out, they are a priori extremely unlikely because the optical depth to such nearby, low-mass, and slow lenses is extremely small. On the other hand, if the lens lies at a distance typical of bulge lenses $D_L \sim 6$ kpc and the source is a bulge star, then $\tilde{r}_E = D\theta_E \sim 17$ AU, which would imply $\Delta u_{\text{pred}}/\Delta u_{\text{obs}} \sim 1\%$. That is, parallax would contribute negligibly to the observed effect. We therefore conclude that most likely parallax does not play a major role in the interpretation of MACHO 99-BLG-47, but that detailed modeling will be required to determine to what extent such a role is possible.

What is the physical reason that parallax must be so much stronger (i.e., $\pi_E$ must be so much bigger) to have a significant effect in the case of MACHO 97-BLG-41 than for EROS BLG-2000-5? Fundamentally, the former is a close binary, and there is consequently a huge “lever arm” between the radial position of the outlying caustic, $u_c \sim 1.7$ and the binary separation, $d \sim 0.5$. That is, $u_c/d \sim 3.2$. This is almost an order of magnitude larger than for EROS BLG-2000-5, for which $u_c \sim 0.8$ (Fig. 3.8), $d \sim 1.9$, and $u_c/d \sim 0.4$. Consequently, parallax has to be an order of magnitude larger to reproduce the effects of the same amount of the projected binary orbital motion.
3.13. Summary

We have presented here results from two seasons of photometric $I$-band monitoring of the microlensing event EROS BLG-2000-5, made by the PLANET collaboration. The lightcurve exhibits two peaks which are well explained by a finite source crossing over a fold-type – inverse-square-root singularity – caustic, followed by a third peak which is due to the source’s passage close to a cusp. We find no geometry involving a rectilinear source-lens relative trajectory and a static lens that is consistent with the photometry. However, by incorporating both parallax and binary orbital motion, we are able to model the observed lightcurve. In particular, the detection of the parallax effect is important because it enables us to derive the microlens mass, $M = 0.612 \pm 0.057 \, M_\odot$, unambiguously by the combination of the projected Einstein radius – measured from the parallax effect – and the angular Einstein radius – inferred from the source angular size and the finite source effect on the lightcurve. The source size is measurable from the magnitude and the color of the source. The kinematic properties of the lens/source system derived from our model together with the lens-source relative parallax measurement as well as an additional radial velocity measurement indicate that the event is most likely caused by a (K3) giant star in the bulge being lensed by a disk binary (M dwarf) system about 2 kpc from the Sun. Additional information on the specific density function along the line of sights, differential reddening across
the field, and a metallicity measurement of the source could further constrain the source location.
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<th>seeing cut (arcsec)</th>
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<td>$\cdots$</td>
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<td>$\leq 3.1$</td>
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<td>3.63</td>
<td>2.44</td>
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Table 3.1: PLANET $I$-band Photometry of EROS BLG-2000-5

$^a$The formal value reported by DoPHOT.

$^b$The difference imaging analysis result has been used.
Table 3.2: Relations between Parameterizations. For simplicity, the reference times, $t_c$, for both systems are chosen to be the same: the time of the closest approach to the cusp, i.e., $(u_{tc} - u_{cusp}) \cdot \dot{u}_{tc} = 0$. The additional transformation of the reference time requires the use of equations (3.17) and (3.19). Figure 3.2 illustrates the geometry used for the derivation of the transformation. The unit vector $\hat{e}_n$ points toward the NEP while $\hat{e}_w$ is perpendicular to it and points to the west (the direction of decreasing ecliptic longitude).
<table>
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<tr>
<td>$t'_E$</td>
<td>99.8 days</td>
<td>1.5 day</td>
</tr>
<tr>
<td>$t_c$</td>
<td>1736.944$^b$</td>
<td>0.005 day</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>$4.80 \times 10^{-3}$</td>
<td>$4. \times 10^{-5}$</td>
</tr>
<tr>
<td>$\pi_{E,</td>
<td></td>
<td>}$</td>
</tr>
<tr>
<td>$\pi_{E,\perp}$</td>
<td>0.222</td>
<td>0.031</td>
</tr>
<tr>
<td>$\dot{d}$</td>
<td>0.203 yr$^{-1}$</td>
<td>0.016 yr$^{-1}$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.006 rad yr$^{-1}$</td>
<td>0.076 rad yr$^{-1}$</td>
</tr>
<tr>
<td>$\mu_{E,w}$</td>
<td>3.83 yr$^{-1}$</td>
<td>0.49 yr$^{-1}$</td>
</tr>
<tr>
<td>$\mu_{E,n}$</td>
<td>-2.82 yr$^{-1}$</td>
<td>0.44 yr$^{-1}$</td>
</tr>
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<td>$\mu_E$</td>
<td>4.76 yr$^{-1}$</td>
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<td>$\alpha_{ec}$</td>
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<tr>
<td>$\phi$</td>
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<td>3.6</td>
</tr>
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</table>

Table 3.3: PLANET Model Parameters for EROS BLG-2000-5

$^a$1-$\sigma$ errorbar. The uncertainties of fit parameters are determined by fitting $\chi^2$ distribution to a quadratic hypersurface. For more details, see Appendix D

$^b$Heliocentric Julian Date – 2450000.

$^c$The angle of $\mu_E$ with respect to ecliptic west.

$^d$The angle of $d_{t_e}$ with respect to ecliptic west.
<table>
<thead>
<tr>
<th>coefficient</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_I$</td>
<td>0.452 ± 0.075</td>
</tr>
<tr>
<td>$\Lambda_I$</td>
<td>0.011 ± 0.137</td>
</tr>
<tr>
<td>$\Gamma_I \cos \Phi + \Lambda_I \sin \Phi$</td>
<td>0.207 ± 0.156</td>
</tr>
<tr>
<td>$\Lambda_I \cos \Phi - \Gamma_I \sin \Phi$</td>
<td>0.402 ± 0.003</td>
</tr>
<tr>
<td>$c_I$</td>
<td>0.552 ± 0.090</td>
</tr>
<tr>
<td>$d_I$</td>
<td>0.011 ± 0.139</td>
</tr>
<tr>
<td>$c_I \cos \Psi + d_I \sin \Psi$</td>
<td>0.290 ± 0.166</td>
</tr>
<tr>
<td>$d_I \cos \Psi - c_I \sin \Psi$</td>
<td>0.470 ± 0.003</td>
</tr>
</tbody>
</table>

Table 3.4: Limb-Darkening Coefficients for EROS BLG-2000-5. The error bars account only for the uncertainty in the photometric parameters restricted to a fixed lens model, determined by the linear flux fit.

\(^a\)rotational transformation that maximizes the variance
\(^b\)\(\Phi = -61.32^\circ\)
\(^c\)rotational transformation that minimizes the variance
\(^d\)\(\Psi = -57.14^\circ\)
<table>
<thead>
<tr>
<th>location</th>
<th>( \langle v_y \rangle )</th>
<th>( \sigma_x )</th>
<th>( \sigma_y )</th>
<th>( \sigma_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{v}_{L,p} ) disk</td>
<td>-18</td>
<td>38</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>( \mathbf{v}_{S,p} ) disk</td>
<td>-11.5</td>
<td>31</td>
<td>21</td>
<td>17</td>
</tr>
<tr>
<td>bulge</td>
<td>( \cdots )</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 3.5: Kinematic Characteristics of the Lens and the Source. The \( x \)-direction is toward the Galactic center from the LSR, the \( y \)-direction is the direction of the Galactic rotation, and the \( z \)-direction is toward the north Galactic pole. The lens is assumed to be an M dwarf while the source is a K3 giant. The quoted values for the disk components are derived from Binney & Merrifield (1998).

\( ^a \)asymmetric drift velocity
Chapter 4

Nonplanetary Microlensing Anomaly: MACHO 99-BLG-47

4.1. Introduction

The hallmark of planetary microlensing events is a short deviation from an otherwise normal, PSPL event. Mao & Paczyński (1991) showed that extrasolar planets could be detected from such events, and Gould & Loeb (1992) gave an explicit prescription for how the planet/star mass ratio $q (\ll 1)$ and the angular separation $d$ (in units of the angular Einstein radius $\theta_E$) could be reconstructed by decomposing the event lightcurve into its “normal” and “perturbed” components.

Work during the ensuing decade has elucidated many additional subtleties of planetary lightcurves, but their fundamental characterization as briefly perturbed PSPL events has remained intact. Of particular importance in the present context,
Griest & Safizadeh (1998) showed that events with small impact parameter
($u_0 \ll 1$; where $u_0$ is the minimum separation between the source and the lens
center of mass in units of $\theta_E$) probe the so-called “central caustic” of the lens
geometry, making them much more sensitive to the presence of planets than the
larger impact-parameter events analyzed by Gould & Loeb (1992), which probe the
outer “planetary caustic”.

These central-caustic events are of exceptional importance, even though
they are intrinsically rare. They are rare simply because the central-caustic is
much smaller than the planetary caustic, so the great majority of planet-induced
deviations (of fixed fractional amplitude) are due to planetary caustics. However,
the probability of detecting a planet is much greater in small impact-parameter
events, partly because the source is guaranteed to pass close to the central caustic,
which almost by definition is near the center of the lens geometry ($u = 0$) and
partly because even the sensitivity of planetary caustics is enhanced for $u_0 \ll 1$.
(Here $u$ denotes the source position on the sky, normalized to $\theta_E$, with respect to
the lens center of mass.) By contrast, higher impact-parameter events miss the
central caustic, and they are likely to miss the planetary caustic as well because
it lies in a random position relative to the source trajectory. Because of their
higher sensitivity to planets, and because they can be recognized in real time, low
impact-parameter events are monitored more intensively than typical events by
microlensing follow-up networks, which in turn further enhances their sensitivity to planets.

Central-caustic events, like their planetary-caustic cousins, involve a short deviation from an otherwise normal PSPL lightcurve. The major difference between these two classes of planetary events is that central-caustic anomalies always occur near the peak, whereas planetary-caustic perturbations can occur anywhere on the lightcurve, and are typically expected on the wings of the lightcurve. Of particular importance, for central-caustic events, there is no simple prescription for extracting \( d \) and \( q \) by decomposing the lightcurve into “normal” and “perturbed” components and it is unclear to what degree these two parameters are degenerate.

Another important, albeit accidental, discovery was that planets could give rise to perturbations that are not short compared to the event timescale. In the course of their search for planetary perturbations among 43 approximately PSPL events, Gaudi et al. (2002, see also Albrow et al. 2001b) found one event, OGLE-1999-BUL-36, that was asymmetric in a way that was consistent with the presence of a planet. They argued, however, that the asymmetry was also consistent with parallax effects induced by the Earth’s motion around the Sun of the type analyzed by Gould et al. (1994), and that in general it would be extremely difficult to distinguish between the two possible causes of such an asymmetry. They concluded that, in most cases, microlensing searches are not able to
distinguish between parallax and a weak, asymmetric planetary perturbation, and consequently, all such “detections” should be ignored. This reduces the sensitivity of microlensing searches to planets, but only by an extremely small amount since, as Gaudi et al. (2002) showed, long-timescale asymmetric perturbations account for less than $\sim 1\%$ of all planetary events. Hence, the long-timescale asymmetric events also confirm in a way the basic paradigm: planetary perturbations have short durations relative to the parent lightcurve, and in the rare cases for which they do not, they are not recognizable as a planetary perturbation anyway.

However, not all short timescale deviations are due to planets and therefore the mere detection of such an anomaly does not prove the presence of a planet. Gaudi & Gould (1997) showed that close binaries ($d \ll 1$) give rise to lightcurves that are virtually identical to PSPL events, except when the source comes very close to the lens center of mass ($|u| \ll 1$). Hence, for events with $u_0 \ll 1$, the lightcurve looks “normal” except for a brief deviation near the peak. Qualitatively, this is exactly the same behavior as that of central-caustic planetary events. Similarly, lightcurves of wide-binary ($d \gg 1$) events can also take the same form if one – and only one – of the caustics lies very close to the source’s passage. Indeed, a close correspondence between a certain pair of close-binary and wide-binary events was discovered both theoretically (Dominik 1999) and observationally (Albrow et al. 1999c; Afonso et al. 2000). It remains an open question under what conditions these various types
of events can be distinguished from one another. If central-caustic planetary events
could not be distinguished from close- and/or wide-binary events, it would seriously
undermine planet searches in high-magnification events and hence would call into
question the basic strategy adopted by microlensing follow-up groups (e.g., Albrow et al. 1998).

Here we analyze the lightcurve of the microlensing event MACHO 99-BLG-47,
the first intensively monitored event with a short-lived, high S/N deviation from an
otherwise normal PSPL lightcurve. We identify the lens as an extreme-separation
binary rather than a planetary system, thereby showing that, at least in this case,
the two classes can be clearly distinguished. We also show that both wide- and
close-binary solutions fit the data equally well, implying that although planetary
perturbations can be distinguished from those arising from extreme-separation
binaries, the discrimination between very close and very wide binaries may be
difficult in practice.

4.2. MACHO 99-BLG-47

The initial alert for the microlensing event MACHO 99-BLG-47 was issued
The event was located about
18° from the Galactic center along the disk ($l = 17^\circ59'10'', b = -1^\circ57'36''$;
RA = 18h30m49.4, Dec = −14°10′53″) and reported to be rather faint at baseline (V = 21.5, R = 20.3). PLANET began monitoring MACHO 99-BLG-47 right after the electronic alert with the expectation that it would be a very high magnification event. PLANET detected anomalous behavior in the lightcurve during the first week of August, and subsequently issued an anomaly alert on 1999 August 4.²

4.2.1. Observations and Data

The PLANET lightcurve of MACHO 99-BLG-47 consists of observations made from three different southern sites: the Elizabeth 1 m at SAAO, Sutherland, South Africa; the YALO 1 m at CTIO, La Serena, Chile; and the Canopus 1 m near Hobart, Tasmania, Australia. The observations were carried out in two bands: I (at all three sites) and V (at SAAO only). The event was most intensely monitored in 1999 August during and following the photometric peak and the anomaly, but we obtained data through mid-September and also when the event was close to baseline early in the 2000 season as the source came out from behind the Sun. After the usual reductions, we perform photometry by two independent methods: a direct fit to the PSF using DoPHOT (Schechter et al. 1993) and difference imaging using ISIS (Alard & Lupton 1998; Alard 2000). For details, see also Albrow et al. (2000a). Because of the different characteristics of the two


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methods and varying frame qualities, we recover a different number of photometric measurements. Before any elaborate efforts to clean and select the data, we have 303 points from DoPHOT photometry and 297 from ISIS photometry (Table 4.1) for the 1999 observations, as well as 8 points observed from YALO during early 2000 season (DoPHOT photometry only).

The photometric observations taken at SAAO are calibrated to the standard Johnson-Cousins system with respect to E-region stars (Menzies et al. 1989) observed contemporaneously with MACHO 99-BLG-47 at SAAO on 1999 July 31. The instrumental magnitudes of observations from Canopus and YALO are not calibrated explicitly, but rather we allow independent source fluxes for different sites and bands in our modeling, which provide the relative photometric scalings among them.

We also include in our analysis the publicly available MACHO photometry, which comprises 77 $V_{\text{MACHO}}$ and 90 $R_{\text{MACHO}}$ points from the 1999 season as well as 213 $V_{\text{MACHO}}$ and 227 $R_{\text{MACHO}}$ points taken between 1995 and 1998. The latter were used to constrain the baseline and to check for source variability. We find no evidence for such variability.
<table>
<thead>
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<th>Telescope</th>
<th>Filter</th>
<th># of total points</th>
<th># of points analyzed</th>
<th>Median seeing (arcsec)</th>
<th>Error rescaling factor</th>
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</thead>
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<td>I</td>
<td>98</td>
<td>88</td>
<td>1.81</td>
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</tr>
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<td>...</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canopus 1 m</td>
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<td>51</td>
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<td>YALO 1 m</td>
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<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>Mt. Stromlo 50'</td>
<td>R_MACHO</td>
<td>90</td>
<td>90</td>
<td>2.59</td>
<td>1.04</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
|                 | V_MACHO| 77                | 76                   | 0.709                  | 0.730 \(^a\) \(0.730^{b}\)

| Table 4.1: Photometric data of MACHO 99-BLG-47. |

\(^a\) with respect to PLANET/DoPHOT model
\(^b\) with respect to PLANET/ISIS model
The difference photometry is placed on an absolute scale by a linear regression from the PSF-based flux (DoPHOT) to the differential flux (ISIS), after removing obvious faulty points from the two data sets based on the reports by each reduction/photometry package. We find that, except for the Canopus data, the residuals of the PSF-based flux from the regression line are strongly correlated with the size of the seeing disk, but that most of this correlation is removed by adding a linear seeing correction. For example, for the SAAO $J$-band observations, we detect the presence of a nonzero linear seeing dependence term of $\sim 1.7 F_{\text{base}} \text{arcsec}^{-1}$ (or $\sim 10 F_s \text{arcsec}^{-1}$), with the S/N being as large as $\sim 30$. Here $F_s$ and $F_{\text{base}}$ are the net flux of the lensed source star alone and the total flux in the same PSF when there is no magnification.

In order to minimize the effects of known systematics, we optimize the data set by rejecting outliers and reevaluating the sizes of the photometric error bars. We also include a correction term for the correlation between the seeing and the blended flux that enters the same PSF with the lensed source when we fit the observed flux to a specific magnification model. These procedures are described in detail and fully justified in the previous chapters as well as several earlier PLANET papers (Albrow et al. 2000a,b, 2001b; Gaudi et al. 2002). Briefly, we first construct an initial clean subset of the data by rejecting 3-$\sigma$ outliers with respect to the linear regression from DoPHOT to ISIS flux, and then determine a reference model based
on this subset. Once we have a reference model, we include all the data and follow the same iterative procedure of outlier removal and error rescaling described in the previous chapters. The final data set used for the subsequent analysis reported in the present study contains 276 DoPHOT-reduced and 266 ISIS-reduced PLANET points as well as 166 MACHO points (one \( V_{\text{MACHO}} \) point rejected), all from the 1999 season (Table 4.1). The data obtained during seasons other than 1999 (the 2000 season YALO \( I \), and the 1995 – 1998 season MACHO \( V_{\text{MACHO}} \) and \( R_{\text{MACHO}} \) bands) are included in the analysis as a combined single point for each set. In doing so, 4 data points (all \( R_{\text{MACHO}} \) in the 1998 season) are eliminated as outliers.

4.3. Analysis and Model

The lightcurve of MACHO 99-BLG-47 (Fig. 4.1) is that of a normal high-magnification PSPL event with a short-lived \((\sim 3 \text{ days})\) deviation near the peak, of the type predicted for planets (Griest & Safizadeh 1998) or binaries with extreme separations. We therefore develop a method for probing the space of lens geometries of this type and then search for the minimum \( \chi^2 \) within this space.
Fig. 4.1.— Observations and models of MACHO 99-BLG-47, all scaled to calibrated Cousins $I$-band with blending as registered by SAAO DoPHOT photometry. Shown are PLANET $I$-band from SAAO, Canopus, YALO, PLANET $V$-band from SAAO, and MACHO $V_{\text{MACHO}}$ and $R_{\text{MACHO}}$. The ISIS reduced PLANET points are plotted after applying the transformation to the absolute scale derived from the linear regression between difference photometry and PSF-based photometry. The solid curve shows the final close-binary lens model fit (Table 4.2) to the data while the dotted curve shows the “best” degenerate form of PSPL lightcurve (eq. [4.1]) fit to a high-magnification subset of the data that excludes the anomalous points near the peak. The half-magnitude offset between this curve and the data is the main observational input into the algorithm to search for the final model (see § 4.3). Note that on the scale of the figure, the wide-binary solution is completely indistinguishable from the close-binary solution, i.e., the solid curve can represent both the close-binary and the wide-binary lens models equally well.
We begin by excising the anomalous points near the peak and fitting the remaining lightcurve to the degenerate form of the Paczyński (1986) profile that obtains in the high-magnification limit (Gould 1996),

\[ F(t) = \frac{F_{\text{peak}}}{\sqrt{1 + (t - t_0)^2/t_{\text{eff}}^2}} + F_{\text{base}}. \]  

(4.1)

Here \( F(t) \) is the instantaneous flux, \( F_{\text{peak}} \) is the peak flux above the baseline of the model, \( F_{\text{base}} \) is the baseline flux, \( t_0 \) is the time of the peak, and \( t_{\text{eff}} \) is the effective width of the lightcurve. These degenerate parameters are related to the standard parameters by \( F_{\text{peak}} = F_s/u_0, \ F_{\text{base}} = F_s + F_b, \) and \( t_{\text{eff}} = u_0 t_E, \) where \( t_E \) is the Einstein timescale. We find \( t_0 \sim 2451393.6 \) (Heliocentric Julian Date), \( t_{\text{eff}} \sim 1 \) day, and that \( F_{\text{peak}} \) roughly corresponds to \( I_{\text{peak}} \sim 15.3 \) (Fig. [4.1]). However, the exact values of those parameters are quite dependent on which points we excise. Note that though the baseline magnitude is measured to be \( I_{\text{base}} = 19.1, \) there is at this point essentially no information of how the baseline flux divides into source flux \( F_s \) and blended flux \( F_b, \) which is why it is necessary to fit the lightcurve to the degenerate profile (eq. [4.1]).

Next, we reinser the excised points near the peak and note that the maximum (overmagnified) deviation from the degenerate fit is \( \sim 0.5 \) mag (Fig. 4.1). We then establish a set of initial trial event geometries as follows. For each diamond-shaped caustic produced by various geometries of \((d,q)\) pairs, we examine the magnification

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as a function of distance from the “caustic center” along each of the three directions defined by the four cusps of the caustic (one direction is redundant due to the reflection symmetry with respect to the binary axis). Here for computational simplicity, we use an analytic proxy point for the “caustic center,” defined as follows. For close binaries, we adopt the center of mass of the lens system for the caustic center. On the other hand, for wide binaries, the position of one component of the binary shifted toward the other component by $[d(1 + q^{-1})]^{-1}$ is chosen as the caustic center. [These choices can be understood from the approximation developed in 4.6. See also Dominik (1999).] Then, at each point $u_\bullet$, the point of the cusp-axis crossing, we take the ratio of the actual magnification to that predicted for the corresponding PSPL lens: a point mass lens located at the caustic center with the same mass as either the whole binary (close binaries) or the nearest lens alone (wide binaries). We proceed with the search only if this ratio lies in the interval $[1.2, 1.8]$. We then choose a source trajectory perpendicular to the cusp axis as an initial trial model, with $u_0 = |u_\bullet|$ and $t_E = t_{\text{eff}}/|u_\bullet|$, and use four-parameter $(\alpha, t_E, t_0, \text{and } u_0)$ downhill simplex (Press et al. 1992) to search for a best-fit model to minimize $\chi^2$, which is determined by a linear-flux fit to the model of lens magnifications (and thus the corresponding $F_s$ and $F_b$ are found immediately), subject to the constraint of fixed $(d,q)$. Figure 4.2 shows the result of this search (based on ISIS solutions) in a contour plot of $\chi^2$-surface over the $(d,q)$-space. The $\chi^2$ is rising around all the boundaries shown in Figure 4.2, except toward lower
While the $\chi^2$-surface flattens with $\Delta \chi^2 \simeq 32$ as the mass ratio $q$ becomes very small ($q \lesssim 0.01$) for $d \sim 0.065$ and $d \sim 15$, we find no evidence of a decrease of the $\chi^2$ as $q$ becomes smaller than 0.01. Rather it asymptotically approaches $\Delta \chi^2 \simeq 32$.

We find well-localized minima of $\chi^2$ over the searched $(d,q)$-space, one of close binaries ($d \simeq 0.13$) and the other of wide binaries ($d \simeq 11.3$), whose exact parameters depend slightly on whether we use the ISIS or DoPHOT photometry (Tables 4.2 and 4.3). We adopt the ISIS solutions in the subsequent discussion (and they are also what is shown in Fig. 4.2) because the lightcurve shows less scatter and consequently, the errors for the model parameter determinations are smaller. Despite the combination of the dense coverage near the peak by the PLANET data and the extensive baseline coverage by the MACHO data, the final two ISIS models (i.e., the close binary and the wide binary) are essentially indistinguishable: $\Delta \chi^2 = 0.6$ for 412 degrees of freedom, with the close binary having the lower $\chi^2$ of the two. In § 4.6, we discuss this degeneracy in further detail.

Finally, we also note that $u_0 t_E$ and $F_s/u_0$ from the best-fit binary-lens model parameters are not quite the same as the $t_{\text{eff}}$ and $F_{\text{peak}}$ derived from the initial degenerate form of the lightcurve fit. These discrepancies are traceable to small differences between a true Paczyński (1986) curve and its degenerate approximation (eq. [4.1]), and are not due to differences between the binary and corresponding PSPL event, which are very small except around the peak. Since the parameters
Fig. 4.2.— Contour plot of $\chi^2$-surface over $(d,q)$ space based on solutions for PLANET (ISIS) and MACHO data. The binary separation $d$ is in units of the Einstein radius of the combined mass, and the mass ratio $q$ is the ratio of the farther component to the closer component to the source trajectory (i.e., $q > 1$ means that the source passes by the secondary). Contours shown are of $\Delta \chi^2 = 1, 4, 9, 16, 25, 36$ (with respect to the global minimum). We find two well-isolated minima of $\chi^2$, one in the close-binary region, $(d,q)=(0.134,0.340)$, and the other in the wide binary-region, $(d,q)=(11.31,0.751)$ with $\Delta \chi^2 = 0.6$ and the close binary being the lower $\chi^2$ solution. Also drawn are the curves of models with the same $\dot{Q}$ as the best-fit close-binary model and the same $\gamma$ as the best-fit wide-binary model (see 4.6).
Table 4.2: PLANET close-binary model of MACHO 99-BLG-47. We note that the fact that $\chi^2 \sim$ dof results from our rescaling of the photometric errorbars. However, we use the same scaling factor here and for models described in Table 4.3 so the indistinguishableness between two models is not affected by this rescaling. The uncertainties for parameters are “1-σ error bars” determined by a quadratic fit of $\chi^2$ surface.

<table>
<thead>
<tr>
<th>parameters</th>
<th>PLANET (ISIS) + MACHO</th>
<th>PLANET (DoPHOT) + MACHO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>0.134 ± 0.009</td>
<td>0.121 ± 0.009</td>
</tr>
<tr>
<td>$q$</td>
<td>0.340 ± 0.041</td>
<td>0.374 ± 0.058</td>
</tr>
<tr>
<td>$t_E$</td>
<td>163 ± 26 days</td>
<td>183 ± 32 days</td>
</tr>
<tr>
<td>$t_0$</td>
<td>1393.9331 ± 0.0071</td>
<td>1393.9309 ± 0.0075</td>
</tr>
<tr>
<td>$u_0$</td>
<td>$(8.6 \pm 1.1) \times 10^{-3}$</td>
<td>$(7.5 \pm 1.3) \times 10^{-3}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>294°99 ± 0°25</td>
<td>294°45 ± 0°25</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>412.13</td>
<td>420.45</td>
</tr>
<tr>
<td>dof</td>
<td>412</td>
<td>423</td>
</tr>
</tbody>
</table>

$^a$the closest approach to the binary center of mass  
$^b$the Heliocentric Julian Date – 2450000  
$^c$the binary center of mass lying on the righthand side of the moving source
<table>
<thead>
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<th>PLANET (DoPHOT) + MACHO</th>
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<td>$0.751 \pm 0.193$</td>
<td>$0.917 \pm 0.288$</td>
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<td>$1393.9133 \pm 0.0074$</td>
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<td>$u_0^{a, b}$</td>
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Table 4.3: PLANET wide-binary model of MACHO 99-BLG-47.

$^a$with respect to the Einstein radius of the combined mass

$^b$The closest approach to the caustic center. See § 4.3 for the definition of the caustic center.

$^c$the Heliocentric Julian Date – 2450000

$^d$the caustic center lying on the righthand side of the moving source
derived from equation (4.1) function only as seeds for simplex, and since the final $\chi^2$-surface is very well-behaved, these discrepancies in input values do not influence the final result.

4.4. Discussion

The mass ratios of the best fit models are $q = 0.340 \pm 0.041$ (close binary) and $q = 0.751 \pm 0.193$ (wide binary), which are well away from the regime of planetary companions ($q \lesssim 0.01$). By comparison, the ratio of the duration of the anomalous portion of the lightcurve ($t_{\text{anom}} \sim 3$ days) to the Einstein timescale ($t_E \sim 160$ days$^3$) is $r = t_{\text{anom}}/t_E \sim 0.02$. For lightcurves perturbed by planetary caustics, one typically finds $q \sim r^2$ (Gould & Loeb 1992). This relation clearly does not apply to the caustics of extreme-separation binaries.

Models with $q \leq 0.01$ (and $q \geq 100$) are formally rejected at $\Delta \chi^2 \simeq 32$, which is significant but not in itself an overwhelming rejection of the planetary hypothesis. Hence, we examine the least $\chi^2$ model for $q = 0.01$ ($\Delta \chi^2 = 32.3$) for its plausibility and the origin of statistical discriminating power. We discover that the difference of $\chi^2$ is mostly from the MACHO data between HJD 2451310 and

---

$^3$ While the value here is for the close binary, the value for the wide binary is essentially the same because the relevant mass of the corresponding point-mass lens for the wide binary is not the combined mass but the mass of the single component that the source passes by.
2451360, approximately 2 months prior to the photometric peak. However, we also
find that the “planetary” model exhibits an extremely high peak magnification
($A_{\text{max}} \sim 15000$), and consequently, requires the event to be much longer
($t_E \simeq 40 \text{ yr}$) and more extremely blended ($I_s \simeq 25.7$, $F_s/F_{\text{base}} \simeq 0.2\%$) than the
already unusually long and highly blended best-fit models. In addition, there is a
clear trend of increasing peak magnification (and thus timescale and blend as well)
as $q$ is lowered beyond 0.01. This follows from the fact that the observed timescale
of the anomaly in the lightcurve essentially fixes the source movement relative
to the caustic. However, as $q$ becomes smaller, the size of the caustic relative to
the Einstein ring shrinks, and therefore, the timescale of the event, that is the
time required for the source to cross the Einstein ring, increases. (Note that this
behavior causes a mild continuous degeneracy between $q$ and the blending.) These
parameters determined for the “planetary” model are extremely contrived and
highly improbable a priori. Furthermore, the timescale associated with the “planet”
component, $t_p = q^{1/2}t_E \sim 4 \text{ yr}$ is much longer than that of typical stellar lenses,
which further reduces the plausibility of the planetary interpretation. In summary,
while in simple statistical terms, the star/planet scenario is not overwhelmingly
disfavored relative to extreme-separation binaries, we nevertheless can conclude
that it is highly unlikely that this event is due to a star/planet system.
From the standpoint of refining microlensing planet detection strategies, it is important to ask how one could have discriminated between the planetary and extreme-separation binary solutions with greater statistical significance. As noted above, most of the discriminating power came from the MACHO data points on the rising wing of the lightcurve, even though (or in a sense, because) these had the largest errors and the lowest density of the nonbaseline coverage. That is, the precision PLANET photometry over the peak and falling wing “predicts” the rising wing for each of the models, but the noisier MACHO data can only roughly discriminate between these predictions. Hence, the key would have been to get better data on the rising part of the lightcurve. In practice this is difficult: the MACHO data are noisier exactly because they are survey data, and one does not know to monitor an event intensively until the lightcurve has actually started to rise.

The long duration of the event, \( t_E \approx 160 \text{ days} \) (close binary) or \( t_E \approx 220 \text{ days} \) (wide binary), may lead one to expect that the event would show some sign of parallax effects (Gould 1992; Mao 1999; Soszyński, et al. 2001; Bennett et al. 2001). Similarly, the close passage of the source to a cusp could in principle give rise to finite source effects (Gould 1994; Nemiroff & Wickramasinghe 1994; Witt & Mao 1994), as for example is given in Chapter 2 for OGLE-1999-BUL-23 and Chapter 3 for EROS BLG-2000-5. Indeed, in the case of EROS BLG-2000-5, the
combination of parallax and finite-source effects permitted us to measure the mass of a microlens for the first time. We have therefore searched for both parallax and finite source effects, but find no significant detection of either.

Finally, we examine the position of the source on the CMD (Fig. 4.3). The most prominent feature found in the CMD is a track of stars running diagonally in the same direction as would the main sequence. Since the field is in the Galactic disk, this feature is probably not a true main sequence, but rather likely to be a “reddening sequence”, that is, an ensemble of mostly disk turnoff stars at progressively greater distances and correspondingly greater reddenings. The baseline “star” (combined light of the source and blended light) lies within this sequence toward its faint/red end although its position is seeing-dependent because of the seeing-correction term. The position indicated in the CMD is plotted assuming median seeing. However, the source itself \( I_s = 20.9, (V-I)_s = 1.94 \) lies \( \sim 2 \) mag below the baseline “star,” in a region of the CMD that is well below the threshold of detection. If the baseline were mainly composed of source light, and the “blended light” were simply source light that had been falsely attributed to blending by a wrong model, then the real source would lie within the well populated “reddening sequence”, and the timescale would be much shorter, \( t_E \sim 30 \) days. We therefore searched for solutions with little or no blending. However, we find that these are ruled out with \( \Delta \chi^2 = 2440 \). Furthermore, the best-fit models with no
Fig. 4.3.— CMD of $3' \times 3'$ field surrounding MACHO 99-BLG-47 ($l = 17^\circ 99$, $b = -1^\circ 96$) derived from SAAO observations. Shown are the positions of the baseline “star” (S+B) and the lensed source (S) in the absence of lensing (close-binary model). The position of the lensed source in the wide-binary model differs from this by substantially less than the size of the symbol. The errors are also smaller than the symbols. The majority of stars in the CMD are most likely turnoff stars seen at increasingly greater distances in the Galactic plane, and hence at correspondingly greater reddenings.
blended light are still extreme-separation binaries and not planetary systems. Since we have only a crude understanding of the CMD, no strong argument can be made that the position of the source is implausible. If, in fact, the observed track of stars in Figure 4.3 is really the reddening sequence of disk turnoff stars, the source position is consistent with a low-mass main sequence star lying \( \gtrsim 2 \) mag below the turnoff. This would also explain the lack of finite source effect. Moreover, we note that in disk fields, long events are not at all uncommon (Derue et al. 2001) because the observer, source, and lens are all moving with roughly the same velocity.

Regarding the large amount of blended light required to fit the observed lightcurve, we note that the highly significant seeing effect in the PSF-based photometry (see § 4.2.1) is also evidence that the event is strongly blended. In addition to the direct confirmation of the strong seeing effect from the comparison between the differential flux and the PSF-based flux measurements, we independently detect a significant seeing correction when we fit the observed flux to the magnification model. For SAAO \( I \)-band observation, the seeing correction determined from the model fit using DoPHOT flux is slightly smaller \((\sim 9F_s \text{ arcsec}^{-1})\) than the value derived from the regression between DoPHOT and ISIS flux. The seeing correction derived by fitting ISIS data to the model is basically consistent with zero.
4.5. Binary Lens vs. Binary Source

Multiple-peak events like the one seen in Figure 4.1 can be caused by a binary source (Griest & Hu 1992) rather than a binary lens. In general, one expects that such events will be chromatic, since the colors of the two sources will not usually be the same. By contrast, MACHO 99-BLG-47 is achromatic: the difference in color of the two wings of the lightcurve is $\Delta V - I = 0.01 \pm 0.05$. In the limit that one component of the binary is completely dark, the event will be achromatic, and yet can have still have multiple peaks caused by orbital motion of the binary during the event. In this case, however, there will be a series of roughly equally spaced peaks that gradually die out as the event declines (Han & Gould 1997), contrary to the distinct double-peaked behavior seen in Figure 4.1. Hence, a binary-source explanation appears a priori implausible.

Nevertheless, it is possible in principle that the source has two components of nearly identical color. We search for binary-source models, with either static or slowly moving components, but find that the best fit model has $\Delta \chi^2 = 1167$, which clearly rules out such models. The basic problem is that the observed second peak is very sharp given its height and the timescale of the declining lightcurve. Binary-source lightcurves that fit these latter features tend to have a width that is closer to that of the dotted curve in Figure 4.1. Recall that this curve represents a single-lens degenerate fit to the nonpeak data. We conclude that the explanation
for the double-peaked behavior of the lightcurve is that it is a binary-lens rather than a binary-source event.

4.6. Close and Wide Binary Correspondence

Dominik (1999) noticed that the Chang & Refsdal (1979, 1984) lens (CRL; see also Schneider et al. 1992) well approximates the binary lens system in the vicinity of each individual lens component of extreme wide-separation \((d \gg 1)\) binaries and in the vicinity of the secondary of extreme close-separation \((d \ll 1)\) binaries while the quadrupole lens (QL) approximation works nicely near the center of mass of extreme close-separation binaries. He further showed that the size and shape of the caustics and the behavior of the magnification field associated with them are very similar among the true binary lenses, the CRL approximation, and the QL approximation if the shear, \(\gamma\), of the CRL and the absolute eigenvalue of the quadrupole moment tensor, \(\hat{Q}\) (for simplicity, hereafter \(\hat{Q}\) will be just referred to “the quadrupole moment”), of the QL are both very small and their numerical values are close to each other \((\gamma \simeq \hat{Q} \ll 1)\). These findings also imply the existence of a correspondence between a wide binary with \(\gamma = d_w^2 q_w (1 + q_w)^{-1}\) and a close binary with \(\hat{Q} = d_c^2 q_c (1 + q_c)^{-2}\), sometimes referred to as a \(d \leftrightarrow d^{-1}\) correspondence. (Here and throughout this section, we use the subscript \(c\) and \(w\) to distinguish the
parameters associated with the close and wide binaries.) Here we rederive the basic result for this correspondence, and explore it more thoroughly.

The general form of the binary lens mapping equation can be expressed in notation utilizing complex numbers (Witt 1990),

\[ \zeta = z - \frac{\epsilon_1}{\bar{z} - z_1} - \frac{\epsilon_2}{\bar{z} - z_2}, \]  

(4.2)

and the magnification associated with a single image can be found by,

\[ A = \left(1 - \left| \frac{\partial \zeta}{\partial \bar{z}} \right|^2 \right)^{-1}, \]  

(4.3)

where \( \zeta, z, z_1, \) and \( z_2 \) are the positions of the source, the image, and the two lens components normalized by the Einstein radius of some mass, and \( \epsilon_1 \) and \( \epsilon_2 \) are the masses of lens components in units of the same mass. For the extreme close-binary case, if one chooses the center of mass as the origin and the binary axis as the real axis and sets the combined mass to be unity, then \( z_1 = d_c \epsilon_2, \) \( z_2 = -d_c \epsilon_1, \) \( \epsilon_1 = (1 + q_c)^{-1}, \) and \( \epsilon_2 = q_c(1 + q_c)^{-1}, \) and the lens equation (4.2) becomes

\[ \zeta = z - \frac{\epsilon_1}{\bar{z} - d_c \epsilon_2} - \frac{\epsilon_2}{\bar{z} + d_c \epsilon_1} \]

\[ \approx z - \frac{1}{\bar{z}} - \frac{d_c^2 \epsilon_1 \epsilon_2}{\bar{z}^3} + \frac{d_c^3 \epsilon_1 \epsilon_2 (\epsilon_1 - \epsilon_2)}{\bar{z}^4} + \cdots \quad (d_c \ll |z|). \]

(4.5)

Here the monopole term \((\bar{z}^{-1})\) is the same as for a PSPL and the first non-PSPL term is the quadrupole \((\bar{z}^{-3}),\) which acts as the main perturbation to the PSPL
if the quadrupole moment is small; \( \hat{Q} = d_c^2 \epsilon_1 \epsilon_2 = d_c^2 q_c (1 + q_c)^{-2} \ll 1 \). For a
given source position \( \zeta \), let the image position for the PSPL be \( z_0 \). Then, under
the perturbational approach, the image for the QL approximation is found at
\( z = z_0 + \delta z_c \), where \( |\delta z_c| \ll |z_0| \sim 1 \). Using the fact that \( z_0 \) is the image position
corresponding to \( \zeta \) for the PSPL (i.e., \( \zeta = z_0 - \bar{z}_0^{-1} \)), one obtains
\[
\delta z_c \approx \hat{Q} \left( 1 - \frac{1}{|z_0|^4} \right)^{-1} \left[ \left( \frac{1}{z_3^{\bar{z}_0}} - \frac{1}{z_3^{2 z_0}} \right) + \left( \frac{1}{z_4^{z_0 \bar{z}_0}} - \frac{1}{z_4^{2 z_0}} \right) \frac{1 - q_c}{1 + q_c} d_c + \cdots \right], \quad (4.6)
\]
from equation (4.5) and taking only terms linear in \( \delta z_c \) and \( \hat{Q} \). Here, we note that
\( \hat{Q} \sim \mathcal{O}(d_c^2) \) so that the second term in the bracket is lower order than \( \hat{Q}^2 \). In order
to find the magnification for this image, we differentiate equation (4.5), and find,
\[
\frac{\partial \zeta}{\partial \bar{z}} \approx \frac{1}{z^2} + \frac{3 \hat{Q}}{z^4} \left[ 1 - \frac{4(1 - q_c) d_c}{3(1 + q_c)} \bar{z}^4 + \cdots \right]. \quad (4.7)
\]
Then substituting \( z = z_0 + \delta z_c \), for which \( \delta z_c \) is given by equation (4.6), into
equation (4.7), we obtain,
\[
\frac{|\partial \zeta|^2}{|\partial \bar{z}|} \approx \frac{1}{|z_0|^4} \left[ \hat{Q} \left( \frac{3 |z_0|^4 - 2 |z_0|^2 - \frac{1}{z_0^2} + \frac{1}{z_0^4}}{|z_0^8 - |z_0|^4} \right) \right.
\frac{1 - q_c}{1 + q_c} d_c + \cdots \right] \quad \text{and} \quad (4.8)
\]
Note that the total magnification for the given source position is usually dominated
by one or two images found close to the critical curve. Thus, we consider only
the case for which the non-perturbed PSPL images lie close to the unit circle, so
we have $|z_0| = 1 + \Delta$ and $\Delta \ll 1$. Then, we find the expression for the inverse
magnification for the QL approximation (up to the order of $d_c^3$),

$$A^{-1} \approx \left|4\Delta - 2\tilde{Q}\left(\frac{1}{z_0^2} + \frac{1}{\bar{z}_0^2}\right) + 3\tilde{Q}\left(\frac{1}{z_0^3} + \frac{1}{\bar{z}_0^3}\right) \frac{1 - q_c}{1 + q_c}d_c\right|$$
$$= 4\left(|z_0| - 1\right) - \tilde{Q}\Re(z_0^{-2}) + \frac{3(1 - q_c)}{2(1 + q_c)}d_c\tilde{Q}\Re(z_0^{-3})\right| . \quad (4.9)$$

For the extreme wide-binary case, one can rewrite the lens equation (4.2) as

$$\zeta = z - \frac{1}{\bar{z}} - \frac{q_w}{\bar{z} + d_1} , \quad (4.10)$$
$$\approx z - \frac{1}{\bar{z}} \frac{q_w}{d_1} \frac{1}{\bar{z}_0} + \frac{q_w}{d_1^2} \bar{z} + \cdots \quad (d_w \gg |z|) . \quad (4.11)$$

Here the position and the mass of the first lens component are the origin and the
unit mass so that $z_1 = 0$, $z_2 = -d_1$, $\epsilon_1 = 1$, $\epsilon_2 = q_w$, and $d_1 = (1 + q_w)^{1/2}d_w$. We
note that, apart from the constant translation, the first non-PSPL term here is
essentially the shear, $\gamma = q_w d_1^{-2} = q_w d_w^{-2}(1 + q_w)^{-1}$ for the CRL approximation.
Analogous to the extreme close binary, if $\gamma \ll 1$, the image position for the
CRL approximation can be found by the perturbational approach, but here the
corresponding PSPL source position would be $\zeta + q_w d_1^{-1} = z_0 - \bar{z}_0^{-1}$. Then, the
image deviation $\delta z_w = z - z_0$ of CRL from PSPL is

$$\delta z_w \approx \gamma \left(1 - \frac{1}{|z_0|^4}\right)^{-1} \left[\left(\frac{z_0}{\bar{z}_0} - \bar{z}_0\right) + \left(\frac{z_0^2}{\bar{z}_0^2} - \frac{z_0}{\bar{z}_0}\right) \frac{1}{(1 + q_w)^{1/2}d_w} + \cdots \right] . \quad (4.12)$$

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Using this result and the derivative of equation (4.11),

\[
\frac{\partial \zeta}{\partial \bar{z}} \approx \frac{1}{\bar{z}^2} + \gamma \left[ 1 - \frac{2}{(1 + q_w)^{1/2}} \frac{\bar{z}}{d_w} + \cdots \right],
\]

(4.13)

we find

\[
\left| \frac{\partial \zeta}{\partial \bar{z}} \right|^2 \approx \frac{1}{|z_0|^4} + \gamma \left[ \frac{|z_0|^4 + 2|z_0|^2 - 3}{|z_0|^4 - 1} \left( \frac{1}{z_0^2} + \frac{1}{\bar{z}_0^2} \right) - \frac{2|z_0|^6 + 2|z_0|^4 - 4|z_0|^2}{|z_0|^4 - 1} \left( \frac{1}{z_0^3} + \frac{1}{\bar{z}_0^3} \right) \frac{1}{(1 + q_w)^{1/2}d_w} + \cdots \right].
\]

(4.14)

From the same argument used for the QL approximation of the extreme close binary, we can set \(|z_0| = 1 + \Delta\), and then the inverse magnification for the CRL approximation (up to the order of \(d_w^{-3}\)) is

\[
A^{-1} \approx \left| 4\Delta - 2\gamma \left( \frac{1}{z_0^2} + \frac{1}{\bar{z}_0^2} \right) + 3\gamma \left( \frac{1}{z_0^3} + \frac{1}{\bar{z}_0^3} \right) \frac{1}{(1 + q_w)^{1/2}d_w} \right|
\]

\[
= 4 \left| (|z_0| - 1) - \gamma \Re(z_0^{-2}) + \frac{3}{2(1 + q_w)^{1/2}} \frac{1}{d_w} \gamma \Re(z_0^{-3}) \right|. 
\]

(4.15)

By comparing equations (4.9) and (4.15), we therefore establish the magnification correspondence (up to the order of \(d_c^2 \sim d_w^{-2}\)) between the close binary with \(\hat{Q} = d_c^2q_c(1 + q_c)^{-2}\) and the wide binary with \(\gamma = d_w^{-2}q_w(1 + q_w)^{-1}\) when \(\hat{Q} \approx \gamma \ll 1\).

For the two PLANET ISIS models for MACHO 99-BLG-47, we obtain \(\hat{Q} = 3.40 \times 10^{-3}\) for the close binary solution \((d_c, q_c) = (0.134, 0.340)\) and
\( \gamma = 3.35 \times 10^{-3} \) for the wide binary solution \((d_w, q_w) = (11.31, 0.751)\). Hence, the observed degeneracy between the two PLANET models (see § 4.3 and Tables 4.2 and 4.3) is the clearest example of this type of correspondence observed so far. While Afonso et al. (2000) reported that two different binary lens models, one of a close binary and the other of a wide binary, can fit the observed lightcurve of the (caustic-crossing) binary lens event, MACHO 98-SMC-1, one can infer from their figure 8, that the degeneracy between the two models exists only for the specific lightcurves (which is essentially a particular one dimensional slice of the magnification field over the source plane) but not for the magnification field in the neighborhood of the caustic as a whole. This is obvious from the relative rotation of the two caustics. In addition, the source magnitudes for the two models of MACHO 98-SMC-1 differ by \( \sim 0.18 \) mag. By contrast, the difference of the predicted \( I_s \) between the two models of MACHO 99-BLG-47 is only \( \sim 0.02 \) mag. In fact, we get \( \hat{Q} = 6.5 \times 10^{-2} \) (the close binary) and \( \gamma = 1.8 \times 10^{-2} \) (the wide binary) for the two models of MACHO 98-SMC-1, and thus, although the degeneracy of the MACHO 98-SMC-1 lightcurve is somehow related to the correspondence of \( d \leftrightarrow d^{-1} \), it cannot be completely explained simply by the argument in this section, and it should be investigated further for its origin. On the other hand, the degeneracy of MACHO 99-BLG-47 is the first definitive observational case of the correspondence between extreme separation binaries. This can be also seen in
Figure 4.4, which illustrates the similarity between the magnification fields for the two models of MACHO 99-BLG-47.

The very low $\Delta \chi^2$ between the two solutions despite the excellent data implies that it is extraordinarily difficult to break this degeneracy with photometric data. Gould & Han (2000) showed that for MACHO 98-SMC-1 the two solutions were also astrometrically degenerate, at least for data streams lying within a few $t_E$ of the peak. They argued that this astrometric degeneracy, like the corresponding photometric degeneracy, was rooted in the lens equation. However, as shown in this section, the correspondence between the equations describing close- and wide-binaries is purely local (See also Dominik 2001). For example, there is a constant-offset term in equation (4.11), which does not give rise to any local photometric or astrometric effects, but which must “disappear” at late times. Hence there must be a latetime astrometric shift between the two solutions. Such a shift was noted explicitly by Gould & Han (2000) for the case of MACHO 98-SMC-1, and these are likely to be a generic feature of close/wide corresponding pairs of solutions.

We also plot the lines of $(d,q)$ pairs that have the same shear $\gamma$ as the best-fit wide-binary model or the same quadrupole moment $\hat{Q}$ as the best-fit close-binary model on $\chi^2$-surface contour plot shown in Figure 4.2. While the iso-shear line for wide binaries lies nearly parallel to the direction of the
Fig. 4.4.— Grayscale plot of the difference of the normalized flux fields, $2|F_w - F_c|/(F_{s,w}A_w + F_{s,c}A_c)$ of the two PLANET models around the caustic. The two fields are scaled and aligned so that the source trajectories coincide with each other. The resulting shape and the relative size of the caustics are remarkably close to each other [c.f., fig. 8 of Afonso et al. (2000) and Fig 2.6] Also drawn are the contours of zero difference (dotted line) and 5% difference (solid line). While the actual source trajectory ($y = 0$) naturally traces well the zero-difference line, the flux difference would also be extremely small for other trajectories through the region, except very near the caustic.
principal conjugate near the best-fit model, it is clear from the figure that the condition of \( \hat{Q} \simeq \gamma \ll 1 \) alone does not define the observed well-defined twofold degeneracy, which involves the additional correspondence between higher order terms beyond the quadrupole moment (\( \sim d_c^2 \)) and the pure shear (\( \sim d_w^{-2} \)). Further comparison between equations (4.9) and (4.15) indicates that there exists a magnification correspondence up to the order of \( d_c^3 \sim d_w^{-3} \) if the condition
\[ d_c(1 - q_c)(1 + q_c)^{-1} = d_w^{-1}(1 + q_w)^{-1/2} \]
is also satisfied in addition to \( \hat{Q} = \gamma \ll 1 \). We find that, for the two PLANET models, \( d_c(1 - q_c)(1 + q_c)^{-1} = 6.6 \times 10^{-2} \) (the close binary) and \( d_w^{-1}(1 + q_w)^{-1/2} = 6.7 \times 10^{-2} \) (the wide binary). Hence, we conclude that, in fact, these two conditions,

\[ d_c d_w (1 + q_w)^{1/2} = \frac{1 + q_c}{1 - q_c} \]

(4.17)

define a unique correspondence between two extreme separation binaries. We also note that the images (not shown) of the iso-\( \Delta \chi^2 \) contours for the close binary models of MACHO 99-BLG-47 under the mapping defined by the above two relations follows extremely closely the corresponding iso-\( \Delta \chi^2 \) contour for wide binary models, except for the difference of \( \Delta \chi^2 \simeq 0.6 \) offset between two solutions.
4.7. Summary

MACHO 99-BLG-47 is the first microlensing event with a short-lived, high S/N anomaly, characteristics that could betray the existence of a planet around the lensing star. Nevertheless, we conclude that the lens of MACHO 99-BLG-47 is not a planetary system, but an extreme-separation (very close or very wide) binary composed of components of similar mass, based on the result of the lightcurve fit as well as the extreme value of event duration and blending fraction required for any plausible “planetary” fit.
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Appendix A

Fold-Caustic Magnification of Limb-Darkened Disk

If the point-source magnification due to a linear caustic is

\[ A_r = \left( \frac{u_r}{\delta u_\perp} \right)^{\frac{1}{2}} \Theta(\delta u_\perp), \]  

(A.1a)

\[ \delta u_\perp = \frac{\theta_x - \theta_c}{\theta_E}, \]  

(A.1b)

where the coordinate system is chosen so that the caustic lies parallel to the \(y\)-direction with \(x\)-coordinate of \(\theta_c\) and the \(x\)-coordinate of the point source is \(\theta_x\), then the magnification of the finite source with limb darkening at the origin is

\[
A = F_{s,\lambda}^{-1} \int_D d^2 \theta \ A_r S_\lambda \\
= \int_{-\infty}^{\infty} d\theta_x \ A_r \int d\theta_y \ \frac{S_\lambda(\theta_x, \theta_y)}{F_{s,\lambda}}
\]
\[
\begin{align*}
= & \int_{-\infty}^{\infty} d\theta_x A_r \int d\theta_y \delta(\theta'_x - \theta_x) \frac{S_\lambda(\theta'_x, \theta_y)}{F_{s,\lambda}} \\
= & \int_{-\infty}^{\infty} d\theta_x A_r \int d^2\theta \frac{S_\lambda(\theta)}{F_{s,\lambda}} \delta(\theta \cos \phi_y - \theta_x) \\
= & \int_{-\infty}^{\infty} d\theta_x A_r \int_{\theta_0}^{0} \theta d\theta S_\lambda(\theta) \int_{-\pi}^{0} d\phi_y \delta(\theta \cos \phi_y - \theta_x) \\
= & \int_{-\infty}^{\infty} d\theta_x A_r \int_{\theta_0}^{0} \theta d\theta S_\lambda(\theta) (\theta^2 - \theta_x^2)^{-\frac{1}{2}} \Theta(\theta - |\theta_x|) \\
= & \frac{2(u_r\theta_E)^\frac{1}{2}}{\pi \theta_x^2} \int_{-\infty}^{\infty} d\theta_x (\theta_x - \theta_c)^{-\frac{1}{2}} \Theta(\theta_x - \theta_c) \Theta(\theta_x - |\theta_x|) \\
\times & \int_{|\theta_x|}^{\theta_0} \theta d\theta (\theta^2 - \theta_x^2)^{-\frac{1}{2}} \left\{ 1 - \sum_m \Gamma_{m,\lambda} \left[ 1 - \frac{m + 2}{2} \left( 1 - \frac{\theta^2}{\theta_x^2} \right)^\frac{m}{2} \right] \right\} \\
= & \frac{2(\frac{u_r}{\theta_x})^\frac{1}{2}}{\pi} \Theta(\theta - \theta_c) \int_{\max(-\theta_c, \theta_x)}^{\theta_0} \frac{d\theta_x}{\theta_x^2 \theta_c^2} \left( \frac{\theta_x - \theta_c}{\theta_x} \right)^{-\frac{1}{2}} \\
\times & \int_{|\theta_x|}^{\theta_0} \theta d\theta \left( \frac{\theta^2 - \theta_x^2}{\theta_x^2 - \theta_c^2} \right)^{-\frac{1}{2}} \left\{ 1 - \sum m \Gamma_{m,\lambda} \left[ 1 - \frac{m + 2}{2} \left( 1 - \frac{\theta^2}{\theta_x^2} \right)^\frac{m}{2} \right] \right\} \quad (A.2a)
\end{align*}
\]

Here, \(\delta(x)\) is Dirac delta distribution, \(\Theta(x)\) is Heaviside step function, and \(S_\lambda(\theta)\) is the axis-symmetric surface brightness profile taken the form of equation (2.3).

Then, defining

\[ G_n(\eta) \equiv \frac{2(n + 1)}{\pi} \Theta(1 - \eta) \int_{\max(-1, \eta)}^{1} \frac{dx}{(x - \eta)^\frac{1}{2}} \int_{|x|}^{1} \frac{(1 - \rho^2)^n \rho}{(\rho^2 - x^2)^\frac{1}{2}} d\rho, \quad (A.2b) \]

one obtains

\[ A = \left( \frac{u_r}{\rho_*} \right)^\frac{1}{2} \left\{ G_0 \left( -\frac{\Delta u_\perp}{\rho_*} \right) + \sum_m \Gamma_{m,\lambda} \left[ G_m \left( -\frac{\Delta u_\perp}{\rho_*} \right) - G_0 \left( -\frac{\Delta u_\perp}{\rho_*} \right) \right] \right\} \quad (A.2c) \]

Here \(\Delta u_\perp = -\theta_c/\theta_E\) is the distance to the caustic from the source center in units of the Einstein ring radius and \(\rho_* = \theta_s/\theta_E\). Note that \(\Delta u_\perp\) is positive when the source
center lies inside the caustic and vice versa. The second integral in equation (A.2b) can be evaluated by using a new variable $\nu$,

$$\nu = \arcsin \sqrt{\frac{\rho^2 - x^2}{1 - x^2}} ; \quad \rho^2 = \sin^2 \nu + x^2 \cos^2 \nu,$$

(A.3a)

$$\int_{|x|}^{1} \frac{(1 - \rho^2)^n \rho}{(\rho^2 - x^2)^{\frac{1}{2}}} \, d\rho = (1 - x^2)^{n+\frac{1}{2}} \int_0^{\pi/2} d\nu \,(\cos \nu)^{2n+1}$$

$$= \frac{\sqrt{\pi}}{2} \frac{n!}{(n + 1/2)!} (1 - x^2)^{n+\frac{1}{2}}, \quad (A.3b)$$

and therefore,

$$G_n(\eta) = \frac{1}{\sqrt{\pi}} \frac{(n + 1)!}{(n + 1/2)!} \Theta(1 - \eta) \int_{\max(\eta, -1)}^{1} dx \frac{(1 - x^2)^{n+\frac{1}{2}}}{(x - \eta)^{\frac{1}{2}}}. \quad (A.3c)$$


**Appendix B**

**Determination of $\varsigma$**

If $s$ is the Sun’s position vector with respect to the Earth normalized by 1 AU, then the projection of $s$ onto the plane of the sky, $\varsigma$, is

$$\varsigma = s - (s \cdot \hat{n})\hat{n},$$  \hspace{1cm} (B.1)

where $\hat{n}$ is the line-of-sight unit vector toward the position of the event on the sky, while the projection of $\hat{p}$, the unit vector toward the north ecliptic pole (NEP), is given by $\hat{p} = \hat{p} - (\hat{p} \cdot \hat{n})\hat{n}$. Then, $(\varsigma_w, \varsigma_n)$, the ecliptic coordinate components of $\varsigma$, are

$$\varsigma_w = \frac{(\hat{p} \times \varsigma) \cdot \hat{n}}{|\hat{p}|} = \frac{(\hat{p} \times s) \cdot \hat{n}}{\sqrt{1 - (\hat{p} \cdot \hat{n})^2}},$$  \hspace{1cm} (B.2a)

$$\varsigma_n = \frac{\hat{p} \cdot \varsigma}{|\hat{p}|} = -\frac{(\hat{p} \cdot \hat{n})(s \cdot \hat{n})}{\sqrt{1 - (\hat{p} \cdot \hat{n})^2}},$$  \hspace{1cm} (B.2b)
where we make use of $\hat{p} \cdot s = 0$. One can choose three-dimensional coordinate axes so that the $x$-axis is the direction of the vernal equinox, the $z$-axis is the direction to the NEP, and $\hat{y} = \hat{z} \times \hat{x}$. Then,

$$
\hat{p} = (0, 0, 1) ; \quad (B.3a)
$$

$$
s = (r_\oplus \cos \lambda_\odot, r_\oplus \sin \lambda_\odot, 0) ; \quad (B.3b)
$$

$$
\hat{n} = (\cos \lambda_0 \cos \beta_0, \sin \lambda_0 \cos \beta_0, \sin \beta_0) , \quad (B.3c)
$$

where $r_\oplus$ is the distance to the Sun from the Earth in units of AU, $\lambda_\odot$ is the Sun’s ecliptic longitude, and $(\lambda_0, \beta_0)$ are the ecliptic coordinates of the event. By substituting equations (B.3) into equations (B.2), one finds that

$$
\varsigma_w = -r_\oplus \sin(\lambda_\odot - \lambda_0) , \quad (B.4a)
$$

$$
\varsigma_n = -r_\oplus \cos(\lambda_\odot - \lambda_0) \sin \beta_0 . \quad (B.4b)
$$

In general, one must consider the Earth’s orbital eccentricity ($\epsilon = 1.67 \times 10^{-2}$) to calculate $\varsigma$ for any given time. Then,

$$
r_\oplus = 1 - \epsilon \cos \psi ; \quad \lambda_\odot = \xi - \phi_T , \quad (B.5a)
$$

$$
\psi - \epsilon \sin \psi = \Omega t , \quad (B.5b)
$$

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\[
\sin \xi = \frac{(1 - \epsilon^2)^{1/2} \sin \psi}{1 - \epsilon \cos \psi}; \quad \cos \xi = \frac{\cos \psi - \epsilon}{1 - \epsilon \cos \psi},
\]

where \(\psi\) and \(\xi\) are the eccentric and true anomalies of the Earth, \(\phi_Y = 77^\circ 86\) is the true anomaly at the vernal equinox (March 20, 07h35m UT for 2000; Larsen & Holdaway 1999), \(t\) is the time elapsed since perihelion, and \(\Omega = 2\pi \text{ yr}^{-1}\). Note that the Earth was at perihelion at January 3, 05h UT for 2000 (Larsen & Holdaway 1999). Although equations (B.5) cannot be solved for \(r_\oplus\) and \(\lambda_\odot\) in closed form as functions of \(t\), one can expand in series with respect to \(\epsilon\) and approximate up to the first order (epicycle approximation) so that

\[
r_\oplus = 1 - \epsilon \cos(\Omega t); \quad \lambda_\odot = \Omega t - \phi_Y + 2\epsilon \sin(\Omega t).
\]
The angular position of a celestial object observed from an observatory on the surface of the Earth is related to its geocentric angular position, $\varphi_X$ by

$$\varphi'_X = \varphi_X - \frac{\gamma R_{\oplus}}{D_X}, \quad (C.1)$$

where

$$\gamma = r - (r \cdot \hat{n}) \hat{n} \quad (C.2)$$

is the projection of the position vector, $r$, of the observatory with respect to the center of the Earth, onto the plane of the sky and normalized by the mean radius of the Earth, $R_{\oplus}$; and $D_X$ and $\hat{n}$ are the distance and the line-of-sight vector to the object of interest. If one observe a microlensing event, the actual dimensionless
lens-source separation vector therefore differs from \( \mathbf{u} \) (eq. [3.17]) by

\[
\mathbf{u}' = \frac{\varphi_S' - \varphi_L'}{\theta_E} = \mathbf{u} + \frac{\mathbf{R}_E}{r_E} \gamma
\]

\[= \mathbf{u} + (t - t_c) \mu_E - \pi_E (\zeta - \frac{\mathbf{R}_E}{\text{AU}} \gamma). \tag{C.3}\]

To find the algebraic expression for ecliptic coordinate components of \( \gamma \), we choose the same coordinate axes as for equations (B.3). Then, the position vector, \( \mathbf{r} \) is expressed as

\[
\mathbf{r} = (\cos \delta g \cos \tau_\Upsilon, \cos \delta g \sin \tau_\Upsilon \cos \varepsilon + \sin \delta g \sin \varepsilon, \sin \delta g \cos \varepsilon - \cos \delta g \sin \tau_\Upsilon \sin \varepsilon), \tag{C.4}\]

where \( \delta_g \) is the geographic latitude of the observatory, \( \tau_\Upsilon \) is the hour angle of the vernal equinox (i.e. the angle of the local sidereal time) at the observation, and \( \varepsilon = 23^\circ 44 \) is the angle between the direction toward the north celestial pole and the NEP (here we also assume that the Earth is a perfect sphere). Then, following a similar procedure as in B,

\[
\gamma_w = \frac{(\mathbf{p} \times \gamma) \cdot \hat{n}}{|\mathbf{p}|} = \frac{(\mathbf{p} \times \mathbf{r}) \cdot \hat{n}}{\sqrt{1 - (\mathbf{p} \cdot \hat{n})^2}}, \tag{C.5a}\]

\[
\gamma_n = \frac{\mathbf{p} \cdot \gamma}{|\mathbf{p}|} = \frac{\mathbf{p} \cdot \mathbf{r} - (\mathbf{p} \cdot \hat{n})(\mathbf{r} \cdot \hat{n})}{\sqrt{1 - (\mathbf{p} \cdot \hat{n})^2}}, \tag{C.5b}\]

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one obtains the ecliptic coordinate components of $\gamma$, 

$$
\gamma_w = -\sin\delta_g \sin \varepsilon \cos \lambda_0 + \cos\delta_g (\cos \tau_Y \sin \lambda_0 - \sin \tau_Y \cos \varepsilon \cos \lambda_0); 
$$  \hfill (C.6a)

$$
\gamma_n = \sin\delta_g (\cos \varepsilon \cos \beta_0 - \sin \varepsilon \sin \lambda_0 \sin \beta_0) \\
- \cos\delta_g [\cos \tau_Y \cos \lambda_0 \sin \beta_0 + \sin \tau_Y (\sin \varepsilon \cos \beta_0 + \cos \varepsilon \sin \lambda_0 \sin \beta_0)] . \hfill (C.6b)
$$
As we described in § 3.6.2, our $\chi^2$ minimization procedure is effective at fixed $(d, q)$ (with the nine other geometrical parameters allowed to vary), but does not work when all 11 geometrical parameters are allowed to vary simultaneously. We therefore find the global minimum by evaluating $\chi^2$ over a $(d, q)$ grid. How can the errors, and more generally the covariances, be determined under these circumstances?

In what follows, the parameters will be collectively represented by a vector $a_i$ and the indices $i, j$ will be allowed to vary over all $p = 11$ parameters. We assign $a_1 = d$ and $a_2 = q$, and designate that the indices $m, n$ will be restricted to these two parameters. What we seek to evaluate is $c_{ij} = \text{cov}(a_i, a_j) \equiv \langle a_i a_j \rangle - \langle a_i \rangle \langle a_j \rangle$.

First we note that it is straightforward to determine $c_{mn}$: simply evaluate $\chi^2$ at a series of points on the $(a_1, a_2)$ grid, and fit these to
\( \chi^2 = \chi^2_{\text{min}} + \sum_{m,n=1}^{2} \hat{b}_{mn}(a_m - a_m^{\text{min}})(a_n - a_n^{\text{min}}) \). Then \((\hat{c}_{mn}) = (\hat{b}_{mn})^{-1}\) is the covariance matrix restricted to the first two parameters, i.e., \(\hat{c}_{mn} = c_{mn}\).

Next, at fixed \((a_1, a_2) = (a_1^\dagger, a_2^\dagger)\), we evaluate the restricted covariance matrix of the remaining \(p - 2 = 9\) parameters by varying 0, 1, or 2 parameters at a time and fitting the resulting \(\chi^2\) hypersurface to \(\chi^2 = \chi^2_{\text{min}} + \sum_{i,j=3}^{11} \tilde{b}_{ij}(a_i - a_i^{\text{min}})(a_j - a_j^{\text{min}})\). Then, using equations (E.5), one can show that \((\tilde{c}_{ij}) = (\tilde{b}_{ij})^{-1}\) is related to the full covariance matrix by

\[
\tilde{c}_{ij} = c_{ij} - \sum_{m,n} \hat{b}_{mn}c_{mi}c_{nj} \quad (i, j \neq 1, 2),
\]

and the parameters, \(\tilde{a}_i\), at the constrained minimum are

\[
\tilde{a}_i = a_i^{\text{min}} - \sum_{m,n} (a_m^{\text{min}} - a_m^\dagger) \hat{b}_{mn}c_{ni} \quad (i \neq 1, 2).
\]

Differentiating equation (D.2) with respect to \(a_m^\dagger\) yields

\[
\frac{\partial \tilde{a}_i}{\partial a_m^\dagger} = \sum_n \hat{b}_{mn}c_{ni},
\]

\[
c_{mi} = \sum_n \hat{c}_{mn} \frac{\partial \tilde{a}_i}{\partial a_n^\dagger}.
\]

The partial derivatives can be determined simply by finding the change in \(a_i\) as one steps along one axis of the \((d, q)\) grid. Since \(\hat{c}_{mn}\) is already known from the
first step, above, the $c_{mi}$ are also known. Finally, the remaining covariances can be found by substituting equation (D.4) into equation (D.1),

$$c_{ij} = \tilde{c}_{ij} + \sum_{m,n} \tilde{c}_{mn} \frac{\partial \tilde{a}_i}{\partial a_m} \frac{\partial \tilde{a}_j}{\partial a_n}.$$  

(D.5)
Appendix E

Minimization with Constraints

We use Lagrange multipliers to evaluate the $n$-dimensional vector $\{\tilde{a}_i\}$ that minimizes the quadratic function $H(\{a_i\}) = \sum_{i,j=1}^{n} b_{ij} (a_i - a_i^0)(a_j - a_j^0) + H_0$, subject to the $m$ linear constraints $\sum_{i=1}^{n} a_i \alpha_i^k = z^k$ ($k = 1, \ldots, m$). Here the $a_i^0$, the $b_{ij}$, and $H_0$ are constants. At this minimum, $\nabla H$ must lie in the $m$-dimensional subspace spanned by the constraint vectors $\{\alpha_i^l\}$, i.e., $\sum_{j=1}^{n} b_{ij} (\tilde{a}_j - a_j^0) + \sum_{i=1}^{m} \alpha_i^l D^l = 0$, or

$$\tilde{a}_i = a_i^0 - \sum_{l=1}^{m} D^l \kappa_i^l ; \quad \kappa_i^l \equiv \sum_{j=1}^{n} c_{ij} \alpha_j^l,$$

(E.1)

where $(c_{ij}) \equiv (b_{ij})^{-1}$, and the $D^l$ are constants to be determined. Multiplying equation (E.1) by each of the $\alpha_i^k$ yields a set of $m$ equations,

$$\sum_{l=1}^{m} C^{kl} D^l = \sum_{i=1}^{n} a_i^0 \alpha_i^k - z^k ; \quad C^{kl} \equiv \sum_{i=1}^{n} \alpha_i^k \kappa_i^l = \sum_{i,j=1}^{n} c_{ij} \alpha_i^k \alpha_j^l,$$

(E.2)
which can be inverted to solve for the $D^k$,

$$D^k = \sum_{l=1}^{m} B^{kl} \left( \sum_{i=1}^{n} a_{i}^{0} \alpha_{i}^{l} - z^{l} \right) ; \quad (B^{kl}) = (C^{kl})^{-1} . \quad (E.3)$$

Note that for the special case where $\mathcal{H}$ can be interpreted as $\chi^2$, $c_{ij}$ is the covariance matrix of the unconstrained parameters $a_i$, i.e.,

$$c_{ij} = \text{cov}(a_i, a_j) = \langle a_i a_j \rangle - \langle a_i \rangle \langle a_j \rangle .$$

One then finds by direct substitution that $\text{cov}(D^k, a_{i}^{0}) = \sum_{l=1}^{m} B^{kl} \kappa_{i}^{l}$, so that the covariances of the constrained parameters $\tilde{c}_{ij} = \text{cov}(\tilde{a}_i, \tilde{a}_j)$, are given by

$$\tilde{c}_{ij} = c_{ij} - \sum_{k,l=1}^{m} B^{kl} \kappa_{i}^{k} \kappa_{j}^{l} . \quad (E.4)$$

To further specialize to an important subcase, let $\alpha_{i}^{k} = \delta_{ik}$. Then $\kappa_{i}^{k} = c_{ik}$ and $C^{kl} = c_{kl}$, hence $B^{kl} = \hat{b}_{kl}$, where $(\hat{b}_{kl}) = (\hat{c}_{kl})^{-1}$ and $\hat{c}_{kl}$ is the (unconstrained) $m \times m$ covariance matrix restricted to the $m$ parameters that are to be constrained. Thus, for this special case, equations (E.1), (E.3), and (E.4) become

$$\tilde{a}_i = a_{i}^{0} - \sum_{k,l=1}^{m} c_{ik} \hat{b}_{kl} (a_{i}^{0} - z^{l}) ; \quad \tilde{c}_{ij} = c_{ij} - \sum_{k,l=1}^{m} \hat{b}_{kl} c_{ik} c_{jl} . \quad (E.5)$$