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CROSS SECTION AND ASTROPHYSICAL S-FACTOR MEASUREMENTS OF RADIATIVE PROTON CAPTURE ON $^{102}$Pd, $^{116}$Sn, AND ALPHA CAPTURE ON $^{112}$Sn AT LOW ENERGIES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * *

The Ohio State University

2001

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Abstract

The cross section and astrophysical S-factor measurements for $^{102}$Pd(p,γ)$^{102}$Ag and $^{116}$Sn(p,γ)$^{117}$Sb reactions have been performed in the proton energy range 2.6 to 4.25 MeV using the 11MV FN Tandem Van de Graaff accelerator at the University of Notre Dame. In addition, the $^{112}$Sn(α,γ)$^{118}$Te reaction has been measured over the alpha beam energy range 7.0 to 10.5 MeV. The results have been compared to those of statistical model calculations generated by the NON-SMOKER code. Since low cross sections are involved, an activation technique has been used, whereby gamma rays have been detected off-line by two HPGe detectors in a low background environment. These measurements will be used to judge the applicability of the NON-SMOKER code to calculations of nucleosynthesis in the γ- and rp- processes in explosive astrophysical environments, where currently almost no experimental data exist in the mass region beyond A=100.
To Kuzey,

who means everthing to me!
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CHAPTER 1

INTRODUCTION

The Universe as a whole is the best laboratory for studying nuclear physics. Where else can you find the $10^9$K temperatures and a 15 billion year old experiment?

Nuclear Science Division, Berkeley Lab.

In the last few decades, nuclear astrophysics has grown into one of the major subfields in nuclear physics. In the past, the understanding of hydrostatic burning in stars and related stellar evolution has involved mainly nuclear physics along the valley of stability [Fowler (1984)]. Now, studies of the origin and the abundances of the elements, astrophysical sites and scenarios which require knowledge of nuclear processes and properties at two extreme sides of the nuclear chart, very neutron rich and proton rich nuclei, have become the motivation for many on-going and future developments worldwide. Important progress has been achieved in many aspects of nuclear astrophysics due to new observational tools, improvements in laboratory facilities, and detectors. Progress in computer hardware and software and improved theoretical models have contributed to better simulations of astrophysical scenarios, but have also raised new questions which will keep the field exciting in the future.
Although the origin of the abundances of the elements is generally well understood, certain discrepancies remain. In particular, the observed abundances of p-process elements, that is, nuclei that are blocked from production via the r- and s-processes by stable nuclides, are frequently inconsistent with their theoretical predictions. These abundances are based on complex network calculations within a given astrophysical environment and involve many reactions. In the case of p-process calculations especially, many of the reactions involve proton-rich nuclei far from stability. Consequently, hardly any reaction rates in the region of interest either can be or have been measured, and so p-process studies are forced to rely on statistical models to estimate the reaction rates. The few relevant reactions that have been investigated in the mass region beyond A=100 include (p,γ) reactions on targets of $^{112}\text{Sn}$ [Chloupek et al. (1999)], $^{118}\text{Sn}$ [Chloupek et al. (1999)], and $^{142}\text{Ce}$ [Verdieck & Miller (1967)]. Also, (α,γ) reaction measurements on targets of $^{139}\text{La}$ [Verdieck & Miller (1967)], and $^{144}\text{Sm}$ [Somorjai et al. (1998)] have been investigated.

Nucleosynthesis of the p-process nuclei is thought to be due to a combination of several nuclear processes, such as the rp-process, the γ-process, the α-process, and the ν-process [Boyd (1999)]. In an effort to help constrain the theoretical models of the first two sub-processes, we have made cross section measurements of three different reactions. The rp-process is a rapid series of (p,γ) reactions and β⁺-decays on existing light seed nuclei in a high temperature H-rich environment ($\sim 0.2-2.0\times10^9$K); the alternative γ-process proceeds through a series of photodissociation reactions, (γ,p),
(γ,n), and (γ,α), on existing heavy seeds at a high temperature of 2-3\times10^9 K. The high temperatures, together with the fact that the site of the rp-process and γ-process is often a degenerate object such as a white dwarf or a neutron star, means that the possible astrophysical sites for these processes are explosive environments.

In this thesis, the measurements of the cross sections for the $^{102}$Pd(p,γ)$^{103}$Ag, $^{116}$Sn(p,γ)$^{117}$Sb and $^{112}$Sn(α,γ)$^{116}$Te reactions are presented. Incident proton energies ranged from 2.6 MeV to 4.25 MeV and incident alpha energies ranged from 7.5 MeV to 10.5 MeV. These energies are particularly interesting since they are relevant to the Gamow window at high temperature environments. Cross sections are measured, and then astrophysical S-factors are deduced, so as to test the applicability of the leading statistical model, NON-SMOKER [Rauscher & Thielemann (2000)], widely used in p-process simulations. In Chapter 2, the star formation, the burning stages in stars, and three major processes (s-, r- and p-processes) for synthesis of heavy nuclei (A\geq60) are discussed separately. In Chapter 3, the formalism used in calculating the reaction rates and the NON-SMOKER code is presented. In Chapter 4, the operation of the FN tandem Van de Graaff accelerator used for the measurements is described. The apparatus and the methods used in this study are also discussed in this chapter. Then, the data, the analysis of the data, and the method used for cross section and astrophysical S-factor determinations and the results of our measurements are presented and compared with those of NON-SMOKER code in Chapter 5. Conclusions are presented at the end.
CHAPTER 2

ELEMENT NUCLEOSYNTHESIS

The main motivation in nuclear astrophysics is to explain the observed abundances and the origin of the elements. Stellar nucleosynthesis has become a very successful theory for the element formation, and assumes that only the abundant light elements, $^1$H and $^4$He produced in the big bang, are its source elements. These elements are converted to the heavier nuclei by burning processes in stars. H-burning, He-burning, C-burning, Ne-burning, O-burning, Si-burning are the main burning stages which produce the elements up to around iron group. Those are the most stable nuclei with the highest binding energy per nucleon. Thus, beyond that point, the models for the creation of most of the heavy nuclides involve successive neutron capture reactions and $\beta$-decays. The s- and r-processes are the models of how this is done. However, there is a family of massive stable nuclei of about 34 members, the p-process nuclei, from $^{74}$Se up to $^{198}$Hg which are relatively rich into protons and which cannot be produced by the neutron capture processes, the s- and r-processes. So, some other mechanisms than a sequence of neutron captures and $\beta^-$-decays must be responsible for synthesizing these 34 rare proton rich isotopes. The process (or
collection of processes) responsible for nucleosynthesis of proton rich nuclei shielded from s- and r-processes is called the p-process.

The star formation, the burning stages in stars, and three major processes for synthesis of heavy nuclei ($A \geq 60$) are discussed in the next sections separately.

2.1 Stellar Evolution

As the interstellar material condenses to form a star, the kinetic energy of the individual atoms increases, thereby the temperature and density within the star increase until hydrogen burning within the core proceeds, producing energy that increases the pressure within the star. Eventually the gravitational collapse is balanced by the pressure provided by energy produced in these exothermic reactions. The equilibrium continues until the hydrogen in the core is depleted, or until the energy released is insufficient to prevent further gravitational collapse. Then the star begins to collapse again, the temperature and density increasing until helium burning within the core starts once again stabilizes the star against further collapse. The helium burning taking place in the core produces carbon and oxygen. However, hydrogen burning now starts in the shell surrounding the core, since the collapse also elevates the temperature and density throughout the surrounding regions of the star. This process repeats when the core helium is consumed, and carbon burning leads to production of more oxygen, neon and silicon. All these burning stages happen through fusion reactions producing heavier nuclei until the core has been converted to nuclei in the iron region, $A \sim 60$ (most tightly bound nuclei) and the star has an onion-like
structure; an iron core surrounded by a silicon and sulphur shell, then oxygen, car-
The gravitational collapse then continues until the core density becomes roughly the density of nuclear matter, releasing a huge amount of energy. Then, the outer envelopes of the star are ejected into the interstellar medium by a giant explosion known as a supernova, leaving the core as either a neutron star if $M < 1.8M_\odot$ ($M$ and $M_\odot$ are the masses of the star and Sun), or a black hole if $M > 1.8M_\odot$ [Rolfs & Rodney (1988)].

2.2 Nucleosynthesis of Nuclei $A<60$

Main sequence stars with masses below $\sim 1M_\odot$ generate most of their energy via the pp-chain while stars with heavier mass create most of their energy via the CNO cycle. Since hydrogen is the most abundant element (more than 90%) in the universe, one expects hydrogen burning to play an important role in stellar burning and nucleosynthesis. There are two alternative reaction sequences in H-burning; the pp-chain which converts $^1\text{H}$ into $^4\text{He}$ initiated by $^1\text{H}(p,e^+\nu)^2\text{H}$ and the CNO cycle which converts $^1\text{H}$ into $^4\text{He}$ by sequence of $(p,\gamma)$ and $(p,\alpha)$ reactions on C, N, and O with interceding $\beta$-decays.

2.2.1 Hydrogen Burning in the pp-Chains

In a main sequence star's hydrogen burning stages, proton-proton chains (pp-chains) and CNO cycles, are the main energy sources. In the pp-chain, since the probability of the simultaneous fusion of four protons into helium is vanishingly small, two protons form a deuterium nucleus ($d=^2\text{H}$) which then reacts with another proton
and forms $^3\text{He}$.

\[ p + p \rightarrow d + e^+ + \nu \quad [Q = 1.44\text{MeV}] \quad (2.1) \]

\[ d + p \rightarrow ^3\text{He} + \gamma \quad [Q = 5.494\text{MeV}] \quad (2.2) \]

The proton-proton chain (pp-chain) is named after its first reaction, which proceeds via the weak interactions. It is $\sim 20$ orders of magnitude weaker than typical nuclear interactions. Typically, low energy cross section values of exothermic reactions mediated by strong forces are $\sim 10^{-2}$ b (1b = $10^{-24}\text{cm}^2$), those of reactions mediated by electromagnetic forces are $\sim 10^{-6}$ b, and those of reactions mediated by weak forces are $\sim 10^{-20}$ b. For that reason the lifetime of protons within the sun is extremely long, about $1 \times 10^{10}$ years. Since the later reactions in pp-chains are all governed by electromagnetic and nuclear interactions, reaction rates are much faster than deuterium production.

The $^4\text{He}$ nucleus can be produced by one of three alternative branches (pp1, pp2, pp3 chains) starting with $^3\text{He}$. The pp1 branch requires two $^3\text{He}$ nuclei (the first two reactions have to be completed twice). The other two branches, pp2 and pp3, require $^4\text{He}$ nuclei either produced in this burning or primordial. Then $^7\text{Be}$ can react with an $e^-$ for the pp2-chain to produce $^7\text{Li}$ or with a $p$ for the pp3-chain to produce $^8\text{B}$, as shown in Figure 2.2.
Figure 2.2: The pp-chains of reactions involved in the conversion of four protons into a $^4\text{He}$ nucleus [Boyd (1999)].

### 2.2.2 Hydrogen Burning in the CNO Cycles

While the pp-chains determine the energy generation in main sequence stars with $M \leq 1.5 M_\odot$, the CNO cycles dominate in more massive stars. The reactions involving C and N are important reactions since the elements between He and C have very low abundances. The sequence of reactions involved in the conversion of four protons into helium with an energy release of 26.73 MeV is known as the primary CNO cycle,

$$^{12}\text{C}(p, \gamma)^{13}\text{N}(e^+\nu)^{13}\text{C}(p, \gamma)^{14}\text{N}(p, \gamma)^{15}\text{O}(e^+\nu)^{15}\text{N}(p, \alpha)^{12}\text{C}. \quad (2.3)$$

Note that this process begins with proton capture on $^{12}\text{C}$. It ultimately returns the starting point, so that $^{12}\text{C}$ nuclei act as catalysts. When $^{12}\text{C}$ is destroyed by a
proton capture, it can also be created by a $^{15}\text{N}(p,\alpha)^{12}\text{C}$ reaction. The fact that the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction is the slowest reaction in this cycle results in an enrichment of $^{14}\text{N}$, the most abundant nucleus in CNO cycle hydrogen burning.

There are a few alternate cycles that are offshoots of the primary CNO cycle, and these cycles are less important for the energy production but significantly affect the nucleosynthesis of the CNO isotopes. A sequence of reactions known as the secondary cycle (CNO Bi-cycle) proceeds by

$$^{15}\text{N}(p,\gamma)^{16}O(p,\gamma)^{17}F(e^+\nu)^{17}O(p,\alpha)^{14}\text{N},$$

and eventually returns to the primary CNO-cycle at $^{14}\text{N}$. This cycle is important for the nucleosynthesis of $^{16}\text{O}$ and $^{17}\text{O}$ isotopes. However, if the $^{17}\text{O}$ isotope captures a proton, then another cycle proceeds through

$$^{17}O(p,\gamma)^{18}F(e^+\nu)^{18}O(p,\gamma)^{19}F(p,\alpha)^{16}O.$$  

All these cycles are indicated in Figure 2.3. As discussed in Section 2.3.3 the CNO cycles are also important for nucleosynthesis of heavier nuclei in the outer hydrogen burning shells during the later stages of stellar evolution.

### 2.2.3 Helium Burning

The fusion of protons into $^4\text{He}$ continues until hydrogen in the star's core is exhausted. When the star collapses and the core temperature rises to $T_s \sim 1$-$2$, the fusion of $^4\text{He}$ into heavier nuclei begins, increasing the radiation pressure. Then,
star's periphery is heated by the helium core, and expands. The star increases in volume and the surface temperature decreases, and the star becomes a red giant.

The helium burning stage consists of the gradual fusion of several $^4\text{He}$ into $^{12}\text{C}$ and $^{16}\text{O}$. Three $^4\text{He}$ nuclei form $^{12}\text{C}$, which is called the triple alpha reaction, in two steps as

$$^4\text{He} + ^4\text{He} \rightleftharpoons ^8\text{Be}, \quad \text{(2.6)}$$
$$^8\text{Be} + ^4\text{He} \rightarrow ^{12}\text{C} + \gamma. \quad \text{(2.7)}$$

These two reactions dominate He-burning, in which two $^4\text{He}$ temporarily form $^8\text{Be}$ which decays back into two $^4\text{He}$ after $\sim 10^{-16}$s until the rate of production is equal to that of destruction. When the density of $^8\text{Be}$ reaches an equilibrium, further
\(^4\)He capture proceeds to form \(^{12}\)C as in Equation 2.7. The rate of \(^{12}\)C production is significantly enhanced by a resonance of the cross section corresponding to the second excited state of \(^{12}\)C (\(\sim 7.65\) MeV) near the \(^8\)Be+\(^4\)He threshold.

Once a sufficient \(^{12}\)C abundance is obtained by the triple alpha reaction, further \(\alpha\) captures may occur, producing \(^{16}\)O. The \(^{16}\)O(\(\alpha,\gamma\))\(^{20}\)Ne cross section, however, is so small that very little \(^{20}\)Ne is made.

\[
^{12}\text{C + }^4\text{He} \rightarrow ^{16}\text{O} + \gamma \quad \text{(2.8)}
\]

\[
^{16}\text{O + }^4\text{He} \rightarrow ^{20}\text{Ne} + \gamma \quad \text{(2.9)}
\]

To form nuclei beyond \(^{20}\)Ne is rare in a typical stellar environment.

### 2.2.4 Carbon-Neon-Oxygen Burning

If the mass of the star is greater than \(8\,M_\odot\), the core will contract further, achieving the required temperature and density (\(T_9 \sim 0.8, \rho \sim 10^5\) g cm\(^{-3}\)) for C-burning. The most probable reactions involved are

\[
^{12}\text{C} + ^{12}\text{C} \rightarrow ^{20}\text{Ne} + ^4\text{He} \quad \text{(2.10)}
\]

\[
\rightarrow ^{23}\text{Na} + p \quad \text{(2.11)}
\]

After C-burning of a massive star, the density of \(\sim 10^7\) g cm\(^{-3}\) and \(T_9 \sim 1\) are achieved, and the Ne-burning can be ignited by the reactions

\[
^{20}\text{Ne} + \gamma \rightarrow ^{16}\text{O} + ^4\text{He} \quad \text{(2.12)}
\]

\[
^{20}\text{Ne} + ^4\text{He} \rightarrow ^{24}\text{Mg} + \gamma \quad \text{(2.13)}
\]
The first reaction ejects a $^4\text{He}$ by photonuclear reactions. Then the liberated $^4\text{He}$ reacts with another $^{20}\text{Ne}$, producing $^{24}\text{Mg}$. The primary nuclei produced will be $^{16}\text{O}$ and $^{24}\text{Mg}$.

The necessary temperature for oxygen burning is $T_9 \geq 2$ due to the high Coulomb barrier. The oxygen burning can proceed via several reactions

$$^{16}O + ^{16}O \rightarrow ^{28}\text{Si} + ^4\text{He} , \quad (2.14)$$

$$\rightarrow ^{31}\text{P} + p , \quad (2.15)$$

$$\rightarrow ^{31}\text{S} + n , \quad (2.16)$$

$$\rightarrow ^{30}\text{P} + d , \quad (2.17)$$

$$\rightarrow \text{many others} . \quad (2.18)$$

At the end of the O-burning, 90% of the mass of the core is $^{28}\text{Si}$ and $^{32}\text{S}$ [Boyd (1999)].

### 2.2.5 Silicon Burning

After oxygen burning, the core consists mainly of $^{28}\text{Si}$. Direct Si burning ($^{28}\text{Si} + ^{28}\text{Si}$) does not occur in stars since it requires extremely high stellar temperatures ($T_9 > 3.5$) due to high Coulomb barrier. At these high temperatures photodisintegration of nuclei should be considered. The photodisintegration is, of course, endothermic, but the ejected particles can be immediately recaptured producing the original nucleus, even heavier nuclei.
Due to the high temperatures, $^{28}\text{Si}$ starts to photodisintegrate, ejecting $\alpha$-particles, protons, and neutrons. The free light particles are then captured by the remaining heavier nuclei, thus building up heavier nuclei in nuclear statistical equilibrium (NSE), until $^{56}\text{Fe}$ is synthesized. In NSE, the strong and electromagnetic interactions are in equilibrium, but the weak interactions are not.

For more detailed discussion of burning stages see [Rolfs & Rodney (1988)] [Wallerstein et al. (1997)] [Boyd (1999)].

2.3 Nucleosynthesis of Nuclei $A \geq 60$

The $s$-, $r$-, and $p$-processes, which produce the nuclei heavier than iron, are distinct nucleosynthetic mechanisms, and they occur in different environments with quite different conditions. For each process (in more detail for the $p$-process), the mechanism and the current thinking on where these processes occur are summarized in the following sections.

2.3.1 The s-Process

The $s$-process, or slow-neutron-capture process, assumes that the time elapsed between two successive neutron captures $\tau_n$ is much longer than the typical $\beta$-decay times $\tau_\beta$ ($\sim 10^5 - 10^7$ sec) of the nuclei along the s-process path [Meyer (1994)]. This means that the s-process path is near the valley of stability because, when a nucleus captures a neutron and becomes $\beta$-unstable, it decays before capturing another
neutron, as shown in Figure 2.4. The s-process pathway can continue up to $^{209}$Bi, but it terminates at $^{210}$Bi, which $\alpha$-decays back to $^{206}$Pb. The characteristic time for neutron capture is given by $\tau_n = \frac{1}{n_n \langle \sigma v \rangle}$ where $n_n$ is the neutron number density, $\langle \sigma v \rangle$ is the product of the neutron capture cross section times velocity averaged over the Maxwell-Boltzmann velocity distribution. The s-process neutron density is in the range of $10^8$ to $10^{10}$ cm$^{-3}$ [Boyd (1999)] and the temperature is about $\sim 2.5 \times 10^8$K [Meyer (1994)].

The s-process abundance peaks are observed near $A \sim 90, 138,$ and 208 because of exceptionally small neutron capture cross sections for nuclei with a magic number of neutrons ($N=50, 82,$ and 126) as seen in Figure 2.6. It is reasonable to expect a buildup of the abundances of the isotopes at these points, providing a bottleneck in the s-process path, shown in Figure 2.4.

The s-process takes place in stable astrophysical environments since it is a slow process (it can operate over thousands of years). The most basic concern is the source of the neutrons. There are two candidates: the $^{13}$C($\alpha,n)^{16}$O reaction and the $^{22}$Ne($\alpha,n)^{25}$Mg reaction, both of which have positive Q-values.

The $^{13}$C($\alpha,n)^{16}$O reaction requires a hydrogen burning region that can mix with the helium burning core of a red giant. $^{13}$C is produced in hydrogen burning region by $^{12}$C($p,\gamma)^{13}$N($\beta^+)^{13}$C. It is then mixed down into the helium burning core, and results in the $^{13}$C($\alpha,n)^{16}$O reaction thus providing the necessary neutron density to be used in an s-process.
Figure 2.4: Pathways of s- and r-processes. p-Nuclei in this region (shaded) are also shown [Rolfs & Rodney (1988)].
The possible site for the $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$ reaction may occur in the helium burning region, and the source of $^{22}\text{Ne}$ depends on the existence of $^{14}\text{N}$ produced in the CNO cycle of the hydrogen burning. The successive $\alpha$ capture reactions and one $\beta^+$-decay convert the $^{14}\text{N}$ to $^{22}\text{Ne}$, and then to $^{25}\text{Mg}$, by the $^{14}\text{N}(\alpha,\gamma)^{18}\text{F}(\beta^+)$ $^{18}\text{O}(\alpha,\gamma)^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$ reaction chain. The initial CNO abundances determine the number of neutrons available for an s-process [Ulrich (1982)]. The pre-existing seed nuclei then capture neutrons to produce the s-process nuclei. That means the s-process is clearly a secondary process, i.e., it depends on preexisting seed nuclei.

The most promising site for the s-process is low mass AGB stars, in which $^{13}\text{C}$ is more promising as the source of s-process neutrons, and s-process calculations show good agreement with solar abundances [Käppeler et al. (1990)]. Reviews of the s-process have been provided by [Rolfs & Rodney (1988)] [Käppeler et al. (1989)] [Meyer (1994)] [Wallerstein et al. (1997)].

2.3.2 The r-Process

A different neutron capture process is needed to synthesize neutron-rich nuclei that are by-passed by the s-process, as well as nuclei heavier than $^{209}\text{Bi}$. The possible process is the r-process or rapid-neutron-capture process, which is responsible for the synthesis of roughly half of the nuclei heavier than $A\sim70$ [Cowan et al. (1991)]. In contrast to the s-process, the r-process is based on the hypothesis that neutron captures are much faster than $\beta$-decays ($\tau_\beta < \tau_n$). The r-process can be completed in a matter of a few seconds.
How the r-process occurs depends on whether it is a primary process, which has no pre-existing seed nuclei, or a secondary process, which has pre-existing seed nuclei [Meyer (1994)]. However, astronomical data on very old stars suggest that the r-process is primary [Sneden et al. (1996)]. In both cases neutron captures continue until an equilibrium between \((n,\gamma)\) neutron capture and \((\gamma,n)\) neutron disintegration reactions is reached. When the nuclei reach a “waiting point” nucleus, they must wait for a \(\beta^-\)-decay to increase their proton number \(Z\). These waiting points determine the r-process pathway which is typically \(\sim 15\) or more neutrons to the neutron-rich side of the valley of stability, although it depends on the instantaneous neutron number density and temperature. Once the neutron capture reactions cease, the nuclei produced decay back to the valley of stability, as indicated in Figure 2.4. Very heavy nuclei \((A \sim 270)\) can be produced by the r-process. Eventually, it terminates when the products fission into lighter nuclei.

The r-process requires a huge density of neutrons, typically \(n_n > 10^{20} \text{ cm}^{-3}\) as well as a high temperature environment about \(10^9 \text{ K}\). The r-process nuclei have a distinctly different abundance pattern from those of the s-process abundance, as can be seen in Figure 2.6. The abundance distribution of r-process nuclei shows peaks at mass numbers 80, 130, and 195 because of neutron closed shell effects. The r-process peaks do occur in the abundance curve at masses displaced to the low side of the s-process peaks and they are much broader (involve more mass numbers) than those obtained for the s-process abundance peaks.
Several astrophysical sites for the r-process have been proposed in violent events such as supernovae (probably the hot neutrino bubble of type II supernovae), and colliding neutron stars. More detailed information can be found in [Schramm (1982)] [Cowan et al. (1991)] [Meyer (1994)] [Wallerstein et al. (1997)].

2.3.3 The p-Process

General Considerations

The abundances of the p-nuclei are usually smaller by more than one order of magnitude compared to those synthesized by the other two neutron capture processes. Because of the large Coulomb barriers of the heavy nuclei, charged particle capture reactions such as proton capture reactions are not efficient. High temperature environments are needed to accomplish this. However, high temperatures also increase the photodissociation rate. In any event, the p-process will at least involve both of these processes.

The generally accepted list of p-nuclei and their isotopic abundances are given in Table 2.5 [Wallerstein et al. (1997)]. The abundance $N_A$ is given in the usual meteoritic scale ($N_{Si} = 10^8$). A few nuclides can also be synthesized in part by s-, r-, and/or $\nu$-processes, but they do not contribute much to the abundance of most of the p-nuclei. Most p-nuclei are one to two orders of magnitude less abundant than the more neutron-rich isotopes (see Table 2.5).
<table>
<thead>
<tr>
<th>Element</th>
<th>Z</th>
<th>A</th>
<th>$N_A$</th>
<th>$(N_A/\Sigma N_A) \times 100$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Se</td>
<td>34</td>
<td>74</td>
<td>0.55</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>Kr</td>
<td>36</td>
<td>78</td>
<td>0.15</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>Sr</td>
<td>38</td>
<td>84</td>
<td>0.13</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>Nb</td>
<td>41</td>
<td>92</td>
<td>0.0</td>
<td>0.0</td>
<td>$\beta$ decays to $^{129}$Mo; $(T_{1/2})^b = 3.5 \times 10^7$ y</td>
</tr>
<tr>
<td>Mo</td>
<td>42</td>
<td>92</td>
<td>0.38</td>
<td>14.84</td>
<td></td>
</tr>
<tr>
<td></td>
<td>94</td>
<td>0.24</td>
<td>9.25</td>
<td></td>
<td>Also produced by $s$ process</td>
</tr>
<tr>
<td>Ru</td>
<td>44</td>
<td>96</td>
<td>0.10</td>
<td>5.52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>98</td>
<td>0.035</td>
<td>1.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pd</td>
<td>46</td>
<td>102</td>
<td>0.014</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>Cd</td>
<td>48</td>
<td>106</td>
<td>0.020</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>108</td>
<td>0.014</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In</td>
<td>49</td>
<td>113</td>
<td>$7.9 \times 10^{-3}$</td>
<td>4.3</td>
<td>Also produced by $r$ and $s$ processes</td>
</tr>
<tr>
<td>Sn</td>
<td>50</td>
<td>112</td>
<td>0.037</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td>114</td>
<td>0.025</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Te</td>
<td>52</td>
<td>120</td>
<td>$4.3 \times 10^{-3}$</td>
<td>0.09</td>
<td>Also produced by $s$ process</td>
</tr>
<tr>
<td>Xe</td>
<td>54</td>
<td>124</td>
<td>$5.7 \times 10^{-3}$</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>126</td>
<td>$5.1 \times 10^{-3}$</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ba</td>
<td>56</td>
<td>130</td>
<td>$4.8 \times 10^{-3}$</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>132</td>
<td>$4.5 \times 10^{-3}$</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>La</td>
<td>57</td>
<td>138</td>
<td>$4.1 \times 10^{-3}$</td>
<td>0.09</td>
<td>$\beta$ decays to $^{134}$Ce; $(T_{1/2})^b = 1.5 \times 10^{11}$ y</td>
</tr>
<tr>
<td>Ce</td>
<td>58</td>
<td>136</td>
<td>$2.2 \times 10^{-3}$</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>138</td>
<td>$2.8 \times 10^{-3}$</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sm</td>
<td>62</td>
<td>144</td>
<td>$8.0 \times 10^{-3}$</td>
<td>3.10</td>
<td>$\alpha$ decays to $^{142}$Nd; $(T_{1/2})^b = 1.03 \times 10^8$ y</td>
</tr>
<tr>
<td></td>
<td>146</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dy</td>
<td>66</td>
<td>156</td>
<td>$2.2 \times 10^{-4}$</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>158</td>
<td>$3.8 \times 10^{-4}$</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Er</td>
<td>68</td>
<td>162</td>
<td>$3.5 \times 10^{-4}$</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>Yb</td>
<td>70</td>
<td>168</td>
<td>$3.2 \times 10^{-4}$</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Hf</td>
<td>72</td>
<td>174</td>
<td>$2.5 \times 10^{-4}$</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>Ta</td>
<td>73</td>
<td>180</td>
<td>$2.5 \times 10^{-6}$</td>
<td>0.01</td>
<td>Actually $^{189}$Ta$^m$; $(T_{1/2})^b = 1.2 \times 10^{15}$ y</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\beta^-$ decays to $^{189}$W or $e^-$ captures to $^{189}$Hf</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Also produced by $s$ process</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Also possibly produced by $s$ process</td>
</tr>
<tr>
<td>W</td>
<td>74</td>
<td>180</td>
<td>$1.7 \times 10^{-4}$</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Os</td>
<td>76</td>
<td>184</td>
<td>$1.2 \times 10^{-4}$</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Pt</td>
<td>78</td>
<td>190</td>
<td>$1.7 \times 10^{-4}$</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Hg</td>
<td>80</td>
<td>196</td>
<td>$5.2 \times 10^{-4}$</td>
<td>0.15</td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\)Anders and Grevesse (1989).
\(^{b}\)Tuli (1995).

Figure 2.5: The p-process nuclides [Wallerstein et al. (1997)].
Figure 2.6: The Solar System abundance curves for s-process (dashed line), r-process and p-process nuclei (solid line), relative to Si=10^6 [Meyer et al. (1996)].
All p-nuclei are even-even nuclei except $^{113}$In, $^{138}$La, and $^{180}$Ta. The peaks at $^{92}$Mo and $^{144}$Sm and a smaller peak at $^{112}$Sn show the effects of the neutron magic numbers $N=50$ and 82 and that of the proton magic number $Z=50$, respectively. The rise in the abundance curve beyond $A=190$ is due to the locally high binding energies at magic numbers $N=126$ and $P=82$. Note the low abundances of the two odd-odd p-nuclides $^{138}$La and $^{180}$Ta since they are extremely rare. The abundance curve of the p-nuclei is roughly parallel to those of s- and r-nuclei. This behaviour might be because the s- and/or r-nuclei served as seeds for the synthesis of p-nuclei [Lambert (1992)].

There are several astrophysical settings which have appropriate conditions to form p-nuclei from pre-existing s- and r- nuclei. The main problem is really which site contributes the bulk of the p-nuclei in the different regions of the periodic table.

The p-process was originally thought to be a proton capture process which was imagined to occur in a hydrogen-rich region in massive stars undergoing a supernova explosion [Burbidge et al. (1957)]. The supernova shock wave passing through the hydrogen-rich region would increase the temperature and the p-nuclei would be produced by proton capture reactions on seed nuclei formed in the s- and/or r-processes. However, the densities, temperatures, and timescales are not realistic for the hydrogen-rich envelope to produce higher mass nuclei due to the possibility of photodisintegration reactions [Woosley & Howard (1978)].

The conditions required for the p-process are summarized by Figure 2.7 and 2.8, which show the timescale for a proton capture reaction by the most stable proton.
rich isotope of each element for different proton mass density values ($\rho Y_p$) at the temperature of $T_9=1$ (where $T_9$=temperatures in billions K) and the temperature of $T_9=3$, respectively. In the case of an astrophysical site with a proton mass density $\rho Y_p$ of 1 g cm$^{-3}$, $10^4$s is needed for a proton capture reaction by $^{93}$Mo ($Z=42$). The higher charge needs an even larger time scale to capture a proton. The timescale for a proton capture reaction will decrease if the environment has higher proton mass density. For example, 10 s are needed for $^{93}$Mo to capture a proton when $\rho Y_p$ is $10^3$ g cm$^{-3}$, as seen in Figure 2.7 [Meyer (1994)] while $\sim 10^{-2}$ s for $\rho Y_p$ is $10^6$ g cm$^{-3}$.

![Figure 2.7: Timescales for the proton capture reaction at the fixed temperature of $T_9=1$K. The solid curve is for the proton mass density of $\rho Y_p = 1$ g cm$^{-3}$, the long dashed-dotted curve is for $10^3$g cm$^{-3}$, and the short dashed-dotted curve is for $10^6$g cm$^{-3}$ [Meyer (1994)].](image_url)
What happens to the timescale if the temperature increases? Figure 2.8 shows the answer. Not only the timescale for the proton capture reaction decreases, but also the timescale for the \((\gamma,n)\) photodisintegration reaction decreases. The fact that the timescale for a \((\gamma,n)\) reaction is even less than that for a \((p,\gamma)\) reaction after about \(Z \sim 40\) \((\rho Y_p = 1 \text{ gcm}^{-3})\) tells us a photodissociation reaction is more likely to occur than a proton capture reaction under these conditions. As a result the dilemma is obvious for \(p\)-nuclei synthesis. In order to produce \(p\)-nuclei by proton capture reactions at low temperatures without their being impeded by photodissociation
reactions, a high proton density is needed for a long time. However, such conditions are not realistic. In the case of higher temperatures, photodissociation reactions will dominate, and proton capture reactions will not be a good candidate for producing p-nuclei.

Photodissociation reactions (γ-process) are responsible for synthesizing the most massive p-nuclei while proton capture reactions (rp-process) are more likely to occur in the relatively low mass range. However, a few p-nuclei are so difficult to make by these processes that their synthesis seems to necessitate some other specific processes [Wallerstein et al. (1997)]. All these processes will be discussed in the following sections.

The γ-Process

The γ-process is a series of photodisintegration reactions, i.e. (γ,n), (γ,p), and (γ,α) reactions, induced on heavier seed nuclei synthesized in the s- and r-processes. The typical parameters for the γ-process are temperatures of $2 \leq T_9 \leq 3$, densities $\rho \approx 10^7$ g/cm$^3$, and time scales $\tau$ of the order of seconds. Several astrophysical sites for the γ-process have been proposed; the oxygen- and neon-rich layers of type II supernovae seem to be good candidates.

At the beginning, (γ,n) reactions dominate the synthesis of p-nuclei instead of charged particle photodissociation reactions, since the neutrons emitted do not need to surmount the Coulomb barrier. As the nuclei become neutron deficient, the neutron separation energy increases. At some point, a branching point, (γ,n) is
deflected by \((\gamma,p)\) and \((\gamma,\alpha)\) reactions. Heavy (even-even) nuclei will tend to lose an \(\alpha\) particle whereas, for odd heavy nuclei and light nuclei, proton ejection will be more important. An example of photodisintegration flows is shown in Figure 2.9.

The photodissosiation reaction rates slow down and matter tends to accumulate at the waiting points, especially for nuclei with a closed neutron or proton shell; the higher binding energies of these nuclei produce this effect. For odd-odd \((Z-N)\) particle nuclei, very little material will accumulate, since they tend to be weakly bound. After the freezing-out of nuclear reactions, \(\beta\)-decays to stable isobars, the \(p\)-nuclei, will occur.

Woosley et al. ran the network of nuclei and \((\gamma,n)\), \((\gamma,p)\), and \((\gamma,\alpha)\) reactions for 12 different temperatures \((2.1 \leq T_9 \leq 3.2)\) to compute the resulting abundances \(N_i\) [Woosley & Howard (1978)]. The total abundance then for a given nucleus is

\[
N_{tot}(Z,A) = \sum_i C_i N_i(Z,A)
\]

where \(C_i\) is a weighting function \((\sum_i C_i = 1)\). Even the simplest case \((C_i = 1/12)\) gives good results for heavy \(p\)-nuclei \((A \geq 100)\). For each \(p\)-nucleus, an average overproduction factor \((\vartheta \equiv \langle F \rangle)\) relative to mean average overproduction \((\vartheta_{av} \equiv F_o)\) is calculated and the results from different studies are shown in Figure 2.10. Many of them are produced within a factor of 3 of their solar abundance except for \(^{113}\text{In}\), \(^{115}\text{Sn}\), \(^{118}\text{La}\), \(^{152}\text{Gd}\), and \(^{180}\text{Ta}\). However the results are not so good for the light \(p\)-nuclei, especially the Mo and Ru \(p\)-nuclei as seen in Figure 2.10(a). They investigated whether they could get more efficient production of lighter \(p\)-nuclei by
Figure 2.9: $\gamma$-Process pathways [Woosley & Howard (1978)].
a modified distribution of seed nuclei, and the results are shown in Figure 2.10(b) and (c). The trend has been eliminated, but $^{92,94}\text{Mo}$ and $^{96,98}\text{Ru}$ are underproduced.

This study was extended by [Rayet et al. (1990)] who investigated explosive nucleosynthesis in an O-burning layer of a massive star. The reaction network was improved by including particle induced reactions, and the results for relative overproduction factors are in Figure 2.10(e) and (f). Their results are generally similar to those of [Woosley & Howard (1978)]. [Prantzos et al. (1990)] attempted to produce p-nuclei with a realistic model of the Type II supernova SN 1987A, and they were able to reproduce the p-nuclei abundances within a factor of 3 of solar abundances for $\sim 50\%$ of the p-nuclei. In this model, the light p-nuclei are still underproduced as seen in Figure 2.10(d). However, $^{180}\text{Ta}$ appears for the first time to be significantly produced in a p-process calculation.

According to Howard et al., the failure of these models is their inability to produce the abundance pattern of the light p-nuclei such as Mo and Ru due to insufficient seeds with $A \geq 96$ [Howard et al. (1991)]. They suggested that the explosion of a C-O white dwarf is a possible site. The s-process nuclei from several flashes of He-burning prior to the formation of the white dwarf produce the seeds for p-nuclei. The p-nuclei can be made near the surface of the C-O core which has a region between $2.4 - 3.2 \times 10^9$ K. The bulk of the p-nuclei are reproduced within a consistent narrow production range of about 9. $^{92,94}\text{Mo}$, $^{98}\text{Ru}$, $^{102}\text{Pd}$, $^{106}\text{Cd}$, and $^{108}\text{Cd}$ are coproduced within a factor of 3. However, $^{180}\text{Ta}$ is not produced well, $^{152}\text{Gd}$ is underproduced and $^{74}\text{Se}$, $^{78}\text{Kr}$, and $^{84}\text{Sr}$ are overproduced as seen in Figure 2.10(g).
Figure 2.10: Overproduction factors relative to the average overproduction as a function of mass for different studies: (a),(b),(c) [Woosley & Howard (1978)]; (d) [Prantzos et al. (1990)]; (e) and (f) [Rayet et al. (1990)]; (g) [Howard et al. (1991)].
It appears that the γ-process explains the abundance of heavy p-nuclei (A > 10^8) quite well, but always severely underproduces the lighter p-nuclei (A ≤ 10^8) by factors of the order of 10 to 100. The reason the light p-nuclei were overproduced in the [Howard et al. (1991)] study was that another process, the rp-process, was also included in this work. That is discussed below.

The rp-Process

The alternative rp-process is the most successful explanation for the nucleosynthesis of the lighter p-process nuclei with a transition point occurring specifically around ^{92,94}_{2}Mo and ^{96,98}_{2}Ru.

The rp-process is a rapid series of rapid proton captures and β⁺-decays on existing light seed nuclei in high temperature (Tg ~ 0.2 to 2.0 K) and high density (~10^5 g/cm³) explosive environments, most typically in novae and X-ray bursts.

At low temperatures, there are independent cycles such as the CNO cycle and the NeNa cycle. The rp-process begins by breaking out of one cycle to the next, giving a higher energy production as well as heavier element synthesis, i.e., Hot-CNO(HCNO) cycle → NeNa cycle → MgAl cycle → ... [Kubono (1995)]. For more details see the reference [Boyd (1999)].

Figure 2.11 shows an example of the rp-process pathway [Schatz et al. (1998)]. The reaction path therefore follows a series of fast proton capture reactions until further proton capture is not allowed by either a negative proton capture Q-value or photodisintegration of the daughter nucleus because of a small positive proton
Figure 2.11: rp-Process pathways [Schatz et al. (1998)].
capture $Q$-value. Then, the reaction flow must wait for a $\beta$-decay at this nucleus, known as a "waiting point", and the abundances are built up at these waiting points. The speed of nucleosynthesis towards heavier nuclei, the energy generation, the produced isotopic abundances and how much fuel is left in an astrophysical scenario are determined by the total lifetimes of the waiting points [Schatz et al. (1998)].

Accreting neutron stars or white dwarfs in binary systems and Thorne-Żytkow objects have also been suggested as possible sites for the rp-process. In the neutron star scenario, hydrogen rich material flows from the partner (usually a red giant) to the neutron star and reaches high enough temperatures during the accretion process, driving the rp-process the higher nuclei up to $A \sim 100$. In a Thorne-Żytkow object (TZO), a neutron star is merged with a red giant. There will be a strong convective hydrogen-rich environment at high temperatures directly above the surface of the neutron star. The problem is here it is not known if there have been enough or any TZOs. Serious doubt has been cast on the possibility for TZOs to even exist [Chevalier (1996)].

Unfortunately, neither of these processes, $\gamma$- and rp-processes, is capable of producing $^{138}\text{La}$ and $^{180}\text{Ta}$. $^{138}\text{La}$ and $^{180}\text{Ta}$ are not produced in adequate amounts by the $\gamma$-process in SNII ejecta and SNIa. However, the $\nu$-process in the neutrino wind generated by stellar collapse in supernovae may synthesize these isotopes by neutrino-induced neutron or proton removal from nearby abundant nuclides [Woosley (1990)]. $^{138}\text{La}$ is produced in a SNII's Ne-shell by neutron evaporation ($\nu, n$) from $^{139}\text{La}$ and it may also be synthesized from $^{138}\text{Ba}$ by ($\nu, e^-$) reactions. Both $^{139}\text{La}$ and $^{138}\text{Ba}$ are
produced in the s-process. These two reactions appear capable producing the observed value of $^{138}$La [Wallerstein et al. (1997)]. $^{180}$Ta would be made from $^{181}$Ta by ($\nu$,n) reactions, which appears to produce an abundance consistent with observation.

Another possibility for some p-nuclei that cannot be produced by either of these processes is the "a-rich freezeout" suggested by [Woosley & Hoffman (1992)]. Some light p-nuclei ($^{74}$Se, $^{78}$Kr, $^{84}$Sr, $^{92}$Mo) with some r-process nuclei actually would be coproduced in a neutrino-driven wind following the explosion of a supernovae type II (for more information, see [Boyd (1999)]).

Recently, the repetitive rp-process, (rp)$^2$-process, in successive accretion-explosion events from a red giant companion onto a neutron star or black hole, or in Thorne-Żytkow objects was proposed by [Boyd (1999)] to synthesize the light p-nuclei ($^{74}$Se to $^{112}$Sn and higher). They showed that it is capable of producing huge amounts of the Mo and Ru p-nuclei, as well as other p-nuclei in that general mass region.

Although much work has been devoted to these nucleosynthetic processes and much progress has been achieved in our understanding of them, many uncertainties on the astrophysical conditions and sites exist, especially for the precise determination of the main nuclear parameters for the r- and p-processes, predominantly due to lack of experimental data for the cross sections and reaction rates at astrophysically relevant energies. All reaction rates have been derived theoretically using statistical model calculations explained in the next chapter. Future studies are needed to synthesize all the p-nuclei in the abundances that are found in the Solar System, and especially to understand the underproduction of some specific light p-nuclei.

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CHAPTER 3

THEORY

The knowledge of astrophysical reaction rates is important to understand and follow the nucleosynthesis and energy productions in stellar environments. The standard treatment of nuclear reaction rates in a stellar environment are introduced in this chapter. Then, basic concepts of a statistical model code, NON-SMOKER, which we have used to make comparisons to our data, are discussed.

For convenience, reactions are often denoted by

\[ x + y \rightarrow C \rightarrow X + Y + Q, \]  

(3.1)

where \( x \) and \( y \) are projectile and the target nucleus with relative velocity \( v \); together they constitute the entrance channel \( (x + y) \), while \( X \) is the residual nucleus and \( Y \) is the outgoing particle which together constitute the exit channel \( (X + Y) \). \( C \) is the compound nucleus, and \( Q \) is the energy equivalent to the difference in mass between the entrance channel and the exit channel (known as the \( Q \)-value of the reaction). A positive \( Q \)-value means that energy is given off while a negative \( Q \)-value requires energy in order for the reaction to proceed.
3.1 Reaction Rates Under Astrophysical Conditions

In typical stellar environments, the stellar gas is nondegenerate, and the motion of the nuclei is nonrelativistic. When the gas is in thermodynamic equilibrium, the velocities of the nuclei can be described by a Maxwell-Boltzmann velocity distribution,

\[ \phi(v) = 4\pi v_i^2 \left( \frac{m_i}{2\pi k T} \right)^{3/2} \exp \left( -\frac{m_i v_i^2}{2k T} \right), \]

where \( T \) is the temperature of the stellar gas, \( m \) and \( v \) are the mass and velocity of the nucleus of interest, \( k \) is the Boltzmann constant, and \( i \) denotes the various elemental components of the gas.

The relative velocity between particles of type \( x \) and \( y \) in a stellar gas, i.e. a mixture of gasses in thermodynamic equilibrium, varies over a wide range. If \( \phi(v) \) is the relative velocity spectrum in this range, which is defined such that \( \int \phi(v)dv=1 \), \( \phi(v)dv \) is the probability that the relative velocity of the pair of particles has a magnitude \( v \) in the range \( dv \). Calculation of the total reaction rate involves the integral over all velocities. However, the reaction rate involves a double integral over both velocity distributions since there are two types of particle with different masses:

\[ \langle \sigma v \rangle = \int_0^\infty \int_0^\infty \phi(v_x)\phi(v_y)\sigma(v)v dv_x dv_y, \]

where \( v \) is the relative velocity of \( m_x \) and \( m_y \), \( v=v_x-v_y \). The two velocities \( v_x \) and \( v_y \) can be transformed into the variables \( v \) and \( V \) (the center of mass velocity) by using the usual kinematic relations as follows

\[ v_x = V + \frac{m_y}{m_x + m_y}v, \]
\[ v_y = V - \frac{m_x}{m_x + m_y}v. \]
The double integral is changed to two independent integrals:

\[
< \sigma v > = \int_0^\infty \phi(V) dV \int_0^\infty \phi(v) \sigma(v) vdv,
\]

where the transformed velocity distributions \( \phi(V) \) and \( \phi(v) \) are

\[
\phi(V) = 4\pi V^2 \left( \frac{M}{2\pi kT} \right)^{3/2} \exp \left( -\frac{MV^2}{2kT} \right),
\]

\[
\phi(v) = 4\pi v^2 \left( \frac{\mu}{2\pi kT} \right)^{3/2} \exp \left( -\frac{\mu v^2}{2kT} \right),
\]

where \( \mu \) is the reduced mass of the interacting particles, \( \mu = m_x m_y / (m_x + m_y) \), and \( M \) is the total mass, \( M = m_x + m_y \). Performing the integration over the variable \( V \) and using the center of mass energy \( (E = \frac{1}{2} \mu v^2) \), the reaction rate per particle pair is

\[
< \sigma v > = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \exp \left( -\frac{E}{kT} \right) dE.
\]

Therefore, the reaction rate can be determined from a measurement of cross section \( \sigma(E) \).

If a stellar gas contains \( N_i \) particles per \( cm^3 \) of type \( i \), with relative velocities \( v \) between particles of type \( x \) and type \( y \), the rate of nuclear reaction \( r \), which is in units of reactions per \( cm^3 \) per second, can be given by [Rolfs & Rodney (1988)]

\[
r = N_x N_y < \sigma v >.
\]

where \( < \sigma v > \) is the average value of the \( \sigma(v)v \) over the velocity distribution, and is referred to as the reaction rate per particle pair. If the two particles are identical, the reaction rate, \( r \) is divided by two in order to eliminate double-counting. The cross
section can be separated into two parts: non-resonant and resonant, as discussed in what follows.

### 3.1.1 Charged-Particle Induced Nonresonant Reaction Rates

In nuclear astrophysics, the cross section is parametrized in terms of its known energy dependence. For charged particle induced nonresonant reactions, the major energy dependence is due to the Coulomb penetrability factor, which for energies well below the Coulomb barrier can be approximated by the Gamow factor. Because of the exponential behavior of the probability for tunneling, and another non-nuclear energy-dependent term which always contains a factor $\lambda^2 \propto \frac{1}{E}$, the cross section can be written as

$$\sigma(E) = \frac{1}{E} \exp(-2\pi\eta)S(E),$$  \hspace{1cm} (3.9)

where $\eta$ is the Sommerfeld parameter given by

$$\eta = \frac{Z_x Z_y e^2}{\hbar v},$$  \hspace{1cm} (3.10)

where $Z_x$ and $Z_y$ represent the charges of the interacting nuclei. The function $S(E)$, therefore, represents the nuclear contribution to the energy dependence of the cross section, and is defined by equation 3.9. This function is referred to as the nuclear or astrophysical S-factor, which is expected to be a slowly varying function of energy in the absence of any resonances. Since the astrophysical S-factor varies much less
rapidly with energy than the cross section, it is much more useful in extrapolating measured cross sections to astrophysical energies, as illustrated in Figure 3.1.

\[ \sigma(E) \text{ and } S'(E) \]

Figure 3.1: Energy dependence of the cross section \( \sigma(E) \) and the astrophysical \( S'(E) \)-factor. Extrapolation to lower energies is more reliable when the astrophysical \( S'-factor was used [Rolfs & Rodney (1988)].

The reaction rate per particle pair, \( < \sigma v > \) in terms of \( S(E) \), can be obtained as

\[ < \sigma v > = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp \left[ -\frac{E}{kT} - \frac{b}{E^{1/2}} \right] dE, \quad (3.11) \]

where

\[ b = \frac{(2\mu)^{1/2} \pi e^2 Z_x Z_y}{\hbar} = 0.989 Z_x Z_y \mu^{1/2} \quad (MeV)^{1/2} \quad (3.12) \]

which comes from the barrier penetrability.
Figure 3.2: The dominant energy dependent functions and the Gamow peak, given by the product of the Maxwell-Boltzmann distribution and the Coulomb penetration factor, are shown for nuclear reactions between charged particles [Rolfs & Rodney (1988)].

Since for non-resonant processes $S(E)$ is a slowly varying function of energy, the behavior of the integrand in Equation 3.11 is dominated by the exponential fall off at high energies of the Maxwell-Boltzmann distribution and the low energy fall off of the charged particle cross section; their product is non-vanishing only over a small energy region. The energy dependence of this exponential term is very nearly Gaussian, as illustrated in Figure 3.2 [Rolfs & Rodney (1988)], and this peak is known as the Gamow peak at the effective energy

$$E_0 = 1.22[Z_x^2Z_y^2\mu T_6^{31/2}] , keV$$ (3.13)
with a full width at $1/e$, $\Delta$

$$\Delta = 0.749[Z^2_x Z^2_y \mu T^5_\beta 1/6], keV$$ (3.14)

where $T_\beta$ is the temperature in millions Kelvin and $\mu$ is the reduced mass (in amu). In the absence of resonances, the astrophysical $S(E)$-factor can often be taken constant with its value at $E_0$, $S(E_0) = S_0$.

### 3.1.2 Resonant Reaction Rates

The stellar reaction rate can have contributions from resonant processes. A resonance in the cross section may occur when the energy in the entrance channel corresponds to a quasibound state in the compound nucleus.

The resonant cross section depends on the spins, partial widths and the resonant energy, and can be described by the well known Breit-Wigner expression [Breit & Wigner (1936)] for a single level resonance as

$$\sigma_{BW}(E) = \frac{\hbar^2}{2\mu E} \frac{(2J + 1)}{(2J_x + 1)(2J_y + 1)} \frac{\Gamma_a \Gamma_b}{(E - E_R)^2 + (\frac{\Gamma}{2})^2}$$ (3.15)

where $\mu$ and $E$, as mentioned before, are the reduced mass and the center of mass energy, respectively, $J$ is the spin of the resonant state at the resonant energy $E_R$, $J_x$ and $J_y$ are the spins of the interacting particles, $\Gamma_a$ and $\Gamma_b$ are the partial widths of the entrance and exit channel, respectively, and $\Gamma$ is the total width ($\Gamma = \Gamma_a + \Gamma_b + \cdots$).

The Breit-Wigner formula, above, is described only for isolated resonances, that is, when the level widths are small compared with the energy separation of the levels,
and the resonance is assumed narrow if the total width $\Gamma$ is small compared with
the resonance energy $E_R$. In this case, $\Gamma \ll E_R$, the Maxwell-Boltzmann term
changes very little over the resonance, and can be taken outside of the integral in
Equation 3.7 as

$$
< \sigma v > = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} E_R \exp \left( -\frac{E_R}{kT} \right) \int_0^\infty \sigma_{BW}(E) dE,
$$  \hfill (3.16)

and the energy dependence of the partial and total widths can also be ignored. Then,
performing just the cross section integral gives

$$
\int_0^\infty \sigma_{BW}(E) dE = 2\pi^2 \frac{\hbar^2}{2\mu E_R} \frac{\Gamma_a \Gamma_b}{\Gamma}, \hfill (3.17)
$$

and the stellar reaction rate per particle pair for a narrow resonance is written as

$$
< \sigma v > = \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 \omega \gamma \exp \left( -\frac{E_R}{kT} \right).
$$  \hfill (3.18)

Here, $\omega$ is the statistical factor given by

$$
\omega = \frac{2J + 1}{(2J_z + 1)(2J_y + 1)}, \hfill (3.19)
$$

and the product $\omega \gamma$ is the resonance strength given by

$$
\omega \gamma = \omega \frac{\Gamma_a \Gamma_b}{\Gamma}. \hfill (3.20)
$$

If there are several non-overlapping narrow resonances that contribute to the reac­tion, the reaction rate per particle pair is summed to give

$$
< \sigma v > = \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 \sum_i (\omega \gamma)_i \exp \left( -\frac{E_{Ri}}{kT} \right). \hfill (3.21)
$$
For broad resonances, $\Gamma \geq 10\% \ E_R$, neither the Maxwell Boltzmann term nor the energy dependence of widths nor can be assumed to be constant over the resonance, and Equation 3.15 is integrated numerically. However, the energy dependence of widths can often be calculated [Rolfs & Rodney (1988)], and the cross section of broad resonance can then be calculated using the measurable parameters $E_R$, $J$, $\Gamma$, $\Gamma_a$ and $\Gamma_b$ at resonant energy.

For the medium and heavy nuclei many resonances usually contribute to the reaction rate per particle pair due to high density of resonances. When the level density in the compound nucleus is high, the cross section is then dominated by a multiple of overlapping resonances, and appears non-resonant. Then, the cross section can be described using the Hauser-Feshbach approach, which is a statistical model of compound nuclear reactions.

### 3.2 Inverse Reaction Rates

So far only the forward reaction has been considered. This usually refers to the direction which has positive $Q$-value for low stellar temperatures. As the temperature rises, however, the inverse reaction with negative $Q$-value may also proceed

\[
\text{forward reaction}: \quad x + y \rightarrow X + Y, \quad Q > 0 ,
\]

\[
\text{inverse reaction}: \quad X + Y \rightarrow x + y, \quad Q < 0 .
\]

Since the laws governing reactions do not change when the direction of the reaction is reversed (the principle of detailed balance [Blatt & Weisskopf (1962)]), the inverse
reaction rate can be written as

\[ <\sigma v>_{XY} = \frac{(2J_x + 1)(2J_y + 1)}{(2J_X + 1)(2J_Y + 1)} \left( \frac{\mu_{xy}}{\mu_{XV}} \right)^{3/2} \exp\left(-\frac{Q}{kT}\right) <\sigma v>_{xy}. \] (3.22)

If one of the exit channel particles is a photon \((x + y \rightarrow X + \gamma)\), it is called a radiative capture reaction, an example of which is

\[ ^{102}\text{Pd} + p \rightarrow ^{103}\text{Ag} + \gamma, \quad Q = 4.15 \text{ MeV}. \]

Radiative capture reactions are important in stellar nucleosynthesis. They contribute both to nucleosynthesis and to the stellar energy production. The intensity of thermal photons increases at high stellar temperatures, and the inverse reaction could also occur

\[ ^{103}\text{Ag} + \gamma \rightarrow ^{102}\text{Pd} + p, \quad Q = -4.15 \text{ MeV}. \]

This process, known as photodisintegration, converts a heavier nucleus to a lighter nucleus, and the reaction rate per particle pair is written

\[ <\sigma v>_\gamma = \left( \frac{\mu kT}{2\pi\hbar^2} \right)^{3/2} \frac{(2J_x + 1)(2J_y + 1)}{N_\gamma(2J_X + 1)} \exp\left(-\frac{Q}{kT}\right) <\sigma v>_\gamma \] (3.23)

where \(N_\gamma\) is the total number of photons per unit volume.

### 3.3 Nuclear Abundances in Stellar Environments

The rate of change of the abundance of a particular nucleus is equal to the difference of the reaction rates which produce the nucleus and those that destroy the nucleus. If there are \(m\) possible production reactions, \(n\) possible destruction reactions, \(p\) possible decays, and the decay of the particle \(X\) itself (if it is unstable), the rate of change of

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The abundance of particle X is

\[ \frac{dN_X}{dt} = \left( \sum_{i=1}^{m} \frac{N_i}{1 + \delta_{iX}} <\sigma v \gamma_{iX} - \sum_{k=1}^{n} \frac{N_k}{1 + \delta_{kX}} <\sigma v \gamma_{kX} - \frac{1}{\tau_X} \right) N_X - \sum_{j=1}^{p} \frac{N_j}{\tau_j}. \] (3.24)

As can be seen in Equation 3.24 the time evolution of nuclidic abundances in a reaction network involve a set of coupled, non-linear, ordinary differential equations which can be solved using a computer program.

In general, the cross section can be calculated by summing the cross sections resulting from compound reactions via an average overlapping resonances (Hauser-Feshbach $HF$), single resonances (Breit-Wigner $BW$), direct reactions ($DI$) and interference ($int$) terms [Rauscher (1998)] as follows

\[ \sigma(E) = \sigma^{HF}(E) + \sigma^{BW}(E) + \sigma^{DC}(E) + \sigma^{int}(E). \] (3.25)

Different reaction mechanisms will dominate depending on the density of levels per energy interval since different regimes of level densities are probed at the various projectile energies.

### 3.4 Compound Nuclear Reactions

When two nuclei collide they can coalesce to form a compound nucleus (CN) which is a highly excited and unstable nucleus. In a CN, the energy of the bombarding particle is distributed among several nucleons of the target nucleus. This may occur in a large number of ways, producing many closely spaced resonances. After a relatively long period (typically $\sim 10^{-15}$-$10^{-19}$ sec) compared to the time interval for
the bombarding particle to travel across the diameter of the target nucleus (about $10^{-22}$ sec), the compound nucleus may decay, usually into an ejected small particle and a product nucleus or $\gamma$ decay to ground state, as seen Equation 3.1.

\[ A_{X}Z + Z \rightarrow A+1_{X^{*}}^{Z+1}(C) \rightarrow A+1_{X}^{Z+1} \]

It is assumed that the probability of various decay modes, i.e., exit channels $(X + Y)$, are independent of the way the compound nucleus formed. $C$ lives long enough and loses its "memory" of how it was formed through the entrance channel $(x + y)$. Generally $x$ and $X$ will be nucleons or light nuclei. In our case $x$ is a proton or a helium nucleus, and $X$ is a $\gamma$-ray, in which case the reaction is called a radiative capture reaction. For example, the compound nucleus of $^{103}\text{Ag}$ is formed by bombarding $^{102}\text{Pd}$ with protons. This is highly excited and may $\gamma$-decay to a lower lying $^{103}\text{Ag}$ state. Since $^{103}\text{Ag}$ is $\beta^{+}$-unstable, it then decay into $^{103}\text{Pd}$. This decay can populate any one of many excited states of $^{103}\text{Pd}$, which then decays to the ground state, producing $\gamma$-rays, see Figure A.1. These were the $\gamma$-rays in which we were interested, and which we detected, for this work.
3.5 NON-SMOKER

Astrophysical investigations of the nucleosynthesis in stellar environments require the knowledge of thermonuclear reaction rates involving unstable nuclei most of which currently are not accessible experimentally, especially the p-process nuclei explained in the previous chapter. In most cases the statistical model approach (Hauser-Feshbach, HF) is used to calculate the cross sections and reaction rates due to high level densities in the compound nucleus.

The statistical model HF [Hauser & Feshbach (1952)] assumes that the reaction proceeds via a compound nucleus which decays into the reaction products. The average cross sections can be calculated by factorizing into a cross section for the formation of the compound nucleus ($\sigma_{\text{form}}$) and a branching ratio ($br_{\text{dec}}$) which describes the probability of the decay into the exit channel of interest compared with the total decay probability into all possible exit channels [Rauscher et al. (2000)]. Therefore, the average cross section is written as

$$\sigma_{\text{HF}} = \sigma_{\text{form}} br_{\text{dec}} = \sigma_{\text{form}} \frac{\Gamma_{\text{final}}}{\Gamma_{\text{tot}}}. \quad (3.26)$$

Energy averaged transmission coefficients $T$ can be used when a high level density in the compound nucleus is available at the energy of the incident projectile. Then, the well known expression can be written

$$\sigma_{i}^{n\nu}(j, \sigma; E_{ij}) = \frac{\pi h^2/(2\mu_{ij} E_{ij})}{(2J_i^\mu + 1)(2J_j + 1)} \times \sum_{J, \pi} (2J + 1) T_{ij}(E, J, \pi, E_i^\mu, J_i^\mu, \pi_i^\mu) T_{\nu}(E, J, \pi, E_{m}, J_{m}, \pi_{m}) \frac{T_{\text{tot}}(E, J, \pi)}{T_{\text{tot}}(E, J, \pi)} \quad (3.27)$$
for the cross section $\sigma^{\mu\nu}$ of the $i^\mu(j,\nu)m^\nu$ reaction, where $i^\mu$ is the target state, $m^\nu$ is the excited state of the final nucleus, $E_{ij}$ is the center of mass energy, $\mu_{ij}$ is the reduced mass, $J$ is the spin, $E$ is the excitation energy, and $\pi$ is the parity of the excited states. The compound nucleus parameters are those without subscripts, the nuclei participating in the $i^\mu(j,\nu)m^\nu$ reaction are shown with subscripts, and the specific excited states are indicated with superscripts. Experiments measure $\sum_\nu \sigma_i^{\mu\nu}(j,\nu;E_{ij})$, summed over all excited states of the final nucleus, with the target in the ground state [Hoffman et al. (1999)]. In an astrophysical plasma of temperature $T^*$, the astrophysical cross section $\sigma_1^i(j,\nu;E_{ij})$ of thermally populated target states $\mu$ is given by

$$\sigma_i^1(j,\nu;E_{ij}) = \frac{\sum_\nu (2J_i^{\nu} + 1)e^\nu(-E_i^{\nu}/kT^*) \sum_\nu \sigma_i^{\mu\nu}(j,\nu;E_{ij})}{\sum_\nu (2J_i^{\nu} + 1)e^\nu(-E_i^{\nu}/kT^*)}$$

(3.28)

where $k$ is the Boltzmann constant. $T_0^\nu(E, J, \pi)$ can be replaced by the total transmission coefficient, $T_0(E, J, \pi)$, by summation over $\nu$ in Equation 3.27 and is given by

$$T_0(E, J, \pi) = \sum_{\nu=0}^{\nu_m} T_0^\nu(E, J, \pi, E_m, J_m, \pi_m)$$

$$\times \int_{E_m}^{E-S_{m,\nu}} \sum_{J_m,\pi_m} T_0(E, J, \pi, E_m, J_m, \pi_m)\rho(E_m, J_m, \pi_m)dE_m,$$

(3.29)

where $S_{m,\nu}$ is the channel separation energy. Note that the summation over excited states above the highest experimentally known state $\nu_m$ is changed to an integration over level density $\rho$. Similarly, the summation over target states $\mu$ is applied.

For statistical model calculations, the particle and $\gamma$-transmission coefficients $T$ and the level density, $\rho$, of excited states are the important parameters as seen in
Equations 3.27 through 3.29. Thus, the accuracy of these components determines the reliability of such model calculations.

In recent calculations, the nuclear level density has shown the highest uncertainties [Rauscher, et al. (1997)]. In this reference, a highly improved fit to experimental level densities was obtained by a back-shifted Fermi-gas formalism utilizing an energy dependent level density parameter and employing microscopic corrections from a recent Finite Range Droplet Model (FRDM) mass formula [Möller et al. (1995)].

To describe the nuclear level density, $\rho(U)$, the backshifted Fermi gas formalism [Gilbert & Cameron (1965)] with an energy dependent level density parameter $a$ and the backshift $\delta$ in the energy levels with microscopic corrections from nuclear mass models may be used. Compared to other formalisms, this leads to improved fits in the mass range $20 \leq A \leq 245$ [Rauscher et al. (1997)]. This is given by

$$\rho(U, J, \pi) = \frac{1}{2} f(U, J) \rho(U)$$  \hspace{1cm} (3.30)

with

$$\rho(U) = \frac{1}{\sqrt{2\pi}\sigma} \frac{\sqrt{\pi}}{12a^{1/4}} \frac{\exp(2\sqrt{aU})}{U^{5/4}},$$  \hspace{1cm} (3.31)

$$f(U, J) = \frac{2J + 1}{2\sigma^2} \exp \left( \frac{-J(J+1)}{2\sigma^2} \right),$$  \hspace{1cm} (3.32)

$$\sigma^2 = \frac{2m_uAR^2}{5} \frac{\sqrt{U}}{a},$$  \hspace{1cm} (3.33)

where $U = E - \delta$. The spin dependence $F$ is determined by the spin cut-off parameter $\sigma$. Since the level density is dependent on the level density parameter $a$ and the backshift $\delta$, level density predictions depend on the reliability of $a$ and $\delta$. However,
the backshifted Fermi gas approach diverges for $U=0$ (i.e. $E = \delta$). At low energies, it is found experimentally to be well represented instead by

$$\rho(U) \propto \frac{e^{U/T}}{T}.$$  

Then, these two formula are matched and the density of states available for the compound nucleus can be determined.

After calculating level densities, the range of validity of the Hauser-Feshbach approach can be determined. Even though it is often stated that it is only applicable for intermediate and heavy nuclei, the only necessary condition is a large number of resonances at the appropriate bombarding energies. Therefore, an average over resonances can be used to calculate the cross section. Recent numerical test calculations shows that the sufficient average number of resonances is 5-10 [Rauscher et al. (1997)].

In this study, experimental measurements are compared with the recently developed statistical model code NON-SMOKER which is derived from the well known code SMOKER [Thielemann et al. (1987)]. For neutrons and protons the optical potential of Jeukenne, Lejeune and Mahaux is used to calculate the individual particle transmission coefficients by solving the Schrödinger equation [Jeukenne et al. (1977)]. The potential of McFadden and Satchler is used [McFadden & Satchler (1966)] for alpha particles.

The cross section and the reaction rate calculations using NON-SMOKER and more information about the code can be found on [NON-SMOKER Website].
The relevant cross sections have been calculated from the proton dripline to the neutron dripline for neutron, proton, and alpha induced reactions and their inverses for $10 \leq Z \leq 83$ in the temperature range $10^8 \leq T(\text{K}) \leq 10^{10}$ [Rauscher & Thielemann (2000)]. These reaction rates are purely theoretical rates except for nuclear masses and excited state information where available. These rates should be supplemented with experimental rates as they become available, although some of them involve such short-lived nuclei that they will never be measured. Further details on calculations of the cross sections and reaction rates using NON-SMOKER code are presented at [REACLIB website].
CHAPTER 4

EXPERIMENT

Our experiments were performed at the Nuclear Structure Laboratory of the University of Notre Dame using the FN tandem Van de Graaff accelerator, which is a very suitable accelerator device for this experiment. The following chapter describes the operation of the FN tandem Van de Graaff accelerator and discusses the apparatus and the methods used in the present experiment.

4.1 FN Tandem Accelerator Facility

An overhead floor plan of the Notre Dame Nuclear Structure Laboratory is shown in Figure 4.1. The FN tandem accelerator is capable of a terminal voltage in excess of 10.5 MV, allowing up to 20 MeV proton and deuteron beams, 30 MeV helium 3 and helium 4 beams, and higher energy beams for heavier ions. A Van de Graaff accelerator was an appropriate choice for this experiment because of the extremely precise energy and stability of the accelerated beam. Beam energy resolution of a few kilo electron-volts for beams of several million electron-volts are achievable.
Figure 4.1: Schematic floor plan of the Notre Dame Nuclear Structure Laboratory [ND University].
Another advantage not shared with other types of accelerators is that the energy may be changed very rapidly allowing repeated irradiations in a short time.

After a target was irradiated in the East target room, it was taken to the counting area where the radioactively hot target was placed between two hyperpure germanium detectors (HPGe). The counting setup was built in a separate room from the target room so the detectors would not be saturated by the inevitable large flux of radiation associated with the beam. Another advantage of having a separate counting area is that we were able to obtain a larger solid angle by utilizing a close geometry for the target and the detectors.

4.1.1 Ion Source and Beam Formation

The negative ion beam of most elements can be produced in a Sputtered Negative Ion Cesium Source (SNICS). A reservoir of cesium is heated so that cesium vapor rises from the reservoir into an enclosed area between a cooled cathode and a heated ionizer. Some of the cesium condenses onto the cool cathode surface while some of the cesium condenses on the hot ionizer and becomes positively charged. The positively charged ionized cesium accelerates towards the cathode, sputtering material from the cathode through the condensed cesium layer. Some of the sputtered ions pick up electrons as they pass through the condensed cesium layer, and form the negatively charged ion beam. Since the entire source is operated at below ground potential (~ 80 KeV) this negative beam is accelerated out of the source and enters the beam pipe. This is schematically shown in Figure 4.2.
A helium ion source, known as HIS, is used to produce negatively charged $^3\text{He}$ and $^4\text{He}$ ion beams (see the reference [ND Website] for details). The main part of the HIS is the duoplasmatron as seen in Figure 4.3. Electrons produced by a heated filament ionize the source gas (helium), and positively charged helium ions emerge from a small aperture. In order to extract the positive helium beam, the extraction electrode is held at approximately -20 kV. The beam is then focussed into the charge exchange region which is filled with lithium vapor. Some of positively charged helium beam will gain electrons resulting in some negatively charged helium beam. Since the exchange region is maintained at about -22 kV, the negatively charged helium
beam, with an energy of 44 keV, is accelerated from the charge-exchange region into the FN tandem Van de Graaff accelerator.

Figure 4.3: HIS ion source used to obtain $^4$He beams [ND Website].

4.1.2 Tandem Accelerator

These negative ions are deflected in a magnetic field into the accelerating tube of the first stage of the tandem accelerator which has both ends at ground potential and a high positive voltage (V) terminal in the middle as shown in Figure 4.4. The negative ions are accelerated towards the terminal and gain an energy of $e \times V$ electron-volts (eV). In the terminal, the beam passes through a thin carbon foil. Electrons are stripped during passage through this foil so that a positive ion beam is obtained. This ion beam is repelled away from the positive terminal and is provided with a second stage of acceleration as it travels down the acceleration tube to ground at the high energy end of the beam. The name “tandem” arises from these two
Figure 4.4: FN tandem Van de Graaff accelerator beamline component diagram (provided by University of Notre Dame). A negative ion beam is changed into a positive ion beam to accomplish a double acceleration.
accelerations: one before the foil and one after the foil. Clearly if the charge of the ion emerging from the foil is \( +ne \), the total energy of the ions reaching the high energy end of the accelerator is \((1+n)\) eV electron volts plus the small energy the beam had at injection (\(~ 80\) keV). The fully accelerated beam exits from the accelerator, is bent through 90° in an "analyzing magnet". Collimating slits at the exit of the analysing magnet assure that only a certain chosen energy beam is extracted. Furthermore, precise regulation of the beam energy is achieved by measuring the difference between the amounts of beam hitting the two slits. Then this difference is used to regulate the terminal voltage, hence the beam energy. The beam is then deflected by a switching magnet into any one of six different beam lines in the east target room.

4.2 Activation Method

In order to measure the radiative capture cross sections of the reactions studied in this work, targets were bombarded with either a proton or alpha particle beam. Since the targets chosen for this experiment were such that the residual nuclei were unstable, cross sections could be determined by measuring the number of nuclei created via the residual radioactivity (in our case \( \beta^+ \)-decay). This is known as an activation method and involves two stages. The first stage is the irradiation of the targets by projectiles and the second stage is the measurement of the residual radioactivity after the irradiation is stopped.
4.2.1 Irradiation of the Target

After the beam was aligned, and the projectile energy was arranged, the target was attached to a holder, and placed in the target chamber as shown in Figure 4.5. The beam was collimated before entering the target chamber by a tantalum collimator. The beam intensity was chosen to avoid target degradation, and the beam current hitting the target was in the range of 75 nA and 160 nA. The beam intensity was kept low at lower projectile energies since the energy loss in the target for a lower energy beam is higher and there could have been a possibility of melting holes in the targets. In order to get enough activity and to optimize the time in irradiation and counting, the irradiation time was chosen to be at least 1.5 times longer than the half-life of the reaction product.

It was essential to have an accurate measurement of the total number of charged particles hitting the target during the irradiation runs. Since the vast majority of the beam passes through the target unscattered, the beam incident on the target was determined by stopping the beam down-stream of the target with a Faraday cup, which was isolated from the rest of the chamber and the beam pipe. It was placed behind the target chamber as shown in Figure 4.5.

When the charged particle strikes the target foil, secondary electrons are knocked out from the target material. Two Neodymium magnets (PASCO EM-8621) which had a residual flux density of ~11000 Gauss were placed on the beampipe between the Faraday cup and the target to prevent the knocked out electrons from the target.
from hitting the Faraday cup, and to prevent knocked out electrons from escaping from the Faraday cup itself.

An alternative method of suppressing false beam current measurements was attempted in previous experiments; that of holding the Faraday cup at a negative potential. Despite our precautions, and those used in previous experiments [Chloupek et al. (1999)], we observed systematic differences in beam current ranging from 15% at the lowest energies to less than 5% at the highest energies between target-in and target-out runs. To correct for this we performed target-out runs before and after every reaction-data run and normalized the 1 minute interval readings to these. Using this method we estimate a contribution to the overall uncertainty in
the cross section measurements ranging from 5% at the lowest energies to 2% at the highest energies.

The ion beam current incident upon the Faraday cup during the irradiation was then recorded by integrating in real time with a Brookhaven Current Integrator (BCI) allowing fluctuations in the beam to be monitored. LabView software [ND University] was used to get the total charge accumulation by the Faraday cup for every minute interval as explained in Chapter 5. The beam current profile during the irradiation for one of the projectile energies are shown in Figure 4.6.

After each irradiation, the activated target was taken to a separate low-background counting area, and placed between two HPGe detectors to measure the induced (β+-delayed) γ-ray activity.

4.3 Measurement of the Residual Radioactivity

Even when there was no radioactive source between the detectors, they registered counts because of cosmic radiation that continuously bombards the Earth's atmosphere and the existence of natural radioactivity in the environment. Also in a laboratory where numerous radioactive sources are kept and accelerators are in use, the background rate will of course be somewhat higher than elsewhere.

HPGe detectors were placed in stainless steel cylinders of 1.9 cm thickness and surrounded by lead bricks of at least 10 cm thickness as passive shielding to ensure low background. These were then covered by six plastic scintillators as an active cosmic ray veto. This counting setup is shown in Figure 4.7.
Figure 4.6: An example of the beam current profile. The short runs at the start and end were taken with no target present and allowed a normalization for the effects of false beam current measurement due to dispersion of the beam by the thick target.
In order to reduce the positioning uncertainty in the counting setup, a lead brick with a customized groove designed to accept the base of the target holder was placed appropriately when the setup was built. Aluminum was chosen as the target frame and the target holder material since it has good heat conductivity which allows for effective cooling during the irradiation.

Figure 4.7: A schematic diagram of counting setup. The plastic scintillators covered both the top and the sides of the counting setup.

In principal, the detection of positrons is easier than that of $\gamma$-rays because of their higher energy deposition rate in a detector. However, several factors meant that we did not wish the $\beta^+$ particles to enter our detectors. Firstly, the detectors had an aluminum protective cover, and this would make it difficult to estimate the effect of this on the positron detection efficiency. Secondly, if the positrons were not stopped before reaching the detector their remaining kinetic energy plus the positron-electron
annihilation would broaden the peaks, making definition of the peaks extremely difficult. By stopping the positrons before they get to the detector, one has only the 511 keV peak in each detector.

Therefore, aluminum plates were placed between the target and detectors to stop the positrons before they reached the detector. Since a positron slows down in aluminum plate until its kinetic energy approaches zero, it annihilates in the plate with an electron producing two 511 keV γ-rays with an angle of 180° at exactly the same time due to the momentum conservation. Thus, the positron emissions are detected by counting the 511 keV γ-rays or even better by counting the coincident 511 keV γ-rays in two HPGe detectors, which are placed in a straight line with the target between them, as seen in Figure 4.7.

4.4 Aluminum Plates

The 511 keV γ-rays were detected instead of positrons for the reasons discussed in the previous section. The aluminum plates of the size (14cm x 16cm x 6mm) were used to stop the positrons before they entered the detector.

The penetration depth of positrons into the aluminum were determined by the graph of penetration depth from [Marion (1968)] shown in Figure 4.8 to find the appropriate thickness of aluminum to stop the most energetic positrons.

The 511 keV γ-rays from the electron-positron annihilations could either pass through one or both aluminum plates depending on where the positrons were stopped.
This geometrical effect was taken into account in a Monte Carlo Simulation explained in the next chapter.

Figure 4.8: Penetration of electrons in aluminum [Marion (1968)].

To a small degree, the $\gamma$-rays also were affected by aluminum plates. They are quite penetrating and will, for the most part, pass through the aluminum plates. The decrease in the intensity of $\gamma$-rays of energy $E$ going through aluminum of thickness $d$ can be written

$$I = I_0 \exp(-\mu d)$$  \hspace{1cm} (4.1)
where $I_0$ is the incident beam, and $\mu$ is the absorption coefficient. The absorption coefficient which is the absorption probability per unit length can be calculated by

$$\mu = N \sigma = \sigma (N_A \rho / A)$$

(4.2)

here $N$ is the density of atoms, $\sigma$ is the gamma-ray interaction cross-section, $N_A$ is the Avagadro's number, $\rho$ is the aluminum density, and $A$ is the molecular weight.

### 4.5 Target Properties

Isotopically enriched $^{102}\text{Pd}$, $^{112}\text{Sn}$, and $^{116}\text{Sn}$ targets were used in this work. They were borrowed from the Indiana University Cyclotron Facility (IUCF). The thicknesses and the major contaminants of the targets are listed in Table 4.5. The targets were in the form of thin self-supporting foils of 2.0 mg/cm$^2$ for $^{102}\text{Pd}$, 2.7 mg/cm$^2$ for $^{112}\text{Sn}$, and 2.1 mg/cm$^2$ for $^{116}\text{Sn}$. Small thin foil targets were used to get well defined energy values at which reactions occurred.

In choosing the target material for this work, some criteria needed to be considered. First of all, the radiative capture reaction cross section was required to be high enough to have a realistic chance of measuring it. The $(p,\gamma)$ reaction was also required to be the dominant channel at the projectile energy used for the $^{102}\text{Pd}$ and $^{116}\text{Sn}$ targets. Otherwise the observations would be affected by other reactions. The $(p,\gamma)$ reaction cross section drops when the other channels open, especially the $(p,n)$ channel due to its threshold value.
<table>
<thead>
<tr>
<th>Target</th>
<th>Thickness</th>
<th>Isotope</th>
<th>Atomic Percent</th>
<th>Daughter</th>
<th>Half-life</th>
</tr>
</thead>
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<tr>
<td>$^{102}$Pd</td>
<td>2.0 mg/cm²</td>
<td>$^{102}$Pd</td>
<td>77.89 %</td>
<td>$^{103}$Ag</td>
<td>65.7 m</td>
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<tr>
<td></td>
<td></td>
<td>$^{104}$Pd</td>
<td>5.49 %</td>
<td>$^{105}$Ag</td>
<td>41.29 d, 7.23 m</td>
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<tr>
<td></td>
<td></td>
<td>$^{106}$Pd</td>
<td>6.41 %</td>
<td>$^{106}$Ag</td>
<td>23.96 m, 8.28 d</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{108}$Pd</td>
<td>5.41 %</td>
<td>$^{107}$Ag</td>
<td>44.3 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{110}$Pd</td>
<td>3.51 %</td>
<td>$^{109}$Ag</td>
<td>39.6 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.28 %</td>
<td>$^{111}$Ag</td>
<td>7.45 d, 64.8 s</td>
</tr>
<tr>
<td>$^{112}$Sn</td>
<td>2.7 mg/cm²</td>
<td>$^{112}$Sn</td>
<td>98.90 %</td>
<td>$^{116}$Te</td>
<td>2.49 h</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{114}$Sn</td>
<td>1.00 %</td>
<td>$^{118}$Te</td>
<td>6 d</td>
</tr>
<tr>
<td>$^{116}$Sn</td>
<td>2.1 mg/cm²</td>
<td>$^{116}$Sn</td>
<td>95.60 %</td>
<td>$^{117}$Sb</td>
<td>2.8 h</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{117}$Sn</td>
<td>1.63 %</td>
<td>$^{118}$Sb</td>
<td>3.6 m, 5 h</td>
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<tr>
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<td></td>
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<td>$^{119}$Sb</td>
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<td></td>
<td></td>
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<td>$^{120}$Sb</td>
<td>15.89 m, 5.76 d</td>
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<tr>
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<td></td>
<td>$^{120}$Sn</td>
<td>0.63 %</td>
<td>$^{121}$Sb</td>
<td>stable</td>
</tr>
</tbody>
</table>

Table 4.1: Properties of the targets used in this experiment. Half-lives are given in units of seconds (s), minutes (m), hours (h), and days (d).

The half-life of the reaction product should be appropriate (neither too short nor too long) to perform the activation method for several different energies. Long half-lives require the use of longer irradiation times to approach saturation and longer counting times to get enough yield. Short half-lives give difficulties in transferring the target to the counting setup after each irradiation and potentially problems in counting high activities due to dead time effects. A half-life of 10 minutes to a few hours was ideal for the setup used in this experiment.

The high purity of the primary target material was required to avoid interference from other reactions of secondary target materials.
Finally, the reaction product had to have gamma-ray or positron emission or both to be detected by the HPGe detectors.

After all these considerations, we determined that $^{102}\text{Pd}$ and $^{116}\text{Sn}$ targets for $(p,\gamma)$ reaction and $^{112}\text{Sn}$ target for the $(\alpha,\gamma)$ reaction would be appropriate choices. The half-lives of the nuclei populated in the reactions studied are $^{103}\text{Ag} : 1.095\text{h}$, $^{117}\text{Sb} : 2.8\text{h}$, and $^{116}\text{Te} : 2.49\text{h}$. 

4.6 Gamma-Ray Interactions

Gamma rays interact with matter via three major processes which lead to the partial or complete transfer of the gamma ray energy to electron energy: photoelectric absorption, Compton scattering, and pair production. The electrons produced by these processes in turn deposit their energy in the detector, which thus generates a voltage pulse that is then processed in the electronics, digitized, and analyzed.

In the photoelectric absorption process, an incident $\gamma$-ray energy, $E_{\gamma}$, is completely absorbed by an atom in a detector causing a photoelectron to be ejected by the atom from one of its bound shells (usually from the K-shell) with an energy equal to the incident $\gamma$-ray energy minus the binding energy of the photoelectron, $E_e^- = E_{\gamma} - E_B$. The vacancy left by the electron is quickly filled through capture of a free electron from the medium or atomic rearrangement producing an Auger electron or a characteristic X-ray which is reabsorbed. So this type of interaction results in full energy deposition, and in a detector's energy spectrum will produce a characteristic peak called the photopeak at the energy of the $\gamma$-ray. In Compton scattering, the
incoming γ-ray scatters off an atomic electron giving the electron a fraction of its energy. Applying energy and momentum conservation, the energy of the scattered γ-ray, $E'_\gamma$, in terms of its scattering angle $\theta$ is given by

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_0c^2}(1 - \cos \theta)}$$

(4.3)

where $m_0c^2$ is the rest mass energy of the electron (0.511MeV). The kinetic energy of the recoiling electron is thus the difference in the incident and the scattered γ-ray energy, which is

$$E_{e^-} = E_\gamma - E'_\gamma = \frac{E_\gamma^2}{\frac{m_0c^2}{1 - \cos \theta} + E_\gamma}$$

(4.4)
For small scattering angles, \( \theta \approx 0 \), the energy transfer to the recoil electron is very small while for a head-on collision, \( \theta = \pi \), a maximum energy transfer occurs. Since all scattering angles may occur in the detector, a continuum of energies is transferred producing a spectrum called the Compton background up to the maximum value called the Compton edge.

Finally, pair production process is possible in addition to first two processes if the incident \( \gamma \)-ray energy is at least twice the electron mass energy \( (E_\gamma \geq 1.022 \text{ MeV}) \). In this process, an incident \( \gamma \)-ray is converted into an electron-positron pair. The total kinetic energy of the electron-positron pair is

\[
E_{e^-} + E_{e^+} = E_\gamma - 2m_0c^2
\]  

(4.5)

where the recoil energy is neglected. Being charged, the electron and positron quickly deposit their kinetic energy to the material, and the positron annihilates with an electron producing back to back \( \gamma \)-rays with the energy equal to \( m_0c^2 \) (0.511 MeV) each. However, pair production is not significant for \( \gamma \)-ray energies less than a few MeV. So, photoelectric absorption becomes the most important process for the \( \gamma \)-rays we are interested in.
4.7 HPGe Detectors

The gamma-ray radiation was detected by means of two hyperpure germanium (HPGe) detectors of size of 66.5 mm diameter and 64.2 mm length coaxial crystals. These were used because of their high energy resolution, which allowed them to identify the γ-rays of interest with sufficient accuracy, especially for the $^{102}$Pd($p, \gamma$)$^{103}$Ag reaction.

The HPGe detectors are semiconductor diode type detectors which are formed by setting up two different type regions in the semiconductor, p-type (excess hole concentration) and n-type (excess electron concentration). A charge balance exists between these two regions, called the depletion region, in which electron-hole pairs

Figure 4.10: A schematic diagram of HPGe detector [ORTEC Website].
form from the incident radiation. A bias voltage causes these electron-hole pairs to drift, producing the electric signal.

The HPGe detectors used in this experiment were n-type coaxial detectors in which the outer face has a thin p+ layer and the applied biases were negative ($\sim -3000$ V and $\sim -3500$ V). Electrons were the main charge carriers.

The room temperature operation of any type of germanium (Ge) detectors is impossible due to the large thermal induced leakage current, and therefore HPGe detector crystals must be continuously cooled with liquid nitrogen ($\text{LN}_2$) to keep the leakage currents low. A major advantage of HPGe detectors is that they can be stored at room temperature, then cooled when they are used.

4.8 Electronics and Data Acquisition

The raw data signals obtained from the detectors were processed with the electronics setup shown in Figure 4.11. Two output signals from the preamplifier of each detector were fed separately into a main (slow) amplifier for the data acquisition system and into a fast amplifier for the event strobe and anticoincidence requirement.

For the data acquisition system, the negative analog signals, with an amplitude of around a few hundred mV, were amplified to 1-2 V and inverted to give positive signals using a slow amplifier (ORTEC 572 Amplifier) with a gain of around 10 and a shaping time of $3\mu\text{sec}$. These were then fed into Channel 1 and 2 of an Analog to Digital Converter (ORTEC 811 ADC) to digitize them.
The other output signals from the two preamplifiers of the HPGe detectors were shaped by timing filter amplifiers (ORTEC 579 TFA and ORTEC 454 TFA) to give outputs with a fast signal in the range of 10-30 ns for timing signals.

These negative output signals were then fed into the Constant Fraction Discriminators (EG&G ORTEC QUAD CFDs), which maintain good timing properties, and thus were converted into logic pulses. The START signal from the CFD of one detector and a STOP signal from the other detector, delayed by ~ 0.5μsec, were fed into a Time to Pulse Height Converter (ORTEC 467 TPHC or TAC). The TAC converted the time difference between them into an output pulse whose height was proportional to this time period. The TAC was set to operate on a maximum time difference of 2μsec. The positive output signal of the TAC was then fed into Channel 0 of the ADC. In the case where there was a signal on one detector and not the other, no TAC output signal was obtained.

The logic pulses from the CFDs were also combined ("OR"ed) and sent to the coincidence box with a veto signal from the six plastic scintillators. When the ionizing radiation strikes the plastic scintillator (24" x 8" x 5"), it excites an atom which then deexcites emitting scintillation photons. In a photomultiplier coupled to the scintillator, these scintillation photons are collected and converted into a weak current of photoelectrons which is then amplified to an electrical pulse that can be used for anticoincidence measurements.

The signals from the photomultipliers of the plastic scintillators were first amplified and shaped by means of TFAs, then were changed into logic signals by EG&G
Figure 4.11: Electronic diagram of the experiment.
ORTEC QUAD CFD's to be able to use them for anticoincidence measurements. All these logic pulses were combined by two LSR Model 429 Logic Fan In Fan Out units (FIFOs), creating one logic pulse representing an OR of all the plastic scintillator detectors. After using a GG 8000 Model Gate and Delay Generator (Octal Gate Generator) to change the polarity and to insert the correct delay, this signal was sent to a coincidence unit as a veto signal. An output signal of the coincidence box then represented an event in which either E1 or E2 detected a γ-ray, while there was no signal received in any of the plastic scintillators. This was then sent to the ADC gate to tell the ADC module to analyze the analog signals from the slow amplifiers and the TAC, and also to the “Event In” of the CAMAC crate to initiate the recording process.

Finally, an accurate 1 kHz quartz oscillator was used for two purposes; firstly it was scaled down by a rate divider (1/2000) and fed into a scalar. The numerical value of this scalar was recorded as a fourth parameter for each event, thus allowing every event to be time-stamped. Secondly, the clock and the clock vetoed by the data acquisition busy signal were recorded in scalers to obtain live-time information. These were written to file every five minutes giving live-times for each five minute data set. Therefore four parameters were recorded for each HPGe event; the time difference between detector signals (TAC signal), the energy signals from both detector (E1 and E2 signals), and a clock value.

The data acquisition system at the University of Notre Dame’s Nuclear Structure Laboratory was designed by the Holifield Radioactive Ion Beam Facility at Oak
Ridge, and described in [Meißner (1995)]. It consists of three pieces of equipment; the CAMAC modular crate, the VME front-end processor, and the DECstation. The CAMAC modular crate holds the modules needed for the data acquisition and provides an interface between the modules and the other two components of the data acquisition system. The VME is triggered by an event signal, and tells the CAMAC to start taking data which then travels through the VME to the DECstation by a network connection. Then, DECstation provides the software for acquiring, processing, and viewing the data.

At the workstation, the data for individual experimental runs were stored as event-by-event data on 8 mm magnetic tapes during the whole experiment for off-line data analysis. At the same time, the data were used to construct spectra in the memory of the workstation, and these spectra were displayed on the monitor providing the real time data visualization to guide the progress of the experiment. Then, off-line analysis of data recorded on tapes was performed on VMS workstations at the Ohio State University by using the CERN Physics Analysis Workstation software package (PAW).
The data, the analysis of the data, and the method used for cross section and astrophysical S-factor determinations are presented below. Each data set was divided into time intervals coinciding with the known dead time intervals. Then, four histograms were produced for each interval; a spectrum of event-singles (i.e. a spectrum of events in one detector regardless whether or not the other detector saw an event) in detector 1 (hereafter E1), a spectrum of event-singles in detector 2 (hereafter E2), a coincidence event spectrum in E1 which had a positive TAC signal and coincided with 511 keV event in E2, and a coincidence event spectrum in E2 which had a positive TAC signal and coincided with 511 keV event in E1. The event-singles peak measurements including 511 keV were deduced from the first two spectra, and coincidence measurements of 511 keV γ-rays in both detectors simultaneously were deduced from last two spectra.
5.1 Energy Calibration

Evaluating the energies of the peaks in the spectrum was accomplished by calibrating the HPGe detectors using the standard $\gamma$-ray sources of $^{22}$Na, $^{60}$Co, $^{133}$Ba, and $^{137}$Cs. These sources produce $\gamma$-ray lines at known energies and intensities over the entire experimental energy range of interest.

A least-square fitting of the calibration data relating energy to channel number gave us a good fit to a straight line of the form of

$$ E_1 = [(1.076 \pm 0.001) \times Ch_1 - (30.6 \pm 0.7)] \text{ keV} \quad (5.1) $$

for detector 1 and

$$ E_2 = [(1.0882 \pm 0.0009) \times Ch_2 - (33.8 \pm 0.6)] \text{ keV} \quad (5.2) $$

for detector 2 (by using the mathematical Analysis and Data Visualization Software PHYSICA [Chuma (1994)]). The $\gamma$-ray energy values for photopeaks from the reaction data, corresponding to the channel numbers ($Ch_1$ and $Ch_2$) in the spectra for detector 1 ($E_1$) and detector 2 ($E_2$), were obtained from Equation 5.1 and Equation 5.2, respectively. The energy versus channel number calibration curve for one of the detectors is shown in Figure 5.1. The straight line is the result of the linear fit to the data with $\chi^2 = 13.6$ for 9 degrees of freedom.
Figure 5.1: The energy calibration curve with standard γ-ray sources for one of the detector (E1).
5.2 Efficiency Calibration of HPGe Detectors

One of the important parameters in the experiment is the efficiency of the HPGe detectors. If detectors saw every charged particle (proton, alpha or beta) which entered its active volume, the detector would have a counting efficiency of 100 percent. On the other hand, uncharged radiations, such as γ-rays, must first undergo a significant interaction in the detector as described in the previous chapter. Since γ-rays can travel large distances between interactions, γ-ray detectors typically have less than 100 percent efficiencies. It then becomes crucial to have a precise figure for the detector efficiency in order to relate the number of pulses counted to the number of photons incident on the detector.

The absolute or total efficiency of the experiment is the quantity that gives the ratio (fraction) of the number of γ-rays detected to the number of γ-rays emitted by the source. One should remember that to be detected the γ-rays must interact with the crystal and produce a pulse higher than the threshold of the discriminator.

The photopeak efficiencies of the HPGe detector for different γ-line energies were determined using standard sources with known intensities in μCi (1 μCi = 3.7 × 10⁴ decays/second) as measured at a time, t₀. Standard sources of ²²Na, ⁶⁰Co, ¹³³Ba, and ¹³⁷Cs were placed at the same location where the irradiated target would be placed to detect the induced activity, as seen in Figure 4.7. The γ-rays from the standard sources were detected to perform the efficiency calibrations. The intensity of the sources, I(t), at the time t which is the time when the present experiment was
undertaken can be calculated from

\[ I(t) = I(t_0) \times e^{-\ln 2(t-t_0)/t_{1/2}}. \]  

(5.3)

Here \( I(t_0) \) is the intensity at time \( t_0 \), the time when the source was calibrated, and \( t_{1/2} \) is the half-life of the \( \gamma \)-ray source. The calculated \( \gamma \)-ray intensities of the sources on the day of the experiment are shown in Table 5.1. The number of \( \gamma \)-rays of a certain energy emitted by the source is

\[ N_{\text{emitted}} = I(t) \times \frac{BR}{100} \times t_{\text{run}}, \]  

(5.4)

where \( BR \) is the branching ratio of the particular \( \gamma \)-ray emission, and \( t_{\text{run}} \) is the counting time. The number of detected \( \gamma \)-rays, \( N_{\text{detected}} \), may then be obtained by calculating the area under each specific \( \gamma \)-line in the spectrum (for more detail, see Section 5.5). If the counting rate is high, the live-time correction \( (LT) \) should be included to calculations as

\[ N_{\text{detected}} = \frac{\text{Area}}{LT}. \]  

(5.5)

<table>
<thead>
<tr>
<th>Sources</th>
<th>Half-life (y)</th>
<th>Activity ((\mu\text{Ci})) at ( t_0 )</th>
<th>Time ( t_0 )</th>
<th>Activity ((\mu\text{Ci})) at ( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{23}\text{Na} )</td>
<td>2.604</td>
<td>10.27</td>
<td>1 Sep 1990</td>
<td>0.83</td>
</tr>
<tr>
<td>(^{60}\text{Co} )</td>
<td>5.271</td>
<td>10.80</td>
<td>1 Sep 1990</td>
<td>3.12</td>
</tr>
<tr>
<td>(^{133}\text{Ba} )</td>
<td>10.53</td>
<td>11.65</td>
<td>1 April 1981</td>
<td>3.37</td>
</tr>
<tr>
<td>(^{137}\text{Cs} )</td>
<td>30.07</td>
<td>10.95</td>
<td>1 April 1981</td>
<td>4.75</td>
</tr>
</tbody>
</table>

Table 5.1: Standard \( \gamma \)-ray sources used for the detector efficiency calibration, their half-lives (years) and activities \((\mu\text{Ci})\) at time \( t_0 \) and time \( t \) which are times of the last measurements done and the efficiency calibration experiment we performed on 01-31-2000, respectively.
The efficiency for detecting the specific $\gamma$-line is given by

$$\text{eff}_\gamma = \frac{N_{\text{detected}}}{N_{\text{emitted}}}.$$  \hspace{1cm} (5.6)

This efficiency is the total efficiency which is the product of the detector geometry (since the detectors do not cover $4\pi$ steradians), and the intrinsic efficiency of the detector. For each photopeak obtained from the HPGe detectors, the efficiency was calculated separately and an energy dependence determined. Thus for the reaction data at some other energy the appropriate efficiency could be interpolated. The measured total efficiencies for both detectors are shown in Figure 5.2.

Note that for most of our experimental data, a measurement of the total detector efficiency is sufficient, since we have measured the efficiency with the calibration sources placed at the same positions as the reaction data targets. However, the intrinsic detector efficiency is important in the Monte Carlo studies addressed in section 5.8. This is because these studies involve estimating how uncertainties in the geometry, such as the source to detector distance, affect the efficiency. One time this becomes evident is in the observed efficiency of 511 keV $\gamma$-rays that are coming not from the source position, but from the aluminum plates where electron-positron annihilation of $\beta^+$ particles is occurring.
Figure 5.2: Efficiency versus γ-ray energy of standard sources of known activities.
5.3 Spectrum Analysis

Each irradiated target was placed between two back to back HPGe detectors to get \( \beta^+ \)-delayed \( \gamma \)-ray emission spectra. The next sections describe the analysis of the data from each of the three irradiated targets.

5.3.1 Spectrum Analysis of \(^{102}\text{Pd}(p,\gamma)^{103}\text{Ag}\) Data

A typical energy spectrum recorded from the irradiated \(^{102}\text{Pd}\) for one of the HPGe detectors is shown in Figures 5.3 and 5.4. The energy values of \( \gamma \)-ray peaks for detector 1 and detector 2 were identified using the energy calibration equations 5.1 and 5.2, and they are summarized in Table 5.2. Note that 511 keV \( \gamma \)-ray peak is due to the annihilation radiation of positrons in the aluminum plate whereas the \( \gamma \)-rays of interest here occur from the decay of excited states of \(^{103}\text{Pd}\). Since the \( \gamma \)-decays of states populated by the \( \beta^+ \)-decay from \(^{103}\text{Ag}\) formed in the \(^{102}\text{Pd}(p,\gamma)\) reaction to \(^{103}\text{Pd}\) occur very quickly, all of these photopeaks show a dependence on time consistent with the 2.8 hour half-life of \(^{103}\text{Ag}\). Figure A.1 shows the decay scheme of \(^{103}\text{Ag}\) [Firestone et al. (1996)]. The relevant \( \gamma \)-ray energies and branching ratios are given in Table 5.3 [NNDC Website].

From the decay spectrum of the irradiated \(^{103}\text{Ag}\) target, 13 \( \gamma \)-ray peaks including the 511 keV peak and 511 coincidence peak were identifiable and had enough statistics for the relevant cross section calculations. In general, the HPGe detectors
performed well, with typical energy resolutions for photopeaks of ~ 2.3 keV FWHM, as expected for such a detector.

<table>
<thead>
<tr>
<th>Channel-1</th>
<th>Energy (keV)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>139</td>
<td>119</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>166</td>
<td>148</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>201</td>
<td>186</td>
<td>Background</td>
</tr>
<tr>
<td>250</td>
<td>238</td>
<td>Background</td>
</tr>
<tr>
<td>256</td>
<td>244</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>277</td>
<td>267</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>503</td>
<td>511</td>
<td>Cosmic rays</td>
</tr>
<tr>
<td>523</td>
<td>532</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>718</td>
<td>742</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>799</td>
<td>829</td>
<td>Background</td>
</tr>
<tr>
<td>963</td>
<td>1007</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>984</td>
<td>1030</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>1101</td>
<td>1155</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>1127</td>
<td>1183</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>1211</td>
<td>1274</td>
<td>$^{103}\text{Ag}$</td>
</tr>
<tr>
<td>1384</td>
<td>1460</td>
<td>Background</td>
</tr>
</tbody>
</table>

Table 5.2: Identifiable peaks in the irradiated $^{102}\text{Pd}$ spectrum [Firestone et al. (1996)].

Further qualitative assessment of Figure 5.3 reveals there is a threshold at about 150 keV. This is due to the CFD threshold level required in the electronics. Events below this energy were only recorded because there was a coincident event in the other detector that resulted in the data acquisition process occurring for the event. An example of this can be seen from Figure 5.3. Even though 119 keV and 148 keV lines were below the detection threshold energy, they were observed because the other detector fired. This effect was also seen in the decay spectrum of the $^{133}\text{Ba}$
Figure 5.3: Decay spectrum obtained from $^{102}\text{Pd}(p,\gamma)^{103}\text{Ag}$ reaction for 200 min counting time and 67 min irradiation time at 3.75 MeV proton beam.
Figure 5.4: Expanded decay spectrum of irradiated $^{102}$Pd target for the $\gamma$-ray energy range of interest.
Table 5.3: γ-ray energies and branching ratios greater than 1.0% for $^{103}$Ag. The branching ratios and uncertainties are given in % [NNDC Website].

<table>
<thead>
<tr>
<th>γ Energy</th>
<th>γ Uncertainty</th>
<th>Branching Ratio</th>
<th>BR Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>118.74</td>
<td>0.05</td>
<td>31.2</td>
<td>2.0</td>
</tr>
<tr>
<td>148.20</td>
<td>0.04</td>
<td>28.3</td>
<td>1.8</td>
</tr>
<tr>
<td>243.96</td>
<td>0.05</td>
<td>8.5</td>
<td>0.5</td>
</tr>
<tr>
<td>266.86</td>
<td>0.05</td>
<td>13.3</td>
<td>0.9</td>
</tr>
<tr>
<td>511.</td>
<td>0.0</td>
<td>54.12</td>
<td>0.0</td>
</tr>
<tr>
<td>531.92</td>
<td>0.06</td>
<td>8.8</td>
<td>0.6</td>
</tr>
<tr>
<td>742.11</td>
<td>0.08</td>
<td>2.54</td>
<td>0.17</td>
</tr>
<tr>
<td>1007.08</td>
<td>0.08</td>
<td>3.24</td>
<td>0.22</td>
</tr>
<tr>
<td>1029.97</td>
<td>0.08</td>
<td>1.30</td>
<td>0.09</td>
</tr>
<tr>
<td>1155.27</td>
<td>0.10</td>
<td>3.05</td>
<td>0.20</td>
</tr>
<tr>
<td>1182.77</td>
<td>0.15</td>
<td>1.52</td>
<td>0.10</td>
</tr>
<tr>
<td>1273.83</td>
<td>0.12</td>
<td>9.4</td>
<td>0.7</td>
</tr>
</tbody>
</table>

calibration source shown in Figure 5.6. Note that the 356 keV line could be detected in one detector and 81 keV line in the other detector.

Although the 244 and 267 keV lines were above the threshold, we note that the efficiency calibration in this energy range was suspected to be poor since the calibration in this range relies only on the $^{133}$Ba source calibration data. Unfortunately, having only a rather strong $^{133}$Ba source, which generates several γ-rays of similar energy, resulted in significant pile up and consequent poor energy resolution and peak shape; the $^{133}$Ba calibration data photopeaks typically had energy resolution of ~3.5 keV FWHM.

To confirm these suspicions, a simple simulation of how this pile up might affect the peak shapes was performed. This simulation was not extensive enough to include
Compton scattering or similar effects. However, the simulation result shown in Figure 5.5 is clearly consistent with the poor resolution of the $^{133}$Ba data seen in Figure 5.6. For these reasons we are expecting to have difficulties calculating cross sections below the 511 keV $\gamma$-line.

Figure 5.5: A simulation of pile up effects for the $^{133}$Ba source.
Figure 5.6: Decay spectrum of $^{133}$Ba calibration source.
5.3.2 Spectrum Analysis of $^{116}\text{Sn}(p,\gamma)^{117}\text{Sb}$ Data

An example $^{116}\text{Sn}(p,\gamma)^{117}\text{Sb}$ reaction spectrum is shown in Figure 5.7 and the decay scheme of $^{117}\text{Sb}$ is shown in Figure A.2. After the energy calibration of the $^{116}\text{Sn}$ target, the relevant peaks identified with enough statistics were 158 and 511 keV $\gamma$-ray lines for proton beam energies below 3.25 MeV. For higher proton energies, three more $\gamma$-energy lines (861 keV, 1004 keV, and 1021 keV) had enough statistics, see Table 5.4. Note as described in previous section, the 158 keV line is not reliable enough to be included in the calculation of cross section. The $\gamma$-ray energies and the corresponding branching ratios used for cross section measurements are presented in Table 5.5.

<table>
<thead>
<tr>
<th>Channel-1</th>
<th>Energy (keV)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>176</td>
<td>158</td>
<td>$^{117}\text{Sb}$</td>
</tr>
<tr>
<td>828</td>
<td>861</td>
<td>$^{117}\text{Sb}$</td>
</tr>
<tr>
<td>961</td>
<td>1005</td>
<td>$^{117}\text{Sb}$</td>
</tr>
<tr>
<td>975</td>
<td>1021</td>
<td>$^{117}\text{Sb}$</td>
</tr>
</tbody>
</table>

Table 5.4: Identifiable peaks in the irradiated $^{116}\text{Sn}$ spectrum [Firestone et al. (1996)].

5.3.3 Spectrum Analysis of $^{112}\text{Sn}(\alpha,\gamma)^{116}\text{Te}$ Data

The decay scheme of $^{116}\text{Te}$ is shown in Figure A.3 in Appendix [Firestone et al. (1996)]. We were able to identify the $\gamma$-rays of 629 keV and 638 keV originating from the $\beta^+$-decay of the parent ($^{116}\text{Te}$) with enough statistics to measure the cross sections...
Figure 5.7: Decay spectrum obtained from $^{116}\text{Sn}(p,\gamma)^{117}\text{Sb}$ reaction for 181 min counting time and 178 min irradiation time at 3.75 MeV proton beam.
Figure 5.8: Expanded decay spectrum of irradiated $^{116}$Sn target for the $\gamma$-ray energy range of interest.
<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Uncertainty (keV)</th>
<th>Branching Ratio</th>
<th>BR Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>158.562</td>
<td>0.015</td>
<td>85.9</td>
<td>0.4</td>
</tr>
<tr>
<td>511.</td>
<td>0.0</td>
<td>3.40</td>
<td>0.0</td>
</tr>
<tr>
<td>861.35</td>
<td>0.05</td>
<td>0.31</td>
<td>0.04</td>
</tr>
<tr>
<td>1004.51</td>
<td>0.15</td>
<td>0.21</td>
<td>0.03</td>
</tr>
<tr>
<td>1020.6</td>
<td>0.5</td>
<td>0.21</td>
<td>0.018</td>
</tr>
</tbody>
</table>

Table 5.5: \(\gamma\)-ray energies and branching ratios greater than 0.2% for \(^{117}\)Sb [NNDC Website].

at the highest alpha beam energy of 10.5 MeV. In addition, 931 keV and 1294 keV \(\gamma\)-rays produced from the \(\beta^+\)-decay of the daughter (\(^{116}\)Sb) were used for calculations at alpha beam energies of 8.75 MeV and 10.5 MeV. The \(^{112}\)Sn(\(\alpha, \gamma\))\(^{116}\)Te reaction spectrum is presented in Figure 5.9.

Table 5.6 shows the relevant \(\gamma\)-ray energies and the corresponding branching ratios in the \(^{116}\)Te spectrum. Note that here, since the \(\beta^+\)-decay daughter of the \(^{112}\)Sn(\(\alpha, \gamma\))\(^{116}\)Te reaction is also \(\beta^+\)-unstable, we are able to use \(\gamma\)-rays observed from the \(^{116}\)Sn decay scheme to determine the reaction cross section. It has been assumed that there was no previous activation on the target at both alpha beam energies since the 8.75 MeV data was the first energy to show any evidence of activation by the \(^{112}\)Sn(\(\alpha, \gamma\)) reaction, and the 10.5 MeV run occurred about 4 days later when all activation would have decayed away (\(T_{1/2}=2.49\) hours).
Figure 5.9: Expanded decay spectrum of irradiated $^{112}$Sn target for the $\gamma$-ray energy range of interest.
<table>
<thead>
<tr>
<th>γ Energy (keV)</th>
<th>γ Uncertainty (keV)</th>
<th>Branching Ratio</th>
<th>BR Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>93.70</td>
<td>0.10</td>
<td>33.1</td>
<td>2.1</td>
</tr>
<tr>
<td>103.00</td>
<td>0.10</td>
<td>1.98</td>
<td>0.15</td>
</tr>
<tr>
<td>511.00</td>
<td>0.0</td>
<td>1.20</td>
<td>0.0</td>
</tr>
<tr>
<td>628.70</td>
<td>0.10</td>
<td>3.21</td>
<td>0.16</td>
</tr>
<tr>
<td>637.90</td>
<td>0.20</td>
<td>0.75</td>
<td>0.05</td>
</tr>
<tr>
<td>931.84</td>
<td>0.05</td>
<td>24.8</td>
<td>1.9</td>
</tr>
<tr>
<td>1293.56</td>
<td>0.15</td>
<td>85.0</td>
<td>7.0</td>
</tr>
</tbody>
</table>

Table 5.6: γ-ray energies and branching ratios greater than 1.0% for $^{116}$Te [NNDC Website].

5.4 Background

In order to reduce to the potential contributions of the natural background, active and passive shieldings were used as mentioned in Section 4.3. The use of plastic scintillators in coincidence with the HPGe signals as a veto of cosmic rays reduced the overall count rate about 30%. The lead and stainless steel coverage of the counting setup reduced the natural background by about a factor of 10 from that of unsheilded HPGe detectors. The major remaining background sources were $^{40}$K, which produces 1460 keV γ-rays and the interactions of the ambient neutrons with Cu components inside the HPGe detector that produce 186 keV and 660 keV γ-rays (see Table 5.2).

To separate γ-ray lines from unwanted background, the activity spectrum of each target was taken before doing any irradiations. The background spectrum obtained from the $^{102}$Pd target is shown in Figure 5.10. Note that the dominant
Figure 5.10: Background spectrum of $^{102}$Pd target before any irradiation. This spectrum took 140 minutes to accumulate.
overall background besides the room background was due to the Compton scattered photons.

5.5 Peak Area Calculations

In order to calculate the cross sections from the experimental data, the area under each \(\gamma\)-ray photopeak had to be determined. In general, two different methods could have been applied to calculate the peak areas: simple summation (integration) of the counts in the appropriate energy range including a background substraction, or, a fitting procedure using a Gaussian or similar shape, and then determining the area from the fit parameters. It was wise to use the integration procedure in our case due to the excellent resolution (well separated peaks and the smooth background) obtained as seen in Figure 5.11, which shows a schematic of the integration and background subtraction process. The peak areas were calculated by a simple PAW program as described below.

If \(A\) is the sum of all counts recorded between limits chosen at the low and high energy limits of the peak, and \(B = B_1 + B_2\) is the number of counts of the background (see Figure 5.11), the net count of the photopeak is \(C = A - B = A - (B_1 + B_2)\). Note that the width of \(B_1 + B_2\) is chosen to be equal to the width of \(A\). The standard error of the total count of the photopeak, \(\sigma_C\), is then

\[
\sigma_C = \sqrt{\left(\frac{\delta C}{\delta A}\right)^2 \sigma_A^2 + \left(\frac{\delta C}{\delta B_1}\right)^2 \sigma_{B_1}^2 + \left(\frac{\delta C}{\delta B_2}\right)^2 \sigma_{B_2}^2},
\]

(5.7)
where \( \sigma_A = \sqrt{A}, \sigma_{B_1} = \sqrt{B_1}, \) and \( \sigma_{B_2} = \sqrt{B_2}. \) Using Equation 5.7, one obtains for the standard error of the total count as

\[
\sigma_C = \sqrt{A + B_1 + B_2}.
\]  

The standard error of the total yield for different \( \gamma \)-line peaks was found to range for \( \sim 1 \% \) up to \( 20 \% \).

### 5.6 Cross Section Calculations

In this experiment, we wished to measure the cross section for \( ^{102}\text{Pd}(p,\gamma)^{103}\text{Ag}, \)
\( ^{116}\text{Sn}(p,\gamma)^{117}\text{Sb}, \) and \( ^{112}\text{Sn}(\alpha,\gamma)^{113}\text{Sb} \) at energies below the Coulomb barrier. Measurement of the reaction directly would have proved very difficult, since it would be hard to distinguish reaction produced \( \gamma \)-rays from the intense flux of \( \gamma \)-rays inevitably resulting from interactions of the beam in the target chamber. We therefore
chose to use an activation method. This is made possible because the products of the reactions of interest are themselves unstable to $\beta^+$-decay, with half-lives of a duration that make measurement of the residual $\gamma$-activities possible. Detection of the residual activity combined with knowledge of the decay scheme provides a measurement of the cross section. The experiment therefore may be considered as having two stages; the first stage is the irradiation of the targets by projectiles and the second stage is the measurement of the residual radioactivity after the irradiation is stopped.

As the target is irradiated, the radioactive nuclear species that is formed also undergoes radioactive decay. The rate of decay is given simply by $\lambda N$, where $\lambda$ is the decay constant and $N$ is the total total number of radioactive nuclei present. The rate of change in $N$ is given by the difference between the rate of formation, $r(t)$, and rate of decay

$$\frac{dN(t)}{dt} = r(t) - \lambda N(t).$$  \hspace{1cm} (5.9)

This equation assumes that $r$ is constant with time, that is, the proton flux does not vary during the irradiation period, and burnup or decrease in the number of target nuclei over the measurement is negligible. The solution of the differential Equation 5.9 for the condition $N=0$ at $t=0$ and $N = N_{\text{ere}}$ at $t = t_{\text{irr}}$ is

$$N_{\text{ere}} = \frac{r}{\lambda}(1 - e^{-\lambda t_{\text{irr}}}).$$  \hspace{1cm} (5.10)

However, in our experiment $r$ was not constant because the proton flux was found to vary during the irradiation period as shown Figure 5.12. Therefore, the production
Figure 5.12: An example of the unstable beam current profile. The short data sets before and after the main run were taken with no target. The variation observed in the beam current through the run as recorded here was taken into account for the off-line analysis.
rate as a function of time for a thin target is given by

\[ r(t) = \sigma(E)I_p(t)n_T \]  

(5.11)

where, \( \sigma(E) \) is the energy dependent cross section of the reaction, \( I_p(t) \) and \( n_T \) are the beam current (number of protons per time) and the total number of active target nuclei per unit area, respectively. As discussed in Section 4.2.1, the beam current \( I_p(t) \) was recorded with one minute intervals during the irradiation period, \( t_{irr} \). The total number of radioactive isotopes present in the target after the irradiation period, \( N_{cre} \), was calculated by the relation

\[ N_{cre} = \sum_i \frac{\sigma n_T I_{pi}}{\lambda} (1 - e^{-\lambda(60sec)})e^{-\lambda(t_{irr} - t_i)} \]  

(5.12)

where, \( I_{pi} \) is the constant beam current in the \( i^{th} \) one minute interval, \( t_i \) is the time at the end of the \( i^{th} \) one minute interval, and \( \sum_i \) is the sum over the one minute intervals. Thus, the cross section was calculated from

\[ \sigma = \frac{N_{cre}}{N_p n_T} \]  

(5.13)

where,

\[ N_p = \sum_i \frac{I_{pi}}{\lambda} (1 - e^{-\lambda(60sec)})e^{-\lambda(t_{irr} - t_i)}. \]  

(5.14)

The total number of active target nuclei per cm\(^2\), \( n_T \), was obtained as follows,

\[ n_T = \rho w \frac{N_A}{A} \times \frac{\text{purity}}{100} \]  

(5.15)

where, \( \rho \) is the density of the target element in g cm\(^{-3}\), \( w \) is the thickness of the target in cm, \( N_A \) is the Avagadro number, \( A \) is the effective atomic mass of target
in atomic mass units, and the purity is the atomic percent of the target isotope of interest.

The total number of nuclei created during the irradiation, $N_{cre}$, was obtained from the total number of decays in a counting interval, $N_{dec}$, between time $t_1$ and time $t_2$. The activities at time $t_1$ and time $t_2$ (see Figure 5.13) are given by

$$N(t_1) = N_0 e^{-\lambda t_1},$$

$$N(t_2) = N_0 e^{-\lambda t_2},$$

$$N_{dec} = N_0 (e^{-\lambda t_1} - e^{-\lambda t_2}).$$

Thus,

$$N_{cre} = N_0 - N_{left} = \frac{N_{dec} e^{\lambda t_1}}{1 - e^{-\lambda (t_2 - t_1)}} - N_{left}.$$
Here, \( N(t_1) \) and \( N(t_2) \) are the activities at time \( t_1 \) and time \( t_2 \), \( N_0 \) is the number of nuclei present at the end of the irradiation, \( t_{irr} \), and \( N_{efi} \) is the number of daughter nuclei that were left over from previous irradiations. The number of decays that occurred during the counting interval was calculated from the number of counts under the area of the each peak in the spectrum, \( N_{peak} \), and combined with the detector efficiency (\( \epsilon \)), the branching ratio (\( BR \)) and the acquisition system live time (\( LT \)) as

\[
N_{dec} = \frac{N_{peak}}{\epsilon LT BR}.
\]  

(5.18)

The cross section determined from each photopeak and time interval was combined using Equation 5.13 to get an overall cross section value for the \(^{102}\text{Pd}(p,\gamma)^{103}\text{Ag}\), \(^{116}\text{Sn}(p,\gamma)^{117}\text{Sb}\), and \(^{112}\text{Sn}(\alpha,\gamma)^{116}\text{Te}\) reactions.

The measurements of the cross sections were made in the laboratory coordinate system, where the target is at rest, but we are interested in analysing them relative to the center of mass of the projectile and target in the CM system as given by

\[
E_{CM} = \frac{M}{m + M} E_{lab}.
\]  

(5.19)

The difference between the center of mass energy (\( E_{CM} \)) and the laboratory energy (\( E_{lab} \)) is a factor of \( \frac{M}{m + M} \), where \( m \) and \( M \) are the mass of the projectile and the target, respectively.

The energy loss of the projectile in the target material was calculated by the program STOPX [ND University], assuming the reactions occurred at the center of the target, this was then the value of the energy used in the calculation of the
Proton Energy (MeV) & Energy Loss in $^{102}$Pd (keV) & Energy Loss in $^{116}$Sn (keV) \\ 
| 2.60 | 62.0 | 60.7 | 
| 2.70 | 60.6 | 58.0 | 
| 2.80 | 59.2 | 56.7 | 
| 3.00 | 56.7 | 54.4 | 
| 3.25 | 54.0 | 51.8 | 
| 3.50 | 51.5 | 49.5 | 
| 3.75 | 49.3 | 47.4 | 
| 4.00 | 47.3 | 45.6 | 
| 4.25 | 45.5 | 43.8 | 
| 4.75 | 42.3 | 40.9 | 
| 5.25 | 39.6 | 38.3 | 

Table 5.7: Proton energy losses in $^{102}$Pd and $^{116}$Sn targets

cross sections. Table 5.7 and 5.8 shows the proton and alpha energy losses in specific targets.

5.7 Errors

The various contributions to uncertainty in final cross section and S-factor values have been analyzed as follows. Firstly, there are the purely statistical contributions due to the nature of the counting statistics explained in Section 5.5. Next, uncertainties which would affect the individual cross section measurements, e.g., uncertainties in the proton exposure or in the placement of the target were considered. These were added in quadrature to get the purely statistical error bars and called run to
run error bars hereafter. Finally, an extra component to the error bars was due to sources such as the size of the detectors, and the target thickness. The level of these uncertainties are indicated in the Table 5.9. These were deduced using the Monte Carlo simulation; see Section 5.8. Error such as these would affect all the energy cross section measurement we make.

The value of systematic uncertainties throughout the experiment due to the geometric setup of the experiment, uncertainties in the $\gamma$-ray detection efficiency, uncertainties in branching ratios for the $\gamma$-rays, and uncertainties in the absorption of $\gamma$-rays in the aluminum and steel were determined and added in quadrature as in Equation 5.20

$$\Delta(f(x,y,z))^2 = \left(\frac{\delta f(x,y,z)}{\delta x} \Delta x\right)^2 + \left(\frac{\delta f(x,y,z)}{\delta y} \Delta y\right)^2 + \left(\frac{\delta f(x,y,z)}{\delta z} \Delta z\right)^2, \quad (5.20)$$

where $f$ is a function of $x, y, z...$ The weighted averages of the measured cross sections from different $\gamma$-lines of both detectors for each projectile energy were calculated by

$$\sigma(E) = \frac{\sum_i \sigma_i(E)[\Delta\sigma_i(E)]^{-2}}{\sum_i [\Delta\sigma_i(E)]^{-2}}, \quad (5.21)$$

<table>
<thead>
<tr>
<th>Alpha Energy (MeV)</th>
<th>Energy Loss in $^{112}$Sn (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.00</td>
<td>41.4</td>
</tr>
<tr>
<td>7.50</td>
<td>39.5</td>
</tr>
<tr>
<td>8.00</td>
<td>37.8</td>
</tr>
<tr>
<td>8.75</td>
<td>35.7</td>
</tr>
<tr>
<td>9.50</td>
<td>34.0</td>
</tr>
<tr>
<td>10.50</td>
<td>32.0</td>
</tr>
</tbody>
</table>

Table 5.8: Alpha energy losses in the $^{112}$Sn target
and the errors in the weighted average were

$$\Delta \sigma(E) = \left( \frac{1}{\sum_i [\Delta \sigma_i(E)]^{-2}} + \sum_i \left[ \frac{\sigma_i(E) - \sigma(E)}{i} \right]^2 \right)^{1/2}. \tag{5.22}$$

Error bars for the weighted averages of the cross section measurements include statistics and appropriate run-to-run uncertainties such as fluctuations in the proton exposure and uncertainties in the exact placement of the target between the detectors. An additional purely systematic error of 15%, due to uncertainties in the detector size and performance, is not shown. This is not shown because it shifts all the data points up or down simultaneously.

### 5.8 Monte Carlo Simulation

A Monte Carlo simulation of the experiment, including a detailed geometry of the setup, was performed and the efficiency normalized to the experimental measurements. In order to investigate the systematic uncertainties associated with the geometry, simulations were performed while varying the geometry from the default values. These variations were at a level equal to the estimated uncertainty, for example, 2 mm in the placement of the target. Around $10^7$ γ-rays originating from the target were then simulated, with a spherically isotropic angular distribution assumed, and the change in efficiency noted. In this way, the geometric parameters (such as target-to-detector distance, which resulted in the largest uncertainty) could be determined. A summary of these findings is given in Table 5.9.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \Delta \text{Eff1} ) (%)</th>
<th>( \Delta \text{Eff2} ) (%)</th>
<th>( \Delta(\text{Eff1+Eff2}) ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DETD</td>
<td>10.7459</td>
<td>10.4518</td>
<td>10.5988</td>
</tr>
<tr>
<td>DETL</td>
<td>3.4631</td>
<td>3.0075</td>
<td>3.2355</td>
</tr>
<tr>
<td>RSEP</td>
<td>14.7031</td>
<td>14.798</td>
<td>14.7507</td>
</tr>
<tr>
<td>TGTH</td>
<td>0.0173</td>
<td>0.0863</td>
<td>0.0345</td>
</tr>
<tr>
<td>DT1H</td>
<td>0.1176</td>
<td>0.3628</td>
<td>0.1231</td>
</tr>
<tr>
<td>DT2H</td>
<td>0.3438</td>
<td>0.4325</td>
<td>0.3882</td>
</tr>
<tr>
<td>TGTX</td>
<td>0.4238</td>
<td>0.6641</td>
<td>0.1196</td>
</tr>
<tr>
<td>DT1X</td>
<td>0.1780</td>
<td>0.6467</td>
<td>0.2333</td>
</tr>
<tr>
<td>DT2X</td>
<td>0.6119</td>
<td>0.6549</td>
<td>0.6335</td>
</tr>
<tr>
<td>ALCO</td>
<td>2.0975</td>
<td>2.9889</td>
<td>2.1182</td>
</tr>
<tr>
<td>TGTZ</td>
<td>14.8233</td>
<td>14.1995</td>
<td>0.3113</td>
</tr>
</tbody>
</table>

Table 5.9: Systematic sources of uncertainty for the 742 keV line. DETD is the detector diameter; DETL is the detector length; RSEP is the separation between detector and target; TGTH is the target height; DT1H is the detector 1 height; DT2H is the detector 2 height; TGTX is the target offset to the side; DT1X is the detector 1 offset to the side; DT2X is the detector 2 offset to the side; ALCO is the aluminum absorption efficiency; TGTZ is the target offset between detectors.

The simulations of the 511 keV \( \gamma \)-ray and the 511 keV coincidences were slightly different than other decays. This time, it was assumed that around \( 10^8 \) positrons were emitted randomly with energies depending on the \( \beta^+ \)-decay spectrum of the daughter nucleus. These then propagated until they hit either the aluminum plates or the lead dog-house. They then penetrated a depth appropriate to their energy. The final resting place for the \( \beta^+ \) was then used as the origin for two back-to-back 511 keV \( \gamma \)-rays. These were then tracked. In this way the coincidence efficiency, and its uncertainty, could be determined.
<table>
<thead>
<tr>
<th>γ Energy (keV)</th>
<th>Detector-1 Eff (%)</th>
<th>Detector-2 Eff (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>118.74</td>
<td>0.964</td>
<td>1.10</td>
</tr>
<tr>
<td>148.20</td>
<td>1.56</td>
<td>1.58</td>
</tr>
<tr>
<td>243.96</td>
<td>1.66</td>
<td>1.92</td>
</tr>
<tr>
<td>266.86</td>
<td>1.75</td>
<td>2.02</td>
</tr>
<tr>
<td>511</td>
<td>1.86</td>
<td>2.12</td>
</tr>
<tr>
<td>531.92</td>
<td>2.36</td>
<td>2.73</td>
</tr>
<tr>
<td>742.11</td>
<td>1.40</td>
<td>1.62</td>
</tr>
<tr>
<td>1007.08</td>
<td>1.01</td>
<td>1.16</td>
</tr>
<tr>
<td>1029.97</td>
<td>0.989</td>
<td>1.15</td>
</tr>
<tr>
<td>1155.27</td>
<td>0.934</td>
<td>1.07</td>
</tr>
<tr>
<td>1182.77</td>
<td>0.918</td>
<td>1.07</td>
</tr>
<tr>
<td>1273.83</td>
<td>0.901</td>
<td>1.04</td>
</tr>
<tr>
<td>511c</td>
<td>0.452</td>
<td>0.452</td>
</tr>
</tbody>
</table>

Table 5.10: Photopeak efficiency measurements of HPGe detectors for different γ-ray energy lines. 511c refers to 511 keV γ-ray coincidences in both detectors.

Table 5.9 shows a typical breakdown of the geometric contributions to the error bars for one of the γ-energy lines; the 742 keV line. Different parameters contributing uncertainties are listed in the first column.

Note that the biggest contributions are coming from parameters of the separation between the detector and the target (RSEP) and the target offset between detectors (TGTZ), which are essentially the same sources.
5.9 The Cross Section Results

5.9.1 $^{102}\text{Pd}(p,\gamma)^{103}\text{Ag}$ Reaction Cross Sections

As we expected cross section measurements deduced from $\gamma$-ray emissions below 511 keV were found to be inconsistent with those of higher energy $\gamma$-ray lines which were in agreement with each other as seen in Figure 5.14. This figure shows the ratio of cross section measurements of individual $\gamma$-energy lines for each detector and the average cross section of the individual lines above 511 keV.

The weighted averages of the cross sections as measured from 9 different $\gamma$-ray lines from 511 keV to 1274 keV, including 511 keV coincidences from annihilations, for both detectors are shown in Figure 5.15. The theoretical calculations using the NON-SMOKER code are shown by the solid lines, and the experimental results are given by the individual points with error bars. An additional purely systematic error-bar for the experimental points of $\sim$ 15% is not shown.

5.9.2 $^{116}\text{Sn}(p,\gamma)^{117}\text{Sb}$ Reaction Cross Sections

Clearly we had a problem with the lowest proton beam energy measurements. Namely, the cross section measurements for individual $\gamma$-ray lines are consistent with one another. Furthermore, the cross section values of 511 keV singles and coincidences are dramatically inconsistent with the excitation function trend. We
Figure 5.14: The ratio of individual gamma line cross sections and the average cross section at an incident proton energy of 2.6 MeV for detector 1 (filled squares) and detector 2 (filled circles).
Figure 5.15: Comparision of measurements of the $^{102}$Pd(p,$\gamma$)$^{103}$Ag reaction with NON-SMOKER cross section calculations. Experimental points are filled circles with error bars and the line shows the calculated values.
Figure 5.16: Comparision of measurements of the $^{116}\text{Sn}(p,\gamma)^{117}\text{Sb}$ reaction with NON-SMOKER cross section calculations. Experimental points are filled circles and the line shows the calculated values.
note that at this very low beam energy we had severe problems tuning the beam. Thus, one possible explanation is that the beam may have partly missed the target and hit the target holder producing an extra source of $\beta^+$-decays for this run. We analysed the half-lives of the events in the 511 keV peaks and observed them to have a half-life of $\sim 80$ min. Although we were unable to identify the exact source, this time constant suggests it would not have contributed to subsequent runs. Indeed measurements of the decay constant for other runs gave the expected result. Given these inconsistencies in this data point we have chosen not to include these data in further analysis.

5.9.3 $^{112}\text{Sn}(\alpha,\gamma)^{116}\text{Te}$ Reaction Cross Sections

The calculation to deduce the cross section from the 932 keV and 1294 keV $\gamma$-ray lines produced by the $\beta^+$-decay of the daughter nuclei ($^{116}\text{Sb}$) was different compared to the previous calculations where $\gamma$-rays were observed directly following the decay of the radiative capture reactions.

Let $N_{Te}(t)$ be the number of parent nuclei, which are $\beta^+$-decay unstable with a half-life of 2.49 hours at time $t$ producing $^{116}\text{Sb}$, and let $N_{Te}(0)$ be the number at $t = 0$, defined to be the time the irradiation began. $N_{Sb}(t)$ is the number of daughter nuclei, which are also $\beta^+$-decay unstable with a half-life of 15.8 minutes at time $t$ producing stable granddaughter nuclei $^{116}\text{Sn}$. The relevant decay constants are presented by $\lambda_{Te}$ and $\lambda_{Sb}$. If the irradiation lasts from $t=0$ to $t_o$ with cross-section $\sigma$ by a beam flux $I_o(t)$ (number of incident alpha particles per unit time), and $n_T$ is

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the number of target nuclei per unit surface, the variations in time $dt$ of the number of parent, daughter and granddaughter nuclei are given as

\begin{align}
\frac{dN_{Te}(t)}{dt} &= \sigma N_{Te}(t) I_{\alpha}(t) - \lambda_{Te} N_{Te}(t), \quad (5.23) \\
\frac{dN_{Sb}(t)}{dt} &= \lambda_{Te} N_{Te}(t) - \lambda_{Sb} N_{Sb}(t), \quad (5.24) \\
\frac{dN_{Sn}(t)}{dt} &= \lambda_{Sb} N_{Sb}(t). \quad (5.25)
\end{align}

During the irradiation ($0 < t < t_o$), the alpha beam current was known for each one minute interval. Assume that the irradiation period has $n$ one minute intervals, then by using the solutions of the differential equations above, the number of $^{116}$Te and $^{116}$Sb at the end of the irradiation can be found by numerically following the following relation for the $n^{th}$ one minute interval until the end of the irradiation

\begin{align}
N_{Te}(n) &= N_{Te}(n - 1)e^{-\lambda_{Te} \Delta t} + I_{\alpha}(n) \rho \sigma \frac{N_A}{A}, \\
N_{Sb}(n) &= N_{Sb}(n - 1)e^{-\lambda_{Sb} \Delta t} + \frac{\lambda_{Te}}{\lambda_{Sb} - \lambda_{Te}} N_{Te}(n - 1) \left( e^{-\lambda_{Te} \Delta t} - e^{-\lambda_{Sb} \Delta t} \right). \quad (5.27)
\end{align}

The last values generated thus give us the number of $^{116}$Te and $^{116}$Sb at the end of the irradiation in units of $\sigma$; $N_{Te}(t_o) = C_1 \sigma$ and $N_{Sb}(t_o) = C_2 \sigma$ where $C_1$ and $C_2$ are coefficients. After the irradiation, $^{116}$Te continues to $\beta^+$-decay producing $^{116}$Sb. However these $^{116}$Sb nuclei also $\beta^+$-decay producing $^{116}$Sn which give the $\gamma$-rays we detect in the interval $t_1$ to $t_2$. So, the number of decays of $^{116}$Sb to $^{116}$Sn between $t_1$ and $t_2$ gives the number of $^{116}$Sn nuclei created between $t_1$ and $t_2$ which can be calculated by $N_{peak}/(\epsilon LT BR)$, and the relationship is

\begin{align}
\frac{N_{peak}}{\epsilon LT BR} &= \int_{t_1}^{t_2} \lambda_{Sb} N_{Sb}(t) dt, \quad (5.28)
\end{align}

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Figure 5.17: Comparison of measurements of the $^{112}\text{Sn}(\alpha,\gamma)^{116}\text{Te}$ reaction with NON-SMOKER cross section calculations. Experimental points are filled circles and the line shows the calculated values.
where $N_{sb}(t)$ can be written in terms of $N_{Te}(t_o)$ and $N_{sb}(t_o)$ as

$$N_{sb}(t) = N_{sb}(t_o)e^{-\lambda_{sb}t} + \frac{\lambda_{Te}N_{Te}(t_o)}{\lambda_{sb} - \lambda_{Te}} \left( e^{-\lambda_{Te}t} - e^{-\lambda_{sb}t} \right). \quad (5.29)$$

The cross section value can be found since everything is known except $\sigma$ in these equations. The weighted averages of the cross sections as measured from relevant $\gamma$-ray lines for both detectors are presented in Figure 5.17.

5.10 Astrophysical S-factor Results

The astrophysical S-factors and the Gamow window were described in Section 3.1.1. The astrophysical S-factor values were deduced from cross section measurements and compared with the results of the NON-SMOKER code in this section. The astrophysical S-factor is defined by

$$S(E) = \sigma(E)E[exp(2\pi\eta)] \quad (5.30)$$

where $\eta = Z_1Z_2e^2/\hbar v$ is the Sommerfeld parameter, $Z_1$ and $Z_2$ are the proton numbers of the interacting particles and $v$ is their relative velocity. We are particularly interested in the astrophysical S-factor values within the Gamow window. The calculated Gamow peak values, $E_0$, were shown in Table 5.11 for relevant temperatures.
Table 5.11: Gamow peak ($E_o$) and width values ($\Delta$) in MeV at selected temperatures for relevant reactions.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$T$ (in $10^9$ K)</th>
<th>$E_o$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{102}$Pd($p,\gamma$)$^{103}$Ag</td>
<td>0.5</td>
<td>0.98</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.56</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>2.48</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>3.25</td>
<td>2.12</td>
</tr>
<tr>
<td>$^{116}$Sn($p,\gamma$)$^{117}$Sb</td>
<td>0.5</td>
<td>1.04</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.65</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>2.62</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>3.43</td>
<td>2.85</td>
</tr>
<tr>
<td>$^{112}$Sn($\alpha,\gamma$)$^{116}$Te</td>
<td>0.5</td>
<td>2.60</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>4.12</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>6.55</td>
<td>2.45</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>8.58</td>
<td>3.44</td>
</tr>
</tbody>
</table>
Figure 5.18: The measured and calculated astrophysical $S$-factors of the $^{102}\text{Pd}(p,\gamma)^{103}\text{Ag}$ reaction.
Figure 5.19: The measured and calculated astrophysical $S$-factors for the $^{116}$Sn(p,γ)$^{117}$Sb reaction.
Figure 5.20: The measured and calculated astrophysical $S$-factors of the $^{112}$Sn($\alpha,\gamma$)$^{116}$Te reaction.
CHAPTER 6
CONCLUSION

The measured data, obtained using an activation technique, extend the experimental knowledge of the cross sections and the astrophysical S-factors in the mass region beyond $A=100$. It should be noted that our measurements on all targets were extended to energies down to well inside the Gamow window for the explosive environments as shown in Section 5.10.

We have found cross sections which are generally significantly higher than the predictions of the NON-SMOKER statistical model code, especially at the lower energies at which we measured where the differences were around a factor of three or more. One possibility of this might be an incorrect description of the density of states. Such differences are not too worrying given that the width and density of levels probed here are not as high as one would expect in the nuclei far from stability where the rp- and $\gamma$-processes operate, and hence where one might expect the statistical model approach to perform better. However, further measurements on heavy nuclei are warranted to determine if a modification to the NON-SMOKER code is needed.
It was shown that the activation technique was a successful method for measuring the cross sections of radiative capture reactions. So, it can be used to study \((p, \gamma)\) reactions for other nuclei especially those of higher masses. It is limited, however, to targets that produced reaction product nuclei with half-lives. In addition, finding isotopically pure targets on the proton rich-side of valley of stability is difficult since they have low abundances. We have also shown that this technique works well for \((\alpha, \gamma)\) reactions, although in general more reaction channels are likely to contribute to the observed decays because of the higher excitation energy needed to overcome the Coulomb barrier of the desired reaction.

Other improvements to future experiments would be to consider ways of improving the efficiency calibration of the detectors. Our desire to achieve a high efficiency led to a very close geometric setup. In turn, this meant high dead-times and consequent poor resolution with the relatively high activity of the calibration sources. Furthermore, the close geometry made exact placement of the irradiated target a major source of systematic uncertainty. Lastly, more isotopes may be possible targets for this technique if a quicker (automated) change over from irradiation to counting could be achieved.
Appendix A

Decay Schemes

Figure A.1: Part 1 of the decay scheme of $^{103}$Ag. All energies are given in keV, level energies are shown in boldface type [Firestone et al. (1996)].
Figure A.2: Decay Scheme of $^{117}$Sb [Firestone et al. (1996)].
Figure A.3: Decay Scheme of $^{116}$Te [Firestone et al. (1996)].
BIBLIOGRAPHY


[ND University] “ND Nuclear Structure Laboratory Facilities”

[ND Website] “ND Nuclear Structure Laboratory” http://www.nd.edu/~nsl/.


