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EXPERIMENTS WITH FLOWING SOAP FILMS: INTERMITTENCY AND STRUCTURE IN TWO-DIMENSIONAL TURBULENCE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

* * * * *

By

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ABSTRACT

Turbulence is one of the great unsolved problems of physics. One of the few clean results, known as Kolmogorov's 4/5th law, proposes an exact relation between the third moment of the velocity difference and the distance between the two points at which the velocities are measured. An extension of this law predicts the scaling behavior of the higher order moments as well. Experiments, however, have measured unquestionable deviations. Extended filaments of vorticity, their strength magnified by the vortex-stretching term of the Navier-Stokes equation, have often been deemed the culprit. Since Kolmogorov's theory does not account for these intermittent structures, their existence brings one of the few rigorous results into question.

In two dimensions (2D) things should be simpler. These extended vortex filaments (a strictly 3D phenomena) are wholly absent, implying that the 2D equivalent to the 4/5ths law ought to be rigorously correct. Nonetheless, controversy has arisen over whether intermittency is in fact observed in 2D as well. This matter is studied in detail using a relatively new experimental technique which employs rapidly flowing soap membranes as a model 2D fluid. Surprisingly, our measurements indicate that under certain circumstances intermittency appears as prominently in 2D as it does in 3D.
Since the vortex tubes deemed responsible for intermittency in 3D cannot exist in 2D, however, a second fundamental issue comes into question: what flow structures are responsible for intermittency in 2D turbulence? By decomposing the flow into a set of distinct topological structures, longitudinal intermittency, as well as energy and enstrophy transfer, is found to be the result of saddle-like regions in the flow field and not the vortex centers historically associated with intermittency.

Finally, the extent to which the soap film deviates from an ideal classical 2D fluid is quantified. Specific areas of investigation include: air drag, thickness fluctuations, film compressibility, and thickness dependent viscosity effects. None of these are found to contribute significantly to the intermittency.
For my family
I would like to thank Fernand Hayot and Ciriyam Jayaprakash for their stimulating discussions over the course of this work; David Andereck for the use of the laser Doppler velocimetry system (LDV); John Harrison for his assistance in collecting some of the LDV data sets; J. D. Wear for his gracious computing assistance; The Ohio State University and the Petroleum Research Foundation for their financial support; and Maarten Rutgers for his always insightful, always candid leadership.
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FIELDS OF STUDY

Major Field: Physics
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A significant body of literature has accumulated over the last two decades on 2D turbulence, both as a result of its relevance to large scale atmospheric weather phenomena (Fig. 1.1), and because of the relative computational feasibility of simulations in 2D as compared to those in 3D. However, even in the lesser dimension, the intricate complexity that is the hallmark of turbulence, is still in ample abundance. The investigation of turbulence, therefore, generally proceeds within a statistical framework. Additionally, a number of simplifying assumptions are often made in turbulence theory, the most basic being that the flow is spatially homogeneous and isotropic (that is, displays a translational and rotational symmetry). Kolmogorov, in his 1941 work on turbulence, made the further assumption that the structures in a turbulent flow have a characteristic appearance which is independent of their scale. Under this assumption he arrived at the conclusion that the moments of the difference in velocity between two points in the flow should scale with an exponent proportional to their order, \( \langle \delta v^p \rangle \propto r^p \). However, empirical evidence eventually showed that there were im-
important deviations from these expected scaling laws in 3D turbulence (see Anselmet, *et al.* [2] for example). Such deviations were attributed to the presence of coherent vortical tubes (i.e., existing much longer than the characteristic time-scale of the background turbulent dynamics).

There are a couple of significant differences between two- and three-dimensional turbulence. The vortex stretching term in the governing equations, the very term that creates these thin tornadic filaments, is conspicuously absent in 2D. As a result, not only must the energy in the system be conserved by the turbulent dynamics, but also each of the even order moments of the vorticity, most notably, the mean square vorticity, or enstrophy, must be conserved. Kraichnan, therefore, postulated that energy and enstrophy would be transferred between eddies of different scales through two separate cascades [3], one transferring energy from the scale at which it is injected toward larger scales, the other transferring enstrophy toward smaller scales.

These differences, between turbulence in 2D and 3D, have left room for an ongoing debate concerning the existence of intermittency in 2D and the mechanism by which intermittent structures might form. Indeed, experiments on magnetohydrodynamically driven turbulence in a thin stratified layer showed no sign of intermittency in either the individual energy or enstrophy cascades [4, 5, 6]. However, 2D numerical simulations have repeatedly found intermittency in temporally evolving flows, both at long times in freely decaying turbulence [7, 8, 9, 10, 11, 12, 13, 14], and when finite-size effects lead to a condensation of energy in the largest eddies [15, 5].
Figure 1.1: Atmospheric von Karman vortex street trailing (a) Guadalupe Island, Mexico (image credit: NASA/GSFC/JPL, MISR Team, and is available at http://nix.nasa.gov), and (b) Juan Fernandez Islands, Mexico (taken from LandSat 7, and available at http://climate.gsfc.nasa.gov/ cahalan/KarmanVortices.html).
In the latter case, there is a corresponding formation of coherent structures at much smaller scales [16, 17]. Intermittency has also been observed in steady-state simulations when both the energy and enstrophy cascades are present [18, 19]. It is the experimental corollary of this final situation which will be reported in this thesis. This investigation encompassed not only the statistics of a variety of 2D turbulent flows but also the various structures present within such flows. Specific types of structures were found to be responsible for intermittency. Furthermore, the various structures played differing roles in the longitudinal and transverse statistics.

The investigation was carried out using soap films as a platform for studying two-dimensional fluid dynamics. These films have a spatial extent in two dimensions which is roughly five orders of magnitude larger than in the third. The result is that, to an excellent approximation, the turbulent dynamics are constrained to interact in a plane. There are, however, important ways in which the dynamics of these thin films may deviate from classical 2D fluid mechanics. There is, first of all, a coupling of the turbulent dynamics within the film to the surrounding air. Such a coupling has been approximated by a linear (or nearly linearly) drag term which removes energy at the largest length scales [20]. Spatial variations in the film thickness may also lead to fluctuations in the local mass density, or if dynamic, to an effective compressibility of the 2D fluid. Furthermore, the preference of the surfactant to concentrate at the surfaces leads to an effective film viscosity which is dependent on the thickness. These complications, specific to the use of soap membranes, will be investigated to determine
what impact, if any, they have on 2D intermittency.

1.1 TURBULENCE THEORY

1.1.1 THE GOVERNING EQUATIONS

There is essentially one equation which governs all incompressible fluid flow. Since this equation is of such importance to the field, and since it can be done with some brevity, I will supply a short derivation following Feynman [21]. The equation of motion of a fluid element consists of three terms resulting from acceleration, pressure, and viscosity. The pressure term can be derived by considering a fluid element with spatial dimensions $\Delta x$, $\Delta y$, and $\Delta z$. Considering only the $x$ direction at first: if the pressure on the left side of the element is given by $p$, then the pressure on the right is given by $p(x + \Delta x) = -(p + \partial p/\partial x \Delta x)$. The net force on the element along the $x$-axis is thus $-(\partial p/\partial x \Delta x) \Delta y \Delta z$. The other directions follow from similar considerations resulting in a pressure term of the form, $-(\partial p/\partial x \hat{i} + \partial p/\partial y \hat{j} + \partial p/\partial z \hat{k}) \Delta V = -\nabla p \Delta V$, where $\Delta V$ is the element volume.

Setting this equal to the element mass times its acceleration, we have the most basic form of the equation. However, simply setting $ma = m \partial v/\partial t$ is incorrect, as it gives us the rate of change of the velocity at a given spatial location. What is needed of course, is the change in the velocity of the fluid element (which has moved). It is
initially moving at $v(x, y, z, t)$, and at time $\Delta t$ later at

$$v(x + \Delta x, y + \Delta y, z + \Delta z, t + \Delta t) =$$

$$= v(x + v_x \Delta t, y + v_y \Delta t, z + v_z \Delta t, t + \Delta t)$$

$$= v(x, y, z, t) + \left( \frac{\partial v}{\partial x} \right) v_x \Delta t + \left( \frac{\partial v}{\partial y} \right) v_y \Delta t + \left( \frac{\partial v}{\partial z} \right) v_z \Delta t + \left( \frac{\partial v}{\partial t} \right) \Delta t. \tag{1.1}$$

The acceleration is then given by $\Delta v/\Delta t$, or in vector notation by

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v. \tag{1.2}$$

Setting the acceleration and pressure terms equal and dividing each by the element volume, we arrive at the equation Euler derived in 1755,

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{1}{\rho} \nabla p. \tag{1.3}$$

Though most of us intuitively associate turbulence with viscous flows, it turns out that the above equation, in which the viscous term is conspicuously absent, is generally quite adequate. The full Navier-Stokes (N-S) equation, rigorously derived about 90 years later,

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{1}{\rho} \nabla p + \nu \nabla^2 v. \tag{1.4}$$

is not necessary within some limits (to be discussed shortly).

First, however, it will be helpful to gain some understanding of how the problem of turbulence is approached. Working even with the inviscid equation above, is anything
but trivial. It has been almost 250 years since this result was achieved. During that
time, reasonably comprehensive theories about E&M, general relativity, quantum me­
chanics, quantum electrodynamics, etc., have been developed and thoroughly tested.
Yet very few rigorous results exist for turbulence. Fluid mechanics, as a classical field
theory, runs into a number of difficulties in the turbulent limit [22]. First, it doesn’t
respond well to perturbative expansions because the Reynolds number, as the relevant
expansion parameter, is much greater than one for turbulent flows. Second, there are
nonlinear interactions in the system which couple the motion of fluid elements, both
on nearby and vastly different scales. And finally, in the incompressible limit, the
sound speed is infinite, implying that the pressure at any given location is determined
by the velocity field everywhere. That is, in r- as well as k-space, the equation is
highly non-local.

One might thus wonder how any theoretical progress can be made since, beyond
the limit of very slow flows with simple geometries, analytic solutions for the actual
velocity field are impossible. Some insight can be gained by analogy with other
complex systems. For example, when we wish to solve a thermodynamics problem,
we set off, not by writing down the initial states and interactions of the $\sim 10^{23}$
particles in the system, we instead begin by assuming that, precisely because of the
system’s size and complexity, averaging over enough particles or for a long enough
period of time will remove any trace of these initial conditions and give an adequate
representation of any given particle’s likely behaviour. The same approach can be
used when considering the infinitude of interacting fluid elements in a fully turbulent flow. Instead of looking for exact solutions, one can make predictions about average flow characteristics, and the magnitude of fluctuations about these mean values.

Under this guise, one may explore the region of applicability of Euler's equation with respect to turbulent flows. Since the key ingredient missing is the viscosity, it is reasonable that the equation's applicability be related to the relative magnitude of the viscous dissipation term. A cursory examination suggests that the magnitude is given by $\nu U/L^2$, where $U$ and $L$ are the pertinent velocity and length scales. Comparing this with the inertial term $((\mathbf{v} \cdot \nabla) \mathbf{v} \sim U^2/L)$, we find that the ratio of the inertial to the viscous term is of order $Re \equiv LU/\nu$, where $Re$ is known as the Reynolds number [23]. In general, when viscous forces are dominant (that is, $Re \ll 1$), any fluctuations or disturbances in the flow are quickly dampened. However, when inertial forces dominate ($Re \gg 1$), little dampening occurs and the flow may be quite turbulent.

This approach can be applied not only to the turbulent flow as a whole but to the individual eddies of which a turbulent flow is composed. A particularly good explanation is given by Landau and Lifshitz [24] from which I will summarize a few of the most relevant points. In fully developed turbulent flows (in which we are specifically interested), a continuous spectrum of eddy sizes exists, from the system size $l$ (also known as the external scale) all the way down to the smallest scales present (which we will call $\lambda_0$). Furthermore, each eddy of size $\lambda$ has some characteristic velocity $v_\lambda$ associated with it. At the largest scales (i.e. $\lambda \sim l$), this velocity is
comparable to the average magnitude of the fluctuations in the flow velocity ($\Delta v$).

However, as the eddy scale decreases so must the magnitude of the associated velocity fluctuations so that when $\lambda = 0$, $v_\lambda = 0$. As a result, for large scale eddies both $\lambda$ and $v_\lambda$ are comparably large resulting in a large eddy Reynolds number. As the size of the eddies decreases so, too, does the corresponding eddy Reynolds’s number, $Re_\lambda$. This implies that viscous effects are only important at the smallest scales, that is, when $Re_\lambda \sim 1$ and below.

1.1.2 ENERGY AND ENSTROPHY CASCADES

An interesting implication of the absence of a viscous term through a large portion of the turbulent scales is that, without such a term, energy cannot be dissipated by the larger eddies. In fact, it can only be removed at the smallest scales. This suggests that some form of cascade must take place to transfer energy from the injection scale to the scale at which it can be dissipated. Because this cascade actually starts with the largest eddies, the energy transfer rate per unit mass $\epsilon$ can only depend on quantities relevant at those scales (e.g., $\Delta v$, $l$, and the fluid density $\rho$). The only combination of these variables with the correct dimensions is $\epsilon \sim (\Delta v)^3/l$. We can use this quantity (again with dimensional arguments) to define the inner scale of the flow, that is, where the viscosity becomes important, as $\lambda_0 \sim (\nu^3/\epsilon)^{1/4}$, or substituting $\epsilon$, $\lambda_0 \sim l/Re^{3/4}$ (which we estimate to be about about 0.1 mm). For scales much larger than this, the corresponding Reynolds number is large or, equivalently, the viscosity can effectively
be taken to be very small, suggesting that, in this regime, Euler's equation is an 
adequate representation of the turbulence.

Similarly, dimensional arguments can be used to obtain the energy spectrum $E(k)$ 
as a function of wavenumber $k$ in the inertial range (that is, the range where viscous 
dissipation is unimportant). Kolmogorov's assumption that this quantity depends 
only on the wavenumber and energy transfer rate gives [3] $E(k) \sim \varepsilon^{2/3}k^{-5/3}$. We 
can verify, albeit by slightly hand-waving arguments, that the previously suggested 
direction of this cascade (i.e., toward smaller scales) is correct by examining the rate 
of change of the average kinetic energy per unit mass of the flow. This can be derived 
from the Navier-Stokes equation by taking the inner product with the velocity. The 
result is [25]

$$\frac{1}{2} \frac{d\langle v_i^2 \rangle}{dt} = \nu \langle \nabla^2 v_i \rangle = -\nu \langle \omega^2 \rangle,$$

where $\omega = \nabla \times v$ is the vorticity and $i$ denotes the velocity component. This has 
an interesting implication. If we rewrite the N-S equation in terms of vorticity (by 
taking the curl), we find $(\partial \omega/\partial t) = \nabla \times (v \times \omega) + \nu \nabla^2 \omega$. Using a well-known vector 
identity, we can rewrite the term involving both the velocity and vorticity as

$$\nabla \times (v \times \omega) = (\omega \cdot \nabla)v - (v \cdot \nabla)\omega + v(\nabla \cdot \omega) - \omega(\nabla \cdot v).$$

Working from the right, the fourth term disappears as a result of incompressibility; 
the third disappears since, in general, $\nabla \cdot (\nabla \times a) = 0$. The second term is present 
in both two and three dimensions. The first term, however, drops out in two, since
the only nonzero component of vorticity is \( \omega_z \), while \( \mathbf{v} \) is independent of \( z \). It is this first term that, in 3D, leads to an increase in the velocity gradients in regions where the vorticity is already high. This results in vortex stretching, or magnification of the vorticity. As a result, the enstrophy, given by \( \langle \omega^2 \rangle \), diverges as \( \nu \to 0 \), keeping \( -\nu \langle \omega^2 \rangle \) finite. That is, energy can still be effectively dissipated at small scales in a 3D flow even when \( Re \gg 1 \).

However, as this term is absent in the 2D N-S equation the enstrophy is conserved, eliminating any divergence as the viscosity goes to zero. Therefore, in the large Reynolds number limit of 2D turbulence, no energy can actually be dissipated by the viscous term in the Navier-Stokes equation. This suggests that the energy cascade in 2D must be in the opposite direction, away from the dissipation scale and, instead, toward smaller wavenumbers.

On the other hand, the rate of change of the average enstrophy is given by [25]

\[
\frac{1}{2} \frac{d\langle \omega^2 \rangle}{dt} = \nu \langle \omega \nabla^2 \omega \rangle = -\nu \langle (\partial \omega / \partial x_i)^2 \rangle. \tag{1.7}
\]

If we assume that material lines can be extended in 2D turbulence, a hypothesis which is empirically quite reasonable, then it is quite plausible that \( \langle (\partial \omega / \partial x_i)^2 \rangle \) diverges as \( \nu \to 0 \). This implies that enstrophy can be dissipated at small scales, even in the large Reynolds number limit. The corresponding cascade is, therefore, toward large \( k \).

In the inertial range we can again use dimensional analysis to obtain the form of the squared-vorticity, or enstrophy cascade as [3], \( E(k) \sim \eta^{2/3} k^{-3} \). Here the enstrophy transfer rate per unit mass \( \eta \) replaces \( \epsilon \).
The picture that thus emerges is that in 2D, energy and enstrophy are injected about some wavenumber $k_i$. The two then cascade away from this point with energy going to larger scales and enstrophy to smaller. Enstrophy is pulled out of the system by viscous dissipation at the smallest scales. Energy, on the other hand, may be extracted from the largest scales through the addition of a velocity-dependent drag term (e.g. a term that experimentally would couple the flow to some external medium). In simulations where such a term is not present, energy piles up in the lowest mode available to it in a process that has been likened to Bose condensation.

1.1.3 VELOCITY CORRELATIONS

The correlation between velocities at different scales in the flow is of central importance to our study of turbulence. The separation between such points provides a variable length scale with which the structure of the flow can be probed. For example, dimensional analysis can be used to quickly tell us something about how the velocity fluctuations vary with length scale [24]. For a distance of order $\lambda$ in the energy cascade (that is, $\lambda \gg l_i$, with $l_i$ the injection scale corresponding to $k_i/(2\pi)$), the fluctuations scale in general as $v_\lambda^p \sim (\epsilon \lambda)^{p/3}$, where $p$ is an integer denoting the moment order. Similarly, for the enstrophy cascade ($\lambda \ll l_i$), we find $v_\lambda^p \sim (\eta^{1/3} \lambda)^p$.

A rigorous derivation can be made only in the case of the cube of the velocity difference between two points,

$$\delta \mathbf{v}(\mathbf{r}) = \mathbf{v}(\mathbf{x} + \mathbf{r}) - \mathbf{v}(\mathbf{x}), \quad (1.8)$$
where \( x \) is the spatial location and \( r \) the separation. The resulting \textit{longitudinal moment}, or structure function, of order 3 is given in the energy cascade of 2D turbulence by [26]

\[
\langle (\delta v(r))^3 \rangle = \frac{3}{2} \epsilon r, \tag{1.9}
\]

where \( \parallel \) implies that the relevant velocity component and separation are parallel. A similar relation holds in the enstrophy cascade as well, and, in agreement with our dimensional arguments, is given by [26]

\[
\langle (\delta v(r))^3 \rangle = \frac{1}{8} \eta r^3. \tag{1.10}
\]

Both results are independent of the presence of external drag on the system (for example, from air), although such a dissipative term will potentially limit the extent of the inertial range at large scales.

In the future, unless specifically noted otherwise, we will be referring to the longitudinal structure functions and will, in general, denote them by \( S_p(r) = \langle (\delta v(r))^p \rangle \).

It is also possible, however, to imagine the \textit{transverse} case, where the separation and velocity are taken perpendicular. The transverse moments and velocity differences will, in that case, be denoted by \( S^T_p(r) \) and \( \delta v_T^p(r) \) respectively.

### 1.1.4 Intermittency

A further prediction, that the moments of higher order will, in general, scale according to \( S_p(r) \propto r^{\zeta_p} \) with \( \zeta_p = p/3 \) in the energy cascade and \( \zeta_p = p \) in the enstrophy
cascade, is arrived at if one assumes that the structures in the flow field have a characteristic appearance that is independent of scale. *Intermittency* then refers to flows in which this assumption is invalidated by the sparsely distributed appearance of small scale structures which are unusually strong for their size and may exist relatively unchanged for times much longer than the characteristic scale of the background dynamics. It is generally agreed upon that such structures exist in 3D (for a lengthy discussion, see Frisch [27]). Evidence from both experiments and simulations suggests the existence of abnormally strong small scale structures which cause the exponents $\zeta_p$ to fall below those predicted under the assumption of self-similarity (see, for example, Refs. [28, 2, 29, 30, 31, 32, 33]). Long concentrated filaments of vorticity (Fig. 1.2) are the most often-cited cause.

Perhaps the best theoretical explanation available at this point is that these coherent structures lead to an intermittency in the energy dissipation field. The exponents of the moments of such a field have been constructed in such a way that they are independent of any adjustable parameters [34]. The corresponding velocity-difference exponents, given by $\zeta_p = p/9 + 2[1 - (2/3)^{p/3}]$, are in excellent agreement with the bulk of the available data. The applicability of this idea to 2D flows, however, is not immediately obvious. The vortex-stretching term which creates these structures is absent in 2D and, as a result, it is not clear that there should, in general, be corresponding intermittency corrections.

Indeed, theoretical arguments presented for both the individual energy [35] and
Figure 1.2: Spontaneous vortex filament observed in three-dimensional turbulence. The tank was backlit with diffusive light and the flow seeded with small bubbles that preferentially concentrate in low pressure regions at the centers of vortices [31].
enstrophy [36] cascades of *steady-state* 2D turbulence suggest that no intermittency is possible (steady-state systems are those for which drag or dissipation term(s) remove energy at one or both ends of the spectrum at the rate at which it is input). This is in agreement with experiments done by Paret *et al.* [5, 6] in stratified magnetohydro-dynamically driven 2D flows.

There are three configurations, however, in which intermittency corrections have been seen in 2D numerical simulations. If an initially turbulent flow is allowed to decay without additional forcing, coherent vortices appear in the flow after a sufficiently long time. It is these coherent structures that are thought to lead both to deviations of the scaling exponents from the self-similar prediction and to a steepening of the energy spectrum [37, 7, 8, 13, 14]. The process for strong vortex formation, in this case, is likely the successive merger of weaker vortices of like sign. However, we will show in a later chapter that these vortices alone are not directly capable of accounting for the intermittency found in longitudinal statistics.

Deviations from the self-similar prediction have also been seen as a finite-size effect. In such non-steady-state simulations, energy is injected at one scale and the cascade of that energy followed to larger and larger scales [16, 17]. No mechanism for extracting energy at large scales is typically included in the simulation (e.g. a velocity-dependent drag term), and, as such, when the cascade of energy reaches scales on the order of the box size, it begins to accumulate in this lowest mode, creating a particular scale of eddy containing an anomalously large amount of energy. These very large vortices
then interact through a mechanism which is not yet well understood to create strong intermittent structures at much smaller scales.

Finally, intermittency has been seen to varying degrees in simulations when both cascades are simultaneously present in the energy spectrum (see Refs. [17, 18, 19]). The current thesis will report both on steady-state and decaying turbulence for two different magnitudes of external drag.

1.2 EXPERIMENTAL GEOMETRIES

1.2.1 APPARATUS

The medium used in this investigation of 2D intermittency was the soap film. The notion of using soap films to study 2D flows was first explored by Couder [38, 39, 40]. Fluid motion was initiated in these first experiments by dragging an obstacle through a stationary horizontal film (e.g., a small cylinder to study vortex shedding, or an array of cylindrical teeth to study turbulence). However, the static nature of the soap film allowed only for the study of decaying systems, and not those in which a steady-state of energy transfer was achieved. Shortly thereafter, Gharib and Derango[41] constructed a horizontal apparatus in which the soap film itself flowed past a static obstacle. This allowed for the study of steady-state turbulence. However, the intensity of the turbulence was limited by the small velocity of the film (roughly 30 cm/s).
Kellay, Wu, and Goldburg [42] then designed an apparatus in which the soap film was oriented vertically and driven by gravity, allowing for fluid velocities which were effectively limited only by the asymmetric wave speed of the film (on the order of several hundred cm/s). The apparatus in current use differs in a number of details but follows roughly the design of this last system.

**Geometry**

The basic elements of the apparatus and soap film geometry are shown in Fig. 1.3. The soap solution (roughly 1.5\% by volume of Dawn liquid detergent in purified water) starts its journey in the upper reservoir (a) before passing through a short section of silicone tubing (b) on its way to the injection nozzle (d). Flow is regulated by a valve (c) which varies the cross-sectional area of the supply tube.

The end of the tube is then simply capped with a syringe needle, the tip of which is placed high along the interior of the nozzle. Solution then runs smoothly down the wall. This eliminates an earlier problem with bubble formation when the end of the tube itself was pressed as tightly as possible into the nozzle. The seal, in this case, was never perfect, allowing air to enter the fluid path and form bubbles in the base of the nozzle (particularly at low flow rates), leading to an almost periodic oscillation of the mean flow velocity.

Two separate pressure heads are then involved in determining the solution flow rate. A pressure drop of $\Delta P_R$ between the water level in the upper reservoir and the

18
Figure 1.3: Schematic diagram of current soap film apparatus, including: (a) upper reservoir, (b) supply tube, (c) computer-controlled valve, (d) injection nozzle, (e) main guide wires, (f) side pull lines, (g) tensioning weight or spring, and (h) lower reservoir. [43]
valve determines the rate of flow through this resistance point. As the solution then flows into the nozzle itself, it forms a second pressure head which increases until $\Delta P_N$ balances the viscous resistance in the nozzle tip.

Solution is then injected into the expansion section of the apparatus (I), where the film spreads between two nylon guide wires (e). We have found no limit to the actual size of the soap film that can be generated with this geometry (as much as 100 feet tall has been done). However, the aspect ratio of the expansion section (height to final width) must be kept large so that surface tension and Laplace pressure forces have ample opportunity to smooth and spread initial thickness variations in the film. If the aspect ratio is too small, a thick jet of fluid is left in the center of the film. In the present experiment, the expansion section is 66 cm long with a maximum width of between 6 and 8 cm.

The side pull lines (f) (which allow for control of the film width) mark the beginning of the test section (II). In this 100 cm region, the guide wires remain essentially parallel (except for a very slight curvature due to the surface tension of the film). All experiments take place in this section and, in general, consist of some obstacle(s) being placed near the top of the section to generate a turbulent flow. Measurements can then be made to characterize various aspects of the flow in the remainder of the test section.

The film width is contracted in the remaining height (III), so that the two guide wires meet at the lowest point in the channel. Early versions of the apparatus relied
on a small weight (g) at this point to keep tension on the wires. The present apparatus is tensioned by springs instead. In either case, the solution is collected at this point in a lower basin (h).

A number of details have been added to the apparatus to improve various aspects of experiment control and film longevity. A diagram of the present system is shown in Fig. 1.4 and is to scale except for the vertical distance from injection nozzle to pull wires (it has been shortened significantly here to conserve space). Reservoirs are visible at both the top and bottom of the frame (constructed of 1 inch square aluminum extrusion from 80/20 Inc.). The solution is recirculated between them using a magnetically coupled pump (Cole Parmer Inc.). Recirculating the solution has a number of advantages:

- it conserves potentially costly tracer particles introduced into the solution for flow tracking;

- it allows for runs of extended length with an essentially unlimited supply of solution; and,

- it maintains a constant pressure head $\Delta P_R$ to eliminate changes in the mean velocity associated with a reduction in the amount of solution in the upper reservoir.

To avoid contaminating the solution during the recirculation process, all parts of the pump that are wetted are either stainless steel or Teflon. As well, the recirculation
Figure 1.4: Side view of vertical soap film tower.
and overflow tubing is Teflon lined. Additionally, both the pull wires and valve are computer-controlled for accuracy and repeatability.

**Vacuum Cylinder**

The entire soap film tower is further encased in a cylindrical acrylic chamber 8 feet high and 14 inches in diameter. By varying the pressure within the chamber, the effect of the coupling of the turbulent dynamics to the surrounding air can be studied. The chamber is constructed of three separate sections of 0.25” thick acrylic (from Dayton Plastics), the ends of which are protected by rubber gaskets with an L-shaped profile (Kurt J. Lesker, LG14B). As a slippery gasket can easily be sucked into the vacuum, small guards run around the inside of the cylinders just above each of the gaskets to keep solution from collecting around the seal when the film breaks. The sections are then separated by 5/16” thick aluminum rings with a square outer perimeter of side 16”. In addition to a central 13.5” diameter hole, an array of 1/4”-20 thru holes is drilled in each corner (Fig. 1.5). Steel rods can then be run between the rings to allow the cylinders to be temporarily lifted by winch, or removed altogether.

The necessary thickness of the cylinder walls can be calculated from the bulk modulus of the acrylic [44],

\[
\sigma_\theta = -\frac{r_o^2\Delta P}{r_o^2 - r_i^2} \left(\frac{r_i^2}{r_o^2} - 1\right),
\]

where \(r_i\) and \(r_o\) are the inner and outer radii of the cylinder respectively, and \(\Delta P\) is the pressure drop across the cylinder wall. Using 1/4” acrylic, the requisite circumferential
stress, $\sigma_\theta$, is surpassed by a factor of 10. This allows a safety margin for variations in the material properties and manufacturing process. As a secondary precaution, the entirety of the cylinder is enclosed by a 1/2" mesh of 12 gauge steel.

As gravity and the vacuum itself are essentially all that seals the chamber, some air leakage into the system is not surprising. In fact, a fairly large throughput Stokes pump (model 212H-11) is necessary to outpace the leaks initially (if this is not adequate, a few hundred pounds of lead bricks provides sufficient assistance). Even after a reasonable vacuum has been achieved and the air pressure itself is sufficient to significantly reduce the amount of air flow into the chamber, a regulator is necessary to maintain a constant air pressure to within a fraction of a Torr over the length of a data run. This regulator sits in parallel with a large through-put valve to quickly pump down the system initially. Feed-throughs in the aluminum base of the apparatus allow cables controlling the stepper motors and recirculation pump to pass from ambient pressure to vacuum. Film width and valve settings are controlled by software written in LabVIEW.

Velocity data can be collected by either laser Doppler velocimetry (LDV) or particle imaging velocimetry (PIV) (for more information on these data acquisition methods see Chapter 6). The LDV head sits on a three-axis positioning system with more than a meter of vertical travel, 20 cm horizontally along the plane of the film, and 3 cm into the film. Control is accomplished by an integrated LabVIEW package developed in-house which allows for automated horizontal and vertical scans, as well as
focusing of the LDV. As long as the interior of the cylinder is kept clean, the decrease in data rate working through the acrylic is minimal.

PIV, on the other hand, is complicated somewhat by the presence of the cylinder wall. The brightest scattering from the TiO$_2$ seed particles is achieved when the strobe, particle, and camera all lie in-line. Some deviation from this is necessary to keep the strobes from direct view of the camera (an aperture placed near the film is useful). However, with the chamber in place, light from the strobes scatters from imperfections in the cylinder surfaces, saturating the image. It is, therefore, necessary to keep a large angle between the strobes and the camera-film line (Fig. 1.5). It is also beneficial to place the strobes as close to the cylinder as possible, minimizing the area that they illuminate, and keeping it from view of the camera.

### 1.2.2 Forcing

Two different comb geometries are used in the generation of the turbulent dynamics (Fig. 1.6): either one comb is set across the soap film perpendicular to the mean flow direction, or two combs are arranged nearly parallel to the mean flow direction. The distinction between the dynamics is that, in one, a flow is forced once and immediately begins to decay, and, in the other, energy is continually added. The horizontal comb used in the present experiment consists of an array of teeth, each with square cross-section of dimension 1.5 mm, and a 1.5 mm spacing between teeth. Such a comb is long enough to completely span the width of the flow. The construction of the
Figure 1.5: Top view of soap film tower. The arrangement of strobes, camera, and LDV head for doing PIV and LDV is shown. The entire apparatus is surrounded by a vacuum chamber to allow the pressure of the air surrounding the film to be varied between ambient laboratory pressure and the vapor pressure of the solution.
angled combs differs slightly. They are composed of cylindrical teeth with an average spacing of 1.6 mm and a diameter of 0.22 mm. Two significantly different lengths of angled comb were used, one set being 64 cm long, the other approximately half that. The longer combs were used for the generation of the turbulent flows investigated in Chapter 2, the shorter for the data sets presented in Chapters 3, 4, and 5, where more significant limitations were placed on the vertical extent of the forcing area.

Figure 1.6: Two different comb geometries used to incite turbulence.
1.3 SOAP FILMS

Since this investigation of 2D intermittency will be performed in the context of a flowing soap membrane, some background on the considerations specific to this particular medium will prove useful. We will examine briefly in this section the cause of surface tension and examine the mechanism by which the aforementioned may be modified to give the film some elasticity (the root of a soap film's tremendous stability). We will then look at how deformations in the film may, through the surface tension, affect internal pressure gradients, potentially driving flow. Lastly, we will turn our attention to two areas related to the compressibility of the film: the wave speed of thickness variations in the film, and the rate at which these thickness variations may be dissipated by Laplace pressures.

1.3.1 SURFACE TENSION

Because of its polarity, a single H$_2$O molecule in bulk can make numerous attractive hydrogen bonds with nearby molecules. Taking that single molecule from the bulk and moving it to the surface breaks a fraction of those bonds. The energy of the system is increased by the work necessarily done while stretching those bonds. Therefore, in order to populate new interfacial area, work must be done to bring a molecule from the bulk solution to the interface, severing some of these nearest-neighbor interactions. It is for this reason that water seeks to minimize the area of an exposed surface.
The characteristic quantity describing the free energy at the interface of two bulk phases is the surface tension. The surface tension depends intimately on the specifics of the microscopic interactions both between molecules of the two phases and between the molecules in bulk, and is best, if not only, determined empirically. The behavior of the water in a capillary tube, for instance, is a function of the individual surface tensions between the three possible pairings of water, glass, and air (in addition to gravitational and Laplacian considerations).

Surface tension can be envisioned, on the one hand, as the contractile force per unit length acting normal to the boundary of an interface and tangential to its surface. Imagine a static wire framing three sides of a thin film. The fourth is held by a wire which can freely slide but is sustained by an external force. To extend the slide and create a larger film, it is necessary to overcome the contractile force of the film. The total energy change during the process will be \( dE = \delta q - \delta w \), where \( \delta q \) is the change in the heat, and \( \delta w \) the work done,

\[
\begin{align*}
\delta w &= F_{\text{ext}} \, dx \\
\delta w &= 2L \gamma \, dx \\
\delta w &= \gamma \, dA.
\end{align*}
\]

Here, the magnitude of the external force must be large enough to overcome the surface tension \( \gamma \) acting along the entire length of the slide \( L \) on both sides of the film. Further, we have taken \( 2L \, dx \) equal to the change in the surface area \( dA \) (the 2 being the result of having to consider the surface tension as acting along both
sides of the film). The total work done on the system then is \( \delta w = P \, dV - \gamma \, dA \).

Using this, along with the fact that \( \delta q = T \, dS \) and \( G = E - TS + PV \), leads to
\[
dG = V \, dP - S \, dT + \gamma \, dA.
\]

From this, we are in the position of being able to interpret the surface tension in its thermodynamic context. That is [45],
\[
\gamma = \left( \frac{\partial G}{\partial A} \right)_{T,P}.
\] (1.13)

We see now that it is quite valid to interpret the surface tension as being the energy cost of creating new interfacial area.

1.3.2 SURFACTANTS

It is possible to vary the surface tension of a bulk solvent (water in our case) with the addition of a surface active component (surfactant). These are compounds which, in general, are amphipathic and preferentially concentrate at the interface between two phases. Amphipathic molecules are a composite of two very different parts, a hydrophilic head and a hydrophobic tail. They prefer to concentrate at the interface between two phases so that the polar head can remain surrounded by water, while the hydrophobic tail may rest comfortably in the other phase. It should be noted that, for soluble surfactants, the hydrophobic nature of the tail must not be too overwhelming, as the molecules must still be water-soluble [46].

The addition of these molecules at the interface reduces the surface free energy. The polar heads of the surfactant bond readily with the surrounding surface water molecules (so it is to them much like they remain in bulk), while the tails are free
to lay across the surface and bond much more weakly where they can. The cost of adding new surface area is thus significantly reduced.

The simplest technique for measuring surface tension involves a basin of pure solvent with a barrier stretching across the surface which divides the basin into two sections. The barrier need not, in fact, must not, impede the flow of pure liquid beneath it (Fig. 1.7). A sensitive balance is attached to the barrier and surfactant added to the surface on one side so that the sides have surface tensions $\gamma$ (with surfactant) and $\gamma_0$ (pure solvent). Because of the difference in surface tension, it will be necessary to apply a force $\pi L$ to the barrier to prevent its movement (where $L$ is
the length of the barrier). The force per unit length $\pi$ is called the surface pressure [47],

$$\pi = \gamma_0 - \gamma.$$  \hfill (1.14)

To make the analogy of viewing $\pi$ as a pressure clearer, we note that there is a 2D analog to the Ideal Gas Law (which we will soon find useful). At small concentrations, the individual surfactant molecules can be thought of as non-interacting. Imagine one on the surface of a solvent bouncing between two opposing walls. The change in momentum with each collision is $\Delta p = mv - (-mv) = 2mv$, and a collision with a given wall occurs every $\Delta t = 2l/v$ seconds (where $l$ is the distance between walls and the two is a consequence of the molecule having to travel both down and back before hitting again). The force is given by the rate of change of the momentum, or $\Delta p/\Delta t = mv^2/l$. The 2D pressure is then the force per unit length $\pi = F/l = mv^2/l^2$. For a 2D gas of $N$ molecules there are $N/2$ bouncing in each of the two orthogonal directions. Therefore, $\pi A = Nmv^2/2$, where $A$ is the area, and $mv^2/2$ is the kinetic energy. From basic statistical mechanics, we know that the energy for each degree of freedom is equal to $k_B T/2$. We, therefore, find the result [47],

$$\pi = \frac{N k_B T}{A}$$

$$= \frac{k_B T}{\sigma}$$

$$= N_A k_B T \Gamma$$

$$= R T \Gamma,$$  \hfill (1.15)
Figure 1.8: Stretching of a soap film element. (a) Unstretched arrangement, (b) Marangoni limit of rapidly stretched film, and (c) Gibbs limit of film stretched over a time scale large compared to that of the diffusion of surfactant to the surfaces.

where we have taken $\sigma = A/N$ as the surface area per surfactant molecule, and the surface excess, or surface concentration of surfactant, as $\Gamma = (\sigma N_A)^{-1}$. For small concentrations, the surface tension, therefore, increases linearly with a decrease in the surface concentration of surfactant. This dependence is at the root of the tremendous stability of soap films. If a region of the film is stretched, the surface excess goes down, leading to a proportional increase in the local surface tension, an increase which opposes further stretching. The process is illustrated in Fig. 1.8.

The response of the film to this sort of distortion is quantified by its elasticity,

$$E = 2A \frac{\partial \gamma}{\partial A}. \quad (1.16)$$

Using Eq.(1.15) along with the relationships

$$c_0 = c_1 + \frac{2\Gamma}{\hbar} \quad \text{and} \quad \Gamma = Kc_1, \quad (1.17)$$
we find the elasticity in the Marangoni limit (that is, on time scales short compared with the diffusion time of new surfactant to the surface) to be given simply by $E = 2\pi = 2RT c_0 hK/(h + 2K)$. Here, $c_0$ is the total concentration of surfactant, $c_1$ the bulk concentration, and $K$ an intrinsic property of the particular surfactant, equivalent to the virtual thickness of the surfactant layer at the film surface. The characteristic time scale for the diffusion of an individual surfactant molecule from the bulk to the surface can be calculated under the assumption that it makes a Gaussian random walk [48]. It should theoretically be on the order of 0.01 s; however, empirically it is found to be on the order of a second, owing, most likely, to the existence of impurities in the solution.

A second definition of the elasticity can be used to determine $E$ in the Gibbs limit (that is, on time scales large compared to that of surfactant diffusion). The previous definition given in Eq. (1.16), takes into account only the instantaneous variation in surfactant concentration at the surfaces through $\partial A$. The contribution of the surfactant in bulk can effectively be added by assuming incompressibility (thus not allowing the surfaces to vary independent of the bulk solution). We then have

$$E_G = -2h \frac{\partial \gamma}{\partial h} = 4RT \frac{c_0 h K^2}{(h + 2K)^2} = \frac{2E_M K}{h + 2K}. \quad (1.18)$$
1.3.3 **LAPLACE PRESSURE DIFFERENCE**

A final concept will prove to be useful in understanding thickness fluctuations. The Laplacian pressure difference may be most familiar for the role it plays in capillary action. To understand the origin of this pressure, imagine a small, initially planar, section of film. What is necessary to impart it with some small curvature? Any deviation from the plane will necessarily increase the surface area of the segment. This increase will result in a change in the energy of the system,

\[ dG = \gamma dA. \]  

(1.19)

To so increase the energy, we must do an equivalent amount of work. This is accomplished by the application of an additional pressure across the surface of the segment,

\[ dw = \Delta p A dz, \]  

(1.20)

where \( dz \) is the deflection. Setting these equal, it is possible to solve for the pressure difference in terms of the radii of curvature of the surface in two orthogonal directions [47],

\[ \Delta p = \gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right). \]  

(1.21)

This implies that the pressure is always higher by an amount \( \Delta p \) on the concave side of a curved surface (even for solids). In a capillary with the tip held in a reservoir of bulk solution, for example, the pressure is the same at the surface of the fluid in the tube as it is at the surface of the reservoir. This implies that the pressure on
the convex side of the interface is less than atmospheric pressure. This results in a fluid level which increases until the pressure \( pgz \) balances the pressure drop across the curved surface.

1.3.4 WAVE SPEEDS

Thick films consist of a layer of bulk fluid sandwiched between twin interfaces of surfactant. The thickness (0.1μm to 100μm) is great enough that there is no appreciable interaction between the surfactant molecules of the two opposing faces. Two types of vibrational modes are possible in these films. In the first, the two surfaces of the film remain parallel but oscillate out of the film plane. Such deformations of the film geometry are termed asymmetric oscillations. The second disturbance is characterized by propagating variations in the film thickness, and are known as symmetric oscillations. The two modes are illustrated in Fig. 1.9.

Asymmetric waves are a result merely of the fact that deformations in the plane geometry of the film increase its area and, thus, are driven by the surface tension. This is true even in regions where the surfactant concentration is uniform. The propagation velocity thus depends on the surface tension \( \gamma \) of the solution [40],

\[
  v_{\text{asym}} = \sqrt{\frac{2\gamma}{\rho h + 2\pi \rho_{\text{air}}/k_0}}. 
\]  

(1.22)

The second term in the denominator effectively accounts for the accompanying motion of the air entrained by the wave, where \( \rho_{\text{air}} \) is its density and \( k_0 \) the magnitude of the corresponding wave vector. An empirical approximation, that the thickness of the
column of air affected by the film motion is of order the magnitude of the oscillation wavelength, was used here.

Symmetric waves, on the other hand, can arise as the result of two different effects. The first are spatial variations in the Laplace pressure due to the surface curvature. This plays a role in films composed both of pure liquids and surfactant solutions. This effect, however, is much smaller in magnitude than the forces originating from variations in the surface excess. As we saw in the previous section, changes in $\Gamma$ are related to the film’s elasticity and, therefore, have wave speed given by [40]

$$v_{sym} = \sqrt{\frac{2E_M}{\rho h}}.$$  \hspace{1cm} (1.23)

Increasing the amount of surfactant (and thereby reducing the surface tension), thus has the effect of increasing the symmetric wave speed (effectively by allowing for
Figure 1.10: Flow past a cylindrical obstacle at velocities near the asymmetric wave speed [43]. (a) Slightly sub-sonic flow with waves forming just upstream of the obstacle. (b) Slightly supersonic flow. (c) Fully supersonic flow with clearly identifiable shock waves. Visualization is under monochromatic light so that interference fringes denote changes in film thickness. Channel width is 2 cm.
larger surface tension gradients). Experiments must be done well below $v_{\text{sym}}$ to avoid significant compressibility effects. One finds, however, that such an assumption is usually quite valid. Estimating the ratio $v_{\text{asym}}/v_{\text{sym}}$ with $\rho_{\text{air}}/\rho_{\text{sol}} \approx 10^{-3}$, $\lambda_0 \approx \text{cm}$, $h \approx 10^{-3} \text{ cm}$, $\gamma_0 \approx 70 \text{ dynes/cm}$, and $\gamma \approx 30 \text{ dynes/cm}$, we find that $v_{\text{asym}}/v_{\text{sym}} \approx 1/2$.

The implication is that asymmetric oscillations out of the film plane will occur long before we approach the speed where thickness fluctuations may readily be propagated. These lower velocity asymmetric oscillations are shown in Fig. 1.10. All experimental measurements were made below $v_{\text{asym}}$ and, therefore, well below $v_{\text{sym}}$.

1.3.5 Thickness Variations

Although the fluid velocity in the third dimension of a soap film is highly constrained, the flow may still couple to a third degree of freedom in the form of thickness variations. If the magnitude of such variations is small, they should have little effect on the flow, merely existing as passive tracers [49]. However, for larger fluctuations, the density of the film (mass per unit area) varies in proportion to the thickness, and the effective film viscosity in inverse proportion. This situation, therefore, may significantly complicate the equation of motion of the flow. The magnitude of such fluctuations, as well as a pertinent time scale for their dispersion, is thus of some relevance.

One can imagine two limiting cases for the dispersion of an initial thickness variation: plug (that is, extensional) flow where the walls themselves play an active role
in the refilling process, and Poiseuille flow between surfactant 'walls' that are taken to be immobile. The former case is particularly relevant where a dimple is formed on a short Marangoni time scale and quickly allowed to relax. New surfactant will not have a chance to diffuse to the surface and, hence, a surface tension gradient will act to force flow in the same direction as the Laplace pressure difference (a result of the curved surfaces of the dimple walls). However, if the dimple is created over a longer Gibbs time scale, or in such a way that the surfactant surface concentration (or surface excess Γ) initially remains uniform, any movement of the film walls will be opposed by increasing the surfactant concentration in the dimple (and thus decreasing the local surface tension). The mobility of the walls is, therefore, expected to be significantly restricted. This case will be covered in more depth in Chapter 5.

Let us here examine the case of immobile walls and Poiseuille flow more closely. The driving force for dimple refill is, in this case, simply the pressure difference across the curved surface of the walls, in general given by Eq.(1.21). If the dimple is taken to be a spherical section (Fig. 1.11), this reduces to

$$\Delta p = \frac{2\gamma}{r_d},$$  \hspace{1cm} (1.24)

where $r_d$ is the radius of the spherical dimple, given by,

$$r_d = \frac{1}{2} \left( \frac{R^2 + \Delta h^2}{\Delta h} \right).$$  \hspace{1cm} (1.25)

Combining these two equations and taking the $\Delta h \ll 1$ limit, we find the pressure in
Figure 1.11: Cross-section of a soap film with uniform surface excess in which a dimple is initially present. Refilling is accomplished primarily by Poiseuille flow between the surfactant walls. Film thickness is denoted by $h$, dimple depth by $\Delta h$, spherical dimple radius by $r_d$, and dimple cross-sectional radius by $R$. 

\[ P - \Delta P \]
the dimple to be

\[ p = 4 \gamma h_{rr}. \]  

(1.26)

For slow flows, the net force due to the pressure on a fluid element is balanced by the force of the frictional stress. That is, \((dp/dr) = (d\tau/dz)\), where \(\tau = \mu (dv/dz)\). Thus, for the mean flow, the equation of motion becomes

\[ \nabla p = \mu \nabla^2 \bar{v} \simeq -\frac{6\mu}{h^2} \bar{v}. \]  

(1.27)

We finally write a continuity equation for the flow,

\[ h_t + h \nabla \cdot \bar{v} = 0. \]  

(1.28)

Combining Eqs. (1.26), (1.27), and (1.28), a partial differential equation for the time-dependent refill of the dimple is achieved,

\[ h_t = \frac{4h\gamma}{\mu} \frac{\partial}{\partial r} (h^2 h_{rrr}). \]  

(1.29)

If we then take the pertinent radial scale to be \(R\), and the pertinent thickness scale to be \(\Delta h\), we find the time scale for dimple refill to be on the order of

\[ T = \frac{\mu R^4}{4\gamma \Delta h^3}. \]  

(1.30)

For example, a 4 mm dimple 3 \(\mu\)m deep in a film with surface tension 30 dynes/cm will have a refill time on the order of \(10^5\) s (or about a day), much longer than the 2-3 s lifetime of any particular element of film. Such dimples then are, for all practical purposes, static and do not contribute to the compressibility of the flow.
CHAPTER 2

INTERMITTENCY IN 2D TURBULENCE

Experiments done on 2D turbulent flows exhibiting either an isolated energy or enstrophy cascade have yielded results in excellent agreement with theoretical expectations [5, 6]. That is, the statistics of the velocity differences were found to be Gaussian and independent of the scale. However, to our knowledge, no group has yet performed an extensive experimental investigation of the turbulent statistics of a flow in which both cascades are simultaneously present (though evidence of dual cascades was previously reported by Rutgers [50]). Simulations done under these conditions have, in varying circumstances, found intermittent results. These include freely decaying simulations with a linear friction term by Smith and Yakhot [17], steady-state simulations with a linear friction term by Babiano, et al. [18], and steady-state simulations by Borue [19] without such a term. It is not completely clear, however, especially in the first and last cases, how significant a role turbulent decay or finite-size effects may have played in the development of intermittency (recall §1.1.4). It is the experimental investigation of intermittency in the simultaneous energy and enstrophy cascades that is the
central purpose of this chapter. The effect of external drag on the flow statistics, spectra, and scaling laws, will also be explored.

Intermittency in two dimensions, at least in decaying turbulence, is thought to be a result of the appearance of long-lived coherent structures superimposed on what is an otherwise turbulent background flow [14]. It has been suggested that such structures can be modeled in the simplest approximation by point vortices interacting on a background field which otherwise obeys the statistics and scaling laws consistent with the self-similar approximation [11, 12, 51]. However, because of the difference in structure between these coherent vortices and the random background field, the overall statistics of the flow field are altered by their presence, particularly at length scales on the order of the vortex sizes. Thus, by examining the statistics of a scale-dependent statistical quantity, it is possible to detect the presence of such structures. One such quantity is the velocity difference between two points where the separation \( r \), effectively introduces a pertinent length scale,

\[
\delta v(x, r) = v(x + r) - v(x).
\]  

\[ (2.1) \]

A scale dependence of the shape of the \( P(\delta v(r)) \), typically with tails broader at small \( r \) than those of the Gaussian distribution, then suggests the presence of intermittent structures [14]. Though we will argue in a later chapter that the presence of anomalously strong vortices is not alone sufficient to explain intermittency in the longitudinal statistics, use of the velocity differences (at least when both the longitudinal and transverse forms are used) is sufficient to detect a much broader class of
intermittent structures than merely the vortex center. The scale dependence of the characteristics of the flow structures will also have an effect on the moments of the velocity differences. The assumption that the flow field is self-similar (i.e. that the statistics are independent of scale), leads to $S_p \equiv \langle \delta v(r)^p \rangle \propto r^{\zeta_p}$, where $\zeta_p = p/3$ is the scaling exponent. The presence of small scale structures in the flow, however, breaks the self-similarity, and leads to scaling exponents which are not linear in $p$.

A further indicator of intermittency is given by the normalized moments,

$$H_p(r) = \frac{\langle \delta v^p \rangle}{\langle \delta v^2 \rangle^{p/2}}$$

the inverse of which can be viewed as a measure of the fraction of time that the velocity signal is ‘active’ [27]. Intermittent flows show a positive divergence of the normalized moments at small $r$, suggesting that at small length scales the flow remains reasonably inactive a large portion of the time. This inactivity is only occasionally punctuated by sharp changes in the velocity, suggesting the passing of a localized structure. These events can be identified in the raw velocity field. We find that they are, indeed, not the result of individual spurious data points but rather represent smooth changes in the velocity over a number of measurements, exceptional only because of their steep slope.

Simulations and experiments have looked at these quantities for clues to the presence of intermittency in both steady-state and decaying systems and those with and without the addition of a drag term in the 2D Navier-Stokes equation which dissipates
energy at the largest scales \[20\],

\[
\rho_s \frac{D\mathbf{v}}{Dt} = -\nabla P + \mu_s \nabla^2 \mathbf{v} + \lambda |\mathbf{v}|^2 \mathbf{v}
\]  

(2.3)

where \( \mathbf{v} \) denotes the 2D velocity, \( P \) the pressure, \( \rho_s \) the solution density, \( \mu_s \) the solution viscosity, and \( \lambda \) the magnitude of the drag term. Many simulations choose for \( \alpha \) the value unity. This turns out to account reasonably well for the effect of air drag in soap film experiments \[20\] but is not the only possibility. To quantify what effect such a drag term has on intermittency, the present experiment was performed both at atmospheric pressure and near the solution vapor pressure. This amounts to a reduction in the surrounding air density \( \rho_a \) to about 1/25th its original value.

However, to estimate the effect of this reduction on the turbulent dynamics it is necessary to construct some sort of theoretical framework. We do this by approximating the surrounding air as a laminar boundary layer. Imagine, for example, a semi-infinite plane moving through an initially quiescent fluid. The boundary layer thickness \( \delta \) a distance \( y_1 \) from the leading edge of the plane can be approximated by setting the magnitude of the inertial and drag terms equal, \( v^2 / y_1 \approx \nu_a v / \delta^2 \), where \( \nu_a = \mu_a / \rho_a \) is the kinematic viscosity. Rearranging, we find that, up to an empirical constant \[52\],

\[
\delta \propto \left( \frac{\nu_a y_1}{v} \right)^{1/2}.
\]  

(2.4)

The drag force on a fluid element is then given by

\[
\frac{F}{A} \propto \mu_a \frac{v}{\delta} \propto \left( \frac{\mu_a \rho_a}{y_1} \right)^{1/2} v^{3/2}.
\]  

(2.5)
The relative importance of the air drag may then be determined from the ratio of Eq. (2.5) to the inertial term \( \rho_s h v (\partial v/\partial y) \), giving,

\[
\left( \frac{\mu_a}{\rho_s^2 y_1} \right)^{1/2} \left( \frac{\rho_a v}{h^2 (\partial v/\partial y)^2} \right)^{1/2}.
\]  

(2.6)

The factor on the left is unchanged by the reduction in air pressure. However, when the air drag on the film is reduced, there is a resulting increase in the acceleration of fluid leaving the nozzle. This makes for thinner, faster moving films on average, which reduces the net effectiveness of the pressure reduction. The increase in acceleration is so spectacular as to make a significant reduction in the valve setting necessary to keep the film from greatly exceeding the wave speed for motions out of the film plane. For the particular experimental settings used, we then find that, in addition to a decrease of \( \rho_a \) by a factor of 25, there is an increase in the mean downstream velocity by 25%. We further estimate \((\partial v/\partial y)^2\) by the second moment at the largest scales, finding an increase by a factor of 7 in vacuum, and allow the film thickness to decrease by a factor \( \beta \). The relative change in the importance of air drag at the solution vapor pressure is, therefore, given by,

\[
\left( \frac{(1/25)(5/4)}{7(1/\beta)^2} \right)^{1/2} \approx \frac{\beta}{10}.
\]  

(2.7)

That is, if the film thickness were to remain constant, the effects of air drag would be roughly an order of magnitude less at the solution vapor pressure, \( P_{vap} \), than at atmospheric pressure, \( P_{atm} \). This result, however, is tempered by a corresponding decrease in the film thickness. Measurements made at \( P_{vap} \) indicate that \( h \) decreases
by a factor of roughly 2.5. The relative importance of the air drag, therefore, decreases by a factor of 4.

2.1 EXPERIMENTAL DETAILS

To study the simultaneous energy and enstrophy cascades, an angled comb geometry first investigated by Rutgers, et al. [50] is used. The two angled combs consist of a staggered row of cylindrical teeth, 0.22 mm in diameter, and 1.6 mm apart. The 64 cm combs are arranged at the top of the test section so that they are 1.2 cm apart at the top and 7 cm apart at the bottom. Measurements are then made in the center of the flow, level with the bottom teeth.

2.2 ENERGY SPECTRUM

Two-dimensional simulations and experiments have studied both the inverse energy and the forward enstrophy cascades suggested by Kraichnan [3]. As we saw in §1.1.2, energy conservation suggests that there be an inertial cascade of energy in which $E(k) = C\varepsilon^{2/3}k^{-5/3}$, where $k$ is the wavenumber and $\varepsilon$ is the energy transfer rate per unit mass. This followed the same dimensional arguments first applied to 3D turbulence by Kolmogorov [53]. In 2D, however, there is a further constraint. The mean square vorticity, or enstrophy ($\Omega = 1/2 |\nabla \times \mathbf{v}|^2$), is also conserved. This suggests the existence of a second inertial range in which $E(k) = C' \eta^{2/3}k^{-3}$, where $\eta$
Figure 2.1: Energy spectrum at (a) $P_{\text{atm}}$ and (b) $P_{\text{vap}}$. Gray lines denote the theoretical expectations of $k^{-5/3}$ in the energy cascade (unshaded region), and $k^{-3}$ in the enstrophy cascade (shaded region). A laminar flow spectrum is included to estimate the magnitude of the experimental noise.

is the enstrophy transfer rate per unit mass.

The spectrum in Fig. 2.1 shows simultaneous evidence of these two scaling regimes, suggesting that the dynamics are indeed largely 2D. Comparing the curves at ambient pressure (a), and near the vapor pressure (b), one sees the dramatic effect of the five-fold reduction in drag. Though the two spectra are otherwise very similar, the turbulence at $P_{\text{vap}}$ contains nearly an order of magnitude more energy than at $P_{\text{atm}}$. 

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In the inverse energy cascade (associated with driven turbulence), the energy is proportional to $k^{-1.8}$, differing slightly from the expected $k^{-5/3}$. In the forward enstrophy cascade (associated with decaying turbulence), the energy scales with $k^{-3.35}$ (at $P_{\text{atm}}$) and $k^{-3.2}$ (at $P_{\text{vap}}$). A scaling exponent of magnitude greater than 3 in the enstrophy cascade is not unexpected. Such a result is well documented [9, 37, 51, 54] and suggested to be a result of the formation of coherent vortices. The size distribution of these vortices determines the slope of the spectrum [7, 8] and likely contributes to small-scale intermittency [14].

Also included in the figure is the spectrum of a laminar flow (taken without forcing) to estimate the relative magnitude of the experimental noise. Deviations at small $k$ from the white noise spectrum are most likely the result of changes in the velocity caused by laboratory air currents.

The location of the bend between the two inertial ranges gives one estimate of the effective injection length scale. In both the $P_{\text{atm}}$ and $P_{\text{vap}}$ cases, it is much larger than the comb tooth diameter of 0.22 mm. However, since the data sets are taken at the center of the flow, a temporal evolution of the turbulence occurs between the forcing and collection points. It is, therefore, not surprising that the injection length scale has also evolved [55] (this progression is investigated in more depth in §3.2.2).

This transition region can also be seen in Fig. 2.2 where the dependence of the spectra on $k$ is explicitly multiplied out. The transition between the two cascades lies between 1 and 2 cm at $P_{\text{atm}}$, and between 0.6 and 2 cm at $P_{\text{vap}}$. It turns out that there
Figure 2.2: Energy spectra at $P_{\text{atm}}$ and $P_{\text{vap}}$ with the explicit dependence on the wavenumber $k$ multiplied out. The bend in the spectrum is apparent here as the transition region between the two scaling regimes. Arrows indicate the effective injection length scales estimated from the ESS structure functions: approximately 1.6 cm at $P_{\text{atm}}$, and 0.64 cm at $P_{\text{vap}}$. The enstrophy cascade is shaded, while the energy cascade is not.
is a corresponding transition between the two scaling regimes in the ESS structure functions. We estimate this transition to lie at 1.6 cm at $P_{\text{atm}}$, and at 0.64 cm at $P_{\text{vap}}$. Subsequent figures will differentiate between the two inertial ranges deduced from the above figures. The enstrophy cascade will be shaded while the energy cascade will remain white.

2.3 Velocity-difference PDFs

Using the longitudinal velocity differences, that is, taking both $v$ and $r$ along the direction of the mean flow, it is possible to probe the velocity statistics as a function of length scale. A self-similar velocity field should have probability distribution functions $P(\delta v(r))$ with a shape independent of $r$, as was the case in the experiments of Paret and Tabeding on the individual cascades [5, 6]. Intermittent events break the self-similarity of the flow field, however, and subsequently lead to velocity difference statistics which are scale dependent. In fact, in classical 2D turbulence, some believe that deviations of the velocity-difference PDFs from Gaussian must be caused by intermittency [56] (though this is somewhat at odds with other experimental results [57]).

Figure 2.3 shows $P(\delta v(r))$ for a number of $r$ spanning both the forward enstrophy and inverse energy cascades at $P_{\text{atm}}$. To accentuate relative shape changes, the width of each distribution was rescaled by its second moment. The distributions with $r$ in the energy cascade are close to Gaussian and differ substantially from distributions
Figure 2.3: Normalized velocity difference PDFs for a few separations spanning the energy and enstrophy cascades at $P_{\text{atm}}$. The thin dotted line is a Gaussian fit.
with $r$ in the enstrophy cascade, which are decidedly more exponential in nature. This dependence of the velocity-difference PDFs on scale suggests a fundamental change in the characteristics of the flow at small scales and a breaking of the self-similarity.

The variation of the shape can be quantified by fitting the distributions to a stretched exponential function [30, 58], $P(\delta v) \propto \exp \left\{ -|\delta v|^\alpha \right\}$. Since the central region of the PDF consistently approaches a Gaussian curve independent of the separation, only the tails of the PDFs beyond $2(\delta v^2)^{1/2}$ are fit [30, 58]. The trend toward a slower than Gaussian falloff at small length scales in data taken both at $P_{\text{atm}}$ and at $P_{\text{vap}}$, is shown in Fig. 2.4 (where the error bars show a 95% confidence interval). The
general features of the two curves (i.e., slope and magnitude) are remarkably similar, differing only by an apparent shift in the length scale \( r \). This offset corresponds to the difference in the effective injection scales seen in the turbulent spectra.

For larger \( r \), the exponent \( \alpha \) surpasses the value that would denote Gaussian statistics. This is a common feature of the turbulence data we have, whether angled or horizontal combs are used for forcing, though we do not yet understand its significance (the same deviation was seen in magnetohydrodynamically driven films as well [59]). The dependence of the stretching exponent on \( r \), however, indicates a flow-field which is not self-similar. This basic change in the characteristics of the flow at small length scales is an indication that the turbulence is intermittent (note that a deviation from Gaussian statistics needs not explicitly imply a scale dependence [57]).

Figure 2.3 also suggests that the \( \alpha \) for negative \( \delta v \) are different than for positive, reflecting an inherent asymmetry. This asymmetry can be quantified by the function \( q(r) = P(\delta v) - P(-\delta v) \) shown in Fig. 2.5. This result agrees very well with the qualitative form of previous experimental [5] and numerical [60] results. The structure functions of odd order can, in fact, be calculated in such a way as to demonstrate explicitly their dependence on this asymmetry,

\[
S_p(r) = \int_0^\infty \delta v(r)^p q(\delta v(r)) \, d(\delta v(r)),
\]

(2.8)
demonstrating immediately that the asymmetry in the velocity-difference PDFs is responsible for the energy and enstrophy transfer in the system through equations (1.9) and (1.10).
Figure 2.5: Asymmetry of the velocity-difference PDFs, \( q(\delta v) = P(\delta v) - P(-\delta v) \), for separations \( r \) of: (a) 0.50, (b) 0.79, (c) 1.66, and (d) 3.50 cm at \( P_{atm} \).
2.4 Intermittent ‘events’

If isolated coherent structures are indeed present in the flow, one might hope to isolate the instances where such structures passed the measuring point. This can be done by noting that the deviation of the statistics from Gaussian occurs most significantly at small scales, and in the tails of the velocity-difference probability distributions. Individual events can then be singled out by examining sections of the raw velocity data which correspond to points in the tails of the PDFs.

Typical events are shown in (a) and (b) of Fig. 2.6, demonstrating a characteristic negative slope. Part (c) shows a much rarer event with positive slope. This reflects the direction of the inherent asymmetry in the 2D velocity difference PDF. An average of all such events is shown in part (d) for \( r = 0.5 \) cm (a relatively small scale) and \( \frac{\delta u(r)}{(\langle \delta u(r)^2 \rangle)^{1/2}} \geq 5.35 \) (to select only the tails of the PDF). There are 317 in total. Note that the events have been plotted as raw time traces from the LDV, as has been done in previous 3D work [32, 61]. The application of the frozen turbulence assumption leads to a spatial shock that has, on average, a positive slope and, therefore, a positive \( S_3(r) \). The shock-like nature of these events bears a strong qualitative resemblance to intermittent events measured in 3D turbulence. However, in 3D, the slope of the events is in the opposite direction, further indicating that the soap film turbulence is, in fact, 2D. Each of these events is the result not of a single spurious data point but rather of a smooth slope comprised of 10 to 15 velocity measurements. However, the steepness of the slope in each case is uncharacteristic of the rest of the
Figure 2.6: (a) and (b) show typical events which correspond to velocity differences in the tails of the $r = 0.5$ cm PDF with $|\delta v(r)/\langle \delta v(r)^2 \rangle^{1/2}| \geq 5.35$ at $P_{\text{atm}}$. (c) is a much rarer event with positive slope. The average of all such events is displayed in (d). The events are plotted as a function of time for easy comparison with 3D results [32, 61]. The mean local velocity is subtracted out in each event for easier comparison.
flow field and, as such, suggests the passing of the isolated structures which are the root of intermittency.

2.5 STRUCTURE-FUNCTION SCALING LAWS

Deviations of the flow from self-similarity are further manifest in the moments of the velocity differences,

\[ S_p(r) = \langle \delta v^p(r) \rangle = \int_{-\infty}^{\infty} \delta v^p P(\delta v(r)) d(\delta v). \]  

(2.9)

We also define a second function,

\[ G_p(r) = \langle |\delta v(r)|^p \rangle, \]

(2.10)

which is often calculated for odd \( p \) when a constant scaling range in \( S_p(r) \) is narrow or absent. Though the theoretical implications of \( G_p(r) \) with \( p \) odd are not yet completely clear, we include it here as it will allow us to effectively extend the width of the scaling regime. It is reassuring that results derived from \( G_{odd}(r) \) follow the same trend as those derived from \( G_{even}(r) \equiv S_{even}(r) \).

Though no analytic relation has been derived for structure functions of orders other than three (recall Eqs. (1.9) and (1.10)), self-similarity implies that the structure functions should scale like \( S_p(r) \propto r^{\zeta_p} \), with \( \zeta_p = p/3 \) in the energy cascade. Recall that similar relations hold in the enstrophy cascade as well [37, 26], such that \( S_3(r) \propto r^3 \) and \( \zeta_p = p \).
Theory suggests that the scaling of the structure functions with separation should be consistent throughout each of the cascades. However, the extent of the constant power-law scaling of the structure functions can be quite small because of the limited width of the inertial range [5]. The spectrum shows the combination of the two cascades to be only a decade at $P_{atm}$, and a decade and a half at $P_{vap}$. Furthermore, since the structure functions should scale like $\zeta_p = p$ in the enstrophy cascade and $\zeta_p = p/3$ in the energy cascade, some transitional region between the two limits is not unexpected. Fig. 2.7 shows the structure functions at $P_{atm}$ with their theoretical dependence on $r$ explicitly multiplied out (the same qualitative result is found at $P_{vap}$). The gray lines show the slope of the corresponding structure functions expected under the self-similarity hypothesis.

The standard remedy for the small size of the scaling regime follows a technique introduced by Benzi, et al. [62] where $S_p(r)$ is instead plotted against $G_3(r)$, leading to a much cleaner scaling throughout the inertial range. This procedure is referred to as extended self-similarity (ESS) and is used in most studies of intermittency in 2D and 3D turbulence. We will refer to the scaling exponents derived from this technique as $\zeta_{p,ESS}$.

The ESS structure functions are shown in Fig. 2.8 at $P_{atm}$ with the numerical fit in the enstrophy cascade explicitly multiplied out. The increase in the smoothness of the curves over the structure functions plotted against the separation is obvious. The structure functions are also now predicted to scale in the same way, whether in
Figure 2.7: Structure functions of orders 2, 3, 4, and 6 with their expected theoretical dependence on the separation in the energy cascade explicitly multiplied out (respectively \( \zeta_p = 2/3, 1, 4/3, \) and 2). Gray lines show slopes of the corresponding structure functions if self-similarity is assumed.
Figure 2.8: ESS structure functions at $P_{\text{atm}}$. The $\zeta_{p,\text{ESS}}$ dependence as determined by fitting to the enstrophy cascade range of the ESS structure functions is explicitly multiplied out. Horizontal rules are simply guides to the eye.
Figure 2.9: Scaling exponents $\zeta_{p,\text{ESS}}$ from the energy ($\circ$) and enstrophy ($\triangledown$) cascades at (a) $P_{\text{atm}}$ and (b) $P_{\text{vap}}$. The solid line is the self-similar prediction of $\zeta_{p,\text{ESS}} = p/3$ and the dashed line is the log Poisson theory [34].

The clear bend in the curves near $G_3(r) = 20,000$ corresponds to a separation of 1.6 cm (near the center of the spectral region separating the two inertial ranges), and suggests that there are different scaling exponents in the energy and enstrophy cascades. A similar bend is seen in the ESS structure functions at $P_{\text{vap}}$ at a separation of 0.64 cm.

The scaling exponents from nonlinear fits to the ESS structure functions are reported in Fig. 2.9 for each cascade (with error bars denoting a 95% confidence interval). Points in the enstrophy cascade at either pressure ($\triangledown$), deviate significantly from
Figure 2.10: Flatness \((p = 4)\) and hyper-flatness \((p = 6)\) in the energy and enstrophy cascades for \(P_{\text{atm}}\) (solid line) and \(P_{\text{vap}}\) (dashed line). The constant values expected for Gaussian velocity difference PDFs are shown as horizontal lines.

the self-similar expectation of Kolmogorov, denoted by the solid line. A somewhat lesser degree of deviation is also found in the energy cascades \((\circ)\) as well. These deviations inherently reflect the scale dependence of the velocity-difference statistics, and, therefore, further indicate that the flow is intermittent.

2.6 Normalized Moments

As we found in §2.3, the existence of intermittent structures in the flow field results in a broadening of the tails of the velocity difference PDFs [14]. Such a change is
amplified by the higher order structure functions since, as can be seen in Fig. 6.5, the moments of higher order are weighted increasingly by the tails of the $P(\delta v)$. By normalizing the higher order moments, it becomes possible to effectively remove the effect of the central portion of the $P(\delta v)$ and highlight the impact of the tails. This is done using

$$H_p(r) = \frac{\langle \delta v^p \rangle}{\langle \delta v^2 \rangle^{p/2}}.$$  

(2.11)

The normalized moment of order 4 is called the flatness and of order 6 the hyper-flatness. If the probability distributions are Gaussian and scale independent, we expect the normalized moments to have the constant values $H_4 = 3$ and $H_6 = 15$. However, we instead find that both the flatness and hyper-flatness show a non-trivial dependence on the separation (Fig. 2.10). For larger $r$, we find the hyper-flatness approaches the Gaussian prediction, suggesting that intermittency may be present only weakly in the energy cascade. However, the power-law growth of the $H_p(r)$ at small $r$ reflects the increasing deviation of the statistics in the wings of the PDFs from Gaussian, signaling the presence of small-scale intermittency in the enstrophy cascade. Again, notice that what difference exists between the curves at $P_{vap}$ and $P_{atm}$ appears to amount only to an offset in the scale $r$, and corresponds to a difference in the effective injection scale seen in the corresponding spectra.
2.7 Summary

An experimental investigation of turbulent flows in which both the energy and enstrophy cascades were simultaneously present was undertaken and each of the major signs of turbulent intermittency examined. In each case, the indicators of intermittency were found to strongly suggest the existence of anomalously strong intermittent structures in the flow. These indications were specifically:

- A scale-dependence of the shape of the velocity-difference probability distributions, tending toward exponential tails at small $r$, and toward Gaussian at large $r$;

- A scale-dependent deviation of the hyper-flatness from the constant value predicted for Gaussian statistics; and,

- Scaling exponents which fell significantly below the self-similar prediction of $p/3$.

The differences that existed between these quantities at atmospheric pressure and the solution vapor pressure appeared to amount substantially to an offset in the pertinent length scale $r$, corresponding to a shift in the effective injection scale in the turbulent spectra. We were further able to isolate the events in the tails of the probability distributions responsible for each of these signs of intermittency, and verify that they, in fact, corresponded to smooth but rapid changes in the velocity over a period of $10^{-12}$ data points, rather than single anomalous points dominated by noise. Lastly, we
showed that the energy transfer within the inertial cascades was a direct consequence of the asymmetry in the velocity-difference probability distributions.
CHAPTER 3

FREELY-DECAYING 2D TURBULENCE

In the previous chapter, all of the evidence indicated that the 2D turbulent flow field was intermittent when both the inverse energy and forward enstrophy cascades were simultaneously present. Such is not the case when only the enstrophy cascade governs the dynamics between eddies of different scales. The structure-function scaling exponents then hover about the self-similar prediction $\zeta_{p,\text{ESS}} = p/3$ (in agreement with theoretical predictions and the experimental results of Paret, et al. [6]). This is true throughout the short temporal evolution that can be affected by observation of the flow at locations further downstream of the forcing.

The case is dramatically different in the evolution of the turbulent flow in which both cascades are initially present. The corresponding spectra show that the energy cascade begins to diminish immediately when forcing is arrested. Furthermore, its disappearance coincides with a smooth shift in the scaling exponents from the large deviations indicative of intermittency to a nearly precise agreement with the self-similar expectation. We will draw two conclusions from this result: first, the presence
of both cascades is a necessary prerequisite here for intermittency; and second, the fundamental mechanism responsible for intermittency, in this case, is different than that at long times in decaying turbulence.

We further will show that the energy cascade asserts its presence in the structure function of third order as well. When only the enstrophy cascade is present in the turbulent spectrum, only one scaling regime is found in the third moment. The inclusion of driven as well as decaying processes in the dynamics (that is, the presence of both cascades), however, leads to a second scaling regime in $S_3(r)$. Not only is this second range not present when only the enstrophy cascade is observed, but the disappearance of the spectral energy cascade in the evolution of turbulence in which both cascades are initially present coincides with the elimination of this second scaling range in $S_3(r)$.

3.1 EXPERIMENTAL DETAILS

To study the decay of turbulent flows and the effects of air drag, data was taken with two different forcing arrangements (horizontal and angled combs) and at two different surrounding air pressures ($P_{\text{vap}}$ and $P_{\text{atm}}$), resulting in a total of four data supersets. In each of these cases, data was taken at six different locations downstream of the combs, corresponding to six different decay times. The result is a total of 24 independent data sets, each containing approximately 33.4 million points (a total of roughly 800 million velocity measurements).
The horizontal comb used in the present experiment consists of an array of teeth with square cross-section of dimension 1.5 mm, and a spacing of 1.5 mm between teeth. The two angled combs, however, are identical to those used in Chapter 2 (cylindrical teeth with an average spacing of 1.6 mm and a diameter of 0.22 mm), with the exception that they are 33 cm in length rather than 64 cm. The reduction in distance over which the turbulence is forced was necessary because of the limitation on the vertical size of the test section. There was not enough room to make measurements to 50 cm below the longer combs. As in Chapter 2, the combs were placed in the film so that they formed an inverted wedge 1.2 cm apart at the top and 7 cm apart at the bottom. Measurement locations are given as distances below the last teeth.

### 3.2 Gross Flow Qualities

Before moving on to considerations specific to intermittency, the downstream evolution of some of the basic flow characteristics is presented.

#### 3.2.1 Velocity and Turbulence Intensity

One of the most basic attributes of a vertically falling soap film is its mean downstream velocity \( \langle v_y \rangle \). The evolution of this quantity, shown in Fig. 3.1, suggests, for one thing, that the film is still accelerating. We see also through a comparison of (a) and (c) with (b) and (d), that, not surprisingly, the acceleration is higher in flows examined at \( P_{\text{vap}} \).
Figure 3.1: Evolution of mean velocity $\langle v_y \rangle$ below horizontal comb at (a) solution vapor pressure and (b) atmospheric pressure; and below angled combs at (c) vapor pressure and (d) atmospheric pressure.
than those at $P_{atm}$. The fact that the flows are indeed accelerating, however, means
that a simple one-to-one correspondence between the spatial and temporal evolution
of the flow will not hold. This is an important fact since the temporal dependence of
the quantities that we will consider is the more universally reported measure of the
turbulent evolution. We shall, therefore, compute the corresponding time at each of
the locations $y_j$ from
\[ t_j = \sum_{i=1}^{j} \frac{2(y_i - y_{i-1})}{\langle v_y \rangle_i + \langle v_y \rangle_{i-1}}, \tag{3.1} \]
with $t_0 = 0$.

The variance in the velocity at each downstream location may also be calculated,
allowing for the determination of the turbulence intensity $I_t = \langle (v'_y)^2 \rangle^{1/2}/\langle v_y \rangle$ (where
$v'_y = v_y - \langle v_y \rangle$). The magnitude of said quantity decreases with downstream position
(Fig. 3.2). This corresponds essentially to a decrease in the turbulent energy of the
flow due to viscous dissipation and air damping. For more information on the role
that the turbulence intensity plays in the validity of LDV data, please refer to §6.1.2.

3.2.2 EVOLUTION OF THE INTEGRAL SCALE

In addition to changes in the downstream velocity and turbulence intensity, there is
an evolution of the systematic length scales. This progression can be seen in Fig. 3.3,
where the structure function of second order is shown at a number of locations below
a horizontal comb at $P_{vap}$. The overall shape of the $S_2(r, y)$ is remarkably consistent,
demonstrating that the functional dependence of the moments on $r$ is essentially
Figure 3.2: Devolution of turbulence intensity $I_t$, below horizontal comb at (a) solution vapor pressure and (b) atmospheric pressure; and below angled combs at (c) vapor pressure and (d) atmospheric pressure.
Figure 3.3: Spatial evolution of $S_2(r, y)$ at: (a) 5, (b) 8, (c) 15, (d) 25, (e) 35, and (f) 50 cm below a horizontal comb at $P_{\text{vap}}$.

independent of downstream position. In fact, a scaling of the form

$$S_2(r) \to c_0 S_2(c_1 r)$$  \hspace{1cm} (3.2)

suffices to map the second moment at a given location onto the second moment at any other. The $c_1(y)$ then contain the information relevant to the evolution of the integral, or most energetic, scale in the inertial range.

A numerical determination of the $c_1(y)$ (i.e., by some least-squares type method), however, requires an algebraic approximation of $S_2(r)$ at one location (the particular location is not important as far as the numerical method is concerned). A series
expansion of the form

\[ S_2(r) \simeq \sum_{n=0}^{12} a_n r^{n/3}, \]  

(3.3)

where the \( a_n \) are determined by a least-squares fit at a location \( y_{\text{fit}} \), turns out to provide the best fit to the empirical data across the full range of separations. The substitution given in Eq. (3.2) is then made, and the \( c_0 \) and \( c_1 \) determined at each of the other locations using the \texttt{NonlinearRegress} function available in \textsc{Mathematica}.

The results for turbulence decaying below a horizontal comb at \( P_{\text{vap}} \) are plotted in Fig. 3.4. The agreement between the actual data points and the rescaled series...
expansion is quite good. The corresponding $c_1$, shown in Figs. 3.5 and 3.6, suggest that the turbulent length scales exhibit a roughly power-law increase with downstream distance $l/l_{fi} \propto y^{0.23\pm0.11}$ and time $l/l_{fi} \propto t^{0.30\pm0.19}$. As was alluded to in §3.2.1, the difference in the exponents is merely a reflection of the fact that the film is still accelerating. It should also be noted that the error in the determination of the scaling exponents here is quite large, essentially as a result of the limited number of temporal data points.

However, even given the sizeable error bars present here, the empirically derived exponent lies well below the value of unity theoretically predicted by Batchelor [25] and experimentally confirmed by Martin, et al. [55]. It further appears that an appeal to the presence of air drag is not a sufficient explanation for this discrepancy since (a) the scaling exponent reported here is for $P_{vap}$, and (b) Martin, et al., found said exponent to be only very weakly dependent on the surrounding air density. This result is, therefore, still not completely understood.

The same procedure was followed for data taken below a pair of angled combs as well (again at $P_{vap}$). The agreement between the empirical data and rescaled analytic approximation is again quite reasonable (see Fig. 3.7). A plot of the corresponding temporal evolution of $l/l_{fi}$ is shown in Fig. 3.8. However, the dispersion of the data points in this case is great enough to make a determination of the scaling exponent impossible.
Figure 3.5: Spatial evolution of integral scale below horizontal comb at $P_{\text{vap}}$. 
Figure 3.6: Temporal evolution of integral scale below horizontal comb at $P_{\text{vap}}$. 
Figure 3.7: Empirical values of the second moment obtained at the solution vapor pressure, distances of (a) 0, (b) 2.5, (c) 10, (d) 20.8, (e) 34.8, and (f) 45.9 cm below the angled combs. The solid lines are the rescaled fit of Eq. (3.3) (itself originally determined by fitting to (c)).
Figure 3.8: Temporal evolution of integral scale below angled combs at $P_{vap}$. 
Additionally, the evolution of the Reynolds number ($Re = u_{rms} l_{int}/\nu$) may be calculated from the progression of the integral, or most energetic, scale (estimating the mean velocity of the eddies by $u_{rms} \simeq (S_2(l_{int}))^{1/2}$). Plotting the Reynolds number below both horizontal and angled combs in Fig. 3.9 (both at $P_{vap}$), we see that $Re$ either remains essentially constant (below a horizontal comb), or is decreasing (below the angled combs). This result is again at odds with the earlier work of Martin, et al., which found $Re$ either constant (at atmospheric pressure), or increasing (near the solution vapor pressure). This discrepancy is perhaps not unexpected, however, in light of the fact that the progression of the integral scale in the present case is sub-linear.

3.3 SPECTRUM

There is a fundamental difference between the turbulence generated by a horizontal comb and that generated by a pair of angled combs. In the first case, the turbulence is forced at only one vertical location in the flow (i.e., at only one instant in time). Beyond this initial forcing, the turbulence is forever in a state of decay. Correspondingly, an examination of the energy spectra shows only the presence of an enstrophy cascade. On the other hand, when a pair of combs is inserted into the flow at an angle which deviates from the perpendicular to the mean flow, there is a region in which the turbulence continues to be forced by contact with new comb teeth, even while flow structures generated earlier have begun to decay. As a result, the two cascades
Figure 3.9: Temporal evolution of Reynolds number below horizontal (□) and angled (●) combs (both at $P_{\text{vap}}$). Lines are merely guides to the eye.
associated with driven and decaying turbulence (energy and enstrophy respectively) are simultaneously present in the flow.

If the energy and enstrophy transfer at a given wavenumber is indeed local, that is, depends only on the magnitude of the energy spectrum at that wavenumber, the fact that the two cascades are simultaneously present should have no effect on the statistics of the flow in either range (e.g., the smaller scales of the enstrophy cascade would in no meaningful way interact with the larger scales in the energy range but only be advected along by them). However, Kraichnan [63] showed that, quite to the contrary, the energy cascade in 2D turbulence is not only non-local but significantly more so than the corresponding cascade in 3D, and, furthermore, that enstrophy transfer acts over an even broader range of scales. As a result, it may actually be somewhat surprising that the two distinct scaling regimes with their very different dynamics can coexist at all [64]. Our limited understanding of the mechanisms of energy and enstrophy transfer in either of these ranges [65, 66] (let alone at the vastly different scales involved in interactions between the ranges), leaves open the possibility that some such interaction may intermittently result in the formation of an unusually strong structure by a mechanism not available when only one cascade is present.

3.3.1 CASCADES

Figures 3.10 and 3.11 report the energy spectra below a horizontal comb at $P_{\text{vap}}$ and $P_{\text{atm}}$ respectively. The two are qualitatively virtually identical. The major quanti-
Figure 3.10: Spectrum taken at solution vapor pressure, distances (a) 5, (b) 8, (c) 15, (d) 25, (e) 35, and (f) 50 cm below horizontal comb.

tative differences lie in the greater total energy exhibited at $P_{\text{vap}}$ (a consequence of the reduction in air drag), and a correspondingly larger inertial range. Notice that no energy cascade is present. Only an $E(k) \propto k^{-3}$ inertial range transfers enstrophy toward smaller scales.

On the other hand, the spectra for turbulence forced by two angled combs is shown in Figs. 3.12 and 3.13 ($P_{\text{vap}}$ and $P_{\text{atm}}$ respectively). In this case, we initially see two independent cascades, the scaling of which agrees very well with the theoretically predicted exponents, especially at $P_{\text{vap}}$. The discrepancy in the energy cascade at
Figure 3.11: Spectrum taken at atmospheric pressure, distances (a) 5, (b) 10.2, (c) 15.2, (d) 25.1, (e) 35, and (f) 50 cm below horizontal comb.
\( P_{\text{atm}} \) is likely a result of forcing which is insufficiently strong to fully overcome the effects of air drag. This was not an issue with the data taken in Chapter 2 where the significantly longer 64 cm combs were used, nor does it appear to be in vacuum where air drag plays a far less significant role.

As the turbulence evolves downstream of the combs, however, it is no longer continuously forced. As a result, the width of the energy cascade initially diminishes before quickly disappearing altogether. We will see that this progression is reflected in the disappearance of a second scaling regime initially seen in the third moment, and in a shift of the scaling exponents, both of which are initially indicative of an intermittent flow toward the self-similar expectation.

### 3.3.2 Energy Dissipation

The total area under the spectral curves in each case decreases downstream, both as a result of viscous effects at the smallest scales in the flow, and through a coupling of the largest scales to the surrounding air. In general, the total turbulent energy, therefore, appears to decrease roughly exponentially with time (Fig. 3.14). The leveling off of (b) at longer times (if it can be trusted) is a feature previously reported by Martin, Wu, and Goldburg [55]. Yet it is not clear why then, for instance, curve (d) does not appear to show the same tendency.
Figure 3.12: Spectrum taken at solution vapor pressure, distances (a) 0, (b) 2.5, (c) 10, (d) 21, (e) 35, and (f) 46 cm below angled combs.
Figure 3.13: Spectrum taken at atmospheric pressure, distances (a) 0, (b) 2.5, (c) 10, (d) 20, (e) 35, and (f) 50 cm below angled combs.
Figure 3.14: Energy dissipation below horizontal comb at (a) vapor pressure and (b) atmospheric pressure; and below angled combs at (c) vapor pressure and (d) atmospheric pressure.
3.4 Third-order structure functions

3.4.1 Enstrophy cascade

Recall that one of the few exact results of 2D turbulence relates the third moment of the velocity difference to the rate of enstrophy dissipation. This relation was given in Eq. (1.10), and shown to be proportional to the cube of the separation. For comparison, we plot the empirically-obtained third moment at a number of locations downstream of a horizontal comb (recall that in this case only an enstrophy cascade is present; see Fig. 3.10). The plots, Figs. 3.15 and 3.16, show the third moments at $P_{\text{vap}}$ and $P_{\text{atm}}$ respectively. The latter are in very good qualitative and quantitative agreement with the earlier results of Belmonte, et al. [67] (also at $P_{\text{atm}}$). Note, also, that the location of the maximum value of $S_3(r)$ corresponds to the location of the integral scale in Figs. 3.10 and 3.11. As the integral scale increases (recall §3.2.2) so, too, does the separation $r$, for which $S_3(r)$ reaches its peak.

3.4.2 Simultaneous enstrophy and energy cascades

The relatively simple shape of Figs. 3.15 and 3.16 (which persists relatively unchanged for quite some distance downstream) is complicated significantly in the turbulent field below a pair of angled combs (Figs. 3.17 and 3.18). Concentrating on data taken near
Figure 3.15: $S_3(r)$ below horizontal comb at solution vapor pressure. Distances of measurement locations below comb are: (a) 5, (b) 8, (c) 15, (d) 25, (e) 35, and (f) 50 cm.
Figure 3.16: $S_3(r)$ below horizontal comb at atmospheric pressure. Distances of measurement locations below comb are: (a) 5, (b) 10.2, (c) 15.2, (d) 25.1, (e) 35, and (f) 50 cm.
Figure 3.17: $S_3(r)$ below angled combs at vapor pressure. Distances of measurement locations below bottom of combs are: (a) 0, (b) 2.5, (c) 10, (d) 21, (e) 35, and (f) 46 cm.

the solution vapor pressure, we notice first that, at the two positions nearest the comb, the third moment exhibits two separate and clearly independent peaks. Furthermore, as the turbulence decays, this more complicated structure is replaced by the familiar singly peaked shape characteristic of §3.4.2.

Some light may be shed on these observations by inspection of the corresponding spectra (Fig. 3.12) where, initially, an inertial energy cascade is quite prominent. The presence of this second cascade suggests that, in addition to Eq. (1.10) governing the behavior at small $r$, the relation given in Eq. (1.9) should hold at the largest
inertial scales. We, therefore, expect two limiting behaviors for the third moment: a cubic dependence on the separation at small scales, and a linear dependence on the separation at large scales. As a result, some deviation from the shape of the third moments in §3.4.1 is not unexpected. However, the exact form of the $S_3(r)$ appears to be somewhat more complicated than a smooth transition between these two scaling behaviors. One possible explanation for this complication lies in the absence of a constant term in either Eq. (1.9) or (1.10). The implication is that an extrapolation of the functional behavior of the third moment should in each range independently go to zero. Such a requirement is not easily fulfilled if the transition between the two regions is smooth and monotonically increasing. Instead, some more complicated transitional region must exist to connect the two independent behaviors. Again, notice that the value of $r$ at which the third moment peaks marks roughly the maximum length scale contained in the associated spectrum.

Further downstream of the combs, the second peak becomes rapidly weaker before disappearing altogether. A further examination of Fig. 3.12 shows that this disappearance coincides to a large degree with the elimination of the energy cascade from the turbulent spectra, thus confirming the association of this additional behavior with the presence of the second cascade. We further note that, in the corresponding data taken at $P_{\text{atm}}$, the reduction in definition of the two peaks is likely the result of two contributing (and not unrelated) factors. Because of the increased effectiveness of air drag the total amount of turbulent energy in the flow, as well as the width of the
Figure 3.18: $S_3(r)$ below angled combs at atmospheric pressure. Distances of measurement locations below bottom of combs are: (a) 0, (b) 2.5, (c) 10, (d) 20, (e) 35, and (f) 50 cm.
inertial range, is greatly reduced. Even in data taken at the solution vapor pressure, the absence of significant regions of constant scaling in the third moment is likely related to the relatively small width of the inertial range. This is all the more pertinent in the case of simultaneous energy and enstrophy cascades where the inertial range is further divided into separate regions with very different limiting behaviors.

3.5 STRUCTURE-FUNCTION SCALING

EXPONENTS

The scaling exponents below horizontal and angled combs are shown in Figs. 3.19 and 3.20 respectively, along with Kolmogorov’s self-similar prediction and the log-Poisson prediction of She and Leveque [34]. It is an interesting result that in neither case is the intermittency seen to increase with the temporal evolution of the flow (as judged by the deviation of the scaling exponents from the K41 prediction). This may at first seem at odds with the growing volume of numerical results suggesting that intermittency is almost universally seen to develop in freely decaying 2D turbulence through a process of vortex mergers [7, 8, 13, 14, 37]. The dynamics are well approximated in this case by a few strong point vortices interacting on a turbulent background field [11, 12, 51]. It turns out, however, that the appearance of intermittent structures (again, historically associated with strong vortices) does not usually occur until after some characteristic decay time $T_c$ is surpassed. This time scale is reported in Smith
Figure 3.19: Evolution of scaling exponents $\zeta_{p,\text{ESS}}$ below horizontal comb at $P_{\text{vap}}$. See the caption of Fig. 3.10 for location of (a)-(f).

and Yakhot [14] in terms of eddy turnovers (or revolutions of the most energetic vortices). They find $T_c$ to be roughly 15.

Since the size and velocity of these eddies is continually evolving (recall §3.2.2), the transformation to a scale based on the number of eddy turnovers $T$, usually involves an integral of the form [14]

$$T = \int_0^t d\tau \; v_{\text{rms}}(\tau) \, k_l(\tau),$$

(3.4)

where $k_l(\tau)$ is the inertial range wave number corresponding to the integral scale, and $v_{\text{rms}}(2\pi/k_l, \tau) = v_{\text{rms}}(l_i, \tau)$ is the RMS velocity associated with that scale. The former
Figure 3.20: Evolution of scaling exponents $\zeta_{p,\text{ESS}}$ below angled combs at $P_{\text{vap}}$. See the caption of Fig. 3.12 for locations (a)-(f). The curve (f) results from a catastrophic shrinking of inertial range.
may be estimated from Figs. 3.10 and 3.12, and the latter from $(\langle \delta v(l_{\text{int}}, \tau) \rangle^2)^{1/2}$. Upon approximation of this integral, we find that the farthest data point downstream of a horizontal comb corresponds at $P_{vap}$ to only 3.2 eddy turnover times, and below a pair of angled combs (again at $P_{vap}$) to only 0.59 eddy turnover times. Therefore, no significant increase in the intermittency is expected in either case as a result of those mechanisms responsible for the development of intermittency at long times in decaying flows.

Even more interesting, perhaps, is the fact that in turbulence forced by a pair of angled combs, the intermittency initially present in the flow is seen to diminish quite rapidly. This disappearance further coincides with the elimination of the second scaling regime, suggesting that the two are correlated. Furthermore, it suggests that the mechanism behind the intermittency in this case is fundamentally different than that occurring at long times in decaying turbulence. One would otherwise expect the decay process to immediately continue, enhancing the strong vortical structures that would presumably already exist.

However, there is a second mechanism by which intermittency has been found to develop in 2D, even in continually forced turbulence. This is as a finite-size effect. Smith and Yakhot [16, 17] report the appearance of intermittent structures in a developing flow when the integral scale reaches the box size (through the energy cascade process) and an excess of energy accumulates at this particular wavenumber. One might, therefore, assert that the intermittency observed in the present experiment
is indeed a finite-size effect. This possibility may seem plausible in light of the fact that below the horizontal comb the largest scales in the inertial range never reach the channel width of 7 cm (Fig. 3.10), whereas the inertial range below the angled combs initially extends at least this far. Furthermore, the disappearance of the second peak in the third moment suggests that these largest scales are quickly dissipated, collapsing the inertial range back to scales below the channel width, and possibly eliminating the mechanism responsible for the intermittency. However, Fig. 3.12 shows no evidence whatsoever of any anomalous energy concentrated in wavenumbers about the channel width. Neither Smith and Yakhot [16, 17], nor Borue [19], saw any evidence of finite-size intermittency effects as long as the $E(k) \propto k^{-5/3}$ scaling range remained intact.

3.6 SUMMARY

The difference between forcing with horizontal and angled combs is one of fundamental importance. Independent of the decay time, turbulence forced by a horizontal comb is characterized by:

- A single enstrophy cascade in the energy spectra;

- A third moment which contains only one scaling range at small $\tau$; and,

- Structure-function scaling exponents near those obtained under the assumption of self-similarity, $\zeta_p = p/3$. 

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However, when forced by a pair of angled combs, the turbulent energy spectra and statistics were found to be strongly dependent on the decay time and to differ initially in a number of important ways from the turbulence forced by a horizontal comb. First and foremost, the energy spectra initially contained both inverse energy and forward enstrophy cascades. Downstream of the combs, however, in the region where the turbulence was no longer forced, the energy cascade quickly disappeared. The third moment and structure-function scaling exponents were found to undergo coinciding changes. That is, initially:

- The third moment indicated two separate scaling ranges, one at small scales corresponding to the enstrophy cascade, and a second at large scales corresponding to the energy cascade; and,

- The structure-function scaling exponents contained significant intermittency corrections.

At longer decay times, however, the energy spectra came to exhibit only a single enstrophy cascade in which:

- The second scaling regime at large separations in the third moment disappeared; and,

- The scaling exponents evolved smoothly toward the self-similar prediction proposed by Kolmogorov.
These results, in agreement with Chapter 2, indicate that the structures responsible for intermittency in 2D here arise as a result of the simultaneous existence of both the energy and enstrophy cascades.
CHAPTER 4

TWO-DIMENSIONAL FLOW STRUCTURES

We saw in Chapter 2 that there are intermittency corrections to the structure-function scaling exponents of 2D turbulence when both the inverse energy and forward enstrophy cascades are simultaneously present. We further isolated the events in the LDV velocity trace that fell in the wings of the longitudinal velocity-difference PDFs and suggested that it was these events specifically that were responsible for the intermittency in the flow.

To determine the actual 2D structures corresponding to these events, an alternative data acquisition method known as particle imaging velocimetry (PIV) will be used extensively. Unlike LDV, which constrains one to the collection of data along a one-dimensional trace through the flow, PIV allows for the instantaneous acquisition of an entire 2D velocity field. This makes possible the investigation of a number of new flow quantities which cannot be obtained from LDV data.

In the current chapter, we will begin by following in much the same thematic line as in §2.4 by exploring the velocity-difference events that fall in the far wings of
the PDFs but this time with the full 2D velocity field surrounding the event at our disposal. We will then seek to reformulate our investigation of these events in a more quantitative context, one which will ultimately allow us to determine exactly what types of structures play the roles key to both longitudinal and transverse intermittency. Finally, we will show that there is an excess of these structures with very large magnitudes beyond that expected for a turbulent flow with Gaussian statistics.

4.1 Experimental Details

Two combs, each 29 cm long, were placed in the film so that they formed an inverted wedge 1.2 cm apart at the top and 5 cm apart at the bottom. The cylindrical teeth had an average spacing of 1.6 mm and a diameter of 0.22 mm. PIV was used to acquire an instantaneous velocity field across a $1.95 \times 2.7$ cm area (width $\times$ height) located 5.7 cm below the last teeth.

4.2 Empirical Examples of Flow Structures

In the literature, the vortex center (actually, a vortex tube in 3D) has been held almost exclusively responsible for intermittency, in both two and three dimensions. However, it should shortly become apparent that these structures alone are not a
sufficient explanation. In fact, as far as the longitudinal velocity differences and structure functions are concerned, vortical structures are not intermittent at all [68].

4.2.1 Longitudinal Structures

Just as was done in Chapter 2 with the LDV data, the probability distribution of the longitudinal velocity differences can be calculated from the PIV measurements. If the same technique is then used, that of flagging the events that fall into the tails of the velocity-difference PDFs, we can go back and examine the actual corresponding flow structures.

An example of one such structure, from the tails of the longitudinal velocity-difference PDF, is shown in Fig. 4.1(a). The structure is not a vortex at all but rather a region which has a distinctly saddle-like shape. A vertical velocity trace is shown in (b) and illustrates the fluctuation in both the longitudinal (solid line) and transverse (dashed line) velocity components as one moves across the structure. The change in the longitudinal component is actually quite tremendous (though the rescaling of the length of the velocity vectors does not do it justice). In a matter of roughly half a centimeter, \( v_y \) changes by almost 80 cm/s.

The structure of the saddle itself is interesting as well. The incoming and out-going branches represent two independent axes of symmetry across which it may be reflected (see Fig. 4.2). This remains true even if the magnitudes of the incoming and outgoing velocities are not the same. Furthermore, the parity of the longitudinal velocity
Figure 4.1: (a) Saddle structure responsible for events in the wings of the longitudinal velocity-difference PDFs with the mean flow velocity subtracted out (region of interest is circled). (b) Longitudinal (solid line) and transverse (dashed line) velocities along vertical slice (gray line). Note, the length of the velocity vectors has been shortened significantly. The RMS fluctuation is 24 cm/s.
Figure 4.2: Asymmetric saddle structures are responsible for energy and enstrophy transfer. Vortex centers and regions of extended shear, are not capable of demonstrating the necessary asymmetry.

difference under this reflection is even. For example, if the saddle is reflected across the outgoing jets and the velocity difference measured between the incoming jets, the magnitude and sign remain the same as in the unreflected saddle. A rotation through 180 degrees causes no change in the velocity difference either. As a result, even if we assume that right- and left-handed structures are created in equal proportion, an asymmetry in the magnitude of the positive and negative velocity differences can prevail (the former corresponding to the outgoing jets, the latter to the incoming). This is very different than the case of a structure which exhibits an odd parity. The equal probability of finding an oppositely-handed structure then ensures that finding a negative velocity difference of the same magnitude is equally likely.
That the saddle points correspond to the longitudinal velocity differences is, therefore, not in disagreement with theory. Both Eqs. (1.9) and (1.10) can be rearranged to show their explicit dependence on the longitudinal structure function of order three, for example,

$$\epsilon = \frac{2 S_3^L(r)}{3r}. \tag{4.1}$$

Furthermore, the $S_p^L(r)$ for odd $p$ can be written completely in terms of the asymmetry in the probability distribution (recall Eq. (2.8)). As a result, the structures responsible for the energy and enstrophy transfer must exhibit a corresponding asymmetry that the saddle points may, indeed, exhibit.

### 4.2.2 Transverse Structures

In a similar manner, the two flow structures corresponding to the wings of the transverse velocity-difference PDFs can be isolated. An example of one of these structures, the vortex center, is shown in Fig. 4.3. In this case, the change in the transverse component of the velocity $v_x$ is large and rapid across the structure though the longitudinal component barely changes at all. This is a strong qualitative indication that vortex centers really play little role in longitudinal intermittency.

A second type of structure has also been seen to contribute to the wings of the transverse PDFs. These are the extended regions of large shear that occur, for example, between co-rotating vortices (see Fig. 4.4). One sees again that the change in the transverse velocity component across the structure is large and abrupt though
Figure 4.3: (a) Vortex center partially responsible for events in the wings of the transverse velocity-difference PDFs with the mean flow velocity subtracted out (region of interest is circled). (b) Longitudinal (solid line) and transverse (dashed line) velocities along vertical slice (gray line).
the longitudinal component barely changes. Note that the vorticity and strain rate are approximately equal in these regions so that the Jacobian determinant is small or zero (the Jacobian matrix is a way of quantifying the structural form of the flow that will be examined in the following section).

The symmetry of these structures is very different in many respects from that of the saddle points. The parity of both with respect to the transverse velocity differences is odd (as it turns out to be for the saddle structure as well). As a result, the PDFs of the transverse velocity differences should be symmetric. The parity of the extended shear region, on the other hand, turns out to be even for the longitudinal velocity differences (independent of the angle $\theta$ of the axis across which the reflection takes place). However, an identical structure rotated through $2\theta$ results in a longitudinal velocity difference of equal magnitude but opposite sign, ensuring that such structures do not contribute to the asymmetry in the longitudinal PDF (and, therefore, not to energy or enstrophy transfer either).

4.3 The Jacobian Matrix

Flow topology has not historically played an active role in the analysis of fluid dynamical systems. The reason for this is likely that, even today, the collection of entire 3D flow fields is not generally experimentally feasible (though the novel technique of scalar imaging velocimetry is allowing for fully 3D velocity measurements under limited circumstances [69, 70, 71]). Flow patterns have, therefore, been analyzed mainly...
Figure 4.4: (a) High shear region partially responsible for events in the wings of the transverse velocity-difference PDFs with the mean flow velocity subtracted out (region of interest is circled). (b) Longitudinal (solid line) and transverse (dashed line) velocities along vertical slice (gray line).
through the use of pathlines traced by smoke or other particulates introduced into the flow. Though advances in computing power are now allowing for fully 3D simulations in which topological structures can be analyzed, comparison with experiment still remains elusive.

Two-dimensional flows, on the other hand, lend themselves exceedingly well to these analyses. It is perhaps the inertia of tradition that has, so far, kept this area of inquiry from widespread investigation. Having now seen a couple of qualitative examples of the types of flow structures that may play a role in 2D turbulent dynamics, we will undertake a more quantitative investigation of the flow topology to see what can be learned about the nature of the intermittent structures in 2D.

Topologically, the points of greatest interest in the flow are the critical points \[72\] where the magnitude of the velocity vanishes. About such points, the flow field can be expanded to first order in a Taylor series,

\[
v(x, y) = (v_x(x, y), v_y(x, y)) =
\]

\[
(v_x|_{x_0, y_0} + (\partial_x v_x)_{x_0, y_0} (x - x_0) + (\partial_y v_x)_{x_0, y_0} (y - y_0),
\]

\[
v_y|_{x_0, y_0} + (\partial_x v_y)_{x_0, y_0} (x - x_0) + (\partial_y v_y)_{x_0, y_0} (y - y_0)). \quad (4.2)
\]

The nature of the flow about this region is thus determined by the four derivatives of the vector field, and is completely defined, up to a vector constant, by the Jacobian
matrix,

\[
\begin{pmatrix}
\partial (v_x, v_y) \\
\partial (x, y)
\end{pmatrix} = \begin{pmatrix}
\partial_x v_x & \partial_y v_x \\
\partial_x v_y & \partial_y v_y
\end{pmatrix}.
\]  \hspace{1cm} (4.3)

The eigenvalues, given by

\[
\frac{1}{2} \left( (\partial_x v_x) + (\partial_y v_y) \pm \sqrt{(\partial_x v_x)^2 + 4(\partial_y v_x)(\partial_x v_y) - 2(\partial_x v_x)(\partial_y v_y) + (\partial_y v_y)^2} \right), \hspace{1cm} (4.4)
\]

are of particular interest as they allow us to infer something about the physical structure of the flow near the critical point. In general, they are complex numbers of the form \(a_j + ib_j\), where \(j \in \{1, 2\}\). The corresponding sign of the \(a_j\) determines whether the critical point is attractive or repulsive. The imaginary components, \(b_j\), correspond to a circulation of the flow about the point.

A diagrammatic representation of a few of the possible flow structures [72] is given in Fig. 4.5. For real eigenvalues, the flow about the critical point is irrotational, as in (a)-(c). For \(a_1, a_2 < 0\) one finds an attracting node, (a); and for \(a_1, a_2 > 0\), a repelling node, (c). For \(a_1 a_2 < 0\), the flow takes the form of a saddle point, as in (b). For complex eigenvalues, the flow is either an attracting focus, \(a_1, a_2 < 0\), as in (d), or a repelling focus, \(a_1, a_2 > 0\), as in (f). A purely imaginary eigenvalue denotes a vortex center, (e).

The number of structures that can be imagined is dramatically reduced if the flow is incompressible. In this case the eigenvalues are simply given by

\[
\pm \sqrt{(\partial_x v_x)^2 + (\partial_y v_x)(\partial_x v_y)}. \hspace{1cm} (4.5)
\]
The disappearance of the terms outside of the square root implies that the eigenvalues can no longer be complex. The existence of attracting and repelling foci in incompressible flows is, therefore, ill-fated. The two eigenvalues must now be equal in magnitude and have opposite sign, thus eliminating the attracting and repelling nodes as well. The structure of 2D incompressible flows is left to a combination of saddle points and centers—with one exception. An eigenvalue of zero may correspond to one further structure, that characterized by a region of linear shear.

Such a view is further confirmed by examination of the determinant of the Jacobian matrix, denoted $J(v_x, v_y)$. It is readily verified that $J(v_x, v_y) = (\partial_x v_x)(\partial_y v_y) -$
\((\partial_x v_y)(\partial_y v_x)\) can be expressed equivalently as [73]

\[ J(v_x, v_y) = \frac{1}{4}(\omega^2 - \sigma^2), \]  

(4.6)

where the squared vorticity is given by \(\omega^2 = \sum_{i,j}(\partial_i v_j - \partial_j v_i)^2/2\), and the squared strain rate by \(\sigma^2 = \sum_{i,j}(\partial_i v_j + \partial_j v_i)^2/2\). This particular representation makes it clear that regions of large positive \(J(v_x, v_y)\) are related to vortex centers, while regions of large negative \(J(v_x, v_y)\) to saddle points (where the strain rate is highest). The Jacobian determinant field corresponding to the saddle-shaped and linear-shear structures in Figs. 4.1 and 4.4 respectively, are shown in Figs. 4.6 and 4.7. Note that the central region of the saddle is indeed very dark (i.e., negative and of large magnitude) and the extended-shear region is quite neutral, as far as the Jacobian determinant is concerned.

4.4 Velocity differences and the Jacobian determinant

4.4.1 Longitudinal velocity differences

The correspondence between the velocity differences and the various structures may be quantified by examining the conditional probability distributions \(P((\delta v)_{\|}, J)\) and \(P((\delta v)_{\perp}, J)\). The longitudinal case is shown in Fig. 4.8. The velocity difference varies along the horizontal axis, such that the events closest to the sides of the PDF cor-
Figure 4.6: Jacobian determinant calculated from the velocity field of Fig. 4.1. Light is positive, dark negative.
Figure 4.7: Jacobian determinant calculated from the velocity field of Fig. 4.4. The principle thing to note here is the absence of any large $J$ in the neighborhood of the shear structure. Light is positive, dark is negative.
respond to the far tails of $P(\delta v)_||$. The asymmetric (top to bottom) and roughly triangular shape demonstrates a strong correlation between the value of the Jacobian determinant and the velocity difference. The tails of the longitudinal velocity-difference PDFs are, in fact, heavily populated by structures with a large negative Jacobian determinant, implying that the strain rate far outweighs the vorticity in importance; that is, the structures are more saddle-like in nature.

Points with relatively large vorticity are not found in the wings. These structures predominate to inhabit the region about $(\delta v)_|| = 0$. This demonstrates that the vortex centers do not contribute appreciably to longitudinal intermittency. One may notice, however, that there is an unexpectedly large concentration of saddle points at particularly small velocity differences. A quick examination of Fig. 4.9 explains why. The symmetry of the saddle points is such that, when the structure is rotated by 45 degrees from the major axes, the longitudinal velocity component vanishes. The various angles of rotation represent an additional variable which continuously scales the magnitude of the longitudinal velocity difference between zero and the value measured along the actual jets. This explains the equal likelihood of finding such structures at any point in the distribution. Notice also, that saddle-like structures with very large strain rate appear to be much less prevalent than centers with comparable magnitudes of squared-vorticity. This asymmetry will be confirmed in the probability distribution of the Jacobian determinant ($\S$4.5).
Figure 4.8: Correlation between longitudinal velocity difference (for a separation of 0.5 cm) and the Jacobian determinant. Intensity denotes $\log P((\delta v)_\parallel, J)$. 
Figure 4.9: The symmetry of the saddle structures is such that at certain angles of rotation the longitudinal velocity component vanishes. This angle then represents a third variable which scales the magnitude of the longitudinal velocity difference between zero and the maximum taken directly along one of the jets.

4.4.2 TRANSVERSE VELOCITY DIFFERENCES

The appearance of the conditional transverse probability distribution, $P((\delta v)_\perp, J)$ (Fig. 4.10), is more complicated than that derived by a simple interchange of the roles played by the center- and saddle-like structures. The shape is more akin to a chevron than a triangle, with sides that appear to bulge slightly.

The concave portion of the PDF (i.e. at positive $J$) demonstrates that vortex centers are not found about $(\delta v)_\perp = 0$. The symmetry of the vortex centers is very different than that of the saddles. Regardless of the orientation of a vortex center, the transverse velocity difference across it remains the same. The scaling of the longitudinal velocity differences across the saddle is completely absent here.
Figure 4.10: Correlation between transverse velocity difference (for a separation of 0.5 cm) and the Jacobian determinant. Intensity denotes $\log P(\delta v_1, J)$. 
The vortex centers are most prevalent in the wings of the transverse PDF where the saddle structures contribute relatively little. Yet, the centers do not appear to be the largest contributor to the far tails. The structures corresponding to the largest transverse velocity differences actually have $J(u_x, u_y) = 0$ (i.e., the strain rate and vorticity contributions are equal, though this indicates nothing about their actual magnitudes). This indicates that the largest contribution to transverse intermittency comes from the extended regions of large shear.

4.4.3 Transverse versus longitudinal scaling exponents

Theoretical predictions for the scaling behavior of the transverse structure functions in 3D are generally absent. The literature contains a number of contradictory experimental and numerical results, some suggesting that the transverse moments scale in an identical fashion to the longitudinal (e.g. [74, 75]), others showing that the transverse moments are significantly more intermittent, and have smaller scaling exponent $\zeta_p$ (e.g. [76, 77, 78, 79]).

An attempt to explain the conflicting results was put forward by L’vov, et al. [80]. They recognized that the relevant symmetry group for a homogeneous isotropic medium was the rotation group SO(3) and, correspondingly, constructed the longitudinal and transverse structure functions from a basis of spherical harmonics, $Y_{i,m}$.
Figure 4.11: Ratio $R_p = S_p^L(r)/S_p^T(r)$ showing the difference in scaling exponents of the longitudinal and transverse structure functions.
The various $S_p^L(r)$ and $S_p^T(r)$ may then be formed as a sum of scalar functions of $r$ which, in principle, can have different scaling exponents, as well as different weights. As a result, though the asymptotic behavior is expected to be the same, the exponents for a limited scaling range may be quite different. There are two notable exceptions: incompressibility suggests that the second order structure functions scale the same, and a combination of incompressibility and constant energy dissipation suggest that the third order structure functions with absolute value (i.e. $G_p(r)$) do as well [80].

The difference in the scaling exponents can be determined empirically by calculating the separation dependence of the ratio [76]

$$R_p = S_p^L(r)/S_p^T(r).$$ (4.7)

The resulting scaling is shown in log-log form in Fig. 4.11. Fits to the $R_p$ demonstrate that the transverse structure functions are, indeed, different from the longitudinal, with mean scaling exponents given by $\zeta_p^T = \zeta_p^L - \Delta \zeta_p^T$ (this quantity itself, is likely $r$ dependent), where $\Delta \zeta_2^T = 0.0071 \pm 0.0036$, $\Delta \zeta_4^T = 0.091 \pm 0.011$, and $\Delta \zeta_6^T = 0.20 \pm 0.023$. The scaling exponents determined using extended self-similarity are shown independently in Fig. 4.12.
Figure 4.12: Longitudinal (●) and transverse (○) ESS scaling exponents. The transverse moments are indeed found to be more intermittent. Upper line is self-similar K41 prediction; lower, the log Poisson prediction. Recall that in Chapter 3, the energy cascade at $P_{\text{atm}}$, when forced with the shorter angled combs, was less well formed than that examined in Chapter 2. It is, therefore, not unexpected that the longitudinal intermittency here, under the same forcing conditions, is less significant than in Chapter 2 as well.
4.5 Probability distribution of the Jacobian determinant

Some measure of explanation for the difference in the longitudinal and transverse intermittency can be achieved through an examination of the probability distribution of the Jacobian determinant $P(J/J_{\text{RMS}})$, shown in Fig. 4.13. Two features stand out immediately. First, the PDF is non-analytic at $J = 0$, a characteristic that arises from the fact that saddles and centers are topologically distinct. Second, the probability distribution is clearly asymmetric. The direction of this asymmetry is in agreement with the recent results of Rivera, Wu, and Yeung [73]). Furthermore, they found that the form of the PDF in a magnetohydrodynamically-driven horizontal soap film was, in general, independent of the forcing configuration when $J(v_x, v_y)$ was rescaled by $J_{\text{RMS}}$ (i.e. $J' = J/J_{\text{RMS}}$).

That the scaling exponents of the transverse moments deviate more strongly than the longitudinal from the self-similar prediction of $\zeta_p = p/3$ proposed by Kolmogorov, is, therefore, not surprising [68]. Structures having a very large vorticity appear to be considerably more prevalent than structures with a strain rate of comparable magnitude. Since such structures contribute significantly only in the wings of the transverse velocity-difference PDFs, their existence is manifest only in the corresponding structure functions. Furthermore, events with equal, but very large magnitude, vorticity and strain rate are doubtless hiding about $J(v_x, v_y) = 0$ but contribute only to the tails of the transverse PDFs.
Figure 4.13: Probability distribution of the Jacobian determinant. The solid line is the theoretical prediction for $b = 1/3$. The upper and lower dashed lines show predictions for $b = 0.46$, and $b = 0.20$ respectively ([73]).
Rivera, et al. further derived an integral form for $P(J')$ by assuming that the flow was a translationally invariant Gaussian random field. The shape of the resulting PDF depends on only one free parameter [73],

\[ b = -\frac{\langle (\partial_x v_y)(\partial_y v_x) \rangle}{\sqrt{\langle (\partial_x v_y)^2 \rangle \langle (\partial_y v_x)^2 \rangle}} = \frac{\partial^2_x \partial^2_y M}{\sqrt{\partial^2_x M \partial^2_y M}}, \]  

(4.8)

where $M(u, w) = \langle \psi(x, y) \psi(x', y') \rangle$, $u = |x - x'|$, and $w = |y - y'|$. The stream function $\psi$ is a scalar field that is often used to represent 2D flows. The velocity is related to the stream function by $v_x = \partial_y \psi$ and $v_y = -\partial_x \psi$ (the contours of the stream function give the direction of the velocity at all points in the flow).

In general, the $P(J')$ must be numerically integrated. However, if a further assumption is made that $M$ is elliptically symmetric, that is $M(u, w) = M(u^2 + cw^2)$ with some constant $c$, then $b$ reduces to 1/3. The $P(J')$ then becomes integrable, taking the form [73]

\[ P(J') = \frac{2C\pi^{3/2}}{3} e^{3J'/2} \times \left\{ \begin{array}{ll} 1 & \text{for } J' < 0, \\ \Phi_c \left( \frac{3}{2} \sqrt{J'} \right) & \text{for } J' > 0, \end{array} \right\}, \]  

(4.9)

where $\Phi_c$ is the complementary error function.

Although we see the same basic features as Rivera, et al. (namely, the non-analytic zero point and overall asymmetry), our results deviate more strongly from the theoretically predicted curve than theirs. One possibility is that, though empirically we find $b = 0.339$ (in excellent agreement with the elliptic assumption), the RMS deviation is actually quite large, $\sqrt{(\langle b^2 \rangle - \langle b \rangle^2)} = 0.135$. However, we have included in Fig. 4.13 the curves predicted for $b = 0.46$ and $b = 0.20$ (the upper and lower
Figure 4.14: Probability distributions of the Jacobian determinant from top 0.26 cm of imaged region (dashed line) and bottom 0.26 cm of imaged region (dotted line). In each case the shape is essentially identical suggesting that vertical inhomogeneities in the flow are not the cause of the differences from theory.

dashed lines respectively), and find that this variation accounts for neither the greater curvature of the empirical result nor the excessively large tail.

These findings suggest that one or both of the other assumptions, must be violated. Either the velocity field is inhomogeneous or it is not strictly represented by a Gaussian random field. Because the turbulence evolves as it moves downstream, the flow field is not strictly homogeneous. The mean downstream velocity increases by 0.59% over the 2.7 cm high region of film that we image, while the magnitude of the fluctuations decreases by a more significant 6.8%. However, the shape of $P(J')$
Figure 4.15: Probability distributions of the Jacobian determinant taken from a 0.12 cm vertical stripe about the region where the velocity reached its maximum in Fig. 6.16 (dashed line), and from the left 0.12 cm of the imaged region (dotted line).

appears to be unaffected by this inhomogeneity. For example, a PDF computed from the top 2.6 mm of the imaged region can be compared to one computed from the bottom 2.6 mm. The root mean square (RMS) fluctuation in the Jacobian determinant decreases from 46.7 s⁻² at the top, to 36.0 s⁻² at the bottom. However, the shape of scaled $P(J')$ is virtually identical (Fig. 4.14). Inhomogeneity in the flow due to this small amount of temporal evolution is, therefore, not likely the cause of the discrepancy between experiment and theory.
The same sort of comparison can be made between PDFs computed from different vertical stripes in the flow. The greatest difference is between regions where the three quantities in Fig. 6.16 differ the most. We, therefore, compare $P(J')$ computed at $x = 0.6$ and $x = 1.95$ cm. The two curves are shown in Fig. 4.15. In this case, the difference is no longer negligible. Compared to the flow nearer the film boundaries, the central region of the film has a deficiency of structures with large strain rate, and a corresponding excess of structures with large vorticity. Both deviate significantly on one side or both from the theoretical prediction. This suggests that the additional curvature of the empirical $P(J')$ is, at least, not simply the result of combining probability distributions from different regions of the flow which have, for example, different mean values or fluctuation magnitudes. It, however, cannot eliminate the possibility that such inhomogeneities in the flow combine through the non-local Navier-Stokes equation in such a way as to fundamentally alter the Gaussian nature of the turbulence.

4.6 **VORTICITY PROBABILITY DISTRIBUTION**

The anomalously large tail of $P(J')$ for large positive values of the Jacobian determinant (that is, for vortex center type structures) is further confirmed by an examination of the vorticity probability distribution itself. As it, like the velocity differences, is a derivative-related variable, unusually broad tails in the PDF (i.e. wider than Gaussian) are a further indicator of intermittency [14]. The particularly strong coherent
structures which are at least in part responsible for intermittency, dominate these wings.

The probability distribution of the vorticity in the current flow is shown in Fig. 4.16. The tails of the PDF are significantly broader than Gaussian, falling off in an essentially exponential fashion. The slight shift in the central region of the PDF toward positive $\omega$ is thought to be the result of an effective net circulation of the flow. This arises from an asymmetric variation in the mean downstream velocity across the imaged region of the flow.
4.7 Summary

In the present chapter, the structures of largest magnitude, those responsible for intermittency, were qualitatively identified as belonging to one of three basic categories: vortex centers, extended regions of high shear, or saddle-like structures. The Jacobian matrix and its determinant were introduced to quantify the structural nature of the flow field. Regions where the Jacobian determinant was large and positive corresponded to vortex centers; regions with large negative Jacobian determinant correspond to saddle-like points. In extended regions of large shear, the determinant nearly vanished.

The conditional probability distributions relating the Jacobian determinant to the longitudinal and transverse velocity differences indicated that:

- Saddle-like structures are solely responsible for the intermittent longitudinal statistics;

- Strong vortex centers, previously thought to be at the root of longitudinal intermittency, play no direct role; and,

- Intermittency in the transverse statistics is the result of contributions both from strong vortex centers and extended regions of large shear, the latter being most intermittent.

Intermittency in the transverse velocity differences was stronger than in the longitudinal statistics. This was likely the result of an asymmetry in the probability dis-
tribution of the Jacobian determinant, with strong vortex centers being significantly more likely than saddle-like structures of comparable magnitude. Anomalously strong vortices were found in the wings of the vorticity PDF, as well, with tails decaying more slowly than expected under the assumption of self-similarity.
CHAPTER 5

COUPLING OF FILM THICKNESS TO TURBULENT DYNAMICS

Illumination by monochromatic light is a visualization technique often used with soap films. Stunning patterns result from the interference of light reflected from the front and back surfaces of the film. Yet, the fact that it works so well suggests that not only must there be some variation in the film thickness, but that such variations are associated to some degree with the flow structure (see Fig.5.1).

In this chapter, we will see that there is some correlation between the thickness and vorticity fields, confirming both previous experimental and theoretical results. From a look at the conditional probability distribution of the thickness and Jacobian determinant, however, we will see that the thickness is correlated only to vortex-center-like structures in the flow and not to the structures responsible for longitudinal intermittency. That thickness fluctuations do not play a role in the intermittent statistics is further confirmed by the conditional probability distributions comparing the lon-
Figure 5.1: Soap film illuminated by monochromatic light source. The four circular objects across the top are the tips of comb teeth used to incite turbulence. Mean flow direction is down.
gitudinal velocity difference to both the absolute film thickness and the thickness difference.

We will further show that variations in the effective mass density of the film, which is linearly proportional to the film thickness, play no role in modifying either the downstream or cross-stream velocities. Additionally, we will argue that, although there is an inverse relation between the effective film viscosity and the film thickness, such variations are not significant for the scale of structure in which we are interested. Dynamic thickness fluctuations will also be investigated. We will show that numerical error plays a particularly large role in the computation of the divergence of the velocity field; however, even with that being the case, the computed compressibility of the film appears to be negligibly small. This is further confirmed by our findings that no correlation exists between the film compressibility and the intermittent structures in which we are interested.

5.1 Correlation between thickness and vorticity fields

The prevalence of circular shapes in the thickness field of Fig. 5.1, gives one the impression that a correlation between the thickness and vortex centers may exist. Following in this vein, we more closely examine the vorticity field (Fig. 5.2) of the vortex center previously shown in §4.2.2 (Fig. 4.3). A comparison with the thickness
field (Fig. 5.3) shows that the location and major features of the central vortex appear to be echoed in the thickness field. However, the fact that the thickness field is always positive, while the vorticity may be negative, indicates that the thickness field is more likely related to the modulus of the vorticity or to the squared vorticity.

This observation is not in disagreement with theoretical expectations. Chomaz suggests two limiting behaviors [82, 83, 49] depending on whether the flow dynamics are fast (Marangoni) or slow (Gibbs) compared to the diffusion time of new surfactant to the film surfaces (recall §1.3.2). If the film velocity is taken to be small compared to the symmetric wave speed of Eq. (1.23), the dynamic variables can be expanded in a series about the corresponding Mach number. At second order the dimensionless thickness field is given by [82]

$$h = \Gamma + \frac{e}{2k},$$

where $h$ and $K$ have been rescaled by the mean thickness $\langle h \rangle$, and $\Gamma$ by the typical surface concentration, $c_0 K \langle h \rangle / (\langle h \rangle + 2K)$, computed from Eq. (1.17). In the Marangoni limit, the dominant contribution to the thickness field is from the surface excess which plays a role much akin to a pressure field [82]. This is not surprising in light of Eq. (1.15), where a 2D equivalent of the ideal gas law was derived, demonstrating that the 2D pressure is, at least in the limit of low concentration, proportional to the surface excess. The same is true even in the Gibbs limit, if $k$ is large (i.e., $\langle h \rangle \ll 1$). That is, for small thicknesses, little surfactant is contained in the bulk 'reservoir' and a correspondingly small relaxation of the surface excess takes place.
Figure 5.2: Vorticity field containing a strong vortex center (see corresponding velocity field in Fig. 4.3).
Figure 5.3: Thickness fields corresponding to the same region of the flow as Figs. 4.3 and 5.2. The thickness field was constructed in two different manners. In (a) the field was computed as described in §6.3.2. The individual elements in (b), however, correspond to the total average intensity including the TiO₂ particles and have been rescaled by the corresponding local average over many separate images. This has the effect not only of eliminating apparent thickness changes due to variations in the lamp intensity but also of dividing out any thickness inhomogeneity. The agreement between the two is quite good.
For thicker films and longer times, however, the $e$ field can play the dominant role. In such limit, the thickness field begins to "look like" the modulus of the vorticity $[82]$. 

Such a correlation was experimentally verified by Rivera, Vorobieff, and Ecke $[84, 85]$. They further suggest that this correlation may be strengthened as a result of the thick miniscus about the comb teeth being peeled off and wrapped into the shedding vortices, thereby creating fluctuations in the thickness and vorticity fields simultaneously. Results from the present experiment show a correlation between the vorticity and thickness fields as well. The probability, $P(\omega, h)$, shown in Fig. 5.4, suggests that regions of high vorticity (e.g., vortex centers) are primarily composed of thinner film whereas regions of low vorticity are generally thicker.

The mean cross-correlation between two fields is given by $[85]$ 
\[
\chi(\alpha, \beta) = \frac{\langle (\alpha - \langle \alpha \rangle)(\beta - \langle \beta \rangle) \rangle}{\sigma_\alpha \sigma_\beta}.
\] (5.2)

Substituting the thickness $h$ and enstrophy $\omega^2$, we find $\chi(h, \omega^2) = -0.296$ where, in agreement with the aforementioned probability distribution, the negative sign implies that regions of higher enstrophy are related to thinner films. The magnitude of $\chi$ is in reasonable quantitative agreement with other results at very short decay times $[84, 85]$. Note for comparison that the dimensionless decay time here is given by $y\sqrt{\langle \omega^2 \rangle}/\langle u_y \rangle = 3.2$, where $y = 7.5$ cm is the distance below the bottom of the combs. In these experiments, the magnitude of the cross-correlation was seen to initially increase as the film further decayed. This somewhat surprising fact was attributed to a finite compressibility of the film. As we do not have $\chi$ as a function of the decay...
Figure 5.4: Conditional probability distribution $P(h, \omega)$. The lobed structure indicates that the film is typically thinner in the central regions of vortices where the vorticity is highest. The thickest regions of the film all have little or no accompanying vorticity.
time, it is impossible to tell whether a similar increase in the correlation would be seen here. We will show in a later section that the compressibility of the present film is at least an order of magnitude smaller than that reported in the experiments of Rivera, Vorobieff, and Ecke [84] (and even this value is thought to be largely dominated by numerical error).

The asymmetry that exists between the lobes of positive and negative vorticity in Fig. 5.4 is a result of the inhomogeneity in the flow (Fig. 6.18), both with respect to the vorticity itself and, further, through a similarity in the spatial inhomogeneity of the vorticity and thickness fields (Fig. 6.16). This is, however, not the sole cause of the particular shape of \( P(\omega, h) \). A PDF constructed from a thin vertical stripe about \( d\langle h \rangle/dx = \langle \omega \rangle = 0 \) shows the same lobed structure, indicating that a correlation between the vorticity magnitude and film thickness exists independent of any inhomogeneity.

5.2 TURBULENT STRUCTURES AND THE FILM THICKNESS

As the intermittent 'center' structures are regions of particularly large vorticity, one might ask more generally if the structures responsible for longitudinal and transverse intermittency are as a rule correlated to particularly thin or thick regions of the film. The results of such an investigation are shown in Fig. 5.5.
Figure 5.5: (a) Probability distribution $P(h, J)$ showing correlation between the Jacobian determinant and film thickness (intensity is the log of the probability). (b) Thin black lines are a contour plot of the empirical distribution above. The thick gray line is the distribution $P(h)P(J)$, which in the limit that $h$ and $J$ are completely independent variables, should be equivalent to $P(h, J)$. 

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Recall that the Jacobian determinant, given by $J(v_x, v_y) = 1/4(\omega^2 - \sigma^2)$, indicates something about the prevailing nature of the local flow structure; large positive $J$ indicating vortex center-like regions and large negative $J$ suggesting saddle-shaped structures. An examination of Fig. 5.5(a) suggests that for regions of the flow where the vorticity outweighs the strain rate (that is, for vortex centers), there appears to be a slight skewing of the distribution suggesting that these structures are more often associated with thinner regions of the film. This is not necessarily indicative that thin regions of the film are the cause of particularly large vorticity structures, as would have to be the case if absolute film thickness were to play a causative role in the transverse intermittency, but rather it is likely just a confirmation of the findings of §5.1.

In contrast, the portion of (a) corresponding to negative $J$ (that is, corresponding to regions of the flow which are predominantly saddle-like), is significantly less biased, if at all. The degree to which the probability distribution is skewed can be directly visualized by comparing the empirical distribution $P(h, J)$ with the distribution $P(h)P(J)$ (contour plots are shown in Fig. 5.5(b) for easier comparison). In the case that the thickness and Jacobian determinant fields are completely independent, the shape of these two distributions should be identical. However, there are significant variations between the two distributions. Most noticeable is the decrease in film thickness in regions of large vorticity (though this depletion persists even to relatively small squared-vorticity magnitudes). We also see a weak correlation be-
tween saddle-like structures and film thickness, these structures being slightly more prevalent than expected in thicker regions of the flow. In neither case are the structures of largest magnitude correlated either to the very thinnest or thickest regions of the film, indicating that variations in the absolute film thickness are not the cause of intermittency.

5.3 Intermittent velocity differences

We may verify this result more directly through the conditional probability distribution relating the velocity differences to the thickness field. If rare occurrences of anomalously thick or thin film (that is, regions of very large or small mass density), are indeed the cause of the intermittent velocity differences, there should be a direct correlation between the velocity differences of particularly large magnitude and the local film thickness, or perhaps thickness difference.

A plot of \( P(h, (\delta v(r))) \) is shown in Fig. 5.6 for a separation of \( r = 0.5 \) cm. The shape of the PDF is essentially Gaussian, exhibiting none of the lobed structure expected of velocity differences that are strongly correlated to the thickness field (e.g., as the conditional vorticity distribution was). The same is true when the conditional probability of the velocity and thickness difference between the two points in question is plotted (Fig. 5.7). These two results, \( P(h, (\delta v(r))) \) and \( P(\delta h, (\delta v(r))) \), directly show that intermittent events are not directly related to either particularly thick or thin regions of the film, nor to particularly sharp fluctuations in film thickness.

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Figure 5.6: Probability distribution $P(h, (\delta v(r)|_\parallel))$, for $r = 0.5$ cm. The essentially Gaussian shape suggests that large magnitude velocity differences are not correlated to, for instance, particularly thick or thin regions of film. The intensity varies logarithmically with $P$. 

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Figure 5.7: Probability distribution $P(\delta h, (\delta v(r\hat{y}))_\parallel)$, for $r = 0.5$ cm. The essentially Gaussian shape again suggests that large magnitude velocity differences are not correlated to large differences in film thickness between the two measurement locations. The intensity varies logarithmically with $P$. 
5.4 FILM THICKNESS AND THE VELOCITY MAGNITUDE

Film thickness fluctuations and, hence, density fluctuations may play a role in the turbulent dynamics through corresponding variations in the momentum and gravitational forces. Figure 5.8(a) demonstrates a correlation which is apparently quite strong between the downstream velocity, $v_y$, and the film thickness. There are two possible causes. It may be that small regions of thick film fall faster than other areas within the same region of interest (i.e., within a distance $r$) as a result of density fluctuations. However, it turns out that the majority of the correlation between the thickness and downstream velocity arises simply because of a similarity in the spatial inhomogeneity of the two fields (Fig. 6.16). When the corresponding conditional probability distribution, $P(h, v_y)$, is constructed only from data points along a single vertical stripe in the field about $\partial(v_y)/\partial x = \partial(h)/\partial x = 0$, the apparent correlation between thickness and downstream velocity disappears (Fig. 5.8(b)). The implication is that the downstream velocity, where both momentum and gravitational effects play a role, is not strongly affected by variations in the film thickness.

That the cross-stream velocities as well are independent of film thickness can be seen in the probability distribution $P(h, v_x)$ (Fig. 5.9). Unlike in the case of the downstream velocity, the relative absence of inhomogeneities in the mean cross-stream velocity (as we will see in Fig. 6.17) ensures that an accurate description of
Figure 5.8: Correlation $P(h, v_y)$, between thickness and downstream velocity. (a) Using the entire velocity field, and (b) using only a single vertical stripe of the velocity and thickness fields to eliminate the effects of similarities in the inhomogeneity of the two fields. The general symmetry of the latter suggests that fluctuations in the gravitational force on the fluid elements due to density variations play a negligible role in the turbulent dynamics. Intensity is log of $P$. 
the correlation between $h$ and $v_x$ can be achieved using data from the entire width of the velocity field.

5.5 EFFECTIVE FILM VISCOSITY

Soap films are composed of two different species with vastly different rheological properties. The viscosity of the surfactant interfaces, $\nu_s$, greatly exceeds that for the bulk solution, $\nu_b$. For very thin films, the largest contribution to the effective viscosity, $\nu_{\text{eff}}$, comes from the film surfaces, whereas for thicker films, the bulk solution plays the
dominant role. The result is an added complexity in the equation of motion for the soap film in that the kinematic viscosity is dependent on film thickness. Specifically, we have [86]

\[ \nu_{\text{eff}} = \frac{2\nu_s}{h} + \nu_b. \]  

(5.3)

Vorobieff and Ecke [1] find an asymptotic value \( \nu_b \) only 1.5 times the viscosity of pure water (extracted from the vortex-shedding frequency of a cylindrical tooth). However, at the mean film thickness in the present experiment \( \langle h \rangle = 4.9 \mu m \), the viscosity may be between 6 and 8 times that of the bulk. This may, at first glance, appear catastrophic; however, recall that the range of turbulent scales in which we are interested is precisely that for which the effect of the viscosity is negligible. Throughout the inertial range, the momentum term dominates the turbulent dynamics to the extent that the Euler equation suffices to describe the fluid motion (recall §1.1.1). The viscous term serves only to dissipate energy from the smallest scales in the system. The turbulent dynamics are independent of the exact form of this term to the extent that it is often replaced in numerical simulations by a hyper-viscosity term of the form \( \nu \nabla^{2n} \mathbf{v} \), where \( n \) is taken to be much larger than 1 (often 4 or even 8). As a result, variations in the effective viscosity have a negligible effect on the turbulent dynamics. What effect does exist likely contributes mainly to a blurring of the transition between the inertial and dissipation ranges.
5.6 Dynamic thickness fluctuations, film compressibility

So far the assumption of incompressibility has implicitly been made throughout. This is not necessarily a poor assumption, even if sizeable thickness variations exist in the film (Fig. 5.11). Compressibility itself is only an issue if these thickness fluctuations are dynamic on the time scales pertinent to the system. We found in §1.3.5 that this was certainly not true if the Laplace pressure was the only driving force behind dimple refill. If such thickness variations are carried along merely as a passive scalar, the divergence of the flow will, in fact, vanish.

In the following sections we will show that:
Figure 5.11: Probability distribution of film thickness. Superimposed humps are the result of digital quantization. See §6.3.3 for more details.

- the major contribution to the apparent divergence in the velocity field is numerical error, rather than an actual compressibility of the film;

- the magnitude of the mean-square divergence is small, both compared to a measure of the turbulent dynamics in the flow, and to earlier measurements of this same quantity in thicker films [84]; and,

- the computed compressibility is correlated neither to particular structures in the flow, nor to particularly thick or thin regions of the film.
5.6.1 **Numerical Error**

Unlike the vorticity, which is inherently quite large in a turbulent flow, the compressibility of a fluid element is expected to be small, perhaps vanishingly small. Such an outcome, however, relies on the exact cancellation of four independent and empirically-obtained velocity vectors. As a result, any error inherent in the determination of these vectors has the potential to play the significant role in the computation of the divergence. A plot of $\nabla \cdot v$ (Fig. 5.12) shows that the divergence, indeed, tends to fluctuate more rapidly than the vorticity or Jacobian determinant computed from the same velocity field. In fact, a calculation of the ratio of the RMS difference between neighboring points to the RMS value yields 0.41 for the vorticity, 0.66 for the Jacobian determinant, and 1.11 for the divergence.

Furthermore, a plot of the spatial variation of the mean square divergence in the imaged region of the film is somewhat intriguing. Though some cross-stream inhomogeneity is found in most of the measured quantities, they tend to monotonically decay downstream in a relatively smooth fashion. The mean square divergence, however, is not monotonically decreasing (Fig. 5.13). One explanation lies in the variation of the intensity with which the film is illuminated. Though a dispersive element is used on both of the strobe lamps, some variation between the light intensity at the center and edges of the imaged region remains. The calculation of the velocities in the fainter perimeter may, therefore, be less robust. Some background on the principles used in the determination of these vectors, as well as a rudimentary discussion of
Figure 5.12: Divergence field $\nabla \cdot \mathbf{v}$ for the velocity field in Fig. 4.3.
computational error, is given in §6.2 (a full discussion of the error in particle-imaging velocimetry calculations is beyond the scope of this thesis, being itself the focus of a sizeable body of literature).

A comparison of the spatial variation of the strobe intensity and mean square divergence can be made to quantify their correlation. Since the calculation of the velocity vectors requires two images which are each illuminated by a separate strobe, we first construct a quantity of the form,

\[ I_{12}(x, y) = \frac{I_1(x, y)I_2(x, y)}{\langle I_1 \rangle \langle I_2 \rangle}, \]

where \( I_1 \) and \( I_2 \) are the illumination intensity fields from the first and second strobe respectively. The correlation between the product of the intensities and mean square divergence fields is then given by \( \chi(I_{12}, \langle (\nabla \cdot \mathbf{v})^2 \rangle) = -0.59 \), suggesting that regions of particularly large divergence are indeed found in areas of the film where the illumination is faintest.

### 5.6.2 Magnitude and Spatial Correlation

Since the contribution to any given velocity vector from numerical error is a random variable with magnitude and sign that are reasonably independent of the error in nearby vectors, the spatial correlation of the divergence provides an indicator of flow compressibility that is reasonably free of the effects of noise. An investigation of the spatial correlation of the divergence, \( \chi((\nabla \cdot \mathbf{v})_y, (\nabla \cdot \mathbf{v})_{y+r}) \) (Fig. 5.14), does indeed indicate that the film is, in actuality, at least slightly compressible.
Figure 5.13: Spatial variation of the mean square divergence $\langle (\nabla \cdot \mathbf{v})^2 \rangle$. 
One notable exception to the spatial independence of the noise contributions arises as a result of the fact that calculations of the divergence at locations on a grid include terms of the form \((v_{i+1} - v_{i-1}) \cdot \hat{y}\). The computation of the divergence at \(i\) and \(i + 2\), therefore, includes one velocity vector in common. The result is that, independent of any real film compressibility, a calculation of the divergence of randomly generated noise would result in an anti-correlation of \(-1/4\) between these two spatial locations. Each of the other points in the spatial divergence correlation is free of this contribution. To reduce the effect of any inhomogeneity in the divergence field, we took \(r\) parallel to \(\hat{y}\), and only performed the average in \(\chi\) over \(y\) in a vertical stripe about \(x = 0.6\) cm.

The spatial extent of the correlation (Fig. 5.14) indicates that the film compressibility is most significant at scales on the order of 0.25 cm. This is roughly half the size of the smallest intermittent structures we have investigated (\(~0.5\) cm and up), suggesting that the role compressibility plays is small. Furthermore, even if numerical error does not make as significant a contribution as we suspect it does, the magnitude of the divergence is generally quite small though some variation certainly does exist (Fig. 5.15). However, we are primarily interested in the degree to which this affects, or is caused by, the turbulent structures in the flow. As such, a more useful quantity is one which characterizes the relative compressibility of the film with respect to the intensity of the turbulence. One such dimensionless ratio compares the mean square divergence \(D = \langle(\nabla \cdot \mathbf{v})^2\rangle\) to the enstrophy \(\Omega = \langle\omega^2\rangle\). This quantity was formerly re-
Figure 5.14: Mean correlation $\langle \chi \rangle$ of the divergence at locations separated by a distance $r$ (the $\langle . \rangle$ here indicates an average over points along a vertical stripe in the film at $x = 0.6$ cm). The data point with $\chi \sim -0.2$ includes an excess anti-correlation due to noise effects which is not present at other scales (see text for explanation). The gray line is merely a guide to the eye.
Figure 5.15: Probability distribution of the velocity divergence.

ported by Rivera et al. [84] for much thicker soap films (e.g., 30 \( \mu m \)). They found the ratio \( D/\Omega \) to be between 0.1 and 0.2, independent of the downstream position (i.e., decay time). For the thinner films used in the present experiment ((\( h \)) = 4.9 \( \mu m \)), the relative compressibility is roughly an order of magnitude smaller, \( D/\Omega = 0.025 \). This further indicates that the film compressibility may be quite negligible.

It was recently suggested [87] that a more pertinent comparison might be between the term representing momentum dissipation due to compressibility (that is, the gradient of the velocity divergence term, \( \eta' \nabla (\nabla \cdot \mathbf{v}) \), present in the compressible N-S
equation), and the inertial term, $\rho (v \cdot \nabla) v$, thereby directly comparing the importance of the compressive and turbulent forces in the flow. The resistance of the soap film to compression arises from two separate mechanisms, given that the solution itself remains incompressible. First, there is a Laplace pressure due to the induced interfacial curvature of the compressed region (Fig. 5.16) and, second, there is a retarding surface tension gradient that arises as a result of an increase in the surface excess $\Gamma'$ when the area of the film element is decreased.

Looking first at the Laplace pressure, if a film element with sides of length $l$ and thickness $h$ is compressed slightly along one axis so that the thickness changes in the center by $\Delta h$, the excess volume contained in the two (assumed) conical slivers is given by

$$dV = h \, dA = 2 \int_{-l/2}^{l/2} \sqrt{r_d^2 - x^2} \, dx - (r_d - \Delta h) l \approx \frac{4}{3} l \Delta h, \quad (5.5)$$

where $r_d$ was defined in Eq. (1.25) (with the substitution $R \rightarrow l/2$). The result may be rearranged to give an expression for the thickness change $\Delta h$ as a function of small
variations, \( dA \), in the area of the film element. This result is substituted back into \( r_d \), and that expression into \( \Delta p = \gamma/r_d \) (from Eq. (1.21)) to obtain an expression for the force per unit length of side necessary to overcome the Laplace pressure,

\[
h \Delta p \simeq \frac{6\gamma h^2 dA}{A^{3/2}}. \tag{5.6}
\]

With regards to the second mechanism of resistance, the force per unit length necessary to overcome the surface tension gradient due to a small change \( dA \) in the film element area is simply given by the elasticity (recall Eq. (1.16)). Plugging in reasonable values for the pertinent quantities in both results, we find that the resistance to compression from the Laplace pressure is 5 to 6 orders of magnitude smaller than that from the film elasticity. The effect of the former can, therefore, be neglected.

Though one may expect that the soap film analog to the second coefficient of viscosity (which we will denote \( \eta_\gamma \)) involves the elasticity of the film, the exact form of the coefficient is perhaps not so obvious. We begin by noting that the force on a fluid element due to surface tension effects is given by

\[
F = \left( \frac{\partial \gamma}{\partial x} \right) \Delta x \Delta y. \tag{5.7}
\]

Furthermore, the coefficient \( \eta_\gamma \), which indicates the film’s resistance to compressive forces, should be determined by an equation of the form,

\[
\frac{F}{A} = \eta_\gamma \left( \frac{\partial v_x}{\partial x} \right). \tag{5.8}
\]
Rearranging Eq. (5.7) we find

\[ \frac{F}{A} = \left( \frac{\partial \gamma}{\partial x} \right) \]

\[ = \left( \frac{\partial \gamma}{\partial v_x} \right) \left( \frac{\partial v_x}{\partial x} \right) \]

\[ = \left( \frac{\partial \gamma}{\partial h} \right) \left( \frac{\partial h}{\partial t} \right) \left( \frac{\partial t}{\partial v_x} \right) \left( \frac{\partial v_x}{\partial x} \right). \]  

(5.9)

However, \( E = -2h(\partial \gamma/\partial h) \) and \( \partial h/\partial t + h \nabla \cdot v = 0 \) (where the first equation defines the film elasticity and the second just implies that the solution itself is incompressible).

Making these substitutions gives

\[ \eta_\gamma = \frac{1}{2} E (\nabla \cdot v) \left( \frac{Dv}{Dt} \right)^{-1}, \]  

(5.10)

where the total derivative is necessary here since \( v \) indicates the velocity of an individual film element, not the velocity at a static location in the flow (recall §1.1.1).

The issue of compressibility in a soap film is thus somewhat more complicated than for simple fluids. This is a result of the soap film effectively having some memory of the dynamics that have occurred in the past. For instance, a large velocity divergence in a film element at one instant results in additional surfactant accumulating on the element interfaces, thereby increasing the surface tension gradient and making further compression more difficult. As well, the very inertial term to which we would ostensibly like to compare the compressibility also appears explicitly in \( \eta_\gamma \). Its presence is not fully understood, but may imply that the compressibility actually increases in importance in a nonlinear way as the intensity of the turbulence increases. Unfortunately, because of the time derivative of the velocity, it is not possible for us to
actually determine this coefficient. Doing so requires a series of three PIV images (i.e.,
two separate velocity fields, each separated by a very small time difference) which is
not experimentally feasible at present.

5.6.3 RELATION TO FLOW STRUCTURES AND THICKNESS

If compressibility is a fundamental cause of intermittency, it is not unreasonable
to suspect that the particularly intense structures associated with longitudinal and
transverse intermittency will show a strong correlation to regions of large divergence.
A plot of the conditional probability distribution, \( P(\nabla \cdot v, J) \), however, shows no
indication that such is the case. Some asymmetry, top to bottom for instance, does
exist, but it is merely the result of the asymmetry inherent in the distribution of the
Jacobian determinant. There is no asymmetry off of the major axes. The anomalously
large strength of the structures responsible for intermittency is, therefore, not the
result of film compressibility.
Figure 5.17: Conditional probability distribution $P(\nabla \cdot \mathbf{v}, J)$. Top to bottom asymmetry is a consequence of the asymmetry inherent in the distribution of $J(u_x, u_y)$ itself. There is no off-axis asymmetry indicative of a correlation between the divergence and Jacobian determinant fields.
Figure 5.18: Conditional probability distribution $P(h, \nabla \cdot \mathbf{v})$. The absence of any off-axis asymmetry indicates that the two are not strongly correlated.

Though it was mentioned earlier that the much thicker films used in the experiments of Rivera, et al., [84] exhibited significantly more relative compressibility than the films investigated here, we find no evidence of a correlation between the divergence and thickness fields for the range of thicknesses investigated here (Fig. 5.18). The absence of any such correlation may simply reflect the fact that the dependence of the divergence on the thickness is very weak, or it may reflect differences in the forcing of the turbulence. The turbulence investigated here was forced with comb teeth 0.22 mm in diameter, while theirs was forced by comb teeth 2.7 mm in diameter.
5.7 **SUMMARY**

A correlation between the thickness and vorticity fields in soap film turbulence lead to stunning visualization possibilities. Regions of the flow associated with the strong saddle-like structures responsible for longitudinal intermittency, however, were not preferentially related to either particularly thin or particularly thick regions of the film. Neither the largest nor most rapid thickness fluctuations corresponded to:

- The intermittent structures with largest magnitude vorticity or strain rate; or,
- The largest velocity differences.

As well, passive variations in the effective mass density of the film were not significant enough to alter the distribution of downstream or cross-stream velocities. Furthermore, though some dynamic fluctuations in the film thickness did take place, the relative compressibility of the film was very small. Such fluctuations were not correlated to the structures responsible for intermittency. We, therefore, conclude that film thickness variations are not the cause of intermittency in 2D soap film turbulence.
CHAPTER 6

EXPERIMENTAL METHODS

Having now spent a considerable time exploring the physics of intermittent 2D flows, we will take a more in-depth look at the experimental techniques and equipment used in the collection of the data.

6.1 LASER DOPPLER VELOCIMETRY

6.1.1 PRINCIPLE

Laser Doppler velocimetry (LDV) relies on the interference of two coherent laser beams. The beams are aligned so that they are focused at the same location on the soap film, creating a small region of either horizontal or vertical interference fringes. The flowing film is seeded with a small concentration of tracer particles. Such particles should be large and reflective enough to be useful, though, their size must, at the same time, be small in comparison to the scale of the turbulent dynamics,
Figure 6.1: Side view of LDV probe. Two coherent laser beams are focused at the same point on the soap film resulting in interference fringes.

lest they significantly effect the very motion that is being studied. Furthermore, as we would like to minimize the magnitude of thickness fluctuations in the film, the size of the tracer particles should be small compared to the thickness of the film. Titanium dioxide (TiO₂) particles were chosen for this experiment, as they are both highly reflective and have a mean diameter of only 0.22 μm. Each time one of these particles passes through the measurement area, a time series of reflections is received by the sensing hardware (connected fiber-optically to the same probe that projects the beams). The signal processing unit then determines the frequency (f) of the reflections and calculates the period (T = 1/f). Using the fringe spacing (in our case λ = 3.52 μm), the velocity is readily determined (v = λ/T).

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6.1.2 Error

Hardware

The numerical resolution of the digitizing hardware (TSI Inc., IFA-750) limits the resolution of the velocities that can be achieved using LDV. The precision of the equipment used in the present experiment is 11 bits. This limits the maximum resolution to \(1/2^{11} \approx 5 \times 10^{-4} = 0.05\%\) of the magnitude of the measurement. For flow speeds on the order of 200 cm/s, this translates into a velocity resolution of approximately 1 mm/s.

The LDV hardware also places constraints on the maximum spatial resolution that can be achieved. A maximum data rate of 120kHz implies a minimum separation of 17 \(\mu\)m for flows moving at 200 cm/s. However, the actual location at which the two coherent laser beams cross has a finite measurement volume [43] with diameter on the order of 0.1 mm. Correspondingly, data rates above roughly 20 kHz are not useful at the flow speeds in which we are interested.

Laminar Flow

In addition to the simple limits placed on the numerical resolution of the measurements by the digitizing hardware, random noise may be introduced into the velocities through the data acquisition and spectral analysis processes. To quantify the magnitude and impact of this noise, a data set was taken for a flow without any forcing (i.e.,
a laminar flow). The corresponding energy spectrum is shown in Fig. 6.2. For wave vectors $0.3 < k < 5$, the energy is essentially constant, indicating that the observed velocity field at the corresponding scales $3 > r > 0.2$ is dominated by random noise.

An examination of the velocity-difference PDFs at a number of separations (Fig. 6.3) indicates that the width and shape of the distributions is independent of the scale $r$, for $r < 3.1$ cm. The velocity differences at these scales are dominated by the independent random noise contributions to the individual velocity measurements.
Figure 6.3: Velocity difference probability distributions for a laminar flow. The 34 curves shown span a range of separations $r$, from 0.07 cm to 3.1 cm. These are not normalized the their individual standard deviations. Both the shape and width here are independent of $r$. The gray line is the theoretically predicted distribution for the sum of two independent random Laplace variables (see text).

The maximum separation for which the width of the PDFs is scale-independent corresponds to the wavenumber less than which the spectrum shows a $k$-dependence. Thus, at larger scales, long-range correlations exist in the actual film velocity, likely as a result of laboratory air currents.

Quantifying the significance of the noise inherent in the LDV data acquisition process is important for the purpose of eliminating it as a possible cause of the intermittent statistics. Recall that the definition of the velocity difference is $\delta v(x, r) =$
\( v(x + r) - v(x) \), where \( v(x) \) is the observed velocity. In light of the above results, we no longer rigorously assume the validity of \( v(x) \) and, instead, rewrite it as a sum of the actual velocity \( u(x) \) and a random variable \( \xi \). The observed velocity difference then becomes

\[
\delta v(x, r) = (u(x + r) + \xi_j) - (u(x) + \xi_i)
\]

\[
= (u(x + r) - u(x)) + (\xi_j - \xi_i),
\]

(6.1)

and includes contributions both from the actual velocity difference (expected to be Gaussian if the turbulent flow is Gaussian), and the difference in the instantaneous value of the random variable \( \xi \) at the two locations.

Let us then assume \textit{a posteriori} that \( \xi \) has a Laplace probability distribution of the form

\[
p_\xi(\delta v) = \frac{1}{2\beta} \exp \left( -\frac{|\delta v|}{\beta} \right).
\]

(6.2)

Further, if the distribution of such a random variable is symmetric and with zero mean, the resulting probability distribution of the difference, \( \xi_j - \xi_i \), is equivalent to the distribution of the sum, \( \xi_j + \xi_i \).

The PDF of the sum of two independent random variables can be computed either through a convolution of the two probability distributions, or through the use of their characteristic functions [88],

\[
K(s) = \int_{-\infty}^{\infty} \exp(is \delta v)p(\delta v) \, d(\delta v).
\]

(6.3)
The advantage to using the latter, at least conceptually, is that the characteristic function of a sum of independent random variables is the product of their individual characteristic functions. For example, performing the above operation on the Laplace distribution \( p_\xi(\delta v) \), we find \( K_\xi(s, \beta) = (1 + s^2 \beta^2)^{-1} \). The PDF of the sum \( \xi_j + \xi_i \), is then given by

\[
p_{\text{noise}}(\delta v) = \int_{-\infty}^{\infty} \exp(-is\delta v)(K_\xi(s))^2 \, ds
= \int_{-\infty}^{\infty} \exp(-is\delta v)(1 + s^2 \beta^2)^{-2} \, ds
= \frac{\beta + |\delta v|}{4\beta^2} \exp\left(-\frac{|\delta v|}{\beta}\right),
\]

where the inverse Fourier transform has been used to recover the resulting PDF. The gray curve in Fig. 6.3, is a plot of the resulting \( p_{\text{noise}}(z) \), with \( \beta = 1.5 \), which reproduces the functional form of the noise-dominated velocity-difference PDFs extremely well.

We then make the assumption that the actual turbulent flow field is non-intermittent, with Gaussian velocity-difference probability distributions. The characteristic function of the observed velocity differences should then be of the form \( K_{\text{obs}}(s, \sigma, \beta) = K_{\text{Gauss}}(s, \sigma)(K_\xi(s, \beta))^2 \), where \( K_{\text{Gauss}}(s, \sigma) = \exp(-s^2 \sigma^2/2) \). The inverse Fourier transform of \( K_{\text{obs}} \) (needed to recover the actual PDF) must be performed numerically. For comparison, we use an empirical data set taken at the bottom of a pair of angled combs in vacuum, and compute the velocity-difference PDF for a separation \( r = 0.4 \) cm (the corresponding spectra and scaling exponents were presented in Chapter 3). Using \( \beta = 1.5 \) (the width of the laminar flow velocity-difference PDFs...
found above), we find that the central region of the observed $P(\delta v(\tau = 0.4))$ is best fit by $\sigma \approx 9$. A comparison between the theoretical prediction and the empirical PDF is shown in Fig. 6.4. The effect of the noise in the LDV measurement process is not sufficient to account for the full width of the extended tails in the observed velocity-difference PDFs at the separations in which we are interested. We, therefore, must conclude that the broad tails are a real characteristic of the turbulence, and not an artifact of LDV noise.
As a further verification that the results indicative of intermittency were independent of experimental noise, we varied the minimum signal-to-noise ratio that the signal processing hardware accepted for included data points, changed the bandpass filter settings, and adjusted the number of cycles required by the laser Doppler velocimeter for each measurement. None of these had a significant effect on the results. For the final runs, however, we required a high signal-to-noise ratio and a minimum of eight cycles per data point. We also found that slight asymmetries were inherent in the velocity PDF, but that, regardless of the particular asymmetry, the implications regarding intermittency were always the same.

It should also be noted that the time between consecutive measurements in the raw velocity data has an essentially exponential distribution. To allow for the calculation of the Fourier transform and increase the speed and accuracy with which other analyses are done, a field of evenly spaced velocities was computed by linear interpolation between points in the raw data set. The full range of analyses was also done using a binning method which avoided the necessity of first doing such a linear interpolation. However, the error in the statistics incurred by this method was greater than that using linear interpolation [89]. Further, the time required to do the analyses increased by more than an order of magnitude.
Figure 6.5: Structure function integrands of orders 2 through 18 for a separation $r$ of 0.4 cm at $P_{\text{atm}}$.

STRUCTURE-FUNCTION INTEGRANDS

There is an inherent limitation on the maximum order of structure function that can be accurately calculated from a finite data set. As the order of the structure function is increased, weight is given increasingly to the tails of the PDFs. Theoretically, these tails extend to infinity; however, in practice we have points only out to a few root mean square (RMS) deviations.

For example, the velocity distribution at $P_{\text{atm}}$ below the angled combs is nearly Gaussian with an RMS deviation of $\sigma = \langle v^2 \rangle^{1/2} = 21$ cm/s. The farthest excursion of the data from the mean is 100 cm/s, or $4.8\sigma$. This increases to $5.1\sigma$ at $P_{\text{vap}}$. For
Figure 6.6: Tenth order structure function integrand, $\delta v^{10}P(\delta v)$ for $r = 0.4$ cm at $P_{\text{atm}}$.

A representative velocity difference PDF with $r = 1.69$ cm, the RMS deviation is $\sigma = 23$ cm/s and the farthest excursion is 140 cm/s, or $6.2\sigma$. To confirm that the rare occurrences in the wings of the velocity difference PDFs are, in fact, associated with real events, we examined portions of the velocity field surrounding a number of them. In each case the large velocity difference is not caused by a single extraneous data point, but rather by a continuous change in the velocity over eight to twelve measurements (see §2.4 for more detail).

For the lower order structure functions the finite width of the PDFs is not a problem. Most of the weight of the integral in Eq. (2.9) comes from well within these
experimental limits so that

\[ S_p(r) = \int_{-\infty}^{\infty} \delta v^p P(\delta v) \, d(\delta v) \]

\[ \approx \int_{-n(\delta v^2)^{1/2}}^{n(\delta v^2)^{1/2}} \delta v^p P(\delta v) \, d(\delta v), \]  

(6.5)

where \( n \) is the number of standard deviations to which we have data. However, as
the order is increased and the integrals are weighted more and more by the tails of
the PDFs, this relation will hold less and less precisely. One can see this process
occurring in Fig. 6.5 for our smallest separation (which tends to be the noisiest).
The location at which \( \delta v^p P(\delta v) \) reaches its maximum moves steadily outward with
increasing order. At some point the maximum moves so far out that the function fails
to decrease again significantly before reaching the edge of the data. The integrand
of order ten is plotted in Fig. 6.6. A large portion of the weight is coming from the
tails of the PDFs, and the curve is thus starting to become slightly noisy. It is at this
point that we begin to incur errors due to the finite spread of our data set.

**Convergence of Structure Functions**

As suggested by Anselmet, Gagne, and Hopfinger [2], the convergence of the structure
functions with an increase in the size of the data set also provides a check on the
quality and quantity of the data. If each small subset of the data is statistically
identical to all other subsets, we expect to arrive at the same value for the \( S_p(r) \)
regardless of the size of the subset used in the calculation, or which subset in particular
is chosen.
Figure 6.7: Convergence of (a) sixth and (b) twelfth order moments as a function of data set size for separations: $r = 0.40$ cm (dotted lines), $r = 1.7$ cm (dashed lines), and $r = 7.4$ cm (solid lines) at $P_{atm}$. Thin horizontal lines denote 5% deviations.
However, an arbitrarily small subset cannot possibly contain all of the information about very rare events that is necessary to make accurate determinations of the higher order structure functions. We have plotted in Fig. 6.7 the running averages of the structure functions of sixth and twelfth order for various separations. The result is rescaled by the final value so that each independently converges to one. For a subset of \( N \) samples from a data set of length \( N_{\text{max}} \), we are thus plotting

\[
\frac{\langle S_p(r) \rangle_N}{\langle S_p(r) \rangle_{N_{\text{max}}}}.
\]

(6.6)

For all separations, the variation of the sixth order structure functions is below 10% for data sets greater than 7 million points. At twelfth order, it takes considerably longer for the statistics to converge, particularly at the smallest separations. This is expected since the events which dominate the higher order calculations are very rare and will not be adequately characterized by the smaller data sets. However, even at twelfth order, we find that beyond 21 million points the values of the structure functions have significantly stabilized, and their values settled to within 5% of the final. This leads to the conclusion that the threshold number of points necessary for an accurate calculation at twelfth order is below the 33.6 million points used.

TAYLOR'S HYPOTHESIS

It is appropriate at this point to mention that the mere use of LDV data implicitly assumes the validity of a weak version of Taylor's frozen turbulence assumption [90]. Imagine that we could freeze the turbulent velocity field in the soap film at a given
instant in time. If we were then to translate this frozen picture of the turbulence at a velocity $U$ past some measurement location, reading off the velocities as we went, the time trace of the velocities would be precisely equal to a spatial trace through the velocities where the distance between any two points was given by $\Delta x = U \Delta t$. This would imply a simple and direct connection between time series measurements and a theory based solely on spatial separation.

However, in reality, the turbulent dynamics in the film are anything but static. As the film falls past some measurement location $A$, the distance between a pair of fluid elements separated by $\Delta x$ when the first element passes $A$, may be closed or expanded by the time the second element passes depending on their relative velocities. In fact, the second element may not even be in line with $A$ by the time it passes the same vertical location. Taylor’s frozen turbulence assumption, in its strongest sense is, therefore, immediately violated.

However, results gleaned from such measurement techniques are not immediately invalidated. It is merely necessary to realize that one’s results may incur an increasing amount of error as the magnitude of the turbulent fluctuations grows relative to the mean downstream velocity of the flow. This ratio, called the turbulence intensity \cite{27} (recall §3.2.1), is given by $I_T = \nu_{RMS}/\langle v_y \rangle$. The weaker assumption, that the temporal statistics of the velocity fluctuations remain the same as the spatial ones, appears to be satisfied at least up to $I_T \approx 0.14$ in 2D soap film turbulence \cite{91}. Furthermore, results are routinely reported in 3D for $I_T$ between 0.2 and 0.3 \cite{2, 58}. 

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One other aspect of Taylor's hypothesis must be taken into consideration in the context of a vertically falling film. If the measurement location is downstream a distance \( dx \) from the forcing location, a time of roughly \( \tau = dx/(v_y) \) elapses between the turbulent forcing and the velocity measurement. During this period the turbulence continues to evolve. This provides us with an opportunity to explore the temporal evolution of 2D turbulence simply by moving the measurement location farther downstream of the forcing location. Since all of the velocities in an LDV data set are measured at the same spatial location and, hence, at the same point in the temporal evolution of the flow, there is little effect on LDV measurements as a result of this evolution.

The case, however, is somewhat different for the measurement technique described in the following section. In particle imaging velocimetry, entire 2D velocity fields are acquired at once. As a result, our earlier worry about the validity of using temporal correlations in place of spatial ones disappears completely, since the spatial correlations themselves can directly be measured. However, if we are imaging a macroscopic vertical region of the flow, there will be an overall temporal evolution of the turbulence within the imaged region. The flow field, as perceived by this measurement technique can, therefore, never be completely homogeneous in vertically falling films.
6.2 PARTICLE IMAGING VELOCIMETRY

6.2.1 PRINCIPLE

As in LDV, the flow is initially seeded with tracer particles (e.g., TiO$_2$). Particle imaging velocimetry (PIV) consists of taking two snapshots of the positions of these particles at slightly different times (Fig. 6.8). Each of the resulting images is then divided into a grid-work of small squares, as in parts (a) and (b). A local 2D correlation is performed between each corresponding pair of regions. The result is a function that is strongly peaked about the average distance that fluid in that region moved in the time between the two images (parts (c) and (d)). Finding the center of this peak and dividing by the time difference between the images gives the velocity.

The actual determination of these 2D correlations can be very computationally intensive. If the regions each have side $N$, it is necessary to do $O(N^2)$ multiplications at each of $N^2$ different offset positions (i.e., calculate a magnitude for the correlation at one location, then offset the two regions by an additional pixel and redo the calculation, etc.). The result is that each correlation requires $O(N^4)$ multiplications. However, a dramatic improvement in the computational efficiency can be achieved by noticing that the correlation (shown here in one dimension for simplicity) can be rewritten as a combination of Fourier transforms [92],

$$\text{Corr}(g, f) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(y) f(x + y) dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) G^*(k)e^{-ikx} dk, \quad (6.7)$$

where $* \text{ denotes the complex conjugate.}$ The advantage is that each of the three 2D
Figure 6.8: Illustration of the velocity field calculation from tracer particle locations at different times. (a) A random arrangement of particles that has been translated by some small amount in (b) a time $dt$ later. (c) Result of doing a 2D correlation between grayed regions. (d) Side-view of correlation showing relative magnitudes.
Fourier transforms can be done in $O(2N^2 \log_2 N)$ computations using a Fast Fourier Transform algorithm (FFT) [93, 94]. The time savings is significant. The regions used in the present analysis are of size $32 \times 32$ pixels, and zero padded out to $64 \times 64$ pixels. The difference between the two methods is then 150,000 multiplications versus 17 million, or the difference between doing the analysis of the 2239 pairs of images in an afternoon instead of two weeks.

6.2.2 Error

The Pulnix TM-9701 CCD camera used for the PIV measurements has a resolution of $768 \times 472$ pixels, and a corresponding field of view of $2.70 \times 1.95$ cm at the soap film plane. Each CCD pixel, therefore, represents a spatial area of $35 \times 41$ \(\mu\)m in the final image. However, the TiO$_2$ particles used for tracking are of mean diameter $0.22$ \(\mu\)m and, as such, any given particle stimulates only a single pixel, eliminating the possibility of achieving sub-pixel resolutions by Gaussian fits to the pixel intensity. As a result, there is a quantization of the computed velocities. An offset of a single pixel in the downstream direction corresponds to a difference in velocity of $\Delta v_{\text{offset}} = (35 \times 10^{-4})/dt = 11.1$ cm/s, where $dt = 318$ \(\mu\)s is the time between successive images. This quantization is somewhat smoothed as a result of variations in the film velocity within the $32 \times 32$ pixel blocks; however, some evidence of this quantization remains in related PDFs (Fig. 6.9).
Figure 6.9: Probability distribution of the downstream velocity $v_y$. 
6.3 THICKNESS MEASUREMENTS

PIV is becoming a standard method for exploring 2D turbulence in flowing soap films. An additional aspect of the turbulent flow problem, specific to working with soap membranes, can be included by simultaneously measuring the film thickness. Such information can be extracted either from measurements of the scattering intensity, if fluorescent dye is added to the solution, or from number density measurements of the PIV seed particles [85]. In either case, the results must first be calibrated by some external means.

6.3.1 CALIBRATION

Calibration has historically been somewhat difficult in flowing films since illumination with a monochromatic light source can indicate only relative thickness changes, not absolute values. One often resorts to first imaging some external cell of known thickness for calibration. However, we have found that certain types of light sources can be used in concert with numerical calculations to allow an accurate measurement of absolute film thickness to be made from the color of the reflected light.

A derivation of the reflected intensity $I_r$ from a thin film is easily found [95]. To first order, it varies with incident intensity $I_i$, reflectivity $R$, wavelength $\lambda$, index of refraction $\mu$, angle of refraction $\theta$, and thickness $h$, as

$$I_r = 4I_i R \sin^2 \left( \frac{2\pi}{\lambda} \mu h \cos \theta \right).$$  \hspace{1cm} (6.8)
Figure 6.10: Spectrum of light reflected normal to a film of thickness (a) 500 nm, and (b) 2000 nm. Illumination is by uniform white light ($I_i = 1$).

We will not be concerned here with the explicit form of $R$, as it depends only on the angle of the incident light and the index of refraction of the solution and, thus, doesn’t change the spectral characteristics of the reflection.

The spectrum is shown for a couple of film thicknesses in Fig. 6.10 (illumination is by uniformly white light, $I_i = 1$). However, it is not immediately obvious what a film of this thickness actually looks like. To ascertain this, it is necessary to first understand something about how the human visual system works. The retina, or back wall of the eye, is covered with two types of specialized neurons, rods and cones. The rods lie outside of the central region of vision and respond primarily to brightness.
Their response is strongly peaked toward the blue end of the spectrum (the reason red light does not destroy night vision) and under optimal conditions can perceive individual photons of light [96].

The cones are responsible for color vision. They contain various forms of proteins called opsins which are sensitive only to certain regions of the spectrum. The principle sensitivity [97, 98] for blue opsin is at 445 nm, for green at 535 nm, and for red at 575 nm. The population of each type of cone varies dramatically (blue accounting for only 2%), though the overall sensitivity of the visual system to each is nearly equal [96]. In the present paper we will assume that the spectral response of the cones
is approximately Gaussian with width $\sigma = 30 \text{ nm}$ (narrower implies more saturated colors, wider more washed out).

Our perception of color is, therefore, not a detailed map of the spectral characteristics of an object, but rather a weighted average which compresses this information into three variables,

$$C_j(t) = \int_{-\infty}^{\infty} K_j(\lambda) I_\lambda(\lambda, t) \, d\lambda,$$

where,

$$K_j(\lambda) = \frac{1}{\sigma \sqrt{\pi}} \exp \left\{ - \left( \frac{\lambda - \lambda_j}{\sigma} \right)^2 \right\},$$

and $j \in \{ \text{RED, GREEN, BLUE} \}$. The resulting color components $C_j(t)$ for a given thickness are then sufficient information to generate the perceived color using any number of utilities (in our case MATHEMATICA).

The variation in the color of light reflected from films of different thickness is illustrated in Isenberg [95]. For films of thickness less than roughly 50 nm, there is an almost complete cancellation of the light across all wavelengths. Soap films in this thickness range are called common black films. For slightly thicker films there is a range where the colors become very saturated. However, for thicker films still, the oscillations in the reflected light intensity (as a function of $\lambda$) occur on a scale much shorter than the width of the response curves for the various opsins (compare Figs. 6.10(b) and 6.11). The effect occurs first at shorter wavelengths. The result is that the blue contribution to the perceived color remains nearly constant and, thus, for
films thicker than roughly a half micron, the reflected color oscillates only between green and red. For films thicker than a couple of microns, even the longer wavelength portion of the spectrum begins to wash out noticeably, thus severely limiting the usefulness of the method for obtaining accurate measurements of films thicker than a micron or so.

One solution rests in the use of light with non-uniform spectral characteristics. This can be accounted for by letting $I_i \rightarrow I_i(\lambda)$. Typically, the form of this factor is dictated by the characteristics of various plausible laboratory light sources. Atomic spectra data [99] can be used to create the emission profile for various lamps singly
or in combination. The functional form of \( I_i(\lambda) \) is modeled by a sum of narrow Gaussian distributions with centers determined by the location of the spectral lines, and amplitudes set by their relative strength.

Emission spectra for sodium and xenon lamps are shown in Fig. 6.12. The sodium lamp is monochromatic. The oscillation between almost completely destructive and completely constructive interference, thus, leads to stunning visual topographic maps of the film thickness. However, it provides no information about the absolute thickness, only relative changes.

Calculations were done with various other light sources as well, including neon, helium, and krypton. Neon and helium suffer the same fate as sodium. Krypton oscillates between bands of red, blue, and purple. Only xenon appears a likely candidate for absolute thickness measurements. In addition to short wavelength oscillations between color pairs (in thickness-space), there is a longer wavelength change in the color of the bands which can be used to make a determination of the absolute film thickness. One drawback to xenon, however, is a decrease of roughly two orders of magnitude in brightness when compared to sodium.

Fluorescent lamps are another possible source of film illumination. We noticed two interesting things about the film when viewed under ordinary laboratory lights (Philips TL735 fluorescent lamps). First, the oscillation between green and pink bands in the reflected light persisted to much greater film thicknesses than expected under illumination with uniformly white light (where the saturation of the colors appears
Figure 6.13: Spectral lines for Philips TL735 fluorescent lamps. Magnitude of peaks only shows relative intensities.
to wash out after only a few oscillations) and, second, there were occasional bands of purple among the pink-green pairs which were not accounted for by an assumption of uniformly white light.

The spectrum for the Philips TL735 lamp is shown in Fig. 6.13. It has three prominent peaks situated near the regions of peak sensitivity for the three opsins. This maximizes the effective brightness of the lamps for a given power output and ensures that we perceive the lamps to be nearly white in color (the slightly higher magnitude of the red and green lines makes for a warmer light). However, since the contributions to the white light come from narrow peaks rather than from uniformly across the spectrum, the washing out that occurs with truly white light does not occur until much later. Furthermore, a prominent purple band is observed at a film thickness of 3 μm (when the film is viewed at a 75 degree angle to the normal). This band, therefore, provides a recognizable marker at a thickness which is easily obtainable in vertically falling soap films. Calibration at other specific thicknesses can be accomplished by counting pink-green oscillations from this known marker.

6.3.2 FLUORESCENCE

The ability to compute the absolute film thickness from interference fringes allows for the calibration of other methods by which the thickness can more readily be determined experimentally. The addition of a small quantity of fluorescent dye to the solution, for example, allows for the determination of the film thickness from the
fluorescent intensity of the film when illuminated by the PIV strobes. A series of calibration measurements, done at a number of different interference-fringe determined thicknesses, allows for the computation of the gradient \((dI/dh)\) (where \(I\) is the measured fluorescent intensity, and \(h\) the film thickness). This calibration process, however, is complicated by two factors. First, a spatially uneven illumination of the film by the strobe lamps may indicate variations in the film thickness which do not really exist and, second, thicker films really do vary in thickness across their width, a fact which is readily seen from the interference fringes of light reflecting from a laminarly flowing film and quantified by Wu, et al. [100].

These calibration measurements consist of averaging ten to twelve images of a film in laminar flow together at each of six known film thicknesses. Since spatial variations in the thickness are possible, the measurements are each calibrated at the center of the film. The resulting average intensity fields were further convoluted with a narrow Gaussian kernel to smooth any small scale variations in the intensity. A lack of interference fringes in the thinnest film indicated that there was essentially no variation in thickness across the entire width. It was, therefore, assumed that what intensity variations did exist were the result of uneven illumination by the strobes. To eliminate this effect from the actual thickness measurements, all of the measured intensity fields are divided by the smoothed intensity variations across the thinnest laminar flow.
Figure 6.14: Fluorescent intensity of film of thickness $h$, relative to intensity of thinnest film. Gray line is best linear fit by least squares method.
A plot of the relative intensity versus the interference fringe derived film thickness is shown in Fig. 6.14. A fit of the form \( I_r = I_b + \left( \frac{dI_r}{dH} \right) h \), where \( I_r \) is the intensity relative to that of the thinnest film, and \( I_b \) the background intensity, finds \( I_b = 0.874 \pm 0.010 \) and \( \left( \frac{dI_r}{dH} \right) = 0.0568 \pm 0.0028 \). By rearranging, the absolute film thickness can be determined from the relative intensity at any point in the flow. The resulting thickness profiles of the laminar flows used in the calculation are shown in Fig. 6.15, and are in reasonable agreement with profiles derived from infrared absorption measurements [100]. The slight asymmetry in the thinner films is likely the result of an irregularity in the nozzle opening.

### 6.3.3 Error

The camera used for making the thickness measurements (see §6.2.2) is an 8-bit gray scale CCD camera. As a result of its finite color depth, one expects a quantization of the thickness measurements corresponding to a single bit change in the pixel intensity. For the particular concentration of fluorescent dye used, the relationship between pixel intensity and film thickness is given by \( I = 47.9 + 3.13h \), with \( \Delta I/\Delta h = 3.13 \), implying that the change in film thickness corresponding to a unit change in pixel intensity is \( \Delta h/\Delta I = 0.320 \ \mu m \).

The effect of this quantization is apparent in the probability distribution of the film thickness (Fig. 5.11) which includes a number of superimposed humps. Computing the spectrum of the PDF, we find that it has a particularly large spike corresponding
Figure 6.15: Thickness profiles of soap films in laminar flow.
to a thickness ‘period’ of 0.652 μm, which somewhat surprisingly is almost precisely

double the one-bit thickness change computed above. The implication is that the camera registers not single bit changes in intensity but a minimum of two-bit changes. This picture was confirmed upon closer examination of the raw image data. The cause is, as yet, unknown.

6.4 HOMogeneity

As the homogeneity of a turbulent flow is either explicitly or implicitly assumed in almost all theoretical work, the degree to which the present flow satisfies this ideal is of some interest. Note that what we are referring to here is the statistical homogeneity of the flow, that the magnitude of the mean value and fluctuations of a given turbulent quantity is independent of the spatial location at which it is measured. This is only true in the limit of a large data set (i.e., many ensembles). What suffices as ‘large enough’ is determined from convergence-type calculations in the spirit of §6.1.2, but with the quantity of interest replacing the computed moments. As the higher order moments are particularly sensitive indicators of the statistics, much more robust quantities such as the mean velocity are sufficiently determined from much smaller data sets.

In addition to the downstream temporal evolution of the flow mentioned in §6.1.2, there are two further aspects of homogeneity with which we need to be concerned. The first is a change in the local flow characteristics due to a much larger scale in-
homogeneity. This type of effect is a result of the non-local Navier-Stokes equation, which implies that any slight disturbance in the flow anywhere can potentially affect the characteristics of the flow everywhere. Perhaps the most reasonable, and maybe only, way to proceed theoretically in this case is to neglect the infinite array of special cases, achieving a result which is as universal as possible. The existence of some degree of inhomogeneity experimentally, however, is a forgone conclusion. The question, then, becomes one of the sensitivity of the various statistical quantities to this inhomogeneity. This particular aspect of the flow homogeneity is the only one that affects LDV measurements, making them the most robust in vertically falling soap films.

Because of the spatial variation in the turbulent forcing when angled combs are used, the potential exists for some inhomogeneity in the cross-stream direction. Specifically, we will examine the case of two combs, each 29 cm long, placed in the film in an inverted wedge 1.2 cm apart at the top and 5 cm apart at the bottom. The cylindrical teeth have an average spacing of 1.6 mm and a diameter of 0.22 mm. PIV was used to acquire 2239 instantaneous velocity fields across a 1.95 \times 2.70 \text{cm} area, the top of which was located 5.7 cm below the last comb teeth.

The cross-stream profiles of the mean downstream velocity, turbulence intensity, and film thickness are shown in Fig. 6.16. In each case, the quantities vary between 25 region of the peak (or valley) in each case likely corresponds to the apex of the angled combs. Those areas of the flow that were ‘forced’ most recently (i.e., off to the
sides), have had the least chance to accelerate since moving through the combs and, therefore, have lower mean velocity. The turbulence intensity is highest here both as a result of the recent forcing and the lower mean velocity. The inhomogeneity in the thickness field is likely an artifact of the experimental geometry. Solution is injected into the central region of the film and, for slower, thinner films, quickly spreads evenly edge to edge. For thicker, faster films, however, there is both more solution to spread and less time to do it. As a result, larger volume fluxes of solution typically result in a distortion of the thickness profile [100]. The homogeneity of the flow under translations in the vertical direction is much greater. From the top of the field to the bottom, the mean velocity increases by only 0.59%. The RMS fluctuation along the direction of the mean flow decreases by a more significant 6.8%. The mean thickness decreases linearly by 4%.

Cross-stream profiles of the cross-stream velocity and turbulence intensity are shown in Fig. 6.17. The variation in \( \langle v_x \rangle \) is slight but non-zero. Though the mechanism behind this velocity gradient is not understood, it may serve to homogenize the thickness variations in the flow. The cross-stream turbulence intensity also shows a roughly 30% variation across the film width. However, the change, higher in the center and lower toward the edge, is in the opposite direction of the change in the downstream intensity. This may be a consequence of the cross-stream fluctuations nearer the edges being restricted by the film boundaries. Further, the cross-stream and downstream turbulence intensities measured at \( x = 0.6 \text{ cm} \) differ by 30%, sug-
Figure 6.16: Horizontal profiles of the mean downstream velocity $\langle v_y \rangle$ (●), turbulence intensity $I_T$ (○), and film thickness $\langle h \rangle$ (■).
Figure 6.17: Horizontal profiles of the mean cross-stream velocity $\langle u_x \rangle$ (●) and cross-stream turbulence intensity $I_T$ (○).

gesting that the flow is not rigorously isotropic.

An inhomogeneity in the mean vorticity $\langle \omega \rangle$ is also found and thought primarily to be a consequence of the variation in the downstream velocity. Also, since the gradient $(\partial \langle v_y \rangle / \partial x)$ is not centered in the velocity field, there is a corresponding shift in the mean vorticity when averaged over all spatial locations, as was seen in Fig. 4.16.

The second, and less subtle, aspect of flow homogeneity of relevance is that some of the quantities of interest to us are themselves non-local (e.g., the velocity difference). If the gross characteristics of the flow change dramatically on a scale comparable to the separation $r$, for instance, the most important contribution to said quantity may
Figure 6.18: Spatial variation of the mean vorticity.
Figure 6.19: The various longitudinal and transverse velocity difference probability distributions for a separation of 0.52 cm: $(\delta v(y))_{\parallel}$ (solid), $(\delta v(y))_{\perp}$ (dashed), $(\delta v(x))_{\parallel}$ (dotted), $(\delta v(x))_{\perp}$ (dash-dotted).

come from the flow's inhomogeneity. The effect of measuring non-local quantities on a scale similar to that of the inhomogeneity is illustrated in Fig. 6.19, where various longitudinal and transverse velocity-difference PDFs are shown. In most cases, the inhomogeneity is directly of little consequence. However, in the case of the transverse velocity difference with separation perpendicular to the mean flow $(\delta v(x))_{\perp}$, we are measuring the difference in a quantity which itself has a significant average gradient in the direction of the separation. The results are, therefore, catastrophic. None of these quantities played any role in our investigation of intermittency.
6.5 Summary

The underlying principles and limitations of the methods used in the measurement of 2D velocity fields and film thickness are described. With regard to the LDV measurements that are the crux of our assertion of 2D intermittency in soap films, we find that:

- The quantity and quality of the data are sufficient for the investigation of structure functions up to order 12;
- The statistical noise introduced by the measurement hardware does not account for the extended tails of the velocity-difference PDFs;
- A weak version of Taylor’s hypothesis is valid at the relevant turbulence intensities;
- Homogeneity is only a concern to the extent that distant disturbances in the flow may affect the local statistics through the non-local N-S equation, since the LDV statistics include measurements at only one spatial location and at only one instant in the temporal evolution of the flow.

Resolution limitations in the PIV-determined velocities and the fluorescent-intensity-determined thicknesses are more significant with the current experimental hardware than in LDV measurements, both in qualitative and quantitative analyses, but not to the extent that they curtail their usefulness. A novel method for calibrating absolute film thickness is further described.
APPENDIX A

The MATHEMATICA code to calculate the dependence of film color on thickness is reasonably succinct, and I shall, therefore, include it for reference. The code which defines to first order the reflected intensity from a thin film is (see equation (6.8))

\[ I_{r}[\lambda, t, \theta] := 4 I I R \sin \left( \frac{2\pi}{\lambda} \frac{h}{\mu \cos[\theta]} \right)^2 \]  \hspace{1cm} (6.11)

Since we are not interested here in the specific magnitude of the reflected intensity but only in a comparison between the intensities of different wavelengths, we set the reflectance R to 1. We also approximate the index of refraction of the solution as that of bulk water,

\[ R = 1; \mu = 1.33; \]  \hspace{1cm} (6.12)

The response of the various opsins to light is then approximated by

\[ k[\lambda, j] := \frac{1}{\sigma \sqrt{\pi}} \text{Exp} \left[ -\left( \frac{\lambda - \lambda c[j]}{\sigma} \right)^2 \right] \]  \hspace{1cm} (6.13)

where

\[ \sigma = 30 \times 10^{-9}; \]  \hspace{1cm} (6.14)
denotes the width of the response curve in meters and

\[
\lambda_c = \{575, 535, 445\} \times 10^{-9}; \tag{6.15}
\]

holds the peak response wavelengths of the red, green, and blue opsins, respectively.

Calculation of the response to uniform white light is achieved by simply setting the incident intensity \(I_i\) constant. An alternative is to construct an appropriate form for the incident light from known atomic emission spectra. We shall use sodium as an example due to its relative simplicity,

\[
\text{sodiumlines} = \\
\{(3302, 1.2), (5890, 80), (5896, 40)\};
\]

\[
\sigma 1w = 6 \times 10^{-9}; \tag{6.16}
\]

where the first term in each pair denotes the wavelength in angstroms and the second term the relative intensity. We also define a line width \(\sigma 1w\) which is set to the smallest value that will yield reliable results during numerical integration. The actual spectrum is then constructed by mapping a Gaussian function to each of the spectral lines and summing the result,

\[
I_i = \text{sodium} = N[ \\
\text{Plus}\@\@ \\
\left( \frac{\[2\]}{\sigma 1w} \right) \exp \left[ - \left( \frac{\lambda - [1] * 10^{-10}}{\sigma 1w} \right)^2 \right] \& \\
/\@ \text{sodiumlines}) \\
]; \tag{6.17}
\]

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Determination of the relative contributions to the color from each of the opsins is accomplished by numerical integration over the visible spectrum for each of the ocular response curves,

\[
\text{GetColor}[t_, \theta_] :=
\begin{align*}
\text{Table}[
N\text{Integrate}[k[\lambda, j] \text{Ir}[\lambda, t, \theta],
\{\lambda, 300 \times 10^{-9}, 700 \times 10^{-9}\}],
{j, 1, 3}]
\end{align*}
\]

(6.18)

The first term of the resulting three-element list corresponds to red, the second to green, and the last to blue. The numbers, however, are not, as yet, scaled to the \([0, 1]\) range required by the \text{RGBColor} function. This is to allow for different methods of normalization. For example, if the relative intensity between color bands is of primary importance, the RGB values for the entire thickness range can be rescaled by the maximal value of the entire set (useful for visualizing results from a monochromatic light source). Or, if an accurate determination of color is most important, each RGB triplet can be rescaled by its individual maximum (particularly useful for observation under nearly white light, or a complex spectrum such as xenon),

\[
\text{T}[x_] := \text{Transpose}[x]
\]

(6.19)

We first make a kind of shorthand notation for a function we will use repeatedly, then set the thickness range and resolution for which we wish to calculate the color.
of reflected light,

\[ t_{\text{min}} = 10^{-8}; \quad t_{\text{max}} = 6 \times 10^{-6}; \quad t_{\text{step}} = 10^{-8}; \quad (6.20) \]

The result is placed in a table which contains both the color information and a suitable Graphics object,

\[
\text{colors} = \text{Table}[
\{ \text{GetColor}[t, 0],
\text{Rectangle}[[t, 0], [t + t_{\text{step}}, 1]]
\},
\{t, t_{\text{min}}, t_{\text{max}}, t_{\text{step}}\}]
\]

(6.21)

The final output involves applying the RGBColor function to the color triplets after a suitable rescaling, and then displaying using Show,

\[
\text{Show}[\]
\quad \text{Graphics}\_\@T[\{\]
\quad \quad \text{Apply}[\text{RGBColor}, \frac{T[\text{colors}][[1]]}{\text{Max}[T[\text{colors}][[1]]]}, \{1\}],
\quad \quad T[\text{colors}][[2]]
\quad \}]
\quad \text{Frame} \rightarrow \text{True}
\]

(6.22)
APPENDIX B

It is certainly relevant to place the computer code written in-house for the analysis of the LDV and PIV data sets here so that its validity may be verified. However, the current working versions of the three C++/MFC (Microsoft Foundation Class) software packages alone contain nearly 15,000 lines of code (or roughly 250 pages of single-spaced type). Significant bodies of Mathematica and LabVIEW code also played roles at least as critical to the results presented in this dissertation. As I do not believe the inclusion of such a large volume of tedious code is either reasonable or useful, I will omit it with the understanding that I will happily furnish it in a more appropriate medium if the need or interest arises.
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