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Calibration of Airborne and Spaceborne Laser Altimeters Using Natural Surfaces

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * *

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ABSTRACT

Laser altimetry is an evolving technology in mapping and many other geoscience fields. As the potential of this technology is realized, issues of accuracy, registration and calibration of the data receive growing attention; this can be noticed from the number of publications discussing these issues. The measuring of range instead of reflectivity and the geometric realization of the data acquisition system carry the registration of laser data away from the traditional self-calibration models and theories that were derived in photogrammetry through the years. In fact, they give rise to many questions, for example, how should the calibration of the system be approached, what parameters can be recovered, how reliable can their recovery be, and what affects their reliability. Besides, finding the correspondence between a non-calibrated laser point and the surface element it was reflected from may be at times very difficult.

This research aims towards deriving an integrative data registration approach for laser altimetry data. The proposed approach utilizes natural surfaces or man-made objects as the control for registration of laser data. However, the essence of this research lies in using the derived model to answer the questions that were raised above, including a solution for the correspondence problem. The algorithm is derived to handle both airborne and spaceborne systems. With airborne systems
one, issues of calibration of different system configurations are investigated, and with
spaceborne systems, difficulties due to the increasing effects of systematic errors and
sparse sampling of the terrain are dealt with.
Dedicated to my parents, Daniel and Rachel, and to Yael
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CHAPTER 1

INTRODUCTION

Airborne and spaceborne laser ranging is an emerging technology for capturing data of physical surfaces. The availability of commercial airborne systems is rapidly growing [8, 29] and so are the reported types and number of applications of airborne laser scanning (ALS). On a more global scale, satellite altimetry systems have proved successful, e.g., Mars Orbital Laser Altimeter (MOLA) [80, 91, 92], and Shuttle Laser Altimeter (SLA) [33], a precursor to the Geoscience Laser Altimeter System (GLAS) [77] and the Vegetation Canopy Lidar (VCL) [20]. Several properties make laser ranging advantageous compared to other active sensing techniques such as radar interferometry. Among them are the well-controlled size and small footprint on the ground, the accurate pointing capability of the beam, and the potential accuracy that can be achieved [47, 51, 54, 58, 59].

Laser altimetry is mostly associated in the mapping community with, (i) the rapid and autonomous generation of Digital Surface Models (DSM) [47, 52], (ii) its contribution towards autonomous surface reconstruction either as a sole sensor [57, 35] or within an integrated sensor framework [34, 46], and (iii) autonomous object recognition [87, 89]. It is used, however, in a wider range of applications in different geoscience
fields. An important application is the measurement of canopy and surface elevation in wooded areas [65, 66]; the VCL mission [20], for example, is dedicated to this purpose. The textured pattern of the canopy and the occluded ground pose difficulties in elevation measurement by conventional mapping techniques. With lasers, height is measured directly, and as some of the transmitted energy is backscattered from the ground generating elevation models becomes a feasible task. Optimizing the characteristics of laser altimeters for vegetation and canopy measurement is reported in [9, 10, 11]. At the technical university in Vienna [56, 57, 58] algorithms for the extraction of surface topography over wooded areas have been developed. Utilizing the returned waveform analysis for distinguishing the surface from the canopy is reported in [43, 82].

Another active field in which laser altimetry is used extensively consists in mapping the uniform surfaces of snow and ice for determining the mass balance of the polar regions [32, 53, 55, 84]. Laser altimetry surveys are used to map changes on the West Antarctic ice streams [81], on mountain glaciers in Alaska [2, 23] and in the Swiss Alps [25]. Detailed surface relief (sastrugi) is measured to estimate the contribution from spatial noise to stratigraphic record [85]. Yet other applications include monitoring erosion of coastal areas, change detection [64], and study of surface roughness and albedo from the analysis of the waveform [71, 76].

In machine vision close range laser ranging is an active field. Algorithms for segmentation of range images received considerable attention [4, 6, 15, 44, 45], and
in [6, 18] object recognition from range data is reported. Issues of point location or accuracy of the positional data are not addressed, especially as range data are usually given in a matrix format (range images).

The accuracy potential of laser altimeters is high. Krabill et al. [54] report elevation differences of the order of 10–15 cm of surfaces that were surveyed by repeat flights. Knabenschuh and Petzold [52] report elevation differences of less than 30 cm for 95 percent of the data when testing laser points versus surveyed ground points. Kraus and Rieger [58] provide an empirical form for the accuracy of DEM points derived from laser altimetry over wooded areas (α slope angle):

\[ \sigma_H[cm] = \pm(18 + 120 \tan \alpha) \] (1.1)

The authors mention that eliminating the systematic errors improved the expected accuracy to ±10 cm. Another indication for the potential accuracy of laser altimeters is the science requirements for the GLAS mission. The goal consists in detecting elevation changes of 1.5 cm/yr over 10,000 km² where surface slopes are < 0.6°, and of 0.5 cm/yr over 200,000 km² where surface slopes are < 0.2° [77].

Reaching the accuracy potential requires the elimination of systematic errors in the data. Many reports indicate that such errors are still left in the data. Huising and Gomes Pereira [47] report systematic errors of 20 cm in elevation and of several meters in position between overlapping laser strips. Crombaghs et al. [19], and Maas [62] also identify systematic trends between overlapping strips. Eliminating systematic errors requires suitable calibration procedures that combine laboratory
calibration [54, 70] and in-flight calibration. Performing in-flight system calibration is imperative not only for more accurately locating the laser footprint, but also for removing systematic effects that distort the reconstructed surface form.

Calibration of laser altimetry systems is a nontrivial problem. Clearly it involves more than the definition of an analytical form of the sensor biases, as in the case of an in-flight calibration of aerial cameras. In laser mapping, no unique correspondence can be established between laser points and control points. Furthermore, the points derived from laser altimetry are not necessarily on the physical surface but rather determined as a function of the distribution of backscattered energy [27]. Thus, a control-point based calibration faces lots of difficulties. Another major challenge is rooted in the geometric realization of the data acquisition system. From each firing point only one beam is being transmitted. This is a rather weak configuration for recovery of systematic biases as it leaves no intrinsic redundancy. One potential effect of this configuration is an increased correlation of the calibration parameters that implies that not all the systematic biases may be recovered separately. Therefore, the question is which biases can be resolved, and how to solve for them.

As can be noticed, the calibration is a strategy and not just a formulation of the calibration equation. The identification of reference objects is very different from what is common practice in photogrammetry. Finding the location of a reference point is practically impossible, even if conducted manually. An extreme situation exists when the sampling is sparse, the data acquisition mode is of a profiler, and the calibration has to be performed over natural terrain. This scenario is, however,
exactly the anticipated one with the calibration of GLAS. Correspondence is a difficult problem to solve especially when the search space is big and the attributes that define the correspondence are difficult to assess. Therefore, it was usually left untreated or was simplified in manners that cannot be applied in general. Nevertheless, in-flight calibration of laser systems and the registration of laser data receive growing attention in recent years. Many of the fundamental papers that discuss laser systems devote a considerable part to the aspect of “tying the laser points to the ground” and others are solely dedicated to this subject. The reported solutions rely heavily on photogrammetric concepts of self-calibration and/or block adjustment; consequently they ignore the properties of the laser data acquisition system and of the laser data. Little analysis has been carried out regarding the reliability of the calibration parameters, their recoverability, and optimal configurations of control entities for the calibration. Not surprisingly then, the conclusions and interpretations are sometimes questionable or unfounded.

1.1 Scope of this research

The research presents a registration approach for laser altimetry data. The motivation lies in finding an adequate representation for the calibration of this type of data, not by imposing existing concepts. The algorithm described here utilizes natural or man-made objects as the control for registering the laser data.
The algorithm addresses the calibration of both airborne and spaceborne laser altimeters. With airborne systems a variability in data acquisition system configurations, e.g., profilers, conical scanners or line scanners, have different implications on the system calibration. With spaceborne systems the relatively large footprint size, low data acquisition rate, the profiler data acquisition mode, and unknown correspondences between the laser points and the natural surface, pose lots of difficulties. This research aims at presenting a solution that is applicable for both airborne and spaceborne systems without imposing preliminary constraints.

Systematic effects can be already noticed with the current low altitude flying systems; as the flying altitude increases the effect of the errors increases as well. To minimize this effect, an analysis of the error sources is performed, their effect on the reconstructed surface and conditions for their recoverability are studied in this research.

The research also addresses questions regarding the configuration and the existence of adequate calibration sites. Little work has been done in relation to the analysis of the recovered parameters and how they are affected by the configuration of control features. The importance of such an analysis is clear – eliminating the systematic errors reduces the needed preprocessing time and increases the degree of automation, see [47, 19, 62]. This analysis becomes more important for the selection of calibration sites for satellite laser altimeters. As with geodetic network design [73], obtaining an optimal control configuration is the objective. Ideas about the optimal shape of the calibration sites are difficult to quantify, and visual inspection of
DEM’s and geometrical intuitions have limited use in determining adequacy of sites. Consider, for example, the choice between flat, mountainous or hilly terrain for calibration sites, or determining what is the role of natural landmarks in “locking” the laser beam. Analysis of control feature configurations and the expected accuracy of the calibration parameters are therefore carried out here.

1.2 Organization of this work

This work is organized as follows.

Chapter 2 provides information about laser altimetry. System concepts and description of several data acquisition systems are presented first, followed by the derivation of the laser geolocation equation and the analysis of potential error sources.

Chapter 3 reviews existing calibration procedures and outlines the contribution of this dissertation to the research in this topic.

Chapter 4 presents the proposed calibration model. After describing the general scope of the model, several aspects are studied in detail. First the application of the outlined model for laser altimetry systems is presented; then a recoverability analysis of the different systematic errors in the system is studied. A solution to the correspondence problem is provided with an analysis. An additional aspect to be examined is the reliability of the calibration parameters. Similar to the Von-Gruber point distribution in relative orientation, different configurations provide more robust solutions. The focus here is on deriving
analytical criteria for assessing the reliability of the recovered parameters. Additionally, incorporation of waveform knowledge (see section 2.1) is studied.

Chapter 5 presents experimental results. Calibration of the NSF-SOAR system (National Science Foundation Support Office for Aerogeophysical Research) enables one to evaluate the calibration algorithm. Aspects of site selection and evaluation of the correspondence algorithm are presented for the GLAS (Geoscience Laser Altimeter System) configuration.

Chapter 6 concludes this research with comments and recommendations for future research on this topic.
CHAPTER 2

BACKGROUND

2.1 System concepts

A laser can be regarded as a smart amplifier that is physically based on the concept of stimulated emission. The word laser is an acronym for light amplification by the stimulated emission of radiation. Lasers use atoms or molecules to store energy and to emit that energy as light. This is accomplished by energizing (pumping) the electrons in the atoms of a laser medium to an excited state by an energy source. The excited atoms are then "stimulated" by external photons to emit the stored energy in the form of photons (stimulated emission). The emitted photons have the frequency characteristics of the atoms and they travel in phase with the stimulating photons. These photons in turn stimulate other excited atoms, which release more photons. Light amplification is achieved as the photons move back and forth in the laser cavity, triggering further stimulated emissions. The photons generated in this fashion are emitted in the form of an intense, directional, and monochromatic laser beam through the partially reflective mirror.

Two principles are used to perform laser range measurement. The most common principle is pulse ranging. Pulsed laser ranging is based on the measurement of the
time interval between the pulse transmission and its return. A short pulse with high peak power is transmitted from the system, the travel time is measured by counting returned photons. The relationship between time and range is

\[ \rho = \frac{cT}{2} \]  

(2.1)

with \( T \), the round-trip travel time; \( \rho \), the range; and \( c \), the velocity of light.

The alternative ranging principle is measuring the phase difference between the transmitted and received signal. This principle is applied to lasers that continuously emit light, it is therefore called continuous wave (CW) ranging [88]. In order to avoid phase ambiguities, the transmitted signal is modulated to multiple frequencies so that long wavelengths enable one to determine the coarse range, and shorter ones the precise range. The relation between phase and range is

\[ \rho = \frac{\lambda}{4\pi} \Phi \]  

(2.2)

with \( \lambda \), the laser wavelength, and \( \Phi \), the measured phase.

Pulse lasers are usually solid-state lasers that are based on a Neodymium Yttrium Aluminum Garnet (Nd:YAG) laser. For Nd:YAG lasers the fundamental wavelength is 1064 nm (NIR range); with a double frequency wavelength it is 532 nm (green range).

A typical laser altimeter system is composed of three major components, a transmitter subsystem, a receiver subsystem, and a position and attitude subsystem.
The transmitter subsystem is composed of a diode-pumped laser and the transmitter optics [16]. To achieve high precision ranging with pulsed laser altimeters, the energy built in the laser system is transmitted as a short pulse (typical time interval of 5–10 nsec). A beam expander telescope controls the transmitted laser divergence angle. Divergence angles, \( \Theta \), as small as 0.1 mrad with a beam diameter of the order of few centimeters can be reached. The size of the footprint on the ground is a function of the flight altitude and the divergence angle:

\[
D_b = 2Z \tan \Theta
\]  
\[\text{(2.3)}\]

with \( D_b \), the beam diameter; \( \Theta \), the beam divergence angle; and \( Z \), the distance to the surface. The divergence angle is a function of the wavelength and the transmitter aperture, \( D_a \):

\[
\Theta = 2.44 \frac{\lambda}{D_a}
\]  
\[\text{(2.4)}\]

The energy distribution across the laser beam (transverse mode) is not uniform. The commonly used transverse mode is the so-called TEM00 mode that has a smooth intensity profile given by a Gaussian [39].

\[
I(r) = \frac{2P}{\pi d^2} \exp\left(-\frac{2r^2}{d^2}\right)
\]  
\[\text{(2.5)}\]

with \( I(r) \), the intensity function; \( P \), the total energy; \( d \), the laser footprint size (measured between the \( \pm 2\sigma \) points); and \( r \), the distance from the center of the laser beam.
Equation 2.5 implies that the energy decreases as a function of the distance from the center of the beam. Therefore, less energy returns from the outer part of the footprint.

The receiver subsystem includes a telescope for collecting the returned photons and electronics for counting the photons and generating the returned waveform [16]. The receiver aperture (8–50 cm diameter) is designed to capture the desired signal from the backscattered radiation. The aperture is mounted to share the same optical path as the transmitter. Optical interference bandpass filters are mounted on the optical path to filter out background radiation (e.g., solar radiation). Two types of devices are employed for pulse detection. Silicon avalanche photodiodes (Si-APD) are typically used at NIR laser wavelength, while photomultiplier tubes (PMT) are more effective at shorter wavelengths [16]. Detecting (discriminating from the background noise), analyzing and digitizing the returned waveform is carried out by receiver electronics.

The third subsystem measures the position and attitude of the laser system. For airborne systems, the position and the orientation are usually determined by an integrated differential GPS and INS system. A combination of star cameras and INS is employed for achieving the high accuracy needed for attitude determination of satellite systems [79].

When ignoring signal-to-noise ratio (SNR) aspects, the accuracy of the travel time measurement is mainly affected by the laser transmitted pulsewidth and the sampling
rate of the waveform. Therefore, the ranging accuracy is almost independent of the flight altitude; it depends mostly on the altimeter characteristics.

2.1.1 Waveform analysis for range determination

The signal strength variation during one laser ranging cycle is illustrated in Figure 2.1. The three intervals of the laser ranging are the signal transmission, an intermediate interval when only optical and electronic noise are present, and a rise in power indicating the return of the transmitted pulse from the surface. Relief variation within the footprint increases the time interval in which the backscattered energy (photons) will be received. The shape and width of the received pulse are also affected by other surface characteristics. For example, surface roughness broadens the pulse and albedo affects the amount of reflected energy. As a result, the laser altimeter waveform contains information about the surface structure, roughness, and albedo. More detailed information about this issue can be found in the report by Filin and Csathó [27].
The broadening of the returned pulse increases the uncertainty of the time measurement and makes the question of where and how to measure the return time more difficult. Different timing estimators, such as Constant Fraction Discrimination (50% Risetime Point), Pulse Centroid (Center of Area), Pulse Mid-Point and Pulse Mean, are used [1]. For regular surfaces the best range estimation method is the pulse centroid, but it involves recording the whole waveform, and therefore it is less frequently used. Based on pulse centroid range determination, Gardner [31] derives a closed form expression for estimating the travel time and the mean-square pulse width for a uniformly sloped terrain.

\[
E\{T_p\} = \frac{2z(1 + \tan^2 \theta_T)}{c \cos \phi} \left[ 1 + \left(1 + 2 \tan^2 (\phi + S_x)\right) \frac{\text{Var}(\Delta \phi_x)}{2} \right]
\cos^2 \phi \left(1 + \frac{2 \tan^2 S_y \cos^2 S_x}{\cos^2(\phi + S_x)}\right) \frac{\text{Var}(\Delta \phi_y)}{2}
\] (2.6)

\[
E\{\sigma_p^2\} = (\sigma_i^2 + \sigma_h^2) + \frac{4 \text{Var}(\Delta \xi) \cos^2 S_x}{c^2 \cos^2(\phi + S_x)} + \frac{4 z^2 \tan^2 \Theta_T}{c^2 \cos^2 \phi} \left[ \tan^2 \Theta_T + \tan^2 (\phi + S_x) + \frac{\tan^2 S_y \cos^2 S_x}{\cos^2(\phi + S_x)} \right]
\] (2.7)

with \(T_p\), travel time estimated by pulse centroid; \(\sigma_p\), received RMS pulse width; \(\sigma_i\), transmitted RMS pulse width; \(\sigma_h\), RMS width of the receiver impulse response, \(c\), velocity of light; \(\Delta \xi\), surface roughness; \(\phi\), off-nadir pointing angle; \(S_x\), surface slope in the \(xz\) plane; \(S_y\), surface slope in the \(yz\) plane; \(z\), altimeter altitude; \(\Theta_T\), half width of the divergence angle; \(\Delta \phi_x\), pointing error parallel to the pointing direction; and \(\Delta \phi_y\), pointing error perpendicular to the pointing direction.
Other range determination methods may introduce a bias to the ranging. For example, thresholding depends on the amount of energy reflected from the surface (an effect called "range-walk"), the CFD depends on the pulse broadening. Calibrating the range for these effects is therefore mandatory [54, 70].

Complex surfaces require different range determination techniques. Figure 2.2 shows a distribution of the reflected energy over a wooded area. The return from the ground is the narrow peak at the end of the waveform, the broad part is energy reflected from the canopy. Processing such waveforms by conventional methods will provide elevation that has no physical meaning. Some systems provide first and last
returns. Algorithms that are based on digitizing the full waveform and analyzing it, e.g., by waveform decomposition, are described in [43, 82, 93].

2.2 Data acquisition systems

Different data acquisition systems generate different spatial distributions of the laser points on the ground. Laser altimeters can be roughly divided into two categories – profiling systems and scanning systems.

For profiling systems the beam direction is fixed (usually close to a nadir-looking direction), and with an aircraft forward motion a profile point pattern is generated. As simple as this configuration is, it is still relevant, for example, if the point density should not be too big. Airborne profilers are still in use, e.g., the NSF-SOAR (National Science Foundation Support Office for Aerogeophysical Research) laser altimetry system [13, 14]. Furthermore, spaceborne laser altimeters (MOLA, SLA, GLAS) operate in this mode. An extension of this concept is introduced in the design of VCL [20] where five altimeters will be mounted on the same satellite to increase the coverage of one swath.

Scanning systems are common for airborne laser altimeters. Their advantages are clear, the relatively wide area covered by each swath and the high density of laser points are optimal for mapping projects. The swath width is a function of the flying altitude and the scan angle. Three factors determine the point density along and across the flying direction – the pulse transmission rate (number of pulses per
second), the scanning rate, and the swath width (field of view). Several scanning concepts are implemented resulting in different spatial patterns. The common scanning configuration is a line scanning. The laser beam is deflected across the flight direction generating a linear pattern of points. A popular implementation is based on using an oscillating mirror that deflects the transmitted beams as the mirror rotates. The result is a zigzag-like spatial point pattern (including the effect of aircraft motion).

Two different concepts enable the generation of parallel lines. The first one is based on a rotating polygonal (multifacet) scan mirror [69]. This can be extended by some tuning to achieve a grid distribution of the laser points on the ground (ignoring variation in the aircraft velocity, and relief variation effects). The second concept is based on a fiber scanner [88] that deflects the beam without the need of rotating mirrors. Therefore, the dependency on mechanical parts is minimized.

A different pattern is achieved by a somewhat different concept that results in a conical shape [54]. A nutating mirror (also called Palmer scanner) rotates around its axis and generates an “egg-shape” point pattern that closely resembles an ellipse. Together with the aircraft forward motion a spiral shape is formed by the laser points. With the conical scanning pattern each element on the ground is scanned twice, once by a forward looking beam and then by a backward looking beam. The spiral shape distribution of the laser points produces an irregular point distribution (compared to the line scanning). Moreover, the point density is changing and at the edges of the swath laser points are denser than at the center of the swath.
2.3 Geolocation of the laser beam

The laser geolocation equation incorporates the different components of the laser altimeter system. Section 2.1 showed that a laser altimeter measures only the range between the laser firing point and the footprint, the position and the attitude of the system should be obtained from external sources, mostly GPS receivers and inertial mapping units (IMU), or star cameras. The integration of the three components involves three different reference frames – the laser altimeter reference frame in which the laser ranging is measured, the inertial frame in which attitude angles are measured, and the earth centered reference frame in which position is measured. Figure 2.3 illustrates the different reference systems.
Laser points are measured in the laser-altimeter reference frame. The origin is at the altimeter firing point. The z-axis is defined by the zero position of the altimeter. The definition of the other two axes is not strict. For a profiling system the concept of the x, y-axes is not as meaningful. For a line scanner the yz-plane is defined by the scanning plane, and for a conical scanner the y-axis is the reference orientation of the laser system in the aircraft. In general, the system is aligned so that the x-direction coincides with the body x-axis, the z-axis points towards the zenith and the y-axis completes a right-hand side system. The coordinates of a laser point in the altimeter reference frame are given by eq. 2.8

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}_\text{altimeter} = R_{\text{scanner}} \begin{bmatrix}
  0 \\
  0 \\
  -\rho
\end{bmatrix}
\] (2.8)

The altimeter reference frame is a local frame. Transforming it to a global frame requires its alignment with the body frame (determined by the INS alignment) and its translation to the GPS receiver, more precisely to the phase center of the GPS receiver antenna. The transformation involves the rotation of the altimeter frame into the body frame, \( R_m \), and an origin shift of the altimeter frame to the phase center by an offset vector, \( [\delta_x \quad \delta_y \quad \delta_z]^T \). An equivalent result would be obtained by first translating the altimeter frame and then rotating it.

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}_\text{body} = \begin{bmatrix}
  \delta_x \\
  \delta_y \\
  \delta_z
\end{bmatrix}_\text{body} + R_m R_{\text{scanner}} \begin{bmatrix}
  0 \\
  0 \\
  -\rho
\end{bmatrix}
\] (2.9)

The body frame enables one to transform the laser point to the topocentric reference frame.
Converting the topocentric laser point into a global coordinate system is carried out by using the GPS receiver position in the geocentered WGS-84 frame. This conversion involves the transformation of the topocentric reference frame to the ellipsoidal reference frame, and further transformation to the Earth-centered geodetic reference frame. The final form is given in eq. 2.11. The laser point geolocation equation is also given in [60, 86].

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}_{\text{topocentric}} = R_{\text{INS}} \begin{bmatrix}
\delta_x \\
\delta_y \\
\delta_z
\end{bmatrix}_{\text{body}} + R_m R_{\text{scanner}} \begin{bmatrix}
0 \\
0 \\
-\rho
\end{bmatrix}
\]  \hspace{1cm} (2.10)

where:

- \(x, y, z\) - location of the footprint in the WGS-84 geocentric coordinate system.
- \([X_0 \ Y_0 \ Z_0]^T_{\text{WGS-84}}\) - location of the phase center of the GPS receiver antenna in the WGS-84 system.
- \(\rho\) - range as measured by the laser altimeter system.
- \(R_{\text{WGS}}\) - rotation from the local ellipsoidal system at the position of the GPS antenna into the WGS-84.
- \(R_{\text{GEO}}\) - rotation from the topocentric reference system, defined by the local vertical, to the local ellipsoidal system centered at the phase center of the GPS antenna. The angle between the local vertical (points along the local gravity vector) and the vector normal to the WGS-84 ellipsoid is also called “deflection of the vertical.”
- \(R_{\text{INS}}\) - rotation from body system to the topocentric system centered at the phase center of the GPS antenna, defined by the INS attitude angles.
- \(\delta_x, \delta_y, \delta_z\) - offset vector between the phase center of the GPS antenna and laser reference point, defined in the body system.
$R_{\text{scanner}}$ – rotation between laser beam and the laser altimeter system, defined by the scanning angles.

$R_m$ – rotation between the altimeter and the body system.

Some further considerations concerning the conversions between the International Celestial Reference Frame (ICRF) and the IERS Terrestrial Reference Frame (ITRF) are needed, and so is the atmospheric delay effect. Schütz, [78] and Herring and Quinn [41] address these problems.

A more convenient reference frame to work with is a local reference frame. Coordinates in the local reference frame are computed by

$$
\begin{bmatrix}
x \\
y \\
z_{\text{local}}
\end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_{\text{local}} + R_{\text{INS}} \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{\text{Body}} + R_m R_{\text{scanner}} \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix}
$$

(2.12)

where:

$$
\begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix}_{\text{local}} = (R_{0\text{WGS-84}} R_{0\text{GEO}})^T \begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix}_{\text{WGS-84}} - 
\begin{bmatrix}
\bar{X} \\
\bar{Y} \\
\bar{Z}
\end{bmatrix}_{\text{WGS-84}}
$$

(2.13)

$R_{0\text{WGS-84}}, R_{0\text{GEO}}$ – are constant rotation matrices, defined by the origin of the local reference frame.

$\bar{X}, \bar{Y}, \bar{Z}$ – is the origin of the local reference frame in the WGS-84 coordinate system.

The quantity $R_{\text{WGS-84}} R_{\text{GEO}}$ can be approximated by a constant for the expected size of calibration sites. Therefore, $(R_{0\text{WGS-84}} R_{0\text{GEO}})^T (R_{\text{WGS-84}} R_{\text{GEO}}) \cong I_3$. The origin of the local system is arbitrary. However, it is very likely to be located at the centroid of the surveyed area with elevation set to sea level or near the surface elevation.

In the following, the laser points are assumed to be in the local reference frame, unless otherwise mentioned.
2.4 Potential error sources

The three components of the laser system, namely, position, navigation and ranging, are sources for systematic errors. Errors may be intrinsic properties of each component, and additional errors are the consequence of their integration. A review of the literature concerning calibration shows that there is no standard error modeling in existence; the type of modeled errors vary from one author to another, and usually only few error sources are modeled. Little has also been done in terms of a comprehensive analysis of the potential error sources in the system. Both [59] and [75] analyze the potential errors sources, but a modeling of their effect appears only in [75]. This section introduces the different types of error sources; their modeling and analysis is treated in detail in Chapter 4 when recoverability is discussed.

It is agreed upon by many authors that the mounting bias between the laser altimeter reference system and the aircraft/spacecraft body frame is a major error source [17, 61, 70, 86, 88]. A mounting bias implies that the assumed pointing direction of the laser beam is incorrect, a modeling of the mounting bias is given in eq. 2.10. Schenk [75] separates the pointing bias into the measured part and the unknown part. Here, the mounting bias is treated as single entity, $R_m$, the measured part is considered as an initial approximation.

A range bias, $\delta \rho$, results in a measured range that is either systematically shorter or longer than the true one [59, 61, 78, 81, 75]. Surprisingly enough, the range bias is mentioned in the geoscience community and mostly ignored in the mapping one. Calibration of the range bias is mentioned in [61, 81], Hofton et al. [42] reports
about a range bias of the order of +12 m. This is an extreme value, but it indicates the importance of estimating this potential error source. Schütz [79] analyzes the potential sources that may affect the range bias during the lifetime of GLAS. The author mentions a potential dependency on the orbiting altitude, thermal factors, and variation with time. Lemmens [59] models systematic dependency on the ranged distance through

$$\rho' = \lambda \rho + \delta \rho$$

with $\rho'$, the corrected range; $\rho$, the observed range; $\lambda$, the systematic scale dependency on the range; $\delta \rho$, a range offset bias.

With this modeling the range correction is composed of an offset bias component and a scale factor. Other factors mentioned by Lemmens [59] that affect the range error are slope, roughness, and albedo. Krabill et al. [54] and Ridgway et al. [70] are more explicit in this regard. They describe a dependency on the amplitude of the returned energy, which is a function of the components mentioned by Lemmens [59]. This dependency is rooted in the range determination algorithm, namely thresholding. Determining the range by the waveform centroid provides a more or less unbiased estimate of the range [31]. Yet, either simple thresholding, or a percentage of the rising power are usually used instead. Both depend on the amplitude and require either laboratory or in situ calibration.

Other potential error sources are supposed to create less effect. They include an error in determining the offset vector, $[\delta_x \quad \delta_y \quad \delta_z]^T$, between the GPS and the laser
optical system (see eq. 2.9), time synchronization error between the laser altimeter, INS and GPS systems, INS initialization error, position offset, and scanning system errors.

Corrections for time synchronization errors between the INS angles and the ranging time are evaluated in [42, 70]. The time correction is modeled as

\[ \omega_c = \omega + \dot{\omega}t \]  

with \( \omega \), the measured angle, \( \dot{\omega} \), the angular velocity, and \( t \), a time delay. The reported corrections are of the order of a few hundredths of a second. Schenk [75] describes another type of potential time related error that is due to an interpolation error. An aliasing, caused by a higher rate of transmitted laser pulses than the INS and GPS rate, causes errors in position and attitude determination of the system at the ranging time. However, the high sampling rate of the INS system is expected to reduce the effect of this type of error.

Schenk [75] also describes systematic errors of the scanning system. The errors include

- An angular error between the assumed null direction and the true z-axis (an index error).

- A swath angle error that models an incorrect swath angle.

- A non-perpendicular angle between the scan plane and the x-axis (a scan plane error).
In regard to position errors, Burman [17] mentions tropospheric delay or a datum transformation error as potential sources of this error. As for the tropospheric delay, it is a function of the distance between the airborne system and the GPS base station. Solutions that incorporate multiple base stations over a mission may reduce the problem. The author does not provide numerical values to indicate any substantial datum shift. Kilian et al. [51] model, instead of the mounting bias, an INS bias that is an angular offset between the INS system and the inertial frame. The authors model a positional error and, in addition, a change in the offset over time.

In the next chapter various methods that have been proposed to quantify the biases are reviewed. Prior to that, in the coming section, definitions and symbols that are used in the following are presented.

2.5 Definitions

2.5.1 Terms

Footprint – the area on the ground that was illuminated by the laser beam. Footprint will also define the actual point location on the ground \((x, y, z\)-coordinates) the laser beam illuminated.

Laser point – the ground position \((x, y, z\)-coordinates) that was measured by the laser altimeter system. Position is a function of the measured range, attitude, position, and the system biases (when estimates exist) of the laser altimeter at the time of ranging.
Search space — the area on the ground in which the footprint of a given laser point can be located. The search space is defined as a function of the expected magnitude of the systematic errors.

Solution space — a multi-dimensional space that contains the potential values of the system biases.

“Modified” slopes — a normal to the surface that was rotated (see also page 42).

Correspondence problem — In the current problem formulation, which uses control surfaces, correspondence is defined as finding the surface in which the footprint is located.
2.5.2 Symbols

\( x, y, z \) — location of the laser point.

\( \mathbf{l} \) — vector of the laser point coordinates.

\( X_0, Y_0, Z_0 \) — location of the phase center of the GPS receiver antenna.

\( \delta_x, \delta_y, \delta_z \) — offset vector between GPS antenna and laser firing point, defined in the body system.

\( \rho \) — measured range.

\( \Omega, \Phi, \Psi \) — the INS rotation angles.

\( \dot{\Omega}, \dot{\Phi}, \dot{\Psi} \) — time derivatives of the angular variations.

\( R \) — designate rotation matrices. For example, \( R_{INS} \) is the INS rotation matrix.

\( \delta \rho \) — range bias.

\( \lambda \) — scaling correction for the range.

\( R_m \) — mounting bias in the form of rotation matrix.

\( \omega, \phi, \kappa \) — the mounting bias angles.

\( s \) — the surface slope vector.

\( c \) — the “modified” surface slope vector.

\( t \) — time synchronization bias. Subscripts denote the component to which the time synchronization bias relates.

\( \Xi \) — the systematic errors vector.

\( w \) — the transformed observation vector.

\( A \) — the coefficient matrix.

\( B \) — the conditions matrix.

\( \xi \) — the vector of unknowns.

\( e \) — observation noise.

\( P \) — weight matrix.

\( \Sigma \) — dispersion matrix.

\( \sigma_0^2 \) — the variance component.

\( := \) — denotes, “defined as”.
CHAPTER 3

MOTIVATION

A variety of procedures have been developed for recovering the calibration parameters and solving for the unknown correspondence between the laser points and the illuminated spots on the ground. The first section in this chapter reviews these approaches, the section that follows outlines the contribution of this research to this field.

3.1 Review of previous work

Vaughn et al. [86] perform in-flight calibration by flying over a locally leveled surface such as a water body. Pitch and roll maneuvers are carried out to determine the calibration parameters. The only systematic error recovered is the mounting bias between the laser altimeter and the INS. To compute the calibration parameters the authors use the fact that with pitch or roll maneuvers over a flat surface, the range variation as a function of the pitch and roll angles takes a concave shape. Second-degree polynomials are fitted to both these shapes. Their minima are considered as the mounting bias. The biases are then used to update the pitch and roll angles; the process is repeated until convergence is
reached. According to the authors, convergence is reached whenever the mounting bias is less than \(1^\circ\). The correspondence problem is bypassed by using a flat locally horizontal plane as a reference surface.

**Ridgway et al.** [70] also consider the pitch and roll components of the mounting bias as the main systematic error sources. The in-flight procedure is similar to Vaughn et al. [86], namely, flying over a flat locally horizontal surface and performing pitch and roll maneuvers. The target function that is minimized is the norm of the differences between the elevation of the reconstructed laser points and the plane elevation. They report on adjusting small timing errors between the INS and the laser; no numerical experiments confirm their claim.

**Hofton et al.** [42] describe a calibration of NASA’s Scanning Lidar Imager of Canopies by Echo Recovery (SLICER) [9], a line scanner with medium footprint size. They consider six systematic errors – the three angles of the mounting bias and three time errors. Pitch and roll maneuvers over a flat surface are also used here. The solution is obtained via search in the 6-D space. Time errors are of the order of a few hundredths of a second. However, no accuracy estimate validates these values. A similar concept is used for calibrating the NSF-SOAR laser system, see Spikes et al. [81]. Instead of a grid search, the authors use a trial and error approach. The quality of fit is evaluated by comparing the reconstructed terrain with the surveyed one.
Kilian and others in Stuttgart [30, 50, 51] have developed an approach that views the calibration of the laser strips as a block adjustment. The approach is point-based, thus the correspondence is established between laser points and distinct landmarks such as building corners. The algorithm is divided into two parts. Firstly, tie zones from neighboring strips are matched via least-squares matching (LSM), the adjusted parameters are the offsets—\(dx, dy, dz\)—between the tie zones. Distinct landmarks, e.g., building corners are used as 3-D control points and flat surfaces as elevation control (namely, by introducing a correction to the \(z\)-component). These offsets are then "explained" by the calibration parameters. Test statistics are used to evaluate the statistical significance of the adjusted values. Some aspects of this approach may affect the accuracy of the recovered parameters. For example, in the matching phase the matched surfaces originate from the reconstructed surface, which is deformed; thus the differences that are used are inaccurate. Another drawback is rooted in the objects that are chosen as control points. Laser altimetry is a sampling device and the chance that a corner or an edge will be sampled is low; consequently the differences are based on interpolation. Furthermore, as indicated in [58] and by many others, corners of objects are less accurate laser points.

Burman's model [17] is based on adjusting the DSM points and the calibration parameters at the same time. Clearly, the large number of parameters involved in adjusting the calibration parameters and the DSM points makes this approach infeasible. In practice, the author incorporates control points like Kilian et al. [51]. Some analysis about the recovery of angular biases based on overlapping flights is presented, but it is limited. In addition, it is not clear how
the control points are obtained, but the correspondence is between corners of objects. Instead of LSM the data are resampled into a grid, and corners are extracted via an interest point operator. A dense grid around the matched interest points is then defined (the grid size is $21 \times 21$, the cell size is about the resolution of the laser data) and the laser points then participate in the adjustment. Control points are most likely the coordinates of the given corners. The approach seems to be a refined version of Kilian et al. [51].

Luthcke et al. [61] present an algorithm for calibrating satellite laser altimeters. The systematic errors include the mounting error and the range bias. Calibration is performed over the ocean surface via pitch and roll maneuvers (vary between $-3^\circ$ and $+3^\circ$). Ocean surfaces are flat; therefore, the correspondence problem is circumvented. However, more complicated surface modeling is involved (e.g., tides, effect of atmospheric pressure, etc.), and flatness of the surface is only true in an average sense.

A different view of the registration of laser data considers the registration as a transformation between two Cartesian coordinate systems. One coordinate system is represented by the cloud of 3-D laser points and the other by the ground coordinate system. The transformation is performed via the similarity transformation where translation, rotation and scale can be applied; the scale factor is usually assumed to be unity. The similarity transformation maps all the laser points together into the ground control system by determining the transformation parameters such that:

$$ |T(x_l, y_l, z_l) - (x_w, y_w, z_w)| = \min $$

(3.1)

The subscripts $l$ and $w$ denote laser point and ground point, respectively.
Postолов et al. [68] apply a variation on the similarity transformation, adopted from [56], to tie the points to the surfaces. The transformation is divided into two parts, a 2-D planimetric transformation followed by a transformation of the z-component. The research focuses on the transformation of the data, the correspondence between the laser surface and control surfaces is not treated here.

Habib and Schenk [36] tackle the correspondence between the cloud of points and the control surfaces simultaneously with the transformation problem. This is achieved by using the Generalized Hough Transform [3] – a voting procedure in the solution space. To alleviate the high dimensionality of the original problem, namely, a search in a 7-D space, a relaxed approached is used. The parameters are solved sequentially, one at a time, and the procedure is performed iteratively until convergence is reached. Final refinement is then performed using the algorithm described in [74].

Crombaghs et al. [19] encounter disagreements in overlapping zones of neighboring laser strips. Their model assumes a vertical translation and two rotations for each strip. The observations include tie points, i.e., relative height differences between laser points and control observations yield differences with respect to ground elevations. Points are selected over relatively flat areas so that no interpolation error will be introduced. Some remaining nonlinear errors are reported, but not explained. They are corrected by applying a second-degree polynomial correction.
Maas, 2000 [62] also proposes a method for tying overlapping laser strips. The transformation between the overlapping strips is carried out via local translations of the laser points. The proposed solution is based on LSM; convergence is reached in two or three iterations. The DSM of one strip is considered as the reference and the distances between the points of the other strip and the DSM are to be minimized. Problems occur near breaklines due to occlusions and discontinuities, or due to sampling resolution. Filtering of the data is therefore applied to eliminate these problems.

Filin et al. [28] show that the similarity transformation is an inaccurate modeling of the registration problem. For example, some of the deformations that are caused by the systematic errors are nonlinear (e.g., the effect of the range bias on the reconstructed surface) and cannot be compensated by a linear transformation. Furthermore, the effect of the errors changes between forward and backward flights. Thus, not all of the systematic effects can be removed with this approach. The parameters that are recovered might, therefore, be inaccurate, resulting in less accurate laser points.

3.2 The contribution of this dissertation

The literature review in the preceding section reveals that the research about calibration of laser data is far from complete. The adaptation of photogrammetric concepts into the calibration of laser systems is noticeable and not always appropriate; it suggests that a solution that originates from an analysis of the data acquisition model is needed. In addition, solutions are in most cases limited and domain specific;
for example, the correspondence is addressed only in [51, 17]. But even here the solution is tied to urban scenes. The alternative of calibration over flat locally horizontal surfaces is clearly limited, for example, when data is acquired over rolling terrain.

These limitations motivate an effort to devise a solution that is less restrictive in regards to the type of control information that is used. In this research, the focus of the first part is on finding an adequate representation of the calibration of laser altimeter systems; an analysis of the type of control information that suits the data and the system characteristics follows. The representation leads to the derivation of an analytical model and to the solution of the correspondence problem. A general yet efficient solution for finding the correspondence between the laser data and the control information is presented.

In addition, realization of the calibration concept involves several aspects that were not studied in detail in the past, for example, the recoverability of the different error sources in the system, the expected reliability of the estimated parameters and the configuration of control features that accommodate this. The importance of such analysis is clear; the goal is eliminating as many systematic errors in the system so that the overhead involved in preprocessing the laser data is reduced. The analysis of the recoverability and the expected reliability becomes more instrumental for the selection of calibration sites for satellite laser altimeters. Analysis of control feature configurations is essential for deriving such criteria.
In summary, the major contribution of this dissertation to the research of laser altimetry is in introducing a general calibration strategy that is based on system characteristics and on the data properties. The analysis of the various related aspects such as recoverability, reliability, and criteria for calibration site selection makes this work a more complete study of this problem.
CHAPTER 4

THE PROPOSED CALIBRATION STRATEGY

The calibration strategy comprises the calibration model and the solution for the correspondence between the laser points and the ground control. The previous chapters have indicated that the solution for the calibration is more involved than what it may seem at first. The first section of this chapter presents an outline of the proposed model with an emphasis on the motivation for adopting such an approach.

The realization of the model involves different types of data acquisition systems and different error sources. Section 2.4 has shown that there are numerous biases. The presentation begins with a model that assumes few biases, followed by the analysis of additional effects. The effects of using different system configurations are also studied here. The recoverability analysis is followed by the solution for the correspondence problem.

4.1 Outline of the proposed model

The question that motivates the derivation of the calibration strategy is how the correspondence between the laser points and the spots on the ground they illuminate
can be established. When the correspondence is established, the calibration becomes an ordinary adjustment problem that, with all of its complexities, is still rooted in a well established domain. Using simplified assumptions, such as calibration over flat surfaces or using distinct landmarks, are not always applicable. A general purpose solution should handle cases such as sparse sampling, high flying or orbiting altitude, and/or no distinct landmarks. Therefore, it becomes obligatory to analyze what type of knowledge exists about this problem at the outset.

The basic fact is that the disagreement in position between the laser points and the physical surface is due to systematic and random errors; by correcting the systematic errors the differences between the laser points and the surface are reduced. This realization leads to an alternative look at the calibration problem, instead of "forcing" correspondence (e.g., corner points) and computing the calibration parameters, the likelihood that a certain set of calibration parameters represent the true ones is evaluated. This evaluation can be carried out, for example, by measuring the norm of the residuals vector. The motivation for this approach is that even though the true incident point of the laser beam on the ground is unknown, a configuration exists under which the association between measured ranges and their locations on the terrain can be reestablished.\(^1\) This configuration is a function of the calibration parameters; using these parameters as variables the difference between the laser points and the surface is minimized in a chosen metric.

\(^1\)Degenerate cases may lead to multiple solutions. The goal is, however, to avoid them.
This formulation is rooted in an inferential framework. In general, instead of explaining the parameters by the data, the reversed question is asked how the data are best explained by the parameters. Realization of this concept involves evaluating the disagreement between the laser point coordinates and the surface. The proposed approach models this via two spatial relations. One is captured by the laser geolocation equation and incorporates the ground coordinates of the laser-point as a function of position, attitude, and range of the laser altimetry system. The second relation is between the planimetric position of a ground point and its elevation. The goal is to merge these two relations. This can be carried out by parametrically expressing a surface as

\[ f(x, y, z) = 0 \]  
(4.1)

The footprint coordinates can be viewed as a vector valued function \( g \) of the observations \(- Y\), the systematic errors \(- \Xi\), and the random errors \(- \varepsilon\) (augmented for simplicity), that can be written in the following form \( \mathbf{l} = g(Y, \Xi, \varepsilon) \). Consequently, the following relation can be written

\[ f(x_l, y_l, z_l) = h(Y, \Xi, \varepsilon) = 0 \]  
(4.2)

with \( h \) the parametric representation of the surface as a function of the observations, and the systematic and the random errors.

The obvious target function is minimizing the \( l_2 \) norm of the residuals vector, which also has the property of providing the best linear uniformly unbiased estimate for the parameters.
Many advantages of this problem formulation can be revealed already in this brief introduction. From a more theoretical standpoint this formulation models the essence of the problem; it ties the two relevant elements – the 3-D laser points and the surface. Furthermore, an explicit surface model incorporates additional information about the terrain, for example, trends, into the calibration model. In addition, with this formulation, no restriction on the surface type, e.g., flat surfaces, is needed. The approach is also independent of the laser system sampling rate. The inferential flavor turns the focus from the reconstructed surface to the laser points themselves. The traditional "feature based" approach depends on the rate of the sampling for detecting characteristic properties; here, in contrast, the focus is on minimizing the distances between the points to the ground, the density of the laser points is, therefore, not as significant.

An element of great importance that follows from the proposed approach is that the correspondence to be established between the laser points and the control information is simplified. Notice that the association here is between the laser points and the surface elements that contain the footprint. That is a subtle but important difference between association of the laser points and their corresponding ground position. The current association is less restrictive insofar as it does not require well-defined 3-D landmarks for calibrating the system, such as the approaches proposed by [17, 51]. It also implies that no distinct features are needed for solving the parameters or for establishing correspondence. In fact, laser points that fall "inside" ordinary surfaces (i.e., not "on" or near breaklines) are as good or even better in terms of accuracy than any other points.
The following sections go into greater detail in realizing the model and the concepts that have been outlined here. Sections 4.2 and 4.3 discuss the actual implementation of this approach and analyze the recoverability of the systematic errors. Section 4.4 discusses the solution for the correspondence problem.

4.2 Recoverability Analysis

In this section the above concept is realized. Two issues are treated in detail, the implementation of the calibration model, and the analysis of the system properties and parameter recoverability. In addition, the merits of different system configurations (profiling, or the different types of scanning) are studied.

4.2.1 Profiling system configuration

The parameters that are investigated first include the mounting bias expressed by the matrix $R_m$; the offset vector between the GPS receiver and the laser system, $[\delta_x \delta_y \delta_z]^T$; and the range bias, $\delta \rho$. Their incorporation into the model in eq. 2.12, with $R_{\text{scanner}} = I_3$, leads to

$$
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + R_{\text{INS}} \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} + R_m \begin{bmatrix} 0 \\ 0 \\ -(\rho + \delta \rho) \end{bmatrix}
$$

(4.3)

The mounting bias is expected to be small, therefore the analysis uses the rotation matrix approximation for small angles, given by equation 4.4. A general form is presented later in the chapter.

$$
R_m = \begin{bmatrix} 1 & -\kappa & \phi \\ \kappa & 1 & -\omega \\ -\phi & \omega & 1 \end{bmatrix}
$$

(4.4)
Introducing the correction explicitly and ignoring second-order effects leads to equation 4.5.

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix} = \begin{bmatrix}
  X_0 \\
  Y_0 \\
  Z_0
\end{bmatrix} + R_{INS} \begin{bmatrix}
  \delta_x \\
  \delta_y \\
  \delta_z
\end{bmatrix} + \begin{bmatrix}
  -\rho \phi \\
  \rho \omega \\
  - (\rho + \delta \rho)
\end{bmatrix}
\] (4.5)

As can be seen, the effect of the heading bias \( \kappa \) is unnoticeable in a profiler.²

Another spatial relation exists between the ground position and the terrain elevation. In reality, the function of the terrain is not known. It is more realistic to assume that the surface consists of a set of surface patches, each with its analytical form given. For the analysis of the calibration model the following planar representation is assumed although any other surface model can be used.

\[
s_1 x + s_2 y + s_3 z + s_4 = 0
\] (4.6)

The plane equation in its first part can be viewed as a scalar product between the normal to the surface element and the vector of the ground point coordinates.³ The vector notation simplifies the derivation and the analysis compared to an explicit representation. With this representation, the relation in eq. 4.6 can be written as:

\[
s \cdot x + s_4 = 0
\] (4.7)

with \( x_{3 \times 1} = [x \ y \ z]^T \) and \( s_{1 \times 3} = [s_1 \ s_2 \ s_3] \).

²Due to the order of rotations the heading bias does affect the system, but this effect is absorbed by the two other angles.

³An alternative representation is achieved by expressing the plane and the points in homogeneous coordinates.
Using eq. 4.7 in combination with 4.5 leads to eq. 4.8. The unknowns are \( \delta_\rho, \delta_x, \delta_y, \phi, \) and \( \omega. \)

\[
\Theta := \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + R_{INS} \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} + \begin{bmatrix} -\rho\phi \\ \rho\omega \\ -(\rho + \delta\rho) \end{bmatrix} + \begin{bmatrix} \bar{e}_x \\ \bar{e}_y \\ \bar{e}_z \end{bmatrix} + s_4 = 0 \tag{4.8}
\]

where \( \bar{e}_x, \bar{e}_y, \bar{e}_z, \) are the random error sources in the \( x, y, z \) directions propagated from the noise in the system. Equation 4.8 can also be written as

\[
-(s_1X_0 + s_2Y_0 + s_3Z_0 + s_4) = c_1\delta_x + c_2\delta_y + c_3\delta_z - c_3\delta\rho - (c_1\phi - c_2\omega + c_3)\rho + s_1\bar{e}_x + s_2\bar{e}_y + s_3\bar{e}_z \tag{4.9}
\]

where

\[
c_{1 \times 3} := \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix} R_{INS} \tag{4.10}
\]

The relation in eq. 4.10 essentially describes a coordinate transformation of the surface slope \( s \) by the altimeter attitude angles into the body system. The vector \( c \) thus defines a "modified" slope. One example of this is to fly over a flat horizontal surface, \( s = [0 \ 0 \ -1] \) while performing aircraft maneuvers; the right-hand side of eq. 4.10 becomes the last column of \( R_{INS} \) (namely, \( c = [r_{13} \ r_{23} \ r_{33}]_{INS}^T \)). Therefore, for flat surfaces the "modified" slope is fully determined by the aircraft attitude.

Returning to the derivations, equation 4.9 shows that the correction for the \( z \)-component, \( \delta_z, \) and for the range, \( \delta\rho, \) are equivalent; therefore \( \delta_z \) is constrained to zero. Rearranging 4.9 as a function of the unknowns leads to 4.11;
\[ w := c_3 \rho - (s_1 X_0 + s_2 Y_0 + s_3 Z_0 + s_4) = \\
\frac{c_1 \delta_x + c_2 \delta_y - c_3 \delta \rho - c_1 \rho \phi + c_2 \rho \omega + s_1 \bar{e}_x + s_2 \bar{e}_y + s_3 \bar{e}_z}{5} \quad (4.11) \]

The model parameters are recovered via the Gauss-Helmert model,

\[ w_n = A_{n \times m} \xi_m + B_{n \times 3n} e_{3n}, \quad e \sim \{0, \sigma_0^2 P^{-1}\} \quad (4.12) \]

with \( w \), the transformed observation vector; \( A \), the coefficient matrix; \( B \), the conditions matrix; \( \xi \), the vector of unknowns; \( e \), the observation noise; \( P \), the weight matrix; \( \sigma_0^2 \), the variance component; \( n \), the number of laser points; and \( m \), the number of unknowns. The least-squares criterion, which also provides the best linear uniformly unbiased estimator for \( \xi \), results in

\[ \hat{\xi} = (A^T (BP^{-1}B^T)^{-1} A)^{-1} A^T (BP^{-1}B^T)^{-1} w \quad (4.13) \]

with:

\[ D(\hat{\xi}) = \sigma_0^2 (A^T (BP^{-1}B^T)^{-1} A)^{-1} \quad (4.14) \]
\[ \sigma_0^2 = \frac{(B\bar{e})^T (BP^{-1}B^T)^{-1} (B\bar{e})}{n - m}, \quad B\bar{e} = w - A\hat{\xi}. \quad (4.15) \]

Grouping the different error components into a single error term \( \bar{e} = s_1 \bar{e}_x + s_2 \bar{e}_y + s_3 \bar{e}_z \) equation 4.11 can be written in matrix form as

\[
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_n
\end{bmatrix} =
\begin{bmatrix}
  c_{11} & c_{21} & -c_{31} & -c_{11} \rho_1 & c_{21} \rho_1 \\
  c_{12} & c_{22} & -c_{32} & -c_{12} \rho_2 & c_{22} \rho_2 \\
  \vdots & \vdots & \vdots & \vdots & \vdots \\
  c_{1n} & c_{2n} & -c_{3n} & -c_{1n} \rho_n & c_{2n} \rho_n
\end{bmatrix}
\begin{bmatrix}
  \delta_x \\
  \delta_y \\
  \delta \rho \\
  \phi \\
  \omega
\end{bmatrix}
+ \begin{bmatrix}
  \bar{e}_1 \\
  \bar{e}_2 \\
  \vdots \\
  \bar{e}_n
\end{bmatrix} \quad (4.16)
\]
Equation 4.16 provides a convenient form for the analysis of the calibration of a profiling system and the recoverability of the system biases. Notice that the parameters depend only on the "modified" slope of the surface patch and the measured range, the position of the system \((X_0, Y_0)\) at the time of the ranging has no effect on the coefficient matrix \(A\). Assuming a basic profiler configuration with no aircraft/spacecraft maneuvers \((c = s)\), which is the common configuration for satellite altimeters, reveals that horizontal surfaces or surfaces tilted only in one direction (either \(s_1 = 0\) or \(s_2 = 0\)) turn some of the columns to zero. Therefore, they cannot recover all parameters independently. Using a single surface, \(s_{11} = s_{12} = \cdots = s_{1n} = s_1\) and \(s_{21} = s_{22} = \cdots = s_{2n} = s_2\), makes the first three columns linearly dependent, and also columns four and five. Therefore, a single surface is unsuitable to recover all the parameters regardless of the number of observations. Recovering all the parameters or even part of them (such as the mounting and the ranging biases) requires at least two known surfaces.

An analysis of the relations between the first two columns and the fourth and fifth shows that the difference between them is the multiplication with the range component in columns four and five. With small relief variation, the similarity between the two pairs of columns becomes high. High similarity among columns means that the parameters in question also have a very similar effect on the system and it is difficult to identify the actual effect of each of them independently. The high similarity is inherent to the calibration of profilers and means that, by using direct methods, only three parameters can be resolved simultaneously – the range bias and either the \(x\), \(y\) components of the offset vector or the mounting bias along the \(x\), \(y\)-axes. The high
similarity also means that solving for either two is nearly equivalent. Good approximations for the offset vector are usually obtained by measuring the distance between the GPS receiver antenna and the laser altimeter; their measurements should be introduced as stochastic constraints.

Adding aircraft maneuvers to the system is analyzed with the help of equation 4.10. The effect of slope "modification" implies that performing aircraft maneuvers is equivalent to flying over changing topography. Theoretically, calibration of profilers over horizontal surfaces with flight maneuvers is equivalent to calibration over various tilted surfaces without maneuvers.

The next subsection extends the modeling and analysis to a scanning pattern. The focus again is on the recoverability of the calibration parameters.

### 4.2.2 Scanning system configuration

Scanning systems add variability in the pointing direction of the laser beam. The pointing direction of the beam in the laser altimeter reference frame is given by

\[
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix} + \begin{bmatrix}
  \delta_u \\
  \delta_v \\
  \delta_w
\end{bmatrix} = R_{\text{scanner}} \begin{bmatrix}
  0 \\
  0 \\
  -(\rho + \delta\rho)
\end{bmatrix} \approx R_{\text{scanner}} \left( \begin{bmatrix}
  0 \\
  0 \\
  -\rho
\end{bmatrix} + \begin{bmatrix}
  0 \\
  0 \\
  -\delta\rho
\end{bmatrix} \right)
\]

Following eq. 2.12, the laser equation for scanning systems that model the same biases as in 4.3 is
\[
\begin{bmatrix}
    x \\
    y \\
    z \\
\end{bmatrix} = \begin{bmatrix}
    X_0 \\
    Y_0 \\
    Z_0 \\
\end{bmatrix} + R_{\text{INS}} \begin{bmatrix}
    \delta_x \\
    \delta_y \\
    \delta_z \\
\end{bmatrix} + R_m R_{\text{scanner}} \begin{bmatrix}
    0 \\
    0 \\
    -(\rho + \delta \rho) \\
\end{bmatrix}
\] (4.18)

The major difference between the scanning and the profiling configurations is that the mounting bias effect is shifted to be between the scanning component and the INS rotation. One consequence is that the heading bias effect (along the z-axis) becomes noticeable. Equation 4.18 shows that the scanning component has no effect on the offset bias vector, therefore, the focus is on the derivations for the mounting biases. The part in eq. 4.18 that has changed with respect to eq. 4.3 due to the scanning is

\[
R_{\text{INS}} \begin{bmatrix}
    1 & -\kappa & \phi \\
    \kappa & 1 & -\omega \\
    -\phi & \omega & 1 \\
\end{bmatrix} R_{\text{scanner}} \begin{bmatrix}
    0 \\
    0 \\
    -\rho \\
\end{bmatrix} = R_{\text{INS}} \begin{bmatrix}
    1 & -\kappa & \phi \\
    \kappa & 1 & -\omega \\
    -\phi & \omega & 1 \\
\end{bmatrix} \begin{bmatrix}
    u \\
    v \\
    w \\
\end{bmatrix} =
\]

\[
R_{\text{INS}} \left( \begin{bmatrix}
    u \\
    v \\
    w \\
\end{bmatrix} + \begin{bmatrix}
    -v & w & 0 \\
    u & 0 & -w \\
    0 & -u & v \\
\end{bmatrix} \begin{bmatrix}
    \kappa \\
    \phi \\
    \omega \\
\end{bmatrix} \right)
\] (4.19)

where the second term in equation 4.19 is a function of the mounting bias components. Introducing the surface property (directly as a "modified" surface slope) leads to the final form, as given in eq. 4.20

\[
[c_1 \; c_2 \; c_3] \left( \begin{bmatrix}
    u \\
    v \\
    w \\
\end{bmatrix} + \begin{bmatrix}
    -v & w & 0 \\
    u & 0 & -w \\
    0 & -u & v \\
\end{bmatrix} \begin{bmatrix}
    \kappa \\
    \phi \\
    \omega \\
\end{bmatrix} \right) =
\]

\[
c_1 u + c_2 v + c_3 w + (-c_1 v + c_2 u) \kappa + (c_1 w - c_3 u) \phi + (-c_2 w + c_3 v) \omega
\] (4.20)

The complete form, after constraining \( \delta_z \) to zero (comparable to eq. 4.9), is given in eq. 4.21
\[ c_1u + c_2v + c_3w - (s_1X_0 + s_2Y_0 + s_3Z_0 + s_4) = \]
\[ c_1\delta_x + c_2\delta_y - c_3\delta\rho + (-c_1v + c_2u)\kappa + (c_1w - c_3u)\phi + (-c_2w + c_3v)\omega + \]
\[ s_1\tilde{e}_x + s_2\tilde{e}_y + s_3\tilde{e}_z \quad (4.21) \]

The coefficients of the mounting bias are a function of the INS angles, the slopes and the scanner angle. With no scanning \((R_{\text{scan}} = I_{3\times3})\), the laser beam pointing direction becomes \([u \ v \ w] = [0 \ 0 \ -\rho]\) and equation 4.21 is reduced to eq. 4.9. For line scanner \((R_{\text{scanner}} = R_x(\omega_i))\), the laser beam pointing direction is \([u \ v \ w] \approx [0 \ -\omega_i\rho \ -\rho]\), and for a conical scanner with rotation along the z-axis and an off-nadir pointing angle (in either the x- or y- axes) the vector \([u \ v \ w]^T\) will have a full form.

In terms of the recoverability of the parameters, the scanning pattern alleviates the dependency between the offset vector and the mounting angles. For a conical scanner, the scan in all directions generates coefficients with all signs (as it points in all directions, \(u\) and \(v\) are changing signs, \(w\) is fairly constant) with respect to the mounting bias parameters. This property makes the relevant columns linearly independent and enables one to identify both the mounting bias and the offset vector simultaneously. For a line scanner \((u = 0, w \approx -\rho)\), equation 4.21 shows that \(\delta_x\) and \(\phi\) have the same behavior as the profiler, making them nonseparable. The scanning pattern does not allow separating the z-component of the offset vector and the range.
bias. That is so since the off-nadir scanning angle is not big enough to have a significant effect. The important point is that the scanning geometry extends the number of parameters that can be resolved from merely three to six parameters for a conical scanner and five for a line scanner.

Having developed the basic equations for a profiling and scanning system, the next subsection extends the current analysis to other potential systematic error sources.

4.2.3 Inclusion of additional parameters

The following section models additional error sources in the system and investigates their recoverability. The focus is on outlining the analytical approach and considerations involved in handling this issue.

The extended form models additional parameters including the INS bias, \( \Delta R_{INS} \); the INS time synchronization bias, \( t_{INS} \); the GPS time synchronization bias, \( t_{GPS} \); a position offset, \( [\delta X_0 \ \delta Y_0 \ \delta Z_0] \); and the range scale change, \( \lambda \). Their effect is modeled as follows

\[
\begin{bmatrix}
x
\end{bmatrix} = 
\begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix} + 
\begin{bmatrix}
\delta X_0 \\
\delta Y_0 \\
\delta Z_0
\end{bmatrix} + 
R_{INS} 
\begin{bmatrix}
v_x \\
0 \\
0
\end{bmatrix} t_{GPS} + 
\Delta R_{INS} R_{\omega + \hat{\omega} t_{INS}, \phi + \hat{\phi} t_{INS}, \kappa + \hat{\kappa} t_{INS}} \left( \begin{bmatrix}
\delta_x \\
\delta_y \\
\delta_z
\end{bmatrix} + R_m R_{scanner} \begin{bmatrix}
0 \\
0 \\
-(\lambda \rho + \delta \rho)
\end{bmatrix} \right)
\]

The formal form for solving the parameters that are modeled here is given, after linearization, in eq. 4.23.
\[ w_i = s \left( I_3 \begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \end{bmatrix} + R_{INS} \begin{bmatrix} v_x \\ 0 \\ 0 \end{bmatrix} \right) dt_{GPS} + \]

\[
\frac{\partial \Delta R_{INS}}{\partial \omega_I} \begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} d\omega_I + \frac{\partial \Delta R_{INS}}{\partial \phi_I} \begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} d\phi_I + \frac{\partial \Delta R_{INS}}{\partial \kappa_I} \begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} d\kappa_I +
\]

\[
\frac{\partial R_{INS}}{\partial t_{INS}} R_m R_{scanner} \begin{bmatrix} 0 \\ 0 \end{bmatrix} dt_{INS} + R_{INS} \frac{\partial R_m}{\partial \omega} R_{scanner} \begin{bmatrix} 0 \\ \rho \end{bmatrix} d\omega +
\]

\[
R_{INS} \frac{\partial R_m}{\partial \phi} R_{scanner} \begin{bmatrix} 0 \\ \rho \end{bmatrix} d\phi + R_{INS} \frac{\partial R_m}{\partial \kappa} R_{scanner} \begin{bmatrix} 0 \\ \rho \end{bmatrix} d\kappa +
\]

\[
R_{INS} R_m R_{scanner} \begin{bmatrix} 0 \\ 0 \end{bmatrix} d\lambda + \bar{\varepsilon} \quad (4.23)
\]

where:

dt_{INS} – INS incremental time synchronization bias.

dt_{GPS} – the incremental GPS time synchronization bias.

d\omega_I, d\phi_I, d\kappa_I – change in the angles of the INS bias.

d\lambda – range scale change.

\[
\begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} = R_{INS} R_m R_{scanner} \begin{bmatrix} 0 \\ 0 \end{bmatrix} (4.24)
\]

The extended form provides one row in the Gauss-Helmert model (eq. 4.12) whose parameters are solved by eq. 4.13. Equation 4.23 is not too informative for the recoverability analysis of the calibration parameters. To follow the analysis that was
carried out until now, some assumptions, such as angles being small, or ignoring non-linear effects, are made. With these assumptions in mind, much more of information about the effects and the recoverability of the parameters is gained.

**Range correction**

The range has been assumed so far to be correct up to a given constant bias. It was indicated in Section 2.4 that the range depends on additional parameters. These dependencies require modeling a first-order correction to the range either as a function of the measured distance or the amplitude. Information about the amplitude is usually not available. However, it is a reasonable assumption that under constant flying altitude and homogeneous albedo, the amplitude will be highly correlated with the "modified" slope. For small relief variation in the calibration area, amplitude and range will also be highly correlated. In general, both can be modeled as a scale (linear) factor. Following eq. 4.22, the effect is modeled by

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
= \begin{bmatrix}
  X_0 \\
  Y_0 \\
  Z_0
\end{bmatrix} + R_{INS} \begin{bmatrix}
  \delta_x \\
  \delta_y \\
  \delta_z
\end{bmatrix} + R_m R_{scanner} \begin{bmatrix}
  0 \\
  0 \\
  -(\lambda \rho + \delta \rho)
\end{bmatrix}
\]

(4.25)

with \( \lambda \), the range scale. Without repeating the derivations, the range scale change has the following contribution

\[
c' \rho - (s_1 X_0 + s_2 Y_0 + s_3 Z_0 + s_4) = c_1 \delta_x + c_2 \delta_y - c_3 \delta \rho + \cdots - c_4 \rho d \lambda + s_1 \delta x + s_2 \delta y + s_3 \delta z
\]

(4.26)

with \( \lambda = 1 + d \lambda \), and \( c' = c R_m R_{scanner} \), the surface slope in the laser altimeter system. The analysis that was carried out in regards to eq. 4.16 holds here also. Without significant changes in elevation it is difficult to distinguish between the trend and the constant effect. However, this dependency can be circumvented using the relation
\[ \lambda \rho_i + \delta \rho = \rho_i + d\lambda (\bar{\rho} + \Delta \rho_i) + \delta \rho = \rho_i + \Delta \rho_i d\lambda + \delta \rho' \]  

(4.27)

with \( \bar{\rho} \) the average range, and \( \delta \rho' := \delta \rho + d\lambda \bar{\rho} \), a modified constant bias.

By applying the scale correction to the difference in range and not to the whole quantity, the range scale coefficients have positive and negative values and thus it is not influenced by the range bias itself. This shows that in addition to the parameters resolved so far a range scale change can also be recovered.

A similar “modification” can be applied to isolating the \( \delta_x \), and \( \delta_y \) components of the offset vector and the mounting bias for a profiling system

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix} + R_{INS} \begin{bmatrix}
0 \\
0 \\
-(\bar{\rho} + \Delta \rho_i + \delta \rho')
\end{bmatrix} = \begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix} + R_{INS} \begin{bmatrix}
\delta_x' \\
\delta_y' \\
\delta_z - \bar{\rho}
\end{bmatrix} + R_m \begin{bmatrix}
0 \\
0 \\
-(\Delta \rho_i + \delta \rho)
\end{bmatrix} \quad (4.28)
\]

where \( \delta_x' := \delta_x - \bar{\rho} \phi \), \( \delta_y' := \delta_y + \bar{\rho} \omega \) are modified offsets in the \( x \)-, \( y \)-directions.

Incorporation of the range scale change and the modified offset vector is straightforward, following the derivations in eq. 4.26 and 4.28.

**INS bias**

Errors in initializing the INS system cause systematic biases in the directions read by the navigation system. This results in an incorrect conversion of the body reference.
frame to the inertial reference frame. The INS bias is modeled in eq. 4.22 and its effect on the data is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \Delta R_{INS} R_{INS} \left( \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} + R_m R_{scanner} \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix} \right) \quad (4.29)$$

Recovering the INS bias depends on aircraft maneuvers. Assuming little aircraft maneuvers and ignoring the flying direction (namely, assuming without loss of generality a north flying direction), the INS rotation matrix is close or equal to the identity matrix, and therefore

$$\Delta R_{INS} R_{INS} \approx R_{INS} \Delta R_{INS} \quad (4.30)$$

The INS bias multiplies the offset vector and the mounting bias matrix. Consequently it is absorbed by these parameters. To recover the INS bias, the relation in 4.30 should be altered. The analysis of the INS bias effect for profiling and scanning systems is similar. It is therefore performed here for a profiling system, while assuming first that $R_m = I_3$.

$$\begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix} \begin{bmatrix} 0 & -\kappa_I & \phi_I \\ -\kappa_I & 0 & -\omega_I \\ -\phi_I & \omega_I & 0 \end{bmatrix} R_{INS} \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix} =$$

$$\begin{bmatrix} 0 & -\kappa_I & \phi_I \\ -\kappa_I & 0 & -\omega_I \\ -\phi_I & \omega_I & 0 \end{bmatrix} \begin{bmatrix} -\Phi \rho \\ \Omega \rho \\ -\rho \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix} \begin{bmatrix} -\Omega & -1 & 0 \\ -\Phi & 0 & 1 \\ 0 & \Phi & \Omega \end{bmatrix} \begin{bmatrix} \kappa_I \\ \phi_I \\ \omega_I \end{bmatrix} \rho =$$

$$-(s_1 \Omega + s_2 \Phi)\rho \kappa_I - (s_1 - s_3 \Phi)\rho \phi_I + (s_2 + s_3 \Omega)\rho \omega_I \quad (4.31)$$

with $\Omega$, $\Phi$ – the INS attitude angle along the $x$- and $y$-axes respectively (denoted here by uppercase letters for distinction). Substituting the coefficients into the linearized form in eq. 4.11, thus allowing additional mounting bias angles, $\phi$ and $\omega$, yields
\[ c_3 \rho - (s_1 X_0 + s_2 Y_0 + s_3 Z_0 + s_4) = \]
\[ c_1 \delta_z + c_2 \delta_y - c_3 \delta \rho - (s_1 \Omega + s_2 \Phi) \rho \kappa_I - (s_1 - s_3 \Phi) \rho \phi_I + (s_2 + s_3 \Omega) \rho \omega_I - (s_2 + s_3 \Omega) \rho \omega - c_1 \rho \phi + c_2 \rho \omega + \tilde{e} \] (4.32)

Notice that the coefficients for the INS heading bias with small maneuvers will be small (a product of the slope and the INS angles). Since the heading bias is also small, the overall effect is then of second order. The dependency on the aircraft maneuvers is also visible. With no maneuvers, the "modified" slopes are the slopes themselves just as for the INS biases. Notice, however, that the mounting bias coefficients depend on the modified slope, \( c \), and not the surface slope, \( s \), therefore, flying in different heading directions can help in reducing the similarity between the two types of biases. The modifications for scanning systems involve modifying the slopes, \( c \rightarrow c' \) and updating both \( \Omega \) and \( \Phi \) by replacing \( R_{INS} \) with \( R_{INS} R_{Scanner} \).

**Time synchronization error**

Laser altimeter systems are composed of three independent components – ranging, attitude and position. Therefore, two types of time errors are possible. Fixing the ranging time suggests that the INS time and the GPS time can be shifted. The aliasing, mentioned in section 2.4 is not random in the true sense, but cannot be modeled better than the fitting error, in case that a smooth function is fitted for interpolation.

**The GPS synchronization error** — The GPS time synchronization error effect is an offset in position. The relation in 4.33 shows that the synchronization error has
the same effect as the offset vector between the GPS antenna and the laser system. With no dramatic changes in the aircraft velocity, the product of the offset and the velocity is constant.

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix} + R_{INS} \begin{bmatrix}
v_x \\
0 \\
0
\end{bmatrix} t_{GPS} + R_{INS} \begin{bmatrix}
\delta_x \\
\delta_y \\
\delta_z
\end{bmatrix} + R_m R_{scanner} \begin{bmatrix}
0 \\
0 \\
-\rho
\end{bmatrix}
\]

with \( \delta_x := \delta_x + v_z t_{GPS} \), a modified offset in the \( x \)-direction.

For calibration of satellite laser altimeters, Martin and Thomas [63] propose rotating the satellite by 90° (moving from a “sailboat” orbiting mode to an “airplane” mode [79]). In contrast to an airborne calibration, performing yaw maneuvers does not imply that the flying/orbiting direction is changing. In this way, the effect of the GPS time error and the offset vector are not equivalent and both can be recovered separately. For airborne calibration this concept cannot be realized.

The INS synchronization error — The possible time offset between the INS and the laser altimeter causes a different effect. The INS time bias is modeled as follows:
R_{INS} = R[\Omega + \dot{\Omega}_{INS}, \Phi + \dot{\Phi}_{INS}, K + \dot{K}_{INS}] \quad (4.34)

with \dot{\Omega}, \dot{\Phi}, \dot{K}, the time derivatives of the angular variations, computed by

\begin{align*}
\dot{\Omega}_i &= \frac{\Omega_{i+1} - \Omega_{i-1}}{2\Delta t} \\
\dot{\Phi}_i &= \frac{\Phi_{i+1} - \Phi_{i-1}}{2\Delta t} \\
\dot{K}_i &= \frac{K_{i+1} - K_{i-1}}{2\Delta t}
\end{align*} \quad (4.35)

with \Delta t is the sampling interval.

Using the small angle assumption, the modified rotation matrix has then the following form:

\begin{align*}
R_{INS} &= \begin{bmatrix}
1 & -(K + \dot{K}_{INS}) & (\Phi + \dot{\Phi}_{INS}) \\
(K + \dot{K}_{INS}) & 1 & -(\Omega + \dot{\Omega}_{INS}) \\
-(\Phi + \dot{\Phi}_{INS}) & (\Omega + \dot{\Omega}_{INS}) & 1
\end{bmatrix} = \\
&= \begin{bmatrix}
1 & -K & \Phi \\
K & 1 & -\Omega \\
-\Phi & \Omega & 1
\end{bmatrix} + \begin{bmatrix}
0 & -\dot{K}_{INS} & \dot{\Phi}_{INS} \\
\dot{K}_{INS} & 0 & -\dot{\Omega}_{INS} \\
-\dot{\Phi}_{INS} & \dot{\Omega}_{INS} & 0
\end{bmatrix} \quad (4.36)
\end{align*}

Hofton et al. [42], and Spikes et al. [81] model three different time offsets, one for every axis.\(^4\) This is a more general derivation since a single time-offset encapsulates the three coefficients. Using the subdivision into three offsets, the coefficient form for a profiling system is

\begin{align*}
d\Theta &= \begin{bmatrix}
s_1 & s_2 & s_3
\end{bmatrix}
\begin{bmatrix}
0 & -\dot{K}_K & \dot{\Phi}_\Phi \\
\dot{K}_K & 0 & -\dot{\Omega}_\Omega \\
-\dot{\Phi}_\Phi & \dot{\Omega}_\Omega & 0
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
-\rho
\end{bmatrix} = \\
&= 0t_K + s_1 \dot{\Phi}\rho\dot{\Phi} - s_2 \dot{\Omega}\rho\dot{\Omega} \quad (4.37)
\end{align*}

\(^4\) Vaughn et al. [86] mention time lags to each parameter due to internal processing of INS data.
The surprising result is that for a profiling system the time bias for the heading direction is not recoverable. The form for the two other biases indicates that the recovery of the time offsets is a direct function of the aircraft maneuvers and that recovering these parameters over a flat or nearly flat surface is impossible. The direct dependency on the angular derivatives means that these biases are independent of the other angular biases, and thus recoverable under aircraft maneuvers. Here, with the time derivatives available, they can be recovered together with the INS bias and the mounting bias. For a scanning system, the analysis in section 4.2.2 shows that the rotated range vector (eq. 4.17) forms an either full or partially full vector.

\[
\begin{bmatrix}
0 \\
0 \\
-\rho
\end{bmatrix}
\cong
\begin{bmatrix}
\cos \kappa_s & -\sin \kappa_s & 0 \\
\sin \kappa_s & \cos \kappa_s & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
0 \\
\omega_s \rho \\
-\rho
\end{bmatrix}
\]

conical scan

\[
\Rightarrow d\theta \cong [s_1 \ s_2 \ s_3] dR_{INS} \begin{bmatrix}
-(\sin \kappa) \omega_s \rho \\
(\cos \kappa) \omega_s \rho \\
-\rho
\end{bmatrix}
\]

(4.38)

with \(\omega_s\) as off-nadir scanning angle, and \(\kappa_s\), the conical scanner heading angle. Therefore, the heading angle time offset can be recovered here.

**Position offset**

A position offset may be caused by an error in the datum transformation or by a tropospheric delay, if the GPS base station and the laser system are far from one another. The modeling of the constant position offset effect is given in equation 4.39

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix}
+ \begin{bmatrix}
\delta X_0 \\
\delta Y_0 \\
\delta Z_0
\end{bmatrix}
+ R_{INS} \begin{bmatrix}
\delta_x \\
\delta_y \\
\delta_z
\end{bmatrix}
+ R_{scanner} \begin{bmatrix}
0 \\
0 \\
-\rho
\end{bmatrix}
\]

(4.39)
The constant offset is independent of the system flying direction. Therefore, flying in opposite directions (a roundtrip pattern) should set the constant position offset independent of the offset vector, and by the same token from the mounting bias. The derivation begins from a form similar to equation 4.3, where the pitch and roll INS angles, \( \Omega \) and \( \Phi \), are set to zero.

\[
\begin{bmatrix}
  x \\
y \\
z
\end{bmatrix} = 
\begin{bmatrix}
  X_0 \\
  Y_0 \\
  Z_0
\end{bmatrix} + 
\begin{bmatrix}
  \delta X_0 \\
  \delta Y_0 \\
  \delta Z_0
\end{bmatrix} + 
\begin{bmatrix}
  \cos K & -\sin K & 0 \\
  \sin K & \cos K & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  \delta x \\
  \delta y \\
  \delta z
\end{bmatrix} + 
R_{INS} \begin{bmatrix}
  0 \\
  0 \\
-\rho
\end{bmatrix} + 
\begin{bmatrix}
  \tilde{e}_x \\
  \tilde{e}_y \\
  \tilde{e}_z
\end{bmatrix}
\]

(4.40)

The mounting bias is assumed to be small. Therefore,

\[
\begin{bmatrix}
  x \\
y \\
z
\end{bmatrix} = 
\begin{bmatrix}
  X_0 \\
  Y_0 \\
  Z_0
\end{bmatrix} + 
\begin{bmatrix}
  \delta X_0 \\
  \delta Y_0 \\
  \delta Z_0
\end{bmatrix} + 
\begin{bmatrix}
  \cos \kappa & -\sin \kappa & 0 \\
  \sin \kappa & \cos \kappa & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  \delta x \\
  \delta y \\
  \delta z
\end{bmatrix} + 
\begin{bmatrix}
  -\rho \phi \\
  \rho \omega \\
-\left(\rho + \delta \rho\right)
\end{bmatrix} + 
\begin{bmatrix}
  e_x \\
  e_y \\
  e_z
\end{bmatrix}
\]

(4.41)

Introducing the surface constraint in eq. 4.7 into eq. 4.41, when \( \delta_z \) is constrained to zero, leads to

\[
w = s_1 \delta X_0 + s_2 \delta Y_0 + s_3 \delta Z_0 + (s_1 \cos K + s_2 \sin K) \delta x + (s_2 \cos K - s_1 \sin K) \delta y -
\rho(s_1 \cos K + s_2 \sin K) \phi + \rho(s_2 \cos K - s_1 \sin K) \omega - s_3 \rho + \tilde{e}
\]

(4.42)

As with the analysis that followed eq. 4.16, the effect of the mounting bias angles and the offset vector is similar. Notice that the coefficients for the \( z \)-component of
the position offset and the range offset bias are different only by a sign and, therefore, have a similar effect; consequently, $\delta Z_0$ is constrained to zero.\(^5\)

To analyze eq. 4.42, assume the heading angle, $K$, is close to 0 for one direction and to $\pi$ in the other direction. With this configuration $\sin K \sim K$ for small angles, $\cos K \sim 1$ for $K \sim 0$ and $\cos K \sim -1$ for $K \sim \pi$. Using observations from both tracks will have the following form

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_1\rho & -s_2\rho & s_3 \\ s_1 & s_2 & -s_1\rho & s_2\rho & s_3 \end{bmatrix} \begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \phi \\ \omega \\ \delta \rho \end{bmatrix} + \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$$  \hspace{1cm} (4.43)

Consequently the positional constant offsets as well as the other biases are recoverable, both for a profiling and for a scanning system. Notice that the repeat flight does not necessarily have to cover the same surface. However, both surfaces should be modeled in the same coordinate system.

4.2.4 Summary

The recoverability analysis shows a remarkable fact. In spite of the relatively weak geometric configuration of the system, many parameters can be resolved by simple means such as aircraft maneuvers and varying topography. None of the parameters require distinct 3-D landmarks as control information.

It has also been shown that the effect of parameters which cannot be recovered is usually absorbed by other parameters to a large extent. In case of secondary errors,\(^5\) as a side remark, it can be shown that working in a global coordinate system and using sites at both sides of the hemisphere, the $\delta Z_0$ can also be recovered.
such as the range scale change and the offset vector, methods for circumventing the
high collinearity between the parameters can be developed.

The analysis has also revealed that, in terms of system calibration, a scanning
laser system is useful in separating the laser altimeter biases from the other com­
ponents. Consequently, the offset between the GPS receiver and the laser altimeter
can be recovered. Another aspect is that a scanning pattern can save the need for
performing pitch and roll maneuvers to recover the more serious biases. Thus less
uncontrolled effects are introduced to the system. The analysis has also shown that
a conical scanning pattern is preferable to line scanning, at least in terms of recover­
ability of the different parameters.

A question that arises concerns model selection, namely, how to determine which
parameters absorb most of the systematic errors in order to eliminate those that do
not contribute. This topic is discussed later together with the experimental results.

4.3 Uncertainty in the surface slopes

In the preceding section, an implicit assumption has been made that the surface
parameters are error free. In reality they carry their own uncertainty. The random
errors in the measurements of the ground coordinates (e.g., from photogrammetric
or GPS surveys), propagate into the surface parameters. These uncertainties can be
modeled as stochastic constraints. In case of a subdivision of the surface into triangu­
lar cells, an adjustment of the coordinates will directly modify the surface parameters.
However, if the surface elements have more complicated boundaries (e.g., surfaces extracted from photogrammetry, or after applying a segmentation algorithm), adjusting the points will obviously deform the surface. The modeling proposed here is based on adjusting the surface parameters, and constraining them (in a stochastic sense) to contain the boundary points. This way there is no need to introduce the reference points into the adjustment but only the boundary points of the surface elements; their accuracy can be estimated via error propagation. First-order continuity is addressed insofar as neighboring surfaces share a similar constraint that “forces” (up to the limit of stochastic constraint) the adjacent surfaces to pass through the corner points (nodes). Finally, the surface property is maintained, i.e., a plane remains a plane no matter how complex its boundary may be. The form in 4.23 is thus extended as follows

\[
\begin{align*}
\theta_i = & \frac{\partial \theta_i}{\partial s_{i_1}} ds_{i_1} + \frac{\partial \theta_i}{\partial s_{i_2}} ds_{i_2} + \frac{\partial \theta_i}{\partial s_{i_3}} ds_{i_3} + \frac{\partial \theta_i}{\partial s_{i_4}} ds_{i_4} + \ldots \\
& \underbrace{\ldots + \frac{\partial \theta_i}{\partial \omega} d\omega + \ldots + \frac{\partial \theta_i}{\partial \delta \rho}}_{A_1, \xi_1} \ldots + \frac{\partial \theta_i}{\partial \delta \rho} (4.44)
\end{align*}
\]

The stochastic constraints are introduced as

\[
E\{0\} = \begin{cases} 
X_j s_{i_1} + Y_j s_{i_2} + Z_j s_{i_3} + s_{i_4} 
\end{cases}_{A_2, \xi_2}, \quad D\{0\} = \sigma_0^2
\]

(4.45)

with \(X_j, Y_j, Z_j\), the coordinates of the \(j\)th node; \(j = 1, 2, \ldots, n\); \(0\), a vector of zeros with random properties. \(E\) denotes the statistical “expectation”; \(D\) the “dispersion” with \(\sigma_0^2\) as the a-priori variance.
Notice that this representation is underconstrained as the four surface parameters are solved simultaneously. Methods that handle such cases exist [73]. However, fixing one parameter (usually $s_3 = -1$) is the simplest way to overcome this deficiency. Alternatively, normalizing the normal vector, $\|s\| = 1$, can be used as well. The model presented below fixes $s_3 = -1$.

Integration of the stochastic properties of the ground points into the model can be carried out within the extended Gauss-Markov model with pseudo-observations. The formal set-up is given by

$$
\begin{bmatrix}
    w & 0 \\
    0 & 0
\end{bmatrix} =
\begin{bmatrix}
    A_1_{q \times 3n} & A_2_{q \times m} \\
    A_3_{3n \times 3n} & 0
\end{bmatrix}
\begin{bmatrix}
    \xi_{1 \times 3n} \\
    \xi_{2 \times m}
\end{bmatrix} +
\begin{bmatrix}
    \bar{\varepsilon} \\
    \bar{e}_0
\end{bmatrix},
\begin{bmatrix}
    \bar{\varepsilon} \\
    \bar{e}_0
\end{bmatrix} \sim \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma' := BB^T & 0 \\ 0 & \Sigma_0 \end{bmatrix} \right)
$$

(4.46)

with $\Sigma_0$ the dispersion matrix of the stochastic constraints as propagated from the dispersion matrix of the ground points; $m$ as the number of unknowns, $n$ as the number of nodes, and $q$ as the number of laser points.

The normal equations follow as

$$
\begin{bmatrix}
    N_{33} + N_{11} & N_{12} \\
    N_{21} & N_{22}
\end{bmatrix}
\begin{bmatrix}
    \hat{\xi}_1 \\
    \hat{\xi}_2
\end{bmatrix} =
\begin{bmatrix}
    A_3^T \Sigma_0^{-1} 0 + c_1 \\
    c_2
\end{bmatrix}
$$

(4.47)

Based on [73] the solution that provides the best linear uniformly unbiased estimate for $\hat{\xi}_2$ is thus obtained as
\[ \hat{\xi}_2 = N_{22}^{-1}c_2 - N_{22}^{-1}N_{21}(N_{33} + S_1)^{-1}(A_3^T\Sigma_0^{-1}A_3 + c_1 - N_{12}N_{22}^{-1}c_2) \]  \hspace{1cm} (4.48)

where:

\[ N_{ij} := A_i^T(\Sigma')^{-1}A_j \hspace{0.5cm} (i, j \in \{1, 2\}) \]
\[ N_{33} := A_3^T\Sigma_0^{-1}A_3 \]
\[ c_j := A_j^T(\Sigma')^{-1}y \hspace{0.5cm} (i, j \in \{1, 2\}) \]
\[ S_1 := N_{11} - N_{12}N_{22}^{-1}N_{21} \hspace{0.5cm} \text{is the 1st Schur complement.} \]

Notice that the uncertainty in the ground control can be of use also in determining the partition into surface elements. Unless the surface was predetermined (e.g., the result of a photogrammetric measurement of surface elements) the uncertainty propagates into the surface parameters. Using the F-test, the statistical significance of the difference between neighboring surface elements can be used for grouping neighboring cells, resulting in larger surface elements.

4.4 Correspondence

The underlying assumption for the analytical model so far is that the surface elements illuminated by the laser beams are known. In practice the situation is different. Modest assumptions about the biases and the flying altitude show that the beam can be shifted away from the surface element it illuminated. The deviation increases as a function of the error in the pointing direction and the flying/orbiting altitude. For example, with an orbiting altitude of 600 km, a 1 mrad mounting bias results in a search window of the order of $1.2 \times 1.2$ km on the ground for each laser point. As the only information carried by a laser point is the $x, y, z$ coordinates that were deformed by the biases, finding the correspondence between a laser point and the spot on the ground, or even the surface element it illuminated, is practically impossible. To establish a correspondence several laser points are needed, but even then solving the
correspondence can be difficult. System constraints, such as a profiler mode, sparse sampling, and a general type of terrain, do not enable one to assume that correspondence can always be established by matching distinct objects. Furthermore, even if limiting the search space by the expected size of the errors and searching for correspondence between surfaces and laser points, the number of surface combinations that should be evaluated can be very big.

Common methods for finding correspondence are relational matching, relaxation labeling, and maximum cross correlation. They aim at minimizing the difference between data from both domains; in this case it is the reconstructed laser surface and the control surface. These methods will face numerous difficulties with the current data. Deriving distinct features/attributes for evaluating correspondence between 2-D objects and a 3-D object is difficult. In addition, these methods ignore the underlying deformation process, due to the biases, and therefore, can result in meaningless solutions, for example, rotating the surface in a way that has no physical meaning in terms of the flight trajectory.

Solutions that have been proposed can be categorized into three groups, calibration over flat surfaces, scanning the solution space, and feature extraction based approaches.

With calibration over a flat locally horizontal surface [61, 86, 70], the constraint that can be imposed is simple; the elevation difference between the laser points and
the surface should be minimized. Since the elevation is constant the difference depends on the laser point; therefore, no additional knowledge is needed. Equation 4.8 shows how this approach can be extended to any flat surface, horizontal or tilted. However, flat or tilted surfaces that cover the whole calibration site cannot generally be supposed.

The second class of strategies is based on exploration of the solution space. A grid search is based on a systematic scanning of the solution space. It is performed first by partitioning the solution space into a hyper-dimensional grid, followed by evaluation of each entry to find the one with minimum norm of the residuals. The computational complexity of such an approach is of a polynomial order ($O(x^n)$), with $n$ as the grid dimension. The relaxation version of the generalized Hough transform where one parameter is evaluated at a time in an iterative fashion [36] reduces the computational complexity to ($\sim O(x^3)$). These methods handle the correspondence implicitly. Each set of parameters defines a unique correspondence between the laser points and the ground. The advantages of the systematic search are obvious; the scan of the solution space guarantees the finding of the best fitting solution as long as it is within the solution space bounds and no degenerate cases arise. The approach is practically a “brute force” technique, therefore little modeling is needed. In addition, the implementation of this method is very simple. However, the dependence on the number of parameters in terms of the computational efforts limits the use of this approach, especially in view of the evaluation of additional parameters [42].
The third class is based on matching the two domains. Kilian et al. [51] detect correspondence via LSM and use of corner points as control points. A similar concept is used in [17, 62] where the authors rasterize the laser data and search for corner points directly via corner operators. Proximity is implicitly assumed since no relative relations are being measured. On a more global scale, two approaches have been proposed for the calibration of GLAS (a profiling system) in [79]. The first one involves tracking the laser beam by ground reflectors, the other proposes matching the profile to the DEM by using cross-correlation. The tracking of the laser beam solves the correspondence directly, but it has a very sensitive setup that can fail if the errors exceed their assumed values. The cross-correlation is perhaps the only method that views the reconstructed dataset as a whole; furthermore, it does not assume distinct features. Regardless of the matching of a "true" surface with a deformed version derived from the reconstructed laser points, between a 2-D "surface" and an actual 3-D surface it is unclear how unique and accurate the correspondence can be established, especially across the profile line.

In this dissertation the problem is approached differently. The properties of the model show that identifying the surface element from which the laser point was reflected is sufficient for obtaining the solution for the correspondence problem. This is already a significant improvement over matching 0-D object points; the question is how the set of surface elements can be identified. The following subsection presents an algorithm for solving this problem. The algorithm approaches the problem from different perspectives by taking advantage of the local smoothness of the surface and its effect on the solution space.
4.4.1 The template approach

An analysis of the calibration model shows that the dependency on the surface makes the problem difficult to invert. The surface component is involved and it is impractical to have it modeled analytically. The proposed solution originates from existing concepts for finding a solution in a nonlinear environment that is difficult to model parametrically, e.g., deformable templates [90] or the Iterative Closest Point (ICP) algorithm [7, 24]. The underlying concept is that a template is realized in a mathematical model (involving transformation), which is a function of the domain to which it is related. The template variables are updated at each iteration (shifted, rotated, etc.) using, e.g., gradient-decent, until convergence is reached. Here the template is the analytical model that was derived. The domain of concern is more abstract; it is not the surface itself, but the augmentation of the search windows of all the laser points in the model. The iterative convergence here is performed using Newton’s algorithm ([83], e.g.).

The setup of the problem involves a given set of observations, y, and a model, g. The model is a function of a parameter vector, Ξ and the surface slopes, S. The surface slopes are also a function of Ξ, since a certain vector, Ξ, defines the given slopes directly via ray tracing; this function is difficult to model. The problem can be stated as finding a parameter vector, Ξ* that minimizes the target function

\[ \Phi(Ξ^*) = \arg\min_Ξ \{[(y - g(Ξ, S(Ξ)))^T P [y - g(Ξ, S(Ξ))]] \} \]  \hspace{1cm} (4.49)

For an initial vector Ξ_m the surface slopes are constant, therefore derivatives for g with respect to Ξ can be taken
as was done previously. Estimates are computed using

\[
A_{\|s} = \frac{\partial g_{\|s}(\Xi)}{\partial \Xi^T} |_{\Xi = \Xi_m}
\]  

(4.50)

The updated parameter vector \( \hat{\Xi}_{m+1} = -m + Cm + i \) updates the function.

\[
\hat{\xi}_{m+1} = (A_{\|s}^T P A_{\|s})^{-1} A_{\|s}^T P y
\]

(4.51)

The updated parameter vector \( \Xi_{m+1} = \Xi_m + \hat{\xi}_{m+1} \) updates the \( g_{\|s} \) function.

Convergence is reached when no update for \( \Xi \) is needed, namely, \( \hat{\xi} = 0 \). With this, a steady correspondence had been found (since \( g \) was not updated) and the solution converged to a minimum.

Figure 4.1 illustrates the iterative steps of the algorithm. Based on the initial approximations point 1 is associated with surface A, point 2 with surface B, and point 3 with surface D. The association of points 1 and 3 is wrong; however, the calibration parameters that are recovered with this association transform the beams to illuminate the true surfaces - 1-B, 2-C, and 3-E. In the next iteration, which uses the correct correspondence, the actual parameters will be recovered.

The dependence on surface slopes can make the solution sensitive to small fluctuations, especially when the initial approximations for the solution are far from the actual one. A hierarchical representation of the terrain is proposed to help avoid this, meaning that the solution is approached in a coarse-to-fine strategy. First, a simplified surface representation is used that removes local fluctuations (or noise) and leaves the trend of the terrain. Simplification of the terrain can be approached in
Initial estimation

Penultimate iteration

Figure 4.1: The iteration process – first level
many ways, one way might be via scale-space theory by convolving the surface with a set of Gaussian filters of different \( \sigma \)'s. Alternatively the terrain can be segmented into analytical surfaces, e.g. [6, 15], according to a given tolerance, or by using a mesh simplification algorithm [40]. The implementation here is based on local least-squares fitting of a planar surface for a given surface patch that becomes smaller between iterations. Surface fitting is chosen for several reasons, for once it has close resemblance to surface segmentation — the computation of an average slope over the region of interest suppresses local phenomena — then the local nature of the operator reduces the computational burden by being applied only around the laser points.

The rate of convergence is greatly enhanced by the coarse-to-fine approach. If the surface is relatively smooth, being close to the solution enables convergence in few iterations. Figure 4.2 demonstrates this. In the illustrated example the initial correspondence at the coarser level is the actual one (namely, 1–A', 2–B', and 3–C'); therefore, after the first iteration the update locations of the laser points are on the surfaces they illuminated. With larger surface patches some accuracy is lost at the coarser level, but the global behavior of that area is measured; fast convergence close to the actual solution is thus achieved, even when begun from a more distant surface element.

Knowledge about the parameters can be transferred from iteration to iteration as prior knowledge. The extended Gauss-Helmert model accommodates this inclusion as follows
Initial estimation

Result after First iteration

Figure 4.2: The iteration process – second level
\[
\begin{bmatrix}
y_n \\
0
\end{bmatrix}
= \begin{bmatrix}
A_{n \times m} & B e_m
\end{bmatrix}
\xi_m + \begin{bmatrix}
0 \\
e_0_m
\end{bmatrix},
\begin{bmatrix}
e \\
e_0
\end{bmatrix}
\sim \begin{bmatrix}
0 & \Sigma \\
0 & 0
\end{bmatrix}
\]

(4.52)

and is solved in the least squares sense by

\[
\hat{\xi} = [A^T (B \Sigma B^T)^{-1} A + \Sigma_0^{-1}]^{-1} (A^T \Sigma^{-1} y + 0)
\]

(4.53)

and

\[
D\{\hat{\xi}\} = \sigma_0^2 [A^T (B \Sigma B^T)^{-1} A + \Sigma_0^{-1}]^{-1}
\]

(4.54)

with \( \Sigma \) as the covariance matrix of the current set of observations, and \( \Sigma_0 \) as the covariance matrix of the prior knowledge about \( \xi \).

A question that follows is how initial values for the parameter vector \( \Xi \) are obtained (clearly, using \( \Xi = 0 \) is not as legitimate as in the case when the correspondence was given); and how local minima for \( \Phi(\Xi^*) \) are avoided. With no good intuition for the approximate location, initial approximations are obtained using the Monte-Carlo method. The solution can either converge to local minima or to the global minimum, all of which are measured in terms of the mean squared distance between the transformed laser points and the surface. Since the surface is smooth, not too many initial guesses are needed to converge to the global solution. Notice that putting efforts in obtaining good first approximations is not advisable; the labor involved in this effort is far greater than having few samples from the solution space computed.

\[\text{It is possible to have only one peak, convergence is then reached within one iteration.}\]
A more robust version of the algorithm works as follows:

1. **begin** initialize $i = 0$, $d =$ surface patch size, $\Delta d =$ amount of decrease between subsequent iterations,

2. Choose $n$ random pairs of mounting bias angles, $(\omega_i, \phi_i)$.

3. do $i \leftarrow i + 1$;

4. for each laser point, $l_j$, do

5. compute the incident point $\{x_j, y_j, z_j\} = g(l_j, \hat{\Xi}_i)$ by ray tracing

6. compute $s_j = f(x_j, y_j, z_j)$

7. Solve for $\hat{\xi}_i$

8. $\hat{\Xi}_i = \Xi_0 + \hat{\xi}_i$ using eq. 4.13

9. if $\hat{\xi}_i \equiv 0$, then if $d =$ size of one DEM cell

10. then declare convergence,

11. else $d \leftarrow d - \Delta d$

12. until $i = n$

13. select $\hat{\Xi}_i$ with the minimum norm of the residuals

14. Perform small perturbations around the solution for verification
The $n$ random pairs of mounting bias angles can be sampled from the whole space or, alternatively, can be chosen by dividing the solution space and selecting a pair at random from each cell. In addition, if a distribution of the expected biases is known a priori this can be incorporated into the sampling of the pairs. The solution is not exhaustive; however, simulations show that the global minimum has always been found, and that in calibration sites, which had the slope distribution property, the differences between global and local minima are very distinct.

4.4.2 Discussion

The correspondence strategy originates from the inferential calibration concept as was outlined earlier. In this regard it is an integral part of the algorithm. The concept is based on the exploration of the solution space. The template based approach utilizes the smoothness of the object space, and, in turn, the solution space to convergence to the true solution.

This method belongs to the class of approaches that explore the solution space to find the global minimum; it can, therefore, be associated with the grid-search solution and the generalized Hough Transform. As the two approaches are based on a similar concept, convergence to the true solution should be guaranteed. The main difference is the way the proposed method uses the properties and the formulation of the calibration model to reduce the dependency on the solution space dimensionality.
The equivalence enables one to illuminate the advantages of the proposed approach. The major one is the independence of the number of parameters that are recovered simultaneously. Notice that with the proposed approach, the solution space is used to obtain the initial (proposed) correspondence between the laser beams and the surface elements; accurate determination of the set of parameters can, therefore, be relaxed without losing generality. Furthermore, the analytical model implies that the proposed approach is not restricted to the predefined search space. Since convergence is sought the solution can drive itself outside of the original search space boundary when necessary. Therefore, it is a "safer" method than a conservative scan of the assumed space. Notice also that a possible existence of degenerate cases, where multiple solutions provide the same norm of the residuals, is a problem that is shared with the traditional approaches. If multiple solutions exist both approaches will fail to distinguish between them.

4.5 Optimization of the calibration configuration

Several questions regarding the calibration of laser altimeters have still not been answered. For once, it is not clear how reliably the estimation of the calibration parameters can be done. Furthermore, it is of interest to know what configuration yields better calibration parameters; and, besides, what does a configuration mean? A question that follows is how can such configurations be identified. The analysis of reliability is presented in the two following sections. Then the concept is generalized to determine criteria for site evaluation.
In general, “configuration” is an abstract concept, particularly for calibration of laser altimeters. While in photogrammetry spatial point configuration for image re-section or block adjustment is simple to perceive, for calibrating laser altimeters it is more involved. It is not clear, for example, what a configuration implies for a profiling system; should the beam point off-nadir for calibrating the system, and if so, how much and why, and what if maneuvers are limited? Similar questions arise for scanning systems. Efforts have been made, nevertheless, to apply photogrammetric concepts to the calibration of laser altimeter systems; see, for example, block adjustment procedures and concepts in [51, 62]. No justifications are being made, however, for using such procedures.

The approach taken here stems from an analytical perspective. The objective is to find out what enhances the recovery of the calibration parameters. This way the abstract concept of configuration is grounded in a more solid reasoning.

Section 4.2.1 has shown that the calibration parameters are largely independent of the altimeter position, and in some regards, of the position of the footprint. Equation 4.11 indicates that the surface slopes and the laser altimeter pointing angle have great impact on the coefficients of the calibration parameters. Therefore, surface properties are summarized by their slopes. To demonstrate the role of slopes, consider a flat surface with small deviations from a horizontal plane. The dependency between the mounting bias and the range bias is very high because the mounting bias can also be explained by the range bias. Hence, this is a weak configuration for calibration.
Three criteria are applied to optimize the configuration for the calibration – minimization of the correlation between the calibration parameters, minimization of their trace of the dispersion matrix, and minimization of the condition number of this matrix. The correlation minimization increases the degree of independence in estimating the different parameters; high correlation implies a strong similarity between parameters and less confidence in the obtained values. The minimization of the dispersion matrix trace

\[ tr(D\{\hat{\xi}\}) = \min \]  

is the overall goal because it implies a smaller error hyperellipsoid. The condition number – the ratio between the largest and the smallest eigenvalue – is an indication of the relative significance of the parameters. As the ratio approaches 1 all parameters have a similar significance; as it approaches \( \infty \), the system of equations becomes rank-deficient (singular). An additional aspect of robustness is the sensitivity with respect to gross errors. It is analyzed both in terms of detecting outliers and in terms of the effect of undetected outliers on the recovered calibration parameters.

The analysis begins with a profiling system and the three major error sources, namely, the two mounting bias angles and the range bias. An analysis of other types of systems is presented later. The equivalence between surface slope changes and aircraft maneuvers was established in Section 4.2.1. In the following, the term "surface slope" is used, even though it can be interpreted as a "modified" slope (see definition on page 42).
The normal equation matrix, \( N \), encapsulates the information about the system parameters. Therefore, the constraints can be imposed on this form. Following eq. 4.11, a closed matrix form of \( N \) is given in eq. 4.56, thereby neglecting the rows and columns for \( \delta_x \) and \( \delta_y \).

\[
N_{3\times 3} = \begin{bmatrix}
\sum_{i=1}^{n} c_{i1}^2 - \sum_{i=1}^{n} c_{i2} c_{i3} \rho_i & \sum_{i=1}^{n} c_{i1} c_{i3} \rho_i & \sum_{i=1}^{n} c_{i1} c_{i2} \rho_i^2 \\
\sum_{i=1}^{n} c_{i2}^2 \rho_i & \sum_{i=1}^{n} c_{i2} \rho_i^2 & \sum_{i=1}^{n} c_{i1} c_{i2} \rho_i^2 \\
\sum_{i=1}^{n} c_{i3} \rho_i & \sum_{i=1}^{n} c_{i1} c_{i2} \rho_i & \sum_{i=1}^{n} c_{i1} \rho_i^2 \\
\end{bmatrix}
\] (4.56)

Notice that the elements of \( N \) are functions of the surface slopes. An analysis of the normal equations matrix shows that if the surface elements have slopes of positive and negative directions, the values of the off-diagonal elements \( N_{12} \) and \( N_{13} \) are reduced; if the total magnitude \( \Sigma c_{i(2)}^+ \) and \( \Sigma c_{i(2)}^- \) are similar the sum \( \Sigma c_{i(2)}^+ + \Sigma c_{i(2)}^- \) is close to zero. The other off-diagonal element involves both the slopes in the \( x \)- and \( y \)-directions. A combination of slopes that have similar signs (i.e., positive-positive or negative-negative) and opposite signs (i.e., positive-negative or negative-positive) reduces this value.

The analysis of the correlation between the calibration parameters is verified by the cofactor matrix \( N^{-1} \)

\[
N_{3\times 3}^{-1} = \frac{1}{\det[N]} \begin{bmatrix}
C_1^2 C_2^2 - (C_{12})^2 & C_1^2 C_2 - C_{12} C_1 & C_2 C_{12} - C_2^2 C_1 \\
C_1^2 C_2 - C_{12} C_1 & n C_1^2 - (C_1)^2 & n C_{12} - C_2^2 C_1 \\
C_2 C_{12} - C_2^2 C_1 & n C_{12} - C_2^2 C_1 & n C_2^2 - (C_2)^2 \\
\end{bmatrix}
\] (4.57)

where:

\[
C_1 := \sum_{i=1}^{n} c_{i1} \rho_i \\
C_2 := \sum_{i=1}^{n} c_{i2} \rho_i 
\]
\( C_1^2 := \sum_{i=1}^{n} \rho_i^2 \)
\( C_2^2 := \sum_{i=1}^{n} \rho_i^2 \)
\( C_{12} := \sum_{i=1}^{n} c_i c_{i2} \rho_i^2 \)

The square of the correlations are

\[
\text{corr}\{\delta \rho, \phi\}^2 = \frac{(C_1^2 C_2 - C_{12} C_1)^2}{(C_1^2 C_2^2 - (C_{12})^2)(nC_1^2 - (C_1)^2)} \tag{4.58}
\]

\[
\text{corr}\{\delta \rho, \omega\}^2 = \frac{(C_2^2 C_1 - C_{12} C_2)^2}{(C_1^2 C_2^2 - (C_{12})^2)(nC_2^2 - (C_2)^2)} \tag{4.59}
\]

\[
\text{corr}\{\omega, \phi\}^2 = \frac{(nC_{12} - C_2 C_1)^2}{(nC_1^2 - (C_1)^2)(nC_2^2 - (C_2)^2)} \tag{4.60}
\]

Small values for \( C_1, C_2, \) and \( C_{12} \) reduce the correlation between the calibration parameters. Notice that as \( C_1, C_2, \) and \( C_{12} \) decrease (approach zero) the elements that involve a product or that are squared are reduced faster than the ones that appear alone (thus decreasing in a linear rate). Therefore, as \( (C_1, C_2, C_{12}) \to 0 \) the correlations become

\[
\text{corr}\{\delta \rho, \phi\} \approx \frac{C_2}{\sqrt{nC_2^2}} \tag{4.61}
\]

\[
\text{corr}\{\delta \rho, \omega\} \approx \frac{C_1}{\sqrt{nC_1^2}} \tag{4.62}
\]

\[
\text{corr}\{\omega, \phi\} \approx \frac{C_{12}}{\sqrt{C_1^2 C_2^2}} \tag{4.63}
\]

It can be seen that small values for \( C_1 \) and \( C_2 \) reduce the correlation between the range bias and the angles of the mounting bias. To reduce the correlation between the angles of the mounting bias, \( C_{12} \) should be reduced as well.

Assuming that surfaces which reduce the correlation are used, the analysis of the trace follows easily. With big surface slopes the diagonal elements decrease and the
variances become smaller because. The element in $N_{11}^{-1}$ tells that as the number of observations increases, the parameter estimate approaches its actual value if no correlation is present.

The decrease in the correlation between the parameters due to the surface normal trends illustrates the dominant effect of surface topography on the sensitivity of the solution. Under an adequate topography a strong solution with minimum dependency between the parameters and small variances can be achieved. Notice that calibration over flat surfaces via pitch and roll maneuvers is a good strategy; it involves variation in the slopes in the $x$- and $y$-directions, and, by the very nature of the aircraft maneuvers, achieves symmetry. Notice that the pattern of the maneuvers is not as important as the magnitude of the pointing angle and the symmetry of the pattern.

The reliability of the recovered parameters is affected both by more accurate observations, i.e., reduced variances in $\Sigma_y$, and by an improved calibration configuration [73]. The accuracy of the observation depends on the quality of the instruments, therefore it is not considered further. The following reliability analysis focuses on the effect of the configuration on the reliability. Reliability is measured in terms of the detectability of gross errors and the effect of undetected gross errors on the parameter estimation. It is analyzed using the relation in

$$\bar{e} = Q_{\bar{e}}P_y y$$

with $Q_{\bar{e}} := P_y^{-1} - AN^{-1}A^T$
The redundancy matrix, $Q_{eP_y}$, allows evaluating the robustness. A good configuration for observations will follow the relation

$$ (Q_{eP_y})_{jj} \approx \frac{n - m}{n} \quad \text{for all} \quad j, j = 1, 2, \ldots, n \quad (4.65) $$

with $m$, the number of estimated parameters.

Equation 4.64 shows that only the form $AN^{-1}A^T$ is sensitive to changes in the configuration. Using a configuration that fulfills the slope criteria, the cofactor matrix $N^{-1}$ becomes diagonally dominant and primarily exerts a scaling effect on the coefficients in the design matrix $A$. Therefore the form $AN^{-1}A^T$ can be interpreted as a sum of dyadic forms with the columns of the matrix $A' = A(N^{-1})^{1/2}$. The diagonal elements $(A'A^T)_{jj}$ are scaled functions of the "modified" slopes, and for the recovery of the range and the mounting bias $(Q_{eP_y})_{jj}$ is, in fact, a scaled version of the surface vector. Analysis of several types of surface combinations show that high redundancy numbers (about 70 percent and higher) are usually obtained. The redundancy number is mostly affected by the relative magnitude of the surface slope, compared to other surfaces that are used, and by the number of observations. Configurations which result in a small redundancy number are possible, e.g., small number of surfaces and a single observation on a surface with larger slopes compared to the others. However, such combinations are not advisable, even based on intuition.
The effect of an undetected outlier on the parameter estimate (exterior reliability) is modeled by

\[ \Delta \hat{\xi}_j = N^{-1} A^T P \eta_j \delta_j \]  

(4.66)

with \( \delta_j \) as an outlier in observation \( j \), and \( \eta_j \) as a \( j^{th} \) unit vector.

The analysis also looks into the number of surfaces required and their trend. A small number of surfaces with reasonable surface slopes makes the configuration more realistic and attainable. The effect of different surface configurations is demonstrated in Table 4.1.

### Table 4.1: Effect of slope distribution

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Slopes</th>
<th>Correlation</th>
<th>Condition Number</th>
<th>Redundancy number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( s_1 )</td>
<td>( s_2 )</td>
<td>( \phi, \delta \rho )</td>
<td>( \omega, \delta \rho )</td>
</tr>
<tr>
<td>I</td>
<td>0.01</td>
<td>0</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>II</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>III</td>
<td>-0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>IV</td>
<td>-0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>V</td>
<td>-0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Configurations I and II do not follow the recommended slope property, while configuration III–V do. The first configuration is a very clear illustration of the results expected for a calibration over a flat terrain, the second one shows that with an increase in the surface slopes a better solution is achieved. Notice, however, that the second configuration has weaker results than the following three even though the second configuration has greater slopes. The increase in the condition number presents another implication of the decrease of slopes; at the limit when the slopes are 0 the condition number approaches infinity. Evaluating the effect of the slopes on the accuracy can be carried out using the diagonal elements of the cofactor matrix. The results show that the values for mounting bias angles for configurations II–V are ten times smaller than the values for the first one.

The results also indicate that the number of surface elements does not play an important role. Section 4.2.1 has indicated that two surfaces are sufficient for solving the system; comparing configuration II with configuration III–V shows that three surfaces can provide a better solution than multiple ones. The number of points needed for the recovery is also small. Equation 4.9 shows that each laser point contributes one observation, therefore, the minimal number of observations is the number of parameters. Exceeding this number with an airborne system is clearly not difficult, and so is the selection of points which illuminate surfaces that satisfy the slope requirement. Equation 4.11 shows that for spaceborne systems a small number of points is sufficient for calibrating the system. For example, with a 160 m distance between consecutive laser shots (for the GLAS configuration), a surface segment of the order of 2 km covers 18 laser shots leading to a redundancy of 15 observations. The
time needed for acquiring data for covering 2 km is very short (~ 1/2 sec). Therefore, variation in the calibration parameters over time can be identified independently.

The analysis shows that the accuracy increases when greater slopes are used. The dependency of the ranging accuracy on the waveform tempers this trend. Section 2.1.1 has indicated that, as surface slopes increase, the signal-to-noise ratio (SNR) decrease, causing an increased uncertainty in the range determination. Equation 2.7 provides an analytical expression that models this relation. Ignoring aspects of surface roughness, the expected variance is a function of the squared tangent of the slope angle in degrees. The slope effect on the variance is also a function of the flying altitude, $z$, and the laser beam divergence angle, $\Theta$. Therefore, the effect of the slope on the uncertainty can change between different systems and missions. A detailed analysis of the slope effect on the ranging accuracy can also be found in [37], and the empirical measure of accuracy in [58] is also a function of the tangent. The weight matrix therefore has the form

$$P_y = \begin{bmatrix}
\frac{1}{\sigma_{s11,s21}^2} & 0 & \ldots & 0 \\
0 & \frac{1}{\sigma_{s12,s22}^2} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \frac{1}{\sigma_{s1n,s2n}^2}
\end{bmatrix}$$

(4.67)

with $\sigma_{s11,s21}^2$ the ranging variances as functions of the slope in the $x$ and $y$ directions. The modified $N$ is given in eq. 4.68
\[
N = \begin{bmatrix}
    n & \sum_{i=1}^{n} \frac{c_{12} \rho_{i}}{\sigma_{s_{11},s_{22}}^{2}} & \sum_{i=1}^{n} \frac{c_{1i} \rho_{i}}{\sigma_{s_{11},s_{21}}^{2}} \\
    \sum_{i=1}^{n} \frac{c_{22} \rho_{i}}{\sigma_{s_{11},s_{22}}^{2}} & \sum_{i=1}^{n} \frac{c_{12} c_{1i} \rho_{i}}{\sigma_{s_{11},s_{22}}^{2}} & \sum_{i=1}^{n} \frac{c_{1i} \rho_{i}}{\sigma_{s_{11},s_{21}}^{2}} \\
    \text{sym.} & \sum_{i=1}^{n} \frac{c_{22} \rho_{i}}{\sigma_{s_{11},s_{22}}^{2}} & \sum_{i=1}^{n} \frac{c_{i2} \rho_{i}}{\sigma_{s_{11},s_{21}}^{2}} \\
\end{bmatrix}
\]

(4.68)

Notice that the weight matrix penalizes observations with steep slopes. Therefore, a compensation is achieved while some of the physical properties of the system are incorporated into the model. Section 2.1.1 has shown how the surface slopes (or "modified" slopes), roughness, and albedo affect the SNR and, in turn, the ranging accuracy (see eq. 2.7). Their effects can be incorporated into the calibration model in a similar way. Information encapsulated in the waveform enables one also to propose a calibration procedure over nonconventional sites such as wooded areas. Figure 2.2 on page 15 shows that some return arrives from the surface. An analysis of the returned portion from the ground can be performed by waveform decomposition. The ranging accuracy is then assigned by a concept that is similar to the above. This way, waveform information is also introduced into the calibration model.

### 4.6 Extension of the configuration analysis

Better estimation of the calibration parameters requires observations with more information about the biases. The improvement is measured both in terms of increased confidence in estimating the different parameters and in terms of reduced correlation between the parameters. Section 4.5 has indicated that a configuration is largely determined here in terms of slopes and maneuvers; the analysis is therefore about the relations between flight maneuvers and surfaces, and the recovery of the various parameters.
Mounting bias

Section 4.5 has shown that for recovering the mounting bias the x and y components of the “modified” slope, $c_1$, $c_2$, should be large. Subject to the decrease in ranging accuracy, laser points with a big “modified” slope carry more information than observations with small ones. Notice that aircraft maneuvers are not required, ranging over sloping surfaces is equivalent to an off-nadir pointing. Therefore, a calibration of a profiling system, e.g., GLAS, does not require spacecraft maneuvers. From a practical point of view this is an important fact; it supports the use of small calibration sites (assuming that maneuvers cannot be performed over a few seconds) and enables one to separate the variation over time from the particular calibration of the parameters.

Conditions for recovering the heading mounting bias $\kappa$ vary between the different types of systems. For profiling systems the heading bias cannot be recovered. With line scanners, equation 4.21 indicates that the “modified” slope $c_2$ has no effect on the recovery; the dependency is, therefore, on the “modified” slope along the flight direction $c_1$. For a conical scanner, either or both “modified” slopes enable one to recover the heading bias. The concept of reducing the correlation by using slopes in opposite pointing directions (e.g., a saddleback rooftop) also holds here.

Ranging bias

Recovering of the range bias depends on $c_3$ the z-component of the “modified” slope. In general, this value is close to −1, therefore, any observation carries as much information as others for recovering the range bias.
Range scale change and “range-walk”

Equation 4.26 shows that the correction for the range scale change, is also a function of $c_3$. The modification for range difference in eq. 4.27 suggests that bigger range differences facilitate better estimates. This can be achieved by a calibration over a varying topography (man-made or natural), by changing the flying altitude, and with less impact, by an off-nadir pointing. Notice that isolating the range scale change and the “range-walk” requires both relief variation and attitude maneuvers.

In terms of similarity to different parameters, variation about the mean range implies that the similarity with the range bias estimate is small. Regarding the mounting bias, unless $\Delta \rho$ and $c_1$ or $c_2$ are dependent the range scale correction and the mounting bias angles will be independent. Configurations where such dependency is high are possible although unlikely.

INS errors

The INS errors comprise the INS bias and the INS time synchronization errors. Equation 4.32 shows that information about the INS errors is a function of the aircraft flight pattern. Section 4.2.3 has indicated that the correlation between the INS bias and the mounting bias is high. Without different flight patterns, the mounting bias absorbs most of the errors.

The INS synchronization error also depends on aircraft maneuvers. In contrast to the INS bias, the time errors gain more information from the smaller angles. Equation 4.37 shows that the time error depends on the first derivatives and the surface
slopes. The first derivatives grow as the aircraft is moving between the extremes, namely, at the smaller angles the derivatives are big. The similarity with the mounting bias is also expected to be small. Equation 4.31 shows that the coefficients for recovering the INS time synchronization errors depend on the product of the slope and the first derivative. The mounting bias angles depend on the modified slope (which is, generally speaking, a function of the slope, and the aircraft pointing angle), and therefore is likely to be independent of the INS coefficient. The dependency on the first derivative suggests that the similarity with other type of biases should be small as well.

**Position offset**

Some aspects about the position offset was implicitly addressed in Section 4.2.3. As was indicated, a roundtrip flight enables one to recover the position offset. The structure of eq. 4.43 indicates that the mounting bias and the position offset decorrelate one another when estimated. As similar parameters are used in both cases, the analysis is comparable to the mounting bias analysis. Notice that the datum error is not affected by the aircraft maneuvers. Therefore, surfaces with positive and negative slopes are needed for recovering this type of error.

### 4.7 Site selection

The analysis in Section 4.5 shows that an evaluation of the expected accuracy of the calibration parameters can be performed given the illuminated surface elements. The problem is that, as the parameters are not known a priori, it is unclear which surface elements are illuminated. One approach to evaluate the adequacy of a calibration site consists in simulating different bias combinations and verifying that the expected
accuracy is achieved for the different combinations. In this section an alternative approach that saves the need for simulation is presented.

Notice that the calibration algorithm is not terrain sensitive but rather slope sensitive; the organization of the surface elements in the site is not per se of great interest. The concern is about the directions and magnitude of the surface slopes that the beam illuminates. This property has the notion of a slope distribution. It suggests that a calibration site can be evaluated by using a more general concept of the distribution of slopes inside of it.

The slopes generate a bi-variate distribution with the $x$-slopes and $y$-slopes as the two spatial variables. A symmetric distribution is optimal for achieving the desired slope properties. It reduces the similarity between the range and the mounting bias as well as the mounting bias angles themselves. However, further analysis of the normal equations matrix in (eq. 4.56) shows that a requirement can be derived that is easier to assess. Notice that the independence between the range and the mounting bias estimates relates to the mean of the distribution via

$$ n\hat{\mu}_x = \sum_{i=1}^{n} s_{1i} $$

$$ n\hat{\mu}_y = \sum_{i=1}^{n} s_{2i} $$

(4.69)

with $n$ – the number of surface patches. A good configuration will have $\mu_x$, and $\mu_y$ close to zero.

$^7$Recall that the pointing errors are the most affecting types of errors, so the full space should not be explored.
Elimination of the correlation between the estimates for the mounting bias angles is achieved by independent variations of the $x$-slopes and the $y$-slopes; this is so since the element $N_{23}$ in eq. 4.56 transfers into the "spatial" covariance between these two slopes under a zero mean assumption (denoted $\bar{0}$).

\[ nC\{s_1, s_2\} = \sum_{i=1}^{n} (s_{1i} - \bar{0})(s_{2i} - \bar{0}) \]  

(4.70)

The expected accuracy of the recovered parameters can then be determined from an analysis of the variances $\sigma_x^2$ and $\sigma_y^2$ according to

\[
 n\tilde{\sigma}_x^2 = \sum_{i=1}^{n} (s_{1i} - \bar{0})^2 \\
 n\tilde{\sigma}_y^2 = \sum_{i=1}^{n} (s_{2i} - \bar{0})^2
\]  

(4.71)

Consequently, the analysis of the first two moments of the bi-variate distribution is sufficient for characterizing this distribution and for deriving conclusions about the adequacy of the calibration sites. A "safer" analysis will separate the calibration site into strips. The slope distribution of each strip will then be analyzed separately. If a significant landform appears in only part of the site, this partition will indicate this.

Notice how criteria that arrive from a purely analytical analysis establish a test that avoids any simulations for calibration site selection, and merely rely on a relatively basic analysis of the digital elevation model.
CHAPTER 5

EXPERIMENTAL RESULTS

The experiments presented in this chapter aim at verifying the proposed approach and at introducing additional aspects of the calibration of laser altimeters. The chapter describes the calibration of the NSF-SOAR system (National Science Foundation Support Office for Aerogeophysical Research) in the interior of the West Antarctica Ice Shelf (WAIS) and experiments with a spaceborne configuration. The calibration of the NSF-SOAR system is different from previous work as no flat surfaces or distinct landmarks were available near the camp site where the system was flown. In addition to testing and demonstrating the algorithm, issues of model selection and data analysis are studied. With the spaceborne configuration the site selection model is applied for several sites followed by a validation of the results. Demonstrating the whole calibration strategy, the Monte-Carlo based correspondence approach is then tested and analyzed.

5.1 Implementation

Before describing the experiments and the results, several implementation aspects are discussed.
Control information involved in the experiments was given as a set of \( x, y, z \) coordinates. Surfaces were created by subdividing the plane into cells according to the given set of points and then fitting planes to each cell. Verifying the laser points inside a surface was included as part of the correspondence algorithm. In general, the parametric surfaces are boundless, and, therefore, the recovered parameters can transform the beam away from the surface boundaries. In the correspondence algorithm, prior to each iteration the point-to-surface correspondence is established by using ray-tracing; the ray-tracing algorithm assures the inclusion of the point inside the surface. When convergence is reached, the calibration parameters are not updated (i.e., zero increments), and the correspondence that was established in the previous iteration has not been changed. The inclusion of the laser point in the surface is therefore guaranteed. Notice that the inclusion does not imply convergence to the true surface, a false convergence that leads to a local minimum is also possible.

**Coarse-to-fine implementation.** To enhance the rate of convergence and to reduce the number of trials before convergence to the actual solution is reached, a more global representation of the terrain was used in the first iterations. The algorithm that was implemented is based on a local plane fitting around the position of each laser point. The coarse-to-fine implementation was applied only to the GLAS configuration where the search space was big. At the coarsest level a surface was fitted to points included in ±150 m geographical window around each laser point, in the next iteration the window size was reduced to ±90 m, and the last one was at the level of the cell size (which was 30 m). These values were the result of several trials, concerned with evaluating how far the first approximations can be from the actual
solution so that convergence is reached within one guess. Increasing the surface size at the coarsest level did not affect the convergence while beginning with the ±90 m window required at times better approximations to lead to convergence. It is not unlikely that other values can be more suitable, and perhaps even using only two levels. However, the focus in the experiments conducted here was on testing the overall algorithm. Notice that the size of the surface patches that was given is in the search space units (i.e., meters); these values are not meant to be generalized to airborne systems. With these, the search space is relatively small and it is more questionable if and when a coarse-to-fine implementation is needed. In the NSF-SOAR example, where the surface was smooth and the search space small, the algorithm was applied directly to the surfaces derived from the data.

The algorithm that was used in the experiments with the GLAS configuration was based on drawing samples from the solution space until the variance component was under a preset threshold. This setup was used to assess the number of trials needed before convergence to the actual solution is reached. Nevertheless, a version that draw \( n \) samples based on a uniform distribution was tested as well. The majority of trial converged to the true solution. The implementation also integrated pointing jitter and ranging noise into the system.

5.2 NSF-SOAR

The approach was applied to calibrating the NSF-SOAR (National Science Foundation Support Office for Aerogeophysical Research) laser altimetry system. The
NSF-SOAR system is a unique suite of geophysical, mapping and navigational instruments, mounted in a ski-equipped aircraft. The combination of laser profiling, ice penetrating radar, airborne gravity, and magnetic measurements was designed to investigate the geologic control on the ice flow [14]. The flight mission used for testing the calibration procedure aimed at mapping surface elevation changes on the WAIS ice streams [81]. The area flown is in the interior of the WAIS where no flat horizontal surfaces or distinct landmarks were available. Precise calibration of the NSF-SOAR laser system challenges the traditional calibration approaches [86, 70, 42, 51].

5.2.1 System characteristics

The laser profiling system of NSF-SOAR is an Azimuth Model LRY 500 pulsed-laser transceiver. The system has the following characteristics:

- Laser – a diode-pumped Nd:YAG pulsed, near infrared (1064 nm) laser.
- Beam divergence angle – 4.5 mrad. For a nominal flight altitude of 300 m the footprint size is 1.5 m.
- Quoted range single pulse accuracy – 10 cm.
- Laser altimeter frequency – 1000 Hz. 64 pulses were averaged eight times per second providing ranges approximately every eight meters.
- INS system – A Litton Aero Products LTN92 INS unit, a laser gyroscope.
- Position determination – a differential carrier-phase GPS. Receivers were operating at 2 Hz on the ice sheet at the base of the operation and on the aircraft.
System calibration was performed over the aircraft landing strips ("skiways"). To provide ground control, snowmobile-mounted GPS surveys were conducted along the skiways and their surroundings. The ground survey pattern at the station is presented in Fig 5.1.

Several passes over the calibration site were performed for calibrating the system. Flying a profiler system in a simple flight pattern provides very limited information for recovering the system biases; it was necessary to perform aircraft pitch and roll maneuvers over the calibration site. The control surface model was formed by triangulating the points collected by the ground survey. The relatively high density of points facilitated a good description of the terrain, and the properties of the Delaunay
Triangulation (DT) algorithm provided an optimal partition of the calibration site. The irregular network of triangles is a less convenient pattern to process, especially as access time is longer compared to a grid pattern. However, it enables working with the original observations instead of a resampled set. Planar surface parameters were computed by plane fitting.

For a more convenient analysis of the results, the data were converted into a local coordinate system, based on equation 2.12. The transformation to a local coordinate system facilitates a more natural analysis of the improvement in the laser point geolocation.

5.2.2 The calibration procedure

NSF-SOAR is a low flying altitude system; system biases cannot shift the laser points more than a few tens of meters away from their assumed location on the ground. However, not knowing the biases in advance, the correspondence between laser points and surface elements is not certain. To resolve the uncertainty, the algorithm adopted the surface-update part from the “template approach” (see Section 4.4). The Monte-Carlo part was omitted since the beam cannot be too far from the true surface element.

The calibration procedure works as follows: first an initial correspondence is computed according to given approximations about the biases (or assumed zero biases), then the calibration parameters are solved. The correspondence is then updated and
a new adjustment follows. When no update is needed the algorithm terminates; this usually happens within one or two iterations.

5.2.3 The basic three-parameter model

With laser profilers the effect of the heading bias is absorbed in the other two pointing biases, therefore, it cannot be recovered separately. The recovered biases included the pitch and roll mounting bias and the range bias.
Figure 5.3: Distribution of residuals after removal of systematic errors
Estimated calibration parameters
\[ \delta \rho = -0.60 \pm 0.004 \text{m} \]
\[ \omega = 0.1843 \pm 0.015^\circ \]
\[ \phi = 1.8649 \pm 0.014^\circ \]

Correlation matrix
\[
\begin{bmatrix}
1.000000 & -0.291862 & 0.114840 \\
-0.291862 & 1.000000 & 0.069682 \\
0.114840 & 0.069682 & 1.000000 \\
\end{bmatrix}
\]

\[ \sigma_0 = \pm 0.065 \text{ m} \]

---

Table 5.1: NSF-SOAR calibration – three parameters

Figure 5.2 depicts the distribution of the residuals before calibrating the system. The recovered parameters are listed in Table 5.1

Figure 5.3 depicts the residuals after the system calibration. Evaluating the statistical characteristics shows that the majority of the errors were removed. This is indicated by the small variance component and the narrow distribution of the residuals after adjustment. The results confirm that flight patterns with varying pitch and roll angles decorrelate the estimated parameters. Also the reliability of the solution increases with large angles. A condition number of 776.5 indicates that the geometry of this calibration procedure is fairly strong. In summary, the solution for the calibration parameters has very desirable properties of low variance, low correlation between the parameters, and a relatively low condition number.
\[
\begin{align*}
\text{corr}\{\tilde{e}, \omega\} &= -0.08 \\
\text{corr}\{\tilde{e}, \phi\} &= 0.09 \\
\text{corr}\{\tilde{e}, \rho\} &= -0.28 \\
\text{corr}\{\tilde{e}, s\} &= 0.04
\end{align*}
\]

Table 5.2: NSF-SOAR – cross correlation after removal of the mounting and range biases

The effect of the calibration on the cross correlation between the residuals and different observations in the system is also evaluated. The numbers are listed in Table 5.2

There is a relatively low correlation between the pointing angles and the residuals and also between the surface slope and the residuals. Nevertheless, the correlation between the residual vector and the range shows that not all of the systematic effects were removed.

The correlation between the residuals and the range indicates the existence of a range scale factor (or range-walk correction). Nevertheless, additional error components were also modeled (see section 4.2.3) and the error might also originate from any of them. The potential existence of additional errors raises the question of how to analyze their significance. The following subsection addresses this issue.

5.2.4 Model selection

The correlation with the range suggests that evaluating the effect of the range scale (or waveform processing algorithm) should be carried out first. The modeling of the range scale follows the model in eq. 4.26 that corrects the range difference. The revised calibration results are listed in Table 5.3
Estimated calibration parameters
\[ \delta \rho = -0.60 \pm 0.0041 \text{m} \]
\[ \omega = 0.181 \pm 0.0151 \degree \]
\[ \phi = 1.851 \pm 0.0133 \degree \]
\[ \lambda \rho = 0.997 \pm 0.0005 \]

Correlation matrix
\[
\begin{array}{cccc}
1.000000 & -0.284599 & 0.109006 & 0.043147 \\
-0.284599 & 1.000000 & 0.093653 & -0.161439 \\
0.109006 & 0.093653 & 1.000000 & -0.033188 \\
0.043147 & -0.161439 & -0.033188 & 1.000000 \\
\end{array}
\]

\[ \hat{\sigma}_0 = \pm 0.0609 \text{m} \]

Table 5.3: Calibration with range scale

Notice that the correlation among the four parameters is very low; this indicates that using relative differences is an appropriate modeling to recover the range scale effect. The RMS was reduced from 6.5 cm to 6 cm. This may not be a significant reduction, but considering the original small variance it is a significant one. A stronger indication to the effect of the range scale correction can be realized from Table 5.4. Notice the decrease in the cross correlation between the range and the residuals from 28% to 1%. Furthermore, both the cross correlation between the roll and pitch, and the residuals were halved compared to their previous values. This indicates that most if not all of the systematic effects were removed from the system.

Introducing different parameters did not decrease the variance and the cross-correlation as the range scale did; implying that not all error sources are significant. Significance implies eliminating further systematic errors or reducing the RMS of the residuals. A parameter contribution is, therefore, measured in terms of the decrease in
\begin{align*}
\text{corr}\{\tilde{e}, \omega\} &= -0.05 \quad \text{corr}\{\tilde{e}, \phi\} = 0.04 \\
\text{corr}\{\tilde{e}, \rho\} &= -0.01 \quad \text{corr}\{\tilde{e}, s\} = 0.07
\end{align*}

Table 5.4: NSF-SOAR – cross correlation after removal of the mounting and range biases and range scale

the variance component. Assuming the model is composed of two sets of parameters, 
\(\xi_m = [\xi_{1 \in m-r} | \xi_{2 \in m}]\), where \(\xi_1\) are parameters believed to be significant and \(\xi_2\) parameters believed to be insignificant, the test

\[ H_0: \xi_2 = 0 \]

versus

\[ H_1: \xi_2 \neq 0 \]

evaluates the contribution of the additional set of parameters. This is tested via the likelihood ratio test [49] by using the \(F\) distribution. The test statistic \(T\) is given by

\[ T = \left( \frac{\hat{\Sigma}_e}{\hat{\Sigma}_f} \right)^{-\frac{n}{2}} = \frac{R/r}{\Omega/(n-m)} \sim F_{(r,n-m)} \]  

(5.1)

with \(R := \tilde{e}_1^T \tilde{e}_1 - \tilde{e}_0^T \tilde{e}_0\); \(\tilde{e}_0\), the residual vector under the null hypothesis; \(\tilde{e}_1\), the residual vector under the alternative hypothesis; and \(\Omega := \tilde{e}_1^T \tilde{e}_1\).

Using the \(F\) test shows that the range scale correction is statistically significant. Therefore, the null hypothesis is rejected. The fact that the redundancy in the system of equations is big is a major contributor to this.
Table 5.5 presents the results of calibrating the system using eight parameters – the previous four, the INS bias, \(\omega_I, \phi_I\), and the time synchronization bias, \(t_\omega, t_\phi\). The results show that additional values have a negligible contribution to the system (comparing the RMS or the results with the range scale to the current one). Notice that the time correction to the rotation along the \(y\)-axis, \(t_\phi\), is of the order of few hundredths of a second, which is similar to the magnitude reported by [70, 42]. The effect of such small time errors is not big however. The particular significance of this example is, therefore, in demonstrating the great number of parameters that can be recovered simultaneously. It can be seen that the correlation between the INS bias and the mounting bias is indeed big. The results show that, in fact, the INS bias is practically zero.

The solution is also analyzed by viewing the ranging process as a time series. Laser altimetry is a dynamic system in which measurements are ordered along the time scale; particularly this is a suitable representation of a profiling system where no spatial pattern exists.\(^8\) Testing for an autoregression is testing the assumption that the errors in the observations are random, suggesting uncorrelated random variables

\[
p(\bar{e}_i|\bar{e}_{i-1}) = p(\bar{e}_i)
\]  

(5.2)

Dependency between the residuals can be attributed to unmodeled or neglected physical phenomena in the system (i.e., ignoring the range scale correction when solving for the three parameters), or to phenomena that do not have an adequate physical modeling, e.g., the aliasing caused by different and nonsynchronous sampling.

\(^8\) It is noted that the spatial distribution of the residuals was also analyzed. No systematic phenomenon was noticed.
Estimated calibration parameters

\[ \delta \rho = -0.60 \pm 0.0049 \text{m} \]
\[ \omega = 0.188 \pm 0.0410 ^\circ \]
\[ \phi = 1.867 \pm 0.0150 ^\circ \]
\[ \lambda \rho = 0.998 \pm 0.0005 \]
\[ \phi_I = -0.0095 \pm 0.0211 ^\circ \]
\[ \omega_I = -0.0115 \pm 0.0379 ^\circ \]
\[ t_{\phi} = 0.0330 \pm 0.0053 \text{[sec]} \]
\[ t_{\omega} = -0.0079 \pm 0.0056 \text{[sec]} \]

Correlation matrix

\[
\begin{pmatrix}
1.000 & -0.179 & 0.116 & 0.035 & -0.118 & -0.027 & -0.047 & -0.109 \\
-0.179 & 1.000 & 0.404 & -0.218 & -0.469 & -0.379 & -0.014 & -0.002 \\
0.116 & 0.404 & 1.000 & -0.134 & -0.710 & -0.883 & -0.052 & -0.105 \\
0.035 & -0.218 & -0.134 & 1.000 & 0.116 & 0.140 & 0.229 & -0.016 \\
-0.118 & -0.469 & -0.710 & 0.116 & 1.000 & 0.746 & 0.062 & -0.091 \\
-0.027 & -0.379 & -0.883 & 0.140 & 0.746 & 1.000 & 0.043 & -0.094 \\
-0.047 & -0.014 & -0.052 & 0.229 & 0.061 & 0.043 & 1.000 & 0.000 \\
-0.109 & -0.002 & -0.105 & -0.016 & -0.091 & -0.094 & 0.000 & 1.000
\end{pmatrix}
\]

\[ \sigma_0 = \pm 0.0608 \text{ m} \]

Table 5.5: Calibration with all parameters

rates (see Section 2.4). With the time series concept, the calibration process can be viewed as a detrending operator aimed at eliminating the systematic phenomena from the signal so that only a random noise remains. The autoregressive relation is modeled as

\[
z_t = \tilde{e}_t - \alpha \tilde{e}_{t-1}
\]

(5.3)

with \( z_t \), as the uncorrelated noise; \( \tilde{e}_t \), the residuals at time \( t \); and \( \alpha \), the autocorrelation parameter.
The autoregression can be extended further in time to $t - 2$, $t - 3$, \ldots. Evaluating the existence of an autoprocess in the system is therefore not trivial. Independence is evaluated by the Durbin-Watson statistic given by

$$d = \frac{\sum_{i=2}^{n}(\ddot{e}_t - \ddot{e}_{t-1})^2}{\sum_{i=2}^{n} \ddot{e}_t^2} \quad (5.4)$$

The results for the current case are $d = 1.32$ for the three parameter case, $d = 1.43$ for the model with the range scale correction, and $d = 1.44$ for the model including the eight parameters. For a 5% level of confidence (Type I error) having the statistic $d > 1.67$ suggests no positive autocorrelation, values between $1.44 < d < 1.67$ suggest an inconclusive case, and value $d < 1.44$ indicates an autocorrelation between the parameters. In case of a strong evidence for autocorrelation between the residuals, methods for removing the autoregressive effect from the parameter estimation can be found in [67].

The small values of the additional parameters puts in question the logic behind modeling them from the beginning. Notice, however, that as the flying/orbiting altitude rises, the effect of these errors increases and so does the importance of their recovery. Furthermore, with no appropriate error modeling and data analysis procedures, unnecessary biases are likely to remain in the system and show up as systematic effects in the laser data. For example, consider the recovery of a range scale/range-walk bias. In conventional in-flight calibration procedures this type of error is not modeled at all.
5.3 Satellite Altimeters – GLAS

Reference to calibration of satellite altimeters were made throughout this work. In fact, several aspects of the proposed calibration strategy originated from problems posed by a spaceborne configuration. The two prominent examples are the site selection and the general solution for the correspondence problem. Both of them are discussed in this section.

The site selection is a rather abstract idea. It suggests that a calibration site can be characterized using only a few numbers and without knowing the shape of the terrain. To validate this concept several sites are tested according to the criteria derived in Section 4.7, and a simulation of a calibration over these sites is used to verify the results. Even though a simulation is a synthetic experiment, similar results were obtained by independent processing in different centers (NASA-GSFC [72], and NASA-Wallops [63]) suggesting that the results presented here are not too optimistic.

The solution for the correspondence enables demonstrating the whole calibration strategy. It also enables analyzing several aspects concerning the algorithm and the site characterization. It is applied to several of the sites that are analyzed.

Even though other issues of calibration of satellite laser altimeters are not explicitly addressed in this section, they have received attention throughout this work. As an example, the analysis evaluated the number of parameters that can be recovered,
the expected robustness of the recovered parameters, and the properties of a calibration site, from which the site selection concept emerged.

5.3.1 Site selection

The sites evaluated here are part of a set of sites that was considered by the GLAS science team. The sites are located at crossovers of the GLAS orbit during the calibration phase of the satellite. Most of the crossovers are located in the western part of the U.S. and one is in Antarctica. The test areas include the Dry Valley region in Antarctica, a crossover located at the northern part of the U.S (N1), and two other sites located at the southern part of the U.S. (S2, S3). S2 is a mixture of flat and mountainous terrain, and S3 is located at the Colorado plateau and is characterized by a very distinct drop in elevation at the Grand Canyon. The relevant statistical properties of the analyzed sites is summarized in Table 5.6.

<table>
<thead>
<tr>
<th>Site</th>
<th>$\mu_x$</th>
<th>$\mu_y$</th>
<th>$\text{Cov}(x, y)$</th>
<th>$\sigma_x^2$</th>
<th>$\sigma_y^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry Valley</td>
<td>-6.44</td>
<td>-1.33</td>
<td>-0.05</td>
<td>22.44</td>
<td>27.06</td>
</tr>
<tr>
<td>N1</td>
<td>0.42</td>
<td>-0.11</td>
<td>0.03</td>
<td>13.17</td>
<td>12.45</td>
</tr>
<tr>
<td>S2</td>
<td>-3.04</td>
<td>0.02</td>
<td>-0.04</td>
<td>19.07</td>
<td>18.27</td>
</tr>
<tr>
<td>S3</td>
<td>1.53</td>
<td>-0.25</td>
<td>0.46</td>
<td>9.88</td>
<td>8.37</td>
</tr>
</tbody>
</table>

Table 5.6: Statistical properties of the calibration sites

Ranking the sites by the criteria derived in Section 4.7 suggests that the first three are considered adequate sites for a calibration. The correlation is low and the ratio
between the variance and the mean is big enough to suppress deviation from a zero mean. All three sites have a relatively high relief variation; they are also characterized by varying topography with rising and descending surfaces. In terms of ranking the sites, Dry Valley is expected to generate the results with the smallest variance. S3, in contrast, is supposed to generate less reliable estimations. Notice that crossing the Grand Canyon, a distinct natural landmark, does not help in turning S3 into a good site.

Evaluating the adequacy of the ranking is carried out via a simulation of GLAS passing over the sites. To make the simulation realistic, the expected GLAS orbit was used as the trajectory; the expected random errors were based on the GLAS specifications [79]. The pointing jitter was set to 1", the range error was set to 50 cm to accommodate both the ranging error and surface roughness. The results presented here used similar biases (15 mrad mounting bias, and 20 cm range bias); other parameters were tested as well, providing similar results. The recovered parameters are listed in Tables 5.7-5.10.

As can be seen, the statistics manage to predict the most important aspects. Notice, for example, that the standard deviation for the roll bias in the Antarctica site is a bit bigger than the pitch bias. The standard deviation for the S2 parameters is bigger than those derived over Dry Valley. S3 is indeed the least favorable calibration site, the calibration parameter RMS is five times bigger than the other sites.
Estimated calibration parameters
\[ \delta p = 0.17507898 \pm 0.022\text{m} \]
\[ \omega = 0.00014979 \pm 1.52\times10^{-7}\text{[rad]} \]
\[ \phi = 0.00015001 \pm 1.26\times10^{-7}\text{[rad]} \]

Correlation matrix
\[
\begin{pmatrix}
1.0000 & -0.0396 & -0.1521 \\
-0.0396 & 1.0000 & -0.0406 \\
-0.1521 & -0.0406 & 1.0000 \\
\end{pmatrix}
\]

\[ \hat{\sigma}_0 = \pm 0.498 \text{ m} \]
Condition Number = 18.96

Table 5.7: Dry Valley Antarctica (orbit 116)

---

Estimated calibration parameters
\[ \delta p = 0.21734168 \pm 0.027\text{m} \]
\[ \omega = 0.00015000 \pm 1.52\times10^{-7}\text{[rad]} \]
\[ \phi = 0.00014988 \pm 1.49\times10^{-7}\text{[rad]} \]

Correlation matrix
\[
\begin{pmatrix}
1.0000 & 0.0090 & 0.0865 \\
0.0090 & 1.0000 & 0.0170 \\
0.0865 & 0.0170 & 1.0000 \\
\end{pmatrix}
\]

\[ \hat{\sigma}_0 = \pm 0.511 \text{ m} \]
Condition Number = 11.81

Table 5.8: N1 (orbit 67)
Estimated calibration parameters
\[ \delta \rho = 0.18834583 \pm 0.031 \text{ m} \]
\[ \omega = 0.00014995 \pm 2.49\times10^{-7} \text{ [rad]} \]
\[ \phi = 0.00014978 \pm 2.61\times10^{-7} \text{ [rad]} \]

Correlation matrix
\[
\begin{bmatrix}
1.0000 & 0.0528 & -0.1822 \\
0.0528 & 1.0000 & 0.0045 \\
-0.1822 & 0.0045 & 1.0000 \\
\end{bmatrix}
\]

\( \delta_0 = \pm 0.561 \text{ m} \)
Condition Number = 28.808

Table 5.9: S2 X-over, Mojave Desert, W of Las Vegas, (Orbit 37, ascending)

---

Estimated calibration parameters
\[ \delta \rho = 0.22116896 \pm 0.036 \text{ m} \]
\[ \omega = 0.00146339 \pm 9.49\times10^{-7} \text{ [rad]} \]
\[ \phi = 0.00152471 \pm 8.36\times10^{-7} \text{ [rad]} \]

Correlation matrix
\[
\begin{bmatrix}
1.0000 & -0.6618 & 0.2244 \\
-0.6618 & 1.0000 & -0.1815 \\
0.2244 & -0.1815 & 1.0000 \\
\end{bmatrix}
\]

\( \delta_0 = \pm 0.643 \text{ m} \)
Condition Number = 410.481

---

Table 5.10: S3 X-over, Arizona, SE of Grand Canyon, (Orbit 22, ascending)
The results indicate that potential sites are available and that terrain characterized by varying topography is likely to generate good calibration sites. The correlations between the different calibration parameters as taken from flight over real surfaces are small and under the prevailing assumptions indicate accurate recovery of the parameters. For the sake of completeness it is worth mentioning other factors that should be integrated into a site selection process – among them are the aridity of the site, level of cloudiness, and the susceptibility for erosion during the lifetime of the mission.

### 5.3.2 Correspondence

Obtaining the solutions in Tables 5.7–5.10 requires identifying the surface elements the beams illuminate. This section elaborates on this part. Solution for the correspondence is demonstrated by using three sites, N1, O116, and S3. Each of them reveals an aspect about the algorithm.

The setup that was used for the testing was a 2 mrad search space (equivalent to 2.4 km, with 600 km orbiting altitude) that was chosen at random. The algorithm that was applied was not the robust version that computes \( n \) initial guesses, but an algorithm that terminated when the expected RMS was achieved. Testing if the convergence is to the true solution was simple, as the biases were known.

The first result is for the calibration site N1, Table 5.11 summarizes the trails. The results present the initial guesses, the final convergence parameters, and the variance component, for each trial until true convergence is achieved (the last row). Having
more than one trial suggests that local minima (false convergence) exist in the solution space. Figure 5.4 depicts the solution space as was evaluated using the pitch and roll
bias. The z-axis is the RMS in negative sign (for better graphical representation). The prominent fact is that local minima exist in the solution space and that there are several of them. Local minima are equilibrium points in which no further update of the solution can be achieved. They result from a complex search space for the laser points participating in the calibration. The distinction of the global minimum with respect to the local ones is of great importance. A big difference suggests that it is very unlikely to interpret a local minima as the global one. The current site shows that the difference in standard deviation is of the order of several tens of meters. Misinterpretation is therefore very unlikely.

The solution space for S3 and O116 demonstrates different characterizations of the solution space. The graphical representation of the solution space for O116 shows that only one peak exists in this space and that it is a very distinct one; there are no local minima inside this domain. This property suggests that beginning with any initial guess as far as it is from the true value still leads to the true solution, and that one initial guess is sufficient for obtaining convergence to the true solution. The relatively smooth pattern of the terrain is one reason for the existence of one global minimum, the high relief variation resulting in big slopes makes the global minimum distinctive. In contrast S3 represents the other extreme. Here the solution space is very flat. It is, therefore, difficult to distinguish between the global minimum and the local ones. The shallow solution space demonstrates another problem with the

\(^9\)Notice that this graph is the graphical representation of the grid-search approach for two variables.
calibration of sites like S3. Even though the calibration parameters can be recovered over a site with a relatively small relief variation (and smaller slopes), when the correspondence is unknown it is difficult to distinguish between convergence to the true solution and to a false one.
Figure 5.5: Solution space for O116

Figure 5.6: Solution space for S3
CHAPTER 6

CONCLUSIONS AND FUTURE WORK

This research studied the calibration of a laser altimeter system. An analysis of the data characteristics and the data acquisition concept has indicated a need for a model that is different from the traditional data registration concepts, e.g., the ones applied in photogrammetry. It was identified that the two prevailing problems are the nonredundant determination of laser points and the unknown correspondence between laser points and the spot they illuminate on the ground.

This dissertation proposed a strategy that utilizes natural surfaces or man-made objects to resolve the calibration parameters. This formulation is advantageous to the existing methods as it neither limits the solution to flat surfaces nor requires distinct control features. Using natural surfaces as control removes the need for preliminary knowledge about the correspondence between laser points and the ground; the proposed formulation solves for the calibration parameters and the correspondence simultaneously. The presented research has also supplemented the calibration model with a general solution to the correspondence problem. The derived solution is an integral part of the calibration model. An adaptive coarse-to-fine segmentation of the terrain is used to improve the rate of convergence.
An analysis of the recoverability of the potential error sources shows that in spite of the poor initial configuration and the unknown correspondence, many of the parameters can be solved simultaneously. Configurations that generate solutions with a minimum correlation and small variance can be generated by fairly simple means. By analyzing the properties of the proposed method, it has demonstrated that moderate slopes are sufficient to generate reliable solutions. The only requirement consists in having the surface elements oriented in different directions. The compelling conclusion is that natural terrain will yield results that are accurate and reliable. The results for calibrating the NSF-SOAR system and the experiments with the calibration sites for a spaceborne configuration have demonstrated how effective these properties are.

Several aspects will benefit from further work. The following section outlines in some detail several research directions.

6.1 Future work

Analysis of potential error sources. The dynamic properties of the system and the dependency on position and navigation components suggests that the error modeling and understanding will benefit a lot from experimenting with the calibration of different systems, or with the same ones, but in different campaigns. Analyzing of autodependencies as well as obtaining better knowledge about temporal changes in the course of a laser data survey, are some natural steps in this direction.

A better evaluation of the system accuracy. Besides the work by Gardner [31], little has been done in this regard. The only study of system accuracy so far is given in [58] which is carried out in a wooded area without solid theory behind it.
6.1.1 Calibration with no control surfaces

In this dissertation, an underlying assumption has been made that the surface parameters are known. However, many systematic errors can be recovered without knowledge of the surface parameters at all.

A geometric interpretation of the effect of the biases on the reconstructed surface demonstrates some aspects that can be used for the derivation of an analytical model. For example, the analysis below reveals that the recovery of the roll bias can be carried out with no surface knowledge if a roundtrip flight surveys the same surface. The configuration in Figure 6.1 illustrates how each flight over a surface with a slope angle $\alpha$ generates different reconstructed surfaces; in this example one with slope angle $\alpha'$, and the other with slope angle $\beta'$. Constraining the two surfaces to have a similar slope after the roll bias $\Delta \omega$ is removed enables recovering the roll bias as is demonstrated by the following derivations

$$
\begin{align*}
\alpha &= \alpha' - \Delta \omega \\
\alpha &= \beta' + \Delta \omega
\end{align*}

\implies \quad \Delta \omega = \frac{\alpha' - \beta'}{2}

(6.1)
$$
A model that follows this concept is based on the idea that, after applying the transformation, the different surfaces should coincide with the actual one. The modeling proceeds from eq. 4.8. Here, the surface parameters are also considered as unknowns. The linearized form of the modified model is given in eq. 6.2

\[ w := \bar{c_3} \rho - (\bar{s}_1 X_0 + \bar{s}_2 Y_0 + \bar{s}_3 Z_0 + \bar{s}_4) = - \bar{c_3} \delta \rho - \bar{c_1} \rho \phi + \bar{c_2} \rho \omega + x s_{1i} + y s_{2i} + s_{4i} + \bar{e} \quad (6.2) \]

with \( \bar{c} \), the approximation for the "modified" slopes and, \( s_i \), the surface parameter increments. The current model sets \( s_3 = -1 \), alternatively the constraint, \( ||s|| = 1 \), can be used.

Some modifications of eq. 6.2, which are not discussed in this brief review, enhance the reliability of the recovered parameters. A geometric interpretation of the effect of the biases (see, e.g., [28]) shows that both the surface constants and the range bias can be recovered simultaneously.

Beside experimenting with different surface configurations that enhance the reliability of the parameter recovery when knowledge about the surface parameters is not available, the definition of the surfaces (boundaries etc.,) is a problem that should be studied. The latter issue is especially involved when no subdivision of the surfaces/terrain is given a priori. Another type of constraint to be studied is a relational one, which could impose surface smoothness, for example.
6.1.2 Additional types of control features

Other type of features that can be utilized as control information are control points and control lines. Existence of such control entities reduces the dependency between the surface and the calibration parameters when calibrating without knowing the surface parameters. In addition, lines and points are easier to measure than surfaces, therefore their use simplifies the procedure for calibrating laser systems. The lines of interest are breaklines between two surfaces. This type of line is very common, for example, roof gables, breaklines for ramps, etc.

Control points introduce information about the surface they belong to. The contribution and incorporation of control points can modeled as:

\[ 0 = s_1 x_p + s_2 y_p + s_3 z_p + s_4 - (s_1 e_{x_p} + s_2 e_{y_p} + s_3 e_{z_p}) \]  \hspace{1cm} (6.3)

with \( x_p, y_p, z_p \) the coordinates of the control point.

With a breakline structure as shown in Figure 6.2, where \( \mathbf{n}_1, \mathbf{n}_2 \) are the two surface normals \( \mathbf{n}_1 = \{s_{1x}, s_{1y}, s_{1z}\}; \mathbf{n}_2 = \{s_{2x}, s_{2y}, s_{2y}\} \) and \( \mathbf{l} \) is the line describing the ridge in the 3-D space.

Describing the line by a reference point \( x_0, y_0, z_0 \) and a pointing vector \( u_x, u_y, u_z \), four constraints can be written: two geometric constraints that are based on the fact that the dot product between the normal and the ridge should be zero, and two constraints for the two planes to include the ridge line origin \( \{x_0, y_0, z_0\} \) so that
Figure 6.2: Roof gable model

\[ n_1 \cdot l = 0 \]
\[ n_2 \cdot l = 0 \] (6.4)
\[ s_{11}x_0 + s_{12}y_0 - z_0 + s_{14} = 0 \]
\[ s_{21}x_0 + s_{22}y_0 - z_0 + s_{24} = 0 \]

Four constraints are insufficient to recover all the parameters with one overflight. However, it is likely that by using ridge lines that are oriented in different directions, the calibration parameters can be recovered without the need for a roundtrip flight pattern over the surveyed area. Study of the criteria to recover parameters by using a linear constraint involves experimenting with the effect of different configurations, and as before, their effect on the reliability of the recovered parameters.
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