THE ROLE OF INSTRUCTIONAL AIDS
IN ARITHMETIC EDUCATION

DISSERTATION
Presented in Partial Fulfillment of the Requirements
for the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By
WILLIAM PATTON EIDSON, B. A., ED. M.

The Ohio State University
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Approved by:

[Signature]
Adviser
Department of Education
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CHAPTER I

INTRODUCTION TO THE STUDY

Statement of the Problem. This is a study of the role of instructional aids in the elementary-school arithmetic program. The investigation centers largely around the three problems of what instructional aids to use and when and how to use them. Three subsidiary purposes are to develop:

1. criteria for selecting and using arithmetic instructional aids.

2. an objective instrument for ascertaining, in part, prospective-teacher understanding of principles and processes involved in selecting and using instructional aids in arithmetic education, with special reference to the elementary school.

3. a checklist for ascertaining, in part, prospective-teacher understanding of the use of instructional aids in arithmetic education, as evidenced by student-teacher performance when faced with arithmetic-teaching situations.

Unquestionably the most appropriate index of understanding of arithmetic instructional aids on the part of prospective teachers is their performance in arithmetic instruction following graduation. It appears justifiable to make use of the less desirable but more practical alternative of developing a valid and reliable objective instrument for ascertaining certain aspects of prospective-
teacher understanding of the role of arithmetic instructional aids, there being no instrument of such nature known to the author.

To ascertain certain aspects of prospective-teacher understanding of the role of arithmetic instructional aids on a performance basis, it was felt that the development of a checklist would be a desirable and practical second alternative. On the checklist would be recorded the prospective-teacher's reactions to arithmetical situations confronted during student-teaching.

**Significance of the Study.** Instructional aids are many and varied. Some are intangible, such as the teacher's voice, her personality, her knowledge of subject matter and methods. Others are tangible, such as the chalkboard, the textbook, projection equipment and supplies, the bulletin board, the dairy on a nearby farm, and the wealth of resource persons found in every community. It is generally agreed that the use of instructional aids has potential worth in the teaching of arithmetic. Moreover, when instructional aids are used in arithmetic methods courses, they tend to become an inseparable part of the process of making arithmetic concepts meaningful to prospective teachers as well as laying a methods foundation whereby such pre-service teachers may in the future make arithmetic concepts meaningful to their pupils.
In evaluating the understanding of arithmetic concepts and processes possessed by prospective teachers, the great tendency has been to place almost total emphasis on "pure arithmetic," omitting the part played by instructional aids in bringing about such understanding. In reference to enrichment materials in mathematics education, Schaaf\(^1\) remarked:

It might frankly be pointed out that new entrants into the teaching profession, while generally well prepared both in pure mathematics and in educational theory, are regrettably weak in these "background aspects of mathematics." This is understandable, but scarcely excusable; if conventional courses in mathematics and in education do not deal adequately with such material, then provisions for its inclusion are in order. Drawing upon backgrounds of mathematical teaching is extremely significant; it can make a maximal contribution only if the teacher has an adequate mastery of a sufficiently wide variety of material.

Colleges of education have been repeatedly criticized for their failure to teach prospective teachers the efficient and intelligent use of instructional aids. Dale\(^2\) stated in 1944 that he knew of no college of education or teachers college which insisted upon a mastery of such competencies as using the blackboard, handling excursions, skillfully demonstrating the meaning of fractions, and reading a map. At the same time he warned against a


\(^2\) Edgar Dale, "When Do We Start?" Educational Screen, XXIII (May, 1944), p. 200.
teacher-education program that attempts to develop such competencies out of context. Wheeler and Perkins declared: "One of the greatest faults in teaching arithmetic is in the improper selection of object-lessons that do not illustrate, and in not selecting enough of them of different types."

Hoban contended that the cure for verbalism lay in the effective use of visual-sensory aids in both the instructional and learning processes. He emphasized, however, that effective use of visual sensory aids in instruction requires preparation on the part of teachers, so that they might know these tools of teaching, where to get them and how to use them. He maintained, also, that the responsibility for this knowledge and technique rested in teacher-training institutions.

A survey of 113 teachers colleges made by McConnell in 1947 revealed that such institutions were giving too little attention to the use of audio-visual instructional aids and to teacher-preparation in their use. McConnell

3 Raymond Holder Wheeler and Francis Theodore Perkins, Principles of Mental Development, p. 486.


declared that practically all of the 113 teachers colleges figuring in the survey were aware of the situation. As a result of the study, he made the following recommendations:

1. Instruction in the use of teaching aids should be offered to all teaching candidates.

2. Student teachers should be required to use audiovisual materials in their student teaching.

According to Gnaedinger, it is generally wrongly presumed that prospective teachers have an adequate understanding of the use of instructional aids upon graduation. He remarked that as late as 1946 superintendents of city school systems reported that one of the chief obstacles to the development of classroom teachers was disinterest or inability on their part to select and use instructional aids in an effective way. A study of devices used by arithmetic teachers in selected schools in Massachusetts indicated that there was not a need for more devices but rather a better understanding by teachers of how to use more effectively those already in existence.

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A survey made in 1932 by Brueckner and Dickeman revealed that teachers were concerned with their lack of understanding of the use of arithmetic instructional aids. Following are some of the items on which the highest per cents of teachers indicated a felt need of help:

1. Using excursions, projects, exhibits.
2. Training pupils in the use of reference materials.
3. Using school equipment such as the mimeograph.
4. Preparing improved types of supplementary materials.

Further evidence of the significance of the present investigation was revealed in a questionnaire study made in 1946 by the Research Division of the National Education Association, involving all cities above 2500 population. It was found that among curriculum fields in which elementary teachers were making the most effective use of film teaching aids, mathematics ranked last. The tabular results of this study are presented on page 7.

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TABLE 1
CURRICULUM FIELDS IN WHICH ELEMENTARY TEACHERS ARE MAKING MOST EFFECTIVE USE OF TEACHING FILMS

<table>
<thead>
<tr>
<th>Curriculum Field</th>
<th>Number of Times Field Is Mentioned</th>
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<tbody>
<tr>
<td>Social studies</td>
<td>730</td>
</tr>
<tr>
<td>Science</td>
<td>487</td>
</tr>
<tr>
<td>Health</td>
<td>139</td>
</tr>
<tr>
<td>English</td>
<td>82</td>
</tr>
<tr>
<td>Safety</td>
<td>47</td>
</tr>
<tr>
<td>Music</td>
<td>30</td>
</tr>
<tr>
<td>Art</td>
<td>26</td>
</tr>
<tr>
<td>Practical arts</td>
<td>19</td>
</tr>
<tr>
<td>Physical education</td>
<td>12</td>
</tr>
<tr>
<td>Mathematics</td>
<td>3</td>
</tr>
</tbody>
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Definitions of Basic Terms Used in the Study.

1. Instructional aids. As used in the present study, an instructional aid may be defined as anything which the teacher or pupil uses to aid the pupil in learning. Kinder\(^{10}\) states: "Anything that increases the ease of learning may be called a teaching aid, not with the connotation of crutch, but of tool or material." As examples of teaching aids he mentions pictures, specimens, maps, globes, charts, diagrams, books, and models. Grossnickle,\(^{10}\)

Junge, and Metzner11 also define instructional materials as any materials which contribute to the learning process. Among arithmetic instructional materials they include any picture, model, book, real activity, or teaching aid which provides experiences to the learner for purposes of (a) introducing, enriching, classifying, or summarizing abstract arithmetic concepts, (b) developing desirable attitudes toward arithmetic, and (c) stimulating further interest and activity on the part of the learner in the subject.

Not all educators are in agreement with the above points of view. Brown and VanderMeer12 defined the term "audio-visual instructional materials" as "Those (materials) which do not depend exclusively upon comprehension of words or similar symbols." Dale,13 too, included as audio-visual materials of instruction all "materials that do not depend primarily upon reading to convey their meaning."


13 Edgar Dale, Audio-Visual Methods of Instruction, p. 3.
Noel and Leonard\textsuperscript{14} likewise excluded from their definition of audio-visual education all learning materials such as books and pamphlets.

The difficulty of arriving at a consensus of opinion relative to an acceptable definition of "instructional aids" obviously lies in the attempt of some educators to divide learning materials or aids into two distinctive categories: One class consisting of the so-called sensory aids or materials (those of a concrete or semi-concrete nature), the other of abstract symbolic or verbal materials such as books and magazines. It is evident that this group of educators places abstract learning materials on a pedestal, consigning to the concrete and semi-concrete aids a subsidiary role--supplementary to the learning of abstract subject matter. This writer feels that such a point of view is indicative of the mind-reservoir theory of learning, the primary aim of education being the accumulation of a body of knowledge.

The writer favors the point of view that learning involves "forked-stick" situations, the facing and solution of problems. To resolve problematic situations, to bring about desired changes of behavior in the learner, the teacher and pupils reach out on all sides to gather

\textsuperscript{14} Elizabeth Goudy Noel and J. Paul Leonard, \textit{Foundations for Teacher Education in Audio-Visual Instruction}, p.l.
pertinent data. The data might conceivably come from instructional films, from the abacus, from magazine illustrations, or from abstract material contained in the arithmetic textbook. No aid is considered supplementary to another. Rather, all aids are complementary. If an available aid is needed and is meaningful, it is used, regardless of whether it is of a concrete, semi-concrete, semi-abstract, or abstract nature.

2. Arithmetic Education. By arithmetic education is meant that form or portion of teacher education which is aimed specifically at the development of competency in teachers and prospective teachers in the teaching of arithmetic. This competency consists of understandings and abilities in regard to the acceptable objectives of arithmetic as outlined in a later section of this study. In the present study the term, arithmetic education, refers specifically to its application to the elementary school.

3. Teacher Understanding. As used in this study, the term, teacher understanding, refers to the capability of the teacher or prospective teacher to form reasoned judgments with respect to what arithmetic instructional aids to use, when to use them, and how to use them, when faced with arithmetic-teaching situations. Van Engen\textsuperscript{15} stated:

To discuss the "meaning" of "understanding" by reviewing all the various ways that the word "understanding" can be used would be of little value for the teacher of mathematics. The main problem centers around how the word may be used by children in the elementary- and secondary-schools.

He maintained that "understanding" refers to something that is in the possession of an individual. The individual who understands is in possession of the cause and effect relationship. Understanding, in brief, is in the process of integrating concepts—placing them in a certain sequence according to a set of criteria.

Brownell and Sims¹⁶ remarked:

A technically exact definition of "understanding" is not easily found or formulated. . . . Quite apart from the difficulty of the undertaking, it would seem to be unnecessary, for the terms are employed by most people, school people included, with considerable agreement as to meaning.

They presented the following as being the essential characteristics of understanding:

a. A pupil understands when he is able to act, feel, or think intelligently with respect to a situation.

b. Rather than being all-or-none affairs, understandings vary in degree of definiteness and completeness.

c. The completeness of understanding to be sought varies from situation to situation and varies in any learning situation with a number of factors.

d. Typically, the pupil must develop worth-while understandings of the world in which we live as well as of the symbols associated with this world.

e. Most understandings should be verbalized, but verbalizations may be relatively devoid of meaning.

f. Understandings develop as the pupil engages in a variety of experiences rather than through doing the same thing over and over again.

g. Successful understanding comes in large part as a result of the methods employed by the teacher.

h. The kind and degree of the pupil's understanding is inferred from observing what he says and does with respect to his needs.17

4. Evaluation. Evaluation is the process of making value judgments. It includes not only subject-matter achievement but also attitudes, interests, ideals, ways of thinking, work habits, and personal and social adaptability. Traditionally evaluation has been associated with appraisal of the learner's work for the purpose of obtaining a "mark" or measure of success. The writer favors the point of view that the real and most important function of evaluation is that of facilitating learning. In terms of specific evaluation in arithmetic, Sueltz18 declared:

17 Ibid., pp. 28 ff.

Thus it is apparent that evaluation must include all phases of our program in arithmetic. It must include understandings of concepts and principles, knowledge of facts and socio-economic information, understandings and ability to perform operations, and ability to recognize and to think through the mathematics in a problem or situation as all of these are used in the functional life of an intelligent citizen.

5. Learning. As conceived of by modern psychologists and educators, learning consists of something entirely different from the process of training "faculties of the mind," of packing fact after fact into little storehouses in the brain, of inscribing factual information on a "tabula-rasa" mind, or of making neural connections and conditioning reflexes. Rather, modern psychologists and educators appear to be in more or less agreement that learning is any change of behavior which is a result of experience as distinguished from changes in behavior brought on by maturation alone.\(^\text{19}\) Cole and Bruce\(^\text{20}\) remarked:

> It (learning) sets the whole person into motion, challenges him with problems, opens up vistas of growth, structures his world so that his energies may direct themselves accurately upon important targets. In addition to the facts which come into the classroom discussion, there is the social setting, the conditions of learning and

\(^{19}\) For instance, see: Lawrence E. Cole and William F. Bruce, *Educational Psychology*, p. 12; Ernest R. Hilgard, *Theories of Learning*, p. 4; and Glenn Myers Blair and Others, *Educational Psychology*, p. 93.

\(^{20}\) Cole and Bruce, *loc. cit.*
discovery, the interpersonal roles which develop between children and between teacher and class; and these conditions are all-important in determining the kind of person who matures as knowledge is acquired, the use that will be made of the knowledge, the effectiveness of the learning process.

6. Procedures. "Procedure" means the manner or method of proceeding in a process or course of action. For example, the term might refer to the established way of administering tests, or to the manner in which a controlled experiment is ordinarily conducted. In the present study the term "procedure" refers to any order or system of performing, conducting, or proceeding employed by the writer or others with reference to the investigation.

Basic Assumptions. In a study of this nature it is essential that the writer start with certain basic assumptions pertinent to the problem. In a certain sense such assumptions serve as guide lines. They again may be thought of as focal points which aid in channelling the thinking of the writer while he works toward the major objectives of the study. With reference to the reader, they tend to serve as an aid in orienting him to the writer's point of view. In the next few pages some of the major assumptions made in this study will be stated and discussed.

1. Learning is basically sensory and developmental in nature. This assumption will be discussed only briefly in this section, due to the fact that it is more fully
developed in Chapter III, in the section entitled "Psychological Bases for the Use of Instructional Aids in Arithmetic Education." It should be observed that educators and psychologists are far from being in general agreement in regard to this assumption. There are essentially two varying points of view: (a) Sensory learning is essential to primary children only, and (b) Sensory learning is essential to persons of all ages and grade levels. There seemingly is no disagreement among educators and psychologists that young children must learn through their "senses." The disagreement is in regard to whether it is possible for older, more experienced, or higher-grade-level children and adults to learn in the abstract without having previously experienced a specific concept in the concrete. It would appear, however, that if educators and psychologists are in general agreement that sensory experiences are basic to learning among young children, the assumption is sufficiently justified.

The second part of the assumption, relating to the developmental nature of the learning process, has two primary aspects: (a) As the pupil matures physically, mentally, emotionally, he becomes more effective in dealing with instructional aids of a more abstract nature; and (b) the person (regardless of age) who is naïve in a given concept or process tends to learn that concept or process most effectively through the use of aids of a concrete
nature; as the learner becomes less naive in the concept or process (that is, as he becomes more capable of dealing with it), he tends to learn more effectively through the use of learning aids of a more abstract nature. These theories will be more fully developed in the later section mentioned above.

2. **Teacher understanding of the use of instructional aids in arithmetic education is imperative.** Arithmetic instructional aids possess no magical qualities which make them inherently useful. Whatever value they have is dependent upon how they are used by the teacher, or better yet, how they are used by the pupils. The teacher with a lack of pertinent understanding would be apt to accomplish little even with the best of aids. Rather, she would tend to do much harm instead. On the other hand, the teacher with an adequate understanding of arithmetic instructional aids would be able to accomplish much with common objects, such as beans, grains of corn, and buttons.

In order to utilize the broad curriculum of the school of today, Larson\(^{21}\) maintains that the teacher needs a knowledge of the unique contributions of different types of materials, of fundamental principles and techniques of

utilization, and of reliable methods of evaluation. This responsibility of the teacher in the classroom requires specific added competencies on her part. She must know what instructional aids are available and must be able to select the ones which are most appropriate for her needs, as well as make optimum utilization of those employed. According to Larson, proper selection of instructional materials requires a knowledge of:

1. sources, methods of securing, and methods of evaluating materials.
2. the relation of materials to the curriculum and to method.
3. successful use of the potentialities of other various audio-visual media.
4. the different materials available in specific fields (as for arithmetic in the present study).22

In addition, the proper utilization of those materials or aids selected requires knowledge and skills related to:

1. optimum conditions for the presentation of materials.
2. the operation and routine maintenance of different types of equipment.
3. the psychology of learning and the philosophical concepts underlying the use of audio-visual materials (as in the teaching of arithmetic).
4. the use of the proper audio-visual material at the proper time.

22 Ibid., p. 251.
5. techniques of communication through media other than print and oral language.\(^{23}\)

In like manner, Noel and Leonard\(^ {24}\) are of the opinion that to realize the values inherent in the tools of audio-visual instruction, the teacher must have certain knowledges, understandings, skills, and abilities. They present the following list as what they consider the basic requirements for teacher competency in this field:

I. Knowledges and understandings

1. Philosophical and psychological factors underlying the use of audio-visual materials and equipment in the classroom.

2. Results of research studies, past and present, in the field and their implications for instruction.

3. Physical characteristics and nature of the common types of audio-visual materials and equipment, and the educational values and limitations of each.

4. Types of audio-visual materials available in the specific area of the teacher's interest and their potential educational worth and uses.

5. Sources of materials and equipment—local, national, and international.

6. Principles of good teaching that affect the selection and use of these materials.

7. Processes involved in the production of some of the simpler materials, such as mounted prints, handmade slides, filmstrips, and photographs.

\(^{23}\) Ib\(d\)., p. 251.

\(^{24}\) Noel and Leonard, op. cit., pp. 2-3.
8. Methods of procuring, storing, filing, and maintaining the various kinds of materials and equipment.

9. Principles and procedures for setting up an audio-visual education service in a single school or in a school district; how a teacher can use that service; and the teacher's responsibility for cooperating with the department.

10. Background and development of audio-visual education that have a relation to current trends and practices in the field.

II. Skills and Abilities

1. Appraising the educational worth, technical quality, photographic characteristics, and commercial aspects of audio-visual materials.

2. Selecting audio-visual materials to meet pupils' needs and the purposes of instruction.

3. Using audio-visual tools effectively in a classroom situation.

4. Evaluating the effectiveness of the use of these materials in teaching situations; modifying and improving future instructional practices on the basis of such evaluation.

5. Assembling and operating various kinds of equipment and performing simple servicing operations such as lubrication and the replacement of lamps.

6. Providing and arranging the best physical conditions possible for using these materials.

7. Planning and successfully executing a field trip or excursion.

8. Producing simple materials, such as mounted prints, slides, posters, charts, graphs, models, collections of natural science materials, and preparing exhibits and displays.

9. Displaying materials effectively on the bulletin board in the classroom, and in other appropriate locations.
3. **Instruments may be devised to measure certain aspects of teacher understanding of principles and processes involved in the selection and use of arithmetic instructional aids.** Despite the fact that typical standardized tests are based on factual knowledge, Douglass and Spitzer\(^\text{25}\) assert:

> It is possible to devise instruments and procedures to assess meaningful learning outcomes. . . . Nevertheless, it remains a fact that the development of means to judge understanding more adequately will for a long time continue to be an important aspect of educational research.

In every subject-matter area there are available at present many well-known procedures for the evaluation of understanding.\(^\text{26}\) Moreover, Findley and Scates\(^\text{27}\) maintain:

> The evaluation of understanding does not, in general, require new devices and procedures. The teacher should depend upon normal classroom opportunities, the examination of pupils' work products, written tests of different kinds, pupil interviews, and the systematic observation of pupil behavior.

What is true of the evaluation of children's learning would, by the same token, be true of an evaluation program.

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\(^{27}\) Ibid.
for prospective teachers. Again, it seems reasonable to assume that if evalua
tive instruments have been devised to measure understandings in the traditional subject areas, they might also be devised to ascertain teacher understand-
ing of principles and processes involving the selection and use of arithmetic instructional aids. Educators are in rather general agreement that, of the several instruments used to determine student understanding in the various subject areas, the objective test of understanding serves a very worthwhile purpose. Douglass and Spitzer\textsuperscript{28} stated: "... Too frequently objective testing has become exclu-
sively fact-testing. This trend is unfortunate, and it is unnecessary. The new types of test items can be utilized to measure meaningful outcomes, and they should be so used." Thus, it seems reasonable to conclude that if valid and reliable tests of understanding have been devised in other subject fields, the same might as readily be true in the field of pre-service teacher understanding of the use of arithmetic instructional aids.

\textbf{Limitations of the Study.} This investigation repre-
sents an effort to deal only with the role of tangible instructional aids in arithmetic education; it was not the intention to consider the aggregate of intangible qualities

\textsuperscript{28} Douglass and Spitzer, \textit{op. cit.}, p. 19.
essential to successful teaching of elementary-school arithmetic. Likewise, the validity of the objective instrument developed (An Inventory of Teacher Understanding of the Use of Instructional Aids in the Teaching of Arithmetic) should be judged by the accuracy with which it measures only the factor it was designed to measure. It cannot ascertain the aggregate of arithmetic-teaching qualities (intangible instructional aids) involving such factors as personality, social characteristics, intellect, cultural traits, and drive.

Problems related to the control of data made it more practical to limit the investigation to prospective teachers. Likewise, the investigation was limited to arithmetic in the elementary-school program. It was felt that while there are certain instructional-aids understandings applicable at any level of teaching, there are also certain understandings peculiar to specific levels. In order to develop an objective testing instrument which would obtain an adequate sample of the more specific understandings, it was necessary to limit the study to one general level.

Plan of the Study. The plan of the writer in the following portions of this report is as follows:

Chapter II deals with a review of the literature of selected related studies pertaining to: (a) the effectiveness of instructional aids in accomplishing the objectives
of the elementary-school program in general, and of the elementary-school arithmetic program specifically; (b) the attempts made by psychologists and educators to measure "general" insight, and understanding in specific subject areas; and (c) studies that have been made relative to measuring understandings and competencies of teachers in the use of instructional aids in the various subject areas, the focus being on arithmetic.

Chapter III attempts to determine the role of instructional aids in arithmetic education. The resolution of this problem constitutes the major objective of the study and hence receives considerable coverage. Specifically, the chapter deals with the philosophical and psychological bases for the present study. It includes the overall objectives of the elementary-school program, as well as the specific objectives of the elementary-school arithmetic program. Also, the chapter presents classifications of arithmetic instructional aids, with relation to the various operational levels of learning. Advantages and limitations accompanying the use of instructional aids are listed and discussed briefly, and finally dramatized procedures illustrating the effective use of aids in arithmetic instruction are proposed.

Chapter IV is concerned, in the main, with the three subsidiary objectives of the study. The first portion of the chapter relates to the development of criteria for the
selection and use of instructional aids, with special reference to selected topics in arithmetic. The second portion regards the development of an objective instrument to ascertain certain aspects of prospective-teacher understanding of the use of aids in arithmetic instruction in the elementary school. The last part of the chapter is concerned with the development of a checklist to ascertain certain aspects of prospective-teacher understanding of the role of arithmetic instructional aids on a performance basis.

Chapter V is a summary of the preceding chapters, with conclusions and recommendations pertinent to the study.

Chapter Summary. Chapter I is an introduction to the study. The first section deals with the statement of the problem, while the second portion discusses the significance of the investigation. The succeeding section attempts to define certain concepts basic to the study, especially those of a more or less controversial nature. The next portion considers the major assumption made by the writer in regard to the investigation, with attempts to justify them. Then the limitations of the study are stated and discussed. Finally, a plan of the succeeding chapters of the study is presented.
Plan of the Chapter. First, findings are offered which relate to the effectiveness of instructional aids in accomplishing the objectives of the elementary-school program in general, and of the elementary-school arithmetic program specifically. This aspect of the literature is basic to the study. Unless instructional aids are worthwhile in facilitating the learning process, there is no point in being concerned with whether or not prospective teachers understand principles relating to their selection and use. There can be no worthwhile object in attempting to measure understandings of concepts and processes which have no basic value. More specifically, there can be no point in attempting to devise objective-test items to ascertain prospective-teacher understanding of the use of the abacus, for example, unless evidence can be produced that this teaching aid possesses potential value in facilitating the teaching-learning process in arithmetic.

Second, the writer reviews the scanty bit of literature relating to attempts that have been made by psychologists and educators to measure "general insight" and understanding in specific subject areas, the focus being on
arithmetic. Third, studies, likewise sparse, that have been made relative to determining teacher understanding of the use of instructional aids in general, are presented. Last, the literature is reviewed to ascertain whether any attempts to measure prospective-teacher understanding of the use of arithmetic instructional aids have been made.

**Effectiveness of Instructional Aids in the Learning Process: Direct Experiences.** Research findings indicate that the more direct the experience on which a concept is built, the greater will be the pupil's knowledge and understanding of that concept.¹ In order to build concepts it is necessary to provide experiences upon which to establish them. Such initial concepts may then be combined and manipulated to form more complex ones. Concepts based solely on verbalism can result only in verbalism.²

Although Bedwell's³ study established the fact that it is possible to have verbal or factual knowledge without having a functional concept, the author at the same time


² Ibid.

³ Margaret Bedwell, "Comprehension of Concepts of Quantity in Third Grade Social Studies Reading Materials," Unpublished Master's thesis, University of Iowa, 1932, as reported by Serra in the study above.
pointed out that practical experience is most important in concept development. As a result of an investigation of the comprehension difficulties of 30 third-grade pupils, Herbers concluded that concrete materials and personal experiences are necessary to overcome verbalism. Sachs, too, pointed out that children do not acquire concepts by merely meeting words in context. In like manner Sims, Stolte, and Wiederfeld each concluded as the result of analytical and experimental studies that experience is of the utmost importance in building concepts.

Effectiveness of Instructional Aids in the Learning Process: Manipulative Aids. Obviously there is no clear-cut difference in many respects between this classification of instructional aids and the preceding classification.


7 Helen B. Stolte, "The Ability of Fourth-Grade Children to Comprehend Certain Geographical Concepts," Unpublished Master's Thesis, State University of Iowa, as reported by Serra. (See Footnote No. 1.)

8 M. Theresa Wiederfeld, An Experimental Study in Developing History Reading-Readiness with Fourth Grade Children, as reported by Serra. (See Footnote No. 1.)
direct experiences. The learner may have direct experiences, in other words, with manipulative aids; or the reverse, he may manipulate aids in the process of having a direct experience. It might help to clarify the two classifications somewhat, however, if direct experiences are thought of as experiences of a functional nature, experiences that come about in the regular course of the school life; for example, taking the milk count, taking attendance, or putting sufficient stamps on a letter. On the other hand, manipulative aids may be thought of as being more of the nature of contrived experiences, experiences that are set up for the expressed purpose of teaching a specific concept or process. For example, blocks, pegs, acorns, and buttons used in the process of teaching number concepts are regarded as manipulative aids. However, this distinction does not always hold true, especially not in the case of manipulative aids of a social nature, such as nails, measuring cups and spoons, toothpicks, or milk bottles. Nevertheless, to think in terms of the distinction given above should help somewhat in distinguishing between the two classifications.

Much of the research relating to direct, functional experiences in arithmetic at the same time refers to manipulative aids used in those experiences. In an experimental study involving grade IV children, made to determine the effect of the use of a functional program of arithmetic
instruction, employing concrete experiences as well as many concrete aids, Harding and Bryant found that the experimental group made significantly greater gains than did the control group, which used vicarious experiences and drill procedures, in both computation and reasoning-problem solving ability. The results of the study, as measured by the Stanford Achievement Test, indicated clearly that, "concrete materials and 'alive' problems are more effective than abstract examples and 'artificial' problems for learning." Furthermore, evidence indicated that, "within certain limits, methods and materials of instruction may be more important than I. Q. in the development of certain skills and abilities."

In an experimental study, involving grade V pupils, made to determine what effect the use of real experiences (in which were used manipulative aids having a social significance, such as measuring cups and spoons, thermometers, scales, and tape measure) had upon the mastering of denominate numbers and the multiplication and division of common fractions, Harap and Mapes found that the pupils

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mastered 14.2 of the 17 fundamental processes included in
the test on denominate numbers and on the multiplication
and division of common fractions, being equivalent to an
average mastery of 84 percent of the processes. In a
later study involving the same pupils, in an essentially
similar program, made to determine the effectiveness of
such a program on the learning of decimal fractions, Harap
and Mapes\textsuperscript{11} found that the pupils mastered 96 percent of
the 27 basic processes, as compared with 67 percent for
the control group. A re-test a year later revealed a
"final" achievement of about 97 percent.

In still another experimental study in which third-
grade children used meaningful experiences such as measur-
ing, weighing, and the manipulation of money, Harap and
Barrett\textsuperscript{12} found that the class mastered 93 percent of the
steps set up as a goal for the grade and attained a final
score of 4.1, or slightly above the normal grade standing.

Mulholland\textsuperscript{13} found that the use of "fraction pies"

\textsuperscript{11} Henry Harap and Charlotte E. Mapes, "The learning
of Decimals in an Arithmetic Activity Program," Journal of

\textsuperscript{12} Henry Harap and Ursula Barrett, "Experimenting with
Real Situations in Third-Grade Arithmetic," Educational

\textsuperscript{13} Vernie Mulholland, "Fifth Grade Children Discover
Fractions," School Science and Mathematics, LIV (January,
(discs) by her fifth-grade pupils promoted (a) understanding of the common-fraction concept and (b) understanding and ability to solve problems requiring the addition, subtraction, multiplication, and division of common fractions. Howard\textsuperscript{14} discovered in an experimental investigation involving low fifth-grade pupils that the use of concrete objects and explanatory charts plus emphasis on the "why" of arithmetic processes, followed by practice on computation exercises, resulted in significantly greater long-time retention than did instruction without the use of such aids and without follow-up practice on computation exercises. As a result of a study made to determine the fractional concepts possessed by 266 pupils of the kindergarten and the first, second, and third grades of a Chicago elementary school, Polkinghorne\textsuperscript{15} concluded that, "the teaching of fractions should be in the form of concrete experiences with real objects since this was the way in which the children tested had gained their understanding of fractions." Schott\textsuperscript{16} reported that in an experimental


study involving fourth, fifth, and sixth grade pupils, practice in the use of the adding machine resulted in tremendous gains in scores made on the basis of the California Arithmetic Test, both when using the adding machine to take the test as well as when using paper and pencil only.

In an experimental study made to determine whether the use of the "borrowing crutch" in subtraction has a detrimental effect on the user, Brownell\(^\text{17}\) concluded that (a) the "crutch" did not interfere with correctness of work, that it even increased it, and that (b) it did not interfere with the rapidity of work (except in the later stages when it probably should have been abandoned). Of especial significance to the present study is Brownell's concluding paragraph in which he states:

Still, one can hardly justify the space in this article (to say nothing of the space in the original monograph) if the only purpose is to sponsor a particular device in teaching subtraction. There is, however, further justification. The borrowing device should be regarded as typical of many other devices as yet not accepted and perhaps not even known which, when discovered and put to work, will make learning easier and more meaningful to children. If the meeting of this responsibility requires the abandonment of much that has passed as learning theory, then false learning theory

should be cheerfully discarded in the assurance that a sounder theory will be found to take its place.\textsuperscript{18}

Studies reported by Ramsey\textsuperscript{19} and Powell\textsuperscript{20} give evidence of the possibilities for learning inherent in museum materials and close cooperation between the museum and the school.

**Effectiveness of Instructional Aids in the Learning Process: Field Trips or Excursions.** Atyeo\textsuperscript{21} concluded after summarizing a number of individual studies dealing with excursions that the excursion technique is superior to class discussion for teaching material requiring comparisons and knowledge of concrete objects which can be more easily visualized. Raths,\textsuperscript{22} too, concluded that carefully planned experiences may result in clarifying the beliefs which students hold. However, Clark\textsuperscript{23} held that while the school excursion or field trip was important in development of interests, ideals, and appreciations in students, it was

\begin{itemize}
  \item \textsuperscript{18} Ibid., p. 424.
  \item \textsuperscript{19} Grace F. Ramsey, *Educational Work in Museums of the United States*.
  \item \textsuperscript{20} Lydia Powell, *The Art Museum Comes to the School*.
  \item \textsuperscript{21} H. C. Atyeo, *The Excursion as a Teaching Technique*.
  \item \textsuperscript{23} Ella C. Clark, "An Experimental Evaluation of the School Excursion," *Journal of Experimental Education, XII* (September, 1943), pp. 10-19.
\end{itemize}
of little value in the acquisition of factual knowledge.

Effectiveness of Instructional Aids in the Learning Process: Pictorial Aids. Pictorial aids are commonly referred to as visual aids. However, this writer feels that the term "visual aids" is much more inclusive than is the term "pictorial aids," strictly speaking. He considers pictorial aids as only those visual aids such as flat and projected still pictures, motion pictures, and stereoscopic scenes. He conceives of pictorial aids as being a bridge between concrete aids and abstract aids. Osburn and his associates, after experimenting with vicarious experiences between the levels of direct experiences and verbalism, concluded that experiences with pictures, illustrations, models, etc., should be used to make concrete the relationships implied in language.

More research studies have been done with pictorial aids than with any other classification of instructional aids, this fact being especially true in the case of motion-picture films. Among the first of such research were the Sumstine study in 1918 and the pioneer work of


Joseph J. Weber, who investigated the "Comparative Effectiveness of Some Visual Aids in Seventh Grade Instruction." The early studies dealing with instructional aids of this type set a "which-is-more-effective?" pattern of study which has been rather generally followed by similar succeeding studies, at least until of rather recent times. That is, generally speaking they were concerned with such problems as the following:

1. Are motion-picture films more effective with dull or with bright children?
2. Which are more effective -- silent or sound films?
3. What is the optimum grade level and age to profit from films?
4. What is the optimum frequency of use and the optimum techniques to use with films?
5. Which are more effective -- motion or still pictures?
7. The effectiveness of films in learning facts, in producing the ability to think and reason, on the development of attitudes and appreciations, and in developing interests and skills.

Since the pioneer studies of Sumstine and Weber, not only have numerous investigations relating to the

effectiveness of instructional films been made but also have quite a compilation of summaries of such individual studies, among them being those by Long,27 Hoban,28 Dale and Hoban,29 Kinder,30 Dale, Finn, and Hoban,31 and many others. In view of this fact, the many individual investigations will not be rehashed in the present study. In general, the findings and conclusions reached by those making such experimental studies were as follows:

1. Use of an instructional film tends to give the child clear-cut notions of the objects and actions in the world about him.

2. Students like classroom motion pictures and the majority would like even more.

3. The use of films as an integral part of classroom procedure arouses and maintains pupil interest and increases the amount of voluntary reading and class discussion.

4. The use of the film is superior to the use of verbal material alone or to the unorganized use of other visual aids in effecting permanence of learning.

27 Ibid.
28 Charles F. Hoban, Motion Pictures in Education.
5. Instructional films are more effective if shown in the regular classroom than when shown in the auditorium, or in a special projection room.

6. In terms of informational responses, films appear to be more effective for dull pupils than for bright; in terms of ability to make generalizations, films appear to be more effective on bright than on dull pupils.

7. There seems to be no general agreement relative to which form of pictorial aid is the more effective—silent or sound films, or filmstrips and slides or motion pictures. Implications were for the selection and use of specific forms of pictorial aids for different educational purposes.

8. There is not sufficient evidence to generalize in regard to whether pictorial aids are more effective at one grade- or age-level than at another.

9. In presenting tables, maps, and charts, the film is no better than the actual tables, maps, and charts presented as such.

It is extremely doubtful whether today many of these experimental findings should be taken at face value. For example, Smith concluded on the basis of a recent study made by him (as well as on recent studies by several others) that little evidence was found to support the position that the use of motion pictures in the classroom greatly affects the ranking of students with respect to the amount which they learn. Evidence also indicated that bright students profit more from films in terms of actual learning, as measured by the tests employed. In an earlier

32 Herbert A. Smith, "Intelligence as a Factor in the Learning Which Results from the Use of Educational Sound Motion Pictures," Journal of Educational Research, XLVI (December, 1952), pp. 249-61.
study Smith, investigating the relationship between test-intelligence and pupil achievement in three situations: (a) films as the sole means of presentation, (b) teacher demonstration as the sole means of presentation, and (c) a combination of films and teacher demonstrations, found positive correlations between intelligence and achievement, the correlation being higher for the section in which films were used exclusively. However, the differences were not statistically different.

In an experimental study made to determine the comparative effectiveness of lessons on the slide rule presented via television and in person, Anderson and Vandermeer concluded that teaching the slide rule by television was practically as effective as teaching it in person, no significant differences in test results being found. However, this study also revealed that students taught by television tended to forget more readily than did those taught in person. At the same time, it was concluded that for all practical purposes, due to timing, financial

33 Herbert A. Smith, "The Relationship Between Intelligence and the Learning Which Results from the Use of Educational Sound Motion Pictures," *Journal of Educational Research*, XLIII (December, 1949), pp. 241-49.

considerations, and personal contact, it was very unlikely that television would replace the in-person teacher except in an emergency.

Dale, Finn, and Hoban,\(^\text{35}\) too, are rather skeptical of the findings of early studies dealing with instructional films. They remarked: "... It must be remembered that these conclusions are based on films now outmoded and that, since these data were gathered, greatly improved films have been made available for teaching skills."

Moreover, according to them, some of the studies made were not experimentally sound and some were considerably handicapped by limitations of techniques of measurement.\(^\text{36}\)

 Furthermore, the major emphasis today in experimental investigations involving instructional aids is on all types of aids rather than on whether one aid is superior to another; it is on the potential effectiveness of the totality of aids to bring about desired changes in behavior. Moreover, emphasis is placed on the aid only to the extent that it is applicable to specific problematic situations encountered by the pupil. Finally, emphasis is placed on how and when an instructional aid is used.

\(^{35}\) Dale, Finn, and Hoban, op. cit., p. 264.

\(^{36}\) Ibid., p. 271.
Effectiveness of Instructional Aids in the Learning Process: Graphic Aids. Graphic aids are those consisting of the various types of graphs, illustrations, maps, charts, bulletin boards, posters, and cartoons. It is obvious from these illustrations that there is a considerable difference of difficulty among them. Thomas found that slow fourth-grade pupils derive little meaning from graphs, but that superior fourth-grade pupils understand the meaning of simple graphs and can read simple facts from them. She also found that picture graphs, two-dimension diagrams, and circle graphs are easiest for pupils of all grades to read while line graphs are hardest. Washburne concluded as the result of an experimental study, that the bar graph is the most favorable graph in regard to recalling relative amounts (static comparisons called for involving a fair degree of difficulty). Wrightstone discovered that with junior high school pupils, pictorial graphs proved to be more effective than bar, circle, and line graphs and


that students preferred pictorial graphs. Dale, Finn, and Hoban,\textsuperscript{40} however, are of the opinion that the effectiveness of pictorial graphs in conveying quantitative information has been exaggerated. Wrightstone\textsuperscript{41} detected a gradual growth in the ability of students to read maps and graphs from the seventh through the twelfth grade.

Malter\textsuperscript{42} disclosed that diagrams are probably the most complex and abstract of graphic aids. In an experimental study designed to determine the ability of elementary-school pupils of grades IV-VIII to interpret process-diagrams, specifically to follow a diagram depicting the flow of wheat through a flour mill, he found that the majority of the two hundred twenty-seven pupils tested were unable to follow the flow of materials when definite clues such as "Start Here," "End Here," "Follow Arrows," and directive arrows were omitted. Upon inclusion of such clues, it was revealed that a majority of the grade VI pupils tested (the only grade involved in the second phase

\textsuperscript{40} Dale, Finn, and Hoban, \textit{op. cit.}, p. 283.


of the experiment) were helped in their attempts to read the diagram. The implications, obviously, are that since the difficulty of diagrams in general is granted, every reasonable means should be used to make them meaningful to the learner.

In another study Malter concluded that most children are unable to read many of the cross-sections (a special kind of diagram found in their reading materials). He maintained that it is reasonable to believe that cross-section diagrams are meaningful to children only if based on a more concrete foundation, even though, as he admitted, there is little evidence to support this point of view. In still a further study Malter concluded that children might read satisfactorily such symbols as arrows, designating motion; circles, framing enlargements; and possibly wavy, serrated lines, indicating cutaways, but that they experienced difficulty in interpreting such symbols as dash-lines, indicating displacement or movement, and straight lines, relating members of families.

As a result of such studies as the foregoing, it is


obvious that some forms of graphic aids are rather difficult and must be used only when the operational level of pupils is sufficiently high.

**Effectiveness of Instructional Aids in the Learning Process: Audio-Aids.** These aids consist of the radio, transcriptions, recordings, and film sound tracks. They are aids which stimulate learning primarily through the sense of hearing. Bathurst\(^45\) found that transcriptions proved to be a good stimulus to further study. However, studies by Rulon\(^46\) resulted in findings opposed to those of Bathurst. Recordings were found to be of little value in motivating students to further study and failed to exhibit any superior effectiveness in the teaching of informational material. A slight shift in attitudes was achieved by the use of recordings, but the results were in no way


conclusive. In general, Rulon's studies showed that there is no sound basis for the belief that youth learn best through the ear. In a somewhat different type of investigation in which 91 sixth-grade pupils were used as subjects, Mitchell found that variety radio programs had a detrimental effect upon pupil achievement but that musical programs had no such adverse consequence. Pupils of higher ability showed gains with the musical program background.

Lowdermilk discovered that students' social attitudes were affected by transcribed radio broadcasts but not uniquely. Also, students appeared to regard materials presented over the loudspeaker somewhat less critically than they did material read. Miles detected that while interests in some aspects were heightened by supplementary recordings used in science classes, students' breadth of interest was decreased.

Effectiveness of Instructional Aids in the Learning Process: Abstract Aids. Although a wide variety of

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48 Ronald R. Lowdermilk, Reading, Radio, and Attitudes, Ohio State University, Bulletin 1942, No. 63.

49 Robert J. Miles, Auditory Aids and the Teaching of Science, Ohio State University, Bulletin 1943, No. 57.
instructional aids have been developed and used over the past several years, it is generally agreed that the textbook remains the number one teaching aid. W. T. Harris pointed out sixty years ago: "You can take your book wherever you please. . . . You cannot select the time for hearing the great teacher talk as you can for reading the book. . . . Nearly all the great teachers have embodied their ideas in books." Buckingham declared: 

. . . . We realize that direct experience with things does not cover the ground. . . . Direct experience will carry a student only a short distance toward mathematical insight. . . . There is too much to know and to appreciate, there are too many attitudes to acquire and too many judgments to be reached. Our civilization is incurably a civilization of the book.

One study relating to the inefficacy of mathematics textbook problems was that of Risinger who conducted a 10-weeks experiment with eighth-grade classes to compare growth in problem-solving ability, the experimental groups being taught consumer information about housing while the


51 Ibid.

control groups used traditional materials. Risinger concluded as a result of the investigation that, (a) use of experimental materials may result in pupil ability to solve problems superior to that expected from the use of traditional materials; (b) teachers may replace traditional text material with consumer information on housing with confidence that pupil ability to solve problems will be enhanced; and (c) actual and current data may be substituted for abstract and hypothetical data with reasonable assurance that pupil ability to solve problems will be augmented.

Although the remainder of the references presented in this section are philosophical rather than experimental in nature, the writer feels that they are valuable points of view and worthy of presentation. Barnes justifies the dearth of experimental studies involving the textbook in these terms: "So, the textbook problem becomes one wherein reasonable questions on the efficacy of the textbook in teaching remain in an implied state with little indication of becoming explicit and subject to study." The import of Barnes' statement becomes more and more evident in view of the challenging question put by the textbook publishers, who asked: "Are the authors (of textbooks), most of them leaders in education, and the editors who work with them

mistaken in (their) conception of the role of the textbook? ... Unfortunately we have little scientific evidence to support their convictions."

Chase,55 who emphasized that the effectiveness of the textbook, as in the case of other instructional aids, is very largely dependent upon the effectiveness of the teacher, declared:

The way textbooks are used is indicative of the whole educational philosophy of a school. The school that turns up its nose in disdain at the use of any textbook in any study area is one which fails to capitalize upon a very valuable instrument in the learning process. The school whose whole learning program in any so-called subject area is determined by a single textbook, or even multiple texts, is one which fails to understand that a textbook is only one useful tool among many which needs to be used to make learning and growth rich and fruitful.

Burton,56 in agreement, expressed the point of view that,

The chief investigation relating to textbooks should be performed, not on the textbooks but on those who use them! Research might be extended to those who educate teachers in the use of textbooks! The problem: How to eliminate the equally grotesque ...
belief that children and youth may be introduced to the complex real world through texts alone, particularly one text alone? An extensive ration for meeting the real world. This is not to say that texts are useless. Far from it. Their number will increase and, we hope, their nature improve.

Attempts Made by Psychologists to Measure Insight.
The question of insight in arriving at meaningful solutions to thought problems has been investigated by a rather large number of psychologists, but only a few attempts have been made to measure it directly and objectively. According to Fehr, the psychologist and the educator are for the most part concerned with different aspects of the concept. Whereas psychologists are interested in the scientific aspects of the process, in finding out what it is that goes on in the mind or the organism as they solve problems, educators, on the other hand, have been concerned with how to apply the findings of psychologists to the teaching process so that teachers may develop a greater problem-solving ability on the part of students.


59 Fehr, loc. cit.
Attempts to measure insight directly and objectively were made by Steinmetz, the main emphasis being placed on the appraisal of the ability of the testee to perceive the world as others see it. Kimber, who used the California Test of Personality in an attempt to measure insight, seemingly considered the concept as the ability of the testee to comprehend the meaning signified by an item. Gross attempted to measure "self-insight" by the use of a number of rating scales which were concerned mainly with the various common defense mechanism or "lie" items to which most people would agree. While this approach proved promising, the items have been criticized as being too general for clinical or research purposes. Dymond attempted to measure insight as defined by the role theory or empathy. The extent to which subjects took the role of characters mentioned in Thematic Apperception Test Stories was taken as a measure of insight.


Attempts Made by Educators to Measure Mathematical Understandings Possessed by Pupils. Brownell stated in 1941 that exceedingly little had been done either informally or systematically to find practicable and valid procedures for evaluating "mathematical understandings."

In fact, he cited only two existent examples of such standardized tests and they, according to him, had only limited coverage and functions. The two exceptions named were: (a) two sections of the Analytical Scales of Attainment, one section devoted to testing "Quantitative Relationships," the other to "Arithmetic Vocabulary," and (b) a short section in the Iowa Every-Pupil Test. However, Schaaf reported eleven years later that considerable progress had been made in improving the quality of tests and evaluation procedures, particularly along the following lines: (a) a notable shift in emphasis from measuring habits and skills to a recognition and appraisal of understanding and power; (b) a more determined effort to evaluate the quality of problem-solving reasoning, the use


of judgment, critical thinking, attitudes, and apprecia-
tions; and (c) a frank admission that "paper and pencil" 
techniques must be supplemented by observation, discussion 
and interviews in order to secure valid evaluation.

In line with the newer philosophy of evaluation, 
Brownell, with the help of other educators, devised test 
items which purported to measure mathematical understand-
ings, the object being to serve as a model for teachers who 
desired to measure the mathematical understandings of their 
pupils. A few years later the entire Forty-Fifth Yearbook, 
Part I, of the National Society for the Study of Educa-
tion was devoted to the measurement of understanding, one 
chapter being reserved to the measurement of understanding 
in elementary-school mathematics. Krout, expressing a 
new point of view in testing, reasoned on psychological 
grounds that education involved three stages of develop-
ment, one of which was concept formation, judgment, and 
reasoning. Thus, he reasoned psychologically that to test 
this developmental stage of learning, multiple-choice and 
matching tests should be employed.

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66 Brownell, "The Evaluation of Learning in Arith-

67 The Measurement of Understanding, op. cit.

68 Maurice H. Krout, "Psychological Standards in 
Measuring Achievement," School Science and Mathematics, 
XLVI (December, 1946), pp. 803-806.
Mathematics teachers belonging to the Educational Records Bureau, in cooperation with the Cooperative Test Service of the American Council on Education, in 1939 devised two forms of a multiple-choice test (with a third form to be completed in 1940) which purported, among other accomplishments, to measure understandings of seventh, eighth, and ninth grade pupils in an integrated program of arithmetic-algebra. The test items were based on twelve objectives for general mathematics at these grade levels agreed upon by various schools. Hastings, too, devised a test to measure mathematical concepts in the junior high school. After having used six different testing devices, he concluded that no single technic or testing device, such as the multiple-choice test, is a sufficient index of the behaviors measured by all the other types of tests.

To measure understandings and judgments in arithmetic, Sueltz used objective materials such as pieces of paper

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69 See: Margaret Seder, "An Experimental Study of a New Mathematics Test for Grades 7, 8, and 9," *Mathematics Teacher*, XXXII (October, 1939), pp. 259-264.


to be divided in half by lines and circles whose areas were to be compared. He also devised written tests to measure understanding and judgment of fourth, fifth, and sixth grade elementary-school pupils and of seventh, eighth, and ninth grade junior-high school pupils, using exercises that were both pictorial and verbal. The written tests were followed by personal interviews of testees in order to determine the validity of the written tests. Sueltz concluded on the basis of such tests of understanding as he devised that, "There is almost no pure guessing." Sueltz later applied the junior-high-school tests to college students to determine what understandings and judgments had been retained or augmented since their initial period of study.

Further attempts to measure mathematical understandings were made by Ebert who attempted to construct a test which would ascertain the extent to which eighth-grade pupils comprehend selected generalizations in the field of

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elementary-school mathematics, and by Spache\(^7^4\) who constructed an arithmetic test which purported to measure not only the actual solution of problems, but also the pupils' ability to: (a) recognize or understand the facts given, (b) decide what facts are to be found in solving the problem, (c) choose the appropriate arithmetic computations to be employed, (d) estimate a probable answer, and (e) execute the solution. Other tests which appeared to resemble that of Ebert were the New York State Progress Tests in Arithmetic,\(^7^5\) which measured not only computational skills but also mathematical ideas and concepts through choosing the correct process and judging the sufficiency of data given to solve a problem.

**Attempts to Measure Teacher Understanding of Mathematics.** On the assumption that teachers themselves must possess the understandings they are trying to transmit to their pupils if arithmetic is to be taught so that children acquire real understanding of arithmetic processes and

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concepts, Orleans and Wandt\textsuperscript{76} developed two tests which purported to measure how much teachers understand about arithmetic processes, concepts, and relationships. The tests devised for this purpose were made up of both free-choice and multiple-choice questions. An earlier study had been made by Orleans\textsuperscript{77} in 1950 to ascertain the extent to which teachers and prospective teachers of arithmetic understand the processes and concepts which are represented by the shortcuts they teach, this study, like the later one, being justified on the assumption that if teachers do not themselves understand underlying processes and concepts they cannot get pupils to learn with understanding.

The procedure used in the Orleans study involved the construction and administration first of a free-answer type test to determine teachers' understanding of various arithmetic principles and processes, and second the construction and administration of a multiple-choice test based on data used in the free-answer test. These tests were administered to 722 persons (193 undergraduates taking courses in


\textsuperscript{77} Jacob Orleans, The Understanding of Arithmetic Processes and Concepts Possessed by Teachers of Arithmetic, Publication No. 12, Office of Research and Evaluation, College of the City of New York, as reviewed in \textit{Mathematics Teacher}, XLVI (May, 1953), pp. 346-347.
methods of teaching in the elementary school, including the teaching of arithmetic; 340 undergraduate student teachers; 43 graduate students in education; 80 classroom teachers, including 26 in primary grades, 45 in grades 4-6, and 9 in junior-high mathematics; and to 66 other persons). Some of the most important findings of this investigation were:

1. About 2 out of 3 teachers understand partial products in multiplication.

2. About 2 out of 3 teachers know that division is a shortcut for subtraction.

3. About 2 out of 5 understand partial products in long division.

4. About 2 out of 5 know the meaning of remainder in division.

5. About 3 out of 5 know that a fraction can mean a quantity as well as a ratio.

6. About 3 out of 10 understand the changing of fractions to lower or higher terms.

7. About 1 out of 5 understands the inverting process in the division of fractions.

8. About 4 out of 5 understand the moving of the decimal point in the division of fractions.

9. About 1 out of 7 understands the meaning of remainder in the measurement type of division, involving division of fractions.

10. About 1 out of 3 understands the meaning of percent.

11. About 3 out of 5 know what the base of our number system is.

12. About 4 out of 5 know the meaning of decimal fractions.

13. The amount of understanding possessed by the four groups was not largely different.
14. Those who had had no college mathematics understood almost exactly as much as those who had had one or two courses in college.

15. Those who had had three or more courses of college mathematics seemed to understand considerably more than the other two groups.

Based on the findings of this study, Orleans concluded: (a) Teacher-education institutions may have only indirect influence on the program of number work in the schools; (b) teacher-education institutions can directly influence the prospective teacher's knowledge and understanding of arithmetic and his preparation for his responsibility as the person who is to get children to learn about numbers; and (c) teacher-education programs have not done an adequate job in this respect.

Glennon carried out a study in 1947 to determine the extent of growth and mastery of certain basic mathematical understandings possessed by teachers college freshmen, teachers college seniors, and teachers-in-service, all on the elementary-school level. Data were gathered by the administration of a Test of Basic Mathematical Understanding, an objective test of the multiple-choice type covering five areas: (a) decimal system of notation, (b) basic mathematical understanding.

understandings of integers and processes, (c) basic understandings of fractions and processes, (d) basic understanding of decimals and processes, and (e) basic understanding of the rationals of computation. Among the most pertinent findings of this investigation were the following:

1. No significant difference in achievement of basic mathematical understandings existed between a teachers college freshman and a teachers college senior. In other words, senior students had not grown in mathematical understanding as a result of four years of teachers' education program.

2. No significant difference in achievement of basic mathematical understanding existed between teachers' college seniors who had taken a course in the Psychology and Teaching of Arithmetic and those who had not.

3. No significant difference existed between teachers-in-service who had taken a graduate course in the Psychology and Teaching of Arithmetic and those who had not.

Based on the findings of this study, Glennon concluded:

1. Teachers' college freshmen possess about 44% of the mathematical understandings.

2. Teachers' college seniors possess about 43% of these basic mathematical understandings.

3. Teachers-in-service possess about 55% of these basic mathematical understandings.

Significant implications of these findings and conclusions are stated by Glennon as follows:

Since both careful research studies and other less-precise methods of rational inquiry have proven the worthwhileness of teaching for understandings, we must accept the responsibility for doing so. We are now in a position to create an "all-out" effort to discover new methods and materials for teaching and evaluating for growth in basic mathematical understandings.
In 1954 Tomlinson completed a study concerned with the determination of professional understandings possessed by prospective elementary-school teachers in ten elementary-school curriculum areas, one of which was mathematics. To determine certain aspects of such understandings, he developed valid and reliable objective tests, together with a rating scale to be used in working with student teachers. In the specific area of mathematics, Tomlinson was concerned with professional understanding of the purposes, principles, and methods of teaching mathematics in the elementary school. It was found that professional understandings as defined and measured in this investigation showed no significant relationship with achievement marks assigned by supervisors to indicate success in student teaching.

One of the most comprehensive attempts to develop testing instruments to ascertain teacher understanding of elementary-school arithmetic concepts, processes and practices has been effected by Lowry W. Harding, Professor of Education, The Ohio State University. Several forms of each of four tests have been devised for this purpose. For teacher self analysis of skill in arithmetic and insight

into possibilities for teaching arithmetic to children, the following three tests have been provided:

1. Computation in Digital Tasks (to ascertain teacher computational skill).

2. Reasoning in Prose Tasks (to ascertain teacher problem-solving ability).

3. Perception of Arithmetic Teaching Situations (to ascertain teacher insight into the value and utilization of various experiences and situations for helping pupils learn arithmetic, as well as to determine teacher creativity and originality in teaching arithmetic).

A fourth test, Inventory of Professional Understandings in Arithmetic, has been devised to determine teacher comprehension of the nature of arithmetic, principles and practices in teaching it, the use of materials of instruction, adjustment of arithmetic to other parts of the curriculum, and related aspects of the program for "teaching children arithmetic." In regard to these tests, Harding stated:

Results from the complete series of tests should provide evidence on all aspects of one's ability as a teacher of arithmetic except that of actual classroom performance and rapport with pupils. No paper-and-pencil instrument or personal interview procedure has yet been devised which yields reliable predictive data on this factor in the teacher's competence.

80 Lowry W. Harding, Explorations in Arithmetic, p. v.
Attempts Made to Measure Teacher Understanding of the Use of Instructional Aids. While no studies specifically relating to attempts to measure teacher understanding of the selection and use of instructional aids, either in arithmetic or in any other specific elementary-school subject, were found recorded in the literature, the writer feels that the following studies are rather pertinent to the present investigation.

A pioneer questionnaire study made in 1917 by the chairman of the Committee on Practice Teaching for Secondary Teachers to determine methods of selecting and supervising practice teachers, revealed (together with other findings) that among 111 teacher-training institutions reporting, 43 required that their student teachers be able to use apparatus (instructional aids), while 40 did not.

In a study made to determine what skills and knowledges are regarded as important for classroom teachers (based on a composite summary of opinions of (a) experienced teachers, (b) audio-visual supervisors, (c) general supervisors, and (d) administrators, DeBernardis

and Brown\textsuperscript{82} found, by means of an audio-Visual Instruc-
tional Aids Check Sheet, that the values presented in
Tables 2 and 3, following, were assigned to the various
aspects of a program of audio-visual education. Since all
respondents were also asked to indicate each item in which
they possessed skill or knowledge, the list of "audio-
visual" items, together with the corresponding weight given
to each item, became in a very real sense both a self-
checking scale as well as a scale of value to neutral
observers.

**TABLE 2**

**AVERAGE RATINGS ON IMPORTANCE OF SKILLS IN USE
OF AUDIO-VISUAL AIDS**

<table>
<thead>
<tr>
<th>Skill</th>
<th>Average Rating on Importance*</th>
<th>Percentage of Respondents Possessing Skill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanics:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Operate 16 mm. silent and</td>
<td>1.6</td>
<td>90</td>
</tr>
<tr>
<td>sound projector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Operate lantern slide</td>
<td>1.8</td>
<td>81</td>
</tr>
<tr>
<td>projector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Set up portable screens</td>
<td>1.9</td>
<td>87</td>
</tr>
<tr>
<td>4. Operate opaque projector</td>
<td>1.9</td>
<td>76</td>
</tr>
</tbody>
</table>

\textsuperscript{82} Amo DeBernardis and James W. Brown, "A Study of
Teacher Skills and Knowledges Necessary for Use of Audio-
Visual Aids," \textit{Elementary School Journal}, XLVI (June, 1946),
pp. 550-556.
**TABLE 2 (continued)**

**AVERAGE RATINGS ON IMPORTANCE OF SKILLS IN USE OF AUDIO-VISUAL AIDS**

<table>
<thead>
<tr>
<th>Skill</th>
<th>Average Rating on Importance*</th>
<th>Average Percentage of Respondents Possessing Skill</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Operate electric phonograph</td>
<td>2.2</td>
<td>81</td>
</tr>
<tr>
<td>6. Operate duplicating equipment (mimeograph, Ditto, etc.)</td>
<td>2.2</td>
<td>81</td>
</tr>
<tr>
<td>7. Operate 35 mm, sound and silent (filmstrip projector)</td>
<td>2.3</td>
<td>56</td>
</tr>
<tr>
<td>8. Change lamps, clean and oil projector</td>
<td>2.4</td>
<td>57</td>
</tr>
<tr>
<td>9. Operate radio receiver</td>
<td>2.6</td>
<td>56</td>
</tr>
<tr>
<td>10. Check and splice 16 mm. film</td>
<td>2.6</td>
<td>54</td>
</tr>
<tr>
<td>11. Operate transcription player</td>
<td>3.1</td>
<td>42</td>
</tr>
<tr>
<td>12. Operate recording machine</td>
<td>3.3</td>
<td>28</td>
</tr>
<tr>
<td>13. Operate public-address system</td>
<td>3.4</td>
<td>45</td>
</tr>
<tr>
<td>14. Operate micro-projector</td>
<td>3.4</td>
<td>20</td>
</tr>
<tr>
<td>15. Operate wire recorder</td>
<td>3.7</td>
<td>11</td>
</tr>
</tbody>
</table>

**Utilization of Visual Aids:**

<table>
<thead>
<tr>
<th>Skill</th>
<th>Average Rating on Importance*</th>
<th>Average Percentage of Respondents Possessing Skill</th>
</tr>
</thead>
<tbody>
<tr>
<td>16. Select appropriate audio-visual material for curriculum</td>
<td>1.1</td>
<td>83</td>
</tr>
<tr>
<td>17. Use films effectively in teaching</td>
<td>1.2</td>
<td>81</td>
</tr>
<tr>
<td>18. Make effective use of exhibit materials in teaching</td>
<td>1.4</td>
<td>77</td>
</tr>
<tr>
<td>19. Use graphic materials effectively in teaching</td>
<td>1.4</td>
<td>75</td>
</tr>
<tr>
<td>20. Use slides and filmstrips effectively in teaching</td>
<td>1.4</td>
<td>73</td>
</tr>
<tr>
<td>21. Evaluate results of use of audio-visual aids</td>
<td>1.6</td>
<td>61</td>
</tr>
<tr>
<td>22. Organize and conduct field trips successfully</td>
<td>1.7</td>
<td>71</td>
</tr>
<tr>
<td>23. Prepare and use study guides for use of audio-visual aids</td>
<td>1.8</td>
<td>59</td>
</tr>
<tr>
<td>Skill</td>
<td>Average Rating on Importance</td>
<td>Average Percentage of Respondents Possessing Skill</td>
</tr>
<tr>
<td>-----------------------------------------------------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>24. Use radio and transcriptions effectively in teaching</td>
<td>1.9</td>
<td>49</td>
</tr>
<tr>
<td>Production of Visual Aids:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25. Mount, file, and classify flat pictures</td>
<td>2.0</td>
<td>74</td>
</tr>
<tr>
<td>26. Prepare maps and charts</td>
<td>2.2</td>
<td>65</td>
</tr>
<tr>
<td>27. Prepare exhibits and dioramas</td>
<td>2.3</td>
<td>57</td>
</tr>
<tr>
<td>28. Make Ditto master-copies</td>
<td>2.5</td>
<td>64</td>
</tr>
<tr>
<td>29. Cut mimeograph stencils</td>
<td>2.8</td>
<td>62</td>
</tr>
<tr>
<td>30. Mount and bind lantern slides</td>
<td>2.8</td>
<td>45</td>
</tr>
<tr>
<td>31. Operate 16 mm. motion picture camera</td>
<td>2.8</td>
<td>44</td>
</tr>
<tr>
<td>32. Make handmade lantern slides</td>
<td>2.8</td>
<td>38</td>
</tr>
<tr>
<td>33. Use various construction materials (wood, plastic, clay, metal)</td>
<td>2.9</td>
<td>37</td>
</tr>
<tr>
<td>34. Use a still camera in making indoor and outdoor pictures in color and in black and white</td>
<td>3.0</td>
<td>59</td>
</tr>
<tr>
<td>35. Edit and splice 16 mm. film</td>
<td>3.1</td>
<td>38</td>
</tr>
<tr>
<td>36. Make photographic lantern slides</td>
<td>3.5</td>
<td>21</td>
</tr>
<tr>
<td>37. Make photographic copies</td>
<td>3.6</td>
<td>23</td>
</tr>
<tr>
<td>38. Develop and print photographic films</td>
<td>3.7</td>
<td>27</td>
</tr>
<tr>
<td>39. Operate enlarger</td>
<td>3.7</td>
<td>27</td>
</tr>
</tbody>
</table>
TABLE 2 (continued)

AVERAGE RATINGS ON IMPORTANCE OF SKILLS IN USE
OF AUDIO-VISUAL AIDS

<table>
<thead>
<tr>
<th>Skill</th>
<th>Average Percentage of Respondents Possessing Skill</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rating on Importance*</td>
</tr>
<tr>
<td>Facilities:</td>
<td></td>
</tr>
<tr>
<td>40. Arrange classroom for best seating</td>
<td>1.4</td>
</tr>
<tr>
<td>41. Select and place screen to fit classroom</td>
<td>1.4</td>
</tr>
<tr>
<td>42. Improve room acoustical conditions within limits of facilities available</td>
<td>1.8</td>
</tr>
</tbody>
</table>

*The ratings are based on the following scale: 1 = of great importance, 2 = of considerable importance, 3 = of average importance, 4 = of importance only in special cases, and 5 = of little importance.

TABLE 3

AVERAGE RATINGS OF IMPORTANCE OF KNOWLEDGE
OF ITEMS IN USE OF AUDIO-VISUAL AIDS

<table>
<thead>
<tr>
<th>Knowledge</th>
<th>Average Percentage of Respondents Possessing Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rating on Importance*</td>
</tr>
<tr>
<td>Mechanics:</td>
<td></td>
</tr>
<tr>
<td>1. Types and advantages of various types of projection equipment</td>
<td>1.9</td>
</tr>
<tr>
<td>2. Fundamentals of sound and silent projector operation</td>
<td>2.0</td>
</tr>
</tbody>
</table>
### TABLE 3 (continued)

**AVERAGE RATINGS OF IMPORTANCE OF KNOWLEDGE OF ITEMS IN USE OF AUDIO-VISUAL AIDS**

<table>
<thead>
<tr>
<th>Knowledge</th>
<th>Average Rating on Importance*</th>
<th>Average Percentage of Respondents Possessing Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Fundamentals of sound recording</td>
<td>2.9</td>
<td>38</td>
</tr>
<tr>
<td>4. Types and advantages of various types of recording equipment</td>
<td>2.9</td>
<td>33</td>
</tr>
<tr>
<td>5. Types and values of various cameras</td>
<td>3.1</td>
<td>29</td>
</tr>
<tr>
<td>6. Fundamentals of optics</td>
<td>3.2</td>
<td>37</td>
</tr>
<tr>
<td><strong>Utilization of visual aids:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Effective utilization techniques</td>
<td>1.3</td>
<td>63</td>
</tr>
<tr>
<td>8. Best aids for specific purposes</td>
<td>1.4</td>
<td>61</td>
</tr>
<tr>
<td>9. Criteria for evaluating effectiveness of audio-visual materials</td>
<td>1.7</td>
<td>64</td>
</tr>
<tr>
<td>10. Psychological basis for use of audio-visual instructional materials</td>
<td>1.7</td>
<td>61</td>
</tr>
<tr>
<td>11. Sources of audio-visual materials and costs</td>
<td>1.8</td>
<td>65</td>
</tr>
<tr>
<td>12. Findings of scientific studies on use of audio-visual instructional materials</td>
<td>2.5</td>
<td>50</td>
</tr>
<tr>
<td><strong>Production of visual aids:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. Types of mounts, sources, and costs</td>
<td>2.5</td>
<td>40</td>
</tr>
<tr>
<td>14. Kinds, sources, costs of construction materials</td>
<td>2.5</td>
<td>37</td>
</tr>
<tr>
<td>15. Basic tool processes for wood, metal, plastic and paper</td>
<td>3.0</td>
<td>22</td>
</tr>
<tr>
<td>16. Fundamentals of photography, lighting, printing, developing, and composition</td>
<td>3.3</td>
<td>28</td>
</tr>
<tr>
<td>17. Types and characteristics of photographic films and papers</td>
<td>3.3</td>
<td>22</td>
</tr>
</tbody>
</table>
TABLE 3 (continued)

AVERAGE RATINGS OF IMPORTANCE OF KNOWLEDGE OF ITEMS IN USE OF AUDIO-VISUAL AIDS

<table>
<thead>
<tr>
<th>Knowledge</th>
<th>Average Rating on Importance</th>
<th>Average Percentage of Respondents Possessing Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facilities:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18. Safety practices to be observed in handling electrical equipment</td>
<td>1.6</td>
<td>52</td>
</tr>
<tr>
<td>19. Methods and materials for darkening rooms</td>
<td>1.8</td>
<td>70</td>
</tr>
<tr>
<td>20. Methods and equipment for ventilating rooms</td>
<td>1.6</td>
<td>48</td>
</tr>
<tr>
<td>21. Methods and conditions for storage of audio-visual materials</td>
<td>2.1</td>
<td>51</td>
</tr>
<tr>
<td>22. Types of screens and advantages of each</td>
<td>2.2</td>
<td>52</td>
</tr>
<tr>
<td>23. Fundamentals of acoustics</td>
<td>2.5</td>
<td>34</td>
</tr>
</tbody>
</table>

*The ratings are based on the same scale as in Table 2.

Based on the foregoing data, the following implications for pre-service and in-service training of teachers were suggested:

1. All items receiving average ratings of 2.9 or higher could be considered the important skills and knowledges for effective use of audio-visual materials. Skills or knowledges for effective ratings below 2.9 while not considered really unnecessary in the use of audio-visual aids, were to be included in in-service or pre-service training of teachers only if time permitted after the more important items were cared for.
2. Instructors conducting in-service or pre-service courses in audio-visual instruction may have class members fill out similar check sheets and use the results as a guide to topics to be covered in the course.

3. The check sheet should be used to obtain from teachers in pre-service or in-service courses a vote on the items or topics to be included in the course and those to be given greatest emphasis.

4. The check sheet could be used as a personal-progress chart by individual teachers to take an inventory of audio-visual instructional skills and knowledge which they possess, thus helping teachers to ascertain their strengths or weaknesses in the use of audio-visual aids.

Certain phases of the National Teacher Examinations, developed by the National Committee on Teacher Examinations of the American Council on Education, might be considered significant for the present study. Some of the areas tested by these examinations which are more or less related to the present attempt to measure teacher understanding in the use of instructional aids in arithmetic were listed by Ryans (Associate Director of the National Committee on Teacher Examinations of the American Council on Education) as follows:

1. Knowledge of educational trends and development.
2. Understanding of child development.
3. Understanding of the learning process.
4. Knowledge of methods and practice of individual and group analysis.

6. Knowledge of appropriate principles and procedures in instruction.
7. Present understanding of teaching fields.
8. Reasoning and problem-solving ability.

Of direct relation to the present study also are the many individual attempts made by instructors, in both arithmetic education courses and in general courses in audio-visual education in colleges of education throughout the country, to construct instruments to determine teacher understanding of the selection and use of instructional aids. While the typical audio-visual education course is rather general in nature, many of the understandings gained in such courses may be applicable to arithmetic education.

**Chapter Summary.** In Chapter II the writer attempted to review the literature relating to the several major aspects of the subject under study. First, experimental studies and a few points of view were presented relating to the effectiveness of instructional aids in general and in arithmetic instruction specifically, in relation to the learning process, with specific reference to the following classifications of instructional aids: (a) direct experiences, (b) manipulative aids, (c) field trips or excursions, (d) pictorial aids, (e) graphic aids, (f) audio-aids, and (g) abstract aids.

Second, studies were presented in which educators and psychologists attempted to devise instruments with which to
measure "general insight," and studies in which educators attempted to devise instruments to measure mathematical understandings possessed by both pupils and teachers.

Third, although no studies were found in which specific attempts were made to measure teacher understanding of the use of instructional aids, a few studies of a related nature were presented.
CHAPTER III

A SEARCH FOR AN ADEQUATE ROLE OF INSTRUCTIONAL AIDS IN ARITHMETIC EDUCATION

Plan of the Chapter. The primary aim of this chapter is to attempt to determine the legitimate role of instructional aids in arithmetic education. It is evident that this part of the study is basic to the rest. Unless it can be shown that the use of instructional aids has a legitimate place in the teaching of arithmetic, there is certainly no point in carrying out the remainder of the investigation, that is in being concerned with whether prospective teachers possess an understanding of such aids in the program of elementary-school arithmetic.

The problem of ascertaining the role of instructional aids in arithmetic instruction is approached philosophically by first presenting points of view of various commissions, committees, and other organizations, as well as those of various individual educators, with regard to the general objectives of the elementary school in the United States. Next, points of view of various commissions, committees, and other organizations, as well as those of various individual educators, are stated with respect to the specific objectives of the elementary-school arithmetic.
program. From the many statements of the various educational organizations and individual educators, an attempt is made by the writer to arrive at an acceptable list of objectives for the elementary school and for a program of elementary-school arithmetic.

The writer next presents his findings with respect to psychological bases for the use of instructional aids in arithmetic education. The nature of the learning process is discussed, and out of this discussion implications are made relative to the advisability of using or not using arithmetic instructional aids. It is hoped that the writer's analysis of the nature of the learning process throws some light on the question of the relationship between the various classifications of instructional aids and the different operational levels of learning. Considerable treatment is given to an attempt to consolidate some of the philosophical points of view and the psychological findings for the purpose of determining more specifically the role of instructional aids in arithmetic education.

A portion of the chapter is devoted to various classifications of instructional aids, as designated by divers educators, and examples of arithmetic instructional aids representative of such classifications. The purpose is to aid the reader in gearing the use of various classifications
of aids to operational levels of individual pupils. Next, a section is devoted to findings and points of view relative to the advantages, as well as the disadvantages, attending the use of instructional aids. This section is somewhat related to an earlier section in Chapter II which deals with the literature regarding the effectiveness of instructional aids. However, the present emphasis is more of a general nature than is the earlier treatment. Finally, suggestive lessons involving specific developmental concepts of arithmetic are presented in which various instructional aids are used at different operational levels of learning.

The Objectives of the Elementary School

From time to time in the history of American education certain important national committees, commissions, and other bodies have made pronouncements that have been widely accepted as bases for educational planning at various levels. An example of such a widely-accepted statement of objectives is that of the Commission on the Reorganization of Secondary Education which in 1918 issued the report, the "Cardinal Principles of Secondary Education," which gained national acceptance as a statement of the aims of not only secondary education but of elementary education as well. These aims, to which both the secondary and the elementary schools increasingly directed their programs, were:
The pronouncement of the Commission on the Reorganization of Secondary Education was not the first instance of attempts made by educational bodies to define the function of the school. It is reasonable to believe that from the earliest days of formal schooling, man has been concerned with the functions the school would serve. Otto\(^2\) stated:

Since education cannot proceed intelligently in any country unless its purposes, its objectives, the direction in which it is going, have been clearly defined, and, since the question of clearly defined purposes of education is particularly difficult in a complex democratic society, it is not surprising that the problem of general purposes in education has received much attention in the professional literature of this country.

The functions of the elementary school (as well as those of the secondary school) were considered by both the Committee of Ten and the Committee of Fifteen.\(^3\) In the early part of the twentieth century, the desire of educators

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2 Henry J. Otto, Elementary School Organization and Administration, p. 63.

to define the aims of the elementary school, as well as to economize time in the accomplishment of these aims led to the appointment in 1911 by the Department of Superintendence of the National Education Association of a Committee on Economy of Time in Education.\(^4\) This Committee made four reports which were embodied in yearbooks of the National Society for the Study of Education. Concerning these reports, Wilson\(^5\) maintained that educational leaders were agreed on the following aims: "To insure that all children acquire those habits, skills, knowledges, ideals, and prejudices which should be the property of all citizens in our democracy."

Under the leadership of Morrison,\(^6\) a committee of supervisors and teachers in 1933 formulated the cardinal objectives for a modern and progressive statewide educational program. The cardinal objectives indicated that it is the function of the elementary school to help each child to: (a) understand and practice desirable individual aptitudes; (b) discover and develop his own desirable


\(^6\) Committee on Elementary Education of the New York Council of Superintendents, *Cardinal Objectives in Elementary Education*. 
individual aptitudes; (c) cultivate the habit of critical thinking; (d) appreciate and desire worthwhile activities; (e) gain command of the common integrating knowledge and skills; and (f) develop a sound body and normal mental attitudes.

While the foregoing objectives were the work of only a state committee, the writer feels that they reflected the contemporary educational thought of the nation with regard to the elementary school. Too, this set of objectives reveals the continued effect of the earlier "Seven Cardinal Principles of Education."

In 1938 the Educational Policies Commission⁷ defined the general end of education as, "the fullest possible development of the individual within the framework of our present industrialized democratic society." The objectives of education set forth by the Commission are as follows:

I. The Objectives of Self-Realization, which are centered on the personal development, growth, and learning of the individual.

a. The Inquiring Mind. The educated person has an appetite for learning.

b. Speech. The educated person can speak the mother tongue clearly.

c. Reading. The educated person reads the mother tongue efficiently.

d. **Writing.** The educated person writes the mother tongue effectively.

e. **Number.** The educated person solves his problems of counting and calculating.

f. **Sight and Hearing.** The educated person is skilled in listening and observing.

g. **Health Knowledge.** The educated person understands the basic facts concerning health and disease.

h. **Health Habits.** The educated person protects his own health and that of his dependents.

i. **Public Health.** The educated person works to improve the health of the community.

j. **Recreation.** The educated person is participant and spectator in many sports and other pastimes.

k. **Intellectual Interests.** The educated person has mental resources for the use of leisure.

l. **Esthetic Interests.** The educated person appreciates beauty.

m. **Character.** The educated person gives responsible direction to his own life.

II. **The Objectives of Human Relationship,** which are concerned with the relationships of the individual with his home and family neighbors, and community.

a. **Respect for Humanity.** The educated person puts human relationships first.

b. **Friendships.** The educated person enjoys a rich, sincere, and varied social life.

c. **Cooperation.** The educated person can work and play with others.

d. **Courtesy.** The educated person observes the amenities of social behavior.
e. **Appreciation of the Home.** The educated person appreciates the family as a social institution.

f. **Conservation of the Home.** The educated person conserves family ideals.

g. **Home-making.** The educated person is skilled in home-making.

h. **Democracy in the Home.** The educated person maintains democratic family relationships.

### III. The Objectives of Economic Efficiency, which include the education of the individual both as a producer and a consumer, as well as an investor.

a. **Work.** The educated producer knows the satisfaction of good workmanship.

b. **Occupational Information.** The educated producer understands the requirements and opportunities for various jobs.

c. **Occupational Choice.** The educated producer has selected his occupation.

d. **Occupational Efficiency.** The educated producer succeeds in his chosen vocation.

e. **Occupational Adjustment.** The educated producer maintains and improves his efficiency.

f. **Occupational Appreciation.** The educated producer appreciates the social value of his work.

g. **Personal Economics.** The educated consumer plans the economics of his own life.

h. **Consumer Judgment.** The educated consumer develops standards for guiding his expenditures.

i. **Efficiency in Buying.** The educated consumer is an informed and skillful buyer.

j. **Consumer Protection.** The educated consumer takes appropriate measures to safeguard his interests.
IV. The Objectives of Civic Responsibility, involving the individual's relationships with his government and with the people of other nations.

a. Social Justice. The educated citizen is sensitive to the disparities of human circumstance.

b. Social Activity. The educated citizen acts to correct unsatisfactory conditions.

c. Social Understanding. The educated citizen seeks to understand social structures and social processes.

d. Critical Judgment. The educated citizen has defenses against propaganda.

e. Tolerance. The educated citizen respects honest differences of opinion.

f. Conservation. The educated citizen has a regard for the nation's resources.

g. Social Applications of science. The educated citizen measures scientific advance by its contributions to the general welfare.

h. World Citizenship. The educated citizen is a cooperating member of the world community.

i. Law Observance. The educated citizen respects the law.

j. Economic Literacy. The educated citizen is economically literate.

k. Political Citizenship. The educated citizen accepts his civic duties.

l. Devotion to Democracy. The educated citizen acts upon an unswerving loyalty to democratic ideals.

The most recent general pronouncement of elementary-school objectives was made by the Mid-Century Committee on Outcomes in Elementary Education, the report being prepared
by Nolan C. Kearney. The "Kearney Report" presents the specific objectives of elementary education as outlined by a distinguished group of consultants and evaluated by carefully selected critics. The Report states the problem of elementary-school objectives in the following manner:

Why do we send our children to school? What do we expect them to learn in school? What do they learn?

We Americans spend a great deal of effort and money on education. It is our second largest social enterprise. Yet, when we ask each other what we expect our young people to gain from education, particularly that part of schooling which is not directly preparation for a job, our answers are often vague and fuzzy. What do we expect children to learn in school—in the elementary school, for instance? Some interested people say that the elementary schools should limit their goals to learning of "the fundamentals," but what are the fundamentals to be learned by children in our time?

Clear-cut and definitive answers to these questions would be helpful to almost everyone concerned with education, including persons whose job it is to devise ways of measuring educational growth, but, since most of the questions do not have clear-cut or definitive answers, the next best thing is a summary of what competent educators and informed citizens think the goals of education ought to be. This book reports, and provides the setting for, just such a summary.

Accordingly, the "Kearney Report" has outlined the objectives of the elementary school in terms of a grid. Objectives are expressed in terms of behavioral changes in:

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8 Nolan C. Kearney, Elementary School Objectives.

9 Ibid., p. 13.
(1) knowledge and understanding, (2) skill and competence, 
(3) attitude and interest, and (4) action pattern as 
applied to the curriculum areas of: (a) physical develop­
ment, health, and body care, (b) individual social and 
emotional development, (c) ethical behavior, standards, 
and values, (d) social relations, (e) the social world, (f) 
the physical world, (g) esthetic development, (h) communi­
cation, and (i) quantitative relationships. Cross­
sections of the educational grid are made at the end of the 
primary level (third grade—age 9); at the end of the 
intermediate level (sixth grade—age 12), and at the upper 
grade level (ninth grade—age 15) in terms of the expected 
outcomes of the "average child" at those points.

In answer to the criticisms of many consultants, 
critics, and others who felt that the goals of the "Kearney 
Report" are set too high, Kearney replied:

It is difficult in a statement of goals to 
distinguish among those that are now being achieved, 
those that are possible of achievement though not 
now being achieved, and those that it would be 
desirable to achieve though we are not sure of the 
difficulties that may stand in the way. We know, of 
course, that some children have very limited abili­
ties and that goal-achievement for them will be 
limited in consequence. Other children will be 
highly intelligent and even the most difficult 
goals will be attained. It is in determining what 
average children may be expected to do in the 
achievement of goals that difficulties arise.11

10 Ibid., pp. 38 and 40.
11 Ibid., p. 46.
Kearney\footnote{12} reminded readers that the idea is stressed in the report again and again that each child is entitled to conditions under which he can reach his optimum development, rather than the thought that all children of a specific grade should be expected to master certain skills, memorize certain facts, read certain books, spell certain words, or compute with certain symbols.

In addition to pronouncements of objectives made by commissions, committees, and other bodies, several such pertinent statements have been made by educators throughout the history of education. Herbert Spencer\footnote{13} declared in 1859 that the purpose of education is to "prepare us for complete living." Herbart\footnote{14} stressed the development of "many-sided interests, so that the individual might go on into the world equipped with interests and abilities in many lines." To Dewey\footnote{15} the aim of education was:

\begin{quote}
... to secure a progressive development of capacities, having due regard for individual
\end{quote}


\footnote{13} Cubberley, \textit{op. cit.}, p. 777.


\footnote{15} John Dewey, "The Duties and Responsibilities of the Teaching Profession," School and Society, XXXII (August 9, 1930), pp. 188-191.
differences, and including a physical basis of vigorous health, refined esthetic taste, power to make worth-while use of leisure, ability to think independently and critically, together with command of the tools and processes that give access to the accumulated products of past cultures; on the social side, this personal development is to be such as will give desire and power to share in cooperative democratic living including political citizenship, vocational efficiency, and social good-will.

H. A. Overstreet,16 Professor of Philosophy, College of the City of New York, stated that the specific objective of education is:

To arouse interest in and understanding of the factual world, including the student himself; to arouse interest in and understanding of the potential world, including the student himself; to equip the student with the tools of communication, exploration and invention; to develop habits of initiative and self-help; to awaken a thinking consideration of conflicting points of view; to develop a habit of suspending judgment until the facts are in.

The objectives of education, as stated by Washburne,17 were:

1. To develop in every child physical and emotional well being.

2. To guide every child to a personally and socially satisfying fulfillment of his own individual development pattern.

16 Beulah Amidon, Democracy's Challenge to Education, p. 28.

17 Ibid., pp. 31-32.
3. To help every child to acquire mastery of the means of social intercourse, means which include the functional parts of the three R's, and the most commonly used knowledge and concepts of our culture.

4. To give to every child a realization that his well-being is inextricably bound up with the well-being of his community, his state, his nation, and humanity the world over.

Koehler maintained that in general there are three purposes of education:

1. It aims to give young men and women the sort of training which will enable them to become self-supporting.

2. To develop the personality of the individual.

3. The social purpose. Public education is the process of training in good citizenship.

What do all these statements of the objectives of education "add up to?" One logical conclusion certainly is that to define the objectives of education in a democracy is much more difficult than in a totalitarian state. One would unquestionably have to agree with Barr, Burton, and Brueckner, that it is next to impossible to summarize in a single, all-inclusive statement the general purposes of education, that better direction can ordinarily be secured


19 A. S. Barr, William H. Burton, and Leo J. Brueckner, Supervision, p. 163.
from short lists of statements. Of a number of statements of the objectives of education, the following concepts were given emphasis:

1. Developing, equipping
2. Self-sustained persons
3. Individual mind and character
4. Individual responsibility
5. Habits of tolerance
6. Every child
7. Making democracy work
8. Intelligent participation
9. The right method
10. Understanding, appreciation, knowledge, ideals
11. Freedom from class stratification
12. To make free men
13. Civic responsibility

A further observation which might be made from the foregoing statements of objectives of education is that, although the writer referred originally to objectives of the elementary school, the discussion persisted in reverting to a treatment of general objectives of education. Otto explains this trend as follows:

Although the elementary school, the high school, and the college were each originally established for its own particular purpose and with little if any thought of having each of these units contribute its share to a common goal, yet in the process of time something like a common purpose has emerged.

Otto goes on to say that there is a decided trend in recent years toward the belief that education at all levels

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20 Amidon, op. cit., pp. 28-38, passim.
should concern itself with the wholesome growth and
development of each pupil, thus giving greater communality
of focus to elementary and secondary education. In other
words the elementary school does not have any unique func­
tions. Its objectives lie within the framework of general
purposes of education in American democracy. The general
goals toward which the elementary school should strive are
the general purposes of education. Its uniqueness consists
of its curriculum, method, and organization because it is
dealing with a particular age-group of children, rather
than in uniqueness of objectives. With regard to the
communality of educational objectives, the basic implica­
tions are for a more closely integrated and articulated
program of education, rather than one segmented by separate
purposes for each respective type of vertical school
organization.

Objectives of the Elementary-School Arithmetic Program

During the colonial period, and until about 1625, the
aim of arithmetic education was utilitarian. Most problems
related to trade and commerce, emphasis being placed on
"commercial arithmetic."22 Starting about 1821, due

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22 Guy M. Wilson, "Arithmetic," Encyclopedia of
Educational Research, p. 44.
largely to the influence of Warren Colburn (who himself was influenced by Pestalozzi), the practical commercial viewpoint was abandoned and discipline of the mind became the chief aim of the subject, great emphasis being placed on mental arithmetic. Starting about the beginning of the twentieth century, the aim of arithmetic education began to shift to social usage. Frank M. McMurry proposed that content in arithmetic be restricted to what can "be shown to have a plain relation to some real need of life."24

The Commission on Postwar Plans,25 created by the Board of Directors of the National Council of Teachers of Mathematics, stated in 1945 that we must conceive of arithmetic as having both a mathematical aim and a social aim, but that the fundamental reason for teaching arithmetic is represented in the social aim. If arithmetic does not contribute to more effective living, it has no place in the elementary curriculum. The Commission reminded, however, that it is not a matter of having to choose between the mathematical aim and the social aim—rather, that both aims

23 Ibid., p. 45.
24 Ibid.
must be realized. A number of theses were presented by the Commission for improving the mathematics programs in grades 1 to 14. The first of these theses, which are presented below, is applicable to all the grades mentioned above, whereas the others are specifically applicable to grades 1 to 6 inclusive.

Thesis 1. The school should guarantee functional competence in mathematics to all who can possibly achieve it.

Thesis 2. We must discard once for all the conception of arithmetic as a mere tool subject.

Thesis 3. We must conceive of arithmetic as having both a mathematical aim and a social aim.

Thesis 4. We must give more emphasis and much more careful attention to the development of meanings.

Thesis 5. We must abandon the idea that arithmetic can be taught incidentally or informally.

Thesis 6. Readiness for learning arithmetic ideas and skills is primarily the product of relevant experience, not the effect of becoming older.

Thesis 7. We must learn to administer drill (repetitive practice) much more wisely.

Thesis 8. We must evaluate learning in arithmetic more comprehensively than is the common practice.

The Committee for Part II of the Fiftieth Yearbook of the National Society for the Study of Education expressed their belief that competence in quantitative

26 The Teaching of Arithmetic, op. cit., p. 3.
thinking is of first-order importance in education and that it warrants purposeful teaching of the most effective type that can be devised. They, too, however, expressed belief in a second objective of arithmetic education, relating to the fact that arithmetic is a product of thought and is concerned with the mathematics of social thinking. Furthermore, they also reminded that there is no dichotomy between mathematical and social arithmetic, rather that they are two aspects of the same thing.

More specifically, the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, on the Place of Mathematics in the Secondary Schools, stated:

The mathematics program of the elementary schools is the indispensable foundation of all the pupils' later mathematical work. If that foundation is weak, the pupil's subsequent progress is likely to be permanently handicapped. . . . The following attainments may be regarded as normal mathematical equipment of the American pupil who has satisfactorily completed the work of the sixth grade.

1. A familiarity with the basic concepts, processes, and the vocabulary of arithmetic.

2. Understanding of the significance of the different positions that a given digit may occupy in a number, including the case of a decimal fraction.

3. A mastery of the basic number combinations in addition, subtraction, multiplication, and division.
4. Reasonable skill in computing with integers, common fractions, and decimal fractions.

5. An acquaintance with the principal units of measurements, and their use in everyday life situations.

6. The ability to solve problems involving computation and units of measurement.

7. The ability to recognize, to name, and to sketch such figures as the rectangle, the square, the circle, the triangle, the rectangular solid, the sphere, the cylinder, and the cube.

8. The habit of estimating and checking results.\(^{27}\)

The Mid-century Committee on Outcomes in Elementary Education\(^{28}\) outlined the objectives of arithmetic in terms of behavioral changes in: (1) knowledge and understandings, (2) skill and competence, (3) attitude and interest, and (4) action pattern in regard to quantitative relationships. Emphasis is placed on giving the child an understanding of how our number system works and why, so that he will have greater competence in using numbers. Since mathematics is the language of quantity, it could be included as another means of communication, but it is so important and specialized that it is considered separately.


\(^{28}\) Kearney, op. cit., pp. 38 and 40.
Emphasis is also placed upon the ability to analyze and solve problems on the basis of the particular problem, the information needed to solve it, and how to get the information.

In addition to mathematics commissions, committees, and other such bodies, a number of individual educators have from time to time stated personal points of view relative to the objectives of the elementary-school program of arithmetic. In 1948, Thiele and Brueckner,29 reporting for the New York Regent's Inquiry, stated that an up-to-date program of arithmetic education has the following four aims, in addition to the common objectives of all education:

1. To help students assimilate a body of information that will give an understanding and appreciation of social application of numbers in affairs of life.

2. To make pupils aware of ways in which numbers have facilitated human progress and of problems now faced by institutions thus developed, in short, the sociology of numbers.

3. To develop in pupils resourcefulness and ingenuity in dealing with numbers in social situations they encounter, and efficient modes of quantitative thinking.

4. To develop essential skill in computation and number manipulation.

Moser, Kinney, and Purdy\textsuperscript{30} concluded that the primary purpose of all mathematical instruction is to help children attain growth in the ability to think quantitatively, this ability depending upon:

1. An understanding of the nature of the number system and especially of principles and relationships which serve to make it an instrument for quantitative thinking.

2. A command of number symbolism—the language of mathematics.

3. A command of the fundamental processes.

4. The ability to correlate grouping arrangements described for specific social situations with the principles of mathematical grouping studied earlier.

Brueckner\textsuperscript{31} stated that in the modern arithmetic program two major objectives are recognized, namely, (a) the mathematical phase (the development of the ability of the learner to perform the various number operations intelligently and skillfully), and (b) the social phase (the development of the ability of the learner to apply quantitative procedures effectively in the social situations encountered in life outside the school).


Bathurst expressed the point of view that from the first grade through the sixth or eighth grade there are three major objectives in helping boys and girls use arithmetic:

1. To help children understand the number concepts in daily living.

2. To teach children to compute or figure with the skill required in their everyday activities.

3. To help children improve their ability to recognize in their life problems the situations that require computation or other use of number.

According to Brownell, "The only sound justification for giving arithmetic a place in the elementary curriculum is that it contributes directly to more effective, more intelligent, and more complete living." He agrees that arithmetic has a mathematical as well as social aim, but warns that we cannot emphasize one to the exclusion of the other; rather, both are essential to a functional curriculum in arithmetic. Buckingham, who holds a similar point of view, remarked:

The teacher who emphasizes the social aspect of arithmetic does well. But in the sense in which

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the term is here used, he is not teaching the meaning of arithmetic. He may, and indeed he should, use a socially significant approach, but his teaching of a given unit is not complete until the goal of mathematically meaningful ideas has been reached . . . . We must, therefore, do two things. We must teach arithmetic as a social study, and we must teach it as mathematics.

Clark\(^{35}\) declared that the overall objective of teaching arithmetic is to develop the ability to think one's way through arithmetic problem situations— in brief, growth in power to do quantitative thinking— and to develop confidence in the pupil to solve problems.

Harding\(^{36}\) maintains that the modern school's responsibility for instruction in arithmetic goes beyond the traditional objectives of skill in computation and formal procedures in solving abstract problems. Emphasized is the conviction that in addition to mechanical accuracy in routine operations, goals include adaptation or transfer of principles, generalizations, and individualized application. He remarked:

> Accepting Bode's definition of learning as "the finding and testing of meaning in experience," objectives of arithmetic instruction have been broadened in many schools. In such schools, the purposes of teaching arithmetic include helping


\(^{36}\) Lowry W. Harding, Functional Arithmetic; Photographic Interpretations, p. 8.
children understand the uses of number to:
(1) record accurately the quantitative rela-
tionships of objects; (2) communicate ideas and
facts about quantity; (3) aid in understanding
fields of knowledge such as science, social
studies, art, music, and health; (4) classify,
organize, and record results of research and
other work; (5) help in gaining understanding of
present culture through the history of number
and measurement in earlier civilizations.

This view of the objectives of arithmetic instruction,
which Harding refers to as the "functional-meaning theory,"
places emphasis upon the function of number as a means of
teaching its structure, mathematical meaning, and use
through an indirect approach. In terms of specific
objectives of arithmetic education, Harding has listed the
following developmental concepts as the major arithmetic
topics in the elementary school:

1. Approximation or estimation
2. Average
3. Counting and ranking
4. Formulas and algebraic symbolization
5. Fractions, common
6. Fractions, decimal
7. Geometric form
8. Graphic representation
9. Grouping and number combinations
10. Identification: meaning of number symbols
11. Measurement: linear, quantity, temperature, time
12. Money and business practice
13. Number system organization, serial idea, decimal
nature
14. Operations: addition, subtraction, multiplication, division
15. Percentage
16. Place value and face value

37 Ibid., p. 28.
17. Problem solving
18. Proof and generalization
19. Quantitative thinking: mathematical judgment
20. Ratio and relationship
21. Reading and writing of number ideas
22. Vocabulary of number and quantity
23. Zero representation

Professor Harding has kindly granted the writer permission to use the foregoing list of developmental concepts as the mathematical basis of the present study. Just as there is no one general understanding possessed by the learner which is applicable to all concepts or processes of arithmetic, likewise, there is probably no one instructional aid which is effective in promoting understanding of all arithmetic concepts or processes. Accordingly, there can be no such thing as a prospective-teacher understanding of the use of arithmetic instructional aids, "in general." Rather, it is necessary to consider arithmetic, arithmetic instructional aids, and understanding of the use of instructional aids in arithmetic in terms of specific objectives. For example, the prospective teacher might very conceivably possess an understanding of the use of instructional aids in the concept of "counting" and yet prove to be extremely naive in using instructional aids to promote pupil understanding of "common fractions." Accordingly, the above list of arithmetic developmental concepts

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38 Ibid., pp. 44 ff.
furnishes an excellent component inventory of specific objectives of the whole of the elementary-school arithmetic program. The matter of determining prospective-teacher understanding of the use of instructional aids with specific reference to the twenty-three developmental concepts of arithmetic is considered in detail in Chapter IV. It probably should be stated at this point that, since learning is looked upon as being developmental in nature, none of the foregoing twenty-three concepts is regarded as being peculiar to a specific grade-level. It is not a matter of having some of the topics presented in one grade, followed by other topics in succeeding grades. Rather, all twenty-three topics are presented in all the elementary grades. For instance, the concept "approximation or estimation" will be presented in the kindergarten; it will also be presented in the sixth grade, but obviously on a higher level of operation.

What conclusions may be reached in view of the foregoing statements of objectives of the elementary-school arithmetic program? In summing up the statements of overall objectives of the elementary school, it was emphasized that there are no objectives of education which are unique to the elementary school. Rather, the elementary school is one of three or four educational institutions working toward common objectives. This concept of "wholeness" is
equally applicable to the elementary-school arithmetic program. The point of view commonly expressed today is that the mathematics program cannot live apart to itself. Caswell and Foshay\textsuperscript{39} maintained that

A teacher cannot teach arithmetic alone, or spelling, or reading. Subject matter cannot be considered apart from children, and in every experience the whole child is affected. A teacher may ignore attitudes, effects on character, and the like, when teaching arithmetic, but the effects are there nevertheless.

In the same vein of thought Brueckner and Grossnickle\textsuperscript{40} remarked that the teacher should recognize the possible contributions instruction in arithmetic can make to the social objectives of all education. They pointed out the fact so commonly overlooked by the classroom teacher that

Many of the experiences pupils have in school that are rich in application of number can be designed as experiences in democratic living. Here the teacher can so conduct the learning program that intelligence . . . forms the basis of action. Actual practice in solving problems of daily life that are of concern to the pupils is a most valuable type of experience in democratic living. In most instances arithmetic makes valuable contributions to these experiences.

\textsuperscript{39} Hollis L. Caswell and Wellesley A. Foshay, \textit{Education in the Elementary School}, p. 37.

\textsuperscript{40} Leo J. Brueckner and Foster E. Grossnickle, \textit{How to Make Arithmetic Meaningful}, p. 3.
Psychological Bases for the Use of Instructional Aids in Arithmetic Education

In the foregoing portion of the present chapter the writer has attempted to present some philosophical bases for elementary education and for the elementary-school arithmetic program in particular. Now the matter of the psychological nature of learning will be considered. As was stated previously, these two sections dealing with the philosophical and psychological bases of education, of arithmetic instruction, and of the use of instructional aids in arithmetic education are basic to the remainder of the study.

Learning has been defined in an earlier chapter as a change in behavior in the learner. In a broad sense, learning may be considered as a process of adaptation, the acquiring of new ways of behaving or performing in order to make better adjustments to the demands of life. While granted that there is no complete consensus among psychologists and educators as to how children learn, there does seem to be rather general agreement relative to several principles of learning. Brown mentions the following as


being among the factors of learning most commonly agreed upon: (a) Readiness is a factor in learning; (b) learning proceeds from the familiar to the new; (c) learning is the result of experience; and (d) learning is developmental.

Readiness as a Factor in Learning. Readiness is generally conceded to be of a specific nature. There is, for example, presumably a readiness stage for learning to count meaningfully, a readiness state for grouping, for working with common fractions, for making approximations—a state of readiness for each of the twenty-three basic developmental concepts of arithmetic. Woody defined educational readiness as the preparation which an instructor consciously makes in getting the student ready to learn the things to be taught. In an arithmetic readiness program experiences are provided in which the pupils contact informally and concretely, processes and concepts that are later to be taught more systematically. Furthermore, readiness to learn a given concept is related to a variety of factors such as maturation, experience, and emotional and personal adjustment. According to Hilgard, readiness has three chief components: (a) sufficient physiological


44 Ernest R. Hilgard, Introduction to Psychology, p. 252.
maturity, (b) appropriate preparatory experiences, and (c) aroused interest or the desire to learn. If a pupil lacks any one of the components, his learning will not go on efficiently. Thus, a pupil is not ready to add numbers until he is old enough, until he has built up an adequate arithmetical background, and until he desires to add numbers.

Arithmetic readiness is not limited to primary children. It operates throughout life and at all stages of the learning process. College prerequisite courses furnish one common example. The college student's readiness to study engineering depends partly upon appropriate preparatory experiences in mathematics and upon aroused interest.

As was mentioned above, one of the major factors influencing readiness to learn is maturation, or genetically determined growth—growth which "takes place without express efforts to promote it or even in fact of efforts to prevent it." Maturation and learning operate as dual forces in almost all cases of behavior. Although the two may be separated in idea, "they can not be separated in actual performance any more than we can separate the speed


of a pitched ball from its direction, though we can cer­
tainly distinguish them.\textsuperscript{47} Because of the interdepend­
ence of maturation and learning, learning cannot transcend
maturation.\textsuperscript{48} A child who has not reached a sufficient
stage of mental and physical development cannot perform
school tasks which require a higher level of development.

A second major factor mentioned as basic to readiness
to learn is the child's previous experience. As Blair\textsuperscript{49}
stated, "The whole program of prerequisite courses and
sequences of learning are predicated upon the assumption
that basic skills are necessary before complex tasks are
tackled." The child is not ready to work with quantitative
sums until his previous experience has provided him a
proper background for thinking out and interpreting such
concepts. However, background experiences do not guarantee
readiness to learn. Such experiences tend to promote
readiness only if they are meaningful to the learner. Such
experiences must be intellectualized, rather than merely
carried out in a routine mechanical manner, if they are to
promote readiness.

A third big factor basic to the readiness of the child

\textsuperscript{47} English, \textit{loc. cit.}

\textsuperscript{48} Blair, Jones, and Simpson, \textit{loc. cit.}

\textsuperscript{49} Ibid., p. 120.
to learn is his emotional attitude and personal adjustment. Many children who have sufficient maturation and background experiences are yet not in a state of readiness for learning specific concepts because of such factors as emotional blocks brought on by any number of causes, such as home pressures, earlier failures, or other factors which tend to promote a general disinterest in the topic under study. Furthermore, the child must have his interest aroused—must have the desire to learn. As stated by Harding, \(^50\) "... You can lead a horse to water, but you can't make him drink—if you want to be sure that he drinks you must first manage to get him thirsty... ." 

With specific reference to readiness, one commonly expressed statement needs clarification. It is rather customary to hear teachers remark that such-and-such a child is not ready to work with numbers. What is generally meant is that the child is not ready to work with abstract numbers. The modern elementary school takes the view that pupils of any school age or level are ready to work with arithmetic concepts. Obviously beginners are able to work with such concepts only on a much lower level of operation than after having had more gainful experiences and attained greater maturity. Some pupils will be found ready to

\(^50\) Harding, Functional Arithmetic: Photographic Interpretations, op. cit., p. 29.
operate with a given concept at one level, while others will be capable of working at varying higher and lower levels. One of the basic jobs of the teacher is to ascertain by appropriate methods and instruments the varying readiness levels of her respective pupils in specific concepts and processes.

Building a Background of Experiences. A second basic aspect of the learning process has been referred to as building up a background of experiences. This feature of learning is dependent upon how the learner has perceived the world about him and at the same time is basic to how he will perceive the world about him in the future. That is, background experiences are both dependent upon and basic to perception. By definition, perception is the act of perceiving the world about us. It is the process by which one observes the world of objects and other people. According to Hilgard, no one sees the world exactly as it is; no two persons see it exactly alike; each individual's perception depends partly on what he himself is. What each individual perceives is influenced by his past experiences as well as, to an extent, by what he wants to see. Perception, obviously, is essential to learning, motivation, and thinking. Furthermore, while perception is a function of

51 Ernest R. Hilgard, Introduction to Psychology, p. 281.
the sense organs, it cannot be related definitely to one specific sense organ. The sense organs always function in a mutually dependent fashion, each perception of a single stimulus involving a response of the total organism. When the eye perceives an object, when the taste buds are stimulated, when the ear perceives a sound, for instance, a whole chain of reactions are "set off" in the body. Wheeler and Perkins\textsuperscript{52} stated: "Because objects are presented to one of the so-called senses, say hearing, is no guarantee that the child's awareness will involve only auditory processes." Several sense organs commonly contribute to a total perceptual experience. In other words, a deaf person would not perceive an atomic explosion in the same manner as a person of normal hearing. The blind men of Indostan did not "see" the elephant as people of normal vision were accustomed to doing. To the person whose sense of smell has been impaired, food does not taste the same as to a person with a normal sense of smell.

Furthermore, even though different persons do possess normal senses, their perceptions of the world about them still tend to be different. As explained by Blair, Jones, and Simpson,\textsuperscript{53} "Although everyone is equipped with an

\textsuperscript{52} Wheeler and Perkins, \textit{op. cit.}, p. 147.

\textsuperscript{53} Blair, Jones, and Simpson, \textit{op. cit.}, p. 107.
elaborate sensory system which orients him to his environment, this system operates in a manner dictated not altogether by its structure, but as a result of experience. The way we see, feel and hear the world in which we live is largely a matter of learning." Wheeler and Perkins reminded us that

This situation is not so surprising when we reflect that the organism responds as a whole even to so-called isolated stimulations impinging separately upon the nervous system through the eye, ear, and other organs of sense. The ear is necessary but not sufficient in learning. The ear alone does not determine what the nature of the auditory experience will be. A field of experience and a total nervous system are also necessary. Not only that, but the quality that emerges from a ground . . . must have a field property of its own.54

The Formation of Concepts. According to Hilgard,55 the starting point for perception is the ability to discriminate, first in regard to differences, secondly in regard to similarities. He alleged:

Were we unable to discriminate among stimuli we could not find our way about, we could not tell the edible from the inedible, we could not control temperature, we could not read. Hence perception is closely related to action. . . . Hence perception is essential to learning, motivation and thinking.

It is rather commonly agreed that much of what is called learning really involves a change in ways of looking

54 Wheeler and Perkins, _op. cit._, p. 140.

55 Hilgard, _Introduction to Psychology, op. cit._, p. 281.
at one's environment. Additional background experiences tend to cause the learner to become more discriminative. That is, he tends to become better able to point out similarities and differences in objects perceived. Perception (using the various senses) continues until in time a background of experiences is built up, enabling the child to form judgments or to recognize differences in objects, and still later to recognize similarities between objects. As the background of experiences is built up, the learner in time will become proficient in recognizing the relation between the whole of a situation and its parts as well as the interrelationship of the parts. In other words, the learner has formed a concept, a generalized abstraction. He has discovered a property common to all objects in a specific category. Examples of concepts are rules, principles, definitions, axioms, and scientific laws. According to Wheeler and Perkins, 56 "Judgments are recognitions of concrete relations between specific objects while concepts are recognitions of abstract relations."

The value of concepts becomes evident at once when one realizes that without the ability to form concepts (generalized abstractions) the learner would have to face each new situation anew. He would have to resort to the

use of concrete instructional aids in solving every problem. However, by responding to various objects as both similar and different, the learner is able to organize his environment into meaningful categories. The learning of concepts, then, requires a reorganization of already existing ideas and experiences.

A further important aspect to note in the formation of concepts is: "Conceptualization is not an all or none proposition."57 In other words, the kindergarten child may have concepts in regard to counting; likewise, the college student will have concepts of counting. This modern view of the nature of concepts discredits the not-so-modern practice of waiting until the fourth grade to deal with long division, the sixth grade to take up decimal fractions, or until junior high school to deal with formulas. These concepts may be readily and realistically dealt with in the lower elementary-school grades, the difference, however, being one of operational rather than conceptual level. Concepts increase in depth of meaning with gainful experience.

The transfer of Learning. In connection with a discussion of the perceptual and conceptual aspects of the learning process, the writer feels that consideration must

57 Blair, Jones, and Simpson, op. cit., p. 223.
be given to the nature of the transfer of learning.

According to Blair, Jones, and Simpson,\textsuperscript{58}

Transfer may occur when there is a similarity between two activities either in substance or procedure. Anything which can be learned can be transferred. \ldots Transfer may be quite specific, as when elements of one learning situation occur in identical or similar form in another. \ldots Also transfer may be general, in that a given learning such as a principle, a set, or method has influence upon any number of later learning situations.

Fehr\textsuperscript{59} pointed out that new learning is in part a matter of transference of past learning. The degree to which transference takes place depends on the degree of similarity of the new situation to the original learning situation, the learner's ability to analyze relationships, and the amount of varied experience in previous learning. Hilgard\textsuperscript{60} emphasized that for transfer to operate, not only must similarities be present, but must be perceived by the pupils. Blair, Jones and Simpson\textsuperscript{61} laid stress on the fact that good teaching always involves teaching for transfer. Teachers who stress understanding are apt to stimulate learning of a useful and enduring nature, whereas emphasis

\textsuperscript{58} Ibid., p. 261.

\textsuperscript{59} Fehr, \textit{op. cit.}, p. 31.

\textsuperscript{60} Hilgard, \textit{Introduction to Psychology}, \textit{op. cit.}, p. 256.

\textsuperscript{61} Blair, Jones, and Simpson, \textit{op. cit.}, pp. 256-57.
upon facts and memorization of rules and procedures which are not fully understood by pupils tends to result in a superficial lip service which has neither permanence nor utility.

Hendrix\(^\text{62}\) expressed the point of view, commonly held by modern educators, that, the way in which a child learns a generalization will affect the probability of his recognizing a chance to use it. Sueltz\(^\text{63}\) stated in effect that too great a dependence upon formalized paper and pencil problems robs the student of many of the opportunities for transfer which exist in other kinds of mathematical experiences. Luchins\(^\text{64}\) discovered that the employment of teaching methods whereby children arrive at their own formulas (generalizations) is more conducive to transfer than the method of starting with generalizations. As explained by Blair, Jones, and Simpson,\(^\text{65}\)


\(^{63}\) Sueltz, "Mathematical Understandings and Judgments Retained by College Freshmen," \textit{op. cit.}, p. 19.


\(^{65}\) Blair, Jones, and Simpson, \textit{op. cit.}, p. 261.
Teaching for transfer requires that the objectives of schooling be clearly defined, that teachers study content and method to find interrelationships among materials and the applicability to other situations of learned skills, that the teaching method be such that students are given practice in transfer.

Hilgard emphasized that in regard to the use of arithmetic instructional aids by the pupils, it is important in view of the theory of transfer that there be similarities between the use of concrete aids, semi-concrete aids, and aids of an abstract nature to clarify a given concept—similarities not based on identities among them but on common patterns or parts.

There are two ways of considering the developmental nature of the learning process. One consideration relates to the observation that at the primary level the attention span is short, physical activity incessant, interest spontaneous, curiosity high, concepts of time and space limited, much desire to manipulate and handle things, and a limited ability for cooperative work. Because of these characteristics, there should be ample opportunity for large muscle activity and rest; opportunities to use manipulative materials, blocks, toys, simple construction materials, free rhythms, dramatic play, discussion, and conversation; and experiences should relate to the "here and now" world,

employing familiar objects and concepts. As the pupil becomes more mature physically and as he builds up an adequate background of experiences, he will become better able to use the finer muscles of the body, to work with more remote concepts of time and space, and to employ materials of a more abstract nature.

A second way of considering the developmental nature of learning relates to the theory that any individual naive in a given process, topic, or concept proceeds to learn it best through a series of steps ranging from the process of getting the learner in a state of readiness to learn the process, topic, or concept, to the top operational level of dealing with pertinent materials of an abstract nature. This process is explained by Grossnickle, Junge, and Metzner67 as follows: Learning number consists in an orderly series of experiences beginning with concrete objects and progressing toward abstractions. There are different stages, levels, or steps which may be identified in the learning of arithmetic. These levels are:

1. Readiness for learning
2. Laboratory period for discovery
3. Verbal and symbolic representation of a quantitative situation
4. Systematic verbal presentation
5. Adult level of operation

The point is emphasized, however, that there are no

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67 Grossnickle, Junge, and Metzner, op. cit., p. 156.
definite lines of demarcation among the different steps, instead, they may be overlapping.

In a separate article, Grossnickle presented a partial explanation of how the learning process operates in each of the above stages of learning. In the laboratory period for discovery, the pupil may first find a roundabout way of solving a problem, then proceed to discover for himself the most economical way of arriving at the solution. In this period, the pupil learns or discovers the conventional method for performing a given process. During the first part of this stage of learning, the pupil works with manipulative aids until he is able to verbalize the solution. This ability to make a verbal explanation of his manipulative-aids solution to the problem signifies to the teacher that the pupil is ready to advance to the next stage of the learning process— the verbal and symbolic representation of a quantitative situation—in which pictorial and graphic aids are used. The pupil has hence advanced from the use of concrete aids to the use of those of a semi-concrete nature. This stage of learning was explained very thoroughly by Williams who remarked that


This level of the learning process is so frequently slighted. In other words, while many teachers do an effective job using concrete manipulative materials during the initial stages of the formation of arithmetic concepts, they too frequently shift so rapidly from these concrete visual experiences to abstract verbal symbols that children are unable to breach the gap. What Professor Williams proposes is the use of intermediary aids which lessen the abruptness of the transition, specifically the use of charts as one type of visual aid to serve this need. Emphasized, however, is the warning that the use of such aids should not be introduced until after concepts have been developed through rich first-hand experiences.

As soon as the pupil displays his ability to verbalize solutions by the use of semi-concrete aids, he is permitted to work with abstract symbols. In other words, he is ready to work on an adult level. At this stage of learning, he works most economically in terms of abstract aids.

In regard to the developmental nature of learning, Wheat remarked:

Learning at the later stages depends upon learning at the earlier stages. The higher levels of learning are not reached at a single bound. They are not brought down to the student. They

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70 Harry Grove Wheat, Foundations of School Learning, p. 378.
are not attained except from the necessary preliminary steps from the lower levels. The student moves up to the later steps after he has taken the earlier steps.

With regard to the use of direct experiences, Wheat stated:

There is need for a reasonable amount of direct experience in the solutions of problems which are within the ability of the student to solve. . . . Direct experience in problem-solving is a means to an end. When the end--namely, vividness--is gained, direct experience should give way to those ends which give breadth. 71

Hilgard 72 remarked that one of the key problems of economical learning lies in the answer to the question: "Who should learn what?" The right answer, according to him, depends partly on both the general level of ability and on specialization of ability but is also a matter of level of development. How intelligent is the child? What are his special gifts? How far has he matured? It was revealed experimentally that children who had not yet acquired an understanding of concrete number facts were actually retarded in their learning of arithmetic by being drilled on arithmetic operations--that is, on the use of abstract materials. 73

71 Ibid., p. 283.
72 Ernest R. Hilgard, Introduction to Psychology, p. 252.
In regard to the problem of using appropriate instructional aids at the various stages of the learning process, two notes of warning might well be sounded. Grossnickle, Junge, and Metzner\textsuperscript{74} reminded teachers that the function of good instruction in arithmetic is to have the pupil operate at the highest level of difficulty at which work is meaningful to him, and that obviously there are pupils who develop sufficient insight into the meaning of number that the use of manipulative and visual aids is not needed to introduce a new process. A second caution related to the fact that a child might practically be working at different stages of development at the same time and that a child who is working at the abstract stage might conceivably have to return to the concrete or semi-concrete stage to solve some problem even after being able to do some work in the abstract.

\textbf{The Sensory Nature of the Learning Process.} One of the basic assumptions made in Chapter I of this study was to the effect that all learning is basically sensory in nature. Considerable evidence to support this assumption has already been presented in the foregoing portion of the present chapter. Additional evidence will be presented now relative to this point of view. Van Engen\textsuperscript{75} asserted:

\textsuperscript{74} Grossnickle, Junge, and Metzner, \textit{op. cit.}, p. 172.

\textsuperscript{75} Van Engen, \textit{op. cit.}, p. 86.
"Reactions to the world of concrete objects are the founda-
tion stones from which the structure of abstract ideas
arises." In other words, it is probable that all mental
life has at its roots the actions or manipulations per-
depend on a learning situation. The present-day emphasis
on multi-sensory aids has a secure foundation according to
the point of view expressed by Heibreder, Smoke, and other
psychologists of the operationalists' school of thought.
Gesell, too, holds a similar point of view. He
maintained:

It is probable that all mental life has a
motor basis and a motor origin. The non mystical
mind must always take hold. Even in the rarefied
realm of conceptual reasoning we speak of intel-
lectual grasp and of symbolic apprehension.
Thinking might be defined as a comprehension and
manipulation of meaning. . . . Counting is based
on serial motor manipulations.

Weaver and Bollinger, also, feel that all learning
is the result of sensory experiences. They reasoned that
not only are sensory experiences necessary to more complete
comprehension, but that they are the key to pupils' mental
activity and future learning. Sensory experiences aid in

76 Ibid., p. 88.
77 Ibid., p. 86.
78 Arnold Gesell, Infant Development: The Embryology
of Early Human Behavior, p. 58.
79 Gilbert C. Weaver and Elroy W. Bollinger, Visual
Aids, p. 4.
creating the proper mental image for the learner, and that is the true basis of learning. Oral and printed words and symbols usually mean nothing to the pupil until translated into a mental image. Reading (whether a literature book or an arithmetic textbook) is properly defined as "getting" the meaning from the printed symbols on the page. However, the meaning is not contained in the pupil himself, rather in the sensory background experiences of the pupil.

Overstreet remarked:

The plain fact is that whatever we think about, whatever we believe in, whatever we appreciate, is something that was originally introduced to our consciousness by one or more of our five senses: sight, hearing, touch, smell, and taste. Even the most abstract ideas that we cherish—ideas that seem to be above physical limitations—came to us in the first place through sight or hearing or one of the other senses. Even the creative insights that seem entirely the product of our own minds are in the final analysis, a reordering into new and vivid imaginative patterns of materials already collected through our senses.

According to Syer, the senses and their sensations (defined as any experiences which result from stimulation of the senses) are the building blocks from which perceptions, concepts, ideas, thoughts, and learning are

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80 Bonaro W. Overstreet, How to Stay Alive as Long as You Live, as quoted by Edgar Dale, Audio-Visual Methods in Teaching, p. 44.

constructed. He remarked that any attempt to trace "meaning" back to its origin, or any attempt to clarify "meaning" by explanatory examples and applications eventually leads us to concrete objects and experiences which have involved the senses. Although granting that abstractions may be built from other abstractions, he reasoned that somewhere back in the formulation of these ideas is a foundation in sensory perceptions, and that often we rush back to these perceptions when the abstractions become vague and confusing.

To guard against the possibility of being misinterpreted, the writer has repeatedly referred to the interrelationship of the various senses and to the interrelationship of "sensory" learning and abstract learning. It is hoped that no one will interpret the statement that, "all learning is basically sensory," as a claim that the learner must return to the use of objective aids whenever faced with a new situation or new problem. The statement has been made or inferred at several points in the preceding pages that abstractions may be learned from other abstractions. What the writer does contend, however, is that the original abstractions upon which later abstractions are built are dependent upon sensory experiences. Syer\(^2\) cautioned that sensory learning cannot discount the

\(^{2}\text{Ibid.}, pp. 103-104.\)
importance of motivation, memorizing, the use of mental concepts, problem-solving, emotional activities, and imagery. Moreover, to those who believe that sensory experiences are dependent exclusively upon the organs of sense, Wheeler and Perkins replied:

Images have been defined as cortically or centrally aroused sensory experiences, to differentiate them from sensory experiences, proper, which latter, it has long been thought, were dependent exclusively upon the organs of sense. This distinction was supposed to explain the greater stability and vividness of sensory experiences. Now we know that this distinction between imagination and sense perception is arbitrary, and that the one, as well as the other, involves the cortex and peripherally aroused processes.

Points of View Regarding the Role of Instructional Aids in Arithmetic Education

Following the two preceding sections of this chapter dealing with the philosophical bases of elementary-school education and of the elementary-school arithmetic program specifically, and the nature of the learning process, the writer feels it proper next to present his, as well as other, points of view regarding the role of instructional aids in the program of elementary-school arithmetic. The major emphasis on instructional aids in the early literature was on their motivating values—on their ability to stimulate and maintain interest—and even on their

sensational nature. With no intent to discount the value of instructional aids as motivating instruments, the writer feels that they serve an even more basic function. In a modern program of arithmetic—as well as other curriculum fields—we do not restrict the pupil to any one text or to any one reference source. His sources of data are, so to speak, as extensive as the universe itself. In a very real sense, the universe becomes his workshop, his schoolroom. He is free, in fact obligated, to gather his data from whatever source produces pertinent information. The source may consist of textbooks, pamphlets, realia, models, specimens, charts, graphs, films, slides, field trips, direct experiences—anything applicable to the problem or center of interest of concern to the learner.

The educator having such a point of view would regard with considerable disfavor any reference to instructional aids as supplementary materials. No aid is considered supplementary to an other aid so long as it furnishes applicable data. However, although the universe becomes the "source-shop" for data, the writer would not be so naive as to assume that beginning primary children would utilize such aids as line graphs or slide rules any more than they would go to the Encyclopedia Britannica to gather data. In other words, the developmental level of the pupil becomes a real factor not only in terms of age or
grade-level, but also in terms of naivete of persons of any age in specific concepts, processes, or topics.

Again, instructional aids cannot be considered "extra" if accepted is the organismic and integral nature of learning which considers the curriculum as all the experiences (and materials or aids) over which the school has control. Furthermore, one cannot segregate learning to the extent of saying that the child learns this bit of knowledge or understanding from the textbook, this bit from manipulative objects, and this bit from another source. The child learns from all aspects of a very inclusive curriculum.

Witt, too, emphasized that instructional materials should not be thought of as something different, something extra that is not a part of the regular educational program. He stated that films, filmstrips, radio programs, field trips, objects, exhibits, and similar items are all instructional materials and are used for much the same purpose that teachers use books and other printed materials. He warned that any hindrance to this concept should be eliminated and that a separation of audio-visual instruction from other aspects of the instructional program tends to keep the emphasis focused on materials as such rather

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than on the needs and concerns of the learners. Noel and Leonard\textsuperscript{85} declared in effect that instead of audio-visual materials being regarded as a mere supplement or enrichment to textbooks, under certain conditions the textbook may supplement the other instructional aids.

Educators generally agree that there is no one best way of using instructional aids in arithmetic education. Clark and Fehr\textsuperscript{86} maintained that to one teacher (perhaps in the primary grades) sensory experiences may be to help pupils build concepts, discover facts or relationships, that they may be the initial step in proceeding from the concrete to the abstract. To another teacher the purpose may be to illustrate, to make concrete that which the pupil has already learned. It becomes obvious that many instructional aids may serve more than one purpose and the teacher whose objective for using instructional aids in a particular learning situation is clearest tends to make the best selection and use of such aids. Clark and Fehr asserted that the skillful teacher

\[\ldots\] balances each step from concrete experience to semi-concrete representations, to

\textsuperscript{85} Noel and Leonard, \textit{op. cit.}, pp. 4-5.

words and symbols, to generalized abstract theorems, through proper evaluation and by appropriate attainable challenges. He also recognizes that not all learning may start in concrete material objects, but that much new learning may start in already learned abstractions. Thus algebraic fractions are referred back to the abstractions learned in the study of arithmetic fractions and not back to parts of concrete objects. Geometry is related to geometrical drawings, trigonometry is related back to geometry and algebra.87

It is commonly agreed that the good teacher does not use aids in arithmetic instruction for the sake of the aid itself. That is, arithmetic instructional aids are not ends in themselves, but only means to desired goals. Educational purposes are the real bases for the selection of arithmetic instructional aids. According to Corey,88 "Any general claim that all teachers should spend a certain amount of class time using films, or pictures, or transcriptions, or recordings, misses the point. The kinds of instructional materials to be used depend upon what the teacher is trying to teach." Witt89 remarked that too often materials are selected first and then utilization becomes a matter of finding ways of interesting pupils in the materials—a case of putting the cart before the horse.

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87 Ibid., p. 347.
89 Witt, op. cit., p. 110.
Pupil needs and interests should be identified first, and then arithmetic instructional aids selected which can be used to help meet those needs or satisfy those interests. Under these circumstances there can be no one standard pattern of utilization. Methods of selection and use will be many and varied.

To the extent that there is no uniqueness of educational objectives for the elementary school; to the extent that the objectives of the elementary-school arithmetic program are in a sense synonymous with the objectives of the elementary-school curriculum in general; to a similar extent there are no psychological principles that are uniquely pertinent to the selection and use of arithmetic instructional aids. Psychological principles affecting the selection and use of arithmetic instructional aids are the same principles which have been emphasized again and again in courses in educational psychology, child growth and development, and educational methods. Some of these principles are listed by Noel and Leonard as follows:

1. The individuals in any class are in different stages of growth.
2. Each individual has his own rate of growth.
3. Each individual has certain basic drives.
4. Each individual has certain special interests.
5. Each individual acts as a whole and not with separate intellectual, emotional, and physical responses.

Noel and Leonard, op. cit., p. 25.
6. Learning must be purposeful.
7. Learning usually persists to the extent that it is used. A person learns what he practices.
8. Activities must be appropriate to the learning desired.
9. There is no learning without interest.
10. A close relationship exists between interest and readiness to learn.
11. There must be readiness for an experience before a pupil will be able to profit from it.
12. Interest in and enthusiasm for the thing to be learned will develop a favorable attitude toward what is being learned.
13. Difficulties in learning may be caused by many factors, some of which are outside the pupil and beyond his control. For instance, difficulties may be caused by:

   a. the teacher
   b. the materials
   c. the problem under discussion
   d. the environment
   e. home conditions
   f. poor health

Sueltz⁹¹ cautioned that devices (instructional aids) themselves seldom teach arithmetic. Learning depends upon how they are used. The role of the teacher is still paramount in stimulating interest, in directing attention to the essential ideas and principles, and in clinching learning. It becomes immediately obvious that not all arithmetic teaching can be done with objective aids. As a warning, Sueltz⁹² stated: "Remember that it is the way you use a model or device that determines whether you are an artist

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⁹²Ibid.
teacher or merely an artisan doing a job." Furthermore, it cannot be overemphasized that a modern program in arithmetic "... employs a variety of instructional materials to be used in a learning process which begins with real experiences and culminates in mathematical abstractions."\(^93\)

Much of the criticism aimed at the use of instructional aids in arithmetic education seemingly stems from the inexpertness of the teacher in their use. The skillful teacher will bring to the learning situation at the proper time aids most useful in mastering a skill, in providing informational backgrounds, in building concepts, or in developing attitudes, relationships, interests, and appreciations. The writer is in agreement with Badgley and Maaske\(^94\) that, "The teacher needs to be as familiar with the content of the material (instructional aids) used as he or she would be with the usual textbook material relating to the subject."

Reeve\(^95\) cited the following five main reasons why

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\(^{93}\) Grossnickle, "Instructional Materials in Arithmetic," \emph{op. cit.}, p. 36.


\(^{95}\) W. D. Reeve, "Multi-Sensory Aids in Teaching Arithmetic," \emph{Teachers College Record}, XLVI (April, 1945), p. 420.
teachers of mathematics, like teachers of other school subjects, have been slow to utilize multi-sensory aids to the utmost:

1. Teachers tend to rely too much on traditional practices in solving educational problems; in other words, they cling to the old question-and-answer type of recitation.

2. Too many teachers are afraid to try new ways of teaching. They lack appreciation, imagination, and ingenuity; they are indifferent or apathetic; and in many cases their training has not qualified them to do creative teaching.

3. They do not always understand the purpose of multi-sensory aids, and understanding is essential if these aids are to facilitate instruction.

4. Unsympathetic administrative attitudes may play a part in discouraging teachers from trying out new ideas.

5. Many teachers and administrators are fearful of the expense involved in providing proper classroom equipment for the use of multi-sensory aids.

The writer feels that teacher education in the use of arithmetic instructional aids should be an integral part of the regular course in arithmetic education, rather than an attempt to make a separate course of it. Furthermore, he believes that teachers tend to teach as they themselves were taught. The implication in this view is the need for instructors of prospective teachers to make effective use of arithmetic instructional aids themselves. Moreover, the writer feels that instructors of pre-service classes in arithmetic education are obligated to see to it that
prospective teachers possess an adequate understanding of how to select and use arithmetic instructional aids as a partial requirement of the regular arithmetic-education course. Justification of this point of view implies further the necessity for developing methods and instruments to determine whether such understanding on the part of prospective teachers does exist.

Classifications of Instructional Aids with Reference to the Developmental Nature of the Learning Process

As one might readily expect, many different classifications of instructional aids have been presented by various educators since the beginning of their conception of the educative process as being a function of all the senses rather than of only an isolated brain. Most educators have attempted to base such classifications upon an increasing progression from aids of a concrete nature to those which are abstract.

One of the early classifications of instructional aids was that of Ernest L. Crandall, Director of Lectures and Visual Instruction in New York City in 1923. Crandall divided aids into the following four classes: (1) real objects (consisting of (a) excursions, (b) specimens,

96 National Education Association Addresses and Proceedings, 1923, pp. 540-43.
(o) experiments); (2) loose pictures; (3) stereopticon views; and (4) motion pictures. He was convinced that aids should be applied in the learning process in the order given. For example, he felt that motion pictures should come later than slides, both in terms of the child's development as well as later in the lesson. To Crandall, aids should come earliest which involve some physical activity because, according to him, the motor impulses are dominant in the earlier years. Those aids should come latest which exact the most mental activity. At the same time, however, he admitted that his views of the role of instructional aids were applicable only in general terms—that under proper conditions all visual aids might be used interchangeably at all ages and stages.

An example of a different method of classifying aids was afforded by Kinder, who, it would seem, made no especial attempt at classifying in terms of the developmental nature of learning. His classification is as follows:

I. Repetitive Materials
   a. blackboards
   b. bulletin boards
   c. duplicating devices

II. Pictorial and Graphic Representations
   a. photographs
   b. textbook illustrations
   c. prints and etchings

d. cutouts
e. post cards
f. newspaper clippings
g. drawings and sketches
h. charts, graphs, and tables
i. cartoons
j. pictorial statistics
k. posters
l. maps and globes
m. diagrams and schematics

III. Still Projected Pictures
a. stereographs (unprojected)
b. lantern slides
c. filmstrips
d. opaque projections
e. positive transparencies
f. micro-slide projections
g. tachistoscopes

IV. Projected Motion Pictures
a. silent motion pictures
b. sound-on-film motion pictures

V. Auditory Materials
a. phonograph records
b. electrical transcriptions
c. radio broadcasts
1. AM
2. FM
d. centralized sound systems

VI. Audio-Visual Aids in Combination
a. sound motion picture
b. television
c. sound filmstrips

VII. The School Journey

VIII. The Museum

IX. Representations and Relief Displays
a. models
b. objects
c. specimens, collections, samples
d. relics
e. dioramas
f. sand tables, miniature sets, floor representations
g. mock-ups
h. miniature dolls, etc.
X. Dramatizations

XI. Demonstrations

XII. Miscellaneous
   a. flash cards
   b. albums
   c. illustrated booklets, scrapbooks

Dale\textsuperscript{98} classified instructional aids in terms of a "Cone of Experience," proceeding from a base consisting of "direct, purposeful experiences" through a progression of aids of a more and more abstract nature to the apex, consisting of "verbal symbols," of which the most abstract instructional aids are composed. Dale cautioned: "You must see that the cone classifies sensory aids in terms of greater or less concreteness and abstractness as learning experiences, and not with respect to the difficulty of the ideas they communicate."\textsuperscript{99} The complete classification of aids, as portrayed on the "Cone of Experience," is as follows:

1. direct, purposeful experiences
2. continued experiences: models, mockups, object specimens
3. dramatized experiences: plays, puppetry, sociodrama, role-playing
4. demonstrations: apparatus, chalkboard
5. field trips
6. exhibits: school-made displays, bulletin boards, posters

\textsuperscript{98} Dale, \textit{Audio-Visual Methods of Instruction}, \textit{op. cit.}, p. 43.

\textsuperscript{99} Ibid., p. 54.
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7. television
8. motion pictures
9. recordings, radio, still pictures
10. visual symbols: maps, cartoons, graphs
11. verbal symbols

One of the most sincere efforts to classify arithmetic instructional aids specifically with reference to the developmental nature of learning is that of Grossnickle, Junge, and Metzner¹⁰⁰ who grouped them into the following four classes:

1. real experiences
2. manipulative materials
3. pictorial materials
4. symbolic materials

To these educators, instructional materials are "Anything which contributes to the learning process." As was true in the case of Dale, they cautioned that the classification cannot be regarded as rigid and inflexible. Rather, the various instructional aids overlap and blend into each other. The instructional aids must be viewed as existing on a continuum in which various materials appear in increasing abstractness as the learner proceeds from direct experiences to symbolic materials. The lowest level of quantitative thinking results from dealing with real objects whereas the highest level results from working with abstract symbols.

"Real experiences" are defined as "tangible, direct, first-hand experiences . . . active participation in a real-life situation with responsibility for the outcome." While the authors grant that "real experiences," considered by themselves, are not actually instructional materials, they feel justified in giving them that connotation here because they provide a medium through which objects and materials become available to children for study purposes. Technically, the things used or met in the experience, and not the experience itself, are the instructional aids.

Manipulative aids are defined as those aids which the pupil is able to feel, touch, handle, and move. Such aids may or may not have a social significance. For example, a measuring cup is a manipulative aid with a social significance, whereas the manipulative aid, place-value pocket, does not have any social value as such.

Pictorial materials include pictures of all types, including charts, graphs, and diagrams. Pictorial materials may be both flat and projected.

Symbolic materials include written or printed aids, such as textbooks, workbooks, instructional tests, and pamphlets. Such materials comprise the most abstract of instructional aids.

101 Ibid., p. 162.
Significance of the Classification of Instructional Aids. What is the point, if any, of dividing instructional aids into different categories? Is such a distinction purely an artificial arrangement, or does it serve a real purpose? These questions may be better answered if classifications, such as those of Grossnickle, Junge, and Metzner, are related to the following operational levels or stages of learning, as identified by the same educators:

1. Readiness for learning
2. Laboratory period for discovery
3. Verbal and symbolic representation of a quantitative situation
4. Systematic verbal presentation
5. Adult level of operation

Although authorities generally agree that there is no set sequence of stages of learning or of a set pattern for the application of instructional aids, prospective teachers might be helped considerably in their teaching procedures if they consider the use of the various classes of instructional aids in relation to the operational stages or levels of learning. The relationship might be diagrammed as follows:

102 Ibid., p. 156.
<table>
<thead>
<tr>
<th>Operational Stages of Learning</th>
<th>Classifications of Instructional Aids to Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Readiness for learning</td>
<td>1. Direct, purposeful experiences, including excursions</td>
</tr>
<tr>
<td></td>
<td>2. Pictorial materials (flat and projected) as an overview</td>
</tr>
<tr>
<td>2. Laboratory period for</td>
<td>1. Manipulative aids</td>
</tr>
<tr>
<td>discovery.</td>
<td>2. Instructional film for demonstration</td>
</tr>
<tr>
<td>a. exploratory (pupil)</td>
<td></td>
</tr>
<tr>
<td>b. developmental</td>
<td></td>
</tr>
<tr>
<td>1. class demonstration:</td>
<td></td>
</tr>
<tr>
<td>2. pupil discovers for:</td>
<td></td>
</tr>
<tr>
<td>himself.</td>
<td></td>
</tr>
<tr>
<td>3. Representation of process</td>
<td>1. Pictorial aids</td>
</tr>
<tr>
<td>or concept</td>
<td>2. Semi-abstract aids</td>
</tr>
<tr>
<td></td>
<td>1. Examples and problems in arithmetic</td>
</tr>
<tr>
<td></td>
<td>textbook and workbook</td>
</tr>
<tr>
<td></td>
<td>2. Actual &quot;arithmetic&quot; problems encountered in life</td>
</tr>
</tbody>
</table>

It may be observed that the operational stage systematic verbal presentation has been omitted in the above diagram. The omission was intentional. In practice the step operates during each of the stages presented in the outline above. For example, when the child is able to present a reasonably adequate verbal explanation of the direct, purposeful arithmetic experiences he has been having during the readiness for learning operational stage, the teacher is thus given a rather valid indication of the child's readiness to proceed to the next operational stage, the laboratory period for discovery, during which arithmetic is first presented in a formal, systematic manner. As
soon as the child is able to present an adequate verbal explanation of his manipulations and discoveries during the laboratory period, he is permitted to work with instructional aids peculiar to the next operational level. The same approach is followed during the next stage of learning. The procedure as outlined above should not be misinterpreted as inferring that every child in the class is required to proceed in a goose step through each operational level from the lowest to the highest. Rather, each child is encouraged to work at his highest efficient operational level. If a pupil gives adequate indications of understanding of various lower-level operational stages, he certainly is not required to work with the instructional aids peculiar to those levels. In the typical classroom there will be children working at each operational level on a specific concept or process. Moreover, it may be expected that there will be some children in the classroom who will never advance beyond the lower levels of learning.

It should be repeated that the readiness stage of learning might be thought of as operating, as indeed it does, throughout all stages of the learning process. That is, with specific reference to each arithmetic concept, we might conceive of the pupil as being "ready" to work with concrete instructional aids, with semi-concrete instructional aids such as pictorial representations, with semi-abstract instructional aids such as dots and x's, and with
systematized symbolic materials such as the examples and problems found in arithmetic workbooks and textbooks. However, in terms of operational stage No. 1 in the diagram, readiness refers to preparedness of the pupil to manipulate concrete aids in order to "discover" solutions or to make meaningful a specific concept. It has been shown that the learner works more economically with manipulative aids if he has been made ready for them in terms of a background of direct, purposeful, and related experiences. For instance, if the child has had the direct, purposeful (but informal) experience of passing out sheets of paper to the children in his row, he has thereby gained an initial understanding of the one-to-one relationship which will help to "ready" him for working formally with concrete manipulative counting devices on a one-to-one basis.

Although this part of the study is presented by the writer with many reservations and cautions, he feels that it does have inestimable value in helping the prospective teacher to clarify his arithmetic thinking. With these qualifying conditions in mind, let us see how in theory the teacher would go about presenting a specific developmental concept taken from the list of twenty-three topics selected as the mathematical bases of the study.

The arithmetic concept "counting and ranking" is a logical starting place. The unlikely supposition is made, just for the sake of example, that all pupils in the group
are naive with respect to the concept. Diagrammatically (and in theory, of course) the teacher would relate instructional aids to the operational stages of learning as follows:

Concept: Counting and Ranking

<table>
<thead>
<tr>
<th>Operational Level</th>
<th>Instructional Aids</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Readiness for Learning</td>
<td>1. Direct, purposeful (informal) experiences, such as:</td>
</tr>
<tr>
<td></td>
<td>a. passing out paper, crayons, and other supplies</td>
</tr>
<tr>
<td></td>
<td>b. taking roll</td>
</tr>
<tr>
<td></td>
<td>c. taking &quot;milk count&quot;</td>
</tr>
<tr>
<td>2. Exploratory-Discovery Stage of Learning</td>
<td>2. Manipulative Aids, such as:</td>
</tr>
<tr>
<td></td>
<td>a. fact finders</td>
</tr>
<tr>
<td></td>
<td>b. counting blocks, discs, coins, children, splints</td>
</tr>
<tr>
<td></td>
<td>c. abacus, counting frames</td>
</tr>
<tr>
<td>3. Verbalization and representation of manipulation processes</td>
<td>3. Visual aids, such as:</td>
</tr>
<tr>
<td></td>
<td>a. counting charts, employing:</td>
</tr>
<tr>
<td></td>
<td>(1) pictorial representations</td>
</tr>
<tr>
<td></td>
<td>(2) semi-abstract representations</td>
</tr>
<tr>
<td></td>
<td>(3) symbolic representations</td>
</tr>
<tr>
<td>4. Performance at adult level</td>
<td>4. Abstract examples and problems involving &quot;counting and ranking,&quot; such as:</td>
</tr>
<tr>
<td></td>
<td>a. arithmetic textbooks and workbooks</td>
</tr>
<tr>
<td></td>
<td>b. life situations</td>
</tr>
</tbody>
</table>
Now to return to the qualifying conditions stated earlier in regard to this method of relating the operational levels of learning to the classifications of instructional aids, it should be remembered that in actual practice, the learning process does not progress in any such made-to-order manner. For instance, the classroom of beginners would not all be naive in the matter of being able to count and rank. Some would be; others would be at the exploratory-discovery stage of learning; others would be scattered along the scale. One of the basic jobs of the teacher is to determine the operational level of each pupil in order to know which classification of instructional aids might be used most efficiently. It should be kept in mind that the diagrammatic presentation of instructional aids is intended to be only suggestive, not exhaustive. It would be an impossible task to list all applicable instructional aids. The schools of each section of our country possibly have access to some such aids peculiar only to that region.

Another irregularity the teacher in all probability will discover is that some of his pupils are able to work with abstract numbers, yet are unable to verbalize, to explain a given process or concept. Such pupils usually need to revert to the use of aids of a concrete, semi-concrete, or semi-abstract nature to make the operation or concept more meaningful. This condition tends to exist in any unselected group, whether it be composed of beginners
in arithmetic or of students in a senior mathematics course.

In spite of the shortcomings of the diagrammatic presenta­tion of the relationship shown between operational levels and classifications of aids, the writer feels that the diagram as presented has real worth if the teacher will accept it with a realization of its limitations. It should tend to keep the operational levels and their relationship to the various classes of instructional aids in perspec­tive. Also, it should tend to result in a systematic utilization of instructional aids instead of an indiscrim­inate, unorganized use. In Appendix A is presented a listing of representative instructional aids which tend to be applicable in promoting understanding of each of the twenty-three basic developmental concepts of arithmetic.

Advantages in the Use of Instructional Aids

in Arithmetic Education

To preface this subtopic, it should be stated that the use of instructional aids is not an all-or-none proposition; rather, it is one of degree. Indeed, it would be rather difficult to conceive of instruction in arithmetic or any other subject without the use of instructional aids of some sort and in some degree. Hence, to speak of the advantages or disadvantages of the use of instructional aids in a program of arithmetic education implies an ade­quate and intelligent use of such aids as contrasted with
such a program in which a paucity of instructional aids is used or in which such aids, as are employed, are used without adequate understanding.

The following values for instructional materials adequately used in the teaching situation have been supported by research evidence:

1. They supply a concrete basis for conceptual thinking and hence reduce verbalistic response of students.

2. They have a high degree of interest for students.

3. They supply the necessary basis for developmental learning and hence make learning more permanent.

4. They offer a reality of experience which stimulates self activity on the part of pupils.

5. They develop a continuity of thought, this being especially true of motion pictures.

6. They contribute to the growth of meaning and hence to vocabulary development.

7. They develop experiences not easily secured in other materials, and hence they contribute to the depth and variety of meaning.\textsuperscript{103}

Research has shown definitely that visual aids are a distinct asset to effective teaching in all or practically all the curriculum subjects.\textsuperscript{104} Moreover, pupils learn better and forget less rapidly when they can see and


\textsuperscript{104} Kinder, \textit{op. cit.}, p. 338.
handle something instead of merely building mental images of it.\textsuperscript{105} It has been stated that the Army training period was shortened by forty percent as a result of the use of the training films.\textsuperscript{106} Some of the advantages of sensory aids, as stated by McKown and Roberts\textsuperscript{107} are as follows:

1. They are novel, and variety is always attractive to the child as well as to the adult.
2. Sensory aids allow some freedom from the formal restrictions of the traditional recitation.
3. Sensory aids are relatively easy to understand and master because they are concrete.
4. Sensory aids cause the child immediately to focus his entire attention on them. The child cannot successfully look at a motion picture, examine an object, or operate a working model and think about something else at the same time.
5. Many of the aids provide the child with opportunities to handle and manipulate them. Both children and adults are manipulators by original nature, both enjoying firsthand experiences.
6. Sensory aids satisfy immediate curiosity. Even temporary curiosity in the child is important because it is a starting point to learning.
7. An aid makes an appeal because it does not satisfy completely. The more the child sees, the more he wants to see.


\textsuperscript{106} Reeve, "Multi-Sensory Aids in Teaching Arithmetic," \textit{op. cit.}, p. 420.

\textsuperscript{107} Harry C. McKown and Alvin B. Roberts, \textit{Audio-Visual Aids to Instruction}, pp. 33 ff.
According to McClusky, the values in audio-visual instruction apply to all four major types of learning, namely: the development of motor skills, building associations, solving problems, and creating attitudes and appreciations. McClusky listed the following as some of the major values in audio-visual instruction:

1. Audio-visual instruction provides the concrete experiences which are essential to enriched learning. A common fallacious notion in teaching is that the human ability to recall verbalized experience is synonymous with real learning. Evidence supports, however, that a pupil's verbal expressions are not always reliable evidence of a real thorough understanding. In providing experiences essential in learning, audio-visual materials are valuable, on the associative level, to:

   a. give meaning to words and symbols, such as numbers and map signs
   b. clarify ideas involving higher abstractions
   c. combat the temptation to over verbalize instruction.

2. Audio-visual materials are of value to the teacher in the accurate communication of ideas. The first step in good instruction is the successful intercommunication of ideas between teacher and pupil. Audio-visual materials accomplish this purpose by:

   a. particularizing word and number concepts. Since concepts are generalizations, they must be pinned down by specific examples and illustrations, if true understanding is to result.
   b. making demonstrations in teaching more effective. Pupils make little progress in developing behavior patterns by being told how or by reading how. They learn to do by doing and by imitating the behavior of others who are skilled.

---

o. "speaking" a universal language. They serve to cut across language differences and barriers.

3. Audio-visual instruction aids accurate thinking. Both verbal and non-verbal clues are essential to most reflective thinking. Audio-visual instruction contributes to accurate thinking by:
   a. furnishing some of the basic raw materials for thought.
   b. directing attention. The instructional aid may serve to keep the attention centered on the problem.
   c. clarifying symbolic concepts by relating them to concrete situations. A verbal "clue" tested in a practical situation may be found not to work and vice versa.

4. Audio-visual instruction contributes to the development of attitudes by:
   a. arousing the emotions and feelings, as through motion pictures and posters.
   b. employing the "seeing-is-believing" concept. Teachers deal daily with attitudes both positive and negative in their students. There is no better way to develop desirable attitudes than by the audio-visual approach.

Wittich and Schuller\(^{109}\) portrayed in tabular form a general view of the advantages (as well as the limitations) of certain types of instructional aids. Their findings are reproduced as Table 4 on page 146. The x's indicate that designated instructional aids possess the advantages (or "Contributions to Learning") opposite which they appear. Likewise, a blank indicates a limitation with regard to that specific "Contribution to Learning" opposite which it

## TABLE 4
### ADVANTAGES AND LIMITATIONS OF INSTRUCTIONAL AIDS

<table>
<thead>
<tr>
<th>Contributions to Learning</th>
<th>16 mm.</th>
<th>Sound Motion Picture Film</th>
<th>Filmstrip</th>
<th>Slides</th>
<th>Flat Pictures and Text</th>
<th>Posters</th>
<th>Chalk Board</th>
<th>Field Trips</th>
<th>Radio</th>
<th>Recordings and Transcriptions</th>
<th>Models and Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>VISUAL</td>
<td></td>
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<tr>
<td>Visually re-creates situations involving motion which occur anywhere</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<tr>
<td>Visually recreates the past</td>
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<td>X</td>
<td>X</td>
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<td>Visualizes theoretical ideas and microscopic life</td>
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<td>X</td>
<td>X</td>
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<tr>
<td>Visualizes with natural color</td>
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<td>X</td>
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<td>Visualizes natural dimensions (three dimensional)</td>
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<tr>
<td>AUDIO</td>
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<tr>
<td>Re-creates characteristic or environment sounds</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Re-creates events through dramatization</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
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<tr>
<td>UTILIZATION</td>
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<tr>
<td>Sequence fixed</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
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<tr>
<td>Flexible organization permits rearrangement</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
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<tr>
<td>Permits restudy</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Permits leisurely examination, discussion, etc.</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Control of time and place of use</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Can usually be produced locally</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
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</tr>
</tbody>
</table>


* Sound Strips only.
appears. On the strength of research findings plus authoritative points of view, one might justifiably conclude with Weaver and Bollinger¹¹⁰ that

... teaching aids have a wide range of application, extending from the development of a factual memory to the recognition of things that flash on the screen for a fraction of a second. They serve to inspire action, create desirable attitudes, expedite the development of skills, orient people to new situations, and in general explain how things work.

The time evidently has passed when the teacher can rightfully regard himself or be regarded with any great degree of bases as the source of all knowledge to be imparted to his pupils. The teacher encounters great difficulty in keeping abreast of the ever-increasing expense of knowledge even in his specialized subject field. Instructional aids, such as films and opaque projection materials can do much to enrich and keep up to date the offerings of the teacher. Research has presented evidence that many concepts in the curriculum and many of its objectives can be presented and attained more effectively with audio-visual materials than through verbal methods alone. Furthermore, the use of instructional aids such as films makes it possible to "reach" larger audiences than do the more traditional methods, and there is reason to believe that this point, with especial reference to

¹¹⁰ Weaver and Bollinger, op. cit., p. 8.
television, might become more important in view of the critical teacher shortage.

Disadvantages in the Use of Instructional Aids in Arithmetic Education

Although Dewey gives evidence throughout his writings that he is a staunch advocate of the liberal use of instructional aids, at the same time he warns against using them promiscuously. Specifically he pointed out:

While direct impression has the advantage of being first-hand, it also has the disadvantage of being limited in range. Direct acquaintance with the natural surroundings of the home environment, so as to give reality to ideas about portions of the earth beyond the reach of our senses and as a means of arousing intellectual curiosity, is one thing. As an end-all and be-all of geographical knowledge it is fatally restricted. . . . Just as the race developed especial symbols as tools of calculation and mathematical reasonings, because the use of fingers as numerical symbols got in the way, so the individual must progress from concrete to abstract symbols—that is, symbols whose meaning is realized only through conceptual thinking. 111

Before the turn of the century, Foster 112 pointed up some of the dangers in the overuse of the objective method. She granted that the child should see that four straws and three straws are seven straws, before he learns that $4 + 3 = 7$, but she at the same time pointed out that four


splints and three splints should not be set before the child every time he is asked how many four and three make. She emphasized that not all number work even of young children should be done by looking at and counting objects. The objective method is a good thing as an initiatory measure; it is the push that starts the sled downhill but it should not be expected to push the sled all the way. Illustration, in other words, can be carried too far.

In regard to this last point, however, Foster was referring to the operational level of learning rather than to chronological age or grade level. She, for example, brought out the point that the objective method should not be confined to the lowest grades. Instead, it should be used throughout one's life schooling, but in each case only as an initiatory measure. It should not continue to be used as a crutch in steps that ought to be taken without it, but should be supplemented generously and in many lines of thought replaced by processes of a mental nature. The habit of dependence upon sensation for complete apprehension is a distinct inconvenience in daily life.

Hoban\textsuperscript{113} mentioned the following five major characteristics of audio-visual materials which act as deterrents

to their wider use in the curriculum:

1. Many of the materials are expensive.
2. They are difficult to obtain when they can be used to best advantage.
3. Projection equipment is expensive.
4. Projection equipment requires manual skill for operation and technical skill for maintenance.
5. Some form of building modification is often necessary for the effective use of the equipment.

An ironical fact that Hoban brought out was that the high cost of such instructional aids as films, as well as the projection equipment, is due in large measure to the fact that there is a relatively small market for them, and until the demand increases, the high cost must obviously be borne by the relatively few users.

Effective use cannot be made of instructional aids unless they are readily available to the classroom teacher. Only a few unsuccessful attempts to obtain aids needed must be experienced to cause teachers to throw up their hands in frustration and disgust at the futility of such further attempts. Although availability does not guarantee utilization, it is certainly one of the prerequisites.

Not only can the unavailability of instructional aids become frustrating to teachers intent on their use, but also can the lack of such essentials as dark blinds and usable wall outlets in the classroom. Such equipment as the opaque projector requires a rather dark room for successful operation. Unless the conditions necessary for successful use of instructional aids can be secured rather
readily and with comparative ease, the normal tendency is for the teacher to shy away from further attempts to use aids of this nature and resort to the exclusive use of abstract materials which require little physical exertion.

A further deterrent to the wider use of instructional aids such as projection equipment is the too common feeling of inadequacy and incompetence on the part of many teachers, in spite of the fact that general courses in audio-visual methods are today a common part of the curriculum of colleges of education. Many teachers make use of pupil services, but obviously the teacher herself has the overall responsibility in regard to the proper use of such equipment and materials.

In addition to the mechanical obstacles encountered in using instructional aids (especially manipulative and visual aids), many teachers voice the objection that they simply do not have time to use films, filmstrips, manipulative aids, and the like. This objection seemingly is most frequently expressed by teachers who in reality or in fancy feel that they have a definite course of study—a definite body of subject matter—to cover. As stated by Hoban, \(^{114}\)

"Under these circumstances it is understandable that teachers ask when and where they will find time to use

\(^{114}\) Ibid., p. 67."
enriching materials." Hoban further remarked, however:

To their relief and amazement, more and more teachers are discovering that the crowded curriculum can be more meaningful in many of these areas only through the use of films, slides, and recordings and that the crowded curriculum remains relatively meaningless to many students without these materials. 115

In view of the recognized and experimentally determined advantages, as well as disadvantages, of the various instructional aids used in arithmetic education, the writer feels that prospective teachers of elementary-school arithmetic should obtain clear concepts regarding the psychological implications, as well as authoritative points of view, regarding their use. They should be thoroughly familiar with the advantages of, as well as the dangers and limitations in, the use of such aids. Prospective teachers should know the standards and criteria for selecting them and the approved methods for incorporating them in the arithmetic program. Moreover, they should be cognizant of the scope and types of arithmetic instructional aids and the purposes and levels of operation for which each is appropriate. Finally, the writer feels that teacher-education institutions should assume their rightful share of responsibility for prospective-teacher understanding of how to select and use such aids.

115 Ibid.
Dramatized Procedures Illustrating the Use of
Instructional Aids in Arithmetic Education

Originally the writer had in mind to present dramatized illustrations of the use of instructional aids in teaching each of the twenty-three basic concepts of arithmetic. However, as he delved more deeply into this portion of the study, he realized that to illustrate the entire list of concepts in this manner would add approximately one hundred pages to the dissertation. While granting the value of dramatized procedures embracing the entire group of twenty-three concepts, the writer felt that a representative sampling consisting of one arithmetic topic or process ordinarily emphasized in each of the elementary-school grades one to six would suffice. The selection of arithmetic topics was based upon the recommendations of Bathurst¹¹⁶ and the Committee of Seven,¹¹⁷ and does not in any sense represent the only topics which the writer might have selected to represent the respective grades. The arithmetic topics and processes selected by the writer upon which the dramatized illustrations are based are:


(1) Grade 1 — Counting
(2) Grade 2 — Grouping
(3) Grade 3 — Subtracting tens, specifically the concept of "borrowing"
(4) Grade 4 — The addition and subtraction of simple common fractions
(5) Grade 5 — The multiplication of a common fraction by a whole number
(6) Grade 6 — The measurement of area

The six dramatized illustrations are presented in Appendix B.

Chapter Summary. In Chapter III the writer has attempted to define the role of instructional aids in arithmetic education in both philosophical and psychological terms. The philosophical aspects of the problem were presented with reference to: (a) statements of objectives of elementary education, as proposed by both educational organizations as well as individual educators, and (b) statements of objectives of the elementary-school arithmetic program as proposed likewise by both educational bodies and individual educators. A listing of the twenty-three developmental concepts of arithmetic proposed by Professor Lowry W. Harding of the College of Education, The Ohio State University, was selected as the mathematical bases for the present study.

The psychological bases for using instructional aids in arithmetic instruction were next discussed, with
emphasis being given to such aspects of the problem as: (a) readiness for learning, (b) building up a background of experiences, (c) the developmental and operational nature of the learning process, (d) the transfer of learning, and (e) the sensory nature of basic learning. Considerable treatment was given to attempting to consolidate certain of the philosophical points of view with psychological findings to define an acceptable role of instructional aids in the teaching of elementary-school arithmetic.

A portion of the chapter deals with the presentation of diverse classifications of instructional aids proposed by various educators, with emphasis on the classification suggested by Grossnickle, Junge, and Metzner. The writer attempted to exemplify how various classifications of instructional aids might be geared to the operational levels of learning (as defined by the above educators) in teaching a specific arithmetic topic or process.

Brief attention was then given to the advantages and disadvantages usually accompanying the use of instructional aids in arithmetic education as well as in other subject areas. Finally, suggestive dramatized lesson procedures were presented to exemplify the use of various instructional aids in developing certain specific arithmetic concepts customarily emphasized in each of the six elementary-school grades.
CHAPTER IV

DEVELOPMENT OF INSTRUMENTS FOR ASCERTAINING TEACHER UNDERSTANDING OF USES OF INSTRUCTIONAL AIDS IN ARITHMETIC EDUCATION

Plan of the Chapter. Chapter III was concerned with the role of instructional aids in the teaching of elementary-school arithmetic. Chapter IV deals with the three subsidiary aspects of the study. The first portion of the chapter treats of the development of criteria for the selection and use of aids in arithmetic instruction in the elementary school, with specific reference to the twenty-three developmental concepts of arithmetic selected as the mathematical bases of the study. The second portion of the chapter concerns the construction and validation of an objective instrument, a multiple-choice test, for ascertaining certain aspects of teacher understanding of the principles and practices involved in selecting and using aids in arithmetic instruction in the elementary school. The third section of the chapter pertains to the development of a checklist, intended primarily as a self-check by teachers and student-teachers to promote personal growth in the proper use of arithmetic instructional aids.
The Development of Criteria

It has been emphasized in numerous places throughout the preceding pages of this study that the amount of instructional aids possessed by the school or on display in the classroom is not in itself significant. The availability of aids is important, but only to the extent that they are appropriate to the purpose for which they are intended and to the degree that the classroom teacher possesses an adequate understanding of their use. An excellent aid wrongly used may do irreparable harm. On the other hand, the teacher with an adequate understanding of how to select and use instructional aids may be able to put to excellent instructional use objects which the less informed would not utilize in a teaching situation. Common examples are bottle caps, acorns, buckeyes, pine cones, pebbles, nuts, and broomhandle discs.

In view of the existence of an overabundance of cases of the latter sort, it would seem essential that the prospective teacher of elementary-school arithmetic, as well as other subjects, possess an adequate understanding of how to select and use instructional aids appropriate to the needs of a specific situation. While no one scheme or device can guarantee such understanding, without a measuring stick by which to gauge the appropriateness and adequacy of such efforts, even the most sincere attempt at selecting and using instructional aids properly tends to be
pretty much a hit-or-miss procedure. Such a measuring stick, or set of criteria, may prove to be of inestimable value, a guide by which prospective teachers may pattern their efforts in selecting and using instructional aids, a measure of efforts already made.

The Selection of General Criteria for the Use of Instructional Aids. Numerous excellent sets of criteria of a general nature for the selection and use of instructional aids have been developed and adjudged to be valid by various authorities. By "general nature" is meant not specifically geared to any one subject. A number of sets of such criteria are presented by the writer, since they are applicable in a general way to the elementary-school arithmetic program. After presentation of these sets of criteria, the writer then proceeds with the development or adaptation of criteria more specifically geared to the selection and use of instructional aids in the elementary-school arithmetic program. The sets of general criteria presented embrace the following aspects of the instructional-aids program:

1. Direct, purposeful experiences
2. Contrived experiences
3. Manipulative aids
4. Field trips
5. Still pictures (both projected and non-projected)
6. Motion pictures
7. Non-commercial aids
8. Free and inexpensive instructional materials
9. Radio broadcasts
10. Instructional aids for group use
11. Instructional aids for development of the individual
12. Selection of instructional aids on a teacher-pupil cooperative basis
13. Selection of teaching aids that will lead to a balanced development in children

**Criteria for the use of direct, purposeful experiences**:¹

1. The experiences should be interesting.
2. The experiences should be obtained in a context which promotes further learning.
3. The experiences should be experimental or creative in nature.
4. The experiences should be varied enough to meet the needs of all the pupils.
5. Opportunities should be given pupils to talk about, to infer from, and to verbalize about the experiences.
6. Possibilities for transfer and generalization should be made use of in solving other problems.
7. Individual differences in the ability to generalize should be taken into account.
8. The experiences should be economical in time, money, and effort.
9. The experiences should be safe and healthful.

**Criteria for the use of contrived experiences**:²

1. A model should be used only when it serves the purpose better than does the original.
2. The model used should portray the idea desired more effectively than would any other device.
3. The idea should be appropriate for representation in a model. It should not be too elementary or too complicated.
4. Each part of the model should be made to the same scale. Human figures should be of proportionate size.

5. The important details of construction should be correct.
6. Care should be taken that wrong impressions of size, color, and shape do not result from using the model.
7. The model should not oversimplify the idea.
8. The model, if workable, should stand up under frequent use.
9. The model, if made by the pupils, should be worth the time, effort, and money involved.
10. The model, if purchased, should be used often enough to justify its cost.
11. The model should act as a stimulant to further learning.

Criteria for the use of manipulative aids

1. The aid should be of value for individual study, group study, or both.
2. The aid should be simple to manipulate.
3. The aid should be small enough to be wieldy.
4. The aid should be psychologically and philosophically sound.
5. The aid should be economical in the use of time.
6. The aid should be relatively inexpensive or easy to make.
7. The aid should tend to arouse interest.

Criteria for the use of field trips

1. Sufficient time should be available for the trip.
2. The group should be the proper size.
3. Transportation and the route should be satisfactory.
4. The individual or group expense should be satisfactory.
5. The group should know what it is to see.
6. The trip should be interesting and worthwhile educationally.
7. The trip should not encounter any unreasonable delays or waste of time.

8. A satisfactory guide service should be provided for.
9. Ample provision should be made for teacher-pupil planning and for teacher-pupil evaluation of the trip.

Criteria for the use of still pictures (projected and non-projected)¹⁴

1. They should help achieve your teaching purpose.
2. They should convey a generally true impression.
3. They should give an accurate impression of relative size.
4. They should add to the pupil’s fund of knowledge.
5. They should stimulate the imagination.
6. They should focus attention upon one main idea.
7. They should have the proper amount of detail.
8. They should be purposeful, relevant, significant.
9. They should be truthful, authentic, up-to-date, and complete.
10. They should be easily understood. They should be simple in composition, proper in emphasis.
11. They should be interesting. They should be such that the pupils will study them seriously.
12. They should be suggestive of reality, actions, contrast, comparison, and continuity.
13. They should be appropriate to the age and grade levels of the pupils.
14. They should be artistic in elements, composition, lines, color and finish.
15. The picture should be a good print. It should be clear and distinct and free from blemishes or soiling.

16. The picture should be of practical size. It should be large enough to show details and small enough to be easily handled and used.

17. If colored, the coloring should be truthfully used for purposes of identification.

Criteria for the use of instructional motion picture films

1. The film should relate to your specific teaching purpose.
2. The film should contribute more effectively than would some other material.
3. The content should be truthful and unbiased.
4. The content should be properly suited to the needs, age, and abilities of the pupils.
5. The mechanical quality of the film should be satisfactory.
6. The film should be such that it can be correlated and integrated with a particular subject and a specific grade level.
7. The film should be produced for a particular purpose.
8. The pictures should be of good quality.
9. The content of the film should be limited to pertinent facts.
10. The film should meet reasonable standards of technical excellence.
11. The film should possess good motivating qualities.

Criteria for the use of non-commercial aids

1. The aid should be such that it can be easily made by the teacher, by the pupils, or by both.

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2. The aid should be such that it can be made with materials on hand or with inexpensive materials that are readily available.

3. The aid should have a multiplicity of use. It should be such that it can be used to explain or give experience with more than one concept or age group.

4. The aid should be worth the time and effort involved in making it.

5. Directions should be available for its construction and use.

6. The physical qualities of the aid should be satisfactory.

7. The aid should be easily manipulated.

8. Whenever possible, it should be made true to life and so constructed that it will actually function.

9. It should really aid learning. It should be meaningful and easily understood.

10. It should be appropriate to the topic studied. It should be such that it can be used again for a different emphasis or expanding knowledge.

11. The cost in both time and energy should be less than the cost of comparable purchased materials.

Criteria for the use of free and inexpensive instructional materials

1. The subject matter should be accurate. There should be a freedom from exaggerated statements.

2. The subject matter should be timely. The information should add to that available in most textbooks.

3. The subject matter should be presented in a factual manner.

4. The subject matter should be unbiased. There should be a clear-cut educational purpose. Posters and charts should be free from obtrusive advertising.

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7 List compiled partly from criteria set up by Jean D. Grambs in, Using Current Materials to Study Current Problems, p. 27.
5. The agency that produced the material should be clearly stated.
6. Sources should be given for quoted facts. The sources should be reputable and authoritative.
7. The date of publication should be given.
8. The materials should single out no particular groups or individuals for derogatory portrayal that is likely to give offense.
9. Pictures, illustrations, and language should be in good taste.
10. The cost, if any, should be commensurate with the educational value of the material.
11. The material should be such that it can be filed for future use.
12. If films and filmstrips, they should be reviewed in standard guides to films. If reviewed, the reviews should be favorable for the use contemplated.
13. The commercial aspects of the material should not outweigh the educational contribution.
14. The material should not support a special-interest point of view without due regard to objectivity.
15. Use of the material should not obligate the school in any way.
16. The material should be in accordance with school policy regarding the use of non-textbook material.
17. The reading level and idea content should be within the range of ability of the class.
18. The material should be such that the mores of the community will not be offended by its use.

Criteria for the use of radio broadcasts

1. The materials presented should contribute to the attainment of the objectives of the educational program.
2. The materials presented should be authentic and accurate in all details.

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8 From a list of criteria compiled by Harry C. McKown and Alvin B. Roberts, *Audio-Visual Aids to Instruction*, pp. 230-33.
3. The program should have unity.
4. The program should be suitable for the grade level for which it is being considered.
5. The program should arouse interest and motivate the listener to read more about the subject presented.
6. The program should summarize the main points and hence fix them in the student's mind.
7. The program should be presented in an interesting manner.
8. The program should be intellectual rather than emotional.
9. The length of the program should be suitable for the listening group.

Criteria for selecting teaching aids for group use

1. The material should benefit all members of the group in their development at this time.
2. The material should develop concepts and deepen insights that are functional in group enterprises.
3. The material should offer rich possibilities for group thinking and discussion.
4. The material should lead the group on to further purposeful research, creative endeavor, experimentation, and the like.
5. The material should further help the group grow in its social orientation and in its processes of working together harmoniously.

Criteria for selecting teaching aids to develop the individual

1. The material should be gauged to the developmental level of the child with whom it is being used.
2. The material should be sufficiently appealing in content, arrangement, and appearance that the child will want to work with it.

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3. The material should be so organized and presented that the child can gain new concepts, get further insight, and make his new learnings relatively permanent.

4. The use of this material should encourage the child to seek further new experiences with such material and to want to extend his learnings in this area of experiences.

5. The material should be such that, in the using of it, the child increases his understanding of processes as well as achieving satisfactory products.

6. The material should be such that the child can work with it with a satisfying sense of accomplishment.

7. The use of this material should improve the child's social adjustment with other children.

Criteria for the selection of instructional aids on a teacher-pupil cooperative basis:

1. The material should be such that it will help the teacher and pupil now better than at a later time.

2. The material should be useful as well as interesting.

3. The material should be such that it will be understood, without being too easy or too difficult.

4. The material should be such that it will give us some new ideas that will push our study ahead.

5. The material should be such that it will be of value to the group as a whole rather than to only one individual or to a small group.

6. The material should give the pupils a contact with a kind of instructional material with which they have need for further experiences.

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11 Ibid., p. 221.
Criteria for the selection of teaching aids that will lead to balanced development in children

1. The material should better help the child in his learning than another material would.
2. The instructional materials chosen should be such that they facilitate learning and the promotion of a friendly and helpful attitude among the group even when selected for the particular needs of an individual child.
3. The instructional material selected should be of a kind that has not been used too frequently or to the exclusion of other valuable materials.
4. Consideration should be given to the types of teaching aids the children need to extend further their experiences with various kinds of available instructional materials.
5. The material should be chosen in the interests of a well-balanced and integrative program rather than upon the basis of purely personal interests of the teacher or a child.
6. The material should be such that it will promote the integration of the whole program of the group.
7. Attractive and helpful materials should be secured for each area of the school program.

The Development of Specific Criteria for Selecting and Using Arithmetic Instructional Aids. One of the three subsidiary purposes of the present study was the development of criteria for the selection and use of arithmetic instructional aids. More than one possibility presented itself in this regard. Serious consideration was given, for instance, to examining the foregoing and other lists of criteria and selecting those which the writer considered to be especially pertinent in terms of their specific relation

\[12\text{ Ibid., p. 219.}\]
to the instruction of elementary-school arithmetic. This plan was not used, however, because it was felt that there was no really significant benefit to be derived from repeating criteria already capably stated.

The emphasis throughout this study has been largely on the psychological aspects surrounding the selection and use of instructional aids related to the elementary-school arithmetic program. Great stress has been placed on the selection and use of aids appropriate to the operational and developmental levels of the learners. In view of this circumstance, and the fact that criteria embracing these features of the study are seemingly least referred to in the literature, the writer felt that most benefit would be derived from the development of criteria for selecting and using arithmetic instructional aids which were based primarily upon the operational and developmental aspects of the instructional-aids program.

Webster defined criterion as "a standard of judging; a rule or test by which facts, principles, opinions, and conduct are tried in forming a correct judgment respecting them." Criteria, it is generally conceded, must start with objectives. A criterion can be valid only in terms of a specific objective. Sloan\(^{13}\) stated:

Any discussion of validity must recognize that there is no such thing as "validity" without some qualification. No test or technique is valid except in the sense that it is valid for some purpose. A given test may be valid for several different purposes and the validity in each instance may differ. Thus, we speak of different "validities" and should specify the kind of validity which we have in mind.

Accordingly, a criterion, just as any other measuring stick, must be created to measure a specific thing. A carpenter's rule ordinarily does a satisfactory job in ascertaining the length of a board. However, it would be entirely worthless in measuring the temperature of a room. The specific objectives used as the mathematical bases of the present study, the reader is reminded, is the list of twenty-three developmental concepts of arithmetic proposed by Professor Harding. Accordingly, these concepts formed the bases for the development by the writer of criteria by which teachers might gauge their attempts to select and use instructional aids in the teaching of elementary-school arithmetic. In general, the application of instructional aids to the attainment of the twenty-three developmental concepts might be stated as follows:

1. Approximation or estimation. The instructional aid should tend to make meaningful the concept of approximation and estimation; it should help pupils grow in the ability to make approximations and estimations.

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14 Lowry W. Harding, Functional Arithmetic: Photographic Interpretations, pp. 54 ff.
2. Average. The instructional aid should promote pupil understanding of the concept of averages; it should help pupils grow in the ability to compute averages.

3. Counting and ranking. The instructional aid should promote pupil growth in an understanding of the concept of counting and ranking; it should help pupils grow in the ability to count and rank quantitative amounts and numbers.

4. Formulas and algebraic symbolization. The instructional aid should promote pupil growth in understanding formulas and algebraic symbols; it should help pupils grow in the ability to use formulas and symbols effectively.

5. Fractions, common. The instructional aid should help pupils grow in an understanding of, and the ability to work with, common fractions.

6. Fractions, decimal. The instructional aid should help pupils grow in an understanding of, and the ability to work with, decimal fractions.

7. Geometric forms. The instructional aid should help pupils grow in the understanding of, and the ability to work with, geometric forms.

8. Graphic representation. The instructional aid should help pupils grow in an understanding of graphic representations and the ability to work with them effectively.

9. Grouping and number combinations. The instructional aid should help make the concept of grouping and number combinations meaningful to pupils; it should help them grow in the ability to group numbers more effectively.

10. Identification; Meaning of number symbols. The instructional aid should help make the various number symbols meaningful to pupils; it should help them grow in the ability to identify them quickly, accurately, and meaningfully.

11. Measurement: linear, quantity, temperature, time. The instructional aid should make meaningful the concept of measurement as applied to linear, quantity, temperature, and time concepts; the aid
should promote growth in pupils of the ability to compute such measurements effectively.

12. **Money and business practices.** The instructional aid should promote a developmental understanding of money and business practices; it should help pupils grow in the ability to solve "money" problems effectively.

13. **Number system:** organization, serial idea, decimal nature. The instructional aid should help to make meaningful the concept of our number system with reference to its organization, serial idea, and decimal nature.

14. **Operations** (addition, subtraction, multiplication, division). The instructional aid should help pupils to grow in an understanding of, and the ability to work with, the "fundamental processes" of arithmetic.

15. **Percentage.** The instructional aid should promote growth in understanding of the concept of percentage; it should help pupils grow in the ability to compute percentages.

16. **Place value and face value.** The instructional aid should promote growth in understanding of place value and face value of numbers.

17. **Problem solving.** The instructional aid should promote understanding of the concept of problem solving; it should help pupils to solve problems in a more meaningful way.

18. **Proof and generalization.** The instructional aid should promote understanding of the nature of proof and generalization; it should help pupils to verify their calculations.

19. **Quantitative thinking:** Mathematical judgment. The instructional aid should promote growth of understanding of the nature of quantitative thinking and the ability to make mathematical judgments.

20. **Ratio and relationship.** The instructional aid should promote growth in understanding of the nature of the concepts of ratio and relationship; it should promote growth in the ability of pupils to solve problems involving ratio and relationship.
21. **Reading and writing of number ideas.** The instructional aid should promote understanding of the concept of reading and writing of number ideas.

22. **Vocabulary of number and quantity.** The instructional aid should promote understanding of the various concepts of number and quantity.

23. **Zero representation.** The instructional aid should promote understanding of the function of zero in our number system.

To develop criteria for the selection and use of instructional aids in arithmetic education, based on the operational and developmental aspects of the learning process, the writer proceeded as follows: The first of the twenty-three developmental concepts of arithmetic, "approximation or estimation," was considered, a thorough search of the literature being made to find psychological bases for the development or adaptation of criteria applicable to the learning process with regard to this specific concept. Pertinent criteria were recorded. The same procedure was then used with the second-named developmental concept of arithmetic, "average." The writer then discovered, what had not occurred to him at first, that the same criteria from the operational and developmental point of view, were as equally applicable to the concept "average" as to "approximation or estimation." A close examination of criteria of a psychological nature applicable to the remainder of the twenty-three developmental concepts further substantiated this discovery. The criteria applicable to the selection and use of instruc-
tional aids with reference to the first-named developmental concept were as equally applicable to all twenty-three concepts. All the criteria developed or adapted by the writer, with the exception of numbers 1, 9, and 10, relate to the psychological aspects of the learning of arithmetic. No. 1 criterion is necessary in order to relate the application of instructional aids to the specific developmental concepts of arithmetic, it having been emphasized earlier that a criterion can be termed valid only in terms of a specific objective. The list of criteria developed or adapted by the writer follow.

Criteria for the Selection and Use of Instructional Aids in Arithmetic Education

1. The instructional aid should tend to help the pupil grow in understanding of one or more of the twenty-three developmental concepts of arithmetic.

2. If possible, the operational level of the pupil with regard to a specific concept or process of arithmetic, should be ascertained before making use of instructional aids.

3. In working with instructional aids related to a specific arithmetic concept, the teacher should utilize the principle of "starting where the child is."

4. Instructional aids should be appropriate to the operational and developmental level of the pupils using them.

5. The use of instructional aids should progress from the familiar to the less familiar.

6. With regard to a specific arithmetic concept or process, pupils should signify their understanding of instructional aids used at one operational level before
proceeding to the use of instructional aids at the next higher operational level.

7. Pupils should not be required to continue working with instructional aids of a lower-level operational nature after having proved themselves ready to work with aids appropriate to a higher level.

8. With regard to a specific arithmetic concept or process, a variety of appropriate instructional aids should be used at each operational level of learning.

9. Instructional aids should be used as an integral part of the instructional procedure rather than as an added activity or as an activity set apart for certain days or occasions.

10. Appropriate instructional aids should be readily accessible when needed.

Validity of the Criteria. It has been stated in an earlier portion of the study that there are no principles of learning peculiar to the use of teaching aids. That is, the same principles apply to the utilization of instructional aids in the teaching-learning process as in all good teaching procedures in general. Upon examination of the preceding criteria developed or adapted by the writer, it may be readily observed that each, with the exception of the first, is in reality only a restatement of an already commonly-accepted principle of learning, with specific reference to the use of instructional aids in arithmetic. The criteria have evolved as a result of the philosophical point of view that instructional aids are not extrinsic to the pupil's elementary-school arithmetic experiences, but instead are an integral part of it. As such, the use of
instructional aids in arithmetic are ruled by the same principles governing the whole teaching-learning act. These principles, stated in other forms, are emphasized throughout the literature of modern education, and several books dealing with this aspect of education were perused by the writer preparatory to compiling the foregoing list. To the writer's knowledge, this is the only attempt made to compile "principles-of-learning criteria" specifically pointed to the use of instructional aids in the elementary-school arithmetic program.

Building the Objective Testing Instrument

Any attempt at constructing a test of understanding in a subject area must obviously be considered as representing only a very restricted sampling of all the items involving pertinent subject matter. No single test could be constructed which would measure each of its many elements directly and independently in separate test items.

Determining What to Test. The first step taken by the writer in devising test items to ascertain prospective-teacher understanding of the use of instructional aids in arithmetic was to make an outline of topics which, in his judgment, based on an analysis of pertinent literature, should be included in such a test. In making the outline, the writer was faced with a dual task. First, it was necessary to select a valid list of objectives for the
elementary-school arithmetic program, it being generally agreed that the test constructor should start with objectives (values which individuals or groups hold). In an earlier chapter the writer has listed and discussed a number of excellent statements of elementary-school arithmetic objectives, as conceived by both individuals and various groups. Once objectives are set up, it is necessary to think of them in terms of measurable behaviors. Accordingly, because of the measureability of each component concept in the list, as well as the comprehensive coverage of the complete list, the twenty-three "developmental-concepts" of elementary-school arithmetic proposed by Professor Harding of the College of Education, The Ohio State University, have been selected because they meet these conditions. Greene, Jorgensen, and Gerberich justify the use of such a list of objectives in these terms: "Reports of national committees and the writings of subject and test specialists often provide excellent foundations for standardized and informal objective test construction."

The second task of the writer was to investigate the various types of instructional aids which tend to contribute to the attainment of the stated arithmetic objectives.

15 Harry A. Greene, Albert N. Jorgensen, and Raymond J. Gerberich, Measurement and Evaluation in the Elementary School, p. 68.
For this purpose, classifications of aids commonly referred to by authorities in this subject area were selected. For the sake of more specific treatment, the classifications used by the writer in constructing the Test-Item Outline are in some instances broken down more specifically than are usual classifications of aids. In essence, however, no classifications are added or deleted. Briefly, the process of devising initial test items followed somewhat the following pattern:

1. Select a valid list of arithmetic objectives for the elementary-school program.

2. Select a valid list of types of instructional aids which may contribute to the attainment of the stated objectives.

3. Attempt to pattern test items in terms of a relationship between a stated objective of the elementary-school arithmetic program and a specific instructional aid (or a specific classification of instructional aids) which may contribute to the attainment of the stated objective.

A specific example of the above procedure is presented below.

1. One of the objectives of the elementary-school arithmetic program (that is, one of the twenty-three "developmental concepts" proposed by Professor Harding) is an understanding of the concept of common fractions.

2. It has been experimentally revealed that the use of "fraction pies" (discs) tends to promote an understanding of the concept of common fractions.

3. Therefore, a test item based on teacher understanding of the use of "fraction pies" in clarifying the concept of common fractions has "face validity," and may justifiably be included.
However, not all items could be devised from such a neatly dovetailed relationship as in the above example. Some of them were based upon the testimonials of teachers' classroom experiences rather than upon findings from controlled experiments. Others were based upon philosophical studies, it being generally understood, as expressed earlier in this investigation, that a number of instructional aids do not lend themselves to experimental study.

Table 5 presents a skeletal outline devised by the writer to serve as a guide in making test items. The Y axis consists of the arithmetic topics (the twenty-three developmental concepts) on which the items were based. In addition to the twenty-three specific developmental concepts, the writer has added, for use in the present study only, two other topics of a general nature. The first, "Application to arithmetic concepts in general," was included because some instructional aids may have a multiple use in attaining the stated arithmetic objectives. The second, "General nature of learning in arithmetic," has been added because some instructional aids apply to the process of learning in general. The X axis of the outline consists of various classifications of teaching aids which tend to contribute to an understanding of certain arithmetic topics located on the Y axis. As has been previously mentioned, the types of instructional aids listed do not follow the usual classifications of such aids but are
### TABLE 5

**TEST-ITEM OUTLINE**

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<thead>
<tr>
<th>Arithmetic Topics</th>
<th>Classifications of Aids</th>
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<td></td>
<td>Direct Experiences</td>
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<td>Manipulative</td>
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<td>Field Trips</td>
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<td>Miscellaneous</td>
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<td>1. Approximation or estimation</td>
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<td>5. Common fractions</td>
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<td>9. Grouping and number combinations</td>
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<td>10. Identification: Meaning of number symbols</td>
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<td>11. Measurement: Linear, quantity, temperature, time</td>
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<td>12. Money</td>
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<td>13. No. system: Organization, serial idea, decimal nature</td>
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<td>14. Operations: Addition, subtraction, multiplication, division</td>
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<td>17. Problem solving</td>
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<td>18. Proof and generalization</td>
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<td>19. Quantitative thinking, mathematical judgment</td>
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<td>24. Application to arithmetic concepts in general</td>
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<td>25. General nature of learn. in arith.</td>
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prepared more specifically for the purpose of the Test-Item Outline only.

The Test-Item Outline having been completed, the problem of the type or types of questions to be constructed was next considered. The literature revealed that test specialists generally favor the multiple-choice type of objective test item. Although not easily constructed, it offers several advantages over other varieties of objective items. Travers\textsuperscript{16} maintained: "Nearly every function which the true-false item or the completion item can perform is better executed by the multiple-choice question." Greene, Jorgensen, and Gerberich\textsuperscript{17} also emphasized that

It is readily, although not easily adaptable to the measurement of discriminating power, inferential reasoning, interpretative ability, reasoned understanding, generalized ability, and other types of outcomes deriving from the pupil's ability to apply and use facts.

A further point sometimes expressed is that the multiple-choice type of item is valuable because it tends to present a life-like situation in that, real life frequently involves making a choice of several available answers rather than being a matter of "either . . . or."

\textsuperscript{16} Robert M. W. Travers, \textit{How to Make Achievement Tests}, p. 60.

\textsuperscript{17} Greene, Jorgensen, and Gerberich, \textit{op. cit.}, pp. 177-78.
In view of the foregoing attributes of the multiple-choice type of test item, and in spite of the fact that such items are difficult and time-consuming to devise, the writer felt that this kind of item was best suited to the construction of a test to ascertain teacher understanding of the use of aids in arithmetic instruction.

**Preparation of the Items.** The most fundamental problem in preparing test items is to obtain satisfactory criteria by which to validate them. Test items can be only as good as the criteria by which they are measured. Finding a number of applicable criteria which experts had already adjudged to be valid, the writer took the liberty of selecting and adapting certain of these for the present study. Only criterions No. 1 and 2 below were created *in toto* by the writer. The others have been adapted from a number of sources.18 Criteria which appeared to be most applicable to the devising of test items to be used in the study are:

1. Initial test items should relate to all topics possible listed in the Test-Item Outline; an attempt should be made to give equal coverage to each topic.

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2. Initial test items should relate to all classes of aids possible listed in the Test-Item Outline; an attempt should be made to give equal coverage to each class.

3. Insofar as possible, the items should require the application of understandings to realistic school situations and not be limited to isolated or abstract concepts and principles.

4. The items should refer to meaningful and significant understandings which are peculiar to the elementary-school arithmetic program (that is, they should have "face value") and not be merely measures of general intelligence.

5. The items should be developed in keeping with the accepted technical procedures of multiple-choice item construction such as: (a) the use of correct grammar, (b) the avoidance of clues and suggestions, (c) the avoidance of "a" and "an" to introduce the options, (d) the avoidance of "catch words," (e) the inclusion of as much of the statement as possible in the stem, and (f) a random distribution of the correct responses among the option positions.

6. Insofar as possible, the items should be concisely stated.

7. There should be only one option that provides the indisputably correct answer to the item.

8. All items should have five alternate answers.

9. Items should be based upon both discriminatory power and difficulty level.

10. Insofar as possible, the options offered as possible answers should be plausible and within the same general context.

11. Items should not deal with matters of opinion unless there is general agreement among authorities regarding the correct answer.

The preparation of initial items was begun by the writer during the Winter Quarter, 1955, as a partial requirement of the advanced graduate course, "Evaluation in
Higher Education." As items were constructed, duplicate copies were furnished the instructor and each student. Thereon each item submitted was critically analyzed by students and instructor, with the emphasis on the curricular validity of the item, clarity and conciseness of the statement of both stem and options, the suitability of the optional answers, and other points. The criticism and the suggestive recommendations received were of inestimable value to the writer, not only with reference to the specific items receiving criticism but also in regard to other items yet to be devised. One half of the items used in the completed test forms were devised while the writer was a student of this course.

The immediate goal of the writer was the devising of a large number of multiple-choice items from which 120 of the best would be selected to construct two preliminary 60-item forms of a test to be administered to students in Education 510, an undergraduate course in arithmetic education for prospective elementary-school teachers, offered by the College of Education, The Ohio State University.

Sources utilized for the construction of items were principally arithmetic education and visual-aids textbooks, experimental studies involving the use of arithmetic aids reported in various periodicals, reports of classroom teachers' first-hand experiences with various arithmetic aids, and the philosophical writings of arithmetic and
instructional aids specialists. Every available report of pertinent experimental investigations especially was examined to ascertain whether or not a specific instructional aid or class of instructional aids had proved to be effective (or ineffective) in attaining a stated objective of the elementary-school arithmetic program, thereon giving rise usually to one or more test items.

It was the writer's original intention to devise test items covering every arithmetic topic and instructional aid listed in the Test-Item Outline. Frequently, however, the writer found himself in possession of, what he considered, a good question, constructed independently of the Test-Item Outline. The result was a case of attempting to determine whether on the basis of the outline he could justify using such an item. In other words, it was an instance of the means coming before the objective. Thus, making items was in reality an open-ended matter—at times starting with objectives and working toward the construction of pertinent questions, at other times starting with test items and justifying or rejecting them on a basis of the outline objectives. It may be readily deduced that the coverage of arithmetic topics and instructional aids was not uniform. However, the writer felt that it was reasonably so.

After having constructed a large number of items, the next procedure was to select the best ones to compile into
the two proposed preliminary test forms. The items were allotted to Form A or Form B on the basis of the Test-Item Outline. At this stage of the process there were no bases for allotting items to one form or the other in terms of difficulty level and discriminatory power. Rather, it was more a matter of attempting to select items for each test form which pertained to each stated arithmetic topic and to a varied application of instructional aids. The writer had the impression in the beginning that a completed preliminary test form would contain an approximately equal number of items covering the various arithmetic topics and classifications of aids. However, the completed forms did not evolve in such a uniform manner. Some of the arithmetic topics lend themselves better to the employment of one aid than to another. On the other hand, some of the aids, as might be expected, have a general use in attaining the stated arithmetic objectives. Table 6 presents a scattergram of the classifications of items used in Test Form A, while Table 7 does likewise for Test Form B. It needs to be mentioned that the categorization assigned the various items is not iron-clad by any sense of the word.

Administration of the Preliminary Test Forms. After the writer had edited and revised the test items numerous times during the process of their evolvement into preliminary test forms and had checked them against the criteria, they were submitted to advanced graduate students
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<th>Arithmetic Topics</th>
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<td>21. Reading and writing of number ideas</td>
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**TOTALS**

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in elementary education and to staff consultants from whom helpful suggestions were received. When it was felt that the test forms were reasonably well prepared, sufficient copies were duplicated to administer to three sections of Education 510, the object being to obtain statistical data upon which to construct one evolving test form, which in addition to content validity, would possess statistical validity in terms of difficulty level and discriminatory power of the various items used.

The reader is reminded that the specific aim of this portion of the study was to construct one valid and reliable form of a multiple-choice type test which would ascertain certain aspects of prospective-teacher understanding of the use of instructional aids in the teaching of elementary-school arithmetic. Being valid implies that the test possesses both curricular or content validity, as deduced by the logical analysis of the author and the opinion of experts in the subject, as well as statistical or objective validity. "Face-Validation" was used by the writer in selecting items from various sources to include in the test forms. Whenever items are based upon the writer's own logical analysis of what is to be evaluated or measured, without subjecting them to comparison with external standards, "face-validity" is being used as the
criterion. In other words, "face-validity" means that the item appears to measure what it is supposed to measure or to measure what is popularly understood from the title. A test item which asks the testee to select the name of the President of the United States from among 5 alternate answers would, for example, have no "face-validity" in a test which supposedly is testing prospective-teacher understanding of the use of aids in the teaching of elementary-school arithmetic. It would not be testing what it was supposed to test.

The writer has already discussed ways in which he attempted to establish curricular validity of the various items used in the two preliminary test forms. Now he will explain procedures employed to establish objective or statistical validity of the various items, based on the resulting scores secured during the initial tryouts. It is generally agreed by test specialists that curriculum or content validity is of foremost importance. No amount of statistical validity can compensate for an objective test item which is basically inferior in content. On the other hand, statistical validity is the objective means of determining the degree to which success or failure on an item indicates possession of the ability which is being

measured. There is no one generally acceptable method of validating a test item. Instead, there are several methods, some of which have several variations.

Since objective evidence of the validity of test items may be determined only after the test has been administered, the next step was to have preliminary tryouts of the completed forms on a typical population. Since the final form was to be an inventory of the understanding possessed by prospective teachers, it appeared logical that the preliminary test forms would produce most valid results if the initial tryouts were made in terms of undergraduates who were preparing to become elementary-school teachers. At the time, the writer was Graduate Assistant to Professor Lowry W. Harding of the College of Education, The Ohio State University, among his duties being assisting with prospective elementary teachers in Education 510, an undergraduate course in the teaching of elementary-school arithmetic. In view of these facts, it was decided to use these groups as testees for the preliminary tryouts. Accordingly, in the Autumn Quarter, 1955, the two preliminary forms were inter-piled and administered to 105 undergraduate prospective elementary-school teachers comprising three sections of Education 510. The purpose of this preliminary tryout, it should be borne in mind, was to obtain objective evidence upon which to base the selection of the 60 best items (of the 120 used in the two preliminary
forms) with which to construct one form of a test possessing both content and statistical validity.

The tests were administered by the writer personally during the regular class period. The students were advised to attempt each item, whether or not they were certain of the answer. This advice was given because some of the items were descriptions of specific situations which called for the application of general principles rather than specific facts in determining the best alternate answer. It was felt that even though the student may not be able to react immediately to the question, the application of his general understandings with regard to the situation would constitute more than a chance guess.

**Preparation of the Final Test Form.** Following the scoring of the preliminary tests, the writer proceeded to run an item analysis on each of the 120 items used in order to select the 60 best items to develop a third improved test form. Test specialists seem to be in rather general agreement that, statistically speaking, the best test items have the following characteristics:

1. They have been answered correctly by a majority of the "good" students and incorrectly by a majority of the "poor" students.

2. They discriminate significantly between the "good" students and the "poor" students.

3. They have an average difficulty at or near the 50 per cent level, with a spread from the very easy to the very difficult.
The procedure used to determine which of the 120 items used in the tryout tests met the above requirements was as follows: The resulting scores on each test form were arranged in descending order, after which the Upper-Lower 27 Percent Technique was applied. This procedure consists of selecting the test papers of the 27 percent of students making the highest scores and comparing their performance on each test item with the 27 percent of students making the lowest scores. For example, an item which is answered correctly by the upper group and incorrectly by the lower group is one which differentiates between the groups. If 70 percent of the upper group answer an item correctly while only 30 percent of the lower group do so, the item meets both requirements No. 1 and 2 above. That is, it was answered correctly by a majority of the "good" students and missed by a majority of the "poor" students and at the same time has significant discriminating power. An item might very conceivably meet requirement No. 1 and not meet requirement No. 2, or vice versa. For example, an item answered correctly by 52 percent of the upper group and by 49 percent of the lower group would meet requirement No. 1 but not No. 2 because it has a low discriminating power. According to Smith, a items with a discriminating power...

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around zero or negative are of little use in testing. On the other hand, an item might meet requirement No. 2 and not No. 1. For example, an item answered correctly by 40 percent of the upper group and by 5 percent of the lower group has significant discriminating power even though it does not meet the requirement relating to being answered correctly by a majority of the upper group.

It is generally agreed that the term "significant discriminating power" has no set interpretation. In different circumstances and with different sets of data it may have widely different meanings. A discriminating power of 25 might be considered extremely low in one instance and high in another. Generally speaking, when the writer of the present study mentions that a discriminating index is significant, it should be interpreted as meaning that it is among the highest of such discriminations relating to the 120 items used in the two test forms.

Various other divisions of "good" and "poor" students have been used by test specialists in determining the discriminatory power of test items. Among such divisions have been the upper and lower 50 percent, the upper and lower 33 1/3 percent, the upper and lower 25 percent, the upper and lower 10 percent, and others. However,
Kelley maintained:

... If no distinction is made among the members of our upper and lower groups separately, when studying the performance of items, that we should, in general, select the twenty-seven percent highest on the criterion measure for the upper group and the twenty-seven percent lowest for the lower group.

Forlano and Pinter, after having made a study of upper and lower groups based on 50 percent, 33 1/3 percent, 27 percent, 16 percent, and 7 percent, concluded that, "... for a simple and rapid, rough and ready method of item validation of test items of the inventory type, the Upper and Lower 27 Percent Method is to be preferred, even though the distributions are more or less normal."

After selecting the upper and lower 27-percent groups for each form of the test, the performance of these groups on each item in the test was determined with reference to the following data:

1. No. of students in the upper group who answered each item correctly.
2. No. of students in the upper group who answered each item incorrectly.
3. No. of students in the upper group who omitted each item.


22 George Forlano and Rudolf Pinter, "Selection of Upper and Lower Groups for Item Validation," Journal of Educational Psychology, XXXII (October, 1941), pp. 544-549.
4. No. of students in the upper group who did not reach each item.

Comparable information was also secured for the lower group. Using the above data, the proportions of successes in the highest and lowest 27-percent groups were then determined in terms of a percentage score which was corrected for chance guesses and to take into account the fact that some items were omitted by the testees and that some of the slower testees did not reach every item in the test. The formula used to secure the corrected scores is as follows:

\[
P = \frac{R - W}{K - 1} \frac{N - NR}{N}
\]

where:

- \(P\) = the corrected percentage of the group responding correctly to the item,
- \(R\) = the number of testees who answered the item correctly,
- \(W\) = the number of testees who answered the item incorrectly,
- \(K\) = the number of alternate choices in the multiple-choice item,
- \(N\) = the number of testees in the group,
- \(NR\) = the number of testees who did not reach the item in the test

Specifically, with reference to one item used in Test Form B of this study, of 14 testees in the group, 10

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answered the item correctly, 2 answered it incorrectly, 1 omitted it, and 1 slow reader failed to reach the item. All items had 5 alternate answers. Computation of the corrected percentage of success of the group on this specific item was as follows:

\[
P = \frac{10 - \frac{2}{4}}{13 - 1} = \frac{10 - 2/4}{13} = \frac{19}{26} = .73 = 73\%
\]

Davis\(^{24}\) pointed out that item-analysis data that do not include such adjustments as the above are often highly misleading. For example, a difficulty index based only on the performance of individuals who marked an item with testees who omitted the item and testees who didn't reach the item, both being dropped from the computations, will be spuriously large (except in the special case when it is zero).

If an item is so difficult for the members of the group in which it is tried out that fewer subjects in both upper and lower groups answer it correctly than would be expected by chance, the corrected percentages of success of both groups would be negative. However, upon the recommendation of Davis,\(^{25}\) no negative corrected scores were recorded, there being no straightforward means of using Davis' Item-Analysis Chart if for a given item the signs

\(^{24}\) Ibid., p. 32.

\(^{25}\) Ibid., pp. 35-36.
of the two proportions are different. According to Davis, "This is a limitation of all similar short-cut methods of item analysis when this correction for chance is employed, but it is not a serious limitation; in most situations the advantages of using the correction for chance outweigh its disadvantages." Accordingly, he suggested that when the percentage of successes in the lowest 27 percent of the sample is 0 or negative, a 4 percent success be recorded instead (that is, in cases where the number of testees in the group is comparable to that used in the present study). Likewise, when the percentage of successes in the highest 27 percent of the sample is closest to 100 he recommended that a 96 percent corrected percentage be recorded for a group of the size used in the present study. Otherwise, if the percentage of successes in both the highest and lowest percent of the sample is 100, the discrimination index is 0. If both percentages are zero, the discrimination index is 0.

After computing the corrected percentages of successes of the upper and lower groups on each item, the next step was to consider the problem of the relationship of the successes of the upper and lower group, that is in terms of the discriminatory power of each test item. Various methods have been employed to secure such a discrimination

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26 Ibid., p. 38.
index. Smith\textsuperscript{27} simply subtracted the percentage of successes on each item by the lower group from that of the upper group (or vice versa), using the difference as the discrimination index for that item. For instance, if 80 percent of the upper group answered an item correctly whereas only 30 percent of the lower group answered it correctly, the discrimination index would be 50 (since $80 - 30 = 50$). Thorndike\textsuperscript{28} stated that the most satisfactory item validity index based on the upper and lower 27 percent is the estimate of the coefficient of correlation between item and test obtainable from tables prepared by Flanagan.\textsuperscript{29} Flanagan's table, which was reproduced by Thorndike,\textsuperscript{30} makes it extremely simple to compute item validity coefficients from the percentages of success in the upper and lower 27 percent by following the appropriate row and column of the table to the point of intersection, that figure representing the discrimination index of the groups with respect to that item.

\begin{itemize}
\item \textsuperscript{27} Smith, \emph{op. cit.}, p. 21.
\item \textsuperscript{28} Robert L. Thorndike, \emph{Personnel Selection}, p. 242.
\item \textsuperscript{29} J. C. Flanagan, "General Considerations in the Selection of Test Items and a Short Method of Estimating the Product-Moment Coefficient from the Data at the Tails of the Distribution," \emph{Journal of Educational Psychology}, XXX (December, 1939), pp. 674-680.
\item \textsuperscript{30} Thorndike, \emph{op. cit.}, Appendix B, pp. 347-351.
\end{itemize}
However, Thorndike pointed out that one limitation in the use of any correlation coefficient as a measure of degree of relationship between two variables is the fact that units on the scale of correlation values do not have the same significance as one goes from small to large coefficients. For example, the change in correlation as one goes from 0.20 to 0.25 is not comparable to the change as one goes from 0.90 to 0.95. In an effort to compensate for this limitation in the units in which correlation coefficients are expressed, Davis developed an item index which is based on Fisher's z-transformation of the correlation coefficient. Flanagan's correlation values are converted into z values and multiplied by a constant such that the index becomes 100 when 99 per cent of one group and only 1 percent of another succeed with an item. This procedure yields indices in which the units are approximately equal throughout the scale. The indices also are convenient in that there are no decimal points to cause confusion or error. The procedure for determining the discrimination index from the Item-Analysis Chart devised by Davis is similar to that employed with Flanagan's Table. That is, simply follow the appropriate row and column of the chart to the point of intersection and note the discrimination-index number located at that point.

31 Davis, op. cit.
In addition to computing the successes of the upper and lower groups in terms of each item, it is necessary to determine the difficulty level of each item, it being essential to know whether the difficulty of an item used in a test is suited to the group for which it is intended. By item difficulty is meant the percentage of individuals able to answer the item correctly. It is the general opinion of test specialists that in practice an item should not be so easy that all persons can pass it; nor should it be so difficult that none are able to pass it. However, Freeman\(^2\) remarked:

Theoretically, it would be desirable that the test be so scaled that there is at least one item which can be passed by all for whom the test is intended. For zero scores on a particular test do not necessarily mean absolute zero capacity in the process being measured; nor will all zero scores necessarily signify the same condition. Conversely, it would be desirable that a test be scaled upward to a level where no one for whom the test is intended is able to pass the highest item.

While it can be shown statistically that an item of 50 percent difficulty discriminates between more pairs of persons than does an item passed by a smaller or larger group, it is obvious that items used should not be restricted to the 50 percent difficulty level. Flanagan\(^3\) remarked: "We may dismiss the notion that all items should

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\(^2\) Freeman, *op. cit.*, p. 25.

\(^3\) Flanagan, *op. cit.*, p. 676.
be 50 percent difficulty as one based on a hypothetical situation which is contrary to fact." It is the general feeling of test specialists that some items should be included that have been passed by a large percentage of testees and some by only a small percentage, with many degrees between. Freeman\textsuperscript{34} maintained:

There is no formula for determining the exact distribution of item difficulties; but the common practice is to select those items the average difficulty of which is at or near the level of 50 percent, with a wide degree of scatter. The final consideration will be the highest predictive value when compared with the criteria.

The common method employed to determine the difficulty level of an item is to divide the number of testees answering the item correctly by the total number of testees attempting the item. Thus, if 105 testees attempted an item, but only 55 answered it correctly, the difficulty level of that item would be .53 (since 55 ÷ 105 = .53).

For convenience, however, Davis has included in his Item-Analysis Chart the estimated difficulty level of items along with the discrimination index, both computed on the basis of percentages of successes of the upper and lower groups after correction for chance guesses and to account for omissions and items not reached. The difficulty levels assigned to the various items in the two preliminary test forms used in this study were computed from this chart.

\textsuperscript{34} Freeman, \textit{op. cit.}, p. 25.
and may differ somewhat from difficulty-level indices based on a division of correct answers by the attempts made.

The percentages of successes of the upper and lower groups with reference to each test item in Form A and Form B, corrected to account for chance guesses and for omissions and items not reached, are presented in Tables 8 and 9, pages 203 and 204. Also included is the difficulty level and the discrimination index, computed on the basis of Davis' Item-Analysis Chart, and a column in which is designated whether an item has been adjudged significantly discriminatory to justify its inclusion in the final test form to be constructed. It has been stated earlier that the term "significant," as applied to discrimination, has no set interpretation. However, Davis\textsuperscript{35} declares that items with discrimination indices above 20 (as computed on the basis of his Item-Analysis Chart) will ordinarily be found to have sufficient discriminating power for use in most achievement and aptitude tests. All items discriminating to this extent were indicated by "yes" in the appropriate column mentioned above. Sixty-five items were found to have a discrimination index above 20. Actually only 37 of the 65 items having significant discriminatory power met the statistical requirement, mentioned earlier.

\textsuperscript{35} Davis, \textit{op. cit.}, p. 15.
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* Based on corrected percentages of success of upper and lower 27 percent.
relative to the criterion that the best test items are those which have been answered correctly by a majority of the "good" students and incorrectly by a majority of the "poor" students. It was felt, however, that this failing did not constitute a serious statistical defect in view of the fact that all items met the minimum significant index of discrimination. Moreover, the average difficulty level of all items used in the final test form was 49, with a difficulty range from 76 to 25, thereby satisfying adequately the requirement that the item-difficulty level of test items used should be at or near 50 percent, with a wide spread.

To devise a 60-item final form of the test, 5 additional items were eliminated because their content was similar to that of other items used. In this manner the 60 items which made up the final form were selected.

Having selected the items to be included in the final form, the next step was to arrange the items in the form according to their difficulty level. Remmers and Gage stated: "Within each type of test item, the order of presentation should be from 'easy' through 'moderately difficult' to 'most difficult.'" Accordingly, the 60 items were ranked according to their difficulty level, from the

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least difficult to the most difficult (that is, from those items having a difficulty index of 76 to those having a difficulty index of 25), and were entered in the final test form in that order.

Since the test items in the improved Test Form C were comprised of a varied assortment of items from both Forms A and B, it was thought advisable to re-randomize the "best answer" positions. The procedure followed was simply a repetition of the randomization process initially used for Forms A and B. Five marbles, each a different color, were placed in a bag. A numerical "positional value" (1 to 5) was assigned each marble. A paper was numbered 1 to 60 inclusive to correspond to the sixty test items of Form C. The bag was then shaken, after which one marble was drawn out. The number which had been assigned to the marble of that color was placed after number 1, after which the marble was returned to the bag and the procedure repeated. The process continued until the "best answer" of each of the 60 test items had been assigned a random position. The completed test Form C has been placed in Appendix C. The categorization of the component items with reference to the arithmetic topics and the instructional aids embodied, is presented in Table 10.
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</tr>
<tr>
<td></td>
<td>Miscellaneous</td>
</tr>
<tr>
<td></td>
<td>Total</td>
</tr>
<tr>
<td>1. Approximation or estimation</td>
<td></td>
</tr>
<tr>
<td>2. Averages</td>
<td></td>
</tr>
<tr>
<td>3. Counting and ranking</td>
<td></td>
</tr>
<tr>
<td>4. Formulas and algebraic symbolization</td>
<td></td>
</tr>
<tr>
<td>5. Common fractions</td>
<td></td>
</tr>
<tr>
<td>6. Decimal fractions</td>
<td></td>
</tr>
<tr>
<td>7. Geometric form</td>
<td></td>
</tr>
<tr>
<td>8. Graphic representation</td>
<td></td>
</tr>
<tr>
<td>9. Grouping and no. combinations</td>
<td></td>
</tr>
<tr>
<td>10. Identification: Meaning of number symbols</td>
<td></td>
</tr>
<tr>
<td>11. Measurement: Linear, quantity, temperature, time</td>
<td></td>
</tr>
<tr>
<td>12. Money</td>
<td></td>
</tr>
<tr>
<td>13. Number System: Organ., serial idea, decimal nature</td>
<td></td>
</tr>
<tr>
<td>15. Percentage</td>
<td></td>
</tr>
<tr>
<td>16. Place value and face value</td>
<td></td>
</tr>
<tr>
<td>17. Problem solving</td>
<td></td>
</tr>
<tr>
<td>18. Proof and generalization</td>
<td></td>
</tr>
<tr>
<td>19. Quantitative thinking, mathematical judgment</td>
<td></td>
</tr>
<tr>
<td>20. Ratio and relationship</td>
<td></td>
</tr>
<tr>
<td>21. Read and writ. of no. ideas</td>
<td></td>
</tr>
<tr>
<td>22. Vocab. of no. and quantity</td>
<td></td>
</tr>
<tr>
<td>23. Zero representation</td>
<td></td>
</tr>
<tr>
<td>24. Application to arithmetic concepts in general</td>
<td></td>
</tr>
<tr>
<td>25. General nature of learning in arithmetic</td>
<td></td>
</tr>
<tr>
<td><strong>TOTALS</strong></td>
<td>18 16 2 3 3 18 2 0 2 0 360</td>
</tr>
</tbody>
</table>
Ascertaining the Reliability of the Completed Test.

Form C of the test having been constructed, the next matter of importance concerned the ascertainment of its reliability. According to Freeman, 'The term reliability refers to the extent to which a test is internally consistent and the extent to which it yields consistent results on testing and retesting.' Estimating the reliability of a test involves some means of securing at least two measures with the same testing instrument or with different forms of the same instrument and determining the agreement or the reliability coefficient between them. The closer the agreement, the greater is its reliability. The three methods of determining the reliability of a test usually mentioned by testing specialists are:

1. The test-retest method, in which case the same testing instrument is applied more than once to the same group, its reliability being based on the consistency of resulting scores.

2. The equivalent-forms method, in which case two test forms supposedly as similar as possible in content, mental processes required, length, difficulty, and all other respects, are administered to the same group, the reliability of the tests being dependent upon the agreement of scores obtained.

3. The split-test method, in which case the items of a single test are split into halves, usually by pooling odd-numbered items for one score and even-numbered items for another score, the method providing essentially a measure of the

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37 Freeman, op. cit., p. 15.
test's internal consistency, assuming an equal level of performance throughout the test by each person.

Since the writer had in mind to ascertain the reliability of only one test form, consideration was given to only the test-retest and the split-test methods. A search of the literature revealed that the test-retest method has the following disadvantages: (a) Repeating the test at too short an interval introduces the memory factor, thereby tending to make the self-correlations of the test too high, (b) the practice effect from having had the mechanical experience of taking the test once tends to have a similar result, and (c) repeating the test after too long lapse of time permits such factors as growth, intervening learning, and unlearning, thereby tending to make the correlation lower than it should be. Moreover, due to the crowded schedule of students' work in Education 510, the writer felt it unfair to usurp a disproportionate amount of their class time for this one aspect of the course. Hence, it was decided to use the split-test method, administering Test Form C to two sections of Education 510 students during the Winter Quarter, 1956, in order to obtain objective data basic to ascertaining the reliability coefficient of the completed form as well as to determine the coefficient of correlation between it and selected external criteria.
According to Remmers and Gage, the odd-even method of splitting test items usually makes the two scores obtained from a single testing reasonably equivalent in such respects as practice, fatigue, distractions, boredom, mental set, item difficulty and content. This condition obviously prevails only when the test has been built with due regard for such matters as item discrimination and difficulty, curricular validity, type of items used, and a progression of items in the test from the easy to the difficult. A further caution is, the split-half method should not be applied to tests for which speed is an important factor. According to Adkins and her associates, the use of the split-half method for speed tests, plus the application of the Spearman-Brown formula, lead to spuriously high estimates of reliability.

Essentially the same procedure was followed in administering Test Form C as that previously described for the two preliminary forms A and B. The subjects were 79 students comprising two sections of Education 510 during the Winter Quarter, 1956. Again instructions were given both verbally and in writing relative to the procedure to follow in indicating their choice of the "best answer" to

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38 Remmers and Gage, op. cit., p. 134.
39 Dorothy Adkins and Others, Construction and Analysis of Achievement Tests, p. 152.
each of the 60 items comprising the test. The students were requested to attempt every question for the same reason previously given. Essentially the only significant variation in testing conditions this time from those during the preliminary testing was that of motivation. At the time of the administration of Test Forms A and B the students were informed that the results would not apply toward their course mark. During the second administration, however, the students were told that this test represented a substitute for an oral-report assignment which had been previously made. Hence, presumably the students were motivated to "do their best."

After the test was given, three scores were obtained for each student: the odd-numbered items answered correctly, the even-numbered items answered correctly, and the total score (since the reliability of the test was to be based on the agreement of the standard deviations of the two halves and that of the whole test). A distribution of the three sets of scores was then prepared in order to facilitate computation of the standard deviations. These distributions, the mean scores, standard deviations, and ranges are presented below in Table 11.
<table>
<thead>
<tr>
<th>Half-Test</th>
<th>Whole Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>49-50</td>
<td>3</td>
</tr>
<tr>
<td>47-48</td>
<td>8</td>
</tr>
<tr>
<td>45-46</td>
<td>8</td>
</tr>
<tr>
<td>43-44</td>
<td>9</td>
</tr>
<tr>
<td>41-42</td>
<td>13</td>
</tr>
<tr>
<td>39-40</td>
<td>13</td>
</tr>
<tr>
<td>37-38</td>
<td>6</td>
</tr>
<tr>
<td>35-36</td>
<td>2</td>
</tr>
<tr>
<td>33-34</td>
<td>6</td>
</tr>
<tr>
<td>31-32</td>
<td>3</td>
</tr>
<tr>
<td>29-30</td>
<td>3</td>
</tr>
<tr>
<td>27-28</td>
<td>3</td>
</tr>
<tr>
<td>25-26</td>
<td>1</td>
</tr>
<tr>
<td>23-24</td>
<td>1</td>
</tr>
<tr>
<td>21-22</td>
<td>1</td>
</tr>
<tr>
<td>19-20</td>
<td>1</td>
</tr>
<tr>
<td>17-18</td>
<td>1</td>
</tr>
<tr>
<td>15-16</td>
<td>1</td>
</tr>
<tr>
<td>13-14</td>
<td>1</td>
</tr>
<tr>
<td>11-12</td>
<td>1</td>
</tr>
</tbody>
</table>

Total 79 79 79
Mean 20.4 19.5 39.9
S. D. 3.6 3.4 6.1
Range 27-12 26-11 50-25

A visual picture of the correlation of odd-numbered scores with those even-numbered is presented below in Table 12. It may be readily observed that the correspondence between the two sets of scores, while far from perfect,
TABLE 12

SCATTER-DIAGRAM OF TEST FORM C ODDS-EVENS SCORES

<table>
<thead>
<tr>
<th>11</th>
<th>13</th>
<th>15</th>
<th>17</th>
<th>19</th>
<th>21</th>
<th>23</th>
<th>25</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>24</td>
<td>26</td>
<td>28</td>
</tr>
<tr>
<td>25-26</td>
<td></td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23-24</td>
<td></td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21-22</td>
<td></td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19-20</td>
<td></td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17-18</td>
<td></td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>15-16</td>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13-14</td>
<td></td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11-12</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

does have a distinct tendency in that direction. To have had a perfect correlation coefficient of 1.00, all scores would have fallen in one of the blocks traversed by the diagonal line. In other words, each student would have had to make the same score on odd-numbered items as on even-numbered.

The procedure used to determine the statistical relationship of the odd-even numbered items to the whole test was as follows: First, the standard deviations were computed for each set of scores obtained. To ascertain standard deviations a number of formulas are available. The writer chose the following formula because it makes a correction for chance guesses and is relatively simple to compute.
S. D. = \sqrt{\frac{\text{Summation of } f d^2}{N} - \left(\frac{\text{Summation of } f d}{N}\right)^2}

where S. D. = Standard Deviation  
da = deviation from assumed mean  
N = number of cases  
f = frequency  
i = interval

Substituting numerical values statistically obtained from the data presented in Table 11 in the above formula for each set of scores results in the following computations:

(1) S.D. of odd-numbered items = \frac{2\sqrt{271} - \frac{39^2}{79}}{79} = 3.6

(2) S.D. of even-numbered items = \frac{2\sqrt{224} - \frac{2^2}{79}}{79} = 3.4

(3) S.D. of whole test = \frac{2\sqrt{736} - \frac{64^2}{79}}{79} = 6.1

These standard deviations are relatively small, the implication being that the variability of the students taking the test was relatively slight. In other words, they comprised a rather homogeneous group.

The problem of determining the reliability coefficient of the test still remained. Such a coefficient based only on the relationship of the odd-numbered items of the test with the even-numbered items would be valid for only half of the test. A means of determining the reliability of the whole test must be used, due to the fact that the reliability of tests varies with the number of functioning items they contain. Therefore, the reliability of a
half-test is lower than that of a whole test.

A frequent means for stepping up the reliability of a half-test to that of a whole test is the Spearman-Brown technique which, according to Remmers and Gage, has been found experimentally to give results in agreement with the actual reliability of whole tests; that is, predicted and obtained whole test reliabilities have been found to be the same. Briefly, the Spearman-Brown formula, as applied to the writer's situation, is as follows:

\[
\text{Reliability of whole test} = \frac{2 \times (\text{reliability of half-test})}{1 + \text{reliability of half-test}}
\]

The writer did not use the Spearman-Brown formula, however, due to the fact that after computing the standard deviations, it is less complicated to use the Cronbach formula which, according to Remmers and Gage, does not require computing the correlation or correcting by the Spearman-Brown Formula. It requires only the standard deviations of each half-test and the whole test. This formula is as follows:

\[
\text{Cronbach's formula}
\]

---

40 Remmers and Gage, op. cit., p. 134.


42 Remmers and Gage, op. cit., p. 135.
\[ r = 2 \left( 1 - \frac{S_0^2 + S_e^2}{S_t^2} \right) \]

where \( r \) = reliability coefficient of whole test
\( S_0^2 \) = standard deviation of odd-numbered test half
\( S_e^2 \) = standard deviation of even-numbered test half
\( S_t^2 \) = standard deviation of whole test

By substituting the standard deviations obtained earlier, the formula becomes:

Reliability of whole test = \( 2 \left( 1 - \frac{12.96 + 11.56}{37.21} \right) = .68 \)

Significance of a Coefficient of Reliability of .68.

Coefficients of reliability may range from 1.00 (that is, perfect correlation) through .00 to -1.00 (that is, perfect negative correlation). To have had a perfect and positive correlation, each of the 79 students to whom Test Form C was administered would have had to answer exactly the same number of odd-numbered items as even-numbered. Obviously, obtaining such results is extremely unlikely. In this instance only 14 of the 79 students tested received such scores. A coefficient of reliability of .00 would have indicated the complete absence of relationship between the students' scores on odd- and even-numbered items, while a perfect negative coefficient of -1.00 would have signified that the students made top scores on one type of items (either odd or even) and bottom scores on the other. It is evident that a testing instrument is never perfectly reliable. In view of this fact, a pertinent question is,
how unreliable might a test be and still be useful for evaluation purposes? As expected, opinions of educational and testing experts vary. The best answer seems to be that the test should have the highest reliability possible. Thorndike advised the use of the most available measure even if it has a reliability coefficient of only .40 or .50. Remmers and Gage remarked: "For research purposes, psychologists may find tests useful if their reliability coefficients are as low as .50." Guilford, too, maintained that many useful tests exist having reliability coefficients below .80.

A coefficient of reliability of .68 for Test Form C indicates that the agreement between the scores on the odd- and even-numbered items is neither perfect nor completely absent, rather that there is a distinct tendency, on the whole, for the size of scores on odd-numbered items to be associated with scores of like size on even-numbered items. Indicated, further, is the presumption that Test Form C, having a reliability coefficient of .68, is adequately reliable to justify its use as one of several measures to ascertain teacher understanding of principles and practices.


involving the selection and use of instructional aids in the elementary-school arithmetic program. Further justification for its use would seem to lie in the fact that, to the writer's knowledge, there is existent no other test of this nature having a higher reliability coefficient.

Comparison of Test Form C with External Criteria. Having established the reasonable adequacy of the coefficient of reliability of Test Form C in terms of its internal consistency, there remained the question of how the results obtained on this test compared with those of other tests and means of evaluation involving the same students. In other words, it was desirable to know whether there is a relationship, and if so to what extent, between the scores students obtained on Test Form C and on other tests they have taken.

It was felt by the writer that comparison of Test Form C results with those of at least four external criteria of a satisfactory nature would suffice to give reasonable evidence of presence or absence of such relationship. The population taking the test had previously taken the Ohio State Psychological Examination, presumably at the time of their enrollment.\(^{45}\) In view of this fact, the writer felt

\(^{45}\) One student involved in the Test Form C testing program had not yet met this requirement. Also, at least one student failed to take one or more of the three other tests with which Test Form C was correlated. It was necessary, therefore, to omit the consideration of these students in computing the intercorrelations.
that the results obtained on this examination would furnish one satisfactory basis for comparison. In addition, the students involved had taken a series of three other Education 510 course tests during the same quarter, and it was felt that their scores on these would furnish further satisfactory bases for comparison. These three testing instruments consisted of (1) Computation in Digital Tasks, (2) Reasoning in Prose Tasks, and (3) Inventory of Professional Understandings in Arithmetic, all having been constructed and semi-standardized by Professor Lowry W. Harding of the College of Education, The Ohio State University. The comparative scores obtained by individual students on these five testing instruments are presented in Table 16 in Appendix D. Intercorrelations of these testing instruments, computed by the Pearson formula from grouped scores, are presented in Table 13 on page 220.

Significance of the Intercorrelations of Test Form C and the Other Variables. Examination of Table 13 reveals that Test Form C correlated from .44 to .20 with the other four variables. To state that such correlations are either high or low would be without meaning without considering all factors involved. As might be reasonably expected, the test correlated highest with the Ohio State Psychological Examination and the Inventory of Professional Understandings in Arithmetic, these two tests obviously requiring a greater degree of understanding on the part of the testees.
TABLE 13

INTERCORRELATIONS OF FIVE TESTING INSTRUMENTS

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Test Form C</td>
<td>.44</td>
<td>.20</td>
<td>.25</td>
<td>.43</td>
<td></td>
</tr>
<tr>
<td>B. Ohio State Psychological Examination</td>
<td>.28</td>
<td>.13</td>
<td>.57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. Computation in Digital Tasks</td>
<td>.69</td>
<td>.31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D. Reasoning in Prose Tasks</td>
<td></td>
<td></td>
<td></td>
<td>.28</td>
<td></td>
</tr>
<tr>
<td>E. Inventory of Professional Understandings in Arithmetic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Likewise, it was reasonable to believe that Test Form C would correlate least with the test Computation in Digital Tasks, in view of the fact that college students are normally able to solve a number of the examples included in the latter test in a more or less mechanical manner without really having an understanding of the processes involved.

A comparison with intercorrelations of other tests might serve to keep the above intercorrelations in proper perspective. Freeman\(^4\) revealed that the intercorrelations of the sub-tests comprising The Chicago Tests of Primary Abilities were as follows:

\(^4\) Freeman, *op. cit.*, p. 38.
Freeman also disclosed that the intercorrelations of the sub-tests of the American Council on Education Psychological Examination: College Edition, 1948, were as follows:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Arithmetic</td>
<td>.432</td>
<td>.558</td>
<td>.427</td>
<td>.297</td>
<td>.450</td>
<td></td>
</tr>
<tr>
<td>2. Figure Analogies</td>
<td>.488</td>
<td>.349</td>
<td>.285</td>
<td>.529</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Number Series</td>
<td>.323</td>
<td>.253</td>
<td>.498</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Completion</td>
<td>.658</td>
<td>.574</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Same-Opposite</td>
<td>.563</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Verbal Analogies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It may be readily observed that the intercorrelations obtained by the writer compare rather favorably with those of the sub-tests of The Chicago Tests of Primary Mental Abilities but less so with those of the sub-tests comprising

47 Ibid., p. 231.
the ACE Psychological Examination. One factor tending to result in lower intercorrelations than would have otherwise been the case was the relative homogeneity of the group of testees. The students were all elementary-education majors and were, for the most part, in their sophomore year in the College of Education at The Ohio State University. The coefficients are lower, therefore, than they would have been if an entirely unselected group of students had been used. This fact is borne out by the intercorrelations of the Army General Classification Test.\footnote{Ibid.} It was revealed that the overall AGCT scores correlated with other tests as follows: Army Alpha, .90; Otis Higher Mental Ability Examination, .83; American Council on Education Psychological Examination, .79. However, when the test scores were correlated with individual training-school marks, the correlations were: clerical trainees, .40; airplane mechanic trainees, .32; officer candidates, .40.

Performance by Experienced Teachers on Test Form C.

While the primary emphasis in this portion of the study is on the development of a valid and reliable testing instrument which might be used to ascertain prospective-teacher understanding of principles and practices involved in selecting and using arithmetic instructional aids, it was
desirable to know whether the completed Test Form C might not also be worthwhile if applied to a group of experienced teachers. More specifically, would the test items be too elementary for a group of teachers who had already been teaching elementary-school arithmetic for a number of years? Obviously if their scores tended to run out the top, the testing instrument would be of little use in their case. It was felt that a relatively small sampling would suffice to shed light on this question.

Accordingly, during the Winter Quarter, 1956, Test Form C was administered to 14 experienced teachers who were currently enrolled in The Ohio State University Twilight School, in the graduate course, Mathematics in Elementary Education. Motivation was presumably comparable to that of the undergraduate group, since the students were informed that the test results would apply to their course mark. The resulting scores, the mean score, the standard deviation, and range are presented in Table 14.

It may be observed in the table below that the test results of the 14 experienced teachers were essentially comparable to those of the pre-service teachers. While slight gains were made, they were not at all significant. It was further found that the Test Form C scores of the experienced-teacher group correlated .26 with their scores on the test Computation in Digital Tasks and .39 with their scores on the test Reasoning in Prose Tasks. It is rather
TABLE 14
TEST SCORES OF IN-SERVICE TEACHERS ON TEST FORM C

<table>
<thead>
<tr>
<th>Scores</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>51-52</td>
<td>2</td>
</tr>
<tr>
<td>49-50</td>
<td>0</td>
</tr>
<tr>
<td>47-48</td>
<td>1</td>
</tr>
<tr>
<td>45-46</td>
<td>1</td>
</tr>
<tr>
<td>43-44</td>
<td>0</td>
</tr>
<tr>
<td>41-42</td>
<td>0</td>
</tr>
<tr>
<td>39-40</td>
<td>0</td>
</tr>
<tr>
<td>37-38</td>
<td>0</td>
</tr>
<tr>
<td>35-36</td>
<td>1</td>
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<tr>
<td>33-34</td>
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<tr>
<td>31-32</td>
<td>1</td>
</tr>
<tr>
<td>29-30</td>
<td>0</td>
</tr>
<tr>
<td>27-28</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>14</strong></td>
</tr>
</tbody>
</table>

Mean 41.1
S. D. 7.5
Range 52-28

evident, therefore, based on this small sampling, that Test Form C may justifiably be used as one instrument to ascertain the degree of understanding possessed by experienced teachers as well as pre-service teachers with regard to principles and practices involving the selection and use of instructional aids in the elementary-school arithmetic program.

Development of the Checklist

Among the major evaluations made during the pre-service training of a teacher is the evaluation of his
performance as a student teacher. This evaluation, to be most effective, should be of an external nature as well as by the student-teacher himself. Troyer and Pace stated that

... evaluation of what students learn in courses of professional and general education is not complete until we have determined how well they can use their knowledge and insight in their own teaching. The building of closer relationships between theory and practice has been increasingly a major objective of programs of teacher education. ... It is at the time of student teaching, however, that the best opportunities to evaluate the application of theory to practice ordinarily occur.

Of the two evaluations, however, it is generally agreed that self-evaluation is the more important. Dickson emphasized that, "Self-analysis is the key to good evaluation." He further maintained:

It is essential that student teachers be able to think critically about their performances. Unless they can see themselves much as others see them, they lack a sound foundation on which to build for growth. This means that each student teacher must become extremely objective in his self-analysis, and must develop a sincere desire to improve in the quality of his practices. Rationalizing, excuse making, or "passing the buck" has no place in the program of any student who desires to become a competent teacher.

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51 Ibid., p. 25.
It was the purpose of this portion of the study to explore the possibility of constructing a checklist by use of which prospective teachers might gauge their use of arithmetic instructional aids when confronted by arithmetic-teaching situations during their student teaching. It is intended primarily as a self-evaluation instrument rather than a score-card by which student-teachers are to be rated by observers. At the same time, however, the nature of the instrument is such that it may very well serve the latter purpose, although not primarily designed as such.

With regard to rating scales, Mersereau\textsuperscript{52} remarked:

It should be said here that the writer recognizes the generally accepted unreliability of rating schemes. A number of studies have demonstrated that practically all rating schemes are more or less unreliable. However, until some more accurate method of measurement is devised, the supervisor of practice teaching must do the best he can with the measuring instruments at hand.

Currently, Strang\textsuperscript{53} made essentially the same statement, that although rating scales are not at present highly dependable instruments for appraising, they are better than the haphazard and biased appraisals that people are continuously making. They generally are conceded to have the


following values: (a) they stimulate more accurate and
close observation, and (b) they serve as a springboard for
discussion to supplement other information obtained rela-
tive to student-teacher performance. Strang maintained:

Rating has a place in personnel work. As
in the case of other techniques, one must under-
stand its purposes, values, and limitations in
each situation. No important decisions should
be based on ratings alone; all available data
should be used in gaining understanding of an
individual. 54

According to Adkins55 and her associates, "In general, any
test that saves money, time, materials, and manpower is
worth using as long as it cannot be replaced by a better
test. A test of comparatively low validity is preferable
to the alternative of purely chance or random selection."

Self-evaluation can be accomplished in more than one
way. The method used in one situation might not be effec-
tive if used in another. Since people and situations are
different, it is obvious that they must evaluate and be
evaluated in different ways. One of the frequently used
self-evaluation techniques is the checklist. Douglass56
stated twenty-five years ago:

54 Ibid., p. 110.
55 Adkins and Others, op. cit., p. 162.
56 Harl R. Douglass, "Methods of Student-Teacher
Rating," Educational Administration and Supervision, XVII
I am much more enthusiastic about the use of the detailed check list for the purpose of diagnosis of teaching procedures and effectiveness than the use of such a list in the form of a score card. Such a check list should include under each larger division of aspects of teaching procedure, sub-divisions accompanied by standards either expressed or implied.

Torgerson and Adams\(^{57}\) declared: "The checklist has the advantage of being a simple way of reporting a great deal of information with a minimum of time and effort." They further remarked that self-rating scales possess the following values:

1. They may be used to help pupils clarify some of the less easily measured goals of the educational program and to appraise their progress toward such goals.

2. Results of self-rating scales may help the teacher to understand the pupil's ideas about himself.

3. Self-rating scales may be used to help pupils toward improved self-knowledge and self improvement.

4. Self-rating scales can serve as a basis for teacher-pupil conferences.\(^{58}\)

Development and Validation of the Self-Evaluation Checklist. A search of the literature disclosed a number of sets of criteria for the development of checklists and


\(^{58}\) Ibid., pp. 140-142.
various other kinds of self-rating scales. There seems to be rather general agreement that such instruments should possess the following characteristics:

1. The number of items should be few, preferably less than twenty.

2. The scale should be accompanied by suitable explanations.

3. The scale should meet the general criteria of validity and reliability.

4. A majority of the items should be of a diagnostic nature rather than in the form of a rating scale.

5. No item should be included in the scale which calls for an answer the person does not know, unless it reveals the need of his knowing the answer.

6. The statement should be as clear, concise, and short as possible, to convey full meaning.

7. Items should be used which reveal understanding rather than mere factual knowledge.

8. Items should measure behavior rather than introspective analysis.

9. The scale should be based upon the objectives of the program in which the student-teachers are engaged.

10. The scale should be a cooperative effort of all those concerned.

To possess curricular or content validity, the items included in a self-evaluation checklist, just as the items which comprise any form of evaluation instrument, must be grounded in the objectives of the program in terms of which the individual is to be evaluated. The mathematical objectives of the present study are the twenty-three developmental concepts of arithmetic proposed by Professor Harding. After due consideration, the writer decided to devise the checklist items from the criteria developed earlier and presented on pages 173-174. Thus, the columnar portion of the instrument was to be comprised of the twenty-three basic developmental concepts of arithmetic, and the horizontal part of adaptations of the proposed criteria. It seems reasonable to conclude that the checklist possesses validity in more or less direct proportion to that possessed by the criteria and the mathematical objectives. The completed Self-Evaluation Checklist is presented as Table 15, page 231.

Applicability of the Checklist. The purpose of the self-evaluation checklist devised by the writer is to reformulate the objectives of arithmetic with relation to instructional aids in terms which imply the development of "doing ability" in addition to that of understanding. The checklist could serve as an instrument by means of which the student-teacher, as well as the in-service teacher could record many of the procedures used with instructional
<table>
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<tr>
<th>Concept Introduced</th>
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<td>Skills Developed</td>
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<tr>
<td>Attitudes Toward Learning</td>
<td>❌</td>
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</tbody>
</table>

**Arithmetic Concepts**

- Use of any instructional aids
- Self-assessment of understanding
- Self-assessment of instruction given
- Concept introduced at appropriate level
- Instruction given for understanding
- Instruction given for skill development
- Instruction given for attitude development
- Concept introduced at appropriate level
- Instruction given for understanding
- Instruction given for skill development
- Instruction given for attitude development
- Concept introduced at appropriate level
- Instruction given for understanding
- Instruction given for skill development
- Instruction given for attitude development
- Concept introduced at appropriate level
- Instruction given for understanding
- Instruction given for skill development
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- Concept introduced at appropriate level
- Instruction given for understanding
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- Instruction given for attitude development
- Concept introduced at appropriate level
- Instruction given for understanding
- Instruction given for skill development
- Instruction given for attitude development
- Concept introduced at appropriate level
- Instruction given for understanding
- Instruction given for skill development
- Instruction given for attitude development
aids to attain the stated arithmetic objectives. The instrument would suggest the instructional-aids procedures they might use, as well as help them to note various gaps in the use already made of instructional aids. If certain activities were seldom or never engaged in, certain arithmetic concepts or processes never touched upon, certain instructional aids or classes of instructional aids never made use of, these facts might be indicative of the quality of arithmetic teaching done.

With specific reference to the use of the checklist by student-teachers, various interpretations of the recorded data should be discussed in regular conferences between the student-teacher and the supervisor of student-teachers as well as in the regular student-teaching seminars. If the checklist is to be used as a rating scale, the teacher should be a co-partner in its administration. It should not be considered an instrument to be forced upon teachers or student-teachers. Their suggestions for revision and improvement should be welcomed and utilized at all times. In fact, considerable leeway should be given teachers in the actual use to be made of the checklist. Some teachers, for example, might merely use check marks in the appropriate places while others might make use of the key symbols to indicate the extent of their utilization of aids with regard to the various arithmetic concepts and processes.
While not expected that a student-teacher will encounter necessarily all stages of learning with reference to the twenty-three arithmetic concepts, or not necessarily all the twenty-three concepts, during the quarter of student-teaching, it is felt that his "arithmetic experiences" should be rich and varied. Periodic discussions between the student-teacher and the cooperating teacher, based on the checklist, can facilitate intelligent planning with regard to enrichment of such experiences during the remaining student-teaching period.

Chapter Summary. In Chapter IV the writer attempted to accomplish three objectives. The first portion of the chapter is concerned primarily with the development of criteria for selecting and using instructional aids in the elementary-school arithmetic program. The writer justified these criteria on a basis of their apparent identity to generally-accepted principles of learning. In connection with this aspect of the study, several other lists of criteria were presented relating to significant and pertinent categories.

The major portion of the chapter deals with the development and validation of an objective test which might be used as one means of ascertaining certain aspects of teacher understanding of principles and practices involved in selecting and using instructional aids in the
elementary-school arithmetic program. The writer explained in some detail the process used in selecting test items to be included in two preliminary test forms, how these tests were administered, and how on a basis of an item analysis a third improved form was built from the two preliminary forms.

Considerable treatment was given to the procedures followed in administering the improved form of the test and the comparison of the resulting scores by the odds–evens method to determine its reliability in terms of internal consistency. Test Form C was found to have a coefficient of reliability of .68 (corrected), thus being apparently adequate for the purpose intended. It also was found to correlate with four external criteria as follows: The Ohio State Psychological Examination, .44; Computation in Digital Tasks, .20; Reasoning in Prose Tasks, .23; Inventory of Professional Understandings in Arithmetic, .43.

A small portion of the chapter deals with a discussion of the administration of Test Form C to a small sampling of experienced elementary-school teachers in order to determine whether the instrument was suitable for ascertaining their understanding of arithmetic instructional aids. It was found that the same instrument might as equally be used with in-service teachers as with those who have never taught.
The last portion of the chapter deals with the development of a checklist intended primarily as a self-check by teachers and student-teachers to guide their use of instructional aids in attaining the objectives of the elementary-school arithmetic program. Considerable treatment was given to the procedures used in building the instrument and to the uses which it might serve.
CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary and Conclusions

Purposes of the Study. The major and subsidiary purposes of the present study were to:

1. attempt to define an acceptable role of instructional aids in the elementary-school arithmetic program.

2. develop criteria for selecting and using arithmetic instructional aids.

3. develop an objective instrument for ascertaining, in part, prospective-teacher understanding of principles and processes involved in selecting and using arithmetic instructional aids.

4. develop a checklist for ascertaining, in part, prospective-teacher understanding of the use of arithmetic instructional aids, as evidenced by student-teaching performance when faced with arithmetic-teaching situations.

Significance of the Study. While the values of instructional aids in the teaching and learning processes have been definitely established through scientific investigations, a large number of teachers still continue to make inadequate and unintelligent use of them. Despite the huge successes effected by instructional aids during and since World War II, their use in the teaching of arithmetic is neither so effective or so extensive as might
be expected.

A common ailment of pupils of today's schools is verbalism; a common remedy for verbalism is the intelligent use of appropriate instructional aids. A National Education Association survey in 1946 revealed that teachers of arithmetic in the elementary school were making the least use of instructional aids of any curriculum area. Teachers have been slow to make full use of aids in the teaching of arithmetic because of a lack of understanding on their part of the role such aids should play in the teaching-learning process. This lack of understanding tends to result in one of two things: (a) teachers make only a minimum use of arithmetic aids, or (b) they make improper use of such aids.

A large part of the responsibility for the inadequacy of teachers in the proper use of arithmetic instructional aids must be assumed by the teacher-preparation institutions and specifically by the instructors of courses in the teaching of elementary-school arithmetic. Both the institutions and the instructors are giving too little attention to the instruction of students in the selection and use of arithmetic aids, a disproportionate amount of emphasis instead being devoted to "pure-arithmetic" methods on the abstract level. It has been generally wrongly presumed that prospective teachers have an understanding of the use of arithmetic instructional aids upon graduation.
Instructors of arithmetic methods courses have the dual responsibility of using instructional aids in their arithmetic-education classes and of teaching their students to do likewise.

The major assumptions made in this study were:

1. Learning is basically sensory and developmental in nature.

2. Teacher understanding of the use of instructional aids in arithmetic education is imperative.

3. Instruments may be devised to measure certain aspects of teacher understanding of principles and processes involved in the selection and use of arithmetic instructional aids.

The major limitations of the study are:

1. It attempts to deal with only the one factor "role of instructional aids in arithmetic education," rather than the aggregate of qualities essential to successful teaching of elementary-school arithmetic.

2. The instruments developed are valid only in terms of the one factor they were designed to measure.

3. The validity of the objective test developed by the writer was determined by data obtained exclusively from pre-service teachers.

Defining the Role of Arithmetic Instructional Aids.

To determine the role of instructional aids in arithmetic education, it is essential first to examine (a) statements of objectives of the elementary school, and (b) statements of objectives of the elementary-school arithmetic program. Several widely-accepted lists of such objectives made by both national and state educational bodies were examined
and analyzed. Particular emphasis in the study was given to the statement of the "Cardinal Principles of Secondary Education," made in 1918 by the Commission on the Reorganization of Secondary Education; to the Cardinal Objectives in Elementary Education formulated by New York in 1933; the objectives of education as set forth by the Educational Policies Commission in 1938; and to the report of the Mid-Century Committee on Outcomes in Elementary Education or as it is more commonly known, the "Kearney Report." In addition, reference was made to views on the purposes of education held by Spencer, Herbart, Dewey, Overstreet, Washburne, Kehler, and Otto. In such statements of objectives of education, the following concepts are rather generally emphasized:

1. developing, equipping
2. self-sustained persons
3. individual mind and character
4. individual responsibility
5. habits of tolerance
6. every child
7. making democracy work
8. intelligent participation
9. the right method
10. understanding, appreciation, knowledge, ideals
11. freedom from class stratification
12. to make free men
13. civic responsibility

A further generalization with regard to these statements of objectives relates to the fact that the elementary school does not have any unique functions; its objectives lie within the framework of the general purposes of education in American Democracy. The general goals toward
which the elementary school should strive are the general purposes of education. The elementary school is unique only with regard to its curriculum, methods, and organization because it is dealing with a particular age-group of children.

The same procedure was followed with regard to the objectives of the elementary-school arithmetic program. Particular emphasis was given to the statements of objectives made by the following national bodies: (a) The Commission on Postwar Plans, created by the Board of Directors of the National Council of Teachers of Mathematics; The Committee for the Fiftieth Yearbook, Part II, of the National Society for the Study of Education; the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, on the Place of Mathematics in the Secondary Schools; and the Mid-Century Committee on Outcomes in Elementary Education. In addition, the individual views held by the following educators were likewise discussed: Thiel, Bruckner, Moser, Kinney, Purdy, Bathurst, Brownell, Buckingham, Clark, and Harding. Examination of the above and other pronouncements of objectives revealed that there are no unique functions of the arithmetic program; its purposes are instead the objectives of the entire educational program. The mathematics program cannot live apart to itself.
There is no general comprehension of arithmetic; an understanding of this curriculum area relates instead to each of the component concepts and processes which comprise the arithmetic program. Likewise there is no general insight into the use of instructional aids in teaching arithmetic; each concept or process must be considered separately. Hence, the elementary-school arithmetic program must be broken down into workable component parts. Professor Lowry W. Harding of The Ohio State University has furnished such a breakdown with his list of twenty-three developmental concepts which the writer secured permission to use as the mathematical bases of the present study.

Ascertainment of the role of instructional aids in arithmetic must also consider the psychological bases of learning. There is rather general agreement relative to several principles of learning, among them being: (a) Readiness is a factor in learning; (b) learning proceeds from the familiar to the new; (c) learning is the result of experience; and (d) learning is developmental. Considerable coverage was given to these and other aspects of the learning process, the purpose being to indicate how teaching aids operate in terms of each aspect. There are no principles of education peculiar to the utilization of arithmetic teaching aids. Instead, their proper use falls
within the framework of principles in which all good teaching procedures operate.

Considerable attention was given to the formation of concepts, this aspect of learning being the keystone to the aforementioned assumption that all learning is basically sensory. The learner's background experiences are both dependent upon and basic to perception, tending ultimately to result in the formation of concepts. The ability to form concepts makes it unnecessary for the learner to use concrete instructional aids in solving every problem. Contrariwise, if the learner has not built up an adequate background of experiences, the use of concrete aids when facing a new situation is highly essential. With regard to transfer, the way in which a child learns a generalization (or concept) will affect the probability of his recognizing a chance to use it. Too great a dependence upon formalized paper and pencil problems robs the student of many of the opportunities for transfer which exist in other kinds of mathematical experiences afforded by a variety of teaching aids rather than only those of an abstract nature.

Considerable treatment was given to the postulate that learning is developmental in nature. The implications are for the use of instructional aids that are appropriate to the optimum learning level of the pupil. Readiness is an aspect of the developmental nature of learning; it operates throughout life and at all learning levels. Readiness to
learn is usually considered to be dependent upon (a) sufficient physiological maturity, (b) appropriate preparatory experiences, and (c) aroused interest or the desire to learn. The writer used the following classification of learning levels as the basis for the utilization of instructional aids: (a) readiness for learning, (b) exploratory-discovery stage of learning, (c) verbalization and representation of manipulative processes, and (d) performance at an adult level. However, there are no definite lines of demarcation separating the different steps; instead, they may be overlapping. An understanding of the learning levels is prerequisite to the use of instructional aids, since the teacher must be aware of the type of aids to use at each learning level.

Various classifications of instructional aids were presented, including those of Crandall, Kinder, Dale, and Grossnickle, Junge, and Metzner. A very important aspect of teacher understanding of the use of instructional aids concerns his ability to gear appropriate classifications of aids to the various operational stages of learning. Examples were presented to illustrate how these two aspects are related.

Formerly the major emphasis on instructional aids in arithmetic was placed on their ability to stimulate and maintain interest—and even on their sensational nature.
In contrast, today the good school places the major emphasis on teaching aids as sources of data. Moreover, they are not considered of secondary importance; no aid is looked upon as being supplementary to another or as something extra. Arithmetic instructional aids are not ends in themselves, but only means to desired goals. Furthermore, instructional aids themselves seldom teach arithmetic; the role of the teacher in their use is still paramount. In addition to their importance in promoting basic learning, the proper use of instructional aids in arithmetic tends to clarify processes which pupils are able to do in a routine, abstract manner, but the procedure of which they are unable to explain.

A majority of the disadvantages accompanying the use of arithmetic instructional aids appear to be factors of a variable nature which might easily be removed or minimized as a result of better understanding on the part of the classroom teacher and improved administration of the instructional-aids program.

To illustrate the use of instructional aids in an elementary school arithmetic program, the writer presented six dramatized classroom procedures embodying the use of instructional aids in teaching a different concept or process in each of the grades 1 to 6. The concepts illustrated were: (a) counting, (b) grouping, (c) subtracting
tens, specifically the concept of "borrowing," (d) the addition and subtraction of simple common fractions, (e) the multiplication of a common fraction by a whole number, and (f) the measurement of area.

The Development of Criteria. Chapter IV deals with the three subsidiary purposes of the study: (a) the development of criteria for the selection and use of instructional aids in the elementary-school arithmetic program, (b) the building of a valid and reliable objective instrument, a multiple-choice test, for ascertaining certain aspects of teacher understanding of arithmetic instructional aids, and (c) the development of a checklist, intended primarily as a self-check by teachers and student-teachers to promote personal growth in the proper use of arithmetic instructional aids.

Of the above three subsidiary purposes of the study, criteria for the selection and use of instructional aids in arithmetic were developed first, since they served as a foundation for the construction of the test and the checklist. Preparatory to the process of developing criteria specifically geared to arithmetic instructional aids, the writer presented a number of sets of criteria of a general nature embracing the following aspects of the instructional-aids program: (d) direct, purposeful experiences, (2) contrived experiences, (3) manipulative aids, (4) field trips, (5) still pictures (both projected and non-
projected), (6) motion pictures, (7) non-commercial aids, (8) free and inexpensive instructional materials, (9) radio broadcasts, (10) instructional aids for group use, (11) instructional aids for development of the individual, (12) selection of instructional aids on a teacher-pupil cooperative basis, and (13) selection of teaching aids that will lead to a balanced development in children.

In view of the fact that a major emphasis in this study has been on the philosophical and psychological aspects of the selection and use of instructional aids in arithmetic, the writer reasoned that criteria which he developed should be grounded in generally accepted philosophical and psychological principles of learning. Since there are no principles of learning unique to the use of instructional aids, the development of criteria was merely a process of relating generally-accepted principles of learning to arithmetic instructional aids. In so doing, major emphasis was placed on the developmental and operational levels of the learning process. The ten criteria developed or adapted by the writer are equally applicable to the attainment of each of Professor Harding's twenty-three developmental concepts of arithmetic. Since the criteria are in reality only variations of commonly-accepted principles of learning, the writer reasoned that they were valid for the purpose intended, it having been previously accepted that principles of learning apply to
all aspects within the framework of learning. The writer's only contribution in the matter was readapting them to apply to arithmetic aids specifically.

Constructing the Objective Test. A third purpose of the study was the construction and validation of an objective test which would measure certain aspects of teacher understanding of the selection and use of arithmetic teaching aids. The foundation of a good test obviously lies in its objectives. There is no such thing as a general understanding of arithmetic. Understanding must be measured in terms of specific objectives of the elementary-school program. As previously stated, Professor Harding's twenty-three developmental concepts of arithmetic furnished such a basis. The next step of the writer was to select a valid list of the types of instructional aids which tend to contribute to the attainment of one or more of the stated objectives. Test items were then patterned in terms of the tendency of a specific aid or class of aids to contribute to the attainment of a specific arithmetic objective. The decision of what test items to use was based upon the findings of controlled experiments, testimonials of classroom teachers, and philosophical studies. To guide the writer in devising test items an outline was constructed in terms of both arithmetic concepts and classifications of teaching aids.
It was decided to construct the test of 5-option multiple-choice test items since this type of item is sufficiently discriminating to minimize the guessing and chance element. Criteria by which the mechanics of the test items were guided and measured were selected from miscellaneous sources. Sources utilized for the construction of test items consisted of arithmetic education and visual-aids textbooks, pertinent experimental studies reported in the literature, reports of classroom teachers' first-hand experiences with arithmetic aids, and philosophical writings of arithmetic and instructional-aids specialists.

After a large number of items had been devised, critically examined by graduate students and staff consultants, and revised and refined a number of times, they were allotted to two separate test forms, 60 items to each, on a basis of the aforementioned Test-Item Outline. Sufficient copies of the two test forms were duplicated to administer during the Autumn Quarter, 1955, to three sections of students in Education 510, an undergraduate course in elementary-school arithmetic methods and materials offered by the College of Education, The Ohio State University. The purpose was to select the 60 "best" items from the two preliminary test forms in order to construct a third form which, in addition to content or curricular validity, would possess statistical validity in terms of discriminatory power and difficulty level.
After the results made on the initial tryouts were ascertained, the upper-lower 27 percent technique was applied to the resulting scores in order to ascertain which items on the two preliminary test forms possessed significantly great discriminatory power to include in the third test form. The students who made the highest 27 percent of the test scores on each test form and the 27 percent who made the lowest scores were noted. An item-analysis was then made to determine the performance of the upper and lower 27 percent groups with reference to the number of students in each group who (a) answered each item correctly, (b) answered each item incorrectly, (c) omitted each item, and (d) did not reach each item. The scores obtained by the upper and lower groups were then corrected for chance guesses, omissions, and to take into account items not reached. The corrected percentages of successes of each group on the respective items were then referred to the Davis Item-Analysis Chart to ascertain their discriminatory index in terms of the relationship of successes of the upper group to those of the lower group on each test item. The estimated difficulty level of each test item was also secured from the chart. It was found that of the 120 items used on the two preliminary test forms 65 met Davis' statistical requirement that a good test item has a discriminatory index of above 20 (as computed on the basis
of his Item-Analysis Chart). Five of these items were eliminated due to their similarity of content to other items used. The range of difficulty of the "best" 60 items was from 76 to 25 with an average difficulty level of 49, thereby satisfying the commonly-accepted requirement that the items used in a test should have a wide spread of difficulty with the average being at or near 50.

The 60 best items were compiled into a third test form, which the writer designated as Test Form C. The items were arranged in the test in the order of their difficulty level, the easiest item (the one having a difficulty index of 76) being entered as item No. 1 and the most difficult item (the one having a difficulty index of 25) being entered as item No. 60. The positions of the "best" answers of the respective test items were re-randomized due to the fact that the 60 items were comprised of an assortment of items from both forms A and B.

The curricular and statistical validity of Test Form C having been reasonably established, the writer proceeded to ascertain whether the test possessed significant reliability, both in terms of its internal consistency and to the extent that it would yield results consistent with other forms of evaluation when used on the same population.

The split-test method (odds-even) was used. Test Form C was administered to two sections (a total of 79 students) during the Winter Quarter, 1956, in order to obtain
objective data with which to ascertain the reliability coefficient of the test and the coefficient of correlation between it and selected external criteria. The only essential difference in testing conditions between the administration of Test Form C and that of the two preliminary Test Forms A and B was in the motivation element. During the preliminary tryouts, the students were told that the scores made would not count for or against their course grade. During the second administration, however, the students were informed that the test represented a substitute for an oral-report assignment which had been previously made. Hence, presumably the students were motivated to "do their best."

Each student's score was divided into three categories: (a) the number of odd-numbered items answered correctly, (b) the number of even-numbered items answered correctly, and (c) the total score (since the reliability coefficient of the test was to be computed on the basis of the agreement of the standard deviations of the two test halves and that of the whole test). The standard deviation of the odd-numbered test half was found to be 3.6, that of the even-numbered items 3.4, while that of the whole test was 6.1. The test was found to have a reliability coefficient of .68 (corrected).

The relatively small size of the standard deviations of the two test halves and of the whole test indicates a
rather homogeneous group of testees with a rather slight
deviation of ability from the mean score. A coefficient of
reliability of .68, while not as high as the writer might
have hoped for, seemed to be adequate, nevertheless.
Instances were cited in which test specialists recommended
the use of tests having reliability coefficients as low as
.40 or .50.

Test Form C was correlated with four other testing
instruments which had been administered to the students
involved in the study. Using the Pearson formula for
grouped scores, the writer found the test to correlate as
follows: The Ohio State Psychological Examination, .44;
Computation in Digital Tasks, .20; Reasoning in Prose Tasks, .28; Inventory of Professional Understandings in Arithmetic, .43. The correlations, while not high,
appeared to be adequate when gauged by intercorrelations
of sub-tests comprising certain tests of national repute.
Moreover, homogeneity of groups tested tends to result in
low correlations.

The primary emphasis in this study has been on pre-
service teachers. An added aspect of the investigation,
however, was to try out Test Form C on a sampling of 14
experienced elementary-school teachers to determine whether
the items were too elementary to result in a desirable
spread of scores. It was found that the test results of
the experienced teachers were essentially similar to those
of undergraduates. Hence, the writer concluded that, based on the small sampling, Test Form C is an equally valid instrument whether applied to experienced or pre-service teachers.

Development of the Self-Evaluation Checklist. A fourth purpose of the study was to develop a checklist intended primarily as a self-check for student-teachers and teachers to guide their selection and use of arithmetic instructional aids. The vertical portion of the checklist was comprised of the twenty-three basic developmental concepts of arithmetic. The horizontal part was made up of adaptations of the criteria developed earlier by the writer. The checklist was adjudged by the writer to be valid on the grounds that the individual items comprising it consist of adaptations of generally-accepted principles of learning as applied to the equally-accepted concepts and processes of elementary-school arithmetic.

Recommendations

1. In arithmetic education courses full use should be made of instructional aids. The instructor himself should use arithmetic aids in teaching his classes. Also, he should see to it that his students have ample opportunities to make wide use of them and to discuss their use. Students should demonstrate in class the appropriate use of aids in teaching each of the basic developmental concepts
of arithmetic. This procedure might be carried out as group projects, one student of each group taking the role of teacher, the others acting as elementary-school pupils. For example, in this way students would demonstrate the use of such aids as the place-pocket, the abacus, and coins to teach the process of carrying and borrowing; "fraction pies," folded sheets of paper, and measuring cups to teach the common-fraction concept; and egg cartons, pop cases, muffin pans, dominoes, and the counting frame to teach the concept of grouping. The group-work procedure would afford students ample opportunities to verbalize their utilization and demonstration of teaching aids. Students also should be given opportunities to act as commentators during the projection of filmstrips in connection with the teaching of specific arithmetic concepts.

2. Colleges of education should cooperate with arithmetic-education instructors to build up adequate supplies of instructional aids to be housed in classrooms where arithmetic-education courses are taught. If such courses are to be of a laboratory nature, instructional aids used in them must be readily available. It is virtually an impossibility for instructors to transport an adequate supply of arithmetic instructional aids from one building to another.
3. The teacher-education staff in elementary education should make reasonably certain that pre-service teachers possess an adequate understanding of the use of arithmetic instructional aids. They should exhibit such understanding through adequate performance in terms of appropriate evaluative instruments and methods. Administration of Test Form C affords one valid and reliable means of obtaining pertinent data. The Self-Evaluation Checklist devised by the writer may be used in working with student-teachers. These instruments were intended in large measure to be merely suggestive. Instructors of arithmetic-education courses should develop additional objective tests of a comparable nature. Test Form C is far from perfect. Every administration of a test tends to reveal certain weaknesses in the instrument which need to be corrected. Items may be better worded or better items substituted.

4. Cooperating teachers should be encouraged to permit student-teachers in their charge to spend a proportionate amount of time in arithmetic instruction. Far too few pre-service teachers are afforded this opportunity. The typical cooperating teacher is extremely distrustful of the capability of student-teachers to handle competently classes in arithmetic. On the other hand, the typical student-teacher is customarily pictured as being "scared to death" of having to teach the subject. One solution to
this problem is the adequate preparation of pre-service teachers in principles and processes involving the selection and use of arithmetic instructional aids and the subsequent introduction of a wealth of such aids into arithmetic instruction during the student-teaching experience. Having tangible materials with which to work tends to instill added confidence in student-teachers, if they have adequate understanding of their use.

5. In-service teachers who are inadequately prepared in the selection and use of arithmetic instructional aids should be aided to become competent in such techniques through graduate courses, workshops, and other means. Action research geared to teacher understanding of the use of arithmetic instructional aids should be carried on in specific school systems. The projects might be effected by means of testing (or inventory) programs to ascertain weaknesses in teacher understanding and followed up by improvement programs in the form of workshops conducted by the school system and by curriculum committee meetings in the individual schools.
APPENDIX A

A PARTIAL LIST OF INSTRUCTIONAL AIDS WHICH TEND TO PROMOTE UNDERSTANDING OF THE TWENTY-THREE BASIC CONCEPTS OF ARITHMETIC
A PARTIAL LIST OF INSTRUCTIONAL AIDS WHICH TEND TO PROMOTE UNDERSTANDING OF THE TWENTY-THREE BASIC CONCEPTS OF ARITHMETIC

Approximation or Estimation

1. ropes, ribbon, string, sticks, etc. of various lengths
2. objects of various weights
3. containers of various sizes
4. pictures of containers of various sizes
5. pictures of objects of various sizes and weights
6. pictures of objects of various lengths

Games

1. "Pin the Tail on the Donkey" (magnetic)
2. darts (magnetic recommended)

Average

1. ribbons, strings, sticks, ropes, etc. of various lengths
2. objects of various weights
3. containers of various sizes
4. pictorial representations of "averages"
5. counters of various kinds to distribute among members of a group
6. grocer's scales
7. balance scales

Counting or Enumeration, and Ranking

Basic (other than films)

1. pieces of paper, tickets, tongue depressors, wooden discs, coupons, sales
Sources of Certain Commercial Aids

tax stamps, clothes pins, poker chips, dominoes, spools, toothpicks, acorns, checkers, milk bottle caps, beans, grains of corn, buckeyes, iron washers, marbles, pop bottle caps, buttons, pebbles, counting blocks (1" cubes), counting sticks
2. cutouts of animals, fruit, etc.
3. toy money, real money
4. number sequence box
5. fact finder (counting frame)
6. abacus
7. peg board
8. pictured counting discs
9. counting chart (hundred chart)
10. counting chart (removable numbers)
11. egg cartons, pop cases, milk cases, muffin pans, pop cartons
12. flannel board (or felt board), magnetic board
13. Counting Rhymes (a book of counting rhymes)

Games
1. dominoes
2. "jacks"
3. "Pin the Tail on the Donkey" (magnetic)
4. dart board (magnetic recommended)
5. ring toss (magnetic recommended)

Sound Film
1. "Let's Count"

Filmstrips
1. "Counting to 5"
2. "Counting to 10"
3. "Counting from 10 to 15"
4. "Counting from 15 to 20"
5. "Counting from 20 to 40"

1 Numbers refer to commercial firms listed on pages 277, 278, and 279.
Sources of Certain Commercial Aids

6. "Counting from 40 to 100" 10
7. "Counting by 10's to 30" 10
8. "Counting by 10's to 50" 10
9. "Counting by 10's to 80" 10
10. "Counting by 10's to 100" 10

Steropticon Slides
1. "Arithmetic" 37

Formulas and Algebraic Symbolization
1. lists of familiar abbreviations
2. scrapbook of recipes and formulas
3. collection of bus and train timetables

Common Fractions

Basic (other than films)
1. apples, oranges, candy bars (sectioned), crackers
2. yard of ribbon, lace, tape, string
3. strips of paper
4. empty rectangular and cylindrical boxes, cut horizontally into fractional parts
5. glass measuring cups
6. measuring spoons
7. paper pie plates, circular paper discs (from bottom of bakery cakes)
8. rectangular pieces of paper
9. circles of tagboard to represent pies (cut into fractional parts)
10. chairs, books, pads, pencils, milk bottle caps, buttons, and other counting devices (to divide into fractional groups)
11. rulers (showing fractional parts of inch to 1/16")
12. toy and real coins
13. dissected squares of cardboard
14. sheets of paper folded into parts
15. fraction wheel
16. fractional cutouts to match pictorial representations
17. fraction cards
18. fraction discs (dissected)
19. fraction chart
20. circular and rectangular illustrations
21. scales (avoirdupois), balance and spring
22. "Fruit Plate" (mock-up of fruit, bisected, trisected, and quartered) 36
23. adjustable fraction discs
24. flash cards: "Krest Fraction" 42
25. adjustable fraction discs: "Parts Imparter" 12
26. "Teaching Aid Ruler" 31
27. Scales: "Wa-it" 2
28. fraction pies 7
29. block tile 22
30. fraction board 14
31. common-fraction adding device 14
32. "pie" diagrams
33. standard reference circle (1' diameter)
34. fraction-equivalent chart
35. fraction-equivalent board
36. fraction counter (a form of abacus)
37. foot-fraction board
38. various measures: cup, half-pint, pint, quart, gallon, peck, bushel, half-bushel
39. diagrams of above measures
40. "Fraction Trainer" 38
41. scrapbook of recipes (to be increased or decreased)
42. gasoline gauge marked in fractions
43. scaled drawings
44. dominoes

Sound Films
1. "Parts of Things" 53
2. "We Discover Fractions" 6
3. "What Are Fractions?" 13
4. "Simple Fractions" 25
5. "How to Add Fractions" 20
6. "How to Subtract Fractions" 20
7. "How to Change Fractions" 20
8. "How to Multiply Fractions" 20
9. "Multiplying Fractions" 25
10. "How to Divide Fractions" 20
11. "Division of Fractions" 25
12. "Introduction to Fractions" 20
Filmsstrip

1. "What Is a Fraction?" 8
2. "Writing Fractions" 8
3. "Units and Fractional Parts" 35
4. "How Large Is a Fraction?" 8
5. "Multiple Fractions: Numerator and Denominator" 35
6. "Fractions of a Group" 8
7. "Adding Fractions" 8
8. "Comparing Fractions: Adding and Subtracting" 20
9. "Addition and Subtraction of Fractions" 20
10. "Common Denominators" 8
11. "Multiple Fractions: Improper Fractions" 35
12. "Mixed Numbers" 8
13. "Using Mixed Numbers" 8
14. "Improper Fractions: Mixed Numbers" 35
15. "Reducing and Changing Fractions" 35
17. "Multiplying Fractions by Fractions" 8
18. "Multiplying Fractions" 35
19. "Dividing Fractions" 35
20. "Reciprocals: The Rule of Division" 35
21. "Multiplication and Division of Fractions" 20
22. "Adding Like Fractions and Mixed Numbers" 46
23. "Adding Unlike Fractions and Mixed Numbers" 46
24. "Changing the Terms of Fractions" 46
25. "Comparing Fractions" 20
26. "Fractional Parts of a Group" 20
27. "Fractional Parts of a Whole" 20
28. "Fractional Parts of a Whole and Groups" 20
29. "The Meaning of Fractions" 46
30. "Subtracting Like Fractions and Mixed Numbers" 46
31. Subtracting Unlike Fractions and Mixed Numbers" 46
32. "Multiplying Fractions and Mixed Numbers" 46
33. "Non-Unit Fractions of a Whole and Groups" 20
Source of Certain Commercial Aids

Decimal Fractions

Basic (other than films)

1. abacus
2. Flash Cards: "Krect Fraction" 42
3. "Fraction Trainer" 38
4. "Parts-Imprter" 12
5. speedometer
6. cardboard ruler graduated to tenths of a foot
7. circular discs divided into tenths
8. circular diagrams divided into tenths
9. micrometer
10. vernier
11. place-pocket
12. meters of various kinds with readings in decimals
13. coins: pennies, dimes, silver dollars

Sound Films

1. "What Are Decimals?" 13
2. "Decimal Fractions" 20
3. "Decimals Are Easy" 6

Filmstrips

1. "Introduction to Decimals" 8
2. "Adding and Subtracting Decimals" 8
3. "Comparing Decimals" 8
4. "Decimals and Common Fractions" 8
5. "Decimal and Percentage Series" 8
6. "Multiplying Decimals" 8

Geometric Form

1. scrapbook of assorted geometric shapes
2. concrete objects having various geometric shapes
3. clay (to construct various geometrical forms)
4. compass (for drawing circles, etc.)
5. flannel board, felt board, magnetic board
6. matching designs (cutouts to match countersunk designs)
7. blocks having various geometric shapes
8. pictures of various geometric forms
9. assortment of tagboard cutouts of various geometric forms
10. "Fruit Plate" (mockup fruit, bisected, trisected, and quartered) 36
11. protractors
12. transparent flexible Ruler-Scale Protractor
13. geometric surfaces and solids 31
14. design blocks 31
15. block tile 22
16. chalkboard compass
17. carpenter's square

Graphic Representation

Basic (other than films)

1. milk chart (showing milk used in classroom)
2. collection of graphs secured from newspapers, magazines, etc.
3. clock face colored to represent time for play, work, sleep, etc.
4. charts showing pint and quart bottles of milk
5. illustrations of line graphs, bar graphs, and pictographs

Sound Film

1. "Maps Are Fun" 6

Films

1. "Graphs" 46

Grouping and Number Combinations

Basic (other than films)

1. counters of all kinds (previously listed)
2. egg cartons, pop cases, milk cases, pop cartons, muffin pans
3. dominoes
Source of Certain Commercial Aids

4. number sequence box
5. weaving frame
6. peg board
7. chart of pictorial, semi-abstract, and corresponding number groupings
8. pictorial-grouping charts (showing various ways to group object to total same number)
9. semi-abstract grouping charts (showing various ways to group dots, x's, etc. to total same number)
10. Arithmetic Readiness Cards (in grouping) 40
11. counting frame (fact finder)
12. abacus
13. toy and real money
14. Structural Arithmetic Board 15
15. pictured counting discs
16. "Number-Ite" (a type of peg board) 48
17. "Slidex" (a visual arithmetic chart) 45
18. grouped counting blocks (5-blocks, 9-blocks, 10-blocks, etc.)

Games

1. Pig Dice Game 33
2. dominoes
3. "jacks"

Sound Film

1. "What Is Four?" 53

Filmstrip

1. "A Number Family in Addition" 37

Stereopticon Slides

1. "Arithmetic" 37

Identification: Meaning or Significance of Number Symbols

Basic (other than films)

1. collection of empty containers with labels indicating contents
2. picture-symbol cards and charts
3. semi-abstract-pictorial-number cards and charts
4. semi-abstract symbolic cards and charts
5. number books
6. pictures designating various quantities

Sound Film
1. "What Is Four?" 53

Filmstrip
1. "What Numbers Mean" 37

Stereopticon Slides
1. "Arithmetic" 37

**Measurement: Areas and Volumes, Liquid and Dry, Size and Weight, etc.; Denominate Numbers**

Basic (other than films)
1. bottles: half-pint, pint, quart
2. containers: cup, gallon, peck, five-gallon, bushel
3. measuring spoons and cups
4. pictures of containers of various sizes
5. cubes of various sizes
6. pedometer
7. foot rulers, yard sticks, tape measures
8. pictures portraying use of various linear measuring instruments
9. "Teaching-Aid Ruler" 31
10. measure charts
11. square-measures (1 sq. in. and 1 sq. ft., for clarifying purposes)
12. block tile 22
13. wooden disc (1' diameter)
14. carpenter's square
15. dozen-egg carton
16. 1/2-dozen muffin pan
17. height and weight charts
18. balance scales (avoirdupois weight)
19. spring scales (avoirdupois weight)
20. Play-Weight Scales
21. pictures portraying uses of scales
22. Scales: "Wa-it"
23. Balance Scale: "Counting House"
24. clock dials, adjustable hands
25. felt-board clockfaces
26. puzzle clock face
27. Judy Clock Face
28. paper plates (for making clock dials)
29. conventional clocks and watches
30. stop watch
31. egg timer
32. sandglass
33. time-belt map of United States
34. time-belt map of world
35. clock dial colored to represent time for play, work, school, sleep, etc.
36. booklet: Set the Clock
37. centigrade thermometer
38. fahrenheit thermometer
39. manipulative thermometer
40. pictures of daily activities, together with clockdials
41. pictures of thermometers indicating temperatures under varying conditions
42. calendars (with removable numbers)
43. calendars (conventional)

Games
1. "Tell Time Quizmo"
2. "Pin the Tail on the Donkey" (magnetic)

Sound Films
1. "Measurement"
2. "Areas"

Filmstrips
1. "History of Area Measures"
2. "History of Our Calendar"
3. "The Story of Measurement"
4. "History of Linear Measures"
5. "History of Our Number System"
6. "History of Telling Time"
7. "History of Weight and Volume Measures"
### Source of Certain Commercial Aids

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### Money and Business Practice

#### Basic (other than films)

1. real coins and paper money  
2. toy money  
3. abacus  
4. place-pocket  
5. money-value charts  
6. price lists  
7. mail order catalogs  
8. sales slips  
9. sales stamps  
10. checks (blank and negotiated)  
11. receipts (blank and negotiated)  
12. money order forms  
13. order blanks  
14. bus transfers  
15. tickets, tokens  
16. change maker  
17. change tray  
18. cash register  
19. toy cash register  

### Games

1. "Let's Go Shopping"  
2. "Rich Uncle"  
3. "Monopoly"
Sound Films

1. "The Story of Money" 17
2. "About Money" 4
3. "Fred Meets a Bank" 6
4. "Using the Bank" 10
5. "What Is Money?" 6

Filmstrip

1. "Time and Money" 11

Number System: Organization, Serial Idea, Decimal Nature

Basic (other than films)

1. 100-peg board
2. dart board (having numbers arranged from 0 through 9, 10 through 19, etc.)
3. abacus
4. place-pocket
5. toy and real coins
6. number cards
7. counting frame (fact finder)
8. number chart (hundred chart--having numbers arranged from 0 through 9, 10 through 19, etc.)
9. number chart (with removable numbers)
10. tens block
11. tens square (or hundred board)
12. 1000-peg board

Game

1. "Quiz Me" 31

Sound Film

1. "The Teen Numbers" 53

Filmstrip

1. "History of Our Number System" 53
Source of Certain Commercial Aids

Operations: Addition, Subtraction, Multiplication and Division

Basic (other than films)

1. all sorts of counting devices (previously listed)
2. coins
3. cutouts of animals, fruit, and other objects
4. abacus (conventional)
5. abacus (19-bead type)
6. counting frame (fact finder)
7. charts "objectifying" the various operations
8. charts employing dots, x's, o's, stars, etc. to illustrate the various operations
9. pictured counting discs
10. Combination Slide Board
11. "Slidex" (a visual arithmetic chart)
12. place-pocket
13. Tangible Additive Board
15. flash cards (of the various operations)
16. calculating device: "Junior Arithmetic"
17. adding machines (various kinds)
18. slide rule (elementary)
19. Rapid Easy Drill Charts
20. Ad-O-Master Jr. (Magic Calculator)
21. Toy cash register
22. Math-O-Block
23. Work book: Discovering Arithmetic
24. Rotary scales
25. sliding scales
26. grouped number blocks
27. subtraction box
28. multiplication-division board
29. multiplication-division table
30. finger board multiplier
31. multiplication wheel
32. addition wheel
33. Add-A-Count Scale

Games

1. "Arithmetic Quiz"
2. Pig Dice Game
Source of Certain Commercial Aids

3. Magnetic "Fish Pond"
4. dominoes
5. "jacks"
6. "Addit"
7. "Lotto"
8. "Keeno"
9. "Quizmo"
10. "Sorry"
11. "Hot Numbers"
12. "Over the Rainbow"
13. "Rich Uncle"
14. "Imma Whiz"
15. "Contack"
16. "Monopoly"
17. Rook
18. "Sum Fun"
19. "Tally It"
20. Magnetic dart board
21. calculating device: "Junior Arithmetic"
22. puzzle: "Attaboy"
23. magnetic ring toss

Sound Films
1. "Parts of Nine"
2. "Addition Is Easy"
3. "Subtraction Is Easy"
4. "Borrowing in Subtraction"
5. "The Meaning of Long Division"
6. "Arithmetic for Beginners"
7. "Division Is Easy"
8. "Multiplication Is Easy"

Filmstrips
1. "Multiplication and Division"
2. "Addition and Subtraction Concepts"
3. "Work and Play with Numbers" (5 and 6, 7 and 8, 8 and 9, 9 and 10)
4. "Order of the Operations"
5. "A Number Family in Addition"
6. "Compound Subtraction"
7. "Addition and Subtraction"
8. "The Three's"
9. "The Two's in Division"
Stereopticon Slides

1. "Arithmetic"  37

Tachistoscope Slides

1. "Number Combinations"  24

Percentage

Basic (other than films)

1. Hundred Board  33
2. "Fraction Trainer"
3. 100-peg board
4. collection of illustrations employing concept "percentage" (e.g. newspaper advertisements indicating a certain percent off)
5. lists of batting averages
6. standings of various ball clubs

Sound Films

1. "Percentage"  18
2. "Per Cent in Everyday Life"  6
3. "The Meaning of Percentage"  53

Filmstrips

1. "Introduction to Percentage"  8
2. "Fractions, Decimals, and Percentage"  18
3. "Problems in Percentage"  8
4. "Using Percentage"  8

Place Value and Face Value

Basic (other than films)

1. abacus
2. place-pocket
3. speedometer
4. meters of various kinds
5. "Counting Meter"  48
6. pennies, dimes, silver dollars
Source of Certain Commercial Aids

Sound Film
1. "The Teen Numbers"

Problem Solving
Basic (other than films)
1. counters of various kinds (as previously listed)
2. abacus
3. counting frame (fact finder)
4. coins and currency
5. pictorial illustrations pertinent to problems
6. illustrations using dots, o's, x's, stars, tally marks, etc. to "objectify" problems
7. measuring instruments of various kinds to concretize abstract problems

Filmstrip
1. "Problem Analysis"

Proof and Generalization
1. counters of various kinds (as previously listed)
2. abacus
3. number frame (fact finder)
4. coins and currency
5. foot rulers marked in eighths of inch; others in sixteenths of inch
6. spirit level
7. tape measure
8. bathroom scales, grocer's scales
9. measuring cups, measuring spoons
10. containers of various sizes
11. micrometer
12. plumb line
13. centigrade thermometer
14. fahrenheit thermometer
15. pedometer
16. standard reference circle
Quantitative Thinking, Mathematical Judgment

These concepts require the use of essentially the same instructional aids as listed in the two preceding sections.

**Ratio and Relationship**

**Basic**

1. scale models
2. pictures of objects of contrasting size (tall buildings, low buildings; large animal, small animal; etc.)
3. maps with different scale of miles
4. clay (to construct objects of relative size)
5. balance scales
6. speedometer (to show relationship of tenths of miles to miles)
7. containers of various sizes
8. wooden blocks of various sizes and thicknesses
9. rope, string, ribbon, sticks, etc. of various lengths
10. objects of various weights
11. pictures of containers partially filled
12. pantograph
13. telescope, field glasses, photographic lens
14. "Mek-N-Ettes" (mockup of intermeshed gears) 48
15. Play-Weight Scales 34
17. "Math-O-Block" 8
18. reading glass
19. Lincoln Logs (construction logs) 26
20. Tinker Toys

**Games**

1. "Pin the Tail on the Donkey" (magnetic)
2. ring toss
3. horse shoes (rubber recommended)
4. dart board (magnetic recommended)
Reading and Writing of Number Ideas

Basic (other than films)

1. large calendar (conventional type)
2. calendar (with removable numbers)
3. collection of stories and poems involving quantities and shapes
4. collection of illustrated advertisements indicating prices
5. scrapbook of recipes
6. class diary
7. log of trips taken
8. attendance chart
9. milk chart
10. flannel board, felt board, magnetic board
11. toy dial telephone
12. telephone directory
13. clock dial
14. thermometer
15. grocer's scales
16. bathroom scales
17. speedometer
18. meters of various kinds
19. collection of house numbers
20. pictures of street numbers
21. number chart (conventional type)
22. number chart (with removable numbers)
23. mail order catalogs
24. price lists
25. Booklet: Follow the Stars 49
26. "Number Builder" 21
27. picture-number books
28. picture-number cards

Games

1. "Lotto" 51
2. "Contack" 33

Filmstrips

1. "Reading Numbers to 10" 10
2. "Writing Numbers to 10" 10
3. "Reading Numbers to 50" 10
4. "Reading Numbers to 100" 10
5. "Working with Numbers to 100" 10
6. "Writing Numbers to 100" 10
7. "Writing Fractions" 8
8. "We Learn Numbers," Part I (1 through 5) 11
9. "We Learn Numbers," Part II (6 through 10) 11

Vocabulary of Number and Quantity

Basic (other than films)

1. collection of pertinent arithmetical terms
2. scrapbook of familiar mathematical concepts, pictorially portrayed

Filmstrip

1. "Arithmetical Concepts" 11

Zero Representation

Basic (other than films)

1. measuring devices to indicate fractional concepts between 0 and 1
2. number chart to structure ideas of decade organization of numbers
3. abacus
4. place-pocket
5. speedometer
6. pennies, dimes, silver dollars
7. thermometer

Filmstrip

1. "Zero a Place Holder" 35
Manufacturers or Distributors of Certain Arithmetic
Instructional Aids Referred to in Foregoing List

2. Archer Plastics, Elmhurst 73, New York, N. Y.
4. Children's Productions, Box 1313, Palo Alto, Cal.
5. Cook and Company, Minneapolis 12, Minnesota.
8. Curriculum Films, Inc., 41-47 Crescent St., Long Island City, N. Y.
11. Eye Gate House, Inc., 330 West 42nd Street, New York 18, N. Y.
12. Exton Aids, Milbrook, N. Y.
13. Films Incorporated, 330 West 42nd Street, New York 18, N. Y.
16. Ideal School Supply Company, 8312 Birkhoff Avenue, Chicago 20, Ill.
17. International Film Bureau, 57 E. Jackson Blvd., Chicago 4, Ill.
23. Kenworthy Educational Service, Buffalo 3, N. Y.
26. Lincoln Logs, Chicago 47, Ill.
27. L. R. Posey, Southern University, Baton Rouge, La.
29. Maxim Games Company, New York, N. Y.
32. Mystoplane Company, Inc., 19 W. 24th St., New York, N. Y.
33. Parker Brothers, Inc., New York, N. Y.
34. Paul Bon Hop, Inc., New York 57, N. Y.
35. Photo and Sound Production, 116 Natoma Street, San Francisco 5, Cal.
39. Samuel Gabriel Sons and Company, New York, N. Y.
40. Scott, Foresman, and Company, 433 E. Erie Street, Chicago 11, Ill.
41. Selchow and Righter Company, New York, N. Y.
42. Self-Teaching Flashers, Lincoln 6, Neb.
43. Sifo Company, St. Paul, Minn.
44. Simon and Schuster, New York, N. Y.
45. Slidex Company, Providence, R. I.
47. Teaching Film Custodians, Inc., 25 W. 43rd Street, New York 18, N. Y.
48. The Judy Company, Minneapolis, Minn.
49. The Saalfield Publishing Company, Mansfield, Ohio
50. Visual Sciences, Suffern, N. Y.
51. Whitman Publishing Company, Racine, Wis.
53. Young America Films, Inc., 18 E. Forty-First Street, New York 17, N. Y.
APPENDIX B

DRAMATIZED PROCEDURES ILLUSTRATING THE USE OF INSTRUCTIONAL AIDS IN ARITHMETIC EDUCATION
DRAMATIZED PROCEDURES ILLUSTRATING THE USE OF INSTRUCTIONAL AIDS IN ARITHMETIC EDUCATION

Concept: Rational Counting

Grade Level: First

Purpose of the Lesson: To give the pupils practical experiences in counting and ranking, to stimulate an interest in counting, and to introduce the idea of grouping.

Previous Readiness Experiences: The pupils have had extensive informal rational counting experiences in identifying the number of objects in a group. They have not yet attempted to represent the number of objects contained in such groups by the use of other counting objects and devices.

Problem Setting for the Lesson: The faculty at Hatcher Elementary School had each received a memo from the principal requesting a physical inventory of the supplies and equipment of their classrooms. Most of the teachers complied in a matter-of-fact manner and dismissed the matter from mind. However, Miss James, the first-grade teacher, saw in the request a golden teaching opportunity.

Materials Provided for the Lesson: Objective counters of various kinds such as counting blocks, discs, sticks, pennies, pebbles, bottlecaps, acorns, checkers, and
counting frame, and the sound film, "We Learn to Count."

Procedure in Teaching: The lesson was approached through the request the principal had made.

Teacher: "Boys and girls, Mr. Smith, our principal, has asked us to make a list of the number of objects in our classroom. Do you think we can help him?"

Pupils: "Yes-s-s-s!"

Sue: "But, how? We don't know how to write."

Jane: (Reassuringly) "But Miss James does. We can count how many things there are, and Miss James can write them down."

Teacher: (Smiling) "We shall all write the number down. There is more than one way to write. Now, I'm going to give each of you boys and girls some counters. Johnny, here are some counting sticks; Jimmy, some wooden blocks for you; Sally, would you like to use these acorns? John, how about these pebbles? Buckeyes for Julia, checkers for Sam, and bottle caps for Bill. May, would you and Bobby like to work together on the counting frame?" (Continue in this manner until each child has at his disposal some form of manipulative counter)

"Now, if everyone is ready, we shall start counting the objects in our room."

Tommy: "Let's count the tables first, Miss James. I know how many tables we have."
Teacher: "A fine place to start. Will each of you boys and girls at your seats, place on your desks as many of the counters you have as we have tables in our room. And, May, will you and Bobby push out on the wire of your counting frame as many beads as we have tables in our room."

Sue: (Excitedly) "I have mine, Miss James!"

Others: "Mine, too! Look at mine, Miss James!"

Teacher: "In a moment. But first, will you see whether your neighbor is having any trouble? Perhaps you can be of help to him."

Joe: "I need some help, Miss James. May Bill help me with mine?"

Teacher: "Yes, Joe, I'm certain Bill will gladly help you. In counting we must be very careful. We want to be certain that the number we give Mr. Smith is the correct one, don't we, boys and girls?"

Pupils: "Yes-s-s-s-s-s!"

Teacher: (After a few moments) "Now, Jimmy, will you tell the other boys and girls the story of your counters?"

Jimmy: "I placed 6 counting blocks on my desk. That showed that we have 6 tables in our room."

Sam: (Volunteering) "I put 6 checkers on my desk. That means our room has 6 tables."

May: "Bobby and I showed we have 6 tables by pushing 6 beads out on one of the wires of our counting frame."
Teacher: "John's second pebble is larger than the others. I wonder whether he placed it there on purpose."

John: "Yes, Miss James. The second table is larger than the others."

Each pupil was given an opportunity to verbalize his manipulation of the concrete counters on his desk before the teacher proceeded to the next objects to be counted.

Several Pupils: "This is fun! Now we know how to write with counters."

In later lessons the pupils were given opportunities to record the number of objects in the room by the use of simple drawings, then by semi-abstract symbols such as marks, o's, and x's, and finally by the use of abstract numerals. To summarize and review their various experiences in counting, the group were then shown the instructional film, "We Learn to Count."

Concept: Combining Groups

Grade Level: Two

Purpose of the Lesson: To give the pupils practical experiences in combining groups of objects and finding the total. To lay the foundation for systematic addition and subtraction.

Previous Readiness Experiences: The pupils have had extensive meaningful experiences in rational counting.

Problem Setting for the Lesson: As was customary each
year, the principal had requested of each faculty member a physical inventory of the contents of his classroom. The second-grade teacher relayed this request to her group.

**Materials for the Lesson:** Concrete counting aids, such as counting blocks, discs, sticks, pennies, acorns, and the abacus, the place-value pocket, and the filmstrip, "A Number Family."

**Procedure in Teaching:** The lesson was approached through the request the principal had made.

**Teacher:** "Boys and girls, at our last faculty meeting Mr. Smith asked all the teachers to hand in to him a list of the number of objects in their classrooms. I wonder if you boys and girls would like to help me make out the list for our room."

**Pupils:** "Yes, yes!"

**Sammy:** "We did it last year, Miss Brown. We took a count of the things in our room for Mr. Smith. It was fun."

**Sue:** "We used counters like buttons and sticks at first. Then we used pictures and dots."

**Mary:** "Then we used real numbers. We learned how to write the number of things using real numbers."

**Teacher:** "Goodness! Such enthusiasm! Now, there is just one thing more. Mr. Smith not only would like for us to report the number of objects in our room, but he also wants to know how many different kinds of each object
there are."

Tommy: "You mean, Miss Brown, that he would like to know how many folding chairs and how many small chairs we have separately, then how many altogether?"

Teacher: "That's right, Tommy. Do you boys and girls think we can do this for Mr. Smith?"

Pupils: (Eagerly) "Yes, yes!"

Teacher: "There's no need to tell you boys and girls that we must be very careful to count correctly. Mr. Smith certainly doesn't want us to give him the wrong number, does he?"

Pupils: "No-o-o-o!

Teacher: "In order to count correctly, we shall count in each of the four ways that we have learned. To check our count, we shall ask Johnny's group to use the counters they have at their seats; Grace's group may draw pictures to show their answers; Sammy's group may use dots or marks to represent their answers. There is room for a few of you at the chalkboard. The others may use paper and pencil. Sue's group may use numerals to represent their count. Some of you may work at the chalkboard, also, the others at their seats."

Sue: "We understand, Miss Brown. We can do it. May we start with the chairs?"

Teacher: "Fine! Now, will Johnny's group place on their
The various members of the group responded by placing 8 sticks, 8 blocks, 8 acorns, 8 checkers, and the like on the desks in front of them. The second group sketched 8 chairs on their papers or on the chalkboard. The third group made 8 dots, marks, o's, or the like on their papers or at the chalkboard. Sue's group responded by writing the numeral 8; a few wrote the word "eight," while one or two others did both.

Jimmy: "I don't believe anybody needs help, Miss Brown. We've done this before."

Teacher: "You certainly learned well. Now Mr. Smith also wants to know the number of folding chairs."

Joe: "Aw, that's easy. We just do the same thing for the folding chairs as we did for the little chairs."

Sue: "But, shouldn't we put this group close to the other because Mr. Smith wants to know how many chairs there are altogether."

Teacher: "An excellent idea, Sue. Suppose all of you boys and girls try putting this group of blocks, sticks, pictures, dots, numerals, or whatever you are working with just underneath the group with which you represented the number of small chairs."

Joe: "Oh, I see now! Then we can shove the two groups together, count the new big group, and that will be the
total number of chairs. Boy, how simple!" (Accordingly, the respective groups placed 5 counters similar to those with which they represented the small chairs underneath the first group)

Jane: "We can tell Mr. Smith that we have 8 small chairs and 5 folding chairs or 13 chairs in all."

Bill: "We forgot Miss Brown's chair! It's a straight-backed chair. We will have three groups of chairs instead of just two."

Bobby: "Then I will place 8 bottle caps in the top group, 5 bottle caps in the second group, and 1 bottle cap in the third group. That makes 14 bottle caps in all when I shove them together into one group. That means we have 14 chairs altogether in our room."

Davy: (Who was working with the abacus) "I've got troubles, Miss Brown. How in the world can I show all that with only 9 beads on this wire to do it with? I shoved up 8 beads to show that we have 8 small chairs, but that only leaves one bead for the rest of the chairs. I sure don't get it."

Teacher: "Perhaps the place-value pocket would help, Davy. Jane and Sally have been working with it and maybe they will explain to us their procedure."

Jane: "Well, we put 8 markers in the top pocket of the ones column since there are 8 small chairs. Then we put
5 markers in the second pocket down in the ones column since there are 5 folding chairs. Then we placed 1 marker in the third pocket underneath the others to stand for Miss Brown's chair."

Sally: "Then we took the markers out of the separate pockets and put them all in one pocket in the ones column to see how many there were in all. When we discovered there were 14 markers altogether, we knew we would have to give 10 of them to the tens pocket because 9 is the largest number you can show in any one column."

Jane: "So, we bundled up ten of the markers, put a rubber band around them and put the bundle over in the tens column. Then instead of having 14 single markers we had 1 bundle of 10 markers and 4 single ones. It's a lot easier to keep in mind than trying to think 'fourteen ones,' besides we can write it in columns then."

Joe: "Now I see. I shove up 8 beads to show that we have 8 small chairs. I was right that far. But then when 5 more chairs are to be added, I should think 1 ten and 3 ones instead of 13 ones. So, I would shove back all the 8 beads in the ones column except 3 and shove up one bead in the tens column. Then to show Miss Brown's chair I should shove up one more bead in the ones column. Altogether then I should have 1 bead in the
tens column which stands for 10 chairs and 4 beads in the ones column which stands for 4 more chairs. Boy, arithmetic is fun when you can make discoveries like that."

Grace: "I drew 8 chairs in one row, 5 chairs in the second row, and 1 chair in the third row. Then I drew a line and below it drew 1 group of fourteen chairs to show that altogether we have 14 chairs."

Sammy: "On my paper I put 8 dots in the top group, 5 dots in the next group, and 1 dot in the bottom group. Then I ran a ring around all the dots. Then I counted all the dots inside the ring and found there were 14."

Sue: "I wrote 8 chairs, 5 chairs, and 1 chair in a column. I added the three numbers together and got 14. I checked my answer by making marks on the board and counting them. I also counted the three sums on my fingers."

A similar procedure was then followed in inventorying other objects in the classroom, each pupil being given ample opportunities to verbalize the process used. At the end of the experience, the group were shown the filmstrip, "A Number Family," which served to summarize and review the steps which the pupils themselves had been following.

Concept: Borrowing in Subtraction

Purpose of the Lesson: To extend the pupils'
experiences in subtraction, introducing in a meaningful way the concept of "borrowing." To relate subtraction to practical every-day experiences of the pupils.

Previous Readiness Experiences: Pupils have had ample experiences in subtracting both objectively and abstractly quantities not requiring "borrowing."

Problem Setting for the Lesson: A pupil is faced with the problem of separating the money his mother gave him, donating an undisclosed amount to the Heart Drive, but "holding back" enough to pay for his lunch.

Materials Provided for the Lesson: Pennies and dimes, other objective counters, place-value pocket, abacus, chalkboard, pencil and paper, and the sound film, "Borrowing in Subtraction."

Procedure in Teaching: The lesson was approached through the uncertainty of the pupil as to how much money he might donate to the Heart Drive.

Johnny: "Mother was in such a hurry this morning that she just handed me this bit of change and said to give some of it to the Heart Drive, just keeping out enough to buy my lunch. I don't know how much to give to the Heart Drive, Miss White, and still have enough money left for lunch."

Ann: "Mother did that same thing to me last week. I had the same trouble as Johnny. She left me with 35¢ and
told me to buy my lunch and give the rest to the Jr. Red Cross."

Teacher: (Sensing a possible functional lead to the concept of "borrowing" in subtraction, which she had planned to introduce at this time) "Well, that is a coincidence. How did you decide how much you might give to the Jr. Red Cross, Ann, since that money was taken up during the opening exercises? Maybe you can help Johnny solve his problem."

Ann: "What I did, Miss White, was to write down 35¢, what Mother gave me and subtract the 25¢ that I had to save out for lunch from it, like this." (demonstrating on the blackboard) "I found out I could give 10¢ to the Jr. Red Cross. My answer was right because I had enough money left to buy my lunch afterwards."

Teacher: "That was proof enough that your answer was correct. If it worked in your case, it should work in Johnny's. Suppose we try it, Johnny." (Inwardly hoping that the solution of his problem would involve borrowing.)

Joe: "We'll have to know how much money he has first. How much money did your mother give you in all, Johnny?"

Johnny: "Here it is." (laying the change on his desk) "Three dimes, 2 nickels, and 3 pennies—43¢ in all."

Ann: "Well, all there is to do is write down 43¢ (doing so on the chalkboard), then put 25¢ under it and
subtract. The answer will be the amount of money Johnny can give to the Heart Drive. 5 from 3 is -- oh! There's something wrong here, Miss White. 5 is bigger than 3. You can't take 5 from 3."

**Teacher:** (Inwardly pleased with the turn Johnny's problem had taken) "That's right, Ann. We can't take 5¢ from 3¢. However, we do know that we can subtract because 25¢ is smaller than 43¢, and we know that we can subtract whenever the smaller number is on the bottom. Let me ask you boys and girls this question: when your father is downtown and wants to put a nickel in the parking meter but has only, say, a half-dollar, what does he do?"

**Jimmy:** "Dad was in that very fix just last Saturday. What he did was to go to the drug store and get change."

**Teacher:** "That's right; he changed the large coin for other smaller coins. Now that's exactly what we do in subtracting when we can't take one number from another."

**Billy:** "We know he can subtract the two sums, Miss White, 'cause his 43¢ is made up of 3 dimes, 2 nickels, and 3 pennies. If he picks up 2 dimes and one nickel, what is left on the desk is how much he can give to the Heart Drive."

**Mary:** "I could work it with blocks. I would simply place 43 blocks on my desk to represent Johnny's 43¢. Then I would pick up 25 of them to represent the price of
lunch. The blocks that remained on the desk would show the amount of money Johnny would be able to give to the Heart Drive."

Alice: "I could do the same thing on paper or at the board. I would make a group of 43 marks, then 'pen up' 25 of them to represent the 25¢ Johnny has to save for lunch money. The marks outside the 'pen' would show the amount of money Johnny might give to the Heart Drive."

Teacher: "These explanations are fine, boys and girls, but are we answering Ann's question? She is faced with the problem of taking 5¢ from 3¢. How can she do it? From past experiences you will remember that the amount 43¢ may be read another way to denote the place value of the digits."

Joe: "I know, Miss White. It is 4 dimes and 3 pennies. And the 25¢ may be read 2 dimes and 5 pennies. Is that right, Miss White?"

Teacher: "Right, Joe. Now with this way of naming the amounts in mind, does anyone of you boys and girls see what we can do when we find we are unable to subtract 5¢ from 3¢ in a problem such as we have here?"

Sally: "We might take one of the 4 dimes and change it into pennies in the same way Jimmy's daddy got change to put into the parking meter. Johnny would still have the same amount of money, but instead of having 4 dimes and
3 pennies, he will now have 3 dimes and 13 pennies."

Teacher: "That's certainly excellent thinking, Sally.

Let's make those changes you have mentioned in the 43¢
which Ann has written on the chalkboard. We'll cross
out the 4 and put a small 3 above it to show that we
took away one of the dimes leaving only 3. Then we'll
put a small 1 in front of the 3 in the ones column to
denote that it is 13 now instead of 3. We say that one
of the dimes was "borrowed" and changed into 10 pennies."

Ann: "Oh, I see! Now I have 13 pennies on top and only 5
on the bottom. So, I can subtract 5 from 13. That
gives me 8¢, and 2 dimes from 3 dimes leaves 1 dime.

So, my answer is 1 dime and 8 cents or 18¢.

Sue: "I believe I could work that out on the place-value
pocket. It's just like carrying in addition, only back­
wards; it's 'carrying backwards,' isn't it, Miss White?"

Teacher: "Why, Sue, I think that's a very good name for
the procedure, although arithmetic books ordinarily
speak of it as 'borrowing.' You may wish to use the
customary terms."

The place-value pocket was set up, with Sue placing 4
bundles, each having 10 strips, in the 10's place and 3
strips in the units place. Then she removed one of the
bundles of 10 strips from the tens pocket, removed the
rubber band, and placed the 10 separate slips of paper in
the units pocket with the other three units slips. The place-value pocket then held 3 bundles of 10 strips and 13 single strips. 2 bundles of 10 strips and 5 single strips were then removed to represent the subtraction of 25¢. Remaining in the place-value pocket were 1 tens bundle and 8 units strips, representing the 18¢ which Johnny might donate to the Heart Drive.

Jane: "May I show how to do it on the abacus, Miss White?"

After a few typical errors, Jane demonstrated the subtraction process in the following manner: She represented 43¢ by shoving up 4 beads on the tens wire and 3 beads on the units wire. To subtract 5 ones from 3 ones, she shoved back one tens bead and shoved up 5 units beads. Then to subtract 2 tens from the remaining 3 tens, she drew back two of the tens beads, leaving 1 tens bead and 8 units beads up on the two respective wires, these two groups of beads representing 13¢, the remainder.

In time, other examples and problems involving borrowing were introduced and solved in a similar manner. To summarize and review the operation for the group, the sound film, "Borrowing in Subtraction," was shown. "Borrowing" and the use of the "borrowing crutch" to this group had real significance.
Concept: The Addition and Subtraction of Simple Common Fractions

Grade Level: Fourth

Purpose of the Lesson: To introduce in a meaningful way some of the main ideas and procedures involved in the addition and subtraction of simple common fractions and to consider the significance of the terms numerator and denominator.

Previous Readiness Experiences: The pupils have a good understanding of the processes of addition and subtraction of whole numbers and have had rich experiences talking and writing about concepts such as 1/2 of the pupils, 1/4 of the books, and the like.

Problem Setting for the Lesson: The class had been discussing the matter of how the 24-hour day is divided differently for people having different customs and occupations.

Materials Provided for the Lesson: Sheets of paper 11 x 8½, pencils, chalkboard, rulers, crayons, the sound film, "How to Add Fractions," and the filmstrip, "Addition and Subtraction of Fractions."

Procedure in Teaching: The lesson was approached

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1 This illustration was adapted from a paper prepared by Mary Jenkins, an undergraduate student in Education 510 during the Autumn Quarter, 1955, in the College of Education, The Ohio State University.
through an informal discussion of how the pupils' own 24-hour day is divided.

**Teacher:** "Since half of our day has passed, how much of the day do we have left to finish our planned work?"

**Susie:** "One half a day."

**Teacher:** "How do you know that?"

**Susie:** "Well, if half of the whole day is gone, there's just one more half left in the day."

**Johnny:** "I know something else. There are two fourths left."

**Jane:** "We go to school 3 hours in the morning and 3 hours in the afternoon. The half-day of school we have yet to go is 3 hours long, but that's only 1/8 of the whole day."

**Tim:** "Why don't we draw a picture of a day divided into eighths?"

**Teacher:** "A good idea, Tim. What materials will you need?"

**Tim:** "To get it exactly right we will have to use a ruler and measure it off into eight parts. Some of us could work at the board and the rest at our seats."

**Teacher:** "We might see the hands of those who want to work at their seats. (Hands go up.) Is that a majority, Joe?"

**Joe:** "Yes, it's over half of the pupils and that's a majority."
Teacher: "How do you know it is over half?"

Joe: "Because half of 25 is $12\frac{1}{2}$, and we had two more hands than 12. You can't cut a hand in half."

Teacher: (After papers and rulers are made ready) "How long is your paper?" (Pause while pupils measure)

Voices: "Eleven inches."

Teacher: "How may we find $\frac{1}{2}$ of it, Joan?"

Joan: "Fold it in half."

Tommy: "Measure it down to half."

Teacher: "Tommy, what would your measurement be in inches?"

Tommy: "About five because five is half of ten and the paper is just a little over that."

Teacher: "How can we help Tommy find exactly how long $\frac{1}{2}$ of the paper is, Joan?"

Joan: "We could fold it in half and measure the half. Like this, Tommy." (She helps Tommy fold and measure the folded half.)

Tommy: "It's $5\frac{1}{2}$ inches, Mrs. Jones." (In the meantime most of the other pupils had also folded their papers in half.)

Teacher: "How many halves do you have?"

Voices: "Two!"

Susie: "Two halves or one whole. Two halves equal one whole."

Teacher: "Right. Now will you color the top half your
favorite color and mark it 1/2. (after a lapse of time.) Now will you color one half of the bottom in your second favorite color and mark it ---"

Voices: "1/4!" (Time is allowed for the coloring and marking.)

Steve: "Now we have one half and one half of a half colored."

John: "That is 3/4 of the whole sheet of paper. If we divided the top half into fourths, we would have 3/4 colored in all. 1/2 of anything is the same as 2/4, and 2/4 and 1/4 equal 3/4.

Teacher: "That's right, John. When we speak of coloring 3/4 of our sheet of paper, we really mean that we divided our sheet of paper into four equal parts and colored 3 of them. The bottom part of the fraction, or the denominator, tells us into how many parts a thing has been divided. The top part, or numerator, tells us the number of such parts we have or are concerned with at the time. Common fractions cannot be added or subtracted unless they have the same denominators. John told you a moment ago how he added 1/2 + 1/4. (writing the algorism on the chalkboard.) In order to add the two fractions he changed 1/2 to 2/4. 2/4 and 1/4 may then be added or subtracted because they have the same denominator."
Joe: "At first we colored 1/2 of our sheet of paper. The 2 is the denominator and 1 is the numerator. The 2 means we divided our paper into two equal parts. The 1 means we colored one of the two parts."

Teacher: "Right, Joe. Now if we have colored 3/4 of our sheet of paper, someone might want to know what part we have not colored."

Ann: "It is 1/4, Mrs. Jones, since we might say we divided our paper into 4 equal parts and one of the 4 parts has not been colored. One part of 4 equal parts is called 1/4."

Susie: "4/4 - 3/4 = 1/4."

Teacher: "Right. Now, as I remember it, we started out to divide our day into eighths. Does anyone see how we can show 1/8 on our paper? (Hands in air) Ann."

Ann: "We can color half of the 1/4 of our sheet of paper that is not yet colored. That will be 1/8 of the whole sheet."

Jane: "After Ann's suggestion has been followed) "7/8 of our sheet of paper is colored now. 1/8 of it is not colored."

Teacher: "Can you 'see' 8/8 in your sheet of paper? Drawing some dotted lines might help you. (a pause) Can some of you tell me the story of what you see now on your paper?"
Joan: "By making dotted lines it is easy to see that \( \frac{1}{2} = \frac{4}{8} \), that \( \frac{1}{4} = \frac{2}{8} \), and that our whole sheet of paper contains \( \frac{8}{8} \)."

Joe: "It's easy to see that \( \frac{1}{2} + \frac{4}{8} = 1 \) whole."

Susie: "I can see that \( \frac{8}{8} - \frac{1}{2} = \frac{1}{2} \)."

Similar statements continued until each child had had an opportunity to verbalize his reactions. In later lessons the addition and subtraction of common fractions concepts were expanded. Before leaving these processes, the pupils were shown the filmstrip, "Addition and Subtraction of Fractions" as well as the sound film, "How to Add Fractions," in order to summarize the various steps and facets connected with the addition and subtraction of common fractions.

**Concept:** Multiplication of a Common Fraction by a Whole Number

**Grade Level:** Fifth

**Purpose of the Lesson:** (1) To guide pupils in an understanding of the process of multiplying a common fraction by a whole number, (2) to develop the generalization that to multiply a common fraction by a whole number affects only the numerator of the fraction.

**Previous Readiness Experiences:** The pupils have had rich experiences in adding and subtracting common fractions and in the multiplication of whole numbers. They have also
been introduced to the terms, numerator and denominator, and have had experiences in the reduction of fractions to their lowest terms.

**Problem Setting for the Lesson:** The pupils had just had their September physical checkup and were comparing heights, weights, and other findings with those for the previous May checkup.

**Materials Provided for the Lesson:** Yardstick, rulers, chalkboard, paper, pencil, glass measuring cups, sound film, "How to Multiply Fractions," and the filmstrip, "multiplying Fractions."

**Procedure in Teaching:** The lesson was approached through the speculation of some of the pupils concerning how much they would grow in a year, based on their rate of growth during the three summer months.

**Tommy:** "The nurse told me that I had gained 3/8 of an inch since getting measured last May."

**John:** "She told me I had gained 1/2 inch. I wonder how much I will grow in a year."

**Teacher:** "Seizing the opportunity to approach the multiplication of common fractions functionally) "It might be interesting to find out, assuming that you grow for the entire year at the same rate as during the summer."

**Sue:** "If we did grow at the same rate, we would grow 4 times as much in a year as we did during the summer."
Our summer vacation is 3 months long, and that's 1/4 of a year."

Joe: "So, in a year Tommy would grow four times 3/8 inches, but I don't know how much that would be."

Teacher: (Writing the algorism on the chalkboard) "Yes, that would be 4 x 3/8 inches or 4 x 3/8". What can we do now to find out how much Tommy would grow in a year at that rate?"

Mary: "Why not measure it on the yardstick? It's marked off in eighths of inches. Remember, we used it when we were adding fractions. We could measure 3/8 of an inch on it 4 times and see what the total length would be."

Teacher: "A good idea, Mary. A few of you might use our two yardsticks; others may use your own rulers."

Tommy: (After a minute spent in combining four 3/8" on his ruler) "It comes out twelve-eighths inches or one inch and a half, Miss Smith. Twelve-eighths inches are the same as one inch and a half. I would grow 1 inch and a half in a year."

Teacher: "Perhaps we could draw a picture of it, too. A few of you might work at the chalkboard, the remainder at your seats."

Accordingly a number of diagrams depicting a section of a ruler are marked off in 1/8ths, 1/4ths, 1/2's, and inches. In her drawing at the chalkboard Miss Smith
bracketed each of the 4 groups of 3/8" markers, then bracketed these four groups with 1 large bracket.

Joe: "Four groups of 3/8" in each group make a total of 12 eighths inches, and that makes an inch and a half in all. That's just like multiplying any number."

Teacher: (Completing the algorism which she had placed on the chalkboard) "Yes, Joe, 4 x 3/8 inches = 12/8 inches = 1 1/2 inches."

Mary: "The yardstick also shows that 4 groups of 3/8 inches in each group are equal to 6/4 inches or 3/2 inches before we reduce it down."

John: "I figured out how much I would grow in a year if I grew 1/2 inch during the summer. It would be 4 x 1/2 inch. That would be 4 halves or 2 inches. I would grow 2 inches in a year's time."

Sue: "I could measure how much Tommy would grow in a year with the measuring cup." (laughter)

Teacher: "I think that's a wonderful idea, Sue. Would you demonstrate it for the class?"

Thereon, Sue procured three glass measuring cups from the cupboard. She filled one with water to the 3/8 mark and poured the contents into one of the other cups. She repeated this procedure three times, thereby filling the second cup and half of the third.

During the period each pupil computed on his ruler and
diagrammed pictorially on paper or at the chalkboard several other examples of a somewhat similar nature.

Joe: "We seem to be multiplying the upper number of the fraction in each case. Finding the product of a whole number times a fraction seems to be just a matter of multiplying the whole number times the number of parts in the fraction."

Teacher: "You are right, Joe. In multiplying a common fraction by a whole number, we multiply the whole number by the parts taken of the fraction, or as you have already learned to call it, the numerator. We have shown here today that to multiply a fraction by a whole number is simply a shortcut to adding the fraction as many times as stated by the whole number."

The processes of the reduction of fractions and cancellation, which had been taken up to a certain extent during the process of teaching the addition of fractions were reviewed and expanded, following these introductory lessons. In later lessons the matter of multiplying fractions by fractions was taken up, first objectively, later abstractly. To clarify certain difficulties as well as to review the process of multiplying a fraction by a whole number, the class were shown a portion of the sound film, "How to Multiply Fractions" and the filmstrip, "Multiplying Fractions." Later they were shown the entire films.
Concept: The Measurement of Area

Grade Level: Sixth

Purpose of the Lesson: To help the pupils develop an understanding of square measure.

Previous Readiness Experiences: The pupils have had rich background experiences with linear measurements as well as with the "four fundamental processes."

Problem Setting for the Lesson: A group of sixth-grade boys approached their teacher at recess to have her settle a dispute. Two of the boys had been arguing about the respective size of their fathers' farms. John said that his dad's farm was the largest in the community since it was almost half a section. Eddie maintained that his father's farm, containing over 300 acres, was the larger.

Materials Provided for in the Lesson: Tape measure, rulers, yardsticks, chalkboard, rectangular pieces of paper, and the filmstrip, "Area of Rectangles."

Procedure in Teaching: The teacher approached the lesson through a reference to the playground argument. After some discussion the class decided they did not have enough information to solve the problem. No one in the class knew for certain how large either an acre or a

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2 This illustration was adapted from a paper prepared by Dean Oloott, an undergraduate student in Education 510 during the Autumn Quarter, 1955, in the College of Education, The Ohio State University.
section is, nor the relationship between the two measurements. Some of the pupils, not being sure of the size of their fathers' farms, decided to gather that data as well as information relative to acres, sections, and the means of measurement. The discussion of the problem was to be resumed during arithmetic class the next day.

The next day

Teacher: "Let's list on the board all the information that we've gathered. Did anyone find out how large a section is?"

John: "640 acres."

Teacher: "And how large is your father's farm?"

John: "I said almost half a section and that's 320 acres."

Teacher: "Eddie, did you find out exactly how large your father's farm is?"

Eddie: "Dad says it's 315 acres."

Teacher: "Well then the two farms must be nearly the same size. Let's finish listing the data we've gathered."

Paul: "An acre is 43,560 square feet, but Dad says it would be too hard to measure land by the foot. They use rods instead. A rod is the same as 16 1/2 feet."

Ann: "In the back of my dictionary it says that an acre is also 160 square rods, or it can be 4,840 square yards."
Teacher: "How do square feet, square yards and square rods differ from ordinary feet, yards and rods?"

Jimmy: "I've seen a yardstick, but I've never seen a square yardstick."

Teacher: "Well actually, Jimmy, we don't have a square-yard measuring instrument. We could make one, but it would be rather awkward to use. It might help us to visualize square measurement better, however, if we did make a square-inch measuring device and use it some."

Joe: "We could just cut out an inch square of construction paper. That would be a square inch. (hesitating) Or does a square inch have to be square?"

Teacher: "No, a square inch does not have to be a square, but it probably would be less confusing if we used a square piece of paper to start with. Suppose we each try Joe's suggestion. Then use your square inch of paper to determine the area or the number of square inches contained in one of the backs of your arithmetic textbook."

A lapse of time

Jimmy: "It's awfully hard to measure with a ruler like this. You have to watch too many sides at a time."

Sally: "I couldn't measure the book cover itself either without marking on it. I made an outline of the book back on a sheet of paper, then measured that. I made a
mark around the inch square of paper every time I put it down. There were 35 squares in all. So the book cover must contain 35 square inches."

Ann: "Our arithmetic book back is 7 inches long and 5 inches wide. There are 7 rows of inch squares one way and 5 rows the other."

After measuring several other small rectangles with their square-inch measuring instrument, the pupils "discovered" that the area of a rectangle might be determined more readily by multiplying the length by the width. From measuring small rectangles, some of the pupils decided to find the area of their classroom in order to see how its size compared with an acre of land. The process was accomplished by measuring two adjoining sides with yardsticks and multiplying the two dimensions.

Eddie: "Our room has an area of 588 square feet. An acre is a lot bigger; it contains 43,560 square feet."

Teacher: "That's right. Paul told us that ground is measured in rods. We've found that a rod is 16 1/2 feet. Can we use the tape measure you brought to measure off 16 1/2 feet at a time and keep track of the number of rods as we go along?"

Class: "Yes."

Teacher: "In the problems we've had so far, we've measured the length and the width, then multiplied to find the
area. Now we have the area that we want to measure, since we know that an acre is 160 square rods. How can we find out what length and width to measure off? It's something like working the problem backwards."

John: "Well, the length and width have to turn out to be 160 when they're multiplied. We'll have to think of two numbers that equal 160 when we multiply them together. 2 times 80 will."

Ann: "4 times 40 will give us 160."

Joe: "10 times 16 will do it."

Teacher: "We've got three quick correct measurements we could use to measure off an acre. Which do you think would be the best to use?"

John: "Well, I said 2 times 80, but that would be only 2 rods on the end and 80 rods long; that would be awfully long and narrow."

Eddie: "We ought to use 10 times 16 then, 'cause they're more nearly the same measurements, and the acre wouldn't be so long and narrow."

Teacher: "Very good thinking, Eddie. Do the rest of you agree?"

Class: "Yes."

Subsequently the class moved outside where they proceeded to measure off an acre of land. Starting from the corner of the building, they measured off 10 rods and 16
rods at right angles, then completed the rectangle. Members of the class stood at the corners and along the sides to see and to demonstrate how large an acre is.

At a later class meeting the group were shown the filmstrip, "Area of Rectangles," thereby helping to summarize and review the procedures which they had followed.
APPENDIX C

AN INVENTORY OF TEACHER UNDERSTANDING OF THE USE OF
INSTRUCTIONAL AIDS IN THE TEACHING OF ARITHMETIC

FORM C
AN INVENTORY OF TEACHER UNDERSTANDING OF THE USE OF INSTRUCTIONAL AIDS IN THE TEACHING OF ARITHMETIC

Form C

This test represents an attempt to secure one type of evidence relating to your understanding of the use of instructional aids in the teaching of arithmetic in the elementary school.

Directions: For each of the "questions" which follow, you are requested to indicate in the blank provided the number of the "best answer."

1. Classroom teachers should guard against the selection and use of free and inexpensive instructional aids, to be used in problem solving, that are:
   (1) attractive and easy to read. 1.____
   (2) of the sales-pressure type 2.____
   (3) unbiased and authentic 3.____
   (4) timely and relevant 4.____
   (5) interestingly written 5.____

2. It might be reasonably expected that one valid appraisal of the filmstrip, "Learning About Using Pennies, Nickels, and Dimes," would be:
   (1) the filmstrip should not replace the use of actual coin manipulation. 1.____
   (2) such concepts could be made more meaningful on motion-picture film. 2.____
   (3) such a filmstrip can guarantee pupil understanding of the concept. 3.____
   (4) such concepts may be made more meaningful by use of the arithmetic textbook illustrations. 4.____
   (5) children will learn coin values without the use of aids. 5.____

3. Certain manipulative aids are valuable in clarifying the concept of place and face value. Such aids tend to be more meaningful to pupils if used:
   (1) by the teacher and pupils simultaneously. 1.____
   (2) only by the teacher. 2.____
   (3) only by the pupils. 3.____
   (4) by a specially-trained supervisor. 4.____
   (5) by the principal. 5.____

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4. In using manipulative aids to clarify the meaning of number symbols, the teacher should avoid using aids that are:
   (1) simple to manipulate. 1. 
   (2) economical in the use of time. 2. 
   (3) psychologically and philosophically sound. 3. 
   (4) inexpensive. 4. 
   (5) small enough to be wieldy. 5. 

5. To acquaint her primary pupils with the vocabulary of quantity and number facts, the teacher should avoid the use of radio broadcasts that are:
   (1) intellectual rather than emotional. 1. 
   (2) concerned with interesting the learner. 2. 
   (3) accurate and authentic in detail. 3. 
   (4) concerned with motivating the listener. 4. 
   (5) unsuitable for the grade level. 5. 

6. In view of the developmental nature of the learning process, the best aid to use initially in teaching a child to tell time would most likely be:
   (1) cardboard clock faces. 1. 
   (2) the filmstrip, "How To Tell Time." 2. 
   (3) the motion picture film, "How to Tell Time." 3. 
   (4) pictorial illustrations of how to tell time. 4. 
   (5) a written explanation of how to tell time. 5. 

7. In the teaching of arithmetic, misconceptions are quite common as to what instructional aids actually are. The best answer is, they are:
   (1) motion pictures which clarify abstract concepts for the learner. 1. 
   (2) manipulative aids which enrich learning experiences. 2. 
   (3) graphic materials which introduce and build up abstract concepts. 3. 
   (4) any concrete objects which serve to clarify semi-concrete concepts. 4. 
   (5) anything of a concrete, semi-concrete, semi-abstract, or abstract nature which contributes to the learning process. 5. 

8. The teacher is sometimes confronted with the problem of whether or not to use free arithmetic instructional aids furnished by private firms. Authorities recommend all of the following criteria for the selection and use of such aids, except the statement that the aids should be:
9. A common problem faced by the arithmetic teacher is to determine on what level of learning individual pupils are capable of operating in a new topic to be taken up. One recommended means of determining this operational level is to use:

- group intelligence tests
- individual intelligence tests
- aptitude tests
- readiness tests
- sociometric tests

10. Mrs. Green needs manipulative aids to make the concept of various formulas meaningful to her elementary-school group. She is aware that it is generally best for pupils to produce their own aids when the:

- school cannot afford to buy them
- educational benefits gained therefrom are worth the time and cost involved
- quantity of school-made and commercial aids becomes low
- experience in making aids is needed
- materials needed making the aids are available

11. When using a motion-picture film to teach a concept, such as approximation or estimation, the teacher should refrain from using a film that is:

- limited to pertinent facts
- produced for a particular purpose
- unbiased
- unrelated to her specific teaching purpose
- concerned with the correlation and integration of school subjects

12. A practical purpose served by the showing to pupils of a motion-picture film such as, "The Concept of Our Number System," is to:
13. The type of graph ordinarily used to teach simple common-fraction concepts is the:
   (1) pictorial graph.  1.
   (2) line graph.  2.
   (3) "pie" graph.  3.
   (4) bar graph.  4.
   (5) diagram.  5.

14. In selecting an arithmetic textbook, the most important factor to look for is the:
   (1) list of illustrations.  1.
   (2) list of suggested aids.  2.
   (3) size.  3.
   (4) content.  4.
   (5) format.  5.

15. A common criterion concerning instructional aids relates to whether the aid is appropriate for the age, intelligence, and experience of the learner. On this basis, the following film which would likely be least suitable for an elementary-school group is:
   (1) "Addition Is Easy."  1.
   (2) "Subtraction Is Easy."  2.
   (3) "Two Plus Two Does Not Always Equal Four."  3.
   (4) "Borrowing in Subtraction."  4.
   (5) "Let's Count."  5.

16. In view of the developmental nature of the learning process, the algorism below which should normally be presented first is:
   (1) \[
   \frac{2}{5} \text{ of anything} + \frac{1}{5} \text{ of anything} = \frac{3}{5} \text{ of anything}
   \]
   (2) \[
   \frac{2}{5} \text{ of a mile} + \frac{1}{5} \text{ of a mile} = \frac{3}{5} \text{ of a mile}
   \]
   (3) \[
   \frac{2}{5} \text{ of an apple} + \frac{1}{5} \text{ of an apple} = \frac{3}{5} \text{ of an apple}
   \]
   (4) \[
   \frac{2}{5} \text{ of a mile} + \frac{1}{5} \text{ of a mile} \]
   (5) \[
   \frac{2}{5} \text{ of a mile} + \frac{1}{5} \text{ of a mile} = \frac{3}{5} \text{ of a mile}
   \]
17. Mrs. Martin wishes to take her pupils on a field trip to provide functional experiences in reading and writing number facts. She realizes that she should guard against involving her group in a trip that must be concerned with:

(1) individual and group expense.  
(2) chaperone and guide service.  
(3) teacher-pupil planning and follow-up.  
(4) unreasonable delays and waste of time.  
(5) transportation and routing.

18. Arithmetic workbooks may furnish a very worthwhile source of examples for practice in addition, subtraction, multiplication, and division. In view of this fact, the teacher should realize that each of the following is considered a valid criterion for the selection and use of such workbooks, except the statement that the workbook should:

(1) contain instructions and directions for each lesson.  
(2) provide for a definite system of drills, reviews, and tests.  
(3) provide for a diagnosis of individual weaknesses.  
(4) contain charts to make it possible for each child to compare his scores with scores of other pupils in the class.  
(5) contain material of practical usefulness for the maturity level.

19. Mrs. Brown is planning to use the opaque projector to help clarify certain difficulties the pupils are experiencing in understanding the common-fraction concept. She should be cognizant of the fact that the opaque projector:

(1) will not project colored pictures.  
(2) tends to break down the emulsion on photographs used.  
(3) will not project black-and-white pictures.  
(4) accommodates only a small "audience."  
(5) tends to distort characters projected on the screen.

20. A valid appraisal one might expect of the motion picture film, "Let's Count," portraying a number of elementary situations in which the counting of objects is necessary, might be:
(1) rote counting obviates the necessity of using instructional aids.
(2) children will learn to count more successfully without the use of aids.
(3) the concept of counting may be made more meaningful if presented on a filmstrip.
(4) this film should not supplant the use of concrete aids in teaching meaningful counting.
(5) this film may obviate the necessity of using concrete aids in teaching meaningful counting.

21. In meaningful teaching of abstract number facts to pupils, the following algorithm which should normally be presented first is:

(1) 2 miles
(2) 2 of anything
(3) 2
(4) 8 chairs
(5) 2 + 4 = 6

22. Mrs. White frequently uses audio aids to give her primary group common experiences in the vocabulary of number and quantity. She has discovered that a great advantage the record player has over the radio in this respect is its:

(1) cheaper per pupil cost.
(2) exercise of the visual imagination.
(3) adaptability to a varied time schedule.
(4) varied content of the programs.
(5) importation of good music to the school.

23. The proper use of concrete and semi-concrete instructional aids in regard to teaching the meaning of number symbols, tends to:

(1) guarantee meaningful instruction.
(2) hinder meaningful instruction.
(3) obviate the need for abstract number symbols.
(4) result in more meaningful instruction.
(5) make the teacher's job easier.

24. Verbalism in regard to children's vocabulary of number and quantity concepts is rather commonly caused by overuse in the elementary school of:
(1) manipulative instructional aids. 1.
(2) abstract instructional aids. 2.
(3) visual instructional aids. 3.
(4) concrete instructional aids. 4.
(5) realia. 5.

25. Mrs. White feels that her primary group will profit by direct experiences involving counting and ranking. However, being a good teacher, she will guard against involving her class in such experiences that:
(1) are experimental in nature. 1.
(2) are uneconomical in terms of time, money, and effort. 2.
(3) vary in nature from previous experiences. 3.
(4) are creative in nature. 4.
(5) result in verbalized follow-up discussions. 5.

26. Misconceptions are quite common as to who is capable of dealing with arithmetic materials of an abstract nature. Research reveals that this ability is:
(1) limited to highly intelligent individuals. 1.
(2) equally shared by all individuals. 2.
(3) possessed in proportion to the age of the individual. 3.
(4) possessed in degree by both child and adult. 4.
(5) possessed in proportion to the grade-level of the individual. 5.

27. It tends to be easier for a child to understand basic number concepts if he first works with:
(1) symbolic aids. 1.
(2) pictorial aids. 2.
(3) graphic aids. 3.
(4) manipulative aids. 4.
(5) verbal aids. 5.

28. Selection of arithmetic textbooks becomes a serious matter in view of all their potential advantages and limitations. In selecting arithmetic textbooks, little weight should be given to the:
(1) title. 1.
(2) date of copyright. 2.
(3) author's reputation. 3.
(4) book-review comments. 4.
(5) size of the textbook. 5.
29. To discover in what different ways a number may be grouped, elementary-school pupils initially should ordinarily use:
(1) symbolic instructional aids. 1.____
(2) graphic instructional aids. 2.____
(3) visual instructional aids. 3.____
(4) pictorial instructional aids. 4.____
(5) concrete instructional aids. 5.____

30. To make more meaningful the concept of formulas, the teacher had an option of using a motion-picture film or using a filmstrip. She should be aware that:
(1) if the mastery of factual content is the major activity and motion is not essential, the filmstrip will suffice. 1.____
(2) in presenting projected materials, there is no substitute for motion pictures. 2.____
(3) even the silent picture is superior to the filmstrip. 3.____
(4) even when dramatization is important, the filmstrip is equal in value to the motion-picture film. 4.____
(5) there is no substitute for the filmstrip when motion is not important. 5.____

31. In view of the developmental nature of the learning process, the last of the following concepts to be introduced would normally be:
(1) one-half of an apple. 1.____
(2) 1/2 of a circle. 2.____
(3) 1/2. 3.____
(4) one-half. 4.____
(5) one-half of anything. 5.____

32. Arithmetic teachers should bear in mind that the most widely used and efficient instructional aid is the:
(1) motion-picture film. 1.____
(2) textbook. 2.____
(3) excursion. 3.____
(4) filmstrip. 4.____
(5) still picture. 5.____

33. Mrs. Smith is considering using the stereoscope to help present to her elementary-school group a more realistic concept of distance when viewing pictures. She should be aware that a great disadvantage of the stereoscope as a teaching aid is its:
34. The first grade pupils at Hatcher School used aids such as beans, buttons, and acorns to assist them in the process of counting and ranking. This practice is best justified at this learning level because:

1. it prevents children from counting on their fingers.
2. counting is a difficult operation.
3. for young children, learning is essentially sensory.
4. children learn to recognize objects as well as to count.
5. children like to handle objects.

35. Some printed teaching aids in arithmetic are intended to be used exclusively by teachers. Accordingly, it usually is not considered essential that they be:

1. written on the reading level of the grade in which they are to be used.
2. built upon the experiences, needs, and interests of the learners.
3. an integral part of the in-service education program.
4. physically attractive.
5. co-operatively selected and developed by all concerned.

36. The visual-aid instrument so valuable in arithmetic instruction for the three-dimensional effect obtained, as when viewing specially-photographed pictures of geometric forms, is the:

1. slide-film projector.
2. film-strip projector.
3. opaque projector.
4. lantern-slide projector.
5. stereoscope.

37. To bring about a balanced development in the mathematical judgment of children, the teacher should refrain from using arithmetic instructional aids that:

1. are based solely upon the personal interests of the teacher and child.
2. are conducive to integration of the whole school program.
38. An arithmetic aid especially designed to make the concept of percentage more meaningful is the:
(1) counting frame. 1.
(2) abacus. 2.
(3) place-value pocket. 3.
(4) twenty-board. 4.
(5) hundred-board. 5.

39. Mrs. Smith feels that she needs an opaque projector to give her elementary-school pupils practice in such concepts as the rapid identification of number symbols. She should be aware that a great disadvantage of the opaque projector is:
(1) typewritten material does not reproduce well. 1.
(2) it is unreasonably complicated to operate. 2.
(3) it will not reproduce color on the screen. 3.
(4) it is rather bulky and cumbersome. 4.
(5) it must be refocused each time material is inserted. 5.

40. Mrs. Brown's pupils are naive in regard to the concept of percentage. Accordingly, she realizes that they will tend to learn that topic most efficiently, in terms of time and effort, through the initial use of:
(1) concrete instructional aids. 1.
(2) semi-concrete instructional aids. 2.
(3) pictorial instructional aids. 3.
(4) symbolic instructional aids. 4.
(5) verbalized instructional aids. 5.

41. The most simplified level of quantitative thinking tends to result from the use of:
(1) abstract symbolic aids. 1.
(2) semi-concrete aids. 2.
(3) pictorial aids. 3.
(4) concrete aids. 4.
(5) abstract verbalized aids. 5.

42. In view of the fact that the learning process proceeds from the concrete to the abstract and from the familiar to the less familiar, the last of the following concepts to be introduced to children would normally be:
(1) four apples. 1.
(2) four chairs. 2.
(3) 4 rods. 3.
(4) four books. 4.
(5) four rods. 5.

43. As soon as a pupil is able to give meaningful verbal statements concerning the solution to an arithmetic problem, he should be encouraged to work with:
(1) manipulative aids. 1.
(2) pictorial aids. 2.
(3) symbolic aids. 3.
(4) auditory aids. 4.
(5) direct experiences. 5.

44. When the point to be emphasized in arithmetic instruction is the relationship of individual quantities to a whole, the graph commonly used is the:
(1) "pie" graph. 1.
(2) bar graph. 2.
(3) line graph. 3.
(4) pictorial graph. 4.
(5) diagram. 5.

45. Use of a "Teacher's Guide," which accompanied a set of filmstrips relating to our number system, could be best justified on the ground that it:
(1) tells the teacher what to say when acting as commentator. 1.
(2) obviates the necessity for teachers to preview films. 2.
(3) serves as a source book for pupils. 3.
(4) provides teachers with a detailed description of films they are unable to preview. 4.
(5) provides a detailed explanation relative to the operation of the projector. 5.

46. As revealed by a variety of sources, the pupil was capable of working at an adult level in the reading and writing of number ideas. Accordingly, for this child the teacher should see to it that he works primarily with:
(1) readiness aids. 1.
(2) manipulative aids. 2.
(3) abstract aids. 3.
(4) pictorial aids. 4.
(5) concrete aids. 5.
47. To build up a readiness for the additive concept, the children have had a number of successful direct experiences and are now ready for the exploratory-discovery stage of learning. The type of instructional aids most appropriate for this stage of learning would be:
(1) more direct experiences to strengthen the pupils' readiness. 1.
(2) visual symbolic aids. 2.
(3) manipulative aids. 3.
(4) verbalized symbolic aids. 4.
(5) field trips. 5.

48. The children displayed a thorough readiness to begin formal work on the concept of grouping. However, during the ensuing exploratory-discovery stage of learning in which manipulative aids were used, the children simply could not "see the light." In so far as teaching aids are concerned, the next step taken by the group should be the:
(1) use of verbalized aids. 1.
(2) re-use of the same manipulative aids. 2.
(3) use of more readiness experiences. 3.
(4) use of pictorial aids. 4.
(5) use of different manipulative aids. 5.

49. Some manipulative aids are specifically designed to help the pupil understand some one phase of arithmetic. One such aid is the:
(1) measuring cup. 1.
(2) place-value pocket. 2.
(3) thermometer. 3.
(4) yard stick. 4.
(5) Avoirdupois scales. 5.

50. In introducing a new process in arithmetic, the teacher should first:
(1) determine the operational level of the child. 1.
(2) use real experiences. 2.
(3) use manipulative aids. 3.
(4) use pictorial aids. 4.
(5) use symbolic aids. 5.

51. Although arithmetic textbooks have many advantages, they also have some disadvantages. One such disadvantage is, they:
(1) tend to guarantee uniformity of instruction. 1.
(2) are written by someone other than the classroom teacher. 2.
(3) often serve as more of a help to the teacher than for the pupil. 3.
(4) oftentimes are not relied on by the teacher. 4.
(5) are frequently arranged logically rather than psychologically. 5.

52. The concept "vicarious experience" is quite commonly used in arithmetic textbooks. However, an example of an experience that is not vicarious is:
(1) reading the batting averages of leading baseball players. 1.
(2) listening to a radio program about track records of Olympic stars. 2.
(3) watching the television program, "The Football Scoreboard." 3.
(5) baking a cake by following a printed recipe. 5.

53. An aid which has proved very practical in arithmetic instruction in reproducing drawings to scale is the:
(1) planisphere. 1.
(2) hypsometer. 2.
(3) pantograph. 3.
(4) sextant. 4.
(5) stethoscope. 5.

54. It is important that the teacher be aware that arithmetic activities may be on different levels of learning. Of the following activities, the one on the lowest level of learning is:
(1) recording graphically scores made by pupils on a test involving the whole-number concept. 1.
(2) using manipulative aids in learning whole-number concepts. 2.
(3) dramatizing a story employing whole-number concepts. 3.
(4) reading a story using whole-number concepts. 4.
(5) using an instructional film to clarify whole-number concepts. 5.

55. The pupil who is at the adult stage of learning in working with decimal fractions, tends to benefit most by the use of:
(1) concrete instructional aids.  
(2) verbalized instructional aids.  
(3) direct experiences.  
(4) pictorial instructional aids.  
(5) semi-concrete instructional aids.  

56. To be most useful, pictures in arithmetic textbooks which relate to problem solving, should be:  
(1) functional.  
(2) compositional.  
(3) informational.  
(4) illustrative  
(5) associative.  

57. An elementary-school group which has been successfully working with manipulative aids in order to gain insight into whole-number concepts, would now, other things being equal, tend to profit most by:  
(1) watching the motion-picture film "What Is Four?"  
(2) watching the filmstrip "A Number Family in Addition."  
(3) watching the filmstrip "Subtraction Is Easy."  
(4) watching the motion-picture film "Addition Is Easy."  
(5) dividing objects such as apples into fractional parts.  

58. The type of graph ordinarily used to show quantitative trends is the:  
(1) bar graph.  
(2) pictorial graph.  
(3) area graph.  
(4) line graph.  
(5) diagram.  

59. During the exploratory-discovery stage of learning in reference to determining how many pieces of ribbon 2/3 yd. long could be cut from a strip 4 yds. long, which of the following solutions would you be least likely to receive from elementary-school pupils?  
(1) Add 2/3 to 2/3 and continue until the sum is 4.  
(2) Subtract 2/3 from 4 and continue until there is no 2/3 remaining.  
(3) Divide 4 by 2/3 by inverting the divisor and multiplying.
(4) Multiply $2/3$ by 2, 3, 4, etc. until the product is 4.
4.____
(5) There are 3 pieces in 2 yards; hence in 4 yards, there are 6 pieces.
5.____

60. If the pupil can learn in a meaningful way that $5 + 4 = 9$ as a result of having experienced that $5 + 3 = 8$, this is a case of:
(1) wrong procedure because it is impossible to have meaning in the abstract without having experienced it in the concrete.
1.____
(2) correct procedure because it is inefficient and may be harmful to resort to concrete and semi-concrete instructional aids when unnecessary.
2.____
(3) correct procedure because this new experience will become more meaningful as the pupils work with it in the abstract.
3.____
(4) wrong procedure because every new learning situation should make use of all types of instructional aids.
4.____
(5) correct procedure because the two examples are so similar that anyone who has correctly worked the first should be able to do the second.
5.____
APPENDIX D

RECORDS OF UNDERGRADUATE STUDENTS PARTICIPATING
IN ESTABLISHING RELIABILITY OF TEST FORM C
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RECORDS OF UNDERGRADUATE STUDENTS PARTICIPATING
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A. BOOKS


334


B. PERIODICAL ARTICLES


Dale, Edgar. "When Do We Start?" Educational Screen, XXIII (May, 1944), pp. 200-201.


Forlano, George, and Pinter, Rudolf. "Selection of Upper and Lower Groups for Item Validation," Journal of Educational Psychology, XXXII (October, 1941), pp. 544-49.


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Smith, Herbert A. "Intelligence as a Factor in the Learning Which Results from the Use of Educational Sound Motion Pictures," *Journal of Educational Research*, XLVI (December, 1952), pp. 249-51.


Williams, Catherine M. "The Function of Charts in the Arithmetic Program," Arithmetic Teacher, II (October, 1955), pp. 72-76.


______. "Tests and Measurements," Nation's Schools, XLIII (March, 1949), pp. 39-41

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Lowdermilk, Ronald R. *Reading, Radio, and Attitudes*. Evaluation of School Broadcasts. The Ohio State University, Bulletin 1942, No. 63. Columbus, Ohio: The University, 1942.


Miles, Robert J. *Auditory Aids and the Teaching of Science*. Evaluation of School Broadcasts. The Ohio State University, Bulletin 1943, No. 57. Columbus, Ohio: The University, 1943.


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E. ENCYCLOPEDIA ARTICLES


F. UNPUBLISHED MATERIAL

Dickson, George E. "Handbook for Off-Campus Student-Teaching." Mimeographed pamphlet, Division of Education and Psychology, Central Washington College of Education.


AUTOBIOGRAPHY

I, William Patton Eidson, was born at Catlettsburg, Kentucky, October 7, 1910. I received my secondary education in the public schools of Catlettsburg and Fairborn, Ohio. My undergraduate training was obtained at Morehead State College, Morehead, Kentucky, from which I received the degree Bachelor of Arts in 1936. From Duke University, I received the degree Master of Education in 1941. I was a teacher and principal in the public schools of Boyd County, Kentucky, from 1929 to 1940 and teacher at the Cheyenne River Indian Boarding School, Cheyenne Agency, South Dakota from 1940 to 1942. After military service during World War II, I was a high school teacher from 1946 to 1949 and an elementary-school principal from 1949 to 1954 in the public schools of Ashland, Kentucky. I attended the Graduate School of The Ohio State University during the summers of 1952 and 1953 and returned to the campus as a resident student in 1954. While in residence I was Graduate Assistant successively to Dr. George E. Dickson, Dr. Lowry W. Harding, and Dr. Loren R. Tomlinson, while completing the requirements for the degree of Doctor of Philosophy.