AN ANALYSIS AND SYNTHESIS OF RESEARCH RELATING TO
SELECTED AREAS IN THE TEACHING OF ARITHMETIC
Volume I

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CHAPTER I

A PRELIMINARY VIEW OF THE PROBLEM

Arithmetic is one of the essential "three R's" that has been proclaimed in both song and verse, and it is one of the essential aspects of elementary education stressed by both laymen and educators. Yet, as teachers endeavor to teach arithmetic in the classrooms of the elementary schools, much confusion exists concerning the best procedures to follow in meeting the many problems met. To add to the confusion, recognized authorities in the field are far from agreement on many aspects of teaching arithmetic.

The Problem

The purpose of this study is to make an analysis and synthesis of significant research relating to each of the major areas of arithmetic instruction, to the end that professionally interested persons may be aided in their critical examination of teaching practices currently being followed. It is only through the findings revealed by scientific investigations that one can hope to find his way intelligently through the midst of conflicting opinions. By presenting research evidence in some detail, and by integrating the findings, a foundation is laid which will help
those interested in teaching arithmetic to judge the worth of research evidence for themselves and thus be enabled to choose more validly those practices to follow in arithmetic instruction.

In developing this study, several problems arose. Two of the most important ones may be noted as follows: (1) What should be the major areas in the teaching of arithmetic? In this study, areas of arithmetic instruction are considered to be those major divisions or categories of content into which the total subject of teaching arithmetic may be divided, e.g., readiness, methods of teaching, instructional materials, etc. It is essential to know what the major areas should be so that important segments of content will not be slighted or by-passed. (2) What are the major points of view concerning each of the areas of teaching arithmetic? Points of view are considered to be the theories or beliefs held pertaining to a given area in arithmetic instruction. In order to evaluate the various points of view, it is essential first to identify them.

Importance of the study.—This study may be justified from several important aspects. It is one means of making available to those concerned with arithmetic instruction important pieces of research dealing with the major areas in the teaching of arithmetic. Reports of research in arithmetic are so numerous and scattered that it is practically impossible for interested persons to obtain them or to have
the time to sift through the many journals to find answers to their specific questions. The task is further complicated by the difficulty of selecting the most reliable studies from the many that have been reported.

Much of the research which is available in the literature is reported in technical language which is probably not understandable to those who have not had an adequate background in interpreting complex statistics. The classical study by Thorndike,¹ which shows that arithmetic does not strengthen the mind any more so than do physical education or other less "brainy" subjects, is an example of the many reported investigations which are so clothed in statistical procedures and terminology that the ordinary reader is likely to miss the true significance of the research evidence.

It may further be pointed out that much of the available research deals with such narrow specifics that students and teachers are likely to fail to see the implications which the findings have for teaching. By integrating the data from a series of related studies, teachers should be helped to see the relatedness of the evidence to the total picture of teaching. Further, much of the research in arithmetic is often reported without giving clues for implementation. Teachers need more specific guidance in see-

ing what the findings mean for classroom teaching or for the use of instructional materials.

The reported results from some investigations are in serious conflict. Unless these conflicts are pointed out, many students of arithmetic education will possibly fail to see the true picture. Many findings which were once considered to be true have been found to be untrue when experiments have been repeated under different conditions.

Most of the authors writing on the teaching of arithmetic make only brief mention of research findings. Such a presentation of research does not give the student enough of the details to enable him to comprehend fully the significance of the study. The summaries of research which will be presented in the present analysis will be more detailed than the brief references usually given to research evidence, and at the same time they will be more concise and to the point than the original reports.

As will be shown later when a review will be made of the research related to the problem being considered in the present study, the research summaries in arithmetic which are now available are either limited in scope or present data in support of one or a limited number of points of view. Since many of the reviewers have not been selective in presenting their studies, the reader must consider the irrelevant research along with the more significant. As a result of this situation, students are left in a quandary as they
try to find direction through the maze of research. Another factor to consider is that most of the research summaries are now out of date and badly in need of being brought up to date.

Procedure

In developing the present study, the plan of attack was divided into five steps, as follows: (1) selecting areas, (2) identifying the major points of view relating to each area, (3) locating research, (4) selecting studies, and (5) treatment of research. Each of these steps will be discussed briefly in the following sections.

Selecting areas.--The first task in conducting this study was to select the areas of arithmetic education which should be considered in this analysis. As was mentioned earlier, areas are considered to be those major divisions or categories of content into which the total subject of teaching arithmetic may be divided, e.g., readiness, methods of teaching, instructional materials, etc.

In an effort to determine which areas should be used in this study, a survey was made of the literature to see which aspects of arithmetic instruction recognized leaders were stressing. For this purpose the books by Buckingham,²

²Burdette R. Buckingham, Elementary Arithmetic--Its Meaning and Practice.
Brueckner and Grossnickle,\textsuperscript{3} Spitzer,\textsuperscript{4} Stokes,\textsuperscript{5} Wheat,\textsuperscript{6} and Wilson\textsuperscript{7} were representative of the many books scanned. In addition to text and reference books, numerous articles in professional magazines were surveyed.

Several issues of \textit{Education Index} were examined carefully to see what classifications were used in listing articles having to do with the teaching of arithmetic. From this information a table was constructed to show the number of articles appearing for each of the major classifications. This afforded some clues as to the relative importance attached to the various aspects of arithmetic.

Use was made of the sixty-seven problems isolated by Brownell\textsuperscript{8} from the researches summarized by Buswell and Judd.\textsuperscript{9} The summaries provided by Buswell and Judd were

\textsuperscript{3}Leo J. Brueckner and Foster E. Grossnickle, \textit{Making Arithmetic Meaningful}.

\textsuperscript{4}Herbert F. Spitzer, \textit{The Teaching of Arithmetic}.

\textsuperscript{5}Newton Stokes, \textit{Teaching the Meanings of Arithmetic}.

\textsuperscript{6}Harry Grove Wheat, \textit{The Psychology and Teaching of Arithmetic}.

\textsuperscript{7}Guy M. Wilson, Mildred B. Stone, and Charles C. Dalrymple, \textit{Teaching the New Arithmetic}.


\textsuperscript{9}G. T. Buswell and C. H. Judd, \textit{Summary of Educational Investigations Relating to Arithmetic}.
analyzed by Brownell to determine the problems about which the researchers had been concerned. These summaries were read very carefully by a group of five persons. When all of the summaries had been read, the five members of the group independently prepared complete lists of the problems which had been investigated in arithmetic. A total of 127 such problems was thus secured. Then, in joint session, the members of the group reclassified and reduced the number of problems from 127 to 67.

After approximately 100 small separate classifications had been made on the basis of the above surveys and reading, an attempt was made to group the smaller segments of content into larger related areas. For example, many single aspects such as errors, difficulties, and diagnosis have been considered phases of remedial teaching, and finally, remedial teaching has been considered along with grouping, failure, and promotion as aspects of the larger concern, providing for individual needs.

Through grouping smaller segments into larger related areas, the final selection of areas of concern in arithmetic instruction was made as follows: (1) Readiness, (2) Scope of the Arithmetic Curriculum, (3) Grade Placement of Content, (4) Curriculum Patterns, (5) General Methods of Teaching, (6) Special Procedures in Performing the Fundamental Operations, (7) Special Procedures in Problem Solving, (8) Instructional Materials, (9) Providing for Individual Needs,
and (10) Evaluation. As will be explained later, a few of
the above areas will be considered in the same chapter.

The treatment of "special procedures" in topic 6,
"Special Procedures in Performing the Fundamental Operations," was limited to the addition, subtraction, multiplication, and division of whole numbers. This was done because the separate processes and skills in arithmetic are so numerous that it would be impractical to consider all of them in this study. Analyzing the research having to do with all of the computational operations in arithmetic could well form the basis for another entire study.

It is recognized that the above areas could have been arranged or titled differently. The selection does appear to be very complete, however, for as the writer read the many research reports in collecting data for this analysis, he did not find any investigations that could not be classified according to one of the above broad areas.

Identifying the major points of view relating to each of the broad areas. Once the areas were selected, a return was made to the literature on the teaching of arithmetic to ascertain the points of view held by recognized authorities in the field. As was defined earlier, points of view are considered to be the theories or beliefs held pertaining to a given area of arithmetic instruction. No attempt has been made to present every shade of difference with which the various aspects of the areas are held, but rather to point
out the major points of view, for it is recognized that for many of the categories there are as many shades of thinking as there are individual teachers.

Although practically every reference used in this study was useful in identifying a point of view, the writer wishes to make special acknowledgement of the help secured from Dr. Lowry W. Harding in his Functional Arithmetic: Photographic Interpretations, Dubuque, Iowa: Wm. C. Brown Company, 1952, 196 pages. In that book, Harding identified five distinct theories in the teaching and learning of arithmetic. His identification assisted the present writer in classifying points of view relating to several of the divisions of content used in this study.

Locating research.--From the bibliographies of research summaries which are related to the present analysis were secured valuable references. Among such summaries to be reviewed later were those by Wilson,10 Buswell and Judd,11 Monroe and Engelhart,12 Knipp,13 Glennon and Hunnicutt,14 and others. The Education Index gave references to

11 Buswell and Judd, op. cit.
14 Vincent J. Glennon and C. W. Hunnicutt, What Does Research Say About Arithmetic?
many pertinent experimental studies. Much assistance was gained from the yearly bibliographies relating to research in arithmetic provided by G. T. Buswell in each November issue of *The Elementary School Journal*. Another valuable source of references was the reviews of research pertaining to the teaching of arithmetic appearing from time to time in the *Review of Educational Research*. Other sources were Dissertation Abstracts (A Guide to Dissertations and Monographs Available in Microfilm), Ann Arbor, Michigan: University Microfilms, and Doctoral Dissertations Accepted by American Universities, Compiled for the Association of Research Libraries, New York: The H. W. Wilson Company. These last two sources were most valuable for the references made to dissertation studies which have been made in the field of arithmetic education.

Selecting studies.--It is not the purpose of this study to use all the research that has ever been made relating to the teaching of arithmetic, but rather to select that which offers interested professional persons the most valid direction in carving a path through the confusion which exists in teaching arithmetic. It is recognized that research may have been omitted from this analysis which is as significant as some of that which has been included. In some cases it will have been omitted because of choice between closely comparable studies. In other cases, perhaps, studies will have been overlooked. To the extent that such
omissions occur, it is regretted, because they impair the usefulness of this report.

In this analysis, books and articles which contain nothing but opinion or unsupported recommendations will not be considered as contributing to research evidence. Such materials will be used in defining points of view, and in the event that an aspect of arithmetic does not lend itself to research, or research has not been done in a particular area of arithmetic, ideas and recommendations by people recognized as authorities may be given in order to provide a picture of the situation as it is currently seen.

When several similar researches have been reported with regard to one of the selected areas of arithmetic instruction, an attempt will be made to select the research which more nearly meets the criteria which have been established. When several closely related researches are available, one or two will be summarized in detail with only a brief mention made of the contributions of the others. In the event that only one research study relating to a segment of an area is available, that one will be reported, even though it may not meet satisfactorily the selected criteria. Any interpretations which are made concerning that research will be developed with reference to any weakness or limitations which the study may have.

In order to help select the most significant and valid research, the following criteria have been established. As
was mentioned previously, it is recognized that much of the research may not meet each criterion in every detail, but the research will be selected according to the degree to which it does meet the criteria as a whole. The six criteria that were developed for this purpose are stated and explained below.

1. Is the experimental factor clearly defined and restricted? If the experimental factor is not clearly defined and restricted, one cannot tell whether the results have been brought about by the experimental factor or by other possible contributing factors.

2. Are variations in pupil factors, such as intelligence, age, etc. controlled so as to assure dependability of the experiment? When pupil groups which are being compared are not properly equated, the results can be misleading and invalid. If such factors as age and intelligence are not controlled, one is at a loss to know how much the results were influenced by the differences in age or intelligence.

3. Were non-experimental factors, such as instructional methods, instructional materials, and time spent in learning activity controlled? Here again, unless these factors are the same for all groups participating in the study, it is impossible to determine to what extent the results were due to the operation of the experimental variable or due to the influence of other non-experimental factors.
4. In measuring the results or differences in outcomes, were the measuring instruments and procedures valid and reliable? For tests or other measuring instruments to be valid they must actually measure what they are purported to measure. Testing devices to be reliable must guarantee consistent results.

5. Was the sample large and representative enough to warrant the conclusions that were reached? A sampling should be representative of the population in general for the particular age group being studied. It is obvious that some types of studies demand more participating subjects than others. According to Garrett:

A small sample is often satisfactory in an intensive laboratory study in which many measurements are taken upon each subject. But if $N$ is less than 25, say, there is often little reason for believing such a small group of persons to be adequately descriptive of any population.

The larger the $N$ the larger the SD of the sample and the more inclusive (and presumably representative) our sample becomes of the general population.

6. Were the conclusions made in terms of the statistical significance of the findings? Conclusions cannot be very convincing when they are based on differences so small as to be unreliable or statistically insignificant. In discussing levels of significance, Garrett explains: "The answer to the question of when a difference is to be taken

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as statistically significant depends upon the probability of the given difference arising 'by chance'. In a further explanation, Garrett says: "Two sets of accuracy limits are in general use and have been accepted as standard by most investigators. These limits define what are called the .05 and .01 levels of confidence." In illustrating the meaning of .05 and .01 levels of confidence, Garrett writes: "We can expect to be wrong 5% of the time if we take the .05 level and 1% of the time if we take the .01 level. These levels .05 and .01 reflect degrees of assurance, therefore, the .01 level deserving greater respect than the .05." 

Treatment of the research.---Once the research relating to each given broad area has been selected, most of the studies will be summarized in detail, giving: (1) the complete bibliographical reference, (2) the problem or purpose, (3) research procedure, (4) findings, and (5) conclusions. The purpose of summarizing the research in detail is to give the reader a picture of the details of the investigation so that he may more adequately evaluate the study and thus be enabled to draw more competently his own conclusions. When the findings are presented out of the context from which they were derived, they can be misleading and misinterpreted.

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16 Ibid., p. 216.
17 Ibid., p. 186.
18 Ibid., p. 187.
Of the research studies available pertaining to each division, the most complete ones will be selected for full treatment. Other related research studies which add significant information will then be summarized in less detail. A degree of repetition will be found in the present analysis, for many of the researches will have strong implications for several of the areas to be considered. The studies will be summarized only once in detail, however. When they are referred to in connection with other classifications, they will be discussed only briefly and a cross reference given to the chapter where they will be reviewed in greater detail.

Review of Related Research

As will be shown, many of the research summaries which have been made concerning the teaching of arithmetic have been limited to a small aspect of the total problem of teaching arithmetic, or have touched on the broader aspects of the problem very lightly.

One of the first and most exhaustive summaries of research in arithmetic was that made by Buswell and Judd.¹⁹ In that summary the authors listed 307 titles, which included practically all of the research done in arithmetic in the United States prior to 1925. The survey is valuable for some of its references and for the findings and conclusions given. Many of the references included, however, can only vaguely be termed research and would not be able to meet

¹⁹Buswell and Judd, op. cit.
rigid research criteria. Since the report was so complete and exhaustive, interested persons are still faced with the problem of separating the chaff from the wheat. Also, most of the truly significant investigations in arithmetic have been made since that summary was compiled.

Buswell later made a critical survey of research in arithmetic up to January 1, 1929. The purpose of that report was to summarize the facts concerning arithmetic which were based upon objective data, to note the trends which characterize research in the various phases of arithmetic, and to present what might be called a list of accepted generalizations relating to arithmetic. No attempt was made to present a detailed summary of research studies in arithmetic.

Since the main emphasis in the above survey was placed upon indicating the trends of research up through 1929, the findings were not integrated in an effort to answer questions or solve problems which may confront teachers of arithmetic. No guide appears to be given for helping one find his way through the many existing researches.

Buswell continued his efforts to make available research studies in arithmetic. His "Summary of Arithmetic Investigations" appeared in the June issue of The Elementary Education Quarterly.

School Journal from 1926 through 1932. Those summaries are valuable mainly for their references, for no attempt was made to synthesize the research findings.

Beginning with Volume XXXIV of The Elementary School Journal, November, 1934, and continuing with each subsequent November issue through Volume XXXVIII of 1947, Mr. Buswell presented "Selected References on Elementary Instruction (Arithmetic)". This selected annual bibliography provides one of the best sources of important investigations and offers an efficient means of keeping acquainted with the developments in arithmetic. In regard to the bibliography, Buswell made the following statement:

The following bibliography differs from the preceding "Summaries of Arithmetic Investigations," compiled by the writer annually since 1925, in that the present bibliography makes no pretense of completeness, but rather selects from the material in arithmetic those studies which seem most deserving in a limited list.\(^1\)

Since November of 1948, Maurice L. Hartung has continued to provide a selected bibliography in each November issue of The Elementary School Journal. This listing continues to be a valuable source for locating pertinent research studies in the teaching of arithmetic.

A significant contribution to compiling research in arithmetic was made by Monroe and Engelhart.\(^2\) Their purpose


\(^2\) Monroe and Engelhart, op. cit.
was to present a summary and an evaluation of the research relating to instructional methods employed in the teaching of arithmetic in Grades I through VIII. Most of their 128 references were selected from G. T. Buswell and C. H. Judd, *Summary of Educational Investigations Relating to Arithmetic*, which was referred to above.

Their justified conclusions are valuable since the authors tell the reader what can be accepted with confidence and indicate which conclusions have been made on the basis of unsound research techniques. However, since the purpose of the report was to evaluate research, many investigations have been included which do not meet the criteria of good research. The summary is further limited, for recent investigations have found many of the earlier research findings untenable. For example, research on drill techniques are of less importance today, because research has shown that when attention is given to meanings and generalizations, the role of drill is changed.23

Knipp24 made an analysis of experimental investigations reported between 1911 and 1940 that compared methods of teaching arithmetic in grades one through nine. Sixty-four

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24Knipp, op. cit.
experiments were carefully analyzed in order to ascertain whether there existed any trends in experimental interests, procedures, and results, and also to discover whether there seemed to be any relationship between interest in the specific fields of investigation and reported results. The analysis was tabulated in such a way that it indicated for each experiment the aspect of subject matter investigated, the grade level in which the experiment took place, the methods compared, the kind of experimental control employed, the length of the experimental period, and the statistical tests of significance applied to the reported difference.

It can readily be seen that Knipp was more interested in analyzing the details of research techniques than in providing research to help teachers find answers to the problems which they face everyday in the teaching of arithmetic.

One of the most valuable summaries of research in arithmetic has been prepared by the Research Service of the Silver Burdette Company. That review refers to 116 separate references, which include both experimental investigations and theoretical discussions. Only a few of the researches are reported in detail. In most instances, only the major findings or conclusions are given. The summary is further limited, since its purpose was specifically to "see what research has to offer in support of the meaning theory."

26 Ibid., p. 1.
Presenting research in support of this one view appeared to prevent an objective look at other points of view.

Another key summary of research investigations in arithmetic has been prepared by Wilson. In that report the researches were grouped and reported according to three broad topics, namely: (1) The Curriculum, (2) The Psychology of Arithmetic, and (3) Methods of Teaching. Although Wilson refers to 133 separate studies, he does not go into much detail in reporting them. The summary does little to resolve the conflict in findings and to integrate the reported results. Clues for implementation are generally missing.

A recent research summary has been prepared by Glennon and Hunnicutt. As stated by the authors the purpose of that report was:

. . . to present to the teacher, to other school personnel and to the interested parent, a summary of the theoretical and scientific knowledge of the place of arithmetic and of the teaching and learning in the modern elementary classroom in the hope that it will bring about improved methods on the part of the teacher and improved growth on the part of the learner.

As can be seen from the statement of the purpose, the study was not limited to research, but contains much theoretical

\[27\text{Wilson, "Arithmetic," op. cit.}
\]

\[28\text{Glennon and Hunnicutt, op. cit.}
\]

\[29\text{Ibid., pp. 3-4.}
\]
discussion as well. An attempt was made to provide research data and theoretical foundations in answer to specific questions related to the curriculum and to the teaching of arithmetic. The research data that is provided is limited to a brief mention of the findings.

Morton, through discussion, answers many questions about which classroom teachers are frequently concerned. He bases his discussion around such topics as the understanding of the different processes in fractions and whole numbers, grade placement, the use of crutches, and evaluation. Although most of his general discussion is probably based on valid research data, he fails in most instances to refer to specific research to give validity to his statements. In addition to his general discussion, Morton lists seven questions which he answers by giving the findings of a specific investigation. That Morton's report is limited in scope is evidenced by the fact that his bibliography of selected references contains only seventeen entries.

Much credit goes to the Review of Educational Research for its periodic reviews of research dealing with arithmetic. The references cited are most valuable, and this


31See the following issues: January, October, December, 1931; April, 1934; February, 1935; April, December, 1937; June, 1939; October, 1942; October, 1945; October, 1948 and subsequent issues.
remains one of the best sources for locating significant investigations. The reviews are limited, however, since they are concerned with reporting research for only the three preceding years. This being true the interested reader fails to grasp the full weight of the total research evidence available in arithmetic.

A few significant articles in the professional literature have summarized research findings having to do with arithmetic instruction. Beatty surveys briefly sixty-seven key researches in a timely article which places emphasis on the role of child development, understanding, and rich, meaningful experiences in the learning of arithmetic.

In another pertinent article, Fehr reports research findings to provide teachers, supervisors, and administrators with the data that may aid them in their respective duties. The reviews were reported according to the following categories: problem solving, evaluation of learning, individual differences, training of teachers, and common learnings. Although Fehr's bibliography contains only twenty-five titles, they appear to be most significant. Most of the studies referred to are comparatively recent, having been done in the past ten years or so. In most instances, however, Fehr does not go into much detail in reporting the research. Frequently,

32 Leslie Beatty, "Re-Orienting to the Teaching of Arithmetic," Childhood Education, XXXVI (February, 1950)
33 Howard F. Fehr, "Present Research in Teaching Arithmetic," Teachers College Record, LIII (October, 1950) pp. 11-23.
rather than referring to the original source, he refers to secondary sources such as the Encyclopedia of Educational Research or the Review of Educational Research.

Several summaries have been made which have been limited to a single phase of arithmetic. Examples of these are: Brownell on the number of abilities of children when they enter school; Johnson, problem solving; Wilson, social usage of arithmetic; and Riess, arithmetic readiness.

Digest: Related research.—The major surveys which have attempted to summarize investigations in arithmetic have been reviewed briefly. It has been noted that each of these surveys has not met the purpose of the present study for one or more of the following reasons: (1) many of the summaries are now out of date, (2) they have been confined to a limited aspect of arithmetic, (3) they have been presented from one point of view, (4) the significant research studies have not been sifted from the many that have been reported, (5) the

34 William A. Brownell and others, Arithmetic in Grades I and II: A Critical Summary of New and Previously Reported Research.


36 Guy M. Wilson, What Arithmetic Shall We Teach?


37 Anita Riess, Number Readiness in Research.
essential details relating to research procedures and conditions have not been given, (6) the findings have not been integrated and synthesized, and (7) clues for implementing the findings have been missing.

Overview of the Remainder of the Study

Most of the selected areas of arithmetic education will be presented in separate chapters. An exception to this procedure will be found in Chapter III. That chapter will include the following categories which are related aspects of the curriculum: (1) Scope of the Arithmetic Curriculum, (2) Grade Placement of Content, and (3) Curriculum Patterns.

The presentation in each chapter will be somewhat as follows: A brief definition of the subject or topic being considered in the particular selected area will be given. This will be followed by a discussion or description of the various points of view pertaining to the area of arithmetic instruction under consideration. Following this discussion, the research which pertains to the various subdivisions into which the larger area may be broken will be summarized. After the research has been presented relating to each subdivision, a brief summary or digest will be given in which an attempt will be made to synthesize the findings from the several studies, and to offer conclusions and general implications for teaching. Finally, a concluding summary for the entire chapter will be made in which an effort will be made to bring together the major experimental results and to
offer conclusions and implications having to do with the specific area being considered. It is hoped that this summary will serve to integrate the findings so that interested professional persons may see the relationships among the separate investigations and see possibilities for implementing those findings in actual classroom practices.

In the concluding chapter an attempt will be made to combine the research evidence and conclusions from the entire study. Along with the conclusions will be indicated possible implications for teaching, for the curriculum, for the use of instructional materials, and for evaluation.
CHAPTER II

ARITHMETIC READINESS

The term "readiness" in education is of comparatively recent origin. It appeared first in connection with reading. During the early twenties educators became concerned over the large number of failures in the first grade. Since reading was considered to be a major cause of the failures, educators began focusing their attention on the improvement of reading. Many concluded that formal reading was being introduced too early, and programs were accordingly established to provide pre-reading experiences and to postpone the more direct approach in the teaching of beginning reading. Educational literature began offering suggestions for preparing children for the more complex act of reading. Representative of such literature was the Yearbook of the National Society for the Study of Education for 1925. Concerning the readiness period, the following statement is made in that Yearbook:

This period includes the pre-school age, the kindergarten, and frequently the early part of the first grade. Its primary purpose is to provide the training and experience which prepare pupils for instruction in reading.

It was not until about 1930 that the term "arithmetic readiness" began to appear in the literature. The term was first applied to beginning formal instruction in arithmetic,

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1 The Twenty-Fourth Yearbook of the National Society for the Study of Education, Part I, Report of the National Committee on Reading.

2 Ibid., p. 24.
but has gradually been expanded to include the teaching of all phases of arithmetic in all grades. Sueltz gives a general definition of arithmetic readiness which would probably be accepted by most recognized authorities in the teaching of arithmetic. His definition is: "Readiness in arithmetic is the stage of a child's development where it is opportune for him to proceed into a new experience or phase of learning." Various points of view, however, are in evidence as to what constitutes readiness "to proceed into a new experience or phase of learning."

Major Points of View Relating to Arithmetic Readiness

As one examines the various theories held concerning the teaching and learning of arithmetic, it is possible to identify several distinct conceptions of what constitutes arithmetic readiness. A discussion of these several conceptions will follow.

Faculty psychology concept. — Even before the actual use of the term "arithmetic readiness," one can gather from the literature rather definite viewpoints or conceptions relating to the ability to learn arithmetic. Prior to 1900, and to a limited extent today, the learning and teaching of arithmetic was based on the concepts of faculty psychology. The mind was thought to be divided into faculties with each

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faculty related to the learning of specific subject matter. All people did not have the same faculties, so some people were not expected to be able to learn arithmetic, while others who had the appropriate faculty could be expected to acquire great facility in the use of number. The mind was considered to be very similar to a muscle. By exercising the arithmetic faculty through working difficult problems, the mind was expected to become stronger, and consequently one could work increasingly harder problems. Readiness or the ability to do addition or other processes consisted in knowing the rule for the given process. One did not need to learn through separate drill or to practice the mechanics of the process. If the rule was learned and understood, it could be applied as needed. Wilson quotes the following rule for addition from Pike's _New System of Arithmetic_ which was published in 1802:

> Having placed units under units and tens under tens, etc., draw a line underneath and begin with units. After adding up every figure in that column, consider how many tens are contained in their sum and placing the excess under the units and carrying so many as you have tens to the next column of tens or row and set down the whole amount in the last row.\(^4\)

**Mental maturity concept.**—Another point of view, which places strong emphasis on mental maturity, is implied in the following statement by Osborne and Gillet in discussing the

\(^4\)Guy M. Wilson, Mildred B. Stone, and Charles C. Dalyrmple, _Teaching the New Arithmetic_, p. 120.
findings of the Committee of Seven:

In general the results indicate that there is a stage of mental growth for each topic in arithmetic before which it is wasteful of time and energy to attempt to teach it, and a stage beyond which there is little to be gained by further postponement.5

Brownell termed this view of maturation as the "inner growth concept."6 In a critical analysis of this point of view, as he conceived it from the writings of the Committee of Seven, Brownell wrote:

Here maturation is thought of as some kind of inner ripening—a form of growth which is quite unaffected by experience. The ability to multiply fractions, for example, is a special function which matures by reason of internal compulsion. There is little that can be done to influence the rate of its growth, for such function is not susceptible to external influences. We can but wait until it has reached the required stage of development.7

A second conception of maturation is interpreted by Brownell as he further criticizes the concept of readiness which he feels was implied by the Committee of Seven's studies. Brownell continued:

When the committee asserts that the addition of decimals cannot economically be taught before the mental age of twelve years and six months, it says, in effect, that arithmetical maturation is a condition of general intellectual development. The child

5Raymond W. Osborne and Harry O. Gillet, "Mental Age Is Important Factor in Teaching Arithmetic," Nations Schools, XII (July, 1933) p. 20.


7Ibid., p. 503.
can learn to add decimals when he has attained a mental status of twelve years and six months. Less mental maturity materially limits the success of any attempt to teach the process. In some way general intelligence is the essential element to successful learning in arithmetic. Let us call this conception of maturation the "M. A. conception."8

In connection with the maturation point of view, very little value seems to be given to experience. Experience is considered in the limited extent that it is recognized that the ability to do a given computational process is dependent upon the ability to use the prerequisite skills required in doing the new process. For example, it is recognized that children need to be able to subtract and multiply in order to perform long division. The ability to get the correct answer and to successfully do all the mechanical steps required in the algorism seems to be the criterion which is used to determine whether or not a child is ready to move to a new process. Little attention is given to the thought processes of the child in arriving at his answer. Since the adult level of achievement is the only objective sought or considered, little significance is attached to cruder means of arriving at answers. If the correct answer can be obtained with speed and accuracy, it makes little difference whether the process is understood or not. Each skill in arithmetic is to be learned separately, and it is not expected that the learning of one addition fact, for example, can contribute to the learning of another closely related fact.

8Ibid., p. 504.
Incidental learning concept.—Under what is commonly termed the incidental theory of learning and teaching arithmetic, another point of view concerning arithmetic readiness is observed. In this view no specific attention is given to readiness. It is assumed that since children have developed considerable number skills before coming to school, they will continue to do so as they meet number situations in their daily experiences. A rich background of natural number experiences and increased mental maturity plus the intrinsic motivational force of the present purposeful problem situation will provide the readiness to solve the problem at hand. It makes little difference whether or not skills are learned in a sequential order, for the motivation of the moment will cause the learner to learn whatever skills are necessary.

Experience concept.—A number of professional persons object to the theories represented by each of the previously mentioned views concerning arithmetic readiness. Consequently we find emerging another point of view concerning number readiness. This theory may appropriately be termed the "experience concept". The advocates of this concept object to the maturation concept on the following points: (1) the emphasis given to mental maturity, and (2) the idea that a correct answer or overt behavior alone is sufficient evidence of readiness to proceed to a new phase of learning. They object to the incidental point of view which maintains that skills can be learned out of sequential order.
In the experience concept of readiness, the role of innate mental maturity is not completely ignored, but greater emphasis is placed on experience. Experience, as a basis for readiness, includes not only facility in giving correct answers, but also includes vocabulary understanding, meaning and understanding of concepts and computational processes, and the understanding of relationships inherent in the number system. Basic to the experience concept of readiness is the belief that each specific in arithmetic does not need to be learned separately. Instead, it is recognized that there is transfer of learning from one phase of arithmetic to another related phase as the learner sees the relationships in the situation. Seeing relationships and being able to use such relationships in solving untaught materials is vital to arithmetic readiness.

The experience concept of readiness, like the maturation concept, places importance on the sequential order of learning skills. Woody clarifies this point when he writes: "Readiness for the teaching of any element in arithmetic presupposes facility in handling all of the subsidiary parts constituting the new element."\(^9\)

Rather than accepting the criterion of a correct answer or the correct manipulation of a computational process, as indicative of readiness, this concept of readiness demands

more. It is recognized that a child may arrive at a correct answer by several ways. If he is attempting to add \( \frac{4}{5} \) he may count: 1, 2, 3, 4, 5, 6, 7, 8, 9; or he may do partial counting by starting with 5 and counting 6, 7, 8, 9. He may add combinations which he knows like \( \frac{4}{4} = \frac{8}{1} = 9 \). The most mature way, of course, would be to give the answer directly as 9 without resorting to more immature procedures of counting or partial counting. Stages of readiness are considered within a given process. For readiness it is considered important to know the child's level of performance within the process. Rosenquist sums up the significance of the maturity level of performance when she says: "The criterion of the progress a pupil makes in the understanding of number ideas should not only be the accuracy of his responses, but also the maturity level at which he works to arrive at his responses."\(^{10}\)

Organismic concept.—This view of readiness embodies the organismic age concept of growth as presented by Olson. According to Olson and Hughes, organismic age is the average of the mental, dental, reading, weight, height, carpal, and grip ages.\(^{11}\) These authors recognize the influence of all physical and emotional factors on learning. They point this

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\(^{10}\)Lucy L. Rosenquist, *Young Children Learn to Use Arithmetic*, p. 9.

out through the following explanation:

Theoretically the measures taken should represent an inclusive theory of the organism. A complete account might include measures of emotionality, social adjustment, gross bodily development, circulation, efficiency of sense organs, development of educational and physical skills, and measures of metabolic function. . .12

In the organismic concept of readiness one sees recognition being given to the possible influence of both experience and maturation, rather than taking an either or position. Burton points out the relationship of maturation, experience, and social demand in promoting readiness when he writes:

It is the pedagogical counterpart, so to speak, of maturation but includes social and intellectual maturity as well. For example, we say at a certain time a child is ready to read, ready for formal arithmetic... His physical and neurological maturity and his experiential background are such that he could read, could do abstract arithmetic... if the situation arose which demanded these things.13

Advocates of the organismic concept of readiness recognize, like the advocates of the experience concept, the importance of meaning and understanding, and the importance of sequential learning of skills. The organismic group, however, go a step further and recognize the significance of the dynamic goal seeking purposes of the learner and the role of emotion. The learner is of more importance than the subject matter to be learned. Meanings and understandings are important, but the meanings which the learner has come from his

12Ibid., p. 526.
interaction with concrete objects and the abstract number symbols. The meaning is not in the number symbols themselves. Since the meanings are derived as the individual interacts with the numerical situations, the experience derived from such interaction is of vital importance in providing readiness. Harding illustrates the organismic view of meaning when he says:

...meaning is not in objects, but is stimulated by them; meaning is not in the symbols representing objects or ideas, but may be communicated and suggested by them; meaning is not in the persons viewing and using objects or symbols, but may be developed, understood and conveyed by them. Meaning is the effective qualitative interrelationship between persons, objects, and symbols. Symbols are named third in the hierarchy because they are abstract creations of man to represent, mean, signify functional relationships between himself and objects.\(^\text{14}\)

The criterion for measurement of readiness is the facility with which the individual is able to handle quantitative situations which he meets in solving problems which are real to him; not merely his facility in solving textbook problems. The indications are that if he can solve textbook problems but cannot apply the solution to his own problems, he does not have a full understanding of the processes involved.

**Indications of Readiness**

It is clear from the discussion of the above conceptions of arithmetic readiness that there is not complete agreement among educators as to what constitutes arithmetic readiness.

Due to such differences of opinion, students and teachers are certain to be confused as they try to form a concept of arithmetic readiness for themselves. The findings of research can do much to clear up such confusion by affording a reliable guide to be followed in formulating one's concept of arithmetic readiness.

The arithmetic children know when they enter school. Due largely to the emphasis on the postponing of instruction in arithmetic, many investigators began determining what arithmetic children know when they come to school. The assumption was that if children have been learning arithmetic on the outside and indicate that they have a background of experience in number, they should have instruction in arithmetic on entering school in order to continue their progress in numerical achievement.

Perhaps the classical study in determining what children know about arithmetic when they enter school is that of Buckingham and MacLatchy. They endeavored to measure the number knowledge of young children with respect to counting, number concepts, and number combinations. The pupils who participated in this study were between the ages of six and six and one half years of age and had first entered first grade in September, 1928. The children, the majority of whom

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were from sixteen cities and villages in Ohio, were selected at random from all the children in the various classes represented. In all, 1,356 children were interviewed by first grade teachers before they had given the children any number instruction.

Six interview type tests were used by the teachers in gathering the data. The tests, identified briefly, were as follows: (1) Test I, rote counting by ones and tens; (2) Test II, counting concrete objects; (3) Test III, number selection, in which the examiner began by saying, "Give me five buttons," and followed with the other numbers through 10; (4) Test IV, number identification, in which examiner exhibited a certain number of objects to the pupil and asked: "How many are there here?"; (5) Test V consisted of ten verbal problems of one form, namely: "If you have ---- and get ----, how many will you have then?"; and (6) Test VI, in which the examiner sought to ascertain whether the pupil had a functional knowledge of selected number combinations presented by means of objects.

The major results of the tests for each of the areas tested were:

1. **Rote counting by ones**: Ninety per cent of the children could count to 10; seventy-five per cent counted to 14 or 15; sixty per cent counted to 20; and twelve per cent counted to 100.
2. **Rote counting by tens**: Slightly more than twenty-five per cent of the children failed altogether; fifty per cent counted at least as far as 40; twenty-five per cent counted to 100.

3. **Counting objects**: Over ninety per cent of the children counted at least 10 objects correctly; seventy per cent counted at least 15; and the average counted 20 objects, the highest number of objects asked to be counted.

4. **Reproducing numbers**: According to the criterion of three times in three trials, sixty-four per cent of the children were able to reproduce 5; fifty-six per cent, 6; fifty-three per cent, 7; forty-eight per cent, 8; and fifty per cent, 10.

5. **Naming numbers**: Forty-two per cent of the children succeeded three times out of three on the hardest number, namely 10; whereas, sixty-two per cent of the children succeeded with the number 5.

6. **Combinations in verbal problems** (Test V): Seven per cent of the children gave correct answers to all of the combinations used. Almost half the children got five combinations right, and only eleven per cent of them failed to get any right. The combinations ranged in difficulty from $5/1$, which seventy-one per cent of the children answered correctly, to $4/5$, which only twenty-two per cent answered correctly.

7. **Combinations with objects** (Test VI): Half of the children answered five of the ten combinations correctly when
the objects were concealed. When the objects were visible, more than half the children answered all the combinations correctly. The combination, $6 \div 4$, the hardest one for these children, was answered correctly by nearly one third of the children when the objects were concealed and by fifty-eight per cent when the objects were uncovered.

It was concluded that since children have already acquired much facility in the use of number when they enter school, they are ready to be given more systematic instruction in number.

In an attempt to give a critical review of both new and previously reported research on the arithmetical skill and knowledge of children just entering first grade, Brownell reports an original study and brings together the findings of other significant studies on the problem. Specifically, Brownell tried to provide such data as was needed in order to answer the question, "Is the primary grade pupil intellectually capable of profiting from systematic instruction in arithmetic?"

In the fall of 1938 and the fall of 1939, Brownell collected new data on the readiness of first grade children for arithmetic. He tested thirty-two classes from twenty-four schools, using the series of pretests published in connection with

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16William A. Brownell and others, Arithmetic in Grades I and II: A Critical Summary of New and Previously Reported Research.
17Ibid., p. 8.
with "Jolly Numbers" (primary materials of the Daily Life Arithmetics). 18

In connection with writing up his own study, Brownell summarized the data from the investigations of Buckingham and MacLatchy, 19 Grant, 20 MacLatchy, 21 Polkinghorne, 22 Russell, 23 Wheeler, 24 Woody (two studies), 25 and Yokum. 26 His treatment of the data was topical, that is to say, all research which related to the skill, item of knowledge, or

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19 Buckingham and MacLatchy, op. cit.


other category in question was brought together and discussed jointly. The topics considered were: (1) rote counting; (2) enumeration; (3) identification; (4) reproduction; (5) crude quantitative comparison; (6) exact quantitative comparison, or matching; (7) number combinations or facts, and their use in problems; (8) fractions; (9) ordinals; (10) reading and writing numbers; (11) recognition of geometric forms; and (12) time, U. S. money, and measures.

After compiling the findings of the above mentioned studies, Brownell indicated that the following skills and concepts were quite well developed when children start to school: (1) rote counting by 1's through 20, (2) counting up to 20 objects, (3) number combinations in simple situations up to sums of 6 or 7, (4) halves and fourths applied to single objects, (5) recognition of simple geometric figures like circle and square, (6) telling time at the hour.

It was indicated that the following skills were developed among a reasonably large per cent of the children: (1) rote counting to 100 by 1's and 10's, and rote counting to 20 or 30 by 2's, (2) number combinations with sums of 9 or 10 in verbal problems.

It was pointed out that less than a third of the children entering school had acquired competency in: (1) reading or writing numbers beyond 10, (2) telling time at the half and quarter hours, (3) knowing the relative values of coins other than pennies, and (4) counting by 3's to 30.
The conclusion implied was that since children have acquired much number ability when they enter school they are ready to receive additional number experience when they enter school.

Several studies not dealing directly with the problem of determining number knowledge of beginning first grade children have provided data to supplement the above studies relating to the number knowledge of beginning first grade children. Such investigations were made by Gesell and Ilg, Mott, Mott, McLaughlin, Riess, Stotlar, and Carper. The findings from these studies are consistent with the findings summarized by Brownell in that they indicate that preschool and school age children have acquired some ability in regards to number before entering school.

Helping to support the conclusion that children

27 Arnold Gesell, and Frances L. Ilg, The Child From Five To Ten, pp. 399-402.


beginning first grade are ready for planned quantitative experiences is research which shows that kindergarten and first grade children have profited from such experiences. Brownell, Koenker, and Wilburn found that kindergarten and first grade children gained from a systematic program in arithmetic when the teaching program included the use of concrete materials. The children participating in the studies not only gained an understanding of the number concepts, but were also greatly interested in and enthusiastic about the number program. Mott and Martin found that with the exception of counting by rote to one hundred, the average kindergarten child will carry into first grade after a lapse of three months the number experiences he learned in kindergarten.

Summary: The arithmetic children know when they enter school. — The most significant statement that can be made on basis of the evidence reported above relating to the number knowledge of beginning first graders is that many children

33 William A. Brownell and others, Arithmetic in Grades I and II: A Critical Summary of New and Previously Reported Research.


36 Sina Mott and Mary Elizabeth Martin, "Do First Graders Retain Number Concepts Learned in Kindergarten?" Mathematics Teacher, XL (February, 1947) pp. 72-78.
entering first grade have made a good start on the road toward quantitative competency. However, the data indicate that there is a wide range in the numerical ability of beginning first grade children. The evidence indicates that children are ready to continue their numerical growth, with not all children being at the same level. Judging from the variations in accomplishment revealed by the findings of the various investigators, it cannot be said that all children are ready for the same experiences.

The sequential order of number development.—Knowing the sequential order in which number develops gives teachers a clue to arithmetic readiness. After a child has developed one ability, the teacher, from the findings of research, can know in general what can logically be expected next.

Russell,37 using as subjects pupils in the kindergarten, first, and second grades during the school year 1932-1933 at Lawrence, Kansas, sought to discover in children four to eight years of age: (1) the responses to quantitative situations of more and less; (2) the understanding and use of words denoting more, less, many, equal, and same; (3) the limits, perceptually, beyond which these children cannot make distinctions between quantities; and (4) the manner in which these children go about the task of making distinctions between quantities. Asking children to respond to a test situation in which different sized blocks were used in various ways,

37Russell, op. cit.
Russell found that the child's first concept of number is a manyness from which the quantity and serial aspects of number differentiate. He found further that cardinal and ordinal number concepts develop simultaneously, and concluded that the ability to count, in itself, is not a reliable measure of the development of ability in cardinal and ordinal use of number. The findings indicated that the child, four and a half to five years of age, readily understands the terms "most," "both," and "biggest." Words denoting same and equal are not understood. The child can compare a visual notion only of three or perhaps four. The seven year old child uses such terms as "many," "most" and "more." The words "same" and "equal" are not fully comprehended. Counting by ones is a difficult method for differentiating groups and is not accurate above five. The child will form subgroups first which have unequal value mathematically. At a later stage in the differentia-process, counting by one is used. Judd,\(^\text{38}\) too, indicated that young children first see the group as a whole and later develop the ability to differentiate the group into its individual components.

In an uncontrolled experimental situation, McLaughlin\(^\text{39}\) analyzed in preschool children the development of three phases of quantitative experience, namely, counting, recognition of

\(^{38}\) Charles Hubbard Judd, *Psychological Analysis of the Fundamentals of Arithmetic.*

\(^{39}\) McLaughlin, op. cit.
number aggregates, and combinations of number aggregates. The data were secured by means of two tests given to 125 children ranging in age from 36 to 72 months. In order to be able to compare attainment at successive age levels, the children were selected to form three age groups of comparable intelligence. For rote counting McLaughlin found that the range was from 4.5 at three years to 33.4 at five. Ability in rational counting was found to be slightly less than in rote counting. Throughout the three successive years there was a steady advance in ability to enumerate correctly the items in the concrete series, though, in general, achievement in rational counting was less than in rote until the first three or four decades were reached.

In regard to the findings concerning the development of number aggregates, McLaughlin summarized her findings by saying:

... the order of development is from perception of simple spatial forms of aggregates toward analysis, first by counting single unities, later by recognition of small numbers as 2 or 3 and counting on the other unities in the group number. Still later these small groups may be combined by counting by twos, or by combining "doubles," i.e., 2 and 2; 3 and 3, etc. Finally, a mature stage is reached, characterized by the prompt recognition and naming of aggregates as groups or cardinal numbers.40

McLaughlin further concluded: "The awareness of cardinal number involves a higher stage of abstraction than is the case in serial counting."41 Consistent with the conclusion

40Ibid., p. 351.
41McLaughlin, loc. cit.
reached by McLaughlin is that reached by Judd following experimentation involving the counting process. Judd stated as follows:

A group of five things is a reality distinct from the individual objects which compose the group. Five people are not merely five separate experiences; they are a group. The group has a kind of reality which is broader and more inclusive than the individuals of which it is composed.  

Carper discovered that with young children the limits attained in rote counting and in enumeration are usually the same. She found that a relatively small per cent of the subjects counted further by rote than by enumeration, and a still smaller per cent counted further by enumeration than by rote.

In studying the number concepts of small children, Mott found that about half of the five-year olds tested were counting objects beyond their rote counting. She concluded like Russell that rote counting was not a necessary prerequisite to object counting.

Riess made a fundamental examination of the process of number development in young children both by reference to previous studies and on the basis of her own observations of children's reactions in quantitative situations, On the basis

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42 Judd, op. cit., p. 108.
43 Carper, op. cit.
44 Mott, op. cit.
45 Riess, op. cit.
of her observations she set forth the pattern in number development. The first stage in the process is referred to as pre-numerical behavior, in which children do not differentiate between two and three, but between the familiar pattern of this and that and a vague many. The second stage that Riess sets forth is that in which the naming function is most characteristic. In this stage the child established a one-to-one correspondence between a sequence of performances and a set of number signs. The young child does not know how many objects he has. All he knows is that as one separates objects, one repeats number words. Riess maintains that this is not counting, but naming or matching. She suggests that only when the last number name becomes the identification of the group of objects named in order can the child be said really to count. The third stage is termed the ordinal or ordering function. When the child has to choose between objects for which the exact order or succession is important, his numbering comes to imply a positional or sequential reference and thus the transition from naming to ordering is done. The fourth and last phase is called the grouping function which indicates a mature understanding of the significance of cardinal number. The development of the grouping or cardinal function is brought about through the use of successive ordinal development of number.

The studies just referred to have been concerned with the sequential development of number largely within pre-school
children. A significant study has been made by Gesell and Ilg in which the sequential developmental aspects of number are reported from one year of age to ten. The number aspect was only a small phase of a large comprehensive study of all aspects of child growth and development. The approach was longitudinal in that many of the same children were studied from infancy to ten years of age at the Yale Clinic of Child Development at Yale University.

The investigators used standardized procedures and observation for collecting data, but these means were supplemented with naturalistic observations stenographically recorded. The findings relating to number development indicate the sequential development of number as well as the general level of children's reactions to quantitative situations at the ages of from one to ten. The findings are briefly summarized below.

At one year of age the child indicates the one-by-one pattern of manipulating one object after another consecutively, while at three years of age the child can count two objects. At four and one half years of age the child can count "just one," "two" or "three" cubes on request. When the child is five years old he can count and point to thirteen objects, and can do simple addition and subtraction by using crude and

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46 Gesell and Ilg, op. cit.

47 The interested reader is referred to pages 399-402 of the above reference for a detailed account of the findings relating to number development.
immature procedures. At six years the child shows greater maturity in counting and can count to 30 or more. By the time the child is seven years old he can count to 100 by 1's, 5's, 10's, and by 2's to 20. Increasing growth is shown, so that by the time the child is nine years of age, he has a good command of most of the addition and subtraction combinations as well as the multiplication and division combinations. The child at this time uses multiplication as a short form of addition.

Summary: The sequential order of number development.-- The findings from the above studies, considered collectively, indicate that, in general, very young children first see a group of objects as a whole and described as "many". This manyness becomes more refined as the child begins to differentiate "more" and "less", and begins to have a conception of "just one". According to most investigations, although the evidence is frequently contradictory, development in rote counting appears to precede growth in rational counting, with the two abilities becoming about equal as the child begins counting in the second and third decades. Following growth in rational counting, children gradually begin to use partial counting and grouping in apprehending groups.

Even though children seem to learn rote counting a little in advance of rational counting, it is doubtful if rote counting has any particular value as far as readiness is concerned other than the learning of number names in serial order.
By the time the child enters school he has already begun to count rationally. It is suggested here that number names can be learned more meaningfully in connection with rational counting without the primary grade teacher devoting time to teaching rote counting.

Most of the studies indicate that there is not a sharp division line where one developmental stage in numerical competency ends and the next begins. Until a mature level of apprehending numerical situations is reached, evidence is that a new ability begins as an older one is continued and refined. For example, a child will continue to count as he begins to use grouping. Skills do not seem to be mastered all at once, but develop gradually from crude beginnings to the level of adult achievement.

Children's thought patterns in dealing with number. — A clue to arithmetic readiness is afforded by the maturity level at which a child deals with number. A child may be able to give correct answers by using very immature and crude methods. Research has done much to indicate the thought patterns followed by children in dealing with number.

In a series of related investigations, Brownell attempted to determine how children apprehend the total number of concrete objects in small groups, and how they add abstract number combinations. More than one thousand children in the

48 William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.
first five grades in six Illinois schools participated in the study. The data were collected by means of group tests and personal interviews. It was found that a variety of procedures such as counting one by one, partial counting, and grouping are used by children with both concrete and abstract numbers. On the basis of the individual analyses of how children apprehend number groups, Brownell concluded:

...the development of ability to deal effectively with concrete number is not to be thought of as a series of simple stages through which a child passes easily, naturally, and expeditiously once he has been taught to count. Counting as a method of dealing with concrete numbers is an exceedingly simple, immature method and yet a method with which a child may rest content long after he should have begun to think of numbers in more abstract terms .... The last stage in the development of ability to apprehend concrete number, that stage in which the objective representation of a number serves merely to initiate the most abstract methods of apprehension, is far removed from counting and is to be reached only by taking several intervening steps.\(^49\)

In regard to the relative importance of accuracy, speed, and mental processes, Brownell concluded on the basis of the findings in the study: "The mental processes of the pupils, not simply the accuracy and speed with which results are produced, should become the chief concern of the teacher."\(^50\)

Another significant study was made by Brownell and Carper\(^51\) in which they endeavored to determine as much as

\(^{49}\)Ibid., p. 81.

\(^{50}\)Ibid., p. 83.

\(^{51}\)William A. Brownell and Doris V. Carper, Learning the Multiplication Combinations.

For a detailed summary of this study see Chapter V.
possible about the way in which children learn the multiplication combinations. More than 4,000 pupils from the end of the third grade to the end of the fifth grade participated in the experiment. Group tests and individual interviews were the means by which the data were secured. The investigators found eleven different ways in which children reacted in obtaining solutions to multiplication combinations.

Strong implications for readiness relative to maturity of thought processes were indicated from the findings of an important study by Carper. The purpose of this investigation was to discover the methods used by children entering the first grade in dealing with various types of number activity and to evaluate their success in using the different methods. The subjects in the study were two hundred seventy children who had been in school less than three weeks. The data were secured from a test which included the following items: counting (rote and enumeration), recognition, identification, and reproduction of concrete groups, and addition and subtraction combinations in concrete, verbal and abstract forms. All of the sections of the test, except one, were handled by one of three procedures: grouping, counting, or partial counting. The section involving reproduction of groups by drawing was done by various forms of counting, grouping, or by non-quantitative matching or drawing.

Children were found to use both counting and grouping

52Carper, op. cit.
as methods in obtaining solutions to quantitative situations. Of significance was the finding that children who used a grouping process in the simplest concrete situations were very successful in using grouping in obtaining answers for abstract combinations and for verbal problems. Children who counted in the concrete situation seldom found their method of solution effective in the abstract situation. Improved ability in counting did not carry over to improved facility in arriving at solutions to abstract problems or combinations. The children who counted in concrete situations and were successful in the abstract situation resorted to counting fingers or imaginary marks in dealing with the abstract number situation. Further, it was found that the grouping method was quite as accurate as the counting method in dealing with single concrete groups and, of less importance, not as time consuming as counting. It was concluded that the grouping method was the most efficient method to learn from the standpoint of wide applicability as well as from the standpoint of accuracy in dealing with verbal and abstract problems.

Of significance in showing the value of considering the maturity with which children deal with number is a study of Deans. In this study, Deans\textsuperscript{53} investigated the effectiveness.


For a more detailed summary of this study, see Chapter IV.
of guiding children through certain temporary intermediate procedures in learning the four fundamental processes in arithmetic. The investigator found that such a method was of definite advantage in helping children to progress from one maturity level to the next. An investigation by Brownell and Chazal\textsuperscript{54} shows that drill alone is not sufficient to cause children to progress from one maturity level of performance to the next.

Findings which substantiate those reported above relating to the thought processes used by children in dealing with numerical situations are revealed by additional evidence reported by Brownell,\textsuperscript{55} and by McLaughlin.\textsuperscript{56}

**Implications: children's thought patterns in dealing with number.**—The significant point to be made from the data reported concerning the thought processes of children is that in evaluating children's ability in a given phase of arithmetic, teachers must look deeper than merely determining whether or not the child has obtained the correct answer. This calls for evaluation techniques based on individual


\textsuperscript{56}McLaughlin, \textit{op. cit.}
interviews and close observation of children as they work. Methods of instruction are needed which will guide children as they progress from using immature thought processes to using meaningful habituation. Teachers, temporarily, at least, need to accept crude methods used by children and not to expect them to adopt more mature methods immediately. To help children use more mature methods of working, teachers must use instructional materials which will cause children to use grouping rather than counting in apprehending groups of objects.

**Major Factors Constituting Arithmetic Readiness**

Much controversy has arisen among educators as to which factors constitute readiness or the ability to do arithmetic. Some have placed their emphasis on mental maturity, while others have considered experience to be the major factor. More recently the trend has been to recognize the interrelatedness of experience, maturation, intelligence, and such factors as reading ability and the social purposes which the learner may have. Research has done much to show the relative contribution of the many factors constituting readiness.

**Mental Age.**—The classical study in which mental age was considered of prime importance as the readiness factor was conducted by the Committee of Seven under the leadership
of Carleton Washburne. The committee sought to determine the mental age level at which the various topics in arithmetic could be taught to completion. Following an experimental period in which topics were taught in the usual grade, and in one or two grades immediately preceding and following, the mental age was determined at which three-fourths of the children could show 80 per cent mastery of a given topic. When this standard was reached the topic was designated to be taught at that given mental age level. For example, ten years and two months was the suggested level for teaching to completion compound multiplication (two-place multiplier, not over three places in the multiplicand).

This study was severely attacked by Brownell because of the strong emphasis given to mental maturity as a readiness factor and because of the lack of consideration given the experience factor of readiness. Johnson, who followed the recommendation of the Committee of Seven in reorganizing the elementary arithmetic program in the Chicago schools,


For a detailed summary of this study see Chapter III.


found significant gains in achievement. On the other hand, Grossnickle found that division was taught successfully at an earlier mental age than that recommended by the Committee of Seven.

In a significant investigation, Cruickshank has shown that children of the same mental age are not equally capable in arithmetic. The same has been shown to be true in other areas of learning by Aldrich and Doll, Merrill, and Fox.

**Experience.**—Not accepting mental maturity as the


For a more detailed summary of this study see Chapter VIII.


63 M. A. Merrill, "On the Relation of Intelligence and Achievement in the Case of Mentally Retarded Children," Comparative Psychology Monographs, II (1924) pp. 1-100.

primary factor in readiness, Moser\textsuperscript{65} reports a learning experiment in primary grade arithmetic in which he attempted to re-evaluate the relative contributions of maturation, as measured by mental age, and of environment as determined by the educational experiences provided. Through a method of teaching fractions which provided a rich background of experience in developing fractional concepts through the use of manipulative materials, Moser found that children in second grade learned fractions at mental ages one to two years lower than the age proposed by the Committee of Seven. The investigator concluded that "optimal" mental age as described by the Committee of Seven for introducing fractional symbolization is not independent of the learning experiences provided.

Smith\textsuperscript{66} made a significant contribution to the professional understanding of arithmetic readiness when he made a study to clarify the relative contribution to arithmetic readiness of three factors: experience, mental maturity, and I.Q. In a controlled experiment, Smith sought to discover the relative reliability of the three above factors in the prediction of success in two digit-division. The subjects who participated in the study included approximately 500 fifth


grade pupils enrolled in twenty classes in the schools of three communities in the Chicago area. At the beginning of the experiment, none of the children had had any experience with dividing with a two-digit divisor, although they had completed the study of one-digit division.

An intelligence test and a foundations test covering multiplication, subtraction, and division by one-digit divisors were given at the beginning of the experiment. At the conclusion of the experiment an end test in division by two-digit divisors was given. This test was followed at a later date by a retention test in division by two-digit divisors.

A correlation technique was used to study the relationship between the scores made on the end test and each of the other three factors, namely, mental age, I.Q., and foundations test scores, the latter factor indicating experience. Although a positive relationship was found between the scores on the end test and each of the other three factors, a higher positive relationship was found between the end test scores and the foundations test scores than between the end test scores and either of the other two factors. The same relationship was found between the retention test scores and mental age, I.Q., and foundations test scores. It was further discovered that the children who made few errors in subtraction, multiplication, or division with one-digit divisors tended to make few errors in the corresponding processes on the end test. Those who made many errors in the processes...
used in division tended to make many errors in those same processes on the end test.

From the findings presented, the investigator concluded that successful attainment in division by two-digit divisors is more closely related to experiential background than it is to either I.Q. or to the mental age. He concluded further that if children are to be expected to master division by two-digit divisors, they must first have mastered subtraction, multiplication, and division by one-digit divisors.

The findings by Koenker\(^67\) that both mental age and experiential background are significantly associated with good achievement in two-figure division, are consistent with the findings reported by Smith above. Koenker concluded that since poor achievement in two-figure division is influenced by an inadequate mastery of one-figure division, multiplication, subtraction, and vocabulary terms used in division, a more adequate readiness program for two-figure division is needed.

General maturation.—Research in the area of arithmetic has not fully measured the contribution of maturation to learning. The well known study by Benezet\(^68\) is often referred

\(^{67}\)Robert H. Koenker, "Certain Characteristic Differences Between Excellent and Poor Achievers in Two-Figure Division," Journal of Educational Research, XXXV (April, 1942) pp. 578-86.

to as a study which shows the influence of maturation. In this study children who did not begin the formal study of arithmetic until the sixth grade are compared with a group of sixth grade children who had had specific instruction in arithmetic since the beginning of the first grade. A testing program which measured the achievement of the two groups at the end of the sixth grade, indicated that the group who had had delayed instruction did as well as the group who had had instruction from the beginning.

The above study is frequently criticized. A close examination of the program outlined for the group who were not taught arithmetic as such shows that many functional experiences in arithmetic were provided for the children. On examination of the program, both Thiele and Buswell maintained that Benezet had merely delayed the teaching of formal arithmetic, for a functional program was in evidence all the time.

Suggesting the influence of maturation is a study by Bemis and Trow. Eighteen retarded children from Grade 6A were given special remedial help in arithmetic. The average

69 C. Louis Thiele, "An Incidental or an Organized Program of Number Teaching?" Mathematics Teacher, XXXI (February, 1938) pp. 63-7.


gain was nine months instead of a normal gain of five months for the five months of instruction. In order to determine whether the gain was of permanent improvement, the remedial group was compared over an additional two year period with a control group who had had the same amount of regular instruction in arithmetic, but no special remedial work. Pupils in the two groups were paired on the basis of chronological age, I.Q., and amount of retardation. Although in general the gains at the end of the two-year period favored the experimental group, for certain individual pupils the remedial work appeared of little or no value when compared with the progress of the paired controls who had not had special work.

Outside the area of arithmetic some significant experiments have been made to show the role of maturation in learning. Such investigations have been conducted using both animals and children as subjects. In a study on which chicks were used, Breed and Shepard\(^7\) prevented one group of chicks from obtaining practice in pecking for several days while the chicks in another group were permitted to peck naturally from the time they were hatched. When the chicks in the former group were given a chance to peck, they showed as much efficiency within a couple days as those who had received practice from the start. When this experiment was repeated later by

Bird under varying conditions, findings consistent with those reported by Breed and Shepard were reported.

Carmichael's evidence from a study of tadpoles likewise reflects the influence of maturation in early development. Tadpoles that had developed in water containing chlortone, which prevented motion without inhibiting growth, were found to swim quite as freely when placed in plain water as other tadpoles which had been permitted to develop normally.

In an important study by Gesell and Thompson the method of co-twin control was used. One of two identical twins was given daily training in climbing stairs for a period of six weeks, beginning at the age of forty-six weeks. In the meantime, the other twin was deprived of specific training in this skill. At the end of the six-weeks' period, it was found that the trained child climbed stairs somewhat more expeditiously than the control child but the difference was quickly reduced. After three weeks of opportunity, the performances of the control child was much superior to that


75 Arnold A. Gesell and Helen Thompson, "Learning and Growth in Identical Infant Twins: An Experimental Study by the Method of Co-Twin Control," Genetic Psychology Monographs, VIII (September, 1930) pp. 209-319.
shown by the other child at the end of the six weeks' training period.

In an experiment relative to the efficacy of early and deferred vocabulary training, Strayer used as subjects the same twins which were used in the experiment just reported by Gesell and Thompson. Beginning at the age of eighty-nine weeks, C was given intensive training for four weeks and returned to a normal environment. C reached a higher level of achievement in language development at the end of twenty-eight days of training than twin T had attained after thirty-five days of training begun when she was five weeks younger. At the end of C's training period T had better pronunciation but this difference rapidly disappeared during the following weeks.

Implications: experience, intelligence, and maturation. On the basis of evidence revealed by the investigations concerned with the contribution of maturation, experience, and intelligence as factors contributing to arithmetic readiness, it can be said that all of these factors seem to play a significant role in arithmetic readiness. Neither of the factors can be considered as contributing to arithmetic readiness, except as the factors are considered in relationship to each other. Learning cannot take place without mental and

76Lois Curry Strayer, "Language and Growth, the Relative Efficacy of Early and Deferred Vocabulary Training, Studied by the Method of Co-Twin Control," Genetic Psychology Monographs, VIII (September, 1930) pp. 209-319.
physical maturity, without sufficient native intelligence, and without experience. Any single factor cannot make its full contribution to readiness in the absence of the other factors. Over emphasis on experience or training frequently seems to give temporary gains, but with greater maturity the gains can be achieved in less time. Perhaps social demand for a skill in arithmetic should determine whether emphasis be given to experience or whether to wait for the advantages to be derived from increased maturity.

Social demand and intrinsic purpose.—When arithmetic is presented in the narrow sense of a subject to be taught rather than as a tool in helping children solve their social problems, little attention is given to the purposes which children have, and consequently such purposes are not considered as influencing readiness. Several studies, which are to be summarized in greater detail in other chapters, have reflected the importance of the learner's purposes in influencing ability to learn.

Harap and Mapes, recognizing the importance of the goal seeking purposes of children, found that as children's purposes were carried out through the medium of planned activities, the achievement of the pupils in computational skill in fractions was equal or superior to that of children who were taught in ways that did not consider their goal

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seeking purposes. Passehl\textsuperscript{78} substantiated the findings reported by Harap and Mapes when he conducted a similar experiment in the teaching of fractions when attention was given to the children's intrinsic purposes.

In a controlled experiment with fourth grade children, Harding and Bryant\textsuperscript{79} found that when activities were planned to carry out children's purposes, the gains made by the experimental group were statistically superior to those made by the control group. Not only were the experimental children superior to the control group in arithmetic achievement, but they were rated higher in emotional and social adjustment.

For six years in sixth grade arithmetic, Williams\textsuperscript{80} built her arithmetic program on the questions raised by the children as they carried on various experience units which the children helped to plan and execute. Williams found that the children made gains in arithmetic far in advance of that normally indicated by standardized tests. Further, the children were rated better emotionally and socially adjusted than were comparable children who had been in a more traditional program.


In an uncontrolled study, Conner and Hawkins\textsuperscript{81} showed that pupils in the Cleveland schools reached the national norm as established by standardized tests when the instructional program was based on the problems which the pupils had a real purpose for solving. When ordinary textbook problems which had no intrinsic purpose for the pupils had been used, the school had been unable to reach the national norm.

\textbf{Significance: social demand and intrinsic purpose.---Regardless of the intelligence, maturity, or experience one may have, learning is certainly facilitated by the so-called will to learn. Teachers, in order to promote the fullest readiness for learning, should plan to capitalize on children's purposes as much as possible in selecting instructional materials and learning activities.}

\textbf{The role of meaning and understanding.---In terms of readiness it was once assumed that each number fact or process needed to be learned separately without any consideration given to an understanding of relationships in the number system or to the understanding of the mechanics involved in performing the algorism of a computational process. Much research has been done to show that seeing relationships facilitates learning by increasing the chances of transfer to untaught situations. Also, when meanings are established, skills}

and processes are remembered longer than when facts alone and not meanings are stressed.

MacLatchy, in an individual case study, found that an understanding of our ten base number system helped to eliminate the difficulty the child had in bridging the tens in addition. Buckingham showed that when children saw the relationship between a given addition fact and the corresponding subtraction fact, both combinations were learned more readily than if the related facts were taught so that the relationship was not discovered.

McConnell conducted an experiment with second grade children in learning the 100 addition and the 100 subtraction facts. One group had been taught the combinations mechanically through repetition, whereas the other group was given an opportunity to develop an understanding of the facts through concrete and semi-concrete experiences with objects. On the tests used, the group taught mechanically had a better mastery on the criterion of immediate and automatic response, while the other group was superior in those tests which put a premium on deliberate work and more thoughtful manipulation.


84T. R. McConnell, Discovery vs Authoritative Identification in the Learning of Children.
In a controlled experiment, Thiele\textsuperscript{35} found that second
grade children who were taught the 100 addition facts by a
method which stressed generalizations, number meanings, and
relationships were significantly superior in the mastery of
the combinations to the children who had been taught the com-
binations without attention being given to understanding.
Perhaps of more significance, the group who was taught mean-
ings and understandings did significantly better in untaught
number situations. Evidence from a study by Grener and
Raths\textsuperscript{36} supports Thiele's findings.

Of significance is the data presented by Nicholson\textsuperscript{37}
which show that children who had the understanding that zero
is a place holder made fewer zero errors than did children
who had learned separately the combinations involving the
zero in the 100 addition facts.

Swenson\textsuperscript{38} discovered that second grade children who
were taught the basic addition combinations by a method which
stressed meanings and generalizations were significantly

\textsuperscript{35}L. Thiele, The Contributions of Generalization to
the Learning of the Addition Facts.

\textsuperscript{36}Norma Grener and Louis E. Raths, "Thinking in Grade
pp. 38-42.

\textsuperscript{37}Ethel Lodge Nicholson, "A Comparison of Two Methods
of Teaching Arithmetic, Grades I, II, and III," (Unpublished

\textsuperscript{38}Esther J. Swenson, "Organization and Generalization
as Factors in Learning, Transfer, and Retroactive Inhibition,"
superior to children taught by less meaningful methods in
the number of combinations learned, in retention, and in
transfer to untaught addition and subtraction items.

Of particular concern to readiness is the study by
Hendrix. She showed that frequently children have grasped
an understanding, and are able to apply the generalization,
even though they are unable to verbalize the understanding or
generalization.

Inference: meaning and understanding.—Due to the ad-
vantages to be derived from meanings and understandings in
promoting the ability to learn, transfer, and retain skills,
it is imperative that recognition be given to the role of
meaning and understanding in the development of a concept of
arithmetic readiness. In order to take full advantage of
meanings and understandings, the teacher must use methods
which will facilitate children in acquiring these attributes
to learning and readiness. In view of the findings reported
by Hendrix, teachers need to observe carefully in order to de-
tect signs of understandings which the child cannot always
verbalize. Such tests as prepared by Glennon for measuring

89Gertrude Hendrix, A New Clue to Transfer of Training,"
Elementary School Journal, XLVII (December, 1947) pp. 197-
208.

90J. Vincent Glennon, "A Study of the Growth and Mastery
of Certain Basic Mathematical Understandings on Seven Educa-
University, 1948).

For a detailed summary of this study see Chapter IX.
meanings and understandings has much to promise in the way of evaluating understandings that are not measured by ordinary tests which are concerned primarily with rate and accuracy.

Vocabulary and reading ability.—Vocabulary and reading ability have long been associated with ability in arithmetic. Arithmetical vocabulary is closely associated with children's ability to comprehend number concepts and mathematical terminology.

A principal study dealing with number vocabulary was made by Buswell and John. The purpose of the first part of the study was to study children's reactions to arithmetical terms and the development of their concepts of these terms from grade to grade. According to the criterion of frequency of use, one hundred terms were selected from the arithmetical terms used in twenty-seven different textbooks for grades one through six. The terms were grouped according to the following five divisions: (1) words or signs which are strictly technical; (2) terms relating to time, space, or quantity; (3) terms relating to measurement; (4) commercial terms; and (5) terms relating to spatial figures.

Four group tests were administered to measure understanding of vocabulary as presented in a variety of situations. The subjects were 1,500 pupils, 500 in the second half of each of Grades IV, V, and VI. The pupils were from twelve

91 Guy T. Buswell, and Lenore John, The Vocabulary of Arithmetic.
school systems in the states of Illinois, Ohio, Kansas, Alabama, Michigan, Iowa, Kentucky, and Oklahoma.

Data provided by the tests indicated that pupils in a given grade differ widely in the size of their arithmetical vocabularies. Although the number of terms known increases from grade to grade, the distribution in the three groups show a large amount of overlapping. The difficulty of the classes of terms into which the list is divided indicated that, in general, the technical terms are the most difficult, and that the terms relating to time, space, or quantity are the least difficult. The terms relating to spatial figures, the terms of measurement, and the commercial terms were found to be of average difficulty. Analysis of the incorrect responses revealed that incorrect meanings are frequently associated with terms. In some cases the number of pupils who had misconceptions regarding the terms was greater than the number of pupils who understood them, and the amount of misunderstanding did not decrease materially from Grade IV to Grade VI. It was found that when words were presented in various situations, as they were in the test situation, children responded to a word correctly in one situation and not in others. The investigators concluded from this evidence that the ability to respond to a word correctly in one situation does not necessarily show that understanding is complete. It is implied that further experience with the word may be needed for complete understanding.
In the second part of the study, Buswell and John tried to discover the changes in the understanding of arithmetical terms as children pass from grade to grade in the elementary school. A group of 240 children, 40 from each of the first six grades from three Illinois elementary schools, was selected. Children were tested individually. For each of the twenty-five terms used in the test, the children were asked: "Can you tell me what this word means in arithmetic?" Responses for each term were classified into four general divisions: (1) satisfactory; (2) doubtful; (3) incorrect; and (4) omissions or non-arithmetical meanings.

It is impossible in this summary to report the findings for each separate word. The words "spend", "number", "enough", "both", "count", and "smaller" were known by half or more than half of the first grade pupils, whereas for such words as "quotient", "area", and "decimal" there were no satisfactory responses in any of the first three grades.

When in Grades V and VI the word "count" was defined by eight pupils as meaning to add and by four as meaning to say numbers, and when nineteen other pupils defined the word in terms of itself, the investigators concluded that at least thirty-one of the eighty pupils in these two grades had not refined their concepts of the word "count" much beyond the level of the concepts held by pupils in Grades I and II.

On the basis of the evidence revealed by the study, the investigators have the following to say concerning the
development of new concepts:

In studying their elementary school pupils' responses through the first six grades, one may observe the gradual manner in which concepts are developed. In the case of semi-technical words which are used to some extent outside of school one finds beginnings of understanding even in the first grade. Pupils react to such words out of the background of their experience, and in the process of developing full understanding they use and later reject many erroneous meanings. From the entire body of experience, patterns of understanding are finally developed which constitute true concepts of the terms. In the case of more strictly technical words, such as quotient, which do not appear in the out-of-school experience of pupils there is less evidence of an accumulated background of understanding and more evidence of a process of learning which begins with the end product, the definition, unsupported by sufficient experience. The wrong responses to such words as quotient and product are very different from the wrong responses to words which have a background of out-of-school experience.

If the pupils are to learn technical terms, the first demand on the school is that it supply a gradually increasing body of experience which will provide a meaningful background for the terms that must be learned. The fulness of meaning will depend on the maturity of the pupil...

If pupils are to think clearly in arithmetic, they must have valid concepts of the terminology of the subject. The words borrow, cancel, carry, and reduce must not be associated simply with textbook definitions which can be memorized. 

A recent study by Harrison has pointed out the need for clear understanding of words, signs, and symbols in being able to cope with number in textbook problem situations. She points out:

When meanings are taken into account, common words, for instance, are not as easy as their

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92 Ibid., pp. 103-104.
93 Irene Gertrude Harrison, Survey of Words and Signs in Two Arithmetic Textbook Series.
frequency of use seems to indicate. Words like "face," "root," "yard," "round," and "carry," are common words. Their ordinary meanings may be familiar to the general reader, but their arithmetical meanings may still be unfamiliar. In fact, the familiar appearance of such words may even contribute to an element of difficulty, when they are used with specialized meanings. Technical words such as "product," "dividend," "radius," and "perimeter," generally regarded as more difficult than common words because of their lower frequency of occurrence, may actually present a simpler semantic problem than common words. . . . When the field of meanings is considered broadly enough, it is found that arithmetical words and signs are rarely, if ever, mono-meaning.

The investigator made a semantic count of the arithmetical signs, words, and collocations occurring in approximately 6000 problems in current arithmetic textbooks for grades III-VIII. In addition to words, she found a significant number and variety of arithmetical signs and concluded that they should be included as an integral part of vocabulary comparable in importance to the verbal vocabulary. She found variant meanings for most of the arithmetical words. It was concluded that numerous terms, notably abbreviations and hyphenated words, are potentially ambiguous. That investigator found that semantic variation, ambiguity, and introduction of new meanings through collocation characterize equally the vocabularies of textbooks in arithmetic for Grades III-VIII.

The influence of experiences designed to improve number vocabulary upon growth in problem solving skills is borne out
by research conducted by Johnson. That investigator evaluated the effect of specific instruction in vocabulary on success in problem solving in seventh grade mathematics. The experiment, involving 898 pupils in twenty-eight seventh grade classes, was conducted during 1941-43 in three junior high schools of Niagara Falls, New York. The various classes were assigned to either the experimental or the control group. The two groups were statistically equated on the basis of chronological age, mental age, arithmetic achievement, reading ability, and knowledge of arithmetic vocabulary. The experimental variable, used with the experimental group, was the provision for a variety of experiences designed to develop a meaningful understanding of vocabulary beyond that which was provided by the textbook itself. The time devoted to vocabulary instruction in the experimental group averaged from five to eight minutes per class period. The vocabulary study was taken from the regular class period; no extra time was allowed. The control classes had to rely entirely upon the textbook and regular class discussion for their learning of the mathematical terms.

At the end of the fourteen week experiment, it was found that when the control and experimental groups were matched statistically on mental age and initial status in vocabulary,
the experimental group achieved significantly greater gains than did the control group in both vocabulary and problem solving. This superiority of the experimental group held true for pupils on practically all levels of mental ability and initial status in the area under study.

In another phase of the experiment, a retention test, administered three months after the close of the experiment, revealed the fact that the experimental group was still statistically superior to the control group in the knowledge of the vocabulary included in the special instruction.

On the basis of the findings, the investigator concluded that the value of the special vocabulary experience, as provided in the experiment, was independent of the class in which they were used. Using well planned experiences designed to improve number vocabulary will result in significant gains in both vocabulary development and problem solving skill.

Bringing out further the influence of reading and vocabulary on ability in arithmetic is a study conducted by Treacy,96 who investigated the relationship of certain reading skills to ability to solve verbal problems in arithmetic. He sought specifically to determine whether general reading level or specific reading skills are significantly related to ability to solve problems in arithmetic.

Through an extensive testing program, the mental ability, problem solving status, and ability in fifteen specific reading skills were determined for 244 7B pupils in two Milwaukee junior high schools. The eighty pupils having the highest T-Score on problem solving were designated as "good achievers," and the eighty pupils having the lowest combined T-Score were designated as "poor achievers". The good and poor achievers were then compared on each of the fifteen reading skills.

With mental age and chronological age statistically controlled by the Johnson-Newman technique, it was discovered that good achievers were significantly better than poor achievers in the following reading skills: (1) vocabulary in context; (2) vocabulary, isolated words; (3) arithmetic vocabulary; (4) quantitative relationships; (5) perception of relationships; (6) integration of dispersed ideas; (7) drawing inferences from context; and (8) retention of details.

No significant differences were found between the two groups in: (1) prediction of outcomes; (2) understanding of precise directions; (3) rate of comprehension; (4) general information; (5) grasp of central thought; and (6) interpretation of content.

Since good achievers were found significantly superior to poor achievers in four reading skills associated in one way or another with vocabulary, the investigator suggested the importance of stressing the meaning of terms, general and
mathematical, as an approach to improving pupils' ability in problem solving.

On the basis of the findings, the investigator concluded that specific reading skills, rather than general reading ability, is significantly related to problem solving ability. Treacy recommended that research on the relationship of reading and problem solving be in terms of specific reading skills rather than in terms of general reading ability. In planning procedures for helping pupils through reading instruction to improve their ability in problem solving, the investigator recommended that reading should be regarded as a composite of skills, rather than as a generalized ability.

Evidence comparable to that reported by Treacy was reported by Hansen. That investigator studied factors associated with successful achievement in problem solving in sixth grade arithmetic.

Implications: vocabulary and reading.--In view of the evidence revealed by the vocabulary and reading studies reported above, it may be said that children need a rich background of experience in order to extend word and concept meanings used in coping with number in both oral and written problem situations. Words can have many meanings. These meanings grow from faint ideas to extended and refined

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understandings. The ordinary definitions which words have frequently have a different and technical meaning in arithmetic; therefore, children become confused when they try to apply the ordinary understanding to the technical situation.

The investigations indicate that improvement in arithmetic ability can be expected to result from activities designed to improve vocabulary and reading skill. In promoting readiness to cope with number in an ever increasingly mature way, vocabulary and reading development needs to be an integral part of the readiness program at all instructional levels.

Summary

In this chapter have been presented the various points of view concerning arithmetic readiness. These views ranged from practically no consideration given to readiness to a comprehensive view in which multiple interrelated factors were considered such as mental and physical maturity, intelligence, experience, intrinsic purpose on the part of the learner, meaning and understanding, and reading ability. Important experimental investigations have been summarized pertaining to two topics: (1) indications of readiness, and (2) major factors constituting arithmetic readiness.

In the two following sections will be reviewed briefly the research findings related to each of the above mentioned topics. In the third and final section conclusions together with suggested implications for teaching will be offered.
Indications of readiness.--The well known study by Buckingham and MacLatchy\textsuperscript{98} revealed that beginning first grade children have acquired much facility in the use of number when they enter school. Wide variations, however, were found in the children's ability to cope with number. Brownell,\textsuperscript{99} who reviewed new and previously reported research on the arithmetical skill and knowledge of children entering first grade, substantiated the findings reported by Buckingham and MacLatchy. Studies reported by Koenker,\textsuperscript{100} Wilburn,\textsuperscript{101} and Mott and Martin\textsuperscript{102} indicate that children can and do learn arithmetic in kindergarten and first grade.

In regard to the sequential order of number development, evidence presented by Russell,\textsuperscript{103} Judd,\textsuperscript{104} McLaughlin,\textsuperscript{105} Carper,\textsuperscript{106} Mott,\textsuperscript{107} Riess,\textsuperscript{108} and Brownell\textsuperscript{109} shows

\textsuperscript{98}Buckingham and MacLatchy, \textit{op. cit.}

\textsuperscript{99}William A. Brownell and others, \textit{Arithmetic in Grades I and II: A Critical Summary of New and Previously Reported Research.}

\textsuperscript{100}Koenker, "Arithmetic Readiness at the Kindergarten Level," \textit{op. cit.}

\textsuperscript{101}Wilburn, \textit{op. cit.}

\textsuperscript{102}Mott and Martin, \textit{op. cit.}

\textsuperscript{103}Russell, \textit{op. cit.}

\textsuperscript{104}Judd, \textit{op. cit.}

\textsuperscript{105}McLaughlin, \textit{op. cit.}

\textsuperscript{106}Carper, \textit{op. cit.}

\textsuperscript{107}Mott, \textit{op. cit.}

\textsuperscript{108}Riess, \textit{op. cit.}

\textsuperscript{109}William A. Brownell, \textit{The Development of Children's Number Ideas in the Primary Grades.}
that number development proceeds from first seeing a group of objects as a whole and described as "many". From this manyness the child begins to make crude comparisons of "more" and "less" until finally he is able to identify the individual objects composing the group. Development in counting progresses from counting one-by-one, to partial counting, to grouping. The studies are somewhat contradictory in regard to the beginning of rote and rational counting. Most of them, however, indicate that rote counting precedes rational counting in the early stages. By the time the child is counting in the second or third decades, his rational counting appears to surpass his rote counting.

Significant data provided by Brownell and Carper, Judd, Carper, Brownell, and Deans reveal that children do not suddenly acquire adult competency in performing number processes, but rather progress through a series of crude immature procedures before reaching mature facility in number reaction. Brownell and Chazal have shown that even though a teaching procedure may by-pass crude and immature

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110 Brownell and Carper, op. cit.
111 Judd, op. cit.
112 Carper, op. cit.
113 William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.
114 Deans, op. cit.
115 Brownell and Chazal, op. cit.
procedures for working with number, children will use those procedures anyway.

Major factors constituting arithmetic readiness.--Many factors have been shown to contribute significantly to readiness to learn arithmetic. The relationship between innate mental maturity and number readiness has been revealed by Washburne.\(^{116}\) On the other hand, Moser\(^{117}\) has shown that a background of rich experiences makes it possible for children to learn aspects of arithmetic younger than can ordinarily be expected. Smith\(^{118}\) and Koenker\(^{119}\) have demonstrated that both experience and maturity are positively correlated with the ability to learn arithmetic skills.

Evidence presented by Harap and Mapes,\(^{120}\) Harding and Bryant,\(^{121}\) Passehl,\(^{122}\) Williams,\(^{123}\) and Conner and Hawkins\(^{124}\) reveal that intrinsic purpose and the will to learn on the part of the learner are vital readiness factors.

\(^{116}\)Washburne, op. cit.

\(^{117}\)Moser, op. cit.

\(^{118}\)Smith, op. cit.

\(^{119}\)Koenker, "Certain Characteristics Differences Between Excellent and Poor Achievers in Two-Figure Division," op. cit.

\(^{120}\)Harap and Mapes, op. cit.

\(^{121}\)Harding and Bryant, op. cit.

\(^{122}\)Passehl, op. cit.

\(^{123}\)Williams, op. cit.

\(^{124}\)Conner and Hawkins, op. cit.
The importance of meaning and understanding in arithmetic as readiness factors has been borne out by research findings provided by Swenson, Thiele, McConnell, and Nicholson. These investigators have shown that when a child understands what he is doing, sees relationships in the number system, and is able to form generalizations, he learns more readily, retains longer what he learns, and is more capable of transferring his learnings to related untaught number situations.

The contribution of vocabulary development to number readiness is brought out clearly by Buswell and John, and Harrison. Johnson showed the influence of specific vocabulary experiences on problem solving in seventh grade arithmetic. Treacy found a positive correlation between specific reading skills and problem solving ability.

Conclusions and implications for teaching.—After considering the research evidence presented above, the following

125 Swenson, op. cit.
127 McConnell, op. cit.
128 Nicholson, op. cit.
129 Buswell and John, op. cit.
130 Harrison, op. cit.
131 Johnson (Harry C.), op. cit.
132 Treacy, op. cit.
conclusions together with implications for teaching are offered:

1. It can be concluded that since children have acquired much number competency when they enter school, they are ready to continue such learning when they enter school. However, since a wide range in the development of number concepts and skills is found in beginning school children, the need is evident for differentiation to be made in the instructional procedures followed, in number experiences provided, and in the use of instructional materials. Teachers need to use both group and individual evaluation techniques in determining the number abilities of the children, and then to provide for small group and individual instruction on the basis of the information revealed by the readiness tests.

Even though it is concluded that first grade children are ready for number experiences, it is not to be implied that they are ready to learn adult procedures in dealing with number. Children need to use their immature procedures as they progress toward mature competency in dealing with number. Some children will need many experiences in using concrete materials to build concepts and to develop skills. More mature children will be able to work with abstract materials sooner.

2. Number development does proceed in a rather definite order as was outlined above. Knowing the order in which number develops should be of value to teachers in selecting
and guiding number experiences for children. Such information should help to eliminate the presentation of number experiences before previous skills and concepts have been developed. Beginning with the child at his level of development should avoid much of the need for remedial instruction which is too frequently found in upper elementary grades.

3. Even though rote counting appears to precede rational counting in the early pre-school stages, it is not recommended that rote counting be considered as part of the instructional program in the first grade. Since rote counting is more or less meaningless, the value of rote counting is doubtful other than in helping the child learn the sequential order of number and the number names. For practical purposes, kindergarten and first grade teachers can incorporate those values through teaching rational counting which is both more meaningful and functional.

4. Whether in basic counting, addition, or multiplication, children do not suddenly acquire mature adult procedures in dealing with number. Instead, children go through a series of crude or immature procedures before reaching the adult level of quantitative competency.

For teaching this means that teachers need to use readiness tests to determine the child's procedure in meeting quantitative situations, and to proceed from there in helping the child progress to more mature ways of working with number. The teacher must probe deeper in evaluating readiness
than merely determining whether or not the child has the correct answer, for the correct answer can frequently be secured by using the most immature procedure. Temporarily, at least, the teacher must accept the child on the level where he is, and provide experiences to facilitate his progress in coping with number more effectively. Children may remain on an immature level unless the motivation is such as to cause him to want to move to a more mature level.

5. Each of the factors, namely, experience, intelligence, and maturation, may be considered to contribute significantly to readiness to learn arithmetic. Neither of the factors can be considered as contributing to arithmetic readiness, except as the factors are considered in relation to each other. Any single factor cannot make its full contribution in the absence of the other factors. Over emphasis on experience or training frequently seems to give temporary gains, but with greater maturity the gains can be achieved in less time.

In view of the evidence, perhaps it should be said that social demand for skills in arithmetic should determine whether emphasis be given to experience or whether to wait for the advantages to be derived from increased maturity. It is encouraging that teachers do not need to sit by and wait for maturity to take place. Teachers can do much to provide experiences so that children can learn skills earlier if needed. Promoting readiness through experience calls for
providing children with as much firsthand experience as possible in meeting numerical situations. Children's own problem situations can be used to promote arithmetic learning. Number concepts and skills can be developed, enriched, and extended through the use of concrete materials.

6. It can definitely be said that intrinsic purpose and the will to learn on the part of the learner facilitate readiness to learn. Teachers can provide this type of motivation by using as fully as possible problem situations which are personal to the learner. In such situations, intelligence, maturity, and experience can make their fullest contribution to learning.

7. Meaning and understanding are significant readiness factors, for when such factors are present, the child learns more readily, retains longer what he learns, and is enabled to transfer his learning to related untaught number situations.

Since meanings and understandings are of such importance, it is necessary in evaluating readiness at any stage of learning for the teacher to ascertain how well the child understands what he is doing in arithmetic. This calls for evaluation techniques which will measure such understandings.133

8. Vocabulary and reading ability should be considered as affecting number readiness. To promote number readiness,

133For a discussion of such techniques, see Chapter IX.
the teacher needs to provide experiences so that children will extend and enrich their concepts of words. Developing skill in reading needs to be considered as part of arithmetic instruction; reading cannot be left exclusively to the so-called reading period.

Vocabulary and reading are factors of a readiness program that need to be considered particularly by the teachers of the upper elementary grades. A heavy demand is made on vocabulary and reading in meeting the problem situations at the more advanced levels.
CHAPTER III

THE CURRICULUM

This chapter will be divided into four main divisions. The first section will be devoted to the scope or content of the curriculum. An attempt will be made to present the various points of view relating to the purposes of arithmetic in the school curriculum. Research will be presented to assist one forming a sounder conclusion as to what should constitute the scope or content of the elementary school program.

In the second section the sequence or grade placement of topics in arithmetic will be considered. Much controversy has arisen as to the best and most logical place to present the various phases of the arithmetic content in the elementary school. A look at research should do much to eliminate some of the controversy and afford a sounder basis for distributing the arithmetic content throughout the elementary grades.

Curriculum patterns will constitute the content of the third section. Again, research will be presented to help one form a better conclusion as to which type of curriculum pattern has the most to offer, both in regard to the total elementary school program and in regard to arithmetic itself.

The concluding section will serve as a summary for the entire chapter. From all the research that will be presented, major conclusions will be drawn and implications for teaching
offered. This section should help one gain a more comprehensive view of the total elementary school curriculum.

**Scope of the Arithmetic Curriculum**

Scope is considered here to be the depth and breadth of content which constitutes the elementary school arithmetic curriculum. A look at the various points of view relating to the scope of the arithmetic curriculum and at the research bearing on the topic should do much to give one a much broader view and understanding of the scope of the elementary arithmetic program. The following section will point out the major concepts concerning the purposes and content of the arithmetic curriculum.

I. Major Concepts Relating to the Purposes and Content of Arithmetic

Various opinions exist as to what should constitute the scope or content of the elementary school arithmetic curriculum. These opinions are largely formed in terms of what individuals consider the purposes of arithmetic to be. From the literature on the subject of arithmetic, several pronounced views may be found as to the purposes of arithmetic in the elementary school and consequently of the content to be taught. A discussion will follow concerning these various view points or concepts.

**Disciplinary and transfer values.**--Arithmetic in the United States, especially during the eighteenth and nineteenth
centuries, was treasured and valued because it was believed that working arithmetic problems strengthened the mind. Such strengthening was considered, not only because it enabled one to work practical problems as needed in later life, but was considered valuable because it was supposed to have a big transfer effect in enabling one to do better reasoning and thinking in other areas which required a strong mind. In order to achieve the full effects from working arithmetic, the content of the arithmetic program consisted of many theoretical, puzzle type problems. An example of such a problem as given by Smith and Eaton is as follows:

When first the marriage-knot was ty'd
Between my wife and me,
Her age did mine as far exceed
As three times three does three
But when seven years, and half seven years
We man and wife had been,
My age came then as near to hers
As eight is to sixteen.
What was each of our ages when we married?¹

Consistent with the disciplinary point of view, the content of the program was selected entirely by adults. Motivation was almost entirely extrinsic.

Social utility concept.--With the recognition of the fact that arithmetic did not possess the mental disciplinary values it was once supposed to have, an effort was made to eliminate from the curriculum all the content that did not show possibility of being needed either by children or adults

In everyday life. At first, attention was centered on adult computational use of arithmetic, but was gradually broadened to include adult informational use of arithmetic as well. For the most part, the purpose of arithmetic was centered on meeting the needs of adults. In regard to the purpose of and adult needs for arithmetic, Wilson writes:

The basic and dominating aim of arithmetic in the schools is to equip the child with the useful skills for business. Business calls for decisions in terms of money. It calls for estimating and comparisons. Figuring, much of it simple, but some very complicated, is the regular accompaniment of business; arithmetic is 'the tool of business.' Yet 90 per cent of adult figuring is covered by the four fundamental processes, addition, subtraction, multiplication, and division. Simple fractions, percentage, and interest, if added to the four fundamental processes, will raise the percentage to over 95 per cent. Mastery of these essentials becomes the drill load in arithmetic for the grades. Beyond that, the work is information problem work adjusted to child interests.

In another connection Wilson places emphasis on restricting the content of the elementary school arithmetic curriculum to adult usage. He says:

. . . Common adult usage should be the limit of any arithmetic undertaken for drill mastery in the grades and general high school. Work beyond this should be recognized as different; if it is recreational in nature, it can be left to choice; if it is highly vocational, it can be left to later learning on the job by the few who need it; and if it is in the nature of encyclopedic information, it can be left in the encyclopedia for reference when needed, if it ever is.

2Guy M. Wilson, Mildred B. Stone, and Charles O. Darrymple, Teaching the New Arithmetic, p. 7.

3Ibid., p. vii.
Since adult usage is the criterion for selecting content, specific units of comparatively separate and unconnected subject matter are selected. Little attention is given to selecting content so that advantage may be taken of the relationships within the number system. Proponents of this point of view select the number skills which they consider will be needed by adults and strive to teach that content by the most direct way possible.

Immediate needs of children.--The advocates of this point of view would base the content of the curriculum on those numerical situations which arise naturally in children's on-going experiences both in school and out of school. Little recognition is given to the sequential order of number development. It is assumed that the gaps will be filled in as problems are met in recurring situations.

A description of the use and selection of arithmetic content consistent with the immediate needs of children concept is given by Harding. He points out that the proponents of the "incidental-learning theory," as he terms it, appear to:

... use as materials of instruction the activities, ideas, and questions arising in a natural or relatively undirected setting. Since there is no situation in the life of the child that is purely arithmetical, matters of number and quantity are studied as they arise in childlike situations. Number work is a casual part of activities, rather than being planned as the core or main goal of any work. The assumption is that, while there may not be any arithmetic present in some activities, other activities
will involve sufficient number experiences to provide adequate understanding and skill.  

The science of number.—One group considers the structure and organization of arithmetic as a science. It is assumed that the structure and organization of number relationships are so arranged that each phase of arithmetic is logically and inherently connected with preceding and following phases. The system itself becomes the content of the curriculum and the purpose is primarily to teach the system. Regarding the science of number and its places in the elementary school, Wheat writes:

The science of number finds logical formulation and description in the school subject of arithmetic, which is offered in the elementary school as a means of training the pupils in methods of exact thinking. Through the study of arithmetic the pupil of the public school is introduced to the most fundamental of the sciences and to one of the most outstanding intellectual achievements of the human race.  

Wheat further elaborates on the purposes of arithmetic in the elementary school by saying:

The purpose of instruction in arithmetic is to provide them with methods of thinking, with ideas of procedures, with meanings inherent in number relations, with general principles of combination and arrangement, in order that the quantitative situations of life may be handled intelligently and without doubt and uncertainty. The purpose is so to order and systematize the child's methods of dealing with combination and arrangement of objects that he may go through life freed from the necessity of confronting problems of an arithmetical nature. The purpose rests


upon the assumption that the individual has a higher function to perform in life than to expend his energy in solving what were once problems in arithmetic but are problems no longer. He must be set free from the necessity of ever having problems in arithmetic to solve.\(^6\)

Since seeing relationships inherent in the number system is considered of such great importance, it is inconsistent with this point of view to base the selection of the arithmetic content on social usage alone. When the criterion of social usage is used, content may be eliminated which would weaken important links in the sequential chain of number relationships. An intelligent grasp of the number relationships is considered more important than mere facility in mechanical computation. An understanding of such relationships enables one to see through computational processes and the individual is not left to the mercy of rote memory.

Considering that the science of number itself is held to be of such supreme importance, little attention is given to solving problems which are real to children. Problems used for practice are usually contained in textbooks. The content is almost entirely selected by adults and administered to the children through formal lessons.

**Planned functional use concept.**—This point of view includes basic ideas held in several of the foregoing theories pertaining to subject matter selection. The values of both social usage and the science of number are considered important. Arithmetic is regarded as a science and consideration

\(^6\text{Ibid.}, \ pp. \ 140-41.\)
is given to the sequential nature of number and to the relationships therein. The system, however, is not the dominating aim, but is used to serve the children as a tool in solving their own problems now and later. The first emphasis is placed on the child, his purposes and his personal development. Children's real quantitative situations are selected as the medium through which to teach arithmetic. Two procedures are frequently followed in carrying out children's purposes, and at the same time avoiding violation of the sequential nature of number. One way is through teacher-pupil planned activities such as operating a grocery store. Another way is through integrated units of work out of which grow many purposeful numerical situations on which to base the content of the curriculum. The teacher is alert to the needs of the children and is alert to see that arithmetic learning is not left purely to chance as is done in the immediate needs concept.

II. Direction from Research

The points of view discussed above relating to the selection of content for the elementary school arithmetic curriculum, indicate that there is far from agreement as to what should constitute the content or scope of the program. Research findings give some basis for selecting the content as will be shown.

Mental discipline and transfer values.—The idea that mathematics, language or some other "difficult" subjects
trained the mind was rather effectively proven incorrect by the classic experiment by Thorndike. In 1923, Thorndike\textsuperscript{7} investigated the relative contribution of the commonly taught high school subjects such as mathematics, Latin, French, history, biology, cooking, bookkeeping, shopwork, etc. to the improvement of general training or reasoning ability. About 8564 pupils, who were in grades nine, ten, and eleven in May, 1922, participated in the study. Tests were given in May, 1922, and a reexamination given in May, 1923 to the subjects participating in the study. The examinations used were alternate forms made of a composite of general intelligence tests in common use, plus certain ones added in order to have a measure with spatial as well as verbal and numerical content.

The school subjects studied during the school year from September 22, 1922 to June 23, 1923 were recorded for each pupil participating in the investigation. Gains made by the pupils on the two tests were studied in relation to the subjects studied. By equating such factors as initial ability and special training, it was possible, through a special statistical procedure, to evaluate gains in relation to each of the subjects offered in the curriculum. The relative influence of each subject in the curriculum on general ability to think and reason was thus determined.

The results indicated that language and mathematics did

not contribute any more to the general improvement in ability to think than did other so-called "easier" subjects. Thorndike concluded: "Those who have the most to begin with gain the most during the year. Whatever studies they take will seem to produce large gains in intellect." 8

In further elaboration of the findings Thorndike continues:

By any reasonable interpretation of the results, the intellectual values of studies should be determined largely by the special information, habits, interests, attitudes, and ideals which they demonstrably produce. The expectation of any large differences in general improvement of the mind from one study rather than from another seems doomed to disappointment. The chief reason why good thinkers seem superficially to have been made such by having taken certain school studies, is that good thinkers have taken such studies, becoming better by the inherent tendency of the good to gain more than the poor from any study. When the good thinkers studied Greek and Latin, these studies seemed to make good thinking. Now that the good thinkers study Physics and Trigonometry, these seem to make good thinkers. If the able pupils should all study Physical Education and Dramatic Art, these subjects would seem to make good thinkers. These were, indeed a large fraction of the program of studies for the best thinkers the world has produced, the Athenian Greeks. After positive correlation of gain with initial ability is allowed for, the balance in favor of any study is certainly not large. Disciplinary values may be real and deserve weight in the curriculum, but the weight should be reasonable. 9

Implications: mental discipline.--Relying on the evidence produced by Thorndike, one cannot justify placing arithmetic in the curriculum purely for its mind strengthening

8Ibid., p. 95.
9Ibid., p. 98.
values. This helps to rule out the content built around nonsense and impractical problems and paves the way for considering more practical and socially useful content.

**Socially useful content.**--Becoming increasingly alarmed with the highly theoretical and impractical content which made up much of the arithmetic curriculum during the nineteenth century, leaders in the field of education began to look for more practical content, and to consider the elimination of the impractical from the curriculum. The need for such elimination was suggested especially by McMurray. Increasingly an effort was made to include in the arithmetic curriculum only the arithmetic used by adults in everyday social activities.

Wilson, who appears to have been one of the foremost researchers in determining what arithmetic is actually used by adults, made a synthesis of research which had been made by himself and others to determine the socially significant arithmetic. In his own original study, Wilson used a method termed the "school-pupil-survey method." Children in the upper elementary grades and high school students were requested to report daily the figuring actually done or the numbers actually used by their parents. The returns were analyzed as to the process involved, the difficulty of the process, and the topic with which the problem dealt.

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10 Frank M. McMurray, "What Omissions Are Desirable in the Present Course of Study?" *Yearbook of the National Department of Superintendents*, 1904, p. 194.

11 Guy M. Wilson, *What Arithmetic Shall We Teach?*
Adaptations of the above procedure were used and applied to a variety of social situations by the other researchers whose studies were analyzed and summarized by Wilson. Among the investigations included in Wilson's summary were:

1. Wise's study of adult usage, covering a widely extended area (Wisconsin, Iowa, Illinois, Nevada, Missouri, Texas, and California);
2. Woody's study of arithmetic usage in wholesale and retail stores;
3. Charter's study of department store arithmetic;
4. Noon's study of the child's use of numbers;
5. Smith's study of arithmetic used by first-grade children in out-of-school life; and
6. Bobbitt's study in which he determined the informational arithmetic needed by adults by analyzing all pages of one issue of each of twenty newspapers and periodicals.

On the basis of the information provided by the many investigations made to determine what arithmetic is needed in


everyday activities, Wilson made specific suggestions pertaining to that content which should be included in and to that which should be omitted from the arithmetic curriculum.

Wilson's recommendations concerning the arithmetic that should receive main emphasis in the elementary school may be summarized briefly as follows: (1) the main emphasis in the addition of whole numbers should be limited to 1, 2, 3, and 4 place numbers with not over 5 addends; (2) emphasis in subtraction limited to 4 place minuends; (3) main practice in multiplication confined to 1 and 2 place multipliers; (4) main practice in division limited to 1 and 2 place divisors; (4) fractions, main emphasis on 1/2, 1/4, 3/4, 1/3, 2/3, 8ths, 5ths, 10ths, 12ths, and 16ths; (6) addition and subtraction of fractions concerned mainly with halves, 4ths, 3rds, 6ths, and 12ths, with these same fractions used in mixed numbers; (7) multiplication of fractions concerned mainly with 1/2 and 3/4 as 3/4 of $2.00 or 4 1/2 X $3.00; (8) very little attention to be given to division of fractions except when a real situation arises; and (9) decimals, simple work in addition, subtraction, multiplication of decimals, and dividing simple decimals by whole numbers.

Limits beyond those outlined above would come in for minor and incidental emphasis and practice. Wilson lists and recommends the following to be taught as informational arithmetic:

Informational teaching

Reading large numbers, addresses, telephone numbers, etc.
Reading decimals.
The tables of weights and measures (for reference).
Roman numerals to XII and beyond as needed.
Fractions with odd and large denominators.
Graphs and how to read them.
Blue-prints and how to read them (for brighter pupils only).
Longitude and time, standard time (correlated with geography).
Foreign moneys, ...
Squares and roots (for brighter pupils only)
Through life situations--family budget, marks of good bad investments, various kinds of investments and their merits, public expenditures, personal accounts, savings accounts, insurance expenses of travel, upkeep of an automobile, etc.
The market page of the daily paper, how to read market quotations and their meaning. ...
Calculating devices and machines used in stores and offices.13

Among recommended omissions, Wilson included such items as: carpeting; papering; puzzle problems in percentage; intricate and unusual problems in square or cubic measure; addition, subtraction, multiplication, and divisions of denominate numbers; stocks and bonds, except in investment studies.

The findings from an additional investigation by Wilson19 supplements his findings and recommendations reported above. A study by Willey20 to be reported later in this chapter brings out some of the social uses children have for

18Guy W. Wilson, What Arithmetic Shall We Teach? p. 127.


arithmetic.

Implications: socially useful content.--The type of studies referred to in the preceding section have made a valuable contribution in eliminating deadwood from the arithmetic curriculum and suggesting content of a more practical nature. By the elimination of impractical content, pupils should be able to concentrate more fully on the practical, and thus be more competent in meeting everyday problem situations.

Objections that can be made to basing the curriculum strictly on the basis of the evidence summarized above is that we are living in a changing society, and what is socially useful today is very likely to change with the times. Another objection may well be that since most of the investigations were pointed primarily at adult needs, problem situations may tend to be artificial and remote for elementary school children.

Content based on the immediate needs of children.--A significant study by Hanna,²¹ which is to be reviewed later in this chapter, indicates that when an arithmetic curriculum is based on the immediate needs of children alone, number situations do not appear in sufficient quantity to promote adequate number learning. The data he provided indicate that number content needs to be more specifically planned for.

²¹Paul R. Hanna, "Opportunities for the Use of Arithmetic in an Activity Program," Tenth Yearbook of the National Council of Teachers, pp. 85-120.
Content based on the logic of the number system.---Many investigations have shown the importance of presenting number so that advantage may be taken of the relationships in the number system. Most of those studies have been presented in more detail in other chapters of the present report, so they will be mentioned only briefly here.

Wilburn\(^22\) found that an understanding of the ten idea in the number system facilitated learning. Swenson\(^23\) showed the advantage of teaching the addition combinations in such a way that children could see relationships and form generalizations. Children so taught were significantly superior to children who had been presented the combinations in such a way that relationships were not seen. The children who saw the relationships were superior in the number of combinations learned, in retention of skills, and in the ability to work untaught addition and subtraction combinations. Findings reported by McConnell,\(^24\) Thiele,\(^25\) and Nicholson\(^26\) largely


\(^{24}\)T. R. McConnell, Discovery versus Authoritative Identification in the Learning of Children.


substantiate the findings reported by Swenson.

Relating to the sequential order of presenting skills, both Smith and Koenker found that a good experiential background in prerequisite skills was significantly associated with good achievement in two-figure division. The findings indicated that an inadequate mastery of one-figure division, multiplication, and subtraction resulted in poor achievement in two-digit division.

Inference: content based on the logic of the number system.—The evidence presented by the studies above point to the values of selecting content so that advantage may be taken of inherent relationships in the number system and of presenting skills in a sequential order so that one skill can be built logically upon the preceding skill. Even though arithmetic may be presented through a more informal approach by means of activities and experience units as will be discussed later in this chapter, it will be well for the teacher to arrange learning activities so that advantages to be derived from the logic of the number system will not be lost sight of.

Content based on planned functional use.—The significance of using real problem situations as the content for


28Robert H. Koenker, "Certain Characteristic Differences Between Excellent and Poor Achievers in Two-Figure Division," Journal of Educational Research, XXXV (April, 1942) pp. 578-86.
arithmetic is borne out by several important experimental studies. Harap and Mapes report an important research in which they attempted to determine the extent to which the fundamentals are learned in an arithmetic activity (non-integrated) program. The study was limited to the content taught in the second half of Grade V, namely, multiplication and division of fractions and denominate numbers.

The investigators summarize the conditions under which the experimental course was conducted as follows:

(1) The units were based on socially real situations or activities. (2) The situations were selected because they were rich in the use of fractions. (3) No attention was given to the order of the occurrence of the steps in fractions. (4) No external practice or drill was introduced. (5) No more than the usual time allotted to arithmetic was devoted to computation. (6) The experimental class was an ordinary class of 37 children having an average intelligence quotient of 113 on the National Intelligence Tests, Scale A, Form I. (7) The teacher was one of a group especially selected for this school because she excelled in the teaching of arithmetic.

Seventeen fundamental process steps in the multiplication and division of fractions and denominate numbers formed the basis of a preliminary and a final test. The evidence revealed that, even though the children possessed little ability in the use of the processes as measured by the initial test, the average number of steps learned by the class was

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30 Ibid., p. 519.
14.2 out of a possible 17. This achievement is equal to a mastery of 84 per cent of the processes.

The findings indicate that the learning of the steps in random order as they naturally occurred did not affect the learning process nor did the degree of mastery of the steps depend upon the number of repetitions. The investigators reached the following conclusion:

... under the conditions of good teaching, an arithmetic activity program based on real situations in school and social life incorporating the basic arithmetical steps will be mastered.31

In a later investigation which was reported in 1949, Passehl32 conducted an experiment similar to the Harap and Mapes study just reported. The investigation was carried out with sixth grade children in the learning of addition, subtraction, multiplication, and division of common fractions. Throughout the study emphasis was placed upon real problem situations in school and in life. The results were found to be consistent in every important detail with those reported by Harap and Mapes.

Additional research which has shown the significance of basing arithmetic content on problem situations which are real to the learner has been reported by Harap and Barnett.33

31ibid., p. 525.


Harding and Bryant,\textsuperscript{34} Conner and Hawkins,\textsuperscript{35} White,\textsuperscript{36} and Williams.\textsuperscript{37}

Conclusions: content based on planned functional use. -- The conclusion that may be drawn from the findings reported above is that learning is facilitated when children are dealing with content which is basic to carrying out their own ongoing life experiences. To utilize fully such numerical situations, the teacher must be aware of the real quantitative problems children are meeting, and further, the teacher must be alert to utilize those experiences. Without being fully alert to the quantitative situations children are meeting, and planning to use those situations to the fullest, number will very likely not appear in sufficient quantity to supply all of the content for the elementary school arithmetic curriculum. This possibility is especially pointed out by Hanna.\textsuperscript{38}

Even though the content of the elementary arithmetic curriculum


\textsuperscript{36}Helen M. White, "Does Experience in the Situation Involved Affect the Solving of a Problem?" \textit{Education} (April, 1934) pp. 451-55.


\textsuperscript{38}Hanna, \textit{op. cit.}
is being selected from children's real problems, the teacher needs to be alert to the logic of the number system so that advantage may be taken of the relationships inherent in the number system and of the sequential nature of number.

An interesting and profitable study could be made to determine to what extent the number needs of adults are automatically met through a curriculum based on content selected to meet the quantitative needs of children. It seems sensible to suggest that arithmetic content be selected to both meet the needs of children now as well as contributing toward helping children meet their later adult needs.

Vocabulary a part of content.—Vocabulary should be an important part of the content of arithmetic. Buswell and John,39 and Harrison40 have pointed out the wide array of mathematical terms and signs with their many meanings that are demanded in coping with mathematical situations. The significance of giving special attention to the development of vocabulary was revealed by Johnson,41 who compared the achievement of an experimental and a control group of junior high school pupils. It was shown that the pupils who had had special vocabulary experiences were superior to children in


40Irene Gertrude Harrison, Survey of Words and Signs of Two Arithmetic Textbook Series.

the control group who had not had such experiences. This finding was found to be true for children of practically all levels of mental ability and initial status in arithmetic achievement.

Revealing data were presented by Treacy\textsuperscript{42} who compared eighty good achievers with eighty poor achievers in problem solving in arithmetic. Among other findings, that investigator found that good achievers were significantly superior to poor achievers in four reading skills associated in one way or another with vocabulary. On the basis of the data Treacy suggested the importance of stressing the meaning of terms, both general and mathematical, as an approach to improving ability in problem solving.

III. Summary and Implications for Content

The present section has been concerned with the scope or content aspect of the curriculum. Several distinct concepts were presented concerning the content of arithmetic in the elementary school. A look at the disciplinary concept revealed that arithmetic was valued for its mind strengthening qualities, and accordingly, the content was selected for the contribution it could make toward strengthening the mind. Relative to the social utility concept, it was shown that arithmetic is looked at from a more practical point of view. An

effort is made to select that content needed by children and adults in meeting their societal needs. In considering the science of number concepts, it was indicated that due to the great emphasis placed on not violating the internal relationships in the number system, the system itself largely becomes the content. The planned functional use concept emphasizes the importance of basing the content on children's real problem situations.

A look at research did much to show the degree of validity of the various concepts. Thorndike\textsuperscript{43} showed that arithmetic did not have the mind strengthening qualities it was once believed to have. Studies made by Wilson\textsuperscript{44} and others helped to determine more adequately the aspects of number needed in meeting the everyday number demands of man. Swenson,\textsuperscript{45} Thiele,\textsuperscript{46} Nicholson,\textsuperscript{47} and others have revealed the values to be gained by stressing the relationships in the number system. Those values are lost when number skills are taught out of relation to each other. Findings by Wilson\textsuperscript{48} and others show that man needs number for informational purposes and not for computational purposes alone. Buswell and

\begin{footnotesize}
\begin{enumerate}
\item \textsuperscript{43}Thorndike, \textit{op. cit.}
\item Guy M. Wilson, \textit{What Arithmetic Shall We Teach?}
\item Swenson, \textit{op. cit.}
\item Thiele, \textit{op. cit.}
\item Nicholson, \textit{op. cit.}
\item Wilson, \textit{op. cit.}
\end{enumerate}
\end{footnotesize}
John,^49 Harrison,^60 and Treacy^51 reveal the importance of considering number vocabulary as a significant and essential part of the arithmetic content to be included in the elementary school curriculum. Finally, Harap and Mapes,^52 Harding and Bryant,^53 Williams,^54 and Conner and Hawkins^55 show the values to be derived from selecting content based on children's own on-going experiences.

In view of the evidence presented it seems prudent to say that the arithmetic curriculum is best when it is geared to facilitating children in meeting their quantitative experiences now and later. Children's real problems can serve as the medium through which to teach arithmetic. It is essential that content be so selected and presented that important relationships in the number system are not lost sight of. Even though teachers need to keep in mind the arithmetic skills needed by adults in so far as those can be determined, those skills cannot always be presented and used with children in the way that adults use them. Research evidence presented in Chapter II dealing with readiness shows that children grow to

^49 Buswell and John, op. cit.
^50 Harrison, op. cit.
^51 Treacy, op. cit.
^52 Harap and Mapes, op. cit.
^53 Harding and Bryant, op. cit.
^54 Williams, op. cit.
^55 Conner and Hawkins, op. cit.
adult competency by first using crude and immature procedures. This means that skills needed by adults cannot be determined and then taught to children by the most direct means possible. Since we need to consider relationships in the number system, we cannot select adult skills and teach them in isolation.

Surveys made of adult usage can be used to advantage in helping to eliminate much of the deadwood from the curriculum. Much of this can be eliminated without in any way endangering the values to be derived from treating number as a science.

Grade Placement of Content in Arithmetic

By grade placement of content in arithmetic is meant the placing of the various phases of arithmetic subject matter in the elementary grades where they can be learned most effectively. Many factors such as mental maturity, social demand, and difficulty of arithmetical processes have been used as criteria in determining where the various topics in arithmetic should be taught. There is much disagreement among educators as to the best procedure in distributing arithmetic content among the grades. A look at the several theories concerned with the grade placement of topics in arithmetic should prove helpful.

I. Theories Relating to the Grade Placement Of Content in Arithmetic

Several pronounced theories are in evidence concerning
the placing of topics in the teaching of arithmetic. Two extreme views are reflected in what are sometimes called the "stepped-up" curriculum and the "stretched-out" curriculum. A discussion will follow pertaining to these two extreme views and also relating to views which may be considered to fall between the two major views.

The stepped-up curriculum.--In the "stepped-up" curriculum topics are presented at the grade level at which they can be taught to completion. The idea of completion implies the degree of mastery equivalent to efficient adult performance. Since it is assumed that children learn directly without going through crude intervening steps, no additional teaching is planned for other than periodic reviews to maintain skills. In this concept of placing topics, it is considered that mental maturity is the primary requisite for arithmetic readiness. The successful mechanical performance of preceding skills is thought necessary in order to master succeeding skills, but little attention is given to the meaning and understanding of the processes. Although social usage is considered as it is applied to adult usage of number, little recognition is given to the immediate needs for number which the children themselves may have.

The stretched-out curriculum.--Basic to the "stretched-out" curriculum is the belief that children learn gradually from crude beginnings to an ever increasing level of mastery and understanding. Consequently, topics are presented over
a period of several years before a reasonable degree of mastery is expected. The degree to which topics are taught depends upon the readiness of the children to learn. A broad comprehensive view is taken of readiness. Recognition is given to the big contribution of many factors influencing readiness, namely, mental and physical maturation, experience, I.Q., vocabulary, and the degree to which number has meaning for the individuals involved. Also, recognition is given to the social needs which children have for number in solving their own quantitative problems. Emphasis may be given to a particular topic earlier or later, depending on the need the child may have for learning the topic.

Several other theories may be placed on a continuum, with the "stepped-up" curriculum and the "stretched-out" curriculum at the two extremes. Some are based almost completely on the basic principles of the "stepped-up" curriculum, while others are more in line with the concepts of the "stretched-out" curriculum.

Unitary treatment.—"Unitary treatment" is a term used by Wilson. It means that as each fundamental operation is taught, all of the beginning, intermediate, and advanced stages in the fundamental operation are taught, one right after the other, so that the entire operation is taught on one grade level. For example, in teaching addition, a group of ten primary facts are taught, followed in succession by the

56Wilson, Stone, and Dalrymple, op. cit., p. 38.
more advanced addition skills, until the whole process of addition through all of its stages have been taught. The more complicated phases of addition as well as the simple phases are taught to complete mastery on a given grade level without postponing the more advanced stages to higher grade levels. The more difficult addition combinations are frequently taught before the easier combinations on the assumption that the harder combinations will require more drill. Wilson states in this connection: "There is some advantage in bringing hard facts into the drill early, in order that the hard facts may have more drill." 57

A similar approach is taken in teaching subtraction, multiplication, and division. Pupils are expected to master processes completely before going to new topics.

Telescoped reteaching.—This view is based on the belief that even though phases of topics are taught to completion in a given grade, it is not to be expected that children will have completely mastered them, so that more than a mere review to maintain skills is required. Morton 58 uses the term "telescoped reteaching" in referring to this point of view. Concerning the teaching of skills again which children have been taught the previous year, Morton asserts:

They [pupils] need a chance to learn or re-learn. They need also an opportunity to learn more

57 Ibid., p. 107.

than they have previously learned, to give a greater degree of understanding of numbers and the processes with numbers. They need a program which we are here calling "telescoped reteaching".

... We say "reteaching" because there is an abundance of evidence supporting the contention that much learning is at first partial and incomplete; that pupils differ greatly in what they acquire from their experiences in previous years; that there is considerable forgetting, especially during the long summer vacations; and that more mature degrees of understanding become possible as the pupils become more mature.

We say "telescoped" because, obviously, the reteaching can be briefer and less detailed than was the first teaching. The reteaching of a topic common to all of these grades, such as addition, will be briefer in Grade 4 than in Grade 3, briefer in Grade 5 than in Grade 4, briefer in Grade 5.

Spiral overlap.--Rather than reteaching the work of the previous grades at the beginning of the year and then going ahead with the work of the new grade for the remainder of the year, in the spiral overlap concept of grade placement, topics are returned to at varying intervals throughout the year. Unlike unitary treatment of topics where all phases of addition are taught at once, in spiral overlap topics are treated at varying intervals in an ever expanding way throughout a given school year and throughout the following years. Perhaps during one year the addition facts are taught, and during the following year the child will refine his mastery of the facts as he at the same time expands his acquaintance with addition by learning to do column addition and other processes in addition. Throughout the elementary school years,

59 Ibid., p. 227.
the child, by returning to addition at varying intervals, gradually achieves adult competency in using the skill. The idea in spiral overlap is that each of the operations (addition, subtraction, multiplication, and division) are in turn developed to a given point, left for a time while something else is taught, picked up again and developed to a further stage of refinement.

Saturation.--The term "saturation" is used by Harding, meaning that all topics in arithmetic are presented on every grade level, with the children on each grade level learning as much of each topic as they can. Harding has identified twenty-three topics in the teaching of arithmetic, and he illustrates pictorially and verbally how children from kindergarten through the eighth grade gradually acquire understanding and use of number in an ever increasingly mature way.

Consistent with the saturation point of view, some topics are stressed and treated somewhat more systematically at times than are others, but situations present themselves almost daily wherein increased growth in the use of all topics is made possible. This being true, it is not necessary to shelve some phases of arithmetic while other phases are receiving the limelight.


61Ibid., pp. 44-196.
II. Conditions Affecting Grade Placement

With such diverse points of view, all those concerned with the grade placement of arithmetic content are certain to meet confusion as they try to make decisions regarding this problem. Research dealing with the problem has been done from several perspectives as will be shown.

Mental age.--The most extensive investigations in the area of grade placement have been the experiments conducted by the Committee of Seven of the Northern Illinois Conference on Supervision under the leadership of Carleton Washburne.62 These investigations consisted of controlled, cooperative experiments in 255 cities and towns in sixteen states. At least 1190 teachers and 30,744 children were involved. The committee sought to determine the mental age at which three-fourths of the children could show 80 per cent mastery for the various topics to be taught in arithmetic. In order to find out when the various topics in arithmetic should be taught, the committee tried to determine the mental age at which three-fourths of the children could show 80 per cent mastery as determined by a retention test given several weeks after the final teaching of the topic. Initial testing included intelligence tests to determine mental age, pretests to determine existing knowledge of the topic to be taught, and tests to discover whether or not the children had

the prerequisite knowledge and skill, and in some cases the prerequisite experience and concepts for learning the new topic.

In locating the grade at which topics could be taught most advantageously, some schools taught a topic at the customary grade level, with other schools teaching the topic one grade lower, or one or two grades higher. After the designated teaching time for each topic, a final test was given to determine the immediate learning of the children. A retention test was given six weeks later. Recommended grade placement for the various topics was made on the basis of the scores made on the retention test.

Following are summarized briefly some of the recommendations made by Washburne for the grade placement of topics according to mental age levels. For a full list and discussion of the many recommendations made, the reader should refer to the original source.63

1. Mental Age 6-7. (1) Addition and subtraction facts with sums of ten and under; (2) this year and possibly the next should be devoted largely to informal experiences and activities to give children real concepts of numbers and space relations without any systematic drills.

2. Mental Age 7-8. (1) Addition facts with sums of 10 and under; (2) make simple comparisons of sizes and make simple measurements.

63 Ibid., pp. 309-16.
3. Mental Age 8-9. (1) Multiplication facts with products of 20 and less; (2) understand standardized time units, such as minutes, hours, and days.

4. Mental Age 9-10. (1) Column addition with columns not more than three digits high and three digits wide; (2) simple multiplication involving no partial product over 20; (3) easy division facts with dividends of 20 and less; (4) the meaning of simple common fractions.

5. Mental Age 10-11. (1) Column addition four digits high and three digits wide; (2) meaning and use of simple decimals; (3) addition and subtraction of fractions and mixed numbers with like denominators.

6. Mental Age 11-12. (1) Multiplication facts with products over 20 mastered; (2) all division facts completed; (3) long division with a two-place divisor and a one-place quotient; (4) division of decimals by integers; (5) multiplication of decimals.

7. Mental Age 12-13. (1) Long division with a two-place quotient involving naught, remainder, and trial divisor difficulties; (3) multiplication and division of fractions.

8. Mental Age 13-14. (1) Long division with a three-place quotient; (2) in linear measure, division of feet and inches by a whole number when both feet and inches are evenly divisible, and division of yards by inches.

and, in the case of subtraction of mixed numbers, involving borrowing; (2) the manipulation of denominate numbers, involving the more complicated forms of applying the four fundamental processes to them and involving the changing of denominators.

In support of the findings and recommendations of the Committee of Seven were the results of a study made by Johnson. Johnson endeavored to determine the effects of using a curriculum based upon the Committee of Seven recommendations under the typical situation as found in the common run of Chicago schools. A course of study was introduced in 1937 in which the skills were placed so as to agree as closely as possible with the recommendations set forth by the Committee of Seven.

Three equivalent forms of a survey test which covered all of the facts and skills in arithmetic were used in 1938, 1939, and 1940 with children in grades 3B through 8A. The number of children tested for the successive years was 25,000, 30,000, and 20,000 respectively. The group tested in 1938 was considered the control group, since this group had been under the traditional program for a number of years. The results of the 1940 test were considered to measure the experimental group after using the new grade placement schedule for two years.

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It was found that the average gain in improvement throughout grades 3B to 8A for the two year period (1939, 1940) was over 21.2 per cent above the status of the grades in 1938.

In interpreting the results of the study, Johnson points out that since mental ages of from 9 through 13 years are represented in the average fifth grade, the teaching of the past had been suited to the two upper groups, the 12 and 13 year olds mentally, or the upper 30 per cent of the class. By moving the requirements up about a year to the level of the 11 year olds mentally, suitable instruction is provided for an additional 40 per cent of the class or 70 per cent in all.

As a conclusion, Johnson says:

In conclusion may it be said that despite the recognized limitations in the experiment of the Committee of Seven, the total effects of readjusting a course of study in accordance with the Committee of Seven recommendations is a substantial gain in arithmetic efficiency, resulting in the curriculum fitting about 70 per cent of the children to the work instead of about 30 per cent.65

Several investigations have given findings which indicate that topics can be taught at an earlier mental age than that recommended by the Committee of Seven. Grossnickle66 conducted an experiment on the fourth grade level in which he specifically tried to determine the relative merits of two methods of estimating the quotient when a two-figure divisor

65 Ibid., p. 173.
is used. Two groups of fourth graders (216 pupils in one group and 233 in the other) were equated on the basis of intelligence, ability in subtraction, multiplication, and ability in the use of a one-figure divisor in long division. One group was taught division by the "apparent method" while the other group used the "increase-by-one method". Although no significant differences were found relative to the merits of the two methods, it was found that both groups showed evidence of having learned division with a two-figure divisor. The groups made an average retention test score of about 96 per cent. On the basis of this evidence, the investigator concluded that division of this type could be taught in the last half of the fourth grade when the average mental age was found to be ten years and three months. This was in contradiction to the standard of twelve years and seven months recommended by the Committee of Seven.

Experience.--A significant contribution related to the professional understanding of grade placement was made by Moser. In his investigation great emphasis was placed upon the importance of experience as a factor in grade placement of arithmetic content as opposed to the mental age factor which had been stressed so greatly by the Committee of Seven.

With the social environment as the experimental variable, Moser endeavored to reappraise the relative contributions

of maturation as measured by mental age, and of environment, as influenced by the educational experiences provided. Fractional grouping and computation was the subject matter of that investigation.

The original investigation was conducted in the second grade of the Linda Lee Tall School, the campus laboratory school for Maryland State Teachers College at Towson, Maryland. Thirty-three children participated in the original study. The investigation was conducted on the second grade level because it was on this level that children could be used for subjects who had not had previous systematic instruction in fractions. Further, the children in the second grade had mental ages far below the level recommended by the Committee of Seven for the teaching of fractions.

The method employed for teaching fractional concepts was that of permitting children to discover fractional understandings and relationships through manipulating concrete objects. After understandings were acquired on a sub-symbolic level, the children were gradually encouraged to use abstract symbols in solving problems. Fractional symbols and fractional exercises were introduced through the medium of number stories. Emphasis throughout the instruction was placed on meaning and understanding as opposed to rote mechanical learning. The relationship between grouping with fractions and grouping with whole numbers was stressed throughout. Once children indicated that they could obtain solutions on the abstract level, instruction was stopped.
An End Test, a Post-Experimental Test, and a Retention Test were given to obtain three measures, namely, (1) achievement, (2) ability to transfer principles learned to untaught situations, and (3) ability to retain learnings in the absence of further instruction. The data revealed that 73 per cent of the children had more than 80 per cent of the test items correct on the End Test. This same standard was reached by 88 per cent of the children on the Post-Experimental Test. Ninety-one per cent of the children reached the standard of 80 per cent achievement on the Retention Test, given sixteen weeks after instruction had ceased. The ability to transfer taught principles to untaught situations was considered highly significant with the efficiency in the new situation being over 90 per cent of that while working in taught situations.

The findings were in sharp contrast to the recommendations of the Committee of Seven in regard to the mental age at which children can learn fractions. It was revealed that nineteen of the thirty-three children successfully learned skills in fractions two years before the mental age recommended by the Committee of Seven. These skills were learned by twelve other children one year before the level recommended by the Committee.

Following very much the same procedure as used in the Linda Lee Tall School, the experiment was repeated in three rural centers in Durham County, North Carolina, and in a new
class under a different teacher in the Linda Lee Tall School. Data were thus available for a total of 153 second grade children. Except for minor variations, the findings from these repeated studies largely substantiated the findings from the original study.

The investigator concluded that the optimal mental age for teaching fractional processes is not independent of the learning experience. This is especially true when the teaching is related to previous experiences with whole numbers and when the teaching of the abstract processes is preceded by special manipulatory activities. Moser further concluded that learning based on meaningful experience is resistant to extinction and that such learning actually improves in the majority cases.

Helping to give a more comprehensive view regarding the factors which determine when content should be taught, are the studies by Smith,68 and by Koenker,69 which were summarized in detail in the previous chapter which dealt with arithmetic readiness. These investigators gave evidence to show that experience, mental maturity, and I.Q. are each significant factors determining when content can best be taught.

Significance: mental maturity and experience.--One should avoid taking an either or attitude toward considering mental maturity or experience as factors in determining when

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68 Smith (Lawrence Joseph), op. cit.

69 Koenker, op. cit.
arithmetic can and should be taught. Both factors play a significant role in providing readiness for number learning. One factor, considered in the absence of the other, cannot make its fullest contribution to learning. As suggested in Chapter II, "Arithmetic Readiness," it seems logical that social demand should determine whether to place emphasis on experience or whether to wait for the advantages to be derived from increased maturity. The role of social demand in influencing grade placement of arithmetic content will be considered later in this chapter.

Regardless of any weaknesses which may be pointed out concerning the Committee of Seven studies, they have made a highly important contribution to the grade placement of arithmetic topics. The result has been that content has been moved up in the curriculum to a level where a higher percentage of children can deal with it more effectively. Evidence of this value has been presented by Johnson. On the other hand, Moser has made a significant contribution in showing that teachers can do much to promote readiness. It is not necessary to sit by and wait for readiness to take place of its own accord.

Beginning arithmetic instruction.—Due to the emphasis placed on teaching topics in arithmetic to completion and mastery, there was a general movement of topics up in the curriculum.}

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70 Johnson (J. T.), op. cit.
71 Moser, op. cit.
curriculum to the level where such mastery could take place. As a result of this movement, there began a trend toward the elimination of systematic instruction in arithmetic from the curriculum in the early primary grades.

One of the best known studies providing data in support of arguments for postponing instruction in arithmetic was the experiment conducted by Benezet. The results of using two types of arithmetic programs for the elementary grades were compared. In one group of schools the traditional drill method based on textbook problems was used from the very beginning, while in another group of schools formal arithmetic was not taught for the first five years. The experimental schools (no formal arithmetic) devoted their main efforts to teaching the children to read, to reason, and to recite.

Beginning gradually in 1929, the new program was first introduced in five rooms--three third grades, a combined third and fourth grade, and one fifth grade. The experimental programs were extended, so that by the fall of 1932 about one-half of the third, fourth and fifth grade rooms in the city were working under the new curriculum. The experimental schools began using textbooks in grade six.

Much informal testing and observation was made by Benezet himself. A more formal evaluation of the program was made by Benezet himself. A more formal evaluation of the program was

made under the direction of Guy Wilson of Boston University. Ninety-eight sixth grade children from the experimental program and 102 from the traditional groups were tested. On the earlier computational test, the traditional group excelled. By April the two groups were about equal. When the last test was given in June, it was found that one of the experimental groups led the city.

From his personal observation, Benezet found that the experimental groups were superior in ability to do critical thinking and in oral and written expression. The experimental groups developed more interest in reading. Benezet concluded that formal instruction in arithmetic could well be left until the sixth grade or later.

This study is highly suggestive of the role of maturational in learning. However, as one examines the curriculum outline as given by Benezet for the experimental groups, one finds evidence of much provision for the children to learn arithmetic functionally. Provision was made for the children to do purposeful counting, to develop vocabulary and number concepts, to tell time, to use money, and to use various measuring instruments. Games were used involving the use of number. When both Thiele and Buswell studied the experiment, they found much provision for systematic instruction in

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73c. Louis Thiele, "An Incidental or an Organized Program of Number Teaching?" *Mathematics Teacher*, XXXI (February, 1938) pp. 63-7.

the curriculum for the experimental groups.

Many investigations have been devoted to showing that first grade and kindergarten children do profit significantly from a systematic instructional program. One of these studies was made by Koenker, who attempted to determine the value of a rich arithmetic readiness program at the kindergarten level.

The investigation was conducted in two similar elementary schools, both of which had morning and afternoon kindergarten sections. In each school, both kindergarten sections were taught by the same teacher. One section in each school was called the experimental group, while the other section was called the control group. After giving an initial readiness and intelligence test, it was found that the average readiness score and the average I.Q. of the children in the two schools were practically equal.

The regular kindergarten program, which made little provision for number experiences, was used with the control group. In addition to the regular kindergarten program, the two experimental groups were given a rich arithmetic readiness program. Through realistic activities, the children were given number experiences including counting and grouping, measuring and weighing, using money, keeping records, telling time, reading numbers, and developing number vocabulary.

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When the experiment was concluded at the end of the year, twenty-seven children from each group were equated on the basis of intelligence and the initial readiness scores. The readiness test was repeated at this time. The differences in gains between the two groups was highly significant and favored the experimental group.

Koenker reached the following conclusions:

1. The kindergarten children in this study profited from a rich and meaningful arithmetic readiness program.

2. The kindergarten children in this study showed a great deal of interest in number readiness activities based on their needs and experiences.

3. The two teachers in this study were of the opinion that an arithmetic readiness program at the kindergarten level was of great value.76

Findings from other studies showing that kindergarten and first grade children do profit from a systematic program in arithmetic have been consistent with the findings reported by Koenker. Such research has been reported by Brownell,77 Gunderson,78 Hooper and Stratton,79 Mott and Martin,80 and

76Ibid., p. 223.
77William A. Brownell and others, Arithmetic in Grades I and II: A Critical Summary of New and Previously Reported Research.
80Sina Mott and Mary Elizabeth Martin, "Do First Graders Retain Number Concepts Learned in Kindergarten?" Mathematics Teacher, XL (February, 1947) pp. 75-78.
Wilburn. 81

Social demand, a clue to grade placement.--Helping to substantiate the idea that instruction in arithmetic should have a place in the early primary grades, have been investigations which show that young children do have a social need for learning arithmetic. Such research further indicates the social demand for arithmetic at the various grade levels and should afford another clue for grade placement.

Willey 82 made such an investigation when he endeavored to determine what number experiences arise in the life of the child from spontaneous and natural situations. Such problem situations were recorded as observed by urban, rural, and cadet teachers in Santa Clara County, California. The teachers were asked to record the exact computation as it was used by the pupils for an eighteen weeks period. Problems thus used were recorded for children enrolled in kindergarten through grade six.

For treatment of most of the data, kindergarten, and grades one and two were considered as Group I; grades three and four as Group II; and grades five and six as Group III. A total of 639 number problems were recorded for Group I; 658 for Group II; and 1,187 for Group III. This was a total of 2,484 number problems for all grades combined. Table I indicates the percentage distribution among the processes as

81 Wilburn, op. cit.
82 Willey, op. cit.
used by each of the three grade groups.

TABLE I
PROPORTION AMONG PROCESSES USED IN PROBLEMS
BY GRADE GROUPS

<table>
<thead>
<tr>
<th>Process</th>
<th>Group I</th>
<th>Group II</th>
<th>Group III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting</td>
<td>52.88</td>
<td>8.97</td>
<td>5.08</td>
</tr>
<tr>
<td>Common fractions</td>
<td>4.29</td>
<td>11.19</td>
<td>18.48</td>
</tr>
<tr>
<td>Subtraction</td>
<td>6.45</td>
<td>18.95</td>
<td>11.50</td>
</tr>
<tr>
<td>Denom. no.</td>
<td>4.45</td>
<td>11.58</td>
<td>12.62</td>
</tr>
<tr>
<td>Division</td>
<td>1.92</td>
<td>9.95</td>
<td>14.07</td>
</tr>
<tr>
<td>Multiplication</td>
<td>2.97</td>
<td>9.46</td>
<td>11.37</td>
</tr>
<tr>
<td>Mensuration</td>
<td>8.55</td>
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<td>7.75</td>
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<td>Addition</td>
<td>8.00</td>
<td>11.25</td>
<td>5.59</td>
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<td>Decimal fractions</td>
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<td>7.32</td>
<td>9.61</td>
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<tr>
<td>Reading and writing no.</td>
<td>9.94</td>
<td>1.40</td>
<td>1.17</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>.38</td>
<td>.63</td>
<td>1.10</td>
</tr>
<tr>
<td>Per cent and average</td>
<td>.00</td>
<td>0.50</td>
<td>1.66</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.00</strong></td>
</tr>
</tbody>
</table>

Adapted, Willey, Ibid., p. 361.

Willey summarizes the data concerning grade level and its relation to the processes used as follows:

(a) Counting is used most by kindergarten, first and second grade pupils.
(b) Common fraction are used most by fifth and sixth grades.
(c) Subtraction is used most by the third and fourth grades.
(d) Denominate numbers are used equally by the two upper grade groups (Group II and III).
(e) Division is used most by pupils in the fifth and sixth grades.
(f) Multiplication has most usage in the fifth and sixth grades.
(g) Mensuration receives equal emphasis in all grade groups.
(h) Addition is used most by the third and fourth grades.
(i) Decimal fractions are used about equally in the third and fourth grades and in the fifth and sixth grades.
(j) Reading and writing of numbers, when considered as a problem in itself is most used by the kindergarten, first and second grades.\textsuperscript{83} From the data presented it can be concluded that kindergarten and first grade children indicate a need for number in handling quantitative situations which arise naturally from their on-going activities. Also, even though actual practice indicates that certain processes are used more in some grades than in others, most of the processes are used to an extent on every grade level. In order to help children have greater facility in meeting their problem situations, it may be concluded that most, if not all, processes need to be given some attention on all grade levels.

Many other reported investigations substantiate Willey's findings that young children do have a social need for learning arithmetic. Among such investigations are those by Culver,\textsuperscript{84} Reid,\textsuperscript{85} and Smith.\textsuperscript{86}

Pointing out further that young children have already acquired much facility with number when they enter school and are consequently ready to continue learning arithmetic, is the research summarized relating to this point in Chapter II, "Arithmetic Readiness". Two principal studies showing

\textsuperscript{83}Ibid., p. 361


\textsuperscript{85}Florence E. Reid, "Incidental Number Situations in First Grade," Journal of Educational Research, XXX (September, 1936) pp. 38-43.

\textsuperscript{86}Smith (Nila B.), \textit{op. cit.}
that children have acquired much arithmetic ability when they enter school are those by Buckingham and McLatchy,\textsuperscript{87} and Brownell.\textsuperscript{88}

\textbf{Implications: beginning instruction in arithmetic.---} The evidence points rather conclusively that children entering first grade are ready for instruction in arithmetic. They have acquired some facility in the use of number before entering school, they have a special need for learning arithmetic, and indications are that children in the early primary grades do profit from instruction in arithmetic. This does not mean that they are ready to learn arithmetic processes to completion, but they can develop number concepts, and gain facility in using immature procedures in finding solutions to problems which arise. Children profit from instruction when instructional procedures make use of concrete materials and realistic problem situations which arise in the social environment of the child.

\textbf{How children learn arithmetic.---} Much reported research evidence reveals that learning does not take place all at once. The implication is that the expectation of mature mastery of processes immediately or in a single grade level is illogical.


\textsuperscript{88}William A. Brownell and others, Arithmetic in Grades I and II: A Critical Summary of New and Previously Reported Research.
Brownell and Carper\textsuperscript{39} have indicated the stages through which children pass in learning the multiplication combinations. In separate studies, Brownell\textsuperscript{90} and Carper\textsuperscript{91} identified the stages through which children pass in apprehending the number of objects in a group. In another investigation, Brownell and Chazal\textsuperscript{92} showed that drill for automatic response does not eliminate less immature procedures for dealing with number, but merely causes children to become more efficient in the procedure they are already using. The advantages of using concrete manipulative materials as an immature procedure in dealing with number before attempting to master abstract algorithms have been demonstrated by Moser,\textsuperscript{93} Deans,\textsuperscript{94} Wilburn,\textsuperscript{95} and others. The use of a crutch in helping children to progress from immature procedures to more mature

\textsuperscript{39}William A. Brownell and Doris V. Carper, Learning the Multiplication Combinations.

\textsuperscript{90}William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.


\textsuperscript{93}Moser, op. cit.


\textsuperscript{95}Wilburn, op. cit.
procedures has been demonstrated by Brownell. 96

One of the pioneer researches showing that learning is a gradual process requiring several years to reach the adult level of mastery was made by Judd. 97 In a series of experimental situations, Judd revealed the developmental stages through which pupils progress in learning to count, a process which is generally considered to be the easiest in dealing with number. Adults were used in some phases of the experiments, while children were used in others. In most of the situations, the subjects were asked to count series of sounds or series of flashes of light. The speed at which the sounds or flashes of light were presented was controlled by special laboratory apparatus designed for this purpose.

Table II is revealing in that it shows the progressive ability of children to count series of sounds from grade one through grade six. Table III shows the ability of pupils in grades one through six in counting flashes of light.

In regard to the data presented in Table II, Judd says:

Throughout the period of elementary schooling, pupils improve in ability to count series of sounds. First-Grade pupils know the number names and can use them with some degree of success when the number of sounds to be counted is small, but they can deal successfully only with sounds that are given at a very slow rate. By the time pupils reach the third grade, they may be said to have become fully proficient in dealing with the slow rates. By the


97 Charles Hubbard Judd, Psychological Analysis of the Fundamentals of Arithmetic.
time pupils reach the fifth or sixth grade, some of them exhibit a degree of proficiency comparable to that shown by adults.\textsuperscript{98}

### TABLE II
TOTAL NUMBER OF ERRORS MADE BY TWENTY PUPILS IN EACH GRADE IN COUNTING SERIES OF SOUNDS

<table>
<thead>
<tr>
<th>Number of Sounds Given Per Second</th>
<th>Grade</th>
<th>Two</th>
<th>Three</th>
<th>Four</th>
<th>Five</th>
<th>Six</th>
<th>Seven</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>56</td>
<td>75</td>
<td>76</td>
<td>100</td>
<td>130</td>
<td>136</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>30</td>
<td>37</td>
<td>61</td>
<td>86</td>
<td>120</td>
<td>114</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>15</td>
<td>19</td>
<td>35</td>
<td>76</td>
<td>93</td>
<td>112</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>10</td>
<td>17</td>
<td>36</td>
<td>54</td>
<td>75</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>7</td>
<td>25</td>
<td>32</td>
<td>43</td>
<td>62</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>VI</td>
<td>5</td>
<td>4</td>
<td>15</td>
<td>35</td>
<td>44</td>
<td>64</td>
</tr>
</tbody>
</table>

\[\text{Judd,}\ \text{Ibid.},\ \text{p. 39.}\]

Concerning the developmental progress in ability to count light flashes, Judd writes:

Here, again, steady progress throughout the elementary-school period is shown, but this progress is far behind that shown. . . for counting sounds. In this case, the fifth-grade pupils were less proficient by far than were the adults. Apparently, development in this kind of counting is not only less complete for all individuals but much slower than is development in counting sounds.\textsuperscript{99}

From this evidence it may be concluded that even with a simple skill like counting, mature competency is not obtained in a short period of time, but instead several years are required.

\[\text{\textsuperscript{98}Ibid.},\ \text{p. 39.}\]

\[\text{\textsuperscript{99}Ibid.},\ \text{p. 40.}\]
TABLE III
TOTAL NUMBER OF ERRORS MADE BY TWENTY PUPILS IN EACH GRADE IN COUNTING SERIES OF FLASHES OF LIGHT

<table>
<thead>
<tr>
<th>Grade</th>
<th>Two</th>
<th>Three</th>
<th>Four</th>
<th>Five</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>75</td>
<td>86</td>
<td>124</td>
<td>...</td>
</tr>
<tr>
<td>II</td>
<td>51</td>
<td>79</td>
<td>121</td>
<td>148</td>
</tr>
<tr>
<td>III</td>
<td>33</td>
<td>65</td>
<td>96</td>
<td>134</td>
</tr>
<tr>
<td>IV</td>
<td>23</td>
<td>55</td>
<td>107</td>
<td>143</td>
</tr>
<tr>
<td>V</td>
<td>20</td>
<td>65</td>
<td>89</td>
<td>127</td>
</tr>
<tr>
<td>VI</td>
<td>9</td>
<td>47</td>
<td>92</td>
<td>122</td>
</tr>
</tbody>
</table>

Judd, Ibid., p. 40

Another study which shows that specific skills need to be considered on several grade levels was made by Brueckner and Melbye. These investigators indicate that long division needs to be taught on several grade levels rather than trying to teach the process to completion in a given grade.

That investigation, the purpose of which was to determine the relative difficulty of the various elements in long division with two-place divisors, was made at the University of Minnesota. The method used in teaching long division was the increase-by-one method for all divisors ending in 6, 7, 8, and 9. Data were available for 474 pupils in grades 5B through 6A.

Leo J. Brueckner and Harvey O. Melbye, "Relative Difficulty of Types of Examples in Division with a Two-Figure Divisor," Journal of Educational Research, XXXIII (February, 1940) pp. 401-14.
Data were gathered by administering two tests in long division. Test A consisted of thirty-two examples. In these examples, only a one-place quotient was needed. The divisors were selected so that all decades were covered, and the second figures in the divisors were selected so that all symbols from 0 through 9 were included. All major difficulties in selecting quotients were included in the test. Test B, containing thirty-six examples, involved two- and three-figure quotients, zero difficulties in quotients, remainders, and the difficulties in naming quotients that were tested in Test A.

The per cent of error by pupils who had been taught the process was assumed to be the best index to the difficulty of the various examples. The level set up as a standard for gradation was placed at 25 per cent error.

The findings revealed that the average per cent of error for all types of examples ranged from 46.0 in Grade 5B to 28.7 in Grade 6B. Fewer errors were made by children at each grade level in the examples in which the apparent quotient was the true quotient. Errors were more numerous in the examples in which the apparent quotient was not the correct quotient. The difference between the means for the two major types was found to be greatest in Grade 5B where there was a difference of 39.2 per cent. The least difference between the two types was found in Grade 6A where the difference was 22.9 per cent.
When the second type was broken down into subtypes, it was found that some subtypes were much more difficult than others. Some phases of division were learned at lower mental ages than were other phases. The investigators concluded that since types of division examples with two-figure divisors vary greatly in difficulty, they should be taught on different grade levels.

Implications: how children learn arithmetic.--The way children learn arithmetic has great implications for grade placement of arithmetic content. According to the research findings which have just been presented, children do not learn or gain directly and immediately adult procedures and facility for dealing with number. Rather, the indications are that children go through immature procedures in dealing with number before reaching the adult level of competency. Further, some phases of processes, for example, types of division examples with two-figure divisors, are much more difficult than others and should therefore be placed at different grade levels.

From these indications it seems logical that arithmetic topics should be spread out over several years rather than expecting children to achieve complete mastery within one school year. The procedure should be to place content so that children move from immature procedures to mature procedures, and so that easy phases of processes are presented first, followed later by the more difficult.
III. Summary and Conclusions for Grade Placement

In considering the grade placement of content in elementary school arithmetic, several diverse theories have been presented. The two extreme views are those held by the "stepped-up" curriculum idea and the "stretched-out" curriculum concept. The advocates of the "stepped-up" curriculum postpone the introduction and teaching of the various phases of arithmetic until the child has the mentality to master fully all aspects of the process. On the other hand, the followers of the "stretched-out" curriculum introduce arithmetic topics early and consider the topics for several years before reasonable mastery is expected. Variations of the two extremes are represented by the theories of unitary treatment, telescoped reteaching, spiral overlap, and saturation.

A look at research has done much to offer practical guidance in placing arithmetic content. Studies by Washburne\(^{101}\) pointed up the needs and values of considering mental age in placing topics, whereas, Moser\(^{102}\) has done much to show the significance of experience in determining when arithmetic content can appropriately be taught. Showing that one should consider both experience and mental maturity in determining when to teach portions of arithmetic has been research evidence presented by both Smith\(^{103}\) and Koenker.\(^{104}\) These

\(^{101}\)Washburne, op. cit.
\(^{102}\)Moser, op. cit.
\(^{103}\)Smith (Lawrence Joseph), op. cit.
\(^{104}\)Koenker, "Certain Characteristic Differences Between Excellent and Poor Achievers in Two-Figure Division," op. cit.
two investigators have shown that both mental maturity and experience are positively correlated with success in arithmetic.

There do not appear to be any research data which specifically point to the relative worth of the intervening points of view, such as spiral overlap, telescoped reteaching, or saturation. However, studies reported by Brownell and Carper, Brownell, Carper, Deans, and Judd give conclusive evidence that children do not learn arithmetic processes all at once, but rather progress to the adult level of performance by passing through intermediate stages: Helping to support the saturation point of view are the findings given by Willey who has shown that most arithmetic processes are needed to an extent by pupils on all grade levels. These findings indicate that the major arithmetic topics need to be considered almost daily in helping children meet their quantitative needs.

The evidence from research revealed by Culver.

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105 Brownell and Carper, op. cit.
106 William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.
107 Carper, op. cit.
108 Deans, op. cit.
109 Judd, op. cit.
110 Willey, op. cit.
111 Culver, op. cit.
Smith, Reid, and Willey indicate that young children do have a social need for learning arithmetic. Brownell and Koenker present data to show that kindergarten and first grade children do profit from number instruction.

In view of the research evidence referred to above, several conclusions may be drawn as follows:

1. Both experience and mental maturity must be considered in deciding when to teach processes in arithmetic. It is suggested here that social demand for number determine whether to place the chief emphasis on mental maturity or experience. If social demand is great, suitable experiences may be provided so that the child may learn the arithmetic process early. If social demand is not great, teachers should wait for the advantages to be derived from the child's greater maturity. Willey and others have done much to indicate the social need which the various age groups have for number.

2. The "stretched-out" curriculum concept or a variation of that concept is more in agreement with how children

112 Smith (Nila B.), op. cit.
113 Reid, op. cit.
114 Willey, op. cit.
115 William A. Brownell, and others, Arithmetic in Grades I and II: A Critical Summary of New and Previously Reported Research.
116 Koenker, "Arithmetic Readiness at the Kindergarten Level." op. cit.
117 Willey, op. cit.
learn arithmetic than is the "stepped-up" curriculum concept which advocates that children master topics on one grade level. It is recommended here that topics be spread over several grade levels rather than trying to teach them to completion on one grade level.

3. Instruction in number should be started when children enter school. For young children especially, it is recommended that instructional procedures make use of concrete materials and realistic problem situations which arise in the social environment of the child.

4. Since children on all grade levels differ greatly in experiential background and mental maturity, there is a need for differentiation of content. This implies that all children within a given grade are not ready for uniform number experiences. It is recommended here that teachers at all grade levels consider content of varying degrees of difficulty and complexity in order to meet the diverse needs of the children. This is in opposition to rigidly placing arithmetic topics so that all children consider them with the same degree of emphasis.

Curriculum Patterns

In the present analysis, the term "curriculum pattern" refers to the way in which subject matter or experience are organized. The specific consideration in connection with this topic is to see how arithmetic fits into the curriculum. Even though the specific concern here is with arithmetic, it
is recognized that it is almost impossible to consider arithmetic and its place in the curriculum completely separate from other phases of the curriculum.

I. Types of Curriculum Patterns

Following will be presented the major ideas relating to the more general curriculum patterns followed in this country. It is seldom that one will find the true types in operation as described below, but rather it is more probable that one will find blends of the patterns. The basic factors regarding the different curriculum patterns are pointed out, in order to provide a background against which to view the findings of research in arithmetic concerned with these patterns.

Isolated subject organization.—Perhaps the most prevalent pattern of curriculum organization today is that in which each subject is presented as an organized body of subject matter without reference to the other subjects in the curriculum. Arithmetic is taught only during the arithmetic period. Even though arithmetic is taught as an isolated subject, many different methods may be used in presenting the material. The most frequent means of presentation, however, is through the formal textbook approach. Occasionally arithmetic is presented through purposeful activities which are carried on during the arithmetic period without specifically relating the subject to the other areas of the curriculum.

Correlated curriculum.—In this pattern of organization, two or more subjects are taught during separate periods, with
each dealing as nearly as possible with related topics. In a more informal situation the teacher may take time from the reading, geography, or science lesson to teach whatever arithmetic is needed to give greater clarity and understanding of the subject matter at hand. Likewise, during the arithmetic period, the teacher may take some of the problem situations arising from the other subjects to form the basis for part of the arithmetic lesson.

In more formal correlation, the subjects are taught in isolation, but with each contributing to a central theme. For example, in a study of transportation, the history class may study the history of transportation, while in the arithmetic class the children may deal with transportation problems based on such factors as speed, cost, mileage, etc.

Broad fields curriculum.—A term frequently used that appears to be almost synonymous with broad fields is "fusion". This term is used especially in secondary education. This particular means of organizing the curriculum appears to have had its inception in the social studies in which an effort was made to create a new synthesis of subject matter by uniting history, geography, sociology, political science, and economics around topics considered appropriate for study by elementary and high school students. Concerning this move in curriculum organization, Caswell and Foshay say:

This procedure was not merely one of extracting parts from these various fields and combining them. It was

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rather an effort to establish a new and different organization with subject matter drawn from the various social sciences. 

Following the lead given by the social studies, we find that it is common to find reading, writing, and language drawn together as language arts, and health and nature study as general science. The result is that rather than having twelve or more separate subjects being taught, we have only five or six broad areas with each having its place in the schedule. Even in this arrangement, arithmetic, in the majority of instances, retains its identity and is considered one of the broad areas. Pertaining to the broad fields represented in the curriculum, Otto writes: "Language arts, social studies, arithmetic, science, and the creative and re-creative arts are commonly represented as 'broad fields'." Caswell and Foshay indicate the prevalence of a similar representation, indicating that arithmetic is commonly considered as one of the broad areas.

Integrated curriculum.—This means of curriculum organization is termed differently by different authorities. Caswell calls it "areas-of-living curriculum". Otto refers to is

119 Hollis L. Caswell and A. Wellesly Foshay, Education in the Elementary School, p. 239.
121 Caswell and Foshay, loc. cit.
122 Ibid., p. 240.
as "activities-of-living curriculum".\textsuperscript{123} It is referred to by others as the "experience curriculum". In this type of organization an attempt is made to break with an organization which emphasizes subject matter to be taught, but stresses instead, areas of living, social functions, developmental tasks, or problems of children.

Regardless of the point of emphasis, basically the organization is the same in that subject lines are broken down as problems related to the larger unit are considered. Organized subject matter is drawn on as needed in solving problems and in providing information related to the unit or study in progress. To assure that the child will have sufficient breadth in his educational experience, certain topics are frequently pegged for given grades. In some instances units are specifically prescribed, but it is usually left to the teacher and pupils to plan the specific details.

Authorities differ as to how arithmetic and the other so-called skill subjects fit into this type of program. Some maintain that the arithmetic skills can be taught adequately as they occur naturally in pursuit of the main unit study. Others assert that arithmetic can be taught all through the unit, but that a separate period must be provided in which special attention is given to the skills needed in carrying on the larger unit.

\textsuperscript{123} Otto, \textit{op. cit.}, p.228.
Emerging curriculum.--Some educators agree with the advocates of the integrated curriculum that subject matter is not the all important concern. They object, however, to pegging topics for different grades, maintaining that the teacher and children should be free from such restriction. It is stressed that the activities and experiences provided should be based entirely on children's felt and immediate needs. In this situation arithmetic is taught entirely as it arises in meeting the immediate needs of the children.

II. Clues from Research Pertaining to Curriculum Design

Research has much to offer educators who are concerned with the place of arithmetic in the total curriculum framework. Research findings are available from small individual studies as well as from larger studies involving several schools or a whole city school system. Most of the investigations have been concerned with comparing achievement of pupils in a more traditional subject organization with pupils in a more democratic and less rigid type of organization. Much of the research has been concerned with methods, materials, and other phases of arithmetic rather than being concerned specifically with the problem of curriculum design.

Single subject organization.--Harding and Bryant report a study in which it is possible to compare two ways of dealing with arithmetic within the framework of the single

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124 Harding and Bryant, op. cit.
subject curriculum organization. A comparison was made of the achievement of two groups of fourth grade pupils. One group was taught through functional procedures, while the other group was taught by formalized drill methods. Related problems as given by the authors were:

Could concrete experiences involving mathematical procedures be introduced into one classroom of a public school without disturbing other teachers or disrupting the school program? Is a vitalized program possible within the limits of instructional materials regularly provided or obtainable without special expenditure by the teacher or school treasury? Is it possible to cover the material included in the American Course of Study for one group by means of concrete experience and is such a method practical in terms of teacher time and energy expended? What are the comparative achievements from the two methods of instruction, in social adjustment and emotional stability?125

The school was organized on a departmental basis, which meant that each subject was taught without relation to other subjects in the curriculum and that children's number experiences were confined to the "arithmetic" period. Grouping, marking, and promotion were on the basis of subject-matter achievement. The experiment covered one semester of work with each group, both of which were taught by the same teacher.

Personal experience projects, instead of conventional drill, were used with the experimental group. In selecting activities for the experimental group, due consideration was given to experiences which were rich in mathematical possibilities, ease of execution, interest appeal to the children, and adaptability to the situation. Children participated in

125 Ibid., p. 323.
selecting and planning the learning experiences. Typical of the activities engaged in by the children were weighing and measuring themselves for health records, keeping temperature and weather charts, and constructing kites. Special help with skills and processes were given as needed. The textbook was used occasionally as one would use a general reference book.

With the control group, page by page assignments were made from the adopted textbook in arithmetic. The conventional drill method was used in teaching computational mechanics, and every effort was made to exemplify the more efficient "typical" procedures.

Alternate forms of the Stanford Achievement Test were used to supply data regarding arithmetic achievement. Anecdotal records supplied data relative to emotional and social adjustment.

In computational skills, the experimental group proved to be as capable as the control group. In reasoning, the experimental group proved to be definitely superior. As evidenced by the anecdotal records, the experimental children showed more desirable social characteristics and emotional stability than did the children from the control group. As indicated by the rest of the staff, the activities of the functional program were undisturbing to them and they found ideas from the program for enriching their own programs. Records kept by the teachers related to the topics covered through the experience activities revealed that the regular
course of study requirements were covered through the functional program.

The investigators concluded that the following could be accomplished as effectively, if not more so, through the functional program as through the more traditional program: (1) gain in both computation and problem solving skills; (2) meeting course of study requirements; (3) providing practical ideas for other staff members; (4) providing growth in social and emotional adjustment; and (5) meeting objectives within the limits of the instructional materials regularly provided or procurable without personal expenditure of money by the teacher.

A conclusion, not specifically made by the investigators, is that even though arithmetic is organized as a separate subject and limited to the arithmetic period, functional procedures can do much to raise the quality of outcomes above that ordinarily achieved by the more traditional drill type "class" in arithmetic.

Many other experimental investigations have shown that even though arithmetic is organized as a separate subject, improved methods of instruction will bring about significant changes. Among such studies are those by Harap and Mapes,126

126 Harap and Mapes, op. cit.
Implications: single subject organization.--Although the curriculum pattern does call for a strict separate subject organization or a departmentalized organization with a different teacher for each subject, much can be done to raise the effectiveness of the program. Improved teaching methods and better selection and use of instructional materials can add greatly to improving the quality of outcomes in both arithmetic achievement and in the total development of boys and girls. An individual teacher can do much to promote more favorable learning regardless of the curriculum framework within which he may be working.

Correlated curriculum: Great implications for the need for correlating arithmetic with other subjects in the curriculum has been brought out in a study by Woody. In that

127McConnel, op. cit.
129Swenson, op. cit.
130Nicholson, op. cit.
investigation Woody attempted to ascertain the contribution of arithmetic to the intelligent reading of various types of material designed for use in the elementary school in Grades III to VIII, inclusive. The study was concerned primarily with the arithmetic needed for understanding and appreciating the ordinary reading which children in the elementary schools are asked to do.

Approximately 13,298 pages of material contained in thirty-eight textbooks and nine issues of magazines written for juvenile readers were checked to ascertain the arithmetic needed in order to understand this material. The material examined included thirteen readers designed for Grade V; ten hygiene and health books written for use in Grade V; twelve geographies prepared for use in Grades III, IV, and V; twelve home economics textbooks designed for use in the junior high school; and nine copies of magazines commonly read by junior high school age children. Tabulations were made showing the frequencies of numerals, measures, dates, mathematical signs, or any terms denoting arithmetic in any way.

The tabulations revealed that Arabic numerals were encountered 90,417 times; numerals expressed in words 22,721 times; use of ordinals 5380 times; Roman numerals 1775 times; time measures 3180 times; square measure 825 times; reference to various units of liquid measures 1081 times; measurement of weight 733 times; 318 different mathematical concepts such as "much," "center," "little," "budget," etc. were used 49,280 times.
In view of the many quantitative situations appearing in elementary school reading materials, Woody offered the following conclusions:

1. The numbers (Arabic and Roman), fractions (simple and decimal), and various units of measurement encountered in the reading are in general very similar to those found in the studies designed for the purpose of ascertaining the knowledge of arithmetic needed for making the necessary calculations involved in business and social transactions of life.

2. The subject-matter designed for use in the elementary school contains many large numbers, fractions with large denominators, and strange units of measure, especially those involving electricity and foreign money which are not included in the subject-matter of arithmetic designed for giving skill in those types of calculations needed for meeting the demands of business and social life.

3. The subject-matter checked includes many arithmetical terms which must be understood if the material read is to have meaning and be appreciated. Actual manipulation involving the arithmetical processes underlying the terms may often be necessary to give adequate understanding and appreciation of the processes under consideration.\(^{134}\)

Another significant study has been made which points out how the correlation of arithmetic and geography can further the learning of both. After tabulating statements in texts of fifth and sixth grade geography, Blouch\(^{135}\) indicated the frequent needs for thinking in terms of number in geography studies. Number material in the geography tests were found to fall into the following categories: area, population, 

\(^{134}\)Ibid., p. 74.

distance, latitude, climate, elevation, population, trade, and subjects related to them.

Blouch gives the following excerpts from several textbooks to show some of the number situations dealing with area and population:

Atwood and Thomas, The American Nations, page 248
"Canada is a huge country. It is larger than the continental United States and Alaska together, and yet it has only about one-twelfth as many people."

Barrows, Parker and Sornesen, The American Continents, page 212.
"Puerto Rico has too many people and too little land. About 500 people must make a living from each square mile of land."

Whipple and James, Neighbors on Our Earth, page 156.
"When Buenos Aires was opened as a port, only about 24,000 people lived there. A hundred years ago the population was 90,000. Today it is counted at over 3,000,000, having gained about 2,000,000 in the last forty years."

Carpenter, Our South American Neighbors, page 232.
"Peru's population is composed of: 8 per cent whites; 45 per cent mestizos; 47 per cent Indians."

Other pertinent examples were given to show how number was used in the geography textbooks dealing with distance and size, latitude, climate, elevation, time, production and trade.

Blouch points out that frequently geography textbooks contain excellent charts, graphs, or diagrams which help to give meaning to the numerical statements. On the other hand, she indicates that many times when quantitative ideas are expressed, the authors of textbooks seem to take for granted

136 Ibid., p. 689.
that ten and eleven year olds have the arithmetical background to make the facts meaningful, or they are confident that the teacher will grasp the opportunity to provide activities which will make the thinking as concrete as possible.

Illustrations are cited by Blouch to show how she has correlated geography and arithmetic in dealing with each of the areas in geography requiring arithmetic. The following example is cited to show how arithmetic and geography are correlated in dealing with the topic of area and population.

Since population densities are frequently referred to in texts, it is well for pupils to compute the density in their own state and county as well as that of the United States as a means of comparison. A very simple but striking way to show population densities of the states, is a dot map which the children can make. Using an outline map, a dot is placed on for each 200,000 people within its borders. This calls for the latest census information, and involves some figuring in determining the number of dots each state needs. Care must be taken to keep the dots uniform in size. This may be a cooperative activity with a large map, or individual projects with smaller desk maps. Using dots to represent numbers of people, can motivate an investigation into latitude, climate, topography, occupations, and other geographic relationships.

Inference: correlated curriculum.--The two above studies point the need for correlating arithmetic with the other aspects of the curriculum. In order to teach the other subjects in the curriculum adequately, all arithmetic cannot possibly be left to be taught during a scheduled arithmetic period. Either informal or formal correlation can be used to meet the need for arithmetic demanded for adequate understanding of

137Ibid., p. 699.
reading materials in the so-called content subjects. Correlating arithmetic with other subjects can give new meaning and purpose to arithmetic content, and at the same time can give depth of meaning and understanding to other subjects in the curriculum.

Unplanned number experiences.—Hanna\textsuperscript{138} made an investigation to determine to what extent opportunities for arithmetic were present in activity programs which did not make provision for a supplementary period in which arithmetic was considered more systematically. Six teachers of Grade III and six teachers from Grade VI from public and private schools in New York City, from suburban communities adjacent to New York City, and from rural communities of New Jersey participated in the program. In these schools most of the arithmetic was taught through worthwhile interest activities rather than as a separate organized subject.

The teachers were asked to record every situation requiring quantitative thinking and manipulation which was encountered by individuals or by the class as a whole in the pursuit of child-selected activities. In recording such problems the teachers made a record of the problem situation and the computation involved in solving the problem. Information of this type was recorded for the last two months of the spring term and for the first two months of the fall term of 1933.

\textsuperscript{138}Hanna, \textit{op. cit.}
The majority of the problems recorded in the third grade arose in connection with seven units of work, namely:
(1) Story of Old New York; (2) New York, the Wonder City; (3) The Story of Clocks; (4) Life in Anne Hutchinson's Time; (5) Life in Holland; (6) Food Study; and (7) Beginnings of the Earth.

For the sixth grade, the majority of the problems were found in connection with ten units of work, namely: (1) Study of New York's Water Supply; (2) The Solar System and Life Beginnings; (3) Health Work; (4) Natural Wealth of the United States; (5) Hallowe'en; (6) A School Store; (7) Study of Carelessness; (8) Anne Hutchinson's Time; (9) Western Movement; and (10) Club Activities.

The findings may be summarized as follows: (1) a total of 439 problems were found by pupils in Grades III and VI growing out of their activities. Of these, 279, or 63%, involved computations, while 160, or 37%, involved no computations. Of the problems found for the third grade pupils, 56% involved computation, while it was found that 72% of the problems for the sixth grade children could be classified as computation. (2) Of a total of 279 computational problems for the two grades, 145 problems, or 52%, were one-step problems; 72, or 26%, were two-step problems; and 62, or 22%, were three-or-more-step problems. (3) The total problems involving computation required 640 computational manipulations for solution--24% were in addition, 23% in subtraction, 38% in
multiplication, and 15% in division. (4) Of the computations completed in the two grades, a total of 286, or 45%, were integers; 87, or 1%, were fractions; 43, or 7%, were mixed numbers; 277, or 42%, were decimals (almost entirely money), while decimal fractions and other types accounted for 4% of the total.

In considering the number of computational problems per grade that grew out of the unit work, it was found that for Grade III only five computational problems per room per month were met. Only about six computational problems per month for each sixth grade room were found.

The major conclusion reached was that even though the activities contained a richness of arithmetic experience, they did not provide sufficient quantitative situations to develop fully number skills. The teacher must supplement the number experiences coming from the activities with number experiences provided through additional instructional periods.

Implications: unplanned number experiences.--It seems logical to say that a curriculum pattern which does not make more specific plans for arithmetic learning than that provided in the study reported above by Hanna cannot assure adequate arithmetic competency on the part of children to meet their own immediate needs nor their future adult needs. The research by Harding and Bryant, Harap and Mapes, and

139 Harding and Bryant, op. cit.
140 Harap and Mapes, op. cit.
Williams\textsuperscript{141} reported elsewhere in this chapter show that even though arithmetic is specifically planned for in arithmetic activity and integrated experience units, other important values such as improvement in emotional and social stability are not detracted from.

**Planned number experiences in experience and arithmetic activity units.**—In a pertinent investigation, Williams\textsuperscript{142} showed that an integrated experience curriculum can provide sufficient quantitative content on which to base the arithmetic program in the sixth grade. Williams states the purpose of her study by saying: "The purpose of the study was to determine the mathematical learning which took place in nine successive sixth-grade groups (1935 through 1944) under experience curriculum and to develop the implications for curricular and instructional practices."\textsuperscript{143}

As part of a more detailed description of the experience curriculum, Williams writes:

\ldots the experience curriculum is a planned way of learning subject-matter functionally in life content. The teacher plans with the children so that the content and use of arithmetic—as well as the language arts and the social studies—are integral parts of living which contribute to promoting communication and critical thinking, developing social insight, and increasing computational skills and mathematical insight.\textsuperscript{144}

In regard to arithmetic, that investigator was concerned

\begin{itemize}
\item[$\text{141}$] Williams, op. cit.
\item[$\text{142}$] Williams, op. cit.
\item[$\text{143}$] Ibid., p. 154.
\item[$\text{144}$] Ibid., p. 156.
\end{itemize}
with the sum total of situations to which mathematical con­cepts and principles could contribute significantly. The
teacher endeavored to help children grow in ability to dis­cover where and how arithmetic could contribute to a better control of numerical situations. Children were encouraged to use arithmetic to achieve their own purposes.

The real problems which grew out of the units of study and which were utilized in meeting the needs of the children in carrying forward the curricular activities were listed. It was discovered that there was a wealth, rather than a dearth, of arithmetic, inherent in, and essential to, the daily living of children when they were carrying forward group and individual purposes. In order not to detract from the unit in progress, an arithmetic period was scheduled in which attention was given to developing the skills in solving the number problems met in carrying forward the unit experi­ences.

Each year Stanford Achievement Tests were used to mea­sure achievement in arithmetic. Concerning achievement, Williams summarized the findings as follows:

Analysis of these standard test scores for the nine successive groups revealed that they compared very favorably with the norms. In all but 7 of the 208 cases involved, the composite score showed growth of more than 0.6 in the school year. The average gains for the nine successive years were: 1.3, 1.25, 1.4, 1.89, 2.0, 1.69, 1.8, 1.9, and 1.8 respectively. At the beginning of Grade VI, the majority of the com­posite scores fell in Grades IV and V; at the end of the year they fell in Grades VI and VII. For four of the nine years, the median score at the end of Grade VI
was 7.0; during the other years it was 6.3, 6.4 (twice) and 6.6 (twice). The children consistently scored higher in the reasoning than in the abstract examples test. On the whole, whether a child entered Grade VI ranking somewhere in Grade III on the test or ranking somewhere in Grade VI or above, he made significant growth during the year.\textsuperscript{145}

The investigator found that the functional arithmetic program of the experience curriculum was successful in bringing about significant gains in such qualities as initiative, critical thinking, independent work habits, a sense of responsibility, disposition and ability to carry an activity through to completion, disposition to think quantitatively in a life situation in which such thinking is needed, and to do so flexibly and naturally in terms of the situation. When three groups of the experimental children and three groups from a traditional program were equated and compared, it was found that the experimental group had significant advantages over the traditional children in respect to the above qualities.

Williams concluded that the experience curriculum provided sufficient functional problem situations on which to base the arithmetic program. It was further concluded that such a program was more than adequate when based on the criteria of achievement in arithmetic and growth in personal development.

The experimental evidence referred to earlier in this chapter by Harap and Mapes,\textsuperscript{146} Harap and Barnett,\textsuperscript{147} and

\textsuperscript{145}Ibid., p. 159.

\textsuperscript{146}Harap and Mapes, \textit{op. cit.}

\textsuperscript{147}Harap and Barnett, \textit{op. cit.}
Passehl\textsuperscript{148} indicates that arithmetical learnings were superior when arithmetic was introduced through functional situations which were selected because they were rich in the possibilities for arithmetic.

Another key investigation which affords an appraisal of the plan of organizing arithmetic experiences through the medium of activities was made in 1941. Under the direction of J. Cayce Morrison\textsuperscript{149} an important survey was made of a curriculum experiment with the activity program in the elementary schools of New York City.

The curriculum experiment under survey began in 1934, when the Committee on Educational Problems of the New York Principals' Association voted to make the activity movement the committee's major topic of study for the year. In 1935 the activity program was started in at least one grade in each of thirteen different schools, and the Superintendent of Schools approved the extension of the experimental program to approximately seventy schools for a period of five or six years. By 1939-40 more than 80 per cent of the teachers assigned to the activity schools were reported as participants in the activity program.

In 1940 the New York State Legislature appropriated funds for the State Education Department to survey the experimental curricula in New York City Schools. After making

\textsuperscript{148}Passehl, \textit{op. cit.}

\textsuperscript{149}J. Cayce Morrison, \textit{The Activity Program}. 
preliminary plans, the field work on the survey began in December, 1940, and was completed in April, 1941.

Evidence revealed that, as indicated by practice, the different schools held varying concepts of the activity program. The survey committee indicated that the following statement was generally applicable to the activity program as followed by the New York City Schools:

As conceived in the New York City Experiment, the development of an activity program was primarily an effort to shift the emphasis of teaching in the elementary school from subject matter to the child. It was an attempt to make the child an active participant rather than a passive recipient in the educational process.

The activity program placed special emphasis upon the development in children of self control, critical thinking, creative expression, and desirable social relationships. These were to be attained in part through children's participation in planning their own work, learning by actual experience, and helping to keep records and to evaluate work done. It emphasized adapting materials and methods to the needs and abilities of the individual pupils; the wider use in the school program of opportunities for creative work in art, music, dramatics and construction; the cultivation of elementary research skills, and the creation of school and classroom atmosphere conductive to democratic living, and of friendly active, cooperative relations between school, home and community.150

In order to determine the effect of the activity program on the development of children with special reference to knowledge and skills in the areas of reading, arithmetic, language usage, work-study skills, and social studies, the survey committee conducted a testing program. More than 2100 pupils in Grade 6A were included in part of the testing

program; about 1040 pupils were in activity schools and about 1090 in the regular schools. The children from the two types of schools were equated on the basis of age, grade, sex, and mental ability.

The activity schools were found to be significantly superior in four areas, namely, critical reading, use of index, use of references, and civic attitudes. The data were contradictory for three areas, namely, reading for word meaning, reading graphs and charts, and the use and interpretation of knowledge in elementary science.

In the remaining areas the differences were inconclusive. The regular schools were found to excel slightly in spelling and grammatical usage, reading for detailed understanding and for the central thought of a paragraph, knowledge of children's literature, problem analysis in arithmetic, map reading and use of the dictionary. Skills in which the activity schools tended to excel were: analyzing arithmetic problems for sufficiency of data, knowledge of current events, use and interpretation of knowledge in the social events, use and quality of expression in written letters. It was found, however, that as the samplings of activity pupils more nearly approached attainment of the activity concept the margin of advantage to the regular schools tended to decrease for problems solving as well as for several of the other skills.

An additional testing program which evaluated children's progress consecutively through the fourth, fifth, and sixth
grades tended to confirm the major findings as revealed by the sixth grade testing program. A slight contradiction was found in the case of arithmetic. Whereas, in the sixth grade testing program, the regular schools appeared to have a slight advantage in the analysis of arithmetic problems, in this phase of the study the pupils of the activity program appeared to have a slight advantage in arithmetic reasoning. Since one test favored the activity program slightly and the other test favored the regular program, the survey committee concluded:

. . . it may be safely concluded that neither program has demonstrated as yet any superiority over the other in helping children to master that important area of arithmetic known as problem solving. Neither does the analysis of the subtest data of the Modern Test indicate that either program has any marked advantage over the other in reading comprehension, in speed in reading, in arithmetic computation, or in language usage. 151

In view of the evidence presented by the data, the following summary conclusions were reached by the committee:

In the mastery of knowledge and skills, three conclusions are supported by the data. . . . (1) the only statistically reliable differences between the two programs favor the activity program and are found in those areas affected by the latter's emphasis upon developing skill in critical reading, in the use of elementary research techniques, and in the development of civic attitudes and understanding of social relationships; (2) in all but three of those areas of knowledge and skills wherein the differences are found favored the regular program, improving the sampling of the activity program tended to lessen or eliminate the difference; (3) the activity program may be continued and improved with reasonable assurance that children will gain as thorough a mastery of knowledge and skills as they would in the regular program.

151 Ibid., p. 93.
In the appraisal of the school's influence upon the attitudes and behavior of pupils outside the classroom, certain conclusions are evident: (1) the pupils in the activity schools like school better, find it more interesting, and tend more to carry its influence into their life outside the school; (2) pupils in the activity schools tend to excel and in some cases by reliable differences in such qualities as cooperativeness or working together, self-confidence or poise, lack of subservience, creative abilities, self-discipline and scientific outlook; (3) in developing respect for the authority of home and school, the two programs are almost evenly matched in the outcomes.152

Other appraisals made of the New York City activity program have revealed findings which largely substantiate the findings and conclusions reported by the survey. Such evidence has been reported by Wrightstone,153 Hunnicutt,154 Thorndike,155 and Jersild.156

An appraisal: planned number experiences in experience and arithmetic activity units.—In making an appraisal of how arithmetic experiences are provided through an experience or activity approach in the teaching of arithmetic, it is important to keep in mind that the conception of such programs

152 Ibid., pp. 162-63.
varies from school situation to situation. In an integrated experience curriculum as interpreted by Williams, functional number experiences were found in such quantity as to be more than adequate in promoting quantitative growth on the part of children. If number experiences are to be found in sufficient quantity in experience and arithmetic units, teachers must look for and be fully aware of the possibility of the contribution of number to the unit in progress. Without specifically looking for and providing for growth in number development in connection with units of work, number situations will very likely not appear in sufficient quantity to promote number competency adequately.

The evidence revealed by research indicates that, as a whole, arithmetic achievement by children in activity schools in both computation and problem solving is equal to or superior to such achievement by pupils in schools that have been more rigidly organized. When one considers that children from activity schools are found superior in social and emotional adjustment, then the activity plan of organizing subject matter and experiences has even more to recommend it.

To gain full effects from improved plans of organizing and presenting curricular experiences, it seems practical to recommend that colleges preparing teachers give increased attention to such procedures in their teacher education programs. Of equal importance in this respect, is the need for

157Williams, op. cit.
in-service education for teachers currently teaching in the public schools.

III. Summary and Conclusions for Curriculum Patterns

It has been shown that many approaches are used in organizing number experiences for children in the elementary school. The usual procedure is to teach arithmetic through an isolated subject approach; however, much effort has been directed toward correlating arithmetic with other phases of the curriculum or toward teaching arithmetic through problem situations occurring in experience and activity units.

The findings from research have been presented which offer help in determining how best to organize number experiences. Harding and Bryant have shown that even though arithmetic is organized as a separate subject with instruction limited to the scheduled arithmetic period, functional procedures can do much to raise the quality of outcomes above that ordinarily achieved by the more traditional drill type arithmetic class. The need for correlating arithmetic with reading in the content subjects was indicated by both Woody and Elouch. Significant evidence has been presented by Hanna to show that number does not appear in

158 Harding and Bryant, op. cit.
159 Clifford Woody, Nature and Amount of Arithmetic in Types of Reading Material for the Elementary Schools.
160 Elouch, op. cit.
161 Hanna, op. cit.
sufficient quantity in an integrated activity program to assure number learning on the part of the children. On the other hand, Williams\(^{162}\) has revealed that when teachers are conscious of the possibility of number in an experience unit, quantitative situations appear in such quantity to be more than adequate to meet the number needs of children. Even so, Williams found it practical to schedule a supplementary period in order to teach the skills needed to meet the quantitative demands of the experience units. Morrison\(^{163}\) presents evidence to show that children will gain as thorough a mastery of knowledge and skills through an activity program as they would in a regular program. Morrison shows further, along with Harding and Bryant,\(^{164}\) and Williams,\(^{165}\) that in addition to learning skills in an experience or activity program, children from such programs are superior in emotional and social adjustment.

In view of the evidence revealed above, the following conclusions are offered:

1. Curriculum patterns cannot be evaluated adequately without considering teaching methods and materials. A given curriculum pattern can be improved by improving teaching methods and the use of instructional materials.

\(^{162}\)Williams, op. cit.
\(^{163}\)Morrison, op. cit.
\(^{164}\)Harding and Bryant, op. cit.
\(^{165}\)Williams, op. cit.
2. In order to understand the quantitative nature of other subjects in the curriculum, arithmetic instruction cannot be left exclusively to the so-called arithmetic period. Further, content drawn from other subjects can add life and vitality to the scheduled arithmetic class.

3. Although the evidence is sometimes contradictory and inconclusive, it may be said that arithmetic skills are learned as well if not better in an experience or arithmetic activity curriculum as in the traditional single-subject type curriculum. If we are to consider the development of the whole child, we should definitely follow an experience or activity program, for such programs contribute more to the emotional and social development of the child.

4. Even though an experience or activity program is followed, arithmetic must be definitely planned for. When left purely to chance arithmetic learnings cannot be assured.

Summary

Three aspects of the elementary school curriculum as it relates to arithmetic have been considered in this chapter. The three aspects have been: (1) the scope of the arithmetic curriculum, (2) grade placement of content in arithmetic, and (3) curriculum patterns. In connection with each of these three divisions, the major points of view pertaining to them were discussed and analyzed. Finally, for each aspect, research findings which have a bearing on the problem were presented.
Below will be summarized the major research evidence relating to each of the phases of the curriculum considered. The major conclusions together with implications for teaching will be presented in a concluding section.

The scope of the arithmetic curriculum.--The classical study by Thorndike\textsuperscript{166} ruled out the selection of arithmetic content for its supposed mind strengthening qualities. Thorndike provided rather conclusive evidence that arithmetic cannot be expected to contribute any more to strengthening the mind than can the so-called less difficult school subjects.

The survey studies made by Wilson\textsuperscript{167} and others pointed out the type of arithmetic needed by man in meeting his everyday social needs. Bobbitt\textsuperscript{168} presented evidence to show the informational aspects of number needed by man in order to understand quantitative situations referred to in newspapers. The number needs of children have been indicated by Willey\textsuperscript{169} and others.

The findings from several experiments reveal the importance of presenting number so that vital relationships inherent in the number system are not lost to view. These studies

\textsuperscript{166}Thorndike, \textit{op. cit.}

\textsuperscript{167}Guy M. Wilson, \textit{What Arithmetic Shall We Teach?}

Guy M. Wilson, "Social Utility Theory As Applied to Arithmetic, Its Research Basis, and Some of Its Implications, \textit{op. cit.}

\textsuperscript{168}Bobbitt, \textit{op. cit.}

\textsuperscript{169}Willey, \textit{op. cit.}
further indicate the value of adhering to the sequential nature of number. Such investigations were made by Swenson, Thiele, McConnell, Nicholson, and Wilburn.

The significance of basing content on problem situations which are real to the learner are borne out by important evidence presented by Harap and Mapes, Harap and Barnett, Fassehl, Harding and Bryant, Conner and Hawkins, White, and Williams. The necessity of considering vocabulary as part of the content to be used in elementary arithmetic has been revealed by Buswell and John, Harrison, Johnson, and Treacy.

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Swenson, op. cit.


McConnell, op. cit.

Nicholson, op. cit.

Wilburn, op. cit.

Harap and Mapes, op. cit.

Harap and Barnett, op. cit.

Fassehl, op. cit.

Harding and Bryant, op. cit.

Conner and Hawkins, op. cit.

White, op. cit.

Williams, op. cit.

Buswell and John, op. cit.

Harrison, op. cit.

Johnson (Harry C.), op. cit.

Treacy, op. cit.
Grade placement of content in arithmetic.—Much dispute has arisen as to whether or not mental maturity or experience is the important factor to consider in determining when aspects of arithmetic content can appropriately be taught. The role of mental maturity has been shown by Washburne, while Moser has revealed the significance of experience. Johnson found that children in the Chicago schools made significant gains in arithmetic learnings when a curriculum was adopted in keeping with the mental age recommendations made by Washburne. Investigations made by Smith and Koenker have shown the futility of emphasizing one of the factors alone, and have revealed the importance of considering both factors. These last two investigators have shown that success in arithmetic is positively correlated with both experience and mental maturity. Willey pointed out the social demand for number which different grade groups have.

Studies reported by Brownell, Buckingham and

186 Washburne, op. cit.
187 Moser, op. cit.
188 Johnson, (J. T.), op. cit.
190 Koenker, "Certain Characteristic Differences Between Excellent and Poor Achievers in Two-Figure Division," op. cit.
191 Willey, op. cit.
192 William A. Brownell and others, Arithmetic in Grades I and II: A Critical Summary of New and Previously Reported Research.
MacLatchy, and others have indicated that children have acquired much number competency when they enter school and consequently are considered ready to receive further number experiences on arrival at school. Both Brownell and Koenker have provided evidence that kindergarten and first grade children profit from number instruction.

Strong evidence has been provided by Brueckner and Melbye to show that a process like long division needs to be taught on several grade levels rather than trying to teach the entire process to completion in a given grade. Pointing out further that mature facility in the use of number is not acquired all at once are data reported by Brownell, Brownell and Carper, Carper, Deans, and Judd. Their evidence is conclusive that children progress through a series of intermediate steps in arriving at adult competency in the use of number.

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193 Buckingham and MacLatchy, "Number Abilities of Children When They Enter Grade One," op. cit.
194 Brownell, op. cit.
195 Koenker, "Arithmetic Readiness at the Kindergarten Level," op. cit.
196 Brueckner and Melbye, op. cit.
197 William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.
198 Brownell and Carper, op. cit.
199 Carper, op. cit.
200 Deans, op. cit.
201 Judd, op. cit.
Curriculum patterns.--Practice shows that arithmetic learnings are organized and presented through various curriculum patterns. These patterns range all the way from a strict isolated subject organization to unplanned use of number. The latter procedure implies that children will learn number as it naturally occurs in meeting their own on-going life experiences. Harding and Bryant, Harap and Mapes, and Passehl have provided evidence to show that even when arithmetic is presented more or less as an isolated subject, learning outcomes can be greatly improved by following better teaching methods, and by the use of content which is real to the learner.

The importance of correlating arithmetic with other aspects of the school curriculum has been pointed out by Woody and Blouch. These investigators have shown that number is needed in reading in the content subjects of the curriculum, and they have indicated further how content from other areas of the curriculum can be used to add vitality to the arithmetic program itself.

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202 Harding and Bryant, op. cit.
203 Harap and Mapes, op. cit.
204 Passehl, op. cit.
206 Blouch, op. cit.
Hanna\textsuperscript{207} presents strong evidence which reveals that number does not appear in sufficient quantity in most integrated activity units to assure adequate number learning on the part of children. Williams,\textsuperscript{208} on the other hand, shows that when teachers are conscious of the possibility of number in experience units, the number of quantitative situations available will be more than adequate to meet the number needs of children. Williams found it practical, however, to schedule an arithmetic period in which to teach the number skills needed in coping with the number problems appearing in the units. Harding and Bryant,\textsuperscript{209} Harap and Mapes,\textsuperscript{210} and Passehl\textsuperscript{211} show that number can be presented through the medium of arithmetic activity units which are selected because of their possibility for affording rich quantitative experiences. Morrison\textsuperscript{212} reports evidence that children gain as thorough a mastery of knowledge and skills through an activity program as they do in a regular program.

Aside from the value of learning skills through an activity program, evidence is provided by Harding and Bryant,\textsuperscript{213}

\begin{itemize}
\item[\textsuperscript{207}]Hanna, \textit{op. cit.}.
\item[\textsuperscript{208}]Harding and Bryant, \textit{op. cit.}.
\item[\textsuperscript{209}]Harding and Bryant, \textit{op. cit.}.
\item[\textsuperscript{210}]Harap and Mapes, \textit{op. cit.}.
\item[\textsuperscript{211}]Passehl, \textit{op. cit.}.
\item[\textsuperscript{212}]Morrison, \textit{op. cit.}.
\item[\textsuperscript{213}]Harding and Bryant, \textit{op. cit.}.
\end{itemize}
Williams, and Morrison to indicate that children participating in activity programs reflect more desirable emotional and social behavior than do children from more rigidly organized programs.

Conclusions and implications for teaching.--In view of the research evidence presented above, the following conclusions together with implications for teaching appear justified:

1. Arithmetic content should not be selected for its mind strengthening qualities, but rather for its value in meeting the number needs of children now as well as their later adult needs. This means using practical problems as opposed to using theoretical and nonsense problems.

2. Problems should be used which are real and purposeful to the learner. Teachers must be alert to the quantitative situations appearing in the everyday experiences of children and use those to promote number learning.

3. Computational skills should be so presented that the sequential nature of number and the relationships inherent within the number system are not violated nor destroyed. This is in opposition to selecting isolated bits of content and teaching those through mechanical drill techniques. Instead, related factors should be taught together so that children

214Williams, op. cit.
215Morrison, op. cit.
can see relationships, form generalizations, and thus be enabled to transfer principles learned to untaught number situations. Number relationships and the sequential nature of number can be adhered to without violating the principle of using those aspects of number which are socially useful.

4. Number vocabulary must be considered to be a part of the content to be taught in arithmetic. In order for children to meet adequately problem situations, teachers must help children develop concepts implied by number vocabulary. Such concepts can be developed best through concrete experiences.

5. Many aspects of number need to be taught for their informational worth. Even though social usage implies that many phases of number need not be taught as computational processes, it is often necessary that children have an acquaintance with such aspects in order to comprehend general reading materials. Informational phases of number can be developed through regular arithmetic instruction and through practical situations as they occur in content reading.

6. Both mental maturity and experience must be considered as important factors in determining when the various phases of arithmetic content can best be taught. It is suggested here that social demand largely determine whether or not to place emphasis on mental maturity or experience. If social demand is great, teachers should provide the necessary experience so that children can be assisted to learn the topic early. On the other hand, if social demand is not great,
teachers should wait for the advantages to be derived from greater maturity.

7. In view of the way children learn, arithmetic topics should be spread out over a period of several years rather than expecting children to achieve full mastery within a given school year. This implies that children will first use crude processes in arriving at many of their solutions before finally reaching full maturity in performing mathematical computations.

8. Children should receive number instruction when they enter school. This instruction should take place through informal procedures and deal with quantitative situations which are real to the learner. It is not expected, however, that all children will be ready for the same number experiences.

9. Due to the fact that children on all grade levels differ in experiential background and mental maturity, there is a need for differentiation of content. Teachers must consider many aspects of a process on each grade level rather than expecting all children in a grade to be ready for the same phase of a process.

10. In order to evaluate adequately a given curriculum pattern, it is important to consider the teaching methods and instructional materials being used. A curriculum pattern can frequently be good or bad depending on the teaching method and the instructional materials being used.
11. It is impractical to consider arithmetic only during a scheduled arithmetic period. Arithmetic must be considered in connection with other aspects of the curriculum in order that children be helped to understand the quantitative phases of the other areas of the curriculum. Content drawn from other divisions of the curriculum can do much to add purpose and significance to the scheduled arithmetic period.

12. Even though the evidence is frequently contradictory and inconclusive, it may be said that arithmetic skills are learned as well if not better through the medium of an activity program as in a more rigidly organized program. The activity program definitely has more to offer if one is to consider the development of the whole child rather than his intellect alone, for such programs contribute more to the social and emotional adjustment of the child.

13. Regardless of the curriculum pattern followed, provision for arithmetic instruction must be definitely made. There is greater assurance that children will learn arithmetic when number experiences are definitely planned and provided, than when such experiences are left to chance.
CHAPTER IV

GENERAL METHODS

In the broadest sense, general methods may be considered to be those practices which are applicable in teaching any of the different subjects in the curriculum or which may be followed in guiding children as they pursue purposeful problems or activities. In a more narrow sense, the term is applicable to those methods which may be used in teaching any of the various aspects of a given subject, for example, the different processes in arithmetic. It is with the latter meaning in mind as it is applied to the teaching of arithmetic that the term is used in this chapter.

In the first section of this chapter will be discussed the several types of general methods that are sometimes followed in arithmetic instruction. Following the presentation of the different types of general methods, research will be reviewed in the second section so that interested professional persons may more intelligently form conclusions concerning the method or methods to be used in the teaching of arithmetic. In conclusion, a summary of the chapter will be provided. This summary will include the major conclusions to be drawn from the research evidence.

Types of General Methods

From the past and present practices, several general
methods relating to the teaching of arithmetic are in evidence. In actual classroom situations there is usually much overlapping between methods, so it is doubtful if one will often find a given method being followed exclusively. A discussion will follow of the several types of general methods.

The cypher book method.--The cypher book method could also be called the rule method. This was the popular method used in teaching arithmetic in the United States prior to 1821 and to a lesser extent afterward. The cypher book method was used by those who were strong believers in faculty psychology. It was assumed that if a given rule were presented, very little additional instruction needed to be given. If one had the appropriate mental faculty, he could apprehend the meaning embodied in the rule and could apply the rule to the specific mathematical process to which it applied. Rules were memorized, and "sums" to which the rule applied were worked and copied in the cypher book.

The teacher's role was that of giving rules, setting examples to be worked in which the rule could be applied, and examining the work to see if the answer checked with that which the master had derived years before when he made his own cypher book. Instruction was almost entirely individual. At first textbooks were not used, but later books were used in which the rule and the examples to be worked were given, so that the teacher had only to examine the work. Monroe sums up the situation when he writes: "... there was practically no attempt to instruct pupils, but in so far as there was any
plan for guiding the pupil in his learning, it was deductive, i. e., from rule to problem.  

Monroe quotes as follows from Joseph T. Buckingham in volume 13 of Barnard's American Journal of Education in giving one pupil's experience in using the cypher book method:

At length, in 1790 or 1791, it was thought that I was old enough to "Cypher," and accordingly was permitted to go to school more constantly. I told the master I wanted to learn to cypher. He set me a "sum" in simple addition—five columns of figures and six figures in each column. All the instruction he gave me was, add the figures in the first column, carry one for every ten, and set the over-plus down under the column. I supposed he meant by the first column the left-hand column, but what he meant by carrying one for every ten was as much a mystery as Samson's riddle was to the Philistines. I worried my brain for an hour or two, and showed the master the figures I had made. You may judge what the amount was when the columns were added from left to right. The master frowned and repeated his former instruction, add up the column on the right, carry one for every ten, and set down the remainder. . . .

Lecture or telling method.—Two variations may be discovered in the lecture or telling method. In one case, the teacher identifies the correct procedures and responses and provides practice for stamping in the responses. Since learning is assumed to take place all at once, the children are taught the direct procedures as used by adults. This means that children are discouraged from using manipulative materials, or crude procedures, for such practices would


2Ibid., p. 44.
merely retard the child in achieving adult competency in dealing with number. Steps to be followed in doing abstract computations are demonstrated to the children as the teacher works examples on the chalkboard, or the teacher may refer the children to the examples worked out in the textbook. Children are then assigned practice problems so that they may fix facts and learn to follow through on the mechanics of the algorithms as outlined by the book or teacher. The teacher does not use concrete or pictorial materials to help the children gain insight into why a fact is true, or help the children form generalizations applicable to other situations.

In teaching problem solving, the children are given cue words or formal steps as aids in attacking problems. The children are given rules or formulas for doing certain computations, or verbal problems. Deductive reasoning is stressed. The pattern is from the general to the specific. Consistent with this method is the recitation in which the children "cite" back to the teacher what the book or teacher has told them.

In the second type of the lecture or telling method, the teacher also identifies correct responses or outlines the steps to be followed in doing a computational process. The teacher, however, goes a step further than the teacher who is using the more authoritative approach. He wants the children not merely to accept his word that a given fact is true, but he wants them to understand why the fact is true.
He uses concrete materials, such as pennies, or pencils, or less concrete materials, such as charts or diagrams, and demonstrates to the children why a given fact is true. The children's attention is called to relationships in the number system. The teacher states rules or generalizations and demonstrates why they are true. The children are expected to accept facts, relationships, and generalizations thus identified and demonstrated by the teacher. Once the teacher is certain that he has caused the children to understand, practice materials are provided to aid the children in becoming automatic in their response to number situations.

The discovery or inductive method. — Like the methods discussed previously, the discovery or inductive method relies largely on an organized class situation for the learning and teaching of arithmetic. But in this situation the children are given the opportunity to discover and verify number facts and relationships for themselves. From such facts and relationships thus discovered and found to be true, the children are encouraged to form generalizations which can be applied to other number situations which are different in detail but not in principle. The children are encouraged to do inductive reasoning, that is to go from the specific to the general.

The teacher who is aware of relationships inherent in the number system, organizes and presents materials in such a way that children are aided in discovering relationships and in forming generalizations. The teacher guides the children's thinking by skillful questioning which helps the children to
channel their thinking in discovering a fact, relationship, or generalization. The best teachers do not press the children into verbalizing the generalization but look for more subtle behavior on the part of the child which indicates that he has grasped the full import of the generalization. Learning is not believed to take place all at once, so children are encouraged to use immature procedures in dealing with number as they gradually progress toward the adult level of number competency.

Activity methods.--In the exact sense, activities may be considered to include all the experiences in which children engage in the process of learning. In that sense the learning experiences outlined above are activities. The term "activity methods" as used here, however, refer to more informal procedures for the teaching of arithmetic. Generally, the child is given more opportunity to share in planning and in setting up objectives. The classroom procedures are usually less rigid and formal, and the general atmosphere may be described as being social. As stated by the Survey Committee of the New York City activity program: "The formal recitation is modified by conference, excursion, research, dramatization, construction and sharing, interpreting and evaluating activities."

Activity methods may be considered from two points of view. One view is that the activities of boys and girls are

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utilized for teaching desired portions of organized subject matter. The other view is that subject matter is drawn on as necessary in helping children carry out their chosen activities. In the first case, it is the subject matter that is important, and in the second case, it is meeting children's purposes that is important.

Activities may involve projects in which children construct or make something. Collings defines four types of projects, namely, play, excursion, story, and hand. The projects are defined as follows:

Play Projects represents those experiences in which the purpose is to engage in such group activities as games, folk dancing, dramatization, or social parties. Excursion Projects involve purposeful study of problems connected with environments and activities of people. Story Projects include purposes to enjoy the story in its various forms—oral, song, picture, phonograph, or piano. Hand Projects represent purposes to express ideas in concrete forms—to make a rabbit trap, to prepare cocoa for the school luncheon, or to grow cantaloupes.4

Through these projects, arithmetic as well as other fundamental subject matter is used and drawn on as needed in helping children carry their project to completion. The project, rather than the subject matter, is the important concern.

Closely related to the projects mentioned above are arithmetic activity units. These units are usually selected because they involve quantitative situations which are of immediate concern to children and because they are rich in

4Ellsworth Collings, An Experiment with a Project Curriculum, p. 46.
possibilities for the application of number. In regard to the activity units which were followed in a sixth grade in the teaching of decimals, Harap and Mapes say: "The activities which make up the sixth grade course were studied because they were rich in the application of decimals. Each unit was based on a meaningful situation in the child's experience in school and in daily life."^5

The units, which comprised the work of the year for the sixth grade in learning decimals, are listed by Harap and Mapes as follows:


In the activity method implied by Harap and Mapes, skills were considered as they arose in the pursuit of activities. This means that unit skills were taught out of their natural sequential order. At no time was any special practice or drill provided. It was assumed that skills would be learned as they occurred in solving the problems arising in connection with the unit.

The unit approach followed by Williams seems to be a little more comprehensive than the approach followed by Harap


^6Ibid., p. 687.
and Mapes. Williams\footnote{Catherine M. Williams, "Arithmetic Learning in an Experience Curriculum," Educational Research Bulletin, VIII (September, 1949) pp. 154-62.} was not any more concerned with arithmetic than she was with the other subject areas of the curriculum. However, Williams and the children were conscious of the contribution that arithmetic could make to the experience unit, so arithmetic was not left purely to chance. A socially significant problem was selected by the teacher and children. As they sought information related to this problem, they drew on all organized subject matter for whatever contribution the subject matter could make to the solution of the problem. In addition to the unit work, Williams did have a scheduled arithmetic period in which the children were given special help in doing whatever arithmetic was needed in carrying forward the main unit.

A variation of the experience unit method as used by Williams, is the unit approach followed in an emerging type curriculum. In this case no special attention is given to the consideration of the contribution that arithmetic can make to the unit in progress. Neither is any special period provided for helping children with skills in arithmetic. At anytime, when the children happen to meet a numerical problem, the children are given whatever immediate help they need in meeting the problem at hand. It is assumed that children will learn all the arithmetic that need as they meet problem situations in pursuing whatever activity they happen to be engaged in.
Contributions to an Understanding of General Methods

To help one formulate for himself a general method to be used in the teaching of arithmetic, one should weigh the possible influence of several factors. Among the many possible contributing factors, one should consider teacher personality, the influence of types of leadership, instructional procedures, and the way children learn. A composite view of the role of these several factors should help to give one an understanding of general method. Research, which has been made in regard to these various factors, has much to offer educators in developing a general method to be used in the teaching of arithmetic.

Drill methods versus inductive or generalization methods.--The value of isolated drill techniques in the teaching of arithmetic has long been questioned. One of the pioneer studies reflecting the weakness of drill as a teaching method was made by Kirkpatrick. That investigator presented data to show that memorization in the absence of use is a poor and inefficient teaching method. It was revealed that the utilization of the pupil's previous knowledge is much superior to reliance on memorization alone.

A more recent investigation showing the inadequacy of

drill as a teaching method was made by Nicholson. Her purpose was to determine the relative effectiveness of two methods of teaching the four fundamental processes in Grades I, II, and III. One method, called Method G, was a method whereby the children were encouraged to discover the solution to number combinations for themselves. Children were to draw their concepts of abstract number through the use of concrete and semi-concrete materials. Children were to study the inherent relationships in groups of concrete materials by making comparisons, by taking groups apart and putting the objects together again to form new groups. Basic ideas relative to relationships learned in addition were extended to subtraction, multiplication, and division. After pupils had learned basic facts and relationships themselves, they were given opportunities to practice the facts. When a pupil failed to remember a combination, he was presented with manipulative materials so that he could develop again the meaning and relationship of the number fact.

The second method, called Method R, depended mainly on drill and repetition for the pupils to acquire a knowledge of number facts. The teacher herself used manipulative materials to demonstrate the verification of the facts in the initial instruction. Following demonstration, the facts were learned

by the pupils through repetition. Even though addition and subtraction facts were taught together, and multiplication approached through addition, and division facts derived from multiplication facts, relationships were stressed only in a very limited sense. Combinations involving the zero were taught as separate combinations.

This experiment was carried out over a period of three years, from September 1938 to June 1941. The study was conducted in two Philadelphia schools, one of which was the experimental school and taught by Method G, and the other was the control school and taught by Method R.

Complete data were secured for 359 pupils in Method G and 333 pupils in Method R. In order to compare the results of the two methods, experimental and control pupils were matched by pairing on the basis of grade, sex, chronological age, and raw score on the Philadelphia Mental Ability Test.

Most of the findings were definitely in favor of Method G. The superiority of Method G was shown for Grades I and II in achievement related to the addition and subtraction combinations, and in Grade II in zero addition and zero subtraction. Superiority for Method G was shown for achievement in multiplication and division in the end results in Grade 3B. This superiority for multiplication and division was not consistently maintained, however. The ability to transfer principles to untaught situations favored pupils taught by Method G. In Grade III, Method G pupils were found superior in applying the combinations to problem situations as measured
by the Philadelphia Standard Tests. When a follow-up test was administered in Grade 4B, it was found that Method G still maintained its superiority.

In general, it was concluded that Method G, which stressed discovery on the part of pupils for forming basic principles and generalizations, was superior to Method R, which relied mainly on repetition.

Other studies, carried on before and since the study made by Nicholson, have given support to the superiority of inductive methods through which pupils discover and verify number facts and relationships, and form generalizations.

In a controlled experiment, McConnell\(^\text{10}\) compared the effectiveness of two methods of teaching the one hundred basic addition and the one hundred subtraction facts in the second grade of selected schools in Toledo, Ohio. The experiment lasted more than seven months of the school year.

In summarizing the basic points of each of the two methods, McConnell writes:

Method A, in summary, is characterized by the following procedures: (1) The number combinations, considered as S-R connections, are identified authoritatively, and supposedly without meaning. The pupil takes the word of the teacher that his work is wrong and depends on the teacher to supply him with the right answer. (2) The child learns these combinations by the process of literal repetition.

Method B, in summary, is characterized by the following procedures: (1) The number combinations are identified by the child through the active process

\(^{10}\text{T. R. McConnell, Discovery vs Authoritative Identification in the Learning of Children.}\)
of discovery or verification. The pupil discovers his errors for himself and finds the correct answer on his own initiative. (2) Learning is a configurational, i.e., a meaningful, not a mechanical process, involving the development of insight, which is facilitated by repetition of stimulus-situations, not literally, but fundamentally, the same. (3) Learning is a relational process.  

Method A was used with part of the second grade children, while Method B was used with the remaining second graders. After equating the two groups of pupils on the basis of arithmetic pretest scores and intelligence quotients, 441 pupils remained in Group A, and 422 in Group B.

Test data revealed that Group A, taught by mechanical repetition, was superior to Group B, taught by a meaningful discovery and verification method, on the tests which stressed automatic and immediate responses to the number facts. Pupils from Group B, however, were found to be superior on those tests which put a premium on ability to transfer principles to untaught number situations and to do more thoughtful manipulations.

It was concluded that if immediate response was the main objective, then Method A was superior, but if meaningful thinking was the objective, then Method B was superior.

Following methods similar to those used by Nicholson and McConnell, Thiele found that children who were taught the 100 addition facts by an inductive method were

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11ibid., p. 16.

12C. L. Thiele, The Contributions of Generalization to The Learning of the Addition Facts.
significantly superior in both achievement and transfer ability to children taught by a method stressing repetition.

Swenson\(^\text{13}\) investigated the relative effectiveness of three different teaching methods for teaching second grade children the basic addition combinations. The generalization method encouraged the comprehension of relationships and the discovery and formulation of meaningful generalizations. The drill method emphasized the formation of specific bonds or connections directly without helping children to rationalize the correct responses. This method discouraged children from attempting to derive answers meaningfully.

The drill-plus method, a compromise between the generalization and the drill method, gave the children a limited opportunity to verify the answers and the combinations were grouped by sum. The findings revealed that the children taught by the generalization method were significantly superior in the number of combinations learned, in retention, and in ability to transfer principles to untaught addition and subtraction combinations.

In a similar study, Anderson\(^\text{14}\) investigated the differential effect of two instructional procedures upon achievement in arithmetic. One procedure emphasized discovery,


understanding, and generalization, whereas the other method emphasized the relative discreteness of the elements of knowledge and skill learned by formal drill. In the generalization method, the broad principles of field theory of learning were applied, while in the drill method, the principles of connectionism were stressed. His findings were similar to those reported by McConnell in that when the criterion was ability to transfer what had been learned to untaught situations, the pupils who had been encouraged to understand what they were learning were significantly superior. On the other hand, when the criterion was automatic response, there was little difference between the pupils taught by the two methods.

The above findings were not found to be true for all pupils, however. In general the drill method tended to favor the pupils who were low in ability but high in achievement, whereas, the generalization method tended to favor more the pupils of high ability and low achievement. Anderson concluded that methods which emphasized relational learning, discovery, and generalization were more productive when ability to do mathematical thinking and to transfer principles to new number situations were considered as major objectives.

In a controlled investigation Howard\(^{15}\) showed that a


For a more detailed summary of the above study see Chapter VII.
method which embodied both meaningful instruction and purposeful practice was superior to methods which placed their sole emphasis on either meaning or drill. The superiority of the meaning plus practice method was found especially evident when the subjects participating in the study were tested after a long vacation period.

Having a significant bearing concerning the most effective method for teaching generalizations is a study made by Hendrix. Hendrix found that a method which made it possible for the subjects to discover generalizations for themselves was superior to a method in which the teacher identified the generalization or to a method in which the subjects discovered the generalization for themselves and then were pressed into verbalizing the understanding.

Overview: drill methods versus inductive generalization methods. -- The above evidence, considered as a whole, points to the superiority of a method which makes it possible for children to discover and verify number facts and relationships and to form generalizations for themselves. This self-discovery or inductive method is superior to a method in which the teacher identifies number facts and rules and has the children memorize these facts and rules through repetitive drill. The findings definitely favor the inductive method when the major objective is to have children do mathematical thinking.

There are some inconsistencies in the reported results relative to the criterion of immediate response. On the basis of the present evidence, however, one may conclude that if blind immediate response is the objective sought for, it may not make too much difference which method is followed, but if the objective is to have children do meaningful thinking in using and applying number, the inductive method definitely has more to offer. The inductive method is definitely superior in promoting retention of skills and in transfer effect in dealing with new number situations.

It seems prudent to follow the suggestion made by Anderson\(^{17}\) that further research be made concerning the relative value of drill and inductive methods for pupils of varying levels of mental ability. Costello\(^{18}\) and others have shown that pupils of low mental ability are particularly weak in the ability to generalize, an ability on which inductive methods place a big premium.

**Activity methods**—Activity methods are characterized as being more informal and indirect than are the formal drill methods of teaching arithmetic. Research has done much to evaluate the effectiveness of activity methods in the teaching of arithmetic.

\(^{17}\)Anderson, *op. cit.*

In a pioneer study, Collings sought the answer to the following questions: "Can the country school curriculum be selected directly from the purposes of boys and girls in real life. If so, to what extent, with what effect, and under what conditions?"

For this experiment, three rural schools located in McDonald County, Missouri, were selected. One school, called the Experimental School, had an enrollment of forty-one children ranging from six years of age to fifteen. The other two schools were known as the Control Schools and had a total enrollment of sixty children ranging in age from six to sixteen years. The schools were considered equal in all aspects except for the curriculum which was the chief variable in the study. The children in the Experimental School followed a curriculum selected directly from their purposes in real life. The traditional subject curriculum was used with the children in the Control Schools.

Four types of projects—play, excursion, story, and hand—were used to afford the children an opportunity to realize their own purposes during the operation of the Experimental School. Arithmetic and other subject matter was used and drawn on as needed in helping children carry their projects to completion.

In 1917, the children in the three schools were given standardized tests to determine their status in the "Three R"

\[\text{Collings, op. cit.}\]

\[\text{Ibid., p. 4.}\]
subjects. Four years later, in 1921, the children were tested again. In order to compare the achievement of the two groups in the common facts and skills, the children were paired (one from each group) on the basis of intelligence, chronological age, number of years' schooling, and number of years spent in the schools if the experiment. Also, the achievement of the children from the Experimental School was compared with the standards set by national norms.

The findings indicated that the Experimental School, when compared with the Control Schools, was definitely superior in everyone of sixty-eight median achievement scores. The Experimental School was superior in thirty-six out of fifty-seven median achievement scores when compared with the national standards. In regard to arithmetic, the data revealed that when the children from each grade from the first through the eighth were compared as to achievement in the four fundamental operations, the difference favored the Experimental School in every instance.

It was found further that both the children and parents from the Experimental School showed far greater improvement in attitudes toward the school and education.

The following conclusions were reached by Collings:

The foregoing data would seem unquestionably to indicate that the curriculum can be selected directly from the purposes of boys and girls in real life.

The Experimental School outcomes and its general practicability in the other schools of the county
seem to justify the conclusion that the curriculum can be selected entirely from the purposes of boys and girls in real life.\textsuperscript{21}

Additional studies have borne out the advantages to be derived from activity methods. Through arithmetic activity units, which were selected because they were rich in the application of number and because they were of real concern to the children, Harap and Mapes,\textsuperscript{22} and Harap and Barnett\textsuperscript{23} found that children learned skills out of sequential order and without special drill as effectively as did children who were taught by more formal methods which stressed drill. Findings reported by Passehl\textsuperscript{24} were consistent with those reported by Harap and Mapes.

Harding and Bryant,\textsuperscript{25} in a controlled experiment with fourth grade children, taught arithmetic through the medium of purposeful units. Skills were taught as needed. The teacher worked a few examples to demonstrate a process and the children worked a few examples to clear up any misunderstanding.

\textsuperscript{21}Ibid., pp. 339-340.


but drill as a teaching technique was not stressed. Textbooks were used as one would use a good general reference book. It was found that the experimental children were superior in most respects in arithmetic to children who had been taught by a method which stressed drill and the following of a textbook from day to day. The experimental children showed greater evidence of emotional and social growth than did the control children.

Williams taught arithmetic through integrated experience units, supplemented by a scheduled arithmetic period in which she gave special help to the children in learning number skills needed in pursuing the unit in progress. She found that the children over a period of nine successive years made gains of 1.3, 1.24, 1.4, 1.89, 2.0, 1.69, 1.8, 1.9, and 1.8, respectively. It can be seen that the gains were superior when the normal gain expected is only 1.0. Not only did the children show significant gains in both computation and problem solving, but when they were compared with comparable children who had been taught by more traditional methods, the children were found superior in desirable emotional and social characteristics.

Each of the above investigators (Harap and Napes, Harap and Barnett, Fassehl, Harding and Bryant, and Williams) concluded that arithmetic can be taught as successfully through informal activity methods as through the more direct formal methods.

\(^{26}\text{Williams, op. cit.}\)
A comprehensive survey was made of the activity program in New York City under the leadership of J. Cayce Morrison. The results of this survey were described in full detail in Chapter III, "The Curriculum". The findings concerning arithmetic were generally inconclusive. The conclusion reached by the survey committee, however, was:

"... the activity program may be continued with reasonable assurance that children will gain as thorough a mastery of knowledge and skills as they would in the regular program."^28

Hanna^29 gave evidence to show that when arithmetic is taught purely through an incidental approach, number situations do not appear in sufficient quantity to provide a basis for all of the arithmetic needed. He concluded that incidental methods could not be relied on completely for the teaching of arithmetic, indicating that number experiences need to be planned for more specifically than merely leaving them to chance.

**Digest: activity methods.**--Research findings pertaining to arithmetic achievement resulting from activity programs or methods have not been entirely consistent. The evidence from some studies has favored traditional programs slightly,

^27Morrison, op. cit.

^28Ibid., p. 162.

^29Paul R. Hanna, "Opportunities for the Use of Arithmetic in an Activity Program," Tenth Yearbook for the National Council of Teachers of Mathematics, pp. 85-120.
while in others the advantages have been for the activity methods. It is safe to say that arithmetic skills have not suffered due to such methods. The general conclusion has been that arithmetic has been learned as well, if not better, through activity methods as through more traditional methods. Aside from arithmetic achievement, children in the activity schools have been found to be superior to children from regular schools in leadership qualities, and in emotional and social stability.

Most of the studies indicate that arithmetic, even in activity programs, needs to be specifically planned for in the curriculum and not left to chance. Incidental methods, although high in motivational values, cannot be relied on completely for arithmetic teaching.

Crude procedures versus direct procedures.—Of value in helping one form a concept of general method have been experimental investigations which have compared the relative merits of methods stressing direct learning with methods which have recognized learning to be a gradual process and have thus encouraged children to use immature procedures in their treatment of number as they gradually progress toward more mature performance. The foundation for assuming that learning does not take place all at once is based on the evidence presented by Judd, who showed that the adult level in counting ability was not reached before the fifth grade.

even though children had begun to count on the preschool level. Additionally, Brownell, and Carper presented evidence that children pass through several stages in learning to apprehend the number of objects in a group. Also, Brownell and Carper have indicated the stages through which children pass in learning to multiply.

One significant study has been made which reveals the values to be derived from a teaching method which takes into consideration children's maturity level for working with number. Deans conducted an experiment in which she recognized a child's maturity level of thinking in dealing with number situations as having great implications for the method to be used in teaching the child. She investigated the effectiveness of a method of teaching arithmetic in which children's immature and inadequate procedures in dealing with number were accepted as normal and valuable steps toward their achievement of mature behavior with reference to number.

The subjects were children from six second grades in the

31 William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.


33 William A. Brownell and Doris V. Carper, Learning the Multiplication Combinations.

Cincinnati Public School System. After using group tests and interviews to determine the number abilities of the children at that time, an instructional program was launched which was designed to encourage children to progress toward more mature methods of thinking than they were found to be using at the beginning of the study. Deans used manipulative materials to help the children progress from one maturity level to the next. This was in opposition to common drill procedures which take the direct approach of trying to get children to respond to number directly as the adult would.

Interviews were held at intervals throughout the fourteen weeks of the study in order to evaluate the progress of the children from one maturity level to the next. Analysis of the interview data gave evidence of decided growth in maturity of thought processes. A t-test for significance of difference between the number responses in Level 4 (meaningful habitual recall) in the initial and final interviews revealed that differences were significant for all pupil groups and highly significant for four of the groups. The investigator concluded that such a method was of decided advantage in helping children to progress from one maturity level to the next.

The above study is especially significant when viewed in the light of the findings reported by Brownell and Chazal. 35

They found that drill as a teaching method is not sufficient to cause children to progress from one maturity level of performance to the next, but that it merely causes the child to become more proficient in using his present thought process.

The advantage in first arriving at solutions to problems on the sub-symbolic level have been demonstrated by Moser,\(^{36}\) Wilburn,\(^{37}\) Thiele,\(^{38}\) Nicholson,\(^{39}\) Swenson,\(^{40}\) and others. Brownell\(^{41}\) has shown the advantages of using a crutch in subtraction to help children progress from immature to more mature procedures.

Evaluation: crude procedures versus direct procedures.—The evidence shows clearly that children do not learn immediately the direct procedures as used by adults in dealing with number. The evidence shows further that children make significant gains when a teaching method is used which recognizes their level of thinking relative to number and provides instructional materials which help them progress toward the


\(^{38}\)Thiele, op. cit.

\(^{39}\)Nicholson, op. cit.

\(^{40}\)Swenson, op. cit.

level of meaningful habitual recall. A general method for teaching arithmetic should make provision for facilitating children in their progress toward using more mature thought patterns. Direct drill procedures are ineffective in doing this.

Teacher personality and types of leadership.—Regardless of what method a teacher may follow, the results are influenced by the environment in which the learning takes place. The teacher's personality and the type of leadership under which children work contribute to the classroom climate.

A key study showing the influence of teacher personality on children was made by Anderson and Brewer. Fifty-nine second grade children and their respective teachers in two separate classrooms were observed. There were twenty-nine children in one room and thirty in the other. The children in the two rooms were considered comparable in all important variables.

A total of two hours of non-consecutive five minute observations were made of each child by a trained observer. During the five minute observation periods the observer recorded the behavior of a particular child as well as all the dominative and social integrative contacts of the teacher which were directed at that child either as an individual or

as a member of a group.

It was found that the degree of dominative and integrative action on the part of the teacher had an effect on the children's behavior. The children, under the guidance of the teacher who evidenced less dominative but more integrative action toward the children, refrained to a greater extent from annoying disturbances and indicated more voluntary participation in learning experiences than did the children under the teacher who made more dominative and fewer integrative contacts.

On the basis of the evidence presented by Anderson and Brewer, it may be concluded that integrative behavior on the part of the teacher contributes greatly to children's cooperativeness and to their emotional health and stability. Domination by the teacher causes children to become more resistant and is detrimental to their emotional health and development.

Having great implications for teaching methods are the experiments conducted by Lewin, Lippitt, and White. They investigated the effect of different types of leadership upon group behavior. In one experiment the behavior of one group of five ten year old children who were under democratic leadership was compared with the behavior of a comparable group under autocratic leadership. In the second experiment,

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the behavior of four comparable clubs of ten year old boys was studied as each group experienced three types of leadership—democratic, autocratic, and laissez-faire.

Four clubs of boys were organized on a voluntary basis, but the boys were carefully equated on patterns of interpersonal relationships, intellectual, physical and socio-economic status, and personality characteristics, so that the important variables, except that of leadership, were the same in each group. Four adult leaders were selected to conduct club meetings on authoritarian, democratic, and laissez-faire principles. The clubs met weekly for five months, with a different leader and technique of leadership every six weeks. Each leader at some time played each of the three different types of leadership. Observations of each club meeting were made by observers from behind a burlap wall in a darkly shaded area. Stenographic records were made of the social interaction and behavior observed for each club meeting. In summarizing the principal findings the investigators say:

In the first experiment, hostility was 30 times as frequent in the autocratic as in the democratic group. Aggression... was 8 times as frequent.

In the second experiment, one of the five autocracies showed the same aggressive reaction as was found in the first experiment. In the other four autocracies, the boys showed an extremely non-aggressive, "apathetic" pattern of behavior.

Four types of evidence indicate that this lack of aggression was probably not caused by lack of frustration, but by the repressive influence of the autocrat; (a) out bursts of aggression on the
days of transition to a freer atmosphere; (b) a sharp rise of aggression when the autocrat left the room; (c) other indications of generalized apathy, such as an absence of smiling and joking; and (d) the fact that 19 out of 20 boys liked their democratic leader better than their autocratic leader, and 7 out of 10 also like their "laissez-faire" leader better.44

Even though this was only a preliminary report, it seems justified to conclude that group behavior is closely related to the type of leadership under which the group is working. Democratic leadership is best for contributing to the emotional and social behavior of children.

Inference: teacher personality and types of leadership.--The evidence presented indicates that teacher personality and the type of leadership under which boys and girls work influence greatly their emotional and social behavior. If schools are to have objectives larger than subject matter achievement, then it is logical that a general teaching method give consideration to the learning climate generated by teacher personality and type of leadership.

The evidence indicates that integrative behavior on the part of the teacher is more conducive to fostering cooperation and emotional health on the part of the learner than is dominative behavior. The learning climate is best when the teacher guides his class through democratic procedures as opposed to autocratic procedures. For a general teaching method, democratic procedures indicate the need for teacher-pupil planning in executing the details of the work in

44Ibid., p. 298-99.
progress. This is in contrast to the teacher dominating and indicating every move and step to be taken. Democratic leadership does not imply a laissez-faire type of leadership under which the children do as they please.

Summary

General methods have been defined as those methods which may be applied in teaching any of the various phases of arithmetic. Several such methods have been identified and discussed. The most pronounced methods appear to be the cypher book method, the lecture or telling method, the discovery or inductive method, and various activity methods.

Contributions from research have been presented in shedding light on the relative merit of the several methods. Practically any reliable research points to the absurdity of the cypher book method. Kirpatrick\(^45\) made one of the pioneer studies revealing the weakness of isolated drill as a teaching method. Later studies, especially those made by Nicholson,\(^46\) McConnell,\(^47\) Thiele,\(^48\) Swenson,\(^49\) and Anderson,\(^50\) have reflected the superiority of inductive generalization.

\(^{45}\)Kirpatrick, *op. cit.*
\(^{46}\)Nicholson, *op. cit.*
\(^{47}\)McConnell, *op. cit.*
\(^{48}\)Thiele, *op. cit.*
\(^{49}\)Swenson, *op. cit.*
\(^{50}\)Anderson (G. Lester), *op. cit.*
methods as opposed to drill methods. For the most part, these studies have revealed that inductive methods are superior when the criteria are critical thinking, retention of skills, and the ability to transfer principles learned to untaught number situations. Howard\textsuperscript{51} shows the value of an inductive method which provides for meaningful practice as opposed to an inductive method without practice, or of drill by itself. Hendrix\textsuperscript{52} indicates the superiority of a method which promotes the learning of generalizations inductively as opposed to the more direct identification of generalizations by the teacher. No additional advantages were found from pressing children to verbalize their understandings.

Research findings reflect some inconsistencies when the criterion of immediate response is used. Much of the evidence, especially that reported by McConnell,\textsuperscript{53} indicates that if immediate response is the objective sought, a drill method has more to offer, while if meaningful and critical thinking is valued, the inductive method has more to offer. The data reported by Anderson\textsuperscript{54} indicate the possible desirability of drill for children of low mentality. Costello\textsuperscript{55} indicates that children of low mentality are especially weak in the ability to generalize, suggesting the practicability

\textsuperscript{51}Howard, op. cit.
\textsuperscript{52}Hendrix, op. cit.
\textsuperscript{53}McConnell, op. cit.
\textsuperscript{54}Anderson (G. Lester), op. cit.
\textsuperscript{55}Costello, op. cit.
of using more direct methods with such children.

Collings,\textsuperscript{56} Harap and Mapes,\textsuperscript{57} Harap and Barnett,\textsuperscript{58} Passehl,\textsuperscript{59} Harding and Bryant,\textsuperscript{60} and Morrison\textsuperscript{61} report studies in which arithmetic is taught through the medium of functional activities. In such procedures learning by doing is stressed as opposed to more formal procedures involving drill. Although there are slight inconsistencies in the reported findings, for the most part the evidence shows that arithmetic computational skills are learned as effectively through the medium of actitivities as through more formal procedures. The research indicates a definite superiority for activity methods in respect to critical thinking and problem solving ability. Further, research data reflect the superiority of activity methods in promoting emotional and social development of children. Hanna\textsuperscript{62} presents evidence that arithmetic learnings cannot be assured through activity methods which rely completely on an incidental or unplanned approach.

\textsuperscript{56}Collings, \textit{op. cit.}
\textsuperscript{57}Harap and Mapes, "The Learning of Fundamentals in an Arithmetic Activity Program," \textit{op. cit.}
\textsuperscript{58}Harap and Barnett, \textit{op. cit.}
\textsuperscript{59}Passehl, \textit{op. cit.}
\textsuperscript{60}Harding and Bryant, \textit{op. cit.}
\textsuperscript{61}Morrison, \textit{op. cit.}
\textsuperscript{62}Hanna, \textit{op. cit.}
Studies reported by Judd,®³ Brownell,®⁴ Carper,®⁵ Brownell and Carper,®⁶ and Deans®⁷ show the values of a general teaching method which makes it possible for children to use immature procedures as they gradually progress toward mature competency in coping with number. The advantages of a general method which encourages children to arrive at solutions on the sub-symbolic level through the use of concrete materials has been demonstrated by Moser,®⁸ Wilburn,®⁹ Thiele,®⁰ Nicholson,®¹ Swenson,®² and others.

Research further indicates that the learning climate has an influence on children's behavior and consequently a bearing on general method. Anderson and Brewer⁷³ reveal that integrative behavior on the part of the teacher as opposed to dominative behavior contributes more to cooperation, emotional health and stability on the part of the child.

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®³ Judd, op. cit.
®⁴ William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.
®⁵ Carper, op. cit.
®⁶ Brownell and Carper, op. cit.
®⁷ Deans, op. cit.
®⁸ Moser, op. cit.
®⁹ Wilburn, op. cit.
®¹ Thiele, op. cit.
®² Nicholson, op. cit.
®³ Swenson, op. cit.
®⁴ Anderson and Brewer, op. cit.
Strong evidence has been presented by Lewin, Lippitt, and White\textsuperscript{74} to show the superiority of democratic leadership as opposed to autocratic and laissez-faire types of leadership.

Several conclusions, together with implications for teaching, may be offered in view of the research evidence cited above. These conclusions are as follows:

1. The value of any method can best be assessed in terms of the desired outcomes. If blind immediate response is the objective sought, drill methods have about as much to offer as any other method. If meaningful response, critical thinking, retention of skills, and the ability to transfer principles learned to untaught number situations are the objectives valued, then inductive or discovery methods should be followed.

2. Inductive methods are superior to deductive methods for teaching rules, principles or generalizations. Further, a teaching method which recognizes understanding on an unverbalized level is superior to a method which presses for verbalization of an understanding once such understanding has been acquired on the unverbalized level. This implies that teachers must be alert to subtle indications that a child understands, even though he cannot verbalize the understanding. The value of using observation of children as they work as an evaluation technique is strongly implied.

3. There is some possibility that drill methods may be

\textsuperscript{74}Lewin, Lippitt, and White, \textit{op. cit.}
of more value than methods stressing generalizations for children of lower mentality. This possibility suggests the need for using different teaching methods with children of varying abilities. Further research is needed in this area.

4. Although inductive or discovery methods are used, children still profit from meaningful practice. Once children have discovered that certain number facts are true, practice can be used to advantage in helping to strengthen and maintain such learnings. The implication is that practice should follow the acquisition of understanding.

5. Methods which make it possible for children to use crude procedures, concrete materials, and crutches as they gradually progress to mature procedures in working with number are superior to methods which short cut such procedures and insist that children adopt more competent procedures directly. This calls for identifying children's thought patterns used in coping with number and providing appropriate experiences in helping children advance to more mature procedures in reacting to number.

6. Although there are slight inconsistencies in the research findings, it may be said that computational skills are learned equally as well through activity methods as through more formal and direct methods. Activity methods are definitely superior in promoting problem solving ability than are more traditional drill approaches. Activity methods have much to commend them if one considers the development
of the whole child, for such methods have more to offer in promoting the total adjustment of children. Here, again, the outcomes desired will influence the methods selected.

7. It is impractical to consider general methods without considering the general learning climate. Academic learning as well as outcomes related to cooperation and emotional health and stability are fostered in a situation resulting from democratic leadership. This calls for democratic procedures which make use of teacher-pupil planning in setting up and executing the details of the immediate program. This does not imply a situation in which children do as they please.

8. It does not appear practical nor sensible that a teacher follow a given teaching method exclusively, but rather on occasion use aspects of the several methods. Purposeful activities should be used to provide functional learning situations. Skills needed in carrying forward the activities should be taught through inductive or discovery procedures. The teacher should work with children on their respective levels and use appropriate materials in helping children discover number facts and relationships for themselves. Purposeful practice should be used to help the child maintain the skills learned, although the amount of practice needed will be much less than when isolated practice alone is used. Finally, the teacher's general method should embody the best principles of democratic leadership.
AN ANALYSIS AND SYNTHESIS OF RESEARCH RELATING TO
SELECTED AREAS IN THE TEACHING OF ARITHMETIC

Volume II

DISSERTATION

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CHAPTER V

SPECIAL PROCEDURES IN PERFORMING THE
FOUR FUNDAMENTAL OPERATIONS

This chapter will be concerned with the special procedures used in performing and teaching the algorithms of the four fundamental operations. The treatment of this topic will be limited to the addition, subtraction, multiplication, and division of whole numbers. This limitation is made because the separate processes and skills in arithmetic are so numerous that it would be impractical to consider all of them in the present study. Analyzing the research relating to each of the computational operations in arithmetic could well form the basis for another entire study.

With each of the fundamental operations dealing with whole numbers there are specifics about which educators are frequently in disagreement. In the following sections, the major disagreements which occur in regard to each of the fundamental operations will be mentioned briefly and followed by a presentation of the research which has a bearing on the problem.

Addition

The principal questions in the teaching of addition appear to center around the following issues: (1) direct or indirect procedures in teaching addition, (2) the order of
teaching the addition combinations, (3) upward versus downward addition, and (4) whether or not to teach the addition and subtraction combinations together. Each of the above issues will be considered individually.

**Direct or indirect procedures in teaching addition.**—Many teachers assume that children can learn to respond to addition combinations directly and immediately as adults do, while other teachers assume that learning addition facts do not take place all at once. Instead, they believe that children progress through stages in learning to add or to put groups together.

There is much evidence to show that children need to develop addition concepts slowly and from crude beginnings. Brownell\(^1\) has shown that children progress through several stages before learning to apprehend groups in the mature way as used by adults. He demonstrated that in apprehending the number of objects in a group, children first count one-by-one, then do partial counting, and finally achieve habitual response. Other investigators who have substantiated the findings reported by Brownell have been Carper,\(^2\) Riess,\(^3\) and

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\(^1\)William A. Brownell, *The Development of Children's Number Ideas in the Primary Grades.*


McLaughlin. Through a teaching method which used concrete materials and recognized as natural children's crude procedures in apprehending groups, Deans has shown that children can be aided significantly to achieve a mature level of adding or combining groups.

In view of the evidence presented, it may be concluded that addition combinations are best presented by a method which facilitates children to develop addition concepts slowly and from crude beginnings. The pattern of presenting addition combinations should be to proceed from the concrete to the abstract. This is in opposition to presenting the abstract combinations in the initial instruction.

Order of teaching addition combinations.--Another problem which arises in connection with teaching the addition combinations is determining the order in which to present the combinations. Early investigations were geared to determining the comparative difficulty of the various number combinations so that they could be presented in the order of difficulty. The criterion of difficulty was the amount of isolated drill required for children to master the combination. Other investigators have been concerned with presenting number

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combinations so that advantage may be taken of the relationships inherent in the number system.

One important investigation has pointed to the weakness of studies which have tried to determine the comparative difficulty of addition combinations when difficulty was measured by the amount of drill required to learn the combinations. Swenson\(^6\) endeavored to determine whether or not learning method had any effect on the order of difficulty of the addition facts as determined by drill-type learning procedures. The 100 addition facts were taught to three groups of second-graders. Three hundred thirty two second-grade children were divided into three groups by a stratified randomization procedure.

Each group was taught by a different instructional method, termed D, X, and G methods. In the D method, the combinations were taught in the order of difficulty as determined by several difficulty studies. In the G method the combinations were presented in such a way that advantage could be taken of the relationships between the various combinations. Pupils were helped to discover such relationships and form generalizations. The X method was designed to correspond as nearly as possible to the general practice in teaching arithmetic combinations. In the latter method, even though the combinations were presented in an organized pattern according

to size of sum, emphasis was placed upon drill.

After an instructional program lasting fifteen weeks, an attempt was made to evaluate the progress of the three groups. It was found that there occurred a significant difference in relative difficulty of the number combinations studied during the learning period. It was shown that teaching methods which varied chiefly in their degree of emphasis upon organization and generalization had a significant influence on the relative difficulty of the number combinations. The investigator concluded that teaching method had a great influence on the relative difficulty of the number combinations. She recommended that research workers plan and interpret their results in terms of the learning method children are using.

Other key studies have shown the value of presenting addition combinations in such an order that children may discover relationships and form generalizations. Such presentation cuts down on the amount of repetitive drill, helps to maintain learnings, and aids children in dealing with untaught number combinations. Evidence of such values has been demonstrated by McConnell, Thiele, and Wilburn. It was

7T. R. McConnell, Discovery versus Authoritative Identification in the Learning of Children.


shown by Nicholson\textsuperscript{10} that children gained greater control
over the use of the zero combinations when the zero was taught
as a place holder than when the zero facts were taught as
separate number combinations.

It may be concluded that number combinations are best
presented in an order which will enable children to take ad-
vantage of relationships inherent in the number system. Such
a procedure will reduce the amount of practice needed, will
facilitate the retention of facts learned, and will promote
transfer ability to untaught combinations.

Teaching addition and subtraction combinations to-
gerther.--Closely related to the question of deciding the best
order for presenting addition combinations is the question of
whether or not to present related addition and subtraction
facts together or separately. In order to find the answer to
this question, Buckingham\textsuperscript{11} turned to classroom experimen-
tation.

Second grade children from seven school centers partic-
ipated in the experiment. The children in each center were
divided into groups--one group to be taught the combinations
together and the other group taught the addition combinations

\textsuperscript{10}Ethel Lodge Nicholson, "A Comparison of Two Methods
of Teaching Arithmetic, Grades I, II, and III," (Unpublished

\textsuperscript{11}B. R. Buckingham, "Teaching Addition and Subtraction
Facts Together or Separately," Educational Research Bulletin,
VI (May, 1937) pp. 228-29, 240-42.
and then the subtraction facts. The groups were equated and paired on the basis of the Pressy Primary Classification Test. At the end of the experiment six of the seven groups who were taught the combinations together were found to be superior in achievement to those who had learned the combinations separately. The investigator concluded that the evidence was further conclusive in favor of teaching related subtraction and addition combinations together.

Upward or downward addition.—Teachers are frequently faced with the problem of deciding whether to teach children to add upward or downward. The research considering this problem has been limited. One such study was made by Buckingham.\(^\text{12}\) That investigator conducted a controlled experiment in which he attempted to find out whether it is better to teach children to add upward or downward. In seven school centers, a total of 244 second and third grade children were divided into two groups of equal ability on the basis of a preliminary test designed to determine readiness for beginning the study of column addition. The test included the basic addition facts as well as the higher-decade facts.

Both groups in each school center was taught by the same teacher. One group was taught to add upward, while the other group was taught to add downward. The teacher devoted twenty minutes each day to each group to the teaching of column addition.

The duration of the experiment varied with the different groups. When the teacher had taught column addition to the stage where the pupils were fairly proficient in adding columns of one-place numbers, a final test was given and the experiment was terminated. The final test, consisting of fifty examples, was constructed so that each example on Form B when added upward was the same as an example on Form A when added downward. The children who were taught to add upward took Form A of the test, while the children who had been taught to add downward took Form B.

It was found that in six of the seven school centers, the difference in accuracy favored the children who were taught downward addition. Four of these differences were at least 1.5 times the probable error. This was interpreted to mean that if the experiment were repeated under the same conditions, the chance of obtaining similar results could be expected in four cases out of five.

Recognizing possible limitations of the experiment, Buckingham concluded: "... we feel that the results strongly indicate the wisdom of teaching children to add downward. When the verdict at seven different centers is six to one in favor of this method, it is nothing more than good management to follow the figures where they lead."13

In view of the evidence, it is recommended that children be taught to add downward. Although not borne out by research, some additional reasons for adding downward may be

13 Ibid., p. 322.
cited. Downward addition is performed in the opposite direction from the usual subtraction procedure, thus tending to emphasize that one process is the opposite of the other. Adding downward makes it possible to record the answer at the point where the addition is completed. Even though children are taught to add downward, it is recommended that they add upward as a check on the accuracy of their work.

Digest: addition.—Research findings indicate that addition combinations are best taught by a procedure which enables children to develop concepts slowly and from crude beginnings. This means that children should be permitted to count objects one-by-one and to group objects in other ways as they verify number facts and arrive at solutions. Only after such crude beginnings, should children be expected to progress to the stage where they can be automatic in their response to abstract addition combinations.

Additional findings from research point to the definite value of grouping number combinations so that children discover relationships. An example of such a pattern of presentation is that after doubles, such as 3 plus 3 or 4 plus 4 are taught, the near doubles such as 3 plus 4 or 4 plus 5 are taught. Children should be led to discover that the latter facts are only one more that the familiar double. Children can discover the relationship between 4 plus 5 and 5 plus 4, and thus cut down on the number of specific facts to be learned. Grouping combinations so that relationships may be found, cuts down on the degree of difficulty
traditionally assigned to given combinations when taught through isolated drill.

There are definite values to be derived from teaching the related addition and subtraction combinations together. If a child has worked with concrete materials in forming groups, he has had a chance to discover that when four objects and five objects are combined he has nine objects, and likewise, when he removes four objects from the group, he has five. Such a procedure gives a more complete understanding of the total group, and enables the child to discover the relatedness of addition and subtraction as well as permitting him to see that they are opposite processes of working with groups. Seeing the relatedness of addition and subtraction facts cuts down on the amount of practice required to learn the facts.

The advantage appears to be slight in favor of teaching children to add columns downward. Even though downward addition is taught, it seems logical that children be taught to add upward as a check on the accuracy of his work. The social need for such accuracy should do much to motivate the need for checking.

Subtraction

The main controversy in the teaching of subtraction centers around the best procedure to follow in performing the subtractive process. There appears to be several possible methods of subtraction.
Osburn\textsuperscript{14} has identified twelve possible ways of subtracting. In regard to the methods identified by Osburn, Wilson says: "These are combinations of borrowing and equal additions; take-away, additive, and complementary; downward and upward."\textsuperscript{15}

Even though Osburn has identified twelve possible methods of subtraction, Johnson\textsuperscript{16} has given the three principal methods of subtraction used in the United States as the decomposition method, the equal addition method, and the Austrian method. Johnson explains: "It is conservative to say that they cover more than 95 per cent of the subtraction in whole numbers as taught in the United States today, for no other method is given in the arithmetic texts of the last twenty years."\textsuperscript{17}

Each of these three methods may be illustrated by showing the thought process involved as the subtraction is carried out. The thought process for each of these methods for the example 73 follows:


2. The Equal Addition Method.--7 from 13; 3 from 7.


\textsuperscript{15}Guy M. Wilson, Mildred B. Stone, and Charles J. Dalrymple, \textit{Teaching the New Arithmetic}, p. 138.

\textsuperscript{16}John Theodore Johnson, \textit{The Relative Merits of Three Methods of Subtraction}.

\textsuperscript{17}Ibid., p. 2.
3. The Austrian Method.—7 and 6 are 13; 3 and 4 are 7.

Most of the research has compared the relative merits of the decomposition method and the equal addition method. Summaries relating to methods of performing subtraction have been made by Ruch and Mead,\(^{18}\) Johnson,\(^{19}\) Wilson,\(^{20}\) Morton,\(^{21}\) and others. Although the research findings are in conflict, it has generally been shown that when the decomposition method and the equal addition method are both taught by drill procedures and the criterion of success is measured by rate and accuracy, pupils taught by the equal addition method have had a slight advantage.

Of significance in evaluating the relative merits of the decomposition and the equal addition methods of teaching subtraction was the study by Brownell.\(^ {22}\) That investigator objected to the previous research which had attempted to evaluate these two methods of teaching subtraction on the grounds that in most instances the two procedures had been taught by drill techniques and that the comparison of the


\(^{19}\) Johnson, op. cit.

\(^{20}\) Wilson, Stone, and Dairymple, op. cit., pp. 141-45.


merits of the two methods had been limited to the outcomes of rate and accuracy. Brownell felt that previous research had given insufficient attention to the relative ease or difficulty of teaching or learning the two procedures. In Brownell's study the basis of comparison was extended from rate and accuracy to include also understanding on the part of the children as well as their ability to transfer principles learned to untaught situations. In the phase of the study reported here there were approximately 328 third grade children from twelve classes in North Carolina.

The two methods of subtraction were taught in two different ways. One method stressed rational explanation, while the other stressed mechanical performance. There were four experimental groups, two for each of the two subtraction procedures. Half of the children, who were learning the decomposition method, were taught the procedure in a mechanical way (DM-subjects), while the other half of the decomposition subjects were taught the procedure in a rational way (DR-subjects). In a similar way, there were two groups being taught the equal addition method. The two groups consisted of the EAM-subjects (equal addition, mechanical) and the EAR-subjects (equal addition, rational).

In order to help the EAR-children rationalize the subtraction process they were taught the principle that the difference between two numbers is unchanged if the same amount is added to both of the original numbers. With both rational groups, manipulative objects were used to help the children
grasp basic understandings before progressing to the abstract algorism. Also, the groups were permitted to use a crutch to help record the thought process as carried out in the "borrowing" process. On the other hand, the mechanical groups were given the bare essentials as far as understanding was concerned. They were shown how to perform the process and given drill in performing the mechanics involved in the process.

Following the experimental teaching period which lasted for three weeks, computational tests were given and individual interviews were held. The tests gave measures of rate and accuracy in taught skills in "borrowing", as well as skill in untaught or more advanced "borrowing" skills. The interviews were concerned primarily in securing evidence related to the degree of understanding. Six weeks later, following a period when no special attention was given to subtraction, a retention test was given and another interview was held.

The principal findings may be summarized as follows:
(1) In comparing DM-children with DR-children, the differences were significantly in favor of the DR-children. (2) There was no significant difference between EAR and EAM-children. (3) The findings favored both EAR and EAM as opposed to DM-procedures. (It may be observed that the present results agree perfectly with those of previous research which has almost always favored EA). (4) With regard to rate or accuracy, the relative advantages between DR and EA were not all in
the same direction. The DR-children, in the specific skill taught, were more accurate at the end of the instructional period, but not six weeks later. Except on "non-borrowing" subtraction, the DR-subjects were reliably more accurate on the skills as measured by the transfer test.

In view of the findings reported, the following conclusions were offered: (1) If the decomposition method of subtraction is to be taught at all, it had best be taught rationally. (2) If the equal additions method is taught, it would make little difference whether it was taught mechanically or rationally. (3) The values one assigns to the various learning outcomes should influence the choice between DR and EAR. If immediate speed and accuracy are desired, then EAR should be chosen. If retention, and transfer ability are valued, then DR would be chosen.

The findings reported by Brownell and Moser\textsuperscript{23} from a larger study are largely consistent with the findings reported above. The larger study included about 1200 subjects.

In another study, which is reported in detail in Chapter VII, "Instructional Materials", Brownell\textsuperscript{24} shows the advantage of permitting children to use a crutch as they "borrow" in subtraction. The purpose of the crutch was to help the child record his thought process as he decomposed or

\textsuperscript{23}William A. Brownell and Harold E. Moser, \textit{Meaningful versus Mechanical Learning: A Study of Grade III Subtraction}.

changed the minuend as is necessary when "borrowing" takes place. The conclusion was that such a procedure facilitated the learning of subtraction, and its use was recommended.

**Inference: methods of subtraction.**—The early research generally gave a slight advantage to the equal addition method. The method of teaching was mechanical and competency was measured in terms of rate and accuracy. More recent research, especially that reported by Brownell, shows that it makes a difference whether the two principal subtraction methods are taught meaningfully or mechanically. The equal addition method, when taught meaningfully, has a slight advantage in regard to rate and accuracy. On the other hand, the decomposition method, when taught meaningfully, has more to offer when one is considering understanding on the part of children as well as the ability to transfer principles to new situations.

In terms of the larger values, it seems practical that teachers teach the decomposition method, and teach it meaningfully. The process can be demonstrated visually with concrete objects, so that children can actually see what is happening as the subtraction process is performed. The equal addition method cannot be demonstrated through the use of concrete materials, and consequently cannot be made as meaningful.

\[25\]Brownell, "An Experiment on 'Borrowing' in Third-Grade Arithmetic," *op. cit.*
The principal issue in connection with multiplication seems to be whether or not to teach the multiplication tables. This issue appears to stem from the trend in the development of education when multiplication tables were taught as an isolated, almost meaningless, drill procedure. Revolting against such a procedure, educators began considering multiplication only as it incidentally appeared in connection with on-going events in the child's learning environment. This being the case multiplication tables were practically abandoned.

Currently the trend is to teach multiplication combinations or tables, but they are taught meaningfully in connection with quantitative situations which are real to the learner. The big consideration, perhaps, should be, not whether or not multiplication tables will be taught, but rather whether to teach them directly as a competent adult knows then, or whether to teach them more indirectly, recognizing that the learner will go through intermediate growth stages in achieving meaningful automatic response.

Research has done much to indicate the best procedures to follow in teaching the multiplication combinations. Several studies have shown that when multiplication combinations are taught meaningfully, children are aided in learning the combinations. In an informal experiment, Thiele\textsuperscript{26} used an

inductive approach in teaching the combinations. The children built the multiplication tables by addition and then studied the completed tables for the relationships which they could find. Once children had verified the correctness of the facts, they used drill devices to promote automatic response. Although Trice does not present his quantitative data, the teachers who participated in the experiment were convinced that much less repetitive practice was needed than in cases where the combinations were taught without giving consideration to inherent relationships.

Both Fowlkes\(^{27}\) and Wheat\(^{28}\) present evidence that children learn the multiplication combinations with greater ease when they are taught by a method which places emphasis on meaning and understanding. Further, the controlled experiment by Nicholson,\(^{29}\) which was summarized in detail in a previous section of this chapter, bears out the conclusion that the learning outcomes are greater when the multiplication combinations are taught through inductive methods in which children discover multiplication facts and relationships for themselves than when the combinations are taught by a method


\(^{29}\)Nicholson, op. cit.
based on repetitive drill.

Perhaps the most significant investigation to be made concerned with learning the multiplication combinations has been made by Brownell and Carper. These investigators made a study to discover as much as possible about the way in which children learn the multiplication combinations from the end of the third grade to the end of the fifth.

In collecting their data, both group testing and individual testing or interviews were used. The group tests revealed data relative to speed and accuracy. Individual interviews were then held with as many as possible of the children who had taken the group tests for the purpose of determining the thought processes of the children in arriving at their answers.

One phase of the study (the first study) involved four schools, two in Burlington and two in Raleigh, North Carolina. The larger or extended study included children from nineteen school systems in eight different states. The number of subjects in the two investigations totaled more than 4,000. Enough children were interviewed twice to yield a total of 2,887 interview records.

To secure data with respect to thought processes, interviews were arranged with individual pupils within two weeks of the group tests. Eleven categories were selected through a preliminary study for classifying pupils' thought processes

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30William A. Brownell and Doris V. Carper, Learning the Multiplication Combinations.
in dealing with the combinations. The eleven thought processes with descriptions as given by Brownell are as follows:

**H Habituation**, meaningful. The correct answer is confidently given at once, with every indication of understanding.

**M Memory, Rote.** The answer is given at once and with apparent confidence, but there is no evidence of understanding.

**G Guessing.** The answer, usually wrong, is given rather promptly, but it is evident that the child is guessing.

**S Solution.** The child starts with a familiar combination, and adds to, or subtracts from, the product to get the answer. There are many kinds of solution, for example:

- \(3 \times 4 = 2 \times 4 = 8, / 4 = 12\).
- \(3 \times 4 = 4 \times 4 = 16, - 4 = 12\).

**C Counting.** The child adds or counts the same unit several times.

**R Reversal.** The child interchanges multiplier and multiplicand, in order to get a more familiar order.

- \(6 \times 3 = ?; 3 \times 6 = 18\), so \(6 \times 3 = 18\).

**T Tables.** The child starts with a lower combination in the particular table involved and recites the table to the required point.

- \(4 \times 6 = ?; 4 \times 1 = 4; 4 \times 2 = 8; 4 \times 3 = 12, \ldots\) or, \(4 \times 4 = 16; 4 \times 5 = 20; 4 \times 6 = 24\).

**V Visualization.** The child reproduces groups of objects or figures in clear imagery and works with these.

- \(4 \times 5 = ?; 4 \times 5\) (nickel) = 20 or 20.

**N No Attempt.** The child makes no attempt to state the answer.

**I Indeterminate.** The child gives an answer for the combination, but you cannot ascertain how he got it (that is, his processes).

**X Miscellaneous.** The child reports a process not included in the preceding list—or you are uncertain as to its proper classification.\(^{31}\)

At the time of this investigation, drill (repetitive practice) of various kinds was the principal method for teaching the combinations. Only limited use was made of means for giving the children an understanding of the process of

\(^{31}\text{Ibid.},\ p.\ 52.\)
multiplication and a grasp of the relationships among the combinations.

The findings from the group tests for the local investigation (North Carolina) revealed that out of a possible score of 81, the median score for Grade IIIA was 64; Grade IVB, 80; Grades IVA, VB, and VA, 81. The median number of minutes required to complete the 81 combinations were as follows: Grade IIIA, 6:20; IVB, 4:40; IVA, 2:45; VB, 3:25; VA, 2:50. On the basis of this data the investigators concluded that after Grade IVA little improvement takes place in either rate or in accuracy of work. Mastery when measured by these two indices is practically attained by the end of Grade IVA.

By means of Table IV the investigators present the data to show how children in the interviews indicated they arrived at their answers to the combinations.

Concerning the data resulting from the personal interviews, the authors say:

. . . the interview data show that children in the local study did not learn the combinations as they were supposed to learn them. They were taught the combinations by drill, but they did not learn them by repetition—or at least by repeating what they were expected to repeat. Under conditions of drill, sight of "3 X 7" . . . is expected invariably and immediately to set off the response "twenty-one." Drill is supposed to cause the immediate establishment of such direct associations, between factors and products and only the products. In place of using the expected associations, they tended to arrive at answers by a variety of processes. The expected process, H, or possibly M, was, it is true common from Grade IIIA on, but it had not entirely supplanted other processes even at the end of Grade VA.
In a word these children did not learn the facts all at once—at one jump, as it were. Instead, they seemed to learn by a series of jumps. This series carried them from undesirable, or inefficient, or immature processes through processes intermediate in desirability, efficiency, and maturity, to the final stage of meaningful habituation. 32

TABLE IV

LOCAL STUDY: PER CENTS OF ANSWERS OBTAINED
BY VARIOUS PROCEDURES

<table>
<thead>
<tr>
<th>Percentage Distribution, by Half-Grades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Process</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Habituation, Meaningful</td>
</tr>
<tr>
<td>Memory, Rote</td>
</tr>
<tr>
<td>Guessing</td>
</tr>
<tr>
<td>Solution</td>
</tr>
<tr>
<td>Counting</td>
</tr>
<tr>
<td>Reversal</td>
</tr>
<tr>
<td>Table</td>
</tr>
<tr>
<td>Visualization</td>
</tr>
<tr>
<td>No attempt</td>
</tr>
<tr>
<td>Indeterminate</td>
</tr>
<tr>
<td>Miscellaneous</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

Brownell and Carper, Ibid., p. 67.

Variations were found in the thought processes followed by the same child. Seventeen selected subjects were studied individually. Of the seventeen selected subjects, only two employed the same process with all fifteen combinations under special investigation. One other subject used rote memory with fourteen facts and meaningful habituation with one. The

32Ibid., pp. 69-70.
rest of the fourteen subjects used three or more processes, four using three, four using four, five using five, and one using six. The median number of different processes was four. The findings revealed that in general children may employ relatively advanced processes with certain combinations and relatively immature processes with others.

In further reference to the results of individual interviews, the writers expressed the conclusion:

It is only when learning is viewed in terms of accuracy or in terms of promptness of answer that the illusion of quick, sudden learning arises. Actually children do not rapidly come to mature and efficient thought processes and hence to true mastery of the combinations. Instead, they move through a series of stages in thinking on the way to those final processes. In this connection pronounced individual differences were found in the local study, but they were differences in rate of progress. Some children seemed to move considerably faster than others and probably skipped a stage or telescoped two stages. Others moved slowly and probably stopped overlong at some intermediate stage. But such individual differences were only to prevent an oversimplification of the picture of development; the picture itself is unchanged. It takes children a relatively long time to traverse the series of thought states; but traverse them they must, if they are eventually to have useful and intelligent control over the combinations.

When the findings from the larger study are considered, it was found that the additional data corroborated the data from the local study in every important detail. Further evidence was presented to indicate that even though children may give the correct answers to multiplication combinations in group tests, many children use methods which are less

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33 Ibid., p. 83.
mature than meaningful habituation in arriving at their answers.

That children do not learn immediately the most mature way of responding to number has been demonstrated with processes other than multiplication. Judd\textsuperscript{34} found this to be true with counting, and Brownell\textsuperscript{35} and Carper\textsuperscript{36} found this to be true in regard to how children apprehend groups of objects.

**Implications:** multiplication tables.—Social demand makes it imperative that children and adults be competent in being able to multiply with ease and accuracy. To do so, it is necessary that one know the multiplication combinations. The big question concerns how best to learn or teach those combinations.

The research evidence just presented shows that children do not learn the combinations immediately and directly even though they are presented that way. Teachers should recognize that children go through intermediate stages in arriving at meaningful habituation and make provision to help children progress from stage to stage. Children can be facilitated in achieving the ultimate goal (meaningful habitual response) by helping them to develop understanding for

\textsuperscript{34}C. H. Judd, *Psychological Analysis of the Fundamentals of Arithmetic*.

\textsuperscript{35}William A. Brownell, *The Development of Children's Number Ideas in the Primary Grades*.

\textsuperscript{36}Carper, *op. cit.*
the multiplication concepts as they progress from the knowledge that multiplication is a short cut for addition. Children should be helped to build and verify multiplication combinations by the use of crude or immature procedures.

Prompting children to discover relationships among the various combinations cuts down on the amount of repetitive drill which is required when the combinations are studied in isolation. Relationships are seen as children build the so-called tables for themselves. This inductive approach to constructing the tables is in contrast to the older procedure of presenting the tables to children ready made and asking them to memorize them.

Division

Two principal issues need to be considered relative to the teaching of division. One is whether to teach short or long division in the introductory stages. The other has to do with ascertaining the best method for finding the quotient figure with two-figure or larger divisors. These two issues will be considered in turn.

**Short division.**—Short division may be considered as that system of dividing which records only the quotient or answer, and does not make any provision for recording intermediate steps such as partial products or subtractions. The intermediate steps are done mentally. In contrast is long division, which makes provision for recording intermediate steps. Of the examples at the right, the first is worked by
short division, while the second is worked \( \frac{3 r 1}{2} \) by long division. Research has attempted to determine whether to teach short or long division.

John compared the relative value of teaching long or short division. One group of fifth grade pupils was taught to divide by a one-figure divisor by using long division, while another group was taught to use the short form in the same type of division. Following one week's instruction for each group in division with a one-figure divisor by each of the respective forms, both groups were taught to divide by two-figure divisors. John summarizes the results from using the two division forms as follows:

1. Before the pupils were taught to divide by two-digit numbers, Group 1, the group dividing by one-digit numbers by means of the short division form, showed greater speed, and Group 2, the group using the long-division form, showed greater accuracy.

2. After division involving divisors of two or more digits had been taught, the pupils who were taught to use the long-division form in dividing by one-digit numbers were able to solve examples with either kind of divisor more accurately than the pupils who were taught to use the short-division form in dividing by one-digit numbers.

3. The pupils who were taught both forms tended to use the long division form in all examples regardless of the fact that they had been taught to solve examples with one-digit divisors by means of the short-divisor form.38

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38Ibid., pp. 682-83.
John concluded that long division should be taught in preference to short division.

Olander and Sharp\textsuperscript{39} found that three out of four pupils from Grades IV to XII chose to work difficult examples in division with one-figure divisors by long division. Pupils using long-division were also found to be more accurate.

In another study, Grossnickle\textsuperscript{40} endeavored to determine the relative speed and accuracy of pupils when using the short form and the long form of division with a one-figure divisor. A special test was constructed in which easy and difficult divisions were included. In the test was a sampling of 100 different facts out of a possible 450 division facts with a one-figure divisor. Part of the examples contained four figures in the quotient, while part contained three figures. Alternate forms (A and B) of the tests were prepared.

The subjects used in the investigation were pupils in Grades V-XII from the public schools of two school systems near Jersey City, and students in each year of a three year normal school representing Grades XIII, XIV, and XV. About 100 students from Grades V-XV took the test. Alternate pupils in each grade took form A, while the remainder took Form B.

\textsuperscript{39}Herbert T. Olander and E. Preston Sharp, "Long Division Versus Short Division," \textit{Journal of Educational Research}, XXVI. (September, 1932) pp. 6-11.

\textsuperscript{40}Foster E. Grossnickle, "An Experiment with a One-Figure Divisor in Short and Long Division," \textit{Elementary School Journal}, XXXIV (March, April, 1934) pp. 496-506, 590-99.
Those pupils using Form A used short division, while the group using Form B used long division. The law of chance was relied upon to equate the groups. All subjects participating in this investigation had been taught to use short division when dividing by a one-figure divisor.

The investigator summarized the findings as follows:

1. In the case of both easy and the difficult division examples, the subjects using the long-division form divided much more accurately than did those who used the short-form.

2. The subjects using the short-division form completed the entire test in less time than did those who used the long-division form. However, there was no significant difference in the time required to work the difficult division examples by the two different division notations. In the case of easy division examples less time was consumed by the subjects who used the short-division form.

3. The further the grade level of the subjects tested was removed from the grade in which short-division was taught, the more pronounced was the difference in favor of the long-division form.

4. Intelligence, as measured by standard tests of mental ability had no appreciable effect on the accuracy of the subjects. In the case of short-division, the group superior in mental ability solved the examples in less time than did the group inferior in mental ability. In the case of the long-division form there was no significant difference in either speed or accuracy of the upper and the lower halves of the group ranked according to mental ability.41

In view of the evidence, Grossnickle concluded as follows: "The results of this study point unerringly to the conclusions that the long-division form is superior to the short-division form for a one-figure divisor."42

41Ibid., p. 598.
42Ibid., p. 599.
In a later study, Grossnickle presented data which largely substantiate the findings reported above.

Inference: long or short division.--The available research evidence indicates that the long form of division is preferable to the short form. This seems sensible, at least during the initial stages of learning, for it avoids burdening the child with having to keep a mental record of his performance while he is learning to perform division. Further, the long form is needed in the more complicated division problems which the child will be working later.

Much of the research measures the effectiveness of the two procedures in terms of rate and accuracy. More intensive learning studies would be valuable in revealing which form results in greater understanding and insight.

Methods of estimating the quotient figure in division.--Another point of disagreement in division has to do with the method to be used in estimating the quotient figure when the divisor is a two-figure number. Even though Grossnickle identified ten methods for estimating the quotient in long division when the divisor is a two-figure number, there are two methods which appear to be most frequently used and about

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Foster E. Grossnickle, "The Incidence of Error in Division with a One-Figure Divisor When Short and Long Forms of Division Are Used," Journal of Educational Research, XXIX (March, 1936) pp. 509-11.

which most disagreement centers. These two methods are the apparent method and the increase-by-one method.

In the apparent method the tens' figure of a two-figure divisor is not changed in estimating the quotient figure regardless of the value of the units' figure. In the increase-by-one method the tens' figure of the divisor is not changed when the units' figure is 5 or less, but, when the units' figure is 6 or more, the tens' figure is increased by one. Much of the research related to division has been centered around determining the relative merit of the two methods.

Grossnickle attempted to determine the relative worth of the apparent method and the increase-by-one method of estimating the quotient when a two-figure divisor is used. One group of fourth graders was taught by one method, while a comparable group of fourth graders was taught by the other method. No significant differences were found regarding the relative merit of the two methods.

However, on the basis of a further study Grossnickle favors the apparent method. Grossnickle made a study to compare the relative accuracy of the apparent and the increase-by-one methods for estimating the quotient when the divisor is a three-figure number. The data indicated that the percentage of accurate estimations is about the same for a

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46 Foster E. Grossnickle, "Estimating the Quotient by Two Methods in Division with a Three-Figure Divisor," Elementary School Journal, XXXIX (January, 1939) pp. 352-56.
three-figure divisor as for a two-figure divisor. Grossnickle, however, recommends the apparent method because of the possibility of interference by the increase-by-one method due to the pupil not knowing when to increase the guide figure. In this connection Grossnickle writes:

The possibility of interference for a three-figure divisor is much greater for the increase-by-one method than for the apparent method. When the increase-by-one method is used with a two-figure divisor and the units' figure is 6 or more, the guide figure is increased by one. When the divisor is a three-figure number, the tens' figure is used for deciding whether the guide figure is to be increased. This change in position of figures may cause interference. If the divisor is 218, the pupil may change the guide figure to three because of the value of the units' figure, 8. In that case most of the estimations will have to be corrected. On the other hand, if the divisor is 281, the pupil may not increase the guide figure because of the units' figure, 1. The chief benefit which accrues from the increase-by-one method will, therefore, be neglected. For that reason it seems to be a matter of prudence to teach a one-rule procedure. The writer strongly recommends that only the apparent method be used for teaching estimation of the quotient in division when the divisor contains more than one figure.47

Unlike Grossnickle, Morton appears to favor the increase-by-one method. Morton's evidence for favoring the increase-by-one method is backed up by a statistical study in which he has shown that when all possible divisors and dividends are used the chances of getting the correct quotient by the increase-by-one method is greater than by using the

apparent method. In summarizing part of his data Morton says:

If the apparent method is used with the 18,594 examples in which the two-figure divisors end in 6, 7, 8, or 9, correct quotient figures will be obtained in only 35.8 per cent of all cases. The increase-by-one method, however, yields the correct quotient figure in 79.0 per cent. For these 18,594 examples in which a two-figure divisor ends in 6, 7, 8, or 9, the ratio of successes with the increase-by-one method to successes with the apparent method is more than 2.2 to 1.\(^{49}\)

Moser\(^{50}\) considered that the quantitative evidence alone as presented by Morton should not be considered as a valid reason for teaching the increase method. He maintained that if the evidence presented by Morton had practical value that people using the increase-by-one method should be more efficient than those people who use the apparent method. In a test of this hypothesis, Moser presents the following evidence as gathered from a research study in progress. He writes:

Preliminary research with college freshmen at the Towson State Teachers College (Maryland) has failed to indicate differences in division efficiency when students of comparable ability are compared in rate and accuracy. Taken as a group students who used Rule I (Apparent Method) exclusively, on a test of divisions with divisors ending in 6-9, made scores equal to those produced by students employing the two-rule procedure (increase-by-one method). Although incomplete, the data seem to show that, with the one-rule procedure, learning reorganizations are possible

\(^{49}\)Ibid., pp. 146-47.

which can contribute to a rather considerable degree of expertness in the use of the rule.51

Osburn52 concurs with Moser in the conviction that the statistical margin favoring the increase-by-one method is offset by the interference and frustrations resulting from teaching the increase-by-one method in addition to teaching the apparent method.

Conclusions: methods of estimating the quotient figure in division.—Experimental evidence does not present any significant difference between the results secured by the two methods. Statistical evidence reveals that the increase-by-one method gives the true quotient figure more often than does the apparent method. On the other hand, statistical evidence reveals that it is more complicated to correct the estimated quotient to find the true quotient when the increase-by-one method is used. Further, there is likely to be confusion when the increase method is used with three-figure divisors. Also, against the increase-by-one method is the point that the child must learn and use the apparent method when the guide figure in the divisor is less than six. Using the increase-by-one method necessitates the child learning two methods and necessitates his switching back and forth between the two methods.

In view of the lack of evidence showing any significant

51Ibid., p. 519.

difference between the use of the two methods and in view of the disadvantages which can be pointed out from using the increase-by-one method, it is recommended that the apparent method be taught to children.

None of the investigations have revealed any difference in the relative ease of learning and teaching the two methods, nor has a study been made to indicate which method promotes more understanding and insight on the part of the pupil. Studies which have been made have measured success in terms of rate and accuracy. A study should be made in which an evaluation would point out which method has, not only greater advantage in rate and accuracy, but, also, greater advantage in understanding, retention of skills taught, and in transfer possibilities.

Summary

In the foregoing part of this chapter have been presented the major issues pertaining to procedures in performing each of the four fundamental operations. Research was presented to serve as a guide in helping one select the most appropriate procedure to teach children.

Strong evidence is presented by Brownell, 53 Riess, 54 and McLaughlin 55 to show that children need to develop

53 William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.

54 Riess, op. cit.

55 McLaughlin, op. cit.
addition concepts slowly and from crude beginnings. Swenson has shown that teaching method has a significant influence on the relative difficulty of the number combinations. Data presented by McConnell, Thiele, Wilburn, and Nicholson reflect the value of presenting addition combinations in such an order that children discover relationships and form generalizations. Indicating the advantage of presenting related addition and subtraction combinations together has been research made by Buckingham. Also, Buckingham has presented evidence that there is a slight advantage in teaching children to add downward rather than upward.

Most of the research findings, evaluated in terms of rate and accuracy, have shown that pupils taught subtraction by the equal addition method have had a slight advantage. Brownell, however, presents significant data to show that the decomposition method when taught meaningfully is more

56 Swenson, op. cit.
57 McConnell, op. cit.
59 Wilburn, op. cit.
60 Nicholson, op. cit.
61 Buckingham, "Teaching Addition and Subtraction Facts Together or Separately," op. cit.
62 Buckingham, "Upward Versus Downward Addition," op. cit.
63 Brownell, "An Experiment on 'Borrowing' in Third-Grade Arithmetic," op. cit.
conducive to promoting retention and transfer ability. On the other hand, equal addition taught meaningfully is more conducive to fostering speed and accuracy.

Thiele, Fowlkes, and Wheat present evidence which reveals that multiplication combinations are learned with greater ease when they are taught meaningfully. Brownell and Carper indicate that children do not learn the multiplication combinations directly, even though they are taught that way. Instead, children progress through a series of intermediate steps on the road to meaningful habitual response.

John, Olander and Sharp, and Grossnickle reflect the superiority of teaching long division as opposed to teaching short division. Grossnickle did not find any significant differences concerning the relative merit of the apparent and increase-by-one methods of estimating the quotient figure in division. Morton, however, presents statistical

65Fowlkes, op. cit.
66Wheat, op. cit.
67Brownell and Carper, op. cit.
68John, op. cit.
69Olander and Sharp, op. cit.
70Grossnickle, "An Experiment with a one-Figure Divisor in Short and Long Division," op. cit.
71Grossnickle, "An Experiment with Two Methods of Estimation of the Quotient," op. cit.
72Morton, "Estimating Quotient Figures When Dividing by Two-Place Numbers," op. cit.
evidence that the increase-by-one method yields the correct quotient more often than does the apparent method. On the other hand, both Moser\textsuperscript{73} and Osburn\textsuperscript{74} show that in spite of the statistical margin favoring the increase-by-one method, adults using the apparent method are equally as capable as those using the increase-by-one method.

After considering the weight of the research evidence summarized above concerning procedures in performing the four fundamental operations, the following conclusions and implications for teaching are offered:

1. Addition combinations are best presented by a method which facilitates children's development of addition concepts slowly and from crude beginnings. This implies that in presenting addition combinations teachers should proceed from the concrete to the abstract.

2. Number combinations are best presented in an order which will enable children to take advantage of relationships inherent in the number system. Further, a teaching method which stresses meanings and relationships has a significant influence on the relative difficulty of the number combinations. Such presentation cuts down on the amount of repetitive practice needed, helps to maintain learnings, and facilitates children in transferring principles learned to new number combinations. This means of presentation is in opposition to

\textsuperscript{73}Moser, \textit{op. cit.}

\textsuperscript{74}Osburn, "Levels of Difficulty in Long Division," \textit{op. cit.}
teaching the combinations as isolated facts to be learned through repetition and memorization.

3. Related addition and subtraction combinations are best taught together. To avoid confusing the two processes, it is possible that initial instruction should stress addition and be followed by subtraction. After any danger of confusing the two processes has passed, the related combinations for the two processes should certainly be taught together. Additional research might reveal whether or not children are handicapped by confusing the two processes in initial instruction.

4. Children should be taught to add downward. Aside from research evidence, it may be pointed out that downward addition is performed in the opposite direction from the usual subtraction procedure, thus helping to emphasize that one process is the opposite of the other. Adding downward makes it possible to record the answer at the point where the addition is completed. Even though children are taught to add downward, it is recommended that they add upward as a check on the accuracy of their work.

5. In terms of the larger values, it is recommended that teachers teach the decomposition method of subtraction and teach it meaningfully. Through the use of concrete objects or manipulative aids it is possible to demonstrate visually what is happening as the decomposition process is performed, while the equal addition method cannot be so
demonstrated.

6. Social demand makes it essential that one know the multiplication combinations. As children are taught the multiplication combinations they should be aided in seeing the relationships among the various combinations, and thus led to develop the combinations inductively for themselves. Children do not learn the combinations directly, even though they are taught that way.

7. In terms of rate and accuracy, the long form of division is preferable to the short form for the initial teaching of division. This is also very likely true when understanding and insight are considered. More intensive learning studies would be valuable in indicating which form results in greater understanding and insight.

8. In view of the lack of evidence showing any significant difference between the use of the apparent method and the increase-by-one method of ascertaining the quotient figure in two-figure division, it is recommended that the apparent method be used. This avoids confusing children with two methods.

It is recommended that research be made to determine the relative ease of learning and teaching the two methods, and to indicate which method promotes more understanding and insight on the part of the pupil. Previous research on the topic has limited itself almost completely to rate and accuracy outcomes.
Problem solving appears more difficult to teach than do computational processes. Most teachers have had the experience of finding children who were able to perform the mechanics of computational processes, and yet were unable to solve verbal problems. Of big concern to teachers is how best to teach problem solving.

This chapter will be divided into three main parts. In the first part the more pronounced methods or procedures which have been used in teaching problem solving will be identified.

The second part will be devoted to presenting the research which has attempted to evaluate the relative merits of the several methods. Finally, in a concluding section, the major research findings will be summarized and significant conclusions offered.

**Distinct Problem Solving Methods**

Several distinct methods or procedures have been used in helping children gain facility in problem solving. Much controversy has arisen concerning the value of the various methods. A description will follow of the specific means used in helping children gain competency in problem solving.
Individual method.--In the individual method the child is allowed to use whatever method of analysis and solution he chooses in solving problems. It is assumed that if the child understands the problem, he can find his own means of analyzing and solving the problem.

Dependencies method.--This method stresses the development of a thought pattern to be followed in the analysis and solution of problems. Hanna describes and illustrates this method as follows:

This method directs the pupil to follow a particular thought pattern in each solution. The pupil determines first what is to be found in the problem. The answer depends upon certain factors of the problem and each factor is likewise dependent upon other factors, and so on, until the pupil has unraveled the essential facts and dependencies in the problem. The following problem will illustrate:

John had 16 cents. He earned 10 more and then spent 15 cents for ice cream. How much money did John have left:

The pupil is directed to think what is asked for:
I am to find the money John has left. To find how much money is left I would have to know the total amount of money he had and the amount spent. To find the total money he had, I would have to know the amount he had at the start and the amount of money he earned.1

Conventional formula method.--The conventional formula method is similar to the dependencies method described above in that it too outlines a plan of formal analysis. In this method the pupil asks himself questions as an aid in analyzing

and solving problems. The questions listed by Hanna to be used in this method are:

1. What is asked for in the problem?
2. What is given in the problem?
3. How should these facts be used to secure the answer?
4. What is the answer?^2

Cue word method.—In the cue word method the children are directed to look for cue words in the wording of the problem which will give a clue to the computational process to be used in solving the problem. The following problem is used for illustration: "John has five apples, and Mary has six apples. How many apples do they have altogether?" In solving a problem of this type, the children are to look for the cue word "altogether" which is supposed to tell them to add. Such words as "more" and "less" indicate subtraction.

Analogy method.—The analogy method is used to teach the children to see the analogy or relationship between a difficult problem and an easy problem involving the same basic ideas or principles. After deciding how to work the easy problem, the children are encouraged to use the same procedure in solving the correspondingly difficult problem.

Pattern method.—The pattern method is a variation of the analogy method. The child is taught the formula for working a particular problem. In order to fix the formula several similar problems are presented in which the child applies the new formula.

^2Ibid., p. 444.
Evaluation of Problem Solving Methods

A look at research should do much to help one select a method to be used in teaching problem solving in arithmetic. Much of the research has compared methods or procedures, while other research has been concerned with ascertaining factors affecting problem solving ability.

Independent versus formal analysis methods.--Hanna compared the merits and defects of the dependencies method, the conventional-formula method, and the individual method of problem solving. (These three methods are identified in the foregoing section). He attempted to measure the gains in ability to solve problems following an extended period of drill on each of the three methods. The subjects were children from twenty-four classes--twelve seventh grades and twelve fourth grades. The findings revealed that superior and average children did almost as well with one method as with another. Pupils of inferior arithmetic ability, however, did significantly better when taught by the dependencies method.

Washburne and Osborne compared the relative merits of the following three methods for teaching problem solving:

1) no special technique, but children given many problems


to solve; (2) formal analysis; and (3) analogy method. Concerning the findings, the investigators say:

Training in the seeing of analogies appears to be equal or slightly superior to training in formal analysis for the superior half of the children; analysis appears to be decidedly superior to analogy for the lower half; but merely giving many problems, without any special technique of analysis or the seeing of analogies, appears to be decidedly the most effective of all.  

Thiele\(^5\) checked the validity of Wasburne and Osborne's conclusion by comparing the relative effectiveness of three methods of teaching problem solving. The methods were: Association Method, Analysis Method, and Vocabulary Method. In the Association Method, sample problems were explained fully and were worked by the teacher and pupils together. These problems served as models. The children were given various types of problems presented in random order. If they failed to work a problem, they were to refer back to the model set of problems to find one like it. In the Analysis Method, the children were given practice in determining what was given, what they must find out, and what procedure to follow in solving the problem. In the Vocabulary Method, the children were aided in working problems by selecting important key words omitted from the wording of the problem.

\(^{5}\)Ibid., p. 304.

Approximately 1200 high fourth grade children in the Detroit schools were used as subjects. Three groups were matched on the basis of intelligence, home conditions, age, and sex. Each group was taught by one of the three different methods. The gain for the three groups over a period of fifteen weeks were: Association Method, 2.86 problems; Analysis Method, 1.63 problems; and Vocabulary Method, 1.50 problems. The difference favoring the Association Method was found to be statistically significant.

The study was repeated with a smaller group, and it was found that the findings were in substantial agreement with those of the first study. Thiele concluded: "Both studies bear out Washburne and Osborne's conclusion that 'children who were taught no special technic but simply solved many problems surpassed those who spent time learning a method of solving problems.'"

Thiele states further:

May we not argue further that if children are left to make their own generalizations about problem solving from repeated successful experiences that these generalizations are more valuable to them than ready-made types which adults think they should make? If these conclusions are valid, the teaching of technics of problem solving in the lower grades should be discontinued.

Digest: independent versus formal analysis methods.

The research reported thus far concerning methods of problem solving...
solving have given somewhat conflicting results. The general findings, however, appear to favor the independent method. There is also some evidence to suggest that children of different abilities may profit from somewhat different methods.

Even though the studies reported to this point have been concerned with comparing special methods in teaching problem solving, drill has been the general method used in applying each of the various problem solving patterns. The additional studies to be reported should give further insight into the relative worth of the different problem solving procedures, for in these studies insight and understanding are emphasized as opposed to mechanical drill.

The effects of following a set pattern in problem solving.—A significant contribution was made to the professional understanding of problem solving by Luchins.8 That investigator refers to the situation in which several problems, all solvable by one somewhat complex procedure, are presented in succession. In respect to this situation, Luchins tried to determine whether or not the individual will be blinded to using a more direct method when a similar task is given which can be solved by a more direct and simple method.

The subjects used in the various phases of the experiment were from college, adult education, high school, and

elementary school classes. These different groups were used in order to determine whether or not the different age and educational groups reacted in the same way to the problem situations.

The subject matter of the experiment consisted of eleven problems, in which the subjects were to obtain a required volume of water with the use of certain empty jars for measuring. For example, in problem number two the individual was given three empty jars of 21, 127, and 3 quart capacities, respectively. The problem was to measure exactly 100 quarts of water. The solution was obtained by filling the 127-quart jar-(A) and from it was filled the 21-quart jar-(B) once and the 3-quart jar-(C) twice. The formula for working the problem was thus \( B - A - 2C \). The next four problems (numbers 3, 4, 5, and 6) were worked by exactly the same formula. Problems 7, 8, 10, and 11 could be worked by this formula, but they could be worked more efficiently by a shorter, more direct formula. Problem 9 could not be solved by the regular formula but could be solved by the short cut formula by which number 7 could be worked most directly.

In an explanation of terminology, Luchins says: "In line with tradition, the habituation to the repeatedly used procedure (in this case the \( B - A - 2C \) method) will be called an *Einstellung*."\(^\text{10}\) Using the definition from H. C. Warren, *Dictionary of Psychology*, New York: Houghton Mifflin Company,
1934, page 371, Luchings gives the definition of Einstellung as: "'the set which immediately predisposes an organism to one type of motor or conscious act.'" Luchins explains further:

Problems 2, 3, 4, 5, and 6 are then "Einstellung (E) problems" which may generate an "Einstellung Effect" for the subsequent "critical test problems" (7 and 8), which, if the Einstellung operated, would be solved in the Einstellung (E) method, B - A - 2C, and not in the more simple and direct fashion designated as the D-Method.  

Problems ten and eleven were also considered as additional critical problems since the D-Method could be used in solving them too.

The subjects were presented the problems one after the other at two and one-half minute intervals. After completing problem six, one group of subjects were asked to write on their papers, "Don't be blind!" This group was referred to as the DBB Group. The group not given this warning was called the Plain Group. A third group was called the Control Group. For the latter group, problems 2-5 were eliminated, because it was in these problems that the habitual formula pattern was established.

The principal results may be summarized as follows:

(1) The Plain and DBB groups, whether young or old, whether with little education or much, showed large E-Effects. (2) The Control groups showed little or no E-Effects. (3) With

11 Ibid., p. 3.
12 Ibid., p. 4.
the adult groups there were considerable decrease of E-solutions and an increase in D-solutions after both the "Don't be blind!" instruction and the presentation of problem nine. With the elementary children there was found to be little recovery effect—no consistent reliable increases were brought about by problem nine or by the DBB instruction. (4) The E-Effect had a damaging effect on problem nine as shown by the quantitative results. Very few of all control groups failed to solve problem nine. A very high percentage of the Plain Groups (as high as 87\% for some groups) failed to work problem nine. The percentage was not quite so high for the DBB group.

Concerning the principal findings, Luchins writes:

Every group's results... point directly to the fact that the Einstellung—the tendency to repeat the E method in subsequent problems, was clearly a hindrance, preventing a large number of subjects from solving a problem which Control Group subjects solved. On the basis of these results... this formulation seems appropriate: Einstellung—habituation—creates a mechanical state of mind, a blind attitude toward problems; one does not look at the problem on its own merits but is led by a mechanical application of a used method. Thus the habituation—Einstellung—produced in Problem Nine a surprising failure to solve a simple problem, in the same way as it blinded subjects to direct solution in the previously discussed critical problems. /problems 7, 8, 10, and 11/.

Significance: following a set pattern in problem solving.—The study summarized above points clearly to the damaging effect on problem solving skill when problems are presented in such an order that one problem after the other can

\[\text{Ibid. pp. 14-15.}\]
be worked by a given formula. Such a procedure stifles thinking to the extent that when a problem arises the learner is handicapped in solving the problem.

This study has great implications for teaching and for the preparation of instructional materials. For teaching it means that teachers should avoid using a drill approach in teaching problem solving. That is, teachers should avoid having children work over and over problems which can be solved in identically the same way. Instead, teachers should present problems so that each problem varies enough to call for new thinking.

For instructional materials, the findings from this study indicate that problems should be selected and arranged so that each individual problem is presented in a new perspective. Such materials should go a long way toward preventing problem solvers from being blinded to using different and new approaches in solving problems.

The use of cue words versus understanding.—Another approach to gaining an understanding of the conditions leading to effective problem solving was made in an important study by McEwen. The purpose of the investigation was to determine the effect of selected cue words in problem solving. The data were collected by making an analysis of group tests results and information secured through personal interviews.

Two group tests which contained selected cue or identifying phrases were used to determine the extent to which children use such cues in solving verbal problems. The interviews revealed data relative to the ways in which children used the cues.

Test I of the group tests contained ten different identifying phrases which seemed to be associated most closely with one or another of the four fundamental operations. Such cue words, grouped according to the process with which they are most generally associated, are listed as follows: Addition ("in all," "both...together." "total"); Subtraction ("difference between," "fewer," "needed,"); Multiplication ("apiece," "times"); Division ("divided," "shared...equally").

Test II contained cue words used in problems so that they were used with a different meaning than ordinarily associated with the words. In each test the children were merely to indicate whether they would add, subtract, multiply, or divide in working the problems. The underlying purpose was to select processes rather than to find numerical answers.

The first group test was administered to about 2,400 children in Grades IIIA-VIA in seventy different classrooms. Test II was given to 591 pupils in the high division only of Grades IIIA-VIA in sixteen different classrooms. A total of 202 representative children were interviewed concerning how they had decided which process to use in the solutions of the
problems in Test I. In the case of Test II, fifty-five children were selected to be interviewed.

The findings revealed that: (1) Although there was a gradual decline in the use of cues from Grades IIIA-VIA, cues were used extensively in all grades. In Test I, 31 per cent of the children in Grade VIB used one or more verbal cues. The per cent in Grade IIIA was 89 per cent. (2) Certain cues seemed to be associated consistently with each of the four fundamental operations. (3) In test II, where the ordinary meaning of the cue was not used, it was found that many errors were caused by children selecting the process with which the word is ordinarily associated. For example, the cues "in all," "both...together," and "products," caused as many as 40 per cent erroneous addition answers in the four grades. In Grade IIIA, 91 per cent of the pupils responded regularly to one or more of the special kind of cues in Test II. In Grade VIA, the per cent was 41. (4) Poor achievers in problem solving were more affected by cues than were good achievers. (5) Pupils who used cues tended to do so with both "expected" and "unexpected" meanings for those cues. (6) Children of superior problem solving ability were prone to ignore practically all cues. (7) When a test was administered twice within a short period, the pupils of superior ability were more consistent than were the poor achievers.

On the basis of the data, the investigator concluded
that the practice of teaching children to solve problems by having them identify cues with particular problems was unjustified. Such cues were considered to interfere with sound quantitative thinking. McEwen would stress the meaning of the separate processes as well as the interrelationships between processes so that children might think intelligently in selecting the process to be used in problem solving rather than blindly following cue words. In view of the findings, McEwen would also stress the correct principles of procedures based on true understanding as opposed to over emphasis on correct answers, however obtained.

In a study similar to that reported above, Doty\textsuperscript{15} found evidence to substantiate McEwen's conclusion that true understanding is essential to effective problem solving ability. Doty recommended that a teaching program emphasize complete understanding and comprehension of problem situations. This recommendation was made in view of the fact that he found that good problem solvers used procedures characterized by understanding and comprehension. On the other hand, the investigator recommended that undesirable procedures such as formal analysis, the use of cue words, etc., which are based on drill methods of teaching be prevented and broken up.

\textit{Evaluation: the use of cue words versus understanding.} -- The two foregoing studies point clearly to the superiority

of understanding as opposed to using cue words, formal analysis, or other procedures based on drill techniques in promoting problem solving ability on the part of children. An understanding of the separate processes together with an understanding of the interrelationships between the processes will aid children to select intelligently the process to be used in problem solving rather than blindly following cue words. A teaching program should emphasize complete understanding and comprehension of problem situations.

Summary

Several distinct ways of teaching problem solving have been considered in this chapter. These ways have varied from individual methods which stress understanding on the part of the child to more formal methods which stress definite procedures to be followed in teaching problem solving.

In comparing the relative merits of the dependencies method, the conventional-formula method, and the individual method, Hanna found that the average and superior children did about as well with one method as they did with another. It was found, however, that pupils of inferior arithmetic ability did significantly better when taught by the dependencies method. Washburne and Osburn, and Thiele discovered

16Hanna, op. cit.
17Washburne and Osburn, op. cit.
18Thiele, op. cit.
that children who were taught no special method but simply solved many problems surpassed those who spent time learning a method of solving problems.

Luchins\(^{19}\) presents convincing evidence that working similar problems over and over by the same procedure creates a mechanical state of mind so that the learner does not look at a problem on its own merit but is led by a mechanical application of a used method. Both McEwen\(^{20}\) and Doty\(^{21}\) reflect the importance of true understanding of problem situations and procedures as opposed to relying on cue words or other drill techniques for teaching problem solving.

Certain inconsistencies appear concerning the worth of the several methods of teaching problem solving when drill is the general method used in applying the methods. In spite, of inconsistencies, the research findings summarized above justify several conclusions.

1. It is possible that different methods should be used with children of varying abilities. This is a possibility that needs further research.

2. Most of the research supports the independent method. This is especially true if understanding is stressed.

3. Problems should be presented in such an order that one problem after another cannot be worked by a given

\(^{19}\)Luchins, op. cit.

\(^{20}\)McEwen, op. cit.

\(^{21}\)Doty, op. cit.
formula. For teaching this means that teachers should avoid using a drill approach in teaching problem solving by having children work repeatedly problems which can be solved in identically the same way. Each problem should be so presented that new and critical thinking is demanded.

4. Cue words should not be used in teaching problem solving. Instead, in helping children select the process or processes to be used in problem solving, teachers should stress the meaning of the separate processes as well as the interrelationships between processes so that children can think rationally in selecting the process to be used in problem solving.

5. In promoting problem solving skill, understanding should be valued above correct answers, which are frequently obtained by faulty means. Understanding is basic to true problem solving ability.
CHAPTER VII

INSTRUCTIONAL MATERIALS

Instructional materials as used in this study refer to anything which facilitates learning. Grossnickle, Junge, and Metzner give a comprehensive interpretation of instructional materials when they write:

"Materials. . . include any picture, model, book, real activity, or teaching aid which provides experiences to the learner for purposes of (a) introducing, enriching, classifying, or summarizing abstract arithmetic concepts, (b) developing desirable attitudes toward arithmetic, and (c) stimulating further interest and activity on the part of the learner in the subject."¹

The above writers further classify instructional materials into four classifications. They say: "There are four kinds or classes of instructional materials: (a) real experiences, (b) manipulative materials, (c) pictorial materials, and (d) symbolic materials."² They define each of the types as follows:

Real experience. . . means tangible, direct, firsthand experience.

Manipulative materials are those which the pupil is able to feel, touch, handle, and move. They may be real objects which have social applications in our everyday affairs, or they may be


²Ibid., p. 161.
objects which are used to represent an idea or a characteristic of number or of the number system. Such objects as a measuring cup, ruler, scales, thermometer, and milk bottles of different size are used in our daily affairs. Materials designed specifically to help the pupil understand some phase of arithmetic include such objects as an abacus, factfinder, place-value pocket, markers, and fractional parts. The materials in the second group have little or no social significance. . . .

Pictorial materials include such things as charts, graphs, diagrams, and materials which may be projected on a screen. . . .

Symbolic materials constitute the fourth classification. The name implies that written or printed materials are in this group. The sources of symbolic materials are twofold. First, systematic study materials, such as textbook, workbook, and instructional tests; and second, quantitative situations which arise in other fields, as in science or in social studies. . . .

A fifth classification of instructional materials is used in the present study. It will be called crutch materials. In the broad sense all instructional materials, especially manipulative materials, may be considered a crutch. As the term "crutch" is used here, specific reference is made to marks or numerals which a child may use in column addition to record the amount carried from one column to the next, or which he may use in subtraction to help record his thought process as he indicates the change in the numbers in the minuend to show that "borrowing" has taken place. Also, cue words which are used to give hints and suggestions in the solution of verbal problems may be considered as a form of a crutch.

3Ibid., pp. 162-63.
It is doubtful if any recognized authorities in the field of education would object to the general way in which instructional materials are defined and classified above. Serious differences do exist, however, concerning the relative importance of the various types of instructional materials. Differences also exist concerning how and when such materials should be used. In the following section the major points of view pertaining to the role of instructional materials in the teaching of arithmetic will be presented.

Major Points of View Concerning Arithmetic Instructional Materials

Differences which arise concerning arithmetic instructional materials generally arise from the theories held concerning how learning takes place and from the purposes held concerning the role of arithmetic in elementary education. Several distinct points of view may be identified.

Mental discipline concept.—The mental discipline concept of arithmetic was most prominent in this country during the eighteenth and nineteenth centuries, although traces of it may be noted occasionally today. For those who held this concept, arithmetic was valued principally for the mind strengthening effects which were believed to result from the study of arithmetic. Consistent with the mental discipline point of view, instructional materials consisted largely, if not entirely, of purely abstract and impractical computational examples and verbal problems. Practically no
attention was given to real experiences, or to manipulative or pictorial materials.

An example of the type of problem used during the eighteenth and nineteenth centuries for its mind strengthening qualities is the following as given by Smith and Eaton:

A lady was asked her age, who replied thus:
My age, if multiplied by three,
Two-sevenths of that product tripled be;
The square root of two-ninths of that is four--
Now tell my age, or never see me more.  

Few examples were used for providing practice in the use of skills required in doing the computational processes of addition, subtraction, multiplication, or division. It was assumed that if the rule governing these operations was learned, it could be applied in doing the operation when needed.

Drill concept.--Basic to the drill concept of instructional materials is the belief that learning is achieved directly without going through intervening steps. It is assumed further that each skill or specific of learning is learned without being related in any way to other specifics. Little value is given to the background of number experience which either grows out of real quantitative situations or which results from concepts derived from seeing numbers as represented by concrete manipulative materials.

The main purpose of arithmetic is considered to be

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preparation for the needs of adult life. Those skills needed by adults have been determined by surveys, so the objective now is to teach those skills by the most direct means possible. This means using abstract materials almost entirely, for the adult use of number is largely with abstract number. Textbooks and workbooks provide the problems and practice materials. These materials are selected by adults and have little relation to children's immediate concerns. Motivation is occasionally provided through the use of games, flash cards, and other drill devices for the primary purpose of "stamping in" each individual skill or fact.

Little emphasis is placed upon reasoning and understanding. Instead of stressing reasoning in problem solving, cue words such as "more," "less," or "altogether" are emphasized for the hints they give in selecting the fundamental operation to be used in solving the problem at hand.

Emerging curriculum concept.—In the emerging curriculum no pre-planned use is made of any type of arithmetic instructional materials. Problems are solved as they are met in on-going experiences or activities. This means that all problems will be real and supposedly of high motivational value to the children.

Concrete manipulative materials and abstract materials are used as the particular situation demands. With due consideration given to the child's development and understanding, the pattern for using instructional materials will
usually be from real problems as they arise, to manipulative materials, to pictorial materials, to abstract materials. Textbooks will be on hand and used as one would use a reference book.

Little or no use is made of practice materials in developing either computational or problem solving skills. It is assumed that these skills will be learned adequately as they are used in solving firsthand problems.

Science of number concept.—Consistent with the science of number concept of arithmetic, the content to be taught is preplanned and determined. It is the purpose of the school and teachers to see that the content is learned, so the most effective instructional materials and procedures in meeting this objective are used.

Learning is assumed to be a gradual process, which develops from crude beginnings to the level of adult efficiency. Competency consists of more than being able to obtain correct answers and perform the mechanics of the computational processes. Knowing the "why" of the various steps in algorithms and seeing and understanding relationships basic to the number system are considered essentials. This being true, concrete manipulative materials are used by the teacher to demonstrate number relationships and meanings. Progressing from manipulative materials the teacher uses charts, diagrams, and illustrations to aid the child in getting further meaning. Gradually the child is introduced
to abstract illustrations through the use of examples which have been worked out in the textbook.

A textbook is usually the basic piece of instructional material. Problems in the text are usually based on social situations about which the child is familiar, although they are not his immediate problems. Attempts are increasingly being made to point out the social significance of the textbook problems by calling to the child's attention how the problems can be applied to daily problems now and in later adult life.

The pattern in the use of instructional materials is from materials prescribed by a course of study, to pre-planned demonstration with manipulative materials, to pictorial materials, to abstract materials, to real problems. After understanding has been fully achieved, practice materials, such as flash cards, or workbooks are used to promote greater speed and facility in the use of the fact or process being studied.

Planned use of real problems.--In this concept of the use of instructional materials, real problem situations growing out of children's personal number experiences are used as the basic instructional material. Real problems which children have in carrying on home and school activities are used for two reasons. First, they are used in helping children seek answers to their own problems, and second, they are used as the medium through which to teach children needed
arithmetic skills. The teacher keeps his eye on the children's on-coming events which will demand the use of number. Through such purposeful events, the teacher plans to teach arithmetic and plans to see that the sequential nature of number is not violated.

Like the science of number concept, learning is recognized as being a gradual process. Learning is assumed to be best when it is rooted in a rich background of experience. This being true, children from kindergarten on through school are encouraged to use concrete materials or crutches in arriving at solutions to their problems. As children use such materials, they discover for themselves that given number facts and relationships are true. After such experiences in verifying that number facts and relationships are true, children have a better readiness for arriving at solutions to their own problems through the use of more abstract methods.

As the child gains in experience and maturity, he uses abstract information as presented in a textbook to aid him in solving his problems. A textbook is used in the same way that one would use any good general reference book.

Practice materials are used as needed. Children are encouraged to use such materials for the increased advantage to be derived from having a better command over the use of number. After the children have indicated a clear understanding of a fact or skill, practice materials are used on an individual or small group basis.
An Appraisal of Instructional Materials

This section will be devoted primarily to a presentation of research concerned with instructional materials in the teaching of arithmetic. As mentioned earlier in this chapter, a consideration will be given to manipulative materials, pictorial materials, symbolic materials, real experiences, and the use of crutches. Several of these will be considered together, for research has generally not been confined exclusively to one.

Teaching methods using manipulative and pictorial materials. — As defined earlier, manipulative materials refer to objects which may be handled, while pictorial materials include such things as charts, graphs, diagrams, and materials which may be projected on a screen. Pictorial representations may be made of real objects such as apples, children, etc., or of less realistic elements such as circles, squares, or triangles. Considered as a whole, manipulative and pictorial materials may be referred to as audio-visual materials or multi-sensory aids.

In most areas of learning it is generally taken for granted that teaching aids of one kind or another contribute significantly to learning. Dale, Finn, and Hoban\(^5\) made a comparatively recent survey of the research which shows the

worth of audio-visual materials in instruction. This survey included over one-hundred research studies concerning the value of multi-sensory aids in all areas of learning. In view of the evidence revealed by that review of research, the authors sum up the value of such aids by saying:

1. They supply a concrete basis for conceptual thinking and hence reduce meaningless word responses of students.
2. They have a high degree of interest to students.
3. They supply the necessary basis for developmental learning and hence make learning more permanent.
4. They offer a reality of experience which stimulates self-activity on the part of the pupils.
5. They develop a continuity of thought; this is especially true of motion pictures.
6. They contribute to growth of meaning and hence to vocabulary development.
7. They provide experiences not easily secured by other materials and contribute to efficiency, depth, and variety of learning.*

The authors state further in commenting on the value of multi-sensory aids:

Significant gains have been reported in informational learning, retention and recall, thinking and reasoning, activity, interest, imagination, better assimilation, and personal growth and expression; and these results have indicated a time-saving, both in preparation of work, and in completion of minimum essentials.7

Several studies, which have been reviewed in other chapters of this study,8 have shown that when a teaching method is used which makes use of manipulative or pictorial

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6Ibid., p. 255.
7Dale, Finn, and Hoban, loc. cit.
8For more detailed summaries of the studies referred to above see Chapters III and IV.
materials in helping children verify number facts and relationships, children learn more easily, show better retention of skills taught, and show greater ability to transfer principles to untaught number situations. Among such studies which have shown in one way or another the contribution of such materials to learning arithmetic were those made by Moser, McConnell, Deans, Swenson, Thiele, Nicholson, and Wilburn. Morfitt found that children who were taught by a method which made use of concrete


10T. R. McConnell, Discovery versus Authoritative Identification in the Learning of Children.


Materials in aiding children to discover and verify mathematical principles for themselves were superior in both speed and accuracy to children who were taught mathematical processes strictly on the abstract level.

Howard\(^1\) has been one of the few investigators who has tried specifically to determine the relative effectiveness of the use of multi-sensory aids in the field of arithmetic. That investigator endeavored to study the effectiveness of three methods of instruction in the teaching of all processes in fractions on the fifth and sixth grade level. A total of 363 fifth and sixth grade pupils from nine public schools in San Francisco participated in the study during the months of February, March, April, and May, 1947. Each of the three methods was used to teach a group of pupils at four different levels in the fifth and sixth grades, namely, low fifth grade, high fifth grade, low sixth grade, and high sixth grade. The groups using the different methods were equated as to mental age, general arithmetical ability, and ability in the processes of fractions.

The three teaching methods were designated as: (1) drill, (2) meaning, and (3) combination. They are described briefly as follows:

1. **Drill Method.** The children were shown how to

compute fractional processes by the use of abstract symbols only. The children followed the pattern of computation as demonstrated by the teacher without being given an understanding of the reason for using the particular pattern in securing the solution. Once the children could perform the mechanics of a given process, they were given exercises involving computation of a similar nature. No use was made of concrete materials to introduce fractional concepts on a pre-symbolic level.

2. Meaning Method. Concrete materials and charts were used to help children discover fractional concepts, to see relationships, and to form generalizations. Children were to learn the fractional processes through solving verbal problems, rather than through doing drill exercises which stressed the mechanics involved in the particular computational process.

3. Combination Method. Concrete materials were used as in the meaning method described above. After children had developed an understanding of fractional concepts and relationships through using concrete materials, they were given drill exercises similar to those given to the children in the drill group. About half of the time was spent using the meaning method, and the other half of the time was spent using the drill method.

The initial and final tests consisted of the Brueckner Diagnostic Test in Fractions and special tests in problems
involving fractions that were prepared by the investigator. The growth of the pupils in ability to deal with fractions was determined by finding the difference between the initial and final scores for each pupil on each test, and calculating the mean gains for each group from these differences.

The findings revealed that no one of the three experimental groups consistently surpassed the other two groups on all tests at all grade levels. At the low fifth grade level the drill method group surpassed the other groups in achievement on the test of computation, while the combination method group surpassed the meaning method group on the problem tests. At the high fifth grade level the drill method group made greater gains than the other two groups in addition and subtraction of fractions. The drill group was found to be statistically significant in three of four comparisons on tests of computation.

In regard to learning multiplication of fractions at the low sixth grade level, no significant difference was found among the mean gains made by the three groups. In the review of fractions in the sixth grade, the difference among the means of the gains was not significantly different in ten of twelve tests. Most of the correlations computed to determine the relationship of mental age to gains were found to be negative.

On the basis of the evidence, the investigator offered
the two following general conclusions:

1. Three factors influencing the effectiveness of developing children's concepts of fractions in the fifth and sixth grades are: (a) the mental ages of the children, (b) the nature of the mathematical processes, and (c) the nature of the subject matter involved in the initial learning or review.

2. Since the three factors mentioned above vary throughout the school year, a combination method should be employed in which emphasis may be placed at times upon the "drill" method and at times upon the "meaning" method.

Concerning the use of the instructional materials, Howard continues:

Although the use of sets of concrete objects and charts did not yield significant differences, they were very helpful to the teachers of Groups B and C, who were stressing the meaning of fractions. The interest of the pupils in using the materials was very apparent to the investigator. The teachers frequently expressed their satisfaction with the materials supplied, stating that the materials focused the attention of the pupils upon the meaning of fractions. Hence they affected the results by a measurable amount, but to no greater degree than the use of the drill method.

In a follow up study, which took place during the autumn after the intervening summer vacation, Howard repeated the tests given in the original study reported above in the low fifth grade to see what arithmetic the children had remembered from the arithmetic they had been taught the

18 Ibid., p. 86.
19 Ibid., p. 98.
previous spring. At this time a significant difference was found in favor of the group taught by the combination method. The average for the drill group had dropped considerably indicating that the children had forgotten a great deal of the arithmetic they had learned before beginning the summer vacation. The children taught by the meaning and combination methods had lost very little. A few children taught by the combination method actually scored higher than they had on the test when it was given the previous spring. The mean scores for the children taught by the meaning method were consistently lower than the mean scores of the children taught by the combination method which stressed both meaning and practice.

In view of the new evidence revealed by the fall testing, Howard concluded:

1. At the low fifth grade level, children will retain better what they learn in arithmetic if extensive use is made of audio-visual aids and considerable emphasis placed upon teaching the meaning or the "why" of arithmetic. These gains may not be apparent after the first steps have been first taught. In fact, those children taught by a drill approach may, as in Group A of this experiment, score considerably higher on tests given directly at the end of the learning period.

2. A varying relationship should be obtained between the time spent in developing meaning and the amount of practice or drill given through extensive computation. . . . This study indicates clearly that if the teacher omits either the development of the meaning of arithmetic or the provision for adequate practice in computation there is a likelihood that the child will not retain what he has learned, irrespective of how well he appears to answer questions
given directly at the end of the learning situation.  

Synopsis: teaching methods using manipulative and pictorial materials. — For the most part the research referred to above points to significant evidence in favor of using a teaching method which makes use of manipulative or pictorial instructional materials. Moser\(^{22}\) showed that such materials could be used to promote readiness for learning fractions. Deans\(^{23}\) found that using concrete materials helped children to progress to higher maturity levels in their response to number. Evidence was presented by Swenson\(^{24}\) to show that the use of such materials facilitated learning, promoted retention of skills, and improved the ability to transfer learnings to untaught situations. The study by Howard\(^{25}\) shows that less forgetting occurs on the part of children during a long summer vacation when a teaching method is used which uses multi-sensory aids in the teaching of fractions. This was shown to be especially true when the teaching method supplemented the use of such materials with suitable practice materials.

It seems imperative that teachers use such materials in

\(^{21}\) Ibid., p. 29.
\(^{22}\) Moser, op. cit.
\(^{23}\) Deans, op. cit.
\(^{24}\) Swenson, op. cit.
\(^{25}\) Howard, "Three Methods of Teaching Arithmetic," op. cit.
helping children to develop number meanings. As revealed by the research evidence, learning is best when children are encouraged to use the material for themselves in discovering the basis of number facts and relationships.

The value of types of materials.--It has generally been accepted that manipulative materials contribute more to learning than do pictorial materials. Further, it has been generally accepted that pictorial representation of real objects such as apples, children, etc. promote learning more effectively than do pictorial representations of less realistic objects such as circles, squares, or triangles. Several investigations have been concerned with testing the above assumptions.

Carper presented evidence that children who used a grouping process in the simplest concrete situations were successful in obtaining answers for verbal problems and abstract combinations with the same method. It was shown also that children who counted in the concrete situation seldom found their method of solution effective in the abstract situation. After Carper concluded that grouping was a more effective means than counting in apprehending groups, she proceeded to determine which types of materials would be most effective in promoting grouping on the part of children as a way of apprehending groups.

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The methods of forty-eight kindergarten children were observed individually as they used many types of concrete materials in apprehending groups. Test materials were prepared to reveal the effects of variation in: (1) kind of context in which objects were presented; (2) the form or shape of objects used; and (3) the pattern or arrangement of the objects in the group.

The findings indicated that the perceptual form of units, the arrangement of the units, and the kind of context or embeddedness in which the units are presented are important determiners of whether units are dealt with as a whole group or as separate units. It was revealed that children used grouping more frequently when simple pictures were used in a regular pattern and without background or embedding. It was found that concreteness of meaning, such as familiar figures of children as opposed to less concrete figures such as simple geometric figures, was an irrelevant factor in determining whether units would be grasped as a group or considered as separate units. The simplicity of the figure, the regularness of the pattern, and the absence of background were found to be more important than concreteness of meaning.

In explaining the findings, Carper says:

... when the test materials was constructed with geometric figures arranged in a regular pattern on a plain background, a large portion of the children recognized the units immediately as an organized whole. When the material was constructed with pictures of children, etc., with units arranged in an irregular pattern and placed in a pictorial setting, few children were able to grasp groups even with the
smallest number of units. 27

The conclusion reached was that: "... arithmetic situations cannot be classified as to structure in the simple dual categories—abstract and concrete, but the concrete situation may be varied widely by changing the factors which determine its perceptual organization." 28

Dawson 29 has made a further contribution to the professional understanding of the type of instructional material to be used to prompt children to use grouping rather than counting as a means of apprehending groups. He formed the hypothesis that the degree of complexity of the perceptual field was the critical factor in the apprehension of number as a group. This hypothesis is in opposition to the usually accepted view that complexity of representation necessarily moves from the use of real objects, to pictures, to semi-concrete materials (geometric representation) and finally to abstract number symbols. In regard to complexity, Dawson writes: "Complexity is created by a lack of symmetry, by the component elements, by heterogeneity of group elements, and by poor figure-ground relation." 30

To test his hypothesis, Dawson constructed a testing

27 Ibid., p. 150.
28 Ibid., p. 151.
30 Ibid., p. 37.
device consisting of sixteen cards which were used to present each of the numbers 4, 5, 6, and 7 in four levels of complexity. The levels, given in the order of their increasing complexity, were: Type I, geometric, represented by squares, circles, and diamonds; Type II, uniform semigeometric, shown by flower faces or butterflies; Type III, mixed, represented by geometric forms, flower faces, birds, flags, and butterflies; Type IV, complex pictorial, represented by dragon, chair, doll, lamb, and bear. In Type IV, full use was made of the complexity factors, namely, lack of symmetry, heterogeneity of group elements, and poor figure-ground relation. On each card the pictures were arranged to facilitate grouping, so that the pattern of arrangement was not a factor in the study.

A total of one hundred twenty-eight children from the first, second, and fifth grades participated in the study. The test cards were presented to the children individually. Each child was asked to tell as quickly as possible how many things he saw on each card as it was presented. The child's level of response in apprehending the groups pictured on each card was recorded. Listed according to the maturity level of response, the four levels are as follows: Level A, counting; Level B, grouping and counting; Level C, subgrouping; and Level D, total grouping.

The findings indicated that counting as a means of responding increased as the complexity of number presentation
increased. Grouping (Level D) decreased as the complexity of presentation increased. The difference in maturity of response and in time for response between the materials of Type I (simple geometric) and Type II (complex pictorial) was found to be significant.

Dawson concluded that the complexity of the perceptual field was the critical factor in the apprehension of number as a group, and not whether or not the representations were of pictures or geometric figures. The use of complicated group pictures tends to produce counting, not grouping; whereas, the use of materials composed of simple elements promotes grouping as a means of response.

Heard\textsuperscript{31} made a significant contribution toward a better understanding of the relative effectiveness of manipulative and pictorial materials in the teaching of arithmetic. Two groups of third grade children were equated on the basis of ability in arithmetic, intelligence, age, and sex. That investigator taught both groups by a method which placed emphasis on pupil discovery. In addition to using the same textbook materials, both groups used supplementary pictorial and symbolic materials prepared by the teacher for the purpose of making arithmetic meaningful. In this study, pictorial or symbolic materials were considered to be pictures,

The experimental variable in the experiment was the use of manipulative materials by the experimental group in addition to the use of materials which were common to both groups. Manipulative materials were considered to be those teaching aids which a child could move about such as coins, clocks, egg cartons, etc.

Following an instructional period of three and one-half months, the children were tested in order to compare the relative achievement of the two groups in arithmetic. Even though both groups had made significant gains in achievement, the findings did not reveal any significant differences between the two groups as a whole, nor between the upper thirds, the middle thirds, or the lower thirds of the two groups.

The conclusion reached by the investigator was that within the limits of the study, the use of manipulative materials in addition to the use of pictorial materials did not prove to be any more effective than when pictorial and symbolic materials were used alone.

Appraisal: types of multi-sensory aids.—The above studies show that the conventional order of ranking multi-sensory aids according to their supposed contribution to learning is not necessarily the order of their true value. The evidence shows that other factors can influence the effectiveness of such materials.
Carper\textsuperscript{32} has revealed that children use grouping more frequently when simple picture elements are used in a regular pattern without background or embedding. This means that representations of simple geometric shapes are more effective in promoting grouping than are representations of more complicated realistic objects. This same finding was borne out by Dawson.\textsuperscript{33}

Writers of textbooks and workbooks for children need to consider the findings reported by Carper and Dawson. In preparing pictorial materials, consideration needs to be given to the factors of complexity of figures in pictures, the arrangement of figures, and the context in which the figures are presented. Units pictured in groups should be composed of simple picture elements, arranged in a regular pattern as opposed to an irregular pattern, and presented in a plain setting. Such consideration should do much to promote grouping, rather than counting, as a means of responding to quantity.

Heard\textsuperscript{34} has done much to challenge the assumption that manipulative materials are more effective arithmetic instructional aids than are pictorial materials. That investigator has shown that manipulative materials are no more

\begin{itemize}
  \item Carper, \textit{op. cit.}
  \item Dawson, \textit{op. cit.}
  \item Heard, \textit{op. cit.}
\end{itemize}
effective than pictorial materials in the teaching of arithmetic.

The findings reported by Heard would have been more convincing had manipulative materials been used exclusively with one group and pictorial materials with the other group. As it was pictorial materials were used with both groups. Also, the results would be more convincing if the experiment were repeated with larger groups and on different grade levels.

Use of crutches.—As used in the present analysis crutches mean supplementary marks or numerals used by a child to help keep a record of his thought process as he performs computational processes. Those who conceive of learning as being a direct process object to the use of such aids, while those who conceive of learning as a gradual process place much value on crutches as an aid to learning.

Brownell\(^{35}\) conducted an experiment to determine the advantages, if any, in the use of a crutch in borrowing in subtraction. In explaining and demonstrating what is meant by a crutch in borrowing in subtraction, Brownell writes:

One device which has been used consists in having children change the form of subtraction as \(\begin{array}{c} 5 \\ 4 \end{array} \)
illustrated at the right. The tens' figure is reduced by 1, and the new tens' digit (5) is actually written on the paper. Likewise, evidence of the borrowed ten is visibly shown by the insertion of a small 1 above and to the left

of the ones' figure, thus making 17 instead of 7\(^{36}\)

The subjects who participated in the study consisted of 590 third grade pupils in the white schools of North Carolina. The participants were divided into four experimental groups, each of which was composed of four third grade classes.

Group one (NC, or non-crutch group) was not taught the crutch at all. Under varying conditions, groups two, three, and four were taught the borrowing device. After teaching the use of the device, the procedure with each of these three groups was as follows: (1) Group two (R-Class) was required to use the crutch in all daily work. (2) Group three (O-Class) was denied or forbidden to use the crutch any longer. (3) Group four (O-Class) was taught the short form of borrowing without the crutch. From this point on the children in the O-Class were given the option of using or not using the crutch.

A well planned testing program was used to measure the abilities of the experimental groups in subtraction as they used the crutch under the various conditions outlined above: Test I was administered after Groups two, three, and four had been taught to use the crutch and before they proceeded with their changed procedure in the use of the crutch. This test revealed that Groups two, three, and four did much better than did Group one which had not been taught to use the

\(^{36}\text{Ibid.}, p. 415-16\)
crutch at all.

For further comparison, the non-crutch children were compared with an equal number of crutch children selected from all crutch groups so that the two groups were equivalent in average I.Q., M.A., and rate and accuracy scores on the pretests. This comparison revealed that the crutch children had fewer incorrect answers on both borrowing and non-borrowing examples. This superiority of the crutch children seemed to be true for children of all intelligence levels, as well as for children of all levels of ability in rate and accuracy as revealed by the pretest.

The crutch children failed to borrow eleven times when they should have borrowed as compared with 234 times for the non-crutch children. Further, the crutch children made only ninety-eight combination errors as compared with 168 for the non-crutch children. The findings also revealed that the crutch children were not penalized as far as speed was concerned at any stage in learning to subtract.

The data revealed that even though the children were taught to use the crutch, they tended to use it less and less as they gained greater facility in subtraction. In this respect, Brownell explains:

... it appears clear that if use of the crutch is consistently opposed, the device quickly disappears or rapidly approaches the vanishing point (Section D); that if its use is required, many children will nevertheless insist on abandoning it (Section R); and that if given permission to discard the device, children will abandon it in steadily growing numbers (Section O). It also appears that ... some
children (a relatively small number) will continue to rely on the crutch long after it would seem to have lost any large amount of usefulness.\textsuperscript{37}

The conclusion reached in the study was that the crutch does help children learn subtraction, and that most children gradually abandon the use of the crutch, especially if they are encouraged to do so.

The value of the use of the crutch in subtraction is brought out in an additional investigation by Brownell.\textsuperscript{36} The children in this study who used the crutch were aided in learning subtraction.

\textbf{Inference: use of crutches.--}The evidence presented by Brownell in the study summarized above is convincing in showing that the use of the crutch as a learning aid in subtraction facilitates learning. The findings point out further that the use of such an aid is readily dropped by most pupils after they no longer need the device. This is especially true when children are motivated to drop the use of the crutch.

Using the crutch does much to make the process of subtraction meaningful. Brownell and Moser\textsuperscript{39} have shown that when subtraction is taught meaningfully children are more

\textsuperscript{37}Ibid., p. 422.


\textsuperscript{39}William A. Brownell and Harold E. Moser, Meaningful versus Mechanical Learning: A Study of Grade III Subtraction.
capable in doing subtraction than are children taught by less meaningful procedures.

The research evidence concerning the use of the crutch as a learning aid is consistent with the evidence reported earlier in this chapter relative to the values resulting from the use of manipulative materials. Interpreted broadly, manipulative materials may be considered to be crutches. It was shown that the use of such materials greatly promotes learning.

Real experiences.—Real experience, as used in the present analysis, means personal, direct, firsthand experience. In arithmetic, real experiences refer to personal quantitative situations which appear in the lives of children either in or out of school. The value of using such experiences as instructional materials have been borne out by much research evidence.

Most of the research considering real experiences has been reviewed in detail in other chapters of this study, so it will be mentioned only briefly here. In a controlled experiment, Harding and Bryant\(^{40}\) showed that children who solved their own purposeful problems which originated in connection with various school activities were superior in problem solving ability to children who solved only textbook

problems. Of equal importance, the children who solved their own problems were found to be better emotionally and socially adjusted than were the children who used more traditional instructional materials.

Williams\textsuperscript{41} found that children made significant gains in arithmetic when their arithmetic content was selected from problems growing out of quantitative situations met in carrying through experience units. Harap and Mapes\textsuperscript{42} revealed that under the conditions of good teaching, fractions, through all process steps, were learned adequately as problems growing out of real situations in school were solved. Harap and Mapes\textsuperscript{43} found the same to be true with learning decimals. In a study concerned with teaching fractions in the sixth grade, Passehl\textsuperscript{44} reported findings consistent in every important detail with those reported by Harap and Mapes pertaining to fractions.


White made an investigation to determine whether or not experience in the situation involved affects the solving of a problem. The data were based on the solutions given to twelve pairs of problems by a thousand children in Grade IV. Half of the problems were based on situations within the experience of the majority of the children, while the other half of the problems were based on situations foreign to most of the children. The problems were presented in pairs, each pair having one problem based on a situation within the child's experience, and one problem outside the experience of the child. Each pair of problems was equated as to processes involved, difficulty, and common usage of numbers. When a comparison of the scores made on the problems within the child's experience and scores made on the problems outside the child's experience, it was found that the children made significantly higher scores on the problems which were based on situations with which they had had experience.

Going on the assumption that children learn arithmetic best when the child enjoys what he is doing, or when he is so strongly motivated by an alluring purpose as to be unaware of the monotony involved in the enterprise, Harap and Barnett attempted to determine whether integers could be

45 Helen M. White, "Does Experience in the Situation Involved affect the solving of a Problem?" Education, LIV (April, 1934) pp. 451-55.

learned in an activity program for Grade III. Forty-three third grade children with an average I.Q. of 100.6 were given the opportunity to learn arithmetic as they endeavored to carry out their dynamic goal seeking purposes through the medium of ten activity units which were selected because they offered abundant opportunity for computation with integers. No special practice materials were used and new steps were taken up as it became necessary to use them.

As indicated by test results, the children attained a mastery of 96 per cent of the steps set up as the work of the grade. Also, the class attained a final grade equivalent score of 4.1 indicating that the class was a little above the expected normal grade standing for the end of the school year. On the basis of the findings, the investigators concluded that the fundamentals can be learned very effectively through activity units based on real situations.

Conner and Hawkins\(^{47}\) found that children in the Cleveland Junior High Schools did better on standardized tests after the problem materials used in the instructional program became the problems which children needed to solve in everyday affairs. In another study, Lyda\(^{48}\) discovered that experience in the problem situation eliminates approximately


50 per cent of the common errors in problem solving.

Significance: real experiences.--The significance of using real experiences as a part of the instructional materials in arithmetic is borne out most clearly in the contribution that such experiences made to the improvement of computational and problem solving skills. The use of purposeful problem situations makes full use of the advantages to be derived from intrinsic motivation. When such instructional materials are used, the learner has purposes which are real to him as opposed to the more extrinsic purposes which he has for solving problems which are of no immediate concern to him.

Teachers should be alert to the possibilities afforded by children's real numerical problems for the teaching of arithmetic. Using such situations meets the immediate needs of the learner in solving his own problems, and also, affords a means for teaching arithmetic skills which will be needed now and later in adult life.

Use of cue words.--As used in the present study, cue words refer to the use of words which are used to give hints and suggestions in the solution of verbal problems. Examples of such words are "altogether," which ordinarily indicates that addition is called for, and "more," or "less," which are usually interpreted to mean that subtraction is implied.
In a special study, McEwen endeavored to evaluate the effect of selected cue words in problem solving. He found that the use of such words interfered with intelligent and effective problem solving on the part of children. McEwen concluded that the use of such words should not be permitted. He recommended instead that teachers stress the meaning of the separate processes as well as the interrelationships between the processes so that children could think intelligently in selecting the process to be used in problem solving rather than blindly following cue words.

The implications from this study are that teachers should not encourage children to use cue words in determining which fundamental process to use in solving problems. If children are to understand fully the use of the operations in problem solving, the operations should be taught meaningfully. Children should have an understanding of what the processes mean in terms of grouping or regrouping quantities.

Use of textbooks or printed materials.—Textbooks and other printed materials are probably more widely used than any other materials in the teaching of arithmetic. Findings from several research studies have implications for such materials. These studies have been summarized in other chapters of this study, so they will be mentioned only briefly.


For a detailed summary of this study see Chapter VI.
The vocabulary studies by Buswell and John,\textsuperscript{50} and Harrison\textsuperscript{51} point out the many meanings that arithmetical terms have. These investigations pointed out that children fail to understand many technical terms used in arithmetic textbooks because children have not had a meaningful background for such terms. The implication is that schools need to provide concrete experiences in order to give meaning to many of the terms ordinarily used in textbook materials.

These vocabulary studies reflect further the need for textbook writers to make an increased effort to control the vocabularies used in their arithmetic books. Further, such writers need to take into consideration the multiple meanings that many terms have and make provision to help children grasp the special meaning of terms implied in the particular setting in which they are used.

The need for giving increased attention to number vocabulary development is especially significant in view of the data presented by Treacy.\textsuperscript{52} That investigator revealed that

\begin{itemize}
  \item \textsuperscript{50} Guy T. Buswell and Lenore John, \textit{The Vocabulary of Arithmetic}.
  \item \textsuperscript{51} Irene Gertrude Harrison, \textit{Survey of Words and Signs in Two Arithmetic Textbook Series}.
  \item \textsuperscript{52} J. P. Treacy, "The Relationship of Reading Skills to the Ability to Solve Arithmetic Problems," \textit{Journal of Educational Research}, XXXVIII (October, 1944) pp. 86-96.
\end{itemize}
good achievers in arithmetic were significantly superior to poor achievers in four reading skills associated in one way or another with vocabulary.

Implications for the presentation of verbal problems in arithmetic textbooks may be drawn from the experimental investigation conducted by Luchins. In that study, Luchins showed that when problems are designed and presented in such an order that they can all be worked by a set formula, the learner becomes blind to the possibility of using more direct procedures in solving problems. The findings from that study indicate clearly that verbal problems should be so selected and presented that each problem is presented in a new perspective. Without so doing, the learner fails to think and is handicapped in using more effective problem solving procedures.

The surveys made by Woody and Blouch point to the great possibility of promoting number competency through considering number situations appearing in other school texts. Woody examined approximately 13,298 pages of material contained in thirty-eight elementary school textbooks and nine issues of


For a detailed summary of this study see Chapter VI.

54 Clifford Woody, Nature and Amount of Arithmetic in Types of Reading Material for the Elementary Schools.

For a detailed summary of this study see Chapter III.
magazines written for juvenile readers to ascertain the arithmetic needed to understand this material. He found almost innumerable quantitative situations in these elementary reading materials which demanded an understanding of arithmetic.

Blouch, after tabulating statements in texts of fifth and sixth grade geography, found frequent needs for thinking in terms of number in geography studies. Many of these needs occurred in connection with studying area, population, distance, latitude, climate, elevation, and trade.

The two surveys referred to above point to the need for including numerical situations appearing in other elementary texts as part of the instructional materials to be used in the teaching of arithmetic. It is imperative that such materials be used in furthering the understanding of other phases of the elementary school curriculum and in promoting a better understanding of arithmetic itself.

Harding and Bryant showed that an arithmetic textbook can be used as one would use a general reference book in helping children solve problems which are real to them. The children participating in the study referred to the textbook only when they needed specific information in helping

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For a detailed summary of this study see Chapter III.

56 Harding and Bryant, op. cit.
them solve their own problems. Those children were found to be more effective in problem solving than were children who used the textbook daily as the primary source of instructional materials.

A significant experiment was made by Jones\(^57\) in which she discovered that children made far greater gains when textbooks and other instructional materials were adjusted to the achievement level, needs, interests, and rates of work of individuals. With an experimental fourth grade group, the texts and workbooks used in spelling, reading, and arithmetic ranged all the way from second grade to sixth grade in level of difficulty. The materials used by the control group adhered much more closely to the basic fourth grade texts than those used by the experimental group. It was found that children, whether superior, average, or dull, profited from the adaptation of instructional materials to their level of ability. The investigator concluded that in order to provide for the best learning situation, instructional materials, especially text materials, need to be adapted to the learner's ability and needs, regardless of what school grade he may be in.

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For a detailed summary of this study see Chapter VIII.
Evidence reported by Guller and Edwards support the findings reported by Jones in every significant detail. Their evidence strengthens the conclusion that instructional materials need to be adapted to the individual learner.

Conclusions: textbooks and printed materials.—In view of the data presented above, the following conclusions are warranted: (1) Teachers and writers of arithmetic textbooks need to become increasingly aware of the importance of vocabulary appearing in arithmetic printed materials. Provision should be made to help children gain increasing competence in control of the number vocabulary which they meet in printed materials. (2) Verbal problems appearing in texts need to appear in varied settings so that the learner will not be blinded to the most effective problem solving procedures. (3) In order for children to handle general school reading materials more effectively and in order to improve their arithmetic competency, number situations appearing in other school texts and reading materials should be considered as part of the instructional material in arithmetic. (4) Consideration needs to be given to using a text as a general reference book in helping children solve problems which are real to them. (5) Instructional materials, especially text materials, should be adapted to the ability and needs of the


For a detailed summary of this study see Chapter VIII.
learner regardless of what grade he may be in. (6) Picture materials used in textbooks for the purpose of promoting the ability to group number should be composed of simple picture elements, arranged in a regular pattern, and presented in a plain setting.

Summary

In the present study instructional materials have been defined as any materials which facilitate learning. Five types of materials have been considered, namely, real experiences, manipulative materials, pictorial materials, symbolic materials, and crutch materials. The major points of view relating to the use of arithmetic instructional materials have been presented and discussed. It was shown that the differences which arise concerning arithmetic instructional materials generally arise from the point of view held regarding how learning takes place and from the purposes held concerning the role of arithmetic in the elementary school.

The major research findings pertaining to the value and use of various instructional materials will be summarized briefly below. In a concluding section, conclusions and suggested implications for teaching will be offered.

Digest of research findings.--Investigations made by
Moser,\textsuperscript{59} McConnell,\textsuperscript{60} Deans,\textsuperscript{61} Swenson,\textsuperscript{62} Thiele,\textsuperscript{63} Nicholson,\textsuperscript{64} Wilburn,\textsuperscript{65} and Howard\textsuperscript{66} indicate that when a teaching method makes use of manipulative and pictorial materials, children learn more easily, show better retention of skills learned, and display greater ability to transfer learnings to untaught number situations.

Carper,\textsuperscript{67} and Dawson\textsuperscript{68} have revealed that children use grouping as opposed to counting when instructional materials feature simple picture elements used in a regular pattern without background or embedding. They showed further that representations of simple geometric shapes are more effective in promoting grouping than are representations of more complicated realistic objects. Heard\textsuperscript{69} found that manipulative materials were no more effective than pictorial materials in the teaching of arithmetic.

\textsuperscript{59}Moser, \textit{op. cit.}
\textsuperscript{60}McConnell, \textit{op. cit.}
\textsuperscript{61}Deans, \textit{op. cit.}
\textsuperscript{62}Swenson, \textit{op. cit.}
\textsuperscript{63}Thiele, \textit{op. cit.}
\textsuperscript{64}Nicholson, \textit{op. cit.}
\textsuperscript{65}Wilburn, \textit{op. cit.}
\textsuperscript{66}Howard, "Three Methods of Teaching Arithmetic," \textit{op. cit.}
\textsuperscript{67}Carper, \textit{op. cit.}
\textsuperscript{68}Dawson, \textit{op. cit.}
\textsuperscript{69}Heard, \textit{op. cit.}
Brownell\textsuperscript{70} has shown that the use of a crutch in subtraction assists children in learning the borrowing process, and that the use of the crutch need not persist after greater competency in subtraction is acquired. The value of real situations for improving problem solving ability has been borne out by evidence reported by Harding and Bryant,\textsuperscript{71} Williams,\textsuperscript{72} Harap and Mapes,\textsuperscript{73} Passehl,\textsuperscript{74} White,\textsuperscript{75} Harap and Barnett,\textsuperscript{76} Conner and Hawkins,\textsuperscript{77} and Lyda.\textsuperscript{78} It has been revealed by McEwen\textsuperscript{79} that the use of cue words in problem solving is detrimental to effective problem solving ability.

Findings from several research studies have implications for the use of textbook or printed materials. Buswell and John,\textsuperscript{80} Harrison,\textsuperscript{81} and Treacy\textsuperscript{82} have indicated how

\textsuperscript{70}Brownell, "Borrowing in Subtraction," op. cit.
\textsuperscript{71}Harding and Bryant, op. cit.
\textsuperscript{72}Williams, op. cit.
\textsuperscript{73}Harap and Mapes, "The Learning of Fundamentals in an Arithmetic Activity Program," op. cit.
\textsuperscript{74}Passehl, op. cit.
\textsuperscript{75}White, op. cit.
\textsuperscript{76}Harap and Barnett, op. cit.
\textsuperscript{77}Conner and Hawkins, op. cit.
\textsuperscript{78}Lyda, op. cit.
\textsuperscript{79}McEwen, op. cit.
\textsuperscript{80}Buswell and John, op. cit.
\textsuperscript{81}Harrison, op. cit.
\textsuperscript{82}Treacy, op. cit.
important it is for textbook writers to make provision for the proper development of a thorough understanding of vocabulary terms pertaining to number. The importance of presenting problem materials so that each problem is presented in a new perspective is borne out by evidence presented by Luchins. The surveys by Woody and Blouch point to the great possibility of promoting number competency through considering quantitative situations appearing in other school texts. Harding and Bryant have demonstrated that an arithmetic textbook can be used as one would use a general reference book in helping children solve problems which are real to them. Research by Jones and Guiler and Edwards discloses that in order to provide the best learning situation, instructional materials, especially text materials, should be adapted to the learner's ability and needs, regardless of what school grade he may be in.

Conclusions and Implications. The research findings summarized above justify several conclusions. Also, in view of the findings, certain implications for teaching and for the preparation and use of instructional materials may be

83 Luchins, op. cit.
84 Woody, op. cit.
85 Blouch, op. cit.
86 Harding and Bryant, op. cit.
87 Jones, op. cit.
88 Guiler and Edwards, op. cit.
offered. These conclusions and implications are as follows:

1. Teaching methods should utilize manipulative and pictorial materials in helping children learn and develop number facts, concepts, relationships, and generalizations. These materials are used most effectively when children are encouraged to use such materials themselves in discovering number learnings. The use of such materials is consistent with the principle of learning which stresses that learning is facilitated when we proceed from the concrete to the abstract. For teaching this means that manipulative and pictorial materials should be used to help children acquire basic number facts and understandings before having children work with the more abstract number symbols. However, as children gain maturity in working with number, they should be encouraged to use more abstract symbols and processes.

2. There does not appear to be as big a gap between the relative merit of manipulative and pictorial materials as once believed. It is possible that the quality of given materials is of more importance than whether or not the materials are manipulative or pictorial. This is a point that needs further investigation with children of different ages and with the teaching of different phases of arithmetic.

3. For promoting growth in grouping, pictorial materials should feature simple elements used in a regular pattern without background or embedding. Also, simple geometric shapes are more effective in promoting grouping than
are representations of more complicated realistic objects.

Great implications may be drawn for those who prepare children's textbooks or workbooks, or for teachers who prepare much of their own pictorial materials. If children are to use grouping rather than counting one-by-one in apprehending groups, then quantity must be so presented that children are prompted to group rather than count.

4. In teaching borrowing in subtraction, children should be permitted to use a crutch in helping to record the thought process as the minuend is decomposed. When a teaching method makes it possible for children to use such a crutch in the initial stages of learning, the process can be learned more quickly, retained longer, and untaught subtraction situations are more competently met. Children, however, should be encouraged to refrain from using the crutch once it has served its original purpose.

5. Those preparing textbooks and other printed instructional materials should consider carefully the selection and presentation of number vocabulary. Number terms used in such materials need to be more adequately clarified and defined. Teachers should supplement textbook presentations of vocabulary by providing concrete experiences to help children expand and clarify vocabulary meanings.

6. Verbal problems appearing in textbooks or other practice materials should be so presented that each problem calls for new thinking. Textbook writers and teachers
should avoid presenting a series of problems which can be worked by an identical formula or procedure. Such procedures, rather than promoting problem solving ability, actually stifle true critical thinking.

7. Quantitative situations appearing in reading materials used in other areas of the curriculum should form a part of the instructional materials used in the teaching of arithmetic. In order for children to comprehend fully much of the reading materials in the content phases of the curriculum, teachers should help children meet the quantitative aspects of those materials. This means that arithmetic instructional materials cannot be selected exclusively from those ordinarily assigned to the scheduled arithmetic period.

8. An arithmetic textbook can be used effectively as a general reference book in the teaching of arithmetic. This is especially true when problems are drawn from the real life experiences of the children. This means that children use the textbook in finding out how to perform various computational processes needed in performing problems which are needed in advancing their activities. Further, this implies that only those aspects of a text are used which are needed by the learners, rather than covering all material in the text page by page just because it happens to be in the book.

9. Instructional materials, especially text materials, are used best when they are adapted to the learner's ability and needs. This implies that materials on several different
grade levels should be used with children of a given grade, rather than using the same material with all the children in the grade. Using materials in this way calls for using a grouping procedure whereby it is possible to work with individuals and small groups within the classroom.

10. Problems which are real to the learner are more conducive to meeting the immediate needs of the learner and also to promoting problem solving ability than are problems which are unrelated to the learner's life experiences. For instruction, this means that teachers need to be alert to and use the on-going quantitative experiences of children which demand the use of number.

11. The use of cue words in teaching problem solving are ineffective and actually destructive to promoting true problem solving ability. Rather than stressing the mechanical use of cue words, teachers should stress true understanding of the fundamental processes and also of the immediate problem situation.

12. Practice materials should be used only after children have developed an understanding of basic number facts and skills. This implies that after children have acquired such understanding, practice materials are used to help children become more efficient in the use of the skill or process. Such materials are used best with small groups as needed rather than being used on a whole class basis. When
the latter procedure is followed, many children are forced to use the materials when they have already mastered the process, and other children are forced to use the materials before having acquired sufficient background and understanding.
CHAPTER VIII

PROVIDING FOR THE NEEDS OF INDIVIDUAL PUPILS

How best to meet the needs of individual pupils is a problem with which educators are constantly being faced. This is the problem, especially as it pertains to arithmetic, about which this chapter will be concerned.

In the first section of this chapter the various practices followed in providing for the needs of individual pupils will be presented. The research which can offer guidance in helping one formulate a plan or policy for meeting individual needs will be considered in the second section. Finally, in a summary section will be given the major conclusions and implications for teaching.

Major Practices in Meeting Needs of Individual Pupils

Most teachers have made the observation that there is wide variation in the ability and achievement levels of pupils in a given grade. This variation is pointed out clearly by Lindquist after evaluating the results of the Iowa Every-Pupil Tests of Basic Skills which had been administered to hundreds of pupils. Lindquist writes: "On Test A (silent reading comprehension) of the battery, for example, the range from the 1st to the 99th percentile may be 60 score

330
points at the end of the sixth grade.\textsuperscript{1}

Spitzer\textsuperscript{2} presents data secured from the scores made by children in grades three, four, five, and six on the Arithmetic Test of Every-Pupil Tests of Basic Skills. It was shown that in the third grade the high score was 5.0; median score, 3.9; and low score, 2.3. In the sixth grade the high score was 8.9; median score, 7.2; and low score, 4.7.

There would be few who would doubt the existence of wide variation in individual abilities and needs within a given group of children. Grave differences of opinion do exist, however, as to the best procedure for caring for the individual needs of children in the elementary school.

A discussion will follow concerning these various opinions or procedures for meeting individual needs. Of the practices discussed below, it is doubtful if one will find a given practice being followed exclusively in many schools. In most situations one practice is likely to predominate, although most schools on occasion will use portions of the several practices.

Taught as one group.—Perhaps the most frequent pattern is to group children heterogeneously according to grades. In this grouping arrangement, children will vary somewhat in chronological age as well as in mental ability.

\textsuperscript{1}E. F. Lindquist, Manual for Administration and Interpretation of 1938 Iowa Every-Pupil Tests of Basic Skills, p. 23.

\textsuperscript{2}Herbert F. Spitzer, The Teaching of Arithmetic, p. 400.
The common factor is that all children are considered to be in the same grade. Only incidental attention is given to individual needs. Generally the instruction is aimed at the level of the average with the less capable children getting what they can and the more capable children doing the work without being really challenged.

**Grouping within the classroom.**—As in the grouping plan discussed above, the children of the same grade, but of varying abilities, are placed in the same room. In order to meet the needs of the different children, both individual and small group instructional procedures are followed. Usually the teacher groups the children into about three groups and gives instruction according to the needs of the separate groups. Provision is made for individual instruction within the framework of the smaller groups. The emphasis is placed on developmental growth as opposed to remedial instruction. Here each child proceeds as he is ready, making real and solid progress as he goes and continually succeeding as he goes. Usually groupings are flexible so that children find themselves in different groups for different purposes.

Not only is instruction differentiated, but instructional materials are differentiated as well. Instead of giving all children the same instructional materials, many materials of varying degrees of difficulty are used so that children may use those materials which are in line with their ability.

**Arithmetic activities.**—Several schools have endeavored
to meet individual needs through the medium of activities or projects or one type or another. Such activities are frequently selected because of the great demand for the use of number in doing the activities. Children work both as committees and as individuals in carrying out the activities. Individual differences are cared for largely on a basis of interest, for children work on those phases of the activities which are most purposeful to them. In group projects each individual is encouraged to make a contribution which is in keeping with his ability. Formal instruction and practice materials are provided on an individual and small group basis as the need arises. Since the ability to work with number is highly necessary in order to carry to completion the activity in progress, children are supposedly highly motivated to learn and practice the needed skills.

Homogeneous grouping.--When there are enough children in a school to make up several groups of the same grade, they are frequently divided into groups according to mental age, intelligence quotient, achievement, or some other criteria. It is assumed that since the children are similar in ability, the teacher will be able to teach the class as one group and will not have as much need for differentiating instruction as would be true in a more heterogeneous group where the range in abilities is considered to be larger.

Individualized instruction.--Revolting against the evils of non-promotion, overageness, and drop outs, which
result from the rigid graded plan of organization in which children of diverse abilities are expected to meet the same standards, educators made various attempts to individualize instruction within the framework of the graded plan. These revolts found expression in such plans as the Pueblo Plan, the Dalton Plan, and the Winnetka Plan. Although each of these plans differs in specific details, each makes provision for the child to progress at his own rate in completing academic assignments. For example in arithmetic, as a child completes a given assignment or lesson he progresses to the next. The teacher helps each child individually as he needs help.

Special instruction for remedial and slow learning children.--In some situations remedial and slow learning children go to a special room for instruction in arithmetic and other skills. These children work with the larger group in the more general phases of the curriculum.

Failure.--A common means of attempting to meet the needs of individuals is through failing or retaining children who fail to reach a given grade standard in achievement. It is assumed that the extreme needs of slow achieving children can best be met by having them repeat the work of a given grade. Also, by retaining low achieving children, it is assumed that the range in ability represented in the room will be reduced so that the teacher will not need to differentiate instruction in meeting the needs of the group.
Guidance from Research Concerned with Providing for Individual Needs

In view of such diverse means of meeting the needs of the individual learner, those who are faced with the problem of formulating a plan or policy for meeting the varying arithmetic needs of children are likely to be confused. Fortunately, the findings from research can do much to help one formulate such a plan.

Highly individualized instruction.--Perhaps the best known of the highly individualized plans for meeting the needs of individual children have been the Dalton and Winnetka Plans. Research evaluating these strictly individualized procedures has not been too plentiful. In regard to an evaluation of the Dalton Plan, Otto says: "There are no published studies evaluating the Dalton Plan."

Washburne, Vogel and Gray report the results of a survey study which was made to determine the effectiveness of the Winnetka technique of individual instruction. These investigators give five basic principles underlying the Winnetka technique as follows:

... (1) a clear definition of the essentials of the fundamental subjects in terms of units of achievement; (2) self-instructive, self-corrective

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4Carleton Washburne, Mabel Vogel, and William S. Gray, A Survey of the Winnetka Public Schools.
practice materials in these subjects; (3) diagnostic tests to measure achievement; (4) individual subject promotions, within certain limits, on the basis of achievement in the fundamental subjects; (5) and large emphasis on group and creative activities during certain periods of the day.  

In a specific phase of the study an effort was made to ascertain whether the academic subjects which were taught on an individual basis in Winnetka were learned more effectively or less effectively than in schools using the class method. The Winnetka schools were compared with three other schools or systems as follows: (1) a public school system of about the same size and social composition as Winnetka, (2) a university laboratory school, and (3) a progressive experimental private school. In making the comparison and evaluation 28,000 tests were administered in the participating schools.

The results from the tests indicated that the Winnetka children did better in reading, language, and arithmetic than did the children from the other schools. The investigators write:

The results indicate that the mastery of the fundamental facts in arithmetic, reading, and language, as measured by standardized tests, is facilitated somewhat for most pupils by the Winnetka technique. . . . the results of the studies reported in this monograph seriously challenge the validity of group instruction which fails to make adequate provision for individual differences.

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5Ibid., p. 4.
6Ibid., pp. 6-7.
It was concluded that the drill phases of reading, arithmetic, spelling, and formal language as measured by the tests used are better adapted to the varied capacities of individual children through the practice of individualizing instruction than is possible under the traditional class method. It was concluded further:

The burden placed upon the teacher as a result of the individual technique is not onerous; it is somewhat greater than in the typical public school system, but decidedly less than in the private experimental school and the university laboratory school studied.

In a later investigation, Washburne and Raths attempted to determine how well graduates of the Winnetka elementary schools did in high school. The children from Winnetka attend a high school composed of youth from three other villages. In making the study, the separate villages were considered comparable except for the individualized program of the Winnetka schools. It was found that the pupils from Winnetka were the only group above the average of the high school in scholarship in all five major subjects, namely, English, mathematics, history, science, and languages. This was found to be true for each of the four grade levels in the high school. It was concluded that

7 Ibid., p. 133.
Winnetka graduates did better in high school than did graduates from elementary schools which did not offer an individualized program.

Other studies which are to be summarized later in this chapter have shown the significant gains children have made when given a limited amount of individualized instruction. Such evidence is reported by Guiler and Edwards, Tilton, Thompson, and Jacquelin.

**Evaluation: highly individualized instruction.**—From the evidence presented above, it may be concluded that a program of individualized instruction more adequately meets the needs of individual children than does a program which uses a class method without making any provision for individual needs. This conclusion, however, does not eliminate the possibility of meeting individual needs through procedures which are not so highly individualized. Washburne,  

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Vogel and Gray admit that their study does not answer conclusively the following questions:

Can the so-called "fundamentals" be learned more rapidly and effectively as drill exercises apart from their natural setting?

Do pupils learn when to use facts and do they recognize their social significance as well when the facts are taught in individual self-corrective exercises as when introduced in their natural setting?

Do some pupils learn more effectively under the stimulus of group activities than when working alone?  

The research evidence to be reviewed in the following section indicates the dynamics of group action as opposed to individual action. Other data will point to the possibility of meeting individual needs through a combination group and individual technique.

The value of working in groups.—Even though the previous section reported certain academic values resulting from individualized instruction, other studies cause one to doubt the merit of complete individualized instruction by showing the values of working in groups. Such values are not limited to arithmetic or subject matter achievement, but are reflected through better cooperative efforts, group spirit, etc. If one is concerned with the development of the whole child, and not with just the development of his intellect, working in groups has much to offer.

After considering research evidence, Smith and Dolio 

state: "It now seems clear that changes in attitudes and in actual ways of behaving result more readily from participa-

tion of individuals in the process of group decision than

from mere individual activities..." 14

The force of group action, as opposed to the force of individual action, is shown by several studies. Lewin15
cites evidence to show that a group decision was more effec-
tive in changing the food habits of women than was individ-

dual decision. One group was given lectures on why they should use such foods as hearts, kidneys, and brains during the war period. Through group discussion another group was helped to see the problem and to assume responsibility for doing something about it. Only four out of forty-one women de-
cided on their own to use the recommended foods, whereas, twenty-three out of forty-four women in the group decision group used the new foods.

Lewin16 reports another research in which an effort was made to change students from the consumption of white bread to the consumption of whole wheat bread. It was re-

vealed that a group decision was far more effective than


individual decision in promoting eagerness to change to the new bread.

A few studies in arithmetic have indicated values of children working together. Klugman\textsuperscript{17} has shown that children actually achieve more when working together than when working alone. That investigator sought a solution to the problem of determining whether two children, working together, could do more problems correctly and in a shorter period of time than could each child working alone.

One hundred thirty-six school children in Grades IV, V, and VI participated in the study. To these children were administered both Forms A and B of the Otis Arithmetic Reasoning Test, which is made up of twenty arithmetic problems graduated in difficulty. The children were asked to complete all problems.

The children were divided into equated pairs. The first pair was known as I-P, the second as P-I, the third as I-P, etc. I-P indicated that the pair was to be tested individually with Form A, and then on the following day paired to do Form B cooperatively. P-I indicated paired testing with Form A followed by individual testing with Form B. The individual children were asked to work the problems on their own while the paired children were asked to work together in solving the problems.

\textsuperscript{17}Samuel F. Klugman, "Cooperative Versus Individual Efficiency in Problem Solving," \textit{Journal of Educational Psychology}, XXXV (February, 1944) pp. 91-100.
The results indicated that on the average, the 136 children working individually got about six problems correct out of the possible twenty. Their mean scores jumped to about seven when working in pairs. This difference was found to be statistically significant. The time working together was 184 seconds longer than when working individually. This time was reliably longer. The investigator considered that the longer time was primarily due to the presentation, discussion, rejection, and acceptance of a greater number of possible solutions.

In a related study, Brasch investigated the effects of competition, both individual and group, on the achievement of sixth-grade pupils in arithmetic. The children were asked to do a simple addition test consisting of fifty-six items. Two competitive situations were set up. In one situation (individual competition) the pupils worked for individual prizes, and in the other situation (group competition) the group worked for a group prize. On the basis of chance selection, the forty schools which participated in the study were assigned to one of the competitive situations.

Although significant improvement occurred with both forms of motivation, it was found that group competition was more effective in promoting improvement than was

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Irving Brasch, "The Relative Effects of Individual Competition and Group Competition as Motivational Factors in Achievement at Three Levels of Intelligence," Abstracts of Graduate Theses in Education, University of Cincinnati, 1944, pp. 65-78.
individual effort. The difference was statistically significant.

Brasch concluded that under the conditions of this investigation, group competition was greatly superior to individual competition.

Inference: the value of working in groups.—The evidence summarized above points to the effectiveness of group action in promoting behavior of one type or another. Working in groups for the purpose of achieving goals seems to be more effective than does working as individuals to achieve goals. Group effort appears to generate more motivation than does individual effort. The evidence, however, does not seem convincing enough to say that individuals should always work in groups and not alone on many occasions. The following sections of this chapter will reflect the merit of using a combination of group and individual instruction for meeting the needs of individual children.

Grouping within the classroom and providing limited individualized instruction.—One means of meeting the needs of individual children in a given grade is to group the children within the classroom for instructional purposes. Provision is made for individual instruction within the framework of the smaller groups. An experimental evaluation of such a procedure was made by Jones.
Jones sought to determine the difference between the progress in skill development of two groups of children at the fourth grade level when adaptation to individual needs was made in two ways. One group of children, the experimental group, was taught on their individual levels of accomplishment regardless of grade placement. This group followed a curriculum designed to meet individual needs in respect to level and rate of progress. The other group, known as the control group, was taught as a group the curriculum prescribed for their grade. This latter group pursued the traditional course of study in the conventional manner with only minor and incidental provision made for individual differences.

A total of 448 fourth grade children who were beginning their fourth year in school participated in the study. At the beginning of the study, the intelligence quotient, mental age, and the level of achievement in skills were determined for each pupil. At the conclusion of the school year, tests were given to measure the amount of growth for each child for the year.

The instructional materials used with the experimental group varied with the achievement level, needs, interests, and rates of work of the individual pupils. In spelling, reading, and arithmetic, texts and workbooks were provided

on various levels ranging all the way from the second to the sixth grade. The materials used by the control group adhered much more closely to the basic fourth grade texts.

With the control group the most common practice was to work with the class as a whole. By directing the same instruction to all, an effort was made to strike the level of the average. In order to meet the individual needs of the children in the experimental group, the plan of grouping the children within the classroom was followed. The findings are summarized in Table V.

### Table V

**Comparative Growth of Control and Experimental Groups by Levels of Ability and by Subject-Matter Areas**

<table>
<thead>
<tr>
<th>I. Q. Level</th>
<th>N</th>
<th>Reading</th>
<th>Arithmetic</th>
<th>Spelling</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>110 and above</td>
<td>24</td>
<td>1.27</td>
<td>1.38</td>
<td>1.60</td>
<td>1.45</td>
</tr>
<tr>
<td>90-109</td>
<td>82</td>
<td>.66</td>
<td>.91</td>
<td>1.02</td>
<td>1.17</td>
</tr>
<tr>
<td>Below 90</td>
<td>19</td>
<td>.27</td>
<td>.63</td>
<td>.40</td>
<td>.72</td>
</tr>
<tr>
<td>Total</td>
<td>125</td>
<td>.73</td>
<td>.96</td>
<td>1.03</td>
<td>1.19</td>
</tr>
</tbody>
</table>

Jones, Ibid., p. 264.

On the basis of the evidence, the investigator reached the following conclusion:

... children taught on their individual levels regardless of grade placement make a greater amount of growth than comparable pupils taught as a group the curriculum prescribed for their grade with only minor
and incidental provisions for individual differences.\(^{20}\)

Jones pointed out that the difference which favored the experimental group was true in reading, arithmetic, and spelling and was true for superior, normal, and dull children. The investigator states, however: "... the differences in growth as a result of individualization of instruction are more significant for normal and dull children than for superior children."\(^{21}\)

In a further connection, the author concludes:

There is sufficient evidence to support the conclusion that each child is an individual and has a right to materials and procedures within his capacity.

If given such a program each pupil will find himself in a learning situation where he can succeed, where he is not repeating tasks already mastered, and where he can compete favorably with fellow-workers, where he has something to contribute to the group, and where he can gain through participation in the group. This type of teaching-learning situation eliminates failure as such and breaks down any artificial grade barriers. Gaps in learning are eliminated and repetition becomes unnecessary. Education becomes a series of steps toward maturity.\(^{22}\)

Related to the study made by Jones was an investigation made by Guiler and Edwards,\(^{23}\) who attempted to discover the extent to which computational habits of junior high school pupils may be improved by means of a systematic program of

\(^{20}\)Ibid. p. 270.

\(^{21}\)Ibid. p. 270.

\(^{22}\)Ibid. p. 271.

\(^{23}\)Guiler and Edwards, *op. cit.*
The pupils in Grades VII and VIII in the public schools of Shelby County, Ohio, and Lucas County, Ohio, were the subjects in the program. The experiment extended over a period of twenty-three weeks, during which time the experimental pupils received for two periods a week systematic instruction based on individual diagnosis of their difficulties. For the same two periods a week the control pupils followed conventional group-instruction procedures. During the three other arithmetic periods each week, both the experimental and control pupils pursued their regular course work. In each of the three control schools, the class periods were forty-five minutes in length. In two of the experimental schools, the class periods were thirty minutes in length; in the four other experimental schools, they were forty minutes in length.

The pupils were paired into two equivalent groups on the basis of the results of the Guiller-Christoffenson Analytical Survey Test in Computational Arithmetic. Individual needs were discovered by an analysis of the pupils' initial test papers. Diagnostic charts were made showing the particular weaknesses of individual pupils in each of the experimental classes.

The work for the control groups was organized on the traditional group-instruction basis. The instructional material used by this group consisted of a textbook in which
no provision was made for discovering the computational skills in which the pupils were deficient. Lessons were assigned and prepared in the conventional way with no attempt to make the pupils aware of their difficulties.

With the experimental groups, teaching and practice work were organized on an individual basis, and grade lines were disregarded. Both the teacher and each pupil was made fully conscious of the individual learner's particular computational difficulties. In addition to individual instruction, the class work was organized on the basis of specific pupil needs. The entire class received instruction and practice in the areas which caused difficulty for a majority of the pupils.

At the conclusion of the study, Form 2 of the Guiler-Christoffenson Analytical Survey Test in Computational Arithmetic was used. The final test scores revealed that the mean point gains were 8.3 for all the experimental schools and 5.9 for all the control schools. The mean increase in the number of skills in which the difficulties were overcome were 4.3 for the experimental groups and 2.6 for the control groups. In regard to the findings, the investigators say:

In both the experimental and the control groups there are marked decreases in the percentage of gain from one achievement classification to the classification below . . . . the experimental pupils in each achievement classification made significantly greater progress than did the control pupils in the corresponding classification.  

24 Ibid., p. 359.
Guiler and Edwards reached the following conclusions:

1. Individual group instruction based on individual diagnosis is more effective than is conventional group instruction in the improvement of computational habits.

5. Initial status in computational ability has a marked effect on progress in computation.

6. Increasing the length of the class period does not produce a corresponding change in achievement.

The work of Tilton\(^\text{26}\) bears out the value of individualized instruction. In a controlled experiment, Tilton endeavored to determine whether a small amount of individualized instruction can produce demonstrable results. Arithmetic achievement tests were given to 138 pupils in four fourth grades. Thirty-eight of the pupils who tested low were chosen for special attention in an experiment in which nineteen were given twenty minutes of individual instruction per week for a period of four weeks. The other nineteen children, who were carefully equated and paired with the experimental group, were kept as controls. The control children were not given the individual instruction. It was found that the experimental group had made five months more progress than the control group. Since this superiority is significant at the 1 per cent level, Tilton concluded that a small amount of individualized instruction can be very worthwhile.

\(^{25}\)Ibid., p. 360.

\(^{26}\)Tilton, \textit{op. cit.}\
Results from similar remedial programs have indicated that pupils make significant gains when pupils are given special individual instruction in addition to regular classroom work. Among such programs are those tried by Thompson and Jacqueline.

Digest: grouping within the classroom and providing limited individualized instruction.--The studies reviewed above indicate clearly that it is possible and practical within the framework of the graded plan of organization to adjust materials and instruction to individual children by dividing the larger class group into smaller groups. Such a procedure is conducive to providing small group and individual instruction, thus making it possible to meet more adequately the needs of all children—the superior, the average, and the below average. Under such circumstances it is possible for children to succeed as they go, and thereby eliminate the necessity for failure or remedial teaching.

The research programs further reflect the advantages of providing a limited amount of individual instruction to supplement regular large group instructional procedures. It has been shown that even a small amount of individualized help can bring about significant achievement for slow learning children.

The evidence indicates the desire for a wholesome

27 Thompson, op. cit.
28 Jacqueline, op. cit.
balance between complete individualized instruction and large group instruction where no effort is made to meet individual needs. A combination of group and individualized instruction capitalizes on both the advantages to be derived from working in groups and the advantages to be derived from individualized instruction.

Providing for individual needs through activity and experience units.—Several experimental programs have indicated the possibility of meeting individual needs through the medium of activity and experience units. These programs have been summarized in detail in other chapters of the present study, so they will only be mentioned here. The results from such programs were reported by Harding and Bryant, 29 Harap and Mapes, 30 and Passehl. 31 Through the evidence presented, it was shown that children of varying levels of ability gained significantly in number competency as they worked with number in meeting the quantitative aspects of the various activities. 32


32 For a more detailed summary of the studies by Harding and Bryant, Harap and Mapes, Passehl, and Williams, see Chapter III.
Williams revealed that sixth grade children of varying levels of ability gained in number skill when their arithmetic problems were selected from the quantitative situations met in their on-going experience units. In regard to this, Williams writes: "On the whole, whether a child entered Grade VI ranking somewhere in Grade III... or ranking somewhere in Grade VI or above, he made significant growth during the year."  

Significance: providing for individual needs through activity and experience units.—There appears to be great possibility for meeting needs through activity and experience units. Since the problems met by children in carrying through the various activities are real to them, the purpose for solving the problems becomes a personal matter. Children are motivated to learn skills, for they can see where skills and instruction are necessary in carrying forward their proposed activities.

Arithmetic activity and experience units are usually broad enough in scope so that many opportunities are afforded each child to make a contribution regardless of his ability. Contributions can be made through both group and individual effort.

Meeting the needs of children through this means calls

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34Ibid., p. 159.
for real challenge on the part of the teacher. He must understand the needs and capability of each child in order to guide the child into those experiences which will be most profitable for the child. In order not to leave arithmetic instruction to chance, the teacher must be aware of an capitalize on the quantitative situations involved in the ongoing activities. Further, the teacher must be aware of the sequential nature of number so that gaps are not left in the child's learning.

Homogeneous grouping. As standardized achievement and intelligence tests gained in popularity in this country, educators became increasingly aware of the wide variation in abilities to be found in children. In an effort to teach children of such varying abilities more effectively, it became popular to group children of the same grade into different classrooms according to mental ability, achievement level, or some other criteria. Although the school enrollment largely determines the number of groups into which children of a given grade will be divided, the usual pattern is to have high, average, and low ability sections.

Several studies have been made to compare the relative average achievement of such groups. One such study was made by Theisen. The findings from this study were consistent with the findings of other reported studies showing that the

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greatest gains were made by the high sections, followed by the average and low sections, respectively.

Objecting to evaluating the effectiveness of homogeneous grouping on the criterion of average achievement, Burr attempted to evaluate homogeneous grouping by considering, not average achievement, but individual variations in homogeneous groups.

From standardized tests, scores were obtained for approximately 3,400 pupils. Grade groups were considered from Grade 4B through 6A. The variability was indicated graphically by the overlapping of achievement of homogeneous groups within a grade and statistically by per cent of overlapping of the total range of achievement.

The findings revealed that groups that are homogeneous with respect to average achievement or intelligence, or a combination of both, are not homogeneous with respect to single subjects such as arithmetic and reading. It was further shown that groups that are homogeneous in one subject are not homogeneous in other subjects. Of significance was the finding that groups that are homogeneous in a certain phase of one subject, such as arithmetic reasoning, are not homogeneous in another phase of the same subject, such as arithmetic computation.

In regard to the above findings, Burr concluded:

The conclusion that can be drawn here is that

36Marvin Y. Burr, A Study of Homogeneous Grouping in Terms of Individual Variations and the Teaching Problem.
groups in these field situations, as they are taught at the present time, are not homogeneous and there is no way of forming homogeneous groups except with respect to one subject at a time, and even then the groups will not be entirely homogeneous in the different phases of that one subject.37

In connection with another phase of the study, Burr writes:

The writer postulated a range of achievement within which the classroom teacher need make no gross adjustments to individual differences. The number of pupils falling outside the postulated range, when grouped "homogeneously" as in the three cities studied, was compared with the number falling outside the range when these same pupils were thrown into the random groups. The average size of the groups studied was 31.2 pupils. Of this number, on the average, 10.4 pupils needed adjustment (were outside the postulated range) when grouped "homogeneously" and 13.5 needed adjustment when grouped at random. While this indicated some merit for the grouping in eliminating need for individual adjustment, on the average ten or eleven pupils in a class of about thirty one still represent individual problems.38

The investigator concluded: "... the problem of meeting the individual needs of children is only slightly reduced by 'homogeneous grouping.' "39

West made a study to determine to what extent the practice of ability grouping reduces the variability in

37 Ibid., p. 41.
38 Ibid., pp. 55-56.
39 Ibid., p. 56.
40 Parl West, A Study of Ability Grouping in the Elementary School in Terms of Variability of Achievement, the Teaching Problem, and Pupil Adjustment.
achievement of classes in elementary schools when an attempt is made to adapt instruction and content to each group. With these conditions in mind, he sought further to determine to what extent the practice of ability grouping reduces the number and difficulty of the adjustments necessary for the pupils. After considering the evidence, West concluded:

1. The indications are that, by and large, the variability in achievement in ability groups in grades which have three groups each, is about 83% as great as in unselected groups. In grades having two groups each the variability in ability groups is indicated to be, typically, about 93% as great as in unselected groups.

2. The indications are that, by and large, the practice of ability grouping eliminates the need of adjustment in separate subjects for a number of pupils equal to about 7.2% of the average class enrollment, when there are three groups per grade. When there are two groups per grade it is indicated that, on the average, the need of adjustment is thus eliminated for a number of pupils equal to about 4.0% of the class enrollment.41

A detailed analysis or evaluation of homogeneous grouping was made by Keliher.42 She made her analysis by drawing on the statistical findings and thinking from the fields of education, philosophy, psychology, and sociology. After considering the evidence as revealed by the analysis, Keliher reached the following conclusion:

Homogeneous grouping, as we now have it,

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41Ibid., p. 61.

42Alice V. Keliher, A Critical Study of Homogeneous Grouping with a Critique of Measurement as the Basis for Classification.
appears undesirable. The measurement bases requisite for such grouping presuppose its major concern with the partial, academic phases of life. Acceptance of the philosophy that education is to concern itself with the whole child means rejection of a device which selects for consideration only certain of the individual's abilities and traits. In the light of sound theory and science of education homogeneous grouping should not be employed. In the light of the evidence concerning the results proposed for grouping, it does not achieve those results. Therefore, the major conclusion is that homogeneous grouping is not desirable in our elementary schools.

Conclusions: homogeneous grouping.--From the evidence presented by the studies just summarized concerning homogeneous grouping, it can be seen that it is almost impossible to group children so that they are homogeneous in all respects. If they are homogeneous in one subject, they will not necessarily be homogeneous in others. Also, it can be said that homogeneous grouping reduces only in a very small way a teachers need to consider individual differences. It is evident that such a grouping practice does not foster the best development of the whole child. Considering the whole picture, it can be concluded that homogeneous grouping is not a satisfactory means of meeting the individual needs of children.

Meeting the number needs of mentally retarded children.

In considering how to meet the individual needs of children, one is constantly faced with the special problem of how best to help the mentally retarded child. It is frequently assumed that the intelligence of the mentally retarded differs

43Ibid., p. 162.
from that of the normal only in amount. It is further assumed that the mentally retarded child's arithmetical ability differs from that of the normal child only in his slow rate of learning. Research has done much to help us better understand the mentally retarded child and has given us clues concerning how to meet his needs more adequately.

Cruickshank made a significant investigation to find out whether or not basic differences existed in the responses of mentally retarded and normal boys, matched according to arithmetic and mental ages, to simple arithmetic problems involving the four fundamental operations. Two groups of fifteen boys each were used in the investigation. The experimental group consisted of mentally retarded boys with a mean C.A. of 14.29 years, a mean M.A. of 10.06, and a mean I.Q. of 73.33. The boys in the control group had a mean C.A. of 9.09 years, a mean M.A. of 9.96 years, and a mean I.Q. of 110.4. The mean arithmetic age of the control group was 9.84, while the experimental group had a mean arithmetic age of 9.73. Both groups had arithmetical abilities comparable to third grade children.

The experimental test consisted of ten parts, almost all of which was administered individually. The results of the test indicated that the mentally retarded children were

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inferior to the normal children of the same mental age in: (1) solving abstract problems, (2) solving concrete problems, (3) in ability to isolate unnecessary material in arithmetic problems, (4) in understanding the operation required in an arithmetic problem, (5) in ability to deal with practical arithmetical situations, (6) in responses given to the test of logical analysis, and (7) in arithmetic vocabulary. Most of the above differences were statistically significant.

The principal conclusion reached was that mentally retarded subjects of the mental ages of eight, nine, and ten with equivalent arithmetic ages are inferior to normal subjects of the same mental and arithmetic ages in most distinct arithmetic abilities. The implication is that mentally retarded children need a type of instruction and instructional materials different from that required of normal children who have the same mental age as the retarded children.

In other aspects of child growth and development, research evidence has borne out the conclusions reached by Cruickshank. Aldrich and Doll, and Merrill have shown that mentally retarded children are inferior to normal children.


46M. A. Merrill, "On the Relation of Intelligence and Achievement in the Case of Mentally Retarded Children," Comparative Psychological Monographs, II (1924) pp. 1-100.
children of the same mental age in language comprehension and development. Fox revealed in a comparative study that mentally retarded children are inferior to normal children of the same mental age in the ability to form relationships, to make discriminative judgements, and in the development of insights.

An investigation by Costello is of great value in helping one decide how best to help retarded children grow in number competency. Costello investigated the problem of determining the responses of mentally retarded children to specialized experiences in arithmetic. Specifically, that investigator attempted to apply, under controlled conditions, three different methods of instruction in arithmetic and to observe and evaluate the consequences resulting from such methods.

The subjects included in the study were two hundred seventy-one pupils of the E. S. Miller School of Philadelphia. This school is a special school for the training of mentally retarded children.

Three equated and comparable groups were established. The groups were equated on the basis of grade (or section placement), mental age, and previous school experience. The median pupil had an I. Q. of 74.2. The range of mental ages

was from a mental age of four years six months to thirteen years and five months. The median mental age for the children was eight years and nine months.

The experiment was conducted over a period of sixty-five school days. At the beginning of the study the children were given a composite inventory-type test. The test was administered again at the conclusion of the experiment. During the study all pupils were subjected to individualized procedures, and an attempt was made to stimulate the progress of each.

Three methods of instruction, which may be considered the variables in the study, may be described briefly as follows:

1. The Verbalization Method. This method may be referred to as the method of substitute experience. Words or symbols, either written or spoken, were used to describe a situation through recalled imagery based upon prior experiences. In arithmetic the child was told relationships between various measurements or other arithmetical facts.

2. The Socialization Method. This method was considered as the active, experiencing type of endeavor. The activity received the main emphasis with the development of number ability being subservient or incidental to the main theme or activity. The operation of a miniature store within the classroom would have been typical of this approach. The child learned concepts in relation to a need.
3. The Sensorization Method. This method emphasized the concreteness or realism of presentation. The use of sense organs (sight, hearing, taste, and smell) were used in responding to the requirements of a situation immediately present. The objects or details of the situation were to be examined by one or more of the senses. The child was told the relationships of measurements in conjunction with concrete demonstration.

The data revealed that gains in subject matter by the group using the socialization method were substantially in excess of those occurring through the use of other methods. The poorest method of instruction for the improvement of attention, association, vocabulary, comprehension, and judgment was the verbalization method. Generally the best method was the socialization method closely followed, and in some cases exceeded, by the sensorization method. Relative to teaching method, the general conclusion reached was that a purposeful, cooperative group endeavor enlisting the active interest and participation of pupils is, on the average, most effective for mentally retarded children.

Pupil responsiveness was studied at three levels, namely: (1) 4 years 6 months to 7 years 5 months, (2) 7 years 6 months to 9 years 5 months, and (3) 9 years 6 months to 11 years 5 months. This analysis revealed that as mental age increased there was a steady increase in attention span, and in association, comprehension, vocabulary, and judgment.
skills. The findings revealed that mentally retarded children are generally inferior in ability to generalize. Almost sixty-eight per cent of the entire group were rated as being inferior in this quality. There was only a slight decline in inferior ratings in the ability to generalize as the mental age levels increased for the retarded children. From this evidence, it was concluded that teachers should not make too great demands on mentally retarded children as far as generalizing is concerned.

The evidence presented by Costello concerning the merit of an activity method in the teaching of arithmetic is substantiated by other studies which were not dealing directly with mentally retarded children. The studies reported by Harding and Bryant, Harap and Mapes, and Passehl indicate the importance of purposeful social activities in the teaching of arithmetic. Studies reported by Thiele,

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48 Harding and Bryant, op. cit.
49 Harap and Mapes, op. cit.
50 Passehl, op. cit.
Swenson, Moser, and Wilburn point to the significance of using concrete sensory materials with all children. Anderson indicated that slow learning pupils did not seem to profit significantly from a method which placed emphasis on the ability to generalize. This point is in keeping with the finding reported by Costello above relative to the lack of ability to generalize on the part of mentally retarded children.

Implications: Meeting the needs of mentally retarded children.--The implications are that mentally retarded children who have an I. Q. of approximately 70 can be taught arithmetic. Even so, the indications are that mentally retarded children of the same mental age as normal children differ in the specific ability to cope with number. Although all children profit from real experiences and the use of concrete sensory materials in the learning of arithmetic, mentally retarded children need such experiences even more so.


Further, since such children lack ability to generalize, teachers cannot expect mentally retarded children to transfer principles to untaught number situations as normal children are capable of doing. This implies that specific experiences need to be provided for teaching each of the separate number facts and skills.

Mentally retarded children appear to profit most from a teaching method which uses an activity approach. This method uses number situations which are most concrete and real to the children. This is as it should be, for such children cannot use abstractions as effectively as normal children.

Remedial instruction.---The term "remedial" when applied to learning indicates that something has gone wrong and needs to be changed or remedied. Used in arithmetic it means that given pupils have not learned, or have learned incorrectly, certain things which have been presented to them. Certain of the studies reviewed earlier in this chapter reveal the significant gains made in achievement by children when they are given remedial instruction. The value of remedial help was borne out in the studies by Guiler and Edwards,\textsuperscript{56} Tilton,\textsuperscript{57} Thompson,\textsuperscript{58} and Jacquelin.\textsuperscript{59}

\textsuperscript{56}Guiler and Edwards, \textit{op. cit.}
\textsuperscript{57}Tilton, \textit{op. cit.}
\textsuperscript{58}Thompson, \textit{op. cit.}
\textsuperscript{59}Jacquelin, \textit{op. cit.}
Bemis and Trow⁶⁰ have made a study which causes one to question the long range value of remedial work for all pupils. They cite two views of remedial work. By some it is assumed that remedial work brings pupils up to a level of achievement which will make it possible for them to continue with their class group. Others maintain that pupils can achieve only at their own individual rate of growth even though remedial measures are employed.

In order to test the validity of the two opposing points of view, the investigators set up an experimental situation. Eighteen pupils from Grade 6A were selected for special remedial instruction in arithmetic computation. These pupils were selected because they were most retarded in arithmetic, and yet were high enough in intelligence to make a successful outcome seem probable.

The remedial phase of the study lasted for one semester. Throughout the experiment, each child in the remedial group was considered as an individual. After locating each child's defects in unit skills, remedial measures were applied.

At the end of the experimental period, the average level of achievement for the group was 6-3, whereas, at the beginning of the semester it had been only 5-4. This represented an average gain of nine months instead of five months

which would normally be expected in the time given to the study. Individual pupils had made gains considerably above or below the average.

The investigators continued the investigation over a two year period in order to determine whether or not these gains for the experimental children could be considered permanent. No additional remedial work was given. During the two year period the remedial group was compared with a control group of equally retarded children who had had the same amount of instruction in arithmetic but no remedial instruction. The experimental and the control pupils were paired on the basis of age, I. Q., and amount of retardation in arithmetic.

Even though the average gains for the remedial children for the two year period were found to be higher than for the control children, there were individual remedial children who gained slightly less than the controls. Some control children without having had special remedial help had made gains comparable to the gains made by the remedial children.

In regard to permanency of gains, the investigators concluded that remedial work was of value to certain pupils and of little value to others. Since all children did not profit equally from remedial instruction, and some children made considerable growth without remedial help, the investigators gave the following interpretation:
It is probable that the retarded pupils studied, or any others, could be divided into three categories: (1) Those whose development is beginning to accelerate. These would show improvement with or without instruction. (2) Those whose development is slow, with the possibility of a later acceleration. These would not profit by any special instruction. (3) Those whose development had progressed above their achievement, but who for one reason or another--absence from school, lack of adjustment, unfavorable attitude, etc.--have not acquired certain needed instructional skills. These would profit by remedial work.61

The investigators point out that the children of the third category cannot be distinguished from those in the other classifications by the usual testing procedures. They propose that a means of evaluation which would record physiological growth over a period of years might be found useful in selecting children who would profit from special remedial instruction.

Inference: remedial work.---Ideally the first emphasis should be placed on prevention and good first teaching. This ideal is more nearly reached when provision is made for small group and individual instruction. Under such an arrangement, emphasis can be placed on individual needs from day to day to that the child is not permitted to become educationally ill. The child progresses as rapidly or as slowly as he needs to.

However, under the most ideal conditions some children will need special help due to absence, illness, etc. As

61Ibid., pp. 451-52.
indicated by Tilton\textsuperscript{62} and others children do profit significantly from even a small amount of remedial help. Bemis and Trow,\textsuperscript{63} however, reveal that remedial instruction may be only of temporary value for certain pupils. Even so, the question is here raised whether or not remedial instruction might be advocated on the grounds that it helps the child with his immediate social needs for number. An investigation of this point would help to give a better evaluation of remedial instruction.

Promotion and failure.--Research has made a big contribution in helping one evaluate the place of failure as a means of adjusting instruction to the individual learner. A common argument in favor of retaining children is that children profit from such a procedure by showing a greater increase in achievement. It is further argued that retaining children reduces the range in ability and achievement with which an individual teacher will need to cope.

Cook\textsuperscript{64} reports several research studies to show the effects of non-promotion on pupil achievement and on the reduction of variability in achievement within a given class. In one study, which shows that non-promotion does little to reduce variability, echo different promotion policies

\textsuperscript{62} Tilton, op. cit.

\textsuperscript{63} Bemis and Trow, op. cit.

\textsuperscript{64} Walter W. Cook, Grouping and Promotion in the Elementary School.
were compared. It was found that the achievement for the school with the low retardation rate was significantly higher than it was in the school that tends to retain the low ability pupils longer. Cook concluded:

The variability of achievement with which each teacher has to cope is not significantly different in the two schools. The most important single generalization that may be drawn... is that in any grade above the fourth a teacher may expect that almost the complete range of elementary school achievement will be represented. The only thing we can say with assurance regarding a seventh-grade pupil is that he is in the seventh grade. His achievement in any subject may be anywhere from that of an average second grader to that of an average senior in high school.

Another study which Cook reports shows the effects of high standards of promotion. The seventh grade pupils of two groups of schools having different promotion policies were compared in several respects. One group of schools was made up of schools approximating universal promotion, while the other group of schools had rather rigid promotion requirements and consequently a high ratio of retardation. The schools were matched on all important factors except that of promotion policy. Cook summarized the findings and conclusions as follows:

1. The evidence supports rather conclusively the contention that schools with a relatively high percentage of failures tend to have a relatively high proportion of over-age, slow-learning pupils.

2. In these schools the high percentage of over-age pupils who have been retained through

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ibid., pp. 29-30.
failure reduces both the average intelligence of the classes and the achievement averages of the grades, when compared with schools having more lenient standards of promotion.

3. When the achievements of pupils of equal chronological age and mental age in the two groups of schools are compared, no difference is found.

4. When the variability of classes is compared with respect to mental age and eleven educational achievement fields no significant difference is found between the two groups of schools.

5. The evidence supports the conclusion that by failing slow-learning pupils a school cannot increase its grade achievement averages or reduce the variation of achievement found in individual classes.

Consistent with the findings and conclusions reported by Cook are those reported by Caswell and Foshay. In the investigation reported by these authors it was found that the school which had the lowest rates of non-promotion had the highest achievement levels.

In another study, Caswell found only a chance relationship between the rates of non-promotion and the amount of variability in achievement. Caswell concluded: "... a school with even a large amount of slow progress might be re-organized so as to eliminate retardation without materially affecting the variability of instructional groups."

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66 Ibid., p. 41.
68 Ibid., p. 360.
69 Ibid., p. 361.
Akridge made an investigation to determine whether non-promotion reduces the variability within grade groups. In this study nine schools with low rates of non-promotion were compared with eight schools with high rates of non-promotion. The data revealed that rather than decrease variability, there was a slight tendency for non-promotion to increase the variability within a given group of pupils after they entered grade one.

The findings reported by Farley and Buckingham have shown that non-promotion does not assure subject matter mastery nor the improvement of slow learning rate. Their evidence substantiates the findings reported in the studies reported above.

Too frequently when the question of promotion or failure has been considered, it has been thought of only with the idea of subject matter achievement in mind. Many educators have failed to consider the outcomes in terms of the effects on the child's emotional or social adjustment.

The emotional and social effects of non-promotion were brought out effectively in a study by Sandin. In this study,

70 Garth H. Akridge, Pupil Progress Policies and Practices.


Sandin\textsuperscript{73} considered some of the characteristics of social adjustment, behavior, and attitudes of slow-progress children as compared with their regularly promoted classmates. He further made an appraisal of some of the benefits and disadvantages of non-promotion in view of the similarities and differences noted between the two groups.

The data were provided by 227 regular-progress children and 139 slow-progress children who were enrolled in sixteen classes of five elementary schools in Connecticut. Two classes were selected from each of the grades from one through eight.

Techniques used to secure data were: (1) sociometric tests, (2) behavior ratings of pupils, (3) interviews, (4) general observations, (5) identification by pupils of their own classmates' behaviors ("Who is It Test"), and (6) a questionnaire which was designed to explore the feelings and attitudes of children.

A statistical comparison which was made between the two groups of children revealed that the slow-progress children were: (1) invariably older, (2) generally taller, and (3) in many cases more mature physically than their regularly promoted classmates.

The non-promoted child generally indicated that his friends were in a higher grade, that he wished to be in an

\textsuperscript{73} Adolph A. Sandin, \textit{Social and Emotional Adjustments of Regularly Promoted and Non-Promoted Pupils.}
upper grade partly out of a desire to be with his friends. The slow-progress child was pointed out by his classmates as one who associates with older children. Usually the non-promoted child did not receive the social approval or acceptance of the regularly promoted children.

Both classmates and teachers associated less commendable behavior characteristics to a significantly greater extent with slow-progress pupils. These behavior characteristics denoted unfavorable reactions toward school and school work and antisocial behavior.

The conclusion was that non-promotion is inconsistent with the philosophy of the modern elementary school which aims to promote the development of the "whole" child—physical, social, emotional, and intellectual development. It was concluded further that non-promotion works as a detriment to social and emotional development.

In addition to the study reported above by Sandin, the evidence reported by Barker,74 Bassett,75 and Stoke76 documents the fact that children suffer severe and adverse mental hygiene effects from school failure.


Digest: promotion and failure.--In view of the evidence presented above concerning the value of non-promotion as a means of meeting individual needs, it can be said that non-promotion has little to commend it. It has been shown that most children do not achieve more by being retained, nor does non-promotion reduce the range in ability with which the teacher needs to cope. Also, when one considers the factors of emotional and social adjustment, the practice of failing pupils does not have much to offer.

Although the above statements are true in general, they are not necessarily true for every child. It seems feasible to suggest that some children may profit from retention. When such factors as general maturity, physical build, etc. are taken into consideration, it is likely that a given child will be better emotionally and socially adjusted with children who are younger than himself chronologically. More careful means of evaluation would be demanded to determine which children would profit from being retained.

When teachers made provision to meet each child on his respective level regardless of the grade the child may be in, there is little reason for the teacher to fail the child for academic reasons. Teaching children on their respective levels fosters continuous growth on the part of each child.

Summary

A discussion has been presented showing the major practices which are followed in meeting the needs of individual
pupils. These practices range from complete individualization to large group instruction in which practically no provision is made for meeting individual needs. Significant research data has been reviewed so that reliable guidance could be offered in helping one formulate a plan for meeting the varying arithmetic needs of children.

In the following section will be summarized briefly the major research findings having a bearing on the problem of meeting the individual needs of children. In a final section, conclusions and implications for teaching will be offered.

Synopsis of research evidence. - Washburne, Vogel and Gray,77 and Washburne and Raths78 present evidence that the highly individualized Winnetka Plan of instruction more adequately met the needs of individual children than did class methods without any provision being made for individual needs. On the other hand, Smith and Dolio,79 Lewin,80 Klugman,81 and Brasch82 show that group action is more effective than individual action in promoting desirable behavior of one

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77 Washburne, Vogel and Grey, op. cit.
78 Washburne and Raths, op. cit.
79 Smith and Dolio, op. cit.
80 Lewin, "Forces Behind Food Habits and Methods of Change," op. cit.
81 Klugman, op. cit.
82 Brasch, op. cit.
type or another.

Jones,\textsuperscript{83} and Culler and Edwards\textsuperscript{84} indicate that learnings are improved through a grouping arrangement whereby children are taught as individuals and small groups within the framework of the larger class group. Although Thompson,\textsuperscript{85} Jacqueline,\textsuperscript{86} Tilton\textsuperscript{87} and others show that children profit from remedial instruction, Bemis and Trow\textsuperscript{88} reveal that remedial instruction is of value to certain pupils and of little value to others.

Evidence reported by Harding and Bryant,\textsuperscript{89} Harap and Mapes,\textsuperscript{90} Fassehl,\textsuperscript{91} and Williams\textsuperscript{92} show that children of varying levels of ability gained significantly in number competency as they worked with number in meeting the quantitative aspects of activity and experience units.

\begin{itemize}
\item \textsuperscript{83} Jones, \textit{op. cit.}
\item \textsuperscript{84} Culler and Edwards, \textit{op. cit.}
\item \textsuperscript{85} Thompson, \textit{op. cit.}
\item \textsuperscript{86} Jacqueline, \textit{op. cit.}
\item \textsuperscript{87} Tilton, \textit{op. cit.}
\item \textsuperscript{88} Bemis and Trow, \textit{op. cit.}
\item \textsuperscript{89} Harding and Bryant, \textit{op. cit.}
\item \textsuperscript{90} Harap and Mapes, \textit{op. cit.}
\item \textsuperscript{91} Fassehl, \textit{op. cit.}
\item \textsuperscript{92} Williams, \textit{op. cit.}
\end{itemize}
Burr, 93 West, 94 and Keliher 95 reveal that it is almost impossible to group children so that they are homogeneous in all respects, that homogeneous grouping reduces only in a very small way a teacher's need to consider individual differences, and that such grouping does not foster the development of the whole child.

It has been indicated by Cruickshank 96 that mentally retarded children of the same mental age as normal children are not equally capable in arithmetic. Costello 97 found that mentally retarded children profit most from an activity approach.

Studies reported by Cook, 98 Caswell and Foshay, 99 and Akridge 100 show that non-promotion reduces very little the range in abilities with which teachers have to cope and does not promote increased learning on the part of the retained pupil. Sandin 101 shows that failure is not conducive to the emotional and social development of children.

93Burr, op. cit.
94West, op. cit.
95Keliher, op. cit.
96Cruickshank, op. cit.
97Costello, op. cit.
98Cook, op. cit.
99Caswell and Foshay, op. cit.
100Akridge, op. cit.
101Sandin, op. cit.
Conclusions and implications for teaching.--In view of the evidence reviewed above, several conclusions are justified. Also, certain implications for teaching may be offered. These conclusions, together with implications, are given as follows:

1. Considering (1) that it is almost impossible to group children so that they are homogeneous in all respects, (2) that homogeneous grouping reduces only in a very small way a teacher's need to consider individual needs, and (3) the negative effects that homogeneous grouping has on the development of the whole child, it may be concluded that homogeneous grouping is not a very satisfactory means of meeting the individual needs of children. This implies that children should be grouped heterogeneously into classrooms by grades and other means employed to meet the varying needs of children.

2. Individual difference are so great in most classrooms that it is next to impossible to meet the varying needs of children by teaching the entire class or grade as one group without making special provision for individual needs.

3. A completely individualized program is not recommended since it stresses academic learning alone. Working in groups with limited individual instruction is more conducive to the development of the whole child.

4. A combination of the activity method and grouping within the classroom appears to be the most satisfactory
means of meeting the individual needs of children. Through
the medium of activities, it is possible for children to de­
rive the values from working together in small groups and at
the same time it is possible to meet the individual needs of
the children by permitting each child to make the contribu­
tion which is in line with his ability. The activity ap­
proach is valuable in providing a purpose for learning num­
ber. The teacher can offer direct instruction through indi­
vidual or small group procedures in helping children develop
skills needed in meeting the quantitative situations pre­
seated by the activity. Further, the teacher can adjust ma­
terials and methods to the needs of the different children.
The teacher must be alert to the quantitative situations
which children are meeting and utilize those situations for
teaching number.

Meeting the needs of children through this approach
is a real challenge on the part of the teacher. He must
understand the needs and capability of each child, so that
he can guide the child into those experiences which will
be most profitable for him. Under such circumstances it is
possible for children to succeed as they go, and thereby e­
liminate the necessity for failure or remedial teaching. A
combination of group and individualized instruction capita­
lizes on both the advantages to be derived from working in
groups and the advantages to be derived from individualized
instruction.
5. Failure is inadequate as a means of meeting individual needs. Since continuous progress rather than failure is implied, schools must adjust their programs to the needs of the children within each classroom. Continuous progress cannot be followed in a situation where all children are given the same instruction and materials and expected to meet the same standards.

When we consider the development of the whole child, it seems feasible to suggest that some children may profit from retention. When such factors as general maturity, physical build, etc. are taken into consideration, it is likely that a given child will be better emotionally and socially adjusted with children who are younger than himself chronologically.

If teachers and schools make provision to meet each child on his respective level, there is little reason for the teacher to fail the child for academic reasons. Teaching children on their respective levels is conducive to the best development of the whole child.

6. Even though most children who have failed to learn for one reason or another profit from remedial instruction, it may be only of temporary value for certain pupils. This being true, evaluation techniques are needed to determine which children should receive such instruction. Small group and individual procedures are effective for remedial instruction.
Even though the i ains are only of immediate value for some children, the question is here raised if such help is not of value to the child in helping him meet his immediate social needs for number. Research considering this point would help to give a better basis for evaluating remedial teaching. In any case, the first emphasis should be placed on prevention and good first teaching.

7. Mentally retarded children are inferior to normal children of the same mental and arithmetic ages in most distinct arithmetic abilities. The implications are that mentally retarded children need a type of instruction and instructional materials different from those required of normal children who have the same mental age as the retarded children. Although all children profit from an activity method, such a method is especially valuable for mentally retarded children, for this method uses number situations which are most concrete and real to the children. This is as it should be, for such children cannot use abstractions as effectively as normal children. Since mentally retarded children are especially weak in the ability to generalize, specific experiences need to be provided for teaching each of the separate number facts and skills.
Regardless of the type of curriculum being followed, the method being used in teaching, or the use being made of instructional materials, evaluation is a factor which must be considered in one way or another by all teachers. It is with evaluation as it relates to arithmetic that this chapter is being concerned.

In the first section will be discussed the various concepts of evaluation held by those who are concerned with the teaching of arithmetic. The research findings relating to evaluation in arithmetic will be presented in the second section. Finally, the major conclusions along with suggested implications for teaching will be given in a summary section.

**Major Concepts of Evaluation in Arithmetic**

Various concepts may be noted which are held relating to evaluation in arithmetic. These concepts are closely related to the objectives or outcomes held by those who teach arithmetic. These concepts are also closely related to the learning theory held.

The traditional concept of evaluation. Traditionally, evaluation in arithmetic has been concerned primarily with the rate and accuracy with which children perform the mechanics of arithmetic computations or solve verbal textbook
problems. This concept of evaluation is consistent with the drill point of view in the teaching of arithmetic. For those who follow the drill theory, the immediate objective is to have pupils secure correct answers, so evaluation is used to determine if that objective has been met. Since speed and accuracy are the major outcomes considered, evaluation is seldom used for determining how children arrive at their answers or for measuring children's understanding of principles and processes. Although tests are designed and used primarily to measure achievement, they do have a slight diagnostic value in that they point out the types of processes or skills on which children need to be drilled more intensively.

Consistent with the authoritative philosophy of the drill theory, evaluation is considered to be a process which is administered by and belongs exclusively to the teacher. Pupils have no part in setting up objectives nor in determining whether or not the objectives have been met.

The meaning concept of evaluation—in conjunction with the emphasis that has been placed on meaning and understanding in arithmetic, a new point of view in evaluation has evolved. Evaluation is assigned the role of determining the understanding that children have of the processes they are using in arriving at their answers in addition to the customary role of determining the number of correct answers. Children are not considered to be very competent in performing
mechanical computations if they do not have an understanding of the principles and logic involved in the computational processes.

Consistent with the meaning concept, evaluation begins with determining readiness before arithmetic processes are taught, and continues throughout the teaching-learning situation until finally tests indicate whether or not the child has a reasonable understanding of the particular process and is able to apply the process with accuracy in solving everyday problems. Evaluation is used to determine whether or not the child can apply generalizations gained from learning a specific to untaught number situations. For example, evaluation is used to determine whether a child can, after gaining the understanding that zero added to a given number does not change the value of the number, find the answer to untaught combinations involving the zero. It is further considered to be the function of evaluation to indicate the thought processes or patterns used by the child in arriving at his answers. It is considered that the thought process used by the child is more indicative of the child's mathematical maturity than mere correct answers.

In order to make the type of evaluation indicated above, it is necessary to use many techniques of evaluation, including observations, personal interviews, teacher made tests, and standardized tests designed for diagnostic purposes or for measuring readiness or achievement. No one
technique is considered to be adequate to reveal all the information needed.

In agreement with the view of evaluation described above, is the statement made by Sueltz:

... the real and most important function of evaluation is that of facilitating pupils' learning. Hence evaluation must be a continuous job of both teacher and pupil. It should serve to foster all stages of a meaningful program, ranging from initial stages of encountering new materials to diagnosis of special difficulties and to the final appraisal of success.

Thus it is apparent that evaluation must include all phases of our program in arithmetic. It must include understanding of concepts and principles, knowledge of facts and socio-economic information, understanding and ability to recognize and to think through the mathematics in a problem or situation as all of these are used in the functional life of an intelligent citizen.¹

The whole child concept of evaluation.--Closely related to the above point of view is what may be termed the "whole child point of view". With this point of view one is concerned that the child have the mathematical understanding and competency suggested by the preceding point of view. The advocates of the whole child concept go a step further, however. They are concerned that as the child grows in maturity and competence in the use of number, he is also improving in emotional and social behavior. It is considered that arithmetic learning has not been truly successful if such learning has had a negative effect on the child's emotional and social behavior.

Consistent with the whole child concept, consideration is given to the dynamic goal seeking purposes of children. Children participate with the teacher in making plans and in setting up objectives. Since children participate in setting up objectives, they also have a share in determining how well those objectives have been met.

Implications for Evaluation from Research

Of value in helping teachers and others who are concerned with evaluation in arithmetic are the findings from research. Even though the research dealing directly with evaluation in arithmetic is limited, other arithmetic research which has not been concerned directly with evaluation offers valuable clues for evaluation.

Much of the research having implications for evaluation has been reviewed in the earlier chapters of the present analysis, so that research will only be mentioned briefly here. Investigations which have a more direct bearing on evaluation will be summarized in greater detail.

As pointed out by research findings, several factors need to be considered in evaluation, regardless of whether one is considering evaluation from the point of readiness, diagnosis, or achievement. A consideration of these factors along with evaluation techniques will follow.

Overt behavior.—It is comparatively easy to measure the overt behavior of children as far as number competency is concerned. The child is merely given a task to perform such
as counting objects, adding simple combinations, performing more complex computations, or solving verbal problems of one type or another. This type of evaluation was used by Buckingham and MacLatchy in determining the number abilities of children when they first enter school. Teacher-made tests and a number of standardized tests are useful for this type of evaluation. Even though ascertaining the overt behavior of children has a place in determining readiness and achievement in arithmetic, other facets of behavior need to be evaluated, too. To measure other phases of behavior calls for the use of various evaluation techniques, as will be shown.

Intelligence and past experience. -- Evaluation in arithmetic at any level must take into consideration the intelligence and past experience of the learner. This statement is made on the basis of the findings from several research studies. Washburne presents strong evidence showing the value of mental maturity in learning arithmetic, while Moser

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indicates the significance of experience. Smith and Koenker reveal the importance of both intelligence and experience in promoting number growth.  

Since both intelligence and past experience are closely correlated with the ability to succeed in arithmetic, it is necessary that tests be used to indicate the intelligence of the learner as well as tests to measure past experiences or achievement in arithmetic. Such evaluation will indicate to teachers the type of number experiences to provide for children.

Vocabulary and reading ability.--Evaluation in arithmetic should give consideration to the vocabulary and reading ability of the learner. This need is borne out particularly by Treacy who showed that ability to succeed in arithmetic is correlated positively with vocabulary and reading skills. Johnson revealed that definite vocabulary


6 Robert H. Koenker, "Certain Characteristic Differences Between Excellent and Poor Achievers in Two-Figure Division," Journal of Educational Research, XXXV (April, 1942) pp. 578-85.  

7 For a detailed summary of the four above studies see Chapter III.  


experience improved problem solving ability of children at the seventh grade level.

Among the many possible ways of measuring the understanding of arithmetic vocabulary, Sueltz, Boynton, and Sauble\(^\text{10}\) offer several representative questions for measuring the understanding of vocabulary. One such question used by them is:

Mary read that New York is the largest city in the United States. What does "largest city" mean?

a) has the tallest buildings b) has the most people
c) has the most land d) is worth the most money\(^11\)

Questions used by Buswell and John\(^\text{12}\) are also promising means of evaluating how well children understand number vocabulary.

Maturity of thought processes.—Evidence reported by Brownell,\(^\text{13}\) Brownell and Carper,\(^\text{14}\) Carper,\(^\text{15}\) Deans\(^\text{16}\) and


\(^{11}\) Ibid., p. 153

\(^{12}\) Guy T. Buswell and Lenore John, The Vocabulary of Arithmetic.

\(^{13}\) William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.

\(^{14}\) William A. Brownell and Doris V. Carper, Learning the Multiplication Combinations.


Judd\(^{17}\) indicate that children do not suddenly acquire mature adult patterns in dealing with number. These studies\(^{18}\) indicate that children, whether in basic counting, addition, or multiplication go through a series of crude or immature procedures in dealing with number.

Teachers need to know children's procedures in dealing with number so that they may be helped to progress to higher levels of competency. Deans\(^{19}\) illustrated how children were helped to advance to more mature procedures in handling number after she had ascertained their present level of working with number. This information gave a clue as to where to start with the children in their progress in number growth. Brownell and Chazal\(^{20}\) have shown the futility of using premature drill in trying to hasten children to acquire mature competency in working with number.

The above evidence reflects the need for personal interviews to supplement regular group tests in determining how children think in regard to number. Ordinary tests which ask for correct answers only cannot be used for this purpose, for the studies referred to above indicate that

\(^{17}\)Charles Hubbard Judd, *Psychological Analysis of the Fundamentals of Arithmetic.*

\(^{18}\)For a detailed summary of these studies see Chapter IV.

\(^{19}\)Deans, op. cit.

correct answers may be secured by the most immature procedures.

The significance of personal interviews as a diagnostic testing technique had been revealed by research. Brownell and Watson\(^2\) made a comparative study of the value of two kinds of diagnostic procedures, namely, (1) personal interviews, and (2) written records of pupils' performances. Each child was given a copy of the test and asked to respond in two ways: (1) he was asked to write his work and his answers, and (2) he was asked to "think out loud" as he worked. The interviewer kept a written record of the child's verbal responses.

Usable records were available for 245 fifth grade children. The test used was a modified form of the Brueckner Test in the Addition of Fractions. From the interview records a total of fifty-five categories of faulty procedures were isolated. Ten faults were identified as being responsible for eighty per cent of the total number of errors.

The findings indicated that the personal interview and the analysis of test papers were equally satisfactory for the grosser types of diagnosis. It was discovered, however, that personal interviews were noticeably more consistent or reliable with respect to particular faults. Brownell writes:

"... in this study there were collected many illuminating

examples of pupil difficulties, — difficulties open only to the intimate relations of the interview, in which the language of the child's thought is open to observation."

Brownell cites the following examples of children's thinking in regard to describing their processes in finding a common denominator:

a. "4 is the common denominator because I will go in 3 and 12."

b. "18 is in 6 tables 3 times, and in 9 tables 2 times."

c. "Find something to go into both."

d. "I times 'em."

e. "I multiplied them together when I couldn't think."

f. (For 3/6 and 4/5) "Ain't no number will exactly divide 3 and 5."

g. (For 1/2 and 2/4) "Two is contained exactly into 1, and you drop the two out."

The investigators concluded as follows:

... the conclusions seem to be justified that PI [personal interviews] and AT [analysis of test papers] are equally satisfactory only for the grosser types of diagnosis (total examples missed, total number of faults detected, total number of faults of particular types). For these types of diagnosis AT is to be preferred, because of its convenience values. Then, however, diagnosis is that of the processes and difficulties of individual children PI is both more reliable and more valid.

Largely substantiating the findings and conclusions reported by Brownell and Watson are those given by Williams.

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22 Ibid., p. 374.

23 Ibid., p. 375.

24 Ibid., p. 376.
and Whitaker. These investigators tested 516 pupils who were having difficulties in the four fundamental operations. By use of an individual test the children were asked to verbalize their thinking as they worked the problems. The investigators concluded that diagnosis is an individual matter and that group testing is inadequate for disclosing which specific skills are causing difficulty.

The value of using the personal interview as an evaluation technique is illustrated through numerous studies. The use of interviews in determining how children work with number was demonstrated by Brownell, Carper, Deans, and Buswell and John.

Inference: Maturity of thought patterns.—In view of the fact that research so strongly points out the importance of knowing how children think in regard to number, the obvious inference is that determining thought patterns is a significant part of evaluation in arithmetic. Since paper and pencil tests point out only the more obvious abilities and deficiencies of pupils, it is necessary to supplement this type of evaluation with observations and personal interviews in order to determine more specifically how children work.

25 William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.
26 Carper, op. cit.
27 Deans, op. cit.
with number. These techniques reveal the child's maturity level of determining his answers as well as indicating the types of errors he is making. With this information in hand, the teacher can proceed to provide experiences to help the child progress to more mature ways of dealing with number as well as providing a basis for helping the child overcome the errors he is making.

**Meaning and understanding.**—Research indicates that when a child understands what he is doing, sees relationships in the number system, and is able to form generalizations, the child learns more readily, retains longer what he learns, and is more capable of transferring his learning to related untaught number situations. These facts are borne out particularly by research conducted by Swenson. Other studies supporting these facts in whole or in part were made by McConnell, Thiele, and Nicholson.

Having a significant bearing on the evaluation of


31T. R. McConnell, Discovery vs Authoritative Identification in the Learning of Children.


understanding in arithmetic is a study conducted by Hendrix. In that study, Hendrix investigated the relative effectiveness of three methods of teaching generalizations. In Method I, the generalization was given by the teacher or textbook, and then the subjects were guided in applying the generalization to several examples. In Method III, the subjects were given the opportunity to discover the generalization for themselves. After there were indications that the generalization was understood on the unverbalized level, the subjects were pressed into stating the generalization verbally. After the subjects in Method II gave indication on the unverbalized level that they had discovered the generalization for themselves, they were not asked to verbalize it.

Then the groups were later presented with situations in which the generalization could be applied, it was found that those who had only the unverbalized awareness of the generalization had the highest transfer ability. It was concluded that teaching should cease once children have indicated by their unverbalized behavior that they have grasped the significance of the generalization.

From the findings of the above investigation, strong inferences may be drawn concerning evaluating understanding in arithmetic. Teachers need to observe closely the behavior of children to detect signs of understanding on the

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unverbalized level. This calls for further use of observation as an evaluation technique.

The value of using interviews as a means of getting a clue to how well children understand was indicated by Brownell and Watson's study pertaining to evaluating the maturity of thought processes. It was shown that the interview revealed much more information concerning the child's understanding than could be revealed by ordinary pencil and paper tests.

Of particular importance in the evaluation of understanding in arithmetic was research conducted by Glennon. The purpose of the investigation was to measure the degree to which the growth of understanding was being accomplished in the field of arithmetic on seven educational levels—seventh grade, eighth grade, ninth grade, twelfth grade, teachers college freshmen, teachers college seniors, and teachers-in-service.

The data were collected through a Test of Basic Mathematical Understandings, a test designed by Glennon as part of the study. The test consisted of eighty items of the multiple choice variety and was designed to measure only mathematical understandings that are basic to computational

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35 Brownell and Watson, *op. cit.*

processes commonly taught in the first six grades. Since the purpose was to measure understanding and not the mechanics of computation, no computations were required. An example of a test item included in the test is illustrated by the following example:

At the right is an example in subtraction of figures. Which of the following statements about the example is true?

A. AFGB and CXU added together equal TMXY.  
B. CXU and TMXY added together equal AFGB.  
C. AFGB and TMXY added together equal CXU.  
D. TMXY subtracted from CXU equals APGB.  
E. CXU subtracted from TMXY equals AFGB.  

The findings revealed that the average scores in regard to the understandings basic to computational processes taught in grades one through six for each of the grade groups were as follows: (1) seventh grade, 12.5%; (2) eighth grade, 14%; (3) ninth grade, 10%; (4) twelfth grade, 37%; (5) teachers college freshmen, 44%; (6) teachers college seniors, 43%; and (7) teachers-in-service, 65%.

On the basis of the above findings Glennon concluded that the persons tested had not acquired a satisfactory knowledge of the understandings involved in elementary school arithmetic. That investigator concluded further that the findings point up the need for teachers to emphasize the development of meanings and understandings, principles and generalizations inherent in the number system. This new emphasis calls for a decreasing emphasis on the mastery of

Ibid., p. 122.
isolated computational skills.

In addition to the test used by Glennon, similar tests for measuring understanding in arithmetic have been suggested by Sueltz, Boynton, and Sauble, 39 Spitzer, 38 and Sueltz. 40

Emphasizing the need for using many techniques for measuring understanding in arithmetic, Sueltz, Boynton, and Sauble say:

The usual procedures for evaluation include (1) the use of written tests, (2) observation of the daily work of pupils, (3) interview of the pupils during and after their work, and (4) self-evaluation by the pupils themselves. For the measurement of understanding, a combination of all of these methods is most fruitful. 41

Direct: meaning and understanding.—Considering the values pointed out by research to be derived from children understanding their arithmetic, it is imperative that evaluation assure as part of its function to measure outcomes associated with understanding. Even though ordinary tests indicate whether or not the child has secured the correct answer, such tests do not reveal the child's understanding of the process. Tests as suggested by Glennon, 42 Sueltz, 39 Sueltz, Boynton, and Sauble, op. cit.


41 Sueltz, Boynton, and Sauble, op. cit., p. 141.

42 Glennon, op. cit.
Boynton, and Sauble, \textsuperscript{43} Spitzer, \textsuperscript{44} and Sueltz \textsuperscript{45} are suggestive of types of tests which may be used to measure understanding. Since these tests are designed to measure understanding and not the mechanics of computation, no computation is required.

In view of the fact that Hendrix\textsuperscript{46} has revealed that children frequently understand even though they may not be able to verbalize the understanding, it is necessary for teachers to observe closely the behavior of children in order to detect signs of understanding on the unverbalized level. Brownell and Watson\textsuperscript{47} have shown the significance of using personal interviews in order to gain insight into the child's understanding of number.

The conclusion seems justified that in order to evaluate children's understanding in arithmetic, teachers need to use observation, personal interviews, and pencil and paper tests specifically designed to measure understanding. For fullest value, these several techniques should be used in conjunction with each other, rather than depending on one exclusively.

\textbf{Ability to solve problems real to the learner.---}Much evaluation has been made to determine how well children meet

\textsuperscript{43}Sueltz, Boynton, and Sauble, \textit{op. cit.}
\textsuperscript{44}Spitzer, \textit{op. cit.}
\textsuperscript{45}Sueltz, \textit{op. cit.}
\textsuperscript{46}Hendrix, \textit{op. cit.}
\textsuperscript{47}Brownell and Watson, \textit{op. cit.}
number situations as given in textbooks or standardized tests of one type or another. Frequently children solve problems in these situations but fail to solve problems which they meet in their own daily activities. Most tests indicate that children can add, etc., but they do not necessarily indicate that children can solve their own on-going social quantitative problems.

Evaluation of this type calls for close observation to see with what facility children solve problems which are real to them. Gesell and Ilg,48 and Ries50 used observations effectively in determining how children react to number in functional situations.

Teaching procedures based on problems which are real to children afford teachers an opportunity to observe children in meeting such quantitative situations. An arithmetic activity program as suggested by Harding and Bryant,50 and by Harap and Wapes51 makes it possible to observe children as they solve problems which are real to them.

48 Arnold Gesell and Frances L. Ilg, The Child From Five to Ten, pp. 399-402.


Consideration of social and emotional behavior.--Most evaluation in arithmetic has not considered the child's emotional and social adjustment as being related to the outcomes to be achieved in arithmetic. A few studies, however, in evaluating outcomes in the learning of arithmetic have considered the emotional and social adjustment as part of the outcomes to be evaluated.

Harding and Bryant\(^\text{52}\) (see Chapter III) gave consideration to these factors in comparing two methods of teaching arithmetic. One group of fourth grade pupils were taught arithmetic through functional procedures, while a comparable fourth group was taught by formalized drill methods. The group taught by functional procedures proved to be definitely superior in arithmetic reasoning and equally as capable as the control group in computational skills. Further, the evidence revealed by anecdotal records showed that the experimental children possessed more desirable social characteristics and better emotional stability than did the children from the control group.

The above findings are substantiated by the evidence presented by Williams.\(^\text{53}\) That investigator found that a functional arithmetic program was more successful than the traditional curriculum in bringing about significant gains.

\(^{52}\)Harding and Bryant, op. cit.

In such qualities as initiative, critical thinking, independent work habits, a sense of responsibility, disposition and ability to carry an activity through to completion, leadership, and other qualities reflecting desirable emotional and social behavior. When three groups of the experimental children and three groups from a traditional program were equated and compared, it was found that the experimental children had significant advantages over the traditional groups in respect to the above qualities.

Another important investigation gave consideration to qualities other than academic achievement in evaluating the effectiveness of two types of instructional programs. A comprehensive survey of the New York City Schools, which was made under the leadership of J. Cayce Morrison (see Chapter III), indicated that children who were taught through an activity program were superior to children from more traditional schools in qualities related to desirable emotional and social adjustment.

Significance: social and emotional behavior in evaluation.--The above research evidence indicates a need for a more comprehensive view of evaluation. Most evaluation has stopped when academic achievement has been measured. The above studies, however, mark an important move in evaluation. This move is consistent with the view that "the whole child" needs to be considered and not merely his intellect.

54 J. Cayce Morrison, The Activity Program.
Anecdotal records used in conjunction with observation are suggestive of techniques to use in obtaining information relating to the emotional and social behavior of children. The use of such techniques was illustrated by Harding and Bryant,\textsuperscript{55} and Williams\textsuperscript{56} in securing data concerning the emotional and social behavior of children. Such techniques reveal much about how children work with number, as well as how children react socially and emotionally.

**Immediate versus postponed evaluation.**--The value of measuring achievement sometime after the completion of the immediate teaching-learning situation was pointed out by Howard.\textsuperscript{57} He investigated the effectiveness of three methods of instruction in the teaching of all processes in fractions on the fifth and sixth grade level. The three methods were the drill method, the meaning method, and a combination drill and meaning method. The immediate results favored the drill approach in several important respects. However, when the tests were repeated after the summer vacation, it was found that the children taught by drill had forgotten a great deal of the arithmetic they had supposedly learned before beginning the summer vacation. The children with whom both

\begin{itemize}
  \item\textsuperscript{55}Harding and Bryant, op. cit.
  \item\textsuperscript{56}Williams, op. cit.
\end{itemize}
meanings and practice had been stressed actually improved
during the summer vacation.

The implications are that later evaluations need to be
made to determine whether learnings are lasting and retained
or of only a temporary nature. Repeating tests some time af­
ter the learning period seems especially needed in measuring
the effectiveness of teaching methods.

Multiple use of tests.--The main purpose of readiness
tests is usually considered to be that of determining the
learner's ability to proceed with a new phase of learning.
Aside from this value, research indicates that readiness
tests may also have great diagnostic value. Brueckner de­
serves much of the credit for revealing this additional value
of readiness tests.

A study was made by Brueckner5 in which he endeavored
to test the validity of his assumption that the most useful
function of readiness tests in the field of arithmetic was
not prediction of success but the diagnosis of factors likely
to interfere with learning at any level of the school.
Brueckner constructed a readiness test for children who were
about to begin the study of division with two-figure divi­
sors. First he made an analysis of basic skills pupils should
have before work with two-figure divisors is begun. The test

5Leo J. Brueckner, "The Development of Readiness Tests
in Arithmetic," Journal of Educational Research, XXXIV
(September, 1940) pp. 15-20.
measured the following items:

**Section A**

**Part I.** Knowledge of place value in three place numbers. (4 minutes)

**Part II.** Multiplication used in division in the form of quotient times divisor. (4 minutes)

**Part III.** Knowledge of process with one-figure divisors and dividends of two to four figures. (10 minutes)

**Section B**

**Part V.** Judging the correctness of quotient figures, some of which were too large, others correct, others too small. (4 minutes)

**Part VI.** Completing the work to determine correctness of given quotient figures. (6 minutes)

**Part VII.** Subtraction used in division. (5 minutes)

**Part VIII.** Mental multiplication of two-place numbers by a one-place number. (5 minutes)

**Part IX.** Comparing product of a two-place number multiplied by a one-place number with a given number—the dividend. (5 minutes)

The readiness test was administered to the 58 pupils in six schools of Minneapolis, who were about to begin the study of division with two-figure divisors. The teachers who participated in the investigation taught division as it was presented in the basic textbook and without knowledge of the results of the readiness test, since it was the purpose of the investigation to indicate what the relationship was between the results of the readiness test and success in learning division when no attempt was made to correct difficulties that would be revealed by the results of the readiness test.

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59 Ibid., p. 17.
readiness tests. After the teaching of the steps involved in the division process, a comprehensive test was given to secure a measure of the pupils' success in learning the process. The results of the end test were compared with the results of the readiness test.

An analysis of the readiness tests results indicated that on every part of the test except Part IV, the range of scores was from no errors to every item incorrect. The total scores ranged from 1 to 87 errors. Large numbers of the children for whom complete records were available made very little progress as revealed by the end test. The median percent of accuracy on the end test was about 55 per cent.

Breacchner concluded:

If one can assume that the tests are in fact valid measures of the simple skills they test, it is obvious that many of these pupils had such serious deficiencies in basic essentials that failure to succeed in the new work was a foregone conclusion, unless steps were taken by the teacher to correct shortcomings. 50

Breacchner ran correlations on each part of the readiness test with each of the other parts of the test, with the total scores on the readiness tests and the end tests, and the correlation of the total readiness test scores with the end test score. He found that all correlations were positive. His conclusion was: "... the readiness test we are considering has validity both as a diagnostic device and as a  

50 Ibid., p. 13.
predicative device.\textsuperscript{61}

The investigation conducted by Souder\textsuperscript{52} strengthens and substantiates the findings and conclusions reported by Brueckner. Souder constructed readiness tests for addition and subtraction of fractions. Two equated groups of 5A pupils were used in the study. With the experimental group the diagnosis afforded by the results from the readiness tests formed the basis for teaching, while with the control group the results were not used for instructional purposes. As revealed by the end test, it was shown that the differences for both addition and subtraction of fractions were in favor of the experimental section and were statistically significant at all levels of performance. The conclusion reached by Souder was:

The use for instructional purposes of certain readiness tests in common fractions differentially affects the learning of pupils of varying levels of intelligence, and of varying levels of performance on the readiness test as measured by scores on the end or criterion test of the arithmetical processes in question. It is possible that all pupils at all levels of ability, \ldots may profit significantly by the use of readiness test results for instructional purposes.\textsuperscript{63}

Another value to be derived from tests used in evaluation

\textsuperscript{61}Ibid., p. 20.


\textsuperscript{63}Ibid., pp. 133-34.
was indicated by Podlich, who showed that as much gain was derived from taking the test as from the learning period. It was further revealed that recall in the form of a test prevents forgetting. Pupils taught under a method which provided for periodic testing maintained their original learning over a period of six weeks, while pupils without the benefit of the recall provided by tests showed statistically significant losses.

**Inference: multiple use of tests.**—It may be said in view of the evidence presented above, that tests, which are one of the primary instruments of evaluation, if properly constructed and used may have more value than merely meeting the purpose for which they were originally designed. Readiness tests may be used for diagnostic purposes in pointing out to teachers pupils' specific difficulties so that corrective measures may be used. Children will profit from an instructional program based on the deficiencies indicated by tests. Also, periodic evaluations promote further learning and help the pupil maintain his original learning.

**Summary**

A consideration has been given to the major concepts of evaluation in arithmetic which are held by those who are concerned with such evaluation. An examination of these

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concepts has revealed that traditionally evaluation in arithmetic has been concerned primarily with the rate and accuracy with which children perform the mechanics of arithmetic computations or solve verbal textbook problems. Other concepts include such additional factors as understanding and emotional and social adjustment.

Research was presented to point out: (1) some of the factors which need to be considered in evaluation, and (2) various evaluation techniques to be used for evaluation purposes. In the following section will be summarized briefly the major research findings, while in a concluding section will be indicated the significance of the research evidence for evaluation.

Digest: research findings.—Several important studies have revealed or suggested factors which should enter into evaluation, whether it be for readiness, diagnosis, or achievement. Washburne\textsuperscript{65} shows the value of mental maturity in learning arithmetic, while Moser\textsuperscript{63} reflects the significance of experience. On the other hand, Smith\textsuperscript{67} and Koenker\textsuperscript{63} reveal the importance of both intelligence and experience in

\begin{itemize}
\item Washburne, op. cit.
\item Moser, op. cit.
\item Smith, op. cit.
\item Koenker, op. cit.
\end{itemize}
promoting success in number growth. Treacy\textsuperscript{69} and Johnson\textsuperscript{70} indicate that ability to succeed in arithmetic is closely related to vocabulary understanding.

Studies reported by Brownell,\textsuperscript{71} Brownell and Carper,\textsuperscript{72} Carper,\textsuperscript{73} Deans,\textsuperscript{74} and Judd\textsuperscript{75} reveal the importance of determining children's thought processes in working with number. Evidence presented by Swenson,\textsuperscript{76} McConnell,\textsuperscript{77} Thiele,\textsuperscript{78} and Nicholson\textsuperscript{79} reflect the importance of understanding and meaning in arithmetic, and hence point to the need for evaluating and measuring understanding. Hendrix\textsuperscript{80} shows that teachers need to look for understanding on the unverbalized level.

\textsuperscript{69} Treacy, op. cit.
\textsuperscript{70} Johnson, op. cit.
\textsuperscript{71} William A. Brownell, The Development of Children's Number Ideas in the Primary Grades.
\textsuperscript{72} Brownell and Carper, op. cit.
\textsuperscript{73} Carper, op. cit.
\textsuperscript{74} Deans, op. cit.
\textsuperscript{75} Judd, op. cit.
\textsuperscript{76} Swenson, op. cit.
\textsuperscript{77} McConnell, op. cit.
\textsuperscript{78} Thiele, op. cit.
\textsuperscript{79} Nicholson, op. cit.
\textsuperscript{80} Hendrix, op. cit.
Studies by Harding and Bryant, Williams, and Morrison indicate the trend to evaluate outcomes in terms of the effect on the whole child, not only intellectual growth, but emotional and social growth as well.

The importance of evaluating the retention of skills taught sometime after the immediate teaching period is over has been shown by Howard. This delayed evaluation is especially important when evaluating the relative worth of different teaching methods.

Several investigations have shown that tests can be used for multiple purposes. Brueckner and Souder show that readiness tests can also be used for diagnostic purposes. Podlich indicates that tests have a teaching value in themselves and help to promote the retention of skills taught.

From several studies the values of certain evaluation techniques are brought out. Seltz, Boynton, and Sauble

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31 Harding and Bryant, op. cit.
32 Williams, op. cit.
33 Morrison, op. cit.
34 Howard, op. cit.
35 Brueckner, op. cit.
36 Souder, op. cit.
37 Podlich, op. cit.
38 Seltz, Boynton, and Sauble, op. cit.
offer representative multiple choice questions for measuring the understanding of vocabulary. Glennon presents a multiple choice type test designed to measure mathematical understandings. Similar tests for measuring understanding in arithmetic have been suggested by Sueltz, Boynton, and Sauble, Spitzer, and Sueltz.

The merit of interviews as an evaluation technique was shown by Brownell and Patson, and by Williams and Whitsaker. Studies by Brownell, Carper, Deans, and Buswell and John have illustrated further the value of personal interviews in obtaining a type of information not readily obtained by regular pencil and paper tests. The need for using observation to detect understanding has been indicated by Hendrix.

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96 Glennon, op. cit.
97 Sueltz, Boynton, and Sauble, op. cit.
98 Spitzer, op. cit.
99 Sueltz, op. cit.
95 Brownell and Patson, op. cit.
94 Williams and Whitsaker, op. cit.
96 Carper, op. cit.
97 Deans, op. cit.
98 Buswell and John, op. cit.
99 Hendrix, op. cit.
while Gesell and Ilg, and Riess used observations effectively in determining how children react to number in functional situations. The use of anecdotal records has been illustrated by Harding and Bryant, and Williams in obtaining evidence of social and emotional adjustment.

Significance of research evidence for evaluation.--The research evidence referred to above strongly points to the conclusion that in making a comprehensive evaluation, whether for readiness, diagnostic, or achievement purposes, one needs to consider the following factors: (1) overt behavior (rate and accuracy) in meeting quantitative situations, (2) level of mental maturity and background of experience, (3) the degree of meaning and understanding, (4) maturity of thought processes, (5) the vocabulary and reading ability of the learner, (6) the ability to solve personal quantitative problems, and (7) the adjustment of the whole child—intellectual, social, and emotional. The learner's status in regard to the above factors needs to be determined before instruction begins, with a continuous appraisal during the learning process, until finally, tests indicate that the learner has achieved mature competency and adjustment in his reaction to number.

100 Gesell and Ilg, op. cit.
101 Riess, op. cit.
102 Harding and Bryant, op. cit.
103 Williams, op. cit.
In order to appraise adequately the various factors to be considered in evaluation, many techniques must be used. Standardized tests can be used to measure certain aspects of readiness, diagnosis, and achievement. Such tests need to be supplemented by pencil and paper tests prepared by the teacher. Standardized tests and teacher made tests should be designed not only to measure rate and accuracy, but to measure understanding as well. Interviews, observations, and anecdotal records should be used to reveal much about the learner's quantitative behavior. Such techniques reveal how the child thinks and works with number. Each of the above techniques--standardized tests, teacher made tests, interviews, observations, and anecdotal records--has a significant place in evaluation. No one technique should be relied on completely. For fullest effects, the various techniques should be used in conjunction with each other.

In appraising outcomes, one should not only consider the immediate results, but should also consider the long range effects. Such a procedure may frequently reveal that the immediate results are temporary and misleading. Long range evaluation is especially important in determining the relative worth of different teaching methods.
CHAPTER X

SUMMARY AND RECOMMENDED TEACHING PRACTICES

In this concluding chapter, a brief review will be given of the problem considered in the present study and of the general procedure followed in the development of the study as a whole. The principal findings reported in the entire analysis will be summarized briefly in a second section. The major part of the chapter will be devoted to a description and discussion of a recommended arithmetic instructional program which is based on those valid research findings which have been presented in the present study.

The Problem

It has been the purpose of this study to make an analysis and synthesis of significant research relating to selected areas in the teaching of arithmetic as a basis for critically examining teaching practices. The areas selected for consideration in this analysis are as follows: (1) Arithmetic Readiness, (2) Scope of the Arithmetic Curriculum, (3) Grade Placement of Content, (4) Curriculum Patterns, (5) General Methods of Teaching, (6) Special Procedures in Performing the Fundamental Operations, (7) Special Procedures in Problem Solving, (8) Instructional Materials, (9) Providing for Individual Needs, and (10) Evaluation. Three closely related areas (Scope of the Arithmetic Curriculum, Grade Placement...
of Content, and Curriculum Patterns) were considered together in Chapter III, "The Curriculum". Except for the introductory chapter, a similar plan of presentation was followed in each chapter. After giving a brief definition of the particular area being considered, an analysis of the various points of view pertaining to that area followed. Research was then presented concerning each subdivision into which the larger area was broken. A brief summary or digest was then given, bringing together the major results from the several studies reviewed. A general chapter was provided in which an effort was made to integrate the major experimental results and to draw conclusions related to the specific area being considered.

Findings

In the foregoing chapters of this study, significant investigations concerned with the teaching of arithmetic have been reviewed. The more significant reported findings are summarized as follows:

1. Several interrelated factors (experience, intelligence, maturation, degree of understanding, and intrinsic purpose) influence readiness to learn arithmetic.

2. Most children are not ready to achieve full mastery of a given arithmetic skill (the complete process of division, for example) within a given school year.

3. Children at each grade level show a wide range in the ability to cope with given aspects of arithmetic.
4. Whether in basic counting, addition, or multiplication, children do not learn directly mature adult patterns in reacting to number.

5. During the initial stages of learning the use of crude procedures, concrete materials, and crutches contribute significantly to learning efficiency.

6. Arithmetic does not strengthen the mind any more so than do the so-called easier subjects.

7. Many arithmetical processes (square root, cube root, etc.) are used very little in everyday life.

8. General reading materials present numerous situations calling for an understanding of many phases of number that are seldom needed as computational skills.

9. When arithmetic skills are presented so that the sequential nature of number is not violated, children learn those skills more readily, show better retention of the skills, and show more ability to transfer principles learned to untaught number skills.

10. Arithmetic problem solving is learned better and computational skills are learned equally as well through the medium of an activity program as through one more rigidly organized.

11. Desired arithmetic learning outcomes are greater when arithmetic learning experiences are definitely provided and planned than when such experiences are left to chance.

12. Inductive methods are superior to deductive methods
13. Children profit from meaningful practice. The greatest advantage is derived from practice when it is motivated by a social need and when it follows the acquisition of understanding.

14. The understanding of the function of the processes of addition, subtraction, multiplication, and division, as well as an understanding of the interrelatedness of those processes contributes significantly to promoting problem solving skill on the part of children.

15. The use of cue words in teaching problem solving is detrimental to the development of true problem solving ability.

16. Problems which are purposeful and real to the learner are more effective in promoting problem solving ability than are problems which are unrelated to the learner's life experiences.

17. Instructional materials, especially textbooks, are used best when they are adapted to the learner's ability and needs.

18. An arithmetic textbook can be used effectively as a general reference book in the teaching of arithmetic.

19. A combination of the activity method and grouping within the classroom appears to be the most satisfactory means of meeting the individual needs of children.
20. Many evaluation techniques and devices (standardized tests, teacher made tests, interviews, observations, and anecdotal records) have a key place in arithmetic evaluation.

**Suggested Teaching Practices**

The research findings reported in the present analysis provide a firm basis on which to build an arithmetic program. By relying on the guidance offered by this research, teachers should feel secure in selecting instructional practices to follow. A recommended arithmetic instructional program which is based on valid research findings is described and discussed as follows:

**Philosophy.**—The school should first of all think through its general school philosophy as well as its purposes for teaching arithmetic. This approach is essential, for the teaching practices that a given school follows in the teaching of arithmetic are very closely interwoven with both its conceived school philosophy and its purposes held for including arithmetic in the curriculum. After all of those concerned have an understanding and acceptance of the school's general objectives and its purposes for providing instruction in arithmetic, a stronger foundation is laid for selecting content and practices to meet those purposes most advantageously. If the school considers its function to be that of promoting the total development of children--physical, emotional, social, and intellectual--its teaching practices will be different from the school which is concerned solely with children's intellectual
If the school follows the guide of valid research evidence, it will decide that it should concern itself with the development of the whole child, for it is only as children develop fully in each of the separate aspects of growth that they become truly integrated individuals. Stress should not be put on one aspect of development at the expense of the others. Unless the child is physically well, emotionally secure, and socially adjusted, he cannot be expected to reach his maximum potential in number development or in other areas of the academic program.

Being guided further by research, school authorities should decide to place arithmetic in the elementary school curriculum in order to provide children with a tool to be used in meeting both their immediate and their later needs for number. Schools cannot justify stressing arithmetic as an end in itself, nor can they justify including arithmetic in the curriculum for purposes of strengthening the mind.

Arithmetic readiness.—An essential step in the arithmetic instructional program is that of considering children's readiness to proceed with new learning experiences. It should be recognized that readiness to learn arithmetic is not a magical line with part of the children on one side of the line and part on the other. Readiness is a matter of degree and is operative at all grade levels and in connection with all number learning experiences. The varying degrees of readiness may be
illustrated in part by the indicated performance level of the different children in a given first grade. Some children in the class may have the most immature counting ability which is indicated by the fact that they can give sheets of paper to their classmates one-by-one, although they may not be able to tell how many sheets were given out. Other children will possibly be found who can count the children individually and go and get the required number of sheets of paper. Still other children may be able to count the children in the group by two's or by three's. It should be concluded that each of these separate groups of children are ready for number experiences to help them become more efficient at their present performance level as well as to help them move to a higher level of performance.

Since many interrelated factors (experience, intelligence, maturation, degree of meaning and understanding, and intrinsic purpose) influence readiness to learn arithmetic, teachers at every grade level should be cognizant of these factors. All of the emphasis should not be placed on any one of the factors and the others ignored. It should be recognized that generally the older the child is the more readily he learns, and that the degree of intelligence possessed by the child plays a vital role. It should be recognized, however, that experience also plays a significant part. General maturation and intelligence cannot make their full contribution to learning in the absence of experience. Building
readiness through providing experience to supplement the child's maturation and intelligence should be considered an integral part of the arithmetic instructional program.

Experience can be used to provide readiness for learning any of the arithmetical processes. The process of division will be used here for illustrative purposes. Traditionally division has been introduced as an abstract process in the late third grade or the early fourth grade with the more advanced aspects of division taught in the fifth or sixth grades. The first grade teacher should provide readiness for division by utilizing functional problem situations to develop the meaning of division. The teacher may say, "There are twenty children in our room. We are going to play a game requiring four groups. How many children will be in each group?" The teacher would possibly have the children move into the different groups one-by-one in order to find the answer. More experienced children may substitute counters for children to find the solution.

In further promoting readiness to learn arithmetic, schools should give increased attention to the role of understanding. The understanding the child has of mathematical principles and relationships determines to a large extent his ability to cope with related untaught number situations. For example, the child may be attempting to learn to subtract an example like 605 - 289. If his previous experience in solving an example such as 75 - 24 has been taught meaningfully
through the use of appropriate instructional materials so that he understands that one ten is "borrowed" from the seven tens and regrouped with the five ones so that he now has fifteen ones from which to subtract eight ones, he should have little trouble in transferring the principle involved in borrowing to solving the more complicated subtraction indicated in the first example. On the other hand, if the child has learned to borrow as a mechanical process without understanding, he is much less likely to be able to solve related untaught examples.

Considering the fact that intrinsic purpose or the will to learn plays such a vital role in determining the child's readiness to learn arithmetic, teachers should plan to utilize fully those real situations which children meet demanding the use of number. The utilization of such experiences is discussed more fully in connection with curriculum patterns.

In order to ascertain children's number readiness, teachers should use a combination of several evaluation techniques and devices, namely, observations, interviews, anecdotal records, commercial tests, and teacher made tests. Illustrations of the role and use of these several ways are given later in this report in connection with the treatment of evaluation as an aspect of the arithmetic instructional program.

Selection of content.—Since the purpose of teaching arithmetic in the elementary school should be considered to
be that of providing children with a useful tool in helping them meet their social needs, arithmetic content should be selected with that purpose in mind. This approach makes it possible to eliminate from the curriculum those processes and skills that are little used in the daily activities of man. Among such phases of arithmetic which research has indicated to have little general use are fractions having odd or excessively large denominators, problems involving carpeting or papering, and highly theoretical and nonsense problems. Arithmetic needed for specialized vocational trades should not be taught in the elementary school, but left to be included in the special training of those who are preparing for the particular vocations.

The specifics of what has been found to be socially useful are so detailed that schools considering this problem should refer to the original sources for specific guidance.\(^1\) Although this research can be helpful as a guide, the recommendations should not be followed rigidly. It is an impossibility to determine exactly what arithmetic all children need now or will need later as adults. Changing times no doubt will affect children's future needs for number. As for the present, particular groups of children may find uses for some aspects of arithmetic not ordinarily used by most children.

As one endeavors to determine what arithmetic should be taught, several points should be kept in mind. One is that

\(^1\)For references treating this topic, see Chapter III.
teachers should be alert to the special needs for number which their children indicate and make provision to meet those needs. Another is that since it is impossible to know specifically what arithmetic one will need as an adult, mathematical principles and understandings should be stressed so that one can cope effectively with new content even though it may not have been taught directly. For example, although fractions having odd or large denominators are not used for practice materials, one should be able to transfer principles used in working more commonly used fractions to these special cases. Teaching methods, which will be suggested later, should be selected to facilitate transfer ability.

A definite principle which should be adhered to in selecting socially useful content is that content should not be so selected and presented that the sequential nature of the number system and the relationships inherent within it are violated or destroyed. This is to say that schools should not select isolated bits of socially useful arithmetic and present them so that significant number relationships may not be seen. For example, it would be a poor practice to present addition combinations in such random order as 9 \neq 9, 6 \neq 2, 3 \neq 4, so that children would have little opportunity to see the relationship between one combination and another.

Much arithmetic content should be included in the curriculum for its informational value even though it is not included in computational arithmetic. A case in point may be
the understanding of extremely large numbers as reported in government expenditures. Children will find much need for informational arithmetic in order to read with understanding social studies, science, and general reading materials.

The principal source of content for problem solving should be boys' and girls' real life experiences, both in school and out of school. This approach gives greater assurance of meeting the social needs of children and also provides the most dynamic type of motivation for learning arithmetic. This approach will be discussed further in connection with curriculum patterns.

Grade placement of content.--In an effective arithmetic instructional program, aspects of most processes or skills (fundamental operations with whole numbers, common fractions and decimals; telling time; approximation; measuring; use of money, etc.) should be considered and taught at all grade levels in order that children be able to meet their immediate social needs. The kindergarten or first grade children will use those skills on the most immature level, while the older elementary children should be achieving competency at a higher abstract level. Problems requiring the use of addition, subtraction, multiplication, or division can be solved by a very simple counting process or by the use of complex algorithms. For example, a first grade child may bring 48 cookies to give to the 24 children in his class. He gives the cookies to the children one-by-one in order to determine how many
cookies each child will get. The sixth grade child, of course, will use the abstract algorism to solve his problems calling for division.

Teachers should be alert to recurring quantitative situations arising in the child's total school day so that there is a constant reoccurrence of the need to use each of the arithmetic processes. This means that as number skills are applied in purposeful situations, children are constantly being given the opportunity to practice, expand, enrich and refine those skills. Even though skills will share in receiving major emphasis from time to time, none are temporarily shelved and forgotten.

Although arithmetical processes are considered at every grade level, particular aspects should receive more emphasis at some grade levels than at others. For example, learning to use fractions at the highly abstract level will ordinarily receive major emphasis at the upper elementary grade levels. Social demand should determine to an extent whether to place emphasis on a given aspect of arithmetic early in the grades and stress rich experiences or whether to delay the emphasis until children are in higher grades and rely on maturation. For example, if it is found that a given group of children indicate a greater need for fractions at the second or third grade level than is ordinarily found there, these children can be hastened in their learning of simple fraction manipulation through heavy emphasis on the use of first hand
experiences and concrete manipulative materials. On the other hand, if the particular social situation is not great for the extra use of fractions, undue emphasis on fractions should not be given at this level, but delayed until the children are more mature and have had more experience.

Rather than teaching skills before children have much demand for them, teachers should dwell more on the skills which children are currently needing. By providing extra enriching experiences related to the skills being taught, teachers will be helping children build a stronger and more meaningful foundation on which to build later skills. An example in point is the teaching of the one hundred addition and the one hundred subtraction combinations in the first grade. If pressured most children can learn to respond rather automatically to those combinations at that level. A stronger foundation is laid, however, if only part of those combinations (those with sums of ten or less, for example) are stressed at this level. By confining the major part of the instruction to a limited number of combinations, more time can be taken to guide children in discovering the relatedness of the various combinations, in forming significant generalizations, and in experiencing the use of the combinations in many realistic problem situations. If children are given these enriching experiences, they should find little difficulty in solving a problem involving $\frac{8}{7}$ by the use of counters or objects even though that combination has not
been taught directly.

Closely associated with the grade placement of content is the question of whether or not arithmetic should be taught in the kindergarten and first grade. Since research indicates that children at this level can learn arithmetic, have a social need for number, and retain the arithmetic learned, it should be concluded that arithmetic should be included in the kindergarten and first grade program. It should not be so much a question of whether or not to teach arithmetic at this level, but rather a question of the type of program.

The number program should be planned so that the teacher can systematically assist the children in expanding and increasing their use of number skills needed in meeting more effectively the quantitative demands of their environment. At this level heavy emphasis should be placed on the informal use of concrete and realistic experiences related to the simple aspects of most arithmetical processes. Growing from such experiences, the children should begin using abstract numbers in performing simple operations such as basic addition and subtraction. Because of the emphasis placed on real experiences and the limitation of skills to simple processes, there will be little need at this level for formal practice.

Curriculum patterns.—Because of the larger resulting values in the development of children—better emotional and social adjustment, greater ability to do critical thinking and problem solving, and no loss in the achievement of
computational skills—arithmetic instruction should be provided within the framework of a program organized around arithmetic activity units or integrated experience units. Arithmetic activity units are selected because they are rich in the opportunity for the use of number and at the same time deal with activities which are purposeful to the learners. For example, the class may decide to make kites, a project which will involve the use of number in many meaningful ways as the kites are designed, as materials are purchased, and as the necessary measurements are made. Arithmetic skills will need to be taught so that children can carry out effectively the required mathematical processes.

Integrated experience units deal with a larger social problem, such as "How does man travel today?" In considering this topic, organized subject matter is drawn on and needed to contribute to a better understanding of the problem. Arithmetic makes its contribution to a unit of this type by enabling the children to meet more efficiently the quantitative aspects of the unit.

Considered in view of the outcomes related to arithmetic, these two approaches have not been experimentally compared. About the same favorable results have been reported for both types of programs when they have been compared individually with more traditional approaches. Until research reports differences between these two plans, schools can feel secure that a definite contribution can be made to the total
development of children by adopting either of the plans. It is highly possible that both approaches could be used to advantage to supplement each other.

In the specific use of these plans, research has some definite guidance to offer. In order for children to acquire satisfactory progress in number development, arithmetic instruction and experiences must be definitely planned. If left to chance or incidental happenings, number will very likely not appear in sufficient quantity to provide the necessary experience for children to make desirable progress in arithmetic. This means that in organizing learning experiences through integrated experience units, teachers must be alert to the contribution that arithmetic can make to the unit. If properly surveyed, it will usually be found that most units afford many situations to which number can make a significant and vital contribution. In the case of the transportation unit referred to previously, number can make a contribution to the understanding of reading content when such statements are read as, "Airplanes now travel 600 miles per hour." Computational processes will be needed in considering cost of travel, mileage, speed, etc. Further, many uses for arithmetic will be found as the construction activities, which are usually an essential part of such units, are carried out. For example, the class may decide to construct a miniature railroad which would require that measurements be made to scale, materials purchased, etc.
The teacher should not limit the planned use of number to those situations directly associated with the unit, but should plan to utilize as well those real quantitative situations occurring as school routines and special events are planned and executed. Children can be given many opportunities to advance in their use and understanding of number as they participate in taking attendance, taking lunch orders, counting money contributed to the Red Cross, etc. Special occasions as planning a Halloween party are rich in the possibility for the use of number.

Even though problem situations and the demand for number skills grow from the unit, the teacher usually will not be able to give sufficient instruction at the time quantitative situations occur to enable the children to acquire the skills needed in carrying forward the number aspects of the unit. Special time should be provided in which children can be given specific instruction. This scheduled period should be flexible so that on occasion more time would be taken for arithmetic than at other times. This should depend on the needs and progress of the children. The teaching of skills is treated more fully in the following topic dealing with the teaching of computational skills.

**Teaching computational skills.**—As was indicated earlier in the treatment of curriculum patterns, the need for computational skills should grow from the quantitative situations arising in connection with unit experiences or in
connection with the events of daily school living. Specific guidance in the learning of these required skills should be provided by the teacher.

To be consistent with research findings an inductive or discovery method should be followed in the teaching of arithmetic computational skills. Through the process of discovery the child can best become aware of the relationships between related number facts and processes and thereby lessen his need for practice and increase his ability to cope with untaught number skills. Rather than directly telling the child the answer to a new mathematical fact, the teacher should present the child with manipulative materials so that he can find the answer for himself. In teaching the combination $\frac{3}{4}$, for example, the teacher can help the child find the answer by having him select three objects and then four objects. When these two groups are combined and the child counts the objects in the new grouping, he can see that three and four make seven. The grouping can further be broken down into $\frac{3}{3} \frac{1}{4}$ so that the child can see that $\frac{3}{4}$ is only one more than the combination $\frac{3}{3}$ which he possibly already knows. To add greater meaning the child may be guided to find the relationship between $\frac{3}{4}$ and $\frac{4}{4}$. In this case, the child removes one object from the grouping representing $\frac{4}{4}$ and discovers that $\frac{3}{4}$ is only one less than the former grouping.

As the inductive approach is followed teachers should
look for subtle indications that the child sees and understands relationships between the different facts. He may understand, although he may not be able to verbalize the understanding. He may indicate his understanding more indirectly by saying, "Oh, I see". Further understanding is indicated as the child begins to rationalize the correctness of other near doubles by relating them to the known doubles. According to research evidence there is little to be gained by pressuring the child into verbalizing the understanding once he indicates at the unverbalized level that he understands.

The inductive and meaningful approach outlined in the case of teaching a simple addition combination should be followed in the teaching of all mathematical processes. Once children have verified the truth of the number facts being presented and have grasped the relationships between the related facts, practice should be provided so that children may become more proficient and mature in their response. The motivation for this practice should result from the need for competency in meeting real quantitative situations. Practice should be adapted to individual needs and provided through a variety of ways—games, flash cards, teacher prepared exercises, and commercial materials. More will be said concerning individual needs and practice materials as these topics are treated more fully later in this discussion.

Some adaptation should be made in the application of
the inductive approach with children of lower mentality, for these children have been found to be especially weak in the ability to generalize. This being the case, less emphasis should be given to the relationship between number facts. Children of this type should be given much meaningful experience in using concrete materials to verify that \( \frac{3}{4} = 7\), but it should not be stressed that they see how this fact is related to \( \frac{3}{3}\). With these children each specific number experience should be built up as meaningfully as possible and then appropriate practice materials provided so that they can become as efficient as possible in their response.

**Teaching problem solving.**—The teaching of computational skills and problem solving should be very closely interrelated. It is of little practical value for children to be highly competent in the use of computational skills, if they cannot apply those skills in solving problems of a quantitative nature. In order to solve a problem one has to be able to decide whether to add, subtract, multiply, or divide the expressions of quantity given in the problem. It has been the consensus of research evidence reviewed in the present analysis that when children truly understand the meaning of the processes of addition, subtraction, multiplication, and division, as well as understanding the interrelatedness of those processes, they have little trouble in determining which of the processes to use in a particular situation.

Fundamental operations should be so taught that children
understand that addition is a means of combining groups of quantity, and that subtraction is a process of separating groups. They need to understand further that multiplication is a faster way of combining groups than is addition, and that division is a more efficient means of separating a group into a given number of equal sized groups than is subtraction. These understandings should be stressed as the basic addition and subtraction combinations are taught and later as multiplication and division are receiving emphasis.

Children can be helped to discover the above meanings as they dramatize the solution of functional problem situations. For example, five children are in a group and four more children are invited to join the group. As the children move into the new grouping, they should be encouraged to count to see that there are now nine children. They should be further prompted to see that there are not more children in existence, but that they have merely been regrouped. As children leave the group to indicate subtraction they should be guided to see that there are not really fewer children, but that they have been regrouped or separated. In addition to dramatization, children should use manipulative materials such as wooden discs to combine groups as they add or to separate groups as they subtract.

Through similar dramatizations of functional problems, children should be helped to see that multiplication is a short cut and a more efficient means of combining a number of
equal sized groups into a larger group than is addition. As in the case of addition, children should see that when we multiply that we do not get more but that quantities are combined into one larger group.

Children should be assisted to discover the two basic meanings of division, that of finding the number of equal sized groups contained in a given quantity and that of finding the size of the groups when one knows the number of groups into which the larger group is to be broken or divided. An example of each of the above meanings is implied in each of the following problems. (1) "If one apple costs 4¢, how many apples can be bought for 24¢?" In this case we know the size of the group and need to determine the number of such groups contained in 24. (2) "Six apples cost 24¢. How much will one apple cost?" Here we know the number of groups, and need to find the size of the groups.

Children should be helped to grasp each of the meanings of division as they dramatize the solution or as they use manipulative materials to perform the required division. Pupils should be given the experience of seeing that each of the problems could be solved through subtraction. In the first example the child should make repeated subtractions of four and then count the number of subtractions made to tell him the number of groups. In the latter example, the child should discover that six is contained in 24 four times by subtracting six the necessary times.
In addition to helping children solve problems through stress given to the meaning of the fundamental operations, children should be assisted in understanding problem situations by helping them improve their reading and number vocabularies. Children will encounter many problems requiring that they be able to read with understanding. One way of helping children read such materials with better understanding is to utilize concrete experiences in helping them acquire meaning of vocabulary terms. For example, children can enrich their meaning of the technical term "perimeter" through actually measuring the distance around things, such as the distance around a table, the classroom, or the playground. After such concrete experience, most children will have little difficulty in understanding the meaning of the term when they meet it in a written problem situation. This type of experience will also help the children formulate for themselves and truly understand the abstract formula for finding perimeter, \( p = 2l \neq 2w \).

As a means of gaining competency in problem solving, strong emphasis has been given in the foregoing paragraphs to the importance of understanding the meaning of the fundamental operations so that they may be applied intelligently in the solution of arithmetical problems. Stress has also been placed on the importance of understanding mathematical terms. This approach to helping children become effective problem solvers is the opposite to the approach which relies
on the mechanical use of cue words to indicate which of the fundamental operations to use. The use of such cues as "together", suggesting addition, "less", implying subtraction, "times", indicating multiplication, and "shared"... "equally", meaning division, has been found to be detrimental to promoting true problem solving ability. Such cues should not, therefore, be included as part of the elementary arithmetic instructional program.

**Instructional materials.**—In an effective arithmetic instructional program real experiences should be considered the most useful type of instructional materials. For instance, the children in a sixth grade are making a tool cabinet in which to keep their tools. The situation itself, calling for the use of mathematical measurements and processes, is the most effective type of arithmetic instructional materials that a teacher can use. The situation can be used to assist children in the discovery of the meaning of foot, inches, yard, perimeter, area, etc. As pointed out earlier, arithmetic activity and integrated experience units should be considered a source for many of these real situations.

In order for children to acquire the necessary understandings and skills, it is essential that during the initial stages of introducing new number skills, schools plan to use manipulative materials such as abaci, fractional discs, counters, etc., and pictorial materials such as charts and illustrations. Children should, however, be encouraged to
proceed as rapidly as their development will permit to using more mature procedures in working with number. As children begin working with abstract number, they will frequently need to return temporarily to the use of manipulative materials. For example, a child may use manipulative materials to discover that \( \frac{5}{4} = 9 \), and then proceed to work with abstract symbols. In a day or so he may forget the correct response and need to return to the use of manipulative materials to find the answer again. This should be accepted as natural, for research has shown that learning is a very gradual process and does not take place directly and all at once.

In the classroom use of manipulative teaching aids, teachers should plan to let the children use the materials themselves in discovering number facts and relationships. The teacher's role should be that of guiding the children by skillfully questioning them as they use the materials. For example, the children may discover fractional relationships by comparing different fractional discs. They find out through this process that two halves are the same as one whole, and that two-fourths equal one-half, or that four-fourths make one whole. The teacher may ask, "Which would you rather have, one whole or two halves?" or "Mary has one whole disc, and Charles has four-fourths. Who has more?" Through further questioning the teacher can help the children discover that in the case of unit fractions the larger the denominator the smaller the fractional piece is in size.
Teachers will want to prepare much of their own pictorial instructional materials, and here again, research offers valid guidance. Let us suppose the teacher is wanting to prepare pictorial materials which will prompt children to use the more mature process of grouping rather than counting one-by-one in apprehending the objects in a group. The teacher should arrange the pictured objects in regular patterns such as triangles, diamonds, or the domino, for such arrangements enable children to identify at a glance, three, four, or five objects, whereas, if the objects are arranged in an irregular pattern the child will be practically forced to count the objects individually as he determines the quantity represented. To assist children further to use grouping rather than detailed counting, the objects pictured should be composed of simple elements free of complex details. Simple geometric figures such as circles or squares are more easily identified as groups than are more complex realistic objects like Indians with their detailed attire.

Problem solving practice materials should be so prepared and presented that one problem after the other cannot be solved in identically the same way. When problems are so presented that one problem after the other can be solved in the same way, children are very likely to become blinded to the use of more efficient means of solving problems. A case in point may be that in which children are solving simple interest problems by using the formula \( i = prt \). Practice
problems of this type should be mixed up so that in some cases rate of interest is called for and in others the amount of principal, etc. If formulas are developed with children so that they understand the relationship among the different factors, they can be expected to apply that understanding in solving problems when different factors as principal, rate, or interest are to be found. Each practice problem should be so presented that new and critical thinking is demanded.

Textbooks and other commercially printed materials can have a significant place in the classroom. It should not be so much a question of whether or not to use arithmetic textbooks, but more a question of how to use them. They should be used as one would use any good reference material. For example, the children may be following recipes for making bread or they may be making measurements involving the use of fractions. The children will usually find it advantageous to refer to the book to see how the particular topic is treated. The explanations and illustrations will frequently be found helpful in supplementing discoveries made through the use of more concrete instructional aids. Also, some of the practice material provided by the book will be found useful in helping children become more proficient in the use of the desired skill.

Since children differ in needs and abilities, all children in a given grade should not be expected to use the same book. Copies of books from several series and on several
grade levels should be available. Some of the books will treat certain aspects of arithmetic more adequately than others. The textbook certainly should not become the sole prescriber of what is to be taught. An arithmetic instructional program which limits number experiences to those provided in the text is a very inadequate program. Teachers should remember that no matter how good the textbook may be, first hand experiences cannot be put into the book.

Individual needs.--Children in any grade differ in their ability to learn arithmetic and in their rates of development in the use of number. This being true, schools should make specific provision to meet the individual arithmetic needs of the children in any given grade. To be consistent with the varying progress children make in arithmetic, schools should not establish rigid grade standards which all children are expected to meet. The fourth grade teacher, for example, should plan to teach more arithmetic than that usually assigned or expected at the fourth grade level. Some of the children in this grade will require experiences normally provided at the second or third grade level, while other children should be provided supplementary enriching experiences over and beyond those ordinarily provided in the fourth grade. A principle that should be adhered to in meeting the individual needs of children is that each child should be permitted to progress at his own rate of development and materials and instruction presented
accordingly.

In terms of the best development of the whole child, schools can meet best the varying arithmetic needs of children through the medium of arithmetic activity or integrated experience units as was discussed earlier and through individual and small group instruction provided during the scheduled skills period. Through the medium of activities it is possible for children to derive the values from working together in small groups and at the same time it is possible to meet the individual needs of children by permitting each child to make the contribution which is in line with his ability. To illustrate, let us suppose that children are making stage scenery. The children who are less advanced in number development may be carefully guided in making simple measurements, while the more advanced children may do the more complex figuring in compiling measurements and in figuring costs. As children carry out the various activities associated with units, there are many occasions for children to work in committees or small groups. Since children work as members of many different groups, there is less chance of their becoming branded as belonging to a definite group.

In the skills period the teacher should offer direct instruction by working with the children individually and in small groups. While some children are doing practice work independently, the teacher may work with another group to introduce new skills. When all of the children are doing work
individually, the teacher should move around the room giving help as the different children need it.

Since schools should be concerned with the total development of children, academic achievement should not be the sole criterion on which schools base their promotion policy. If schools make the necessary adjustment for meeting individual needs, there will be little reason for retaining children for purely academic reasons. Even though the research considering the problems of promotion or non-promotion has favored promotion in every instance, it should be remembered that most of the research has been made in terms of averages. What is best for the average may not necessarily be best for the individual child. All available evaluation techniques should be used to secure data relating to the child's physical, emotional, social and intellectual status. After considering the total data, schools should make decisions in terms of what will be best for the individual child. It may be found that because of physical, emotional, social or intellectual immaturity, an individual child will be better adjusted with a younger group of children.

Evaluation.—Since the school should have as its objective the development of the whole child, any evaluation which is made of the arithmetic instructional programs should be made with that objective in mind. Also, since the specific purpose of arithmetic instruction should be to provide children with a tool to be used in meeting the quantitative
aspects of their environment, the final evaluation of children's arithmetic learning should be made in terms of the effectiveness with which they meet their personal arithmetic problems.

In order for the teaching practices described in the foregoing discussion to be truly effective, evaluation must be made an integral part of those practices. In the initial stages of instruction an evaluation must be made of the child's readiness to learn the new material. Evaluation must be made during the teaching-learning process to determine how well the child is achieving and to indicate where he is having difficulty. With this information, the teacher has a basis for adjusting instruction and materials to children's needs.

To be consistent with the research evidence presented in the present analysis, evaluation in arithmetic must be used to gain information relating to the pupil's status in respect to several factors, namely, (1) overt behavior (rate and accuracy) in meeting quantitative situations, (2) level of mental maturity and background of experience, (3) the degree of meaning and understanding, (4) the vocabulary and reading ability of the learner, (5) maturity of thought processes, (6) the ability to solve personal quantitative problems, and (7) the adjustment of the whole child—physical, intellectual, social and emotional.

In order to obtain data relating to the above factors, teachers should use a combination of several evaluation
techniques, because no one technique can be relied on to give information relating to all of the factors. Teachers should plan an evaluation program in which they use observations, interviews, anecdotal records, teacher made tests, and standardized tests. The place of each of these will be illustrated in turn.

Observation is one evaluation technique which the teacher should plan to use daily. The use of observation may be illustrated as follows: The fifth grade teacher has a group of children at the chalkboard practicing the process of long division. As the teacher watches the children work, he can note the efficiency with which each works and he can see where several children or individuals are having particular difficulty. It may be discovered that some children are having specific difficulty in the use of the zero in the quotient, in multiplication, or in subtraction. On other occasions the children may be working at their seats, while the teacher moves about the room observing each child as he works. Information so secured should aid the teacher in providing children with the necessary instruction.

Anecdotal records are written reports of children's observed behavior and should be made in conjunction with observations. The teachers should have records indicating situations in which the child used arithmetic to meet his personal arithmetic problems. This means provides one of the essential indications of how well children are able to use
arithmetic as a useful tool.

Teachers should make consistent use of the personal interview in ascertaining the child's level of performance and understanding. Some of these can be made informally as the teacher works with the children individually in guiding daily work. On other occasions, the teacher may interview the child concerning his thought processes used in taking a written test.

Through the personal interview the child is asked to tell how he is thinking in performing mathematical processes. For illustration, let us say that all of the children in the classroom are responding to the multiplication combination $7 \times 8$. If we are to make an evaluation merely in terms of accuracy, we may decide that all of the children were very proficient, but if we ask the children individually how they arrived at their solutions, we shall probably find that the children are operating at different performance levels. One child may indicate that he really understands the meaning of the combination and habitually responds correctly. Another child may be found who has to add seven eights, while another child may remember that $8 \times 8$ is 64 and subtract 8 from that. Still another child may be found who answers 56 from memory but has no conception of how the solution may be obtained by the use of concrete materials or by one of the procedures followed by the other children. If this child happens to forget the combination, he is at a loss to know how to find
the solution by some other means. The teacher should use this information to help the child increase his understand-
ing of what multiplication means. This may be done through the use of concrete materials and dramatizations referred to earlier.

Teachers should prepare many tests themselves. The teacher knows better than anyone else what specific content has been presented to the children, and he will want to test to ascertain with what degree of success the children are learning. Some of these tests should be designed to measure rate and accuracy; others to measure understanding. The following is an example of a question a teacher may use in determining the children's understanding of the multiplication algorism:

In the example at the right, why is the second partial product (1728) moved one place to the left when we multiply by 3?

1. Because the partial product should always begin under the figure by which you are multiplying.

2. Because 3 is the second figure.

3. Because the 3 means 3 tens or 30 ones.

4. Because the answer will be wrong if you don't.

5. Because that is the way we were taught.

Standardized tests should have a place in the evaluation program. Usually once a year is often enough to give a standardized achievement test in arithmetic. If the test is
given late in the spring it will offer a measure of the achievement for that particular grade. When the information is passed on to the next teacher, the new teacher will have an approximate measure of each child's achievement at the beginning of the school year. By giving the test the following spring, the progress the child has made during the year can be noted. This affords the teacher one indication of how children are progressing in the different computational skills as well as in problem solving.

Standardized tests should also be used in determining number readiness or in making a diagnosis of children's performance with specific content such as long division, for example. Tests of this type usually have much to commend them since they have been more scientifically prepared than are ordinary teacher prepared tests. It should be remembered, however, that no one test can do everything. Additional data should be secured by other means to supplement that revealed by standardized tests.

Synopsis of suggested teaching practices.--In summary it may be said that an arithmetic instructional program which is based on reliable research findings will employ many interrelated practices. One essential procedure is to determine each child's readiness for learning at every grade level. After knowing children's readiness to proceed with new learning experiences, the teacher should work with the children on their respective levels by using a combination of activity
methods and group and individualized instruction within the classroom. Purposeful activities should be used to provide functional learning experiences. Skills needed in carrying forward the activities should be taught through inductive or discovery procedures which make use of appropriate instructional materials in helping children discover and verify number facts and relationships for themselves. Purposeful practice should be used to help children maintain and strengthen skills learned, although the amount of practice needed will be much less than when isolated practice in the absence of understanding is used. Finally, the teacher's methods should embody the best principles of democratic leadership.
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I, Chester Enoch Bartram, was born in Nolan, Mingo County, West Virginia, February 14, 1917. I received my elementary and secondary school education in the public schools of Mingo County, West Virginia. I was graduated from Chat­taroy High School in 1936. My undergraduate training was ob­tained at Marshall College, from which I received the degree Bachelor of Arts in 1946. From that same school I received the degree Master of Arts in 1947. My public school teaching experience was gained in the elementary schools of Mason Coun­ty, West Virginia, where I was a classroom teacher for five years, an elementary school principal for five years, and a county elementary school supervisor for one year. During World War II, I spent forty-three months in the Army Air Corps as a cryptographer. Thirty-two of those months were spent overseas in the European Theatre of Operations. In the summer of 1951, I entered The Ohio State University, where I began working toward the degree Doctor of Philosophy. During the two years spent in residence at The Ohio State University (1953-55), I was graduate assistant to Dr. Lowry W. Harding, Professor of Education. Currently I am Assistant Professor of Education at Mount Union College, Alliance, Ohio, having joined the staff of that school in June, 1955.

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