Coupled Transmission Line Based Slow Wave Structures for Traveling Wave Tubes Applications

Dissertation

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By

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Abstract

High power microwave devices especially Traveling Wave Tubes (TWTs) and Backward Wave Oscillators (BWOs) are largely dependent on Slow Wave Structures for efficient beam to RF coupling. In this work, a novel approach of analyzing SWSs is proposed and investigated. Specifically, a rigorous study of helical geometries is carried out and a novel SWS “Half-Ring-Helix” is designed. This Half-Ring-Helix circuit achieves 27% miniaturization and delivers 10dB more gain than conventional helices. A generalization of the helix structures is also proposed in the form of Coupled Transmission Line (CTL). It is demonstrated that control of coupling among the CTLs leads to new propagation properties. With this in mind, a novel geometry referred to as “Curved Ring-Bar” is introduced. This geometry is shown to deliver 1MW power across a 33% bandwidth. Notably, this is the first demonstration of MW power TWT across large bandwidth. The CTL is further expanded to enable engineered propagation characteristics. This is done by introducing CTLs having non-identical transmission lines and CTLs with as many as four transmission lines in the same slow wave structure circuit. These non-identical CTLs are demonstrated to generate fourth order dispersion curves. Building on the property of CTLs, a ‘butterfly’ slow wave structure is developed and demonstrated to provide degenerate band edge (DBE) mode. This mode are known to provide large field enhancement that can be exploited to design high power backward wave oscillators.
Dedicated to all the freedom fighters of Bangladesh who sacrificed their lives for the sake of freedom of Bangladeshi people, language, culture, rights and existence. Also, I would like to dedicate this dissertation to my parents (Md. Khairat Hossain and Nilufa Yeasmin) and brothers (Md. Shafiul Bari Shohag and Md. Safiquz Zaman Sumon) without whom I would not be able to come this long way.
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Chapter 1: Introduction

Slow wave propagation has been of interest to electromagnetics as it can lead to 1) smaller antennas, filters and RF circuits, 2) novel propagation phenomena and 3) realization of high power RF and optical devices. Among the applications are 1) intense light source [8, 9], 2) miniature antennas [4] with improved directivity, and 3) high power microwave devices such as oscillators and amplifiers [10]. Typically, slow wave properties are realized using periodic layered media that can even exhibit anisotropy. Alternately, slow waves can be formed using Coupled Transmission Lines (CTLs). In this case, one of the transmission lines propagates a strong forward wave, whereas the other adjacent line supports a weak backward wave [3].

To understand the physics of engineered slow wave structures, it is necessary to analyze Coupled Transmission Lines (CTLs), as shown in Fig. 1.1. These CTLs, inherently periodic in nature, are loaded with coupled \((L_M, C_M)\) elements in periodic manner. As is well-known, any periodic loading supports Bloch waves [11] and exhibits passband and stopband behavior. The corresponding \(\omega - \beta\) dispersion curves are of second order and even show the supported waves with nearly zero group velocity at the band edge. By changing the material properties or loading of the CTLs as in the Fig. 1.1(a), the order of the dispersion curve can be altered, a process called dispersion engineering. Specifically, higher order dispersion curves (third or
Figure 1.1: Various slow wave structures. Top: Printed coupled microstrip lines that can realize the 3rd and 4th order dispersion curves [1–3] Middle: Field strength within the coupled transmission lines showing strong field intensities in the middle due to wave velocity slowdown, Bottom: Wave amplitude representing field growth, (b) Slow wave propagation within a helical wire structure placed in a waveguide, (c) Slow wave propagation within a curved ring-bar structure placed in a waveguide, the latter refer to Traveling Wave Tube applications.
fourth) can be attained. Indeed, use of anisotropic material in periodic stacks, instead of isotropic ones, can lead to third or fourth order dispersion relation \cite{12} (Fig. 1.2). The fourth order dispersion relation leads to maximally flat curves, and is associated with Degenerate Band Edge (DBE) modes that are inherently narrowband in nature \cite{1,13–15}. Nevertheless, DBE modes provide the means for large field enhancement as the group velocity drops to nearly zero \cite{1,13}. This reduction in group velocity also implies device miniaturization \cite{16} and increased antenna directivity \cite{4,17–19}. In addition, recent studies of silicon based electro-optical waveguides also demonstrated DBE modes \cite{20,21}

For many applications, wider bandwidth is preferred, and this can be achieved by working near the third order region of the dispersion diagrams \cite{12,22}. These third order curves are associated with an inflection point within the Bloch diagrams. The associated modes are referred to as Magnetic Photonic Crystals (MPC) modes \cite{2,18,19,23–25}. DBE and MPC modes were also realized and demonstrated by fabricating periodic volumetric stacks of dielectrics \cite{26}, as shown in Fig. 1.2. But these stacks are bulky.

In this dissertation, we focus on applications of coupled transmission lines for high power microwave sources. High power microwave sources require slow waves to harness RF energy from intense electron beams \cite{27}. Specifically, traveling wave tubes (TWTs) and backward wave oscillators (BWOs) directly use these slow waves to couple the electron beam energy into RF modes and deliver power on the order of Megawatts. The operation of TWTAs is based on the Cerenkov radiation. Basically, an electron beam is injected into a waveguide where a slow wave structure provides the coupling mechanism. The amplification of the RF wave occurs inside a waveguide
Figure 1.2: Periodic material structures to realize MPC and DBE modes. **Top:** Photonic Crystals using simple dielectric stacking. Forward and backward waves are depicted using blue and green arrows, respectively. **Bottom:** Magnetic Photonic Crystals using volumetric stacks. Anisotropy is realized using metal strips printed on dielectric layers [4]
forming the TWTA. In particular, we focus on TWTAs and design of several SWSs for low to high power applications. It is shown that all the conventional SWSs can be interpreted as transmission line (TL) loading of the waveguides [27]. In fact, the SWS act as artificial dielectrics in modifying the group velocity without affecting the modal properties of the waveguide. A classic example among these SWSs is helix [28]. Helical SWS is inherently wideband in nature. However, helices have power handling limitations [29]. Thus, there is interest to optimize power and bandwidth characteristics of the helical structures.

Some conventional SWSs such as double-helix [30,31] or ring-bars [30,32,33] are known to deliver higher power. Hence, these SWSs can be improved via advanced dispersion engineering to deliver even more power. Even though previous work of Shiffler et. al. [34,35] demonstrated MW range TWT amplifier, they provided very narrow bandwidth (< 1%). Therefore, there is interest for wideband MW power TWTs. Typically, corrugated waveguides [35,36] and coupled cavities [37] are associated with such high power levels. However, they are inherently narrowband resonant structures. In contrast, helical family geometries are associated with large bandwidth although some of them are known to deliver power on the order of kilowatts [38]. Hence, a combination of these structures is explored. In this dissertation, we will pursue CTL analysis to predict suitable \( \omega - \beta \) for improved SWSs. With this in mind, we carry out a design of advanced SWS that combine up to four transmission lines as shown in Fig. 1.3. These CTLs can support DBE-like mode for higher power handling. Utilization of these modes is expected to lead to new paradigms for future high power microwave sources.
This dissertation is organized as follows. In Chapter 2, a brief introduction to
traveling wave tubes parameters are given. Further, an introduction to slow wave
theory and coupled mode theory is also provided. The basic formulation of disper-
sion curves from coupled mode theory is the building block for higher order dispersion
calculations. Chapter 3 carries out a study of the helical family SWS including the
Half-Ring-Helix for low power TWTs. Subsequently, in Chapter 4, we focus on wide-
band high power TWTs. In this regard, we propose a novel geometry to achieve
wideband high power TWT. A transmission line model supported by the coupled
mode calculations is also provided. In Chapter 5, we propose a new theoretical anal-
ysis of CTLs for dispersion engineering. CTLs formed by dual pair of non-identical
TLs to realize higher order dispersion engineering. It is shown that fourth order dis-
ersion relation can be realized using coupled mode theory. These have DBE-like
behavior. A novel geometry referred to as ‘butterfly’ is also introduced for a prac-
tical realization of the 4th order dispersion. This ‘butterfly’ SWS can be used for
delivering high power in miniature TWTs and BWOs. A characteristic of the 4th
order dispersion is the support of waveguide modes that have very strong $E$–field at
the center of the guide. As such, coupling between the beam and RF mode is highly
enhanced.
Figure 1.3: Top: The ‘butterfly’ geometry for realizing the DBE mode. Bottom: Circuit model representing non-identical coupled TLs corresponding to ‘butterfly’ slow wave structure.
Chapter 2: TWT Parameters and Coupled Transmission

Line Theory

2.1 TWT Fundamentals

Slow wave structures (SWS) lie at the core of Traveling Wave Tube Amplifiers (TWTA) [39]. Their operation is based on Cerenkov radiation requiring the coupling of an electron beam to a RF wave supported by the waveguide forming the TWTA [39]. To achieve strong coupling between the electron beam and the RF wave, it is necessary for the TWT to support a slow RF mode [40].

The operational principle is shown in Fig. 2.1. Indeed, SWS is an integral part of TWTs. It influences its performance and defines the power handling capability and bandwidth. Several slow wave structures have already been considered for RF wave slowdown in the past [41]. Among them are the helix [42], contra-wound helical structure [30], ring-bar [32], ring-loop [43], and folded waveguide [44]. In addition to slowing down the RF wave, SWS must also exhibit low dispersion and decent interaction impedance.

Generally, high power TWTs require stronger beam current and beam with large diameter. However, space-charges typically reduce gain for an electron beam when larger cross sections are used, making it difficult to achieve a linear relation between
Figure 2.1: **Left**: A flow chart demonstrating TWT working principles. **Right**: Different choice of high power SWS is shown.
gain and power [27]. As is the case with amplifiers, gain and bandwidth are typically inversely proportional. This also holds for TWTs. This is due to the fact that TWTs gain is dependent on two different parameters at the two edges of a band. For example, at low frequencies, the TWTs small electrical length is a limiting factor. On the contrary, at higher frequencies, even though \( N = \text{number of guided wavelength along the length of the TWT} \) is higher, RF amplification causes excessive electron acceleration and deceleration. Thus, the generated waves lose their synchronism with the electron beam implying loss of coupling and lower gain [28]. In fact, at higher frequencies, the TWTs gain can drop down to zero. The overall TWT gain is sensitive to velocity detuning parameter, \( b \) [45]. This implies that good dispersion characteristics can provide larger bandwidth. Based on the above, to achieve good TWT gain, a goal is to also reduce the phase velocity and effective wavelength of the traveling wave. This is also essential to maintain good dispersion and allow good coupling between electrons and the generated RF wave. For any SWS, lower pitch yields low RF wave phase velocity which in turn increases the electrical length of the TWT. Thus, gain increases for lower pitch guided structures. However, as the electron beam velocity is also reduced (for reduced pitch tubes), the available kinetic energy is correspondingly lower due to being proportional to \( v_e^2 \). Therefore, traditional slow wave structures for TWTs cannot generate high power even though they can deliver high gain (Fig. 2.2).

### 2.1.1 TWT Parameters

To quantify the basic gain parameter of a TWT, we proceed to introduce the various TWT amplifier parameters. The gain of the TWT can take the simple form
Figure 2.2: A schematic showing overview of possible slow wave structures for TWT. Low power TWTs rely on helical based geometries typically associated with large bandwidth. On the contrary, high power TWTs rely on coupled cavity based geometry with narrow bandwidth.
as theorized by J. R. Pierce [27]

\[ G = -9.54 + 54.6xCN \]  \hspace{1cm} (2.1)

Here, the following parameter definitions apply:

\( G = \) Gain
\( x = \) Exponential growth factor of growing wave
\( C = \) Beam-wave coupling parameter
\( I_0 = \) Electron beam current
\( V_0 = \) Cathode voltage
\( K_0(\omega) = \frac{E_{axial}^2}{2\beta^2W_{\nu_g}} \) = On axis interaction impedance of the fundamental mode
\( E_{axial} = \) Axial electric field component
\( \beta = \) Wavenumber of the axially propagating wave
\( \nu_g = \) RF wave group velocity
\( W = \) RF wave energy density
\( N = \) Number of guided wavelengths along the chosen TWT of length \( L \)
\( \lambda_g = \) Guided wavelength of the TWTs RF wave
\( f = \) frequency of RF wave supported by TWT
\( \nu_p = \) RF wave phase velocity along the axial directions in absence of electron beam = \( f\lambda_g \)
\( \nu_e = \) electron velocity
\( L = \) Tube length
\( r = \) Beam radius
\( b = \frac{\nu_e-\nu_p(\omega)}{C(\omega)\nu_p(\omega)} \) Velocity detuning parameter
From the above, we observe that $C$ affects both the gain and bandwidth of the TWT. This is due to the interaction impedance, $K_0$, and phase velocity, $\gamma$, being functions of frequency. However, the gain in (2.1) can be improved by increasing $N$ without changing $C$ significantly. This is a key observation, and is exploited here to achieve higher gain and power. Increasing $N$ can be done in two different ways:
(1) by increasing the length of the tube, and/or
(2) by reducing the guided wavelength of the electron beam through the tube.

The second approach is attractive as it leads to higher gain without increasing the overall physical length of the TWT. In next chapter, we will follow this approach to introduce a novel slow wave structure, Half-Ring-helix that increases the electrical length of the TWT by reducing the guided wavelength of the RF wave.

### 2.1.2 Typical Challenges in TWT Design

Two basic properties of the SWSs are important to design the TWT. One of them is dispersion (phase velocity profile) and another is the interaction impedance. To understand the effect of these parameters, gain of the TWT needs to be understood.

From the Pierce gain theory, a linear gain equation was derived from a simplified transmission line model to represent any SWS. For relativistic beams, it is expressed as [27,34]:

$$G = -9.54 + 54.6xCN(dB)$$  \hspace{1cm} (2.2)

$$C = [K_0]^{\frac{4}{3}}\left[\frac{eI}{4\gamma^2m^2}\right]^{\frac{1}{3}}, K_0 = \frac{E_{axial}^2}{2\beta^2P}$$  \hspace{1cm} (2.3)

$$x = x(4QC), 4QC = \frac{R^2\omega_p^2}{C^2\omega^2}, \omega_p = \sqrt{\frac{e\rho_0}{m\epsilon_0}}$$  \hspace{1cm} (2.4)

In the above, $x$ is the exponential growth factor of the growing wave, $\beta$ is the propagation constant of the RF wave, $C$ is the coupling parameter between the electron
beam and RF wave, $K_0$ is the interaction impedance of the SWS within one period, $\gamma$ relativistic parameter of the electron beam, $m$ is the mass of electron, $e$ is the charge of electron, $v_e$ is the velocity of electron, $N$ is the number of guided wavelengths that electron travel throughout the tube and $P$ is the time-averaged, axially directed RF power flowing along the tubes (usually calculated from the Poynting vector), $4QC$ is the Pierce space-charge parameters, $\omega_p$ is the plasma frequency, $\rho_0$ is the beam charge density and $R$ is plasma frequency reduction factor.

As observed from the definition of $G$, the gain of a TWT is directly proportional to growing factor $x$ and to the cubic root of the interaction impedance [27], $K(\omega)$. These two factors are attributed to two basic properties of the SWS mentioned above. For instance, (2.1)-(2.3) shows that large interaction impedances imply larger gain. Typically, resonant circuits provide large interaction impedance at the expense of narrow bandwidth. An example of such TWTs is the rippled wall waveguides and iris-loaded waveguides, both known to deliver high power across a narrow bandwidth [34].

In addition, velocity detuning affects the growth factor, $x$. To illustrate this, $x$ is plotted as a function of Pierce velocity detuning factor, $b(\omega)$ and Pierce’s space charge parameter, $4QC$, where, $b(\omega) = \frac{v_e - v_x(\omega)}{Cv_x(\omega)}$ (see Fig. 2.2). It can be noted that for electron beams with limited space-charge effects (i.e. small $4QC$), beam velocity and axial RF velocity must be nearly identical (i.e. small $b(\omega)$) in order to maximize gain (large $x$). Also, at the band edge, as axial RF phase velocity deviates from the beam velocity significantly, detuning parameter ($b(\omega)$) increases gradually. For large $b(\omega)$, gain becomes extremely sensitive to $b(\omega)$, leading to sharp drop of gain to zero [45] (see Fig. 2.3). In terms of bandwidth, helices are known to support constant phase velocity across large bandwidth leading to maximum growth, $x$. However, they lack

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strong axial electric field at the center, implying low $K_0$ and low gain. Hence, an SWS which supports a pure $TM$ mode with minimal dispersion would be preferred in order to achieve maximum wideband coupling.

### 2.2 Slow Wave Theory

TWTs and BWOs are dependent on the slow RF wave interaction with electron beams. Therefore, it is important to understand the slow wave phenomena and periodic structures very well since periodic geometries had been used for a long time to generate slow wave metallic structures. In this section, a brief description and concept of slow waves supported by periodic structures are given.

Slow waves refer to the propagation of electromagnetic waves with phase velocity much smaller than the speed of light, $c$ [46]. This wave behavior can be achieved by changing the constitutive $(\epsilon, \mu)$ parameters of the medium properties. Of course, the...
traditional way is to load the medium with appropriate dielectric properties. But, this is an expensive and cumbersome process, and often implies use of exotic and low loss materials, not available in practice. Thus, engineering techniques and synthetic materials need to be considered to realize slow wave phenomena. These techniques often involve the construction of periodic stacks of dielectric layers for the goal to emulate the behavior of artificial dielectrics. The concept of metamaterial is based on periodic structures and leads to engineered media with unusual properties. Indeed, periodic media have received considerable attention due to their usefulness in dispersion engineering. In this section, we introduce the concept of slow wave propagation using periodic loading of coupled transmission lines followed by experimental demonstration.

2.2.1 Periodic Structures

Periodic stack of dielectric material arrangements are often called Photonic Crystals. They support waves that exhibit multiple reflections as they propagate through photonic crystals. The superposition of all these reflections leads to various modes that can be exploited. Propagation through periodic media is typically modeled by invoking the well-known Floquet theorem [47]. As stated in Collins [47], “in a periodic system, if a given mode of propagation exists, fields inside a given cross-section of the system differs from fields that are a period distance away (or integer multiple of period distance away) by a complex constant.” The fields \( \mathbf{E}, \mathbf{H} \) in such a periodic system can be described by the function \( F(x, y, z) \). \( F(x, y, z) \) is periodic along \( z \), the direction of material periodicity. Using \( F(x, y, z) \), the \( \mathbf{E} \) field can be written as:
Figure 2.4: Typical dispersion diagram of the propagation frequency vs. $K = \beta_n p$ in a periodic medium. When $\beta_n p$ is not given for a band of $\Delta f$, this band is referred to as stopband.

$$E(x, y, z) = F(x, y, z)e^{-j\beta_0 z} \tag{2.5}$$

In this, $\beta_0$ is the propagation constant of the mode being described, and because $F(x, y, z)$ is periodic, it can be written as the sum of all possible supported modes. We can write $E$ as,

$$E(x, y, z) = \sum_{n=-\infty}^{+\infty} E_n(x, y)e^{-j\beta_n z} \tag{2.6}$$

where $p$ refers to the periodicity of the medium. In the above, each summation represents the nth mode or harmonic and is associated with the propagation constant:

$$\beta_n = \beta_0 + \frac{2\pi n}{p} \tag{2.7}$$

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The associated phase and group velocity for each of the modes in (2) are given by:

$$v_{pn} = \frac{\omega}{\beta_n} = \frac{\omega}{\beta_0 + \frac{2\pi n}{p}} = \frac{1}{v_{p0}} + \frac{2\pi n}{\omega_p}$$  \hspace{1cm} (2.8)

$$\beta_n = \beta_0 + \frac{2\pi n}{p}$$ \hspace{1cm} (2.9)

Thus, depending on the value of $n$, the phase velocity of the $n$–th harmonic, $v_{pn}$, can be greater than or less than the phase velocity, $v_0$, of the non-periodic medium. A typical dispersion diagram is shown in Fig. 2.4, a curve referred to as Brillouins diagram.

### 2.2.2 Second Order Dispersion

Three consequences can be identified when propagating in linear, homogeneous and isotropic layers:

1) Periodic structures slow down waves in a similar manner as high contrast dielectrics.
2) Backward waves (associated with negative $n$) are a consequence of periodicity.
3) Coupling of forward and backward wave leads to passbands and stopbands in a manner similar to Bloch waves in semiconductors.

Photonic crystals formed by simple isotropic dielectric stacks is the simplest example of these properties. They were originally motivated by the natural phenomena. Usually, periodic arrangement of atoms in crystal lattice form energy bandgaps. The spatial periodicity contributes to the formation of potential wells. Similar idea is exploited in RF wave propagation. Specially, periodic arrangement of dielectric stacks i.e. photonic crystals can also form bandgaps. These stacks cause reflections from the
Figure 2.5: Top: Periodic arrangement of Fe and O atom in FeO lattice form potential well for electrons. Bottoms: Similar phenomena is emulated in periodic stack of dielectrics.
interfaces between two different materials that contribute to form backward wave [48]. The resulting dispersion is only of second order, but clearly shows the formation of passbands and stopbands. The stacking of dielectrics can be simulated by a circuit model using inductance and/or capacitance \((L, C)\) lumped elements to emulate the relative dielectric constants \((\epsilon_r, \mu_r)\) forming the periodic cell. Such circuit analysis provides for an intuitive understanding of the overall dispersion behavior. Also, design can be achieved by changing the circuit parameters. The coupling of backward and forward waves that can create passbands and stopbands can be more easily understood, calculated and illustrated using circuit theory. With this in mind, the next section derives a circuit model of Coupled Transmission Lines (CTLs) and discusses its dispersion behavior using a simple second order dispersion.

### 2.3 Coupled Transmission Line Analysis for Slow Wave Theory

Coupled Transmission Lines (CTLs) are the one of many approaches to realize artificial dielectrics. Using the property of coupled elements i.e.(\(L_M, C_M\)) inside a periodic structure, significant wave slow down can be achieved. They have been extensively used in antenna applications before. In fact, high power microwave sources use contrawound helices that are essentially coupled transmission lines. The modified form of these double helices are the ring-bar geometry that can support greater wave slow down and high impedance profile. Therefore, conventional helical family geometries can be analyzed in much easier way using CTL theory. In addition, if coupled lumped parameters \((L_M, C_M)\) are defined, any complex geometry can be analyzed using the CTL theory. Hence, CTL theory can be a very powerful tool for slow wave structure analysis. In this chapter, we introduce two different classifications of the
CTL analysis based on their property. They can be of 2 different types:

1) Identical Coupled TLs (ITLs)
2) Non-Identical Coupled TLs (NITLs)

Below, we formulate and analyze these two different TLs.

2.3.1 Dispersion Analysis of Identical Coupled Transmission Lines

Coupled TL analysis can serve as a powerful tool to derive dispersion diagrams of slow wave structures. Previously, field analysis using analytic equations had been employed to understand the basic $\omega - \beta$ property of the SWSs. However, the introduction of computer codes made it easier to derive those curves relevant to SWSs. Although, computer simulations are reliable and accurate, it does not provide any insight to the problem and property of the design process. Therefore, circuit models serves a simpler way to derive dispersion diagrams and design steps more clearly. Another advantage of circuit model is that it does not require extensive and rigorous field analysis and can predict dispersion of a relatively complex geometry without much effort. In this subsection, we present simple analytic procedure to derive dispersion diagram of periodically coupled transmission lines.

Circuit Model formulation

In this subsection, we focus on the simple theoretical analysis of a ITLs loaded with inductive and capacitive elements and derive the associated dispersion relation. To formulate the problem, two identical transmission lines with similar constitutive
parameters \((L, C)\) are considered as illustrated in Fig. 2.6. For the moment, only inductive coupling is considered. The specific CTL is illustrated in Fig. 2.6.

Let us assume that the two identical transmission lines (i.e. straight wires) have the same per unit inductance of \(L\), and per unit capacitance, \(C\). Thus, individually, each transmission line supports wave that has the same phase velocity, \(\frac{1}{\sqrt{LC}}\), ideally the speed of light for free space propagation. When the pair of transmission lines is close to each other, they exhibit coupling. This coupling can be described by an inductor and a capacitor placed between the two TLs when forming their equivalent circuit. Using this unit cell model, the basic TL equations for voltage and current wave can be written:

\[
\frac{\partial V_1}{\partial z} = -L \frac{\partial I_1}{\partial t} - L_M \frac{\partial I_2}{\partial t} 
\]

\[
\frac{\partial V_2}{\partial z} = -L_M \frac{\partial I_1}{\partial t} - L \frac{\partial I_2}{\partial t}
\]
The telegrapher's equations take this form:

\[ \frac{\partial I_1}{\partial z} = -(C + C_M) \frac{\partial I_1}{\partial t} + C_M \frac{\partial I_2}{\partial t} \quad (2.10c) \]

\[ \frac{\partial I_2}{\partial z} = C_M \frac{\partial V_1}{\partial t} - (C + C_M) \frac{\partial V_2}{\partial t} \quad (2.10d) \]

\[ \frac{\partial^2 V_1}{\partial z^2} = (L + LC - L_M C_M) \frac{\partial^2 V_1}{\partial t^2} + (L_M C - LC + L_M C_M) \frac{\partial^2 V_2}{\partial t^2} \quad (2.11a) \]

\[ \frac{\partial^2 V_2}{\partial z^2} = (L_M C - LC_M + L_M C_M) \frac{\partial^2 V_1}{\partial t^2} + (L + LC_M - L_M C_M) \frac{\partial^2 V_2}{\partial t^2} \quad (2.11b) \]

\[ \frac{\partial^2 I_1}{\partial z^2} = (L + LC - L_M C_M) \frac{\partial^2 I_1}{\partial t^2} + (L_M C - LC_M + L_M C_M) \frac{\partial^2 I_2}{\partial t^2} \quad (2.11c) \]

\[ \frac{\partial^2 I_2}{\partial z^2} = (L_M C - LC + L_M C_M) \frac{\partial^2 I_1}{\partial t^2} + (L + LC_M - L_M C_M) \frac{\partial^2 I_2}{\partial t^2} \quad (2.11d) \]

In the above, the voltages \( V_{1,2} \) and currents \( I_{1,2} \) refer to the excitation values as measured between each TL and the ground lines. We also note the presence of additional terms due to \( L_M \) and \( C_M \). These terms do not exist in telegrapher's equations and will be responsible for the more complex dispersion curve. To solve (2.11a)-(2.11d), we shall assume a time dependence of \( e^{j(\omega t - \beta z)} \), where, \( \omega = \) angular frequency and \( \beta = \) wavenumber along the direction (\( z \)-axis) of propagation. Applying the time dependence on a pair ((2.11a),(2.11b)) of equations, the resulting dispersion relation takes the form:

\[ (\beta^2 - A \omega^2)^2 - \omega^4 B^2 = 0 \quad (2.12) \]

where \( A = LC + LC_M - L_M C_M \) and \( B = L_M C - LC_M + L_M C_M \)

\[ \beta_1 = \pm \omega \sqrt{A + B} = \pm \omega \sqrt{(L + L_M)C} \quad (2.13a) \]

\[ \beta_2 = \pm \omega \sqrt{A - B} = \pm \omega \sqrt{(L - L_M)(C + 2C_M)} \quad (2.13b) \]
The corresponding phase velocities are:

\[ v_1 = \pm \frac{\omega}{\beta_1} = \pm \frac{1}{\sqrt{(L + L_M)C}} > c \]  \hspace{1cm} (2.14a)

\[ v_2 = \pm \frac{\omega}{\beta_2} = \pm \frac{1}{\sqrt{(L - L_M)(C + 2C_M)}} < c \]  \hspace{1cm} (2.14b)

if \( \frac{C}{C_M} < 2\left(\frac{L}{L_M} - 1\right) \)

Clearly, the presence of \((L_M, C_M)\) leads to slow wave formation as depicted in Fig. 2.7. But more importantly, this \(\beta_2\) implies a much slower wave, and therefore more control to phase velocity, and therefore more control in designing smaller antennas, couplers, RF signal dividers, smaller TWTs, and even low and high power BWOs. Clearly, the added mutual inductance/capacitance has led to the realization of slow waves using circuit parameters rather than actual dielectrics. Therefore this analysis demonstrates that the artificial dielectric behavior can be emulated by CTLs.

**Coupled Mode Theory**

In reality, two oppositely propagating waves (the \(n = 0\) or forward mode and \(n = -1\) or backward mode) couple together to realize slow wave propagation. This leads to passbands and stopbands \([46, 49]\), and coupling of the TLs is important in controlling the dispersion relation.

To better understand the coupling between the \(n = 0\) and \(n = -1\) modes, we consider a simple periodic transmission line system supporting forward wave (FW) and backward wave (BW) modes. The FW is denoted as \(P_n\) and has a propagation constant \(\beta_p\). Similarly, the BW is denoted as \(Q_n\) and has propagation constant \(\beta_q\). The latter \((Q_n)\) is coupled to the forward wave \((P_n)\) with a coupling constant, \(\kappa\). This coupling is assumed to be linear and the resulting combined wave is assumed to have
Figure 2.7: Coupled Transmission Lines can support slow waves. The plotted dispersion lines (red and blue lines) provide an approach for artificial dielectrics. The CTL parameters are: \((L, C) = (1.61722\mu H, 6.8798pF)\) and \((L_M, C_M) = (0.673\mu H, 5.9pF)\)
a propagation constant, $\beta_c$. We note that the uncoupled forward waves differ from one cell to other via a phase delay, $e^{-j\theta_p}$. Similarly, the uncoupled backward wave differs from cell to cell via the delay, $e^{-j\theta_q}$. The pair of modes supported by the CTL system can be expressed as

$$P_{n+1} = P_n e^{-j\theta_p} + \kappa Q_n e^{-j\theta_p}$$  \hspace{1cm} (2.15a)  

$$Q_{n+1} = \kappa P_n e^{-j\theta_p} + Q_n e^{-j\theta_p}$$  \hspace{1cm} (2.15b)  

Rewriting these equations as a function of the differential distance $dz$, we have:

$$P_n \rightarrow P(z)$$  

$$Q_n \rightarrow Q(z)$$  

$$\theta_p \rightarrow \beta_p dz$$  

$$\theta_q \rightarrow \beta_q dz$$  

$$P_{n+1} \rightarrow P(z) + dP(z)$$  

$$Q_{n+1} \rightarrow Q(z) + dQ(z)$$  

$$\kappa \rightarrow K_c dz$$  

With these replacements, (2.15a) and (2.15b) give the differential equations:

$$P_{n+1} = P_n e^{-j\theta_p} + \kappa Q_n e^{-j\theta_p}$$  \hspace{1cm} (2.16a)  

$$Q_{n+1} = \kappa P_n e^{-j\theta_p} + Q_n e^{-j\theta_p}$$  \hspace{1cm} (2.16b)  

$$\frac{dP}{dz} = -j\beta_p P + K_c Q$$  \hspace{1cm} (2.17a)  

$$\frac{dQ}{dz} = K_c P - j\beta_q Q$$  \hspace{1cm} (2.17b)
To solve (2.17), we assume that the solutions are of the form: \( P = P_0 e^{-j\beta z} \) and \( Q = Q_0 e^{-j\beta z} \). As a result, (2.17) can be rewritten as

\[
\begin{bmatrix}
  j(\beta_c - \beta_p) & K_c \\
  K_c & j(\beta_c - \beta_q)
\end{bmatrix}
\begin{bmatrix}
P_0 \\
Q_0
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\] (2.18)

The determinant of this \( 2 \times 2 \) matrix leads to the quadratic equation

\[
\beta_c^2 - (\beta_p + \beta_q)\beta_c + (\beta_p\beta_q + K_c^2) = 0
\] (2.19)

For computing the propagation constants of the supported CTL waves, the solutions/roots of (2.19) are

\[
\beta_c^1 = \frac{\beta_p + \beta_q}{2} + \sqrt{\left(\frac{\beta_p - \beta_q}{2}\right)^2 - K_c^2}
\] (2.20)

\[
\beta_c^2 = \frac{\beta_p + \beta_q}{2} - \sqrt{\left(\frac{\beta_p - \beta_q}{2}\right)^2 - K_c^2}
\] (2.21)

Here, \( \beta_c^1 \) and \( \beta_c^2 \) are the propagation constants of the coupled wave and are a result of the coupling between the \( P \) (forward) and \( Q \) (backward) modes. We note that \( P \) and \( Q \) have the individual propagation constants \( \beta_p = \omega \sqrt{(L + L_M)C} \) and \( \beta_q = \frac{2\pi}{p} - \omega \sqrt{(L + L_M)C} \). Similar results can be found if the other wave solutions are taken i.e. \( \beta_p = \omega \sqrt{(L - L_M)(C + 2C_M)} \) and \( \beta_q = \frac{2\pi}{p} - \omega \sqrt{(L - L_M)(C + 2C_M)} \).

The final dispersion diagram obtained from the above relations is given in Fig. 2.8. The dispersion curve derived is of second order and the band edge is called Regular Band Edge (RBE) near the \( \beta_p = \pi \) point.
Figure 2.8: CTL modes due to coupling enabled by $L_M$ of two identical TLs. Coupling is demonstrated via formation of passband and stopband, a unique feature of periodic structures. The coupling co-efficient for calculation was $K_c = 20.85$. 
Chapter 3: Design of a SWS for for Low Power TWT

The performance of the TWTs are highly dependent on SWSs. Starting from low power TWTs to high power TWTs, SWSs play the key role in the beam-wave interaction. Therefore, understanding the basic properties of SWSs is vital for design process. In this chapter, we discuss some basic properties of the classic helix type slow wave structures. After discussing its features e.g. symmetry, bandwidth and interaction impedance property, we present brief description of advanced helical geometries like contra-wound helices, ring-bars and ring-loops. In the end, we present a novel SWS design called Half Ring-Helix for low power TWT design. The presented structure achieves 10dB gain improvement than helical structure of same size and dimensions. Also 27% miniaturization was also possible using the novel geometry.

3.1 Introduction to Helical Geometry

Helices are the most famous and widely used slow wave structure in high power microwave applications. The geometry itself is associated with nearly perfect symmetry for dispersion-free behavior. In practice, helices are not dispersion free slow wave structure. However, the cut-off frequency of this guided structure is very low and it maintains almost constant phase/group velocity for large band. That is why they are associated with maximum bandwidth any slow wave structure can ever provide.
Figure 3.1: A helical Slow wave Structure: $a =$ radius of the helix, $p =$ pitch/spatial periodicity, $w =$ width of the helix, $\delta =$ thickness of helix

In this section, we first present a brief parametric study of helices and discuss some properties. For more detailed theoretical analysis of helices, readers are encouraged to scour more depth from [42], which has overall a review of this popular slow wave structure.

3.1.1 Helix Geometry and Symmetry

Helices (Fig. 3.1) can be considered as the center cable of any simple co-axial cable except the fact that, the center wire in twisted to form helical shape (Fig. 3.1). The advantage of this twisting is two fold:

(1) It slows down the wave supported by it.

(2) It imparts $TM$ propagation mode.

The slow wave phenomena and modal behavior is directly attributed to its geometrical properties. From an intuitive point of view, the twisting increases the length
of the ‘delay line’ that translates into slowing down of the wave. In addition, the twisting in one period causes partial cancellation of axial magnetic field components. Therefore, axial electric field partially enhanced and that was the key discovery for the vacuum tubes. This axial E-field of helices was utilized to modulate linear electron beam and linear beam tubes e.g. traveling wave tubes and backward wave oscillators came into being.

The most important property of the helices are their almost dispersion free behavior. The underlying reason of this wonderful dispersion property can be explained in many different way. In full wave field calculation point of view, anisotropic boundary condition of helical currents in on transverse plane causes this constant $\omega - \beta$ relation for helices [42]. However, such calculation is much complicated and requires long field calculations at hand. Another way to observe its dispersion is by its symmetry property. Helices are associated with “screw” symmetry i.e. the combination of reflection and translation symmetry [50]. For instance, a small translation ($dz$) replicates as small angular variation ($d\theta$) or vice versa. This is depicted in Fig. 3.2

Next, we discuss some geometric parameters and property of helices.

### 3.1.2 Geometrical Parameters

In general, an approximation of the phase velocity of helix can be written as:

$$v_p = c \sin(\psi)$$

where, $\psi = \tan^{-1}\left(\frac{p}{2\pi a}\right)$ pitch-angle. As seen in (3.1), the phase velocity is approximated independent on frequency. Also, there are two geometrical parameters $p$ and $a$ that can control the overall dispersion of the helical geometry. Above, parametric study of helix parameters are shown in Fig. 3.3 As expected, reducing the pitch
Figure 3.2: The symmetry of helical slow wave structure. Combination of translational and rotational symmetry is depicted. Also, the current in helices is the combination of axial ($J_z$) and angular ($J_\phi$) current. Also, reflection translation symmetry is also shown in Ring-Bar structure although this is also present in helices.
(spatial periodicity) slows the wave more since reduction in $p$ results in increasing the current path for electrons. Therefore, phase velocity is reduced almost linearly. The effect of $a$ can be approximated from the relation (3.1).

Another important property of helix is its field profile. As seen in Fig. 3.3, helix fields are mostly concentrated near the metallic edges. That essentially provide strong traverse $E$-field and rather weak longitudinal $E$-field. As frequency increases, fields are strongly attached to the metals. As a consequence, the longitudinal field diminishes as frequency goes up. This phenomena eventually impacts the interaction impedance of the helical structure. The decaying profile of the interaction impedance is credited to this field profile.

### 3.1.3 Advanced Helical Geometries: Ring-bar, Ring-Loop

As seen above, helices cannot provide large interaction impedance since they have decaying field profile at the center. Also, helices are associated with strong harmonic components that carries a significant amount of power in TWTs and also causes oscillations in vacuum tubes. Therefore, to enhance axial field profile, two helices with opposite orientation were designed and fabricated known as “contra-wound helix” (Fig. 3.4). The mutual cancellation of axial magnetic field causes enhancement of axial electric field on the profile. In addition, two counteractive rotation of two helices cancels some angular harmonics that are responsible for harmonic power. Therefore, the harmonic field strength also reduced. But one disadvantage of this geometry is that it loses the overall symmetry of the helix. Thus, it is dispersive than any regular helix. Another issue with this “contra-wound helix” is that it is extremely fragile.
Figure 3.3: Top: (a) Normalized phase velocity, $v_p$ and (b) Interaction impedance Impedance, $K_0$ profile Vs. frequency for a helix with dimensions: $a = 4.2\text{mm}$, $p = 5, 8, 10\text{mm}$, $\delta = 0.7\text{mm}$, $w = 1\text{mm}$, $b =$waveguide radius $= 11\text{mm}$. Bottom: E-field profile of a helix showing strong traverse field dependence and poor axial components.
and sensitive to the proper relative orientation of the helices. Hence, it is not easily realizable.

To overcome these issues, a modified version of “contra-wound helix” was introduced. The modified slow wave structure is called “Ring-Bar” structure (Fig. 3.4). The rings and corresponding connecting bars emulates the same behavior as “contra-wound helix”. However, this is more dispersive than previous ones since it has only reflection symmetry and partial translation symmetry. However, ring-bar is associated with strong axial $E$-field leading to large interaction impedance. In fact, ring-bar TWTs were a good replacement of couple-cavity TWTs which are inherently narrowband TWTs.
3.2 Half-Ring Helix Design

With the goal of designing a slow wave TWT for low power TWT, having the combined properties of the helix and ring-bar structure, this chapter introduces a new slow wave structure referred to as half-ring-helix (HRH). HRH combines low dispersion and moderately large enough interaction impedance by combining the geometrical features of the helix and ring-bar structures. Concurrently, additional wave slowdown is achieved as compared to previous structures. The goal is to improve the saturated output power up to 500W across the S-band.

As mentioned in chapter 2, if the number of guided wavelengths, $N$, can be increased while keeping all other parameters constant, then gain is increased. One way to increase $N$ is to slow down the wave by making the guided wavelength smaller. However, in doing so, we must ensure that all other parameters, for example dispersion and interaction impedance are not altered significantly. In the following, we discuss how we can design the HRH to achieve high gain, low dispersion and moderate interaction impedance.

To minimize dispersion, velocity detuning parameter, $b$ (referred to section 2.1), of the TWT should be close to zero implying $v_p \rightarrow v_e$. Concurrently, by maximizing the growth factor $x$, the gain can be also increased. These two interrelated parameters ($x$ and $b$) are very important for gain and bandwidth respectively. As already noted, the simple helix is almost dispersion free structure that can generate large gain with good bandwidth. The underlying reason for its dispersion-free behavior is its skew symmetry (viz. simultaneous translational and reflection symmetry) [50].
Figure 3.5: Geometry and dimensions of the proposed HRH structure. a) HRH structure b) Unit cell. Dimensions: \( a = 3.75 \text{mm}, p = 8 \text{mm}, b = 11 \text{mm}, w = 1 \text{mm}, \delta = 0.74 \text{mm}, d = 4 \text{mm} \).

To retain the helix properties, we propose the HRH structure in Fig. 3.5. This TWT is motivated by the wideband behavior of the helix, but is concurrently modified to achieve a smaller guided wavelength. Just like the conventional helix, the HRH also possesses skew symmetry to maintain almost constant axial phase velocity (\( \nu_p \)) inside the TWT across the S-band. However, to increase gain, we need to realize a slow wave structure that can increase \( N \). To do so, the HRH in Fig. 3.5 contains two half-rings within its unit cell. As such, it provides for additional RF delay within the unit cell. We find that the proposed HRH improves small signal gain without appreciably changing bandwidth performance as compared to standard helix TWT. Additional geometrical parameters associated with the HRH unit cell are:

\[ p = \text{unit cell length (period)} \]
\(2b = \) diameter of the cylindrical waveguide \\
\(2a = \) diameter of the HRH \\
\(w = \) width of the HRH bars. \\
\(\delta = \) thickness of the HRH.

After extensive computational experiments, we chose the unit cell values of \(p = 8\text{mm}, \ a = 3.75\text{mm}, \ w = 1\text{mm}, \delta = 0.74\text{mm}.\) Below, we use these parameters to evaluate the performance of HRH and compare it to previous TWT structure.

### 3.2.1 Cold and Hot Test Simulation Results

In this section, we evaluate the cold test performance via full wave simulations. Also, the hot test performance of the HRH TWT is presented using particle-in-cell simulations. These are compared to standard helix TWT performance. The cold test results are presented first and comparison between analytical [27] and numerical data is provided.

#### 3.2.2 Dispersion

To minimize dispersion, it is necessary to achieve nearly constant phase velocity across the band. The standard helix is already known to have this property. Therefore, it is important to build upon its structure and modify it to attain further wave slowdown for large power delivery. Much like the helix, the proposed HRH controls the phase velocity, \(\upsilon_p\), by changing the pitch angle [42], \(\psi\). It was found that the phase velocity of HRH is similar to the standard helix and deviates only \(0.03c\) across the S-band (see Fig. 3.6), where \(c\) is the speed of light. That is, \(|\upsilon_{p0} - 0.03c| < \upsilon_p, \upsilon_{p0} = 0.32c\) across the operational band, a desirable feature for achieving good bandwidth.
Figure 3.6: Normalized wave phase velocity (vertical axis on left) and on-axis interaction impedance (vertical axis on right) as a function of frequency of HRH.

3.2.3 Interaction Impedance

As mentioned earlier, the interaction impedance of the TWT is the primary parameter that controls gain. It is well known that standard helical structures are associated with low interaction impedances and this limits their power handling [27]. The lower value of $K_0$ is due to its rapidly decaying axial electric fields away from the edges and the low field amplitude at the center of the helix [46]. We note that similar behavior is observed in the HRH structure. We also observe that the HRH TWT has the lowest axial electric field intensity ($E_z$) at the center (Fig. 3.7). This explains its decaying interaction impedance in the proposed HRH. However, the HRH dimension parameters can be further optimized to increase the interaction impedance without
affecting dispersion. In addition, dispersion within the TWT can be significantly improved by reducing the radius of the cylindrical waveguide containing the HRH [31]. To do so, we optimized the parameters $a$, $b$ and $p$ using a full-wave simulation in a cold circuit setting [5].

### 3.2.4 Beam-wave Interaction

The hot test of the TWT (i.e., in presence of the electron beam) introduces additional parameters. These include the beam current, space-charge, power level of the beam and collector efficiency among others [40]. Here, the hot test simulation was carried out for two different lengths of $p$, $p = 8\text{mm}$ and $p = 4\text{mm}$.

These design parameters are given in Table-3.1 and Table-3.2. The commercial-grade software CST Particle Studio [6] based on the Finite-Integration technique...
Table 3.1: Design specifications of the HRH TWT amplifier with $p = 8\text{mm}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (GHz)</td>
<td>$2.5 - 3.25 \text{GHz}$</td>
</tr>
<tr>
<td>Peak Output Power</td>
<td>$1.014 \text{kW}$</td>
</tr>
<tr>
<td>Saturated gain</td>
<td>$28 \text{dB}$</td>
</tr>
<tr>
<td>Maximum electronic efficiency</td>
<td>$38.7%$</td>
</tr>
<tr>
<td>Beam current (circular cross section of 2mm radius)</td>
<td>$100 \text{mA}$</td>
</tr>
<tr>
<td>Cathode voltage</td>
<td>$26.208 \text{kV}$</td>
</tr>
<tr>
<td>Focusing magnetic field</td>
<td>$0.5 \text{T}$</td>
</tr>
<tr>
<td>Tube length ($L$)</td>
<td>$814 \text{mm}$</td>
</tr>
</tbody>
</table>

Table 3.2: Design specifications of the HRH TWT amplifier with $p = 8\text{mm}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (GHz)</td>
<td>$2.25 - 2.75 \text{GHz}$</td>
</tr>
<tr>
<td>Peak Output Power</td>
<td>$220 \text{W}$</td>
</tr>
<tr>
<td>Saturated gain</td>
<td>$46 \text{dB}$</td>
</tr>
<tr>
<td>Maximum electronic efficiency</td>
<td>$37%$</td>
</tr>
<tr>
<td>Beam current (circular cross section of 2mm radius)</td>
<td>$100 \text{mA}$</td>
</tr>
<tr>
<td>Cathode voltage</td>
<td>$6.308 \text{kV}$</td>
</tr>
<tr>
<td>Focusing magnetic field</td>
<td>$0.5 \text{T}$</td>
</tr>
<tr>
<td>Tube length ($L$)</td>
<td>$457 \text{mm}$</td>
</tr>
</tbody>
</table>

[51,51,52] combined with the particle-in-cell (PIC) methodology [53–56] was used for hot test simulations. Results are given in Fig. 3.8 and indicate that the maximum output power is $1.014 \text{kW}$. Also, the corresponding maximum gain was $28 \text{dB}$ at $3 \text{GHz}$, and the $3\text{dB}$ bandwidth was $25\%$ (see Fig. 3.8) for a TWT of length $80\text{cm}$. This is a $10\text{dB}$ gain increase as compared to the standard helix TWT of the same length (see Fig. 3.9). In fact, to achieve a gain of $28\text{dB}$ using a standard helix, the TWT must be $110\text{cm}$ long, viz. $27\%$ longer. Therefore, the HRH is suitable for miniaturizing low power TWT amplifiers.
Figure 3.8: Calculated gain and saturated output power levels versus frequency of the HRH TWTA with $p = 8$mm.

Figure 3.9: Gain comparison as a function of frequency for the $p = 8$mm is shown above. Both Helix and HRH TWT amplifier is driven with RF power level of 0.5W.
We note that, in Fig. 3.9, the difference (2.5dB) between our theoretically calculated gain using Pierces formula and the simulated curve (for the helix case) is due to the Eulerian model (adopted by Pierce) and Lagrangian model (used in CST PIC studio) of the space-charges. Specifically, when using Pierces formula, the electron velocity was assumed to be uniform along the tube (fluid assumption) \[57\], i.e. electrons were assumed to have the same velocity over the cross-section of the beam. However, the CST PIC code uses multivalued velocity function to model the electron trajectory across the beam diameter. That is, the CST PIC numerical data are expected to be more realistic. Further, in Pierce’s formula, the interaction impedance was based on the axial electric field along the axis \((r = 0)\). However, in practice, the beam has finite width and the interaction impedance also vary with \(r\), away from the center of beam. This implies that the electron velocity varies away from the center \((r = 0)\). As the numerical results using the CST PIC code allow for variable electron densities and velocity, they can be considered more accurate.

To achieve the maximum gain afforded by the TWT, \(N\) in (2.1) should be very large. To do so, we scaled the structure by a factor of two \((p = 4\text{mm})\) and simulated the structure again. Consequently, the axial phase velocity, \(v_p\), was low and \(N\) was very large, viz. the dominant parameter in (2.1). Concurrently, the gain achieved for the TWT is much higher if its length or the beam current is increased. However, at some specific length or current values, the electron beam within the tube is slowed down to the point that the average electron velocity is below the axial phase velocity of the RF wave. As a consequence, some energy from the amplified wave is transferred back to the beam, implying some gain reduction. This is referred to as hard saturation and can only be observed from a non-linear analysis of the TWT \[40\]. Beyond this
Figure 3.10: Electron trajectory and zoom into end section for 45.7cm long HRH traveling wave tube. The color ramp indicates normalized electron velocity ($\beta(\text{abs}) = \frac{v_z}{c}$).

Figure 3.11: Electron particle trajectory from CST PIC simulation of the proposed TWT (45.7cm). Here, $\beta(\text{abs}) = \frac{v_e(z)}{c}$. 
Figure 3.12: Gain and Saturated output power level of the HRH TWT amplifier with $p = 4\text{mm}$.

...hard saturation point, we cannot observe gain. Moreover, the gain drops significantly after hard saturation. For high frequency of the band, hard saturation is reached early and sudden drop in gain observed (see Figs. 3.8 and 3.12).

The simulated beam-wave interaction, particle preview and electron trajectory are shown in Figs. 3.10 and 3.11. Fig. 3.12 indicate that a maximum gain of the HRH, $G = 46\text{dB}$ was achieved with $p = 4\text{mm}$ (1/2 of $p = 8\text{mm}$). The associated length for maximum gain was 45.7cm (see Fig. 3.12). Also the maximum saturated power of this ($p = 4\text{mm}$) tube is 225W and the corresponding 3dB bandwidth is 23.43%. By comparing the unfolded length of the TWT (metal used in this HRH structure), we can also compare how much metal is introduced in designing the HRH TWT. For the HRH case, using the same pitch, $p$, to achieve the same gain, the unfolded length was
329 cm, whereas that of the original typical helical structures was 287.5 cm. That is, more metal (about 41 cm long) was introduced for the same pitch to design the HRH. This improves wave slowdown and also leads to higher gain.

In the above, it was shown that the proposed HRH TWT achieved 27% size reduction and was nondispersive across the entire S-band. Two designs were proposed corresponding to pitch values of \( p = 8 \text{ mm} \) and \( p = 4 \text{ mm} \). The proposed TWTA was designed to operate in the S-band with a bandwidth of 25% and a maximum gain of 46 dB (Fig. 3.12). In addition, it provides peak power of 1 kW power with power levels above 600 W (Fig. 3.8) across the band (2.53 to 2.5 GHz). We note that the bandwidth and gain performance is expected to improve significantly if tapering of the HRH is implemented [30]. Further, backward waves can be suppressed via attenuator circuits to increase the maximum operating power level.
Chapter 4: Curved-Ring-Bar SWS for Wideband MW Power TWT

Generally, high-power TWTs require stronger beam current and a large diameter electron beam [58]. Since the kinetic energy of the electron beam \( \frac{1}{2}mv_e^2 \) is quadratically proportional to the beam’s velocity, high power TWTs also require high beam velocity. That is, electron velocity should be closer to the speed of light \( c \) to transfer large amounts of energy to RF. On the contrary, good beam-wave coupling requires synchronism of the RF wave phase velocity with the electron beam’s velocity. Since, electron velocity is higher for high energy electrons, RF wave phase velocity should be higher as well for synchronism. That is, high power TWTs require a slow wave structure that supports RF waves having large phase velocity that match that of the electrons. Concurrently, high phase velocity implies lower coupling to the electron beam due to the smaller electrical length \( \beta_z l = \frac{2\pi l}{\lambda_g} = \frac{2\pi f l}{v_p l} \).

From the above, it is apparent that a compromise between strong coupling and velocity synchronism must be pursued to achieve good gain. Specifically, for a relativistic gain, we have [34]

\[
G = -9.54 + 47.3CN \text{(dB)} \quad (4.1)
\]
with

\[ C = [K]\cdot\left[\frac{eI}{4\gamma^3 m \nu_e^2}\right]^{\frac{1}{2}}, \quad K(\omega) = \frac{E_{axial}^2}{2\beta^2 P} \quad (4.2) \]

where, \( \beta = \) propagation constant of the RF wave, \( K = \) interaction impedance of the SWS for one period, \( \gamma = \) electron beam relativistic parameter, \( m = \) electron mass, \( e = \) electron charge, \( \nu_e = \) electron velocity, \( E_{axial} = \) peak axial electric field at the center, and \( P = \) total RF power flow in the structure. From the definition of \( C \), it is observed that higher electron velocity (\( \nu_e \)) results in lower coupling parameter, \( C \) due to scaling with \( \nu_e \) and \( \gamma \). Thus, high velocity electrons reduce the gain of the TWT in (4.2).

The bandwidth of the Traveling Wave Tube Amplifier (TWTA) is limited due to space charges generated by the electron beam [45]. For stronger beam current, intense space charges cause excessive electron acceleration and deceleration. Hence, the generated waves may lose their synchronism with the electron beam, implying loss of coupling and lower gain. At higher frequencies, the TWTs gain can actually drop to zero. For this reason, a wideband and high power TWT is possible only if the electromagnetic wave maintains good beam-wave velocity synchronism along the length of the tube i.e \( \nu_p \approx \nu_e \). Helical circuit structures such as standard helices, double helices, half-ring helices, ring-bars and ring-loops are all suitable to achieve \( \nu_p \approx \nu_e \) [29], [59].

As mentioned in the previous chapter, helical structures have low interaction impedance at higher frequencies [42]. As an alternative, in this chapter, we present a Curved Ring-Bar Structure (CRB) that alleviates the aforementioned issues and achieve high power microwave generation. A coupled TL model of the proposed CRB is also presented to explain the new TWT passband characteristics. We note that the
presence of the mutual inductance ($L_M$) and mutual capacitance ($C_M$) in the coupled TLs are responsible for wideband behavior as compared to corrugated structures.

### 4.1 Curved Ring-Bar Analysis Using Coupled TLs

The CRB structure is essentially a modified Ring-loop type structure [43]. Specifically, the CRB serves elliptic wires/bars to increase the inductance per unit cell to achieve slow $TM_{01}$ waveguide mode. The axial ratio, ($m = h_2/h_1$) of the ellipse is capable of controlling the phase velocity of the propagating wave without changing its periodicity, $p$. The geometry of the structure is shown in Fig. 4.1.

![Figure 4.1: CRB Structure with dimensions: pitch, $p = 22$mm, radius, $a = 4.5$mm, width, $w = 2$mm, thickness, $\delta = 2$mm, $h_1 = 8$mm, $h_2 = 12.8$mm.](image)

The geometry in Fig. 4.1 can be interpreted as two inductively coupled but identical TLs. As noted, the elliptic curves serve to increase the inductance per
unit length and consequently the interaction impedance. The overall layout of the waveguide system is shown in Fig. 4.2.

![Diagram of CRB placed inside a circular waveguide](image)

Figure 4.2: CRB placed inside a circular waveguide. The dimensions are: $d = 57\text{mm}$, $d_m = 11\text{mm} = \text{diameter of the rings}$, $p = 22\text{mm}$

The structure lying at the center of the waveguide (Fig. 4.2) can be interpreted as a pair of coupled TLs separated by a distance of $2a$ from each other. The rings in the middle serve to generate strong coupling inductance between the TL pair. A rigorous analysis of this coupled TL model can be used to explain the underlying reasons for the CRB’s ability to achieve wave slow-down and $TM_{01}$ mode enhancement. A similar circuit model of Ring-Bar structure was presented by Datta et al. [60]. But, a different analysis is considered here as compared to Datta [60] et. al. Our analysis is presented in sections 4.1.1, 4.1.2 and 4.1.4.
4.1.1 Circuit Model Formulation

The lumped TL circuit model of the CRB is difficult to realize because of the curved features of the structure leading to non-trivial mutual inductances, $L_M$. For our case, $L_M$ can be obtained by evaluating elliptic integrals [61]. Below, we first derive a model for a simpler coupled TLs pair using two straight wires (Fig. 4.3). Then we proceed to approximate the solution for curved models with circular or elliptic features (Fig. 4.1-4.2).

Referring to Fig. 4.3, the bars are straight wires and provide an intuitive model for obtaining the circuit parameters [62], $(L,C)$. From transmission line theory, the per unit length $(L,C)$ of a transmission line can be derived from static fields analysis [62]. A similar procedure can be applied for the CRB to derive the dispersion curves [63]. The derivation is given below:

4.1.2 Derivation of the Circuit Model

The two identical TLs are depicted in Fig. 4.4. Each of the TL has inductance and capacitance of $(L,C)$ per unit length. Individually, each transmission line supports a wave whose phase velocity is $v_p = 1/\sqrt{LC} = c$. However, when the TLs are coupled, using the circular rings, there is also a coupling inductance, $L_M$. Due to this mutual inductance, Telegrapher’s equations take the form [64]:

\[
\frac{\partial^2 V_1}{\partial z^2} = LC \frac{\partial^2 V_1}{\partial t^2} + L_M C \frac{\partial^2 V_2}{\partial t^2} \tag{4.3a}
\]

\[
\frac{\partial^2 V_2}{\partial z^2} = L_M C \frac{\partial^2 V_1}{\partial t^2} + LC \frac{\partial^2 V_2}{\partial t^2} \tag{4.3b}
\]

\[
\frac{\partial^2 I_1}{\partial z^2} = LC \frac{\partial^2 I_1}{\partial t^2} + L_M C \frac{\partial^2 I_2}{\partial t^2} \tag{4.3c}
\]

\[
\frac{\partial^2 I_2}{\partial z^2} = L_M C \frac{\partial^2 I_1}{\partial t^2} + LC \frac{\partial^2 I_2}{\partial t^2}. \tag{4.3d}
\]
Figure 4.3: Simpler ring-bar TL comprised of (a) straight wire and (b) rings that couple these wires

Figure 4.4: Equivalent circuit model of the CRB TLs.
To solve (4.3a)-(4.3d), we may assume that the wave solution is of the form $e^{j(\omega t - \beta z)}$. Introducing this wave into (4.3a)-(4.3d) leads to the dispersion relation:

$$(\omega^2 LC - \beta_z^2)^2 - \omega^4 L_M^2 C^2 = 0.$$  \hspace{1cm} (4.4)

The four roots of the dispersion relation are the propagation constants denoted by $\beta_1$ and $\beta_2$ in (4.5). Among them, (4.5a) and (4.5b) are degenerate solutions:

$$\beta_1 = \pm \omega \sqrt{(L - L_M)C} \hspace{1cm} (4.5a)$$

$$\beta_2 = \pm \omega \sqrt{(L + L_M)C} \hspace{1cm} (4.5b)$$

The corresponding phase velocities are:

$$v_1 = \pm \frac{\omega}{\beta_1} = \pm \sqrt{\frac{1}{(L - L_M)C}} > c \hspace{1cm} (4.6a)$$

$$v_2 = \pm \frac{\omega}{\beta_2} = \pm \sqrt{\frac{1}{(L + L_M)C}} < c \hspace{1cm} (4.6b)$$

It is noted that (4.6b) predicts a slower velocity than $c$ and is responsible for the slow wave property of the TL. A similar phenomena can be observed if dielectrics were used to fill the region between TLs. That is, coupled identical TLs can behave as artificial dielectrics. We observe that a fast wave is also generated i.e. presumably due to the presence of a $TE$ wave mode in (4.6a). But, since fast waves do not couple with the electron beam energy, we will ignore this fast wave altogether. However, for certain geometries, this second wave can be dominant. For example, depending on the mutual coupling, a $TM_{01}$ mode maybe dominant. Further, as compared to the regular ring-bar model [60], an absence of mutual capacitive element can lead to larger phase velocity. In fact, the presence of $L_M$ only is responsible for large interaction impedance of the CRB structure. This property can be used to match the
phase velocity of the guide with the electron beam’s velocity. Nevertheless, the lack of capacitive coupling reduces the overall interaction impedance of the SWS.

### 4.1.3 Constitutive \((L,C)\) parameters

The lumped inductance \(L\) and capacitance \(C\) per unit length of two straight wires refer to Fig. 4.6 can be derived by solving Amperes circuit law and Gauss’s divergence equation [62]. The inductance per unit length, \(L\) of a simple straight wire transmission line can be written as:

\[
L = \frac{\mu}{\pi} \ln\left(\frac{d}{r}\right) \tag{4.7}
\]
Figure 4.6: Equivalent circuit model of the simplified straight Ring-Bar TLs. Magnetic field cancellation at the center is used as a model for calculation of mutual inductance, $L_M$. The dimensions are: $d = 57\text{mm}$, $d_m = 10\text{mm} =$ diameter of the rings, $p = 22\text{mm}$. 
where, $d =$ distance between two bars/wires, $r =$ radius of the wire. Similarly, the capacitance per unit length can be written:

$$C = \frac{\pi \epsilon}{\ln(d/r)} \quad (4.8)$$

The calculated lumped inductance and capacitances per unit length derived from (4.7) and (4.8) : $L = 1.34 \mu H$, $C = 8.3034 \text{pF}$. Also, the coupled circuit parameter, $L_M$, can be derived using the equation:

$$L_M = \frac{\mu}{2\pi} \ln\left(\frac{d_m}{r}\right) \quad (4.9)$$

We note that the rings connect the two wires/bars and make them electrically equipotential. But since the currents on the bars/wires are flowing in the same directions, they cancel the transverse magnetic field at the center (Fig. 4.6). It is further noted that the rings carry very small amount of current. Therefore, inductive coupling between them is neglected.

For the specific design at hand, this mutual inductance per unit length for the straight wires/bars as shown in Fig. 4.6 is $L_{M0} = 0.3409 \mu H$. Therefore, from (4.6b), the phase velocity of the wave is: $\nu_p = 0.89c$, where $c = 3 \times 10^8 \text{ms}^{-1}$.

In the above, we calculated lumped circuit parameters for the slow wave structure referred to Fig. 4.6. However, our proposed model is CRB (Fig. 4.2) which is different than Fig. 4.6. Therefore, the lumped circuit parameters are also expected to be different. In CRB structure (Fig. 4.2), the curved features are not straight, rather elliptical or circular. The circular or elliptical wires are expected to provide greater slowdown of wave since they require longer paths for currents to travel on the slow wave structure. This observation is the basis for our model of lumped parameters. Since the waveguide wall is quite far from the slow wave structure, the change in
lumped parameters \((L,C)\) are assumed to be trivial. However, as mentioned before, it is nontrivial to calculate mutual inductance for ellipses since they involve elliptic integrals to solve the Biot-Savart’s law [61]. Therefore, we prefer to choose a simple intuitive method. We compare the current path, \(l\) and flux linkage area, \(A\), covered by elliptic wires of the the CRB to the corresponding straight wires Fig. 4.7. Since current(\(I\)) in wires is inversely proportional to its length, \(l\) and flux(\(\phi\)) linked with the wires are directly proportional to area, \(A\), the increment in ratio of \((\phi/I \leftrightarrow A/l)\) will provide a relative increment of mutual inductance compared to the \(L_{M0}\). This is due to the definition of inductance recalled as \(L = \phi/I\). To do this, let us redefine the geometry of CRB again as shown in Fig. 4.7.

For a moment, we consider the wire as a semi-circular one i.e. \(h_1 = h_2\). The length of the semi-circular arc \((E(m) \times 2h_1 = \pi h_1)\) is \(\pi/2\) times longer than the length of straight bar/wire, \(2h_1\). Thus, the increment of length compared to straight wires is \(\pi/2\). In addition, the bending of the straight wires increases the flux linkage area corresponding to the straight wires by a factor of \((1 + \frac{\pi h_1}{4a})\). Thus, the mutual inductance per unit length, \(L_M\), of the curved bar is \((1 + \frac{\pi h_1}{4a})/(\frac{\pi}{2})\) times larger compared to \(L_{M0}\). This is because the circular path increases the total flux linkage by the amount of \((1 + \frac{\pi h_1}{4a})/(\frac{\pi}{2})\). The corresponding calculated inductance is 0.5011\(\mu H\) implying the phase velocity of 0.85\(c\) The latter is much closer to the full wave simulated phase velocity of 0.88\(c\). We note that the calculation approach is not exact. Nevertheless, the error is marginal in terms of accuracy and understanding the physics of problem. For elliptical wires instead of circular ones, the arc length of the elliptic section can be written as \(E(m)h_1\), where \(m = \) axial ratio of ellipse formed by the bars/wires, \(2h_1\)
Figure 4.7: (Top) Geometrical overview of curved section of a CRB/Ring-loop. (Middle) The length and flux area are shown for straight wires. (Bottom) The relative increment of length and flux area is shown for elliptic wires. For convenient calculation, $h_1 = h_2$ is considered, and $E(m) = \pi/2$ is found from (4.10)

$= \text{length of the current path of the straight wires and } E(m) \text{ is the elliptic integral of second kind [65] written as:}$

$$E(m) = \int_0^{\pi/2} \sqrt{1 - (m^2 - 1)\sin^2(\theta)}d\theta,$$

(4.10)

where, $m = \frac{h_2}{h_1} = \text{axial ratio of ellipse.}$
For special circular case, \( h_1 = h_2 \), and \( E(m) = \pi/2 \) = same as discussed above. Thus, for elliptical case, the mutual inductance can be written as:

\[
L_M = \left(1 + \frac{\pi h_2}{4a}\right) \times \frac{E(m)}{E(m)} \times L_{M0} \tag{4.11}
\]

In subsequent sections, we will use this formula to calculate mutual inductance and apply to coupled mode calculations.

### 4.1.4 Coupled Mode Analysis

Since every periodic structure support forward and backward waves, and the coupling of the waves provide passbands and stopbands, in this section we provide a coupled mode analysis for the CRB slow wave structure for the derivation of dispersion equation. In the above, the mode(s) derived in analysis (4.6b) is a TEM mode since it presumes propagation through two conductor transmission lines lying on free space. However, the dual transmission lines are located inside a metallic circular waveguide and thus the propagating modes are supposed to be regular circular waveguide modes. Further, since the mutual inductance and/or capacitance \( (L_M, C_M) \) mimic the loading of dielectrics (4.5) inside the waveguide, they can be considered as constitutive parameters \( (\mu_r, \epsilon_r) \) forming the dielectric medium inside the waveguide. The dispersion relation of circular dielectric loaded waveguide can be written as [66]:

\[
(\beta_z)_{mn} = \pm \sqrt{\omega^2 \epsilon_0 \epsilon_r \mu_0 - \frac{\chi_{mn}}{a}} \tag{4.12}
\]

where \( \chi_{mn} \) is the \( n \)th \( (n = 1, 2, 3, \ldots) \) zero of the Bessel function \( J_m \) of the first kind of order \( m \) \( (m = 0, 1, 2, 3, \ldots) \), \( (\beta_z)_{mn} \) is the propagation constant along the \( z \)–direction for mode \( (m, n) \) and \( \epsilon_r = \text{relative permittivity} \) and of the medium filling the interior of the circular waveguide.
Since in the waveguide dispersion relation [66], the free space propagation characteristics are incorporated in the term written as $\omega^2 \mu_0 \epsilon_0 \epsilon_r$. In circuit analysis, the equivalent constitutive parameters corresponding to $\omega^2 \mu_0 \epsilon_0 \epsilon_r$ can be written as $\omega^2 LC$ [67]. By capitalizing the dielectric-like property of coupled transmission lines (Fig. 4.5), coupled inductances ($L_M$) are incorporated in waveguide equation to model a forward and backward wave propagation of the waveguide modes (4.13):

\[
(\beta_z)_{mn} = \pm \sqrt{\omega^2 (L + L_M) C - \frac{X_{mn}}{a}}
\]  

(4.13)

Equation (4.13) provides for equations of two oppositely propagating waves inside circular waveguide. From the coupled mode analysis of Pierce [63], it was shown that the two waves propagating in opposite directions with similar field profile can undergo co-directional and contradirectional [49] coupling. It is noted that this coupling is an artifact of periodic structures and has no direct relation with mutual coupling of circuit elements. Following similar calculation steps as shown in [49], we proceed to find a coupled mode equation for contradirectional coupling of two slow forward and backward waves. The coupled mode equation can be written as:

\[
\beta_c^2 - (\beta_p + \beta_q)\beta_c + (\beta_p \beta_q + K_c^2) = 0
\]  

(4.14)

where, $\beta_p = \sqrt{\omega^2 (L + L_M) C - \frac{X_{mn}}{a}}$ and $\beta_q = \frac{2\pi}{p} - \sqrt{\omega^2 (L + L_M) C - \frac{X_{mn}}{a}}$.

In the analysis, we choose one forward wave as fundamental mode ($\beta_p = \beta_0$) and one backward wave as one of space harmonic ($\beta_q = \beta_{-1} + \frac{2\pi}{p}$) in range of same wavenumber (0 to $2\pi$). This provides us two resultant ($\beta_c$) second order dispersion relation:

\[
\beta_{c1} = \frac{\pi}{p} + \sqrt{(\beta_0 - \frac{\pi}{p})^2 - K_c^2}
\]  

(4.15a)
\[ \beta_{c2} = \frac{\pi}{p} - \sqrt{(\beta_0 - \frac{\pi}{p})^2 - K_c^2} \]  

(4.15b)

where, \( \beta_0 = \sqrt{\omega^2 (L + L_M) C - \frac{\chi_0}{a}} \), and \( K_c \) is the coupling parameter between two modes. \( K_c \) can be expressed as:

\[ K_c = \frac{(1 + \frac{\pi h_1}{4a}) E(m)}{E(m)} \sqrt{\frac{\beta_p \beta_q}{|\beta_p - \beta_q|}} \]  

(4.16)

Figure 4.8: Coupled mode dispersion diagram derived for the SWS using (4.15a) and (4.15b) shown in Fig. 4.4

The final coupled line equations derived in TL analysis are given in (4.15) above. These coupled modes demonstrate the existence of passband and stopband (see Fig. 4.8) which is typical in periodic structures. However, for the proposed CRB structure, provided the dispersion diagram derived from circuit analysis in Fig. 4.11. A
A comparison is provided to demonstrate the accuracy of the circuit analysis for complex geometries like CRB structure. Full wave simulation results were obtained using HFSS.

comparison with full wave simulations using the commercial grade software High Frequency Simulation Software (HFSS) [5] based on the finite-element method [68–70] in the Fig. 4.9 validates the accuracy of the circuit model. Specifically, we observe the error of less than 5%.

### 4.1.5 Cold Test Simulation Results

In the above, it is evident that coupled and identical transmission lines emulate artificial dielectrics by lowering the phase velocity of the propagating waves to being smaller than \( c \). Further, geometric parameters i.e. \( p, d, d_m, a, h_1, h_2 \), can be used to modify the circuit parameters \((L, C)\) leading to our control of the wave slowdown. For instance, the bending of the elliptic wires renders greater slowdown of the wave
Figure 4.10: Cold test simulation results of the CRB structure. The $\omega - \beta$ diagram of the proposed CRB structure with 262kV beam line corresponding to 0.75$c$ electron speed (left). Interaction impedance ($K_0$) and phase velocity profile of the proposed CRB as reflected in Fig. 4.10. Clearly, some optimization is needed to provide desired dispersion.

The geometrical parameters and dimensions ($a, b, d, d_m, p, h_1, m$) for the final design of the CRB structure are given in Fig. 4.1 and Fig. 4.2). The phase velocity and interaction impedance profile of the CRB structure using these parameters and based on full wave simulation are shown in Fig. 4.10. We observed that the phase velocity profile of CRB structure shows little deviation of phase velocity $0.1c$ across the $2 - 3$ GHz bandwidth. That is, $0.05c$ deviation of phase velocity across the $2 - 2.5$ GHz, a desirable feature for achieving good bandwidth in S-band. The interaction impedance
profile shows the impedance of above 50Ω in S-band which is reasonable for TWT design.

As mentioned above, corrugated waveguides e.g. rippled wall structures, were used to develop very high power (100MW) TWTs. However, the instantaneous bandwidth associated with the TWTs is very narrow. A comparison of dispersion diagrams for the CRB and rippled wall structures is given in Fig. 4.11b. As seen, dispersion in CRBs is comparatively better than the rippled wall waveguide, implying wide bandwidth. In fact, the rippled wall structure does not support slow wave mode across the entire band. Hence, CRBs are promising for high power and wideband TWT designs as they have large impedance profile with reasonable bandwidth. To verify this, in the next section, we present an example of TWT design using the CRB as the SWS inside the waveguide and evaluate its performance.

4.2 A Design of 1MW TWT with Curved Ring-Bar

A TWT design was simulated using CST PIC code. The CRB was placed within a circular waveguide to form the TWT. The TWT draws 12A beam current from a circular cathode ray tube biased with 262kV. The CRB is placed at the center of the waveguide and using a RF excitation, it supports a wave of phase velocity 0.75c. The TWT generated output power of 1MW. The corresponding input power was 0.625kW, implying a gain of 29dB at 2GHz. The beam-wave coupling of the designed TWT using the CRB structure is shown in Fig. 4.13. The associated design specifications based on PIC simulations are shown Table-4.1.

To verify the CRB’s wideband performance, we proceeded to simulate and plot output power Vs. frequency. In all cases, the input power was 0.625KW. The plot is
Figure 4.11: (a) Top left: Interaction impedance, $K_0$ (left vertical axis) and coupling parameter, $C$ (right vertical axis) as a function of frequency. Top right: CRB dispersion diagram using full wave analysis and 262kV beam line. (b) Comparison of the $\omega - \beta$ diagrams for the CRB and Rippled wall structures. In both cases, the waveguide radius was 63.5mm. All results were derived using full wave simulation.
given in Fig. 4.13 We observe that the output power remains nearly constant for 1.8-2.5GHz. The associated bandwidth is 33% centered at 2.15GHz which is larger than any reported high power TWT in MW power range. The simulated results shows the potential of the CRB as a promising SWS for wideband TWT.

We note that our hot test simulation shows that the backward wave power can be as large as 7% of the forward wave power. To suppress this backward wave power, multiple sections of attenuators/severs can be used near the RF input terminals. However, this will concurrently reduce the forward wave gain as well. To overcome
Figure 4.13: PIC simulated TWT power as a function of frequencies. As shown, 1MW power is achieved over 0.6 GHz bandwidth

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1.8-2.4GHz</td>
</tr>
<tr>
<td>Peak output power</td>
<td>1.02MW</td>
</tr>
<tr>
<td>Gain</td>
<td>29dB</td>
</tr>
<tr>
<td>Maximum electronic efficiency</td>
<td>25%</td>
</tr>
<tr>
<td>Beam current (solid beam of 4mm diameter)</td>
<td>12A</td>
</tr>
<tr>
<td>Cathode Voltage</td>
<td>262kV</td>
</tr>
<tr>
<td>Focusing magnetic field</td>
<td>1.4T</td>
</tr>
<tr>
<td>Tube length(L)</td>
<td>74cm</td>
</tr>
</tbody>
</table>

this issue, inhomogeneous sections with different periodicity can be used and therefore, maximum electronic power can be extracted from the electron beam. These techniques are recommended for further research.

A coupled transmission line analysis to realize slow wave structures was presented. The analysis is based on coupled dual, but identical, transmission lines with mutual coupling inductance between them. These coupling elements are responsible for wave
slowdown. It was noted that dispersion and interaction impedance can be controlled by modifying the the structure geometry. The coupled mode analysis provided propagation constants that described the wideband aspects of the new design. The coupled TL lines were then used to design CRB geometry. Theoretical modeling and full wave analysis verify the new coupled concept. A sample TWT design was presented as well to show CRB’s wideband performance as a slow wave structure. It was found that the TWT-based on CRB structure can deliver maximum power of 1MW with 25% electronic efficiency.

4.3 Characterization of Curved Ring-Bar

CRB slow wave structure has been presented before in order to design a slow wave structure for ‘high power wideband’ TWT in S-band. It was analyzed using the coupled transmission lines theory and predicted to deliver 1MW output power across 1.8-2.4GHz bandwidth. However, the slow wave characteristics were not experimentally validated through measurements. In this section, we present $\omega - \beta$ measurement results to experimentally demonstrate that CRB structure is providing the predicted slow wave characteristics at S-band. A novel synthetic technique is used to determine the $\omega - \beta$ relation using 6-period fabricated CRB structure. The measurement results exhibits phase velocity of 0.7$c - 0.75c$ across 2-2.5GHz. In addition, the measured on-axis interaction impedance was (> 43$\Omega$) across the specified frequencies. The measurement results closely matches the full wave simulation results with maximum error less than 5%.
4.3.1 Mode Purity Property of the CRB Geometry

Considering the challenges to attain high power TWTs across large bandwidths, CRB structure was introduced for improved electron beam to wave coupling [71]. The CRB is composed of two elliptically flared wires/bars connected with two rings forming a unit cell as in Fig. 4.14. This structure (at the center of the waveguide) can be interpreted as a pair of coupled Transmission Lines (TLs) separated by a distance $2a$ from each other (see Fig. 4.15).

The coupled TL analysis provided a simpler way to derive the dispersion diagram of such a complex geometry. The analytically derived dispersion relation in (4.15) provides the $\omega - \beta$ dispersion diagram of the CRB structure. As seen in Fig. 4.15c,
Figure 4.15: (a) CRB structure with straight bars/wires for modeling purposes. (b) Equivalent circuit model of the CRB with a unit cell model (c) Comparison of the full-wave dispersion analysis to the coupled TL model.
it accurately verifies the coupled TL concept and physics of the slow-wave formation inside CRB geometry providing a new parameter \((m)\) to control concurrently the dispersion and the interaction impedance. One of the most attractive feature of this structure is that it can achieve large phase velocities \((0.7c – 0.75c)\) that can couple to high energy electron beams. The wave slow down is achieved through the mutual inductances \((L_M)\) that plays the same role as dielectrics. To illustrate this, CRB design steps are shown in Fig. 4.16. As seen, two coupled TLs are evolved to form the resultant elliptical CRB geometry. Also, the mutual capacitances between these two lines are suppressed by the presence of the rings. Only mutual inductances \((L_M)\) prevails at the center of each unit cell. Therefore, the CRB structure can support phase velocities as large as 0.75c. In addition, the cancellation of \(H\) – fields enhances the \(E\) – field at the center of the geometry. The ellipses in the resultant CRB structure provide uniform \(E_z\) field that translates to \(TM_{01}\) mode purity and moderate interaction impedance. In fact, the fundamental mode of the circular waveguide is reversed from \(TM_{01}\) to \(TE_{11}\) due to this mode purity. Even if the guided wave structure supports \(TE_{11}\) mode, it is very weak compared to the \(TM_{01}\) mode. We will observed it later experimentally. Also, the TL loading inside the waveguide provides comparatively larger bandwidth as compared to the corrugated waveguides due to the inherent broadband property of any transmission line geometry (i.e. helix, double-helix, ring-bar etc.) compared to any resonant corrugated waveguide geometry.

### 4.3.2 Full-Wave Analysis of The Final Design

The \(\omega \sim \beta\) diagram along with the beam line and interaction impedance profile of the CRB structure is shown in Fig. 4.17. These were obtained from full wave
simulations using HFSS [5]. It is seen that the designed SWS has a phase velocity greater than $0.7c$ and can couple to very high energy ($\geq 260$ KeV) electron beams. This feature is very important for high power operation. Specifically, we observed that the phase velocity profile of the CRB structure is less dispersive showing only $0.05c$ deviation of the phase velocity across the $2 - 2.5$ GHz band. It is noted that other SWSs, e.g. helices, ring-bar, ring-loop etc. can also provide high phase velocities. However, they typically deliver low interaction impedance, in those cases implying small gains. To illustrate this, a comparison of interaction impedance between the CRB and regular ring-bar structure is provided in Fig. 4.18. As seen, the forward wave interaction impedance of the CRB is larger than regular ring-bar structure. In addition, the backward wave interaction impedance of the CRB is lower than to that of the regular ring-bar structure. Therefore, CRB is more suitable for high power operation since there is lower possibility of backward wave interaction with electron
beam in the TWT. Also, the CRB provides moderate interaction impedance values while maintaining high phase velocity. The interaction impedance profile shows an impedance of $K_0 > 45\Omega$ in the S-band, reasonably good for high power TWT designs.

![Graph showing frequency vs. phase constant and interaction impedance profile.](image)

Figure 4.17: Left: $\omega - \beta$ diagram of the proposed CRB structure with 262kV beam line (corresponding to 0.75$c$ electron speed). Right: Interaction impedance $K_0(\omega)$ profile of the proposed CRB.

### 4.3.3 CRB Fabrication and Characterization

In this section, we describe the measurement process for the characterization of the designed CRB structure. At first, the fabricated 6-period CRB is shown in Fig. 4.14c. A highly conductive copper sheet (99.99% copper) was cut to form the elliptic bars using a high precision water jet cutter. Then rings had the dimensions mentioned in Fig. 4.14, and were soldered onto the elliptic bars. For the characterization of the fabricated CRB, we used a similar procedure to that described by Guo et. al. [72].
But instead of a cage wheel launcher, a small co-axial probe was used to excite the $TM_{01}$ mode [36].

A schematic of the experimental setup for measuring the dispersion relation is shown in Fig. 4.19. As seen, the CRB structure was inserted in a circular cavity and was braced by Styrofoam having a relative permittivity of $\epsilon_r = 1.3$. Therefore, the Styrofoam had negligible effect on the measurement. As shown in Fig. 4.19, a probe was placed at the center of the cavity side to ensure that only symmetrical modes are excited. This probe was then connected to a Vector Network Analyzer (VNA) to measure the $|S_{11}|$. The measurement setup is described in the next paragraph.

The circular cavity shown in Fig. 4.19 is supposed to exhibit resonances at phase differences: $0, \pi, 2\pi, 3\pi.....n\pi$. These are the natural resonances of the cavity. However, when the $n$ periods SWS structure is inserted, the guided wavelength shrinks, and an additional $(n - 1)$ resonances are expected to appear between the natural
resonances. These resonances \(((n + 1)\text{ in total})\) are \(\pi/n\) apart and are given by the equation:

\[
f = \sum_{n=0}^{\infty} a_n \cos(n\beta p)
\]  

(4.17)

where, \(a_n\)'s are the weight coefficients for the trigonometric series used to derive the dispersion diagram.

The expression (4.17) can be expanded into the matrix form:

\[
\begin{bmatrix}
f_0 \\
f_1 \\
f_2 \\
\vdots \\
f_n \\
\end{bmatrix} =
\begin{bmatrix}
cos0 & cos0 & \cdots & cos0 \\
cos0 & cos(\pi/n) & \cdots & cos(\pi/n) \\
cos0 & cos(2\pi/n) & \cdots & cos(2\pi/n) \\
\vdots & \vdots & \cdots & \vdots \\
cos0 & cos(n\pi/n) & \cdots & cos(n\pi/n) \\
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
a_2 \\
\vdots \\
a_n \\
\end{bmatrix}
\]

(4.18)

At measured resonant frequencies, \(f_i\), this matrix can be used to determine the weight coefficients \((a_0, a_1, \ldots, a_6)\) for the known electrical lengths, i.e. \(\beta p = 0, \pi/6, 2\pi/6, 3\pi/6, 4\pi/6, 5\pi/6\)
and $\pi$. The extracted weight coefficients are then inserted in equation (4.17) to find the final $\omega - \beta$ diagram.

$\omega - \beta$ Diagram Measurement

The measured $\omega - \beta$ diagram is shown in Fig. 4.20. As seen, the experimental dispersion curve is in excellent agreement with the numerical data exhibiting same pattern as the simulated one. We note that the probe length was kept short, and equal to $\lambda/5$ to avoid undesired mode excitation. It is also worth mentioning that the measurement is dependent on the resonant behavior of the overall cavity. Near the
π-point, the resonant frequencies are densely populated. Also, there are weak resonances observed corresponding to the fundamental TE$_{11}$ mode of the circular cavity. Nevertheless, the CRB supports strong TM$_{01}$ mode behavior with the TE$_{11}$ modes mostly suppressed. This property of CRB made it possible to measure dispersion using a simple co-axial probe instead of using a cage-wheel launcher.

**Interaction Impedance Measurement**

To measure $K_0$, we used a non-resonant perturbation method as proposed by Lagerstrom [73]. In this method, the dispersion ($\omega - \beta$) diagram was determined by measuring the phase ($\phi$) of the propagating wave inside the SWS loaded waveguide. The waveguide was then perturbed by placing a dielectric rod at the center of the slow-wave structure [74,75] and the $\omega - \beta$ measurement was conducted again. The interaction impedance is computed by measuring the difference in propagation constants with and without the perturbing rod to give [75]:

$$K_0(\omega) = \frac{2}{\beta_0 \pi \epsilon_0 (\epsilon_r - 1) r_b^2} \frac{\Delta \beta}{\omega} \quad (4.19)$$

where, $\Delta \beta$ is the change in $\beta_0$ due to the perturbation caused by the rod in the axis of SWS. The other parameters in (4.19) are: $K_0(\omega)$ is the Pierce impedance of the fundamental space harmonic, $\omega$ is the angular frequency, $\beta_0$ is the axial propagation constant of the fundamental space harmonic without the perturbing rod, $\epsilon_r$ is the dielectric constant of the perturbing rod, $r_b$ is the radius of the rod.

To obtain $\Delta \beta$, we measured $|S_{11}|$ with the dielectric rod present using the similar procedure described in section – 4.3.3. After obtaining the $\omega - \beta$ relation for with/without the dielectric rod, $\Delta \beta$ was calculated for each frequency. Next, $K_0(\omega)$
was calculated using (4.19). For the dielectric rod, we used a 4mm radius Alumina $(\epsilon_r = 9.5 - 9.8)$ rod due to its large relative permittivity.

The measured $\Delta \beta$ and interaction impedance is shown in Fig. 4.21 exhibiting that $K_0(\omega) \geq 43\Omega$ across $1.8 - 2.5$ GHz. Further, the $K_0(\omega)$ profile is following the same pattern verifying the accuracy of the measurement method.

Figure 4.21: (a) Measurement of $\Delta \beta$ using perturbing Alumina rod. (b) Measured $\Delta \beta$ and (c) $K_0(\omega)$ as a function of frequency.
Clearly, this measurement procedure differs from conventional method in two ways. First, there is no need for phase measurements. Thus, conventional sliders techniques with precise measurement tools for sliding are not needed here. Second, a long waveguide structure is not required and results can be calculated through post-processing of $\Delta \beta$ data. However, this method demonstrates maximum error of 9% near the band edge (> 2.5GHz). Therefore, the results are more accurate in the passband region (1.8 – 2.5GHz). For cold test measurements, it is recommended to used finite length waveguide structure with phase measurements since cavity method is not appropriate for $\omega - \beta$ measurement inside waveguides.

The CRB structure was fabricated and tested via synthetic cavity resonance method. The CRB was placed in a cavity to measure its $\omega - \beta$ diagram and a simple cost effective approach of non-resonant perturbation method was chosen to measure the interaction impedance. Excellent agreement was achieved between the measured dispersion diagram and the simulated one. In fact, resonant frequency measurement results were also presented and compared with simulated ones. Measured interaction impedance clearly follows the simulated impedance profile although the measured impedance was lower than the latter. It is noted that these measurement results are presented to validate the slow wave phenomena and property of the CRB at hand. However, to design a full-length TWT, detailed cold-test measurements are recommended with waveguides, support layout and coupler designs.
Although, CRB provides Megawatts power across 33% of maximum bandwidth, there are limitations to its power level. The maximum output power that can be extracted from this TWT is 1.5MW. This is due to the fact that beam-wave power transfer become non-linear beyond the output power level of 1.5MW. The over-bunching of the electrons with such high energy (> 300keV) and dense beam (20A) generates harmonics and non-linear distorted signal at the output port. Thus, the CRB TWT can not be used to generate power more than 1.5MW maintaining the same bandwidth. Increasing the power can drastically reduce the bandwidth significantly.
Chapter 5: Non-identical Coupled TLs for Degenerate Band Edge Modes

An example of coupled transmission line based slow wave structure was described in detail in Chapter 4. A circuit model was provided to predict dispersion diagram more accurately. However, each TL component of the coupled TL system were identical to each other. In this chapter, we introduce the analysis of the coupled transmission lines formed by non-identical TL components. They provide special modes called Degenerate Band Edge (DBE) modes that are suitable for high power microwave generation. Finally, an example of DBE mode is demonstrated in a design of a SWS called ‘butterfly’ SWS.

5.1 Introduction

Periodic and/or metamaterial structures are routinely used to control propagation characteristics of electromagnetic waves. They have been successfully used in numerous applications, including antenna arrays [76], leaky wave antennas [77], electromagnetic bandgap structures [78] and artificial magnetic conductors [79], frequency selective surfaces [80] and slow wave realization [1, 2, 81]. Negative refractive index metamaterials [82,83] have also led to a number of exotic applications including cloaking [84], subwavelength imaging [85] and electrically small antennas [86, 87], among
others. In addition, periodic stack of dielectrics have been routinely used to achieve bandgaps [88](Fig. 5.1).

Periodic stack of dielectrics are often referred to as photonic crystals as in Fig. 5.1. The presence of two misaligned anisotropic layers forming the unit cell of the periodic stack can also lead to the realization of anisotropic media [89]. The latter have been shown to exhibit Degenerate Band Edge (DBE) [26]. However, insertion of an additional magnetic layer into the unit cell generates Magnetic Photonic Crystal (MPC) modes [2,3].

As can be understood, the realization of such artificially anisotropic media is difficult. As a simpler alternative, Locker et. al. [1] introduced the concept of Coupled Transmission Lines (CTLs) to emulate photonic crystals. This approach has also been used to realize miniaturized antennas [81] and to achieve frequency independent beams-canning [77] in leaky wave antennas. Although the concept of coupled TLs has been successfully used [90], its potential has yet to be exploited. For example, if the coupled TLs are not printed, they can also be inserted inside waveguides to realize a new class of modes. This approach can open possibilities for new applications (Fig. 5.2). For example, Othman et al. [91] proposed a novel medium that can realize DBE modes inside circular waveguides. These modes can be useful to amplify RF wave, interacting with an electron beam [92] in Traveling Wave Tubes (TWTs) and/or Backward Wave Oscillators (BWOs). Typically, the efficiency of these microwave devices is low and dependent on the coupling between the electron beam and waveguide modes. Therefore, the introduction of new modes within the waveguide can provide a mean for improved beam-to-RF mode interactions.
In this chapter, we present a new class of CTLs to realize fourth-order dispersion diagrams. In previous works [1, 2], only a single pair of coupled lines was considered. Further, the coupling was not characterized in terms of \((L, C)\) parameters. Also, the mechanism of mode coupling leading to higher-order dispersion curves was not explained. In this paper, we build upon the concept of CTLs and proceed to generate higher-order dispersion curves using a new ‘coupled mode’ technique for dual pairs.
of non-identical coupled TLs (Fig. 5.2). The key characteristics of the new coupled TLs are:

1. Non-identical TLs (NITLs)

2. Coupled \((L, C)\) parameters

3. Coupling co-efficients, later defined as \(K_{c1}, K_{c2}, \text{ and } K_{c3}\);

Specific example applications will be given for applications using four CTLs. These application will focus on TWTs and BWOs where the coupled TLs are inserted within circular waveguide to improve beam-RF coupling. Overall, we believe that the findings and analysis in this paper will provide a basis for dispersion engineering pertaining to other applications such as resonator antennas [17], waveguides [21] and cavities [93].

5.2 Dispersion Engineering Using Dual Non-Identical Pair of TLs

5.2.1 Background

Transmission lines are inherently periodic structures as they can be modeled with periodically spaced lumped elements \((L, C)\) of period \(p\). TLs support both forward (FW) and backward waves (BW). By controlling the \((L, C)\) parameters of the TLs, slow waves \((v \ll c)\) can be realized. Indeed, this property has been used to enhance coupling of the electron beam to RF waves in TWT and BWO applications. Examples of CTLs include double helix [42], ring-bar [33, 71] and ring-loop [43] etc. For these cases, the TL pair had identical lumped elements \((L, C)\) and supports Regular Band Edge (RBE) modes. This mode is observed at the frequency corresponding to \(\beta p = \pi\)
in the dispersion diagram and where $\beta$ is the propagation constant and $\omega$ is the angular frequency. We remark that RBE resonances are a consequence of the coupling between the forward and backward wave modes [49,71,94].

When the coupled TLs are not identical viz. composed of different lumped elements ($L, C$) as shown in Fig. 5.3a, the forward and backward waves have unequal phase and group velocities and give rise to double band edge (DbBE) modes seen in Fig. 5.3b(bottom left). In fact, DBE modes can also be achieved if the lumped parameters of the non-identical coupled TLs are chosen appropriately (Fig. 5.3b(bottom right)). Actually, more than second-order dispersion can be achieved using a pair of non-identical CTLs. Below, we provide a theoretical analysis for the coupled TLs to generate higher-order dispersion curves.
5.2.2 Theoretical Analysis

The presented analysis follow the coupled mode theory [49, 63]. Accordingly, the coupled pair of TLs are associated with the propagation constants [49]:

$$\beta_{\pm} = \frac{\beta_m + \beta_n}{2} \pm \sqrt{\left(\frac{\beta_m - \beta_n}{2}\right)^2 - K_c^2}$$  \hspace{1cm} (5.1)

Here, $\beta_m = \omega \sqrt{LC}$, $\beta_n = \frac{2\pi}{p} - \omega \sqrt{LC}$, and $\beta_{\pm}$ refer to the forward (FW) and backward (BW) modes for the coupled TLs system. Also, the coefficient, $K_c$ represents the
coupling between the $\beta_{mn}$ modes. We remark that (5.1) is the building block of our analysis. Specifically, by choosing appropriate modes to replace the $(\beta_m, \beta_n)$ pair, fourth-order dispersion curves can be generated.

To begin, let us consider two uncoupled TLs associated with different lumped inductances and capacitances $(L_1, C_1)$ and $(L_2, C_2)$. These lines are depicted in blue and red color in Fig. 5.4a. Each TL supports forward and backward waves associated with unequal velocities, $v_1 = \frac{1}{\sqrt{L_1 C_1}} \neq \frac{1}{\sqrt{L_2 C_2}} = v_2$. The propagating constants of these four waves are:

$$\beta_a = \omega \sqrt{L_1 C_1} = \frac{\omega}{v_1} \quad (5.2)$$
$$\beta_b = \frac{2\pi}{p} - \omega \sqrt{L_1 C_1} = \frac{2\pi}{p} - \frac{\omega}{v_1} \quad (5.3)$$
$$\beta_c = \omega \sqrt{L_2 C_2} = \frac{\omega}{v_2} \quad (5.4)$$
$$\beta_d = \frac{2\pi}{p} - \omega \sqrt{L_2 C_2} = \frac{2\pi}{p} - \frac{\omega}{v_2} \quad (5.5)$$

Each of the above $\beta$’s, gives rise to the linear dispersion curves (5.2)-(5.5) in Fig. 5.4b. Notably, unlike identical TLs, each line supports non-overlapping FW and BW modes represented by solid and dashed lines, respectively. We denote the FW propagation constants as $\beta_a$ (solid blue line) and $\beta_c$ (solid red line). Similarly, $\beta_b$ (dashed blue line) and $\beta_d$ (dashed red line) represent the BW propagation constants.

When the TLs are coupled, they couple through the forward and backward mode pairs. This gives rise to second-order dispersion curves as in Fig. 5.4c (bottom right). Specifically, the coupling between forward and backward mode pairs e.g. $\beta_a(\omega)$, $\beta_d(\omega)$ and $\beta_b(\omega)$, $\beta_c(\omega)$ gives rise to second-order dispersion curves. The associated
propagation constants are given by

\[
\begin{align*}
\beta_u &= \frac{\pi}{p} - \frac{\omega}{2} \left( \frac{1}{v_2} - \frac{1}{v_1} \right) + \sqrt{\left( \frac{\pi}{p} - \frac{\omega}{2} \left( \frac{1}{v_2} + \frac{1}{v_1} \right) \right)^2 - K_{c1}^2} \quad (5.6) \\
\beta_v &= \frac{\pi}{p} - \frac{\omega}{2} \left( \frac{1}{v_2} - \frac{1}{v_1} \right) - \sqrt{\left( \frac{\pi}{p} - \frac{\omega}{2} \left( \frac{1}{v_2} + \frac{1}{v_1} \right) \right)^2 - K_{c1}^2} \quad (5.7) \\
\beta_w &= \frac{\pi}{p} + \frac{\omega}{2} \left( \frac{1}{v_2} - \frac{1}{v_1} \right) + \sqrt{\left( \frac{\pi}{p} - \frac{\omega}{2} \left( \frac{1}{v_2} + \frac{1}{v_1} \right) \right)^2 - K_{c2}^2} \quad (5.8) \\
\beta_x &= \frac{\pi}{p} + \frac{\omega}{2} \left( \frac{1}{v_2} - \frac{1}{v_1} \right) + \sqrt{\left( \frac{\pi}{p} - \frac{\omega}{2} \left( \frac{1}{v_2} + \frac{1}{v_1} \right) \right)^2 - K_{c2}^2} \quad (5.9)
\end{align*}
\]

The coefficients, \( K_{c1} \) and \( K_{c2} \) represent the coupling between the oppositely traveling modes for the TL pairs. They are given by [71]:

\[
\begin{align*}
K_{c1} &= \frac{(1 + \frac{\pi h_2}{4a}) E(m)}{E(m)} \sqrt{\frac{\beta_a \beta_d}{|\beta_a - \beta_d|}} \quad (5.10a) \\
K_{c2} &= \frac{(1 + \frac{\pi h_2}{4a}) E(m)}{E(m)} \sqrt{\frac{\beta_b \beta_c}{|\beta_b - \beta_c|}} \quad (5.10b)
\end{align*}
\]

where, \( E(m = \frac{h_2}{h_1}) = \int_0^{\pi/2} \sqrt{1 - (m^2 - 1) \sin^2(\theta)} \, d\theta \) is the elliptic integral of the second kind.

It is noted that (5.6)-(5.9) represent second-order dispersion curves and are associated with an RBE resonances at \( \beta p = 2 \) (rad) and \( \beta p = 4.3 \) (rad), respectively (Fig. 5.4c). Their associated dispersion curves are given in Fig. 5.4c (bottom right).

To characterize and observe higher-order coupling, we introduce a new coupling parameter, \( K_{c3} \). This quantity represents coupling between \( \beta_v \) and \( \beta_w \). Due to being in proximity to each other, these pairs couple further inside waveguide and form higher-order dispersion curves. The corresponding propagation constants, \( \beta_2 \) and \( \beta_3 \) are fourth-order dispersion curves. Indeed, fourth-order dispersion curves can be derived using the process described in [49].
When (5.11)-(5.14) are plotted in Fig. 5.5, DbBE and DBE modes are observed subject to appropriate choices for $K_{c1}$, $K_{c2}$ and $K_{c3}$. That is, $K_{c3}$ is important in realizing higher-order dispersion curves.

$$\beta_1 = \frac{\pi}{p} + \sqrt{\left[\frac{\omega}{2} \left(\frac{1}{v_1} - \frac{1}{v_2}\right) - \sqrt{\left\{ \frac{\pi}{p} - \frac{\omega}{2} \left(\frac{1}{v_1} + \frac{1}{v_2}\right) \right\}^2 - K_{c1}^2} \right] - K_{c3}^2}$$  \hspace{1cm} (5.11)

$$\beta_2 = \frac{\pi}{p} - \sqrt{\left[\frac{\omega}{2} \left(\frac{1}{v_1} - \frac{1}{v_2}\right) - \sqrt{\left\{ \frac{\pi}{p} - \frac{\omega}{2} \left(\frac{1}{v_1} + \frac{1}{v_2}\right) \right\}^2 - K_{c1}^2} \right] - K_{c3}^2}$$  \hspace{1cm} (5.12)

$$\beta_3 = \frac{\pi}{p} + \sqrt{\left[\frac{\omega}{2} \left(\frac{1}{v_1} - \frac{1}{v_2}\right) - \sqrt{\left\{ \frac{\pi}{p} - \frac{\omega}{2} \left(\frac{1}{v_1} + \frac{1}{v_2}\right) \right\}^2 - K_{c2}^2} \right] - K_{c3}^2}$$  \hspace{1cm} (5.13)

$$\beta_4 = \frac{\pi}{p} - \sqrt{\left[\frac{\omega}{2} \left(\frac{1}{v_1} - \frac{1}{v_2}\right) - \sqrt{\left\{ \frac{\pi}{p} - \frac{\omega}{2} \left(\frac{1}{v_1} + \frac{1}{v_2}\right) \right\}^2 - K_{c2}^2} \right] - K_{c3}^2}$$  \hspace{1cm} (5.14)

Above, $K_{c1}$, $K_{c2}$ and $K_{c3}$ signify different mode formation mechanisms. For example, the parameters $K_{c1}$ and $K_{c2}$ represent natural coupling between the forward and backward modes of two non-identical TLs. The effect is ‘in-plane anisotropy’ due to non-linear coupling between the non-identical TLs pairs. The derivation of $K_{c3}$ is a cumbersome process and is beyond the scope of this paper. Indeed, a numerical approach can be employed to compute $K_{c3}$.

We note that special choices for $K_{c1}$, $K_{c2}$ and $K_{c3}$ lead to the realization of DbBE (Fig. 5.5a) and DBE modes (Fig. 5.5b). A strong flat top fourth-order resonance (DBE mode) is observed for $K_{c1} = K_{c2} \neq K_{c3}$ (Fig. 5.5b). Recently, Othaman et al. [92] demonstrated that DBE modes can be realized by imposing angular anisotropy using elliptic irises in circular WG. In the following section, we present an example of such DBE mode realization using TLs.
5.2.3 Realization of DBE Mode Using ‘Butterfly’ Geometry

Above, we proposed a coupled pair of non-identical TLs (quad TLs) to realize higher-order dispersion curves. In this section, we present an example of such CTLs placed inside a waveguide. As already stated, strong coupling among the TLs is necessary to achieve DBE modes and concurrently realize anisotropic medium. Othman et. al. [91] demonstrated such a medium by using misaligned elliptic irises placed on the axis of a circular waveguide.

In this chapter, we propose to realize DBE modes using two pairs of free standing wire TLs placed orthogonally to each other. One such structure is demonstrated in Fig. 5.6a. The associated structure is formed by a ‘butterfly’ unit cell. This unit cell is composed of two non-identical pairs of TLs represented by elliptic wires/bars, marked as blue and red in Fig. 5.6. Notably, the four TLs are placed circularly among a set of rings. These rings connect them and serve to realize coupling among the four TLs. The coupling rings are marked with green color and allow control of mutual inductances/capacitances between the TL pairs. It is noted that each identical pair (blue or red) in Fig. 5.6 serves as a single TL component of the non-identical TL model as shown in Fig. 5.4a. This coupling mechanism serves to:

1. Provide a medium that support slow waves.

2. Lower the cut-off frequency of waveguide modes

3. Facilitates coupling of the lower order modes to form higher-order such as DBE mode.

To illustrate the above mechanism, we refer to Fig. 5.7. Indeed, the introduction of ‘butterfly’ geometry lowered the cut-off frequency of each mode forming slow waves.
inside the circular waveguide as shown in Fig. 5.7a. As depicted in Fig. 5.7b, the degenerate $TE_{11}$ mode coupled to the $TE_{21}$ mode to form the DBE $TM_{01}$ resonance. We remark that coupling is achieved via the mutual $H_z$ fields supported by the $TE_{11}$ and $TE_{21}$ modes. The detailed dispersion diagram for DBE mode was obtained via full-wave simulation using Ansoft High Frequency Simulation Software (HFSS) package, 2015 [5]. They are shown in Fig. 5.8. The resonant frequency of the DBE $TM_{01}$ mode is observed at approximately $\omega_d = 3.52$ GHz.

DBE mode can be useful for electron beam-RF wave interaction for BWOs. To illustrate the BWO interaction, a BWO design was simulated using Computer Simulation Software (CST) Particle in Cell (PIC) [6] code. The ‘butterfly’ SWS is placed at the center of the waveguide as shown in Fig. 5.9 (bottom left). The bunching of electrons verifies the beam-wave interaction (Fig. 5.9)(bottom left). The BWO draws 15A current from a circular cathode biased with 52kV. The tube is 220mm long and generates 83kW power with 11% electronic efficiency. It is noted that the designed BWO is composed of homogeneous sections i.e. all the unit cells have same spatial periodicity. Typically, the homogeneous section BWOs are associated with electronic efficiency around 10-15% [95]. Inhomogeneous ‘butterfly’ SWS can significantly improve the electronic efficiency of the BWO.

In conclusion, the presence of fourth-order DBE mode verifies the concept of dual pair of non-identical TLs as an effective medium to support higher-order dispersion engineering. The same concept can be extended further to design couplers, filters and printed circuits utilizing the higher-order resonances and mode coupling.
We introduced a new class of TLs that can generate unusual DbBE and DBE modes. It was demonstrated that dual pair of non-identical TLs can generate higher-order dispersion curves, specially DBE modes. Fourth-order dispersion equations were derived using the coupled mode analysis for the non-identical pair of coupled TLs. Further, it was shown that the order of the dispersion curves are dependent on the choice of coupling parameters, $K_{c1}$, $K_{c2}$ and $K_{c3}$ and dispersion can be controlled by them. Practical realization of the anisotropy required for the DBE modes was demonstrated via a simple ‘butterfly’ geometry used to construct the four coupled TLs. DBE modes can be exploited to engineer new class of TWT and BWOs. An example of BWO design was presented to verify the beam-wave interaction with DBE $TM_{01}$—like mode. This theory and example are expected to serve as tools to engineer dispersion curves for more practical applications.

Although ‘butterfly’ structure is suitable for generating high power and high efficiency BWO, it does not provide much tunable bandwidth. Typically, RBE resonances provide 1-2% of tuning bandwidth for the oscillation frequency. The tuning can be changed by changing the beam voltage, viz. moving the beam-line in the dispersion diagram. However, ‘butterfly’ SWS support flat dispersion curve that curbs this property and almost 0.01% tuning is expected from the interaction.
Figure 5.4: (a) A pair of uncoupled and coupled non-identical TLs supporting FW and BW whose propagation constants are defined in (5.2)-(5.5). (b) $\omega - \beta$ diagram of the uncoupled TLs for each of the supported modes given in (5.6)-(5.9). c) RBE resonances realized by the non-identical coupled TLs are found due to unequal velocities $(\nu_1, \nu_2)$. These curves refer to circuit parameters $(L_1, C_1) \equiv (16.17\mu H, 68.8pF)$ and $(L_2, C_2) \equiv (6.5\mu H, 27.52pF)$. 
Figure 5.5: $\omega - \beta$ diagrams associated with coupled pairs of TLs. The $(L, C)$ parameters of the coupled TLs of each pair are: $(L_1, C_1) \equiv (16.17\mu H, 68.8pF)$ and $(L_2, C_2) \equiv (6.5\mu H, 27.52pF)$. (a) DbBE dispersion curves viz. weak coupling of the dual TL pair and (b) DBE dispersion curves. These are fourth-order curves and higher-order dispersion condition: $\frac{\partial^3 \omega}{\partial \beta^3} \neq 0$. 
Figure 5.6: (a) The ‘butterfly’ slow wave structure placed within a circular waveguide for realizing DBE modes. Unit cell is shown below the circular waveguide. Each of the four TLs is formed of a series of elliptical loops. Also, a ring at the center of the TLs serve to achieve coupling among the TLs. The dimensions of the elliptical and circular rings are: $h_a = 50.8\text{mm}$, $h_b = 36.4\text{mm}$, $p = 22\text{mm}$, $r_b = \text{ring radius} = 4.5\text{mm}$, $r_g = \text{WG radius} = 45\text{mm}$. Notably, the unequal pairs emulate in different planes emulate non-identical coupled TLs. (b) ‘butterfly’ geometry and its equivalent TL structure.
Figure 5.7: (a) Comparison of the dispersion diagrams with (Right) or without (Left) the ‘butterfly’ geometry inside the circular waveguide. As seen, the introduction of the ‘butterfly’ TL structure lowered the cut-off frequency of each mode by splitting the $TE_{11}$ degenerate modes. (b) Illustration of coupling to form the DBE $TM_{01}$ mode by coupling of $TE_{21}$ and $TE_{11}$ degenerate is shown.
Figure 5.8: $\omega - \beta$ diagram of 'butterfly' structure using HFSS [5]. DBE mode is depicted along with its field profile. The DBE resonance is observed at $\omega_d = 3.52$ GHz.
Figure 5.9: Top: $\omega - \beta$ diagram of the first 20 modes of ‘butterfly’ SWS using HFSS [5]. A 52kV 15A beam line is drawn to show resonant point (marked by circle) where beam-wave interaction takes place in a simple ‘butterfly’ BWO design. Bottom: Demonstration of beam-wave interaction in a BWO loaded with ‘butterfly’ SWS simulated using CST PIC code [6]. The presence of bunching verifies the electron modulation process, essential for wave-particle coupling and power transfer.
Chapter 6: Summary and Future Work

6.1 Summary

TWT amplifiers have been used as high power microwave sources for decades. They have also been reliable microwave sources due to their durability, high power delivery and linearity. However, they are traditionally narrowband (1-5%) devices, particularly at high power (Megawatts) applications. This dissertation focused on increasing the bandwidth of high power TWTs by improving electron beam to RF interactions. This was done by introducing several concepts of slow wave structures to enhance the axial electric field. The dissertation explored several SWSs.

1) Helical SWSs due to their large bandwidth. They are known to have skew symmetry (combination of translational and rotational symmetry) and provide large bandwidth. However, their axial electric fields are weaker particularly at high frequencies, implying lower interaction impedance and smaller efficiencies than the Ring-bar SWS.

2) Half-Ring-Helix (HRH) SWS [59]. These were demonstrated to have 25% bandwidth at S-band and delivered a gain of 46dB and peak power of 1kW (see Fig.3.8). The HRH SWS improved gain by 10dB over that of traditional helix. Also, it allowed for miniaturization.
3) **Ring-bar SWS.** These are typically dispersive and high power SWS. However, they cannot support waves with very high phase velocity \((\geq 0.7c)\) and large bandwidth.

4) **Curved Ring-Bar** [71]. These were also introduced for improved bandwidth and gain. Indeed, as much 33\% bandwidth centered at 2.1GHz with more than 1MW power were observed. The elliptic bars in the structure provided mode degrees of freedom in design to control the phase velocity and interaction impedance. Specifically, we achieved phase velocity as high as \(0.7c - 0.75c\) across 0.5GHz bandwidth. The tapering of the wave velocity is also possible by changing the axial ratio of elliptic parameter, \(m\), without changing periodicity, \(p\). A 6 period CRB SWS was fabricated and tested at the Ohio State ElectroScience Laboratory (cold test). For testing, the CRB was placed inside a cavity and all 7 resonances were observed. These resonances were observed to construct the \(\omega - \beta\) diagram for comparison with simulations. Using the measured measured \(\omega - \beta\), we extracted the interaction impedance values showing \(K_0 \geq 43\Omega\) across S-band.

Notably, SWS circuits based on helices, ring-bar, ring-loops, CRB etc. are forms of coupled transmission lines (CTLs) [60, 71]. With this in mind, we introduced a generalization of the CTL concept to derive new class of SWSs for TWTs.

A variety of CTLs were considered, including TL pairs with unequal \((L, C)\) values. It was found that controlling the TLs leads to higher order dispersion curves. Further, it was found that multiple CTLs can be used to achieve even greater control in the dispersion of the \(\omega - \beta\) diagrams. It was demonstrated mathematically that dual pair of non-identical TLs can generate higher order dispersion curves (4th order (DBE)). It was shown that the order of the dispersion curves for the dual pair of TLs can be
characterized by 3 parameters: $K_{c1}$, $K_{c2}$ and $K_{c3}$. Practical realization of the dual pair CTL was demonstrated via a simple ‘butterfly’ geometry. Overall, the higher order modes can be explained to engineer new class of TWTs and BWOs.

### 6.2 Future Work

#### 6.2.1 A Backward Wave Oscillator design Using Inhomogeneous ‘Butterfly’ Structure

In Chapter 5, we presented a ‘butterfly’ slow wave structure. The structure utilized novel DBE modes. Since DBE resonances are strong compared to second order regular band edge resonances, ‘butterfly’ SWS are expected to deliver more power. Therefore, a 6-period BWO was designed using this geometry to realize a high power microwave source. The designed BWO is shown in Fig.6.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>3.38 GHz</td>
</tr>
<tr>
<td>Peak Output Power</td>
<td>62.5kW</td>
</tr>
<tr>
<td>Maximum electronic efficiency</td>
<td>33%</td>
</tr>
<tr>
<td>Beam current (circular cross section of 2mm radius)</td>
<td>4A</td>
</tr>
<tr>
<td>Cathode voltage</td>
<td>52kV</td>
</tr>
<tr>
<td>Focusing magnetic field</td>
<td>1T</td>
</tr>
<tr>
<td>Tube length ($L$)</td>
<td>16cm</td>
</tr>
</tbody>
</table>

Hot test PIC simulation was conducted using CST PIC code. The hot test parameters are given in Table-6.1. As seen from Table-6.1, the BWO delivered peak power of 62.5kW at 3.38GHz with maximum electronic efficiency of 33%. It is noted that the designed BWO is composed of homogeneous sections i.e. all the
unit cells have same spatial periodicity. Typically, the homogenous section BWOs are associated with electronic efficiency around 15-20% [36]. Therefore, the preliminary simulation results exhibit great potential for advanced design. The time domain output signal is shown in Fig.6.2.

Although the BWO shows promising result in terms of efficiency and high power delivery, there are several issues that need to be resolved to improve the output power and efficiency. One of the issue is the presence of harmonics and intermodulation product (IMP). Harmonics generally carry significant amount of RF power resulting
in poor gain performance and low efficiency. However, IMP cannot be observed from time domain output. To observe the problem, Fourier transform of the output signal are shown in Fig.6.3. As seen, there are multiple frequency components near the 2GHz point in the spectral analysis as shown in Fig.6.3. It is clear that desired interaction at at 3.38GHz is strong compared to others (2.2GHz and 1.8GHz) and it carries 60% of the output power. Among these, the component 2.2GHz is present due to interaction of beam line to the second mode as depicted in Fig.5.9.

![Time Domain Output](image)

Figure 6.2: Time domain simulation results using CST PIC code. The results show existence of almost constant 250V amplitude level implying 62.5kW output power

There are several reasons that might cause undesirable level of frequency component power. One of the reason is the harmonic strong harmonic impedance. They are inevitable in any periodic structures. This harmonic components can be suppressed by introducing attenuators/severs or harmonic injection [96]. Typically, the severs
caused significant loss of output power. Therefore, harmonic injection can be investigated more to suppress the harmonic power. Injection of the harmonic signal into the BWO shifts the harmonic power generated from beam-wave interaction very far in the spectrum and the output power level is restored at the desired frequency. In Fig.6.3, we observed IMP caused by original signal and interaction of beam line with second hybrid mode at 2.2GHz. To overcome this issue, the second mode needs to be suppressed. This is a design problem and is left for future consideration.

It is not only periodicity of the SWS that causes generation of large harmonic power. The most crucial reason for strong harmonic component is the non-linear interaction of electron beam to RF wave. Typically, it happens when the beam current of the BWO is two to three times larger than the starting current of the BWO. The non-linear interaction happens in the over-bunching regime. This is shown in Fig. 6.5. As shown, the over-bunching causes multi-valued velocity distribution in electron beam leading to poor output power and efficiency. It is desired to operate BWO in steady state regime where bunching is regular and steady.

To solve this overbunching issue, inhomogenous BWO can be designed. In fact, this efficiency of a BWO can be improved significantly if inhomogenous sections are introduced. It was shown in [97] that efficiency of the BWO can be improved by three fold by introducing inhomogenous sections in it. Actually, the starting current of inhomogenous section BWO is two to three time higher than that of homogenous BWO. Therefore, inhomogenous BWO operate usually in steady state regime leading to improved efficiency.

To create inhomogenous BWO, typically spatial periodicity is altered and tapered. However, ‘butterfly’ geometry can leverage from its phase velocity control property.
The phase velocity of the BWO can be tapered and make inhomogenous without changing the pitch. The dimensions $h_a$ and $h_b$ in Fig.5.6 can be optimized to find right phase velocity along the tube different sections. One of the possible velocity profile can be chosen by creating a log-periodic velocity profile [98] using the ellipses as shown in Fig.6.5

6.2.2 mm-Wave TWTs Using Photonic Bandgap Structure

High power TWTs have been designed and realized at low frequencies. However, due to the increasing demand of high data rates and high speed satellite communications, mm-Wave RF amplifiers are of increasing interest now. Typical challenges of mm-Wave TWTs are poor power handling, low efficiency and reduced coupling to waveguides. In addition, traditional RF circuits scaled to mm-Wave frequencies suffer from mode competition that limits electronic efficiency. Currently, mm-Wave TWTs are limited to < 100W output power and below 5% electronic efficiency. These are the major challenges for the realization of mm-Wave TWTs. To improve output power and efficiency of the mm-Wave TWTs, enhanced interaction between the RF circuit and the electron beam is mandatory. But typical RF circuits e.g. helices, folded waveguides face increasing fabrication challenges that will likely limit their ultimate performance. Therefore, a new class of interaction circuit is required to build high-performance mm-Wave TWTs.

To address the aforementioned challenges, photonic-band gap (PBG) structures have been studied in recent years [7,99]. These are periodically spaced metallic or dielectric lattices extensively used in optics before and are very promising in microwave region. One of the major advantages of PBGs is that they offer large bandgaps (6.6)
between RF modes that solve the mode competition issue arising from scaling conventional TWT technology to mm-Wave. Hence, PBGs appear to be a very promising emerging technology for advanced TWT development. However, PBGs are difficult to couple to waveguides that carry RF power into and out of the structure. To solve this problem, a modal theoretical and numerical analysis of the RF circuits and waveguide must be conducted and appropriate geometries need to be developed. These can be the key goals of the future research effort. This research has the promise of developing a new class of mm-Wave TWTs with significantly improved performance for communications and RADAR applications.

6.2.3 Multipactor Analysis

Secondary emission is novel phenomenon in high power microwave sources. It is basically caused by any impact of electrons into metal or dielectric interfaces. With sufficient impact energy and angle, these primary electron ejects more electron from the surface. If there is an RF signal in the background and the energy of the RF signal matches with the electron energy, a resonance condition is met and avalanche of electrons are emitted [100]. This is called multipactor current. This currents can be initiated by simple sparks caused by the sharp edges in waveguides. Specially, in space applications where RF components require special standards, multipactor current can seriously damage the components like filters, couplers, waveguides [101]. Therefore, understanding the phenomena is very important. To do this, a simulation was conducted in CST PIC studio. The 'Secondary Emission Yield (SEY) simulation was conducted twice.
First a simple tapered waveguide was drawn as shown in Fig.6.7. The waveguide is 440mm long with 238mm tapered along the axis. The electron beam is injected to the waveguide region with opening cross section of 10mm×2.5mm. The global mesh size chosen for the simulation is λ/30 which made the number of total mesh cells as 4 million. The waveguide wall material chosen for simulation is copper and the SEY model chosen for secondary particle is Furman. Please note that the particle initial velocity was chosen as 0.8c and it takes 1.83ns for electrons to reach on the other side of the waveguide. Two different simulation results are presented in the Fig.6.8. The secondary emission results show that Frequency Selective Surface (FSS) can reduce SEY as time progresses. The maximum number of emitted secondary electron is 1.8 × 10^8 when no FSS was used. However, the number came down to 1.4 × 10^8 when FSS was used and the trend diminishes as time progress. As seen, inclusion of suitable FSS can suppress the SEY significantly.
Figure 6.3: Top: Fourier spectrum of the RF output signal of the designed BWO. Multiple frequency components are present at 3.38GHz, 2.2GHz and 1.8GHz. Middle: The zoomed view of the output signal around 0-5GHz. A minor peak was observed before 1.8GHz which is the result of over-bunching of electrons. Bottom: The interaction of beam line with hybrid TE mode. The interaction region is marked by the black circle.
Figure 6.4: Particle Phase Space of the designed BWO. As shown, multivalued velocity distribution is present due to over bunching and significant amount of electrons remain uncoupled.

Figure 6.5: A log periodic ‘butterfly’ SWS for inhomogenous BWO tube. The profile is useful to increase the starting current leading to steady-state operation.
Figure 6.6: A PBG based slow wave structure Top for mm-Wave TWT [7] and the corresponding dispersion diagram Bottom. A clear bandgap is observed, convenient for mode coupling and efficient beam-wave interaction.
Figure 6.7: Top: A secondary emission yield simulation inside a tapered waveguide. The dimensions are given as shown and the waveguide is made of copper. Bottom: FSS was introduced in tapered section of the waveguide to suppress the SEY.

Figure 6.8: A comparison of SEY with or without the inclusion of FSS. As shown, FSS reduces SEY significantly as time progresses.
Appendix A: Matlab Code for Evaluating Pierce growth factor, $x$

Below is the MATLAB code for evaluating Pierce gain parameter, $x$. The small signal growth factor ($x$) of a signal is a non-linear function of space charge parameter, $QC$ and velocity detuning parameter, $b$. The definition of these parameters are given in the text. Readers are encouraged to go through the reference [27] for detail information about the property and nature of the parameters. Both the parameters are the input of this function expressed as $QC$ and $b lin$ respectively and the output is the exponential growth factor, $x$ expressed as $x lin$. Please note that $QC$ is unit-less parameter specifying intensity of the debunching force of space charge [28] whereas growth factor, $x$ is also unit-less.

The code is given below:

```matlab
function [x lin] = Piercegrowthx(QC, b lin)
y = -4 : 0.001 : 0;
y sqr = y.^2;
num1 = -(8*y sqr + 3*QC) + sqrt((40*y sqr*QC) - (8*y) + (9*QC*QC));
den1 = 4*y;
b1 = num1./den1;
num2 = -(8*y sqr + 3*QC) - sqrt((40*y sqr*QC) - (8*y) + (9*QC*QC));
```

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den2 = 4 * y;

b2 = num2./den2;

b1 = b1(imag(b1) == 0);
y1 = y(imag(b1) == 0);

b2 = b2(imag(b2) == 0);
y2 = y(imag(b2) == 0);

x1 = sqrt((3 * (y1.^2)) + (2 * b1 .* y1) - QC);
x1in = spline(b1(1 : end - 1), x1(1 : end - 1), b1in);

end
Appendix B: Matlab Code for Evaluating Pierce Space Charge Parameter, $Q$ for Helix

This MATLAB function written below is the numerical solution for evaluating one the space charge parameter, $Q$ described by Pierce. $Q$ depends on the geometry of the slow wave structure. Here, helix was chosen as SWS and the functions were used from reference [27]. The input of this function depends on four parameters namely spatial periodicity, $p$ (in mm), velocity detuning parameter, $b$ (unit-less), beam radius, $a$ (in mm) and RF frequency, $f$ (in GHz). All the input parameters are written with same notation. The output $Q$ is also written same as definition. The code is given below:

\[
\text{function}[Q] = \text{PierceQfunction}(p, b, a, f)
\]

\[
f = f \times 1e9;
\]
\[
w = 2 \times \pi \times f;
\]
\[
lambda_0 = (3e8/f);
\]
\[
beta_0 = (2 \times \pi / \lambda_0);
\]
\[
tanphi = (p / (2 \times \pi \times a));
\]
\[
cotphi = 1 / tanphi;
\]
\[
betae = w / ve;
\]
\[
gamma_0 = 0 : 0.01 : 1000; \text{num1} = \text{besseli}(0, gamma_0 \times a) \times \text{besselk}(0, gamma_0 \times a);
\]
`den1 = besseli(1, gamma0 * a) .* besselk(1, gamma0 * a);`

`f1 = (((gamma0 * a) .^ 2) .* (num1 ./ den1));`

`f2 = ((beta0 * a * cotphi)^2);`

`f3 = f2 - f1;`

`mm = find(f3 < 0);`

`gamma0sol = gamma0(mm(1) - 1);`

`gammap = 0 : 0.01 : 1000;`

`X1 = ((beta0 * a * cotphi) ./ (gammap * a))^2;`

`Y1 = (X1 .* besseli(1, gammap * a) .* besselk(1, gammap * a)) - (besseli(0, gammap * a)) .* besselk(0, gammap * a));`

`Z1 = (besselk(0, gammap * a))^2;`

`RHS = (-1) * Y1 ./ Z1;`

`LHS = (besseli(0, gammap * b) ./ besselk(0, gammap * b));`

`f4 = RHS - LHS;`

`mmm = find(f4 < 0.001 & f4 > -0.001);`

`gammapsol = gammap(mmm(1));`

`M1 = gamma0sol ./ ((gammapsol^2) - (gamma0sol^2));`

`M2 = (beta0^2) ./ (gamma0sol^2);`

`M2 = (1 + M2)^(-0.5);`

`Q = 0.5 * betae * M2 * M1;`

`gamma0sola = gamma0sol * a;`

`Qeff = Q * M2 * (gamma0sol/betae);`

`end`
Bibliography


