Multi-GNSS Precise Point Positioning Using GPS, GLONASS and Galileo

THESIS

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Abstract

A Global Navigation Satellite System (GNSS) refers to a global, satellite-based, all-weather, 24-hour operational radio-navigation and time transfer system that is designed to provide positioning, timing and navigation (PNT) services primarily for military as well as civilian applications. In recent years, a positioning method known as Precise Point Positioning (PPP) has attracted broad interest in scientific research and engineering applications, as it does not require a reference station, reduces labor and equipment costs and simplifies field work. PPP, however, requires a long convergence time of 30 minutes or more in order to ensure centimeter level positioning accuracy, as PPP provides float ambiguity resolution due to the fact that ambiguity terms are not integer numbers because of satellite and receiver un-calibrated hardware delay biases. Although the main issue with the PPP method is its long convergence time, further improvements in the positioning accuracy especially for short observing-session durations is also expected. In this thesis, the impact of combining GPS, GLONASS and Galileo on the solutions of the positioning accuracy and convergence time problems is investigated. For this purpose, specific experiments are conducted using GPS, GLONASS and Galileo measurements collected at Multi-GNSS Experiment (MGEX) stations. First, the performance of the PPP method is analyzed in both static and kinematic modes for the following scenarios: GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo. Secondly, the
positioning solutions obtained with different precise satellite orbit and clock products of three IGS MGEX analysis centers are compared since at the time of writing, there was no combined precise product available. Thirdly, the performance of GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo PPP solutions are demonstrated under different elevation cutoff angles (15°, 25° and 35°) to simulate constrained environments. Furthermore, the performance of GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo PPP solutions from 1-hour, 2-hour and 3-hour observing-session durations are compared to analyze how the observing-session duration affects the positioning accuracy. Lastly, the performance of PPP and long baseline Differential GPS (DGPS) with ionosphere and troposphere elimination are compared through the use of combined GPS/GLONASS/Galileo measurements in order to compare the impact of the multi-GNSS combination on the PPP and DGPS methods. According to the numerical results, it is found that combined GPS/GLONASS improves both the positioning accuracy and convergence time over GPS-only while combined GPS/GLONASS/Galileo may either improve or worsen the positioning accuracy and convergence time over combined GPS/GLONASS, which depends on the number of Galileo satellites used.
Dedicated to my little sister
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<th>Full Form</th>
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<tbody>
<tr>
<td>AS</td>
<td>Authorization Service of BeiDou</td>
</tr>
<tr>
<td>BDT</td>
<td>BeiDou Time</td>
</tr>
<tr>
<td>BKG</td>
<td>Bundesamt fur Kartographie und Geodasie</td>
</tr>
<tr>
<td>CDDIS</td>
<td>Crustal Dynamics Data Information System</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>CNES</td>
<td>Centre National d’Etudes Spatiales</td>
</tr>
<tr>
<td>CODE</td>
<td>Center for Orbit Determination in Europe</td>
</tr>
<tr>
<td>CORS</td>
<td>Continuously Operating Reference Station</td>
</tr>
<tr>
<td>CS</td>
<td>Commercial Service</td>
</tr>
<tr>
<td>DGPS</td>
<td>Differential Global Positioning System</td>
</tr>
<tr>
<td>DOY</td>
<td>Day of Year</td>
</tr>
<tr>
<td>EGNOS</td>
<td>European Geostationary Navigation Overlay Service</td>
</tr>
<tr>
<td>FCB</td>
<td>Fractional Cycle Bias</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>FOC</td>
<td>Full-Operational-Capability</td>
</tr>
<tr>
<td>GDOP</td>
<td>Geometric Dilution of Precision</td>
</tr>
<tr>
<td>GEO</td>
<td>Geostationary Orbit</td>
</tr>
<tr>
<td>GFZ</td>
<td>GeoForschungsZentrum Potsdam</td>
</tr>
<tr>
<td></td>
<td>xxx</td>
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GIOVE
Galileo In-Orbit Validation Experiment

GLONASS
Russian Global Navigation Satellite System

GLONASST
GLONASS Time

GPS
Global Positioning System

GPST
GPS Time

GST
Galileo System Time

GTRF
Galileo Terrestrial Reference Frame

GNSS
Global Navigation Satellite System

HDOP
Horizontal Dilution of Precision

HEO
High Elliptical Orbit

IERS
International Earth Rotation and Reference Systems Service

IGS
International GNSS Service

IGSO
Inclined Geosynchronous Orbit

IGN
Institut Geographique National

IOC
Initial Operational Capability

IOV
In-Orbit Validation

ISB
Inter-System Bias

ITU
International Telecommunication Union

JGS
Japanese satellite navigation Geodetic System

MCS
Master Control Station

MEO
Medium Earth Orbit

xxxix
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>MGEX</td>
<td>Multi-GNSS Experiment</td>
</tr>
<tr>
<td>NAVSTAR</td>
<td>US Navigation System with Timing and Ranging</td>
</tr>
<tr>
<td>OS</td>
<td>Open Service</td>
</tr>
<tr>
<td>PNT</td>
<td>Positioning, Navigation and Timing</td>
</tr>
<tr>
<td>PPP</td>
<td>Precise Point Positioning</td>
</tr>
<tr>
<td>PRS</td>
<td>Public Regulated Service</td>
</tr>
<tr>
<td>PTF</td>
<td>Precise Timing Facility</td>
</tr>
<tr>
<td>QZSS</td>
<td>Quasi Zenith Satellite System</td>
</tr>
<tr>
<td>QZSST</td>
<td>QZSS Time</td>
</tr>
<tr>
<td>SAR</td>
<td>Search and Rescue</td>
</tr>
<tr>
<td>SoL</td>
<td>Safety-of-Life</td>
</tr>
<tr>
<td>STD</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>UPD</td>
<td>Uncalibrated Phase Delay</td>
</tr>
<tr>
<td>UTC</td>
<td>Universal Time Coordinated</td>
</tr>
<tr>
<td>UTC(NTSC)</td>
<td>UTC time maintained by the National Time Service Center, Chinese Academy of Science</td>
</tr>
<tr>
<td>UTC(SU)</td>
<td>UTC time maintained by the National Time Scale of Russian Federation</td>
</tr>
<tr>
<td>UTC(USNO)</td>
<td>UTC time maintained by the US Naval Observatory</td>
</tr>
<tr>
<td>WAAS</td>
<td>Wide Area Augmentation System</td>
</tr>
<tr>
<td>VDOP</td>
<td>Vertical Dilution of Precision</td>
</tr>
<tr>
<td>WGS-84</td>
<td>World Geodetic System-84</td>
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CHAPTER 1: INTRODUCTION

1.1 Background

A Global Navigation Satellite System (GNSS) refers to a global, satellite-based, all-weather, 24-hour operational radio-navigation and time transfer system which is designed to provide positioning, timing and navigation (PNT) services primarily for military as well as civilian applications. GNSS is the collective name for the US Navigation System with Timing and Ranging (NAVSTAR) Global Positioning System (GPS), the Russian Global Navigation Satellite System (GLONASS), European Galileo, and the Chinese BeiDou. At the time of writing, only GPS and GLONASS were fully operational. Galileo was in the Initial Operational Capability (IOC) phase and BeiDou only had regional convergence for the Asia-Pacific region. Table 1 shows at the time of writing and full constellation satellite numbers of these systems. More details about the systems will be provided in Chapter 2.

<table>
<thead>
<tr>
<th>System</th>
<th>Origin</th>
<th>At the time of writing</th>
<th>Full Constellation</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td>USA</td>
<td>30</td>
<td>24</td>
</tr>
<tr>
<td>GLONASS</td>
<td>Russia</td>
<td>26</td>
<td>24</td>
</tr>
<tr>
<td>Galileo</td>
<td>European Union</td>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>BeiDou</td>
<td>China</td>
<td>14</td>
<td>35</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>78</td>
<td>113</td>
</tr>
</tbody>
</table>

Table 1. A summary of at the time of writing and full constellation satellite numbers of Global Navigation Satellite Systems
The basic concept of GNSS positioning is based on measuring the ranges between the satellites and a GNSS receiver to use the multilateration concept which determines a position by using the distances from at least four known points. However, these range measurements are affected by some error sources, such as satellite and receiver clock errors, satellite ephemeris error and atmospheric errors. In order to improve the positioning accuracy, a positioning method should be implemented, that eliminates or at least mitigates these error sources. For this purpose, Differential Global Positioning System (DGPS) method is usually preferred, which mitigates the error sources by differencing the simultaneous measurements of a rover (user) station and a reference station with known coordinates. However, the mitigation of orbital and atmospheric (ionospheric and tropospheric) errors is highly correlated with the baseline length between the reference station and the rover station. Even under normal atmospheric conditions, the baseline length should not be larger than 5-10 km and 50-60 km to eliminate ionospheric and tropospheric errors, respectively. Precise Point Positioning (PPP) is another popular positioning method, which uses publicly available Global Navigation Satellite System (GNSS) precise orbit and clock products provided by International GNSS Service (IGS) to improve the positioning accuracy with only a single GNSS receiver (Zumberge et al., 1997; Kouba and Héroux, 2001).

In comparison with DGPS, PPP reduces the labor and equipment costs and simplifies field work, as it does not require a reference station for its simultaneous measurements. As a result, it has attracted wide interest within scientific research and engineering applications. However, PPP requires a long convergence time of 30 min or
more to ensure centimeter level positioning accuracy (Geng et al., 2011). This time is actually required for the ambiguity terms in the carrier-phase measurements to converge to constant numbers in the estimation filter. In DGPS applied in double difference mode, the ambiguity terms are integer numbers. These ambiguity terms can therefore be identified quickly through the use of their integer nature instead of waiting for their convergence to constant numbers. However, in PPP, the ambiguity terms are non-integer numbers because of the satellite and receiver un-calibrated hardware delay biases which are canceled out in the differencing process (Rizos et al., 2012). For this reason, they cannot be found as is the case in DGPS, as there are infinite possibilities for the non-integer ambiguity terms. Hence, it is required to wait until they converge to constant numbers. The convergence time mainly depends on the geometry of the satellites, because the design matrix used to estimate the unknowns in the estimation filter, changes with the satellite coordinates. Due to the slow velocity of the satellites which is about 4 km/s, the geometry of the satellites changes very slowly with respect to the user on the earth, which actually leads the convergence time to be too long. More details about the ambiguity terms and the estimation filter are given in Chapter 3 and Chapter 4, respectively.

Although the main issue with the PPP method is its long convergence time, further improvements in the positioning accuracy especially for short observing-session durations is also expected.

An integration of GPS with other satellite navigation systems such as GLONASS and Galileo can solve the issues of the long convergence time and positioning accuracy (Bisnath and Gao, 2009; Rizos et al., 2012; Seepersad and Bisnath, 2014). The
improvements in terms of the positioning accuracy and convergence time were demonstrated by integrating GPS with GLONASS (Cai and Gao, 2007; Melgard et al., 2009; Cai and Gao, 2013).

Through the deployment of the new navigation satellite systems, such as Galileo and BeiDou, IGS launched a new project known as Multi-GNSS Experiment (MGEX) to provide precise orbit and clock products for these new constellations (Montenbruck et al., 2014). As a result of this project, the investigation of multi-GNSS PPP with these new constellations has become feasible.

Tegedor el al. (2014) conducted the first investigation in which daily static datasets of four MGEX stations were processed in the kinematic mode for different configurations: GPS-only, GPS/GLONASS, GPS/GLONASS/Galileo and GPS/GLONASS/Galileo/BeiDou. They found that the addition of Galileo and BeiDou does not significantly improve the positioning accuracy and the contribution of BeiDou is notable only in the Asia-Pacific region as a result of the fact that BeiDou currently only has regional convergence.

Li et al. (2015) conducted a study to analyze the performance of multi-GNSS PPP in both static and kinematic modes using daily static datasets for several constellation combinations which do not include GPS/GLONASS/GALILEO combination. They concluded that the addition of BeiDou, GLONASS and Galileo to GPS improves the positioning accuracy and convergence time.

Cai et al. (2015) evaluated the performance of multi-GNSS PPP in both static and kinematic modes. The datasets were processed in the different constellation combinations
which do not include GPS/GLONASS/GALILEO combination. From their work, they concluded that the quad-constellation improves the positioning accuracy and convergence time over single-constellations and dual-constellations in both static and kinematic modes. Furthermore, the contribution of Galileo is not significant.

Rabbou and El-Rabbany (2015) conducted a kinematic experiment to evaluate the performance of the between-satellites single difference multi-GNSS PPP method. The kinematic dataset was processed with different configurations: GPS-only, GPS/GLONASS and GPS/GLONASS/Galileo. They concluded that the combination of GPS and GLONASS improves the positioning accuracy over GPS-only and that the contribution of Galileo is not significant due to the limited number of Galileo satellites.

White and Langley (2015) evaluated the performance of GPS-only, Galileo-only and GPS/Galileo PPP solutions in static mode using simultaneous observations of GPS and Galileo. They stated that while GPS-only can provide centimeter level accuracy, Galileo-only achieves decimeter level positioning accuracy and the integration of Galileo with GPS slightly decreases the positioning accuracy.

In conclusion, among these studies, the performance of the un-differenced multi-GNSS PPP method was not evaluated using the simultaneous measurements of GPS, GLONASS and Galileo satellites in both static and kinematic modes. Therefore, the main focus of this thesis is to make a comprehensive performance investigation of the multi-GNSS PPP method on both static and kinematic modes using simultaneous GPS, GLONASS and Galileo measurements for scientific research and engineering applications, such as crustal deformation and landslide monitoring (Gao and Wang, 2007;
Wang, 2013; Capilla et al., 2016), offshore resource exploration, sea floor mapping (Bisnath and Gao, 2009; Geng et al., 2010; Tegedor et al., 2014) and precision farming (Bisnath and Gao, 2009; van Bree and Tiberius, 2012).

1.2 Thesis Objectives

- Review and present the state-of-the-art functional and stochastic model of the multi-GNSS PPP method.

- Investigate the performance of the multi-GNSS PPP method in static mode to find how the combination of GPS, GLONASS and Galileo affects the positioning performance for scientific research and engineering applications, such as structural health monitoring (Ehiorobo and Irughe, 2012), crustal deformation monitoring (Gao and Wang, 2007) and the establishment of Continuously Operating Reference Station (CORS) network (El-Hattab, 2014).

- Investigate the performance of the multi-GNSS PPP method in kinematic mode to find how the combination of GPS, GLONASS and Galileo affects the positioning performance for scientific research and engineering applications, such as landslide monitoring (Gao and Wang, 2007; Wang, 2013; Capilla et al., 2016), seafloor mapping (Bisnath and Gao, 2009; Geng et al., 2010; Tegedor et al., 2014), airborne mapping (Yuan et al., 2009) and sea floor crustal deformation monitoring (Fujita et al., 2006; Tadokoro et al., 2006).

- Compare the impact of different precise orbit and clock products from three MGEX analysis centers on the positioning accuracy since at the time of writing, these three analysis
centers were providing their own GPS/GLONASS/Galileo precise satellite orbit and clock products and there was no combined precise satellite orbit and clock product available.

- Demonstrate the performance of GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo PPP solutions under different elevation cutoff angles (15°, 25° and 35°) to simulate constrained environments for engineering applications, such as mobile mapping in urban areas (El-Mowafy, 2007) and deformation monitoring and positioning in open-pits mines (Cai et al., 2014).

- Demonstrate the performance of GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo PPP solutions from 1-hour, 2-hour and 3-hour observing-session durations to find how the positioning performance is affected from different observing-session durations since the positioning accuracy is highly related to the observing-session durations (Eckl et al., 2001; Grinter and Janssen, 2012; Li et al., 2015). Rapid-static GNSS applications, such as rapid-static landslide monitoring may benefit from this experiment.

- Investigate the impact of combining GPS, GLONASS and Galileo measurements on PPP and long baseline DGPS with ionosphere and troposphere elimination. PPP is the most commonly used method in remote areas where the separation between a reference station and a rover station could be several hundred kilometers. DGPS is not preferred for several reasons. Firstly, the satellite orbit errors, the ionospheric and tropospheric delays as well as some special errors such as solid earth tide cannot be sufficiently mitigated by differencing techniques. Secondly, DGPS method requires the simultaneous observations of at least one reference station with known coordinates. Lastly, the DGPS method requires
that at least four common satellites be tracked simultaneously by both the reference and receiver stations. For long baselines, even this minimum number of common satellites may not be ensured. The error sources, however, can be mitigated by either estimating them as unknowns or applying correction models. Additional to the precise satellite orbit and clock products, IGS also provides observations of its reference stations with their precise coordinates even in real time online for free. Although the network density of the current IGS reference stations is low, it is still sufficient for the long baseline DGPS. The observations of these reference stations may therefore be used instead of setting up a reference station. Lastly, with the combination of GPS, GLONASS and Galileo, DGPS may solve the issue of the minimum number of common satellites for long baselines. This comparison is completed because in the multi-GNSS combination, PPP needs to handle the satellite and receiver related clock errors and also the hardware delay biases. DGPS on the other hand is capable of eliminating them. Note that the GLONASS hardware delay biases are not completely eliminated because it uses FDMA technique. See Section 2.2, for more details. In this experiment, DGPS solutions are obtained with two different processing settings. In the first setting, the ionospheric and tropospheric errors are estimated as unknowns. Additionally, the precise satellite orbit and clock products are used and some special error corrections are applied. In the second processing settings, the only difference lies in that the ionospheric delay is mitigated by the ionosphere-free linear combination. The possible applications which may benefit from this experiment are airborne mapping (Yuan et al., 2009), offshore resource exploration, sea floor mapping (Bisnath and Gao, 2009; Geng et al., 2010; Tegedor et al., 2014), in which positioning
accuracy is currently sufficient while convergence time is expected to be improved and sea floor crustal deformation monitoring applications (Fujita et al., 2006; Tadokoro et al., 2006), in which both the positioning accuracy and convergence time are expected to be improved.
CHAPTER 2: GLOBAL NAVIGATION SATELLITE SYSTEMS

This chapter introduces the Global Navigation Satellite Systems by providing information about their nominal constellations, current constellations and signals. In addition, their time and coordinate reference systems are provided.

2.1 Global Positioning System (GPS)

GPS is operated and maintained by the U.S. Air Force. The nominal constellation of GPS consists of 24 satellites in six evenly spaced orbital planes with 55° inclination to the equator. Each plane contains four satellites, launched into a near-circular orbit with the altitude of about 20200 km above the earth and the orbital period of 11 hours 58 minutes. This nominal constellation provides global converge with at least four satellites at any time of day.

There are many types of GPS satellites. These are Block I, Block II, Block IIA, Block IIR, Block IIR-M, Block IIF, and Block III satellites. Eleven Block I satellites were launched between 1978 and 1985. The lifetime of these satellites is about 4.5 years and they can only provide the positioning services for 3 or 4 days without any contact with the control segment. They transmit a civil code called as C/A code on the L1 frequency and a military code called as P code on both L1 and L2 frequencies. L1 and L2 frequencies are 1575.42 MHz and 1227.60 MHz, respectively. The first Block II satellite was launched on February 14, 1989. The lifetime of these satellites is about 7.5 years and they can provide
the positioning services for 14 days without any contact with the control segment. The first Block IIA satellite was launched on November 26, 1990 (A denotes advanced). They are able to provide positioning services for 180 days without any contact with the control segment. The first Block IIR satellite was successfully launched on July 23, 1997 (R denotes replacement). The lifetime of these satellites is about 10 years and they can provide the positioning services for about half a year without any contact with the control segment without any degradation in orbit accuracy thanks to the capability to autonomously determine their orbits and generating their own navigation messages. They are able to measure the distances between them and to transmit observations to other satellites or to the control segment. The first Block IIR-M satellite was launched on September 25, 2005 (M denotes modernized). Compared to the previous satellite types, Block IIR-M satellites transmit a new civil code called as L2C code on L2 frequency, which allows for mitigating of the ionospheric effects, and also a new military code called as M code on both L1 and L2 frequencies for improved accuracy, enhanced encryption and anti-jamming capabilities. The GPS modernization process started with Block IIR-M satellite. The first Block IIF satellite was launched on May, 28 2010 (F denotes follow on). Compared to the previous satellite types, Block IIF satellites transmit a new civil code known as L5C on a new L5 frequency, which is 1176.45 MHz protected for the safety-of-life applications. The lifetime of these satellites is about 15 years. Block-III satellites is the future phase of the GPS modernization. Block-III satellites will transmit a fourth civil code called as L1C code on L1 in order to interoperate with international GNSS. Its lifetime will be also longer.
The GPS time (GPST) is on a continuous atomic time scale without the leap second corrections, which increments from a reference epoch starting at midnight on the night of 5th January 1980 and morning of 6th January 1980 in the universal Time Coordinated as maintained by the U.S. Naval Observatory (UTC (USNO)). In other words, the start epoch of GPST was at 00.00.00 on 6th January 1980 in UTC (USNO). The GPST is maintained by the Master Control Station (MCS) (IS-GPS-200G 2012). At the time of writing this thesis, the GPS time was exactly 17 seconds ahead of Universal Time Coordinated (UTC). UTC is widely used for international timekeeping and as the reference for civil time in most countries. It is obtained using data from about 230 atomic clocks in 60 world-wide laboratories (BIPM, 2013). Because of irregularities in the Earth's rate of rotation, an integer leap second adjustment is performed on UTC upon a recommendation by the International Earth Rotation and Reference Systems Service (IERS) based on astronomical observations of the Earth’s rotation. UTC (USNO) is a local UTC. The UTC (USNO) is kept by an ensemble of cesium standards and hydrogen masers with a difference to the UTC in the order of some nanoseconds.

GPS uses the World Geodetic System-84 (WGS-84) reference system for the position of satellite antenna phase centers. Please refer to Hofmann-Wellenhofer et al. (2007: Chapter 9) for more details.

At the time of writing this thesis, there were 30 operational GPS satellites, a combination of old and new satellites. This constellation was made up of 2 Block IIA, 12 Block IIR, 7 Block IIR M and 9 Block IIF GPS satellites.
2.2 GLONASS

GLONASS is operated and maintained by the Russian Military Space Forces. The nominal constellation of GLONASS consists of 24 operational in three evenly spaced orbital planes with 64.8° inclination to the equator. Each plane contains eight satellites launched into a near-circular orbit with the altitude of about 19100 km above the earth and the orbital period of 11 hours, 15 minutes and 44 seconds.

The first prototype satellite of GLONASS was launched in 1982 and the number of the launched satellites reached to 18 in 1985. All these satellites transmit the navigation signals on two frequency bands. Since GLONASS uses the FDMA (Frequency Division Multiple Access) technique, each satellite transmits on the different frequencies by the frequency channel number as follows:

\[
L_1 = 1.602 + 0.5625 \, k \, (\text{MHz}) \\
L_2 = 1.246 + 0.4375 \, k \, (\text{MHz})
\]  

where \(k\) indicates a channel number (\(k = -7, \ldots, +6\))

Between 1985 and 1990, the first generation GLONASS satellites with longer lifetime and improved time and frequency standards were launched. The number of GLONASS satellites decreased to seven in 2001 due to insufficient funds (Zinoviev, 2005). Russian government approved a Federal GLONASS Program to modernize and rebuild the system for the period of 2002-2011 (Gibbons, 2006). The second generation GLONASS satellites called as GLONASS-M where M denotes modernized was launched in 2001. The lifetime of these satellites was increased to about 7.5 years. These satellites transmit a new civil code on L2 frequency band. The first third generation GLONASS satellites known as
GLONASS-K was launched on 26 February 2011 with an increased lifetime. GLONASS-K satellites transmit a new civil code on the new L3 frequency band. GLONASS-K signal follows the Code Division Multiple Access (CDMA) technique in which each satellite transmits a different code on the same frequency similarly to GPS and Galileo. The GLONASS constellation reached its full orbit capacity (FOC) on 8 December 2011.

The GLONASS time (GLONASST) is not on a continuous atomic time scale. It requires the leap second corrections simultaneously with the UTC. It is synchronized with the Universal Time Coordinated as maintained by the National Time Scale of Russian Federation (UTC (SU)) which is a local UTC and kept by an ensemble of cesium standards and hydrogen masers with a difference to the UTC in the order of some nanoseconds. Its difference to the UTC is in the order of some microseconds (Lewandowski et al., 1996). The GLONASS system time is maintained by the GLONASS Central Synchronizer (CS) time. There is a constant offset of three hours between the GLONASS time and UTC (SU) due to the difference between Moscow time and Greenwich Time. In addition to the leap second corrections and the 3 hour offset, the difference between GPST and GLONASST is usually 100 or several 100 ns level.

GLONASS uses the PZ-90.11 reference system for the position of satellite antenna phase centers. Please refer to Hofmann-Wellenhof et al. (2007: Chapter 10) for more details.

At the time of writing this thesis, there were 26 operational GLONASS satellites, a combination of old and new satellites. This constellation was made up of 24 GLONASS-M and 2 GLONASS-K satellites.
2.3  Galileo

Galileo is operated and maintained by the European Space Agency. The nominal constellation of Galileo consists of 27 operational and 3 spare satellites in three evenly spaced orbital planes with 56° inclination to the equator. Each plane contains ten satellites launched into a near-circular orbit with the altitude of about 23222 km above the earth and the orbital period of 14 hours 4 minutes and 45 seconds. This nominal constellation provides global converge with at least six satellites in view from anywhere (almost) on the Earth at any time of day.

Galileo system development plan can be divided into three phases which are Galileo In-Orbit Validation Experiment (GIOVE) phase, Galileo In-Orbit Validation (IOV) phase and Galileo Full-Operational-Capability (FOC) phase. GIOVE phase has been completed with two experimental satellites, GIOVE-A and GIOVE-B, were launched on 28 December 2005 and on 27 April 2008, respectively. The purposes of launching these satellites were to secure the frequencies with the International Telecommunication Union (ITU) and to validate the technologies used in the nominal constellation. IOV phase has been completed with four IOV satellites were launched as pairs on 21 October 2011 and 12 October 2012. The purpose of this phase was to make extensive orbit and control segment tests. Now, Galileo system is in the FOC phase in which the nominal Galileo constellation will be provided by 2019-2020. At the time of writing this thesis, the total number of the launched FOC satellites was four. The first two FOC satellites were launched on 22 August 2014, but unfortunately to the wrong orbits. The next two FOC satellites were launched on 27 March 2015.
IOV and FOC satellites are fully representative of each other. In other words, they are same type of satellites although their roles are different in the development process of Galileo system. They transmit the navigation signals on E1 (1575.42 MHz), E5a (1176.45 MHz), E5b (1207.14 MHz) and E6 (1278.75 MHz) frequencies. Galileo system provides Open Service (OS), Public Regulated Service (PRS), Commercial Service (CS), Search and Rescue (SAR) and Safety-of-Life (SoL) services using the navigation signals on these bands. OS is provided on E1 (1575.42 MHz), E5a (1176.45 MHz) and E5b (1207.14 MHz) frequencies.

Galileo System Time (GST) is on a continuous atomic time scale without the leap second corrections, which increments from a reference epoch starting at midnight on the night of 21st August 1999 and morning of 22nd August 1999 in UTC. It is maintained by the Precise Timing Facility (PTF) at the Galileo Control Centre in Italy (ESA 2013b, Inside GNSS 2013). At the start epoch, the GST was ahead of UTC by 13 seconds (Galileo ICD 2010). The GST is aligned to GPST except for the 1024 weeks difference of the time system origin and a small time offset.

Galileo uses the Galileo terrestrial reference frame (GTRF) for the position of satellite antenna phase centers. Please refer to Hofmann-Wellenhof et al. (2007: Chapter 11) for more details.

At the time of writing this thesis, while there were 4 IOV and 4 FOC total 8 satellites in the orbit, only 4 IOV satellites were transmitting the navigation signals, the 4 FOC satellites were under commissioning.
2.4 BeiDou

BeiDou is developed by the Chinese Academy of Space Technology for primarily the military missions. The nominal constellation of Galileo consists of 35 satellites including 5 Geostationary (GEO) satellites, 3 Inclined Geosynchronous Orbit (IGSO) satellites and 27 medium orbit (MEO) satellites.

The GEO satellites are positioned at 58.75° E, 80° E, 110.5° E, 140° E and 160° E respectively at an altitude of 35,786 km. The IGSO satellites are located in orbit at an altitude of 35,786 km at an inclination of 55°. The MEO BeiDou satellite orbits are at an altitude of 21,528 km with an inclination of 45° and have an orbital period of 12 hours and 53 minutes (Leick et al., 2015).

Frequencies for BeiDou are allocated in three bands, which are 1575.42 MHz (B1), 1191.795 MHz (B2) and 1268.52 MHz (B3). Both Open Service (OS) and Authorization Service (AS) are provided in the B1 band, while only OS in the B2 band and AS in the B3 band.

BeiDou Time (BDT) which is on a continuous atomic time scale without leap second corrections. The BDT is related to the Universal Time Coordinated through UTC (NTSC) which is a UTC time maintained by National Time Service Center, China Academy of Science. The start epoch of BDT was at 00:00:00 on 1st January 2006 UTC (NTSC).

BeiDou uses the China Geodetic Coordinate System 2000 (CGCS2000) for the position of satellite antenna phase centers.
At the time of writing this thesis, there were 5 GEO, 5 IGSO and 4 MEO operational satellites with the regional coverage only in the Asia-Pacific region.

2.5 QZSS

The Quasi Zenith Satellite system (QZSS) is a regional navigation satellite system which covers regions in East Asia and Oceania centering on Japan. The plan is to place the satellites in High Elliptical Orbit (HEO), which helps to overcome the objects (building) interception for the satellites signals. The system also improves positioning accuracy by transmitting signals L1C/A, L1C, L2C and L5 that are equivalent to modernized GPS signals. QZSST (QZSS Time) is aligned to GPST. It has the same origin as GPST and the same definition of one-second rate of GPST (Takasu, 2013). QZSS uses the Japanese satellite navigation Geodetic System (JGS) for the position of satellite antenna phase centers. At the time of writing this thesis, only one QZSS satellite was operational.

There are also satellite based augmentation systems (SBAS) such as the Wide Area Augmentation System (WAAS), the European Geostationary Navigation Overlay Service (EGNOS) and GPS Aided Geo Augmentation Navigation (GAGAN) system implemented by the Indian government. They are used to maintain high positioning accuracy by providing additional information such as an ephemeris correction and localized ionospheric delay information. This additional information is calculated using the GNSS satellite data collected the ground stations. Then, the correction data is broadcast to users through the three geostationary satellite communication links.

WAAS consists of two geostationary satellites and a network of 38 ground stations across the US. EGNOS infrastructure comprises a ground network of 44 station in Europe
and 3 geostationary satellites. The signal coverage area includes most European countries. GAGAN consists of three geostationary satellites and a network of 15 ground stations across India. These three satellite based augmentation systems provide GPS signal corrections.
CHAPTER 3: GPS OBSERVABLES AND ERRORS

The basic concept of GNSS positioning is based on measuring the ranges between the satellites and a GNSS receiver and apply multilateration concept which is determining a position by knowing the distances from at least four known points. However, these range measurements are affected by some error sources, such as the satellite and receiver clock errors, satellite ephemeris error and atmospheric errors. This chapter presents these range measurements and error sources with the explanations of their mitigation techniques.

3.1 GPS Measurements and Observables

There are two type of range measurements: pseudo-range (code) and carrier-phase measurements. The pseudo-range is the distance between the antenna phase centers of the satellite at the signal emission time and the receiver at the signal reception time. It is determined by multiplying the signal traveling time with the speed of light in a vacuum. The signal travelling time is the difference between the reception time at the receiver and the emission time at the satellite. Due to the fact that the receiver and satellite clocks are not perfectly synchronized to the GNSS system time, the pseudo-range measurement contains satellite and receiver clock offset errors. The pseudo-range measurement for GPS can be formulated as follows:
\[ P_i^G = c\left[ (t_{r(G)} + dt_G) - (t^{s(G)} + dT^G) \right] \]
\[ = c(t_{r(G)} - t^{s(G)}) + c(dt_G - dT_G) \]
\[ = \rho^G + c(dt_G - dT_G) \]

where

\( G \) - Refers to GPS

\( i \) - Refers to frequency \( (i = 1, 2, 3) \)

\( c \) - Speed of light in a vacuum

\( t_{r(G)} \) - True signal reception time in GPS system time

\( t^{s(G)} \) - True signal emission time in GPS system time

\( dt_G \) - Receiver clock offset from GPS system time

\( dT^G \) - Satellite clock offset from GPS system time

\( \rho^G \) - True geometric range between the antenna phase centers of satellite and receiver

\[ \rho^G = \sqrt{(X^G - X_r)^2 + (Y^G - Y_r)^2 + (Z^G - Z_r)^2} \]

\( X^G, Y^G, Z^G \) - Satellite coordinates

\( X_r, Y_r, Z_r \) - Receiver coordinates

In addition, other errors such as satellite ephemeris error, ionospheric and tropospheric errors, sagnac effect, multipath, etc. should be taken into account. The full mathematical expression for GPS pseudo-range measurement on \( L_i \) frequency is as follows:
\[ P^G_i = \rho^G + c[dt^G - dT^G] + d\rho^G + \frac{I^G}{f^2_i} + T^G + cb^G_{r, \ p_i} - cb^G_{s, \ p_i} + M^G_{p_i} + \varepsilon^G_{p_i} + \delta^\text{rel}_{sag} \] (4)

where

- \( P^G_i \) - GPS pseudo-range measurement on \( L_i \) frequency
- \( d\rho^G \) - Orbital error of GPS satellites
- \( \frac{I^G}{f^2_i} \) - Ionospheric delay between GPS satellites and receiver on \( L_i \)
- \( T^G \) - Tropospheric delay between GPS satellites and receiver
- \( b^G_{r, \ p_i} \) - GPS receiver code hardware delay bias on \( L_i \)
- \( b^G_{s, \ p_i} \) - GPS satellite code hardware delay bias on \( L_i \)
- \( M^G_{p_i} \) - Multipath on GPS pseudo-range measurements on \( L_i \)
- \( \varepsilon^G_{p_i} \) - Measurement noise on GPS pseudo-range measurements on \( L_i \)
- \( \delta^\text{rel}_{sag} \) - Sagnac effect correction applied to receiver clock in meter

The precision of the pseudo-range measurements has been about 1\% of the chip length of a code. Therefore, the precisions of the pseudo-range measurements with C/A code and P code are 3 m and 0.3 m, respectively (Hofmann et al., 2001).

The carrier-phase measurement is the phase difference between the signal’s replica generated by the receiver and the signal received from the satellite at the instant of the measurement (Misra and Enge, 2006). The carrier-phase measurement is expressed as the sum of the fractional carrier-phase recorded by the receiver and an unknown integer number of phase cycles at the starting epoch between the satellite and the receiver. The unknown integer number of phase cycles is also known as an integer ambiguity and exists
because the receiver has no way of knowing when the carrier wave left the satellite. The integer ambiguity will remain constant for a satellite, as tracking of that satellite is continued without a loss of lock (Hofmann et al., 2001).

A loss of lock or a phase cycle slip will introduce a new unknown ambiguity. A cycle slip is a sudden jump in the carrier-phase observable, generally, by an integer number of cycles due to signal blockage by buildings, trees, severe ionospheric distortion etc. A cycle slip results in all subsequent measurements being offset by a constant integer number of cycles.

Although the availability of the integer ambiguity and the cycle slips, the carrier-phase measurements are more accurate than pseudo-range measurements. The precision of the carrier-phase measurements is in millimeter level since the carrier-phase can be measured with the resolution of 0.01 cycle (Hofmann et al., 2001; Misra and Enge, 2006). The full mathematical expression for GPS carrier-phase measurement on \( L_i \) frequency is as follows:

\[
\Phi_i^G = \rho^G + c \left[ d_{tg} - d_T^G \right] + d\rho^G - \frac{I_i^G}{f_i^2} + T^G + cb_{r, \phi_i} - cb_{s, \phi_i} + M_{\phi_i} + \epsilon_{\phi_i} + \lambda_i \left[ N_i^G + \phi_{r_0, i} - \phi_{s_0, i} \right] + d_{pco} + d_{set} + d_{oc} + d_{rel} + \lambda_i \phi_{pw}
\]

(5)

where

- \( \Phi_i^G \) - GPS carrier-phase measurement on \( L_i \) frequency
- \( b_{r, \phi_i} \) - GPS receiver phase hardware delay bias on \( L_i \)
- \( b_{s, \phi_i} \) - GPS satellite phase hardware delay bias on \( L_i \)
- \( N_i^G \) - GPS integer ambiguity term on \( L_i \)
\( \phi^G_{r_0, i} \) - GPS receiver initial fractional phase bias on \( L_i \)

\( \phi^G_{s_0, i} \) - GPS satellite initial fractional phase bias on \( L_i \)

\( \lambda_i \) - Wavelengths of \( L_i \) signal

\( M^G_{\Phi_i} \) - Multipath on GPS carrier-phase measurements on \( L_i \)

\( \varepsilon^G_{\Phi_i} \) - Measurement noise on GPS carrier-phase measurements on \( L_i \)

\( d^s_{pc\text{o}} \) - Satellite antenna phase center offset correction explained in Section 3.3.1

\( \phi_{pw} \) - Phase wind-up correction explained in Section 3.3.2

\( d_{\text{set}} \) - Solid earth tide correction explained in Section 3.3.3

\( d_{oc} \) - Ocean tide loading correction explained in Section 3.3.4

\( d_{rel} \) - Relativistic effect corrections explained in Section 3.3.5

Note that for the sake of simplicity, the timing group delays (TGD) which are known satellite hardware delay biases are not written separately. They are lumped to the corresponding satellite code and phase hardware delay biases on \( L_i \). TGD corrections are transmitted in the navigation message. In addition, they are canceled out by ionosphere-free linear combination.

The possible sources of the unknown satellite and receiver hardware delay biases on \( L_i \) are delay on the antennas and cables used in the satellites and receivers. Furthermore, the initial phase biases in Equation 5 are also called uncalibrated phase delay (UPD) and fractional cycle bias (FCB).

The following part explains the error sources identified in the above equations and their mitigation strategies in DGPS and PPP methods. The errors are classified two groups: common and special to precise point positioning method.
3.2 Common Error Sources

3.2.1 Satellite Orbit and Clock Errors

The satellite orbits should be known for any arbitrary epoch to be able to obtain Positioning, Navigation and Timing (PNT) information using GNSS (Remondi and Hoffman-Welenhof, 1989). These satellite orbits can be obtained from the broadcast orbits or the precise orbits (Hofmann-Wellenhof et al., 2001).

The broadcast orbits can be computed in a system-related Earth-Centered, Earth-Fixed (ECEF) coordinate system using the orbital parameters transmitted by the satellites in real-time as a part of their navigation messages in the accuracy range of about 1-6 meters while the precise orbits can be obtained in International Terrestrial Reference Frame (ITRF) coordinate system from International GNSS Service (IGS) via Internet free of charge in different latencies and accuracies, but generally in the range of about 5 centimeters.

The satellite orbit error is the difference between its actual and predicted orbits. In DGPS, the satellite orbit errors are significantly mitigated by differencing techniques provided in Section 2.1.3. However, the mitigation success depends on the separation between the reference and rover stations (Misra and Enge, 2006; Sanz Subirana et al., 2013). In PPP, the satellite orbit errors are mitigated using the precise orbits to be able to provide high positioning accuracy. Note that the precise orbits can also be used for DGPS to increase the positioning accuracy.

The satellite clock error refers to the offset between GNSS reference time and satellite clock time due to a lack of synchronization of the satellite clock with respect to
GNSS reference time. The satellite clock error can be mitigated using the broadcast clock corrections in the navigation messages with the precision of 7 nanoseconds, or precise clock corrections with precision of about 0.1 nanoseconds depending on the latency from IGS. Note that 1 nanosecond clock error causes a range error of about 30 cm.

In DGPS, these satellite clock errors are eliminated completely by differencing techniques without depending on the separation between the reference and rover stations (Parkinson et al., 1996; Sanz Subirana et al., 2013). However, in PPP, it should be mitigated using the precise clock products of IGS.

3.2.1.1 IGS Organization

IGS is a volunteer organization consisting of more than 200 individual agencies and institutions that maintain a global network of monitoring stations for GNSS to provide precise satellite orbit and clock products and also data to the scientific and engineering communities (Dow et al., 2009). Until recently, only the GPS and GLONASS precise orbit and clock products could be provided (Rizos et al., 2013). With the rise of satellite navigation systems such as Galileo and BeiDou, IGS has initiated the Multi-GNSS Experiment (MGEX) to provide data and precise satellite orbit and clock products for these new systems, too. For this purpose, a new network of multi-GNSS monitoring stations has been deployed around the world with the contributions of the volunteer agencies and institutions (Montenbruck et al., 2014). Figure 1 shows the MGEX station distribution and supported constellations as of 2014.
At the time of writing this thesis, five analysis centers compute multi-GNSS precise orbit and clock products using the measurements of the MGEX network stations (http://igs.org/mgex/products):

- Centre National d'Études Spatiales (CNES)
- Center for Orbit Determination in Europe (CODE)
- GeoForschungsZentrum Potsdam (GFZ)
- Technische Universität München (TUM)
- Wuhan University

An overview of the products provided by these analysis centers as of August 2015 is provided in Table 2.
Table 2. An overview of MGEX products and their providers (http://igs.org/mgex/products)

<table>
<thead>
<tr>
<th>Analysis Center</th>
<th>Constellations</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNES</td>
<td>GPS/Galileo</td>
<td>Orbit 15 min, Clock 5 min</td>
</tr>
<tr>
<td>CODE</td>
<td>GPS/GLONASS/Galileo/BeiDou/QZSS</td>
<td>15 min, 5 min</td>
</tr>
<tr>
<td>GFZ</td>
<td>GPS/GLONASS/Galileo/BeiDou/QZSS</td>
<td>5 min, 5 min</td>
</tr>
<tr>
<td>TUM</td>
<td>Galileo/QZSS</td>
<td>5 min, 30 sec</td>
</tr>
<tr>
<td>Wuhan University</td>
<td>GPS/GLONASS/Galileo/BeiDou/QZSS</td>
<td>15 min, 5 min</td>
</tr>
</tbody>
</table>

The precise orbit and clock products of these analysis centers are available at the Crustal Dynamics Data Information System (CDDIS) MGEX product archive as well as mirror sites hosted by Institut Géographique National (IGN) and Bundesamt für Kartographie und Geodäsie (BKG).

In this thesis, the precise products of GFZ is used to mitigate satellite orbit and clock errors. In addition, the effects of the precise products of CODE, GFZ and Wuhan University to the positioning accuracy are investigated. An overview of the processing strategies of these analysis centers are given in Table 3 (Steigenberger et al., 2015; Guo et al., 2016; Uhlemann et al., 2015).
In the all experiments, the positioning solutions are obtained at each 30 seconds. However, as can be seen from Table 2, the sampling interval of the clock product is 30 seconds for only GFZ. Because of this, the interpolation of the remaining satellite orbit and clock products is required to obtain precise satellite orbit and clock corrections at the sampling of 30 seconds. RTKLIB interpolates the satellite orbit products with the Lagrange interpolation \((n=10)\) given in Equation 6 (Hofmann-Wellenhof et al., 2001; Guochang, 2007; Takasu, 2013).

\[
f_{10}(t) = \sum_{j=0}^{10} f(t_j)l_j(t)
\]

where

- \(f_{10}(t)\) - 10 degree interpolated polynomial function which can be solved for epoch \(t\)
- \(f(t_j)\) - Known functional values at epochs \(t_j, j = 0, ..., 10\)
- \(l_j(t)\) - Base function of 10 degree related to epoch \(t\). It can be written as follows:
RTKLIB interpolates the satellite clock products linearly with the formula given in Equation 8 (Takasu, 2013). However, these interpolations may decrease the accuracy of the precise products, which depends on the degree of the interpolations.

\[
l_j(t) = \prod_{\substack{i=0 \atop i \neq j}}^{10} \frac{t - t_i}{t_j - t_i}
\]  

(7)

where

\[
dT^G(t) = \frac{(t_{i+1} - t)dT^G(t_i) + (t - t_i)dT^G(t_{i+1})}{t_{i+1} - t_i} \quad (t_i \leq t < t_{i+1})
\]  

(8)

3.2.2 Ionospheric Delay

The ionosphere is the layer of the atmosphere at a height of 50 km to 1000 km above the earth. It contains free electrons due to the sun’s radiation, the solar activity and geomagnetic disturbances. In normal atmospheric conditions, its delay is about 1-3 m at night and about 5-15 m in the mid-afternoon at mid-latitudes. Furthermore, its magnitude increases at the equator (Misra and Enge, 2006). The ionosphere advances the carrier-phase measurements and delays the pseudo-range measurements as they travel through the ionosphere. Therefore, the carrier-phase measurements are measured longer and the pseudo-range measurements are measured shorter than they should be. The ionospheric delay term given in Equations 4 and 5 for pseudo-range and carrier-phase measurements of GPS can be written in Equations 9 and 10, respectively.

\[
dT^G(t_i), dT^G(t_{i+1}) \quad - \text{Known GPS satellite clock correction at epochs } t_i \text{ and } t_{i+1}
\]
\[
\frac{I^G}{f_i^2} = \frac{40.33}{f_i^2} TEC
\] (9)

\[-\frac{I^G}{f_i^2} = -\frac{40.33}{f_i^2} TEC\] (10)

where

\[f_i\] - Frequency on \(L_i\)

\[TEC\] - Total electron content

As can be seen from these equations, the ionospheric delays for the pseudo-range and carrier-phase measurements depend on the total electron content (TEC) along the propagation path of a signal and the frequencies of the measurements due to the dispersive medium property of the ionosphere. TEC depends on the geographic location of the receiver (well behaved in the mid-latitude), time (active at noon and quiet at night) and the solar activities.

In DGPS, for the short baselines (5-10 km), the ionospheric error is mitigated by differencing techniques due to fact that the reference and the rover stations are most likely affected by the same magnitude ionospheric delay. For the long baselines, the ionospheric error can be mitigated through a linear combination of the \(L_1\) and \(L_2\) frequencies, known as an ionosphere-free linear combination, although it destroys the integer nature of the ambiguity terms due to the float number coefficients in the ionosphere-free linear combination, and increases the noise by three times due to the error propagation. In PPP, the ionospheric error is mitigated through the ionosphere-free linear combination for dual frequency GNNS users (Kouba, 2009). The ionosphere-free linear combination for GPS pseudo-range and carrier-phase measurements on \(L_1\) and \(L_2\) are as follows:
3.2.3 Tropospheric Delay

The troposphere is the layer of the atmosphere from the surface of the earth up to 40 km (Hoffmann-Wellenhof et al., 2001). It can be separated into dry (0-40 km) and wet (0-11 km) components. The dry component consists of dry gas molecules and represents about 90% of the total tropospheric error while the wet component consists of the water molecules and represents about 10% of the total tropospheric error. The troposphere is a non-dispersive medium for the frequencies below 15 GHZ and delays both code and carrier-phase measurements. Therefore, it cannot be eliminated by using dual-frequency measurements. The dry tropospheric error can be modeled successfully at zenith direction, but the wet tropospheric error cannot be modelled easily due to the irregular variation of the water molecules over time. Once they are computed at zenith direction, they should be projected to the receiver-satellite direction. This projection is done using a mapping function. The tropospheric error term for GPS which is given in Equations 4 and 5 can be formulated as follows:

\[ T^G = m_{dry}(E^G_r)Z_{dry} + m_{wet}(E^G_r)Z_{wet} \]  

where

- \( m_{dry} \) - Dry mapping function
- \( m_{wet} \) - Wet mapping function
- \( Z_{dry} \) - Zenith dry tropospheric delay

\[ p_{IF}^G = \frac{f_1^2}{f_1^2 - f_2^2} p_1^G - \frac{f_2^2}{f_1^2 - f_2^2} p_2^G \]  

(11)

\[ \Phi_{IF}^G = \frac{f_1^2}{f_1^2 - f_2^2} \Phi_1^G - \frac{f_2^2}{f_1^2 - f_2^2} \Phi_2^G \]  

(12)
The tropospheric error term given in Equations 4, 5 and 13 can also be written as follows to estimate the zenith total tropospheric delay instead of the zenith wet delay in the estimator filter:

\[ T^G = m_{dry}(E^G_r)Z_{dry} + m_{wet}(E^G_r)[Z_{total} - Z_{dry}] \]  \hspace{1cm} (14)

where

\[ Z_{total} \] - Zenith total tropospheric delay which is only unknown parameter in Equation 14. The computation of the remaining parameters will be explained in the following part.

\[ Z_{total} = Z_{dry} + Z_{wet} \]  \hspace{1cm} (15)

In this study, in both PPP and DGPS methods, the zenith dry tropospheric delay is estimated using Saastamoinen model while the zenith total tropospheric delay is estimated as an unknown parameter. The Saastamoinen model derived using gas laws and making some assumptions regarding changes in pressure, temperature and humidity with altitude (Saastamoinen, 1973; Hoffmann-Wellenhof et al., 2001; Misra and Enge, 2006). In order to project them to the receiver-satellite direction, the Neill Mapping Function is used (Neill, 1996). RTKLIB estimates the zenith dry tropospheric delay using Saastamoinen model by assuming zero zenith angle and zero relative humidity as follows (Takasu, 2013):

\[ Z_{dry} = \frac{0.002277}{\cos z} \left\{ p + \left( \frac{1255}{T} + 0.05 \right) e - \tan^2 z \right\} \]  \hspace{1cm} (16)

where
\( z \) - Zenith angle of a satellite [rad]

\[ z = \frac{\pi}{2} - E_r^G \]  \hspace{1cm} (17)

\( p \) - Total surface pressure in [millibar]. It can be calculated with Equation 18.

The only required input is the ellipsoidal height of a station.

\[ p = 1013.25 \times (1 - 2.2557 \times 10^{-5} h)^{5.2568} \]  \hspace{1cm} (18)

\( h \) - Ellipsoidal height of a station in [km]

\( T \) - Absolute temperature of the air for the location of a station [K]. It can be calculated with Equation 19. The only required input is the ellipsoidal height of a station.

\[ T = 15.0 - 6.5 \times 10^{-3} h + 273.15 \]  \hspace{1cm} (19)

\( e \) - Partial pressure for the location of a station [millibar]. It can be calculated with Equation 20. The only required input is the relative humidity for the location of a station.

\[ e = 6.108 \times \exp\left\{\frac{17.15T - 4684.0}{T - 38.45}\right\} \times \frac{h_{rel}}{100} \]  \hspace{1cm} (20)

\( h_{rel} \) - Relative humidity for the location of a station which is assumed to be zero by RTKLIB (Takasu, 2013).

Neill’s wet and dry mapping functions are given in Equations 21 and 22, respectively (Neill, 1996; Leick et al., 2015):
\[ m_{\text{wet}}(E_r^G) = \frac{1 + \frac{a}{b}}{1 + \frac{1 + c}{\sin(E_r^G)}} + \frac{\sin(E_r^G)}{\sin(E_r^G) + \frac{b}{\sin(E_r^G) + c}} \] (21)

where

- \( a, b, c \) - Station’s latitude dependent coefficients (Leick et al., 2015)
- \( E_r^G \) - Elevation angle of a satellite

The coefficients \((a, b, c)\) for the wet mapping function are listed in Table 4 for some specific latitudes, \( \varphi \). If \( \varphi < 15^\circ \), the same coefficients for \( \varphi = 15^\circ \) can be used. If \( \varphi > 75^\circ \), then the same coefficients for \( \varphi = 75^\circ \) can be used. If \( 15^\circ \leq \varphi \leq 75^\circ \), then linear interpolation should be applied (Leick et al., 2015).

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>( a \cdot 10^4 )</th>
<th>( b \cdot 10^3 )</th>
<th>( c \cdot 10^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>5.8021897</td>
<td>1.4275268</td>
<td>4.3472961</td>
</tr>
<tr>
<td>30</td>
<td>5.6794847</td>
<td>1.5138625</td>
<td>4.6729510</td>
</tr>
<tr>
<td>45</td>
<td>5.8118019</td>
<td>1.4572752</td>
<td>4.3908931</td>
</tr>
<tr>
<td>60</td>
<td>5.9727542</td>
<td>1.5007428</td>
<td>4.4626982</td>
</tr>
<tr>
<td>75</td>
<td>6.1641693</td>
<td>1.7599082</td>
<td>5.4736038</td>
</tr>
</tbody>
</table>

Table 4. Coefficients for Niell’s wet mapping function. \( \varphi \) refers to the latitude of a station (Leick et al., 2015)

\[ m_{\text{dry}}(E_r^G) = \frac{1 + \frac{a}{b}}{1 + \frac{1 + c}{\sin(E_r^G)}} + \frac{h}{\sin(E_r^G) + \frac{a}{\sin(E_r^G) + c}} + h \left[ \frac{1}{\sin(E_r^G) + \frac{b}{\sin(E_r^G) + c}} + h \left[ \frac{1 + \frac{b_h}{1 + c_h}}{1 + \frac{a_h}{1 + c_h}} \right] \right] \] (22)

where
\( a, b, c \)  - Station’s latitude dependent coefficients

\( a_h, b_h, c_h \)  - 2.53 \( \times 10^{-5} \), 5.49 \( \times 10^{-3} \) and 1.14 \( \times 10^{-3} \), respectively

\( h \)  - Ellipsoidal height of a station in [km]

In the Neill’s dry mapping function, the coefficients \( (a, b, c) \) should be corrected as follows since their magnitudes depend on time due to the atmospheric changes:

\[
a(\varphi, DOY) = \tilde{a} - a_p \cos(2\pi \frac{DOY - DOY_0}{365.25})
\]

(23)

\[
b(\varphi, DOY) = \tilde{b} - b_p \cos(2\pi \frac{DOY - DOY_0}{365.25})
\]

(24)

\[
c(\varphi, DOY) = \tilde{c} - c_p \cos(2\pi \frac{DOY - DOY_0}{365.25})
\]

(25)

where

\( DOY \)  - Day of year which is day since January 1st of the current year

\( DOY_0 \)  - 28 for the stations at northern hemisphere and 211 for the stations at the southern hemisphere

\( \tilde{a}, \tilde{b}, \tilde{c} \)  - Station’s latitude dependent coefficients

\( a_p, b_p, c_p \)  - Station’s latitude dependent coefficients

The coefficients \( (\tilde{a}, \tilde{b}, \tilde{c}) \) and \( (a_p, b_p, c_p) \) are listed in Table 5 for some specific latitudes. For the remaining latitudes, if \( \varphi < 15^\circ \), the same coefficients for \( \varphi = 15^\circ \) can be used. If \( \varphi > 75^\circ \), then the same coefficients for \( \varphi = 75^\circ \) can be used. If \( 15^\circ \leq \varphi \leq 75^\circ \), then linear interpolation should be applied (Leick et al., 2015).
Table 5. Coefficients for Niell’s dry mapping function. $\varphi$ refers to the latitude of a station (Leick et al., 2015)

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>$\tilde{a} \cdot 10^3$</th>
<th>$\tilde{b} \cdot 10^3$</th>
<th>$\tilde{c} \cdot 10^3$</th>
<th>$a_p \cdot 10^5$</th>
<th>$b_p \cdot 10^5$</th>
<th>$c_p \cdot 10^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1.2769934</td>
<td>2.9153695</td>
<td>62.610505</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>1.2683230</td>
<td>2.9152299</td>
<td>62.837393</td>
<td>1.2709626</td>
<td>2.1414979</td>
<td>9.0128400</td>
</tr>
<tr>
<td>45</td>
<td>1.2465397</td>
<td>2.9288445</td>
<td>63.721774</td>
<td>2.6523662</td>
<td>3.0160779</td>
<td>4.3497037</td>
</tr>
<tr>
<td>60</td>
<td>1.2196049</td>
<td>2.9022565</td>
<td>63.824265</td>
<td>3.4000452</td>
<td>7.2562722</td>
<td>84.795348</td>
</tr>
<tr>
<td>75</td>
<td>1.2045996</td>
<td>2.9024912</td>
<td>64.258455</td>
<td>4.1202191</td>
<td>11.723375</td>
<td>170.37206</td>
</tr>
</tbody>
</table>

The total zenith tropospheric delay can reach about 2.5 m and increases up to about 25 m as the elevation angle decreases to 5°. In addition, the total zenith tropospheric delay decreases as the station height increases due to the fact the path length of a signal through the troposphere gets shorter (Misra and Enge, 2006; Grejner-Brzezinska, 2015).

In DGPS, the tropospheric error under normal atmospheric conditions for the baselines up to 50-60 km, is mitigated by differencing techniques due to fact that the reference and the rover stations are most likely affected by the same magnitude tropospheric delay. For the longer baselines, it may be estimated as an unknown. Furthermore, when there is important height difference between the reference and rover stations, it may also be estimated as an unknown even for the short baselines (Misra and Enge, 2006).

3.2.4 Receiver Clock Error

The receivers are generally equipped with the inexpensive crystal clocks which are not set exactly to GNSS reference time and also they can drift easily over time (Hofmann-
Wellenhof et al., 2001). This offset between the GNSS reference time and the receiver time is called the receiver clock error. In PPP, it can be mitigated by estimating as an unknown parameter, while in DGPS, it can be eliminated by the between satellite differences techniques without depending on the separation between the reference and rover stations (Sanz Subirana et al., 2013). See Chapter 4 for more details on forming DGPS equations.

3.2.5 Multipath and Noise

The multipath is the phenomenon of a signal reaching a GNSS antenna via two or more paths (Misra and Enge, 2006), which is illustrated in Figure 2.

![Figure 2. Illustration of the multipath error. Un-reflected signals (red) are the desired ones. However, reflected signals (purple) being reflected from buildings cause the receiver to calculate the distance between satellite and itself incorrectly since the reflected signals (purple) travel longer than the un-reflected signals (red) which is desired. This unwanted effect is known as multipath (Kumar, 2014)](image)

The maximum multipath error could be approximately half the code chip length and one-quarter of carrier-phase wavelength. Therefore, the magnitude of the multipath...
error could be maximum 150m for C/A, 15 meter for P(y) code and about 5 cm for the carrier-phase measurements (Cai, 2009; Hoffmann-Wellenhof et al., 2001). However, its magnitude is generally about 1-5 m in the pseudo-range measurements and 1-5 cm in carrier-phase measurements (Misra and Enge, 2006).

The effects of multipath cannot be removed through modeling or by differencing techniques. However, its effect may be decreased by using a choke ring antenna which can decrease the multipath error and setting up this antenna away from reflecting objects. In addition, the satellites at low elevations angles could be discarded by setting an elevation cutoff angle.

3.3 Special Error Sources

3.3.1 Satellite Antenna Phase Center Offset

The pseudo-range and carrier-phase measurements refer to the distance between satellite and receiver antenna phase centers. Broadcast orbits are given for these satellite antenna centers, while precise orbits refer to satellites center of mass due to force models used for satellite orbit modeling (Spofford and Remondi, 1999; Kouba, 2009). The offset between the satellite antenna center and the satellite center of mass can be obtained from ANTEX files provided by IGS in the satellite body-fixed coordinate system with respect to the satellite center of mass (Mader, 1999). In order to apply the satellite antenna phase center offset correction to carrier-phase measurements, first, satellite PCO offset vector in the satellite body-fixed coordinate system is converted to the Earth-centered Earth-fixed (ECEF) coordinate system and then it is projected into the satellite-receiver direction as follows (Sanz Subirana et al., 2013; Takasu, 2013):
\[ d_{pc}^s = ( \begin{bmatrix} \hat{e}_x^s & \hat{e}_y^s & \hat{e}_z^s \end{bmatrix}^{-1} \Delta_{APC}^s )^T \hat{e}_s^r \] (26)

where

\[ d_{pc}^s \] - Satellite antenna phase center offset correction applied to carrier-phase measurements in meter

\[ \hat{e}_z^s \] - Unit vector from the satellite center of mass to the earth center

\[ \hat{e}_z^s = - \frac{\overline{r}_{MC}^s}{\| \overline{r}_{MC}^s \|} \] (27)

\[ \overline{r}_{MC}^s \] - Satellite center of mass coordinate vector from precise orbit products in ECEF coordinate system

\[ \hat{e}_y^s \] - Unit vector of cross product of \( \hat{e}_z^s \) and \( \hat{e}_s \)

\[ \hat{e}_y^s = \frac{\hat{e}_z^s \times \hat{e}_s}{\| \hat{e}_z^s \times \hat{e}_s \|} \] (28)

\[ \hat{e}_s \] - Unit vector from the satellite center of mass to the Sun

\[ \hat{e}_s = \frac{\overline{r}_{sun}^s - \overline{r}_{MC}^s}{\| \overline{r}_{sun}^s - \overline{r}_{MC}^s \|} \] (29)

\[ \overline{r}_{sun}^s \] - Sun’s coordinate vector obtained from the planetary ephemeris in ECEF coordinate system

\[ \hat{e}_x^s \] - Completes the right-handed satellite body-fixed system

\[ \hat{e}_x^s = \hat{e}_y^s \times \hat{e}_z^s \] (30)

\[ \Delta_{APC}^s \] - Satellite antenna phase center offset vector in the satellite body-fixed coordinate system with respect to the satellite center of mass obtained from ANTEX file

\[ \hat{e}_s^r \] - Unit vector pointing from satellite to receiver in ECEF coordinate system
\[ \hat{e}_s^r = \frac{\vec{r}^s - \vec{r}_r}{||\vec{r}^s - \vec{r}_r||} \]  

(31)

\( \vec{r}^s \) - Satellite coordinate vector in ECEF coordinate system

\( \vec{r}_r \) - Receiver coordinate vector in ECEF coordinate system

In this study, satellite antenna phase center offset vector in the satellite body-fixed coordinate system with respect to the satellite center of mass is obtained from ANTEX file called igs08_1861.atx.

3.3.2 Phase Wind-Up

The satellite antenna rotates due to the reorientation of its solar panels toward the Sun, while the receiver antenna can rotate due to the dynamic motion of the antenna in the kinematic mode. This rotation of the satellite or receiver antenna around its vertical axis change carrier-phase measurements up to one cycle (one wavelength). This effect is called phase wind-up (Wu et al., 1993; Kouba, 2009; Cai, 2009). In order to determine the orientation of receiver and satellite antennas, Wu et al (1993) represented the receiver and satellite antennas as effective dipole vectors \( \vec{D}_r \) and \( \vec{D}^s \), respectively (Figure 3).
Figure 3. Layout of dipole vectors to compute the wind up correction. \((\hat{e}_x^s, \hat{e}_y^s, \hat{e}_z^s)\) are the unit vectors in the satellite body-fixed coordinate system. \((\hat{e}_x^r, \hat{e}_y^r, \hat{e}_z^r)\) are the receiver unit vectors in local coordinate system along north, east and up directions. \(\hat{e}_s^r\) is the unit vector pointing from satellite to receiver. \(\vec{D}^s\) and \(\vec{D}^r\) are the effective dipole vectors representing satellite and receiver antennas, respectively.

The effective dipole vectors representing satellite and receiver antennas can be found as follows:

\[
\vec{D}^s = \hat{e}_x^s - \hat{e}_s^r (\hat{e}_s^r \cdot \hat{e}_x^s) - \hat{e}_s^r \times \hat{e}_y^s
\]

\[
\vec{D}^r = \hat{e}_x^r - \hat{e}_s^r (\hat{e}_s^r \cdot \hat{e}_x^r) + \hat{e}_s^r \times \hat{e}_y^r
\]  

(32)  

(33)  

where
\( \vec{D}^s, \vec{D}_r \) - Effective dipole vectors representing satellite and receiver antennas, respectively.

\( \hat{e}_s^r \) - Unit vector pointing from satellite to receiver in ECEF coordinate system given in Equation 31

\((\hat{e}_x^s, \hat{e}_y^s, \hat{e}_z^s)\) - Unit vectors in the satellite body-fixed coordinate system explained in Section 3.3.1

\((\hat{e}_x^r, \hat{e}_y^r, \hat{e}_z^r)\) - Local receiver unit vectors in the local coordinate system along north, east and up directions

The phase wind-up correction can be computed as follows:

\[
\phi_{pw} = \text{sign}[\hat{e}_z^r \cdot (\vec{D}^s \times \vec{D}_r)] \arccos \frac{\vec{D}^s \cdot \vec{D}_r}{||\vec{D}^s|| ||\vec{D}_r||}
\]  

(34)

where

\( \phi_{pw} \) - Phase wind-up correction applied to carrier-phase measurements in cycles

\[
\text{sign}[x] = \begin{cases} 
-1 & \text{if } x < 0 \\ 
0 & \text{if } x = 0 \\ 
1 & \text{if } x > 0 
\end{cases}
\]

\( \vec{D}^s, \vec{D}_r \) - Effective dipole vectors representing satellite and receiver antennas, respectively.

\( \hat{e}_s^r \) - Unit vector pointing from satellite to receiver in ECEF coordinate system

In PPP, the phase wind-up effect can degrade the carrier-phase measurements up to one wavelength. Therefore, it should be applied to the carrier-phase measurements. In DGPS, the phase wind-up effect can be negligible when the baseline separation between
reference and rover receiver is up to a few hundred kilometers. However, it can be reach up to 4 cm for a baseline of 4000 km (Kouba, 2009).

3.3.3 Solid Earth Tide

The gravitational forces of the Sun and the Moon cause some variation on the crust of the Earth due to the fact that the earth is not real solid object and acts as an elastic body. This phenomenon is called solid earth tide and cause up to 5 centimeters horizontal site displacement and up to 30 centimeters vertical site displacement (Abdel-salam, 2005; Cai, 2009; Kouba, 2009). The solid earth tide correction is as follows (IERS, 1989):

\[
\Delta \vec{r} = \sum_{j=2}^{3} \frac{GM_j |\vec{r}_r|}{GM |R_j|^3} \left\{ [3l_2(\hat{R}_j \cdot \hat{r}_r)]\hat{R}_j + [3\left(\frac{h_2}{2} - l_2\right)(\hat{R}_j \cdot \hat{r}_r)^2 - \frac{h_2}{2}]\hat{r}_r \right\} + \left[-0.025 \sin \varphi \cos \varphi \sin(\theta_0 + \lambda)\right]\hat{r}_r
\]

(35)

where

- \(\Delta \vec{r}\) - Site displacement vector in ECEF coordinate system
- \(GM\) - Gravitational parameter of the Earth
- \(GM_j\) - Gravitational parameters of the Moon \((j = 2)\) and the Sun \((j = 3)\)
- \(\vec{r}_r, \hat{r}_r\) - ECEF coordinate vector of the station and its unit vector, respectively
- \(\vec{R}_j\) - ECEF coordinate vectors of the Moon \((j = 2)\) and the Sun \((j = 3)\)
- \(\hat{R}_j\) - Unit vectors of ECEF coordinate vectors of the Moon \((j = 2)\) and the Sun \((j = 3)\)
- \(l_2\) - Nominal second degree Love number \(0.6090\)
- \(h_2\) - Nominal Shida dimensionless number \(0.0852\)
- \(\varphi\) - Latitude of the station
\( \lambda \) - Longitude of the station

\( \theta_g \) - Greenwich mean sidereal time

In order to apply the solid earth tide correction in meter to carrier-phase measurements, site displacement vector in ECEF coordinate system given in Equation 35 can be projected into the satellite-receiver direction as follows:

\[
d_{set} = (\Delta \vec{r})^T \hat{e}_{s}^c
\]

\( d_{set} \) - Solid earth tide correction applied to carrier-phase measurements in meter

\( \Delta \vec{r} \) - Site displacement vector in ECEF coordinate system given in Equation 35

\( \hat{e}_{s}^c \) - Unit vector pointing from satellite to receiver in ECEF coordinate system given in Equation 31

For long baseline DGPS, it may be applied to the carrier-phase measurements to increase positioning accuracy while it is negligible for short baselines.

### 3.3.4 Ocean Tide Loading

The gravitational forces of the Moon and Sun cause ocean tides. This ocean tides redistribute the seawater mass which deforms the sea floor and a surface of an adjacent land. This phenomenon is called ocean tide loading.

The ocean tides can be modelled as a harmonic series of the main tidal constituents. The main tidal constituents include the semi-diurnal, the diurnal and the long-period tidal constituents. Each main tidal constituent has its own amplitudes and phases (Petit and Luzum, 2010). The magnitude of the amplitudes and the phases depends on the station’s location. In order to estimate the amplitudes and phases of these main tidal constituents, a global ocean tide model is used such as FES2004 (Kouba, 2009). A global ocean tide model
can be developed using the sea surface height data provided by tide gauges and satellite altimeters. Using the Green’s function obtained by the earth’s elastic models, the station specific amplitudes and phases of these main tidal constituents can be estimated (Yuan, 2010). However, PPP users do not need to make these estimations because the station specific amplitudes and phases of the main tidal constituents (see Equation 35) can be obtained directly using the online ocean loading service of the Onsala Space Observatory at froste.oso.chalmers.se/loading for free of charge (Kouba, 2009). With the obtained station specific amplitudes and phases of these main tidal constituents, the site displacement due to the ocean tide loading can be estimated as follows (Petit and Luzum, 2010):

\[
\Delta \vec{c} = \begin{bmatrix} \Delta c_1 \\ \Delta c_2 \\ \Delta c_3 \end{bmatrix} = \sum_{j=1}^{11} A_{cj} \cos(\chi_j(t) - \Phi_{cj})
\]  

(37)

where

\( \Delta \vec{c} \) - Station displacement vector in local coordinate system (up, west, south)

\( j \) - Represents 11 main tidal constituents \((M_2, S_2, N_2, K_2, K_1, O_1, P_1, Q_1, M_f, M_m, S_{sa})\)

\( A_{cj} \) - Station specific amplitude of tidal constituent \( j \) obtained from the online ocean loading service of the Onsala Space Observatory at froste.oso.chalmers.se/loading for free of charge

\( \Phi_{cj} \) - Station specific phase of tidal constituent \( j \) obtained from the online ocean loading service of the Onsala Space Observatory at froste.oso.chalmers.se/loading for free of charge
\( X_j(t) \) - Astronomical argument of tidal constituent \( j \) at epoch \( t \) calculated as follows:

\[
X_j(t) = \omega_j t + X_j + u_i
\]  

(38)

where

\( \omega_j \) - Angular velocity of tidal constituent \( j \) which can be found in Table 26 of Doodson (1928)

\( X_j \) - Astronomical argument of tidal constituent \( j \) at epoch \( t = 0 \) which can be found in Table 26 of Doodson (1928)

\( u_i \) - Constant depending on tidal constituent \( j \) which can be found in Table 26 of Doodson (1928)

In order to apply the ocean tide loading correction in meter to carrier-phase measurements, the site displacement vector in up, west, south local system given in Equation 37, first, is arranged as east, north, up local system and then it can be projected into the satellite-receiver direction as follows:

\[
d_{oc} = (\Delta \bar{c}, enu)^T \hat{e}_{s,enu}
\]  

(39)

where

\( d_{oc} \) - Ocean tide loading correction applied to carrier-phase measurements in meter

\( \Delta \bar{c}, enu \) - Station displacement vector in (east, north, up) local coordinate system

\[
\Delta \bar{c}, enu = \begin{bmatrix}
-\Delta c_2 \\
-\Delta c_3 \\
\Delta c_1
\end{bmatrix}
\]  

(40)
\( \hat{e}_{s, \text{enu}} \) - Unit vector pointing from satellite to receiver in ENU local coordinate system

\[ \hat{e}_{s, \text{enu}} = E_{rot} \hat{e}_s \]  

(41)

where

\( \hat{e}_s \) - Unit vector pointing from satellite to receiver in ECEF coordinate system given in Equation 31

\( E_{rot} \) - Rotation matrix from ECEF coordinate system to local coordinate system

\[ E_{rot} = \begin{bmatrix}
- \sin \lambda & \cos \lambda & 0 \\
- \sin \varphi \cos \lambda & - \sin \varphi \sin \lambda & \cos \varphi \\
\cos \varphi \cos \lambda & \cos \varphi \sin \lambda & \sin \varphi
\end{bmatrix} \]  

(42)

where

\( \varphi \) - Latitude of the station

\( \lambda \) - Longitude of the station

For both PPP and DGPS methods, the ocean tide loading correction may be neglected for the stations which are far from oceans (Kouba, 2009). Furthermore, in DGPS, it is canceled out, but it may be applied to carrier-phase measurements for long baselines.

3.3.5 Relativistic Effect

The difference of the gravitational field at the satellite and the observing site and the motion of the satellite influence the frequency of the satellite clock according to the general and special relativity. According to the general relativity theory, clocks at high altitudes above the Earth run faster than clocks on its surface due to the less gravity and according to the special relativity theory, clocks moving with high velocity run slower than clocks with smaller relative velocity. The relativistic effect on the satellite clock can be
divided as a constant effect and a periodic effect which varies with the satellite position (Ashby, 2003). The constant effect is corrected by the satellite clock manufacturers by changing the nominal frequency of satellite clock. As an example, the GPS nominal frequency of satellite clock, 10.23, is changed to 10.229999955 MHz due to the fact that the satellite clocks run 38 $\mu$s $\text{day}^{-1}$ faster than on the ground. Note that the frequency of satellite clock is observed on the earth surface as 10.23 MHz instead of 10.229999955 MHz due to the relativistic effect. While the periodic part is corrected as follows (Ashby, 2003; Hofmann-Wellenhof et al., 2008).

$$\Delta t_{rel} = -2 \frac{\vec{r} \cdot \vec{v}}{c^2}$$

(43)

where

- $\Delta t_{rel}$ - Relativistic effect correction applied to the satellite precise clock correction in second
- $\vec{r}$, $\vec{v}$ - Satellite coordinates and velocity vectors derived from precise orbit products in ECEF coordinate system
- $c$ - Speed of light in a vacuum

This relativistic effect should be added to the precise satellite clock product $dT_s$ (Kouba, 2009; Takasu, 2013). In DGPS, the relativistic effect is canceled (Zhu and Groten 1988).

Furthermore, in the presence of the Earth gravitational field, the speed of light changes. Therefore, the signal propagation is affected (Ashby, 2003). As a result, the
magnitude given by Equation 44 should be added to the geometric range (Sanz Subirana et al., 2013).  

\[ \Delta \rho_{rel} = 2 \frac{GM}{c^2} \ln \left( \frac{|\vec{r}_s| + |\vec{r}_r| + |\vec{r}_s - \vec{r}_r|}{|\vec{r}_s| + |\vec{r}_r| - |\vec{r}_s - \vec{r}_r|} \right) \]  

(44) 

where  

\( \Delta \rho_{rel} \) - Relativistic effect correction applied to geometric range in meter  

\( G \) - Earth gravitational constant  

\( M \) - Mass of the Earth  

\( \vec{r}_s \) - Satellite coordinate vector in ECEF coordinate system  

\( \vec{r}_r \) - Receiver coordinate vector in ECEF coordinate system  

\( c \) - Speed of light in a vacuum 

The maximum range error is about 18.6 mm for the PPP (Hofmann-Wellenhof et al., 2008). In DGPS, its effect is in mm level (Zhu and Groten 1988). 

In addition, during the signal propagation from the satellite to the receiver on the earth, the earth rotates and this introduce another relativistic effect known as Sagnac Effect that can be formulated by (Su, 2001):  

\[ \delta_{sag}^{rel} = \frac{1}{c} \left( \vec{r}_r - \vec{r}_s \right) \cdot (\omega_e \times \vec{r}_r) \]  

(45) 

where  

\( \delta_{sag}^{rel} \) - Sagnac effect correction applied to receiver clock in meter  

\( \omega_e \) - Earth’s rotation vector  

\( \vec{r}_s \) - Satellite coordinate vector in ECEF coordinate system  

\( \vec{r}_r \) - Receiver coordinate vector in ECEF coordinate system
\( c \) - Speed of light in a vacuum

The Sagnac effect correction should be applied to the receiver clock in the pseudo-range and carrier-phase measurements (Hofmann-Wellenhof et al., 2008).

In addition, a GNSS receiver clock located on the Earth surface is rotating with 0.5 Km/s at the equator. This cause 10 ns receiver clock error (1ns causes 30 cm range error) after 3 hours (Hofmann-Wellenhof et al., 2008). However, this effect is generally corrected by the receiver software or absorbed by the receiver clock offset. Therefore, no correction is needed.

For the sake of simplicity, the relativistic effect corrections given in Equations 43, 44 and 45 can be combined to represent the relativistic effect corrections as only one term as follows:

\[
d_{\text{rel}} = c \Delta t_{\text{rel}} + \Delta \rho_{\text{rel}} + \delta_{\text{sag}}^{\text{rel}}
\]

where

\( d_{\text{rel}} \) - Relativistic effect corrections applied to carrier-phase measurements in meter

\( \Delta t_{\text{rel}} \) - Relativistic effect correction applied to the satellite precise clock correction in second

\( \Delta \rho_{\text{rel}} \) - Relativistic effect correction applied to geometric range in meter

\( \delta_{\text{sag}}^{\text{rel}} \) - Sagnac effect correction applied to the receiver clock offset in the pseudo-range and carrier-phase measurements in meter

\( c \) - Speed of light in a vacuum
Table 6 shows a summary of the magnitudes of the error sources and whether they should be accounted for in PPP and DGPS or not.

<table>
<thead>
<tr>
<th>GNSS Error</th>
<th>Magnitude [m]</th>
<th>PPP</th>
<th>DGPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite orbit error</td>
<td>1-6</td>
<td>✓</td>
<td>✓/x</td>
</tr>
<tr>
<td>Satellite clock error</td>
<td>2</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Ionospheric delay error</td>
<td>1-100</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Tropospheric delay error</td>
<td>2-25</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Multipath error</td>
<td>1-5</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Satellite antenna phase center offset error</td>
<td>&lt;2</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Phase wind-up error</td>
<td>&lt;0.2</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Solid earth tide error</td>
<td>&lt;0.3</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Ocean tide loading error</td>
<td>&lt;0.05</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Relativistic Effect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In satellite clock</td>
<td>&lt;1</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>In geometric range</td>
<td>&lt;0.02</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>In receiver clock (Sagnac Effect)</td>
<td>23</td>
<td>✓</td>
<td>x</td>
</tr>
</tbody>
</table>

Table 6. A summary of the magnitudes of the error sources and whether they should be accounted for in PPP and DGPS or not. Note that satellite antenna phase center offset, phase wind-up, solid earth tide error, ocean tide loading and relativistic effect are applied to the carrier-phase measurements since their magnitudes are smaller than the noise in the pseudo-range measurements except sagnac effect which is applied to the pseudo-range measurements, too.
CHAPTER 4: DIFFERENTIAL GLOBAL POSITIONING SYSTEM AND PRECISE POINT POSITIONING METHODS

This chapter presents the estimator filter, DGPS and PPP methods. First, a brief summary of Kalman Filter is presented since RTKLIB uses Extended Kalman Filter. Secondly, DGPS method is introduced by explaining the differencing techniques. Last, the detailed state-of-art mathematical model of multi-GNSS PPP and its corresponding stochastic model is provided.

4.1 Kalman Filter

Kalman Filter is an optimal recursive estimator that minimizes the mean square error of the estimated parameters using a priori knowledge about system and measurement models and also their corresponding stochastic models (Gelb, 1976; Welch and Bishop, 1995). System and measurement models of discrete-time linear Kalman filter is shown as:

System Model

\[ x_k = F_{k-1} x_{k-1} + w_{k-1} , \quad w_{k-1} \sim N(0, Q_{k-1}) \]  \hfill (47)

Measurement Model

\[ z_k = H_k x_k + v_k , \quad v_k \sim N(0, R_k) \]  \hfill (48)

where

\[ x_k \]  - State vector at epoch k

\[ F_{k-1} \]  - Transition matrix at epoch k-1
\[ x_{k-1} \] - State vector at epoch \( k-1 \)

\[ w_{k-1} \] - System noise vector at epoch \( k-1 \)

\[ Q_{k-1} \] - Covariance matrix of system noise vector at epoch \( k-1 \)

\[ z_k \] - Measurement vector at epoch \( k \)

\[ H_k \] - Design matrix at epoch \( k \)

\[ v_k \] - Measurement noise vector at epoch \( k \)

\[ R_k \] - Covariance matrix of measurement noise vector at epoch \( k \)

The random noise vectors \( w \) and \( v \) are assumed to be distributed according to the normal (or Gaussian) distribution with known statistics and they are independent from each other.

Kalman Filter consists of two major steps. The first one is the prediction, and the second one is the update. In the prediction step, the forward state vector and its covariance matrix are predicted using the system model and the current updated state vector and its covariance matrix. In the update step, these predicted state vector and its covariance matrix are updated with the new measurements.

**Prediction Step**

\[
\hat{x}_k(-) = F_{k-1} \hat{x}_{k-1}(+) \quad (49)
\]

\[
P_k(-) = F_{k-1}P_{k-1}(+)F_{k-1}^T + Q_{k-1} \quad (50)
\]

**Update Step**

\[
K_k = P_k(-)H_k^T[H_kP_k(-)H_k^T + R_k]^{-1} \quad (51)
\]

\[
\hat{x}_k(+) = \hat{x}_k(-) + K_k[z_k - H_k\hat{x}_k(-)] \quad (52)
\]

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\[ P_k(+) = [I - K_k H_k] P_k(-) \]  

(53)

where

\( \hat{x}_k(-) \) - Predicted state vector

\( \hat{x}_k(+) \) - Updated state vector

\( P_k(-) \) - Covariance matrix of predicted state vector

\( P_k(+) \) - Covariance matrix of updated state vector

\( K_k \) - Gain matrix

In order to initialize the Kalman filter, the initial values of the state vector \( x_0 \) and its covariance matrix \( P_0 \) should be provided.

### 4.2 Extended Kalman Filter

In this thesis, the system model is linear as follows:

\[ x_k = F_{k-1} x_{k-1} \]  

(54)

where

\( x_k \) - State vector at epoch k

\( x_{k-1} \) - State vector at epoch k-1

\( F_{k-1} \) - Transition matrix at epoch k-1

\[ F_{k-1} = I \]  

(55)

However, the measurement model is nonlinear. Therefore, an extension of the Kalman filter called Extended Kalman filter is used. Extended Kalman filter can be used when the system model and/or the measurement model are non-linear.

The nonlinear measurement model of Extended Kalman Filter is shown as:
\[ z_k = h_k(x_k) + v_k, \quad v_k \sim N(0, R_k) \]  \hspace{1cm} (56)

where

\[ h_k(\bullet) \quad \text{- Nonlinear measurement model function} \]

This nonlinear measurement model function is linearized using the predicted state vector \( \hat{x}_k (-) \).

\[ h_k(\hat{x}_k (-)) = H_k \hat{x}_k (-) \]  \hspace{1cm} (57)

where

\[ H_k \quad \text{- Design matrix at epoch k obtained as taking partial derivative of nonlinear measurement model function } h_k(\bullet) \text{ with respect to predicted state vector } \hat{x}_k (-). \]

It can be shown as follows:

\[ H_k = \left. \frac{\partial h_k(x)}{\partial x} \right|_{x=\hat{x}_k(-)} \]  \hspace{1cm} (58)

4.3 **Differential Global Positioning System**

Differential Global Positioning System (DGPS) approach is used to eliminate or at least mitigate the errors in measurements. In addition to the measurements of a rover station, DGPS requires the simultaneous measurements of at least one reference station with known coordinates. These simultaneous measurements are used to form difference of the measurements, which reduces or eliminates most of the common errors in the measurements at both stations forming a baseline, such as orbital error, satellite clock error, receiver clock error, and atmospheric errors. It is important to note that the multipath and receiver noise errors are not eliminated by DGPS.
Two most commonly used differencing methods are discussed in the following section, which are single and double differencing. Note that these equations are shown on \( L_1 \) frequency since differencing in additional frequencies is performed in the same manner. For more details about the DGPS method, refer to Teunissen and Kleusberg (1997) and Takasu (2013).

### 4.3.1 Single Difference

A single difference is obtained by differencing the simultaneous measurements of the rover and the reference station from one satellite. The concept of the single difference is illustrated in Figure 4.

![Figure 4. Concept of the single difference between receivers i and j and satellite k (Richardson, 2015)](image)

The observation equations for pseudo-range and carrier-phase measurements between receivers \( i \) and \( j \) and satellite \( k \) on \( L_1 \) frequency as follows:
\[ P_{i,1}^k = \rho_i^k + c[dt_i - dT^k] + d\rho_i^k + \frac{l_i^k}{f_1^2} + T_i^k + cb_i, \ p_1 - cb_k, \ p_1 + M_{i,p1}^k + \varepsilon_{i,p1}^k \]
\[ + \delta_{sag_i}^k \]

\[ P_{j,1}^k = \rho_j^k + c[dt_j - dT^k] + d\rho_j^k + \frac{l_j^k}{f_1^2} + T_j^k + cb_j, \ p_1 - cb_k, \ p_1 + M_{j,p1}^k + \varepsilon_{j,p1}^k \]
\[ + \delta_{sag_j}^k \]

\[ \Phi_{i,1}^k = \rho_i^k + c[dt_i - dT^k] + d\rho_i^k - \frac{l_i^k}{f_1^2} + T_i^k + cb_i, \ \phi_1 - cb_k, \ \phi_1 \]
\[ + \lambda_1 \left[ N_{i,1}^k + \varphi_{i0,1} - \varphi_{k0,1} \right] + M_{i,\phi1}^k + \varepsilon_{i,\phi1}^k \]
\[ + d_{pco_i,1}^k + d_{set_i}^k + d_{oc_i}^k + d_{rel_i}^k + \lambda_1 \Phi_{pw_i,1}^k \]

\[ \Phi_{j,1}^k = \rho_j^k + c[dt_j - dT^k] + d\rho_j^k - \frac{l_j^k}{f_1^2} + T_j^k + cb_j, \ \phi_1 - cb_k, \ \phi_1 \]
\[ + \lambda_1 \left[ N_{j,1}^k + \varphi_{j0,1} - \varphi_{k0,1} \right] + M_{j,\phi1}^k + \varepsilon_{j,\phi1}^k \]
\[ + d_{pco_j,1}^k + d_{set_j}^k + d_{oc_j}^k + d_{rel_j}^k + \lambda_1 \Phi_{pw_j,1}^k \]

The terms in Equations (59-62) were defined and explained in Equations (2-5). On \( L_1 \) frequency, single-differenced pseudo-range and carrier-phase measurements of the receivers \( i \) and \( j \) and satellite \( k \) can be expressed with Equations 63 and 68, respectively.

\[ P_{ij,1}^k = P_{j,1}^k - P_{i,1}^k \]

\[ P_{ij,1}^k = \rho_{ij}^k + cd_{ij} + \frac{l_{ij}^k}{f_1^2} + T_{ij}^k + cb_{ij}, \ p_1 + M_{ij,p1}^k + \varepsilon_{ij,p1}^k \]
\[ + \delta_{sag_{ij}}^k \]
\[ P_{ij, 1}^k = (\rho_i^k - \rho_j^k) + (c dt_i - c dt_j) + \left( \frac{i_1^k - i_1^j}{f_1^k} \right) + (T_i^k - T_j^k) \]
\[ + (c b_{i, p_1} - c b_{j, p_1}) + (M_i^k - M_j^k) \]
\[ + (\varepsilon_{i, p_1}^k - \varepsilon_{j, p_1}^k) \]
\[ + (\delta_{rel, i}^k - \delta_{rel, j}^k) \]
\[ \Phi_{ij, 1}^k = \Phi_j^k - \Phi_i^k \]
\[ \Phi_{ij, 1}^k = \rho_{ij}^k + c dt_{ij} - \frac{t_{ij}^k}{f_1^k} + T_{ij}^k + c b_{ij} + \lambda_1 [N_{ij, 1}^k + \phi_{ij, 1}] + M_{ij, \Phi_1}^k + \varepsilon_{ij, \Phi_1}^k \]
\[ + d_{pcy}^k + d_{set}^k + d_{ocj}^k + d_{rel}^k + \lambda_1 \phi_{pw}^k \]
\[ \Phi_{ij, 1}^k = (\rho_i^k - \rho_j^k) + (c dt_i - c dt_j) - \left( \frac{i_1^k - i_1^j}{f_1^k} \right) + (T_i^k - T_j^k) \]
\[ + (c b_{i, \Phi_1} - c b_{j, \Phi_1}) + (\lambda_1 [N_{ij, 1}^k - N_{ij, 1}^k + \phi_{ij, 1} - \phi_{j, 1}]) \]
\[ + (M_i^k - M_j^k) + (\varepsilon_{i, \Phi_1}^k - \varepsilon_{j, \Phi_1}^k) \]
\[ + (d_{pcy}^k - d_{pcy}^k) + (d_{set}^k - d_{set}^k) + (d_{ocj}^k - d_{ocj}^k) \]
\[ + (d_{rel}^k - d_{rel}^k) + (\lambda_1 \phi_{pw}^k - \lambda_1 \phi_{pw}^k) \]

Single differencing eliminates satellite clock error, the satellite initial phase bias, the satellite phase and code hardware delay biases and the relativistic effect correction applied to the satellite precise clock correction. The satellite orbit error is also eliminated or at least mitigated as a function of the baseline length between receivers. In addition, the ionospheric and tropospheric delays, satellite phase center offset, solid earth tide, ocean tide loading, relativistic effect and phase wind-up can be significantly reduced over short baselines. Refer to Sections 3.2 and 3.3 for more details.
4.3.2 Double Difference

A double difference is obtained by subtracting two single differences. It requires the simultaneous measurements of the rover and the reference station from two satellite. The concept of the double difference is illustrated in Figure 5.

![Figure 5. Concept of the double difference between receivers i and j and satellite k and l (Richardson, 2015)](image)

According to Equations (63-68), the single differenced equations for satellite, \( k \) and receivers, \( i \) and \( j \), and satellite, \( l \) and receivers, \( i \) and \( j \), are formed on \( L_1 \) frequency as follows:

\[
P_{ij, 1}^k = \rho_{ij}^k + cdt_{ij} + \frac{l_{ij}^k}{f_1^2} + t_{ij}^k + cb_{ij} + p_1 + M_{ij,P1}^k + \epsilon_{ij,P1}^k + \delta_{sag ij}^k
\]

\[
P_{ij, 1}^l = \rho_{ij}^l + cdt_{ij} + \frac{l_{ij}^l}{f_1^2} + t_{ij}^l + cb_{ij} + p_1 + M_{ij,P1}^l + \epsilon_{ij,P1}^l + \delta_{sag ij}^l
\]
\[
\Phi_{ij,1}^k = \rho_{ij}^k + cdt_{ij} - \frac{l_{ij}^k}{f_1^2} + T_{ij}^k + cb_{ij}, \quad \Phi_1 + \lambda_1 \left[ N_{ij,1}^k + \varphi_{ij,0} \right] + M_{ij,\Phi_1} + \varepsilon_{ij,\Phi_1}^k
\]
\[+ d_{sco,ij,1}^k + d_{set,ij}^k + d_{oc,ij}^k + d_{rel,ij}^k + \lambda_1 \phi_{pw,ij,1}^k \tag{71} \]

\[
\Phi_{ij,1}^l = \rho_{ij}^l + cdt_{ij} - \frac{l_{ij}^l}{f_1^2} + T_{ij}^l + cb_{ij}, \quad \Phi_1 + \lambda_1 \left[ N_{ij,1}^l + \varphi_{ij,0} \right] + M_{ij,\Phi_1} + \varepsilon_{ij,\Phi_1}^l
\]
\[+ d_{sco,ij,1}^l + d_{set,ij}^l + d_{oc,ij}^l + d_{rel,ij}^l + \lambda_1 \phi_{pw,ij,1}^l \tag{72} \]

The double-differenced pseudo-range and carrier-phase measurements on \(L_1\) frequency can be formulated by subtracting of the two single differences as follows:

\[
p_{ij,1}^k = p_{ij,1}^k - p_{ij,1}^l = (p_{ij,1}^k - p_{ij,1}^l) - (p_{ij,1}^l - p_{ij,1}^l) \tag{73} \]

\[
p_{ij,1}^l = \rho_{ij}^l + \frac{l_{ij}^l}{f_1^2} + T_{ij}^l + M_{ij,\Phi_1} + \varepsilon_{ij,\Phi_1}^l + \delta_{sag,ij} \tag{74} \]

\[
p_{ij,1}^l = \rho_{ij} - \rho_{ij}^l + \frac{l_{ij}^k}{f_1^2} - \frac{l_{ij}^l}{f_1^2} + (T_{ij}^k - T_{ij}^l) + (M_{ij,\Phi_1} - M_{ij,\Phi_1}^l) \tag{75} \]

\[
+ (\varepsilon_{ij,\Phi_1}^k - \varepsilon_{ij,\Phi_1}^l) + (\delta_{sag,ij} - \delta_{sag,ij}) \]

\[
\Phi_{ij,1}^k = \Phi_{ij,1}^k - \Phi_{ij,1}^l = (\Phi_{ij,1}^k - \Phi_{ij,1}^l) - (\Phi_{ij,1}^l - \Phi_{ij,1}^l) \tag{76} \]

\[
\Phi_{ij,1}^l = \rho_{ij}^l + \frac{l_{ij}^k}{f_1^2} + T_{ij}^l + \lambda_1 N_{ij,1}^l + M_{ij,\Phi_1} + \varepsilon_{ij,\Phi_1}^l \tag{77} \]

\[
+ d_{sco,ij,1}^k + d_{set,ij}^k + d_{oc,ij}^k + d_{rel,ij}^k + \lambda_1 \phi_{pw,ij,1}^k \]

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\[ \Phi_{ij}^{kl} = (\rho_{ij}^k - \rho_{ij}^l) - \left( \frac{t_{ij}^k}{f_i^k} - \frac{t_{ij}^l}{f_i^l} \right) + (T_{ij}^k - T_{ij}^l) + (\lambda_1 N_{ij,1}^k - \lambda_1 N_{ij,1}^l) \]

\[ + (M_{ij,\phi_1}^k - M_{ij,\phi_1}^l) + (\varepsilon_{ij, \phi_1}^k - \varepsilon_{ij, \phi_1}^l) \]

\[ + \left( d_{pco,ij,1}^k - d_{pco,ij,1}^l \right) + \left( d_{set,ij}^k - d_{set,ij}^l \right) + \left( d_{oc,ij}^k - d_{oc,ij}^l \right) \]

\[ + (d_{rel,ij}^k - d_{rel,ij}^l) + (\lambda_1 \phi_{pwl,ij,1}^k - \lambda_1 \phi_{pwl,ij,1}^l) \]

(78)

Double differencing eliminates satellite and receiver clock errors, the satellite and receiver initial phase biases, the satellite and receiver phase and code hardware delay biases, the relativistic effect corrections applied to the satellite precise clock correction and receiver clock. The satellite orbit error, the ionospheric and tropospheric delays, satellite phase center offset, solid earth tide, ocean tide loading, relativistic effect and phase wind-up are also eliminated or at least mitigated as a function of the baseline length between receivers. As seen in Equation 78, the ambiguity term is an integer number. Refer to Sections 3.2 and 3.3 for more details.

4.4 Functional Model of Multi-GNSS Precise Point Positioning

For the sake of convenience, GPS pseudo-range and carrier-phase measurements on \( L_i \) frequency are provided here again in Equations 79 and 80, respectively. Note the tropospheric delay term, \( T_G \), is written as in Equation 14 given and explained in Section 3.2.3.

\[ P_i^G = \rho^G + c [d_{t_G} - d T_G^G] + d \rho^G + \frac{I_G^G}{f_i^G} + m_{dry}(E^G_R)Z_{dry} + m_{wet}(E^G_R)[Z_{total} - Z_{dry}] \]

\[ + c b_{r,pi}^G - c b_{s,pi}^G + \delta_{sag}^r + M_{pi}^G + \varepsilon_{pi}^G \]

(79)
\[ \Phi_i^G = \rho^G + c[d_t G - d T^G] + d \rho^G - \frac{f_i^G}{f_i^2} + m_{dry}(E_r^G)Z_{dry} + m_{wet}(E_r^G)[Z_{total} - Z_{dry}] \]

\[ + cb_{r, \Phi_i} - cb_{s, \Phi_i} + \lambda_i \left[ N_i^G + \phi_{r_o, i} - \phi_{s_o, i} \right] \]

\[ + d_{pco} + d_{set} + d_{oc} + d_{rel} + \lambda_i \phi_{pw} + M_{\Phi_i} + \varepsilon_{\Phi_i} \tag{80} \]

where

\( P_i^G \) - GPS pseudo-range measurement on \( L_i \) frequency

\( \Phi_i^G \) - GPS carrier-phase measurement on \( L_i \) frequency

\( G \) - Refers to GPS

\( i \) - Refers to frequency ( \( i = 1, 2, 3 \) )

\( \rho^G \) - True geometric range between the antenna phase centers of satellite and receiver

\( c \) - Speed of light in a vacuum

\( d_t G \) - Receiver clock offset from GPS system time

\( d T^G \) - Satellite clock offset from GPS system time

\( d \rho^G \) - Orbital error of GPS satellites

\( f_i^G \), \( f_i^2 \) - Ionospheric delay between GPS satellites and receiver on \( L_i \)

\( m_{dry} \) - Dry mapping function

\( m_{wet} \) - Wet mapping function

\( Z_{dry} \) - Zenith dry tropospheric delay

\( Z_{total} \) - Zenith total tropospheric delay

\( E_r^G \) - Elevation angle of a GPS satellite
$b_{r, pi}^G$ - GPS receiver code hardware delay bias on $L_i$

$b_{s, pi}^G$ - GPS satellite code hardware delay bias on $L_i$

$b_{r, \Phi_i}^G$ - GPS receiver phase hardware delay bias on $L_i$

$b_{s, \Phi_i}^G$ - GPS satellite phase hardware delay bias on $L_i$

$M_{P_i}^G$ - Multipath on GPS pseudo-range measurements on $L_i$

$\varepsilon_{P_i}^G$ - Measurement noise on GPS pseudo-range measurements on $L_i$

$M_{\Phi_i}^G$ - Multipath on GPS carrier-phase measurements on $L_i$

$\varepsilon_{\Phi_i}^G$ - Measurement noise on GPS carrier-phase measurements on $L_i$

$\bar{N}_{i}^G$ - GPS integer ambiguity term on $L_i$

$\varphi_{r_{0}, i}^G$ - GPS receiver initial fractional phase bias on $L_i$

$\varphi_{s_{0}, i}^G$ - GPS satellite initial fractional phase bias on $L_i$

$\lambda_i$ - Wavelengths of $L_i$ signal

$d_{pco}^s$ - Satellite antenna phase center offset correction explained in Section 3.3.1

$\phi_{pw}$ - Phase wind-up correction explained in Section 3.3.2

$d_{set}$ - Solid earth tide correction explained in Section 3.3.3

$d_{oc}$ - Ocean tide loading correction explained in Section 3.3.4

$d_{rel}$ - Relativistic effect corrections explained in Section 3.3.5

$\delta^G_{sag}$ - Sagnac effect correction applied to the receiver clock given in Equation 45

Some error terms in Equations 79 and 80 can be eliminated by modeling or a linear combination.
The satellite orbit and clock offset error terms are eliminated using precise satellite orbit and clock products. Furthermore, the satellite code and phase hardware delay bias terms can also be removed with these precise satellite clock products (Defraigne and Baire, 2011; Cai and Gao, 2013; Chen et al., 2015).

The receiver hardware delay biases depend on the frequency. Due to the fact that GPS satellites transmit on the same frequencies, the receiver hardware delay biases are the same for all GPS channels. As a result, the receiver code hardware delay bias term can be lumped into the receiver clock offset error term and then they can be written as a one combined GPS receiver clock error term (Abdel-Salam, 2005; Cai and Gao, 2013).

In order to maintain consistency between code and phase measurements in the estimation of a common receiver clock offset term, the receiver code hardware delay bias term is included in carrier-phase measurements. This receiver code hardware delay bias term and the receiver phase hardware delay bias term can be lumped into the integer ambiguity term since they are very stable over time (Cai and Gao, 2013; Afifi and El-Rabbany, 2015). Furthermore, the receiver and satellite initial phase bias terms can also be lumped to the integer ambiguity term since they are stable, too. As a result, the combination of them can be written as a one non-integer ambiguity term in the unit of meter.

Ionospheric delay error term can be eliminated with the ionosphere-free linear combination as explained in Section 3.2.2.

The tropospheric delay term, $T^G$, is written as Equation 14. As explained in Section 3.2.3, in this equation, the only unknown term is zenith total tropospheric delay, $Z_{total}$. Therefore, the remaining terms can be eliminated as explained in Section 3.2.3.
Satellite antenna phase center offset is eliminated as explained in Section 3.3.1. Phase wind-up effect is eliminated as explained in Section 3.3.2. Site displacement caused solid earth tide is eliminated as explained in Section 3.3.3. Site displacement caused ocean tide loading is eliminated as explained in Section 3.3.4. The relativistic effects are eliminated as explained in Section 3.3.5. Sagnac effect is eliminated as given in Equation 45. The multipath terms can be eliminated for the sake of simplicity since it cannot be removed through modeling. Please see Section 3.2.5 for suggestions to decrease its effect.

Based on these eliminations, the new simplified equations for GPS ionosphere-free pseudo-range and carrier-phase measurements on $L_i$ frequency are as follows:

$$P_{IF}^G = \rho^G + c\tilde{d}t_G + m_{wet}(E^G_r)[Z_{total}] + \epsilon_{PIF}^G$$ (81)

$$\Phi_{IF}^G = \rho^G + c\tilde{d}t_G + m_{wet}(E^G_r)[Z_{total}] + \tilde{N}_{IF}^G + \epsilon_{\Phi IF}^G$$ (82)

where

$P_{IF}^G$ - Ionosphere-free linear combination of GPS pseudo-range measurements provided in Section 3.2.2

$\Phi_{IF}^G$ - Ionosphere-free linear combination of GPS carrier-phase measurements provided in Section 3.2.2

$\rho^G$ - True geometric range between the antenna phase centers of satellite and receiver

$m_{wet}$ - Wet mapping function

$Z_{total}$ - Zenith total tropospheric delay

$E^G_r$ - Elevation angle of a GPS satellite

$\epsilon_{PIF}^G$ - GPS ionosphere-free pseudo-range measurement noise
\( e_{\phi, IF}^G \) - GPS ionosphere-free carrier-phase measurement noise

\( c \tilde{d}_G \) - Combined GPS receiver clock error in meters

\[
c \tilde{d}_G = c (d_t_G + [\alpha b^{r, p_1}_r - \beta b^{r, p_2}_r])
\]

where

\( c \) - Speed of light in a vacuum

\( d_t_G \) - Receiver clock offset from GPS system time

\( b^{r, p_i}_r \) - GPS receiver code hardware delay bias on \( L_i \) in seconds

\( b^{r, IF}_r \) - GPS ionosphere-free differential code bias (DCB) in seconds

\( \alpha = \frac{f_1^2}{f_1^2 - f_2^2} \)

\( \beta = \frac{f_2^2}{f_1^2 - f_2^2} \)

\( \bar{N}_{lIF}^G \) - Combined GPS ionosphere-free non-integer ambiguity term in meter

\[
\bar{N}_{lIF}^G = \alpha \left( \lambda_1 \left[ \bar{N}_1^G + \phi_{r_0, 1}^G - \phi_{s_0, 1}^G \right] \right) - \beta \left( \lambda_2 \left[ \bar{N}_2^G + \phi_{r_0, 2}^G - \phi_{s_0, 2}^G \right] \right) + \alpha (c b^{r, \phi_1}_r - c b^{r, p_1}_r) - \beta (c b^{r, \phi_2}_r - c b^{r, p_2}_r)
\]

where

\( \bar{N}_l^G \) - GPS integer ambiguity term on \( L_l \) in cycles

\( \phi_{r_0, i}^G \) - GPS receiver initial fractional phase bias on \( L_i \) in cycles

\( \phi_{s_0, i}^G \) - GPS satellite initial fractional phase bias on \( L_i \) cycles

\( cb^{r, p_i}_r \) - GPS receiver code hardware delay bias on \( L_i \) in meters

\( cb^{r, \phi_i}_r \) - GPS receiver phase hardware delay bias on \( L_i \) in meters
For Galileo measurements, these equations can be written in the same manner. However, they cannot be written in the same manner for GLONASS measurements. The reason is that the receiver code and phase hardware delay biases which depend on the frequency are not the same for all GLONASS satellites since GLONASS satellites transmit on different frequencies except the new modernized GLONASS-K satellites, with two operational satellites at the time of writing.

Cai and Gao (2013) expressed GLONASS receiver code and phase hardware delay biases as follows:

\[
\begin{align*}
\bar{b}^R_{r, p_i} &= b^R_{r, p_i} + \bar{b}^R_{r, p_i} \\
\bar{b}^R_{r, \phi_i} &= b^R_{r, \phi_i} + \bar{b}^R_{r, \phi_i}
\end{align*}
\]

where the superscript \( R \) refers to GLONASS

- \( b^R_{r, p_i} \) and \( b^R_{r, \phi_i} \) - Average receiver code and phase hardware delay biases, respectively. They are same for all GLONASS satellites.

- \( \bar{b}^R_{r, p_i} \) and \( \bar{b}^R_{r, \phi_i} \) - Satellite-dependent receiver code and phase hardware delay biases, respectively. They are different for each GLONASS satellite.

Note that these satellite-dependent receiver hardware delay biases, \( \bar{b}^R_{r, p_i} \) and \( \bar{b}^R_{r, \phi_i} \), are also known as inter-frequency or inter-channel biases (Cai and Gao, 2013; Kozlov et al. 2000).

As a difference from GPS, only the GLONASS average receiver code hardware delay bias term can be lumped into the GLONASS receiver clock offset term because the GLONASS satellite-dependent receiver code hardware biases are not the same for all GLONASS satellites. This satellite-dependent receiver code hardware bias term is lumped
into the GLONASS measurement noise term. Since its effect shows up in the GLONASS pseudo-range measurement noises, GLONASS measurements are assigned a much smaller weight compared to GPS and Galileo measurements (Geng et al., 2010a; Cai and Gao, 2013; Chen et al., 2015).

In order to maintain consistency between the GLONASS code and phase measurements in the estimation of a common receiver clock offset term, the GLONASS receiver average code hardware delay bias term is included in the carrier-phase measurements. This receiver average code hardware delay bias term and the receiver phase hardware delay bias term can be lumped into the integer ambiguity term since they are very stable over time (Geng et al., 2010a; Dach et al., 2010; Cai and Gao, 2013; Afifi and El-Rabbany, 2015; Chen et al., 2015). Furthermore, the receiver and satellite initial phase bias terms can also be lumped to the integer ambiguity term since they are stable, too. As a result, the combination of them can be written as a one non-integer ambiguity term in the unit of meter.

Based on these facts, the new simplified equations for GLONASS ionosphere-free pseudo-range and carrier-phase measurements can be written as follows:

\[
P_{IF}^R = \rho^R + c\tilde{d}_R + m_{wet}(E_r^R)[Z_{total}] + (c\tilde{d}_r, P_{IF}^R + \varepsilon_{P_{IF}^R})
\]

\[
\Phi_{IF}^R = \rho^R + c\tilde{d}_R + m_{wet}(E_r^R)[Z_{total}] + \tilde{N}_{IF}^R + \varepsilon_{\Phi_{IF}^R}
\]

where the superscript \( R \) refers to GLONASS

\( P_{IF}^R \) - Ionosphere-free linear combination of GLONASS pseudo-range measurements

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\( \Phi_{IF}^R \) - Ionosphere-free linear combination of GLONASS carrier-phase measurements

\( \rho^R \) - True geometric range between the antenna phase centers of GLONASS satellite and receiver

\( m_{wet} \) - Wet mapping function

\( Z_{total} \) - Zenith total tropospheric delay

\( \varepsilon_{PIF}^R \) - GLONASS ionosphere-free pseudo-range measurement noise

\( \varepsilon_{\Phi IF}^R \) - GLONASS ionosphere-free carrier-phase measurement noise

\( c\bar{b}_{r, PIF}^R \) - GLONASS ionosphere-free combination of satellite-dependent receiver code hardware delay biases in meters

\[
c\bar{b}_{r, PIF}^R = c(\alpha_R \bar{b}_{r, P1}^R - \beta_R \bar{b}_{r, P2}^R)
\]

where the superscript \( R \) refers to GLONASS

\( c \) - Speed of light in a vacuum

\[
\alpha_R = \frac{f_{k, 1}^2}{f_{k, 1}^2 - f_{k, 2}^2}
\]

\[
\beta_R = \frac{f_{k, 2}^2}{f_{k, 1}^2 - f_{k, 2}^2}
\]

\( f_{k, 1}, f_{k, 2} \) - Frequencies of a GLONASS satellite \( k \) on \( L_1 \) and \( L_2 \), respectively. Refer to Equation 1 in Section 2.2 to see how to calculate \( f_{k, 1} \) and \( f_{k, 2} \).

\( \bar{b}_{r, P_i}^R \) - GLONASS satellite-dependent receiver code hardware delay biases on \( L_i \). They are different for each GLONASS satellites.

\( c\bar{d}t_R \) - Combined GLONASS receiver clock error in meters
\[ c \ddot{t}_R = c \left( d t_R + \left[ \alpha_R b_{\phi_1, R, 1}^{R, \text{avg}} - \beta_R b_{\phi_2, R, 2}^{R, \text{avg}} \right] \right) \]
\[ = c \left( d t_R + b_{\phi_1, R, \text{PIF}}^{R, \text{avg}} \right) \]

(90)

where the superscript \( R \) refers to GLONASS.

- \( d t_R \) - Receiver clock offset from GLONASS system time
- \( b_{\phi_1, R, \text{avg}}^{R, \text{avg}} \) - GLONASS average receiver code hardware delay biases on \( L_i \)
- \( b_{\phi_2, R, \text{PIF}}^{R, \text{avg}} \) - GLONASS ionosphere-free combination of average receiver code hardware delay biases
- \( \bar{N}_{iF}^R \) - Combined GLONASS ionosphere-free non-integer ambiguity term in meters

\[ \bar{N}_{iF}^R = \alpha_R \left( \lambda_1 \left[ N_1^R + \varphi_{\phi_1, R, 1}^R - \varphi_{\phi_2, R, 1}^R \right] \right) - \beta_R \left( \lambda_2 \left[ N_2^R + \varphi_{\phi_1, R, 2}^R - \varphi_{\phi_2, R, 2}^R \right] \right) \]
\[ + \alpha_R \left( c b_{\phi_1, R, 1}^{R, \text{avg}} - c b_{\phi_2, R, 1}^{R, \text{avg}} \right) - \beta_R \left( c b_{\phi_1, R, 2}^{R, \text{avg}} - c b_{\phi_2, R, 2}^{R, \text{avg}} \right) \]

(91)

where
- \( N_i^R \) - GLONASS integer ambiguity term on \( L_i \) in cycles
- \( \varphi_{\phi_1, R, i}^R \) - GLONASS receiver initial fractional phase bias on \( L_i \) in cycles
- \( \varphi_{\phi_2, R, i}^R \) - GLONASS satellite initial fractional phase bias on \( L_i \) cycles
- \( c b_{\phi_1, R, \text{avg}}^{R, \text{avg}} \) - GLONASS receiver average code hardware delay bias on \( L_i \) in meters
- \( c b_{\phi_1, R, \text{avg}}^{R, \text{avg}} \) - GLONASS receiver phase hardware delay bias on \( L_i \) in meters

Since each GNSS system has different time scale, system time difference parameters with respect to a reference time scale may be introduced to maintain the consistency in the receiver clock offset parameter of the GNSS systems (Cai and Gao,
2013). If the GPS system time is chosen as the reference time scale, GPS, GLONASS and Galileo ionosphere-free pseudo-range and carrier-phase equations can be expressed as follows:

\[
P_{IF}^G = \rho^G + c\tilde{\tau}_G + m_{wet}(E_r^G)[Z_{total}] + \varepsilon_{\Phi IF}^G \tag{92}
\]

\[
\Phi_{IF}^G = \rho^G + c\tilde{\tau}_G + m_{wet}(E_r^G)[Z_{total}] + \tilde{N}_{IF}^G + \varepsilon_{\Phi IF}^G \tag{93}
\]

\[
P_{IF}^E = \rho^E + c\tilde{\tau}_G + c dt_{G-E}^{sys} + m_{wet}(E_r^E)[Z_{total}] + \varepsilon_{\Phi IF}^E \tag{94}
\]

\[
\Phi_{IF}^E = \rho^E + c\tilde{\tau}_G + c dt_{G-E}^{sys} + m_{wet}(E_r^E)[Z_{total}] + \tilde{N}_{IF}^E + \varepsilon_{\Phi IF}^E \tag{95}
\]

\[
P_{IF}^R = \rho^R + c\tilde{\tau}_G + c dt_{G-R}^{sys} + m_{wet}(E_r^R)[Z_{total}] \left( \varepsilon_{\Phi IF}^R + \tilde{b}_r^{P I F} \right) \tag{96}
\]

\[
\Phi_{IF}^R = \rho^R + c\tilde{\tau}_G + c dt_{G-R}^{sys} + m_{wet}(E_r^R)[Z_{total}] + \tilde{N}_{IF}^R + \varepsilon_{\Phi IF}^R \tag{97}
\]

where G refers to GPS, R refers to GLONASS and E refers to Galileo.

\[
dt_{G-E}^{sys} \quad \text{- GPS-Galileo system time difference parameter}
\]

\[
dt_{G-E}^{sys} = (\tilde{\tau}_E - \tilde{\tau}_G)
\]

\[
= [(dt_E + b_{r, P I F}^E) - (dt_G + b_{r, P I F}^G)] \tag{98}
\]

\[
= [(dt_E - dt_G) + (b_{r, P I F}^E - b_{r, P I F}^G)]
\]

where

\[
dt_E \quad \text{- Receiver clock offset from Galileo system time}
\]

\[
dt_G \quad \text{- Receiver clock offset from GPS system time}
\]

\[
b_{r, P I F}^E \quad \text{- Galileo ionosphere-free differential code bias (DCB) in seconds}
\]

\[
b_{r, P I F}^G \quad \text{- GPS ionosphere-free differential code bias (DCB) in seconds}
\]

\[
dt_{G-R}^{sys} \quad \text{- GPS-GLONASS system time difference parameter}
\]
\[ \text{dt}_{\text{G-R}}^{\text{sys}} = (\tilde{dt}_R - \tilde{dt}_G) \]
\[
= [(dt_R + b^{R,\text{avg}}_{r, PIF}) - (dt_G + b^{G, PIF}_{r, PIF})] \\
= [(dt_R - dt_G) + (b^{R,\text{avg}}_{r, PIF} - b^{G}_{r, PIF})]
\]

where

\( dt_R \) - Receiver clock offset from GLONASS system time

\( dt_G \) - Receiver clock offset from GPS system time

\( b^{R,\text{avg}}_{r, PIF} \) - GLONASS ionosphere-free combination of average receiver code hardware delay biases

\( b^{G}_{r, PIF} \) - GPS ionosphere-free differential code bias (DCB) in seconds

The remaining terms in Equations (92-99) were defined and explained in Equations (79-80).

The differences between the receiver clock offsets from different systems \((dt_E - dt_G)\) and \((dt_R - dt_G)\) are corrected by receiver itself using time system difference corrections in GLONASS and Galileo navigation files (Takasu, 2013; Petrovski, 2014).

\((b^{E}_{r, PIF} - b^{G}_{r, PIF})\) and \((b^{R,\text{avg}}_{r, PIF} - b^{G}_{r, PIF})\) terms are known as inter-system bias (ISB) and can be eliminated by using ISB products of IGS (Takasu, 2013).

As a result, the final GPS, GLONASS and Galileo ionosphere-free pseudo-range and carrier-phase equations can be expressed as follows:

\[
P_{IF}^{G} = \rho^{G} + c \tilde{dt}_G + m_{\text{wet}}(E_r^{G})[Z_{total}] + \varepsilon_{PIF}^{G}
\]
\[
\Phi_{IF}^{G} = \rho^{G} + c \tilde{dt}_G + m_{\text{wet}}(E_r^{G})[Z_{total}] + \bar{N}_{IF}^{G} + \varepsilon_{\Phi IF}^{G}
\]
\[
P_{IF}^{E} = \rho^{E} + c \tilde{dt}_G + m_{\text{wet}}(E_r^{E})[Z_{total}] + \varepsilon_{PIF}^{E}
\]

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\[
\Phi_{IF}^E = \rho^E + c\bar{t}_G + m_{wet}(E^E_F[Z_{total}]) + \tilde{N}_{IF}^E + \varepsilon_{\Phi_{IF}}^E \tag{103}
\]

\[
P_{IF}^R = \rho^R + c\bar{t}_G + m_{wet}(E^R_F[Z_{total}]) + (\varepsilon_{\Phi_{IF}}^R + \bar{b}_{IF}^R) \tag{104}
\]

\[
\Phi_{IF}^R = \rho^R + c\bar{t}_G + m_{wet}(E^R_F[Z_{total}]) + \tilde{N}_{IF}^R + \varepsilon_{\Phi_{IF}}^R \tag{105}
\]

The unknown parameters in the above given equations (Equation (100-105)) are three receiver coordinates, one GPS receiver clock offset, one zenith tropospheric total delay parameter and non-integer ambiguity terms whose number equals to the number of observed satellites. If the number of observed satellites is \( n \) at epoch \( t \), the total number of unknowns is \((5+n)\). Therefore, at least 5 satellites are required since each satellite brings 2 measurements which are one ionosphere-free code measurement and one ionosphere-free phase measurement.

According to the information given above, the state vector for RTKLIB is given in Equation 106.

\[
\begin{bmatrix}
\bar{x} = [x, y, z, c\bar{t}_G, Z_{total}, \tilde{N}_{IF}^G, \tilde{N}_{IF}^R, \tilde{N}_{IF}^E, \ldots \ldots]^T_{1x(5+n)}
\end{bmatrix}
\tag{106}
\]

where \( n \) refers to the total number of observed satellites

\( \bar{x} \) - State vector of unknown parameters

\( x, y, z \) - Receiver station coordinates

\( c\bar{t}_G \) - Combined GPS receiver clock error in meters

\( Z_{total} \) - Zenith total tropospheric delay

\( \tilde{N}_{IF}^G, \tilde{N}_{IF}^R, \tilde{N}_{IF}^E \) - Combined ionosphere-free non-integer ambiguity terms of the first GPS, GLONASS and Galileo satellites, respectively (in meters)

An example of a measurement vector and a design matrix explained in Section 4.1 for one satellite from each GNSS system are given in Equations 107 and 108, respectively.
\[
\tilde{z} = [P_{IF}^{G1}, \Phi_{IF}^{G1}, P_{IF}^{R1}, \Phi_{IF}^{R1}, P_{IF}^{E1}, \Phi_{IF}^{E1}, \ldots \ldots ]^T_{1 \times 2n}
\]

where \(n\) refers to the total number of observed satellites

\(\tilde{z}\) - An example of a measurement vector

\(H\) - An example of a design matrix

\(P_{IF}^{G1}, P_{IF}^{R1}, P_{IF}^{E1}\) - Ionosphere-free pseudo-range measurements of the first GPS, GLONASS and Galileo satellites, respectively

\(\Phi_{IF}^{G1}, \Phi_{IF}^{R1}, \Phi_{IF}^{E1}\) - Ionosphere-free carrier-phase measurements of the first GPS, GLONASS and Galileo satellites, respectively

Note that the design matrix given in Equation 108 is the corresponding design matrix of the state and measurement vectors given in Equations 106 and 107, respectively. It is created by taking partial derivatives of the unknown parameters in the measurement model. Refer Section 4.2 for more details.
4.5 Stochastic Model of Multi-GNSS Precise Point Positioning

In order to apply Kalman Filter, the stochastic models of the measurements and parameters should be provided. The stochastic model of measurements is related to the accuracy of the measurements and the stochastic model of the parameters are related to the initial accuracy of the parameters and their variations with time.

4.5.1 Stochastic Model of Measurements

In this section, the covariance matrix of measurement noise vector, $R$, explained in Section 4.1 is defined. It states the statistics of the noise on the measurements and also relationship between them. The pseudo-range and carrier-phase measurements have different accuracies, as explained in Section 3.1. Therefore, they should be weighted according to their accuracies. Besides, satellite signals can travel through different geometric paths. A long path can cause attenuation for the GNSS signals, which makes the measurements noisy. This can be quantified though either the elevation of the satellite, the signal to noise ratio or a combination of both. (Collins and Langley, 1999; Witchayangkoon, 2000).

RTKLIB used in the experiments and discussed next weights the pseudo-range and carrier-phase measurements according to the elevation of the satellite. It is given for the pseudo-range and carrier-phase measurements in Equations 109 and 110, respectively.

$$\sigma_{P_{IF}}^2 = 3\tau \left[ \sigma_p^2 + \frac{\sigma_p^2}{\sin(EL_r^S)} \right]$$  \hspace{1cm} (109) 

$$\sigma_{\Phi_{IF}}^2 = 3\tau \left[ \sigma_\phi^2 + \frac{\sigma_\phi^2}{\sin(EL_r^S)} \right]$$  \hspace{1cm} (110)
where the superscript $s$ refers to satellite, the subscripts $P_{IF}$ and $\Phi_{IF}$ refer to ionosphere-free linear combinations of pseudo-range and carrier-phase measurements, respectively.

\[ \sigma_{P_{IF}}^{s^2} \quad - \text{Variance of ionosphere-free pseudo-range measurement noise} \]

\[ \sigma_{\Phi_{IF}}^{s^2} \quad - \text{Variance of ionosphere-free carrier-phase measurement noise} \]

\[ \sigma_P, \sigma_\Phi \quad - \text{Standard deviation of pseudo-range and carrier-phase measurements noises, respectively} \]

3 - It comes from the ionosphere-free linear combination since the noise in the ionosphere-free linear combination is 3 times larger than the noise in the pseudo-range and carrier-phase measurements (Misra and Enge, 2006).

\[ \tau \quad - \text{The scale factory is included in Equations 109 and 110 because of the existence of inter-frequency biases in GLONASS, which makes GLONASS measurements nosier than GPS and Galileo measurements. It equals 1 for GPS and Galileo and 1.5 for GLONASS} \]

\[ EL_r^s \quad - \text{Elevation angle of satellite} \]

The covariance matrix of measurement noise vector, $R$, can be written as follows:

\[
R = \begin{bmatrix}
\sigma_{P_{IF}}^{s^2} & 0 & 0 \\
0 & \sigma_{\Phi_{IF}}^{s^2} & 0 \\
0 & 0 & \sigma_{P_{IF}}^{s^2} & 0 \\
& & & \ddots \\
0 & 0 & \sigma_{\Phi_{IF}}^{s^2} & 0 \\
& & & & \ddots \\
& & & & & \sigma_{P_{IF}}^{s^2} & 0 \\
& & & & & 0 & \sigma_{\Phi_{IF}}^{s^2}
\end{bmatrix}_{2n\times2n} (111)
\]
where \( n \) refers to the total number of observed satellites

\( R \) - Covariance matrix of measurement noise vector

\( \sigma_{F1F}^{s,1} \), \( \sigma_{F1F}^{s,n} \) - Variance of ionosphere-free pseudo-range measurement noise of satellites number 1 and \( n \), respectively

\( \sigma_{\phi1F}^{s,1} \), \( \sigma_{\phi1F}^{s,n} \) - Variance of ionosphere-free carrier-phase measurement noise of satellites number 1 and \( n \), respectively

### 4.5.2 Stochastic Model of Parameters

In this section, the matrixes \( P \) and \( Q \) explained in Section 4.1 will be defined. \( P \) is the covariance matrix of the state vector. It represents the uncertainties in the estimated state vector. \( Q \) is the covariance matrix of system model noise vector. It represents the variation in the true values of the unknown parameters with time (Paul, 2008).

As stated in Section 4.1, Kalman filter requires the initial values of the state vector \( x_0 \) and its covariance matrix \( P_0 \). After the filter is initialized, the state vector and its covariance matrix for the following epochs will be estimated though the filter.

The initial covariance matrix of the state vector can be written as follows:

\[
P_0 = \text{diag}(\sigma_x^2, \sigma_y^2, \sigma_z^2, \sigma_{\Delta\tau_G}^2, \sigma_{ztot}^2, \sigma_{NIF}^2, \ldots, \sigma_{NIF}^2) \quad (5+n)(5+n)
\]  

(112)

where \( n \) refers to the total number of observed satellites

\( \sigma_x^2, \sigma_y^2, \sigma_z^2 \) - Initial variance of receiver coordinates

\( \sigma_{\Delta\tau_G}^2 \) - Initial variance of combined GPS receiver clock error

\( \sigma_{ztot}^2 \) - Initial variance of zenith total tropospheric delay
\[ \sigma_{N_{IIP}}^2, \sigma_{N_{IIP}}^2 \] - Initial variances of combined ionosphere-free non-integer ambiguity terms of the satellite number 1 and n, respectively.

The diagonal elements of the initial covariance matrix of the state vector are the initial variance of the corresponding elements of the state vector. For example, if the uncertainty of initial zenith tropospheric total delay value is 0.3\( m \), then its corresponding initial variance in the initial covariance matrix of the state vector, \( \sigma_{Z_{total}}^2 \), is 0.3\(^2\) \( m^2 \). The reason of using 0.3\( m \) as an uncertainty of initial zenith tropospheric total delay is that the initial zenith tropospheric total delay is estimated with Saastamoinen model whose uncertainty is assumed to be 0.3\( m \). The initial variances of the remaining parameters given in Equation 50 will be provided in the explanation parts of the experiments.

\[ Q \] is the covariance matrix of system model noise vector. It represents the variation in the true values of the unknown parameters with time (Paul, 2008).

If the variations of an unknown parameter at epoch k-1 and k are assumed to be independent from each other, then the unknown parameter can be modeled as white noise. If it is assumed that variation in the true value of an unknown parameter depends on the time, then it can be modeled as Random Walk in which error grows with time. Furthermore, it can be assumed that there is no variation in the true values of an unknown parameter such as static position coordinates. In this case, unknown parameters are considered as constant and thus no model is required.

In RTKLIB, kinematic positioning coordinates and receiver clock offset error are modeled as white noise. Furthermore, zenith tropospheric total delay parameter is modeled
as Random Walk while static positioning coordinates and non-integer ambiguity parameters are considered as constants (Takasu, 2013).

According to this information for RTKLIB, the covariance matrix of system model noise vector $Q$, which is divided into sub-blocks can be shown as follows:

$$
\begin{bmatrix}
Q_{xyz} \\
Q_{cdtG} \\
\vdots \\
Q_{Z_{total}} \\
\vdots \\
Q_{N_{IF}}
\end{bmatrix}
\begin{bmatrix}
3x3 \\
x1 \\
\vdots \\
x1 \\
\vdots \\
(5+n)x(5+n)
\end{bmatrix}
$$

where $n$ refers to the total number of observed satellites

- Covariance matrix of system model noise vector $Q$

$$Q_{xyz} = \begin{cases}
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} & \text{for static mode} \\
\begin{bmatrix}
q_x & 0 & 0 \\
0 & q_y & 0 \\
0 & 0 & q_z
\end{bmatrix} & \text{for kinematic mode}
\end{cases}$$

(114)

$$Q_{cdtG} = q_{cdtG}$$

(115)

$$Q_{Z_{total}} = q_{Z_{total}} \Delta t$$

(116)

$$Q_{N_{IF}} = 0$$

(117)

$q_x, q_y, q_z$ - Spectral density of receiver kinematic position coordinates in X, Y, Z components, respectively. Spectral density represents the rate of change of a parameter.
\[ q_{\text{cùI}_G} \] - Spectral density of receiver clock offset error

\[ q_{Z_{\text{toto}}t} \] - Spectral density of zenith tropospheric total delay

\[ \Delta t \] - Time increment being time difference between epoch \( k \) and epoch \( k-1 \)

According to the rate of change of a parameter, its spectral density value can be chosen. For example, if it is assumed that the rate of change of the zenith tropospheric total delay as 1 cm/hour then the spectral density of the zenith tropospheric total delay can be formulated as follows (Abdel-Salam, 2005):

\[
q_{\text{trop}} = \left( \frac{1 \text{ cm}}{1 \text{ hour}} \right)^2 = \left( \frac{0.01 \text{ m}}{3600 \text{ sec}} \right)^2 = 7.7 \times 10^{-12} \text{ m}^2/\text{sec} \quad \text{(118)}
\]

The spectral density values of the remaining parameters given in Equation 106 will be provided in the explanation parts of the experiments.
CHAPTER 5: EXPERIMENTS

This chapter presents the experiments objectives, the data selection and the statistical evaluation strategy. Furthermore, it introduces the RTKLIB software used to process datasets and gives data processing settings. Finally, the experiments results are discussed in detail.

5.1 Station and Data Selection

In order to conduct the experiments, 24-hour daily observation datasets collected at four globally distributed MGEX stations on seven consecutive days from August 7, 2015 to August 13, 2015 are downloaded from Crustal Dynamics Data Information System (CDDIS) website (ftp://cddis.gsfc.nasa.gov/pub/gps/data/campaign/mgex/daily/). Due to the absence of the dataset on August 9, 2015 at WIND station, the collected dataset on August 14, 2015 is used. The data sampling rate is 30 seconds. In addition to this general data introduction, the specific data for each experiment is also provided before the experiment results are provided in their related sections. Figure 6 shows the distribution of these selected MGEX stations.
These selected MGEX stations are equipped with dual frequency JAVAD TRE_G3TH DELTA multi-GNSS receivers and JAV_RINGANT_G3T multi-GNSS antennas.

There are several reasons to choose these stations. First reason is to provide global distribution. The second reason is to choose stations which are equipped with the same brand of receivers and antennas. The last reason is to choose specific time window in which there is no significant absence of observed datasets. At the time of writing, there were some problems in the availability of observed datasets since MGEX project was not completed yet.
From each 24-hour daily observation dataset, a 3-hour dataset which contains the maximum average number of Galileo satellites is windowed to analyze the direct effect of Galileo observations. As an example, this process is illustrated in Figure 7 and 8 using the 24-hour daily observation dataset collected at POTS station on August 8, 2015.

Figure 7. The visible satellites for the 24-hour daily observation dataset collected at POTS on August 8, 2015. Green denotes the satellites transmitting data in L1 and L2, yellow denotes the satellites transmitting data in only L1, blue denotes the satellites transmitting data in L1, L2 and L5 and red denotes the satellites transmitting data in L1 and L5. In y axis, G denotes GPS, R denotes GLONASS and E denotes Galileo. The unit of time is GPS Time (HH:MM)
As it can be seen from Figure 7, the 3-hour dataset containing the maximum average number of Galileo satellites is collected between 15:00 and 18:00 GPS Time. Therefore, this session is used for the experiments. Figure 8 shows the visible satellites with 15° elevation cutoff angle for this 3-hour dataset.

Figure 8. The visible satellites for the 3-hour dataset collected at POTS at 15:00-18:00 GPS Time on August 8, 2015. Green denotes the satellites transmitting data in L1 and L2, yellow denotes the satellites transmitting data in only L1, blue denotes the satellites transmitting data in L1, L2 and L5 and red denotes the satellites transmitting data in L1 and L5. In y axis, G denotes GPS, R denotes GLONASS and E denotes Galileo. The unit of time is GPS Time (HH:MM)
However, the average number of Galileo satellites is not always three as in above given example because of the orbital period of Galileo satellites. The orbital period of Galileo satellites is 14 hours 22 minutes, which equals $\frac{10}{17}$ sidereal days (1 sidereal day $\approx 23$ hours 56 minutes). This orbital period is related to the orbital altitude of Galileo satellites according to the Kepler’s 3rd law, which states that the square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit. If the earth did not rotate, the stationary GNSS user would observe the same Galileo satellites at the same points in the sky every $\frac{10}{17}$ sidereal days. However, as a result of the fact that the earth rotates, the stationary GNSS user can observe the same Galileo satellites at the same points in the sky every 10 sidereal days. The time required to observe the same GPS satellites and the same GLONASS satellites is 1 sidereal day and 8 sidereal days, respectively. The explanation of this phenomenon is that their orbital periods are $\frac{1}{2}$ and $\frac{8}{17}$ sidereal days, respectively.

The precise satellite orbit and clock products of the three analysis centers used in this thesis are downloaded from the CDDIS website (ftp://cddis.gsfc.nasa.gov/gnss/products/mgex/). For more details about these analysis centers and their precise satellite orbit and clock products, refer to Section 3.2.1.1.

5.2 Software Introduction

All of the experiments within this thesis are conducted through the use of a well-known open source program package RTKLIB (http://www.rtklib.com/) (Rizos et al., 2012). RTKLIB is designed by Tomoji Takasu from Tokyo University of Marine Science
and Technology. It supports standard and precise positioning algorithms with
GPS, GLONASS, Galileo and QZSS both in real-time and post-processing methods such
as DGPS, RTK and PPP. At the time of writing, it was also the only available open source
software that is able to process combined GPS/GLONASS/Galileo measurements, which
was the main reason for choosing this software to process measurements. In addition,
RTKLIB is recommended by IGS and have been successfully used for a few studies
(Stempfhuber and Buchholz, 2011; Crawford, 2013; Chen, 2015).

The positioning performance of RTKLIB was also validated by comparing its
results with those of the similar studies being Cai and Gao (2013) who investigated the
performance of GPS-only and combined GPS/GLONASS solutions using 3-hour datasets
and Grinter and Janssen (2012) who compared the performance of GPS-only solutions
from different observing-session durations using Canadian Spatial Reference System
(CSRS) PPP tool. It is found that the results obtained with RTKLIB in this thesis are
slightly better than those presented in these two studies.

5.3 Statistical Evaluation Strategy

In this thesis, the positioning solutions were obtained in the Earth-Centered Earth-
Fixed (ECEF) reference system. However, the positioning errors are analyzed in the east,
north and up directions, based on the reference station coordinates from IGS SINEX files
which contain weekly station coordinate solutions and can be downloaded from the CDDIS
website (ftp://cddis.gsfc.nasa.gov/pub/gnss/products/). The precision of these solutions is
less than 2 mm (Altamimi and Collilieux, 2009).
Firstly, the estimated ECEF coordinates of the stations are subtracted from the reference station coordinates to obtain the positioning errors ($\Delta X, \Delta Y, \Delta Z$) in the ECEF reference system. Afterwards, the positioning errors ($\Delta X, \Delta Y, \Delta Z$) in the ECEF reference system are transformed to the positioning errors in the North, East, Up (n, e, u) coordinate system using the transformation matrix in Equation 119, where $\lambda$ and $\phi$ are the reference station’s longitude and latitude, respectively.

$$\begin{bmatrix} n \\ e \\ u \end{bmatrix} = \begin{bmatrix} -\cos \lambda \sin \phi & -\sin \lambda \sin \phi & \cos \phi \\ -\sin \lambda & \cos \lambda & 0 \\ \cos \lambda \cos \phi & \sin \lambda \cos \phi & \sin \phi \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$ (119)

In the experiments, the statistical values of the static and kinematic positioning errors are calculated for each direction, separately. The statistical values of the kinematic positioning errors are calculated using the last one-hour positioning solutions (Cai, 2009). The static positioning solutions are obtained as a single solution (Abdel-salam, 2005; Grinter and Janssen, 2012; White and Langley, 2015).

The Equations 120, 121 and 122 provide the estimation of the mean ($\mu$), the standard deviation (STD) and the root-mean-square (RMS) of the positioning errors in the east direction. For the north and up directions, they can be estimated in the same manner.

The mean ($\mu$) of the positioning errors in the east direction is estimated as follows:

$$\mu = \frac{1}{n} \sum_{i=1}^{n} e_i$$ (120)

where

- $e_i$ - Positioning error in the east direction for the solution $i$
- $n$ - Number of the positioning solutions
The standard deviation (STD) of the positioning errors in the east direction are estimated as follows:

\[
STD = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (e_i - \mu)^2}
\]  

(121)

Root-mean-square (RMS) of the positioning errors in the east direction are estimated as follows:

\[
RMS = \sqrt{\frac{1}{n} \sum_{i=1}^{n} e_i^2}
\]  

(122)

For the PPP method with float ambiguity resolution, the convergence time refers to the time required for each coordinate component to reach a certain level accuracy (Li and Zhang, 2014). This certain level accuracy can be adopted differently by different users. In this study, in order to agree with the related studies (Cai, 2009; Cai and Gao, 2013; Li, 2014; Cai et al., 2015), it is adopted as ±10 cm which is also the standard PPP accuracy (Collins et al., 2010). Therefore, the convergence time, in this study, refers to the number of epochs in which the positioning errors reach ±10 cm and stay within ±10 cm. In addition, the availability, in this study, refers to the total number of epochs with the position solutions whose accuracy is better than ±10 cm. The convergence times and availabilities are analyzed in east, north and up directions, separately.
5.4 Experiments

5.4.1 Experiment 1: Performance of Static Precise Point Positioning

The performance of the static PPP method is analyzed by comparing GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions using a total of 28 three-hour datasets windowed from 24-hour daily datasets collected at four MGEX stations including POTS, WIND, RIO2 and ULAB on seven consecutive days from August 7, 2015 to August 13, 2015. Note that due to the absence of the dataset on August 9, 2015 at WIND station, the dataset on August 14, 2015 at WIND station is used. The distribution of these selected MGEX stations are shown in Figure 6. The session details of these three-hour datasets are listed in Table 6. Note that the datasets are also numbered to identify them easily in the experiments, figures and tables. All observations have a sampling interval of 30 seconds. The default satellite elevation cutoff angle of 15° is maintained (Cai, 2009; Takasu, 2013). The precise satellite orbit and clock products of GFZ are used to mitigate the satellite orbit and clock errors. The precise satellite orbit and clock products are available at a sampling interval of 5-min and 30-sec, respectively. Extended Kalman filter is used for the estimation. Refer to Equation 106 for more details about the parameters. The static receiver position and the float number ambiguity parameters are considered as constants. Therefore, their spectral density values are set to zero. The receiver clock offset error is modelled as white noise with a spectral density setting of $10^4 \text{m}^2/\text{sec}$. The zenith tropospheric total delay (ZTD) is modelled as a Random Walk process with a spectral density setting of $10^{-8} \text{m}^2/\text{sec}$. The initial variances of the static receiver position, the ambiguity parameters, the receiver clock offset error are assumed to be $100^2 \text{m}^2$ while the
initial variance of ZTD is assumed to be $0.3^2 m^2$. The phase and code observation precisions are set to 0.003 m and 0.3 m, respectively. GLONASS observations are down-weighted by the scale factor of 1.5 due to the existence of the inter-frequency biases in the GLONASS measurements (Takasu, 2013). It is also important to note that the special error corrections are also applied to the carrier-phase measurements, including ocean tide loading, solid earth tide, phase wind-up, relativistic effect and satellite antenna phase center offset. The satellite and receiver antenna phase center offsets are corrected using igs08_1861.atx file provided by IGS. The multi-GNSS processing software RTKLIB is used to process the datasets. The setting configurations of RTKLIB are given and explained in Appendix. The positioning solutions are compared with the corresponding IGS weekly solutions in order to find the positioning errors. Refer to Section 5.3, for more details about the statistical evaluation strategy.
<table>
<thead>
<tr>
<th>Date</th>
<th>Start and End Time (GPS Time)</th>
<th>Dataset Number</th>
<th>Average Number of Galileo Satellites</th>
<th>Date</th>
<th>Start and End Time (GPS Time)</th>
<th>Dataset Number</th>
<th>Average Number of Galileo Satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 7, 2015</td>
<td>09.00 – 12.00</td>
<td>1</td>
<td>2</td>
<td>August 7, 2015</td>
<td>09.00 – 12.00</td>
<td>8</td>
<td>2.64</td>
</tr>
<tr>
<td>August 8, 2015</td>
<td>15.00 – 18.00</td>
<td>2</td>
<td>3</td>
<td>August 8, 2015</td>
<td>18.00 – 21.00</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>August 9, 2015</td>
<td>20.00 – 23.00</td>
<td>3</td>
<td>2.84</td>
<td>August 10, 2015</td>
<td>03.00 – 06.00</td>
<td>10</td>
<td>2.52</td>
</tr>
<tr>
<td>August 10, 2015</td>
<td>06.00 – 09.00</td>
<td>4</td>
<td>1.80</td>
<td>August 11, 2015</td>
<td>09.00 – 12.00</td>
<td>11</td>
<td>2.69</td>
</tr>
<tr>
<td>August 11, 2015</td>
<td>12.00 – 15.00</td>
<td>5</td>
<td>2.74</td>
<td>August 12, 2015</td>
<td>21.00 – 24.00</td>
<td>12</td>
<td>2.10</td>
</tr>
<tr>
<td>August 12, 2015</td>
<td>18.00 – 21.00</td>
<td>6</td>
<td>3</td>
<td>August 13, 2015</td>
<td>03.00 – 06.00</td>
<td>13</td>
<td>1.43</td>
</tr>
<tr>
<td>August 13, 2015</td>
<td>06.00 – 09.00</td>
<td>7</td>
<td>1.68</td>
<td>August 14, 2015</td>
<td>06.00 – 09.00</td>
<td>14</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 7. Session details of the datasets collected at POTS, WIND, RIO2 and ULAB stations and used in the static PPP experiment (see Figure 6)

5.4.1.1 Analysis of Positioning Accuracy

Since the average number of Galileo satellites are not same in all datasets, the RMS of the positioning errors for GPS-only, combined GPS/GLONASS and combined
GPS/GLONASS/Galileo solutions for each dataset is demonstrated separately in Figures 9, 10 and 11 for east, north and up directions, respectively.

Figure 9. The RMS of the positioning errors [meters] in the east direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for each dataset. The datasets were collected at POTS, WIND, RIO2 and ULAB stations. See Table 7 for session details.
Figure 10. The RMS of the positioning errors [meters] in the north direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for each dataset. The datasets were collected at POTS, WIND, RIO2 and ULAB stations. See Table 7 for session details.

Figure 11. The RMS of the positioning errors [meters] in the up direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for each dataset. The datasets were collected at POTS, WIND, RIO2 and ULAB stations. See Table 7 for session details.
As seen in Figures 9, 10 and 11, the positioning accuracy in the north direction is better than the positioning accuracy in the east direction. It is related to the orientation of satellite orbits and the motion of satellites which moves to northward and thus changes the satellite geometry faster in the north direction (Blewit, 1989; Rizos, 1997; Cai, 2009).

Furthermore, the positioning accuracy in the north direction is also better than the positioning accuracies in the up direction. The reason is that receivers cannot observe the satellites under the horizon due to the fact that the Earth is not transparent to the GNSS signals, which worsens the satellite geometry in the up direction (Langley, 1999; Grejner-Brzezinska, 2015).

It is also seen that the RMS of the positioning errors for combined GPS/GLONASS/Galileo solutions are not always smaller than those for combined GPS/GLONASS solutions. If the average number of Galileo satellite in a combined GPS/GLONASS/Galileo dataset is more than two, it improves the positioning accuracy or at least provides similar positioning accuracy over combined GPS/GLONASS. For example, the 2nd dataset collected at POTS station on August 8, 2015 has the maximum level of improvement, as this dataset contains three Galileo satellites. However, if the average number of Galileo satellites in a combined GPS/GLONASS/Galileo dataset is equal or less than two, it decreases the positioning accuracy over combined GPS/GLONASS. For example, the 18th dataset collected at RIO2 station on August 10, 2015 has the maximum level of decline, as it contains only one Galileo satellite.

The cause of this inconsistency in the positioning accuracy with the variation of the average number of Galileo satellites would be that the addition of Galileo satellites to
GPS/GLONASS brings new hardware delay biases as well as new unknown parameters, such as inter-system bias (ISB) and non-integer ambiguity terms, as detailed in Section 4.3 (Dennis, 2015). According to this, while one Galileo satellite brings one ISB, one non-integer ambiguity term and two measurements (one ionosphere-free code measurement and one ionosphere-free carrier-phase measurement), three Galileo satellite brings one ISB, three non-integer ambiguity term and six measurements (three ionosphere-free code measurements and three ionosphere-free carrier-phase measurements). As a result, it is obvious that if the average number of Galileo satellites is less than two, it may not compensate its new hardware biases and unknowns.

There are, however, three exceptions within a total of 28 datasets. In the 15th dataset collected at RIO2 station on August 7, 2015, combined GPS/GLONASS/Galileo improves the 3D positioning accuracy over combined GPS/GLONASS although the average number of Galileo satellites in this dataset is 1.73. The cause of this increase would be the noticeable improvement in the overall satellite geometry despite the small average number of Galileo satellites, which can be seen from the sky plot given in Figure 12. Although the average number of Galileo satellites in the 16th and 17th datasets are 2.63 and 2.82, respectively, this aspect does not increase the 3D positioning accuracy over combined GPS/GLONASS solution since the Galileo satellites tracks almost same trajectory with GPS and GLONASS satellites and thus do not improve the overall satellite geometry, which can be again seen from the sky plots given in Figure 12.
Figure 12. Sky plots of the 15th, 16th and 17th datasets collected at RIO2 station. See Table 6 for session details
The mean RMS of the positioning errors for all datasets will be discussed in the next section. However, since the RMS of the positioning errors changes with the average number of Galileo satellites, the mean RMS of the positioning errors is calculated in two different cases. In case 1, all datasets are used while in case 2, the datasets in which the average number of Galileo satellites is more than two are used. See Table 7 for session details.

5.4.1.2 Overall Analysis

- Case 1

This case presents the mean RMS of the positioning errors for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for all datasets collected at POTS, WIND, RIO2 and ULAB stations, which is given in Table 8.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>0.018</td>
<td>0.014</td>
<td>0.022</td>
</tr>
<tr>
<td>North</td>
<td>0.006</td>
<td>0.006</td>
<td>0.008</td>
</tr>
<tr>
<td>Up</td>
<td>0.013</td>
<td>0.011</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Table 8. Mean RMS of the positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for all datasets. The datasets were collected at POTS, WIND, RIO2 and ULAB stations. See Table 7 for session details.

As seen in Table 8, one can state that combined GPS/GLONASS improves the positioning accuracy by 22% and 15% over GPS-only in the east and up directions, respectively. However, it does not change the positioning accuracy in the north direction.
The reason for this would be that the satellite geometry of GPS is already good in the north direction due to the north-south ground tracks of GPS satellites, which changes the satellite geometry faster in the north direction (Blewit, 1989; Rizos, 1997; Cai, 2009).

Additionally, combined GPS/GLONASS/Galileo worsens the positioning accuracy by 57%, 33% and 9% over combined GPS/GLONASS in east, north and up directions, respectively. The reason for the variation of the magnitudes of declines among the directions would be that the Galileo satellites effect the overall satellite geometry in east, north and up directions differently, as given some examples in Figure 12. The reason for the reduction in the positioning accuracy would be that if the average number of Galileo satellites is small, it may not compensate its new hardware biases and unknowns, as explained before.

In addition, it is important to note that the relationships among the positioning errors in east, north and up directions may seem strange since it is expected that the positioning errors in the east and north directions are similar and one in the up direction is worst. However, in the PPP method with float ambiguity resolution especially for short datasets, it is not the case since the positioning solutions in the east direction may not converge to their correct values in short durations. This is why the positioning error in the east direction worse than those in the north and up directions which are less related to the ambiguity resolution (Blewitt, 1989). The results presented by Geng et al. (2009) demonstrate this phenomenon very well. In their study, the hourly datasets are, first, processed in the PPP method with float ambiguity resolution and then processed in the PPP method with integer ambiguity resolution which requires a network of reference stations.
The results changed from (3.8, 1.5 and 2.8) centimeters to (0.5, 0.5 and 1.4) centimeters in east, north and up directions, respectively.

- **Case 2**

This case presents the mean RMS of the positioning errors for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for the datasets in which the average number of Galileo satellites is more than two, which is given in Table 9. The observations are observed at POTS, WIND, RIO2 and ULAB stations.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>0.018</td>
<td>0.014</td>
<td>0.016</td>
</tr>
<tr>
<td>North</td>
<td>0.005</td>
<td>0.005</td>
<td>0.007</td>
</tr>
<tr>
<td>Up</td>
<td>0.016</td>
<td>0.011</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Table 9. Mean RMS of the positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for the datasets in which the average number of Galileo satellites is more than two. The datasets were collected at POTS, WIND, RIO2 and ULAB stations. See Table 7 for session details.

As seen in Table 9, combined GPS/GLONASS/Galileo again worsens the positioning accuracy by 14%, 40% and 9% over combined GPS/GLONASS in east, north and up directions, respectively. However, in this case, the magnitude of the 3D decline between combined GPS/GLONASS and combined GPS/GLONASS/Galileo is 3 millimeters while it was 7 millimeters in case 1. This result supports the explanation given above since it states that if more Galileo satellites are used, the positioning accuracy may improve. In addition, it is important to note that this 4 millimeters improvement is so
significant for precise engineering and scientific applications, such as structural health monitoring (Ehiorobo and Iruge, 2012), earthquake crustal deformation monitoring (Gao and Wang, 2007), sea floor crustal deformation monitoring (Fujita et al., 2006; Tadokoro et al., 2006) and geodetic positioning applications whose accuracy requirements range from 0.1 to 0.5 millimeters (Seepersad and Bisnath, 2013).

The explanation for why the integration of Galileo still decreases the positioning accuracy over combined GPS/GLONASS although more Galileo satellites are used would be the accuracy of Galileo precise products at the time of study since the network of MGEX reference stations was still developing and did not have a good global distribution. White and Langley (2015) also found this decline over GPS-only solutions as they combined GPS and Galileo satellites.

5.4.1.3 Conclusion

In this study, the performance of static PPP is analyzed by comparing GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for scientific research and engineering applications where the positioning accuracy is important in post-processed static mode, such as structural health monitoring (Ehiorobo and Iruge, 2012), crustal deformation monitoring (Gao and Wang, 2007) and the establishment of Continuously Operating Reference Station (CORS) network (El-Hattab, 2014).

According to the numerical results, combined GPS/GLONASS improves positioning accuracy over GPS-only solutions. Additionally, combined GPS/GLONASS/Galileo may either improve or worsen positioning accuracy over combined GPS/GLONASS solutions, which depends on the number of Galileo satellites.
used. However, it is also shown that the magnitude of reduction in the positioning accuracy may be reduced using more Galileo satellites from the comparison of case 1 and case 2. As the number of Galileo satellites and the accuracy of Galileo products are increased, further improvement in the positioning accuracy will be expected.

5.4.2 Experiment 2: Performance of Kinematic Precise Point Positioning

Rabbou and El-Rabbany (2015) stated that combined GPS/GLONASS/Galileo improves the positioning accuracy slightly over combined GPS/GLONASS based on the between-satellites single-differenced multi-GNSS PPP method. They obtained this result by using an only single dataset that contains a maximum two Galileo satellites, but drops to one satellite during the observation duration. Cai et al. (2015) concluded that combined GPS/GLONASS/BeiDou/Galileo slightly improves the positioning accuracy over combined GPS/GLONASS/BeiDou. They reached this result by also using an only single dataset in which the average number of Galileo satellites is three. In this experiment, the performance of kinematic PPP is analyzed by comparing GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions using a total of 7 three-hour datasets windowed from 24 hour daily datasets collected at POTS stations on seven consecutive days from August 7, 2015 to August 13, 2015 (see Figure 6). The session details of these datasets are listed in Table 10. All the observations have a sampling interval of 30 seconds. The default satellite elevation cutoff angle of 15° is maintained (Cai, 2009; Takasu, 2013). The precise satellite orbit and clock products of GFZ are used to mitigate the satellite orbit and clock errors. Extended Kalman filter is used for the estimation. Refer to Equation 106 for more details about the parameters. The kinematic position coordinates
are modelled as white noise with a spectral density setting of $10^2 \text{m}^2/\text{sec}$ (Takasu, 2013). The other parameter settings and the special error source corrections are the same as the previous static PPP processing in Section 5.4.1. The multi-GNSS processing software RTKLIB is used to process the datasets. The setting configurations of RTKLIB are given and explained in Appendix. The positioning solutions are compared with the corresponding IGS weekly solutions in order to find the positioning errors. Refer to Section 5.3, for more details about the statistical evaluation strategy.

<table>
<thead>
<tr>
<th>Date</th>
<th>Start and End Time (GPS Time)</th>
<th>Dataset Number</th>
<th>Average Number of Galileo Satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 7, 2015</td>
<td>09.00 – 12.00</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>August 8, 2015</td>
<td>15.00 – 18.00</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>August 9, 2015</td>
<td>20.00 – 23.00</td>
<td>3</td>
<td>2.84</td>
</tr>
<tr>
<td>August 10, 2015</td>
<td>06.00 – 09.00</td>
<td>4</td>
<td>1.80</td>
</tr>
<tr>
<td>August 11, 2015</td>
<td>12.00 – 15.00</td>
<td>5</td>
<td>2.74</td>
</tr>
<tr>
<td>August 12, 2015</td>
<td>18.00 – 21.00</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>August 13, 2015</td>
<td>06.00 – 09.00</td>
<td>7</td>
<td>1.68</td>
</tr>
</tbody>
</table>

Table 10. Session details of the datasets collected at POTS station and used in the kinematic PPP experiment (see Figure 6)

5.4.2.1 Analysis of Positioning Accuracy and Convergence Time

The processing solution of the 4th dataset which is the three-hour dataset windowed from the 24-hour daily dataset collected at POTS station on August 10, 2015 is chosen for
an illustration purpose. The sky plots of this three-hour dataset under 15° elevation cutoff angle for GPS, GLONASS, Galileo and combined GPS/GLONASS/Galileo are given in Figure 13.

Figure 13. Sky plots for GPS, GLONASS, Galileo and combined GPS/GLONASS/Galileo for the 4th dataset collected at POTS on August 10, 2015. Green denotes the satellites that transmit data on L1 and L2, yellow denotes the satellites that transmit data on only L1, blue denotes the satellites that transmit data on L1, L2 and L5 and red denotes the satellites that transmit data on L1 and L5.
The number of visible satellites, GDOP, HDOP and VDOP values of this dataset given in Figure 14 for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo.

Figure 14. The number of satellites, GDOP, HDOP and VDOP plots for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo for the 4th dataset collected at POTS station on August 10, 2015. G denotes GPS, G/R denotes combined GPS/GLONASS and G/R/E denotes combined GPS/GLONASS/Galileo.

The average numbers of satellites are 6.41, 12.93 and 14.79 for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo, respectively. The average GPS HDOP value is 1.43, while the average combined GPS/GLONASS HDOP value is 0.82. One may state that adding GLONASS to GPS decreases HDOP by 43%, which is significant. However, the addition of Galileo to combined GPS/GLONASS improves the average HDOP value by only 10% over combined GPS/GLONASS and makes it 0.74. For VDOP values, the addition of GLONASS to GPS improves the average...
VDOP value by 42% over GPS-only and makes it 1.32 from 2.27. The addition of Galileo to combined GPS/GLONASS improves the average VDOP value by 3% over combined GPS/GLONASS and makes it 1.28 from 1.32.

The positioning errors of GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for the 4th dataset collected at POTS on August 10, 2015 are given in Figure 15 in east, north and up directions, respectively.

![Figure 15](image)

Figure 15. The positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for the 4th dataset collected at POTS on August 10, 2015. See Table 10 for session details.

Table 11 shows the mean, STD and RMS of the positioning errors for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo for the 4th dataset collected at POTS on August 10, 2015 in east, north and up directions, respectively.
<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
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<tr>
<td>East</td>
<td>Mean</td>
<td>0.267</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.109</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.288</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>-0.055</td>
<td>-0.002</td>
</tr>
<tr>
<td>North</td>
<td>STD</td>
<td>0.023</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.060</td>
<td>0.004</td>
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<tr>
<td></td>
<td>Mean</td>
<td>-0.113</td>
<td>-0.021</td>
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<tr>
<td>Up</td>
<td>STD</td>
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</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.131</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Table 11. Statistics of the positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for the 4th dataset collected at POTS on August 10, 2015. See Table 10 for session details.

As seen in Table 11, combined GPS/GLONASS improves the RMS of the positioning accuracy by 87%, 93% and 82% over GPS-only in east, north and up directions, respectively. However, combined GPS/GLONASS/Galileo worsens the positioning accuracy by 14%, 40% and 9% over combined GPS/GLONASS in east, north and up directions, respectively. The reason of the reduction in the positioning accuracy would be the average number of Galileo satellites in this dataset collected at POTS on August 10, 2015, which is 1.80. As explained before, the new biases and parameters introduced by Galileo satellites may not be compensated, if the average number of Galileo satellites is less than two. Furthermore, as seen in Figure 13, one visible Galileo satellite tracks almost the same trajectory with a GLONASS satellite, which does not contribute to the overall satellite geometry especially in the east and north directions. The reason for the variation of the magnitude of the decline among the directions would be that the Galileo satellites
effect the overall satellite geometry in east, north and up directions differently, as seen in Figure 13.

Table 12 gives the convergence times for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for the 4th dataset collected at POTS on August 10, 2015 in east, north and up directions, respectively.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>356</td>
<td>25</td>
<td>59</td>
</tr>
<tr>
<td>North</td>
<td>239</td>
<td>41</td>
<td>25</td>
</tr>
<tr>
<td>Up</td>
<td>319</td>
<td>116</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 12. The convergence times [the number of 30-sec epochs] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for the 4th dataset collected at POTS on August 10, 2015. See Table 10 for session details.

As seen in Table 12, it can be stated that with the combination of GPS and GLONASS, the convergence time is reduced by 93%, 83% and 64% over GPS-only in east, north and up directions, respectively. In addition, combined GPS/GLONASS/Galileo further reduces the convergence time by 39% and 85% over combined GPS/GLONASS in the north and up directions, respectively. However, it worsens the convergence time in the east direction by 136% over combined GPS/GLONASS. The reason of the improvements in the north and up directions would be the contribution of Galileo satellites to the overall satellite geometry since the convergence time is related to the satellite geometry (Bisnath and Gao, 2009). As seen in Figure 13, the Galileo satellites improve the satellite geometry
in the north direction rather than in the east direction. Furthermore, the addition of Galileo satellites also improves the satellite geometry in the up direction since the satellite altitude of Galileo satellites is different than those of GPS and GLONASS, which leads to improve the satellite geometry. The reason of the worsening of the convergence time in the east direction should be further investigated whether it is related to this dataset or not.

Since the average number of Galileo satellites are not same in all datasets, the RMS of the positioning errors and the convergence times for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for each dataset is demonstrated separately.

Figures 16, 17 and 18 provide the RMS of the positioning errors for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions in east, north and up directions, respectively.

![Figure 16. The RMS of the positioning errors [meters] in the east direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 10 for session details]
Figure 17. The RMS of the positioning errors [meters] in the north direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 10 for session details.

Figure 18. The RMS of the positioning errors [meters] in the up direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 10 for session details.
As seen in Figures 16, 17 and 18, in the 1st, 4th and 7th datasets collected at POTS station, combined GPS/GLONASS/Galileo solutions are slightly worse than combined GPS/GLONASS. The reason for this is the average numbers of Galileo satellites in these datasets which are 2, 1.8 and 1.68, respectively. In the remaining datasets, the average number of Galileo satellites is more than two and thus combined GPS/GLONASS/Galileo improves the positioning accuracy or at least provides similar positioning accuracy over combined GPS/GLONASS. Since the explanation for the effect of the average number of Galileo satellites to the positioning accuracy is given before, it is not repeated here. The reason of the difference between the maximum and minimum RMS values is the variation on the DOP values of the datasets.

Figures 19, 20 and 21 provide the convergence times for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions in east, north and up directions, respectively.
Figure 19. The convergence times [the number of 30-sec epochs] in the east direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 10 for session details.

Figure 20. The convergence times [the number of 30-sec epochs] in the north direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 10 for session details.
Figure 21. The convergence times [the number of 30-sec epochs] in the up direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 10 for session details.

As seen in Figure 19, GPS-only solutions require longer convergence time than combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions due to having higher DOP values. However, if the observed GPS satellites provide a good satellite geometry, this convergence time can be close to those of combined GPS/GLONASS (Cai, 2009). For example, in the 3rd dataset collected at POTS station on August 9, 2015, the convergence time of GPS-only solution in the east direction is similar with the convergence time of combined GPS/GLONASS solution since the GPS satellites are observed in the east-west sides of the station as seen in Figure 22.
As seen in Figure 19, the worsening of the convergence time in the east direction is not related to the number of Galileo satellites used. Even if the average number of Galileo satellites is more than two, combined GPS/GLONASS/Galileo may still worsen the convergence time in the east direction over combined GPS/GLONASS. Note that we also saw this phenomenon in the 4th dataset collected at POTS on August 10, 2015, which was given for an illustration purpose. The reason for this would be the hardware biases which are lumped to the integer ambiguity term as detailed in Section 4.4. The magnitudes of these hardware biases change day to day (Cai, 2009). As their magnitudes are large, they may worsen the float ambiguity resolution. As the ambiguity resolution worsens, the convergence time of positioning solutions in all directions also increases (Ge et al., 2008; Geng et al., 2010; Collins et al., 2010) while the biggest worsening is seen in the east
direction since the east direction is up to five times more affected from the ambiguity resolution than the north and up directions due to the orbit design and motion of satellites (Blewitt, 1989). In our results, we see the worsening in the east direction, which supports the explanation given above.

From Figures 20 and 21, one can state that GPS-only solutions require again longer convergence time than combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions in both the north and up directions due to the having higher DOP values. In addition, combined GPS/GLONASS/Galileo improves or at least provides similar convergence time in the north and up direction over combined GPS/GLONASS.

The mean RMS of the positioning errors and mean convergence time for all datasets will be discussed in the next section. However, these values change with the average number of Galileo satellites, the mean RMS of the positioning errors and mean convergence time are calculated in two different cases. In case 1, all datasets are used while in case 2, the datasets in which the average number of Galileo satellites is more than two are used. See Table 10 for session details.

5.4.2.2 Overall Analysis

- Case 1

This case presents the mean RMS of the positioning errors and mean convergence time for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for all datasets collected at POTS station, which are given in Tables 13 and 14, respectively.
As seen in Table 13, combined GPS/GLONASS improves the positioning accuracy by 79%, 67% and 66% over GPS-only in east, north and up directions, respectively. However, if Galileo is added to combined GPS/GLONASS, the positioning errors worsen by 13%, 25% and 26% over combined GPS/GLONASS in east, north and up directions, respectively.

Table 13. Mean RMS of the positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for all datasets collected at POTS station. See Table 10 for session details.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>0.119</td>
<td>0.025</td>
<td>0.029</td>
</tr>
<tr>
<td>North</td>
<td>0.027</td>
<td>0.009</td>
<td>0.012</td>
</tr>
<tr>
<td>Up</td>
<td>0.104</td>
<td>0.035</td>
<td>0.047</td>
</tr>
</tbody>
</table>

As seen in Table 14, combined GPS/GLONASS improves the convergence time by 81%, 77% and 75% over GPS-only in east, north and up directions, respectively. In

Table 14. Mean convergence times [the number of 30-sec epochs] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for all datasets collected at POTS station. See Table 10 for session details.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>216</td>
<td>42</td>
<td>79</td>
</tr>
<tr>
<td>North</td>
<td>121</td>
<td>28</td>
<td>25</td>
</tr>
<tr>
<td>Up</td>
<td>247</td>
<td>61</td>
<td>42</td>
</tr>
</tbody>
</table>
addition, combined GPS/GLONASS/Galileo improves the convergence time by 11% and 31% over combined GPS/GLONASS in the north and up directions, respectively, while it worsens the convergence time by 88% in the east direction.

The mean results in the positioning accuracy and convergence time for all datasets collected at POTS station agree with the results of 4th dataset given and explained above for an illustration purpose. Therefore, the explanations are not repeated here.

- **Case 2**

This case presents the mean RMS of the positioning errors and the mean convergence time for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for the datasets in which the average number of Galileo satellites is more than two, which are given in Table 15 and 16. The observations are observed at POTS station.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>East</strong></td>
<td>0.047</td>
<td>0.023</td>
<td>0.017</td>
</tr>
<tr>
<td><strong>North</strong></td>
<td>0.010</td>
<td>0.011</td>
<td>0.009</td>
</tr>
<tr>
<td><strong>Up</strong></td>
<td>0.081</td>
<td>0.038</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Table 15. Mean RMS of the positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for the datasets in which the average number of Galileo satellites is more than two. The datasets were collected at POTS station. See Table 10 for session details.
Table 16. Mean convergence times [the number of 30-sec epochs] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for the datasets in which the average number of Galileo satellites is more than two. The datasets were collected at POTS station. See Table 10 for session details.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>144</td>
<td>41</td>
<td>74</td>
</tr>
<tr>
<td>North</td>
<td>56</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>Up</td>
<td>200</td>
<td>47</td>
<td>35</td>
</tr>
</tbody>
</table>

As seen in Table 15, in the case 2, combined GPS/GLONASS/Galileo improves the positioning accuracy by 26%, 18% and 11% over combined GPS/GLONASS in east, north and up directions, respectively. However, in the case 1, combined GPS/GLONASS/Galileo worsened the positioning accuracy by 13%, 25% and 26% over combined GPS/GLONASS in east, north and up directions, respectively. In addition, it is important to note that combined GPS/GLONASS/Galileo improves the 3D positioning accuracy by 7 millimeters which is so significant for precise engineering and scientific applications, such as deformation monitoring (Fujita et al., 2006; Tadokoro et al., 2006; Gao and Wang, 2007) and seismology for earthquake and tsunami studies (Xu et al., 2013). These results support the explanation that if the average number of Galileo satellites is more than two, it can compensate its new biases and parameters with its improvement in the overall satellite geometry.

From the comparison of the mean convergence time in case 1 and 2, one can see that in case 2, the differences between convergence times of combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions are decreased by 2 and 3.5 minutes over
the case 1 in the east and up directions, respectively. Therefore, it is expected that the convergence time would be further decreased if more Galileo satellites are used because the convergence time is quite related to the satellite geometry (Bisnath and Gao, 2009). However, there is no direct relation was found between the convergence time in the north direction and the average number of Galileo satellites since in the both case 1 and 2, the differences between the convergence time of combined GPS/GLONASS and combined GPS/GLONASS/Galileo are 1.5 minutes. The reason for this would be that the convergence time of combined GPS/GLONASS in the north direction is already short for the kinematic PPP solutions which is 12 minutes and thus the contribution of Galileo satellites at the time of study may not be enough to improve the overall satellite geometry in the north direction.

5.4.2.3 Conclusion

In this study, the performance of the kinematic PPP method is analyzed by comparing GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions for scientific research and engineering applications, such as landslide monitoring (Gao and Wang, 2007; Wang, 2013; Capilla et al., 2016), offshore resources exploration, seafloor mapping (Bisnath and Gao, 2009; Geng et al., 2010; Tegedor et al., 2014), precise farming (Bisnath and Gao, 2009; van Bree and Tiberius, 2012), airborne mapping (Yuan et al., 2009) and sea floor crustal deformation monitoring (Fujita et al., 2006; Tadokoro et al., 2006).

According to the numerical results, combined GPS/GLONASS improves both the positioning accuracy and convergence time over GPS-only solutions. Furthermore, it is
also found that combined GPS/GLONASS/Galileo may either improve or worsen both the positioning accuracy and convergence time over combined GPS/GLONASS solutions, which depends on the number of Galileo satellites used. It is also shown that improvement in the positioning accuracy and convergence time using more Galileo satellites. Therefore, these results are promising in that when the number of operational Galileo satellites increases, the positioning accuracy and convergence time will be further improved. It is important to note that the kinematic tests in this experiment are conducted for the European area. Therefore, further investigation may be required for other areas since even full operational GNSS constellation does not provide a uniform satellite sky distribution which depends on the latitude of an observer (Santerre, 1991).

5.4.3 Experiment 3: Impact of Different Multi-GNSS Precise Satellite Orbit and Clock Products

In this experiment, the positioning solutions obtained with the precise satellite orbit and clock products of three IGS MGEX analysis centers are compared to evaluate the impact of different precise satellite orbit and clock products on the positioning accuracy. These analysis centers are:

- Center for Orbit Determination in Europe (CODE)
- German Research Centre for Geosciences (GFZ)
- Wuhan University

For more details about these analysis centers, refer to Section 3.2.1.1.

The positioning solutions obtained with the precise satellite orbit and clock products of three IGS MGEX analysis center are compared using a total of 14 three-hour
datasets windowed from 24 hour daily datasets collected at two MGEX stations called POTS and RIO2 from seven consecutive days, i.e. August 7-13, 2015 (See Figure 6). The session details of the datasets are listed in Table 17. Combined GPS/GLONASS/Galileo observations with a sampling interval of 30 seconds are used. The default satellite elevation cutoff angle of 15° is maintained (Cai, 2009; Takasu, 2013). Extended Kalman filter is used for the estimation. Refer to Equation 106 for more details about the parameters. The parameter settings and the special error sources corrections are the same as the previous static PPP processing in Section 5.4.1. The datasets are processed using the precise satellite orbit and clock products of CODE, GFZ and Wuhan University separately in post-processed static PPP mode. The multi-GNSS processing software RTKLIB is used to process the datasets. The setting configurations of RTKLIB are given and explained in Appendix. The positioning solutions are compared with the corresponding IGS weekly solutions in order to find the positioning errors. Refer to Section 5.3, for more details about the statistical evaluation strategy.
<table>
<thead>
<tr>
<th>Date</th>
<th>Start and End Time (GPS Time)</th>
<th>Dataset Number</th>
<th>Average Number of Galileo Satellites</th>
<th>Date</th>
<th>Start and End Time (GPS Time)</th>
<th>Dataset Number</th>
<th>Average Number of Galileo Satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 7, 2015</td>
<td>09.00 – 12.00</td>
<td>1</td>
<td>2</td>
<td>August 7, 2015</td>
<td>03.00 – 06.00</td>
<td>8</td>
<td>1.73</td>
</tr>
<tr>
<td>August 8, 2015</td>
<td>15.00 – 18.00</td>
<td>2</td>
<td>3</td>
<td>August 8, 2015</td>
<td>09.00 – 12.00</td>
<td>9</td>
<td>2.63</td>
</tr>
<tr>
<td>August 9, 2015</td>
<td>21.00 – 24.00</td>
<td>3</td>
<td>2.33</td>
<td>August 9, 2015</td>
<td>12.00 – 15.00</td>
<td>10</td>
<td>2.82</td>
</tr>
<tr>
<td>August 10, 2015</td>
<td>06.00 – 09.00</td>
<td>4</td>
<td>1.80</td>
<td>August 10, 2015</td>
<td>18.00 – 21.00</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>August 11, 2015</td>
<td>12.00 – 15.00</td>
<td>5</td>
<td>2.74</td>
<td>August 11, 2015</td>
<td>06.00 – 09.00</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>August 12, 2015</td>
<td>18.00 – 21.00</td>
<td>6</td>
<td>3</td>
<td>August 12, 2015</td>
<td>12.00 – 15.00</td>
<td>13</td>
<td>2.72</td>
</tr>
<tr>
<td>August 13, 2015</td>
<td>06.00 – 09.00</td>
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<td>1.68</td>
<td>August 13, 2015</td>
<td>15.00 – 18.00</td>
<td>14</td>
<td>2.02</td>
</tr>
</tbody>
</table>

Table 17. Session details of the datasets collected at POTS and RIO2 stations and used in the impact of different precise products experiment (see Figure 6)

5.4.3.1 Analysis of Positioning Accuracy

Since the average number of Galileo satellites are not same in all datasets, the RMS of the positioning errors for combined GPS/GLONASS/Galileo solutions with the precise satellite orbit and clock products of CODE, GFZ and Wuhan University for each dataset is demonstrated separately in Figures 23, 24 and 25 for east, north and up directions, respectively.
Figure 23. The RMS of the positioning errors [meters] in the east direction for combined GPS/GLONASS/Galileo solutions with the precise satellite orbit and clock products of CODE, GFZ and Wuhan University for each dataset collected at POTS and RIO2 stations. Blue refers the solutions with the precise products of CODE, green refers to the solutions with the precise products of Wuhan University and red refers to the solutions with the precise products of GFZ.

Figure 24. The RMS of the positioning errors [meters] in the north direction for combined GPS/GLONASS/Galileo solutions with the precise satellite orbit and clock products of CODE, GFZ and Wuhan University for each dataset collected at POTS and RIO2 stations. Blue refers the solutions with the precise products of CODE, green refers to the solutions with the precise products of Wuhan University and red refers to the solutions with the precise products of GFZ.
Figure 25. The RMS of the positioning errors [meters] in the up direction for combined GPS/GLONASS/Galileo solutions with the precise satellite orbit and clock products of CODE, GFZ and Wuhan University for each dataset collected at POTS and RIO2 stations. Blue refers the solutions with the precise products of CODE, green refers to the solutions with the precise products of Wuhan University and red refers to the solutions with the precise products of GFZ.

As seen in Figure 23, the positioning accuracy for combined GPS/GLONASS/Galileo solutions with the precise satellite orbit and clock products of GFZ is better than those of CODE and Wuhan University.

Figures 24 and 25 indicates that the solutions obtained with the precise satellite orbit and clock products of GFZ provides better or at least similar positioning accuracy with those of CODE and Wuhan University.

5.4.3.2 Overall Analysis

This section presents the mean RMS of the positioning errors for combined GPS/GLONASS/Galileo solutions with the precise satellite orbit and clock products of CODE, GFZ and Wuhan University for all datasets collected at POTS and RIO2 stations, which is given in Table 18.
<table>
<thead>
<tr>
<th></th>
<th>CODE</th>
<th>Wuhan Uni.</th>
<th>GFZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>0.052</td>
<td>0.047</td>
<td>0.023</td>
</tr>
<tr>
<td>North</td>
<td>0.012</td>
<td>0.013</td>
<td>0.011</td>
</tr>
<tr>
<td>Up</td>
<td>0.016</td>
<td>0.013</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Table 18. Mean RMS of the positioning errors [meters] for combined GPS/GLONASS/Galileo solutions with the precise satellite orbit and clock products of CODE, GFZ and Wuhan University for all dataset collected at POTS and RIO2 stations. See Table 17 for session details.

As seen in Table 18, the solutions obtained with the precise satellite orbit and clock products of GFZ improves the positioning accuracy by 56%, 8% and 25% over those of CODE and by 51%, 15% and 8% over those of Wuhan University in east, north and up directions, respectively. Table also states that 3D positioning accuracy which can be obtained with the precise products of CODE, Wuhan Uni and GFZ at the time of writing are 5.6, 5.0 and 2.8 cm, respectively. However, further improvement in the positioning accuracy is expected since at the time of writing, the network of MGEX reference stations was still developing and did not have a good global distribution, which degrades the accuracy of the precise products.

The main reason of the positioning accuracy differences among the solutions obtained with the precise products of these three analysis centers would be the differences among the sample intervals of their precise products (Kouba and Herous, 2001). The precise satellite orbit and clock products should be interpolated at the corresponding time of observations to eliminate satellite orbit and clock errors. If the precise satellite orbit products are interpolated, the error from the interpolation is not significant due to the fact
that the satellite orbits have a very smooth behavior. However, if the precise satellite clock products are interpolated, the error from the interpolation is significant because of the high level of irregularity on the satellite clocks (Guo et al., 2010; Andrei and Chen, 2010). It is important to clarify that these errors from the interpolations are not attributed to the detect of the interpolation method but to the nature of the satellite orbit and clock products. Furthermore, these errors are also not related to the RTKLIB software since it uses linear interpolation method for the clock product interpolation, which provides better accuracy than other interpolation methods as shown by Yuan et al. (2008), and Lagrange interpolation for the orbit product interpolation, which provides mm level accuracy (Guo et al., 2010).

These interpolation errors can be absorbed into the ambiguity term (Han, 2000). As the ambiguity resolution worsens, the positioning accuracy in east, north and up directions worsens, too. The biggest decline is seen in the east direction, which supports the given explanation since the east direction is up to 5 times more effected from the ambiguity resolution than the north and up directions due the orbit design and motion of satellites (Blewit, 1989).

As given before, CODE and Wuhan University provides orbit products with 15-minute intervals and clock products with 5-minute intervals while GFZ provides the orbit products with 5-minute intervals and the clock products with 30-second intervals. Because of this, GFZ provides better positioning accuracy than CODE and Wuhan University. Furthermore, another reason of the positioning accuracy differences among the solutions obtained with the different precise products would be the different strategy of these analysis
centers to calculate precise orbit and clock products. For example, Wuhan University estimates inter-frequency bias (IFB) as an unknown parameter while CODE ignores it. More details about the models of the analysis centers were given Section 3.2.1.1.

5.4.3.3 Conclusion

In this experiment, the positioning solutions obtained with the precise satellite orbit and clock products of three different IGS MGEX analysis centers are compared to evaluate the impacts of different precise products on the positioning accuracy for precise static applications, such as crustal deformation monitoring (Gao and Wang, 2007) and the establishment of Continuously Operating Reference Station (CORS) network (El-Hattab, 2014). It is found that combined GPS/GLONASS/Galileo with the precise satellite orbit and clock products of GFZ are better than those of CODE and Wuhan University especially in the east direction, which is explained above.

5.4.4 Experiment 4: Impact of Different Elevation Cutoff Angles

The performance of GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo PPP solutions are demonstrated under different elevation cutoff angles (15°, 25° and 35°) in terms of the positioning accuracy and availability in the post-processing kinematic mode using a total of 7 three-hour datasets windowed from 24-hour daily datasets collected at POTS stations on seven consecutive days (see Figure 6). It is important to note that here, the availability refers to the total number of epochs with precise position estimates whose position accuracy is better than 10 cm. Refer Section 5.3 for the reason of choosing 10 cm as a threshold. The session details of the datasets are listed in Table 19. All observations have a sampling interval of 30 seconds. The precise satellite
orbit and clock products of GFZ are used to mitigate the satellite orbit and clock errors. Extended Kalman filter is used for the estimation. Refer to Equation 106 for more detail about the parameters. The parameter settings and the special error sources corrections are the same as the previous kinematic PPP processing in Section 5.4.2. The multi-GNSS processing software RTKLIB is used to process the datasets. The setting configurations of RTKLIB are given and explained in Appendix. The positioning solutions are compared with the corresponding IGS weekly solutions in order to find the positioning errors. Refer to Section 5.3, for more details about the statistical evaluation strategy.

<table>
<thead>
<tr>
<th>Date</th>
<th>Start and End Time (GPS Time)</th>
<th>Dataset Number</th>
<th>Average Number of Galileo Satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 7, 2015</td>
<td>09.00 – 12.00</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>August 8, 2015</td>
<td>15.00 – 18.00</td>
<td>2</td>
<td>3</td>
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<td>August 9, 2015</td>
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<td>2.84</td>
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<td>August 10, 2015</td>
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<td>1.80</td>
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<td>August 11, 2015</td>
<td>12.00 – 15.00</td>
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<td>2.74</td>
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<td>August 12, 2015</td>
<td>18.00 – 21.00</td>
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<tr>
<td>August 13, 2015</td>
<td>06.00 – 09.00</td>
<td>7</td>
<td>1.68</td>
</tr>
</tbody>
</table>

Table 19. Session details of the datasets collected at POTS station and used in the impact of different elevation cutoff angles experiment (see Figure 6)
5.4.4.1 Analysis of Positioning Accuracy and Availability

The processing solution of the 5th dataset collected at POTS station on August 11, 2015 is chosen for an illustration purpose. The number of visible satellites, HDOP and VDOP plots of the 5th dataset collected at POTS station on August 11, 2015 under 15°, 25° and 35° elevation cutoff angles for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo are given in Figure 26.

![Figure 26](image)

Under 25° and 35° elevation cutoff angles, the average number of GPS satellites are 6.2 and 4.8, respectively while it is 7.7 under 15° elevation cutoff angles. As a result, the average HDOP at 15° elevation cutoff angle increases from 1.15 to 2.91 and 3.83 at 25° and 35° elevation cutoff angles, respectively. The average VDOP at 15° elevation...
cutoff angle increases from 1.88 to 4.94 and 7.59 at 25° and 35° elevation cutoff angles, respectively. Fortunately, HDOP and VDOP values for combined GPS/GLONASS and combined GPS/GLONASS/Galileo do not increase significantly, which are around two even with 35° elevation cutoff angle.

The positioning errors for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for the 5th dataset collected at POTS on August 11, 2015 are given in Figures 27, 28 and 29 in east, north and up directions, respectively. Note that these figures are given without windowing or smoothing the positioning solutions in order to illustrate the impacts of different cutoff angles on the positioning accuracy and availability better.

Figure 27. The positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15° elevation cutoff angle for the 5th dataset collected at POTS station on August 11, 2015. See Table 19 for session details.
Figure 28. The positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 25° elevation cutoff angle for the 5th dataset collected at POTS station on August 11, 2015. See Table 19 for session details.

Figure 29. The positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 35° elevation cutoff angle for the 5th dataset collected at POTS station on August 11, 2015. See Table 19 for session details.
As seen in Figures 27, 28 and 29, if the elevation cutoff angle increases, the availability of the positioning for GPS-only drops significantly due to having high DOP values. However, even at 25° and 35° elevation cutoff angles, both combined GPS/GLONASS and combined GPS/GLONASS/Galileo provide positioning estimates whose accuracy is better than 10 centimeters.

Table 20 shows the RMS of the positioning errors for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for the 5th dataset collected at POTS on August 11, 2015 in east, north and up directions, respectively.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>East</strong></td>
<td>15°</td>
<td>0.008</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>25°</td>
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<tr>
<td></td>
<td>35°</td>
<td>0.326</td>
<td>0.016</td>
</tr>
<tr>
<td><strong>North</strong></td>
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<td>0.009</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>0.013</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>0.084</td>
<td>0.022</td>
</tr>
<tr>
<td><strong>Up</strong></td>
<td>15°</td>
<td>0.022</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>25°</td>
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<tr>
<td></td>
<td>35°</td>
<td>0.183</td>
<td>0.072</td>
</tr>
</tbody>
</table>

Table 20. The RMS of the positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for the 5th dataset collected at POTS station on August 11, 2015. See Table 19 for session details.
As seen in Table 20, at 15° elevation cutoff angle, combined GPS/GLONASS provides better positioning accuracy than GPS-only and combined GPS/GLONASS/Galileo, as explained in Section 5.4.2.

At 25° elevation cutoff angle, combined GPS/GLONASS improves the positioning accuracy by 5.2, 0.7 and 2.1 cm over GPS-only in east, north and directions, respectively. The minimum improvement is seen in the north direction where GPS has already a good satellite geometry. However, combined GPS/GLONASS/Galileo worsens the positioning accuracy by 0.5, 0.1 and 1.8 cm over combined GPS/GLONASS in east, north and up directions, respectively.

At 35° elevation cutoff angle, combined GPS/GLONASS improves the positioning accuracy significantly by 31.0, 6.2 and 11.1 cm over GPS-only in east, north and directions, respectively, due to fact that at 35° elevation cutoff angle, GPS has very high DOP values. Furthermore, the difference between the positioning accuracy for combined GPS/GLONASS/Galileo and combined GPS/GLONASS decreases compared to the those at 25° elevation cutoff angle since the combination of Galileo to GPS/GLONASS improves HDOP.

The availability of the positioning solutions for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for the 5th dataset collected at POTS station on August 11, 2015 are given in Table 21.
Table 21. Availability of the positioning solutions whose accuracy is better than 10 centimeters [the number of 30-sec epochs] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for the 5th dataset collected at POTS station on August 11, 2015. See Table 19 for session details.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>15°</td>
<td>348</td>
<td>345</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>284</td>
<td>346</td>
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<tr>
<td></td>
<td>35°</td>
<td>8</td>
<td>302</td>
</tr>
<tr>
<td></td>
<td>15°</td>
<td>347</td>
<td>351</td>
</tr>
<tr>
<td>North</td>
<td>25°</td>
<td>267</td>
<td>359</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>93</td>
<td>316</td>
</tr>
<tr>
<td></td>
<td>15°</td>
<td>324</td>
<td>341</td>
</tr>
<tr>
<td>Up</td>
<td>25°</td>
<td>181</td>
<td>293</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>53</td>
<td>221</td>
</tr>
</tbody>
</table>

As seen in Table 21, for all the scenarios, if the elevation cutoff angle increases, availability of the positioning solutions drops. For GPS, the magnitude of this decrease is very significant due to having high DOP values at high elevation angles.

Combined GPS/GLONASS solution is better than GPS-only solution since it doubles the number of visible satellites and improves the DOP values as can be seen in Figure 26. Furthermore, even at 35° elevation cutoff angle, combined GPS/GLONASS can still provide positioning solutions whose accuracy is better than 10 centimeters in the horizontal components, but the availability of the positioning solutions whose accuracy is better than 10 centimeters for combined GPS/GLONASS/Galileo solutions decreases in the east and up directions where Galileo has higher potential to improve the overall satellite geometry since in the north direction, combined GPS/GLONASS has already good satellite
geometry. The reason of these decreases at 35° elevation cutoff angle would be that the average number of Galileo satellites at 35° elevation cutoff angle decreases to 2.22 from 2.74.

Since the average number of Galileo satellites are not the same in all datasets, the RMS of the positioning errors for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for each dataset is demonstrated separately in Figures 30, 31 and 32 for east, north and up directions, respectively.

Figure 30. The RMS of the positioning errors [meters] in the east direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for each dataset collected at POTS station. See Table 19 for session details
Figure 31. The RMS of the positioning errors [meters] in the north direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for each dataset collected at POTS station. See Table 19 for session details.

Figure 32. The RMS of the positioning errors [meters] in the up direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for each dataset collected at POTS station. See Table 19 for session details.
As seen in Figures 30, 31 and 32, for all the scenarios, if the elevation cutoff angle increases, the RMS of the positioning errors worsens. For GPS, this decrease is very significant due to having high DOP values.

It is also seen that at all elevation cutoff angles, if the average number of Galileo satellites is more than two, combined GPS/GLONASS/Galileo improves the positioning accuracy or at least provides similar positioning accuracy over combined GPS/GLONASS, such as 2<sup>nd</sup>, 5<sup>th</sup> and 6<sup>th</sup> datasets collected at POTS station on August 8, 11 and 12, 2015, respectively.

Furthermore, at 25° and 35° elevation cutoff angles, even if a dataset contains less than two Galileo satellites, GPS/GLONASS/Galileo may still improve the positioning accuracy over combined GPS/GLONASS, such as 1<sup>st</sup> and 4<sup>th</sup> collected at POTS station on August 7 and 10, 2015, respectively. The reason would be that even small number of Galileo satellites may improve the overall satellite geometry since at 25° and 35° elevation cutoff angles, the number of GPS and GLONASS satellites decreases significantly compared to the one at 15° elevation cutoff angle.

In the 7<sup>th</sup> datasets collected at POTS station on August 13, 2015, combined GPS/GLONASS/Galileo worsens the positioning accuracy over combined GPS/GLONASS since the average number of Galileo satellites in this dataset is around one.

At 25° and 35° elevation cutoff angles, in the 3<sup>rd</sup> dataset collected at POTS station on August 9, 2015, combined GPS/GLONASS/Galileo worsens the positioning accuracy
over combined GPS/GLONASS since the average numbers of Galileo satellites at these 
elevation cutoff angles are 1.97 and 1.68, respectively.

The mean RMS of the positioning errors and the mean availability of the 
positioning solutions whose accuracy is better than 10 centimeters for all datasets will be 
discussed in the next section. However, these values change with the average number of 
Galileo satellites, the mean RMS of the positioning errors and the mean availability of the 
positioning solutions whose accuracy is better than 10 centimeters are calculated in two 
different cases. In case 1, all datasets are used while in case 2, the datasets in which the 
average number of Galileo satellites is more than two are used. See Table 19 for session 
details.

5.4.4.2 Overall Analysis

- Case 1

This case presents the mean RMS of the positioning errors and the mean availability 
of the positioning solutions whose accuracy is better than 10 centimeters for GPS-only, 
combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 
25° and 35° elevation cutoff angles for all datasets collected at POTS station, which are 
given in Table 22 and 23, respectively.
**Table 22.** Mean RMS of the positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for all datasets collected at POT S station. See Table 19 for session details.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>15°</td>
<td>0.119</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>0.313</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>0.999</td>
<td>0.165</td>
</tr>
<tr>
<td>North</td>
<td>15°</td>
<td>0.027</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>0.154</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>2.010</td>
<td>0.180</td>
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<tr>
<td>Up</td>
<td>15°</td>
<td>0.104</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>0.251</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>4.333</td>
<td>0.234</td>
</tr>
</tbody>
</table>

**Table 23.** Mean availability of the positioning solutions whose accuracy is better than 10 centimeters [the number of 30-sec epochs] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for all datasets collected at POT S station. See Table 19 for session details.

<table>
<thead>
<tr>
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<th>GPS/GLONASS/Galileo</th>
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<tbody>
<tr>
<td>East</td>
<td>15°</td>
<td>168</td>
<td>322</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>118</td>
<td>267</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>7</td>
<td>174</td>
</tr>
<tr>
<td>North</td>
<td>15°</td>
<td>263</td>
<td>337</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>190</td>
<td>334</td>
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<tr>
<td></td>
<td>35°</td>
<td>23</td>
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<tr>
<td>Up</td>
<td>15°</td>
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<td></td>
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<tr>
<td></td>
<td>35°</td>
<td>14</td>
<td>144</td>
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</tbody>
</table>
As seen in Table 22, the mean RMS of the positioning errors also states that the positioning accuracy decreases as the elevation cutoff angle increases. 3D RMS of the positioning errors for GPS-only are 16, 43 and 488 cm at 15°, 25° and 35° elevation cutoff angles, respectively. For GPS-only solutions, the 3D positioning accuracy decreases significantly as the elevation cutoff angle increases.

The integration of GLONASS to GPS improves the positioning accuracy significantly and the magnitude of the improvement increases as the elevation cutoff is set to higher angles. 3D RMS of the positioning errors for combined GPS/GLONASS are 4.3, 6.6 and 33.8 cm at 15°, 25° and 35° elevation cutoff angles, respectively. Even under 25° elevation cutoff angle, combined GPS/GLONASS provides sub-decimeter 3D positioning accuracy.

However, combined GPS/GLONASS/Galileo decreases the positioning accuracy slightly over combined GPS/GLONASS. 3D RMS of the positioning errors for combined GPS/GLONASS/Galileo are 5.7, 8.4 and 41.1 cm at 15°, 25° and 35° elevation cutoff angles, respectively. Under 25° elevation cutoff angle, combined GPS/GLONASS/Galileo also provides sub-decimeter 3D positioning accuracy, but its positioning accuracy is less than the positioning accuracy of combined GPS/GLONASS. This result does not agree with the result of the 5th dataset chosen for an illustration purpose. The reason would be that the average number of Galileo satellites in the 5th dataset is more than two while in this case, we average all datasets collected at POTS station to calculate the mean RMS of
the positioning errors. As seen in Table 19, in three datasets, the average number of Galileo satellites is less than two, which decreases the mean RMS of the positioning errors.

Table 23 states that if the elevation cutoff angle increases, the availability of the positioning solutions drops. Combined GPS/GLONASS solutions are better than GPS-only solutions. As a difference to the results of 5th dataset chosen for an illustration purpose, at 35° elevation cutoff angle, combined GPS/GLONASS/Galileo improves the availability of the positioning solutions over combined GPS/GLONASS in the up direction. The reason for this would be the different altitude of Galileo satellites, as explained before.

- **Case 2**

This case presents the mean RMS of the positioning errors and the mean availability of the positioning solutions whose accuracy is better than 10 centimeters for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for the datasets in which the average number of Galileo satellites is more than two, which are given in Table 24 and 25, respectively. The observations are collected at POTS station.
### Table 24. Mean RMS of the positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for the datasets in which the average number of Galileo satellites is more than two. The observations were collected at POT8 station. See Table 19 for session details.

<table>
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<tr>
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<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>East</strong></td>
<td>15°</td>
<td>0.047</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>0.110</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>0.501</td>
<td>0.081</td>
</tr>
<tr>
<td><strong>North</strong></td>
<td>15°</td>
<td>0.010</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>0.035</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>0.175</td>
<td>0.030</td>
</tr>
<tr>
<td><strong>Up</strong></td>
<td>15°</td>
<td>0.081</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>0.149</td>
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<tr>
<td></td>
<td>35°</td>
<td>0.309</td>
<td>0.113</td>
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</table>

### Table 25. Mean availability of the positioning solutions whose accuracy is better than 10 centimeters [the number of 30-sec epochs] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions under 15°, 25° and 35° elevation cutoff angles for the datasets in which the average number of Galileo satellites is more than two. The observations were collected at POT8 station. See Table 19 for session details.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>East</strong></td>
<td>15°</td>
<td>244</td>
<td>323</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>206</td>
<td>291</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>10</td>
<td>221</td>
</tr>
<tr>
<td><strong>North</strong></td>
<td>15°</td>
<td>310</td>
<td>340</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>291</td>
<td>341</td>
</tr>
<tr>
<td></td>
<td>35°</td>
<td>48</td>
<td>322</td>
</tr>
<tr>
<td><strong>Up</strong></td>
<td>15°</td>
<td>184</td>
<td>311</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>110</td>
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</tr>
<tr>
<td></td>
<td>35°</td>
<td>27</td>
<td>169</td>
</tr>
</tbody>
</table>
Table 24 states that 3D RMS of the positioning errors for combined GPS/GLONASS are 4.6, 6.0 and 14.2 cm at 15°, 25° and 35° elevation cutoff angles, respectively, while 3D RMS of the positioning errors for combined GPS/GLONASS/Galileo are 3.9, 6.5 and 12.7 cm at 15°, 25° and 35° elevation cutoff angles, respectively. From these results, it can be stated that combined GPS/GLONASS/Galileo provides better or at least similar positioning accuracy over combined GPS/GLONASS if the average number of Galileo satellites is more than two.

In the separate analysis of the directions, it is seen that combined GPS/GLONASS/Galileo improves or at least provides same positioning accuracy over combined GPS/GLONASS in the north and up directions, respectively. However, in the east direction, as the elevation cutoff angle increases, the positioning accuracy for combined GPS/GLONASS/Galileo is getting worse. The reason would be the reduction in the average number of Galileo satellites at high elevation cutoff angles. This reduction is seen in the east direction instead of the north direction since Galileo has higher potential to improve the satellite geometry in the east direction rather than the north direction since GPS and GLONASS satellites have already good satellite geometry in the north direction (Blewit, 1989, Rizos, 1997; Cai, 2009). Therefore, the reduction in the average number of Galileo satellites effects the east direction more than the north direction.

Table 25 confirms that if the elevation cutoff angle increases, the availability of the positioning solutions drops. Furthermore, combined GPS/GLONASS solutions are better than GPS-only solutions. As same as in case 1, in this case, combined
GPS/GLONASS/Galileo again improves the availability of the positioning solutions over combined GPS/GLONASS in the up direction at 35° elevation cutoff angle.

5.4.4.3 Conclusion

In this experiment, the performance of GPS-only PPP, combined GPS/GLONASS PPP and combined GPS/GLONASS/Galileo PPP are demonstrated in terms of positioning accuracy and availability under different elevation cutoff angles (15°, 25° and 35°) to simulate constrained environments. From the numerical results, it can be stated that combined GPS/GLONASS improves both the positioning accuracy and availability over GPS-only. Therefore, it is highly recommended to use combined GPS/GLONASS observations for engineering applications in constrained environments, such as mobile mapping in urban areas (El-Mowafy, 2007) and deformation monitoring and positioning in open-pits mines (Cai et al., 2014).

Furthermore, it is also found that combined GPS/GLONASS/Galileo may either improve or worsens both the positioning accuracy and availability over combined GPS/GLONASS, which depends on the number of Galileo satellites used and the satellite geometry of GPS and GLONASS satellites. This result is promising in that when the number of operational Galileo satellites increases, the positioning accuracy and availability will be further improved.

5.4.5 Experiment 5: Impact of Different Observing-Session Durations

The performance of the static PPP method is analyzed by comparing GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions from 1-hour, 2-hour and 3-hour observing-session durations of the datasets collected at POTS, WIND,
RIO2 and ULAB stations on seven consecutive days from August 7, 2015 to August 13, 2015. Note that due to the absence of the dataset on August 9, 2015 at WIND station, the dataset collected at WIND station on August 14, 2015 is used. The distribution of these selected MGEX stations and the session details of the datasets are given in Figure 6 and Table 7, respectively. All observations have a sampling interval of 30 seconds. The precise satellite orbit and clock products of GFZ are used to mitigate the satellite orbit and clock errors. Extended Kalman filter is used for the estimation. Refer to Equation 106 for more detail about the parameters. The parameter settings and the special error sources corrections are the same as the previous static PPP processing in Section 5.4.1. The multi-GNSS processing software RTKLIB is used to process the datasets. The setting configurations of RTKLIB are given and explained in Appendix. The positioning solutions are compared with the corresponding IGS weekly solutions in order to find the positioning errors. Refer to Section 5.3, for more details about the statistical evaluation strategy.

5.4.5.1 Analysis of Positioning Accuracy

Since the average number of Galileo satellites are not same in all datasets, the RMS of the positioning errors for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions from each 1-hour, 2-hour and 3-hour datasets collected at POTS, WIND, RIO2 and ULAB stations are demonstrated separately in Figures 33, 34 and 35 for the east, north and up directions, respectively.
Figure 33. The RMS of the positioning errors [meters] in the east direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions from each 1-hour, 2-hour and 3-hour datasets collected at POTS, WIND, RIO2 and ULAB stations. See Table 7 for session details.

Figure 34. The RMS of the positioning errors [meters] in the north direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions from each 1-hour, 2-hour and 3-hour datasets collected at POTS, WIND, RIO2 and ULAB stations. See Table 7 for session details.
Figure 35. The RMS of the positioning errors [meters] in the up direction for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions from each 1-hour, 2-hour and 3-hour datasets collected at POTS, WIND, RIO2 and ULAB stations. See Table 7 for session details.

As seen in Figures 33, 34 and 35, if the observing-session duration is shortened, the RMS of the positioning errors worsens. However, there is few exceptions especially in the up direction. The reason for this would be the sudden change in the atmosphere especially in the troposphere which is highly related to the accuracy of the up direction (Ugur, 2013). As the 3D positioning errors from 1-hour, 2-hour and 3-hour datasets are compared, it is seen that only 12th dataset does not agree with the generalization that if the observing-session duration is shortened, the RMS of the positioning errors worsens because of its large magnitude of error in the up direction.

The reason for the reduction in the positioning accuracy as the observing-session duration is shortened would be that the PPP method provides the float ambiguity resolution which depends on the change of satellite geometry in order to make unknown parameters
converge to their correct values. Since the change of satellite geometry in short observing-session durations is less than that in long observing-session durations, the positioning accuracy worsens as the observing-session duration is shortened (Kouba and Herous, 2001). Since the variation of the RMS of the positioning errors was explained in the static PPP experiment (Section 5.4.1), it is not repeated here again.

The mean RMS of the positioning errors will be discussed in the next section. However, since the RMS of the positioning errors changes with the average number of Galileo satellites, the mean RMS of the positioning errors is calculated in two different cases. In case 1, all datasets are used while in case 2, only datasets in which the average number of Galileo satellites is more than two are used. See Table 7 for session details.

5.4.5.2 Overall Analysis

- Case 1

This case presents the mean RMS of the positioning errors for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions from all 1-hour, 2-hour and 3-hour datasets collected at POTS, WIND, RIO2 and ULAB stations, which is given in Table 26.
<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>East</strong></td>
<td>1-hour</td>
<td>0.037</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>2-hour</td>
<td>0.022</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>3-hour</td>
<td>0.018</td>
<td>0.014</td>
</tr>
<tr>
<td><strong>North</strong></td>
<td>1-hour</td>
<td>0.015</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>2-hour</td>
<td>0.008</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>3-hour</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td><strong>Up</strong></td>
<td>1-hour</td>
<td>0.036</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>2-hour</td>
<td>0.017</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>3-hour</td>
<td>0.013</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 26. Mean RMS of the positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions from all 1-hour, 2-hour and 3-hour datasets collected at POTS, WIND, RIO2 and ULAB stations. See Table 7 for session details.

As seen in Table 26, the positioning accuracies of all scenarios decrease as the observing-session duration is shortened. Fortunately, the difference between the positioning accuracies of combined GPS/GLONASS solutions from 3-hour and 2-hour datasets is small. Therefore, according to this numerical results, 2-hour GPS/GLONASS observations may be used instead of 3-hour observations, which reduces labor and equipment costs and simplifies field work.

Furthermore, the difference between the positioning accuracies of combined GPS/GLONASS/Galileo and combined GPS/GLONASS increases as the observing-session duration is shortened. The reason for this would be that in short dataset, the compensations of the new biases and parameters introduced by Galileo satellites with their
contribution to the overall satellite geometry get harder since the satellite geometry changes less than that in long observing-session durations.

- **Case 2**

  This case presents the mean RMS of the positioning errors for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions from the 1-hour, 2-hour and 3-hour datasets in which the average number of Galileo satellites is more than two, which is given in Table 27. The datasets were observed at POTS, WIND, RIO2 and ULAB stations.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GPS/GLONASS</th>
<th>GPS/GLONASS/Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>1-hour</td>
<td>0.040</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>2-hour</td>
<td>0.025</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>3-hour</td>
<td>0.018</td>
<td>0.014</td>
</tr>
<tr>
<td>North</td>
<td>1-hour</td>
<td>0.014</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>2-hour</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>3-hour</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Up</td>
<td>1-hour</td>
<td>0.036</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>2-hour</td>
<td>0.018</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>3-hour</td>
<td>0.016</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 27. Mean RMS of the positioning errors [meters] for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions from the 1-hour, 2-hour and 3-hour datasets in which the average number of Galileo satellites is more than two. The datasets were collected at POTS, WIND, RIO2 and ULAB stations. See Table 7 for session details.

As seen in Table 27, the positioning accuracies in all scenarios again decrease as the observing-session duration is shortened. However, the difference between the
positioning accuracies of combined GPS/GLONASS/Galileo and combined GPS/GLONASS becomes smaller as compared to the RMS values in case 1. This result confirms the explanation given above.

5.4.5.3 Conclusion

The performance of the static PPP method is analyzed by comparing GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions from 1-hour, 2-hour and 3-hour observing-session duration datasets. From the numerical results, it can be stated the positioning accuracies in all scenarios decrease as the observing-session duration is shortened. However, the positioning accuracies of combined GPS/GLONASS solutions from 3-hour and 2-hour datasets are similar. Therefore, 2-hour GPS/GLONASS observations may be used instead of 3-hour observations in the rapid-static GNSS applications, such as rapid-static landslide monitoring.

Additionally, the difference between the positioning accuracies of combined GPS/GLONASS/Galileo and combined GPS/GLONASS increases as the observing-session duration is shortened. However, it is also shown that this difference between the positioning accuracies of combined GPS/GLONASS/Galileo and combined GPS/GLONASS decreases as the more Galileo satellites are used. This result is promising in that when the number of operational Galileo satellites increases, the positioning accuracy will be further improved.

5.4.6 Experiment 6: Comparison of PPP and DGPS

The comparison of PPP and DGPS methods is conducted using a total of 7 three-hour datasets windowed from 24-hour daily datasets collected at POTS and WROC stations.
on seven consecutive days, i.e. August 7-13, 2015 (see Figure 36). POTS station is equipped with JAVAD TRE_G3TH DELTA receiver and JAV_RINGANT_G3T antenna while WROC station is equipped with LEICA GR25 receiver and LEIAR25.R4 + LEIT antenna. In DGPS method, POTS station is used as a rover station and WROC station is used as a reference station. The separation length between them is 309 km. The reason of choosing this separation length is that as time of writing, the network of MGEX stations was still developing and thus station density was so low, furthermore, the observation datasets of some stations were not available on the Internet. PPP solutions are obtained using the observations collected at POTS station. In both methods, the solutions are obtained using combined GPS/GLONASS/Galileo observations. All observations have a sampling interval of 30 seconds. The multi-GNSS processing software RTKLIB is used to process the datasets. In both methods, the default satellite elevation cutoff angle of 15° is maintained (Cai, 2009; Takasu, 2013) and extended Kalman filter is used for the estimation. In the PPP method, the parameter settings and special error sources corrections are the same as the previous kinematic PPP processing given in Section 5.4.2.

DGPS solutions are obtained with two different processing settings. In the first processing setting, the ionospheric and zenith total tropospheric delays are estimated as an unknown. The precise satellite orbit and clock products of GFZ are used to mitigate the satellite orbit and clock errors. Furthermore, the solid earth tide and ocean tide loading corrections are applied to the measurements. In the second processing setting, the only difference is that the ionospheric delay is mitigated using the ionosphere-free linear combination. The processing setting configurations of RTKLIB are given and explained in
Appendix. The positioning solutions are compared with the corresponding IGS weekly solutions in order to find the positioning errors. Refer to Section 5.3, for more details about the statistical evaluation strategy.

The session details of the datasets and also Kp values for the corresponding days are listed in Table 28. Note that Kp index represents the overall level of ionospheric activities. It is derived from ground-based magnetic field measurements and ranges from 0-9. A quiet day will have Kp values less than three. Kp values between three and five correspond to a moderately disturbed field, while Kp values of six or greater occur during a major magnetic storm. This values are downloaded from National Oceanic and Atmospheric Administration (NOAA) website (http://www.swpc.noaa.gov/products/planetary-k-index).

Figure 36. The geographical location of POTS and WROC stations
Datasets

<table>
<thead>
<tr>
<th>Date</th>
<th>Start and End Time (GPS Time)</th>
<th>Dataset Number</th>
<th>Average Number of Galileo Satellites</th>
<th>Kp Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 7, 2015</td>
<td>09.00 – 12.00</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>August 8, 2015</td>
<td>15.00 – 18.00</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>August 9, 2015</td>
<td>20.00 – 23.00</td>
<td>3</td>
<td>2.84</td>
<td>3</td>
</tr>
<tr>
<td>August 10, 2015</td>
<td>06.00 – 09.00</td>
<td>4</td>
<td>1.80</td>
<td>4</td>
</tr>
<tr>
<td>August 11, 2015</td>
<td>12.00 – 15.00</td>
<td>5</td>
<td>2.74</td>
<td>2</td>
</tr>
<tr>
<td>August 12, 2015</td>
<td>18.00 – 21.00</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>August 13, 2015</td>
<td>06.00 – 09.00</td>
<td>7</td>
<td>1.68</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 28. Session details of the datasets collected at POTS station and used in the comparison of PPP and DGPS methods experiment

5.4.6.1 Analysis of Positioning Accuracy and Convergence Time

The processing solution of the 5th dataset which is the 3-hour dataset windowed from the 24-hour daily dataset collected at POTS station on August 11, 2015 is chosen for an illustration purpose.

Figure 37 shows the positioning errors for combined GPS/GLONASS/Galileo solutions with the PPP and DGPS methods for the 5th dataset collected at POTS station on August 11, 2015 in east, north and up directions, respectively.
Table 29 gives the RMS of the positioning errors for combined GPS/GLONASS/Galileo solutions for the 5th dataset collected at POTS station on August 11, 2015 in east, north and up directions, respectively.
<table>
<thead>
<tr>
<th></th>
<th>PPP</th>
<th>DGPS w/ iono estimation</th>
<th>DGPS w/ iono-free</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>0.007</td>
<td>0.012</td>
<td>0.013</td>
</tr>
<tr>
<td>North</td>
<td>0.005</td>
<td>0.015</td>
<td>0.022</td>
</tr>
<tr>
<td>Up</td>
<td>0.029</td>
<td>0.050</td>
<td>0.058</td>
</tr>
</tbody>
</table>

Table 29. RMS of the positioning errors [meters] for combined GPS/GLONASS/Galileo solutions for the 5<sup>th</sup> dataset collected at POTS station on August 11, 2015. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination.

As seen in Table 29, the positioning solutions of PPP method improves the positioning accuracy by 42%, 67% and 42% over those of DGPS with ionospheric error estimation in east, north and up directions, respectively. The reason for this would be that PPP uses more observations (Geng, 2011) and estimates less unknown parameters than DGPS since DGPS can use only the observations of the satellites observed by reference and rover stations simultaneously and estimates the ionospheric delays for each satellites and the zenith tropospheric total delays at both the reference and rover stations. Refer to Takasu (2013), for more details. In order to illustrate the explanation that PPP uses more observations than DGPS, the visible satellites for POTS and WROC stations are given in Figures 38 and 39, respectively.
Figure 38. The visible satellites for the 3-hour dataset collected at POTS station at 12:00-15:00 GPS Time on August 11, 2015. Green denotes the satellites transmitting data in L1 and L2, yellow denotes the satellites transmitting data in only L1, blue denotes the satellites transmitting data in L1, L2 and L5 and red denotes the satellites transmitting data in L1 and L5. In y axis, G denotes GPS, R denotes GLONASS and E denotes Galileo. The unit of time is GPS Time (HH:MM)
Figure 39. The visible satellites for the 3-hour dataset collected at WROC station at 12:00-15:00 GPS Time on August 11, 2015. Green denotes the satellites transmitting data in L1 and L2, yellow denotes the satellites transmitting data in only L1, blue denotes the satellites transmitting data in L1, L2 and L5 and red denotes the satellites transmitting data in L1 and L5. In y axis, G denotes GPS, R denotes GLONASS and E denotes Galileo. The unit of time is GPS Time (HH:MM)

As seen in Figure 38 and 39, PPP uses more observations than DGPS. Some examples of the satellites which provides more observations to PPP are G4, G8, G27, R9, R10, R20. The reason for the differences between the observations of POTS and WROC stations would be the different geographical locations of these stations since a GNSS constellation does not provide a uniform satellite sky distribution (Santerre, 1991). Note
that even if the satellite elevation cutoff angle was set to 0°, this phenomenon was again seen.

Furthermore, Table 29 also states that the positioning solutions of the DGPS with ionospheric error estimation improve the positioning accuracy by 8%, 32% and 14% over those of DGPS with ionosphere-free linear combination in east, north and up directions, respectively. The reason of this improvement would be the noise in the DGPS with ionosphere-free linear combination, which is three times larger than the noise in the DGPS with ionospheric error estimation. For more details, refer to Section 4.5.1.

Table 30 shows the convergence time of the positioning errors for combined GPS/GLONASS/Galileo solutions for the 5th dataset collected at POTS station on August 11, 2015 in east, north and up directions, respectively.

<table>
<thead>
<tr>
<th></th>
<th>PPP</th>
<th>DGPS w/ iono estimation</th>
<th>DGPS w/ iono-free</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>71</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>North</td>
<td>15</td>
<td>22</td>
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</tr>
<tr>
<td>Up</td>
<td>14</td>
<td>86</td>
<td>86</td>
</tr>
</tbody>
</table>

Table 30. The convergence times [the number of 30-sec epochs] for combined GPS/GLONASS/Galileo solutions for the 5th dataset collected at POTS station on August 11, 2015. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination.
As seen in Table 30, PPP improves the convergence time by 32% and 84% over the DGPS methods in the north and up directions, respectively. However, it worsens the convergence time by 29 and 28.5 minutes over the DGPS with ionospheric error estimation and the DGPS with ionosphere-free linear combination in the east direction, respectively. The reason of the worsening of the convergence time in the east direction should be further investigated whether it is related to this dataset or not.

Figures 40, 41 and 42 provide the RMS of the positioning errors for combined GPS/GLONASS/Galileo solutions with PPP and DGPS methods for each dataset collected at POTS station in east, north and up directions, respectively.

Figure 40. The RMS of the positioning errors [meters] in the east direction for combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 28 for session details. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination.
Figure 41. The RMS of the positioning errors [meters] in the north direction for combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 28 for session details. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination.

Figure 42. The RMS of the positioning errors [meters] in the up direction for combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 28 for session details. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination.
In Figures 40, 41 and 42, one can see that if the average number of Galileo satellites is less than two, DGPS is better than PPP since PPP may not compensate the new biases and parameters introduced by Galileo satellites while DGPS can eliminate the receiver and satellite clock offset errors and also the hardware delay biases except for GLONASS inter-frequency bias (IFB). See Section 4.3 for more details. For example, the RMS of the positioning errors for PPP solutions are bigger than those of the DGPS solutions in the 4th and 7th datasets in which the average numbers of Galileo satellites are 1.80 and 1.68, respectively. If the average number of Galileo satellites is more than two, then PPP provides better positioning accuracy than DGPS.

Figures 43, 44 and 45 provide the convergence times for combined GPS/GLONASS/Galileo solutions with PPP and DGPS methods for each dataset collected at POTS station in east, north and up directions, respectively.
Figure 43. The convergence times [the number of 30-sec epochs] in the east direction for combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 28 for session details. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination.

Figure 44. The convergence times [the number of 30-sec epochs] in the north direction for combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 28 for session details. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination.
Figure 45. The convergence times [the number of 30-sec epochs] in the up direction for combined GPS/GLONASS/Galileo solutions for each dataset collected at POTS station. See Table 28 for session details. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination.

As seen in Figure 43, the worsening of the convergence time in the east direction is not related to the number of Galileo satellites used. Even if the average number of Galileo satellites is more than two, PPP may worsen the convergence time in the east direction over DGPS. Note that we also saw this phenomenon in the 5th dataset collected at POTS on August 11,2015, which was given for an illustration purpose. The reason for this would be the hardware biases which are lumped to the integer ambiguity term as detailed in Section 4.4. As explained before, the magnitudes of these hardware biases change day to day (Cai, 2009). As their magnitudes are large, they may worsen the float ambiguity resolution. As the ambiguity resolution worsens, the convergence time of positioning solutions in all directions also increases (Ge et al., 2008; Geng et al., 2010; Collins et al., 2010) while the
biggest worsening is seen in the east direction since the east direction is up to five times more affected from the ambiguity resolution than the north and up directions due to the orbit design and motion of satellites (Blewitt, 1989). In our results, we see the worsening in the east direction, which supports the explanation given above. In DGPS applied in double difference mode, these hardware biases are canceled out thus the changes in their magnitudes do not affect the ambiguity resolution in the DGPS method except GLONASS inter-frequency bias (IFB) as explained before.

As seen in Figure 44, in the north direction, it is seen that both the DGPS and PPP methods require almost same convergence times. Figure 45 states that in the up direction, the convergence time of PPP is slightly better than DGPS methods.

The mean RMS of the positioning errors and mean convergence time for all datasets will be discussed in the next section. However, these values change with the average number of Galileo satellites, the mean RMS of the positioning errors and the mean convergence time are calculated in two different cases. In case 1, all datasets are used while in case 2, the datasets in which the average number of Galileo satellites is more than two are used. See Table 28 for session details.

5.4.6.2 Overall Analysis

• Case 1

This case presents the mean RMS of the positioning errors and the mean convergence time for combined GPS/GLONASS/Galileo solutions with the PPP and DGPS methods for all datasets collected at POTS station, which are given in Table 31 and 32, respectively.

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Table 31. Mean RMS of the positioning errors [meters] for combined GPS/GLONASS/Galileo solutions for all datasets collected at POTS station. See Table 28 for session details. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination

<table>
<thead>
<tr>
<th></th>
<th>PPP</th>
<th>DGPS w/ iono estimation</th>
<th>DGPS w/ iono-free</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>0.029</td>
<td>0.023</td>
<td>0.036</td>
</tr>
<tr>
<td>North</td>
<td>0.012</td>
<td>0.011</td>
<td>0.013</td>
</tr>
<tr>
<td>Up</td>
<td>0.047</td>
<td>0.033</td>
<td>0.039</td>
</tr>
</tbody>
</table>

Table 32. Mean convergence time [the number of 30-sec epochs] for combined GPS/GLONASS/Galileo solutions for all datasets collected at POTS station. See Table 28 for session details. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination

<table>
<thead>
<tr>
<th></th>
<th>PPP</th>
<th>DGPS w/ iono estimation</th>
<th>DGPS w/ iono-free</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>79</td>
<td>80</td>
<td>89</td>
</tr>
<tr>
<td>North</td>
<td>25</td>
<td>24</td>
<td>32</td>
</tr>
<tr>
<td>Up</td>
<td>42</td>
<td>64</td>
<td>72</td>
</tr>
</tbody>
</table>

Table 31 states that the mean positioning accuracies of the DGPS with ionospheric error estimation are better by 21%, 8% and 30% than those of PPP in east, north and up directions, respectively, due to fact that if the number of Galileo satellites is less than two, the positioning accuracy of PPP decreases, as explained before. In addition, it can be stated
that the average result confirms that the DGPS with ionospheric error estimation improves the positioning accuracy by 36%, 15% and 15% over the DGPS with ionosphere-free linear combination in east, north and up directions, respectively.

As seen in Table 32, the differences between the mean convergence time of the PPP and DGPS with ionospheric error estimation methods are 0.5, 0.5 and 10 minutes in east, north and up directions, respectively. Furthermore, the DGPS with ionospheric error estimation method improves the convergence time by 10%, 25% and 11% over the DGPS with ionosphere-free linear combination method in the east, north and up directions, respectively.

- **Case 2**

  This case presents the mean RMS of the positioning errors and the mean convergence time for combined GPS/GLONASS/Galileo solutions with the PPP and DGPS methods for the datasets in which the average number of Galileo satellites is more than two, which are given in Table 33 and 34. The observations are collected at POTS station.
Table 33. Mean RMS of the positioning errors [meters] for combined GPS/GLONASS/Galileo solutions for the datasets in which the average number of Galileo satellites is more than two. The datasets were collected at POTS station. See Table 28 for session details. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination.

<table>
<thead>
<tr>
<th></th>
<th>PPP</th>
<th>DGPS w/ iono estimation</th>
<th>DGPS w/ iono-free LC</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>0.017</td>
<td>0.028</td>
<td>0.042</td>
</tr>
<tr>
<td>North</td>
<td>0.009</td>
<td>0.012</td>
<td>0.013</td>
</tr>
<tr>
<td>Up</td>
<td>0.034</td>
<td>0.035</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Table 34. Mean convergence time [the number of 30-sec epochs] for combined GPS/GLONASS/Galileo solutions for the datasets in which the average number of Galileo satellites is more than two. The datasets were collected at POTS station. See Table 28 for session details. PPP refers to the positioning solutions obtained with PPP method. DGPS w/ iono-estimation refers to the positioning solutions obtained DGPS with ionospheric error estimation as an unknown. DGPS w/ iono-free refers to the positioning solutions obtained with DGPS method with ionosphere-free linear combination.

<table>
<thead>
<tr>
<th></th>
<th>PPP</th>
<th>DGPS w/ iono estimation</th>
<th>DGPS w/ iono-free LC</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>74</td>
<td>77</td>
<td>105</td>
</tr>
<tr>
<td>North</td>
<td>27</td>
<td>32</td>
<td>37</td>
</tr>
<tr>
<td>Up</td>
<td>35</td>
<td>60</td>
<td>70</td>
</tr>
</tbody>
</table>

Table 33 shows that the mean positioning accuracies of PPP are better by 39%, 25% and 3% than those of the DGPS with ionospheric error estimation method in east,
north and up directions, respectively. In addition, the DGPS with ionospheric error estimation method improves the positioning accuracy by 33%, 8% and 22% over the DGPS with ionosphere-free linear combination method in east, north and up directions, respectively. These results confirm the results of 5th dataset given for an illustration purpose above.

As seen in Table 34, the mean convergence time of PPP are better by 4%, 16% and 42% than those of the DGPS with ionospheric error estimation method in east, north and up directions, respectively. In addition, the DGPS with ionospheric error estimation method shortens the mean convergence time by 27%, 14% and 14% over the DGPS with ionosphere-free linear combination method in east, north and up directions, respectively.

5.4.6.3 Conclusion

In this experiment, the performance of the PPP and DGPS methods are compared using combined GPS/GLONASS/Galileo observations in order to investigate the possible benefit of multi-GNSS combination on the PPP and DGPS methods for remote area applications, such as offshore resources exploration, sea floor mapping (Bisnath and Gao, 2009; Geng et al., 2010; Tegedor et al., 2014) and sea floor crustal deformation monitoring (Fujita et al., 2006; Tadokoro et al., 2006).

According to the numerical results, it is found that if the average number of Galileo satellites is more than two, PPP provides better positioning accuracy than DGPS. However, if the average number of Galileo satellites is less than two, DGPS provides better positioning accuracy than PPP. Additionally, the DGPS with ionospheric error estimation method provides better positioning accuracy and convergence time than the DGPS with
ionosphere-free linear combination method. PPP may either provide better or worse convergence time than DGPS, which may depend on the magnitude of the hardware biases lumped to the integer ambiguity term.
CHAPTER 6: CONCLUSIONS and RECOMMENDATIONS

This thesis focuses on the multi-GNSS PPP method using simultaneous GPS, GLONASS and Galileo observations. For this purpose, both the functional and stochastic models of the state-of-art multi-GNSS PPP method are provided in details and the performance of the multi-GNSS PPP method was assessed with various experiments presented in Chapter 5 using the static datasets collected at MGEX stations. The conclusions obtained from these experiments are given in the following:

The performance of the static PPP method is analyzed by comparing GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions. According to the numerical results, combined GPS/GLONASS improves the positioning accuracy over GPS-only. In addition, combined GPS/GLONASS/Galileo may either improve or worsen the positioning accuracy over combined GPS/GLONASS, which depends on the average number of Galileo satellites used. However, it is also shown that positioning accuracy can be improved using more Galileo satellites.

The performance of the kinematic PPP method is analyzed by comparing GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions. According to the numerical results, combined GPS/GLONASS improves both the positioning accuracy and convergence time over GPS-only. It is also found that if the average number of Galileo satellites is more than two, combined GPS/GLONASS/Galileo
can improve the positioning accuracy and convergence time over combined GPS/GLONASS. However, the convergence time in the east direction would also be related to the magnitude of hardware biases. These results are promising in that when the number of operational Galileo satellites increases, the positioning accuracy and convergence time will be further improved.

The impact of different precise satellite orbit and clock products of three IGS MGEX analysis centers on the positioning accuracy are analyzed. It is found that the solutions obtained with the precise satellite orbit and clock products of GFZ are better than those of CODE and Wuhan University at 30-second solution sampling interval.

The performance of GPS-only PPP, combined GPS/GLONASS PPP and combined GPS/GLONASS/Galileo PPP are demonstrated in terms of positioning accuracy and availability under different elevation cutoff angles (15°, 25° and 35°) to simulate constrained environments. From numerical results, it can be stated that both the positioning accuracy and availability decrease as the elevation cutoff angle increases. Combined GPS/GLONASS improves both the positioning accuracy and availability significantly over GPS-only. Furthermore, it is also found that combined GPS/GLONASS/Galileo may either improve or worsen both positioning accuracy and availability slightly over combined GPS/GLONASS, which depends on the number of Galileo satellites used.

In addition, the positioning accuracies of GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions worsen as the observing-session duration decreases from 3-hour to 1-hour. However, it is also found that the positioning accuracy of combined GPS/GLONASS solutions from 2-hour and 3-hour are very similar. Therefore,
2-hour GPS/GLONAS observations can be collected instead of 3-hour in order to reduce labor and equipment costs and simplify field work.

The performance of the PPP and DGPS methods are compared using combined GPS/GLONASS/Galileo measurements in order to investigate the possible benefits of multi-GNSS combination on the PPP and DGPS methods. It is found that the positioning accuracies of the DGPS with ionospheric error estimation method are better than those of PPP if the average number of Galileo satellites is less than two. However, if the average number of Galileo satellites is more than two, the positioning accuracies of PPP are better than those of the DGPS with ionospheric error estimation in east, north and up directions, respectively. PPP may either provide better or worse convergence time than DGPS, which may depend on the amount of the hardware biases lumped to the ambiguity term and cancel out through double-differencing. No relationship is found between the number of Galileo satellites and the convergence time in the PPP method. In addition, it is also found that the DGPS with ionospheric error estimation method improves both the positioning accuracy and convergence time over the DGPS with ionosphere-free linear combination method.

For future research works, the following recommendations are provided:

1) The real-time precise satellite orbit and clock products for Galileo satellites will be expected to be available in the future. Therefore, the performance of the multi-GNSS PPP method may be evaluated for real-time applications.

2) When BeiDou has a global convergence, it can also be combined with GPS/GLONASS/Galileo PPP and its effect in terms of positioning accuracy and convergence time can be analyzed.
3) As the time of writing, three analysis centers were proving their precise products and there was not any available combined precise product. With the expected combined precise products and more visible Galileo satellites, the performance of multi-GNSS PPP can be further assessed with GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo solutions.

4) In the PPP model, instead of using ionosphere-free linear combination, the ionospheric error can be mitigated by estimating it as an unknown.

5) The performance of the multi-GNSS PPP method can be evaluated using between-satellite single difference equations in both the static and kinematic modes.
References


Guo, Q. (2015). Precision comparison and analysis of four online free PPP services in static positioning and tropospheric delay estimation. GPS Solutions, 19(4), 537-544.


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Appendix: The Processing Settings of RTKLIB
This appendix describes how GNSS measurements can be processed to obtain positioning solutions with RTKLIB software by providing and explaining the processing settings used in this thesis. RTKLIB consists of several application programs (APs) and a portable program library. In this study, application program known as RTKPOST with RTKLIB ver.2.4.3 b5 is used to process the datasets. Figure 46 shows the main graphical user interface (GUI) of RTKPOST.

![RTKPOST GUI](image)

Figure 46. The main GUI of RTKPOST

First, the setting configurations of RTKLIB is explained for experiment 1 which evaluates the performance of static multi-GNSS PPP. After that, only differences in the
setting configurations of RTKLIB between experiment 1 and the remaining experiments are explained.

In Figure 46, the observation files are uploaded to the field of RINEX OBS, whereas the navigation files, the precise satellite orbit files and the precise satellite clock files are uploaded to the fields of RINEX *NAV/CLK, SP3, IONEX or SBS/EMS, respectively.

Due to the fact that the datasets are daily, the start and the end times of three-hour datasets are introduced in the field of Time Start (GPST) and Time End (GPST), respectively.

By pushing Options button, the processing options are set. Figure 47 shows the Setting 1 tab of the Options menu.

![Options Menu](image)

Figure 47. Setting 1 tab of the RTKLIB Options menu used in experiment 1
**Position Mode** is set to PPP Static. For GPS and GLONASS, L1/L2 and for Galileo L1/L5 frequencies are used automatically. **Filter Type** is maintained as Forward in order to analyze the performance of the convergence time.

**Elevation Mask** (°) refers to the elevation cutoff angle that is maintained at 15°.

Both the solid earth tide and ocean tide loading corrections with their formulas are given and explained in Section 3.3.3 and 3.3.4 are then applied by choosing Solid/OTL in the field of **Earth Tides Correction**.

In the field of **Ionosphere Correction**, Iono-Free LC is chosen to mitigate ionospheric delay using the ionosphere-free linear combination explained in Section 4.4.

In the field of **Troposphere Correction**, Estimate ZTD is chosen to estimate total zenith delay as an unknown as explained in Section 4.4.

In the field of **Satellite Ephemeris/Clock**, precise is chosen in order to mitigate the satellite orbit and clock errors that use the precise satellite orbit and clock products.

By checking **Sat PCV** and **Rec PCV**, the satellite and receiver antenna phase center offsets are corrected, which is formulated and explained in Section 3.3.1.

The phase wind-up correction, as explained and formulated in Section 3.3.2 is applied to the carrier-phase measurements by checking **PhWindup**.

The navigation satellite systems desired for the positioning solutions are introduced by checking them at the bottom of Setting 1 tab of the RTKLIB options menu illustrated in Figure 47. Figure 48 shows the Setting 2 tab of the RTKLIB Options menu.
The setting 2 tab of the RTKLIB options menu are related to the ambiguity resolution. Since the ambiguity terms in PPP are not integer numbers as a result of the uncalibrated hardware delays, as explained in Section 4.4, within the field of Integer Ambiguity Res (GPS/GLO/BDS), it is set as OFF to estimate ambiguities as float numbers. The remaining options are related to the integer ambiguity resolution and the thresholds for the processing. Due to the fact that integer ambiguity resolution cannot be used in this thesis, they are not related to the processing of the datasets. The recommended default settings are used for the thresholds. Output tab of the RTKLIB Options menu is shown in Figure 49.
In the field of **Solution Format**, the output solution format is set to X/Y/Z-ECEF in order for the solutions to exist in the Cartesian coordinate system. In order to write the headers and the processing options to the output files, the fields of **Output Header** and **Processing Options** are set to ON. **Time Format** is chosen as hh:mm:ss GPST to label solutions with the time format set to yyyy/mm/dd hh:mm:ss in GPS time. By setting single in the fields of the **Solution for Static Mode**, single solution is obtained. Figure 50 shows the stats tab of RTKLIB options menu.
This menu is related to the stochastic model menu of the PPP method which is explained in Section 4.5. **Measurement Errors (1-sigma)** section is related to the covariance matrix of the measurement noise vector of the Extended Kalman Filter. Please refer to Section 4.5.1 for more details and the meaning of these numbers. **Process Noises (1-sigma/sqrt(s))** section is related to the covariance matrix of the system noise vector of the Extended Kalman Filter. Refer to Section 4.5.2 for the meaning of these numbers and their determination. The positions tab of the RTKLIB Options menu is illustrated in Figure 50.

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<table>
<thead>
<tr>
<th>Measurement Errors (1-sigma)</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Code/Carrier-Phase Error Ratio L1/L2</td>
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<td>100.0</td>
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<tr>
<td>Carrier-Phase Error a+b/sinEl (m)</td>
<td>0.003</td>
<td>0.003</td>
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<tr>
<td>Carrier-Phase Error/Baseline (m/10km)</td>
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<td></td>
</tr>
<tr>
<td>Doppler Frequency (Hz)</td>
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<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process Noises (1-sigma/sqrt(s))</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiver Accel Horiz/Vertical (m/s2)</td>
<td>1.00E+01</td>
<td>1.00E+01</td>
</tr>
<tr>
<td>Carrier-Phase Bias (cycle)</td>
<td>1.00E-04</td>
<td></td>
</tr>
<tr>
<td>Vertical Ionospheric Delay (m/10km)</td>
<td>1.00E-03</td>
<td></td>
</tr>
<tr>
<td>Zenith Tropospheric Delay (m)</td>
<td>1.00E-04</td>
<td></td>
</tr>
<tr>
<td>Satellite Clock Stability (s/s)</td>
<td>5.00E-12</td>
<td></td>
</tr>
</tbody>
</table>
Antenna Type (*: Auto) is checked and * is written to the related field of Rover in order to allow the program to automatically recognizes the antenna type and antenna deltas from RINEX observation files. Figure 52 shows the Files tab of the RTKLIB Options menu.
Figure 52. Files tab of the RTKLIB Options menu used in experiment 1

In this tab, the required files for the special error sources, as explained in Section 3.3, may be uploaded. The same IGS ANTEX file being igs08.1861.atx is uploaded to the fields **Satellite/Receiver Antenna PCV File ANTEX/NGS PCV** for the satellite and receiver antenna phase center corrections, respectively. In the field of **EOP Data File**, the earth orientation parameter (EOP) data files are uploaded and in the field of **OTL BLQ File**, the ocean tide loading (OTL) coefficients files are uploaded.

For experiment 2, the performance of multi-GNSS kinematic PPP, the only **Positioning Mode** in Figure 47 is changed to Kinematic PPP. Refer to Section 4.5.2 to see the change in the stochastic model of the position coordinates.
For experiment 3, the comparison of the precise satellite orbit and clock products, the same processing settings of the static PPP are used by changing the precise satellite orbit and clock products on the main GUI of RTKPOST, as provided in Figure 46.

For experiment 4, which is the impact of the different elevation cutoff angles, only the elevation cutoff angles are changed in the field of Elevation Mask (°) within the processing settings of kinematic PPP.

For experiment 5, the impact of different observing-session duration, the processing settings of the static PPP are used.

For experiment 6, the comparison of PPP and DGPS, the PPP solutions are obtained with the processing settings of kinematic PPP provided above. The DGPS solutions are estimated with two different processing settings. In the first processing setting, the setting 1 tab of the RTKLIB options menu given in Figure 47, the Position Mode is set to kinematic. Additionally, as explained in Section 4.3, the atmospheric, orbit and special errors are not fully eliminated by the differencing techniques for the long baselines, hence, and Iono-free LC is chosen in the field of Ionosphere Correction. This allow to mitigate ionospheric delay using the ionosphere-free linear combination, as explained in Section 4.4. Estimate ZTD is set in the field of Troposphere Correction to estimate total zenith delay as an unknown as explained in Section 4.4. The field of Satellite Ephemeris/Clock is set to Precise, in order to use precise satellite orbit and clock products. Both solid earth tide and ocean tide loading corrections whose formulas provided and explained in Section 3.3.3 and 3.3.4 are applied to the carrier-phase measurements by selecting Solid/OTL in the field of Earth Tides Correction. Since RTKLIB does not allow integer ambiguity
resolution if the ionosphere-free linear combination is used, the field of **Integer Ambiguity Res (GPS/GLO/BDS)** is set as OFF to estimate ambiguities as float numbers. The coordinates of the reference station are introduced in the field of **Base Station** illustration in Figure 51.

In the second processing setting, all settings are as explained above except, the ionospheric error is mitigated through the estimation as an unknown instead of using the ionosphere-free linear combination.