Three Essays Regarding the Economics of Resources with Spatial-Dynamic Transition Processes

DISSERTATION

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By

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Abstract

This dissertation considers questions about the economics of resources exhibiting spatial-dynamic transition processes, specifically, invasive species and wildfires. These topics are increasingly important due to their large damage potentials and the increasing management budgets by governments, but the economics literature has only recently begun to incorporate both space and time into its analysis. The three essays in this manuscript tackle problems using empirics and numerical modelling in order to continue to expand the economic understanding of spatial-dynamic processes.

The first chapter empirically analyzes the decision of land developers to build single family homes near lakes invaded with Eurasian watermilfoil, an aquatic invasive plant. A duration model of land conversion is utilized to discover the change in likelihood that a single-family housing unit will be developed after the arrival of Eurasian watermilfoil. The results show a significant decrease in the likelihood that both lakefront and near lake properties will be developed into homes following a Eurasian watermilfoil invasion. This is shown to have a sizable
impact on the number of houses constructed near invaded lakes and suggests that uninvaded lakes are most likely being overdeveloped.

The second chapter develops a spatial-dynamic model of the optimal control of a general invasive species which is actively spreading throughout a landscape. Along with explicitly allowing the invasion to spread over space, this model permits the population to grow in intensity as well. It is discovered that optimal intensities of invasion vary over space and that ignoring the natural heterogeneity in the landscape leads to suboptimal management decisions. The inefficiency of these decision is shown to increase as the heterogeneity in ecological carrying capacity increases. The model is then applied to the Asian cap invasion in the Mississippi River basin and shows that the current strategy of preventing Asian carp from entering the Great Lakes may be optimal. Overall, this project showcases the importance of recognizing spatial heterogeneity in ecological carrying capacities along with delivering strong evidence that control efforts need to be spatially targeted to most efficiently manage a spreading invasive species.

Finally, the third chapter considers the fact that some spatial-dynamic processes, like wildfires, spread at different rates across landscapes that contain a variety of economically valued patches. Spread rates effect the costs of management, while the values of different properties influence the damages associated with a burn. This chapter
incorporates these complexities into a spatial-dynamic model of optimal wildfire management and finds efficient solutions for controlling multifaceted fire scenarios. Suppression of wildfires is shown to be more likely in areas where the economic values are spatially clustered, thereby allowing a minimum amount of control to have the maximum benefit. It is also shown that spatial location with respect to the initial ignition location is important in determining optimal control strategies in heterogeneous spread scenarios.
Dedication

To my wonderful wife, Beth, and my son, Jamie.

When you are surrounded by people who believe in you, it is impossible not to believe in yourself.
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None of this would be possible without the support of my wife, Beth, and the constant smiles from my 11 month old son, Jamie. Their unconditional love has brought me joy even on the toughest of days.

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Publications

Eurasian Watermilfoil Effects the Development of Lakefront

Fields of Study

Major Field: Agricultural, Environmental and Development Economics
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Chapter 1: The Effects of Eurasian Watermilfoil on the Development of Properties near Lakes

1.1 Introduction

The economic losses from invasive species in the United States are large. In estimating these losses, Pimentel, Zuniga, and Morrison (2005) identify over 50,000 invasive species in the United States, and report damages totaling $120 billion dollars per year. While this number is large, it looms even larger when compared to the trivial $459 million and $556 million spent by the federal government on invasive species prevention in 1999 and 2000. (Lovell and Stone 2005). With large damages and government spending on the rise, it is unsurprising that economists have begun to investigate the effects of optimal control strategies, policy measures, and management practices on limiting the spread of invasive species (Epanchin-Niell and Wilen 2012; Timar and Phaneuf 2009; MacPherson et al. 2006).

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1 Recently, several states have begun to recognize that invasive species are a serious threat to their natural resources and tourism sectors. This has led some to calls for substantial increases in invasive species prevention spending (Lynch 2014).
Some of the most infamous invasive species are aquatic. Zebra mussels, Silver Asian carp, and Eurasian watermilfoil plague freshwater lakes and rivers across the United States, and have been doing so for decades. These underwater invaders are a major concern for policy makers, especially in states where tourism relies heavily upon lakes and rivers. Since the unique characteristics of each invading species affects ecosystems in different ways, empirical studies concerning specific species are critically important to policymakers. Providing accurate information on the scope and scale of an invading species’ negative ecological and economic impacts is vital as researchers continue to find ways to combat them.

For more than a century, biologists and ecologists have studied invasive species and the ecosystem modifications stemming from their presence. Whether these modifications are driven by the invasive species (Clavero and García-Berthou 2005) or by other exogenous factors that make ecosystems vulnerable to invasion (i.e. habitat destruction (MacDougall and Turkington 2005)), the introduction of an invasive species often results in a lower quality environment than the one that existed prior. A large body of economic literature finds that individual decision makers respond to changes in their environment in order to avoid or mitigate economic losses and improve their well-being. This suggests
that an invasive species introduction will likely result in changes in economic behavior.

The existing economics literature offers surprisingly little empirical evidence of changes in human behavior driven not by invasive species policy, but by the presence of an invasive itself. Since invasive species often cause significant environmental change, their presence alters how humans interact with their environment; potentially causing unintended consequences on ecosystems through altered behavior. Understanding both the behavioral responses of individuals and the potentially large spillover impacts of this behavior is vital for the future development of invasive species policies aimed at reducing their negative impacts. This chapter provides the first evidence of altered human behavior in the presence of Eurasian watermilfoil as reflected through the supply side of new single family housing.

1.2 Context

Eurasian watermilfoil is an aquatic invasive plant native to Europe, Asia, and northern Africa. Though the exact date of arrival to North America is unknown, the plant was reported in several states by 1950 (Smith and Barko 1990). As of 2003, Eurasian watermilfoil was present in nearly every state.
The growth and propagation characteristics of Eurasian watermilfoil are incredible, and help to explain why it is such a nuisance as well as why it has so successfully spread over the last 60 years. Growing up from the bottom of a lake, the plant branches out after reaching the surface, forming a thick canopy of leaves and vines known as a milfoil bed. Milfoil beds deter those who recreate, because the vines tangle in boat motors and cling onto swimmers\(^2\). Milfoil beds also change the ecology of the lake, shading out native plants (Madsen, et. al. 1991) and changing predation patterns by providing new hiding places for invertebrates and small fish. Removing an invasion is nearly impossible given Eurasian watermilfoil’s remarkable reproductive characteristics. It is able to reproduce from stem fragments, therefore, pulling plants out or mowing them down only furthers its spread (Smith and Barko 1990). These stem fragments can also attach to, and be spread by boats, which explains the high incidence of milfoil in lakes with public water access and larger size.

Lakefront homeowners are also affected by the presence of milfoil. Along with depleted utility from lake recreation, several hedonic studies have shown that homeowners on invaded lakes face reduced property values. In one such study done in New Hampshire, Halstead and Michaud (2003) use a dummy variable to identify lakes invaded by milfoil as well as

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\(^2\) Eurasian watermilfoil has even been known to be a drowning hazard.
an interaction term between the size of the lake and the presence of milfoil. Using ordinary least squares estimation, the authors conclude that the presence of milfoil leads to decreased lakefront property values of 20%-40%.

More recently, Horsch and Lewis (2009) examine the impact of milfoil in a hedonic study of lakes in Wisconsin using a difference in differences methodology to account for time-invariant neighborhood characteristics. They find a 13% reduction in land values associated with lakefront properties on milfoil invaded lakes. Zhang and Boyle (2010) examine a measure of aquatic macrophyte cover on five Vermont lakes. Their study is unique in that measurements of submerged macrophyte are found for each individual property, which is more accurate than a simple dummy variable for each lake. They show that as milfoil increases and adds to the macrophyte already present in the lake, property values decrease by 1% - 16% for each incremental increase. Taken together, these studies provide clear evidence that milfoil substantially impacts housing markets, although no existing studies have specifically addressed the supply side of the market.

This chapter provides evidence of a housing market supply side response to milfoil invasions. Because milfoil is aquatic, it is expected that responses to a milfoil invasion will be strongest for homes nearer to lakes and that this response will decrease as the distance to lakes increases.
Since developers often decide to convert multiple individual parcels to housing concurrently, the impacts of milfoil on lakefront and near lake homes are both considered. A 400 meter buffer from publicly accessible lakes is used to define near lake homes since the amenities of these lakes are most likely to spill over into adjacent neighborhoods. This approach differs from the majority of the hedonic literature which focuses almost exclusively on lakefront properties. With this strategy, the effects of milfoil on both lakefront and non-lakefront development are estimated.

This analysis finds that both lakefront and near-lake development is substantially influenced by the presence of milfoil. This impact is negative, meaning that the probability of near lake properties being developed into single-family housing units decreases significantly after a milfoil invasion. This result holds even when properties are not located directly adjacent to a lake. To examine the extent to which this decreased likelihood might affect the total number of developed parcels, counterfactual simulations are performed in which the spread of Eurasian watermilfoil is assumed to have been halted in 1990. These simulations suggest, that in the absence of milfoil, an additional 2,128 single family homes would have been constructed near the lakes where invasion was prevented. This result may suggest that the presence of milfoil has accelerated development on and near non-invaded lakes, potentially leading to additional urban sprawl.
1.3 Duration Model

Duration models provide answers to the question of “how long until some event will occur?” by modeling the time it takes (the duration) for a transition (the failure) to occur. In a land conversion context, the baseline hazard is the portion of the hazard function which is identical across parcels, covariates are any variables that may influence the development of parcels (neighborhood characteristics, public policies, etc.), and coefficients are estimates of the change in duration caused by shifts in the covariates. The focus here is on the conversion of undeveloped properties into developed single-family housing units with an emphasis on this conversion for properties near to lakes.

Irwin and Bockstael (2004) are an early example of this type of supply-side analysis. They examine whether a parcel’s proximity to open space plays a role in the timing of the development decisions made by landowners. They estimate a Cox proportional hazards model and find that parcels in the vicinity of more open space are more likely to be developed than those near less open space. Using a large variety of parametric and semi-parametric duration models, Towe et al. (2008) address the issue of accelerating encroachment of urban areas onto agricultural land. Whether using a Weibull, Cox proportional hazard or a semi-parametric piece-wise exponential model, they find that an option of a farmland preservation program (i.e. farmers receiving payments to not develop their land)
significantly delays the conversion of previously undeveloped farmland. These results suggest that specification of the baseline hazard function may not be of great importance in duration analysis applied to land conversion. In the analysis performed in this chapter, a similar indifference to model specification is found.

The econometric foundation\(^3\) of duration analysis is based on a measure of the time to failure. This is represented by the random variable, \(T\), which has a continuous probability distribution \(f(t)\), where \(t\) is a realization of \(T\). The cumulative probability of an observation surviving until at least \(t\) is given by the survival function

\[
S(t) = 1 - \int_0^t f(s) \, ds = \text{Prob}(T \geq t).
\]

While this function is useful, analysts often wish to know the probability that an observation fails in a particular time period. For the land conversion problem addressed in this paper, the previous statement is equivalent to asking “What is the probability that a property will be developed in time period \(t\) given that it has not been developed in all time periods before \(t\)?” The hazard rate provides an answer to this question and is given by

\[
\lambda(t) = \frac{f(t)}{S(t)},
\]

\(^3\) The notation in this section largely follows the notation used in Greene (2012)
or equivalently,

\[ \lambda(t) = \lim_{\Delta t \to 0} \frac{\text{Prob}(t \leq T \leq t + \Delta t \mid T \geq t)}{\Delta t}. \]  

(3)

Several assumptions are required to estimate duration models. A primary assumption of concern is non-informative censoring. The implication of this is that data censoring must not be related to the probability that parcels are developed. This study contains right censoring, meaning that there are parcels that remain undeveloped from throughout the entire study period. This analysis includes over 31,000 right censored parcels to account for this concern. Another form of censoring is left censoring, which refers to all parcels developed before 1990. This study follows much of the land conversion literature and assumes that all parcels that become developed will stay developed throughout the length of the study, and thus, there can be no redevelopment. For this reason, all left censored parcels are dropped from the analysis.

Several approaches exist for modelling the hazard function (parametric, semi-parametric, and nonparametric), all of which have pros and cons. Parametric methods assume a parametric form of the hazard function, the most common of which is Weibull, due to the computational

\[ \text{Prob}(t \leq T \leq t + \Delta t \mid T \geq t) \]
simplicity of estimation and assumed monotonic changes in the baseline hazard over time. For much of the land conversion literature, especially prior to the housing collapse in 2008, this assumed increase in baseline hazard over time reflects to a large extent the U.S. housing market. An additional benefit of a parametric baseline hazard is the ease at which counterfactual simulations can be conducted.\(^6\)

The hazard function for the parametric Weibull model takes the form

\[ \lambda(t) = \lambda p(\lambda t)^{p-1} \]  

where

\[ \lambda_i = e^{x_i'\beta} \]  

The sign on the shape parameter, \( p \), determines whether the hazard is increasing or decreasing over time. When \( p = 1 \), the hazard remains constant over time and the Weibull model is equivalent to an exponential model. Using the survival and density functions for the Weibull model,

\[ S(t) = e^{(-\lambda t)^p} \]  

\[ f(t) = \lambda p(\lambda t)^{p-1} e^{(-\lambda t)^p} \]  

The likelihood function can be calculated.

\[ L_i = \prod_{i=1}^{N} \left( \lambda p(\lambda t)^{p-1} e^{(-\lambda t)^p} \right)^{d_i} \left( e^{(-\lambda t)^p} \right)^{1-d_i} \]  

---

\(^6\) While the simplicity of estimation that results from the parametric assumptions is appealing, several authors have examined semi-parametric approaches that avoid explicitly defining a baseline hazard function for sensitivity. In the land conversion literature, including initial analysis in this paper, there appear to be few qualitative differences across parametric and semi-parametric models.
$d_i$ is an indicator variable signaling whether observation $i$ is censored in time $t$. This likelihood forms the basis for the analysis that follows.

1.4 Data

The study area for this project consists of the seven county region encompassing the Twin Cities metro area of Minnesota\(^7\) (see figure 1). There are several reasons why this area is ideal for studying how milfoil affects housing supply decisions, especially around lakes. First, there are a large number of lakes which are well dispersed throughout the study area, and many of these lakes have been affected by Eurasian watermilfoil for decades. Watermilfoil was first discovered in Minnesota in 1987, and had spread to the vast majority of large lakes by 2005. More than half of the invaded lakes in the state are found within this seven county region. The Minnesota Department of Natural Resources collects data on the state’s lakes, which allows for rich economic analysis when it is paired with housing data from the area.

\(^7\) The counties in the study are Anoka, Ramsey, Dakota, Scott, Washington, Hennepin, and Carver.
1.4.1 Data on Lakes

A dataset on lakes was assembled and made available from the MetroGIS Datafinder (datafinder.org). This dataset contains information on lake area, clarity, and whether lakes have public parks or public water access. To supplement this dataset, a list of Eurasian watermilfoil invaded lakes and the year of each invasion are merged with the lake characteristics. This is a publically available list constructed by the Minnesota Department of Natural Resources (DNR)\(^8\). It should be noted that only the bodies of water that are classified as a “lake” by the Minnesota DNR and that have an official Department of Wildlife (DOW) lake number are considered lakes in this study\(^9\).

The initial sample of lakes consists of 1,435 individual freshwater lakes averaging approximately 68 acres in size. After removing bodies of water without DOW lake numbers, the final estimation set consists of 1,304 lakes averaging an area of 71 acres. Table 1 shows the number of invaded lakes from 1990 through 2005 for both the full sample of lakes and the estimation sample. Of the lakes used in estimation, 20 are invaded prior to 1990. This number increased to 92 by 2005. While these numbers...

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\(^8\) The full list is available as of 5/26/13 at the following URL: http://files.dnr.state.mn.us/eco/invasives/infestedwaters_newmilfoil.pdf

\(^9\) The list of waters invaded by milfoil only lists lakes with DOW lake numbers. Thus, it is not known whether lakes without these numbers are invaded or not. Consequently, all such lakes are dropped. These dropped bodies of water tend to be very small and are most likely marshy areas rather than open water.
may seem low, they represent some of the largest lakes in the region. The average size of an invaded lake is 409 acres whereas non-invaded lakes averaged just under 43 acres in size. In addition, 92% of invaded lakes had public water access compared to only 35% of non-invaded lakes. A map of the invaded lakes in the study region is shown in figure 2.

1.4.2 Price Index Data

In addition to the data on lakes, a complete dataset of new housing transactions is assembled for the entire study area. This dataset of housing transactions was originally compiled by Klaiber (2008), and subsequently used by Klaiber and Phaneuf (2010) in a study of open space valuation in the Twin Cities area. This dataset contains 448,209 observations of property sales from the year 1990 to 2005. The data cleaning steps for the housing transaction dataset are described in the appendix.

Descriptive statistics for key housing attributes of all transactions are shown in table 2. The expected trends for attributes of home transactions are found across near lake and non-lake homes. In particular, near lake homes are consistently larger, sold for a higher values, and located on more acreage. While the focus of this study is not

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10 Descriptive statistics for transactions broken divided by pre and post 1990 construction can be found in Table A.1 in the appendix.
on the determinants of housing price, as would be the case for a hedonic analysis, these trends suggest that the data conforms to prior expectations.

With this data, a spatially and temporally varying price index is created. This housing price index captures the location value of homes, net of their structural attributes, at the census tract level for each year in the study period. Importantly, the within-tract varying lake covariates are omitted from his model to ensure that the value of spatial proximity to lakes are not “netted-out” in the price index.

1.4.3 Creating a Price Index

Following a similar strategy to the one developed in Bayer, Keohane, and Timmins (2009), let $P_{i,l}$ denote the sale price of a single-family home in location $i$ on within tract $l$. For each individual year between 1990 and 2005, the hedonic model is specified as

$$P_{i,l} = \rho_l e^{X_i \beta + \epsilon_{i,l}}$$

(9)

where $X_i$ is a vector of observable housing characteristics and $\rho_l$ is a scaling parameter for tract $l$. By taking the log of both sides, the following regression is estimated for each year in the dataset allowing the hedonic price slope of different structural attributes to potentially change across time.

$$\ln P_{i,l} = \ln \rho_l + X_i \beta + \epsilon_{i,l}.$$  

(10)
Using a fixed effects estimation, this model allows for the recovery of an index for the price of a homogenous house in a particular census tract during a specific year. This model is estimated individually for each of the 16 years in the time period. Estimation results for a single year, 2000, are shown in table 3.

1.4.4 Developable Parcels and Links to Invasives

Duration modeling requires a dataset of all potentially developable parcels at each step of the study period. These potentially developable parcels can be categorized as either parcels that are developed during the study period or parcels that were developable but were not developed during the study period (right censored). Both subsets of the duration data must be constructed, and the process by which this is done is discussed in the ‘Data Cleaning’ section in the appendix. Table 4 shows a frequency distribution of new housing construction during the study period by year. The locations of this new housing construction are shown in figure 3. Note that fairly widespread development is prevalent during this period. In particular, locations further away from downtown and nearer to lakes experienced heavy development. Of the 153,912 new homes constructed over this period, 26,971 were located within 400 meters of a lake which they had access to. Including the right censored data, the total number of
vacant and developable parcels at the beginning of the study period is 184,937\textsuperscript{11}.

To serve as proxies for developer revenues and costs a variety of landscape variables are created and attached to each parcel in the dataset.\textsuperscript{12} Arc-GIS measurements of steep slopes and prime farm land (SSURGO) are created, and distances to the nearest metro center and highways are calculated for each residential parcel. In addition, time-varying municipal services boundaries, as well as local land use measures like the location of parks and golf courses, are overlaid. Summary statistics for each of these measures is presented in table 5.

Since milfoil is likely to have the largest impact on near lake properties several lake-distance classes of potentially impacted homes are created. First, spatial data on parcel maps identify lakefront parcels, which results in a total number of parcels identified as lakefront of 6,995, or approximately 4\% of the total parcels. Second, 23,774 non-lakefront homes are identified that are located within 400 meters of a publically accessible lake. Finally, a variable indicating, for each home, whether their nearest publically accessible lake is invaded or not is created. Figure 4 shows the number of residential developments that occur near invaded

\textsuperscript{11} Robustness checks for increased numbers of right censored data are shown in the appendix table A.3.

\textsuperscript{12} Duration models of land conversion are not limited to the study of residential property development. The competing risks duration model used by Hite et al. (2003) looked at all developments of parcels, including industrial and commercial.
and uninvaded lakes throughout the study period. Development is consistently higher on uninvaded lakes; however, this declines over time as fewer uninvaded lakes remain due to development and the spread of milfoil. Descriptions of all the covariates are displayed in table 6.

1.5 Econometric Results

This section shows econometric results for two models which control for different levels of time-constant hazard impacts using fixed effects. The seven county region used in this study is, geographically, very large. Therefore, it is likely that unobserved heterogeneity contributes to the land conversion decision. Using census tract level spatial fixed effects controls for some of this heterogeneity by allowing differences in the baseline conversion probabilities across space.

It is also possible that specific features of individual lakes are unobserved, yet, are in part driving conversion decisions. To account for this, the baseline census tract fixed effects specification is extended to include 1,304 lake specific fixed effects in addition to 650 tract fixed effects. These account for any features of the lakes that are time-constant (e.g. size) as well as any unobserved features that are also time-constant. To attach lake fixed effects to parcels, 1000 meter buffers are created around each lake. Parcels within each buffer are assigned a value of 1 for that particular lake fixed effect. Some parcels are located within close
proximity of multiple lakes, in which case they are associated with multiple lake fixed effects.

Table 7 presents results from a Weibull model including 650 census tract fixed effects to capture differences in baseline hazards across space.\textsuperscript{13} Estimates are reported as hazard ratios, which are measures of the probability of land conversion due to a change in a covariate (assuming that the parcel has not yet been developed). Hazard ratios are the exponential of the estimated coefficients; therefore, hazard ratios greater than one imply an increase in the likelihood of development (failure), whereas a hazard ratio less than one implies a decrease in the likelihood of the development.

The results into two groups for ease of discussion. The top panel of the table presents the key results showing the impact of proximity to lakes, indicators for lakefront, location of a parcel within 400 meters of public access and riverfront. As discussed previously, the lakefront, 400 meter access, and riverfront variables capture the joint effect of both the revenue and cost impacts of near-water development because they are omitted from the price index regression. Overall, they take on the expected signs that

\textsuperscript{13} Using a Weibull model makes a parametric assumption about the functional form of the baseline hazard. As a robustness check, a Cox proportional hazards model (Cox 1972) is also run (shown in Appendix table A.2). As in much of the existing land conversion literature, these results are qualitatively similar across specifications. As a result, the discussion is restricted to the Weibull model which has the byproduct of being significantly easier to estimate computationally.
lakefront, near lake and riverfront parcels are more likely to be developed than non-water homes.\textsuperscript{14}

The primary focus is on the interactions of lakefront and 400 meter access with an indicator of a time-varying measure of milfoil invasion. Additional interactions involving lake size, clarity, and public access are also included as the invasion of a lake could have a differential impact on homeowners if a lake is larger, has a higher clarity or has public water access. Examining the results in table 7, the estimates show that invaded lakes negatively affect the likelihood of development for all parcels near them. Parcels whose nearest accessible lake is invaded are nearly 9\% less likely to be developed that parcels whose nearest accessible lake was not invaded. If those parcels are within 400 meters of an access point they are 32\% less likely to experience development post invasion and if they are lakefront parcels they are 60\% less likely to develop.

Turning attention to additional interactions between invaded lakes and lake attributes, the estimates show that clarity moderates the negative impact of milfoil on development, however, this effect is only 21\% for lakefront parcels which falls far short of overcoming the 60\% decrease in probability of development resulting from invasion. Similar effects for

\textsuperscript{14} In exploratory analysis, it is found that netting out the revenue components of near-water homes by inclusion of these variables in the price index regression to form spatially varying price indices within tracts resulted in a significant decrease in hazard ratio for these covariates in the duration model as the variation remaining net of price was largely capturing expected cost components of near water development.
clarity are found for homes near lake access, however, this effect is less than 12% which once again falls far short of overcoming the 32% decrease resulting from invasion. Larger lakes experience less of an effect for lakefront properties and the interaction with access is not significant. Overall, these results suggest significant behavioral changes in response to invasion.

The bottom panel of results are associated with parcel specific measures that approximate traditional revenue and cost measures for firms and are common in much of the land conversion duration literature. Among significant covariates, the direction of almost all the hazard ratios follow prior expectations, with parcels being more likely to be developed if they are within view of a park, further from highways, in an area with undeveloped or agricultural land, and served by metropolitan/urban services. Also, parcels further from a metro center, on steep slopes, and parcels in areas with more industrial or commercial development are less likely to be developed, all else equal.15 Finally, the results show that higher priced locations have a higher likelihood of being developed, likely due to revenue considerations on the part of developers.

Table 8 presents results that largely mirror those from table 7; however, 650 census tract and 1,304 lake fixed effects are included in the

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15 Land use covariates are identified by forming these at the block group level, which varies within tracts. These measures also vary over time.
model. The inclusion of lake specific controls in addition to tract controls has two key implications for the results. First, no time-constant lake attributes such as clarity and size can be identified as standalone covariates. Second, the parcels used to identify the variable Invaded is a now a subset of those used for identification in the previous model, and this alters the significance of some of the interaction covariate estimates, including Lakefront*Invaded. The reason for this decrease in power is because over 20% of the invaded lakes in this data were invaded prior to 1990 and therefore contain no intertemporal variation with the variable Invaded. The result is that the effect of invasion for parcels near these lakes is not captured by the Invaded variable but rather by the 1000m lake dummy variable. This causes some changes in the magnitudes of hazard ratios, but does not alter the qualitative direction of the effects for the key covariates.

Overall, a 9% decrease in development is seen for parcels who’s nearest accessible lake is invaded. This effect increases to 27% if located within 400 meters of an invaded lake. The effect for lakefront homes is a decrease of 18%, however, this result is no longer statistically significant due to the reduced power for identification discussed above. A moderating effect still exists for lake clarity. Overall, these results are quite similar to the prior set of results that omitted lake fixed effects and suggest that
unobserved heterogeneity at the lake level is not a primary driver of the previous results and provides additional robustness to the key findings.

In the bottom panel of results similar impacts are once again seen for spatially and time-varying covariates that proxy for revenues and costs. There is a large and significant increase in development probability associated with views of parks, increasing distance from highways, and the availability of urban services while a decreasing likelihood of development is found for homes further from the city center, and a decrease from industrial development, steep slopes, and distance to the metro center all lower the likelihood of single family development.

1.6 Impact of altered spread

One implication of these findings is that Eurasian watermilfoil decreases the likelihood that lakefront and near lake parcels will be developed, or conversely that a relative increase in development is likely on non-invaded lakes. Using the Weibull model presented in table 8, two versions of a hypothetical development scenario are examined where milfoil spread was prevented soon after its arrival into the Twin Cities area of Minnesota.\textsuperscript{16}

Figure 5 shows the extent of milfoil invasion prior to 1990 which is

\footnotesize{\textsuperscript{16} While a reduced form model is used for this exercise, it should be noted that general equilibrium effects which are unaccounted for are likely to be an important consideration. To account for these directly, a structural model of new housing supply would be required which is left for future research.}
assumed to be constant and not to spread further for the purposes of these scenarios. The first scenario simulates the change in development patterns that would have existed by eliminating the spread of milfoil. The second scenario builds upon the first by increasing the price of lakefront homes which are not invaded based on the hedonic literature showing that invaded homes lose value. In this scenario, the price of lakefront home prices increases 13%, as estimated by Horsch and Lewis (2009), in addition to eliminating the milfoil invasion.

Table 9 presents simulation results for each scenario in terms of parcel development at the census tract level. In general, there are few differences across scenarios in the number of census tracts where housing development increases. In terms of magnitude of development within the altered tracts, when house prices do not vary there is an average increase of 17 housing units per tract. This average increases to 19 when prices fluctuate as well. Figure 6 shows the tracts experiencing new development as a result of this scenario where prices increase.\footnote{There is no significant visual difference between the maps of the two scenarios, therefore the map for the non-price increasing scenario is omitted.} In terms of the distribution of this increase, nearly 41\% of tracts receiving new development experienced at most 10 units while only 2 tracts experienced more than 100 new homes constructed. The extent to which these additional homes are likely to be near lakes suggests that milfoil invasion
led to a relatively large decrease in near lake development and likely increased development on uninva
ded lakes in the area.

1.7 Discussion

Invasive species are a growing concern in the United States due to the large damages they cause to ecosystems, biodiversity, and businesses. While hedonic studies have shown that one of the damages of milfoil manifests itself in the form of diminished lakefront property values and existing recreation studies show that individuals avoid areas where invasive species have degraded ecosystems, there is little evidence examining how a broader range of human behavior is impacted due to the presence of an invasive. This study is the first to directly link the presence of an invasive species to changes in behavior of housing developers.

This analysis uses a rich dataset of land, lake, and parcel characteristics from the Twin Cities region of Minnesota to estimate a duration model of land conversion. The results provide substantial evidence that land developers avoid milfoil invaded lakes, both in terms of lakefront development as well as nearby property development. For near lake homes, the probability of development decreases 37% after a milfoil invasion. These large decreases in the probability of development can result in significantly fewer developments near lakes, which has important implications for public finance and policy. One such implication is that
developers are likely to increase the rate of development on non-invaded lakes. This will likely lead to excess sprawl as well as potential congestion concerns.

For local municipalities, milfoil management also raises large budgetary concerns. *The New Hampshire Union Leader* newspaper in an article titled “Towns overburdened with combating state’s milfoil infestation” explains that the state only provides an average of 18 percent of the needed milfoil management funding to local communities, with the remainder largely provided by municipalities through local tax receipts. Given that waterfront property taxes are a large, if not the largest, portion of many small municipalities tax revenue (Montgomery 12), these results have substantial implications for this revenue stream. Land developers’ avoidance of milfoil invaded lakes will lead to fewer lakefront and near lake properties from which municipalities collect property taxes. With fewer resources available to combat the spread of milfoil, the propagation of milfoil will likely worsen, causing a negative feedback loop in which land developers continue to avoid invaded lakes leading to further reductions in the tax base. In addition, a rush by developers to develop, uninvaded lakes is likely to exacerbate the spread of milfoil as recreation and lake usage are likely to increase with new development, thus providing additional opportunities for milfoil to spread to new areas.
## 1.8 Tables

### Table 1.1: Lakes Invaded by Year

<table>
<thead>
<tr>
<th>Year</th>
<th>Frequency</th>
<th>Percent</th>
<th>Cummulative</th>
<th>Frequency</th>
<th>Percent</th>
<th>Cummulative</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Full Dataset</td>
<td></td>
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<td>Estimation Dataset</td>
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<tr>
<td>Pre-1990</td>
<td>21</td>
<td>21.9%</td>
<td>21.9%</td>
<td>20</td>
<td>21.7%</td>
<td>21.7%</td>
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<tr>
<td>1990</td>
<td>8</td>
<td>8.3%</td>
<td>30.2%</td>
<td>8</td>
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<tr>
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<td>9</td>
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<td>39.6%</td>
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<td>9.8%</td>
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<tr>
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<td>68.8%</td>
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<td>2000</td>
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<td>83.3%</td>
<td>4</td>
<td>4.3%</td>
<td>83.7%</td>
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<tr>
<td>2002</td>
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<td>2003</td>
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<td>90.6%</td>
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<td>5.4%</td>
<td>91.3%</td>
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<td>2004</td>
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<td>3.3%</td>
<td>94.6%</td>
</tr>
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<td>2005</td>
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<td>100.0%</td>
<td>5</td>
<td>5.4%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Total</td>
<td>96</td>
<td>100%</td>
<td></td>
<td>92</td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>
Table 1.2: Housing Transaction Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>All Transactions (N=448,209)</th>
<th>Near Lake Transactions (N=80,324)</th>
<th>Non Lake Transactions (N=367,885)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acres</td>
<td>0.4267 0.9823 0.05 20</td>
<td>0.4308 0.7457 0.05 20</td>
<td>0.4258 1.0267 0.05 20</td>
</tr>
<tr>
<td># Baths</td>
<td>2.0234 0.8220 1 8</td>
<td>2.2457 0.8641 1 8</td>
<td>1.9749 0.8044 1 8</td>
</tr>
<tr>
<td># Bedrooms</td>
<td>3.2203 0.9032 1 8</td>
<td>3.3458 0.9297 1 8</td>
<td>3.1929 0.8950 1 8</td>
</tr>
<tr>
<td># Fireplaces</td>
<td>0.3194 0.6369 0 6</td>
<td>0.4689 0.7615 0 6</td>
<td>0.2868 0.6014 0 5</td>
</tr>
<tr>
<td>Square Footage</td>
<td>1743.0 844.16 500 7978</td>
<td>1971.0 916.50 500 7978</td>
<td>1693.2 803.80 500 7933</td>
</tr>
<tr>
<td># Stories</td>
<td>1.3919 0.4144 1 3</td>
<td>1.4534 0.4186 1 3</td>
<td>1.3784 0.4123 1 3</td>
</tr>
<tr>
<td>Garage</td>
<td>0.8954 0.3060 0 1</td>
<td>0.8893 0.3138 0 1</td>
<td>0.8967 0.3043 0 1</td>
</tr>
<tr>
<td>Age</td>
<td>36.122 29.844 1 120</td>
<td>29.887 25.670 1 120</td>
<td>37.483 30.511 1 120</td>
</tr>
<tr>
<td>Log Price</td>
<td>11.93 0.5920 10.13 14.04</td>
<td>12.086 0.5991 10.13 14.04</td>
<td>11.901 0.5852 10.13 14.04</td>
</tr>
</tbody>
</table>
Table 1.3: Year 2000 Housing Price Index Estimation Results with Tract Fixed Effects

| Variable      | Coefficient | Std Err | P>|t| |
|---------------|-------------|---------|------|
| Acres         | 0.0305      | 0.0049  | ***  | <.001|
| # Baths       | 0.0526      | 0.0041  | ***  | <.001|
| # Bedrooms    | 0.0251      | 0.0032  | ***  | <.001|
| # Fireplaces  | 0.0846      | 0.0044  | ***  | <.001|
| Square Footage| 0.0002      | 0.0000  | ***  | <.001|
| # Stories     | 0.0036      | 0.0080  |      | 0.653|
| Garage        | 0.1513      | 0.0113  | ***  | <.001|
| Age           | -0.0027     | 0.0002  | ***  | <.001|

N = 22,142
R-Squared = 0.9994
Table 1.4: Developed Residential Parcels by Year

<table>
<thead>
<tr>
<th>Year Built</th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>9,222</td>
<td>5.0%</td>
<td>5.0%</td>
</tr>
<tr>
<td>1991</td>
<td>9,323</td>
<td>5.0%</td>
<td>10.0%</td>
</tr>
<tr>
<td>1992</td>
<td>11,870</td>
<td>6.4%</td>
<td>16.4%</td>
</tr>
<tr>
<td>1993</td>
<td>12,153</td>
<td>6.6%</td>
<td>23.0%</td>
</tr>
<tr>
<td>1994</td>
<td>10,457</td>
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<td>28.7%</td>
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<tr>
<td>1995</td>
<td>8,433</td>
<td>4.6%</td>
<td>33.2%</td>
</tr>
<tr>
<td>1996</td>
<td>9,223</td>
<td>5.0%</td>
<td>38.2%</td>
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<tr>
<td>1997</td>
<td>8,103</td>
<td>4.4%</td>
<td>42.6%</td>
</tr>
<tr>
<td>1998</td>
<td>9,773</td>
<td>5.3%</td>
<td>47.9%</td>
</tr>
<tr>
<td>1999</td>
<td>10,759</td>
<td>5.8%</td>
<td>53.7%</td>
</tr>
<tr>
<td>2000</td>
<td>9,757</td>
<td>5.3%</td>
<td>59.0%</td>
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<tr>
<td>2001</td>
<td>9,114</td>
<td>4.9%</td>
<td>63.9%</td>
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<tr>
<td>2002</td>
<td>9,211</td>
<td>5.0%</td>
<td>68.9%</td>
</tr>
<tr>
<td>2003</td>
<td>9,672</td>
<td>5.2%</td>
<td>74.1%</td>
</tr>
<tr>
<td>2004</td>
<td>9,154</td>
<td>4.9%</td>
<td>79.1%</td>
</tr>
<tr>
<td>2005</td>
<td>7,688</td>
<td>4.2%</td>
<td>83.2%</td>
</tr>
<tr>
<td>Total Developed</td>
<td>153,912</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Undeveloped</td>
<td>31,025</td>
<td>16.8%</td>
<td>100.0%</td>
</tr>
<tr>
<td>All Parcels</td>
<td>184,937</td>
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<td></td>
</tr>
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</table>
Table 1.5: Summary Statistics for All Parcels

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Near Lake (N=30,769)</th>
<th></th>
<th></th>
<th>Not Near Lake (N=154,168)</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Min</td>
<td>Max</td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>Park 50m</td>
<td>0.065</td>
<td>0.246</td>
<td>0</td>
<td>1</td>
<td>0.038</td>
<td>0.190</td>
</tr>
<tr>
<td>Golf 50m</td>
<td>0.021</td>
<td>0.144</td>
<td>0</td>
<td>1</td>
<td>0.023</td>
<td>0.150</td>
</tr>
<tr>
<td>Log Metro Center Distance</td>
<td>10.04</td>
<td>0.428</td>
<td>6.87</td>
<td>10.97</td>
<td>10.11</td>
<td>0.478</td>
</tr>
<tr>
<td>Log Highway Distance</td>
<td>7.379</td>
<td>1.198</td>
<td>0</td>
<td>9.79</td>
<td>7.422</td>
<td>1.217</td>
</tr>
<tr>
<td>Steep Slope</td>
<td>0.047</td>
<td>0.211</td>
<td>0</td>
<td>1</td>
<td>0.013</td>
<td>0.111</td>
</tr>
<tr>
<td>Prime Farm Land</td>
<td>0.333</td>
<td>0.471</td>
<td>0</td>
<td>1</td>
<td>0.354</td>
<td>0.478</td>
</tr>
<tr>
<td>% Commercial (Block Group)</td>
<td>0.012</td>
<td>0.030</td>
<td>0</td>
<td>0.62</td>
<td>0.012</td>
<td>0.027</td>
</tr>
<tr>
<td>% Ag/Undeveloped (Block Group)</td>
<td>0.503</td>
<td>0.264</td>
<td>0</td>
<td>0.97</td>
<td>0.629</td>
<td>0.228</td>
</tr>
<tr>
<td>% Industrial (Block Group)</td>
<td>0.013</td>
<td>0.031</td>
<td>0</td>
<td>0.55</td>
<td>0.017</td>
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<tr>
<td>% Metropolitan Urban Services (Block Group)</td>
<td>0.623</td>
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<td>Covariate</td>
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<td>Description</td>
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<td></td>
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<td>--------------------</td>
<td>----------------------------------------------------------------------------</td>
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<tr>
<td><strong>LAKE COVARIATES</strong></td>
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<tr>
<td>Lakefront</td>
<td>Dummy</td>
<td>Signals that a property resides within 50 meters of a lake</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lakefront * Invaded</td>
<td>Interaction</td>
<td>Signals that a property resides within 50 meters of an invaded lake</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lakefront * Invaded * Area</td>
<td>Interaction</td>
<td>Area of lake if property is lakefront on invaded lake</td>
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<td></td>
<td></td>
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<tr>
<td>Lakefront * Invaded * Clarity</td>
<td>Interaction</td>
<td>Clarity of lake if property is lakefront on invaded lake</td>
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<td></td>
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<tr>
<td>Lakefront * Invaded * Access</td>
<td>Interaction</td>
<td>Signals that a property resides within 50 meters of an invaded lake with public water access</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>400m Access</td>
<td>Dummy</td>
<td>Signals that a property's closest lake is within 400 meters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400m Access * Invaded</td>
<td>Interaction</td>
<td>Signals that a property's closest lake is within 400 meters and is invaded</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400m Access * Invaded * Area</td>
<td>Interaction</td>
<td>Area of the closest lake within 400 meters if that lake is invaded</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400m Access * Invaded * Clarity</td>
<td>Interaction</td>
<td>Clarity of the closest lake within 400 meters if that lake is invaded</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Invaded</td>
<td>Dummy</td>
<td>Signals that a property's closest lake is invaded</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Riverfront</td>
<td>Dummy</td>
<td>Signals that a property is within 50 meters of a river</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Access</td>
<td>Dummy</td>
<td>Signals that a property's closest lake has public water access</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area</td>
<td>Continuous</td>
<td>Area of closest lake with public water access</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clarity</td>
<td>Continuous</td>
<td>Clarity of closest lake with public water access</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>NON-LAKE COVARIATES</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Park 50m</td>
<td>Dummy</td>
<td>Signals that a property is within 50 meters of a park</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Golf 50m</td>
<td>Dummy</td>
<td>Signals that a property is within 50 meters of a golf course</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Metro Center Distance</td>
<td>Continuous</td>
<td>Log distance to the nearest metro center</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Highway Distance</td>
<td>Continuous</td>
<td>Log distance to the nearest highway</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steep Slope</td>
<td>Dummy</td>
<td>Signals that a property contains a slope of more that 18 degrees</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prime Farm Land</td>
<td>Dummy</td>
<td>Signals that a property's soil is composed of land deemed high quality for farming</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Commercial</td>
<td>Continuous (Time Varying)</td>
<td>Percent of the parcel's census block used for commercial purposes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Ag/Undeveloped</td>
<td>Continuous (Time Varying)</td>
<td>Percent of the parcel's census block used for agricultural purposes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Industrial</td>
<td>Continuous (Time Varying)</td>
<td>Percent of the parcel's census block used for industrial purposes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Metropolitan Urban Services</td>
<td>Continuous (Time Varying)</td>
<td>Percent of the parcel's census block that recieves metropolitan urban services</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price Index</td>
<td>Continuous (Time Varying)</td>
<td>Price index</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 1.7: Weibull Model with Tract Fixed Effects

| Covariate                          | Hazard Ratio | Std Err \(^a\) | P>|z|  |
|------------------------------------|--------------|----------------|-------|
| **LAKE COVARIATES**                |              |                |       |
| Lakefront                          | 1.15794      | 0.03089        | ***   | 0.000 |
| Lakefront * Invaded                | 0.40351      | 0.08045        | ***   | 0.000 |
| Lakefront * Invaded * Area         | 1.00002      | 0.00001        | **    | 0.012 |
| Lakefront * Invaded * Clarity      | 1.21002      | 0.05498        | ***   | 0.000 |
| Lakefront * Invaded * Access       | 0.97673      | 0.20166        |       | 0.909 |
| 400m Access                        | 1.07482      | 0.01183        | ***   | 0.000 |
| 400m Access * Invaded              | 0.68635      | 0.05109        | ***   | 0.000 |
| 400m Access * Invaded * Area       | 1.00000      | 0.00000        |       | 0.810 |
| 400m Access * Invaded * Clarity    | 1.11599      | 0.03950        | ***   | 0.002 |
| Invaded                            | 0.91379      | 0.01209        | ***   | 0.000 |
| Riverfront                         | 1.34497      | 0.09448        | ***   | 0.000 |
| Access                             | 1.02101      | 0.03743        |       | 0.571 |
| Area                               | 0.99998      | 0.00000        | ***   | 0.000 |
| Clarity                            | 1.01952      | 0.00430        | ***   | 0.000 |
| **NON-LAKE COVARIATES**            |              |                |       |
| Park 50m                           | 1.14373      | 0.01848        | ***   | 0.000 |
| Golf 50m                           | 1.00226      | 0.01897        |       | 0.905 |
| Log Metro Center Distance          | 0.64236      | 0.04057        | ***   | 0.000 |
| Log Highway Distance               | 1.03343      | 0.00379        | ***   | 0.000 |
| Steep Slope                        | 0.85218      | 0.01938        | ***   | 0.000 |
| Prime Farm Land                    | 0.94923      | 0.00641        | ***   | 0.000 |
| % Ag/Undeveloped                   | 1.23824      | 0.02865        | ***   | 0.000 |
| % Commercial                       | 0.68842      | 0.09462        | ***   | 0.007 |
| % Industrial                       | 0.11051      | 0.01300        | ***   | 0.000 |
| % Metropolitan Urban Services      | 5.14772      | 0.10540        | ***   | 0.000 |
| Price Index                        | 1.43721      | 0.01572        | ***   | 0.000 |

\(^a\)Robust standard errors clustered by parcel
N = 184,937 (153,007 uncensored, 31,930 censored)
650 Census Tract Fixed Effects
Table 1.8: Weibull Model with Tract Fixed Effects and 1000m Lake Dummies

| Covariate                          | Hazard Ratio | Std Erra | P>|z| |
|-----------------------------------|--------------|----------|-----|
| **LAKE COVARIATES**               |              |          |     |
| Lakefront                         | 1.06373      | 0.02324  | *** | 0.005 |
| Lakefront * Invaded               | 0.82486      | 0.17586  | 0.366 |
| Lakefront * Invaded * Area        | 0.99999      | 0.00001  | 0.170 |
| Lakefront * Invaded * Clarity     | 1.15525      | 0.06467  | *** | 0.010 |
| Lakefront * Invaded * Access      | 0.60246      | 0.13456  | ** | 0.023 |
| 400m Access                       | 1.16118      | 0.01546  | *** | 0.000 |
| 400m Access * Invaded             | 0.63053      | 0.05593  | *** | 0.000 |
| 400m Access * Invaded * Area      | 0.99999      | 0.00001  | *** | 0.007 |
| 400m Access * Invaded * Clarity   | 1.12510      | 0.04686  | *** | 0.005 |
| Invaded                           | 0.91435      | 0.01238  | *** | 0.000 |
| Riverfront                        | 1.13177      | 0.08996  | 0.119 |
| **NON-LAKE COVARIATES**           |              |          |     |
| Park 50m                          | 1.07256      | 0.01830  | *** | 0.000 |
| Golf 50m                          | 0.92654      | 0.01947  | *** | 0.000 |
| Log Metro Center Distance         | 0.64778      | 0.05741  | *** | 0.000 |
| Log Highway Distance              | 1.09728      | 0.00526  | *** | 0.000 |
| Steep Slope                       | 0.86862      | 0.02306  | *** | 0.000 |
| Prime Farm Land                   | 0.98506      | 0.00715  | ** | 0.038 |
| % Ag/Undeveloped                  | 1.33377      | 0.03610  | *** | 0.000 |
| % Commercial                      | 1.25926      | 0.20106  | 0.149 |
| % Industrial                      | 0.13917      | 0.01863  | *** | 0.000 |
| % Metropolitan Urban Services     | 5.11797      | 0.12203  | *** | 0.000 |
| Price Index                       | 1.65610      | 0.01921  | *** | 0.000 |

a Robust standard errors clustered by parcel

N = 184,937 (153,007 uncensored, 31,930 censored)

650 Census Tract Fixed Effects, 1304 1000m Lake Identifiers
Table 1.9: Increased Development Due to Completely Preventing Spread After 1989

<table>
<thead>
<tr>
<th>Increased Parcel Development</th>
<th>Stop Invasion, Constant Price</th>
<th>Stop Invasion, Price Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>66</td>
<td>59</td>
</tr>
<tr>
<td>11-20</td>
<td>17</td>
<td>22</td>
</tr>
<tr>
<td>21-30</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>31-40</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>41-50</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>51-60</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>61-70</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>71-80</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>81-90</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>91-100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&gt;100</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Number of Tracts with Increased Development: 112

Average Increase per Affected Tract: 17 (Stop Invasion, Constant Price) 19 (Stop Invasion, Price Increase)

Total Number of Tracts: 650 650

Average Increase in Development per Tract: 2.9 (Stop Invasion, Constant Price) 3.3 (Stop Invasion, Price Increase)
1.9 Figures

Figure 1.1: Seven County Study Region in Minnesota
Figure 1.2: All Invaded Lakes within Study Region
Figure 1.3: Housing Development After 1990
Figure 1.4: Near Lake Residential Developments on Invaded and Uninvaded Lakes
Figure 1.5: Lakes Invaded Before 1990
Figure 1.6: Increased Development Due to Policy to Prevent Milfoil Spread After 1989
Chapter 2: Spread Externalities and Management of Invasive Species: Implications of Heterogeneous Carrying Capacities

2.1 Introduction

Invasive species are a subset of non-native species that can affect existing environments through damages to property values, agricultural productivity, native fisheries, recreation, and tourism (Lockwood et al. 2006). Annual damages from invasive species have been estimated as high as $120 billion. The rapid growth in invasive populations and the increasing need for management policy raises concern about effective use of public funds to determine both when and where to invest in control policy.

Invasive species populations simultaneously grow and spread across space. Management of invasive species, therefore, presents a challenging problem in which economic decisions and natural resource stocks are linked through spatial-dynamic processes (Smith et al 2009).

The individual components of space and time have been widely studied by resource economists. However, research on the spatial-temporal links is relatively nascent. Spatial-dynamic problems generally
involve resource stocks that are governed by biophysical mechanisms following diffusion or dispersal processes. Examples of spatial-dynamic processes include the spread of an invasive species (Epanchin-Niel and Wilen 2012; Fenichel et al. 2014), the diffusion of fish biomass (Sanchirico and Wilen 2005), or the transmission of diseases. An important distinction between spatial effects, which can be incorporated in models that allow for spatial dependence, and spatial-dynamic effects that make spatial outcomes endogenous in the model. For example, decisions to control invasive populations in one location affect the growth and intensity of invasion in other locations. Control decisions at one point in time will affect decisions at all other locations and all future time periods.

This chapter adds to the spatial-dynamic literature by developing a model of optimal invasive species management that simultaneously considers spatial heterogeneity across patches. Heterogeneity is incorporated by modeling a landscape with discrete ecological patches that has spatially varying carrying capacities. Ecological carrying capacities influence both the intensity and spread of invasion, which leads to heterogeneity in costs and damages across patches depending upon the intensity witnessed at a given time period. The resulting model is one which closely mimics the invasion manager’s problem in which she must decide where, when, and how much control is appropriate. The stylized model is applied to the management of Asian Carp invasion and
parameterized to reflect damages and control costs in the Great Lakes region of the United States.

Spatial-dynamic models present analytical challenges because determining the optimal policy requires that a complex system of partial differential equations be solved. The solution to these problems often depends upon the spatial geometry of the landscape (Brock and Xepapadeas 2008; Smith et al 2009). In the context of Asian Carp, carrying capacity constraints gives rise to ecological bottlenecks where optimal control policy should target spread while using suppression to moderate damages in the river system. This policy finding is consistent with ongoing efforts to block Asian Carp from entering the Great Lakes using high cost electric barriers which are placed at an ecological bottleneck near Chicago, Illinois.

The model developed in this paper, uses integer programming to solve for the optimal management of an invasive species in discrete-time, discrete-space, ecologically constrained landscapes where a social planner minimizes the discounted sum of heterogeneous damages and control costs. This two-dimensional model demonstrates tradeoffs between localized control and spread externalities when controlling invasive species with spatially heterogeneous ecological carrying capacities. Model results show that failure to account for both ecological constraints and gradients in the intensity of invasive populations results in average inefficiencies as
high as 17% of the expenditures associated with invasive species management.

2.2 Optimal Management of Invasive Species

Optimal management of invasive species is particularly challenging because of the tradeoffs between computational complexity and realistic representations of ecological and economic characteristics that influence policy outcomes. Prior work examining the economics of invasive species management largely focuses on either dynamics or on space, with very few studies truly combining the two.

Studies that focus upon the dynamics of the system examine the intertemporal tradeoffs in determining when to invest in controlling the invasive. An important economic tradeoff is between different management strategies for preventing (Leung et al. 2002) or detecting (Mehta et al. 2007) the invasion versus allowing the population to spread initially and then determining the optimal time to control. In other cases, the passing of time can correspond with the intensity growth of the invasive, in which case the intertemporal decision to control can be thought of as choosing an optimal level of intensity management (Eiswerth and Johnson 2002). While the strength of these early studies is in examining when it is optimal to control, it ignores the often complex spatial dimensions of real-world policy.
Among studies that address spatial concerns directly many have adopted a simplified spatial context often consisting of two distinct spatial locations for analytical tractability. In models that examine more complex spatial interactions computational challenges often limit the analysis of transition dynamics to determine optimal paths that lead to steady state outcomes (Potapov et al. 2007, Sanchirico et al. 2010, Albers et al. 2010, Fenichel et al 2014). A simple model examining steady state outcomes can then be used to explore ecologically and economically complex interactions, and the social planner is afforded multiple avenues by which to best manage invasive species. More recently, resource economic models have examined spatial-dynamic processes with a generalized spatial landscape (Sanchirico and Wilen 1999, Costello and Polasky 2008, Smith et al. 2009). These models highlight the need for spatially explicit policy instruments for optimal resource management (Sanchirico and Wilen 2005; Brock and Xepapadeas 2010).

Recent advances in optimal management of invasive species have adopted more complex characterizations of the landscape using discrete-time, discrete-space models where a central planner optimally controls a spreading invasion process over a simulated landscape (Epanchin-Neill and Wilen 2012). This method allows to solve for both the optimal transition paths and steady state outcomes. Building on this literature, this chapter develops a spatial-dynamic model that incorporates
heterogeneity in ecological conditions and in the intensity of invasion in a landscape with multiple ecological patches. The model is applied to the problem of managing invasive Asian carp populations in the Great Lakes. However, the numerical model developed here is generalizable to a range of applications such as forest fires, epidemics, or other processes with spatial dynamic spread characteristics where heterogeneous intensity across space is a central concern for policymakers.

2.3 Model

This section develops a discrete-space, discrete-time model of optimal invasive control in a landscape with multiple ecological patches. The general model representing a social planner’s problem can be written as:

$$\min_{s_{it}} \{ \sum_t \sum_i \beta^t (D(x_{it}; k_i) + C(s_{it}, x_{it}; k_i)) \}$$

(11)

Subject to:

$$x_{i(t+1)} = x_{it} + \max \left( F(x_{it}; k_i), G(x_{(i-1)\ell}, x_{(i+1)\ell}; k_i) \right) - s_{it}$$

(12)

In equations (1-2) the state variable, $x_{it}$, represents the intensity of invasive population in location $i$ at time $t$. $s_{it}$ is a control variable that represents economic behavior – decisions to control the invasive species through suppression mechanisms, $t \in [0, T]$ indexes time and $i \in [1, N]$ indexes space where $N$ is the number of discrete patches. $\beta$ is the discount factor. Each location $i$ is influenced by exogenous ecological processes that determine its carrying capacity, $k_i$. $D(x_{it}; k_i)$ is the instantaneous damage...
caused by the invasive species and the cost function, \( C(s_t,x_{it};k_i) \), represents control costs. The function \( F(x_{it};k_i) \) represents invasive population dynamics within patch \( i \) that depend on the intensity of invasion at time \( t \), \( x_{it} \), and the ecological carrying capacity in each patch, \( k_i \). The function \( G(x_{(i-1)t},x_{(i+1)t};k_i) \) represents spread from the adjacent patches \( (i-1) \) and \( (i+1) \) into patch \( i \). A social planner’s problem is to choose actions simultaneously over time and across space to minimize the stream of discounted damages and control costs.

Growth of the invasive population within a patch is linear. In the absence of any control, intensity in an invaded patch \( i \) increases by 1 unit in each time step until the invasive population reaches the carrying capacity. Therefore, \( F(x_{it};k_i) = 1 \forall 0 < x_{it} < k_i; 0 \) otherwise. We model spatial-dynamic interaction across patches through a gradient-dependent spread of the invasive such that the difference in intensity between adjacent patches determines the quantity of spread from higher to the lower intensity patches. \( G(x_{(i-1)t},x_{(i+1)t};k_i) = \max \{0,(x_{(i-1)t} - x_{it}),(x_{(i+1)t} - x_{it})\} \).\(^{18}\)

In the absence of any control following an exogenous introduction in the initial time period \( (t=0) \), the invasive population grows within each invaded location and spreads to adjacent patches. For example, consider

\(^{18}\)Results are qualitatively similar in a model with a density-dependent diffusion process for the spread of invasive populations (See Appendix B). However, computational complexity limits the spatial extent of the landscape.
a landscape of 7 patches and carrying capacities, $k_i$, for each patch represented by the vector $K = [3, 1, 4, 5, 2, 2, 5]$. In time period zero, the location of initial invasion is discovered by the social planner, $x_{4,0} = 1$. In subsequent time periods intensity in invaded patches grows by 1 unit, and the invasion spreads from patches with higher intensities to adjacent patches with lower intensities. This spatial-dynamic growth process is constrained by carrying capacity as illustrated in Figure 1. Zero-flux boundary conditions are assumed, which is equivalent to assuming a carrying capacity of zero outside of the spatial domain.

The social planner then chooses the timing, location, and quantity of control. It is possible to suppress invasion intensity to any non-negative quantity, but the cost of suppression increases as the level of intensity decreases due to increased search costs associated with more limited stock. To solve the model using integer programming, the state and control variables are constrained to be integers; $x_{it} \in [0, k_i], s_{it} \in [0, x_{it}]$.

### 2.4 Damage and Cost specifications

Quantifying the damages inflicted by an invasive species is critical for determining optimal control, since damages avoided measure the benefits from management. The damage function specified here is one in which marginal damages from invasion depend on three landscape characteristics: carrying capacity, intensity of invasion, and a baseline
value of the patch (which could reflect proximity to recreation sites or population centers). The damage associated with patch $i$ being invaded in time period $t$ is given by:

$$D(x_{it};k_i) = p_i \times \sum_{n=1}^{N} \frac{x_{it}^n}{k_i}$$

(13)

where $p_i$ is the baseline value of patch $i$. Damages are increasing in patch value $\left(\frac{\partial D}{\partial p_i} \geq 0\right)$ and in intensity of invasion $\left(\frac{\partial D}{\partial x_{it}} \geq 0\right)$ but are decreasing in carrying capacity $\left(\frac{\partial D}{\partial k_i} \leq 0\right)$.

Suppression costs depend on the degree of suppression employed, $s_{it}$, the carrying capacity of the patch, and the intensity of invasion. Suppression is a control strategy that reduces the intensity of the invasive or prevents growth of the invasion within a patch. As the intensity of a patch decreases relative to its carrying capacity the amount of effort required to find and suppress additional units of the invasive increases. Therefore, the suppression cost function is increasing in $k_j$ and decreasing in $x_i$. $\alpha$ is a scaling parameter. Eradicating an invasive species is very difficult and costly to do. To account for this difficulty, the cost function is piecewise at the point where suppressing to zero intensity occurs. $\gamma$ is a scaling parameter.

$$C(s_{it}) = \begin{cases} 
\alpha \times \sum_{n=1}^{N} \frac{k_i}{(x_{it} - n)} & \text{for } s_{it} < x_{it} \\
\gamma \times k_i & \text{for } s_{it} = x_{it}
\end{cases}$$

(14)
2.5 Numerical Methods

To this point, the model here has been described as a discrete dynamic programming problem. However, to find a solution, the problem is solved as a binary integer programming problem. Binary integer constrained optimization problems, have very large sets of feasible solutions and are analytically intractable when the state space is large. Therefore, the numerical ‘branch and bound’ method (Land and Doig 1960) is employed to solve the model in MATLAB (See the numerical appendix in section 2.9 for a detailed account of the solution methodology).

In a binary integer model, this method first ignores the integer constraints and solves the resulting linear programming problem. If the solution is composed entirely of integers then the problem is solved. However, for the majority of problems it is not be this simple. Instead, some, or all, of the elements in the solution vector will be non-integers. In this case, the problem is split into two new problems by selecting one of the non-integer valued variables and creating a problem where this variable is assumed equal to zero and another where this variable is assumed to be equal to one. This process is repeated on each of the resulting problems until an all integer solution is found. The objective value of this solution is calculated and compared to each other active node in the solution ‘tree’. All strictly dominated nodes may be eliminated or ‘pruned’, thereby, eliminating portions of the solution set and reducing the computational
intensity of the problem. This process yields a globally optimal solution to the binary integer-programming problem. The ability to eliminate some of the solutions is extremely important in this study because the problem involves a very large numbers of variables. Consider, for example, that a standard grid-space in this analysis is 10x25 and is simulated over a 150 period time horizon. In this scenario, there are 37,500 binary variables, resulting in a solution set contains $2^{(37,500)}$ possible solutions.

2.6 Results

To analyze the impact of carrying capacities on optimal control strategies, several simulated landscapes are considered. First, steady states for a variety of homogeneous landscapes are shown and discussed. These steady states serve as the baseline for comparison to the steady states in heterogeneous environments. The differences in optimal strategy between the homogeneous and heterogeneous landscapes, and the cost savings associated with these differences, represent the inefficiencies of a ‘naïve’ central planner who either fails to account for spatial heterogeneity or assumes the landscape to be homogeneous when making control decisions. Numerical results show that these inefficiencies increase as the heterogeneity or variance in carrying capacity within a landscape increases. When the variance of carrying capacity is very large, neighboring patches may have dramatically different abilities to sustain
the invasive. In these cases, it is possible to find optimal control strategies in which the steady-state level of suppression varies across the landscape to prevent both spread and growth. This is shown via a stylized example of the Asian carp invasion in Mississippi River Basin spreading towards the Great Lakes. Parameter values used in the simulations described below are shown in Table 1.

2.6.1 Homogeneous Carrying Capacity

Consider a base case in which a central planner is faced with a single, low-intensity invasion, in a landscape of 25 patches with uniform carrying capacities. Because carrying capacities are identical across space, the cost of suppression at a given level of intensity, does not vary over space. As a result of this, the optimal steady states are also homogenous across space. Three steady states (shown in figure 2) exist for every parameterization with a homogenous carrying capacity landscape: no management, immediate eradication, and suppression to a uniform intensity level.\(^{19}\)\(^{20}\) While carrying capacity plays a role in all three of these, its influence on the interior solution with suppression to a positive intensity level, is most relevant for policy analysis. In landscapes with

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\(^{19}\) The optimal intensity level resulting from the ‘suppression to a uniform intensity level’ strategy may vary with different parameterizations of the model.

\(^{20}\) The realization of one of these three outcomes depends on (1) the location of the initial invasion, and (2) the intensity of the initial invasion.
higher carrying capacities, optimal intensity levels are higher than those with lower carrying capacities, as shown in figure 3. This is because, at a given level of intensity, damages decrease with carrying capacity, while suppression costs increase. Therefore, in the face of increasing costs and decreasing damages, the central planner opts to suppress at a higher level of intensity. However, it is important to note that homogeneity in optimal long run intensity levels is a consistent result across all landscapes with homogeneous carrying capacity. This result does not always emerge when carrying capacities are allowed to vary across space.

2.6.2 Heterogeneous Carrying Capacity

In the real world landscapes are rarely homogeneous. Therefore, to introduce ecological realism it is necessary to consider heterogeneous ecological constraints across space. Ecological constraints, such as natural fluctuations in food supply or competing populations, can alter the ecosystem capacity to sustain an invasive species population.

Similar to the homogeneous landscapes discussed above, landscapes here are divided into 25 discrete patches. Now, however, carrying capacities are randomly drawn from a normal distribution centered at 10 with a standard deviation of 2. An example of one such landscape is shown in figure 4. Centering the distribution at 10 ensures the total area of the landscape is identical to a homogeneous landscape.
with carrying capacities of 10. Therefore, direct comparisons can be made between these heterogeneous landscapes and the homogeneous landscapes in the previous section.

Unlike outcomes in a homogeneous landscape (Section 2.6.1), the optimal intensity level at which the invasion is suppressed is no longer uniform across space, as seen in figure 5. The cause for this non-uniform optimal suppression pattern is similar to the intuition for varying levels of optimal suppression with changes in the uniform carrying capacities in the homogeneous cases seen in figure 3. Damages are relatively cheaper at a given intensity level when the carrying capacity of a patch is larger, while suppression costs at that same level of intensity are higher. Thus, the central planner allows the intensity to grow before suppressing in high carrying capacity patches.

If individual carrying capacities were the sole driver of the optimal intensity of invasion, then all patches with identical carrying capacities would be suppressed at the same level. This, however, is not the case, as can be observed by comparing patch 2 with patch 6 in Figure 5. Both have 13 units of carrying capacity, yet the optimal level of intensity for patch 2 is 5, while patch 6 has an optimal intensity of 4. This is because optimal levels of steady-state suppression are determined by two factors –

\[21\] Figure 6 provides a robustness check for initial intensity level. With an initial intensity level of 6, it is optimal to first suppress the invasion down to 1, 2, 1, but then treat it exactly like the case where initial intensity level is 1.
carrying capacities and the relative gradients in carrying capacity and invasion intensity across patches. This result highlights that a central planner who ignores the spatial complexities of landscapes may implement inefficient control strategies, as discussed in more detail in the following section.

2.6.3 The Inefficiency of a Naïve Central Planner

This section considers two naïve strategies that a central planner could pursue if she chose to ignore the spatial heterogeneity within a landscape. The first is to treat the landscape as if it were homogeneous, and the second is to acknowledge heterogeneity but ignore the effects it has upon optimal control strategies. Comparison of total discounted costs demonstrate that both are inefficient strategies, and this inefficiency grows as the variance of carrying capacity heterogeneity increases.

As shown in section 2.6.1, homogeneous landscapes have optimal control strategies which do not vary across space. Therefore, if a central planner were to treat a heterogeneous landscape (like the one in section 2.6.2), as if it were homogeneous, they would pursue the control strategy shown figure 7(b). This uniform suppression is not optimal, because the intensity of the invasion is kept too low in some locations (Patches {2, 8, 13, 17}) and too high in other locations (Patches {5, 20}). The total present value of the damages and control costs associated with the
invasion in this scenario is higher than the cost-minimizing value function, and therefore, the homogeneous strategy is inefficient. This magnitude of inefficiency depends on the variability of carrying capacities in the landscape. 10 model runs were simulated to compare costs of a naïve uniform suppression strategy with optimal control strategies for landscapes with randomly drawn carrying capacities with mean 10 and standard deviation 2. Results show that, on average, the present value of the damages and control costs of implementing the naïve strategy are 1.9% greater than those of the optimal control strategy. This inefficiency grows when landscapes are more heterogeneous, as shown in figure 8; the inefficiency costs increase to as much as 4.3% when the standard deviation of carrying capacities is 4.

Even if the central planner does recognize heterogeneity in the landscape, failing to account for corresponding spatial spillovers in making control decisions also results in inefficient control schemes. This result is shown in figure 8. The central planner’s optimal control strategy is shown in figure 9(a), while 9(b) shows a scenario where the central planner ignores spatial spillover caused by heterogeneity. Notice that patches \{3, 6, 16\} all require multiple units of suppression to maintain a lower level of intensity in those patches while also preventing spread from entering from the much more intensely invaded neighboring patches. This suppression strategy is very costly relative to the mitigated damages.
Therefore, these control strategies can be very inefficient, as shown in figure 10. 10 model runs were simulated to compare costs of a naïve suppression strategy that ignore spatial spillovers with optimal control strategies for landscapes with randomly drawn carrying capacities with mean 10 and standard deviation 2. Results show that the present value of the damages and control costs of implementing the naïve strategy are on average 8.7% greater than those resulting from the optimal control strategy. The inefficiency grows to 16.7% when the standard deviation of randomly carrying capacities is increased to 4.

2.6.4 Application to the control of invasive Asian carp in the Mississippi River Basin

The previous results have shown that introducing spatial heterogeneity in carrying capacities alters the optimal levels of suppression, and that as the heterogeneity increases, a homogeneous suppression response becomes more inefficient. However, when heterogeneity increases, there are situations where a control strategy using both suppression and eradication is optimal. In this section, the spatial-dynamic model is applied to the management of invasive Asian carp and parameterized to reflect costs and damages from Asian carp in the Mississippi River Basin that can spread into the Great Lakes.
Ecological bottlenecks will be discussed as a way of potentially stopping the spread of an invasive species.

Asian Carp have existed in the Mississippi River Basin since they escaped from aquaculture facilities in the late 1970s or early 1980s (Chick and Pegg 2001). They have since spread through many of the Midwest’s rivers including the Ohio, Missouri, and Illinois. In some locations their numbers have grown to staggering levels and threaten to crowd out native species and disrupt ecosystems. On the central Illinois River, reports of 13 tons of Asian Carp per mile of river are common (Invasive Asian Carp Threaten Ecosystems in Midwestern Waterways, 2010). While the presence of Asian Carp in the river systems has already caused some damages in the river basin, mainly by increasing competition for native sport fish, the more pressing concern is that they will enter into the Great Lakes and potentially wipe out the $7 billion fishing industry that resides there.

The only viable mechanism to stop the spread of Asian carp is with an electric barrier that prevents movement of fish from one side to the other. Costs of operating an electric barrier are on average $2.4 million dollars per year (Stern, Upton, & Brougher, 2014). Setting the parameter $\gamma = 2,400$ in the cost function in equation 5 reflects this high cost of spread prevention. Suppression strategy for Asian Carp populations is simply to harvest them. However, a substantial market does not exist in the United States for Asian Carp. Because of this, some states have started hiring
fisherman to harvest the carp. In 2012, $1.2 million was spent on these programs, resulting in 284 tons of carp being removed from the river systems (Asian Carp Control Strategy Framework, 2012). The average cost per ton of carp, $4,225, is likely a lower bound of employed suppression costs so we set our suppression parameter to a small but slightly higher value of $\alpha = 10$ in the control cost equation. Asian Carp damages in rivers are difficult to parameterize because no empirical estimates exist at this time. Damages for other aquatic invasives have been quantified, however. $12$ million has been spent annually to restore lake trout populations in response to the destruction by the sea lamprey (DNR: Aquatic Invasive Species, n.d.). The Asian Carp could have similar effects on fish populations so this seems like a good upper bound for the total damages. Dividing this into the 25 patches, the damage parameter $p = 200$ reflects a spatially uniform baseline impact of the Asian Carp in a river. The results below are extremely robust to alternative parameterizations, as can be seen in Table 1.

Results presented up until this point focused on interior solutions with optimal suppression strategies that result in maintaining a positive invasive population in the steady state. This is because the vast majority of solutions that use eradication are trivial; either damages are extremely high and eradication occurs immediately, or eradication costs are low and eradication occurs immediately. But there are scenarios in which
eradication and suppression are implemented simultaneously across the landscape. This section explores one such scenario based on the application to control the Asian carp.

Because rivers are heterogeneous environments, some areas will be better suited for sustaining Asian carp populations than others. One example is when the river narrows significantly resulting in less area for Asian carp to reside. In this type of environment, carrying capacities will be low. Also, eradication becomes cheaper because electric barriers can be smaller and yet still effective. Such locations will be referred to as ecological bottlenecks. There are many ecological bottlenecks along the rivers that connect to the Great Lakes and under the parameterization described above, such bottlenecks may be good locations to stop the spread of Asian carp before they reach the Great Lakes.

Figure 11 represents a landscape containing an ecological bottleneck that separates a river (left side) from a large lake (right side). The lake has higher carrying capacities than the river and also is assigned greater baseline values. Once again, the baseline value of each river patch is set equal to $200,000 (p_{1-13} = 200). The damage parameter is different in the Great Lakes region of the landscape, where a $7 billion dollar fishing industry is at risk of being decimated by Asian carp. Dividing this total potential damage across the 12 columns representing the lake, each column is worth $583,333,000. Therefore, $p_{14-25} = 583,333$. 
Examining the optimal pathway, in $t = 0$, an invasion is already present within the river. It can be seen that completely eradicating the invasion is infeasible because the population is already too well established. Once the discovery is made, immediate suppression is the best response. In $t = 1$, the Asian Carp population is harvested down to lower intensity levels in the three left-most patches. The carp are allowed to spread until the ninth time period. At $t = 9$, when the carp reach the bottleneck, an electric barrier is utilized to prevent them from crossing, thereby saving the Great Lakes from invasion. Once this steady state is reached, the landscape will be maintained in the long run.

Even with a small carrying capacity like that of a bottleneck, it may be surprising that paying the high cost in every time period to prevent the spread of an invasion is an optimal strategy. However, the intuition driving this decision is straightforward. When a bottleneck separates an invasion from a segment of the landscape that has a high potential damage, the cumulative discounted costs of eradicating the bottleneck in each time period can be less than the damages from allowing the invasion to spread through the entire landscape. The other alternative, eradicating immediately, is not optimal because the total cost occurs right away, whereas blocking at the bottleneck has a lesser repeated cost that is eventually discounted to an infinitely small amount. For these reasons, repeatedly preventing the spread of an invasive will only be an optimal
strategy if it prevents the invasion from entering into a highly susceptible area, such as the Great Lakes, where immense damages will occur.

It is worth noting that this is the strategy currently being pursued by policy makers to keep Asian carp out of the Great Lakes. Electric barriers have been set up in the Chicago, Illinois area to attempt to halt their advance\textsuperscript{22}. One notable difference between these results and the real world is that real world intensity levels in the rivers are not being suppressed to the very low levels seen in this solution. This model suggests that more effort should be put into reducing population levels of Asian carp within the river systems in order to minimize the damages being inflicted.

2.6.5 Spatial Diffusion Process

Incorporating density dependent spread into a binary integer problem is challenging, and the resulting problems are extremely computationally intensive. Constraints are no longer as simple as those for linear spread and growth because the intensity of invasion within each patch now relies upon the total intensity of the invasion in neighboring patches. Therefore, to determine when a given cell is invaded, a constraint must be created for every possible pattern of invasion that results in the

\textsuperscript{22} Unfortunately, real world electric barriers do not seem to be 100% effective in preventing spread. This uncertainty is not accounted for in this example.
invasion of the cell. The result of this complexity is that it is not possible
to simulate invasions within large landscapes over long time horizons.
Only small problems can be simulated, and therefore, the results shown
here are only utilized as robustness checks for the larger problems in the
paper.

Consider the following problem:

\[
\min \{ \sum_t \sum_i \beta^i \left( D(x_{it}; k_i) + C(s_{it}, x_{it}; k_i) \right) \}
\]  

(15)

Subject to:

\[
x_{i(t+1)} - x_{it} = \min \left\{ x_{it} + \frac{(x_{i(t-1)t} - x_{it}) + (x_{i(t+1)t} - x_{it})}{2}, k_i \right\} - x_{it}
\]  

(16)

This is the same objective function as seen earlier, however, it is now
subject to a different growth function. Under this growth function, a
patch’s intensity today depends not only upon its intensity in the previous
time period, but also upon the gradient of intensity that exists between it
and its neighbors. The growth resulting from this function is shown in
figure 12.

Notice that growth occurs much more rapidly under this function
than it does in a linear growth world, and one might wonder whether or
not the results in this more complicated growth scenario match mirror the
linear case. In the small scenarios that can be simulated, the answer is
yes.

When the previous landscape of 5 homogeneous patches with
carrying capacity 8 is simulated over a 13 period time horizon with \( \alpha = \)
1, the optimal management pathway is shown in figure 13. Just like the linear model, the optimal control strategy in this homogeneous landscape is to maintain a uniform level of intensity across space. Compare this to the optimal pathway of a random heterogeneous landscape with carrying capacities drawn from normal distribution centered at 8 (shown in figure 14). This is the same type of control strategy seen in the linear results of, and therefore some credence can be given to the idea that control of an invasive species in a heterogeneous environment should be spatially targeted.
2.7 Discussion

Managing invasive species presents a natural resource problem in which the processes governing the system are inherently spatial and dynamic in nature. Spatial units, or patches, are heterogeneous in their ecological carrying capacities and damages from spatial dynamic externalities grow within a region and spread across regions. This chapter develops a spatial dynamic model of optimal control with two distinct control strategies to manage invasive species and shows that heterogeneous carrying capacities and intensities of invasion affect the central planner’s optimal control strategies.

This chapter contributes to the existing literature on managing spatial dynamic externalities in three ways. First, this research relaxes the assumption of homogeneous landscapes, thereby allowing for ecologically complex landscapes, and develops a model in which damages and costs of control depend upon spatially varied ecological carrying capacities and intensities of affliction. This allows for two distinct control mechanisms to be used; eradication and suppression. Second, the optimal level of intensity in each patch is not necessarily identical (like is seen in the homogeneous carrying capacity case). Instead, intensities in patches with larger carrying capacities should be allowed to increase more than those in smaller carrying capacity patches. This results in optimal management paths with varying degrees of tolerance for the externality across different
patches. These results complement existing work in spatial-dynamic management of renewable resources (Sanchirico et al 2010, Smith et al 2009, Epanchin-Neill and Wilen 2012, Sanchirico and Wilen 1999) and highlight the need for spatially targeted policies to optimally control spatial dynamic externalities. Finally, it is shown that assuming homogeneity when implementing control strategies may lead to policies that are inefficient. These inefficiencies exist when heterogeneous landscapes are treated as if they are homogeneous or when spatial spillovers from the heterogeneity are ignored. Furthermore, as the heterogeneity increases these inefficiencies grow. This implies that planners must pay extra close attention when attempting to devise control strategies in patchy environments or environments that have been heavily altered by human activity in certain areas.

Another insight from this model is that unless the decision to eradicate is made as soon as the invasive is discovered, it will never be optimal to eradicate. Whereas previous work has explored tradeoffs between prevention and optimal cleanup of invasive pests (Finnoff et al 2007, Costello et al 2007), this work focuses on post-invasion inter-temporal tradeoffs and show that if elimination of the invasive is optimal, it is cost effective to eradicate it as soon as possible. Any waiting period allows the affliction to grow and spread, making it more costly and therefore suboptimal. Also, with the exception of ecological bottlenecks,
stopping the spread is almost never optimal because it is very expensive to eradicate high carrying capacity patches over and over again. A much more likely scenario is to allow the affliction to spread across the landscape and to suppress to a tolerable intensity level.

Finally, it is almost always optimal for a central planner to minimize the variation of intensities between neighboring patches. In this model, unless there are drastic differences in the baseline values of neighboring patches, the optimal control strategies will always force the intensity levels of neighboring patches to be within one unit of each other. Implications of this are that suppression activity should take place not only in a particular high value area, but also in the patches surrounding it. This ensures that populations do not build in areas near those being controlled and that cost efficiency is maintained.
### 2.8 Tables

**Table 2.1: Parameterizations of Models**

<table>
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<th>Value</th>
<th>Robustness of Results</th>
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<tr>
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<td></td>
<td>$\tau$</td>
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<tr>
<td></td>
<td>$\gamma$</td>
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</tr>
<tr>
<td></td>
<td>$\tau$</td>
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</tr>
</tbody>
</table>
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Chapter 3: The Control of Wildfires with Heterogeneous Spread Rates in Complex Economic Landscapes

3.1 Introduction

Wildfire is a powerful force which, for millennia, has swept through landscapes, recycling nutrients, preserving biological diversity, reducing biomass, and controlling insect and disease populations (Mutch 1994). Its power is not restricted to nature, as there is evidence of economic effects as well. Wildfires dissuade hikers and mountain bikers from recreating in affected areas (Hesseln et al. 2003), they lower the nearby residential property values (Loomis 2004, Donovan et al. 2007), and they can affect the harvest timing of merchantable timber (Reed 1984).

It was the economics of the fire threatened timber industry that originally led to a nationwide wildfire suppression campaign nearly a century ago. The goal for most of this time period was the immediate suppression of any wildfire in order to protect the forest preserves, the timber industry that harvested from them, and the United States citizens who relied on a cheap and dependable lumber supply (Pinchot 1905). This operation picked up steam over the next several decades with technological
improvements in firefighting, such as fire suppressing chemicals and airplanes, as well as educational campaigns like Smokey the Bear. Over the years, suppression efforts became incredibly effective at containing wildfires before they became large, and over 97% of wildfires are now controlled by initial attack crews (Calkin et al. 2005).

While suppression efforts have significantly decreased the number of wildfires, there is well established evidence that the absence of fire has led to abnormally dense forests with high levels of fuel accumulation (Dodge 1972, Covington and Moore 1994, Bond and Keeley 2005). Wildfires in these high fuel settings burn more intensely and are more difficult to manage than those in locations that burn more regularly\(^{23}\) (Arno and Allison-Bunnell 2002, Reinhardt, et al. 2008, Bradshaw 2012). Therefore, increased effort has been placed upon dealing with these fuel loads before wildfires begin.

Currently, the two most common ways of addressing fuel accumulation are mechanical fuel reduction and prescribed burning. Both methods have been shown to lower fire severity, reduce tree mortality (Wagle and Eakle 1979, Martin et al. 1988, Pollet and Omi 2002, Fernandes and Botelho 2003), and create areas where wildfire suppression is more effective\(^{24}\) (Moghaddas and Craggs 2008). Mechanical thinning can

\(^{23}\) There is also evidence that suggests that increased temperatures and drier conditions caused by climate change is playing a role in the intensification of wildfires (Balling et al. 1992, Westerling et al. 2006).

\(^{24}\) This may not be the case in extreme weather conditions (McCarthy and Tolhurst 2001).
also be used by forest managers to mitigate potential losses from wildfire (Daigneault et al. 2010) and can be spatially targeted to best account for the net value of current timber stands with heterogeneous characteristics (Konoshima et al. 2010). However, the feasibility of nationwide fuel treatment is doubtful due to its incredibly high cost of implementation and the requirement that treatment be repeated on a 5-35 year cycle depending on the ecosystem (Gorte and Bracmort 2012).

One alternative is to allow more wildfires to burn (Kauffman 2004) when weather conditions and the initial ignition locations are favorable. This is a more cost efficient way of obtaining the ecological benefits from burning than fuel treatment efforts, and allowing wildfires to burn may also result in significant decreases in the suppression costs of future wildfires in the same areas (Houtman et al. 2013).

There are some significant questions that come up when talking about allowing wildfires to burn. When should fires be allowed to burn and when should they be suppressed? Which areas should be protected? And how does the spatial distribution of economically valuable area’s change the fire management strategy? These are complicated questions that require a model with the ability to accommodate a variety of economic values across a spatially explicit landscape.

In this chapter, a spatial-dynamic, numerical model of optimal control is developed for wildfire suppression. It uses, as an input, the fire
spread dynamics found using FARSITE (Finney 2004), a wildfire spread simulation software, then solves for the optimal control solution to the predicted fire pattern. This model can be used for fires with homogenous spread rates, as seen with many surface fires, or for fires whose spread rates are more heterogeneous, like in areas with patchy fuel build up. It can accommodate very complex spatial distributions of economic values and therefore provides an accurate representation of the wildland-urban frontier, or multi-aged forests. By incorporating these complexities, this model provides real-world applicable solutions to wildfire management problems.

3.2 Literature on Wildfire Control

Economic decision making takes place both before and after the ignition of a wildfire, and bodies of literature exist in both cases. Prior to a wildfire, an economic agent only experiences it as a form of risk. Reed (1984), who was among the first to investigate the effects of this risk on the optimal rotation of a forest stand, shows that wildfire risk acts like an increase in the discount rate within the optimal rotation problem and therefore decreases the optimal rotation period. This concept inspired more work on the managerial response to specific wildfire risk situations such as that of old-growth forests (Reed 1993), forests with both merchantable timber and recreational potential (Englin et al. 2000), forests
undergoing fuel management practices (Amacher et al. 2005), and forests with the potential of earning carbon sequestration credits (Daigneault et al. 2010). By investigating how optimal management fluctuates in different scenarios, this literature helps determine the net present value of current forest stocks and future rotations that is at risk from wildfire.

Once a wildfire is ignited, risk becomes reality and the time for harvesting decisions has passed. Economic decision makers must now choose whether or not to protect forest stocks, or other economically valuable areas, from the approaching flames. If protection is appropriate, decisions must be made on whether to immediately suppress the fire or to actively manage its spreads. Immediate suppression is pursued by initial attack crews and is the strategy most heavily utilized by today’s fire management teams. The initial attack is “the action taken to halt the spread or potential spread of a fire by the first firefighting force to arrive at a fire (Merrill and Alexander 1987).” The ability to contain a fire depends upon the effectiveness of the initial attack crews, and how quickly they can construct the first fire lines (Hirsch and Martell 1996, Hirsch et al. 1998) This effectiveness partially depends upon the timeliness of the initial attack force’s arrival, and therefore, the optimal spatial distribution of firefighting resources is crucial to being able to contain one or sometimes multiple wildfires (Lee et al 2013). If a wildfire breaks the containment of the initial attack crews, or if immediate suppression is not the optimal
management strategy, fire managers must prepare for a longer battle against a larger fire that could stretch on for weeks or even months.

The literature on managing a large active wildfire is small and theoretical. One of the first papers attempting to theoretically model the management of an active forest fire is Parks (1964). In this fully deterministic model, the objective is to choose a level of suppression force in order to minimize the total cost of suppression and damages from the fire. The model assumes the area burned to be a linearly increasing function of time against which the suppressing force combats with a constant marginal effectiveness level. This suppressing force is time constant, meaning no reinforcements can be sent later in the timeline. Other works object to this time constant assumption and have adapted models that allow for a time varying suppression force size that can ebb and flow as necessary in a dynamic model of optimal control (Parlar and Vickson (1982), Parlar (1983)). These early models provide good economic intuition for battling wildfires, but do not fully tackle the problem. They do not explicitly account for space or the heterogeneity that can exists across it in economic values or ecological characteristics. The model developed in this study will explicitly account for these heterogeneities and will provide evidence for when and where to control wildfires.
3.3 Methods

3.3.1 Landscape Setup

The landscapes constructed in this project are made up of many individual parcels of land managed by a single central planner. For ease of visualization, these patches are arranged as cells within a grid space. It is assumed that landscapes are isolated from the outside world, and therefore fires within the landscape cannot spread beyond the outside borders. Fires spread from patch to rook-adjacent patch in accordance to the spread coefficients which are preassigned to each patch. Spread coefficients specify the number of time periods that a patch burns before the fire spreads into other adjacent patches. For example, a patch with a spread coefficient of 3 will burn for 3 time periods before the fire spreads into its neighboring patch. This allows for many real world complexities to be deterministically incorporated into the model such as fuel composition or forecasted weather and wind conditions. Once a patch has burned for its allotted amount of time, it is said to be “burned out”, and cannot be reignited. Each patch may also be assigned its own individual economic value, which allows for highly valued locations, such as timber stands of different ages, to be differentiated from the rest of the landscape. These
values are representative of the economic loss that would stem from a patch burning.

To determine the fire spread coefficients, the fire spread simulation software, FARSITE (Finney 2004), is utilized. FARSITE deterministically simulates a wildfire’s spread from supplied data on a landscape (such as elevations, slopes, fuel composition, and canopy cover), weather (such as precipitation, daily high and low temperatures, and humidity), and wind characteristics (such as speed and direction). FARSITE can facilitate extremely complex wildfire scenarios with many different fuel patterns, topographies, and weather conditions. This project will not utilize the full potential of FARSITE, but rather will focus upon using the output from simple wildfires to calibrate an economic model of optimal control.

The landscape used in this project is called Flatland, and is included with FARSITE when it is downloaded. True to its name, Flatland is a very homogeneous landscape. It measures approximately 146,000 hectares, has no changes in elevation, and thus no slopes, and has a fuel composition limited to just one type, ‘Pine/Grass.’ This fuel type tends to experience surface fires fueled mostly by herbaceous material, litter, and dead-down stemwood (Anderson 1982). Surface fires, like these, are simple

---

25 It is assumed that the net effect of a fire in a patch is negative, meaning that the economic damages outweigh any ecological benefits that the patch may experience.

26 FARSITE is publically available and can be downloaded at http://www.firelab.org/project/farsite
to model because they are usually not accompanied by blowing lit litter which can cause spot fires far beyond the fire perimeter\textsuperscript{27}.

Two simple fire spread models are run in FARSITE to calibrate the spread rate coefficients in this project. The ignition point for each fire is in the center of the landscape. Both fires spread through homogeneous landscapes, but one fire is fueled by a 5 mile per hour wind from the South, whereas the other is not. This leads to spread patterns that look like figures 1 and 2. The radial lines seen rippling from the center of each figure are representative of the fire perimeter in 12 hour time steps. Each of these lines will represent one time period in this analysis. Therefore, determining the spread coefficients for each patch is a matter of determining the number of time steps that occur within each patches bounds. The spread coefficients for figures 1 and 2 are shown in the grid spaces in figures 3 and 4.

\subsection{3.3.2 Spatial-Dynamic Model of Optimal Control}

The central planner’s problem is to minimize the total discounted damages and management costs associated with a wildfire spreading through a landscape made up of discrete patches with heterogeneous spread rates.

\textsuperscript{27}The stochastic nature of blowing leaf litter is often found in conjunction with high intensity crown fires and will not be considered here.
The problem is set up as a binary integer programming problem, and therefore, all variables must either take a value of 0 or 1. This makes incorporating heterogeneous spread rates tricky since some patches will burn for more than one time period. To allow for these heterogeneous spread rates, the two-dimensional grid spaces representing the landscapes are transformed into three-dimensional grid spaces where the height of each column (which represents a single patch) corresponds to the spread coefficient for that cell. Now, as a fire burns through a patch, the height of the column acts as a timer counting the time periods until the fire spreads into its adjacent patches.

Damages from a patch burning are a function of a patch specific constant multiplier, \( d_{i,j} \), and the spread coefficient for a given patch.

\[
D_{i,j} = \frac{d_{i,j}}{sc_{i,j}}
\]  

(17)

As the fire burns through a patch, the damages gradually accumulate until the patch is fully burned out, at which point the full value of the patch, \( d_{i,j} \), has been lost. For forested patches, \( d_{i,j} \) is be calculated as follows:

\[
d_{i,j} = PV_{Current} - PV_{Replant} + c_{restore} - \text{salvage}
\]  

(18)

\( PV_{Current} \) is the present value of the revenue stream resulting from harvesting trees every, \( T^* \) years, where \( T^* \) is determined according to a Faustman model so that the marginal value of the trees at the time of harvest equals the opportunity cost of holding those trees (Clark 1976). \( PV_{Replant} \) is calculated in the same way, and is equal to the present value.
of the revenue stream resulting from planting today. The difference between these two present values is the loss of revenue that would result from wildfire destroying the trees on the patch. $c_{\text{restore}}$ is the cost of restoring the patch after the fire before trees are replanted. Any trees surviving the burn can be salvaged and sold at a discount. $PV_{\text{current}}$ and $PV_{\text{replant}}$ are both calculable as follows:

$$PV = e^{-\delta T_1}[V(T_1) - c] + e^{-\delta T_2}[V(T_2 - T_1) - c] + e^{-\delta T_3}[V(T_3 - T_2) - c] + \cdots \quad (19)$$

$V(T)$ is the revenue from the timber sale at time period $T$, $c$ is the cost of harvest and replanting, $\delta$ is the discount rate (Clark 1976). Salvage revenue is calculated as follows:

$$\text{salvage} = (Q - mQ) \times \varphi \times p - c_{\text{salvage}} \quad (20)$$

$Q$ is the pre-fire quantity of timber within the patch, $m$ is the tree mortality rate of the fire, $\varphi$ is the fraction of the stumpage price paid for salvaged timber, $p$ is the stumpage price, and $c_{\text{salvage}}$ is the cost of salvage. While $d_{i,j}$ is determined by solving a dynamic optimization problem with a time horizon of hundreds of years, the parameter enters the model as a static value due to the fact the time scale of a wildfire is drastically shorter than the time scale within forest growth problem.

The central planner can seek to control the fire by spending resources on suppression, which prevents the wildfire from spreading from a burning adjacent into the suppressed patch for a single time period. The cost of suppression in a given time period is the value of suppressing the
entire patch, \( e_{i,j} \), divided by the number of periods that the patch can burn, the spread coefficient, \( sc_{i,j} \).

\[
S_{ij} = \frac{e_{ij}}{sc_{i,j}} \tag{21}
\]

As the spread coefficient increases for a patch, the rate of spread within that patch decreases, therefore, the cost of suppression is increasing with the spread rate, which is consistent with previous literature. For this project \( e_{i,j} = 200 \) for each patch except for those directly surrounding the initial ignition location. For these patches \( e_{i,j} = 500 \) to represent the higher cost required to maintain a rapid response initial attack team. Additional heterogeneity is created by varying the spread coefficients over space.

The central planner may also choose to burn patches prior to the arrival of the wildfire. In real-world wildfire scenarios these operations are known as suppression fires, but they will be called managed burns in this work. Managed burns are used to burn fuel in a controlled manner in order to break up the fuel load across the landscape. Patch specific constant costs of managed burnings, \( mb_{i,j} \), are predefined and divided by the spread coefficient for each patch.

\[
B_{ij} = \frac{mb_{ij}}{sc_{i,j}} \tag{22}
\]

When suppression fires are utilized within a patch, the costs are accrued over the spread coefficient time horizon, but the damages associated with burning cell \((i,j)\) are accruing as well. Therefore, it is more expensive to
use a managed burn to clear fuel from a patch than it is to allow the wildfire to burn the patch. $mb_{i,j}$ is held equal to 10 in all patches from this point forward.

The stage is now set to consider the spatial-dynamic numerical optimal control problem where the variables are described in table 1. Equations (24) through (27) are the initial conditions for the optimization problem. They say that the initial fire must be discovered in the bottom most cell of a column and that no cells can be suppressed or burned out initially. The idea is that the fire must first be discovered before it can be suppressed and a patch must be burned before it can be burned out. That the fire must start from the bottommost cell is akin to saying that the count of the number of time periods for which a cell will burn must start at one.

Equations (28) through (32) dictate how the fire spreads through the landscape over time. (28) says that if a cell is burning today and is not the topmost cell in its column then the cell above will burn tomorrow unless it is suppressed tomorrow. This ensures that the built in count function executes properly to allow for heterogeneous spread. Equation (29) specifies that if a cell is burning and is the top most cell of its column, then the bottom most cell of the column to the left will burn tomorrow unless that cell is suppressed tomorrow. Equations (30), (31), and (32) dictate similar spread pattern, but in the right, back, and front directions.
respectively. Equation (33) states that if a managed burn is used in a cell, that cell is on fire. Equation (34) establishes how it is possible for a cell become burned out. It says that a cell is burned out today if it has ever been on fire in the past.

Equations (36) through (38) are binary integer constraints that force all $x_{i,j,l,t}$, $y_{i,j,l,t}$, and $b_{i,j,l,t}$ to either take the value of 0 or 1.
Model 3.1

\[
\min_{y,z} \left\{ \sum_{t=1}^{T} \beta^t \left( \sum_{i,j,l,t \in \Omega} (x_{i,j,l,t} * D_{i,j}) + \sum_{i,j,l,t \in \Omega} (y_{i,j,l,t} * S_{i,j}) + \sum_{i,j,l,t \in \Omega} (z_{i,j,l,t} * B_{i,j}) \right) \right\}
\]

Subject to,

\[
x_{i,j,l,0} \in \begin{cases} \{0\} & \text{if } l > 1 \\ \{0,1\} & \text{if } l = 1 \end{cases} \quad \forall (i,j,l,0) \in \Omega
\]

\[
y_{i,j,l,0} = 0 \quad \forall (i,j,l,0) \in \Omega
\]

\[
z_{i,j,l,0} = 0 \quad \forall (i,j,l,0) \in \Omega
\]

\[
b_{i,j,l,0} = 0 \quad \forall (i,j,l,0) \in \Omega
\]

\[
-1 \leq (x_{i,j,l,t} - x_{i,j,(l+1),1,(t+1)} - y_{i,j,(l+1),1,(t+1)}) \leq 0 \quad \forall (i,j,l,t) \in \Omega
\]

\[
-1 \leq (x_{i,j,l,t} - x_{i,j,(l-1),1,(t+1)} - y_{i,j,(l-1),1,(t+1)} - b_{i,(l-1),1,(t+1)}) \leq 0 \quad \forall (i,j,l,t) \in \Omega
\]

\[
-1 \leq (x_{i,j,l,t} - x_{i,(l+1),j,1,(t+1)} - y_{i,(l+1),j,1,(t+1)} - b_{i,(l+1),j,1,(t+1)}) \leq 0 \quad \forall (i,j,l,t) \in \Omega
\]

\[
-1 \leq (x_{i,j,l,t} - x_{i,(l-1),j,1,(t+1)} - y_{i,(l-1),j,1,(t+1)} - b_{i,(l-1),j,1,(t+1)}) \leq 0 \quad \forall (i,j,l,t) \in \Omega
\]

\[
-1 \leq (z_{i,j,l,t} - b_{i,j,l,t}) \leq 0 \quad \forall (i,j,l,0) \in \Omega
\]

\[
0 \leq \sum_{t=0}^{t^*} x_{i,j,l,t} \leq 0 \quad \forall (i,j,l,t) \in \Omega
\]

\[
x_{i,j,l,t} \in \{0,1\} \quad \forall (i,j,l,t) \in \Omega
\]

\[
y_{i,j,l,t} \in \{0,1\} \quad \forall (i,j,l,t) \in \Omega
\]

\[
b_{i,j,l,t} \in \{0,1\} \quad \forall (i,j,l,t) \in \Omega
\]
3.4 Solution Method

This problem was coded using Matlab, and solved with a numerical integer programming solver called SCIP\textsuperscript{28}. The numerical methods used here are similar to those utilized in the second chapter of this dissertation, which are described in detail in section 2.9 of the appendix. The difference in methods is that this problem deals with an even greater number of variables due to the inclusion of a third dimension in the grid space. The ‘branch and bound’ method (Land and Doig 1960) is used to solve for the optimal solutions. This method ignores the integer constraints, solving the as though it is a linear programming problem. The solution is then analyzed to determine whether it is composed entirely of integers. Most of the time it is not, so two more problems are created; one where one of the non-integer variables is assumed to be equal to one, and another where the same variable is assumed to equal zero. This process is repeated on each of the resulting problems and iterations continue in this manner until an all integer solution is found. The objective value of this solution is calculated and compared to the objective values of each other active node in the solution ‘tree’. If a node is strictly dominated by the integer solution’s objective value then that node is ‘pruned’, thus eliminating part of the

\textsuperscript{28} SCIP is available for download with the Opti Toolbox package found here: http://www.i2c2.aut.ac.nz/Wiki/OPTI/
solution set. Eventually, this process yields a globally optimal solution to the binary integer-programming problem.

3.5 Results

A series of simulations are now performed to solve for the optimal management solutions in a variety of different landscapes. In section 3.5.1, forestry scenarios are analyzed in a landscape with homogeneous spread rates (Figure 3). For the simulations shown in this section, damages are calculated in accordance with the methodology described in section 3.3.2 and the assumed parameter and functional forms shown in Figure 5. The Faustman problem is solved yielding an optimal rotation of 48 years. This is followed by demonstrations of the influence and importance of spatial location within landscapes with heterogeneous spread rates (Figure 4).

3.5.1 Homogeneous Spread Rates with Forestry Based Economic Values

The economic value of parcels naturally vary across space. In a forestry context, these values are tied into the ages of the stands of trees. As trees age, the present value of their existence increases as do the potential damages from a fire. For this reason, it is logical that older stands of trees should often be protected during a wildfire while younger trees may be left to burn (as shown in the optimal control scenario in Figure 6).
But the way these values are distributed across space is also an important determinant in the optimal control of wildfires. When high values are concentrated in a smaller geographical area they are easier to protect with fewer resources. This can be seen by comparing Figure 7 to Figure 8. Each landscape contains 9 stands of 15 year old trees, but they have different spatial configurations. In Figure 7 the stands are stretched out along two of the edges, which creates a very difficult control wildfire situation where each stand would need to be suppressed individually. For this reason, the option strategy is to allow the entire landscape to burn with not attempt at control. The stands in Figure 8, however, are clustered together. This allows four units of suppression to protect 8 stands of trees. Thus it is important to acknowledge that knowing the total value of the timber at risk is not always sufficient for determining the optimal control strategy for a wildfire. The spatial distribution of that value must be known as well so that the costs of control may be weighed against the benefits.

It is also possible for damages to vary within a cluster of high value timber. Different areas may be at different points in the harvesting schedule and therefore are worth different amounts today. Figure 9 investigates a variety of these scenarios, and a multitude of different optimal control strategies are found. Scenario 1 begins with a group of stands with different ages. Each subsequent scenario is the same group of stands 5 years later. It can be seen that control scenarios evolve with the
When all the trees within the cluster are relatively young, like in scenario 1, the benefits of protecting are not large enough to warrant any control, but as the stands age different control strategies become optimal. The protected area becomes larger as the stands towards the middle of the landscape age and act as a kind of shield. Their high value requires protection and the lesser valued stands behind wind up protected in the process. In scenario 8 and then 9, the front trees are harvested and the optimal control strategy shifts back to protecting a smaller area and allowing the young trees to burn. The implication of these results is that different control strategies are optimal at different times, even within the same forest. Managers should therefore be constantly updating their wildfire control strategies as their forests age and change in value.

3.5.2 Spatially Heterogeneous Spread Rates

In the results up to this point, wildfires have spread with a constant rate. Real world spread rates, however, vary for many reasons like fuel composition, weather, or the time of day. While it is difficult to incorporate all the dynamic complexities that push and pull fires across landscapes, it is important to consider some of the deviations in control strategies that may arise when simple heterogeneity is incorporated. This subsection will integrate a 5 mile per hour sustained wind out of the South. The wind propels the fire Northward at a faster rate than was previously
experienced. On the other hand, the fire spreads slower to the South. The spread rates for each patch are shown in figure 4. Even with this simple heterogeneity in spread rate, the optimal control solutions are different depending upon the location of high value areas relative to the initial ignition location of the wildfire. The main differences in control strategies stem from the fact that the cost of suppression is directly related to the rate of spread. This means that as a fire spreads more rapidly it is more costly to suppress than if it were spreading at a slower pace.

The heterogeneous economic values for each scenario are presented in Figure 10. Figure 12 shows a landscape with a single high-value home. This is representative of a home built in the Wildland-Urban Interface (WUI). An identical landscape is also shown in Figure 13. The only difference between these two scenarios is that figure 12 is subject to heterogeneous spread rates described above, whereas Figure 13 is not. First looking at Figure 13, the central planner discovers the wildfire in the centermost patch in time period 0 and immediately uses a managed burn in the patches to the left, to the right, and below the high value patch in order to ensure that only one time period’s worth of suppression is utilized. Compare this with figure 5 and notice that only the patches to the right and to the left of the high-value patch are burned using a managed burn. In this case the central planner has taken note that the fire will spread at a slower rate in the patch below the high-value patch than it will in the
high value patch and therefore suppression will be cheaper in the patch. Therefore, it is optimal for the central planner to utilize this cheaper suppression and in doing so she prevents the high-value patch and the low value patch below it from being burned.

Optimal control strategies can also be influenced by the location of the high-value area relative to the heterogeneous spread rate. An example of this is shown in figures 14 and 15. In figure 14, a row of high-value patches stretches along the top of the landscape. A wildfire is discovered in the centermost patch and is propelled by the 5 mile per hour wind out of the South. A similar scenario is shown in Figure 15, however, now the row of high-value patches stretches along the bottom of the landscape, where the fire approaches at a slower rate and the fuel composition lends itself to a slower burn and therefore cheaper suppression. The differences in control strategy between these two scenarios is stark. When the high-value patches are being encroached upon by a rapidly spreading fire the optimal strategy is to suppress it immediately in order to prevent spread from occurring since suppressing a rapidly moving fire is very expensive. When the high-value patches are on the opposite side of the landscape where the wind and fuel composition aids suppression efforts by keeping spread rates low, the optimal strategy is to allow the fire to burn and use suppression to control the fire in the high-value patches. Once again, as is seen in the previous results, it is best to minimize the number of time
periods in which suppression forces are utilized. This requires a managed burn to be used directly to the left and the right of the high value area in time periods 7 and 8. These results suggest that control efforts must be flexible enough to accommodate the exogenous conditions that may affect spread rates. If it can be anticipated that a fire will spread rapidly towards a high value area then the fire should be suppressed right away. But if weather conditions or other environmental factors will help suppression efforts as the fire approaches the high value area, then protection should be concentrated upon the high value area.

3.5.3 Conclusion

Wildfires are a natural part of forest ecosystems and have been regulating the system dynamics for millennia. Because of this, tree species have adapted to their particular fire regimes and have learned to thrive. In the last 100 years, however, human suppression efforts have reduced the number of wildfires quite dramatically. One unintended consequence of this action has been that fuel loads have built up in locations where they historically have been burned away regularly. This has led to more intense fires and larger budgets for forest fire management organizations. An alternative to physically removing fuel by mechanical means is to allow more wildfires to burn under optimal conditions. But this raises many questions about when to let fires burn and how best to control an active
wildfire. This paper develops a spatial-dynamic model of optimal management of wildfires in landscapes with heterogeneous economic values as well as heterogeneous spread rates to attempt to try to answer some of these questions.

The results provide several insights for the future management of wildfires. First, the total economic value of the landscape means less, when it comes to determining the optimal management strategies, than the distribution of the economic value across space. Landscapes with spatially-concentrated values are more likely to see strategies where the wildfire burns throughout the landscape, but the high value patch is protected, whereas landscapes with dispersed value are more likely to witness immediate eradication. This has implications for fires that begin in the wildland-urban interface where human development continues to push into areas that are more fire-prone. Another result is that even when valuable areas, such as forests, are clustered together the optimal control strategy will change over time as the ages of the forest change. This suggests that forest managers need to constantly update their wildfire response plans to keep up with the changing economic values of their stock. Finally, these results imply that it is important to account the effect that spatially heterogeneous spread rates will have upon suppression strategies. High value areas are more likely to be individually protected if they exist in locations with relatively low fire spread rates, which allow for
cheaper suppression. If instead, high value areas are in parts of the landscape where spread rates are very high it may be more efficient to suppress the fire immediately.
3.6 Tables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{i,j,l,t}$</td>
<td>Binary integer variable taking the value of 1 if cell $(i,j,l)$ is on fire in time period $t$ and 0 if not.</td>
</tr>
<tr>
<td>$y_{i,j,l,t}$</td>
<td>Binary integer variable taking the value of 1 if cell $(i,j,l)$ is suppressed in time period $t$ and 0 if not.</td>
</tr>
<tr>
<td>$z_{i,j,l,t}$</td>
<td>Binary integer variable taking the value of 1 if a suppression fire is utilized in cell $(i,j,l)$ in time period $t$ and 0 if not.</td>
</tr>
<tr>
<td>$b_{i,j,l,t}$</td>
<td>Binary integer variable taking the value of 1 if cell $(i,j,l)$ is burned out in time period $t$ and 0 if not.</td>
</tr>
<tr>
<td>$D_{i,l}$</td>
<td>Damage function for cells within column $(i,l)$.</td>
</tr>
<tr>
<td>$S_{i,j}$</td>
<td>Suppression cost function for cells within column $(i,j)$.</td>
</tr>
<tr>
<td>$B_{i,j}$</td>
<td>Patch specific constant managed cost.</td>
</tr>
<tr>
<td>$C$</td>
<td>Set of all cells within the grid space.</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Set of all cells within grid space $C$ which are also the topmost cell in their column</td>
</tr>
<tr>
<td>$\Omega^c$</td>
<td>Set of all cells within grid space $C$ which are not the topmost cell in their column</td>
</tr>
<tr>
<td>$T$</td>
<td>Time horizon</td>
</tr>
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Figure 3.1: Wildfire with Homogeneous Spread Rates
Figure 3.2: Wildfire with Heterogeneous Spread Rates
Figure 3.3: Spread Coefficients for Homogeneous Spread Rates

Figure 3.4: Spread Coefficients for Heterogeneous Spread Rates
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Assumed Value/Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stumpage Price</td>
<td>$6.00/ft³</td>
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<tr>
<td>Replanting Cost</td>
<td>400/hectare</td>
</tr>
<tr>
<td>Salvage Cost</td>
<td>500/hectare</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>0.02</td>
</tr>
<tr>
<td>Mortality Rate</td>
<td>0.25</td>
</tr>
<tr>
<td>Salvage Price</td>
<td>$4.25/ft³</td>
</tr>
<tr>
<td>Post-Fire Recovery Cost</td>
<td>500/hectare</td>
</tr>
</tbody>
</table>

Volume Growth Function: $\rho \left( e^{10 - \frac{75}{t}} \right)$

Figure 3.5: Parameterization of Forestry Model
Figure 3.6: Protecting the Old while Letting Young Trees Burn
Figure 3.7: Optimal Control Strategy with Sprawled Out 15 Year Old Stands
Figure 3.8: Optimal Control Strategy with Clustered 15 Year Old Stands
Figure 3.9: Alternate Stand Age Configurations within a Cluster

(Continued)
Figure 3.9 (Continued)

Scenario 4

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<th>Stand Ages</th>
<th>Stand NPV</th>
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<td>20 20 25</td>
<td>74 74 92</td>
</tr>
<tr>
<td>20 25 30</td>
<td>74 92 111</td>
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<tr>
<td>25 30 30</td>
<td>92 111 111</td>
</tr>
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</table>

Optimal Fire Control

Scenario 5

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<th>Stand Ages</th>
<th>Stand NPV</th>
</tr>
</thead>
<tbody>
<tr>
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<td>92 92 111</td>
</tr>
<tr>
<td>25 30 35</td>
<td>92 111 133</td>
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<td>30 35 35</td>
<td>111 133 133</td>
</tr>
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Optimal Fire Control

Scenario 6

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</tr>
<tr>
<td>30 35 40</td>
<td>111 133 158</td>
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<tr>
<td>35 40 40</td>
<td>133 158 158</td>
</tr>
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</table>

Optimal Fire Control

(Continued)
**Figure 3.9 (Continued)**

### Scenario 7

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<th>Stand NPV</th>
<th>Optimal Fire Control</th>
</tr>
</thead>
<tbody>
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<td>35 35 40</td>
<td>133 133 158</td>
<td></td>
</tr>
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<td>35 40 45</td>
<td>133 158 186</td>
<td></td>
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<td>158 186 186</td>
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</tr>
</tbody>
</table>

### Scenario 8

<table>
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<th>Stand NPV</th>
<th>Optimal Fire Control</th>
</tr>
</thead>
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<td>158 158 186</td>
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### Scenario 9

<table>
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<th>Stand NPV</th>
<th>Optimal Fire Control</th>
</tr>
</thead>
<tbody>
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<td>45 45 2</td>
<td>186 186 8</td>
<td></td>
</tr>
<tr>
<td>45 2 7</td>
<td>186 8 25</td>
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<tr>
<td>2 7 7</td>
<td>8 25 25</td>
<td></td>
</tr>
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Figure 3.10: Economic Value Parameterizations by Patch for Each Model in Section 3.5.2
Figure 3.11: Single High-Value Patch with Heterogeneous Spread Rates
Figure 3.12: Single High-Value Patch with Homogeneous Spread Rates
Figure 3.13: Row of Mid-Value Patches Across Top of Landscape with Heterogeneous Spread Rates
Figure 3.14: Row of Mid-Value Patches Across Bottom of Landscape with Heterogeneous Spread Rates
References


Reed, W. J. (1993). The decision to conserve or harvest old-growth forest. *Ecological economics, 8*(1), 45-69.


Appendix A: Chapter 1 Appendix

A.1. Housing Transaction Cleaning.

The original dataset consisted of 663,001 individual property transactions spanning the years 1990 to 2007. Transactions are observed from the entire seven county study region surrounding the Twin Cities metro area. Along with housing transactions, this dataset also contains many initial land transactions as well, where a developer purchased the property before constructing a home. To eliminate as many land transactions as possible, a multi-step process is implemented that is similar to that proposed by Klaiber (2008), who first used this dataset.

First, only transactions occurring between 1990 and 2005 are kept since these are the years of analysis for this project. Also, any transactions where the selling price was less than $150,000 are eliminated, as most small, residential-sized lots will be sold below this price. Also, only transactions occurring after the house was built are considered, therefore, any observations where the year built is later than the year of sale are eliminated. The next cleaning step looks at the grantee’s name for each transaction and removes observations where the name contains any
keywords that might indicate that the grantee is a housing developer. The keywords used in this process are: “Homes”, “Construction”, “Builder”, “Development”, “LLC”, “Inc”, “Corp”, “Const”, “Properties”, “Develop”, “Develo”, “Investm”, “Construc”, “Bldrs”, “Company”, “Assoc”, and “Co”.29 Not all the transactions in this dataset contain information on grantee name, therefore a second strategy is employed to separate land transactions from housing transactions within this subset using repeated transactions that exist. An observation is eliminated if its price increased by more than 40% each year in between transactions. As expected, almost all the observations for which this holds true are within one year of the property being built, thereby indicating that the transaction is likely a land transaction. However, it is also likely that some homes have had significant remodeling and therefore experienced large increases in value. Even if this is not the case, all transactions prior to a remodel are dropped because the structural characteristics of the properties are taken from the most recent sale, and therefore are not reflective of the homes structure at the time of previous sales. Finally, a cleaning filter is used which eliminates any outliers as roughly identified by the 1st and 99th percentiles. This is accomplished by removing transactions that do not satisfy the following:

$25,000 \leq \text{Sale Price} \leq 1,250,000; \ 0.05 \leq \text{Acres} \leq 20; \ 1 \leq \text{Baths} \leq 8; \ 1 \leq$

---

29 It should be noted that these keywords exist in a variety of capitalizations, so several different capitalization constructions were checked. For example, “Corp”, “CORP”, and “corp.”
Bedrooms ≤ 8; 500 ≤ Square Feet ≤ 8,000; 1 ≤ Stories ≤ 3; Age ≤ 120; Fireplaces ≤ 6; Price Per Square Foot ≤ $500. After these cleaning steps the final dataset contains 448,209 housing transactions.

A.2. Duration Data Cleaning

In its original form, the dataset on new housing construction and potentially developable parcels contains 962,653 parcels of residential, commercial, industrial, vacant, and agricultural properties in the seven county study region. There are many discrepancies in this dataset, therefore, significant cleaning is warranted. Often, the issue is missing data, however, there are instances where problems clearly occurred in the geocoding process. Identifying and eliminating as many errors as possible is critical before an accurate analysis can be performed.

The cleaning process begins by identifying potentially developable single family residential or agricultural parcels for the time periods of interest for this study. Any properties built before 1990 are eliminated. The remaining properties are either built after 1989, or do not contain data on the year built. Not containing year built data indicates either that the property does not contain a building, and therefore could have been developed during the study period, or that the data is simply missing, in which case the observation should be eliminated from the dataset. Many steps are taken to try to distinguish between the two.
First, bad observations are identified by looking for irregular County ID’s and PIN numbers. County ID’s indicate which of the seven counties each parcel resides and should contain a 3-digit number representing the county. Observations with any values that do not represent one of the seven counties are eliminated. PIN numbers are a unique identifying variable and contain two parts. The first is the 3-digit county identifier identical to the County ID, and the second is the unique parcel identifier. Any observations containing only the 3-digit county identifier and no unique parcel identifier are eliminated. These observations also contain a disproportionately large amount of missing data. The next step is to eliminate any properties identified as agricultural preserves, predominate landmark, or businesses, as these cannot be developed into single family housing units. Also, any properties with a total assessor value of less than $200 are eliminated. Manually comparing these parcels from ArcGIS to satellite imagery data shows that these observations are clearly irregularities. Often, they are oddly shaped and wedged in between existing structures, clearly not suitable for development into houses, which is likely the reason for their low assessor value. Observations are kept only if they are identified as some sort of ‘home style’ indicating that the property has, or has the potential to have in the future, some sort of single family detached home (not including townhomes or apartments). This can include standard home types such as ‘Ranch’, but may also include vacant
land (which is potentially developable), and can even be left blank (indicating that it has not yet been developed). Lastly, a property is kept only if the area of the ArcGIS parcel is greater than 464.5 square meters, which is the minimum area a property can be and still be allowed to be developed into a single family home according to the building code for the city of Minneapolis.

At this point, the dataset is split into an ‘undeveloped’ subset and a ‘developed’ subset to facilitate additional cleaning. In the ‘undeveloped’ data, all observations with a building assessor value greater than zero are dropped. This indicates that a building already exists on the property and therefore the property is not truly undeveloped. Finally, while manually observing the ArcGIS data, it became apparent that many observations appeared to be flawed. Many of these parcels appeared only as sliver of land in ArcGIS. These slivers are most likely geocoding errors and are taking on the variable values of neighboring patches. To eliminate these errors it is first necessary to identify them by taking advantage of their odd shapes. Very long and skinny shapes have extremely large perimeters and small areas, therefore, they can be identified with ratios of these two measurements. Parcels are eliminated if the area divided by the perimeter is less than five or if the perimeter squared divided by the area is greater than 50. These cutoffs are based on manual observation of the shapes of
the eliminated parcels and comparisons to satellite imagery data. This procedure almost exclusively eliminates extremely small parcels.

At this point the undeveloped and developed parcel datasets are merged back together to create the final data set of 184,937 observations.

A.3. Appendix Tables

Table A.1: Housing Transaction Summary Statistics by Pre and Post 1990 Development

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pre-1990</th>
<th></th>
<th></th>
<th>Post-1990</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std Dev</td>
<td>Min</td>
<td>Max</td>
<td>Mean</td>
<td>Std Dev</td>
</tr>
<tr>
<td>Acres</td>
<td>0.3805</td>
<td>0.9189</td>
<td>0.05</td>
<td>20</td>
<td>0.5685</td>
<td>1.1436</td>
</tr>
<tr>
<td># Baths</td>
<td>1.8632</td>
<td>0.7650</td>
<td>1</td>
<td>8</td>
<td>2.5155</td>
<td>0.7954</td>
</tr>
<tr>
<td># Bedrooms</td>
<td>3.1270</td>
<td>0.8805</td>
<td>1</td>
<td>8</td>
<td>3.5068</td>
<td>0.9118</td>
</tr>
<tr>
<td># Fireplaces</td>
<td>0.3649</td>
<td>0.6714</td>
<td>0</td>
<td>6</td>
<td>0.1799</td>
<td>0.4913</td>
</tr>
<tr>
<td>Square Footage</td>
<td>1572.9</td>
<td>724.10</td>
<td>500</td>
<td>7978</td>
<td>2265.3</td>
<td>963.94</td>
</tr>
<tr>
<td># Stories</td>
<td>1.3007</td>
<td>0.3770</td>
<td>1</td>
<td>3</td>
<td>1.6718</td>
<td>0.3985</td>
</tr>
<tr>
<td>Garage</td>
<td>0.9179</td>
<td>0.2744</td>
<td>0</td>
<td>1</td>
<td>0.8261</td>
<td>0.3790</td>
</tr>
<tr>
<td>Age</td>
<td>46.613</td>
<td>26.982</td>
<td>2</td>
<td>120</td>
<td>3.9041</td>
<td>3.7625</td>
</tr>
<tr>
<td>Log Price</td>
<td>11.827</td>
<td>0.5470</td>
<td>10.127</td>
<td>14.039</td>
<td>12.261</td>
<td>0.6050</td>
</tr>
</tbody>
</table>

*338,106 Transactions  b110,103 Transactions
Table A.2: Cox Model with Census Tract Fixed Effects

| Covariate                                      | Hazard Ratio | Std Err $^a$ | P>|z| |
|-----------------------------------------------|--------------|--------------|------|
| **LAKE COVARIATES**                          |              |              |      |
| Lakefront                                     | 1.16480      | 0.02912      | ***  |
| Lakefront * Invaded                           | 0.38689      | 0.07786      | ***  |
| Lakefront * Invaded * Area                    | 1.00002      | 0.00001      | ***  |
| Lakefront * Invaded * Clarity                 | 1.21057      | 0.05327      | ***  |
| Lakefront * Invaded * Access                  | 1.03384      | 0.21375      | 0.872|
| 400m Access                                   | 1.05539      | 0.01096      | ***  |
| 400m Access * Invaded                         | 0.66049      | 0.04751      | ***  |
| 400m Access * Invaded * Area                  | 1.00000      | 0.00000      | 0.679|
| 400m Access * Invaded * Clarity               | 1.13236      | 0.03826      | ***  |
| Invaded                                       | 0.99603      | 0.01262      | 0.754|
| Riverfront                                    | 1.33795      | 0.09501      | ***  |
| Access                                        | 0.98970      | 0.03417      | 0.764|
| Area                                          | 0.99998      | 0.00000      | ***  |
| Clarity                                       | 1.00961      | 0.00409      | **   |
| **NON-LAKE COVARIATES**                       |              |              |      |
| Park 50m                                       | 1.11952      | 0.01632      | ***  |
| Golf 50m                                       | 1.01069      | 0.01871      | 0.566|
| Log Metro Center Distance                     | 0.65002      | 0.04006      | ***  |
| Log Highway Distance                          | 1.03547      | 0.00369      | ***  |
| Steep Slope                                   | 0.85759      | 0.01879      | ***  |
| Prime Farm Land                               | 0.94454      | 0.00610      | ***  |
| % Commercial                                  | 0.91976      | 0.02167      | ***  |
| % Ag/Undeveloped                              | 0.84164      | 0.10901      | 0.183|
| % Industrial                                  | 0.15308      | 0.01530      | ***  |
| % Metropolitan Urban Services                  | 4.28532      | 0.08460      | ***  |
| Price Index                                   | 1.10284      | 0.02438      | ***  |

$^a$Robust standard errors clustered by parcel

N = 184,937 (153,007 uncensored, 31,930 censored)
<table>
<thead>
<tr>
<th>Covariate</th>
<th>Tract FE</th>
<th>Tract and Lake FE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LAKE COVARIATES</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lakefront</td>
<td>1.05555</td>
<td>0.99781</td>
</tr>
<tr>
<td>Lakefront * Invaded</td>
<td>0.60636</td>
<td>1.07418</td>
</tr>
<tr>
<td>Lakefront * Invaded * Area</td>
<td>1.00003</td>
<td>1.00000</td>
</tr>
<tr>
<td>Lakefront * Invaded * Clarity</td>
<td>1.13562</td>
<td>1.13665</td>
</tr>
<tr>
<td>Lakefront * Invaded * Access</td>
<td>0.74856</td>
<td>0.49654</td>
</tr>
<tr>
<td>400m Access</td>
<td>1.02950</td>
<td>1.05782</td>
</tr>
<tr>
<td>400m Access * Invaded</td>
<td>0.84539</td>
<td>0.84581</td>
</tr>
<tr>
<td>400m Access * Invaded * Area</td>
<td>0.99999</td>
<td>0.99998</td>
</tr>
<tr>
<td>400m Access * Invaded * Clarity</td>
<td>0.95237</td>
<td>0.95054</td>
</tr>
<tr>
<td>Invaded</td>
<td>0.97232</td>
<td>0.94841</td>
</tr>
<tr>
<td>Riverfront</td>
<td>0.98936</td>
<td>0.98775</td>
</tr>
<tr>
<td>Access</td>
<td>1.02329</td>
<td>0.95054</td>
</tr>
<tr>
<td>Area</td>
<td>0.99998</td>
<td>0.95054</td>
</tr>
<tr>
<td>Clarity</td>
<td>0.99226</td>
<td>0.95054</td>
</tr>
<tr>
<td><strong>NON-LAKE COVARIATES</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Park 50m</td>
<td>0.99349</td>
<td>0.92272</td>
</tr>
<tr>
<td>Golf 50m</td>
<td>1.07519</td>
<td>1.00012</td>
</tr>
<tr>
<td>Log Metro Center Distance</td>
<td>1.27968</td>
<td>1.53508</td>
</tr>
<tr>
<td>Log Highway Distance</td>
<td>1.04741</td>
<td>1.11389</td>
</tr>
<tr>
<td>Steep Slope</td>
<td>0.69232</td>
<td>0.70005</td>
</tr>
<tr>
<td>Prime Farm Land</td>
<td>0.96842</td>
<td>0.98840</td>
</tr>
<tr>
<td>% Ag/Undeveloped</td>
<td>1.52250</td>
<td>1.60417</td>
</tr>
<tr>
<td>% Commercial</td>
<td>0.48004</td>
<td>0.83478</td>
</tr>
<tr>
<td>% Industrial</td>
<td>0.12806</td>
<td>0.19263</td>
</tr>
<tr>
<td>% Metropolitan Urban Services</td>
<td>5.23921</td>
<td>5.37604</td>
</tr>
<tr>
<td>Price Index</td>
<td>1.29567</td>
<td>1.45266</td>
</tr>
</tbody>
</table>

*a Robust standard errors clustered by parcel

N = 195,576 (153,007 uncensored, 42,569 censored)

650 Census Tract Fixed Effects, 1304 1000m Lake Identifiers
Appendix B: Appendix for Chapter 2

B.1 Numerical Appendix Introduction

This numerical appendix explains the coding intricacies for solving the binary integer programming simulations in the paper. Coding is done in MATLAB R2015a because of its intuitive array and matrix organizational structure and the quality of its numerical solvers.

Work on this project stretched over three years and made use of three separate binary integer program solvers. Under a previous version of MATLAB, \textit{bintprog}, which exclusively solves binary integer programs, lent itself to solving small landscape problems and simulations over short time horizons. While this worked in the initial analysis, it quickly became apparent that larger landscapes, with additional state and control variables, made for a more robust analysis of the problem. Fortunately, in a new edition, MATLAB retired \textit{bintprog} and replaced it with the more efficient solver, \textit{intlinprog}. \textit{intlinprog}, is capable of solving both integer and mixed integer programming problems and is currently the default solver for these types of problems in MATLAB. Quicker computation was the greatest benefit of \textit{intlinprog}, and the shorter run times allowed for
problems to be solved over time horizons as long as 150 time periods. Over
time horizons this long, all cost and damage values fall well below 1% of
their original values, making a strong case that any constant pattern of
invasion and control seen over this time horizon is a steady state. While
\texttt{intlinprog} made quick work of big problem with linear spread and grow,
increasing the complexity of the problem to include gradient based growth
proved to be too computationally intensive. Therefore, one final solver,
\textsc{SCIP}, provided the computational efficiency increase necessary to attempt
these problems in the robustness check section. \textsc{SCIP}, which bills itself as
the most efficient non-commercial constraint integer programming solver,
is available with a MATLAB interface when downloaded as a part of the
‘Opti Toolbox’. Even with this very efficient solver, only small problems
with gradient based growth can be solved.

Each of these solvers approaches the binary integer problem in
roughly the same way. First, they reduce the size of the set of solutions as
much as possible using a variety of heuristics and cuts. Once the solution
set has been reduced, the solvers begin a brute force solution method
known as branch and bound, which is described in the paper above. While
slight differences in coding have made some solvers more efficient than
others, a series of trials, conducted by the authors, found that each of the
three solvers finds the same solutions to identical problems. This inspires
confidence that each solver is truly finding the global solution to each problem. The preferred solver, strictly, due to its efficiency, SCIP.

B.1.1. A Basic Binary Integer Programming Problem

The basic binary integer programming problem is as follows:

\[
\begin{array}{cl}
\min_{x} & f^T x \\
\text{subject to} & Ax \leq b \\
& Aeq \times x = beq \\
& x \in \{0,1\}
\end{array}
\]  \hspace{1cm} \text{(39)}

Where \( f \) is a \((1 \times N)\) vector of damage and cost coefficients, \( A \) is an \((M \times N)\) matrix of coefficients for the linear inequality constraints, \( b \) is an \((M \times 1)\) vector of coefficients corresponding to the right hand side of the inequality constraints, \( Aeq \) is an \((L \times N)\) matrix of the coefficients for the linear equality constraints, and \( beq \) is an \((L \times 1)\) vector of coefficients corresponding to the right hand side of the equality constraints. \( N \) is the total number of state and control variables within the vector \( x \), \( M \) is the total number of inequality constraints, and \( L \) is the total number of equality constraints. Each of these elements must be individually constructed in MATLAB in such a way that the resulting simulation is representative of a problem in which an invasive species spreads and grows within a discrete landscape. The construction of each element is discussed in the sections below.
B.1.2. Creating the Elements of the Problem

The easiest way to picture this problem is to think of the grid spaces, shown in the paper, as matrices where each element is indexed by a number. For example, a three patch, homogeneous landscape with a carrying capacity of three is represented as follows:

\[
\begin{pmatrix}
  x_1 & x_4 & x_7 \\
  x_2 & x_5 & x_8 \\
  x_3 & x_6 & x_9 \\
\end{pmatrix}
\]

This, of course, only represents a single time period. When the problem has multiple time periods, multiple matrices are necessary to represent the landscape over time. A three time period horizon is represented as

\[
\begin{pmatrix}
  x_1 & x_4 & x_7 & x_{10} & x_{13} & x_{16} & x_{19} & x_{22} & x_{25} \\
  x_2 & x_5 & x_8 & x_{11} & x_{14} & x_{17} & x_{20} & x_{23} & x_{26} \\
  x_3 & x_6 & x_9 & x_{12} & x_{15} & x_{18} & x_{21} & x_{24} & x_{27} \\
\end{pmatrix}
\]

Note that elements \{x_1, x_{10}, x_{19}\} represent the same cell in the landscape in different time periods.

The elements \{x_1, x_2, ..., x_{27}\} shown above, are the state variables in the problem that indicate whether or not a specific cell in the problem is invaded. Control variables, which indicate whether the central planner chooses to suppress in a cell, must also be modelled in a similar fashion.

\[
\begin{pmatrix}
  z_1 & z_4 & z_7 & z_{10} & z_{13} & z_{16} & z_{19} & z_{22} & z_{25} \\
  z_2 & z_5 & z_8 & z_{11} & z_{14} & z_{17} & z_{20} & z_{23} & z_{26} \\
  z_3 & z_6 & z_9 & z_{12} & z_{15} & z_{18} & z_{21} & z_{24} & z_{27} \\
\end{pmatrix}
\]

Note that \(z_1\) is the control variable that corresponds to the state variable \(x_1\).

To create the single vector of state and control variables that exists in the binary integer programming problem shown above, the matrices are
vectorized as follows with the state variables stacked in order above the control variables:

\[
\mathbf{x} = \begin{pmatrix}
\mathbf{x}_1 \\
\mathbf{x}_2 \\
\vdots \\
\mathbf{x}_{27} \\
\mathbf{z}_1 \\
\mathbf{z}_2 \\
\vdots \\
\mathbf{z}_{27}
\end{pmatrix}
\]

All the necessary constraints are now created by predetermining the relationships between these state and control variables. These relationships (rows) are stacked to form the matrices \( A \) and \( A_{eq} \).

**B.1.3. Creating the matrix \( A_{eq} \) and the vector \( b_{eq} \)**

\( A_{eq} \) is a matrix of the coefficients for the linear equality constraints. The only equality constraints necessary are those which determine the cells that are initially invaded. Once again, suppose the landscape consists of three homogeneous patches, with a carrying capacity of three.

\[
\begin{pmatrix}
\mathbf{x}_1 & \mathbf{x}_4 & \mathbf{x}_7 \\
\mathbf{x}_2 & \mathbf{x}_5 & \mathbf{x}_8 \\
\mathbf{x}_3 & \mathbf{x}_6 & \mathbf{x}_9
\end{pmatrix}
= \begin{pmatrix}
\mathbf{x}_{10} & \mathbf{x}_{13} & \mathbf{x}_{16} \\
\mathbf{x}_{11} & \mathbf{x}_{14} & \mathbf{x}_{17} \\
\mathbf{x}_{12} & \mathbf{x}_{15} & \mathbf{x}_{18}
\end{pmatrix}
= \begin{pmatrix}
\mathbf{x}_{19} & \mathbf{x}_{22} & \mathbf{x}_{25} \\
\mathbf{x}_{20} & \mathbf{x}_{23} & \mathbf{x}_{26} \\
\mathbf{x}_{21} & \mathbf{x}_{24} & \mathbf{x}_{27}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\mathbf{z}_1 & \mathbf{z}_4 & \mathbf{z}_7 \\
\mathbf{z}_2 & \mathbf{z}_5 & \mathbf{z}_8 \\
\mathbf{z}_3 & \mathbf{z}_6 & \mathbf{z}_9
\end{pmatrix}
= \begin{pmatrix}
\mathbf{z}_{10} & \mathbf{z}_{13} & \mathbf{z}_{16} \\
\mathbf{z}_{11} & \mathbf{z}_{14} & \mathbf{z}_{17} \\
\mathbf{z}_{12} & \mathbf{z}_{15} & \mathbf{z}_{18}
\end{pmatrix}
= \begin{pmatrix}
\mathbf{z}_{19} & \mathbf{z}_{22} & \mathbf{z}_{25} \\
\mathbf{z}_{20} & \mathbf{z}_{23} & \mathbf{z}_{26} \\
\mathbf{z}_{21} & \mathbf{z}_{24} & \mathbf{z}_{27}
\end{pmatrix}
\]

In time period zero, let the central planner discover a one unit initial invasion in the middle patch. This must be specified in advance to MATLAB and the constraint row in \( A_{eq} \) must force \( x_6 = 1 \) and all other state
variables to equal zero. Also, as will be discussed later, all control variables must equal zero since the invasion must be discovered before it can be suppressed. Therefore constraint is as follows.

\[(0 0 0 0 1 0 0 0 \ldots 0 0 0) \times \begin{pmatrix} x_1 \\ \vdots \\ x_6 \\ \vdots \\ z_{27} \end{pmatrix} = 1\]

The row vector made up of zeros and a one is a single row within the matrix \(A_{eq}\), and in this case it is the only row because only one cell is initially invaded. The one on the right hand side is the lone element in the vector \(b_{eq}\). Notice that because this is an equality constraint, there is no decision to be made. The sixth element of the row vector is the only non-zero element, and therefore, in order for the constraint to bind, \(x_6\) must equal one.

**B.1.4. Creating the matrix A and the vector b with linear spread and growth**

A is a matrix of coefficients for the linear inequality constraints and it is formed in a very similar fashion to \(A_{eq}\). One major difference is that there are multiple rows which describe multiple types of constraints. All the constraints dictating how invaded cells spread, grow, and persist through time are part of \(A\).
A concept central to the problem at hand is that an invaded cell remains invaded until it is suppressed or until the end of the time horizon arrives. The constraints that dictate this must, of course, be pre-specified for each state variable in each time period other than the last. Therefore, in the simple problem discussed in previous sections, 18 constraints are required to specify that an invaded cell remains invaded unless it is suppressed in the next time period. An example of one of these is if \( x_6 = 1 \), then \( x_{15} = 1 \), unless \( z_{15} = 1 \). Or written in vector notation

\[
\begin{pmatrix}
    x_1 \\
    x_6 \\
    \vdots \\
    x_{15} \\
    \vdots \\
    z_{15} \\
    \vdots \\
    z_{27}
\end{pmatrix}
\begin{pmatrix}
    0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & (-1) & 0 & \cdots & 0 & (-1) & 0 & \cdots & 0 & 0 & 0
\end{pmatrix}
\leq 0
\]

Note that if \( x_6 = 1 \) then either \( x_{15} = -1 \) or \( z_{15} = -1 \) must be true in order to satisfy the inequality constraint. The row vector becomes a single row in the matrix \( A \) and the zero on the right hand side becomes its corresponding element in the vector \( b \). A more general formulation of these persistence constraints is

\[
If \ x_c = 1 \quad Then, \ x_{c+i+j} = -1
\]

\[
Unless, x_{c+i+j+(T+1)} = -1
\]
Where $i$ is the highest carrying capacity in the landscape is, $j$ is the number of patches in the landscape, and $T$ is the time horizon. For the sake of simplicity, all the following constraints will not be written in vector form, but in this fashion instead.

Specifying spread and growth is nearly identical to specifying persistence constraints except that the cell to which the invasion is transmitted lies to the left, to the right, or above the original. For cells spreading to the left, the general formulation of constraints is

If $x_c = 1$

Then, $x_{c+i* (j-1)} = -1$

Unless, $x_{c+i* (j-1)* (T+1)} = -1$

For cells spreading to the right,

If $x_c = 1$

Then, $x_{c+i* (j+1)} = -1$

Unless, $x_{c+i* (j+1)* (T+1)} = -1$

And for cells growing upwards in intensity,

If $x_c = 1$

Then, $x_{(c-1)+i* j} = -1$

Unless, $x_{(c-1)+i* j* (T+1)} = -1$

When creating spread and growth constraints, special attention must be paid to the location of each cell within the landscape. For example, a cell in the left most patch cannot spread to the left, a cell in the right most
patch cannot spread to the right, and a cell in the upper most row cannot
grow upward in intensity. Therefore, no spread or growth constraints
should be made for these cells for directions which they cannot go.

The last set of constraints within the matrix $A$ force the central
planner to suppress from the top down within patches. This might seem
strange, since in a real world scenario there is no up/down, but these
constraints are vital to the functionality of the model. The only way that
this model knows how much intensity of invasion exists within a patch is
with the height of the continuously invaded cells within a column.
Therefore, to reduce intensity, suppression must be accomplished from
the top down. The constraint’s general formulation is

$\text{If } x_c = 1$

$\text{Then, } x_{c+1} = -1$

These constraints are only applicable to cells that are not in the bottom
most row of the landscape.

From all of the inequality constraints created here, the matrix $A$
would consist of 72 constraints. Only the continuity constraints must be
made for state variables in the final time period.

**B.1.5. Creating the vector $f$**

$f$ is a vector of coefficients that specifies how much damage is caused
when a cell is invaded within the landscape and how much it costs the
central planner to prevent this damage. Therefore, each state and control variable within \( x \) has a corresponding cost or damage parameter within \( f \).

The elements representing damages are created by dividing the intensity level which the cell represents by the carrying capacity of the patch and then multiplying by the baseline value of the patch. Normalize the baseline values of each patch to one, therefore in the previous three patch landscape the damages associated with the state variables in the first time period,

\[
x = \begin{pmatrix} x_1 & x_4 & x_7 \\ x_2 & x_5 & x_8 \\ x_3 & x_6 & x_9 \end{pmatrix} \Rightarrow f = \begin{pmatrix} 1 & 1 & 1 \\ .66 & .66 & .66 \\ .33 & .33 & .33 \end{pmatrix}
\]

For future time periods these damages are discounted

\[
x = \begin{pmatrix} x_{10} & x_{13} & x_{16} \\ x_{11} & x_{14} & x_{17} \\ x_{12} & x_{15} & x_{18} \end{pmatrix} \Rightarrow f = \begin{pmatrix} .95 & .95 & .95 \\ .627 & .627 & .627 \\ .314 & .314 & .314 \end{pmatrix}
\]

\[
x = \begin{pmatrix} x_{19} & x_{22} & x_{25} \\ x_{20} & x_{23} & x_{26} \\ x_{21} & x_{24} & x_{27} \end{pmatrix} \Rightarrow f = \begin{pmatrix} .903 & .903 & .903 \\ .596 & .596 & .596 \\ .298 & .298 & .298 \end{pmatrix}
\]

Then these matrices are vectorized to create the top half of the vector \( f \).

\[
x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{26} \\ x_{27} \end{pmatrix} \Rightarrow f = \begin{pmatrix} 1 \\ .66 \\ \vdots \\ .596 \\ .298 \end{pmatrix}
\]

Suppression costs associated with each control variable are created in a similar fashion to the damages shown above. For each element, the carrying capacity is divided by the intensity level represented by that element then multiplied by the parameter alpha. If alpha is normalized to
one for each patch and suppression costs are discounted over time, the elements of $f$ associated with the control variables are as follows,

$$x = \begin{pmatrix} z_1 & z_4 & z_7 \\ z_2 & z_5 & z_8 \\ z_3 & z_6 & z_9 \end{pmatrix} \Rightarrow f = \begin{pmatrix} 1 & 1 & 1 \\ 1.5 & 1.5 & 1.5 \\ 3 & 3 & 3 \end{pmatrix}$$

$$x = \begin{pmatrix} z_{10} & z_{13} & z_{16} \\ z_{11} & z_{14} & z_{17} \\ z_{12} & z_{15} & z_{18} \end{pmatrix} \Rightarrow f = \begin{pmatrix} .95 & .95 & .95 \\ 1.425 & 1.425 & 1.425 \\ 2.85 & 2.85 & 2.85 \end{pmatrix}$$

$$x = \begin{pmatrix} z_{19} & z_{22} & z_{25} \\ z_{20} & z_{23} & z_{26} \\ z_{21} & z_{24} & z_{27} \end{pmatrix} \Rightarrow f = \begin{pmatrix} .903 & .903 & .903 \\ 1.354 & 1.354 & 1.354 \\ 2.708 & 2.708 & 2.708 \end{pmatrix}$$

Once again these matrices are vectorized and represent the bottom half of the vector $f$. The entire vectors of state and control variables, $x$, and the associated cost and damage vector, $f$, are as follows,

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{26} \\ x_{27} \\ z_1 \\ z_2 \\ \vdots \\ z_{26} \\ z_{27} \end{pmatrix} \Rightarrow f = \begin{pmatrix} 1 \\ .66 \\ \vdots \\ .596 \\ .298 \\ 1 \\ 1.5 \\ \vdots \\ 1.354 \\ 2.708 \end{pmatrix}$$

B.1.6. Creating a heterogeneous landscape

So far, the landscapes discussed have been homogeneous in carrying capacity, meaning that the columns in the matrices are of equal height. To create spatially heterogeneous carrying capacities, rectangular matrices are still used, however, the constraints that dictate how the
invasion spreads are omitted for some of the state variables. For example, a landscape with carrying capacities {3, 2, 3} that exists over a three period time horizon still consists of 27 state variables and 27 control variables.

\[
\begin{pmatrix}
 x_1 & x_4 & x_7 \\
 x_2 & x_5 & x_8 \\
 x_3 & x_6 & x_9 \\
\end{pmatrix}
\begin{pmatrix}
 x_{10} & x_{13} & x_{16} \\
 x_{11} & x_{14} & x_{17} \\
 x_{12} & x_{15} & x_{18} \\
\end{pmatrix}
\begin{pmatrix}
 x_{19} & x_{22} & x_{25} \\
 x_{20} & x_{23} & x_{26} \\
 x_{21} & x_{24} & x_{27} \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
 z_1 & z_4 & z_7 \\
 z_2 & z_5 & z_8 \\
 z_3 & z_6 & z_9 \\
\end{pmatrix}
\begin{pmatrix}
 z_{10} & z_{13} & z_{16} \\
 z_{11} & z_{14} & z_{17} \\
 z_{12} & z_{15} & z_{18} \\
\end{pmatrix}
\begin{pmatrix}
 z_{19} & z_{22} & z_{25} \\
 z_{20} & z_{23} & z_{26} \\
 z_{21} & z_{24} & z_{27} \\
\end{pmatrix}
\]

However, the right spread constraints for \{x_1, x_{10}\}, the left spread constraints for \{x_7, x_{16}\}, and the upward growth constraints for \{x_5, x_{14}\} are omitted. Without these constraints, it is impossible for \{x_4, x_{13}, x_{22}\} to be invaded. Therefore, the central planner will never choose to control with cells \{z_{13}, z_{22}\}. Essentially, the problem pretends that the variables \{x_4, x_{13}, x_{22}, z_4, z_{13}, z_{22}\} do not exist.

**B.1.7. Requirements specific to the solver SCIP**

Different binary integer programming solvers require slightly different inputs before performing simulations. The matrices and vectors described above are sufficient for running the solvers *bintprog* or *intlinprog*, however, some additional requirements are necessary before running the solver SCIP. SCIP requires a single matrix of linear constraints, therefore, \(A_{eq}\) needs to be appended onto \(A\). SCIP also requires a vector of upper bounds and a vector of lower bounds for these constraints. \(b_{eq}\) should be appended onto \(b\), and the resulting vector is the required vector of upper
bounds. While a vector of lower bounds is necessary for SCIP, it is not critical to the problem in general. These lower bound values will, however, change the efficiency of the computational time, so authors should customize these values to suit their own needs. Finally, SCIP requires that all variables be identified as binary so that it knows that the problem is a binary integer one. With these requirements in place, it is possible to run SCIP and find the optimal solution to the problem.
B.2 Code Appendix

The following is MATLAB code for the Asian Carp application shown in section 2.6.4.

```matlab
T = 150;  % Time Horizon
i = 20;   % Height of Grid space
j = 25;   % Number of Patches
r = .05;  % Discount Rate
alpha = 10;  % Suppression cost parameter
gamma = 2400;  % Eradication cost parameter

k = [14 9 11 14 13 8 15 12 14 12 9 13 4 20 20 20 20 20 20 20 20 20 20];

lake = 500000;
```
river = 200;
p = [river river river river river river river river river river river river river ...  
lake lake lake lake lake lake lake lake];

% Create the carrying capacity configuration in the initial time period

% Cells specified in this matrix cannot be invaded, eradicated, or suppressed in the first time period.
% With this, we can create heterogeneous columns from the rectangular grid space.
Persistence_blanks_0 = [1:i-k(1) i+1:(2*i-k(2)) 2*i+1:(3*i-k(3)) 3*i+1:(4*i-k(4))...  
4*i+1:(5*i-k(5)) 5*i+1:(6*i-k(6)) 6*i+1:(7*i-k(7)) 7*i+1:(8*i-k(8)) 8*i+1:(9*i-k(9))...  
9*i+1:(10*i-k(10)) 10*i+1:(11*i-k(11)) 11*i+1:(12*i-k(12)) 12*i+1:(13*i-k(13))...  
13*i+1:(14*i-k(14)) 14*i+1:(15*i-k(15)) 15*i+1:(16*i-k(16)) 16*i+1:(17*i-k(17))...  
17*i+1:(18*i-k(18)) 18*i+1:(19*i-k(19)) 19*i+1:(20*i-k(20)) 20*i+1:(21*i-k(21))...  
21*i+1:(22*i-k(22)) 22*i+1:(23*i-k(23)) 23*i+1:(24*i-k(24)) 24*i+1:(25*i-k(25))];

% Extend Persistence_blanks_0 through all time periods
for m=1:T
    Persistence_blanks(:,m) = Persistence_blanks_0+i*j*(m-1);
end

%The next step is to construct the matrix of constraints. This will be %done in steps.

%PERSISTENCE CONSTRAINTS
%NOTE: These are inequality constraints (<= 0)
B = spalloc((i*j*(T-1)),(i*j*T*2),(3*i*j*(T-1)));
for c = 1:((T-1)*(i*j))
    if c+i*j ~= Persistence_blanks
        B(c,c) = 1;
        B(c,c+i*j) = -1;
        B(c,(c+i*j)*(T+1)) = -1;
    end
end
% Identify cells in the right most and left most columns
ints = 1:i*j*T;
for m = 1:i
  right(m,:) = ints(i*j-(m-1):i*j:end);
  left(m,:) = ints(m:i*j:end);
end

% RIGHT SPREAD CONSTRAINTS
% NOTE: These are inequality constraints (<= 0)
Cright = spalloc((i*j*(T-1)),(i*j*T*2),(3*i*j*(T-1)));
for c = 1:i*j*(T-1)
  if c ~= right
    if (c+i)+(i*j) ~= Persistence_blanks
      Cright(c,c) = 1;
      Cright(c,(c+i)+(i*j)) = -1;
      Cright(c,((c+i)+(i*j)*(T+1))) = -1;
    end
  end
end

% LEFT SPREAD CONSTRAINTS
% NOTE: These are inequality constraints (<= 0)
Cleft = spalloc((i*j*(T-1)),(i*j*T*2),(3*i*j*(T-1)));
for c = 1:i*j*(T-1)
  if c ~= left
    if (c-i)+(i*j) ~= Persistence_blanks
      Cleft(c,c) = 1;
      Cleft(c,(c-i)+(i*j)) = -1;
      Cleft(c,((c-i)+(i*j)*(T+1))) = -1;
    end
  end
end

% Identify cells in top most row
up = 1:i*i*j*T;
%UPWARD SPREAD CONSTRAINTS
%NOTE: These are inequality constraints (<= 0)
Cup = spalloc((i*j*(T-1)),(i*j*T*2),(3*i*j*(T-1))); for c = 1:i*j*(T-1)
    if c ~= up
        if (c-1)+(i*j) ~= Persistence_blanks
            Cup(c,c) = 1;
            Cup(c,(c-1)+(i*j)) = -1;
            Cup(c,((c-1)+(i*j)*(T+1))) = -1;
        end
    end
end
end

%CONTINUITY OF GROWTH CONSTRAINTS
%The point of these constraints is to ensure that suppression occurs %from the top of a column downward. The invaded cells within a %column must be continuous.
%NOTE: These are inequality constraints (<= 0)
Ccont = spalloc((i*j*(T-1)),(i*j*T*2),(2*i*j*(T-1))); for c = 1:((T-1)*(i*j))
    if all(rem(c,i) ~= 0 & c ~= Persistence_blanks)
        Ccont(c,c) = 1;
        Ccont(c,c+1) = -1;
    end
end
end

%Matrix of Spread Constraints
A = [B;Cright;Cleft;Cup;Ccont];

%INITIAL INVASION CONSTRAINT
% Specify which cells are initially invaded
% NOTE: If multiple cells in a column are initially invaded, only the % top cell needs be specified here because of the continuity
% constraints
x0 = zeros(1,size(A,2));
x0(17) = 1;
x0(38) = 1;
x0(59) = 1;
x0(80) = 1;

A1 = [A; x0];

% Create discount vector 'beta'
beta = zeros(i*j*T,1);
for t = 1:T
    for c = 1:i*j*T
        if c <= i*j*t && c > (i*j)*(t-1)
            beta(c) = (1+r)^(1-t);
        end
    end
end

% Suppression Cost
k_new = repmat(k,i,1);
s = zeros(i,j);  % Specify size of s
for c = 1:j
    for r = 1:i
        s(r,c) = alpha*(k_new(r,c)/((i-(r-1))));
    end
    s(i,c) = gamma*k_new(r,c);
end
s = s(:);
s = repmat(s,T,1);

% Damages
p_new = repmat(p,i,1);
for c = 1:j
for r = 1:i
  d(r,c) = p_new(r,c)*((i-(r-1)))/(k_new(r,c));
end
end
d = d(:);
d = repmat(d,T,1);

%COST AND DAMAGE VECTORS
damage = beta.*d.*ones(i*j*T,1);
suppress_cost = beta.*s.*ones(i*j*T,1);

% Vector containing the coefficients of the linear objective function
f = [damage; suppress_cost];

ru = [zeros(size(A,1),1); 5*ones(size(x0,1),1)]; % Upper bound of each constraint in matrix A
rl = ru-1; % Lower bound of each constraint in matrix A

% Run integer program
for c = 1:i*j*T*2
  xtype(c) = 'b'; % Specify all variables as binary
end
lb = zeros(i*j*T*2,1); % Lower bound for variables
ub = ones(i*j*T*2,1); % Upper bound for variables

opts = optset('maxtime',1000000,'maxnodes',100000000);
[x(:,;,:),fval,exitflag,stats] = scip([],f,A1,rl,ru,lb,ub,xtype,[],[],[],opts);

% Convert x vector to matrices
x = reshape(x,i,j,2*T);

% Create matrix X (optimal spread matrix) and Z (optimal suppression matrix)
X = x(:,1:T);
Z = x(:,T+1:2*T);