Green Building Evaluation of the Roman Pantheon Using Fuzzy Set Concept.

THESIS

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By

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Abstract

This research focuses on the ancient green building evaluation of the Roman Pantheon, use the fuzzy set concept. In today’s society and in the future, there will be pressure from society to continue to make improvements on green building design and construction, and this research aims to advance current green building evaluations. An ancient building, such as, the Roman Pantheon was used for this research because it can provide great insight to ancient design and construction practices, many of which have been forgotten over the years.

The green building evaluation first begins with the evaluation of specific green building categories including Site Selection, Use of Water, Energy Conservation, Materials and Resources, Air Quality, and Innovation of Design and Construction. Each category is rated based on the subcategories, which are assigned linguistic rating values. The linguistic values range from extremely non-green to extremely green. After all linguistic ratings are assigned, a weight average is calculated to determine the category rating and the total greenness rating of the Pantheon.

While the weighted average can provide a total green building rating for a structure, it does not take into account the unclearly defined linguistic expressions. In order to provide a more realistic green building evaluation, the fuzzy set concept was implemented to calculate the overall greenness rating. The results of this research were that the weighted
average and the fuzzy rating did differ in their final rating values. Based on the assessment using the weighted average the Pantheon has a total greenness rating of 1.29 or between fairly green and green, and based upon the fuzzy set concept, the Pantheon has a total rating of [-0.61, 2.01] or between neutral and fairly green.
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Fields of Study

Major Field: Civil Engineering
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Chapter 1 Introduction

1.1 Background

Green buildings are nothing new in today’s architectural and construction landscape. The driving factor of the green building movement is sustainable development, which impacts the design, construction, and life span of the buildings and those who build and occupy them.\(^1\) Much effort is put into future plans of new rating systems, better methods of building design, construction and the understanding of impacts our buildings have on the environment. Little time and research has explored ancient techniques of possible green building design and construction. Whether or not our ancestors understood greenness\(^2\) or thought about sustainability can be debated, but based on their accomplishments they understood design and construction considerably well and were efficient in their methods.

Ancient construction and engineering can be found all over the world, but one in particular culture contributed a large amount of knowledge to the engineering of structures; the Romans. One of the most significant advancements in architectural and construction history is the discovery of concrete (opus caementicium) and the Romans are given credit

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\(^1\) Kibert 2008:1.

\(^2\) Instead using “green” for both adjective and noun that indicate a subset of the sustainability of a constructed facility, in this study, the author uses “green” as an adjective and “greenness” as the noun associated with a sustainable constructed facility.
for its discovery. Not only are they credited for its discovery, but they mastered its use through trial and error and years of building experience. Concrete as a new building material allowed for architects to design with less limits, engineers to build with more ease, and labors with less manpower. One of Rome’s most well-known and researched examples of concrete use is the Pantheon and its monolithic dome.

The Romans have gone down in history as one of the most successful civil engineering cultures of all time just as the Roman Pantheon is historically known as one of the longest standing architectural wonders of the world. Today the Pantheon is over 2000 years old and has been carefully maintained to nearly original conditions allowing for researchers to gain a full understanding of the design, construction process, and materials used to build it, making it an ideal structure to use for an ancient green building evaluation.

1.2 Goal and Objective

In today’s building industry there is a huge push for buildings to use green methods for design and construction, but very little is known about the greenness of historical structures. It is the goal of this study to research the history, materials, and construction process of the Pantheon in Rome in order to evaluate the buildings greenness. Because the Pantheon is mostly still true to its original form, with documented changes, it is an ideal structure to use for such evaluation.

There is no question that new buildings in the United States are being evaluated in terms of both its greenness and long-term sustainability by guidelines and rating systems provided by the United States Green Building Council (USGBC). Too much emphasis is put on the future of building design and construction, but not enough has been devoted to
the study of ancient structures. The objective of this research is to use the knowledge obtained through the literature research to aide in the understanding of the structure and its construction process, to then evaluate it under similar greenness guidelines provided by USGBC.

1.3 Scope and Limitations

The scope of this study is to determine the greenness of the Pantheon by studying the construction methods and techniques used to build the structure. The greenness will be evaluated under six categories, Site Selection, Use of Water, Energy Conservation, Materials and Resources, Air Quality and Innovation of Design and Construction. Fuzzy logic will be used to find the total greenness rating for the Pantheon. A computer-aided design software will be used to create a 3D model which will provide a visualization of the building and the various stages of construction. Materials, tools, and equipment used by the Romans will be covered throughout the discussion of construction stages.

The study will not discuss in depth the structural capacities of the Pantheon nor will the study elaborate in depth on the selection of construction materials used to build the Pantheon. While the design and material selection play a vital role in the Pantheon’s existence, this study aims to explore the structure as a building, its constructing process and greenness evaluation. While much of the scope of this study aims to evaluate the green engineering used to build the Pantheon it will not explore the structures sustainability.
1.4 Tasks

In order to provide a visual representation of the Pantheon, a 3D model was developed using Autodesk Inventor. The exploration of construction stages will be developed into a series of computer aided drawings. The detailed drawings will aid in the description of the structure and understanding the construction of the five main parts of the Pantheon: the foundation, portico, transition block, rotunda and the dome. A rating system to evaluate the greenness of the Pantheon will be created based upon similar guidelines provided by USGBC. The greenness rating is determined and reviewed individually by category and used to calculate a total rating. A subjective assessment tool called fuzzy logic is used to calculate and graph the ratings in order to find the total greenness rating of the Pantheon.

1.5 Thesis Organization

The remainder of the thesis will be divided into five additional chapters. Chapter 2 will provide a literature search of historical overview of both Roman architectural achievements and a historical background on the Pantheon. It is important to understand the recent history leading up to the design and construction of the Pantheon, as well as understanding the meaning and use behind the structure.

Chapter 3 begins to discuss the details of the Pantheon, the components and their dimensions and material composition. In order to adequately cover the resources, techniques and methods used during construction it is important to know the relationship of each component to the other and to have a general understanding the materials used. Images of the 3D model will be used to provide a visual representation of the structure.
Chapter 4 will discuss in great depth the details of construction. Beginning with the surveying and site layout, and ending with the final touches of the portico and decor. In order to gain a better understand on the methods used by the Romans, and in order to evaluate the greenness of them, the tools and equipment utilized during the construction will be discussed in Chapter 4.

Chapter 5 will take the knowledge learned and described in the previous chapters to evaluate the entire structures greenness. It will include eight sections, the first section will discuss the definition of greenness and how it will be evaluated throughout the chapter, one section for each of the six evaluation topics, as previously mentioned, and one section to summarize the overall greenness.

Following the initial greenness evaluation Chapter 6 will discuss the definitions of fuzzy logic and its importance to this research. A sample of graphical representations and calculations of the rating categories as well as a discussion of the total greenness rating will be provided in this chapter. Appendix C will provide the models and rating calculations for all categories.

Chapter 7 will include an overall summary. Final conclusions of the study will be discussed and suggestions for continued research in similar topics will be provided.
Chapter 2 Literature Review

2.1 Introduction

Understanding the relevant history of the Pantheon and Roman construction technologies aides in the understanding of what resources were available and what design influences impacted to creation of the Pantheon. It is this understanding that will later be used to evaluate the Pantheon’s greenness relative to its time. In order to provide context to the time frame of when the Pantheon’s construction took place, a historical overview is provide within the following sections.

2.2 Architectural History of Rome from Augustus to Hadrian

Beginning at the time Caesar Octavian was granted the title of Augustus, in 27 BC, Rome began to see a profitable and stable government. Not only did Augustus bring the city of Rome peace, and prosperity, he also invested in a large building campaign throughout his reign. The most significant work took place during the first half of his reign with the help of his friend and partner, Marcus Agrippa.³ Agrippa is well known and documented for his campaign of public works to improve the water supply along with other infrastructure needs of the city, but he also managed to build a new quarter in an area of

Rome called Campus Martius. Agrippa’s construction campaign in Campus Martius includes, but is not limited to the Pantheon (the original Pantheon), Basilica of Neptune, Laconica of Agrippa, Saepta Julia⁴, and Diribitorium di Agrippa⁵.

The Augustan Period also brought the Temple of Mercury at Baia, known as one of the first large domed structures in Roman history. The Temple of Mercury is dated to the Augustan Period because the construction technique known as *opus incertum* was used for the walls.⁶ *Opus incertum* is the use of irregular small stones as the exterior concrete wall covering and was common practice used before it progressed into *opus testaceum*, which is a more uniform concrete wall covering made of bricks. The first major public structure in Rome to consist of nearly all brick-faced construction was a Tiberian outer wall structure for a military camp of the Praetorian Guard in A.D. 21-23. The camp covered over 14 acres and the wall stood nearly 9 meters tall making it one of the largest masonry projects of its time.⁷

Nearly a century after the original Pantheon and the Temple of Mercury were built, and 40 years after the construction of the Tiberian brick-faced wall Nero constructed his Domus Aurea, also known as Nero’s Golden House in A.D. 64. Nero’s Golden House contained barrel-vaulted chambers and an octagonal domed hall. Similar to the Pantheon the rotunda is made of brick and concrete. The span of the octagonal dome only measures at 14.7 meters, which does not surpass its domed predecessors. The Golden House is

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⁴ Julius Caesar’s building later used for public voting.
⁵ A public hall for voting purposes.
⁶ Waddell 2008:55.
significant because of the combination of a domed hall and the series of arched and vaulted chambers.\(^8\)

The combination of arches and vaults is not unique to Nero’s Golden House. One of Rome’s most iconic structures, the Colosseum, utilizes arches and vaults at a large quantity. The Colosseum was started by Vespasian (ruled A.D. 69-79) and work continued under the rule of Titus (ruled A.D. 79-81) and completed by Domitian (ruled A.D. 81-96).\(^9\)

The superstructure includes a series of concrete barrel vaults along with both stone and concrete archways. The Colosseum is a shining example of how the Romans quickly perfected construction technologies such as archways and vaults (Figure 2.1). It is the experience of such technologies that eventually led to the implantation of concrete domes.

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\(^8\) Hemsoll 1989:2.
Figure 2.1: The Colosseum, Rome, Italy ca. A.D. 72 – 96 (photo courtesy Dr. Fabian H. Tan, 2015).

Twenty years after the inauguration of the Colosseum and only 5-10 years before the Pantheon, the Roman emperor Trajan commissioned the Markets of Trajan in A.D. 100-112. Trajan’s Market was built on the hillside of Quirinal Hill and structurally took advantage of the slope (Figure 2.2). Having been built on a hill the structure has a unique layout. The market was full of barrel-vaulted shops and several different levels built on the changing elevation of the hill. Brick-faced concrete was a main construction technique, but the market was full of travertine details. The main level of the Trajan’s Market included a semicircular design, which radiated throughout the structure. The Markets of Trajan are an important architectural achievement because of the complex multi-level design and the use
of common building materials such as brick, concrete and travertine. It may very well be the first covered shopping center in history.

![Image of Trajan's Market in Rome, ca. AD 100-110](photo courtesy Dr. Fabian H. Tan, 2015)

Figure 2.2: Trajan’s Market in Rome, ca. AD 100 - 110 (photo courtesy Dr. Fabian H. Tan, 2015).

2.3 Predecessors of the Pantheon

After its completion the Pantheon was one of a kind, yet still had a strong connection to many pre-existing building types. The Pantheon's main building components; rotunda, dome and portico provide an insight to its predecessors and other related structures. The rotunda has a circular plan and its walls are cylindrical, the dome stems from the invention of arches and vaults, while the portico was a common temple front in Roman architecture. To gain a better understanding of pre-Hadrian construction
accomplishments and design inspirations two categories of earlier Roman structures are discussed: circular and domed structures.

2.3.1 Circular Structures

Circular structures were not new to Roman architecture when the Pantheon was designed. While there may be many pre-Pantheon examples of circular buildings, what makes the Pantheon stand out against structures within the same design category is its size and scale. The circular design allows for a large enclosed area and it was an easy plan to layout during pre-construction. Circular architecture stems from two common sources; religious and funerary buildings, such as temples and tholoi (plural of tholos, beehive-shaped structure or tomb).

Circular temples that pre-date the Pantheon are typically much smaller in scale and did not have porticos. The Temple of Vesta (ca. 7th century BC shown in Figure 2.3) in Forum Romanum and Temple of Hercules Victor in Forum Boarium (ca. 6th century BC), both in Rome, are circular structures consisting of an inner cylindrical cella or naos (interior central room) that was closed off to the public and was surrounded by a ring of columns. Like temples in Greece, those in Italy have cellae (plural of cella) or nai (plural of naos). The Romans were inspired by the architectural designs found in Greece including the use of columns.

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10 Rectangular-shaped temples in Greece were often furnished with an adyton (a sacred inner chamber next to naos)
Figure 2.3: The remains of the Temple of Vesta (photo courtesy Dr. Fabian H. Tan, 2015). First built by Numa Pompilius, the second king of Rome, 716-673 BC. It was likely the second earliest and most sacred structure in Rome. The eternal fire dedicated to the goddess Vesta must be kept alive by the vestal virgins.
Figure 2.4: Temple of Hercules Victor, ca. 579 – 534 BC (photo courtesy Dr. Fabian H. Tan, 2015). This round temple has several names including Temple of Mater Matuta and Temple of Vesta before specialists settled down with Temple of Hercules Victor, whose original construction was attributed to the Roman king Servius Tullius, ca. 579 – 534 B.C..

The Pantheon was not the first circular structure designed and constructed; many pre-existing Roman structures, such as the Mausoleum of Augustus (ca. 28 BC) and Laconica (baths) of Agrippa (ca. 1st century BC), both in Rome, and the Temple of Echo (also known as Temple of Mercury, ca. 1st century BC) in Baiae on the Bay of Naples, could have been design inspirations for Hadrian.

Funerary architecture was often times cylindrical in shape and even had dome coverings. Circular funerary structures found in Greece were *tholoi*. An example of a *tholos* is the the Treasury of Atreus (ca. 13th century BC), a corbelled
dome and the predecessor of later Roman domes, is found in Mycenae, west of Athens (Figure 2.5).

Figure 2.5: Treasury of Atreus in Mycenae, ca. 13th century B.C. (photo courtesy Dr. Fabian H. Tan, 2015). The tholos (corbelled dome) of the Treasury of Atreus, seen from outside. Note the corbelled archway above the entrance.

2.3.2 Arches, Vaults, and Domes

Even before the widespread use of concrete the Romans had mastered the arch and vault technology. The voussoir arch is an archway made of wedge-shaped stones and relied on the downward weight and outward thrust of the stones to remain intact. Stone arches were commonly implemented in the design of aqueducts and bridges throughout Roman history because of the span distance and
strength they could provide. Once concrete became a common building material, arches and vaults began to take new forms. Rather than relying on the downward weight and outward thrusts of the stones to build an arch, the Romans could build arch-shaped formwork and make concrete arches and vaults that are often times decorated with stone facings to resemble a true stone arch and vault. The concrete arches and vaults eventually transformed into semi-domes, half-domes and full domes.

Domes provide the largest enclosed open space possible for a structure. The Romans did not only use domes because of the large interior space is created, but also because it was the most logical method to roof a circular space without center supports.

Before the development of the concrete dome known today, an ancient technique called corbelling was used to create a dome like structure. It is a method that involved stacking stones horizontally in the shape of an arch that meet at a center point and counter balanced to support itself. Corbelled domes can be best described as beehive shaped. The Middle Bronze Age tholos tombs such as the Treasury of Atreus mentioned above is one of the earliest examples of a domed structure (Figure 2.6).
2.4 Historical Overview of Pantheon

Originally built by Marcus Agrippa in 28-24 B.C. and later rebuilt during Hadrian reign in A.D. 118 the Pantheon is still standing over 2000 years later. The Pantheon was originally a single building in a large building campaign completed in the Campus Martius in Rome, Italy. Agrippa’s construction projects, including the original Pantheon, in his building campaign described earlier were later destroyed in a fire in A.D. 80. All that remained of the Pantheon’s original structure was the foundation and was later incorporated into the new Pantheon built by Hadrian.\textsuperscript{11} The real architect of the existing Pantheon is unknown, but it is believed that Hadrian played an in-depth and vital role in

the design and construction. Hadrian had a strong belief of honoring the creators and leaders before him and because of this belief he kept Agrippa’s inscription, “M·AGRIPPA·L·F·COS·TERTIVM·FECIT” – Marcus Agrippa the son of Lucius, three times consul, built this.  

![Figure 2.7: The Pantheon’s dome and porch (photo courtesy Dr. Adrian Tan, 2015).](image)

Having been built in Campus Martius the Pantheon is surrounded by several large structures; in front of the Pantheon was a long forecourt, to the west was the Baths of Nero and to the east of the forecourt stood a temple for Hadrian’s mother-in-law Matidia. East

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of the Pantheon was Saepta Julia, a voting precinct. Also nearby on the east was Agrippa’s Baths. Campus Martius is a low-lying, frequently flooded area in Rome and was not an ideal building location for the vast scale of the Pantheon, but Hadrian kept the original location and adopted to same alignment as the original structure.\(^{13}\) Agrippa’s Pantheon followed the alignment of the surrounding buildings, which happened to be 5 degrees west of true north.

Not only did Hadrian keep the location and alignment, he also kept the structures name. The word Pantheon is believed to mean ‘all gods’ and because the Pantheon is a temple\(^{14}\) it is believed to be a temple dedicated to all of the gods. At one time the Pantheon was home to the statues of Mars, Venus and many others supporting the theory that the Pantheon was named after the gods.\(^{15}\) The function of the Pantheon relates to astronomy and the sun, moon, planets and stars and theory has it that the Pantheon’s dome represents the heaven and the oculus is the sun.\(^{16}\)

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\(^{13}\) Waddell 2008:67.
\(^{14}\) The Pantheon is not a traditional, rectangular-shaped temple, but it has similar features such as the high podium with steps leading up to only one side of the structure (MacDonald 1982:119-127).
\(^{15}\) MacDonald 1976:77.
\(^{16}\) Grant 1960:285.
Figure 2.8: The dome of the Pantheon (courtesy Dr. Fabian H. Tan, 2015)
Chapter 3 The Pantheon

3.1 Introduction

The structure of the Pantheon can be described by its five main components: foundation, transition block, rotunda, dome and porch (commonly termed as portico) as shown in Figure 3.1. To better understand the structure as a whole each component has been fully examined and modeled after the complete collection of sketches done by Giovanni Battista Piranesi (1720–1778) and his son Francesco (1790). Drawings by both Piranesi make up the most detailed etches ever provided for the Pantheon (Figures 3.2 and 3.3). The construction process explanation and the greenness evaluations are dependent upon the understanding of the structure and its individual components. More importantly how they relate to one another. The following sections provide a brief description of each component and their relationships to one another as well as each component’s overall dimensions and the materials used for its construction.

17 Attached on the south side of the Pantheon is Agrippa’s Bath and could be considered the sixth component. Agrippa’s Bath was not included because the original purpose of the Pantheon was a temple of which the Bath was not a part of the temple structure; therefore it is not included in this research.
Figure 3.1: Isometric view of the Pantheon from the Northwest highlighting the five main components covered by this research.
Figure 3.2: View of the Pantheon by Giovanni Battista Piranesi (1720 –1778). Note the existing towers at the back of Agrippa porch that are no longer extant.

Figure 3.3: Rear view of the Pantheon by Giovanni Battista Piranesi (1720 –1778) shows the remains of Agrippa’s bath that is still extant.
3.2 Foundation

A structure’s foundation is in many ways the most critical component because the foundation bears the weight of the entire structure and if the foundation fails the entire structure will fail. The Romans knew the structural importance of the foundation and put careful thought into its design to ensure it would be more than adequate to support the entire structure, but at the same time they were able to minimize materials and labor by using ring foundations. Through the observations of Vitruvius (3.4.1) the typical foundation was built to be at least one half wider than the walls it supports and rammed earth was used to fill the space between the foundation walls. The Romans would implement the ring foundation technique for the Pantheon. The Pantheon’s foundation can be broken down into two separate sections, the foundation of the portico and that of the transition block and rotunda.

The foundation of the portico reuses the original ashlar (stone) foundation of Agrippa’s structure. The north side of Hadrian’s rotunda is located against the south walls of Agrippa’s foundation. The current portico lies on the north side of the rotunda. The ashlar foundation was held together using iron clamps which provides horizontal stability, but not vertical; therefore ashlar foundations are not as stable as concrete foundations. The stones could easily shift vertically, creating an uneven foundation to build a structure on top. From examining the portico excavation sketches of Beltrami from 1898, the stones were placed in strips running north to south and spaced to that they would lay underneath

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the location of the columns.\textsuperscript{20} When the ashlar foundation was reused concrete was added to create a more stable and uniform structure of which to build the portico atop. Partially resting on the portico foundation is the very large transition block (see Figure 3.7), meaning more weight rests on the south side than the north side of the portico’s foundation. Because of this, concrete had to be added to the portico foundation after settling had occurred. Overall the portico’s foundation measures nearly 16 meters by 35 meters as shown below in Figure 3.4\textsuperscript{21}.

The foundation of the rotunda is a slab foundation, but in a circular form or ring, and made of all Roman concrete rather than blocks of stone. The Roman concrete known as \textit{opus caementicium} is a mixture of mortar and aggregate. The mortar is made of water, lime, and sand or pozzolana, a volcanic powder.\textsuperscript{22} As suggested by Vitruvius strip foundations were built one half wider than the walls they were designed to hold, which is also true for the ring slab of the rotunda’s foundation. Two rings make up the total thickness of more than 10 meters with an outer overall radius of roughly 30 meters and inner radius of 20 meters. The original ring foundation had a thickness of only 7.2 meters. The second ring with an additional thickness of 3 meters was added after the first foundation ring of concrete cracked.\textsuperscript{23} Together the two rings make up the foundation of the rotunda, the main building component.

\textsuperscript{20} Waddell 2008:297-302.
\textsuperscript{21} Dimensions estimated using details provided by the authors of sources such as Moore, Waddell and Wilson Jones.
\textsuperscript{22} Moore 2005:24-26.
\textsuperscript{23} Moore 2005:7.
Figure 3.4: Portico foundation top view shown in red and the ring foundation outlined in blue (a) and side view of the portico foundation (b).

Connecting the foundation of the portico to the ring foundation is the foundation beneath the transition block. This part of the foundation is made of concrete and built in
conjunction with the ring foundations. The details of each part of the portico foundation are listed below in Table 3.1.

<table>
<thead>
<tr>
<th>Part</th>
<th>Materials</th>
<th>Material Amount</th>
<th>Dimensions$^{24}$</th>
</tr>
</thead>
</table>
| Portico Foundation/Agrippa’s (8 stone strips, 7 strips slab) | Travertine blocks, iron clamps and rammed earth. Strip style foundation. | 960 m$^3$ stone 840 m$^3$ rammed earth                           | 16 m x 35 m (overall)  
 Strip width = 2.5 m  
 Strip length = 16 m  
 Strip depth = 3 m |
| *Concrete added to portico foundation for additional support. |                                                     |                          |                                                                 |
| Foundation Ring 1           | Roman concrete.                                     | 5,168 m$^3$ concrete     | Inner radius = 20 m  
 Outer radius = 27 m                                              |
| Foundation Ring 2           | Roman concrete.                                     | 2,686 m$^3$ concrete     | Inner radius = 27 m  
 Outer radius = 30 m                                              |
| Block Foundation            | Roman concrete.                                     | 428 m$^3$ concrete       | Area between portico foundation and ring foundations.            |

Table 3.1: Foundation Details

3.3 Rotunda

The main body of the Pantheon is the rotunda, a round building typically covered by a dome. The rotunda is the largest component of the Pantheon having a height of roughly 30 meters and an outer wall radius of 28 meters (Figure 3.5a). The exterior of the Pantheon’s rotunda has three sections marked by cornices at 13, 22 and 30 meters above the ground (Figure 3.5b). The interior has two sections matching the heights of the first

$^{24}$ All major dimensions were obtained by the author from sources such as Waddell, Wilson Jones and Moore. Minor dimensions such as the hidden voids within the Pantheon’s walls are fragmented and scarce; thus, for visual illustration purposes, in order to show the readers the existence of these voids, these dimensions were estimated.
two on the exterior at 13 and 22 meters above floor level (Figure 3.5b). From an interior view the dome looks as though it rests upon the top of the second section, but the dome actually beings at 30 meters, which measures at the top of the rotunda.


All major dimensions were obtained by the author from sources such as Waddell, Wilson Jones and Moore. Minor dimensions such as the hidden voids within the Pantheon’s walls are fragmented and scarce; thus, for visual illustration purposes, in order to show the readers the existence of these voids, these dimensions were estimated.
Figure 3.5: Images of the Pantheon’s Rotunda. The three exterior sections of the rotunda show in (a) and the dimensions of the interior and exterior sections in (b).
The walls of the rotunda are 6 meters thick and are made of brick facing and concrete filler, a common construction practice by the Romans. The average brick size used for the Pantheon measures 28.75 centimeters long and 3.45 centimeters thick (Table 3.2). With bricks that size and with mortar joints averaging 1 centimeter thick on all sides, more than 4500,000 bricks would have been used to build the outside brick-faced wall of the rotunda alone. Roman concrete as discussed in the previous section was used for the wall construction, however the aggregates placed in the walls varied in size and weight. In the case of the Pantheon the contractors strategically placed the larger and heavier aggregate materials in the bottom layers of concrete and the aggregate pieces become progressively smaller and lighter with each rising section of the walls.27

<table>
<thead>
<tr>
<th></th>
<th>Middleton cm)</th>
<th>Licht (cm)</th>
<th>Blake (cm)</th>
<th>Average (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Brick Length</strong></td>
<td>33</td>
<td>27-28</td>
<td>21.5-30</td>
<td>28.75</td>
</tr>
<tr>
<td><strong>Brick Thickness</strong></td>
<td>3</td>
<td>3.8</td>
<td>2.8-4.3</td>
<td>3.45</td>
</tr>
<tr>
<td><strong>Mortar Thickness</strong></td>
<td>1</td>
<td>1.3</td>
<td>1</td>
<td>1.52 28</td>
</tr>
</tbody>
</table>

Through Course: 58.60cm long/wide and 4.2cm thick

Table 3.2: Pantheon Brick Dimensions given by Middleton, Licht and Blake29

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28 Average brick size used to calculate number of bricks used in construction.
29 Waddell 2008:92.
The thick solid walls of the rotunda would have been more than enough support for the dome. To make the walls lighter and to use less material, a series of voids were incorporated into the design. At the floor level the voids are visible and are used for displaying statues, and the voids in the upper two thirds of the rotunda are almost all hidden from sight. The voids also provided easy access to the interior during construction and aided in the curing time for the concrete by allowing air circulation into the walls. 30

Figure 3.6: Rotunda wall showing the radii of the interior and exterior sides (top view).

Being the largest part of the Pantheon, the rotunda required a significant amount of materials to build. A summary of the rotunda and materials is provided in Table 3.3. The type and amount of materials plays a vital role in determining the structure’s greenness.

30 Wards-Perkins 1981:114.
<table>
<thead>
<tr>
<th>Part</th>
<th>Materials</th>
<th>Material Amount</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotunda Outer Wall</td>
<td>Bricks and mortar.</td>
<td>450,000 bricks</td>
<td>Radius = 28 m</td>
</tr>
<tr>
<td>Rotunda Inner Wall</td>
<td>Bricks and mortar.</td>
<td>350,000 bricks</td>
<td>Radius = 22 m</td>
</tr>
<tr>
<td>Rotunda Level 1</td>
<td>Bricks, mortar and concrete.</td>
<td>350,000 bricks</td>
<td>Top elevation = 13 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13,232 m³ concrete (without voids)</td>
<td>Overall height = 13 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Outer radius = 28 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Inner radius = 22 m</td>
</tr>
<tr>
<td>Rotunda Level 2</td>
<td>Bricks, mortar and concrete.</td>
<td>240,000 bricks</td>
<td>Top elevation = 22 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8,482 m³ concrete (without voids)</td>
<td>Overall height = 9 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Outer radius = 28 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Inner radius = 22 m</td>
</tr>
<tr>
<td>Rotunda Level 3</td>
<td>Bricks, mortar and concrete.</td>
<td>216,000 bricks</td>
<td>Top elevation = 30 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7,540 m³ concrete (without voids)</td>
<td>Overall height = 8 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Outer radius = 28 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Inner radius = 22 m</td>
</tr>
<tr>
<td>Rotunda Wall Total</td>
<td>Bricks, mortar and concrete.</td>
<td>812,000 bricks</td>
<td>Outer radius = 28 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>28,274 m³ concrete (without voids)</td>
<td>Inner radius = 22 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Height = 30 m</td>
</tr>
</tbody>
</table>

Table 3.3: Rotunda Details

3.4 Transition Block

The transition block connects the portico to the rotunda. It also has three similar exterior sections as seen on the rotunda and are marked by exterior cornices (Figure 3.7). Based on the discovery that the lowest section of the block and rotunda have interlocking bricks which indicate that the two components were built together for at least a portion of the Pantheons construction. While the transition block has many similarities with the rotunda it has one distinct difference, the only stairways in the structure are within the wall of the transition block. The stairways not only could be used to move materials to

---

31 All material calculations performed by author. Material calculation made by assuming rotunda wall thickness is same throughout the height and no voids. See Appendix B.
33 Waddell 2008:92.
the upper levels during construction, but they provided air flow to help advance the curing time of the concrete and dematerialized the structure.

Figure 3.7: Three levels of the transition block.
Figure 3.8: The transition block (center) as seen from the northeast side of the Pantheon (photo courtesy Dr. Adrian Tan).
<table>
<thead>
<tr>
<th>Part</th>
<th>Materials</th>
<th>Material Amount</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block Level 1</td>
<td>Roman concrete and bronze.</td>
<td>59,800 bricks 4,576 m³ concrete</td>
<td>Top elevation = 13 m Overall height = 13 m Width = 32 m Length = 10 m</td>
</tr>
<tr>
<td>Block Level 2</td>
<td>Roman concrete and bricks.</td>
<td>41,896 bricks 2,880 m³ concrete</td>
<td>Top elevation = 22 m Overall height = 9 m Width = 32 m Length = 10 m</td>
</tr>
<tr>
<td>Block Level 3</td>
<td>Roman concrete and bricks.</td>
<td>37,240 bricks 2,816 m³ concrete</td>
<td>Top elevation = 30 m Overall height = 8 m Width = 32 m Length = 11 m</td>
</tr>
<tr>
<td>Block</td>
<td>Roman concrete and brick.</td>
<td>138,900 bricks 10,272 m³ concrete</td>
<td>Height = 30 m</td>
</tr>
</tbody>
</table>

Table 3.4: Transition Block Details

3.5 Dome

One of the most famous and most largely studied components of the Pantheon is the roof of the rotunda, the dome, shown in Figure 3.8. Spanning nearly 50 meters the dome of the Pantheon has an overall radius of 25 meters and consists of seven step rings, a saucer dome, and an oculus with a diameter of 6 meters. For the purpose of this study the dome, when referred, includes the seven step rings and the saucer dome as shown below in Figure 3.10. Resting upon the walls of the rotunda the dome has an overall height of 13 meters. The entire dome itself is not monolithic, but each of the eight parts is believed to be monolithic.\(^{35}\)

\(^{34}\) All major dimensions were obtained by the author from sources such as Waddell, Wilson Jones and Moore. Minor dimensions such as the hidden voids within the Pantheon’s walls are fragmented and scarce; thus, for visual illustration purposes, in order to show the readers the existence of these voids, these dimensions were estimated.

\(^{35}\) Waddell 2008:105.
The seven step rings all vary in depth, width and height, but all have downward slopes. Just as the concrete walls of the rotunda and block, the steps rings were built of concrete, but without brick-facing walled formwork. The rings used timber formwork that
was later removed. Each of the step rings functions as a supporting wall below the next and all together with the rotunda wall supports the saucer dome that rests at the top. The saucer dome with a height of nine meters has varying thicknesses, at the base it has a thickness of six meters and near the oculus has a thickness of 1.5 meters.\(^{36}\) The exterior view of the dome does not seem nearly as impressive as the view from inside the Pantheon (see Figure 3.12).

![Figure 3.11: Pantheon’s exterior wall as seen from its southwest side. Notice the portico on the left of the dome. The structure on the right was the remain of the Bath of Agrippa (photo courtesy Dr. Adrian Tan).](image)

As previously mentioned the interior of the dome looks as though it starts at 22 meters above the floor, but it actually starts at 30 meters above the floor. The interior of the dome is lined with 140 coffers that start just above the top of the second section of the rotunda wall, which creates an illusion that the dome is larger than its true form (Figure 3.12).

Figure 3.12: Pantheon’s interior with its numerous alcoves (photo courtesy Dr. Adrian Tan).
Part | Materials | Material Amount | Dimensions
---|---|---|---
Saucer Dome | Roman concrete and bronze. | 737 m³ | Height = 9 m  
Radius = 16.76 m
Step Rings | Roman concrete and bronze. | 1488 m³ | Varying sizes for each step ring.  
Total height = 3.8 m
Oculus | Roman concrete and bricks. | minimal | Radius = 3 m

| Part | Materials | Material Amount | Dimensions
---|---|---|---

Table 3.5: Dome Details

Figure 3.13: Comparison of dome (a) and coffers (b) starting levels.

---

37 All major dimensions were obtained by the author from sources such as Waddell, Wilson Jones and Moore. Minor dimensions such as the hidden voids within the Pantheon’s walls are fragmented and scarce; thus, for visual illustration purposes, these dimensions were estimated.
3.6 Portico

Having a stunning height of 22 meters, the portico catches the attention of anyone who approaches the Pantheon (Figure 3.14b). As a traditional temple front the portico is linear in shape. Rectangular in form, 35 meters by 16 meters, and the portico is home to 16 marble columns each 16 meters tall. Eight columns span the front of the portico with seven intercolumniations, with the middle being the widest and the remaining six equally spaced. The remaining eight columns make up two additional rows, four each. A layout of the columns is shown below in Figure 3.15.

![Figure 3.14: Portico of the Pantheon. (a) Side view from the West and (b) front view from the North. Continued.](image-url)
Figure 3.14: Continued

Figure 3.15: Portico column layout.
The portico is covered by a roof that extends from the transition block. The original portico design had the pitch of the roof reaching 28 meters, the height of the rotunda wall, but instead it reaches only 22 meters. The change in height is why there are two triangular pediments on the front of the Pantheon rather than only one. The intended columns were not able to be quarried, transported and positioned at the height required, therefore during the construction the design was adjusted to incorporate shorter columns. The upper pediment was built directly on the transition block, which was constructed before (at the back of) the portico. Because the roof height of the portico did not reach the full 28 meters the two triangular pediments do not properly align (Figure 3.16 below).\(^{38}\)

\[\text{Figure 3.16: Upper and lower pediments on portico.}\]

\(^{38}\) Wilson Jones 2000:204-212.
Figure 3.17: The portico with its granite columns supporting the lower front pediment (courtesy Dr. Adrian Tan)
Many materials were resourced to complete the portico. The 16-meter tall monolithic columns are made of granite. The bases of the capitals along with pilasters, the door casing, parts of the entablature, tympanum and cornices, are made of marmor Pentelicus\textsuperscript{39}. Another marble, marmor Lunense\textsuperscript{40}, was used for the column capitals because it allowed of the details of the Corinthian style to be carved more easily due to the softer nature of the marble.

\textsuperscript{39} Pentelic marble, quarried from Mount Pentelicus in Attica, Greece
\textsuperscript{40} Lunense marble, quarried from Carrara, transported through the ancient port of Luna (now Luni), northwest Italy.
Figure 3.19: Corinthian capitals made of Lunense marble support the entablature made of Pentelic marble. On top timber trusses and rafters were installed to support the portico roof (photo courtesy Dr. Adrian Tan).

Resting atop of columns is the porticos entablature. The entablature consists of several parts, architrave, frieze and cornice that would be placed individually. Parts of the entablature are made of stone and incorporate finely carved details. The portico of the Pantheon has both interior and exterior entablatures. The exterior supports the triangular pediment and raking cornice that it trims, while the two interior entablatures support two piers of which the truss rest upon.

Materials other than granite and marble were used to build the portico. The interior supports for the roof were made of stone piers and concrete arches covered in brick and
The pediment of which was later covered by marble was made of stone. The roof structure was primarily made of bronze, including the support beams and roof tiles, which was a preferred building material because of its fire proof quality. After two devastating fires in Rome that pre-date the Pantheon contractors and designers became more inclined to use fireproofing materials for construction. While the portico may not be the largest component, it most certainly has the most detail out of the five components covered in Chapter 3.

<table>
<thead>
<tr>
<th>Part</th>
<th>Materials</th>
<th>Material Amount</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pediment</td>
<td>Stone, Marble facing</td>
<td>243.2 m³ of stone block</td>
<td>Height = 7.6 m, Width = 32 m, Depth = 2 m</td>
</tr>
<tr>
<td>Roof</td>
<td>Wood, bronze and stone</td>
<td>Not calculated.</td>
<td></td>
</tr>
<tr>
<td>Entablature</td>
<td>Stone, Marble facing</td>
<td>407 m³ of stone</td>
<td>Total Length = 76 m, Height = 2.68 m, Depth = 2 m</td>
</tr>
<tr>
<td>Columns (16)</td>
<td>Granite</td>
<td>50 m³ of granite</td>
<td>Height = 16 m, Average thickness (diameter) = 2 m</td>
</tr>
</tbody>
</table>

Table 3.6: Portico Details

3.7 Summary of Components

Each of the five components, foundation, rotunda, transition block, dome and portico, have their own unique characteristics, but together they create one cohesive

---

41 Waddell 2008:124.
42 First fire took place in 27 B.C. and the second in A.D. 80. Both fire destroyed large portions of Rome and it lead to the use of more fire-proof materials.
43 All major dimensions were obtained by the author from sources such as Waddell, Wilson Jones and Moore. Minor dimensions such as the hidden voids within the Pantheon’s walls are fragmented and scarce; thus, for visual illustration purposes, these dimensions were estimated.
structure. As discussed in the previous sections the Pantheon may appear to be a simple rotunda with a traditional temple front, but within the design many complex components are hidden inside. Not only is it important to know and understand the components and their dimensions and materials for a structure greenness evaluation, but to also understand the construction process and required tools, equipment and methods. Chapter 4 will discuss the construction process of the Pantheon’s substructure and superstructure.
Chapter 4 Sequence of Pantheon’s Construction

4.1 Pre-Construction

By A.D. 118 the Romans had decades of experience in pre-construction activities such as planning, building layout, and mobilization. Pre-construction includes any and all activities completed before the excavation process, including: surveying, soil samples, earth moving and leveling, building layout, and material procurement. The location of the Pantheon, in the Campus Martius, adds a level of complexity to the pre-constructions phase, but nothing the Romans have not already accomplished in the past.

4.1.1 Soil and Water Table

Little has been documented or recorded on the soil and water tables of Rome during the second century A.D., but what is known is that Rome was strategically placed along the Tiber River in order to have sufficient and easy access to water, and additional security. Because Rome was constructed along the side of a river, certain areas of Rome, such as Campus Martius were susceptible to flooding. The flooding would naturally bring in layers of slit and as a result the ground level in
Campus Martius would rise with each flood.\textsuperscript{44} It is with this in mind that the foundations for buildings in this area would require to be well-designed to adequately support the superstructure.

According to a study completed in 1981 by S.A.R.A. (Societa Aerofotografie e Rilevamenti Aerofotogrammetrici), the average altitude of Campus Martius is 16.50 meters. To provide a better idea of the positioning of the Pantheon below are the altitude findings provided by S.A.R.A. \textsuperscript{45}

<table>
<thead>
<tr>
<th>ALTITUDES OF CAMPUS MARTIUS AND THE PANTHEON</th>
<th>(METERS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Campus Martius (in 1981)</td>
<td>16.50</td>
</tr>
<tr>
<td>water table in 1981 (south side of the drum)</td>
<td>13.28</td>
</tr>
<tr>
<td>portico floor level and average interior floor level</td>
<td>13.23</td>
</tr>
<tr>
<td>S.A.R.A. datum (raised sidewalk on east side of Pantheon)</td>
<td>13.10</td>
</tr>
<tr>
<td>upper level of travertine pavement in front of portico</td>
<td>11.93</td>
</tr>
<tr>
<td>level of travertine on each side of the portico</td>
<td>12.00-11.80</td>
</tr>
<tr>
<td>top of reticulate wall on south side of drum</td>
<td>11.34</td>
</tr>
<tr>
<td>base of ashlar wall at the northeast corner of the portico</td>
<td>9.74</td>
</tr>
<tr>
<td>height of the base of the reticulate wall on the south side of the portico</td>
<td>9.38</td>
</tr>
<tr>
<td>upper surface of the tufa bases on the south side of the drum</td>
<td>9.18</td>
</tr>
<tr>
<td>Oculus</td>
<td>58.30</td>
</tr>
<tr>
<td>top of drum (east side)</td>
<td>43.60</td>
</tr>
<tr>
<td>top of drum (west side)</td>
<td>44.70</td>
</tr>
<tr>
<td>top of transitional block</td>
<td>43.60</td>
</tr>
</tbody>
</table>

Table 4.1: S.A.R.A. Altitude Findings for Campus Martius and Pantheon

\textsuperscript{44} Waddell 2008:68.
\textsuperscript{45} Waddell 2008:69.
The Pantheon is only one mile from the Colosseum; therefore it is hypothesized that they share similar soil profiles.⁴⁶ The Colosseum rests on a bed made up of Pleistocene sedimentary deposits (gravels, sands and clays), recent Alluvial deposits (clays, silts and sands) and at ten meters below ground level lies bedrock (consolidated clay).⁴⁷ A soil profile made up of mostly sand, silts and clays would only add to the complexity of constructing a foundation in a frequently flooded area.

4.1.2 Surveying and Site Layout

Surveying consists of three tasks: establishment of bearings, measurement of distances, and the estimation of heights.⁴⁸ Even 2000 years ago surveying would be an important component of the pre-construction process. In the case of the Pantheon the surveyors would not have a blank plot of land to divide and mark for a new structure, they would be using the same site of the original Pantheon. Typically well before the construction process would being a surveying team would measure the site, determine legal boundaries, and take soil samples called soundings.⁴⁹ The samples would provide the designers and contractors with useful information to make any structural changes to foundation ensure a stable building. Because Hadrian reused a former site little if any earth moving was required, but would be done prior to the building layout.

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⁴⁶ Google Earth 4/18/2015.
⁴⁷ Funiciello et al. 1995:935.
The Romans commonly used three surveying tools: *groma, dioptra, and chorobates*. The *groma* and *dioptra* were used to extend a measured line or mark off lines at right angles in the horizontal direction. The *dioptra* while being complex was widely used by the Romans and it is very likely it would have been used during the layout of Pantheon. Because the *gromae* primary use is to form right angles and there are not many right angles in the Pantheon’s layout, it is very unlikely the Romans used the *groma* for the surveying of the Pantheon. The *chorobates* is a bench shaped leveling tool with sight holes and plumb-lines on each end, and a water level across the top. The *chorobates* is a simple leveling tool and would have been utilized to level the site during the earth moving process. It is very likely that a simple cord and stake system was used to draw out the circular plan of the Pantheon’s ring foundation.

While the measurements were being made the plan would be marked and laid out using a cord and stake method. The cords would extend well beyond the edge of the foundation because the ground below the cords will be excavated for the foundation. As previously mentioned in Chapter 2, several structure including Agrippa’s Baths, Saepta Julia and others surrounded the Pantheon’s site, making it a little more complicated to run cords far beyond the building layout. The cords and attached plumb-bobs hanging from them would act as guides; therefore it was extremely critical that they remain in place until the exaction process was complete. Based on the building floor plan and the likely hood that the Romans

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50 O’Connor 1993:59-60.
used a stake and cord layout system the Pantheon’s layout can been seen in the drawing below (Figure 4.1).

Figure 4.1: Possible cord and stake layout of the Pantheon.

Using the existing ashlar foundations of Agrippa’s Pantheon, the surveyors would begin with finding the centerline and carry it out to the center point of the rotunda. The center point would be marked roughly 27 meters from the south side of the portico foundation. Second, the surveyors would mark the distance away from the center point for the inner side of the ring foundation, 20 meters from the center, and again for the outer side of the ring foundation at 27 meters from the
center point. Once the foundation ring was laid out the surveyors would complete the layout by connecting the ring and the portico foundation to mark sections the transition block.

4.1.2 Material Procurement

During the ground-plan layout the contractor would begin gathering required materials, tools, transportation devices and equipment for the excavation process and foundation erection. Excavators would need digging tools such as shovels, hoes and picks, and lifting devices, such as ore baskets and simple derrick cranes. The Romans made their shovels of wood or iron and were commonly used during the excavation process. The ore basket is a common lifting and holding device made of reed and would be used to lift excavated soil from the trench. The simple derrick crane consisted of two wooden legs, four ropes and a pulley system that was used to lift the basket.52 Traveling and utility wagons were commonly used during construction activities and would be pulled by horse or bull.

The wagons were used to both bring and remove materials to and from the job site. The clay-baked bricks used for the walls and dome of the Pantheon were fabricated in Rome and are therefore considered local material and easily transported to the site.53 Pozzolana would be brought to the site in wagons from the nearby Alban Hills, 17 miles from the Pantheon’s site.54 Travertine was also

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54 Google Earth.
brought to the site in wagons from Tivoli. Barges and ships would also be required to bring some materials, such as lime from Naples to the site of the Pantheon. ⁵⁵ Below is a map showing the proximity of Naples, Tivoli and the Alban Hills in relation to the Pantheon’s site (Figure 4.2).

![Map of Italy showing the original location of materials used for the construction of the Pantheon](image)

**Figure 4.2:** Map of Italy showing the original location of materials used for the construction of the Pantheon ⁵⁶

4.2 Substructure Construction

The substructure consists of the foundations for three sections: the portico, the transition block and the rotunda. The portico’s foundation is separated from that of the

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⁵⁵ Moore 1995:175-179
⁵⁶ Google Earth.
transition block and the rotunda foundation, which were constructed together. Each foundation type required laborers, tools, equipment and transportation of materials and the following sections will describe the construction process of both foundations.

4.2.1 Portico Foundation

As discussed in the previous chapter the foundation of the portico is a combination of the existing ashlar foundation and the new concrete addition. In order for the contractors to reuse Agrippa’s ashlar foundation an inspection of the stone blocks would need to confirm the materials were adequate to reuse. At this time any broke stone or weak connections would be fixed and replaced.

4.2.2 Rotunda and Transition Block Foundation

After surveying and laying out of the building as described above in the pre-construction section, the laborers would excavate the soil using the guides laid out by the surveyors. After reaching a desired depth, determined by the contractor, laborers would begin placing the layers of concrete and travertine pieces. The ring foundation of the Pantheon measures at roughly 5 meters deep. The desired foundation depth was reaching *solidum* (solid ground) which is considered to be compacted earth or ideally bedrock. The excavation and timber lagging construction is shown below in Figures 4.3-4.6.

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59 Figures 4.3-4.6 are provided for this research courtesy of Dr. Adrian Tan (2015).
First the vertical posts (solider piles) would be driven into the ground using a pulley pile-driver. Once the vertical posts are in place the excavation process would begin by remove earth in layers 0.296 meters deep and place boards across the vertical posts. The excavation and board placement would take place until a depth of 5 meters was reached. Horizontal struts were placed between the vertical posts to keep the 10 meter wide trench from caving inward. The wooden boards would be left in place and the Roman concrete, with heavy aggregates would be placed in layers until the floor elevation was reached.

Lifting devices, cranes and baskets that were utilized during the excavation process to remove earth would also be used to lower the concrete materials down into the trench. Laborers would need to construct wooden ladders to provide easy access into the depths of the trench. The boards not only provided protection from possible cave-ins, but the Romans could build platforms on the horizontal struts and attach platforms to the wood boards. The platforms provided a stable working area for placing concrete.
Figure 4.3 Drive soldier piles (vertical posts) to a depth of about 5 meters (Tan 2015).

Figure 4.4 Excavate to a depth of 0.296 meters and place first layer of boards behind posts (Tan 2015).
Figure 4.5 Continue excavation and placing timber boards to reach 5 meters deep (Tan 2015).

Figure 4.6 (d): Begin placing *caementicium* into the ring foundation to the desired floor elevation (Tan 2015).
The rotunda foundation also included a drainage system. Because the dome has an open (oculus), with a diameter of 6 meters, water easily enters the Pantheon; therefore a drainage system would be extremely necessary to maintain a clean and healthy interior environment. The drainage holes seen from the interior after completion and a diagram of the entire system is shown below in Figures 4.7 and 4.8.
Figure 4.7: Drainage Holes - These holes on the floor inside the Pantheon were installed for drainage purposes; conduits were constructed under the floor to channel the water to the outside perimeter of the rotunda (photo courtesy of Dr. Adrian Tan).
Figure 4.8 A broken section of the Pantheon shows a drainage pattern under the floor. Note that water ponding on the floor enters drainage pipe and exits outside the concrete ring foundation. This photo was taken by Dr. Adrian Tan from a display (unknown author, December 2011) inside the Pantheon.

4.3 Superstructure Construction

The portico, transition block, rotunda, and dome make up the superstructure of the Pantheon. The superstructure includes to most complex construction practices and each of its four major components has unique challenges and building practices, but many construction techniques across the components. In this section the superstructure components are explored in the order of which they were constructed.
4.3.1 Rotunda and Transition Block

The superstructures first constructed parts were the walls of the block and rotunda. The walls of the Pantheon are faced with triangular shaped, baked clay bricks called *bessales*. The bricks act as permanent formwork for the *opus caementicium*, which would be placed in layers as the wall rose up from the ground. The Romans used heavier material for the aggregate, such as travertine, in the bottom layers of concrete. The brick-faced concrete wall structure is referred to as *opus testaceum* and was common practice in Roman architecture during the second century A.D.

As discussed in Chapter 3 the walls of the rotunda and transition block can be subdivided into three levels marked externally by cornices. The three levels may very well have been architectural, but they provided convenient stopping points during the walls construction (Figure 4.7). The rotunda wall would be built in three phases following the three levels.

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*Bessales* are square bricks that Romans would cut in half along the diagonal to create two triangular shaped bricks that would be used to make the outer layers of the wall. Roman *bessales* measure on average 19.7cm (Adam 1994:147).
Figure 4.9: Rotunda and Transition Block Wall Levels (a) level one of wall up to first cornice, (b) level two of wall up to second cornice, and (c) level three of wall up to third cornice.
The first phase of wall construction would be building the permanent brick formwork. Layers of bricks and mortar were placed in two rings, an outer and inner, roughly six meters apart on top of the foundation. Masons would work side by side to ensure the walls rose uniformly around the entirety of the structure. If for example, each mason were responsible for constructing 5 meters of the wall 13 masons working simultaneously would be required to build the outer wall and 10 masons for the inner brick wall, for a total of 23 masons. The bricks walls would be built 1.2 meters at a time, filled with Roman concrete and topped off with through-courses. Though-courses (also referred to as bonding-courses) are made of flat brick titles and extend through the width of the wall.\(^{61}\) They provided convenient stopping places for the masons and work could easily be continued from the through-course surface. Figure 4.5 shows the relationship of the brick wall, layers of concrete and through-courses.

\(^{61}\) Harris 1996.
After the wall reached a height of 13 meters an interior and exterior cornice would be added and the first level of the Pantheon would be complete. While working to the height of 13 meters the masons would be building the entrance, apse, niches, doorways and hallways, and other voids within the walls (Figure 4.9). The voids added complexity to the building process, which required attention to detail and created a lengthy construction process. The voids vary in space, height and width, some are even rounded and circular in form. The apse, and two semi-circular niches were topped with semi-domes in the middle level, which required temporary formwork to hold the brick and concrete until it hardened.

The lower and middle levels are clearly defined by their cornices on the interior and exterior walls. A major distinction between the construction process of the first two levels and the third is that the upper level wall and dome construction

Figure 4.10: Rotunda and transition block wall construction.
took place simultaneously. The dome and upper level wall both begin at the same elevation, the top of the middle level wall as previously discussed in Chapter 3. The construction process of the upper level wall and dome will be discussed in the next section.

Figure 4.11: Comparison of the voids in the three levels of the rotunda and block wall. (a) Lower level. (b) Middle level. (c) Upper level. Continued
The Pantheon has no evidence of putlog holes on the exterior; therefore independent (free standing) scaffolding would have been employed to construct the walls (Figure 4.7). The timber scaffolding was constructed alongside the height of the wall. Materials would be hoisted up and down in baskets using a pulley system similar to the method used for the excavation process. The masons and laborers would use timber ladders to access the higher levels of the scaffolding and working platforms were built at various levels.

Figure 4.12: Independent Scaffolding for interior and exterior wall construction. Right front view. Right side view. (Illustration: N. Parshall after Adam)

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Various techniques were employed during the wall construction. An important feature to mention are the relieving arches that can be found throughout the rotunda walls, above any door, window or cavity. Relieving arches only carried the load around the cavities while the concrete hardened. After the concrete hardened the arches no longer provided structural support unless they were through-wall arches. Arches that extend through the entire wall improve ductility and support lateral loads in the wall and are clearly present and visible on the exterior of the Pantheon.\textsuperscript{64}

The apse and niches were designed for aesthetic purposes and were used to display status of Roman gods. The cavities found within the three levels of the rotunda wall were constructed for easy access in and out of the building during construction and they provided ventilation for the curing process. All of the voids and cavities meant less materials were required during construction; however they made the construction process more complex. Beginning at the first level of the rotunda wall the voids and cavities would be measured and temporary centering and scaffolding would be built to support the rising walls of the voids. Many of the voids have curved or even semi-domed ceilings or covers (Figure 4.11).

\textsuperscript{64} Harris 1996.
Figure 4.13: Interior View of the Pantheon highlighting the semi-domes over the niches.

The transition block built along side of the rotunda wall would rise in the same way, brick-facing and filled in layers of concrete and wooden independent scaffolding would be utilized. The block like the wall so of the rotunda also consist of many voids and cavities, including two staircases (shown in Figure 4.6 above).
4.3.2 Dome

After the construction of the first two levels of the wall were complete, the third level of the rotunda wall and the first level of the dome would begin. The first level of the dome includes the first row of coffers and in order to account for the rounded interior wall with coffers a complex centering system was employed.\textsuperscript{65} The centering would be built in large parts on the ground. Arched ribbed formwork and coffer frames of specific sizes were designed and built to construct the first section of the dome. Additional arched ribbed and coffer formwork would be built and used for the step rings and the saucer dome. Due to the formworks complexity a trial run construction of the formwork was conducted at ground level. The trail run allowed for the laborers to gain a better understanding of the materials, methods and construction process of the centering.

After a successful trial run the first level of ribbed centering parts would be placed, by several cranes, on top of the second level of the rotunda wall. Each ribbed formwork would be held in place by a crane until the Pantheon was surrounded by a ring of cranes.\textsuperscript{66} Twenty-eight ribs were constructed, one for each column of coffers and one crane could support only one rib of formwork; therefore 28 cranes were required to support the first phase of centering. As the first level of centering rose, laborers would be securing them together and attaching the coffer frames. After the first phase of centering was secured the wall would begin being built.

\textsuperscript{65} There were 28 coffers in each row and decreased in size with each rising row.
\textsuperscript{66} Taylor 2003:204.
Because this portion of the dome is also part of the rotunda’s wall it is constructed similar to how the lower two levels of the wall were built, using brick faced concrete. The bricks were only used as exterior wall formwork (permanent) and the centering served as the interior wall formwork (temporary). Concrete would be placed in layers however instead of heavy aggregate (travertine, tufa and clay brick) used in the lower levels of the wall, the third level has aggregate made of only clay brick. The first phase of centering also includes the first and largest step ring, which is the elevation of the second row of coffers.

Once the first level of the dome and third level of the wall was complete and properly cured the second phase of centering could be attached to the first. Because the first level of the dome was designed to support itself and the weight of the centering most of the cranes could be removed and broken down. Some cranes remained to secure the centering while the second phase of ribbed formwork was being hoisted and attached to the existing centering. Repeating similar processes before, the coffering formwork would begin being attached up to the fifth row of coffers and regular boards would be attached up to the oculus.

After the center was finalized concrete would be laid in layers to build the next six step rings. Each step ring is monolithic and built individually. Structurally the step rings carried their own weight and would be carried directly downward onto the next step ring or wall. As long as the step rings were properly cantilevered the dome would support itself and transfer its load to the center of the
rotunda wall. The step ring construction is very similar to the methods used in corbelling methods mentioned in Chapter Two. The centering scheme for the dome construction discussed above was proposed in 2003 by R. Taylor and is shown below in Figure 4.11.

![Figure 4.14](image)

Figure 4.14: Pantheon’s centering. (a) Left shows phase one ribbed centering and right shows phase two ribbed centering. (b) Combined centering scheme for both phases. The coffering details show that the formwork for the coffers would have been attached to the centering (after Taylor).

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4.3.3 Portico

The first and largest task to be completed for the portico is the placement of the granite columns. Cranes and pulley systems would be erected in specific locations for the placement of each column. Working from the two interior rows outward placing the columns would require a tremendous amount of coordination, equipment, manpower and energy. Once the 16 columns were in place the entablature would be constructed next. Because the entablature consists of several parts (architrave, frieze and cornice) it was not placed as one large piece, but rather several smaller individual pieces.

After the entablature was secured in place the interior piers and arches along with the pediment would be constructed next. The stones for the piers would be lifted into place and wooden centering would be erected to hold the concrete arches while they cured. In similar fashion the stone pediment would begin to take form one block of stone at a time. The pediment was finished off with a decorative cornice.

The truss roof was constructed similar to today’s wooden trusses, consisting of horizontal beams that rest upon the two side entablatures and the two interior walls (arch/pier supports). Including horizontal beams, king posts and rafters, the portico roof was simple in design and was easily constructing in place rather being preassembled and lifted into place. The bronze roof tiles would ensure a water and fireproof structure.
4.3.4 Interior and Exterior Finishes

Just as important as the structural designs were the buildings finishes. Finishes include wall coverings (interior and exterior), decorative carvings, paintings, and floor veneers. The finishing and decorating process is just as time consuming and difficult as the structural construction process. Many construction techniques were employed during the decorating process, such as scaffolding and the use of lifting devices. The interior height of the Pantheon brought many challenges in the decorating phase.

The interior wall covering consisted of seven different types of marbles. Geometric shaped marble panels would be hung on the walls using a fastening and mortar system. A series of hole would be dug into the wall for each panel and the holes would be filled with mortar. The marble panels would have holes cut into the edges and a metal fastener would be embedded into the marble and into the mortar filled holes. Because the walls consisted of intricate designs varying in different colors and sized panels, this process would be extremely time consuming.

Similar to the wall, the interior floor also has a detailed design and consists of circular, squared and rectangular shapes. Unlike the wall fastening technique, the floor tiles could easily be prefabricated off site and quickly placed onto a bed of mortar without the use of clamps. The only time consuming task with laying the floor tiles was to ensure the pattern was laid correctly and that the floor was level.

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69 The seven types of marble used on the interior of Pantheon: white marble, Pavonazzetto, Verde Antico, red porphyry, Giallo Antico, green basalt, and Bigio Africano (Waddell 2008:104).
70 The cut tiles patterns on the walls and floors is referred to as *opus sectile* (Taylor 2003:232-233).
The Pantheon’s exterior was covered by a combination of marble and stucco. Marble was primarily reserved for the face of the Pantheon and covered the stone portico and parts of the rotunda. The marble would be hung in similar fashion as the interior, using a hole and fastener method. Stucco was a common covering technique for brick exteriors, however it cannot be confirmed weather or not the Pantheon’s brick was covered or left exposed.

4.4 Conclusion and Discussion of Construction Process

The construction process of the Pantheon is complex and required many years to complete and the contractors in charge had a tremendous amount of detailed work to coordinate. The Romans were able to use design and construction techniques that were successful in the past, this helped eliminate trial and error practices. Even though many of the methods used for the Pantheon were common practice, new innovative methods were also employed. The circular rotunda with wall full of cavities meant less materials and lighter walls, but were designed to maintain strength to hold the dome. Centering schemes were commonly built for arches and vaults, but nothing in comparison to the centering built for the dome had been attempted.

Beginning with the foundation the Romans reused and recycled many items and materials. The original ashlar foundation was utilized in the new structure, broken stone and bricks were used as aggregate throughout the different levels of the wall, and wooden formwork and scaffolding were reused during construction. Not only were materials reused and recycled, but demolition and construction waste were kept to a minimum, yielding an environmental friendly construction process.
Details of the construction process are key components that are considered when evaluating an ancient structure’s greenness. Aspects such as the site location, water use, energy, materials, resources, air quality, and innovation help provide a better understanding on if a structure is indeed green or if environmental cautious decisions were made during the construction process. Chapter 5 will discuss the greenness rating system as well as the Pantheon’s greenness rating.
Chapter 5 Green Building Evaluation of the Pantheon

5.1 Introduction

When discussing the greenness of a structure the term green is often confused with sustainable; therefore it is important to understand the differences between the two terms. Sustainability when discussing construction refers to the ecological (environmental), social and economic issues of a building, with a focus on the community, while a green building is a healthy facility designed and built in a resource-efficient manner, using ecologically (environmental) based principles.\textsuperscript{71} In other words a green building is evaluated on its environmental impacts alone, while a sustainable building must meet criteria in terms of its environmental, economic and social impacts.\textsuperscript{72} The greenness can be viewed as only a small part or a subset of a buildings overall sustainability, also meaning that a building can be green and not sustainable, but in order for a building to be sustainable it must be green.

The United State Green Building Council (USGBC) is a leader in the green and sustainability realm for building design and construction. USGBC as created in 1993 is an organization that promotes sustainability in the building and construction industry.\textsuperscript{73} USGBC has a number of ways that the help support the United States community and the

\textsuperscript{71} Kibert 2008:9.
\textsuperscript{72} Yanarell et al 2009: 296-302.
\textsuperscript{73} USGBC website – www.usgbc.org/about/history: accessed 9/7/2015.
even larger global community that we all share. One of their most widely known programs is called LEED or Leadership in Energy and Environmental Design.\textsuperscript{74} LEED has a rating system that services as the basis for the greenness evaluation of the Pantheon. There are eight categories used for the LEED rating system: Location and Transportation, Sustainable Sites, Water Efficiency, Energy and Atmosphere, Materials and Resources, Indoor Environmental Quality, Innovation and Regional Priority. Because many of the current categories and their subcategories are too modern they cannot be applied to ancient structures; therefore LEED is only used as a guide for the ancient structure greenness evaluation developed for the Pantheon.

With LEED as a guide six categories have been developed for the ancient structures green building evaluation: Site Selection, Use of Water, Energy Conservation, Materials and Resources, Air Quality, and Innovation of Design and Construction. Table 5.2 below shows each of the subcategories used to evaluate the Pantheon in terms of greenness. Rather than a rating system use for LEED, the Ancient Greenness Evaluation will evaluate each subcategory on a scale. The scale will range for \textit{-4 for extremely non-green} to \textit{+4 for extremely green}.\textsuperscript{75} The full scale is provided below in Table 5.1. Zero will represent \textit{neutral} for any items that may not add or take away from the Pantheon being a green building. Once each subcategory is provided a numerical value the main category will be summarized by adding each subcategories value and taking the weighted average, which will provide the category with a final numerical value. After each value for the six main

\textsuperscript{74} USGBC website - www.usgb.org/leed\#rating: accessed 9/7/2015.

\textsuperscript{75} The term non-green is used for lack of a better term for not green. Terms very not green and not very green can easily be confused, thus very non-green will be used to mean very not green.
categories has been determined the Pantheon will be given an overall greenness value which will be translated into a statement ranging from *extremely green* to *extremely non-green* in terms of greenness.

Each of the six evaluation categories does not carry equal weight in the greenness evaluation processes. For example the site Selection plays a larger role in the overall greenness than does the Use of Water; therefore the Site Selection category carries a heavier weight. Weights between 0 and +4 are given for each main category and shown in Table 5.9. The values assigned in the following section are for illustration purposes and were subjectively assigned and may vary.

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Table 5.1: Green Building Evaluation Rating Scale
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<tr>
<td>Construction Methods</td>
</tr>
</tbody>
</table>

Table 5.2: Ancient Green Building Evaluation Table of Categories
5.2 Site Selection

The selection of site is one of the most important factors of constructing a building or structure. In some cases the site is selected for specific reasons, such as surrounding environments, local resources, or historical purposes. Hadrian did not choose the original location, which was chosen in 28 B.C. by Agrippa, but Hadrian did choose to reuse the same site. For restoring an existing site a greenness rating of 2 (green) is added to Site Selection. It can be debated whether or not the Romans considered the environment when building and if restoration was more than a kind gesture of recognizing the compliments of former emperors and architects, but in today’s society restoration is highly encouraged and is considered environmental friendly and green.

The high priority location of the Campus Martius adds a high rating of 4 (extremely green) for site selection. The prime location of Campus Martius comes at a high price, not in terms of cost, but in terms of the soil conditions that had to be worked around to building a suitable foundation. Being less than a mile from Tiber River the soil beneath the Pantheon is not ideal for large building construction; therefore the Site Assessment does not yield a green result and is given a rating of -2 (non-green). While the soil conditions are poor the location does provide convenient and easy access to road and the river, which were both utilized for materials transportation to the site. Having materials easily delivered directly to the site made the construction process more efficient and because of that a rating of 3 (very green) was given for Access to Quality Transit.

The transportation of materials did not cause much pollution because the equipment to move materials consisted of wagons, trailers, and boats. The Pantheon pre-dates the
industrial revolution therefore the pollution thought of today did not yet exist. The only true pollution for construction would have been smoke from fires and animal waste, both of which can be considered minimal; therefore Construction Activity Pollution is given a rating of 3 (very green).

The last subcategory covered under Site Selection is Rain Water Management. By A.D. 118 and the start of construction for the Pantheon the Romans had mastered drainage systems within buildings, and building aqueducts. As mentioned in Chapter 4 the Pantheon does indeed have a drainage system from the interior to the exterior to remove any rain water that enters the structure. Because of their knowledge of drainage systems and aqueducts it is hypothesized that the Romans would have controlled as much rain water as best they could; therefore Rain Water Management is given a rating of 2 (green).

Table 5.3 below highlights each of the five subcategories and their assigned ratings for Site Selection. Overall the ratings for site selection were mostly positive. From the high priory site location and ease of access transit to restoring a historic structure the Pantheon’s Site Selection has an overall numerical rating of 1.667 meaning the Site Selection has a rating between fairly green (1) and green (2), but slightly more towards green, on the rating scale.
### Subcategories of Site Selection

<table>
<thead>
<tr>
<th>Subcategories of Site Selection</th>
<th>Rating</th>
<th>Linguistic Expression</th>
<th>Reason for Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Priority Site</td>
<td>SR₁</td>
<td>4</td>
<td>Extremely Green</td>
</tr>
<tr>
<td>Access to Quality Transit</td>
<td>SR₂</td>
<td>3</td>
<td>Very Green</td>
</tr>
<tr>
<td>Site Assessment</td>
<td>SR₃</td>
<td>-2</td>
<td>Non-Green</td>
</tr>
<tr>
<td>Site Development – Protect or Restore</td>
<td>SR₄</td>
<td>2</td>
<td>Green</td>
</tr>
<tr>
<td>Rain Water Management</td>
<td>SR₅</td>
<td>2</td>
<td>Green</td>
</tr>
<tr>
<td>Construction Activity Pollution</td>
<td>SR₆</td>
<td>3</td>
<td>Very Green</td>
</tr>
</tbody>
</table>

#### Average Greenness Rating for Site Selection

<table>
<thead>
<tr>
<th>Rating</th>
<th>linguistics</th>
<th>Reason for Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Green</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3: Site Selection Evaluation

Average Rating Calculations for Site Selection⁷⁶:

\[ SR_T = \Sigma (SR_i \cdot W_i) / \Sigma (W_i) \]

\[ SR_T = [(SR_1 \cdot W_1) + (SR_2 \cdot W_2) + (SR_3 \cdot W_3) + (SR_4 \cdot W_4) + (SR_5 \cdot W_5) + (SR_6 \cdot W_6)] / [W_1 + W_2 + W_3 + W_4 + W_5 + W_6] \]

\[ SR_T = [(4 \cdot 1) + (3 \cdot 1) + (-2 \cdot 1) + (2 \cdot 1) + (2 \cdot 1) + (3 \cdot 1)] / [1+1+1+1+1+1] = 2 \]

---

⁷⁶ All subcategories were assigned an equal weight of 1.
5.3 Use of Water

The Use of Water for a building varies depending on the type of construction required and the building type itself. In the case of the Pantheon the construction activities required a mild use of water. The Tiber River’s water was used for transporting materials by boat, but this was a very efficient method to get materials close to the construction site in a very easy manor. Other uses of water would be for the concrete and mortar mixtures, cleaning of construction tools and equipment, along with other minimal uses. Because of the amount of water required for the concrete and mortar along with other construction activities the Construction Activity Water Use subcategory earns a rating of 1 (fairly green).

Once the construction is completed the water use is nearly nothing. The Pantheon would not require any outdoor water use unless the citizens would use water to clean their buildings; therefore Outdoor Water Use earns a rating of 3 (very green). Because the Pantheon was built as a temple it did not require any indoor plumbing or water use. The oculus at the peak of the dome is not covered and therefore rain comes through the opening, but the architect and contractor incorporated a drain into the design; therefore the Indoor Water Use has a rating of 2 (green).

Overall as show in Table 5.4 the Use of Water category has a greenness rating of 2 (green). The Romans valued their water and were fortunate to have the Tiber River within close proximity to the site of the Pantheon. Because of the low water use from construction through the lifetime of the building it is rated as green (2).
<table>
<thead>
<tr>
<th>Subcategories of Water Use</th>
<th>Rating Value</th>
<th>Linguistic Expression</th>
<th>Reason for Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outdoor Water Use</td>
<td>WR_1</td>
<td>3</td>
<td>Very Green</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Minimal outdoor water use once building was in use.</td>
</tr>
<tr>
<td>Indoor Water Use</td>
<td>WR_2</td>
<td>2</td>
<td>Green</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>No known plumping. Built-in drains for rainwater that enters through the oculus.</td>
</tr>
<tr>
<td>Construction Activity Water Use</td>
<td>WR_3</td>
<td>1</td>
<td>Fairly Green</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Water used for concrete and other construction activities.</td>
</tr>
</tbody>
</table>

**Average Greenness Rating for Use of Water**

| 2 | Green |

Table 5.4: Water Use Evaluation

Average Rating Calculations for Use of Water\(^77\):

\[
WR_T = \frac{\sum (WR_i \times W_i)}{\sum W_i}
\]

\[
WR_T = \frac{[(WR_1 \times W_1) + (WR_2 \times W_2) + (WR_3 \times W_3)]}{W_1 + W_2 + W_3}
\]

\[
WR_T = \frac{[3 \times 1 + 2 \times 1 + 1 \times 1]}{1+1+1} = 2
\]

\(^77\) All subcategories were assigned an equal weight of 1.
5.4 Energy Conservation

Construction of structures both small and large require tremendous amount of energy; therefore when efforts are put into place to use less energy those efforts can be described as green. The Pantheon is by no means a small structure, but moderate in size compared to buildings and structures of its time. Due to the lack of modern construction technology the Romans had to use a tremendous amount of manpower to complete a project, but because they didn’t have the technology we have today they did indeed implement methods that required the least amount of effort as possible and in turn conserved energy. Because of their experience in construction they knew what was and wasn’t worth the extra energy, such as the redesign of the portico. The columns required for the original portico were too large to quarry in one piece and not to mention transport and put into place; therefore less energy was used for the construction of the portico. Other construction short cuts would be taken throughout the project; therefore a rating of 2 (green) is given for the Minimal Energy Use subcategory.

A large part of the overall energy use was material procurement. The Pantheon required quarrying of stone, cutting of timber, importing and transporting materials to site, manufacturing bricks, and obtaining enough materials for the aggregate. For the rotunda alone nearly 900,000 bricks and 30,000 cubic meters of concrete were used. A lot of energy would be required to manufacture such a large quantity of bricks and produce such a large amount of concrete. Because of the high demand of materials the Energy Use of Materials is given a rating of -2 (non-green).

---

78 Calculations highlighted in Chapter 3 and shown in Appendix B.
During the Pantheons construction energy was spent by man power. The masons extracted large amounts of time and energy placing the bricks and mortar and laborers laying the concrete, building scaffolding, transporting materials, constructing formwork. Heavy machinery and power tools are a modern luxury. Labors and animals had to produce the energy to lift materials to high elevations, which was made slightly easier using the lifting technologies of the time. Overall the amount of energy extracted to complete the construction tasks was extremely high and for that reason Construction Activities Energy Use is given a rating of non-green (-2).

Two subcategories that score positively within the energy conservation are the Heating and Cooling Systems Energy Use. Both heating and cooling are rated green (2). The Romans may not have had heating and cooling systems we know and enjoy today, but they did have their own methods to heat and cool a structure; however the Pantheon was not designed to include either of those technologies. Because no energy is used to heat or cool the Pantheon it is considered green.

The last subcategory for energy conservation is Lighting Energy Use. The only original lighting methods for the Pantheon were natural light and candlelight/fire. No openings can be found on the structure except for the oculus and entrance; therefore very little natural light can enter the Pantheon. The Romans only other source of light would be candles/fires and fire requires energy to burn. Because of the lack of natural light and the use of fire for additional light the Lighting Energy Use is given a rating of fairly non-green (-1).
Energy Conservation is an important aspect of a structure’s greenness. Not only is it important to reduce energy use during construction activities, but to also reduce the use of energy for the lifetime of the structure. The Pantheon is no doubt a large construction project requiring extremely large amounts of materials, manpower, and energy. The short term energy use for the Pantheon is poor, but long term the Pantheon does not require much at all. Overall the Energy Conservation is rated between neutral (0) and fairly green (1), but most closely rated to neutral. All the ratings are shown below in Table 5.5.
### Subcategories of Energy Conservation

<table>
<thead>
<tr>
<th>Subcategories of Energy Conservation</th>
<th>Rating</th>
<th>Linguistic Expression</th>
<th>Reason for Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal Energy Use</td>
<td>ER₁</td>
<td>2</td>
<td>Green</td>
</tr>
<tr>
<td>Energy Use for Materials</td>
<td>ER₂</td>
<td>-2</td>
<td>Non-Green</td>
</tr>
<tr>
<td>Construction Activity Energy Use</td>
<td>ER₃</td>
<td>-2</td>
<td>Non-Green</td>
</tr>
<tr>
<td>Heating System Energy Use</td>
<td>ER₄</td>
<td>2</td>
<td>Green</td>
</tr>
<tr>
<td>Cooling System Energy Use</td>
<td>ER₅</td>
<td>2</td>
<td>Green</td>
</tr>
<tr>
<td>Lighting Energy Use</td>
<td>ER₆</td>
<td>-1</td>
<td>Fairly Non-Green</td>
</tr>
</tbody>
</table>

#### Total Greenness Rating for Energy Conservation

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Greenness Rating</strong></td>
<td>0.167</td>
<td>Between Neutral and Fairly Green</td>
</tr>
</tbody>
</table>

Table 5.5: Energy Conservation Evaluation

---

**Average Rating Calculations for Energy Conservation**

\[
ER_T = \frac{\Sigma (ER_i \times W_i)}{\Sigma W_i} \\
ER_T = \frac{[\Sigma (ER_1 \times W_1) + (ER_2 \times W_2) + \ldots + (ER_6 \times W_6)]}{W_1 + W_2 + \ldots + W_6} \\
ER_T = \frac{[(2 \times 1) + (-2 \times 1) + (-2 \times 1) + (2 \times 1) + (2 \times 1) + (-1 \times 1)]}{1+1+1+1+1} = 0.167
\]

---

79 All subcategories were assigned an equal weight of 1.
5.5 Materials and Resources

A structure materials and the use of resources during construction is an important aspect of the greenness evaluation. The reuse of recycled materials is easily implemented in today’s construction project in the finishing touches, such as flooring, interior woodwork, lighting and décor. Even in ancient times contractors used reused and recycled materials for construction. The Pantheon reused foundations, used broken stone and bricks as aggregate, and wooden scaffolding was commonly reused. The Pantheon earns a green (2) rating for Reuse of Materials and Resources and a very green (3) rating for Recyclable Materials Use.

Not only is it environmentally conscious to reuse and use recycled materials, but also to use local materials. The meaning of local largely depends on the transportation access. In modern times local could easily be over 120 miles away, while in ancient times 10 miles is not considered local. As discussed in Chapter 4 under Materials Procurement, several key materials had to be transported from Tivoli, Naples and Alban Hills, none of which would be considered local to Rome. Because of the long distances the materials traveled and the lack of current transportation methods the Pantheon earns a negative rating of very non-green (-3) for Local Material Use.

Because materials were not local and readily available and also considering nearby buildings that were demolished or destroyed, the reuse of materials and the use of recycled materials was a common practice in construction. Aggregate materials such as broken brick, tufa, and travertine were used in the foundation and wall construction. The Romans would also reuse scaffolding wood to build new scaffolding or even for the centering for
the dome construction. A rating of 3 (very green) was given for Recyclable Materials Use. Minimal construction and demolishing waste was accumulated because of the recycled materials use and reuse of construction materials; therefore the Construction and Demolition Waste Management was given a rating of green (2).

As previously discussed in Chapter 3 there was large amount of materials required to construct the Pantheon. Totaling over 975,841 bricks, 47,217 m³ of concrete and 1,610 m³ of stone. There is no denying that there was a negative impact made by the construction materials. Construction Materials was given a -2 rating (non-green). The large amount of materials is due to the large scale of the project, which also means a large amount of labor was required to build the Pantheon. Laborers varied from slaves, freedmen and the poor to skilled laborer and soldiers. The free and cheap labor provided by the slaves, freedmen and poor, would be used to do more of the backbreaking work such as transport and cutting materials, while the skilled laborer and soldiers would use their expertise to do the more complex tasks such as building centering and the masonry work. Due to the free and cheap labor tactics and the large quantity of overall labor needs the Labor category is given a rating of very non-green (-3).

The erection of large structures not only require a tremendous amount of manpower, but also equipment and tools. As discussed in Chapter 4 the construction of the dome alone required 28 cranes to be assembled and used at one time and required an extremely demanding centering scheme.\(^\text{*}\) The construction of the walls required

\(^\text{*}\) The estimate of 28 cranes used to build the dome is theorized by Taylor and based upon the dome construction rehearsal. Only 28 cranes of average size could fit around the Pantheon. This is also the number of columns of coffers (Taylor 2003:203-204).
scaffolding to be built around the entire building and for lifting devices to be implemented in several locations to keep the masonry and concrete workflow steady. Shovels, baskets, leveling devices, drills and cutting tools and many more were also required throughout the construction process. By no means were the equipment and tool demands low and for that reason the Construction Equipment and Tools category is rated non-green (-2).

Overall the Materials and Resources category is given a negative rating of -0.429, between fairly non-green and neutral (Table 5.6 below). An average negative rating is earned largely due to the lack of consideration for using local materials, using free and cheap labor, and being wasteful in the construction material selection and equipment and tools use. The Romans are credited for the reuse of and recycling of materials that they had readily available, as well as reusing some of the construction materials from other projects to reuse later in the construction process.
### Subcategories of Materials and Resources

<table>
<thead>
<tr>
<th>Subcategories of Materials and Resources</th>
<th>Rating</th>
<th>Linguistic Expression</th>
<th>Reason for Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reuse of Construction Materials</td>
<td>MR₁</td>
<td>2</td>
<td>Green</td>
</tr>
<tr>
<td>Local Material Use</td>
<td>MR₂</td>
<td>-3</td>
<td>Very Non-Green</td>
</tr>
<tr>
<td>Recyclable Material Use</td>
<td>MR₃</td>
<td>3</td>
<td>Very Green</td>
</tr>
<tr>
<td>Construction and Demolition Waste</td>
<td>MR₄</td>
<td>2</td>
<td>Green</td>
</tr>
<tr>
<td>Management</td>
<td>MR₅</td>
<td>-3</td>
<td>Very Non-Green</td>
</tr>
<tr>
<td>Construction Materials</td>
<td>MR₆</td>
<td>-2</td>
<td>Non-Green</td>
</tr>
<tr>
<td>Construction Equipment and Tools</td>
<td>MR₇</td>
<td>-2</td>
<td>Non-Green</td>
</tr>
</tbody>
</table>

**Agrippa’s Pantheon was partially reused for the foundation of current portico. Reuse of temporary formwork and scaffolding.**

**Materials used for the Pantheon were not considered local materials.**

**Recycled broken brick, stone and tufa into aggregate for concrete. Recycled wooden formwork throughout project.**

**Minimal construction and demolishing waste was accumulated because of the recycled materials use and reuse of construction materials.**

**Free and cheap labor tactics are not sustainable and there was large labor need throughout the project.**

**Large amount of materials were required for project completion.**

**High demand for construction tools and equipment.**

### Average Rating Calculations for Materials and Resources

\[
MR_T = \frac{\sum (MR_i \times W_i)}{\sum W_i}
\]

\[
MR_T = \frac{[(MR_1 \times W_1) + (MR_2 \times W_2) + (MR_3 \times W_3) + (MR_4 \times W_4) + (MR_5 \times W_5) + (MR_6 \times W_6) + (MR_7 \times W_7)]}{W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7}
\]

\[
MR_T = \frac{[(2 \times 1) + (-3 \times 1) + (3 \times 1) + (2 \times 1) + (-3 \times 1) + (-2 \times 1) + (-2 \times 1)]}{1+1+1+1+1+1+1} = -0.429
\]

---

81 All subcategories were assigned an equal weight of 1.
5.6 Air Quality

The modern world we live in today there is a growing concern for reducing pollution and maintaining quality air. Perhaps the Romans did not have the same invested interest in air quality that we do today, but nonetheless they understood the need for clean air. The city of Rome was not a clean city and the quality of air was poor. The contractors had little control over the surrounding outdoor air quality, but they did have control over any additional pollution created by construction activities or unhealthy work environments for the laborers. The very first construction activity, excavation, did not provide a healthy environment and would have had very poor air quality for the excavator to work in; however overall and in regards to ancient times the air quality was adequate for its time. For this reason the Outdoor Air Quality is given a rating of 2 (green).

One the other hand the Pantheon would have had good indoor air quality. The building didn’t include any plumbing, and it had the proper drainage required for the rainwater. The interior also had a very large open space, but without many openings. The indoor air would have been more than adequate for the time and has a rating of 2 (green). The indoor air quality is rated green, but the building does lack a true ventilation system or even a method to properly vent the indoor space. While the oculus and door do provide ventilation, the building would be better ventilated with windows; however with walls being 6 meters thick it would not be easy to include windows into the design while keeping the building structurally sound. Considering the design restrictions, the Building Ventilation System earns a rating of -1 (fairly non-green).
In general the air quality surrounding the Pantheon would have been poor, while the indoor air would have been of good quality considering the lack of ventilation other than the oculus and doorway as the only openings to the building. The Air Quality has an overall rating of 1, meaning the Air Quality is *fairly green* (Table 5.7).
### Subcategories of Site Selection

<table>
<thead>
<tr>
<th>Subcategories of Site Selection</th>
<th>Rating</th>
<th>Linguistic Expression</th>
<th>Reason for Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outdoor Air Quality</td>
<td>AR₁</td>
<td>2</td>
<td>Green</td>
</tr>
<tr>
<td>Indoor Air Quality</td>
<td>AR₂</td>
<td>2</td>
<td>Green</td>
</tr>
<tr>
<td>Building Ventilation System</td>
<td>AR₃</td>
<td>-1</td>
<td>Fairly Not Green</td>
</tr>
</tbody>
</table>

#### Table 5.7: Air Quality Evaluation

| Total Greenness Rating for Air Quality | 1 | Fairly Green |

Average Rating Calculations for Materials and Resources\(^{82}\):

\[
AR_T = \frac{\sum (AR_i \times W_i)}{\sum W_i}
\]

\[
AR_T = \frac{[(AR_1 \times W_1) + (AR_2 \times W_2) + (AR_3 \times W_3)]}{W_1 + W_2 + W_3}
\]

\[
AR_T = \frac{[2 \times 1 + 2 \times 1 + (-1 \times 1)]}{1+1+1} = 1
\]

---

\(^{82}\) All subcategories were assigned an equal weight of 1.
5.7 Innovation of Design and Construction

The Pantheon’s design is unique because it is not a traditional temple style for its time. For example, the portico does not fit with the large rotunda and domed roof and the redesign of the portico to accommodate shorter columns wasn’t intended, but with aesthetics aside the Pantheon is full of engineering influences. Working from the ground up the strip style foundation was not new or innovative, but the double ring foundation that supports the rotunda is extraordinary because it supports a large structure while resting on the least ideal soil conditions possible and for that reason it is grand example of the engineering knowledge and understanding of the Romans. The rotunda itself is innovative because it incorporates a complex system of voids and passageways that served three functions. First the voids meant less materials required, which is benefit to the structures overall greenness evaluation. The voids also provide accesses to the inside of the structure during the construction process and they allowed for air to filter within the walls and increased the curing time of the concrete. The design of the walls was very innovative and shows the engineering expertise the Romans possessed at the time. Moving to the top of the structure the dome comprised of seven step rings and a saucer dome take the spotlight and really expresses the innovative design required to build the Pantheon. The span of the Pantheon had never before been attempted and wasn’t surpassed until more modern times in the 19th century. The Romans understood the engineering behind the cantilevered step rings and also realized that a dome without the step rings would never have withheld the internal pressures required to span such a distance. Because of the ingenuity put into the
design of the entire structure, from the foundation to the dome, the Design subcategory earns a rating of 4, *extremely green*.

It is hard to debate if the Romans understood the environmental impacts caused by their construction activities and buildings, but the design of the structure did have impacts on the environment and are considered in the greenness evaluation. The dematerialization found in the walls because of the voids and cavities meant that the Pantheon had less of a negative impact on the environment, but with a structure over 30 meters tall and spanning nearly 50 meters there will also be a negative impact on the environment. Even though the building is large the Romans were able to incorporate recycled materials into the structure which means less new materials were required. Overall the Pantheon’s design did not necessarily consider the environment, but the Romans did implement methods to prevent negative impacts on the environment. Environmental Impact of Design was given a rating of 1 (*fairly green*).

Many of the construction techniques employed through the Pantheon’s building process did require a large amount of materials and therefore had a poor impact on the environment; however their techniques were innovative. Utilizing brick-faced concrete construction that prevented the need for temporary formwork during wall construction was very successful. The bricks doubled as a suitable wall covering that could easily be decorated in stucco and paintings. Without the expert scaffolding and centering systems the walls and dome of the Pantheon would never have been built. The scaffolding could quickly be assembled and doubled as a support structure for pulleys and lift systems for hoisting materials to high elevations. Centering schemes for arches and vaults were
commonly built, but the centering required to build the dome was complex and well thought out. Rehearsing the centering construction could have easily saved the Romans much difficult if trying to build it for the first time on the top of the rotunda wall. Because the Romans exercised a tremendous amount of strategic planning and enhanced former building techniques into new innovative solutions the subcategory of Construction Methods was given a rating of 4, *extremely green*.

Innovation of Design and Construction was given the highest rating of all the categories. It earned the highest rating due to the complex and creative designs implemented throughout the structure as well as the use of current and new construction techniques. The Innovation of Design and Construction earned a total rating of 3 (*very green*).
<table>
<thead>
<tr>
<th>Subcategories of Site Selection</th>
<th>Rating</th>
<th>Linguistic Expression</th>
<th>Reason for Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
<td>IR₁</td>
<td>4</td>
<td>Extremely Green</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>The Pantheon was well designed and the Romans used the expertise learned from past projects and enhanced the overall effectiveness of the design.</td>
</tr>
<tr>
<td>Environmental Impact of Design</td>
<td>IR₂</td>
<td>1</td>
<td>Fairly Green</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Methods to prevent negative impacts on the environment were implemented such as reuse of materials.</td>
</tr>
<tr>
<td>Construction Methods</td>
<td>IR₃</td>
<td>4</td>
<td>Extremely Green</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Romans exercised a tremendous amount of strategic planning and enhanced former construction techniques into new innovative solutions.</td>
</tr>
<tr>
<td>Average Greenness Rating for Innovation of Design and Construction</td>
<td>3</td>
<td>Very Green</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.8: Innovation of Design and Construction Evaluation

Average Rating Calculations for Materials and Resources\(^{83}\):

\[
IR_T = \frac{\sum (IR_i \times W_i)}{\sum W_i}
\]

\[
IR_T = \frac{[(IR_1 \times W_1) + (IR_2 \times W_2) + (IR_3 \times W_3)]}{W_1 + W_2 + W_3}
\]

\[
IR_T = \frac{[(4 \times 1) + (1 \times 1) + (4 \times 1)]}{1 + 1 + 1} = 3
\]

---

\(^{83}\) All subcategories were assigned an equal weight of 1.
5.7 Conclusion of Green Building Evaluation

Each of the main categories, Site Selection, Use of Water, Energy Conservation, Materials and Resources, Air Quality, and Innovation of Design and Construction, are rated based upon their subcategories rating averages. Each of the subcategories carries a weight of one, meaning each subcategory is evaluated equally. When determining the overall greenness of the Pantheon the six main categories carry a different weight of importance in terms of greenness. The weights range from 0 to 4 (positive weights only), where 0 means the category carries no weight in terms of greenness and a 4 meaning it carries the highest. For example Use of Water has a weight of 0.5, which is the lowest weight provided; therefore in terms of the Pantheon’s greenness, water provide to least amount of impact on the overall greenness. The weights used are provided below in Table 5.9.

After factoring in the weights the overall greenness rating of the Pantheon is 0.669, between fairly green and green. The largest contributing factors were the Energy Conservation and Materials and Resources, because they have the most impact on the environment; therefore directly impact the greenness of the structure. In order to calculate the overall rating the sum of the categories multiplied by their weights is taken and divided by the sum of the weights, as shown below.
<table>
<thead>
<tr>
<th>Evaluation Categories</th>
<th>Rating</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site Selection</td>
<td>SRₜ</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>WS</td>
<td>2</td>
</tr>
<tr>
<td>Use of Water</td>
<td>WRₜ</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>WW</td>
<td>.5</td>
</tr>
<tr>
<td>Energy Conservation</td>
<td>ERₜ</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td>WE</td>
<td>4</td>
</tr>
<tr>
<td>Materials and Resources</td>
<td>MRₜ</td>
<td>- 0.429</td>
</tr>
<tr>
<td></td>
<td>WM</td>
<td>4</td>
</tr>
<tr>
<td>Air Quality</td>
<td>ARₜ</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>WA</td>
<td>1</td>
</tr>
<tr>
<td>Innovation of Design and Construction</td>
<td>IRₜ</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>WI</td>
<td>3</td>
</tr>
<tr>
<td><strong>Total Greenness Rating</strong></td>
<td>GRₜ</td>
<td>1.29</td>
</tr>
</tbody>
</table>

Table 5.9: Total Greenness Evaluation

\[
GRₜ = \sum (GR_i * W_i) / \sum (W_i)
\]

\[
GRₜ = [(SRₜ * WS) + (WRₜ * WW) + (ERₜ * WE) + (MRₜ * WM) + (ARₜ * WA) + (IRₜ * WI)] / [WS + WW + WE + WM + WA + WI]
\]

\[
GRₜ = [(2 * 2) + (2 * 0.5) + (0.167 * 4) + (-0.429 * 4) + (1 * 1) + (3 * 3)] / (2 + 0.5 + 4 + 4 + 1 + 3) = 1.29
\]

5.8 Discussion

The above greenness rating system utilizes several terms which cannot be clearly defined. Rating options such as green, fairly green and non-green, are dependent upon the individual/expert that used the above rating system to rate the greenness of an ancient structure. Fairly green could very likely have different meanings depending on the various
experts, these terms or ratings would therefore be fuzzy. Fuzzy meaning there is not clear definition and the meaning of the term directly relies on an individual’s perception of the said term. In order to account of the fuzziness of the rating system, fuzzy logic can be utilized to provide a mathematical approach to the greenness evaluation process. Chapter 6 will discuss the fuzzy logic approach to evaluating the Pantheon’s greenness.
Chapter 6 Green Building Evaluation Using Fuzzy Set Concept

6.1 Introduction

The greenness evaluation process discussed in Chapter 5 is determined by the research opinions of one subject expert; however the ratings can easily differ from one expert to another. The difference in ratings can be due to the difference in opinion or because of the fuzziness or imprecise definition of the rating terms (green, very green, etc.). The greenness ratings used in this research are simple linguistic expressions. For example “the Pantheon is a green building” is a set of information provided in words. The term green is fuzzy, meaning there is not a clear definition of what green actually means and it largely depends on who is using the term, yet it is useful. Fuzzy sets, a concept first introduced by Zadeh in 1965, can be used to capture a group of unclearly defined terms and transform them into mathematical expressions.\(^\text{84}\) The benefit of using the fuzzy set concept is that each set can be quantified, graphed and developed into a fuzzy set model. Once the model has been developed a ranking index\(^\text{85}\) can be calculated and provides a

\(^{84}\) Zadeh 1965:338-353.
numerical value for each linguistic expression or linguistic value. Having individual linguistic values can allow researchers (users) to better define previously unclearly defined terms and in turn make useful conclusions on the set of data.

The goal of the green building evaluation is to determine the Pantheon’s overall greenness rating or the total rating. The six greenness evaluation categories (Site Selection, use of Water, Energy Conservation, Materials and Resources, Air Quality and Innovation of Design and Construction) and their subcategories will be developed into fuzzy set models and later used to calculate the total greenness rating for the Pantheon. Models for overall ratings as discussed by Tan will be utilized to combine the six rating categories and their weights to calculate the total greenness rating. Certain evaluation categories play a larger and more important role in the overall greenness, this importance factor is defined as a weight \( W_i \). The weights of the categories range from 0 to 4. Zero meaning that category does not carry any weight and 4 meaning that category carries the most amount of weight. For the purpose of this study \( R_T \) is the overall rating, \( R_i \) represents the individual ratings and \( W_i \) represents the weights of each individual rating. In order to mathematically obtain the total greenness rating of the Pantheon each category is evaluated and a total rating is found (\( R_T \)). The total rating can be calculated by using the following equation:

\[
R_T = \frac{\Sigma (R_i \times W_i)}{\Sigma W_i} \quad \text{.......................................................... (1)}
\]

---

88 All subcategories are given and equal weight of 1.
For example, if there are two ratings, *green* \( (R_1 = [2, 4]) \) and *extremely green* \( (R_2 = [3, 5]) \), with weights of \([1, 3]\) \( (W_1) \) and \([2, 4]\) \( (W_2) \) respectively, then the total rating would be calculated by the following:

\[
R_T = \frac{[2, 4] * [1, 3] + [3, 5] * [2, 4]}{[1, 3] + [2, 4]} = \frac{\min(2, 6, 4, 12), \max(2, 6, 4, 12)}{[3, 7]} = \frac{[2, 12]}{[0.67, 1.71]}
\]

Fuzzy models can be used to provide a visual representation of the fuzzy sets (linguistic rating expressions) being used to calculate the total greenness rating. For the rating of the Pantheon’s greenness the fuzzy models will have an area of 10 with the x-axis (ratings) ranging from -5 to 5, and the y-axis (degree of belief) ranging from 0 (0% degree of belief) to 1 (100% degree of belief). The standard greenness ratings developed specifically for this research are shown below in a fuzzy model format (Figure 6.1). Each linguistic expression is represented as an equilateral triangle, commonly called membership function, with the base at \( y = 0 \) and the tip of the triangle along the top at \( y = 1 \). For example *fairly green* is given a numerical value of 1 and is shown by the triangle with coordinate points of \((0, 0), (1, 1) \) and \((2, 0)\). For this research the membership function of each fuzzy set (linguistic expression) is expressed in the form \([x_1, x_2]\), for example *fairly green* is represented by \([0, 2]\) (see Table 6.1). As the degree of belief (y-axis) increases the less variance exists in the rating (x-axis).
Figure 6.1: Fuzzy Model of Standard Greenness Ratings

<table>
<thead>
<tr>
<th>Numerical Value</th>
<th>Linguistic Expression of Rating Terms</th>
<th>Fuzzy Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 4</td>
<td>Extremely Non-Green</td>
<td>[-5, -3]</td>
</tr>
<tr>
<td>- 3</td>
<td>Very Non-Green</td>
<td>[-4, -2]</td>
</tr>
<tr>
<td>- 2</td>
<td>Non-Green</td>
<td>[-3, -1]</td>
</tr>
<tr>
<td>- 1</td>
<td>Fairly Non-Green</td>
<td>[-2, 0]</td>
</tr>
<tr>
<td>0</td>
<td>Neutral</td>
<td>[-1, 1]</td>
</tr>
<tr>
<td>1</td>
<td>Fairly Green</td>
<td>[0, 2]</td>
</tr>
<tr>
<td>2</td>
<td>Green</td>
<td>[1, 3]</td>
</tr>
<tr>
<td>3</td>
<td>Very Green</td>
<td>[2, 4]</td>
</tr>
<tr>
<td>4</td>
<td>Extremely Green</td>
<td>[3, 5]</td>
</tr>
</tbody>
</table>

Table 6.1: Greenness Evaluation Rating Scale and Fuzzy Sets
6.2 Fuzzy Set Models for the Pantheon

A series of fuzzy set models were created in order to find the total greenness rating of the Pantheon. This section explains the process of creating such models and how the total rating for each individual category was calculated as well as the Pantheon’s final greenness rating. The Pantheon’s greenness evaluation is based upon the ratings of the greenness six categories as discussed in Chapter 5. Each of the six categories includes its own subcategories. For example the first category is Site Selection, which has six subcategories such as High Priority Site (SR₁) and Access to Quality Transit (SR₂). Table 6.2 below provides a full list of the subcategories for Site Selection and Appendix C includes all the rating categories. Each subcategories is given a rating label. For Site Selection the rating labels (Rᵢ) consist of SR₁–SR₆, which stand for Site Selection Rating 1 to 6. Pulling for the ratings provided and discussed in the previous chapter, rating labels are assigned their values. Table 6.2 lists the Rᵢ values in both forms, linguistic expressions and fuzzy set numerical values. In this study, all subcategories are given the weights, which may or may not vary. These weight can also be represented by triangular membership functions, Wᵢ [x₁, x₂], similar to the rating values. For the purpose of this study, the weight membership functions for all subcategories of Site Selection are given a fuzzy set of [1, 1] or in other words they will all carry an equal weight of importance. In this case, the triangular membership function becomes a vertical line called fuzzy singleton.
Using Eq. 1 the total rating for Site Selection (\(SR_T\)) was calculated (calculations shown below). Site Selection was given an overall rating of \([0.67, 2.67]\), which when converted back to linguistic expressions is between *fairly green* and *green*. The Site Selection Fuzzy Set Model is shown below in Figure 6.2

\[
SR_T = \frac{\Sigma (SR_i \times W_i)}{\Sigma W_i}
\]

\[
SR_T = \frac{[3, 5] 	imes [1, 1] + [2, 4] 	imes [1, 1] + [-3, -1] 	imes [1, 1] + [1, 3] 	imes [1, 1] + [-1, 1] 	imes [1, 1] + [2, 4] 	imes [1, 1]}{[1, 1] + [1, 1] + [1, 1] + [1, 1] + [1, 1] + [1, 1]}
\]

\[
SR_T = \frac{[3, 5] + [2, 4] + [-3, -1] + [1, 3] + [-1, 1] + [2, 4]}{[1, 1] + [1, 1] + [1, 1] + [1, 1] + [1, 1]}
\]

\[
SR_T = \frac{[4, 16]}{[6, 6]} = [0.67, 2.67]
\]
Figure 6.2: Site Selection Fuzzy Set Model

The sample discussed above highlights only one of the six greenness rating categories (Site Selection) while Appendix C includes the models and calculations for all categories. Table 6.3 below summarizes the six categories by rating ($R_i$) and their weights ($W_i$) and lists the Pantheon’s Overall Greenness Rating of $[-0.61, 2.01]$. The Fuzzy Model of the overall greenness rating of the Pantheon is shown in Figures 6.3-6.5. Because there are six ratings and six weights, the two are graphed in separate models (Figures 6.3 and 6.4) and Figure 6.5 combines all 12 models and includes the total rating ($GR_T$) which falls between neutral and fairly green.
<table>
<thead>
<tr>
<th>Rating Categories</th>
<th>$R_i$</th>
<th>$W_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR$_T$ Site Selection Rating</td>
<td>[1,3]</td>
<td>[1,3]</td>
</tr>
<tr>
<td>WR$_T$ Water Use</td>
<td>[1,3]</td>
<td>[.5,.5]</td>
</tr>
<tr>
<td>ER$_T$ Energy Conservation</td>
<td>[-.83,1.167]</td>
<td>[3,5]</td>
</tr>
<tr>
<td>MR$_T$ Materials and Resources</td>
<td>[-1.43,.57]</td>
<td>[3,5]</td>
</tr>
<tr>
<td>AR$_T$ Air Quality</td>
<td>[0, 2]</td>
<td>[0,2]</td>
</tr>
<tr>
<td>IR$_T$ Innovation of Design and Construction</td>
<td>[2,4]</td>
<td>[2,4]</td>
</tr>
<tr>
<td>GR$_T$ Pantheon’s Overall Greenness Rating</td>
<td>[-0.61, 2.01]</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.3: Final Greenness Fuzzy Ratings and Weights

$$GR_T = [.67, \ 2.67]*[1, \ 3] + [1, \ 3]*[.5, \ .5] + [-.83, \ 1.167]*[3, \ 5] + [-1.43, \ .57]*[3, \ 5] + [-1, \ 1]*[0, \ 2] + [2, \ 4]*[2, \ 4] / [1, \ 3] + [.5, \ .5] + [3, \ 5] + [3, \ 5] + [0, \ 2] + [2, \ 4]$$

$$GR_T = [.67, \ 8.01] + [.5, \ 1.5] + [-2.49, \ 5.835] + [-4.29, \ 2.85] + [0, \ 2] + [4, \ 16] / [1, \ 3] + [.5, \ .5] + [3, \ 5] + [3, \ 5] + [0, \ 2] + [2, \ 4]$$

$$GR_T = [-1.61, \ 36.195] / [9.5, \ 19.5] = \mathbf{[-0.61, \ 2.01]}$$
Figure 6.3: Greenness Ratings for Six Evaluation Categories
Figure 6.4: Fuzzy Model of Weights for each Category.
Figure 6.5: Fuzzy Model of Total Greenness Rating for the Pantheon (GR_{\tau})
6.3 Rating Index of Fuzzy Set Models

Another method to conceptualize the fuzzy set models is to calculate the rating index as discussed by Juang and Kalidindi. The rating index (I) calculation puts a single numerical value on each of the ratings and a total rating. Once the models are created the following equation can be used to calculate the rating index (I):

\[ I = A_R - A_L + C \]

(2)

\[ A_R \] is the area to the right of the rating (triangle) being indexed and \[ A_L \] is the area to the left. The constant, \( C \), is the total area of the universe of discourse. When calculating the rating index for the Total Greenness Rating of the Pantheon (GR\textsubscript{T}), which is \([-0.61, 2.01]\), simple arithmetic can be used. Figure 6.6 below shows the areas that are calculated in order to find \( A_R \) and \( A_L \).

\[ A_R = (1/2) \times (\text{base width} + \text{top width})_{\text{right}} = (1/2) \times [(5 - 0.61) + (5 - 0.70)] = 3.65 \]

\[ A_L = (1/2) \times (\text{base width} + \text{top width})_{\text{left}} = (1/2) \times [(5 + 0.61) + (5 + 2.01)] = 5.05 \]

\[ C = 10 \times 1 = 10 \]

\[ I_G = 3.65 - 5.05 + 10 = 8.6 \]

\[ \text{Juang and Kalidindi 1987:124-130.} \]
Figure 6.6: Calculating the Rating Index (a) shows $A_L$ for the Greenness Rating Total, (b) shows $A_R$ for the Greenness Rating Total.
Rating indices for all the standard linguistic rating expressions and main categories are shown below in Table 6.4 and 6.5 the calculations are listed in Appendix C. The smaller the rating indices correlate to the more positive rating terms. Index calculations were carried out for all six main greenness evaluation categories and for the total greenness rating of the Pantheon and are shown on the rating index scale in Figure 6.7. The index scale allows easy comparison between the standard rating index values. The total greenness rating labeled as $I_G$ in Figure 6.7 is easily understood be fall between index values 8 (*fairly green*) and 10 (*neutral*), but is much closer to 8 than 10; therefore the Total Greenness Rating of the Pantheon is between *fairly green* and *neutral* but more towards *fairly green* than neutral.

<table>
<thead>
<tr>
<th>Rating Index</th>
<th>Rating Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>Extremely Not Green</td>
</tr>
<tr>
<td>16</td>
<td>Very-Not Green</td>
</tr>
<tr>
<td>14</td>
<td>Not Green</td>
</tr>
<tr>
<td>12</td>
<td>Fairly-Not Green</td>
</tr>
<tr>
<td>10</td>
<td>Neutral</td>
</tr>
<tr>
<td>8</td>
<td>Fairly Green</td>
</tr>
<tr>
<td>6</td>
<td>Green</td>
</tr>
<tr>
<td>4</td>
<td>Very Green</td>
</tr>
<tr>
<td>2</td>
<td>Extremely Green</td>
</tr>
</tbody>
</table>

Table 6.4: Standard Rating Index Values
### Rating Indices for All Categories

<table>
<thead>
<tr>
<th>Rating Index</th>
<th>Rating Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS</td>
<td>Site Selection</td>
</tr>
<tr>
<td>IW</td>
<td>Use of Water</td>
</tr>
<tr>
<td>IE</td>
<td>Energy Conservation</td>
</tr>
<tr>
<td>IM</td>
<td>Materials and Resources</td>
</tr>
<tr>
<td>IA</td>
<td>Air Quality</td>
</tr>
<tr>
<td>II</td>
<td>Innovation of Design and Construction</td>
</tr>
<tr>
<td>IG</td>
<td>Total Greenness of Pantheon</td>
</tr>
</tbody>
</table>

Table 6.5: Rating Indices for All Categories

![Diagram of评级指数](image)

Figure 6.7: Final greenness rating index values shown on a standard rating scale.

### 6.4 Conclusion of Fuzzy Logic

The concepts of fuzzy logic, fuzzy sets and models, and rating index take the uncertainty of definitions of the rating terms and allow them to be simplified to numerical values. When evaluating the greenness of an ancient structure, such as the Pantheon, often times terms such as *green* and *very green* can be distinctly different between opinions or very much share a similar meaning. Because of the fuzzy nature of the greenness evaluation

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90 Calculations provided in Appendix C.
fuzzy sets and models were developed to provide a mathematical process that takes into account the fuzziness of the ratings.

Comparing the results of the fuzzy approach and the non-fuzzy approach in Chapter 5 it is clear that the fuzzy sets to adjust the final total rating. The Total Greenness Rating of the Pantheon using non-fuzzy logic is between fairly green and green, with a rating of 1.29, slightly more towards fairly green. When using fuzzy sets the Total Greenness Rating of the Pantheon is [-0.61, 2.01], which is between fairly green and neutral. In conclusion, the two methods used (non-fuzzy weighted average and fuzzy logic) gave a slightly different rating values. The difference is produced due to the fact that fuzzy logic takes into account the subjective linguistic ratings whereas the weighted average does not; these fuzzy ratings allow the crisp non-fuzzy numbers to be expressed in a range of numbers, which represent the imprecise, yet realistic, nature of our subjective judgment. Therefore, we deem the use of fuzzy logic provide a more realistic total rating.
Chapter 7 Conclusion

7.1 Summary

This study provides a state-of-the-art tool to assess the greenness of the Roman Pantheon based on the exploration of the history of relevant structures, materials and construction techniques. To begin the research a historical overview of Roman architecture and construction extends back to AD 27. Following the historic architecture overview, relevant predecessors of the Pantheon and their construction practices were discussed. The historical information provides a basis of knowledge and skillset that could be expected when evaluating the Pantheon’s greenness.

Then shifting the focus completely on to the Pantheon, the Pantheon is described by its five main components: foundation, rotunda, transition block, dome and portico. Each component’s materials, dimensions, purpose and relationship to one another are discussed and shown through a series of photographs and images of the 3D model. Calculations were made to estimate the amount of materials used to build the Pantheon and the total amounts play a large role in rating the Materials and Resources category of the greenness evaluation.

While the basic information about the Pantheon is helpful in the greenness evaluation, the study of the Pantheon’s construction is what mostly impacts the total
greenness rating. The construction process was broken out into three main sections, pre-construction, substructure and superstructure. Pre-construction discusses the soil and water table, surveying and site layout, and material procurement. The substructure consists of the foundations of the portico and the rotunda, while the superstructure includes the rotunda, transition block, dome and portico.

With this in mind, none of the earlier explored research discussed a green building evaluation or the Pantheon’s sustainability. Green buildings and sustainability are at the forefront of modern construction practices. Even though ancient structures and technologies led us where we are today in terms of building success, they have not been continually recognized or thought of in terms of their greenness. The United State Green Building Council’s modern rating system\(^\text{91}\) was used to model the rating categories for the ancient greenness evaluation process. Six categories were developed: Site Selection, Use of Water, Energy Conservation, Materials and Resources, Air Quality, and Innovation of Design and Construction. As discussed in Chapter 5 each category has subcategories, which all earn their own individual rating used to calculate a total greenness rating. Note that these ratings were presented based on the author’s own subjective knowledge based on currently available information. These ratings may also vary from person to person; and as new information emerges regarding any greenness variable of this ancient monument, an update can be made accordingly.

Since one’s assessment is almost always characterized by useful but imprecise judgment, the author introduces a new approach to handle such imprecision, i.e., through

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\(^{91}\) Leadership in Energy and Environmental Design (LEED)
the use of fuzzy logic. A concerning factor in the ancient greenness rating system is the use of hedges such as extremely, very and fairly. These linguistic expressions too do not have clear definitions; yet, they reflect certain degree of belief of those who express them. The fuzzy logic was employed to provide method of calculations that take into account the varying definitions of these terms. Thus, these hedges and expressions are represented by membership functions. The total greenness rating of the Pantheon was not determined by a quick measure of the current state of the structure, but rather using an extensive knowledge of the historical information on materials, dimensions and construction practices used originally.

7.2 Conclusions

The study of ancient structures and their construction process provides society with knowledge not only on the construction technologies, materials and methods, but with knowledge on how ancient societies lived. Modern construction advancements have been able to be made due to the advancements in materials, technology, and equipment, but credit also must be given to learn from the past. The same can be true for the green building programs that we see making great strides in today’s construction projects. Ancient green building evaluations can provide the research today’s civil engineers and architects can utilize to make improvements in the current rating systems.

Understanding the Pantheon’s greenness rating requires the knowledge of history in terms of architecture and construction practices. The literature search for this study provides knowledge on the architect and the reasoning behind the structure. Hadrian was a
leader that believed in historical significance and importance, which is why he rebuilt the Pantheon. While the Pantheon was rebuilt, it is not believe to be an exact replica of Agrippa’s structure, but nonetheless, was dedicated to Agrippa. The Pantheon was rebuilt as a Temple and shared a purpose with Agrippa’s Pantheon. The main difference between the two structures is the building styles and they differed because of the advancements in construction materials and techniques. By the time Hadrian built his Pantheon, roman engineers had mastered the used of concrete as a common building materials and brick-faced structures were very common. Advancements in vaults and dome construction had also been made between the time Agrippa’s Pantheon was built in 28 B.C. and A.D. 118 when Hadrian began his construction.

Hadrian’s Pantheon was unique in terms of design and construction. This study concludes that the design of the Pantheon is complex and required detailed plans to construct. Throughout the study an estimate of materials were calculated for each of the components and the estimated totals include: over 970,000 bricks, 47,000 m$^3$ of concrete and 1,600 m$^3$ of stone. The construction required extensive amount of timber for scaffolding and centering for the construction of the walls and dome. The dome also required a large amount of cranes to erect. Because of the large scale of the Pantheon the construction also required a tremendous amount of manual labor. Labor includes the materials procurement, excavation, formwork, scaffolding and centering erection, masonry work, concrete mixing and placement, and many more other construction activities.

It is with this historical analysis in mind that the Pantheon was evaluated in terms of its greenness. A rating system unique to ancient structures had to be developed due to
the fact that today’s rating systems include categories that do not apply to ancient structures. The ancient rating system consists of 6 main categories and 28 subcategories. Each subcategory was carefully considered and assigned a linguistic expression ranging from extremely non-green to extremely green that correlates to a numerical value. The ratings of the subcategories were then used to calculate the total rating for the 6 main categories and the main categories ratings were used to calculate the Pantheon’s total greenness rating. It was determined by the rating system (weighted average) the Pantheon is indeed green, but on the scale of extremely non-green to extremely green, the Pantheon falls close to fairly green (1.29).

In order to provide a reliable final rating, fuzzy logic was used to attempt to capitalize on the subjective judgment expressed in linguistic rating values. This study aims to provide a method to calculate the total greenness rating of an ancient structure and fuzzy logic has proven to be the most appropriate method in which to make that calculation. Fuzzy logic provides a clearer understanding of the fuzzy rating terms used in this study. A total rating, using fuzzy logic, yields [-0.61, 2.01], which falls between neutral and fairly green, but closer to fairly green. While there is not much different between the two rating values, it is the conclusion of this study that the fuzzy logic provides a method that takes into account the unclear rating terms.

Based on the author’s subjective judgment, it can be concluded that overall the Pantheon was not found to be an extremely green ancient structure, but was rated on the positive side of the rating spectrum which means current construction practices cannot gain a substantial amount of knowledge on green building practices. Even though the Pantheon
has not proven to be a structure to be modeled for its greenness, the study still provides a starting point for future research in ancient green building evaluations. A standard rating system has proven to provide sufficient rating totals, but fuzzy logic provides a more realistic method to produce a final numerical value. In conclusion this study successfully completed its goal in determining the Pantheon’s total greenness rating by modeling a new approach to green building evaluations.

7.3 Recommendations

Provided with the conclusions above, it can be stated that there is room for improvements and progression in the realm of ancient green building evaluations. While this study provides a successful attempt at rating an ancient structure in terms of its greenness, it is only a starting point. After the completion of this study some recommendations for future research have been compiled.

One area of which this study could make improvements would be gathering ratings on the Pantheon from several other subject experts and this is the first recommendation for future research. The basic knowledge outlined in this study could easily be transformed into an intelligent system, which could share the historical knowledge base about the Pantheon with external individuals and capture their onions of the Pantheon’s greenness. Gaining rating values from more than one subject expert will strengthen the argument that the Pantheon is green.

The second recommendation would be to use the same or very similar rating process for research on other ancient structures. If the same rating system is used for a
number of ancient structures a database of green building ratings on ancient structures could be compiled and the database could then provide a general idea of which cultures, geographical locations, and building styles were rated highest in terms of greenness. The database could provide current architects and engineers a resource of which to turn to for motivation on future projects if greenness is a design or construction factor.

A third recommendation is to continue similar research with respects to a building category, such as ancient temples, baths, or amphitheaters. A total greenness rating could be calculated to a building type, which would provide researchers with areas of which can be improved upon in terms of green building design and construction. At the same time this continued research could also provide information on which ancient building types to use to model future construction projects.

All major dimensions were obtained by the author from numerous sources. Minor dimensions such as the hidden voids within the Pantheon's walls are fragmented and scarce; thus, for visual illustration purposes, in order to show the readers the existence of these voids, these dimensions were estimated. Thus, as a final recommendation, a refinement of these dimensions would also be pertinent to reach more accurate graphical images, especially for, say, the development of augmented reality or virtual reality models.
**bessales**: square bricks that measure 19.6 cm x 19.7 cm (on average)

**bipedales**: square bricks that measured 59.2 cm x 59.2 cm (on average)

**cella**: interior central room of a temple (also referred to as naos)

**chorobates**: An ancient surveying tool that resembles a bench used to level

**Corinthian**: the third of the three Greek orders with a slender fluted column and decorated capital

**dioptre**: an ancient surveying tool used to extend a measured line or mark off right angles in the horizontal plane

**groma**: an ancient surveying tool primarily used to measure and form right angles

**non-green**: the complement of green

**opus caementicium**: Roman concrete made of lime based mortar and stone aggregate

**opus incertum**: the use of irregular shaped small stone as a wall facing

**opus testaceum**: brick faced wall filled with Roman concrete (*opus caementicium*)

**portico**: a porch like structure, commonly found on temple fronts

**pozzolana**: volcanic ash commonly used in ancient Roman concrete (*opus caementicium*)

**rotunda**: a round building; especially: one covered with a dome
solidum: solid ground

tholos: beehive-shaped structure or tomb (plural: tholoi)

tympanum: semi-circular or triangular decorative wall surfaced over an entrance (plural: tympana)

voussoir arch: archway made of wedge-shaped stone and relies on the downward weight and outward thrust of the stone to remain intact
For the purpose of this research all major dimensions (rotunda height, wall radii, etc.) were obtained by the author from sources such as Moore, Waddell, and Wilson Jones.

Minor dimensions such as the hidden voids within the Pantheon's walls are fragmented and scarce; thus, primarily for visual illustration purposes, these dimensions were estimated.

The calculations provide in this research are estimates based on the major dimensions, as well as the minor estimated dimensions.

**Stone Strip Foundation**

<table>
<thead>
<tr>
<th>Strip width</th>
<th>Strip length</th>
<th>Strip depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 m</td>
<td>16 m</td>
<td>3 m</td>
</tr>
</tbody>
</table>

8 stone strips

Length x width x depth x qty. = (2.5m x 16m x 3m) x 8 strips of stone = **960 m³ of stone material**

**Rammed Earth Strip Foundation**

<table>
<thead>
<tr>
<th>Strip width</th>
<th>Strip length</th>
<th>Strip depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 m</td>
<td>16 m</td>
<td>3 m</td>
</tr>
</tbody>
</table>

7 earth stone

Length x width x depth x qty. = (2.5m x 16m x 3m) x 7 strips of earth = **840 m³ of rammed earth material**

**Foundation Ring 1**

<table>
<thead>
<tr>
<th>Inner radius</th>
<th>Outer radius</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 m</td>
<td>27 m</td>
<td>5 m</td>
</tr>
</tbody>
</table>

\[(\pi r^2 x \text{depth})_{\text{outer}} - (\pi r^2 x \text{depth})_{\text{inner}} = [\pi(27^2)(5)] - [\pi(20^2)(5)] = 5,168 \text{ m}^3 \text{ of concrete}\]

**Foundation Ring 2**

<table>
<thead>
<tr>
<th>Inner radius</th>
<th>Outer radius</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>27 m</td>
<td>30 m</td>
<td>5 m</td>
</tr>
</tbody>
</table>

\[(\pi r^2 x \text{depth})_{\text{outer}} - (\pi r^2 x \text{depth})_{\text{inner}} = [\pi(30^2)(5)] - [\pi(27^2)(5)] = 2,686 \text{ m}^3 \text{ of concrete}\]

**Block Foundation**

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.5 m</td>
<td>5 m</td>
<td>5 m</td>
</tr>
</tbody>
</table>

1/2 x length x width x depth x qty. = (1/2)(17.5)(5)(5)(2) = **438 m³ of concrete**

**Foundation Total**

960 m³ stone, 840 m³ rammed earth and 8,292 m³ concrete

Table B.1: Foundation Material Calculations
### Rotunda Level 1
Outer Wall Radius = 28 m, Inner Wall Radius = 22 m  
Height = 13 m, Brick Size = 0.29 m x 0.04 m  

\[(\pi r^2 \times \text{height})_{\text{outer}} - (\pi r^2 \times \text{height})_{\text{inner}} = [\pi(30^2)(13)] - [\pi(24^2)(13)] = 13,232 \text{ m}^3 \text{ of concrete}\]

\[
((\text{Outer Wall Circumference} / \text{Brick Length}) \times (\text{Wall Height} / \text{Brick Height})) + ((\text{Inner Wall Circumference} / \text{Brick Length}) \times (\text{Wall Height} / \text{Brick Height})) = [((\pi \times 56)/.29) \times (13/.04)] + [((\pi \times 44)/.29) \times (13/.04)] = 352,075 \text{ bricks}\]

### Rotunda Level 2
Outer Wall Radius = 28 m, Inner Wall Radius = 22 m  
Height = 9 m, Brick Size = 0.29 m x 0.04 m  

\[(\pi r^2 \times \text{height})_{\text{outer}} - (\pi r^2 \times \text{height})_{\text{inner}} = [\pi(28^2)(9)] - [\pi(22^2)(9)] = 8,482 \text{ m}^3 \text{ of concrete}\]

\[
((\text{Outer Wall Circumference} / \text{Brick Length}) \times (\text{Wall Height} / \text{Brick Height})) + ((\text{Inner Wall Circumference} / \text{Brick Length}) \times (\text{Wall Height} / \text{Brick Height})) = [((\pi \times 56)/.29) \times (9/.04)] + [((\pi \times 44)/.29) \times (9/.04)] = 243,743 \text{ bricks}\]

### Rotunda Level 3
Outer Wall Radius = 28 m, Inner Wall Radius = 22 m  
Height = 8 m, Brick Size = 0.29 m x 0.04 m  

\[(\pi r^2 \times \text{height})_{\text{outer}} - (\pi r^2 \times \text{height})_{\text{inner}} = [\pi(28^2)(8)] - [\pi(22^2)(8)] = 7,540 \text{ m}^3 \text{ of concrete}\]

\[
((\text{Outer Wall Circumference} / \text{Brick Length}) \times (\text{Wall Height} / \text{Brick Height})) + ((\text{Inner Wall Circumference} / \text{Brick Length}) \times (\text{Wall Height} / \text{Brick Height})) = [((\pi \times 56)/.29) \times (8/.04)] + [((\pi \times 44)/.29) \times (8/.04)] = 216,660 \text{ bricks}\]

### Rotunda Total
812,478 bricks and 28,274 m$^3$ concrete

Table B.2: Rotunda Material Calculations:
Block Level 1
Length = 32 m, Width = 11 m, Height = 13 m
Brick Size = .29 m x .04 m

Length x width x height = 32 x 11 x 13 = **4,576 m³ of concrete**

\[
\frac{\text{Block Wall Length}}{\text{Brick Length}} \times \frac{\text{Block Wall Height}}{\text{Brick Height}} + 2 \times \left(\frac{\text{Block Wall Width}}{\text{Brick Length}} \times \frac{\text{Block Wall Height}}{\text{Brick Height}}\right) = \left(\frac{32}{.29}\right) \times \left(\frac{13}{.04}\right) + 2 \left(\frac{11}{.29} \times \left(\frac{13}{.04}\right)\right) = 59,800 \text{ bricks}
\]

Block Level 2
Length = 32 m, Width = 11 m, Height = 9 m
Brick Size = .29 m x .04 m

Length x width x height = 32 x 11 x 9 = **2,880 m³ of concrete**

\[
\frac{\text{Block Wall Length}}{\text{Brick Length}} \times \frac{\text{Block Wall Height}}{\text{Brick Height}} + 2 \times \left(\frac{\text{Block Wall Width}}{\text{Brick Length}} \times \frac{\text{Block Wall Height}}{\text{Brick Height}}\right) = \left(\frac{32}{.29}\right) \times \left(\frac{9}{.04}\right) + 2 \left(\frac{11}{.29} \times \left(\frac{9}{.04}\right)\right) = 41,896 \text{ bricks}
\]

Block Level 3
Length = 32 m, Width = 11 m, Height = 8 m
Brick Size = .29 m x .04 m

Length x width x height = 32 x 11 x 6.8 = **2,816 m³ of concrete**

\[
\frac{\text{Block Wall Length}}{\text{Brick Length}} \times \frac{\text{Block Wall Height}}{\text{Brick Height}} + 2 \times \left(\frac{\text{Block Wall Width}}{\text{Brick Length}} \times \frac{\text{Block Wall Height}}{\text{Brick Height}}\right) = \left(\frac{32}{.29}\right) \times \left(\frac{8}{.04}\right) + 2 \left(\frac{11}{.29} \times \left(\frac{8}{.04}\right)\right) = 37,240 \text{ bricks}
\]

Block Total
138,936 bricks and 10,272 m³ concrete

Table B.3: Transition Block Material Calculations:
### Saucer Dome

<table>
<thead>
<tr>
<th>Material</th>
<th>Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius = 16.76 m, Height = 9 m, Thickness = .5 m</td>
<td></td>
</tr>
<tr>
<td>((r \times r \times h/2 \times \pi)<em>{outer} - (r \times r \times h/2 \times \pi)</em>{inner} = (16.76 \times 16.76 \times (9/2) \times \pi) - (16.26 \times 16.26 \times (8.5/2) \times \pi))</td>
<td>(= 737 \text{ m}^3) of concrete</td>
</tr>
</tbody>
</table>

### Step Rings (seven)

<table>
<thead>
<tr>
<th>Ring</th>
<th>Radius</th>
<th>Height</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.6 m</td>
<td>2.28 m</td>
<td>2.3 m</td>
</tr>
<tr>
<td>2</td>
<td>23.5 m</td>
<td>.8 m</td>
<td>1.2 m</td>
</tr>
<tr>
<td>3</td>
<td>22.4 m</td>
<td>.8 m</td>
<td>1.2 m</td>
</tr>
<tr>
<td>4</td>
<td>21.24 m</td>
<td>.8 m</td>
<td>1.2 m</td>
</tr>
<tr>
<td>5</td>
<td>20 m</td>
<td>.8 m</td>
<td>1.2 m</td>
</tr>
<tr>
<td>6</td>
<td>19 m</td>
<td>.8 m</td>
<td>1.2 m</td>
</tr>
<tr>
<td>7</td>
<td>17.9 m</td>
<td>.8 m</td>
<td>1.2 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ring</th>
<th>Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((\pi r^2)<em>{outer} - (\pi r^2)</em>{inner}) \times \text{ height} = [\pi(24.6^2)] - [\pi(24.6 - 2.3)^2] \times 2.28 = 763 \text{ m}^3\text{ of concrete}</td>
</tr>
<tr>
<td>2</td>
<td>((\pi r^2)<em>{outer} - (\pi r^2)</em>{inner}) \times \text{ height} = [\pi(23.5^2)] - [\pi(23.5 - 1.2)^2] \times 0.8 = 138 \text{ m}^3\text{ of concrete}</td>
</tr>
<tr>
<td>3</td>
<td>((\pi r^2)<em>{outer} - (\pi r^2)</em>{inner}) \times \text{ height} = [\pi(22.4^2)] - [\pi(22.4 - 1.2)^2] \times 0.8 = 131 \text{ m}^3\text{ of concrete}</td>
</tr>
<tr>
<td>4</td>
<td>((\pi r^2)<em>{outer} - (\pi r^2)</em>{inner}) \times \text{ height} = [\pi(21.24^2)] - [\pi(21.24 - 1.2)^2] \times 0.8 = 124 \text{ m}^3\text{ of concrete}</td>
</tr>
<tr>
<td>5</td>
<td>((\pi r^2)<em>{outer} - (\pi r^2)</em>{inner}) \times \text{ height} = [\pi(20^2)] - [\pi(20 - 1.2)^2] \times 0.8 = 117 \text{ m}^3\text{ of concrete}</td>
</tr>
<tr>
<td>6</td>
<td>((\pi r^2)<em>{outer} - (\pi r^2)</em>{inner}) \times \text{ height} = [\pi(19^2)] - [\pi(19 - 1.2)^2] \times 0.8 = 111 \text{ m}^3\text{ of concrete}</td>
</tr>
<tr>
<td>7</td>
<td>((\pi r^2)<em>{outer} - (\pi r^2)</em>{inner}) \times \text{ height} = [\pi(17.9^2)] - [\pi(17.9 - 1.2)^2] \times 0.8 = 104 \text{ m}^3\text{ of concrete}</td>
</tr>
</tbody>
</table>

### Oculus

<table>
<thead>
<tr>
<th>Material</th>
<th>Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius = 3 m</td>
<td>No calculations performed for the oculus. Minimal materials used.</td>
</tr>
</tbody>
</table>

### Dome Total

<table>
<thead>
<tr>
<th>Material</th>
<th>Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dome Total</td>
<td>(2,225 \text{ m}^3) of concrete</td>
</tr>
</tbody>
</table>

Table B.4: Dome Material Calculations
<table>
<thead>
<tr>
<th><strong>Columns</strong></th>
<th>Height = 16 m, average thickness = 2 m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((\pi r^2 \times \text{height}) = \pi \times 1^2 \times 16 = 50 \text{ m}^3 \text{ of granite})</td>
</tr>
<tr>
<td><strong>Entablature</strong></td>
<td>Length = 76 m, Height = 2.68 m, Width = 2 m</td>
</tr>
<tr>
<td></td>
<td>(\text{length} \times \text{width} \times \text{height} = 76 \times 2 \times 2.68 = 407 \text{ m}^3 \text{ of stone})</td>
</tr>
<tr>
<td><strong>Pediment</strong></td>
<td>Wall Radius = 28.9 m, Height = 6.8 m, Brick Size = .29 m (\times .04) m</td>
</tr>
<tr>
<td></td>
<td>(\frac{1}{2} \times \text{length} \times \text{width} \times \text{height} = \frac{1}{2} \times 32 \times 2 \times 7.6 = 243.2 \text{ m}^3 \text{ of stone})</td>
</tr>
<tr>
<td><strong>Portico Total</strong></td>
<td>(50 \text{ m}^3 \text{ granite and 650.2 m}^3 \text{ stone})</td>
</tr>
</tbody>
</table>

Table B.5: Portico Material Calculations
Continuing from Chapter 6 (Fuzzy Logic) Appendix C includes the fuzzy models and calculations of all greenness rating categories. Each of the main categories (Site Selection, Use of Water, Energy Conservation, Materials and Resources, Air Quality, and Innovation of Design and Construction) has a table summarizing the ratings ($R_i$) and weights ($W_i$) of each subcategory and the total rating for the main category, calculations for the total rating ($R_T$), and a visual model.

![Figure C.1: Standard Greenness Ratings Fuzzy Model](image-url)
Site Selection

<table>
<thead>
<tr>
<th>Rating Categories</th>
<th>$R_i$</th>
<th>$W_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Priority Site</td>
<td>SR$_1$</td>
<td>Extremely Green = [3, 5]</td>
</tr>
<tr>
<td>Access to Quality Transit</td>
<td>SR$_2$</td>
<td>Very Green = [2, 4]</td>
</tr>
<tr>
<td>Site Assessment</td>
<td>SR$_3$</td>
<td>Non-Green = [-3, -1]</td>
</tr>
<tr>
<td>Site Development</td>
<td>SR$_4$</td>
<td>Green = [1, 3]</td>
</tr>
<tr>
<td>Rain Water Management</td>
<td>SR$_5$</td>
<td>Green = [1, 3]</td>
</tr>
<tr>
<td>Construction Activity Pollution</td>
<td>SR$_6$</td>
<td>Very Green = [2, 4]</td>
</tr>
<tr>
<td>Site Selection Total Rating</td>
<td>SR$_T$</td>
<td></td>
</tr>
</tbody>
</table>

Table C.1: Site Selection Fuzzy Ratings and Weights

$$SR_T = \Sigma (SR_i * W_i) / \Sigma (W_i)$$

$$SR_T = [3, 5][1, 1] + [2, 4][1, 1] + [-3, -1][1, 1] + [1, 3][1, 1] + [1, 3][1, 1] + [2, 4][1, 1] / [1, 1] + [1, 1] + [1, 1] + [1, 1] + [1, 1]$$

$$SR_T = [[\min(3, 3, 5, 5), \max(3, 3, 5, 5)] + [\min(2, 2, 4, 4), \max(2, 2, 4, 4)]] + [\min(-3, -3, -1, -1), \max(-3, -3, -1, -1)] + [\min(3, 3, 5, 5), \max(3, 3, 5, 5)] + [\min(1, 1, 3, 3), \max(1, 1, 3, 3)] + [\min(2, 2, 4, 4), \max(2, 2, 4, 4)]] / [6, 6]$$

$$SR_T = [3, 5] + [2, 4] + [-3, -1] + [1, 3] + [1, 3] + [2, 4] / [6, 6]$$

$$SR_T = [4, 16] / [6, 6] = [1, 3]$$
Figure C.2  Site Selection Fuzzy Set Model
### Use of Water

<table>
<thead>
<tr>
<th>Rating Categories</th>
<th>$R_i$</th>
<th>$W_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outdoor Water Use</td>
<td>WR&lt;sub&gt;1&lt;/sub&gt;</td>
<td>Very Green = [2, 4]</td>
</tr>
<tr>
<td>Indoor Water Use</td>
<td>WR&lt;sub&gt;2&lt;/sub&gt;</td>
<td>Green = [1, 3]</td>
</tr>
<tr>
<td>Construction Activity Water Use</td>
<td>WR&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Fairly Green = [0, 2]</td>
</tr>
<tr>
<td>Water Use Total Rating</td>
<td>$WR_T$</td>
<td>[1, 3]</td>
</tr>
</tbody>
</table>

Table C.2: Use of Water Fuzzy Ratings and Weights

\[
WR_T = \frac{\sum (WR_i \cdot W_i)}{\sum W_i}
\]

\[
WR_T = [2, 4] \cdot [1, 1] + [1, 3] \cdot [1, 1] + [0, 2] \cdot [1, 1] / [1, 1] + [1, 1] + [1, 1]
\]

\[
WR_T = \left[\min(2, 2, 4, 4), \max(2, 2, 4, 4)\right] + \left[\min(1, 1, 3, 3), \max(1, 1, 3, 3)\right] + \left[\min(0, 0, 2, 2), \max(0, 0, 2, 2)\right] / [3, 3]
\]

\[
WR_T = [2, 4] + [1, 3] + [0, 2] / [3, 3]
\]

\[
WR_T = [3, 9] / [3, 3] = [1, 3]
\]
Figure C.3: Use of Water Fuzzy Model
## Energy Conservation

<table>
<thead>
<tr>
<th>Rating Categories</th>
<th>( R_i )</th>
<th>( W_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal Energy Use</td>
<td>( \text{ER}_1 )</td>
<td>Green = [1, 3]</td>
</tr>
<tr>
<td>Energy Use for Materials</td>
<td>( \text{ER}_2 )</td>
<td>Non-Green = [-3, -1]</td>
</tr>
<tr>
<td>Construction Activity Energy Use</td>
<td>( \text{ER}_3 )</td>
<td>Non-Green = [-3, -1]</td>
</tr>
<tr>
<td>Heating System Energy Use</td>
<td>( \text{ER}_4 )</td>
<td>Green = [1, 3]</td>
</tr>
<tr>
<td>Lighting Energy Use</td>
<td>( \text{ER}_5 )</td>
<td>Green = [1, 3]</td>
</tr>
<tr>
<td>Fairly Non-Green</td>
<td>( \text{ER}_6 )</td>
<td>Fairly Non-Green = [-2, 0]</td>
</tr>
<tr>
<td>Energy Conservation Total Rating</td>
<td>( \text{ER}_T )</td>
<td>[-0.83, 1.167]</td>
</tr>
</tbody>
</table>

Table C.3: Energy Conservation Fuzzy Ratings and Weights

\[
\text{ER}_T = \frac{\sum (\text{ER}_i \times W_i)}{\sum W_i}
\]

\[
\text{ER}_T = \left[ \min(1, 1, 3, 3), \max(1, 1, 3, 3) \right] + \left[ \min(-3, -3, -1, -1), \max(-3, -3, -1, -1) \right] + \left[ \min(-3, -3, -1, -1), \max(-3, -3, -1, -1) \right] + \left[ \min(1, 1, 3, 3), \max(1, 1, 3, 3) \right] + \left[ \min(-2, -2, 0, 0), \max(-2, -2, 0, 0) \right] / [6, 6]
\]

\[
\text{ER}_T = \left[ 1, 3 \right] + \left[ -3, -1 \right] + \left[ -3, -1 \right] + \left[ 1, 3 \right] + \left[ 1, 3 \right] + \left[ -2, 0 \right] / [6, 6]
\]

\[
\text{ER}_T = [-5, 7] / [6, 6] = [-0.83, 1.167]
\]
Figure C.4: Energy Conservation Fuzzy Model
Materials and Resources

<table>
<thead>
<tr>
<th>Rating Categories</th>
<th>Ri</th>
<th>Wi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reuse of Construction Materials</td>
<td>MR₁</td>
<td>Green = [1,3]</td>
</tr>
<tr>
<td>Local Material Use</td>
<td>MR₂</td>
<td>Very Non-Green = [-4,-2]</td>
</tr>
<tr>
<td>Recyclable Materials</td>
<td>MR₃</td>
<td>Very Green = [2,4]</td>
</tr>
<tr>
<td>Construction and Demolition Water</td>
<td>MR₄</td>
<td>Green = [1,3]</td>
</tr>
<tr>
<td>Management</td>
<td>MR₅</td>
<td>Very Non-Green = [-4,-2]</td>
</tr>
<tr>
<td>Construction Materials</td>
<td>MR₆</td>
<td>Non-Green = [-3,-1]</td>
</tr>
<tr>
<td>Construction Equipment and Tools</td>
<td>MR₇</td>
<td>Non-Green = [-3,-1]</td>
</tr>
<tr>
<td>Materials and Resources Total Rating</td>
<td>MR₇</td>
<td>[-1.43, 0.57]</td>
</tr>
</tbody>
</table>

Table C.4: Materials and Resources Fuzzy Ratings and Weights

\[ MR_T = \frac{\sum (MR_i \times W_i)}{\sum W_i} \]


\[ MR_T = [-10, 4] / [7, 7] = [-1.43, 0.57] \]
Figure 6.C: Materials and Resources Fuzzy Model
Air Quality

<table>
<thead>
<tr>
<th>Rating Categories</th>
<th>$R_i$</th>
<th>$W_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outdoor Air Quality</td>
<td>$AR_1$</td>
<td>Green = [1, 3]</td>
</tr>
<tr>
<td>Indoor Air Quality</td>
<td>$AR_2$</td>
<td>Green = [1, 3]</td>
</tr>
<tr>
<td>Building Ventilation System</td>
<td>$AR_3$</td>
<td>Fairly Non-Green = [-2, 0]</td>
</tr>
<tr>
<td>Air Quality Overall Rating</td>
<td>$AR_T$</td>
<td>[0, 2]</td>
</tr>
</tbody>
</table>

Table C.5: Air Quality Fuzzy Ratings and Weights

$$AR_T = \frac{\sum (AR_i \times W_i)}{\sum (W_i)}$$

$$AR_T = [-2, 0] \times [1, 1] + [1, 3] \times [1, 1] + [-2, 0] \times [1, 1] / [1, 1] + [1, 1] + [1, 1]$$

$$AR_T = [[\min(1, 1, 3, 3), \max(1, 1, 3, 3)] + [\min(1, 1, 3, 3), \max(1, 1, 3, 3)] + [\min(-2, -2, 0, 0), \max(-2, -2, 0, 0)]] / [3, 3]$$

$$AR_T = [1, 3] + [1, 3] + [-2, 0] / [3, 3]$$

$$AR_T = [0, 6] / [3, 3] = [0, 2]$$
Figure C.6: Air Quality Fuzzy Model
Innovation of Design and Construction

<table>
<thead>
<tr>
<th>Rating Categories</th>
<th>Ri</th>
<th>Wi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
<td>IR₁</td>
<td>[1, 1]</td>
</tr>
<tr>
<td>Environmental Impact of Design</td>
<td>IR₂</td>
<td>[0, 2]</td>
</tr>
<tr>
<td>Construction Methods</td>
<td>IR₃</td>
<td>[3, 5]</td>
</tr>
<tr>
<td>Innovation of Design and Construction Total Rating</td>
<td>IRₜ</td>
<td>[2, 4]</td>
</tr>
</tbody>
</table>

Table C.6: Innovation of Design and Construction Fuzzy Ratings and Weights

\[
\text{IR}_T = \frac{\sum (\text{IR}_i \times \text{W}_i)}{\sum \text{W}_i}
\]

\[
\text{IR}_T = [3, 5] \times [1, 1] + [0, 2] \times [1, 1] + [3, 5] \times [1, 1] / [1, 1] + [1, 1] + [1, 1]
\]

\[
\text{IR}_T = \frac{\min(3, 3, 5, 5), \max(3, 3, 5, 5) + \min(0, 0, 2, 2), \max(0, 0, 2, 2) + \min(3, 3, 5, 5), \max(3, 3, 5, 5)]}{[3, 3]}
\]

\[
\text{IR}_T = [3, 5] + [0, 2] + [3, 5] / [3, 3]
\]

\[
\text{IR}_T = [6, 12] / [3, 3] = [2, 4]
\]
Figure C.7: Innovation of Design and Construction Fuzzy Model
Total Greenness

<table>
<thead>
<tr>
<th>Rating Categories</th>
<th>$R_i$</th>
<th>$W_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site Selection Rating</td>
<td>$SR_T$</td>
<td>[1, 3]</td>
</tr>
<tr>
<td>Water Use</td>
<td>$WR_T$</td>
<td>[1, 3]</td>
</tr>
<tr>
<td>Energy Conservation</td>
<td>$ER_T$</td>
<td>[-0.83, 1.17]</td>
</tr>
<tr>
<td>Materials and Resources</td>
<td>$MR_T$</td>
<td>[-1.43, 0.57]</td>
</tr>
<tr>
<td>Air Quality</td>
<td>$AR_T$</td>
<td>[0, 2]</td>
</tr>
<tr>
<td>Innovation of Design and Construction</td>
<td>$IR_T$</td>
<td>[2, 4]</td>
</tr>
</tbody>
</table>

Pantheon’s Overall Greenness Rating | $GR_T$ | [-0.61, 2.01]

Table C.7: Final Greenness Fuzzy Ratings and Weights

$$GR_T = \Sigma(GR_i * W_i) / \Sigma(W_i)$$

$$GR_T = [1, 3]x[1, 3] + [1, 3]x[.5, .5] + [-0.83, 1.17]x[3, 5] + [-1.43, 0.57]x[3, 5] + [0, 2]x[0, 2] + [2, 4]x[2, 4] / [1, 3] + [.5, .5] + [3, 5] + [3, 5] + [0, 2] + [2, 4]$$

$$GR_T = \text{min}(1, 3, 3, 9) + \text{max}(1, 3, 3, 9) + \text{min}(.5, .5, 1.5, 1.5) + \text{max}(.5, .5, 1.5, 1.5) + \text{min}(-2.49, -4.15, 3.51, 5.85) + \text{max}(-2.49, -4.15, 3.51, 5.85) + \text{min}(-4.29, -7.15, 1.17, 2.85) + \text{max}(-4.29, -7.15, 1.17, 2.85) + \text{min}(0, 0, 0, 4) + \text{max}(0, 0, 0, 4) + \text{min}(4, 8, 8, 16) + \text{max}(4, 8, 8, 16) / [9.5, 19.5]$$

Figure C.8: Fuzzy Model for Six Main Greenness Categories
Figure C.9: Fuzzy Model for Weights of Six Main Categories
Figure C.10: Fuzzy Model of Total Greenness Rating for the Pantheon (Total Greenness)
Rating Index Calculations

Site Selection Rating Index:

\[ S_R = [1, 3] \]

\[ A_L = \frac{(5+1) + (5+2)}{2} = 6.5 \]

\[ A_R = \frac{(5-3) + (5-2)}{2} = 2.5 \]

\[ C = 10 \times 1 = 10 \]

\[ I_S = A_R - A_L + C = 2.5 - 6.5 + 10 = 6 \]

Use of Water Rating Index:

\[ W_R = [1, 3] \]

\[ A_L = \frac{(5+1) + (5+2)}{2} = 6.5 \]

\[ A_R = \frac{(5-3) + (5-2)}{2} = 2.5 \]

\[ C = 10 \times 1 = 10 \]

\[ I_W = A_R - A_L + C = 2.5 - 6.5 + 10 = 6 \]

Energy Conservation Rating Index:

\[ E_R = [-0.83, 1.17] \]

\[ A_L = \frac{(5 - 0.83) + (5+1.17)}{2} = 4.67 \]

\[ A_R = \frac{(5-1.17) + (5-0.17)}{2} = 4.33 \]

\[ C = 10 \times 1 = 10 \]

\[ I_E = A_R - A_L + C = 4.33 - 4.67 + 10 = 9.66 \]
Materials and Resources Rating Index:

\[ M_R = [-1.43, 0.57] \]

\[ A_L = [(5 -1.43) + (5+0.57)] / 2 = 4.07 \]

\[ A_R = [(5-0.57) + (5+0.43)] / 2 = 4.93 \]

\[ C = 10 * 1 = 10 \]

\[ I_M = A_R - A_L + C = 4.93 - 4.07 + 10 = 10.86 \]

Air Quality Rating Index:

\[ A_R = [0, 2] \]

\[ A_L = [(5+0) + (5+2)] / 2 = 5.5 \]

\[ A_R = [(5-2) + (5-1)] / 2 = 3.5 \]

\[ C = 10 * 1 = 10 \]

\[ I_A = A_R - A_L + C = 3.5 - 5.5 + 10 = 8 \]

Innovation of Design and Construction Rating Index:

\[ A_R = [2, 4] \]

\[ A_L = [(5+2) + (5+4)] / 2 = 7.5 \]

\[ A_R = [(5-4) + (5-3)] / 2 = 1.5 \]

\[ C = 10 * 1 = 10 \]

\[ I_A = A_R - A_L + C = 1.5 - 7.5 + 10 = 4 \]
Total Pantheon Greenness Rating Index:

\[ G_R = [-0.61, 2.01] \]
\[ A_L = [(5-0.61) + (5+2.01)] / 2 = 5.05 \]
\[ A_R = [(5-2.01) + (5-0.70)] / 2 = 3.65 \]
\[ C = 10 * 1 = 10 \]
\[ I_G = A_R - A_L + C = 3.65 - 5.05 + 10 = 8.6 \]
References


