A STUDY OF STELLAR SCINTILLATION

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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Approved by:

[Signature]

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I. INTRODUCTION

The theoretical telescopic image of a star is for all practical purposes that predicted by diffraction theory for a point source at an infinite distance. The fact that the image does not behave according to prediction is well known. The image is in general much larger than predicted and shows changes in shape, position and brightness which cannot be explained in terms of the telescopic optics.

The fluctuations or changes in the image fall into three categories: motion of the center of light of the image, termed dancing; change of shape of the image, termed pulsation; and change of brightness of the image, termed scintillation. The twinkling of the stars is merely that part of scintillation which occurs within the time resolution of the human eye, that is, more than one tenth to one sixteenth of a second.

The astronomer often combines all effects, including the mean image size, into a semiquantitative measure termed seeing. The actual quantities referred to when the term seeing is used depends to a considerable extent upon the type of observations being made at the time. For example, image size and pulsation are particularly disturbing in the measurement of the separation and of position angle of close double stars; scintillation, on
the other hand, is most disturbing in the measurement of stellar brightness. This report will be concerned only with the observation of scintillation except where observational attempts to relate seeing as a whole to scintillation have been made.

Of the three fluctuating quantities associated with the image of a star, scintillation is the easiest to measure, since it is possible, by means of modern photoelectric techniques, to transform the brightness variations into an electrical signal which is capable of being rather accurately measured. Stellar scintillation can, in fact, be considered as noise superposed upon the mean brightness level of a star. Such noise has frequency components whose periods are too short to be associated with an object as massive as a star. The noise must, then, be introduced by disturbances in the path of the starlight as it proceeds from the star to the photodetector. It will become evident later, from some of the functional relationships established for scintillation, that the disturbances causing the brightness fluctuations are associated with the earth's atmosphere and not with any extra-terrestrial cause or anything within the telescope itself.
II. INSTRUMENTATION

Two types of observation of scintillation have been made: (1) recording the variations in image brightness of the star directly on a photographic oscilloscope\textsuperscript{1, 2, 3} or an inking type recorder\textsuperscript{4}; and (2) the frequency analysis\textsuperscript{5, 6} of the variations in image brightness. The latter technique is capable of giving the most complete description of the noise signal arising from scintillation, although by its very nature (since it is really a method of finding the time average power spectrum of the signal) it rejects some of the instantaneously available information present in the scintillation signal, such as the maximum deviation from the mean level. The data


\textsuperscript{2} Ellison and Wilson, "Photoelectric Recording of Stellar Scintillation," The Observatory, 71, 26, 1951.


\textsuperscript{4} Hosfeld, Progress Reports 3 to 9 incl., Contract No. AF 19(604)-41.

\textsuperscript{5} Goldstein, Naval Research Lab. Reports N3462 (1949) and N3710 (1950).

given by the frequency analysis technique are, on the other hand, more easily handled and consequently was the technique chosen for this work.

The major drawback to the frequency analysis method is the long period of time necessary to complete a representative measurement. As will be pointed out later in the discussion, it is necessary to observe for a considerable length of time, four to five minutes, at any one frequency. Thus, if any appreciable number of points are desired to fix the frequency curve of the scintillation, too long an interval of time is needed to complete one observation. During this time certain of the parameters affecting the scintillation can change markedly, in which case the frequency curve will no longer give a true representation. Although it is possible to select observations that will minimize this effect, such as observing Polaris which undergoes only a slight diurnal motion, it restricts the number of parameters affecting scintillation which can be accurately studied. To circumvent this difficulty, a tape recorder has been utilized. This permits the analysis of a homogeneous sample of scintillation to be made to any degree of thoroughness desired while permitting at least some control over the scintillation parameters.

Also since these analyses were to be carried out for daytime observations, further modifications in the usual
equipment were necessary to reduce the effects of background illumination of the sky.

1. DESCRIPTION OF INSTRUMENTATION

The following equipment has been used in the stellar scintillation observations. The assembled apparatus is shown in Fig. 1. The optical system consists of the McMillin Observatory 12.5-inch refractor and an astronomical photometer modified for daytime use. The detector is either an RCA 1P21 or 6199 photomultiplier. The high-voltage source for the photocell is an Atomic Instruments No. 306P high-voltage supply. A Tektronix No. 112 dc Amplifier or a dc Amplifier of local design and construction is used for amplification of the anode signal of the phototube. A low frequency tape recorder and a General Radio No. 762-B Vibration Analyzer are used in determining the scintillation frequency curves. Two Brown recorders are used to continuously record the output of the signal amplifier and of the analyzer. A Krohn Hite No. 420-A oscillator and a General Radio Microvolter are used for calibration purposes. Various other circuit elements were constructed at the McMillin Observatory. These items of equipment will be discussed individually and finally collectively as they are used for an observation and subsequent analysis.
Fig. 1. Photograph of Equipment
A. Photometer

The photometer is a standard astronomical type modified to handle the special observational problems encountered in this work. Basically it consists of the following: a photomultiplier, either a 1P21 or 6199 tube, and associated voltage dividing network used to derive operating potentials; a filter wheel, permitting various spectral ranges to be studied; a diaphragm wheel, permitting larger or smaller areas of the telescope field to be observed; a finder eyepiece, which permits the observer to center the desired object in the photometer field; and a Fabry or field lens, which images the objective of the telescope in the light of the object being studied upon the photocathode. This latter feature minimizes the effects of guiding errors which, otherwise, could give rise to considerable erroneous fluctuations due to a concentrated star image striking regions of varying sensitivity on the cathode.

Two modifications were made to the basic photometer. First, a constant light, operated from a heavy-duty storage battery, was incorporated, such that it could be placed in the field and adjusted to give an illumination level equal to that of the object being observed.

7Progress Report No. 2, Project 480, Ohio State University Research Foundation, Contract No. AF 19(604)-41
Second, an auxiliary eyepiece was installed behind the photometer diaphragm wheel. The light for this eyepiece is removed from the normal light path by an unsilvered glass flat. This reduces the photocathode illumination by about 4%, but permits simultaneous visual and photoelectric observations of the object being studied. This is used during the daytime to facilitate guiding.

B. High-Voltage Supply

This is a standard positive grounded No. 306P Atomic Instruments High-Voltage Supply. The output voltage of this instrument is sufficiently stable that variations introduced into the photomultiplier output are inappreciable. The supply was modified to permit the continuously variable range of the instrument to extend down to 300 volts instead of the 500 volts for which it was designed. This was necessary to handle the high light levels encountered in the daytime observations without causing fatigue or more serious damage to the photocell.

C. Signal Amplifiers and Associated Circuitry

A Tektronix No. 112 wide band amplifier was used for part of the observations. This is a direct-coupled balanced type amplifier having a band pass of 0 to 1 megacycle/sec. and a useful gain, for the present work, which is variable from less than unity to 250. This amplifier developed some minor instabilities which could not
be tolerated in the scintillation measurements and consequently was replaced by an amplifier designed and constructed at the McMillin Observatory. The circuit is shown in Fig. 2. This amplifier has fixed gains of 1, 2.5, 5, 7.5, and 10. The output is linear from 0 to 1,000 cps in all ranges.

Each of these amplifiers were connected by a resistive impedance matcher to the input of the tape recorder and to a Brown Recorder. The fluctuating portion of the signal is recorded on the magnetic tape while the Brown Recorder gives a permanent record of the light level.

The signal amplifier is preceded by an input selector and bias control. The input selector allows the input of the amplifier to be switched to either the collector plate of the photomultiplier or to a calibration circuit. The collector plate switch position also has provision for the introduction of a dc bias. This bias is delivered by a flashlight cell through a compensated resistance network which maintains the input resistance at 0.5 megohm, while the bias is varied from 0.45 to 0.50 volt by 0.05-volt steps. For daytime work this biasing arrangement permits the amplification of the "star over sky" signal to a usable value while keeping the phototube output current within a safe operating range. It is also useful in the study of sources such as planets, which show a small percentage scintillation,
Fig. 2. Circuit of D.C. Amplifier
since the variable signal, although a small percentage of the half volt maximum output of the phototube, may be amplified to an analyzable level after biasing out the dc component.

D. Dc Level Recorder

The average level, or dc component, of the observed signal is determined by means of a Brown Recorder and a coupling preamplifier. The recorder is a standard model with a two-second full scale pen travel time and is operated at a chart speed of one inch per minute. The coupling amplifier is a balanced modified cathode follower.

E. Magnetic Tape Recorder

The recorder in its present form consists of the circuitry from an Amplifier Corporation of America No. 821 FM Tape Recorder and a Magnecorder PT6-A 15 inch/second tape transport mechanism. The circuitry is essentially that developed by Green. In order to avoid difficulties arising from magnetic tape surface variations in the recording of very low frequency signals, it is necessary to resort to a frequency modulation recording technique. The circuit used in this recorder is shown schematically in Fig. 3.

Fig. 3. Schematic Circuit of Magnetic Tape Recorder
In the "record" position the input signal is fed to a constant-gain, two-stage, one-sided preamplifier. The output of the preamplifier is presented to the record level and coupling amplifier through a potentiometer which gives control of the recording level. This amplifier consists of one stage of plate amplification and a cathode-follower output stage. The quiescent point of the output is determined by a potentiometer. This quiescent point determines the center frequency of the modulation unit. The modulation unit is a multivibrator having an output frequency proportional to the voltage on its grids. The output from the modulator is presented to the recording head.

In playback the signal impressed on the tape is picked up by the magnetic head. The signal is amplified by the same preamplifier used in the record circuitry. The output of this amplifier is clipped; that is, for all pulses recorded on the tape from the record multivibrator an approximate square wave is developed at the output of the clipper. The clipped signal is differentiated, the negative pulses are discarded, and the positive pulses are used to operate a "one shot" multivibrator. The signal at this point should be an exact duplicate of the output of the record multivibrator before being impressed upon the tape. The signal is then demodulated by the filter and should be a reproduction of the original signal.
The main source of noise in such a system, when operating correctly, should be in the tape transport mechanism, since variations in tape speed result in an apparent modulation of the carrier frequency. In order to minimize this type of noise a Magnecorder Tape Transport was incorporated in the system, because the initial mechanism proved too erratic in its drive characteristics. The low impedance record-playback head of the Magnecorder mechanism was matched to the record multivibrator by means of a transformer. The playback circuit incorporates a one-stage amplifier with cathode feedback in connection with the magnetic head. This reduces undesirable frequency effects found in the magnetic head. The amplifier cathode resistor is shunted by a capacitor chosen such that the negative feedback for higher frequencies is reduced, thus compensating for high frequency losses in the head.

Several minor, although important changes, were made in the original circuit. The feedback in the preamplifier was increased, reducing harmonic distortion and permitting larger signals to be used without cutoff due to overloading. It was also necessary to short the input with a condenser having little effect at 1,000 cps and below but becoming quite effective at the FM frequencies, around 10,000 cps, used in the recorder, since the pre-amplifier was susceptible to pick-up of the record
multivibrator signal when high input impedences were used.

F. Analyzer

A General Radio No. 762B Vibration Analyzer was used to determine the frequency spectrum of the recorded noise signal. This instrument has a frequency range of 2.5 to 750 cps. The analyzing circuit uses a capacitance-resistance feedback circuit in conjunction with a wide pass amplifier to develop very sharp tuning with a bandwidth which is approximately a constant percentage of the center frequency. The signal from the analyzing section is then measured by a logarithmic voltmeter. The output of this meter is half-wave rectified and by suitable filtering can be presented as a measurable dc signal. Over-filtering is used by the addition of a large condenser, 2,000 microfarads across 20,000 ohms, to partially smooth variations due to changes in the scintillation.

G. Analyzer Recorder

A Brown two-second full-scale recorder is used to record the output of the analyzer. Since the signals being analyzed are variable, the analyzer output is also variable. The Brown recordings, after calibration,

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afford the only practicable method for determining the average energy at a given frequency.

H. Signal Calibration Equipment

A General Radio No. 546-C Microvolter, a Krohn-Hite No. 420-A low frequency oscillator and a dry cell are used to develop the ac and dc calibration signals used in determining the output voltages at the collector plate of the photomultiplier tube. The microvolter gives a continuously variable output from 1 microvolt to 1 volt when supplied with an input voltage of 2.2 volts. The Krohn-Hite Oscillator has a frequency range from 0.35 to 52,000 cps.

I. Arrangement of Equipment

A schematic diagram of the equipment arrangement is given in Fig. 4.

2. REDUCTION AND OBSERVING TECHNIQUE

The observing procedure using the equipment is as follows. The star is first centered in the diaphragm of the photometer and its light is permitted to fall upon the photocathode. The high-voltage supply and the sensitivity of the signal amplifier are then adjusted until a readable signal is produced at the tape recorder. The record level control is then adjusted to give approximately the same level for all recordings. Both the recorder and signal amplifier are operated well below cutoff in
Fig. 4. Schematic Diagram of Entire Instrumentation
order not to clip the larger peaks appearing in the scintillation signal. If it is not possible to adjust the high voltage supply and the signal amplifier to give a usable ac signal without damage to the photocell, as in the case of a star during the daytime, the output signal of the cell is adjusted to about one-half volt by means of the high-voltage supply. The major portion of the dc signal is then biased out by means of the input bias control and the signal amplifier gain is increased until the ac portion of the signal becomes usable. The scintillation signal is then recorded for a period of four minutes. Next a sky reading is taken in order to correct the dc reading on the star for background light.

The photometer is reoriented so that the star is no longer in its field and the standard source, a battery-operated lamp, is placed in the optical path of the photometer. With identical settings on the equipment, the illumination level of the standard source is increased to match the mean level shown by the star plus the background illumination just observed. This signal is then recorded for a two-minute period to permit a reduction for shot noise and instrumental noise to be made.

The input-signal amplifier is next changed over to the calibration circuit and a known signal at 100 cps and of one minute duration is impressed upon the tape. After this, known dc voltages are applied to the amplifier
to determine the average voltage level shown by the dc recorder. This completes the observation. Two such observations can be placed upon one 1250-foot tape. In the daytime work only one observation can be placed on a tape since it becomes necessary to increase the time interval spent recording the standard source signal to four minutes in order to reduce for the high shot noise present in the signal (caused by the large background illumination) with an accuracy comparable to that gained at night.

The analysis of the signal consists of playing back the entire signal once for each of the standardized frequency settings on the analyzer. There are seventeen such settings. The calibration signal is then played back with the analyzer accurately tuned for the signal on that tape. The analyzer output recording is later examined to determine the average signal at each frequency. The relative voltages at each frequency are taken from the analyzer calibration curves. These data are then reduced for shot noise by the relation:

\[ \text{shot noise} = \sqrt{(\text{total noise})^2 - (\text{standard source noise})^2}. \]

The calibration signal at 100 cps is used to convert these relative voltages to true voltages. These voltages are then reduced to unit bandwidth, and frequency effects introduced by the system are removed by means of a
correction factor. The resultant value represents the
noise voltage at a particular frequency per unit band-
width. The per cent equivalent sine wave modulation\textsuperscript{10,11}
is then given by the relation: \[ M(\%)_f = 1.414 \frac{E_f}{E_{dc}} \]
where \( M(\%) \) equals the per cent equivalent sine wave
modulation per unit bandwidth at frequency \( f \), \( E_f \) equals
the average noise voltage at frequency \( f \), and \( E_{dc} \) equals
the average dc level of the signal.

The average noise voltage, \( E_f \), should be measured
in rms units if the above expression for \( M(\%)_f \) is to hold
exactly. Since, however, the output stage of the vibra-
tion analyzer is an averaging-type meter, it does not
indicate true rms values but reads low on noise voltages,
such as stellar scintillation signals, by a factor
of 12.8\% when it is calibrated using a sine wave
voltage.\textsuperscript{12,13} The frequency analyses made in this work

\textsuperscript{10}Goldstein, \textit{loc. cit.}

\textsuperscript{11}Mikesell et al, \textit{loc. cit.}


\textsuperscript{13}The output stage of the vibration analyzer was
shown to be an averaging-type voltage indicator by simul-
taneously measuring the output of the filter section of
the analyzer with a Hewlett-Packard Model 400 D Vacuum-
tube voltmeter and the output stage of the analyzer. The
Hewlett-Packard meter is a full-wave averaging-type instru-
ment. The analyzer output stage and the Hewlett-Packard
voltmeter gave the same voltage indications for a random
noise signal over the entire frequency range used in the
scintillation frequency analyses.
were not corrected for this effect and consequently give the average component amplitudes to be expected if a scintillation signal is measured using an averaging type meter. If the "true" values for the per cent equivalent sine wave modulation or the root mean square deviation are desired the values given here must be increased by 12.8%, the mean square deviation values presented must be increased by 27%.

3. CALIBRATION OF THE SYSTEM

The system as used has several sources of nonlinearity which must either be shown to be negligible or for which correction must be made in the reductions of the observational data. The sources of such nonlinearity are:

1. Logarithmic output of analyzer;
2. Variable bandwidth of analyzer;
3. Frequency response of analyzer;
4. Frequency characteristics of magnetic tape recorder;
5. Attenuation due to distributed capacitance of the input lead across the high impedance input of the signal amplifier;
6. Frequency response of signal amplifier;
7. Nonlinearity of microvolter in the ac and dc ranges used; and

The last three sources of nonlinearity are either negligible or are removed by the voltage calibration applied during each observation. The others are of sufficient magnitude to merit more detailed treatment.

Since the output of the analyzer as seen by the
analyzer recorder is approximately logarithmic with linear input, it is necessary to determine the relationship between its input and output. Furthermore, because of frequency effects in the analyzer this relationship is not quite the same for all frequencies. It is therefore necessary to calibrate at a sufficient number of points in the frequency range so that interpolation between these points is valid. A representative calibration, at 100 cps is shown in Fig. 5.

The calibration of the analyzer in conjunction with its recorder was determined for 28 settings of the analyzer. The 28 settings represent 24 distinct frequencies, the number of settings exceeding the number of frequencies since the analyzer frequency spread is broken into five overlapping ranges. The endpoints of the overlapping ranges must be determined twice to permit interpolation between points within a given range. Since the frequency range is logarithmic the calibrated points are closely spaced at the low frequency end, 1 cps apart, and are more distantly spaced at the high frequency end, 100 cps apart. The calibration consisted of tuning the analyzer to a given oscillator frequency and plotting the recorder deflection versus relative input voltages. Checks at various frequencies made by applying known voltages while changing the sensitivity settings of the system show that calibration is valid to approximately 1%.
Fig. 5. Calibration of Frequency Analyzer at 100 CPS
The next source of incorrect reading is that of bandwidth. Since the analyzer used has a bandwidth which changes with frequency it is necessary to reduce the voltage readings for bandwidth by the relation:

$$E_{\text{unit bandwidth}} = \frac{E_{\text{measured}}}{\sqrt{\text{bandwidth}}}$$

Furthermore, since this analyzer, although having approximately a constant percentage bandwidth, shows deviations from this condition, it is not possible to measure the bandwidth at one frequency, express the result in per cent of the center frequency, and use this value for all frequencies. To determine the relative bandwidth effect, a white noise (shot noise) generated by illuminating the photocell with the standard source was analyzed at each calibration frequency, directly, without the tape recorder. Several precautions were necessary: the signal was analyzed for 10 minutes at each frequency to eliminate statistical fluctuations in the signal; the signal being analyzed was made large compared to instrumental noises; the standard source was operated for 48 hours before starting the calibration readings to permit it to stabilize; and the average values were taken from the analyzer recordings with a planimeter to remove personal reading errors. As is evident, several other effects are incorporated in this calibration: first, the input attenuation, caused
primarily by the coaxial signal leads used; second, the frequency effects in the signal amplifier; and third, frequency effects in the analyzer circuitry.

Since the measures give only a relative comparison between different frequency settings, it was necessary to normalize the data at one frequency, in this case 100 cps, and multiply the resulting numbers by the square root of the measured bandwidth at this frequency. Figure 6 shows a plot of the response curve of the analyzer at 100 cps. The effective bandwidth, determined by taking the area under the curve, is 2.23 cps. A plot of the relative bandwidth effect as modified by the input attenuation versus frequency is shown in Fig. 7. The discontinuities in this curve occur at the end points of the analyzer frequency ranges.

The last major correction term remaining is the frequency response of the magnetic tape recorder. To determine this correction, signals of equal amplitudes, which were large compared to the tape recorder background noise, were recorded at the calibration frequencies. The playback voltages were then measured using the analyzer to reduce further the effects of background noise. The resulting values were normalized at 100 cps. The plot of these data is shown in Fig. 8.

At each frequency, the reciprocal of the product of the data given in Figs. 7 and 8 is the total correction to
Fig. 6. Bandwidth of Analyzer at 100 CPS
Fig. 7. Relative Bandwidth Effect and Input Attenuation Vs. Frequency
Fig. 8. Frequency Response of Magnetic Tape Recorder
be made to the raw data to correct for bandwidth and frequency effects in the system. The correction factor as a function of frequency is shown in Fig. 9. The non-linearity of the analyzer output is removed in the process of reducing the analyzer recorder deflections to relative voltages by the curves represented in Fig. 5. The final correction factor is good to about 2%.

4. DISCUSSION OF SHOT NOISE

There remain several points which need discussion before the presentation of observed data. One of these points is the "shot noise" observed when the photocathode is illuminated by a lamp of constant intensity.

The discussion of shot noise is possibly best covered by the consideration of a specific observation. A 1P21 photomultiplier was illuminated by a dc lamp at a light level such that, with 360 volts across the tube, an anode current of $2.3 \times 10^{-7}$ ampere was measured. This corresponds to a light flux on the cell cathode of the order of $2 \times 10^{-6}$ lumen. The noise in the signal, measured directly with the analyzer, was 7.36 microvolts at 20 cps. The measured noise could have arisen from one or more of the following causes:

a. Thermal noise in the input resistor;
b. Current noise in the input resistor;
c. Amplifier and circuit noise;
d. Shot noise arising from anode current electrons;
Fig. 9. Total Correction Factor Vs Frequency
e. Shot noise caused by discrete pulses arising from photoemission or thermal emission from the cathode.

The sources of noise are best discussed separately.

(a) The magnitude of the thermal noise can be calculated using the familiar formula of Nyquist:

\[ E^2 = 4RKT\Delta f \]

where

- \( E \) = rms voltage
- \( R \) = resistance, in ohms
- \( K \) = Boltzmann constant
- \( T \) = Resistor temperature (° Kelvin)
- \( \Delta f \) = frequency bandpass.

For the present case \( T = 300° K \), \( R = 0.62 \times 10^6 \) ohm, and \( \Delta f = 0.64 \) cps. The values give a noise of the order of 0.08 microvolt. This value is insignificant compared to the measured noise, indicating that the thermal noise is contributing essentially nothing.

(b) and (c) The above argument is exactly correct only when the resistor is not carrying any current, since another phenomenon, current noise, may add to the noise signal. Current noise arises from slight fluctuations in the value of a resistor when the resistor carries a current. When a dc signal from a battery equal to that given by the photomultiplier was applied to the input resistor of the amplifier, the noise output was so low that a reliable reading could not be made. Current noise is therefore negligible.

Furthermore, this measurement also indicates that the amplifier and associated circuitry are not
responsible for the noise.

(d) and (e) Shot noise is now left as the remaining possibility. This noise arises because of the random arrival of discrete pulses at a measuring device. When applied to the case of discrete electrons in a current flow, it is often called the Schottky effect. The analysis of the effect, in which one assumes pulses of charge $M$ and arrival rate $K$, leads to the expression:\cite{goldman}

$$I^2 = 2M^2K\Delta f,$$

where $I$ represents the root mean square current fluctuations in the bandpass $\Delta f$. If one considers a pulse to be a single electron, this expression becomes the Schottky formula: $I^2 = 2eIT\Delta f$, where $I$ is the average dc current and $e$ is the charge on the electron.

For the current measured in the case under discussion, the noise caused by discrete electrons in the anode current is of the order of $2.2 \times 10^{-13}$ amperes, corresponding to a voltage of 0.14 microvolt. Again this is too small to be a major contributing factor to the measured noise.

The shot noise expression can be written with $M$, the pulse height, replaced by "me" where $m$ is the multiplication of the phototube and $e$ is the charge on the electron. The product $me$ is the average pulse height at

the anode caused by a single emission at the cathode. With this substitution and noting that meK is the average current \( \overline{I} \), the general shot noise expression becomes

\[ I^2 = 2me\overline{I}Af. \]

The randomness in the electrical pulses is determined by the randomness in the arrival of "effective" photons at the cathode. In the present case, for a 1P21 photomultiplier operated at 38 volts per stage, the RCA tube manual indicates an average multiplication, \( m \), of the order of 3,000. Calculating the noise current gives

\[ I = 1.16 \times 10^{-11} \text{ ampere}. \]

This corresponds to a noise voltage of 7.19 microvolts and agrees with the measured noise. For the sake of completeness, it should be mentioned that the contribution of the dark current amounts to only 0.01 microvolt in this case.

Since all the noise sources which were considered were random, with the possible exception of instrumental noises, they add quadratically. Thus, all but the shot noise associated with the arrival of photons at the cathode are, in effect, negligible. To emphasize this point it can be stated that the measured noise in properly functioning photomultiplier equipment is predominately a function of the photon arrival rate.

The agreement between the noise calculation and observation has been checked at other frequencies and illumination levels; the agreement persists.

In order to correct the scintillation observations
for the shot noise effect, that is the noise which would be present in the star signal if no scintillation were present, a dc operated lamp, adjusted to the mean level of the object being measured, is employed to generate a noise signal to be used in correcting the raw noise measurement for the shot effect. Since the signals are random, they add in quadrature, the correction being made as indicated on page 11. The shot noise becomes the predominant effect in the noise signal in the higher frequencies where the scintillation drops to zero or at least values considerably less than those of the shot effect.

5. ACCURACY OF EQUIPMENT AND TECHNIQUE

Another point of discussion needed before the presentation of the data is the one of what length signal is necessary to remove the statistical fluctuations in the noise signal to a point where the average value becomes stationary and meaningful as a measure of the noise voltage at a given frequency. Theory predicts that the averaging time should be long compared to the reciprocal of the noise frequency or of the bandwidth being covered, whichever gives the largest value. For the lowest frequency observed, 2.5 cps, the first criterion gives a value of about one minute if a factor of 100 times the period is adopted. Since the bandwidth
is in the neighborhood of 0.05 cps for a center frequency of 2.5 cps, the second criterion gives a value of the order of 10 minutes or greater. Experimentally, the average settles down to a variation of about \( \pm 1\% \) after a 10-minute interval, and to within \( \pm 3\% \) after 4 minutes. This was determined by observing the output of the analyzer at a frequency setting of 2.5 cps while using a signal derived from the standard source as the input.

A further check on this effect plus one on the calibration of the entire system in the lower frequency ranges, where the greatest difficulty is usually encountered, is illustrated in Fig. 10. Two four minute signals were impressed upon the magnetic tape from the standard source adjusted to two levels, the first giving 0.535V and second giving 0.487V. The per cent sine wave modulation measured is very small, of the order of 0.05\%, when compared to the values of about 1\% or greater derived from stellar sources in this range. As the curves indicate, there is no pronounced preference for any frequency. The average deviation from the mean value falls within \( \pm 5\% \). These deviations are considerably greater than those indicated above. This is due mainly to the fact that the light source used for the latter case was much weaker and therefore gave rise to greater statistical fluctuations. The observing time in this case should have been of the order of 10 minutes to gain
Fig. 10. Standard Source Noise Analysis.
equivalent results.

Since the noise sources used for the above discussion were battery-operated lamps, the question of the scatter in the case of stellar scintillation is still open. This was checked by recording a star signal for a 13-minute period. The star was Vega at the zenith and the full 12.5 inch telescope aperture was used. The signal on playback was analyzed by two-minute intervals, giving the result shown in Fig. 11.

Numbering of the points in time sequence has not been attempted. It suffices to say that the deviations from the mean in the measures do not all change with the same sense for all frequencies at any one instant. The root mean square deviations about the average value at a given frequency is of the order of 10%, while the spread is as high as 30% in some cases. This is much larger than would be expected from the measures on the standard source and certainly must be attributed to the scintillation signal and not to the equipment or the reduction techniques.

As is evident from Fig. 11 the higher frequencies show less variability than the low frequencies. Furthermore, if larger bandpasses are used, the fluctuations decrease. For example the average per cent sine wave modulation per unit bandwidth when taken over the range from 2.5 to 9.5 cps is found to be 0.82% with a root
Fig. 11. Scintillation Frequency Analyses for Six Consecutive Two-Minute Intervals
mean square deviation of 4.2% as compared to the deviation of about 10% for single points within this range.

Before setting the limits of accuracy of the system the reproducibility of results using the tape recorder should be investigated. Fig. 12 shows the reproducibility of scintillation signals by the recorder. The first tracing is the output of the analyzer when the signal from the phototube was presented directly to it. A smaller time constant than usual was used on the output side of the analyzer in order to show all fluctuations in the scintillation. The analyzer was set at 35 cps. A magnetic tape recording of the scintillation was made simultaneously along with this analysis. On playback to the analyzer the recorded signal gave the second tracing. Although the levels of the two tracings are not identical, because of the difficulty in adjusting the output of the recorder to be exactly the same as the original signal, the two tracings show almost identical fluctuations, indicating a faithful reproduction of the star signal by the tape recorder. The reproducibility of results is further demonstrated in Fig. 13, in which is plotted the analysis of the same recording made at two separate times and with slightly different playback conditions. The root mean square deviation of the per cent equivalent sine wave modulation between the two curves for all frequencies measured was 2.40%. In other words the
Fig. 12. Comparison of Direct and Recorded Scintillation Signal
Fig. 13. Comparison of the Results of Two Separate Frequency Analyses Made on the Same Recorded Scintillation Signal
equipment and reduction technique combined are capable of determining a single point to about ±2.5%.

It is therefore possible to conclude from the experiments on the reproducibility of results and those on the standard source that a single point on the frequency curve can be determined with an accuracy of about ±3% using a 4-minute signal. But this point is representative only of that particular 4-minute period. That is to say, since scintillation itself is variable, that point is determined only to within ±10% as far as the scintillation is concerned. This latter value depends on the particular night of observation, that is, upon the "steadiness" of the scintillation itself. If the area under the curve is considered, or if the average per cent equivalent sine wave modulation is computed over an extended range of frequencies, the result is determined to about ±4%.

In summarizing, it can be said; the results have an internal consistency of about ±3% while the external consistency can be about ±10% or ±4% on a given night, depending upon whether single frequency measures are considered or whether extended ranges are considered, respectively. Thus it appears safe to conclude that the errors present in the equipment and the reduction technique combined are well within the limits set by the scintillation itself.
III. MANNER OF PRESENTATION AND ACCURACY OF RESULTS

The final product of the reduction described in Section II is a scintillation versus frequency curve in which the per cent equivalent sine wave modulation per unit bandwidth is given as a function of frequency. As a general procedure, this curve is determined from 17 readings at fixed frequencies taken on a single 4-minute record of a scintillating star. An example of such a curve is shown in Fig. 14.

Detailed analysis and interpretation of a great number of these curves in their original form would be extremely difficult. It, therefore, becomes helpful to define various parameters of the curve which are more easily handled in a statistical treatment. These parameters, if possible, should describe some unique characteristic of the curve. There are, at least in the first simplification, three primary characteristics of a scintillation curve: the amount of scintillation, the shape of the curve, and the frequencies present in the scintillation. The last two characteristics of the curve are the most difficult to separate and may be lumped into a category which can be called the type of scintillation.

1. DEFINITION OF SCINTILLATION PARAMETERS

The amount of scintillation at each frequency is given directly by the curve. In order to simplify the
% EQUIVALENT SINE WAVE MODULATION
VS FREQUENCY
VEGA
8/27/53
12.5'' APERTURE 00:47 E.S.T.
$\varphi = 45.4^\circ$

Fig. 14. A Representative Frequency Analysis of Stellar Scintillation
description of the amount of scintillation, the frequency range covered has been broken into three subranges; 2.5 to 10.5 cps, 10 to 50 cps, and 50 to 450 cps. The average per cent equivalent sine wave modulation per unit bandwidth is computed as an index of the amount of scintillation in each of the frequency subranges. The average over the full range 2.5 to 450 cps is also computed and is referred to as the average total scintillation. It should be pointed out at this time that total refers to the frequency range normally measured in this work and not to the total frequency range possible in the scintillation noise signal, since components in excess of 450 cps are sometimes present in the scintillation signal. For a good part of the measurements made, however, the scintillation has either decreased to a small value or has disappeared completely at 450 cps, especially for large telescope apertures, so that the average total scintillation can for all practical purposes be considered as the total, over all scintillation frequencies. In the presentation of the results, whenever the total, over all frequencies, is indicated the scintillation measures have been extrapolated and averages computed on this basis.

Since the scintillation is a noise signal, that is, shows no fixed phase relationship between frequency components, the amount of scintillation cannot be computed
directly from the scintillation curves but must be computed from the square of these curves. In other words, the averaging is made, not with respect to "amplitude" but with respect to "power". The square root of the "power" over a specified range gives the root mean square deviation of the signal in that range. The average per cent equivalent sine wave modulation in a given range can be converted to root mean square deviation of the signal in that range by multiplying it by the square root of the frequency interval covered. The average values were used in order to keep the figures for all frequency ranges about the same order of magnitude. A per cent equivalent sine wave modulation per unit bandwidth of 1% over a frequency range from 50 to 450 cps results in a much larger rms value over the entire range than does a like value over a frequency range from 2.5 to 10.5 cps.

The various average per cent equivalent sine wave modulation values are symbolized by $\bar{M}(\%)_{2.5-10.5}$, $\bar{M}(\%)_{10-50}$, $\bar{M}(\%)_{50-450}$, and $\bar{M}(\%)_{\text{total}}$ where the subscripts refer to the frequency range and the bar designates average. If the bar is deleted, the total modulation or the root mean square deviation for the frequency range indicated by the subscripts is inferred.

The type of scintillation can be characterized in two fashions, both of which depend upon the shape of the curve and the frequency content of the scintillation.
Mikesell has characterized the shape of the curve by two parameters; the frequency at which the scintillation begins to drop off and the frequency at which the scintillation signal is lost in shot noise. A variation of these two frequency determinations is shown in Fig. 14. An average line is drawn for the low-frequency components of the scintillation, and another for the mid- and high-frequency components, neglecting the slowly changing tail which seems to be a characteristic of most of the scintillation curves. The intersection of the two average lines determines the "rollover frequency", which is similar to Mikesell's "drop-off parameter"; the intersection of the higher frequency average and the zero axis determines the "crossover frequency". Determination of these frequencies, however, depends to a certain extent upon the manner in which the scintillation curves are plotted. All values given in this report were read from semilog plots similar to Fig. 14.

A more precise determination of the shape of the curve can be gained by defining scintillation moduli. These are simply the ratio of the average per cent

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15 Mikesell, A. H., "Scintillation At Very Low Frequencies and At Different Seasons of The Year," paper presented at the Symposium on Stellar Scintillation and Astronomical Seeing held at The Ohio State University, October, 1953. An abstract of this paper appears in Quarterly Report No. 9 issued under Project 480, Ohio State University Research Foundation, Contract No. AF19(604)-41.
equivalent sine wave modulation value in the midfrequency range, 10-50 cps, and of the value in the high-frequency range, 50-450 cps, to the value in the low-frequency range, 2.5-10.5 cps. In symbols these moduli are represented as:

$$\text{mid-frequency modulus} = \text{Mod}_{10-50} = \frac{\overline{M}(\%)}{\overline{M}(\%)}_{10-50}$$

$$\text{and high-frequency modulus} = \text{Mod}_{50-450} = \frac{\overline{M}(\%)}{\overline{M}(\%)}_{50-450}$$

The numbers so derived will be independent of the fashion in which the data are plotted. Since the separation between frequency content and shape of the curve is not complete, if such could ever be the case, both the rollover and crossover frequencies and the scintillation moduli will be considered indices of the type of scintillation, the scintillation modulus being a preferred index because of greater precision of determination.

2. SCINTILLATION SCATTER AND ACCURACIES

The possibility that one or more of the parameters defined above might show less scatter than the others has been investigated. For example, in the discussion of the equipment it was shown that the higher frequencies
tend to show less scatter over a thirteen-minute interval of time than do the low frequencies. (See Fig. 11). It may well be asked whether this scatter becomes greater when longer periods of time are utilized. To answer this question, a sequence of observations were made on Polaris, the only bright star available which shows negligible diurnal motion and consequently can be studied without introducing a changing path length through the earth's atmosphere. Fig. 15 shows the results for one night of a sequence of 11 standard observations (4-minute signal) made at 10-minute intervals. The values of $\overline{M}(\%)$ for the ranges 2.5 to 10.5, 10 to 50, 50 to 450 and the total range are shown versus time. This plot again indicates that the strength of the low-frequency components can be more variable than that of the high-frequency components. It is evident from the plot of the Polaris observations that over short periods of time, of the order of 5 or 10 minutes, the scintillation can remain quite steady. An arbitrarily selected observation could lead to rather erroneous results. Thus, since large peak deviations occur, 21% for the low frequencies, a peak-to-peak deviation of 42% is indicated, that is, if the lowest value in this two-hour sequence were compared to the highest value, the conclusion would be that the scintillation had almost doubled. This, of course, is erroneous, if the mean condition is under study. In fact with such
Fig. 15. A Comparison of the Amount of Scintillation for a Sequence of Observations on Polaris
large fluctuations it would not be out of the question to ask whether a single observation on such a variable quantity can characterize the mean condition at all. Fig. 16 shows the results for type of scintillation criteria for the same data.

This study was made for a total of 17 nights, all other observing sequences except the one depicted in Fig. 15 and 16 consisted of 5 determinations made over an interval of one hour. The results of these observations are shown in Table I.

**TABLE I**

Summary of Results of the Study of the Variability of Scintillation

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Minimum rms Deviation</th>
<th>Average rms Deviation</th>
<th>Maximum rms Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{M}_{2.5-10.5}$</td>
<td>2.3%</td>
<td>3.3%</td>
<td>14.7%</td>
</tr>
<tr>
<td>$\bar{M}_{10-50}$</td>
<td>3.4%</td>
<td>9.5%</td>
<td>26.0%</td>
</tr>
<tr>
<td>$\bar{M}_{50-450}$</td>
<td>3.9%</td>
<td>15.9%</td>
<td>41.0%*</td>
</tr>
<tr>
<td>$\bar{M}_{\text{total}}$</td>
<td>4.5%</td>
<td>10.3%</td>
<td>20.3%</td>
</tr>
<tr>
<td>$\text{Mod}_{10-50}$</td>
<td>2.1%</td>
<td>6.7%</td>
<td>16.6%</td>
</tr>
<tr>
<td>$\text{Mod}_{50-450}$</td>
<td>0.0%</td>
<td>13.5%</td>
<td>41.1%*</td>
</tr>
</tbody>
</table>

This table is based on 17 sequential observations on Polaris. The starred quantities occurred on a night in which a progressive change in the scintillation occurred thus giving an abnormally high reading. More realistic values would be 28.3% for $\bar{M}_{50-450}$ and 21.9% for $\text{Mod}_{50-450}$. 
Fig. 16. A Comparison of the Shape of the Scintillation Curve for a Sequence of Observations on Polaris
On the basis of these results it appears possible to raise the probable error in a single scintillation measurement above that indicated in Section II, part 5. The probable error is of the order of 10% rather than the 4% quoted earlier for a single extended observation (10 minutes). The shape of the curve as evidenced by the $\text{Mod}_{10-50}$ shows the least scatter and consequently is the best quantity to use if an attempt to correlate scintillation with atmospheric phenomenon is made using single scintillation measurements of normal length (4 minutes) for the scintillation parameter.

It is of considerable interest to note that no significant trends between the magnitude of the scintillation and the amount of scatter can be found. That is to say, it is not possible to estimate the amount of scatter to be expected from the magnitude of a single scintillation measurement. Consequently applying the average scatters noted above to any scintillation measurement is exceedingly dangerous since the true scatter is a parameter of that particular night and not necessarily of either the amount or type of scintillation.

It is also certain that the scatter to be expected for observations made using smaller apertures will be greater. The above results were determined for a 12.5 inch aperture in order to give the most favorable picture of scintillation scatter and thereby give a lower bound
to be expected for accuracy of a single observation.

3. EFFECTS OF NONUNIFORMITY OF THE PHOTOCATHODE

Another important point which must be clarified before presenting the observations concerns the uniformity of response of the photocathode surface. The fact that photocathodes show irregular response with microscopic changes in position over the surface is well known. Indeed this is precisely the reason that a Fabry lens is used to spread the image over a larger cathode area and consequently average out this effect. Photocathodes also show a progressive change in sensitivity with microscopic position changes across the surface. The 1P21 photomultiplier has a cathode which has a maximum sensitivity along the edge which is closest to the first dynode. The sensitivity seems to remain relatively constant along lines parallel to this edge. The effect of the grid wires across the short dimension of the cathode, however, causes the sensitivity to show large deviations of comparatively narrow width along this direction of "constant" response. The RCA 6199 "end on" photomultiplier also shows a similar, though apparently smaller, change in sensitivity with position along a radius from the cathode center. The grid wire effect is absent since the focusing grid is behind rather than in front of the cathode.
The macroscopic changes in sensitivity, on the other hand, could give trouble in that the effective photo-electric aperture of the telescope would not necessarily agree with the physical aperture. This results from the fact that a photometer using a Fabry lens sets up a one-to-one relation between a specific elemental area on the cathode and on the objective of the telescope. Thus the "effective" aperture of the system would be a function of the integral of the weighted responses taken over the proper area. This need not be the same as that which would result from a completely uniform surface under otherwise identical conditions. Such an effect, if significant, would invalidate to a considerable extent the study of the effect of aperture size upon scintillation.

The effect of the nonuniformity of the cathode upon these measures has been investigated in a set of observations represented by Figs. 17 and 18. Figure 17 shows a comparison between the response of the photometer (using a 1P21 tube) and the expected response as the aperture size is changed while looking at a constant blue sky. The photometer was placed in two positions, one position was a rotation about the optical axis of 90° from the other. The measured values and predicted values agree within 6% on the average. Considering the great range in illumination level, a factor of 156 for the
Fig. 17. The Effect of Nonuniformity of a 1P21 Cathode Upon the Relation Between Cell Output and Surface Area Used for Constant Illumination Level
Fig. 18. An Experimental Test of the Effect of Nonuniformity of the Photo-Cathode upon Scintillation Measurement.
1-inch to 12.5-inch aperture, this agreement is very satisfactory.

It is still possible, on the other hand, that the nonuniformity has fortuitously been averaged for the dc level while the nonuniformity will still change the effective "scintillation aperture". Figure 18 shows the results of an experimental test, which shows that this is not the case. The maximum and minimum gradients of the photosensitivity changes for the 1P21 photomultiplier were first found. Then a 1-inch by 12-inch aperture was placed over the telescope objective. The photometer was next oriented in two positions, one with the slit along the maximum gradient then along the minimum gradient. Two observations were made on Vega with the photometer in each position. Since these are the two most extreme conditions under which the nonuniformity of the cathode could operate, any effect present should certainly become evident. The averages of the two maximum and minimum gradient observations plotted in Fig. 18 show no significant differences. In fact, there is less than 1% difference in the total scintillation between the two observations. Therefore nonuniformity of the cathode introduces effects in the scintillation curves of a magnitude less than that caused by the normal scatter in the scintillation as a function of time.

The nonuniformity effects should be even further
reduced for the 6199 phototube since there the surface sensitivity change is less and tends to show circular symmetry. Consequently no distinction is made in the presentation of the results between measures made with the 1P21 tubes and 6199 tubes. Furthermore on the basis of these measures the "effective" scintillation aperture has been taken to be the same as the physical aperture used.
IV RESULTS

1. PRELIMINARY DISCUSSION

The phenomenon of scintillation depends upon a number of parameters. This number is large enough that, even with the 635 observations available for this discussion, the grouping of the data for the purpose of isolating variables must be relatively coarse in order to have a statistically significant sample. Furthermore the parameters are not all independent. For example, it is demonstrated later that the relationship between scintillation and zenith distance varies with meteorological conditions; namely, with the wind velocity around the 200 mb level.\textsuperscript{16} Thus, if all observations are lumped together in order to reduce the effects of the inherent scatter already noted, another type of scatter is introduced, since observations with a variety of wind velocities would then be arbitrarily grouped together. There are not a sufficient number of observations available on days having any one particular wind velocity to permit an accurate determination of the zenith distance effect as a function of wind.

\textsuperscript{16}The 200 millibar pressure level occurs at an average height of 40,000 ft. in the earth's atmosphere and lies approximately at the average location of the tropopause.
Stellar scintillation has been defined as the fluctuation in the intensity of the light of a star measured with conventional telescopic optics regardless of the distribution of the light of the star in the image field of the telescope. Scintillation may also be defined in terms of the so-called "shadow bands".

The objective of the 69-inch reflector of the Perkins Observatory photographed\textsuperscript{17} in the light of the star, Capella, is shown in Fig. 19. It is evident that the distribution of the light of the star falling upon the objective is not uniform. It is the variation in this distribution which gives rise to the fluctuation of the stellar intensity as measured in the focal plane of the telescope. It is clear from Fig. 19 that the percent variation in the integrated light of the star will decrease with larger apertures because of the averaging effect of the aperture upon the pattern. The high-frequency components in the signal should decrease faster than the low-frequency components in the signal since the averaging effect of the aperture will be more pronounced for the small-dimensioned fluctuations in the shadow band pattern.

Studying the shadow band pattern directly is a problem of considerable difficulty. It is much simpler,

\textsuperscript{17}These photographs were taken by Mr. Roger Hosfeld.
PHOTOGRAPHS OF THE 69 INCH PERKINS TELESCOPE TAKEN IN THE LIGHT OF CAPELLA

Fig. 19. Shadow Band Patterns
and just as valid, to study scintillation after the fluctuations of the pattern have been integrated by the telescope. The integrated fluctuations are represented by Fig. 20, which presents oscillograph photographs of samples of the output of the tape recorder for recordings of the scintillation given by a star at the zenith as viewed by 12.5-, 3-, and 1-inch apertures. The $M(\%)$ value or the rms deviation of the signal, shown on the diagram, was determined from the frequency analysis of the entire signal (4-minute duration for each aperture). The peak-to-peak variation compared to the mean level is computed by means of the ac calibration signal on the tape and the mean level of the signal as measured on the dc level recorder, since the mean level cannot be determined directly from the magnetic recordings of the scintillation. These photographs show that the frequency components increase with decreasing aperture; the $M(\%)$ and peak-to-peak deviation computations show that the amount of scintillation likewise increases.

Similar photographs for a constant aperture of 3 inches with varying zenith distance are shown in Fig. 21. They show a decrease in high frequency components and an

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18 The amplitudes shown in Fig. 21 do not give a direct comparison of the true scintillation amplitudes since the recording level used with the magnetic tape recorder is always adjusted to give approximately the same recorded amplitude for all scintillation signals.
### SCINTILLATION VS APERTURE

(0.3 SECOND SAMPLE)

<table>
<thead>
<tr>
<th>APERTURE</th>
<th>PEAK TO PEAK/D.C.</th>
<th>M(%) TOTAL</th>
<th>ZENITH DISTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5&quot;</td>
<td>16%</td>
<td>4.2%</td>
<td>1.5</td>
</tr>
<tr>
<td>3&quot;</td>
<td>40%</td>
<td>15.6%</td>
<td>2.7</td>
</tr>
<tr>
<td>1&quot;</td>
<td>102%</td>
<td>29.8%</td>
<td>3.6</td>
</tr>
</tbody>
</table>

VEGA: 8/25-26/53

Fig. 20. Oscilloscope Tracings of Stellar Scintillation for Various Apertures
SCINTILLATION VS ZENITH DISTANCE

(0.3 SECOND SAMPLE)

Fig. 21. Oscilloscope Tracings of Stellar Scintillation at Various Zenith Distances

<table>
<thead>
<tr>
<th>ZENITH DISTANCE</th>
<th>8.5</th>
<th>70.6</th>
<th>79.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SECANT Z</td>
<td>1.01</td>
<td>3.01</td>
<td>5.39</td>
</tr>
<tr>
<td>PEAK TO PEAK/D.C.</td>
<td>68%</td>
<td>125%</td>
<td>213%</td>
</tr>
<tr>
<td>M (%) TOTAL</td>
<td>30.0%</td>
<td>38.9%</td>
<td>51.2%</td>
</tr>
<tr>
<td>STAR</td>
<td>CAPELLA</td>
<td>RIGEL</td>
<td>RIGEL</td>
</tr>
</tbody>
</table>

3" APERTURE: 12/2-3/53
increase in the amount of scintillation with increasing zenith distance.\textsuperscript{19}

The oscilloscope photographs display quickly to the eye the qualitative features of scintillation, the quantitative features are best examined by the frequency analyses.

2. THE ZENITH DISTANCE EFFECT

One of the most obvious observations which can be made on scintillation is that stars close to the horizon scintillate more than those overhead. This is certainly evident to the eye. The complete description of this effect, however, is not simple. The manner in which the scintillation varies with the secant of the zenith angle \( Z \) depends upon the aperture, the frequency range under consideration and meteorological factors. The latter, which is discussed in detail elsewhere in the report (page 84), complicates the determination of the zenith distance effect considerably.

Figures 22 and 23 show the cumulative plot of \( \log \bar{M}(\%)_{2.5-10.5} \) and \( \log \bar{M}(\%)_{\text{total}} \) for the 12.5-inch aperture.

\textsuperscript{19}The peak-to-peak variation increases more rapidly than the rms deviation. Since the high frequency components decrease more rapidly than the low frequency components it becomes evident that the peak-to-peak deviation is roughly a measure of the low frequency components of the signal.
Fig. 22. The Variation of Low-Frequency Stellar Scintillation With Zenith Distance for A 12.5-Inch Aperture
Fig. 23. The Variation of Stellar Scintillation Taken Over All Frequencies With Zenith Distance for A 12.5-Inch Aperture
versus log sec Z, respectively. The data can be approximated by two straight lines for both the $M(\%)_{2.5-10.5}$ and $M(\%)_{total}$. A change in slope can be seen at a value of log sec Z around 0.4 ($Z \approx 65^o$) in both cases.

Approximate functional relationships between $M(\%)$ and sec Z of the form $a(sec Z)^n$ follow:

\begin{align*}
M(\%)_{2.5 \ to \ 10.5} & = 1.0 \ (sec \ Z)^{1.6} \quad Z < 65^o \\
& = 3.6 \ (sec \ Z)^{0.1} \quad Z > 65^o
\end{align*}

and

\begin{align*}
M(\%)_{total} & = 0.3 \ (sec \ Z)^{1.5} \quad Z < 63^o \\
& = 1.0 \ (sec \ Z)^{0.0} \quad Z > 63^o
\end{align*}

The other frequency ranges, 10 to 50 and 50 to 450 cps, are so strongly affected by the meteorological conditions when treated in this manner that they give almost meaningless results. The mid-frequency range does show behavior somewhat similar to the low-frequency range except that the change in slope is more pronounced and occurs at a smaller zenith distance, the slope becoming negative with large Z. The high-frequency range shows a decrease with zenith distance throughout almost the entire span of zenith distance.

Examination of the zenith distance effect when allowance is made for meteorological conditions indicates
that the powers of $\sec Z$ given above are of the correct order of magnitude, the power up to $Z = 65^\circ$ should be around $3/2^{20,21}$.

Similar studies were made for apertures of 6, 3 and 1 inch. The cumulative plots of the 3-inch observations are shown in Figs. 24 and 25 in a fashion similar to those for the 12.5-inch observations. These plots demonstrate the effect of the smaller apertures upon the secant $Z$ relationship. The amount of scintillation is increased but the overall rate of change with zenith distance is in general decreased. The corresponding results for the 6-, 3- and 1-inch apertures are:

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20. Megaw, from an analysis of observations made by Eilison, has found a slope of $3/2$ for smaller values of secant $Z$ with a change of slope to $1/2$ and less with larger secant $Z$. The observations used were expressed in terms of peak deflections of the scintillation using a 15-inch aperture and consequently refer to the low-frequency components. "Interpretation of Stellar Scintillation," Quarterly Journal of the Royal Meteorological Society, 80, 248, April, 1954.

21. Hosfeld, using a technique which measures the scintillation components from 0 to 10 cps, has found an exponent of 1.4 for a 12.5-inch aperture. See Progress Report No. 5, Project 480, Ohio State University Research Foundation, Contract No. AF 19(604)-41.
Fig. 24. The Variation of Low-Frequency Stellar Scintillation With Zenith Distance for A 3-Inch Aperture
Fig. 25. The Variation of Stellar Scintillation Taken Over All Frequencies With Zenith Distance For A 3-Inch Aperture
The point at which the curve changes slope occurs slightly earlier in the run of zenith distance with decreasing aperture. Also $\overline{M}(\%)_{\text{total}}$ becomes almost constant with secant $Z$ for small apertures.
The values given above represent a first order attempt at the description of the zenith distance effect upon the amount of scintillation in that wind velocity effects are neglected. It is also possible to study the effect of zenith distance upon the moduli and the crossover and rollover frequencies. These quantities are even more strongly affected by meteorological factors. In general, however, it is found that the moduli decrease for larger zenith distances. The crossover and rollover frequencies also decrease with increasing zenith distance. These two effects are expected on the basis of the $M(\%)$ relationships mentioned above, and show that as the zenith distance increases the high-frequency components decrease while the low-frequency components increase. A plot of the zenith distance effect upon the moduli and crossover and rollover frequencies is given in the next section where a partial separation of the meteorological factors has been made.

3. SEASONAL VARIATION OF SCINTILLATION

A seasonal variation in scintillation was first discovered by Mikesell. Mikesell found that the crossover frequencies were in general higher in winter than in summer. This effect is demonstrated in Figs. 26 and 27

\[22\text{ Mikesell, A. H., loc. cit.}\]
Fig. 26. The Seasonal Effect Upon the Crossover Frequencies for A 12.5-Inch Aperture
Fig. 27. The Seasonal Effect Upon the Rollover Frequencies For A 12.5-Inch Aperture
where the rollover and crossover frequencies for the 12.5-inch observations are plotted versus $Z$. The open circles are observations made from October, 1953 through May, 1954, called the "winter" period and the solid circles are observations made during the "summer" period, or from June through September. A rough separation of the winter from summer observations occurs, becoming best in the case of the rollover frequencies.

The same effect holds true for the smaller apertures as indicated in Figs. 28 and 29 where the 3-inch aperture observations are depicted. The scatter and possible intermixing of values for the two "seasons" becomes greater for smaller apertures. The crossover frequencies can exceed 1,000 cps for apertures of smaller size.23

The moduli likewise tend to show a separation into two seasonal groups. A complete separation by season is not to be expected since, as is demonstrated later, the effect depends basically on wind velocities and hence only secondarily on season.

Since the shape of the scintillation curves appear to show a seasonal variation it is probable that the total amount of scintillation is likewise affected. That this is true is demonstrated by the Figures in the

23Values of the crossover frequency in excess of 450 cps are found by extrapolation where possible.
Fig. 28. The Seasonal Effect Upon the Crossover Frequencies for a 3-Inch Aperture
Fig. 29. The Seasonal Effect Upon the Rollover Frequencies for a 3-Inch Aperture
Appendix, where a cumulative plot of frequency analyses broken down by "season" and zenith-distance intervals is given. The total scintillation is greater in the winter, on the average, although the lower frequency components are greater in the summer. The scatter in the amount of scintillation also shows a greater spread in winter than in summer. The average amounts of scintillation for \(1 \leq \text{sec } Z < 1.5\) are given in Table II for various apertures.

**TABLE II**

<table>
<thead>
<tr>
<th>Aperture (inches)</th>
<th>Season</th>
<th>(\bar{M}(%)) (^{24})</th>
<th>Crossover frequency (cps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
<td>summer</td>
<td>8.2</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>winter</td>
<td>9.8</td>
<td>315</td>
</tr>
<tr>
<td>6</td>
<td>summer</td>
<td>16.1</td>
<td>180</td>
</tr>
<tr>
<td></td>
<td>winter</td>
<td>18.9</td>
<td>340</td>
</tr>
<tr>
<td>3</td>
<td>summer</td>
<td>24.8</td>
<td>205</td>
</tr>
<tr>
<td></td>
<td>winter</td>
<td>34.2</td>
<td>405</td>
</tr>
<tr>
<td>1</td>
<td>summer</td>
<td>36.8</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>winter</td>
<td>47.1</td>
<td>700</td>
</tr>
</tbody>
</table>

The average crossover frequency is roughly doubled between summer and winter. It should be pointed out in passing that the crossover frequency defined earlier is actually a fictitious frequency in that it does not.

\(^{24}\)All frequencies were used from 2.5 to "infinity", rather than 2.5 to 450 cps, by extrapolating the scintillation curves.
really give the intersection of the scintillation curve with the zero axis. It ignores completely the tail on the end of the curve where the scintillation is gradually lost in the shot noise.

While the separation into winter-summer-scintillation is a helpful one, seasonal differences are not the basic cause of the variations in scintillation. This is clarified in the section on the correlation of scintillation with upper air winds.

4. SLIT OBSERVATIONS

An obvious question, when thinking of scintillation in terms of the shadow band pattern, is to ask whether the pattern has any preferential motion. Direct photography of the pattern does not seem to yield the answer conclusively, possibly because of the transitory nature of the pattern. Increasing the rate at which pictures of the pattern are taken, in an attempt to freeze any motion leads, unfortunately, to a decrease in the amount of light available and, consequently, to photographs which cannot be properly analyzed.

The problem of pattern motion can be studied photo-electrically. The technique utilizes a slit aperture.

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25Mikesell, Hoag and Hall, loc. cit.

26See Progress Report No. 2, Project 583, Ohio State University Research Foundation Contract No. AF19(604-964.
which can be rotated. The effect of an 80° rotation of a 1 x 12 inch slit on the scintillation curve is shown in Fig. 30.

In one position of the slit, the curve shows the character of a large aperture observation; in the slit position at roughly 90° from this one, the curve shows a character more like that for a small aperture observation. This can be explained in terms of pattern motion. If the slit is along the motion, the small elements of the shadow band pattern, which give rise to the high frequencies, suffer from the aperture integration effect with a subsequent loss in high-frequency components. When the slit is across the pattern motion, the integration effect is greatly lessened for the small elements; consequently more high-frequency components are generated.

The process of determining that slit orientation which coincides with the pattern motion is too time-consuming when the entire frequency spectrum is used. A simpler technique is, therefore, applied. The output of the signal amplifier is presented directly to the analyzer without the tape recorder. The strength of the scintillation component at 200 cps is measured using a wider bandpass, 16 cps, than is normally used in order to give a more stable reading. The slit is then rotated from 0°, corresponding to the N-S direction, to 180° by 22.5° intervals. The approximate slit position giving
% EQUIVALENT SINE WAVE MODULATION

VS

FREQUENCY

1"X12" SLIT

3/2/54

ARCTURUS

157.5

05:43

Z=29.1

78.75

05:49

Z=30.2

Fig. 30. Scintillation Observed With A 1-Inch X 12 Inch Aperture.
the minimum signal at 200 cps is determined. A few more runs are made with the slit position near and about the expected minimal position. Upon plotting of the reduced data the minimal position of the slit with respect to the 200 cps component can be read to about ±5°, depending on the particular shape curve encountered. One such plot is shown in Fig. 31.

The length of the slit in the position giving the minimum signal, if the discussion on the shadow band motion just given is correct, is parallel to the direction of motion of the pattern but does not give the sense of the motion.

5. CORRELATION OF SCINTILLATION WITH UPPER AIR WINDS

Since stellar scintillation arises in the earth's atmosphere it follows that a correlation of scintillation with meteorological parameters is to be expected. The present data show no strong correlation with any of the more obvious surface conditions. In fact, nighttime scintillation appears to be almost independent of surface weather phenomenon. A correlation has been found between the shape of the curve and the wind speed and also between the slit orientations and wind directions at the 200 mb level.

The correlation between slit orientation and wind direction is presented in Fig. 32. The dashed lines
Fig. 31. Variation of the 200 CPS Scintillation Component With Angular Position of A 1-Inch X 12-Inch Slit
Fig. 32 Correlation of Slit Position for Minimum 200 cps Scintillation Component with Wind Direction at the 200-mb Level
Fig. 33  The Relation of Scintillation Moduli to Wind Speed at 200 mb for a 12.5-inch Aperture
represent deviations of $\pm 20^\circ$, which are estimates of the extremal error in reading the wind directions from plotted weather maps. The points, 29 in number, have a correlation coefficient of 0.99 about a line of regression having a slope of $44.2^\circ$ and an intercept of 4.2. This agrees well with the desired line having a slope of $45^\circ$ and a zero intercept. The computed probable error is $\pm 6.7^\circ$. Similar correlations at the surface, 700 mb, and 500 mb levels do not exhibit so good an agreement.

The shape of the curve appears to be related to the wind speed at the 200 mb level. Figure 33 shows a plot of $\text{Mod}_{10-50}$ and $\text{Mod}_{50-450}$ for a 12.5-inch aperture versus the wind speed in knots at the 200 mb level. The moduli are taken for observations within $30^\circ$ of the zenith. The wind speeds are estimated to be good to $\pm 10$ knots since they are measured by a geostrophic wind scale, then corrected on the basis of actual velocities measured at weather stations nearby to Columbus, Ohio such as Dayton and Cincinnati. Similar correlations at other pressure levels are not so favorable, especially when large values of the moduli (around unity) are encountered.

The correlation of moduli versus wind speed also holds for smaller apertures. The three-inch observations are shown in Fig. 34. The relation between the moduli and wind speed is not so pronounced however because of the particular choice of the moduli which has been made.
Fig. 34 The Relation of Scintillation Moduli to Wind Speed at 200 mb for a 3-inch Aperture
The mid-frequency modulus for the three-inch aperture is already near unity even on days showing low velocities and therefore cannot show much change as the velocity increases. The scatter is also slightly greater, a characteristic of the small aperture observations in general. A more advantageously defined modulus could show the wind velocity effect more favorably for the small apertures.

The amount of scintillation, and the crossover and rollover frequencies, also correlate with wind velocity. The strength of the low-frequency components tend to show a decrease with increasing wind velocities while the high-frequency components tend to show an increase. The total scintillation, on the average, also shows an increase with wind velocity. The amount of scatter is much greater when a correlation is attempted between the amount of scintillation and wind velocity, probably caused by the fact that the amount of scintillation is itself more variable than the shape of the scintillation-frequency curves as shown in Section III. This is probably due to the fact that the strength of the turbulence and the wind velocity do not have a one to one correspondence.

Gifford and Mikesell have also correlated

scintillation with upper air wind velocities. They carried their work only up to about 30,000 feet, however. Furthermore, their correlations were made using the amount of scintillation and thus would not be expected to give the best correlation possible.

The fact that correlations, although of less strength, are found at almost all levels in the atmosphere is to be expected. The winds aloft are generally correlated with each other, the greater the separation between levels the poorer the correlation.

The 200 mb level has a feature which makes it physically preferable as the site of the turbulence causing scintillation. The height of this pressure level, around 40,000 feet, is near the mean position of the tropopause. The tropopause, defined as the inversion layer at the top of the troposphere, is quite well suited for the generation of turbulence and has been shown to have considerable turbulence by aircraft crossing it.

The relationship of scintillation to the wind velocity at the 200 mb level can be used to clarify some of the observational results already presented. The seasonal variation falls out almost immediately. The average wind speeds are higher at the 200 mb level over Columbus during the winter months and, therefore, one expects the scintillation to show more high-frequency
components along with a decrease in low-frequency components during the winter as compared to summer. The total scintillation shows an increase. Furthermore, since the winds tend to show a wider range of velocities in the winter than in the summer, an increase in the scatter of the scintillation measures is found. These statements are all confirmed by the seasonal variation results presented earlier. It appears then, that the term seasonal variation of scintillation is, in a sense, a misnomer since the seasonal variation actually is in the wind velocities.

One cause of the scatter in the cumulative plots of the amount of scintillation versus secant Z, presented in the earlier part of this section, can now be explained. Fig. 35 is a linear plot of the low-frequency scintillation for a 12.5-inch aperture versus secant Z for two wind velocity ranges at the 200 mb level. Velocities of 26 to 50 knots are represented by open circles, and velocities greater than 100 knots by solid circles. The figure shows the manner in which the scatter tends to decrease when the data are grouped by wind velocity.

The wind velocity is even more effective in introducing scatter into the relation between scintillation and zenith distance for the high-frequency components. An illustration of this is given in Fig. 36 where
Fig. 35. Comparison of the Low-Frequency Scintillation-Zenith Distance Effect for High and Low Wind Velocity Observations
Fig. 36. Comparison of the High-Frequency Scintillation-Zenith Distance Effect for a Day of High and One of Low Wind Velocity
\(\bar{M}(\%)_{50-450}\) for a 12.5-inch aperture is plotted versus zenith distance. The observations for two nights are given, one night of low velocity, 15 knots, the other of high velocity, 100 knots.

The determination of an exact relationship between the moduli and zenith distance for the data accumulated prior to the present is essentially impossible because of the strong dependence of the moduli upon the wind velocity. This is demonstrated in Fig. 37 where \(\text{Mod}_{10-50}\) is plotted versus zenith distance for the data observed on the same days as in Fig. 36.

A sufficient amount of data is not yet available to permit the scintillation to be expressed as a function of the wind velocity and zenith distance simultaneously.

6. APERTURE EFFECT

It has already been noted that the amount of stellar scintillation is a function of the aperture size. The relationship between scintillation and aperture size is also a function of the zenith distance, the frequency component under measurement, and the wind velocity at the 200 mb level.

The general manner in which the scintillation curve changes with aperture, at the zenith, is shown in Fig. 38. Next the plot of the ratio of the \(\bar{M}(\%)\) in the four standard frequency intervals, for various aperture sizes
Fig. 37. Comparison of the Mid-Frequency Modulus-Zenith Distance Effect for a Day of High and One of Low Wind Velocity
Fig. 38. Variation of the Scintillation Curves With Aperture for Stars Near the Zenith
with respect to the 12.5-inch aperture is shown in Fig. 39. The curves shown represent the relationship between scintillation and aperture size for low wind velocity.

The combined effects of zenith distance and wind velocity upon the ratio of the amount of scintillation for a 3-inch compared to a 12.5-inch aperture is indicated in Figs. 40 and 41. The scintillation ratios for the range 2.5 to 10.5 cps are shown in Fig. 40, and for the scintillation ratios over the total frequency range in Fig. 41.

7. EXTENDED AND MULTIPLE SOURCES

Planets and double stars offer opportunity for another type of observation, that in which the source area is varied as opposed to that in which the aperture, or receiver, area is varied.

The scintillation from Jupiter has been measured on two nights. A comparison of the scintillation for Jupiter and the star Aldebran with the same aperture, taken on August 31, 1953, is presented in Fig. 42. The ratios of the amounts of scintillation of Jupiter to that of a star for this night and the other night, January 15, 1955, upon which the same type of measurements were made, are shown in Table III.
Fig. 39. The Relation Between the Amount of Stellar Scintillation for Various Aperture Sizes Expressed in Terms of the Scintillation From A 12.5-Inch Aperture.
Fig. 40. The Effect of Zenith Distance Upon the Ratio of the Low-Frequency Scintillation Components for a 3-Inch to a 12.5-Inch Aperture for Different Wind Velocities at 200 MB.

RATIO OF LOW FREQUENCY
SCINTILLATION OF 3"
TO 12.5" APERTURE

VS
SECANT Z

M (%) 2.5-10.5

- 26 ≤ V ≤ 50 KNOTS
- V > 100 KNOTS
Fig. 41. The Effect of Zenith Distance Upon the Ratio of the Scintillation Taken Over All Frequencies for a 3-Inch to a 12.5-Inch Aperture for Different Wind Velocities at 200 MB
Fig. 42. A Comparison of Planetary and Stellar Scintillation for a 12.5-Inch Aperture
TABLE III

Comparison of $\bar{M}(\%)_{\text{total}}$ for Jupiter and a Star as a Function of Aperture Size

<table>
<thead>
<tr>
<th>Date</th>
<th>Aperture</th>
<th>1&quot;</th>
<th>3&quot;</th>
<th>6&quot;</th>
<th>12.5&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>8/31/53</td>
<td></td>
<td>0.135</td>
<td>0.040</td>
<td>0.047</td>
<td>0.087</td>
</tr>
<tr>
<td>1/15/55</td>
<td></td>
<td>0.047</td>
<td>0.042</td>
<td>0.049</td>
<td>0.097</td>
</tr>
</tbody>
</table>

It is evident that the amount of scintillation for Jupiter is considerably less than that for a star using the same size aperture. It is possible however to compare the scintillation of Jupiter through a small aperture to that of a star through a large aperture. A plot of the scintillation curves for Jupiter and Aldebran using a 1-inch and 12.5-inch aperture respectively is shown in Fig. 43 for the night of 8/31/53. The comparison for both nights is also presented in Table IV.

TABLE IV

Comparison of the Scintillation of Jupiter for a Small Aperture to that of a Star for a Large Aperture

<table>
<thead>
<tr>
<th>Date</th>
<th>Aperture</th>
<th>$\bar{M}(%)_{\text{total}}$</th>
<th>Mod10-50</th>
<th>Mod50-450</th>
</tr>
</thead>
<tbody>
<tr>
<td>8/31/53</td>
<td>Jupiter</td>
<td>1&quot;</td>
<td>0.206</td>
<td>0.786</td>
</tr>
<tr>
<td>Star</td>
<td>12.5&quot;</td>
<td></td>
<td>0.300</td>
<td>0.438</td>
</tr>
<tr>
<td>1/15/55</td>
<td>Jupiter</td>
<td>1&quot;</td>
<td>0.124</td>
<td>0.823</td>
</tr>
<tr>
<td>Star</td>
<td>12.5&quot;</td>
<td></td>
<td>0.329</td>
<td>0.798</td>
</tr>
</tbody>
</table>
Fig. 43. A Comparison of Planetary Scintillation for a 1-Inch Aperture to Stellar Scintillation for a 12.5-Inch Aperture
On the first night the small to large aperture ratio was 0.69 while on the second night it was 0.38. From Fig. 43 and the moduli given in Table IV it is apparent that although the amount of scintillation for the small aperture observation of Jupiter is comparable to that for the large aperture observation of the star, of the order of 2/3, the relative amount of high frequencies present in the Jupiter observation is greater. On the other night the scintillation for the small aperture observation of Jupiter was considerably less when compared to the large aperture observation of the star. If the aperture were decreased for the Jupiter observation in order to make the scintillation more comparable to that of the 12.5-inch observation of the star, the high frequency content would undoubtedly be increased as indicated by the data presented in the section on the effects of aperture size on scintillation. Since Table IV shows that the high frequency content of the scintillation of Jupiter for a 1-inch aperture is already comparable to that of the star, decreasing the aperture for the Jupiter observation would increase the relative high frequency content over that of the large aperture observation for the star in much the same fashion which occurred on the other night. Thus it appears evident, at least from the limited data available at this moment, that although the scintillation from a planet may be made equal to that of a star by the
simple expedient of decreasing the aperture size used for the planet observation, the shape of the scintillation curves for the planet compared to the star will not be the same, the planet having an excess of high frequency components.

A planetary surface can, of course, be regarded as a continuum of point sources taken over the disc of the planet. Presumably, the lessened scintillation from a planet, that is a continuous surface, as compared to a star can be ascribed to the manner in which the fluctuations in light received from the various elements of the surface combine at the telescope aperture. It would be helpful if pairs of such elements, of various separations, could be isolated and studied in order to determine the manner in which the composition of the fluctuations is effected. Double stars offer opportunities for such observations.

Two double stars of contrasting separations were observed: 6 Serpentis in which the components are separated by 22 seconds of arc, and Castor whose components are only separated by 2.5 seconds of arc. The first star, 6 Serpentis, has a sufficient component separation to permit each component to be measured individually as well as in combination. Component A was observed and then A and B together, using one tape; a second tape was used to record component B and then, A
and B together once again. The average of the analyses for both components observed simultaneously was computed and compared to the combination of the individual component analyses added together on a basis of in-phase scintillation and on the basis of random phase scintillation for the two signals. The contribution of each component was weighted according to its brightness. The per cent scintillation which would result from the in-phase addition of the individual signals is the same as that which would be obtained from a single star having the combined brightness of components A and B. The ratio of the random combination to the in-phase combination for the total scintillation should be 0.743 for this star.

The observed ratio was 0.615. That this value is less than the value given by the random combination of the individual signals can be explained on one of two bases: the inherent scatter in the scintillation is responsible; or some sort of phasing existed between the signals from components A and B. The per cent difference between the two observations of both components together was 2.7%, the per cent difference between the relative scintillation for component A and component B was 5.1%. Using the latter value of scatter and assuming all of the scatter to be in one component, or a deviation of 10%, the maximum effect upon the random combination
would be to reduce it by 9% from that computed assuming no scatter, or to the value 0.677. The observed value was reduced by almost twice this amount, or 17%. It is, therefore, more plausible to accept the phasing explanation.

Because of the close spacing of the components of Castor it is not possible to separate them enough to make individual frequency analyses of the two components. It is possible, however, due to the proximity in the sky of Pollux, a star of comparable brightness, to record on the same tape the scintillation of both the double and the single star with a very short time interval between observations. Pollux should give the scintillation to be expected from Castor if the components of Castor scintillated in phase. The expected scintillation from Castor can be computed on the basis of the assumption of random combination of the scintillation from its components. The ratio of such a random combination to the in-phase combination, where the scintillation from each component is weighted according to the brightness of each component, is 0.765. The observed ratios for the $M(\%)_{total}$ for various apertures on February 21, 1954 and January 15, 1955 are given in Table V.
TABLE V
Ratio of $\bar{M} (%)$ total for Castor to that for Pollux on 2/21/54 and 1/15/55

<table>
<thead>
<tr>
<th>Aperture (inches)</th>
<th>2/22/54</th>
<th>1/15/55</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
<td>0.765</td>
<td>0.891</td>
</tr>
<tr>
<td>6</td>
<td>0.778</td>
<td>0.743</td>
</tr>
<tr>
<td>3</td>
<td>0.843</td>
<td>0.690</td>
</tr>
<tr>
<td>1</td>
<td>0.861</td>
<td>0.782</td>
</tr>
</tbody>
</table>

The double-star observations indicate that the combination of two-star signals is not necessarily random. The double-star signals show a different aperture effect than do single-star signals. Furthermore the exact aperture effect appears to be different on different nights. The component separation is undoubtedly another variable effecting the amount of scintillation from a double star as is indicated when the combined signal for θ Serpentis having a relative strength 0.615, is compared to the two 12.5-inch observations for Castor having relative strengths of 0.765 and 0.891. The fact that the scintillation from a double star is less than that from a single star would be expected; the fact that the scintillation can fall below that to be expected from random combination of the signals from a single star, such as occurred for the θ Serpentis observation or the January 16, 1955 6-inch observation of Castor, would
not necessarily have been expected. The explanation of this can best be given in terms of the pattern structure. This point is taken up in the section on the auto-correlation function.

It would be logical to expect that the effect described above would show variation with the upper air wind velocities, for example; note the difference between the Castor observations for February 22, 1954 and January 15, 1955. The effect will also change with the height and thickness of the turbulent layer. Both the mean size and lifetime of the turbulent elements can be expected to affect the "phasing" of the scintillation signals from the double-star components. There are not enough double-star observations available now to make any conclusive statements regarding this point; only the possibility of trends along the lines noted may be extracted from such limited observations.

8. COLOR SCINTILLATION

That stars appear to scintillate in color is well known. Color scintillation can arise in either of two fashions: scintillation shows a real color dependence,

---

28 This phenomenon has been known for at least 100 years. An excellent historical sketch of this and other scintillation phenomena are given by Nettelblad, Loc. cit.
or the appearance of color scintillation is entirely dependent upon a "phase" relationship in the scintillation at different wavelengths. The phasing would arise due to slightly different air paths traversed by the light of different colors as a result of general atmospheric refraction.

Observations were made on Vega near the zenith and Arcturus low in the sky in restricted wavelength regions, centered at 3720 and 6280 angstroms and with a bandpass of approximately 500 and 600 angstroms respectively. Figs. 44 and 45 show a plot of the frequency analyses for the blue and the red filters along with an analysis made without filters. The large scatter in the points for the color observations is due to the fact that the available signal was very small after restricting the wavelength region, and consequently it became difficult to reduce for the shot noise. This was especially so in the high-frequency region. The comparison of the \( \mathcal{M}(\%)_{\text{total}} \) and the moduli for the observations is given in Table VI.
Fig. 44. Scintillation as a Function of Color for Small Zenith Distance
Fig. 45. Scintillation as a Function of Color for Large Zenith Distance
TABLE VI

Values of $\overline{M}(\%)_\text{total}$, $\text{Mod}_{10-50}$ $\text{Mod}_{50-450}$ For Different Wavelength Regions and at Two Zenith Distances

<table>
<thead>
<tr>
<th>Zenith Distance</th>
<th>Filter</th>
<th>$\overline{M}(%)_\text{total}$</th>
<th>$\text{Mod}_{10-50}$</th>
<th>$\text{Mod}_{50-450}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vega 5.8°</td>
<td>Red</td>
<td>0.22</td>
<td>0.42</td>
<td>0.02</td>
</tr>
<tr>
<td>2.7°</td>
<td>Blue</td>
<td>0.28</td>
<td>0.47</td>
<td>0.05</td>
</tr>
<tr>
<td>1.3°</td>
<td>Clear</td>
<td>0.28</td>
<td>0.48</td>
<td>0.06</td>
</tr>
<tr>
<td>Arcturus 65.8°</td>
<td>Red</td>
<td>0.76</td>
<td>0.41</td>
<td>0.03</td>
</tr>
<tr>
<td>67.5°</td>
<td>Blue</td>
<td>0.52</td>
<td>0.32</td>
<td>0.01</td>
</tr>
<tr>
<td>70.5°</td>
<td>Clear</td>
<td>0.68</td>
<td>0.32</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The two figures and Table VI indicate that there is no great difference between the time average scintillation in the blue and in the red. The values of the moduli given in Table VI do, however, show a tendency for the high frequencies to be relatively low for the red signal compared to the blue signal at the zenith and just the reverse to be true at $Z$ around 65°. It should be pointed out, however, that Vega is an A-type star, rich in blue light, while Arcturus is a K-type star, rich in the longer wavelengths. The red signal for Vega is consequently much weaker and the scintillation curve more uncertain in the high-frequency regions due to the shot noise. Just the reverse is true for the Arcturus observations. The conclusions which can be made from the data in Table VI are therefore somewhat suspect and at best must be considered tentative.
In general, however, the measures do indicate that the variation of the frequency curve of scintillation with wavelength is very small, not at all what might be expected on the basis of visual observation of color scintillation. This would serve to indicate that the primary cause of color scintillation is a phasing effect due to the different paths traversed through the earth's atmosphere by the various wavelengths as a result of differential refraction. This had, of course, been noted earlier by other workers.

9. DAYTIME SCINTILLATION

The number of daytime observations made with the present equipment is small. A number of daytime observations were made before the tape recorder was introduced into the equipment. These observations, all for 12.5-inch aperture, though possibly of less accuracy than later observations using the magnetic tape recordings, are included in this preliminary discussion of daytime scintillation.

Before beginning the discussion of the stellar scintillation it seems appropriate to answer the question of whether scintillation may be expected from the daylight sky.\(^{29,30}\) The noise signal from a 17.5 circular-second

\(^{29}\)Goldstein, loc. cit.

\(^{30}\)Mikesell et al., loc. cit.
section of the blue sky on all days on which daytime observations were made always showed scintillation of less than 0.01 per cent in the frequency range 2.5 to 640 cps. Furthermore, there is a scatter in the observations of "positive" and "negative" values; that is, the dc lamp gave rise to a noise signal greater than the apparent sky noise. This is to be expected, and arises from statistical fluctuations in the noise signals and from the difficulty encountered in adjusting the dc lamp to exactly the same illumination level as the sky. When this scatter is taken into account, the upper limit of sky scintillation over shot noise can be set at 0.005 per cent equivalent sine wave modulation per unit bandwidth. An observation on a 0.6 circular-second sample of the zenith sky shows less than 0.09 per cent average equivalent sine wave modulation per unit bandwidth. It is evident that scintillation by the daytime sky, if any, is extremely small and in any case of no importance compared to stellar scintillation.

An example of a daytime stellar scintillation analysis as compared to a nighttime analysis is shown in Fig. 46. These curves show that there is no striking difference in the shape of the curve, at least for a 12.5-inch aperture, and that the amount of scintillation is not very much greater during the daytime.

In order to be able to give an estimate of the
% EQUIVALENT SINE WAVE MODULATION VS FREQUENCY

ARCTURUS
9/28/53
14:45 E.S.T.
Z = 21.1

VEGA
9/28/53
20:57 E.S.T.
Z = 25.3

Fig. 46. A Comparison of Daytime to Nighttime Stellar Scintillation
increase in daytime over nighttime scintillation, 13 observations made before the magnetic tape recorder was placed in use and 7 observations made subsequent to that time were grouped together. The average values of $\overline{M}(\%)_2.5$ to $10.5$ and $\overline{M}(\%)_{\text{total}}$ for $1.0 \leq \sec Z \leq 1.6$ were computed for the daytime observations. Similar averages were computed for selected nighttime observations. The nighttime data were chosen on the basis of a comparison of the daytime moduli to the nighttime moduli. The daytime moduli were almost identical with those for low velocity nighttime observation, both with regard to the value of the moduli and with regard to the change of moduli with secant $Z$. The comparison of the daytime averages to the averages of the low velocity nighttime data gave ratios of daytime over nighttime of 1.44 for $\overline{M}(\%)_2.5-10.5$, and 1.98 for $\overline{M}(\%)_{\text{total}}$.\footnote{Hosfeld has studied the diurnal variation of scintillation in the low-frequency region (0 to 10 cps). He finds a factor of 2 between daytime and nighttime scintillation for low frequencies. Progress Report No. 5, Project 480, Ohio State University Research Foundation Contract No. AF 19(604)-41.}

If the daytime values are compared to the high-velocity measurements, the ratios are 1.85 for $\overline{M}(\%)_2.5-10.5$ and 1.73 for $\overline{M}(\%)_{\text{total}}$. It is not anticipated that the values of the ratios will exceed the values quoted by a very significant amount since the
daytime observations were made near noon when the maximum scintillation is to be expected. Possibly the most important point to be made on the basis of these observations is not that daytime and nighttime scintillation are different, but rather to point out the amazing similarity, both in shape of curve and amount of scintillation, which exists between them.

10. CORRELATION OF SCINTILLATION AND SEEING

The possibility of using scintillation measurements as an index of astronomical seeing is an intriguing one, since scintillation can be measured quantitatively whereas the main effects of seeing (image size, image dancing and pulsation) do not lend themselves easily to such measures.

Hosfeld has already shown that seeing and scintillation do not necessarily have a one-to-one correspondence. Fig. 47 illustrates a verification, over the total frequency range, of an experiment originally performed by Hosfeld for the low-frequency regions. The experiment consists of observing the scintillation and image quality of a star before and after creating artificial turbulence in the air in the immediate neighborhood of the telescope. This can be accomplished on cold nights merely by opening

Fig. 47. The Effect of Introducing Thermal Turbulence Near the Telescope Upon Stellar Scintillation
the door to the dome of the observatory and permitting warm air to enter the dome from the heated portions of the observatory building, the heated air leaving the dome by way of the observing slit.

On the night on which the observation represented in Fig. 47 was made, a temperature differential of at least 30° F existed between the cold air and the heated air. After admitting the hot air the image was five times its previous size. The image motion was also strongly increased and the image showed considerable boiling. The scintillation, on the other hand, was not increased as shown by Fig. 47. The per cent difference between the two scintillation observations was only 1.75% for the integrated scintillation over the entire frequency range. This experiment demonstrates quite conclusively that the seeing can be quite bad without sensibly affecting the scintillation.

In an attempt to determine whether there actually is any correlation between scintillation and seeing, estimates of the image quality, for the full aperture, were made at the time of the scintillation observations. Image size, bodily motion, pulsation (or "internal" motion), multiplicity of the image and the frequency of image blowups were noted. Of these observables, only the first, image size, is capable of being measured semiquantitatively with the present equipment. The image
diameter was estimated in terms of the diameter of the crosswires of the photometer finder eyepiece.

Figure 48 is a plot of the correlation between image size and scintillation. There seems to be a tendency for large scintillation to accompany large image size. The scatter evident in Fig. 48 could arise in several ways: first, the criterion of image size is rather coarse and somewhat subjective in its nature; second, the seeing estimates and scintillation measurements are not exactly simultaneous, but are separated by as much as 5 or 10 minutes of time; and, third, the amount of scintillation may determine only the minimal image size, other nearby turbulence could enhance the image size without increasing the scintillation. The simultaneity of the seeing estimate and the scintillation measurement is probably essential to the determination of the exact solution of this problem since both scintillation and seeing are known to show almost instantaneous changes of rather large magnitude.

The other parameters of the image condition do not show any strong correlation with the amount of scintillation. If any exists, it will be discovered only by more refined seeing measures than could be used in this analysis.

The type of scintillation, that is, the shape of the scintillation curve, does not seem to show a
Fig. 48. Correlation Between Image Size and Scintillation
correlation with any feature of seeing. Fig. 49, for example, shows a plot of the crossover frequency versus image size, and it is obvious that no marked evidence of correlation is present.

The fact that scintillation is poorly correlated with all seeing parameters except image size can most probably be explained by the fact that scintillation is caused by air turbulence at a relatively large distance from the telescope, as is evidenced by the correlation between scintillation and the winds at the 200 mb level, and the experiment of the introduction of artificial turbulence. What happens near the telescope, then, can virtually destroy the seeing while not sensibly bothering the amount of scintillation. On the other hand, the turbulence layer giving rise to scintillation, even though at a distance, will affect the image size. The size of the image and the amount of scintillation will be determined by the strength of the turbulence; the shape of the scintillation curve, however, will be determined primarily by the wind velocity. If the degree of turbulence and the wind velocity do not necessarily have a one-to-one correspondence, then the conclusion concerning the image size, amount of scintillation and

---

Fig 49. Correlation Between Image Size and Crossover Frequency
shape of the curve becomes evident. The size of the image is affected by nearby turbulence, so that large image size can occur with relatively low scintillation.

11. SHADOW BAND STRUCTURE

As mentioned in part I of this chapter, it is possible to discuss stellar scintillation in terms of the shadow band structure. The study of the shadow bands is quite difficult when carried out by means of photography. It is possible, however, to use scintillation measurements of the frequency analysis type to study shadow bands by the method described by Keller. 34

The time-average structure of the shadow pattern is determined by measuring the average mean square deviation of the combined values of the brightness of a star observed by two telescopes. These quantities, as a function of the separation of the telescopes, are related to the autocorrelation function of the shadow band pattern. The actual measurements described here were made with a single telescope, a set of objective diaphragms having two one inch apertures of variable spacings, in increments of one inch, being used to

simulate two separate telescopes. Since the 12.5-inch McMillin refractor was used the upper limit on the separations was a center to center spacing of 11 inches.

Keller, in his paper, shows that the autocorrelation function of the shadow band pattern, \( \phi(\Delta r) \), is related to a function which he defines as

\[
Q(\Delta r) = \frac{I_{12}^2 - 2I_1^2}{2I_1^2},
\]

where: \( I_{12}^2 \) is the mean square deviation of the combined brightness levels for the star viewed by two telescopes at a separation \( \Delta r \),

and \( I_1^2 \) is the mean square deviation of the star signal for a single telescope.

The \( Q \)-function can be rewritten in terms of the per cent mean square deviation by noting that

\[
I_1^2 = I_{DC}^2 \left[ M_1(\%) \right]^2,
\]

and \( I_{12}^2 = (2I_{DC})^2 \left[ M_{12}(\%) \right]^2 \),

where: \( I_{DC} \) is the mean brightness level of the star through a single aperture.

Then the \( Q \)-function may be written as
\[ Q(\Delta r) = \frac{4\left[ M_{12}(\%) \right]^2 - 2\left[ M_1(\%) \right]^2}{2\left[ M_1(\%) \right]^2} \]

It can be shown that these Q-functions are essentially the same as the true autocorrelation function \( \rho (\Delta r) \) for small apertures. For the measurements described here the aperture was one inch which means that for all features of the pattern larger than about one inch the Q-functions are the same as the autocorrelation function with no further correction needed.

Observations of this type were made on three nights in March of 1954. One set of observations, taken on March 15, 1954, while showing general tendencies similar to the other two nights, has been discarded because the scintillation underwent a major change during the time necessary to complete the set.

The orientation of the line of centers of the double hole aperture with respect to the shadow band pattern is important whenever the pattern has a preferred direction of motion, or non-circular elements. If autocorrelation measures are made along four or more orientations, this direction may be determined directly from the measures themselves. Since, however, the observation time becomes excessive, it is much more advantageous to observe with only two orientations, one along the direction of maximum element size and one at right angles to this direction.
The slit measurements described in Section 4 of this chapter may be used to determine these directions as will become evident from the following discussion.

The type of Q-function given by the double hole observations is illustrated in Fig. 50, which presents the results for the night of March 1, 1954. The distance between aperture centers, measured along the directions parallel and perpendicular to the slit position giving minimal high frequency components, is represented by $\Delta r$. Since the observing apertures are small, the Q-functions will be referred to as autocorrelation functions on the basis of the previous discussion.

Two points are evident from the autocorrelation functions presented in Fig. 50. First, the pattern shows a correlation over distances as great as 11 inches, with the pronounced negative values indicating a rather strong tendency toward a well-defined average structure. Second, the correlation behavior in the two directions is different, the parallel direction autocorrelation having a coarser structure than that for the perpendicular direction.

The size of the elements presented in the pattern can best be found from the Fourier spectrum of the shadow band pattern. Keller\textsuperscript{35} has shown that the Fourier

\textsuperscript{35}Ibid.
Fig. 50 Observed Autocorrelation Functions Over all Frequency Components for the Night of March 1, 1954.
expansion of the $Q$-functions can be expressed as

$$B(w, \phi) = B_0(w) - B_2(w) \cos 2\phi$$

where: $w$ is the wave number equal to $\frac{2\pi}{L}$,

$\phi$ is the angle between the line of centers and the direction of element size elongation,

and $B_0(w)$ and $B_2(w)$ are expressed as follows:

$$B_0(w) = \frac{I_1^2}{\pi K^2} \frac{(1+k^2)}{\alpha} \int_0^\infty d(Ar) Q_0(Ar) J_0(wAr)$$

and

$$B_2(w) = \frac{I_1^2}{\pi K^2} \frac{(1+k^2)}{\alpha} \int_0^\infty d(Ar) Q_2(Ar) J_2(wAr).$$

In the expressions for $B_0(w)$ and $B_2(w)$:

$K^2$ = a constant dependent upon the aperture size,

$I_1^2$ = mean square deviation of brightness level for a single aperture,

$\alpha$ = ratio of response of one telescope system to the other,

$J_0$ and $J_2$ are the zero and second order Bessel functions respectively,

$Q_0(Ar) = Q_{||} + Q_{\perp},$

and $Q_2(Ar) = Q_{||} - Q_{\perp},$

where: $Q_{||} = Q(\Delta x, 0)$

and $Q_{\perp} = Q(0, \Delta y).$

This is actually an approximation, and holds whenever the pattern does not vary too greatly from circular
symmetry. Under this condition $B_0(w)$ gives the circularly symmetric portion of the pattern, while $B_2(w)$ is a measure of the asymmetry. The sum of the two expansions, $B_0(w) + B_2(w)$, gives the spectrum of the pattern in the parallel direction; the difference, $B_0(w) - B_2(w)$, gives the spectrum of the pattern in the perpendicular direction. It is immediately apparent, since the amplitude of the pattern going with a specified $w$ is given by the square root of the value of $B_1(w)$, the complete expansion, that $B_0(w)$ must be everywhere positive and that the absolute value of $B_2(w)$ must be equal to or less than that of $B_0(w)$.

This is not the case in the expansion of the observed autocorrelation functions as shown in Figs. 51 and 52, the $B_0(w)$ and $B_2(w)$ expansions for the nights of March 1, and March 16, 1954. Correct expansions would have to be computed on the basis of an autocorrelation function measured over infinite distance and for all values of the spacing variable. The actual curves are measured over a finite distance and with a spacing variable taken in discrete steps. These two factors give rise to the introduction of spurious components, some of which may have imaginary amplitudes. This effect will become most severe for values of $w$ less than that corresponding to the cutoff point of the measurements, $w < 0.4$, and also
Fig. 51  Fourier Spectrum of the Shadow Band Pattern for March 1, 1954
Fig. 52  Fourier Spectrum of the Shadow Band Pattern for March 16, 1954
for \( w > 2.5 \) where the discrete measuring interval becomes effective. The effect of changes in the level of the scintillation and experimental errors are harder to evaluate and may be expected to introduce some further effects in the \( B_0(w) \) and \( B_2(w) \)-functions for the range of \( w \) presented. An attempt to minimize the effect of the variation of the amount of scintillation was made by selecting the order in which readings for different hole spacings were taken. Because of these factors only the strongest maxima in the \( B \)-curves can be considered reliable.

The significant wavelengths present on the two nights, determined from the sum and difference of the \( B_0(w) \) and \( B_2(w) \)-functions, are given in Table VII.

TABLE VII

<table>
<thead>
<tr>
<th>Date</th>
<th>Direction</th>
<th>Length</th>
<th>Weight</th>
<th>AL</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/1/54</td>
<td>( \perp )</td>
<td>5.6&quot;</td>
<td>0.9</td>
<td>0.55&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.6&quot;</td>
<td>0.6</td>
<td>0.18&quot;</td>
</tr>
<tr>
<td></td>
<td>( \parallel )</td>
<td>9.8&quot;</td>
<td>1.0</td>
<td>1.25&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.8&quot;</td>
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<td></td>
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<td>0.15&quot;</td>
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<tr>
<td>3/16/54</td>
<td>( \perp )</td>
<td>9.8&quot;</td>
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<td>1.00&quot;</td>
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<tr>
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<td></td>
<td></td>
<td>2.6&quot;</td>
<td>0.7</td>
<td>0.02&quot;</td>
</tr>
<tr>
<td></td>
<td>( \parallel )</td>
<td>10.5&quot;</td>
<td>0.8</td>
<td>1.65&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.8&quot;</td>
<td>0.5</td>
<td>0.25&quot;</td>
</tr>
</tbody>
</table>
The weight is a measure of the relative strength of the amplitudes corresponding to the components; \( \Delta L \) is the halfwidth of the peaks and indicates the spread in wavelengths.

The problem of pattern motion can also be studied in terms of autocorrelation function analysis technique. To determine the pattern motion a new function \( Q_q \) is defined.\(^3^6\) This is exactly analogous to the \( Q \)-functions except that it is computed on the basis of the mean square deviation of the stellar signal for a restricted band of frequencies centered at a frequency \( q \). This is the circular frequency, in other words, \( 2\pi \times \) times the frequency in cycles per second. The frequency will be expressed in terms of cycles per second in keeping with the rest of this work.

A plot of these "autocorrelation" functions for each of the two orientations used for the line of centers of the double apertures for the night of March 16, 1954 are presented in Fig. 53. The \( Q_q \)-functions were computed for a band of frequencies 20 cps wide centered at 20, 75, 150 and 250 cps. Only three of these functions are shown in each plot.

The main point of this plot is that while the functions are sensibly the same in the perpendicular

\(^{3^6}\)Ibid.
Fig. 53  Observed "Autocorrelation" Functions for the Night of March 16, 1954. The Functions Measured Parallel to the Pattern Elongation are Presented in A, Perpendicular to it in B.
direction ($\Delta x = 0$) for all frequencies, the functions show a marked change with frequency in the parallel direction ($\Delta y = 0$). A change in the function with frequency will arise if the shadow band pattern has motion in the direction along which the "autocorrelation" is measured. This point can be demonstrated as follows.

Keller has shown that

$$Q_q(\Delta x, \Delta y) = \frac{\alpha K^2}{I_{1q}^2(1+\kappa^2)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dvB'(u,v,q) \cos(u\Delta x + v\Delta y) ,$$

where: $\alpha$ and $K$ have the meaning already noted,

$\Delta x$ and $\Delta y$ are the spacing variable components measured in the parallel, and perpendicular directions respectively,

$u$ and $v$ are the wave numbers associated with the distances $\Delta x$ and $\Delta y$,

and $B'(u,v,q)$ is the Fourier expansion of the "autocorrelation" functions (this corresponds to the $B$-function already defined).

If the aperture used for the measurements is small compared to the details of the pattern, $K$ will be a constant independent of $u$, $v$ and $q$. Furthermore for the case under discussion $\alpha$ can be taken as unity. Then, if it is assumed that the pattern is fixed but has a uniform translation in the $\Delta x$ direction,

$$|u| = \left| \frac{q}{Vx} \right|.$$
Consequently the $Q_q$ functions observed in a direction parallel to the assumed motion will be

$$Q_q(\Delta x, 0) = \cos\left(\Delta x \frac{q}{V_x}\right).$$

Thus, if the assumptions are valid, the functions, $Q_q(\Delta x, 0)$ should be cosine curves with wavelengths $L_x$ which are proportional to the frequency $q$, or in cycles per second, $\frac{q}{2\pi V_x}$.

From Fig. 53 it is evident that the curves are not pure cosine functions, but it is apparent, however, that Fourier analyses of these curves will tend to exhibit maxima at certain characteristic wavelengths. The results of such analyses for March 16, 1954 are presented in Fig. 54 where the positions of the maxima are presented versus frequency. It will be noted that several maxima are observed for each frequency. Weights have been assigned to these maxima as indicated in the figure and are based on (a) the height of the peak in the Fourier spectrum, and (b) on the estimated reliability of the determinations as influenced by the uncertainty of the data. The half width of the peak is also shown in the diagram.

A straight line was fitted to these points as shown in the figure. The slope of this line corresponds to a value of $V_x = 62.5$ knots. On this same night the upper
Fig. 54  Maxima of the Fourier Spectra of the Parallel "Autocorrelations" for the Night of March 16, 1954 Vs. Frequency for which the "Autocorrelations" were Computed
air wind velocities over Columbus, Ohio taken from the 200 and 500 mb weather charts were 60 knots at an angle of 125° and 35 knots at an angle of 150° respectively. The wind velocity at the tropopause measured by radar tracked balloons was 100 knots at an angle of about 115° at Dayton, Ohio. The orientation of the line of centers for the measurements shown in Fig. 53 was 123° as determined from the slit measurement. The time of the meteorological observations does not agree exactly with that of the scintillation measures but differs by about 4 hours.

A similar analysis for March 1, 1954 gives a value of 75 knots for the velocity of the shadow band pattern. The 200 and 500 mb charts give wind values of 42.5 knots at 40° to 100° and 45 knots at 120° respectively. The tropopause data at Dayton gave a value of about 70 knots at an angle of 90°. The line of centers was oriented at an angle of 83°.

While it is evident that the shadow band pattern cannot be described entirely as being a fixed pattern in translation across the line of sight it appears that the evidence is strong that a considerable part of its observed behavior may be accounted for in this fashion. The correlation of $|V_x|$ with the upper air winds suggests strongly that this is the case and that a traveling shadow band pattern does exist and that its origin is
connected with atmospheric conditions in the vicinity of the 200 mb region. This is in keeping with the findings in Section 5 of this chapter. It is, furthermore, in essential agreement with the results of Gifford and Mikesell.\textsuperscript{37} Further support is given by the observations of Butler\textsuperscript{38} and Hosfeld\textsuperscript{39} who concluded from measurements of a different nature that the pattern has motion. Hosfeld also concluded that the pattern elements were elongated in the direction of motion of the pattern.

The autocorrelation curves, $Q_q(0,\Delta y)$, measured in a direction perpendicular to the predominant pattern motion are shown in Fig. 53. The similarity of the curves for different frequencies is striking. Curves of $Q_q(0,\Delta y)$ constructed from the data of March 1, 1954 do not show such striking similarity, and it is not clear at this time whether the degree of similarity is high enough in all cases to represent an important characteristic of all shadow band patterns.

The problem of the scintillation received from a double star may also be discussed advantageously in terms

\textsuperscript{37}Gifford and Mikesell, \textit{Loc. cit.}


of the autocorrelation function of the shadow band pattern. In the first analysis, if the turbulence layer causing the scintillation can be considered to be relatively thin, then the shadow band pattern received from a double star can be considered as the superposition of two identical patterns, each being that received from a single component, and separated by a distance, along the direction of the line joining the components, equal to the height of the turbulent layer times the sine of the component separation in angular measure. The strengths of the two patterns will be proportional to the brightness of the component giving rise to each pattern. It is immediately evident that such a condition corresponds exactly to the autocorrelation measurement on a single star for a \( \Delta r \) equal to the amount of the pattern separations and for two telescopes having apertures equal to that used for the double star observation but with sensing devices having relative sensitivities of unity and \( \alpha \), where \( \alpha \) is the brightness of one component of the double star expressed as a fraction of the brightness of the other component.

Keller has treated this problem in his paper and finds that the autocorrelation function of the shadow band pattern can be found in terms of the double star measurements by the following expression:
\[ \mathcal{P}(\Delta r) = \frac{(1+\alpha^2)}{2 \alpha} Q(\Delta r) - 4 \left( \frac{\Delta w}{w_0} \right) F_1(w_0 \Delta r) F_2(w_0 R_2) \]

where: \( \alpha \) has already been defined

\[ w_0 \] is the predominant element size present (expressed in wave numbers, \( 2\pi \frac{L_0}{L_0} \)),

\[ \Delta w \] is the half width of the peak occurring at \( w_0 \) in the Fourier expansion of the autocorrelation function,

\[ Q(\Delta r) \] is the Q-function as determined from the double star measurements

and \( F_1(w_0 \Delta r) \) and \( F_2(w_0 R_2) \) are two functions dependent upon the separation of the patterns and the observing aperture respectively.

The last two expressions \( F_1(w_0 \Delta r) \) and \( F_2(w_0 R_2) \) are given by

\[ F_1(w_0 \Delta r) = w_0 \Delta r J_1(w_0 \Delta r) \]

and \[ F_2(w_0 R_2) = 1 - \frac{w_0 R_2}{2} \left[ \frac{J_0(w_0 R_2)}{J_1(w_0 R_2)} \right] \]

where: \( J_0 \) and \( J_1 \) are the zero and first order Bessel functions.

This analysis assumes a shadow pattern having a dominant wavelength, corresponding to \( w_0 \), with a spread about this wavelength characterized by a halfwidth \( \Delta w \). The approximation given above is valid to the third order in \( \frac{\Delta w}{w_0} \).
It is quite evident that determining the autocorrelation function of the shadow band pattern from double star observations by means of this expression would be impractical. If, however, the autocorrelation function is known, the Q-function, and consequently the amount of scintillation to be expected from a double star, can be calculated as a function of separation of the patterns arising from each component at the objective and the size of the objective. The results presented for the double stars in Section 7 of this chapter now become more easily accepted since it is apparent that the amount of scintillation is not a simple function of only the component separation but is a complicated function of the component separation, the height of the turbulent layer, the autocorrelation function of the shadow band pattern and the size of the aperture used for the measurements.

It is possible, of course, to compute the height of the scintillation layer from the observed scintillation of a double star of known separation and position angle if the autocorrelation function is known for that night. One such observation is available. On March 1, 1954 both the scintillation from Castor for a 3-inch aperture and the autocorrelation function were determined.

The Q-function from the double star measurement can be determined from the known scintillation for the double star and for a single star using the same telescope.
aperture. This is corrected for the difference in brightness of the components. The corrections for the pattern spacing and aperture size are then made. This complete expression is solved simultaneously with the observed autocorrelation function to determine the separation of the patterns, \( \Delta r \). The height of the layer in terms of \( \Delta r \) is then given by

\[
h = \frac{\Delta r}{1.45}
\]

where: 
- \( h \) = height measured in 10,000 ft. intervals
- \( \Delta r \) = separation of patterns measured in inches.

On the night in question, March 1, 1954, the position angle of Castor agreed with the orientation of the line of centers for the perpendicular case of the autocorrelation function. The solution resulting is ambiguous and gives results of 21,000, 43,000 and 64,000 feet. The height of the tropopause as determined at Dayton, Ohio was 29,000 feet on this night, this is an abnormally low value. As indicated earlier the scintillation measurements and the meteorological measurements were not made simultaneously but differed by four hours. The most important point is not whether the values of the heights agree precisely at the moment but rather, it is the fact that no solution was possible below 20,000 feet, indicating that the turbulence
causing scintillation is a high level phenomenon. This is in agreement with Section 10 of this chapter.

12. CONCLUSIONS

It has been possible to study the phenomenon of stellar scintillation in considerable detail by means of the noise spectrum of the scintillation. The use of the magnetic tape recorder has made it possible to obtain homogeneous samples of the noise signals for the analyses with the result that a more complete separation of variables is possible than could have been obtained if the analyses were made using the signal output at the telescope.

The relationship between the amount of scintillation and the zenith distance of the star has been determined for average conditions. It was found that this relationship is actually not constant but depends upon meteorological conditions in the upper troposphere. There were not enough observations available to define the zenith distance relationship explicitly as a function of these meteorological conditions.

The predominant meteorological condition affecting scintillation appears to be the turbulence and associated wind velocities in the vicinity of the tropopause. A correlation between scintillation and the wind velocities at the 200 mb level (40,000 ft.) has been made. The wind
direction, but not its sense, was determined with a probable error of ±6° using the slit observations. The magnitude of the wind velocity was determined with a probable error of about ±14 knots by means of the scintillation moduli. These moduli define the shape of the curve. While it has been found that the magnitude of the scintillation taken over the total frequency range increases, on the average, with the wind velocities in the 200 mb region, the correlation between the amount of scintillation and the wind velocities shows considerable scatter. This can be explained upon the assumption that the strength of the turbulence, and consequently of the scintillation, does not have a one to one correspondence with the wind velocities.

Although the most favorable correlation between both direction and magnitude of the upper air wind velocities and scintillation occurred at the 200 mb level, it is felt, since this level occurs at the mean level of the tropopause, that the actual correlation between the winds and stellar scintillation occurs at the tropopause. This conclusion becomes more logical when it is considered that the tropopause, defined as a major inversion level, and the atmospheric layers adjacent to it, are regions of constant turbulence. Furthermore it has been possible to demonstrate that turbulence, sufficient to sensibly destroy the stellar image, artificially introduced near
the telescope, has no effect upon the amount of scintillation. Thus, it is inferred that the turbulence causing the scintillation is at some distance from the viewing device. Although only one such observation is available, the double star observation in conjunction with a determination of the autocorrelation function of the shadow band pattern results in a value of the height of the layer in excess of 20,000 feet. The exact level cannot be found at this time from this observation since the solution is ambiguous. Furthermore, it has not been possible to assess the effects of experimental error upon this determination. While experimental error is undoubtedly important, it is thought that its effect cannot be sufficient to invalidate the lower bound of about 20,000 feet for the height of the turbulent layer causing most of the stellar brightness fluctuations.

It was possible to demonstrate that the seasonal variation of stellar scintillation, first found by Mikesell of the Naval Observatory, can be explained as a result of the seasonal variation in the upper air wind velocities.

The effect of aperture size upon the magnitude of the scintillation was demonstrated. The increase in the strength of the scintillation with decreasing aperture is more pronounced for the high frequencies. The aperture relationship is also affected by the upper air wind
velocities.

A comparison of daytime to nighttime scintillation leads to the conclusion that the daytime scintillation exceeds that at night by a factor of two at most. The shape of the frequency spectrum for both daytime and nighttime scintillation is the same.

Measurements in restricted wavelength regions at both a high and low zenith distance indicate that any effects introduced into the scintillation frequency spectrum, both with regard to magnitude and distribution of frequency components, as a function of the wavelength of the light being observed must be a second order effect. The wavelengths used were 3720 angstroms and 6280 angstroms. The effect of color scintillation visible to the eye is probably caused by a "phasing" effect in the scintillation signal arising from different pathlengths traversed through the earth's atmosphere by the various wavelengths of light as a result of general atmospheric refraction.

A positive correlation between astronomical seeing and scintillation has been found only between image size and the magnitude of the scintillation. Large image size accompanies high values of scintillation. Since, as previously noted, the presence of turbulence close to the telescope can affect the image size without increasing the level of the scintillation, it is apparent
that the amount of scintillation defines a lower bound for the image size, any turbulence occurring between the "scintillation" layer and the telescope will increase the image size above this value. While it is almost certain that the negative results gained from the correlation of other phases of astronomical seeing with scintillation are correct, at least to the first order approximation, the possibility of a more subtle relationship between these parameters and scintillation cannot be excluded due to the crudeness of the seeing estimates.

The autocorrelation measurements have shown that the shadow band pattern has a fairly definite structure with the predominant element sizes of the order of 5 to 10 inches. These elements, furthermore, show an elongation in the direction of motion of the pattern. This motion appears to correlate both in direction and magnitude with the winds in the 200 mb region, thus substantiating the conclusions reached in the wind versus scintillation correlation. It is also possible to discuss the results of planetary and multiple star scintillation in terms of the time average autocorrelation function of the stellar shadow band patterns. It is evident from the discussion on the autocorrelation function that the amount of scintillation received from a double star is a complicated function of the separation of the components, the height of the turbulence, the
autocorrelation function and the telescopic aperture used. The case for planetary scintillation is even more complicated since instead of a superposition of two shadow band patterns, as in the case of a double star, the resultant pattern for a planet is best considered as an integration of displaced shadow band patterns carried out over two dimensions, the displacement of the shadow band patterns being a function of position taken on the face of the planet and the height of the turbulence layer causing the scintillation. These conclusions are borne out by the data taken on planetary and multiple source scintillation.
ACKNOWLEDGMENTS

This work could not have been carried out without the aid of a considerable number of persons and organizations. I wish to thank both the Air Forces Cambridge Research Center and the Ohio State University Research Foundation for the opportunities afforded me through their financial support. I would like to thank Dr. J. Allen Hynek, my advisor and supervisor, for his advice and counsel during this study. I also wish to thank Dr. Geoffry Keller for his interest and advice in this work especially with regard to the autocorrelation studies. I am also indebted to Dr. Heinz Fischer of the Air Forces Cambridge Research Center for the encouragement and help which he extended to me while this study was in progress. The many discussions which I have had with Mr. Roger Hosfeld have been very stimulating and of great value to me in carrying out this work. The assistance of Miss Anna Marie Smith, who carried out the tape playbacks and mathematical computations, is greatly appreciated. The drawings in this dissertation were completed by Mr. Donald Herke. I would also like to thank Mr. Carl McWhirt and the employees of the machine shop of the Department of Physics and Astronomy for their kind and ready help in the preparation and repair of equipment.
APPENDIX

AVERAGE SCINTILLATION FOR VARIOUS APERTURES AND SEC Z RANGES

All 12.5- and 3-inch aperture observations made to date are presented point by point. The observations are grouped according to season: summer, June through September, and winter, October through May. They are also grouped according to sec Z ranges. The 1- and 6-inch observations for the first sec Z range are also included. The figures are arranged as shown in Table VIII.

Besides the cumulative plots, the average for the four aperture sizes in the first sec Z range are present in Figs. 80, 81, 82 and 83 in the order of decreasing diameter.

The data for these four aperture sizes are also summarized by plotting the function

\[ M^2(f) = \int_{f}^{\infty} m^2(f) df \]

where: \( M^2(f) \) = mean square deviation of the signal for all frequencies from \( f \) to infinity,

and \( m(f) \) = the per cent sine wave modulation per unit bandwidth as a function of frequency.

These are presented in Figs. 84, 85, 86, and 87, again in the order of decreasing aperture. The rms deviation as a
function of lower cutoff frequency, where the frequency range considered extends from that frequency to infinity, is similarly presented in Figs. 88, 89, 90, and 91.

The amount of scintillation to be expected in any frequency range can be determined from the mean square deviation curves (Figs. 84, 85, 86, and 87). This is accomplished by taking the square root of the difference of the mean square values at the end points of the frequency range being considered.
# TABLE VIII

**ARRANGEMENT OF CUMULATIVE PLOTS OF SCINTILLATION DATA**

<table>
<thead>
<tr>
<th>Aperture (inches)</th>
<th>Season</th>
<th>SEC -Z- RANGE</th>
<th>1.0-1.5</th>
<th>1.5-2.0</th>
<th>2.0-2.5</th>
<th>2.5-3.5</th>
<th>3.5-4.5</th>
<th>4.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
<td>Summer Winter</td>
<td>Fig. 55</td>
<td>Fig. 56</td>
<td>Fig. 63</td>
<td>Fig. 64</td>
<td>Fig. 67</td>
<td>Fig. 71</td>
<td>Fig. 74</td>
</tr>
<tr>
<td>6</td>
<td>Summer Winter</td>
<td>Fig. 57</td>
<td>Fig. 58</td>
<td>Fig. 63</td>
<td>Fig. 64</td>
<td>Fig. 67</td>
<td>Fig. 71</td>
<td>Fig. 74</td>
</tr>
<tr>
<td>3</td>
<td>Summer Winter</td>
<td>Fig. 59</td>
<td>Fig. 60</td>
<td>Fig. 65</td>
<td>Fig. 66</td>
<td>Fig. 69</td>
<td>Fig. 73</td>
<td>Fig. 76</td>
</tr>
<tr>
<td>1</td>
<td>Summer Winter</td>
<td>Fig. 61</td>
<td>Fig. 62</td>
<td>Fig. 65</td>
<td>Fig. 66</td>
<td>Fig. 70</td>
<td>Fig. 73</td>
<td>Fig. 76</td>
</tr>
</tbody>
</table>
Fig. 55. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
% EQUIVALENT SINE WAVE MODULATION

vs

FREQUENCY

12.5" APERTURE
1.0 = SEC Z = 1.5
(0° = Z = 48°)

WINTER

Fig. 56. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 57. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 58. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
% EQUIVALENT SINE WAVE MODULATION VS FREQUENCY

3° APERTURE
SUMMER
40 ≤ SEC Z < 1.5
(0° ≤ Z < 48°)

Fig. 59. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 60. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 61. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
% EQUIVALENT SINE WAVE MODULATION
VS
FREQUENCY

1" APERTURE
LO ≤ SECZ ≤ 1.5
(0° ≤ Z ≤ 48°)
WINTER

Fig. 62. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 63. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 64. Per Cent Equivalent Sine Wave Modulation Vs. Frequency

% EQUIVALENT SINE WAVE MODULATION

vs

FREQUENCY

12.5" APERTURE

L5 = SEC Z < 2.0

(48° < Z < 60°)

WINTER
Fig. 65. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 66. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
% EQUIVALENT SINE WAVE MODULATION
VS
FREQUENCY
12.5° APERTURE
2.0 ≤ SEC Z < 2.5
(60° ≤ Z < 86°)
SUMMER

Fig. 67. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
% EQUIVALENT SINE WAVE MODULATION
VS
FREQUENCY

12.5" APERTURE
2.0 ≤ SEC Z < 2.5
(60° ≤ Z ≤ 66°)
WINTER

Fig. 68. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 69. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 70. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 71. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 72. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 73. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 74. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
% EQUIVALENT SINE WAVE MODULATION
VS
FREQUENCY
12.5° APERTURE
3.5 ≤ SEC Z < 45
(73° ≤ Z < 77°)
WINTER

Fig. 75. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
% EQUIVALENT SINE WAVE MODULATION

vs

FREQUENCY

3° APERTURE
3.5 ≤ SEC Z ≤ 4.5
(73° ≤ Z ≤ 77°)
SUMMER

Fig. 76. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 77. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 78. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 79. Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 80. Average Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 81. Average Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Average % Equivalent Sine Wave Modulation vs Frequency

3° Aperture
1° ≤ sec Z < 4.5
(0° ≤ Z < 48°)

- Summer
- Winter

Fig. 82. Average Per Cent Equivalent Sine Wave Modulation Vs. Frequency
Fig. 83. Average Per Cent Equivalent Sine Wave Modulation
Vs. Frequency
Fig. 84. Mean Square Deviation for the Frequency Range from the Lower Cutoff Frequency to Infinity
Fig. 85. Mean Square Deviation for the Frequency Range from the Lower Cutoff Frequency to Infinity
Fig. 86. Mean Square Deviation for the Frequency Range from the Lower Cutoff Frequency to Infinity
Fig. 87. Mean Square Deviation for the Frequency Range from the Lower Cutoff Frequency to Infinity
Fig. 88. Root Mean Square Deviation for the Frequency Range from the Lower Cutoff Frequency to Infinity
Fig. 89. Root Mean Square Deviation for the Frequency Range from the Lower Cutoff Frequency to Infinity

AVERAGE ROOT MEAN SQUARE DEVIATION
VS
LOWER CUTOFF FREQUENCY
6' APERTURE
1° SEC Z = 45°
(0° ≤ Z ≤ 48°)
- SUMMER
- WINTER

SUMMER - WINTER
AVERAGE ROOT MEAN SQUARE DEVIATION

V S

LOWER CUTOFF FREQUENCY

3" APERTURE
1 = SEC Z = 1.5
(O° ≤ Z ≤ 48°)
- SUMMER
- WINTER

Fig. 90. Root Mean Square Deviation for the Frequency Range from the Lower Cutoff Frequency to Infinity
Fig. 91. Root Mean Square Deviation for the Frequency Range from the Lower Cutoff Frequency to Infinity.
I, William Mansel Protheroe, was born in Maesteg, Wales, October 16, 1925. I am a naturalized citizen of the United States of America. I received my secondary school education in the public schools of Barberton, Ohio. I am a graduate of The United States Merchant Marine Academy; Kings Point, New York. I attended the Academy during the World War II, during which time an accelerated program extending over 21 months was in effect. I received marine engineer licenses covering operation of both steam and diesel propulsion plants. My undergraduate training was taken in two schools, Heidelberg College, Tiffin, Ohio and The Ohio State University. I received the degree of Bachelor of Science in Physics in 1950 from the latter institution. While at Heidelberg College I held a scholarship in Physics, and served as laboratory instructor in physics. After completing my undergraduate work I was accepted as a graduate student in the department of Physics and Astronomy at The Ohio State University. During the first year of graduate work I served as Assistant at the McMillin Observatory under the direction of Dr. J. Allen Hynek. Following this to the present time I held the position of Research Assistant or Associate in the Department of Physics
and Astronomy under an Ohio State University Research Foundation contract. Dr. J. Allen Hynek was the supervisor of this contract. The research which I carried out while in this capacity served as the basis for my dissertation to be used in meeting the requirements for the degree of Doctor of Philosophy.