HEAT TRANSFER TO GRANULAR BEDS
FROM VIBRATING PLATES

DISSERTATION
Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By
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ABSTRACT

Because high rates of heat flow to granular solids, especially in vacuum, are very difficult to achieve, it is of interest to determine whether such heat transfer may be enhanced by vibration of the heat transfer surface. Specifically, this investigation studied the effects of the amplitude and/or frequency of vibration and the depth of the granular bed on the heat transfer coefficient between plate and bed. Two bed depths (1-1/2 and 3 in.) were used, while amplitudes (peak-to-peak) were 0 to 0.04 in. and frequencies were 0 to 3210 cpm.

The heat transfer coefficient between a hot plate and a granular bed may be conveniently measured when the bed consists of a material which will sublime under operating conditions. With no inert gas present, heat flow to the bed will control this process. Naphthalene (6-8 mesh particles) was chosen as the test material, and heat transfer coefficients were calculated from the bulk flow of naphthalene from a vaporizer chamber containing a vibrating hot plate to a separate condenser chamber.

The results of a factorial experiment design indicated that the heat transfer coefficient between the hot plate and the bed was not significantly affected by bed depth in the range studied, but that it was strongly dependent on some interaction between amplitude and frequency. Coefficients were about 15 Btu/ft²hr. F for stagnant beds, increased to above 30 with vibration, and fell off to about 6 at the maximum amplitude and frequency used. The data were found to correlate with the maximum velocity of the heat transfer surface, and changes in
the coefficient with vibration could be explained by relating them to the forces acting in and on the granular bed.
OBJECTIVE

The work done under this research program constitutes an exploratory study of the general problem of whether or not the inherently poor heat transfer between a solid surface and a granular bed in vacuum may be enhanced appreciably by vibration of the heat transfer surface.

The specific objective of this study was to determine whether the amplitude and/or frequency of the vibrating heat transfer surface or the depth of the granular bed significantly affect the heat transfer coefficients between these vibrating plates and granular beds.
ENGINEERING IMPORTANCE

High rates of heat flow to granular solids are very difficult to achieve, and where high vacuum conditions are specified, heat transfer rates are even poorer because the gas convection heat transfer mechanism is virtually eliminated and heat conduction controls. The development of fluidized bed techniques has been a big step towards the solution of this problem, but in vacuum, fluidization is frequently impractical. The chief solution in this case has been agitation of the granular bed. In recent years much work has been done on this problem, but there is still a great lack of information that is necessary for good design of equipment for heating granular beds in vacuum.

Some areas of industrial processing making use of heat transfer to granular solids under vacuum include vacuum drying, prevention of caking during screening operations, regeneration of adsorbents, reactivation of catalysts, and perhaps most importantly, simple sublimation. Commercially, simple sublimation processes are found in two major divisions: vacuum sublimation and freeze-drying.

Vacuum sublimation, while applicable to only a limited number of materials, has certain attributes. It can be used where contamination from air and other gases is not permissible, thereby enabling recovery of an extremely pure product. Also, vacuum sublimation eliminates the hazard of dust explosions (41). Vacuum sublimation (40) can be used 1) where the material is unstable or temperature sensitive, 2) to get the proper physical properties of the product, 3) where volatile impurities are to be removed from a nonvolatile and heat sensitive product,
4) where the material has a high melting point at which temperature corrosion is high, and 5) to separate a mixture of volatile solids. Sublimation is also economical if the volatile material is mixed with a high percentage of nonvolatile material. A fairly large proportion of the materials susceptible to sublimation are now being commercially sublimed. These include (40):

<table>
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<th>Magnesium</th>
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<td>Anthranilic acid</td>
<td>Naphthalene</td>
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<td>Anthraquinone</td>
<td>3-Naphthol</td>
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<tr>
<td>Benzantrone</td>
<td>Phthalic anhydride</td>
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<td>Benzoic acid</td>
<td>o-Pthalimide</td>
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<td>Calcium</td>
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<td>Camphor</td>
<td>Salicylic acid</td>
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<td>Ice (Freeze-drying)</td>
<td>Sulphur</td>
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<td>Iodine</td>
<td>Uranium hexafluoride</td>
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Many of these are small-volume, high-priced chemicals, but sublimation is also used in the separation and purification of certain chemicals (salicylic acid, magnesium, calcium, etc.) in tons/day quantities (41), (57).

Freeze-drying, a relative newcomer to industry, is of prime importance in the manufacture of many pharmaceuticals, especially biologicals and vaccines, and in the processing of foods for indefinite storage without refrigeration. These products include extracts, citrus juices, meats, milk, viruses, serums, bacterial cultures, antibiotics such as penicillin, streptomycin and terramycin, cortisones, hormones, vitamins, blood plasma, and gamma globulin (4). Among the advantages of freeze-drying are 1) potency is retained, 2) the product is sterile, 3) the product reconstitutes readily, 4) volatile constituents are retained, 5) there is little or no coagulation, 6) there is no oxidation, 7) foaming is avoided, and 3) there is no case-hardening of the product (4), (3).
LITERATURE REVIEW

Little or no information is available in the literature concerning heat transfer to granular solids from vibrating plates. At least one manufacturer of vibrating conveyors, and very probably more, has worked on this problem, but no data have been published.

The principle was applied before 1938 to the large-scale drying of molybdenite concentrate (19), and a small amount of work was done on this problem at Battelle Memorial Institute in 1951. In the more recent investigation at Battelle, there was evidence that the sublimation rate of a powder being heated in vacuum on a vibrating plate far exceeded the sublimation rate of a solid cake being heated on a fin tube. Certain advantages in the use of vibratory equipment in general have been listed by Van Antwerpen (50). These are 1) there is little corrosion due to the lack of bearings or rolling surfaces, 2) no internal lubrication is required, and 3) power consumption is not excessive.

A great amount of work has been done on heat transfer to granular solids in fluidized beds (13), (52). A patent has even been granted dealing with the use of sonic vibrations to aid fluidization (5). Resnick and White (44) investigated entrainer sublimation of naphthalene into air, hydrogen, and carbon dioxide. In 1948, Brinn, Friedman, Gluckert, and Pigford (12) studied heat transfer to sand flowing in a settled condition through a single vertical pipe jacketed with a heat transfer fluid. This investigation examined the factors which influence heat transfer in this type of equipment and adapted
existing theory and equations for rod-like flow of fluid to this solids case.

The actual equipment now in use for heating granular solids includes among others, screw conveyors, vibrating conveyors, spray chambers, rotary and tray-type dryers, and ordinary agitated kettles (4), (42). In addition to steam, hot water, electric and direct gas-fired heating, radiant heaters or induction heaters are sometimes used (3), (57). To promote heat transfer in these apparatus, large heat transfer areas, uniform, thin layers of small particles, and agitation are recommended (11), (42), (55).

As explained in the section on Analysis of the Problem, conductive heat transfer is the controlling mechanism in a study of this type. According to Fourier's law (38), heat is conducted at a rate \( \frac{dQ}{dt} \) which is equal to the product of three terms,

1) the area for heat flow taken at right angles to the direction of flow,
2) the temperature gradient in the direction of heat flow, and
3) a proportionality constant, \( k \), known as the thermal conductivity.

Expressing this relationship mathematically gives:

\[
\frac{dQ}{dt} = -kA \frac{dT}{dx}
\]  

(1)

At steady state, the rate of heat transfer is constant and so:

\[
q = -kA \frac{T}{dx}
\]  

(2)
The thermal conductivities of solids vary greatly, with the metals, in general, having the higher conductivities. Solid thermal conductivities are normally independent of pressure but dependent on the temperature. For most homogeneous solids, this temperature relationship is approximately linear, but the temperature coefficient may be either positive or negative (38). For crystals it is normally negative (27).

As shown in Table I, the thermal conductivities of liquids are smaller than those for solids, and gas conductivities are smaller than those for liquids. Accurate thermal conductivity data for fluids are difficult to obtain because of the presence of convection currents in the measuring apparatus. The k's of most liquids decrease as temperature increases with the notable exception of water and certain aqueous solutions. Pressure has relatively little effect on liquid k's and this may be neglected under normal operating conditions.

### Table I. Thermal Conductivities of Various Materials at Room Temperature (38)

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal Conductivity (Btu)/(hr.)(ft.²)(°E/ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metals</td>
<td>5 - 240</td>
</tr>
<tr>
<td>Building and insulating materials</td>
<td>0.02 - 2</td>
</tr>
<tr>
<td>Water</td>
<td>0.35</td>
</tr>
<tr>
<td>Liquids</td>
<td>0.05 - 0.3</td>
</tr>
<tr>
<td>Air</td>
<td>0.015</td>
</tr>
<tr>
<td>Gases and vapors</td>
<td>0.004 - 0.02</td>
</tr>
</tbody>
</table>
The thermal conductivities of gases have been more often examined theoretically because of the existence of the kinetic theory of gases. Prior to 1890 Maxwell (37) had derived the relationship between thermal conductivity, viscosity, and specific heat at constant volume:

\[ k = B \nu C_v \]  

(3)

An empirical formula for \( B \) has been suggested by Eucken (38) which, when combined with Maxwell's equation above, gives:

\[ \frac{C_p \nu}{k} = \frac{4}{9-5/\gamma} \]

This relationship is useful in convection heat transfer and in estimating \( \nu \) or \( k \) if only one of the two is known.

The thermal conductivity of a gas does not vary linearly with temperature. The relationship may be represented by a Sutherland type equation,

\[ k_T = k_492^{\frac{C_s}{T_0R + C_s}} \left( \frac{T_0R}{492} \right)^{3/2} \]

(5)

where \( T_0R \) is the absolute temperature and \( C_s \) is an empirical constant.

According to the kinetic theory, when the mean free path of the gas molecules is small compared with the distance over which heat is being conducted, \( k_{\text{gas}} \) should be independent of the pressure. This condition is met at all except very low pressures, i.e., when mean free path becomes large relative to the container dimensions. At these very low pressures, \( k_{\text{gas}} \) is directly proportional to the pressure (17).
theory of heat conduction at these pressures has been developed from two points of view. Knudsen (32) considered the energy transfer of individual molecules impinging on a surface, while Smoluchowski (47) used the concept of a temperature jump or discontinuity at a gas-solid interface.

Knudsen derived the following equation for energy transfer from a hot to a cold surface in the molecular flow region \( (17), (33), (36), (49) \),

\[
E_0 = \frac{\kappa (Y + 1) P_{\text{mol}}}{T_2} (T_1 - T_2)
\]

Converting this equation to practical units gives,

\[
q_{\text{fmc}} = 22.2 \frac{\lambda_0 A_{\text{fmc}} P_{\text{mm}}(T_1 - T_2)}{\sqrt{T_2} \times 459} \text{ Btu/hr.}
\]

In this equation, \( \lambda_0 \) is called the free-molecule conductivity. It is defined by:

\[
\lambda_0 = \frac{25.9}{\sqrt{M}} \frac{Y + 1}{Y - 1} \text{ Btu/hr.}^0\text{R.} \text{ft.}^2\text{mm.Hg}
\]

The symbol \( \kappa \) represents the "accommodation coefficient". This coefficient indicates the fractional extent to which those molecules that strike the surface and are reflected from it have their mean energy adjusted toward complete equilibrium with the wall temperature. Jeans (28) presents an equation for \( \kappa \) relating it to the masses of the molecules involved:

\[
\kappa = \frac{\lambda_{\text{mm}}}{(m + m')^2} = 1 - \left(\frac{m - m'}{m + m'}\right)^2
\]

Amdur (2) in 1946 claimed that \( \kappa \) is affected by the amount of gas which is adsorbed on the surface and is, therefore, pressure dependent. Thomas and Brown (48) in 1950 disputed this and stated that \( \kappa \) remains constant as pressure is reduced toward zero.
Smoluchowski (47) detected that at low pressures an appreciable temperature jump occurs at a gas-solid interface through which heat is being conducted. He explained that when a hot gas is in contact with a cooler surface, "the molecules going toward the cool plate are endowed with a greater energy than that which would correspond to the temperature of the plate since they are coming from hotter regions. Those going out from it after rebounding have only its (the plate's) exact temperature even if there is a complete equalization of temperature during impact on the plate (i.e., if \( \alpha = 1 \)). Therefore, the mean value of (the temperatures of) both must be greater than the temperature of the plate itself and there must be a temperature jump." This temperature jump at a wall bounding an unequally heated gas is analogous to the phenomenon of viscous slip at low pressures.

Smoluchowski proposed that for two parallel planes separated by a distance, \( s_{mc} \), the heat loss per unit time may be represented by:

\[
q = \frac{kA(T_1 - T_2)}{s_{mc} + sj_1 + sj_2}
\]

(10)

Here, \( sj \), the jump distance is defined by:

\[
\Delta T_j = -sj \frac{dT}{dx}
\]

(11)

Jump distances may be thought of as hypothetical distances behind each wall which, when added to the true distance between the walls, give the wall separation required to produce a constant temperature gradient with no discontinuity at either wall. According to Kennard (31), the jump distance at a surface may be calculated from:
\[ s_j = \frac{2}{\gamma - 1} \cdot \frac{2}{\gamma + 1} \cdot L = \beta' L \]  

(12)

where, \( \gamma = \frac{9\gamma - 5}{4} \)

For the case where \( s_{j1} = s_{j2} \),

\[ q = \frac{kA(T_1 - T_2)}{s_{mc} + 2s_j} = \frac{kA(T_1 - T_2)}{s_{mc} + 2\beta' L} \]  

(13)

Then, since the mean free path, \( L \), is inversely proportional to the pressure, \( 2\beta' L = C/P_{mm} \) and

\[ q = \frac{kA(T_1 - T_2)}{s_{mc} + C/P_{mm}} \]  

(14)

For very low pressures, \( s_{mc} \) becomes insignificant in relation to \( C/P_{mm} \) and

\[ q = \frac{kA(T_1 - T_2)}{C/P_{mm}} \]  

(15)

That is, \( q \) is independent of \( s_{mc} \) and directly proportional to the pressure. This is identical to the conclusion arrived at by Knudsen with his independent approach.

Many papers have been written concerning investigations of the effective thermal conductivities (\( k_{eff} \)) of granular beds. Most of these investigators measure \( k_{eff} \) experimentally, vary certain factors which affect the conductivity, and then express \( k_{eff} \) empirically. Schumann and Voss (45) state the limitation to a theoretical approach to this problem when they say, "For a system of granulated material of undefined shape and where packing is irregular, a strict mathematical analysis is virtually impossible."
Nevertheless, investigators have made considerable headway towards an understanding of this problem. In 1926 Aberdeen and Laby (1) found that measured conductivities were the same for a number of materials and, therefore, concluded that $k_{\text{eff}}$ was controlled by the gas phase. Waddams (51), in his paper, agreed that for low density materials, such as silox powder, conduction through the solid phase is negligible because of poor thermal contact. However, for other solids, where thermal contact is better, the solid phase conductivity is quite important. Waddams describes several methods for the measurement of $k_{\text{eff}}$ and he states regarding the mechanism of heat transfer that 1) convection is small because the gas is divided into very small volumes, and 2) radiation is reduced because the powder acts as a series of lamanae between the walls. He adds that theoretically heat transfer through granular materials should be largely independent of particle size.

Awbery (7) makes this same statement but limits independence of $k_{\text{eff}}$ from particle size to cases where there is no heat transfer between the solid and gas phases.

Schumann and Voss (45) claim that $k_{\text{eff}}$ is independent of particle size as long as the packing remains constant. However, they showed experimentally that $k_{\text{eff}}$ increases rapidly as closeness of packing increases. They say further that the effective thermal conductivity of a granular bed will not be proportional to $k_{\text{solid}}$ unless $k_{\text{solid}}/k_{\text{gas}}$ is constant. They caution that just because $k_{\text{gas}}$ is usually much smaller than $k_{\text{solid}}$, it cannot be taken as effectively zero since the gas phase plays a very important part in conduction of heat through such a bed.
If a bed were in a perfect vacuum, and if the grains touched only at mathematical points, $k_{\text{eff}}$ would be zero (there would actually remain a small, finite conductivity due to radiation). In practice, according to Schumann and Voss (45), as pressure is reduced $k_{\text{eff}}$ decreases and approaches zero, but ultimately levels out at some "residual conductivity" value because grains do not have contact only at mathematical points. Their data show that the gas phase becomes important, i.e., $k_{\text{eff}}$ is greater than the residual conductivity, above a pressure of about 100 microns of mercury.

The reduction in $k_{\text{eff}}$ as pressure is lowered could be caused by a corresponding decrease in heat transfer by convection since convection is affected by changes in gas density. However, Schumann and Voss argue that convection is not important in heat transfer through granular beds since interstices between grains are so small that convection currents could not be set up unless extremely high temperature gradients existed. Instead, they feel that any change in $k_{\text{eff}}$ with pressure is caused by a corresponding change in $k_{\text{gas}}$ as pressure is raised or lowered. Their paper claims that in a granular bed, mean free path is always close to the container dimension in the vicinity of grain contact; therefore, since according to the kinetic theory $k_{\text{gas}}$ is no longer independent of pressure when the mean free path approaches the container dimension, the effective thermal conductivity of a granular bed varies with pressure even at fairly high pressures.

Argo and Smith (6) have investigated the problem of radial heat flow in a packed bed through which a gas is flowing axially.
At normal pressures, only a small fraction of the heat transferred radially is by conduction in the gas and solid phases in the bed. This agrees with conclusions of other authors (13), (14). Argo and Smith propose a method for predicting $k_{\text{eff}}$ in packed beds by summing the contributions of each mechanism by which heat is transferred in that bed. These include 1) conduction in the gas phase, 2) convection in the gas phase, 3) radiation and 4) a series mechanism accounting for the heat flow from particle to particle by radiation, conduction, and convection.

Eian and Deissler (18) measured $k_{\text{eff}}$ of granular beds of magnesium oxide, stainless steel, and uranium oxide in air, helium or argon. The granular material was placed in an annular space whose inner wall was heated electrically while its outer wall was cooled by convection. Temperature gradients were measured by thermocouples placed at various radial positions in the powder. Average bed temperatures were between 120 and 1455°F, and gas pressures were varied from 14 to 264 psia. Under these conditions, $k_{\text{eff}}$s ranged from 0.2 to 1.5 Btu/hr.ft.$^2$(°F./ft.).

Eian and Deissler use the "Knudsen equation" to predict the pressure below which $k_{\text{eff}}$ begins to vary appreciably with pressure, i.e., the pressure below which mean free path becomes appreciable relative to the interstitial dimensions. This "breakaway pressure" is given by:

$$P_b = 6.35 \times 10^{-22} \frac{T \text{ in °R}}{\frac{\text{mm.Hg}}{\sqrt{2D_p}}}$$

(16)
where, \( T_{OR} \) = gas temp., °R.
\( \sigma_N \) = mol. diam. determined from viscosity, ft.
\( D_p \) = particle diam., ft.

Their results indicate that \( k_{gas} \) is more important than \( k_{solid} \) in determining \( k_{eff} \).

Kannuluik and Martin (30) present a similar method for finding thermal conductivities of gases or granular materials. They measured \( k_{eff} \) for carborundum, magnesium oxide, glass, and diphenylamine powders in hydrogen, air, carbon dioxide, and helium. Gas pressures were varied from 5 to 760 mm. Hg giving \( k_{eff} \)'s in the range 0.03 to 0.3 Btu/hr*ft.\(^2\)\(^{(\circ F./ft.)}\). In their equipment an electrically heated silver wire was mounted axially in a powder-filled glass tube. Heat entered the powder from the hot wire and was removed by an ice-water bath in which the glass tube was immersed.

Girton and Potter (21), using equipment of the same general type as Eian and Deissler, employed vibration to pack the powder whose effective thermal conductivity was to be measured. Size segregation of particles from one end of the annulus to the other was possible due to this vibration, but they point out that "if there was a segregation of the particles according to size, the data do not indicate any consistent effect of such segregation on the apparent thermal conductivity". Effective thermal conductivities range from 0.04 Btu/hr*ft.\(^2\)\(^{(\circ F./ft.)}\) for calcium carbonate powder to 0.5 Btu/hr*ft.\(^2\)\(^{(\circ F./ft.)}\) for a copper-copper oxide eutectic powder. The gas phase present was air at atmospheric pressure.
Wilhelm, et. al. (56), list a number of properties which quiescent granular beds seem to have in common. These are:

1. Heat transfer is almost purely conductive provided the particle size, the gas pressure, or the temperature are not too high. Rough upper limits are 3-4 mm. diameter, 8-10 atm., and 300 C respectively.

2. The effective thermal conductivity is more dependent on $k_{\text{gas}}$ than on $k_{\text{solid}}$. Thus, in the case of the usual packed beds of solids, $k_{\text{eff}}$ is low and tends to approach that of the gas in the interstices.

3. The effective thermal conductivity of the bed rapidly approaches $k_{\text{solid}}$ when the per cent voids become small.

4. Granular systems exhibit a pressure dependence of $k_{\text{eff}}$ which sets in at much higher gas pressures than is ordinarily observed for gases in unpacked containers. This effect varies with particle size and is attributed to the mean free path becoming comparable to the interstitial dimensions.
ANALYSIS OF THE PROBLEM

Introduction

The conventional method for expressing the ability of a solid to conduct heat is to assign to the solid a thermal conductivity. Similarly, the ease with which heat will flow across a film is expressed by a heat transfer coefficient. This problem concerns the study of heat flow across an interfacial film between a hot vibrating plate and a granular bed; to assess the efficiency of this heat transfer, a coefficient, \( h_i \), must be measured. This heat transfer coefficient will then be used in an equation of the form:

\[
\frac{Q}{\Theta} = h_i A_s (T_s - T_g)
\]  

(17)

Hence, in order to evaluate \( h_i \), we must measure \( Q, \Theta, A_s, T_s, \) and \( T_g \). For three of these quantities, no difficulty will be encountered, viz., \( \Theta, A_s, \) and \( T_g \). The time period, \( Q \), may be arbitrarily set; the area of the heat transfer surface, \( A_s \), may be measured directly; and the surface temperature of the heat transfer plate, \( T_s \), may be measured with suitable instruments. The determination of \( h_i \) then depends upon evaluating \( Q \) and \( T_g \).

Two general types of experiments involving heat transfer to granular solids are possible. In one, the solid has a negligible vapor pressure under operating conditions, while in the other, its vapor pressure is high enough so that appreciable sublimation occurs. The problems encountered and the techniques required in making these
two types of runs are quite different. Sublimation runs can be made under steady-state conditions (latent heat can leave the bed at the same rate as heat enters the bed from the hot plate), while runs using nonvaporizing materials will be transient in nature (no heat leaves the bed and hence the bed temperature continuously approaches the plate temperature).

As stated above, it is necessary to measure $Q$ and $T_g$ in order to find $h_i$. In sublimation experiments, $Q$ may be obtained from a knowledge of the latent heat of sublimation of the material used and the weight of this material sublimed during some test period, $\theta$. Fortunately too, $T_g$ is easy to define when the bed sublimes since under properly chosen conditions (e.g., no inert gas present, negligible pressure drop through the bed, etc.) the entire bed will be essentially isothermal. In the case of a nonvaporizing bed, some characteristic bed temperature would have to be determined experimentally.

**Heat Transfer to a Vaporizing Particle**

Heat may be transferred from a hot plate to a granular bed by three basic mechanisms: radiation, convection, and conduction. In the temperature region involved in this investigation (under 180 F), radiation will be negligible. Also, according to a number of authors (18), (30), (33), (45), (51), (56), the convection mechanism will be of no importance in a granular bed at low pressure. This leaves conduction as the sole means by which heat can be transmitted into the vaporizing bed.
The conduction of heat, however, may take place over any of three routes. First there is solid-to-solid conduction which may occur across any solid-solid interface between plate and particle. Then, as discussed earlier, there are two types of conduction through the gas phase: ordinary molar conduction and the so-called free molecular conduction. Each of these three will transmit a fraction of the total heat flow from plate to particle. Hence,

\[ q_{\text{total}} = q_{ss} + q_{mc} + q_{fmc} \]  

(18)

The total heat flow is also:

\[ q_{\text{total}} = h_i A_s (T_s - T_g) \]  

(19)

And, therefore,

\[ h_i A_s (T_s - T_g) = q_{ss} + q_{mc} + q_{fmc} \]  

(20)

The amount of heat flowing to an individual particle resting on the hot plate by each of the three conduction routes will be determined by the geometry of the particular particle. The total heat transfer area, \( A_s \), is the sum of the areas for heat transfer by the three routes, i.e., \( A_{ss} + A_{mc} + A_{fmc} \). Where particle and plate are actually in contact, heat flow is naturally by solid-to-solid conduction. It will be assumed that there is no interfacial resistance to heat flow so that the particle surface at the interface will assume the hot plate temperature, \( T_s \). The remainder of the particle surface will remain at equilibrium with the vapor phase. This requires that there be a pressure drop \( p_s^0 - p_g^0 \) from Point 1 to Point 2 in Figure 1.
FIGURE 1. HEAT FLOW TO A SINGLE PARTICLE

This pressure drop will drive vapors which are formed at Point 1 out from under the particle. Heat will be conducted through the particle itself from the surface 1-1 to the remainder of the particle surface where it is removed as latent heat. The rate of this heat flow will be limited by the size and shape of the particle, the temperature gradients within the particle, and the thermal conductivity of the particle itself, $k_{\text{solid}}$. If an average effective distance for this heat flow, $L_{ss}$, can be determined, a heat transfer coefficient, $h_{ss}$, can be calculated from:

$$h_{ss} = \frac{k_{\text{solid}}}{L_{ss}}$$  (21)
Then,

\[ q_{ss} = h_{ss} A_{ss} \Delta T_{ss} \]  

(22)

In Figure 1, it is seen that in certain regions the plate-to-particle separation is less than the mean free path of the molecules in the gas phase. In this region heat flow will occur by free-molecular conduction and may be represented by the equation (17), (32):

\[ q_{fmc} = \frac{22.2 \kappa T_{\text{mm}}}{\sqrt{T_g} + 459} A_{fmc} T_{fmc} \]  

(23)

A heat transfer coefficient for this region may then be defined as:

\[ h_{fmc} = \frac{22.2 \kappa T_{\text{mm}}}{\sqrt{T_g} + 459} \]  

(24)

And,

\[ q_{fmc} = h_{fmc} A_{fmc} \Delta T_{fmc} \]  

(25)

Finally, heat may be conducted from plate to particle by gas conduction in the region of plate-particle separation greater than the mean free path. According to Smoluchowski's temperature discontinuity theory (17), (47),

\[ q_{mc} = \frac{kA_{mc} \Delta T_{mc}}{s_{mc} + s_{j1} + s_{j2}} \]  

(26)

This equation could, of course, be used even in the free molecular conduction region. However, it contains the vapor phase thermal conductivity which is generally not known too accurately and it contains the jump distances, \( s_{j1} \) and \( s_{j2} \) which are also in some doubt. Because of this, Equation (23) probably represents free molecular
conduction more satisfactorily. The heat transfer coefficient for this region is then:

\[ h_{mc} = \frac{k}{smc + sj1 + sj2} \]  

(27)

Since \( sj \) is on the order of the mean free path distance, it is seen that \( h_{mc} = k/sm \) for \( smc \gg mfp \). The heat transferred in this region is then:

\[ q_{mc} = h_{mc}A_{mc} \Delta T_{mc} \]  

(28)

Now, combining Equations (19), (21), (24), and (27) gives,

\[ h_{i}A_{s}(T_s - T_g) = h_{ss}A_{ss} \Delta T_{ss} + h_{fmc}A_{fmc} \Delta T_{fmc} + h_{mc}A_{mc} \Delta T_{mc} \]  

(29)

The temperature driving forces for each route can be arbitrarily defined as equalling \( T_s - T_g \), so that:

\[ h_{i}A_{s} = h_{ss}A_{ss} + h_{fmc}A_{fmc} + h_{mc}A_{mc} \]  

(30)

Dividing by \( A_{s} \),

\[ h_{i} = \frac{A_{ss}}{A_{s}}h_{ss} + \frac{A_{fmc}}{A_{s}}h_{fmc} + \frac{A_{mc}}{A_{s}}h_{mc} \]  

(31)

That is, the measured film coefficient, \( h_{i} \), is equal to the sum of the heat transfer coefficients for each of the three heat flow routes times the fraction of the total heat transfer area used by each route.

Unfortunately, the above analysis depends primarily on a knowledge of the geometry involved. For a vaporizing bed, the geometry
is continually changing, and if the hot plate is vibrating, causing individual particles to bounce, an attack of the problem from this standpoint would be most difficult.

However, two limiting cases may be examined for a single subliming particle resting on a nonvibrating hot plate. In the first of these it may be assumed that a spherical particle at $T_g$ in equilibrium with the vapor phase is brought into contact with the hot plate at $T_S$. With a knowledge of the physical properties of the vapor phase, a value for $h_1$ may be calculated from Equation (31). Such a calculation is shown in Appendix C. Of course, for this case, $q_{ss}$ is zero since there is no solid-to-solid contact area.

The second case is somewhat more complex. Here it is assumed that the flat surface of a hemisphere at $T_g$ is brought into contact with the hot plate at $T_S$. Two sub-cases may be imagined. In one, the contact pressure between particle and plate is greater than the vapor pressure of the particle at $T_g$, while in the other it is less. This determines whether solid-to-solid contact will occur or whether the particle will "float" on a film of vapor. This is made more clear if a force balance is written for the particle. The downward forces on the hemisphere include that due to gravity and that due to the pressure of the surrounding fluid, $p_g$, on the horizontal projected area of the particle. The only upward force is the reaction of the plate against the particle over the same projected area. Hence, at equilibrium,

$$\text{Force downward} = \text{Force upward (contact area x contact pressure)}$$
\[ \frac{m_p g}{g_c} + \frac{p_g D^2 \pi}{4} = \frac{P_{contact} D^2 \pi}{4} \]  

(32)

or,

\[ P_{contact} = \frac{4m_p g}{D^2 g \pi} + p_g \]  

(33)

If the vapor pressure of the solid at the plate temperature, \( p_0 \), is equal to or greater than \( P_{contact} \), there can be no solid-solid contact; on the other hand, if \( p_0 \) is less than \( P_{contact} \), there must exist a finite area of solid-solid contact to support the particle.

If the particle floats, heat transfer will again be entirely by conduction through the gas phase. A pressure drop will occur in the gas film underneath the particle between its center and its periphery so that the average pressure in the film will be between \( p_0 \) and \( p_s \). If the average pressure and an average plate-to-particle separation were known, the heat flow could be calculated. On the other hand, if the particle does not float, the particle must initially vaporize around the circumference of the solid-to-solid contact area until the pressure drop from the perimeter of the solid-solid contact area to the external vapor space is \( p_0 - p_s \). That is, because of the assumption of zero interfacial resistance to heat flow, the particle temperature at Point 1 in Figure 1 must be \( T_s \) and the vapor pressure must be \( p_s \). The particle in this area would, in effect, flash vaporize until the required pressure drop is achieved. The particle would then appear as in Figure 2. If the average length of the undercut were known, the rate of solid-solid heat transfer could be
approximated by the relaxation method for steady-state conduction outlined by McAdams (39).

FIGURE 2. HEMISPHERICAL PARTICLE ON HOT PLATE AFTER FLASH VAPORIZATION

Use of a Subliming Bed

In sublimation processes, such as were carried out in this investigation, any of five different steps may be controlling. These steps are 1) rate of heat flow to the solid in the vaporizer, 2) rate of change from solid to gas, 3) rate of transport of mass from vaporizer to condenser, 4) rate of change from gas to solid, or 5) rate of heat flow from the solid in the condenser. Of these, Steps 2 and 4 are of no practical significance in simple sublimation (42).

For the idealized case where heat transfer in the vaporizer is completely controlling, we may assume that mass transfer between vaporizer and condenser and heat transfer in the condenser are each capable of infinite rates. Because of this, their driving forces, $\Delta T_c$ in the condenser and $\Delta p$ between the vaporizer and the condenser, will be zero. Therefore, it can be seen that the temperature of the granular bed will be $T_c$, the condenser temperature, and that the pressure throughout the equipment will be $p_c^0$, the vapor pressure of
the solid at the condenser temperature. With the vaporizer heat transfer surface held at $T_s$, the $\Delta T$ in the vaporizer will be $T_s - T_c$, and the rate of vaporization will be controlled solely by the value of $h_1$ and the area of the vaporizer heat transfer surface, $A_s$. For a constant $A_s$, $T_s$, and $T_c$,

$$\frac{Q}{\Theta} = h_1 A_s (T_s - T_c) = K_1 h_1$$  \hspace{1cm} (34)$$

and the rate of heat transfer will be a function of only $h_1$. Now, the rate of heat transfer, $(Q/\Theta)$, and the rate of vaporization, $(W/\Theta)$, are related by the latent heat of sublimation, $H$:

$$W/\Theta = (Q/\Theta)/H$$ \hspace{1cm} (35)$$

Hence,

$$W/\Theta = (K_1 h_1)/H = K_1 h_1$$ \hspace{1cm} (36)$$

For the second case, when heat transfer in the condenser completely controls, there will be a zero $\Delta T$ in the vaporizer and again a zero $\Delta p$ between vaporizer and condenser. Here, the temperature and the pressure throughout the equipment will be $T_s$ and $p_s^0$ respectively. The rate of heat transfer in the condenser will be,

$$\frac{Q}{\Theta} = h_c A_c (T_s - T_c)$$ \hspace{1cm} (37)$$

And, for a constant $A_c$, $T_s$, and $T_c$,

$$\frac{Q}{\Theta} = K_2 h_c$$ \hspace{1cm} (38)$$
And,

\[ W/\theta = (K_{2h_c})/H = K_h^i h_c \]  \tag{39}

When the transfer of mass from vaporizer to condenser is assumed to completely control the sublimation process, there will be zero \( \Delta T \)'s in both vaporizer and condenser, but a finite \( \Delta p \) will exist between these two sections. The temperature and pressure in the vaporizer will be \( T_s \) and \( p_s^0 \) while those in the condenser will be \( T_c \) and \( p_c^0 \). Then, \( \Delta p = p_s^0 - p_c^0 \). Now, depending on the temperatures \( T_s \) and \( T_c \) and the size and geometry of the connecting system, we may have any flow conditions ranging from purely molecular flow, through a transition region to turbulent flow. The rate of sublimation will be limited by the proper fluid flow relationships depending on which type of flow or combination of types of flow exist.

In an actual sublimation, none of the above idealized conditions may be reached. Finite driving forces for each of the three steps must exist. Fortunately, however, there is a way in which we can cause the rate of mass transport to be completely controlling even though finite \( \Delta T \)'s occur in both vaporizer and condenser. This may be done by separating the vaporizing and condensing chambers with an orifice and operating under such conditions that sonic flow takes place through this orifice. With this type of apparatus, and choosing proper operating conditions, the heat transfer coefficient, \( h_1 \), may be measured at steady-state.

At this point let us visualize what is occurring in the sublimation chamber. For one thing, heat is flowing into the granular
bed at a decreasing rate as the temperature of the bed increases (the \( \Delta T \) becomes smaller). At the same time, the pressure within the vaporizer is rising (vapor pressure increases as bed temperature rises) so that the flow of vapors, and therefore latent heat, out of the vaporizer increases with temperature. It is readily apparent that an equilibrium condition will be reached in which the heat into the vaporizer equals the heat out. In this steady-state, the heat in and the heat out of the vaporizer may be represented:

\[
\frac{Q}{\theta}_{\text{in}} = h_1 A_s (T_s - T_g) \tag{40}
\]

\[
\frac{Q}{\theta}_{\text{out}} = G_T A_0 H_T \tag{41}
\]

Therefore,

\[
h_1 A_s (T_s - T_g) = G_T A_0 H_T \tag{42}
\]

And,

\[
h_1 = \frac{(G_T A_0 H_T) \sqrt{A_s (T_s - T_g)}}{A_s (T_s - T_g)} \tag{43}
\]

In this equation,

\[
G_T = f_1(T_g)
\]

\[
A_0 = \text{a constant}
\]

\[
H_T = f_2(T_g)
\]

\[
A_s = \text{a constant}
\]

\[
T_s = \text{a constant}
\]

Or, in other words,

\[
h_1 = f_3(T_g)
\]
Therefore, if 1) the pressure and temperature upstream from the orifice are the vapor pressure and corresponding temperature of the solid, 2) the ratio of specific heats, $\gamma$, for the vapor is known, 3) the discharge coefficient for the orifice is known, 4) the latent heat of sublimation of the solid is known as a function of temperature, and 5) the molecular weight of the vapor is known, it is possible to calculate a series of curves for a particular system, solely from theory, which give $h_1$ as a function of only $T_g$ with the orifice diameter and $T_g$ as parameters. Such curves are presented in Figure 3. These particular curves are valid for a sublimate having a molecular weight of 128, a $\gamma$ of 1.2, and a latent heat of sublimation of 220 Btu/lb. mass (see Appendix D for sample calculation and data for these curves).

In addition to the method just presented for the calculation of $h_1$, its value may be calculated in two other ways. Both of these procedures use this form of the basic equation,

$$h_1 = \frac{(Q/\theta)}{\sqrt{A_3(T_s - T_g)}}$$

and differ only in the method of calculation of $Q/\theta$. In one, the electrical power input to the heat transfer plate is metered, and this value converted to $Q/\theta$. By the third method the sublimate is collected during a measured test period, $\theta$. This weight, multiplied by the latent heat of sublimation, gives $Q$. The heat transfer coefficients calculated in this last manner are considered as "actual" values, while those found by the other two short-cut methods are designated "theoretical". In the results of this study the heat transfer coefficients obtained by these three methods are compared.
FIGURE 3. $h_i$ AS A FUNCTION OF THE TEMPERATURE WITHIN THE VAPORIZER
Design of Experiments

The heat transfer coefficient between a vibrating hot plate and a subliming granular bed may conceivably be affected by a rather large number of factors. Of these, however, some will be specific to the particular experimental technique employed. The effects of others can be studied in fixed beds \(^{(1)}\), and only a few remaining factors are basic to the study of heat transfer to a subliming bed from a vibrating plate. Among these are 1) the amplitude of vibration, 2) the frequency of vibration, 3) the type of motion of the vibrating plate, i.e., simple harmonic motion, etc., 4) the depth of the vaporizing bed, and 5) the elasticity of both the hot plate and the granular particles. Of these five factors, three were chosen for study: amplitude, frequency, and bed depth. The other two were held constant by 1) vibrating the hot plate in simple harmonic motion for all runs, and 2) using the same bed material and hot plate for all runs. While it is realized that other factors may affect \(h_1\), it must be assumed that any effects of these other factors will be responsible for the degree of reproducibility.

The experiments chosen to ascertain the effects of the three primary variables were arranged according to a factorial design. According to the factorial design principle, replicate observations

\(1\) These include 1) the size, size distribution, and shape of the particles within the bed, 2) the physical properties of the particles and the gas phase, e.g., specific heat, thermal conductivity, etc., 3) the pressure of any inert gas present, 4) the temperature of the hot plate, 5) the surface characteristics of both hot plate and particles, and 6) the packing of the granular bed.
should not be carried out under exactly the same conditions. One factor should always be altered from one experiment to the next. When this is done, the experimental design is more efficient and more useful results are likely to be obtained (58). When factorial design is utilized, a statistical technique known as "analysis of variance" may be applied for examination of the data to determine the presence of primary effects of the variables as well as the effects of interactions among the variables. The experimental design used is shown in Table III, page 73.

Because only three factors were chosen for study in this investigation, a dimensional analysis approach was not required. All three of the factors to be studied are unidimensional terms and two have the same dimension, length; these two would, therefore, be indistinguishable from one another in such an analysis.
EQUIPMENT AND CHOICE OF TEST MATERIAL

Introduction

To perform the necessary functions for this study, equipment of original design was needed. The apparatus was built to perform a specific purpose, but it could easily be adapted for other types of investigations (e.g., measurement of orifice discharge coefficients, measurement of vapor phase molecular weights, etc.) as well as for other work on heat transfer from vibrating plates.

Choice of Test Material

A number of substances including benzoic acid, camphor, phthalic anhydride, and salysilic acid were considered for use as the test material before the final selection of naphthalene. This choice was made because naphthalene is readily available, cheap, relatively nontoxic, and has a suitable vapor pressure relationship. The vapor pressure of naphthalene is high enough to give adequate sublimation rates below 200 F and yet low enough so that sample weights, etc., would not change appreciably during processing. In addition, since temperatures in the vaporizer can be maintained above 100 F while the temperature in the condenser should never be above 50 F, if ice water is used as the coolant, the maximum pressure ratio which could exist across the orifice is 0.067. This assures us that sonic flow will occur since the critical pressure ratio for naphthalene is 0.56 \((35)\).
The material actually used was Eastman Organic Chemicals Flake Naphthalene P-168. This material has a maximum melting point depression of 1.2°C. From this, it can be shown (see Appendix E) that the maximum deviation in vapor pressure of this material from that for pure naphthalene is 2 per cent. A plot of the vapor pressure of naphthalene vs. temperature (1/T) is presented in Figure 4. Data for this plot appear in Appendix F.

The latent heat of sublimation of naphthalene was calculated by the Clausius-Clapeyron equation as shown in Appendix G. The Clausius-Clapeyron equation should apply rigorously to this case because 1) vapors will obey the perfect gas law at the low pressures involved, and 2) the solid molar volume will be completely negligible compared to the gas molar volume at these pressures.

The Equipment

Introduction

The test equipment for this investigation was constructed in a laboratory at the Battelle Memorial Institute in Columbus, Ohio. A floor space of approximately 5 x 15 ft. was needed for the four major space-taking components: the vacuum chamber and water jacket, the recording null-point galvanometer, the hot water storage tank, and the condenser water storage tank. An over-all view of this equipment is shown in Figure 5.

Figure 6 is a schematic diagram of the equipment. During an experimental run, naphthalene is vaporized from the heat transfer cup within the vaporizer chamber. The naphthalene vapors then pass
FIGURE 4. VAPOR PRESSURE OF NAPHTHALENE FROM 40-176 F.


A-14370
FIGURE 5. OVER-ALL VIEW OF THE EQUIPMENT
The vacuum chamber (vaporizer and condenser chambers) is completely surrounded by a water jacket not shown in this diagram.

The vibrator is located directly under the heat-transfer cup.

FIGURE 6. SCHEMATIC DIAGRAM OF EQUIPMENT
through one of the orifices at sonic velocity and are condensed on the fintubes within the condenser chambers. The fintubes are cooled internally by circulating ice water. Cold traps were installed as a safety precaution to prevent possible passage of naphthalene vapors into the vacuum pump. The McLeod gage indicates the pressure at the vacuum pump and the pressure control manometer and solenoid valve work in conjunction, as described later on, to maintain a desired vacuum. Details of these various units are described on the following pages.

The Heat Transfer Cup

The heat transfer surface, a nickel-plated copper disc 3/4 in. thick and 11-15/16 in. in diameter, composes the bottom of a circular cup 6 in. deep. The wall of this cup is of 1/2 in. thick, Grade LN-5, Formica. Three concentric electric ring heaters of 1250 w. total capacity are clamped to the bottom of the copper disc. A copper-constantan thermocouple buried within 1/32 in. of the top surface of this disc at its center gives indication of the surface temperature and acts as the triggering element for the surface temperature control system described later on in this section. Figure 7 is a sketch of the heat transfer cup, its mounting, and the upper end of the vibration transmission shaft. The photograph in Figure 8 shows the heat transfer cup mounted in the vaporizer chamber.
FIGURE 7  CUTAWAY VIEW OF HEAT-TRANSFER CUP AND MOUNTING
FIGURE 8. HEAT TRANSFER CUP MOUNTED IN VAPORIZER CHAMBER
The Vibrator

To produce vibration the load rides on an eccentric cam. This cam consists of a No. 99511 New Departure ball bearing whose outer race supports the load without revolving while its inner race, mounted on an eccentrically drilled hub, revolves with the drive shaft. The vibratory motion is transmitted to the heat transfer cup through the vibration transmission shaft. Motion is restricted to the vertical direction by a 5-3/4-in. long guide bearing around this shaft.

In order for the results of this study to be reproducible, or in fact to be of any value, the motion of the heat transfer surface had to follow some regular pattern. Simple harmonic motion was chosen as the easiest to produce and the most reasonable. Appendix H presents a mathematical proof that the load, riding as it does on an eccentric cam, will move in simple harmonic motion. Figures 9 and 10 show the drive shaft, its supporting bearings, and the eccentric cam bearing. The amplitude of vibration can be changed by removing one cam and replacing it with another of different eccentricity. Frequencies are varied by using different pulley ratios between the vibrator drive shaft and the vibrator motor as shown in Figure 11.

Springs are required to hold the load down on the rotating cam; otherwise the cam would, in effect, fall out from under the load. The forces which these springs must exert may be calculated from (46):

\[ F_{sp} = \frac{(Ea_1)}{E_c} - \frac{(E_g)}{E_c} \]  \hspace{1cm} (45)
FIGURE 9. VIBRATOR DRIVE SHAFT, ECCENTRIC CAM, SPRING MOUNTINGS, AND LOWER END OF VIBRATION TRANSMISSION SHAFT
FIGURE 10. VIBRATOR DRIVE SHAFT, ECCENTRIC CAM, AND SUPPORTING BEARINGS

A-14355
FIGURE 11. PULLEYS CONNECTING VIBRATOR MOTOR AND VIBRATOR DRIVE SHAFT
\[ a_m = 2\pi^2 sn^2 \]  

(46)

And, therefore,

\[ F_{sp.} = (2\pi^2/g)Esn^2 - (Eg)/g_c \]  

(47)

As an example, the spring force required for a mass of 70 lb., an amplitude of 0.04 in., and a frequency of 3210 cpm. would be 341 lb. force. Three compression springs, in mountings as shown in Figures 9 and 12 and used in the vibrator assembly. Each spring is capable of exerting up to 400 lb. force. The vibrator drive shaft, which must carry this load through the cam, was designed so that its deflection, even under a spring loading of more than 1300 lb. force would be less than 0.001 in. Such precautions in design were necessary in order to be sure that the vibratory movement of the heat transfer cup would be as close to perfect simple harmonic motion as possible.

The vibrator assembly was purposely designed so that a Syntron electric vibrator, Model V15B, could be substituted for the mechanical vibrator just described. This feature will be convenient should electrically produced vibrations be desired for experiments in the future. The Syntron vibrator was actually considered first for use alone on the equipment. However, the electric vibrator could not be successfully adapted to produce more than one frequency (3600 cpm.). An electronic frequency regulator was built for this purpose (see Figure 13). It used a No. EL C3J Thyatron tube to block out one out of every two, or two out of every three, or three out of every four, etc., cycles from reaching the vibrator. However, the fact that each
FIGURE 12. COMPRESSION SPRINGS AND SPRING MOUNTINGS

A-14374
FIGURE 13. ELECTRONIC FREQUENCY REGULATOR FOR SYNTRON VIBRATOR

A-14375
electrical impulse retained a characteristic 60-cycle wave form, combined with the fact that the mass and springs of the vibrator itself were mechanically tuned to 60 cycles per second, resulted in a 60-cycle vibration output regardless of the actual frequency of the electrical pulses fed to the vibrator. Because of this, the mechanical vibrator described above was used during this study.

The Vacuum Chamber

The vacuum chamber consists of two main parts, the vaporizer chamber, and the condenser chamber. Details of the construction of the vaporizer chamber are shown in Figure 14. A neoprene O-ring fitted in the groove shown in the large top flange of the vaporizer chamber acts as an excellent vacuum seal. The vibration transmission shaft, which passes from the atmosphere through the water jacket and into the bottom of the vaporizer chamber was sealed in place by three short lengths of Gooch tubing as indicated in Figure 15.

Orifice holders were installed in each of the outlet pipes leading to the condenser chamber. In these holders, shown in Figure 14, could be mounted sharp-edged orifices of the size desired. Those available included orifices of 2, 1, 1/2, 1/4, 1/8, and 1/16 in. diameter.

Four-inch Keystone vacuum butterfly valves were installed between the vaporizer and each of the two condenser chambers so that either one or both of these chambers could be used. Each condenser chamber consists of a shell surrounding an extruded aluminum fin tube.
FIGURE 14. VAPORIZER CHAMBER
FIGURE 15. SEAL AROUND VIBRATION-TRANSMISSION SHAFT
These components are shown in Figure 16 and again the groove indicated in the fintube flange holds a neoprene O-ring as a vacuum seal. Ice water is pumped through bayonet tubes and into each fintube by a small close-coupled pump of 1-1/4 gpm. capacity. The condenser water is cooled in the condenser water storage tank directly with crushed ice.

Each condenser chamber is followed by an individual, dry ice-cooled cold trap. Behind these traps are installed a tilting McLeod gage having a range of 0.01 to 10 mm. Hg and a Cenco Hypervac vacuum pump.

The Water Jacket

A water jacket of approximately 40-gal. capacity was required. This jacket covers the entire vacuum chamber with a blanket of hot water approximately 2-in. thick during each experimental run. This jacket, shown in Figure 17, was necessary to prevent heat losses from the system and to prevent any condensation of the sublimate on the vacuum chamber walls.

During experimental runs, hot water is circulated through this jacket at approximately 8 gpm. The water is maintained at the desired temperature by three electric immersion heaters of 2300-watt total capacity operated through a relay by a Fenwal temperature controller. Even though the jacket is covered on all sides with aluminum foil-backed Fiberglas insulation, heat losses are rather large because of escaping water vapor. Because of this, the jacket heaters are satisfactory for maintaining a desired temperature but are incapable of
Shell (2 Required) Material: Steel

Shell (2 Required) Material: Steel

Fintube (2 Required) Material: Aluminum

**FIGURE 16. CONDENSER CHAMBER**

A-14342
heating water from room temperature in a reasonable time. To overcome this difficulty and to provide storage for the water from the jacket while runs were not in progress, a well-insulated hot water storage tank was built. The water is ordinarily kept in this tank and only pumped into the jacket during actual runs. An air line running into the jacket itself is used to hasten drying when the jacket is emptied, thereby reducing corrosion. The storage tank is equipped with electric immersion heaters of 3000-watt capacity operated in a similar manner to those in the jacket by a Fenwal controller. Water can be heated from room temperature to 160 F overnight with 1000 watts or in about 3-1/2 hours if all 3000 watts are used.

The Control Panel

The control panel, which is shown in Figure 5, includes controls for the following units: the vacuum pump, the pressure controller, the recording null-point galvanometer, the ring heaters, the condenser water pump, the hot water pump, the water jacket and storage tank heaters, the mechanical vibrator, and the Syntron electric vibrator. Circuit diagrams for the panel board are given in Appendix I. Of these, only two need explanation.

The pressure controller is a device for holding the pressure within the vacuum chamber constant by admitting small amounts of air or other gas into the system through a 3/8-in. solenoid valve (see Figure 6). This valve is opened automatically when the pressure in the vacuum chamber falls below the control setting. As the pressure falls, the mercury level in a closed end U-tube manometer (see Figures
60

6 and 38) rises and comes in contact with a nickel wire, closing the circuit to the solenoid. This device is also used to return the system to atmospheric pressure at the end of a run.

It is also necessary to hold the heat transfer surface temperature at a constant value throughout a run. As stated earlier, the copper-constantan thermocouple buried in the copper hot plate is the triggering element for this control. The emf from this thermocouple is "bucked" with the emf from a portable potentiometer (the temperature equivalent of the setting of this bucking potentiometer is termed the "control point"). The resultant zero emf is indicated by a Brown electronic recording null-point galvanometer. Included in this galvanometer is a micro-switch which can be adjusted to open and close at the zero point of this instrument. This micro-switch shorts out an adjustable shunt resistance when the surface temperature begins to fall. The increased voltage to the ring heaters then causes the surface temperature to rise until the loss has been recovered. When the control point is reached, as indicated by a zero reading of the galvanometer, the micro-switch opens, the current is forced to travel through the additional adjustable resistance, the voltage drop across the ring heaters is decreased, and the surface temperature again begins to fall. By proper adjustment of the total applied voltage with a Variac and by setting the adjustable resistance to a minimum value, cycling of this controller can be practically eliminated.
Equipment Calibration

The accuracy with which the heat transfer surface temperature is known is of great importance in the calculations of the results of this study. Tests were carried out to determine the true heat transfer surface temperatures as closely as possible.

Using a Pyrocon Surface Pyrometer, a traverse of the hot plate at the surface temperature to be used in the experimental runs (approximately 168°F) was made. Surface temperatures could be read with a precision of ± 0.5°F and readings at all locations on the plate were the same. Once this was known, it was evident that temperature gradients, at least greater than 1°F, did not occur on the plate. However, the actual surface temperature of the plate was known no better than before because of the many inaccuracies inherent in ordinary surface temperature measurement. These inaccuracies are discussed by Bailey (8) and by Colburn and Hougen (15). They include:

1. If the measuring element is held against the surface, the gas film causes a temperature drop.

2. If the cross-section of the measuring element is large, the conduction of heat along the element may introduce a large error.

(2) Originally a stainless steel hot plate had been installed in the heat transfer cup. A similar traverse over this plate showed that a gradient of as much as 14°F existed between the central area and a point on the circumference where the electrical connections to the outer ring heater had been made. This gradient was eliminated in the hot plate actually used by 1) increasing the power density (watts per unit area of heat transfer surface) of the outer ring heater, 2) rotating the three concentric ring heaters so that their electrical connectors were not lined up along one radius, 3) increasing the plate thickness from 7/16 in. to 3/4 in., and 4) changing from stainless steel to the more highly conductive material, copper.
3. If the measuring element has appreciable exposed surface and the surroundings are not at the surface temperature, heat radiated may cause considerable error.

4. The thermocouple or other measuring element may act as an insulator itself and change the normal surface temperature.

To overcome these difficulties, a surface temperature measuring device was built based upon an idea presented by Boyer and Buss (10). The device consists of a copper rod, 1/2 in. long and 1/2 in. in diameter, surrounded on all sides except the bottom by at least 1/4 in. of Formica insulation. A small electric heater is laid on the top surface of the copper rod and is electrically insulated from the copper with glycerol-litharge cement. Two copper-constantan thermocouples are imbedded in the copper; one is 1/16 in. from the bottom (uninsulated surface) of the copper rod, and the second is 1/4 in. above the first. The two thermocouples are wired as shown in Figure 18 so that the difference in their readings is indicated on a null-point galvanometer. The emf from either thermocouple may also be read with this arrangement (calibration curves for these thermocouples appear in Appendix J).

After first placing the exposed end of the copper rod on the surface whose temperature is to be measured, the current to the small heater on top of the copper rod is adjusted until the net heat flow in a vertical direction through the rod is cut to zero. The condition at which this zero heat flow occurs will be when there is a zero temperature gradient in a vertical direction through the rod.
FIGURE 18. WIRING DIAGRAM FOR SURFACE-TEMPERATURE-MEASURING DEVICE
This condition is indicated by the null-point galvanometer, and when it has been reached, the readings of the thermocouples should be the true surface temperature. These measurements were quite tedious and time-consuming, but surface temperatures accurate to 0.2 C could be measured. Figure 19, prepared by the use of this procedure, gives the correction to be applied to the control point to find the true plate temperature.

It was also necessary to determine whether or not the surface temperature varies as the heat losses from the heat transfer surface change. To check this, the surface was brought to a control point of 176 F with no cover on the vaporizer chamber and no water in the jacket. The surface temperature was measured with the Pryocon Surface Pyrometer and the power required to hold this control point was metered at 67 watts. Next, hot water was pumped into the water jacket and the cover was placed on the vacuum chamber. After steady-state had been reached, the power input was reduced to 20 watts but the measured surface temperature was exactly the same as before. Therefore, the surface temperature is a function of only the control setting and is not affected by changes in power to the ring heaters.

In order to determine the effective power input to the ring heaters during experimental runs, it was necessary to correct the measured power to these heaters by subtracting any power losses. Such losses were quite small, as would be expected, and were measured at 2.5 watts for a heat transfer surface temperature and water jacket temperature of 168 F. No measurable increase in bed temperature could be found after extended periods of vibration at room temperature.
FIGURE 19. CORRECTION TO BE APPLIED TO CONTROL POINT TO DETERMINE TRUE SURFACE TEMPERATURE
Should heat be generated within the bed in this manner, however, it will be inherent for the particular vibration condition and, because of its small magnitude, it need not be corrected for.

The mercury bulb thermometer used for measuring the temperature within the vaporizer was calibrated against a Bureau of Standards thermometer. The correction curve for this thermometer is presented in Appendix J.
PROCEDURE

Preparation of Test Material

To prepare the naphthalene for use in the experimental runs, it is first melted over an electric hot plate in an aluminum foil-lined porcelain bucket. The solid cake, formed when this naphthalene cools to room temperature, is removed from the bucket in one piece, and broken up with a hammer into lumps no larger than 2 to 3 in. in their greatest dimension. These pieces are then crushed in a jaw crus her after which the naphthalene is screened with Ro-Tap equipment into the desired size fractions. The fines and unwanted size fractions are again melted for reprocessing. Normally, sample preparation was carried out once or twice a week so that an ample supply of screened naphthalene was always on hand.

Making an Experimental Run

Values are chosen for the four independent variables (amplitude, frequency, bed depth and particle size). On the day previous to the experimental run, the proper cam is installed on the vibrator drive shaft to give the chosen amplitude, and appropriate pulleys are placed on the drive shaft and vibrator motor to produce the chosen frequency. The hold-down springs are then adjusted to the compression which will be necessary to hold the load continuously in contact with the eccentric cam. Finally, make-up water is added to the hot water storage tank and the storage tank heaters are turned on so that the water may be heated overnight. The Fenwal controller in the storage tank is adjusted so
that the final water temperature will be approximately the same as the
temperature of the heat transfer surface in the vaporizer. The fintubes
are installed in the condenser chambers and the desired orifices are
placed in the orifice holders between the vaporizer and condenser
chambers. The vacuum butterfly valve leading to the No. 2 condenser
chamber is closed while the valve to the No. 1 condenser is left open.

On the morning the run is to be made, approximately 45 minutes
before the start of the experimental run itself, the recording null-
point galvanometer is turned on. This instrument is allowed to warm
up, with its input short-circuited, for at least one-half hour before
the start of each run; this warm-up period assures that the indicated
null-point will remain constant.

The contents of a 75-lb. bag of crushed ice are dumped into
the condenser water storage tank, and the two air-powered agitators in
this tank are turned on. Dry ice is then charged to each cold trap.
After 10 to 15 minutes, the condenser pump is started to begin the cir-
culation of ice water through the fintubes.

Finally, a weighed amount of granular naphthalene of the
chosen particle size is charged to the vaporizer cup, the top flange is
sealed on the vaporizer chamber, the vaporizer thermometer is placed in
position through this top flange (the bulb of this thermometer is
stationed approximately 1/2 in. above the heat transfer surface), and
the experimental run is ready to begin.

Since all runs of this study are to be made at the lowest
pressure at which the equipment can operate, the pressure controller is
not used during these runs. Instead, the vacuum pump is turned on and
allowed to pump the system down as far as possible (this is normally in
the range 80 to 100 microns Hg). When the pressure has reached about
150 microns, the immersion heaters in the hot water storage tank are
turned off and the water from this tank is pumped up into the water
jacket. When the jacket is full, the water it contains is circulated
through the hot water pump and the temperature controller is set to main-
tain the jacket temperature at about 10 F below the heat transfer surface
temperature. While the water is being pumped into the water jacket, the
"bucking" potentiometer is standardized and crushed ice is added to the
thermocouple cold junction.

Next, the vibrator is turned on. The recording null-point
galvanometer is then switched into the heat transfer surface control
circuit, the ring heaters are turned on, and the hot plate is brought
to the control point as quickly as possible. The voltage to the ring
heaters is periodically adjusted throughout the run to minimize cycling
about the control point. In addition, the water jacket temperature is
regulated to a point 3 to 4 degrees above the temperature within the
vaporizer. During the run, the power to the ring heaters (voltage x
current), the vaporizer temperature, the water jacket temperature, and
the pressure at the vacuum pump are recorded.

When the heat transfer surface temperature is cycling gently
about the control point, the water jacket temperature is slightly above
the temperature with the vaporizer, and the temperature within the
vaporizer becomes constant, the "test period" is started. This is done
by reversing the positions of the two vacuum butterfly valves. This
reversed position (No. 1 closed and No. 2 open) is maintained for a
measured time interval, usually 15 minutes, and then the valves are returned to their original positions. To reach the steady-state condition required during the test period, normally two to three hours are required.

The solenoid valve to the vacuum pump is then opened, the vacuum pump turned off, and the system returned to atmospheric pressure. The equipment is shut down by turning off the vibrator, the ring heaters, the recording galvanometer, and the jacket immersion heaters. The hot water is drained from the water jacket into the storage tank, and the cover is removed from the vaporizer chamber. The naphthalene remaining in the heat transfer cup is screened and weighed, and any naphthalene condensed on the walls of the heat transfer cup is also weighed. The condenser water pump and the agitators in the condenser water storage tank are turned off, and then both fintubes are removed from the condenser chambers. The condensed naphthalene on these fintubes is scraped off each one and weighed.

The heat transfer coefficient, $h_i$, is then calculated from the data of the run (see Appendix K for sample calculation).
RESULTS

The original data for all experimental runs are given in Table X, Appendix L. The results of the calculations for each run, based upon these data, are presented in Table II.

The heat transfer coefficients based upon the actual weight of naphthalene sublimed during each test period were transferred to Table III for ease of comparison with the four independent variables. This transfer could be performed directly for all runs except Nos. 10 and 12. In these two runs, no test period was used, and the values given in Table III were obtained by dividing the heat transfer coefficients based on the measured vaporizer temperatures by the average ratio (1.029) of the heat transfer coefficients found in these two ways for all other runs.

Figures 20, 21, and 22 represent respectively the variation of the heat transfer coefficient, $h_1$, with amplitude of vibration, frequency of vibration, and maximum velocity of the vibrating heat transfer surface (see Table VII, page 90 for data). The values of $h_1$ based on actual vaporization rates are the ones plotted.
<table>
<thead>
<tr>
<th>Run No.</th>
<th>Frequency, cpm.</th>
<th>Frequency, in.</th>
<th>Bed Depth, in.</th>
<th>Nominal Particle Size, mesh</th>
<th>Heat Transfer Surface Temperature, °F</th>
<th>Loss (3) lb.</th>
<th>Actual Vaporization Rate, lb./hr.</th>
<th>Theoretical Vaporization Rate Based On Power Input lb./hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1400</td>
<td>0.04</td>
<td>1 1/2</td>
<td>6-8</td>
<td>171.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1a</td>
<td>1400</td>
<td>0.04</td>
<td>1 1/2</td>
<td>6-8</td>
<td>171.0</td>
<td>0.06</td>
<td>1.137</td>
<td>1.244</td>
</tr>
<tr>
<td>1b</td>
<td>1400</td>
<td>0.04</td>
<td>1 1/2</td>
<td>6-8</td>
<td>167.6</td>
<td>0.12</td>
<td>1.101</td>
<td>1.324</td>
</tr>
<tr>
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<td>1 1/2</td>
<td>6-8</td>
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<td>0.11</td>
<td>1.225</td>
<td>1.770</td>
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<td>0</td>
<td>0</td>
<td>1 1/2</td>
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<td>0.19</td>
<td>1.040</td>
<td>1.228</td>
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<td>0.04</td>
<td>3</td>
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<td>0.943</td>
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<td>0</td>
<td>1 1/2</td>
<td>14-20</td>
<td>167.9</td>
<td>0.08</td>
<td>1.066</td>
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(3) Loss = amount of naphthalene lost during handling (see pages 84-85).
(4) ΔT = temperature driving force for heat transfer.
(5) "Run" indicates the entire period of vaporization; "test period" indicates the period of vapor
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(\text{i}) Indicates the period of vaporization into the No. 2 condenser chamber.
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Denser chamber.
TABLE III. EXPERIMENTAL RESULTS

Dependent variable: \( h_1 \) (Btu/ft.\(^2\)hr.\(^\circ\)F.)
Bracketed figures indicate run numbers

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</tr>
<tr>
<td>Frequency</td>
<td>685</td>
<td>1400</td>
</tr>
<tr>
<td>0.02</td>
<td>17.0 (9)</td>
<td>16.4 (25)</td>
</tr>
<tr>
<td>0.04</td>
<td>20.0 (15)</td>
<td>39.6 (22)</td>
</tr>
<tr>
<td></td>
<td>18.2 (16)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>pre-vibr.</td>
<td></td>
</tr>
<tr>
<td>Particle Size, in.</td>
<td>0.056 [10-14 mesh]</td>
<td>0.040 [14-20 mesh]</td>
</tr>
<tr>
<td>-------------------</td>
<td>----------------------</td>
<td>--------------------</td>
</tr>
<tr>
<td><strong>Initial Bed Depth, in.</strong></td>
<td><strong>Initial Bed Depth, in.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
<td><strong>685</strong></td>
<td><strong>1400</strong></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.1 (4)</td>
<td>16.6 (11)</td>
<td>17.5 (20)</td>
</tr>
<tr>
<td>19.4 (17)</td>
<td>pre-vibr.</td>
<td>31.6 (21)</td>
</tr>
</tbody>
</table>
FIGURE 20. VARIATION IN HEAT-TRANSFER COEFFICIENT WITH AMPLITUDE OF VIBRATION AT CONSTANT FREQUENCY
FIGURE 21. VARIATION IN HEAT-TRANSFER COEFFICIENT WITH FREQUENCY OF VIBRATION AT CONSTANT AMPLITUDE
FIGURE 22. VARIATION IN HEAT-TRANSFER COEFFICIENT WITH MAXIMUM VELOCITY OF VIBRATING
HEAT-TRANSFER SURFACE
DISCUSSION OF RESULTS

Rejection of Runs

A total of 38 experimental runs were attempted during this investigation. Of these, seven were made before the start of an organized experimental program. These "preliminary runs" were made using impure naphthalene of odd particle size, and were carried out solely for the purpose of locating equipment weaknesses and to devise a workable experimental procedure. Of the seven, two were not completed, in another the bed caked completely solid, and in each of the other four a major equipment failure occurred. The remaining 31 runs were carried out according to the factorial design of experiments described in the section on Analysis of the Problem. The results of eight of these 31 had to be rejected for the reasons given in Table IV.

One problem encountered in these runs was caking of the granular bed, at times so solidly that the cake could only be broken with a hammer. This caking evidently occurred during the initial heat-up period, and was caused by the condensation of vapors on cold particles within the bed. Caking appears to become more serious as particle size is reduced (surface to volume ratio for particles increases) and as the bulk density of the bed is increased. It will be noted from Table II that no serious caking was encountered when using 6-8 mesh particles. Caking to a slight degree occurred when the bed bulk density was increased by previbration as in Run Nos. 15, 16, and 17. In Run No. 19, made with a smaller particle size (10-14 mesh), the bed did not cake at all, presumably because of relative movement of particles within the bed.
TABLE IV. REASONS FOR REJECTION OF RUNS

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Reason for Rejection</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Run not completed because McLeod gage failed</td>
</tr>
<tr>
<td>1a</td>
<td>Naphthalene melted on heat transfer surface</td>
</tr>
<tr>
<td>1b</td>
<td>Stem of one vacuum valve broke</td>
</tr>
<tr>
<td>6</td>
<td>Granular bed caked (14-20 mesh particles)</td>
</tr>
<tr>
<td>7</td>
<td>Granular bed caked (14-20 mesh particles)</td>
</tr>
<tr>
<td>8</td>
<td>Granular bed caked (14-20 mesh particles)</td>
</tr>
<tr>
<td>12a</td>
<td>Ring heaters failed and steady-state was never reached</td>
</tr>
<tr>
<td>13</td>
<td>Cold trap cracked during run</td>
</tr>
</tbody>
</table>

due to vibration. Caking predominated, however, in the runs made with 14-20 mesh particles. Only Run No. 5 was completely free of caking, and this run combined high amplitude (0.04 in.) and fairly high frequency (1400 cpm.) with low bed depth (1-1/2 in.). Run No. 18, the same as Run No. 5 except that a 0.02 in amplitude was used, produced only slight caking of the bed. The other three runs with this particle size (Run Nos. 6, 7, and 8) involved higher bed depths and/or no vibration at all, and complete caking resulted. Runs in which serious caking occurred were rejected because information on heat transfer to granular beds was being sought, and once caking developed, the beds were no longer granular.
The losses listed in Table II as occurring during each run are somewhat misleading, and this point should be cleared up. These values indicate the total weight loss of naphthalene from start to finish of each run; all of which occurred either before the naphthalene was sealed in the vacuum chamber or after it was removed, i.e., no weight loss could occur while the naphthalene was sealed within the vacuum chamber, since it was found that negligible amounts of naphthalene passed into the cold traps. A certain amount of spillage occurred during transfer of the naphthalene from one container to another. This type of loss was especially significant during removal of the hot naphthalene from the heat transfer cup after completion of a run. Also, the vaporization of hot naphthalene during this transfer accounted for a part of the total loss. The fact that a weight gain (i.e., a negative loss) occurred in several runs is easily explained. Since little care was exercised in the cleaning of the walls of the heat transfer cup and the No. 1 fin tube between runs, varying amounts of naphthalene were left on these surfaces from one run to the next. Consequently, naphthalene remaining from a previous run was often removed after the following run was made. If this excess were sufficient, a weight gain would be indicated. Since the losses were not continuous throughout the run, relatively large values in the loss column of Table II should not be cause for rejection.

Statistical Evaluation of the Data

The existence or nonexistence of primary effects, and even of interactions of the three major variables studied (amplitude, frequency,
bed depth) seems quite clear-cut. The three experimental runs involving different particle sizes did not give sufficient information to predict anything, except in a most general way, about the effect of particle size on \( h_1 \). Inspection of the heat transfer coefficients obtained (see Table III and Figures 20 and 21) indicates that:

1. A very strong interaction between amplitude and frequency exists.
2. No other interactions appear important.
3. For a constant amplitude, frequency of vibration has a strong primary effect, and this effect causes a "peak" value in \( h_1 \) as frequency increases.
4. The same indication holds for amplitude with respect to frequency.
5. Neither amplitude nor frequency appear to have a large primary effect when considered over a range of values of the other.
6. Bed depth has little, if any, effect on \( h_1 \).
7. It appears from the runs made with varied particle sizes that \( D_p \) might have an effect on \( h_1 \), and \( h_1 \) may be more sensitive to this effect when the heat transfer coefficient is at or near its peak values.

An analysis of variance, which has its greatest value in giving most of the above information when such is not immediately evident in the results, can still be of use. We can verify the indications given above, offer a certain "confidence" for these statements, and
even calculate confidence limits for the experimental data independently of those found by absolute duplication of experimental runs.

In Appendix M are presented several variance analyses of the data. From these we may draw the following conclusions:

1. With a confidence greater than 97.5 per cent, there is a frequency x amplitude interaction affecting the heat transfer coefficient. The analysis of variance does not give the particular function of s and n which is involved.

2. On an over-all basis, there are no other significant primary effects or interactions.

3. For an amplitude of 0.02 in., the confidence is over 99.5 per cent that frequency has a significant effect on $h_1$, while at an amplitude of 0.04 in. the confidence is not quite so high, but is still better than 99 per cent.

4. The error variance for the data is 6.86, i.e., the standard deviation for the values of $h_1$ is

$$\sigma = \sqrt{6.86} = 2.62 \text{ Btu/ft}^2\text{hr.}^\circ\text{F.}$$

Hence, the 95 per cent confidence limits for the data are

$$\pm 1.96\sigma = \pm 5.1 \text{ Btu/ft}^2\text{hr.}^\circ\text{F.}$$

Confidence limits for the data may also be roughly estimated by a short-cut method outlined by Proschan and Babcock (43). From the four pairs of replicate observations (see Table V), the 95 per cent confidence limits may be estimated at $\pm 3.5 \text{ Btu/ft}^2\text{hr.}^\circ\text{F.}$ In addition, since bed depth has been demonstrated to be of negligible effect, runs
made under otherwise identical conditions may be thought of as replicates. In this case, Proschan and Babcock’s method gives 95 per cent confidence limits of $\pm 2.6 \text{ Btu/ft}^2 \text{hr.} ^\circ \text{F.}$ (see Appendix M for calculation of these confidence limits).

Contributing to the experimental error or error variance, which are directly related to confidence limits, are such items as the effects of changes in particle size or bed depth during a run. For instance, normally at the conclusion of each run made with 6-8 mesh particles, the naphthalene remaining in the heat transfer cup was screened. It was found that, in all runs but four, over 90 per cent of the naphthalene was still in the 6-8 mesh range, i.e., average particle size remained effectively constant throughout each run (see Table II). The four exceptions were Run Nos. 10, 12, 23, and 24: the only four runs made at 3210 cpm. The reduction of the 6-8 mesh fraction to about
80 per cent for these runs could have been caused by increased attrition at this high frequency. The fact that the average particle size decreased only slightly during a run even though in some cases over half of the bed was vaporized, indicates a preferential vaporization of those particles near the hot plate. Because of this, the true particle size at the heat transfer surface was not held constant during a run and any changes in \( h_1 \) due to this effect are included in the error variance.

Bed depth also continuously decreased during each run as naphthalene vaporized from the bed. In this case, however, an average depth could be evaluated for each test period, since the final weight of naphthalene in the heat transfer cup was measured immediately after the conclusion of the test period. Because the amount vaporized during the test period had been measured, an average bed depth could be found if necessary. This average depth would be fairly accurate over the entire test period since, as seen from Table II, no more than 11 per cent of the initial bed weight was vaporized during any test period.

**Estimation of Maximum Measurement Error**

It is also possible to calculate a maximum measurement error in \( h_1 \) from estimated errors in individual measurements. In the calculation of a heat transfer coefficient, an equation of the general form below is used in which the quantities within the brackets are the ones actually measured.

\[
h_1 = K \left[ \frac{W'}{\bar{g}D_s^2(T_s - T_g)} \right]
\]  

(48)
Table VI gives the estimated accuracies of the individual measurements. From this table it is evident that other measurement errors are negligible with respect to the errors in \((T_s - T_g)\).

**TABLE VI. ESTIMATED PRECISION OF INDIVIDUAL MEASUREMENTS**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Maximum Error</th>
<th>Magnitude of Quantity</th>
<th>Maximum per cent Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W)</td>
<td>±0.3 g.</td>
<td>max. = 181.8 g.</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>min. = 72.8 g.</td>
<td>0.42</td>
</tr>
<tr>
<td>(G)</td>
<td>±2 sec.</td>
<td>max. = 900 sec.</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>min. = 600 sec.</td>
<td>0.33</td>
</tr>
<tr>
<td>(D_s)</td>
<td>±1/64 in.</td>
<td>11-15/16 in.</td>
<td>0.13</td>
</tr>
<tr>
<td>(T_s)</td>
<td>±0.5 °F.</td>
<td>max. = 169.1 °F.</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>min. = 166.6 °F.</td>
<td>0.30</td>
</tr>
<tr>
<td>(T)</td>
<td>±0.4 °F.</td>
<td>max. = 155.8 °F.</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>min. = 134.2 °F.</td>
<td>0.30</td>
</tr>
<tr>
<td>((T_s - T_g))</td>
<td>±0.9 °F.</td>
<td>max. = 34.3 °F.</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>min. = 10.8 °F.</td>
<td>8.33</td>
</tr>
</tbody>
</table>

Hence, the maximum measurement error in the heat transfer coefficient is approximately:

\[
\text{MME} = \pm \left(0.9\right) \left(100\right) / (T_s - T_g) \text{ per cent}
\]  

\((49)\)

At this point it may be well to emphasize the distinction between confidence limits and maximum measurement error. The **maximum measurement error** gives us the limits within which all measurements of a quantity will lie if the only errors present are those due to...
inadequacies of the methods of measurement. The confidence limits on the other hand, which are obtained by statistical techniques, give us a calculated probability that another replication will yield a value within the stated limits of the average value already obtained.

**Mechanism of Heat Flow**

In the section on Analysis of the Problem it was explained that under the conditions of this investigation, heat transfer into the vaporizing bed would be almost entirely by conduction. Also it was pointed out that the contribution to the total heat transfer by each conduction route (solid-solid conduction, molar conduction through the gas, and free molecular conduction through the gas) is determined by the geometry of the bed packing and the individual particles adjacent to the hot plate.

In Equation 31,

\[ h_i = \frac{A_{ss}h_{ss}}{A_s} + \frac{A_{mc}h_{mc}}{A_s} + \frac{A_{mc}h_{mc}}{A_s} \]  

only \( h_{mc} \) can be directly calculated without a knowledge of the geometry. In Appendix C this is done and \( h_{mc} \) is found to be on the order of 50 Btu/ft.\(^2\)hr.\(^{\circ}\)F. If the average distance for heat transfer through the solid is assumed to \( D_p/8 \), then

\[ h_{ss} = \frac{k_{solid}}{L_{ss}} \]  

\[ h_{ss} = \frac{0.172 \text{ Btu/ft.}^2\text{hr.}(^\circ\text{F}/\text{ft.})}{0.00932 \text{ ft./8}} \]
\[ h_{mc} = \frac{k_{gas}}{S_{mc}} \] (50)

\[ = \frac{0.0056 \text{ Btu/ft.}^2\text{hr.}^{\circ}\text{F.}}{0.00932 \text{ ft.}/8} \approx 5 \text{ Btu/ft.}^2\text{hr.}^{\circ}\text{F.} \]

(see Appendix N for calculation of \( k_{solid} \)). If \( L_{ss} \) is as much as \( D_p/2 \), \( h_{ss} \) would still be on the order of 40 Btu/ft.$^2$hr.°F. Since both \( h_{ss} \) and \( h_{fmc} \) are greater than the \( h_i \)'s obtained, it is evident that \( h_{mc} \) must be smaller than \( h_i \). If an average separation of \( D_p/8 \) is again assumed,

In the preceding paragraph, a value of \( D_p \) was assumed which corresponds to 6-8 mesh particles. However, results indicated preferential vaporization of those particles at the bottom of the bed. Therefore the layer of particles actually in contact with the hot plate was, for the most part, smaller than the 6-8 mesh size. This decreased particle size would, if anything, tend to increase \( h_{ss} \) and \( h_{mc} \), but it would not affect \( h_{fmc} \). A certain amount of naphthalene dust particles would tend to work their way to the bottom of the bed due to the vibration. However, it is not felt that their contribution to the measured \( h_i \) is too great because of their relatively short lives. That is, for a single one of these dust particles, the heat flux to it from the hot
plate should be quite high\(^6\) while its mass is quite small. Such par-
ticles would vaporize rapidly so that a large accumulation of such dust
on the hot plate would not be expected.

Because thermal conductivities are affected by the temperature
(and in the free molecular conduction region by the pressure), the heat
transfer coefficients measured in this study should be temperature de-
pendent (the pressure in the vaporizer is also related to the temperature
by the vapor pressure of the solid). This effect was minimized by hold-
ing the heat transfer surface temperature constant for all runs, but the
average temperature of the interfacial film between bed and plate
\((T_s + T_g)/2\) still varied from 151 to 161 F. \((610\) to \(620\) R.). This de-
viation is less than 2 per cent of the absolute temperature so \(k_{\text{solid}}\)
and \(k_{\text{gas}}\) were essentially the same for all runs. However, from Equation
\((23)\), \(h_{fmc}\) is directly proportional to the pressure, and the vapor
pressure of naphthalene changes from 2.7 mm. at 151 F. to 4.3 mm. Hg at
161 F. Therefore, \(h_{fmc}\) is considerably more affected by the temperature
than either \(h_{ss}\) or \(h_{mc}\). This fact could be of importance in future
studies for use in assessing the relative magnitude of \(h_{fmc}\) as compared
to \(h_{mc}\) and \(h_{ss}\). That is, by changing only the diameter of the sonic
orifice from one run to the next, the pressure within the vaporizer
could be changed while all other factors are held practically constant.
A change in \(h_1\) could then be attributed to a corresponding change in
\(h_{fmc}\). Similarly, the vaporizer pressure may be held constant for all
runs by a proper choice of orifice diameter for each \(h_1\).

\(^6\) The majority of heat transfer to very small particles should be
by free molecular conduction so that the average heat transfer
coefficient to a single particle should approach
\(h_{fmc} \approx 50\) Btu/ft.\(^2\)hr.\(^\circ\)F.
Correlation of the Data

Since the effect of bed depth on the heat transfer coefficient has been shown to be insignificant, and since the analysis of variance indicates a definite correlation between $h_1$ and some function of amplitude and frequency, an attempt was made to find this function.

The correlation one would most expect would be between $h_1$ and some function of $s$ and $n$ having a physical significance. One possibility is that the heat transfer coefficient is directly related to the number of impacts per unit time of the granular bed on the vibrating hot plate. Such a function might be roughly approximated by calculating trajectories of a single, spherical particle of given elasticity bouncing on a flat plate moving in simple harmonic motion. Unfortunately, this possibility must be discarded because the optimum heat transfer coefficients, and hence the regions of greatest interest, occur at vibration conditions under which the maximum acceleration of the vibrating plate, $a_m$, is less than the acceleration of gravity, $g$ (see Appendix O). Theoretically under these conditions, the granular bed, or in a simplified case a single particle, would remain continuously in contact with the plate without bouncing, and a value for "impacts per unit time" could not be calculated.

Efforts to correlate $h_1$ with vibration energy (energy $\propto s^2n^3$) as well as with numerous other simple functions of $s$ and $n$ also failed.

However, there appeared to be a strong relationship between $h_1$ and the maximum velocity of the vibrating plate. This velocity, $v_m$, is related to $s$ and $n$ by:
Table VII presents the converted data using maximum plate velocity and bed depth as the new independent variables. By making use of the plot of these data given in Figure 22, it was possible to construct the curves shown in Figures 20 and 21. The actual experimental data are shown as point values on these plots. As seen in Figure 22, \( h_1 \) has an intermediate value at \( v_m = 0 \) and initially increases slowly with \( v_m \). The heat transfer coefficient then increases more rapidly and reaches a peak value before falling off asymptotically to \( h_1 = 0 \) as \( v_m \) becomes larger.
TABLE VII. CONVERTED DATA USING MAXIMUM PLATE VELOCITY AND BED DEPTH AS INDEPENDENT VARIABLES

Dependent Variable: h₁ (Btu/ft²·hr·°F.)
Bracketed Figures Indicate Run Numbers

<table>
<thead>
<tr>
<th>Maximum Plate Velocity (ft./sec.)</th>
<th>1-1/2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15.7 (2)</td>
<td>14.1 (4)</td>
</tr>
<tr>
<td>17.0 (9)</td>
<td>16.6 (11)</td>
<td></td>
</tr>
<tr>
<td>0.060</td>
<td>16.4 (25)</td>
<td>17.5 (20)</td>
</tr>
<tr>
<td>0.120</td>
<td>39.6 (22)</td>
<td>31.6 (21)</td>
</tr>
<tr>
<td>0.122</td>
<td>32.1 (13a)</td>
<td>32.6 (14)</td>
</tr>
<tr>
<td>0.244</td>
<td>35.3 (14a)</td>
<td></td>
</tr>
<tr>
<td>0.280</td>
<td>19.5 (1c)</td>
<td>21.0 (3)</td>
</tr>
<tr>
<td>0.560</td>
<td>20.7 (10)</td>
<td>19.9 (12)</td>
</tr>
<tr>
<td>0.560</td>
<td>5.9 (24)</td>
<td>5.9 (23)</td>
</tr>
</tbody>
</table>

Interpretation of the Data

Introduction

Unfortunately, there appears to be no physical explanation for a relationship between the heat transfer coefficient and the maximum velocity of the vibrating plate. The apparent relationship must be looked upon as an extremely fortuitous coincidence, which may serve conveniently as a simple artifice for interpolating data of this sort.

The correlation between h₁ and the maximum acceleration of the vibrating plate (aₘ ∝ sₙ²), while not yielding as simple a graphical
relationship, nevertheless presents a curve which more readily lends itself to physical interpretation of the data.

As discussed previously, the heat flux between the hot plate and the granular bed increases rapidly as the separation between the plate and the particles decreases. A maximum heat flux through the gas phase is reached as the distance for heat transfer approaches the mean free path of the gas. Therefore, the majority of the heat transferred may be thought of as flowing through a "true contact area", \( A_t \), which is very nearly the sum of the solid-solid contact area, \( A_{ss} \), and the area for free molecular conduction, \( A_{fmc} \). Considering changes in \( h_1 \) with vibration, it may therefore be reasoned that the heat transfer coefficient from a vibrating plate to a granular bed basically depends on three factors:

1. The "true area of contact" between the hot plate and the bed, \( A_t \).
2. The "true time of contact" between the hot plate and the bed, \( Q_t \).
3. The average "contact pressure" between the hot plate and the bed, \( P_{contact} \).

The true area of contact and the true contact time are related to \( A_s \) and \( Q \) by:

\[
A_{contact} = \frac{A_t}{A_s} = \text{true contact area} \quad \frac{\text{true contact area}}{\text{total area of heat transfer surface}}
\]

\[
Q_{contact} = \frac{Q_t}{Q} = \text{true contact time} \quad \frac{\text{true contact time}}{\text{total time for heat transfer}}
\]
Therefore,

\[ h_1 = f_4(A_{\text{contact}}, Q_{\text{contact}}, P_{\text{contact}}) \]  \hspace{1cm} (52)

True Contact Area

The true area of contact between hot plate and bed depends upon the bulk density of the bed, i.e., the number of contact points between the bed and the hot plate per unit area of the hot plate, and upon the orientation of the bottom layer of particles in the bed, i.e., the average area of actual contact per contact point.

Number of Contact Points. For a constant particle size, changes in bulk density of a granular bed (number of contact points per unit area) with increasing maximum plate acceleration may be qualitatively shown as in Figure 23. That is, a granular bed dumped loosely into a container such as the heat transfer cup will possess a relatively loose packing associated with a bulk density, \( \rho_{bl} \). As vibration of the heat transfer surface is started and gradually increased, no changes in packing occur and the bed, in effect, "does not know it is being vibrated".

At a certain maximum plate acceleration, \( a_{mc} \), the maximum forces on the particles within the bed (from Newton's Law, force = mass x acceleration), exceed a critical value, and the unstable bed configuration begins to shift into a closer packing. When this critical value, which is dependent on the density, surface friction coefficient, etc., of the bed material, is reached, the inertial forces within the bed have become large enough to overcome the frictional forces which have
been holding each individual particle in place. Hence, after these forces are once exceeded, the bed attains a maximum closeness of packing with continuous vibration, and the number of contact points per unit area reaches a maximum level and remains constant. This is shown by the horizontal line at $\rho_{b2}$ in Figure 23.

At $a_m = g$, a new phenomenon occurs as the vibrating plate is actually pulled out from under the bed once during each cycle. In the region of $a_m$ slightly greater than $g$, the bed, because of the close proximity of the individual particles, will absorb all impact energy when the plate again contacts the bed. Hence, the collisions of the bed with the vibrating plate may be assumed to be inelastic, i.e., the bed
does not "bounce". Also, because of the continued downward impacts of particles further up in the bed, it will be assumed that the bed remains in contact with the plate from the instant of inelastic collision until the plate starts its downward movement again in the succeeding vibration cycle. During the period of free fall in each cycle, the bed changes from a maximum closeness of packing to a somewhat looser packing. On collision with the hot plate, the bed will be partially repacked by the impact and will remain in this repacked configuration throughout the remainder of the cycle. The degree of this repacking will depend upon the magnitude of the impulse during collision, which is, in turn, directly proportional to the relative velocity of approach at the instant of contact. Appendix P shows that the relative velocity of approach, \( v_r \), may be either increasing or decreasing with \( a_m \), depending upon the particular amplitude or frequency involved. The bulk density of the bed, therefore, while in contact with the hot plate, can only be shown in Figure 23 by a few of the infinite family of curves in the region \( a_m > g \). Each curve in this family is for a constant amplitude or a constant frequency. The exact shapes of these curves are uncertain and would have to be determined experimentally. Some or all of the curves may reach maximums in this region as the collision impulse reaches a peak and then decreases. It will be noted that each of these curves has a break-point at some value of \( a_m \) beyond which the curve falls off much more rapidly. This break-point, which is again dependent on the particular amplitude or frequency, occurs at \( a_m = a_{m*} \). When \( a_{m*} \) is exceeded, the ability of the bed to absorb impact energy is insufficient, bouncing begins, and the bed begins to take on the familiar "fluidized"
appearance of a vibrating granular bed. When this occurs, bulk density is reduced quite rapidly with increasing $a_m$.

**Area of Contact per Contact Point.** The orientation of the bottom layer of particles in the bed changes in a somewhat similar manner with increasing maximum plate acceleration, as shown in Figure 24. At $a_m = 0$, an average area of contact per point, $A_{pl}$, is associated with the loose packing of an unvibrated bed. With vibration of very low energy, free particles resting on the plate in unstable positions will roll or tip over into a more stable orientation, thus presenting an increased heat transfer area to the hot plate. At $a_m = a_{mc}$, frictional forces within the bed are overcome, and with continuous vibration, as explained previously, a maximum closeness of packing is attained. In this most closely packed configuration, each particle in the bottom
layer of the bed tends to present a maximum contact area to the hot plate as represented by $A_{p2}$.

Until $a_m = g$, no change in particle orientation will occur. At $a_m = g$, however, a sudden change in particle orientation takes place. During the finite time interval during each cycle in which the bed is in free fall, the individual grains are able to rotate, even though but slightly, so that the face presented to the hot plate by each particle is no longer of maximum area. In fact, because the particles should attain random orientations, the average area per contact point touching the hot plate should be constant for $a_m > g$ and should be roughly the same as the average area per contact point presented in a stagnant bed of random orientation. This analysis holds only for particles having a number of faces on their surfaces. With particles having rounded surfaces, such as a bed of spheres, contact area per point is, of course, constant.

**Contact Area per Unit Area of Hot Plate.** Now, true contact area per unit area of heat transfer plate, $A_{contact}$, will be the product of the number of contact points per unit area of the heat transfer surface and the average area per contact point: the product of the curves in Figures 23 and 24. The resultant curve is given in Figure 25.

**True Contact Time**

The contact time per cycle may be evaluated somewhat more quantitatively. By plotting fraction of the cycle time during which plate and bed are in contact, $\theta_{contact}$, vs. $a_m$, the curve in Figure 26
FIGURE 25. CHANGES IN CONTACT AREA PER UNIT AREA OF HEAT TRANSFER SURFACE AS $a_m$ INCREASES

FIGURE 26. CHANGES IN CONTACT TIME BETWEEN BED AND PLATE AS $a_m$ INCREASES
is produced. It will be noted that the bed is continuously in contact with the plate from $a_m = 0$ to $a_m = g$, i.e., $\theta_{\text{contact}} = 1.0$. For $a_m$ greater than $g$, contact time falls off rapidly with increasing $a_m$ (see Appendix P), since plate and bed are completely out of contact from Point $a$ to Point $c$, as shown in Figure 27. Plate and bed remain in contact from Point $c$ to Point $a'$, which is in line with the assumption of inelastic collision in this region. Once again, a family of curves must be shown in the region $a_m > g$, each curve applying to a constant amplitude or frequency. There are no humps as in Figure 23, however, since $\theta_{\text{contact}}$ continuously decreases with increasing $s$ and/or $n$ and is not affected by increased collision impulses. As $a_m$ on each curve in Figure 26 reaches and exceeds $a_{\text{me}}$, however, impact energy can no longer be absorbed by the bed, and bouncing starts. With this, contact time is much more quickly reduced, and it rapidly approaches zero.

![Figure 27](https://example.com/figure27.png)

**FIGURE 27.** RELATIVE MOVEMENT OF VIBRATING PLATE AND GRANULAR BED FOR $g < a_m < a_{\text{me}}$
Contact Pressure

Contact pressure is essentially the force per unit contact area holding a particle in contact with the hot plate; the higher this pressure, the better the efficiency of contact and vice versa. For a stagnant bed, this pressure could be determined, if the contact area were known, since the only force involved is gravity. When the heat transfer surface is vibrating, the situation becomes much more complex.

The force acting between the bed and the plate will be proportional to the mass of the bed and to the relative acceleration of the bed with respect to the vibrating plate. For simple harmonic motion of the plate, its acceleration can be expressed:

\[ a_p = \frac{1}{2} \omega^2 \cos \omega t \]  

(53)

The bed is, of course accelerated downward by gravity at all times, so that the relative acceleration of plate and bed is,

\[ a_r = \left( \frac{1}{2} \right) \omega^2 \cos \omega t - g \]  

(54)

Figure 28 shows the relative acceleration of the bed with respect to the plate for the cases \( a_m < g \) and \( a_m = g \). For these two cases, \( a_m < g \), the \( a_r \) curves are proportional to the instantaneous force existing between plate and bed, and for both of these cases, the time average force is equal to the force of gravity.

For the case where \( a_m > g \), several adjustments have to be made to the \( a_r \) curve for the correct picture. For one thing, bed and plate are not in contact during the initial part of the cycle and hence no
force can act between them for this interval. Secondly, the impulse on collision, which is a function of the relative velocity of approach at the instant of collision, can cause quite high instantaneous forces (very high contact pressures) at impact. Such a curve might appear as in Figure 29.
In previous discussion, it has been pointed out that the relative velocity of approach can range from zero to very high values, depending upon the particular amplitude and frequency of vibration. The peak in Figure 29 represents the change in acceleration of the bed with respect to the plate at the instant of impact, but neither the height of this peak nor the time interval of its duration can be determined (46). Because of this, a time average contact pressure cannot be found for $a_m > g$. However, since the contact pressure is a force per unit area, it is directly proportional to the relative acceleration of the bed and
the plate and to the mass of the bed. It appears, therefore, that contact pressure is not an important factor, since all runs were replicated at two bed depths (the masses of the beds differed by a factor of two between duplicate runs), and no significant change in heat transfer coefficients could be observed.

Relationship of Heat Transfer Coefficient to True Contact Area and True Contact Time

With the above assumption that $P_{contact}$ has no effect on heat transfer, it may be postulated that the amount of heat transferred, $Q$, is proportional to the true contact area and the true contact time:

$$Q \propto (A_t)(\theta_t) = (A_{contact} A_s)(\theta_{contact} \theta)$$  (55)

Hence,

$$Q \propto (A_{contact}) (\theta_{contact})$$  (56)

But, the amount of heat transferred, $Q$, is also proportional to $h_1$ and so,

$$h_1 \propto (A_{contact})(\theta_{contact})$$  (57)

$A_{contact}$ and $\theta_{contact}$ are known graphically as functions of $a_m$ and may be represented,

$$A_{contact} = f_5(a_m)$$

$$\theta_{contact} = f_6(a_m)$$

so,

$$h_1 \propto \left[f_5(a_m)\right] \left[f_6(a_m)\right]$$  (53)

Therefore, $h_1$ is proportional to the product of the curves in Figures
25 and 26. By replotting these two curves on the same graph (see Figure 30), a curve representing their product may be drawn which should give a qualitative representation of \( h_i \) vs. \( a_m \).

![Figure 30. Qualitative representation of \( h_i \) vs. \( a_m \)](image)

A number of similarities are evident when this curve is compared with the experimental data plotted as \( h_i \) vs. \( a_m \) (see Figure 31 and Table VIII for data). At \( a_m = 0 \), the nonvibration condition, heat transfer from a solid surface to a granular bed, especially in vacuum, is known to be quite poor. The values of \( h_i \) for heat transfer to stagnant beds, falling in the range 1 to 17 Btu/ft.²hr.°F, are
FIGURE 31. VARIATION IN HEAT-TRANSFER COEFFICIENT WITH MAXIMUM ACCELERATION OF VIBRATING HEAT-TRANSFER SURFACE
TABLE VIII. CONVERTED DATA USING MAXIMUM PLATE ACCELERATION AND BED DEPTH AS INDEPENDENT VARIABLES

Dependent Variable: $h_i$ (Btu*ft./hr.*F.)
Bracketed Figures Indicate Run Numbers

<table>
<thead>
<tr>
<th>Maximum Plate Acceleration, (ft./sec.$^2$)</th>
<th>Bed Depth, inches</th>
<th>1 1/2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15.7 (2)</td>
<td>14.1 (4)</td>
<td></td>
</tr>
<tr>
<td>17.0 (9)</td>
<td>16.6</td>
<td>(11)</td>
<td></td>
</tr>
<tr>
<td>17.8</td>
<td>16.5 (20)</td>
<td>(20)</td>
<td></td>
</tr>
<tr>
<td>17.7</td>
<td>15.4 (25)</td>
<td>(25)</td>
<td></td>
</tr>
<tr>
<td>17.9</td>
<td>15.6 (13a)</td>
<td>(13a)</td>
<td></td>
</tr>
<tr>
<td>17.8</td>
<td>32.1</td>
<td>(14)</td>
<td></td>
</tr>
<tr>
<td>17.8</td>
<td>35.3</td>
<td>(14a)</td>
<td></td>
</tr>
<tr>
<td>32.1</td>
<td>21.0</td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td>32.1</td>
<td>19.5 (1c)</td>
<td>(1c)</td>
<td></td>
</tr>
<tr>
<td>35.8</td>
<td>19.5</td>
<td>(1c)</td>
<td></td>
</tr>
<tr>
<td>35.8</td>
<td>20.7</td>
<td>(10)</td>
<td></td>
</tr>
<tr>
<td>94.1</td>
<td>19.9</td>
<td>(12)</td>
<td></td>
</tr>
<tr>
<td>188.5</td>
<td>5.9</td>
<td>(23)</td>
<td></td>
</tr>
</tbody>
</table>

Therefore not at all unreasonable. For very low maximum accelerations, $h_i$ increases only slightly with vibration, but at $a_m \leq 6$ ft./sec.$^2$, the coefficient is suddenly increased to a value of about twice that for a stagnant bed. Apparently, 6 ft./sec.$^2$ corresponds to the $a_{mc}$ mentioned in previous discussion. The heat transfer coefficient is essentially constant (experimental points are within 16 per cent of the average) at this level until $a_m$ approaches $g$. As indicated in Figure 31, a change in $h_i$ may actually precede the $a_m = g$ condition. Such an effect may be the result of unavoidable extraneous vibrations embodied in vibrating
equipment of this type. These extraneous vibrations could also affect the magnitudes of $a_{mc}$ and $a_{me}$. However, because of the positive drive vibration employed, it is felt that such harmonics would have no major effects on forces acting within bed.

As discussed previously, a family of curves should exist in the region $a_m > g$, and so two curves must be sketched here. One curve includes Run Nos. 10, 12, 23, and 24 which were all made at $n = 3210$ cpm., while the other curve passes through points from Run Nos. 1c, 3, 23, and 24 which were all made at $s = 0.04$ in. The shapes of these curves cannot be defined because of the limited number of experimental points in this region. However, because of a definite difference in appearance of the granular bed between Run Nos. 23 and 24 and all other runs, it is felt that $a_{me}$ (the point above which impact energy is not absorbed) was exceeded in these two runs at $a_m \approx 188$ ft./sec$^2$. At both bed depths (1-1/2 and 3 in.) the bed was being vigorously agitated and appeared to be fluidized; the granular beds in Run Nos. 10 and 12, the next lower $a_m$ value, appeared much more quiet with none of the fluidized bed characteristics.

For each of the six runs at $a_m > g$, the contact times and the impulse per pound of bed for each vibration cycle may be calculated (see Appendix Q). The contact time, as explained previously, must decrease with increasing $a_m$. For Run Nos. 1c and 3 at $a_m = 35.8$ ft./sec$^2$, $\theta_{contact}$ is about 0.82. In Run Nos. 10 and 12 ($a_m \approx 94$ ft./sec$^2$), $\theta_{contact}$ has been reduced to about 0.46, while $\theta_{contact}$ for Run Nos. 23 and 24 ($a_m \approx 188$ ft./sec$^2$) is less than 0.33 even if the hot plate-
bed collision is still inelastic, i.e., if $a_m$ has not been reached. If the bed bounces, $\Theta_{\text{contact}}$ will be even smaller. Should $\Theta_{\text{contact}}$ be the only important factor, i.e., if the effect of $A_{\text{contact}}$ is insignificant, the values for $h_i$ should have the same ratio to the $h_i$ at $a_m = g$ as the ratio of the $\Theta_{\text{contact}}$'s. Using the average value of $h_i$ at $a_m = g$ as a basis we have,

<table>
<thead>
<tr>
<th>Run Number</th>
<th>$h_i$</th>
<th>$\Theta_{\text{contact}}$</th>
<th>$h_i/h_i \max.$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13a, 14, 14a, 21, 22 average</td>
<td>34.2</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>1c</td>
<td>19.5</td>
<td>0.82</td>
<td>0.57</td>
</tr>
<tr>
<td>3</td>
<td>21.0</td>
<td>0.82</td>
<td>0.61</td>
</tr>
<tr>
<td>10</td>
<td>20.7</td>
<td>0.46</td>
<td>0.61</td>
</tr>
<tr>
<td>12</td>
<td>19.9</td>
<td>0.46</td>
<td>0.58</td>
</tr>
<tr>
<td>23</td>
<td>5.9</td>
<td>0.33</td>
<td>0.17</td>
</tr>
<tr>
<td>24</td>
<td>5.9</td>
<td>0.33</td>
<td>0.17</td>
</tr>
</tbody>
</table>

It is seen from this tabulation that the heat transfer coefficients in Run Nos. 1c, 3, 23, and 24 are lower and those in Run Nos. 10 and 12 are higher than would be anticipated if $\Theta_{\text{contact}}$ alone were causing the change. This indicates a contribution to $h_i$ by another factor, viz., $A_{\text{contact}}$.

Earlier it was pointed out that the number of contact points per unit area of hot plate will depend upon the impulse acting between bed and hot plate during collision. This impulse, the product of the average impact force between bed and plate and the time this force acts,
is only a part of the total impulse acting between bed and plate during each cycle. There is, in addition, an impulse due to the difference in accelerations of bed and plate while they are in contact. Because no significant changes in $h_i$ occurred when bed depth was doubled, the actual impulse, which is directly proportional to the total mass of the bed, is not felt to be of importance, but rather the impulse per pound of bed is felt to be significant. Since impulse is force $\times$ time, it is also mass $\times$ acceleration $\times$ time, and therefore impulse per pound is simply acceleration $\times$ time. For the collision, the impulse per pound is equal to the relative velocity of approach at impact.

In Appendix Q, the impulse per pound during collision is calculated for each of the runs at $a_m > g$. For Run Nos. 1c and 3 it is only 0.03 ft./sec. For Run Nos. 10 and 12, it is more than ten times as large at 0.39 ft./sec. The fact that $h_i$'s from both sets of runs are at approximately the same level is attributed to the balance between $Q_{\text{contact}}$ and $A_{\text{contact}}$ which exists. In one set, $Q_{\text{contact}}$ is high but $A_{\text{contact}}$ is low; in the other, the reverse is true. Run Nos. 23 and 24, in spite of a high collision impulse (0.87 ft./sec.), have lower $h_i$'s than ratioed $Q_{\text{contact}}$'s would predict. This is further evidence that $a_{\text{me}}$ had been exceeded in these runs so that bouncing of the bed reduced the actual $Q_{\text{contact}}$ below the calculated 0.33.

The impulse per pound of bed due to the difference in accelerations of the bed and the vibrating hot plate is also calculated in Appendix Q for each run at $a_m > g$ (for $a_m \leq g$ this impulse per pound is simply $g$/cycle time). These values should not be combined with the
collision impulse per pound because they act over the entire time that plate and bed are in contact during each cycle; the collision impulse, on the other hand, acts only for an unknown time interval after the instant of collision during which repacking takes place.

In certain of the stagnant-bed runs (Run Nos. 15, 16, and 17), the granular bed was vibrated with maximum accelerations greater than \( a_{mc} \) (6 ft./sec.\(^2\)) for a short period (up to 2 min.) previous to the start of the experimental run itself; in the other runs under nonvibration conditions (Run Nos. 2, 4, 9, and 11), this "previbration" was not used. The ranges of the \( h_i \)'s obtained from these two sets of runs are completely separated\(^{(7)}\), and furthermore, statistical methods (see Appendix M) show a significant difference between the ranges. This difference can be attributed to an increase in the area of actual contact between plate and bed taking place as the bed is previbrated above the critical vibration condition. Because these beds were not previbrated for more than two minutes, the coefficients obtained were not as high as those in the region \( a_{mc} < a_m < g \) where vibration was carried on continuously. It is quite possible that heat transfer coefficients for stagnant beds can be raised at least to the level of those in the region \( a_{mc} < a_m < g \) by extended periods of previbration. The heat transfer coefficients from the nonprevibrated runs are those plotted in Figures 20, 21, 22, and 31 because it is felt that these present fairer values.

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\(^{(7)}\) For the nonprevibrated runs coefficients from 14.1 to 17.0 Btu/ft.\(^2\)hr.°F. were obtained. When the beds were previbrated, coefficients from 18.2 to 20.0 Btu/ft.\(^2\)hr.°F. were found.
for comparison of heat transfer coefficients for beds with and without vibration.

Comparison of Heat Transfer Coefficients
Calculated by Three Methods

Finally, the agreement shown between $h_1$'s based on actual sublimation rates and $h_1$'s based only on the measured vaporizer temperature was very good (see Table II). The per cent deviations in $h_1$'s found in these two ways ranged from 0.0 per cent to +6.7 per cent, and averaged only +2.9 per cent. The fact that the deviations were all positive makes it very highly probable that some constant used in the equations for sonic flow was slightly in error. It could be the values used for naphthalene vapor pressure, or the orifice discharge coefficient for which a book value of 0.83 was used, but a more reasonable assumption is that the orifice area was approximately 2.9 per cent larger than that for a 1/2-in. diameter hole. Such an error would result if the orifice diameter were only 0.007 in. oversize. By correcting the theoretical $h_1$'s for this constant error, the theoretical $h_1$'s for all runs agree with the actual $h_1$'s within ±3.7 per cent. Hence, in future investigations, once the constant correction has been found (for this particular 1/2-in. orifice it is -2.8 per cent), the heat transfer coefficient can be accurately calculated from the steady-state vaporizer temperature alone. Because sonic mass velocity is proportional to the square root of the vapor phase molecular weight (see Appendix D), the close agreement between the actual vaporization rates and the rates based on the vaporizer temperature offers strong evidence against appreciable vapor phase association of naphthalene molecules.
Since vaporizer temperature and pressure are related by the vapor pressure curve of the solid when there is a negligible pressure drop through the bed, either variable may be measured and used to calculate $h_1$ (see Appendix R for maximum $\Delta p$ through bed). A temperature measurement is ordinarily easier to make, and this technique was used in this study. However, a method was worked out, after considerable difficulty, for measuring vaporizer pressure directly. This special procedure for finding the total pressure within a vacuum chamber containing condensible vapors became necessary after attempts were made, with no success, to measure this pressure with a tilting McLeod gage connected to a line through which a slight bleed was admitted. Readings here were consistently low, values being about 15-25 per cent of the actual pressure.

A personal communication from Mr. C. P. Holderbaum of the Consolidated Vacuum Corporation indicated that other engineers have experienced this same difficulty. He stated, "A temperature of 70 C (approximate vaporizer temperature) combined with the solubility of naphthalene make the conventional instruments for measuring pressures in your range practically useless", and he suggested two methods which have actually been used for measuring such pressures. Each required a reference chamber separate from the one containing the condensible vapors. Rather elaborate set-ups requiring separate vacuum pumps were recommended. Fortunately, it was discovered that true pressures could be obtained with the simpler arrangement shown in Figure 32.
Valve No. 2 was kept closed except when a reading was being taken. In this way, a pressure of noncondensible gas slightly in excess of the pressure within the vaporizer chamber could be maintained in the McLeod gage. When a reading was desired, Valve No. 2 was opened and the gage pressure was quickly reduced to that in the vaporizer and a reading taken. Valve No. 1 was then cracked to allow a small bleed of air to purge the connecting lines of condensible vapors. Valve No. 2 was then immediately closed, followed by Valve No. 1, leaving the gage in proper condition for the next reading.

This procedure was used in Run No. 17 in addition to the measurement of the vaporizer temperature. During the steady-state test period, a pressure of 2.65 mm. Hg was read in the vaporizer chamber. The vapor pressure corresponding to the indicated temperature was 2.55 mm. Hg. At least a part of the discrepancy of 0.1 mm Hg can be
attributed to the unavoidable inert pressure within the vaporizer. This inert pressure was not known, but it can be assumed to be on the order of the pressure indicated at the vacuum pump, viz., 0.105 mm. Hg.

The heat transfer coefficients for each of the runs, calculated from power input to the ring heaters, are listed in Table II. Agreement is not very good with per cent deviations ranging from -22.2 per cent to +44.5 per cent and averaging +11.9 per cent. Even with the use of a constant correction of -11.9 per cent to bring the averages of the heat transfer coefficients as closely together as possible, agreement would be no better than ±32 per cent. If the one maximum and one minimum deviations are disregarded, agreement is brought within ±17 per cent. The poor agreement here can be attributed primarily to a lack of knowledge of the exact electrical energy input to the heat transfer cup during the test period. The instantaneous power was indicated by a voltmeter and ammeter in the heater circuit, as shown in Figure 38. Since neither a recording wattmeter nor a watt-hour meter was used, the power input was assumed to be the average of the products of the readings of the voltmeter and ammeter taken at several times during each test period. In the intervals between these readings, however, the voltage across the heaters was often adjusted to different values in order to hold the heat transfer surface temperature as constant as possible. In future work, if power inputs alone are to be relied upon for calculation of heat transfer coefficients, it is recommended that a watt-hour meter be installed in the heater circuit so that the energy input may be accurately known.
CONCLUSIONS

The following conclusions may be drawn from the results of this study. It must be remembered that these conclusions are strictly valid only for the specific material and the specific ranges of conditions which were investigated. It is felt that there is justification, however, for extending a number of these conclusions to cover more general cases.

1. Individual heat transfer coefficients between vibrating plates and granular beds of subliming material resting on these plates may be successfully measured by the method outlined in this paper. This involves controlling the rate of this heat transfer by flow of the sublimate through a sonic orifice so that a steady-state condition is attained. With this method, only one variable, the temperature within the vaporizer, need be measured.

2. These heat transfer coefficients depend very strongly on the mutual interaction between amplitude and frequency of vibration. An optimum heat transfer coefficient will be attained in the region $a_{mc} < a_m < g$. Heat transfer coefficients may be conveniently correlated with the maximum velocity of the vibrating plate for interpolation or extrapolation.

3. The depth of the granular bed apparently has no effect on the individual heat transfer coefficient between a vibrating hot plate and this bed. Interactions of bed depth with either amplitude or frequency of vibration are similarly not significant.
4. Heat transfer coefficients to stagnant beds in vacuum may be appreciably increased by previbration of these beds at \( a_m > a_{mc} \), thereby promoting closer bed packing and increased true contact area.

5. There is no appreciable vapor phase association of naphthalene under its own vapor pressure at temperatures of 135 to 155 F.

6. Total pressure within a vacuum chamber containing condensible vapors may be accurately measured with an ordinary McLeod gage by maintaining a slightly positive pressure with respect to the vacuum to be measured in this gage and the connecting lines except while actually taking readings. This may be done with a suitable valving arrangement, as described in this paper on page 112.
RECOMMENDATIONS FOR FUTURE WORK

Having found that increases of 100 per cent or more in the heat transfer coefficient can be produced by vibration of the heat transfer surface, it becomes of immediate interest to determine what improvements in heat transfer accompany other methods of agitation. It is suggested that measurements of $h_1$ be made using primarily paddle-type agitation of the granular bed, the speed of rotation as well as the size and shape of the agitator blade and the clearance between the blade and the heat transfer surface could be varied. A comparison of heat transfer coefficients combined with cost estimates for the various types of equipment should reveal rather quickly whether or not the investigation of vibrating heat transfer surfaces should be continued beyond this point.

If further research is warranted, much more information should certainly be obtained about the effects of the variables already studied, and, in addition, this type of study should be extended to cover effects of other important variables listed on page 32. It is recommended that the region of maximum plate acceleration greater than gravity be investigated much more thoroughly, with emphasis on the use of widely differing amplitudes and frequencies. Also, the study of previbration should be extended, to determine how much improvement in $h_1$ can be obtained and how this improvement varies with the length of the previbration period. A relationship might be anticipated between $h_1$ and the bulk density of the previbrated bed. The use of intermittent vibration, as well as the utilization of vibration and other agitation concurrently might also be investigated.
It is recommended that a study of heat transfer to non-vaporizing granular beds from vibrating plates be made. Using a material such as glass beads, sand, or metal shot, a characteristic bed temperature should first be defined. When much bulk movement occurs, the bed should be practically isothermal; however, when agitation is relatively low, temperature gradients will exist through the bed. A temperature-sensitive element placed in the bed near to the heat transfer surface would register the temperature most likely to be characteristic of the bed. It might be possible to find an average bed temperature calorimetrically, but specialized procedures would be necessary.

Once $T_g$ has been defined so that temperature driving force may be calculated, a method must be determined for evaluating the heat transfer coefficient itself. One possibility would be to refine the technique described in this paper so that the power input to the electric heaters could be converted to the instantaneous flow of heat to the bed. A second possibility would be to install cooling coils in the bed so that a steady-state condition could be reached. Even without achieving a steady-state, the heat transfer coefficient could be calculated by integrating the equation:

$$\frac{dQ}{d\theta} = UA(T_s - T_g)$$

To use this method would still require a knowledge of the characteristic bed temperature, $T_g$, as a function of the time. A third possibility to be considered is that the heat transfer coefficient might be calculated by combining the individual heat transfer coefficients
obtained from subliming beds with measurements of the effective thermal conductivity of the vibrating beds themselves.

Insight into the flow of heat through nonvaporizing beds might be gained through a study of the movement of the particles ("convection currents") through the bed during vibration. Possible tools would include the use of colored particles whose distribution throughout the bed could be observed, or the use of a single radioactive particle whose movement within the bed could be traced with appropriate instruments. For nonvaporizing beds, the heat flow through the bed could be the controlling factor. If this is the case, optimum heat transfer may occur in the region \( a_m > g \) rather than at \( a_m < g \), as in the case of sublimation.

Certain equipment modifications are also recommended. These include:

1. Install a more closely fitted guide bearing around the vibration transmission shaft. This should help to eliminate much of the extraneous vibration presently occurring in both the heat transfer cup and the framework supporting the vacuum chamber and water jacket.

2. Fasten the vaporizer chamber directly to the framework, so that any warpage of the top flange occurring when the cover flange is bolted in place will not disturb the alignment of the vibration transmission shaft and its guide bearing.
3. Use a slotted nut and cotter key to prohibit the locking nut on the end of the vibrator drive shaft from backing off during vibration.

4. Eliminate the electronic recording null-point galvanometer from the heat transfer surface temperature control circuit. It is believed that the use of a simple indicating null-point galvanometer will be sufficient in its place and will not be subject to the sometimes erratic behavior of the electronic instrument.

5. The true surface temperature of the hot plate should be measured at a number of spots over its entire surface. If this is done, any small temperature gradients will become evident, and the necessary corrections can be applied to give more accurate temperature driving forces than could be found in this series of runs.

6. Install a watt-hour meter in the ring heater circuit, so that the total energy input to these heaters may be known in addition to the instantaneous power input.
APPENDIX A

Nomenclature

\( a_m \) = maximum acceleration of vibrating heat transfer surface, surface, ft./sec.\(^2\)

\( a_m' \) = maximum downward acceleration of cam, in./min.\(^2\)

\( a_{mc} \) = maximum acceleration of vibrating heat transfer surface at which forces within the granular bed overcome the frictional forces holding each particle in place, ft./sec.\(^2\)

\( a_{me} \) = maximum acceleration of vibrating heat transfer surface above which the bed can no longer absorb the impact energy of collision between bed and vibrating plate, ft./sec.\(^2\)

\( a_p \) = acceleration of vibrating heat transfer surface, ft./sec.\(^2\)

\( a_r \) = relative acceleration of granular bed with respect to the vibrating hot plate, ft./sec.\(^2\)

\( A \) = area for heat transfer, ft.\(^2\)

\( A_c \) = area of condenser surface, ft.\(^2\)

\( A_{contact} = \frac{A_t}{A_s} \), dimensionless

\( A_{fmc} \) = area for heat transfer by free molecule conduction, ft.\(^2\)

\( A_{mc} \) = area for heat transfer by molar conduction through gas phase, ft.\(^2\)

\( A_o \) = orifice area, ft.\(^2\)

\( A_{PL} \) = average area per contact point for unvibrated bed, ft.\(^2\)

\( A_{P2} \) = average area per contact point for closely packed bed, ft.\(^2\)

\( A_s \) = area of heat transfer surface in vaporizer, ft.\(^2\)

\( A_{SS} \) = area for heat transfer by solid-solid conduction, ft.\(^2\)

\( A_t \) = "true contact area" between hot plate and granular bed, ft.\(^2\)
B = k/\nu C_v, dimensionless

c_o = orifice discharge coefficient, dimensionless

C = 2\pi L P_{mm}, ft. mm. Hg

C_p = specific heat at constant pressure, Btu/lb.mass°F.

C_S = Sutherland constant, °F.

C_v = specific heat at constant volume, Btu/lb.mass°F.

d = differential operator

D_o = orifice diameter, in.

D_p = particle diameter, ft.

D_s = diameter of heat transfer surface in vaporizer, in.

e = logarithmic base, 2.718...

E = mass to be vibrated, lb.mass

f = friction factor, dimensionless

f_1( ) = function of

f_2( ) = function of

f_3( ) = function of

f_4( ) = function of

f_5( ) = function of

f_6( ) = function of

F_{sp.} = spring force required, lb.force

F = average force acting during impulse, lb.force

g = acceleration of gravity, 32.17 ft./sec.^2

g_{c} = conversion factor, 32.17 lb.mass ft./lb.force sec.^2

g_{c}^{'} = conversion factor, 1.390 lb.mass ft./lb.force min.^2

G = mass velocity of fluid, lb.mass/hr.ft.^2

G_{cni} = maximum mass velocity of fluid in a frictionless nozzle under conditions of isothermal flow, lb.mass/hr.ft.^2
\[ P_0 \sqrt{\frac{g_c M}{(eRT_0)}} = \]  

\( G_{\text{cniT}} \) = maximum mass velocity of fluid in a frictionless nozzle under conditions of isothermal flow at \( T_g \), lb.mass/hr.ft.²

\( G_{\text{Sol}} \) = weight of solvent, g.mass

\( G_{\text{T}} \) = sonic mass velocity at \( T_g \), lb.mass/hr.ft.²

\( h_c \) = heat transfer coefficient at condenser surface, Btu/ft.²hr.°F.

\( h_{\text{mco}} \) = heat transfer coefficient for free molecule conduction, Btu/ft.²hr.°F.

\( h_i \) = individual heat transfer coefficient between hot plate and granular bed, Btu/ft.²hr.°F.

\( h_{\text{max}} \) = individual heat transfer coefficient between hot plate and granular bed in the region \( \alpha_{\text{mc}} < \alpha_m < \alpha, \) Btu/ft.²hr.°F.

\( h_{\text{mc}} \) = heat transfer coefficient for molar conduction through gas phase, Btu/ft.²hr.°F.

\( h_{\text{ss}} \) = heat transfer coefficient for solid-solid conduction, Btu/ft.²hr.°F.

\( H \) = latent heat of sublimation, Btu/lb.mass

\( H' \) = latent heat, Btu/lb.mole

\( H_{\text{T}} \) = latent heat of sublimation at \( T_g \), Btu/lb.mass

\( k \) = thermal conductivity, Btu/ft.²hr.°F/ft.

\( k_{\text{eff}} \) = effective thermal conductivity of granular bed, Btu/ft.²hr.°F/ft.

\( k_{\text{gas}} \) = thermal conductivity of gas, Btu/ft.²hr.°F/ft.

\( k_{\text{solid}} \) = thermal conductivity of solid phase in granular bed, Btu/ft.²hr.°F/ft.

\( k_t \) = thermal conductivity at temperature, \( T \), Btu/ft.²hr.°F/ft.

\( k'_t \) = thermal conductivity, 10⁻⁴ watt/cm°C.

\( k_{492°F} \) = thermal conductivity at 32°F, Btu/ft.²hr.°F/ft.

\( K \) = a proportionality constant, Btu sec.in.²/hr.ft.²g.mass
\[ K' = \text{a proportionality constant, Btu/lb\_mass} \]
\[ K_1 = \text{a proportionality constant, } ^\circ \text{F} \cdot \text{ft}^2 \]
\[ K'_1 = \text{a proportionality constant, ft}^2 \cdot \text{lb\_mass} \cdot ^\circ R/\text{Btu} \]
\[ K_2 = \text{a proportionality constant, } ^\circ \text{F} \cdot \text{ft}^2 \]
\[ K'_2 = \text{a proportionality constant, ft}^2 \cdot \text{lb\_mass} \cdot ^\circ R/\text{Btu} \]
\[ L = \text{mean free path, ft} \]
\[ L_b = \text{bed depth, ft} \]
\[ L_{ss} = \text{average effective distance for heat flow within particle, ft} \]
\[ m = \text{mass of molecules in surface, amu} \]
\[ m = \text{mass, lb\_mass} \]
\[ m' = \text{mass of molecules striking surface, amu} \]
\[ m_g = \text{mass of molecules in sphere, amu} \]
\[ m_p = \text{mass of particle, lb\_mass} \]
\[ m_s = \text{mass of molecules in plane, amu} \]
\[ m_{\text{sol}} = \text{molecular weight of solvent} \]
\[ \Delta(mV) = \text{change in momentum, lb\_mass ft}./\text{sec} \]
\[ M = \text{molecular weight of gas, dimensionless} \]
\[ M' = \text{molecular weight of vaporizing solid, dimensionless} \]
\[ MME = \text{maximum measurement error in } h_1, \text{ per cent} \]
\[ M_s = \text{molecular weight of solute, dimensionless} \]
\[ n = \text{frequency of vibration, cpm} \]
\[ \pi = \text{gas density, molecules/cm}^3 \]
\[ n' = \text{frequency of vibration, cps} \]
\[ n_1, \ldots, n_6 = \text{frequencies of vibration, cpm} \]
\[ N = \text{resistance factor, dimensionless} \]
\[ = 0 \text{ for sharp-edged orifice} \]
\( N_{Re} = \text{Reynold's number, dimensionless} \)
\( = \frac{(D_p \nu^2)}{\nu} \)

\( p^0 = \text{vapor pressure, mm. Hg} \)

\( p_b = \text{breakaway pressure, mm. Hg} \)

\( p_c^0 = \text{vapor pressure at } T_c, \text{ mm. Hg} \)

\( p_e = \text{pressure of fluid surrounding particle, lb.force/ft.}^2 \)

\( p_g^0 = \text{vapor pressure at } T_g, \text{ mm. Hg} \)

\( p_{mm}^0 = \text{vapor pressure of naphthalene, mm. Hg} \)

\( p_o^1 = \text{absolute pressure upstream from orifice, lb.force/ft.}^2 \)

\( p_o^0 = \text{absolute pressure upstream from orifice, mm. Hg} \)

\( p_s^0 = \text{vapor pressure at } T_s, \text{ mm. Hg} \)

\( \Delta p = \text{pressure difference between vaporizer and condenser chambers, mm. Hg} \)

\( \Delta p_b = \text{pressure drop through bed, lb.force/ft.}^2 \)

\( P = \text{pressure, dyne/cm.}^2 \)

\( P_{contact} = \text{contact pressure between hot plate and granular bed, lb.force/ft.}^2 \)

\( P_{mm} = \text{pressure, mm. Hg} \)

\( P_{sol} = \text{vapor pressure of solvent from its solution at } T, \text{ mm. Hg} \)

\( P_{sol}^0 = \text{vapor pressure of pure solvent at } T, \text{ mm. Hg} \)

\( q = \text{steady-state rate of heat transfer, Btu/hr.} \)

\( q_{fmc} = \text{rate of heat transfer by free molecule conduction, Btu/hr.} \)

\( q_{mc} = \text{rate of heat transfer by molar conduction through gas phase, Btu/hr.} \)

\( q_{ss} = \text{rate of heat transfer by solid conduction, Btu/hr.} \)

\( q_{total} = \text{total rate of heat transfer from hot plate to granular bed, Btu/hr.} \)

\( Q = \text{heat transferred, Btu} \)

\( (Q/\theta) = \text{rate of heat transfer, Btu/hr.} \)
\( (Q/e)_{in} \) = rate of heat flow into vaporizer, Btu/hr.
\( (Q/e)_{out} \) = rate of heat flow out of vaporizer, Btu/hr.

\( r_1 \) = radius of cam, ft.
\( r_2 \) = radius of vibrator drive shaft, ft.

\( R \) = gas constant, \( \frac{1546 \text{ ft} \cdot \text{lb} \cdot \text{force}}{\text{lb} \cdot \text{mole} \cdot \text{deg} \cdot \text{R}} \).

\( R' \) = gas constant, \( \frac{3.577 \text{ Btu}}{\text{lb} \cdot \text{mole} \cdot \text{deg} \cdot \text{C}} \).

\( R_{cc} \) = center-to-center distance of cam and vibrator drive shaft, ft.

\( s \) = amplitude of vibration, in.
\( s' \) = amplitude of vibration, ft.

\( s_j \) = temperature jump distance, ft.

\( s_{jg} \) = temperature jump distance at surface of sphere, ft.
\( s_{js} \) = temperature jump distance at surface of plane, ft.

\( s_{jl} \) = temperature jump distance at hot surface, ft.
\( s_{j2} \) = temperature jump distance at cold surface, ft.

\( s_m \) = distance for heat transfer by molar conduction through gas phase, ft.

\( s_1 \ldots s_6 \) = amplitudes of vibration, in.

\( t \) = time, sec.
\( \Delta t \) = time interval of impulse, sec.

\( T \) = temperature, °F.

\( T_{abs.} \) = absolute temperature, °K.

\( T_C \) = temperature of condenser surface, °F.

\( T_{oC} \) = temperature, °C.

\( T_g \) = characteristic temperature of granular bed, °F.

\( T_o \) = absolute temperature upstream from orifice, °R.

\( T_{oR} \) = absolute temperature, °R.
\( T_s \) = surface temperature of heat transfer plate in vaporizer, °F.

\( T_1 \) = temperature of hot surface, °F.

\( T_2 \) = temperature of cold surface, °F.

\( \Delta T \) = driving force for heat transfer, °F.

\( \Delta T_c \) = driving force for heat transfer in condenser, °F.

\( \Delta T_{fmc} \) = driving force for heat transfer by free molecule conduction, °F.

\( \Delta T_j \) = temperature jump at surface, °F.

\( \Delta T_{mc} \) = driving force for heat transfer by molar conduction through gas phase, °F.

\( \Delta T_{ss} \) = driving force for heat transfer by solid-solid conduction, °F.

\( U \) = over-all heat transfer coefficient between hot plate and nonvaporizing granular bed, Btu/ft.²hr.°F.

\( v \) = instantaneous velocity of hot plate, ft./sec.

\( v = \) integration substitution, ft.

\( = \frac{(D_p/2)^2 - y^2}{2} \) ft.

\( v_b \) = instantaneous velocity of bed, ft./sec.

\( v_m \) = maximum velocity of heat transfer surface, ft./sec.

\( v_{mol} \) = average molecular velocity at \( T_2 \), cm./sec.

\( v_r \) = relative velocity of approach of bed and hot plate, ft./sec.

\( v_{rl}, \ldots v_{rh} \) = instantaneous relative velocities of approach of bed and hot plate, ft./sec.

\( v_{1}, \ldots v_{4} \) = instantaneous velocities of hot plate, ft./sec.

\( V \) = velocity, ft./sec.

\( w_s \) = weight of solute, g.mass

\( W \) = weight of solid vaporized, lb.mass

\((W/\theta)\) = rate of vaporization, lb.mass/hr.

\( W' \) = weight of solid vaporized, g.mass
\[ \begin{align*}
\Delta x &= \text{distance in direction of heat transfer, ft.} \\
\Delta y &= \text{distance over which heat is transferred, ft.} \\
y &= \text{distance along y axis, ft.} \\
y_{\text{mfp}} &= \text{y value at which } x = \text{mfp, ft.} \\
\mathcal{A} &= \text{accommodation coefficient, dimensionless} \\
\mathcal{A}_g &= \text{accommodation coefficient at surface of sphere, dimensionless} \\
\mathcal{A}_s &= \text{accommodation coefficient at surface of plane, dimensionless} \\
\mathcal{A}' &= \left( \frac{(2 - \gamma)}{\varepsilon} \right)^{1/2} \left( \frac{(2 \varepsilon)}{(\gamma + 1)} \right)^{1/2}, \text{dimensionless} \\
\varepsilon &= \frac{(9 \gamma - 5)}{4}, \text{dimensionless} \\
\gamma &= \frac{C_p}{C_V}, \text{dimensionless} \\
\Theta &= \text{time for heat transfer, hr.} \\
\Theta' &= \text{length of test period, sec.} \\
\Theta_{\text{contact}} &= \Theta_t/\Theta, \text{dimensionless} \\
\Theta_t &= \text{true time of contact between hot plate and granular bed, hr.} \\
\lambda_o &= \text{free molecule conductivity, Btu/ft.2hr.°R. mm. Hg} \\
\nu &= \text{viscosity, lb.mass/ft. hr.} \\
\nu' &= \text{viscosity, lb.mass/ft. sec.} \\
\rho &= \text{density, lb.mass/ft.}^3 \\
\rho_{\text{bl}} &= \text{bulk density of unpacked granular bed, lb.mass/ft.}^3 \\
\rho_{\text{b2}} &= \text{bulk density of granular bed at maximum closeness of packing, lb.mass/ft.}^3 \\
\sigma &= \text{standard deviation, Btu/ft.2hr.°F.} \\
\sigma &= \text{molecular diameter, cm.} \\
\sigma_v &= \text{molecular diameter determined from viscosity, ft.} \\
\omega &= \text{angular velocity of vibrating hot plate, radians/sec.}
\end{align*} \]
APPENDIX B

Bibliography


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Calculation of Conductive Heat Transfer From a Heated Plate to a Sphere Resting on the Plate

Assumptions:

1. The temperature of the plate remains constant at $T_s$.
2. The temperature of the sphere remains constant at $T_g$.
3. All heat flow will occur by conduction through the gas phase. There will be no solid-solid conduction because of the point contact between the sphere and the hot plate.
4. Heat will flow in the vertical direction only from plate to sphere.
5. Heat flow will be by molar conduction where the separation of plate and particle is greater than the mean free path of the gas and by free molecular conduction where the separation is less than the mfp.
In the figure above, heat will be transferred by free molecular conduction within the circular area, $A_{fmc}$, from $y = 0$ to $y = y_{mfp}$; heat will be transferred by molar conduction within the annular area, $A_{mc}$, from $y = y_{mfp}$ to $y = D_p/2$.

The heat transferred by molar conduction may be calculated as follows:

From Fourier's Law,

$$q_{mc} = -k_{gas} dA_{mc} \frac{\Delta T}{\Delta x}$$

(60)
Referring to Figure 33, $\Delta T = T_g - T_s$, $\Delta x = x + s_{js} + s_{jg}$, and $dA_{mc} = 2\pi y dy$.

Therefore,

$$q_{mc} = 2\pi k_{gas}(T_s - T_g) \int_{y_mfp}^{D_p/2} \frac{y dy}{x + s_{js} + s_{jg}}$$

But,

$$y^2 + (x - \frac{D_p}{2})^2 = \frac{D_p^2}{2}$$

and,

$$x = \frac{D_p}{2} - \left[\left(\frac{D_p}{2}\right)^2 - y^2\right]^{\frac{1}{2}}$$

However, we are concerned only with heat flow to the bottom half of the sphere, so,

$$x = \frac{D_p}{2} - \left[\left(\frac{D_p}{2}\right)^2 - y^2\right]^{\frac{1}{2}}$$

Hence,

$$q_{mc} = 2\pi k_{gas}(T_s - T_g) \left(\frac{D_p}{2}\right) \frac{y dy}{x + s_{js} + s_{jg} + \frac{D_p}{2} - \left[\left(\frac{D_p}{2}\right)^2 - y^2\right]^{\frac{1}{2}}}$$

To integrate, let $\left[\left(\frac{D_p}{2}\right)^2 - y^2\right] = y'^2$
·. \[ -2ydy = 2\sqrt{v} \, d\sqrt{v} \]

and, \[ ydy = -\sqrt{v} \, dv \]

Then,

\[
q_{mc} = 2n k_{gas} (T_s - T_g) \left[ \frac{y_{mfp}}{s_j} \right] \int_{s_j + s_{jg} + \frac{D_p}{2}}^{\sqrt{v}} \left( \frac{\sqrt{v} \, dv}{s_j + s_{jg} + \frac{D_p}{2}} \right) \]

\[ \ln(s_j + s_{jg} + \frac{D_p}{2} - \sqrt{v}) \]

\[
= y_{mfp} \]

\[
y = \frac{D_p}{2} \]

\[ - (s_j + s_{jg} + \frac{D_p}{2}) \ln \left( s_j + s_{jg} + \frac{D_p}{2} \left[ \frac{D_p}{2} - y^2 \right] \right) \]

\[ \frac{Y_{mfp}}{2} \]

From integration tables (24),

\[ q_{mc} = 2n k_{gas} (T_s - T_g) \left[ s_j + s_{jg} + \frac{D_p}{2} - \sqrt{v} \right] \]

\[ - (s_j + s_{jg} + \frac{D_p}{2}) \ln \left( s_j + s_{jg} + \frac{D_p}{2} \left[ \frac{D_p}{2} - y^2 \right] \right) \]

\[ \left( \sqrt{v} \right) \]

\[ \frac{Y_{mfp}}{2} \]

The heat transferred by free molecule conduction may be calculated from Equation 7,

\[ q_{fmc} = \frac{22.2 \cdot \mathcal{A} \cdot s \cdot \mathcal{A}_{fmc} \cdot p_{mm} (T_s - T_g)}{\sqrt{T_0} + 459} \]

where,

\[ \mathcal{A}_o = \frac{25.9}{\sqrt{M}} \cdot \frac{\gamma' + 1}{\gamma' - 1} \]

\[ \gamma_s = 1 - \left( \frac{m_s - m'}{m_s + m'} \right)^2 \]
Therefore,

\[ q_{\text{fmc}} = \frac{(22.2)(25.9)(\pi) y_{\text{mfp}}^2 p_{\text{mm}} (T_s - T_g)}{\sqrt{M} \left( \sqrt{T_g + 459} \right) (\gamma - 1) \left( 1 - \frac{m - m'}{m + m'} \right)^2} \]  

(73)

The total conductive heat transfer from a heated plate to a sphere resting on the plate is then equal to the sum of Equations 71 and 73.

For the particular materials and conditions involved in this study, heat transfer coefficients \( h_{\text{mc}} \) and \( h_{\text{fmc}} \) may be calculated for this case. The conditions are:

1. The average gas temperature, \( T_g \), is 150 F.
2. The average gas pressure, \( P_{\text{mm}} \), is 2.6 mm Hg.
3. The gas phase consists of naphthalene vapor.
4. The sphere is naphthalene, and the plane is nickel.
5. The particle diameter is 0.00932 ft. (6-8 mesh)

A heat transfer coefficient, \( h_{\text{mc}} \), for the molar conduction region may be expressed:

\[ h_{\text{mc}} = \frac{q_{\text{mc}}}{A_{\text{mc}} (T_s - T_g)} = \frac{2 \pi k_{\text{gas}}}{A_{\text{mc}}} \left[ s_{js} + s_{jg} + \frac{D_p}{2} - \left( \frac{D_p}{2} \right)^2 - y^2 \right]^{\frac{1}{2}} \]

\[ \left( s_{js} + s_{jg} + \frac{D_p}{2} \right) \ln \left( s_{js} + s_{jg} + \frac{D_p}{2} - \left( \frac{D_p}{2} \right)^2 - y^2 \right)^{\frac{1}{2}} \]

\[ \frac{D_p}{2} \]  

(74)

In this equation:

From Johnson and Huang (29),
where, \( \gamma = 1.2 \)  

\[ c_p = 0.28 \text{ Btu/(lb \cdot mass \cdot F)} \]  

Sarrow and McClellan, (9)  

\( \nu = 1.65 \times 10^{-2} \text{ lb \cdot mass/ft \cdot hr.} \)  

Sarrow and McClellan, (9), and  

Hougen and Watson, (26)  

\[ k_{\text{gas}} = 5.6 \times 10^{-3} \text{ Btu/ft \cdot hr \cdot (°F/ft.)} \]  

From Figure 33,

\[ A_{m_{c}} = \frac{1}{4} \left[ \left( \frac{D_{p}}{2} \right)^{2} - \gamma_{mfp}^{2} \right] \]  

where, \( D_{p} = 0.00932 \text{ ft.} \)  

From Equation 63,

\[ \gamma_{mfp} = \left[ \left( \frac{D_{p}}{2} \right)^{2} - \left( L - \frac{D_{p}}{2} \right)^{2} \right]^{\frac{1}{2}} \]  

where, from Daniels (16),

\[ L = \frac{1}{30.5 \sqrt{2} \pi \sigma^{2} \pi} \]  

where, \( \bar{n} = \frac{6.02 \times 10^{23}}{(22.400)\left(\frac{760}{(338)}\right)\left(\frac{273}{273}\right)} \)  

\[ \bar{n} = 8.5 \times 10^{16} \text{ molecules/cm}^{3} \]
and, from Glasstone (22),
\[ \sigma = 7.4 \times 10^{-8} \text{ cm.} \]
\[ \therefore L = 1.58 \times 10^{-5} \text{ ft.} \]
and, \[ \gamma_{mf} = 3.83 \times 10^{-4} \text{ ft.} \]
and, \[ A_{mc} = 6.77 \times 10^{-5} \text{ ft.}^2 \]

From Kennard (31),
\[ s_{js} = \frac{2 - \alpha_s}{\alpha_s} \frac{2 \epsilon}{\gamma + 1} L \quad (79) \]

where, from Equation 9,
\[ \alpha_s = 1 - \frac{m_s - m'}{m_s + m'} \quad (9) \]

where, \[ m_s = 58.7 \text{ (mol. wt. of Ni)} \]
\[ m' = 128 \text{ (mol. wt. of naphthalene)} \]
\[ \therefore \alpha_s = 0.86 \]

From Kennard,
\[ \epsilon = \frac{2 \gamma - 5}{4} \quad (80) \]
\[ = 1.45 \]
\[ \therefore s_{js} = 2.8 \times 10^{-5} \text{ ft.} \]

Also from Kennard,
\[ s_{jg} = \frac{2 - \alpha_g}{\alpha_g} \frac{2 \epsilon}{\gamma + 1} L \quad (81) \]

where, \[ \alpha_g = 1 - \left( \frac{m_g - m'}{m_g + m'} \right)^2 \quad (9) \]
Since \( m_g = m', \lambda_g = 1 \)

\[ \therefore s_{jg} = 2.1 \times 10^{-5} \text{ ft.} \]

Inserting the above values in Equation 74 gives:

\[ h_{mc} = 8.1 \text{ Btu/ft}^2\text{hr.°F.} \]

The heat transfer coefficient, \( h_{fmc} \), for the free molecule conduction region may be expressed:

\[ h_{fmc} = \frac{q_{fmc}}{A_{fmc}(T_s - T_g)} = \frac{22.2 \lambda s A_0 \rho_{mm}}{\sqrt{T_g + 459}} \quad (24) \]

In this equation:

From Equation 9, \( \lambda_s = 0.86 \)

From Equation 8, \( A_0 = \frac{25.9 \gamma + 1}{\sqrt{M \gamma - 1}} \quad (8) \)

where,

\( M = 128 \)

\( \gamma = 1.2 \)

\( A_0 = 25.2 \text{ Btu/ft}^2\text{hr.°F. mm. Hg} \)

\( P_{mm} = 2.6 \text{ mm. Hg} \)

\( T_g = 150 \text{ F.} \)

Inserting the above values in Equation 24 gives:

\[ h_{fmc} = 50.7 \text{ Btu/ft}^2\text{hr.°F.} \]

Using Equation 31 to calculate \( h_1 \),

\[ h_1 = \frac{A_S}{A_s} h_{ss} + \frac{A_{fmc}}{A_s} h_{fmc} + \frac{A_{mc}}{A_s} h_{mc} \quad (31) \]
where,

\[ A_{ss} = 0 \]
\[ A_s = \pi \frac{D_p^2}{2} = 6.82 \times 10^{-5} \text{ ft.}^2 \]
\[ A_{mc} = 6.77 \times 10^{-5} \text{ ft.}^2 \]
\[ A_{fmc} = A_s - A_{mc} = 0.05 \times 10^{-5} \text{ ft.}^2 \]
\[ h_{fmc} = 50.7 \text{ Btu/ft.}^2\text{hr.}^\circ\text{F.} \]
\[ h_{mc} = 8.1 \text{ Btu/ft.}^2\text{hr.}^\circ\text{F.} \]

Hence,

\[ h_i = 6.4 \text{ Btu/ft.}^2\text{hr.}^\circ\text{F.} \]
APPENDIX D

Calculation of \( h_1 \) as a Function of \( T_g \) at Constant \( D_0 \) and \( T_s \)

Equation 43 is used to construct the curves in Figure 3.

\[
\frac{h_1}{h_3} = \left( \frac{G_{T_g} A_0 H_{T_g}}{A_s(T_s - T_g)} \right)^{1/3}
\]  

(43)

In this equation, \( H_{T_g} \) is assumed to be constant at 220 Btu/lb-mass, and \( A_s \) is constant at 0.785 ft.\(^2\). \( A_0 \) and \( T_s \) are used as parameters, so that for a single calculation,

\[
h_1 = k' \frac{G_{T_g}}{T_s - T_g}
\]

(82)

where,

\[
G_{T_g} = c_0 \left( \frac{G}{G_{cni}} \right) G_{cni} T_g
\]

(83)

\[
= c_0 \left( \frac{G}{G_{cni}} \right) P_0 \gamma \sqrt{\frac{g N}{e R}} \sqrt{\frac{1}{T_g + 459}}
\]

(84)

\[
= c_0 \left( \frac{G}{G_{cni}} \right) \frac{g N}{e R} P_0 \gamma \sqrt{\frac{1}{T_g + 459}}
\]

(85)

From Lapple (35), for a sharp-edged orifice, \( c_0 = 0.83 \).

Also, for \( \gamma = 1.2 \), and \( N = 0 \), \( (G/G_{cni}) = 1.07 \). Therefore,

\[
G_{T_g} = (0.83)(1.07) \sqrt{\frac{(32.17)(128)}{(2.718)(1546)}} P_0 \gamma \sqrt{\frac{1}{T_g + 459}}
\]

(86)

\[
= 0.88 P_0 \gamma \sqrt{\frac{1}{T_g + 459}}
\]

(87)

And, for \( P_0 \) in mm. Hg and \( G_{T_g} \) in lb-mass/hr-ft.\(^2\),

141
Hence, by assuming a $T_g$, $p'_o$ is found from the vapor pressure curve, Figure 4, and $G_{T_g}$ may be calculated.

\[
G_{T_g} = 8810p'_o \cdot \frac{1}{T_g + 4.59} 
\]

This information is plotted in Figure 3A. Now, by selecting values for $A_o$ and $T_s$ for Equation 43, $h_1$ is known as a function of $T_g$. Solving by trial and error gives the data in Table IX, which are plotted in Figure 3.
FIGURE 34. SONIC MASS VELOCITY VERSUS TEMPERATURE FOR NAPHTHALENE UNDER ITS OWN VAPOR PRESSURE
### TABLE IX. DATA FOR PLOT OF $h_i$ VS. $T_g$
WITH $D_0$ AND $T_s$ AS PARAMETERS

<table>
<thead>
<tr>
<th>$T_s$, °F.</th>
<th>$D_0$, in.</th>
<th>$A_0$, ft$^2$</th>
<th>$h_i$ (Btu/ft$^2$hr.°F.)</th>
</tr>
</thead>
<tbody>
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<td>1/4</td>
<td>$3.40 \times 10^{-4}$</td>
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<td>1/8</td>
<td>$8.50 \times 10^{-5}$</td>
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<td></td>
<td>1/16</td>
<td>$2.12 \times 10^{-5}$</td>
<td>122</td>
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</tbody>
</table>
APPENDIX E

Calculation of Maximum Deviation in Vapor Pressure of Test Material from that for Pure Naphthalene

The specifications for Eastman Organic Chemicals Flake Naphthalene P 168 report a maximum melting point depression of 1.2°C for this material. From Lange's Handbook of Chemistry (34), one gram-mole of impurity in 1000 g. of naphthalene will depress the freezing point by 6.9°C. Hence, the maximum impurities in the test material are:

Max. impurities = \( \frac{0.17 \text{ g. mole}}{1000 \text{ g.}} \)

= 0.0222 g. mole impurity/g. mole naphthalene

Also, from Lange's Handbook, the equation for vapor pressure lowering is:

\[
G_s M_s = \frac{w_s m_s}{P_s - P^o_s} \quad (89)
\]

where,

- \( G_{sol} \) = wt. of solvent, g.mass
- \( M_s \) = mol. wt. of solute
- \( w_s \) = weight of solute, g.mass
- \( m_{sol} \) = mol. wt. of solvent
- \( P^o_{sol} \) = vapor pressure of pure solvent, mm. Hg
- \( P_{sol} \) = vapor pressure of solvent from its solution at the same temperature, mm. Hg

Rewriting the equation,
\[
\frac{G_{\text{sol}}}{m_{\text{sol}}} = \frac{w_s}{m_s} \left( \frac{P_{\text{sol}}}{P^0_{\text{sol}} - P_{\text{sol}}} \right) \quad (90)
\]

or, \( m_{\text{sol}} \) = \( m_{\text{solute}} \left( \frac{P_{\text{sol}}}{P^0_{\text{sol}} - P_{\text{sol}}} \right) \quad (91)\)

and, \( \frac{P_{\text{sol}}}{P^0_{\text{sol}} - P_{\text{sol}}} \) = \( \frac{m_{\text{solute}}}{m_{\text{sol}}} \) \( (92) \)

From the melting point calculation,

\[
\frac{P^0_{\text{sol}} - P_{\text{sol}}}{P_{\text{sol}}} = 0.0222 \quad (93)\]

Hence,

\[
P^0_{\text{sol}} - P_{\text{sol}} = 0.0222P_{\text{sol}}
\]

or,

\[
P_{\text{sol}} = 1.0222P^0_{\text{sol}}
\]

and,

\[
P_{\text{sol}} = 0.978P^0_{\text{sol}} \quad (94)
\]

Therefore, the maximum possible deviation in vapor pressure between pure naphthalene and the actual test material is 2.2 per cent.
APPENDIX F

Vapor Pressure of Naphthalene

Volume III of the International Critical Tables (53) gives the following equation for the vapor pressure of naphthalene in the range 32 to 176 F:

$$\log p^o_{mm} = 11.450 - \frac{(71.401)(0.05223)}{T_{abs}}$$

(95)

Using this equation, the following points were calculated:

<table>
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<tr>
<th>Temperature (°F)</th>
<th>Vapor Pressure (mm. Hg)</th>
</tr>
</thead>
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<td>41.0</td>
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<td>0.0189</td>
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<td>68.0</td>
<td>0.0532</td>
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<tr>
<td>86.0</td>
<td>0.140</td>
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<tr>
<td>140.0</td>
<td>1.79</td>
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<tr>
<td>158.0</td>
<td>3.81</td>
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<tr>
<td>176.0</td>
<td>7.73</td>
</tr>
</tbody>
</table>
APPENDIX G

Calculation of the Latent Heat of Sublimation of Naphthalene

Using the Clausius-Clapeyron Equation (25):

\[
\frac{dp^o_{mm}}{p^o_{mm}} = \frac{H'}{R'T^2_{abs}} \frac{dT_{abs}}{T^2_{abs}}.
\]  \hspace{1cm} (96)

or,

\[
H' = \frac{R'T_{abs}}{p^o_{mm}} \frac{dp^o_{mm}}{dT_{abs}}.
\]  \hspace{1cm} (97)

\[
= \frac{R'T_{abs}}{R'T_{abs}} \frac{dp^o_{mm}}{p^o_{mm}} \frac{dp^o_{mm}}{dT_{abs}}.
\]  \hspace{1cm} (98)

\[
= \frac{R'T^2_{abs}}{R'T_{abs}} \frac{dlnp^o_{mm}}{dT_{abs}}.
\]  \hspace{1cm} (99)

Then,  

\[
H = 2.303\frac{R'T^2_{abs}}{M'} \frac{dlogp^o_{mm}}{dT_{abs}}.
\]  \hspace{1cm} (100)

From Appendix F,

\[
\left(\frac{dlogp^o_{mm}}{dT_{abs.}}\right) = \frac{(71.401)(0.05223)}{T^2_{abs}}.
\]  \hspace{1cm} (101)

Therefore,

\[
H = \left(2.303\frac{R'T^2_{abs.}}{M'}\right) \left(\frac{3729}{T^2_{abs.}}\right).
\]  \hspace{1cm} (102)
\[ H = \frac{(2.303)(3729)R'}{M} \]

\[ = \frac{(2.303)(3729)(3.577)}{128} \]

\[ = 240.0 \text{ Btu/lb}\cdot\text{mass} \]
APPENDIX H

Proof That the Heat Transfer Cup, Riding on an Eccentric Cam, Moves in Simple Harmonic Motion

Consider the diagram of the circular cam mounted eccentrically on the circular drive shaft shown in Figure 35:

![Diagram of Cam Mounted on Vibrator Drive Shaft](image)

**FIGURE 35. DIAGRAM OF CAM MOUNTED ON VIBRATOR DRIVE SHAFT**

In this diagram, A is the center of the cam of radius $r_1$, B is the center of the vibrator drive shaft of radius $r_2$, and $R_{cc}$ is the distance between these two centers.

Now, point A will revolve around point B in a circle of radius $R_{cc}$. As shown by Shortley and Williams (46), page 270, the projection, on a diameter, of uniform circular motion is simple harmonic motion. The load, due to the construction of the vibrator, will always rest on
the circumference of the cam, vertically above point A and at a distance \( r_1 \) from A. Therefore, if the projected vertical motion of point A is in simple harmonic motion, the projected vertical motion of the point upon which the load rests will also undergo this same motion. The amplitude of vibration of the load will be the same as that of point A, and this amplitude is twice the center-to-center distance \((2R_{cc})\) of the cam and the drive shaft. It should be noted that this amplitude is independent of the radii of both the drive shaft and the cam.
APPENDIX I

Control Panel Circuit Diagrams
Condenser Water Pump

Hot-Water Pump

Hot-Water Storage—Tank Immersion Heaters

FIGURE 36. CIRCUIT DIAGRAMS
Water-Jacket Immersion Heaters

Vacuum Pump

Mechanical Vibrator

FIGURE 37 CIRCUIT DIAGRAMS
Heat-Transfer Surface—Temperature Control

Pressure Control

FIGURE 38. CIRCUIT DIAGRAMS
Calibration Data

Figure 39 gives the corrections to be applied to the readings of the thermocouples in the surface temperature measuring device.

![Calibration Curves for Thermocouples](image)

**FIGURE 39. CALIBRATION CURVES FOR THERMOCOUPLES IN SURFACE-TEMPERATURE MEASURING DEVICE**

Figure 40 gives the correction curve for the vaporizer thermometer.
FIGURE 40. CALIBRATION CURVE FOR VAPORIZER THERMOMETER
Sample Calculation for Experimental Run

For Run No. 14, the following data were obtained:

Initial weight naphthalene in cup: 3200 g.
Final weight naphthalene in cup: 2158 g.
Naphthalene condensed on cup walls: 133 g.
Naphthalene in No. 1 condenser: 660 g.
Naphthalene in No. 2 condenser: 169.0 g.

Particle size distribution of naphthalene remaining in cup:

<table>
<thead>
<tr>
<th>Mesh</th>
<th>6 - 8</th>
<th>8-10</th>
<th>10-14</th>
<th>&gt;14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wt. (g.)</td>
<td>2008</td>
<td>84</td>
<td>36</td>
<td>30</td>
</tr>
</tbody>
</table>

Length of test period: 15 min.
Pressure at vacuum pump during test period: 0.090 mm. Hg
Temperature in water jacket during test period: 155.3 F
Average power input during test period: 120.9 w.
Average vaporizer temperature during test period: 153.5 F
Heat transfer surface temperature during test period: 167.6 F

From these data, the calculations below were made:

1. Loss of naphthalene = 3200 - 2158 - 133 - 660 - 169
   = 3200 - 3120 = 80 g. = 0.176 lb. mass

2. Actual vaporization rate during test period =
   (169.0/453.6)/(15/60) = 1.490 lb. mass/hr.

3. Theoretical vaporization rate based on power input:
Measured power input = 120.9 W.

Correction for losses = 2.5 W.

Actual power input = 118.4 W.

Latent heat of sublimation at vaporizer temperature = 240.0 Btu/lb.m

Theoretical vaporization rate = \( \frac{104.9}{240.0} \)

\[ = 1.687 \text{ lb.m/hr.} \]

4. Theoretical vaporization rate based on vaporizer temperature:

Sonic mass flow rate at vaporizer temperature (value from Figure 34) = 1110 lb.m/hr. ft.²

Area of orifice = \( 1.36 \times 10^{-3} \) ft.²

Theoretical vaporization rate = \( (1110)(1.36 \times 10^{-3}) \)

\[ = 1.510 \text{ lb.m/hr.} \]

5. \( T_s - T_g = 167.6 - 153.5 = 14.1 \text{ °F} \)

6. \( h_i \) from actual vaporization rate:

\[ h_i = \frac{(W/\theta)(H_{T_g})}{(A_s)(T_s - T_g)} \]

\[ = \frac{(1.490)(240.0)}{(0.777)(14.1)} \]

\[ = 32.6 \text{ Btu/ft}^2\text{hr.°F}. \]

7. \( h_i \) from theoretical vaporization rate based on power input:

\[ h_i = \frac{(1.687)(240.0)}{(0.777)(14.1)} \]

\[ = 37.0 \text{ Btu/ft}^2\text{hr.°F}. \]
8. \( h_1 \) from theoretical vaporization rate based on vaporizer temperature

\[
h_1 = \frac{(1.510)(240.0)}{(0.777)(14.1)}
\]

\[
= 33.1 \text{ Btu/ft}^2\text{hr.}^\circ\text{F.}
\]

9. Ratio of theoretical vaporization rate based on power input to actual vaporization rate = 1.687/1.490

\[
= 1.132
\]

10. Ratio of theoretical vaporization rate based on vaporizer temperature to actual vaporization rate = 1.510/1.490

\[
= 1.013
\]
APPENDIX L

Experimental Data
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<th>Run No.</th>
<th>Date</th>
<th>Freq., cpm.</th>
<th>Ampl., in.</th>
<th>Bed Depth, in.</th>
<th>Particle Size, mesh</th>
<th>Heat Transfer Surface Temp., °F.</th>
<th>Initial Naph. in Cup, g.</th>
<th>Final Naph. in Cup, g.</th>
<th>Naph. Cond'd in Cup, g.</th>
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</thead>
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<td>6-8</td>
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<td>-</td>
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<td>Naph. on No. 1 Fintube, g</td>
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<td>Avg. Power Input, w</td>
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APPENDIX M

Statistical Calculations

1. Variance analysis of heat transfer coefficients from factorial experiment at two bed depths, two amplitudes of vibration, and three frequencies of vibration:

<table>
<thead>
<tr>
<th>Bed Depth, (in.)</th>
<th>Amplitude, (in.)</th>
<th>Frequency, (cm)</th>
<th>Sum</th>
<th>Mean</th>
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<td></td>
<td>685</td>
<td>1400</td>
<td>3210</td>
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</tr>
<tr>
<td>1 1/2</td>
<td>0.02</td>
<td>16.4</td>
<td>32.1</td>
<td>20.7</td>
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<td>0.04</td>
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<td>19.5</td>
<td>5.9</td>
</tr>
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<td>0.02</td>
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<td>34.0 (avg.)</td>
<td>19.9</td>
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<tr>
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<td>0.04</td>
<td>31.6</td>
<td>21.0</td>
<td>5.9</td>
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<th>Mean</th>
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<td>23.43</td>
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<tr>
<td>0.04</td>
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<td>123.5</td>
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<tr>
<td>Mean</td>
<td>22.37</td>
<td>22.01</td>
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\[
X = \frac{(264.1)^2}{12} = 5812.4000
\]

\[
T = (16.4)^2 + \ldots + (5.9)^2 = X = 1231.3100
\]

\[
T_{FD} = \frac{(56.0)^2 + \ldots + (25.8)^2}{2} = X = 1491.3850
\]

\[
T_{FA} = \frac{(33.9)^2 + \ldots + (11.8)^2}{2} = X = 1195.4550
\]

\[
T_{DA} = \frac{(69.2)^2 + \ldots + (58.5)^2}{3} = X = 32.2167
\]

\[
F = \frac{(105.1)^2 + (106.6)^2 + (52.4)^2}{4} = X = 176.4325
\]

\[
D = \frac{(134.2)^2 + (129.9)^2}{6} = X = 13.4317
\]
\[ A = \frac{(140.6)^2 + (123.5)^2}{6} = x = 24.3683 \]

\[ FD = T_{FD} - F - D = 13.4108 \]

\[ FA = T_{FA} - F - A = 694.6542 \]

\[ DA = T_{DA} - D - A = 6.3067 \]

\[ \text{Error} = T - F - D - A - FD - FA - DA = 14.5958 \]

The variances of the primary factors, their interactions, and the experimental error are presented in Table XI.

2. Variance analyses of heat transfer coefficients from factorial experiment at two bed depths and three frequencies of vibration, but at constant amplitude of vibration:

For an amplitude of 0.02 in.:

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<td>3210</td>
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<td>34.0 (avg.)</td>
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<td>86.8</td>
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<td><strong>33.9</strong></td>
<td><strong>66.1</strong></td>
<td><strong>40.6</strong></td>
<td><strong>172.4</strong></td>
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<td><strong>2</strong></td>
<td><strong>2</strong></td>
<td><strong>2</strong></td>
<td><strong>8</strong></td>
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</tr>
<tr>
<td><strong>Mean</strong></td>
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<td><strong>16.95</strong></td>
<td><strong>33.05</strong></td>
<td><strong>20.30</strong></td>
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<td><strong>21.55</strong></td>
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\[ x = (172.4)^2 / 8 = 3715.22 \]

\[ T = (16.4)^2 + / / / + (19.9)^2 - x = 377.02 \]
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<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F-Ratios</th>
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Combining the FD and DA interactions with experimental error:

<p>| Experimental error       | 34.3133        | 5                  | 6.8627      |</p>
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<th>95%</th>
<th>97.5%</th>
<th>99%</th>
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<td>6.7054</td>
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<td>$\sigma^2 + 3\sigma_D^2$</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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<td>$\sigma^2$</td>
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with experimental error:

6.8627
\[ F = \frac{(31.8)^2 + (40.6)^2}{2} - x = 373.79 \]
\[ D = \frac{(85.6)^2 + (86.8)^2}{4} - x = 0.18 \]

Error = T - F - D = 3.05

**TABLE XII. VARIANCES OF PRIMARY EFFECTS AND EXPERIMENTAL ERROR FOR ALL RUNS AT s = 0.02 IN.**

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<th>Mean Square</th>
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<th>99%</th>
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For an amplitude of 0.04 in.

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<th>3210</th>
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<td>2</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>15.90</td>
<td>35.60</td>
<td>20.25</td>
<td>5.90</td>
<td>19.41</td>
<td></td>
</tr>
</tbody>
</table>
\[ X = (155.3)^2/8 = 3014.76 \]
\[ T = (16.4)^2 + \ldots + (5.9)^2 = X = 948.95 \]
\[ F = \frac{(31.8)^2 + \ldots + (11.8)^2}{2} = X = 915.32 \]
\[ D = \frac{(81.4)^2 + (73.9)^2}{4} = X = 7.03 \]

Error = \( T - F - D = 26.60 \)

**TABLE XIII. VARIANCES OF PRIMARY EFFECTS AND EXPERIMENTAL ERROR FOR ALL RUNS AT s = 0.04 IN.**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F-Ratio</th>
<th>95%</th>
<th>99%</th>
<th>99.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>915.32</td>
<td>3</td>
<td>305.11</td>
<td>34.40</td>
<td>9.3</td>
<td>29.5</td>
<td>47.5</td>
</tr>
<tr>
<td>Bed Depth</td>
<td>7.03</td>
<td>1</td>
<td>7.03</td>
<td>0.79</td>
<td>10.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp. Error</td>
<td>26.60</td>
<td>3</td>
<td>8.87</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>948.95</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Calculation of confidence limits by Proschan and Babcock's method (143):

Using the four pairs of replicated runs,

\[
\text{Avg. range} = \frac{[(17.0 - 15.7) + (20.0 - 13.2) + (16.6 - 14.1) + (35.3 - 32.6)]}{4} = 2.1
\]

For four sets of two measurements each, their multiplying factor is 1.67. Hence,
95% confidence limits = \( \pm (1.67)(2.1) \) 
\[ = \pm 3.5 \text{ Btu/ft}^2\text{hr}^\circ\text{F}. \]

Using all pairs of measurements at the two bed depths as replicates,

\[
\text{Avg. range} = \frac{[(16.4 - 15.4) + (17.5 - 16.4) + \ldots + (21.0 - 19.5) + (5.9 - 5.9)]}{8} 
= 1.8
\]

For eight sets of two measurements each, their multiplying factor is 1.42. Hence,

95% confidence limits = \( \pm (1.42)(1.8) \) 
\[ = \pm 2.6 \text{ Btu/hr} \cdot \text{ft}^2 \circ\text{F}. \]

4. Calculation of significance in the separation of the ranges of \( h_j \) for previbrated and nonprevibrated stagnant runs:

Using the method outlined by Proschan and Babcock (43),

\[
\begin{array}{c|c|c}
\text{Nonprevibrated} & h_i & \text{Range} \\
15.7 & 17.0 & 1.3 \\
14.1 & 16.6 & 2.5 \\
\end{array}
\]

\[
h_{\text{avg.}2} = 15.8
\]

\[
\begin{array}{c|c|c|c}
\text{Previbrated} & h_i & \text{Range} & \text{Avg. range} \\
20.0 & 18.2 & 1.8 & \frac{5.6}{3} = 1.9 \\
19.4 & & & \\
\end{array}
\]

From Proschan and Babcock's table, for 3-1/2 sets of two values
each, $\gamma^* = 0.716$ at 95% confidence level.

$$\gamma^* = \frac{h_{\text{avg.1}} - h_{\text{avg.2}}}{2R_{\text{avg.}}} = \frac{19.2 - 15.8}{(2)(1.9)} = \frac{3.4}{3.8} = 0.895$$

Since $\gamma^* > \gamma^1$, there is a significant difference (95% confidence) between previbrated and nonprevibrated runs.
APPENDIX N

Calculation of Thermal Conductivity of Solid Naphthalene

From Volume V of the *International Critical Tables* (5th),

\[
k'_T = 38(1 - 33 \times 10^{-4} T_0) \times 10^{-4} \text{ watt/cm.}\circ\text{C.} \quad (104)
\]

Hence, at 0 \(^\circ\text{C.},\)

\[
k_T = 0.220 \text{ Btu/ft.}^2\text{hr.}(\circ\text{F.}/\text{ft.})
\]

and, at 150 \(^\circ\text{F.} = 66 \circ\text{C.},\)

\[
k_T = 0.172 \text{ Btu/ft.}^2\text{hr.}(\circ\text{F.}/\text{ft.})
\]
APPENDIX 0

Calculation of Frequencies at Which $a_m = g$

From Shortley and Williams (146),

$$a_m = \frac{2\pi n^2 s'}{12} \quad (105)$$

where,

- $a_m = \text{maximum acceleration in simple harmonic motion, ft./sec}^2$
- $n' = \text{frequency, cps.}$
- $s' = \text{amplitude, ft.}$

Therefore, at $s = 0.02$ in., the frequency at which $a_m = g = 32.17 \text{ ft./sec}^2$ is,

$$n' = \frac{\sqrt{a_m}}{\pi s'} \quad (106)$$

$$= \frac{\sqrt{32.17}}{\pi \left(\frac{2}{12}\right)(0.02/12)}$$

$$= 31.3 \text{ cps.}$$

Therefore,

$$n = 1880 \text{ cpm}$$

At $s = 0.04$ in.,

$$n' = \frac{\sqrt{32.17}}{\pi \left(\frac{2}{12}\right)(0.04/12)}$$

$$= 22.17 \text{ cps.}$$

And,

$$n = 1330 \text{ cpm}$$
APPENDIX P

Changes in Relative Velocity of Approach of Hot Plate and Bed and Contact Time With Increasing a m

1. Changes in relative velocity of approach of hot plate and bed with increasing a m:

The maximum acceleration of the vibrating hot plate is proportional to sn^2; that is, a_m increases directly with amplitude and with the square of the frequency. The instantaneous velocity of the plate is given by,

\[ v = \left( \frac{\pi}{120} \right) \text{sn} \sin \left( \frac{2\pi nt}{60} \right) \] (107)

For a falling body (the granular bed), the velocity is,

\[ v_b = -gt \] (108)

Therefore, the relative velocity of approach of plate and bed, v_r, will be:

\[ v_r = \left| \left[ \left( \frac{\pi}{120} \right) \text{sn} \sin \left( \frac{2\pi nt}{60} \right) - gt \right] \right| \] (109)

In this equation, s and n can vary from 0 to \( \infty \), \( \sin(2\pi nt/60) \) can vary from 1 to -1, and t can vary from 0 to 60/n. The relative velocity of approach, therefore, can range from 0 to \( \infty \).

A graphical representation may be used to demonstrate changes in v_r with a_m. First, for a constant frequency, v_r may be shown as in Figure 4.1.

From Figure 4.2 it is seen that a characteristic curve of v_r vs. time has the shape.

173
FIGURE 4.1. CHANGES IN $v_\tau$ AT CONSTANT FREQUENCY
FIGURE 42. CHARACTERISTIC SHAPE OF $v_r$ VS. TIME CURVE

That is, two maxima in $v_r$ occur during each cycle, with $v_r$ dropping to zero at some point between. As shown in Part 2 of this Appendix, the instant of impact between hot plate and bed approaches $60/n$ as either $s$ or $n$ is increased, i.e., as $a_m$ increases. Therefore, it is seen from Figure 42 that $v_r$ may be either increasing or decreasing as $a_m$ increases at constant frequency.

For a constant amplitude, a similar analysis holds. Figure 43 shows curves of $v_r$ vs. time for constant amplitude, and it is noted that, as in Figure 42, a characteristic curve will have two maxima during each cycle. As above, it is evident that $v_r$ at impact may either increase or decrease with increasing $a_m$.

2. Changes in contact time with increasing $a_m$:

$\Theta_{\text{contact}}$ is the fraction of the total time that the granular bed and the hot plate are actually in contact. It is represented in
FIGURE 43. CHANGES IN $v_r$ AT CONSTANT AMPLITUDE
Figure 27 by the time interval c to a' divided by the cycle time, a to a'. In Figure 44 are shown a number of curves representing the motion of the hot plate for a constant frequency, \( n \), and several different amplitudes, \( s_1, s_2, s_3, s_4, s_5, \) and \( s_6 \). In this same figure, the dashed curve represents the motion of the falling bed; the intersection between this dashed curve and each of the various sine curves gives the time at which collision between the bed and the hot plate occurs at each amplitude. The sine curves are designated from \( s_1 \) to \( s_6 \) so that amplitude increases progressively. It is easily seen that the instant of impact approaches \( 1/n \) as amplitude increases and, therefore, the fraction of actual contact time, \( \Theta_{\text{contact}} \), decreases towards zero.

From Figure 45, for a constant amplitude, \( s \), and increasing frequencies, \( n_1 \) through \( n_6 \), the conclusion is reached that \( \Theta_{\text{contact}} \) decreases continuously towards zero as the frequency of vibration of the hot plate is increased at constant amplitude.

Since \( a_m \propto sn^2 \), \( \Theta_{\text{contact}} \) must likewise be reduced as \( a_m \) increases.
FIGURE 44. CHANGES IN TIME OF IMPACT WITH AMPLITUDE AT CONSTANT FREQUENCY
FIGURE 45. CHANGES IN TIME OF IMPACT WITH FREQUENCY AT CONSTANT AMPLITUDE
APPENDIX Q

Calculation of Contact Times and Impulse per Pound of Bed for Runs at am > g

1. Calculation of contact times for Run Nos. 1c, 3, 10, 12, 23, and 24:

Assuming that the hot plate drops out from under the bed at the moment it starts its downward movement, the instant of collision may be calculated by trial and error as the intersection of the curves representing the simple harmonic motion of the plate and the falling body motion of the bed. For the plate,

\[ x = \left(\frac{1}{2}\right)s \cos \omega t - \left(\frac{1}{2}\right)s \]  

(110)

For the bed,

\[ x = \left(\frac{1}{2}\right)gt^2 \]  

(111)

Now, for Run Nos. 1c and 3, \( w = 116.7 \) radians/sec., and \( s' = 0.04/12 \) ft. By trial and error, it is found that the curves intersect at \( t = 0.0077 \) sec. after the start of the cycle. Since the cycle time is \( 60/n \) seconds,

\[ \theta_{\text{contact}} = \frac{60}{11400} - 0.0077 \]

\[ = 0.82 \]

Similarly, for Run Nos. 10 and 12, \( \theta_{\text{contact}} = 0.16 \), and for Run Nos. 23 and 24, \( \theta_{\text{contact}} = 0.33 \).
2. Calculation of collision impulse per pound of bed for runs at $a_m \geq g$:

Impulse is defined as the product of an average force, $F$, times the interval during which this force acts, $\Delta t$. For a collision,

$$\text{Impulse} = \bar{F}g \Delta t = \Delta (mV) = \text{change of momentum}$$

Therefore, even though the duration of the interval in which collision force acts is not known, the impulse can be calculated from the change of momentum on collision. Impulse per pound of bed is then simply the difference in velocities of the bed with respect to the hot plate before and after collision. For an inelastic collision, the relative velocity after impact is zero, and so the impulse of collision per pound is equal to the relative velocity of approach of bed and hot plate at the instant of impact. This relative velocity of approach is given by:

$$v_r = \left\lvert \left(\frac{n}{720}\right) \sin \left(\frac{2\pi nt}{60}\right) - gt \right\rvert$$

Therefore,

<table>
<thead>
<tr>
<th>Run No.</th>
<th>$s$ (in.)</th>
<th>$n$ (cpm)</th>
<th>Time of Collision After Start of Cycle (sec)</th>
<th>$v_r$ (ft./sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1c and 3</td>
<td>0.04</td>
<td>1400</td>
<td>0.0077</td>
<td>0.03</td>
</tr>
<tr>
<td>10 and 12</td>
<td>0.02</td>
<td>3210</td>
<td>0.0101</td>
<td>0.39</td>
</tr>
<tr>
<td>23 and 24</td>
<td>0.04</td>
<td>3210</td>
<td>0.0124</td>
<td>0.87</td>
</tr>
</tbody>
</table>

3. Calculation of impulse per pound of bed due to relative acceleration of bed and plate for runs at $a_m > g$:

This impulse may be found by graphically integrating the area under a curve of relative acceleration of bed with respect to plate vs.
time. Relative acceleration is given by Equation 54,

\[ a_r = \frac{1}{2} st^2 \cos \theta - g \]  

(54)

Using this equation, the following table may be prepared:

<table>
<thead>
<tr>
<th>Run Nos. 1c and 3</th>
<th>Run Nos. 10 and 12</th>
<th>Run Nos. 23 and 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>t (sec.)</td>
<td>( a_r ) (ft./sec.(^2))</td>
<td>t (sec.)</td>
</tr>
<tr>
<td>0</td>
<td>3.6</td>
<td>0</td>
</tr>
<tr>
<td>0.002</td>
<td>3.1</td>
<td>0.001</td>
</tr>
<tr>
<td>0.003</td>
<td>3.1</td>
<td>0.002</td>
</tr>
<tr>
<td>0.005</td>
<td>7.8</td>
<td>0.003</td>
</tr>
<tr>
<td>0.008</td>
<td>17.0</td>
<td>0.004</td>
</tr>
<tr>
<td>0.010</td>
<td>28.5</td>
<td>0.005</td>
</tr>
<tr>
<td>0.013</td>
<td>40.9</td>
<td>0.006</td>
</tr>
<tr>
<td>0.015</td>
<td>53.2</td>
<td>0.007</td>
</tr>
<tr>
<td>0.018</td>
<td>63.3</td>
<td>0.009</td>
</tr>
<tr>
<td>0.020</td>
<td>67.4</td>
<td>0.010</td>
</tr>
<tr>
<td>0.023</td>
<td>67.4</td>
<td>0.012</td>
</tr>
<tr>
<td>0.025</td>
<td>63.3</td>
<td>0.013</td>
</tr>
<tr>
<td>0.028</td>
<td>53.2</td>
<td>0.014</td>
</tr>
<tr>
<td>0.030</td>
<td>40.9</td>
<td>0.015</td>
</tr>
<tr>
<td>0.033</td>
<td>28.5</td>
<td>0.016</td>
</tr>
<tr>
<td>0.035</td>
<td>17.0</td>
<td>0.017</td>
</tr>
<tr>
<td>0.038</td>
<td>7.8</td>
<td>0.018</td>
</tr>
<tr>
<td>0.040</td>
<td>3.1</td>
<td>0.019</td>
</tr>
<tr>
<td>0.043</td>
<td>3.6</td>
<td></td>
</tr>
</tbody>
</table>

From this table, the graphs on the following three pages were prepared and graphically integrated to give:

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Impulse/lb. of bed due to ( a_r ) (ft./sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>lc and 3</td>
<td>1.37</td>
</tr>
<tr>
<td>10 and 12</td>
<td>0.56</td>
</tr>
<tr>
<td>23 and 24</td>
<td>0.60</td>
</tr>
</tbody>
</table>
Total area = 5405
(Unit area = 0.0025 ft./sec.)
Impulse/lb. = 1.37 ft./sec.

FIGURE 46. RELATIVE ACCELERATION OF BED AND PLATE FOR RUN NUMBERS 1C AND 3
Total area = 2220
Impulse/lb. = 0.56 ft./sec.

Figure 47. Relative Acceleration of Bed and Plate for Run Numbers 10 and 12

A-14362
Total area = 2390
Impulse/lb. = 0.60 ft./sec.

FIGURE 48. RELATIVE ACCELERATION OF BED AND PLATE FOR RUN NUMBERS 23 AND 24

A-14360
APPENDIX R

Calculation of the Maximum Pressure Drop
Through the Granular Bed

The maximum flow rate of vapors through the granular bed, assuming all vapors to be generated at the bottom of the bed, was 1.6 lb. mass per hr. This is equivalent to 0.0125 lb. moles of naphthalene per hr. At the maximum bed temperature of 156 F (pressure = 3.5 mm. Hg) the volumetric flow rate through the bed would be 1220 ft.3/hr.

Next, the Reynold's number may be calculated. From Lapple (35),

\[ N_{Re} = \frac{D_p \cdot V \cdot \rho}{\mu} \]  

(113)

where,

\[ D_p = \frac{0.112}{12} = 0.00932 \text{ ft.} \]

\[ V = \frac{1220}{0.777} = 1570 \text{ ft.}^3/\text{hr.} \]

\[ \rho = 0.00131 \text{ lb. mass/ft.}^3 \]

\[ \mu \] was found in Appendix C to be approximately 4.6 x 10^{-6} lb. mass/ft. sec. Therefore,

\[ N_{Re} = \frac{(0.00932)(0.135)(0.00131)}{(4.6 \times 10^{-6})} \]

\[ = 1.16 \]

From Lapple (35),

\[ f = 730 \]
Hence,

\[ \Delta P_b = \left( \frac{4fL_b}{D_p} \right) \left( \frac{V^2}{2g_c} \right) \]  

(114)

The maximum \( L_b \) was 3 in. = 0.25 ft.

\[ \therefore \ \Delta P_b = \left[ \left( \frac{4(730)(0.25)}{0.00932} \right) \right] \left[ \left( \frac{0.00131(0.435)^2}{(2)(32.2)} \right) \right] \]

\[ = 0.3 \text{ lb force/ft}^2 \]

\[ \approx 0.1 \text{ mm. Hg} \]
I, David George Stephan, was born in Columbus, Ohio, February 8, 1930. I received my secondary school education in the public schools of Upper Arlington, Ohio. My undergraduate training was obtained at The Ohio State University, from which I received the degree Bachelor of Chemical Engineering in 1952. By pursuit of a combined curriculum in the College of Engineering and the Graduate School, I received the degree Master of Science on the same date. I was then selected for a Battelle Fellowship in the Department of Chemical Engineering at The Ohio State University. I have held this appointment continuously while completing the requirements for the degree Doctor of Philosophy.