Exploring Influences of Mathematics Coach-Teacher Interactions on the Development of Teacher Pedagogical Knowledge, Effective Mathematical Teaching Practices, and a Classroom Culture of Mathematical Inquiry

Dissertation

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Abstract

This study set out to examine how interactions between a mathematics instructional coach and a teacher influence teacher pedagogical content knowledge, instructional practices, and a classroom culture of mathematical inquiry (CCMI). The research literature on mathematics instructional coaching was limited, but showed promise in supporting teachers’ learning regarding reform-based mathematics teaching and learning. Learning through teaching has been well documented in the literature; however, teaching experience is not sufficient in teacher learning. An instructional coach’s role is to support teacher learning through job-embedded professional development. The study examined (a) the nature of coach-teacher interactions on the development of teacher’s pedagogical content, effective mathematics teaching practices, and the development of a CCMI; (b) the conditions that made such interactions productive; and (c) the manifestations of change in teacher practices regarding teacher pedagogical content knowledge, effective mathematics teaching practices and CCMI.

The study was a qualitative, single-case study. This was a naturalistic study in which the data were collected in the natural setting: the classroom. The researcher was the instructional coach in the study. One first-grade teacher participated in the study as well as twelve of her students. The site of the investigation was a kindergarten through grade five school with approximately 480 students, of which 85% received free or reduced lunch.
The data examined in this study included the components of a coaching cycle, pre- and post-conferences, classroom coaching sessions and coach/researcher reflections. In addition, the researcher conducted interviews and observations at the beginning and end of the coaching cycle and three months after the coaching cycle ended. The interviews and conferences were audiotaped, and the classroom coaching sessions and observations were videotaped. The interview, conference, and observation data were analyzed using the constant comparative method. These data were used to determine the conceptual categories used to analyze the data, while the literature was used to define these categories. The classroom coaching sessions were viewed repeatedly to identify significant episodes based on the conceptual framework used in this study. Using all sources of data, a narrative timeline was written and used to analyze the data for ways in which teaching practices evolved over time.

This study found that PCK, teaching practices, and CCMI were primary themes in the coach-teacher interactions. The coach’s role during mathematics lessons was limited due to the teacher’s comfort level. This required use of spontaneous instead of planned coaching moves. The conditions that contributed to implementation of teacher’s practices included (a) zone of proximal development; (b) visualization of practices; (c) ease of implementation; (d) teacher’s beliefs; (e) messages from the broader community; and (f) the coach’s understanding of PCK, teaching practices, and CCMI. Success of implementation of strategies depended on the coach’s ability to help the teacher visualize practices and to trim and decompress teaching practices. Finally, a model for coach growth-of-knowledge is proposed based on the model by Perks and Prestage (2008) for teacher and mathematics teacher educator (MTE) growth-of-knowledge.
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Chapter 1: Introduction

Overview

Over the past three decades, the mathematics education community has made recommendations regarding a different vision for mathematics teaching and learning. Historically, mathematics teaching and learning focused on computation and related skills. However, societal changes call for a different definition of mathematical literacy, which in turn requires different approaches to teaching. This new vision entails teaching and learning that fosters mathematical reasoning and problem solving and views mathematics as a sense-making subject wherein students are active participants in learning rather than passive receivers of knowledge (National Council of Teachers of Mathematics [NCTM], 1989). Key people in changing the way that mathematics is taught and learned are the classroom teachers (NCTM, 1991). For this to happen, teachers need long-term support and adequate resources. Many documents have been created to support teachers learning of these shifts in teaching (NCTM, 1989, 1991, 2000, 2014). Having these resources available is not sufficient; job-embedded professional development is needed to help teachers make sense of these changes and effectively put them into practice.

Problem Statement

Researchers in mathematics education have attempted to characterize ways in which students learn mathematics with understanding. Many different definitions of
understanding have been provided in mathematics education literature. An early definition of mathematical understanding involved building relationships and making connections among mathematical ideas (Brownell, 1935, p. 370). More recently, the authors of the *Common Core State Standards for Mathematics* (CCSS) wrote, “One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from” (National Governors Association Center for Best Practices, 2010, p. 4). Skemp (2006) distinguished between two types of understanding, relational and instrumental. Traditional forms of teaching and learning focus on instrumental understanding, which Skemp referred to as “rules without reason” (p. 2). Relational understanding refers to knowing what to do and why.

Understanding is also a key aspect of mathematical proficiency. The NCTM (2000) defined mathematical proficiency as having factual knowledge, conceptual understanding, and procedural fluency. The authors of the report *Adding It Up. Helping Children Learn Mathematics* (2001), defined mathematical proficiency similarly but also included adaptive reasoning, strategic competence, and productive disposition. While there are many different ways that understanding mathematics has been defined, they each have similarities. Students who understand mathematics are able connect mathematical ideas, understand why the mathematics works, are able to solve unfamiliar problems, are able to represent and communicate their thinking and are able to reason about mathematics and justify their ideas mathematically. In addition, students will view mathematics as important and valuable.

Supporting this type of learning requires a shift in every aspect of pedagogical
views of teaching and learning mathematics. These shifts include many elements; however, this study will focus on three. These are (a) pedagogical content knowledge needed for teaching mathematics; (b) effective mathematical teaching practices; and (c) a classroom culture of inquiry. Traditionally the focus of instruction has been on mastering facts, skills, and procedures with little or no emphasis on meaning; however, mathematics education reform suggests this knowledge base is not sufficient for mathematical proficiency (National Research Council, 2001; NCTM, 2000; Skemp, 2006).

Mathematics classrooms focused on teaching for understanding and sense making encompass a broader range of components than conventional classrooms, including problem solving, reasoning, mathematical communication, and justification.

Creating classrooms that embody mathematical understanding is possible and requires deliberate action by the teacher. The literature has recommended ways in which teachers can help students make sense of mathematical ideas and recommends that students have opportunities for reflection and communication (Hiebert et al., 1997). Teachers must provide students with tasks that deepen their content knowledge and provide experiences for them to make mathematical connections (NCTM, 2000). In addition, teachers need to provide students with opportunities to reason and communicate about mathematics. The NCTM suggested that “Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills.” (p. 21). These types of experiences are critical in developing students’ mathematical understanding.
While the mathematics education research community has made some progress in defining what and how students learn mathematics, such research-based recommendations have not been transferred to instructional practice in most classrooms (Kawanaka & Stigler, 1999; Staples, 2007; Stigler & Hiebert, 1999). Despite efforts in both pre-service and in-service teacher education programs, little has changed in the teaching and learning of mathematics (Stigler & Hiebert, 2009). Reform-based instructional practice is vastly different than the experiences of most teachers in their own elementary and secondary school education. Teachers’ conceptions of teaching mathematics are largely formed on their own learning experiences in elementary and secondary schooling. The change in thinking recommended by reform regarding what and how children learn and the instructional practices that support learning is neither commonsense nor innate; however, it is teachable (Ball & Forzani, 2010). Yet, determining how to support teachers in their learning about reform-based instructional practices and creating the environment needed to support these practices and how to implement them in the classroom has been a challenge. This is partly due to an “underdeveloped understanding” (Staples, 2007, p. 162) of the practices and the roles of the teacher and students needed to participate in reform-based classrooms.

Traditional forms of professional development have not been successful in supporting teachers in changing practice that is recommended by the mathematics education research community. Teachers need more support from experts in the field in order to implement these changes. Some important characteristics of effective professional development are that it is intensive, ongoing, connected to practice, and focused on the teaching and learning of specific academic content (Darling-Hammond,
Chung Wei, Andree, Richardson, & Orphanos, 2010; Guskey, 2003; Loucks-Horsely, Love, Stiles, Mundry, & Hewson, 2003). Coaching programs such as the Mathematics Coaching Program (MCP) (Brosnan, 2010) have shown promise in implementation of reform-based practices. However, coaching models such as MCP need a deeper research base in how to support coaches in their own learning about mathematics content and pedagogy and how the coaches can support teachers’ learning and implementation of research-based best practices.

**Purpose**

The purpose of this study was to examine the nature of coach-teacher interactions focused on developing teacher pedagogical content knowledge, effective teaching practices, and a classroom culture of mathematical inquiry. Such classrooms embody different characteristics than that of a traditional classroom. Hiebert et al. (1997) defined features of a mathematics classroom that are critical to teaching for mathematical understanding. These include the nature of mathematical tasks, the role of the teacher, the social culture of the classroom, mathematical tools as learning supports, and equity and accessibility. The literature supports the following components of the social culture of the classroom that supports a community of mathematical inquiry (Goos, 2004; Hiebert et al., 1997; Horizon Research, 2000; NCTM, 1991). First, students’ ideas are believed to have merit and can potentially lead to learning with understanding. Second, students need to have autonomy to solve problems in ways that make sense to them. In addition, students need to respect that everyone needs to understand their own ideas. Third, mistakes should be appreciated and seen as an opportunity to learn and an opportunity for analysis. They should not be covered up, but used in the construction of knowledge. Finally, the shift of
authority for correct arguments needs to lie within the logic and structure of mathematics, rather than in the social status of the participants. Students’ logical explanations should be seen as a source of correctness, as opposed to relying on the teacher or a text. This requires a shift in the power dynamic of the classroom. In this type of environment, students feel safe to share, defend and critique ideas, and receive feedback from others. This type of classroom—wherein students learn to speak and think mathematically and participate in mathematical discussions to solve new problems—is what Richards (1991) described as an inquiry classroom. Goos (2004) described such classrooms as communities of mathematical inquiry. In this study, the coach will work with the teacher to share this vision and help create this community.

Research Questions

1. What is the nature of coach-teacher interactions when the objective of the collaboration is to facilitate the development of teacher pedagogical content knowledge, effective teaching practices, and a classroom culture of mathematical inquiry?

2. Under what conditions are coach-teacher interactions productive when the objective of the collaboration is to facilitate the development of teacher pedagogical content knowledge, effective teaching practices, and a classroom culture of mathematical inquiry?

3. What are some manifestations of changes in teacher pedagogical content knowledge, effective teaching practices, and classroom culture of mathematics inquiry resulting from coach-teacher collaboration?
Definition of Terms

Classroom culture of mathematical inquiry. Throughout the literature, the descriptions of inquiry teaching and learning are quite diverse (Towers, 2010). For this study, the researcher defined a community of mathematical inquiry as one that focuses on mathematical sense making and justification of ideas and arguments (Goos, 2004). This type of classroom community requires mutual respect for all members of the learning community regardless of status. All students contribute ideas, and all ideas are respected and considered. Errors are seen as an opportunity for learning. Students feel safe to share ideas. They are encouraged to consider and analyze each other’s ideas, to press each other for clarification, and to persist at making sense of mathematics until they understand. They see others’ ideas as an opportunity to learn and refine their own ways of thinking. Students have intellectual autonomy, which means they are free to use their own thinking to solve a problem. Correct solution strategies are based on mathematically valid logical arguments, not on the status of a particular member. A shared authority of deciding correctness involves the students, not solely the teacher.

MCP mathematics coaching. The Mathematics Coaching Program (Erchick & Brosnan, 2005) is a professional development program for mathematics instructional coaches that includes teaching coaches about mathematics content and mathematics-specific pedagogy as well as general coaching strategies, in order to better support the teachers they coach. This program offers a specific structure that provides the most impact on teacher and student learning as evidenced by the research (Brosnan, 2010). The research states that coaching a teacher requires frequent and consistent teaming with the teacher. This program provides a model in which the coach works with a teacher for six
Eight to eight weeks on a daily basis and includes a pre-conference and a post-conference weekly with the teacher.

The MCP coaching approach is grounded in the concept of the coach and teacher teaming to improve student learning of mathematics. This teaming includes collaboration in the development of lessons, team teaching, collaborative debriefing focused on student learning, and a continuation of this cycle of reflective practice. Coaches bring an expertise from their own professional development experiences to assist teachers in the use of inquiry, guided discovery, and problem-based, student-centered mathematics instruction, as well as in teacher professional learning of mathematics content for teaching. (Brosnan, Bucci, & Erchick, 2006).

**Rationale and Significance**

This study was designed to contribute to the literature on mathematics instructional coaching and its influences on the implementation of reform-based teaching and teacher development. All aspects of a reform-based classroom are integrated and contribute to each other. Each element is altered when one of the other elements is not in place. The author of this study focused on three such elements while coaching a first grade teacher: (a) teacher pedagogical content knowledge, (b) effective mathematical teaching practices, and (c) a classroom culture of mathematical inquiry.

Various studies have made efforts to identify characteristics of effective professional development (Darling-Hammond & Sykes, 1999; Loucks-Horsley et al.,
2003; National Staff Development Council, 2001; NCTM, 1989), but few have included coaching as a professional development model. Mathematics instructional coaching encompasses many of these characteristics which include. Effective mathematical coaching

1. Enhances teachers’ content and pedagogical knowledge;
2. Provides sufficient time and other resources;
3. Promotes collegiality and collaboration;
4. Aligns with other reform initiatives;
5. Models high-quality instruction;
6. Builds leadership capacity;
7. Includes follow-up and support; and
8. Is ongoing and job-embedded. (Darling-Hammond et al., 2010; Desimone, 2002; Guskey, 2003)

In an examination of the research of mathematics teacher professional development, Sztajn (2011), found that mathematics teacher education has only recently become an emerging field of study. The need for increasing teacher knowledge in mathematics and mathematics teaching and learning is becoming increasingly more important as the level and types of mathematical knowledge students are expected to acquire is increasing. For example, students are no longer expected to memorize rules and procedures to produce an answer (Adler et al., 2005). The charge today is to teach in a way that students can learn mathematics with understanding and see it as a sense-making subject (Hiebert et al., 1997; Kilpatrick et al., 2001). Yet, there is limited knowledge about how teachers gain the type of knowledge to teach increasingly higher
levels of mathematics and different processes and practices that have become part of the mathematics curriculum. Longitudinal large-scale studies are needed to answer questions about how teachers learn to teach mathematics and are lacking because the field of study is in its early stages. However, “small-scale qualitative studies make significant contributions for conceptualizing the complexity of teacher education and modeling individual teachers’ learning process” which can in turn be used to develop theories from which to inform large-scale studies (Adler et al., 2005, p. 370).

The last element in this study of the critical features of a reform-based classroom is the social culture of the classroom (Hiebert et al., 1997). Without the appropriate culture, task integrity may be compromised, and the students may not be engaged. This will impact what students will learn from the task, their view of the nature of mathematics and what it means to do mathematics. While the research-base has created descriptions of these classrooms, there is little known about how to support teachers in creating this type of classroom, and even less is known about how a coaching model can support teachers in establishing a culture that promotes learning for understanding. Teachers need support in creating this type of culture, and an intensive job-embedded model such as coaching may provide the support teachers need.

This study is significant in learning whether a mathematics coach can navigate the complex and sensitive terrain of learning about and making appropriate changes in teacher pedagogical content knowledge, teaching practices, and classroom culture. Coaching is difficult work, but it can transform teachers’ and students’ learning through establishing a classroom culture of inquiry.
Chapter 2. Literature Review

In this Chapter, it is important to establish what the current research suggests about related topics in the literature. The author of this study has organized this chapter into three main sections: 1) Mathematics teacher knowledge and practice; 2) Mathematics socio-cultural elements; and 3) Instructional coaching. The mathematics socio-cultural elements include socio-mathematical norms, social culture of the classroom, and classroom culture of inquiry. Instructional coaching discussed various models and findings and ends with a specific coaching model, referred to as Content Focused Coaching. The author concludes this Chapter with a Theoretical Framework that was used to guide this study.

Mathematics Teacher Knowledge and Practice

This section on mathematics teacher knowledge and practice includes what the research suggests about the following topics: (a) reform-based vision of teaching and learning; (b) teacher pedagogical content knowledge; (c) teacher learning; and (d) effective teaching and learning.

Reform-based vision of mathematics teaching and learning. *Agenda for Action* (NCTM, 1980) presented a new vision for mathematics education. This came about as a response to the previous decade’s “back to basics” focus, which did not produce increased achievement in computation facility as the movement predicted (1980). In addition, the current reform movement, as opposed to others, was influenced by research
in cognitive psychology regarding how children learn. This area of research was applied
to mathematics education, and the field has continued to develop over the past three
decades. Along with teaching content, mathematical practices have been emphasized in
the literature. These practices describe activities in which students engage to become
more proficient problem solvers. The *Principles and Standards for School Mathematics*
(NCTM, 2000) included the Mathematical Processes Standards, which are problem
solving, reasoning, communication, connections, and representations. A similar focus in
the *Common Core State Standards for Mathematics* (National Governors Association
Center for Best Practices, 2010), called the Standards for Mathematical Practice, included
the mathematical processes and also the strands of mathematical proficiency conceptual
understanding, procedural fluency, adaptive reasoning, strategic competence, and
productive disposition (National Research Council, 2001).

**Teacher pedagogical content knowledge.** In an examination of research on
teaching, Shulman (1986) found that the existing views of teacher knowledge at the time
were not sufficient. The focus was on content knowledge and general pedagogy, which
was not sufficient. There was a “missing paradigm” (p. 7) or a blind spot regarding the
content in the research literature and in teacher certification and evaluation. The literature
on research on teaching emphasized how teachers managed their classrooms, organized
activities, planned lessons, determined levels of questioning, determined general student
understanding, and assigned praise or blame. Shulman (1986) noticed a lack of research
into areas such as how teachers determine what content to teach and how to teach it, how
questions are asked, how explanations are offered, and how to deal with
misunderstandings (Shulman, 1986). He developed a more extensive framework for
thinking about content knowledge, which addressed the missing paradigms. These included (a) subject matter content knowledge; (b) pedagogical content knowledge; and (c) curriculum knowledge.

According to Shulman (1986) subject matter content knowledge refers to the organization and amount of knowledge in the mind of the teacher. To think correctly about content knowledge requires the organization of basic concepts and procedures, the knowledge of why and how they work, ways in which truth and validity of claims are established, and an understanding of what content is central to the discipline and what is not. Shulman (1986) defined pedagogical content knowledge as the intersection of subject matter knowledge and teaching, or “the ways of representing and formulating the subject that make it comprehensible to others” (p. 9). Pedagogical content knowledge includes the ways of representing and formulating the subject that make it comprehensible to others, including the most useful representations, examples and analogies, knowing what makes learning of particular content easy or difficult, preconceptions and misconceptions regarding specific topics, and the knowledge and strategies to address the misconceptions (Shulman, 1986, p. 10).

Shulman (1986) identified curriculum knowledge as knowledge of the programs designed for teaching subjects and topics, awareness of the instructional materials available to teach the programs, and the ability to decide when and when not to use specific materials for instruction. Further, he distinguished between lateral curriculum knowledge—the teacher’s ability to relate a given concept to issues being discussed in other classes—and vertical curriculum knowledge, otherwise known as the teacher’s
familiarity with concepts being taught within the same subject area in years prior and years following the current year.

Ball, Thames, and Phelps (2008) designed a framework for pedagogical content knowledge and elaborated on the work of Shulman (1986) by making it specific to the discipline of mathematics. Their framework included Shulman’s descriptions and organized them into the following domains of PCK:

1. Knowledge of content and students;
2. Knowledge of content and teaching; and

Ball and associates (2008) suggested including curriculum as part of PCK, and the author of this study used it as part of analysis. Further descriptions for each of the three domains were discussed in their work to provide more specific definitions. Table 1 explains the domains and categories of PCK developed by Ball et al. (2008).

Table 1. Domains of Pedagogical Content Knowledge.

| Knowledge of content and students (KCS) | a) Knowledge of student errors and misconceptions,  
b) Knowledge of mathematically accurate and appropriate content for students  
c) Knowledge of students’ experiences about a topic and how those past experiences may be helpful or confusing |
|----------------------------------------|---------------------------------------------------------------------------------------------------------------|

Continued
**Table 1 continued**

| Knowledge of content and teaching (KCT) | a) Knowledge of how to sequence topics in a unit, examples in a lesson and student solutions in a whole class discussion,  
b) Knowledge of which student responses would be productive to share in a whole class discussion and which should be saved for later,  
c) Knowledge of the advantages and disadvantages of specific mathematical representations and models |
| Knowledge of content and curriculum (KCC) | a) At what grade level particular content is taught  
b) How concepts are connected in the curriculum  
c) The models and representations with which students are familiar |

**Teacher learning.** According to Piaget, the purpose of learning is to develop autonomy, and the main purpose of a coach is to help teachers develop autonomy in developing reform-based instructional strategies and in creating a classroom community that supports that. While it is documented that teachers learn through their own experience of teaching, Mason (2002), suggested that learning through teaching requires more than experience. Mason (2002) wrote, “Teachers, confined to their own classrooms often are trapped in narrow, isolated environments in which experiences and responses to them are predictably guided by the models created by them earlier in similar environments.” Teachers need the support of a knowledgeable other who possesses a deep understanding of content and pedagogy to help facilitate their learning.

Maher and Alston (1990) described the following components to learning. First, learners have prior knowledge, which must be accessed during instruction. Second,
developing understanding is critical to learning, which includes retaining, accessing or utilizing what they learn. Third, successful learners actively monitor their learning, reflect on what they do, and do not understand and use strategies such as asking questions and explaining to themselves and to others to increase their knowledge. In order for teachers to learn from a coach, their prior knowledge about teaching and learning must be understood and used to create new learning. Teachers must have a deep understanding of the mathematics content and pedagogy in order for them to use the learning and become autonomous. And further, they need opportunities to reflect upon their instruction and student learning. A coach can facilitate this during conversations with the teacher. The classroom should be a place for teacher learning, and research has shown it can be such a place (Campbell, 2011). Learning through teaching has been given considerable attention in the mathematics education literature (Leikin & Rota, 2006).

Teacher learning has been shown to occur through analyzing student work and by focusing on student thinking. Teachers have demonstrated the changes recommended in the reform documents including beliefs about the nature of mathematics content, the nature of learning and the nature of teaching (Brosnan, 2010). In addition, (Wood, Cobb, & Yackel, 1991) found that teachers construct their own knowledge in the process of negotiating socio-mathematical norms with students during classroom instruction. By creating a classroom rich in mathematical discourse where students are giving a variety of explanations and justifications, teachers learn what constitutes mathematical sophistication, efficiency, and difference.

**Effective teaching and learning.** Due to changing societal needs NCTM (1989) developed a vision for what mathematics students should learn. In addition to content,
these standards included processes including problem solving, communication, reasoning, and mathematical connections. This change in what students should learn required changes in the teaching of mathematics. As a companion, NCTM also created the Professional Standards for Teaching Mathematics to provide guidance for this shift in expectations of student learning (NCTM, 1991). These standards described four elements of teaching, including tasks, discourse, environment, and analysis of teaching and learning. Since then, other documents have provided guidance to teachers in implementing these shifts in teaching (NCTM, 2000, 2014)

The teaching and learning of mathematics is complex. Because of the symbiotic relationship between teaching and learning, it is important to consider what constitutes each activity with respect to the current research literature.

**Effective mathematics learning.** A more comprehensive view of what it means to learn mathematics was developed in response to the changing needs of people today, as well as the current research in cognitive psychology and mathematics education (Bass et al., 2002; Wagner, 1993). *The Adding It Up Report (2001)* defined learning as mathematical proficiency and included five strands: (a) conceptual understanding, (b) procedural fluency, (c) adaptive reasoning, (d) strategic competence, and (e) productive disposition. Each of these strands is interrelated and insufficient without the others. Students who do not adequately develop each of these competencies are likely to have a fragile knowledge base, have more difficulty retaining mathematics ideas, and have more difficulty learning new content and applying content they have already learned (Bransford, Brown, & Cocking, 2000; Schoenfeld, 1992; Skemp, 2006).

The *Common Core State Standards* include Standards for Mathematical Practice.
(National Governors Association Center for Best Practices, 2010), which are processes in which students engage while learning mathematics. These standards were based upon the strands of mathematical proficiency (National Research Council, 2001) as well as the previous Process Standards (NCTM, 2000). They include

1. Make sense of problems and persevere in solving them;
2. Reason quantitatively and abstractly;
3. Construct viable arguments and critique the reasoning of others;
4. Model with mathematics;
5. Use appropriate tools strategically;
6. Attend to precision;
7. Look for and make use of structure; and
8. Look for and express regularity in repeated reasoning.

The following principles of learning will provide students with experiences that set a foundation for effective teaching:

1. Engage with challenging tasks that involve active meaning making and support meaningful learning;
2. Connect new learning with prior knowledge and informal reasoning and, in the process, address preconceptions and misconceptions;
3. Acquire conceptual knowledge as well as procedural knowledge, so that they can meaningfully organize their knowledge, acquire new knowledge, and transfer and apply knowledge to new situations;
4. Construct knowledge socially through discourse activity and interaction related to meaningful problems;
5. Receive descriptive and timely feedback so that they can reflect on and revise their work, thinking and understandings, and
6. Develop metacognitive awareness of themselves as learners, thinkers and problem solvers, and learning to monitor their learning and performance. (NCTM, 2014, p. 9)

**Teaching Approaches.** Theories characterizing teachers’ approaches to learning have been developed in order to further define teachers’ pedagogical knowledge and to inform the field of teacher education. Based on then-contemporary literature, Kuhs and Ball (1986) identified approaches to teaching. These approaches included (a) learner-focused, (b) content-focused with an emphasis on understanding, (c) content-focused with an emphasis on performance, and (d) classroom-focused. In the learner-focused approach, the teacher provides problem solving experiences and acts as a facilitator of learning by listening, asking questions, probing student thinking, and supporting by agreeing, restating, and encouraging; the learner actively participates in the construction of meaningful ideas. In the content-focused with understanding approach, the teacher organizes content and guides student learning, and the learner discovers knowledge by engaging in problem solving. In the content-focused approach to learning, the teacher provides knowledge by explaining concepts and demonstrating procedures and skills; the role of the student is to practice skills by following the examples given by the teacher. The final approach, classroom-focused did not emphasize content.

Ernest (1989) described models of teaching. These models are the beliefs or mental models regarding the nature of teaching mathematics and are the basis in which teachers model their behavior (p. 22). These models are important because they have a strong
influence on teachers’ teaching mathematics. These models include (a) pure investigational; (b) problem posing and solving; (c) conceptual understanding enriched with problem-solving; (d) conceptual understanding; (e) mastery of skills and facts with conceptual understanding; (f) mastery of skills and facts; and (g) day-to-day survival (p. 22). These models of teaching are also greatly influenced by teachers’ views of student learning, whether they be those of knowledge construction or as the passive reception of knowledge.

The current reform recommendations are based upon a constructivist perspective. The teaching models described by Ernest (1989) and Kuhs and Ball (1986) include those which align to reform-based teaching and those which align with a more traditional approach. Those models in which the teacher is the facilitator of student learning focus on student thinking and emphasize student understanding. As such, these models would align to the reform-based teaching practices. However, research indicates that teachers’ practices continue to be predominantly constituted of practicing mathematical procedures—as opposed to developing conceptual understanding (Stigler & Hiebert, 2009). Furthermore, these practices use a transmission mode of instruction that focuses on repetition, rehearsal, and practice (Anderson & Bird, 1995; Fives & Buehl, 2008; Strauss, 1993). Additionally, Stipek, Givven, Salmon, and MacGyvers (2001) found that these practices were consistent with the teacher taking control of learning and relied on extrinsic rewards rather than the characteristics of a task to motivate students. Teachers with these approaches in these studies would be less likely to use the models that align to reform-based teaching practices.
**Mathematics teaching practices.** The Professional Standards for Teaching Mathematics (NCTM, 1991) defined the following as critical to worthwhile mathematical tasks, teacher and student discourse, the learning environment, and analysis of teaching and learning. In addition, the standards emphasized the importance of teaching mathematical processes including problem solving, reasoning and proof, communication, connections, and representations. These standards, in conjunction with the Strands of Mathematical proficiency, informed the current Standards for Mathematical Practice. Recent efforts to establish a “unified vision” to teach all students, encapsulated in a text document title *Principles to Action. Ensuring the Mathematical Success for All* (2014), synthesized and elaborated on the existing literature. Mathematics instructional coaching involves providing support of teaching practices. The National Council of Teachers of Mathematics recommended a “research-informed framework…for strengthening the teaching and learning of mathematics” (NCTM, 2014, p. 9). The NCTM recommended the following:

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle.
8. Elicit and use evidence of student thinking.
Implementation of these teaching practices proves to be a challenge due to prevalent cultural beliefs held by parents and teachers regarding the nature of mathematics and the teaching and learning of mathematics.

**Mathematics Socio-cultural Elements.**

The research literature provides findings regarding the socio-cultural elements of a classroom, which are important to reform-based mathematics teaching and learning. In this section, the following topics are included regarding this element: socio-mathematical norms, social culture of the classroom, and classroom culture of mathematical inquiry.

**Socio-mathematical norms.** Inquiry-based classrooms depend on students’ ability to verbalize their mathematical ideas to each other. An important component supporting this dialogue is establishing classroom norms that give students a way to participate in this type of community. Social norms in the classroom include explanation, justification and argumentation. These are general norms that cross disciplines. However, norms that are specific to participating in a mathematics classroom are called socio-mathematical norms. These include knowing what counts as mathematically different, mathematically sophisticated, mathematically efficient, mathematically elegant, and what counts as an acceptable explanation and an acceptable justification (Goos, 2004, p. 269). With proper support, students are able to participate in classroom discussions that are a central part of an inquiry-based classroom.

Teachers must be deliberate in how they introduce and foster norms in the classroom, and they must constantly reinforce them. Yackel and Cobb (1996) described the process of establishing norms as a reflexive one. They were influenced by what is deemed as acceptable mathematical activity yet “constrained by current goals, beliefs,
suppositions and assumptions of the participants in the community” (p. 460). Participation in such activity helps students and teachers develop understanding as they negotiate these norms. Reasoning and sense making processes cannot be separated from their participation in the negotiation of “taken-as-shared meaning” (p. 460). Making this agreement allows mathematical ideas to be established. Because of this intertwined relationship, it follows that participation in an inquiry-based classroom cannot exist without teachers taking intentional actions to support students in developing socio-mathematical norms.

**Social culture of the classroom.** A critical element of creating a classroom culture of inquiry involves establishing a specific type of classroom social culture. Descriptions of these classroom environments have been established in research literature (Goos, 2004; Hiebert et al., 1997; Horizon Research, 2000). The following roles are assumed by teachers of such classrooms and are described in Table 2.

Table 2. Roles of the Teacher in Creating a Social Culture that Supports a Classroom Culture of Inquiry.

| 1. Teacher presses for student sense making. | • Teacher provides opportunities to engage in problem solving.  
• Teacher allows students to use their own strategies.  
• Teacher focuses on methods for solving problems, not solely on correct answers.  
• Teacher emphasizes reasoning, explanation, and justification.  
• Teacher helps students make connections.  
• Teacher goes beyond procedures. |

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<td>2. Teacher encourages individual reflection and self-monitoring.</td>
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<td>- Teacher requires that students justify their answers.</td>
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<td>- Teacher encourages students to look for ways to improve their methods of solving problems.</td>
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<td>- Teacher asks questions that encourage students to question their assumptions and locate their errors.</td>
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<td>3. Teacher uses errors as a source of learning.</td>
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<td>- Teacher provides opportunities for students to examine errors in reasoning.</td>
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<td>- Teacher helps students view errors as a contribution to the learning process.</td>
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<td>4. Teacher encourages student-to-student interaction.</td>
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<td>- Teacher expects students to share their solution strategies.</td>
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<td>- Teacher encourages students to analyze the thinking of other students and examine their methods.</td>
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<td>- Teacher provides ample time for students to share solution strategies, clarify explanations, and compare different methods.</td>
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<td>5. Teacher appreciates and examines all ideas in the classroom.</td>
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<td>- Teacher sees ideas as the currency of the classroom.</td>
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<td>- Teacher examines an idea thoughtfully as a sign of respect to the idea and to the author.</td>
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<td>6. Teacher presses students to use the logic and structure of mathematics to determine reasonableness or correctness of a solution.</td>
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<td>- Teacher shares the validation of correctness of solutions with students.</td>
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<td>- Teacher does not assume the role of unquestioned authority.</td>
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<td>- Teacher expects students to propose and defend mathematical arguments.</td>
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<td>- Teacher facilitates class discussions.</td>
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<td>- Teacher sets the expectation that justifications are based on mathematical reasoning.</td>
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**Classroom culture of mathematical inquiry.** The purpose of this study was to examine the nature of coach-teaching interactions focused on developing teacher pedagogical content knowledge, effective teaching practices, and a classroom culture that
supports a community of mathematical inquiry. First, it was important to describe a community of mathematical practice. Traditionally, mathematics has been viewed as certainty and knowing how to get the correct answer quickly, and doing mathematics has meant that students follow the rules laid down by the teacher; lastly, knowing has meant remembering and applying the correct rule (Yackel & Cobb, 1996). However, current reform literature supports a different vision for mathematics learning and is based on a constructivist perspective. A goal of mathematics reform is to teach mathematics with understanding as opposed to memorizing rules.

According to Lampert (1990) teaching and learning “offer challenges which stimulate mathematical thinking and create opportunities for critical reflection on mathematical understanding.” Leikin and Rota (2006), defined inquiry-based mathematics as students learning to speak and act mathematically by participating in mathematical discussion and solving new or unfamiliar problems. In an inquiry-based classroom the process of doing mathematics and how students are thinking about mathematics is at the forefront of the learning process. Richards (1991) defined inquiry mathematics as students learning to speak and act mathematically by participating in mathematical discussion and solving new or unfamiliar problems. In an inquiry-based classroom, the process of doing mathematics and how students are thinking about mathematics is at the forefront of the learning process. Lampert (1990) stated that learning is about both the activity of acquiring knowledge as well as the knowledge that is acquired. Two agendas occur simultaneously during a mathematics class. One is that the students acquire the technical skills and knowledge of the discipline. The second is
that students learn the skills and dispositions needed to participate in mathematical discourse.

**Evolving conventions.** Many aspects of reform-based classrooms differ from those of conventional classrooms. The type of knowledge and the way students attain that knowledge is one aspect that is drastically different. Reform-based classrooms move away from a model of knowledge dissemination and toward a problem-based model. This revised model requires that students actively construct new knowledge from their existing knowledge. Students are introduced to a problem; they work individually or in small groups solving the problem using their own thinking and strategies, and then a whole-group discussion, facilitated by the teacher, takes place. During this open discussion, students share their solution strategies to the problem. In this model, classroom discourse is an essential part of the inquiry-based classroom (Lampert, 1990). During the exploration part of the class, students discuss their solution strategies and analyze the thinking of other students. Whole-group debriefing, while facilitated by the teacher, involves students explaining their solution strategies to the class. The whole-group discussion serves as a vehicle for students to learn about how other students are thinking. By examining peers’ ways of thinking, it is possible for students to adopt more advanced or efficient strategies. Additionally, through this examination, students are provided with opportunities to analyze the thinking of others.

**Teacher and student roles.** A classroom community of mathematical practice requires different roles for both the teachers and the students. According to Leikin and Rota (2006), the teacher has two responsibilities when guiding the development of such a classroom culture. The first is to focus on methods of mathematical problem solving. The
second responsibility is that the teacher must define to him or herself and to the students the positions of authority. Focusing on the methods has both intellectual and social implications. Focusing on the methods provides students the opportunities to reflect on mathematical relationships and builds understanding. Socially, it provides all students opportunities to participate in mathematical activity (Hiebert et al., 1997). Many of the roles assumed by the teacher in a conventional classroom are now shared by the students (p. 39). Traditionally, the teacher or the textbook has been the authority over correct and incorrect answers. Alternatively, within an inquiry-based classroom, the students determine whether a solution strategy has validity, and the source of authority is mathematics content (Yackel & Cobb, 1996). Students also analyze the thinking of others and determine whether a student’s reasoning is sound. They determine what counts as different, insightful, efficient and acceptable. Such an egalitarian paradigm marks a significant shift from the responsibilities of students in traditional mathematics classrooms, and the teacher plays a central role in guiding the development of student interaction in this type of learning environment.

Reform-based instruction requires active participation on the part of the teacher. Teachers need to assist students in their constructive efforts (Boaler, 2003; Hiebert et al., 1997; Lampert, 1990; Yackel & Cobb, 1996). Studies have identified ways in which teachers support their students and help them participate in a community of inquiry. Lampert (1990) discussed three different ways that she thinks it means to know mathematics. At times teachers need to explicitly tell students the kinds of activities that are and are not appropriate. Other times she modeled the roles she wanted to be able to take in relation to themselves and others. Yet other times she did the mathematics with
them. She encouraged students to share their solutions and to analyze the solutions of others. In both cases the expectation is that students justify their reasoning with mathematical arguments. Lampert (1990) and Goos (2004) both used scaffolding to support their students in assuming their new roles. At first students may need extensive, repetitious modeling, but then as the year progresses, the students take increased responsibility.

This literature provides specific instances of how both researchers supported students in an inquiry learning-based environment. They provided students with specific language to use when revising their answers or challenging the ideas of another student. They invited students to comment on or critique other students’ ideas. They asked students to explain their ideas to others, and they structured students’ thinking by guiding them through strategic steps or connecting their ideas to their existing knowledge (Lampert, 1990). These are some of the ways that teachers support students to become participants in a community of mathematical inquiry.

**Instructional Coaching**

Many coaching models have been developed over the past several decades, such as peer coaching (Joyce & Showers, 1980) and the use of aides known as helping teachers. Campbell (2011, p. 431) defined a coach as a “knowledgeable colleague with a deep understanding of mathematics and how children learn as well as pedagogical expertise” as well as an on-site resource and leader for teachers. The coach does not have regular classroom teaching responsibilities, which enables her/him to focus instead on supporting and collaborating with teachers on curriculum, instruction, and assessment. To further define the work of a coach, Campbell (2011) has revised the core conceptual framework
of professional development to specifically reflect mathematics instructional coaching.

The components include

- **Content focus**, whereby the coach facilitates activities in which teachers address mathematics content and pedagogy, as well as how students learn mathematics;

- **Active learning**, whereby the coach not only models instruction and co-teaches but also engages with teachers in the work of teaching via co-planning, assessment design, observation, debriefing reflections addressing pedagogy and learning, and data-driven decision making;

- **Coherence**, whereby a coach supports teachers’ efforts to understand, to examine ideas and relationships, and to connect prior knowledge and beliefs with new learning as well as teachers’ efforts to reconcile state, district, and school policy demands

- **Duration**, whereby a coach is consistently present to provoke and sustain attention toward addressing problems of practice; and

- **Collective participation**, whereby a coach facilitates inquiry, reflection, and experimentation within a community of practice focused on curriculum, instructional approaches, and interpretation of student meaning. (p. 432)

Research on instructional coaching is limited because of its relatively recent appearance in the literature. This research includes impacts on student achievement (Campbell, 2011; Erchick et al., 2007); coaching relationships and levels of engagement (McGatha, 2008); conditions for a successful coaching program (Obara, 2010); and coaching activities and approaches (Olson & Barrett, 2004). Even though this is a new body of research, there is
evidence to suggest that coaching models show promise as a model to support instructional practices; therefore, coaching models merit continued investigation (Campbell, 1996; Race, Ho, & Bauer, 2002). Both studies concluded that other components of professional development impacted instructional practices, but would likely not have continued without the support of an instructional coach.

**Content-focused coaching.** West and Staub (2003) developed a model for mathematics instructional coaching called Content-Focused Coaching. In this model, the coach and teacher collaboratively plan, enact, and reflect on specific lessons. Content-focused coaching involves a pre-conference, a lesson, and a post-conference. The pre-conference typically focuses on setting lesson goals and planning lessons. The lesson conversations are usually brief and focus on student learning and whole-class discussions. The post-conference addresses student learning and challenges. The coach can play a variety of roles during the lesson, such as observing, co-teaching, and modeling. A first step for the coach is to diagnose teacher needs. West and Staub (2003) wrote, “It is essential to consider what teachers know and can do and what they need to learn and be able to do” (p. 12). The teacher can diagnose teacher needs through the pre-conference; however, they also recommended observing a teacher before the actual coaching cycle begins in order to get to know the teacher and students. Some of these areas of need include

- Content knowledge and disposition toward mathematics;
- Pedagogical knowledge and underlying beliefs about learning;
- Pedagogical content knowledge;
- Diagnosing children’s thinking and assessing prior knowledge; and
• Habits of planning and engagement with curriculum materials.

West and Staub (2003) stated that all of these areas are interrelated and are addressed concurrently; however, one of these is usually more in need strengthening than the others (p. 12).

Even though the research on instructional coaching is in early stages, there is evidence to suggest that coaching models show promise as a mode of professional development, and these models merit continued investigation.

Theoretical Framework

**Overarching framework.** The overarching framework for this study was social constructivism. “Understanding learning as a process of individual and social construction gives teachers a conceptual framework with which to understand the learning of their students” (Simon, 1995, p. 117). This study involved interactions between a mathematics coach and a teacher in order to support the knowledge of the teacher. In this case, this theory of learning provided a conceptual framework for the coach to understand the learning of the teacher.

Another useful theory was that learning can be advanced when a knowledgeable person such as a teacher or peer collaborates with a learner at his or her level of understanding. The distance between the developmental level of the learner and the potential level of learner is called the *zone of proximal development* (Vygotsky, 1978). In this study, the coach/researcher was the knowledgeable other, while the teacher was the learner. Through interviews, conferences, and observations of teaching, the coach/researcher determined the teacher’s level of understanding of mathematics content,
teaching, and learning to determine a place to begin to provide support in helping the teacher reach her potential in creating a community of mathematical inquiry.

**Framework for analysis.** Coaching involves a cycle of social interactions between the coach and the teacher. Social interactions were the source for the data collected during the study, and interactions were analyzed through the lenses of pedagogical content knowledge (Shulman, 1986; Ball et al., 2008), mathematics teaching practices (NCTM, 2014), and social culture of the classroom (Goos, 2004; Hiebert et al., 1997; Horizon Research, 2000; Wood et al., 1991).

Before a coaching cycle begins, a coach holds a pre-conference with the teacher and can observe a lesson taught by the teacher to determine the teacher’s needs (West & Staub, 2003). For this study, the coach/researcher conducted a pre-interview, a pre-observation, and an interview consisting of pre-determined questions, which are described in Chapter 3. From the initial interview and pre-conference, the coach/researcher gathered information about the teacher’s knowledge regarding mathematics content and mathematics teaching and learning. The author was also able to assess the teacher’s instructional practices as well as her concerns regarding mathematics teaching and learning. Doing so allowed the coach/researcher to develop an account of her subject’s teaching practice. The coach/researcher then conducted a pre-observation to help further identify the most critical need of the teacher. Figure 1 shows a model of this cycle for social interactions and the lenses in which they were analyzed (Goos, 2004; Hiebert et al., 1997; Horizon Research, 2000; NCTM, 2014; Wood et al., 1991).
Figure 1. Cycle of coaching interactions.

Figure 1 depicts the cycle of coaching interactions. The inner circle describes the cycle of social interactions that took place. Each of the four circles in the cycle represents three sources used to collect data: coach-teacher conferences, classroom coaching, and coach reflections. From the data, the three themes emerged (shown in the boxes outside the circle) as the predominant topics of conversation between the coach and teacher. The data were analyzed after being collected via a constant comparative method of quantitative analysis (Glaser, 1965). This method consists of four phases, and these four phases are (a) coding each instance in the data; (b) integrating categories and their properties; (c) delimiting the theory; and (d) writing the theory (p. 439).
The coach/researcher began with the raw data and assigned each one a specific code such as content, pedagogical assistance, student thinking, curriculum, grounding in theory, relationship building, socio-mathematical norms, etc. The coach/author went through the data several times. Through this process, she continually revised the themes (i.e., codes) by adding additional themes and renaming themes. The coach/researcher also compared pieces of data with the same code to ensure that the coding was consistent. Next, she integrated codes into the three major themes of pedagogical content knowledge, teaching practices, and classroom culture of mathematical inquiry. Another code that appeared in the data was relationship building. This code did not contribute to the theory, and thus was not included in this study. Other codes were also not included because they did not fit into the larger themes and did not constitute a large part of the data. After a thorough investigation of the mentioned data, the coach/researcher generated a theory pertaining to teacher change in practice.
Chapter 3. Methodology

Introduction

This study set out to determine how coach-teacher interactions develop teacher pedagogical content knowledge, affect teaching practices, and aid the creation of classroom cultures of inquiry. Because the study’s emphasis regarded what took place in a complex social setting, a naturalistic inquiry approach was deemed most appropriate. In this study, the researcher was also the mathematics instructional coach in the building in which the study took place. Instructional coaching involved a cycle of pre- and post-conferences along with classroom coaching. Acting as a coach and researcher, the author documented her experiences through coach/researcher notes and audio and video recordings, which served as data sources. A qualitative approach was used in order to describe the interactions and the teaching practices of the teacher. Data collected included teacher/coach reflective notes, teacher interviews, pre- and post-conferences between the coach and the teacher, and classroom observations. These will be discussed in depth in the data collection section. Data were analyzed using the constant comparative method of qualitative analysis.

Setting

The school had approximately 480 students and 20 classroom teachers in kindergarten through Grade 5. Approximately 85% of the students in the school received free or reduced lunch rates. The teachers in the building taught in self-contained classrooms. The district was in a suburb situated adjacent to a large urban city and had a
low socio-economic demographic. In this district, there were 3 elementary schools, 1 middle school, and 1 high school with approximately 3,000 students, and 200 teachers. The district used a research-based mathematics program that the teachers were expected to follow.

**Participants**

**Researcher.** The researcher of this study was a full-time mathematics instructional coach in the elementary school being studied. The author had held this position for two years at the time of the study. Prior to serving as a mathematics coach, the researcher had varying roles in the Mathematics Coaching Program for three years, including designing and delivering professional development materials for the coaches in MCP and serving as a facilitator of a small group of coaches in the program. In addition, the researcher had experience in designing and facilitating mathematics professional development of a more traditional nature for local school districts and the State. Lastly the author had taught high school mathematics for 12 years prior to conducting the study. These experiences led to a deeper understanding of how children learn mathematics based on the mathematics education literature, and they led to questions about how to best support teachers in implementing these practices in their classrooms.

The researcher’s theoretical viewpoint prior to the study was that mathematical ideas are individually and socially constructed. Furthermore, existing knowledge plays a significant role in constructing new knowledge, and the role of the mathematics coach is to identify the teacher’s current knowledge and build upon that. As for the nature of what it means to know and do mathematics, the researcher’s informed conclusion was that mathematical proficiency means that students must make sense of the mathematics they
are learning as opposed to memorizing procedures laid down by the teacher or another source of authority. In a reform-based classroom, students are thinking, solving non-routine problems, justifying their ideas, and critiquing the ideas of others. While these students learn, teachers facilitate by providing tasks that promote reasoning and problem solving and by teaching students how to participate in this type of community.

**Teacher.** The participant in this study was a teacher in the school in which the researcher was a mathematics instructional coach. While the school building had been an MCP school for two years prior to the study, the teacher being studied had not participated in mathematics instructional coaching. The teacher was a first-grade teacher and held a license in early childhood education. She was in her ninth year of teaching at the time of the study.

**Selection process.** Teachers at different grade levels were invited including a combination of teachers the author had coached in the past and those she had not. Both primary and intermediate teachers were invited to participate. Once the author selected the grade levels in which to coach and observe, she approached teachers whom she thought would be willing to partake in the study. Because coach-teacher relationships are critical in the work of the coach, the author first asked teachers with whom she already enjoyed positive professional relationships. The coach/researcher met with those teachers to explain the study and then gave them a consent form. From the pool of solicited teachers, three were interested in taking part in the study. Two of them had already worked extensively with the author, while the other had never been coached by the author. The following section describes the final selection criteria the author used to choose the sole participant in the study.
Selection criteria. The selection criteria used for this study included several factors. The first factor involved previously established relationships. Effective coaching requires a positive working relationship between a coach and a teacher. Therefore, the coach/researcher considered teachers with whom she had previously developed such a relationship. Because the objective of the study was not to examine relationship building, having this in place in the beginning allowed the coach/researcher to focus on other aspects of coaching. A second factor was teacher availability. Coaching requires time for the coach and the teacher to meet. Teachers who were not able to meet before school, during their conference time or after school were therefore not considered for the study. Another factor was number of years experience that the teacher possessed.

The coach/researcher did not include first-year teachers because of the additional time demands already placed upon them including additional meetings and intensive time spent planning. Finally, the author considered which teacher might need the type of support that the study aimed to examine. Given that the coach/researcher had coached in the building for two years prior to the study, she knew the teachers well. She had coached some of them extensively. Some were making great strides, and this study might not have been as rich in findings. Others were resistant to change and also might not have provided much insight into the questions in this study. In all, the author needed a participant who would be open to learning, could devote the time, and who would benefit from the type of support under examination.

Students. The students of the teacher who participated in the study were invited to be participants. Coaching teachers included supporting them in understanding levels of student thinking and in teaching mathematics content with understanding; therefore
conversations about student understanding were a central part of this process. This included talking with students about their thinking and how they solved problems in groups and individually. Conversations also invited students to attend to their thinking in whole-group discussion. A consent form was sent home with the students for the parents to sign and return by the students or by mail. This method was thought to have a higher return rate as opposed to mailing the consent forms. In this case we received 12 consent forms out of a possible 28, which was 43%. The coach/researcher was then able to identify which students to include in the study that involved the coach/researcher collecting copies of student work and their conversations. She was also able to record and study both teacher-student interactions as well as student-student interactions.

In addition, the coach/researcher explained consent forms to the students during a regular class period with the teacher present. She handed out the forms to the students for them to decide whether they wanted to participate. In this case, 22 students signed the form, which was 79% of the students in the class. These forms were compared with those for which the parents also signed.

**Research Design and Rationale**

The research questions in this study lent themselves to qualitative methods. “Qualitative methods attempt to come to an understanding of a situation from the perspective of the participants” (Torrance, 2010, p. xxiii). Under the qualitative mode, the researcher uses the natural setting, spends considerable time in the field, and is interested in the context in which the study took place (Slavin, 1992). Qualitative research involves very detailed observations of a small number of cases that span an extensive period of time. For this particular study, the coach/researcher observed and analyzed cases that
took place in the teacher’s classroom. It was an in-depth analysis involving one teacher that took into consideration events taking place over a five-month period.

**Case studies.** Case studies are an approach to research that attempt to provide in-depth descriptions of social activity. They are “particular, descriptive, inductive and ultimately heuristic” (Stark & Torrence, 2005, p. 33). This study used a single case study to establish understanding of how coach-teacher interactions centered on creating a classroom that supports a community of mathematical inquiry. A particular type of case study, Accounts of Practice, studies teachers’ classroom practice. Simon and Tzur (1999) stated, “It is an attempt to understand teachers' practice in a way that accounts for aspects of practice that are of theoretical importance to the communities of mathematics education researchers and teacher educators” (p. 256). This method is different from others in that it does not use a deficit model, and it does not use teachers’ own accounts of their practice. For this study, the coach/researcher started with the current understandings of the teacher and built upon them throughout the study in order to advance the understanding and skills of the teacher.

The objective was to describe teacher perceptions from a researcher’s perspective instead of only reporting the points of view of participants. Researchers may offer insights to practice that might not be the focus of the teacher during practice or during reflection on practice. Additionally, the coach/researcher created the accounts through a lens grounded in the theoretical perspectives from current research in which the teacher may not share. An Account of Practice, which was the approach taken by the coach/researcher for this study, begins with the researcher lens. The researcher lens impacts the aspects of the teacher’s practice noticed by the researcher. From the
observation, the researcher creates a description of the teacher’s practice using his/her theoretical lens. This account of practice also influences what is noticed next and can impact the researcher’s conceptual framework. Simon and Tzur (1999, p. 256) described this model as “a coemergent and reflexive relationship between the researchers' conceptual framework and their interactions with the data.” Their understanding of An Account of Practice is represented in Figure 2.

![Diagram](image)

**Figure 1. The genesis of an account of practice.**

**Figure 2. The genesis of an account of practice.** (p.256).

**Trustworthiness**

According to (Norris & Walker, 2005), researchers must take steps to create trustworthiness in naturalistic studies. There are four criteria that must be considered when conducting naturalistic inquiry, and these are credibility, transferability,
dependability, and confirmability. In this study, credibility was addressed through the practices of prolonged engagement and persistent observation (Lincoln & Guba, 1985). Prolonged engagement involves being in the setting for a sufficient amount of time to understand the culture, setting or phenomenon of interest. It also requires that the researcher and participants in the study have established trust and rapport (Lincoln & Guba, 1985). Lincoln and Guba wrote that

If the purpose of prolonged engagement is to render the inquirer open to the multiple influences - the mutual shapers and contextual factors - that impinge upon the phenomenon being studied, the purpose of persistent observation is to identify those characteristics and elements in the situation that are most relevant to the problem or issue being pursued and focusing on them in detail. If prolonged engagement provides scope, persistent observation provides depth. (p. 304)

The researcher had been a coach in this building for two years before the start of this study, and she therefore had time to develop an understanding of the culture of the building and to develop rapport and trust with the participants in this study. In addition, the researcher had the flexibility of lengthening the time for observation to avoid leaving the study prematurely since it was the building in which the researcher worked full-time.

Finally, triangulation was achieved by using various data sources in this study, including transcripts of interviews, detailed accounts written from the observations, and coach/researcher notes.
Research Overview and Timeline

A research timeline was developed and is included in Appendix A. The timeline began with startup tasks such as IRB approval, teacher recruitment, and district approval to conduct the study. There were two sets of data collection that are listed in Appendix A. The first set will occur over a six-week period while the coach/researcher is actively coaching the teacher. The first set will also include a pre-interview and an initial classroom observation; ongoing coach/teacher conferences; and classroom observations and a post-interview and observation. The second data collection referred to in the timeline occurred three months after the coach had stopped working with the teacher to identify residual effects. This second data collection also consisted of a classroom observation and an interview. Finally, the timeline ended with further data analysis and draft write-up.

Data Collection

The preferred methods for naturalistic inquiry are observation and interviews (Norris & Walker, 2005). Accordingly, data collection in this study consisted of coach/researcher notes, teacher interviews, coach-teacher conferences, classroom observations, and classroom coaching. Table 3 provides a summary of the types of data collected and the frequency of each type of data.
Table 3. Types and Frequency of Data Collected.

<table>
<thead>
<tr>
<th>Data type</th>
<th>Frequency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher interviews</td>
<td>1 initial interview&lt;br&gt;1 at end of coaching cycle&lt;br&gt;1 final interview 3 months after coaching cycle</td>
<td>3</td>
</tr>
<tr>
<td>Coach/teacher conferences</td>
<td>1 to 2 per week for 6 weeks</td>
<td>10</td>
</tr>
<tr>
<td>Classroom coaching</td>
<td>2 - 3 per week for 6 weeks – 16 total</td>
<td>16</td>
</tr>
<tr>
<td>Classroom observations</td>
<td>1 initial&lt;br&gt;1 at end of coaching cycle&lt;br&gt;1 final observation 3 months after coaching cycle</td>
<td>3</td>
</tr>
<tr>
<td>Coach reflective notes</td>
<td>Throughout the duration of the study</td>
<td></td>
</tr>
</tbody>
</table>

The diagram in Figure 3 shows the data collection process.

Figure 3. Data Collection Process.
The coach/researcher audiotaped each teacher interview and each coach-teacher conference. Each mathematics lesson in which the researcher coached the teacher was videotaped. The video recorder was set up in the classroom in a stationary location that captured whole-group discussions. When individual students and small groups were engaged in solving problems, the coach/researcher focused the camera on their work to capture their thinking.

**Teacher interviews.** Teacher interviews were used to gain insights from the teacher about her own practice. These were conducted three times during the school year. The first was conducted at the beginning of the work with the teacher, the second at the end of the work with the teacher, and the third three months after the coach finished working with the teacher. Each interview lasted approximately 30 minutes. A distinguishing characteristic of the Accounts of Practice methodology is that it starts with the current understandings and practices of the teacher. An initial interview was used as a source to determine which elements of reform-based practice were being implemented, and I made decisions from that information to determine how to build on those practices. The following questions were used for the first two interviews:

- What are elements of an inquiry-based classroom?
- Which elements of an inquiry-based classroom do you find easy to implement? Why?
- Which elements of an inquiry-based classroom do you find difficult to implement? Why?
- Which elements do you feel you have fully implemented?
- Which elements do you feel you have partially implemented?
• Which elements do you feel you have not implemented?

The interview at the end of the six-week period was used to determine the practices that the teacher continued to use. The final interview was conducted approximately three months after the coach finished working with the teacher to determine residual effects of the coaching experience. The final interview used a different set of questions. At this point, the coach/researcher was more interested in learning about the teacher’s perceptions of the impacts of coaching on her practice. Therefore, the coach/researcher used the following questions:

• What did you learn as a result of coaching?
• What do you feel more confident or comfortable doing?
• How did our interactions support your learning?

The nature of the interviews was mostly scripted; however, some clarifying questions were asked to gain more insights into the teacher’s practices.

**Coach-teacher conferences.** According to West and Staub (2003), coach-teacher conferences consist of both pre- and post-conferences. According to the MCP model, a pre- and a post-conference should take place daily; however, this was not possible due to coach and teacher time demands. The coach-teacher conferences were part of the coaching cycle and occurred one to two times per week. Each conference lasted approximately 20 to 30 minutes. The cycle of pre-conference–observe a lesson–post-conference was not realistic; therefore, the coach/researcher combined elements of both types of conference in one meeting. In general, conferences began with a discussion about a previous lesson regarding students’ struggles and challenges. Then both teacher and coach would discuss an upcoming lesson, possible student challenges, and
instructional strategies the teacher intended to use. Additionally, the coach/researcher addressed areas of need based on the accounts of practice that the coach/researcher developed from previous lessons. Discussion topics were both planned and in-the-moment. Planned topics came from the accounts of practice, and in-the-moment topics were in response to teacher questions, ideas, comments, and concerns. These conferences were audiotaped and transcribed, and themes were identified. This is discussed in greater depth in the data analysis section.

**Classroom coaching sessions.** The MCP model recommends that classroom coaching take place daily. However, these sessions occurred only two to three times per week for six weeks due to factors including meetings the coach/researcher attended, coach and teacher absences, holidays, school cancellations due to weather, and testing days when classroom instruction did not take place. Each lesson lasted for approximately one hour. These lessons were videotaped, and notes were taken to document important parts of the lesson. Factors deemed important included teaching practices and content interactions that took place in one of five scenarios.

These scenarios included whole-group discussion, teacher and an individual student, between students, coach and teacher, and coach and students. In addition, the coach/researcher documented coaching moves she made. The roles a coach can assume in a lesson include observing, modeling, and co-teaching (MCP, 2003). The coach/researcher’s role as requested by the teacher was to be an observer in whole-group discussions. The teacher also asked the coach/researcher to interact with students individually or in groups as they were solving problems so that the coach/researcher could model teacher-student interactions for the teacher. The coach/researcher asked
questions during a whole-group discussion as a way of modeling. Because of the nature of the coach/teacher’s role in the classroom, coaching moves were spontaneous and not planned in advance.

**Classroom observations.** The classroom observations took place three times. The first observation occurred before the coaching began, the second at the end of the coaching cycle, and the third three months after the coaching cycle ended. These observations were very similar to the classroom coaching sessions; however, the coach/teacher’s role was almost exclusively as an observer with little modeling. These observations were videotaped and reviewed by the researcher. Videos were coded according to the elements of social culture listed in Table 2, and notes were taken in the same manner as in the classroom coaching sessions where important aspects of the lesson were documented.

**Reflective notes.** Coach reflective notes were taken after the interviews, the conferences, the classroom coaching sessions, and the observations. These notes were brief and included theoretical themes and planned topics of discussion for future conferences. Notes also documented the coach/researcher’s coaching moves and her rationale for making them.

**Data Analysis**

This section explains how the data were analyzed through certain lenses. It also details the different phases of analysis, and it explains how each major theme was analyzed. The constant comparative method of qualitative analysis was used throughout the study (Glaser, 1965). The intent of the study was to examine coach-teacher interactions to support teachers’ pedagogical content knowledge, teaching practices, and
creation of a classroom culture of inquiry. The lens that the coach/teacher used when coaching the teacher was based on her knowledge and understanding of reform-based mathematics teaching and learning—an understanding that was developed through the study of the literature. The literature supported the coach/teacher’s knowledge of reform-based mathematics teaching and learning (NCTM, 1989, 1991, 2000, 2014). The literature also supported her analysis of pedagogy for student understanding (Hiebert et al., 1997; National Governors Association Center for Best Practices, 2010; NCTM, 2000, 2014; Stein, Engle, Smith, & Hughes, 2008.) Additionally, the research informed her interpretation of cognitive-based instruction (Baroody, Bajwa, & Eiland, 2009; Battista, 2012a, 2012b Battista, 2012c; Carpenter, Fennema, Franke, & Levi, 1999; Fennema et al., 1996). It guided her assessment of classroom cultures of mathematical inquiry as well (Goos, 2004; Hiebert et al., 1997; Lampert, 1990; NCTM, 1991).

Also, the literature informed her examination of mathematical content knowledge (Battista, 2012a, 2012b, 2012c; National Governors Association Center for Best Practices, 2010; NCTM, 2000). Lastly, the coach/researcher called upon the available research to shape an understanding of pedagogical content knowledge (Ball, Hill, & Bass, 2005; Ball et al., 2008; Hill, Ball, & Schilling, 2008). In addition to lessons reaped from surveying the literature, various learning experiences contributed to the coach/researcher’s knowledge base. Two of the most profound experiences were the coursework for her doctoral program and her participation in MCP as a curriculum developer, a coach of coaches, and a coach of teachers. This experiential lens influenced how the coach/teacher observed, and it influenced interactions while working in the classroom. The experiences also partially determined the topics of discussion in the
conferences with the teacher. Figure 4 describes the lens through which the researcher conducted her study.

![Researcher's Lens Components](image)

**Figure 4.** Components of the Researcher’s Lens Used During Classroom Coaching Sessions, Coach-Teacher Conferences and in Analyzing the Data.

To analyze the data for emergent themes, the coach/researcher used a constant comparative method, which consisted of four stages (Glaser, 1965; Corbin & Strauss, 2008).

**Stage 1.** The interviews and conferences were transcribed verbatim in an Excel spreadsheet. Each interview and conference was transcribed on a different sheet. The first column of the sheet included the dialogue that took place. The coach/researcher examined each line of data and assigned a code based on the theoretical lens described above. These codes were placed in subsequent columns. As a new code emerged it was added to the spreadsheet. The coach/researcher used as many codes as possible. As she was assigning codes, she compared them with other incidents with the same code to
ensure consistency of coding. When she identified a new code, she examined previously
coded data to determine if there were other instances. She went through the data
repeatedly until all codes through this theoretical lens were identified and consistently
coded. As she coded the data, the coach/researcher identified the themes of (a)
mathematical content knowledge; (b) relationship building; (c) trust building; (d)
pedagogical assistance; (e) pedagogical content knowledge; (f) socio-mathematical
norms; (g) pedagogy for student understanding; (h) student understanding; (i) student
misconceptions; (j) grounding in theory; (k) student thinking; (l) lesson planning; and (m)
curriculum. Each code was tallied for each interview and conference.

Stage 2. After further analysis, these themes were combined into the three broad
themes of (a) relationship building; (b) pedagogical content knowledge; and (c) teaching
practices. Relationship building, although critical to the work of a coach, did not relate to
the research questions and the purpose of this study; therefore, it was not used to analyze
the data. After these themes were categorized, the literature was used to provide more
specific descriptions for PCK (Ball et al., 2008), teaching practices (NCTM, 2014), and
classroom culture of mathematical inquiry that were used to analyze the data with more
specificity (Goos, 2004; Hiebert et al., 1997; Horizon Research, 2000; NCTM, 2014;
Wood et al., 1991).

Stage 3. The third phase consisted of analyzing the videotaped observations and
conferences. The observations and conferences were initially viewed and edited using
iMovie. Instances that were theoretically significant to the study were identified based on
the researcher lens, and the other parts of the video were omitted. Once this process was
completed, the observations were viewed, and instances of the specific codes for classroom culture of mathematical inquiry were tallied.

**Stage 4.** The next phase consisted of writing a narrative timeline, which included all of the data sources: (a) interviews, (b) conferences, (c) classroom observations, (d) classroom coaching sessions, and (e) researcher reflective notes. The coach/researcher used this narrative for two purposes. The first was to identify the components of PCK and teaching practices represented in the data. The second was to see how those specific instances of the themes developed or did not develop over time. For example, the coach/researcher analyzed the Number Talks lessons and determined how the teaching practices, classroom culture of inquiry, and the teacher’s pedagogical content knowledge evolved or did not evolve throughout the coaching cycle. Figure 5 illustrates the data analysis process used for this study.

![Figure 5. Data analysis process.](image-url)
The following describes the data analysis organized by each theme: (a) teacher pedagogical content knowledge, (b) teaching practices, and (c) classroom culture of mathematics inquiry.

**Teacher pedagogical content knowledge.** The coach/researcher used the following sources of data to analyze teacher pedagogical content knowledge: interviews, conferences, classroom coaching sessions, and observations. The transcribed interviews and conferences were analyzed first. They were initially coded line by line, and theoretical themes were identified. Each specific code was tallied to determine the total number of occurrences for each one. Next, the coach/researcher identified the specific codes that were aspects of pedagogical content knowledge, and she combined them into a broader theme and found the total number of occurrences for PCK. The interviews and
conferences were divided into four segments chronologically to show the total number of occurrences of PCK for each one. Segment 1 consisted of the initial interviews and pre-conference along with the first two weeks of coaching. Segment 2 took place in the middle of the coaching cycle, which was comprised of weeks three and four. Segment 3 consisted of the last two weeks of the coaching cycle. Finally, segment 4 consisted of the final interview and observation.

Another way that pedagogical content knowledge was analyzed was by looking for specific instances of PCK in the interviews, conferences, classroom coaching sessions, and classroom observations. The coach/researcher used the domains and categories defined by Ball et al. (2008) to find specific classroom episodes, interviews, and conferences that exemplified each one. Figure 6 shows the domains and categories.

<table>
<thead>
<tr>
<th>1. Knowledge of content and students (KCS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) knowledge of student errors and misconceptions</td>
</tr>
<tr>
<td>b) knowledge of mathematically accurate and appropriate content for students</td>
</tr>
<tr>
<td>c) knowledge of students’ experiences about a topic and how those past experiences may be helpful or confusing</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. Knowledge of content and teaching (KCT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) knowledge of how to sequence topics in a unit, examples in a lesson, and student solutions in a whole-class discussion</td>
</tr>
<tr>
<td>b) knowledge of which student responses would be productive to share in a whole-class discussion and which should be saved for later</td>
</tr>
<tr>
<td>c) knowledge of the advantages and disadvantages of specific mathematical representations and models</td>
</tr>
</tbody>
</table>

Figure 6. Components of Pedagogical Content Knowledge.
3. Knowledge of content and curriculum (KCC)
   a) at what grade level particular content is taught
   b) how concepts are connected in the curriculum
   c) the models and representations with which students are familiar

**Effective teaching practices.** The analysis of data related to effective teaching practices used the same approach as the analysis of pedagogical content knowledge. The coach/researcher found and described specific instances in the interviews, conferences, classroom coaching sessions, and classroom observations. The coach/researcher then analyzed the raw data to identify specific instances of the teaching practices defined by NCTM (2014). Most of these practices were found in the data, while a few were not. The practices analyzed are shown in Figure 7.

1. Establish mathematics goals to focus learning,
2. Implement tasks that promote reasoning and problem solving,
3. Use and connect mathematical representations,
4. Facilitate meaningful mathematical discourse,
5. Pose purposeful questions,
Figure 7 continued

| 7. Support productive struggle, and |
| 8. Elicit and use evidence of student thinking. |

For those practices found in the data, a description of the episode that represented that practice was reported.

**Classroom culture of mathematical inquiry.** The analysis for the social culture of the classroom was conducted in two different ways. First, the interviews and observations were coded according to the following categories taken from the literature (Goos, 2004; Hiebert et al., 1997; Horizon Research, 2000; NCTM, 1991). Figure 8 shows the categories of classroom culture of mathematical inquiry

| 1. Teacher presses for sense making, |
| 2. Teacher encourages individual reflection and self-monitoring, |
| 3. Teacher uses errors as a source of learning, |
| 4. Teacher encourages student-to-student interaction, |
| 5. Teacher appreciates and examines all ideas in the classroom, and |
| 6. Teacher presses to use the logic and structure of mathematics to determine reasonableness or correctness of a solution. |

Figure 8. Components of Classroom Culture of Inquiry
The following section presents specific examples of how the data were coded and analyzed. The first example illustrates how a specific episode was coded through the four phases of data analysis. During the conference on January 29th, a discussion took place about the appropriateness of skip-counting by increments of 25 for her students in first grade.

**Teacher.** We have kind of talked about counting by 25s on the hundreds chart to model counting by 25s to help with that 25, 50, 75, 100, or one dollar and we actually put quarters in [the hundreds chart] similar to what I did today with the nickels.

**Coach/Researcher.** Do you think that…is that too much for them?

**Teacher.** So that is what I want them to walk away first grade with. I don't want them to count $1.25 or anything like that. But if maybe if they see two quarters or a dime, then

**Coach/Researcher.** …it is about students being able to really make sense of the whole thing today versus when we were in school or how we learned. So my concern is that they are going to memorize 25, 50, 75 $1. But not understand how we are getting it because they are not adept at using those size numbers.

This episode was initially coded as pedagogy for student understanding in phase 1 of data analysis. The coach/researcher’s concern was about the students being able to make sense of skip-counting that size number because they were not adding two-digit numbers yet, and had not done much work with place value. In phase 2 this episode was coded as pedagogical content knowledge. In phase 4, this episode was used as an example of
knowledge of content and students, specifically as knowledge of mathematically accurate and appropriate content for students.

The next example illustrates how the data were coded for a specific classroom culture of inquiry using the literature. This interaction also took place in the January 29th conference wherein the coach/researcher commented on elements of classroom culture of mathematical inquiry that she observed in a previous classroom coaching session.

**Coach.** I heard you say, "How did you know?" And, yes, those are the types of questions we should be asking our students to get at their thinking.

**Teacher.** Right and to challenge them.

**Coach.** To challenge them and to have other kids try to find out how other kids are thinking.

This particular interaction was coded as follows: (a) teacher presses for student sense making; (b) teacher encourages student reflection and self-monitoring; and (3) teacher encourages student-to-student interaction using the descriptions from the literature (Hiebert et al., 1997). Pressing for understanding involves teachers emphasizing reasoning, explanation, and justification. Encouraging self-reflection involves asking questions such as “How do you know?” and “Why does that work?” In this episode, the coach/researcher saw evidence of her doing this. This part of the data was coded accordingly. In the interaction, the coach/researcher also discussed that students needed opportunities to analyze the thinking of others, which the literature describes as a component of student-to-student interaction. This particular episode highlights how the data were assigned specific codes using the literature.
The final example shows how the coach/researcher analyzed teacher change in practice over time by examining classroom coaching sessions and coach-teacher conferences. By coding the data, writing the narrative timeline, and examining the data for specific instances of PCK and teaching practices, the coach/teacher became very familiar with the data. Trends and changes in teacher practice began to emerge for each of the categories: (a) PCK, (b) teaching practices, and (c) classroom culture of mathematical inquiry. The coach/researcher was so familiar with the data that she could identify from memory changes that occurred and changes that did not occur. As she reflected on these themes, she referred back to the transcriptions and the narrative to more accurately account for the changes over time.

For example, the coach/researcher traced the conferences and the classroom coaching sessions regarding the content topic regarding finding the value of a collection of coins. She knew from memory that the quality of tasks with respect to this content improved over time, and specific aspects of classroom culture of mathematical inquiry were being used more frequently. She also remembered that the teacher and the coach/researcher had conversations about appropriateness of content and that the coach/researcher was not able to convince the teacher to change her position on that practice. However, the coach/teacher referred to the transcribed conferences and the narrative to more specifically describe the changes or the aspects that did not change.

Limitations

One potential limitation of the study was the role of the researcher in the study. Specifically, the coach/teacher was a coach in the building in which the research was being conducted. Because the researcher worked closely with the participants in the study
for two years, there was concern of “going native” (Lincoln & Guba, 1985, p. 304). Lincoln and Guba (1985) cautioned that being too immersed in a culture can influence personal judgment. No method would guarantee that this would not happen consciously or unconsciously, but the researchers suggested that being aware is the best defense against over-immersion. Throughout the data analysis process, it was important to stay objective and focused on the theoretical and literature bases of the study.

In addition, there are limits to qualitative research methods. Specifically, while such methods have the potential to provide an in-depth understanding, the results are not generalizable due to the small number of cases. In addition, sample selection could have been problematic. The participants were volunteers and not randomly chosen, nor were they necessarily representative of the larger population.

Other elements that could have contributed to the results include but are not limited to the teacher’s subject matter knowledge and her beliefs. Both of these constructs seemed to impact the influences that the coach/researcher could make on the teacher’s practices; however, due to the fact it is not reasonable to study all of these factors, the coach/researcher did not include them in the study.
Chapter 4: Results

Overview

The purpose of this study was to examine the influences of coach-teacher interactions on teacher pedagogical content knowledge, teaching practices, and on the creation of a classroom culture of mathematical inquiry.

During this study, the coach/researcher conducted three interviews. The first interview occurred before working with the teacher, the second immediately after working with the teacher, and the third three months after working with the teacher. The coach-researcher also conducted three classroom observations. The first observation took place one week before she started coaching the teacher. The second took place at the end of the classroom coaching cycle, and the final observation took place two months after coaching the teacher. The coach/researcher also videotaped 16 classroom episodes during the 6-week period she coached the teacher. Additionally, she took coaching notes regarding the classroom observations and interactions with the teacher.

In Chapter 4, the coach/researcher addresses the interview and conference data regarding all of the conceptual categories identified through the analysis of the data. Next, she presents the data for the characteristics of a classroom culture of inquiry using the data from the three interviews and the three classroom observations. Finally, she
discusses the other two categories—pedagogical content knowledge and teaching practices that emerged through the analysis of the interviews, conferences, observations, and the classroom coaching—by providing examples of how these transpired throughout the work with the teacher. Some episodes showed evidence of more than one theme and are necessarily discussed as examples in multiple places throughout this Chapter.

Findings for Conceptual Categories

The study showed that the nature of the conversations with the teacher largely centered around three main conceptual categories: (a) pedagogical content knowledge (PCK), (b) effective teaching practices, and (c) classroom culture. One other theme that emerged in the data was relationship building. This was an element necessary to continue coaching the teacher. Other interactions were coded that each constituted less than 3% of the total interactions. For these interactions, the coach/researcher included such codes as lesson planning, coach grounding in theory, teacher challenges, and resources. Table 4 shows the results of the number of interactions for each of these categories for the interviews and conferences. The work with the teacher was divided into four main phases. Phase 1 of coaching took place in the first two weeks of working with the teacher and consisted of 4 conferences. Phase 2 took place during weeks 3 and 4 of the coaching cycle and included 4 conferences. Phase 3 took place during the final two weeks of the coaching cycle and included 2 conferences. Phase 4 included post-coaching conferences and consisted of 2 conferences. Table 4 also shows the frequencies for each conceptual category found in the data.
Table 4. Frequency of Conceptual Categories During Coach-Teacher Conferences.

<table>
<thead>
<tr>
<th>Conceptual categories</th>
<th>Phase 1 Jan. 14–Feb. 4</th>
<th>Phase 2 Feb. 9–Feb. 19</th>
<th>Phase 3 Feb. 27–Mar. 19</th>
<th>Phase 4 May–June</th>
<th>Total</th>
<th>Percent of Total Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCK</td>
<td>82</td>
<td>58</td>
<td>25</td>
<td>25</td>
<td>190</td>
<td>31%</td>
</tr>
<tr>
<td>Teaching Practices</td>
<td>42</td>
<td>80</td>
<td>11</td>
<td>15</td>
<td>148</td>
<td>24%</td>
</tr>
<tr>
<td>Classroom Culture of Mathematical Inquiry</td>
<td>45</td>
<td>36</td>
<td>8</td>
<td>9</td>
<td>98</td>
<td>16%</td>
</tr>
<tr>
<td>Relationship Building</td>
<td>34</td>
<td>25</td>
<td>6</td>
<td>12</td>
<td>77</td>
<td>13%</td>
</tr>
<tr>
<td>Other</td>
<td>28</td>
<td>43</td>
<td>0</td>
<td>24</td>
<td>95</td>
<td>16%</td>
</tr>
</tbody>
</table>

The findings shown in this table indicate that the pedagogical content knowledge represented the largest percentage of topics of interaction. Pedagogical content knowledge constituted more than one-third of the conversations that occurred in coach-teacher conferences. Effective mathematics teaching practices constituted approximately one-fourth of the interactions.

**Findings for Classroom Culture of Mathematical Inquiry**

**Interviews.** The interviews provided some insight into the teacher’s knowledge and implementation strategies for a classroom culture of inquiry. As Table 5 indicates, the teacher showed the most growth in the post-observation interview; however, the number of elements described decreased in the final interview.
Table 5. Data Schedule and Frequency of Categories.

<table>
<thead>
<tr>
<th>Category</th>
<th>I1 Jan. 14</th>
<th>O1 Jan. 14</th>
<th>I2 Mar. 19</th>
<th>O2 Feb. 28</th>
<th>I3 June 13</th>
<th>O3 May 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher presses for sense making</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>Teacher encourages individual reflection and self monitoring</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Teacher uses errors as a source of learning</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Teacher encourages student to student interaction</td>
<td>1</td>
<td>6</td>
<td>5</td>
<td>11</td>
<td>0</td>
<td>28</td>
</tr>
<tr>
<td>Teacher examines and appreciates all ideas in the classroom</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>Teacher presses students to use the logic and structure of mathematics to determine reasonableness or correctness of a solution</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

In the initial interview on January 14th, the teacher referenced one element of a classroom culture of inquiry, student-to-student interaction. The teacher stated that students were working in groups to solve a problem given by the teacher, whose role was that of a facilitator. When asked about the role of the teacher and the students in an inquiry-based mathematics class, she responded, “The teacher is asking more open-ended questions or allowing kids to discover questions [the teacher] might have posed. And the students are given opportunities as far as math manipulatives, paper to document, working with partners…in order to answer those questions.” (Int. 1) In the second interview, the teacher addressed an additional element, sense making, and the student-to-student interaction was addressed more often and with more detailed descriptions. “I think the biggest revelation for me was to stop focusing so much on teacher-student interaction and focus on more student-to-student interaction, like I agree with, I disagree
with, I think that is going to stick with me forever.” (Int 2) Here the teacher described student-to-student interaction wherein the students were expected to explain solutions and strategies to their peers.

Another element, student sense making, was encouraged as students listened to their peers’ strategies, analyzed them and provided feedback by indicating if they agree or disagree. The teacher continued to say, “Because it was interesting once they started comparing their answers and supporting each others’ answers, what their thinking was. I think that was very insightful.” (Int 2) This is another example of the teacher encouraging student sense making. Another difference in the two interviews was that the teacher personalized her responses to her experiences with students in the classroom and what she saw happening or changing as opposed to general descriptions. The teacher’s descriptions on student thinking in the post interview indicated a shift in focus in classroom instruction.

The final interview took place in June, almost three months after the teacher’s and coach/researcher’s work together ended. This interview used a different line of questioning that focused on what the teacher had learned. The coach/researcher asked the following questions:

- What did you learn as a result of coaching?
- What do you feel more confident or comfortable doing?
- How did our interactions support your learning?

In this interview, the teacher referred to two additional elements of a classroom culture of inquiry—the teacher encourages sense making, and the teacher encourages reflection and self-monitoring. In response to the first question, the teacher reported that
she had a better understanding of why the curriculum program “did certain things” referring to the games that supported students’ development of number sense strategies and fluency with addition and subtraction facts. She said that before she was just “going through the motions,” but now she had a “deeper understanding.” She also reported increasing her own specialized content knowledge. “I understand why 10 is such an important number and it’s an anchor for so many things in second, third, fourth and fifth grade.” (Int 3) The coach/researcher and the teacher discussed the use of 10 frames as a tool to build combinations of numbers, to subitize, discriminating the number of objects in a collection quickly and accurately without counting (Kaufman, Lord, Reese, & Volkmann, 1949), and to anchor 5s and 10s frequently throughout our conversation. Although this is not a direct reference to the elements of a classroom culture of inquiry, it did indicate that she was making sense of the mathematics in a different way.

The teacher did not respond to the second question. In the initial interview, the teacher was asked a parallel question, about what she found easy to implement and what she felt that she was successful implementing, and this question was not answered either, but it was deflected by saying what she liked instead. This could be due to lack of confidence, lack of self-reflection on what she does well, or discomfort talking about her strengths. The response to the third question indicated that she (i.e., the teacher) watched the coach/researcher’s interactions with students and observed the types of questions she (i.e., the coach/researcher) asked them about their thinking. Referring to her observations, she noticed how the coach/researcher interacted one-on-one with students and in small groups asking, “How did you get your answer? Ok, show me what you did. And having them retrace their steps and being very patient and really engaged on what their
mathematical process was. I think that was important for me to remember to do that, because I often don't let them explain themselves.” (Int 3) Three of the elements did not get referenced in any of the interviews. These elements are (a) teacher sees errors as sources of learning; (b) teacher appreciates all ideas in the classroom; and (c) teacher presses students to use the logic and structure of mathematics to determine reasonableness or correctness of a solution. This could be due to time limitations, teacher beliefs, or teacher content knowledge.

In sum, the teacher’s descriptions in the interviews did change throughout the course of this study. The teacher shifted her responses from general descriptions to what she was observing about student thinking. These descriptions became more detailed and focused, and she addressed more elements in the second and the third interview than the first interview, and the frequency of addressing those elements increased.

**Interviews and Observations.** A comparison between the classroom observations and the interview revealed that the teacher was implementing more elements of a classroom culture of inquiry than the teacher described in the interviews. In the initial interview, only one aspect was mentioned, and that aspect was that the teacher encourages student-to-student interaction. However, the coach/researcher noticed four additional elements in the initial observation than what the teacher discussed in the initial interview: (a) teacher encourages sense making; (b) teacher encourages individual reflection and self-monitoring; (c) teacher uses errors as a source of learning; and (d) teacher appreciates all ideas in the classroom.

In the first observation, the teacher had two different activities. The first activity involved finding values of a collection of coins (see Appendix B for details of this
activity). This did not contain any of the elements of a classroom culture of inquiry. It was teacher-centered with the teacher leading the students how to find the value of the coins, and the teacher was doing all of the explaining. The teacher did not provide opportunities for student thinking, reasoning, or communicating their thinking. The second activity required the students to create a representation of data, and this activity showed some elements of encouraging a classroom culture of inquiry. The teacher promoted sense making by providing opportunities in which to engage in problem solving, helping students to connect representations, and allowing students to use their own strategies to create their representation. There was one instance in which the teacher encouraged a student to examine his method of solving the problem.

The most prevalent elements of encouraging a classroom culture of inquiry were the teacher encouraging student-to-student interaction, and the teacher accepting and examining all ideas in the classroom. The teacher had multiple students share their representations and valued each one. While the coach/researcher found instances of many elements, she also identified some missed opportunities to incorporate them. For example, the teacher said in the pre-conference on January 14th that she had modeled a T-chart to represent data, and all of the students used this method to show their data. So, while the task had potential to promote problem solving, and the teacher introduced the task in a way that did not lead the students to a particular representation, modeling T-charts prior to the lesson may have interfered with the students using their own thinking about how to represent the data.

In the second interview, the teacher described two elements of a classroom culture of inquiry: teacher encourages sense making and teacher encourages student-to-student
interaction; however, the coach/researcher observed four elements in the second observation. The goal of the lesson was to make sense of halves and fourths, and understanding that halves represent two equal parts, and fourths represent four equal parts (see Appendix B, February 28th, Classroom Coaching). The teacher encouraged sense making by allowing students to create halves and fourths using any strategy, and the teacher made connections between representations. The teacher also used errors as a source for learning. She had a discussion about how four vertical folds resulted in five parts instead of four. Again, the most-observed elements were the teacher encourages student-to-student interaction and the teacher appreciates and examines all ideas in the classroom.

She had students share solutions and respected the solutions shared. This lesson was still teacher-centered with teacher telling, which reduced the cognitive demand of the task. She invited students to agree and disagree with each other at times and asked them to explain their reasoning. For example, when she asked the students to fold their paper into fourths, she emphasized four equal parts. As with the first interview the use of the elements was implemented inconsistently, meaning sometimes they were present, but there were other opportunities for her to use them in which she did not. Additionally, the elements also were partially observed. Each element had many different components, and the teacher demonstrated some of the components of the elements, but not all. For example, in regards to sense making, the teacher allowed the students to use their own strategies, but the teacher may have still taught procedures to follow.

In the final interview, the teacher described two elements again which included teacher encourages sense making and teacher encourages individual reflection and self-
monitoring. The final observation on May 16th, discussed in detail in Appendix B, had three different content foci. These were creating equations with a sum of 16, creating a collection of coins given a specific value, and developing conceptual understanding of a two-to-one correspondence. This observation included many more elements of a classroom culture of inquiry than the first two observations and more than what the teacher described in the post-interview. This time, all of the elements were present and observed more often.

The teacher used tasks, which were more open-ended. This provided more opportunities for problem solving, reasoning, and justifying. The data for this observation showed that the teacher pressed for sense making more than twice as often as in the other interviews. The teacher used more components of sense making as well, including problem solving, allowing students to use their own strategies, and focusing on methods instead of only correct answers. The teacher also had students reasoning and justifying more. In addition, the student-to-student interaction was more powerful and observed nearly three times as often. In this observation, the teacher invited students to comment on other students’ reasoning for both correct and incorrect solutions. But the teacher still used language that indicated if the student was correct or incorrect instead of having the students determine correctness.

Overall, the teacher showed growth in most elements of a classroom culture of inquiry. The elements that she used most in the beginning showed the most increase in occurrences. Areas of little change were the teacher’s response to errors in the classroom and the teacher’s authority in the classroom. For example, the teacher always told the correct answer at the close of the problem. On February 27th, in the last conference at the
end of the teacher’s and coach/researcher’s work together, the two discussed the interaction the coach/researcher had with the student about fourths. The teacher commented that the student walked away from a lesson not knowing that his answer was incorrect. “It's almost like he needs to be told. You know what I mean? That what he is doing is wrong. Because he still thinks, he explained it to you, and he thinks that he is right.” (Interview February 27th)

**Further Analysis of PCK and Effective Teaching Practices.**

This section of the results discusses each of the nine elements of PCK. The teaching practices are discussed through examples. These examples were taken from data derived from interviews, conferences, observations, and classroom coaching instances.

**A.) Pedagogical content knowledge.** Pedagogical content knowledge was a significant part of the coach-teacher conversations throughout the duration of the study. Ball, Thames, and Phelps (2008) considered pedagogical content knowledge to be comprised of the following domains:

1. Knowledge of content and students
2. Knowledge of content and teaching
3. Knowledge of content and curriculum

Each of the domains was taken into account when identifying codes. Although these codes were helpful in creating a description for pedagogical content knowledge and determining instances of PCK, they are interrelated, and the data could not be easily coded to just one domain. For this reason, the coach/researcher did not differentiate between the types of PCK in coding, but will address this in the discussion of each theme.
For each domain of PCK, the coach/researcher used at least one classroom segment that represented a typical teacher-coach interaction for each of the categories within each domain. Because the classroom examples encompassed several aspects of PCK, some of the episodes are discussed under multiple descriptors.

**A.1. Knowledge of content and students.** According to Hill et al. (2008), knowledge of content and students refers to knowing how students learn particular content. This includes (a) knowledge of student errors and misconceptions; (b) knowledge of mathematically accurate and appropriate content for students; and (c) knowledge of students’ experiences about a topic and how those past experiences may be helpful or confusing. Each of these aspects of knowledge of content and students was discussed in the conferences with the teacher.

*a.) Knowledge of student errors and misconceptions.* Knowledge of student errors and misconceptions was a topic that appeared in the data throughout the conferences with the teacher. The nature of the conversations was about student misconceptions that the coach/researcher observed during lessons, and the coach/teacher tried to address them in a variety of ways. The coach/researcher recommended resources that provided knowledge about the content and ideas to support students learning. Alternatively, she would discuss learning progressions to help identify previous knowledge that students may be lacking and in turn impeding the learning of new knowledge. However, toward the end of the coaching cycle with the teacher, the coach/researcher used another approach to help the teacher more fully understand student misconceptions.

Specifically, the coach/researcher had videotaped an interaction between a student and herself during a classroom coaching session on February 27th. The
coach/researcher’s goal, by showing the teacher the error in student thinking firsthand, was to help her understand the misconception, to see how the coach/researcher uncovered the misconception and how she handled the misconception. The teacher would also see that, while the coach/researcher questioned the student, she did not lead him to the correct answer, nor did she eventually tell the student the correct answer when misconception remained. During this exchange, the coach/researcher uncovered misconceptions about a student’s understanding of fractions. The student did not completely understand the concept of halves and fourths and was over-generalizing halves. When asked if a rectangle, which was divided into fourths with 2 rows and 2 columns shown in, showed halves or fourths, the student reported that it was four halves.

![Figure 9. Square showing fourths.](image)

He counted them one-half, two-halves, three-halves, four-halves. The coach/researcher shared this video with the teacher, and the teacher was able to more fully understand the misconception, but she did not know how to address the misconception. This is one example of the teacher not knowing what to do when students do not understand a topic.
The following conversation took place as the coach/researcher and the teacher watched the video:

**Coach/Researcher.** I was really stuck on trying to get him to … to try to figure out that that was one fourth. But then sometimes he kind of had it.

**Teacher.** Because the vocabulary there, he is saying fourths, he is saying halves.

**Coach/Researcher.** But on that one (the rectangle with two rows and two columns) he kept saying four halves. So I don't know. What do you think?

**Teacher.** I don't know. It is almost like he needs to have people over for a small group. And just talk it through again. ‘Let’s think about it though. If this is a half then this can’t be four halves. So we call this one-fourth.’ It’s almost like he needs to be told. You know what I mean? That what he is doing is wrong. Because he explained it to you and he still thinks that is right. And he didn't see it. I don’t know. That is probably what I would do, but I don’t know if it’s the right way.

What do you think?

**Coach/Researcher.** Well, when we tell students, that doesn’t mean they get it.

**Teacher.** I know.

I offered a suggestion that might help us assess students’ understanding.

**Coach/Researcher.** It would be interesting if you had a rectangle and you asked them to color one half of it blue, what would he do, or one fourth of it red.

**Teacher.** I think he would.

**Coach/Researcher.** You think he would?

**Teacher.** I think it would be a good activity to start us off.
Coach/Researcher. It is just a matter of delving in and trying to ask those question and trying to figure out what they are really thinking.

Teacher. And that is a good activity. I can give them post-its and see if they can color one-half and on another one color one fourth. And maybe for the higher kids color two fourths.

Coach/Researcher. It would be interesting to see. And then I was talking to M and with questioning she seemed to get it.

Teacher. Yeah, you think it is such an easy concept and when you talk to them deeper, you see that. It is interesting.

Coach/Researcher. And we need to do that as much as possible. It’s hard. But sometimes we think.

Teacher. They know it.

Coach/Researcher. Yeah, I was looking at this and I was thinking, what kind of questions do I ask to see if they understand it.

Teacher. Yeah, we were talking about that this week.
Table 6. Summary of Teacher Actions, Coaching Moves, and Outcomes for Student Misconceptions.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students completed a worksheet coloring fourths of rectangles and circles and partitioning rectangles into fourths.</td>
<td>Teacher introduced task to students. Teacher observed students as they were working.</td>
<td>Students worked in small groups coloring fourths of rectangles and circles and partitioning fourths of rectangles.</td>
<td>The coach/researcher videotaped a coach-student interaction about fourths and uncovered misconceptions. She shared this video with the teacher in the next conference. She discussed with the teacher that telling does not ensure understanding.</td>
<td>Teacher was surprised by the student misconception. Did not know what to do other than “telling.”</td>
</tr>
</tbody>
</table>

This example illustrates the teacher’s need for support in how to identify and address specific misconceptions. The only way she knew how to handle student misunderstandings was to tell the student the correct answer and to pull him and other students into a small group and provide direct instruction.

b.) Knowledge of mathematically accurate and appropriate content for students.

Several of the coach-teacher conversations about content and students focused on developmental appropriateness of content. The coach/researcher noticed student confusion on various topics throughout the time she spent with the teacher including. Topics about which students displayed confusion were money, fractions, equations, linear measurement, solving addition word problems, and skip-counting by 25s. The following classroom session, analyzed for this purpose, dealt with a repeating occurrence of the teacher having students skip-count by 25s. The coach/researcher probed with the teacher regarding whether the students had an understanding of skip-counting numbers of this
size or whether they memorized the sequence. The following is the conversation that pursued regarding skip-counting.

**Teacher.** We [she and her grade-level colleagues] have kind of talked about counting by 25s on the hundreds chart to model counting by 25s to help with that 25, 50, 75, 100, or one dollar, and we actually put quarters in [the hundreds charts pockets] similar to what I did today with the nickels.

**Coach/Researcher.** But is that too much for them?

**Teacher.** But they need to know how to count a combination of coins. So they need to understand, right? They need to know 25, 50, 75, $1.00.

**Coach/Researcher.** Ok.

**Teacher.** So that is what I want them to walk away first grade with. I don't want them to count $1.25 or anything like that. But maybe if they see two quarters or a dime then.

**Coach/Researcher.** Is that in the standards?

**Teacher.** Count a collection of coins.

**Coach/Researcher.** Is that in the Common Core?

**Teacher.** But that is the [former state] standards not the Common Core. Still a fine line.

**Coach/Researcher.** Yeah, so that's right, that's right, that is what I am thinking.

**Teacher.** But we can scale it back if you want, if you think we should stop at count a small collection of coins up to 50 and not make it all the way to a dollar.

**Coach/Researcher.** I don't know.

**Teacher.** You can get back to me.
Me. Yeah, yeah because it is about the student being able to really make sense of the whole thing today versus when we were in school or how we learned. … So my concern is that they are going to memorize 25, 50, 75 $1. But not understand how we are getting it, because they are not adept at using those size numbers.

Teacher. Right, I see what you are saying. And I looked at it like they need to memorize those. That is one of those things that they don't need to dive deeper in. You know what I mean? But when they get to the second grade or third grade, maybe that is why they shut down because maybe they have memorized it.

(January 29th Conference)

The coach/researcher wanted the teacher to know that some topics are convention and need to be told and other topics are constructed. The teacher diverted and the topic did not progress. However, in this instance the coach/researcher tried several approaches to help the teacher understand that memorizing this sequence of skip-counting was not an effective teaching strategy. The coach/researcher used a cognitive coaching approach by asking the teacher if she thought the students understood or if they were memorizing the sequence without understanding. She replied that she thought memorizing was appropriate in this instance. The coach/researcher referred to the CCSS and tried to engage in a conversation to help the teacher know when to tell and when not to tell. The coach/researcher also discussed sense making with the teacher and how that needs to be the focus as opposed to the way she and most people learned mathematics prior to the reform movement.
From the conversation, the coach/researcher thought the teacher had an “ah-ha” moment. She seemed to understand that memorization may interfere with future learning and students “shutting down.” However, the teacher did not give up this activity. In the final classroom observation on May 16th, the students were asked to find a collection of coins that totaled 40, and student misconceptions were uncovered. One student had four quarters and counted them 25, 30, 35, 40. Through questioning, the teacher tried to help the student realize he did not count the coins correctly. She asked other students if they agreed that four quarters was equal to 40. Some students agreed and others disagreed, but none could correctly count the four quarters. The teacher used many reform-based teaching strategies including (a) encouraging student-to-student interaction, (b) asking students to analyze the thinking of other students, and (c) asking students to justify their answers. However, their explanations were incorrect. After several attempts, none of the students were able to explain why the student was incorrect, and they were not able to count the four quarters accurately. The teacher became frustrated and had students repeat the counting sequence 25, 50, 75, one dollar three more times in a songlike way.

**Teacher.** So if you have two quarters, you have 25…

**Students.** 50!

**Teacher.** 50 cents! We know what we have to work on.

Table 7 shows the summary of moves and the outcome for this particular topic.
Table 7. Summary of Teacher Actions, Coaching Moves, and Outcomes for Appropriate Content.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skip-counting by 25s.</td>
<td>Teacher says that she and her colleagues are going to have students skip-count by 25s</td>
<td>N/A</td>
<td>Questions whether this is in the CCSS and appropriate for her students. Expresses concern regarding memorization versus understanding.</td>
<td>Teacher said that she thought memorization was ok, but maybe it caused students to shut down later.</td>
</tr>
<tr>
<td>Students practice rote counting 25, 50, 75, $1.</td>
<td>Continues to have students memorize the sequence.</td>
<td>Repeat sequence.</td>
<td>Shared research on student understanding and asked if teacher thought the students learned.</td>
<td>Teacher continues using an inappropriate strategy.</td>
</tr>
<tr>
<td>Find a collection of coins to total 40 cents.</td>
<td>When student was incorrect she tried questioning and inviting other students to provide explanations.</td>
<td>Used four quarters to count 25, 30, 35, 40.</td>
<td>Provided student more time to think about his solution when he was not able to correct his thinking with student questioning</td>
<td>Student was not able to identify his error. Using quarters may not have been accessible for him.</td>
</tr>
</tbody>
</table>

The students were successful learning the content that was appropriate for them. It was when the teacher tried to teach content beyond what was recommended for students at the first-grade level and that which was accessible for the students that they became confused.

_c.) Knowledge of students’ experiences about a topic and how those past experiences may be helpful or confusing._ This particular aspect of knowledge of students and content was not discussed as often. When the teacher was beginning a topic, the coach/researcher asked about her students’ knowledge from kindergarten, from previous
learning in her class and from informal experiences they may have had outside of school. These discussions took place regarding data, length, time, and components of number sense. The following is an example of how the coach/researcher discussed assessing students’ prior knowledge of time concepts. This topic was first introduced in first grade. However, the coach/teacher suggested that students might have informal knowledge from experiences at home and at school, and that the teacher should assess this knowledge. “Because informally I am sure some or maybe most, …parents are talking about time. You even do that here, and have a clock.” (February 11th, Conference) At the beginning of a brief unit on telling time, the coach/researcher recommended that the teacher show the students a clock and ask what they know about it. In class that day, the teacher asked the students what they know about clocks, and she showed a big cardboard clock to the class. “We are going to have a quick conversation about what we know about clocks. What do we know about why we have clocks? What do we do with them? What do they show us?”

Some of the students commented that it tells us the time, when to go to sleep, when to wake up and when to go to lunch. They also know about the hands of the clock, but they do not know how to tell time on a clock. When shown 11:45 on the clock, the students were not able to state what it was, but their answers were not unreasonable. They gave responses such as 9, 55, and 45. This provided some information about students’ knowledge. Asking the students to explain their reasoning could have provided more insights into their thinking. The students seemed to only be attending to one hand, and seemed to have some knowledge about where the hands were located. Table 8 shows a summary of the episode.
Table 8. Summary of Teacher Actions, Coaching Moves, and Outcomes for Knowledge of Students’ Experiences with a Topic.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessed student knowledge about clocks and time.</td>
<td>Teacher led a class discussion to assess students’ understanding about clocks and time.</td>
<td>Students gave examples of what they knew about time and clocks.</td>
<td>The coach/researcher initiated a conversation about students’ prior knowledge of telling time in a pre-conference.</td>
<td>Coach and teacher gained information about students’ existing knowledge of time.</td>
</tr>
</tbody>
</table>

A.2 Knowledge of content and teaching. According to Ball et al. (2008), knowledge of content and teaching involves knowing mathematics content and pedagogy specific to that content that affect student learning (p. 401). Lesson design and implementation require this type of knowledge, which means that teachers not only need to consider these aspects in planning a lesson, but also need to spontaneously use this knowledge when unexpected responses and ideas surface during instruction. Examples of this type of knowledge include (a) knowledge of how to sequence topics in a unit, examples in a lesson, and student solutions in a whole class discussion; (b) knowledge of which student responses would be productive to share in a whole-class discussion (and which should be saved for later); and (c) knowledge of the advantages and disadvantages of specific mathematical representations and models. The following are examples of each of the descriptors of knowledge of content and teaching found in the data from the conferences and classroom coaching with the teacher.

82
a.) Knowledge of sequencing. For the most part, the teacher followed the curriculum materials for the main part of the lessons. These materials provided the teacher with support in appropriate sequencing of topics. It was when the teacher supplemented her own activities that improper sequencing seemed to interfere with student learning. One of the goals for the teacher was to work on number sense strategies through Number Talks. Specifically, she wanted to work toward students’ fluency with combinations of 10s. The coach/researcher asked the teacher if the students were able to make combinations of 5s, and she was not sure, so both teacher and coach/researcher decided to start there. The teacher used 5 frames as a tool. In one lesson, she showed a 5 frame with 0 dots and asked students how many to make 5 and asked them to write the equation for the situation. The equation she was looking for was $0 + 5 = 5$. The coach/researcher could see the students struggling with creating the equation. This particular 5 frame would be more difficult than a 5 frame with 1, 2, 3, or 4 dots. She used questioning to try to help the students, but the students did not understand. They needed more experience with 5 frames that were easier to visualize. During this activity, the coach/researcher asked her to show a 5 frame with 4 dots. The coach/researcher also suggested that the teacher use 5 frames with two colors that show combinations of 5 in future lessons to help students visualize the two parts.
Table 9. Summary of Teacher Actions, Coaching Moves, and Outcomes for Knowledge of Sequencing.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make combinations of 5 and write equations to represent combinations of 5 given 5 frames.</td>
<td>Teacher showed 5 frame with 0 dots and asked how many to 5 and to write an equation.</td>
<td>Students were confused about how many to 5 and how to write the equation.</td>
<td>Coach/researcher suggested that the teacher use a different combination that would be easier for the students and to use two colors to represent the two parts instead of filled spaces and blank spaces.</td>
<td>Teacher showed a 5 frame with 4 dots. Students were able to find the combination of 5 as 4 and 1. The teacher used two-color 5 frames in subsequent lessons. The students were able to make the combinations of 5 and create equations that represented the 5 frame.</td>
</tr>
</tbody>
</table>

The teacher did not consider how the order in which she sequenced the 5 frames could support student learning. However, the suggestion the coach/researcher provided in this lesson did not transfer to the similar lesson on 10 frames. More explicit conversations were needed to have with the teacher about the impact of sequencing the 5 and 10 frames to support student learning.

b.) Knowledge of which student responses would be productive to share in a whole-class discussion and which should be saved for later. Studies have shown that facilitating whole-group discussions that advance student learning is a challenge for teachers (Ball, 1993; Lampert, 2001). Throughout the time the coach/researcher and teacher worked together, the teacher used a problem-based approach. The teacher posed a
problem to the students, let them solve it using the strategies that made sense to them, and then held a class discussion wherein students shared their solution strategies. The first two phases were implemented well, but the share-out was problematic. One reason for this was because the teacher did not carefully monitor students’ work, and she did not decide in advance which strategies she would have shared. She randomly selected students to share responses without knowing how they solved the problem instead of being more intentional. At times she was surprised by how students solved the problem and did not know how to respond in the whole-class discussion. By tracking student responses and deciding in advance which strategies would be shared based on learning goals, she could have avoided inefficiencies. Additionally, learning could have been more productive. Instead, the teacher tried to correct the students’ errors by questioning and eventually telling students the answer or how to solve the problem. During the one classroom coaching session on January 23rd, the teacher gave the students the following missing addend problem.

“I have 8 balloons in a bundle. Four of them are green. How many are yellow?”

The teacher began with having a student explain what was happening in the problem. A student read the problem and then said $8 + 4 = 11$.

**Teacher.** Why is $8 + 4 = 11$?

**Student 1.** Because there are 8 balloons.

**Teacher.** Hmm Hmm. What does the problem say?

**Student 1.** We have 4 yellow.

**Teacher.** Four are green. Right? And how many are yellow? So does it ask you to add them together?
**Student 1.** Yes.

**Teacher.** It says “How many in all?” Does the problem say there are 8 balloons plus 4 more balloons and that is equal to 11? That’s what the problem said? When I read the problem it says that I have 8 balloons in all. So let’s pretend that these are my balloons. [The teacher counts out 8 crayons.] Then I am going to pretend that 4 of them are greens. [Teacher holds up four crayons.] How many are yellow? Did I add any more?

**Student 1.** No

**Teacher.** No. So I am not going to do 8 plus 4 more and add them together. Out of this 8, 4 of them are green, how many are yellow? Does that make sense?

**Student 1.** Yes.

**Teacher.** Alright, S2, why don’t you explain your thinking?

**Student 2.** $4 + 4 = 8$. Four green balloons and four yellow balloons equals eight balloons.

**Teacher.** Ok, so you already knew that $4 + 4 = 8$? So my friend said in his mind ok $4 + 4 = 8$. [Teacher writes equation on the board and revoices student’s solution.] [Teacher points to the first 4.] These 4 must be green so these 4 must be yellow. Ok? Who has a different strategy?

**Student 3.** Umm. One plus 8 plus 2 more is 11.

**Teacher.** [Teacher writes $1 + 8 + 2 = 11$ on the board.] So you did this?

**Student 3.** Uh huh

**Teacher.** Why?

**Student 3.** I did it in my brain.
Teacher. Alright, and what did your brain tell you to do?

Student 3. Yellow, brown, green

Teacher. But does this problem talk about brown?

Student 3. No.

Teacher. So you don't need that. So this is your one brown?

Student 3. This is yellow.

Teacher. You have 1 yellow, plus 8, plus 2. Right? But the problem says I have 8 balloons. So, whatever you do, it should be equal to 8 [Teacher writes “= 8” on the board.] because you only have 8 balloons. How many are green? Four of them are green. [Teacher points to the words in the word problem displayed on the board.] Right? So out of those 8, there are 4 green. [Teacher writes “4G” on the board for 4 green.] Plus they were asking how many are there. [Teacher writes “4G + ? = 8.”] It can’t be 1 plus 8 plus 2 because it doesn’t match the problem. It should be some amount of green plus some amount of yellow is equal to 8. Ok? Let’s see if S4 can help you out with that.

Student 4. Well I did the same thing as S2. I did 4 plus 4 is equal to 8 but also I wrote 4 are.

Teacher. Ok, good so you know 4 + 4 is equal to 8. Is there anyone else that did something different?

Teacher. Ok, S5, what did you do?

Student 5. I made a number line.

Teacher. How did this number line help you? [Teacher draws a number line on the board.]
Student 5. I stopped at 8 and then I went to 12.

Teacher. Ok, so you think the answer is 12?

Student 5. Student nods.

Teacher. Didn’t we talk about it up here? This was S2’s strategy? Right? And we said there must be 8 in all? There were 4 greens and how many yellows were there to equal 8 in all? Ok, so can we do 8 plus 4 more?

Student 5. Yeah

Teacher. Does that match the problem?

Student 5. No response.

Teacher. No, but how can you use this number line a little differently? [Teacher rereads the problem.]

Student 5. Start on 8.

Teacher. Ok and if you start on 8 what should you do next? What’s another important number?

Student 5. 11

Teacher. Ok, so why 11? Does that match the problem?

Student 5. No.

Teacher. Ok. [Teacher rereads the problem.]

Student 5. 4.

Teacher. Ok, so what are you going to do? You have 4. You have 8 in all. What are you going to do? Count backwards from 4, is that going to help you?

Student 5. Count from 4.

Teacher. Count from 4. To? [Teacher points to the 8.]
**Student 5.** 8.

**Teacher.** Ok. So you are going to go 1, 2, 3, 4. Ok, so if you have 4 balloons, 4 of them are green but you have 8 in all, you have 1, 2, 3, 4, more is going to get you to 8. Ok? Four greens counted to 4 more and that is going to get you to 8. Who had something different? Who drew a picture? Who used their counting on strategies? Student 6 what did you do?

**Student 6.** I drew 4 green and 4 yellow.

**Teacher.** [Teacher draws 4 circles labeled with G and 4 more circles and labeled with Y.] Very good. So now what is the answer? How many are yellow?

[Some students say 4. Some say 8.]

**Teacher.** So can the answer be 8 if 8 is already in our problem?

**Students.** No.

**Teacher.** Four of them are green. How many are yellow?

[Some students shout out 4, and some shout out 8.]

**Teacher.** Four of them are green, 4 of them are yellow is equal to 8 of them in all. So the answer is 4. [Teacher draws a 4 on the board and circles it.]

During the discussion, the teacher did not have an intentional order to the sharing-out, nor did the teacher seem to have known the strategies students used to solve the problems or which students had a correct solution or strategy. Table 10 outlines the moves that the teacher and the students made during their mutual attempt to solve the problem and share-out their answers.
Table 10. Summary of Teacher Actions, Coaching Moves, and Outcomes for Knowledge of Which Student Strategies Would be Productive to Share.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole-group discussion of part unknown problem.</td>
<td>Teacher facilitated discussion asking for volunteers to share out solutions.</td>
<td>Students shared solutions. Some students added the two numbers together. Some correctly found the missing part. Others had solutions that did not make sense.</td>
<td>Coach observed during the share-out without intervening.</td>
<td>Students are confused and many still believe they need to add the numbers in the problem.</td>
</tr>
</tbody>
</table>

The issue of selecting the best responses to share to advance learning was not addressed in the conference, which took place six days later. This is due to several reasons. There were other aspects of this problem that the coach/researcher addressed instead, which included, strategies to help students make sense of problems, and knowing whether a task was appropriate for the student’s level of reasoning because the coach/researcher thought these issues were more critical and would need addressed first. Additionally, when working with a teacher, a coach needs to pick which issues to address in order not to overwhelm the teacher. The coach/researcher decided that students’ sense making and student access was most important at the time. Though the coach/researcher did not address sequencing of student solutions in coaching, it does highlight that this is an area in which teachers may not be knowledgeable and could impact the productivity to whole-group discussions and ultimately student learning.
c.) Knowledge of the advantages and disadvantages of specific mathematical representations and models.

An example (February 5th and February 19th Conferences) regarding advantages of representations involved using 10 frames to develop fluency with combinations of numbers. The coach/researcher suggested that 10 frames were an effective representation to support students’ strategies for single-digit addition. This could help students visually break apart numbers to flexibly add numbers together using derived facts. For example, the following example may help students solve the problem $5 + 6$ more efficiently than counting all or counting one. The teacher would show two 10 frames, one with 5 dots and the other with 6 dots. The students may see that they could fill the top frame with 5 from the bottom frame to make 10 and that there was 1 left over. They may then see that 10 and 1 is 11. However, in order to do this, students need some prior number sense strategies including breaking apart numbers into 5 and some more, and knowing how to find 1 more. Figure 10 illustrates how these number frames would be used.

\[
\begin{align*}
5 + 6 & \quad 5 + 5 + 1 \\
5 + 5 + 1 & = 10 + 1
\end{align*}
\]

Figure 10. Ten frames showing $5 + 6$. 
The teacher seemed interested in using 10 frames to help students with addition of single-digit numbers. She said, “I can do that.” She did not indicate whether or not she used this with students. Table 11 shows a coaching move the coach/researcher used to support the teacher’s knowledge of a representation (i.e., 10 frames), to build students’ fact fluency.

Table 11. Summary of Teacher Actions, Coaching Moves, and Outcomes for Advantages of a Mathematical Representation.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Response</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building flexibility and advancing strategies for single-digit addition problems using 10 frames.</td>
<td>Showed interest in using this with students.</td>
<td>Shared 10 frame as a representation to support learning of single-digit addition.</td>
<td>Unknown.</td>
</tr>
</tbody>
</table>

Another discussion the coach/researcher had with the teacher regarding discussion of advantages and disadvantages of representations and models was the use of tools for measuring length of objects. During a conference in which both teacher and coach/researcher were discussing an upcoming lesson, the two discussed the advantages and disadvantages of various objects used to measure length. In a lesson in which students are measuring lengths of objects, students could use paperclips, square-inch tiles, or cubes to measure the length. The coach/researcher thought using straws might be a better tool than tiles or cubes because they are linear in shape. The coach/researcher thought paper clips would be a good choice as well. The coach/researcher’s concern was that tiles are used to find area and cubes are used to find volume. These tools might be
confusing for students, and they may not focus on the appropriate attribute of the object. The coach/researcher also referred to a research-based resource in which the students used straws cut into one-inch pieces to measure length of objects (Battista, 2012a).

The teacher did not agree that these tools were a concern. The teacher responded, “We use the tiles for a lot of things. So I don't know if it would affect the third-graders because they use it in so many different concepts.” The teacher commented that she thought the rounded ends of the paper clips would make it more difficult for students. She liked that the cubes and the tiles had straight edges and that would make it easier to iterate the units without gaps. The teacher did not consider the shape of the objects used to measure length would interfere with students’ learning of area and volume concepts, and the coach/researcher could not convince her that this as a problem with using those tools to measure length. Table 12 shows the coaching move and outcome from this interaction between the coach/researcher and the teacher.

Table 12. Summary of Coaching Moves and Teacher Responses to Advantages of Using Appropriate Tools.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure length of objects by iterating various nonstandard tools, including paperclips, square-inch tiles, and cubes.</td>
<td>Initiated discussion about measuring tools.</td>
<td>The teacher did not understand the possible disadvantages with using cubes and tiles and continued to have students use them.</td>
</tr>
</tbody>
</table>
The teacher provided students with hands-on experiences using nonstandard units and tools to develop concepts of length measurement. However, she did not recognize the advantages and disadvantages that specific tools could have on students’ future learning of measurement concepts such as area and volume. This could be attributed to the teacher’s lack of experience with teaching these measurement concepts in higher grade levels.

**A.3. Knowledge of content and curriculum.** Knowledge of content and curriculum involves knowing (a) at what grade level particular content is taught; (b) how concepts are connected in the curriculum; and (c) the models and representations with which students are familiar. While Shulman (1986) considered curricular knowledge separate from pedagogical content knowledge. Ball et al. (2008) considered including curricular knowledge as part of pedagogical content knowledge.

*a.) The grade level at which particular content is taught.* Some of the issues and challenges that were discussed and addressed during the coach-teacher conversations involved the transition to teaching the Common Core Standards for Mathematics (CCSSM). This included the content and expectations at each grade level, and understanding how the adopted curriculum materials aligned, and both the teacher and the coach/researcher were still learning the first-grade curriculum. Both the teacher and the coach/researcher had several conversations throughout their work together regarding the content and expectations for first grade, including money, linear measurement, counting, and addition and subtraction word problems. These content topics were either new or moved to a higher grade level. The teacher had difficulty letting go of the old
standards, even after the coach/researcher sought out permission from building and district-level administrators.

A mission of the CCSS is focus and coherence, and the teacher’s attempts to teach both the previous standards and the CCSS undermined this mission (National Governors Association Center for Best Practices, 2010, p. 3). This continued to be a conversation throughout the teacher’s and the coach/researcher’s work together. At the beginning of the work with the teacher, the coach/researcher needed to convince the teacher to let go of certain topics that were no longer in the first-grade curriculum. The coach/researcher was successful with some topics but not others. In the January 24th conference, the coach/researcher and the teacher jointly discussed linear measurement expectations regarding standard units of measure (i.e., inches) and standard measuring tools (i.e., ruler) which were no longer in the first-grade CCSS. The coach/researcher convinced the teacher not to use rulers with the class because they needed extensive experience with iterating of units and with nonstandard units of measure before they were introduced to a standard measuring tool.

On January 29th the coach/researcher and the teacher discussed counting a collection of coins. This is not in the standards at first grade, but the coach/researcher and the teacher discussed foundational concepts she could build to support them in second grade, such as knowing the name and value of coins and making an amount in different ways with numbers appropriate for her students. The coach/researcher’s concern was the size numbers the teacher was using, and whether the students really understood the relationships between the coins such as a nickel having the same value as 5 pennies, as well as her pedagogical approach of skip-counting by increments of 25.
By the end of the coaching cycle, the teacher showed she was learning the expectations for her students in CCSSM. She referred to these standards when we discussed student learning about fraction concepts as evidenced by the conversation below. She stated the expectation for students in first grade regarding fraction concepts is to understand halves and fourths, and that they did not need to learn the symbolic representation of one-half or one-fourth and what it meant. Unit fractions do not appear in the CCSSM until third grade. During the March 19th post-conference, the teacher and the coach/researcher talked about how her students performed on an assessment on fractions. Even though she taught lessons on the fraction symbols and identifying one-half and one-fourth, the students did not understand that on the assessment, but they understood identifying rectangles and circle split into halves and fourths.

**Teacher.** I think for the most part they got the concept of the assessment was divide it into one half or divide it into two equal parts.

**Coach/Researcher.** Hmm hmm.

**Teacher.** Or four equal parts. They got that vocabulary.

**Coach/Researcher.** Ok.

**Teacher.** Um and with that being the primary assessment with *Investigations* that was fine, but then when you say show me one-fourth or color one-fourth there is a disconnect there.

**Coach/Researcher.** Oooh.

**Teacher.** But as far as part of Common Core they don't necessarily have to know the fraction. They just have to know equal parts.
Coach/Researcher.  So the symbolic notation. But they understand the pictorial notation and that they know that half...

Teacher.  Equal parts.

Coach/Researcher.  Is two equal parts.

Teacher.  Hmm hmm.

Coach/Researcher.  And that one-half is one out of two equal parts.

Teacher.  They get that but they don't get the actual representation.

Coach/Researcher.  The number? So as far as word-quantity-symbol. They don't get the symbol piece yet. Ok.

Teacher.  But it’s ok though. That was not necessarily a part of Common Core. But it was interesting.

(March 19th Post-conference)

The teacher showed that she was becoming more familiar with the standards for the students at her grade level.

Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. (National Governors Association Center for Best Practices, 2010)

She still continued to teach the symbolic notation of “1/2” and “1/4” until the assessment; however, she realized that students did not understand, and she was able let go of teaching the symbolic representation.
The coach/researcher revisited these conversations about developmentally appropriate material on several occasions throughout the time she spent with the teacher. Ten days after the conversation with having students use a ruler, the teacher said she would not use it because she was told she did not have to. She agreed not to introduce rulers to the students. However, her response seemed compliant as opposed to understanding appropriateness of content. As far as skip-counting by 25s, the teacher was not willing to give that up, even though she said that memorizing could be why students “shut down.” The final classroom observation showed that the teacher still believed that students needed to practice counting by 25s when her students were not able to count a collection of coins that included quarters. Even though she had been working on it since at least January, the students were still not understanding it or applying this knowledge. The coach/researcher tried to use the CCSS as a reason not to teach concepts, and talked to the teacher about student understanding versus memorizing with varied success.

Table 13. Summary of Teacher Actions, Coaching Moves, and Outcomes for Grade-Level Expectations of the First-Grade Curriculum.

<table>
<thead>
<tr>
<th>Topic or Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using nonstandard versus standard measuring tools and units.</td>
<td>Teacher briefly introduced a standard measuring tool and unit.</td>
<td>N/A</td>
<td>Discussed importance of extensive experience with nonstandard units in students’ learning and understanding.</td>
<td>Teacher introduced the students to the ruler and the inch, but did not have students using it. Students were successful in learning measuring concepts for first grade.</td>
</tr>
</tbody>
</table>
Table 13 continued

<table>
<thead>
<tr>
<th>Topic or Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting collection of coins and skip-</td>
<td>Teacher had students practice skip-counting by 25s and counting collections of</td>
<td>Students engaged in rote skip-counting.</td>
<td>Discussed that skip-counting by 25s was not in CCSSM at all and that it was</td>
<td>Teacher continued to have students practice skip-counting by 25s.</td>
</tr>
<tr>
<td>counting by 25s.</td>
<td>coins through a teacher-led activity.</td>
<td></td>
<td>memorization and would not lead to understanding.</td>
<td>Students were not able to apply the memorized sequence of counting</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>quarters.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student understanding of halves and</td>
<td>Teacher assessed students on understanding of halves and fourths and unit</td>
<td>N/A</td>
<td>Asked teacher about students’ performance on assessment.</td>
<td>Teacher became more familiar with first-grade expectations of fraction</td>
</tr>
<tr>
<td>fourths.</td>
<td>fraction and noticed students did not understand the symbolic representation of</td>
<td></td>
<td></td>
<td>concepts.</td>
</tr>
<tr>
<td></td>
<td>the unit fractions.</td>
<td></td>
<td></td>
<td>Teacher did not pursue teaching symbolic notation of fractions since it</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>was not in the CCSS.</td>
</tr>
</tbody>
</table>

b.) How concepts are connected in the curriculum. One major theme while working with the teacher was to focus on student thinking and understanding, which may have led her to making a content connection. During the May 19th conference, the teacher revealed an important content connection she made while focusing on student thinking. The teacher noticed that students that could not count one were the same students that could not count money. The coach/researcher thought this was very
interesting as well. The coach/researcher had not thought about that connection either.

She said:

I realized that this week that the kids that can't count one from 5, 6, 7, 8, 9, are the same kids that can't count money. Like, I have never made the connection before. Until this week, I was like, you were the same kid I was working on with starting at 7 and counting on 3 more….That makes so much sense now.

This was not a connection that the teacher and the coach/researcher had discussed, and the coach/researcher was pleased that the teacher made this connection on her own by attending to student thinking.

Table 14. Summary of Teacher Actions, Coaching Moves, and Outcomes for Connections in the Curriculum.

<table>
<thead>
<tr>
<th>Content and Curriculum</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connection between counting on and finding the value of a collection of coins.</td>
<td>Teacher focused on student thinking.</td>
<td>N/A</td>
<td>Discussed the importance of students’ counting strategies. Emphasized and modeled attending to student thinking.</td>
<td>Teacher made a connection between counting on and finding the value of coins. It made sense to her why some students couldn’t count the value of a set of coins.</td>
</tr>
</tbody>
</table>

c.) Models and representations about which students are familiar. The data showed a few instances of models and representations with which students were familiar. However, there were a few instances in which this was discussed. The first was regarding
the types of data representations in which students were familiar. In the beginning of the coach/researcher’s and the teacher’s work together, the coach/researcher asked which representations students might use to display data, and what experiences students might have had with data representations in kindergarten. Having worked in kindergarten classrooms, the coach/researcher was familiar with their prior experiences, but the coach/researcher wasn't sure of the teacher’s knowledge and which ones she may have introduced to students. The teacher had a difficult time answering this question and referred to contexts in which they represented data but not specific representations. She reported that they used a weather graph in their calendar routine, and she introduced tally marks and pictures. She reported that she modeled a T-chart (i.e., a two-column chart). The coach/researcher did not probe more with the teacher about that, but later, the students only used one representation for their data—the one that the teacher modeled.

The other representation the coach/researcher and teacher discussed about which students were familiar was 10 frames. The teacher said she used 10 frames to provide practice with subitizing quantities. Therefore, the coach/researcher referred to that representation as a suggestion to teach other number sense concepts.

**B. Effective Teaching Practices**

The National Council of Teachers of Mathematics provided recommendations for effective mathematics teaching and learning (NCTM, 1991, 2000, 2014). Recent efforts to establish a “unified vision” to teach all students, *Principles to Action: Ensuring the Mathematical Success for All* (2014), synthesized and elaborated on the existing literature. These eight principles define effective teaching practices: (a) establish mathematics goals to focus learning; (b) implement tasks that promote reasoning and
problem solving; (c) use and connect mathematical representations; (d) facilitate meaningful mathematical discourse; (e) pose purposeful questions; (f) build procedural fluency from conceptual understanding; (g) support productive struggle; and (h) elicit and use evidence of student thinking. These practices were used as a framework to describe the pedagogical assistance the coach/researcher provided the teacher. Each of these are discussed with the exception of “building procedural fluency from conceptual understanding” and “support productive struggle” because the data did not show significant evidence of the discussion or implementation of these practices.

**B.1. Establishing mathematical goals to focus learning.** “Effective teaching of mathematics establishes clear learning goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions” (NCTM, 2014). Typically, each time the teacher and the coach/researcher met they would discuss the content goals for the next lesson. The curriculum materials along with the CCSSM assisted the teacher in setting goals for the students and situating the goals. It was also common practice for the teacher to discuss the learning goals for the day with the students at the beginning of a lesson. During coaching, the teacher most often inquired about how to sequence the learning and set goals for activities that were not part of the curriculum materials. The following is an example about how the teacher needed support with setting learning goals.

Early in the coach/researcher’s and the teacher’s work together, the teacher set goals for the use of 10 frames to develop number sense (January 29th Conference). Previously, she had used them in a Number Talk (Parrish, 2010) to subitize quantities and to “practice equations.” During the conversation, the coach/researcher gave her some
suggestions such as anchoring numbers to 5 and 10 and adding numbers. During a
Number Talk on February 6th, the teacher inquired about whether a student’s answer was
acceptable. The intent was to make combinations of 5 using 5 frames. A 5 frame the
teacher showed had 4 dots as shown in Figure 11.

![Figure 11 Five frame with four dots.](image)

A student replied that he saw three and one more. The teacher asked the
coach/researcher during the lesson, “Does that work though? Should it be equal to five?”
The coach/researcher replied that it depended on the learning goal. While the students’
statement was true, it was not aligned to the learning goal. The coach/researcher and the
teacher discussed the students’ response with the class, and they determined that the
statement was true, but that they were practicing another concept. In the Number Talk,
the teacher was unclear of the learning goal during the lesson. This could be an issue of
the teacher’s own understanding of the learning goal, the content, and /or what constitutes
an acceptable solution.
Table 15. Summary of Teacher Actions, Coaching Moves, and Outcomes for Establishing Mathematical Goals to Focus Learning.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Talk on combinations of numbers up to 5 and writing equations for the combinations of numbers.</td>
<td>Teacher stated the learning goal at the beginning of the lesson. Teacher asked if a solution was correct.</td>
<td>Students gave answers that were not related to the learning goal.</td>
<td>Coach/researcher probed with the teacher during a conference about the learning goals for using a 10 frame.</td>
<td>Teacher continued to have difficulty focusing on learning goals for the Number Talks. Coach needed to provide more explicit support for the teacher.</td>
</tr>
</tbody>
</table>

### B.2. Tasks that promote reasoning and problem solving

Many of the tasks the teacher used provided opportunities for reasoning and problem solving. Most of these tasks came from the district curriculum materials. The tasks that were teacher-created did not always provide these opportunities. The following is an example of how the coach/researcher supported the teacher in her knowledge of using tasks that promote problem solving and reasoning (February 12th Conference). The teacher discussed a
lesson she taught about counting coins, which involved making 60 using 3 coins. The coach/researcher and the teacher discussed whether there was more than one answer for the problem. The coach/researcher suggested that the teacher use a more open-ended problem such as buying a pencil for 10 and having them show how they could make that amount using any combination of coins. She said she had done that and “that was over some of their heads.”

However, the problem the teacher gave them was more difficult and used a much higher number. In the final lesson on May 16th, the coach/researcher observed a task in which the teacher had the students make a combination of coins, which has multiple solutions. The students were able to come up with a variety of ways. In the discussion of the lesson, the coach/researcher suggested to the teacher to have students make the amounts in more than one way. This would challenge the students who came up with an answer quickly. She liked that idea. She responded, “Oh that’s a good idea. I will do that tomorrow.”

Table 16. Summary of Teacher Actions, Coaching Moves, and Outcomes for Tasks that Promote Reasoning and Problem Solving.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creating a collection of coins for a given amount and for a given number of coins. Show 60 cents using 3 coins.</td>
<td>Teacher posed a closed task to the students with numbers that were not accessible to students.</td>
<td>Coach/researcher probed with the teacher to use smaller numbers and to use a problem that had multiple solutions.</td>
<td>Teacher provided a problem involving money with multiple solutions in a later lesson. Students were successful when they used coins with values that were accessible to them.</td>
</tr>
</tbody>
</table>
B.3. Use and connect mathematical representations. According to NCTM (2000), “representation is both a process and a product. It is both the act of capturing a mathematical concept or relationship in some form and the form itself” (p. 67). When students experience a variety of representations for a particular concept, their understanding of that concept will deepen, and the depth of their understanding is related to the strength of the connections children make (Pape & Tchoshanov, 2001). Young children especially will benefit from using physical models and acting out problems (Cross, Woods, & Schweingruber, 2009).

While working with the teacher, the coach/researcher and she held several conversations about representations. One learning goal she wanted for her students was to be able to represent situations using an equation. Throughout the coaching cycle, including conferences and the classroom lessons, this mathematical representation was addressed in a variety of ways, including how to make a representation more useful to student learning and what constitutes different representations.

In a classroom episode (February 2nd, Classroom Coaching), the coach/researcher made a coaching move to support the students and the teacher in making mathematical connections between representations. The teacher was working on making 5 using 5 frames, and she also wanted to have the students write an equation that represented the part-part-whole relationship. The teacher showed the following five frame shown in Figure 12 and asked the students to write an equation.
A student responded that $4 + 1 = 5$. Another student responded with $1 + 1 + 1 + 1 + 1 = 5$. The second equation did not represent the part-part-whole relationship. The teacher accepted both answers. The coach/researcher asked the students what the 1 and the 4 represented in the equation to determine if they were making the connection between the 5 frame and the equation. The coach/researcher did this to help the students develop understanding of the equation, that the single dot represented the 1 in the equation, the 4 represents the 4 empty spaces, and together they represent the total number of spaces.

The coach/researcher’s other goal in this coaching move was to model for the teacher how to help the students make connections between representations. The students’ responses indicated that they did not understand the connection between the 5 frame and the parts of the equation. A student reported, “They show a way to represent 5,” but he did not connect this to the 5 frame. Another student said, “The 1 is one and there is 4. Four is bigger than one so there is more than one and then there is one and four that make five.” This student seemed to demonstrate an understanding of the connection between the 5 frame and the equation; however, his explanation needed some clarification. It is important to note that many students did not make this connection.

In a conference on February 19th, the coach/researcher and the teacher discussed the teacher’s concern that students were solving word problems the same way. She said they were solving the problems using the same strategy, which the coach/researcher
interpreted as counting strategies. After further probing, the coach/researcher found that she was referring to representations. “I am looking to see more what they are doing. I guess I am focusing more on representation like how are they showing their work. Show me how you did this. Show me how you got your answer.” She was concerned that students were all drawing pictures, and when she asked them to show another way, they would draw another picture. They did not know another way to show their work. The coach/researcher thought this was a good observation made by the teacher, and encouraging students to show their work in more than one way was also an effective teaching strategy. The coach/researcher wondered if this issue was related to students’ counting strategies, and the conversation shifted in that direction.

Table 17 summarizes some episodes of coaching that involved using and connecting mathematical models and representations.

Table 17. Summary of Teacher Actions, Coaching Moves, and Outcomes for Using and Connecting Models and Representations.

<table>
<thead>
<tr>
<th>Topic or Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Making representations of class data.</td>
<td>Teacher modeled how to represent class data. Teacher did not encourage other representations.</td>
<td>Students all used the same representation the teacher used.</td>
<td>Coach/researcher did not discuss this with the teacher because the unit ended just after the coach/researcher started working with the teacher.</td>
<td>Teacher did not seem to understand what characterized different representations of data. There was no change in the students’ representations of data. Continued</td>
</tr>
</tbody>
</table>
Table 17 continued

<table>
<thead>
<tr>
<th>Topic or Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Writing equations for a 5 frame.</td>
<td>Teacher had the students create an equation for a 5 frame.</td>
<td>Students created equations that did not represent the part-part-whole relationship shown in the 5 frame.</td>
<td>Coach/researcher asked the students what each part of the equation represented for the 5 frame. Coach/researcher suggested using two-color frames.</td>
<td>Students were not able to connect the equation to the 5 frame.</td>
</tr>
</tbody>
</table>

| Discussion during conference about student representations. | Teacher expressed concern that students were all solving addition and subtraction word problems by drawing pictures. | N/A | Coach/researcher discussed students’ counting strategies. | Teacher was focusing on mathematical representations. Teacher later focused on students’ counting strategies and showed evidence of using student understanding. |

**B.4. Facilitate meaningful mathematical discourse.** “Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments” (NCTM, 2014, p. 30). Not all classroom discourse promotes student learning. Research literature has provided descriptions of types of discourse that are more likely to promote student learning. Some teacher actions that can support learning in orchestrating classroom discussions are
• Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations;
• Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion;
• Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches; and
• Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning. (p. 35)

The following episodes show some of the opportunities the coach/researcher used to initiate classroom discourse and to make it more productive. During the first classroom observation on January 14th with the teacher, the coach/researcher noticed that some tasks given to students did not provide opportunities for reasoning or problem solving, which are necessary to create classroom discourse that promotes conceptual understanding and meaningful learning (Michaels, O'Connor, & Resnick, 2008). The questions were focused on answer gathering, such as “What is this coin?” and “How much is it worth?” which did not promote discourse. The coach/researcher addressed this with a strategy the teacher and students could implement easily. The coach/researcher suggested that students give a hand signal to show if they agreed with another student as a first step in getting students to reflect upon and analyze the thinking of others. This in itself did not facilitate discourse, but it was a way to encourage students to listen to each other.

After being in the class several times and observing patterns in the teacher’s instruction, the coach/researcher decided to address discourse again (February 4th
The lesson she described provided an opportunity to get the students more involved in the lesson. The teacher planned to show examples of measuring that day. The coach/researcher asked the teacher to use the document camera in the fish activity. The fish activity requires students to determine if the fish were measured correctly (see Appendix B, February 4th Classroom Coaching). Students were shown pictures of a fish measured incorrectly with square tiles and had to identify the errors. Figure 13, taken from the curriculum materials, is an example of some of the fish pictures the students analyzed.

![Figure 13. Examples of fish measured with inch tiles. (Russell & Economopoulos, 2004a)](image)

The teacher facilitated a discussion in which the students determined whether the fish were measured correctly, and they had to justify their answer. For each fish picture, the teacher had the students talk to a partner about whether they thought the fish was measured correctly. The teacher asked the students who thought the fish was measured
correctly and who did not. She sometimes asked them to justify their answers, but not always. After the students had determined if the examples showed correct ways to measure, the teacher had a student in the class model how to measure a fish and asked the students determine if the student measured correctly, and the teacher confirmed he measured correctly. Having a student model and explain how to measure the fish was a positive step for the teacher. To this point, she modeled and explained most of the time.

In the next two conferences with the teacher, the coach/researcher introduced Accountable Talk (February 11th and 12th Conferences). The coach/researcher explained that “Ultimately we want kids responding to each other, ‘I disagree because...’ or ‘I agree because...’ So it is not always the teacher having to prompt the entire discussion.” The coach/researcher further explain that mathematics educators are “getting them to listen [to each other] and it [becomes] more of a conversation instead of between teacher-student.] (February 11th Conference). The coach/researcher explained that, at first, teachers need to model and then invite students to rephrase and comment on each other’s ideas. The coach/researcher also explained that it takes time and practice to develop these expectations in the classroom. The following day, the coach/researcher continued telling the teacher about Accountable Talk. The coach/researcher discussed five strategies to support mathematical discussion and create a classroom in which students share their thinking in “respectful and academically productive ways.” These five strategies were (a) wait time; (b) revoicing students’ explanations; (c) having students revoice other students’ responses; (d) having students apply their own reasoning to another student’s reasoning; and (e) prompting for further participation (Stein & Smith, 2011, p. 69).
The coach/researcher discussed each one of these, and the coach/researcher pointed out that wait time was also used after students respond. This encourages students to continue to reflect on the idea and provides more opportunity for students to comment on other students’ ideas. During this conference, the coach/researcher pointed out some of the moves the teacher was making to encourage her to continue that practice, including asking students if they agree or disagree and why. The teacher indicated that it was frustrating to implement these strategies because she did not think the students were comfortable with that; however, the coach/researcher did not notice that. The coach/researcher reminded her that this takes time and encouraged her to continue to work on norms, especially regarding disagreement with another students’ ideas and incorrect ideas. She commented that it did make some of her students think (February 12th Conference). Therefore, the coach/researcher thought she was seeing a benefit to this type of discourse.

In their next conversation, which took place the following week, the coach/researcher told her about a strategy called “Convince Me” (February 19th Conference) in which the students convince themselves, convince a friend, and convince a skeptic. The coach/researcher thought this strategy would be another way for the teacher to shift the focus of the discussion to students explaining to each other. This did not resonate with the teacher. She said she liked the idea but could not visualize it. The coach/researcher provided an example, but she ascertained it was too much for the teacher at that moment.
Table 18. Summary of Teacher Actions, Coaching Moves, and Outcomes for Classroom Discourse.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Various tasks.</td>
<td>Teacher encouraged students to show if they agreed or disagreed with other students.</td>
<td>Students began using this without prompting.</td>
<td>Coach/researcher suggested hand signals for students to agree and disagree.</td>
<td>Students and teacher adopted this and used it frequently.</td>
</tr>
<tr>
<td>Determining correct and incorrect measuring strategies of fish lengths.</td>
<td>Teacher facilitated a whole-group discussion.</td>
<td>Students responded to each other and analyzed the measurement techniques of the fish. A student modeled how to measure. Students commented if they agreed or disagreed and why.</td>
<td>Coach/researcher suggested using the document camera to have students share their solutions.</td>
<td>Teacher invited students to agree or disagree with the measurement strategy. She allowed a student to model and have students discuss if they agreed with the students work. Students were more engaged. Teacher was still the authority of correct answers.</td>
</tr>
<tr>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Coach/researcher shared Accountable Talk moves.</td>
<td>Teacher incorporated some of the moves. Teacher thought students were uncomfortable, but also made some think. Coach/researcher observed more student engagement.</td>
</tr>
</tbody>
</table>

Continued
Table 18 continued

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Coach/researcher shared “Convince Me” strategy. She offered to model it and try to find some examples of it.</td>
<td>Teacher said she was not able to visualize the strategy and did not seem interested in using it with the students.</td>
</tr>
</tbody>
</table>

B.5. Pose purposeful questions. Part of facilitating productive classroom discourse is the use of purposeful questions. Questioning is a common instructional strategy, and it can serve a variety of purposes. These purposes include assessing (a) students’ reasoning, (b) advancing student reasoning, and (c) helping students make sense of mathematics (NCTM, 2014, p. 35). However, the types of questions asked and the pattern of questions used impact the quality of learning that takes place. Various frameworks have been developed that describe types of questions. NCTM (2014) suggested the following question types: (a) gathering information, (b) probing thinking, (c) making the mathematics visible, and (d) encouraging reflection and justification. Each of these types has an important role, but they differ in level of thinking.

Questioning patterns are also a component of purposeful questions. One particular pattern commonly found in classrooms is the Initiate-Respond-Evaluate (IRE) pattern in which the teacher initiates a question, the student responds, and the teacher evaluates. This type of patterning does not require student thinking nor does it provide information about how students are making sense of mathematics. Other types of questioning patterns...
include funneling and focusing. Funneling attempts to get students toward a desired answer and often disregards students’ ways of thinking, while focusing attends to the students’ thinking and encourages them to reflect on their thinking.

One aspect of questioning the coach/researcher addressed with the teacher was the level of questions being asked. Many of the questions the teacher asked were information gathering. These questions were often only searching for the correct answer and not deeper understanding of the mathematics. For example, in the January 14th classroom observation, the teacher asked questions such as “What is this coin?”, “How many dimes do we have up here?”, and “How much is this penny worth?” In a lesson soon after that, the coach/researcher observed the teacher asking students “Why?” and “How did you get your answer?” The coach/researcher commented on that in a conference to encourage her to continue to questions that encouraged reflection (January 29th Conference). In later conferences (February 11th and 12th) the coach/researcher continued to address the IRE pattern by discussing how to create an environment with more student-student interaction during whole-group instruction. This also addressed the questioning types and patterns the teacher often used.

The coach/researcher also encouraged the teacher to continue providing students with opportunities to agree and disagree and explain why. These conversations manifested in the teachers’ instruction as she used these strategies more often. In the interview at the end of the coach/researcher’s work with the teacher, she reported that she wanted to focus more on student-to-student interaction (March 19th, Interview). During the final observation (May 16th), the teacher was doing an activity in which the students had to make a given value of coins using any strategy they wanted. The teacher invited
students to comment on other students’ strategies. The teacher was not using the IRE pattern. She asked for other ways to solve the problems, and invited other students to comment and agree and disagree. She asked the students how they arrived at their answers. She also did not comment on the correctness or incorrectness of the students’ solutions. She asked more questions that required reflection and justification, such as “How did you get your answer?”, “Do you agree and why?”, and “Who did something similar?”

Table 19. Summary of Teacher Actions, Coaching Moves, and Outcomes for Posing Productive Questions.

<table>
<thead>
<tr>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coach/researcher provided information regarding accountability in whole class discussion in order to eliminate the IRE pattern. Coach/researcher pointed out when the teacher used strategies that encouraged more student interaction, analysis, and reflection.</td>
<td>Teacher used some of these strategies such as asking student to comment on other students’ thinking. Students were more engaged and were doing more thinking instead of the teacher leading the thinking.</td>
</tr>
</tbody>
</table>

B.6. **Build procedural fluency from conceptual understanding.** Procedural fluency is built from a strong conceptual foundation. Students who are taught procedures without meaning, or who are rushed into procedures before conceptual understanding is developed, are led to produce “bizarre results” (Martin, 2009; NCTM, 2014). They are also likely to experience decreased confidence, lack of interest, and mathematics anxiety (Ashcraft, 2002; Ramirez et. al., 2013).
For the most part, coaching conversations took place around students’ conceptual understandings. Much of the work done in early grades focused on building concepts to prepare students for later procedural fluency. There were few instances when the teacher did not seem to agree and understand the importance of conceptual understanding. First was rote counting. The coach/researcher was not able to change the teacher’s practice regarding rote counting. The other instance was using key words in solving problems. In the balloon problem (January 23rd Classroom Coaching), when the students were struggling to solve the problem correctly, the teacher relied on key words to try to help the students solve the problem.

Teacher. Hmm Hmm. What does the problem say?

Student. We have 4 yellow.

Teacher. Four are green. Right? And how many are yellow? So does it ask you to add them together?

Student. Yes.

Teacher. It says “How many in all?” “Does the problem say there are 8 balloons plus 4 more balloons and that is equal to 11?

In the conference on January 29th, the coach/researcher and the teacher discussed students’ challenges with word problems. The coach/researcher offered the following recommendations. The curriculum materials suggested that the students close their eyes and visualize what is happening in the problems. The coach/researcher advised, “Investigations emphasizes that the kids should be able to visualize what is happening in the problem instead of memorizing phrases that might lead them to add or subtract.” She continued to explain that key words do not help students make sense of what is
happening in the problem and that visualizing would help. She said that in previous units, she had students paint a picture in their mind. She did not use that strategy this time and did not say why but that she would use it next time. On February 5th, the teacher used strategies to help her students make sense of a separate, result-known problem about apples. She had them visualize the problem, had them decide if their answer would be more or fewer apples than they started with, and asked a student to explain to the class what was happening in the problem. The students were more successful with this problem. Few students simply added the apples together.

Table 20. Summary of Teacher Actions, Coaching Moves, and Outcomes for Building Procedural Fluency from Conceptual Understanding.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skip-counting by 25s</td>
<td>Teacher had the students practice skip-counting by 25s.</td>
<td>Students rote-counted by 25s.</td>
<td>Coach/researcher had discussions with the teacher about the ineffectiveness of rote memorization, the importance of teaching with understanding, using appropriately sized numbers and that it was not in the CCSS.</td>
<td>Teacher continued to have her students count with rote thinking. Students were not able to apply the skip-counting to solve problems involving quarters.</td>
</tr>
</tbody>
</table>

Continued
<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solving part unknown word problem about balloons.</td>
<td>Teacher used key words to get students to the correct answer.</td>
<td>Students did not make sense of the problem. They incorrectly added the numbers together in the balloon problem.</td>
<td>Coach/researcher discussed with the teacher that key words are ineffective. Coach/researcher discussed a visualization strategy to help students make sense of the problem.</td>
<td>Teacher helped students make sense of a future problem (i.e., separate result unknown about apples) by having them visualize what was happening, discuss if they thought they would have more or fewer than the total number of apples and by having them explain in their own words what was happening in the problem. Students were more successful in solving this problem and did not simply add the two numbers.</td>
</tr>
</tbody>
</table>

**B.7. Support productive struggle.** Productive struggle refers to students grappling with mathematical ideas yet making progress toward a solution. The way that teachers respond to students’ struggles impacts their understanding as well as how they perceive and approach struggle in mathematics (Hiebert & Grouws, 2007; NCTM, 2014). Some teachers believe that, if their students are struggling, they have failed them. A common response to struggle is to rescue them, when in fact this denies students the
opportunity to make sense of mathematics and to persevere through problems (Stein et al., 2009).

The teacher in the study “rescued” her students when they were struggling to answer her questions or to come up with a correct answer quickly. When that happened, she carefully walked them through step by step how to arrive at the correct answer by asking leading questions. During the coaching cycle, the coach/researcher noticed this pattern, but she did not explicitly address this due to her lack of specific knowledge about how to support teachers in this way. The coach/researcher tended to focus more on using developmentally appropriate tasks or strategies to help students make sense of the mathematics rather than addressing it as a separate issue. The coach/researcher thought this would eliminate the unproductive struggle, which the coach/researcher frequently observed in her classroom.

**B.8. Elicit and use evidence of student thinking.** Effective teaching requires teachers to elicit student thinking and use that evidence to adjust instruction. It goes beyond determining if students’ answers are correct or incorrect. Eliciting student thinking requires many aspects of pedagogical content knowledge. Teachers must know what to look for in student thinking, know how to elicit that thinking, know how to adjust instruction to advance student understanding, and address any misconceptions.

Student thinking was central to many of the conversations shared between the teacher and the coach/researcher. In order to support the teacher in her understanding of students’ thinking, the coach/researcher tried different approaches. The coach/researcher encouraged her to let the students do as much explaining as possible as a way to understand their thinking and formatively assess her students. “When you said that you
were going to do some examples whole-group, is there a way student could show because
the research shows that that is really powerful when the student is doing it as opposed to
a teacher?” (February 4th Conference). The coach/researcher pointed out instances of
when she focused on student thinking and encouraged her to continue to do that. “And
just today definitely the kids were explaining their thinking more….And you asked ‘How
do you know?’” (January 29th Conference). The coach/researcher explained different
aspects of student thinking that were critical to focus on, including students’ counting
strategies (February 19th Conference).

The coach/researcher discussed various misconceptions. She noticed when
students solved problems, and she provided instructional suggestions. There were many
examples of these misconceptions. One example was the concept of always adding two
numbers in a word problem (February 12th Conference). Two other were finding the
longer measurement in a measurement-compare problem (February 5th); and naming
fractional parts as four halves (February 27th). As discussed previously in this Chapter
and in Appendix B, February 27th Conference, the coach/researcher shared a videotape of
a student’s misconception of how the coach/researcher elicited student understanding on
problems about halves and fourths.

One example regarding student misconceptions and a suggestion the
coach/researcher provided occurred in a conference on February 5th. The
coach/researcher and the teacher jointly discussed student understanding and
misconceptions regarding measurement comparison. The students struggled with finding
“how much longer,” and the coach/researcher asked if students understood how many
more when finding quantities of objects. As the coach/researcher was working with some
students, she wondered about their understanding of “how many more” and how this would impact their understanding of how much longer. The connection between these ideas was not something about which the coach/researcher was familiar; however, she conjectured that if students did not understand or lacked experience with problems in which they compared quantities, then they would struggle with measurement comparison situations. When the coach/researcher identified a student misconception, she would rely on her knowledge of learning progressions, hypothesize what that learning progression might be, or she would use a research-based resource to learn more about the learning progression.

One challenge that teachers have in eliciting student thinking is what to do with the information. In a conference on February 12th, the teacher and the coach/researcher were deciding upon students’ content needs and where to go next in instruction. The coach/researcher suggested working on addition and subtraction word problems. The coach/researcher had noticed that when students had solved these types of problems in previous lessons, a number of students were confused. They often added the two numbers together regardless the problem type. They were not making sense of the problems. The coach/researcher and the teacher discussed this misconception. The teacher said, “Yeah, so that’s showing us that they are not getting a good understanding of what to do in a word problem.” In the CCSSM, there are 12 problem types for addition and subtraction (National Governors Association Center for Best Practices, 2010, p. 88). The coach/researcher suggested working on these types of problems. The teacher said they had tried to teach it in a variety of ways. She said, “It’s like we need to get knowledgeable of different ways of teaching because we feel like we have hit it all
different ways.” Due to time constraints, the coach/researcher was not able to probe deeper about what she had tried.

However, the coach/researcher explained that problems needed to be very open-ended, and the students need to be able to solve problems using their own strategies. Teachers needed to attend to students’ counting strategies to determine what types of problems were accessible to them. In this instance and other similar situations when students were struggling, the teacher did not have a course of action to intervene other than telling. However, in the May 16th observation, the teacher brought an error to the forefront and tried to facilitate a discussion to address the error. In this instance, none of the other students were able to address the error either, which indicated it was not within reach of the students of the class.

The final conference and interview showed that the teacher was focusing more on student thinking. In the May 19th conference the coach/researcher and the teacher jointly discussed the incredible equations activity, and the teacher commented that “You see a lot.” She was referring to the students’ number sense and counting strategies they used. In the final interview, she reported that what she learned from the coaching experience was that she watched how the coach/researcher interacted with students. She answered, “… sitting down with them one-on-one and with a group and saying, ‘How did you get your answer?’…‘Show me what you did.’ And having them retrace their steps and being very patient and really engaged on what their mathematical process was.” The coach/researcher thinks that was important for her to remember to do that because she often does not let them explain themselves.
These instances were encouraging because the coach/researcher saw signs of the teacher beginning to attend to student thinking as opposed to focusing on correct answers. Shows how the teacher’s thinking evolved regarding eliciting student thinking. A challenge that still remains is how to attend to the thinking once it was been uncovered.

Table 21. Summary of Teacher Actions, Coaching Moves, and Outcomes for Eliciting Student Thinking.

<table>
<thead>
<tr>
<th>Task</th>
<th>Teacher Action</th>
<th>Student Action</th>
<th>Coaching Moves</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement comparison of fish activity.</td>
<td>Teacher provided problems for students to solve regarding measurement comparison.</td>
<td>Students used their own strategies to solve the problems.</td>
<td>Coach/researcher noticed student misconceptions when finding “how many more.” Coach/researcher discussed this with the teacher and probed about students’ experiences with and understanding.</td>
<td>Teacher did not have an answer but said she could assess students’ understanding.</td>
</tr>
<tr>
<td>Discussed next steps for instruction.</td>
<td>Teacher said she had taught it in every way she knew how.</td>
<td>N/A</td>
<td>Coach/researcher suggested addition and subtraction word problems. Coach/researcher said the problems need to be open-ended with students using their own strategies. Coach/researcher said the focus needs to be on students’ counting strategies.</td>
<td>Teacher used student thinking more and focused on number sense and counting strategies in future lessons. The teacher said through coaching that she learned how the coach/researcher proved with students about thinking. Continued</td>
</tr>
</tbody>
</table>
Principles to Actions (NCTM, 2014) recommended eight effective teaching practices for teachers to consider in order to “strengthen the teaching and learning of mathematics” (p. 9), and this framework for effective practices of teaching mathematics indicates the complexities of mathematics instruction and mathematics instructional coaching. In the 6 weeks that the coach/researcher worked with the teacher, most of these elements were identified in the data. Some were addressed more explicitly than others. Some were discussed more often than others, due to the coach/researcher’s own understanding of these elements and her knowledge of how to support teachers.

Discrepancies in Teachers’ Reported Practices versus Enacted Practices.

Research has shown that teachers’ reported practices, and their enacted practices are not always consistent (Mason, 1998; Sapkova, 2013). In each of the conceptual categories, the data showed discrepancies between what the teacher said and what she did. One instance regarding classroom culture of mathematical inquiry occurred on January 24th during a conference. The teacher reported that the previous two units were teacher-directed with little student engagement. She
was eager to start the next unit on linear measurement because it would be more student-centered. She stated the following:

With this unit and with unit four I felt like it was, I felt like I talked the whole time. A lot of teacher-led discussions and teacher-led questioning and this stuff they will get hands-on. They will get to work independently.

Um they need more of that versus me.

Her remarks indicated that the teacher preferred to have a more inquiry-based classroom; however, what the coach/researcher observed during these lessons was inconsistent with that. The teacher often modeled measurement techniques, particularly during whole-group discussion. On January 30th, the teacher introduced the unit on linear measurement. The teacher showed the students how to measure the length of a large plastic tub filled with connecting cubes. The teacher had 9 cubes connected together to use as a measuring tool. She asked, “What do you think is going to be the best way to use these cubes to measure the length of the box?” and pointed to the length of the box.

After one student offered an incorrect strategy, she said, “I’m going to show you how to do this.” The teacher placed the connected cubes at one end of the tub and then said, “When you are measuring things, you start at the corner and you go corner to corner.” During this introduction she also modeled how to measure a box of markers and a notebook. Later in this lesson, the students were given an opportunity to measure objects at their desks such as their school box, pencils, scissors, etc. When they were brought back together whole-group, the teacher asked good questions about why the students had different lengths when
they measured their pencils. A student offered a good reason that they were different lengths. The teacher then modeled how to measure a pencil, and they counted the number of cubes together instead of having a student show how to measure.

Another instance involved the teacher’s pedagogical content knowledge, specifically knowing content that is appropriate for students took place about standard units and measuring tools. In the February 4th, conference the coach/researcher and the teacher jointly discussed introducing the inch and the ruler to the students. The two had previous conversations regarding students’ readiness for standard tools, the need for extensive practice with nonstandard units, and the curriculum regarding linear measurement. In this conference the teacher and the coach/researcher had the following exchange:

**Teacher.** Normally I would introduce the inch by showing it on a ruler.

**Coach/Researcher.** Hmm mmm.

**Teacher.** Um but you told me that I don’t really need to focus so much on rulers. Should I still just pull out a ruler, this is standard, this is what we normally use. This is an inch. This is the same inch. This is the same length from 0 to one, from 1 to 2.

The coach/researcher interpreted the teacher’s response as compliance rather than acceptance. Therefore, the coach/researcher suggested that the teacher could assess students’ knowledge by asking them what they know about a ruler and if they had ever used one. The teacher began by asking students what types of measuring tools with which they were familiar. One student said a scale. Then the teacher used a direct instruction
strategy to discuss meter sticks and rulers. Next, she held up a ruler and stated the following:

This is a called a ruler. Most of the time when people are measuring things that are shorter than this, they use a ruler. Ok? Each point on the ruler is an inch long [points to different inch marks on the ruler]. If I wanted to see how long this nickel is [holds up the nickel to the ruler to model how to measure it], it is a little more than one inch long. This is our unit. It is an inch…. We are not going to use rulers so much in first grade, it is a second-grade thing, but you are going to use these tiles as your unit, [holds up an inch tile] and guess how long a tile is.

Even with the conversations the coach/researcher and the teacher held about standard units of measure and that it was not in CCSS, the teacher still decided to introduce these concepts to the students. However, she did not have the students using them.

Another instance involved pedagogical content knowledge, specifically knowledge of appropriate content for students and effective teaching practices, as well as building procedural knowledge from conceptual understanding. In the discussion on January 29th, the teacher indicated that skip-counting without understanding could be why students in later grades “shut down because they have memorized it.” The coach/researcher thought the teacher was considering that rote memorization was not an effective teaching practice; however, she still had students skip-count to help her students count quarters in a lesson on May 16th.
The teacher reported that she taught number and operations and addition well. However, when the coach/researcher observed the first lesson with an addition word problem of the type part unknown (January 23rd Observation), the teacher focused on correct answers instead of counting strategies. The problem she gave was not accessible to many students in her class. Her lack of knowledge of students’ strategies for solving addition type word problems and the levels of difficulty of the different problem types impacted the lesson and student learning. The students who were still counting all or did not have one-to-one correspondence could not access this problem. She introduced the task using a problem-based approach, and she allowed the students to solve the problem using their own thinking. She provided manipulatives for students to use if they wished; however, she lacked knowledge regarding students’ problem solving strategies for addition type word problems and the accessibility of the different problem types based on children’s strategies.

**Summary of Results**

In this Chapter, the coach/researcher first analyzed the data for the three interviews and for the classroom observations by identifying instances of the characteristics of the classroom culture of inquiry. Next, the coach/researcher presented the findings for other emergent themes in the conferences the coach/researcher had with the teacher. These included pedagogical content knowledge, pedagogical assistance, and relationship building. These themes were analyzed through a framework of its components. The components of pedagogical content knowledge were taken from the
work of Hill et al. (2008) and include (a) knowledge of content and students; (b) knowledge of content and teaching; and (c) knowledge of content and curriculum

Pedagogical assistance was analyzed through the NCTM Teaching Principles (NCTM, 2014) which include (a) establish mathematics goals to focus learning; (b) implement tasks that promote reasoning and problem solving; (c) use and connect mathematical representation; (d) facilitate meaningful mathematical discourse; (e) pose purposeful questions; (f) build procedural fluency from conceptual understanding; (g) support productive struggle; and (h) elicit and use evidence of student thinking.
Chapter 5: Findings

Overview

This Chapter interprets findings for this study, the purpose of which was to examine the nature of coach-teacher interactions focused on developing teacher pedagogical content knowledge, effective teaching practices, and a classroom culture of mathematical inquiry. The following were explored:

1. What is the nature of coach-teacher interactions when the objective of the collaboration is to facilitate the development of teacher pedagogical content knowledge, effective teaching practices, and a classroom culture of mathematical inquiry?

2. Under what conditions are coach-teacher interactions productive when the objective of the collaboration is to facilitate the development of teacher pedagogical content knowledge, effective teaching practices, and a classroom culture of mathematical inquiry?

3. What are some manifestations of pedagogy in teacher pedagogical content knowledge, effective teaching practices, and classroom culture of mathematics inquiry resulting from coach-teacher collaboration?

This study had a limited research focus on interactions between coaches and teachers and how those impact teachers’ knowledge and instruction.
Findings for Research Question 1. What is the Nature of Coach-Teacher Interactions When the Objective of the Collaboration is to Facilitate the Development of Teacher Pedagogical Content Knowledge, Effective Teaching Practices, and a Classroom Culture of Mathematical Inquiry?

The topics of interactions between the teacher and the coach/researcher were many and varied. As shown in Chapter 4, Table 4, the conversations centered on many aspects of pedagogy including classroom culture, teaching practices and pedagogical content knowledge, and each has multiple components. The data indicate that the coach and the teacher primarily engaged in topics centered on pedagogical content knowledge, which constituted 36% of the data. The coach/researcher had to make decisions about which aspects were the most critical to address, and whether it was possible to focus on only a few, which are interdependent and interrelated. The teacher’s limited experiences learning about mathematics-specific pedagogy posed a challenge for the coach/researcher to determine which aspects in which to focus.

Additionally, the teacher taught several content topics in a given day. She typically had a few short activities, which focused on concepts regarding time, money, counting, and/or number sense. The main part of the lesson was usually from the *Investigations* curriculum materials. These topics included data analysis, linear measurement, and fractions. The many aspects of pedagogical content knowledge, teaching practices, and classroom culture—along with the various content topics that were taught while the coach/researcher coached the teacher—provided a challenge because she (the coach/researcher) needed to be knowledgeable enough to identify these pedagogical aspects, as well as know if, when, and how the coach/researcher might address issues she noticed to support teacher and student learning.
Topics of interaction in coaching need to be co-constructed. When working with the teacher the coach/researcher identified areas of need for the teacher and constructed an agenda for their work together. However, she naturally asked questions or raised concerns during the conferences, which needed to be respected and discussed. In other instances, the coach/researcher was not able to get the teacher to engage in a particular topic she considered important. Therefore, the nature of interactions and the content of coach-teacher interactions were based on the intersection of what the coach and the teacher deemed important, which is different than other forms of teacher professional development.

In addition to the conversations being co-constructed, the interactions must lie within the knowledge of both the coach and the teacher. This is consistent with the constructivist perspective. The teacher’s knowledge must be built upon existing knowledge. This model shows that the shared understanding of mathematics and the teaching and learning of mathematics can determine an account of practice. The social interactions that take place between the coach and the teacher can expand this region as both the coach and the teacher can increase knowledge. A coach’s knowledge increases through questions that arise while talking with the teacher and observing the teacher’s practice as well as student thinking. This results in the coach’s search for understanding about the ideas and occurrences in question. The teacher’s knowledge can increase through coaching moves in conferences and in the classroom. Figure 14 adapted from Jaworski (2008) represents this relationship.
The coach’s knowledge of mathematics and mathematics teaching and learning represented in region A, and the teacher’s knowledge of mathematics and mathematics teaching and learning in region B, share a common region, C. Through social interactions between the coach and the teacher, region C will increase.

At times, the conversations were not about content or instruction at all due to situational factors in the building, such as frustration with building meetings that involved bureaucratic mandates, behavioral problems with a student, or challenges with parents. While these conversations may not have been productive regarding increasing teacher knowledge or impacting pedagogy, they were important in building relationships.
The teacher seemed comfortable talking to the coach/researcher about issues with colleagues, parents, and school building frustrations and concerns. It was the teacher who would bring the conversation back around to teaching and learning. Relationship building allowed the coach/researcher to have more productive conversations with the teacher by providing the teacher the opportunity to relieve frustrations. It also enabled developing trust so that both teacher and coach/researcher could discuss content and instruction.

In general, the coach/researcher’s coaching was often in-the-moment based on the teaching and student learning she observed in the classroom and the teacher’s concerns and reactions to conversation in the conferences with the teacher. It was not a well-defined path for learning. The coach/researcher was navigating a journey with an ultimate destination, but without every point along the way decided. It required switching directions at times. It required improvisational moves when unexpected teacher actions or student ideas would surface. It was an adventure, and by the end, the coach/researcher was only part of the way to her destination.

Findings for Research Question #2. Under What Conditions are Coach-teacher Interactions Productive When the Objective of the Collaboration is to Facilitate the Development of Teacher Pedagogical Content Knowledge, Effective Teaching Practices, and a Classroom Culture of Mathematical Inquiry?

As previously stated, the conversations between coach/researcher and teacher included many aspects of pedagogy. However, some were productive, and others were not. Some continued throughout the coach/researcher’s and the teacher’s work together. Others were started but did not progress.

Productive conversations. Some of the conversations showed evidence that they were productive, meaning that the teacher seemed interested, and these topics resulted in
change in practice. Several conditions made the conversations productive. One condition was whether the practice made sense to the teacher and fit into her existing knowledge about mathematics teaching and learning. A second was whether conversations fit in with her beliefs and whether they were consistent with messages from others in the broader culture. When the teacher seemed receptive, she would respond with, “I like that,” “Let’s give that a try today,” (February 4th conference) “Thank you,” or “That is a good idea, I will do that tomorrow” (May 16th conference). A specific instance in which an interaction was productive occurred during a lesson in which the coach/researcher was coaching. This lesson involved a *Number Talk*. Critical components to *Number Talks* are establishing a “safe, risk-free environment,” and “a community of learners built on mutual respect.” Additionally, this community “works toward a collective understanding (Parrish, 2010, p.10).

In order to support the teacher in establishing this culture, the coach/researcher introduced the “me too” symbol. This would encourage students to listen to each other, analyze the thinking of other students and provide a respectful way in which to respond to each other. The teacher began using that strategy immediately, and within a short amount of time, the students began to use it without prompting from the teacher. And they used it in other situations besides *Number Talks*. This strategy was easy for her and her students to implement, and her students responded well. Ultimately, the teacher implemented this strategy and continued to use it.

**Unproductive conversations.** When the teacher was unsure about a statement the coach/researcher made, she responded with a question, stated she thought of it differently, or said that she could not visualize it. The teacher responded with a question
when the coach/researcher asked her about skip-counting by 25s. She asked, “But they need to know how to count a combination of coins. So they need to understand, right? They need to know 25, 50, 75, $1.00.” She seemed to be reconciling what the coach/researcher was telling her, with her own knowledge and beliefs about how to teach money concepts and what it means to understand a concept. In another instance, the coach/researcher had mentioned the “Convince Me” strategy to encourage students to explain and justify their thinking and analyze the thinking of others. In these cases, the teacher did not make attempts to implement these ideas; however; those cases were few.

In regards to the first question, the coach/researcher learned that there were many possible topics about which to focus work with a teacher. While this was not something the coach/researcher had not already realized, the analysis of the data provided more specific information regarding teacher needs. In this case, the teacher had general pedagogical knowledge and skills, but she possessed limited pedagogical knowledge and skills specific to mathematics teaching and learning. The coach/researcher did not know which areas warranted greater focus, which ideas were good starting points, and which ideas would have the most impact on student and teacher learning. Another challenge was that the coach/researcher saw these areas as interrelated as opposed to linear and discrete, and the coach/researcher found it difficult to isolate them and choose a focus.

Even though the coach/researcher set out to build a classroom culture with a goal of teaching for student understanding, she felt that these goals required other types of knowledge including PCK and some of the teaching principles (NCTM, 2014) before the coach/researcher could work on classroom culture. For example, before a productive conversation could happen, it seemed to the coach/researcher that the teacher needed to
understand the content she was teaching in order to support student thinking about that content. While the coach/researcher was working on content and aspects of how to teach specific mathematics content, the coach/researcher introduced simple strategies such as the “me too” symbol. The students and the teacher quickly adopted this strategy and continued to use it throughout the school year. The next step was getting her to ask students why they agreed or disagreed. Then the coach/researcher introduced additional ways to get students involved in a classroom community of inquiry.


During the time the coach/researcher worked with the teacher, both jointly discussed many aspects of pedagogy. Some were adopted by the teacher and used on a regular basis. Some were rejected, while others were inconsistently used. This section addresses each of these aspects in addition to different coaching moves the coach/researcher tried and possible reasons why some were adopted and others were not.

Adopted practices. The teacher began using several of the practices as a result of the coach-teacher interactions. The following describes the practices that the teacher adopted.

Tasks. The teacher began using richer tasks during the time the coach/researcher and she teacher worked together. Many of the tasks she used with students were from the district-adopted curriculum materials, which had multiple solution strategies and multiple answers. However, over time, the tasks she created provided more opportunities for problem solving and reason. In the beginning, the tasks the teacher supplemented were
lower-level and focused on rote memorization or recall of facts. In the pre-observation, the teacher showed a collection of coins to the students, and they counted the value of the coins together as a whole class with the teacher leading the counting. At the end of the year, the teacher gave the students a task to find a combination of coins with a given value. This task was a higher-level task for the students. It required non-algorithmic thinking and significant cognitive demand for the students in the class.

*Student-to-student interaction.* Another idea the coach/researcher shared with the teacher was to have her students use hand signals to determine correctness or incorrectness of students’ responses. The coach/researcher did this for two reasons. First, the coach/researcher wanted to change the pattern of questioning from the IRE pattern to class discussion. Second, the coach/researcher wanted to get the students to be more engaged by listening to other students and analyzing the thinking of others. The teacher adopted this strategy and used it frequently in her classroom, and the students soon adopted this way of communicating with each other without prompting of teaching.

The teacher began having the students explain their solution strategies and their reasoning more often. The coach/researcher offered to set up a document camera so that students could share their work with the whole class. The teacher started using this on a regular basis, and this shifted some of the authority to the students, because they were able to explain how they arrived at their solutions instead of the teacher recording what they did. This also encouraged her to let students do more explaining.

The teacher and the coach/researcher decided an area of student need was developing number sense, and they used Number Talks as a tool for doing that. The teacher had already been trying to use 10 frames, and she initiated the conversation about
what she should do with them. At first, she was focused on the tool as opposed to the content that can be taught with the tool. The coach/researcher explained various number sense concepts that could be developed using 10 frames. She also explained the types of questions the teacher should ask to help build the number sense concepts. This was something the teacher continued to work on with her students for the remainder of the school year.

**Rejected strategies/ideas.** Some topics of discussion did not result in pedagogical change. The teacher was not willing to let go of some unproductive strategies and developmentally inappropriate mathematics. For example, she and the coach/researcher had several conversations about teaching for understanding versus rote memorization of ideas. The teacher commented during one conversation about rote counting, saying, “Right, I see what you are saying. And I looked at it like they need to memorize those. That is one of those things that they don't need to dive deeper in you know what I mean.” Chapters 4 and 5 presented specific examples of this. The teacher firmly held the belief that students needed to count by 25s in order to determine the value of a set of coins. She was still having students practice counting by 25s to 100 in the May 16th classroom observation. She insisted that students use rote counting of other numbers such as twos, fives, and tens without connecting meaning to the counting. However, she believed that some content just needed to be memorized.

She also did not have students operating within numbers that were meaningful or accessible to them. Counting needs to be connected to quantity to develop understanding, and the teacher did not make this connection. Other instances included the teacher introducing symbolic notation to students before they were ready. This happened when
she taught fractions. While students only needed to identify halves and fourths, the teacher also taught unit fractions and the fraction symbol.

**Somewhere in-between.** The teacher attempted some ideas, but they did not get fully or consistently implemented. She and the coach/researcher discussed accountability in whole-group discussions including (a) wait time; (b) asking students to restate someone else’s reasoning; (c) asking students to apply their reasoning to someone else’s reasoning; and (d) inviting further participation. The teacher made attempts at incorporating these but they did not get implemented on a regular basis. On her first attempt, she felt that she continued the conversation for too long, and the students became disengaged.

Another strategy implemented inconsistently was providing opportunities for student explanations and justifications. Sometimes the teacher would ask them to explain their answer, but not always. The teacher asked the students to explain their thinking more when the student gave an incorrect answer than when the student gave a correct answer. She may have thought when the answer was correct, the students understood why. Or she may have been focused more on the correct answer, and when the correct answer was said, she was satisfied and didn’t seek further information.

Another example was the teacher honoring students’ thinking. The teacher used a problem-based model of teaching in which she would provide students with a problem, let them solve it individually or in groups using their own strategies and then facilitate a whole-group discussion of the children’s strategies. However, when students had misconceptions or incorrect solution strategies, the teacher would either ask leading questions to get the students to the correct answer or tell the students how to solve the
problem. Her typical response to handling an error or misconception was to work with the student one-on-one and tell her or him how to solve the problem.

Some of the more complex suggestions were frustrating for the teacher. One example was facilitating a productive mathematical discussion. This required more complex thinking, including having to make decisions about what kinds of questions to ask and how to connect ideas to another—all while managing students and introducing new norms and expectations to the students. Toward the very end of their time working together, the coach/researcher showed a video of herself working with a student and just conversations regarding working with students. The coach/researcher didn’t get much deep reflection from the teacher. Instead, she would get surface-level responses from the teacher. The coach/researcher decided to show a video because, if she only talked about it, the teacher wouldn’t acquire deep understanding. But when the teacher could see what the coach/researcher was doing, the questions she asked about student thinking, the coach/researcher got a different reaction. This time she asked the coach/researcher questions probing student understanding, why the student thought what he did. From this, the conversation led to what to do instructionally to address the misconception.

**Coaching moves.** The coach/researcher used a variety of coaching moves to support the teacher’s learning. While working with the teacher, the coach/researcher noticed that the teacher had general pedagogical knowledge, and knowledge of teaching young children; however, her knowledge in mathematics-specific teaching and learning needed support. In the beginning, the coach/researcher felt like she was talking most of the time during their conversations, and she was doing a lot of telling in order to provide some foundational knowledge of how children learn mathematics. The coach/researcher
had an understanding of how to support student knowledge, but she did not understand how to support teacher knowledge. The coach/researcher did not know of any learning progressions for teachers, how to effectively assess teacher knowledge and understanding, or how to develop a learning path for the teacher. She had a framework, which consisted of research-based teaching practices, and was developing pedagogical content knowledge. But the coach/researcher did not have specific ways in which to use this to support teacher learning.

So, in the beginning the coach/researcher shared ideas backed by theory based on her observations of the teacher’s practice, and she shared research-based books and articles with the teacher. These ideas were always directly tied to the content she was teaching and the pedagogy specific to that content. As the coach/researcher was building the knowledge, she would also connect ideas to previous learning or to her classroom. The coach/researcher knew of some of the mathematics professional development she had attended and connected that knowledge to her practice.

Another strategy the coach/researcher used was modeling during classroom instruction. The teacher did not accept the coach/researcher’s offers to specifically model parts of lessons; however, the coach/researcher modeled “in the moment” by asking students questions during whole-group discussions in order to uncover student thinking and provide opportunities for deeper student understanding. In addition, while students were working on a task, the coach/researcher would circulate through the room and ask students questions to determine their understanding and misconceptions. During the exit interview, she learned that the teacher was observing her interaction with students. The coach/researcher was modeling without realizing it.
During conferences, the coach/researcher also used cognitive coaching techniques in which she would ask the teacher questions about the students’ learning and misconceptions. She found that these conversations did not go to the depth that she intended. Discussing what they had observed did not have the same impact as observing together. The most powerful activity was when the two would talk to students together and then conference about that. At times, they would confer during class, but they did not have time to get in-depth with the students. At the end of the work with the teacher, the coach/researcher videotaped an interaction with the student in which she uncovered a surprising misconception. The coach/researcher decided to share that video with the teacher. The conversation led to a different response than she typically received from the teacher. The teacher was able to better understand the misconception and how the coach/researcher responded to it. The coach/researcher questioned the student without trying to lead him to a correct answer. Instead, her focus was on understanding the student’s thinking as opposed to getting at the correct answer. The coach/researcher did not clear up the misconception, nor did she tell the student the correct answer. The teacher was able to see this as opposed to hearing about it, which led to brainstorming ideas of next steps.

Determining the learning style of the teacher took trial and error and it took the entire coaching cycle for the coach/researcher to figure this out. The teacher needed to actually see specific episodes of student thinking and the coach/researcher’s interactions with the students. In addition, the two were able to reflect on this by watching it on video instead of seeing it in the moment when there were many distractions that might prevent her from thinking deeply about student thinking and understanding.
The following figure represents the coaching moves for this teacher in the coaching cycle. This influence is bi-directional. The coaching moves during conferences were both spontaneous and planned. The planning was a result of previous observations of classroom events, and the spontaneous moves resulted from the teacher’s concerns, questions, and responses. The classroom coaching moves were spontaneous. The teacher did not take advantage of the coach/researcher’s offers to model or co-teach. However, she welcomed input during the lessons. Therefore, the coach/researcher was not able to plan for classroom coaching. While working in the classroom, she would model questioning and eliciting student thinking during whole-group instruction and while students were working individually and in groups. At times the teacher and the coach/researcher interacted with students together.
Figure 15. Coaching cycle with coaching moves.
**Possible reasons for teacher changes in practice.** These manifestations could have occurred for a variety of reasons. Teacher-initiated topics often resulted in a manifestation of pedagogy because the teacher was already interested and motivated to make a change. Manifestations also occurred when the strategy or idea was concrete and simple to implement. For example, having students respond to each other through hand gestures was easy for both the teacher and the students to use. In addition, ideas the coach/researcher could suggest and model during instruction—and which the students could pick up on quickly and easily—were more likely to be adopted by the teacher. An example of this was the hand gesture to show agreement. The students started using this strategy immediately and without prompting, which encouraged continued use by the teacher as well.

Also, the ideas that were connected to her prior learning were more likely to manifest in practice. Even if she was not already implementing the strategies, she had a knowledge base from which to draw. The coach/researcher could refer to a particular resource the teacher already had or something discussed in prior professional development, and she was able to make a connection.

The following conditions resulted in manifestations of pedagogical change for the teacher in the study: (a) the pedagogy was connected to a need identified by the teacher; (b) the strategy was reasonable to implement; (c) the strategy connected to her prior learning; (d) the strategy made sense to the teacher; and (e) the teacher was able to see an episode firsthand under conditions in which she could focus and reflect.

**Obstacles that prevented manifestations.** A variety of factors played a role in the conversations that did not manifest in practice. One of these factors was content
topics, and another was some aspects of pedagogical content knowledge. Furthermore, the teachers’ beliefs about what it means to do mathematics and how children learn mathematics presented an obstacle. Additional factors included lack of comfort or confidence in implementing changes, a fear that change would not result in student learning, and pressure from colleagues as well as simply not knowing what else to do. Another reason for lack of manifestation is the short duration of the work with the teacher. The six-week period was not sufficient to make much change in teacher practice. With opportunities to continue to work with the teacher in subsequent school years, the coach/researcher may perhaps build on the knowledge and skills the teacher was developing. Another obstacle that existed was the coach/researcher’s own limitations in knowledge and skills as a coach. These will hopefully continue to grow with time. The challenges the coach/researcher faced coaching the teacher became learning opportunities for her.

The topics discussed in Chapters 4 and 5 regarding the teacher having students skip-count by 25s despite conversations about developmental appropriateness of numbers and memorization versus teaching for understanding is the most significant example of how obstacles impeded a manifestation of practice. The obstacles may include teacher’s content knowledge and pedagogical content knowledge, as well as pressure from colleagues. The teacher may not have understood the importance of place value in operating on double-digit numbers. Additionally, knowledge of content and curriculum and content and students could have impacted her decision to continue with this activity. The teacher may not have understood what size numbers were appropriate for her students and the curriculum expectations. She may not have understood the relationship
between skip-counting and addition, knowing that counting two is the same as adding two. The teacher did not understand how memorizing the counting sequence without understanding was not likely to contribute to student learning.

This example also points to the strong influence colleagues can have on a teacher’s decision-making. The second-grade teachers put pressure on the first-grade teachers to teach money, and the teachers thought that having students memorize this counting sequence would assist them in learning to count quarters. This pressure had more impact on the teacher’s decision making than the conversations, the curriculum, the research base, and reassurance the coach/researcher provided. In addition, the students still showed no evidence of remembering, applying, or understanding, but the teacher worked on this from the beginning of the time the coach/researcher worked with the teacher in January to the end of the school year as evidenced in the final classroom observation in May. While the data did not indicate she had pressure from the building administrator, pressure from other grade-level teachers was evidenced by the data.

Finally, another factor in this particular case was a lack of clarity and focus on the coach/researcher’s part and knowing which aspects of content and pedagogy were most important to student learning. The coach/researcher’s original framework was to support the teacher in creating a classroom culture of inquiry. However, as she began working with the teacher, she noticed many other areas that needed addressed. Again, this is the first time the teacher had the opportunity to work with a mathematics coach, and she had little other learning specific to mathematics teaching and learning. The coach/researcher was overwhelmed with all of the different aspects of pedagogy that she could and should address. So, for this particular teacher, a challenge for the coach/researcher was to narrow
the focus and scope of her learning. One aspect of pedagogy involved many others. For example, to elicit student thinking, teachers need to be able to choose a task that would reveal student thinking.

They need to have a deep understanding of the content that they want to teach through the task, including conceptual understanding and procedural fluency, and the connections between the two. They need to have an understanding of pedagogical content knowledge, which comprises many elements. They need to know where the knowledge fits into the curriculum, how to address student misconceptions, and the ways in which students might solve the problem. They need to understand learning trajectories and how to determine where students are in that trajectory, where they need to go next, and how to get them there. This requires much more depth of knowledge and complexity than a pre-service program can provide. The sheer number of elements, which could be addressed, were overwhelming and therefore provided a challenge for the coach/researcher’s work with the teacher in her study.

Another factor that influenced the productivity of interactions was the continuity and consistency of content topics taught. The topics in the curriculum materials were taught consistently and with continuity. However, the additional topics the teacher included in her lessons were taught infrequently, and that posed challenges for her to provide interventions and ultimately for the teacher to make changes in practices regarding that content. Addition and subtraction word problems were an example of this. Observations during the first lesson revealed student confusion about how to solve the balloon problem. However, the teacher did not pose these types of problems again until
February 5th, and this was the only other instance of her teaching this content during the coaching cycle.

Finally, another factor that influenced the productivity of the interactions was the coach/researcher’s knowledge. She was continuing to develop her own knowledge of content, pedagogical content knowledge, and how to effectively implement reform-based teaching practices. Opportunities were missed to support the teacher and at times were not effectively or accurately addressed due to limitations in her own knowledge and experiences with first-grade content, curriculum, and students.

In sum, the following conditions influenced whether the interactions were productive:

- Fit into teacher’s existing knowledge (assimilation).
- Teacher is able to visualize the practice.
- Reasonable or easy to implement.
- Consistent with teacher’s belief systems.
- Consistent with messages from the broader community.
- Consistency and continuity of topics.

A factor that could impact teacher change in practice is the result of implementation of practices. Along with this was the coach/researcher’s ability to support the teacher’s vision of reform-based mathematics practices. Images of practice are critical for implementation. In order to create these images, a coach may need to use trimming or decompressing (Ferrini-Mundy et al., 2015). Trimming refers to reducing the complexity of a practice without losing important aspects of that practice. Decompressing involves
making the practice more complex than it appears on the surface by unpacking the aspects of that practice.

Figure 16 explains the coaching cycle and its impacts on the teacher’s implementation of a practice. The model begins with a pre-conference in which the coach attempts to help the teacher create an image of a practice. Then the teacher implements the practice. During the post-conference, if the teacher was satisfied with the outcome, the coach encourages and recognizes successful implementation to encourage adoption of that practice. If the teacher was not satisfied with implementation, the coach may use a variety of strategies to support the teacher. At times, it is a matter of the students getting comfortable and familiar with a practice the teacher needs to continue supporting the students as they understand. Other times the coach may need to use trimming or decompressing to support the teacher’s image of the practice.

Figure 16. Impacts of implementation on teacher change.
Theorizing a Model of Growth in Coach Knowledge.

A coach’s knowledge is a mediating factor in the teacher’s knowledge. Perks and Prestage (2008) proposed a model for growth of teacher knowledge and of mathematics teacher educator knowledge. This is shown in Figure 17, which is an exact reproduction from their work.

Figure 17. Teacher-knowledge tetrahedron and teacher educator knowledge tetrahedron (2008, p.p. 270 - 271).

The tetrahedron on the left represents teacher knowledge, which is influenced by learner knowledge, professional tradition, and practical wisdom while planning for and participating in classroom events. The tetrahedron on the right represents mathematics teacher educator (MTE) knowledge. This is influenced by professional tradition, practical wisdom, and all of the components of the teacher knowledge shown in the bottom left vertex of the MTE tetrahedron. MTE knowledge increases through planning and facilitating mathematics education sessions for teachers.
The coach/researcher proposes a model of growth of knowledge for coaches based on the above models. Table 22 describes the aspects of knowledge for teachers, MTEs, and coaches.

Table 22. Description of Aspects of Knowledge for Teachers, MTE’s, and Coaches.

<table>
<thead>
<tr>
<th>Aspects of Knowledge</th>
<th>Context</th>
<th>Teacher</th>
<th>MTE</th>
<th>Coach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learner Knowledge (LK)</td>
<td>Classroom Events</td>
<td>Math Education Sessions</td>
<td>Pre-Post Conferences and Classroom Coaching</td>
<td></td>
</tr>
<tr>
<td>Practical Wisdom (PW)</td>
<td>Acquired in K-12 education, often ill-connected without conceptual understanding</td>
<td>Teacher-level subject knowledge, PW of experienced teacher, PT</td>
<td>Teacher-level subject knowledge, PW of experienced teacher, PT</td>
<td></td>
</tr>
<tr>
<td>Professional Tradition (PT)</td>
<td>Knowledge from classroom experience</td>
<td>Considering what teachers need to know and creating learning experiences</td>
<td>Considering what teachers need to know, assessing individual teacher knowledge and practice and plan interactions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Knowledge from existing school curriculum, practices and research</td>
<td>Knowledge from systems, institutions and culture. Works as outsider and uses theoretical generalized models</td>
<td>Knowledge from existing school curriculum, practices and research. Work as insider. Must be sensitive to insider perspective and main objective.</td>
<td></td>
</tr>
</tbody>
</table>
Table 22 describes each type of knowledge for teacher, MTE, and coach as well as the setting that influences their learning. Coach and MTE knowledge though similar have some differences in regards to the setting, the professional tradition, and practical wisdom. The practical wisdom is due to the nature of the types of relationships and interactions with the teachers. Since the coach works with individual teachers, he/she must consider knowledge of individual teachers rather than groups of teachers and must also plan learning to meet individual needs. The professional tradition differs because MTEs are predominantly outsiders, while coaches are usually working from the inside. The coach shares the professional tradition of the teacher, but must also remain objective. The following figure represents a coach growth-of-knowledge model and is based on the tetrahedrons by Perks and Prestage (2008).

**Summary of Findings.**

Throughout the work with the teacher, numerous aspects of pedagogy were addressed. Some topics of discussion manifested in classroom practice, while others did not. Those that fit into the teacher’s belief system, made sense to the teacher, and were simple to implement tended to manifest in classroom practice. Providing opportunities for the teacher to see and reflect upon student thinking supported the teacher’s learning more than when the coach/researcher simply told the teacher about it. This could happen either by talking with students alongside the teacher or showing a video of coach-student interactions. In contrast, a variety of factors impeded manifestations, including teacher content and pedagogical content knowledge, teacher belief systems, and pressures from other teachers.
Similar to the MTE tetrahedron, the learner knowledge is based upon the aspects of the teacher knowledge. However, coach knowledge grows through the planning of and participation in pre- and post-conferences and the classroom coaching events.

**Implications for mathematics instructional coaches**

This study points to several implications for instructional coaches. First, teacher learning takes place by attending to student thinking (Carpenter et al., 1996; McDuffie et al., 2008). This can occur in a variety of ways. Learning can occur through coach-teacher discussions about student thinking and misconceptions, however, the greatest impact seemed to be when the teacher and coach could witness students’ thinking firsthand by observing student thinking during side-by-side interaction with students or through watching a video with the coach that showed students engaging in mathematical thinking.
By observing the teacher’s reactions throughout the coach/researcher’s and the teacher’s work together, the coach/researcher could sense what seemed to resonate with the teacher and what did not. After time and trying different ways to connect with the teacher, the coach/researcher began to understand her learning style. She only recognized this once she altered the style and representation of topics for interaction. It was later in the coach/researcher’s and the teacher’s work together that the coach/researcher learned why so many in-depth conversations may not have been productive. Simply discussing how a lesson went, what the students learned, or students’ misconceptions did not prove effective for this teacher. By watching the coach-student interaction regarding halves and fourths together, the coach/researcher was able to direct the teacher’s attention to critical aspects of student thinking.

Many teachers’ learner knowledge is based on their K-12 education, which is often ill-connected and lacking conceptual understanding (Perks & Prestage, 1994). Additionally, this also holds true for experienced teachers (Prestage, 1999). Aubrey (1997) stated that “whilst knowledge of learning and teaching increase with experience, subject matter does not” (p. 160). Therefore, a coach needs to help teachers transform their learner knowledge into teacher knowledge.

When ideas fit into a teacher’s existing schema he/she will be better able to construct new knowledge. Additionally, identifying teachers’ greatest concerns about teaching and learning is important because this will be what teachers are initially most interested and motivated to learn about. When the need for change exists, the teacher is already motivated and more likely to make that change. The coach/researcher learned at the beginning of their work together that the teacher was interested in helping her
students develop number sense. The knowledge and pedagogical strategies she provided became a part of her daily teaching.

Areas of knowledge for a coach includes all of the knowledge that teachers need and additional knowledge. This knowledge includes how to teach teachers. Being an effective mathematics teacher is not sufficient for being an effective coach. Learning about teacher learning is essential. In addition, the teacher needs to work within the professional tradition while balancing the needs and desires of the teacher.

The coach needs to listen more and talk less. This can be a difficult skill for the coach, especially when working with a teacher with limited mathematics-specific pedagogy. Listening allows one to assess the teacher’s knowledge, beliefs, and concerns. It helps build relationships by allowing coaches to get to know the teachers whom they are coaching. It allows teachers to reflect, reason, and problem solve. Social constructivist theory stipulates that knowledge is constructed through social interaction and reflection. By allowing the teacher to talk more, it can promote deeper learning and reflection. At the beginning of working with this teacher, the coach/researcher talked more because she needed to build a foundation of mathematics specific pedagogy. After time, the balance of teacher-talk to coach-talk shifted.

Coaches need to be continual learners as the knowledge of the coach is essential in building teacher knowledge. Many aspects of content and pedagogical knowledge are needed for teaching and learning, and a coach needs to learn as much as possible in order to best support the teacher. It is important for coaches to become as knowledgeable as possible in order to be the informed other that can support a teacher’s learning. Areas in which the coach is not knowledgeable will result in missed opportunities for teacher
learning. Coach learning occurs through the coaching interactions. Pre- and post-conferences and classroom coaching will naturally raise awareness of deficits in coach knowledge and problems of coach practice. Coaches should attend quality professional development that focuses specifically on mathematics teaching and learning. They need to read books and articles from reputable sources. They are obliged to research answers to questions that surface when working with teachers and students using research-based literature.

Learning develops over time, and coaching is most effective when it happens on a consistent basis and over an extended period of time. Ideally, the coach needs to work with a teacher for many weeks and multiple days per week. Additionally, conversations need to happen on a regular basis. When the coach/researcher did not have an opportunity to meet with the teacher for a week, important issues would get lost because new ones would arise. Working and meeting consistently and frequently allows the coach to see how topics are developed over time and how instruction and learning progress, and it provides ample opportunity to discuss teaching and learning. Moreover, the coach can see whether and how student misconceptions have been addressed. Having teachers report out on how things went is different from seeing how things progressed. Coach-teacher discussions are an important part of coaching, but the coach/researcher learned that when that was paired with shared experiences such as viewing a video of student learning, the conversations rise to a deeper level. The coach/researcher was better able to assist in problem solving when she saw it herself rather than relying on the teacher’s explanation and interpretation. Experiencing a lesson is more effective in supporting teaching and learning than hearing about it.
**Implications for Classroom Teachers**

Classroom instructional coaching provides job-embedded professional development for teachers. Coaching takes place in the classroom with teachers with their students. It is individually tailored to the teacher’s specific needs, providing an opportunity to hone in on specific skills. It provides an opportunity to learn more mathematics content, learn about how children learn, and about student thinking and misconceptions. It provides opportunities to collaboratively problem solve with another knowledgeable person including what to do when students are not learning. The coach and the teacher can share the instructional load, while the coach can be someone to co-plan with. The coach can help the teacher find resources and gather materials. In essence, the coach can do everything a teacher can do but has the opportunity to research at a greater depth. The coach has more time, opportunities and flexibility for professional growth and reflection and can support teachers with new knowledge and skills.

**Implications for Mathematics Teacher Educators**

As shown in Chapter 4, the knowledge a coach needs to work with teachers of mathematics is specialized and complex. They need to have a deep understanding of the mathematics of the grade levels they are coaching. Teachers need to have a deep knowledge of the content they teach, but coaches need to have a deep knowledge of a wide range of grade levels. In addition, they need to have deep pedagogical content knowledge including knowledge of content and students; knowledge of content and teaching; and knowledge of content and curriculum. They also need to know how to create a classroom culture that supports student learning, and they need to understand
how students learn. Lastly, they must know effective strategies specific to the teaching and learning of mathematics.

One of the findings of this study is that teachers do not know what to do when students do not understand. They often revert to the same unproductive strategies they have always used. This requires the coach to understand learning trajectories, student misconceptions, and instructional strategies so that he/she can handle these misconceptions and support teacher and student learning.

Teacher educators can assist coaches by helping them identify what are the most critical factors of a classroom that supports student learning. For teachers with many needs, deciding which aspects should be addressed is a challenge. Creating a framework for coaches and helping them prioritize the elements of that framework would help them focus their work. Coaches would consequently be enabled to dive deeper into particular aspects of teaching rather than covering topics in a shallow manner.

**Implications for Administrators**

Administrators play a key role in the success of an instructional coach. Without administrative support, the coach cannot effectively support teacher and student learning. Administrators need to make classroom coaching a priority. A coach’s primary role is to provide job-embedded professional development by working consistently in classrooms for an extended period of time. Coaches need time to work with teachers and need to be available when teachers are teaching. When coaches get bogged down with other duties and roles, they cannot consistently work with teachers in their classrooms, and consistency is critical for success. Coaches and teachers also need to have time to conference. Other duties making it difficult for the coach and the teacher to find common
time to meet become a barrier for these conversations to happen. Assigning coaches to
duties before or after school, or during lunch periods, or using them to cover classes
interferes with access to classrooms.

Administrators can support coaches by recognizing and utilizing coaches’
specialized knowledge and expertise. In addition, they need to support research-based
philosophies and strategies that the coach advocates. This requires trust and respect for
these knowledge bases and skills. If the administrator sends opposing messages to
teachers about teaching and learning, the teachers will likely follow the direction of the
administrator and not the coach, because they are the ones who evaluate teachers. This
undermines the work of the coach and will ultimately have no impact on teaching and
learning. Finally, a culture of respect and support among colleagues is critical in
supporting teachers. While communication between grade-level teachers, these
conversations need to be productive, respectful and not intimidating. They need to focus
on problem solving for the sake of student learning. Coaches can provide guidance on
this because they have the bigger picture of the content and curriculum across many
grade levels.

Administrators need to support coaches’ learning and networking by allowing
time to attend professional development and meetings with others in the same role.
Buildings and districts often have one mathematics coach, and it is necessary for a coach
to have opportunities to learn and collaborate with others in the same role. Coaching
requires a specialized and complex knowledge of (a) deep content, (b) pedagogical
content, (c) curriculum, (d) assessment, (e) how children learn mathematics, (f) how to
create a classroom that supports student learning, (g) how to analyze data, (h) adult
learners, and (i) knowledge specific to being a coach, such as gaining entre to classrooms, building relationships, and having coaching conversations. Coaches need to learn from others with specialized knowledge in order to continue to develop their own knowledge and skills. Coaches also need a network of other mathematics coaches to learn new ideas and for collaborative problem solving of coaching roles.

Administrators need to assist the coach in determining a focus for the work. Coaches often have many teachers in their buildings or districts with whom they can work. In addition, there are many aspects that can be a focus for work. The administrator and the coach should work together to create a vision and a focus for work, and the administrator should support work towards that focus as well.

Finally, administrators today are expected to be instructional leaders as well as to carry out the many other responsibilities of running a school building. A coach can support the administrator by being the instructional leader and context specialist. By utilizing the coach in this way, the administrator can attend to other responsibilities.

**Researcher Reflections**

The coach/researcher felt overwhelmed during the data collection phase of this study. She chose three teachers to be in the study. The coach/researcher worked extensively in their classrooms for six weeks. During that time, she held two conferences per teacher, per week and videotaped three to four 75-minute-long class periods per week. The amount of data collection was extensive. Along with that she was coaching full-time while being a researcher, data manager, and making sure the equipment was charged. Much of this happened simultaneously, and it was overwhelming. There was little time for reflection and analyzing the data during the data collection phase because
the collection, management, and coaching was time intensive. Important information may have gotten lost and opportunities for discussion of topics with the teacher were missed. The coach/researcher was busy “doing,” which is a challenge teachers also face. Focusing on two teachers instead of three would have provided more time and reflection, thus making the coach/researcher’s work with the teachers more effective. Because of the amount of data collected, the coach/researcher had to make decisions about which and how many teachers to use in the study. Each would have made interesting studies for different reasons. The three would have made an interesting cross-case analysis, and may be used for future papers.

However, while analyzing the data of the first teacher and the amount of time it was taking, the coach/researcher decided to only use data for one teacher in this study, and the subject was the first teacher with whom she had already begun data analysis. At the beginning of data collection, the coach/researcher questioned how much data she should collect. After data collection, she questioned how much data to use. She considered randomly choosing classroom videos instead of reviewing all of the data. She decided that it would have an impact on the analysis of the nature of the conversations and how they manifested in classroom practice; therefore, the coach/researcher decided to use all of the video and audio data for only one teacher.

Another factor that contributed to the challenges of coaching was the diverse needs, experiences, knowledge and skills of the teachers. Additionally, in this particular study, each teacher taught a different grade level than the other. The coach/researcher needed to have an understanding of the content, pedagogical content knowledge, and curriculum for each grade level. The teachers had different belief systems about how
children learn, had different learning styles, and responded to pressures differently. Each teacher had different amounts of experience working with the coach/researcher. The coach/researcher was switching gears multiple times per day. Chapter 5 discussed the many aspects of pedagogical content knowledge, classroom culture, and pedagogy that are involved in classroom instruction. All of this was operating in each classroom. While the coach/researcher could plan for coaching, a large part was improvisational. Having to think on the spot was challenging because she found herself pulled in many cognitive directions.

The belief system of the teacher in the study seemed to be a contributing factor to the nature of the conversations and the manifestations of those conversations in instruction. While working with the teacher, the coach/researcher thought the obstacles were about understanding how children learn, so she focused her efforts on increasing her knowledge. In the analysis of the data for the teacher in this study, it became clear that her belief systems and the pressure from colleagues who sent conflicting messages were in some cases more powerful than coaching conversations. This led the coach/researcher to question whether she should have collected data on her beliefs about mathematics teaching and learning in order to use that information in shaping the work she did with the teacher. This raises a question about how teachers’ belief systems may impact coaching moves.

The teacher was hard-working, caring, and had strengths in general pedagogy and knowledge of primary grade students. She lacked deeper content knowledge and knowledge about mathematics-specific pedagogy. As a result, she did not know what steps to take when students had misconceptions or were not learning. Often in these
situations, the teacher reverted to unproductive instructional methods including telling how to arrive at the correct answer, or using lines of questioning to guide the students to the correct answer, rather than developing understanding of the mathematics involved. The teacher needed support in understanding of learning trajectories, understanding how to develop concepts, assessing students’ conceptual understanding rather than skills, and instructional strategies to support learning. The teacher used a constructivist approach when teaching as long as students were getting the correct answer, but when students weren’t, the teacher used a behaviorist approach.

Another obstacle for the coach/researcher was to determine which factors to discuss with the teacher. Mathematics teaching and learning is complex. As the coach/researcher collected and analyzed the data this became more evident to her. The teacher had limited experience in learning the content knowledge and the many interrelated mathematics-specific pedagogical factors prior to the coach/researcher’s and the teacher’s work together. Because there was so much, and this teacher was just beginning to learn more about mathematics and teaching it, the coach/researcher was unsure which were the most crucial to address. The coach/researcher acted upon many of them as they would arise and still let go of some. It felt as though the coach/researcher was switching among many different topics and not deeply addressing any of them. This resulted in some small changes in instruction. The coach/researcher believes that, given continued opportunities to work with the teacher, the two would have been able to make more impactful and lasting changes. The changes that were made seemed to be those that were easy to implement, compelling to the teacher, and affective of immediate change in student
behavior. Fundamental changes were not evident in the data in regard to how children learn.

Finally, even as the coach/researcher writes this, she grapples with her own beliefs about adult learning. The coach/researcher believes that knowledge is constructed and that it is necessary to have knowledge of learning trajectories, how to accurately assess student understanding, and how to determine next steps for instruction. The research base has been growing in this area for mathematics teaching and learning. However, the coach/researcher did not have that same knowledge base for supporting teachers’ learning.

What the coach/researcher did learn is that videotaping is a powerful tool for the coach and the teacher. For the coach it can help identify areas to address with the teacher. Reviewing the tapes in a timely manner might have helped with reflecting and planning. Viewing tapes can help eliminate lost opportunities. As the coach/researcher analyzed the video recordings much later, she noticed important ideas that she thought she should have addressed. Reviewing videos during the time she was coaching would have provided for deeper reflection because, while the coach/researcher was in the classroom, she was coaching and not being a researcher. She was actively engaged in the whole-group discussions and interacting with students as opposed to taking researcher field notes. For the teacher, it provided a better picture of student thinking. It was difficult for teachers to reflect as they are teaching because they are focused on the lesson and the students while engrossed in the many interactions and distractions that take place at any given moment. Allowing time to sit and reflect without “doing” provides for deeper reflection.
Finally, as a coach, the researcher wished she had a coach. She is aware that what teachers can talk about or describe is not the same as what is implemented, and the coach/researcher believes this would be the same for coaches. The coach/researcher had monthly professional development through the Mathematics Coaching Program, which increased her knowledge and provided a network of other coaches, but her professional development was not job-embedded. The coach/researcher found the professional development to be invaluable, but not sufficient. Many coaches do not even have this level of support. As the coach/researcher was coaching, she often wondered if what she was doing was effective, as well as what she could improve upon and what she was doing well. She would not have been the coach she was without the professional development she attended, but she could have honed in more on her craft given specific feedback on her own practice and opportunities to specifically reflect on her own practice.

In conclusion, there were many parallels to teachers teaching students and the coach/researcher working with a teacher. The coach/researcher was still learning about how to assess the teacher, about possible learning trajectories for the teacher, and what to do about teacher misconceptions as well as what to do when they were not moving forward in their learning about teaching and learning.

**Future Research**

Coaching is a naturally curious and reflective process. The coach/researcher learned from the problem solving process required in supporting teachers with the challenges they face regarding student learning. This study provided a more critical analysis of coaching that is beyond the typical reflection that occurs while coaching. Although the coach/researcher learned even more about coaching from this study, new
questions emerged that could be answered by further research. The coach/researcher learned that mathematics teaching and learning is very complex. This study highlighted the intricacies of PCK, classroom culture, and content knowledge that arise during coaching. Teacher preparation programs cannot sufficiently prepare teachers for all they need to know to teach mathematics and the depth of knowledge they need to teach it. More research needs to be done to determine how to better prepare pre-service students regarding these complex factors. Which are the most important to highlight in teacher preparation programs?

The coach/researcher also learned about the limiting factors that impede teacher learning. These factors include teachers’ beliefs about the nature of mathematics content, how children learn mathematics, what it looks like when children are learning versus not learning, teacher content knowledge, state testing, pressure from colleagues, and messages from administrators. Each of these areas could be studied more specifically regarding how to support a teacher when these obstacles interfere with teacher learning.

One of the most significant tools the coach/researcher discovered while coaching this teacher was the value of video. At the very end of her time working with the teacher, the coach/researcher videotaped an interaction she had with a student. She made more progress with the teacher because she could see the entire interaction with the content, the exact questions she asked the student, the exact dialogue, and she could see it without distraction. The conversation between the two was deeper than the others in which the coach/researcher had merely talked about her conversation with the student. The teacher asked deeper questions about why the coach/researcher thought the student had that misconception and how the two could address it. In addition, this brought to the forefront
the importance of having these discussions. Had the coach/researcher known the power of videotaping this type of interaction prior to working with the teacher, she may have been able to have more impact on the teacher. More research on this type of coach-teacher interaction as well as other types is needed to support coaches in providing job-embedded professional development.

Finally, more research needs to be conducted on professional development for coaches. The coach/researcher was overwhelmed with the variety of aspects that she could have addressed when working with the teacher. Identifying which aspects are the most important and developing learning trajectories for teachers that coaches could use to guide and focus their work would better support the work of the coach. Along with that, the coach/researcher felt she needed feedback on her own practice. She wanted a coach to provide job-embedded professional development for her. More research is needed to identify the content and type of learning coaches need. This should include the impacts of professional development and job-embedded professional development on coach learning.
References


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the Research Presession of the National Council of Teachers of Mathematics, Atlanta, GA.


Appendix A: Research Timeline

Table 23. Research Timeline.

<table>
<thead>
<tr>
<th>Month</th>
<th>Research Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 2012</td>
<td>• Defend dissertation proposal</td>
</tr>
<tr>
<td>September 2012</td>
<td>• Request district permission to conduct study</td>
</tr>
<tr>
<td>November 2012</td>
<td>• Submit IRB application</td>
</tr>
<tr>
<td>December 2012</td>
<td>• Teacher recruitment</td>
</tr>
<tr>
<td>January – March 2012</td>
<td>• Data collection-phase 1</td>
</tr>
<tr>
<td></td>
<td>• Review video-taped observations with participants</td>
</tr>
<tr>
<td></td>
<td>• Data transcribing, coding and analysis</td>
</tr>
<tr>
<td>March 2013 – May 2013</td>
<td>• Data collection – phase 2</td>
</tr>
<tr>
<td></td>
<td>• Data transcribing, coding and analysis</td>
</tr>
<tr>
<td>June 2013 – December 2014</td>
<td>• Data transcribing, coding and analysis</td>
</tr>
<tr>
<td>January 2015 – June 2015</td>
<td>• Submit draft dissertation to committee</td>
</tr>
</tbody>
</table>
Appendix B: Narrative

Overview

This appendix is a narrative of the coach-teacher conferences and the classroom coaching sessions in the order in which they took place. An analysis of the data using the lens of the theoretical framework is in Chapter 5.

This study took place from the middle of January through the end of February when the classroom culture and environment had already been established. This is the first time I had worked in this teacher’s classroom in a coaching capacity. However, there were other professional interactions in which a positive relationship had been established including professional development, grade level meetings and school leadership committees. Her response to working with me was positive. She indicated that she was excited to learn new ways to teach mathematics and to become a better math teacher. This was important because the teacher already understood that the coaching model is about teacher learning. She also indicated that math was not her strength. Throughout the duration of the study the teacher kept a math coaching notebook and made notes during the pre- and post conferences. She was open to many of the suggestions made by the coach. However, she challenged some ideas, which are described below. I viewed these questions or disagreements as a positive sign, indicating a level of comfort between us and as insights into her knowledge and beliefs about teaching and learning mathematics. I
interpreted this as an indication that she was open to learning, and not just trying to
please me, but that she was actually reflecting and examining her beliefs with the ideas I
discussed with her.

Accounts of Practice (Simon & Tzur, 1999) is a methodology that describes a
teacher’s instructional practices through a researcher’s lens. “…it is an attempt to
understand teachers' practice in a way that accounts for aspects of practice that are of
theoretical importance to the communities of mathematics education researchers and
teacher educators.” I had developed a conceptual framework, which influenced what I
noticed while working with the teacher during classroom instruction and during
conferences with the teacher. I created accounts of practice, a description of the practice
and possible explanations of choices the teacher made during classroom teaching
episodes. From that description I then determined, a learning trajectory for that teacher. I
used this model to conduct an initial conference to create a baseline of teacher’s current
practices. In the initial interview the teacher was asked the following questions.

• Describe an inquiry-based mathematics class.

• What do you find easy about implementing this type of instruction?

• What do you find challenging about implementing this type of instruction?

Looking back, I realize these questions made an assumption that the teacher uses some
aspects of this type of instruction and believes this is how children learn. This assumption
is based on the fact that the district in which the study took place used a research-based
curriculum with the same fundamental goals as the mathematics education research
community. These include teaching mathematics as a sense-making subject, which
engages students in problem solving, reasoning and communicating about mathematics
(Russell & Economopoulos, 2004b). The role of the teacher in this curriculum was to carefully listen to students, understand their thinking and make instructional decisions based on these observations.

**The Story**

The following describes the interaction during the interviews, coach/teacher conferences and the nature of the classroom coaching sessions that took place in the six-week period I coached the teacher. The study ended with a follow-up interview and observation at the end of the school year. For each interview and classroom coaching session, I included researcher reflections. The story contains all of the coach-teacher conferences. In addition, descriptions of six classroom lessons that occurred over time are included. These lessons depict how pedagogy evolved throughout the work together. These include the first and last observations, which took place on January 14th and May 16th, as well as four lessons in between which took place on January 23rd, February 11th, February 13th, and February 27th.

**January 14th, Initial Interview.** The purpose of this initial interview was to gain insights into the teacher’s practice, to create an initial description of her teaching practices and classroom culture and to consider what would be the focus of initial conversations. In the initial interview the teacher had limited explanations to the interview questions. The answer to the first prompt, “How would you describe an inquiry based class?”, included the following responses.

An inquiry based classroom is where the students are able to ask questions. A lot of the questions are open-based and the students are exploring different things in the room in order to answer those questions.
I probed with the teacher, “What are some of the other components, like what is the teacher doing, what are the students doing, what does it look like, sound like? “ The teacher responded,

The teacher is asking more open ended questions…Or allowing kids to discover questions that she might have posed. Um, and the students are given opportunities as far as math manipulatives, umm paper to document, working with partners, things like that in order to answer those questions.

When asked what she found easy to implement she did not answer the question, but instead she responded that she ‘loves[s] that kids are taking ownership of it.” She likes that” it is more independent.” She describes the teacher as more of a facilitator and she is “available to for questioning but necessarily talking and providing information.”

The teacher used vague descriptions and when probed further, the teacher was not able to elaborate more. The teacher reported that one of the challenges of teaching this way was managing students’ learning. “Like it is hard to know if every kid is getting it.”

When she spent more time with one group of students, she was concerned she was “neglecting” others. I asked her to what extent she was able to “To what extent are you able to get this going in the classroom?” The teacher replied that it depended on the lesson and how the math program approached the concepts. She felt that some lessons were more inquiry based, and some are not.

Some math lessons in *Investigations* are all about inquiry based and they have numerous opportunities, and then so I am able to dive in deeper with
that… It depends on what mathematical concept I am teaching at the time.

Some of them lend themselves more to it than others.

She also reported, when she tried to incorporate her own inquiry-based lessons, she got further behind. I continued to probe with her to construct a picture of her mathematics classroom instruction. I asked “When you add in your own kind of activities, when do you find it necessary to do that?” Her response did not answer the question about when she needed to include additional activities, but rather listed the activities she included in her math lesson. She responded that “we have a Number Talks activity normally at the beginning of our math lesson, and then we get into the math like direct lesson. Then we have math workshops.” She described recent classroom activity as conducting a *Number Talk*, and students working in stations on sorting shapes and also a computer program in which the students use independently.

**Researcher reflections.** Several thoughts came to my mind from this interaction. 1) The teacher had a limited description of an inquiry-based classroom in that she was only able to provide a few of the elements described in the research literature, which did not provide much insight into her instruction. When she described an inquiry-based classroom, she had some components that were consistent with a classroom culture of inquiry and others that were not. *Number Talks* are a tool to develop whole number fluency, while the computer program focused on skill and practice and primarily on getting correct answers versus developing mathematical understanding. This indicated that the teacher used a combination of inquiry-based and traditional activities. The teacher used the district curriculum materials for the main part of her lessons. The types
of activities she supplemented were at times consistent with reform based teaching practices and other times were not.

Looking back I could have asked more questions to get further insights into her instructional beliefs, understanding and practices. For example, I could have probed with her about a particular lesson she used to supplement the curriculum materials to determine her understanding of what constitutes a worthwhile task.

A challenge for me as a coach was thinking spontaneously. I missed opportunities to ask questions or offer suggestions and then time passed before we had another opportunity to have a focused conversation again. By that point, additional important topics surfaced while some previous topics were lost.

**January 14th Pre-conference**

The pre-conference took place at the end of the interview. My goal was to learn more about the lesson I observed earlier that day. The teacher said that they were working on representing data, and later in the week the students would collect their own data. She anticipated this would be very difficult for them because “It is a lot of steps,”, they had to organize their thinking and they had difficulty creating questions. “They really have to organize themselves. Organize their thinking. A lot of them have difficulty with what is a question and they have to create a question.” She continued, “And they have to figure out how to ask everybody and manage that. Even though the lesson is very explicit, there are some kids that just still don't get it.”

I asked about the students’ previous experiences with the content and what they learned in kindergarten in regards to data collection and representation. She said they did
a weather graph in the calendar routine, which was represented in a bar graph. She also said they did some “random surveys” which have two response choices, and that she “modeled t-charts” and “introduced tally marks.” She reported that some students would use pictures, while others would use physical models such as cubes. I asked the teacher what role she wanted me to play, and how involved she wanted me to be. “Things that I can tweak. Things that you see the kids are struggling with and maybe pulling them aside and maybe working with them. Those are things that are interesting and then I can kind of observe how you do that and even the vocabulary that you use that may be different than mine.” Next I explained ways in which I typically worked in classrooms so it was not a surprise to her. While students are working on problems, I listen to and probe their thinking. I also like to observe small group work in which the teacher and I talk to students together. I also discussed the use of the math program and how closely to follow it. The district administrators said that teachers need to use the program with “fidelity,” but they also agreed that they needed to teach to students’ needs. Fidelity was not clearly defined and each teacher and administrator interpreted it differently.

**Researcher reflections.** The teacher seemed to be concerned about using higher level thinking tasks with her students. She was also unsure about students’ prior experiences with representing data. Knowing past experiences and what students know about a topic is important in order to meet students’ needs.

Using a scripted curriculum program has both challenges and advantages. It can assist teachers in many aspects of content and pedagogy, but teachers can also over rely on the materials and not teach to students’ needs. “For the last couple of years in investigations I have just been going through the motions with it. But now I have a
A deeper understanding of why they are doing a certain thing.” I needed to support the
teacher in how to use these materials most effectively.

**January 14th Classroom Coaching**

The initial classroom coaching session provided further insight that was not
revealed in the interview.

I made several observations that were a starting point for the coaching
conversations. In the first lesson, the teacher started with a teacher created activity about
counting money. The teacher displayed three dimes and one penny on a magnetic white
board and first had students identify the coins.

**Teacher.** What is this coin? [pointing to a dime]

**Student.** A dime.

**Teacher.** Kiss your brain. This is a dime and a dime is worth ten cents. So how
many dimes do we have up here?

**Student.** “Three.”

**Teacher.** “Three. So this is worth ten cents. This is worth ten cents. This is worth
ten cents [pointing to each dime] What is this coin called?”

**Student.** “A penny.”

**Teacher.** “A penny. And how much is a penny worth?”

**Students.** “Ooone.”

**Teacher.** “One cent. Now how much is this worth altogether?”

**Student.** “Thirty-one cents.” [The teacher does not acknowledge this response.]

**Teacher.** “Now, am I going to count this one, two, three, four?”

**Students.** “Nooooo.”
Teacher. “Because we know this is worth 10 cents. Lets do a really quick count together. 10, 20, 30, 31.” Some students say 40 and some say 31.

Teacher. It would only be 40 if there was another dime. But because there is a penny here we go 10, 20, 30, 31 pointing to each coin as she counts.

The remainder of the lesson focused on a unit about data. This activity came from the district adopted curriculum materials, *Investigations in Number, Data and Space.*

What would you rather be? (Russell & Economopoulos, 2004a). In the lesson, data was collected as a whole class on whether they would rather be an eagle or a whale. They worked in pairs to create a representation and analyzed the data. The teacher conducted a whole group discussion so that students had opportunities to share their representations. All of the students used two column charts to represent the data with various symbols to represent each data point. One student used tally marks, and the teacher had the student explain how she made her tally marks. The teacher restated that she made bundles of five and the teacher modeled how to count them by fives. During the discussion, the teacher consulted with me about a student who used two different marks in his chart. I replied that I had the same question when looking at the student’s work. “That’s a good question. I was thinking about the same thing.” I responded that in my experience I had always seen the same symbol used in one graph, but I was not sure if that was a requirement.

Researcher reflections. The teacher had already established a positive learning atmosphere in her class and positive relationships with her students. The children seemed happy and the feeling of the classroom was relaxed. The teacher had a very calm, friendly yet firm approach with the students. She was caring and had high expectations regarding classroom behavior. This was an important factor to consider in coaching and I
immediately knew that this was not an area that needed addressed. This allowed more time to focus on content and pedagogy.

**Counting money.** The task used for counting money was a low level closed task as were the questions she asked. The teacher positioned herself as the authority of correct answers and did not provide opportunities to share their thinking. The focus was on correct answers as opposed to developing mathematical understanding. The activity did not seem appropriate for the level of the students in the class as evidenced in the next part of the lesson about data where the students were still counting by ones. However the teacher led them through counting coins by tens and then also had to count on. It seemed that she did not make the connection regarding counting strategies and money. The curriculum materials had activities that involved identifying coins, which were more appropriate for the students. A later conversation revealed that the teacher did not know the connection between counting coins and counting on at the time of this lesson. She discovered that connection in May.

**Data collection, representation and analysis.** The task was appropriate and engaging for the students. The students were able to use whatever representation made sense to them. I anticipated that students would use a variety of strategies; however, they all used a two-column chart to represent the data. The teacher used other types of graphs with her students in other contexts, such as a bar graph to represent data about the weather, and the teacher said in the pre-conference that students would use a variety of representations.

Like last week we did T-charts. I modeled how you can show it with the T-chart… I umm introduced tally marks just as an introduction um pictures
because we are familiar with pictures from word problems. Umm and some kids might want to use a hands on manipulative to show it like as far as cubes or counters but then it still needs to be transferred to a piece of paper so it is still ... And a lot times they use, they're familiar with that as a representation and they are using the vocabulary with that.

However, the students did not a variety of representations, nor did the teacher encourage students to use other representations. The only difference between the students’ representations was the types of marks they used in their chart such as x’s and smiley faces. This task could have provided opportunities to compare types of graphs, and discuss organization that clearly communicates the data. I wondered about the teacher’s content knowledge with respect to this topic. Did she know that the types of marks did not constitute different types of graphical representations? Aside from content knowledge I noticed aspects of the social culture, where students were not invited to explain their thinking or analyze the thinking of others. The teacher was the authority of correct answers. Instructional strategies consisted primarily of questioning with an IRE pattern (initiate, respond, evaluate) where the teacher initiated a question, a student responded and the teacher evaluated. This type of questioning tends to focus on correct answers rather than student reasoning (NCTM, 2014).

The teacher seemed comfortable with me and felt safe enough to ask me a content question during a whole group discussion.

From this observation, I identified areas in which I could support the teacher including elements of classroom social culture, pedagogical content knowledge, and
teaching practices. Some of these elements included. socio-mathematical norms, authority of correct answers, patterns of questioning, levels of questions, characteristics of a worthwhile task, student reasoning and justification, developmental appropriateness of content, and content knowledge involving data representation.

The next opportunity to conference with the teacher was nine days later. During that time the lessons kept moving forward, and I did not have an opportunity to address the content about data representation.

January 22\textsuperscript{nd}, Classroom Coaching

This lesson took place one week after the first classroom coaching session, and before the teacher and I had an opportunity to discuss the last lesson. In the next lesson, the teacher talked with the students before an assessment. Students were in the meeting area and the teacher posted four samples of student work on the board for the students to see. The students all had similar representations, but used different symbols. She pointed out the different symbols on each paper to the students. I posed a question to the class to bring out more mathematics content. “Why do you think this group organized their smiley faces in twos?” The teacher also interjected “See if you can make a connection to the tally marks.” A student commented that it is easier to count. I add that when we represent data, we want it to be easy to read. “What if we had our smiley faces all over the place?” A student responded that it would make it hard to count. Some students are trying to count the tally marks inaccurately, 2, 5, 18, 12. The teacher stops them and says, “Let’s count them altogether.” The class orally counts the tally marks by twos as the teacher points to each group of two. I ask how to count the tally marks. The teacher
counts aloud while some students count with her. “Five, six, seven, eight, nine.” Some
students start counting “Five, ten…”

**Researcher reflections.** When there was not an opportunity to conference with
the teacher. This was a concern for me. I only had six weeks to work with the teacher and
this past week was unproductive. I observed the same type of instruction. The teacher
took over the counting instead of asking the students how they might count. The teacher
seemed to believe that teacher telling led to student understanding and that the goal was
getting the correct answer. In addition, the content of the data lesson was not
mathematical, as it focused primarily on the type of symbols student used. I decided to
address mathematics content during the lesson as a coaching move. After I asked my
question, the teacher asked a follow up question encouraging the students to make a
mathematical connection. The modeling I did may have encouraged the teacher to ask
higher-level questions. Time constraints prohibited me from asking more questions, such
as if there were other ways to represent the data. This was the last day of this unit so I did
not probe with the teacher about the content of this unit, and we had new content to
address.

**January 23rd Classroom Coaching**

The following day, the teacher began the lesson with a part-part-whole word
problem with a part unknown. The teacher posed the following task to the students. “I
have 8 balloons in a bundle. Four of them are green. How many are yellow?” Students
were told to solve the problem on their paper and that they could use counters if they
needed them (not that this direction is not quite what we want—should have said, Solve
this problem, then describe how you solved it on paper. Students worked independently
on this problem, and are free to solve the problem using any strategy they wish. The
teacher circulated through the room and asked questions such as “What is this question
asking you?” I observed students while they worked and talked to individual students to
learn about their thinking. The teacher facilitated a whole group debrief with the students
after they had had time to solve the problem. Seven students are asked to share their
solutions. When the students shared their solutions, the teacher recorded their solutions
on the board. The teacher asked a student what the problem was asking. The student read
the problem and then said $8 + 4 = 11$.

**Teacher.** Why is $8 + 4 = 11$?

**Student.** Because there are 8 balloons.

**Teacher.** Hmm Hmm. What does the problem say?

**Student.** We have 4 yellow.

**Teacher.** Four are green. Right? And how many are yellow? So does it ask you to
add them together?

**Student.** Yes.

**Teacher.** It says “How many in all?” “Does the problem say there are 8 balloons
plus 4 more balloons and that is equal to 11? That’s what the problem said?

When I read the problem it says that I have 8 balloons in all. So let’s pretend that
these are my balloons.” [The teacher counted out 8 crayons.] “Then I am going to
pretend that 4 of them are greens. [Teacher held up four crayons] “How many are
yellow? Did I add anymore?”

**Student.** No
Teacher. No. So I am not going to do 8 plus 4 more and add them together. Out of this 8, 4 of them are green, how many are yellow? Does that make sense?

Student. Yes.

I am uncertain that the student understood.

Teacher. [The teacher called on another student] Alright, S2, why don’t you explain your thinking.

Student 2. 4 + 4 = 8. Four green balloons and four yellow balloons equals eight balloons.

Teacher. Ok, so you already knew that 4 + 4 = 8? So my friend said in his mind ok 4 + 4 = 8. [Teacher writes equation on the board and revoices student’s solution]. These four [Pointing to the first 4 in the equation] must be green so these four must be yellow. Ok? Who has a different strategy?

Student 3. Um, one plus eight plus two more is eleven.

Teacher. [Teacher writes 1 + 8 + 2 = 11 on the board.] “So you did this?”

Student 3. Uh, huh

Teacher. “Why?”

Student 3. I did it in my brain.

Teacher. All right, and what did your brain tell you to do?

Student 3. Yellow, brown, green

Teacher. But does this problem talk about brown?

Student 3. No.

Teacher. So you don't need that. So this is your one brown?

Student 3. This is yellow.
**Teacher.** You have one yellow, plus eight, plus two. Right? But the problem says I have eight balloons. So whatever you do, it should be equal to 8 [Teacher writes “= 8” on the board.] because you only have 8 balloons. How many are green? Four of them are green [The teacher points to the words in the word problem displayed on the board.] Right? So out of those eight, there are four green. [Teacher writes 4 G on the board for four green.] Plus they were asking how many are there. She writes “4G + ? = 8.” It can’t be one plus eight plus two because it doesn’t match the problem. It should be some amount of green plus some amount of yellow is equal to eight. Ok? Let’s see if Student 4 can help you out with that.

The teacher invited other students to share. Another student reported that she used a known fact that $4 + 4 = 8$. One student used a number line and got 12 for the answer. Another student used the number line and got 11. The teacher continues to help students understand.

**Teacher.** Didn’t we talk about it up here? This was Student 2’s strategy? Right? And we said there must be 8 in all? There were four greens and how many yellows were there to equal 8 in all? Ok, so can we do eight plus four more?

**Student 5.** Yeah

**Teacher.** Does that match the problem?

**Student 5.** No response.

**Teacher.** Ok. So you are going to go one, two, three, four. Ok, so if you have four balloons, four of them are green but you have eight in all, you have one, two, three, four, more is going to get you to eight. Ok? Four greens counted to four
more and that is going to get you to eight. Who had something different? Who drew a picture? Who used their counting on strategies? Student 6, what did you do?

Another student said he drew four green and four yellow.

**Teacher.** So now what is the answer? How many are yellow?

Some students say four. Some say eight balloons in a bundle.

**Teacher.** So can the answer be eight if eight is already in our problem?

**Students.** No.

**Teacher.** Four of them are green, how many are yellow?

Students. Some shout out four some still shout out eight.

**Teacher.** Four of them are green, four of them are yellow is equal to eight of them in all. So the answer is four. (Teacher writes “4” on the board and circled it.)

Some students cheer.

**Teacher.** It is not about winning anything it is about understanding the problem.

All right, fix your problems now, and turn them in and let’s meet over at the meeting area.

**Researcher reflections.** In this episode, there were many factors to consider from a coaching standpoint. The teacher implemented some aspects of reform-based instruction. The teacher used a problem-based model of instruction in which a problem was presented to the students, the students solved the problem using their own strategy and then the teacher conducted a whole class discussion where the students shared how they solved the problem. The students explained their solution strategies, and the teacher recorded their solutions on the whiteboard. While the students were working, the teacher
observed the students and used questioning to support students as they solved the problem. Manipulatives were available to students and they were reminded of that. In the debriefing of the problem, the teacher called on students that had both correct and incorrect solutions and/or strategies to the problem. The teacher understood the structure of a problem based activity; however, instructional moves in the whole group debrief of the lesson did not advance student learning for many students. Some students solved the problem quickly with a known fact. Others still did not understand the nature of the problem and simply added the two numbers together in the problem. The teacher was surprised by some of the strategies and solutions and did not know how to address them productively. The teacher used questioning to try to get the students to uncover their errors and she modeled the problem with physical materials. Her efforts became more and more teacher led until she finally told the students the answer. In addition, students became disengaged when the teacher spent too much time trying to get one student to understand. This episode exemplifies the complexity of facilitating whole group discussions of students’ solution strategies. The following were some strategies I noted the teacher could use to more effectively facilitate whole group discussion. These included 1) knowing students’ levels of reasoning 2) selecting problems that all students can productively solve and from which all students can learn 3) monitoring and documenting students’ strategies while they work on problems 4) selecting which students’ strategies to share in advance to help eliminate surprises during the debrief 5) deciding whether or not to share a misconception whole group or to address it in another way 6) preparing for possible student misconceptions in advance and how to address...
them and 7) having students analyze the reasoning of other students, and 8) understanding children’s reasoning about addition and subtraction problem situation.

From this episode, I identified some teacher’s needs as 1) understanding learning trajectories 2) identifying students place in the learning trajectory 3) learning to facilitate more productive whole group discussions including knowing what to do when students are not understanding and 4) establishing classroom norms where students were analyzing their own reasoning and the reasoning of others. 5) Understanding the importance of student thinking over solely getting correct answers. This highlights the complexity of teaching mathematics for understanding as well as the complexity of teaching teachers.

In addition to identifying teacher needs, the balloon problem raised many questions about the teaching and learning that had taken place prior to the coach/researcher’s and the teacher’s work together. I was just beginning to learn about the teacher’s instructional strategies and the students’ understanding of addition type word problems so it was difficult to determine the exact causes of students’ misunderstanding. I wondered whether some of the students were able to make sense of part-unknown problems, which are difficult for students who are direct modeling (Carpenter et al., 1999). Were the students overgeneralizing a strategy to add when there are two numbers in the problem? Had the teacher relied on key words or phrases as she resorted to using in this lesson? Was the problem unclear? How many students were counting all and how many was counting on? If they were counting all, does this have implications for students having difficulty solving this problem? Was writing equations developmentally appropriate for the students or were they being exposed to a symbolic
representation before they were ready? Finally, did the teacher believe that the students all have to have the correct answer by the end of the class period? What was the teacher’s goal for this problem? Did the teacher believe that telling would result in understanding or that the goal was for the correct answer is given? How do we get students to move to more advanced strategies of counting and why do they seem to resist using more advanced strategies?

January 24th Conference

In the first conference the teacher invited the other grade level teachers to the meeting. This was a last minute request on the part of the teacher, and I agreed to meet with the entire grade level team. This impacted the nature of the conversations. The data discussed in this section is only that of the teacher in the study. However the following important topics were still discussed. 1) Student understanding of data representations and analysis. I suggested continuing to represent data in various ways and by connecting it to content in other subjects. 2) The teacher expressed a concern regarding the upcoming unit, not enough work with standard units and standard measuring tools, the inch and the ruler. I assured her that they need a lot of experience with nonstandard units before they are introduced to standard units. And that it is not in the first grade curriculum.

The conversation focused on student understanding in the data unit, which was nearing the end. The teacher shared a concern that students did not fully understand creating data representations and analyzing data. I suggested other opportunities where the students could collect data in future units so they could experience more applications that connect to other content or their everyday lives. For example when they collected information about who packed and who bought lunch. They could also connect it to
social studies or science lessons. Then the conversation moved to the next unit on linear measurement. The nature of this conversation included student challenges with length measurement and the content expectations for students in first grade. At the time of the study, schools were in the process of transitioning to a new set of standards, The Common Core State Standard for Mathematics (National Governors Association Center for Best Practices, 2010), and this posed a challenge for teachers in determining important content to teach. The district report cards were still aligned to the former state standards but the expectation was to teach the newly adopted Common Core Standards. I found out that the teacher was teaching both. In addition I learned that teachers did were not familiar with the some learning progressions, what is developmentally appropriate for students for some content. In the upcoming unit on measurement the teacher was very concerned that the curriculum units did not include standard units of measure and standard measuring tools such as a ruler. This concerned me because when students are rushed into content that is at a level too high for them, students tend to memorize procedures without understanding them. The teacher stated, “I don't like that we spend so much time on nonstandard units of measure, and they don't get much time with standard units of measure, like this [referring to the curriculum series] doesn't even talk about pulling out a ruler.” I reassured the teacher that students need a lot of experience with nonstandard units before it is appropriate to expose them to standard units and a ruler. these experiences are necessary to build understanding. The Common Core Standards focused on understanding the process of iterating units.

Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of
same-size length units that span it with no gaps or overlaps (National Governors Association Center for Best Practices, 2010).

This was consistent with the research about children’s developmental stages of length measurement (Battista, 2012a). The teacher says that the lessons in the curriculum use unit tiles to measure objects, which are an inch, a standard unit of measure, “so that is fine.” In this episode a number of topics were addressed including student understanding of data collection and organization, teaching length, developmental levels of student thinking regarding length, the curriculum and the state standards. The intended topic of conversation was to determine student strengths and struggles with the current unit and the anticipated challenges coming up in the next unit, accountable talk, and specific misconceptions about students; however, the conversation included some other topics and naturally flowed in different directions than anticipated.

The nature of pre and post conferences with teachers is that topics are co-constructed. A coach may have a goal or an agenda, but the teacher may also have concerns or questions; therefore; decisions need to be made in the moment about the focus of the conversation. I needed to address the teacher’s concerns first and then if time, and it seemed appropriate, I included my agenda items. Also, because this conference included other teachers, I did not have the opportunity to address specific pedagogical strategies used by the teacher. This conversation needed to be between the teacher and myself.

January 29th Conference

The next conference centered mostly on pedagogical assistance and relationship building but also included pedagogical content knowledge, pedagogy for student
understanding and grounding in theory. This episode highlights some of the coaching moves I made to support the teacher’s understanding.

My lens in coaching was theoretical; therefore, I used research-based resources when coaching teachers to support the claims I made. My goals for this conference, based on the last coaching session, were 1) teaching and learning of money, 2) teaching and learning of addition concepts 3) student to student interaction and 4) to discuss the next lesson. To prepare for first two goals, I researched the teaching and learning of money concepts and the addition problem types (Van De Walle, J., Bay-Williams, J. & Karp, K., 2009; Carpenter, T. et. al., 1999; Pearson, 2008). During this conference, I shared information I found regarding students’ difficulties when learning money concepts. One challenge for students is that counting coins is often the first time they experience correspondences other than one-to-one, for example, one nickel represents a five-to-one correspondence. I also shared developmentally appropriate content for money at first grade; this included identifying coins and making combinations of coins within a range of numbers in which they can easily understand. I also discussed the distinction of when to tell and not tell depending on whether an idea is convention or is built on other mathematical ideas and fits into a mathematical structural system. For example, the value of a coin is convention and students need to be told that. However, counting a collection of coins involves conceptual understanding and requires understanding other concepts. I validated that the only way to teach the value of coins is to directly tell students and have them practice identifying coins and their values. I built on this by suggesting that the teacher pose problems where the students have to show an amount in more than one way using different coins such as twenty-five pennies, two dimes and a nickel or a quarter.
The teacher responded that she had them show 10 cents in two different ways. Some got it, but for some it was “over their heads.” I thought that was interesting that she noticed students struggled with making ten cents, yet she see the connection between that challenges that students would have finding the value of three dimes and one penny.

She also mentioned that she and her other first grade team members wanted students to skip-count by 25s up to a dollar. I was concerned that the numbers were too high for the students and this would lead to memorization without understanding.

… it is about students being able to really make sense … versus when we were in school. So my concern is that they are going to memorize 25, 50, 75, $1. But not understand how we are getting it because they are not adept at using those size numbers.

The teacher seemed to grapple with the idea of memorization versus understanding.

Right, I see what you are saying. And I looked at it like they need to memorize those. That is one of those things that they don't need to dive deeper in you know what I mean. But when they get to the 2nd grade or 3rd grade maybe that is why they shut down because maybe they have memorized it.

It seemed that the teacher was considering the idea that memorization could support future understanding. However, later conversations and classroom observations showed that the teacher continued to have students practice this up to the end of the school year.

The teacher changed the direction of the conversation away from memorizing versus student understanding. She was interested in a book about 10 frames that I had with me at our meeting. She said she used them before and wanted to use them again. We discussed how she used them, which was mostly subitizing. I discussed that the ten-frame
is a tool to develop concepts and suggested using it to build number relationships to five and ten (for example, 7 is 5 and 2 more) and to develop the concept of teen numbers. I connected this to a workshop on number sense concepts we attended. I reminded her that these concepts would help students with place value and breaking numbers apart which would support students advancement of their stages of development with number combinations (Baroody, 2006; Carpenter et al., 1999). The teacher suggested having students write equations for the ten-frames. The teacher said she would start using them the following day. I was concerned about the teacher having students write equations because equality is a challenging topic for students to learn and that rushing students into symbolic representations too quickly can interfere with student understanding.

The conversation then turned to measurement, which they were starting the next day as well. The teacher said they were going to start with nonstandard units, paper clips. I liked that idea because they are more linear than cubes or square tiles, which are commonly used to measure length in early grades (and which is a huge mistake). I was concerned that tiles are also often used to introduce area and wondered if using tiles or cubes may cause misconceptions about linear measurement. The teacher does not agree with my concern, and preferred them to paper clips because of the rounded ends.

**Teacher.** We use the tiles for a lot of things.

**Coach/Researcher.** I know

**Teacher.** So I don't know if it would effect the 3rd graders because they use it in so many different concepts.

**Coach/Researcher.** Yeah

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Teacher. I don't know. It is tricky.

Coach/Researcher. Square inch is tough for them to grasp but um it takes a while so ok, so that is what we will be doing tomorrow.

Teacher. hmm hmm. Right.

Coach/Researcher. It might be a lot easier for them to grab than the straws or the paper clips but as a rule I kind of like a paper clip better because its

Teacher. Right, I think they use the paper clips, but the paper clips are difficult because of the rounded end. Sometimes they do not know where to stop, but with the cubes and with the tiles they have that straight line, that straight edge.

I could tell from the teacher response that she did not believe this would have an impact on student understanding of length and have implications on learning future measurement concepts. I wondered if this could be due to lack of content knowledge and experience with teaching and learning of area and volume concepts. For this reason, I did not press the teacher.

January 30th, Classroom Coaching

The following day, the teacher began the math lesson with a Number Talk as we discussed, which entailed subitizing and making combinations of five using 5 frames. She showed a five frame with two dots and asked them how many they saw, and then “How can we get to five?”

![Five Frame with Two Dots]
The teacher showed the students another five frame with one dot. She followed the same line of questioning. A student responded that the last frame has two dots and there were three missing and this only had one dot so there must be four missing. The teacher commented on the connection the student made. Again she asked for another way to know there were 4 dots. The student showed good reasoning, communication and number sense skills.

In the main part of the lesson, the teacher taught an introductory lesson on linear measurement. She had the students sit in a circle on the floor and used a large plastic storage tub filled with linking cubes to introduce the lesson. She made a train of nine linking cubes and asked the students what would be the best way to use the cubes to measure the length of the tub. A student answered ten. The teacher asked again, “But how can we use the cubes?” She invited a student to come up and show. The student responded to count the cubes inside the tub. The teacher said that they are not going to learn that today. She thanked the student and asked him to go back to his seat.

The teacher explained and demonstrated how to measure the length of the tub with cubes. “When you are measuring, you go from corner to corner [pointing to each corner] to find out how long it is using cubes. Okay? So, so far, I have [counting each cube] 1, 2, 3, 4, 5, 6, 7, 8, 9. Do I need more or less?” The students called out, “More!” The teacher continued to add cubes and they counted as a whole class with the teacher leading until they had enough to span the length of the tub. She explained that “It is a little tricky when you are measuring.” because sometimes we go a little over and sometimes there is a little less. Next, the teacher used a notebook to demonstrate how to measure length. A student shouted out, “I want to measure a person with cubes.” The teacher responded, ”What
would be wrong with measuring a person with cubes?” The teacher did not allow students
to determine if this was a reasonable strategy, but rather told the students that the method
was wrong. A better question would have been to ask students if they that would be a
good way to measure the height of a person. A student responded that the person might
be too tall. The teacher agreed that there might not be enough cubes to measure a person.
She then modeled how to measure a box of markers.

Next the students measured objects at their desk and recorded their answers in
their math book, and she followed this with a class discussion. They discussed why
students had different answers for the length of a pencil. The pencil was between nine
and ten cubes long and the teacher wanted to make the point that some students used
cubes and other used ten cubes. She then modeled how to measure the pencil.

**Researcher reflections.**

The teacher implemented the suggestions I provided regarding the Number Talk
with 5 frames. The teacher also asked the students “why” and asked for other ways to
determine the answer. I saw progress in questioning, and was happy she was trying out
my suggestions. Number Talks provided a way to get students explaining their thinking,
justifying their answers and analyzing the thinking of others. The activity was appropriate
for the students level and they were successful.

In the measurement part of the lesson, she asked the students to come up with a
strategy to measure length with cubes. However when the students could not, the teacher
modeled. She continued to show how to measure length while the students passively
watched. After the students had an opportunity to practice measuring objects, she
modeled a student’s strategy, instead of letting the student show how she did it. This is an
area I knew I needed to support the teacher. However, there were more lessons before we
had an opportunity. I wanted to recommend assessing student’s knowledge of measuring.
Also, I wondered whether the tub was the best object to use as a model to measure since
this was their first classroom experience with linear measurement. The sides were not
perpendicular so the top length had a different measurement than the bottom length. I
wondered about the teacher’s decision regarding the object she chose measure for her
first example. The box had many attributes that could be measured and the students
seemed confused.

**February 4th, Conference**

Several days following this lesson the teacher and I met. One of my goals for the
conference was to discuss providing more opportunities for students to explain their
reasoning and for the teacher to do less modeling. I noticed the teacher doing most of the
modeling and explaining. I asked about the lesson she was going to teach that day. There
was no discussion about the previous lessons; however, I addressed some concerns of the
previous lesson as we discussed the upcoming lesson. The teacher said that she was going
to introduce the vocabulary “inch” to the students. The teacher did not think the students
had any experience with this term or with a ruler. I suggested that the teacher ask the
students what they know about a ruler, what it is used for and if they have ever used one.
After we discussed the upcoming lesson, I found an opportunity to bring student
explanation and thinking into the conversation. “When you said that you were going to
do some examples whole group is there a way student could show, because the research
shows that [it] is really powerful when the student is doing it as opposed to a teacher.” I
supported this with reasons including developing independent thinking, student
engagement, and that it allows the teacher to assess the students understanding. I also revisited when to tell and when not to tell. The teacher struggled with this. “Right, it is hard.” I acknowledged challenges of this way of teaching, specifically the challenges of implementation. It is difficult to know when to step in and how to advance students’ thinking without taking it over. But the teacher seemed more concerned about the time element rather than the implementation. “Yeah, especially when you have the time factor too, sometimes you are against the wire as far as having time to do the mini lesson and do the independent work and have discussion. Sometimes you have to wrap it up.” She added that one of her students was “very long-winded.” I acknowledged the teacher’s struggles, and did not address it any more during this conference.

I asked if the teacher had any other concerns. The teacher was concerned about student engagement during whole group instruction. The teacher thought that using a display tool would help. I agreed, and believed it would also empower students to share their ideas, provide more opportunities to analyze the ideas of others and to determine correctness of each other’s mathematical ideas. This tool could positively impact aspects of the culture in her classroom. I arranged for a document camera for her to use in her classroom.

**Researcher reflections.** One challenge for me was to know when to keep pushing and when to hold off until another time. I believed that allowing students to explain their thinking was something we could address over time, and I was confident that the next lesson would be more focused on trying to get more students involved. I did not want to frustrate and alienate the teacher and I had to try to sense how the teacher was feeling to determine whether or not to keep discussing an idea. I felt that if the teacher tried to get
the students talking and explaining more, it would provide a better platform for future conversations about student discourse. I noticed that as the teacher did more explaining, the students became less engaged. While I thought that getting the students to explain more was a good starting point for now, I suggested the document camera because I thought it could be tool which might shift some authority to the students. At that time there was no tool to show students actual work. While the suggestion of a document tool seemed superficial, I thought a concrete suggestion might lead to change in teacher behavior.

**January 31st, Classroom Coaching**

The following day, the teacher started the lesson counting by tens. She used the hundreds chart as a reference with a transparent blue square on each multiple of ten. The teacher asked the students what they learned the day before about those numbers. A student commented that instead of counting by ones we can count by tens, or nines or eights or ones. [The student meant that you can count 1, 11, 21, 31, 41, 51… but referred to that as counting by ones]. At first the teacher thought the student meant that you can count to one-hundred by ones or that you can count to 100 by tens, and that was a shortcut. The teacher demonstrated what she interpreted the student was saying and counts 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 while pointing to each number and continued the next row, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, pointing to the numbers 11 through 20. A student said again that you can count by nines or eights or ones. She responded, trying to capture the students’ thinking, you mean you can go 1, 11, 21, 31 or 2, 12, 22? The student nodded his head yes. She asked, “If you are doing that, what are you counting by?” The student says two’s. She asks, “If I am at 2 and I go up 2 more, the answer is 12?” The student

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shook his head no. She said, “Ok, we’ll go with that.” Then she told the students, while pointing to ten, that the number right below is ten more and then demonstrates by counting by tens. Let’s all practice that. “What my friend is saying is that you are also counting by tens over here. [pointing to the column starting with one]. “In a column the number that is right below it is ten more.” The teacher then showed the students by counting from one number to the number directly below it. She explained that this works for any number on the hundreds chart. A student then suggests that they count by tens with their eyes closed. “Let’s do it.”

The next segment of the lesson involved practicing counting coins.

Despite our conversation, she continued to have students count coins. There was no significant pedagogical change in this episode. The teacher placed four dimes on the board and used a think pair share strategy to determine the total amount of the coins. She has them whisper the answer to their partner. This is a good pedagogical strategy but it is focused on the answer, not how the student solved the problem. The students were saying a variety of amounts such as one dollar, ten [cents], twenty [cents] and forty [cents]. After assessing the students, the teacher said, “Let’s review.” She used a step-by-step line of questions to walk them to the answer. She then asked the students the value of a dime and explained that she was not going to count it one, two, three, four. She then had them count along with her 10, 20, 30, 40. Then the teacher said, “And it is forty cents.”

The next part of the lesson used the 5 frames again. This time the teacher asked the students how many they saw and how many to make five. She consulted with me in the middle of the lesson to asked how to phrase it, “How many to five?” and “How many to show five?” In this Number Talk, the teacher added in writing equations. The
teacher used a five frame that showed zero dots. This time the teacher asked them to write an equation to represent the five frame. She used leading questions to help them write the equation, \(0 + 5 = 5\). The next five frame showed had five dots. Again she used leading questions to write the equation, \(5 + 0 = 5\). The teacher chose numbers on the 5 frames were connected to each other. One of the students noticed this connection between these 5 frames. “My friend … noticed that all we did was switch it.” The teacher connected the five frame relationships of combinations of 5 and wrote the equation for each as the students gave them. Another student said that \(4 + 1 = 5\), and the teacher acknowledged that and wrote it on the board. This was an equation to make five but it did not represent the five frame five dots. The teacher started to write another equation, \(3 + \ldots\) On the board and asked the students to finish it. She was trying to show a pattern that the first addend decreased by one and the second addend increased by one.

The final part of the lesson focused on linear measurement. The difference in this lesson was that some of the objects had fractional measurements and they had to tell that the object was a little more, a little less, exactly or between two measurements. Students struggled with this language. The teacher told the students they were going to measure objects and that they could use either the inch tiles, the connecting cube or paper clips. She asked them if they thought that one tool would be more difficult to use. The students offered some reasons why they thought the paper clips might be more difficult. Then the teacher models how to use the tiles to measure the length of a book. The teacher continued to model even though students had experiences in previous lessons with using tools to measure object lengths. She also used the other two tools, and the students noticed that a different amount of tiles than paper clips were needed to measure the book.
They had a discussion about what to do when the number of tiles or paper clips was not exactly the length of the book. The students had trouble communicating whether the book was a little more or a little less than the number of tiles or paper clips so the teacher told them. The students were then given time to measure some objects. The teacher brought them back together for a whole group debrief. She selected two students to share how they measured a pair of scissors. The teacher asked the students why they measured scissors they way they did. The students had difficulty articulating their reasoning, so the teacher asked them questions regarding why their strategy worked. The teacher then showed an incorrect way and had students comment on why that way was not correct. I asked another student to share a different strategy that I found interesting. The student created a sort of measuring tool by linking cubes together. He then added or subtracted cubes to that tool in order to make it the same length as the object he was measuring. The student demonstrated that strategy, but the teacher explained it.

**Researcher reflections.** During this class, there was teacher telling and instances of memorization instead of developing understanding. The discussion regarding counting by tens on the hundreds chart did not show evidence that the students understood skip-counting. Skip counting was only taught as a memorized procedure so I was not surprised that the students were not able to answer her question. In addition, the student who thought that counting by tens starting with other numbers such as one or two or nine or ten had an interesting idea that could have been explored further. The student noticed a pattern, but did not understand the connection between skip-counting by tens and adding tens. I noticed a pattern of teacher telling or asking leading questions when the students did not produce the correct answer. I wondered if she believed that mathematics should
be figured out quickly. With the exception of Number Talks, the teacher was doing most of the thinking and explaining. With Number Talks; however, the students were able to answer quickly and correctly, while with the other activities the students had a more difficult time explaining and/or they had incorrect solutions. I also noticed that on some activities, the students seemed to randomly guess, and their answers were not reasonable, like in the case with the four dimes discussed in this lesson above. To me this signifies that the problem was not developmentally appropriate for the students, and the teacher did not recognize this. These were important aspects I wanted to discuss with the teacher.

**February 4th, Classroom Coaching**

This lesson began with finding the value coins of seven nickels. The teacher had the students sing a song about coins. This was still a memorized procedure. The next lesson started out with identifying coins and counting. They first sang a song about the different coins, their characteristics and their value. The teacher placed seven nickels on the board. She asked the students to whisper to a partner the name of the coins that she placed on the board, and she noticed that some students think it is a quarter and some think it is a nickel. She placed each type of coin on the board and had a class discussion to determine if the seven coins were nickels or dimes. The teacher used questioning to lead the discussion and had the students explain how they knew which type of coin it was. Then the teacher asked a student what they were going to count by and a student responded correctly. They then counted the coins together to arrive at the answer.

The teacher had a class discussion about measuring tools. She introduced a ruler and she made a connection between the inches on a ruler and the inch tiles. However, she did not assess students’ knowledge as we had discussed. Then the teacher introduced the
task from the district adopted curriculum unit (Russell & Economopoulos, 2004a). The teacher relied on the teacher manual to conduct the lesson, referring to it several times throughout the lesson. The teacher showed pictures of fish on the document camera, and they needed to determine if they should keep the fish or throw them back based on whether they were 6 inches or longer. The pictures showed a fish with inch tiles superimposed, as shown in figure 19, and the students had to determine if the fish was measured correctly and why.

Figure 19. Examples of fish measured with inch tiles

The teacher gave the students time to talk to a partner about their thinking. Then she asked the class who thought the fish was measured correctly and who did not by a show of hands. She moved on to another example where the fish was measured a different way. She asked students again if they thought the strategy used was a good
strategy. She had the students discuss it with a partner, and this time she had the students explain why or why not. After the students had time to discuss, the teacher showed both pictures of the fish. She asked students what was wrong with the first one. “Most of you agreed that that is not a good strategy. Plus, like [student] said, they are the same fish, should we get two different answers?” Students. “no!” Teacher. “No. If you are using the right measuring technique, you all have the same answer.” The teacher called on students to explain why the fish were not measured correctly, and then revoiced their explanation. The students’ reasoning was correct. She did not address students who thought the fish were measured correctly or ask them why. Then the teacher showed a third example. The students discussed in pairs whether the fish was measured correctly, and then they had a discussion of whether or not the strategy shown was correct, with the teacher allowing the students to determine correctness. The students explained that there were gaps between the tiles. Finally, the teacher asked a student to show how to measure the fish correctly. Fish pictures were displayed using the document camera and a student was chosen to measure the fish with the inch tiles. She explained that the student got to be the teacher. The student placed the inch tiles on the fish in a straight line from end to end without gaps. The teacher asked students to show a thumbs-up if they agreed with the student’s strategy, and then called on a student to explain why he agreed. The teacher restated the students reasoning. The teacher then explained to the students that they were going to measure fish on their own and record their answers.

We observed students as they measured their fish, looking for strategies and misconceptions. I worked on getting students to collaborate by measuring the same fish and comparing their answers. I noticed the teacher had students analyze the solutions of
other students, asking a student, “Do you agree that it is eight?” She also asked a student what he/she thought about the other student’s strategy for measuring. The students continued measuring their fish until the class period was over.

**Researcher reflections.** This lesson was important to highlight regarding the influences I was making on the teacher and those in which I was not. The teacher began incorporating more elements of a classroom culture of inquiry, but was not making much progress with developing her pedagogical content knowledge.

The following aspect of the lesson showed the teacher’s lack of understanding of pedagogical content knowledge, specifically knowledge of content and curriculum and knowledge of content and students. In this lesson the teacher taught concepts that were not appropriate for her students and were not in the first grade standards. These topics included counting a collection of coins, using a standard tool and a standard unit for measuring, specifically the ruler and the inch. Counting coins was not appropriate because the students had not yet developed understanding of skip-counting and place value concepts necessary to build understanding of money concepts. A future curriculum unit begins to develop understanding of skip-counting (Russell & Economopoulos, 2004c), but up to this point skip-counting had only been approached through rote memorization. Additionally, place value concepts also had not been developed. The Common Core State Standards for Mathematics does not include skip-counting until second grade (National Governors Association Center for Best Practices, 2010, p. 19).

Linear measurement was another topic in which the teacher was struggling with PCK. In a previous conference, I suggested finding out about students’ informal
knowledge regarding rulers and inches, however instead she used a teacher-centered approach to introduce these tools, telling them about the relationship between the ruler and the inch tiles. This was not an appropriate pedagogical strategy because the students were in the early development of learning about nonstandard units of measure and iterating units, which precedes the understanding of standard units of measure.

Prior to this lesson the teacher and I had discussions about grade level expectations, and the developmental appropriateness of the concepts of money and standard units of measure. However, her decision to continue to teach these concepts seemed to indicate a strong belief about their importance in the curriculum and in student learning. Therefore, our conversations had not yet influenced her pedagogical content knowledge or beliefs regarding these topics

The teacher did show a variety of changes in supporting a classroom culture of inquiry. In this lesson, as the students were determining if the fish were measured correctly, she gave up more authority than in past lessons. The teacher provided more opportunities for students to explain their reasoning about measurement in this lesson. She had students justifying their reasoning and analyzing the reasoning of others. She had a student model the correct way to measure the fish, instead of modeling herself. While the students were working on measuring their fish individually and in small groups, she invited students to participate in a community of mathematical inquiry by asking the students to analyze each other’s thinking instead of telling or questioning individual students. This was a step forward for the teacher in terms of implementing a classroom culture of inquiry. In small group work, she reported that she noticed students questioning each others’ thinking. This was a result of the teacher inviting them to
participate in these types of interactions during whole class discussions and small group work.

**February 5th, Conference**

The goal of this lesson was to 1) discuss the previous lesson and 2) to make sure I reinforced the effective practices the teacher was using. I started our conversation by commenting on her patience with students. This was one strength of the teacher; she was very calm in regards to student behavior. The teacher asked about my experiences in other classes. “Do you think [the students] are gaining a lot? I mean you work in first grade and then you jump to 3rd and 4th grade where it is more independent and higher level thinking.” I responded that first grade was higher level thinking too. I told her that I noticed students’ explanations and justifications were improving. I pointed out that many of the students in the district came with a language deficit, but that with the efforts of the teachers, oral language and students’ abilities to justify their answers and describe ideas was improving and that we just had to “keep at it.” The teacher commented, “It is a matter of knowing you have to do that.” She meant that teachers had to be explicit about teaching students how to communicate their thinking and justify their answers. However, I said that our students struggle with problem solving and that needs to be a focus in all grade levels. I told her I felt like I had neglected the primary grade teachers in the building because I was told to focus more on grades 3 – 5 because of the required state tests in mathematics in these grade levels. I added that grades K-2 are very important for setting the foundation for the upper grades, and I was glad to finally be working in these grade levels. The teacher responded that she was happy to finally have a math coach to
help them understand the curriculum materials the district used. She felt that she was a better math teacher because she had a better understanding.

The conversation turned to the previous lesson. I asked the teacher what she thought about the whole group discussion about measuring fish. The teacher commented that she thought it went pretty well, but that she thought it went on too long and the students became disengaged. However, she felt that the students grasped the important ideas of measuring,

…knowing to line it up, knowing to go from edge to edge. And as I was walking around and mainly seeing what they are doing I liked how the partner groups were able to say ‘No that is not the right way to measure that’ or ‘You didn't go to the fishes nose. You stopped at the eyeballs.’ or something like that so I did see that.”

I reinforced the importance of having students analyze other students’ strategies. The conversation then turned to student understanding we observed while the students were working. One pair of students disagreed about how to measure the fish. I encouraged the students to talk about why they had different answers. The student who measured correctly was not able to convince the student she needed to start at the nose of the fish and measure all the way to the end of the tail. The teacher noticed that almost all students correctly measured the fish, but some students were having difficulty determining if the fish was more or less than 6 inches, and others were struggling to accurately count the tiles. She noticed that those students lacked one to one correspondence. While the students seem to understand how to measure objects, some students had difficulties with number
concepts. We discussed some ways to support students with these concepts and what may be the underlying misconceptions that may be interfering with students understanding of comparing lengths of objects to a given length. I suspected that students may be using the wrong referent and wondered if students were able to compare quantities, such as 6 and 7. The teacher thought she should assess students’ understanding of comparing numbers. She also commented that she needed to be careful about the language she used with students. That saying a number is bigger is incorrect and that she needs to say that the measurement is a bigger amount.

In the last part of the conference, I commented on some things that the teacher was doing well, including asking the students to show their thinking and documenting the strategies they used to solve addition problems to encourage her to keep using those strategies. I also used this opportunity to start a conversation about students’ strategies for addition problems, how the teacher could recognize the strategies students used and how to support students to use more advanced strategies.

**Researcher reflections.**

This was a productive conference. We spent considerable time discussing student thinking and misconceptions. I saw that the teacher was beginning to attend to student thinking. An area of concern I saw across the building was that a large number of students seemed to use low level strategies to solve addition problems, specifically counting or direct modeling. I had studied Cognitively Guided Instruction (Carpenter et al., 1999) and the stages of development of number combinations. direct modeling, counting, derived facts and eventually known facts, as well as Baroody’s stages which
include counting, reasoning strategies and known facts. The literature states that the conventional method of students memorizing facts is less efficient, more cognitively demanding and takes more time (Baroody, 2006). In addition, research suggests that students from low-income families have less informal knowledge, and students with less informal knowledge tend to use less advanced strategies (Baroody et al., 2009). At the time of the study, I was not familiar with this research but I had noticed this trend in the students, and it was an area I knew needed support. I hoped to work with the teacher on increasing students’ number sense. I recommended a resource with activities that could support students learning of these concepts. I recommended conducting Number Talks (Parrish, 2010) using 10 frames to help students develop flexibility with numbers and to support addition of single digit numbers.

So as much as you can do Number Talks, and I know you have done some 10 frames and some 5 frames but also this year you were going to get them into addition. So if you have a ten frame with five dots and then another ten frame with seven dots, and how many is that?

I explained that this could help students think about 5 and 7 as 5 and 5 and 2 more. Students know 5 and 5, and this can be used as a basis for finding many other number combinations and lessen the cognitive demand of memorizing a lot of isolated facts.

**February 5th, Classroom Coaching**

The teacher started out with a data activity from the curriculum materials called a Quick Survey. The goal of this part of the lesson was “to collect and look at data.” The emphasis on this was analyzing data as opposed to representing data. After the data was collected the teacher recorded it a two-column chart and asked the students what they noticed about the data. She asked questions that required students to communicate and
reason about the data such as. “What kind of conclusion can you come up with?” She asked the students for an equation to represent the total number of people who took the survey. The teacher also asked some leading questions because the students were struggling with the equation. “What do you notice about the number of people that are in this room right now?” She also had them create an equation, which was still a struggle for the students. Again she led them find the equation.

In the next part of the lesson, the students were given separate result unknown word problems to solve. The students solved a word problems of the type separate result unknown. The teacher read each problem to the students and asked the students to close their eyes and paint a picture in their minds. She asked students decide if the answer would be more or less apples than they started with. The teacher asked the students to justify why. The students had a difficult time explaining why. The student restated the problem and the teacher accepted that and explained that if she ate some apples there would be less. The teacher validated that the result will be less in each problem. The students are then sent to their seats to work on the problem. They are told they can use counters if they need them.

While the students worked, we talked to students about their solution strategies. I noticed one student had an equation with the correct answer, 19-8=11. Then he drew a picture of 19 circles and crossed out 8. I found it interesting that the student had the equation before the drawing so I asked the student how he got the equation. The student had used the hundreds chart and counted backwards, but used a different strategy to support his solution. I also noticed that the first problem could be represented as 18-7 and the second problem is 19-8, both having the same solution. I asked the student why the
problems might have the same solution. The student did not know so I had the student model how to use the hundreds chart to find the solution to see if that would help him. He was not able to see the connection and I did not press. I did not know whether the teacher made this connection either. The teacher saw a student that had 7-18. She told the student that you cannot take 18 away from 7 and that it should be written as “18 take away 7.” Then she asked the student to show her how he got his answer, to tell her with words. I worked with a few more students asking how they solved the problems to try to determine students’ strategies. One student used counters and knew to start with 19 counters, but has difficulty counting accurately. He did not know to add one more counter since he used 18 in the last problem. The student then started to count out eight counters using a different color instead of taking away counters. The student did not have time to finish the problem so it is unclear if the student found a strategy. However, it is clear that the student was using direct modeling to solve the problem, and that the numbers in the problem may have been too difficult since he could not accurately count them. After the students had time to work on solutions to the problem the teacher reconvened the class to discuss them. I offered to set up the document camera so the students could show their work.

The teacher conducted a group discussion of the apple problems. She chose one student to explain his solution and display his work using the document camera. She explained to the class that she chose this student’s work because of “how his math journal looks.” She asked the student to explain his work and directed him to the top of his page, which has a drawing. The student drew 18 circles and crossed out 7. He also wrote an equation of 18-7=11. He also showed his solution using a number line with 7 jumps from
18 and ending at 11. The student explained his solution and the teacher voiced the different ways the student solved the problem. After the student explained that he hopped backwards on the number line, the teacher asked why he hopped backwards instead of forward. The student had difficulty articulating why he counted backwards and responded that he would be counting 18, 19, 20. The teacher then asked if that would match the problem. The student said no. “So if he went backwards, what does that mean?” Another student replies, “She ate them up!” The teacher then asked why he used two strategies and the student said that it would help him count. Then the teacher asked the class if this was a way to prove your answer was correct. Some students responded, some yes and some no. The teacher asked a student that said yes why it was a way to prove the answer. The student responded (inaudible) and the teacher encouraged students to use more than one strategy because if you get a different answer for the problem, then you know you need to go back and check the first solution. Then the teacher told the students that she was talking to a fourth grade teacher and they have a “big test” [referring to the state mandated mathematics assessment] coming up and they needed to be able to prove their answers.

In the final part of the lesson, the teacher focused on a measurement compare situation. She asked how the class could find out how much taller one student was than another. The students were not able to come up with a strategy that she thought was acceptable so she modeled by having two students stand back to back and showed how to use cubes to find the difference of the heights of the students. The teacher then modeled how to find the difference in the length of pictures of two fish.
Researcher reflections. During this lesson, I noticed that the teacher began adopting some of the strategies we had talked about. The students used the strategies that made sense to them, and there were a variety of strategies among the students. She had the students share their solution strategies. Many of them were at the direct modeling stage. Some were writing equations, which I was not sure they understood. The teacher incorporated writing equations into as many lessons as she could, and I was unsure if the students had an understanding of equality. She still continued to model, especially when she believed the students could not find a strategy or did not find one quickly enough.

February 6th, Classroom Coaching

I made many coaching moves in this episode to support student understanding during this lesson, including making connections to help students understand the meaning of equations, and introducing a strategy to get students more involved in whole group discussions. The class started with a Number Talk with the same goals of finding how many to five and writing the equation to represent the five frame. The first five frame she showed had zero dots. The students were given this same problem in a previous lesson [January 31st], when two students were able to state correct equations for the five frame. However, on this day, lack of understanding and misconceptions were revealed.

Figure 20. Blank five frame.
As a result, the teacher used leading questions and telling to get a correct equation.

**Teacher.** “How many squares are there?”

**Student.** “Five.”

**Teacher.** “So whatever our equation is going to be it will be something plus something is equal to five.”

**Student 2.** $0 + 5 = 5$.

**Teacher.** “Ok, what is the other one?”

The student provided a correct answer of $5 + 0 = 5$ so the teacher moved on.

I was unsure if the students understood the connections between the five frame and equation so I asked, “What do the 0 and the 5 mean?” I was trying to make sure that there was meaning to the equation and not just rote memorization; therefore, I modeled questions that probed students’ understanding of the connection between the representations. One student knew that the zero meant there were zero dots, and that there were five squares in all, but they could not explain that there needed to be five dots to get to five. The students had difficulty explaining the meaning of the numbers in the equation, and the problem with zero dots seemed to be particularly difficult for students. The teacher also showed a 5 frame with five dots.

![Figure 21. Five frame with five dots](image-url)
One student replied, “4 and 1 more is equal to five.” The teacher accepted that solution. The students struggled to write the equation, $5 + 0 = 5$. I suggested that the teacher show a five frame with four dots; this might make more sense to the students. This time some students were able to make sense of the equation. However, one student responded $3 + 1 = 4$ but the intent is to write a part-part-whole equation ($4 + 1 = 5$). The teacher asked me if that solution was “ok” because they had been working on the concept of one more. I said that it depended on the goal. If the learning goal is to work on part-part-whole then that would not be a correct answer. I suggested that they use 5 frames with two-color dots when they are working on part-part whole relationships. The teacher agreed and said they will do that next week.

Another move I made during this Number Talk was to encourage students to listen to and respond to each other. During the Number Talk I asked the teacher if she had ever used the “me too” hand signal. The teacher responded that she had not. After a student provides a solution or an idea, the students use this silent hand signal to show that he or she agrees. This gesture encourages students to listen to each other and analyze the thinking of others. I demonstrated for the class and had them use it on the next student explanation during the Number Talk.

In the next part of the lesson, the teacher had the students solve a compare word problem difference unknown using the fish context. The teacher asked a student to restate the problem in his own words. The student was able to focus on the important aspects of the problem, which was to find “how many more inches was the ten inch long fish than the seven inch long fish.” The teacher commented on how she liked how the student said “how many more inches” to represent how much longer. She asked another student to
restate the problem and said, “I like how you are thinking”. The teacher asked another student to restate the problem and then told the students to use their inch tiles to figure out this problem. The teacher did not show them how to solve the problem. I worked with students at one table. One student was able to solve the problem mentally. The other students were using the tiles but were struggling to understand the concept of how much longer. I had the student that was able to solve the problem, explain his thinking to the other students and then asked the students to connect his strategy to the tiles. One student constructed a line of seven tiles and a line of ten tiles. The teacher then asked the class to have one person at each table explain to the other students at the table how they solved the problem. The teacher chose the students who would share with their peers. At one table a student made two rows of tiles, one with 7 and one with 10. The student lined the two rows next to each other. The student was able to say which row was longer but was unable to determine the difference in length between the two. I asked the table group if anyone could tell how much longer the fish was, and I connected it to finding how much taller one person was than the other. But the students were still only able to tell which was taller and which was shorter. The teacher brought the class together and asked how they figured out how much longer the fish was. One student says that there are 17 in all. Most of the students are not able solve the problem so the teacher showed how to solve it. The teacher then assigned another problem of the same type with different numbers for the students to work on. She asked them to build the length of the fish with the tiles. Students were still struggling even though the teacher modeled how to solve the last problem. I wondered about the students’ experiences with compare-difference unknown problems. I made up a problem about crackers. I talked to one pair of students who were
struggling with finding the difference in lengths. I have 3 crackers and you have 4 crackers, how many more do you have? The student was able to correctly answer the question. I connected the how much longer problem to the cracker problem. The teacher was working with another group of students. She asked repeatedly how much longer was the fish. The teacher brought the students back together in a whole group. She asked the students how to solve the problem. “Does the problem, tell us to add it together? Does the problem, tell us to add it together, yes or no?” She asked for a raise of hands, who thought we should add the numbers together? Several students raise their hands. The teacher asked why they think they should add them together and the students could not give a reason. She then called on students to explain how they solved the problem, but this time she called on students who solved the problem correctly. She had a diagram on the board with eight tiles and eleven tiles and drew a line to show there were three more. She also matched up the tiles that were the same in each row and circled the extra tiles in the longer “fish.” She tried very hard to help them understand what “how much longer means,” but some of the students were not ready for this type of problem.

**Researcher reflections.** This episode showed the teacher used a number of aspects of research-based pedagogy. She continued to have students explain their thinking. She used worthwhile problems, which provided opportunities for problem solving, reasoning and communication.

The Number Talk provided many opportunities for coaching moves to support the teacher’s pedagogical content knowledge and student learning. I suggested using a different five frame than zero or five dots and then I suggested using two colors to represent the parts of the equation (such as 2 red and 3 blue). The teacher needed more
focused goals for the Number Talk, and she needed to communicate that to the students. The teacher needed support regarding the most effective ways to sequence problems to support students’ learning and knowledge of which problems would be most difficult for students in the Number Talk. The teacher needed to begin with 5 frames that were easier to visualize the part-part-whole relationship, and discuss what each part of the equation represented in relation to the five frame instead of just asking for the correct equation. She could have provided visuals by posting each five frame on the board with the equation could have helped. Also, students needed to understand the concept of equality, and I did not see evidence that the teacher had provided opportunities to develop that. Even with that understanding this context may have been too abstract, especially with the 5 frames with zero and the five dots.

Another area of need for the teacher was learning trajectories. In the case of the compare word problem, it seemed that the students did not understand the concept of “how many more,” I was unsure if this problem was developmentally appropriate for most of the students in the class. We needed to know individual students’ level of development of counting strategies and the implications that might have on compare problems. The next conference is not for five more days, and there is so much to discuss.

**February 11th, Conference**

In this conference we discussed 1) the upcoming lesson, 2) assessing students informal knowledge of time concepts 3) a meeting she attended that morning regarding state mandated paperwork which frustrated the teacher 4) student challenges and misconceptions regarding a previous lesson and 5) additional strategies for developing a classroom culture of inquiry.
The conference began with a discussion about the upcoming lesson. The teacher explained that the students would be making a fish that is a “keeper”, at least 6 inches long. The following is the dialogue that took place regarding this lesson.

Teacher: ... They are going to create a fish and make sure it is 6 inches or more longer

Me: Ok, how do they create it?

Teacher: Um, they just get to design it. They just get a sheet of paper and they get to do whatever they want with it. And then because of common core, there is a new lesson in investigations and I's still trying to figure it out but I guess because we are focus in on measurement they want to throw in time.

The teacher did not elaborate on the details of how this would be enacted, and I assumed the teacher would pose the problem to the students in a way that they would have to use problem solving and reasoning about how to draw the fish and determine if it was at least six inches long, as the curriculum materials suggested (Russell & Economopoulos, 2004a, p. 49). The teacher changed the subject to lessons that focused on time concepts. We discussed the new supplemental curriculum materials that were developed to align with the Common Core State Standards. The teacher was appreciative that they have added lessons on telling time with analog and digital clocks. During the discussion, I engaged the teacher in conversation about assessing students’ prior knowledge of time and the informal knowledge they bring to school. Additionally the teacher referred to time during the day with students. “Because informally I am sure some or maybe most of our parents are talking about what time it is. You even do that here and you have a clock already. So try to see what they have picked up informally.” I
told her that this can save instructional time, if you know in advance what students already know about a topic. The teacher said that she did that today in class. The conversation shifted to a meeting the teacher attended that morning, I was not able to be there, but the teacher was upset about it. Low performing schools were required to complete and submit paperwork to the state, which caused additional meetings and paperwork for the teachers. The teacher expressed her concern about this process being too time consuming and did not find it valuable. The teacher said that she became more reflective on student learning and her instruction. I reinforced that that was the point, but agreed that it was too time intensive. As a coach I needed to balance teacher concerns and supporting learning which was sometimes contradictory. I needed to build relationships through listening and empathy, but I also needed to provide other perspectives. At times I needed to give up my own agenda items in order to support what the teacher needed at the time.

Another part of the conference focused on students’ understanding of measurement. The teacher reported that students were doing well with measuring objects, but she wanted them to work on expressing what they were doing. She was also concerned that students were struggling with measurement comparison. I had noticed this to be a difficult concept for students. I probed with the teacher about measurement comparison in the CCSSM, and what other experiences students have had with comparison such as comparing numbers. The teacher thought that it was “more of an addition strategy.” I missed an opportunity to probe with the teacher to get more insight into her thinking about comparison concepts. Probing deeper with students was much
easier for me than with teachers. Instead I replied that what students were doing in the unit is an application of comparison.

Moving forward, I asked about students’ misconceptions with concepts of time, an upcoming topic in the curriculum. The teacher mentioned differentiating between the hour hand and the minute hand and the vocabulary of digital and analog clocks. I told the teacher the vocabulary is not important, but they need to know how to tell time using both types of clocks. The teacher and myself were still making sense of the Common Core State Standards, understanding the meaning and grade level expectations of the standards. The teacher did not see the complexity of teaching time. While the teacher was focused on the vocabulary, I saw it as a complex system that was very different than the way students used numbers. This provided insights into her content knowledge regarding time.

We move on again in the conversation. I introduced strategies for holding students accountable in whole group discussion. It was from a book I had been studying titled, *5 Practices for Orchestrating Mathematical Discussions* (Stein & Smith, 2011). Stein and Smith refer to these as “Moves to Guide Discussion and Ensure Accountability.” These include

- revoicing,
- asking students to restate someone else’s thinking,
- asking students to apply their own reasoning to someone else’s reasoning,
- prompting student for further participation and
- using wait time.
The teacher seemed interested to find out more about this. She replied, “thank you” and “I’m excited.” The teacher said that she wanted to be able to read them in more detail so she could understand them better. My goal was to further support her in creating a culture of mathematical inquiry. She had already started putting some strategies in place, such as having students agree and disagree with each other, stating why, and having students comment on and adding to each others’ ideas. I hoped that sharing these strategies would help her refine what she is already using and add to them. I explained, “Ultimately … you want kids responding to each other. It is not always the teacher having to prompt the entire discussion. “…getting them to listen and it be more of a conversation instead of between teacher student, teacher student, teacher student.” I explained that at first the teacher is modeling and then inviting students to comment and ask students, “What do you think about what so and so just said?” and “Who can repeat what so and so said?” I explained that it takes a lot of work in the beginning and it needs to be continually reinforced throughout the year. I also made sure to tell her she was already doing some of this. Aspects of the reading were familiar to her, but there were some other things that she could add to her “toolbox.” I also told her that these were strategies she could use in other content areas. It will enhance all of her teaching, not just mathematics, and provide students more opportunities to engage in this type of learning.

**February 11th, Classroom Coaching**

The class started off with a Number Talk. Students continue to struggle with equations. I asked the students what the numbers represent in the equation to continue to help make sense of them.
The teacher began the class with a Number Talk. The teacher conducted a Number Talk using a five frame with 2 dots. The student responded that it is $2 + 2 + 1 = 5$. The teacher asked the student why he did it that way and then asked students if they agreed or disagreed. The teacher asked questions to keep the students engaged, and invited them to add to another students’ solution or explanation. The teacher showed a five frame with one dot. Several students responded with the equation $1 + 4 = 5$. Another student responded with $1 + 1 + 1 + 1 + 1$. The teacher accepted both answers. The goal of this Number Talk was to make combinations of five; however, the teacher was focused on the symbolic representation. Again the teacher was unclear of the learning goals for the Number Talk. I was also unclear about what the teacher wanted the students to learn. Another student responded to the problem 10, 20, 30, 40, 50. The teacher continued to use questioning to try to get students to the correct answer. We still have not had enough time to discuss some of these issues during conferences.

**Teacher.** “What are you talking about? We didn’t count by tens today. We counted by fives.”

**Teacher.** “How does that match this?” referring to the five frame.

**Student.** “You can count by fives.”

**Teacher.** “How is that going to connect to this?”

**Student.** You can count by fives.

**Teacher.** Ok, but what equation can you write?

**Student.** $2 + 3 = 5$?

**Teacher.** Ok, but does that really match this? You could do one plus what? How many more?
Student. Four?

Teacher. Good.

This student knows an equation for five, but it is evident to me that some of the students are not making connections between the five frame to the equation. Additionally the rote counting sequence is memorized without understanding the connection between skip-counting and quantity. Also, when students are counting coins they are counting by fives or tens, and they may be applying this knowledge inappropriately. So, I interjected to support understanding of the meaning of the equation.

Coach/Researcher. “What do the one and four represent?”

Teacher. I like it. When you think about [the] equation. 1 + 4 or 4 +1 = 5. What does the 1 and 4 represent?

Student 2. They show a way to represent five.

Teacher. What else?

Student 3. The 1 is one and there is 4. Four is bigger than one so there is more than one and then there is one and four that make five.

Teacher. Ok, so he is thinking about more and less. And four is more than one. Ok, good.

I interpret what the student is saying differently. I did not interject because I thought the lesson is going on too long. Sometimes it is best to let ideas go for another time. At this point the teacher put the 5 frames away.

The teacher took notes about what the students say. They said, “It tells time.” “It tells us when to go to sleep.” A student said she goes to sleep at 10:30 and the teacher showed 10:30 on the clock. The students provide more examples of events that are at a
given time such as when they get up and what time lunch was and the teacher showed those times on the clock. I asked, “Does anybody know what time the clock is showing?” The clock is showing 11:45. The students say “9”, “55” and “45”, but they don't acknowledge both hours and minutes except for one student. Another student adds that if both of the hands are on the 8 then it is 8:30. The teacher replies, “Not quite.” And shows 8:30 on the clock. The teacher asked, “What do we call those things that are moving around the clock?” One student knew they were called hands. The teacher said, “Very good. And this one is called the minute hand and this one is called the hour hand.” “Are there other kinds of clocks out there?” A student responded, “An alarm clock.” Has anyone else seen a clock like that? Who agrees or disagrees?” Several students showed the agree symbol.

Next, the teacher began a lesson on measurement, which involved having students draw their own fish that was a keeper. The teacher modeled how to draw the fish. As she was drawing she said, “Already, I have a little bit of a problem. How long does my fish need to be?” The students replied, “Six or more.” The teacher asked, “Do I know if this is six or more?” The students replied no. Then the teacher asked, “What should I have done first by making a smart plan?” The teacher modeled a way to make sure their fish was six or more inches. While the students created and measured their fish, the teacher and I observed. We noticed that some students were not measuring correctly. We decided to display a fish drawing using the document camera and show how we saw some students measuring and hold a class discussion about the measurement technique we observed.
We decided that we should have a discussion but we did not discuss the details of the role of the teacher and the students. I envisioned having the students determine whether the measurement technique was accurate. However, the teacher asked the students, “What is wrong with how this is measured?” as opposed to asking whether they agreed this was an accurate method. A student commented on the space between the tiles.

The teacher held a class discussion with students sharing ways to measure the fish correctly and the students analyzed the techniques provided. She supported the students in participating in a classroom culture of inquiry by inviting them to add to another students idea and asking them whether they agreed or disagreed and why. She provided some respectful ways in which students could respond to each other, which she and I discussed that morning. However, this episode illustrates the teacher’s struggles with handling class disagreements and student misconceptions. The class disagreed whether
the measuring technique was accurate, so the teacher asked, “What is the first rule about measuring? If you measure this book what do you measure?” Some students say corner to corner. The teacher confirmed, “Corner to corner, edge to edge.” A student said, “If you don't measure the head and the tail, it is not edge to edge.” Some of the students had difficulties measuring irregular shaped figures, not knowing where to start and where to stop, but they do understand that tiles need to be iterated in a straight line without gaps or overlaps. The teacher confirmed that the student was correct. I asked, “Why do we have to measure edge to edge?” This was a difficult idea for the students to explain. A student said that if you measure the head, the body and the tail you will get the answer correct. The teacher revoiced what the student said in her words. She had one more student comment. The students were eager to keep sharing. The student said, “I agree with…” The student is taking a while to finish his idea. The teacher interjected, showing her authority of correct answers, “I hope you don't disagree with me.” Another student said jokingly, “I disagree with you.” The teacher said with laughter in her voice, “I just gave you the answer you have to measure from the tail to the head.”

**Researcher reflections.** In this lesson, the teacher reduced the cognitive demand of the problem by modeling how to draw the fish. This was not how I anticipated the lesson would be enacted based on her explanation that students would create their own fish. Additionally the curriculum materials did not intend for the teacher to model how to draw and determine the length of the fish. However, the teacher modeled how to draw the fish and a method for determining whether it was at least six inches long. Instead they followed a prescribed method. As a result, many of the students’ fish looked similar to the one the teacher drew, and many students used the same method. During the February
4th lesson, she had the students determine if the fish were keepers, but in this lesson she modeled, which significantly impacted the type of learning that took place. I wondered about the teacher’s decision making process regarding when to model and when to use a more problem-based approach.

I was also learning the differences between the teacher’s view of mathematics teaching and learning and mine and that my suggestions were enacted differently than I anticipated. When we decided during the lesson to have a whole class discussion regarding the incorrect methods we noticed, the teacher modeled an incorrect way to measure the fish instead of showing the student’s actual work. In addition she asked a leading question indicating that the fish was measured incorrectly instead of allowing students to decide whether the method was accurate. I learned that in the moment instructional decisions were not an effective approach to use with the teacher.

In addition, this lesson indicated that the teacher did not seem to know what to do with disagreement and misconceptions. It became even more that the teacher was the authority of correct answers, and made sure the students knew the answer by the end of mathematics class instead of having students continue to think about ideas. She did not seem to recognize that modeling and telling were not necessarily contributing to student learning, and these actions change what is actually taught and the potential for what is learned.

**February 12th, Conference**

I started the conversation checking in with the teacher to make sure she was not overwhelmed. I bridged this discussion with one about grade level expectations of what to teach, and recommended some resources to help support that. I felt like I had been
giving her a lot of things to think about and maybe it was too much and not focused enough. The teacher assured me that I was not overwhelming her. “I don't feel bombarded. I feel like they are all suggestions.” I was glad that the teacher wasn’t feeling overwhelmed, however, my goal was to teach her about what research has shown to be the most effective ways in which children learn and my ‘suggestions” were grounded in research. I tried to find the balance between teachers feeling mandated to do it “my way” and feeling like they can take my recommendations or leave them. I also believed that I interacted with teachers similarly and some teachers think I come off too strong, while others think I am offering a ‘different way.’ I had to be conscious of how teachers felt about the coach/researcher’s and the teacher’s work together and adjust to maintain the relationship and the work together. Without relationships and trust, coaching cannot happen. The teacher discussed all of the different content that she addressed in her lessons. Money, counting, early number sense ideas and measurement. “I think that is why it comes off as a lot because there is so many different things we have to focus on.”

The curriculum materials included a short activity in each lesson to practice skills called “Daily Routines”, and then a core lesson each day. The teacher would include other activities such as the calendar routine, counting on the hundreds chart, counting coins or Number Talks. So it was not unusual for the teacher to address up three four different content topics in one class period. This provided opportunities for me to address the content, and developmental appropriateness and placement in the curriculum. The teachers were teaching the former state standards as well as the new Common Core State Standards for Mathematics (CCSSM) because the former standards were what teacher had to report on the grade card. I told her that other grade levels were only teaching...
CCSSM, and that I would check with building and district administrators to get official permission since there had not been a clear statement made on how to complete the grade cards when some of the standards were not being taught any longer. We discuss money, which is not in the first grade standards, but supporting ideas should be taught for them to be successful for the second grade standards about money. I provided a resource to help them determine what to supplement and to what depth.

We then discussed next steps for instruction. She wanted to work more on time concepts, and I thought that number sense concepts were a greater need based on observations in her classroom, in other classrooms and in other grade levels. I suggested specific aspects of number sense to target, and I also suggested continuing to work on addition and subtraction problem types. “There are 12 different kinds of word problems here.” (Carpenter et al., 1999; National Governors Association Center for Best Practices, 2010) The teacher told me that she and her colleagues have “done a lot” with word problems. I probe with the teacher about students’ understanding, “But how are they in their understanding of that?” The teacher responds, “I know. It is one of those things. It’s like we need to get knowledgeable of different ways of teaching because we feel like we have hit it all different ways.” It was difficult for me to make recommendations because I was not in her classroom during the previous instruction with this, but I saw student confusion when the teacher did use these types of problems in class. I did not really know what exactly how it was taught and the “different ways of teaching” they used. I offered to support them with this topic, but it was too complex to get into at that moment. However, I told the teacher that it needed to be very open ended and they need to focus on student thinking. “I mean it really needs to be pretty open ended. They need to make a
lot of sense out of it in their own heads. Because there are still some [students], … on the comparing the problems, still adding the two numbers together.”

As another idea. I discussed an assessment I gave to all first graders at the end of the previous year, which indicated the students struggled with making tens. Students were shown five red and five yellow counters and then asked to make ten another way. Almost all of the students just rearranged the counters or put them in an alternating red-yellow pattern. I suggested she could extend the work with making fives to making tens using 10 frames as a next step in instruction. The teacher again said they have done that but they can do it again. She still focused on what she had done and not what the students learned. I offered to help the first grade team with ideas and strategies if they decide to do that. I related students’ learning of this to their learning in about linear measurement. The students understood iteration of units, to measure from one end to the other and also understood the inverse relationship between the size of the unit and the number of units needed to measure an object. “That’s a good point. I will have to think about that. What do they have, like yeah, I have taught it but do they have all have a good understanding of it?” She adds, “And I can't honestly say that right now about any of those things.” I offer some resources and some problems she could use with students.

Next, I gave the teacher a handout I created of the strategies to support student accountability in whole group discussions and discuss the different strategies with her. I commented on how I recognized that she was using some of the strategies. She said she wanted to “pull her hair out” and that she thought the students were uncomfortable with that. She commented, “It did make some think.” I told the teacher that she needed to keep working on that and continue to have conversations with the students to let them know
that it is ok to disagree with someone because we all have ideas and we think about things differently. I also explained that students needed to know that it was ok when we have an idea that is not correct because we are learning and “who in the world has not had an idea that was incorrect?” The teacher commented, “That’s true. Exactly and that’s ok.” Creating this aspect of culture takes a lot of practice, a lot of discussion, some modeling, some encouragement and some scaffolding.

**Researcher reflections.** More interesting insights came from this conversation. First, the teacher did not know the most important mathematical ideas for first grade students. The teacher was very concerned about teaching concepts such as time, money and equations, which were not major content foci for first grade. Additionally, our conversations and my observations of her instructional practices indicated that the teacher needed to become more knowledgeable about number sense, it’s importance in learning mathematics with understanding, what it is and how to teach it. I also encouraged her to move forward in Number Talks. She had been using 5 frames for several weeks now. When we first started with Number Talks, I wanted to know if the students knew combinations of five before we started working on combinations of ten. My intentions were that she would only work with making fives for one or two days to assess student knowledge and because there are only three combinations. But the teacher continued with 5 frames and included writing equations for the 5 frames, and for many students it was not making sense.

Given certain topics, such as making tens and addition and subtraction word problems, the teacher said she had used all the ways she knew. However, I believed it was about using learning trajectories and attending to student thinking rather than using a
different strategy. I wished I had more time to investigate what she meant about teaching strategies, what she tried and how she might approach teaching these concepts differently.

Another interesting insight was her interpretation of how the classroom discourse had evolved. She incorporated many of the strategies we discussed. She said she wanted to “pull her hair out” because some students were uncomfortable. I did not notice this, and even if that were the case, discomfort is ok. I noticed that the students were much more engaged than when she was telling and modeling. However student engagement is much easier to notice while observing than while teaching. I did not notice them being uncomfortable. I thought they enjoyed being able to comment on other students’ thinking. They seemed to be listening to each other and eager to comment from my observation, and I noticed an increase in student engagement during these discussions.

**February 12th, Classroom Coaching**

In the next lesson, the teacher started off with a Number Talk. She used a five frame with one dot and asked students how many they saw. Then she asked them how many more to get five. The students knew there was one dot and they needed four more. The students continued to struggle with finding equations to represent the part-part whole relationship.

In the next part of the lesson the teacher taught a short lesson on clocks. She first had a discussion about what students know about clocks. She continued to have students agree or disagree and had them add to what other students were saying. She then showed
an analog and a digital clock both showing 2.00, discussed vocabulary related to clocks even though we discussed that the vocabulary was not important.

In the next part of the lesson, the teacher gave each student a marker and tiles to measure the length of each. While the students were working the teacher and I circulated through the room making observations about their measuring techniques. The marker was between 5 and 6 tiles long and the students are having trouble knowing how it’s measure. They were still working on understanding more or less. After the students had a solution, the teacher showed the marker using the document camera and asked how many students thought the answer was five inches, six inches or something else. All of the students raised their hands for either five or six inches. She then asked a student to come up and measure the object. While the student was lining up the tiles, the teacher asked the students if they agreed with what he is doing so far. They agree. This teacher did not comment. He laid down five tiles, and it was a little shorter than the marker. The teacher then began to take over the explanation. She counted the number of tiles aloud pointing to each one. Then she asked the students if they agreed or disagreed. Several students gave “a thumbs-down symbol.” A student said that it has an extra space at the end.

**Teacher.** “Student 1 is saying it doesn’t go corner to corner. Student 1 is telling us to add one more.” [The teacher added one more tile.] “Look guys, it is six inches? If it were six inches the marker would go all the way down here,” pointing to the edge of the tile. “So how can we describe this measurement? Is it six?”

**Students.** “No.”
Teacher. “And you said it was more than five. So what words can we use to describe it?”

Student 2. “The tiles are bigger than the marker because the marker is littler.”

Teacher. So is it more than five or more than six?

Student 2. It is more than five.

Teacher. More than five, and then I heard you say it was less than six. So if you look up here, it is less than six inches but more than five inches.” She explained that there is another word for it, ‘between.’

The teacher then said that it was between five and six. She counted the tiles. She then gave them a book to measure, which has a length that is a whole number. After they had worked, she had a student model how to measure it. She asked the class if they agreed, which they did, and that ends that part of the lesson. The students demonstrated understanding of measuring the length of an object whose length was a whole number, however many had difficulty with more and less concepts.

In the final part of the lesson, the students were given four measurement compare word problems with the difference unknown. The teacher and I observed students as they were working and asked questions about how they solved the problem. The students were still struggling with the measurement compare concept and some were just stating the measurement of the fish that was the longest. The teacher did not have students share their solutions of these problems during class that day.

February 13th, Classroom Coaching

The lesson started off with the Start With/Get To activity (Russell & Economopouloos, 2004b), where two numbers between 1 and 100 are chosen from a bag
and the students had to count from the first number selected to the second number
selected. The two numbers were 49 and 59. The teacher said she was going to “challenge
[their] brains.” “How many numbers are we going up?” The students shouted out “Ten!”
The teacher followed up the answer without confirming or denying the student’s response
“How do you know we are going up ten numbers?” One student said they could count it
and knew they were going up ten. The teacher asks for another reason. The teacher
responded, “What did we learn about where numbers are? Like if you go from ten to
twenty, this I ten more. Any number that is below another number is ten more.” She gave
several more examples of numbers that were ten more, then she asked the students
“Sixty-six to seventy-six is what?” The students responded “Ten more!” She did this with
four more examples and each time the students shouted out it was ten more. She did this
with examples and then said, “So any number that is right below another number is ten
more.” The students were told this pattern in a discussion about patterns in the numbers
in the same column of a hundreds chart on January 31st. Instead of having the students
make sense of patterns in the hundreds chart, she reduced the thinking to a procedure.

While many of the students remembered that a number below another number on the
patterns chart was ten more, the students could not explain why the pattern worked, nor
did they use place value reasoning to determine the difference between the two numbers.
Therefore, the students most likely memorized what the teacher told them.

The teacher conducted a Number Talk today, however, this time she used 5
frames with two colors and continued to have students explained what they saw and write
an equation. More students were successful with writing equations with the two-color 5
frames I added. However the teacher was still accepting solutions for equations that did
not represent the situation. The teacher asked me if those solutions were acceptable. I responded that what we are trying to do is find two number combinations that make five. “So, … that is just not our goal today.”

The next part of the lesson was on linear measurement. The teacher asked students how they might measure the distance from her to the door with a sentence strip. This was a new measuring tool, and it was the first time they had talked about measuring distance as opposed to the length of an object. Additionally this is the first time where the students did not have enough units to span a distance. As I saw how the lesson unfolded, I did not see the connection between measuring the distance to the door and the goal for the day, which was to measure the sentence strip as opposed to using the sentence strips as a measuring tool. The students had some good ideas about measuring with the sentence strip however. One student said that they could put sentence strips on the floor. Counting off where the strips would go, he walked and said one, two, three, four, five, six, seven, eight, nine. The teacher said, “Ok, so that is your estimation.” She focused more on the answer than the process. She did not press the student to explain his thinking in more detail. He then said that you could also put one strip on the floor and then put it on the second and see how many it takes. The teacher said, “That is very good thinking.” This student understood the concept of iterating the same unit over and over again to find the distance. She did not comment whether his thinking was incorrect or correct. The teacher invited another student to comment on this students thinking, “What would you like to add to that?” The student says “Tiles.” The teacher asks, “Who agrees that we could use tiles?” Many students indicate they agree. The teacher asks, “Would it take a little bit of tiles or a lot of tiles?” Again, the teacher did not comment on whether this
was a good strategy or not. Next, the teacher introduced the task. The students were going to measure different sized sentence strips with their own steps. Before the students got to work, the teacher modeled how to measure with footsteps by walking heel-to-toe along one of the sentence strips. The distance was a little less than four steps. The students still had a difficult time with less and more. A student commented, “It is still four steps. Just a little bit more.” The teacher had four note cards on the magnetic board that read, “more than” “less than” “almost” and “between.” The teacher pointed to each card and used it to describe the length of the strip. She read it slowly so that the students could say it along with her. Most of the students were not able to correctly describe the length using these phrases. Next she had a student measure with her steps. She asked the class a good question, “Do you think you are going to use more or less steps?” asked the students to justify their answer. A student had difficulty explaining and the teacher had him put his foot next to hers and compare them. She said “Student’s foot is smaller so we will need more.” She had another student do the same. The students then were put in groups and measured the sentence strips with their steps. I accompanied the teacher to observe one group of students. The sentence strip was about one and a half steps long. The teacher asked, is it more than or less than 2? One student said more than two. She asked his partner the same thing. She thought it was less than two. The teacher asked her why. The teacher said, “Very good.” confirming her answer instead of having the students come to an agreement. The student said that she and her partner have two different answers. The teacher said, “That’s ok. What does it mean if you have two different answers?” The student responded because they have different sized feet. The teacher responded, “Thank you” meaning she was correct. The teacher and I continued to observe students and
talked with them about their thinking. At the end of the class period, I spoke with the
teacher about the students’ difficulty with describing measurements that were between
two whole numbers. Many of the students struggled when the measurement wasn’t a
whole number length. I asked her if she noticed the same thing. She replied that the
student workbook did not have them write that but that students didn’t understand “if it is
over it, it is less.” It could be that many students were not ready for that. One student,
however, said the strip was 6 ½. This was interesting because the students had not studied
fractions yet this year, so this was probably learned informally. I should have probed with
the student about her knowledge and experiences with halves.

**Researcher reflections**

The part of the lesson with the hundreds chart raised questions for me regarding
the teacher’s knowledge about the patterns on the hundreds chart and place value. She
approached counting from a memorization strategy in this activity as well as in other
instances such as skip-counting by tens and 25s. We had talked about that before, but the
teacher not yet changed her practices regarding teaching of counting. That could be due
to content knowledge or beliefs about how to teach counting, or both.

The teacher took my suggestion about using two colors of dots in the
Number Talk. This seemed to help some students see the connection between the
representations. The teacher still did not know which students’ responses were correct.
The first time, I thought the teacher may have lost sight of the goal. But then I wondered
if this was an issue of content knowledge.

**February 15th, Classroom Coaching**
She started off by asking the students what they learned yesterday. A student responded that they used different units to measure kid steps, basketball player steps and Popsicle sticks. The teacher had a sentence strip taped to the floor and invited a student to measure the strip with craft sticks. As the student measured, she asked the other students to comment on his measuring techniques. The craft sticks do not go all the way to the end of the sentence strip. The teacher said, “Uh oh it looks like we have a little bit of a problem. You see that empty space at the end. So what does this mean about our measurement?” Before the students get to answer her question, the teacher said they need to count the Popsicle sticks. “We are going to count with him as he points to each Popsicle stick.” Then they all count aloud together. The teacher asks, “What does that mean? Is it exactly ten?” A student responds that it is a little bit over. The teacher points to the phrases displayed on the board, and points to “more than” and she says, “It is more than ten.” She asks for another way to describe it. Another student responds that it is less than eleven. The teacher continued to revoice and provide further explanation for students’ responses. The teacher laid down another craft stick and said, “Because if we put it there, the distance will be less than the 11th Popsicle stick.” A student asks, “What is a unit?” The teacher asks the students to answer his question. The first student responds that they are in unit 5 of their math book. Another student responds, “When you want to know what unit it is.” The teacher responds, “Ok, it is what you are using to measure things.” She goes on the explain some of the units they have used to measure such as inch tiles, paper clips, and that today they are going to use kid steps, basketball player steps and craft sticks.
**Researcher reflections.** The weather graph was an opportunity to delve more into comparison, a topic in which students had difficulty. This was a missed opportunity to support students with comparison problems and connect the other problems students have solved involving comparisons. Neither of us thought to make that connection. This is an area that I needed to pay more attention as well.

The teacher involved students more often in explaining ideas, strategies and concepts. In this lesson, she had a student count the number of craft sticks instead of her. I needed to encourage the teacher to use other strategies besides revoicing such as inviting students to restate another students reasoning, and comment on and analyze the thinking of their peers. This would encourage students to listen to each other more and keep them more engaged.

**February 19th, Classroom Coaching**

The students continued measuring the sentence strips with different units, baby steps, basketball player steps and craft sticks. The students put their answers on a class chart for discussion and comparison the next day. Unfortunately, I was not able to attend this class period to see the discussion.

**February 19th, Conference**

This conference started off with a discussion over some notes I took “a while ago” on students’ strategies for solving word problems. The teacher said that the students were using the same strategy, and the teacher seemed frustrated. She commented that we had talked before how the students were not using a counting on strategy. They were using different representations but seemed to want to always draw pictures and were reluctant
to use other strategies. Even when she asked them to show another way, the students “would still draw a different type picture or a number line or something like that.” I probed with the teacher whether or not a number line is considered a different strategy. She said it is more of a representation than a strategy. I asked how many students were counting on. She thought two students were counting all. I probed further and asked if she is looking for counting all vs. counting on. She then said that she was looking more at the representation. “Well I am looking to see more what they are doing, I guess I am focusing more on representation like, how are they showing their work? Show me how you did this. Show me how you got your answer.” The teacher asked if she wanted me to assess who was counting all and who was counting on. I suggested finding out as they are working on word problems. I then discussed the different types of strategies research has identified that children use to solve whole number computation problems (Baroody, 2006; Bass et al., 2002; Carpenter et al., 1999). I had just attended professional development regarding Cognitively Guided Instruction and children’s problem solving strategies, (Carpenter et al., 1999). I discussed the different counting strategies with the teacher, and explained derived facts, the most advanced strategy where students would use known facts to find unknown fact. I thought some students with proper support would were ready for derived facts. I suggested Number Talks (Parrish, 2010) which provide clusters of problems that lend themselves to solve problems mentally using number sense strategies. But that the students might need to develop other concepts 10 frames such as making tens first. So far she had only used 5 frames, and it was time to move forward. I suggested using 10 frames for a while and then start having Number Talks that will further support
learning basic facts. I also recommend that we want to see how students are counting.

Then I introduced a strategy called “Convince Me” where the student convinces him or herself convinces a friend, and convinces a skeptic. I explained that a student’s job is to be able to convince the skeptic of why their solution is correct. This would encourage more student involvement and engagement. It also encourages student-to-student interaction and not just interaction with the teacher. I offered to show her a video clip of this strategy in action. The teacher said she was having trouble visualizing how this looks, so I try to explain step by step what this might look and I also suggest that there might be a video of this online. The teacher did not seem ready to try this strategy. So, I did not press.

The end of the conversation centered around the assessment on fractions and measurement that was coming up. The teacher did not like the unit assessment because it had only one problem that assessed students’ understanding on measuring and focused on measurement with different size units, and required students to use reasoning and communication skills. She said it was not representative of the entire unit. The curriculum program required that teachers think differently about assessment. The teachers had already collected data on students understanding of using non-standard units to measure the length of objects. The teacher has also been informally assessing the students on their ability to measure the length of an object. There was no need to assess students again unless the students did not show understanding of previous measurement concepts.

February 26th, Classroom Coaching

The teacher started off with a Number Talk using 10 frames, She started off with
five orange and five green. I was happy to see the teacher finally advancing the content. The teacher asked students if there is a different way to write the equation. One student said that if you “flip it” meaning switching the addends in the equation, it would still be the same. She then showed a ten frame with 10 orange dots. The following discussion took place.

**Teacher.** “S, why do you disagree?”

**Student.** $10 + 10 = 20$.

The teacher pointed to the incorrect equation and asked the class if it makes any sense. **Students.** “No.”

**Teacher.** “What needs to be changed?”

**Student.** “$5 + 5 = 10.$”

The teacher redirected the student to the ten frame.

**Teacher.** “Does that match what we see here? Ok, what does the orange represent? Which Number?”

**Student.** Ten

**Teacher.** So ten plus what?

**Student** “One.”

**Teacher.** Where’s the one? Where’s the one green? Do you see another color here?

**Student.** Zero.

**Teacher.** Zero represents the other color. There is no other color here.

**Student 2.** $0 + 10 = 10$. 

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The teacher did not know which examples would be easiest for students and the best way to sequence examples.

In the next part of the lesson, the students were learning about equal parts of a shape. This is an activity out of the district curriculum materials (Russell & Economopoulos, 2004a). They focused on halves in a previous lesson and in this lesson they were focusing on fourths. The teacher used the document camera to show circles. She asked a student to show how to divide the circle into four equal parts. The student drew a line that divided the circle in half and then another line going across the circle to make fourths. The teacher asked students to show the agree/disagree symbol. “Do you think that is four equal parts?” Some students say yes and some say no. The teacher said, “I think that is pretty close to four equal parts.” The teacher was quick to confirm that that she agreed with the student. She added, pointing to one part, “If I was to split this with my family, and I got this piece I would think I was getting a fair amount.” I noted that the teacher does not ask students why they agree or disagree. She teacher asked, “Is this the only way you can divide this?” pointing to the circle, but did not have students show another any of the other ways. Next, the teacher showed pictures of circles divided into four parts and the students had to determine if the parts were divided into fourths. (Russell & Economopoulos, 2004b)
The teacher uncovered one circle at a time for discussion. For each circle she had students give a thumbs up or thumbs down signal to show if they thought it was divided into fourths. The following discussions ensued for the shapes. The first picture she showed is shown below.

Students disagreed whether this circle was divided into four equal parts. The teacher called on one student that had her thumb up. Pointing to one of the small pieces at the top, she asked, “You would agree…if you got this part at the top? Would you feel
like that’s fair?” She then called on another student that did not think it was equal. The student stated that there are “two little pieces and two big ones.” The teacher replied, “Yeah, do the parts look equal?” While she does not press the student to use mathematical terminology, she restated the reason using “equal parts.” She went to the next shape.

Figure 25. Circle partitioned into four horizontal parts. (Russell & Economopoulos, 2004a)

She told the students “This is a tricky one.” The teacher called on a student. The student replied, “The ones on the sides are smaller.” The teacher responded, “Go show us.” The student went to the board and pointed to the two parts on the end. The teacher asked who agreed with her and why. To a student with a thumbs down, “Why do you disagree? Do you think those are four equal parts?” The student replied, “There are four people and there will be all equal amounts.” The teacher moved to the board and pointed and said, “So, this is equal to this?” The student replied, “No.” The teacher continued, “So there are four equal parts?” The student then said, “No.” The teacher then said, “No. This is not showing one fourth because it is not divided into four equal parts. Ok?”

The teacher had the students reconvene at the meeting area. The teacher gave each
student a square piece of paper and told the student to fold it into 4 equal parts. After the students folded their post it notes, the teacher demonstrated how most students folded their paper. The teacher hung it on the whiteboard and she proceeded to write the fraction one-fourth in one of the squares of the folded paper. The teacher then showed how another student folded the post-it note, into four triangles. She proceeds to write $\frac{1}{4}$ in each piece. She discusses how the shapes are different with different folds. She then shows the students how to fold it another way, making four rectangles.

**Researcher reflections.** I believed that her students were not ready for the symbolic notation of fractions, as I believed it would interfere with students’ learning of fraction concepts. Writing equations for five and 10 frames were an example of this.

I had this same concern with fractions. The curriculum materials introduced the symbolic form of fractions to students, but the CCSS do not introduce it until third grade. I was interested to see how the symbolic notation would impact their development of fraction concepts.

**February 27th, Classroom Coaching**

The teacher started the lesson with a Quick Images activity from the curriculum materials. The teacher showed the following image for five seconds and the students had to determine if it was divided into fourths.
The teacher asked the students to “Describe what you saw and how you saw it.” The student said he thought he saw five parts. A student disagreed and the teacher asked him to explain. “Say, I disagree because...” He said he disagreed because he saw four equal parts. The teacher again asked who agreed and who disagreed. Another student said he saw three lines. The teacher said, “I don't know guys, three lines, four lines, five lines. Let’s see who is correct.” She shows the image and pointed to the parts as she counts the parts. As she counted each part, she used the word line. “Alright, let’s count our lines.” The teacher seemed unsure, and counted the parts again silently and corrected herself, “There are five parts.”

**Researcher reflections.** The quick images provided students with opportunities to analyze and describe a shape that was partitioned into equal parts. The teacher continued to invite students to comment on the reasoning of others and explain their thinking. The teacher struggled with the use of precise language, which could have caused confusion for the students. She seemed to correct herself but seemed somewhat unsure. This could have been a topic of conversation.

Next, the students were working in the workbook that accompanied the curriculum materials. I talked with a student on naming fractional pieces of rectangles.
The following is the page from the workbook in which the students were working.

(Russell & Economopoulos, 2004a)

Figure 27. Workbook page: Area Rugs: Squares. (Russell & Economopoulos, 2004a).

The following exchange took place between myself and one student in the class.

**Coach/Researcher.** Tell me again, how much is that piece? Pointing to one piece of a rectangle divided into fourths with two rows and two columns as shown in Figure 28
Student. A half of a half. One-halves, two-halves,

Coach/Researcher. One half, two halves...how do you know they are halves?

Student. Because a half is more than a half. A half is …

The student drew a rectangle and said, “A whole big cracker. Like this. There. A whole cracker.”

Coach/Researcher. That’s a whole cracker. What’s a half of a cracker?

The student drew a horizontal line through the rectangle, pointed to the rectangle and said, “That’s a half of a cracker.”

I was unclear if he was pointing to the whole rectangle or just one half.

Coach/Researcher. Ok. Show me what a half is.

Student. How much? One or two or three or four?

Coach/Researcher. One. Show me one half.

He draws a rectangle at the bottom of the page and then a line underneath it. "The other one would be down here. (off the page) But I don't have any room.”

Coach/Researcher. Oh, but that other one would be down here, and that would be one half. Oh, what about one fourth?
**Student.** A fourth? (The student draws a vertical line in the original rectangle).

**Coach/Researcher.** Show me where is one fourth?

**Student.** One fourth is like this. (He draws two diagonals in a rectangle.)

**Coach/Researcher.** Oh. ok. Are there other ways to make one-fourth besides that way?

**Student.** mm hmm.

He drew a rectangle with four vertical lines and then counted 1, 2, 3, 4, and then realized he had an extra piece so he colored out the last piece. I will just color that in.

**Coach/Researcher.** Ok so you don't want to count that part?

**Student.** No.

**Coach/Researcher.** Ok. So you just made fourths.

**Student.** mm hmm

**Coach/Researcher.** Ok. I revisited the first problem (Fig. 3.6) So then tell me about this. Why are these halves?

**Student.** These are halves because they already have 2 like this.

He turns to a page that has several rectangles divided into 2 parts. Then he draws a rectangle with a horizontal line through it. The top part is much larger than the bottom part.

**Student.** Those are halves but they are not equal halves.

**Coach/Researcher.** They're not equal halves? If they are not equal can they still be halves?

**Student.** No.
Coach/Researcher. Ok

Student. If you look at this, they will be equal halves.

The coach returns to the example of the rectangle divided into 2 rows and 2 columns.

Coach/Researcher. What about this one?

Student. That would be two halves.

Coach/Researcher. Would that be halves or fourths?

Student. That wouldn't be fair because if there were 3 people in the class, and you only have 2 pieces of a cracker. It would not be fair.

He knew a rectangle divided into 4 columns is fourths because it has "four lines" and one that is divided into four triangles because you could count them.

Coach/Researcher. Are they equal?

Student. Yes.

I then turned to another student sitting at the table, and pointed to the following rectangles and asked “Can you tell me, are these halves or fourths?”

![Rectangles showing fourths](image)

Figure 29. Squares showing fourths. (Russell & Economopoulos, 2004a)
The student was able to tell me they were fourths. I asked, “How do you know?” He counted the four parts in each one. There are four in all. I asked, “So each piece is called what?” The student responded “A square.” I said, “Well that’s right, it’s a square, but what fraction is it?” The original student said “Four-halves.” The other student was not able to answer the question.

**Researcher reflections.** After working with the student in the above episode, I uncovered a great deal of confusion. The student showed some understanding of fractional concepts but did not recognize some representations of fourths. In some cases he identified fourths correctly, but in other cases he referred to them as four halves. I asked the student questions to try to understand his thinking as best as I could. I also tried to ask questions to cause cognitive dissonance, but I did not try to lead him to the correct answer or fix his thinking. I knew this was not a simple error with an easy fix but a deeper conceptual misunderstanding, and I knew I would have to do some research to determine how to address it. I knew that if I just told him, he may have memorized the information without understanding. I wondered if too many new fraction concepts were introduced to the students before they had time to solidify them. Without talking to this student about his thinking, these misconceptions would have gone unnoticed and possibly never been adequately addressed. This could have interfered with him learning future fraction concepts. I thought the teacher would find this conversation between the student and myself interesting, and I decided I should share it with her at the next conference, Instead of only telling her about it.
Upon further reflection the Cognitive-Based Assessment Learning Progression on Fraction Concepts (Battista, 2012b) could have been useful. Timing of this content prevented me from using this as a resource, since this lesson occurred at the very end of working with the teacher and it was the end of the unit on fractions. My hope regarding these missed opportunities during this coaching cycle was that I would have future opportunities to work with the teacher in which I could continue to support her learning.

**February 27th, Conference**

As planned, I share this exchange with the teacher at our next conference. The teacher was surprised with the student’s response of “How much, one or two or three or four?” when asked to show one half. She replied “hmm.” I went on to say, “This is the part I thought was interesting.” Referring to the part where the student drew half of the rectangle that would not fit on the page.

**Teacher.** “Where did he get that?”

The video continued and the student drew a rectangle with two diagonals, and the student counted the parts, one, two three, four.

**Teacher.** He has seen that somewhere.

**Coach/Researcher.** “I was really stuck on trying to get him to … to try to figure out that that was one fourth. But then sometimes he kind of had it.”

**Teacher.** Because the vocabulary there, he is saying fourths, he is saying halves.

**Coach/Researcher.** But on that one (the rectangle with 2 rows and two columns) he kept saying four halves. So I don't know. What do you think?
**Teacher.** I don't know. It is almost like he needs to have people over for a small group. And just talk it through again. ‘Let’s think about it though. If this is a half then this can’t be four halves. So we call this one-fourth.’ It’s almost like he needs to be told. You know what I mean? That what he is doing is wrong. Because he explained it to you and he still thinks that is right. And he didn't see it. I don’t know. That is probably what I would do, but I don’t know if it’s the right way. What do you think?

**Coach/Researcher.** Well, when we tell students, that doesn’t mean they get it.

**Teacher.** I know.

I offered a suggestion that might help us assess students’ understanding.

**Coach/Researcher.** It would be interesting if you had a rectangle and you asked them to color one half of it blue, what would he do, or one fourth of it red.

**Teacher.** I think he would.

**Coach/Researcher.** You think he would?

**Teacher.** I think it would be a good activity to start us off.

**Coach/Researcher.** It is just a matter of delving in and trying to ask those question and trying to figure out what they are really thinking.

**Teacher.** And that is a good activity. I can give them post-its and see if they can color one-half and on another one color one fourth. And maybe for the higher kids color two fourths

**Coach/Researcher.** It would be interesting to see. And then I was talking to [a student] and with questioning she seemed to get it.
Teacher. Yeah, you think it is such an easy concept and when you talk to them deeper, you see that. It is interesting.

Coach/Researcher. And we need to do that as much as possible. It’s hard. But sometimes we think

Teacher. They know it.

Coach/Researcher. Yeah, I was looking at this and I was thinking, what kind of questions do I ask to see if they understand it.

Teacher. Yeah, we were talking about that this week, how do I even pre-assess this or walk around and monitor.

The teacher had already planned to give an assessment that day, and we discussed the assessment and whether or not it was sufficient. They discussed some concerns with the assessment. The teacher was concerned the vocabulary was not the same as what had been used during instruction, quarter versus fourth and that the assessment provided shapes that were already divided and the students had to color one fourth or one half.

The conversation moved to the instruction in recent lessons and some of the notes I had taken. I commented that the teacher was getting students to use the agree/disagree symbols to comment on other students thinking. I also brought up some of the strategies I recorded students using on an activity that focused on fact fluency. The activity is particularly helpful to encourage students who are still counting all to use counting on. I shared the strategies the students were using, counting all, counting on and known facts. I suggested that we look for some strategies for supporting students that are counting all. The teacher said that in the next unit the students would be working on making tens and the pre-assessment for that unit would be a good way to assess how students are counting.
“our checklist could be counting all, counting on, and the third one.” Early in the coach/researcher’s and the teacher’s work together the teacher documented whether the students got correct or incorrect answers and now she is thinking about students’ counting strategies. This is huge. I discussed another level the students may be using, …the third one is number relationships, and I don’t know if anyone is using that yet. In Number Talks there is probably something in there. They group the problems that build on each other like saying 5+5, I know it's the same example I use all the time, and then 5 + 6 to see if any kids will make that relationship. Um I had found something. …to help focus the for the rest of the year, the number sense things, Van De Walle talked about four concepts] used for number sense. One of them is subitizing, one of them is anchoring 5’s and 10’s, one is one more and one less and I forget what the other one is, but I could send you those and maybe that could just focus the rest of the year.”

At this point we had to end the conversation because school was about to start. We did not come up with a formal plan together but some ideas were suggested.

**Researcher reflections**

Using the video of the discussion I had with the student about fourths and halves seemed to have more impact than when I simply told the teacher about what happened. But the teacher’s response was to pull him into a small group and tell him. She was uncomfortable with the student leaving and thinking he was right, and did not seem to realize or believe that telling the correct answer does not lead to student’s understanding the mathematics. This made sense because the teacher
always made sure she clearly told the answer to the problems by the end of each class period. I think the teacher learned that the student had a misconception, and that we need to listen to students’ to uncover their thinking. In the conference I offered an idea that later, I was not sure was the best way to deal with the students’ misconception, but the teacher seemed to like the idea. I was not sure what to do. I knew telling was the answer, but I was not satisfied with what we decided. I also needed to learn more about addressing students’ misconceptions.

**February 28th, Classroom Observation**

In this lesson, the teacher used my suggestion to have students color one-half and one-fourth of a rectangle. The teacher planned this lesson on her own. The teacher gave the students square post-it notes and asked the students to fold a post-it in half. “I want to see two halves.” The teacher told too much. Again, I felt like I needed to plan in more detail about how the teacher is going to implement a lesson. The students were also told to color one half of the post-it. The teacher is in the front of the room with a large piece of paper and folded it in half. “This is one half. This is one half. [pointing to each half] This is how you write one-half.” We also did not talk about including the symbolic notation, but focusing making sure they understand the concepts of halves and fourths. I thought introducing the symbols and unit fractions at this stage this could be confounding students’ understanding. She told students to look at some of the different ways their classmates have folded the post-it. “What does one-half mean?” That was a great question, and what we needed to find out regarding student understanding! A student responded, “It means you get one-half and another person gets one-half.” The teacher pointed to the fraction bar and asked what does that means? A student responded “half”
and she replied, “Close. It means 1 out of 2.” I was concerned that there was too much focus on the symbolic representation instead of the conceptual understanding that halves mean two equal parts. None of the students in the class had difficulty making halves, however, most of their descriptions are not mathematical, describing halves as two equal parts. The teacher repeats this for fourths. The students are told to fold the papers into this many parts as she writes “fourths” on the board. “Make sure when you unfold it, you see four equal parts.” By telling the students four equal parts,” she reduced the thinking and this did not provide information about what the students knew about fourths. “If you have folded it into four equal parts, color” she writes one-fourth on the board. “I want to see if you know what that means.” Later in the lesson the teacher shared one students’ strategy to address a misconception. The student folded her paper with four folds, accordion-style which created five parts. The teacher showed this and asked the students how many parts were created with four folds. The teacher counted the five parts and asked the students how many times she should have folded it. The students still thought it should be folded four times. This was an interesting idea for students to explore but students need a lot of experience and hands on exploration to grasp this concept. She had a student demonstrate that it should be three folds, but then says “[I] am going to do it a little different.” She proceeds to show them how to fold it. I ask, “How many folds are there?” to try to focus on the number of dividing lines create 4 equal parts. The teacher responds “Oh, thank you.” The teacher counted aloud as she pointed to them “one, two, three.” There are 3 lines and 4 parts. She asked the students how many parts should be colored. Many students say one. Then she asked if the students folded it a different way. The lesson continued with different ways to fold a rectangle into four equal parts and
shading in one fourth. At the end the teacher asked students to color two-fourths of the rectangle and then students shared their rectangles.

**Researcher reflections.** The teacher’s implementation of what we had discussed about folding the paper into halves and fourths took me by surprise. The teacher did a lot of telling and modeling. I was learning that lessons we planned needed to be discussed in more depth. What I envisioned during planning was from a constructivist lens. Even after spending five weeks with the teacher, I did not consider that she would walk them through step by step based on our conversation. Not only was I reconsidering what we planned for this lesson, the teacher reduced the level of thinking when she implemented it by telling the students to fold halves into two equal parts and fourths into four equal parts, and that was what I wanted to assess. Instead she focused on unit fractions and symbolic notation, which was not in the first grade curriculum and not appropriate for the level of the students in the class. At times when I thought we had the same vision, I learned that our conceptions of the implementation of lessons or ideas were different. I needed to ask more specific questions when planning with the teacher to better support the teacher in designing lessons that focused on teaching for understanding.

This was the last time I worked in the teacher’s classroom, and I was disappointed that I was not able to help her with handling student misconceptions. This was due to my own lack of knowledge about how to address specific misconceptions. I knew that telling, modeling or redirecting student thinking into the way I thought about the concept was not likely to result in student understanding. I believed that teacher’s must identify students level of reasoning, and create learning experiences which are accessible to them. But I was still learning about this myself and was not able to address this in the moment. This
raised an interesting point. I knew that telling students does not work. I also believe that
telling teachers does not work, but just like the teacher, I did not have any other
strategies. So, an important question is, how do we use a constructivist approach when
teaching teachers?

The End of the Story

The following section describes the post - interview that took place at the end of
the coaching cycle, a classroom observation that took place ten weeks after the coaching
session and post conference for that observation and a final interview.

March 19th, Post Interview

In the last post-interview, I inquired about the students’ performance on an
assessment that was given soon after the coach stopped working with the teacher that
included fraction and linear measurement concepts. The teacher said that the students had
to “divide into two or four equal parts” and “they got that vocabulary.” The assessment
gave them pictures and they had to identify pictures of circles and squares that showed
halves and fourths. I found it interesting that the teacher used the word vocabulary
instead of concept. I wonder if the teacher saw this only as an application of vocabulary
instead of a conceptual understanding of equal partitioning. This could be a content
knowledge deficit. She went to say “but then when you say show me one-fourth or color
one-fourth there is a disconnect there” referring to the symbolic notation for one-fourth.
She thought that it was interesting that the students would ask, “What does that mean?”
when they would see the fraction written as a number. The teacher said she was willing to
let that go since it was not in the CCSS. I was glad to hear that the teacher was not
concerned about students’ understanding of symbolic notation and that the teacher knew
the expectations of the CCSS; however, I was curious why the teacher emphasized the symbolic notation in her lessons. One reason could have been that the curriculum materials included it in the lessons and she used that to make instructional decisions. It could have been that she taught that in the past and/ or because she thought the symbolic notation was important.

**March 19th Post-Interview.** The same questions were asked during a pre-conference interview.

- Describe an inquiry-based mathematics class.
- What do you find easy about implementing this type of instruction?
- What do you find challenging about implementing this type of instruction?

In the initial interview the teacher’s descriptions were vague. She described it as discovery, where the teacher and the students were asking a lot of questions and students could use manipulatives to solve problems. In this interview, the teacher’s responses shifted toward student thinking. “I think the biggest revelation for me was to stop focusing so much on teacher to student interaction and focus on more student to student interaction, like I agree with, I disagree with, I think that is going to stick with me forever.” The teacher showed evidence that she was noticing student thinking. “…it was interesting once they started comparing their answers and supporting each others’ answers what their thinking was. I think that was very insightful.” She added “that is what I am striving to have.” I made a point to comment that student interaction is crucial in creating an inquiry based classroom in hopes to encourage the teacher to make that a permanent part of her practice. The teacher also reported that students’ working collaboratively in groups was still a challenge. She was struggling with students who
were “dominant” and not allowing other students to comment. She wanted to work more on getting discussion happening in groups as opposed to one person taking over the conversation. I suggested that she structure the conversation in such a way that students have a more structured pair share where both students have a set amount of time to explain their solutions. I also suggested that she could identify a group that work particularly well together and have them model for the other students in the class. Then they could use that model to create a list of group norms. She said she liked those ideas. I emphasized that establishing norms takes deliberate action by the teacher, takes time and needs to be revisited throughout the year.

**May 16th, Final Classroom Observation**

This classroom observation took place 10 weeks after I stopped working in the teacher’s classroom. The purpose was to determine which elements of pedagogical content knowledge, teaching practices and classroom culture the teacher continued to use. The lesson began with an activity she called “incredible equations.” This task that was not in the Investigations materials. The setting for this was whole group and the students were sitting on the floor in the meeting area. The teacher gave the students a number, and the students had to find ways to make that number. The students had to make the number 16.

**Student 1.** Ummm, 5 + 5 + 1, 2, 3, 4, 5, 6. 5 + 5 + 6. (Looking at the hundreds chart).

The teacher wrote 5 + 5 + 5 on the whiteboard. [She misunderstood what the student said.]

**Teacher.** How did you get that number? Is it 5 + 5 + 6?
**Student.** $5 + 5 + 6$

But our number is 16. So where are you now?

**Student 1.** $5 + 5 + 5$ equals 16.

**Teacher.** So, $5 + 5 + 5 = 16$?

A lot of hands went in the air and students shouted for the teacher’s attention.

**Teacher.** Oh, I am so talking to [Student 1] right now. Come prove your answer.

The student goes up to the hundreds chart.

**Teacher.** Let’s give him a chance now.

The student goes to the hundreds chart.

T. You are starting where?

**Student 1.** Five and you are counting five more and [counting 1, 2, 3, 4, 5, 6 until he reached 16] and 6.

**Teacher.** So you are saying it is $5 + 5 + 6$?

**Student 1.** [nodding] Yes.

**Teacher.** Ok. Who agrees with [Student 1]? How did he get his answer?

**Student 2.** He started at 5 and he put 5 more and he went 10.

**Teacher.** So you knew $5 + 5$ was ten plus 6 more? Alright, who agrees with [Student 2]?

**Student 2.** Twenty take away five.

**Teacher.** Twenty take away five is 16?

A student shouts out 4. But the teacher ignores it.

**Teacher.** Come show me how you got your answer.

The student uses the hundreds chart to figure out his answer.
**Student 2.** 20 + 4

**Teacher.** Is it 20 + 4 or 20 take away 4?

**Student 2.** Take away?

**Teacher.** Take away. 20 – 4 is equal to 16. Who agrees with that one? The teacher writes it on the board. Alright, who else has an incredible equation for 16? We’ve got 4 more.

**Student 3.** Ten… and 8?

**Teacher.** Ten and what, 8?

**Student 3.** 6.

**Teacher.** Ok. She writes 10 on the board. And 6 on the board and then what with 6? Ten…

**Student 3.** Plus

The teacher wrote the addition sign between the 10 and the 6 and then writes “= 16” after the 6 for the equation 10 + 6 = 16.

**Teacher.** Ok, who agrees with [Student 3]? Which one does [Student 3]’s [solution] connect to?

**Student 4.** 5 + 5 + 6 = 16.

**Teacher.** You take this 5 + 5 and make it a ten. Ok, and then you can create a new equation 10 + 6 = 16.

**Teacher.** Think about the six. What could we do with that?

She asked for another student volunteer.

**Student 5.** He says 5 + 5 + 5.

**Teacher.** 5 + 5 + 5. Ok, go prove it.
Student 5 goes to the calendar and the teacher says to use the hundreds chart since he cannot reach it. This student is struggling so the teacher tells him what to do.

**Teacher.** “Start at five and count five more.

The student counts forward five more.

**Teacher.** Ok, you are at 10, right?

**Student 5.** Count five more.

**Teacher.** Are you at 16 yet?

**Student 5.** No.

**Teacher.** What you are going to do? Add a whole other five?

**Student 5.** No, one.

**Teacher.** Add one more?

**Teacher.** So what is our equation going to be? $5 + 5 + 5$ plus what? $5 + 5 + 16$?

**Student 5.** $5 + 5 + 5$.

**Teacher.** You are at 15. Then, what did you do? You added…

**Student 5.** One more.

**Teacher.** You added one more and that is equal to 16.

The teacher calls on one more student. The student used the calendar starting at 30.

**Student 6.** $30 - 14$.

**Teacher.** How did you get that?

**Student 6.** If you go up one

Another student shouts out “It would be ten less.”

**Teacher.** If you go up on the calendar would it be ten less?
Student 6. No, 7.

Teacher. I see what you are thinking. If you start and 30 on the calendar and you know that 7 less is 23 [pointing to 23] on the calendar and 7 less would get you to 16. So, 7 plus 7.

The next part of the lesson, focused on making combinations of coins. However, this episode was different than the others. The problem was more open ended, with multiple solutions. Their first task was to make 22 cents. The teacher invited several students to show how they made this amount. The students were successful with this. The teacher repeated this with forty cents. Again the students shared several strategies. The teacher did not confirm or deny whether the solution is correct. A student came up with 3 dimes and 10 pennies and counted 10, 20, 30, 40, 50, 60. and then self corrected, 10, 20, 30, 31…40. One student said that he used four quarters.

Student. I did 25, 30, 35, 40.

Teacher. “But when you were counting, you went 25, 30, 35, 40. So if you are using quarters, aren't they all worth 25 cents?”

Student. “Yeah, but if you have four, then that is 25 cents and if you have 25 more, you have 30.”

Teacher. Think about that a little bit more, if you have 25 and you have 25 more, you are going to have 30?”

The teacher asked another student if she agreed and she nodded her head. I ask the class what they thought about that.

Student. “I mean 35.”

Teacher. “Twenty-five plus twenty-five more is thirty-five?"
Another student is shaking her head yes.

**Coach/Researcher.** “Can you show us how you got that?”

The student had difficulty coming up with a justification to his thinking.

**Coach/Researcher.** “Do you want us to come back you? Do you want to think about it a little bit more?”

The students seemed to be able to make combinations of coins with dimes, nickels and pennies, however, there was considerable confusion regarding counting quarters. I was trying to give the student an opportunity so that he could think about it more. The teacher kept trying to find a student to count the four quarters correctly.

**Teacher.** “Who wants to help him out? Who might be thinking something similar? Do you know what he is thinking?”

**Teacher.** “Do you want to help him out a little bit?”

Some students tried to provide explanations, but they were incorrect. The teacher tried to help them get the correct answer. They had been practicing counting by 25s up to 100 and a dollar. The teacher thought that rote counting would help them with coins.

**Teacher.** He used how many quarters? Four? He said he used four quarters and it got him to forty cents. He did 25, 30, 35, 40.” Does that sound right?

**Student 2.** No.

**Teacher.** Why does it not sound right?

**Teacher.** Every time you count a quarter you should be counting how many more…because a quarter is worth how much?

Students shouted out “Twenty-five cents.”
**Teacher.** A quarter is worth twenty-five cents. So if you are at twenty-five cents. Then how much are you going to count on if you have two quarters? Are you going to count on five more or twenty-five more? When you are counting quarters you should count twenty-five, fifty, seventy-five, one dollar. [The students start to count along]. Let’s do it again twenty-five, fifty, seventy-five, one dollar. So if he has two quarters how much is that?”

A student starts to say twenty-five, then the teacher says “25…. and students say a variety of answers including 25, 30, 40, 45 and 60. The teacher has students repeat the counting sequence twenty-five, fifty, seventy-five, one dollar three more times in a song like way. “So if you have two quarters, you have 25…” The class shouts out “Fifty!” The teacher says, “Fifty cents! We know what we have to work on.”

The next part of the lesson was a lesson the from *Investigations* and introduced counting objects that come in pairs. The intent of this lesson was to start to build a connection between counting by two’s and quantity. The expectation was not that the students skip-count, although some may make that connection. The teacher already had the students rote counting by two’s on the hundreds chart, and this is the first time that students had the opportunity to develop a conceptual understanding of what it means to count by twos and develop the two to one relationship. First, the class started out brainstorming objects that come in twos. Then the teacher posed the following problem, “How many hands are there in a group of four children?” Instead of asking children how they might solve the problem, the teacher focused on the answer. A few incorrect answers were offered. One student said 10, 20, 30, 40. The teacher said “There’s forty
hands for four people? Who agrees with this answer?” Many students show a disagree symbol. The teacher did not ask him how he got that. The student might have been counting fingers or incorrectly applied skip-counting by tens since they had practiced that in class. The teacher did not explore his reasoning and there could have been very good thinking behind the incorrect solution or there might have been a misconception about skip-counting. In addition, other students may have thought the same thing, and the misconception was never brought to the forefront. Though his answer was incorrect, I saw some student understanding. The student knew to skip-count with a number four times. One student gave the correct answer of eight hands. The teacher asked her how she got that, but she was not able to explain her thinking. The teacher had four students stand up in front of class to model the situation. She asked them to hold up their hands. “How many hands are up here for four people?” The class shouts out “Eight!” She asked a student how she got her answer of eight. The student said she counted by tens. “You counted by tens? Are you counting by ten or are you counting by a different number?” The student replied a different number. The teacher asked what number she counted by. The student said, “One.” The teacher asked, “You are counting 1,2,3,4 and you got 8?” [She pointed to each student as she counted.] The student seemed confused and uncomfortable. She answered quietly to the teacher instead of explaining to the whole class. The teacher did not consider that she may have counted each hand by one. It was interesting that some students were skip-counting even though they had not connected skip-counting to objects up to this point, but rather from a rote memorization of the sequence. The teacher explained, “She counted by tens and said that wasn’t right. She counted by ones and said that wasn’t right. What could she have counted by?” A student
responded, two. The teacher then confirmed, “She counted 2, 4, 6, 8 [She pointed to each student as she counted.] Or she could have counted 1 hand, 2, 3, 4, 5, 6, 7, 8.”

Next the students were given another problem to solve in their math workbooks. The students read the problem together. This time they had to find out how many hands eight children have. “I am going to give you 8 minutes of no talking time. Then you are going to come up here and show your work.” The teacher had the students share their work using the document camera. The first student showed his work. He drew 8 groups of 2 circles.

**Student 1.** There are two hands for one child and two hands for the second child, [inaudible]

**Teacher.** So there’s 8 hands in all? Who agrees ... Who disagrees? [Student 2] why do you disagree?

**S2.** Because 8 + 8 doesn’t equal eight.

**Teacher.** If you have eight children and eight hands it doesn’t equal eight.

The teacher interpreted the student’s explanation differently than I did. I thought he meant two sets of eight hands such as eight left hands and eight right hands, but it was unclear since the teacher did not probe with the student.

**Student 2.** It equals 16.

**Teacher.** It equals 16.

**Coach/Researcher.** Can you show us? Can you go up to the board?

**Teacher.** Bring your math journal up too.

Student 2’s work showed 8 people and there were two lines above each person’s head.
**Teacher.** What are you showing us?

**Student 2.** There are eight people and eight hands and I count by twos, and then I counted the person like two times. I counted the person two times. Once for the first hand and once for the other hand.

The teacher went to the board and explained the student’s picture.

**Teacher.** So let’s look at your picture. Here he has two lines, [pointing to the 2 lines above the first person]. The next person he has two lines, two lines, two lines, two lines, two lines, two lines for all the people. Why do you have 2 lines above each person’s head?

**Student 2.** I am counting by twos.

**Teacher.** Why do you have to count by twos?

**Student 2.** Because there’s [inaudible] persons.

**Teacher.** And how many hands does each person have?

**Student 2.** Two.

**Teacher.** Two. So let’s look here. [Student 2] is saying 2, 4, 6, 8, 10, 12, 14, 16. [pointing to each person as she counts] Who agrees with that?

**Teacher.** Now remember what [Student 1] said. [Pointing to each group of two circles] two, two, two, two and he had 8 groups of two. But he said there were only 8 hands.

**Student 2.** But it says up here, how many children’s hands are there. And there are 2 circles for hands.

**Teacher.** So how many hands are there for 8 people?

**Student 2.** There’s eight.
Teacher. There’s only eight hands? But you drew two. [pointing to the first set of two hands]. How many hands does each person have?

Student 2. Two.

Teacher. So how should you count that?

Student 2. 2, 4, 6, 8, 10, 12, 14, 16? [pointing to each set of hands as he counts]

Teacher. Sixteen. Alright, go ahead and clean up your math journals, and we are going to finish this tomorrow.

Researcher reflections. The teacher started off with the “incredible equations” task, which had the characteristics of a quality task and opportunities for students to participate in a classroom culture of mathematical inquiry. It had potential to emphasize reasoning and communication. The teacher asked students to justify or prove their answer. She invited students to agree and disagree with other students’ reasoning and she asked one student to explain what another student said. While the teacher adopted some strategies of a classroom community centered on student understanding there were some features that the teacher did not implement. One of my first concerns was that many students had incorrect thinking. While the task had good characteristics, the students may not have been ready for this size number. The first student had the correct equation but the teacher misunderstood him and the student did not correct her. This could have been a result of the classroom culture established by the teacher where the teacher was the authority of correctness. She had positioned herself to be the authority of correct and incorrect answers and told the students that they should not disagree with her. The students were familiar with the activity so the teacher must have used it in the past. Another suggestion would be to have students come up with as many equations they
could. It also would have been interesting for the teacher to write two expressions that equal sixteen such as $5 + 5 + 6 = 10 + 6$ and or to put the “answer” first such as $16 = 5 + 5 + 6$ to provide a more flexible understanding of equality.

The next part of the lesson where the students had to make a collection of coins also showed some effective pedagogical strategies. Again the task was open-ended, had multiple solutions, and promoted reasoning, justification and communication. The teacher provided coins for the students to use. She invited other students to help out the student who struggled counting the four quarters. While several students were able to create a correct combination of coins, none of the other students in the class were able to find the value of four quarters. The teacher reverted to the only strategy she knew, rote counting by twenty-fives. This was something the teacher and I talked about as far back as January 29th in a conference. The class had been practicing skip-counting since then; however there was no evidence of student learning or applying this knowledge. The students were successful finding a combination of coins when they used pennies, nickels and dimes, so it is possible that they were just not adept at using numbers as high as twenty-five. Knowing when to continue to let students grapple with a problem and knowing when the struggle was not productive is a common challenge for teachers, and that was also a challenge with the teacher in this study. The curriculum materials included lessons on building conceptual understanding for skip-counting, which took place in the next part of the lesson. The teacher should have postponed any skip-counting until after the students had concrete experiences with the concepts of skip-counting, such as in the lesson about counting by twos.
The last part of the lesson was from the *Investigations* unit *Two’s Fives and Tens* focused solving problems for things that come in two’s. The intent of the first part of this lesson where the teacher introduced the problem was implemented differently than the curriculum unit intended which likely affected the outcome of this introduction. The intent was for students to share strategies they could use to solve the problem rather than focus on the correct answer as the teacher did. This would have been a more effective way to introduce this lesson providing students with some ideas from their peers. This problem was challenging for some students; therefore, she ended up taking over the students’ explanations and leading students to the correct answer. In the next part of the lesson, the teacher allowed students to solve the problem of how many number of hands 8 children have using their own strategies and allowed them to share their solutions using the document camera. She invited students to agree or disagree with the first solution and she did not say if it was correct or incorrect. She asked another student to comment on the first student’s work and then had him show his solution. From there she started taking over the conversation by revoicing, rephrasing and explaining. This particular problem started off with a student-centered approach and then became teacher-centered by the end.

The teacher and I had worked together intensively for six weeks. Some of the strategies she implemented were still present in the final lesson. Others still were not present. This episode depicts the complexity and challenge of teaching and learning mathematics in a classroom culture of inquiry for teachers. It also indicates that some progress can be made with intensive work for an extended period of time, and that teacher learning takes time, just like student learning. Over the course of six weeks, small
changes in pedagogy were happening. More time spent with the teacher could have led to more changes.

**May 16th, Conference**

The conference began with the teacher talking about how the games in *Investigations* were a challenge for students. They students are supposed to make connections back to the learning goals in the discussion and “with this population they don’t do that.” The students focused on the activity as opposed to the mathematics. That is natural for students and it takes explicit instruction to assure that does not happen. I asked the teacher if there was something we could do differently to help them make the connections? “Before I would be ready to toss it out and say it doesn't work, I would think about how we need to facilitate that differently.” The teacher responded that the fifth grade teachers told her they need to know their facts. She did not think that these games were getting them there. I explained according to CCSSM students need to know their facts from memory at certain grades. But first they need to develop fluency. I was not sure the teacher was familiar with how the literature defined mathematical fluency. So I explained that it included accuracy, efficiency and fluency, (Bass et al., 2002; National Governors Association Center for Best Practices, 2010; NCTM, 2000; Russell & Economopoulos, 2004b) I discussed how our math program builds the development of fluency and automaticity. I explained that after considerable work with the conceptual understanding of number combinations and multiplication, strategies can be used to build fluency and automaticity. I shared a strategy used in subsequent grade levels that she could also use with the students. I also said that teachers need to have a way to manage the activities and the learning of the students with their facts. Teachers needed
to hold students accountable for how to practice their facts and teachers needed to determine students’ mastery of facts, determine which facts students are struggling with and target instruction to build fluency with those facts. However, students needed to be held accountable for accurately assessing themselves and teachers needed to assess the students as well. From my experience in classrooms and talking to teachers students were not getting to the “knows from memory” stage. The teacher said that she had done that with her students and most of the students got it, but she tutored 3rd and 4th graders and they are still counting. I remind her that in the DMA training, the facilitator discussed by learning number relationships such as the identity property and commutative property, the number of facts children need to know are greatly reduced. I reminded her that every number is one or two away from five or ten so if students know how to compose and decompose numbers, this should help them learn their facts. We discussed why the students were not becoming fluent in their facts. I believed that a lack of pedagogical content knowledge regarding number sense and fact fluency contributed to this. Throughout this conversation the teacher seemed to move back and forth between frustration and efficacy.

And I have some fourth graders doing that in tutoring and we were playing games to get them to be more fluent. I mean they know the answer they just are not fluent at it. And I don't know how to get teachers to manage that and monitor that with everything else.

And then she added,

And I think maybe Common Core will help us get there. Like I was saying this was our first year of incorporating addition and subtraction
strategies fluently and decomposing numbers and we have done that but not as explicitly as we have done that this year so maybe that will help them as they get older. We weren’t doing that before but now we are exposing that to them.”

She seemed conflicted with the different messages she received from different constituents, and she was trying to convince herself that students understanding will get better if we teach to the CCSS initiative.

In this conference, we continued to discuss curriculum alignment and what to teach and what not to teach. The teacher was still concerned about what was on the grade card, and about topics such as time and money that had little emphasis in the CCSS. Additional pressures came from the teachers at higher grade levels.

…like you said, it all works out. It is just a matter of trusting that it works out and not feeling like I have to teach this, I have to teach this, or I am going to hear about it next year that second graders don't know their coins because we never taught it, and then I just say oh it was in Investigations [referring to the district curriculum program] you know what I mean, versus I did a poem and we have mini books. It is a fine line.

I told her that one of the Common Core authors addressed the time and money concepts in his blog, and that may provide her with more insight into teaching those concepts (McCallum). It is interesting that other teachers had such an influence on her, and I was curious the reason why. They seemed to have more impact than I did.

We then proceeded to discuss the lesson I observed that day. We discussed the “incredible equations.” I told her that it was a good open-ended task and that it gave
students an opportunity to use their own thinking, explain their thinking and that it revealed student understanding. The teacher said, “And you get to see a lot. You get to see a lot, even from my lowest kids if they can just do 15+1 and 1+15.” The teacher noticed how asking open ended questions with multiple solutions revealed students’ thinking and understanding and focused on strategies more than correct answers. I suggested to the teacher that a next step was to set the expectation that students come up with multiple solutions or strategies. The teacher said, “Oh that is a good idea. I will do that tomorrow. Using this strategy challenges all students and keeps all students engaged. Some students will find one solution quickly and others take more time. We discussed the student who had difficulty counting the four quarters. I began with what the student was able to do, counting on by 5’s from 25. She said that she showed him a nickel and asked him what it was and he identified it as a nickel. The teacher seems to think this is an issue of coin identification as opposed to a misconception about the meaning of skip-counting and understanding of place value. However with that said, she told me about an interesting revelation she had. She noticed that the same students who could not count on were also the same students who could not count money. “I realized that this week that the kids that can't count on from 5, 6, 7, 8, 9, are the same kids that can't count money. Like I have never made the connection before… You know you can't do 10, 20, 21, 22 if you can't add and count on without using your fingers and counting all.” This was significant moment for the teacher because she was attending to student thinking and making content connections. This is was first time in the coach/researcher’s and the teacher’s work together this had happened. I mentioned that it could also be a place value issue for students when counting 2 dimes and 2 pennies and they might not understand that ten ones
is equal to ten. The teacher replied that they spend an entire unit on that during their intervention block, but that the math program is not explicit enough when teaching place value concepts. The teacher said,

It was even difficult to have them make those connections between place value and adding. We were just trying to get them to compare two numbers and to use place value to assist them and identifying which number and which digit is in each place … but even having them connect it to adding they were like, oh, oh, … it was a matter of pulling teeth to get them to even see that. But it is part of common core so it is something that we are still learning that we have to do.”

The teacher viewed place value as the digit that belongs in the tens place and ones place, and not seeing that it is how many tens and how many ones. There is a drastic difference between asking “Which number is in the tens place?” and “How many tens are there in this number?” The first is a procedural question that does not get at students’ understanding while the second focuses on place and value and has more of a conceptual underpinning. We go back to talking about the part of the lesson where the students had to make 22 cents. I commented on several aspects of the lesson that I wanted to encourage the teacher to continue. I drew particular focus to her inviting other students to comment on another student’s strategy “so to kind of see if somebody else can [explain] instead of the teacher jumping in and trying to explain it. Throwing it out to the class. So I think you are doing some things that are really, really powerful. Maybe you don’t even realize
how powerful they are.” I also added “You don’t know what the student is thinking in direct instruction.” I provide an example of my own teaching,

I did plenty of it and felt that if I explained it clearly enough they were going to get it. You know, so it was more about me being super clear. And then, you know, I would do this lesson, and then I would come back the next day and be disappointed, nobody got anything. I thought for sure I explained it so well yesterday and then”

The teacher interjected, “Nothing happened.” I added that in an inquiry based approach the student thinking and misconceptions come to the forefront whereas in a direct instruction approach the misconceptions go unnoticed because the teacher is doing all of the explaining and the students are doing all of the listening. I hoped this resonated with her.

**June, Final Interview**

In the final interview I asked different questions than those in the initial interview. I wanted to target what the teacher felt she learned through our interactions, and I did not think that asking the same questions would reveal those understandings. The first question was “I would like to talk about what you feel you learned and what you feel more confident or more comfortable doing.” She reported to have a better understanding of number sense. For example she said that the games helped build automaticity and before she was just going through the motions. She also said she understood the importance of making combinations of ten. “I understand how ten is such an important number and how it is an anchor for so many things in 2nd, 3rd 4th and 5th grade.”
Next, I explained that this study was about coaching and how our interactions contributed to student learning. I asked her how our interactions and my coaching may have contributed to student learning. Her first response was about the resources. As a coach and a researcher, this is not what I hoped to hear. While I was happy she was using more research-based resources instead of some of the others that are available, I hoped to have a more powerful impact than providing her with resources. She explained that “The resources that you provided were good for me to give me another, to give me an outlook and changed how I thought about things.” But then she said,

“And the way that you interacted with kids was big for me because it reminded me to sit back and versus they have it they have it check, check, check. But how did you get that answer and making sure they are really thinking it through and that they can articulate that.” She also commented on the questions I asked students such as “How did you get that?” and “Show me what you did.” She said it was important for her to be reminded of that because “I often don't let them explain themselves.”

Finally, I asked her if there was anything I could help her with in the future and she said more resources and differentiating instruction. This conversation highlighted a couple of shifts that the teacher made from the coach/researcher’s and the teacher’s work together. I was pleased to know she was watching my interactions with students and learning from them. I did not realize that I was coaching the teacher when I was interacting with students; I hadn’t noticed the teacher watching my interactions with individual and small groups of students. She recognized that she needed to listen to student thinking instead of only focusing on the correct answer. In addition, she
acknowledged that she did not let students explain enough. However, I hoped to get a more in-depth response regarding future support, for example, a focus on how children learn, the best ways to teach specific content or what to do when students are not learning.

I noted several overall themes throughout the work with the teacher. She embraced some of the strategies I suggested, including having students agree and disagree with each other, explaining why and how they know and analyzing the reasoning of others. She used tasks with students that had the potential for problem solving, reasoning, communication and justification.

Other areas I did not notice progress. There was some inconsistency in use of tasks. At times the teacher reduced the amount of reasoning and problem solving in the way she introduced the problem, and she took over student explanations at times.

The teacher continued to model and tell. This happened at times during the introduction of a lesson and often when the students seemed confused or had misconceptions. She still believed memorization was effective in some instances. She also did not know how to address confusion, incorrect answers and misconceptions. She ensured the end of the lesson knew the correct answer, and she remained the final authority of correct and incorrect mathematics instead of sharing that authority and using mathematics as the ultimate source to determine correctness.

Ultimately, this story revealed the complexity of mathematics teaching and learning and the complexity of teaching teachers. These complexities are complicated by teacher’s content knowledge and pedagogical content knowledge.