LABORATORY AND FIELD WORK
IN PLANE GEOMETRY.

A Thesis Presented for the
Degree of Master of Arts

by

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Approved by:

[Signature]
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I. R. F.
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CHAPTER 1

INTRODUCTION

THE STATEMENT OF THE PROBLEM

It is unlikely that complete agreement will ever be reached among educators as to the objectives for the teaching of mathematics in the schools even for a particular period. Accompanying the diverse purposes for giving instruction in mathematics there is an even greater lack of conformity among teachers concerning the most effective methods for achieving objectives.

The necessity for teaching the fundamental concepts and principles of mathematics is invariably included as one of many objectives approved and recommended by committees on the teaching of this subject and by researchers in the field. An understanding of the fundamental concepts and important principles of mathematics is usually considered essential for education, regardless of the changes in the needs of human society. However, there are wide differences of opinion affecting the selection of these concepts and principles.

Effective methods and procedures for teaching geometry are a part of the larger problem confronting teachers of mathematics today. The efficacy of the traditional methods for learning geometrical principles as found in Euclid have been questioned. The challenge to these methods for teaching geometry, fortunately has been accompanied by the substi-
tuition of different methods which seem to be more acceptable in the light of recent finding in psychological and educational research. Now, we may ask, "What other methods for teaching geometry are possible?" And, we would inquire if these newer methods are separate and distinct from the method of deductive reasoning; or are they used as aids in the traditional procedure?

The geometry of the grades and junior high schools is referred to in the literature by many different names. Betz says:

The long list of descriptive adjectives which have been used from time to time to characterize this subject, such as, concrete, observational, inventional, mensurational, experimental, constructive, propaedentic, preparatory, empiric, informal, reflects sufficiently the lack of clearness that still prevails on the precise function of this phase of mathematics curriculum.¹

Any one of these names that has been mentioned by Betz carries the implication that instruments, devices and materials are used by pupils to study geometric concepts and principles. Very often one kind of pupil activity has been emphasized almost to the exclusion of other possible and needed activities. This is exemplified by the mechanical drawing activity provided in the program by Austin.²

Many instruments and devices for the study of informal geometry are described in the literature along with suggestions and directions for laboratory exercises and field work projects.


² William A. Austin, A Laboratory Geometry 1926.
Many of which could well be used as interesting and useful things to do as a follow-up for the theorems that have been proved deductively.

The trend in the direction of teaching geometry through laboratory methods is reflected in the report of the committee of Ten of the National Education Association 1894. This committee recommended that:

Instruction in concrete geometry with numerous exercises, be introduced into the grammar school to familiarize the pupil with the facts of plane and solid geometry. 3

The report of the National Committee on Mathematical Requirements 1923, recommended that:

No attempt will be made to define completely such terms as space, magnitude, distance, and solid; although the significance of such terms should be made clear by informal explanation and discussion. 4

The report of "A Committee of the Mathematical Association" (Great Britain) 1923 enumerated three stages for the teaching of geometry.

1. Experimental methods in which the unconscious acquisition of geometric knowledge is emphasized.

2. Deductive methods in which obvious truths are accepted intuitively, rather than by proving them.

3. Emphasis on the sequence of theorems as a part of a logical system. 5

When generalizations are attempted by using the data of


4 Ibid. p. 403.

5 Ibid. p. 404.
experiment, a need for planning pupil activity arises, for which procedures, instruments, devices and materials are important. Hence the problem for this thesis is: A selection of the important facts of geometry, accompanied by proposals for teaching them through laboratory exercises and field work projects.

THE NEED FOR THE STUDY

The laboratory exercises and field work projects provide needed experience for learning. Breslich says:

A definition seldom has meaning to the pupil who has not had the preliminary experiences with the term defined that are necessary for understanding.6

Perhaps the teachers of informal geometry are better able to tell of the immediate needs for this kind of teaching than are other educators who have less direct contact with the problem. Gorman has made a survey (by questionnaire) which extended over fifteen states of the East, Middle West, and far West and was directed to forty-six educational leaders.

Among other things, information was sought concerning the use made in the schools of about seventy-five items of equipment. Gorman said that

Comparatively few schools possess or have adequate access to even the commonest measuring equipment. Even more serious is the widespread unfamiliarity of teachers with proper knowledge and use of such equipment. Furthermore, there is little organized information available on effective teaching techniques and procedures in

teaching the units of measure and the instruments of business activities, and in providing the basic experience. Of the number of books written since 1935 on the teaching of mathematics in the elementary and secondary schools, only two were found to contain any suggestions on the use of actual tools of measure and of business operations.\footnote{F. H. Gorman, "What Laboratory Equipment for Elementary and High School Mathematics", School Science and Mathematics. (April 1943), p. 335.}

The teaching of geometry through laboratory activity and field work projects, presupposes that an effort be made, well in advance, to have a planned program for the purpose of teaching particular concepts and principles. This advance preparation does not mean that no change can be made as needs arise. It is important, however, to have a working scheme or framework into which pupil suggestions can mesh for the teaching of specific concepts and principles. This advance preparation often prevents the method of discovery or pupil activity from becoming mere busy work or play.

Of the value of learning through activity, Dewey Says:

Our progress in general knowledge always consists in part, in the discovery of something not understood in what had previously been taken for granted, as plain, obvious, matter-of-course, and in part in using meanings that are directly grasped as instruments for getting hold of obscure and doubtful meanings.\footnote{John Dewey, How We Think. p. 140.}

PREVIOUS STUDIES

The reform movement for the teaching of geometry which had its inception in the United States during the latter part of the nineteenth century acquired definite impetus through
the reforms instituted by John Perry in 1901.

"The Perry movement" as it has been called had its origin in the needs of young people for a kind of training in mathematics that could be used in practical affairs of everyday life. Perry outlined a course for geometry study which he described as a "Course of Study recommended for Training Colleges, and for boys and girls. Preferably taken as a part of the science course." Coleman describes the course of study thus:

In this course of study, demonstrative geometry, based on Euclid was placed in what Perry called the "Advanced Course" which also included some calculus and analytic geometry of three dimensions. Preceding the study of demonstrative geometry were: (1) mensuration, and (2) "Geometry". The instruction in mensuration by experiments, the use of squared paper, arithmetical calculations, on actual line and angle measurements, and inventive methods of measuring the lengths of curves. Perry recommended that the experimental work to be taken up in conjunction with general practice in weighing and measuring. The work in "Geometry" was to consist in illustrating the truth of certain propositions of Euclid (Book VI.) (by the combined use of arithmetical calculations and drawings) in the elements of trigonometry, in the use of radian measure, in the use of the concepts of scalars and vectors, in the use of coordinates in three dimensions, and in the measurement of the angle between two planes, and that between a plane and a line. 10

Moore's address before the American Mathematical Society in 1902 had the effect of giving direction to the movement for change in content and method for teaching geometry. He said,

9 John Perry, "Discussion on the Teaching of Mathematics", British Association meeting at Glasgow 1901.

Would it not be possible for the children in the grades to be trained in the power of observation and experiment, and reflection and deduction, so that always their mathematics should be directly connected with matter of thoroughly concrete character—In particular the grade teachers must make wise use of the foundations furnished by the kindergarten. The drawing and paper folding must lead on directly to systematic study of intuitive geometry, including the construction of models and the elements of mechanical drawing with simple exercises in geometrical reasoning. 11

During the first decade of the present century the leaders in elementary education wanted to enrich their courses in mathematics through the teaching of geometry. At the same time the secondary schools were looking with favor on intuitive geometry for high school pupils as a preparation for demonstrative geometry. These two movements were somewhat distinct since the grade and high school curricula lacked the necessary unifying objectives. Correlated or general mathematics was developed to meet the needs of this situation.

The establishment of the junior high school was accompanied by a reorganization of the courses in mathematics for that age group. The National Committee on Junior High School mathematics gave the following recommendations concerning the teaching of geometry.

1. The systematic development by observation, measurement, and construction of the elementary properties and relations of geometric figures should lead gradually to methods of abstract and formal proof.

2. Intuitive geometry should be introduced before algebra is taken up as a general topic, although algebraic processes should be introduced as needed in work in mensuration.

3. The work in intuitive geometry should make the pupil familiar with the elementary ideas concerning geometric forms, in the plane, and in space with respect to shape, size, and position. It should, moreover, be carefully planned so as to bring out geometric relations and logical connections. Before the end of this intuitive work the pupil should have very definitely begun to make references and draw valid conclusions from the relations discovered. In other words, this informal work in geometry should be so organized as to make it a gradual approach to, and provide a foundation for the subsequent work in formal demonstrative geometry.\(^\text{12}\)

The report of the National Committee on Mathematical Requirements 1923 recommended that eight specific topics be made the direct object of study in intuitive geometry. They were:

1. The direct measurement of distances and angles by means of linear scale and protractor. The approximate character of measurement. An understanding of what is meant by the degree of precision as expressed by the numbers of "significant" figures.

2. Areas of the square, rectangle, parallelogram, triangle and trapezoid; circumferences and area of a circle; surfaces and volumes of solids of corresponding importance; the constructions of the corresponding formulas.

3. Practice in numerical competition with due regard to the number of figures used or retained.

4. Indirect measurement by means of drawings to scale; use of squared rules paper.

5. Geometry of appreciation; geometrical forms in nature, architecture, manufacture, and industry.

6. Principle geometrical constructions with ruler and compasses, T-square, and triangle

such as that of the perpendicular bisector, the bisector of an angle, and parallel lines.

7. Familiarity with such forms as the equilateral triangle, the 30° - 60° right triangle, and the isosceles right triangle; symmetry; a knowledge of such facts as those concerning the pythagorean relation; simple cases of geometric logic in the plane and in space.

8. Informal introduction to the idea of similarity.  

The final report of the joint commission of the mathematical Association of America and the National Council of Teachers of Mathematics shows to what extent the materials for the study of geometry through pupil activity have been organized. According to this report the junior high school geometry course should include: (1) basic concepts; (2) basic skills and techniques; (3) important geometric facts and relations; and (4) discovering and testing geometric relations. The skills that are considered basic include the use of ruler, compasses, protractor and squared paper. The construction of figures congruent or similar to a given figure, and direct and indirect measurement. These are fundamental. There are three types of basic facts and relations. Metric, positional and functional. The metric facts include the sum of the angles of a triangle, angles as related to parallel lines, the equality of two angles of an isosceles triangle, the equality of the angles of an equilateral triangle, the right triangle relation, and the rules of mensuration. The positional relations include the position of a point

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in a plane represented by two numbers, and a straight line can intersect a circle in not more than two points. Functional relations, such as the relation between the diameter and the circumference of a circle are required.

The progress of the movement for the study of geometry through laboratory and field work is further reflected by magazine articles on the subject; by the character of recent textbooks, and by important thesis written on the subject.

Comstock\textsuperscript{15} pioneered in the use of laboratory equipment for teaching geometry. He emphasized the importance of learning geometric concepts and principles through experimental and mechanical drawing. His laboratory program also provided for correlation of mathematics with other areas of school work, particularly in the field of physics.

Bedford\textsuperscript{16} has prepared plans for a mathematics laboratory in which there is a planned arrangement for all material, and devices, work tables and filing cabinets, together with a classified listing of needed instruments, devices and materials.

This movement is strongly reflected in \textit{Field Work in Mathematics} by Shuster and Bedford published in 1935. This text book was prepared for the purpose of enriching the school program in mathematics through exercises in both direct and indirect measurements. The Construction and use of ancient


and modern instruments for measuring are explained with accompanying diagrams and discussions. Important techniques such as the work with the slide rule and scale drawing, are explained in this text.

Douglas and Kinney published a text book in 1945 on general mathematics, called "Senior Mathematics". A scale drawing with needed materials indicated is given for making a home made transit. Directions and exercises are given for learning geometric principles with the aid of mechanical drawing instruments, and scale drawing. The use of the vernier caliper for direct measurement and indirect measurements for field work are also included.

"New Plane Geometry" by Mallory published in 1943, employs intuitive geometry and construction as a vehicle for introducing fundamental definitions. The student is led to see that the conclusion reached through intuition and experimentation should be held tentatively. The applications of geometry are stressed through numerous examples. Three differentiated courses are provided through the selection of theorems; through the selection of exercises; and through the emphasis on practical application.

Drake and Johnson\textsuperscript{17} have prepared a most complete classified list of visual aids for the mathematics classroom. They also have prepared a scale drawing with specifications and directions for making a demonstration slide rule.

\textsuperscript{17} Richard Drake and Donovan Johnson, "Vitalizing geometry with visual aids", The Mathematics Teacher (Feb. 1940) Vol. XXIII No. 2, p. 56.
The Eighteenth Yearbook for The National Council of Teachers of Mathematics is entirely devoted to multi-sensory aids for teaching mathematics through laboratory and field work methods.

Ramseyer made an important study to determine the values to be derived from laboratory work. The investigation is an unpublished master's thesis, "A Study of the Use of Laboratory Activities in High School Mathematics" Ohio State University, 1934. The following questions were posed: (1) Does the use of the laboratory contribute to the knowledge of the principles of Algebra and geometry? (2) Does the laboratory contribute any knowledge of the part that mathematics has played in the progress of civilization, and the mathematical values of the universe? (3) Does the laboratory help students to see the relations of mathematical thinking to the solution of present day problems? (4) Does the laboratory tend to develop interest and desirable attitudes of the student toward mathematics?

The data concerning the effect of the use of the laboratory were derived from tests on application of geometry and the students record sheet. Some of the evidence showed that the student had been benefitted by the laboratory experience. However, the evidence was not sufficiently conclusive to justify the conclusion that the laboratory is an effective instrument in teaching the principles of geometry.

The unpublished master's thesis by Morehead, "Laboratory Procedures in a Junior High School Mathematics Program in a
Study of Measurement*, Ohio State University, 1943, is excellent for suggestions as to study guides related to measurement.

One of the most helpful sources for instruments, with directions for making and using them, is the unpublished master's thesis, "Illustrative Devices and Field Work in Secondary Mathematics" by Baumgartner, Ohio State University, 1935.

LIMITATIONS OF THE STUDY

This study explores the field of mathematics education to determine the fundamental concepts and principles of geometry for use as instructional material for junior and senior high schools.

An effort has been made to gather material from various sources that is pertinent to the problem of providing appropriate pupil activity in the study of geometry. Since only the selected list of geometric concepts and principles of plane geometry, as explained in chapter three has been used, some interesting activities involving other mathematical principles have been omitted.

The limitations of this study arise primarily from the fact that suggested laboratory and field work is limited to selected concepts and principles and the study guides included, cover only a portion of the useful facts and principles. They are meant to be suggestive as to how a complete program covering important and useful ideas in geometry could be
planned and organized.

ORGANIZATION OF THE REMAINDER OF THE STUDY

Chapter two discusses the idea of pupil activity for the study of geometry through the use of instruments, devices, and materials. Generalizations related to form and space were made by the ancient Egyptians from a study of their environment in accordance with their needs. It is proposed that useful geometric ideas be learned in the same way by the modern student.

Chapter three explains how a list of fundamental geometric concepts and important principles was selected from the findings of authoritative studies in the field of geometry.

Chapters four and five are composed of illustrative study guides for laboratory activities and field work projects.

Chapter six is a summary of the work of this study and recommendations for further work in this field.
CHAPTER II

THE EVOLUTION OF THE IDEA OF PUPIL EXPERIENCE IN
MATHEMATICS THROUGH LABORATORY ACTIVITIES AND
FIELD WORK PROJECTS

THE ORIGIN OF INFORMAL GEOMETRY

The environment of prehistoric peoples was a laboratory
where, through long centuries of living, they slowly discovered
many basic mathematical concepts and principles for which they
had need. The Babylonians and Egyptians\(^1\) of antiquity, somehow,
found ways and means to measure the number of days in a year,
the number of degrees in a circle, the area of a triangle and
the area of a rectangle. The Rhind Papyrus and other similar
writings that have been found in the Egyptian tombs, are books
of mathematical rules and problems that involved many concepts
of units of measure and of number together with the needed
skills for making calculations related to the determination of
areas, volumes, calendar events, and amount of taxes. Measure-
ments could not have been made without instruments and applied
mathematical knowledge. The rise and fall of the Nile were
measured with milestones that were installed along the river's
course. Leveling was understood and was accomplished with a
wooden triangle and a plumb bob. The south face of the Great
Pyramid is perpendicular to the rays of light from Sirius at
transit and the north face of this pyramid is perpendicular
to the rays of light from Polaris at lower transit. The light
from Sirius shines down a shaft that is made perpendicular to

the south face of the pyramid, and illustrates the head of the
dead Pharaoh. The knotted rope was the instrument used by the
government surveyors (rope stretchers) in their work of re-
establishing boundaries after the annual flood, or used for
setting corner stories for buildings and monuments.

There is no written record of how the ancient peoples of
Babylonia and Egypt learned to measure and to make general-
izations or rules as a result of their activities. We can only
conjecture that somehow through sense perception, imagination
and intuition many conclusions were reached concerning form
and size. This was the geometry of ancient Egypt or the Gift
of the Nile.

The wisdom of the Egyptians attracted visitors from all
over the civilized world of their day and among these were
students and scholars from ancient Greece. They studied the
Egyptian geometry carefully and gave it the name "ge-metrein"
which means earth measure and is the origin of our word geometry.

The ancient Egyptians made practical use of their system
of "earth measure" and through empirical means established a
number of useful geometric properties and relationships. The
early Greek thinkers were interested in the knowledge of geom-
etry for its intrinsic value. They established schools, and
studied the properties of geometric figures, as well as the
relations of these properties and developed proofs of geometric
facts and principles altogether by pure logic. We must
realize that it was the Greek scholar who cherished the study
for its own sake. However, the Greek artizans builders and
mariners, profited from the pure science of geometry and
applied it in their building, surveying and navigation.

Thales, who was born in the town of Miletus in 640 B.C. is credited with having been the first of the ancient Greeks who brought back from Egypt a knowledge of geometry. He became so interested in geometry that he soon retired from his business career and devoted his time to the study and teaching of geometry and astronomy. The development of logical geometry is the work of many ancient Greek mathematicians and philosophers whose works were collected and organized by Euclid. The compilation of writings of the ancient Greek mathematicians together with the Alexandrian contributions is now known as Euclid's Elements or "The Elements of Mathematics." The work of the Alexandrian scholars came to an end with the destruction of the University of Alexandria including the library, by the Arabs in 641 A.D. However the conquering Arabs did preserve the works of Euclid and Ptolemy and some of the scientific works of the library. Ptolemy is remembered for his great work in applying geometry and trigonometry to astronomy. His important writings of thirteen books did for astronomy what Euclid's work had done for logical geometry and was used as a textbook for the next thousand years.

After the destruction of the University of Alexandria, the learned geometers and mathematicians scattered through Asia Minor, Arabia and to other parts of the Mediterranean region. During the next thousand years known as the Dark Ages, the learning of the ancient Greek scholars was partially preserved in the areas to which the Alexandrian scholars had
fled. The conquering Arabs are credited with the preservation of mathematical science and astronomy. They translated mathematical works and established schools throughout the great Arabian empire which included Spain. Thus with the revival of learning, the Arabic influence was felt through the introduction of mathematics in the schools and universities of Europe.

Translations of Euclid's Elements were made from the original Greek and from the Arabic into the languages of the various countries of Europe. The work of Euclid became a part of liberal education of the universities and the later public school systems of Europe and America. The utility and fitness of Euclid for training in logical thinking and mathematical principles remained almost unquestioned until very recent years.

INFORMAL GEOMETRY IN GERMANY AND ENGLAND

One of the first deviations from the strictly deductive method of Euclid was the works of Pestalozzi, Herbart and Froebel. Pestalozzi searched for the primary elements of human knowledge and reached the conclusion that they consisted of number, form and language. Coleman explains how geometry was used as a means of education through sense perception by Pestalozzi where he says:

It is the emphasis on the study of form that is of primary interest here. Obviously, this phase of the Pestalozzian system implies the utilization of certain geometric concepts and ideas in general education. This geometric instruction was to form one of the cornerstones
of the whole elementary educational edifice. It is clear, however, that this instruction would not remotely approximate the Euclidean ideal. Rather it was to be based on the sensory experience of the learner. Geometric instruction, according to Pestalozzi was important not for its own sake but as a means to an end in general education.\textsuperscript{2}

Herbert reaffirmed Pestalozzi's sense perception but further contended that the cultivation of sense perception or apperception is contained in the sphere of mathematical education. He further asserted that the imagination must sketch the geometrical figures and point to relationships before steps in deductive reasoning become possible and generalizations can be formed. Herein is the thought that since has found expression among teachers who urge that informal geometry be made a preparation for the study of deductive or logical geometry.

Froebel explained that education should be accomplished through sense perception, self activity and artistic expression. This philosophy of education has found a very extended acceptance in the teaching of informal geometry through the use of laboratory aids within the past few decades. Concerning Froebel's influence on the teaching of geometry, Coleman says:

With regard to geometric instruction, in particular, Froebel reaffirms the importance given to form study by Pestalozzi. There are, however, definite and significant differences between the Froebelian geometric instruction and the Pestalozzian. First of all, Froebel replaced the passive observation of drawings by the use

\textsuperscript{2}Robert Coleman Jr., \textit{The Development of Informal Geometry} p. 4
of materials for the construction of geometric forms and designs, so that much geometric knowledge was impacted through such activities as play, modelling, paper folding, paper cutting and the like. Secondly, the Froebelian scheme is not merely a simple-to-the-complex procedure, but proceeds from the solid to the point and thereafter from the point to the solid. The actual working out of these ideas is in the "Gifts and Occupations Designed for the Kindergarten".

The methodology and the objectives for teaching geometry in the schools of Germany were strongly influenced and altered during the nineteenth and the first quarter of the twentieth centuries. Folke\(^4\), whose views are explained by Coleman, contended that pupils in the preparatory course should discover mathematical concepts and principles in a manner similar to that of the Ancient Egyptians who were satisfied with a mathematical principle which seemed sufficiently correct without logical proof. Coleman points to a paper prepared by Dr. Erler\(^5\) on the occasion of a meeting of a German philologists and teachers in 1878. Erler pointed out that the materials and methods of teaching geometry were unfamiliar to beginning pupils and that preparatory work was needed. He argued that introductory instruction should first acquaint the pupils with the most important concepts by sense perception and intuition. The second objective should be to introduce logical proof through simple causes of easy reasoning. And the third objective should be to give

\(^3\)Ibid., p. 10.
\(^4\)Ibid., p. 50.
\(^5\)Ibid., p. 53.
some training in the use of mathematical instruments such as the ruler, compasses and triangle.

At the close of the nineteenth century, there was a strong feeling among many German teachers of geometry that the instruction should place emphasis on the empirical foundations of geometry; emphasis on the relation between geometry and practical life activities such as surveying, architecture and machinery; the use of intuition; and the use of some simple devices such as the ruler, compasses and protractor. Also at this time there was developing the so-called engineering movement in both Germany and Great Britain, against the formal character of the mathematics teaching in the technical schools of both countries.

Felix Klein addressed a meeting of natural scientists at Breslau in 1904 in which he explained his views for the reorganization of mathematical instruction. This convention appointed a committee to make a report of its recommendations for the improvement of instruction in mathematics and the natural sciences. The work of this committee is the now famous Meran report of 1905. This report explains the committee's attitude toward the following principles of teaching geometry.

1. Instruction should be better adapted to the natural processes of intellectual development than heretofore.

2. New knowledge should be brought into organic combination with the knowledge the pupil already has.

3. Correlation should be established in the various parts of a subject and between the subject and other parts of the curriculum.
4. With full recognition of the formal disciplinary value of mathematics, yet with rejection of all one-sided "special" knowledge without practical significance, instruction in mathematics should develop as far as possible the ability of mathematically observing the environment.

5. The special objectives of mathematical instruction should be: (a) the cultivation of the power of space perception and (b) cultivation of the habit of functional thinking.

6. Abstract conception and proofs, so often misunderstood by the beginner, should be moved up to the higher stages of instruction.

7. Geometric instruction should emerge from intuition, perception and practical measurement.

8. Geometric instruction should be careful to avoid proving those things which are immediately evident to the pupil.

9. There should be a propaedeutic instruction in the quinta.  

The efforts of teachers in England, at the turn of the century to improve and harmonize the teaching of mathematics to meet the needs of students is now known as the Perry movement (and over in England it was often denounced as "Perryism").

The Perry movement was the culmination of about forty years of agitation by mathematics teachers to have informal or practical geometry accepted for the secondary schools. A second objective of the Perry movement was aimed at redirecting the teaching of mathematics in the technical schools to meet the mathematical needs of young

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6Ibid., p. 63.
engineering and technical students. Perry said in his address to the British Association at Glasgow in 1901:

But all the same I hold that the study began because it was useful, it continues because it is useful and it is valuable to the world because of the usefulness of its results.\(^7\)

The earnestness with which Perry addressed this meeting is further shown when he said:

The mathematician says that he wants to have nothing to do with us; but it is too late to say that sort of thing. It is he who has fixed how his subject shall be taught to us in schools, and he provides us with teachers of it. We pay these teachers to give us something that will be useful to us in life, useful to us in understanding our position in the universe.\(^8\)

Perry enumerated several objectives for the teaching of mathematics. He said, in substance, that mathematics is useful:

1. In producing the higher emotion and in giving mental pleasure.

2. In brain development and in producing logical ways of thinking.

3. In the aid given by mathematical weapons in the study of the physical sciences.

4. In passing examinations.

5. In providing men with mental tools.

6. In developing the ability to think things out for one's self.

There was much controversy in England both before and after the Perry address concerning the acceptance of informal geometry in the schools. Professor Beman points out:

\(^7\)John Perry, British Association Report, "Teaching Mathematics." p. 4.

\(^8\)Ibid., p. 4.
The opposition to Euclid in England seems first to have taken definite form in the organization in the year 1871 of the Association for the Improvement of Geometrical Teaching, whose object as stated in the Code of Rules "shall be to effect improvements in the teaching of elementary mathematics and mathematical physics and especially of geometry. 9

Professor Beman quoted Professor DeMorgan, who said in effect that the study of Euclid was to be preferred to any other system that had been proposed to take its place since the dependence of conclusions upon premises is more distinct than in any other system. Beman praised the efforts of Professor Sylvester, who was internationally known, for his work in both England and the United States and gave the often quoted statement by Sylvester:

I should rejoice to see mathematics taught with that life and animation which the presence and example of her young and buoyant sister (natural and experimental science) could not fail to impart, short roads preferred to long ones, Euclid honorably shelved or buried "deeper than ever plummet sounded", out of the school-boy's reach, morphology introduced into the elements of Algebra--projection, correlation and motion accepted as aids to geometry--the mind of the student quickened and elevated and his faith awakened by early initiation into the ruling ideas of polarity, continuity, infinity, and familiarization with the doctrine of imagery and the nonconceiveable.

INFORMAL GEOMETRY IN THE UNITED STATES

The Pestalozzian philosophy that had so much influence in Germany in developing informal geometry appeared in the United States in the Oswego movement which

started about 1860. Primary Object Lessons by Calkins provided lessons in form which introduced the children to such elementary geometric figures as straight and curved lines, angles, types of quadrilaterals, triangles, polygons, parallel lines and perpendicular lines.\(^{10}\)

Thomas Hill wrote a little book of 144 pages in 1854, which he called "First Lessons in Geometry". The motto of this fascinating little book reads, "Facts Before Reasoning" (Sensation or imagination, conception and then reasoning.) This same writer, who was a New England clergyman and a former president of Harvard University, issued his "Second Book in Geometry" in 1863 for children from thirteen to eighteen years of age.

New England schools and teachers manifested considerable interest in experimental geometry for a number of years after the Oswego movement was launched. G. A. Hill wrote an admirable geometry for beginners about 1880. By 1893 the grammar schools of Cambridge had begun to include the elements of intuitive geometry in their curriculum. Betz says:

> In that year Professor Paul H. Hanus of Harvard University gave a course of lectures to teachers in seventh, eighth and ninth grades. These lectures "formed part of the plan whereby Harvard University gave instructions to teachers of the grammar schools in certain new subjects introduced into the curriculum".\(^{11}\)

Another early endorsement for intuitive geometry is the Special Report of the Committee of Ten on Intuitive

\(^{10}\)Coleman, op. cit. p. 103.

\(^{11}\)William Betz, "Teaching of Intuitive Geometry", The Eighteenth Yearbook of the National Council of Teachers of Mathematics pp. 64-65.
Geometry:

1. The child's geometrical education should begin in the first year of school or kindergarten.

2. At the age of ten there should be systematic instruction in concrete or experimental geometry.

3. Construction of plane figures, free hand and with the aid of ruler, compasses and protractor. Indirect measurement of heights and distances, figures carefully drawn to scale and elementary mensuration plane and solid.

4. Estimation with the eye, measure length of lines, magnitude of angles, and areas of simple plane figures; to make actual plans of maps for his own and from his own actual measurements and estimates and to make models of simple geometric solids in pasteboard and clay.

5. The pupil should be required to express himself verbally as well as by drawing and modeling in the language of the science.

6. It is the belief of the conference that the course here suggested will not only be of great educational value to all children but will also be a most desirable preparation for later mathematical work.\textsuperscript{12}

During the first decade of the present century two new books appeared in the field of informal geometry. They were: First Steps in Geometry by C. A. Wentworth and G. B. Hill; and Constructive Form Work by W. N. Hailman. In the work by Wentworth and Hill there is explanation of the elementary concepts; exercises for the child's observation of geometric forms in his environment; and exercises in measurement with the ruler, compasses and protractor. Hailman's work emphasized the constructional aspects of geometry for

\textsuperscript{12}Loc. cit.
the purpose of providing practice to achieve skill in construction; to achieve the power of visualizing creatively in geometric design; and to create interest in the study of formal geometry.

Not only were there some changes in the exercises of the geometry text books but there were innovations in the curriculum of some schools to provide activity for pupils to study geometry with material equipment that was far in excess of the hitherto allowed straight edge and compasses. C. E. Comstock said:

There is a widespread feeling, daily growing more insistent that present methods of teaching mathematics are inefficient. Especially do teachers of physics point out to us the fact that pupils who come to them are unable to apply the algebraic and geometric knowledge which they are supposed to have, to the problems presented to them. Those teachers of mathematics who see and think, are becoming awake to the situation and many of them are searching for a better way.13

The objectives to be attained in Comstock's laboratory exercises were:

1. There should be an intimate connection between mathematics and the problem it solves—-.. The divorce of elementary mathematics from the practical problems of laboratory and life is worse than nonsense.

2. Such a program plays havoc with the ordinary division into which mathematics has been dissected—-.. The elements of the so-called branches of mathematics are simple and aid in the understanding of each other—-.. 

3. Modern apparatus should be employed. Physical apparatus are a necessity; machines, balances, force tables, reflection boards, instruments for measuring, the plane table, the equitorial

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squares, parallel rulers, protractors. (This
would make us part company with Euclid and
Plato.)

4. No one outside of geometry ever draws a
perpendicular after the method of the books.

5. The validity of abstract reasoning should
be tested by ear or by eye or by hand----.

6. Practice must be given so that proficiency
may be acquired.

7. Individual work must be encouraged-----.

8. An opportunity should be given for observ-
vation and discovery.

9. Familiarity with phenomena themselves if of
the greatest assistance to the comprehension
of the theories explaining them......14

A few years later in 1911, Young15 reflects the chang-
ing educational philosophy for the teaching of high school
mathematics. He emphasizes the following points:

1. The chief end of mathematical study must be
to make pupils think.

2. The mere memorizing of a demonstration in
gometry has about the same educational value
as the memorizing of a page from the city
directory.

3. A large number of pupils merely memorize things.

4. Our textbooks in geometry are still not much
different from the model of Euclid who "wrote
over two thousand years ago - and whose text
was not intended for boys and girls but for
mature men".

5. The authors of most current textbooks empha-
size the formal logical side to the exclusion
of the psychological side.

6. The interest of the pupil must first be
aroused before learning can be accomplished.

14Ibid., pp. 17-18.

15J. W. Young, Fundamental Concepts of Algebra and Geometry
pp. 164-165
7. The psychological textbook for the teaching of geometry is "apparently still unwritten".

8. The teacher should realize that the science of mathematics is alive and growing.

9. The teacher's problem is far more psychological than logical.

10. A beginning should be made by insistent appeal to geometric intuition.

11. The teacher should try to have each pupil grow in his capacity to think deductively according to his ability.

12. Remove all formal consideration from the beginning of the course in geometry.

There were many individuals who heartily endorsed the teaching of informal geometry either as a part of the general education program or as a conditioning for science education and a helpful background for the study of demonstrative geometry. The work of the National Education Association, Committee of Fifteen on Geometry, clearly shows, however, that there was as yet no widespread acceptance of the teaching of informal geometry. The committee says:

In spite of all the discussion about constructive geometry (intuitive, metrical, etc.) in the first eight grades, carried on in the past half century, no generally accepted plan has been developed to replace the old custom of teaching the most necessary facts of mensuration in connection with arithmetic,16

Then the committee suggested the following:

1. To provide for preliminary (inductive, constructive, observational) work in geometry in the elementary grades.

2. To precede the work in plane geometry by some definite work in geometrical drawing.17

16Report of the National Committee on Geometry, Vol. 5919120, p. 83

17Ibid., p. 84.
The establishment of the junior high school necessi-
tated a revision of the mathematics curriculum for the seventh
and eighth grades. The National Committee on Junior High
School Mathematics emphasized that the mathematics instruction
in the junior high school should be as a unit. However, the
committee failed to clarify the function of informal geometry.
The impression was left that the demonstrative geometry of the
high school should have a large influence in the mathematics
of the junior high school.\(^{18}\)

The status of informal geometry in the third period
of the present century is reflected by Betz. He said:

> Intuitive geometry is neither an inferior
type of geometry, nor is it merely a preparation
for the real geometry of the high school. It has
meaning and importance in its own right, being
essentially the geometry of every day life.\(^{19}\)

The facts observed in the literature reveal that prac-
tically all teachers of mathematics recognize some function
for informal geometry in the scheme of education. It is
further shown that there are marked differences of opinion
concerning the objectives to be attained and the methods
to be used.

The success of informal geometry as a method for
teaching important geometrical concepts and principles is then
necessarily dependent upon the willingness of teachers to
become acquainted with the teaching of mathematics by
experiment and to use it intelligently in their work.

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\(^{19}\) Betz, *op. cit.*, p. 56.
CHAPTER THREE

A SELECTION OF IMPORTANT MATHEMATICAL CONCEPTS AND PRINCIPLES TO BE DEVELOPED THROUGH LABORATORY AND FIELD WORK

The learning of Mathematics through laboratory and field work projects obviously requires that the work be well planned. A good teaching situation in a program of this type is scarcely possible unless the objectives for pupil activities are clearly defined and suitable materials are provided for the work.

One of the first considerations for such a program is the selection of important mathematical concepts and principles which can be illustrated and used in a variety of laboratory situations or field work projects.

The purpose of this chapter is to explain how a list of important geometric principles, appropriate for use in secondary schools, was compiled. The selection of these mathematical principles was made from the findings of several authoritative studies in the field of mathematics education. The studies of Congdon\(^1\), Christofferson\(^2\), Pickett\(^3\), and Fagerstrom\(^4\) were selected since they seemed to provide appropriate research material peculiarly adapted to the needs of this problem.

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1 Allen R. Congdon, Training in High School Mathematics Essential for Success in Certain College Subjects.
2 Halbert Christofferson, Geometry Professionalized for Teachers.
4 William H. Fagerstrom, Mathematical Facts and Processes Prerequisite to the Study of the Calculus.
Congdon's research concerning the training in mathematics essential for successful college work, was undertaken for the purpose of determining what facts, concepts, skills, general mathematical principles, and methods of instruction, could best be used in the teaching of secondary school mathematics. The need for determining the most essential facts, concepts, skills, mathematical principles, and teaching methods for high schools had been clearly recognized by at least two much emphasized criticisms of the character of the training provided. These two familiar criticisms were, first; the dissatisfaction of many progressive teachers with the traditional formal courses and time worn methods common to most high school curricula. The second challenge was the persistent feeling on the part of college teachers that the high schools had failed to provide a truly functional training in mathematical facts, concepts, skills and relational thinking.

The principal implement for the Congdon study was the work provided for a college level program in physics by Stewart⁵. This investigation took the form of a threefold analysis of the mathematical vocabulary, the mathematical symbolism and the mathematical facts, concepts, skills, general processes and methods of procedure.

⁵ Oscar M. Stewart, Physics, A Textbook for Colleges.
A part of the analysis of the mathematical vocabulary used in the study of physics was made by grouping the words into three general classes. Congdon explains that,

1. Words that are not legitimately a part of the vocabulary of high school mathematics. Such words as helix, ellipsoid, spiral, sinusoidal, angular deviation and conjugate foci belong in this group.

2. Words that are so commonly used in high school mathematics that we may safely assume a satisfactory knowledge of their meanings. Angle, point, center, surface and sign are illustrative of this group.

3. Words that appear frequently in this list, but are not sufficiently emphasized in all high school classes and text books to give assurance that their meanings are thoroughly understood by the pupil.

That phase of Congdon's study concerning the symbols used in physics text books, was done by examining five well known textbooks in college physics and five well known text books in general chemistry courses for colleges. Frequency tables were constructed for the purpose of making comparisons and pertinent conclusions. Concerning the students' familiarity with symbols Congdon has this to say:

The repeated and varied uses of subscripts, both literal and numerical, with capital letters, small letters and Greek letters suggest that the student who has had experience in using subscripts will have a distinct advantage. Primes are also used by many of the texts to have a specific meaning.\(^7\)

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6 Congdon, op. Cit. p. 7.

7 Congdon, op. Cit. p. 23.
The facts, concepts and skills of geometry, algebra and trigonometry were also tabulated on the basis of frequency. Seven hundred seven pages of subject matter were read and five hundred seventy two problems were solved. Congdon's findings concerning the need for geometric facts, concepts and skills are:

There are comparatively few geometric facts involved, herein investigated, and these are the more simple and fundamental facts upon which ample drill is usually given in high school.8

He also says:

The ability to make scale drawings including the use of the protractor in making angles of a given size and in determining the size of given angles is essential.9

Pickett's study involved the preparation of a revised list of theorems based on their use in proofs of theorems, their use in the solutions of exercises, frequency of use in examination lists, their appearance in recognized syllabi and their frequency of inclusion in text books. The study also sought to determine to what extent the list of theorems recommended by the National Committee on Mathematical Requirements in 1923 had been used in such extramural examinations as the Annapolis Entrance, the College Entrance Board, selected state and West Point examinations. Another part of this study sought to determine the use of algebra in the study of plane geometry to learn of the possibilities

8 Congdon, op. Cit. p. 90.
9 Congdon, op. Cit. p. 90.

of solving originals by analysis and indirect proof.

The following criteria were used for the selection of an acceptable list of theorems:

1. The theorem is necessary in proofs of subsequent theorems in a certain logical sequence.

2. Since it is generally agreed among teachers of geometry that one of the primary objectives in teaching plane geometry is the development of the ability to make a logical proof, the importance of originals in plane geometry can be readily assumed. Therefore the utility or applicability of a theorem as a basis of proof in the solution of geometric exercises is considered the second most important criterion for selecting a revised list of theorems for teaching purposes.

3. The theorem must have occurred frequently as an original in selected examinations. The element of chance is always present in the selection of test items; however, theorems which have been found to occur repeatedly on various tests become important to teachers preparing pupils for extramural examinations.

4. The theorem must be considered fundamental according to the various lists of theorems set forth in different syllabi, for example, Syllabus in Plane Geometry, published by the University of the State of New York in 1927, or the Syllabus in Plane and Solid Geometry, published by the College Entrance Examination Board in 1933.

5. The theorem must have appeared repeatedly in representative tests.\footnote{Ibid., p. 2.}
Frequency tables were prepared by Pickett from data obtained through examination of proofs of theorems and through investigation of the use of theorems in solving originals as revealed in checking examination papers prepared by secondary school students. Tables of frequency of occurrence were also prepared from data obtained through examination of syllabi and of geometry textbooks. A similar procedure was used to determine to what extent the recommendation of the National Committee on Mathematical Requirements in 1923 had been followed by the College Entrance Board and by selected State and West Point examinations.

Christofferson's study has greatly enriched the concept of professionalized geometry. This contribution has been accomplished by ascertaining certain important needs for teacher training and by providing a program of experience designed to meet those deficiencies. An examination of the objectives of his study show that they are classified according to:

1. The mathematical objectives of the course.
2. The professional objectives of the course.
3. The professional assumption upon which the course is built.\(^{13}\)

The needs of student teachers for training in the direction of these broad objectives is the origin of the problem

\(^{12}\) Halbert Christofferson, *Geometry Professionalized for Teachers.*

\(^{13}\) Ibid., p. 2.
for Christofferson's study. Concerning the problem he says:

There remains then the problem of securing still further professionalization of subject-matter with more emphasis on the fundamental pattern of teaching geometry as well as on the foundations of geometry, more actual contact with high school geometry, and more attention to the system of formulated reasoning and its application to non-geometric as well as geometric situations. 14

In Christofferson's findings is a list of ten geometric constructions and twenty theorems in plane geometry which he designates, "The Essential Constructions and Theorems of Geometry". 15

Concerning this grouping of essential constructions and theorems, Christofferson says:

It should be made clear, however, that the 'Essential Constructions and Theorems' which are selected on the later-usage criterion are essential in a professional sense rather than in a mathematical sense. Their purpose is to 'refresh' the reader's mind with regard to elementary principles and processes, and to establish a pattern of teaching, but not to serve as a new list of the fundamental theorems of geometry. They are important theorems for a prospective teacher of Euclidean geometry to know, and fundamental in a professional sense only. 16

Fagerstrom's 17 research has shown to what extent various mathematical principles are essential for the study of the calculus. This study was made by analyzing the solutions of the problems found in the "Elements of the Differential and Integral

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14 Ibid., p. 2.
15 Ibid., pp. 10-12
16 Ibid., p. 6.
Calculus by Granville, Smith and Longley. Concerning the reasons for his study, Fagerstrom says:

The Calculus was chosen as the basis for this study because it seems to be the goal to which secondary mathematics heads; because it is the foundation for all subsequent work in mathematics; because its fundamental principles are the most scientific and powerful tool of the modern mathematician; and because it still remains, almost exclusively a college subject, although there is a tendency to introduce the elementary notions of the calculus into the high school curriculum. 18

The facts, principles and processes that appeared in the two thousand eight hundred eleven problems of Granville's calculus were compiled in frequency tables for comparison and study. 19 Conclusions made from these tables, clearly show to what extent the concepts, facts and principles of algebra, geometry, trigonometry and analytic geometry are essential for a study of the calculus. It is of interest to observe that Fagerstrom found that twenty two theorems of plane geometry were used in the solutions of the calculus problems. The frequency of these theorems ranged from one for some of them, to fifty seven for the Pythagorean theorem.

The findings in the researches of Christofferson, Fagerstrom, Pickett and Congdon have therefore been used to compile an appropriate list of important geometrical concepts and principles for laboratory illustrations and field work projects.

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18 Ibid., pp. 1-2.
19 Ibid., p. 4.
The recommendations of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics in Secondary Education were studied, particularly as they apply to intuitive or informal geometry, and were also used in selecting an appropriate list of concepts and principles for laboratory and field work.

Twenty theorems of the list of geometrical principles selected through this study appear in at least two of the lists of the important mathematical principles as determined by the authoritative studies.

Therefore the criteria used in this study for the selection of appropriate geometric concepts and principles for laboratory and field work are:

1. The importance of and need for the concept or principle as established by authoritative studies.

2. The recommendations of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics 1940.20

On the basis of these criteria the following list of geometric concepts for laboratory and field work is proposed:

1. Point

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2. Line
   a. Straight
   b. Curved
   c. Broken
   d. Segment of
   e. Slanting
   f. Horizontal

3. Lines
   a. Perpendicular
   b. Parallel

4. Plane

5. Solid

6. Angle
   a. Acute
   b. Right
   c. Obtuse
   d. Straight
   e. Bisect of

7. a. Equal
   b. Adjacent
   c. Vertical
   d. Sum of
   e. Difference of
   f. Complementary
   g. Supplementary

8. Perpendicular
   a. To a line
   b. To a plane

9. Vertex

10. Polygon

11. Triangle
    a. Isosceles
    b. Equilateral
    c. Right
    d. Acute
    e. Obtuse
    f. Altitude of
    g. Median of
12. Quadrilateral  
a. Trapezoid  
b. Parallelogram  
c. Rhombus  
d. Rectangle  
e. Square  
f. Diagonal of

13. Circle  
a. Center  
b. Radius  
c. Circumference  
d. Diameter  
e. Arc  
f. Semicircle  
g. Chord

14. Symmetry  
a. Line  
b. Point  
c. Plane

15. Congruent figures

16. Similar figures

These same criteria also lead to a selection of the following theorems for laboratory and field work:

1. When a transversal cuts two parallel lines, the alternate interior angles are equal.

2. When a transversal cuts two parallel lines, the corresponding angles are equal.

3. If two lines in the same plane are cut by a transversal, such that the alternate angles are equal, then the lines are parallel.

4. Two right triangles are congruent if the hypotenuse and one other side of one are equal respectively to the hypotenuse and another side of the other.
5. Two right triangles are congruent if a leg and an acute angle of one are equal to a leg and an acute angle of the other.

6. If two sides of a triangle are equal the angles opposite these sides are equal.

7. The sum of the angles of a triangle equals a straight angle or 180°.

8. The locus of a point (in a plane) equidistant from two given points is the perpendicular bisector of the line segment joining them.

9. The locus of a point equidistant from two given intersecting lines is a pair of lines bisecting the angles formed by these lines.

10. A diameter perpendicular to a chord bisects the chord and the arc of the chord.

11. An angle inscribed in a circle is equal to half the central angle having the same arc.

12. The tangent to a circle at a given point is perpendicular to the radius at that point.

13. If a straight line is drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally.

14. If a line divides two sides of a triangle proportionally, it is parallel to the third side.

15. Two triangles are similar, if the two angles of one are equal respectively to the two angles of the other.
16. In any right triangle, the perpendicular from the vertex of the right angle on the hypotenuse divides the triangle into two triangles, each similar to the given triangle.

17. In a right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

18. In a right triangle the perpendicular from the vertex of the right angle is the mean proportional between the segments into which it divides the hypotenuse.

19. In terms of the sides $S$ the altitude of an equilateral triangle is $A = \frac{S}{2\sqrt{3}}$.

20. In terms of the sides $S$ the area of an equilateral triangle is $A = \frac{S^2}{4} \sqrt{3}$.

21. The area of a triangle is equal to one-half the base times the altitude.

22. The area of a trapezoid is equal to one-half the sum of its bases times its altitude.

23. The area of a circle is $\pi r^2$.

24. The circumference of a circle is $2\pi r$.

The problem then becomes the task of preparing laboratory and field work project guides for the teaching of these geometric concepts and principles through the use of instruments, devices and materials.
CHAPTER FOUR

PROPOSED LABORATORY PROCEDURE FOR THE TEACHING OF GEOMETRY.

The concept of "laboratory" was originally associated with the study of the natural sciences. A laboratory has been thought of as a place where instruments, devices and materials are used in experiments for the purpose of discovering important scientific generalizations or laws. Laboratory work in educational institutions is no longer confined to the field of the natural sciences, since many other departments now use materials and devices in their study and investigations.

The development of the mathematics laboratory is the result of efforts to provide a more adequate environment in the classroom for learning. Many of the courses in mathematics are, for the most part, problem courses. The problems that are found in most textbooks, provide the data that are needed for the solutions of the problems. Thus the student has little opportunity to feel that such a book exercise is really his problem. Such a problem is a "make believe" situation which invokes little personal interest.

The pupil's immediate environment as a source for valuable and important educational material should be more widely recognized by teachers of mathematics. In the classroom are to be found many opportunities for teaching geometric concepts. Here the pupil can have experiences with many geometric forms, such as lines, surfaces, intersections,
rectangles, triangles, and the like.

Streets, roads, fields, city parks and buildings, other than the school, furnish an abundance of material. We live in a geometric world but our behavior in the classroom tends to make students believe that mathematics comes out of a book. Says Breslich,

One of the objectives of education is to acquaint the pupil more fully with his surroundings. Mathematics may contribute to this knowledge by making use of the many opportunities which geometry offers for observing geometric facts and principles in the classroom and out of doors. It is not difficult to find evidence that such training is badly needed. Some pupils cannot supply even the most elementary information about the height of the building in which they live, the size of the lot on which it is built, the width of the street, and the distance to the school. Partly responsible for this lack of observation are the teachers of mathematics who limit their teaching to the facts in the text books, and fail to call attention to the abundance of illustrations in the familiar forms and objects which pupils may observe all around them. 1

The child's material world is shared with many other people in what we, in the United States, choose to call a democratic society. Our way of life is dependent for its existence upon the intelligent participation of all individuals in matters of common interest. Therefore, certain

important qualities of personal living are essential requirements for all citizens. The personal characteristics that characterize the democratic individual have been named and somewhat defined by the committee on the Function of Mathematics in General Education for the commission on Secondary School Curriculum. They are:

1. Social Sensitivity.
2. Aesthetic Appreciation.
3. Tolerance.
5. Self-direction.
6. Creativeness.
7. The disposition and ability to use reflective thinking in the analysis and solution of problem situations.2

These qualities of personal living that are essential for our democratic way of life must be acquired by children in the home, in the church, at work, in play, and through school and social activities for children must, therefore, be so fashioned by the school that they will provide training in these desirable personal qualities. The problem is not confined to the work of a special history, or social science group, but should pervade all school activities.

Realistic experiences in democratic living should be provided for every pupil, and this can be done, to a large

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2 Progressive Education Association, Mathematics in General Education, pp. 31-43.
extent, in developing mathematical ideas and principles through laboratory methods. Such methods call for the gathering of data, for group discussion, for the "give and take" of ideas, for the interplay of minds for interpreting and generalizing. Through such activity the student comes into contact with the meaning of the broad concepts of problem solving which are essential to intelligent thinking. These are:

1. Formulation and solution.
2. Data.
3. Approximation.
5. Operation.
6. Proof.
7. Symbolism.

An important task for the teacher is to arrive at a full comprehension of the practical significance of these broad concepts for his own way of thinking and for teaching pupils how to think reflectively.

Pupil activity in laboratory work for the study of mathematics should take the form of an experimental investigation and the nature of the experiment to be performed can become a live topic for discussion among the students under the guidance of the teacher. Such a discussion includes the materials and devices to be used in conducting the experiment as well as a consideration of the data to be collected, which of course must be relevant to the requirements of the

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3 Ibid, p. 59.
problem. In addition to its pertinence, the data must be accurate, reliable, and expressed in the appropriate units of measure.

The raw data should be refined so that it can be used effectively in the problem solution. It should be expressed in appropriate units of measure for purposes of precision and facility in handling. The fundamental concepts necessary for work in measurement are precision, accuracy, rounding off, and significant digits.

The concept of function applies to the relationship that exists between two or more variables in a problem situation. The nature of variability and interdependence should have wide application in all the problems that pupils experience - both mathematical and non-mathematical.

Operational techniques are always needed in laboratory work for the solution of problems. When the pupils are lacking in operational skills for making the necessary calculations, the teacher should sense the opportunity to develop the needed ability. The student feels that a knowledge of these operational skills is important to him in the work that he is doing and he, therefore gives them his best and interested attention.

Students arrive at generalizations by deductive, inductive, and intuitive or experimental methods. The checking of these generalizations or conclusions is the students' effort to establish proof that the results of his thinking
are correct.

An important part of problem solving is the use of symbols to express quantity, indicated operations, and other ideas involved in the solution of a problem. The use of effective symbols in mathematics requires much attention and direction from the teacher so that the thought process may be facilitated.

The study guides in this chapter are planned for the teaching of some of the geometric concepts and principles that were selected in chapter three.

These study guides are intended only to be illustrations of what may be done with these and other geometric concepts and principles.

One concept or principle is emphasized in the activity of each guide sheet. There is included a brief discussion of the previous experiences that the pupil may have had in relation to the subject of the guide sheet. The statement of the mathematical concept or principle, and the discussion of its place in the pupil’s previous experience, are prepared for the teacher to use in making plans and preparations for his teaching responsibilities.

The student should not know or be told about the geometric principles to be studied. Otherwise he can form no new generalization from his experiences while following the directions of the guide sheet. The suggested activities that are prepared as guide sheets for the pupil, may be varied to suit the needs of the teaching situation.
LABORATORY WORK

Experiences with Lines

High school students have a materialistic conception of lines and therefore teachers of geometry should capitalize on these student ideas when teaching the line concept. The students know of such things as strands of wire in wire fences, clothes lines, football field markings, rivers and roads on maps, the straight line seams in the flooring of a school room, lines of a spider web and lines in designs and patterns.

LINES

Study Guide I

1. Needed equipment: cord, carpenter's chalk and yardstick or straight edge.

2. Student activities:
2.1 Use enough cord to reach across the blackboard from left to right. Attach one end of the string to a nail or tack at one side of the blackboard, chalk the string, with the thumb and finger at some point near the middle and "flip" the string so that the chalk mark will be left on the blackboard.

2.2 Lay your yardstick along the chalk line and state in writing one property of the line.

2.3 Attach the chalked string to two points along the upper border of the blackboard and let a loop of it hang limp like a noose. Pull the loop out from the blackboard and let it swing back to leave its mark.

2.4 Apply your yardstick to this line and write your conclusions about it.

2.5 Write a statement telling how the two chalk lines are different.
LINES

Study Guide II

1. Needed equipment: Pencil, paper, ruler, chalk and spherical blackboard.

2. Student activities:
   2.1 Draw a line freehand across your paper.
   2.2 Apply your ruler to this line.
   2.3 Tell what kind of line you drew.
   2.4 Draw a line with the aid of your ruler.
   2.5 Tell what kind of line you drew with your ruler.
   2.6 Write a statement describing the result when a pencil point moves over a flat surface.
   2.7 Draw a chalk line over your spherical blackboard.
   2.8 Write a statement explaining the nature of the line that you drew on the spherical blackboard.

LINES

Study Guide III

1. Needed equipment: Pencil, paper, straight edge, compasses, tacks and cord.

2. Student activities:
   2.1 Describe five situations about your home in which a straight line is needed. For example: Your father can mark a straight row with a garden plow by pushing his plow straight ahead as he keeps in line with two stakes; one stake being at the far end and one stake being a few feet from that end and toward him. Draw figures to represent the situation that you describe and state the geometric principle which operates here.
2.2 Describe a method you could use to mark off a place for a circular flower bed. Make a drawing for this.

2.3 Describe a method you could use to make an oval shaped flower bed and make a drawing to illustrate your description.

LINES

Study Guide IV

1. Needed equipment: Pencil, paper and a straight edge or ruler.

2. Student activities:

2.1 Study the table shown below.

2.2 Read the items in the first column and fill in the spaces provided in the other columns with a check mark to indicate the kind of line resulting:

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>Straight</th>
<th>Curved</th>
<th>Broken</th>
<th>Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Folding sheet of paper once.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Walking to school</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Using a drawing compass</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Drawing with a ruler</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. A fly moving in the air</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Ice skating</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Walking around the school building</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Marking the football field.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Standing corn stalks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. A moving bullet</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. A tossed basket ball</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. Twirling a weight on a string.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. Walking around a field.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15. Walking in the dark.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
LINES

Study Guide V

1. Needed equipment: Pencil, paper, ruler, fruit can, drawing board, carton or box and a hemisphere model.

2. Student activities:

2.1 Study the figures that are mentioned in the first column in the accompanying table and determine the number of different kinds of lines formed in each figure. Record your results in the appropriate column like the example shows.

<table>
<thead>
<tr>
<th>NAME OF FIGURE</th>
<th>number of straight lines</th>
<th>number of curved lines</th>
<th>number of broken lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Schoolroom ceiling.</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2. One classroom window.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3. The blackboard.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4. Your desk.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5. A fruit can.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6. Your drawing board.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7. The entrance walk to school.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8. One flight of school stair steps.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9. A basketball board and ring.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10. The floor of your school room.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11. A box or carton.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12. Half of an orange.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13. A fireplace log.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14. Your school lawn.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15. A half moon.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
LINES

Study Guide VI

1. Needed equipment: A parallel ruler, paper, T square, drawing board, right triangle, plumb bob, string and carpenter's chalk.

2. Student activities:

2.1 Draw lines on the blackboard with the parallel ruler.

2.2 Draw five lines on paper with your T square and drawing board. Hold the T square firm to the edge of the board and move it to various positions to get the different lines. Write a statement concerning a property which these lines seem to have.

2.3 Place your T square firmly against the edge of the drawing board. Lay your right triangle on the drawing board with one of the short sides against the T square. Draw a line along the long side of the triangle. Shift the positions of the T square and repeat four more times. Write a statement concerning a property of the resulting lines.

2.4 Attach two plumb bobs to fasteners at the top of your blackboard. Draw chalk lines along the two plumb bob cords. Write a statement about these chalk lines.

2.5 Stretch two chalked strings horizontally across the blackboard and attach the ends of the cord to two small nails at each end. Have the nails six or eight inches apart at both ends of the blackboard. Flip the cords by pulling each one slightly our from the board and letting it go back to the board. Write about these lines and tell how they are related.
LINES

Study Guide VII

1. Needed equipment: Pencil, paper, T square, right triangle, protractor, carpenter's chalk, carpenter's level, string, and plumb bob.

2. Student activities:

2.1 Draw a horizontal line with your T square. Then without moving your T square, place one of your right triangles on the drawing board with one of its short sides along the edge of the T square and draw a line along the other short side. Use your protractor to measure the angle formed at the intersection of these two lines.

2.2 Place a carpenter's level on a chalked string and make a horizontal line on the blackboard. Then attach a plumb line to the top border of the blackboard and draw a chalkline along the string on the blackboard. Measure the angle formed by these two lines.

2.3 Write your conclusion about lines that meet as these lines do in each of these experiments.

LINES

Study Guide VIII

1. Needed equipment: Ruler, paper and pencil.

2. Student activities:

2.1 The first column in the table given below contains a series of statements in each of which there is a relation between two things. For example: The relation between the long and short edges of your desk top is "Perpendicular." Read each situation in column one and check the appropriate relationship in columns 2, 3 and 4.
<table>
<thead>
<tr>
<th></th>
<th>The long and short edges of your desk top.</th>
<th>Parallel</th>
<th>Perpendicular</th>
<th>Slanting</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>Lines drawn on the floor to a suspended plumb bob.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>The long and short edges of your ruler.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Your driveway to your street.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Two streets of a city running north and south.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>An east and west street to a north and south street.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>The upper and lower edges of your blackboard.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>An upright post to boards nailed to it.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Weatherboarding on a house to the corner strips.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>2&quot;x4&quot; studding to the floor plate.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>2&quot;x4&quot; studding to other 2&quot;x4&quot; studding.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>A tree on level ground to paths leading to it.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>Two adjacent edges of your schoolroom floor.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td>Rafters to the plate where one of their ends rests.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.</td>
<td>A &quot;leaning&quot; post to boards nailed to it.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.</td>
<td>A diagonal gate brace to the slats.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.</td>
<td>The spokes of a wheel to the axle.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.</td>
<td>Ropes of a tent to the ground.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.</td>
<td>The limbs of a tree to the trunk.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.</td>
<td>Your body to the floor when you are standing.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
LABORATORY WORK

Experiences with Angles

The beginning student has a much less distinct idea of "angle" than he has of "point" and "line". Perhaps the word angle may mean "turn" to some students and "sharp point" to others. The word angle is often used incorrectly by the high school student. But if he can have a real life experience for learning the angle concept, there will be less likelihood that he will continue to be confused with it or its use.

ANGLES

Study Guide I

1. Needed equipment: Compasses, pencil, paper and straight edge.

2. Student activities:

![Diagram of various angles labeled P, C, B, A, D, G, F, H with measurements like 180, 100, 90, 40, 30, 20 degrees.]

FIG. 1
2.1 Start with the line OE and with the end at O fixed, let OE move counter clockwise or rotate until E gets to D. The amount of rotation, not the space between OE and OD, is called an angle. The angle is read as "the angle EOD." The point O is called the vertex of the angle. And in naming an angle the vertex, in this case O, is placed in the middle. The angle EOD is $42^\circ$.

2.2 Think of OD as continuing to move counter clockwise to each of the positions marked by the dotted lines. Write the names of the angles and observe their size at these positions. Record your results in the appropriate columns of the table provided on this guide sheet.

<table>
<thead>
<tr>
<th>Name of Angle</th>
<th>Size of Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOD</td>
<td>$42^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.3 Make a cardboard protractor.
ANGLES

Study Guide II

1. Needed equipment: Pencil, paper and straight edge.

2. Student activities:

2.1 Estimate the size of the angles in figures 2, 3, 4, 5, 6 and 7 and record your estimates in the appropriate spaces of the table below. Then measure the angles with your protractor and record your results in the table. Compute the differences between your estimated and measured values.

<table>
<thead>
<tr>
<th>Angle</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured Result</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
LABORATORY WORK

Experiences with Triangles

The objective in this series of exercises is to provide the students with experiences from which they can make the important generalization concerning the sum of the angles of any triangle. The data which the students collect will be approximately true since all measurements are approximately only. The geometric principle to be illustrated in the experiments is:

The sum of the angles of a triangle is equal to two right angles or $180^\circ$.

TRIANGLES

Study Guide I

1. **Needed equipment:** Pencil paper, ruler, and protractor.

2. **Student activities:**

2.1 Measure the angles $a$, $b$ and $c$ of the triangle $\triangle XYZ$ and record your results in the appropriate columns of the table below. Also measure the angles $a$, $b$ and $c$ of the triangle $\triangle MNO$ and record them in the table.

![FIG 8](image1)

![FIG 9](image2)

2.2 Draw a triangle and make it any shape you wish. Letter it $\triangle RST$, mark the angles $a$, $b$ and $c$, measure these angles and record your results in the appropriate columns of the table.
### TABLE VI

<table>
<thead>
<tr>
<th>ANGLE</th>
<th>Triangle AYZ</th>
<th>Triangle MNO</th>
<th>Triangle RST</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a+b+c</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.3 Find the average of all the results found by the members of your class for a, b, c.

2.4 To what generalization concerning the sum of the angles of a triangle do these data seem to lead?

### TRIANGLES

#### Study Guide II

1. **Needed equipment:** Ruler, paper, pencil, cardboard, compasses and scissors.

2. **Student activities:**

   \[ \text{FIG 10} \]

   \[ \text{FIG 11} \]

2.1 **Draw a triangle CED**, making the side CE six inches long. Make the sides CD and DE any convenient lengths. With C as center draw the arc MN using a convenient radius. Then using the same radius, describe the arc OP with E as center. Then without changing the radius make the arc QR with D as center.
2.2 Cut along the dotted lines of your triangle and fit the angles \( X, Y \) and \( Z \) together as in Fig. 11.

2.3 Write your conclusion concerning the sum of the angles of a triangle.

LABORATORY WORK

Experiences with Lines and Angles

The geometric principle for the pupils to discover in this experiment is:

When a transversal cuts two parallel lines, the alternate interior angles are equal.

There are two situations for the pupil to investigate. One involves two parallel lines cut by four transversals; while the other involves two non-parallel lines cut by four transversals.

LINES AND ANGLES

Study Guide I

1. Needed equipment: Paper, pencil, T square, ruler and protractor.

2. Student activities:

2.1 Draw the parallel lines \( MN \) and \( XY \) with your T square as in Fig. 12.

2.2 Draw the transversals \( AB, LM, TS \) and \( WP \).
2.3 Measure each of the angles 2 and 6 in the diagram and record your measurements in the following table.

<table>
<thead>
<tr>
<th>Transversal</th>
<th>Angle a</th>
<th>Angle b</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PW</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.4 Draw the lines MN and AX so that they are not parallel to each other.

2.5 Then draw the transversals AB, KL, ST and WP.

2.6 Measure the angles a and b for each transversal and record your results in the table below.
TABLE VIII

<table>
<thead>
<tr>
<th>Transversal</th>
<th>Angle a</th>
<th>Angle b</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WP</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.7 The angles a and b are called "alternate interior" angles. Do you see any reasons why this is an appropriate name? If so, explain.

2.8 From the data of Table VII what conclusion concerning alternate interior angles of parallel lines seems justified?

LABORATORY WORK

Experiences with Right Triangles.

Pupil activity is now directed to the investigations of the Pythagorean theorem. Those pupils who fail to understand the deductive proof for this theorem may realize some degree of satisfaction concerning its validity by doing this experiment. The geometric principle for the directed activity is:

In a right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

RIGHT TRIANGLES

Study Guide I

1. Needed equipment: Squared paper and a right triangle.
2. Pupil activity:

![Diagram](image)

FIG 14

2.1 Draw a right triangle on graph paper and make AC = 3 inches, CE = 4 inches. How long is AB? Then complete the figure.

2.2 Count the small squares in ADEB.

2.3 Count the small squares in CHFG.

2.4 Count the small squares in ACIH.

2.5 Add the number of small squares in ACIH and CHFG.

2.6 Write your conclusion about this experiment.

2.7 Try this when AC = 6 and BC = 7.

LABORATORY WORK

Exercises with an isosceles triangle.

This experiment is for the purpose of discovering the following geometric principle:

If two sides of a triangle are equal the angles opposite these sides are equal.
The isosceles triangle of a gable roof is familiar to students. A property of the isosceles triangle is easily utilized to do very accurate levelling without a spirit level. This form of level is very convenient for levelling large surfaces or long lines, as it can be made in any desired size. It was widely used by surveyors in ancient times and still is used today when a surveyor's level or a carpenter's spirit level is not available.

**ISOSCELES TRIANGLE**

Study Guide I

1. **Needed equipment:** Ruler and protractor.

2. **Pupil activity:**

![Figure 15](image15)

![Figure 16](image16)

2.1 Measure the lengths of the sides AB and BC and the size of the angles A and C of the triangle ABC in figure 15.

2.2 Measure the lengths of the sides AE and EC and the size of the angles A and C of the triangle AEC in figure 16.

2.3 Write your results in the appropriate columns of the following table.
2.4 Write your conclusions about triangles like ABC and EAC.

LABORATORY WORK

Experiences with Similar Triangles

The teaching of the concept of similarity can be greatly aided by measuring the corresponding parts of similar figures. The most simple situation is that of similar triangles. Much practice in finding ratios through measurements and in finding the size of the corresponding angles is necessary for pupil understanding of similarity.

SIMILAR TRIANGLES

Study Guide I

1. Needed equipment: Pencil, paper, protractor, and ruler.

2. Student activity:

![FIG. 17]

![FIG. 18]
2.1 Measure all the angles in triangles ACB and DFE and record your results in the appropriate places in the following table.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Angle</th>
<th>Angle</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACB</td>
<td>A=</td>
<td>C=</td>
<td>B=</td>
</tr>
<tr>
<td>DFE</td>
<td>D=</td>
<td>F=</td>
<td>E=</td>
</tr>
</tbody>
</table>

2.2 Measure the lengths of all the lines in figures 17 and 18 and record your results in the following tables.

**TABLE XI**

<table>
<thead>
<tr>
<th>AC=</th>
<th>AB=</th>
<th>CB=</th>
</tr>
</thead>
<tbody>
<tr>
<td>DF=</td>
<td>DE=</td>
<td>FE=</td>
</tr>
</tbody>
</table>

**TABLE XII**

<table>
<thead>
<tr>
<th>AC= DF</th>
<th>AB= DE</th>
<th>CB= FE</th>
</tr>
</thead>
</table>

2.3 Write a statement telling what you have discovered about the triangles ACB and DFE.

**SIMILAR TRIANGLES**

Study Guide II

1. Needed equipment: Squared paper, mirror, ruler, protractor and tape measure.

2. Student activities:

![Fig. 19](image-url)
2.1 Place a mirror on the floor of your school room at some point M as in figure 19. Stand at some point O so that you can see in the mirror where the ceiling and a wall of the room meet. You may need to move the mirror and you may need to change your position. Keep trying until you can see the line that a wall makes with the ceiling.

2.2 Mark your position on the floor where you were able to see the top of the wall. Measure OM, MA and OE (the distance from the floor to your eyes).

2.3 Make a scale drawing of the triangle EOM. Measure the angle EMO. Complete the figure by drawing the triangle EMA with the angle AMB equal to the angle EMO. What kind of angles are MOE and EAM?

2.4 Determine height of the room AB from your scale drawing.

2.5 Check your results by measuring the height of your room if this is possible. Measure the angles MBA and EOM.

2.6 Record your measurements in the appropriate places of the following tables.

| TABLE XIII |
|------------------|------------------|------------------|
| Triangle | Angle | Angle | Angle | Side | Side | Side |
| EOM     | MOE=  | OEM=  | EMO=  | EM=  | OM=  | OE=  |
| BMA     | BAM=  | MBA=  | AMB=  | MB=  | MA=  | AB=  |

| TABLE XIV |
|------------------|------------------|
| BA=              | MA=              | BM=              |

2.7 How are the corresponding sides of these triangles related?

2.8 How are the corresponding angles related?
SIMILAR TRIANGLES

Study Guide III

1. Needed equipment: Candle, rectangular cardboard, tape measure, yard stick and two holders.

2. Student activities:

![Diagram of similar triangles with labels L, C, O, A, B, E, F, A', B', E', F', O', S, S', C'.]

FIG. 20

2.1 Figure 20 shows a candle (or an electric lamp) and a cardboard rectangle placed in a line CS that is perpendicular to the blackboard. Fasten the cardboard rectangle in a holder two feet from the lighted candle which should also be secured. Darken the room and let the shadow of the object fall on the blackboard or a wall. Draw the outline of the shadow. Raise the blinds but don't move the candle or the object.

2.2 Measure LA, LB, LE, LF, CO, LA', LB', LE', LF' and CS.
2.3 Measure $AB$, $BE$, $EF$, $FA$, $A'B'$, $B'E'$, $E'F'$ and $F'A'$.

2.4 Record the measurements that are needed in the table below.

<table>
<thead>
<tr>
<th>$\frac{LA'}{LA}$</th>
<th>$\frac{B'E'}{BE}$</th>
<th>$\frac{OO'}{CO}$</th>
</tr>
</thead>
</table>

2.5 You compared three ratios of corresponding lines in the table above. How do these ratios compare?

2.6 Make comparisons with other pairs of corresponding lines that are not included in the table.

2.7 Compare the angle $LBE$ to the angle $LB'E'$.

2.8 How are the triangles $LBE$ and $LB'E'$ related?

2.9 How is the object $ABEF$ related to the shadow $A'B'E'F'$?

3.0 Do you think a figure of a different form could be used for this experiment?

Chapter four has been entirely devoted to the preparation of illustrative study guides for learning geometric concepts and principles. The teacher may offer helpful suggestions in the use of the study guides to students who need such assistance. Class discussions should readily follow as a result of experiences that the students will have in the procedure of each of these guided studies. The use of these study guides should contribute to the teacher's evaluation of the student's work so that fewer formal tests need be given.
CHAPTER FIVE

STUDY GUIDES FOR
FIELD WORK PROJECTS IN THE TEACHING OF
HIGH SCHOOL MATHEMATICS

The study of mathematics through field work projects is perhaps the liveliest kind of study in high school. Students like to do this work. Even those students whose advance has been slow in the textbook work, take interest in measurement activities afield.

PLANNING NEEDED.

The class should be organized into groups of three to five pupils with each group selecting a leader. The members of each group should discuss the statement of a problem and its formulation with the teacher. Instruments and apparatus that are needed for a project in the field should be selected with the guidance of the teacher. Also each student will have a definite part of the work to do as his share of the responsibility in the problem solution. Such activities as taking measurements, setting arrows, recording data, holding range poles and returning the apparatus are some of the essential duties that students must share. It is advisable to limit the membership of a group to five so that there will be work for each one to do. If much field work is done, these groups should be dissolved after a few weeks and a new group or committee formed in order to give as many pupils as possible opportunity to exercise leadership and to assume re-
sponsibility. This regrouping prevents clannishness also.

As was pointed out in chapter four, the big problem is that of young students learning the principles of democratic living by practicing those principles as they work together for a common purpose.\(^1\) Field work projects provide many opportunities for pupils to acquire the desired qualities of personal living that are so necessary for the democratic way of life. A group of students at work on a field project is a democratic social unit. A pupil must be sensitive of his position as a member of such a group and prove his acceptability through his attitude of kindliness and solicitude for others.

There are many aesthetic appreciations which can be shared by students when working outdoors in groups. Being at work out of doors on a nice day; the preparation of a display of mathematical work; the making of mathematical apparatus and illustrative forms; and making neat and unique drawings are some examples of sources of common appreciation for aesthetic values that can be enjoyed.

There must be a lot of give and take when planning and working in field projects. Mistakes may be made in taking measurements, incorrect recordings of data may be made, apparatus and material may be lost or damaged, or wrong units of measure may be used. There is ample opportunity in all

such experiences for the pupil to forget his selfish desires and share difficulties with his fellows, even though he is not the one who made the bad measurement or broke the angle mirror.

The field work project is not the responsibility of any one student, but all members of the group work together through all the phases of the problem solving with mutual interests for common ends. Here the student learns how to cooperate with his fellows and also learn the value of cooperation for democratic living.

Each student must think of what he is doing as a member of a group and he must direct his own activity for effective living in that group. He is free to choose his activity but he soon learns that his freedom is intermeshed with the rights of others.

Many new ideas for, and ways of doing things are proposed by individual members when pupils discuss their common difficulties in a project. Thus every pupil is afforded a good opportunity to learn new and different ways of doing things and he thereby grows in creative ability.

There is wide opportunity in outdoor mathematics projects for students to learn about the important phases of problem solving that were discussed in chapter four. Here, too, the student may learn how to think effectively through guided activities that are not so restricted as are those of the classroom or mathematics laboratory.
The work in the field provides the data for the solution of the problem and these data, along with the plan of solution should be reported on a convenient form. One very good field work project report sheet for high school use is the one used by Dr. Harold P. Fawcett in his field work classes at Ohio State University. This form for reporting field work projects has been revised several times by interested teachers with a view of having it serve the best interests and needs of students. This field work report form and a brief discussion of its various items are given herewith in view of its possible usefulness to teachers of field work mathematics.

REPORT ON FIELD WORK PROJECT
(Page one)

Date started Name Members of Group
Date completed Name Members of Group
Time in the field, hours Name Members of Group
Additional Time to Complete Project, hours

The Problem:
Method to be Used Equipment Used

Rough Sketch. (Page two)

Name
Linear measurements to the nearest
Angular measurements to the nearest
Data needed:
Solution:

(Page three)

Name
Scale Drawing:
<table>
<thead>
<tr>
<th>Name</th>
<th>Special conditions causing error:</th>
<th>Probable effect of these &quot;special&quot; errors:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major mathematical ideas used:</td>
<td>New skills acquired, if any?</td>
<td></td>
</tr>
</tbody>
</table>

Comments:

Special difficulties encountered, if any:

What other projects does this suggest to you?

The report gives the names of the students in each group together with their leader. A brief statement of what work each student did is of importance both for purposeful teaching and for evaluation.

The time used to do the field work and to complete the project reflects, to some extent, the interest and efficiency of the student group.

There is provision for the statement of the problem, for a statement of the method used in solving the problem, a description of the equipment that is needed and a rough sketch to provide direction for the working procedure in the field.

The report shows how appropriately the student has chosen linear and angular units of measure. And it also reflects to what extent precision in measurement has been attained.

The nature of the data that the student has gathered is an indication of how clearly he understands the requirements of his problem and its formulation. Students often take measurements and collect other data that are either irrelevant
or superfluous or both. Then too, there are instances where students will omit the gathering and reporting of data that are absolutely essential for the solution of the problem.

The report of the solution reflects the ability of the student to use the data and to employ appropriate operational skills. Many of the students' deficiencies can be observed in this phase of the report.

The scale drawing often helps the student to formulate and to check his conclusions. It is a valuable device for learning relationships of similar figures and for acquiring the concepts of ratio and proportion.

The report on special conditions causing error and the probable effect of these special conditions, reflects how the student observes his problem situation as to the influence of imperfections in instruments, type of physical conditions and weather.

When the student is asked to report the mathematical ideas used, he is thus required to review all of his activity in solving the problem. The formulation of definite statements of the mathematical principles used is therefore an excellent review exercise for the student.

Unusual difficulties such as too little time, objects in the way, broken instruments and the like, may often arise. These things can be reported as a part of the total problem situation.

That part of the report concerning other projects that
were suggested by the problem, shows how the student views the relationship of his problem to other related problems and reveals, to some extent, whether or not he is thinking relatedly.

The guide sheets of this chapter are offered as suggestions for teaching some of the basic concepts and important principles of plane geometry through field work. These guide sheets are written for the teacher to use, and are prepared as suggestions as to what can be done in this field. It is not possible to herein develop guides and plans for all the important geometric concepts and principles found in the selected list in Chapter three. This effort to plan for the learning of important geometric concepts and principles may provide suggestions for futher work of this sort which can be followed by interested teachers.

PROJECTS FOR FIELD WORK.

Study Guide

Proposed projects to illustrate the following theorem:

Two right triangles are congruent if a leg and an acute angle of one are equal to a leg and an acute angle of the other.
1. To find the distance across a river

1.1 The method to be used:
The guide sheet for this field work project explains how to find the distance DP between two points D and P, the latter being inaccessible. There are directions for laying out the two congruent right triangles DPN and NEB in the field. Use the report sheet for a summary of the work done by each student.

1.2 A selection of instruments, devices and materials:
A 90° angle mirror or sextant range poles, arrows, tape measure, note book and graph paper.

1.3 The activity in the field:
Select a point P on the directly opposite side of the river. Then with the use of two arrows, establish a line AB approximately parallel with the river. Have one
student hold a range pole at some point along AB and another student move along AB until he finds the vertex of the right angle at D with the angle mirror or sextant. Place an arrow at D. Then from D measure a convenient distance along AB such as DN and place an arrow at N. Next measure NB equal to the measured distance DN and place an arrow at B. With the angle mirror or sextant lay out the line BF perpendicular to the line AB at B. The poleman should place an arrow on the line BF. Then walk along BF until the point N comes in line with the point P. Place an arrow at E. Measure BE.

2. Find the distance a rock in the middle of a stream is from the bank.

3. Two coast guardsmen observe an object in the water near the shore. One guard is stationed at the top of an overhanging cliff and the other one is at the foot of the cliff. Find the distance the floating object is from shore.

4. A ship is sighted at sea by two boys playing along the beach. How can the boys find the distance the ship is from shore?

Study Guide II

Proposed projects to illustrate the following theorems:

If a straight line is drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally. (Similar triangles are formed.)

1. Find the distance from a cottage, C, along the shore of a lake to a tree, T, on an inaccessible island in the lake. Also find the distance from a lodge, L, along the shore to the same tree.
FIG. 22

1.1 The method to be used:
Preparation is made for finding the two unknown distances by the method of similar triangles.

1.2 A selection of instruments, devices and materials:
A sextant or a three, four, five rope triangle, tape measure and several arrows.

1.3 The field activities:
Sight L from C and mark two points along CL, such as K and H, with arrows. Erect FH perpendicular to CL at F. Also erect EG perpendicular to CL at E. Measure off FK along FH making FK any convenient length. Then measure along EG to S making ES equal to FK. Draw SK and produce SK to meet TC also produced at the points X and Y. XY is constructed parallel to CL. Measure the lengths of the line segments CL,
XY, CX and LY. The line segments CT and TL may now be calculated by proportion.

1.4 Make a scale drawing of this figure and compare the lengths of the line segments CT and TL, as shown on your scale drawing, with the computed values.

1.5 Make a generalization from the data of this experiment.

2. Find the height of a flag pole using a range pole or stake and tape measure.

3. Find the distance across a lake when one end point is inaccessible. Use range poles, arrows and tape measure.

4. Find the height of a tree with the hypsometer.

5. Find the height of a tower by means of shadows and a tape measure.

6. (a) Find the height of a power pole using a quadrant with a 45° triangle.
   (b) Find the height of your school building with a 45° triangle.
   (c) Find the height of a chimney with a 30-60 degree triangle.

Study Guide III

Proposed projects to illustrate the following theorems:

In a right triangle the perpendicular from the vertex of the right angle to the hypotenuse is the mean proportional between the segments into which it divides the hypotenuse.
1. To find the height of a building.

![Diagram of a building with an angle mirror setup](image)

**FIG. 23**

1.1 The method to be used:

The use of the angle mirror enables the student to find the position of the vertex of the right angle. The distance from the vertex of the right angle to the line that is to be measured can readily be found. Practice will be necessary in the use of the angle mirror before measurements can be taken.

1.2 A selection of instruments, devices, and materials:

An angle mirror, tape measure, graph paper and sextant.

1.3 The activities in the field:

Hold the angle mirror so that the handle of the mirror is in the horizontal position. Observe the ground line (base line) of the building at point C through the window of the mirror until the image (this takes practice) of the top edge (point B) of the building comes into coincidence with the point C. Measure the length of AX which is the height of the student making the observation.
Hold the mirror at that point and measure CX along the ground, if that is convenient, to get the length of AD. If two students are working together, the tape may be stretched from the observer's nose or eye position to the wall to get the length of AD directly.

1.4 Make a scale drawing from the data.

1.5 (a) Determine the height of the building from the scale drawing.

(b) Calculate the length of DB since DB is the unknown quantity in the proportion:

\[
\frac{CD}{AD} = \frac{AD}{DB}
\]

Then the height of the building may be found from the formula: Height = CD DB.

(c) Formulate a geometric principle from the data and solution of the problem.

2. Find the distance from a point on the shore of a lake to an island in the lake using a carpenter's square, a stake and a tape measure.

3. Find the distance between two points A and P on opposite sides of a river. Lay out a line through A and perpendicular to AP along one side of the river. Call this line XY. Sight P from two points on XY so that the sum of the two base angles along XY is 90°. Measure the distances of the two points on XY from A. Find AP.
Study Guide IV

Proposed projects to illustrate the following theorem:

An angle inscribed in a circle is equal to half the
central angle having the same arc. (An angle inscribed in a
semicircle is a right angle.)

1. To lay out a circle one hundred feet in diameter.

![Diagram of a circle with points labeled P, P', P'', P_3, P_4, P_5, and C.]  

FIG. 24

1.1 The method to be used:

The end points of the 100 foot
diameter can be sighted with an
angle mirror and the position of
the angle mirror will be the vertex
of the inscribed right angle in a
semicircle. Therefore the points
of the circumference of the circle
can be found by moving to different
positions.
1.2 A selection of instruments, devices and materials: Range poles, several arrows, tape measure, graph paper, protractor and angle mirror.

1.3 The activities in the field: Lay out a line CD on the school lawn one hundred feet long and mark the ends with arrows. One student should hold a range pole at the point C and another student should hold a range pole at D. A third student carries arrows with him to mark the vertices of the right angles when they are located. A fourth student finds the various points that establish the circumference of the circle. To find these points, the observer looks through the window at either the range pole at C or at the range pole at D. The observer located the points (when the image of one coincides with the other pole being observed) by walking along "about" where he thinks the circumference of the circle should be and moves "in and out" until the image of the pole moves to the correct position.

1.4 Prepare a scale drawing from the data: (a) Measure the inscribed angles at the various points from your drawing. (b) How many degrees are there in a central angle of this circle that has the same arc as one of the inscribed angles? (c) Write your generalization for this experiment.

2. Find the center of a large circle with a 45° angle mirror.
Study Guide V

Proposed projects to illustrate the following theorem:

Two triangles are similar if the two angles of one are equal respectively to the two angles of the other.

1. To find the height of a flag pole.

FIG. 25

1.1 The use of the hypsometer to measure heights of objects affords ready means to solve the problem. The use of the hypsometer will no doubt suggest more student activity for studying its construction and the geometric principles that it involves.

1.2 A selection of instruments, devices and materials:

A tape measure, one arrow, a Jacob's staff or stake, plumb bob, a sheet of squared paper or hypsometer.

1.3 The activities in the field:

If an hypsometer is not available one can be easily and quickly provided. Cut out a piece of squared paper ten
inches wide and fifteen inches long. It is best to use coordinate paper graduated to tenths of an inch. Then paste this ten inch by fifteen inch sheet of squared paper on a piece of stiff cardboard or thin plywood cut to the same size as the squared paper. In the figure, ab is ten inches long and be is fifteen inches long. The plumb bob is attached at a. As the upper edge of the instrument is pointed to C, the plumb bob cord moves along be to C. Where C is (along be), depends upon the angle of elevation BAC. Mark the ab scale from 0 to 100 with 0 at a and 100 at b. Then mark the be scale from 0 to 150 with 0 at b and 150 at e. Measure off a one-hundred foot length from B and mark with an arrow. Nest set up the hypsometer at A and sight C along the upper edge of the instrument. Next take reading along be where the plumb bob cord intersects be. This is a direct reading (in feet) and is the length of BC. The height of the flag pole is therefore BC = AE, AE being the height of the instrument.

The angle bac equals the angle BAC since their sides are perpendicular each to each. The angle at b is a right angle and the angle at B is also a right angle. The triangles bac and BAC have two angles of one equal to two angles of the other. The hypsometer can be combined with, clinometer, slope table and graphic sine, cosine and tangent table. This combination of devices is not especially difficult to make.2

1.4 Make a scale drawing of the figure ABC.

1.5 How would the height of an object be determined if the length were eighty feet?

2. Shuster and Bedford, Field Work in Mathematics, p. 47.
1.6 (a) How would the height of an object be found if the distance $AB$ were two hundred feet?

(b) Have the student write a generalization of the geometric principle involved in solving the problem.

(c) What mathematical principle would enable a student to find the height of an inaccessible object with the hypsometer?

2. Find the distance, $AB$, across a swamp by constructing two similar right triangles. Let $AB$ be a leg of each of them.

3. Find the distance, $AB$, across a pond by constructing a line $XY$ parallel to $AB$. Join $A$ and $Y$ with a line and also draw a line through $B$ and $X$.

Study Guide VI

Proposed projects to illustrate the following theorems:

(a) The area of a trapezoid is equal to one-half the sum of its bases times its altitude.

(b) The area of a triangle is equal to one-half the base times its altitude.

1. To find the area of a lake by the offset method.
1.1 The method to be used:
This is the method of off-sets.
Parallel base lines are run. Then
perpendiculars are erected to these
base lines. Find the total area of
the rectangle thus constructed and
from this subtract the sum of all
the land areas to get the area of
the lake.

1.2 A selection of instruments, devices
and materials:
A sextant, surveyor's cross, range
poles, arrows, drawing paper and
measuring tape.

1.3 The activities in the field.
If a lake or pond is not available
one can be "made" by marking off a
sizable area on the school grounds
by using lime to work the "shore line"
of the "lake". Beginning at A con-
struct AD and AB perpendicular to
each other. Measure AD and AB. Then
erect perpendiculars along AB at con-
venient intervals. Measure the lengths
of all the perpendiculars along AB and
DC. All the resulting figures, except
two, are nearly trapezoids in shape.

1.4 Compute the trapezoid areas. The
areas of the figures EAF and GHG can
be found by the formula for the area
of a triangle. Find the area of the
rectangle ABCD and from this area
subtract the sum of the areas of the
small figures on the border of the
lake.

1.5 Make a scale drawing of the lake.

1.6 The student should make two general-
izations of the geometric principles
used.

2. Find the area of a park surrounding a lake.

3. Find the area of an irregularly shaped flower bed.
Study Guide VII

Proposed projects to illustrate the following theorem:

In any right triangle, the perpendicular from the vertex of the right angle on the hypotenuse divides the triangle into two triangles each similar to the given triangle.

1. To find the shortest or perpendicular distance from an inaccessible island in a lake to a straight road near the lake.

![Diagram of island, lake, and road with points A, D, B, and C labeled.]

FIG. 27

1.1 The method to be used:
A right triangle is constructed with the vertex of the right angle at C and the hypotenuse along the road AB.

1.2 A selection of instruments, devices and materials:
A transit, arrows, range poles, tape measure and drawing paper.

1.3 Activities in the field:
Select a point A along the road. Set up the transit and measure the angle BAC. Suppose the angle BAC to be 55°. Then set up the transit and sight the island making the angle 35° at B. Mark the points A and B with arrows. Then move
along \( AB \) and locate the perpendicular line \( CD \) and mark the point \( D \) with an arrow. Then measure the line segments \( AD \) and \( DB \).

1.4 Make a scale drawing of the figure.

1.5 Compute the length of \( CD \). Compare the computed length of \( CD \) with its length as shown on the scale drawing.

2. Divide a field that has a right triangular shape into two similar right triangles.

3. Lay out a circle that is determined by three trees \( A \), \( B \) and \( C \) not in the same straight line. Lay out a diameter of the circle through \( B \). Find the distance from \( A \) to the diameter through \( B \).

**Study Guide VIII**

Proposed projects to illustrate the following theorems:

(a) The tangent to a circle at a given point is perpendicular to the radius at that point.

(b) When a transversal cuts two parallel lines, the corresponding angles are equal.

1. To find the latitude by observation on the sun.
1.1 The method to be used:
A study of the discussion and the figures found in the guide is essential for an understanding of the problem. Student discussions should preside the work of measurement.

1.2 A selection of instruments, devices and materials:
A sextant and drawing paper.

1.3 The activity in the field:
Follow the directions for "shooting the sun" as given by Shuster and Bedford. 3

The most common way of finding latitude is by measuring the altitude of the sun at high noon, the time when the sun reaches its highest point in the sky. The details of the method of "shooting the sun", to make use of the nautical term are as follows: Hold the sextant vertically with the right hand and look through the eye-piece and $M_2$ at the horizon. Then adjust the index arm with the left hand so that the image of the sun, reflected from $M_1$ to $M_2$ and through the eye-piece coincides with the horizon. By moving the index arm, follow the sun as long as its altitude increases and record the altitude at the moment it starts to decrease, (The greatest angle is the sun's altitude at high noon).

1.4 Make a drawing of each of the three possible situations.

1.5 Relations to study:
(a) The angle $X$ = angle $Y$ since they are corresponding angles.

(b) $CP$ is in the plane of the horizon and is perpendicular to the radius at $O$.

3 Ibid., pp. 72-75.
(c) In figure twenty eight:
Sun over the equator.
1. Angle X is the latitude.
2. Angle X = angle Y.
3. Angle ACP = 90°.
4. There is no declination.
5. Latitude = 90° - the observed altitude of the sun.
6. If the observed altitude of the sun is 50°, latitude = 90° - 50° = 40°.

In figure twenty nine the sun is north of the equator:
1. Angle X = Angle Y.
2. The declination must be added to angle X to find the latitude.
3. Therefore latitude = 90° - observed altitude plus declination.
4. If the observed altitude is 42° and the declination is 6°, latitude = 90° - 42° plus 6° = 54°.

In figure thirty the sun is south of the equator.
1. Angle X = Angle Y.
2. But angle X is greater than the latitude; hence the declination must be subtracted from the angle to get the latitude.
3. If the observed latitude of the sun is 48° and the declination is 12°, latitude = 90° - 48° - 12° = 30°.

(d) There are five corrections to make to find the latitude:
1. Declination of the sun. The sun's declination is the angular distance of the sun from the celestial equator or the angle between the plane of the earth's equator and the sun's rays. See figures fourteen and fifteen. The sun has no declination on March 21 and Sept. 23rd. The sun's declination for all other days
of the year can be
gotten from the nautical
almanac or from the
World Almanac.

2. The semidiameter is the
the error made by sight-
ing the "lower limb" or
lowest part of the sun
instead of the exact
center which is diffi-
cult to locate. If the
lower edge of the sun's
disk is made coincident
with the horizon add 16'.

3. The refraction error is
caus ed by the rays of
light being bent toward
the perpendicular on
entering the atmosphere.

4. Dip is the error caused
by the observer being
above the surface of the
water. To correct for
dip, find in feet the
observer's height above
sea level and subtract
from the observed alti-
tude an angle equal to
the square root of the
observer's height.

5. Parallax is the error
introduced by making
observations from the
surface instead of from
the center of the earth
where, in theory, they
are supposed to be made.

2. Find the latitude of your home by making an ob-
servation on Polaris.
Study Guide IX

Proposed projects to illustrate how several geometric concepts and principles may be learned through the experience of making a map. Some of these concepts and principles are:

Point (intersecting lines), two points determining a line, direction, unit of length, drawing to scale, similar figures, level, horizontal, plane, vertex, vertical, perimeter, angle, relation of sides of similar figures, relation of angles of similar figures, and center of radiation.

1. To make a map by the method of radiation.

1.1 The method to be used:
This is a good method to use where there are no large objects. Students should be required to repeat the work in cases where there is disposition not to follow instructions.

![Diagram of a map made by the method of radiation]

FIG. 31

1.2 A selection of instruments, devices and materials:
A plane table, alidade, magnetic compass, large sheet of drawing paper, drawing pencil, thumb tacks, plumb bob, level, range poles, arrows and measuring tape.
1.3 The activities in the field:

The plane table is essentially a drawing board mounted on a tripod. A sheet of drawing paper is fastened on the board with thumb tacks. The plumb bob cord is fastened to the underside of the face of the tripod and it marks the point of radiation. Level the plane table and mark the point of radiation on the drawing paper. Also check the north-south direction with the magnetic compass (correct for magnetic deviation) and draw a north-south line along the edge of the paper or bring one edge of the drawing paper into the north-south position and mark this on the paper. Be sure that the board is clamped so that it will not revolve. If there is no clamp adjustment, care must be exercised not to move the board in any manner once the position of the board for the making of the map has been determined. Stick a pin at the point 0 and bring the alidade or pointer against the pin. Now point the alidade to any corner of the area and draw a line in that direction. One student should hold a range pole at the corners as they are sighted. Two students will measure the distances from the point on the ground beneath the plumb bob to the corners sighted. Arrows should be used to mark all corners and objects. All objects in the area such as trees, rocks, etc. can be shown on the map.

1.4 Make a scale drawing from the data.

1.5 The students should try to point out the geometric concepts and principles. A list of them could be made in a class discussion period. Some of these geometric concepts and principles may seem to be difficult to find. However, the list as given at the first of this guide sheet can be identified.
2. Make a map by the method of intersection.

3. Make a map by the method of progression or traversing.

Many field work projects could be planned for the study of algebraic trigonometric and physical concepts and principles. The illustrative projects that have been suggested in this chapter are limited to the selected list of geometric concepts and principles as explained in chapter three. A compilation of study guides covering a wide range of mathematical principles for secondary school use should prove to be a valuable aid to many teachers.
CHAPTER SIX

SUMMARY AND RECOMMENDATIONS

Students often inquire whether they will ever use the material that they are studying in school. And teachers not infrequently have difficulty to the point of embarrassment in trying to answer such questions. A laboratory and field work program not only provides the student with opportunities for learning important generalizations in subject matter, and methods for rational thinking but also helps to develop a democratic philosophy for daily living. When students have opportunities to participate in effective ways of thinking, and in the steady development of knowledge and skills through their own guided efforts, they are likely to have a more satisfactory appreciation of the worth of their school program. This is perhaps one justification for choosing laboratory and field work as a topic for study.

An important phase of the problem for this study has been the selection of geometric subject matter appropriate for the educational development of students. Authoritative studies in the field of geometry have been examined for the purpose of selecting a suitable list of concepts and principles for laboratory illustrations and field work projects. Fagerstrom's study revealed the relative importance of the geometric facts and theorems that appeared in the solutions of the problems of Granville's calculus. Congdon's investigation showed to what extent certain geometric concepts
and principles for the study of geometry. Christofferson's list of "Essential Theorems" was determined from his investigation of the needs of prospective teachers of geometry. Therefore the selection of an appropriate list of geometric concepts and principles for use in laboratory and field work was largely influenced by the significant findings of these studies.

A number of study guides have been prepared which are intended to be illustrative of how geometric truths may be studied through pupil activity. It is likely that some changes in their structure may be desired to meet the needs of a particular teaching situation.

A guided activity program such as has been suggested herein for the study of mathematics, has provision for the social development of the student for living in a democracy. The nature of the activities provided should enable the teacher to direct students toward the acquirement of personal qualities for living in a society where the rights and responsibilities of the individual citizen are of paramount importance. An effort has been made to direct the pupil's activity so that his learning experiences will be rich in the art of solving problems and complete in the fullness of democratic living.

This program for learning through experience will better enable students to know how to sift the important problems in daily living from the many that arise and to formulate solutions for them. Thinking, then, in terms of observed data and
the relationships that are apparent between them will be more satisfying and reliable for the formulation of conclusions than is the method of relying on unsupported beliefs and prejudices.

RECOMMENDATIONS

This study has shown that a wider range of the content of secondary school mathematics should be included in a program of pupil activity. Associated with this problem are related problems which call for further study. Among these are:

1. Effective ways of educating school executives, boards of education, and parents concerning the values to be derived from a program of pupil participation in the study of mathematics.

2. Acquainting principals, heads of departments and supervisors with the need of longer periods than the traditional class time if laboratory and field work in mathematics is to be an effective part of the program.

3. Effective ways of educating school executives and boards of education concerning the need for apparatus and equipment for the mathematics laboratory.

Further study is also needed to implement the numerous worthy suggestions and recommendations as found in the literature, with appropriate teaching techniques.
BIBLIOGRAPHY

BOOKS


BIBLIOGRAPHY

TEXTBOOKS


BIBLIOGRAPHY

PERIODICAL ARTICLES


BIBLIOGRAPHY

THESSES

