A FREE READING PROGRAM IN HIGH SCHOOL MATHEMATICS

A Thesis Presented for the Degree of Master of Arts

by

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A FREE READING PROGRAM IN HIGH SCHOOL MATHEMATICS

CHAPTER I

THE PROBLEM IN PERSPECTIVE

The recent eras of indifference, criticism, and applause which have attended the teaching of mathematics have all had their influence upon the efforts of those so employed. Having weathered the discouragements of the first two, while still retaining the rudiments of the subject in the curriculum, high school teachers are now beset with the temptation to relax and feel that the war-inspired popularity of mathematics is insurance against the recurrence of the cycle of public vacillation. Such ease, however, is not for those who wish to establish a permanent respect for mathematics in the minds of on-coming students. Only as those who have completed their formal instruction find it applicable to and consistent with their needs can the instructors feel that the wares they have to offer will continue to be considered desirable.

Acquaintance with the modern literature in the field of mathematics must surely inspire the teacher to pass it on to students, even on the high school level, since so many here end their formal contact with the subject. If, as will be shown later, the traditional textbook cannot supply this vital material, how then is the teacher of high school mathematics to reflect in the classroom the wealth of modern mathematical writing? Authors of specialized works are often thought to be far advanced beyond the
high school level of thought and verbal expression. Many teachers ask seriously concerning these works: Can such writers appeal to the high school mind? Can high school students understand the vocabulary? Are these topics within the range of high school comprehension? Are these topics of practical value, in and out of school?

The undertaking of this study was an attempt to find tentative, if not final answers to these questions.

It would seem, certainly, that teachers must first realize, and then transmit to students, a concept of mathematics which is broader than mere computation. One who has real enthusiasm for the subject will enjoy "Jacobi's retort to Fourier," as reported by Eric Temple Bell, to the effect that "... the true end of mathematics is the greater glory of the human mind,"\(^1\) and will scorn

\[ \ldots \text{the doddering tradition} \ldots \text{that mathematics is (merely) the science of number, quantity, and measurement. These things are an important part of the material to which mathematics has been applied but they are no more mathematics than are the paints in an artist's tube the masterpiece he paints. They bear the same relation to mathematics that oil and ground ochre bear to great art.} \] \(^2\)

Teachers of geometry, in particular, will agree with Bell, who further argues:

\(^1\)Eric Temple Bell, The Queen of the Sciences, p. 3.
\(^2\)Ibid., p. 16.
To be certain that we have not shifted the subject of discussion in an involved argument, or to know that our initial assumptions do really contain all that we think we are talking about is the crux of the whole matter. . . . The evolution of this excessively abstract view of mathematics has been slow, and it is a characteristic product of the mathematical activity of the past half century.

While nearly all students can be led to appreciate this idealistic view to some extent, the larger number will expect their courses in mathematics to give them, in addition, more practical concepts and to reveal opportunities for the use of mathematics in the serious business of making a livelihood.

The foregoing discussion suggests that teachers of mathematics need to keep in close contact with the scientific, commercial, and industrial uses of mathematics as they are reflected in modern writings and to incorporate these into their classroom teachings with a discriminatory eye towards their permanent value. The problem of how such writings may best be presented to high school students is the chief concern of this thesis.

Granted that the impetus to the study of mathematics engendered by World War II should be capitalized, it remains to be discovered how new applications of mathematics may be introduced to high school students as a means of re-vitalizing the subject. Inclusion in textbooks seems a natural, but not entirely a satisfactory, approach.

3Ibid., pp. 17-18.
The mechanics of textbook production and adoption cause that approach to be slow in inclusion of newer applications. Revisions require time and expense. Educators are conservative, unwilling to risk their reputations on unproved innovations. Furthermore, adoption is slow even after the textbook is ready for market. Under the Ohio plan of supplied textbooks, for instance, boards of education are limited to a five-year period before changing the adopted text.

Then, too, one-text teaching has long been under fire because of the narrow concept which it fosters. Uncritical acceptance of the printed word by the student is encouraged unless he has opportunity to compare other writings which are contradictory or at variance. Choice of topics and style of presentation in any given text must necessarily be limited by the experience and opinion of the author or authors. Hence, these features of the textbook may not be entirely adapted to the local needs, may not be broad enough to allow for individual differences, may not be equally clear to all students.

Aside from the limitations set by a single author, or even by joint authors, there arises a further narrowing influence in the rigid surveillance by political, commercial, religious, and other interests, preventing digression from traditional content in textbooks. Untried or controversial material is frowned upon by certain of these interests as a matter of principle and by others which allow monetary
returns to take precedence over originality of thought.

Once it was decided that students should become acquainted with mathematical material outside their textbooks, of first importance was the decision as to how this desirable objective might be realized. Two channels seemed open. By the first the teacher would read, select, and relay by word of mouth or by mimeographed page, such material as was deemed valuable.

Though such a method might prove inspirational, it was thought to be unsatisfactory for many reasons. The restricted amount of class time would seem to rule out any very large quantity or wide selection of such material, even if the teacher's limited preparation period did not preclude the value of the offering. These and other limitations would, it was felt, render the teacher's presentation less authoritative than first-hand contact on the part of the students. An added deterrent was to be expected in the undeveloped stage of note-taking among high school students. What teacher has not experienced their habits of "forgetting" the spoken word before notes can be made; of selecting worthwhile points at varying rates of speed, thus hampering class progress; their difficulty with the matter of organizing notes in a form which will be meaningful later?

Other factors re-inforced the trend toward the second channel, by which students would use the books themselves. It was anticipated, for instance, that thorough
students, inspired by the teacher's reference to other authors, would want to "see for themselves." Teachers need sometimes to remind themselves that the potential mental growth of some of their students may surpass their own and that it is often restricted by the use of too much "pre-digested" study material. Why not permit good students, or even those of lesser caliber, the privilege of "studying the masters instead of their pupils"?\(^4\)

A further advantage of direct contact was to be found in the fresh viewpoint which specialized writings present. Democratic goals of education demand acquaintance with varieties of thinking, with freedom to accept or reject. It is in the allowing of that freedom that writers in specialized fields often excel, in that they are more apt to be motivated by self-expression than by indoctrination.

More mundane, but still forceful, in the decision to have students use the books themselves, was the consideration of the difficulties involved in the production of duplicated material with student clerical help, incapable of transcribing mathematical manuscript with precision. Available methods of duplication were also restrictive in the opportunity which they afforded for the use of mathematical symbols found in printed material. Further, it was doubtful whether the schedule limitations of the school

\(^4\) Ibid., p. 12.
would allow for production in sufficient quantities to supply daily needs. Even should all these hindrances be circumvented, there remained the copyright laws restricting the use of much printed matter for purposes of duplication.

When it was decided that the books should be placed in the hands of the students, a new question arose: How could enough books be purchased to supply an entire class and at the same time insure the variety of treatment and application which would fit the needs of all students? Class sets of one title were, therefore, ruled out as too expensive, too narrow in their appeal, or too exhaustive of class time if used in greater variety.

The desire to permit students the freedom of choice in their study of mathematical applications also contributed to make the book collection one of single or duplicate copies and to place the whole project on the basis of a library activity, even though sponsored in the classroom.

At this point the problem in perspective seemed to be threefold:

1. The work of the teacher.
   1.1 What related studies could be found?
   1.2 What plans were to be laid?
   1.3 What guidance ought to be given?
   1.4 How might results be checked?
2. The selection and care of materials.
   2.1 What were the sources of supply?
   2.2 What criteria should be the basis for selection?
   2.3 How should the book collection be built?
   2.4 What routine methods could be established to prevent damage and loss?

3. The benefits to the student.
   3.1 How should he learn of modern writings?
   3.2 How should he use them?
   3.3 What evidence of learning could he show?

The second chapter will deal with these points, although for the sake of continuity the order of discussion will vary somewhat from that listed above.
CHAPTER II
A RECORD OF THE PROCEDURE

Survey of Related Studies

Before setting the project in motion it was desirable to know what others had done in the direction of enriched programs in mathematics. While the subject of enrichment has received much attention in recent years, there seems to have been little or no attempt to implement it through free reading, especially during the class hour. A few years ago Arthur Eugene Wohlers¹ advocated voluntary reading for his mathematics classes but confined it to an out-of-class activity.

Substitution of the word voluntary for free often occurs in contemporary literature on the subject of reading; in fact, one study definitely declares that: "Freedom in choosing what to read should be accompanied by freedom to choose whether to read or not, . . . ."² For the purposes of the present study, however, voluntary reading will not be considered synonymous with free reading, since it seems inconceivable to the author that a reading program of any kind can exist for a student who elects not to read and remains unpersuaded to engage in reading.


A definition more in keeping with the spirit of the program herein described is to be found in the words of La Brant, who speaks of a free reading program "which permitted the children enough freedom in their reading to make that reading vital and significant to them."3 After summarizing many notions of free reading in her master's thesis, Evelyn McCowan concludes that, "A reading program which permits the youngster to read without restriction and yet offers the most understanding type of guidance will probably yield the most desirable results."4 In writing for the American Council on Education La Brant uses further descriptive terms in referring to: "a general trend toward the acceptance of free, wide, extensive, or diversified reading as a part of the English course . . . collateral, home, or leisure reading (as it is variously called)"5(parentheses La Brant's).

Strang comments on extent of reading, only, in the statement:

3Lou L. La Brant and Frieda M. Heller, An Evaluation of Free Reading in Grades Seven to Twelve, Inclusive, p. 10.


5Lou L. La Brant, "American Culture and the Teaching of Literature," pp. 200-201, Reading in General Education.
Many present day teachers in the content fields prefer to develop their courses by means of much reading of supplementary materials rather than through intensive reading of a textbook, and the marked increase in valuable supplementary materials has made it possible for them to develop this tendency.\(^6\)

She draws instances from the fields of history, science, and occupations, but does not mention the use of supplementary materials in mathematics. Other writers, though, have suggested the advisability of encouraging pupils to read mathematical works other than their textbooks. Hartung and Trimble approve such reading but evidently do not consider it common practice, since they refer to it as "unconventional teaching." They hold however, that the objectives which suggest unconventional procedures are more fundamental than many of the conventional objectives, hence deserve greater emphasis. They also stress the student's need for guidance in the selection of books and articles about mathematics and mathematicians.\(^7\)

The guidance element of a free reading program is acknowledged by such authorities as Witty and Kopel but they would give direction to its operation, toward promoting independence among the readers:

\(^6\)Ruth M. Strang, Problems in the Improvement of Reading in High School and College, p. 36.

The (free reading) plan seems to have unusually significant implications . . . (It provides) a setting in which there is freedom for children to follow various interests through printed materials chosen from widely different sources on many levels of difficulty . . . . children require guidance; but to be really successful the entire program should be so devised that increasing amounts of self-guidance will be exercised. A free reading program intelligently conceived is rich in opportunities for this type of development. In this program the teacher's obligation becomes greater (even though his role is less obvious), his insight must be keener, and his interest more sincere and sympathetic . . . . The teacher does not arbitrarily state that some books are "reference or source books of distinction" and that others are "worthy books for collateral reading." The merit of a book lies almost entirely in its suitability in providing the child with information and experience which will be useful for him.  

The plea for guidance is strengthened by Anderson's observation concerning reading interests:

But quantitative studies which show how much and what kind of reading is done do not necessarily reveal true reading interests. Such studies assume that one is interested in what he reads and that one is not interested in what he does not read. Recent investigations show clearly that people's reading choices are conditioned by a number of important factors hitherto overlooked, such as availability, accessibility, and readability.  

Drawing upon all these sources, and with a conscious attempt to use the expression in conformity with the custom of contemporary writers, the author of this study

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8 Paul Witty and David Kopel, *Reading and the Educative Process*, p. 158.

has formulated the following specifications as component parts, to be interpreted as a definition of "A Free Reading Program in High School Mathematics":

1. An attempt on the part of the one responsible for it to supply a sufficient variety of printed material to provide for the present desires and expected development of mathematical tastes of the participating students.

2. A period in which students will exercise these desires and tastes as they develop.

3. A guidance service which will operate not only at the student's request but at the teacher's recognition of a need for it.

4. A means by which the reader may give evidence of the value he has received from the program, both by demonstration of his increased knowledge and by his own evaluation of his activity in reading.

5. A plan of revision aimed at repeating its successes and eliminating its failures as revealed in the practice of the four preceding specifications.

For a number of years Dr. Harold Fawcett has selected and assigned readings to students in his course on "The Nature of Proof," given at the demonstration school of Ohio State University. He found also that some of them read voluntarily references other than those assigned.\(^{10}\)

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It was from acquaintance with his practice that the present study was inspired. The determination to place this project entirely on a free reading basis is not to be construed in any way as a criticism of his policies, but rather a desire on the part of one less experienced to discover what pupils could profitably read and a growing conviction of the truth of a philosophy of education fostered by one school superintendent who advertises his city system as a place "Where Children Educate Themselves with the Help of Good Teachers."\footnote{11}

With particular reference to mathematical education Trimble seemingly holds to that philosophy when he states:

There are many popular books which would help pupils to broaden their experience in the field of mathematics. . . . Such reading may be done quite incidentally by some pupils, and perhaps not related at all in their minds to the courses which they are taking in science or mathematics. Other pupils will never come in contact with books of this type. If we, as teachers, want to provide an opportunity for growth through reading in science and mathematics, I believe we should encourage our pupils to read materials of this sort.\footnote{12}

Some specific procedures for guiding the reading of high school students are suggested by Dora V. Smith when she says:

\footnote{11}{Harold R. Maurer, Faculty Bulletins, Garfield Heights City Schools, Garfield Heights, Ohio, 1937-38.}

\footnote{12}{H. C. Trimble, "Types of Growth that may be Stimulated through Reading in the Secondary School," \textit{READING AND PUPIL DEVELOPMENT}, Proceedings of the Conference on Reading held at The University of Chicago, Vol.II (1940), p. 39.}
I look forward to the time when we shall break up our scheme of daily recitation by having certain days for 'development lessons,' the teacher reading aloud or the pupils reporting to the class upon materials, which would lead to the establishment of standards for evaluating different kinds of books. On these days the lesson hour will become a starting point for individual reading activities to follow. Then there will be 'workshop days,' in which pupils will read or gather evidence or prepare to dramatize, to read aloud, or otherwise share their findings with the rest of the class.  

These suggestions would seem to indicate that most of the actual reading on the part of the students was to be done outside the classroom. However, the books referred to by Miss Smith were not of a mathematical nature and since, in reading mathematical material, pupils accustomed to rapid reading may become confused when asked to act upon earlier sentences in order to comprehend later ones, ... (the) pupil must be taught to try to solve his difficulties in understanding one clause or sentence by returning to prior statements skimmed over in the first reading.  

If this advice is to be heeded, it is the opinion of the writer that pupils would need a great deal of opportunity for fixing the habit under the guidance of the teacher. But, as Trimble says:

Opportunities which different teachers have for making contributions to the growth of different classes and of individuals in those classes will vary greatly. Different approaches work with different groups ... (We are) dealing with a function of many variables ...  

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14 Hartung and Trimble, loc. cit.  
15 Trimble, loc. cit.
Environmental Factors in the Experiment

Since some of the variables with which this project dealt might be said to be the environmental factors which surrounded it, a description of these might well precede any discussion of the mechanics of executing the plan. The high school in which the experimenter taught was one of approximately five hundred students, distributed through the upper six grades. As part of a township system, it was situated in a section of Ohio which was adequately though not generously supplied with school funds from local taxation.

Twenty-eight per cent of the graduates of the school normally continue their education in colleges and universities, as compared with an average of fourteen and nine tenths per cent for the entire state.\(^\text{16}\) With a populace of greater-than-average interest in sending its children to college, the high school offered the traditional mathematics courses required for college entrance. Since the community was agricultural rather than industrial, little need was felt for the modified courses with applications specially adapted to the various trades. One small section of ninth grade mathematics was devoted to those students who did not wish to take algebra or were advised against doing so.

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\(^{16}\)The Lincoln Library of Essential Information. 1944 p. 1674.
The mathematics classes were scheduled with three teachers, full time being devoted by the writer and one other and two classes of first year algebra being taught by a chemistry teacher. The free reading program was introduced only in classes of tenth grade or above, all of which were taught by the writer. These consisted of three sections of second semester plane geometry and one called "Advanced Mathematics" which dealt only with second year algebra and solid geometry. In all, the rolls of participating classes totaled seventy-one names.

It was the practice of the school to give the Ohio State University Psychological Test during the first semester of each school year to all students in the freshman, sophomore, and junior classes. In addition, scores made by certain students were available from the Otis Group Intelligence Test, Intermediate Form D, which had been given in February, 1944, when those students were in the second semester of the eighth grade. The Cumulative Test Record of Participating Students is shown in Table 1, accompanying Chapter III and a discussion of the implications of the tests may be found in that chapter. At least partial test data were available for all but two or three of the participating students, those who had transferred from other schools too late in the year to take the tests.

Other information would be desirable for the teacher undertaking a free reading program to insure the best
guidance in the matter. Recent scores of reading ability, mathematical comprehension, and computational skills through simple algebra would be helpful in grouping roughly the class members and the available reading material for convenience in making recommendations. Hartung suggested four such groups within a class and gave suggestions for guiding them, in a report to the Committee on Fundamentals of the North Central Association of Secondary Schools and Colleges in 1944. This and other guidance techniques will be discussed more fully in the section of Chapter II entitled: "The Role of the Teacher During the Reading Period."

The Book Collection

Long before the reading program becomes a reality, the preparation for it must have been begun. The collection of a number of books, adequate in quantity and content for use by high school classes, is no small task. If free reading is to flood the arid plains of the traditional mathematics platform, the first specification of the reading program, previously stated, must be looked upon as an antediluvian pursuit on the part of the teacher in charge.

Sources of Supply.--When it was decided, during the preceding summer, that the free reading program would be tried out as opportunity offered, early attention was given

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to the book order to assure the classes sufficient material for free choice. Selections were made from class notes and course bibliographies obtained at the Ohio State University, from the Standard Catalog for High Schools to be found in most high school libraries, from the list of visual and teaching aids issued at New Jersey State Teachers College, and from the pamphlets and brochures mailed by the various book companies. A sixteen page pamphlet listing mathematics books which high school students were reading in the University School of Ohio State University, prepared by Frieda M. Heller in 1938, would also have proved helpful, had it been available at the time of the selection. Education Index, when it can be found in the local library, is a fruitful source of supply for both periodic literature and books, since it contains listings of magazine articles and book reviews. In communities where periodic literature is plentiful the bibliography compiled by William L. Schaaf in 1941 will provide extensive references back as far as 1920.


19 Frieda M. Heller, "Mathematics - queen of the Sciences," in Reading For Background Series, No. 8.


21 William L. Schaaf, A Bibliography of Mathematical Education.
Criteria for Selection.--Although the opinion of the person or persons responsible for the book order must, in the final analysis, control its selection, yet certain criteria can be given careful consideration in order to improve that selection. Among these the following were deemed important in the case under discussion:

1. Recommendation by university or library authorities, or both.

2. Non-technical style of text and tables, charts, or graphs.

3. Likelihood of appeal to more than a narrow group of students on the basis of topics treated.

4. Probable permanence of the importance of topics to life outside the classroom.

5. Teacher's familiarity with the general content.

The first criterion was of cardinal importance in drafting a tentative list although it was expected that the other four would often lead to deletion of some items from the list and, in a few instances would augment it. Direct contact with high school students in the classroom, for example, would tend to make teachers more conscious of the need for non-technical style and widely appealing topics than are those not having such contacts. The need for curbing expenditure, as well as the desire to spend class time to advantage, made the fourth criterion one which could not be ignored. It was thought important for the teacher to possess a general notion of the content of all
books used, in order that the necessary guidance might be
given students in locating references without wasted time.
There existed also a belief that books would be used more
extensively and effectively if the teacher could readily
suggest references of which the students were unaware, yet
pertinent to their interests. (As the program progressed
it was gratifying to note the frequent assistance students
gave by publicizing valuable references they had discovered.)

After due consideration of the factors mentioned, the
list was found to contain twenty-three titles, in addition
to those already available, with duplicate copies requested
for some of the standard works. Subsequent censorship re-
duced the combined orders of library and mathematics depart-
ment to ten titles, and for a time it seemed that the plan
for reading as a classroom activity would have to be
abandoned. However, when the collection had been swelled
by the addition of the teacher's personal copies and by
some few alternate textbooks gleaned from the supply room,
the prospects seemed a little brighter. The collection
had become a matter of chance which somewhat nullified the
criteria employed and did not insure inclusion of the best
from the original request, but at least there would be
"enough to go around."  

22 An annotated bibliography of the books available,
together with additional ones which might have proved
desirable, is to be found in Appendix I.
The Growth of the Collection.-- The caution needs to be added here that no one should postpone the adoption of a reading program for mathematics until the collection shall be considered satisfactory. He should begin, no matter how meager the supply, and administrators, librarians, and others who may hold the purse strings will be faced with the necessity for meeting the demands of enthusiastic students in addition to those of the teacher. Neither should the classroom library be allowed to rest upon its laurels no matter what its degree of completeness may be considered to be at any one stage of its development. The very essence of enrichment lies in the constant revision and addition which will insure timeliness to the reference material. "Modern" is not a word of static connotation, nor is "contemporary" applied eternally to the same literature. New titles MUST be added from time to time if free reading is to remain free.

Plans for Carrying Out the Program

Certain difficulties presented themselves in the structuring of the plan. While the solutions reached are not claimed to be ideal, they were the ones which seemed to work best at the time. Discussion of them here is presented merely as a matter of record and suggestive of the problems which will probably face other teachers who may wish to adopt the plan.
Introducing Students to the Program.--During the first semester it seemed doubtful whether the reading could be engaged in as a classroom activity because of the slow accumulation of books. Incidental reference was made by the teacher, from time to time, to books which related to the subject under discussion. Some few students followed suggestions thus given and actually made use of the books during their study periods or at home. The large majority of students were only vaguely conscious, however, that the contents of the bookshelves in the classroom or in the library had any bearing on their daily preparation for the mathematics course for which they had registered.

When the size of the collection, if not the quality, seemed to warrant the initiation of the second specification for the free reading program, the teacher chose the first Monday of the second semester as a time when all books were centered in the mathematics classroom. A number of them were displayed on the teacher's desk and some introductory remarks were made to give an idea of their contents. The purpose of the reading program was explained as being the broadening of mathematical knowledge and viewpoint through acquaintance with writers other than those responsible for the students' textbooks. It was further explained that such

23See page 14.
reading might enable them to learn about the men who had brought our mathematics to its present stage of development.

At this point the students seemed anxious to handle and examine the books and so occupied themselves until it was necessary to use the remainder of the time for discussion of a schedule by which they could further investigate the material. The teacher suggested that one day each week might be used in this manner without shortening appreciably the explanations and discussions concerning the weekly textbook assignments to which they were accustomed. At the same time, she explained the two programs might be found to bear more relationship to each other if they were carried on simultaneously. Students agreed to, or at least were submissive toward, the idea and expressed their opinion that Monday seemed best suited to the local school program. It was evident from the first that they did not wish to be deprived of the privilege of free reading by all-school activities which were apt to occur later in the week. On the rare occasions when one or another class was so interrupted on Monday, its members always insisted upon the devotion of another day in that week to free reading.

Student Use of Books.—Difficulties having to do with schedule interference included the restricted use of the library which accommodated only forty or fifty students in any one period if overcrowding was to be avoided. Prorated among several departments, this number did not allow
for library visitation by more than a few of the twenty-five members of one geometry class whose study hall periods coincided throughout the day. Added to this handicap were the custom of requiring detailed permit slips for students using the halls during class sessions, and the brief interim which often made it impossible for students to get or return books between classes without tardiness resulting.

Thus, little could be expected from students in the matter of free reading, unless the teacher took the responsibility for providing opportunity to get acquainted with the material at hand. Here was added reason for devoting one day each week to actual reading in the classroom, reserving only a brief time for announcement and explanation of the assignment for the ensuing days that week. At the beginning of this reading day, library copies were checked out to the teacher and brought to the classroom by volunteer students who were among the early arrivals. Since the total number of mathematics titles owned by the library included only twelve volumes it was not thought necessary to record in the library the individual titles. Thus, valuable time was saved. The books were retained in the classroom until all four classes had used them. While this practice precluded the use of these books in the library or study hall, it had the advantage of building regular habits, in that students were led to plan
for extra reading time later in the week if they wished. Those with sufficient foresight familiarized themselves with the schedule of classes involved and reserved books of their choice for use on Monday during periods when none of the four classes was in the mathematics classroom. Or they sometimes made arrangements to "visit" other sections in order to use a particular book at times when regular members of the class were not doing so. "Visiting" students either accepted the responsibility for notifying their study hall teachers, in writing, of the reason for their absence or reported to the study hall and came to the mathematics classroom a few minutes later with a permit slip from the study hall teacher. These details are mentioned to bring out the fact that even a rigid schedule may prove adjustable under co-operative efforts of teachers and pupils with evidence of sincere purpose on the part of the latter.

Checking the Results of Reading. -- When, on the initial reading day, the students manifested a desire to delve into the books without further discussion, the teacher occupied herself with placing on the blackboard a skeleton form for keeping a record of the materials read. There was no intent to mislead students into thinking that the reading period was to descend to the level of time squandering for which they would not be expected to account;
therefore it was thought best to acquaint them immediately with a plan for keeping a record of the activity. One of these records was required of each student at the close of each reading period. Students were encouraged to take notes while they read so as to have the record give complete evidence of the progress made during the period.

The particular form to be used may vary and no perfection is claimed for the one shown in Appendix II. Certain features deserve stress, such as complete bibliographic data, name of student, and date of reading. Essay style of reporting content seemed preferable to notes following any specific pattern, since it allowed greater freedom for reporting various types of material and revealed more of what the student had really learned. Questions listed in the skeleton form served merely as guides to students in the matter of organizing notes. Student reactions and opinions were requested whenever time permitted because they were considered to be vital to the evaluation of the program and of the books used in it.

During the first four or five weeks of the semester in which the plan was instituted the students were encouraged to "browse" with no attempt to organize their reading about a central theme. The weekly records were allowed to accumulate in the student’s folder which was checked by the teacher for each entry. Comments were

24 The form used, together with the records made by several students may be seen in Appendix II.
written on the margins and at the end of the record as it was deemed necessary. To this evaluation was added a letter grade since the policy of the school was conducive to an interest in acquiring "A's" and "B's". At the beginning of each reading period it was suggested that everyone try a new book that day unless he had stopped reading in the midst of some topic which he felt must be completed.

It was not until the second month that a term paper was mentioned as the culmination of the project. It was hoped that this method would discourage boys with predilections for model airplanes from confining their acquaintance with mathematical applications to the narrow field of their hobby. The predominence of aviation literature somewhat defeated this purpose in actual fact. At any rate the selection of a central theme for the term paper served to direct the reading throughout the remainder of the semester and the paper itself provided an opportunity for the student to give evidence of his learning in a more formal way than the incidental references which he might make to it in class discussion from time to time.

Independence was encouraged in the choice of topics for the papers. Only a brief explanation was given with the announcement, to the effect that the paper should deal with as much of the reported reading as the student could find related to his selected topic. In other words, the
topic should be chosen with the idea of showing the relationship, if any, between the various passages read, and of the reading as a whole to the mathematics of the course being pursued. Students who experienced difficulty in making a selection upon this basis conferred with the teacher and were sometimes led to recognize relationships which they had previously overlooked and to realize implications of their reading which had not been apparent to them. Presence of the weekly notes in the folder was of material assistance to both parties during conferences of this sort.

Many students, however, arrived at their decisions without advice further than the group discussions which, by the desire of the classes, occupied a few minutes of each reading period throughout the second month. By the end of that time the great majority of students had completely made up their minds as to the nature of their term papers and structured their reading accordingly. It was common from this point on for one or more students from a class to request permission to spend the reading period in the library, so that information might be gathered from encyclopedias and other general reference sources. It became increasingly apparent that nearly all students were now reading with a purpose of their own and extra papers inserted in their folders testified to the greater volume of reading which was being done outside the scheduled weekly period.
In Chapter III the topics chosen for term papers will be treated at greater length, as will the extent of reading achieved by various individuals.

Further Details of the Plan.--Realizing that high school students, especially below senior rank, have written few term papers and are in awe of the prospect, the teacher laid careful plans for guidance as to form, content, habits of work, and substantiation of statements. Length of the paper was specified as ten pages, or a range of from fifteen hundred to two thousand words, and it was called to the attention of all that the weekly notes alone would amount to more than that in one semester.

Instructions were given at the time the first reading record was written so that complete bibliographic information might be preserved, thus saving the necessity for searching for needed items at the last moment. To serve this purpose the author, title, publisher, place and date of publication, and pages read were placed at the top of each record. Some students realized the worth of this information when they attempted to complete notes which had been interrupted by the class bell, or when they desired at the end of the semester to include in the bibliography a book which had been lost. The requirement of five titles in the bibliography was made with the dual purpose of affording training in proper work habits in connection with the writing of term papers and of preventing
the wholesale plagiarism which often results if a single authority is made acceptable. It was felt that much better assimilation of ideas would result from requiring individually organized theme material which should appeal to no less than five authorities to substantiate its statements.

To make sure that the task of writing should not be hurriedly performed in the eleventh hour before the deadline it was planned to devote the Monday periods of the last month of school to actual preparation of the paper, although the students were reminded that they could not hope to do all of the writing in class. On the first of these writing days each student was asked to prepare an outline of the expected paper with the aid of all the reading records in his folder. The extent of the outlines varied with the ability of the writers, ranging from a simple list of four or five topics to a six-page brief.

Another day was given to the compilation of the bibliography, which for some was merely a matter of alphabetizing the complete notations already made, but which for others entailed laborious searching for overlooked items and hence was better done in the classroom.

Many of the students preferred to do the actual writing at times when they could be alone to organize their thoughts. During the class periods of these last weeks it was found that enough problems arose, both common and individual, to warrant using the time for group discussions.
and individual conferences concerning the format and content of the papers, and more economical work-habits.

The use of headings in a paper this size was discouraged, as it was found to result in a series of unrelated "book reports" rather than the development of a unifying theme. Continuity and organization were desired more than rigorous attention to traditional format and so footnotes were allowed to go unused, since they seemed to confuse students at this level.

One detail which must be mentioned here is the method of obtaining an estimate from students at the culmination of the project as to its worth and appeal. This was attempted in a questionnaire which they were asked to complete during the final week of school, after the term papers had been completed. A tabulation of their answers to the various questions is to be found in Table 8 in Chapter III.

Due respect for the process of acquiring an adequate collection of books necessitates some attention to the techniques of preserving it. No amount of clerical notation can compensate for a lack of the proper rapport among students as to the care and prompt return of books belonging to the school. Approaching the problem from the standpoints of the inconvenience to other students dependent upon the same book, the delay and expense involved in replacing losses, and the need for developing habits of responsibility in the use of public property will, in the order listed, appeal to students of high school age.
In addition, however, the teacher must be constantly on the alert to discover and correct negligence in this matter. The pupil folders, remaining in the room unless signed out at the teacher's desk, provided a simple means of tracing the users of a book throughout the day. It was found expedient to make a quick check between periods whenever possible and to attempt to recover any missing book immediately by recalling the student last known to have used it. Since the collection was small, the absence of a book could usually be detected by a survey of the shelves. In cases where this cursory method did not reveal the loss, incoming students often detected it when they sought the book. Thoughtlessness was most often the reason for carrying the book from the room and, since a student realized that most copies were greatly in demand, he commonly returned the one he had immediately upon discovering its presence among his own materials. In such cases a cordial "Thank you," unaccompanied by any suggestion of reproach, was found to be the best means of encouraging the habit.

As the desire to use books at home increased with the approach of the deadline for term papers, the plan of signing a sheet of paper on the teacher's desk was found to be simple and satisfactory. The sheet was dated and it was agreed that all books were to be returned at
the beginning of the next school day. Of course the method was not infallible and it was often necessary to give reminders of overdue books, but in the main, students were conscientious in their efforts to comply with the regulation. As often as not, the oversight was in the matter of scratching out the record and not in failing to return the book.

Even the diligence of all concerned did not prevent the loss of one book of major importance and two small pamphlets. Since their value represented only a very small per cent of the total investment, it was not considered important in comparison with the values derived from the project. All other books were found to be in excellent condition in the final accounting. In fact, the respect for the reference copies was found to exceed that for supplied textbooks in great degree, if evidence of misuse may be taken as an inverse estimate.

The Role of the Teacher in the Reading Period

Although emphasis was necessarily placed upon caring for the books, that duty was not allowed to occupy the teacher’s attention for more than a brief time at the beginning of the reading period and again at the end. The requests for individual guidance in the location of material were usually handled in connection with the initial checking and the whole activity held to a minimum in order to discourage the waste of time in getting settled. As a positive influence in the direction of establishing quiet
the teacher usually found clerical work to be done early in the period. Checking and evaluating reports from the previous reading day was found to be advantageous at this time, because students were available for conference as to deficiencies and discussion of points which seemed vague in their notes. Then, too, timely suggestions for further research could be made and immediately acted upon. References noted in one folder might be mentioned to other students needing the same material. Inspection of folders in the presence of students acted as an incentive to neatness and care on their part.

For all these reasons the teacher spent a great deal of the reading period in examination of the folders, but if the supply of these ran out, due to their use by the students themselves, other useful employment was not lacking. A casual inspection of the books in use served many purposes. It gave insight into the preference of students, in a personal way which was more easily remembered than the acquisition of such knowledge by the perusal of one folder after another. It gave opportunity for counseling in wiser choice whenever a student was found to be experiencing difficulty or disinterest in the material he was using. It allowed for assistance in the interpretation of difficult passages and for instruction in the art of note-taking, as contrasted with copying. It also furnished cues for linking the reading already done by
students with subsequent related topics in the textbook discussions. Likewise it paved the way for the introduction of references not yet discovered by students, yet pertaining to their interests. Such introductions were made, not as an interruption of the current reading, but usually as they entered most naturally into the general class discussions on other days of the week.

The pursuits mentioned left little time for further acquaintance with any of the unused books which remained on the shelves. A superficial survey of these sometimes brought to light references which had gone unnoticed, but careful reading by the teacher was an impossibility while students were present. Both refresher reading and further research were done at other times.

**Salient Features of the Plan**

The free reading program as it was put in motion was thought to be in many ways imperfect, yet certain outstanding values were recognizable. Salient features are worthy of summarization, thus:

1. Standardization of procedure eliminated waste.
   1.1 Volunteer students (early arrivals) borrowed and returned library copies.
   1.2 Students expected "reading day."
      1.21 Selected books without announcement.
      1.22 Read throughout most of the period undisturbed.
1.23 Study hall members knew when books were available in the library.

1.3 The simple report form insured satisfactory coverage but allowed freedom for variety of content.

1.31 Bibliographic information included author, title, publisher, place, date, and pages read.

1.32 Student evaluation was requested as to the ease of reading, interest and usefulness of material.

1.33 Notes on reading were to include the general topic, important facts, and connection with class assignments in mathematics, either past or present.

1.4 Student folders provided a means of preserving all notes for later reference.

2. Requirement of a term paper as culmination of the project furnished a goal and gave impetus.

2.1 Selection of a theme title prevented uncontrolled "browsing."

2.2 Pressure to produce increased the desire to absorb.

2.3 Meeting a deadline acted as a spur to efforts.

3. Freedom of choice as to reading material made the project desirable from the students' standpoint.
3.1 Removed it from the realm of a chore.
3.2 Allowed for individual differences without drawing a line of distinction between bright and dull students.
3.3 Revealed unexpected aptitudes in some.

4. Acquaintance with types of material other than textbooks was afforded students.
4.1 There was an opportunity to compare the viewpoints of different authors.
4.2 Mathematics was presented as a reasoning process.
4.3 The relation of mathematics to life situations seemed more realistic.
CHAPTER III

AN INTERPRETATION OF THE DATA WITH ALLOWANCE FOR THE LIMITATIONS OF THE STUDY

Lord Kelvin once said,

"When you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind."

If one does obeisance to Lord Kelvin's dictum, he must surely decry the synonymous use of "measurement" and "evaluation" which we so commonly find in current educational literature. Perhaps it would be best, then, to state that this chapter will be an attempt at both, rather than either. For it would certainly be impossible to reduce to formula and "express in numbers" some of the intangible effects of the reading program described in Chapter II. So, even at the risk of its being stamped as "of a meager and unsatisfactory kind," the "knowledge" of this chapter will, in part, be presented as the result of such un-numerical practices as "weighing" personal opinions, "estimating" a correlation between term paper and learning, and "calculating" (with "probable error") the worth and strength of the habits formed by the participating students.

It is to be hoped that the more orthodox enumeration of books most used, pages read, topics employed in the writing of term papers, and of students employing those topics,

\footnote{Carl N. Shuster and Fred L. Bedford, \textit{Field Work in Mathematics}, p. v.}
will serve to establish creditable rank for the project in the minds of those who are "Kelvinistically" inclined in their judgments.

**Limitations of the Study**

Admittedly the essence of a free reading program makes it ill-adapted to precision in the measurement of results obtained from it. In addition there are other limitations to the present study, some of which, at least, are apparent. First, one must keep in mind the restricted nature of the book collection whenever an attempt is made to discuss the interests of students as revealed by their choices. A second factor in reaching conclusions about choices will certainly lie in the students themselves. As the test records show, the majority were of sufficiently high learning aptitude to make it impossible to speak of them as an "average" group. The incomplete status of the records, however, gives heavier weight to that aptitude than it deserves, as will be explained in the discussion of Table 1 later in this chapter.

Viewed in retrospect, the term paper, as it was structured, seems to have exercised a restrictive influence upon the freedom of choice in some instances. Although the ten page requirement was intended as a general guide and the insistence upon a theme was meant to accomplish the organization of the semester's reading, student interpretation in many cases caused these two factors to become
determining ones in the matter of selection and extent of reading. Thus, some students, who would have favored a change of topic after a few weeks, felt impelled to continue with their first selections because they feared that they would not have collected enough material to fill ten pages on one theme before the deadline for completing term papers. Others, perhaps, would have chosen to investigate topics with which they felt inadequate to deal in a term paper.

A further limitation to the effectiveness of the program cannot be overlooked. The moderately conservative policy of the school had fostered a somewhat preconceived concept of the nature of a mathematics class and it was difficult for a number of students to accept whole-heartedly the unorthodox activity of reading outside a textbook and especially the unprofessional theft of the term paper prerogative from the English department! The latter sentiment was apparent in the written comment of student S-LM² concerning his paper on the calculus: "This essay was more adapted for English than geometry." And again in an unsigned statement from another student: "I don't think geometry is the type of thing for a term paper." If, as has been said, the child educates himself, it is doubtful how much learning took place in situations like these.

While the very novelty of the experience may have insured remembrance of its associated ideas on the part of

²Symbolism used to represent individual students is explained on pages 54 and 55.
students, willingness to engage in the experience would undoubtedly have increased the benefits derived from it. It is to be hoped that repetition of the plan will provide enough familiarity to allow for more complete acceptance by that element of the student population which hesitates to be guided into untrodden paths.

An enumeration of limiting factors must take into account also the relatively small number of students involved in the program and the fact that only two adults, the teacher and the librarian, were intimately connected with it during the period in which it was in operation. Before any very reliable conclusions can be reached as to what books are valuable, what schedule of reading most conducive to learning, or what form of evaluation most effectual, the plan should certainly be tried with a greater number of students, of wider range of aptitude, and of more heterogeneous environment and prospects. Before any claim can be made of far reaching values to students resulting from the experience, it would be desirable to observe them over a longer period of time and in comparison with other students of like caliber not having shared the experience. It would be desirable to have this observation made by other teachers as, perhaps, those the participating students will have when they enter college. Since many of these desirable measures were impractical or impossible within the scope of this study, the discussion of the data which makes up the remainder of
this chapter must be considered as imperfect, tentative, and in part, subjective in its conclusions.

Instruments of Evaluation

Critics of progressive education, no less than its proponents, perhaps, deserve credit for the increasing number of evaluation devices existing today. The latter school were, no doubt, more assiduous in their efforts to prove the worth of their cause as a result of the attacks upon it. At any rate the teacher who wishes to examine the fruits of labor is confronted with an impressive array of educational microscopes from which to choose those he will train upon his samplings. No elaborateness is claimed for the instruments employed in this study, but perhaps their very simplicity will be their best recommendation to busy classroom teachers.

C.S.U.P.T. as a Basis for Expectation.--It was fortunate that cumulative records were available of scores, centile ratings, and grade equivalents as determined by the Ohio State University Psychological Test. As mentioned in the discussion of environmental factors in Chapter II, it had been the policy of the school to give the examination early in the first semester to all students except seniors. Thus, at the time the free reading program was inaugurated, there were two sets of existent data on sophomores and three each on juniors and seniors. These data, together with the natural screening which occurred through the elective
status of the courses in plane and solid geometry, justified the establishment of the reading program at sophomore level and above. Exclusion of algebra students was due not merely to an arbitrary decision, but to the belief that the class contained too many members of unknown aptitude, with a probable lack of interest in mathematics except as a graduation requirement.

_Reading Records._--The primary purposes of the reading records were:

1. to furnish the student with a means of ready reference to all the reading he had done during the semester,
2. to train him in good habits of research,
3. to form the basis for the writing of his term paper, and
4. to provide the teacher with a means of tracing the reading of individual students as needed for guidance and evaluation.

In connection with each of these chief functions certain secondary ends were served. The students often reviewed their folders before attempting to select a new book or continuing with one previously used. By this means he was able to recover the train of thought which he had shelved the week before. Then, too, the accumulation of papers with their attendant comments and grades acted as a spur to flagging spirits or as a boon to diligent ones.
Most students had never before been asked to note bibliographical information, but became proficient in the matter and learned the convenience of exact reference to a book, instead of vague description such as, "that red one." Knowledge and use of this convenience are not always synonymous, however, as SR-11F proved when she wrote on the opinion questionnaire: "Keniston-Tully Plane Geometry (easiest). Our regular geometry book (hardest)." Not having written the name of Schorling-Clark-Smith so often as Keniston-Tully, she did not remember it! Drill, it would seem, is a factor in learning; but satisfaction with the thing being learned may also have figured heavily in this case, since the girl showed decided pleasure at being able to learn, from the book of her choice, those things at which she had repeatedly failed in class assignment. Witness in her reading record of April 1, when she first discovered Keniston-Tully: "While leafing through this book I'm really amazed at the wonderful way this book has in the way of starting a student into geometry." And again at the close of school when her questionnaire announced in her own quaint style of English and spelling: "I found in my reading that I acquired a amount of knowledge that I should have gotten in class."

Other students gave evidence that the recording of bibliographical facts over and over served to fix them in their minds. It was not unusual for someone to stop at the
teacher's desk after class with a question such as: "May I take Osteyee home tonight?" or "Has anyone asked to use Shuster and Bedford the fifth period?"

The value of the reading record was well demonstrated when term-paper writing began. Folders were carried home in such great numbers that the teacher had difficulty in keeping on intimate terms with them, but since few additional reports were filed after the writing of term papers began, it was relatively easy to check them after students had completed the entire project. At that time the recently scrawled notations on various records attested the use to which these young authors had put them.

From the teacher's standpoint the value of the reading records was inestimable. Since one of the primary aims of the study was to discover what high school students could and would read, the folders were watched for trends of popularity as to topic, author, and particular passage. It had originally been the intent to search out and keep a file of favorite passages but it was found to be impossible since so many students neglected to give complete information concerning pages read. In some instances no reference at all was recorded and in others the initial page was listed but in the haste at the close of the period the page on which the student stopped had not been noted. This partial information often gave a general idea of the coincidence in reading among various pupils, but it was not considered complete enough or valid enough for the purposes
of this study. Even in so short an experience with the free reading program, however, it was found that certain useful passages stood out clearly enough to serve as points of guidance with students. Use of the folders as a basis for guidance was discussed also in Chapter II.

Evaluation of the books was further facilitated by use of the reading records. Examination of them was expected to show the extent and nature of reading done. It was desirable to study the habits of repetition and change in selection of reading matter. Results of such study will be discussed in connection with Table 3 later in this chapter.

Though not directly pertinent to this study, the use of the reading record in the periodic evaluation of the student may be briefly mentioned here. Since research work is not usually considered to lie on the level of high school student, the weekly records were more liberally graded than text-book assignments. No credit was deducted for selection of easy material, but fulfilment of the skeleton requirements laid down for reports, and reliability and clarity of notes were used as a basis for grading. Since one day in five was devoted to the activity, the average from the folder was weighted as twenty per cent of the periodic grade. Although the criteria for arriving at the letter value of that twenty per cent were never discussed with the students, they undoubtedly noted the more liberal policy followed in the matter and appreciation of it was expressed by J-30F, who shrewdly wrote on the questionnaire: "Aside
from being a worthwhile project, it gives the student a rather easy chance to get a good grade in geometry. If his work in geometry seems hard and his grades have not been good, it gives him an easy chance to bring his grades up." Not the highest motive for engaging in free reading, perhaps, but certainly a practical one for many students!

Term Paper as an Instrument of Evaluation.—The quality of the term paper was instrumental in determining the final grade on the project, but it serves a further use for purposes of this study. From comparison and grouping of all term papers it was possible to determine in a general way the reading interests of the group. In individual instances term papers gave insight into the students' understanding of what they read. The manner in which the material was incorporated, whether copied or paraphrased, all reflected the thinking of the student about what he read. Although no generalizations could be made objectively in the matter, the teacher was able to form judgments which will probably prove useful in dealing with those students who intend to continue the study of mathematics another year.

A measure which was considered more to the point of this study was the résumé of the term paper, which the student was requested, without previous announcement, to write in class, about a week after term papers had been handed in. Discussion of the résumés will occupy a later section of this chapter.
Student Opinion a Worthy Instrument.—For those who believe that in large part the student is his own educator, his satisfaction with his educational experience is an important factor for measurement. The questionnaire submitted to the participating students in this case was a simple one. The resume already mentioned was written at the end of the paper on which the following questions were answered:

1. Did you enjoy choosing your own reading material?

2. Would you have preferred a list of readings from which to choose a certain number?

3. Do you think this reading project should be continued with future classes?

4. Do you feel that on Monday you learned (less than, as much as, more than) during other forty minute periods of mathematics study?

5. Name the easiest book you read. What book was hardest to read?

The questions were read to the students and they were requested to use "Yes" and "No" answers for the first three, one of the prescribed phrases for the fourth, and a title followed by the word "easiest" and another by the word "hardest" for the fifth. Objection was raised that the brevity of the answers did not allow for full expression of their opinions and so they were invited to add any remarks
they chose, provided the type answers stood out sharply for ease in tabulation. Cooperation was excellent in this respect and some interesting comments were obtained as well. Discussion of Table 8, later in this chapter will deal further with the results of the questionnaire.

**Opinions of Instructors.**—It was noted under the heading of limitations that only the librarian and the teacher were in daily contact with the students as conductors of the reading program. It is regrettable that other teacher reactions could not be obtained, but it was thought that the librarian could draw comparisons as to the effectiveness of the project beside the usual library assignments. She would also be in position to evaluate the library habits of participating students. The teacher, also, could report critical observations pertinent to the worth of the program, since it had been undertaken in the spirit of a quest and not of a crusade. Accordingly the opinions and observations of these two are placed at the end of the list of instruments of evaluation.

**Significance of the Test Records.**

Most modern schools seek to know something of the general intelligence and learning aptitude of their students as a means of furnishing teachers with a background for judgment and guidance relative to them. In this matter the writer was fortunate to find a record so extensively and carefully kept. Experience teaches that such records may
be rendered unreliable and even worthless through careless transcription by one who does not grasp the import of the material with which he deals. The record in question gave evidence of having been derived in agreement with the norm sheet for the Ohio State University Psychological Test. It was self-consistent and also consistent, with few exceptions, with the observed performance of the students listed in it.

Of the exceptions, certain ones are sufficiently outstanding to need mention here. Student S-5F\(^5\) was known to do work more in keeping with her freshman centile rating of sixty-eight than with the twenty-eight reported for her sophomore year. Likewise, the work of S-8F more nearly resembled her earlier rating of ninety-four than the current one of thirty-one. S-14F was apt to give an erratic performance though not to the extent the centile ratings of eighteen and seventy-eight, respectively, would indicate. Farthest from the test prediction, if valid, was S-60M, who gave average-to-superior performance in the face of twenty-five and thirteen as successive centile ratings. Whether these discrepancies were due to mistakes in counting scores, to errors in the transcription, or simply to the "bad days" which are apt to coincide with tests for some students, it would be impossible to learn.

In Table 1, following this page, are shown the O.S.U.P.T. ratings of all participating students who had

\(^5\)See pages 54-55.
**TABLE I**

Cumulative Test Record of Students Participating in Free Reading in Mathematics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1M</td>
<td>63 93 B</td>
<td>91 95 B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-2M</td>
<td>34 47 C</td>
<td>41 51 C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-3M</td>
<td>45 75 C</td>
<td>51 69 C</td>
<td></td>
<td>119</td>
</tr>
<tr>
<td>S-4M</td>
<td>42 71 C</td>
<td>58 78 B</td>
<td></td>
<td>117</td>
</tr>
<tr>
<td>S-5F</td>
<td>41 68 C</td>
<td>53 88 C</td>
<td></td>
<td>114</td>
</tr>
<tr>
<td>S-6F</td>
<td>33 43 C</td>
<td>32 25 D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-7F</td>
<td>64 93 B</td>
<td>100 97 A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-8F</td>
<td>66 94 B</td>
<td>34 31 C</td>
<td></td>
<td>114</td>
</tr>
<tr>
<td>S-9F</td>
<td>40 66 C</td>
<td>37 42 C</td>
<td></td>
<td>109</td>
</tr>
<tr>
<td>SI-10F</td>
<td>44 56 C</td>
<td>63 82 B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SR-11F</td>
<td>58 78 B</td>
<td>68 88 B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-12F</td>
<td>59 63 C</td>
<td>48 68 C</td>
<td></td>
<td>112</td>
</tr>
<tr>
<td>SI-13F</td>
<td>53 43 C</td>
<td></td>
<td></td>
<td>102</td>
</tr>
<tr>
<td>S-14F</td>
<td>27 18 D</td>
<td>58 78 B</td>
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**Explanation of Table**

O.S.U.P.T.-Ohio State University Psychological Test  
Sc.-Score  
C.R.-Centile Rating  
G.E.-Grade Equivalent  
I.F.-D.-Intermediate Form D  
S-Sophomore  
I-Incomplete Test Record  
R-Retarded  
J-Junior  
G-S-Graduating Senior  
A-Accelerated  
F-Female  
M-Male  
1-61--Alphabetical order on plane geometry roll.  
101-110--Alphabetical order on solid geometry roll.
# TABLE I (Contd.)

Cumulative Test Record of Students Participating in Free Reading in Mathematics

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<td>47 54 C</td>
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</table>

* 1942 Test Record
ever taken the test in the school in which they were then enrolled. Column four, of the same table, shows the intelligence quotients of some students as measured by the Otis Group Intelligence Test, Intermediate Form D. The intelligence quotients were obtained in February, 1944, when the students taking the test were in the second semester of the eighth grade. Other students of the plane geometry classes involved in this study were not at that time enrolled in the school. Neither were Otis scores recorded for any members of the solid geometry class; administration of the test in February, 1944, seems to have been the first and only one in the current history of the school. No discussion of Otis scores is deemed necessary here except to say that in individual cases they are valuable for reinforcement of, or comparison with, the Ohio State University Psychological Test records, and that they cannot be used as a basis for classifying the group as a whole since it is almost certain that many of its members not so tested would have influenced the trend to a lower average of intelligence while some few of these would have augmented the upper levels. At any rate, since thirty-one members possess these scores, as contrasted with forty who do not, it would be dangerous to attempt to interpret from them any valid measure of the intelligence of the group.

The notation used in Table 1 is not so mystifying as it appears at first glance. The combination of letters and
numbers used to designate a given student is intended to
give, in condensed form, all facts about his classification
which are pertinent to the study. The letters, S and J,
are used to denote the sophomore and junior classes, respec-
tively. Since the reading program was in progress only
during the second semester, the appropriate name of "grad-
uating senior," yielding the symbol GS, seemed a convenient
means of distinguishing such upperclassmen from the sopho-
more.

Any reference to the grade equivalent, separate from
Table 1, must be made with appreciation of its weight,
according as it was derived from a complete or an incomplete
test record; hence, the letter I, for Incomplete, serves
to call attention to a student who has not taken the Ohio
State University Psychological Test as many times as others
of his class in school. Similarly, the letter N indicates
that the student has taken NO O.S.U.P.T.

The letters R and A are the symbols for "retarded"
and "accelerated." The former applies to students who were
enrolled in plane geometry for the second time, or who had
repeated first year algebra before entering plane geometry,
and the latter applies to one sophomore enrolled in solid
geometry. Upperclassmen on the plane geometry roll if they
are not marked "R", simply elected the subject later than
their peers.
The numerical part of the student's designation is an indication of his position in an alphabetical list of all students enrolled in his course. Numbers from one to sixty-one are used for the plane geometry roll and those from one hundred to one hundred ten for the solid geometry roll. The letters, M and F, symbols for male and female, are added in order that reading interests of the separate sexes may be studied.

The three major columns of the O.S.U.P.T. cumulative record are dated 1943, 1944, and 1945, respectively, and are carried throughout the table to accommodate the records of those upper classmen who were enrolled in plane geometry. Since only two complete senior records occurred among the data, it was thought impractical to carry a separate column for 1942, but wiser to enter the two sets of 1942 data in the 1945 column, otherwise empty in senior records, and to call attention to these insertions by a footnote. Each of the major columns carries three items of information concerning the test given that year: score (abbreviated Sc.), centile rating (C.R.), and grade equivalent (G.E.). The last is the grade which the student may be expected to maintain according to his centile rating.\(^4\)

\(^4\) As given in "High School Norms for Form 22, Ohio State University Psychological Test." Ohio High School Intelligence Test Survey, 1945-46.
The cumulative record is included in this study for reference, should it be desired to assemble all known facts about an individual participant, or should substantiation of Table 2 be desired. The latter table, summarized from the first, presents at a glance the distribution of grade equivalents of sixty-nine of the seventy-one members on the class rolls. The two students for whom no records were available were transfers from other schools, who entered too late to take the test.

The demarcations between centile ranks for purposes of assigning the grade equivalents A, B, C, and D, are taken from the most recent norms for the Psychological Test, established in 1943. Double letters are used as grade equivalents for those sophomores whose scores on successive tests caused changes to occur in their grade equivalents. The correspondingly wider range of centile ranks is shown directly below the double-letter classes of grade equivalents. In the case of upperclassmen, with three years' records, agreement of two out of three grade equivalents was accepted with no attention being given to the differing one. This practice is in conformity with the regulation of the school which states that in-between averages shall revert to the nearer of the two grades, with the provision that averages of exactly one half shall revert downward. Down-grading of those sophomores mentioned above was finally accomplished in the single-letter classes of grade equivalents from which per cents of enrollment were found.
TABLE 2

Ohio State University Psychological Test

C.-Complete  I.-Incomplete  Rec.-Record

* Correlation between centile range and grade equivalent is taken from "High School Norms (1943) for Form 22, Ohio State University Psychological Test," used in the 1945-46 Ohio High School Intelligence Test Survey.

** Double letters indicate a change in grade equivalent earned by sophomores on successive tests. Upperclassmen are ranked under single letter grade equivalent according to the average of three grades.

*** Includes students in both plane and solid geometry.

---

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<th>B</th>
<th>B-C</th>
<th>B-D</th>
<th>C</th>
<th>C-D</th>
<th>D</th>
<th>No Rec.</th>
<th>Total</th>
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<td>100</td>
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<td>95</td>
<td>95</td>
<td>75</td>
<td>75</td>
<td>75</td>
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<td>76</td>
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<td>6</td>
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<td>7</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>71</td>
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</tbody>
</table>

Per Cent of Enrollment 5.6 23.9 57.7 10.0 2.8 100

Abbreviations: O.S.U.P.T.- Ohio State University Psychological Test
All sixty-nine of the available test records are grouped together in Table 2 without regard for enrollment in plane or solid geometry. Since the chief purpose of the table is to show the central tendency of grade equivalents for the whole group, distinction between the two mathematics courses was considered unnecessary.

Comparison of these percents of enrollment with the centile ranges assigned to the single-letter classes represented by the per cents shows that the group tended to be overweighted in the A, B, and C classes, contained only half the usual number of members in the D class, and included none in the E class. The greatest overweighting occurred in the C class, which exceeded the centile range by 7.7 per cent; next in degree of overweighting was the B class with 3.9 per cent excess, and last, the A class with only 0.6 per cent. The somewhat above-average character of the group cannot be denied, but it may not have been as great as these figures indicate, for reasons which will now be considered.

In consideration of complete and incomplete records, the added weight which should be given to the former seemed to warrant a separation of the two for the purposes of tabulation in Table 2. Confirming this decision, a study of the individual characteristics of the twelve incomplete records revealed that, in no case, could their completion have resulted in upgrading, and that in only one case,
that of an upperclass A, could down-grading have been impossible.

No very valid prediction can be made concerning the probable influence of the two transfer students for whom no test records existed. From observation of their class work they could have been expected to weight the two ends of the curve, but such observation was too limited to merit appreciable attention. Nevertheless, it can reasonably be stated that if all records had been complete, the distribution curve of the grade equivalents in the study could very well have been less skewed from the normal.

**Data Supplied by Reading Records**

It was suggested in a previous section of this chapter that the reading records contained the keys to several of the values of the free reading program. Data gathered from the pupil folders of accumulated records have been concentrated in tables three to six, inclusive.

**Extent of Free Reading.**—Limited as students were by the quality and variety of material at hand, by the number of reading periods, and by the bibliography requirements of the term paper, yet wide variance was, from the early weeks, apparent in the extent and habits of reading among the seventy-one students participating. Some seized immediately upon some book which looked attractive and pursued it without consideration for other volumes in the collection. Others frankly went adventuring and never
succeeded in establishing an abiding interest in any one topic. Still others made use of several books on the same topic but ignored the possibility of any other selection.

Table 3, which follows this page, is intended to show individual trends in these matters as well as the frequency of the trends in the group. Explanation of the notation used to represent individuals was explained on page 55, and is consistent throughout all tables which refer to them. In the second column appears the number of reading records preserved in the individual folders at the culmination of the reading program. In all but rare cases it can be accepted as the number of records made by that individual during the semester. Students were impressed with the necessity for preserving all data, both for convenience in the preparation of term papers and for the sake of forming good habits of research. All but two students were sufficiently convinced in the matter to consider all notes a part of the finished project and it is fairly certain that the contents of the folders, as they were surrendered with the term papers, are valid indices of the extent of reading which occurred.

The summary of the second column reveals that the bulk of the group wrote from nine to eleven records, inclusive. Of the twelve students reporting seven or less readings, four were absent a number of times, one carried an injured hand for some time, and the other seven were
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(Contd. on next page.)
### TABLE 3 (Contd.)

**Extent of Free Reading by Individual Students:**
**Reading Records, Books, Repetitions**

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<th>No. of Books</th>
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| Total   | 639 |   | 639 |

**Summary:**

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<th>Records Made</th>
<th>No. Reporting</th>
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<td>3</td>
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<td>6</td>
<td>4</td>
<td>4 - 17 - 51</td>
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<td>7</td>
<td>5</td>
<td>5 - 17 - 68</td>
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<td>13</td>
<td>6</td>
<td>6 - 5 - 25</td>
</tr>
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<td>10</td>
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<td>7</td>
<td>7 - 5 - 30</td>
</tr>
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<td>11</td>
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<td>8</td>
<td>8 - 5 - 35</td>
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<td>9</td>
<td>9 - 0 - 0</td>
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<tr>
<td>13</td>
<td>1</td>
<td>10</td>
<td>10 - 0 - 0</td>
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</table>

| Total | 69 | 59 | 214 | 328 |
habitually slow about completing notes before the bell rang, sometimes using a portion of the following week's reading period for that purpose, with the cumulative effect of shortening the ultimate number of records made. The boy who reported twelve readings, and the girl who reported thirteen, were both enthusiastic participants who probably added new material after the beginning of the final month of school when other students had contented themselves with the material already collected or added to their papers without making formal record of the additional reading.

The third column of Table 3 is interesting in that it sets the range of the number of books used at from one to ten. A wider range would hardly have been possible under the conditions of the reading program. The clustering of the tendency around four or five books per student may be viewed in a number of ways. The requirement of five items in the bibliography may have caused the cluster but, since thirty-four of the students either did not meet the requirement or did so after the provided class time had expired, it is more probable that a different explanation is the acceptable one. The size of the collection, and the number of general topics pursued, suggest that no more than four or five books dealing with any one topic could be found. Five students commented in the questionnaire, previously mentioned, that material was hard to find.
A third conjecture, that the four to five book average was a natural ratio to the number of reading periods, has no foundation in fact, as column four reveals it. The notation here used indicates the number of times each of the books listed in column three was used by the student, the most popular book being listed first in each case, and the popularity declining progressively from left to right. Thus no relationship appears to exist between the number of reading periods and the number of books used or the number of uses per book, since the table clearly shows a decided preference for one or two books no matter what the total number selected.

The fourth column of Table 3 presents a vivid picture of the tendency to repetition in the use of a book. In no case was the third most popular book, in any student's selection, used more than twice by him. Furthermore, only one student repeated the use of his fourth most popular selection. Viewed in this light, the five book requirement for the bibliography of the term paper, somewhat arbitrarily and intuitively set, seems to have been a little out of keeping with the offerings of the collection.

From a survey of the summary of column four it may be stated that the books were used from one to eleven times each, with single usage (189 cases) far outweighing the 125 cases in which repetition of use occurred. The total of 314 in the "Number of Times Reported" section of column
four may be thought of as the total number of first uses of the forty-seven books available, or the net circulation of the forty-seven books.

At first glance it would seem quite significant that, in 189 cases out of 314, use of the book was not repeated. Were the books predominantly unattractive? Did they, on the average, contain only sufficient material on one topic to occupy one forty minute reading period? Was the range of topics treated in any one book so specialized as to preclude a second use by students not primarily in search of its offerings? Was the great number of single usage cases due to a natural curiosity and a desire for wide sampling? These questions can not be answered conclusively, since they were not presented to students, who may, or may not have been able to answer them. There are certain indications in the facts, nevertheless, which lead to tentative answers.

As will be brought out a little later in the discussion, students voted overwhelmingly in the affirmative when asked if they enjoyed choosing their own reading. This vote may be taken as an indication that the books proved not to be unattractive, but that most students did possess a natural curiosity about them and a desire to sample as many as the time permitted.

Since, in all but six cases (S-7F, SR-22M, SR-26M, J-30F, GSI-31F, and S-39M), students tended to make more
than one repetition in the use of some books, the remaining questions may be answered thus: Many books contained sufficient material to occupy students for from two to five times, but students chiefly tended to use only one or two books repeatedly in the pursuit of a particular topic, perhaps because they preferred to exhaust the resources of one book along that line before selecting another, or perhaps because other books on the same topic were already in use when they reached the classroom. The latter reason was suggested by one student who qualified an affirmative answer on the question of enjoyment by the words: "when I can get the one I want."

The "Repetitions" section of the summary of Column 4, Table 3, serves to balance the seeming weight of the 189 cases of single usage. In the fifty-two cases where a book was used twice there were, of course, fifty-two repetitions. Accordingly, the number fifty-two is entered under "Repetitions." If a given book was used three times by the same student two repetitions occurred. Since there were twenty-two such cases, a total of two times twenty-two or forty-four repetitions occurred, and so on to a total of 325 repetitions. By comparison of the two (325 and 189) one can readily recognize the fact that the tendency to repeat the use of a book was much stronger than the tendency to cast it aside after one use.
Two additional items of interest may be deduced from a study of the line of totals at the bottom of the summary of Table 3. From the 314 first uses of books by the 69 students reporting, the book-per-pupil ratio is found to be a little better than 4.5, in surprising correspondence with the median number for the group, as reported in the "Books Used" column. In other words the group contained a surprising number of nearly "average students" in this respect. The second item is derived from the comparison of the 314 first uses and the 325 repetitions with their total of 639 uses (also shown in the total number of reading records preserved). These figures prove that, of the total time devoted to free reading, almost equal parts were spent in exploring new material and in continuing with that which had been used before.

Student Use and Opinion of Books.--During the writing of notes on their reading, students were constantly requested to include an opinion. Cooperation was excellent on this point, as the final column of Table 4 (immediately following this page) indicates. "Student Use and Opinion of Books, Revealed by Reading Records" is the descriptive title of the table, which is arranged in nine columns. For convenience the forty-three books are arranged in alphabetical order by authors, with the four sets of encyclopedias inserted in their proper places. Authors and titles occupy the first two columns of the table, followed by seven
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<th>Fav.</th>
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### TABLE 4 (Contd.)

Student Use and Opinion of Books, Revealed by Reading Records

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<td>32.</td>
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<td>4</td>
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<td>Potter, M.A. &amp; Beck, H.R.</td>
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<td>34.</td>
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<td></td>
<td>A Short History of Mathematics</td>
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<td>27</td>
<td>16</td>
<td>5</td>
<td>13</td>
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<td>36.</td>
<td>Schorling, R. &amp; Clark, J.R. &amp; Smith, R.R.</td>
<td>65</td>
<td>8</td>
<td>6</td>
<td>1</td>
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* Owned by student.
TABLE 4 (Contd.)

Student Use and Opinion of Books, Revealed by Reading Records

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<tr>
<th>Author</th>
<th>Title</th>
<th>No.of Cop.</th>
<th>T.U.</th>
<th>Us.</th>
<th>Re.</th>
<th>Opinion</th>
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</thead>
<tbody>
<tr>
<td>38. Shields</td>
<td>Principles of Air Navigation</td>
<td>*</td>
<td>1</td>
<td>1</td>
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<td>40. Smith, D.E.</td>
<td>Number Stories of Long Ago</td>
<td>1</td>
<td>21</td>
<td>13</td>
<td>4</td>
<td>9</td>
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<tr>
<td>41. Smith, D.E.</td>
<td>The Wonderful Wonders of One, Two, Three</td>
<td>1</td>
<td>17</td>
<td>13</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>42. Smith, D.E. &amp; Ginsburg, J.</td>
<td>Number and Numerals</td>
<td>2</td>
<td>35</td>
<td>15</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>43. Amer. Counc. on Educ.</td>
<td>The Story of Numbers</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>3</td>
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<tr>
<td>44. Amer. Counc. on Educ.</td>
<td>The Story of Weights and Measures</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3</td>
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<tr>
<td>45. Amer. Counc. on Educ.</td>
<td>The Story of Writing</td>
<td>1</td>
<td>7</td>
<td>6</td>
<td>1</td>
<td>2</td>
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<td>46. Amer. Counc. on Educ.</td>
<td>Telling Time Throughout the Centuries</td>
<td>1</td>
<td>23</td>
<td>10</td>
<td>4</td>
<td>6</td>
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<tr>
<td>47.</td>
<td>The World Book Encyclopedia</td>
<td>1 set</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

No. of Cop. - Number of Copies
T.U. - Times Used
Us. - Users
Re. - Repeaters
Fav. - Favorable
Unf. - Unfavorable
items of information concerning each of the books or sets. These items are, from left to right, the number of copies, the number of times used, the number of users, the number of repeaters, the number registering a favorable opinion, the number reporting unfavorably, and the number not reporting an opinion. Abbreviations for these items, used as space savers, are explained on the third and final page of the table.

The number of copies of each book is listed in order that the uses per copy may be determined. It will be noted that in all cases where more than two copies were available the books were texts, either in current use or left over from previous classes. In each case the number of uses per copy is seen to be rather lower than the average, indicating that by choice, students usually prefer non-textbook material.

In an attempt to evaluate the books four criteria were set up and the books meeting each of these criteria were placed on preferred lists which were then consolidated into one. The criteria are listed below, with reasons for their selection, and the consolidated list of preferred books follows.

1. Was the book in use at least fifty per cent of the time devoted to free reading? Conservation and wise use of school funds prompted this criterion, since, of all
the literature available, one would wish to recommend that which would give the greatest service.

(2) Was the book used by twenty per cent or more of the students? It will be remembered that one of the original criteria for selection was the likelihood of appeal to more than a narrow group of students. Examination of the frequencies in the "Users" column of Table 4 shows that sixteen of the forty-seven books and encyclopedia sets had ten or more users among the sixty-nine whose folders were preserved, or about fourteen per cent of the students. A desire to shorten the lists so that they would contain only especially useful books caused the approximate fourteen per cent, revealed by the first inspection, to be raised to twenty per cent, or fourteen users. Then, too, twenty per cent seemed a less "narrow group of students" than fourteen per cent.

(3) Was a favorable opinion of the book registered by more than fifty per cent of its users? Simple majority was appealed to for the establishment of this criterion. It was in this connection that the student opinions, written while the books were being used, proved so valuable. In the case of several opinions on the same book, most often written without reference to what had been said previously, it was interesting to screen for the general opinion. As mentioned above, students cooperated well in the attempt at evaluation and while some did not register
their opinions at each repetition of use, there were only seventy-five cases (as shown in the final column of Table 4) in which no opinion of a given book appeared in the record. With reference to the 639 records preserved, the per cent of efficiency in compliance with the request for information is seen to be about eighty-eight.

4) Was use repeated by at least fifty per cent of the users? Again simple majority was the deciding factor. Attention to amount as well as variety demands that the small collection be built of books which have a repeat value for most students. Reputation of the book among students ranks higher, also, if those who have chosen it once return to it a number of times.

Preferred List of Books.—By appeal to the use and opinion columns of Table 4 answers were easily obtained for each of the questions set forth in the criteria. The column headed T.U. (times used) furnished the total number of uses to which each book was put. Since there were approximately ten reading days (from the median number of reading records preserved in student folders) and since four classes were participating in the program, it appears that during the total of forty reading periods, a book must be used twenty times in order to meet the first criterion. Thirteen books were found to qualify under this listing. Their titles appear in the consolidated list which follows, on the next page.
In testing for the second criterion division of the sixty-nine students by five set twenty per cent at between thirteen and fourteen students. Frequencies in the "Users" column show a decided break below thirteen and hence the three books used by thirteen students were included in the list of those which qualified under the second criterion. Ten titles appear on that list.

Comparison of the "Users" column with the "Favorable" column under "Opinions" showed that twenty-eight books were favored by fifty per cent or more of their users. Similar comparison of the "Users" and "Repeaters" columns yielded a list of ten titles which were used more than once by the same person.

Since it would be interesting to know to what extent the four lists would overlap, they were consolidated with the result shown below. Figures following the titles correspond to the numbers of the criteria by which they were qualified.

Consolidated List of Preferred Books

Air Navigation Part I (prepared by U.S.Navy) (3)
Air Navigation Part II (prepared by U.S.Navy) (1), (3)
Air Navigation Part III (U.S.Navy) (1), (2), (3)
Buchan and Borthwick, Aviation Mathematics (2), (3), (4)
Cajori, F., A History of Elementary Mathematics (3), (4)
Eddy and Brolly et al., A Wartime Refresher in Fundamental Mathematics (3)
Harrison, Smiley, and Lent, Maps and How to Understand Them (3)
Hawks, E., Astronomy (1), (3), (4)
Hogben, L., Mathematics for the Million (1), (2)
Hooper, A., A Mathematics Refresher (1), (3), (4)
James, G. & James, R.C., Mathematics Dictionary (3)
Johnson, C., Discover the Stars (3), (4)
Karpinski, L.C., The History of Arithmetic (1), (3), (4)
Keniston and Tully, Plane Geometry (3)
Kraitchick, M., Mathematical Recreations (3)
Lieber, H.C. and I.R., The Education of T. C. Mits (1), (2), (3), (4)
McCready, K., Beginner's Guide to the Stars (3)
Osteyee, C., Mathematics in Aviation (1), (2), (3), (4)
Palmer and Leigh, Plane and Spherical Trigonometry (3)
Sanford, V., A Short History of Mathematics (1), (2), (3)
Schorling, Clark, Smith, Modern-School Geometry (3)
The Science of Pre-Flight Aeronautics
(Research Group, T.C.) (1), (5)
Shuster and Bedford, Field Work in Mathematics (1), (2), (3), (4)
Smith, D. E., Number Stories of Long Ago (1), (2), (3)
Smith, D. E., The Wonderful Wonders of One-Two-Three (2), (3)
Smith and Ginsburg, Numbers and Numerals (2), (3), (4)
The Story of Numbers (Amer. Council on Educ.) (3)
The Story of Weights and Measures
(Amer. Council on Educ.) (3)
Telling Time Throughout the Centuries (Amer. Counc.) (1), (3)

This consolidation of the preferred book lists serves to rank the twenty-nine titles included as the most valuable in the collection of forty-three volumes, exclusive of encyclopedia sets. It may be noted that only three books were outstanding enough to merit qualification under all four of the criteria. These were: The Education of T. C. Mits, Mathematics in Aviation, and Field Work in Mathematics. If correlation among the criteria for selection is significant, the third one, favorable opinion of at least fifty per cent of the users, may be considered the weakest of the criteria chosen, since in every case where a book met only one criterion, it was that one. The inference here is plain -- unless the number of users is first found to merit attention the opinion of those users need not be weighted very heavily in the evaluation.
Significance of Term Papers

The writing of term papers as a culmination of the free reading program was originally conceived to provide a channel for the organization of ideas gleaned from reading and for the expression of these ideas in permanent form. Press of time and preconceived notions in the minds of students as to the nature of term papers prevented the teacher's idealization from becoming fact, in many cases.

Examination of Content.--There was evidence of a certain amount of copying without acknowledgment of source but this might well be attributed to the fact that the time reserved for guidance in the writing did not prove sufficient for dealing adequately with the technique of using footnotes. There were also some papers which did not reflect their color from the reading records enclosed in the folders with them, but seemed rather to have been executed in the last analysis as an attempt to fill the required ten pages with material mathematical.

But since any college professor would probably be willing to verify the statement that these flaws are inherent in term papers as a class, even on the higher level, the sixty-odd samples of immature production, here being considered, may perhaps be styled normal. There were, in many instances, passages which were clearly the result of careful thought on the part of the student author. In
other instances, the ambitious nature of the title and its treatment was a revelation of the capacity of the student for self education. It may be stated here without exaggeration that, even in the midst of the closing duties of the school year, the reading of the term papers was a real pleasure to the teacher. For obvious reasons they cannot all be included in this study yet if it were possible, they would better serve to evaluate the project than discussion of them alone can do. The analysis of briefer features, closely related to the term papers, is well within the scope of this thesis and three such features, (A) the choices of topics, (B) the grades assigned to them, and (C) the student resumes of them, will be dealt with in turn.

(A) Choices of Topics.—As stated in Chapter II, the choice of a topic was left to the student unless he sought help in the matter. Table 5, "Comparison of Term Paper Titles and Grades with O.S.U.P.T. Grade Equivalents," is presented as part of the means of tracing the participation of individual students and also as a basis for classifying the mathematical interests of students. Such a classification is made in Table 6, following page 72

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5 Two complete term papers of outstanding quality are given in Appendix III.

6 See page 29.
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<th>Student</th>
<th>G.E.</th>
<th>Title of Term Paper</th>
<th>Grade Given</th>
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<td>Calculus</td>
<td>A</td>
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<td>S-2M</td>
<td>C</td>
<td>Air Navigation</td>
<td>B</td>
</tr>
<tr>
<td>S-3M</td>
<td>C</td>
<td>Mathematics in Aviation</td>
<td>A</td>
</tr>
<tr>
<td>S-4M</td>
<td>B-C</td>
<td>Aircraft Motors of Today and Yesterday</td>
<td>C</td>
</tr>
<tr>
<td>S-5F</td>
<td>C</td>
<td>Air Navigation</td>
<td>A</td>
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<tr>
<td>S-6F</td>
<td>C-D</td>
<td>Telling Time Through the Ages</td>
<td>A</td>
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<td>S-7F</td>
<td>B-A</td>
<td>(Read text books only. No term paper.)</td>
<td>C</td>
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<td>S-8F</td>
<td>B-A</td>
<td>Geometry in Aeronautics</td>
<td>A</td>
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<tr>
<td>S-9F</td>
<td>C</td>
<td>Lives of Men Who Made Mathematics</td>
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<td>SI-10M</td>
<td>B</td>
<td>Air Navigation</td>
<td>C</td>
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<td>SR-11F</td>
<td>B-C</td>
<td>Plane Geometry</td>
<td>C</td>
</tr>
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<td>S-12F</td>
<td>B-C</td>
<td>Early Mathematics</td>
<td>B</td>
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<td>SI-13F</td>
<td>C</td>
<td>Galileo</td>
<td>C</td>
</tr>
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<td>S-14F</td>
<td>B-D</td>
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<td>C</td>
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<td>SI-15F</td>
<td>C</td>
<td>Mathematics in General</td>
<td>C</td>
</tr>
<tr>
<td>S-16M</td>
<td>D</td>
<td>(Read single book on aviation. No paper.)</td>
<td>A</td>
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<tr>
<td>SI-17M</td>
<td>C</td>
<td>From Numbers Start to Logarithm</td>
<td>A</td>
</tr>
<tr>
<td>S-18F</td>
<td>B</td>
<td>Airplane Instruments</td>
<td>A</td>
</tr>
<tr>
<td>S-19F</td>
<td>B-C</td>
<td>Navigation in Aviation</td>
<td>C</td>
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<td>S-20F</td>
<td>C</td>
<td>The Right Triangle</td>
<td>A</td>
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<td>B</td>
<td>The Slide Rule</td>
<td>A</td>
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<td>SR-22M</td>
<td>B-C</td>
<td>Maps</td>
<td>B</td>
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<td>C</td>
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<td>C</td>
<td>Geo-Navigation</td>
<td>A</td>
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<tr>
<td>S-25M</td>
<td>C</td>
<td>(Read on aviation. No term paper.)</td>
<td>C</td>
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<tr>
<td>J-26F</td>
<td>C</td>
<td>Surveying</td>
<td>B</td>
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<td>S-27F</td>
<td>C</td>
<td>The History of Geometry and Algebra</td>
<td>B</td>
</tr>
<tr>
<td>S-28F</td>
<td>B</td>
<td>Mathematics through the Ages</td>
<td>B</td>
</tr>
<tr>
<td>S-29F</td>
<td>B-C</td>
<td>(Varied historical reading. No term paper.)</td>
<td>A</td>
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<td>J-30F</td>
<td>A-B</td>
<td>A Basis for Mathematics</td>
<td>A</td>
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<td>GS-31F</td>
<td>C-D</td>
<td>Early Mathematical Beginning</td>
<td>A</td>
</tr>
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<td>S-32M</td>
<td>B-C</td>
<td>The Education of T. C. Mits</td>
<td>A</td>
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<td>S-33F</td>
<td>A</td>
<td>The Development of Numbers and Numerals</td>
<td>A</td>
</tr>
<tr>
<td>GSI-34M</td>
<td>C</td>
<td>(Scattered reading. No term paper.)</td>
<td>A</td>
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<td>S-35M</td>
<td>B</td>
<td>(Lost his folder. No term paper.)</td>
<td>D</td>
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<td>C</td>
<td>The Review of Mathematics</td>
<td>D</td>
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<td>C</td>
<td>Numbers and Numerals</td>
<td>C</td>
</tr>
<tr>
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<td>B</td>
<td>History of Mathematics</td>
<td>C</td>
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<td>S-39F</td>
<td>D</td>
<td>My Own Version of Field Work in Mathematics</td>
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<td>S-40F</td>
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<td>Numbers and Field Work</td>
<td>D</td>
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TABLE 5 (Contd.)
Comparison of Term Paper Titles and Grades with O.S.U.P.T. Grade Equivalents

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<th>Student</th>
<th>G.E.</th>
<th>Title of Term Paper</th>
<th>Grade Given</th>
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<td>S-41F</td>
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<td>The Beginning of Mathematics</td>
<td>A</td>
</tr>
<tr>
<td>S-42F</td>
<td>B</td>
<td>Aerial Navigation</td>
<td>C</td>
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<td>SI-43F</td>
<td>C</td>
<td>Mathematics in Ancient Time</td>
<td>C</td>
</tr>
<tr>
<td>SI-44F</td>
<td>C</td>
<td>The History of Arithmetic</td>
<td>A</td>
</tr>
<tr>
<td>S-45M</td>
<td>B-C</td>
<td>Mathematics in Aviation</td>
<td>A</td>
</tr>
<tr>
<td>S-46F</td>
<td>C</td>
<td>Changes in Mathematics During Time</td>
<td>C</td>
</tr>
<tr>
<td>SI-47F</td>
<td>C</td>
<td>The Story of Numbers</td>
<td>B</td>
</tr>
<tr>
<td>S-48F</td>
<td>B-C</td>
<td>Telling Time</td>
<td>B</td>
</tr>
<tr>
<td>S-49M</td>
<td>B</td>
<td>Aviation and Air Navigation</td>
<td>A</td>
</tr>
<tr>
<td>SN-50M</td>
<td>B</td>
<td>The Principles of Aerial Navigation and Dead Reckoning</td>
<td>B</td>
</tr>
<tr>
<td>S-51M</td>
<td>A-B</td>
<td>Geometry</td>
<td>D</td>
</tr>
<tr>
<td>S-52F</td>
<td>A-B</td>
<td>Numbers</td>
<td>B</td>
</tr>
<tr>
<td>S-53F</td>
<td>C</td>
<td>Euclid’s Works</td>
<td>C</td>
</tr>
<tr>
<td>S-54F</td>
<td>C</td>
<td>Numbers and Time</td>
<td>B</td>
</tr>
<tr>
<td>SI-55M</td>
<td>C</td>
<td>Astronomy and Mathematics</td>
<td>B</td>
</tr>
<tr>
<td>S-56F</td>
<td>B-C</td>
<td>Ancient Timepieces</td>
<td>C</td>
</tr>
<tr>
<td>SI-57M</td>
<td>C-D</td>
<td>Map Projections</td>
<td>D</td>
</tr>
<tr>
<td>S-58M</td>
<td>C</td>
<td>One, Two, Three</td>
<td>B</td>
</tr>
<tr>
<td>S-59M</td>
<td>B</td>
<td>Astronomy and Mathematics</td>
<td>B</td>
</tr>
<tr>
<td>S-60M</td>
<td>D</td>
<td>History of Surveying</td>
<td>A</td>
</tr>
<tr>
<td>S-61M</td>
<td>B</td>
<td>Mathematics in Aviation</td>
<td>C</td>
</tr>
</tbody>
</table>

J-101M   | C    | A Report on Navigation | B           |
| J-102M   | C    | Mathematics and Everyday Life | D           |
| J-103M   | C-D  | Logarithms             | D           |
| J-104M   | B-C  | Thought Inspired by Math Class | A           |
| J-105M   | A    | Math and our Everyday Lives | B           |
| J-106M   | A    | The Triangle           | A           |
| J-107M   | B    | Surveying Instruments-Past and Present | B           |
| J-108M   | B    | The Use of Numbers in Instruments and Astronomy | B           |
| J-109M   | C    | Instruments Used in Mathematics | B           |
| J-110M   | C    | Geometry in Aeronautics | B           |

See explanation of Table 1 for abbreviations used.
and for that reason Tables 5 and 6 are closely associated in this discussion.

Scanning the list of titles in Table 5, one is apt to note that they fall into several classes, closely related of course, to the variety of topics offered in the book collection. Eighteen such classes were tentatively listed and then by combination of the various phases which were treated in aviation, mathematical history, and surveying, the list of classes, or general topics, was narrowed to twelve. The term papers were classified under these topics with regard for their content and not alone for their titles.

The resulting distribution is shown in Table 6. The decided preference for the topics, "Aviation" and "History of Mathematics," shown in the column of totals, may be attributed in part to the predominance of such material in the collection. It is impossible to separate the influence of this factor from the natural appeal of the two topics. Some weight may be assigned to the latter, though, since the selection of historical material was common to all four of the grade classes and the selection of aviation material was common to all except the A, A-B class.
### Table 6
Reading Choices of Students Classified as to C.S.U.P.T. Grade Equivalent*

<table>
<thead>
<tr>
<th>General Topic</th>
<th>A &amp; A-B (Above Average)</th>
<th>B &amp; B-C (Av.)</th>
<th>C (Below Av.)</th>
<th>C-D &amp; D</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Astronomy</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Aviation</td>
<td>10</td>
<td>7</td>
<td>1</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>Biography</td>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Calculus</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Geo-navigation</td>
<td>1#</td>
<td>1</td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>History of Mathematics</td>
<td>3</td>
<td>5</td>
<td>15##</td>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>Logarithms and Slide Rule</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Maps</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Philosophy of Mathematics</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Surveying</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>9</strong></td>
<td><strong>24</strong></td>
<td><strong>34</strong></td>
<td><strong>7</strong></td>
<td><strong>74</strong></td>
</tr>
</tbody>
</table>

*The Ohio State University Psychological Test sets Grade Equivalents as follows: 100–96 Centile, A; 95–76 Centile, B; 75–26 Centile, C; 25–1 Centile, D. Combination letters are used to indicate change of grade placement in successive tests.

#Transfer student with no test record. Grade placement on the basis of class work.

##Includes one student whose grade equivalent was B-D.
Choices of the eight students\(^7\) in that "best" class allow for an interesting interpretation. Their topics may be seen to lie, with one exception,\(^8\) along the lines of the history, philosophy, and theory of mathematics. That one exception is the topic of geo-navigation, chosen by the transfer student who is included in this group on the basis of his class work and not because of C.S.U.P.T. prediction. No such generalization is apparent in the choices of the "average" and "below average" groups, whose papers dealt with scattered uses of mathematics as well as with its history.

A seeming discrepancy may be noted between the totals of Table 6 and those of previous tables. The mention of eight students in the A, A-B classes is apt to provoke the idea. This variance may be explained by the dual nature of some term papers which were accordingly listed under two topics. Hence totals in Table 6 do not denote students or term papers, but treatments of topics.

A further word of explanation needs to be made about the grouping of the grade classes for the purposes of this table. Although it may seem inconsistent with the school

\(^{7}\)Table 2, page 56A, shows seven students in the A and A-B classes, but one transfer student is here included on the basis of his class work. Seeming discrepancy in the total of nine shown in Table 6 is explained further on in the text.

\(^{8}\)The choice of trigonometry entitled "The Triangle" and shown in Appendix III cannot be considered an exception since it dealt with the theory of logarithms as well as the source of trigonometric functions.
policy of down-grading in-between students, explained earlier, the present grouping was made in order that "below average," "average," and "above average" student choices might be studied. In the terminology of some intelligence test norms the "above average" group may be said to contain the "superior" and the "very superior."

Some significant characteristics of the first two groups, and of the "very superior" students in the third group, have already been noted. The outstanding characteristic of the "superior" or "slightly above average" group is the insatiable thirst for knowledge of aviation, evidenced by nearly half of them.

Reference to Table 5 furnishes the information that one girl is numbered among the ten "above average" students who were interested in aviation and that two other girls, with grade equivalents of 2, also wrote on the subject. Because of the predominance of boys in the enrollment no attempt was made to discover differences between the sexes in the matter of reading selection. It is interesting to note, however, that the air age holds fascination for the girls as well as for the boys. A yearning for air service was vicariously satisfied for S-SF, who after a period of reading about the twenty-four hour clock used by the American Air Service and the Royal Air Force, wrote: "I have only read a page in this so far because I want to get

9 See page 56.
everything out of it that I can . . . . I think in my vocation which I have planned for after I graduate these things will be very useful."

(B) Comparison of Grades with O.S.U.P.T.—Insight into the thinking of the learner, as gained in the case cited, furnishes a far better evaluation of the learning which has taken place than any letter attached to a sample of his work can ever do. For purposes of office records and college entrance, however, such measurements would prove too voluminous. Hence, the first four letters of the alphabet must serve as a condensation of the teacher's estimate of any passing work done by her pupils. Table 5 carries in the final column the distribution of these letters over the faces of the manuscripts, imperfect symbols though they be for indicating the degrees of worth among those manuscripts.

Simple counting was employed to ascertain that in twenty-six cases the grades assigned to the papers surpassed the expectancy set by the Ohio State University Psychological Test, that there were twenty-eight cases of agreement, and that fifteen cases failed to meet the expectancy. No particular significance is attached to the enumeration, especially since the standards used in grading the term papers were somewhat different from those employed in grading weekly assignments or written examinations. Grades given on term papers were C's and D's only in cases where the specifications as to length and items
of bibliography were seriously deficient. The grades are carried in Table 5, not for their significance, but for the interest which they may hold for teachers confronted with a similar problem of measuring the immeasurable.

(C) Resumés Reflect Actual Learning.--Knowing that term papers can sometimes be compiled rather than composed and that little learning may result, the teacher was desirous to know to what extent students could reproduce the chief points of their papers without benefit of reference materials. Hence, at the time when the five-part questionnaires were introduced into the program, the classes were also instructed to write a paragraph or more telling what their "term papers were about." This instruction turned out to be somewhat ambiguous and perhaps the assumption that they would not have understood the words resumé and précis was not a valid one. At any rate the attempts of the teacher to clarify the task were variously interpreted by the students and were reflected in the resumés through such phrases as "the most important idea," "the thing that impressed me," "I learned," and "my paper dealt with." Though some of these writings could not be called resumés or précis, nearly all of them gave evidence of the quality of learning which had taken place and so fulfilled the purpose of the assignment, which was to test what was understood without reference to papers or books.
In attempting to cull the contributions for representative samples it was difficult to exclude any from this discussion. However, the guiding factor was a desire to distribute the demonstration of values throughout the entire list of the twelve general topics carried in Table 6. Accordingly, a number of resúmes dealing with each of the topics is inserted. That number will be found to vary from one to several, depending upon the popularity of the topic and the length and import of the samples used. In cases where the writer was known, his distinguishing cue is given, but in many instances the students preferred to hand in their questionnaires anonymously. The chosen resúmes are intended to show various degrees of learning and scholarship and in order to preserve the genuine high school flavor errors in English and spelling have been retained except in instances where they were not understandable. In these instances, in so far as possible, attempts at translation have been added rather than substituted. Comments have been interspersed to call attention to the various points of emphasis and to the contrasts in extent of actual knowledge indicated.

Astronomy was selected by only three individuals and the two resúmes given here represent the extremes in the quality of its treatment.

Mathematics can and is used in a great variety of subjects. In my paper, I used the
sextant and astronomy as my subjects. In the sextant you need a knowledge of math(ematics) to work the instrument in taking sights on different objects. Astronomy needs math(ematics) in determining the distances from Earth to other planets, or stars. J-108M

I learned a lot more about the writing of the Romans, Egyptians and the cave men and Indians and others. Also what the discovery of the stars has to do with mathematics. Anon.

Aviation received more widespread attention with the expected variations in the exactitude of knowledge gained.

The thing that impressed me the most about the term paper that I made was the geometry used in finding your course and direction (,), the angles of flight and many others. I thought that you just got in a plane and if you wanted to dive you could without calculating the angles. I also studied more about that 24 hour time used by the air services. S-2M

I learned about lift, drag, and resultant on a rectangular solid in a wind tunnel. I learned about the longest flights, highest
speeds maintained, and the highest heights maintained from the time of the first plane up to today. I learned a little about ratio and proportion.

It gave me an idea on how much geometry is needed and used in Aeronautics. The way in which our theorems and our study of ratio and proportion is connected in Aeronautics was an idea which impressed me. The way to read charts and maps was explained and it interested me.

The things I learned was a lot of facts about navigation, dead reconing, drift, and I also learned a lot of definitions. I learned how numbers came to be and how other countries use numbers.

I learned the fundamentals of air navigation, I also learned how to maneuver (maneuver), plot a course and got to know the instruments used in a plane. (Elsewhere in the questionnaire) for my topic was on navigation and it rather messed (missed) mathematics.
Biography was chiefly attractive to girls except as it functioned in the portrayal of contributions to a particular field in which a boy was interested. The examples given below indicate the interest in people above abstractions, which was apparent in the writers themselves.

The thing that impressed me more than anything else was the way Galileo stuck to what he discovered. It (my term paper) tells how many things you can credit one man for discovering, the things he discovered like the stars, theometer (thermometer), pendulum which improved the clock. It shows what mathematicians have done to help the world. What would we do without clocks. SI-13F

Euclid was a great mathematician and he was a great teacher(.) he also wrote a group of books called the elements which are still being used. We have learned many of his theorems and postulates this year. S-53F

Told mostly about Euclid and his axioms, where, and when Geometry started, who contributed mostly to progress of subject. Anon.

Calculus was understandably less appealing as a term paper topic and the one boy who wrote in very creditable
fashion about it did not attempt to report his findings
divorced from his authority.

My paper is hard to condense into a paragraph
since it is in condensed form as it is. I tried
to show the men who were connected with the
development of the Calculus and tell a few
facts about their lives and what aid they
gave to the development of the calculus. S-1M

Geometry itself held the attention of four students,
even when faced with choice, and papers on the subject
varied from specialized treatises to generalized ramb-
lings augmenting the work done in class.

Circles can be rotated without changing the
position of the figure. That is why wheels
are used in industry, because of their shape.

(Elsewhere was evidence of dissatisfaction with
her own narrow interpretation of "one subject".)

... it (the reading) is only over one sub-
ject and it is difficult to find enough material.
I got less than five pages. S-7F

The early Egyptians were about the first to use
Geometry. Then it spread to Greece and they con-
tributed more to it. It went from there into
northern Europe where it got most of what we
have today. S-27F
Pythagoras had a school for men and mathematics. He based his theorem (theorem), the square of the hypotenuse is equal to the sum of the squares of the other two arms, was based upon the Chinese block printing.

My paper told about angles, triangle, tables, great people in the field mathematics.

Geo-navigation was the topic chosen by a gifted student whose older brother was in the merchant marine. The paper itself was so unusual that it is included, complete with tracings of the boy's own plates, in Appendix III. This resumé, written in ten minutes, without previous warning, is proof positive that he really understood what he wrote:

Navigation is divided into many parts. Geo-navigation is determining position by speed, time, and distance, with the aid of fixes. Piloting is used in ports and along shore. Position is determined by taking fixes (bearings) on various objects. When there are reefs, shoals, etc., a safe passage can be found by using danger /a.

In many harbors ranges are provided. These mark the safe channels. The charts used give location of obstructions (rocks, reefs, shoals etc.) Before leaving a port the course (intended path of travel) is plotted. Any deviation from this course;
(due to compass error, weather conditions etc.), and the track (actual path traveled) will not coincide with the course. Geo-navigation is used when other types (celo-navigation) cannot be used. It isn't to accurate but it is good enough for short distances. Other types of navigation are preferred for greater distances because of their greater accuracy. SN-24M

The history of mathematics presented many facets to the twenty-five students who investigated it. Each of the resumes below was included for the different light which it reflects from the facets as they were viewed.

It (the term paper) was rather educational and adds to one's knowledge on mathematics. I learned of some of the authors which have contributed to the teachings of mathematics. Mathematics developed very slowly and it took several men to prove the same thing possible and right before it was ever printed in books. Different steps in mathematics had many different names before it was finally called what it is today. S-46F

Mathematics started in the caveman age with primitive forms of writing. It gradually progressed until the time of the Egyptians. They
improved the numerical system so much that we use some of their methods today. As time progressed mathematics improved. After the industrial Rev.(olution) mathematics were used in machinery in more complicated forms. Today mathematics and numerals are used in almost all things. S-35M

The number system was first begun in a very primitive form, such as saying "ung". Then numberals were begun by using scratches. Calculating was first done with an abacus and gradually became our system of calculating today. The ancient people had a great deal to do with numbers we use today, especially the people who invented the zero. S-33F

The history of numbers and numerials have come a long way since the beginning of man's counting. S-58M

While writing my term paper I found out exactly how Arithmetic started and the rapid progress it made coming up through the years. This impressed me to a great
extent because before this I had often wondered but never had any chance to find out. It also gave me a clearer understanding of Geometry and made me feel as if I knew more why I was taking it than I did at the beginning of the year. SI-44F

I learned how telling time began with cave men and advanced to our modern clocks and watches. They began telling time by the sun, then by sundials, water clocks, sand, fire, and gradually worked up to early clocks and then to modern accurate clocks. S-46F

Logarithms and the slide rule proved fascinating subjects for certain boys, who reported with varying degrees of accuracy in their term papers but could not be expected to have attained a mastery equal to the resume situation. The subject of logarithms is most clearly presented in the complete paper which forms the second part of Appendix III, but other boys' references, both to logarithms and to the slide rule follow here:

I learned that there was a time when people couldn't count above two or three. How usefull
Logarithms is to a machinist or anybody who has to work a lot of mathematics in the form of finding the roots of a number or in multiplying large sums or in dividing large numbers. SI-17M

My term paper included all the fundamental operations of the slide rule. It had no outstandingly important parts—all sharing the importance placed upon them. That is, whatever operation you were most interested in, would naturally be most important in your mind. For me the most important were multiplication, square root, and proportion. S-21M

Maps and projections did not command the share of attention they deserve, it seemed. Though the available materials were recommended a number of times, they seemed to appeal to a small group of students only. These few dealt adequately with the subject in their term papers but gave only brief generalizations in their resumes.

There are 4 types of projections of the earth, Mercator, Polyconic, Gnomic (gnomonic), and Lambert Conformal. Mercator projection is the most used for surface or flying navigation. The others are not used as much as Mercator, but
on Gnomonic Sailing charts Radio Bearings appear as straight lines and therefore (it is) easy to chart courses.  

I increased my knowledge of mapping and (it) may help me in my future life. It made my Geometry more interesting and helped to teach me to find material on my own. It showed how maps are made now. How they were made in the future (past), who made them. How they help us learn to understand the world better.  

The philosophy of mathematics proved intriguing to some although they did not recognize it as such. The beliefs and conclusions with which they came in contact in their reading were quite apparent in their term papers but the language of philosophy seems not to have entered their own vocabularies sufficiently to allow them to express their ideas within the limitations of a precis. My subject dealt with mathematics and everyday life. I gave the history of mathematics to the history of reasoning. I also showed the importance of mathematics to our present day civilization.  

Anon.
The most important part of my term paper is the story of the totem pole because it explains how unity is important in the progress that is to be made in the future. Anon.

Surveying holds appeal along all scholarship levels but again no very technical explanations were found in the hurriedly written resumes.

(My term paper showed) That without surveying men would have a hard time surviving. Men need it to build their roads, buildings, railways and numerous other things. It is also used to lay out land, find height and distance. SR-26M

In surveying there are many important things, things I did not realize that had to be done. I also did not realize that there were so many surveying instruments. In reading the books I did I learn(ed) a lot about surveying that I don't think I would having (have) known. Anon.

Trigonometry was treated at great length and in such excellent style by one exceptional student that his complete paper (already mentioned under "Logarithms") forms a part of Appendix III. The precis which he wrote in class is given here for comparison.
The triangle and the calculations involved are basic in many branches of science. Important things to be known for maximum use of the triangle are: Logarithms, fundamental theorems, trigonometric functions. Triangulation is used by surveyors and navigators for measuring the distance between inaccessible points.

Student Opinion of the Free Reading Program

In addition to the opinions given weekly concerning specific books, the student reaction to the entire reading program, taken in retrospect, was considered vital to the successful revision of the program, called for in the original specifications set forth for it. Attempt was made to study the reactions from five different angles called to their attention by the questionnaire.

Easiest and Hardest Books.—These were named by students and the tabulation of their choices is shown in Table 7. The vote was naturally quite scattered since no one pupil became acquainted with more than a fourth of the collection. Thus thirty books, or about three fourths of the collection, were named and conflicting opinions were evident in ten of the thirty cases. In only one case,

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10 See Table 3, pages 59A and 59B.
# TABLE 7

"Easiest" and "Hardest" Books as Reported by Students

<table>
<thead>
<tr>
<th>Title</th>
<th>Easiest Book</th>
<th>Hardest Book</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Navigation II</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Air Navigation III</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Air Scout Manual</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aviation Mathematics</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>A History of Elementary Mathematics</td>
<td></td>
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</tr>
<tr>
<td>A Wartime Refresher in Fundamental Mathematics</td>
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<td>1</td>
</tr>
<tr>
<td>Encyclopedias</td>
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<td>4</td>
</tr>
<tr>
<td>Maps and How to Understand Them</td>
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<td>1</td>
</tr>
<tr>
<td>Basic Mathematics</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Astronomy</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Mathematics for the Million</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>A Mathematics Refresher</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Discover the Stars</td>
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</tr>
<tr>
<td>Through the Overcast</td>
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<tr>
<td>The History of Arithmetic</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Plane Geometry - Keniston &amp; Tully</td>
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</tr>
<tr>
<td>Mathematical Recreations</td>
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</tr>
<tr>
<td>Education of T. C. Hits</td>
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<tr>
<td>Merchant Seaman's Manual</td>
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<tr>
<td>Mathematics in Aviation</td>
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<td>3</td>
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<tr>
<td>A Short History of Mathematics</td>
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<td>Modern School Geometry</td>
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<td>Science of Pre-Flight Aeronautics</td>
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<td>Principles of Air Navigation</td>
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<tr>
<td>Field Work in Mathematics</td>
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<td>The Wonderful Wonders of One, Two, Three</td>
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<td>Numbers and Numerals</td>
<td>2</td>
<td></td>
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<tr>
<td>The Story of Numbers</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Telling Time Throughout the Centuries</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

| Other Answers                                    |              | 8            |
| All                                              |              | 2            |
| On the level with other sophomore reading        | 2            |              |

89A
that of Shuster and Bedford's *Field Work in Mathematics*, was the vote deadlocked. Yet since Table 4\(^{11}\) revealed that this book was one of two to meet all four of the criteria set up for judging useful books, we may conclude that divided opinion on the difficulty of a book does not preclude its use and enjoyment by all of those tending toward division. The eight people who thought all books were easy (or none were hard) probably had not the same books in mind as the two who thought all books were hard, though the extremes of scholarship among the class members might make this difference of decision consistent, even for the same book. No claim is made, however, of any absolute value resulting from Table 7. It is included for the purpose of suggesting further research which could yield valuable results if it were sufficiently extensive to admit of validity. Records would have to be kept over a longer period of time with reports from a larger number of students, all having used the entire collection, before any valid deductions concerning the relative difficulty of books could be attempted.

**Pleasure in Free Reading.**—Enjoyment in choosing their own readings, as opposed to having them assigned, was measured by the first two sections of the questionnaire. The vote of sixty-four students was taken and the results appear in Table 8, which follows this page. The "Yes"

\(^{11}\) See pages 64A, 64B, and 64C.


<table>
<thead>
<tr>
<th>1. Did you enjoy choosing your own reading material?</th>
<th>Yes</th>
<th>No</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>11</td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. Would you have preferred a list of readings from which to choose a certain number?</th>
<th>Yes</th>
<th>No</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>51</td>
<td></td>
<td>12*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. Do you think this reading project should be continued with future classes?</th>
<th>Favorable</th>
<th>Unfavorable</th>
<th>Neutral</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>26</td>
<td>1</td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. Do you feel that on Monday you learned (less than, as much as, more than) during other forty minute periods of mathematics study?</th>
<th>Less</th>
<th>As Much</th>
<th>More</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>18</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

| 5. Name the easiest book you read. Which was hardest? | See Table 7 for answers. |

| 6. Write a paragraph or more telling what your term paper was about. | See section of Chapter III dealing with resumes of term papers. |

* Includes two neutral votes.
** Includes one neutral vote.
vote on the question of enjoyment of free reading was identical with the "No" vote on the question of assigned readings, indicating that students looked upon the two plans as opposing alternatives, even though it was suggested that choice among the latter would be allowed. All answers on the question of enjoyment were decidedly "Yes" or "No", but on the question of a list of readings, two students gave noncommittal answers. In addition there were several modifications of the answers to these two sections which gave some illuminating points. The question of enjoyment yielded: "I knew what I wanted and liked and if it was picked for me I don't think I would have finished it." Another student wrote, "To a certain extent, but it was difficult to find an interesting one." Another made the contradictory statement: "Yes, I enjoyed it but did not like the topics we had to write on. Yes, [I would have] enjoyed a list." The lukewarm endorsement of one boy read: "No, I didn't enjoy it but it was better than some of the things we do in school."

A List of Readings.—The proposal of a list from which to choose met with varying responses: "No. Although it might be all right," and "That would have made no difference," occupied the middle ground between, "Yes, because my topic did not afford much material," and "No, having a list would not interest me, it gives you more of a responsibility choosing what you want."
Whether to Continue.—The issue of continuance of the program, if left entirely in the hands of past participants, would have been settled in the negative by the narrow margin of one vote unless further investigation of two ambiguous votes had thrown their influence to the affirmative side. The first of these began with a definite, "No," which was then qualified by the words, "I think future geometry classes should be given a chance to vote as to whether they want the reading program or not." The purely neutral vote recorded among the fifty modified ones under section 3, Table 8, also recommended the vote by future students.

The almost universal tendency to modify the straight "Yes" and "No" answers to this section of the questionnaire point it out as a vital issue. Twenty-three arguments were made in favor of continuance and twenty-six against.

The method of intelligence applied to the "reasons" for negative opinions on the matter of continuance, however, casts a doubt over the wisdom of allowing the might of sheer numbers to make right their decision. Whereas arguments in support of continuance were so varied as to permit no classifications among them, the comments of those voting against continuance fell strangely into a pattern of logic and emotionalism. Among the logical comments were several constructive criticisms such as:
(1) Time wasted: "It was a waste of time. These books have mathematics in them but it is not as concentrated as daily work." S-61M
"I feel that if it is to be continued it should be in a separate class." Anon.

(2) Material useless: "Some of the topics the classes [students] have chosen do not have much mathematics in it [them]. If the books do have mathematics in it [them] they do not understand it because it is more advanced work than they were accustomed to doing." Anon.
"In some cases no math was found at all." S-19M

(3) Method unsatisfactory: "I think they would get more from working together in class. I didn't understand more than half of what I read and I think the subject which I read about should have been studied by the whole class because it is one of the most practical uses of geometry and other mathematics." S-42F

(4) Term paper unpopular: "Making a term paper with geometry is a hard task." Anon.
"I don't think I learned very much from writing a term paper." Anon.
"I think term papers should be confined to college or seniors." S-53F

Several students resorted to emotionalized statements as "proofs" of the justice of their negative notes. Without
realizing it they were merely seeking to justify their own shortcomings as to certain factors:

(1) Use of time: "Didn't learn anything. It just took up time . . . . I had to do biology so . . . . I didn't learn anything from it." S-38F

"I think it takes up too much school time to do this project. We could have used those Mondays for learning geometry. We could have used this time studying for our examination;" (which was scheduled for that afternoon.) S-60M

(2) Low grades: "You work real hard and then you just get C. I didn't have time to copy it . . . . I got a C and think it was worth a B: I didn't learn anything from it." S-38F

(Even the argument of S-42F, given above under "Method unsatisfactory," ended on an emotional note aroused by the grade): "It was studied by the whole class last year and I know they learned more than I ever could have alone. I worked very hard on it though and tried to get as much as I could out of it. For that reason I think I deserved more than a C on my term paper." S-42F

(3) Lack of learning: "I didn't learn anything out of geometry as it was without writing a ten page term paper to mix up my thoughts." S-41F

\[^{12}\text{See S-38F under "low grades" for portion omitted here.}\]
"Too much extra study that should be spent on daily work. It's a dumb way to learn the history of geometry. It ought to be in the book we use everyday and ought to be learned in the classroom."

S-27F

"I don't think I learned very much from writing a term paper. I think the time spent should have been used in explaining geometry to us because there are still a lot of things we don't understand."

Anon.

"It takes too much time and you don't learn geometry."

Anon.

"It cuts down the time which should be used in explaining the work which is supposed to be done for that year."

S-46F

Proponents of the reading program also registered comments based upon various degrees of logic but, since they were not the malcontents, emotionalism did not play so large a part in their arguments. As stated previously, it was next to impossible to fit the favorable opinions into a pattern, hence they are arranged from first to last with reference to the respective soundness of their logic.

"It gives them a wider scope on mathematics."

Anon.

"It gives you a chance to study on your own accord."

S-8F
"It made geometry more interesting and helped to teach me to find material on my own."  

SR-22M

"It gets you acquainted with mathematics where it started and how much it had developed to our present time."

S-54F

"It is a good experience and you not only find out more in mathematics, you get a clearer understanding of why you should take it."

SI-44F

"I think you acquire more mathematics this way than simply learning plan (plain) mathematics."  

JI-109M

"It will help them to understand a chosen topic as well as their geometry."

S-5F

"I think it's interesting to know more about the things you want to know about concerning mathematics."

S-9F

"Yes, because it was very interesting."  

S-57M

"I think our term papers were very useful. Mine helped me to learn the history of our number system and the different methods people used to bring it about."

SI-47F

"I feel it is much better to be able to do your own free reading on a topic which interests you."  

Anon.

"It really isn't hard and I enjoyed having a period to read something I really was interested in. You get something that you alone would like to study and learn something about."

S-39
"I acquired a amount of knowledge that I should have gotten in class." SR-11F

"I think it's a very good idea. I never get any enjoyment out of readings I have to do." SI-43F

"Yes, as long as I don't have to write any more term papers."

SI-10M

"Yes, I'm not taking advanced mathematics." Anon.

"Yes, let the rest suffer." S-1M

"They should be made as uncomfortable as we were."

SAI-106M

In all justice to the last two students quoted it must be said that they were admittedly in the facetious mood of the last week of school and could ordinarily have been expected to present more logical arguments to support their beliefs. Their remarks viewed objectively, serve to complete the range of "proofs" presented. Aside from any influence upon the decision to read or not to read, the quotations taken as a whole might serve as the basis for profitable discussion in future geometry classes concerning the "Nature of Proof."

Conclusions in the issue of continuance may well be deferred until consideration has been given to the fourth section of the questionnaire and to the student reactions upon it.

Extent of Learning.—The amount of learning as compared with periods of similar length, spent in conventional study
and discussion of mathematics, was variously estimated by students. Answers of "Less," "As much," and "More" were suggested to achieve sufficient standardization for purposes of tabulation. Table 8 shows opinion to be again evenly divided, if the "as much" and "more" answers are weighed against those of "less." Again some interesting comments were added. One student who felt that he had learned less, specified class discussion as a means of learning more. The same thought was elsewhere indicated by one or two others who felt a certain inadequacy in private study. A highly significant distinction was drawn by one boy who wrote, "Less mathematics but more about mathematics." It would be revealing to inquire his reason for making the distinction, to learn which he considered more important and to get his explanation of how one could be learned without the other. The closing of school prevented a conference on these points.

Justifiable Conclusions.--The drawing of conclusions from Table 8 becomes somewhat difficult in view of all extenuating circumstances. It must be remembered that fifty-one students, out of sixty-four voting, enjoyed the free reading program and would not have preferred a list of readings from which to choose. Yet student opinion was

13 See page 93.
evenly divided over the points of the amount of learning and of continuance. One might almost detect a puritanical note in the replies of those nineteen students who admitted enjoyment but not value, although it is conceivable that some of them were attempting a long range view of the matter. At any rate the analysis of fifty expressed reasons for affirmative and negative votes on the question of continuance seems to warrant submitting the program to more students before committing it to the realm of discarded techniques.

**Adult Opinion**

Progressive education has been criticized from within by a charge to the effect that it has sometimes encouraged youth to render opinions and make decisions in matters on which "he is not sufficiently informed or mature to be able to make decisions of the magnitude under consideration; that is, he is extended freedom to think about the culture beyond his capacity to do so, and this serves primarily to inflate the ego."\(^{14}\) This charge might be laid upon the evaluation measure just cited, if it were to be used as the sole basis for decision as to the continuance of the reading program, and would most certainly apply if the

\(^{14}\)E. J. Kircher, Class lecture, *Education 620*, Ohio State University, Summer, 1946.
decision should follow the suggestion of two students that future classes be allowed to vote whether or not they should enter upon such a program. Coupled with the belief that a school or class controlled entirely by the will of the students, is just as undemocratic as one which is ruled entirely by the instructor, the theory outlined is another good reason for registering adult opinion of the free reading program before reaching final decision of the issue: to read or not to read.

The Librarian Interviewed.—The librarian of the school where the experiment was tried had opportunity for comparing the approaches to reading, the habits developed, and the values received, as they were apparent in all schemes of library usage throughout the school. Accordingly her reactions to the free reading program in mathematics were sought. During an interview shortly after the close of school she stressed the following points in referring to it:

(1) Sophomores have probably been assigned not more than two term papers and juniors not more than three at this point in their school careers and these are not likely to have been extensive over periods of more than six weeks each, compared with the full semester extent of the mathematics assignment.
(2) Subjects of term papers are commonly assigned in school practice and students are left to seek the aid of the librarian in locating source material.

(3) No reference work in mathematics had previously been done, as verified by the circulation cards of mathematics titles in the library at the time this project was begun. Many bore only one or two dates of issue and in some cases the most recent of these were in 1942.

(4) Students engaged in the free reading program knew the sources of material available and seldom needed the assistance of the librarian.

(5) Circulation increased noticeably as the deadline for term papers approached.

(6) Students were sometimes selfish and even quarrelsome in their demands for a book which others also needed to use.

Observations by the Teacher.—Inferences to be drawn from these remarks are in most cases obvious:

(1) The statement of the librarian, reinforced by the opposition of a number of students, seems to justify the revision of the term paper feature of the program along lines more in harmony with the capacity of the mid-high-school years to produce. Yet in consideration of such samples of work as those in Appendix III, the requirement should not be scaled so
low as to keep superior students from performing at their highest level.

(2) Assignment of subjects for term papers, as contrasted with independent choice, was well within the experience of the students involved and so the weight of their answers on the first two sections of the questionnaire (enjoyment of free reading and lists of readings) is seen to be of some importance.

(3) The free reading program brought about the use of books which would otherwise have been neglected, but which undoubtedly satisfied the needs of students, thus bearing out the statement of Anderson\textsuperscript{15} to the effect that accessibility is a determinant of choice.

(4) Engaging in free reading \textit{in the classroom} served to acquaint students with the materials in the field and to encourage habits of independence in research. Students themselves recognized this value in the program, as reported in the section of this chapter dealing with student opinion.

(5) Increase in circulation, varying inversely with the approach of the deadline, strengthens the intuitive theory, used in planning the project, that pressure to produce increases the desire to absorb.

\textsuperscript{15}See Chapter II, page 13.
(6) Altercations between potential book borrowers might become less in the presence of an adequate collection, though limits in this direction might also provide opportunity for character education. Observations of the teacher from the vantage point of the classroom in which much of the free reading took place were numerous and of differing degrees of importance. A few which have not already entered the discussion may be added here for the purpose of focusing attention upon some additional values of the program and upon some possibilities for revision.

It will be recalled that four questions were originally posed for tentative solution in this thesis.\textsuperscript{16} Opinions and occurrences bearing upon those questions were observed from time to time throughout the semester's experience. The question as to the ability of writers of specialized mathematical works to appeal to the high school mind is partially answered by two boys who expressed their pleasure at finding particular passages. The first, J-102M, included in his term paper the following introduction to a description of the famous totem pole employed for The Education of T. C. Mits.\textsuperscript{17} "During my reading I ran across a very interesting and amusing illustration on how

\textsuperscript{16}See Chapter I, page 2.

\textsuperscript{17}H. G. and L. R. Lieber, The Education of T. C. Mits, pp. 51-73.
our present civilization is all dependent on mathematics." The other, SAI-106M, after a series of comments on the worth of the college mathematics text he was currently reading, finally endorsed the material with the words: "The knowledge of this material gives me much personal satisfaction, although, as yet, I have no use for it." In the innocence of youth he did not concede the service to intellectual curiosity to be a "use," yet more mature minds can recognize that in such satisfaction lies the basis for many of the achievements of civilization.

As to the degree to which the vocabulary was understood, S-42F gave one answer when she wrote: "I didn't understand more than half of what I read. . . . I worked very hard at it though and tried to get as much out of it as I could." Her attitude and the quality of her paper indicate that she was able to derive a fair amount of benefit, even in the face of difficulty with vocabulary and style of writing above her level. Remarks of other students testified to the clarity of explanations in many books intended for adult consumption. Hence it may be concluded that, while all vocabulary difficulty cannot be avoided, learning takes place in spite of it. Nor is it confined to the non-textbook variety of literature, if we accept the word of two students that the material of
the book collection was "on the level with other sophomore reading." 

The matter of high school comprehension of the topics presented may be tentatively decided on the basis of the resumés already discussed, where it appeared to be as varied in degree as one usually expects in any class undertaking.

One paper which has already been quoted from another angle was a particular triumph in comprehension on the part of its writer, who openly scoffed, in the beginning of the term, at all efforts of the teacher to introduce a philosophical outlook upon the application of mathematical habits to the non-mathematical reasoning situations of everyday life. While the introductory ideas of the paper may not indicate complete conversion to a new way of thought, they must be recognized as containing a measure of acquiescence:

Anyone looking at the title of my paper would probably say what has mathematics got to do with our everyday life.(?) If you sit down and think about it for a little while (,) you would soon realize the importance of it and how it has improved our civilization . . .

I believe that every person of today would get along a lot better in the world as it is today if he or she would have a better background

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18 See Table 7, following page 89.
of mathematics. By this I do not mean that everyone should be a wiz at mathematics but that they should understand the history of mathematics and know the history of reasoning."\(^{19}\)

Surely these are strange words from one who had begun the year's work with particular impatience for any class discussions which did not immediately reveal to him the right answer. The Tiefers had accomplished what the teacher would very probably have been unable to do unassisted. Such cases are examples in fact of Bell's theory that it is better to study "the masters instead of their pupils."\(^{20}\)

The practicality of the topics investigated was again a subject upon which there was much disagreement. The opinions which made reference to vocational aspirations were frequent and indicative of the tendency of youth to live in the future. Less far-reaching individuals dropped comments here and there to the effect that they were concerned with the immediate demands of class work, as did S-58:1 after choosing to use his geometry textbook in preference to any of the offerings of the book shelves when he realized that his frequent absence was jeopardizing his progress in the course: "I read all the back theorems and tried to get a hold of the facts."

Of course there were those who insisted that their readings bore no relation to their needs for the course,

\(^{19}\) J-102:1, "Mathematics and Everyday Life!"

\(^{20}\) See page 7.
yet continued to read in the same book or on the same topic week after week. Others maintained the worth to themselves of their selections, with the acknowledgment: "though others may not think so." The conclusion may be reached, then, that a given book collection will offer both practical and impractical materials, but so will students continue to select both.

A Revised Reading Program.--Plans for revision of the free reading program must be made with an eye to the guideposts erected by the experiences of the past. Lifting its beckoning finger above the others is the guidepost which calls for the re-structuring of the term paper. The need is seen for a more spontaneous expression of the knowledge gained. The working out of projects such as construction of instruments, charts, and posters to supplement or replace the term paper, might prove a satisfactory solution if appeal to the ingenuity of students should yield any promising results. The fact that the project approach to expression is not common in the school may be due to more of an intellectual than manual aptitude among students or it may be due simply to the fact that project work has never been introduced.

Next in importance among the guideposts to be heeded is the demand for revising the collection, dropping out those books which proved of little value, or at least augmenting it with others on topics about which students complained of the dearth of material.
Despite the students' rejection of the idea of reading lists a modification of the idea might prove valuable in the form of a card file as suggested by Frances A. Mullen. Her plan of appointing a student committee to take charge of listing helpful references and filing them under suggestive headings bears the stamp of actual classroom use to endorse it.\textsuperscript{21} In adapting it to the conditions of the free reading program all students might be invited to assist by supplying the committee with annotated bibliographic references, inscribed on cards, and signed with the name of the donor, for inclusion in the card file of valuable passages.

Of lesser importance but still desirable would be some plan for financing the replacement of lost books. Payment of a small fee at the beginning of the year might prove feasible, with the provision that should no books be lost, the fee would be refunded or used for class parties at the discretion of the several groups. Reasonable wear must be expected but vandalism, if it occurs, should be treated on an individual basis wherever possible, rather than at the expense of the group.

Perfection of the plan cannot be attained even after much experience but by continually approaching a limit, improvement and progress may become a reality.

\textsuperscript{21} Frances A. Mullen, "Fugitive Materials," The Mathematics Teacher, XXXI:5.
CHAPTER IV

SUMMARY AND CONCLUSIONS WITH
SUGGESTIONS FOR ADDITIONAL RESEARCH

A desire to devise a means of making available to high school students some of the wealth of current writings in the field of mathematics and a wish to test the philosophy of self education prompted the establishment of a free reading program in high school mathematics.

Such a program was defined as having four component parts: (1) variety of material, (2) a period provided for reading, (3) an adequate guidance service, and (4) a plan for revision. In conformance with this definition the program was set up in a high school of five hundred students, in a section of the state notably above average in the number of potential college entrants. Only students on the level of plane geometry and above participated in the free reading program, which was intended for those having particular interest and aptitude in mathematics.

Routine was standardized to make the program more efficient, students kept complete records of their readings and as a culmination of the project wrote term papers on subjects of their own choice related to all or part of their readings.

In addition they were asked to help evaluate the semester's experience by registering their opinions as to: (1) their enjoyment of it, (2) the value of a list of
readings from which to choose, (3) the amount of learning which had occurred, (4) the advisability of continuing the program, and (5) the easiest and hardest books in the collection.

A retrospective study was made of all measures employed in the program and implications drawn as to the values received from the experience and future adjustments to be made. The student opinions were augmented by the observations and considered judgments of the librarian and the teacher in charge.

Certain conclusions were reached, which although based upon the best information obtainable, are yet deemed to be tentative. They include: (1) recognition of a need for a form of expression better than the term paper as it was structured, (2) values resulting to students among which are development of independence in habits of research, greater appreciation for the scope of mathematics, opportunity to follow a particular interest or hobby along mathematical lines, keener discrimination in the choice and acceptance of printed materials, strengthening of the habit of self education, and realization of the relationship of textbook mathematics to life situations, (3) a realization of the need for constant revision of the book collection, (4) increased information for the teacher concerning books and topics which appeal to students, and
an annotated list of books for suggested use in the
same and other high schools.

Appreciation for the weaknesses of this study as well
as for certain related features which it did not attempt
provides a basis for suggestions pointing up additional ef-
fort and research in the direction of bettering free read-
ing programs in high school mathematics.

Chief among the recommendations for further effort is
that free reading should be widely established, not only
for the purpose of affording students opportunity to be-
come acquainted with current writings but in order that
more information may be gathered as to the influence of
such writings upon their thinking. Valuable studies might
be made of the evidence of retention and use of the knowl-
dge gained from the reading, both currently and in sub-
sequent college work. Especially interesting would be any
studies aimed at comparing the work of students engaging
in free reading with that of students of similar aptitude
who had not done so.

A great deal of information needs to be assembled
concerning the available materials. More exhaustive, yet
selective bibliographies need to be built and circulated
among teachers in service and among administrators. The
latter need to be made aware of the need and the possi-
bility of revitalizing the mathematics program in this
manner.
By these and other means yet to be devised the reality of mathematics as a mainspring of civilization may be demonstrated to students and to those who are in charge of their education, in such a way that the former will never again need suffer the effects of its near deletion from the curriculum.
APPENDIX I

AN ANNOTATED BIBLIOGRAPHY FOR USE IN HIGH SCHOOL MATHEMATICS CLASSES
BOOKS USED IN THE STUDY


Elementary treatment of latitude and longitude, maps and the global perspective in Chapter I is somewhat overshadowed by the war flavor of later chapters dealing with strategy and objectives now a little outmoded.


More emphasis on the mathematics of longitude, latitude, and mapping than in the first book of the series. Interesting and helpful diagrams provided.


Explanations of bearings, fixes, course and track, and wind vector diagrams are quite simple. Illustrative diagrams and exercises are included.


Mapping on a flat surface by Mercator and Lambert compared. Airplane design discussed from standpoint of geometry and physical force. Flight maneuvers diagrammed.
Same type of material as Science of Pre-Flight Aeronautics, but easier to read.


A well known text for beginning algebra students, useful for reference and remedial work.


Though written on a juvenile level this encyclopedia contains much of a biographical nature that high school mathematics students find interesting and useful.


A text book. Useful reference for items of exact knowledge as to process and principle.


An American revision of a British text used by R.A.F. Organized by branches of mathematics involved, thus guiding students to material directly related to class work.


Though much of this classic work is beyond the scope of high school mathematics, students find the topics
in which they are interested to be discussed understand-
ably.

*Compton's Pictured Encyclopedia.* (Ford, Guy Stanton, Ed.)

Students of lower ability will still find challenging
material in this children’s encyclopedia.

Eddy, W. C., Brolly, A. H., and others. *A Wartime Refresher
in Fundamental Mathematics.* New York: Prentice-Hall,

Confined to arithmetic and algebra, arranged by daily
lessons with answers. Appreciated by senior high school
students who feel a need for remedial work.

Edgerton, E. I. and Carpenter, F. A. *First Course in the
viii & 348 pp.

One of the older texts. For reference and review.

*Encyclopedia Americana.* New York: Encyclopedia Americana
Corporation, 1940. 30 Vol. & Annuals.

Too well known to need comment. Treatments more brief
than in Britannica.

*Encyclopedia Britannica.* New York: The Encyclopedia
Britannica Corporation, 1937 & Books of the Year.

The standard authority. Less readable style and type
than Americana.
A clear, concise explanation of the various global projections. Emphasizes the value of looking at the map in different positions. Employs the technique of teach and test by quiz method. Well illustrated. Very little advertising.

Excellent reference for arithmetic, algebra, geometry, logarithms, trigonometry, slide rule. Intended for self-instruction, among secondary students page headings assist in the location of desired material.

Understandable to high school students of better than average ability. Good mathematical discussions.

Difficult for most high school students. More useful for clubs than for individual consumption.

Comprehensive review yet complete enough to satisfy the slow learner who wishes to reinforce class work.


Affords the fascination of "reading the dictionary." Students become interested through browsing and then pursue that interest in other books. Useful also for ready reference when fine points of definition enter the class discussion.


Non-technical. A radio reprint which does not contain too much mathematics.


A text for those wishing to learn to fly. Mathematics not paramount but present. Readable style with many helpful diagrams, charts, and sketches. "Cloudy Joe" puts emphasis on what not to do.


Somewhat more difficult presentation of material similar to that of Cajori and Sanford.
The most modern and attractive textbook yet to appear in the field from the standpoint of format and application of geometric principles to out-of-school situations, so far as the writer has observed. Students who are apathetic towards the subject will read from this book and announce proudly facts they have overlooked in studying conventional pages.

Kraitchik, Maurice. *Mathematical Recreations.* New York:
Rather difficult for free reading. Would be of more service to mathematics clubs where materials could be discussed.

Lennes, Nils Johann. *Practical Mathematics.* New York:
A textbook affording some easy applications of the geometric principles learned later than its intended use.

Lieber, Hugh Gray and Lillian R. *Education of T. C. Mits.*
230 pp.
A breezy presentation of mathematics from arithmetic to calculus, through the adventures of the hero, The Celebrated Man In The Street. Profusely illustrated with
cartoon drawings which are entertaining even though they are not completely understood. Some high school boys will read all of it and most students will get the morals. Good propaganda for mathematics.


Useful to students in self-remedial work because it contains frequent short tests. One of the older texts, but easily read.


An extract from a larger work. Not written for children, it requires an earnest student for understanding.


"For seamen, by seamen." Illustrated with colored plates and maps. Good geometry in making of simple charts from its directions.


Written for high school students by a member of the Aviation Education Research Group, Teachers College, Columbia University. Appeals to both boys and girls.
Chapters are graded, so that students can find their level. Plenty of arithmetic and algebra. Some geometry, trigonometry, and physics.


A college text, difficult for most high school students, but useful because of extensive tables and explanation of logarithm. Theoretical definitions of functions in terms of abscissa, ordinate, and distance.


A new text intended for slow learning students on the junior high school level, hence useful in remedial work in upper classes since it is attractive and material is easily located through the Table of Contents, which gives sub-headings of chapters, and through the four-page index. Contains a picture dictionary of words and terms.


Second in the series of textbooks for slow learners. Contains a smattering of geometry. Picture dictionary of words and terms, differing from that in *Mathematics for Everyone*, makes a vivid impression.

A classic necessary to any collection of mathematical history. Students find it readable and give it a high information-per-page ratio.


The textbook used by the group in this study and preferred by some to other selections. Supplement on Geometry in Aeronautics has especially wide appeal. Also good chapter on non-mathematical uses of reasoning.


While intended as a complete text for a high school course in aeronautics, this work contains numerous mathematical applications to airplane structure, aerodynamics, aircraft engines, meteorology and air navigation. Senior high school boys find this book very understandable and many girls enjoy the section on air navigation.


Latest of the Introductory Aviation Series. The compass, radio bearings, maps treated as in the U.S. Navy
series but interspersed with more non-mathematical material.


Practical assurance in the construction and use of simple surveying instruments. Problem material in each chapter with answers at back of book. Test, tables, and index. Especially popular with boys.


Written for the elementary school. The background narrative is too juvenile for high school students but they do not object to it and find much of interest in the plates, illustrations, and explanations. The pronouncing vocabulary is useful, even to them.


More juvenile yet than *Number Stories of Long Ago*, but with less story material, so that high school students usually finish it in one period or so. Entertaining but not too informative. Numerical tricks and puzzles are its greatest attraction.

An elementary discussion of various number systems with emphasis upon the cumulative nature of our present practices with respect to counting and computation. Use of the abacus, development of zero, early method of multiplying, invention of decimals, logarithms and graphs are some of the topics briefly presented.


Enlarges the student's concept of kinds of measurement. From body measures of length to tensile strength of a modern bridge the story is told briefly and simply.


Gives a general idea of man's use of symbols for recording purposes. Primitive representation of number through repeated pictures impresses the student with the need for quantitative symbols.


Sundial, water clock, time candles, mechanical clocks, novelty watches, and electric clocks reveal man's universal interest in time-telling. Chart lists all these with approximate dates of their introduction. Standard time zones are also explained.


An encyclopedia compiled especially for schools. Much easier to use than Britannica, because of simplified location and vocabulary, as well as more legible type and fuller explanation.
RECOMMENDED TITLES FOR ADDING TO THE COLLECTION


Use of formulas stressed, derivations omitted. Some problems containing insufficient data. First six units deal with general knowledge useful to all students, last six with applications suited to the machine shop.


A great deal of geometric material. A new approach in the chapter on "The Mathematics of Seeing." Astronomy treated in its relation to geometry, called "Mathematics, Interpreter of the Universe."


"Lives and discoveries of the foremost mathematicians. May be read easily by senior high school students."


Written for "the average person," this book stresses the financial side of life. Bibliographies on some units list inexpensive related materials.

Presents two kinds of pictures to explain motion: Continuity (infinite) and discreteness (finite). Rather difficult reading for most high school students though they may absorb general notions of the meaning intended.


"Novel methods to be used in creating the Senior High School pupils’ enthusiasm for geometry."


Numerous examples of the use of algebra in industry, science and research. Many illustrations and clear, large type.


Helpful to students of solid geometry who experience difficulty visualizing three dimensions in plane figures. Equipped with explanatory notes to teachers and orthoscope for viewing.


Useful in assisting geometry students to adopt sound
habits of reasoning, providing for transfer to non-mathematical material.


Oddities, paradoxes, problems and tricks in great variety, grouped under catchy titles, illustrated amusingly. Story book print.


Mathematics of home-making and of commercial and industrial occupations open to women. Tests and arithmetical dictionary. Elementary sewing problems the most novel feature.


Prefatory note addresses this material to students who have had no previous high school courses in mathematics, but also to those who have. List of starred supplementary problems more difficult and abstract. Example: "Making a Vernier" unstarred, "Vernier Caliper" starred.


"Over one hundred puzzles based on such topics as "Puzzling Times in Solvamhall Castle;" "The Merry Monks
of Riddlewell;" "The Strange Escape of the King's Jester;" and others. For junior and senior high schools.


"Both junior and senior high school students will enjoy these mathematical puzzles and their solution."


Official publication of the society containing many interesting items about the development and use of numbers from antiquity to the present, with emphasis on the movement to count by twelves.

Heath, Royal Vale. Mathemagic. New York: Simon & Shuster, 1933. (Supplied by The Seven Bookhunters, Station O., Box 22, New York City. Out-of-print books.)

Deals entirely with number puzzles, problems, and tricks.


A subject receiving scattered and fragmentary treatment in most books. Should prove helpful in supplying much from one source.

Fascinating but not much more. Jingles and mathematical riddles in rhyme offer materials for assembly and club programs.


Connects primitive counting devices with modern machines for that purpose. Useful in showing the evolution of mathematics.


Non-Euclidean geometries receive brief attention, also rubber sheet geometry (topology). Rather more attention to calculus. Attractive style invites reading.


Can be read and enjoyed by older students on the high school level. Keyser's classroom experience fits him for understandable style of expression.

- Mathematics and the Dance of Life.

Another of his short, entertaining essays which strikes deep into the philosophy of mathematics.

Contains two essays on mathematics, some others on general logic. Fairly easy reading and interesting to the more mature high school students.


"Undertakes to help the reader acquire a sound philosophic understanding of the meaning of the terms mathematics and science."


Another of the short essays suitable for single period reading.


New York: Industrial Arts Cooperative Service, 519 West 121st St., 1943.

How to make and use an oriental abacus, written by a Chinese-born user of it.

Kokomoor, Franklin Wesley. Mathematics in Human Affairs.


Good presentation of assumptions as the basis of geometry. Statistical data also treated in somewhat elementary fashion.

Beginning with a review of arithmetic and algebra this book shows relationships between abstract mathematics and its applications to the field of electricity. Parts, at least, are well within the scope of high school science courses.


An excellent discussion of games of chance, written in a popular style. Makes an impressive case against gambling, particularly forceful with students confronted with the temptations of slot machines.


"Senior high school recreational material based upon elementary geometrical figures."


Good authority on the subject of maps. Should prove helpful to geometry students. Shows nine ways of making world maps.

For the student interested in fine arts. Excellent correlation of geometry and free hand drawing.


Chapter headings couched in phrases bearing out the title. More readable style than Hogben, Kokomoor, or Kasner-Newman, but less scholarly in its fund of information.


Excellent graphs based on state and local expenditures for schools. Of sufficient size for group attention. Aimed at revision of the state foundation program to provide increased funds.


A text book for beginners, which makes a new approach. Explanations are given in a narrative style suitable for use in free reading even with good students who have been taught algebra by more common methods.

Written for adults who are afraid of mathematics, it should prove useful to high school students who elect the subject, without being too difficult for them. Largely geometric in character. One chapter on logarithms.


Over one thousand more or less familiar passages pertaining to mathematics. For general use in the high school classes.

Morley, Christopher. *When We Speak of a Tenth*. Free pamphlet. Lancaster, Pa.; Hamilton Watch Co.

Incites an appreciation for the precise measurements needed in watch making and in other forms of industry.


Prepared for enlisted navy personnel, type is clear and presentation concise. Wide variety of topics following Raleigh Schorling's teaching methods.


Preface lists two prerequisites for enjoyment of the first nine chapters (of the ten included in the book).
These are: an elementary training in mathematics, and interest in matters mathematical.


Profuse with maps, charts and tables, furnishing timely subject matter for mathematical projects.


Demonstrates to students the need for careful attention to accuracy in the industrial world.

Romance of Clocks. Free pamphlet. Thomaston, Conn.; Seth Thomas Clocks, 1940.

Additional material on the fascinating subject of time telling. Appeals to students of lesser ability on the sophomore level. Most useful with junior high students.


An extension of arithmetic methods, including oddities and recreations. Also treats of logarithms, slide rule, algebra, geometry, trigonometry, calculus, astronomy and the calendar.

Some splendid points are made for the case of mathematics in the first essay. Another on "Thomas Jefferson and Mathematics" presents him in a new light to most high school students.


Attempts self analysis on the basis of numerology. Admittedly a source of entertainment, it serves to impress students with the weight of power attached to numbers throughout history. Learning the numerical meanings of names forms fascinating pastime.

**Suggested Unit Course in Blue Print Reading for Beginners in Machine Shop Practice.** Albany, N.Y.: The University of New York, 1943.

Twenty blue prints, graduated in difficulty, with questions on each. Self-teaching.


One of a mathematics series intended for self-study. Practical problems at end of each chapter apply the principles developed. Other titles in the series: *Algebra for the Practical Man, Geometry for the Practical Man, Trigonometry for the Practical Man, Calculus for the Practical Man*. 

"History, principle, and operation of the slide rule. For use chiefly at the high school level." Material much in demand for boys, particularly, and also some girls.


Logical fallacies discussed with a view to avoiding them in practice, hence with many examples. Appendix I lists: "Thirty-Four Dishonest Tricks Which Are Commonly Used in Argument, With the Methods of Overcoming Them," with cross references to fuller discussions in the body of the book.


Preface: "Aim to present some of the most interesting and suggestive phases of the subject." Numerical tricks explained by algebraic analysis. Final chapter, "Alice in Wonderland," appeals to sense of humor while presenting a philosophical approach to mathematics.
APPENDIX II

PROPOSED SKELETON FORM FOR READING RECORD
WITH SAMPLES OF STUDENTS' WORK

(In accord with the policy laid down in Chapter III English
and spelling errors are retained in reproductions of stu-
dent work.)
From the first research period students were required to make written reports of what was accomplished during the activity. A simple form which yet sets good habits of note taking is somewhat as follows:

Student's Name
Date:

Author:
Title:
Publisher:
Date of Publication:
Pages Covered:

What I read: (General topic.)
(Notes for future use. Connection with present or previous mathematics courses.)

Opinion: (Hard or easy?)
(Interesting?)
(Useful?)
(Helpful to other mathematics assignments?)
Author: Hugh Gray and Lillian R. Liever
Title: The Education of T. C. Mits
Pages Read: 74 pages

What I Read: This is an excellent book. It's purpose
seems to be not only to entertain but to be of a real value
to the reader.

In the first three chapters common falacies of thinking
are taken up, such as jumping at conclusions, following
hunches without checking them, etc.

The fourth chapter points out the value of generaliza-
tion in the form of algebra. T. C. Mits (The Celebrated
Man in the Street) shies away from algebra but is soon con-
vinced of its value in this chapter.

In chapter five we take up the uses of math and some
of the people that use it, such as the research scientist,
the "pure" scientist, the mathematician, and the "pure"
mathematician. This chapter points how necessary each is
and brings out a philosophy based on mathematics.
Author: Karpinski
Title: The History of Arithmetic
Pages Read: 145-151

What I read: The Progress of Arithmetic. (A) To assign each word to its proper place is no easy task and at many times authorities have disagreed as to whether a given word entered, for example, through the French or through the Latin.

II. Greek and Roman influence. (A) To the Greeks we owe the separation of mathematics into four great fields of arithmetic, geometry, astronomy, and music.

(B) Early American Universities continued for a time to use textbooks in Latin.

III. Influence of other races. (A) The Hindu and Arabic terms are few in number but significant in meaning and in importance.

(B) French terminology connects most closely with the English, while the Italian offers quite a few terms in business arithmetic.

IV. The origin of number names. (A) The word "number" comes from the Latin numerare, meaning to count, and more directly through the French nombre.

(B) The English numeral words notably "hundred" and "thousand" resemble the German and Scandinavian forms rather than the Latin forms, centum and milia, used in the Romance languages.
(C) "Million", "billion" and "trillion" are terms which have come into popular use largely since the Great War of the fifteenth century.

(D) In America we use "billion" for one thousand millions and "trillion" for one thousand billions.

My Opinion: Today this reading material answered many questions I have always wondered about, as to where the words we used in math come from. I was much interested in this topic more than the others I have read about.
Author: U.S. Navy

Title: Air Navigation (Part II)

Pages: 36-

What I read: Mercator's features are that the Meridians are parallel and equal distant Latitude is parallel and perpendicular to the Meridians.

The advantage is that it's easy to plot and straight lines between two points indicate it is an accurate course. Scale is in nautical miles. The disadvantage is that the latitude and distant scale expand and most straight lines does not appear as the shortest distances. Radio bearing can not be corrected before plotting on Mercator maps.

Use of Mercator projections are for plotting sheets, used for navigation of surface craft, and coastal aviation charts.

My Opinion: My opinion is that it will be very useful for the subjects I plan to take later.
Author: Buchan, Borthwick, Wadden
Title: Aviation Mathematics
Pages: 43-48

What I read: I read about the essentials of graph making using curved and straight lines. Graphs can be made on just about any subject that there is. It is a mathematical picture or diagram to show at a glance what a formula would hide. Any formula can be laid out in a graph.

My Opinion: I think that graphs are the clearest way there is to show formulas because in a formula there are just a lot of numbers and letters but in a graph everything is laid out clearly.
Author: Shuster and Bedford
Title: Field Work in Mathematics
Pages: 55 and 39

What I read: It is necessary in scouting to be able to determine right angles and horizontal and vertical angles. The angle mirror is the best instrument for that. It is used in requirements 1, 2 and 3 for the merit badge.

How to use the Angle Mirror

The angle mirror should be held so that the line of sight from the observer at B passes along the front edge of one mirror and through the window of the opposite side to some object such as a tree or a ranging pole. The second object must be seen in the mirror.

I am telling about the angle mirror to help understand the chapter on scouting.
Author: Lancelot Hogben
Title: Mathematics for the Millions
Pages Read: Chapter VI - pages 234-237
What I read: What we can do with Trigonometry

When Egypt surrendered to Alexander the Great a city was built at the mouth of the Nile River. This city was called Alexandria. This city gathered the knowledge of the world into its university. The men of this university determined the distances of the sun and the moon from the earth, as also they determined the radius and the circumference of the earth, moon, and the sun. The circumference was only about 50 miles off from their guess.

My Opinion: A lot of reading to get this little bit of reporting.
APPENDIX III

TWO OUTSTANDING TERM PAPERS WRITTEN BY GEOMETRY STUDENTS
Navigation is the art of conducting a ship, air or sea, across the surface of the earth from one point to another and being able to determine its position at all times. It is divided into two parts: (1) Geo-navigation, by which means a ship's position is determined by observation of objects on the earth, and (2) Cele-navigation, by which means a ship's position is determined by observation of heavenly bodies such as the sun, moon, stars and planets. Of these the sun is most commonly used.

Geo-navigation is divided into (1) piloting, by which means a ship's position is determined from visible objects such as lighthouses and beacons. In air navigating, mountains, etc. are used. And (2) dead reckoning, by which means a ship's position is "deduced" or determined by applying a ship's progress and direction through the water or air to her last known fix or position.

In air and sea navigating the three elements of dead reckoning appear. These are: (1) course; path plotted from one point to any number of others; (2) time; marked on the course and track at hourly intervals; and (3) speed; recorded on track. The course is plotted and marked on the chart before leaving for your destination.

The track, the actual path of travel is marked on the chart as you proceed from one point to the next. It is usually recorded hourly. Any deviation from your intended
course and the track will not coincide with the course. If for any reason a loss of speed or a change of direction is noted, the speed must be adjusted and the direction corrected to arrive at the next point on time and to get back on the plotted course.

Fixes may be obtained by sighting objects or using radio direction finders. These are necessary to determine position to see whether the course is being followed. (See chart).

Due to many factors dead reckoning has a certain amount of error. Weather conditions, currents and compass error are a few of the common causes of errors.

Piloting is used in rivers, harbors, ports and along shore. The instruments used most commonly are (1) the compass, used primarily for direction and checking fixes in degrees from the lubbers line. The magnetic compass varies in accuracy with its position on earth. The compass may be corrected by using compass deviation and variation charts and adjusting the electromagnetic corrector to the amount of correction. (2) The polaris, used for taking bearings. (3) The deep sea lead line, used for determining the depth of water on entering or leaving a port. (4) The patent or taffrail is a mechanical device used for recording the distance run through water. Accuracy depends upon various factors such as currents, wind, obstructions, etc. Speed may be determined by taking the distance and dividing it by the time. (5) The fathometer, an echo sounding device along
CHART I

NAVIGATION FIXES
the line of radar and is used to determine the depth of water. One unit sends out sound signals (impulses) and another "catches" them when they "bounce back" from the ocean floor. The time taken is recorded and the depth found thereby. (6) The radio direction finder, used to obtain fixes from radio stations to determine position. Usually two fixes are taken. If possible the intersection of the two fixes should be close to a right angle, thus reducing the error at the point of intersection. (7) The sextant, used to determine and check positions by the number of degrees between sun and horizon. These bearings are affected by the seasons. They are taken at noon; bearing may be taken on any recorded celestial object.

In air geo-navigation your position is usually determined by compass, fixes, radio bearings and known objects. Bearings may be taken on almost any object. One clearly visible is best. The object used depends upon the distance to be traveled. Lighthouses, special buoys and other such objects are best for distances. (See chart).

In dangerous places, such as winding rivers and harbors where rocks make passageways necessary, ranges are usually provided. Ranges (see chart) are sets of towers with lights placed so that when a ship is in the safe channel the towers will line up and look like a single tower. At night it is two lights, in line and one above the other.
CHART II
NAVIGATION RANGES
SHORE LINE
SHALLOW S & SAND BARS
ISLANDS
CORRECT BEARING ON RANGE + LINE OF SAFE PASSAGeway
1. 2. B. RANGE
3. 4. A. RANGE
\[ \rightarrow \] DIRECTION OF LIGHT
\[ \leftarrow \] POSITION ABREAST OF:
A., GLUE FACTORY.
B., SARE I.
\[ \circ \] BOUY-MEANING SHIFT TO NEXT RANGE.

GLUE FACTORY

TO LEFT OF SAFE CHANNEL IN THE SAFE CHANNEL TO RIGHT OF SAFE CHANNEL

A. RANGE (\( \perp \)) B. RANGE (\( \perp \)) B. RANGE (\( \perp \))
Radio direction finder stations provide bearings. A fix may be obtained from several stations day and night.

There are many types of bearings. These are used in many ways. In piloting, besides their use as a fix, they can help to determine the safe channels. There are two ways to use these bearings to find safe channels: (1) the horizontal danger angles and (2) the vertical danger angles. (See chart).

When two fixes are available (two lighthouses) the horizontal method is used. Your chart will show you where obstructions (shoals, reefs, shallows and rocks) are. To determine a safe course through them, put two points in line at each edge of the safe channel. (See chart). Connecting these with the two fixes forms two angles. By using a sextent horizontally the number of degrees in the angle at the intersection of the fixes at your position will be determined. If this angle is greater than the smaller angle and less than the larger angle you are in the safe channel.

When only one fix is obtainable the vertical method is used. The height of all prominent objects is given on charts. Using the sextent vertically the number of degrees of the angle whose vertex is your position can be found. Using the right triangle the angle can be found without the sextent. If this angle is less than the larger angle and larger than the smaller angle formed by connecting the fixes
CHART III
HORIZONTAL DANGER ANGLES
CHART IV

VERTICAL DANGER ANGLES
with two points on the edge of the safe zone, then you are in the safe channel.

In navigating an airship the bearings are usually used to determine position, but in bad weather may be used to decide the necessary altitude needed to rise above mountains.

There are many types of charts. Some are used to show current differences, buoy positions, rocks, lighthouses, high cliffs, factories and other prominent features of the landscape. The charts give the position of shallows, shoals, reefs, etc. Heights of lighthouses, cliffs, smoke stacks and other features of the landscape likewise.

A chart containing this information is used when a course is plotted. (See chart). The course is usually the shortest distance between each point. (See chart).

At 9:00 we leave San Jose Island. At 11:45 we are due to arrive at Ricardo Island City. Failure to correct compass error causes us to leave our charted course. This is the track. (The track may coincide with the course. In this case it does only part of the time.) At 11:00 we sight Baldy Mountain on Ricardo Island. It's being to the right instead of the left of us shows us that we are off our course. We change our direction and increase our speed so that we will arrive at Ricardo Island City on time. We leave Ricardo Island at 12:00. At 1:15 we are in the path of a storm from the north. As soon as the storm abates we get radio fixes and find our position. We again change
CHART V

NAVIGATION COURSE AND TRACK
STORM FROM NORTH

OFF COURSE DUE TO COMPASS ERROR

SIGHT EAGLE MT. ON WRONG SIDE, CHANGE DIRECTION AND INCREASE SPEED TO GET BACK ON COURSE.

AT 2:15 SPEED INCREASED AND DIRECTION CHANGED TO GET BACK ON COURSE.
direction and increase speed to arrive at our destination (Lung Island City) on time.

We leave Lung Island City at 5:00. At 6:52 we line up lighthouse A, between lighthouses B and C. We proceed on the new course (towards lighthouse A). This will enable us to miss the reefs and shallows without using danger angles. When we reach the purple bouy we turn to left and proceed up river. At 9:00 we stop and wait until tugboats arrive to dock us.

When a not too accurate plot of a trip is desired Geo-navigation is useful. To increase its accuracy, elements of Celo-navigation (using heavenly bodies) are used to check course and position. Many factors cause the probable error of Geo-navigation. Some of these are storms, winds, currents, compass error and taffrail error. Most small boats use a form of Geo-navigation and Celo-navigation, enabling them to plot a fairly accurate and safe course with the minimum of paper work and figuring.
GEO-Navigation
Reference List

Air Navigation III
U.S. Navy
1943
McGraw-Hill
New York

American Merchant Seaman's Manual
Felix M. Cornell-Allen C. Hoffman
1942
Cornell Maritime Press
New York

Piloting, Seamanship and Small Boat Handling
Charles F. Chapman M. E.
1944
Motor Boating
New York

American Practical Navigator
Nathaniel Bowditch, LL. D.
1939
U.S. Hydrographic Office
Washington, D.C.

Primer of Celestial Navigation
John Farrill, M. P.
1940
Cornell Maritime Press
New York
Everyone is familiar with the triangle. It is a plane figure having three sides and so three corners or angles called its vertices. Now why should there be any more discussion about such a simple figure? The fact is that the triangle and the computations of its parts have entered into almost every phase of the modern world about us. The physicist, for instance, uses a right triangle in determining impedance in his study of alternating currents in electricity, impedance being the total opposition offered to an alternating current in a circuit. Triangulation is a method used by surveyors and others to determine the distance between two points, one of which is inaccessible and thus makes direct measurement impossible. In this method, known parts of a triangle are set up and the unknown parts, which are the inaccessible distances, are computed by certain relations which exist between the known parts and other parts of that triangle. Trigonometry is a branch of mathematics having to do with the measurement of triangles and with the relations of their sides and angles to one another. Astronomy, physics, and engineering would be non-existant or crippled without it.

Having pointed out a few of the uses of the triangle, let us actually see how the triangle does figure in these things. Let's examine these existing relations, discuss the formulae and proof, and trace the development of the topics involved.
Before going into the uses of the triangle, however, it will be necessary to have an understanding of logarithms and trigonometric functions. These will be needed if we are to understand intelligently the methods of dealing with triangles.

Logarithms are exponents used to simplify certain computations in mathematics. When it is desired to raise a certain number to a certain power, as \(5^2\), \(10^2\), \(10^3\), etc., the operation is thought of as involving the base number, and the exponent indicates the number of times the base is used as a factor. Thus, \(4^2\) means \(4 \times 4\), \(7^3\) means \(7 \times 7 \times 7\), \(10^4\) means \(10 \times 10 \times 10 \times 10\) and so on. Since a logarithm is an exponent, it follows the laws for exponents and the preparation of tables of logarithms depends upon an algebraic principle, expressed as follows: \(a^x \times a^y = a^{x+y}\). Thus we see that in Algebra quantities may be multiplied by adding their exponents. Say that \(a^x = l\) and \(a^y = p\). Then \(x\) and \(y\) are the logarithms the numbers \(l\) and \(p\) to the base \(a\). Now, with the aid of a table of logarithms, instead of multiplying two long numbers, the calculator simply adds their logarithms; the number in the table corresponding to the new logarithm thus obtained is the product.

Any number may be used as the base for logarithms. In actual use, 10 is always used as base. The use of 10 as the base was suggested by Henry Briggs when John Napier first proposed logarithms. When we associate the number 10 with decimal numbers, we can readily see the advantages
that base 10 has over other bases. Logarithms to base 10 are called common logarithms.

The trigonometric functions are of fundamental importance in trigonometry, that science of triangles. Therefore our discussion would not be complete without directing some thought toward them.

If we consider two right triangles, ABC and A'B'C', having equal angles we find that although the sides, a, b, c, and a'b'c', respectively, may or may not be equal, the quotient obtained by dividing the length of a by the length of c will be the same as that obtained by dividing length a' by length c'. Also, if we construct another right triangle, larger or smaller, but with the same acute angles, and we measure the corresponding sides we will find that the same quotients will be obtained. In the same way, the quotients or ratios b/c and a/b remain the same for the same angle in a right triangle of any size.

From this it can be seen that the values of the right triangle ratios depend only on the angles and that a different set of values corresponds to each different angle. A number or quantity related in this way to a second number or quantity is called a function of that second number or quantity.

In the triangles mentioned above as examples, \( \frac{a}{c} \) is called the sine of angle A and may be conveniently expressed as being equal to side opposite angle A divided by
the hypotenuse. Usually written: \( \sin A = \frac{\text{opp}}{\text{hyp}} \), "sin A" being the abbreviated form. Similarly, \( b \div c \) is called the cosine of angle A and \( a \div b \) is called the tangent. These are abbreviated \( \cos A \) and \( \tan A \), respectively. The cosine may be expressed as \( \text{(adjacent leg)} \div \text{(hypotenuse)} \) and the tangent as \( \text{(opposite leg)} \div \text{(adjacent leg)} \).

Using the same triangle we can readily see that the functions of angle B are:

\[
\sin B = \frac{b}{c}, \quad \cos B = \frac{a}{c}, \quad \tan B = \frac{b}{a}
\]

Comparing these formulas with those above we can see that \( \sin B \) is the same as \( \cos A \).

Similarly the ratio \( \frac{b}{a} \) which is the tangent of B is called cotangent of A, and \( \frac{a}{b} \) which is the tangent of A is the cotangent of B. This latest function is written "cot A" or "cot B." In the easily remembered form: \( \cot = \frac{\text{adj}}{\text{opp}} \).

There are two more functions which sometimes used for special purposes. These are called the secant and cosecant and are written sec and csc. Referring to the same right triangle they are defined as follows:

\[
\sec A = \frac{c}{a}, \quad \csc A = \frac{c}{a}
\]

and

\[
\sec B = \frac{c}{a}, \quad \csc B = \frac{c}{b}
\]

or in general:

\[
\sec = \frac{\text{hyp}}{\text{adj}}, \quad \csc = \frac{\text{hyp}}{\text{opp}}.
\]

Next in line in our discussion is the consideration of certain fundamental theorems regarding triangles. The ones
we are concerned with most are those for finding area and dimensions of triangles.

Perhaps the most famous theorem in this line is the Pythagorean theorem. Indeed, this is the basis for a number of famous theorems. Pythagoras was a famous Greek philosopher and mathematician who lived about 540 B.C. He is believed to have given the first rigorous proof of the theorem that bears his name: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

The truth of the theorem for special cases was known by the Egyptians centuries before the time of Pythagoras, and was used by them in building their pyramids. They laid out perpendicular lines by stretching a rope around three pegs so placed that the distances between them were proportional to 3, 4, and 5. The same principal is employed today.

One proof of the Pythagorean theorem is the one attributed to the Hindus which is based on this theorem: In any right triangle, if a perpendicular is dropped from the vertex of the right angle to the hypotenuse, thus dividing the hypotenuse into two segments, either side of the triangle is the mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to it.

In the most fundamental theorem of all those concerning dimensions of triangles, that which states that the
area of a triangle is equal to one half the product of the height and base, the proof is built upon the formula for the area of the rectangle and parallelogram.

Hero, an Egyptian mathematician who lived in Alexandria about the beginning of the Christian era gave the following formula for the area of a triangle:

\[ A = \sqrt{s(s-a)(s-b)(s-c)}, \]

where \( s = \) the semi-perimeter of the triangle and \( a, b, \) and \( c \) are its sides. This is based on our previously mentioned formula for the area, \( A = \frac{1}{2} bh \), and another formula, purely algebraic in proof, \( hc = \frac{2}{c} \sqrt{s(s-a)(s-b)(s-c)}. \) In this latter formula \( s = \) semi-perimeter, \( a, b, c \) are sides of the triangle, and \( hc \) is the altitude from angle \( C \) to side \( c. \)

In electricity, it is possible to graph an alternating current from an armature by means of the sine curve. From such a graph it is seen that the reactance, which is the opposition offered to a changing current in a conductor, lags 90° behind the current. The resistance of the circuit however acts in phase with the current. The total opposition set up by these two forces is known as impedance. Now, in computing the impedance in the circuit, it is noted that the two forces mentioned act at right angles to each other and so impedance is determined as the hypotenuse of a right triangle with the two forces acting as sides. Thus, if \( R \) is the resistance, \( X \) is the reactance, and \( Z \) represents the impedance, or resultant of the two
forces, a right triangle with side $X$ and $R$ and hypotenuse $Z$ is set up. The impedance then equals the square root of the sum of the squares of forces $R$ and $X$, or $X = \sqrt{R^2 + X^2}$.

Thus we see an application of the Pythagorean theorem in a situation that we would think had no relation to triangles.

Whenever a survey extends over a large area a method known as triangulation is employed. This is based upon the principle in trigonometry that if one side and two angles of a triangle are known, the other sides and angle can be computed.

Suppose we wished to determine the exact position of a distant tree. We would first measure a base line. For convenience we will call this $ab$. Next, we record the angle which the line from $a$ to the tree makes with $ab$. We do the same at $b$. Lines $ac$ and $bc$ may readily be computed by means of the trigonometric functions.

Problems of this nature, involving lines, distances and angles can generally be represented by a sketch and such a figure can also frequently be laid out in or divided into right triangles by drawing certain additional lines. The solution of the problem is then reduced to a right triangle calculation involving sides and angles and the fundamental trigonometric function formulas.

As we have seen before, if the values of any side and angle or any two sides of a right triangle are known, the
remaining sides and angles can be found. Sides and angles of a triangle are called its parts.

A short consideration of logarithmic solutions of right triangles is in order here. The logarithms of the natural trigonometric functions are obtained directly from the tables of the logarithmic functions, so that when a number representing the length of a side is to be multiplied or divided by one of the functions, the logarithm of the function may be added to or subtracted from that of the number in the usual manner. (Logarithmic multiplication is performed by addition of the logarithms of the factors to obtain the logarithm of the product, and the product is then the anti-logarithm of the sum of these logarithms. Similarly, a quotient is the anti-logarithm of the difference between the logarithms, the logarithm of the divisor being subtracted from that of the dividend.)

In the solution of problems by means of right triangles, if any two sides of a right triangle are known, the third side can be computed by the hypotenuse-square formula. In many cases however actual measurement of only one side is possible. In such cases, an angle may be measured.

For the measurement of angles, three instruments are in common use, each suitable for a different class of measurements. These are the astronomer's transit telescope, the surveyor's transit and the navigator's sextant.
The astronomer's transit telescope is mounted on a shaft or axle which is perpendicular to the length of the telescope tube. The shaft is mounted in bearings at its ends and the telescope may be turned about the shaft as axis so as to point, upward, downward, horizontally, or in any other direction in a vertical plane. A vertical circle marked in degrees and with the shaft passing through its center turns with the telescope and in any position of the telescope a stationary pointer attached to the frame or mounting indicates on the circle the angle through which the telescope is turned. Thus if a telescope is sighted on a heavenly body and the angle is read on the circle, and the telescope is then turned and sighted on another heavenly body and the angle read again, the difference between the two readings is the angle between the two lines of sight to the heavenly bodies. The vertex of this angle is the center of the rotating circle which is called the meridian circle. The entire arrangement is rigidly mounted and can be turned horizontally only by moving the entire mounting.

The surveyor's transit is a small telescope mounted with a graduated circle on a portable tripod in the same manner as the astronomer's transit telescope, and the vertical angles are read with it in a similar manner. In addition, the instrument is provided with a horizontal circle and the telescope can be turned about a vertical axis passing through the center of the circle. By means
of this arrangement, horizontal angles can also be read. The instrument is provided with levels so that the two circles may be adjusted to be exactly vertical and horizontal, and the telescope itself can be leveled so that the line of sight through it is horizontal.

The sextant is used on shipboard where the motion of the ship would cause a change of position of the telescope between two successive readings. The angle between the lines of sight to two distant objects is, therefore, found by sighting both at once. This is done by using two telescopes, both of which cast their images of the distant objects on a single mirror at the same time. When the telescopes are so adjusted that the two images are seen together they are properly sighted and the angle between the two lines sight is read on the graduated circle. This is really a sixth of a circle, giving the instrument the name sextant. Since the sextant is simply held in the hands while taking a reading, it may be turned so as to read horizontal or vertical angles.

If the line of sight to an object is above the horizontal line which passes through the eye of the observer, the angle between the horizontal and the line of sight is called the angle of elevation of the object. If the line of sight is below the horizontal, the corresponding angle is called the angle of depression of the object.

When the angle between the lines of sight to two objects is in a horizontal plane the angle is called the
bearing of one of the objects from the other. If one of the lines of sight points due north, the angle made with it by the other line is called simply the bearing of the second line. It is given by stating the number of degrees east or west from the north and south line.

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