Non-Foster Circuit Design and Stability Analysis for Wideband Antenna Applications

DISSERTATION

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By

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ABSTRACT

In recent years, there has been a great interest in wide-band small antennas for wireless communication in both ground and airborne applications. Electrically small antennas; however, are narrow bandwidth since they exhibit high a quality factor (Q). Therefore, matching networks are required to improve their input impedance and radiation characteristics. Unfortunately, due to gain-bandwidth restrictions, wideband matching cannot be achieved using passive networks unless a high order matching network is used. Fortunately, the so-called non-Foster circuits (NFCs) employ active networks to bypass the well-known gain-bandwidth restrictions derived by Bode-Fano. Although NFCs can be very useful in numerous microwave and antenna applications, they are difficult to design because they are potentially instable. Consequently, an accurate and efficient systematic stability assessment is necessary during the design process to predict any undesired behavior.

In this dissertation, the design, stability, and measurement of two non-Foster matching networks for two different small monopole antennas, a non-Foster circuit embedded within half loop antenna, a combination of Foster and non-Foster matching network for small monopole antenna are presented. A third circuit; namely, a non-Foster coupling network for a two-element monopole array is also presented for phase enhancement applications. It all examples, the NFCs substantially improve the antenna’s performance over a wide frequency band.
First, the stability properties of NFCs are discussed with a time-domain technique that computes the largest Lyapunov exponent for time series signals. In case of instability, it is shown how the circuit can be stabilized with different controllers. The proposed stability approach has been successfully applied to a negative capacitor to match a 3" electrically small monopole receiver antenna. Measured results verified that the system is stable and the non-Foster matching networks improve both the antenna gain and the signal to noise ratio (SNR).

Second, an efficient way to improve the performance of an electrically small antenna by embedding a non-Foster circuit (NFC) within the antenna structure is presented. This technique is based on the theory of Characteristic Modes (CMs). The antenna, loaded with a non-Foster circuit, shows remarkable improvement in gain and SNR. Furthermore, by loading the antenna the omnidirectional pattern is maintained up to a higher frequency by delaying the onset of higher order current modes.

Third, to the best of our knowledge, this is the first time a complete systematic stability analysis of non-Foster systems is introduced. The method was applied to two non-Foster matching networks; namely, a series connection of a negative capacitor-inductor and a combination of Foster and non-Foster circuits for use with a small monopole antenna. Measured results show that the non-Foster matching networks improve both the antenna gain and the SNR for the largest frequency band reported in the literature.

Lastly, a stable system consisting of a non-Foster coupling circuit is introduced to amplify the output phase difference of an array of two coupled monopole antennas over a
wide frequency band. Measured data of the output phase response of the NFC-two-element biomimetic antenna array (BMAA) shows that it has much wider 3-dB bandwidth in comparison with the passive case. The NFC improves the normalized available output power of the BMAA compared to the case when passive coupling networks are used.
Dedication

This document is dedicated to my family.
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CHAPTER 1
INTRODUCTION

1.1 Motivation and Approach

The last few years, there has been great interest in wide-band small antennas and metamaterials for wireless communication in both ground and airborne applications due to the need for miniaturizing multifunction communication devices. Antennas are an important part of a wireless system; however, electrically small antennas (ESA) suffer from narrow bandwidth behavior, exhibit small radiation resistance, high input reactance and high quality factor ($Q$). Several authors [1] - [5] have examined the fundamental lower limit of the radiation quality factor ($Q$). The best known expression for an electrical small antenna is the one derived by Chu [1] which is expressed as

$$Q \geq \frac{1}{(ka)^3} \text{ for } ka < 1$$  \hspace{1cm} (1.1)

where $k$ is the wave-number ($2\pi/\lambda$) in free space and $a$ is the minimum radius of a hypothetical sphere enclosing the antenna. It can be implied that most the input power is stored as a reactive energy in the near-field and very small power is radiated in the far-field. Therefore, matching networks are required to improve the radiation characteristics of an ESA. However, due to gain-bandwidth restrictions, wideband matching cannot be achieved using low-order passive networks [6] - [7]. Higher order passive networks can
be designed, but with added complexity and higher losses. Fortunately, an antenna matched by an active circuit is not bounded by this limit [8]. It is well-known that the so-called non-Foster forms [9] - [10] employ active networks to bypass the restrictions imposed by the Bode-Fano’s gain-bandwidth restrictions. Although non-Foster circuit (NFC) components (negative capacitors, inductors, and some of their combinations) were proposed several decades ago, recently there has been renewed interest in them for their promising application to wideband microwaves and antenna applications [11] - [20]. While NFCs are very attractive in numerous microwave and antenna applications, they are difficult to design because they are potentially instable. Consequently, an accurate, efficient and systematic stability assessment is necessary during the design process to predict any undesired behavior.

In this dissertation, two non-Foster topologies; namely, negative impedance inverter (NII) and negative impedance converter (NIC) are used to obtain broad usable frequency operation band. The negative impedance inverter is embedded within a half loop small antenna; whereas, the negative impedance converters are used either as matching networks at the feed of monopole-type antennas or as a coupling network between a two element antenna array for phase enhancement applications.

The overall goal of this work is to design wideband small to mid-size simple structure antennas using non-Foster circuits for receiving applications. An additional goal of this work is to stabilize non-Foster systems (non-Foster circuits, antenna systems, and internal impedance of the receivers) in a systematic way.
Key contributions of this dissertation:

- **Time domain Stability Assessment**: Introduces the first work that explicitly addresses the stabilization of non-Foster circuits using a time-domain method that computes the largest (maximal) Lyapunov exponent (MLE) for time series signals. The method can be used with computed and/or measured data. The approach has the ability to provide stability margins during the design process, allowing the designer to use the right controller to stabilize the system.

- **Design of Negative Inductor Based on NII**: Designed, fabricated, and measured a non-Foster circuit based on the NII topology. This inductor is embedded within a small half-loop antenna to improve the antenna gain, signal to noise ratio, and maintain the omnidirectional radiation pattern at higher frequencies. The reactance of this load and its location within the antenna are determined based on the theory of Characteristic Modes.

- **A Complete Stability Analysis of Non-Foster Systems**: Introduced the first systematic way to stabilize non-Foster systems (non-Foster circuits, antenna systems, and internal impedance of the receiver). As a case study: Designed, fabricated, and measured a non-Foster circuit based on a floating NIC composed of the series connection of a negative capacitor and a negative inductor to improve the antenna gain, signal to noise ratio of an electrically small monopole. Achieved the highest reported operation frequencies of a NIC built with discrete components.
• The first use of the Pole-Zero Identification method to assess the stability of non-Foster systems.

• Combination of Foster and Non-Foster Networks:Introduced the first designed and fabricated matching network composed of Foster and non-Foster networks to improve the gain and signal to noise ratio of a receiving small monopole antenna.

• Broadband Biomimetic Antenna Arrays using Non-Foster Networks: Introduced the first coupling network designed with non-Foster elements to enhance the direction-finding capability of a BMAA over a broad-bandwidth with a considerable phase-enhancement factor. Designed, fabricated, and tested the amplified output phase response and the signal to noise ratio after a complete stability study ensuring the system is stable.

1.2 Background and Review

This section first reviews the fundamental limitations of electrically small antennas and Gain-Bandwidth restrictions of lossless passive matching networks. Second, a brief review of non-Foster impedances and previous literature on fabricated non-Foster impedances for electrically small antennas is given.

1.2.1 Electrically Small Antennas

The definition of electrically small antennas is usually specified by the value of \( ka \) where \( k \) is the wave-number \((2\pi/\lambda)\) in free space and \( a \) is the minimum radius of a hypothetical
sphere enclosing the antenna, including a finite ground plane (when applicable) [21].

Figure 1.1 shows hypothetical sphere of radius $a$ enclosing an electrically small antenna. This definition is based on the concept of a radian-length ($\lambda/2\pi$) introduced by Wheeler [2] - [3]. When the maximum dimension of an antenna is less than the radian-length, it can be considered an electrically small antenna [2]. The fundamental limitations of the small antennas have been studied by several authors; Wheeler [2] was the first to introduce the limitations of ESAs; one year later Chu [1] derived the well-known formulas for the limitations of ESAs; namely, [4]:

$$Q_{rad} = \frac{1}{(ka)^3} + \frac{1}{ka}$$

(1.2)

Electrically small antennas (ESAs) have extremely high radiation quality factors because most of the input power is stored in the reactive near-field region and little power is radiated in the far-field. In the last decade, several authors [4]- [5] and [22] - [23] have investigated the $Q$ limits introduced by Chu [1]. ESAs have a major problem in transferring power to or receiving power from the system connected to them since their
input impedances are mainly reactive so that it is very difficult to impedance match them. Generally, with conventional matching technology, ESAs suffer from poor gain due to impedance mismatch losses. If a matching network is built entirely with Foster elements, it suffers from losses as the number of elements needs to increase to achieve broadband matching. Keeping in mind that ESAs are narrow band due to their high $Q$ (quality factor), and the fact that they obey Fano’s gain-bandwidth limitation, it is not possible to simultaneously reduce the antenna’s electrical size while increasing its bandwidth and efficiency. Refer to [24] - [26] for more details on definitions, limitations, designs.

1.2.2 Gain-Bandwidth Limitation of Losses Passive Matching Networks

There is a fundamental gain-bandwidth restriction (or the gain-bandwidth product) derived by Bode [6] and Fano [7] between a resistive generator and a complex passive load when lossless passive matching networks are employed. According to the Bode-Fano criterion, there is a tradeoff between the bandwidth and the minimum tolerable reflection coefficient magnitude. This implies that once the maximum reflection coefficient magnitude is determined, the bandwidth is limited, and vice versa. The most useful form of the Bode–Fano criterion may be stated as [27].

$$\frac{\Delta f}{f_0} \leq \frac{\pi}{Q_0 \ln \left( \frac{1}{\Gamma_m} \right)}$$  \hspace{1cm} (1.3)

where $f_0$ is the center frequency, $\Delta f$ is the frequency range of the interest, $Q_0$ is the quality factor of the load at $f_0$ and $\Gamma_m$ is the maximum reflection coefficient within the frequency range of interest. Equation (1.3) is derived assuming the reflection coefficient
versus frequency response is as shown in figure 1.2, and that the fractional bandwidth is small, i.e., $\Delta f \ll f_0$. Equation (1.3) can be re-written as

$$\exp\left(\frac{-\pi f_0}{\Delta f Q_0}\right) \leq |\Gamma_m|$$

(1.4)

Equation (1.4) demonstrates the threshold reflection coefficient can be lowered at the expense of smaller bandwidth.

### 1.2.3 Foster and Non-Foster Reactive Impedances

In 1924, R. M. Foster introduced a theorem stating that for a lossless passive two-terminal device, the slope of its reactance and susceptance versus frequency must be strictly positive [28]. The Foster’s reactance theorem is a consequence of conservation of energy. Equation (1.5) explains the Foster’s reactance theorem,

$$\frac{\partial X(\omega)}{\partial \omega} > 0 \quad \text{and} \quad \frac{\partial B(\omega)}{\partial \omega} > 0$$

(1.5)
Such a two-terminal lossless device (or one-port network) can be realized by ideal inductors, ideal capacitors, or a combination thereof. Figure 1.3 demonstrates the reactance or susceptance function of a Foster element along the frequency axis, and poles and zeros of these networks alternate on the real axis. Since the frequency derivatives of both the reactance and the susceptance are always positive [27], the reflection coefficient (Γ) of Foster elements (positive inductor and/or capacitor) rotates in the clockwise direction on the Smith chart with increasing frequency as shown in Figure 1.4.

![Figure 1.3. Reactance or susceptance functions of a Foster element along the frequency axis.](image-url)
Figure 1.4. Reflection coefficient of (a) an ideal negative and positive capacitor. (b) an ideal negative and positive inductor as frequencies increases.

If a two-terminal lossless device (or one-port network) violates the Foster reactance theorem, then it is called a non-Foster element. Non-Foster elements such as a negative capacitor and a negative inductor have completely opposite characteristics than conventional Foster elements; namely, the non-Foster element is characterized by the negative slope of its reactance or susceptance as frequency increases, i.e.,

$$\frac{\partial X(\omega)}{\partial \omega} < 0 \quad \text{and} \quad \frac{\partial B(\omega)}{\partial \omega} < 0$$

(1.6)

Figure 1.5 demonstrates the reactance or susceptance function of a non-Foster element along the frequency axis. As shown in Figure 1.4, the reflection coefficient (Γ) of a non-Foster element rotates in the counterclockwise direction on the Smith chart with increasing frequency. A non-Foster element must be an active component in the sense that it consumes energy from a DC power supply. Two common active circuit topologies
used to implement non-Foster implements are the negative impedance converters (NICs) and inverters (NIIs).

![Reactance or susceptance functions of a non-Foster element along the frequency axis](image)

**Figure 1.5.** Reactance or susceptance functions of a non-Foster element along the frequency axis

### 1.2.4 Literature Review

One of the key goals of this dissertation is to design wideband electrically small to mid-size simple structure antennas using non-Foster circuits as matching and coupling networks for receiving applications and to systematically stabilize non-Foster systems. Previous work on the use of non-Foster circuits with electrically small and mid-size antennas can be divided into two categories: Non-Foster circuits as MNs at the antennas feed, and Non-Foster circuits embedded (loads) within the antennas structure. Here, only work that has measured data with fabricated non-Foster circuits are reviewed.
Non-Foster Circuits as MNs at the Antenna’s Port

Earlier research of non-Foster impedance MNs for electrically small receiver antennas has been summarized well in [11]. However, only few papers [11], [19] and [29] - [32] show measured data for fabricated non-Foster impedance MNs for ESAs. In one of the earliest works [30], a negative capacitor was connected in parallel with two different antennas, a 2.5” monopole with 2.5” diameter top hat and a 10” monopole with 10" diameter top hat, to reduce the reactance of the two antennas. The improvement of antenna gain with respect to a 16 foot untuned whip antenna over 0.5MHz ~ 10MHz was shown in [30]. Perry [31] has also connected a negative capacitor in shunt with 3 antennas and showed improvement of antenna gain relative to a 12 foot whip antenna over 0.3MHz ~ 2.5MHz. In [11], [29], a 6” monopole and a 12” dipole antenna is connected in series with negative capacitors. The authors have shown improvement of both antenna gain and SNR from 20MHz to 120MHz, when compared to antennas without MNs. In [29], the highest effective frequency range obtained by applying an actual non-Foster impedance matching circuit to a 20”×2” lossy dipole antenna is from 60MHz to 400MHz. It has also shown that there is an improvement in the antenna gain; however, the electrical size of the antenna was around λ/2 at 286MHz. In [14], a 3" monopole sitting on a 3” square ground plane that is connected in series with both negative capacitors and a series combination of negative capacitor-negative inductor. The authors have shown that the improvement in both antenna gain and SNR from 100MHz to 480MHz for the use of only negative capacitor and from 100MHz to 640MHz for the use of a series combination of negative capacitor-inductor, when compared to antennas
without MNs. However, the SNR improvements were the same as the gain improvements. In both [19] and [32], the results have shown improvement in only the transmission coefficients. In [19], the results have shown improvement of a 15cm monopole antenna transmission coefficient using a series negative capacitor over 30MHz-200MHz. The improvement of the transmission coefficient of a small loop antenna using a negative inductor over the frequency range 100Hz to 200KHz is shown [32].

**Non-Foster Circuits Embedded within the Antennas Structure**

In the earliest work [14], a negative inductor based on NIC has been used as a load for a 3” loop-type small antenna. The work in [14] shows improvement of both antenna gain and SNR from 100MHz to 267MHz, when compared to the antenna without loading. In [33], an NF element (negative inductor) imbedded within an Egyptian-axe dipole antenna (7.5cm and 3cm outside and inside radii, where $k_a=0.49$ @ 300MHz) is shown. The results show only a 25.3MHz-10dB bandwidth around 300MHz (3.89 times greater than passive design). In [34], NF element based on NIC is embedded inside printed monopole antenna (4.8cmx4.38cm and 48cmx99.7cm ground plane). The results show a 10MHz-10-dB bandwidth improvement around 375 MHz (non-Foster: 10MHz; passive: 2MHz); max bandwidth of 40MHz up to around 480MHz. In [33] and [34], only the measured reflection coefficient is shown. Unfortunately, that is not a good indicator of improvement of the antenna performance. In [35], a non-Foster element-loaded parasitic array for broadband steerable patterns was introduced. The antenna prototype consists of a two element (monopole) parasitic array. The monopole had a height of 45 cm, width of
24 cm, and the distance between the driven and parasitic element was 18 cm. The antennas are mounted on a circular aluminum ground plane of diameter 1 m. The simulated and measured results show that broadband uniform nulls are generated from 180–350 MHz, providing about twice the null bandwidth of a passive parasitic load.

The review of previous work of the stability analysis and stabilization of non-Foster systems will be provided through the introduction of chapter two and four.

1.3 Organization of the Dissertation

The remainder of this dissertation is organized as follows: Chapter 2 presents a time-domain approach based on the computation of the largest (maximal) Lyapunov exponents (MLEs) from time-series signals (simulated and/or measured) to assess the stability of non-Foster systems. To the best of our knowledge, this is the first time that the stabilization of non-Foster circuits using a time-domain method is explicitly addressed. In this dissertation, a non-Foster system consists of a non-Foster circuit that can be used in various applications, including as a matching network for an ESA, as a reactive load for an ESA or as phase enhancement circuit in an antenna array. The time domain approach is used for a NFC as a matching network for an electrically small monopole-type antenna. Stabilization of the non-Foster system is shown using conventional linear control techniques. After fabricating the stabilized system, measurements of the stability in the time domain are performed to validate the simulated results, which indicate the system is stable. The system performance (antenna gain, and signal to noise ratio (SNR)) is also measured. Based on measured data, it is shown that a non-Foster impedance matching
network can be used to match an electrically small antenna to a receiver with 50 \( \Omega \) internal impedance over a fairly large frequency band (the largest reported frequency bandwidth).

Chapter 3 introduces a method to improve the performance of a receiving half loop ESA. The antenna is a 3’’ radius wire semicircular loop antenna sitting on an octagonal metal “ground plane” (30’’ long along its diagonal). The theory of Characteristic Modes (CM) is used to identify the different CM radiating modes. The theory of CM offers great physical insight since it allows us to determine the natural modes of the radiating structure and how to improve the antenna’s performance by looking into the mode interactions. The simulated results show that the desired omnidirectional TE mode is maintained for higher frequencies by introducing a negative inductor. The impact of transmission lines on the implementation of negative inductor circuit is also studied. A negative inductor based on the negative impedance inverter is then fabricated. Simulated and measured results of the realized antenna gain with and without non-Foster loading at \( (\phi = 0^\circ \text{ and } \theta = 90^\circ) \) show great improvement. By loading the half loop antenna with a negative inductor, the signal to noise ratio is also improved for a wide frequency band.

Chapter 4 introduces the complete stability analysis of non-Foster systems using a frequency domain method. The method, referred to as Pole-Zero Identification, consists of two major steps to ensure the complete stability of non-Foster systems. Two non-Foster systems; namely, a series combination of negative capacitor-inductor and a combination of Foster inductor with a series connection of a negative capacitor-inductor to match electrically small monopole antenna to 50\( \Omega \) receiver, are examined using the
proposed method. To the best of our knowledge, this is the first time that a matching network composed of Foster and non-Foster elements is fabricated and measured. After introducing gain and phase compensations, all the poles of the system are modified and moved to the left hand side of the complex plane (LHP). The non-Foster circuits and the monopole antenna are fabricated, and the measurement of the systems performance (antenna gain, and signal to noise ratio (SNR)) with and without non-Foster matching show great improvement for a wide frequency band.

Chapter 5 introduces the first non-Foster coupling network (negative capacitor in series with negative resistor) for a two-element monopole antenna array. The coupling network is used to amplify the output phase difference between the two antennas for a wide frequency band. This coupling network also improves the normalized available output power compared to the case where a passive Foster coupling network is used. The complete stability analysis of the entire system is demonstrated in several steps ensuring the stability of the non-Foster system. The system is stabilized with the help of a P-controller and Lead compensation; the latter is incorporated to maintain the output phase amplification. The controllers along with the collector resistors help to tune the output phase response. Measured and simulated data of the output phase response of the NFC-two-element biomimetic antenna array (BMAA) has very good agreement. The measured output phase response has a wider 3-dB bandwidth than the simulated 3-dB bandwidth at the expense of sacrificing small phase improvement.

Finally, Chapter 6 summarizes the key contributions of this dissertation and suggestions for future work are given.
CHAPTER 2

STABILITY ASSESSMENT OF NON-FOSTER CIRCUITS BASED ON TIME DOMAIN METHOD

2.1 Introduction

The interest in non-Foster circuits (NFCs) primarily stems from their potential application in several areas such as impedance matching, reactive loading of antennas, transmission lines and/or waveguides because they are not restricted by the well-known gain-bandwidth product [6] - [7]. They are also attracting attention in dispersion control for metamaterial applications. Non-Foster circuits are inherently nonlinear and tend to exhibit unwanted oscillations. As a consequence, an efficient stability analysis is necessary during the design process to predict any undesired behavior. If oscillations are present, efficient control schemes are necessary to eliminate them.

Unfortunately, frequency or time domain stability tests based on linear/nonlinear systems that only consider a single input-output transfer function may incorrectly conclude the stability of a circuit if some internal modes are hidden by zero-pole cancellations, thus making these tests inadequate for the determination of stability in non-Foster systems considered in this work [36]. A common stability test, known as the Normalized Determinant Function (NDF), requires complete access to all the models of the active and passive devices. The NDF method was introduced in [37] - [38] and has
been used in [39]- [40]. A time-domain stability method for non-Foster reactive elements in active metamaterials and antennas was introduced in [41]. The stability analysis was carried out for non-Foster circuits modeled as ideal negative elements accounting for losses and dispersion. That method also considers input/output port characteristics rather than examining the internal structure of the non-Foster circuit that may potentially lead to a faulty conclusion about the circuit’s stability [36]. In addition, suitable nonlinear stability tests have been proposed in [42] - [44], despite some of them being cumbersome or not practical for large or multi-dimensional circuits containing multiple nonlinear elements. Furthermore, a pole-zero identification technique originally introduced in [45] is suitable for the stability assessment of linear and nonlinear circuits based on the Harmonic Balance method. The Harmonic Balance method uses a frequency domain approach for the linear part of the circuit while the nonlinear portions are handled in the time domain [46].

The time-domain approach presented here is based on computing the largest (maximal) Lyapunov exponents (MLEs) from time-series signals. One key advantage of this method is that it can be used with computed and/or measured data. Another advantage of this approach is the ability to provide stability margins during the design process, allowing the designer to use the right controller to stabilize the system. To the best of our knowledge, this is the first time that explicitly addresses the stabilization of non-Foster circuits using a time-domain method. This chapter is a significant extension of the work presented in [47].
The chapter is organized as follows: In section 2.2, the proposed stability analysis is introduced with the definition of the largest Lyapunov exponents. In section 2.3, the proposed stability assessment is used to access the stability of a non-Foster system (non-Foster circuit and other external stages). Section 2.4 shows the key steps of the stabilization technique. The last section depicts measured time-domain signals showing the stability of the system; in addition, measured results of the realized antenna gain with and without the non-Foster matching network and the improved signal to noise ratio are shown as well.

2.2 Stability Analysis and Lyapunov Exponents

A system is locally stable if sufficiently small perturbations from a steady-state solution continuously decay to zero. Any potential steady-state solution has to be subject to a stability analysis to guarantee proper operation.

The stability assessment implemented here consists primarily of detecting the presence of spurious oscillations, chaotic or self-sustained oscillations that are within or outside the operational frequency band. This is done by applying a Gaussian pulse at the input port and computing the largest Lyapunov exponent of the circuit’s dynamics from the time-domain response (time-series) at various ports including the output port as well as ports within the circuit.

In general, the Lyapunov exponents are used to quantify the level of stability of a linear or nonlinear dynamical system. The sign of the largest exponent from the spectrum dictates the stability properties of the system. In most practical cases, the dynamical
equations of the circuit or system are unknown or approximately known; therefore, Lyapunov exponents can be estimated from an experimental or simulated time series. The time-domain response of the circuit to a pulse excitation can be used to estimate the embedding dimension for phase space reconstruction [48] as well as the largest Lyapunov exponent. The ability to determine the largest Lyapunov exponent from the computed or measured time-domain response of the system without explicitly knowing the system of equations is extremely useful for this application. Various algorithms for the computation of MLE are provided by [49] - [51]. Although it is possible to determine all the LEs from dynamical equations of the circuit or system, it is complicated and computationally consuming to determine LEs from time series as explained in [51]. Therefore, it is not practical for this application to determine all LEs, and it is sufficient to accurately determine the largest LE. The largest Lyapunov exponent can be determined without an explicit or a full reconstruction of the phase space by simply recording the average time evolution of neighboring trajectories and taking into account their exponential characteristics as shown in [51]. This makes the approach of this paper superior to the existing methods mentioned in the introduction. The largest Lyapunov exponents in this chapter are calculated mainly based on the algorithm provided in [51], and a summary of that algorithm is a major part of the flow chart shown in Figure 2.1.

By definition, MLEs are computed from the time response of the circuit for a given set of initial conditions. A positive exponent would imply the presence of chaotic or periodic signals. In this work, positive MLEs (closed to zero in some cases) will be generated by a unstable circuit displaying oscillations. The values of the exponents will be used to
gauge the stability margin of each system for fast comparison between related designs. Put it differently, the computational algorithms are re-purposed to extract the stability information from the transient behavior of the systems being analyzed. For accurate comparison between designs, it is crucial that the time series signals are sampled uniformly and for the same duration.

In case of instability (detection of a positive MLE), an attempt to stabilize the system is made by using different controllers. The system is considered stable when negative MLEs are calculated at multiple nodes. Moreover, the effects of small DC bias variations on system stability are also considered. During stabilization, conventional control techniques can be applied to the system while maintaining the desired non-Foster behavior. It will be shown that some non-Foster circuits may not be controllable; therefore, the circuit is modified to make them controllable. The modification includes the addition of a notch filter or compensatory elements followed by an optimization process to maintain a desired performance. Finally, it is worthwhile to note that the potential change of stability properties engendered by the magnitude of the input signal is in general important in non-linear circuits; however, it will not be considered here because it is assumed that the system is weakly nonlinear (for receiver applications) and remains in the small-signal regime as to not cause these additional nonlinear effects. Nevertheless, such an assumption might not always be the case (transmitter applications) and therefore the amplitude of the input signal and other parameters can be used to characterize a bifurcation analysis when necessary.
Figure 2.1. Flow chart shows the key steps of the time domain stability scheme used in this work. The main body shows the algorithm that evaluates LLEs.

2.3 Floating Negative Capacitor to Tune Electrically Small Monopole Antenna

Generic representations of a monopole antenna and a NFC are depicted in Figure 2.2. The monopole is three inches in length and sits on a 3”×3” square copper ground plane [52]. Although this metallic square is not truly a ground plane at the lower frequencies, it is still refer to as a “ground plane”. The input impedance of the monopole
is highly reactive below its first resonance; therefore, a negative reactive element with negative slope with respect to frequency is needed to cancel it over a reasonable frequency band. The measured input impedance of the antenna has a reactance that can be modeled up to the first resonance frequency as a series connection of a capacitor with \( C_{\text{ant}} = 2.4 \, \text{pF} \) and an inductor (8nH).

![Diagram](image)

Figure 2.2. Monopole antenna and NFC (Linvill’s Balanced NIC [9]) [12]. Branches labeled (1) to (6) are the locations of possible stabilizing networks.

Note that a male-to-male connection is needed to connect the antenna to the designed negative element. The NFC was implemented with two cross-coupled transistors BJT (NE68133) as shown in Figure 2.2 [9]. The NFC was designed by implementing a NIC to cancel the input reactance of the monopole antenna. The load of the NIC circuit is chosen to be a capacitor such that the absolute value of \( C_{\text{NIC}} \) is larger than \( C_{\text{ant}} \) to ensure the total reactance of the system has Foster behavior.
Negative Impedance Converter

The floating NICs are the Linvill’s balanced topologies. Linvill’s balanced open circuit stable (OCS) NIC [9] comprised of BJTs are shown in Figure 2.2. The input impedance analysis of an OSC NIC using an approximate of the small signal π-model of the BJTs [53] shown in Figure 2.3 with passive load $Z_L$ is [14]

$$Z_{in} = -Z_L + \frac{2(Z_{\pi} + Z_L)}{Z_S g_m + 1}$$  \hspace{1cm} (2.1)

Equation (2.1) is very good approximation up to the VHF range. Furthermore, for low frequency approximations ($r_{\pi} \gg 1$ and $g_m \gg C_S$), (10) can be approximated by

$$Z_{in} \approx -Z_L + \frac{2}{g_m},$$  \hspace{1cm} (2.2)

Figure 2.3. Approximate small signal π-model of the BJTs.
where \( g_m \) is the BJT trans-conductance. With ideal transistors in the NIC, it can be observed in (2.2) that the impedance converter coefficient is unity. The second term in the right hand side of (2.2) is a parasitic element due to the non-ideal transistors. If the passive load is a capacitor, the input impedance will be a negative capacitance with some losses.

The stability of the entire system is investigated based on the time-domain approach proposed here. A gaussian pulse of finite width is applied to the system. It is chosen so that its spectrum covers not only the frequency range of interest for the system, but also the entire operating frequency band of the active device. Test signals are selected at a couple of critical nodes as indicated in Figure 2.2 to investigate the stability of the entire circuit. The response (voltage) of the initial design at node (1), located between the NFC and antenna, and node (2), within the NFC, have been simulated and the results shown in Figure 2.4. The largest Lyapunov exponents of these two time series signals are then computed to determine the stability as well as the relative stability margin of the system. The time series signal at node (1) is shown in Figure 2.4 (a). The largest Lyapunov exponent obtained from this signal is +0.9977, which indicates the system is unstable. To ensure no hidden modes are missed, the MLE of the time series signal at node (2), shown in Figure 2.4 (b), is obtained and has a value of +0.9587, which also indicates the system is unstable. The Lyapunov exponents are positive and larger than zero because the signals shown in Figure 2.4 contain several spectral components.
Figure 2.4. Unstable output signals at node #1 (a) and node #2 (b), respectively, showing sustained oscillations from a finite input pulse. The largest computed Lyapunov exponents are positive.

2.4 Stabilization Networks

Since the system considered in section 2.3 is unstable, this section shows how to stabilize the NFCs using conventional linear control techniques. Conventional control techniques (CCTs) in linear control theory are based on a combination of proportional (P), derivative (D) and integral (I) controllers. The question that might arise is why linear controllers are being used to control the non-linear NFCs. Although the NFCs are in general non-linear, they are weakly non-linear in this receive-only application. Secondly, CCTs can be easily implemented to make the NFCs stable. In [54], the use of P, I and D controllers to stabilize multi-transistor circuits has been explained. Even though the method introduced in [54] is based on a frequency-domain analysis, these standard control techniques can still be used here. The reason behind that is that the proportional control action can be
easily implemented with series or shunt resistors, while the inclusion of series or parallel reactive elements can be associated to integral and derivative control actions. Moreover, the combination of resistor and reactive elements can represent proportional derivative and proportional integral controllers. The non-Foster circuit topologies used in this paper consists of only two cross-coupled transistors; therefore, the potential locations of the controllers are limited. The controllers cannot be placed between the NFC and the antenna or at the input port for a couple of reasons. Firstly, a proportional controller will introduce losses to the circuit, which has a deleterious effect on the efficiency of the antenna system. Secondly, using derivative and integral controllers can distort and negatively affect the non-Foster behavior of the circuit.

To stabilize the system in Figure 2.2, it is necessary to modify the circuit. The branches where the controllers can be introduced are labeled in Figure 2.2. However, as mentioned previously, stabilization networks should not be introduced in branches (1) or (6). Furthermore, P-controllers should not be inserted in the branch where the load is located to minimize losses and distortion of the transfer function of the circuit (negative capacitor). Therefore, a stabilization network can be introduced in branches (2-5), and to preserve the circuit’s symmetry, it can also be introduced in pairs either in branches (2-3) or (4-5). We found that the stabilization network has the same effect when introduced either at (2)-(3), or at (4)-(5). In this particular case, two identical stabilization networks are inserted in branches (3) and (5). Any stabilization component that is applied at branch (3), it will also be inserted to branch (5) to maintain symmetry.
The proposed time-domain stability approach is demonstrated here when different controllers are added to a antenna system (antenna and NIC-NFC). First, we assess the stability of the system when a P-controller (series resistor) is inserted at branch (3). Any resistor value should maintain the non-Foster behavior of the circuit (negative capacitor) in order to get the advantage of the non-Foster matching. The non-Foster behavior can be lost when a large resistor value is inserted, which lead the circuit to behave as loss resistor and Foster reactance. As shown in Figure 2.5 (a), there is no value of series resistor that can force the system to be stable since the largest (maximal) Lyapunov exponent (MLE) of the time series signal is always positive. Computed MLEs of time series signals for different $R_{\text{stab}}$ values at branch (3) are shown in Figure 2.5(a). The values of the MLE in Figure 2.5 (a) are small when $R_{\text{stab}}$ approaches 30 $\Omega$, so larger values of $R_{\text{stab}}$ are used to check the possibility of stabilizing the system. The system becomes stable when $R_{\text{stab}}$ equals 8 $K\Omega$, but, unfortunately, the non-Foster behavior is lost. Another attempt to stabilize the circuit was done by inserting a series capacitor in branch (3). Although the MLEs at different locations for several capacitor values have been calculated, the MLE at only one location is shown in Figure 2.5(b). It can be noticed that there is no value of capacitance that can drive the system into stability. The stability analysis was also carried out with an inductor (D- controller) and the MLE calculated. Again, the system remains unstable for all the inductance values shown in Figure 2.5(c). PI and PD controllers were also considered, but the system remained unstable. The circuit appears to be uncontrollable under these standard control actions.
After additional simulations, it was determined that inserting an inductor in parallel to the load (capacitor) to generate a notch filter makes the system controllable. With the notch filter in place and the system controllable, the P-controller (identical resistors at branches 3 and 5), the values of the added inductor and resistor can be optimized to obtain a stable system that performs as a negative capacitor. The optimized values of these elements were found to be 1.8 μH for the inductor and 30 Ω for the resistor. Note that a 0805LS-182 Coilcraft inductor was used in this circuit. Since the absolute value of $C_{NIC}$ should be larger than $C_{ant}$, $C_{L}$ has been chosen to be 3.9pF; a smaller value could be chosen, but this value not only determines the negative capacitance but also determines the resonance frequency of the parallel LC negative tank that is actually being converted. The value of the inductor is large enough not to affect the behavior of the NIC in the frequency range of interest. A small resistor of 20 Ω in series with the load capacitor was added to reduce the losses, as can be seen in equation (2.2), introduced by the parasitics of the non-ideal transistors.

The circuit behaves as a negative inductor up to 50MHz, a frequency lower than the band of interest. Above 50 MHz, it behaves as required, namely, a negative capacitor. In Figure 2.6, the computed MLE obtained from the time signal at node (2) are shown for different values of the series resistor. It can be observed that the system becomes stable when $R_{stab}$ is equal to 30 Ω, and continues to be stable for a wide range of $R_{stab}$ values. The largest Lyapunov exponent (MLE) was computed for several time series signals at different locations to confirm that the system is stable.
Figure 2.5. Computed MLE of a time series signal at node (2) after introducing: (a) resistors, (b) capacitance and (c) inductance at branch (3).

Figure 2.6. Computed MLE of a time series signal at node (2) for several resistors values.
The MLE at node (1) is calculated and it had a value of -1.7870, which confirms the system is stable. When the MLE is negative at all nodes within the NIC and at its output (node between NIC and antenna), the system is stable and there is no possibility of any hidden modes.

2.4.1 Transducer Gain for Non-Foster Matching Network

Generally, the introduction of a stabilization network to the system can degrade its performance. It is common for designers to tradeoff between the system performance and the stability requirement [54]. The figure of merit to quantify the effect of the P-controller is to calculate the realized gain (including input impedance mismatches) of the antenna system.

The proper definition of impedance matching is the optimization of a two port device to deliver maximum power to the load for a given available power at the source. Since the transducer gain equals the delivered power to the available power [55] - [56], impedance matching is equivalent to maximizing the transducer gain of the matching network; namely,

\[ G_T = \frac{P_{del}}{P_{avs}} \]  \hspace{1cm} (2.3)

where \( G_T \) is the transducer gain, \( P_{del} \) is the power delivered to the load, \( P_{avs} \) the available power from the source. The realized antenna gain, including the active matching network (see Figure 2.7) can be expressed as [14], [56] \( G_{ant} = G_{IEEE} G_T \) where \( G_{IEEE} \) is the IEEE antenna gain (valid for transmission or reception) and \( G_T \) for the transmission case is
\[
G_T = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_s\Gamma_{in}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}
\]

where

\[
\Gamma_{in} = S_{11} + \frac{S_{12}\Gamma_L S_{21}}{1 - S_{22}\Gamma_L}
\]

When the transducer is not reciprocal, which is the case here, equation (2.4) should be modified for reception. In other words, the transducer gain for reception is

\[
G_T = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{out}\Gamma_L|^2} |S_{12}|^2 \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2}
\]

where

\[
\Gamma_{out} = S_{22} + \frac{S_{21}\Gamma_L S_{12}}{1 - S_{11}\Gamma_s}
\]

Since the port #1 of the transducer is designed to be 50Ω and the receiver (network analyzer) is also 50Ω, the reflection coefficient looking into the receiver is zero; namely, \(\Gamma_s = 0\). Therefore, the transducer gain becomes

\[
G_{ant} = G_{IEEE} \left( \frac{|S_{12}|^2}{|1 - S_{22}\Gamma_L|^2} \right) (1 - |\Gamma_L|^2)
\]

Figure 2.7. Active matching network connected to the antenna.
The gain improvement due to the matching network is defined as the ratio of the antenna gain with MN to the gain without MN; namely,

\[
G_{\text{imp}} = \left[ G_{\text{IEEE}} \frac{|S_{12}|^2 (1 - |\Gamma_L|^2)}{|1 - S_{22} \Gamma_L|^2} \right] / \left[ G_{\text{IEEE}} (1 - |\Gamma_L|^2) \right]
\]

\[
= \frac{|S_{12}|^2}{|1 - S_{22} \Gamma_L|^2}
\]

(2.7)
in which the antenna gain without MN is \( G = G_{\text{IEEE}} (1 - |\Gamma_L|^2) \). Although DC power is required to bias the active elements (i.e., transistors), the calculated realized gain only considers the RF input power. The transistors are biased with a 12V DC voltage and a DC current (collector) of 12mA. To obtain the best (higher) realized antenna gain, equation (2.7) needs to be optimized, but keeping in mind, the system needs to be stable. It is not possible obtain higher gain when the MN loses its no-Foster behavior. The final design (Figure 2.8) is obtained by introducing lead compensation, will be explained in section 4.4.2, instead of P-controllers at branches (3) and (5). The lead compensation is a parallel combination of resistor (30 \( \Omega \)) and inductor (30nH). Moreover, the load capacitance is set to be 4.3pF to go further away from the stability margin. The MLE at node (1) was calculated and it had a value of -1.7642, which confirms the system is stable. Table 2.1 lists the elements’ values of the schematic representation of a negative capacitor including DC biasing and stabilization networks.
Table 2.1. The values of the series negative capacitor schematic’s elements

<table>
<thead>
<tr>
<th>Element</th>
<th>Value</th>
<th>Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Block</td>
<td>6.8nF</td>
<td>RFC</td>
<td>6.5μH</td>
</tr>
<tr>
<td>$R_1$</td>
<td>14.7KΩ</td>
<td>$L_L$</td>
<td>1.8μH</td>
</tr>
<tr>
<td>$R_2$</td>
<td>16.9KΩ</td>
<td>$R_c$</td>
<td>30Ω</td>
</tr>
<tr>
<td>$R_E$</td>
<td>200Ω</td>
<td>$L_c$</td>
<td>30nH</td>
</tr>
<tr>
<td>$R_L$</td>
<td>20Ω</td>
<td>$R_0$</td>
<td>50Ω</td>
</tr>
<tr>
<td>$C_L$</td>
<td>4.3pF</td>
<td>$V_{dc}$</td>
<td>12V</td>
</tr>
</tbody>
</table>

Figure 2.8. Schematic representation of a negative capacitor included DC biasing and stabilization networks, port impedance, and monopole antenna.
2.5 Measured Results

After stabilizing the system, the negative capacitor matching network comprised of two discrete NPN BJTs (NE68133) with inductor-resistor biasing was fabricated using RT/duroid 5880 substrate with $\varepsilon_r = 2.2$, $\tan\delta=0.0009$, and thickness= 62 mil to match a receiver with 50 $\Omega$ internal resistance to a monopole antenna.

![Fabricated floating negative capacitor](image)

Figure 2.9. Fabricated floating negative capacitor

Figure 2.9 shows the fabricated negative capacitor matching network. In this section, the stability measurements, the realized antenna gain with and without the non-Foster matching, and the improvement signal to noise ratio (SNR) based on measurements are provided.

2.5.1 Stability Measurements

Measurements of the stability in the time domain were performed to validate the simulated results described above. An accurate stability test for this non-linear system is
to excite it with a short pulse and measure the response at various locations within the circuit and at its output. The exciting input pulse is shown in Figure 2.10. This pulse was generated using a pulse generator (Model 4015D) with a pulse head and impulse forming network (Model 5208). Several time series signals inside the NFC were measured using a high impedance active probe, and a digital oscilloscope. Figure 2.11 shows measured time signals at point (1) and (2). The largest Lyapunov exponent of the response (time series signal) at points (1) and (2) of Figure 2.2 were calculated and determined to have values of -1.2493 and -0.7554 respectively. Since both values are negative, the measurements confirm the system is stable.

2.5.2 Antenna Gain and Signal to Noise Ratio

The antenna gain and SNR measurements with and without the Non-Foster MN in the frequency range 50 MHz to 500 MHz were performed in an outdoor range. Note that the antenna’s electrical length at 50 MHz, including the finite ground plane, is around $\lambda/78$.

![Figure 2.10. Measured input pulse used to excite the system.](image)
Figure 2.11. Measured stable signals due to finite input pulse.

Figure 2.12. Outdoor setup for antenna gain and SNR measurements.

**Lower Frequency Measurements (50 MHz-300 MHz):**

Figure 2.12 shows the outdoor setup for the antenna gain and SNR measurements with and without a fabricated Non-Foster MN (negative capacitor). The antenna, see inset in Figure 2.12, is placed on the top of a plastic pole perpendicular to the ground. The
distance between the transmitted antenna and the antenna under test is around 20m, and both antennas are each about 10m above the ground. For the antenna gain measurements, time-gating is applied to minimize multi-path reflections from the ground, walls, etc. The procedure followed to perform time-gating can be briefly summarized as follows [14] and [57]. First, the frequency domain data of the measured $S_{21}$ was converted into the time domain by using an inverse discrete Fourier transform. With the $S_{21}$ data in the time domain, a starting and ending time of a rectangular gating window was determined based on the calculated time delays of the direct and multi-path signals. After applying gating to the data, the time domain data was converted back to the frequency domain by means of a discrete Fourier transform. The measured realized gains are plotted in Figure 2.13. The dashed red and solid black curves in Figure 2.13 are measured data for the realized antenna gain with and without the MN, respectively. To confirm whether the measurement is correct, the simulated antenna gain without a MN (dashed black curve) generated by using Ansoft HFSS v.13 is also plotted in Figure 2.13. Note that HFSS is based on the vector finite element algorithm (FEM) [58] - [64]. When comparing the solid and dashed black curves, it can be observed that there is a small discrepancy. To improve the measured results, ferrite beads were used along a 1-foot cable connected to the antenna to minimize common mode currents. Without using the ferrite beads and due to the presence of common mode currents, the measurements were not as good as presented here. It can be observed that the antenna gain with the MN is clearly improved in the range 50MHz to 300MHz. For instance, the improvement with the MN is around 13dB at 100MHz.
To assess the improvement of SNR with this system, the SNR of the antenna with the MN included relative to the case without a MN is used in this paper. To obtain this improvement, the same procedure suggested in [11] and [14] was used. Both received signals with and without the MN are recorded with a Network Analyzer (Agilent N5230A). The difference (in dB) of these two time-gated received signals is calculated; namely, $S_1 - S_0$ where ($S_1$) is the received signal with the MN present, while ($S_0$) is the received signal without the MN. This difference in dB is identical to the improvement of antenna gain (assuming the source is transmitting the same power in both cases). To measure noise, the signal at the antenna terminal with and without a MN is measured with a spectrum analyzer (Agilent E4440A) while the transmitter antenna is turned off. A low noise preamplifier is inserted at the input of the spectrum analyzer to reduce its
effective noise figure thus to improve the accuracy of the noise measurements. Fortunately, Agilent E4440A model has an internal preamplifier with a 27 dB nominal gain and a 7 dB noise figure. When we performed the noise measurement, we enable the preamplifier to increase the accuracy of the noise measurements. When performing noise measurements, it is very important to be aware of the sensitivity of the receiver, namely, its noise floor. Depending on the noise floor, the noise at the terminals of the antenna with and without a MN may not be detected by the spectrum analyzer. In this measurement, the noise floor of a spectrum analyzer is $-98\,\text{dBm}$ in the range 30 MHz-300 MHz with the preamplifier enabled.

In dB scale, the improvement of SNR is evaluated by taking the difference (in dBs) between the received signals (with/without the MN) while the transmitter is on (as well as external RF interference signals), and the difference in noise (with/without the MN), including external RF interference signals while the transmitter is turned off. In equation form, the improvement of SNR (in dBs) can be expressed as

$$\text{Improvement of SNR}_\text{dB} = (S_1 - S_0) - (N_1 - N_0).$$  \hfill (2.8)

where $N_1$ and $N_0$ are, respectively, the signals at the antenna terminals (transmitter off) with and without the MN. Figure 2.14 shows the measured improvement of SNR with the MN relative to the case without a MN. Since the original measured result (dashed black curve) in Figure 2.14 has several fluctuations, a smoothed version (red curve) is also plotted on the same graph. Based on the smoothed curved, it can be observed that the overall improvement of SNR is up to 13.4 dB in the range 50MHz to 300MHz.
High Frequency Measurements (300 MHz-500 MHz):

Measurements at frequencies above 300MHz, where performed in the same outdoor range but using a different reference antenna. The measurement procedure is the same as that of the lower frequency measurements, including time-gating. Simulated and measured antenna realized gains are plotted in Figure 2.15. It can be observed the improvement of the antenna gain with the NC is maintained up to 420 MHz when compared to the antenna without the NC. Finally, measured results for the improvement of SNR with the non-Foster MN relative to the case without a MN are plotted in Figure 2.16. It can be observed that the overall SNR with the non-Foster MN (smoothed curve) is higher than the SNR without a MN up to around 410MHz.

Figure 2.14. Improvement of SNR with a non-Foster impedance matching network relative to the case without a matching network over 50MHz to 300MHz.
Figure 2.15. Measurement of the realized antenna gains for frequencies 300MHz to 500MHz.

Figure 2.16. Improvement of SNR with a non-Foster impedance matching network relative to the case without a matching network over 300MHz to 500MHz.
2.6 Discussion

The proposed stability method is a general method applicable for both linear and nonlinear microwave circuits since it is a time-domain technique. To ensure the accuracy of the time domain integration, we first perform the stability assessment at the schematic level where the circuit consists of only lumped elements. After we stabilized the system, it is then modeled using a full-wave EM solver and circuit CAD package. The integration method used in the transient-convolution simulations was a trapezoidal integration method that successfully converges with low truncated error. Note that the sampling intervals are kept the same for the purpose of comparison.

2.6 Summary

A time-domain stability assessment is implemented here to determine the stability properties of non-Foster circuits. This technique relies on the calculation of the largest Lyapunov exponents of simulated and/or measured data at multiple nodes within the internal structure of the circuits and at their output ports. The ability to monitor the change of the MLEs’ values during the stabilization process provides great insight because it helps us determine the correct path to stabilize the system as well as the controller type to be used. Stabilization networks consisting of a combination of notch filters and lead compensation were introduced that insured the circuits are stable not only in the frequency band of interest, but also in the entire band of operation of the active devices. The proposed approach was successfully validated with experimental results of a negative capacitor used to tune an electrically small monopole antenna.
According to both simulated and measured results, it was shown that a non-Foster MN can be used to tune an ESA to improve the matching a 50 Ω receiver over a large frequency band (50MHz-420MHz), which is a ratio of 8.4:1. It was also demonstrated that the SNR can be improved when compared to the antenna without a MN. The SNR has been improved up to 13.4 dB in the present work.
CHAPTER 3
NON-FOSTER CIRCUIT EMBEDDED WITHIN AN ELECTRICALLY SMALL ANTENNA

3.1 Introduction

It is well-known that electrically small antennas (ESAs) are inefficient radiators because of their high quality factor (Q). Therefore, matching networks are required to improve their radiation characteristics. However, due to gain-bandwidth restrictions [6] - [7], wideband matching cannot be achieved using passive networks. Non-Foster circuits (NFCs), such as negative capacitors, negative inductors, or some of their combinations, are attractive because they can be used in matching ESAs since they violate the gain-bandwidth limitation.

The objective of this chapter is to improve the performance of a receiving half loop ESA. The antenna is a 3” radius wire semicircular loop antenna sitting on an octagonal metal “ground plane” (30” long along its diagonal). A further objective is to maintain an omnidirectional pattern (TE mode) up to higher frequencies by delaying the onset of higher order modes by loading the antenna itself with a reactive load. An additional objective is to improve the realized antenna gain in the frequency range of interest. To achieve these objectives, the theory of Characteristic Modes (CM) introduced in [65] - [70], which allows us to identify the different radiating modes of the antenna, is used to
provide physical insight during the design procedure. As previously mentioned, a key step in the design is to load the antenna itself with reactive loads and, if necessary, impedance match the antenna at the feeding port. A matching network at the feed can only modify the antenna current in the neighborhood of the feed; however, loading the antenna itself at various locations allows us to control the currents on the entire structure of the antenna [13], [71] - [72]. In this chapter, we load the ESA with only one non-Foster element, which is much simpler and practical than using combination of Foster and non-Foster circuits, to improve its bandwidth in terms of realized gain and radiation pattern. The radiation pattern is also controlled because the load allows us to modify the current on the entire structure.

3.2 Characteristic Mode Analysis of Half Loop ESA

The theory of CM [65] - [70] is very useful for antenna analysis and design. It consists in the numerical calculation of a weighted set of orthogonal current modes (eigencurrents) that are supported by the antenna. These characteristic modes are obtained by solving a particular weighted eigenvalue equation that is derived from the Method of Moments (MoM) impedance matrix. The method of moments (MoM)-based code FEKO [73] was used to perform the EM simulations [74] - [78]. After obtaining the MoM impedance matrix (running the CM simulation), the eigenvalues \( \lambda_n \), the modal significance (MS) coefficients, and the characteristic angles \( \alpha \) can be evaluated. A mode is considered resonant when \( \lambda_n = 0 \), MS=1 or \( \alpha = 180 \). Moreover, the mode is inductive (stores magnetic
energy) if the $\lambda_n > 0$ and $90^\circ \leq \alpha < 180^\circ$ and is capacitive (stores electrical energy) if the $\lambda_n < 0$ and $180^\circ < \alpha \leq 270^\circ$.

The half loop antenna is a 3” wire circular loop antenna mounted on an octagonal metal “ground plane” (30” long along its diagonal); the geometry is shown in Figure 3.1. After some study using the FEKO simulator, it was determined that the 30” octagonal metal works as a good ground plane in the frequency range of interest (larger than 100MHz). The CM analysis is applied first to a 3” half loop antenna sitting on infinity ground plane. The characteristic angles of the first three modes are shown Figure. 3.2. It can be seen that mode 1 resonates at 648MHz. At lower frequencies, mode 1 is capacitive; however, it is inductive above 648MHz. Mode 2 has the same behavior as mode 1; however, it resonates at a higher frequency. Mode 0 is an inductive mode; this mode is sometimes referred to as mode 0, since it can exist down to zero frequency (DC current) where it resonates. The radiation pattern of each mode and the total radiation pattern are shown in
Figure 3.3. The total radiation pattern is obtained by feeding the antenna at one port (where wire and ground plane intersect) while the second intersection is connected to the ground plane. Although the radiation pattern for each mode maintains its shape at the two frequencies (125 and 300 MHz), the total radiation pattern changes and does not keep the desired omnidirectional shape. The reason for this change in pattern is that the first mode (mode 1) dominates at higher frequencies, and leads the antenna to resonate around 292 MHz as shown in Figure 3.4 where the imaginary part of the antenna’s input impedance is depicted. The first antiresonance (parallel resonance) around 292 MHz is due to the interaction of modes 0 and 1 [79].

Figure 3.2. Variation of the characteristic angle with frequency for the three characteristic modes of half loop antenna
Figure 3.3. Radiation pattern at (a) 125MHz (b) 300MHz for 1- Mode 0; 2- Mode 1; 3- Total.

Figure 3.4. Imaginary part of the input impedance of 3” radius half loop antenna
As mentioned previously, the objective here is to keep the omnidirectional TE mode up to higher frequencies and to improve the realized antenna gain. To achieve these objectives, mode 0 should dominate for a wider frequency range, and mode 1 and 2 should resonate at higher frequencies so they do not interact with mode 0 at low frequencies. A key step to maintain the omnidirectional TE mode at higher frequencies is to shift the resonance of modes 1 and 2 to higher frequencies. As is well-known, resonance phenomena is the interaction of electric (capacitive) and magnetic (inductive) energies such that the energy transfers completely from electric to magnetic energies and vice versa. Modes 1 and 2 are capacitive at lower frequencies; however, as the frequency increases, they start to become more inductive and each resonates when the reactance becomes zero. At higher frequencies, they both become inductive. Therefore, by reducing the inductance of modes 1 and 2, they will resonate at higher frequencies. Moreover, reducing the inductance in the frequency range of interest also implies that modes 1 and 2 become stronger at higher frequencies, thus, allowing us to keep the desired pattern up to higher frequencies and improving the realized antenna gain. One way to reduce the inductance for a wide frequency range is to introduce a negative inductance. The question is, where should the negative inductance be placed to delay the resonance frequencies of modes 1 and 2? To resolve this issue, the current distribution of the modes on the antenna’ structure can be examined. Figure 3.5 illustrates the current distribution of modes 1 and 2 for different sampled frequencies to show that the current distribution of each mode remains somewhat unchanged for a wide frequency range. It should be noted that mode 1 has the largest impact within the frequency range of interest. Since the
current distribution is stronger at the extremities of the loop, a negative inductor can be loaded either at the feed port or at the second port where the loops meets the ground plane. We choose the latter to avoid possible coupling with the feed terminal and to have better control of the antenna current distribution. The magnitude value of the negative inductor must be smaller than the input inductance of the antenna impedance model to ensure stability since the negative inductor will be implemented with a non-Foster circuit. The input impedance of the antenna has reactance that can be modeled up to its first resonance frequency as an inductor with $L_{\text{ant}}=200\text{nH}$ in parallel with a capacitor with $C_{\text{ant}}=1.5\text{pF}$. A negative inductor with value $-100\text{nH}$ is chosen to load the antenna. Fortunately, the FEKO simulator allows us to include a negative inductor as a load.

Figure 3.5. The current distribution of (a) mode 1 (b) mode 2 at: 1- 125MHz; 2- 200MHz; 3- 300MHz.
The characteristic angles of the first three modes, after loading the antenna with negative inductor, is shown Figure 3.6. It can be seen that the resonance of mode 1 moved from 648MHz to 848MHz while the resonance of mode 2 increased from 1276MHz to 1520MHz. Mode 0 still maintains its inductive behavior with small reduction at high frequencies. The simulated realized gain patterns (includes input impedance mismatch) for the half loop antenna with and without the negative inductor at different frequencies are shown in Figure 3.7.

Figure 3.6. Variation of the characteristic angle with frequency for the three characteristic modes of half loop antenna loaded with negative inductor.
It can be noted from Figure 3.7 (a.1-a.5), that the antenna without the load does not maintain the omnidirectional TE mode after 200MHz. However, as shown in Figure 3.7 (b.1-b.5) the antenna loaded with a negative inductor maintains the omnidirectional TE mode for frequencies higher than 350MHz. The maximum simulated realized gain (includes input impedance mismatch) improves by 5dB from 100MHz to 350MHz. The realized gain can be further improved by choosing a larger negative inductor. Up to this point, the negative inductor was assumed to be an ideal element. Moreover, the ground plane of the half loop was assumed to be of infinity dimensions. In the following sections, the negative inductor is realized and fabricated by the means of negative impedance inverter. Furthermore, the antenna will be simulated with a 30” metallic ground plane. Measured results of the maximum realized gain are included to verify the simulated results.
Figure 3.7. Realized gain patterns (a) antenna without load (b) antenna with negative inductor loading at: 1- 125MHz; 2- 175MHz; 3- 225MHz; 4- 300MHz; 5- 375MHz
3.3 Grounded Negative Inductor based on Negative Impedance

Inverter (NII) Topology

The objective of this work is to design a stable (without oscillations) negative inductor to load a half loop ESA. A schematic representation of a half loop antenna and a non-Foster circuit (NFC) is depicted in Figure 3.8 (a). As mentioned previously, the test antenna used here is a 3’’ radius wire circular loop antenna on an octagonal metal “ground plane” (30’’ long along its diagonal). The antenna is ESA up to its first resonance since its dimension is less than λ/12. Note that this antenna has two ports at opposite sides. As described in the previous section, the antenna should be loaded with a negative inductor (NI). The negative inductor is designed by implementing a negative impedance inverter (NII) [80]. The NII is implemented with two cross-coupled Gallium Arsenide MESFETs (ATF26884) as shown in Figure 3.8 (b).

It is assumed that each of the FET transistors in Figure 8(b) can be represented by a transconductance \( g_m \), a gate-source capacitance \( C_{gs} \), and output conductance \( G_{ds} \).

![Figure 3.8](image_url)

Figure 3.8. (a) Half loop antenna loaded with NFC. (b) Negative impedance inverter NII topology used to design negative inductor NI.
To design a negative inductor, the load has to be a passive lumped capacitor; therefore, $C_L$ has been chosen to be the load. The input admittance can be calculated to be

$$y_{in} = C_{gs1}s - \frac{g_{m1}g_{m2}}{(C_{gs2} + C_L)s + 1 / r_{ds1}} + \frac{1}{r_{ds2}}$$  \hspace{1cm} (3.1)$$

Based on (3.1), the input admittance can be modeled as a negative inductor in series with a small negative resistor, and this series combination is in parallel with a resistor and capacitor. With the proper transistors biasing, $r_{ds1}$ has more impact at low frequencies; however, $r_{ds2}$ has more impact within the frequency range of interest. Therefore, a small resistor value is added in series with the capacitive load to compensate for the loss introduced by $r_{ds2}$. The capacitance of $C_{gs1}$ limits the available bandwidth of the negative inductor; therefore, the best choice are transistors with small gate-source capacitance. Further compensation has been introduced to reduce the effect of the transistors intrinsic parameters as discussed in [81]. Since the negative inductor is implemented using microstrip transmission lines (MTL), these MTL can degrade the performance of these circuits in terms of bandwidth, efficiency, and stability.

### 3.3.1 Impact of Transmission Lines on the Implementation of Non-Foster Circuits

Analysis of the input admittance of the negative impedance inverter including transmission lines is carried out using two port networks for the transistors and the transmission lines as shown in Figure 3.9.

To simplify the analysis and predict the effect of each line, the first step is to use a simple transistor model consisting only of the input capacitor $C_{gs}$ and the
transconductance $g_m$, while also considering the T.L.2 line and ignoring the effect of T.L.1 (replaced with an ideal short).

![Negative impedance inverter including MTL.](image)

Figure 3.9. Negative impedance inverter including MTL.

The input admittance (at port #1) is then

$$y_{in} = C_{gs1} - i \cot \theta / Z_0 + (i / Z_0 \sin \theta - g_m g_{m2} / C_{gs2}) / \cos \theta$$ \hspace{1cm} (3.2)

where $Z_0$ is the T.L.2’s characteristic impedance, $C_2 = C_{gs2} + C_L$, $\theta = \beta l$, $l$ is the length of T.L.2, $\beta = \omega / \nu$, $\nu = 1 / \sqrt{\mu_0 \varepsilon_{0f} \varepsilon_{ef}}$, and $\varepsilon_{ef}$ is the effective permittivity of the MTL. Using the first two terms of the Taylor series expansions of the trigonometric functions, the input admittance (3.2) can be rewritten as

$$y_{in} = \left[ C_{gs1} + i Y_0 / \nu + g_m g_{m2} l^2 / (2 \nu^2 C_{gs2}) \right] S - g_m g_{m2} / C_{gs2} S$$ \hspace{1cm} (3.3)

The last term in (3.3) represents the desired negative inductance. It can be observed that the second and the third terms (within brackets) of (3.3) are proportional to the length of the transmission line T.L.2, and they contribute to the parasitic capacitance $C_{gs1}$. 

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Therefore, the total parasitic capacitance has been increased by T.L.2, reducing the bandwidth of the negative inductance circuit. Now considering only the T.L.1, the simplified input admittance can be expressed as:

\[
y_{in} = C_{gs}S - \frac{g_{m1}g_{m2}}{S\left[C_2 + l/(vZ_0) - C_2\omega^2\left(1/\sqrt{2v}\right)^2\right]}\tag{3.4}
\]

where the effect of the transmission line T.L.1 is included inside the brackets along with \(C_2\). At frequencies below 1GHz, the second term within the brackets is larger than the third term so that the length of T.L.1 adds to the capacitance of \(C_2\). Therefore, the T.L.1 increases the negative inductance, but it is a relatively small effect. Further analysis was carried out including both transmission lines, which led to the same results; namely, T.L.1 increases the negative inductance, while T.L.2 increases the parasitic capacitance. For a more realistic analysis, a transistor model consisting of three intrinsic elements, namely \(C_{gs}, g_m,\) and \(r_{ds}\) along with T.L.1 and T.L.2 can be used to perform a more accurate analysis. The input admittance (at port #1) considering the T.L.2 line and ignoring the effect of T.L.1 (replaced with an ideal short) can be expressed as:

\[
y_{in} = C_{gs}S + \frac{\coth \gamma l}{Z_0} - \left[\frac{1}{Z_0 \sinh \gamma l}\right] + \left[\frac{g_{m1}g_{m2}r_{ds1}}{C_2sr_{ds1} + 1}\right] \left[\frac{1}{Z_0 \sinh \gamma l\left(\coth \gamma l + 1\right) + \frac{1}{r_{ds2}}}\right]\tag{3.5}
\]

Assuming the transmission line is lossless as assumed previously. Equation (3.5) can be simplified as:
\[ y_{in} = C_{g1}s + \frac{(Y_0 I_0)}{3V} s - \frac{Y_0 V}{l_s} + \left[ \frac{1}{((oZ_0 / \nu)^2 / r_{ds2} - s(lZ_0 / \nu))} \right] - \left[ \frac{g_{m1}g_{m2}}{(C_2Z_0 I_0 / r_{ds2} V)^2 + C_2s + s(lZ_0 / r_{ds1}r_{ds2} V) + 1 / r_{ds1}} \right] \] (3.6)

It can be seen from (3.6), the resulting input admittance shows that the transmission lines not only have an impact on the reactance but also have impact the real part of the input impedance. One solution to the deleterious transmission line effect is to reduce its electrical length as much as possible, but it will always have a nonzero electrical length. Therefore, further modification to control the input impedance is required. The layout of the negative inductor was designed in Agilent ADS Momentum, and the effect of the microstrip transmission line has been taken into account. Since the non-Foster circuits are conditionally stable, the design will not be meaningful without considering the external load to which the non-Foster circuit is connected.

3.3.2 Stability Consideration

Non-Foster circuits are potentially unstable because of the positive feedback. Therefore, it is important to analyze the stability of the whole system; namely, NI-NII and its internal load, and the impedance model of the antenna, which is considered an external load to the NII. As it has been emphasized by many authors, the internal stability should be analyzed to assure there are no internal (within the circuit) oscillations. The stability analysis was performed using a time-domain based method [47]. The analysis is accomplished by applying a Gaussian pulse whose spectrum covers the entire frequency
range of interest. Signals were calculated at different locations within the system (NII and antenna) showing that the system was not stable. Stabilization networks were introduced within the NI-NII and time domain signals were calculated again. The stabilization networks are P-controllers that were added along with T.L\textsubscript{1} and T.L\textsubscript{2} depicted in Figure 3.9. After stabilizing the system, a voltage signal between the antenna and NII shown in Figure 3.10, was calculated showing the system is stable. Please note that, stability assessment and systematic stabilization of non-Foster systems based on pole-zero identification method is used in both Chapters 4 and 5. Internal stability analysis has been performed, and the dominant eigenvalues are -6.63e6, ±j1.94e7 which shows that the system is internally stable.

![Figure 3.10. Simulated voltage signal shows the system is stable.](image)

Figure 3.10. Simulated voltage signal shows the system is stable.
3.3.3 Impact of Transmission Tines on Input Impedance of Non-Foster Inductor

The calculated quality factor (Q) and input reactance of the stabilized negative inductor are shown in Figure 3.11 (represented as red sold lines). Moreover, the impact of adding equal extra length to each of the two transmission lines, which is shown in Figure 3.9, is depicted also in Figure 3.11.

Increasing the length of the transmission lines has a negative effect on the quality factor of the non-Foster element causing a reduction of the antenna gain and bandwidth; although, the latter effect is relatively small since the reactance is large. After analyzing the NII and the stability and realizability study, the non-Foster inductor was fabricated on the RT/duroid 5880 substrate with $\epsilon_r=2.2$ and thickness=62 mil. The fabricated negative inductor is shown in Figure 3.12.

![Figure 3.11. Simulated impedance of negative inductor showing the transmission lines impact. (a) Quality factor. (b) The input reactance.](image-url)
3.4 Simulated and Measured Results

After the successful design and fabricating the stabilized system composed of both a loading network comprised of a non-Foster inductor and a 3” half loop antenna, its performance such as input impedance, antenna pattern are simulated and measured.

3.4.1 Input Impedance

The simulated S-parameters of the negative inductor (NL) were generated and imported into the FEKO simulator to be used as a load for the half loop antenna. Moreover, the negative inductor (non-Foster Circuit) has been measured using a Network Analyzer, and the measured S-parameters of the negative inductor were also imported into the FEKO simulator to simulate the antenna with a measured negative inductor. The input impedance of the 3” radius half loop antenna on 30” ground plane with and without negative inductor (non-Foster circuit, NFC) is shown in Figure 3.13. As expected, after the introduction of the negative inductor within the antenna structure, the first resonant
frequency has been shifted to a higher frequency due to the shift of mode 1 as depicted in Figure 3.6. The input reactance has been dramatically reduced as shown Figure 3.13(a). Moreover, the reactance of the antenna with the simulated negative inductor loading (blue curve) has relatively higher parallel resonance at 392MHz than the reactance of the antenna with the measured negative inductor loading (green curve) whose parallel resonance occurs at 375MHz.

3.4.2 Antenna Pattern

The realized antenna gain pattern with and without NFC loading for different frequency samples were simulated in FEKO. Figure 3.14 shows the coordinate system for the antenna gain simulation and measurements. Figure 3.15 shows the normalized gain of the azimuth plane ($-180^\circ \leq \phi \leq 180^\circ$ and $\theta = 90^\circ$) for different frequencies.

![Figure 3.13. Antenna input impedance with and without NFC. (a) Reactance (Imaginary part). (b) Resistance (Real part).](image)
For frequencies above 200MHz, the antenna pattern without loading is zoomed-in to show in more detail the actual pattern without impacting the scale of the loaded antenna pattern.

From Figure 3.15, it can be noted that the antenna without NFC loading maintains its shape up to 250MHz; whereas, the loaded antenna maintains its shape up to frequencies higher than 350MHz. Figure 3.16 shows the antenna realized gain versus frequency for the elevation plane ($\phi = 0^\circ \text{ and } -180^\circ \leq \theta \leq 180^\circ$). From Figure 3.16, it can be noted that the antenna with NFC loading maintains its shape for frequencies higher than the antenna without NFC loading. Moreover, the realized gain has been improved up to 12 dB. It can also be noted that the front to back ratio improved dramatically, especially for frequencies higher than 200MHz.
Figure 3.15. The normalized gain of the azimuth plane
\(-180^\circ \leq \phi \leq 180^\circ \) and \(\theta = 90^\circ \).

Figure 3.16. The antenna realized gain of the elevation plane
\(\phi = 0^\circ \) and \(-180^\circ \leq \theta \leq 180^\circ \).
3.5 Measured Results

Figure 3.17 shows the fabricated 3” wire circular half loop antenna mounted on an octagonal metal “ground plane” (30” long along its diagonal). The fabricated ground plane consists of Styrofoam covered with a Copper sheet.

![Figure 3.17. 3” wire circular half loop antenna mounted on an octagonal metal “ground plane” (30” long along its diagonal)](image)

3.5.1 Realized Antenna Gains

After connecting the fabricated non-Foster inductor to the fabricated half loop antenna, the realized antenna gain (for $\phi = 0^\circ$ and $\theta = 90^\circ$) with and without non-Foster loading was measured in an outdoor range. The measured realized antenna gains are plotted in Figure 3.18. The dashed blue and the dashed red curves in Figure 3.18 are measured data for the realized antenna gain with and without the NFC load, respectively. To confirm whether the measurement is correct, the simulated antenna
gain with and without NFC loading (solid blue and red curves) generated by using FEKO 6.3 is also plotted in Figure 3.18. When comparing these data, it can be observed that there is very good agreement between the simulated and measured results. However, there is a small discrepancy at lower and higher frequencies. This small discrepancy is expected since the antenna is electrically small within this range of frequencies. From Figure 3.18, it can be observed that the antenna gain has been improved up to 11 dB after introducing the NFC. Measurements were performed up to 300MHz because this is the highest operating frequency of the calibrated antenna.

3.5.2 Improved Signal to Noise Ratio (SNR)

Since the loaded half loop antenna is designed for receiving applications, the signal to noise ratio is an important figure of merit. Figure 3.19 shows a smoothed version of the signal to noise ratio (SNR) improvement that equals the SNR of the antenna loaded with non-Foster inductor (in dB) minus the SNR of the antenna without non-Foster inductor (in dB). The noise measurements were performed with a Spectrum Analyzer having a −98 dBm noise floor.
Figure 3.18. Simulated and measured realized antenna gains with and without non-Foster loading.

Figure 3.19. The measured improved signal to noise ratio (SNR).
3.6 Summary

A significant improvement in the performance of a half loop antenna has been achieved using the theory of Characteristic Modes and non-Foster loading. The theory of CM offers great physical insight since it allows the designer to determine individual desired modes and how to improve the antenna performance by looking into the mode interactions. The simulated results show that the desired omnidirectional TE mode is maintained up to higher frequencies. The impact of transmission lines on the implementation of negative inductor is studied as well. The fabricated negative inductor was designed based on a negative impedance inverter topology. The realized antenna gain with and without non-Foster loading at \((\phi = 0^\circ\text{ and } \theta = 90^\circ)\) showed great improvement. There is very good agreement between the simulated and measured results. By loading the half loop antenna with a negative inductor, the signal to noise ratio was improved up 10 dB for a wide frequency band.
CHAPTER 4

DESIGN AND ANALYSIS OF STABLE NON-FOSTER CIRCUITS

USING A FREQUENCY DOMAIN METHOD

4.1 Introduction

As previously mentioned, Non-Foster circuits (NFCs) can be useful in a large number of microwave and antenna applications; however, they are difficult to design because they are potentially instable. Consequently, an accurate and efficient systematic stability assessment is necessary during the design process to predict any undesired behavior. Unfortunately, many spectral stability tests are insufficient for the actual determination of stability in non-Foster systems [36]. Stability tests that measure port characteristics rather than considering the internal structure of a non-Foster circuit may potentially lead to a faulty conclusion about the non-Foster circuit’s stability. In this chapter, a complete stability assessment of non-Foster circuits is introduced as well as a systematic way to stabilize them. Non-Foster circuits based on the cross-coupled transistor topology (Linvill’s balanced topology is known also as floating negative impedance converter NFC-NIC [9]) are considered as an example to demonstrate the stability method. The reason for choosing this topology to demonstrate this stabilization technique is due to the numerous applications that the floating NFC-NIC can be incorporated. Some of the applications are:
Matching electrically small antennas using a series negative capacitor as in [12] and [19]. In this chapter, series negative capacitor-inductor and series negative capacitor-inductor with shunt passive inductor combinations will be used as case studies.

Using non-Foster multiport loading, simple structure broadband antennas (using negative capacitor-inductor or combination of Foster and non-Foster elements) can be achieved [13] and [72].

Fast-wave low-dispersion transmission lines can be achieved by using series negative inductor [15], which can be used for squint-free broadband leaky-wave antenna applications [16] and true time-delay lines for achieving low beam-squinting radiation in series-fed antenna arrays [17].

Reduction of parasitics in integrated circuits can be achieved with the help of non-Foster circuits [18].

Unfortunately, in previous work mentioned above, there is no mention of any systematic way to stabilize these systems. Some of them used ideal negative elements, and the rest did not study the stability properties of their circuits so that the systems may not completely stable. Therefore, in this chapter, the systematic stabilizing of two systems based on floating NFC-NIC is demonstrated.

It is clear now that the objective of this chapter is development of a systematic scheme to stabilize non-Foster circuits. Yet, a further objective is to match an electrically small antenna (monopole-type) using non-Foster circuits. In other words, the objective is to maximize the realized gain of a given non-Foster topology while ensuring the system’s
stability. A matching network consisting of a series negative capacitor-inductor circuit is the key component to be investigated. Moreover, a matching network (MN) consisting of a series negative capacitor-inductor with a shunt passive inductor will be briefly addressed as well. Secondly, the key stability analysis steps will be also discussed. Furthermore, the third section deals with the stabilization of the first MN by introducing a Pole-Zero identification analysis method. Since this chapter discusses the design of MNs, the concept of impedance matching using an active two-port network is reviewed. Measured data for the realized antenna gain as well as the improved signal to noise ratio (SNR) are provided in another section. Finally, the same steps for matching and stability analysis are briefly presented for the second MN.

4.2 Systematically Stabilization of Non-Foster Circuits Designed to Impedance Match an Electrically Small Monopole Antenna

Generally, the stability of NFCs depends on several factors, including

1) The external load that the NFC connects to.

2) NFC topology.

3) The passive load inside the NFC (the internal load).

4) The DC biasing of the active elements (transistors).

In this chapter, the external load is a monopole-type electrically small antenna. Figure 4.1(a) shows the monopole-type antenna used in this chapter, which is a 3-inch wire with radius 0.062-inch sitting on a circular ground plane of 9-inch radius. The measured input impedance of the monopole antenna has a reactance that can be modeled
as $C_a = 2.3\text{pF}$ and $L_a = 10\text{nH}$. The second factor is discussed in the following section. The third and fourth factors will be addressed later in this chapter.

**Choosing the Right NFC Topology**

The right NFC topology that can lead the system to be stable is a floating NFC topology, and that floating NFC is the one often referred to as open circuit stable (OCS). Figure 4.1(b) shows a floating OCS negative impedance converter (NIC) introduced by Linvill [9]. The approximate input impedance, which is derived in section 2.3 using the small signal π-model of the bipolar junction transistor BJTs, is

$$Z_{in} \approx -Z_L + \frac{2}{g_m}$$  \hfill (4.1)

There are a couple of reasons for choosing the OCS topology to design the matching network for the electrically small monopole. First, since the NFC needs to be connected in series (in cascade) with the ES monopole antenna as can be seen in Figure 4.2, which means the input signal of the NFC is mixing in series the input connection should be the emitter of the transistors as explained in [82]. Therefore, the NFC should be an OCS topology. Moreover, the voltage is the signal that is going to be mixing at the input so that the connection should be series. Second, the fact that no negative resistance is required for stability purposes as will be explained in the next section, the NFC OCS topology should be selected because it is easier to obtain a small positive resistance with an OCS than with a Short Circuit Stable (SCS) topology (compare equation 4.1 with equation 5.10). To complete the discussion about the choice of the NFC topology, the passive load within the NFC needs to be determined.
Figure 4.1. (a) Floating OCS NIC. (b) 3-inch monopole antenna mounted on circular ground plan of 9-inch radius.

Figure 4.2. Floating series connected negative capacitor-inductor to match monopole antenna. Labeled branches (1) to (6) are the locations of possible stabilizing networks.

Since the reactance of the electrically small monopole is highly reactive, the approximate impedance model of a monopole antenna is a series combination of a resistor, a capacitor, and an inductor. To mitigate the large reactance of the antenna, the
reactive passive load within the NFC is chosen to be a series combination of a capacitor $C_L$ and an inductor $L_L$. Once the topology of the NFC is chosen, the concept of impedance matching, which is the maximization of power transfer from a source to a load, will be addressed after studying the stability of the entire system.

In the next sections, the stabilization of a NFC-NIC will be discussed in a systematic way considering the schematic representation of a monopole antenna and a NFC-NIC depicted in Figure 4.2. To ensure the complete stability of these non-Foster circuits (NFCs), two main steps should be addressed:

1- Development of an equivalent model of the entire system to accurately describe the behavior of the system to be used in the stability assessment. (Port Stability)
2- Ensuring that all the closed loop modes of the actual system (not the system model) are on the left hand side of the complex plane (LHP). (Internal Stability)

### 4.3 Impedance Equivalent Model of the Entire System

The first step toward the complete stability of non-Foster systems is to accurately model the entire system. The entire system considered here is shown in Figure 4.3. The passive load within the NFC plays a key role determining the input impedance of the NFC. Therefore, determining the optimal NFC that can yield a stable system has a direct relationship to determining the optimal passive load within a NFC. Because the input/output port impedance modeling of the system will not take explicitly into account the internal behavior of the NFC, the stability analysis is known as port stability.

The stability analysis (port stability) will determine the conditions where the system can be stable. This is an important step since it determines constrains that should be taken
into account to finalize the design of a stable system with the required specifications; namely, improving the antenna realized gain and the signal to noise ratio.

The electrically small monopole antenna can be modeled as a parallel combination of a resistor $R_a$ and inductor $L_a$ in series with a capacitor $C_a$; while, the input impedance of the NFC-NIC can be modeled using the π-model of the BJT and the internal passive load as a series combination of a negative capacitor $C_n$, a negative inductor $L_n$ and a small positive $R_n$. For simplicity, the monopole antenna impedance model can be approximated by a series connection of a resistor $R_a'$, inductor $L_a'$ and a capacitor $C_a$. The approximation was done when $\omega^2 L_a^2 \ll R_a^2$, and as a result $L_a' = L_a$ and $R_a' = \frac{\omega^2 L_a^2}{R_a}$.

The equivalent circuit model of the monopole antenna with the non-Foster circuit and port impedance is depicted in Figure 4.3. It can be observed that all the components are in series and the system is a second order system. The governing differential equation can be found using the K.V.L,

$$v_{R_0} + v_{R_n} + v_{C_n} + v_{L_n} + v_{C_a} + v_{L_a'} + v_{R_a'} = 0$$

(4.2)

Substituting the constitutive equation of the elements in (4.2),

$$\left(R_0 + R_n + R_a'\right)i(t) + \left(L_n + L_a'\right)\frac{di(t)}{dt} + \left(\frac{1}{C_n} + \frac{1}{C_a}\right)\int_{-\infty}^{\tau} i(\tau)d\tau = 0$$

(4.3).

We let $R_{eq} = R_0 + R_n + R_a'$, $L_{eq} = L_n + L_a'$, and $C_{eq} = \frac{C_n C_a}{C_n + C_a}$.
Now, differentiating (4.3) and dividing the equation by $L_{eq}$, leads to the second order differential equation:

$$\frac{d^2 i(t)}{dt^2} + \frac{R_{eq}}{L_{eq}} \frac{di(t)}{dt} + \frac{1}{L_{eq} C_{eq}} i(t) = 0$$  \hspace{1cm} (4.4)

Equation (4.4) can be expressed in a general form:

$$\frac{d^2 i(t)}{dt^2} + 2\alpha \frac{di(t)}{dt} + \omega_0^2 i(t) = 0$$  \hspace{1cm} (4.5)

where $\alpha = \frac{R_{eq}}{2 L_{eq}}$, $\omega_0 = \frac{1}{\sqrt{L_{eq} C_{eq}}}$, and $\zeta = \frac{\alpha}{\omega_0}$. Thus, $\zeta = \frac{R_{eq}}{2 L_{eq} \sqrt{L_{eq} C_{eq}}}$. The solution of equation (4.5) is:

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$  \hspace{1cm} (4.6)

where $s_1 = -\alpha + \omega_0 \sqrt{\zeta^2 - 1}$ and $s_2 = -\alpha - \omega_0 \sqrt{\zeta^2 - 1}$. If $R_{eq} > 0$, which is true in here, the system is unstable in the following cases:
There is one necessary condition that can lead the system to be stable (to have exponential decay); namely,

\[ |C_n| > |C_a| and |L_n| < |L_a| \]

(4.7)

Keeping in mind these constrains, we optimize the system to have the best possible impedance matching for the small monopole antenna.

### 4.4 Ensure All the Closed Loop Modes of the Actual System Are on the Left Hand Side of the Complex Plane (LHP)

The second step toward the complete stability of non-Foster systems is ensuring that all the closed loop modes of the actual system (not the system model) are on the left hand side of the complex plane (LHP), which known as the internal stability. To detect all the system modes and reach the correct system stability conclusion one of two common approaches can be used [83] - [84]. The first approach is to use a state variable description of the entire circuit. Any unstable hidden modes (hidden for a given set of input and output ports) will appear as unstable eigenvalues. This approach requires a complete access to the entire model of the active and passive devices. The second approach, which is based on a frequency-domain approach [45], [54], is more practical.
and will be used from here on. In this section, this frequency-domain approach will be used to perform stability analysis and the system will be systematically stabilized with conventional control techniques (CCTs).

4.4.1 Frequency-Domain Approach for Stability Analysis

The analysis is based on the calculation of a single-input single-output (SISO) transfer function $H(s)$ of the circuit linearized about a particular steady state. This transfer function is obtained by applying a pole-zero identification algorithm to a simulated frequency response $H(j\omega)$ of the linearized circuit. The frequency response of the linearized circuit can be obtained as the impedance seen by a small-signal current source $i_{in}(j\omega)$ connected in parallel at a given circuit node $n$ as in Figure 4.4 (a) or as the admittance seen by small-signal voltage source $v_{in}(j\omega)$ inserted in series at a particular branch of the circuit as in Figure 4.4 (b) [54]. The transform functions corresponding to these two types of excitation can be expressed as:

\[
H(j\omega) = Z'(j\omega) = \frac{v_{out}(j\omega)}{i_{in}(j\omega)}, \quad (4.8.a)
\]

\[
H(j\omega) = Y'(j\omega) = \frac{i_{out}(j\omega)}{v_{in}(j\omega)}. \quad (4.8.b)
\]

The stability of the system has been assessed by calculating SISO transfer functions at different locations in the system using both simulated impedance and admittance frequency responses. Figure 4.5 shows the magnitude and phase of the simulated (green crosses) frequency response (impedance seen by current source) obtained at location (2) shown in Figure 4.2.
System identification algorithms\textsuperscript{1} were applied to the simulated frequency response (green crosses in Figure 4.5) to obtain the transfer function associated (identified transfer function) with the circuit frequency response. The magnitude and phase of the identified transfer function is shown also in Figure 4.5 as a solid blue line. Figure 4.5 shows excellent agreement between the simulated frequency response and the identified transfer function for a wide band frequency response.

The pole-zero map of the transfer function obtained at node (2), shown in Figure 4.6, has poles on the right-half of the complex plane (RHP). This indicates the system is unstable even though the necessary condition has been satisfied. It is clear that it is not enough to depend simply on the input and/or output port(s) model to determine the complete stability of non-Foster systems.

\textsuperscript{1} N4SID: Numerical algorithms for subspace state space system identification. In addition, transfer function identification based on nonlinear least squares.
Figure 4.5. Impedance frequency response at location (2) of Figure 4.2 shows the magnitude and phase of the simulated (green crosses) and the identified (solid blue).

Figure 4.6. The pole-zero map of impedance transfer function obtained at node (2) shows the system is unstable.
Determine whether the system has hidden unstable modes

In general, non-Foster systems (systems that include non-Foster circuits) can have unstable hidden poles (modes) which means the system can have an exact pole-zero cancelation. In [32], the author showed a simple example of the possibility of unstable hidden modes. In order to examine whether or not there are unstable hidden modes for the given system, several closed loop transfer functions have been examined. Keeping in mind that unstable (or stable) modes of all the transfer functions (system poles) are the same for a given system, the system does not have unstable hidden modes if all the examined transfer function for all possible nodes don’t show unstable hidden modes. In this particular system, several closed loop transfer functions have been examined. The pole-zeros maps show all the unstable modes are the same for different transfer functions, so fortunately the system does not have unstable hidden mode. In here, Figure 4.7 shows another pole-zero maps of the system but the transfer function obtained at branch (4), using the admittance form, which indicate the system is still unstable and does not have hidden modes. Both, Figures 4.6 and 4.7 not only have the same unstable poles, but also have the same system poles. Having the same poles for different transfer functions agrees with the principle of system theory. It states that the transfer functions (obtained at any branch or at any node) can have different numerators (different zeros), but all should have the same denominator (same poles), except for exact pole-zero cancellations since all the state variables in a linear system share the same dynamics [45], [83].
Figure 4.7. The pole-zero map of admittance transfer function obtained at branch (4) shows the system is unstable.

4.4.2 Series and Shunt Stabilization Networks

The stabilization networks can be included either in series or in parallel at certain locations within the system [54], [85]. In [45], a closed-loop transfer function was presented as a parallel combination of the transfer function $H(j\omega)$ and a stabilization network. More details about the total closed-loop transfer functions in terms the impedance $Z^*(j\omega)$ and the admittance $Y^b(j\omega)$ transfer functions were provided in [54]. The closed-loop impedance transfer function $Z^c_{cl}(j\omega)$ seen by the small-signal current source $i_{in}(j\omega)$ when a stabilization network $Z_{stab}(j\omega)$ is connected in parallel at a node $n$ (see Figure 4.8 (a)) is [45], [54]

$$Z^c_{cl}(j\omega) = \frac{v_{out}^n(j\omega)}{i_{in}(j\omega)} = \frac{Z^*(j\omega)Z_{stab}(j\omega)}{Z^*(j\omega) + Z_{stab}(j\omega)} = \frac{Z^*(j\omega)}{1 + \frac{Z^*(j\omega)}{Z_{stab}(j\omega)}}$$
The closed-loop admittance transfer function \( Y_{cl}^b(j\omega) \) seen by a small-signal voltage source \( v_{in}(j\omega) \) when a stabilization network \( Y_{stab}(j\omega) \) is connected in series at a branch \( b \) (see Figure 4.8 (b)) is [54]

\[
Y_{cl}^b(j\omega) = \frac{i_{out}^b(j\omega)}{v_{in}(j\omega)} = \frac{Y^b(j\omega)Y_{stab}(j\omega)}{Y^b(j\omega) + Y_{stab}(j\omega)} = \frac{Y^b(j\omega)}{1 + \frac{Y^b(j\omega)}{Y_{stab}(j\omega)}}. \tag{4.9.b}
\]

It is clear that equations (4.9.a) and (4.9.b) represent negative feedback transfer functions with feedback networks given by \( 1 / Z_{stab}(j\omega) \) and \( 1 / Y_{stab}(j\omega) \) on the original transfer functions \( Z^*(j\omega) \) and \( Y^*(j\omega) \), respectively.

Figure 4.8. Connection of a stabilization network. (a) In parallel with impedance stability analysis. (b) In series with admittance stability analysis.

As mentioned in section 4.4, conventional control techniques (CCTs) are used to stabilize the systems. Pole placement strategies based on proportional (P), derivative (D),
integral (I), proportional derivative (PD), or proportional integral (PI) controllers are applied to ensure the system stability. The proportional control action can be implemented with series or shunt resistors, while the derivative and the integral control actions can be achieved with series or parallel reactive elements. Lead compensation, which approximates the proportional derivative (PD), and lag compensation, which approximates the proportional integral (PI) controllers are more practical to implement [86].

**Proportional Controller**

The feedback network $B(j\omega)$ of the proportional control is a constant factor $B(j\omega) = K$.

The transfer function $H(j\omega)$ can be represented as a ratio of two polynomials [54]:

$$H(j\omega) = \frac{N(j\omega)}{D(j\omega)} \quad (4.10)$$

The closed-loop transfer function $H_{cl}(j\omega)$ can be represented as

$$H_{cl}(j\omega) = \frac{N(j\omega)}{D(j\omega) + K \cdot N(j\omega)} \quad (4.11)$$

The poles of $H_{cl}(j\omega)$ are provided by the roots of the characteristic equation

$$D(j\omega) + K \cdot N(j\omega) = 0 \quad (4.12)$$

From (4.9.a), in case of an impedance transfer function, the parallel connection of a resistor $R_{stab}$ at a node $n$ corresponds to applying a proportional control on $Z^n(j\omega)$ with

$$B(j\omega) = K = 1 / R_{stab} \quad (4.13)$$
From (4.9.b), in case of admittance transfer function, the series connection of a resistor $R_{stab}$ at a branch $b$ is equivalent to a proportional control on $Y^b(j\omega)$ with

$$B(j\omega) = K = R_{stab}$$  \hspace{1cm} (4.14)

**Derivative Controller**

The feedback network $B(j\omega)$ of the derivative control is

$$B(j\omega) = K_d \cdot (j\omega)$$  \hspace{1cm} (4.15)

The poles of $H_{cl}(j\omega)$ are provided by the roots of the characteristic equation as

$$D(j\omega) + K_d \cdot (j\omega) \cdot N(j\omega) = 0$$  \hspace{1cm} (4.16)

From (4.9.a), in case of an impedance transfer function, the parallel connection of a capacitor $C_{stab}$ at a node $n$ corresponds to a derivative control on $Z^n(j\omega)$ with

$$B(j\omega) = C_{stab} \cdot (j\omega)$$  \hspace{1cm} (4.17)

From (4.9.b), in case of an admittance transfer function, the series connection of an inductor $L_{stab}$ at a branch $b$ is equivalent to a derivative control on $Y^b(j\omega)$ with

$$B(j\omega) = L_{stab} \cdot (j\omega)$$  \hspace{1cm} (4.18)

**Integral Controller**

The feedback network $B(j\omega)$ of the integral control is

$$B(j\omega) = K_i \cdot (j\omega)$$  \hspace{1cm} (4.15)

The poles of $H_{cl}(j\omega)$ are provided by the roots of the characteristic equation as

$$(j\omega) \cdot D(j\omega) + K_i \cdot N(j\omega) = 0$$  \hspace{1cm} (4.16)
From (4.9.a), in case of an impedance transfer function, the parallel connection of a inductor $L_{stab}$ at a node $n$ corresponds to a integral control on $Z_n(j\omega)$ with

$$B(j\omega) = \frac{1}{L_{stab} \cdot (j\omega)}$$ (4.17)

From (4.9.b), in case of an admittance transfer function, the series connection of a capacitor $C_{stab}$ at a branch $b$ is equivalent to a integral control on $Y_b(j\omega)$ with

$$B(j\omega) = \frac{1}{C_{stab} \cdot (j\omega)}$$ (4.18)

**Lead Compensation**

The feedback network $B(j\omega)$ of the lead compensation is

$$B(j\omega) = \frac{K \cdot (j\omega) + z}{(j\omega) + p} \quad \text{where } z < p$$ (4.19)

The poles of $H_{cl}(j\omega)$ are provided by the roots of the characteristic equation as

$$(j\omega + p) \cdot D(j\omega) + K \cdot (j\omega + z) \cdot N(j\omega) = 0$$ (4.20)

One example of lead compensation is provided here. From (4.9.b), in case of an admittance transfer function, the series connection of a parallel combination of a resistor $R_{stab}$ and inductor $L_{stab}$ at a branch $b$ is equivalent to a lead compensation on $Y^b(j\omega)$ with

$$B(j\omega) = \frac{R_{stab} \cdot (j\omega)}{(j\omega) + R_{stab}/L_{stab}}$$ (4.21)

where $z = 0$, $p = R_{stab}/L_{stab}$, and $K = R_{stab}$.

**4.4.3 Stabilization of the System to Ensure All Modes on the LHP**
Since the system is unstable, it is shown in this subsection how to stabilize the NFCs using the aforementioned controllers and compensations. The non-Foster circuit topology used in this chapter consists of only two cross-coupled transistors; therefore, the potential locations of the controllers are limited as shown in Figure 4.2. The controllers should not be placed between the NFC and the antenna or at the input port (location (1) and (6)) in order not to degrade the performance of the system. For example, if a proportional controller is introduced, losses will be added to the system, which has a deleterious effect on the efficiency of the antenna system. Improper use of derivative and integral controllers can also distort and negatively affect the behavior of non-Foster circuits.

To maintain the system symmetry, conventional controllers should be introduced either at branches (2) and (4) or at branches (3) and (5). It was determined that introducing any of the CCTs at both locations has the same impact on the system stability and, as a result, the CCTs are added at branches (2) and (4). It should be kept in mind that the performance of the non-Foster circuit must be maintained at higher frequencies while the system is stabilized. In this particular system, conventional controllers have been introduced in branches (2) and (4).

From Figures 4.6 and 4.7, it can be clearly seen that there are several unstable poles (modes). There is a pair of unstable modes located at \((8e^8, \pm j4.4e^9)\), also there is another pair of unstable modes around \((4.087e^8, \pm j1.43e^9)\). Moreover, there is couple of unstable modes close to the origin. The dominant unstable mode is the one (or the pair) located furthest to the right on the RHP. It is clear that the pair of unstable modes located at
(8e^8, ±j4.4e^9) is the one located furthest to the right on the RHP. However, it seems that the last mentioned pair of modes is just the third harmonics of the pair located at (4.087e^8, ±j 1.43e^9). To verify this assumption, the higher order harmonics should move further to the left when the pole corresponding to the fundamental frequency is stabilized (moved to the LHS of the complex plane). This is true under the assumption that the higher order harmonics carry less power than the fundamental. In theory, stabilizing the dominant modes can lead to a stable system. Therefore, firstly, the stability analysis of the system will be based on stabilizing the dominant modes. The analysis was carried out with a P-controller (series resistor) introduced at branches (2) and (4). Any resistor value to be chosen should maintain the non-Foster behavior of the matching network. As it can be seen from Figure 4.9 (zoomed in to see the effect of the controller on the dominant modes), the dominant modes move toward the left hand side of the complex plane (LHP). From Figure 4.9, the blue crosses is the original location of the unstable dominant modes, and the red squares show the location of the modes after introducing a 10Ω resistor at the load location to mitigate the losses introduced by the transistor parasitic (see equation 4.1). The green rhombus refers to the dominant modes after introducing a P-controller at branches (2) and (4) with R_{stab} equal to 20 Ω. Finally, the blue triangle was determined when R_{stab} is equal to 40 Ω. It was determined that the system becomes stable when R_{stab} is equal to 60 Ω as indicated by the red plus signs.
Figure 4.9. Evaluation of the dominant pair of complex-conjugate modes versus P-controller values.

Figure 4.10. The pole-zero map of the admittance transfer function obtained at branch (4) when $R_{stab}=60$ is introduced at branches (2) and (4).
The pole-zero map of the transfer function obtained at branch (4), using an admittance form, when \( R_{\text{stab}} \) is equal to 60 \( \Omega \) is shown in Figure 4.10. Figure 4.10 (a) shows that the system’s dominant poles and their higher order harmonics are on the left-hand of the complex plane (LHP); however, by zooming in near the origin (Figure 4.10 (b)), there are still some poles on the RHP. Larger values for the P-controller were applied to the system, but there was no P-controller that can stabilize the system while maintaining the non-Foster behavior of the matching network. To determine whether the system can be stable under other controllers, another attempt to stabilize the circuit was done by inserting a series capacitor in branches (2) and (4). It was noticed that there is no value of capacitance that can drive the system into stability. The stability analysis was also carried out with an inductor (D-controller), but the system still remained unstable. It was determined that there was not gain compensation (the aforementioned controllers and compensations in the subsection 4.4.2) that can move the poles, which are close to the origin, to the LHP to make the system completely stable. It is then necessary to introduce another stabilization technique to ensure complete stabilization of the system. The RHP poles, which are close to the origin, have very low oscillation frequencies; therefore, phase stabilization or a notch filter, can be introduced to mitigate these frequencies. Although phase stabilization is susceptible to errors when determining the oscillation frequencies, the uncertainty can be greatly reduced by the low gain at high frequencies and the wider band of the notch filter. Here, a two-phase compensation is introduced at nodes (2) and (4). The phase compensation consists of a series
compensation of an inductor, a capacitor, and a large resistor designed not to contribute a large amount of noise. Finally, the system stability also depends on the optimized transducer gain of the matching network. In other words, it is necessary to optimize for maximum stable transducer gain.

Matching Network Design for Electrically Small Monopole Antenna

As shown in subsection 2.4.1, the realized antenna gain, with the active matching network included (see Figure 4.11), can be expressed for the receiving case as [56]

\[ G_{aw} = G_{\text{IEEE}} \left( \frac{|S_{12}|^2}{1 - |S_{22} \Gamma_L|^2} \right) (1 - |\Gamma_L|^2) \]  \hspace{1cm} (4.22)

where \( G_{\text{IEEE}} \) is the IEEE antenna gain, \( S_{12} \) is the transmission factor from port\#2 to port\#1, and \( \Gamma_L \) is the antenna reflection coefficient.

![Matching Network S2P](image)

Figure 4.11. Active matching network connected to the antenna.
The above result is obtained assuming the receiver (network analyzer) is a 50Ω, so the reflection coefficient looking into the receiver is zero. The gain improvement due to the matching network is defined as the ratio of the antenna gain with MN to the gain without MN as

\[
G_{imp} = \left( \frac{|S_{12}|^2}{|1 - S_{11}\Gamma_L|^2} \right)
\]

in which the antenna gain without MN is \( G = G_{\text{IEEE}} \left( 1 - |\Gamma_L|^2 \right) \). To obtain the best (higher) realized antenna gain, equation (4.23) needs to be optimized, but keeping in mind that the system needs to be stable.

So far, P-controllers were added at branches (2) and (4) of the non-Foster MN. Moreover, a resistor was added at the load location as well as a capacitor and a inductor. After the values of these components are optimized, the system still has poles in the RHP. As mentioned previously, phase compensation is introduced at nodes (2) and (4) to ensure the complete stability of the system. Therefore, the resistors of the phase compensation component are additional variables in the gain optimization. All the added components are shown in Figure 4.12 while their optimized values are depicted in Table 4.1. This MN delivers the best possible power transfer; meaning it transfers the maximum received power by the antenna to the receiver while maintaining the system stable. Results for the realized antenna gain and the improved SNR will be shown in the next section.
The system is completely stable when lead compensations are introduced at branches (2) and (4), and phase compensation are introduced at node (2) and (4). Note that a stabilization attempt using only phase compensation was performed; however, the system could not be stabilized. Several transfer functions were examined and all of them ensure that all the system poles are on the LHP. The pole-zero map of transfer function obtained at node (4) are shown in Figure 4.13. This figure shows that all the system poles are on the left-hand of the complex plane (LHP) which again indicates that the system is completely stable. Table 4.1 lists the value of the schematic representation of series negative capacitor-inductor included DC biasing and stabilization networks. The NIC load reactance was chosen to be \( C_L = 3.3 \text{pF} \) and \( L_L = 12 \text{nH} \). Although \( L_L > L_a \), \(|L(Z\text{NIC})| < L_a\) due to the non-ideal behavior of the NIC.

<table>
<thead>
<tr>
<th>Element</th>
<th>Value</th>
<th>Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Block</td>
<td>6.8nF</td>
<td>RFC</td>
<td>6.5(\mu)H</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>14.7KΩ</td>
<td>( R_{c2} )</td>
<td>75Ω</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>16.9KΩ</td>
<td>( L_{c1} )</td>
<td>30nH</td>
</tr>
<tr>
<td>( R_E )</td>
<td>200Ω</td>
<td>( R_{p1} )</td>
<td>100Ω</td>
</tr>
<tr>
<td>( R_L )</td>
<td>35Ω</td>
<td>( R_{p2} )</td>
<td>1000Ω</td>
</tr>
<tr>
<td>( C_L )</td>
<td>3.3pF</td>
<td>( L_p )</td>
<td>24nH</td>
</tr>
<tr>
<td>( L_L )</td>
<td>12nH</td>
<td>( C_p )</td>
<td>22nF</td>
</tr>
<tr>
<td>( R_{c1} )</td>
<td>30Ω</td>
<td>( R_0 )</td>
<td>50Ω</td>
</tr>
<tr>
<td>( L_{c1} )</td>
<td>30nH</td>
<td>( V_{dc} )</td>
<td>12V</td>
</tr>
</tbody>
</table>
Figure 4.12. Schematic representation of series negative capacitor-inductor, including DC biasing and stabilization networks, port impedance, and monopole antenna.

Figure 4.13. Pole-zero map of the transfer function obtained at node(4) with gain compensation introduced at branches (2) and (4) and phase compensation introduced at nodes (2) and (4).
Figure 4.14. Impedance frequency response at location (4) of Figure 4.2 shows the magnitude and phase of the simulated (green crosses) and the identified (solid blue).

To show the perfect agreement between the simulated frequency response and the identified transfer function for wide band frequency response, Figure 4.14 shows the magnitude and phase of the simulated (green crosses) and the identified (solid blue) admittance frequency response seen by a small-signal voltage source at location (4).

4.5 Measured Results

After stabilizing the system, the series combination of negative capacitor-inductor matching network comprised of two discrete NPN BJTs (NE68133) with inductor-resistor biasing was fabricated using RT/duroid 5880 substrate with $\epsilon_r = 2.2$, $\tan\delta = 0.0009$, and thickness= 62 mil to match a receiver with 50 $\Omega$ internal resistance to a monopole antenna. As mentioned previously, the non-Foster matching network is designed based on a balanced OCS NIC shown in Figure 4.1(a). Figure 4.15 shows the fabricated negative capacitor-inductor matching network. In this section, the realized antenna gain with and without the non-Foster matching and the improvement signal to
noise ratio (SNR) based on measurements are provided. Although the measurement of
the power capability of the non-Foster matching network is not shown, the fabricated
non-Foster matching network maintains linear operation up to 3 dBm.

![Fabricated negative capacitor-inductor matching network](image)

**Figure 4.15. Fabricated negative capacitor-inductor matching network**

### 4.5.1 Antenna Gain and Signal-to-Noise Ratio

The antenna gain and SNR measurements with and without the non-Foster matching
network were performed in an outdoor range for 30MHz to 730MHz. Since two different
calibration antennas are used in the outdoor measurement, each set of measured results is
intentionally addressed in two different subsections (Lower and higher frequencies).

**Lower Frequencies Measurements (30MHz to 300MHz):**

Note that the outdoor range setup and procedure for this measurement were the same as
(including time gating) in section 2.5.2. After calibrating the time-gated data with a
reference calibration antenna, the measured realized gains are plotted in Figure 4.16. The solid red and black curves in Figure 4.16 are measured data for the realized antenna gain with and without the NLC-MN, respectively. To confirm whether the measurement is correct, the simulated antenna gain without a MN (dashed black curve) generated by using FEKO v7.0 is also plotted in Figure 16. There is excellent agreement between the measured and simulated data.

To investigate the improvement of the antenna gain with the NLC-MN, the difference (in dB) between the gains with and without the MN is plotted in Figure 4.17. It can be observed that the antenna gain with the NLC-MN is clearly improved in the range 30MHz to 300MHz. For instance, the improvement with the NLC-MN is around 16 dB at 240MHz.

Herein, the same procedure of section 2.5.2 is used to assess the improvement of the SNR with the non-Foster matching network. Note that the noise floor of the spectrum analyzer used in this measurement is around $-102\text{dBm}$. Figure 4.18 shows the measured improvement of SNR with the NLC-MN relative to the case without a MN.
Figure 4.16. Measurement of the realized antenna gains for frequencies 30MHz to 300MHz.

Figure 4.17. Improvement of measured antenna gain with the non-Foster matching relative to the case without a matching network over 30MHz to 300MHz.
Figure 4.18. Improvement of SNR with a non-Foster impedance matching network relative to the case without a matching network over 30MHz to 300MHz.

**Higher Frequencies Measurements (300MHz to 730MHz):**

Measurements at frequencies above 300MHz, where performed in the same outdoor range but using different reference antenna. The measurement procedure is the same as that of the lower frequencies measurements, including time-gating. Simulated and measured realized antenna gains are plotted in Figure 4.19 and the improvement of antenna gain with the NLC-MN relative to case without a MN is shown in Figure 4.20. Although DC power is required to bias the active elements (i.e., transistors), the calculated realized gain only considers the RF input power. The transistors are biased with a 10V DC voltage and a DC current (collector) of 13mA.

Finally, a measured result for the improvement of SNR with the negative capacitor-inductor MN relative to the case without a MN is plotted in Figure 4.21. It can be
observed that the overall SNR with the NLC- MN (smoothed curve) is higher than the SNR without a MN up to around 730MHz.

Figure 4.19. Measurement of the realized antenna gains for frequencies 300MHz to 730MHz.
Figure 4.20. Improvement of measured antenna gain with the non-Foster matching relative to the case without a matching network over 300MHz to 730MHz.

Figure 4.21. Improvement of SNR with a non-Foster impedance matching network relative to the case without a matching network over 300MHz to 730MHz.
4.6 The Complete Stability of Foster and Non-Foster Circuits to the Match of Electrically Small Monopole

In this section, combination of Foster and non-Foster circuits as a matching network for the same small monopole, in Figure 4.1(a), is addressed. To the best of our knowledge, all fabricated non-Foster impedance matching networks for monopole or dipole-type small antennas have been made up of either only single negative capacitor connected in a series or a parallel to the antenna as in [11], [14], and [19] or as a series combination of negative capacitor-inductor as in [14] and in the previous subsections of this chapter.

![Diagram of a floating series negative capacitor-inductor with shunt passive inductor](image)

Figure 4.22. Floating series negative capacitor-inductor with shunt passive inductor to match small monopole antenna.

In this section, a brief discussion of the complete stability of the system shown in Figure 4.22 will be addressed. Moreover, the measured data of the realized antenna gain and the improved SNR will be shown. As mentioned previously, the first step toward the
stability assessment is to model the entire system, and then obtain the stability conditions. The second step is to ensure that all the closed loop modes of the actual system (not the system model) are on the left hand side of the complex plane (LHP).

4.6.1 Impedance Modeling of the Entire System

As explained in section 4.3, the electrically small monopole antenna can be modeled as a parallel combination of a resistor $R_a$ and inductor $L_a$ in series with a capacitor $C_a$. The input impedance of the NFC-NIC can be modeled using the $\pi$-model of the BJT and the internal passive load as a series combination of a negative capacitor $C_n$, a negative inductor $L_n$, and small positive $R_n$ along with passive inductor $L_{in}$ that is in shunt with the receiver’s internal resistor $R_0$. For simplicity, at low frequencies, the monopole antenna impedance model can be approximated by a series connection of a resistor $R_a'$, inductor $L_a'$ and a capacitor $C_a$. The approximation was done when $\omega^2 L_a^2 \ll R_a^2$, and as a result

$L_a' = L_a$ and $R_a' = \frac{\omega^2 L_a^2}{R_a}$.

Moreover, the parallel combination of the inductor $L_{in}$ and the receiver’s internal resistor $R_0$ can be approximated by a series connection of a resistor $R_{0}'$ and inductor $L_{in}'$. The approximation was done when $\omega^2 L_{in}^2 \ll R_{0}^2$, and as a result $L_{in}' = L_{in}$ and $R_{0}' = \frac{\omega^2 L_{in}^2}{R_{in}}$. The approximated model of the system is shown in Figure 4.23.

According to [6], the system is only stable if the determinant of the impedance matrix (equation) does not have zeros in the right half of the complex plane (RHP). The impedance equation of the system depicted in Figure 4.23 is:
\[
Z_{sys} = R_\alpha' + Z_{L_n} + Z_{C_a} + Z_{L_a} + R_n + Z_{L_{in}'} + R_{sn}
\] (4.24)

Rearranging (4.25), one obtains,
\[
Z_{sys} = R_\alpha' + R_n + R_{sn} + Z_{C_a} + Z_{L_n} + Z_{L_a} + Z_{L_{in}'} + Z_{L_{in}''}
\] (4.25)

If the equivalent resistor, \( R_{eq} = R_\alpha' + R_n + R_a' \), is positive, which is the case in here, there is only one condition that can lead equation (4.25) to not have zeros in the right half of the complex plane (RHP), namely,
\[
|C_n| > |C_a| \text{ and } |L_{n}| < |L_a + L_{in}'|
\] (4.26)

The reactance of the monopole antenna is modeled as \( C_a = 2.3 \text{pF} \) and \( L_a = 10 \text{nH} \). For the system to be stable, the internal load of the NIC and the passive inductor are chosen to be \( L = 6.8 \text{nH} \), \( C_L = 8.2 \text{pF} \), and \( L_{in} = 8.2 \text{nH} \). The port stability assessment of the system model showed that the system is stable; however, when the actual system was designed based on the aforementioned constrains, the system tended to be unstable. Therefore, stabilization networks are introduced to ensure that the system does not have RHP modes.
4.6.2 Ensure All the Closed Loop Modes of the Actual System Are on the LHP

Herein, the same procedure of section 4.4, including the frequency domain method, is used to assess the stability of the entire system. The system is completely stable when lead compensations are introduced at branches (2) and (4), and phase compensation introduced at the passive load of the NIC. The lead compensation is a parallel combination of resistor and inductor, that is $30\Omega/30\text{nH}$. The passive load is a series combination of capacitor-inductor and a series resistor introduced to reduce the losses and help in the stabilization. A parallel inductor of $1.8\mu\text{H}$ is introduced at the load to work along with the load as the phase compensation. Several transfer functions were examined and all of them ensured that all the system poles are on the LHP. The pole-zero map of transfer function obtained at branch (4) is shown in Figure 4.24.

![Pole-zero map](image)

Figure 4.24. The pole-zero map transfer function obtained at branch (4) when gain compensation introduced to the system at branches (2) and (4) and phase compensation introduced at load location showing the system is stable.
4.6.3 Measured Results

After stabilizing the system, the Foster and non-Foster matching network was fabricated using the same Microstrip substrate and the BJTs (NE68133) as in section 4.5. Fig. 4.25 shows the fabricated matching network. In this section the realized antenna gain with and without the non-Foster matching and the improvement of signal to noise ratio (SNR) based on measurements are provided.

![Fabricated Foster-no-Foster matching network](image)

Figure 4.25. Fabricated Foster-no-Foster matching network

**Antenna Gain and Signal-to-Noise Ratio**

The antenna gain and SNR measurements with and without the non-Foster matching network were performed in an outdoor range for 300MHz to 730MHz. Note that the outdoor range setup and procedure for this measurement were the same as that (including time gating) in sections 2.5.2 and 4.5.1. Simulated and measured realized antenna gains
are plotted in Figure 4.26 and the improvement of antenna gain with the NLC-L MN relative to case without a MN is shown in Figure 4.27.

Herein, the same procedure of section 2.5.2 and 4.5.1 is used to assess the improvement of the SNR with the non-Foster matching network. Again, the noise floor of the spectrum analyzer used in this measurement is around $-102\text{dBm}$. Figure 4.28 shows the measured improvement of SNR with the NLC-L MN relative to the case without a MN.

![Figure 4.26. Measurement of the realized antenna gains for frequencies 300MHz to 730MHz.](image)
Figure 4.27. Improvement of measured antenna gain with the non-Foster matching relative to the case without a matching network over 300MHz to 730MHz.

Figure 4.28. Improvement of SNR with a non-Foster impedance matching network relative to the case without a matching network over 300MHz to 730MHz.
4.7 Summary

In this chapter, the complete stability of non-Foster systems was introduced. Two systems have been introduced, designed, and fabricated as case studies. The first system was a series combination of a negative capacitor-inductor used to match electrically small monopole to a $50\,\Omega$ receiver, and the second system was a series combination of negative capacitor-inductor and shunt passive inductor.

The complete stability analysis of the entire systems was demonstrated in several steps demonstrating a systematic way to assess the stability of non-Foster systems. A Pole-zero identification method was used to assess the internal stability and it helped to stabilize the entire system using conventional control techniques (CCT); namely, gain and phase compensations were introduced to the systems to ensure the complete stability.

Measured data of the realized antenna gain and SNR with and without the non-Foster matching networks were performed in an outdoor range. The measurement results of the first system were shown in two bands (30MHz to 300MHz and 300MHz to 730MHz); however, the measured results of the second system, in which combination of Foster and non-Foster networks were used as a matching network for the monopole antenna, was shown in one band (300MHz to 730MHz). Tremendous improvement was shown for both systems in terms of the antenna realized gain and the SNR when using non-Foster MN or Foster-non-Foster MN.
CHAPTER 5
BROADBAND BIOMIMETIC ANTENNA ARRAYS USING NON-FOSTER NETWORKS

5.1 Introduction

It has been previously demonstrated that the mechanism for the hearing system of a type of insect can be used to boost the sensitivity of direction-finding systems. This is accomplished by amplifying the phase difference between two closely spaced small antennas by inserting a passive, coupling network [87] [88]. These types of arrays are referred to in the literature as biomimetic antenna arrays (BMAAs). Moreover, it was also shown that such BMAAs could achieve directional sensitivities significantly larger than those of regular antenna arrays with the same aperture dimensions. Therefore, these antenna arrays are expected to find applications in areas such as compact direction finding systems, high-resolution small-aperture radar systems, and microwave and millimeter-wave imaging systems. The two-element biomimetic antenna array (BMAA) reported in [87] takes two signals with the same magnitude and a small phase difference between them and converts them to two output signals with a notably large phase difference between them. However, in general, the phase enhancement of traditional BMAAs based on passive components is obtained over a relatively narrow frequency range due to gain-bandwidth restrictions derived by Bode and Fano [6] - [7]. In this
chapter, we propose to use non-Foster elements to design and implement a coupling network to enhance the direction-finding capability of a BMAA over a broad-bandwidth with a considerable phase-enhancement factor.

Non-Foster circuits (NFC) can be implemented with negative impedance converters (NIC) and/or Inverters (NII). This class of active circuits can violate Foster’s Reactance Theorem, which is only applicable to passive lossless circuits, and produce a reactance that has a negative slope with respect to frequency. NFCs are usually implemented with active transistor-based circuits. Non-Foster impedance circuit components can be very attractive for microwave and antenna applications for the reasons mentioned above. However, caution needs to be taken when designing such circuits since they tend to be unstable. This chapter is organized as follows: the first part introduces the BMAA architecture that will be used through this chapter. Secondly, the design of a passive-BMAA to show the concept and constraints of using passive networks will be explained. The third part of this chapter is the design of a wideband BMAA using non-Foster circuits. Finally, simulated and measured results based on the fabricated antenna and non-Foster circuits are provided.

5.2 BMAA Architecture

A block diagram of a two-element BMAA composed of two omnidirectional receiving antenna is shown in Figure 5.1 [88]. The outputs of the antennas \( (x_1, x_2) \) are fed to the two inputs of an external coupling network.
The external coupling network has two inputs and two outputs. The external coupling network takes two input signals, \( x_1 \) and \( x_2 \) with roughly the same magnitudes and a small phase difference of \( \Phi_{in}(\theta) \) between them and converts them to two output signals, \( y_1 \) and \( y_2 \), with a large phase difference of \( \Phi_{out}(\theta) \).

\[
\Phi_{in}(\theta) = \angle x_2 - \angle x_1 = \frac{2\pi l}{\lambda} \sin(\theta)
\]

\[
\Phi_{out}(\theta) = \angle y_2 - \angle y_1
\]

In this chapter, the architecture for a two element BMAA, shown in Figure 5.2, is adopted to enhance the phase difference between the array elements [89]. The coupling network between the two antennas is composed of the reactances \( B_1 \) and \( B_2 \). The output loads (ports) are modeled with two parallel conductances \( G_L \). The BMAA consists of a two-element antenna array composed of two 7.5 cm long monopole antennas with diameter 3.15mm spaced at a distance of 4 cm away from each other. The monopoles are placed on top of a ground plane with dimension of 41x 41cm\(^2\). At 300MHz, the length and spacing are \( \lambda/13.34 \) and \( \lambda/25 \), respectively. Moreover, at 600MHz, the length and spacing are \( \lambda/6.67 \) and \( \lambda/12.5 \), respectively. Two-port Y parameters are obtained to use.
them in the design of the external coupling network to maximize the phase enhancement factor. The phase enhancement factor is defined as the ratio of the slope of the output phase difference of the BMAA, $\Phi_{out}(\theta) = \angle V_2 - \angle V_1$, with respect to $\theta$ at $\theta = 0^\circ$ when the coupling network is included to the slope of $\Phi_{in}(\theta) = \angle V_2 - \angle V_1 = \frac{2\pi d}{\lambda} \sin(\theta)$ with respect to $\theta$ at $\theta = 0^\circ$ when there is no coupling network (regular array).

### 5.3 Design of BMAA Using Passive Coupling Network

To demonstrate the concept of phase amplification between the two antennas, passive coupling networks are first considered. The antennas are modeled with Norton equivalent circuit models including the coupling between the antennas. The external coupling network is composed of $B_1$ and $B_2$. The output loads are modeled with conductances $G_L$ as shown in Figure 5.2 (b). An analysis of even and odd modes including the external coupling networks is performed. The voltages $V_1$ and $V_2$ which are indicated in Figure 5.2 can be obtained as:

\[
V_1 = \frac{\cos(\Phi_{in}(\theta) / 2)}{(G_L + G_{11} + G_{12}) + jA} - \frac{j \sin(\Phi_{in}(\theta) / 2)}{(G_L + G_{11} - G_{12}) + jB} \tag{5.1}
\]

\[
V_2 = \frac{\cos(\Phi_{in}(\theta) / 2)}{(G_L + G_{11} + G_{12}) + jA} + \frac{j \sin(\Phi_{in}(\theta) / 2)}{(G_L + G_{11} - G_{12}) + jB} \tag{5.2}
\]
where

\[ A = B_{11} + B_{12} + B_1 \]  \hspace{1cm} (5.3.a) \\
\[ B = B_{11} - B_{12} + B_1 + B_2 \]  \hspace{1cm} (5.3.b)

\( G_{11} \) is the real part of the antennas’ admittances; \( G_{12} \) is real part of the antennas mutual admittances; \( B_{11} \) is the imaginary part of the antennas’ admittances while \( B_{12} \) is imaginary part of the antennas mutual admittances. \( G_{11}, G_{12}, B_{11}, \) and \( B_{12} \) can be obtained.
at different frequencies from the designed antenna. The output phase difference is
\[ \Phi_{\text{out}}(\theta) = \angle V_2 - \angle V_1. \]

The goal is to find \( B_1, B_2, \) and \( G_L \) that maximize the phase enhancement factor (defined in the last section). The center frequency of the design is considered to be 450MHz in which the length and spacing are \( \lambda/9 \) and \( \lambda/16.67 \), respectively. For a phase enhancement factor larger than 10, the admittance value of \( B_1 \) is obtained using an inductor with inductance \( L = 12nH \) and the admittance value of \( B_2/2 \) is obtained using a capacitor with capacitance \( C = 3.3pF \). These values were obtained with a load conductance equal to \( G_L = 1m\Omega^{-1} \). Using these values, the response of the BMAA is simulated in Agilent ADS and the results are presented in Figure 5.3. This figure shows the output phase response of the BMAA as a function of the angle of incidence of the EM wave at 455MHz; the output phase response of the regular two-elements array is also depicted in Figure 5.3. To complete the design, a matching network consisting of a capacitor with capacitance \( C = 1.6pF \) and an inductor with inductance \( L = 82nH \) is implemented as well. All the capacitors and inductors used in the simulation use the S-parameters of the actual elements provided by known vendors (Murata for capacitors and Coilcraft for inductors). As shown in Figure 5.3, the output phase response of BMAA is larger than the output phase response of regular array with the same aperture. However, this comes at the expense of sacrificing the output power level of the antenna [87].
Figure 5.3. Output phase response of two element array without any external coupling network and two-elements BMAA

Figure 5.4. (a) The normalized output power of BMAA as a function of angle of incidence of the EM wave at 455MHz. (b) Output phase responses of BMAA over 425-455MHz showing the 3-dB bandwidth of BMAA.
Figure 5.4 (a) shows the available output power of the BMAA normalized to the output power available from the regular array. The result in Figure 5.4 (a) is similar to what achieved in [89]. The 3-dB bandwidth of the BMAA output phase response is shown in Figure 5.4 (b), which shows a narrow frequency range around 30MHz. Moreover, the narrow bandwidth of the matching network sections, which transforms the load conductance to 0.02 Ω⁻¹, also plays a rule in limiting the BMAA bandwidth. The fact that the phase enhancement of traditional BMAAs based on passive components is obtained over a relatively narrow frequency range is a result of the gain-bandwidth restrictions derived by Bode and Fano [6]- [7]. In the next sections, we propose to use non-Foster elements to design and implement a coupling network to enhance the output phase response of a BMAA over a broad-bandwidth.

5.4 Design of Broadband BMAA Using Ideal Non-Foster Coupling Network

For broadband BMAA design, only the susceptance \( B_2/2 \) of Figure 5.2 (a) has been replaced by the admittance \( Y_2/2 \) to provide a possible extra degree of freedom since the non-Foster network can have either positive or negative real part of it is impedance or admittance when being realized. However, the susceptance \( B_1 \) has been kept the same as in the previous case. An analysis of even and odd modes including the external coupling networks including the \( Y_2 \) instead of \( B_2 \) is performed. The voltages \( V_1 \) and \( V_2 \), which are indicated in Figure 5.2, can be obtained as:
where

\[
A = B_{11} + B_{12} + B_1
\]

(5.6.a)

\[
B = B_{11} - B_{12} + B_1 + B_2
\]

(5.6.b)

\[G_{11}\] is the real part of the antennas' admittances; \(G_{12}\) is real part of the antennas mutual admittances; \(B_{11}\) is the imaginary part of the antennas' admittances; \(B_{12}\) is imaginary part of the antennas mutual admittances. \(G_{11}, \ G_{12}, \ B_{11}\), and \(B_{12}\) have been obtained at different frequencies from the coupled monopole antenna designed using FEKO 6.3. The output phase difference is \(\Phi_{\text{out}}(\theta) = \angle V_2 - \angle V_1\).

The goal is to find \(B_1, B_2, G_2\) and \(G_L\) that maximize the phase enhancement factor.

The design frequency band is chosen to be 380-600MHz. The analysis is carried out at discrete frequencies with step size of 10MHz. For a phase enhancement factor larger than 10, the admittance values of \(B_{2}/2\) can be fitted to a curve that has a negative slope, which means the element has a non-Foster behavior, and the admittance value is obtained using an negative capacitance \(C_p=-12pF\). Moreover, the real part value of \(Y_{2}/2\) is obtained using a negative resistor of value \(R_p=-97\Omega\). Furthermore, the admittance value of \(B_1\) is obtained to be zero which implies that there will not be an extra element for the coupling network. These values were obtained for a load conductance of \(G_L=0.02\Omega^{-1}\), implying there is no need for a matching network (another advantage of the
non-Foster elements over the passive ones found in the literature). The output phase response of BMAA with ideal non-Foster elements (negative capacitance in series with negative resistor) along with the output phase response of regular array is shown in Figure 5.5. If we look at the largest value of the output phase response at $\theta = 90^\circ$ in Figure 5.5, we can see these results are too optimistic compared to the results of Figure 5.4 (b). The S-parameters of the two monopole antennas used in the simulation to generate the output phase response of Figure 5.5 were generated by FEKO.

Figure 5.5. The output phase response of BMAA with ideal non-Foster elements along with the output phase response of regular array.
The fabricated two (coupled) monopole antenna array is depicted in Figure 5.6 (a). The BMAA was redesigned according to the measured Y-parameters of the fabricated antenna. The design frequency band was chosen to be 350-600MHz. As previously mentioned, the analysis is carried out at discrete frequencies with step size of 10MHz. For a phase enhancement factor larger than 10, the admittance values of \( B_{2/2} \) can be fitted to a curve that has a negative slope, which means the element has a non-Foster behavior, and the admittance value is obtained using an negative capacitance with \( C_p=-9pF \). Moreover, the real part value of \( Y_{2/2} \) is obtained using a negative resistor of value \( R_p=-96\Omega \). As in the first design, the admittance value of \( B_1 \) turns out to be zero, which implies that there will not be an extra element for the coupling network. These values were obtained for a load conductance \( G_L=0.02\Omega^{-1} \). The output phase response of the BMAA with ideal non-Foster elements and measured S-parameters of the coupled monopole antennas compared with the output phase response of regular array.
monopole antennas (shown in Figure 5.6 (a)) over the frequency band of 350-590MHz is shown in Figure 5.6 (b). This range of frequencies is the 3-dB bandwidth of the phase enhancement factor. The 3-dB bandwidth of the output phase response of the BMAA designed using a non-Foster coupling network is 240MHz, which is eight times the bandwidth of the BMAA designed using a passive coupling network. From here on, the BMAA designed using a non-Foster coupling network will be referred to as the NFC-BMAA, while the BMAA using a passive coupling network will be referred to as the BMAA.

5.5 System Stability Analysis

While the above discussion shows the advantage of using non-Foster networks, circuit stability has to be addressed since non-Foster networks tend to be unstable. As discussed in the previous chapters, the stability of NFCs depend on several factors, including,

- The external load Connected to the NFC
- Passive load within the NFC (the internal load).
- NFC topology.
- DC biasing of the active elements (transistors).

In this chapter, the external load is the two monopole elements array. The passive load within the NFC has an one to one relation with the input impedance of the NFC. Therefore, determining the ideal NFC that can yield a stable system is directly related to the determination of the passive load inside an actual NFC.
5.5.1 Determine the Ideal NFC that Can Yield a Stable System

Since at this stage only a model of the non-Foster circuit has been incorporated in the design and not the actual non-Foster network, the port stability will be determined based on modeling the entire system (coupled monopole antennas and non-Foster elements). In other words, the stability analysis (port stability) will determine the condition at which the system can be stable. This is an important step since it determines constrains that should be taken into account to finalize designing a stable system.

The equivalent circuit of the two monopole elements array with a non-Foster coupling network is depicted in Figure 5.7 [55]-[90]. $C_{11}$, $L_{11}$, and $R_{11}$, ($C_{22}$, $L_{22}$, and $R_{22}$), are the elements of the impedance model of standalone monopole. $C_{12}$, $L_{12}$, and $R_{12}$ are the mutual impedance elements of the two monopole antennas.

![Equivalent circuit model of two monopole antennas with non-Foster coupling network and port impedances.](image)

Figure 5.7. Equivalent circuit model of two monopole antennas with non-Foster coupling network and port impedances.
The equivalent model of the two monopole array is valid from 250-650MHz where 
\( C_{11} = 2.27 \text{pF} \), \( R_{11} = 350 \Omega \), \( L_{11} = 10.2 \text{nH} \). \( C_{12} = 28 \text{pF} \), \( R_{12} = 13.4 \Omega \), and \( L_{12} = 3 \text{nH} \). According to [6], the system is only stable if the determinant of the impedance matrix does not have zeros in the right half of the complex plane (RHP). The impedance matrix of the system depicted in Figure 5.7 is:

\[
Z_{\text{sys}} = \begin{bmatrix}
R_0 + Z_{11} & -Z_{12} & -Z_{11} + Z_{12} \\
-Z_{12} & R_0 + Z_{11} & -Z_{11} + Z_{12} \\
-Z_{11} + Z_{12} & -Z_{11} + Z_{12} & 2Z_{11} - 2Z_{12} + Z_p
\end{bmatrix}
\] (5.7)

The determinant of matrix (5.7) is:

\[
|Z_{\text{sys}}| = (R_0 + Z_{11} + Z_{12})(R_0 + \frac{Z_p(Z_{11} - Z_{12})}{Z_p + 2Z_{11} - 2Z_{12}})(2Z_{11} - 2Z_{12} + Z_p) 
\] (5.8)

After manipulating equation (5.8), the determinant equals:

\[
|Z_{\text{sys}}| = R_0^2Z_p + 2R_0^2Z_{11} - 2R_0^2Z_{12} + 2R_0Z_pZ_{11} + Z_pZ_{11}^2 - 2R_0Z_{12}^2 - Z_pZ_{12}^2
\] (5.9)

It can be observed from equation (5.9) that there is no one to one relation between \( R_p \), \( C_p \) and the two-antenna elements array since they are in conjunction with each other and have frequency dependent relation. However, after some parametric study, it has been found that the system (two element array and the NFC coupling network) is stable if \( R_p < -88 \Omega \) when the \( C_p \) is in a range that can amplify the output phase properly. To be more specific, \(|C_p| < 50 \text{pF}\). After optimization and ensuring port stability, the response of the BMAA is simulated in Agilent ADS, where \( R_p = -96 \Omega \) and \( C_p = -9 \text{pF} \). Although the system can be stable using ideal non-Foster network, the internal stability of the entire system has not yet been investigated. The internal stability of the entire system depends
on several factors as mentioned previously. The NFC topology should be a floating NFC since the coupling network is located between two ports of array.

5.5.2 Choosing the Correct NFC Topology

The correct NFC topology that can lead the system to stability is a floating NFC topology. This floating NFC topology is often referred to as short circuit stability (SCS). Figure 5.8 shows a floating SCS negative impedance converter (NIC) introduced by Linvill [9]. Analyzing Figure 5.8 (a) using the approximate of the small signal $\pi$-model of the bipolar junction transistor BJTs shown in Figure 5.8(b), the input impedance for a low frequency approximation is

$$Z_{in} \approx -Z_L - \frac{2}{g_m}$$

(5.10)

There are a couple of reasons that led to choosing the SCS topology to be used in the design of NFC-BMAA. First, since the NFC needs to be connected in shunt with the two element array as can be seen in Figures 5.2 (a) and 5.7 the input connection should be either the base or collector of the transistors [82]. Therefore, the NFC should be SCS topology. Moreover, the current is the signal that is going to be splitting at the input so that the connection should be shunt. Second, the fact that a large negative resistance is required for stability purposes and also for getting phase amplification, the NFC SCS topology should be selected as it can get negative resistance more easier than OCS topology (compare equations (5.10) and (2.2)).
5.5.3 Realization and DC Biasing of the NFC Coupling Network

The schematic of the NFC that is being used as a coupling network for the NFC-BMAA, including the DC biasing is shown in Figure 5.9. The resistors $R_f$ with the DC voltages $V_{CC}$ are used to bias the transistors $T_1$ and $T_2$. The resistors $R_E$ has been included to help stabilizing the Q-point. The resistors $R_C$ are equal $750\,\Omega$. The two monopole antennas are connected to the coupling network at ports #1 and #2.

Since the non-Foster network is implemented using microstrip transmission lines (MTL), these MTL can degrade its performance in terms of bandwidth and the output phase response. As can be seen in Figure 5.9, the load capacitance is chosen to be $3\,pF$, not $9\,pF$ as explained in subsection 5.5.1. There are a couple of reasons for this: First, the conversion factor of the non-Foster network is not exactly one due to the non-ideality of the network elements.
Figure 5.9. Schematic of Non-Foster Network SCS Topology as NFC-BMAA Coupling Network. Labeled branches (1) to (6) are the locations of possible stabilizing networks.

This behavior is due to the MTLs since the MTLs shift the network response to a lower frequency range. The length of MTLs has a significant impact on the output phase response of the NFC-BMAA. It is important for the designer to be able to control the effect of the TLs. Here, tuning the controllers and the resistors $R_C$ can control the output phase response as discussed in subsection 5.63. All the RF components of the non-Foster network are implemented on the top layer of the substrate to avoid using vias to connect layers; therefore, long TL had to be used to connect the load, namely 3$pF$ and 98$\Omega$, between the Trs’ emitters.
5.6 Internal Stability Assessment and Stabilization of NFC-BMAA using Actual NFC

So far, the condition that can lead the system to be stable has been determined. Moreover, the right NFC topology has been chosen, and the actual NFC with the proper DC bias has been introduced. The next step is to assess the stability of the entire system, including the actual NFC. In case of instability, a stabilization network (elements) can be introduced within the NFC to ensure the system is stable [91]. Here, the frequency-domain approach introduced in [45] is used as a tool to assess and stabilize the system.

5.6.1 Frequency-Domain Approach for Stability Analysis

The analysis is based on the calculation of a single-input single-output (SISO) transfer function $H(s)$ of the circuit linearized about a particular steady state. This transfer function is obtained by applying a pole-zero identification algorithm to a simulated frequency response $H(j\omega)$ of the linearized circuit. The frequency response of the linearized circuit can be obtained as the impedance seen by a small-signal current source $i_{in}(j\omega)$ connected in parallel at a given circuit node $n$ as in Figure 5.10 (a) [54].

$$H(j\omega) = Z^n(j\omega) = \frac{v_{out}^n(j\omega)}{i_{in}(j\omega)}, \tag{1}$$

or as the admittance seen by small-signal voltage source $v_{in}(j\omega)$ inserted in series at a particular branch of the circuit as in Figure 5.10 (b).

$$H(j\omega) = Y^b(j\omega) = \frac{i_{out}^b(j\omega)}{v_{in}(j\omega)}. \tag{2}$$
The stability of the system has been assessed by calculating SISO transfer functions at different locations using both simulated impedance and admittance frequency responses. Figure 5.11 shows the magnitude and phase of the simulated (green crosses) frequency response obtained at location (4) shown in Figure 5.9. The frequency response of Figure 5.11 was obtained as the impedance seen by a small-signal current source at location (4).

A system identification algorithm was applied to the simulated frequency response (green crosses in Figure 5.11) to obtain the transfer function associated (identified transfer function) with the circuit frequency response. The magnitude and phase of the identified transfer function is also shown in Figure 5.11 as a solid blue line. Figure 5.11 shows excellent agreement between the simulated frequency response and the identified transfer function over a wide band frequency response. The pole-zero map of transfer function obtained at node (4) (Figure 5.12) has poles on the right-half of the complex plane (RHP), which indicates the system is unstable even though the condition for port stability
has been satisfied. It is clear that it is not enough to depend on the input/output port system model to determine the complete stability of non-Foster systems.

Figure 5.11. Results of the magnitude and phase of the simulated (green crosses) and the identified transfer function (solid blue). (a) Magnitude. (b) Phase.

Figure 5.12. The pole-zero map of transfer function obtained at node (4) shows the system is unstable.
Determine whether the system has hidden unstable modes

In general, non-Foster systems (systems that include non-Foster circuits) can have unstable hidden poles (modes) which means the system can have an exact pole-zero cancelation. In [32], the author showed a simple example of the possibility of unstable hidden modes. In order to examine whether or not there are unstable hidden modes for the given system, several closed loop transfer functions have been examined. If the unstable modes of transfer functions are the same, the system does not have unstable hidden modes; otherwise, the system contains hidden modes. In this particular system, several closed loop transfer functions have been examined. The pole-zeros maps show all the unstable modes are the same for different transfer function, so fortunately, the system does not have any unstable hidden mode. Figure 5.13 shows another pole-zero map of the system but the transfer function obtained at branch (2), using the admittance form, which indicate the system is still unstable and does not have hidden modes. Both, Figures 5.12 and 5.13 not only have the same unstable poles, but also have the same system poles.
5.6.2 Stabilization Networks to Ensure All System Modes on the LHP

Since the system is unstable, it will be shown in this subsection how to stabilize the NFCs using conventional linear control techniques. As described in previous chapters, conventional control techniques (CCTs) in linear control theory are based on a combination of proportional (P), derivative (D) and integral (I) controllers [83]-[84]. CCTs can be easily implemented to make the NFCs stable. In [54], the use of P, D and I controllers to stabilize multi-transistor circuits has been explained. The proportional control action can be implemented with series or shunt resistors, while the derivative and the integral control actions can be achieved with series or parallel reactive elements. Moreover, the combination of resistor and reactive elements can represent proportional derivative (PD) and proportional integral (PI) controllers.
The non-Foster circuit topology used in this chapter consists of only two cross-coupled transistors; therefore, the potential locations of the controllers are limited as marked in Figure 5.9. The controllers should not be placed between the NFC and the two elements array (location (1) and (6)) in order not to degrade the performance of the system. To maintain the system symmetry, conventional controllers should be introduced either at branches (2) and (4) or at branches (3) and (5). It was determined that introducing any of the CCTs at both locations have the same impact on the system stability and as a result, introducing the CCTs at branches (2) and (4) is considered in here. It should be kept in mind that the performance of the non-Foster coupling network must be maintained at the higher frequency range of interest while the system is stabilized. In this particular system, conventional controllers have been introduced in branches (2) and (4). First, the stability analysis of the system was carried out when a P-controller (series resistor) is introduced at branches (2) and (4). Any resistor value chosen should maintain the non-Foster behavior of the coupling network. It was determined that the system becomes stable when $R_{\text{stab}}$ is equal to 100 Ω, and continues to be stable for wide range of $R_{\text{stab}}$ values. The pole-zero map of the transfer function obtained at branch (4), using then impedance form, when $R_{\text{stab}}$ is equal to 120 Ω is shown in Figure 5.14. Figure 5.14 shows that all the system poles are on the left-hand of the complex plane (LHP) which indicates that the system is stable. The fact that $R_{\text{stab}}$ is chosen to be equal 120 Ω and not 100 Ω, is to allow a safety margin to ensure that the system is stable even if some deviation of the components may happen. To determine whether the system can be stable under other controllers, another attempt to stabilize the circuit was done by inserting a
series capacitor in branches (2) and (4). It was noticed that there is no value of capacitance that can drive the system into stability. The stability analysis was also carried out with an inductor (D-controller), but the system remains unstable.

To confirm that the system is stable when 120 Ω P-controller was introduced at branches (2) and (4), several transfer functions were examined and all ensure that all the system modes are all LHP. The pole-zero map of the transfer function obtained at branch (2), using an admittance form, when $R_{\text{stab}}$ is equal to 120 Ω is shown in Figure 5.15. It shows that all the system poles are on the left-hand of the complex plane (LHP) which again indicates that the system is stable.

![Figure 5.14](image-url)  
Figure 5.14. The pole-zero maps of transfer functions obtained at branch (4) shows the system is stable.
Figure 5.15. The pole-zero maps of transfer functions obtained at branch (2) shows the system is stable.

Figure 5.16. Results of the magnitude and phase of the simulated (green crosses) and the identified (solid blue) transfer function at branch (2). (a) Magnitude. (b) Phase.
To show the perfect agreement between the simulated frequency response and the identified transfer function for a wide band frequency response, Figure 5.16 is depicted. It shows the magnitude and phase of the simulated (green crosses) and the identified (solid blue) frequency response of the transfer function obtained as the admittance seen by a small-signal voltage source at location (2).

5.7 Simulated and Measured Results

After stabilizing the system, the non-Foster coupling network comprised of two discrete BJTs was fabricated using RT/duroid 5880 substrate with $\epsilon_r = 2.2$ and thickness= 62 mil, is shown in Figure 5.17. This non-Foster coupling network is designed on the basis of a balanced SCS NIC shown in Figure 5.8 (a). The circuit layout has been designed and simulated with the help of a full-wave EM and circuit CAD packages, namely, EM/circuit Co-simulation in Agilent ADS.

Figure 5.17. Fabricated non-Foster coupling network for NFC-BMAA.
5.7.1 Simulated Output Phase Response and Normalized Available Power

The simulated output phase response of the NFC-BMAA with actual non-Foster coupling network and measured S-parameters of the coupled monopole antennas (shown in Figure 5.6 (a)) over the frequency band of 400-580MHz is shown in Figure 5.18. This range of frequencies is the 3-dB bandwidth of the phase enhancement factor. The 3-dB bandwidth of the output phase response of the NFC-BMAA designed using actual non-Foster coupling network is 180MHz. Although the 3-dB bandwidth is relatively small compared to Figure 5.6 (b), it is still much wider than the bandwidth of the passive-BMAA. The small bandwidth reduction is due to the incorporated TLs as mentioned in sub-section 5.5.3. The solid red curves in Figure 5.18 are the output phase response of NFC-BMAA at different frequencies. The largest output phase is around the center frequency band and the smallest output phase is at the two ends of the frequency band (start and end of the frequency band). The maximum output phase of the NFC-BMAA at θ=90° is 120° (Φ_{out}|θ=90°=120°), which is at 490MHz.

Figure 5.19 shows the simulated available output power of the NFC-BMAA normalized to the output power available from the regular array (NFC-BMAA without non-Foster coupling network). At boresight (θ=0°), the normalized available power of the NFC-BMAA is -1dB over the entire frequency band (400-590MHz), which indicates the loss in power is much smaller than the case of the passive–BMAA (see Figure 5.4 (a)). At θ=90°, the normalized available power of the NFC-BMAA varies from -2 to 3.5dB over the frequency band, which indicates the normalized available power of the
NFC-BMAA is larger than zero in most of the frequency band. This shows another advantage (beside wider bandwidth) of the NFC-BMAA over the passive-BMAA.

Figure 5.18. Simulated output phase response of BMAA with actual non-Foster network and measured S-parameters of the coupled monopole antennas along with output phase response of the regular array over 400-580MHz.
5.7.2 Measured Output Phase Response and Normalized Available Power

The measured output phase response of the NFC-BMAA along with the measured output phase response of the regular array over the frequency band of 400-660MHz is shown in Figure 5.20. The measurement was accomplished in an indoor environment using an Agilent N5230A network analyzer. As discussed in the simulation results, the largest output phase is around the center frequency of the band and the smallest output phase is at the two ends of the frequency band (start and end of the frequency band). The maximum output phase of the NFC-BMAA at $\theta=90^\circ$ is $115^\circ$ ($\Phi_{out}\vert_{\theta=90^\circ}=110^\circ$), which is at 520MHz. As it can be seen from Figure 5.20, the output phase response of the NFC-BMAA has a small phase shift, whose average is around $7^\circ$ at $\theta=0^\circ$ (the zero crossing occurs at $\theta=2.8^\circ$) for the entire frequency band.
This small shift is primarily caused by the asymmetric of the exact element values of the NFC-BMAA. The phase shift impact can be compensated for by using a simple phase shifter in line with the input where the phase response being measured. The phase shifter can be implemented with a small piece ($l=2\text{mm}$) of transmission line. The output phase response of the NFC-BMAA with added phase shift along with the original measured and simulated data is shown in Figure 5.21 (a) (see Figure 5.21 (b) zooming around the zero crossing).

The measured available output power of the NFC-BMAA normalized to the output power available from the regular array (NFC-BMAA without non-Foster coupling network) is shown in Figure 5.22. At boresight ($\theta=0^\circ$), the normalized available power of the NFC-BMAA is -1.5dB at the center frequency and deviates around 0.5dB over the
entire frequency band (400-660MHz). At $\theta=90^\circ$, the normalized available power of the NFC-BMAA varies from -1 to 1dB over the entire frequency band.

Figure 5.21. (a) Measurement and simulation of the largest output phase response of two-elements NFC-BMAA showing the phase shift correction of the measured data. (b) Zoom of (a) to show the phase shift compensation.
5.7.3 Measured Normalized Signal to Noise Ratio

In the receiving case, considered this chapter, the signal to noise ratio is the most important figure of merit. In this sub-section, the measured signal to noise ratio (SNR) of the NFC-BMAA normalized to the SNR of the regular array is presented. To measure the received noise, the signal at the antenna input terminal with and without the non-Foster coupling network is measured with a spectrum analyzer (Agilent E4440A). When performing noise measurements, it is very important to be aware of the sensitivity of the receiver, namely, its noise floor. In this measurement, the noise floor of a spectrum analyzer is $-101\text{dBm}$ in the range 300 MHz-700MHz with internal preamplifier enabled.

The measured noise of the NFC-BMAA normalized to the noise of the regular array is shown Figure 5.23. Note that the noise is measured at one port while the other port is
loaded with a 50Ω load. The noise is independent of the incident angle of the EM waves, so that Figure 5.23 shows the noise for all the incident angles. Since the normalized available power of the NFC-BMAA is a function of the incident EM waves (θ), the normalized SNR is shown at θ=0° and θ=90° in Figure 5.24 (a) and (b), respectively. The normalized SNR of the NFC-BMAA shown in both Figure 5.24 (a) and (b) is better than the normalized SNR of the passive-BMAA presented in [89].

![Figure 5.23. Noise added due to the non-Foster coupling network.](image-url)
Figure 5.24. Measured normalized signal to noise ratio of NFC-BMAA. (a) For angle of incidence of the EM wave equal to $\theta=0^\circ$. (b) For angle of incidence of the EM wave equal to $\theta=90^\circ$.

5.8 Summary

In this chapter, a fabricated non-Foster coupling network (negative capacitor in series with negative resistor) for a pair of coupled monopole antennas was introduced. It was used to amplify the output phase response for a wide frequency band and to improve the normalized available output power compared to the case when using a passive coupling network.

The complete stability analysis of the entire system was discussed in several steps demonstrating a systematic way to assess the stability of non-Foster systems. A Pole-Zero identification method was used to assess the internal stability and it helped to stabilize the entire system using conventional control techniques (CCT). The system is stabilized with the help of a p-controller and lead compensation was incorporated to
maintain amplifying the output phase. The controllers along with the collector resistors help to tune the system to obtain the desired output phase response.

Measured and simulated data of the output phase response of the NFC-BMAA shows very good agreement. The measured output phase response has a wider 3-dB bandwidth than the simulated 3-dB bandwidth at the expense of sacrificing small phase improvement. Although the normalized SNR of the NFC-BMAA is less that 0 dB for different incident EM wave angles, it is still larger than the normalized SNR of the passive-BMAA.
CHAPTER 6

CONCLUSIONS AND FUTURE WORK

6.1 Summary and Conclusions

In this work, the systematic design, stability assessment, implementation and measurement of various non-Foster circuits for two different applications is discussed. In chapter 2, the stability properties of NFCs were discussed with a time-domain technique that computes the largest Lyapunov exponent for time series signals. The proposed stability approach was successfully applied to a negative capacitor to impedance match a 3" small monopole receiver antenna. After fabricating the stabilized system (non-Foster network and the antenna), stability measurements were performed verifying that the system is stable. The measured results showed that the non-Foster matching networks improve both the antenna gain and the signal to noise ratio (SNR) for a wide frequency band (50MHz to 420MHz for the antenna gain (8.4:1) and 50MHz to 410MHz for the SNR (8.2:1)). This is the largest improvement among previous published work [14] [29] when using non-Foster elements.

Chapter 3 presented an efficient way to improve the performance of an electrically small antenna by loading the antenna with a non-Foster circuit (NFC). The location of the load as well as its optimal reactance was determined by the use of the theory of
Characteristic Modes (CMs). The NFC was designed based on the NII topology. The impact of the MTL on the NII was studied as well. The antenna loaded with the NFC showed remarkable improvement in antenna gain and SNR for a wide frequency band (100MHz to 300MHz). Furthermore, by embedding the NFC, the antenna’s omnidirectional pattern was maintained up to higher frequencies.

Chapter 4 introduced for the first time the complete stability analysis of non-Foster systems in a systematic way. The proposed method was applied on two non-Foster matching networks; namely, the series connection of negative capacitor-inductor and a combination of Foster and non-Foster circuits, for use with a small monopole antenna. The second system was designed to maintain a constant antenna gain over a wider frequency band. After fabricating the stabilized systems, the measured results showed that the non-Foster matching network for the first system improved both, the antenna gain and the SNR, for the widest frequency band ever reported (30MHz to 730MHz (24.3:1). The second matching network widely improved the antenna gain with a small gain variation along the frequency band of interest (350MHz-730MHz).

Chapter 5 presented a method for amplifying the output phase response of a pair of monopole antennas over a wide frequency band using non-Foster coupling (NFC). The complete stability analysis of the entire system was studied to ensure the system was completely stable. Measured data of the output phase response of the NFC-two-element biomimetic antenna array (BMAA) has a much wider 3-dB bandwidth in comparison with the passive case; namely, the 3-dB bandwidth of the NFC-BMAA is 260MHz whereas the 3-dB bandwidth of the passive-BMAA is 30MHz with small scarfing of the
phase enhancement factor for the NFC-BMAA. The NFC improved the normalized available output power of the BMAA compared to the case when a passive coupling network is used.

6.2 Suggestions for Future Work

6.2.1 Implementation of Distributed Impedance Matching with Foster and Non-Foster Elements

Although the authors in [13] [71] [72] showed a method to control the currents throughout the antenna structure to achieve the desired performance in terms of input impedance, frequency bandwidth and radiation pattern by multi-port loading using a combination of Foster and non-Foster elements, there have been no successful realization of this general technique. To implement such system, it is necessary to study and analyze the stability of the entire structure using the approach introduced in this dissertation to ensure the complete stability of the system so that can be successfully implemented.

6.2.2 Integrated Non-Foster Impedances

As mentioned in [14], it is well known that active circuits composed of discrete transistors and components have limitations at high frequencies due to parasitics and the electrically large size of the layout. It would be highly recommended to design and fabricate integrated-versions of non-Foster impedances for antenna applications, especially, loading non-Foster impedances for compact antennas as mentioned in
previous sections. Therefore, the integrated version of negative impedance converters and inverters are feasible solutions to extend the operation of non-Foster networks to much higher and wider operating frequencies at the expense of the power capability.

6.6.3 Non-Foster Coupling for Phase Enhancement of a Large Number of Array Elements

As shown in chapter 5, a non-Foster coupling network can enhance the output phase response of a two-element array antenna over a wide frequency band compared to the passive coupling. An extension of the use of non-Foster coupling networks for a large number of antenna array elements to enhance the output phase response is recommended. Such a system can find applications in areas such as compact direction finding systems, high-resolution small-aperture radar systems, coupled oscillator arrays, etc. If the coupling between the array elements is not too strong, an extension of presented work is straightforward.
BIBLIOGRAPHY


