Impacts of Vertical Deformation on the Implementation of the National Height Modernization Program

Dissertation

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ABSTRACT

The Height Modernization Program has been designed and implemented by the National Geodetic Survey (NGS) and is an ongoing operation focused on forming accurate, reliable heights using Global Navigation Satellite System (GNSS) technology in conjunction with traditional leveling, gravity, and modern remote sensing information. With the advance and expansion of the space-based technology, GNSS surveying has been used extensively to establish and expand of survey control. For GNSS-derived ellipsoidal heights to have any physical meaning in a surveying or engineering application, the ellipsoidal heights must be transformed to orthometric heights. To assure high accuracy and maintainability of the height system, any change in the location of the reference control mark (horizontal and vertical) must be continuously monitored, and any necessary changes recorded.

Land deformation is a change in shape and form of the Earth’s surface due to an applied force. This can be a result of tensile (pulling) forces, compressive (pushing) forces, shear, bending or torsion (twisting). Many regions around the world undergo such geological changes due to land motion, more specifically vertical land motion (uplifts and subsidence). Vertical deformation or any changes in the shape of the Earth can be caused by both human activities and natural phenomena. These imply major modifications on physical parameters (topographic, geological, hydrological, hydro-geological, etc.),
which result in many environmental implications. Measuring vertical deformation whether due to natural or human related phenomena requires measuring the elevation change over time and whether any change is relative to the geoid or ellipsoid is not particularly relevant. The use of GNSS for deformation monitoring capitalizes on the standard advantages of GNSS surveying, such as the ability to span large distances with high accuracy and the zero requirements for inter-visibility between stations. Also, deformation applications require many repeated observations over time, and GNSS is well suited to automated survey processes that can significantly reduce cost. Given the limitations of GNSS (e.g., orbital errors, atmospheric propagation errors, environmental impact and receiver errors), these issues must be considered in designing a GPS survey. It is possible to account for those issues that are significant over short baselines using commercially available equipment and software and obtain centimeter accuracy heights. However, short baselines are not always practical option and therefore centimeter accuracy is possible even over baselines longer than 100km with occupation times of 24 hours and using specialized data processing.

Firstly, this research proposed a modification to the traditional single control radial network design into a multiple reference radial design and tested its suitability for centimeter level GNSS surveys as a post-processing configuration. Secondly, vertical velocities were computed using GNSS ellipsoid heights derived from multiple repeated campaign surveys. The campaign survey derived vertical velocities were compared to the vertical velocities computed from a multi-year time series of 20 GPS reference stations from the US Continuously Operating Reference Stations (CORS) and Canadian Active
Control System (CACS) national network, the length of each time series spanned from 10 to 16 years based on the availability of the data from each station. This was done to assess the overall capabilities of campaign GNSS survey to detect vertical deformation accurately.

The data processing and analysis were done within the parameters of the NGS-designed software OPUS-Projects and OPUS-Net. These tools have been designated as the future standard for GNSS based three dimensional (3D) coordinate estimations in the United States. A better understanding of a suitable network design will help achieve the accuracy standards of GNSS-derived heights under the National Height Modernization program. The available GNSS surveyed benchmark data from the International Great Lakes Datum of 1985 (IGLD) Height Modernization survey projects (1997, 2005, and 2010) were the selected sample data set used in the research.

The multiple reference radial network performed comparatively well against the Triangle Network Design (TRI) network which represented the closed loop network, with an average difference of 2 mm in the North component; 1 mm in the East component; and -4 mm in the UP component. However, these networks were separated by their computational performance and robustness. The simplicity of the modified radial design is effective for both relative and absolute positioning and is also not as computationally extensive as the TRI network. Due to the interconnectivity of this closed loop network, the number of baselines grows geometrically with the number of stations included in the session which also makes this configuration computationally costly. Also the TRI configuration includes a number of redundant or trivial baselines, in terms of network
adjustment; the inclusion of these trivial baselines may produce resultant solution statistics that are over-optimistic.

Further, vertical velocities between the three Height Modernization surveys projects were produced using ellipsoidal heights generated using the multiple reference radial design. The computed relative vertical velocities assisted in identifying the stations that exhibited uplift and subsidence within the Great Lakes region. These velocities, derived from the three Height Modernization surveys project, were compared to the vertical velocities derived from the multi-year time series analysis of the selected GPS reference stations from the US and Canadian national network. For the stations tested a maximum difference of 4 mm/yr and minimum difference of 0.6 mm/yr was observed between the datasets. The cause of the maximum 4 mm/yr difference is unknown and may be due to some inherent error in the dataset. This leads to the recommendation that multiple consecutive campaign surveys would be needed to determine a continuous pattern of change and assist in verifying random changes in the velocities. However, between the two methods for determining vertical velocity and overall land deformation, the continuous GPS multi-year time series analysis for a time period greater than 2.5 year produced a stable and consistent solution over the campaign surveys.

This study illustrates that through the selection of an optimal network design GNSS coordinates and specifically, ellipsoidal heights can be accurately determined and in turn be used to investigate vertical deformation.
DEDICATION

Dedicated to my mother Marina Patrick Richardson
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VITA

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FIELD OF STUDY

Civil Engineering
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LIST OF ACRONYMS

APC ............................................. Antenna Phase Center

ARP ............................................. Antenna Reference Point

C/A-code .................................... Course/Acquisition-code

CGVD ........................................ Canadian Geodetic Vertical Datum

CGG ............................................. Canadian Gravimetric Geoid

CHAR-ID ..................................... Character Identifier

CO-OPS ...................................... Center for Operational Oceanographic Products and Services

CORS ......................................... Continuously Operating Reference Station

CONUS ....................................... Contiguous United States

DGPS ......................................... Differential GPS

DORIS ........................................ Doppler Orbitography and Radiopositioning Integrate by Satellite

DoD ............................................. Department of Defense

DoY ............................................. Day of Year

ECEF .......................................... Earth-Centered Earth-Fixed

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GEOID12A ............................................Hybrid Geoid Model of 2012

GLONASS .............................................Globalnaya navigatsionnaya sputnikovaya Sistem

GMF .......................................................Global Mapping Functions

GNSS .....................................................Global Navigation Satellite Systems

GPS .......................................................Global Positioning Systems

GPS-IPW ..............................................GPS-Integrated Precipitable Water

GPT .......................................................Global Pressure and Temperature

GRAV-D .................................................Gravity for the Redefinition of the American Vertical Datum

GRS .......................................................Geodetic Reference System

IDB .......................................................Integrated Data Base

IERS .....................................................International Earth Rotation and Reference Systems

IGLD ......................................................International Great Lakes Datum

IGS .......................................................International GNSS Service

ITRF .....................................................International Terrestrial Reference Frame

IPW .......................................................Integrated Precipitable Water

ISL .......................................................Instantaneous Sea Level

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LIDAR ............................................. Light Detection and Ranging

MSL ............................................. Mean Sea Level

NAD83 ............................................. North American Datum 1983

NAVD88 ............................................. North American Vertical Datum 1988

NGS ............................................. National Geodetic Survey

NGVD29 ............................................. National Geodetic Vertical Datum 1929

NHMP ............................................. National Height Modernization Program

NMF ............................................. Niell (New) Mapping Functions

NSRS ............................................. National Spatial Reference System

NOAA ............................................. National Oceanic and Atmospheric Administration

NOS CO-OPS ............................................. National Ocean Service Center for Operational

Oceanographic Products and Services

NRCan ............................................. National Resources Canada

OPUS ............................................. Online Positioning User Service

OPUS-RS ............................................. OPUS Rapid-Static Processing Engine

OPUS-S ............................................. OPUS Static Processing Engine

PAGES ............................................. Program for the Adjustment of GPS Ephemerides
PCV .......................................................... Phase Center Variation

PID .......................................................... Permanent Identifier

SAR .......................................................... Synthetic-Aperture Radar

SLR .......................................................... Satellite Laser Ranging

SST .......................................................... Sea Surface Topography

RMS .......................................................... Root Mean Square

VLBI .......................................................... Very Long Baselines Interferometry
CHAPTER 1: INTRODUCTION

1.1 Background

Land deformation is a change in shape and form of the Earth’s surface due to an applied force. This can be a result of tensile (pulling) forces, compressive (pushing) forces, shear, bending or torsion (twisting) (Mui, 2006). Many regions around the world undergo geological changes due to land motion more specifically vertical land motion, which involves subsidence and uplifts.

Subsidence is the lowering (sinking) of the land surface in response to the removal of subsurface support. Subsidence can lead to costly damage in coastal regions because of the relative rise of sea level, the associated landward shift of the shoreline, and the increased risk of flooding from storm surges. In inland regions subsidence causes several problems, including modifying stream gradients and changing the geomorphology of flood plains. Also subsidence can severely damage property and infrastructure in a developed area. In the United States, for example, more than 17000 square miles in 45 States, have been directly affected by the deformation process of land subsidence (US Geological Survey, 2000). Houston, Texas provides the most extreme case study of subsidence (Wang and Soler, 2013). Compaction of subsurface clay layers owing to withdrawal of ground water is the primary cause of subsidence in the in this region. Accumulated subsidence of over 3 m has been observed within the last 100 years in the
southeastern area of greater Houston including Pasadena, Bay Town, Texas City, and Galveston, Texas (Kramarek, Gabrysch and Johnson 2009).

Uplift, conversely refers to displacement of the Earth’s surface in the direction opposite to the gravity vector (England and Molar, 1990). Hence it is the vertical elevation of the Earth’s surface in response to loading. Uplift of the Earth’s surface has also occurred in response to the removal of last ice sheets through melting earthquakes mountain building (Orogenic uplift) and other tectonic events. The Great Lakes region gives an example of the occurrence of both uplift and subsidence due to the isostatic response of the Earth to removal of the last ice sheet (Sella et al., 2007) and a case of subsidence due to salt mining (Conner, 2013). Another unique case of uplift is drought-induced uplift occurring in the western conterminous United States. This is due to inter-annual changes in crustal loading driven by changes in cool-season precipitation, which cause variations in surface water, snowpack, soil moisture, and groundwater (Borsa, Agnew and Cayan, 2014).

Vertical deformation or any changes in the shape of the Earth can be caused by both human activities and natural phenomena (Nemmour and Chibiani, 2006), that implies major modifications on physical parameters (topographic, geological, hydrological, hydro-geological, etc.), which result in many environmental implications. Therefore, it is important to analyse Earth’s surface deformation. Timely and accurate change detection can provide the foundation to understand relationships and interactions between human and natural phenomena to better manage and use resources (Lu et al., 2004; Muttitanon and Tripathi, 2008). Vertical deformation monitoring can provide valuable information about the dynamic behavior of the Earth surface.
Improving our knowledge of Earth’s deformation in an area of unique geological history or human activities, for example the Great Lakes, is important for shore industries and inhabitants to help in charting and mapping, management of water, shore and land constructions, shore erosion, shipping, hydroelectric dams, basins, power generation, flooding, environmental changes, groundwater resources (their quantity and pollution), as well as in determining crustal stress in regions that are earthquake-prone and extensively mined. Measuring vertical deformation whether due to natural or human related phenomena requires measuring the elevations and elevation changes, therefore it requires accurate height measurements to detect these changes (England and Molar, 1990).

In addition to deformation monitoring, accurate, reliable, and up-to-date heights are essential for a wide range of activities, such as managing construction and infrastructure projects by surveyors and engineers, planning evacuation routes, and there are many applications of accurate heights in flood or inundation mapping and modeling (Veilleux, 2013). Accurate heights are needed to manage restoration projects, complete natural resource inventories, or monitor erosion and/or sea and lake level change but particularly to understand and protect low-lying coastal ecosystems. Finally, accurate monitoring of vertical deformation over time is vital to providing calibration data for modeling and prediction purposes. Scientists need accurate heights to monitor and model changes when studying crustal motion, subsidence, glacial isostatic adjustment (GIA) and seasonal changes like frost heave. The increased awareness of recent climate variations dramatically boosted the accuracy requirement as well as the amount of geospatial data, where height, in particular, is of interest.
The technologies that are most often used for deformation monitoring are Global Positioning System (GPS), or more broadly GNSS, optical levelling, tiltmeter, and satellite radar interferometry. For this research GPS is the main technology used. Using GPS for deformation monitoring capitalizes on the advantages of GPS surveying, such as the ability to span large distances with high precision and the zero requirements for intervisibility between stations. Also, deformation applications require many repeated observations over time, and GPS is well suited to automated survey processes that can significantly reduce cost (Higgins, 1999). Studies conducted by Borsa, Agnew and Cayan (2014); Zilkoski et al. (2003) and Steckler et al. (2010) demonstrated that a network of continuous GPS stations can be used to recover vertical changes due to both wet and dry climate patterns, monsoonal flooding, and compaction of susceptible aquifer systems related to ground-water pumping. Further concluding, that the stability of the GPS monumentation means that the network is also capable of monitoring the current and future effects of surface changes.

1.2 Integrating GPS and Traditional Levelling Techniques

Observed height differences between points on the Earth's surface are traditionally obtained through spirit-levelling (and/or its variants, such as trigonometric, barometric levelling, etc.) (Vaníček, Castle and Balazs, 1980). Levelling involves measuring vertical distances with reference to a horizontal plane or surface. To do this, a levelling rod is needed to measure vertical distances and an instrument known as a level is required to define the horizontal plane. Observations of the levelling rod is taken point by point, data collection is mostly performed along routes easy to access, such as highways/roads,
valleys, etc. This poses a limitation in the spatial distribution of the leveling network, and also a lack of efficiency and cost benefit in data collection.

Although costly and laborious, spirit-levelling is an accurate measurement technique whose procedural and instrumental requirements have evolved to limit possible systematic errors. Associated random errors in levelling originate from several sources, such as refractive scintillation or 'heat waves', refraction variation between readings, vibrations of instrument due to wind blowing, and movement of rod or non-verticality of rod caused by wind, terrain and unsteadiness of surveyor, to name a few. These errors are generally dealt with through redundancy and minimized in the least-squares adjustment process (Vaníček, Castle and Balazs, 1980). However, it should be realized that national networks of vertical control established this way involve large samples of measurements collected under inhomogeneous conditions, such as variable terrain, environments, and instruments, with different observers and over different durations. This result in a number of errors/corrections that must be made to the measurements (see Davis et al., 1981). The problem with using only the elevation differences obtained from spirit-levelling for height-related applications is that the results are not unique, as they depend on the path taken from one point to the other (due to non-parallelism of the equipotential surfaces).

Due to the nature and practical limitations of spirit-levelling most vertical control points are located along roads/railways, which restrict the spatial resolution of control networks and confine the representation of the actual terrain. On the other hand, horizontal control networks have historically been established using triangulation and trilateration, which required that points be situated on hilltops or high points (Davis et al., 1981). As a result,
there exist separate networks for horizontal and vertical control with few overlapping points. However, with the advent of satellite-based positioning systems (GPS, Globalnaya navigatsionnaya sputnikovaya sistema (GLONASS), GALILEO, etc.) and space-borne/airborne radar systems (satellite altimetry, Light Detection and Ranging (LIDAR), Synthetic-Aperture Radar (SAR)) the ability to obtain accurate horizontal and vertical position at virtually any point on land or at sea has become possible (Fotopoulos, 2003). However, the heights obtained from these techniques are referred to a reference ellipsoid, a mathematical surface, and they do not have any physical meaning but only a geometrical one. This type of heights is fundamentally different from the levelling heights which are referred to a specific equipotential surface, namely the geoid, and defined by the Earth’s gravity field.

There is a fundamental relationship that binds the ellipsoidal heights obtained from GNSS measurements and heights with respect to a vertical geodetic datum established from spirit-levelling and gravity data, which is given by (Heiskanen and Moritz, 1967):

\[ h - N = H \]

(1-1)

where \( H \) is the orthometric height obtained by leveling and gravity observations, \( h \) is the ellipsoid height obtained by GNSS observations and \( N \) is the geoid height which is also called geoid undulation obtained from regional gravimetric geoid models or a geopotential model. These terms are explained in more detail in Chapter 3. This relationship is not exact because it ignores the deflection of the vertical; nevertheless, it is close enough for most practical purposes. The above formula can also be investigated in the relative sense given by:
\[ \Delta h - \Delta N = \Delta H \] (1-2)

In this study, the terms geoid, geoid undulation, and geoid height are used interchangeably and refer to the separation between the geoid and the ellipsoid surface (Figure 1.1). Also, GNSS, GPS or geometric heights are used interchangeably and refer to the ellipsoidal height, which is represented by \( h \).

![Diagram](image)

**Figure 1.1. Relationship between the ellipsoid and geoid height of a point on the Earth's surface (NGS, 2014)**

Figure 1.1, illustrates the geometric relationship, given by equation (1-1), which binds the ellipsoidal heights and the orthometric height. Separation between the ellipsoid and geoid
along a perpendicular is known as geoidal undulation \((N)\). The height relative to the geoid measured along a plumb line to a point on Earth's surface is called the orthometric height \((H)\); this is what is usually referred to as elevation. Ellipsoidal height \((h)\) is the distance along a perpendicular above or below the ellipsoid to a point on Earth's surface.

Equations (1-1) and (1-2) can be manipulated to derive the orthometric heights from ellipsoidal heights and a geoid model. Specifically the combination of ellipsoidal height obtained from GPS measurements and the geoid height computed from a geoid model for the determination of the orthometric height is referred to GPS-levelling (Huang and Véronneau, 2004). This method has been adopted in many countries such as United States (US), Canada, New Zealand and Australia (NGS, 2014; National Resources Canada, 2014; Land Information New Zealand, 2009; Featherstone and Kuhn, 2006). In this methodology when two of the heights are known, the third one can easily be computed.

However, the manipulation of that relationship is complicated by several factors that cause inconsistencies when combining the varied heights. Some of these factors include, but are not limited to, the following (Fotopoulos, 2003):

- random errors in the derived heights \(h, H,\) and \(N\)
- datum inconsistencies inherent among the height types, each of which usually refers to a different reference surface
- systematic effects and distortions primarily caused by long-wavelength geoid errors, poorly modeled GPS errors (e.g., tropospheric refraction)
• instability of reference station monuments over time due to geodynamic effects and land subsidence/uplift

Zilkoski, Carlson and Smith (2008) stated that GPS-derived orthometric heights can now provide a viable alternative to traditional levelling techniques for many applications. Traditional methods for establishing vertical control, although accurate, are very laborious, costly and impractical in harsh terrain and environmental conditions. Alternatively, ellipsoidal heights can be efficiently and relatively inexpensively established with dense coverage over land although at a lower accuracy level (Fotopoulos, 2003). Even though it is a viable option and the geometry behind GPS-levelling seems simple, the accuracy of the technique depends on both the accuracy of the ellipsoidal and geoid height data. The accuracy and errors affecting GPS-levelling are discussed in Chapter 2.

1.3 Height Modernization

Height Modernization is an initiative focused on establishing accurate, reliable heights using GNSS technology in conjunction with traditional levelling, gravity, and modern remote sensing information (Veilleux, 2013). It deals with the improvement of height determination and the redefinition of the vertical reference surface to which height are measured. Moreover, the redefinition of the vertical reference system would be by geoid modelling rather than geodetic levelling. This update in the vertical reference system enables measurements of heights with respect to a consistent vertical datum everywhere across the country using GPS and other emerging GNSS technologies. Some of the
practical uses and benefits of a consistent regional vertical datum include, but are not limited to, the following (Zilkoski, Richards and Young, 1992):

- improved coastal/harbor navigation
- accurate elevation models for flood mitigation
- accurate elevation models for environmental hazards
- enhanced aircraft safety and aircraft landing
- accurate models for storm surges and coastal erosion
- improved models for chemical spill monitoring
- improved understanding of tectonic movement
- improved management of natural resources

The National Height Modernization Program (NHMP) designed and implemented by the National Geodetic Survey (NGS) is a nationwide initiative focused on establishing accurate, reliable heights using GNSS technology in conjunction with traditional levelling, gravity, and modern remote sensing information. The goal of NHMP is to assure access to accurate and reliable heights that follow consistent standards across the nation, using data and tools that yield consistent results regardless of terrain complexity and circumstances, and are maintainable over time (Veilleux, 2013).

The role of NGS is to define, maintain and provide access to the National Spatial Reference System (NSRS). The vertical component of the NSRS is the vertical datum, which is a collection of specific points (benchmarks) on the Earth with known heights or
elevations above or below a reference surface approximating mean sea level (NGS, 2014).

NGS has been responsible for defining the official vertical reference system in the United States, since the first general adjustment of the geodetic levelling network in 1900 (NGS, 2014). In 1929, NGS compiled all of the existing vertical benchmarks and created the National Geodetic Vertical Datum of 1929 (NGVD 29). In 1988, the vertical control network, including several thousand kilometers of new levelling, was again mathematically adjusted to remove inaccuracies and to correct distortions, which occurred in the NGVD 29. The new datum, called the North American Vertical Datum of 1988 (NAVD 88), was defined through the levelling network and is presently the official vertical datum in the United States. Also, NGS generates and maintains a gravitational geoid model based on gravity data collected from a variety of sources and a Hybrid model that builds on the gravitational model using GNSS on benchmarks to enable a fit to NAVD 88. Hybrid models provide a practical and accurate transformation from the GPS ellipsoid heights to orthometric heights, called GPS-derived orthometric heights.

As mentioned above, the definition of the NAVD 88 has been realized through conducting various geodetic levelling surveys to vertical benchmarks throughout the country. Over the years the slow movements of the Earth’s crust changed and will continue to gradually change, in relation to the locations and stability of some benchmarks. An example of this is in the Houston-Galveston region, where there aren’t any stable benchmarks in the area as a result of the broad extent of subsidence (Zilkoski et al., 2001). Additionally other benchmarks are inadvertently destroyed through
construction projects, such as road widening. In order to assure high accuracy and the maintenance of the height system, any change in the location of the benchmarks must be monitored. Therefore, the need for a new vertical reference system arises from the dynamic nature of the Earth. The advanced technology of GNSS, can provide a viable alternative to classical geodetic leveling techniques for many applications (Meyer, Roman and Zilkoski, 2006).

The NHMP has been initiated to utilize GNSS technology and it encompasses the principle of GNSS-levelling which deals with the optimal combination of GNSS-derived ellipsoidal heights with gravimetrically-derived geoid undulations for the determination of orthometric heights above mean sea level, or more precisely with respect to a vertical geodetic datum. For GNSS levelling to be feasible it requires a high-accuracy geoid model and cm-level accuracy of ellipsoidal height estimation from GNSS.

NGS has embarked on a project that is driven by the fundamental connection between Earth’s gravity field and the very definition of “height” itself, Gravity for the Redefinition of the American Vertical Datum (GRAV-D) Project. GRAV-D is a proposal by the National Geodetic Survey to re-define the vertical datum of the US by 2022. The specific goal of GRAV-D is therefore to model and monitor Earth’s geoid (a surface of the gravity field, very closely related to global mean sea level) to serve as a zero reference surface for all heights in the nation. The gravity-based vertical datum resulting from this project is expected to be accurate at the 2 cm level for the most part of the country.
1.4 **Research Objectives**

As described in Section 1.2, the use of GNSS measurements to compute orthometric heights depends on the accuracy of the geoid model, as well as the accuracy of the GPS ellipsoidal height. While many applications for GPS surveying need to produce orthometric or normal heights there are some applications where ellipsoidal heights alone are useful. One such application is vertical deformation monitoring where the most important issue is to quantify a change in height over time and whether any change is relative to the geoid or ellipsoid is not particularly relevant (Higgins, 1999).

This research is twofold; firstly, the main focus was placed on attaining the most accurate GPS-derived ellipsoidal height that can be eventually coupled with an accurately developed geoid model for the determination of the orthometric height. This was done through the development and analysis of network designs suitable for GNSS surveys and post-processing configuration, using NGS-designed software OPUS-Projects and OPUS-Net. These tools have been designated as the future standard for GNSS based 3D coordinate estimations in the United States. A better understanding of suitable network design will help achieve the accuracy standards of GPS-derived heights in the National Height Modernization program.

Secondly with the use of the raw GPS data from the International Great Lakes Datum of 1985 (IGLD) Height Modernization survey projects (1997, 2005, and 2010), the accuracy of the GPS derived ellipsoid heights and the performance of the proposed network designs were tested. This research used the computed GPS ellipsoid heights from each campaign survey to compute the vertical velocities. The vertical velocities were used to
identify vertical movement within the Great Lakes. Additionally, a selection of continuous GPS stations was analyzed to assess the overall capabilities of campaign GPS survey to detect vertical deformation precisely.

This data set was selected since the Great Lakes exhibits both subsidence and uplifts associated with natural and related human activities. The Great Lakes are a vast hydraulic system with water levels and flows influenced by engineered channels and control structures. Further, due to its dynamic nature (e.g., glaciation and deglaciation) and human activities (e.g., building construction and salt mining) knowledge of any vertical land deformation within the region is essential to determine and understand the impact on those within the environment.

The reality of surface deformation implies that positional coordinates in many locations change as a function of time. Generally, the computed velocities (vertical/horizontal) are used to update coordinates (affected by movement) measured on one date to corresponding coordinates that would have been measured on another date. Also they allow the users to update (or back date) the values of certain types of surveying observations including interstation GPS vectors (GPS baselines), distances, angles and azimuths from values measured on one date to those that would have been measured on some other date. For example engineering structures (such as dams, bridges, viaducts, high rise buildings, etc.) that serve human life are subject to deformation due to factors, such as changes of ground water level, tidal phenomena, tectonic phenomena, etc. The updated surveying observations based on the velocities are important to ensure that any
movements captured when monitoring these structures are the true movement at the time of survey and to facilitate the comparison of the relative movement over time.

The objectives of the study encompass the following tasks:

1. To investigate the ability to achieve a 2 centimeters accuracy (following National Ocean Survey (NOS) NGS-58 Guidelines) of the GPS derived ellipsoidal height through the use of specific network designs of the GPS tracking stations. This was facilitated by reanalyzing the three International Great Lakes Datum of 1985 (IGLD) Height Modernization projects (1997, 2005, and 2010). These three survey projects represented an A-order project consisting of more than 100 stations with a few hundreds of observed vectors (each project has a different number of stations ranging from 75 to 155). All the raw data from the three IGLD campaigns were reprocessed, in a reference frame (e.g., IGS08 or ITRF08) consistent with the best available orbits and using the associated absolute antenna models. This was done using NGS’ newest web-based GPS data post-processing and management tool, OPUS-Projects. This tool has an access to the NGS continuous operating reference stations (CORS) network and the IGS GPS network, altogether forming a robust nationwide infrastructure allowing for a high accuracy, 3D positioning activities.

2. To investigate the vertical deformation of the Great Lakes exclusively derived from GPS. Using the coordinate solutions computed of the campaign surveys from Task 1 the vertical velocities between the survey epochs were computed. Additionally, a time series of 20 GPS reference stations from the US CORS and
Canadian Active Control System (CACS) national network was evaluated. Obtaining a time series of position derived from these continuous GPS stations allows for a more consistent interpretation of the overall movement or deformation. The multi-year time-series shows the common annual and semi-annual variation. It also provided an independent reference to compare the IGLD Height Modernization surveys project computed velocities.

3. To assess the accuracy of the GPS campaign survey data versus continuous multi-year GPS data for monitoring vertical displacement. Campaign type measurements are temporally intermittent and maybe inefficient in detecting nonlinear crust movements or small leaps which may occur in each new campaign observation. Consequently stable CORS stations may get around such problems, by providing their uninterrupted point positions in a high level temporal resolution.

The research presented in this thesis forms part of the ongoing research at the OSU Satellite Positioning and Inertial Navigation Laboratory (SPIN) in support of the National Height Modernization program. The analysis of vertical displacement presented in this thesis builds upon the results of another phase of the research, documented in Ugur et al. (2013), who examined techniques to improve the accuracy of GPS ellipsoid height with an underlying focus on addressing the effects of tropospheric delay. Improving the determination of GPS ellipsoidal heights is one of the most critical components in the Height Modernization Program and subject to significant errors.
The GPS error sources reduce the accuracy of the derived GPS coordinates and ellipsoidal heights. The ellipsoid height is primarily affected by inherent geometric weakness, but more so by un-modeled part of the neutral atmosphere (troposphere). The effect of the troposphere on the GNSS signals appears as an extra delay in the measurement of the signal traveling from the satellite to receiver. This delay depends on the temperature, pressure, humidity as well as the transmitter and receiver antennas location. Studies have shown that the tropospheric delay, if not accounted for is a critical error source in the determination of GPS ellipsoidal height.

1.5 Overview

Chapter 2 provides background material related to the GNSS. The GNSS observables are described, and different errors affecting GPS observations are discussed. These include orbital errors, multipath errors, receiver noise, and satellite and receiver clock errors and errors due to atmospheric effects (ionosphere and troposphere).

Chapter 3 the necessary background information regarding the height data types and terminology used throughout this research is presented. In particular, it describes the major error sources that affect the geoid undulations, orthometric and ellipsoidal heights. The discussion will provide insight into the problems and challenges encountered when attempting the combination of these assorted height data.

Chapter 4 describes the dataset of the IGLD Height Modernization survey projects (1997, 2005, 2010), the process of acquisition and verification, and give a description of the survey area, the Great Lakes.
Chapter 5 presents the reanalysis of the three (3) campaign surveys 1997, 2005 and 2010 of the IGLD Height Modernization Projects. The reanalysis entailed reprocessing and adjusting the original data from the three IGLD projects with OPUS-Projects (NGS’s newest online processing tool), and testing different tracking station network designs. This was done using a consistent reference frame (IGS08/ITRF08 at the epoch of each survey) with the best available GPS orbits and using the associated absolute antenna (receiver and satellite) models. This chapter includes the description of the software, network configurations, processing settings used and the results obtained from in the reanalysis. The vertical velocities of each of the stations were also computed.

In Chapter 6, the focus is on the time series analysis of a selection of continuous GPS from the United States Continuous Operating Reference Station (US-CORS) and Canadian Active Control System (CACS). The position solutions are from the rerun of data archived at the NGS and include all available data for the 16 year interval, from January 1997 to December 2013. The time series of the GPS observations produced position velocities, which allowed for interpretation of the overall vertical movement or deformation of the Great Lakes. The results and comparisons derived are discussed.

In Chapter 7, the continuous GPS vertical velocity estimations are assessed. The assessment was done by comparing the results from Chapter 4 with an independent vertical velocity data set, the GIA model predictions. In this chapter details the GIA model and the methods used to derive the vertical velocities are defined.

Chapter 8 summarizes the conclusions and provides recommendations based on the results of the experiments presented in the previous chapters.
CHAPTER 2: GPS OBSERVABLES AND ERRORS

This chapter provides the background information regarding the types of data and terminologies used throughout this thesis. In particular, it describes the major error sources that affect the geoidal undulations, orthometric and ellipsoidal heights. The discussion will also provide insight into the problems and challenges encountered when attempting the combination of these assorted height data.

2.1 The Global Navigation Satellite System

The Navigation System with Timing and Ranging (NAVSTAR) Global Positioning System (GPS) is a global, all-weather, satellite-based, 24 hour operational radio-navigation and time transfer system. This system is maintained and operated by the US Department of Defense (DoD). Ultimately the system has been developed to satisfy the requirements of the military forces to accurately determine their position, velocity, and time in a common reference system, anywhere on or near the Earth on a continuous basis (Wooden, 1985). Since the inception of GPS in 1973, the availability of the system has been extended to including new signals for civil use and there’s been an extension of the constellation (Hofmann-Wellenhof, Lichtengegger and Collins, 2001).

GPS was designed to include a constellation of 24 satellites (21 in the full constellation and 3 spares) launched into a near-circular orbit at an altitude of approximately 21000 km.
By design there are four (4) satellites in each of the six (6) orbital planes, which have 60° separation at the equator and 55° inclination to the equator. This configuration allows at least 4 satellites to be in view from (almost) everywhere on the Earth’s surface at all times as it takes a minimum of four observed satellites to determine a unique 3D position with GPS.

The GPS constellation began with the launch of the first experimental Block-I GPS satellite between early 1978 and 1985. Tests with these 11 spacecraft demonstrated the capabilities of the system and resulted in the implementation of an operational system, with the first operational (Block-II) spacecraft launched on February 22, 1989 (McDonald, 2002). The payload of the Block-I and Block-II GPS satellites included two L-band navigation signals at 1575.42 MHz (L1) and 1227.60 MHz (L2).

Throughout the years the Blocks of GPS satellites have been modified. The modifications featured Block-IIA (2nd generation, Advanced), Block-IIR (Replenishment), Block-IIR-M (Modernized), Block-IIF (Follow-on) generally included:

1. Improvements to the constellation of operational satellites
2. Improvements of the navigation accuracy and longer autonomous satellite operation than earlier model GPS satellites
3. A new military signals (M-code) on both the L1 and L2 frequencies for improved accuracy, enhanced encryption, anti-jamming capabilities (Block-IIR-M)
4. A second civil signal L2C on the L2 frequency to provide dual frequency capability and improve resistance to interference (Block-IIR-M)
5. A civil signal (L5) intended for safety-of-life applications (Block-IIF)
Future phases in the GPS modernization program include the Block-III series which will introduce new capabilities to meet the higher demands of both military and civilian users, while also enhancing the satellite design life and adding a new fourth civil signal (L1C) designed to be interoperable with international GNSS. Additionally, this series is also needed to complete the deployment of L2C and L5 signal capabilities that began with the modernized GPS IIR-M and GPS IIF satellites (National Coordination Office for Space-Based Positioning Navigation and Timing, 2014). As of June 25th, 2014, the current GPS constellation included thirty (30) satellites, a mixture of old and new satellites. This constellation is made up of six (3) operational Block IIA, twelve (12) operational Block IIR, seven (7) operational Block IIR (M) and six (8) operational Block IIF GPS satellites.

2.1.1 GPS Measurements and Observables

Range measurements to four (4) satellites are sufficient to resolve the 3D location of the receiver and also the timing error due to the imprecise receiver clock. Each of these satellites broadcasts two separate signals: the L1 (1575.42 MHz) and L2 (1227.60 MHz) microwave carrier signals. In addition, as of June 25th, 2014 there were pre-operational signal broadcasting the L5 (1176.45 MHz) microwave carrier signal from six (6) satellites. All three signals are derived from the fundamental L band frequency (10.23 MHz). Two pseudorandom noise codes are superimposed on the carrier frequencies: the Coarse/Acquisition-code (C/A-code) and the Precision code (P(Y)-code). The P(Y)-code is the precise and protected code reserved for military applications and the C/A-code is a less accurate code reserved for civilian users.
The concept of GPS positioning is based on the range measurements between the satellite and receiver. Pseudo-ranges and carrier phases are important GPS observations used in positioning. The pseudo-range is the distance between the satellite and the receiver’s antennas implied by signal traveling time between emission and reception at the receiver, impacted by timing and other sources of errors. Multiplying the ideal travel time in a vacuum by the light velocity (c), gives the geometric range between the satellite and the receiver (Hofmann-Wellenhof, Lichtengegger and Collins, 2001; Leick, 2004). The two signals are affected by the clock errors, since the receiver and satellite clocks are not perfectly synchronized. P(Y)-code pseudo-ranges can be as good as 20 cm or less, while the L1 C/A code range noise level reaches even a meter or more (Grejner-Brzezinska, 2011). Figure 2.1 below, depicts the relationship between receiver $i$ and satellite $k$. The GPS observables can then be expressed with the following equations (2-1) to (2-4).

![Figure 2.1](image)

*Figure 2.1. GPS observable geometry between satellite $k$ and receiver $i$, where, $\rho^k_i$, is the geometric distance between the satellite $k$ and receiver $i$*
The equation of the pseudo-range observable is developed by first considering geometric distance represented as:

\[ P_i^k = c \cdot (t_i - t^k) = c \cdot (t_{io} + dt_i - t_o^k - dt^k) \]

\[ = c \cdot (t_{io} - t_o^k) + c \cdot (dt_i - dt^k) \]

\[ = \rho_i^k + (dt_i - dt^k) \cdot c \]  

(2-1)

\( c \quad \text{- Speed of light (in a vacuum)} \)

\( t_i; t^k \quad \text{- The measured received and transmitted times, respectively} \)

\( t_{io}; t_o^k \quad \text{- The received and transmitted true (ideal clock) time, respectively} \)

\( dt_i; dt^k \quad \text{- The receiver and satellite clock bias, respectively} \)

\( \rho_i^k \quad \text{- Geometric Distance between the satellite \( k \) and receiver \( i \)} \)

\[ \rho_i^k = \sqrt{(X^k - X_i)^2 + (Y^k - Y_i)^2 + (Z^k - Z_i)^2} \]  

(2-2)

\( X_i, Y_i, Z_i \quad \text{- Receiver coordinates (primary unknowns)} \)

\( X^k, Y^k, Z^k \quad \text{- Satellite coordinates (computed from navigation message information)} \)

The full mathematical expression for the pseudo-range observable also takes into account other systematic and atmospheric errors at the satellite \( k \) and receiver \( i \) on L1 and L2 frequencies:

\[ P_{i,1}^k = \rho_i^k + d \rho^k + \frac{t_i^k}{f_c} + T_i^k + c(dt_i - dt^k) + b_{i,2} + M_{i,1}^k + e_{i,1}^k \]  

(2-3)
\[ p_{i,2}^k = \rho_i^k + dp^k + \frac{t_i^k}{f_2^k} + T_i^k + c(d_t_i - d_t^k) + b_{i,3} + M_{i,1}^k + e_{i,2}^k \] (2-4)

- \( P_{i,1}^k; P_{i,2}^k \) - Pseudo-ranges measured between satellite \( k \) and receiver \( i \) on \( L_1 \) and \( L_2 \), respectively

- \( \frac{t_i^k}{f_2^k}; \frac{t_i^k}{f_2^k} \) - Ionospheric delay between satellite \( k \) and receiver \( i \) on \( L_1 \) and \( L_2 \), respectively

- \( f_1; f_2 \) - Carrier frequencies, above referred to as \( L_1 \) and \( L_2 \)

- \( T_i^k \) - Tropospheric delay between receiver \( i \) and satellite \( k \)

- \( M_{i,1}^k; M_{i,2}^k \) - Multipath error on the pseudo range for the \( L_1 \) and \( L_2 \), respectively

- \( e_{i,1}^k; e_{i,2}^k \) - Measurement noise on pseudo-ranges on \( L_1 \) and \( L_2 \), respectively

- \( b_{i,2}; b_{i,3} \) - The interchannel biases between \( \phi_{i,1}^k \) and \( p_{i,1}^k \), and \( \phi_{i,1}^k \) and \( p_{i,2}^k \), respectively

The carrier phase measurement is calculated by the phase difference between the transmitted signal from the satellite and the signal generated by the receiver oscillator (Hofmann-Wellenhof et al., 2001). The carrier phase is scaled to unit length by multiplying by the carrier wavelength, \( \lambda_1 = \frac{c}{f_1} \). The phase observable is expressed as the sum of the fractional carrier phase and an unknown integer constant representing full waves (Leick, 2004). The fractional component is what is actually recorded by the receiver that is the phase differences between the arriving phases and internally generated receiver phases. The integer component, also known as an integer ambiguity, is an
unknown number of phase cycles at the starting epoch between the satellite and the receiver, which exists because the receiver has no way of knowing when the carrier wave left the satellite. The integer ambiguities are resolved when the carrier-phase data are processed. These integer ambiguities will remain constant for a satellite, as long as the tracking of that satellite is continuous. A positioning solution, in which the ambiguities are approximated by real numbers, is called a float solution and is less accurate than the fixed solutions, where the ambiguities are found in terms of integer cycles.

A phase cycle slip or a loss of lock will introduce a new unknown ambiguity. A cycle slip is a sudden jump in the carrier phase observable, generally, by an integer number of cycles. This can occur due to signal blockage by buildings, trees, etc. (loss of lock), due to receiver malfunction, such as by severe ionospheric distortion or by signal interference. A cycle slip results in all subsequent measurements being offset by a constant integer number of cycles. Despite the additional complications of solving for an integer ambiguity and cycle slips, carrier phase measurements are typically more accurate than pseudo-range measurements, with the typical noise of the measurements being on the order of a few millimeters or less (Grejner-Brzezinska, 2011).

The GPS carrier-phase observables for a given satellite $k$ and receiver $i$ on $L_1$ and $L_2$ frequencies are given as:

$$
\phi_{i,1}^k = p_i^k + d\rho^k - \frac{i_k}{f_i^k} + T_i^k + \lambda_1 N_{i,1}^k + c(d_t^i - dt^k) + \lambda_1 (\phi_{0,1}^k - \phi_{i_0,1}^k) + n_{i,1}^k + \varepsilon_{i,1}^k
$$

(2-5)
\[
\phi_{i,2}^k = \rho_i^k + d\rho^k - \frac{\varphi_{i,0,2}^k}{\lambda_2} + T_1^k + \lambda_2 N_{i,2}^k + c(d_{1} - d_{2}) + b_{i,2} + \lambda_2 (\varphi_{0,2}^k - \varphi_{i,0,2}^k) + m_{i,2}^k + \\
\varepsilon_{i,2}^k
\]

(2-6)

The equations (2-5) and (2-6) differs from the pseudo-range equations (2-3) and (2-4) described above as follows:

\(\phi_{i,1}^k; \phi_{i,2}^k\) - Carrier phase ranges measured between satellite \(k\) and receiver \(i\) on \(L_1\) and \(L_2\), respectively

\(m_{i,1}^k; m_{i,2}^k\) - Multipath error on carrier phase ranges for \(L_1\) and \(L_2\), respectively

\(\varepsilon_{i,1}^k; \varepsilon_{i,2}^k\) - Measurement noise carrier phase ranges for \(L_1\) and \(L_2\), respectively

\(\lambda_1; \lambda_2\) - Wavelengths of \(L_1\) and \(L_2\) phases (\(\lambda_1 \approx 19\) cm and \(\lambda_2 \approx 24\) cm), respectively

\(N_{i,1}^k; N_{i,2}^k\) - Integer ambiguities associated with \(L_1\) and \(L_2\) carrier phase measurements, respectively

\(\varphi_{i,0,1}^k; \varphi_{i,0,2}^k\) - Initial fractional phases at the receiver \(i\) on \(L_1\) and \(L_2\), respectively

\(\varphi_{0,1}^k; \varphi_{0,2}^k\) - Initial fractional phases at the satellite \(k\) on \(L_1\) and \(L_2\), respectively

\(d\rho^k\) - Orbital error of satellite \(k\)

\(b_{i,1}\) - Inter-channel bias between \(\phi_{i,1}^k\) and \(\phi_{i,2}^k\)
Similar pseudo-range and carrier phase equations can be written for the new civil signal on the $L_5$ frequency.

The pseudo-range and carrier phase measurements are affected by systematic errors and random noises. These nuisance parameters (generally unknown) identified in the above equations are orbital errors, satellite and receiver clock errors, tropospheric and ionospheric errors, multipath, inter-channel biases and integer ambiguities. These are usually removed by differential GPS processing or by a proper empirical model (e.g., tropospheric models), and processing of a dual frequency signal (ionosphere).

### 2.1.2 GPS Errors

The GPS code and phase observables, (equation 2-3 to 2-6), for a single frequency contain much more than the range measurement between the visible satellites and an observing antenna. Each term in these equations results in additional error in the GPS measurements, if not accounted for. Effective GPS surveying and by extension GPS-levelling depends on an understanding of the measurement error budget and eliminating or reducing those errors. The errors affecting GPS measurements originate from three sources, namely satellite errors, signal propagation errors and receiver errors. All three types of error sources affect the quality of the estimated ellipsoidal heights and the most significant will be discussed herein (Hofmann-Wellenhof, Lichtengegger and Collins, 2001; Leick, 2004).

The satellite and receiver clock errors occur due to a lack of synchronization between the precise atomic clocks of the satellites (smaller error) and the lower grade receiver clocks
(bigger error) with respect to GPS time. Satellite orbit errors occur when the course of a satellite deviates from its predicted course in a GPS navigation message, used by receivers to predict the position of a satellite at a particular instance in time. These errors can be corrected with precise orbit files for all satellites provided by the International GNSS Service (IGS) or significantly removed by differencing direct observables. The differencing techniques are discussed in some detail in Section 2.1.3 (Grejner-Brzezinska, 2011).

As the GPS signal passes through the atmosphere it is impeded and refracted by the charged particles of the ionosphere and the water vapor and dry gases in the troposphere. This causes deviations to the path of the signal resulting in group delay (code range is measured too long) and phase advance (phase range is measured too short) when the signal passes through the ionosphere. The atmospheric effects, particularly the tropospheric error, directly relate to the accurate determination of the GPS ellipsoid height, which is discussed in more detail in Section 2.3 below.

Multipath is a signal propagation error, which occurs when a signal arrives at a receiver through an indirect path (Braasch, 1996). This is a result of the reflection of the transmitted signal by objects in the area surrounding the receiver antenna (e.g., rooftops, building, trees, etc.). The measurement bias caused by signal multipath act differently, and unlike the other error sources, multipath is normally uncorrelated between antenna locations. Hence, the base and remote receivers experience different multipath interference. There have been numerous improvements to receiver and antenna technology (choke rings, ground planes), which aid in mitigating the effects of multipath.
Despite these technological advances, the best method for most GNSS users to mitigate multipath effects is to simply avoid it by carefully selecting receiver station sites that are free of any reflective interference.

At the receiver level the antenna phase center variations (PCV) are of great concern for accurate GNSS coordinate determination, specifically ellipsoid height determination. GNSS measurements are actually made with respect to the point in the antenna known as the phase center, not the survey mark. Corrections must be applied to reduce the measurement to the unknown point. It has been shown that the antenna phase center is not fixed and varies depending on the elevation of the satellite and also the frequency of the propagated signal (Leick, 2004). Further complications arise from the mixing of different antenna types, which may produce errors in the ellipsoidal heights of up to 10 cm (Rothacher, 2002). Estimated tropospheric parameters are also highly correlated with antenna phase center patterns, which may be incorrectly interpreted in processing software, resulting in amplified errors, especially in the height component. Thus, it is important to use the same antenna make and model for network surveys in order to reduce the errors caused by antenna phase center offsets. Although the mitigation of this error source seems simple compared to the complicated modelling of other error sources, this is a difficult task to manage, particularly for large networks (Fotopoulos, 2003).

Also at the receiver level is the receiver noise, which is a random error generated by the receiver as it processes the received signal to derive pseudo-range and carrier phase measurements. It is considered as white noise because the errors are not correlated over time. There is also no correlation between the code measurements and phase
measurements taken at the same time in a given receiver because these measurements are derived using separate tracking loops. The noise in the code measurements can be isolated from all other errors using a “zero-baseline” concept where two receivers are connected to the same antenna (Shrestha, 2003).

High accuracy GPS positions can only be obtained by mitigating the impact of these error sources. Some errors can be removed or mitigated from *a posteriori* information and modeling. For instance, precise (post-mission) satellite orbits are calculated to remove the error caused by deviations in satellite position. Precise orbits are applied in the post-processing of GPS data and are good up to 5-10 cm and better; available within 24-hour of the observations. Tropospheric error can be mitigated with different types of modeling and corrected for with values calculated from models. The ionospheric error is most commonly mitigated through a linear combination of the L1 and L2 frequencies, known as an ionosphere-free combination.

More troublesome errors are the effects of multipath, which cannot be removed through modeling. Some receivers utilize built-in multipath mitigation techniques, which help reduce the effects of multipath but do not fully remove it from the observables. Antenna ground planes and choke-rings are also employed to mitigate the effects of multipath. Multipath can also result in unpredicted antenna PCV, which can be mitigated through an *in-situ*, or environmental specific, antenna calibration. Additionally, error sources over short baselines can be removed through the use of differential GPS, discussed in the following Section 2.2. A summary of the magnitudes of the common error sources for the standard un-differenced GPS observables is given in Table 2.1.
Table 2.1. Summary of the magnitudes of common GPS error sources between undifferenced GPS and differenced GPS (Grejner-Brzezinska, 2011)

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Standard GPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite Clocks</td>
<td>2.0</td>
</tr>
<tr>
<td>Orbit Errors</td>
<td>2.1</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>5.0</td>
</tr>
<tr>
<td>Troposphere</td>
<td>~ 0.5 (model)</td>
</tr>
<tr>
<td>Receiver Noise</td>
<td>0.3</td>
</tr>
<tr>
<td>Multipath</td>
<td>1.0</td>
</tr>
<tr>
<td>PCV</td>
<td>0.1</td>
</tr>
</tbody>
</table>

In general, the deformation of the crust as a reaction to changing atmospheric pressure is at the level of 1 to 2 cm (Van Dam, Blewitt and Heflin, 1994). The larger displacement is due to ocean loading, which is more difficult to model and may cause height changes of more than 10 cm for stations situated near the coasts (Baker, Curtis and Dobson, 1995). This is important for GPS monitoring of tide gauge stations, which may be incorporated into vertical datum definitions. The best means to deal with these effects is to apply corrections to the estimated heights based on the global models (in conjunction with higher resolution local models, if they are available), which are designed to predict the response to loads. Also, GPS observations over a 24-hour period average out most of the loading error. However, shorter occupation times may lead to significant biases in the estimated ellipsoidal heights if appropriate corrections are not applied (Fotopoulos, 2003; Meyer, Roman and Zilkoski, 2006).
2.2 Differential Global Positioning System

Differential Global Positioning System (DGPS), is based on the use of two or more receivers, where one (stationary) reference or base receiver is located at a known point and the position of the remote (usually roving) receiver is to be determined. At least four common satellites must be tracked simultaneously by both stations. These data are then processed by differencing the respective observables, either pseudo-range or carrier phase, from both stations (Hofmann-Wellenhof, Lichtengegger and Collins, 2001). Differencing is used to reduce or eliminate most of the common errors in the observations at both stations forming a baseline, such as orbital error, satellite clock error, receiver clock error, and atmospheric effects. However, the multipath and receiver noise errors are not eliminated by DGPS. The reduction of orbital and atmospheric (ionospheric and tropospheric) effects is highly correlated to the baseline length between the receivers. The ionospheric effects vary considerably if the baseline length is larger than 5-10 km. Tropospheric effects change less rapidly, and generally under normal conditions, they remain relatively constant for baseline length shorter than 50-60 km (Grejner-Brzezinska, 2011). Three differencing methods are discussed in the following section, single, double and triple differencing.

2.2.1 Single Difference

A single difference is obtained by simultaneously collecting data from one satellite at two receivers separated by a baseline. The concept of the single difference is illustrated in Figure 2.2 between two separate receivers, \( i \) and \( j \), simultaneously receiving measurements from satellite \( k \). Single differencing eliminates satellite clock and orbit
errors, as well as the initial fractional phase term for satellite $k$ at the initial epoch of observation. In addition, atmospheric errors caused by the ionospheric and tropospheric delay can be significantly reduced over short baselines.

![Diagram of single difference mode geometry in differential GPS between receivers $i$ and $j$ and satellite $k$](image)

*Figure 2.2. Concept of the single difference mode geometry in differential GPS between receivers $i$ and $j$ and satellite $k$; where $\rho$, is the geometric distance between the satellite and receiver*

The observation equations for pseudo-ranges and carrier phase ranges can be expressed with the following equations. Note that these equations are shown for a single frequency ($L_1$); additional frequencies can be incorporated in the same manner:

$$P_{i,1}^k = \rho_i^k + d\rho^k + \frac{l_i^k}{f_1^2} + T_i^k + c(dt_i - dt_i^k) + b_{i,2} + M_{i,1}^k + e_{i,1}^k$$  (2-7)
\[ P_{ij,1}^k = \rho_{ij}^k + d\rho^k + \frac{t_{ij}^k}{f_1^k} + T_{ij}^k + c(dt_j - dt_k) + b_{ij,2} + M_{ij,1}^k + e_{ij,1} \]  
(2-8)

\[ \phi_{ij,1}^k = \rho_{ij}^k + d\rho^k - \frac{t_{ij}^k}{f_1^k} + T_{ij}^k + \lambda_1 N_{ij,1}^k + c(dt_j - dt_k) + \lambda_1 (\varphi_{ij,0}^k - \varphi_{ij,1}) + 
\]
\[ m_{ij,1} + \epsilon_{ij,1}^k \]  
(2-9)

\[ \phi_{ij,1}^k = \rho_{ij}^k + d\rho^k - \frac{t_{ij}^k}{f_1^k} + T_{ij}^k + \lambda_1 N_{ij,1}^k + c(dt_j - dt_k) + \lambda_1 (\varphi_{ij,0}^k - \varphi_{ij,1}) + 
\]
\[ m_{ij,1} + \epsilon_{ij,1}^k \]  
(2-10)

The terms in equations (2-7) to (2-10), were defined and explained in equations (2-3) to (2-6). A single difference measurement for the pseudo-range and carrier phase measurements can then be formulated for each pair of observations from satellite, \( k \). The L$_1$ single-differenced pseudo-range and carrier phase measurements from the two receivers, \( i \) and \( j \) and satellite, \( k \), can be expressed with the following equations:

\[ P_{ij,1}^k = P_{j,1}^k - P_{i,1}^k \]  
(2-11)

\[ P_{ij,1}^k = \rho_{ij}^k + \frac{t_{ij}^k}{f_1^k} + T_{ij}^k + c dt_{ij} + b_{ij,2} + M_{ij,1}^k + e_{ij,1}^k \]  
(2-12)

\[ \phi_{ij,1}^k = \phi_{j,1}^k - \phi_{i,1}^k \]  
(2-13)

\[ \phi_{ij,1}^k = \rho_{ij}^k - \frac{t_{ij}^k}{f_1^k} + T_{ij}^k + \lambda_1 \Delta N_{ij,1}^k + c dt_{ij} + m_{ij,1}^k + e_{ij,1}^k \]  
(2-14)

where:

\[ N_{ij,1}^{k*} = N_{ij,1}^k + (\varphi_{ij,0}^k - \varphi_{ij,1}) \]  
(2-15)

\[ N_{ij,1}^{k*} \quad \text{- The non-integer bias, including the ambiguity term} \]

\[ P_{ij,1}^k \quad \text{- The single differenced pseudo-range measurement for the L1 frequency} \]
\[ \phi_{ij}^k - \text{The single differenced carrier phase range measurement for the L1 frequency} \]

Performing single differences eliminates the satellite clock error, and significantly reduces or eliminates orbital errors and atmospheric effects (ionospheric and tropospheric delay) as a function of the length of the baseline between receivers (Hofmann-Wellenhof, Lichtengegger and Collins, 2001).

### 2.2.2 Double Difference

A double differenced measurement is formed by two receivers simultaneously observing two satellites, and is formulated through two steps. To obtain a double difference, two single differences are subtracted. A double-differenced solution eliminates the satellite clock error, receiver clock error, and reduces or eliminates orbital errors and atmospheric effects. Like single differencing, double differencing DGPS may eliminate or reduce the atmospheric errors depending on the length of the baseline. Any remaining atmospheric error after differencing (for long baselines) may be removed through the estimation process by modeling the delays in the GPS observation equation. The canceling effect of the receiver clock biases is the reason why the double differences are preferred. The cancellation results from the assumption of simultaneous observations and equal frequencies of the satellite signals (Hofmann-Wellenhof, Lichtengegger and Collins, 2001).

For the development of the double differenced observations, consider two receivers, \( i \) and, \( j \) and two satellites, \( k \) and, \( l \) as shown in Figure 2.3. The double differenced
observation equations are developed and, once again, considered only the L1 frequency, because development of the observations for the L2 equations is similar.

Figure 2.3. Concept of double difference mode geometry in differential GPS between satellites k and l and receivers i and j; where \( \rho \) is the geometric distance between the satellite and receiver

The single differenced equations for satellite, k and receivers, i and j, and satellite, l and receivers, i and j, are computed for the L1 frequency below, according to equations (2-11) to (2-14).

\[
P_{ij,1}^k = \rho_{ij}^k + \frac{i_{ij}^k}{f_1^k} + T_{ij}^k + c.d\,t_{ij} + b_{ij,2} + M_{ij,1}^k + e_{ij,1}^k
\]  

(2-16)
\[ P_{ij,1} = \rho_{ij} + \frac{i_{ij}}{f_l^2} + T_{ij}^l + c \cdot dt_{ij} + b_{ij,2} + M_{ij,1}^l + e_{ij,1}^l \] (2-17)

\[ \phi_{ij,1}^k = \rho_{ij} + \frac{i_{ij}}{f_l^2} + T_{ij}^l + \lambda_1. N_{ij,1}^k + c \cdot dt_{ij} + m_{ij,1}^l + \varepsilon_{ij,1}^l \] (2-18)

\[ \phi_{ij,1}^l = \rho_{ij} - \frac{i_{ij}}{f_l^2} + T_{ij}^l + \lambda_1. N_{ij,1}^l + c \cdot dt_{ij} + m_{ij,1}^l + \varepsilon_{ij,1}^l \] (2-19)

The double-differenced pseudo-range and carrier phase measurements for the L1 frequency can then be formulated by taking the single difference of the two single differences.

\[ P_{ij,1}^{kl} = \Delta P_{ij,1}^l - \Delta P_{ij,1}^k \Leftrightarrow (P_{ij,1}^l - P_{ij,1}^l) - (P_{ij,1}^k - P_{ij,1}^k) \] (2-20)

\[ P_{ij,1}^{kl} = \rho_{ij}^{kl} + \frac{i_{ij}}{f_l^2} + T_{ij}^{kl} + M_{ij,1}^{kl} + e_{ij,1}^{kl} \] (2-21)

\[ \phi_{ij,1}^{kl} = \Delta \phi_{ij,1}^l - \Delta \phi_{ij,1}^k \Leftrightarrow (\phi_{ij,1}^l - \phi_{ij,1}^l) - (\phi_{ij,1}^k - \phi_{ij,1}^k) \] (2-22)

\[ \phi_{ij,1}^{kl} = \rho_{ij}^{kl} - \frac{i_{ij}}{f_l^2} + T_{ij}^{kl} + \lambda_1. N_{ij,1}^{kl} + m_{ij,1}^{kl} + \varepsilon_{ij,1}^{kl} \] (2-23)

where:

\[ P_{ij,1}^{kl}; \phi_{ij,1}^{kl} - The \ double \ differenced \ pseudo-range \ and \ carrier \ phase \ measurements \ for \ the \ L1 \ frequency, \ respectively. \]

The remaining notations used in equation (2-20 to 2-23), were defined and explained in equations (2-3) to (2-6). Performing double differences eliminates the satellite clock error, receiver clock error, significantly reduces (as a function of baseline length) or eliminates orbital errors and atmospheric effects (ionospheric and tropospheric delay) as a function of the length of the baseline between receivers.
2.2.3 Triple Difference

While the triple difference may seem to be the most ideal of the differential combinations to use, it is often not used in practice when the highest accuracy is needed due to its noise level, but for completeness it is discussed here. A triple difference is obtained by differencing two double differences separated by a time interval, $dt$. For carrier phase observations, a triple difference will cancel the phase ambiguity biases, $N_1$ and $N_2$. A triple difference results in a pseudo-range and carrier phase observation with only the coordinates of the receiver as an unknown.

The noise for each differential combination is amplified by $\sqrt{2}$ for each successive difference, due to mathematical correlations according to the law of error propagation. Thus, the noise in a single difference is increased by a factor of $\sqrt{2}$, a double difference increases the noise by a factor of 2, and a triple difference increases the noise by a factor of $2\sqrt{2}$, when compared to the noise level of a non-differential solution. The advantage of triple differences is the canceling effect for the ambiguities and thus the immunity from changes in the ambiguities. Such changes are referred to as the cycle slips (Hofmann-Wellenhof, Lichtengegger and Collins, 2001). Recall that an integer ambiguity remains constant until a cycle slip occurs. Each cycle slip results in an outlier detectable in the triple difference at the instance it occurs (Leick, 2004).

The triple-differenced pseudo-range and carrier phase observations are given below. As for the other differential combinations, the observations are given only for the $L_1$
frequency, as the formulation of the equations for the L2 frequency is similar. The terms in equations (2-24) to (2-27), were defined and explained in equations (2-3) to (2-6).

\[
P_{ij,1,dt}^{kl} = P_{ij,1,t_2}^{kl} - P_{ij,1,t_1}^{kl}
\]  

(2-24)

\[
P_{ij,1,dt}^{kl} = \rho_{ij,dt}^{kl} + \frac{i_{ij,dt}^{kl}}{f_i^2} + T_{ij,dt}^{kl} + M_{ij,1,dt}^{kl} + e_{ij,1,dt}^{kl}
\]  

(2-25)

\[
\phi_{ij,1,dt}^{kl} = \phi_{ij,1,t_2}^{kl} - \phi_{ij,1,t_1}^{kl}
\]  

(2-26)

\[
\phi_{ij,1,dt}^{kl} = \rho_{ij,dt}^{kl} - \frac{i_{ij,dt}^{kl}}{f_i^2} + T_{ij,dt}^{kl} + m_{ij,1,dt}^{kl} + \varepsilon_{ij,1,dt}^{kl}
\]  

(2-27)

2.3 **Tropospheric Modelling**

Of the atmospheric effect and the overall GPS error budget, the tropospheric delay has the greatest effect on GPS ellipsoidal heights. Brunner and Welsch (1993) stated that the tropospheric delay errors mainly affect the accuracy of height differences. Furthermore, this delay must be considered the main limitation of the attainable height accuracy using GPS. Therefore under the umbrella of height modernization, improving the accuracy of the GPS ellipsoidal height involves addressing the effects of neutral atmospheric (tropospheric) delay on ellipsoidal height.

In this section, a general overview of the structure of the atmosphere is presented with a brief review of the ionosphere and the ionosphere-free linear combination. An introduction of the tropospheric refractive index is given. The effect of the tropospheric components on the path of a GPS signal is derived with mathematical expressions. The estimation of the tropospheric delay components is explained for different models and mapping functions.
2.3.1 The Atmosphere and Its Structure

A radio signal like that of GNSS, encounters different atmospheric conditions as it travels from the radio source (satellite) to the receiver, through various layers as shown in Figure 2.4. The atmosphere affects the traveling signal, which results in changes to the velocity and direction of the propagating signal, as already discussed (Mendes, 1999). This effect bends the signal and introduces some delay in the arrival time of the signal, depending on the refractive index of various atmospheric layers along the actual path (Davis et al., 1985; Mendes, 1999). Additionally, the atmosphere can also be divided into two main regions (Figure 2.4), based on the ionization: the ionosphere, for the ionized region with the presence of free electrons, and the neutral atmosphere for the electrically neutral region (Mendes, 1999). The ionosphere and the neutral atmosphere are two of the most important sources of errors in modern space based geodetic systems, and have a direct impact on the measurements.

The Ionosphere

The ionosphere is the combination of the mesosphere, thermosphere and some parts of the exosphere. The ionosphere contains electrically charged particles and free electrons, and extends approximately from 50 km to 1000 km (Hoyle, 2005). The ionosphere includes a large number of free electrons originating from the sun’s ultraviolet energy. These free electrons change the refraction index of this region, affect the propagating speed, and bend the GPS signals. The magnitude of range error due to bending is negligible when the satellite elevations angle is larger than five degrees. However, the
change in the propagation speed causes a significant range error, and therefore should be accounted for in the GPS observation equations.

Figure 2.4. Atmospheric Profile, illustrating the thermal profiles of the troposphere with appropriate altitude of temperature regions

The ionosphere causes phase advance in carrier phase and group delay in pseudo-range measurements by the same amount (El-Rabbany, 2006). In other words, the pseudo-range is measured longer and the carrier phase range is measured shorter than the true range. Ionospheric propagation group delay effects on GPS signals cause most of the residual receiver error. These delay effects vary considerably depending on random effects, the
time of day, season of the year and the activity state of the 11-year period solar (sunspot) cycle (McDonald, 2002).

The ionosphere is a dispersive medium at microwave frequencies. Hence, the delay, to the first order, is inversely proportional to the square the frequency \( \left( \frac{1}{f^2} \right) \) of the electromagnetic radiation (McDonald, 2002; Dobson et al., 1996). The ionospheric delay can be determined and removed up to the first order with high accuracy (up to cm level) for long baselines by using dual-frequency receivers that allow formulation of ionosphere-free linear combinations (El-Rabbany, 2006). For example the double difference ionosphere free phase measurement, \( \phi_{ij,1}^{kl} \) is:

\[
\phi_{ij,1}^{kl} = \alpha_1 \phi_{ij,1}^{kl} + \alpha_2 \phi_{ij,2}^{kl}
\]  

\[
\phi_{ij,1}^{kl} = \rho_{ij}^{kl} + T_{ij}^{kl} + \alpha_1 \lambda_1, N_{ij,1}^{kl} + \alpha_2 \lambda_2, N_{ij,2}^{kl} + \alpha_1 \epsilon_{ij,1}^{kl} + \alpha_2 \epsilon_{ij,2}^{kl}
\]

Similarly, ionosphere-free pseudo-range \( (P_{1,2}) \) can be obtained:

\[
P_{1,2} = \alpha_1 P_1 + \alpha_2 P_2
\]

In ionosphere free linear combinations, coefficients \( \alpha_1 \) and \( \alpha_2 \) can be defined as:

\[
\alpha_1 = 1
\]  

\[
\alpha_2 = \frac{f_2^2}{f_1^2}
\]  

or

\[
\alpha_1 = \frac{f_1^2}{f_1^2 - f_2^2}
\]  

\[
\alpha_2 = -\frac{f_2^2}{f_1^2 - f_2^2}
\]
Where, equations (2-30) and (2-31), are derived using the assumption $\alpha_1 = 1$ and, equations (2-32) and (2-33), follows the condition, $\alpha_1 + \alpha_2 = 1$.

The disadvantages to the ionosphere-free linear combination are the relatively higher observation noise and the loss of the integer nature of the ambiguities, in the carrier phase equation. As such, the ionosphere-free linear combination is not recommended for short baselines (El-Rabbany, 2006; Hofmann-Wellenhof, Lichtenegger and Collins, 2001). Ionospheric errors, under normal ionospheric conditions if uncorrected, can contribute the largest single propagation error, approximately 5 meters, to GPS operation.

**The Neutral Atmosphere**

The neutral atmosphere consists of several layers defined by characteristics, such as temperature, pressure, and chemical composition. The closest layer to the Earth is the troposphere, extending from the Earth's surface to a height of approximately 15 km, which is composed of dry gases and water vapor. The thin region at the top of troposphere is called the tropopause. The tropopause includes characteristics of both the troposphere and stratosphere. The stratosphere extends from 16 km up to 50 km above the Earth’s surface. The troposphere, tropopause, and stratosphere are considered as the neutral atmosphere because they are electrically neutral. In GPS terminology, the term troposphere generally refers to the neutral atmosphere extending from the Earth’s surface to 50 km altitude.

GNSS signals travelling through the troposphere suffer from the effects of tropospheric attenuation, delay and short-term variations (scintillation). The magnitudes of these
effects are a function of satellite elevation and atmospheric conditions, such as temperature, pressure and relative humidity during signal propagation (Brunner & Welsch, 1993). The troposphere is non-dispersive for GPS (microwave) frequencies, which indicates that the tropospheric delay is not frequency dependent. Therefore, it cannot, unlike its counterpart, the ionospheric effect, be cancelled through the use of dual-frequency measurements. Not compensating for the total tropospheric delay can induce pseudo-range and carrier-phase errors from about 2 m in the zenith direction that increase to more than 20 m for satellites near the horizon (10° elevation) (Leick, 2004).

The subsection 2.3.2 below, concisely reviews the tropospheric theory and effects on GPS derived heights.

The impact of the troposphere delay on a baseline can be divided into two parts (Beutler et al., 1988; Rothacher, 2002):

1. Relative troposphere biases caused by errors of tropospheric refraction at one endpoint of a baseline relative to the other endpoint
2. Absolute troposphere biases caused by errors of tropospheric refraction common to both endpoints of a baseline

The relative tropospheric biases lead primarily to an inaccurate station height, whereas absolute troposphere errors produce scale biases of the estimated baseline lengths. The general estimate of the station height bias due to a relative troposphere error may be computed as (Beutler et al., 1988; Rothacher, 2002):

\[ \Delta h = \frac{\Delta \rho}{\cos z_{\text{max}}} \]  

(2-34)
where:

\[ \Delta h \quad - \text{is the induced station height error} \]
\[ \Delta \rho^0_t \quad - \text{is the relative tropospheric zenith delay error} \]
\[ z_{max} \quad - \text{is the maximum zenith angle of the observation scenario (cutoff)} \]

The above equation indicates that a relative troposphere bias of only 1 cm leads to an error of approximately 3 cm in the estimated relative station height for an elevation cutoff angle of 15°. This error increases in magnitude as the elevation cutoff angle decreases.

The absolute tropospheric biases alternatively are not critical for heights. However, a bias of 10 cm in the troposphere zenith delay at both stations induces a scale bias of 0.05ppm for an elevation cutoff angle of 20°. This is a comparatively small effect compared to the height error caused by a relative troposphere bias. Nevertheless, the effect should be taken into account for baselines longer than about 20 km.

The influence of these biases must be reduced to make full use of the accuracy of the observable by either of the following methods (Rothacher, 2002):

1. Model tropospheric refraction, for example using standard atmosphere, ground meteorological measurements or water vapor radiometers data and apply this correction to the GPS measurement
2. Estimate troposphere parameters (e.g., zenith path delays) in the general GPS parameter estimation process
Ultimately, the best means to deal with tropospheric effects for high precision height determination is to improve measurements and models for water vapor content (Dobson, 1995).

2.3.2 Tropospheric Effects on GNSS Signals and Refractivity

Refractive Index and Refractivity

The refractive index of a medium, \( n \), is defined as the ratio of the speed of propagation of an electromagnetic wave in vacuum, \( c \), to its propagation speed, \( v \), in the medium as:

\[
n = \frac{c}{v}
\]  

(2-35)

The refractive index is more conveniently expressed in terms of refractivity (\( N \)) \(^1\)

\[
N = (n - 1)10^{-6}
\]  

(2-36)

The refractivity can be derived according to Smith and Weintraub (1953). They gave an expression relating the physical parameters of temperature (\( T \) in Kelvin), partial pressure due to dry gases (\( P_d \) in millibars) and partial water vapor pressure (\( e \) in millibars) to refractivity (\( N \)) hence:

\[
N = K_1 \left( \frac{P_d}{T} \right) + K_2 \left( \frac{e}{T} \right) + K_3 \left( \frac{e}{T^2} \right)
\]  

(2-37)

\( K_1, K_2, K_3 \) - Refractivity constants \( \left( \frac{°K}{mb}, \frac{°K}{mb}, \frac{°K^2}{mb} \right) \)

\(^1\) It should be noted that this \( N \) term for refractivity is not related to the geoid undulation term \( \mathbf{N} \) seen in equation (1-1).
The values of $K_1, K_2, K_3$ are empirically determined and cannot fully describe the local situation. An improvement is obtained by measuring meteorological data at the observation site (Hofmann-Wellenhof, Lichtengegger and Collins, 2001).

Equation (2-37) is not the only formula available, Thayer (1974) extended the general refractivity equation given by Smith and Weintraub (1953) with the compression factors for non-ideal gas behavior, and thus, provided a set of refractivity constants for the frequencies below 20 GHz. Additionally, Davis et al. (1985) noted that the integration of the tropospheric refractivity in the form given by Thayer (1974) requires knowledge of the profiles for both the dry and wet components (also referred to as hydrostatic and non-hydrostatic respectively), the mixing ratio of which is highly variable. Therefore, developing an alternative equation for the tropospheric refractivity to create an expression nearly independent of this mixing ratio by using the equation of state, Thayer (1974) extended the general refractivity equation:

$$N = \left[ K_1 \left( \frac{P_d}{T} \right) \right] Z_d^{-1} + \left[ K_2 \left( \frac{e}{T} \right) + K_3 \left( \frac{e}{T^2} \right) \right] Z_w^{-1} \quad (2-38)$$

where $Z_d^{-1}$ and $Z_w^{-1}$ are the inverse compressibility factors for dry air and water vapor, respectively. These terms are empirical factors and are usually modeled as a function of pressure and temperature. The determined refractivity constants and the corresponding uncertainties are:

$P_d$ - Partial pressure of the dry gases (mb)

$e$ - Partial pressure of water vapor (mb)

$T$ - Absolute temperature ($^\circ$K)
\[ K_1 = 77.604 \pm 0.014 \, ^\circ K/mb \]
\[ K_2 = 64.79 \pm 0.08 \, ^\circ K/mb \]
\[ K_3 = (3.776 \pm 0.004) \times 10^5 \, ^\circ K^2/mb \]

From equation (2-38), the terms enclosed in the first bracket depend on the dry gases, and represent the “dry component of the refractivity”, whereas, the terms in second bracket depend on the water vapor content, and represent the “wet component of the refractivity” (Mendes, 1999). Given the above refractivity constants and the corresponding uncertainties for frequencies less than 20 GHz, the refractivity can be determined within 0.02% (Thayer, 1974).

The derivative of the alternative equation for the tropospheric refractivity developed by Davis et al. (1985) will not be shown here. However their final tropospheric refractivity equation from the derivative and the related terms is described below. Equation (2-39) shows the Davis et al. (1985) final tropospheric refractivity:

\[ N = [K_1 R_d \rho] + \left[K'_2 \left(\frac{\rho}{T}\right) + K_3 \left(\frac{\rho}{T^2}\right)\right] Z_w^{-1} \]

\[ K'_2 = \left[K_2 - K_1 \left(\frac{R_d}{R_w}\right)\right] = 17 \pm 10 \, ^\circ K/mb \]

\[ K_1 = 77.604 \pm 0.014 \, ^\circ K/mb \]
\[ K_2 = 64.79 \pm 0.08 \, ^\circ K/mb \]
\[ K_3 = (3.776 \pm 0.004) \times 10^5 \frac{\circ K^2}{mb} \]

\[ \rho = \rho_d + \rho_w - \text{The total density equation (kilogram/Liter (kg/L))} \]

\[ \rho_d, \rho_w - \text{Density of dry air and water vapor (kg/L)} \]

\[ R_d, R_w - \text{Specific gas constant for dry air and water vapor (J kmol}^{-1} \circ K) \]

\[ Z_w^{-1} - \text{The inverse compressibility factor for water vapor} \]

\[ e - \text{Partial pressure of water vapor (mb)} \]

\[ T - \text{Absolute temperature (} \circ K) \]

As previously shown, the refractivity \( N \) of the troposphere can be separated into dry \( (N_d) \) and wet \( (N_w) \) components. Therefore the refractivity, equations (2-37), (2-38) and (2-39) above can be expressed as the sum of the dry and wet components. As an example total refractivity, equation (2-39), can be rewritten as (Davis et al., 1985):

\[ N = N_d + N_w = \left[ K_1 R_d \rho \right]_{N_d} + \left[ K_2 \frac{e}{T} \right]_{N_w} + K_3 \left( \frac{e}{T^2} \right) \]  \hspace{1cm} (2-40)

**Tropospheric Delay**

The electromagnetic (microwave) signal passing through the Earth’s troposphere does not travel along a geometrically straight line \( \rho_a^k \), as it would if it were in a vacuum. Instead the signal traces a curved path \( S \), which not only bends (Figure 2.5) but also delays (Dobson et al., 1996). This generates some excess length along the ray path, which is directly related to the refractive index of the medium.
Figure 2.5. Effect of the atmospheric refraction on GNSS signal propagation in a horizontally stratified atmosphere - S represents the curved ray (true) path, and $\rho_{ak}$ represents geometric distance (direct path) between the satellite and receiver (Dobson et al., 1996)

Combining the geometrically straight line distance ($\rho_{ak}$), equation (2-41), that is conveniently used in GPS data analysis, and the path defined as the minimum signal path length ($L$), equation (2-42), where ($n$) is the varying refractive index of the atmosphere:

$$\rho_{ak} = \int_{\text{path}} d\rho_{ak} \quad \text{(2-41)}$$

$$L = \int_{\text{path}} n \, ds \quad \text{(2-42)}$$

an expression defining the excess path length ($L - \rho_{ak}$) Dobson et al. (1996) and Bevis et al. (1992):
\[ (L - \rho_a^k) = \int_{path} [n - 1] \, ds + [S - \rho_a^k] \]

The “Delay” term in equation (2-43) is the tropospheric delay \( T \) that is directly proportional to the refractive index or refractivity. It can be expressed as a function of atmospheric temperature and pressure (Bevis et al., 1992).

The tropospheric delay can be computed through the integration along the signal path through the troposphere using following expression:

\[ T = \int n - 1 \, ds \] (2-44)

The tropospheric delay expressed in terms of refractivity takes the following form:

\[ T = 10^{-6} \int N \, ds \] (2-45)

The tropospheric delay can also be expressed in terms of the dry and wet refractivity components. Hence the total tropospheric delay is often represented as a linear combination of the hydrostatic delay \( (HD) \) and wet \( (WD) \) delay components:

\[ T = 10^{-6} \int N_d \, ds + 10^{-6} \int N_w \, ds \] (2-46)

These two component effects on the propagation of the GPS signal are different. The dry component accounts for 90% of total tropospheric delay and consists mostly of dry gases. It can be computed from the temperature and pressure measured at the receiver. The variation of water vapor in the atmosphere varies greatly spatially and temporally, making the wet component difficult to model efficiently. As most of the water vapor in the atmosphere occurs at heights less than 4 km, signals from low elevation satellites,
which have a longer propagation path length through the troposphere, are the most affected. The wet delay contributes only 10% to the total tropospheric delay (Hofmann-Wellenhof, Lichtengegger and Collins, 2001; Leick, 2004).

In specific cases the tropospheric delay depends on the distance traveled by the radio wave through the neutral atmosphere, and thus it is also a function of the satellite’s zenith distance. To emphasize this elevation-dependence, the tropospheric delay in equation (2-46) can be written as the product of the delay in zenith direction. The tropospheric delay in the zenith direction, is called the zenith tropospheric delay, and can also be expressed in terms of the dry and wet components; therefore the total zenith tropospheric delay \( T_z \) is expressed as:

\[
T_z = 10^{-6} \int_{ZHD} N_d (H) \, dh + 10^{-6} \int_{ZWD} N_w (H) \, dh
\]

(2-47)

where:

- \( H \) - *Height above earth surface in meters.*
- \( ZHD \) - *Zenith hydrostatic (dry) delay (meter)*
- \( ZWD \) - *Zenith non-hydrostatic (wet) delay (meter)*

Equation (2-47) in its present form does not account for an arbitrary zenith angle of the signal. Considering the line of sight, an obliquity factor must be applied which, in its simplest form, is the projection from the zenith onto the line of sight. The transition of the zenith delay to a delay with arbitrary zenith angle is denoted as the application of a mapping function (Hofmann-Wellenhof, Lichtengegger and Collins, 2001). This
representation allows the use of separate mapping functions for the hydrostatic and wet delay components, as such equation (2-47) becomes:

\[ T(\varepsilon) = ZHD \, m_d(\varepsilon) + ZWD \, m_w(\varepsilon) \]  

(2-48)

\[ T(\varepsilon) \quad - \text{The tropospheric delay at the line-of-sight} \]

\[ m_h, m_w \quad - \text{Hydrostatic and Wet mapping functions} \]

\[ \varepsilon \quad - \text{Elevation angle at the observing site (expressed in degrees)} \]

### 2.3.3 Tropospheric Models

Modeling the tropospheric delay has been considered a critical error source in satellite based positioning and its research has paralleled the development of GPS technology (Mendes, 1999). It is difficult to measure the refractivity directly along the signal path. Various tropospheric models have been developed to represent the integrated tropospheric delay. Generally, surface meteorological parameters, such as pressure, temperature, and humidity are required input for these models. The zenith hydrostatic delay contributes about 90% of the total delay to the tropospheric delay (Hofmann-Wellenhof, Lichtengegger and Collins, 2001; Leick, 2004). Zenith hydrostatic delay models can be estimated with accuracies better than 1% where the zenith hydrostatic delay is considered to be a function of the surface pressure, and in some cases temperature, and hydrostatic equilibrium is assumed. The zenith wet delay contributes about 10% of the total delay, and the zenith wet delay models have accuracies of 10-20%. The wet component depends on water vapor, which is highly variable in the space and time thus difficult to model. Different models for example, the Hopfield (1969),
Saastamoinen (1973), and Goad and Goodman (1974) (modified Hopfield model) are used to estimate the tropospheric error. The Saastamoinen model (1973) is briefly explained below:

**Saastamoinen model (1973)**

The Saastamoinen model is broadly used in GPS data processing. The Saastamoinen (1973) model was used in the OPUS-Projects processing tool in this thesis and is therefore describe in some detail (derivations are not shown here).

Assuming hydrostatic equilibrium, the hydrostatic delay model may be expressed simply as a function of measured surface pressure. Saastamoinen (1973) employed this approach and used the following representation of gravity, $g$, in the zenith hydrostatic model.

\[
g = 9.784(1 - 0.0026 \cos 2\varphi - 0.00000028H) \tag{2-49}
\]

where:

- $\varphi$ - *The latitude of the station*
- $H$ - *The station height above sea level, in meters.*

Saastamoinen (1973) used the tropospheric refractivity constants from Essen and Froome (1951) to determine an expression for the zenith hydrostatic delay ($ZHD$) as follows:

\[
ZHD = 0.0022768 \frac{P_s}{(1-0.0026 \cos 2\varphi-0.00000028H)} \tag{2-50}
\]

- $P_s$ - *Surface pressure at the station (mb)*
In the case of the zenith wet delay model, Saastamoinen (1973) assumed that the water vapor pressure decreases with height, similarly to the total pressure decrease, but more rapidly. Therefore, for average conditions Saastamoinen’s zenith wet delay can be expressed by (Mendes and Langley, 1995):

\[
ZWD = 0.0022768 \left( \frac{1255}{T} + 0.05 \right) e
\]

\[e\] - Partial pressure due to water vapor (mb)

\[T\] - Temperature (°K)

The total Saastamoinen model errors in zenith direction were found to be 0.2 millimeter for the hydrostatic delay and 30 millimeters for the wet delay (Cove, 2005; Mendes, 1999).

2.3.4 Mapping Functions

The mapping function, \((m(\varepsilon))\), is defined as the ratio of the signal path length (also referred to as the delay) through the atmosphere at geometric elevation, \((\varepsilon)\), to the signal path length in the zenith direction. A mapping function is used to map the zenith delay to estimate the slant tropospheric delay. Several mapping functions have been developed. The simplest mapping function is given by, \(sin^{-1}(\varepsilon)\) (Niell, 2000) or the cosecant of the elevation angle. In this derivation, it is assumed that spherical constant height surfaces could be approximated as planar surfaces. This is an accurate approximation only for high elevation angles and with a small degree of bending. More complex mapping functions have been developed, and different mapping functions may be used for the
hydrostatic versus wet delays. Brief descriptions of the main features of various mapping
functions are given in the following:

Marini (1972) showed that the elevation angle dependence of the atmospheric delay for
azimuthally symmetric atmosphere can be expressed in a continued fraction in terms of
$\sin^{-1}(\varepsilon)$ (Marini 1972; Niell 1996).

$$
m(\varepsilon) = \frac{1}{\sin(\varepsilon)+\frac{a}{\sin(\varepsilon)+\frac{b}{\sin(\varepsilon)+\frac{c}{\ldots}}}} \quad (2-52)
$$

where the coefficients, $(a, b, c)$, are constants or linear functions which depend on
latitude and height of the observing site and on temperature, $T$ in Kelvin.

Chao (1972) mapping functions, the continued fraction is truncated to second order terms
and the second order, $\sin(\varepsilon)$ is replaced by, $\tan(\varepsilon)$, and the coefficients, $a$ and $b$, are
determined from empirical data. The hydrostatic ($m_h(\varepsilon)$) and wet ($m_w(\varepsilon)$) mapping
functions are expressed as follows:

$$
m_h(\varepsilon) = \frac{1}{\sin(\varepsilon)+\frac{0.00143}{\tan(\varepsilon)+0.0445}} \quad (2-53)
$$

$$
m_w(\varepsilon) = \frac{1}{\sin(\varepsilon)+\frac{0.00035}{\tan(\varepsilon)+0.017}} \quad (2-54)
$$

Herring (1992) has developed both hydrostatic and wet mapping functions by fitting to
radiosonde data from several North American stations ranging in geographic latitude
from 27° to 65° north for elevation angles down to 3°. The mapping function’s
coefficients depend linearly on surface temperature, the cosine of the station latitude, and the height of the station above the geoid. The expression for the mapping function is:

\[
m(\varepsilon) = \frac{\frac{1}{1 + \frac{a}{1 + \frac{b}{1 + c}}}}{\frac{1}{\sin(\varepsilon) + \frac{a}{\sin(\varepsilon) + \frac{b}{\sin(\varepsilon) + c}}}}
\]

\[m(\varepsilon) = \frac{1}{1+\left(\frac{a}{1+\frac{b}{1+c}}\right)}\]

where:

- \(a\), \(b\), and \(c\) - Constants or linear functions and

- \(\varepsilon\) - Elevation angle at the observing site (expressed in degrees)

The Niell (1996) mapping functions have no parameterization in terms of meteorological conditions, and they provide a better fit and give better accuracy over the latitude range 43° to 75° north for elevation angles down to 3°. This mapping function adopted the form of Marini (1972) expansion with three terms. The coefficients of the continued fraction representation of the hydrostatic mapping function depend on the latitude and height above sea level of the observing site (\(H\)) and on the day of the year. The wet mapping function depends only on the site latitude. The expressions for the hydrostatic and wet mapping functions are given below:

\[
m_h(\varepsilon) = \frac{1}{1+\frac{u_{ht}}{1+\frac{b_{ht}}{1+c_{ht}}}} + h \cdot 10^{-3} \left[ \frac{1}{\sin(\varepsilon)} \right] - \left[ \frac{1}{\sin(\varepsilon) \cdot \frac{b_{ht}}{\sin(\varepsilon) + c_{ht}}} \right]
\]
\[ m_w(\varepsilon) = \frac{1 + \frac{a_w}{b_w} + \frac{c_w}{1 + c_w}}{\sin(\varepsilon) + \frac{a_w}{1 + c_w}} \]

\[ m_h(\varepsilon) \quad - \text{Niell hydrostatic mapping function for the elevation angle of } \varepsilon \]

\[ m_w(\varepsilon) \quad - \text{Niell wet mapping function for the elevation angle of } \varepsilon \]

\[ a_h, b_h, c_h \quad - \text{The coefficients for hydrostatic mapping function} \]

\[ a_w, b_w, c_w \quad - \text{The coefficients for the wet mapping function} \]

\[ a_{ht}, b_{ht}, c_{ht} \quad - \text{Height corrections} \]

\[ H \quad - \text{The station height above sea level, in meters} \]

Boehm et al. (2006) developed a Global Mapping Function (GMF) for both hydrostatic and wet delays that is based on the data from the European Centre for Medium-range Weather Forecasts (ECMWF) Numerical Weather Model (NWM). The numerical weather models attempt to provide the spatial distribution of refractivity throughout the lower atmosphere with high resolution (Boehm et al., 2006).

The coefficients of the GMF were derived from the Vienna Mapping Function (VMF1), which is considered the most accurate mapping function in geodetic applications (Boehm et al., 2006). GMF was developed to be consistent with VMF1, but it does not require external inputs similar to NMF.

The GMF used the same form of the mapping functions as those of NMF, which is the Marini (1972) expression with three terms, but normalized to unity at the zenith direction,
as recommended by Herring (1992). GMF differs from NMF in the coefficients determination technique in the mapping functions (Boehm et al., 2006).

The coefficients of, $a_h$ and $a_w$, were derived from a 15°x15° global grid of monthly pressure, temperature, and humidity profiles provided by ECMWF. Then, the mean values, $a_o$ and the annual amplitudes, $(A)$, of the sinusoidal function were fitted to the time series of the parameters of “$a$” at each grid point. The residuals of the global grids associated with the mean values, $(a_o)$ and annual amplitudes, $(A)$, are in the sub-millimeter level (Boehm et al., 2006).

The hydrostatic ($a_h$) and wet ($a_w$) coefficients can be determined for any site, given the site coordinates and day-of-year, as shown below.

$$a_h = a_{0h} + A_h \cos \left( \frac{D_o Y - 28}{365} \cdot 2\pi \right)$$  \hspace{1cm} (2-58)

$$a_w = a_{0w} + A_w \cos \left( \frac{D_o Y - 28}{365} \cdot 2\pi \right)$$  \hspace{1cm} (2-59)

where:

$$a_{0h} = a_{0w} = \sum_{n=0}^{9} \sum_{m=0}^{n} P_{nm}(\sin \varphi) \left[A_{nm} \cos(m\lambda) + B_{nm} \sin(m\lambda)\right]$$  \hspace{1cm} (2-60)

$a_{0h}$, $a_{0w}$ - Global grid mean value of $a_h$ and $a_w$

$A$ - Annual amplitudes of $a_h$ and $a_w$

$P_{nm}(\sin \varphi)$ - Legendre associated functions (degree $n$ and order $m$)

$A_{nm}, B_{nm}$ - Spherical harmonic coefficients

$\varphi$ - Latitude

$\lambda$ - Longitude
The GMF coefficients, \( b_h \) and \( c_h \), for hydrostatic and wet parts were determined from the VMF1 empirical equations (Boehm et al., 2006). Therefore, GMF and VMF1 use the, \( b_h \) and \( c_h \), coefficients of Isobaric Mapping Function (IMF) from Niell (2002) for the hydrostatic part, and those of Niell Mapping Function (NMF) from Niell (1996) at 45° latitude for wet part (Boehm, Werl and Schuh, 2006).

In the case of hydrostatic part of GMF, the, \( b_h \) and \( c_h \), coefficients are given by (Boehm, Werl and Schuh, 2006), with the following equations:

\[
b_h = 0.0029
\]

\[
c_h = c_0 + \left( \left[ \cos \left( \frac{\text{DoY} - 28}{365} \cdot 2\pi + \psi \right) + 1 \right] \cdot \frac{c_{11}}{2} + c_{10} \right) \cdot (1 - \cos \varphi) \quad (2-61)
\]

\( c_0, c_{10}, c_{11} \) - Constants, obtained from Table 2.2

\( \psi \) - Specifies the northern or southern hemisphere (0 or \( \pi \))

\( \text{DoY} \) - Day of year

\( \varphi \) - Station latitude

Boehm et al. (2006a) provided a look-up table for the parameters, \( c_0, c_{10}, c_{11} \), and \( \psi \), needed for computing the coefficient, \( c_h \), of the hydrostatic part of GMF, as presented in Table 2.2.
Table 2.2. The parameters for $b_h$ to compute $c_h$ (Boehm, Werl and Schuh, 2006)

<table>
<thead>
<tr>
<th>Hemisphere</th>
<th>$c_0$</th>
<th>$c_{10}$</th>
<th>$c_{11}$</th>
<th>$\Psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern</td>
<td>0.062</td>
<td>0.001</td>
<td>0.005</td>
<td>0</td>
</tr>
<tr>
<td>Southern</td>
<td>0.062</td>
<td>0.002</td>
<td>0.007</td>
<td>$\pi$</td>
</tr>
</tbody>
</table>

The, $b_w$ and $c_w$, coefficients of the wet mapping function are fixed to those of NMF (Niell, 1996) at 45° latitude, as follows:

$$b_w = 0.00146$$

$$c_w = 0.04391$$

The hydrostatic and wet GMF can be derived from, equations (2-56) and (2-57), with mapping function coefficients given in this section.
CHAPTER 3: HEIGHT SYSTEMS

3.1 Introduction

Height observations are one of the most fundamental measurement types in geodesy and geodetic science related areas. According to the Geodetic Glossary (NGS, 2014), height is simply defined as, the distance, measured along a perpendicular, between a point and a reference surface (vertical datum). This definition captures the intuition behind heights, but the ambiguity of the reference surface, from which these measurements are to be made, must be clearly defined.

The shape and surface of the Earth are complicated to model, but they must be accurately modeled in order to determine both horizontal and vertical positions of a point on the Earth. As such, there are two main models have been established to approximate the shape, size and orientation of the Earth, the geoid and the ellipsoid.

The ellipsoid is a geometrically defined figure or model of the Earth, whose center is usually assumed to be at the center of mass of the Earth. It is generated by rotating an ellipse around its polar or minor axis. These models account for the slight flattening of the Earth at the poles (Hofmann-Wellenhof and Moritz, 2005; Jekeli, 2000). Conversely, the geoid is a closed, continuous and constant gravity potential surface, formally defined as an equipotential surface of the Earth’s gravity field, which coincides with the mean surface of the oceans. Unlike the ellipsoid, the geoid is not defined
mathematically (Hofmann-Wellenhof and Moritz, 2005; Torge, 2001). There are two practical options for identifying this desired reference surface Vaniček (1991):

1. The abstract option, whereby one specifies a constant value of Earth's gravity potential, \( W = W_0 = \text{const.} \), which defines the geoid as one horizontal surface.

2. The geometrical option, whereby one requires that the chosen horizontal surface (the geoid) approximates in a specific way the mean sea level surface. Traditionally the geometrical option is normally selected.

3.2 Heights

Accordingly, heights (or height differences) fall broadly into two categories based on the related Earth definition models: those that are naturally linked to the equipotential surfaces and plumb lines of the Earth’s gravity field, and those that employ a reference ellipsoid as their datum. The heights associated with these reference surfaces and their relationship are discussed in some detail in the following subsections. Before defining the height types, it is necessary to introduce the geopotential number since it is fundamental in the calculation of the height values.

3.2.1 Geopotential Number

A number of different height systems can be defined, which use the measurements of vertical increments between equipotential surfaces along a plumb lines, from spirit-levelling \((dn)\) and measurements of gravity, \(g\), as given by:

\[
C_p = \int_{P_0}^{P} g \cdot dn
\]  

(3-1)
The geopotential number, $C_P$, represents the difference in potential between the constant value at the geoid, and the potential at the point, $P$, on the surface, as follow (Hofmann-Wellenhof and Moritz, 2005):

$$C_P = W_O - W_P$$  \hspace{1cm} (3-2)

*where:*

- $C_P$ \textit{- Geopotential number}
- $W_O$ \textit{- Potential at the Geoid}
- $W_P$ \textit{- Potential at the point $P$ on the surface}
- $P$ \textit{- A point on the Earth’s surface}
- $P_o$ \textit{- A point on the Geoid}

The geopotential number, $C_P$, is independent of any particular levelling line connecting point $P$ to sea level (geoid). The geopotential numbers are measured in geopotential units (g.p.u.) where $1\text{ g.p.u} = 1\text{ kGal m} = 1000\text{ gal m}$ (Hofmann-Wellenhof and Moritz, 2005). All points have a unique geopotential number with respect to the geoid, and it can be scaled by gravity in order to obtain a height coordinate with units of length. Depending on the type of ‘gravity’ value used to scale the geopotential number, different types of heights can be derived. These height types include dynamic, normal and orthometric heights, which are briefly described and compared in the following subsections 3.2.2 to 3.2.5.
3.2.2 Orthometric Heights

According to Heiskanen and Moritz (1967) the orthometric heights are the natural heights above mean sea level, that is, heights above or below the geoid. Therefore, they have an unequalled geometrical and physical significance. More specifically the orthometric height is the distance of a surface point along the plumb line to the geoid, which is taken as the reference surface (Hofmann-Wellenhof and Moritz, 2005). The plumb line is defined as a line perpendicular to all equipotential surfaces of the Earth’s gravity field that intersects with it (Meyer, Roman and Zilkoski, 2006). Orthometric heights therefore represent the geopotential difference between two points, or the change in the potential of the Earth’s gravity between points. Hence, equation (3-1) along the plumb line becomes:

\[ C_p = \int_{P_0}^{P} g \cdot dH \]  \hspace{1cm} (3-3)

where \( dH \) is the differential element along the plumb line between the geoid and the point on the Earth's surface. Therefore the orthometric height of a point \( P \) on the Earth’s surface denoted as \( H_p \), and can be computed by the following equation (Hofmann-Wellenhof and Moritz, 2005):

\[ H_p = \frac{C_p}{\bar{g}_p} \]  \hspace{1cm} (3-4)

The mean value of gravity along the plumb line, \( \bar{g}_p \), is given by:

\[ \bar{g}_p = \frac{1}{H_p} \int_{P_0}^{P} g \cdot dH \]  \hspace{1cm} (3-5)
The exact computation of \( \bar{g}_P \) would require complete knowledge of the mass density of the crust, which is not practically available. Therefore, the determination of the orthometric heights depends on the approximation used in computing the mean value of gravity. Also, there should be caution taken when combining different types of heights or working with orthometric heights from different national sources, since they can be computed by different approaches (Fotopoulos, 2003).

From equations (3-4) and (3-5) it is evident that any number of orthometric heights systems may be obtained, depending on the selected value of \( \bar{g}_P \) (Fotopoulos, 2003). One of the most common orthometric height systems is Helmert height system, which is based on the Poincaré-Prey reduction model (Hofmann-Wellenhof and Moritz, 2005). In this commonly used approximation, a constant crustal density and a constant gravity gradient are assumed for the terrain point \( P \). The mean value of gravity along the plumb line is computed from the average of the gravity at the endpoints as follows (Hofmann-Wellenhof and Moritz, 2005):

\[
\bar{g}_p = g_p - 2\pi G\rho H_p + \frac{1}{2} \frac{\partial \gamma}{\partial h} H_p \tag{3-6}
\]

where

- \( H_p \) - Orthometric height of point \( P \)
- \( G \) - Newton’s gravitational constant \((66.7 \times 10^{-9} \text{cm}^3 \text{g}^{-1} \text{sec}^{-2})\)
- \( \bar{g}_p \) - Average gravity along the plumb line
- \( g_p \) - Gravity at a point \( P \) on the Earth’s surface
- \( \rho \) - Nominal value for crust density \((2.67 \text{ g/cm}^3)\)
\[ \frac{\partial \gamma}{\partial \ell} \] - Normal gravity gradient (0.3086 mGal/m)

A simplified version of equation (3-6) is given as:

\[ \bar{g}_p = g_p + 0.0424 H_p \]  \hspace{1cm} (3-7)

where the units associated with the coefficient, 0.0424 are mGal/m, and the height, \( H_p \), is given in meters. Substituting equation (3-7) into the orthometric height equation (3-4), the Helmert height is given as:

\[ H_p = \frac{c_p}{g_p + 0.0424 H_p} \]  \hspace{1cm} (3-8)

It should be noted that the computation of the mean gravity along the plumb line in this manner requires \( H_p \), therefore equation (3-7) is usually solved through iteration or by solving the quadratic represented in equation (3-4) with equation (3-6) and neglecting the terms beyond second order (see Jekeli, 2000 for full formulations).

### 3.2.3 Normal Heights

Although not directly used in this work, it is important to provide a brief overview of the normal height system, as it is the basis of heights in many regions worldwide. Normal height is introduced to avoid the hypothesis or modeling of the mass distribution of the topographic masses. This is attained by using the mean normal gravity field, which can be calculated exactly at any point. Hence, if the value for the mean gravity along the plumb line in equation (3-4) is replaced by the mean normal gravity along the plumb line, then the normal heights of a point, \( P \), denoted by \( H_p^* \) can be computed via:
\[ H^*_P = \frac{C_P}{\bar{\gamma}_P} \]  

(3-9)

where

\[ \bar{\gamma}_P = \frac{1}{H^*_P} \int_{P_0}^P \gamma \cdot dH^* \]  

(3-10)

- \( H^*_P \) - Normal height of point \( P \)
- \( \bar{\gamma}_P \) - Mean normal gravity along the plumb line
- \( \gamma \) - Normal gravity
- \( dH^* \) - Normal height difference

Both orthometric and normal heights have a clear geometrical interpretation, with the key difference being that the normal heights refer to the telluroid. The telluroid is defined as that surface where the potential of normal gravity is equal to the actual potential at the Earth's surface along the ellipsoidal normal. The telluroid is not an equipotential surface. The telluroid was proposed by Molodenskii to avoid the complex determination of the topographical density and vertical gradient of gravity, which are necessary components in geoid modelling (NRCan, 2014). Equation (3-10) is the mean normal gravity along the plumb line and \( R \) is a point located on the telluroid, where, \( W_P = U_R \), \( U_R \) being the normal gravity potential at a point \( R \), (see Figure 3.1).

The distance between the telluroid and the Earth’s surface is the height anomaly, \( \zeta_P \) at point \( P \). Often, the distances, \( H^*_P \) and \( \zeta_P \), are reversed along the plumb line; the normal height of the point, \( P \), \( H^*_P \), is represented by the distance between the point on the Earth’s surface and the quasi-geoid. The surface obtained by plotting, \( \zeta_P \), above the ellipsoid is
called quasi-geoid which is closely related to the geoid. However, unlike the geoid, the quasi-geoid lacks the physical interpretation of an equipotential surface (Hofmann-Wellenhof and Moritz, 2005).

![Figure 3.1. The normal height, height anomaly, telluroid and quasi-geoid.](image)

- $P$, is the point on the Earth’s surface; $R$, is a point on the Telluroid; $\zeta_P$, is the height anomaly at point $P$; $H_P$ is the orthometric height, $H_P^*$, is the normal height, $h_P$, is the ellipsoid height; $W_P$, is the potential of the equipotential surface through the point $P$ at the Earth’s surface and $U_R$, is the normal gravity potential at the point $R$ (Ince, 2011)

There are two advantages of using normal heights; firstly the exact value of the normal height can be calculated by using the normal gravity field, and secondly the density information is not required to compute the normal height (Hofmann-Wellenhof and Moritz, 2005; Jekeli, 2000).
3.2.4 Dynamic Heights

The dynamic heights unlike the orthometric, (equation 3-4) and normal heights, (equation 3-9), have no geometrical interpretation and are merely a conversion of the geopotential number to units of length. The dynamic height of a point, \( P \), is defined as:

\[
H_{P}^{\text{dyn}} = \frac{\zeta_P}{\gamma_0}
\]  

(3-11)

where:

- \( H_{P}^{\text{dyn}} \) - Dynamic height of point \( P \)
- \( \gamma_0 \) - Normal gravity value along the plumb line for fixed latitude

The value for the normal gravity along the plumb line in equation (3-4) is replaced by, \( \gamma_0 \), representing the normal gravity for a fixed latitude, usually taken to be 45°. Dynamic heights are merely a conversion of the geopotential number to units of length, relative to the geoid and the points with the same dynamic heights are on the same equipotential surface.

3.2.5 Relationship between \( H, H^* \) and \( H^{\text{dyn}} \)

Essentially, each of the height systems described provides a unique definition of the vertical coordinate of a point on the Earth's surface based on levelling and gravity information. Since they are linked through the geopotential number, it is theoretically possible to convert between any of the three height types. For example, equation (3-12), below is written to express the relationship between the geoid height and height anomaly using equations (3-8) and (3-9):
\[ h_p = H_p + N = H^*_p + \zeta_p \]  \hspace{1cm} (3-12)

and

\[ N - \zeta_p = H^*_p - H_p = \frac{\bar{g}_p - \bar{\gamma}_p}{\bar{\gamma}_p} H_p \approx \frac{\Delta g_B}{\bar{\gamma}_p} H_p \]  \hspace{1cm} (3-13)

where \( \Delta g_B \) is the Bouger gravity anomaly, and \( N \) is the geoid undulation (Hofmann-Wellenhof and Moritz, 2005).

Even though the dynamic heights are not geometrically meaningful like orthometric and normal heights, they are the only type from the three, which are \textit{physically meaningful}. That is, they will indicate the direction of water flow. They are used in the Great Lakes area to determine the lake water level, and can be converted into other height types when required:

\[ H_p = \frac{H^*_p}{\bar{g}_p} = \frac{H^*_p}{\bar{g}_p + 0.0424 H_p} \]  \hspace{1cm} (3-14)

The orthometric and normal heights and the associated reference surfaces discussed above are depicted in Figure 3.2. From the figure, the geoid undulation or geoid height, \( N \), represents the separation between the ellipsoid and the geoid along the ellipsoidal normal, and the height anomaly, \( \zeta_p \), represents the separation between the ellipsoid and the quasi-geoid along the ellipsoidal normal.
Figure 3.2. The reference surfaces (geoid, quasi-geoid, and ellipsoid) and height systems. 
P, is the point on the Earth’s surface; R, is a point on the Telluroid; P₀, is a point on the 
Geoid; Q, is a point on the Ellipsoid; W, is the constant potential value; Wᵢ, is the 
potential of the equipotential surface through the point P at the Earth’s surface; Wₒ, is 
the constant potential value of the geoid; ζᵢ, is the height anomaly at point P; U, is the 
normal gravity potential; Uᵢ, is the normal gravity potential at the point R; Uₒ, is the 
normal gravity potential of the ellipsoid; Hᵢ is the othrometric height, Hᵢᵢ, is the normal 
height; hᵢ, is the ellipsoid height; N, is the geoid undulation (Ince, 2011)

3.2.6 Ellipsoid height

The physical shape of the Earth can be approximated by the mathematical surface of a 
rotational ellipsoid defined by a semi-major axis, a, and flattening, f. All other ellipsoidal 
shape and size defining quantities can be subsequently derived from these parameters 
(semi-minor axis b, eccentricity e, and the radius of curvature in the prime vertical, Rᵦ). 
Because of its smooth well-defined surface, the ellipsoid offers a convenient reference
surface for mathematical operations and is widely used for horizontal coordinates (Seeber, 2003). The geodetic latitude $\varphi$, and longitude $\lambda$, are defined in Figure 3.3, where it is assumed that the center of the ellipsoid coincides with the Earth's center of mass and, the p-axis is the intersection of the meridian plane with the equatorial plane.

![Diagram of reference ellipsoid and geodetic coordinates](image)

**Figure 3.3. Reference ellipsoid and geodetic coordinates.** $xyz$, are the Cartesian coordinates; $\varphi$ and $\lambda$ are the geodetic coordinates, geodetic latitude and longitude, respectively; $h$, is the ellipsoid height; $b$ and $a$, are the semi-minor and the semi-major axis respectively; $R_N$, is the radius of curvature in the prime vertical; $Q$ is a point on the ellipsoid and $P$ is a point on the Earth’s surface (Fotopoulos, 2003)

The straight-line distance between point P on the surface of the Earth and its projection along the ellipsoidal normal onto the ellipsoid, denoted by Q, is the ellipsoidal height $h$. 
The ellipsoidal height is a straight-line distance estimated along the ellipsoidal normal from the geometrical surface of a reference ellipsoid to the point of interest (Zilkoski, 1990). The geometrical surface of the ellipsoid provides the height reference surface as such, the numerical value of the ellipsoidal height of a point is a function of the location, orientation, size and shape of the reference ellipsoid used (Featherstone and Kuhn, 2006). By extension, ellipsoidal heights are purely geometric quantities with no connection to the gravity potential. As such, they cannot account for the local variations in gravity caused by topography to determine a true level surface. With the advancement in technology and techniques, ellipsoidal heights can be obtained from a number of difference systems, such as very long baseline interferometry (VLBI), satellite laser ranging (SLR), and satellite based systems, such as GPS, GLONASS, GALILEO and others (Fotopoulos, 2003).

The computation of ellipsoidal heights using GPS measurements is, in general, more challenging than estimating horizontal coordinates. Although, the common error sources affecting the quality of the positions influence all three coordinates, there are a few key differences, which result in poorer height values (by approximately two to three times), namely (Rothacher, 2002):

- satellite geometry/configuration can only be observed in one hemisphere above the horizon (i.e., there will never be satellites observed below the receiver antenna)
- need to estimate receiver clock corrections at every epoch
- estimation of tropospheric zenith delay parameters
The most limiting factor remains the very high correlation of the receiver clock corrections and tropospheric zenith delay parameters with the ellipsoidal height. The estimation of these effects significantly hinders the achievable accuracy of the height component, even in the absence of other errors and biases (Santerre, 1991). A suggested means for partially decorrelating the height from the receiver clock and tropospheric delay is to take advantage of the zenith dependence and process GPS data at low elevation cut-off angles (Rothacher, 2002). However, lowering the elevation cut-off introduces other problems with data processing as the noise level increases significantly. Therefore, due to the nature of the satellite configuration and the need to estimate receiver clocks (even differences), the height component will always be less accurate than the horizontal positions (Fotopoulos, 2003).

A summary of the heights discussed above is given in Table 3.1. More details can be found in Hofmann-Wellenhof and Moritz (2005) and Jekeli (2000).

Table 3.1. Summary of Height Type. This table summarizes the height types by indicating the definition and characteristics of each of the height types described.

<table>
<thead>
<tr>
<th>Height Type</th>
<th>Definition</th>
<th>Characteristic</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipsoidal Height</td>
<td>( h_p )</td>
<td>Geometrically defined along the perpendicular to the ellipsoid going through point P.</td>
<td>Geometrically meaningful.</td>
</tr>
<tr>
<td>Orthometric Height</td>
<td>( H_p = \frac{c_p}{\theta_p} )</td>
<td>The distance along the plumb line between a point on the geoid and point P on the Earth’s surface. The calculation requires the complete knowledge of the mass density of the crust.</td>
<td>Geometrically meaningful and cannot be determined exactly.</td>
</tr>
<tr>
<td>Normal Height</td>
<td>( H_p^* = \frac{c_p}{\gamma_R} )</td>
<td>The distance between the quasi-geoid and the point P on Earth’s surface, measured along the plumb line. There is no need to make approximations for the density of the Earth's crust.</td>
<td>Geometrically meaningful and can be determined exactly.</td>
</tr>
</tbody>
</table>

Continued
Table 3.1 continued

<table>
<thead>
<tr>
<th>Dynamic Height</th>
<th>$H_p^{dyn} = \frac{C_p}{\gamma_0}$</th>
<th>Dynamic heights are merely a conversion of the geopotential number to units of length, relative to the geoid and the points with the same dynamic heights are on the same equipotential surface</th>
<th>Physically meaningful and associates with a value computed at a fixed latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geoid Height</td>
<td>$h_p = H_p = N$</td>
<td>Used in the conversion of the geometrically defined heights into physical heights.</td>
<td>The separation between the geoid and the reference ellipsoid</td>
</tr>
<tr>
<td>Height Anomaly</td>
<td>$h_p - H_p^* = \zeta_p$</td>
<td>Approximation of the geoid undulation according to the Molodensky’s theory. (Hofmann-Wellenhof and Moritz, 2005)</td>
<td>The separation between the quasi-geoid and the reference ellipsoid</td>
</tr>
</tbody>
</table>

3.3 **Vertical Reference System**

The definition and realization of the vertical reference system is essential in height determination. Associated with the selection of the height system is the selection of the compatible reference surface on which the height is zero. As such, the vertical datum, as defined by Vaníček (1991), is a coordinate surface to which heights are referred. The selection of a vertical datum will be different, depending on the choice of height system and reference surface adopted. There are three different conventional vertical datums: geoid, quasi-geoid, and the reference ellipsoid. There are three kinds of vertical datums used in geodesy: the geoid is a reference surface for orthometric heights; the quasi geoid is a reference system for normal heights, and the reference ellipsoid is the surface for the geodetic (geometric) heights (Vaníček, 1991). Presently, with no official global vertical datum definition, most of the countries around the world use a national vertical datum (Pan and Sjöberg, 1998).
3.3.1 Realization of a Vertical Datum

*Levelling based Vertical Datum*

The common approach for defining vertical datum is to average sea level observations over approximately 18.6 years, (which corresponds to the longest tidal component period) to obtain the mean sea level (MSL) for one or more fundamental tide gauges (Torge, 2001). The MSL is used since it was assumed to approximately coincide with the geoid. This assumption is evidently incorrect, as it is known that the MSL and the geoid differ by approximately 2 meters (Vaniček, 1991; Klees and van Gelderen, 1997). Also, by definition, the geoid is an equipotential surface, whereas MSL is not, due to numerous meteorological, hydrological, and oceanographic effects (Groten and Müller, 1991). This discrepancy between MSL and the geoid is due to the sea surface topography (SST) (Vaniček, 1991).

![Diagram](image)

*Figure 3.4. Establishment of a reference benchmark height. \( H_{\text{MSL}} \) mean value of the local sea level; \( H_{\text{I SL}} \) instantaneous sea level height; \( \Delta H_{\text{BM-TG}} \) height difference between the reference benchmark BM and the tide gauge TG (Fotopoulos, 2003)*
Figure (3.4), depicts a typical scenario for the establishment of a reference benchmark to define a regional vertical datum. The tide gauge records the instantaneous sea level height, $H_{ISL}$, and these values are averaged over a long term in order to obtain the mean value of the local sea level, $H_{ASL}$. The height of the tide gauge is also measured with respect to a reference benchmark that is situated on land, a short distance from the tide gauge station. Then the height of the reference benchmark above mean sea level, $H_{BM}$, is computed by:

$$H_{BM} = H_{MSL} + \Delta H_{BM-TG}$$  \hspace{1cm} (3-15)

Levelling begins from this benchmark and reference heights are accumulated by measuring height differences along levelling lines. The user of this vertical datum conducts differential levelling from the benchmarks on datum to transfer vertical geodetic control for a particular project or application. Vaníček et al. (1980) review the uses of differential levelling.

For highly accurate heights, as needed for a cm-level vertical datum, the tide gauges cannot be assumed to be vertically stable. It is well known that land motion at tide gauges is a source of systematic error, which causes distortion in the height network if it is not corrected for. Land motion at tide gauges and reference benchmarks may be caused abruptly by earthquakes or by erosion or more subtle changes, such as post-glacial rebound and land subsidence. The solution to this problem is to include GNSS in order to estimate the land motion at these tide gauges.
The North American Vertical Datum of 1988 (NAVD 88) was defined through the levelling network. It is the official vertical datum for civilian surveying and mapping activities in the United States (NGS, 2014). The International Great Lakes Datum of 1985 (IGLD85) comes from the same adjustment as NAVD88 except that IGLD85 is expressed in the dynamic height system (NGS, 2014). The current vertical datum in Canada is the Canadian Geodetic Vertical Datum of 1928 (CVGD28), which was realized by leveling measurements. These datums are briefly described in the following subsections.

**Geoid Based Datum**

Geoid modeling came to the forefront with the advances of space-based positioning to relate ellipsoidal heights to orthometric heights. Vaníček (1991) identified an approach for the realization of a "modern" regional vertical datum, using estimates of orthometric heights from satellite-based ellipsoidal heights and precise gravimetric geoidal heights. The main advantage of this approach is that it relates the regional vertical datum to a global vertical reference surface (since the satellite-derived heights are referenced to a global ellipsoid). This approach utilizes the fundamental relationship defined in equation (1.1) in Chapter 1, which converts the ellipsoid heights from GNSS to a physically meaningful height (or height differences) by subtracting the geoid undulation. This avoids the need to level from benchmarks, but does require a GNSS baseline observed to a 3D control point with an ellipsoid height (Featherstone et al., 2012). A major limitation of this approach is that it is dependent on the achievable accuracy of the ellipsoidal height and the geoid model.
3.3.2 Current Vertical Datums in North America

In this section, several vertical reference datums used in North America are discussed. They are the North American Vertical Datum of 1988, NAVD88, the Canadian Geodetic Vertical Datum of 1928, CGVD28, and the International Great Lakes Datum of 1955 and 1985, IGLD55 and IGLD85, respectively. A recent addition to this list is the Canadian Geodetic Vertical Datum of 2013 (CGVD2013), which unlike the other datums listed, is a geoid based vertical datum.

**North American Vertical Datum of 1988 (NAVD 88)**

The United States of America established a National Geodetic Vertical Datum in 1929 by holding MSL fixed at 26 tide gauge stations (five of which were in Canada), and undertaking an adjustment of 106724 km of leveling data (31565 km of which were in Canada). This adjustment was the fourth in a series, the others being undertaken in 1903, 1907 and 1912, with each subsequent adjustment incorporating new data (Zilkoski et al., 1992). The NGVD 29 datum did not use observed gravity data in determining the orthometric corrections, and thus the resulting heights were normal orthometric heights.

The next national adjustment resulted in the definition of a new vertical reference system, known as the North American Vertical Datum, 1988 (NAVD 88). NAVD 88 was developed as a result of varying limitations in the 1929 National Geodetic Vertical Datum (NGVD 29), such as poor coverage, network distortion and destruction of benchmarks due to highway construction, crustal motion, postglacial uplift and subsidence. NAVD 88 was realized by undertaking a minimum constraint adjustment of
the US, Mexican, and Canadian leveling data, holding fixed the primary tide gauge benchmark at Rimouski, located at the mouth of the St. Lawrence River (Zilkoski et al., 1992).

To perform the new general adjustment, approximately 625000 km of leveling was added to the existing 1929 US national network, and 81500 km of re-leveling across the North American continent was also included. The disturbed and destroyed monuments prior to the actual leveling were replaced. This effort also included the establishment of stable “deep-rod” benchmarks, which will provide reference points for “traditional” and GPS leveling techniques (Zilkoski et al., 1992).

NAVD88 provides Helmert orthometric heights defined by, equation (3-8), given by:

$$H_{NAVD88} = \frac{C_{NAVD88}}{g_P + 0.0424 H_P}$$

(3-16)

where:

- $C_{NAVD88}$ - The geopotential number in NAVD88
- $g_P$ - The gravity measured at point P and
- $H_P$ - The orthometric height of point P.

Canada did not adopt NAVD 88; however, NAVD 88 heights are available on a subset of the Canadian levelling network.
Canadian Geodetic Vertical Datum of 1928 (CGVD28)

The reference system of the Canadian Geodetic Vertical Datum 1928 (CGVD28) was established by classical surveying techniques using Mean Water Level (MWL) at five tide gauges. Three of the tide gauges were located on the east coast in Pointe-au-Père, Halifax and Yarmouth and two on the west coast located in Vancouver and Prince Rupert. The leveling network was created from over 80000 benchmark points, spread over approximately 93205 km (CGRSC, 2004; Véronneau, Duval and Huang, 2006).

CGVD28 is not compatible with the geoid since the MWLs at each tide gauge are not on the same equipotential surface and processing of the leveling data includes only normal geopotential numbers. Therefore, to relate the CGVD28 with a geoid model, GPS observations should be collected at benchmarks.

The CGVD28 normal-orthometric heights (approximation to the orthometric heights or normal), were computed by, equation (3-16), where $C_{CGVD28}$ is the geopotential number and $\bar{\gamma}$ is the mean normal gravity.

$$H_{CGVD28}^* = \frac{C_{CGVD28}}{\bar{\gamma}}$$

Consequently, the height reference system of Canada is not compatible with GPS, (CGRSC, 2004). Even though it is an acceptably accurate model regionally, at the national level it does not meet today’s required accuracy mainly due to the distortion introduced by the local sea surface topographies at the defining tide gauges. The system also does not offer the coverage nor the accuracy required for consistent height transfers over the longer baselines made possible through the use of navigation satellite signals.
(Véronneau and Héroux, 2007). With these intrinsic issues the height reference system requires modernization to fully support and realize the substantial benefits of GPS and related modern technologies for accurate height measurement. The realization approach, i.e., a geoid-based vertical datum along with the new terrestrial datasets and the new satellite models are expected to overcome these issues.

**Great Lakes Vertical Datum (IGLD 55, 85)**

In 1953, the International Joint Commission (IJC) of Canada and the United States initiated a program of coordinating basic hydraulic and hydrologic data in the Great Lakes area. The Canadian and United States agencies used heights referenced to different datum, small differences lead to a decision by the IJC to establish the joint International Great Lakes Datum of 1955 (IGLD55) (Zilkoski 1991).

A first-order levelling line was performed along the St. Lawrence River from Point-au-Père (Father's Point), Quebec, to Kingston, Ontario. The datum for IGLD 55 was determined by holding the elevation of local mean water level fixed at Point-au-Père. Normal dynamic elevations, i.e., dynamic elevations using normal gravity values, were adopted as the elevations to be used and published for IGLD 55. The primary reason for adopting dynamic heights for the new datum was to provide a means for the more accurate measurement of geopotential hydraulic head between points (Zilkoski, 1991). Due to crustal movement with respect to sea level, the elevations of the benchmarks shifted not only with respect to the initial reference point but also with respect to each other. These changes prompted the establishment of a new datum of the Great Lakes region, IGLD85. More information about the establishment of IGLD55 can be found in
the report “Establishment of IGLD55, Second Edition” prepared by the Coordinating Committee (The Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data, 1995)

The IGLD85 was established based on the same principle as IGLD55. However due to deterioration of the wharf at Point-au-Père, the gauging station at that site was discontinued and the reference zero moved upstream to Rimouski, Quebec (The Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data, 1995). The development of the IGLD 85 coincided with the development of the NAVD 88. The general adjustment for both these datums, included a minimum constraint adjustment of Canadian-Mexican-US levelling observations. The only difference between IGLD 85 and NAVD 88 is that International Great Lakes benchmark values are given in dynamic height (see equation 2-64)) and NAVD 88 values are given in Helmert orthometric height units. The geopotential numbers for individual benchmarks are the same in both systems (Zilkoski, 1991). The overall differences between dynamic heights referred to IGLD 85 and IGLD 55 ranged from approximately 1 to 40 cm. The IGLD 85 was implemented for use in January 1992 (The Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data 1995).

The additional problems associated with The Great Lakes and St. Lawrence River region are the post glacial rebound or glacial isostatic adjustment (GIA) effect and fluctuations on a short-term, seasonal, and long-term basis (Zilkoski, 1991; Great Lakes Commission, 2010). The GIA effect causes a gradual uplift of the crust and changes in the water level, respectively. The movement related to GIA effects requires careful and accurate
measurements within an accurately established vertical datum. To achieve this, it is recommended that the vertical datum for the Great Lakes, should be updated every 25 to 35 years to reflect continuous and differential changes in land surface elevations across the region. Alternatively, this may also require modernization of the realization of the vertical datum. All this is essential to provide accurate geodetic and water level products and services to the Great Lakes community (Great Lakes Commission, 2010).

**Canadian Geodetic Vertical Datum of 2013 (CGVD2013)**

The Canadian Geodetic Vertical Datum of 2013 (CGVD2013) was officially released in November 2013 and it replaces the (CGVD28). It is a gravimetric datum defined by the equipotential surface \( W_0 = 62,636,856.0 \text{ m}^2\text{s}^{-2} \), that by convention represents the coastal mean sea level for North America. The definition and geopotential value comes from an agreement between Canada and USA. The Canadian Gravimetric Geoid model of 2013 (CGG2013) is the first realization of the vertical datum. CGG2013 is available in NAD83(CSRS) and ITRF2008 for the GRS80 ellipsoid, making it compatible with space-based positioning techniques. Heights in terms of CGVD2013 are orthometric \((H)\).

In 2022, the National Geodetic Survey will replace NAVD 88 with a new geoid-based vertical datum for the United States. To create a geoid of sufficient accuracy, a consistent, updated gravity survey of the US and its territories was needed. The method chosen for this survey was airborne gravimetry, and the project under which this airborne survey falls is known as the Gravity for the Redefinition of the American Vertical Datum or GRAV-D (NGS, 2014). Over the last 20 years, the NGS has published a total of sixteen (16) geoid models that include both gravimetric and hybrid models which cover
regions including the conterminous United States (CONUS) and its territories. These geoid models have improved over the years by implementing better theory; and by conducting gravity surveys to fill large voids of data and new satellite gravity models (Roman et al., 2010).
CHAPTER 4: STUDY AREA AND DATASETS

4.1 The Great Lakes

The Great Lakes are a collection of freshwater lakes located in northeastern North America, on the Canada–United States border. They connect to the Atlantic Ocean through the Saint Lawrence Seaway and the Great Lakes Waterway. They consist of five lakes Superior, Michigan, Huron, Erie, and Ontario, and together form the largest group of freshwater lakes on Earth, containing 21% of the world's surface fresh water (Manninen and Gauthier, 1999). The Great Lakes watershed includes part or all of eight US states (Minnesota, Wisconsin, Illinois, Indiana, Michigan, Ohio, Pennsylvania and New York) and the Canadian province of Ontario (Figure 4.1). Today, more than 33 million people inhabit this drainage basin including more than one-tenth of the US population and one-quarter of the Canadian population. These lakes contain about 23000 km$^3$ of water and cover a total area of 244000 km$^2$.

The channels that connect the Great Lakes are an important part of the system. The St. Marys River is the northernmost of these, a 97 km waterway flowing from Lake Superior down to Lake Huron. The St. Clair and Detroit rivers, and Lake St. Clair between them, form a 143 km long channel connecting Lake Huron with Lake Erie. The Niagara River, 56 km in length links Lakes Erie and Ontario, and sends approximately 1416 to 2832 m$^3$ of water per second over Niagara Falls; the manmade Welland Canal also links these two
lakes, provides a detour around the falls. From Lake Ontario, the water from the Great Lakes flows through the St. Lawrence River all the way to the Atlantic Ocean, about 1609 km away.

Figure 4.1. Map of the Great Lakes basin watershed (Manninen and Gauthier, 1999)

Figure 4.2 shows a profile of the Great Lake systems giving a representation of the channels connecting the lakes, the typical water surface elevations and comparative lake
depths. These lakes are large enough to influence the regional climate, cooling summers and tempering winters, as well as increasing amounts of rain and snow in the region.

Figure 4.2. System profile of the Great Lakes of North America (Conversion: 1 ft = 0.3048 m and 1mi = 1.609km) (Wilby, 2011)

4.1.1 Geological Background

The origins of the North American Great Lakes watershed are a product of multiple glaciations during the late Cenozoic as well as redirected drainage, particularly during retreat of the last ice sheet (Larson and Schaetzl, 2001). The watershed is divided into a southern, lowland region underlain by relatively gently dipping sedimentary rocks of Paleozoic age, and a northern upland region (Canadian Shield) underlain by granite, gneiss, and matavolcanic and metasedimentary rocks of Precambrian age (Hough, 1958; Larson and Schaetzl, 2001).
The lowland region includes the Lake Erie, Lake Michigan and most of the Lake Huron and Lake Ontario. This region is generally blanketed by a continuous mantle of glacial sediments, often greater than 50 m in thickness and in places over 350 m thick. Also there are some areas where low moraine ridges and a few bedrock escarpments exist. The upland region includes most of the Superior and Georgian Bay basins and parts of the Ontario basin. It can be distinguished topographically by a distinct bedrock-dominated topography formed as a result of bedrock structure and differential glacial erosion. Thin, discontinuous glacial sediments blanket this region (Hough, 1958; Larson and Schaetzl, 2001). These glacial sediments indicate that the present geological make-up of the Great Lakes watershed is as a result of both glacial erosion and post-glacial deposition (Hough, 1958). The bedrock formation of the Great Lakes watershed is shown in Figure 4.3.
The basins that contain the Great Lakes are the product of repeated scouring and erosion (Figure 4.4) of relatively weak bedrock by continental glaciers (Laurentide Ice Sheet) that advanced into the Great Lakes watershed beginning perhaps more than 2.4 Million years (Ma) ago (Hough, 1958; Larson and Schaetzl, 2001). There were four distinct glaciations the Nebraskan, Kansan, Illinoian and Wisconsin, which were separated by the interglacial Aftonian, Yarmouth and Sangamon intervals, respectively (Hough, 1958).
The oldest advances, previously called the Nebraskan and Kansan events, have been mapped in Nebraska, Kansas, Missouri, Pennsylvania and New Jersey. The third, the Illinoian, extended over a larger part of Illinois, Indiana and Ohio (Hough, 1958). The last glacial episode, the Wisconsin glaciation, centered in northern and eastern Canada, expanded southward covered the entire Great Lakes watershed extending to the Ohio River to the south and northern Wisconsin and east central Minnesota to the west. This glacial episode began sometime between 65 to 79 thousand years (65–79 kiloannus (ka)) ago.

After 18 ka, the ice margin began to retreat, but experienced a series of re-advances around 15.5, 13.0, 11.8, and 10.0ka. Ice continued its retreat about 10ka and the
watershed was completely ice-free by 9.0 ka, with the deglaciation of the northern rim of the Superior basin (Gillespie, Harrison III and Grammer, 2008). The history of the proglacial lakes that occupied the Great Lakes watershed is summarized in Figures 4.5. During the retreat of ice margins large proglacial lakes were formed in the lake basins between high topography to the south and the ice margin to the north. Their surface elevation and extent varied considerably over time as outlets were either blocked or uncovered by glacier ice. Outlets were also subject to isostatic rebound as well as by channel down cutting, which likewise affected the level of the glacial lakes and the lakes that followed (Larson and Schaetzl, 2001).

Figure 4.5. Glacial Lake Stages: Ice Advances and Retreats (US EPA, 2012)
4.2 Research Data


The Height Modernization GPS survey projects were initiated by National Ocean Service Center for Operational Oceanographic Products and Services (NOS CO-OPS), Geodetic Survey Division of National Resources Canada (NRCan), and the International Joint Commission’s Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data. The purpose of the surveys was to monitor elevation changes across the Great Lakes to facilitate the development of the future release of the International Great Lakes Datum (IGLD). The surveys were conducted in 1997, 2005 and 2010 and included the entire Great Lakes Region from Minnesota and Lake Superior eastward to the St Lawrence Seaway in New York. Figures 4.3 to 4.5 show the spatial locations of the benchmarks for the 1997, 2005 and 2010 IGLD surveys, respectively. Benchmarks in these projects provide the connection between the IGLD85, and NAVD88 water level gauge networks.

Movement of the earth's crust due to isostatic rebound (post glacial land uplift) requires the revision of the vertical datum (elevation reference system) used to define water levels within the Great Lakes-St. Lawrence River system every 25 to 30 years. The update is essential to provide accurate geodetic and water level products and services to the Great Lakes community. A fundamental requirement for coordinated international management is a common elevation reference datum (a defined vertical plane) for water levels. The
revision is targeted for publication as IGLD 2015 based on the final year of the water level data collection portion of the project.

Since these GPS surveys were conducted to monitor small changes in the vertical component, the longest sessions possible were observed. In general, station occupation and observing procedures were carried out according to appropriate sections of the “NGS Operations Handbook” and the current applicable receiver field manuals.

These surveys were observed for session lengths of 24 hours at stations that were secured and permitted use of unattended equipment and 8 hours at stations that were unsecured. All 24-hour and 8-hour sessions, had the same start times and every station was occupied at least twice. For each occupation, the tripod and antenna were removed at least briefly from the mark at the close of a session and reset so as to assure that errors in an earlier setup were not carried forward into the next session meteorological data, such as temperature or pressure, were not collected. Standard occupation log sheets were filled for each occupation of each mark, along with sketches of the mark and photos.

Finally, the surveys were performed using, dual frequency GPS receivers with geodetic antenna and fixed height poles (preferred). The GPS data were collected at a rate of 30 seconds with an elevation mask of 10 degrees. The fixed height poles were checked for length and bubble adjustment before and after the survey, and any other time there was suspicion of a problem. Traditionally tripods and tribrachs were used, when a fixed height tripod was not used. The antenna height, in such cases were made above the mark in both metric and imperial units before and after each session and recorded accordingly.
Figure 4.6. IGLD Height Modernization benchmarks for surveys done in 1997
Figure 4.7. IGLD Height Modernization benchmarks for surveys done in 2005
Figure 4.8. IGLD Height Modernization benchmarks for surveys done in 2010
4.2.2 Data Retrieval

Data for the IGLD Height Modernization Projects 1997, 2005 and 2010 were retrieved from the NGS archives. The data retrieved included: the survey field note and sketches; GPS receiver raw data files; GPS observation and navigation files (.obs and .nav) in a Receiver Independent Exchange (RINEX) format for a total of 214 GPS on benchmark site; NGS Bluebooking files (A-file, G-file, B-file); reports and documentation. Bluebooking is the process of preparing and submitting geodetic data for incorporation into NGS' data base (NGS, 2014). As such, there are specific file formats to adhere to which are, the A-file, which contains the adjustment constraints and processing options file; the B-file, which contains the data from observation logs; equipment codes; station designations; and the G-file, which contains the processed GPS vectors and statistics. The data formats and digital file definitions for these surveys are given in “Inputs Formats and Specifications of the National Geodetic Survey Data Base,” Volume I. Horizontal Control Data, Federal Geodetic Control Subcommittee, September 1994, revised November, 1998. Table 4.1 gives some statistics of data collected:

<table>
<thead>
<tr>
<th>Project Yr.</th>
<th>Project ID</th>
<th>Observation period</th>
<th>Total stations (Benchmarks and CORS)</th>
<th>No. of Benchmarks</th>
<th>Common stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>GPS2379</td>
<td>August 1st – October 1st 2005</td>
<td>117</td>
<td>79</td>
<td>1997-2010</td>
</tr>
<tr>
<td>2010</td>
<td>GPS2824</td>
<td>June 2nd – July 24th 2010</td>
<td>155</td>
<td>77</td>
<td>2005-2010</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1997-2005-2010</td>
</tr>
</tbody>
</table>
The antenna set-up details are critically important to the success of the data processing done in this research. Therefore, with the use of the survey log files and auxiliary report the antenna heights and types were cross reference with that of the header information in the RINEX files before proceeding with any processing.
CHAPTER 5: METHODOLOGY - NETWORK CONFIGURATION ASSESSMENT

5.1 Introduction

This chapter deals with the reanalysis of the three (3) campaign surveys from 1997, 2005 and 2010 of the IGLD Height Modernization project executed by NGS. These projects were initially processed and adjusted individually using the respective available reference frames at the time of the surveys. The reanalysis entailed reprocessing and adjusting the original data from the three (3) IGLD projects using OPUS-Projects in one consistent reference frame (IGS08/ITRF08 at the epoch of each survey) with the best available orbits and using the associated absolute antenna calibration. This will allow for the observation of change over time, without including the change coming from either changes in reference frame or processing software/algorithms. This activity will assure modern, up-to-date evaluation of real movement.

The main focus of this reanalysis is the determination of the most suitable network configuration that can be used to improve the accuracy of the GPS derived coordinates with specific emphasis being placed on GPS derived ellipsoidal heights. Generally there are two traditional network designs used in GPS surveying, the Closed Loop Network Configuration and Single Reference Radial Configuration (Figure 5.1). The closed loop configuration requires that baselines are interconnected to form closed loop geometric
figures, while the single reference network comprises one fixed reference station centered amongst set of unknown stations, all single baselines are formed from the fixed station reference. The closed loop network has been the preferred network of the two networks, as it gives the user a sense of reliability and stability, because of the interconnected baselines and the high level of redundancy. While the single reference network seems less stable due its lack of interconnectivity and geometry, additionally the entire network is hinged to one single reference station. These two configurations are discussed further in Section 5.2.3.

Figure 5.1 – GPS Network Configuration (a) Single Reference Radial and (b) Closed Loop Network Configurations

To determine the suitability of the network design for the accurate determination GPS coordinate solutions, this research proposes and tests a modified single reference radial
design, the multiple reference radial design, against the preferred close loop design, using NGS’ newest online processing tool, OPUS-Projects. OPUS-Projects is a web-based utility, developed by NGS. It is the newest addition to the Online Positioning User Service (OPUS) tools that enable users to process static GPS data and, access NGS’s products and high-accuracy NSRS coordinates. Users can use OPUS-Projects to create and manage projects, upload the GPS data through OPUS, process the data, and publish the project’s results (NGS, 2014). These processing tools are explained in more detail later in this chapter.

In particular, this chapter describes the procedures and models used and, the results obtained from the reanalysis of the GPS data using the close loop and modified single reference radial design. The vertical velocities of each station were also computed which would be used in Chapter 6 for the comparison of campaign surveys to multi-year derived vertical velocities.

5.2 Reprocessing and re-adjusting of the IGLD 2010, 2005 and 1997 Height Modernization surveys

5.2.1 GPS Data Processing

The resulting coordinates were estimated by OPUS-Projects, which is one of the services from the OPUS utilities provided by NGS. OPUS is a suite of web-based GPS processing tools, which provides simple access to high-accuracy National Spatial Reference System (NSRS) coordinates. The suite included OPUS-Static (OPUS-S), OPUS-Rapid Static (OPUS-RS), OPUS-Projects and OPUS-Network (OPUS-Net). These tools, with the exception of OPUS-RS, have been used throughout this research. In this chapter OPUS-S
and OPUS-Projects would be described in some detail, while OPUS-Net would be discussed in Chapter 6. For completeness OPUS-RS would be very briefly described in this section.

The fully automated processing service of OPUS produces a solution after a user enters five inputs: (1) dual frequency ($L_1 + L_2$) data, (2) the antenna type, (3) the antenna height of the Antenna Reference Point (ARP), and (4) the user’s email address (to receive the solution report) and, (5) selection of a processing option. Figure 5.2 below, shows the form used to upload data to OPUS at http://www.ngs.noaa.gov/OPUS/, (NGS, 2014).

![OPUS: Online Positioning User Service](image)

*Figure 5.2. The NGS’s Online User Positioning Service (OPUS) web application used for submitting static GPS observation files to OPUS for processing (NGS, 2014)*

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Final coordinates in the OPUS report are obtained by averaging three separate single-baseline solutions processed using three selected CORS sites as fixed reference stations. The coordinates for each selected CORS reference station referred to the IGS08 reference frame (at epoch 2005.00) are extracted from the NGS integrated database (NGSIDB) and subsequently updated to the mean epoch of the GPS data span collected at the rover (unknown station). OPUS uses the Horizontal Time Dependent Positioning (HTDP) software to update the horizontal coordinates to the appropriate designated epoch (Snay, 1999; Pearson et al., 2010; Pearson and Snay, 2012). The HTDP software uses two approaches to update the CORS coordinates. For those stations that began operating more than 2.5 years before April 16, 2011, a linear velocity for each component has been estimated from a multi-year time series of daily solutions that span a maximum of about 15 years. The original archived coordinates at epoch 2005.00 and their corresponding calculated (true) velocities are used to update the coordinates at any other specified epoch. For stations with a history of less than 2.5 years and for all new stations incorporated into the CORS Network after April 16, 2011, modeled velocities are used. OPUS uses a similar approach to update the vertical position at a CORS site to a specified epoch. A linear vertical velocity computed from the multi-year time series is applied for updating CORS coordinates if they have a history of more than 2.5 years before April 16, 2011. On the other hand, a zero vertical velocity is applied for a CORS with a history of less than 2.5 years if it belongs to the multi-year solution and for all CORS established after April 16, 2011.
**Online Positioning User Service – Static (OPUS-S)**

OPUS-S processes L1 and L2 carrier-phase data in native receiver and RINEX formats. Datasets submitted to OPUS-S must be between two (2) and forty-eight (48) hours in duration and pass several quality control steps before being passed onto the positioning algorithm (Weston, Mader and Schenewerk, 2012). OPUS-S was designed to select three (3) nearby CORS to form individual baselines with the rover that are processed independently. The final set of the coordinates are the average of the best three solutions (NGS, 2014).

Program for Adjustment of GPS Ephemerides (PAGES) is the processing backbone for both OPUS and OPUS-Projects software (NGS, 2014) PAGES is an orbit/baseline estimation software, which uses double-differenced phase as its observable. A variety of parameter types are estimated including tropospheric corrections, station coordinates and linear velocities, satellite state vectors and polar motion. This software runs using the ionosphere-free phase combination, but optionally L1 only, L2 only, or two wide-lane phase combinations can be used. These, in turn, can be used to create partially or completely bias fixed solutions.

The current operational application of OPUS-S uses reference station data from the US National CORS Network and fixed IGS ephemerides to compute independent, double-differenced baseline solutions between the unknown and three neighboring CORS reference stations. OPUS-S selects the reference stations that would create the best geometric distribution at a given location, as an evenly spaced geometric distribution would result in the most stable and reliable solutions (Choi and Weston, 2013). The
system computation uses absolute antenna patterns; carrier phase ambiguity integer fixing where possible or float ambiguities are estimated otherwise; troposphere modeling (Global Pressure and Temperature model (GPT) and Global Mapping Function (GMF) \textit{a priori} models), and are performed in the most recent realization of the ITRF/IGS reference frame (Weston and Ray, 2010).

\textbf{Online Positioning User Service – Rapid Static (OPUS-RS)}

As shown in Figure 3-4, there are two choices of processing models, depending on the duration of the user data. Section 3.2.2 described OPUS-S, in this section the Rapid Station option would be briefly discussed. For short duration datasets ranging from fifteen (15) minutes to two (2) hours, NGS designed OPUS Rapid Static (OPUS-RS), a version that uses up to nine CORS located nearby to estimate the atmospheric delay at the reference stations and predict them at the rover and then compute the coordinates accurately (Choi and Weston, 2013). This processing tool was not used in this study since the datasets used all exceeded the time span requirement. Great details of this method can be found in the work done by Schwarz, Snay and Soler, (2009).

\textbf{OPUS-Projects}

OPUS-S was designed for user simplicity, in that it requires minimum user input and it decides how best to process the data. To give the user more flexibility, NGS developed another OPUS tool, called OPUS-Projects. This processing tool was developed as a natural extension of OPUS. It is a complete web-based, GPS data processing and analysis environment. This edition of the OPUS services gives access to simple visualization,
management, and processing tools for multiple marks and multiple occupations (Armstrong, 2014).

The main idea behind OPUS-Projects is that one or more users can define numerous, independent GPS projects. Each newly defined project is assigned a unique ID which is shared among field personnel assigned to the project. A typical GPS project may include simultaneous occupations that span one or more days, often referred to as a session. After individual GPS data files have been collected in the field, they are submitted through OPUS-S with the project-specific ID to OPUS-Projects. OPUS-S is used as a pre-processor at this stage, to determine if the results for each data file surpass a set of pre-defined tolerances and is acceptable for further analysis. After all the data files for a project have been successfully submitted to OPUS-S, an OPUS-Projects manager can begin to process each of the sessions in a least squares adjustment. Multiple session adjustments are combined using a Helmert blocking normal equation processor, GPSCOM, to estimate a single set of coordinates for each station in the project (Weston, Mader and Schenewerk, 2012).

OPUS-Projects differ significantly from OPUS-S in that the user has the ability to customize data processing. This includes the user’s ability to: (1) choose the reference stations from a selection of any CORS/IGS stations operating on the date the GPS data were collected, (2) customize network designs achieve the accuracy requirements of a particular project, (3) modify processing threshold parameters to regulate the accuracy of a solution (Armstrong, 2014). Furthermore, the various solutions can be combined to perform a full network adjustment to all sites.
At the time of the research, most of the data processing for this study was performed, using OPUS-Projects beta version, which was still undergoing development and revision. Some of the advantages of OPUS-Projects include:

- Data uploading through OPUS-S
- Coordinate results aligned through the NSRS
- Processing using the PAGES and GPSCOM\(^2\) software
- Graphical visualization and management aids including interactive maps powered by Google Maps mapping service.

A flowchart of the basic processing procedures of OPUS-Projects is presented in Figure 5.3 below. The flowchart depicts the series of events which occurs, starting from uploading observation files to OPUS, then processing groups of project mark observations (session processing), and finally adjusting the sessions to get the final network solution. Ultimately, the user may elect to submit and bluebook to the NGS Integrated Database (NGSIDB).

\(^2\) GPSCOM: A program for the combined adjustment of multiple GPS data sets initially processed by the program Pages
5.2.2 OPUS-Projects Processing Settings

Upon creating a project in OPUS-Projects, there are some settings that are selected by the user based on desired preferences, such as solution quality thresholds and data processing settings. The data and solution quality thresholds define how processing results are displayed on the project’s web page. This applies to session solutions, network solutions, and any data uploaded to OPUS-Projects. The data processing settings define the options offered to control the processing. In order to enhance the consistency of the results within
a project however, it is recommended that once these values are set, they should not be changed.

**OPUS-Projects Processing Settings**

Table 5.1 presents the preferred parameters used in this research to set the quality thresholds and processing settings in OPUS-Projects.

### Table 5.1. Quality Indicators and Processing Settings used in GPS Data Processing

<table>
<thead>
<tr>
<th>The Quality Indicators and Data Processing Settings</th>
<th>Data Processing Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data &amp; Solution Quality Thresholds</td>
<td></td>
</tr>
<tr>
<td>Precise Ephemeris</td>
<td>Best Available</td>
</tr>
<tr>
<td>Minimum Observations Used</td>
<td>80 (%)</td>
</tr>
<tr>
<td>Minimum Ambiguities Fixed</td>
<td>80 (%)</td>
</tr>
<tr>
<td>Maximum Solution Root Mean Square (RMS)</td>
<td>0.025 (m)</td>
</tr>
<tr>
<td>Maximum Height Uncertainty</td>
<td>0.020 (m)</td>
</tr>
<tr>
<td>Maximum Latitude Uncertainty</td>
<td>0.020 (m)</td>
</tr>
<tr>
<td>Maximum Longitude Uncertainty</td>
<td>0.020 (m)</td>
</tr>
<tr>
<td>Data Processing Settings</td>
<td></td>
</tr>
<tr>
<td>Output Reference Frame</td>
<td>IGS08/NAD83(2011)</td>
</tr>
<tr>
<td>Output Geoid Model</td>
<td>GEOID12A</td>
</tr>
<tr>
<td>GNSS</td>
<td>GPS-Only</td>
</tr>
<tr>
<td>Troposphere Model</td>
<td>Piecewise Linear</td>
</tr>
<tr>
<td>Troposphere Interval</td>
<td>1800 (s)</td>
</tr>
<tr>
<td>Elevation Cutoff</td>
<td>15 (degree)</td>
</tr>
<tr>
<td>Constraint Weights</td>
<td>Normal</td>
</tr>
<tr>
<td>Network Design</td>
<td>USER; TRI</td>
</tr>
</tbody>
</table>

Under the data and solution quality thresholds column, the best available precise ephemerides are selected, which are the final combinations that are available at twelve (12) days latency. The percentage of observations used and ambiguities fixed are critical for the case of short data spans (data ≈ 2hrs). As an example, if the percentage of observation used is 50% for a two-hour GPS data span, then the solution is generated
based on one-hour data rather than two-hours. Even though the data spans for the IGLD survey data ranged from eight (8) to twenty four (24) hours, the percentage of observations used and ambiguities fixed values were kept as high as possible, and thus the threshold preference was given at 80%. The solution RMS value is related to the baseline length. The RMS value should be as low as possible in a solution; the maximum RMS value was set to 2.5 cm. The uncertainties describe the quality of the coordinates determined by the solution. The minimum uncertainties were set to 2 cm for the height and horizontal components.

Under the data processing setting column, IGS08 was selected as the output reference frame, the coordinates are also presented in NAD83 (2011) derived from IGS 08 at the epoch of survey. The output geoid model used was GEOID12A, a hybrid geoid model, used by OPUS-Projects to convert the ellipsoidal heights to orthometric height. This model was the most recent model available when the processing was performed. OPUS-Projects also calculate orthometric height. Currently OPUS-Projects has the capability to process GPS-only data, and thus files uploaded with GLONASS data are stripped out automatically when processed by OPUS.

OPUS-Projects provides limited user control for tropospheric corrections parameterized (or modeled) in order to remove them. For modeling the tropospheric effect, OPUS-Projects offers two options: (1) piecewise linear and (2) step-offset. Even though these techniques are named “troposphere model” in the preferences, they are actually estimation techniques for the zenith wet tropospheric corrections. The piecewise linear estimations technique was used, as this is a realistic way to represent a time-dependent
effect like this, and to avoid discrete step transitioning from one interval to another (Mader, 2014). The interval to estimate zenith wet tropospheric corrections was set to 1800 seconds, which is the default for the piecewise linear tropospheric correction method. The default elevation cutoff mask of 15° was maintained.

Constraint weights are applied only to project marks or CORS coordinates selected as constraint points in the processing. The option for constraint weights was selected as normal, this allows up to 1 cm of float for the constrained control points to be adjusted to a true best-fit solution (Armstrong, 2014). This level of constraints is very acceptable, as many CORS exhibit seasonal motion and poor velocities, and thus the true value may vary from the published coordinates (Mader, 2014). There were two network designs used in the processing of the dataset. The USER network design option allows the user to create a unique network configuration. The data were also processed using an OPUS-Projects’ predefined network strategy. Descriptions of these network designs are discussed in the following subsection.

5.2.3 Network Configuration

Generally there are two basic types of networks in GPS surveying and processing, which are the single reference radial and closed loop network configurations (Hofmann-Wellenhof, Lichtengegger and Collins, 2001). Firstly, radial configurations comprise a single fixed reference station centered amongst a set of unknown stations; all baselines (single) are formed from this fixed station reference, in such a case, the baselines are not interconnected to form a geometric figure. In this configuration, the baselines are short (approximately 50 km to 100 km) to maximize simultaneous observations. Hofmann-
Wellenhof, Lichtengegger and Collins (2001), indicated that an appropriate use for such radial networks might be to provide positions, where precise coordinates may generally not be needed, (e.g., position of geological features). However, computing the position of each mark relative to the same reference is optimal. The radial network minimizes the relative errors, by making all (independent) GPS marks relative to the same reference. Hence providing substantial relative positions, but the absolute positions depends solely on the single reference. The total error involved in positioning an ‘unknown’ using a single control depends on both the accuracy of the control coordinates and the accuracy of the measurements that relates the unknown.

Secondly, the closed loop network configuration requires that baselines are interconnected to form closed loop geometric figures. This option is the typically preferred configuration by surveyors, because multiple baselines to each point provide a level of redundancy. This type of configuration is fashioned after triangulation or closed traverse methods of conventional optical surveying. In such a case, redundant measurements are important not only to help detect blunders but also to provide a stronger adjustment.

Conventional optical survey measurements involve a relative angle and a distance. It is this relative direction or dependence on a back-sight that requires the survey vectors to be strung together between the beginning and ending points as a traverse. In other words, these direct measurements of distance and angles are inseparable from the marks they connect. This creates vectors free to float in space until constrained to survey controls. Closure errors in this type of survey traverse are typically removed by equally
distributing the angular closure error and then prorating the remaining errors based on the lengths of the traverse lines. This method assumes that the errors occur systematically and propagate evenly through the traverse.

Conversely, GPS measurements are independent and are not connected to each other. Therefore, the back-sight requirement of an optical survey does not hold for GPS surveys. Although possible, artificially creating chains of dependencies among survey marks is not optimal. Hence, GPS vectors/baselines can be put together in any format without geometric restriction.

GPS surveying is by nature radial, meaning measurements are made relative to a base station. Hence, GPS survey projects have inherent characteristics that are very different from those of optically measured networks. This difference creates a way to rethink how GPS projects should be processed. An appropriate post-processing strategy may include using the basic radial network, modified by the inclusion of multiple reference stations to provide stability and reliability, in the network. These additional reference stations are distant from the core radial network (approximately 1000 km), to allow for the absolute tropospheric parameters to be estimated more reliably, as decorrelation of tropospheric effects at both ends of the baselines is achieved with longer baselines as described in Section 2.3.1. Additionally, this modified version improves the absolute positions over the traditional single reference radial network, by the incorporation of additional reference and also removes the single reference control bias.

In general, Duan et al. (1996) suggested that the baseline length for network design studies should be longer than 500 km to determine the tropospheric delay. They
recommend, using remote stations distant from the core network. Schüler (2001) points out that the network should have either a large diameter or at least one baseline that is longer than 700 km to measure tropospheric delay. Ugur et al. (2013), used OPUS-Projects to test various network designs vs. changes in tropospheric delay, and the quality of the positioning results. In their study, a multiple reference network was designed, based on the recommendation of Schüler (2001), to include at least one longer baseline. The enhanced approach selects multiple stations from the IGS network as the distant reference stations aligned the surveyed data to an accurate global reference frame and improved troposphere determinations and resulting heights.

Overall, all these studies concluded that the combination of both short and long baselines optimizes the amount of observations by mutual observation of the satellites and long baselines are needed to dissociate the tropospheric correction adjustment on the short baselines. In this dissertation a modified radial network, the multiple reference radial network design, was further explored to develop this concept of network configuration for GPS processing technique.

OPUS-Projects requires that baselines be defined for each session. Therefore, the session network design provides a basic design strategy for the baselines created during the session processing. The design strategy gives consideration to the baseline length, selection and number of reference stations to include within the network, and the overall configuration. In this study, with the use of the IGLD 85 height modernization project survey data of 1997, 2005 and 2010, and the OPUS-Projects processing system, the multiple reference radial network design will be tested and compared to the closed
geometric network, with special attention placed on the accuracy derived ellipsoid height from each network. The following subsections describe the network designs under investigation.

**Multiple Reference Radial Configuration – (User Defined)**

The configuration is comprised of four reference stations (in some cases 5), and is designed to include one reference station within 100 km of the unknown marks. This reference station is taken from the national CORS network with 24 hours of data and is referred to as a “HUB”. The term “HUB” is NGS jargon meaning a mark that is preferentially selected for inclusion in baselines. In other words, the designated CORS that is a HUB is definitely connected to other survey marks included in the project (Armstrong, 2014). The additional three or four reference stations included are located further from the unknown marks, at least 1000 km away from the core network. These more distant stations are taken from the IGS network, as these stations anchor the surveyed data to an accurate global reference frame. The multiple reference radial configuration is shown in Figure 5.4, where the relative positions are optimized by the single HUB and the absolute positions are optimized by the multiple references.
Figure 5.4. Example of Multiple Reference Radial Network, included are the unknown marks (grey circles), CORS (HUB) (red triangle), IGS reference stations (blue triangles), which radiate from the HUB (modified radial network).

The IGS sites are tightly constrained while the HUB is not constrained (loosely constrained), and is free to be positioned by the IGS sites within the adjustment. The other marks are also adjusted relative to the IGS sites because of their connecting baselines to the HUB.

The multiple reference base approach translates as a “USER” defined network design in OPUS-Projects, as it is independent of the built-in options supplied in OPUS-Projects. The location of CORS station (the designated HUB) was selected based on the location of the surveyed marks and session. The selected IGS stations were ALGO, and NRC1 from
Ontario, Canada, CAGS from Quebec, Canada, MDO1 from Texas, and GODE, from Maryland in the United States of America.

**Closed Loop Network Configuration - OPUS-Projects Defined**

The study data was also processed using an OPUS-Projects defined network design, Triangle Network Design (TRI). The TRI (see Figure 5.5) selects baselines using the Delaunay triangulation algorithm (Armstrong, 2014). This algorithm selects lines connecting points, such that no point falls inside the circumscribing circle of any triangle, i.e. the circle connecting the three vertices of the triangle, other possible lines connecting points are ignored. Delaunay triangulation maximizes the minimum angle of all the angles of the triangles defined, thereby avoiding “skinny triangles” as much as possible. This design may also permit the possibility of a station being several successive stations distant from a constrained station. Planning the location of constrained referenced stations within the overall network design is very important. In this configuration, all surveyed stations and selected reference stations are automatically selected as HUBs (Armstrong, 2014). This network configuration also used the same CORS and IGS station as the USER configuration. Since this is an OPUS-Projects defined network design, the operator can only control the number of reference stations used in the network.
Figure 5.5. Example of Triangle Network Configuration (TRI), included unknown marks (grey circles) and Reference Station (CORS/IGS) (red triangles) are connected to their neighbors forming triangles (closed loop network)

5.3 Data Processing Results and Analysis

The GPS sessions were processed using the selected network designs; the results fell within the preset tolerances described in Section 4.2.2: Table 4.2, following this a network adjustment was performed using OPUS-Projects Network Adjustment suite. Figures 5.6 and 5.7, shows examples of a computed session solution for the IGLD 2010 survey: DOY 160, for the different network configurations. OPUS-Projects allow the user to process sessions repeatedly to derive the best session solution. Consequently, any surveyed station in the session solution that fell outside of the tolerance was individually
inspected and the stations within that session were flagged and eliminated from the session. However, there were situations that required a station to be eliminated from the project because of a low presentation of observations, a large percentage of float vs. fixed solutions, both caused by poor satellite observation. Stations were also eliminated because of the inability to verify antenna details (height and type. Even though there were preset tolerances, the user has the discretion to still accept flagged stations that are not very far off the tolerances.

Figure 5.6, shows the multiple reference radial network configuration for a survey session from 2010 IGLD datasets DOY 160. Shown are the surveyed station (green) and all the observed reference stations (orange) located within the area. The network includes a single HUB (PARY) constrained by three IGS Station (ALGO; GODE; MDO1). Table 5.2, shows the solution quality of the processed survey session.

Figure 5.7, shows the TRI network configuration for a survey session from the 2010 IGLD dataset DOY 160. Shown are the surveyed stations (green) and all the observed reference stations (orange) within the area. The network includes a combination of CORS HUBs (PARY; YOU6; SUP3) and IGS (ALGO; GODE, MDO1) reference stations. Table 5.3, shows the solution quality of the processed survey session. The solution quality indicators of the TRI network (Table 5.3), does not include values for the RMS, this is due to the complexity of the network’s geometry and processing time cost. The baselines in OPUS-Projects, are usually processed individually to prepare for a network solution. Then the "whole" network's data is read to create a normal matrix for the entire network, that matrix is solved, and the data re-read to create post-fit RMSs. The
processing engine of OPUS-Projects, PAGES, does not store the observations internally; hence the time cost factor of reading the data for all those TRI baselines is expensive (Mark Schenewerk, pers. comm.)³.

Figure 5.6. Processed survey session from the 2010 IGLD datasets DOY 160 based on the Multiple Reference Radial Network Design. Shown are the surveyed station (green) and all the observed Reference stations (orange) located within the area. The network includes a single HUB (PARY) constrained by three IGS Station (ALGO; GODE; MDO1)

³ Mark Schenewerk (NOAA/NGS) in E-mail correspondence with the author, March 2015
Figure 5.7. Processed Session solution and the Solution Quality Indicator, for a survey session from 2010 IGLD datasets DOY 160 based on the Triangle Network Design (TRI). Shown are the surveyed station (green) and all the observed Reference stations (orange) located within the area.

Table 5.2. Solution Quality Indicators from the 2010 IGLD datasets DOY 160 based on the Multiple Reference Radial Network Design

<table>
<thead>
<tr>
<th>Marks</th>
<th>Ephemeris. Type</th>
<th>Observation (%)</th>
<th>Fixed (%)</th>
<th>RMS (m)</th>
<th>Std. Dev Latitude (m)</th>
<th>Std. Dev Longitude (m)</th>
<th>Std. Dev Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>053D</td>
<td>Precise</td>
<td>97.8</td>
<td>100.0</td>
<td>0.014</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>079J</td>
<td>Precise</td>
<td>97.2</td>
<td>100.0</td>
<td>0.013</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>085G</td>
<td>Precise</td>
<td>97.8</td>
<td>100.0</td>
<td>0.014</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>090G</td>
<td>Precise</td>
<td>93.5</td>
<td>90.7</td>
<td>0.016</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>9014</td>
<td>Precise</td>
<td>92.5</td>
<td>100.0</td>
<td>0.015</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>A001</td>
<td>Precise</td>
<td>88.4</td>
<td>93.3</td>
<td>0.018</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>FORT</td>
<td>Precise</td>
<td>97.3</td>
<td>100.0</td>
<td>0.015</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>G321</td>
<td>Precise</td>
<td>97.0</td>
<td>100.0</td>
<td>0.014</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>H115</td>
<td>Precise</td>
<td>88.2</td>
<td>100.0</td>
<td>0.021</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
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<tr>
<td>N235</td>
<td>Precise</td>
<td>90.4</td>
<td>89.2</td>
<td>0.020</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>OHMH</td>
<td>Precise</td>
<td>94.7</td>
<td>93.3</td>
<td>0.016</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>RETA</td>
<td>Precise</td>
<td>88.7</td>
<td>100.0</td>
<td>0.019</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>Preferences</td>
<td>Best Available</td>
<td>≥80.0</td>
<td>≥80.0</td>
<td>≤0.025</td>
<td>≤0.020</td>
<td>≤0.020</td>
<td>≤0.025</td>
</tr>
</tbody>
</table>
Table 5.3. Solution Quality Indicator from the 2010 IGLD datasets DOY 160 based on the TRI Network Design

<table>
<thead>
<tr>
<th>Marks</th>
<th>Ephemeris. Type</th>
<th>Observation (%)</th>
<th>Fixed (%)</th>
<th>RMS (m)</th>
<th>Std. Dev Latitude (m)</th>
<th>Std. Dev Longitude (m)</th>
<th>Std. Dev Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>053D</td>
<td>Precise</td>
<td>94.4</td>
<td>96.8</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>079J</td>
<td>Precise</td>
<td>93.5</td>
<td>93.5</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>085G</td>
<td>Precise</td>
<td>94.9</td>
<td>91.8</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>090G</td>
<td>Precise</td>
<td>88.5</td>
<td>93.2</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>9014</td>
<td>Precise</td>
<td>88.2</td>
<td>90.5</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
</tr>
<tr>
<td>A001</td>
<td>Precise</td>
<td>86.3</td>
<td>93.0</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>FORT</td>
<td>Precise</td>
<td>90.6</td>
<td>98.0</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>G321</td>
<td>Precise</td>
<td>94.7</td>
<td>98.1</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>H115</td>
<td>Precise</td>
<td>85.8</td>
<td>96.4</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>N235</td>
<td>Precise</td>
<td>85.3</td>
<td>90.4</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>OHMH</td>
<td>Precise</td>
<td>88.9</td>
<td>96.4</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>RETA</td>
<td>Precise</td>
<td>87.6</td>
<td>95.5</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>Preferences</td>
<td>Best Available</td>
<td>≥80.0</td>
<td>≥80.0</td>
<td>≤0.025</td>
<td>≤0.020</td>
<td>≤0.020</td>
<td>≤0.025</td>
</tr>
</tbody>
</table>

5.3.1 Solution Quality Indicator: Post-Processing Results 1997, 2005, 2010

The Criteria to evaluate "success" of a solution include:

1. The standard error of unit weight this value should be near 1. The acceptable range is 0.1 to 1.1. The standard error of unit weight is a scale factor generated from the least squares adjustment which judges how well you’re a priori weights were set.

2. The overall RMS should be less than the project preference threshold (Section 5.2.2: Table 5.1).

3. The observation count - the number of observations used for each baseline, should be near the expected number based upon data duration and field logs.
4. The percentage of *omitted observations* should be smaller than the implied preference. For example, if the observations used preference (Section 5.2.2: Table 5.1) was set at 80%, then the omitted observations should be less than 20%.

5. The *ambiguities fixed* should also meet the preference set (Section 5.2.2: Table 5.1).

Tables 5.4 and 5.5, gives a summary of the solution quality report produced from the OPUS-Projects results using the multiple reference and TRI network design.

**Table 5.4. Summary of the solution quality report Multiple Reference Radial Network Design**

<table>
<thead>
<tr>
<th>Multiple Reference Radial Network Design</th>
<th>1997</th>
<th>2005</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Error</td>
<td>1.01</td>
<td>0.959</td>
<td>0.847</td>
</tr>
<tr>
<td>Overall RMS (cm)</td>
<td>1.99</td>
<td>1.65</td>
<td>1.56</td>
</tr>
<tr>
<td>Average % Fixed</td>
<td>92.2</td>
<td>90.73</td>
<td>92.24</td>
</tr>
<tr>
<td>Total Observation</td>
<td>3378273</td>
<td>8988362</td>
<td>8578549</td>
</tr>
<tr>
<td>Total Observations Omitted</td>
<td>114991.147</td>
<td>218186.858</td>
<td>459054.106</td>
</tr>
<tr>
<td>Average % Observations Omitted</td>
<td>4.05</td>
<td>2.43</td>
<td>5.23</td>
</tr>
</tbody>
</table>

**Table 5.5. Summary of the solution quality report TRI Network Design**

<table>
<thead>
<tr>
<th>TRI Network Design</th>
<th>1997</th>
<th>2005</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Error</td>
<td>1.07</td>
<td>0.900</td>
<td>0.953</td>
</tr>
<tr>
<td>Overall RMS (cm)</td>
<td>1.88</td>
<td>1.73</td>
<td>1.71</td>
</tr>
<tr>
<td>Average % Fixed</td>
<td>92.95</td>
<td>91.96</td>
<td>92.56</td>
</tr>
<tr>
<td>Total Observation</td>
<td>7861037</td>
<td>18370693</td>
<td>18535143</td>
</tr>
<tr>
<td>Total Observations Omitted</td>
<td>256788.322</td>
<td>646136.886</td>
<td>1174600</td>
</tr>
<tr>
<td>Average % Observations Omitted</td>
<td>3.27</td>
<td>3.52</td>
<td>6.34</td>
</tr>
</tbody>
</table>
5.3.2 Comparison of OPUS-Projects Solutions (Multiple Reference Radial Network vs. TRI)

The solutions generated by the multiple reference radial network design were compared to the solutions generated by the OPUS-Projects built-in TRI network. This involved taking the difference between the coordinate generated from the multiple reference radial network and the TRI configuration. The mean and the standard deviations for the comparison of the multiple reference radial network and TRI network configuration are seen in Table 5.6. Figure 5.8 to 5.10 presents the computed difference in the NEU components.

Table 5.6. Comparison of OPUS-Projects USER to OPUS-Projects TRI

<table>
<thead>
<tr>
<th>OPUS-Projects: Multiple Reference Radial Network (MRR) minus TRI</th>
<th>OPUS-Project MRR minus OPUS-Project TRI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Value</td>
<td>-0.004</td>
</tr>
<tr>
<td>Maximum Value</td>
<td>0.007</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.002</td>
</tr>
<tr>
<td>Mean</td>
<td>0.002</td>
</tr>
<tr>
<td>North (m)</td>
<td>-0.005</td>
</tr>
<tr>
<td>East (m)</td>
<td>0.014</td>
</tr>
<tr>
<td>UP (m)</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The comparison of the coordinates derived from the two configurations (Multiple reference and TRI) showed a mean of 2 mm and a standard deviation of 2 mm in the North component; a mean of 1 mm and a standard deviation of 4 mm in the East component.
component; and a mean of -4 mm and a standard deviation of 8 mm in the UP component. The multiple reference radial network performed comparatively well against the TRI based on the statistics produced. The difference statistics shown in Table 5.6 indicate that the multiple reference is a suitable alternative at a 95% confidence level to the TRI close loop network configuration for the determination of GPS ellipsoidal heights.

However, these networks were separated by their computational performance. From the dataset used, an average processing session may include anywhere from 10 to 60 stations. For the TRI network, the number of baselines grows geometrically with the number of stations included in the session, so, for a session which has 60 stations the number of baselines connecting these stations can be doubled. Due to the close loop geometry of this configuration there are a number of redundant or trivial baseline, which also makes this configuration computationally costly, also in terms of network adjustment, the inclusion of these trivial baselines may produce resultant solution statistics that are over-optimistic. Alternatively, the number of baselines in the multiple reference radial design, are equal to the number of stations included in the session. As a result, the multiple reference radial network design is not as computationally costly to execute a session processing, this is mainly due to the lack of redundant or trivial baselines.
Figure 5.8: Comparison of OPUS-Projects Coordinates: Multiple Reference Radial Network (MRR) vs. TRI (North Component)
Figure 5.9 Comparison of OPUS-Projects Coordinate: Multiple Reference Radial Network (MRR) vs. TRI (East Component) – East difference

2010 East Component: Multiple Reference Radial Network vs. TRI
Figure 5.10. Comparison of OPUS-Projects Coordinates: Multiple Reference Radial Network (MRN) vs. TRI (Ellipsoid Height) – UP difference.
5.3.3 Vertical Velocities Computation – (comparison between epochs)

The vertical velocity computations were carried out using the ellipsoidal heights computed using the multiple reference radial network. The reprocessed and readjusted IGLD surveys were compared between epochs by computing the vertical coordinate difference (epoch 2 – epoch1). These differences were in turn used to compute the relative velocities (vertical coordinate difference / time difference). Figures 5.11 to 5.13 show the spatial location of the benchmarks and the associated relative velocity. The coordinates and the derived vertical velocities for epoch pairs can be seen in Appendix A. The relative velocities assist in identifying uplifts and subsidence within the Great Lakes region. Between 2010 and 1997, the computed velocities had an average of 1.0 mm/yr and dispersion from the mean of 2.0 mm/yr. For the 2005 and 1997 periods the computed velocities had an average of 2.0 mm/yr and dispersion from the mean of 5.0 mm/yr. Finally, for the 2010 and 2005 periods the computed velocities had an average of 3.0 mm/yr and dispersion from the mean of 5.0 mm/yr. Those stations, which exhibited movements opposite to the sites surrounding them may have been affected by an independent factor that is site specific.
Figure 5.11. IGLD 85 Height Modernization Survey Project vertical velocities to the survey periods 2010 – 1997 (13 years). The map depicts the stations that are experiencing uplift (red line) or subsidence (blue line).
Figure 5.12. IGLD 85 Height Modernization Survey Project vertical velocities to the survey periods 2005 – 1997 (8 years). The map depicts the stations that are experiencing uplift (red line) or subsidence (blue line).
Figure 5.13. IGLD 85 Height Modernization Survey Project vertical velocities to the survey periods 2010 - 2005 (5 years). The map depicts the stations that are experiencing uplift (red line) or subsidence (blue line).
5.3.4 Comparison of the computed OPUS-Projects and Natural Resources Canada (NRCan) Solutions

The International Great Lakes Datum of 1985 (IGLD 85) Height Modernization projects (1997, 2005, and 2010) were also reprocessed by the NRCan. NRCan used, the Bernese GPS software (AIUB, 2014), IGS orbits & IGS05 antenna calibrations (Craymer et al. 2012).

Table 5.7. The sample set of benchmarks used in the comparison of the computed OPUS-Projects and Natural Resources Canada (NRCan) Solutions

<table>
<thead>
<tr>
<th>Station Name</th>
<th>Stata</th>
<th>PID</th>
<th>Associated Lake</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESSEX A</td>
<td>MI</td>
<td>AA8053</td>
<td>Huron</td>
</tr>
<tr>
<td>UNIT 10 106</td>
<td>MI</td>
<td>AE8008</td>
<td>Superior</td>
</tr>
<tr>
<td>602</td>
<td>MN</td>
<td>AE8289</td>
<td>Superior</td>
</tr>
<tr>
<td>909 9018 K</td>
<td>MI</td>
<td>AH7272</td>
<td>Superior</td>
</tr>
<tr>
<td>DETOUR MARINA</td>
<td>MI</td>
<td>AH9228</td>
<td>Huron</td>
</tr>
<tr>
<td>LAUNCH SITE</td>
<td>MI</td>
<td>AH9229</td>
<td>Huron</td>
</tr>
<tr>
<td>905 2000 F</td>
<td>NY</td>
<td>AH9230</td>
<td>Ontario</td>
</tr>
<tr>
<td>905 2058 K</td>
<td>NY</td>
<td>AH9232</td>
<td>Ontario</td>
</tr>
<tr>
<td>905 2076 H</td>
<td>NY</td>
<td>AH9233</td>
<td>Ontario</td>
</tr>
<tr>
<td>906 3020 H</td>
<td>NY</td>
<td>AH9234</td>
<td>Erie</td>
</tr>
<tr>
<td>906 3085 G</td>
<td>OH</td>
<td>AH9237</td>
<td>Erie</td>
</tr>
<tr>
<td>906 3090 G</td>
<td>MI</td>
<td>AH9238</td>
<td>Erie</td>
</tr>
<tr>
<td>78U3005</td>
<td>ON</td>
<td>AH9241</td>
<td>Ontario</td>
</tr>
<tr>
<td>913007</td>
<td>ON</td>
<td>AH9244</td>
<td>Superior</td>
</tr>
<tr>
<td>94U9451</td>
<td>ON</td>
<td>AH9247</td>
<td>Huron</td>
</tr>
<tr>
<td>953000</td>
<td>ON</td>
<td>AH9248</td>
<td>Superior</td>
</tr>
<tr>
<td>973006</td>
<td>ON</td>
<td>AH9249</td>
<td>Erie</td>
</tr>
<tr>
<td>973007</td>
<td>ON</td>
<td>AH9250</td>
<td>Huron</td>
</tr>
<tr>
<td>N 235</td>
<td>MI</td>
<td>NE0898</td>
<td>St. Clair</td>
</tr>
<tr>
<td>LSC 5 C 93</td>
<td>MI</td>
<td>OJ0517</td>
<td>Huron</td>
</tr>
<tr>
<td>LAKEPORT RM 2</td>
<td>MI</td>
<td>OJ0599</td>
<td>Huron</td>
</tr>
<tr>
<td>A 293</td>
<td>MI</td>
<td>RJ0586</td>
<td>Superior</td>
</tr>
<tr>
<td>70U672</td>
<td>ON</td>
<td>TY2525</td>
<td>Erie</td>
</tr>
<tr>
<td>81U111</td>
<td>ON</td>
<td>TY5484</td>
<td>St. Clair</td>
</tr>
<tr>
<td>GROS 1</td>
<td>ON</td>
<td>TY5827</td>
<td>Superior</td>
</tr>
</tbody>
</table>
The network adjustments of the IGLD survey campaigns NRCan solutions were retrieved, and the NAD83 coordinates derived from the ITRF 2005 were extracted to facilitate the comparison of the solutions. The results obtained in this study using OPUS-Projects were compared to the NRCan solutions. The OPUS-Projects NAD83 coordinate solutions were derived from ITRF 2008. A sample of the 25 benchmark stations that are common to all 3 survey campaigns were used in the comparison. These stations are located throughout the lakes, Table 5.7 identifies the stations and their associated lake. The absolute mean ellipsoidal height difference were 41.2 mm, 9.3 mm and 10.0 mm for 1997, 2005 and 2010 respectively. These overall differences may be attributed to the data processing methods used to compute the station coordinates.

5.3.5 Summary

This chapter dealt with the reprocessing of the International Great Lakes Datum Height Modernization survey of 1997, 2005 and 2010 using OPUS-Projects and coordinate values produced in the most recent realization of the reference frame (IGS08).

The comparison of the coordinates derived from the two configurations, the multiple reference radial network and TRI, indicated that the former is a viable GPS post-processing network configuration. The simplicity of the modified radial design, is effective for both relative and absolute positioning, also it is not as computationally timely as the TRI network. The combination of both short and long baselines optimizes the amount of observations by mutual observation of the satellites and long baselines are needed to dissociate the tropospheric correction adjustment on the short baselines. Due to
the interconnectivity of the closed loop network of TRI, the number of baselines grows geometrically with the number of stations included in the session which also makes this configuration computationally costly. Also this configuration includes a number of redundant or trivial baselines, in terms of network adjustment, the inclusion of these trivial baselines may produce resultant solution statistics that are over-optimistic.

The comparison of the vertical velocities computed for the time periods 2010 and 1997, and 2005 and 1997, closely agreed with the exception of a few sites. However, the period 2010 and 2005 showed very little comparison. Further investigations of the GPS CORS within the region are needed to detect the possibility of the occurrence seen in 2010 and 2005 periods. These vertical velocities, will be used in the Chapter 6 derived from the ‘campaign surveys’.
CHAPTER 6: CONTINUOUS GPS OBSERVATIONS: VERTICAL VELOCITY ANALYSIS

Chapter 5 focused on the reanalysis of the three (3) campaign surveys 1997, 2005 and 2010 of the IGLD Height Modernization Project executed by NGS. The results of this reanalysis were coordinate values in the IGS08 reference frame for each survey campaign. These coordinates were then used to compute the vertical velocity for each surveyed station. The results of the vertical velocities are illustrated in Figures 5.8 to 5.10. Campaign type measurements are temporally intermittent and may be inefficient in detecting nonlinear crustal movements or small leaps, which may occur in each new campaign observation. Therefore stable CORS stations may get around such problems, by providing their uninterrupted point positions in a high level temporal resolution. This notion will be investigated further in this chapter.

The focus in this chapter is on the time series analysis of a selection of continuous GPS measurements from the US CORS and Canadian Active Control System (CACS). The position solutions are from the rerun of data archived at the NGS, and include all available data for the 16-year interval, covering January 1997 to December 2013. Obtaining a time series of positions (horizontal and vertical) derived from the GPS observations allowed for the analysis of the overall movement or deformation. In particular, this chapter describes the geological makeup of the study area, procedures,
models used and the results obtained from the time series analysis of the selected active control located around the Great Lakes.

6.1 Introduction

GPS has been widely used to measure horizontal deformation of the Earth’s crust with outstanding precision (Bouin and Wöppleman, 2010). In contrast, vertical deformations are more difficult to measure, since GPS does not produce the same accuracy of height determination, compared with horizontal positions, due not only to inherent geometrical weakness, but also to several observational error sources (e.g., signal propagation effects; phase center corrections of the transmitting and/or tracking antennas) which primarily affect the height component (Dobson, 1995; Bouin and Wöppleman, 2010). Provided that error correction models are adopted, high accuracy height measurements can be obtained. As such, GPS technology has been frequently applied to monitor geophysical phenomena through the use of velocities of discrete points estimated from positioning time series of permanent GPS stations (Santamaria-Gómez et al., 2011). GPS vertical velocities have been used to study subduction zones (e.g., Bergeot et al., 2009), to assess postglacial rebound processes (e.g., Nocquet, Calais and Parsons, 2005; Sella et al., 2007) and to correct long term sea level records from tide gauges (e.g., Wöppelmann et al., 2009).

As mentioned in Chapter 2, processed GPS data yield heights above a smooth ellipsoid surface; yet, orthometric height, referring to the local mean sea level, which approximates the surface of the geoid, is often used in practical surveying and engineering applications. Orthometric heights are physically-based quantities. GPS, which is a geometric technique, cannot directly measure orthometric heights. However, for monitoring
purposes such as, the investigation of vertical displacements (subsidence or uplift), temporal changes in ellipsoidal height can be equated to changes in orthometric height (Dobson, 1995). Throughout this chapter, the reference to the vertical means a measurement along the ellipsoidal height, and thus not related to the direction of the plumb line.

This chapter focuses on the vertical displacement of the Great Lakes, which involves the re-analysis of more than a decade of continuous GPS observations. Beginning with January 1997 to December 2013, all available weekly continuous GPS observations were processed with the same software and models, OPUS-Network (OPUS-Net). Every effort was made to maintain a consistence global reference frame (IGS08). This chapter describes the procedure, models used and the results obtained from the analysis of twenty (20) continuous GPS data from the US CORS and CACS network over roughly a 16-year span. Vertical velocities of the GPS stations with time series greater than three (3) years were computed.

6.2 GPS data sets and data processing

6.2.1 Background: The global reference frame

The fundamental parameters for GPS deformation studies are the velocity with which the stations move and the correlated uncertainties of the data. The issue that affects our knowledge of these parameters is connected to the reference frame, to which the GPS positions are tied and the noise properties of the data.
The study of Earth’s deformation requires a global reference system, to which measurements from different locations, times and observation techniques can be uniquely referenced (Nikolaidis, 2002). For such applications, a terrestrial reference frame is generally convenient. The particular reference system chosen is the International GNSS Service (IGS) 2008 (IGS08), this frame is based on GPS observations and was designed to be consistent with the International Terrestrial Reference Frame of 2008 (ITRF2008) (Rebischung et al., 2012). ITRF2008 is the latest frame realization of the International Earth Rotation and Reference Systems Service (IERS). The ITRF is regularly updated to take into account new accumulations and improved analysis strategies (Altamimi, Collilieux and Métivier, 2011). The main elements that determine an ITRF realization are the contributed space geodetic solutions for global sets of coordinates and velocities that are combined with a datum that specify how the frame, origin, orientation, scale and time evolution are materialized (Ray, Dong and Altamimi, 2004). The space geodetic solutions, which contribute to the ITRF construction, are produced using techniques such as Very Long Baseline Interferometry (VLBI), Satellite Laser Ranging (SLR), GNSS, and Doppler Orbitography Radio-positioning Integrated by Satellite (DORIS) (Boucher, Altamimi and Sillard, 1998; Altamimi, Collilieux and Métivier, 2011).

Since 2000, the IGS has used its own realizations of the successive ITRF releases as reference frames for its products. In November 2006, a reference frame based on ITRF2005, called IGS05, was adopted by the IGS, simultaneously with the switch from relative to absolute antenna phase center calibrations (Schmid et al. 2007). As a result of growing velocity propagation errors and of jumps having affected the positions of many reference stations, IGS05 become obsolete. Therefore, a new reference frame based on
ITRF2008, called IGS08, was designed and officially adopted by the IGS starting with GPS week 1632 (April 17, 2011).

Although the ITRF2008 and IGS08 frames are compatible, the latter is based on a significant contribution from the GPS community. The IGS08 frame contains a subset of 232 globally-distributed, stable and well-performing GPS stations from the ITRF2008 network. Figure 6.1 shows the coverage of these sites within the IGS08 network. IGS08 had to be consistent with the latest igs08.atx calibrations. The impact of the receiver antenna calibration update on the IGS08 station positions was thus assessed and turned out to be non-negligible in several cases. Hence, IGS08 was in the end defined as an extraction from ITRF2008 to which position corrections were applied to account for the receiver antenna calibration update (Rebischung et al., 2012).

![Figure 6.1. Full IGS08 network, which contains 232 GNSS stations (Rebischung et al., 2012)](image)
6.2.2 GPS Data

A subset of US National CORS and CACS GPS stations for over the sixteen year period (1997-2013) that encompasses the vertical motions of the Great Lakes was analyzed. Table 6.1 summarizes the data availability records and the characteristics of the stations, and Figure 6.2 illustrates the geographical location of the stations.

Table 6.1. GPS Station Descriptions and Data Availability Record

<table>
<thead>
<tr>
<th>CORS Site</th>
<th>Station</th>
<th>Permanent Identifier (PID)</th>
<th>State</th>
<th>Latitude, N°</th>
<th>Longitude, E°</th>
<th>Years</th>
<th>Period of record</th>
</tr>
</thead>
<tbody>
<tr>
<td>MARBLEHEAD</td>
<td>OHMH</td>
<td>DG9751</td>
<td>OH</td>
<td>41.54368</td>
<td>82.73145</td>
<td>8</td>
<td>2005-2013</td>
</tr>
<tr>
<td>CALUMET HARBOR</td>
<td>CALU</td>
<td>DF5765</td>
<td>MI</td>
<td>41.72985</td>
<td>87.53844</td>
<td>9</td>
<td>2003-2012</td>
</tr>
<tr>
<td>NORTH LIBERTY</td>
<td>NLIB</td>
<td>AF5253</td>
<td>IA</td>
<td>41.77159</td>
<td>91.57490</td>
<td>16</td>
<td>1997-2013</td>
</tr>
<tr>
<td>BUFFALO</td>
<td>BFNY</td>
<td>DE7164</td>
<td>NY</td>
<td>42.87756</td>
<td>78.89045</td>
<td>11</td>
<td>2002-2013</td>
</tr>
<tr>
<td>MILWAUKEE 1</td>
<td>MIL1</td>
<td>AF9485</td>
<td>WI</td>
<td>43.00254</td>
<td>87.88845</td>
<td>9</td>
<td>1997-2006</td>
</tr>
<tr>
<td>FORT GRATIOT</td>
<td>FRTG</td>
<td>DF5360</td>
<td>MI</td>
<td>43.03757</td>
<td>82.48816</td>
<td>10</td>
<td>2003-2013</td>
</tr>
<tr>
<td>PORT WELLER</td>
<td>PWEL</td>
<td>DF5381</td>
<td>ON</td>
<td>43.23673</td>
<td>79.21967</td>
<td>11</td>
<td>2003-2013</td>
</tr>
<tr>
<td>MUSKEGON</td>
<td>MSKY</td>
<td>DG9761</td>
<td>MI</td>
<td>43.23755</td>
<td>86.05458</td>
<td>8</td>
<td>2005-2013</td>
</tr>
<tr>
<td>SAGINAW</td>
<td>BAYR</td>
<td>AJ5551</td>
<td>MI</td>
<td>43.44623</td>
<td>83.89173</td>
<td>10</td>
<td>2000-2013</td>
</tr>
<tr>
<td>KINGSTON</td>
<td>KNGS</td>
<td>DF5369</td>
<td>ON</td>
<td>44.21869</td>
<td>76.51727</td>
<td>10</td>
<td>2003-2013</td>
</tr>
<tr>
<td>STURGEON BAY 1</td>
<td>STB1</td>
<td>AF9553</td>
<td>WI</td>
<td>44.79549</td>
<td>87.31433</td>
<td>12</td>
<td>1996-2008</td>
</tr>
<tr>
<td>ALPENA</td>
<td>NOR3</td>
<td>AJ5563</td>
<td>MI</td>
<td>45.06858</td>
<td>83.56865</td>
<td>12</td>
<td>2001-2013</td>
</tr>
<tr>
<td>PARRY SOUND</td>
<td>PARY</td>
<td>DF5376</td>
<td>ON</td>
<td>45.33855</td>
<td>80.03589</td>
<td>10</td>
<td>2003-2013</td>
</tr>
<tr>
<td>CHEBOYGAN 1</td>
<td>CHB1</td>
<td>AF9491</td>
<td>MI</td>
<td>45.65349</td>
<td>84.46565</td>
<td>12</td>
<td>1996-2008</td>
</tr>
<tr>
<td>ALGONQUIN PARK</td>
<td>ALGO</td>
<td>DE6553</td>
<td>ON</td>
<td>45.9558</td>
<td>78.07137</td>
<td>16</td>
<td>1997-2013</td>
</tr>
<tr>
<td>POINT IROQUOIS</td>
<td>PTIR</td>
<td>DF5377</td>
<td>MI</td>
<td>46.48458</td>
<td>84.63084</td>
<td>10</td>
<td>2003-2013</td>
</tr>
<tr>
<td>MARQUETTE</td>
<td>MIMQ</td>
<td>DG9743</td>
<td>MI</td>
<td>46.54555</td>
<td>87.3787</td>
<td>9</td>
<td>2004-2013</td>
</tr>
<tr>
<td>WISCONSIN POINT 1</td>
<td>WIS1</td>
<td>AF9560</td>
<td>WI</td>
<td>46.70506</td>
<td>92.01521</td>
<td>11</td>
<td>1996-2007</td>
</tr>
<tr>
<td>WISCONSIN POINT 5</td>
<td>WIS5</td>
<td>DJ7858</td>
<td>WI</td>
<td>46.70506</td>
<td>92.01521</td>
<td>5</td>
<td>2008-2013</td>
</tr>
<tr>
<td>UPPER KEWEENAW 1</td>
<td>KEW1</td>
<td>AF9550</td>
<td>MI</td>
<td>47.22707</td>
<td>88.62433</td>
<td>11</td>
<td>1997-2008</td>
</tr>
<tr>
<td>UPPER KEWEENAW 5</td>
<td>KEW5</td>
<td>DK4181</td>
<td>MI</td>
<td>47.22707</td>
<td>88.62433</td>
<td>5</td>
<td>2008-2013</td>
</tr>
<tr>
<td>MICHIPICOTEN</td>
<td>MCHN</td>
<td>DL6502</td>
<td>ON</td>
<td>47.96134</td>
<td>84.90085</td>
<td>4</td>
<td>2009-2013</td>
</tr>
<tr>
<td>ROSSPORT</td>
<td>ROSS</td>
<td>DF5384</td>
<td>ON</td>
<td>48.83373</td>
<td>57.51960</td>
<td>10</td>
<td>2003-2013</td>
</tr>
</tbody>
</table>
Figure 6.2. Location of continuous GPS stations included in the analysis. Combined US CORS and CACS Stations (black) and IGS Reference Stations (red). The stations are labeled according to their four character identification.
The monuments of these stations typically consist of concrete pillars with force-centering brass plates as a reference point. Most monuments are anchored to bedrock, although some are built on large concrete slabs buried in the sandy cover where bedrock is not available. The selection of the CORS/CACS stations was based on the inclusion of these stations in the IGLD Height Modernization survey projects, some of these stations were also collocated with the benchmarks surveyed in those projects.

6.2.3 GPS Observation

This section describes the data collection and processing procedures used to generate the coordinate time series.

All GPS observations and supplementary datasets used in this analysis were collected from the NGS database. NGS manages a network of CORS that provide GNSS data consisting of carrier phase and code range measurements in support of 3D positioning, meteorology, space weather, and geophysical applications throughout the United States, its territories, and a few foreign countries. The NGS CORS network as of January 2014 contained more than 1900 permanently and continuously operating, geodetic quality GPS stations (NGS, 2014).

For each day, measurement files in compressed RINEX format were collected for the twenty (20) CORS/CACS stations listed in Table 6.1, covering the time span of 01 January 1997 to 31 December 2013 (GPS weeks 886 to 1773). The measurement files required to analyze the GPS measurements, such as the broadcast navigation files and observation files were also collected from the NGS archives for each day. All files were
retrieved on a day to day basis prior to processing each solution, as discussed in the section below.

The station metadata, required to interpret the GPS observations, were collected from the NGS CORS website for each station. Each CORS station page includes a site log with the station identification and location, antenna and receiver hardware and firmware changes and antenna heights. The accuracy of this information is critical to the analysis, since errors or omissions in the site history, such as incorrect antenna heights and models, may incorrectly model the phase observables and offsets in the coordinate time series.

6.2.4 GPS Solutions using OPUS-Net Solutions

The weekly solutions were estimated from GPS measurements using OPUS-Net (OPUS-Net). OPUS-Net, is the newest edition of the OPUS suite of GPS processing tool and at this time it is still being beta tested.

This version of OPUS was developed to retain most of the GPS dataset observing requirements of OPUS-S, while incorporating some of the most advanced models and algorithms to extend the positioning capabilities and improve overall accuracy in the new network version known as OPUS-Net (Weston and Ray, 2010). Some of the improvements made, were the replacement of the three independent solutions algorithm in OPUS-S with a network approach where three nearby CORS and the 10 closest IGS reference stations are used in a simultaneous least squares solution. The CORS stations are primarily used to better estimate the troposphere while the position of the unknown station is determined primarily from the more precisely known and monitored IGS
reference stations. Soler, Weston and Foote (2011), stated that the advantage of this procedure is that the unknown systematic errors affecting the coordinates of the non-IGS CORS reference stations (CORS) will be avoided.

Additional enhancements to OPUS-Net include the implementation of absolute antenna patterns and ocean tides (FES2004), using reference station coordinates in IGS08 reference frame, as well as improved phase ambiguity integer fixing and relative tropospheric modeling (GPT and GMF a priori models) (Weston, Mader and Schenewerk, 2012).

Weston and Ray (2010), and Weston, Mader and Schenewerk, (2012), presented the evaluation of the accuracy and performance of the three OPUS versions (OPUS-S, OPUS-Net and OPUS-Projects) and demonstrated that the scatter of daily OPUS-S positions for the stations used in their studies was high due to inhomogeneous solution quality, while the network approach of OPUS-Net showed positions with lower scatter in the north and east components, but sometimes only slightly different in the height produced by OPUS-S. The variable performance for the up component could be due to limitations of the tropospheric modeling or the 3-D ocean tide loading. Moreover, the resulting OPUS-NET coordinate results, agreed very well with the IGS combined SINEX results but differences depend on whether weights are used or not. In addition, OPUS-Projects session processing showed excellent agreement with the coordinates derived using OPUS-Net. As such, to compute the CORS/CACS coordinates in this study OPUS-Net was selected.
Estimated parameters for each weekly solution included the station 3D Cartesian coordinates and integer ambiguities in the IGS08 frame. All datasets were processed using IGS precise (final) orbits, absolute antenna patterns, phase ambiguity integer fixing and relative troposphere modeling (GPT and GMF a priori models). Table 6.2 summarizes the observation data, reference frames, orbital models and processing strategies used by OPUS-Net in this research.

Table 6.2. Summary of OPUS-Net GPS data processing parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Basic observables</strong></td>
<td>Double differenced phase and code pseudo-range observations</td>
</tr>
<tr>
<td><strong>Session and sample rate</strong></td>
<td>24hrs/ 5secs (US-CORS) 30secs (CACS)</td>
</tr>
<tr>
<td><strong>Elevation cut-off angle</strong></td>
<td>15º</td>
</tr>
<tr>
<td><strong>Phase center variation (PCV)</strong></td>
<td>IGS absolute phase center for both tracking and transmitting antenna</td>
</tr>
<tr>
<td><strong>Tropospheric modelling</strong></td>
<td>Empirical Global Pressure and Temperature (GPT) model (Boehm et al., 2007); a priori zenith delays from the model, using standard atmosphere, mapped with the Global Mapping Function (GMF) (Boehm et al. 2006); zenith delay estimated as a Step-wise model with 30 min nodes.</td>
</tr>
<tr>
<td><strong>Ionosphere</strong></td>
<td>Dual frequency combination (First Order IONO-FREE)</td>
</tr>
<tr>
<td><strong>Ocean Tide Correction</strong></td>
<td>Ocean Tide Loading (FE2004)</td>
</tr>
<tr>
<td><strong>Orbits</strong></td>
<td>IGS Precise orbits</td>
</tr>
<tr>
<td><strong>Reference Frame</strong></td>
<td>IGS08 (Altamimi, Collilieux and Métivier, 2011)</td>
</tr>
<tr>
<td><strong>Adjustment</strong></td>
<td>Least squares network, IONO-FREE (L3) solution</td>
</tr>
</tbody>
</table>
The weekly solutions were provided by means of emailed reports. The OPUS-Net report (or output) contains a summary of the processed results from the submitted observation files. In addition to echoing the user input information, the output includes, the processing statistics; the computed Cartesian (X, Y, Z) and latitude longitude and ellipsoid height (φ, λ, h) coordinates for both the IGS 2008 (observation epoch) and the current realization, plate fixed, of the NAD 83(2011) (Epoch 2010.00), along with the NAVD 88 orthometric height based on the Geoid12A model. Also the output provides the processed solutions in the Universal Transverse Mercator (UTM) and the State Plane Coordinate Systems. The results also listed the IGS and CORS stations that were available over the time span. The time span is dependent on the station being processed and can span from 10 to 16 years.

6.3 Station Velocity Estimations

This section details the methods used to analyze the weekly coordinate solutions output from OPUS-Net to determine the station velocities.

To estimate the seasonal and trend terms from the coordinate solutions, two approaches were considered (Dong et al., 2002; Zhang et al., 2003). One approach performs a global network adjustment by estimating these terms from all of the observations simultaneously. The advantage of this approach is that the full covariance matrix is used, where all correlations are taken into account. Apart from the significant computational burden, the primary disadvantage is that any outlier or misfit of a site will affect the estimates of all the other sites. The alternate approach is time series analysis, which involves constructing a position time series for each site separately from daily (or
weekly) solutions and then decomposes the time series to extract the seasonal, trend and random terms. The advantage of this approach is that it is easier to detect and handle the position outliers of each time series. The misfit of one time series will not affect the estimates of the other sites. The weakness of this approach is that the correlation between each time series is neglected. However, Zhang (1996) showed that these correlations are small. The latter approach was adopted, since it is more sensitive to the outliers, which may be the primary error source.

The positions derived for each station are first combined to form the time series of the positions in the Earth Centered Earth Fixed (ECEF) reference system in the IGS08 reference frame. The Cartesian (X,Y,Z) coordinate time series were then rotated to a local topocentric North, East, Up (NEU) coordinate system. Firstly, the mean coordinate (X̄, Ȳ, Z̄) of the stations were subtracted from its weekly coordinate estimates (X,Y,Z), to get a time series of position residuals (ΔX,ΔY,ΔZ) in the ECEF reference system. Then, the time series in the NEU coordinate system were obtained by transforming the residual vector using the transformation in equation (6-1), where λ and φ are the station longitude and latitude, respectively.

\[
\begin{bmatrix}
N \\
E \\
U
\end{bmatrix} =
\begin{bmatrix}
-\cos \lambda \sin \phi & -\sin \lambda \sin \phi & \cos \phi \\
-\sin \lambda & \cos \lambda & 0 \\
\cos \lambda \cos \phi & \sin \lambda \cos \phi & \sin \phi
\end{bmatrix}
\begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z
\end{bmatrix}
\]

The NEU data were cleaned and modeled, independently in each of the 3 coordinate directions. The next section describes the analysis of the time series and estimation of the site motion.
6.3.1 Time Series Analysis on the CORS/CACS Observations

The station coordinates velocities (trend analysis), seasonal and random effects in the time series were investigated by time series analysis after producing the weekly time series of the station coordinates as described in Section 6.3.

The station coordinates $Y(t_i)$ are successive epochs of observations obtained in time $t_i$ ($i = 1, \cdots, n$), which can be represented as an additive decomposition model of trend ($T$), seasonal ($S$) and residual ($e$) components. The general form (Brockwell and Davis, 2002) of the model is:

$$Y_t = T(t_i) + S(t_i) + e(t_i), \ i = 1, \cdots, n. \quad (6-2)$$

The trend component is the long term change which can be modeled by the polynomial function in the time domain:

$$T(t_i) = \sum_{k=0}^{m} \beta_k t_i^k \quad (6-3)$$

In this equation, $\beta_k$ (k = 0 … m) are the parameters depending on the order of the polynomial function (m = 1 for linear trends). Here the trend was treated as linear so the equation (6-3) can be represented as:

$$T(t_i) = \beta_0 + \beta_1 t_i, \quad (6-4)$$

where $t_i$ denotes the year at time index $i$. The reference position ($\beta_0$) and linear rate ($\beta_1$) of the linear model can be used to derive the magnitude and direction of the changes (Verbesselt et al., 2010).
The seasonal component can be modeled using a truncated Fourier series:

\[ S(t_i) = \sum_{k=1}^{p} a_k \sin\left(\frac{2\pi t_i}{f_k}\right) + b_k \cos\left(\frac{2\pi t_i}{f_k}\right), \quad f_k, (k = 1 \cdots p) \]  

(6-5)

where \( f_k \) is the frequency of the signal, while \( a_k \) and \( b_k \) are the periodical components of the series.

Theoretically, function (6-5) can model any shape of oscillation curve in the time series, but in practice, only the two terms for, \( k = 1, f_1 = 1 \) year and \( k = 2, f_2 = 0.5 \) year are sufficient to represent the annual and semi-annual oscillations. Therefore, equation (6-5) can be written as:

\[ S(t_i) = a_1 \cos(2\pi t_i) + b_1 \sin(2\pi t_i) + a_2 \cos(4\pi t_i) + b_2 \sin(4\pi t_i) \]  

(6-6)

where coefficients \( a_1 \) and \( b_1 \) describe the annual periodic motion, and \( a_2 \) and \( b_2 \) describe the semiannual motion.

Therefore, the general form of the additive decomposition model, (equation 6-2), can be rewritten by combining equations (6-4) and (6-5), and including the residual term, so that:

\[ Y(t_i) = \beta_0 + \beta_1 t_i + a_1 \cos(2\pi t_i) + b_1 \sin(2\pi t_i) + a_2 \cos(4\pi t_i) + b_2 \sin(4\pi t_i) + e(t_i). \]  

(6-7)

The unknown trend coefficients \( \beta_0, \beta_1 \) and the unknown seasonal component coefficients, \( a_1, b_1, a_2 \) and \( b_2 \) can be estimated using ordinary least squares. The residual component, \( e(t_i) \), is the remaining variations in the data when the seasonal and trend components are removed from the time series (Brockwell and Davis, 2002).
The decomposition described was facilitated using the R statistics software (Cleveland et al., 1990); the particular function is seasonal-trend decomposition procedure (STL). STL is able to decompose a series into trend, seasonal and residual components based on the locally weighted regression (Cleveland et al., 1990). Each of the three coordinate directions (NEU) was decomposed and each component was analyzed.

Figures 6.3 and 6.4, illustrate examples of the unedited linearly modeled height time series for site KEW1 (Upper Keweenaw, Michigan) and ALGO (Ontario, Canada), respectively. These stations were taken from the US CORS and CACS network.

Figure 6.3(a) shows KEW1 height solutions for each week (black dots) along with the linear model describing the site motion (blue line). The time series plots demonstrate evidence of a gradual upward linear trend, indicating that the height of this station is increasing on average from 1997-2008, at a rate of 1.2 mm/yr. Figure 6.3(b) shows the data residuals or the difference between the data and the linear model from figure 6.3(a). The RMS scatter of the residual is 8.0 mm leading to a velocity standard error of 0.1 mm/yr.
Figure 6.3. Unedited position time series for KEW1 Height. (a) GPS data (black diamonds) with the Linear Trend (blue solid line) and 95% confidence upper and lower bound (blue dashed lines) (b) GPS data residuals
Figure 6.4(a-b) below illustrates ALGO height solutions for each week (black dots) along with the linear model describing the site motion (blue line), for a period of eleven years. Figure 6.4(a) shows the site position for each week (black dots) along with the model describing the site motion (blue line). The time series plots demonstrate evidence of a gradual upward linear trend, indicating that the height of this station is increasing on average from 1997-2014, at a rate of 3.7 mm/yr. Figure 6.4(b) shows the data residuals or the difference between the data and the linear model from figure 6.4(a). The RMS scatter of the residual is 5.7 mm leading to a velocity standard error of 0.01 mm/yr.

Continued

Figure 6.4. Unedited position time series for ALGO Height. GPS data (black diamonds) with the Linear Trend (blue solid line) and 95% confidence upper and lower bound (blue dashed lines). (b) GPS data residuals
There were eight (7) times series generated that exhibit data gaps at different interval throughout the time series, Table 6.3 list those GPS stations and Figures 6.5 and 6.6, shows two sample stations which exhibit these data gaps or offsets. A review of the site logs for these stations did not indicate whether these gaps were related to equipment changes, however it was assumed here that these gaps or offsets in the times series were related to either GPS equipment changes or disruption in the data collection due to equipment failure.
Table 6.3. Observed site offsets and data gaps from the multi-year times series of GPS US CORS/CACS (a subset of stations listed in Table 6.1)

<table>
<thead>
<tr>
<th>CORS Stations</th>
<th>Location</th>
<th>Character Identifier (Char-ID)</th>
<th>EPOCH</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calumet Harbor</td>
<td>MI, USA</td>
<td>CALU</td>
<td>2007.903-2008.214</td>
<td>No site log entry</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2009.923-2010.424</td>
<td>No site log entry</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2012.731-2013.271</td>
<td>No site log entry</td>
</tr>
<tr>
<td>Milwaukee 1</td>
<td>WI, USA</td>
<td>MIL1</td>
<td>2002.808-2003.079</td>
<td>No site log entry</td>
</tr>
<tr>
<td>Marquette</td>
<td>MI, USA</td>
<td>MIMQ</td>
<td>2009.654-2010.616</td>
<td>No site log entry</td>
</tr>
<tr>
<td>Marblehead</td>
<td>OH, USA</td>
<td>OHMH</td>
<td>2009.923-2012.137</td>
<td>No site log entry</td>
</tr>
<tr>
<td>Point Iroquois</td>
<td>MI, USA</td>
<td>PTIR</td>
<td>2008.789-2009.175</td>
<td>No site log entry</td>
</tr>
<tr>
<td>Port Weller</td>
<td>ON, Canada</td>
<td>PWEL</td>
<td>2008.252-2008.271</td>
<td>No site log entry</td>
</tr>
<tr>
<td>Rossport</td>
<td>ON, Canada</td>
<td>ROSS</td>
<td>2009.750-2010.060</td>
<td>No site log entry</td>
</tr>
</tbody>
</table>

Figure 6.5, illustrates the multi-year time series for Port Weller, Ontario which shows two (2) separate linear trends due to an offset in the dataset which, occurred at period 2008.252 and 2008.271. The vertical velocity prior to the change in trend was 0.6 mm/yr for the period 2003 - 2008.252 and 0.04 mm/yr after the change for the period 2008.271 - 2014.000. To analyze the overall movement of this station the offset was removed by subtracting mean of the time series from the offset time series, hence the corrected trend will be 0.01mm/yr for the period 2003-2014.
Figure 6.5. Time series data gap for GPS station Port Weller (PTIR). Change in time series seen at 2008.252 - 2008.271.

Figure 6.6, illustrates the multi-year time series for Marblehead, Ohio, which shows two (2) separate linear trends, which occurred between 2009.923 and 2012.137. The vertical velocity prior to the change in trend was 0.6 mm/yr for the period 2005 - 2009.923 and 0.04 mm/yr after the change for the period 2012.137-2014.000.
Figure 6.6. Time series data gap for GPS station Marblehead (OHMH). Change in time series seen at 2009.923 - 2012.137

Figure 6.7, illustrates the multi-year time series for Calumet Harbor, Illinois which, shows a total of three (3) separate linear trends as a result of two data gaps in the times series which, occurred between 2007.903 and 2008.214, and 2010.424 and 2012.731 respectively. The vertical velocity for the individual series before and after each of the gaps were -1.3 mm/yr for the period 2003.194 - 2007.903 (Series a), -6.3 mm/yr for the period 2008.214 - 2009.23 (Series b) and -1.7 mm/yr for the period 2010.424 to 2012.731 (Series c). Series b in this time series only contained 1 year of data this period is too short and was one considered in the analysis.
Figure 6.7. Time series data gap for GPS station Calumet Harbor (CALU). Change in time series seen at 2007.903 - 2008.214 and 2009.923 - 2010.424

In cases of data gaps as those seen in Figures 6.6 and 6.7, the analysis of the station’s movement were based on the time series which contained data for longer than 2.5 years from which a linear trend could be determined. Therefore any series of data which contained less than 2.5 years for data were not used to compute the velocity.

6.3.2 Outlier Editing

One of the first steps towards obtaining a coherent analysis is the detection of outlaying observations. Detected outliers are candidates for anomalous data that may otherwise adversely lead to model misspecification, biased parameter estimation and incorrect results. It is, therefore, important to identify these errors prior to modeling and analysis (Williams et al., 2004). Outliers are defined as observations that deviate so much from
other observations as to arouse suspicion that it was generated by a different mechanism (Hawkins, 1980).

For data containing more than a small number of isolated outliers, it is necessary to use robust techniques to compute the model parameters (Barnett, 1984). Hence, each position time series \( t_i \) was cleaned using a robust outlier detection algorithm applied to the raw time series data, prior to applying the linear model given in equation (6-7). The detection algorithm is based on the median and interquartile range (IQR) statistic to describe the central values and spread of the data. The IQR of a data sample is equal to the difference between the upper (Q₃) and lower (Q₁) quartiles. Then the outlier was defined to be any observation having:

\[
|t_i - \text{median} (t_{i-w/2,i+w/2})| > k \times IQR(t_{i-w/2,i+w/2})
\]  

(6-8)

for some nonnegative constant \( k = 1.5 \). The median and IQR operates within a window of size (w), chosen to represent one year. For each site outlier epochs were identified separately in each coordinate direction. With this criterion the detection algorithm typically removed 1 to 7 percent of the data from each site. Figure 6.8 shows an example of the edited position time series for site KEW1.
Figure 6.8. Filtered position time series for KEW1, Height. (a) The weekly positions solution (grey circles) are cleaned by excluding data (red circles) (b) GPS data (black diamonds) with the linear trend (blue solid line) and 95% Confidence upper and lower bounds (blue dashed lines). (c) GPS data residuals Results. The solution was cleaned by excluding data that exceeded $1.5 \times \text{IQR}$ in any direction.
6.4 Results

The edited position plots, in the upward direction for all the US CORS and CACS stations used are available in Appendix B. The results highlighted in this section are based on the vertical component. After the GNSS processing, 60 time series were obtained from 20 stations, these time series showed the changes in the North, East and Upward directions of stations. The \((\beta_1)\) parameter, which represents the linear rate in the trend function, (equation 6-7), indicates the yearly increasing (+) or decreasing (-) vertical or horizontal direction change of the station. As it relates to the vertical velocities change in (+) direction, this indicates that the station is rising as seen in station KEW1 (Figure 6.3) and change in (-) direction, indicates the station is subsiding. The North and East coordinate component changes in (+) direction indicates the movement in the North and East direction, while changes in the (-) directions indicates movement in the South and West directions.
6.4.1 Vertical Velocity Solutions

Table 6.4 presents the linear trend component in the upward direction (vertical velocities), which indicates the change or movement of all the stations. The computed vertical velocities are visualized in Figure 6.9. The estimated vertical rates uncertainties were usually between 0.1-0.3mm/yr (with 95% confidence).

Most of the examined time series contained small but significant seasonal variations in both the horizontal and vertical directions. The annual amplitudes reached 1-5 mm horizontally and 3-15 mm vertically. The root mean square (RMS) scatters of residuals for the best fit linear models are at level of 4-6mm (North), 2-3mm (East) and 5-8 mm (Up) (Appendix C).

The linear vertical trends obtained in this study have been compared with the vertical velocities derived by the NGS for the individual stations. Table 6.5 shows the comparison of the vertical velocities. The NGS IGS08 velocities were computed in August 2011 using data through GPS week 1631. The comparison showed that the discrepancy in the vertical velocities for the majority of the stations were below 1 mm/yr.
Table 6.4. Vertical Velocities for the selected US CORS/CACS GPS Stations around the Great Lakes

<table>
<thead>
<tr>
<th>CORS Site</th>
<th>Location</th>
<th>Char-ID</th>
<th>Period of Record</th>
<th>Lake Association</th>
<th>Latitude N°</th>
<th>Longitude E°</th>
<th>Trend mm/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marblehead</td>
<td>OH, USA</td>
<td>OHMH</td>
<td>2005-2013</td>
<td>Erie</td>
<td>41.54368</td>
<td>82.73145</td>
<td>-0.83±0.1</td>
</tr>
<tr>
<td>Calumet Harbor</td>
<td>MI, USA</td>
<td>CALU</td>
<td>2003-2013</td>
<td>Michigan</td>
<td>41.72985</td>
<td>87.53844</td>
<td>-1.30±0.3</td>
</tr>
<tr>
<td>North Liberty</td>
<td>IA, USA</td>
<td>NLIB</td>
<td>2003-2009</td>
<td>N/A</td>
<td>41.77159</td>
<td>91.5749</td>
<td>-0.03±0.1</td>
</tr>
<tr>
<td>Buffalo</td>
<td>NY, USA</td>
<td>BFNY</td>
<td>2002-2013</td>
<td>Erie</td>
<td>42.87756</td>
<td>78.89045</td>
<td>-0.97±0.2</td>
</tr>
<tr>
<td>Milwaukee 1</td>
<td>WI, USA</td>
<td>MIL1</td>
<td>1997-2006</td>
<td>Michigan</td>
<td>43.00254</td>
<td>87.88845</td>
<td>-2.78±0.1</td>
</tr>
<tr>
<td>Fort Gratiot</td>
<td>MI, USA</td>
<td>FRTG</td>
<td>2003-2013</td>
<td>Huron</td>
<td>43.03757</td>
<td>82.48816</td>
<td>-0.62±0.1</td>
</tr>
<tr>
<td>Port Weller</td>
<td>ON, Canada</td>
<td>PWEL</td>
<td>2008-2013</td>
<td>Ontario</td>
<td>43.23673</td>
<td>79.21967</td>
<td>0.10±0.06</td>
</tr>
<tr>
<td>Muskegon</td>
<td>MI, USA</td>
<td>MSKY</td>
<td>2005-2013</td>
<td>Michigan</td>
<td>43.23755</td>
<td>86.05458</td>
<td>-1.00±0.1</td>
</tr>
<tr>
<td>Saginaw</td>
<td>MI, USA</td>
<td>BAYR</td>
<td>2002-2013</td>
<td>Huron</td>
<td>43.4462</td>
<td>83.8917</td>
<td>-0.57±0.3</td>
</tr>
<tr>
<td>Kingston</td>
<td>ON, Canada</td>
<td>KNGS</td>
<td>2003-2013</td>
<td>Ontario</td>
<td>44.21869</td>
<td>76.51727</td>
<td>1.00±0.07</td>
</tr>
<tr>
<td>Sturgeon Bay 1</td>
<td>WI, USA</td>
<td>STB1</td>
<td>1996-2008</td>
<td>Michigan</td>
<td>44.79549</td>
<td>87.31433</td>
<td>-0.26±0.1</td>
</tr>
<tr>
<td>Alpena</td>
<td>MI, USA</td>
<td>NOR3</td>
<td>2001-2013</td>
<td>Huron</td>
<td>45.06858</td>
<td>83.56865</td>
<td>1.00±0.1</td>
</tr>
<tr>
<td>Parry Sound</td>
<td>ON, Canada</td>
<td>PARY</td>
<td>2003-2013</td>
<td>Huron</td>
<td>45.33855</td>
<td>80.03589</td>
<td>1.82±0.08</td>
</tr>
<tr>
<td>Cheboygan 1</td>
<td>MI, USA</td>
<td>CHB1</td>
<td>1996-2008</td>
<td>Huron</td>
<td>45.65349</td>
<td>84.46565</td>
<td>1.37±0.1</td>
</tr>
<tr>
<td>Algonquin Park</td>
<td>Canada</td>
<td>ALGO</td>
<td>1997-2013</td>
<td>N/A</td>
<td>45.9558</td>
<td>78.07137</td>
<td>3.74±0.1</td>
</tr>
<tr>
<td>Point Iroquois</td>
<td>MI, USA</td>
<td>PTIR</td>
<td>2003-2013</td>
<td>Superior</td>
<td>46.48460</td>
<td>84.6308</td>
<td>1.56±0.08</td>
</tr>
<tr>
<td>Marquette</td>
<td>MI, USA</td>
<td>MIMQ</td>
<td>2005-2013</td>
<td>Superior</td>
<td>46.5455</td>
<td>87.3787</td>
<td>0.21±0.1</td>
</tr>
<tr>
<td>Wisconsin Point 1</td>
<td>WI, USA</td>
<td>WIS1</td>
<td>1996-2007</td>
<td>Superior</td>
<td>46.70506</td>
<td>92.01521</td>
<td>-0.79±0.8</td>
</tr>
<tr>
<td>Upper Keweenaw 1</td>
<td>MI, USA</td>
<td>KEW1</td>
<td>1997-2008</td>
<td>Superior</td>
<td>47.22707</td>
<td>88.62433</td>
<td>1.18±0.1</td>
</tr>
<tr>
<td>Rossport</td>
<td>ON, Canada</td>
<td>ROSS</td>
<td>2003-2013</td>
<td>Superior</td>
<td>48.83373</td>
<td>57.5196</td>
<td>3.40±0.1</td>
</tr>
</tbody>
</table>
Figure 6.9. Vertical velocity field for the 20 US CORS/CACS GPS Stations. The map depicts the stations that are experiencing uplift (red arrows) or subsidence (blue arrows).
Table 6.5. Comparison of the US CORS/CACS GPS vertical velocities with NGS IGS08 velocities

<table>
<thead>
<tr>
<th>CORS Site</th>
<th>Location</th>
<th>Char-ID</th>
<th>Latitude N°</th>
<th>Longitude E°</th>
<th>GPS Vertical Velocity mm/yr.</th>
<th>NGS Vertical Velocity mm/yr.</th>
<th>Difference (GPS-NGS) mm/yr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marblehead</td>
<td>OH, USA</td>
<td>OHMH</td>
<td>41.54368</td>
<td>82.73145</td>
<td>-0.83</td>
<td>-0.50</td>
<td>0.33</td>
</tr>
<tr>
<td>Calumet Harbor</td>
<td>MI, USA</td>
<td>CALU</td>
<td>41.72985</td>
<td>87.53844</td>
<td>-1.30</td>
<td>-2.90</td>
<td>-1.60</td>
</tr>
<tr>
<td>North Liberty</td>
<td>IA, USA</td>
<td>NLIB</td>
<td>41.77159</td>
<td>91.5749</td>
<td>-0.03</td>
<td>-1.30</td>
<td>1.27</td>
</tr>
<tr>
<td>Buffalo</td>
<td>NY, USA</td>
<td>BFNY</td>
<td>42.87756</td>
<td>78.89045</td>
<td>-0.97</td>
<td>0.40</td>
<td>0.57</td>
</tr>
<tr>
<td>Milwaukee 1</td>
<td>WI, USA</td>
<td>MIL1</td>
<td>43.00254</td>
<td>87.88845</td>
<td>-2.78</td>
<td>-4.40</td>
<td>-1.62</td>
</tr>
<tr>
<td>Fort Gratiot</td>
<td>MI, USA</td>
<td>FRTG</td>
<td>43.03757</td>
<td>82.48816</td>
<td>-0.62</td>
<td>-0.60</td>
<td>0.02</td>
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<td>Port Weller</td>
<td>ON, Canada</td>
<td>PWEL</td>
<td>43.23673</td>
<td>79.21967</td>
<td>0.10</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td>Muskegon</td>
<td>MI, USA</td>
<td>MSKY</td>
<td>43.23755</td>
<td>86.05458</td>
<td>-1.00</td>
<td>-2.00</td>
<td>-1.00</td>
</tr>
<tr>
<td>Saginaw</td>
<td>MI, USA</td>
<td>BAYR</td>
<td>43.4462</td>
<td>83.8917</td>
<td>-0.57</td>
<td>-1.40</td>
<td>-0.83</td>
</tr>
<tr>
<td>Kingston</td>
<td>ON, Canada</td>
<td>KNGS</td>
<td>44.21869</td>
<td>76.51727</td>
<td>0.97</td>
<td>0.20</td>
<td>0.77</td>
</tr>
<tr>
<td>Sturgeon Bay 1</td>
<td>WI, USA</td>
<td>STB1</td>
<td>44.79549</td>
<td>87.31433</td>
<td>-0.26</td>
<td>-0.60</td>
<td>-0.34</td>
</tr>
<tr>
<td>Alpena</td>
<td>MI, USA</td>
<td>NOR3</td>
<td>45.06858</td>
<td>83.56865</td>
<td>0.97</td>
<td>0.20</td>
<td>0.77</td>
</tr>
<tr>
<td>Parry Sound</td>
<td>ON, Canada</td>
<td>PARY</td>
<td>45.33855</td>
<td>80.03589</td>
<td>1.82</td>
<td>0.80</td>
<td>1.02</td>
</tr>
<tr>
<td>Cheboygan 1</td>
<td>MI, USA</td>
<td>CHB1</td>
<td>45.65349</td>
<td>84.46565</td>
<td>1.37</td>
<td>0.80</td>
<td>0.57</td>
</tr>
<tr>
<td>Algonquin Park</td>
<td>Canada</td>
<td>ALGO</td>
<td>45.9558</td>
<td>78.07137</td>
<td>3.74</td>
<td>4.25</td>
<td>-0.51</td>
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<td>MI, USA</td>
<td>PTIR</td>
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<td>84.6308</td>
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<td>0.16</td>
</tr>
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<td>Marquette</td>
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<td>MIMQ</td>
<td>46.5455</td>
<td>87.3787</td>
<td>0.21</td>
<td>0.70</td>
<td>-0.49</td>
</tr>
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<td>Wisconsin Point 1</td>
<td>WI, USA</td>
<td>WIS1</td>
<td>46.70506</td>
<td>92.01521</td>
<td>-0.79</td>
<td>-0.50</td>
<td>0.29</td>
</tr>
<tr>
<td>Upper Keweenaw 1</td>
<td>MI, USA</td>
<td>KEW1</td>
<td>47.22707</td>
<td>88.62433</td>
<td>1.18</td>
<td>0.90</td>
<td>0.28</td>
</tr>
<tr>
<td>Rossport</td>
<td>ON, Canada</td>
<td>ROSS</td>
<td>48.83373</td>
<td>57.5196</td>
<td>3.40</td>
<td>3.80</td>
<td>0.40</td>
</tr>
</tbody>
</table>
6.4.2 Comparison of the US CORS/CACS and IGLD Height Modernization Survey computed Vertical Velocities

The vertical velocities obtained from campaign type measurements of the IGLD Height Modernization Project were computed from GPS observations executed over short periods of time at one specific time of the year (observation periods: June to October). The campaign surveys are generally discontinuous and uncharacteristic of the entire annual vertical and horizontal movement that may occur. Alternatively, the CORS/CACS provides continuous point positions with a high level temporal resolution. The derived vertical velocities from the IGLD Height Modernization Project (see Section 5.3.5) were compared to those of the multi-year GPS time series (see Section 6.3). The comparison was done for benchmark stations that were collocated with US CORS/ CACS station, by simply matching up the survey time period of the campaign survey and extracting the relevant data from the time series. Figures 6.10 to 6.12, illustrates for three selected stations the comparison between the continuous GPS vertical velocities based on a multi-year time series analysis and the vertical velocities derived from the IGLD Height Modernization campaign surveys.
Figure 6.10. Continuous vs. Campaign GPS Data - Wisconsin Point, WI. The GPS CORS time series (black circle) and the related linear trend (black line); GPS campaign survey results for 1997, 2005 and 2010 (red diamonds) and the related trend line (red line)

Figure 6.10, illustrates the case where the campaign GPS data were used to generate a vertical velocity for the period 1997 to 2010. As shown, the velocity was estimated based on the best fit linear trend through the data for the continuous data, while the individual trends for the 1997-2005 and 2005-2010 were taken from the results of the multiple reference radial data processing carried out in Chapter 5. The computed velocity for the period 1997 – 2005 is -0.6 mm/yr, which is the same vertical velocity as the multi-year
times series, while the velocity for the period 2005-2010 which was 4.6mm/yr, has a difference of 4.0 mm/yr from the multi-year times series.

Due to data availability and decommissioned CORS within the study area, the selection of collocated continuous GPS data may not span over the three years of the campaign surveys, but may cover a specific period, as seen in Figures 6.11 and 6.12.

![Figure 6.11. Continuous vs. Campaign GPS Data – Sturgeon Bay, WI. The GPS CORS time series (black circle) and the related linear trend (black line); GPS campaign survey results for 1997 and 2005 (red diamonds) and the related trend line (red line)]](image_url)
Figure 6.11 illustrates, continuous data extracted for the period 1997 to 2005, with a vertical velocity of 0.3 mm/year which the associated velocity for the related campaign surveys generates a velocity of 1.0 mm/yr which gives a difference of 0.7 mm/yr.

![CONTINUOUS vs. CAMPAIGN GPS DATA - BUFFALO, NY](image)

Figure 6.12. Continuous vs. Campaign GPS Data - Buffalo, NY. The GPS CORS time series (black circle) and the related linear trend (black line); GPS campaign survey results for 2005 and 2010 (red diamonds) and the related trend line (red line)

Figure 6.12, illustrates, continuous data extracted for the period 2005 to 2010, with a vertical velocity of -1.0 mm/year which the associated velocity for the related campaign
surveys generates a velocity of -5.0 mm/yr which gives a difference of 4.0 mm/yr. This difference is quite significant as it is more than twice the average vertical velocity of the surrounding GPS station in that area (Lake Ontario and Lake Erie – Figure 6.9).

6.4.3 Summary

This chapter presented numerical estimates of the vertical motion in the Great Lakes area, which is subject to post glacial rebound. The time-series of weekly positions were analyzed independently for the OPUS-Net solutions using the R statistics software (http://www.r-Projects.org/). For the continuous GPS data series, a simple linear model was fitted to the North, East, and Up local coordinate system using least-squares inversion. The computed results showed the vertical motion occurring in the Great Lakes and illustrated the vertical velocity at the selected CORS/CACS stations. It can be seen that there is a pattern of uplift occurring to the North of the Great Lakes and subsidence to the South (Figure 6.9).

The linear vertical trends obtained in this section were compared to the vertical velocities derived by the NGS for the individual stations. The NGS IGS08 velocities were computed in August 2011 using data through GPS week 1631. The comparison showed that the discrepancy in the vertical velocities for the majority of the stations were below 1 mm/yr.

The comparison of vertical velocities, campaign survey derived vs. multi-year time series analysis, showed that the former can provide a suitable estimation of the vertical velocity. In this dissertation there were only 3 epochs of survey data available for use, 1997, 2005
and 2010 with a span of 5, 8 and 13 years between each pair. However, to be able to verify whether an observed change over time is random or naturally occurring, multiple campaigns survey (more than 3) should be executed for the velocity analysis to be truly effective. Also to investigate the effect of seasonal changes, campaign surveys should be executed at different times throughout the year. Therefore, between the two methods for determining vertical velocity and overall land deformation the continuous GPS multi-year time series analysis for a time period greater than 2.5 year produced a stable and consistent solution over the campaign surveys.
CHAPTER 7: VERTICAL LAND MOTION: USING GLACIAL ISOSTATIC ADJUSTMENT MODEL

In this chapter the main goal is to assess the consistency of the Glacial Isostatic Adjustment (GIA) model. To achieve this, the results of the GPS derived vertical velocities computed in Chapter 6 are compared with the independent vertical velocity predictions from the Glacial Isostatic Adjustment (GIA) model (Peltier, 2004). The following sections define and detail the GIA model and the methods used to derive the vertical velocities.

7.1 Introduction

Vertical motion of the Earth’s crust occurs due to a variety of phenomena including tectonic activity, and postglacial isostatic rebound or glacial rebound (GIA) (Milne et al., 2001), volcanic activity, and geological and anthropogenic subsidence (Bawden et al., 2001; van der Wal et al., 2009). The dominant geological processes causing vertical surface deformation in northeastern United States and Canada is postglacial rebound or GIA (van der Wal et al., 2009), karsting and subsurface mining (United States Geological Survey, 2000).

Another significant process causing vertical motion in western United States is plate tectonics, specifically the subduction at the Cascadia Subduction Zone (CSZ). The CSZ
forms the convergent plate boundary between the Juan de Fuca and North America, plates. This boundary is marked by extensive earthquake activity as well as a belt of active arc volcanoes stretching from northern California to southwest British Columbia (Henton et al., 2006). The plate tectonics within this area has measurable effects on the gradient of the land surface and sedimentation; however the area of interest in this study is away from the subduction zone. The Sedimentation and erosion rates within the CSZ on land and away from coastal areas are <0.3 mm/year and <0.5mm/year respectively and much larger in coastal area. However, land uplift rates due to GIA are generally much larger than sedimentation or erosion rates (van der Wal et al., 2009), therefore these effects are not taken into account in this study.

The observable changes due to the effect of GIA in North America such as sea level rise, vertical land motion, gravity anomalies, have been documented. For instance, Cathles (1975); Peltier and Tushingham (1989); Davis and Mitrovica (1996) have reported that the secular trend in sea levels seen in tide-gauge records can be due to GIA, in the North American region. Lambert et al. (2001) have shown the tilt of the province of Manitoba based on absolute gravity measurements reflecting GIA and associated mass change, and Peltier (2004) has employed these data to further constrain the ICE-5G (VM2) model. Recently, Sella et al. (2007) have reanalyzed a network of GPS stations in North America and have shown that the land uplifted in north-eastern Canada is dominated by GIA. Also observable changes due to the effect of GIA have been documented specifically for the Great Lakes regions. This region is located in the transition zone between uplift and subsidence due to glaciation and deglaciation of the Laurentide ice sheet (Sella, et al., 2007). Mainville and Craymer (2005), looked at the present day tilting of the Great Lakes
region by studying water level records. Braun et al. (2008) compared vertical observation motions derived from GPS solutions and tide gauge and satellite altimetry solutions with predictions obtained from 70 different GIA models.

Knowledge of the GIA process is critical to improving our understanding of crustal deformation and to quantify sea level change. Accurate modelling of this phenomenon is necessary to derive vertical crustal motion at sites where long-term geodetic sensors such as GPS, are not available (Braun et al., 2008). GPS has achieved a level of maturity suitable for the validation of or the comparison of high-precision vertical velocity fields to elastic deformation models in active tectonic zones (e.g., Bergeot et al., 2009); to GIA models (e.g., Johansson et al., 2002; Nocquet, Calais and Parsons, 2005); or to independent vertical rates derived from combined altimetry and tide gauge (TG) measurements (e.g., Kuo et al., 2004). To this end the GPS derived vertical velocities computed in Chapter 6 will be compared with the vertical velocities predictions derived from the ICE-5G (VM2 L90) model version 1.3 (Peltier, 2004).

7.2 Glacial Isostatic Adjustment (GIA)

7.2.1 General Description of Glacial Isostatic Adjustment (GIA)

GIA is the response of the solid Earth to the past changes in surface loading during the glacial cycle (Sella et al., 2007). The last glacial cycle resulted in large volumes of ice accumulated over North America, Scandinavia, Greenland and Antarctica, with thickness as large as 3 km, which was its maximum about 20000 years ago, in the Last Glacial Maximum (LGM). Since then, these ancient ice sheets began to melt, the Earth has responded to the lost ice load by viscous flow of mantle material into the previously
depessed glaciated areas, which drives uplift of the Earth surface of over 1 cm/year in
the center of the former ice sheet (van der Wal et al., 2009). The melt water flowed into
the ocean and caused relative sea-level rise. Overall, sea level has risen approximately
120 meters since the LGM (Peltier, 2004; Sella et al., 2007). The movement of water
over the surface of the Earth, both as water and as ice, during a glacial cycle acts as a
load upon the lithosphere. The Earth elastically deforms in response to this force;
subsiding under the load of an ice sheet or full oceanic basin, and rebounding once the ice
sheets melt or water is removed from the oceanic basins, this deformation is isostatic.

Isostasy refers to a concept whereby deformation takes place in an attempt to return the
Earth to a state of mass equilibrium. The term isostasy was coined by Dutton (1889) to
describe the seemingly compensated state of the Earth’s topography. Originally the
concept of isostasy refered to the compensation of observed external loads (mountains)
by unobservable internal density heterogeneity (their roots). The term applies equally
well to the compensation of applied external surface loads (continental glaciers and ice
sheets) by variations in Earth’s external shape and internal density distribution (Peltier,
2004).

The solid Earth depression beneath an ice sheet results in an internal redistribution of
mantle material to the outside edge of the loaded region. The surface expression of this
mass flow is the growth of a peripheral fore-bulge, generally a few hundred kilometers
wide and up to ~100 m high (Figure 6.1a). Deglaciation results in the unloading of the
Earth in ice-covered areas, thus allowing a return-flow of mantle material to the formerly-
glaciated regions (Figure 6.1b). This not only results in uplift in the formerly-glaciated
regions, but also subsidence in the peripheral regions (Douglas and Peltier, 2002; Tamisiea and Mitrovica, 2011).

Figure 7.1. General postglacial rebound: (a) Around the edges if the ice sheet the crust flexes upward creating a peripheral fore-bulge. (b) After deglaciation, the fore-bulge slowly collapse resulting in subsidence (Henton et al., 2006)

The deformation of the solid Earth is a key process of GIA. However, in order to understand the temporal evolution of GIA it is also useful to make observations, which
relate to past relative sea-level positions. In addition, Earth’s gravity field also changes resulting from this large-scale mass redistribution inside the Earth in the GIA process (Tamisiea and Mitrovica, 2011).

### 7.2.2 GIA Model

The theory of the GIA process was developed in the 1970s in a series of scientific articles. These include the formulation to model the viscoelastic response of the Earth to surface load (Peltier, 1974), in addition to Farrell and Clark (1976) sea level equation for a non-rotating Earth model. The main component of the GIA theory is to solve the sea level equation developed by Farrell and Clark (1976). The initial from of the sea level equation is given as:

\[
S(\varphi, \lambda, t) = C(\varphi, \lambda, t) \left[ \int_{-\infty}^{t} dt' \iint_{\Omega} L(\varphi', \lambda', t') G^{L}(\gamma, t - t') + \frac{\Delta \Phi(t)}{g} \right] (7-1)
\]

where:

- \((\varphi, \lambda)\) - The latitude and longitude of the observation point respectively
- \((\varphi', \lambda')\) - The latitude and longitude of the loading center
- \(\Omega\) - Indicates the entire surface of the Earth

In equation (7-1), \(S(\varphi, \lambda, t)\) is the change in the mean sea level (which is assumed to coincide with the geoid) relative to the deforming surface of the solid Earth at latitude as a function of time \((t)\). The function \(C(\varphi, \lambda, t)\) is the so-called ocean function, which is unity where there is ocean and zero where there is land \((\Omega)\). \(G^{L}(\gamma, t - t')\) is the viscoelastic Green function for the time dependent separation between the surface of the
sea and the surface of the solid Earth. The function $L$, is the surface load per unit area given by:

$$L(\varphi, \lambda, t) = \rho_I I(\varphi, \lambda, t) + \rho_w S(\varphi, \lambda, t).$$  \hspace{1cm} (7-2)

In equation (7-2), $\rho_I$ and $I$ are the ice density and thickness, respectively, and $\rho_w$ and $S$ are ocean density and relative sea level changes, respectively. Note that, after substituting equation (7-2) into equation (7-1), $S(\varphi, \lambda, t)$ exists in both sides of the updated equation (7-1), and thus iterations are needed to solve the sea level equation.

The solution to the sea level equation is related to the computation of the Earth’s response to the loading. To model GIA for accurate predictions, two kinds of input are required (Braun et al., 2008; van der Wal et al., 2009). The first parameter is the Earth model that describes the structure of the elastic and viscosity of the mantle as a function of depth and time. This is used to compute the Earth’s response to the surface loading. The second parameter is the ice loading history, which provides information about the thickness and location of the ice during the last glaciation cycle (van der Wal et al., 2009). The ice loading history is a crucial parameter in GIA modelling, since ice geometry and thickness are directly correlated with Earth parameters (viscosity, lithospheric thickness) and cannot be easily separated. These input parameters cannot be measured directly resulting in large uncertainties (Braun et al., 2008; Huang, 2013). Hence the uncertainties of GIA parameters further affect the accurate determination of the ice mass and the Earth’s deformation processes (Trupin et al., 2002).
The Earth models used in GIA modelling are assumed to be spherically symmetrical, which include an elastic lithosphere of consistent thickness, several layers of viscoelastic mantle, and the outer liquid and inner rigid core. The mantle is usually divided into two layers: upper and lower mantle layers. The viscosity in each layer is assumed to be uniform. (Park et al., 2002; Huang, 2013). The radial elastic and density structure is commonly given by the Preliminary Reference Earth Model (PREM) (Dziewonski and Anderson, 1981). The viscosity is either obtained from geophysical inversion or estimated from independent geophysical studies (Huang, 2013). Continental lithospheric thicknesses typically range between 70 km and 200 km, while mantle viscosities typically range between $10^{19}$ and $10^{24}$ Pascal-second (Pa·s). Wolf et al. (2006), provide a summary of radial viscosity profiles that were inferred from studies using relative sea-level data from the Hudson Bay region.

The ice-loading histories are based on a variety of data sets, the most important being observations during the disintegration of the ice sheets. One of the most important sources relative sea level history is radio-carbon dating of relative sea level changes through the postglacial period (Peltier, 2004). The ice loading history are selected from commonly used global models from the ICE-series; ICE-3G model (Tushingham and Peltier, 1991), ICE-4G model (Peltier, 1994) and the ICE5G model (Peltier, 2004). Methods to solve the sea-level equation include, spectral modelling based on normal mode theory (Clark, Farrell and Peltier, 1978; Peltier, 1974); pseudo-spectral modelling based on normal mode theory (Mitrovica and Peltier, 1991) and finite element modelling (Martinec, 2000; Paulson, Zhong and Wahr, 2005; Wu, and van der Wal, 2003).
The sea-level equation will generate predictions for the relative sea-level change, and rates of relative sea-level change, at any time and any position on the surface of the Earth, also providing prediction of solid Earth deformation (uplift/subsidence). Relative sea level change is a result of perturbations to the gravitational potential, hence present-day gravity anomalies and the rate of change of gravity anomalies may also be predicted (Huang, 2013).

Different GIA models have been developed and tested throughout the years. Braun et al. (2008) and Huang (2013), for example give detailed descriptions and comparisons of contemporary regional and global GIA models. For this research, the predictions from ICE-5G (VM2 L90) model version 1.3 developed by Peltier (2004) are used to compare with the GPS derived vertical velocities, as described below. The version of the of the ICE-5G (VM2 L90) model (version 1.3) model was available at: http://www.atmosp.physics.utoronto.ca/~peltier/data.php.

7.2.3 Comparison with ICE - 5G (VM2 L90) GIA Model

The ICE-5G (VM2 L90) GIA model created by Peltier (2004) is based upon the assumption of spherically-symmetric, self-gravitating and elastically compressible Maxwell visco-elastic Earth model with the VM2 (Peltier, 1996) viscosity structure (Bouin and Wöppplemann, 2010). The thickness of the lithosphere is set to 90 km, the upper and lower mantle have a viscosity of $0.5 \times 10^{21}$ Pa s and $2.6 \times 10^{21}$ Pa s respectively (Braun et al., 2008). This is a refined model that has resulted through correction of the flaws in the previous model ICE-4G (Peltier, 2004). The ICE 5G (VM2 L90) reconstruction of the surface topography and land ice distribution at LGM differs
significantly from its predecessor, with significantly greater land based ice mass (Peltier, 2004).

In general, the seasonal signals present in most GPS position time-series may significantly affect the velocity estimation if the time-series is shorter than 2.5 yr (e.g., Mao, Harrison and Dixon, 1999; Blewitt & Lavallée, 2002; Williams et al., 2004). In this study, only stations with an observation time span longer than three years (20 stations) were used for comparison. The 3 year period was chosen to effectively mitigate rate-biases associated with annual variations (Blewitt & Lavallée, 2002). The uncertainties of the vertical velocities of these stations (Chapter 6, Table 6.4), the ICE-5G (VM2 L90) velocities were provided without uncertainties.

7.3 Results

As stated previously, the ICE-5G VM2 L90 predicted velocities were compared with the derived velocities from the selected US CORS/CACS stations. Table 7.1 shows the GPS and ICE-5G VM2 L90 velocities for these stations.
<table>
<thead>
<tr>
<th>Station</th>
<th>Latitude</th>
<th>Longitude</th>
<th>GPS Vertical Velocity, mm/yr</th>
<th>GIA Vertical Velocity, mm/yr</th>
<th>Difference mm/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>OHMH</td>
<td>41.54368</td>
<td>82.73145</td>
<td>-0.83</td>
<td>-1.62</td>
<td>0.79</td>
</tr>
<tr>
<td>CALU</td>
<td>41.72985</td>
<td>87.53844</td>
<td>-1.30</td>
<td>-1.49</td>
<td>0.19</td>
</tr>
<tr>
<td>NLIB</td>
<td>41.77159</td>
<td>91.5749</td>
<td>-0.03</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>BFNY</td>
<td>42.87756</td>
<td>78.89045</td>
<td>-0.97</td>
<td>-1.05</td>
<td>0.25</td>
</tr>
<tr>
<td>MIL1</td>
<td>43.00254</td>
<td>87.88845</td>
<td>-2.78</td>
<td>-1.39</td>
<td>1.39</td>
</tr>
<tr>
<td>FRTG</td>
<td>43.03757</td>
<td>82.48816</td>
<td>-0.62</td>
<td>-0.98</td>
<td>0.36</td>
</tr>
<tr>
<td>PWEL</td>
<td>43.23673</td>
<td>79.21967</td>
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<td>-0.75</td>
<td>0.65</td>
</tr>
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<td>86.05458</td>
<td>-1.00</td>
<td>-0.92</td>
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</tr>
<tr>
<td>BAYR</td>
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<td>83.89173</td>
<td>-0.57</td>
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<td>0.09</td>
</tr>
<tr>
<td>KNGS</td>
<td>44.21869</td>
<td>76.51727</td>
<td>1.00</td>
<td>0.51</td>
<td>0.49</td>
</tr>
<tr>
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<td>87.31433</td>
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<td>-0.17</td>
<td>0.09</td>
</tr>
<tr>
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<td>83.56865</td>
<td>1.00</td>
<td>0.72</td>
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</tr>
<tr>
<td>PARY</td>
<td>45.33855</td>
<td>80.03589</td>
<td>1.82</td>
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<td>0.67</td>
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<td>CHB1</td>
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<td>84.46565</td>
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<td>1.14</td>
<td>0.23</td>
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<td>ALGO</td>
<td>45.9558</td>
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<td>48.83373</td>
<td>87.5196</td>
<td>3.40</td>
<td>3.19</td>
<td>0.21</td>
</tr>
</tbody>
</table>

There are some disagreements between the modeled and observed value for a few stations. However, a clear distinction between the areas of uplift and subsidence is evident. The GPS data conform least in the east as seen from station ALGO, where the GPS derived vertical velocity is 3.7 mm/yr as compared to the GIA vertical velocity of 1.68 mm/year.
CHAPTER 8: CONCLUSION AND RECOMMENDATIONS

The Height Modernization Program is a nationwide initiative focused on establishing accurate, reliable heights using the GNSS in conjunction with traditional leveling, gravity, and modern remote sensing information. The primary goal of this research was to contribute to the improvement of height estimation using GPS that supports the goals of the National Height Modernization project led by NGS. This was accomplished through the development and analysis of network designs suitable for GNSS surveys and post-processing configuration, using NGS-designed software OPUS-Projects and OPUS-Net. These tools are designated as the future standard for GNSS based 3D coordinate estimations in the United States.

The International Great Lakes Datum of 1985 (IGLD) Height Modernization survey projects (1997, 2005, and 2010), datasets were used for the data processing and analysis done in this research. The computed GPS ellipsoid heights from each campaign survey were used to then compute the vertical velocities. These vertical velocities identified the vertical movement within the Great Lakes. Additionally, a selection of 20 continuous GPS stations was processed using OPUS-Net. A time series analysis was done using the computed coordinates, decomposing the time series in order to define the vertical velocity (trend) of these selected stations. The multi-year data vertical velocities were compared to the velocities of the 3 Height Modernization survey projects (1997, 2005,
and 2010), to assess the overall capabilities of campaign GPS survey to detect vertical deformation precisely.

The Height Modernization survey datasets were reprocessed using the ‘USER’ defined, multiple reference radial network configurations and the TRI network design (closed loop network). The multiple reference radial network is based on the basic radial network, modified by the inclusion of multiple reference stations to provide stability and reliability, in the network These additional reference stations, were distant from the core radial network (approximately 1000 km), to allow for the absolute tropospheric parameters to be estimated more reliably. The comparison of the coordinates derived from the two configurations (Multiple reference and TRI) showed a mean of 2 mm and a standard deviation of 2 mm in the North component; a mean of 1 mm and a standard deviation of 4 mm in the East component; and a mean of -4 mm and a standard deviation of 8 mm in the UP component. The multiple reference radial networks performed comparatively well against the TRI based on the solution quality statistic produced after the processing stages.

However, these networks were separated by their computational performance. Even though there aren’t any physical limitations on data quantity set by OPUS-Projects, there are practical size limits dictated by the fact that OPUS-Projects is a Web-based tool (cloud program). From the dataset used, on average a processing session may include anywhere from 10 to 60 stations. For the TRI network, the number of baselines grows geometrically with the number of stations included in the session. Also this configuration includes a number of redundant or trivial baselines, which also makes this configuration
computationally costly. Alternatively, the number of baselines in the multiple reference radial design is equal to the number of stations included in the session. As a result, the multiple reference radial network design is not as computationally costly and less memory capacity to execute a session processing mainly due to the lack of redundant or trivial baselines.

Further, vertical velocities between the three Height Modernization surveys projects were produced using ellipsoidal heights generated using the multiple reference radial design. These velocities were compared with the vertical velocities derived from the evaluation of the time series of 20 GPS reference stations from the US CORS and Canadian Active Control System (CACS) national network. This was done to determine the reliability of campaign type surveys to precisely model land movement (subsidence or uplifts).

Obtaining the time series of positions derived from the continuous GPS stations allowed for a more consistent interpretation over the campaign surveys of the overall crustal movement or deformation. The multi-year time series showed the common annual and semi-annual variation, while providing an independent reference for the comparison of the IGLD surveys computed velocities. The linear site rates were estimated along with other parameters (seasonality and residuals) for the time series of the weekly averaged positions. The velocities were determined from stations with a time series longer than 2.5 years, from which a linear trend could be determined. This rule was adhered to as well for station that had data gaps due to equipment changes or failure. Therefore any series of data which contained less than 2.5 years for data were not used to compute the velocity. The estimated vertical rate uncertainties were usually between 0.1-0.3 mm/yr. at the 95%
confidence level. These rates were compared to those supplied from the published coordinates for the GPS stations and the NGS IGS08 velocities computed in August 2011 using data through GPS week 1631. The comparison showed that the discrepancy in vertical velocities for most of the stations were within 1 mm/yr. Even with 20 continuous GPS stations, the overall movement pattern is similar to that of Craymer et al. (2012) CACS/CORS vertical velocity field.

The vertical velocities obtained from campaign type measurements of the IGLD Height Modernization Project were computed from GPS observations executed over short periods of time at one specific time of the year (observation periods: June to October). The campaign surveys are generally discontinuous and uncharacteristic of the entire annual vertical and horizontal movement that may occur. Therefore, these velocities were compared to the velocities of the GPS time series, which exhibit high temporal resolution. The comparison was done for benchmark stations that were collocated with US CORS/ CACS stations by simply matching up the survey time period of the campaign survey and extracting the relevant data from the time series. Of the stations tested a maximum difference of 4 mm/yr and minimum difference of 0.6mm/yr was observed between the methods. In the case of the Wisconsin Point station (Figure 6.10), the campaign survey velocities for one period (1997-2005) produced the same vertical velocity as the continuous GPS time series. But over the second period (2005-2010), the velocity showed a 4mm change in velocity (in the same direction), this change was not reflected by the continuous GPS time series. It can be inferred that the GPS campaign surveys can produce suitable results when determining the vertical displacement of a station. However, in cases similar to that of the Wisconsin Point station where the cause
of the 4 mm/yr change is unknown additional campaign survey data may be needed to
detect and verify random changes that may occur. Therefore between the two methods for
determining vertical velocity and overall land deformation, the continuous GPS multi-
year time series analysis for a time period greater than 2.5 year produced a stable and
consistent solution over the campaign surveys.

Generally this work contributed to the overall goal of height modernization by providing:

a. An evaluation of an appropriate post-processing network designs, the multiple
reference “radial” configuration using NGS's latest online processing tool - OPUS-
Projects:

- This network design is the basic radial network, modified by the inclusion of
  multiple reference stations to provide stability and reliability, in the network
  These additional reference stations, are distant from the core radial network
  (approximately 1000 km), to allow for the absolute tropospheric parameters to
  be estimated more reliably, as decorrelation of tropospheric effects at both
  ends of the baselines is achieved with longer baselines An evaluation of the
  reliability of campaign surveys data vs. GPS time series for monitoring land
  movement precisely.

- Providing a time series analysis of continuously operating GPS from the US
  CORS/CACS networks.

b. An evaluation of GPS campaign survey data vs. GPS time series analysis for
monitoring vertical displacement providing:
• Campaign survey results that were consistent with that achieved by continuous GPS time series. Additionally recommending that at least 4 campaign survey must be done to detect the land movement precisely

c. A reanalysis of the 3 IGLD surveys in a consistent reference frame (IGS08/ITRF08) and a processing software (OPUS-Projects) with the best available GPS orbits and using the associated absolute antenna (receiver and satellite) models. This allowed for the observation of land movement over time, without including the change coming from either changes in reference frame or processing software/algorithms.

d. A completely reprocessed IGLD Height Modernization survey dataset yielding:
  • Coordinates that were computed using the NGS's latest online processing tool - OPUS-Projects.

The following recommendations are presented for further work:

An additional output of the time series analysis was the associated seasonal variation. A number of different sources can contribute to the apparent variations in the observed site positions. These sources may stem from the solid Earth and ocean tides to errors in the GPS error correction models. In this study some of the seasonal effects were either modeled or corrected in the solution generated from OPUS-Projects or OPUS-Net, so their contributions should be minimal. This study did not look into the specific causes of the computed seasonal variation however, for future studies this should be investigated to determine how these seasonal periodic effects may be accounted for to achieve cm-level height accuracy, from GPS.
REFERENCES

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APPENDIX A: Reference Tables - Vertical Velocities Computation –
(Comparison between Epochs)

The tables presented in this appendix, shows the coordinate results and the derived vertical velocities for epoch pairs of the IGLD 1997, 2005 and 2010 Height Modernization reanalysis using the Multiple Reference Radial Network configuration. The methodology and analysis are discussed in Chapter 5.
### Table A.1. Coordinate Comparison and Vertical Velocity values for IGLD 2010-1997. Δ Orthometric Height (m) is equal to 1997 Othometric Height subtracted from the 2010 Orthometric Height; Δ Ellipsoid Height (m) is equal to 1997 Ellipsoid Height subtracted from the 2010 Ellipsoid Height

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Table A.2. Coordinate Comparison and Vertical Velocity values for IGLD 2005-1997. Δ Orthometric Height (m) is equal to 1997
Othrometric Height subtracted from the 2005 Orthometric Height; Δ Ellipsoid Height.(m) is equal to 1997 Ellipsoid Height
subtracted from the 2005 Ellipsoid Height
1997 Coordinates
PID

Designation

State

Zone

2005 Coordinates

Orthometric
Northing (m) Easting (m)
Ht. (m)

Ellipsoid
Ht.(m)

Northing (m) Easting (m)

Orthometric
Ht. (m)

Ellipsoid
Ht.(m)

Δ Orthometric
Ht.(m)

Δ Ellipsoid
Ht.(m)

Velocity
Orthometric
Ht.(m/yr)

Velocity
Ellipsoid
Ht.(m/yr)
0.001

214

AA8053

ESSEX A

MI

UT M 17

4834526.383 271027.879

178.315

143.700

4834526.366 271027.882

178.324

143.709

0.009

0.009

0.001

AE8008

UNIT 10 106

MI

UT M 16

5153480.771 702951.176

185.637

149.003

5153480.774 702951.178

185.685

149.051

0.048

0.048

0.006

0.006

AE8289

602

MN

UT M 15

5180532.452 569189.408

184.378

156.113

5180532.440 569189.409

184.373

156.108

-0.005

-0.005

-0.001

-0.001

AH7272

909 9018 K

MI

UT M 16

5154848.777 470970.197

187.911

153.091

5154848.787 470970.197

187.971

153.151

0.060

0.060

0.008

0.008

AH9228

DET OUR MARINA MI

UT M 17

5098021.446 275403.757

177.925

141.114

5098021.447 275403.746

177.977

141.166

0.052

0.052

0.006

0.006

AH9229

LAUNCH SIT E

MI

UT M 17

4947876.825 318790.308

177.561

141.680

4947876.816 318790.294

177.615

141.734

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0.054

0.007

0.007

AH9230

905 2000 F

NY

UT M 18

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91.649

57.449

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91.696

57.496

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0.047

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0.006

AH9231

905 2030 J

NY

UT M 18

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76.745

41.948

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0.003

AH9232

905 2058 K

NY

UT M 18

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75.931

40.013

4794074.728 286726.774

75.961

40.043

0.030

0.030

0.004

0.004

AH9233

905 2076 H

NY

UT M 17

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51.531

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87.692

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0.003

0.003

AH9234

906 3020 H

NY

UT M 17

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176.453

141.299

4749321.408 672342.609

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141.328

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0.004

0.004

AH9235

906 3053 F

OH

UT M 17

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-0.022

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-0.003

AH9236

906 3079 L

OH

UT M 17

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4600539.478 355586.243

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140.584

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-0.003

0.000

0.000

AH9237

906 3085 G

OH

UT M 17

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175.705

140.300

4618644.368 294274.477

175.746

140.341

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0.041

0.005

0.005

AH9238

906 3090 G

MI

UT M 17

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176.502

141.266

4647702.647 312624.358

176.494

141.258

-0.008

-0.008

-0.001

-0.001

AH9241

78U3005

ON

UT M 17

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40.612

4870927.119 727513.224

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40.639

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0.027

0.003

0.003

AH9244

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UT M 16

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156.685

5314159.010 656568.840

193.798

156.759

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0.074

0.009

0.009

AH9247

94U9451

ON

UT M 17

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142.097

5125443.463 303228.266

179.162

142.114

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0.002

AH9248

953000

ON

UT M 16

5363869.858 335528.523

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150.314

5363869.858 335528.518

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150.284

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-0.030

-0.004

-0.004

AH9249

973006

ON

UT M 17

4722928.758 482506.483

176.006

140.350

4722928.752 482506.492

176.008

140.352

0.002

0.002

0.000

0.000

AH9250

973007

ON

UT M17

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172.730

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0.014

0.002

0.002

MB1563

G 321

OH

UT M 17

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143.430

4598897.085 447104.434

177.847

143.450

0.020

0.020

0.003

0.002

ND0194

D 362

PA

UT M 17

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175.514

140.560

4667190.634 576137.296

175.556

140.602

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0.042

0.005

0.005

NE0898

N 235

MI

UT M 17

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177.274

142.632

4704086.026 345269.806

177.286

142.644

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0.002

0.002

NE0963

IBM 55

ON

UT M 17

4760751.369 384142.539

178.605

143.692

4760751.369 384142.536

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143.669

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-0.003

-0.003

OJ0517

LSC 5 C 93

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UT M 17

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143.769

4859590.006 365461.523

178.902

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-0.020

-0.003

-0.003

OJ0599

LAKEPORT RM 2

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145.847

4777492.806 378424.291

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0.052

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0.006

QK0428

J 299

MI

UT M 16

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144.160

5071909.639 676733.745

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144.271

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0.111

0.014

0.014

RJ0586

A 293

MI

UT M 16

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0.006

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UT M 17

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-0.003

T Y5484

81U111

ON

UT M 17

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177.056

142.058

4731831.989 379077.220

177.036

142.038

-0.020

-0.020

-0.003

-0.002

T Y5827

GROS 1

ON

UT M 16

5155525.933 685413.796

184.517

148.019

5155525.930 685413.791

184.525

148.027

0.008

0.008

0.001

0.001


Table A.3. Coordinate Comparison and Vertical Velocity values for IGLD 2010-2005. Δ Orthometric Height (m) is equal to 2005 Orthometric Height subtracted from the 2010 Orthometric Height; Δ Ellipsoid Height (m) is equal to 2005 Ellipsoid Height subtracted from the 2010 Ellipsoid Height

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Northing (m) = Easting (m) + Elliptical Ht.(m) - Elliptical Ht.(m)

Elliptical Ht.(m) = Ht.(m) + Ellipse Ht.(m) - Ellipse Ht.(m)

Δ Ellipse Ht.(m) = Ellipse Ht.(m) - Ellipse Ht.(m)

Velocity Orthometric Ht.(m/yr) = Δ Orthometric Ht.(m) / Year

Velocity Ellipsoid Ht.(m/yr) = Δ Ellipsoid Ht.(m) / Year
Table A.4. Coordinate Comparison and Vertical Velocity values for IGLD 2010-2005. Δ Orthometric Height (m) is equal to 2005 Orthometric Height subtracted from the 2010 Orthometric Height; Δ Ellipsoid Height (m) is equal to 2005 Ellipsoid Height subtracted from the 2010 Ellipsoid Height

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Notes:
- Δ Ellipsoid Height (m) is equal to 2005 Ellipsoid Height subtracted from the 2010 Ellipsoid Height.
- Δ Orthometric Height (m) is equal to 2005 Orthometric Height subtracted from the 2010 Orthometric Height.
- Orthometric Height subtracted from the 2010 Orthometric Height; Δ Ellipsoid Height (m) is equal to 2005 Ellipsoid Height subtracted from the 2010 Ellipsoid Height.
APPENDIX B: Position Time Series Plots (Up Direction)

The figures below represent the edited (outliers removed) time series for the 20 CORS/CACS stations around the Great Lakes discussed in Chapter 6. The plots here identified the vertical displacement of the CORS/CACS stations selected. Several stations such as CALU, MIMQ, OHMH, PWEL and ROSS have time series data gaps or steps, which coincide with the events of GPS antenna or receiver change. In such cases each segment of the time series is model independently and only time series which contains data for more the 2.5 years are used for further analysis and velocity determination.

Figure B.1. Filtered position time series for ALGO, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line)
Figure B.2. Filtered position time series for BAYR, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line).

Figure B.3. Filtered position time series for BFNY, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line).
**Figure B.4.** Filtered position time series for CALU, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line). The data gap in this time series seen at 2007.903-2008.214; 2009.923-2010.424 and 2012.713-2013.271 is due presumably to equipment change or failure.

**Figure B.5.** Filtered position time series for CHB1, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line).
Figure B.6. Filtered position time series for ALGO, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line)

Figure B.7. Filtered position time series for KEW1, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line)
Figure B.8. Filtered position time series for KNGS, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line)

Figure B.9. Filtered position time series for MIL1, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line)
Figure B.10. Filtered position time series for MIMQ, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line). The data gap in this time series seen at 2009.654-2010.616 is due presumably to equipment change or failure.

Figure B.11. Filtered position time series for MSKY, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line).
Figure B.12. Filtered position time series for NLIB, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line).

Figure B.13. Filtered position time series for NOR3, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line).
Figure B.14. Filtered position time series for OHMH, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line). The data gap in this time series seen at 2009.923-2012.137 is due presumably to equipment change or failure.

Figure B.15. Filtered position time series for PARY, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line).
Figure B.16. Filtered position time series for PTIR, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line). The data offset in this time series seen at 2008.789 – 2009.175 is due presumably to equipment change or failure.

Figure B.17. Filtered position time series for PWEL, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line). The data offset in this time series seen at 2008.252 - 2008.271 is due presumably to equipment change or failure.
Figure B.18. Filtered position time series for ROSS, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line). The data offset in this time series seen at 2009.750-2010.060 is due presumably to equipment change or failure.

Figure B.19. Filtered position time series for STB1, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line).
Figure B.20. Filtered position time series for WIS1, UP- direction. GPS data (black diamonds) with the Linear Trend (red solid line)
**APPENDIX C: Reference Tables for Chapter 6**

The following tables are related to the results of the time series analysis of the 20 US CORS/CACS stations discussed in Chapter 6. The tables below provides general statistics for the stations analyzed.

*Table C.1. Data Filtering Summary*

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<th>Location</th>
<th>Char-ID</th>
<th>No. of Observation Points</th>
<th>Remaining Observation point after filter</th>
<th>Percentage of data remover by filter</th>
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Table C.3. Postfit Residual Scatter of the Time Series Analysis for the North and East component

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<th>North (mm)</th>
<th>East (mm)</th>
<th>UP (mm)</th>
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