Sectoral Reallocation and Information Economics

Dissertation

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Abstract

My dissertation is composed of two main topics. The first chapter of my dissertation examines how sectoral reallocation influences fluctuations in U.S. labor markets. The second and third chapters study the role of information in business cycles.

In my first chapter, I note that cyclical U.S. labor markets have seen a reduction in fluctuations since the Great Moderation. Concurrent with this event, I show evidence of reduced cross-sector labor mobility. I quantify the impact of sectoral reallocation in shaping overall labor market volatility by developing a two-sector Diamond-Mortensen-Pissarides model. Calibrated to pre-1984 data, the model allows me to ask how much of the Great Moderation is explained by a rise in frictions sufficient to reproduce the empirical fall in sectoral cyclical reallocation. Fluctuations in all labor market variables diminish following the change. As another distinct result of independent interest, the benchmark two-sector model generates higher labor market volatility, nearer the data, when compared to the predictions of a single-sector version. When one sector faces a negative shock, it sees not only a drop in expected match value but also expected outside value falls only marginally as more workers consider reallocation. These effects combine to produce larger declines in expected surplus, boosting the separation rate and lowering the job-finding rate. Increased barriers to reallocation weaken these additional sources of volatility.
What causes economic fluctuations? A growing literature considers noise shocks, shocks affecting only expectations. Most research in the area has adopted a framework without capital. In my second chapter, I explore the implications of this omission. Incorporating endogenous capital accumulation in a New Keynesian model, I find it reduces the impact of noise shocks, halving the initial output response. When a noise shock hits, investment responds weakly and output is mostly driven by consumption. Usable capital on impact is fixed, so diminishing returns restrain the initial response in labor and output. Subsequent adjustments to the capital stock are gradual in equilibrium, leaving the overall output response muted relative to the labor-only noise shock models.

Are sunspots, information uncorrelated with economic fundamentals, an important source of volatility? Experimental evidence suggests that individuals respond to sunspots, but sunspots are largely the only source of information. My last chapter examines what leads people to use sunspots when they have access to fundamental information. Subjects want to operate in the true (fundamental) state, but achieve lower payoffs for coordination. Fundamental information is provided by a private, imperfect signal. Subjects also see a sunspot in the form of a random public signal and are given no instruction on how to use the information. I find that individuals are more likely to converge on using the sunspot when fundamental information is noisy. When the information set is expanded to include a static and dynamic sunspot signal, convergence is weaker, but focused on the volatile, dynamic option. Together, my results suggest the pull of sunspots is stronger in times of high fundamental uncertainty, and can be responsible for the rise in economic volatility typically associated with these periods.
For my parents
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Chapter 1: Sectoral Reallocation and Labor Markets: A Cyclical Perspective

1.1 Introduction

Around the mid-1980’s, the cyclical properties of US labor markets experienced a shift. Volatilities of key variables such as vacancies and unemployment declined, along with fluctuations in job creation and destruction rates. This leads to a natural question: What drove the high cyclical volatility in labor markets before 1984, and why did that change?

Timed with this event, I also document a fall in cyclical sectoral reallocation through an amended Lilien (1982) measure. There is a large theoretical and empirical literature that suggests the importance of sectoral reallocation in labor market fluctuations. A common argument is that sectoral reallocation is a more costly and time-consuming process than simply staying in one’s last sector of employment.\(^1\)

Because reallocation takes time, sectoral employment shifts can produce large rises in unemployment and thus volatility in a variety of labor market variables. Yet this effect depends on many workers making the simultaneous choice to reallocate. If

\(^1\)Shin and Shin (2008) empirically confirm that not only do reallocating workers face longer unemployment durations than those who stay in their last sector of employment, but they also typically receive a wage decrease.
cyclical bursts of sectoral reallocation were less pronounced after the mid-1980s, as my empirical finding suggests, the short-term fluctuations associated with such bursts would have been moderated. I create a two-sector Diamond-Mortensen-Pissarides (DMP) model with endogenous exit, on-the-job search, and reallocation frictions to isolate and quantify this effect.

My principal question is this: How much of the Great Moderation’s cyclical features can be explained solely through increased impediments to sectoral reallocation? To answer this question, I calibrate my benchmark model to pre-1984 data, and then increase the average cost of reallocation. I choose the new average cost to generate a conservative estimate of the decline in cyclical responsiveness of cross-sector labor movements indicated by my cyclical reallocation measure as the U.S. moved into the Great Moderation.\(^2\) My model predicts that the rise in frictions causes smaller business cycle fluctuations in all labor market variables. The change can account for over 40 percent of the empirical fall in the volatilities of the unemployment, the job-finding rate, and the separation rate. It also explains 33 percent of the reduced cyclical movement in job reallocation rates, the sum of job creation and destruction rates. Furthermore, through its effects on each sector’s idiosyncratic productivity distribution, the dampening of sectoral reallocation lowers the cross-sector average labor productivity correlation, a change also observed in the data.

As an additional result, I find the introduction of an alternative labor market improves the empirical performance in this class of models. Using the canonical Mortensen and Pissarides model, Shimer (2005) discovered shocks to labor productivity alone were not sufficient to reproduce the levels of labor market volatility seen

\(^2\)The target I choose is a conservative estimate based on the raw measure and results from a robustness check.
in the data.³ Therefore, the standard framework is missing some crucial sources of propagation that amplify the impact of exogenous productivity. My environment has three channels alleviating this problem.

Decisions boil down to surplus (matched value minus the value of firm and worker’s outside option) which, through Nash bargaining, determines both wages and profit in a match. Matches dissolve if surplus is too low, vacancies are increasing functions of expected surplus, and workers reallocate based on differences in expected wages. When a negative productivity shock occurs in a single sector model, both matched value and unmatched value fall, mitigating the movement of surplus and consequentially all these events. On the other hand, if a sector experiences a negative shock in my framework, unmatched value falls by less as some worker’s outside option becomes reallocation. Therefore, at the sectoral level, increased volatility is generated through an increased wedge between matched and unmatched value. This is the first channel.

Secondly, sectoral reallocation is a process that takes time. I capture this by assuming that workers changing sectors face a fixed probability of successful reallocation. While in transition, such workers cannot search and are effectively structurally unemployed. Suppose, once again, that one sector experiences a negative shock. Not only does the time required to shift across sectors smooth the reallocation process, it also allows a seemingly small fall in aggregate average labor productivity to produce a large increase in unemployment. In turn, other variables such as measured job-finding and separation rates experience substantial movement.

Lastly, the fact that matches react to reallocation pressures by dissolving marries the previous two channels. As surplus is reduced disproportionately in the less

³Unemployment, vacancies, market tightness, and the job-finding rate volatility of the canonical model were all practically an order of magnitude below the data.
productive sector, job destruction rises. Faced with low sectoral job-finding rates and expected wages, many of those separated immediately enter into the transitional stage of reallocation, leading to its effects. When increase frictions discourage sectoral reallocation, all three channels weaken. This explains my main result.

The model can also account for a pre-1984 recessionary pattern in my cyclical Lilien (1982) measure, which disappeared moving into the Great Moderation and did not reappear until the 2007 recession. The measure is essentially cross-sector logged and detrended employment dispersion. Pre-1984 recessions were characterized by a double spike in the measure; an initial rise at the onset or middle of the recession followed by a second, post-recessionary spike. Considering a variety of alternative shocks in my model, I conclude the pattern is indicative of reallocative shocks. When one sector experiences a large drop in employment relative to the other, the measure rises. A large amount of job destruction occurs in the less productive sector as workers begin to reallocate, decreasing sectoral employment and producing the initial spike. After the shock eases, employment in the low productivity sector stabilizes, but an unusually large group of workers is still in transition. The delayed rise in job creation as they successfully reallocate creates a second spike, fueled by the employment growth in the other sector.

Given the seemingly important role of sectoral reallocation in cyclical labor markets, I end by briefly discussing what we might expect moving beyond the 2007 recession. After the Great Recession, the sectoral reallocation measure reaches a historical low. Additionally, when I compare expansionary volatilities across different periods, the current expansion is characteristic of the Great Moderation. Combining these
observations, it appears that milder labor market fluctuations will be a continuing feature in US labor markets.

The remainder of the paper is organized as follows. Section 1.2 discusses related literature. Section 1.3 presents empirical findings. The model is developed and specified in section 1.4. Section 1.5 presents the solution method, key evolution equations, and calibration. Results are discussed in section 1.6, and section 1.7 concludes.

1.2 Related Literature

To my knowledge, my work is the first to theoretically connect changes in cyclical labor markets to sectoral reallocation, and thus complements other papers that examine the role of reallocation in the macroeconomy. Lucas and Prescott (1974) characterized the stationary equilibrium of an island model wherein workers could choose to reallocate between islands subject to sitting out one period. Rogerson (1986) introduced dynamics into the Lucas-Prescott model by analyzing a two period version. Rogerson found that a dispersion shock reducing one sector’s exogenous productivity relative to the other leads to the number of workers reallocating, and the extent of this grows as dispersion rises. My paper also captures this result. Alvarez and Shimer (2011) examine the equilibrium properties in a variation of Lucas-Prescott with rest unemployment. Instead of automatically engaging in search elsewhere, workers have the possibility to wait for better prospects in their own industry. Their model simultaneously generates persistence in industry wages and high rest unemployment, but requires high average costs of reallocation to match the sectoral wage persistence seen in the data.\(^4\)

\(^4\)I do not examine directly how wages behave in my environment, but I can say average sectoral wages are more persistent than wages in the single sector model. As I assume Nash bargaining,
In work that is the most similar in spirit to my own, Greenwood et al. (1996) examine what cyclical effects sectoral reallocation generates in a Walrasian setting. They find a combination of aggregate and sectoral shocks are important for aggregate unemployment, but can only account for one-sixth of employment volatility. My model delivers levels of unemployment volatility far closer to the data. Moreover, my search and matching environment allows me to explore the behavior of vacancies and other related variables.

I contribute to a set of models that incorporate sectoral reallocation in a Diamond-Mortensen-Pissarides setting by providing a focused look at its impact on volatility in both theoretically and empirically. Chang (2011) approaches sectoral reallocation analytically using a sectoral Mortensen and Pissarides model and concludes that reallocative shocks have a small effect on aggregate unemployment. Pilossoph (2012) develops a model wherein decisions to reallocate are largely based on taste shocks, so there is always a non-zero flow of workers to and from each sector. The model is simulated to match construction and non-construction data, and Pilossoph asks whether or not cross-sector movement played a big role in the Great Recession’s rise in aggregate unemployment. Once again, the conclusion is that sectoral reallocation matters little. A shared assumption between both models is exogenous separation. I argue that this explains the difference in their findings relative to my own.\(^5\) Without endogenous separation, the only margin to increase unemployment is through a gradual process of clogging up workers through low job-finding rates. The inclusion workers are compensated for their outside option. Retention wages then have a role in my setting and rise as a sector’s productivity falls. This in turn dampens the fall of the sector’s average wage dictated by productivity alone.

\(^5\)In Chang (2011), gross sectoral flows were net flows (i.e., workers only reallocated in one direction). Pilossoph (2012) argued the canceling effect of bi-directional flows is what makes sectoral reallocation less important, yet Chang reached the same conclusion without this feature.
of an alternative sector introduces a way for workers to avoid decreases in sectoral job-finding rates by reallocating and prevents reallocative shocks from having any substantial effect on aggregate unemployment.

Mehrotra and Sergeyev (2012) seek to explain rightward shifts in the Beveridge curve, particularly the shift seen past the 2007 recession, as a result of rises in sector-specific shocks. They also derive a sector-specific shock index using factor analysis, and conclude that such shocks have become more important in sectoral employment dispersion since 1984. In my setting, holding the productivity process fixed, a reduction in intra-sectoral mobility leads to less correlated sectoral employment. Carilla-Tudela and Visschers (2013) create a sectoral DMP model and are principally interested in the interaction between three types of unemployment: rest, search, and reallocation. They conclude workers choosing to rest are by far the largest component in unemployment fluctuations. In their model, as aggregate productivity falls, the ratio of unemployment due to reallocation decreases as a fraction of total unemployment. Both findings contradict several studies using PSID data that conclude reallocating workers are a large component in cyclical unemployment, particularly so during recessions.

There are several papers that empirically document the connection between fluctuations and cross-sector movement. On the cyclicality of sectoral reallocation, Lungani and Rogerson (1989) find in PSID data over 1974-1984 that reallocation between goods and service-producing sectors accelerates during recessions. They argue that

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6They find that sector-specific shocks inferred based on employment growth rates have a strong positive correlation with the Lilien measure before 1984, and becomes less correlated moving into the Great Moderation.
that 40 percent of total unemployment weeks during a recession are caused by sectoral reallocation, versus only 25 percent during expansions. While agreeing it is countercyclical, Starr-McCluer (1993) argue that Loungani and Rogerson’s method mismeasures the contribution of reallocation to recessionary and expansionary unemployment weeks. Using PSID monthly data over 1981-1983, Starr-McCluer estimates this contribution at 34 percent and 28 percent, respectively. Through my model, I can measure the number of workers who engage in reallocation, and I find it is negatively correlated (-.28) with detrended output, consistent with these findings. Using more recent PSID data from 1986-1996, Shin and Shin (2008) also conclude reallocating workers’ contribution to unemployment is countercyclical. They also argue that reallocating workers are largely responsible for aggregate unemployment fluctuations. Though unemployment volatility falls in my model as I increase barriers to cross-sector mobility, a large portion is still generated through sectoral reallocation.

Campbell and Kuttner (1996) dissect the contribution of reallocation to employment fluctuations and conclude it accounts for over half. They also show that movement from manufacturing to other sectors is associated with a rise in both job creation and destruction. Similarly, Davis and Haltiwanger (1999) report that allocative (rather than aggregate) shocks are responsible for most cyclical job reallocation (the sum job destruction and job creation). Examining the empirical findings from multiple other papers, Gallipoli and Pelloni (2013) undertake a meta-analysis on the importance of sectoral reallocation in unemployment volatility and find that it accounts for one-quarter to two-thirds of the variability in aggregate postwar unemployment.

A distinct literature concerns itself with measuring sectoral reallocation. The seminal attempt to measure short-term sectoral reallocation was Lilien (1982), which
focused on employment growth dispersion. Abraham and Katz (1986) criticized the measure as attributing sectors’ asymmetric responses to aggregate shocks as reallocation. For example, a negative aggregate shock could lead to a larger fall in employment for construction than education without any cross-sector flows, and Lilien’s measure would record a spike. I base my measure on Lilien’s (1982) specification, and it is subject to the same criticism. This leads me to consider a robustness check on my measure. Results continue to reveal a post-1984 average fall. I also use my model to explore the implications of an Abraham and Katz-style shock. I find such a shock does not reproduce the pre-1984 recessionary pattern uncovered by my measure. Aaronson et al. (2004) adopt an alternative approach using a method from Rissman (1997) that was itself based on Stock and Watson (1989). Controlling for the different sectoral responses to aggregate shocks, they arrive at the same conclusion: cyclical sectoral reallocation has fallen substantially since 1984. The studies above, like my own, seek to measure short-term movement in sectoral reallocation. In contrast, Kambourov and Manovskii (2008) focus on secular changes. Using annual PSID data, they find a secular rise in cross-sector movement since 1968. It is possible that the incentives to reallocate have risen over time, but I argue that, over the cycle, impediments to this have mitigated sudden bursts of sectoral reallocation. Jobless recovery supports this case; if workers are increasingly able to reallocate to productive industries, why then do they linger for longer spells in unemployment?

My study is agnostic as to the source of increased short-term sectoral reallocation frictions; however, there are existing theories. Based on wage data, one theory is that the economy has experienced skill-biased technological change wherein the demand
for workers with college degrees has grown rapidly. This implies higher opportunity costs and time involved in reallocation. A related idea comes from the job-polarization literature. Autor et al. (2003) find the replacement of routine occupations, fueled by computerization, can explain most of the educational demand shift from 1970-1998. More relevant to my story, Jaimovich and Siu (2012) find evidence that jobless recovery, a post-1984 phenomenon, is intimately related to job polarization. Specifically, middle-skilled, routine occupations have experienced permanent job destruction in the last three recessions. Workers formerly employed in such occupations experienced longer unemployment durations and faced very low demand for their skills. If these workers did wish to reallocate, they would need to retool or accept a large wage cut, both of which come at large costs.

1.3 Data

The data section is divided into two parts. The first documents cyclical volatilities over the US post-war period in five key labor market variables: unemployment, vacancies, market tightness (vacancies divided by unemployment), the job-finding rate (JFR), and the separation rate (SR). The second part derives a cyclically-based Lilien (1982) measure of reallocation. I show the measure is positively correlated to long-term changes in cyclical volatilities, cross-sector average labor productivity correlation, and a measure of cyclical job reallocation (job creation plus destruction). I end by performing a simple robustness check on my cross-sector reallocation measure.

1.3.1 Labor Market Volatilities

To examine how labor market volatilities have changed over time, I subdivide the post-war era into the Pre-Great Moderation (1960Q1-1983Q4), Great Moderation (1984Q1-2006Q4), and Post-Great Moderation (2007Q4-2012Q4).\textsuperscript{8} Using monthly, seasonally adjusted unemployment rate data from the BLS CPS, unemployment volatility is the standard deviation of the quarterly averaged, logged, and HP-filtered series. Vacancy data is from the monthly Barnichon (2010) Help-Wanted index and is averaged and detrended in the same manner as the unemployment rate series. This data ends at 2013Q2, so measures of vacancy volatility for the Post-Great Moderation stops at 2011Q4 given adjustment for the HP-filter end-period bias. Market tightness is simply the monthly Barnichon (2010) measure divided by the BLS CPS unemployment series. This measure is then subject to the previous quarterly averaging and detrending process.

The JFR is determined as in Shimer (2005). Let $f_t$ be the JFR at time $t$. Given the short-term ($< 4$ weeks) unemployment series, $U_t^s$, from the CPS, the JFR can be determined through the following relationship between the number of unemployed in this and next period:

$$U_{t+1} = (1 - f_t)U_t + U_t^s$$

Therefore, those who do not find a job or have recently lost a job are the total number of unemployed next month.\textsuperscript{9} Given monthly data, the above equation results

\textsuperscript{8}I begin the Pre-Great Moderation at 1960 to get a symmetric series between it and the Great Moderation.

\textsuperscript{9}As mentioned in Abraham and Shimer (2001), in January 1994 the CPS changed how they interviewed for short-term unemployment, making the measure inconsistent before and after this date. I adjust for this by multiplying post-January 1994 short-term unemployment levels by 1.16. Shimer (2012) describes a more robust method of correction using micro-level observations from the CPS.
in a monthly job-finding rate and is quarterly averaged, logged, and detrended with smoothing parameter $\lambda = 1600$.

I undertake a different method to derive separation rates than the one in Shimer (2005). Shimer (2005) solves for separation rates, $s_t$, through the following relationship:

$$U_{t+1}^s = \left(1 - \frac{1}{2}f_t\right)s_tE_t$$  \hspace{1cm} (1.2)

Next period’s newly separated workers are then comprised of $s_t$ fraction of those employed this period that separated but also an additional adjustment. An individual separated within a given month can, on average, search for half a month before the BLS counts them as unemployed. Consequently, the data on $U_{t+1}^s$ understates the true number of separations. Shimer (2005) adjusts for this by assuming that all individuals separated at rate $s_t$ from $e_t$ engage in search and—given that they on average have half a month—$\frac{1}{2}f_t$ find jobs before being recorded in $U_{t+1}^s$.

I make two changes in measuring separation rates compared to the above method. The derivation of the job-finding rate induces consistency between it and the evolution of unemployment seen in the data. Similarly, I want to do the same with the separation rate and employment series, especially because I will derive job creation and destruction using the series.\(^{10}\) This leads me to:

$$(1 - s_t)E_t + .3f_ts_tE_t + f_tU_t = E_{t+1}$$  \hspace{1cm} (1.3)

\(^{10}\)The employment data had a few sections in the 2000s that were subject to error due to population control changes. For the most part, these seem largely inconsequential, but 2000M1 results in an abnormally large spike. To correct this, I convert the point to an average of 1999M12 and 2000M2 employment levels.
This specification helps to better account for individuals who may separate and not look for employment, and generates a more volatile separation rate process than the one derived from the Shimer (2005) method.

I also assume that not all workers who separate immediately engage in search or, more explicitly, that newly separated workers do not undertake search until the second week of their separation. Therefore, instead of \( .5f_t \) of those newly separated regaining employment before next period, workers have on average 1.2 weeks to search for a new job, reducing the fraction to \( .3f_t \). I make this assumption for an additional reason beyond anecdotal argument. Fujita and Ramey (2009), using data at the individual level from the 1976-2005 CPS, find the contemporaneous correlation of quarterly detrended unemployment and separation rates to be around 0.7. By making the separation rate more dependent on the job-finding rate, its connection with unemployment rates decouple. Restricting my data to 1976-2005, the correlation between the detrended quarterly separation rate series generated by equation (1.3) and detrended quarterly unemployment rate is .37 using \( .5f_t \) versus .48 with \( .3f_t \), with the latter being more in line with evidence from Fujita and Ramey (2009).\(^{11}\)

The cyclical volatility of unemployment \((\sigma_u)\), vacancies \((\sigma_v)\), market tightness \((\sigma_z)\), JFR \((\sigma_{JFR})\), and SR \((\sigma_{SR})\) for my three periods of interest can be seen in Table 1.1. There is a clear drop in volatilities for all variables moving into the Great Moderation; the percentage decrease of each variable’s volatility can be seen in the second part of the table. However, there seems to be a strong reversal moving into the post-Great Moderation. I argue that these high volatilities are largely due to the

\(^{11}\)I am also looking at implementing the Shimer (2012) continuous-time method to correct for this time aggregation bias.
Table 1.1: Post-war US Volatilities

<table>
<thead>
<tr>
<th>Period</th>
<th>$\sigma_u$</th>
<th>$\sigma_v$</th>
<th>$\sigma_{JFR}$</th>
<th>$\sigma_{SR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-GM Data (1960Q1-1983Q4)</td>
<td>0.128</td>
<td>0.15</td>
<td>.08</td>
<td>0.088</td>
</tr>
<tr>
<td>GM Data (1984Q1-2006Q4)</td>
<td>0.091</td>
<td>0.111</td>
<td>.071</td>
<td>0.066</td>
</tr>
<tr>
<td>Post-GM Data (2007Q4-2012Q4)</td>
<td>0.149</td>
<td>0.125</td>
<td>.127</td>
<td>0.099</td>
</tr>
</tbody>
</table>

%Δ Pre-GM to GM  -28.9%  -26%  -44.4%  -11.3%  -25%

Note: All data is monthly data averaged quarterly, logged, and HP-filtered with smoothing parameter $\lambda = 1600$. Unemployment volatility is derived from quarterly averages of monthly, seasonally adjusted BLS CPS data on unemployment rates. Vacancy data is from the monthly Barnichon (2010) Help-Wanted Index and only runs until 2013Q2, resulting in the post-GM data on vacancies and market tightness being measured up to 2011Q4. Market tightness is derived as a ratio between the monthly data on vacancies and unemployment rates. The JFR and SR are derived using data on seasonally adjusted employment, unemployment, and short-term unemployment ($< 4$ weeks) levels from the BLS CPS, and equations (1) and (3), respectively.

Great Recession and, moving forward, fluctuations seem more characteristic of the Great Moderation. I save this discussion for the results section.\(^{12}\)

1.3.2 Reallocation and Correlation Measure

A Cyclical Lilien Measure

Lilien (1982) argued bouts of sectoral reallocation at time $t$ could be observed as dispersion of sectoral employment growth rates:

$$\sigma^L_t = \left( \sum_{i=1}^{N} (s_{i,t}(g_{i,t} - g_t)^2) \right)^{\frac{1}{2}}$$

(1.4)

Where $i$ denotes a sector, $s_{i,t}$ and $g_{i,t}$ is employment share and growth rate, respectively, of sector $i$ at time $t$, and $g_t$ is aggregate employment growth at time $t$.

\(^{12}\)If, instead, I was to compare pre-1984 (1960Q1-1983Q4) to post-1984 (1984Q1-2012Q4), there would still be a notable drop across all labor market variables except for the job-finding rate: $u$ changes by -12.6%, $v$ by -21.6%, $\frac{v}{u}$ by -18.5%, and the separation rate by -15.8%. The job-finding rate becomes more volatile post-1984 by about 9.8%; for some reason, it was disproportionately affected during the Great Recession, skewing it upwards.
As I am interested in the cyclical properties of reallocation, I derive a more fitting measure by logging and HP-filtering ($\lambda=14400$) sectoral and aggregate employment. I then replace the growth rates in Lilien’s measure with these detrended series, $\tilde{e}_t$ and $\tilde{e}_{i,t}$:

$$\sigma^L_t = \left[ \sum_{i=1}^{N} (s_{i,t}(\tilde{e}_{i,t} - \tilde{e}_t))^2 \right]^{\frac{1}{2}}$$

(1.5)

The measure is acquired using monthly sectoral employment data largely at the first level of disaggregation from the CES and covering 1948M1-2014M6.\textsuperscript{13} Table A.1 of the appendix lists the thirteen sectors used. Figure 1.1 shows the quarterly average of the measure.

The most noticeable feature is less movement going into the Great Moderation for both recessionary and expansionary periods with an average drop of 36 percent from .009 (1960Q1-1983Q4) to .0058 (1984Q1-2006Q4). At the start of the series, recessions had a pattern of an initial jump at or during the recession followed by a larger post-recession spike.\textsuperscript{14} Post-recessionary rises of the measure disappear during the Great Moderation, eventually reoccurring in the 2007 recession.\textsuperscript{15} Even though the Great Recession experienced the second highest rise in unemployment over the

\textsuperscript{13}I subdivided manufacturing into durables and non-durables. I also create a “sector” that combines utilities and transport. Individual data for utilities and transportation only began around 1964 and 1972, respectively, but there is data on trade, utilities, and transportation starting at 1948M1. Utilities and transport is the residual employment of this three-sector measure minus trade. I do not disaggregate sectors further for two reasons. Skills are more easily substitutable at the subsector level, which contradicts my view of reallocation being a timely process. Second, Foerster et al. (2011) find industrial production (manufacturing, mining, logging, and utilities) largely was driven by common opposed to sectoral shocks over the postwar period at the 4-digit level. Therefore, I do not view reallocative pressures existing as strongly at the subsector level.

\textsuperscript{14}Later in the paper, I argue this pattern is characteristic of a dispersion shock (when exogenous sectoral productivities diverge): the initial spike is a burst of job destruction in the less productive sector, and the second spike is induced by workers being absorbed into employment of the other sector.

\textsuperscript{15}There is a large, non-recessionary rise at 1978Q1. I would attribute this to issues stemming from the oil crisis as the spike is largely being generated by the mining and logging sector.
Figure 1.1: Quarterly averaged, cyclical Lilien measure. Shaded areas are NBER dated recessions.

postwar era, the measure rises substantially but lower than all except one pre-1984 recession and soon fell to an all-time low. One would think the Great Recession would have induced a disproportionately large need for worker reallocation, especially in areas such as construction. However, in support of this result, Lazaer and Spletzer (2012) conclude that industrial shifts alone cannot account for a large portion of the Great Recession’s unemployment rise.

To make the change in trend of this measure clearer, I plot a 5-year forward rolling average of the monthly measure in Figure 1.2. The last point is 2008M1-2012M12 and contains the beginning and fallout of the Great Recession. The trend seems to move downward beginning around 1975 and falls rapidly at the onset of the Great
Figure 1.2: 5-year moving average of the cyclical Lilien measure.

Moderation. The measure experiences a more substantial rise moving into the 2007 recession, but the level remains depressed compared to the bulk of pre-1984.

Figure 1.3 compares the reallocation measure’s 5-year average to 5-year forward rolling volatilities of unemployment and vacancies. In general, the series have very strong, positive correlations with large concurrent movements. If the measure truly represents cyclical sectoral reallocation, there seems to be an intimate connection with its evolution and changes in cyclical volatilities.

As in the measure, volatilities for unemployment and vacancies are derived from monthly data and face the same smoothing parameter.
Figure 1.3: 5-year moving average of the cyclical Lilien reallocation measure (adjusted to scale) vs. 5-year moving standard deviation of cyclical unemployment and vacancies. Corr(u,σ^L) = .89, Corr(v,σ^L) = .93.

**Sectoral Average Labor Productivity Comovement**

Beyond volatilities, I also find my measure is related to changes in cross-sector average labor productivity correlation. To derive average labor productivity for a sector, I use data on total full-time and part-time sectoral employment and real value added from the BEA’s GDP-by-Industry, which is only at annual frequency.\(^\text{17}\) Additionally, they have made an alteration that makes the 1948-1997 and 1997-2013 data inconsistent. Given I remove the first and last six observations of my detrended series, I can only use data up to 1991. Regardless, we can still take away some implications from examining the series, which importantly includes the Great Moderation transition date.

\(^\text{17}\)I use employment data on total full-time and part-time employment provided by the BEA to help guarantee consistency with their measure of real value added.
Let $V_{i,t}$ and $E_{i,t}$ denote real value added and employment, respectively, for sector $i$ at time $t$ and, then average labor productivity for sector $i$ at time $t$ is:

$$Z_{i,t} = \frac{V_{i,t}}{E_{i,t}}$$

The resulting logged and detrended ($\lambda = 6.5$) series is represented by $\tilde{Z}_{i,t}$. The comovement measure at time $t$, $\hat{c}_t$, is essentially an employment weighted sum of all cross-correlations over an 8-year period starting at time $t$.$^{18}$ More explicitly, letting $E_{i,t}$ denote employment for sector $i$ in year $t$:

$$\bar{E}_{i,t} = \frac{\sum_{t+7}^{t+7} E_{i,t}}{8}$$

$$\bar{E}_{ij,t} = \frac{\bar{E}_{i,t} + \bar{E}_{j,t}}{2}$$

$$E_t = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} E_{ij,t}$$

Cross-sector average labor productivity correlation at time $t$ is:

$$\hat{c}_t = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\bar{E}_{ij,t}}{E_t} \text{corr}(\tilde{Z}_{i,t:t+7}, \tilde{Z}_{j,t:t+7})$$

I want to compare how cyclical sectoral comovement may relate to my reallocation measure, but there is an obvious discrepancy in time aggregation. Therefore, I rederive the measure using an annually averaged employment series, logged and detrended with $\lambda = 6.5$.

Figure 1.4 presents the cyclical measure’s 8-year forward moving average and $\hat{c}_t$ from 1960-1984. Sectoral comovement is fairly positive for most of pre-1984 with an average value of .37. It begins to fall around 1979 and plummets when $\hat{c}_t$ begins to

$^{18}$I choose eight years because at five years the measure becomes too dependent on single observations. Additionally, I wanted to acquire at least one value of the measure strictly in the Great Moderation period.
firmly enter the Great Moderation. Using factor analysis, Foerster et al. (2011) and Mehrotra and Sergeyev (2012) both find sector-specific shocks became more important moving into the Great Moderation. While a lesser role of aggregate shocks could certainly account for the observed fall, I argue that there may be endogenous effects stemming from a decrease in sectoral reallocation.

What workers become separated and reallocate? If a worker is particularly skilled in their sector, they would likely face lower separation rates and higher opportunity costs for reallocation. On the other hand, workers with the lowest productivity are fair game for separation when sectoral productivity falls and, facing poor options within their sector, are more likely to look elsewhere. Removing low productivity

19Foerster et al. (2011) looked at the role of aggregate/sectoral shocks on industrial production while Mehrotra and Sergeyev (2012) examined the impact on sectoral employment fluctuations.
individuals from sectoral employment leads to a rise in average labor productivity and offsets the fall generated by the exogenous drop. On the other hand, the sector that is willing to absorb these particular workers may experience a fall in average labor productivity, even if there was no underlying change in its exogenous process. When workers are able to move more freely, this mechanism will be amplified and result in a higher correlation from the same sized shock.

On that note, going back to Figure 1.4, the cyclical reallocation measure is positively correlated (.48) with $\hat{c}_t$, and provides some basic support to the distributional effect story.

**Job Reallocation**

There is another metric, based on the work by Davis and Haltiwanger (1992) and Davis et al. (1996), that can also give some insight into whether cyclical sectoral reallocation decreased past 1984. Periods of pronounced sectoral reallocation should be associated with higher job destruction and job creation rates. Summing up these two provides the job reallocation rate, a measure of labor market churning. Typically the measure is derived using micro-level data, but the periods of my interest are outside the start of these data sets. Therefore, I acquire job creation and job destruction rates using aggregate data and flow rates from equations (1.1) and (1.3). Therefore, unlike the Davis et al. (1996) method which can capture other flows, aggregately derived job creation is strictly unemployment to employment while job destruction is employment to unemployment. The job creation rate at time $t$ is given by:

$$JC_t = \frac{f_{t-1}U_{t-1} + .3s_{t-1}f_{t-1}E_{t-1}}{.5(E_t + E_{t-1})}$$  \hspace{1cm} (1.11)
And the job destruction rate is similarly obtained:

\[ JD_t = \frac{s_{t-1}E_{t-1}}{0.5(E_t + E_{t-1})} \]  

(1.12)

I use monthly data and take quarterly averages of the resulting monthly rates. Figure 1.5 graphs the 1-year forward rolling average of both rates without any detrending. Decker et al. (2014) find a secular decline in job creation and destruction rates starting around 1982, which the aggregate measure manages to capture. Job reallocation, which is simply the sum of the two, obviously faced the same phenomenon. Of key interest is the rise of job reallocation beginning around 1970, a date that also starts the series of large spikes in my cyclical reallocation measure. For a more suitable comparison, I need to look at detrended rates.
Table 1.2: Job Creation, Destruction, and Reallocation Rates

| Period                         | $\sigma_{\tilde{JC}}$ | $\sigma_{\tilde{JD}}$ | $\sigma_{\tilde{JR}}$ | $|\tilde{JR}|$ |
|-------------------------------|------------------------|------------------------|------------------------|---------------|
| Pre-GM Data (1960Q1-1983Q4)   | .025                   | .042                   | .071                   | .058          |
| GM Data (1984Q1-2006Q4)       | .016                   | .028                   | .046                   | .036          |
| Post-GM Data (2007Q4-2012Q4)  | .018                   | .04                   | .064                   | .052          |
| Davis et al. (2006) (1991Q3-2003Q3) | .02        | .03                   | .017                   | .014          |
| GM Subset Data (1991Q3-2003Q3) | .016                   | .03                   | .048                   | .037          |

Note: Monthly values for JC and JD from equations (11) and (12). JR=JC+JD. Values are quarterly averaged, logged, and detrend with $\lambda = 1600$. For a particular period, $|\tilde{JR}|$ are averages of the detrended job reallocation rate’s absolute value. Davis et al. (2006) is publicly available data derived from the CPS BED.

I log and HP-filter the job creation, destruction, and reallocation rates and present the volatilities in Table 1.2, along with similar statistics from CPS BED-based rates obtained in Davis et al. (2006). Data are subdivided according to the same time periods as in volatilities. The table also includes a measure of cyclical job reallocation fluctuations, $|\tilde{JR}_t|$, which is simply the absolute value of the detrended job reallocation series.

There are a couple of features worth noting, especially regarding the use of aggregates. As in other labor market variables, there is a fall in volatilities and cyclical job reallocation during the Great Moderation. In terms of comparisons between the aggregate measure to Davis et al. (2006), we can see both suggest similar job destruction volatility, but the aggregate measure produces less job creation volatility. The level of cyclical job reallocation and its volatility are also quite high compared to Davis et al. (2006). Checking the correlation of JC and JD reveals the culprit: the aggregate measure results in a positively correlated $\tilde{JC}$ and $\tilde{JD}$ series (.68) compared
Table 1.3: Correlations: Job Creation, Destruction, and Reallocation

<table>
<thead>
<tr>
<th>Period</th>
<th>Corr($\tilde{JC}, \sigma^L$)</th>
<th>Corr($\tilde{JD}, \sigma^L$)</th>
<th>Corr($\tilde{JR}, \sigma^L$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-GM Data (1960Q1-1983Q4)</td>
<td>0.12</td>
<td>0.17</td>
<td>0.16</td>
</tr>
<tr>
<td>GM Data (1984Q1-2006Q4)</td>
<td>-.1</td>
<td>0.04</td>
<td>-.01</td>
</tr>
<tr>
<td>Post-GM Data (2007Q4-2012Q4)</td>
<td>0.37</td>
<td>0.39</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Note: $\sigma^L$ is the quarterly average of the monthly $\sigma^L$.

to the negative correlation (-.23) from micro data.\(^\text{20}\) Because my method is based on aggregate unemployment-employment flows, the aggregate rates ignore job-to-job transitions which may be responsible for this discrepancy.

As my focus is sectoral reallocation, excluding these transitions may not pose much of an issue, as I would argue sectoral reallocation is a process workers mainly undertake during unemployment. Therefore, for my purposes, aggregately derived job creation and destruction rates seem to provide a decent benchmark for conditions faced by reallocating workers.

Table 1.3 provides correlations between the cyclical sectoral reallocation measure and these rates. Job creation and job destruction are positively correlated with the cyclical sectoral reallocation measure pre-1984. Moving into the Great Moderation, the relationship turns negative for job creation and practically zero for job reallocation and destruction, but becomes notably positive during the 2007 recession and beyond. The substantial change in correlation from the Great Moderation to the 2007 recession gives a hint: the cyclical sectoral reallocation measure becomes more correlated with

\(^\text{20}\)Discrepancies between job creation volatility and correlation between measures only widen if separation rates were derived according to equation (1.2), reducing job creation volatility to .015 and raising the correlation to .75. Also, the positive correlation is not a consequence of scaling by employment, as even $s_{t-1}E_{t-1}$ and $f_{t-1}U_{t-1}$ has a strong, positive correlation of .69.
rates during recessionary times. A rise in job reallocation rates occur with large post-recessionary spikes, which can be seen in Figure A.1 of the appendix.

As a last comparison, Figure 1.6 plots the 5-year forward moving averages of both the cyclical job and sectoral reallocation measures. Excluding the meteoric rise that began around 1963 in cyclical job reallocation—going back to Figure 1.5, one can clearly see the origin—both measures seem to track one another across the postwar era.\footnote{Davis et al. (2006) also have quarterly data from 1948Q1-2005Q1 available for manufacturing, a series that Faberman (2008) argues is a good representation of aggregate fluctuations. Comparing cyclical job reallocation from this data to the cyclical sectoral reallocation measure yields a stronger correlation of around .68.} If cross-sector movement is more cyclical and thus associated with violent
bursts, job reallocation should experience similarly large movement. Smaller fluctua-
tions in job reallocation moving into the Great Moderation then further suggests the period also experienced a fall in cyclical sectoral reallocation.

A Simple Robustness Test

When the economy is exposed to an aggregate shock, Abraham and Katz (1986) argued the Lilien (1982) measure confounds asymmetric sectoral employment responses with sectoral reallocation. That is, a negative aggregate shock may induce a larger fall in durables’ employment than educational services, generating a spike in the Lilien (1982) measure. In the previous section, I find the spikes from my sectoral reallocation measure are associated with rises in job reallocation rates, which helps to answer some of this criticism. But, I also now undertake a simple robustness check to try and better control for differences in sectoral responses to aggregate shocks.

The concept behind the exercise is straightforward. Assume the qualitative responses of sectors’ employment to an aggregate shock are the same across time (e.g., a negative (positive) shock always results in a negative (positive) employment response for all sectors). If each sector has the same average level of employment fluctuations across two periods, then a relative change in the cyclical Lilien (1982) measure between these periods is more likely to be capturing sectoral reallocation.

For example, abstracting from my sectoral classification, let us assume we just have services and manufacturing. Before 1984, suppose a negative shock resulted in a fall of -9 percent in manufacturing employment but only -2 percent for services. A smaller aggregate shock post-1984 generates a -3 percent decrease in manufacturing and -1 percent for services. The Lilien (1982) measure would create larger relative spikes simply from differently sized aggregate shocks. However, if I readjusted pre-1984
to generate the same average employment fluctuations between periods, multiplying manufacturing by $\frac{1}{3}$ and services by $\frac{1}{2}$, the measure should say there was no relative difference. Conversely, if a pre-1984 shock resulted in a -6 percent employment drop in manufacturing, but a 3 percent increase in services, the readjustment would still generate a relatively higher value in the Lilien measure for pre-1984. It is important to keep in mind that, as can be seen by the last example, this method can underestimate actual sectoral reallocation.

I subdivide the postwar era into pre-1984 (1960M1-1983M12) and post-1984 (1984M1-2012M12). For each sector, $i$, I find two weights: one equating average employment deviations of both periods during recessions (NBER dated recessions, including the three months prior and after the recession) and the other for expansions (periods in between recessions +/- 3 months). In other words, letting $P$ denote either expansionary or recessionary periods, I find weights $w_{i,P}$ such that the following equation holds:

$$w_{i,P} \frac{\sum_{t \in PR84 \cap P} |\tilde{e}_{i,t}|}{\sum_{t \in PO84 \cap P} 1} = \frac{\sum_{t \in PO84 \cap P} |\tilde{e}_{i,t}|}{\sum_{t \in PO84 \cap P} 1}$$ (1.13)

I then readjust $\tilde{e}_{i,t}$ (and $\tilde{e}_t$) in pre-1984 by multiplying the series by its respective weight, which depends on whether $t$ falls in expansionary or recessionary periods. Table A.2 of the appendix lists the acquired weights. The cyclical sectoral reallocation measure resulting from this readjustment is shown in Figure 1.7. The measure still results in an average value decline (-18 percent) between pre-1984 and the Great Moderation. The 2007 recession now seems more substantial, but was not subject to the same readjustment and still achieves the lowest historical value.

22 I make this division because I view the change in cyclicality of sectoral reallocation as a persistent feature that exists past the 2007 recession.
Figure 1.7: Weighted $\sigma_L^\rho$. Average $\sigma_L^\rho$ (1960M1-1983M12)=.0071, Average $\sigma_L^\rho$ (1984M1-2006M12)=.0058.

Does the readjustment underestimate relative sectoral reallocation before 1984? To check whether or not this may be true, I can examine what sectors are causing a fall in the measure when weights are applied. Practically all of the change is originating just from construction and durables. Weighting only these sectors and aggregate employment, the pre-1984 average of the measure drops from .009 to .0069. Conversely, applying weights to every other sector except durables, construction, and aggregate employment actually results in a small rise to .0091.\(^{23}\)

\(^{23}\)These two sectors are what Cortes et al. (2014) would classify as routine manual occupations. Following all recessions post-1984, jobs destroyed in routine occupations were not replaced during expansions. Faced with skill mismatch, Cortes et al. (2014) and Jaimovich and Siu (2012) find strong evidence these workers could be responsible for the jobless recovery phenomenon. The job polarization literature may then provide some information as to why there was a decline in cyclical sectoral reallocation over the Great Moderation—these workers simply did not have immediate options.
If we look at cyclical monthly employment for all sectors, construction and durables have by far the largest net outflow, with large employment falls and weak gains across pre-1984. Additionally, Loungani and Rogerson (1989) directly examined cyclical effects on employment by using PSID data from 1974-1984. They find the recession of 1975 was timed with a large (-1.8 percent employment share), permanent decrease in durables employment. During the 1980-1983 recessions, they document deep declines in a sector that includes construction, and durables once again experienced notable outflows. Removing only the recessionary weights for these sectors and aggregate employment raises the average pre-1984 value from .0071 to .008, implying a 27 percent average decline moving into the Great Moderation.

Therefore, I conclude the weighted series likely provides, at best, a lower bound for the relative level of cyclical sectoral reallocation pre-1984. At the same time, I will not claim the 36 percent average fall in the original measure is more representative, so I settle for a conservative 24 percent reduction in the measure as a calibration target.

1.4 Model

How do changes in the amount of cyclical sectoral reallocation impact cyclical labor markets, if at all? It is difficult to say exactly how the cyclical properties of sectoral reallocation evolved over the entire postwar era, but there was a clear change between pre-1984 and the Great Moderation. Based on the previous section, I can claim with some confidence that average cyclical sectoral reallocation decreased between these periods. Accompanied with this change was a fall in four components of

$\sum c_{i,t}^{ pre-1984}$ for construction and durables results in -.475 and -.26, respectively.
the cyclical labor market: volatilities, cross-sector average labor productivity correlation, job reallocation, and a cyclical sectoral reallocation measure.

In order to better understand how quantitatively important sectoral reallocation might be in explaining these features, I develop a discrete, two-sector Diamond-Mortensen-Pissarides (DMP) model with endogenous separation and on-the-job search (OTJS). With its non-trivial job-finding rate, vacancy decision, and pool of unemployment, the DMP framework provides a rich environment to examine labor markets. Endogenous separation, based on Mortensen and Pissarides (1994), introduces not just an important margin for separation and worker/firm choice, but also a fluctuating idiosyncratic distribution of productivities. In turn, endogenous separation becomes an important determinant for average labor productivity.

As discussed in Mortensen and Nagypál (2007a), a consequence of endogenous separation without OTJS is countercyclical vacancies, so I view it as a necessary addition.\(^{25}\) OTJS also provides an option beyond unemployment for workers, which helps to dissuade large movements across sectors.

As is typical in DMP environments, there is a unit measure of risk-neutral workers, a measure of risk-neutral firms, and matching is one-to-one. Sectors are distinguished by the index \(k \in \{1, 2\}\).

1.4.1 Matched Output and Separation

A matched firm-worker pair in sector \(k\) produce output—which is equivalent to the labor productivity of the match—given by the following:

\[
\frac{\tau_k}{s_k} = \frac{1 - \tau_k}{s_{l \neq k}} e
\]

\(^{25}\)Other mechanisms that prevent the influx of newly unemployed workers from entering into the vacancy decision, such as rest unemployment, would also produce procyclical vacancies.
$\tau_k$ is a parameter used to target correlation\textsuperscript{26} (as $\tau_k$ rises, correlation decreases) between sectors’ individual labor productivity processes, $s_k$, which follow independent AR(1) processes:

$$log s'_k = \rho_k log s_k + \eta_k$$

(1.15)

where $\eta_k$ is i.i.d. $N(0, \sigma_k^2)$.

$\epsilon$ is the idiosyncratic productivity of a match and it is persistent: with probability $\lambda$ a new $\epsilon$ will be drawn from distribution $F(\epsilon)$ with support $[0, \epsilon^h]$. The surplus of a match—the value of the match minus the value of each worker and firm’s outside option—is monotonically increasing in $\epsilon$. For a low enough $\epsilon$, the match will dissolve as surplus becomes negative. Call the $\epsilon$ where this first occurs $\epsilon^*$. There are then two ways that an existing matched pair can dissolve: exogenously with probability $\chi$ and endogenously when $\epsilon$ reaches at or below $\epsilon^*$. A newly matched pair draw from $F(\epsilon)$, implying that they could draw an $\epsilon \leq \epsilon^*$.\textsuperscript{27} However, I assume a newly matched pair cannot be exogenously dissolved.

1.4.2 Worker Choice

Each worker, employed and unemployed, draw a random fixed cost, $\xi$, to switch sectors from a uniform distribution with support $[0, \xi^h]$. Subject to paying the cost, workers enter into a transitional phase and can only reallocate and search in the other sector with a fixed probability of entry, $\gamma$. Sectoral reallocation then involves

\textsuperscript{26}I could introduce correlation through aggregate productivity, but the state space is already quite large. $\tau_k$ introduces a clear correlation parameter while capturing the concept that seemingly aggregate shocks affect sectors differently.

\textsuperscript{27}One can think of a firm hiring an especially bad worker. Regardless, this makes a very small percentage of new matches in the benchmark calibration. Alternatively, I could assume all individuals start with the same high $\epsilon$, but the resulting labor productivity distribution becomes excessively skewed at this entry point.

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a potentially substantial sitting out cost. If one wishes to reverse their reallocation decision, they must once again pay $\xi$ and are still subject to $\gamma$.

As an accounting assumption, I consider those individuals that switch and have not found their first job in a sector, $u_{i \neq k}^{r}$, still part of sector $k$’s labor force.\(^{28}\) Therefore, the composition of unemployed in sector $k$ is those who stayed in their sector, $u_{k}^{s}$, and those who reallocated to sector $l \neq k$, $u_{l \neq k}^{r}$. If workers in $u_{i \neq k}^{r}$ wished to rejoin their old sector, they would enter into $u_{k}^{s}$ and must still face the switching cost/sitting out probability. For notation, those who reallocated can be subdivided between able and not able ($u_{l \neq k}^{r} = u_{l \neq k}^{r,a} + u_{l \neq k}^{r,n}$) where a worker is considered able if they can engage in search.

Employed workers also have the choice to undertake OTJS subject to a fixed cost $\bar{\xi}$, in which case they still produce, but enter search for that period. A worker can only search within their sector if they choose this option.

### 1.4.3 Sectoral Labor Markets and Search Frictions

Sectoral labor markets are separate in the sense that a worker cannot simultaneously search in both sectors; they must choose to be in one sector versus another. Each sector then has its sectoral matching function and set of searchers. Active searchers in sector $k$ are subdivided into OTJS workers, $o_{k}$, those unemployed whose last job was in sector $k$, $u_{k}^{s}$, and those who reallocated and are able to search in sector $k$, $u_{k}^{r,a}$. Active searchers, $a_{k}$, are then the combined measure of these three worker types: $a_{k} = o_{k} + u_{k}^{s} + u_{k}^{r,a}$. Active searchers face search frictions in the form of a CRS.

\(^{28}\)If a worker reallocates from sector A to B, the CPS counts them as belonging to sector A until they get a job in B.
Figure 1.8: Timing for three types of workers at time $t$: employed, unemployed and within the sector of last employment, and unemployed and reallocating.

The probability of a match is:

$$m_k(a_k, v_k) = m_k = B_k a_k^{\alpha_k} v_k^{1-\alpha_k}$$ (1.16)

Where $v_k$ is vacancies in sector $k$, $B_k$ is the matching efficiency parameter, and $\alpha_k$ is the matching elasticity parameter. Define $\theta_k = \frac{a_k}{v_k}$ as sectoral tightness. Then the probability of a sector $k$ firm finding a match is given by $m_k = B_k \theta_k^{-\alpha_k} = q_k(\theta_k)$. On the other hand, probability of a worker finding a match in sector $k$ is $m_k = B_k \theta_k^{1-\alpha_k} = \theta_k q_k(\theta_k)$.

1.4.4 Timing

Prior to getting into value, it would be helpful to discuss the timing and thus the separation/reallocation process. The model is a weekly model, and we can imagine there are three key time periods: Monday, Wednesday, and Friday. On Monday, everyone in the economy learns their costs and productivities for the week. At the
same time, exogenous separations of existing matches occur, followed by endogenous separation. Therefore, new information is gained and separations occur on Monday. On Wednesday, individuals decide if they want to reallocate. If a person is employed, the reallocation decision is whether or not to look for a new job while an unemployed worker chooses to stay in their sector or reallocate. Lastly, Friday comes and still existing matches produce, and everyone able to search does so. Figure 1.8 provides three simplified timelines with each representing a different type of worker at time $t$. The top timeline is for employed workers, the middle is for unmatched workers who stayed in their last sector of employment ($u_k^*$), and the bottom is for workers currently in transition.

### 1.4.5 Unemployed Worker and Unmatched Firm Value

To ease notation, let $s = (s_1, s_2)$ be the vector of sectoral productivities in the current period. I will also present all value functions from the perspective of sector 1–sector 2’s value functions are symmetric. An unemployed worker receives unemployment benefits/value of leisure given by $b$. They inelastically search, but they make a binary decision to either stay in their current sector or reallocate. Therefore, an unemployed worker has value:

$$U_1(s, \xi) = \max\{U_1^S, U_2^R - \xi\}$$  \hspace{1cm} (1.17)

Let $E$ denote the expectation operator and $W_1(s, \epsilon, \xi)$ matched worker value in sector 1. Then $U_1^S$ is the value of staying in sector 1 and is given by:

$$U_1^S = b + \beta E[\theta_1 q_1(\theta_1) W_1(s', \epsilon', \xi') + (1 - \theta_1 q_1(\theta_1)) U_1(s', \xi')]$$  \hspace{1cm} (1.18)
$U_2^R$ is the value of reallocating to sector 2:

$$U_2^R = b + \beta \mathbb{E} [(1 - \gamma)U_{2,R}(s', \xi') + \gamma U_2(s', \xi')] \quad (1.19)$$

Individuals now face a new choice problem next period with probability $(1 - \gamma)$:

$$U_{2,R}(s, \xi) = \max \{U_2^R, U_1^R - \xi\} \quad (1.20)$$

Unmatched firm value is given by:

$$V_1(s) = -\left(\frac{1 + s_2 - s_1}{s_1}\right)c_1 + \beta \mathbb{E} \left[q_1(\theta_1)J_1(s', \epsilon', \xi') + (1 - q_1(\theta_1))V_1(s')\right] \quad (1.21)$$

$J_1(s', \epsilon', \xi')$ is matched firm value and $c_1$ is the cost of a vacancy. Note that costs are dependent on relative sectoral labor productivity. The main reason for this assumption is to amplify the volatility of vacancies/market tightness. This cost can also be interpreted as efficiency losses in hiring or increased competition in vacancies.\(^{29}\)

As is standard with these models, I assume the free entry condition applies, making $V_1(s) = 0$. Therefore, equation (1.21) is equivalent to:

$$q_1(\theta_1) = \left(\frac{1 + s_2 - s_1}{s_1}\right)\frac{c_1}{\beta \mathbb{E}(J_1(s', \epsilon', \xi'))} \quad (1.22)$$

### 1.4.6 Surplus and Wage Determination

Surplus of a match is:

$$S_1(s, \epsilon, \xi) = W_1(s, \epsilon, \xi) + J_1(s, \epsilon, \xi) - U_1(s, \xi) - V_1(s) \quad (1.23)$$

Wages are determined through an ad-hoc Nash bargaining rule.\(^{30}\) Letting $\eta_1$ denote bargaining power of a worker in sector 1, wages are set such that:

$$\eta_1S_1(s, \epsilon, \xi) = W_1(s, \epsilon, \xi) - U_1(s, \xi) \quad (1.24)$$

\(^{29}\)This assumption has a somewhat similar flavor to the introduction of a fixed/administrative cost that fluctuates with market tightness, as in Pissarides (2009).

\(^{30}\)Due to the introduction of OTJS, the payoff space is nonconvex, implying the standard Nash Bargaining solution doesn’t apply, see Shimer (2006).
\[(1 - \eta_1)S_1(s, \epsilon, \xi) = J_1(s, \epsilon, \xi) - V_1(s) \quad (1.25)\]

### 1.4.7 Matched Value

I present the value of a matched worker/firm in the form of total matched value. Let the total matched value in sector 1 be denoted as:

\[M_1(s, \epsilon, \xi) = W_1(s, \epsilon, \xi) + J_1(s, \epsilon, \xi) \quad (1.26)\]

This form can be further dissected into three components:

\[M_1(s, \epsilon, \xi) = \max\{M_{1,c}(s, \epsilon, \xi), M_{1,o}(s, \epsilon, \xi), U_1(s, \xi) + V_1(s)\} \quad (1.27)\]

Value of a match can then be subdivided into continuing in the match and not engaging in OTJS \( (M_{1,c}) \), engaging in OTJS \( (M_{1,o}) \), or dissolution. I will now elaborate on each.

Value of continuing in the match is given by:

\[M_{1,c}(s, \epsilon, \xi) = s_1^{\tau_1}s_2^{1-\tau_1}\epsilon + \beta\mathbb{E}\left[(1 - \chi_1)\left(\lambda \int_0^{\epsilon_0} M_1(s', \epsilon', \xi') dF(\epsilon') + (1 - \lambda)M_1(s', \epsilon, \xi')\right) + \chi_1(U_1(s', \xi') + V_1(s'))\right] \quad (1.28)\]

The above simply says that continuing matched value is output and the discounted expected value of next period. In the next period, the match could be exogenously dissolved with probability \( \chi_1 \). With probability \( (1 - \chi_1) \), the pair continues and draws a new \( \epsilon \) with probability \( \lambda \) or, with probability \( (1 - \lambda) \), the pair remains with their current \( \epsilon \).
The value of engaging in OTJS, making use of the wage rule in equation (10), is given by:

\[ M_{1,0}(s, \epsilon, \xi) = s_1^{\tau_1} s_2^{1-\tau_1} \epsilon - \bar{\xi} + \beta E\left[ \theta_1 q_1(\theta_1)(W_1(s', \epsilon', \xi') + V_1(s')) + (1 - \theta_1 q_1(\theta_1))(1 - \chi_1)(\lambda \int_0^{\epsilon_h} M_1(s', \epsilon', \xi')dF(\epsilon) + (1 - \lambda)M_1(s', \epsilon', \xi')) + \right. \]

\[ \chi_1(U_1(s', \xi') + V_1(s')) \]  

\[ (1.29) \]

Subject to paying search cost \( \bar{\xi} \), workers can be matched with probability \( \theta_1 q_1(\theta_1) \), in which case the worker will receive matched value and the firm will receive unmatched value. If an individual is not matched, then they face the same possibilities as in equation (1.28).

Given our definition of surplus in equation (1.23), it is straightforward to see the relationship between matched value and surplus:

\[ S_1(s, \epsilon, \xi) = M_1(s, \epsilon, \xi) - U_1(s, \xi) = M_1(s, \epsilon, \xi) - U_1(s, \xi) \]  

\[ (1.30) \]

The last equality results from the zero-profit condition. I will apply this identity directly to individual values in (1.27).

Before that, by substituting the wage rule into \( U_1^S \), I am able to obtain functions of only unmatched value, market tightness, and surplus of sector 1:

\[ U_1^S = b + \beta E\left[ \theta_1 q_1(\theta_1)\eta S_1(s', \epsilon', \xi') + U_1(s', \xi') \right] \]  

\[ (1.31) \]

Given \( U_2^R \) is only unmatched values, \( U_1(s, \xi) \) is then a function of surplus, market tightness, and unmatched values.

Then we have the following:

\[ M_1(s, \epsilon, \xi) - U_1(s, \xi) = \max\{M_{1,e}(s, \epsilon, \xi), M_{1,o}(s, \epsilon, \xi), U_1(s, \xi) + V_1(s)\} - U_1(s, \xi) \]  

\[ (1.32) \]
\[ = \max \{ \min \{ M_{1,c}(s, \epsilon, \xi) - U_1^S(s, \xi), M_{1,o}(s, \epsilon, \xi) - U_2^R(s, \xi) + \xi \}, 0 \} \] (1.33)

By expanding out each component and applying some simple algebra in the above equation, I can show that the following relationship exists between surplus, unemployed value, and market tightness:

\[ S_1(s, \epsilon, \xi) = \max \{ \min \{ f_1^c(S_1, \theta_1, U_1), f_2^c(S_1, \theta_1, U_1, U_2, U_{2,R}) \}, 0 \} \] (1.34)

I then achieve a contraction mapping in surplus that is a function of sector \( k \)'s expected market tightness, unemployed value of reallocation or staying, and surplus. Lastly, I use the wage rule on the zero-profit condition in equation (1.22):

\[ q_1(\theta_1) = \left( \frac{1 + s_2 - s_1}{s_1} \right) \frac{c_1}{\beta E[(1 - \eta_1)S_1(s', \epsilon', \xi')]}. \] (1.35)

Using the four equations for stay and reallocating unmatched values, surplus, and market tightness for each sector (eight in total) enable me to pin down values for surplus, market tightness, and unmatched values. I now discuss the solution methodology.

### 1.5 Solution, Evolution Equations, and Parameters

#### 1.5.1 Algorithm

The state space is discretized. Given the high persistence of productivities required in a weekly model, I discretize \( s_k \) into \( N_s \) points using the Rouwenhorst (1995) method. I assume that \( F(\epsilon) \) is truncated log-normal and discretize \( N_\epsilon \) points linearly in a
method comparable to Tauchen (1986).\(^{31}\) I do the same for \(G(\xi)\), which I had assumed was uniform with \(N_\xi\) linearly spaced points.

The algorithm is based on Fujita and Ramey (2012) wherein they solve the one sector problem using value function iteration. I make initial guesses for \(S_1, S_2, \theta_1, \theta_2, U_1, U_2, U_{1,R},\) and \(U_{2,R}\). Given these guesses, I substitute them into equation (1.34) to acquire new values \(TS_1\) and \(TS_2\), which I then plug into the zero-profit condition to receive \(T\theta_1\) and \(T\theta_2\). With four updated values in hand, I derive \(TU_1\) and \(TU_2\) using unmatched staying value with the adjustment from equation (1.31). I end by substituting \(TU_1\) and \(TU_2\) into reallocating unemployed value, giving me \(TU_{1,R}\) and \(TU_{2,R}\). I then check for pointwise convergence between my new values and previous values, update my previous values, and continue the process until the pointwise absolute distance is under some tolerance level. I set the tolerance level to \(10^{-6}\) in my solution.

### 1.5.2 Evolution and Aggregate Equations

Let me start by introducing some basic notation I will use throughout this section.

The total labor force of sector \(k\) is composed of all employed and unemployed in a sector: \(L_k = e_k + u^s_k + u^r_{i\neq k}\). Recall that those who reallocated and are not able to search in sector \(l\) will be labeled by the set \(u^r_{l\neq k}\) while those who are able and haven’t found a job are labeled as \(u^r_{l\neq k}\). Also, as mentioned previously, active searchers in sector \(k\) is given by \(a_k = o_k + u^s_k + u^r_{k}\). Lastly, let \(M_k = \theta_kq_k(\theta_k)\) be the probability that a worker finds a job in sector \(k\), \(\phi_i\) the probability of drawing

\(^{31}\)Suppose it is discretized into \(N\) points where \(\epsilon_1 < \epsilon_2 < \cdots < \epsilon_N\). Let \(TF(\epsilon)\) denote the truncated lognormal CDF. Then the first point, \(\epsilon_1\), is assigned probability \(TF(\frac{\epsilon_1 + \epsilon_2}{2})\). The last point, \(\epsilon_N\), is assigned probability \(1 - TF(\frac{\epsilon_N + \epsilon_{N-1}}{2})\) and intermediate points, \(\epsilon_i\), have probability \(TF(\frac{\epsilon_i + \epsilon_{i+1}}{2}) - TF(\frac{\epsilon_i + \epsilon_{i-1}}{2})\).
idiosyncratic productivity $\epsilon_i$, and $p(j)$ be the probability of acquiring switching cost $\xi_j$.

Matches in a given sector $k$ are characterized by the worker’s cost of switching $\xi$ and the match’s idiosyncratic productivity $\epsilon$. We can then disaggregate employment into two levels:

$$e = \sum_{k=1}^{2} \sum_{i=1}^{N_k} e(\epsilon_i)_k = \sum_{k=1}^{2} \sum_{i=1}^{N_k} \sum_{j=1}^{N_\xi} e(\epsilon_i, \xi_j)_k$$  \hspace{1cm} (1.36)

For the rest of the section, I will be deriving evolution equations from the perspective of sector 1. $e(\epsilon_i, \xi_j)_1$ evolves differently depending on the region of $\epsilon$: dissolution ($\epsilon \leq \epsilon^*$), on-the-job-search ($\epsilon \in (\epsilon^*, \epsilon^o]$), or no on-the-job search ($\epsilon > \epsilon^o$). Importantly, all of these $\epsilon$ cutoffs are functions of sectoral productivities, $(s_1, s_2)$, and drawn costs $\xi_j$.

A period ends and shocks are realized for next period (Monday). If $\epsilon'_i \leq \epsilon^*(s', \xi_j')$, all matches with $\epsilon'_i$ dissolve:

$$e(\epsilon_i, \xi_j)'_1 = 0$$  \hspace{1cm} (1.37)

Instead, if $\epsilon'_i > \epsilon^*(s', \xi_j')$ and $\epsilon_i \leq \epsilon^o(s, \xi_j)$:

$$e(\epsilon_i, \xi_j)'_1 = p(j)[\phi_i M_1 a_1 + (1 - \chi_1)[(1 - \lambda)(1 - M_1)e(\epsilon_i)_1 + \phi_i \lambda \left( \sum_{i=1}^{N_k} I(\epsilon \in [\epsilon^*, \epsilon^o]) (1 - M_1) e(\epsilon_i)_1 + \sum_{i=1}^{N_k} I(\epsilon > \epsilon^o) e(\epsilon_i)_1) \right]]$$  \hspace{1cm} (1.38)

$I(\epsilon)$ is an indicator function that is 1 when $\epsilon$ falls in the specified range, 0 otherwise. Because this was in the OTJS region in the previous period, it contains new matches that drew $\xi_j$ and $\epsilon_i$, and–conditional on not exogenously dissolving–those who failed to match/redraw a new $\epsilon$. Lastly, there are those existing matches that redraw $\epsilon_i$ as their idiosyncratic productivity (the second line of equation (1.38)). I will let $RD_{1,i}$
denote this second line which represents existing matches in sector 1 that redrew idiosyncratic productivity $\epsilon_i$.

Lastly, if $\epsilon_i' > \epsilon^*(s', \xi_j')$ and $\epsilon_i > \epsilon^o(s, \xi_j)$:

$$e(e_i, \xi_j)'_1 = p(j)(\phi_i M_1 a_1 + (1 - \chi_1)((1 - \lambda)e(e_i)_1 + RD_{1,i}))$$ (1.39)

Therefore, it is equivalent to equation (1.38) except the full measure of those that did not redraw their $\epsilon$ or exogenously dissolve remain in $e(e_i)_1$. Because the evolution of all possible $\epsilon_i$ and $\xi_j$ pairs have been specified, one can derive more aggregated measures of employment using (1.36).

The set of those engaging in OTJS is:

$$o_1 = \sum_{i=1}^{N_s} \sum_{j=1}^{N_m} I(e_i \in (\epsilon^*, \epsilon^o)]e(e_i, \xi_j)_1$$ (1.40)

If one recalls the timing, all existing matches are subject to a probability of exogenous dissolution prior to endogenous separation. Call this set $X_1$:

$$X_1' = \chi_1(e_1 - M_1 o_1)$$ (1.41)

Therefore, all those employed that did not rematch by the end of the period face exogenous separation next period.

Assume that $s_1' < s_2'$, therefore, if they choose to do so, workers will be reallocating towards sector 2 as expected wages are higher. Evolution of $u_2^r,n$ is a somewhat messy object, but can be subdivided into three groups. Those employed in sector 1 this period who decide to dissolve and reallocate($R_2^c$), workers who were in $u_1^c$ and undergo reallocation, $R_2^u$), and lastly individuals who were in $u_2^r,n$ and are still in transition $R_2^s$). Let $\xi^*(s_1, s_2)$ be the $\xi$ where the LHS and RHS of the binary max operator in unemployed staying value (equation (1.17)) are equal. Then any drawn
cost $\xi \leq \xi^*$ would result in reallocation conditional on being unemployed. Looking at the individual components of $u_{2,n}'$, $R_{2}'$ is:

$$R_{2}' = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} I_{\xi' \in [0,\epsilon^*] \cap \xi' \leq \xi^*}(e(\epsilon_i, \xi_j)_1)$$

(1.42)

$I_{\xi' \in [0,\epsilon^*] \cap \xi' \leq \xi^*}$ equals 1 if both conditions are satisfied, zero otherwise. $e(\epsilon_i, \xi_j)_1$ is determined by equation (1.38) if $\epsilon \leq e^*(s, \epsilon_j)$, (1.39) otherwise.

Because costs are realized prior to the endogenous exit decision, any worker that dissolves endogenously but whose costs do not satisfy the condition in equation (1.42) do not switch this period. Keeping this in mind, the evolution of $R_{2}'$ is given by:

$$R_{2}' = \sum_{j=1}^{N_x} I_{\xi' \leq \epsilon^*}p(j)(X_1' + (1 - M_1)u_1^*)$$

(1.43)

Then the composition of $u_1^*$ that switch is workers who exited via exogenous separation and those unmatched from last period. Lastly, the evolution of those who are still in transition is simply:

$$R_{2}' = (1 - \gamma)u_{2,n}'$$

(1.44)

Therefore:

$$u_{2,n}' = R_{2}' + R_{2}' + R_{2}'$$

(1.45)

$u_{2,a}'$ is made up of workers who did not find a match last period and those who escaped transition:

$$u_{2,a}' = \gamma u_{t\neq k} + (1 - M_{t\neq k})u_{t,\neq k}$$

(1.46)

The sum of the above equations gives $u_{2}'$.

Deriving the evolution equations for sector 2 is similar except we need to take into account that some workers in $u_1'$ will return to sector 2. Workers in $u_{2,a}'$ will have the

$^{32}$Put another way, workers who endogenously dissolve do not redraw costs when they first enter unemployment.
same cost cutoff decision as \( u_1^* \); others still in transition will have a different \( \xi \) cutoff determined by equation (1.20). Even though they formally belonged in sector 2, all workers transitioning back from \( u_1^* \) are still subject to \( \gamma \).\(^{33}\)

Going back to using the general index \( k \) for sectors, the equations for other components can now fall into place. The labor force for sector \( k \) is given by:

\[
L'_k = L_k + M_k u_k^{r,a} - M_{l \neq k} u_{l \neq k}^{r,a}
\]  

(1.47)

We can then arrive easily at sectoral unemployment and \( u_k^* \):

\[
u'_k = L'_k - e'_k \tag{1.48}
\]

\[
u_k^* = u'_k - u_l'^{r,a}_k \tag{1.49}
\]

Also, given the above equations, we can derive active searchers (\( a'_k = u'_k + \alpha'_k + u_{k}^{r,a} \)) and then vacancies using sectoral market tightness:

\[
u'_k = \theta'_k a'_k \tag{1.50}
\]

Using sectoral variables, aggregates are straightforward. Aggregate unemployment (\( u \)), vacancies (\( v \)), and employment (\( e \)) are equivalent to \( u_1 + u_2 \), \( v_1 + v_2 \), and \( e_1 + e_2 \), respectively.

To be consistent with my aggregate measure, I ignore job-to-job transitions in my measurement of job creation and destruction. Job creation next period is the sum of all matches from unemployment made on this period’s Friday:

\[
jc' = \sum_{k=1}^{2} M_k (u^*_k + u^{r,a}_k) \tag{1.51}
\]

\(^{33}\)The set of individuals that are ultimately affected by this assumption is quite small. A worker only chooses to engage in reallocation if sectoral productivities have non-trivial dispersion (i.e., for small differences in \( s_k \), even with zero \( \xi \) costs a worker does not reallocate due to \( \gamma \)). So, to affect a notable amount of movers, sectoral productivities would have to trade relative positions rather quickly, but productivities are very persistent given it is a weekly model.
Job destruction next period can be found using aggregate employment and job creation:

\[ j^d' = j^c' + e - e' \]  \hspace{1cm} (1.52)

Job creation and destruction rates are acquired by dividing (1.51) and (1.52) by \(0.5(e + e')\). The job-finding rate, \(f\), and separation rate, \(x\), are derived from job creation and destruction:

\[ f = \frac{j^c'}{u} \]  \hspace{1cm} (1.53)
\[ x = \frac{j^d'}{e} \]  \hspace{1cm} (1.54)

A sector’s average labor productivity is an endogenous object consisting of a weighted sum of each idiosyncratic realization multiplied by the sectoral labor productivity process:

\[ Z_k = \frac{s_k^{1 - r_k} \sum_{i=1}^{N_k} \epsilon_i e(\epsilon_i)_k}{e_k} \]  \hspace{1cm} (1.55)

With aggregate vacancies and unemployment, I acquire the standard measure of market tightness:

\[ \Theta = \frac{v}{u} \]  \hspace{1cm} (1.56)

Concluding, aggregate average labor productivity, \(Z\), is also an endogenous object and derived similarly to the sectoral measure of labor productivity:

\[ Z = \sum_{i=1}^{N} \left( s_1^{1 - r_1} \epsilon_i e(\epsilon_i)_1 + s_2^{1 - r_2} \epsilon_i e(\epsilon_i)_2 \right) \]  \hspace{1cm} (1.57)

1.5.3 Calibration

This section discusses calibration of the benchmark model. I choose to abstract from differences that may introduce sectoral trend growth and assume sectors are
symmetric in their parameters, so I drop the \( k \) subscript. The benchmark model is calibrated weekly to pre-1984 moments when possible.

The sectoral productivity’s persistence and standard deviation are set to \( \rho_s = .9895 \) and \( \sigma_s = .00508 \), respectively. \( \rho_s \) was taken from Hagedorn and Manovskii (2008) and is a standard choice for weekly models. \( \sigma_s \) was chosen to match the average aggregate labor productivity volatility of .016, measured from the quarterly BLS output-per-worker series over 1960Q1-1983Q4. I set \( N_s = 9 \) for my discretization.

The exogenous dissolution probability and idiosyncratic productivity’s standard deviation were simultaneously set to achieve the average 1960Q1-1983Q4 monthly separation rate of 3.1 percent and unemployment rate of 6 percent in the simulated series. These combined goals resulted in \( \chi = .00485 \), which implies a roughly 5.8 percent chance of exogenous dissolution in a given quarter. Recalling the idiosyncratic process is a discretized truncated lognormal, I set \( N_\epsilon = 249 \) and \( \epsilon^h = 1.5 \). \( \sigma_\epsilon \) was set to .75, and the mean of the distribution was assigned such that the expected \( \epsilon \) draw is approximately 1, leading to a choice of \( \mu_\epsilon = .8275 \). With these parameters, the average simulated value of the monthly unemployment and separation rate is 6.06 percent and 2.91 percent, respectively. The probability of redrawing a new \( \epsilon, \lambda \), is .085 as in Fujita and Ramey (2012) and implies the average duration of a match’s \( \epsilon \) is roughly a quarter.

Parameters common to DMP models are set to conservative values. For some context, the steady-state average wage is approximately 1. I chose vacancy costs and unemployment benefits to be values in between Shimer (2005) and Fujita and Ramey (2012). The cost of a vacancy, \( c \), is set to .2 or 20 percent of the average steady-state wage. Unemployment/leisure benefits, \( b \), is .6 which is 60 percent of the average wage.
Table 1.4: Calibrated Values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Target</th>
<th>Value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>4% annual interest rate</td>
<td>.9992</td>
<td>4%</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Sectoral Productivity Persistence</td>
<td>.9895</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Aggregate average labor productivity</td>
<td>.00508</td>
<td>.016</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Average separation rate of 3.1%</td>
<td>.75</td>
<td>2.91%</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>$\sum_{i=1}^{N_c} \epsilon_i g(\epsilon_i) = 1$</td>
<td>.8275</td>
<td>$\sum_{i=1}^{N_c} \epsilon_i g(\epsilon_i) = 1$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Average cross-sector correlation of .37</td>
<td>.85</td>
<td>.36</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Average unemployment rate of 6%</td>
<td>.00485</td>
<td>6.06%</td>
</tr>
<tr>
<td>$B$</td>
<td>Average JFR 45.5%</td>
<td>.098</td>
<td>45.5%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.4 higher u duration for reallocaters</td>
<td>2</td>
<td>1.22</td>
</tr>
<tr>
<td>$\xi^h$</td>
<td>Separation rate volatility of .088</td>
<td>80</td>
<td>.078</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$\epsilon$ persistence</td>
<td>0.085</td>
<td>Avg. $\epsilon$ duration of one quarter</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Bargaining power</td>
<td>0.6</td>
<td>Average steady-state wage 1.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Matching elasticity</td>
<td>0.6</td>
<td>-</td>
</tr>
<tr>
<td>$b$</td>
<td>Unemployment benefits</td>
<td>0.6</td>
<td>60% of steady-state wage</td>
</tr>
<tr>
<td>$c$</td>
<td>Vacancy costs</td>
<td>0.2</td>
<td>20% of steady-state wage</td>
</tr>
<tr>
<td>$\bar{\xi}$</td>
<td>3.2% monthly job-to-job transitions</td>
<td>0.166</td>
<td>3.2%</td>
</tr>
</tbody>
</table>

OTJS fixed cost generates this average monthly percentage in the simulation. The matching efficiency parameter, $B$, is given a value of .098 to generate the average monthly JFR from 1960Q1-1983Q4 of .455 in the simulation. $\tau$ is set to .85 to produce cross-sector average labor productivity correlation of .37 in the simulation, which matches the average value of my comovement measure over 1960Q1-1983Q4.\(^3\) The choice produces a correlation of .36. Lastly $\beta$ is set to .9992, or about an annual interest rate of 4 percent.

The last parameters to assign are $\xi^h$ and $\gamma$, the two reallocation frictions of the model. Shin and Shin (2008) used PSID data from 1986-1996 to explore the contribution of intra- versus inter-sectoral reallocation to unemployment. One conclusion was, on average, workers who reallocate across sectors stay in unemployment for 40 percent longer than those staying within their sector. I do not have any evidence that the relative time of unemployment for reallocaters grew or shrank between pre-1984 and the Great Moderation, so I use 40 percent as the target.\(^5\) Sectoral productivity can take one of nine values given my discretization, with $s(1) < \cdots < s(5) = 1 < \cdots < s(9)$. Searching within each sector, what is the average duration of unemployment when one sector faces $s(3)$ while the other has $s(7)$ (i.e., two productivity realizations that would result in a moderate amount of reallocation)? A recently unmatched worker waits on average 11 ($s(3)$) versus 8.4 ($s(7)$) weeks before becoming employed. $\gamma$ then adds additional time to 8.4 weeks. I select a $\gamma$ of .2, meaning workers typically stay five weeks in transition. Therefore, movers would face an average unemployment duration of 13.4 weeks (assuming they were just unmatched), leading to a 22 percent

\(^3\)The comovement measure reduces to a simple correlation with only 2 sectors.

\(^5\)The average unemployment spell did increase since 1984. However, this may be a phenomenon that equally affected both stayers and reallocaters, making relative durations invariant.
longer length of unemployment compared to stayers. Because this calibration method ignores other factors that may increase relative duration (e.g., a portion of those who reallocate may have already been unemployed), I remain with $\gamma = .2$ and note that targeting a higher duration would only strengthen my results. $\xi^h$ is chosen to try and generate the average 1960Q1-1983Q4 separation rate volatility of .088 in the simulation. Another target could have been the average of my cyclical sectoral reallocation measure over pre-1984, but the mapping between levels of the measure is not clear given my two-sector setting. My choice of 80 produces an average separation rate volatility of .078. Table 1.4 summarizes my calibration and parameterization choices.

1.6 Results

The results section begins with an examination of the quantitative performance of the benchmark model compared to a single sector version with the same ingredients. My principal exercise is to raise labor mobility frictions to target a 24 percent decrease in the model’s cyclical sectoral reallocation measure. By only altering this component, I ask to what extent the fall in the cyclicality of cross-sector reallocation can account for other changes in the labor market moving into the Great Moderation. Next, I discuss the mechanisms and intuition of the model, including an examination of how the cyclical Lilien (1982) measure responds to a variety of shocks. I end by arguing the economy is still facing low labor market volatility past the 2007 recession.

1.6.1 Quantitative Performance

Shimer (2005) found a standardly parameterized, canonical DMP model failed to match empirical volatilities in unemployment, vacancies, and market tightness through productivity shocks alone. In my framework, I still use a flexible Nash
bargaining solution for wage determination and a conservative parameterization. I find the model can perform notably better across several different margins than its single sector counterpart, perhaps providing a piece of the Shimer (2005) puzzle.

For comparison, I create a single sector version of my model by raising the lower bound of reallocation costs arbitrarily high and assuming one sector has the entire labor force. Therefore, the single sector model has the same ingredients and labor productivity specification (equation (1.14)) as my benchmark. I recalibrate the model to hit the same targets as in Table 1.4. Table A.3 of the appendix summarizes the calibration.

At the start of the simulation, I assume the labor force of each sector is equal at .5. I run 5000 simulations with each simulation having 4250 weeks. I remove the first 1000 weeks and average the remaining weeks into 250 quarters. Model results are averages across all 5000 simulations. Table 1.5 shows US pre-1984 data, benchmark results, and single sector results for several different moments. Red variables were targeted, while the other variables are overidentifying restrictions. The sectoral model performs substantially better on most all fronts except vacancies. Sectors’ vacancies tend to move in opposite directions, washing each other out when aggregated. Additionally, the Beveridge curve is more flat in the model than in the data as vacancies in the other sector rise in response to unemployed movers.36

The cyclical sectoral reallocation measure is higher than in the data, but what about actual reallocation? I can directly measure how many individuals on average

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36There is an easy and realistic fix for both vacancies and market tightness. Shin and Shin (2008) noted reallocaters on average receive a lower wage upon moving. Therefore, instead of facing stayer’s expected matched value, reallocaters only can produce a fraction of output initially, so vacancies would not respond strongly to movers. This assumption introduces two more labor markets into the model and would also strengthen endogenous cross-sector labor productivity correlation.
### Table 1.5: Performance of the Benchmark Model

<table>
<thead>
<tr>
<th>Data Source</th>
<th>$\sigma_u$</th>
<th>$\sigma_v$</th>
<th>$\sigma_Z$</th>
<th>$\sigma_{JFR}$</th>
<th>$\sigma_{SR}$</th>
<th>$\text{Corr}(Z_1, Z_2)$</th>
<th>Lilien</th>
<th>$\tilde{J}_R$</th>
<th>$\sigma_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-GM Data (1960Q1-1983Q4)</td>
<td>0.128</td>
<td>0.15</td>
<td>0.275</td>
<td>0.08</td>
<td>0.088</td>
<td>0.37</td>
<td>0.009</td>
<td>0.058</td>
<td>0.016</td>
</tr>
<tr>
<td>Benchmark Model (1960Q1-1983Q4)</td>
<td>0.101</td>
<td>0.076</td>
<td>0.128</td>
<td>0.046</td>
<td>0.078</td>
<td>0.36</td>
<td>0.019</td>
<td>0.048</td>
<td>0.016</td>
</tr>
<tr>
<td>One Sector Model</td>
<td>0.042</td>
<td>0.081</td>
<td>0.117</td>
<td>0.03</td>
<td>0.023</td>
<td>-</td>
<td>0</td>
<td>0.017</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Note: Based on 5000, 4000 period model simulations. Variables in red were targeted. Single sector model is the benchmark model with $\xi^l$ set restrictively high and recalibrated. $\sigma_\epsilon$ is also doubled to 1.5.
Table 1.6: A Look at the Sectoral Level

<table>
<thead>
<tr>
<th>Data Source</th>
<th>$\sigma_{\bar{z}}$</th>
<th>$\sigma_{\bar{u}}$</th>
<th>$\sigma_{z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-GM Data (1960Q1-1983Q4)</td>
<td>0.128</td>
<td>0.150</td>
<td>0.275</td>
</tr>
<tr>
<td>Benchmark Model (1960Q1-1983Q4)</td>
<td>0.144</td>
<td>0.186</td>
<td>0.313</td>
</tr>
<tr>
<td>One Sector Model</td>
<td>0.042</td>
<td>0.081</td>
<td>0.117</td>
</tr>
</tbody>
</table>

Note: Values in the benchmark are at the sectoral level, data is aggregate. Unemployment and vacancies are normalized by the labor force of the sector.

decide to reallocate monthly, and find it to be .2 percent of the labor force. As another metric, I can look at the portion of unemployment due to reallocating workers, finding it to be only 8.8 percent of total unemployment on average. With PSID data from 1974-1984, Loungani and Rogerson (1989) estimated the contribution of reallocaters to total unemployment weeks were 40 percent and 25 percent for recessions and expansions, respectively. Though the Lilien measure is high, directly measuring reallocation suggests the amount of cross-sector movement is within plausible upper bounds.

Vacancies do somewhat poorly when aggregated, but the sectoral level paints a very different picture. Table 1.6 shows model-generated volatilities for unemployment, vacancies, and tightness at the sectoral level. The ratio between vacancy and unemployment volatility is close to perfect, and ultimately shows there may even be a complete solution to the Shimer (2005) puzzle if the washing-out effect could be counteracted.\footnote{See footnote 37.}

\footnote{37See footnote 37.}
1.6.2 A Rise in Frictions

To generate a 24 percent fall in the cyclical sectoral reallocation measure, should $\gamma$ or $\xi^h$ rise? As mentioned in the calibration section, I do not have any evidence that relative durations between reallocaters and stayers have increased moving into the Great Moderation. Additionally, altering $\gamma$ does more than simply lowering the degree of reallocation; $\gamma$ introduces transition unemployment which creates meaningful implications for job-finding and separation rates. Though perhaps reducing $\gamma$ would help to explain the jobless recovery phenomenon and would be an interesting component to study later, I want to focus solely on inhibiting reallocation without introducing secondary effects. Therefore, I only alter $\xi^h$ for this exercise and keep $\gamma$ fixed.

I raise $\xi^h$ from 80 to 123 to hit my target. How does this affect workers’ choices? Suppose we have a roughly 10 percent dispersion shock where one sector’s $s_k$ raises by about 5 percent above steady state and other below by an equal amount. 38 In the benchmark, 4.3 percent of all employed and unemployed in the less productive sector would find reallocating more attractive ($U_{R_l\neq k} > U_{S_k}$). $\xi^h = 123$ lowers the fraction to 3 percent. Alternatively, recalling the average amount of monthly reallocaters in the benchmark was .2 percent of the labor force, raising frictions reduces the amount to .16 percent or a 20 percent decrease. Diminishing sectoral reallocation by 20 percent is then sufficient to account for a 24 percent fall in the measure.

Table 1.7 presents the results of the exercise. The first section provides values from pre-1984 data and the benchmark. The second section is data from the Great Moderation and the model with higher reallocation frictions. For each variable, I

38The shock is simply choosing $s(3)$ and $s(7)$ from my discretized sectoral productivities.
break down the percentage change seen in the data and the model moving from pre-1984 to the Great Moderation. Comparing the two, I can account for the percentage change seen in the data that is attributable to increased frictions to labor mobility.

Increasing reallocation frictions leads to a fall in all my variables of interest: cross-sector average labor productivity correlation, labor market volatilities, and cyclical job reallocation. Most notably, the alteration can account for over 40% of the data’s change in unemployment and separation rate volatility. The effect on vacancies and consequentially market tightness is smaller, though still significant. On the other hand, the rise in frictions generates too large of a fall in job-finding rate volatility. Comparing the job-finding rate to other variables in the data, for some reason its volatility did not decrease by nearly the same magnitude moving past 1984. It could be possible something else occurred post-1984 to counteract the fall in volatility induced by smaller bursts of reallocation.

Lower sectoral reallocation also leads to diminished correlation in cross-sector average labor productivity as well as 33 percent of the empirical drop in cyclical job reallocation. I cannot say to what extent sectoral comovement fell in the Great Moderation due to data continuity issues, but the result is impressive given I still have $\tau$ in place and unchanged.

I view the decrease in the cyclical job reallocation measure as theoretical support that higher fluctuations in job reallocation rates are associated with reallocative shocks, as was empirically suggested by Davis and Haltiwanger (1999).
Table 1.7: Effect of Reallocation Frictions

| Data Source                  | $\sigma_u$ | $\sigma_v$ | $\sigma_{\tilde{z}}$ | $\sigma_{JFR}$ | $\sigma_{SR}$ | $\text{Corr}(Z_1, Z_2)$ | Lilien | $|JR|$ |
|------------------------------|------------|------------|----------------------|----------------|----------------|-------------------------|--------|------|
| Pre-GM Data (1960Q1-1983Q4)  | 0.128      | 0.15       | 0.275                | 0.08           | 0.088         | 0.37                    | 0.009  | 0.058|
| Benchmark Model (1960Q1-1983Q4) | 0.101      | 0.076      | 0.128                | 0.046          | 0.078         | 0.36                    | 0.0192 | 0.048|
| GM Data (1984Q1-2006Q4)     | 0.091      | 0.111      | 0.197                | 0.071          | 0.066         | -                       | 0.0058 | 0.036|
| Less Reallocation Model     | 0.088      | 0.071      | 0.119                | 0.04           | 0.067         | 0.32                    | 0.0145 | 0.042|

| %Δ to GM Data               | -28.9%     | -26%       | -44.4%               | -11.3%         | -23.3%        | -                       | -35.6% | -37.9%|
| %Δ to GM Model              | -12.9%     | -6.6%      | -7%                  | -13%           | -14.1%        | -11.1%                  | -24.3% | -12.5%|

| %Δ Explained by Reallocation| 44.6%      | 25.3%      | 15.8%                | 115%           | 60.5%         | -                       | -      | 33%  |

Note: Less reallocation model has the same parameterization as the benchmark except $\xi^h = 123$ instead of $\xi^h = 80$. Target for this choice is in red.
1.6.3 Inspecting the Mechanism

A High Outside Option

Sectoral reallocation can produce large fluctuations primarily by incorporating an outside option that helps to reinforce downward matched value movements.

Recall the breakdown of surplus for sector 1:

\[ S_1(s, \epsilon, \xi) = M_1(s, \epsilon, \xi) - U_1(s, \xi) \]  (1.58)

In the single sector DMP model, a negative productivity shock is accompanied by a fall in matched value, lowering expected surplus. However, at the same time, unmatched value also decreases, counteracting the drop in matched value that ultimately leads to smaller fluctuations in surplus. Looking at my setting, a negative shock to a sector produces the same fall in matched value but the outside option lowers by less for some workers. Unmatched value is the maximum between reallocating or staying, and some workers begin to consider reallocation. Expected surplus then becomes more downward elastic at the sectoral level, with an impressive fall as sectoral productivities become even more dispersed.

Figure 1.9 graphs unmatched value from the benchmark. The figure assumes one sector remains in steady state while varying cost, \( \xi \), and the other sector’s productivity. As can be seen, the region of costs where individuals price reallocation grows as the other sector becomes relatively more productive, translating to large downward movement in expected surplus.

Prior to continuing with the discussion of mechanisms, I should clarify what I mean by single sector henceforth. It is simply my benchmark where the lower bound
Figure 1.9: Keeping $s_t \neq k$ at steady state, the figure shows unmatched value for sector $k$ as one varies $s_k$ and reallocation costs $\xi$. $\xi$ and $s_k$ are truncated.
of reallocation costs is restrictively high—not the recalibrated single sector model discussed in the quantitative performance section. I do this to emphasize differences within the same parameterization.

That said, figure 1.10 plots expected surplus across productivities for sector $k$ in the single sector and benchmark model when $s_{l\neq k} = s(5)$ (steady state) and $s_{l\neq k} = s(9)$ (the maximum value of the discretized series). The single sector falls nearly linearly as sector 1’s productivity decreases, but the benchmark’s expected surplus reflects the potentially large effect of a weakly falling outside option.

A negative shock of sufficient size to one sector then begins a domino effect that starts in the less productive sector: the job-finding rate plummets, job destruction/separation rates rise, and more workers decide to engage in the time-consuming
process of sectoral reallocation, leading to increased aggregate unemployment.\textsuperscript{39} Productivities positively relate through $\tau$, so the other sector will also experience a fall in expected surplus and job-finding rate. However, this sector’s job creation will see a notable rise as reallocating workers enter and search.

What happens to vacancies? Vacancies are increasing in both expected surplus and active searchers through the zero-profit condition in equation (1.35). Active searchers and expected surplus take a big hit in the less productive sector, and the more productive sector will also see a fall in expected surplus. However, active searchers in the higher productivity sector will rise from workers reallocating, offsetting the fall in expected surplus and increasing sectoral vacancies. Aggregate vacancies will then see an initial fall but an eventual rise as workers reallocate.

The entire process can be shown conveniently by applying a sectoral dispersion shock to the model and examining the responses. Figure 1.11 gives what I view as a series of events induced by the dispersion shock. I normalize sectoral-level variables by their respective labor force, $L_k$. Starting at the top left we see roughly a 10 percent dispersion shock was applied, with one sector’s productivity rising to about 1.05 and the other falling to .95. After half a year, dispersion drops before completely disappearing a year since the initial shock. Moving right, we see the impact on expected surplus in each sector, with a disproportionate fall in the less productive sector. The next image shows sectoral job-finding rates. When the shock initially hits, the low productivity sector sees a fairly large instantaneous drop which continues to fall as the sector’s unemployment rises (the job-finding rate is a sector’s job creation divided by its unemployment).

\textsuperscript{39}For a small enough negative shock, workers will not consider reallocation even if their drawn random fixed cost is zero. The sitting-out cost, $\gamma$, is still too high.
Figure 1.11: A dispersion shock to initially equal-sized sectors. A + marker was hit by a negative shock and the unmarked by a positive shock when not an aggregate measure. Y-axis is the percentage change from steady-state values.
Looking at the next row, a larger portion of \((\epsilon, \xi)\) pairs now has surplus below zero in the less productive sector, leading to a considerably higher destruction rate. Workers who separate face two options—staying or reallocating—that both lead to increased sectoral and aggregate unemployment. Staying builds up sectoral unemployment from the lower probability of a match; reallocating leads to more workers entering transition unemployment as they wait to search elsewhere. Combined, these two sets of unemployed lead to a nearly 60 percent rise in aggregate unemployment above steady-state. Most of the increase in aggregate unemployment is being generated by the rising pool of reallocating workers which grows until the shock begins to diminish. Because \(\gamma\) is fixed, the amount of workers being able to search in the positively shocked sector is also increasing for half a year, and produces a rise in the job creation rate of this sector.

Figure 12’s last row shows the gradual rise in the aggregate job creation rate, which simply reflects what was occurring at the sectoral level. The aggregate job reallocation rate experiences a large initial spike from job destruction and gradually rises as job destruction remains persistently high, and job creation is steadily increasing. Lastly, sectoral reallocation induces a strong change in the idiosyncratic distribution of the less productive sector: endogenous separation removes the lowest \(\epsilon\) matches leading to a rise in average idiosyncratic productivity. It is through this process that reallocation alone can produce positive comovement in average labor productivity.

Raising the average cost to reallocate leads to a smaller portion of workers experiencing below zero surplus and a lesser fall in expected surplus. Consequentially, the entire process in Figure 1.11 becomes diminished.
The Importance of Endogenous Separation

Unlike alternative search and matching models that conclude sectoral reallocation contributes little to aggregate unemployment fluctuations, the previous exercise showed reallocative pressures can generate substantial movement. A common factor between most other models is an exogenous separation assumption. Endogenizing dissolution turns out to be a lynchpin for the bulk of my results.

Endogenous separation allows employed workers to act on the desire to reallocate, producing sudden movement in job destruction (and consequentially job creation as they trickle into the other sector) and a persistent increase in separation rates. Conversely, separation rates in the exogenous version remain fixed at $\chi$ when facing an exogenous shock and unsurprisingly lead to tepid volatility in flow rates.

Given separation rates are constant, the exogenous model depends on a fall in the job-finding rate to raise unemployment. Workers circumvent this mechanism by simply looking elsewhere. Therefore, to the extent aggregate unemployment may rise in the exogenous version depends on the reallocation period’s length, with short periods being insufficient to get any real effect (because of flexible separation rates, the endogenous version still sees a large rise if $\gamma = 1$).\(^\text{40}\)

Aside from certain volatilities, endogenous separation is also responsible for producing positive correlation between sectoral labor productivity. When a match can decide if it wants to dissolve, it will only do so if idiosyncratic productivity is poor.

\(^{40}\)I do solve an exogenous version of the model, but exclude the results for brevity. They are available if requested.
Cutting the fat, so to speak, works to offset exogenous changes in productivity. Conversely, a fixed probability of separation makes no discretion for which match separates, so the idiosyncratic mean would remain unchanged and mechanical assumptions would be the only way to generate changes in average labor productivity comovement.

1.6.4 Reallocative or Aggregate Shocks?

Pre-1984 recessions generated a particular pattern in the cyclical sectoral reallocation measure: a double spike, with the second spike occurring at the end of the recession and associated with a rise in cyclical job reallocation rates. This feature disappeared during the Great Moderation but occurred once again in the 2007 recession.

In this section, I want to examine if my model can reproduce these characteristics in $\tilde{\sigma}_L$ and to what extent they may be unique to reallocative shocks.

I focus on three types of shocks to my benchmark. My first shock is a dispersion of sectoral productivities, which induces sectoral reallocation in my setting. So, responses in $\sigma^L$ represent what the model suggests we could expect if sectoral reallocation was high. The second shock is simply an aggregate shock where both sectors face the same changes in sectoral productivities. Lastly, I apply a shock that I call the Abraham and Katz (1986) (A&K) shock. For this exercise, I assume there is a volatile and non-volatile sector, with the volatile sector experiencing a more rapid fall and recovery to a negative aggregate shock. There is no sectoral reallocation. Though I do not model aggregate shocks, I can simply assume sectoral productivities behave differently and in the same direction. I then raise the lower bound of costs in the benchmark to prevent reallocation.
Because I want to make a comparison to the movement in pre-1984 $\sigma^L$, I make a couple of assumptions to better train the exercise. Perhaps the size of a sector’s labor force prior to the shock matter, so I calibrate the starting labor force for the dispersion and A&K shock differently. I first find the average employment share over pre-1984 for the five sectors with the highest employment volatility. With an employment weighted volatility of 2.65 times that of the other sectors, the five are construction, durables, non-durables, information, and utilities+transportation. Combined, they comprised .438 of the average employment share over pre-1984, which I set as the initial labor force for the volatile sector (with the non-volatile labor force equaling 1-.438). I do the same for my dispersion shock except I look for the five sectors with the largest net cyclical employment outflows over pre-1984, leading to construction, durables, wholesale trade, utilities+transportation, and mining and logging. Averaging the employment share in these five sectors over pre-1984 suggests a labor force share of .363. A dispersion shock generates an outflow from the less productive to more productive sector. Thus, I assign $L_k = .363$ to the negatively shocked sector.

Figure 1.12 shows my three shocks and responses of interest. A dispersion shock creates an initial rise in cyclical output, but it soon falls as unemployment rises. We can see the shock generates a double spike. It rises initially as job destruction induces a large drop in employment for one sector, leaving the other relatively unscathed. Eventually, the less productive sector’s employment stabilizes, but the other sector experiences rapid employment growth as reallocating workers can search. This generates the second spike. Both spikes also have comparable timing as what was in

41To generate these figures, I assume the model is in steady state for 74 weeks. On the 75th week and year-long shock process occurs and returns to steady state on the 127th week. The model then runs until 228 total weeks have passed. Series are time aggregated, logged, and detrended as in my data section with the first and last six observations removed.
Figure 1.12: Three shocks applied to the benchmark model. Dashed lines were calibrated with starting labor forces of $L=.363$ for the dispersion shock and $L=.438$ for A&K shock. x-markers are job creation, + job destruction, and unmarked the cyclical sectoral reallocation measure (adjusted for scale).
the data, with the first spike occurring at the start of the process and the last when output sees a recovery. Lastly, a rise in job creation is more closely correlated with the second spike than job destruction.

Aggregate shocks that affect sectors equally produce no change in $\sigma^L$. On the other hand, an A&K shock does generate a rise in the measure but contains only one spike that peaks when output is at its lowest. Unlike the reallocative shock, as sectors recover they do not experience a notable discrepancy in employment growth, preventing a second rise in the measure.

Reality is more complex and likely a combination of all three shocks, yet–according to the above exercise–the story of only aggregate shocks seems inconsistent with a large post-recessionary rise in $\sigma^L$.

1.6.5 After the Great Recession

If diminished cyclical sectoral reallocation played a notable role in the Great Moderation, as my analysis suggests, then we can form some expectations of labor market dynamics past the 2007 recession. There are a few pieces of circumstantial evidence that point to a continuing Great Moderation, at least in terms of labor markets.

Recalling my sectoral reallocation measure, shortly after the Great Recession it achieved a series low, even in my robustness exercise; though there may have been an incentive to reallocate, it simply was not occurring.

In corroboration of this claim, the current expansion is also characterized by jobless recovery. I view jobless recovery as a phenomenon produced by the inability to move to sectors where demand for one’s set of skills is high. In support of this view, Jaimovich and Siu (2012) argue permanent job destruction in middle-skilled, routine
occupations is responsible for jobless recovery. Therefore, though routine jobs may be highly substitutable across sectors, there were no jobs being created. The only option would be retraining or a large wage decrease, both of which entail either a high explicit or perceived opportunity cost.

We can also just simply look at the data itself to derive some conclusions. Table 1 subdivided volatilities between pre-Great Moderation, Great Moderation, and post-Great Moderation periods. Looking at this table alone, it seems volatilities after the Great Moderation are at or beyond pre-1984 levels. However, a large portion of the observations for this period is the 2007 recession which was a particularly large event. Alternatively, what if we exclude recessions and compare only expansions across different periods?

I further subdivide each of the three periods—Great Moderation (GM), pre-GM, and post-GM—into NBER dated recessions and expansions. For a given variable, I measure the volatility for each expansionary subperiod and create a measure of expansionary volatility. The measure is the sum of expansionary subperiod volatilities where I weight a particular subperiod’s volatility by its duration divided by the total expansionary quarters of that period.\(^42\)

Table 1.8 presents expansionary volatility for key labor market variables and conveys two important points. First, the relatively low volatility of the Great Moderation was not simply due to less frequent recessions; expansionary periods also experienced an average decrease in volatility. Second, looking at data past the Great Recession,

\(^{42}\)For example, there were three expansionary subperiods during the Great Moderation. The subperiod between 1991Q2-2000Q4 lasted for 39 quarters, and there was a total of 85 expansionary quarters during the Great Moderation. Then for expansionary unemployment volatility, the contribution of this subperiod would be \(\frac{39}{85} \sigma_{u, 1991Q2-2000Q4}\) where \(\sigma_{u, 1991Q2-2000Q4}\) is derived from 1991Q2-2000Q4 unemployment rate data.
Table 1.8: Expansionary Volatility

<table>
<thead>
<tr>
<th>Period</th>
<th>$\sigma_u$</th>
<th>$\sigma_v$</th>
<th>$\sigma_{\hat{z}}$</th>
<th>$\sigma_{JFR}$</th>
<th>$\sigma_{SR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-GM Data (1961Q2-1983Q4)</td>
<td>0.11</td>
<td>0.127</td>
<td>0.234</td>
<td>.07</td>
<td>0.077</td>
</tr>
<tr>
<td>GM Data (1984Q1-2006Q4)</td>
<td>0.084</td>
<td>0.094</td>
<td>0.174</td>
<td>.066</td>
<td>0.062</td>
</tr>
<tr>
<td>Post-GM Data (2009Q3-2012Q4)</td>
<td>0.1</td>
<td>0.098</td>
<td>0.188</td>
<td>.092</td>
<td>0.079</td>
</tr>
</tbody>
</table>

Note: All data is monthly data averaged quarterly, logged, and HP-filtered with smoothing parameter $\lambda = 1600$. Expansionary volatility measures a variable’s volatility during expansions (quarters in between NBER dated recessions) of a particular period and is the weighted sum of each expansionary subperiod. Weights are the duration of the expansionary subperiod, divided by the total duration of all subperiods during a period. 1960Q1 was an expansionary period but was excluded due to 1960Q2 being the start of an NBER dated recession. Vacancy and market tightness volatilities are only up to 2011Q4.

which still includes residual effects, expansionary volatility of unemployment, vacancies, and market tightness are more characteristic of the Great Moderation. On the other hand, flow rate volatilities are still abnormally high. If we look back to Table 1.1, the variability of the JFR was exceptionally affected in the Great Recession, being the only series whose value far outmatched pre-1984 levels. Because of this disproportionate impact, it is likely that the residual effect of the Great Recession is especially high for the JFR. The same logic could similarly be applied to the separation rate, which was also partially derived using the JFR.

In conjunction, all the above evidence suggests low labor market fluctuations will continue moving forward.43

43There have been several more thorough explorations into whether or not the Great Moderation will continue moving past the Great Recession. In general, the consensus seems to be that the Great Moderation, at least in terms of low volatility, has not ended. See Clark (2009), Gadea et al. (2013), or Furman (2014).
1.7 Conclusion

My model establishes cross-sector movement is a potentially important source of labor market fluctuations. I show empirical evidence that sectoral reallocation has become less of a cyclical occurrence after 1984. Timed with this event is a fall in variation of other key labor market variables. By raising average moving costs in my model to capture the prior fact, I can account for a substantial portion of the Great Moderation’s drop in cyclical labor market variation.

When the ability to reallocate is high, more workers consider the alternative sector as a viable option if they lose their jobs. Through endogenous separation and a larger inclusion of this outside option in surplus, reallocative shocks can become a sizable force in job destruction, creation, and unemployment movement.

As a final thought, I target a decrease in cyclical sectoral reallocation through a rise in average moving costs, but remain non-committal as to what this reduced-form cost is ultimately meant to represent. In my opinion, this is a valuable topic to pursue and warrants an even more micro-founded approach. However, priority should be given to developing a cyclical sectoral reallocation measure based on micro data as I view it as a necessity to support any conjecture. I leave this for future work.
Chapter 2: The Role of Capital in Noise Shocks

2.1 Introduction

Agents in an economy use information to form beliefs that are used in decisions. Their decisions, in turn, in some way affect aggregates in the economy. How are different types of information interpreted by agents, and what sort of aggregate responses do these interpretations lead to? These questions have been at the heart of a large portion of recent macroeconomic literature, and an important subset of this literature is focused on the response of agents to noise, or expectational, shocks. Noise shocks can be seen as shocks only to an agent’s expectations; they are not driven by the fundamentals of the economy but are extrinsic to the system. The importance of noise shocks in aggregate fluctuations is debatable, and one must ask if results are overly dependent on model restrictions. A particularly large restriction is the absence of capital in the model. Opposed to labor, capital choice today cannot be used in today’s production. Therefore, agents cannot simply expand hours to meet demand and face decreasing returns in labor due to a fixed usable capital stock. This paper seeks to explore the differences capital leads to when under an environment subject to noise shocks.
I use a New Keynesian model based upon the example model in Lorenzoni [61], but incorporate endogenous capital. This model is a no-money economy with a representative household, sticky prices via Calvo [20] pricing, and a simple interest rate rule. Current productivity is made up of a temporary and permanent component and is observed by all agents. However, the permanent component follows a unit root process and is not observable. The household receives an imperfect signal and uses this signal to create expectations about permanent productivity. When I shock this signal, without changing the underlying permanent productivity, I produce a noise shock as it only affects expectations. I first look at the response agents have to a permanent productivity shock and then how they respond to a noise shock. I then compare these responses to the equivalent model where production relies only on labor and apply some sensitivity analysis to my benchmark.

In my benchmark parameterization, a positive permanent shock leads to the same directional movement in aggregates between the capital and no-capital model, but the rate of convergence is considerably more sluggish in the model with capital. Output, consumption, and capital all increase but respond less than the true underlying productivity would suggest in the perfect information equivalent. Capital accumulation responds slowly to the shock, but this is largely due to the gradual nature of investment rather than imperfect information. In fact, removing the noise in agent’s expectations makes a practically insignificant difference in the rate of accumulation. Slow capital accumulation in turn leads to slow convergence to the new balanced growth path which takes nearly three times as long as its no capital equivalent. The prolonged capital accumulation gives some prediction as to what we may expect to happen when a noise shock is applied to the economy. If informational frictions lead
to no significant change in the accumulation of capital under a permanent shock, then what could one expect when we only shock the noise of this friction?

A noise shock leads to positive responses in all the aggregates in both capital and no-capital models. Consumption, inflation, output, investment, etc. all increase on impact and slowly return to previous levels. Positive output responses from a noise shock occur through two vehicles: the household’s Euler equations and sticky prices. Nominal rigidities cause the real interest rate to react more slowly than it would in the case of flexible prices. Through the bond Euler, current consumption—and thus output—depends positively on future expected output and responds negatively to the real interest rate. As expectations increase about the underlying permanent productivity (a good indicator of future productivity), expected output becomes higher. With nominal rigidities, the real rate responds slowly, otherwise it would respond in such a way to completely mitigate any changes in expectations. Between the real rate’s slow response and the increased expectations of future output and consumption, current output increases leading to many other aggregates following suit.\textsuperscript{44} Output is predominantly driven by an increased consumption response; investment responds weakly when not reinforced with observed productivity increasing. As capital makes up a large proportion of production ($\alpha = .34$ in a Cobb-Douglas production function), and this factor is essentially fixed due to its minimal response, labor faces diminishing returns to scale. Diminishing returns causes agents to supply less labor than they would have if they faced constant returns as in previous models.\textsuperscript{45} My benchmark

\textsuperscript{44}Greater detail on the phenomenon, especially the role of nominal rigidities, can be found in Lorenzoni [61] and Barsky [13].

\textsuperscript{45}Khan and Thomas [58] briefly discussed this property under a model with inventories.
model’s lackluster capital response to a noise shock then leads to a weak output response.

On impact, a noise shock in the model without capital causes a response in output that is more than double my benchmark. On the other hand, because capital stock has increased and it is costly to adjust (the household faces convex adjustment costs), the persistence of a noise shock is more significant in the model with capital. The level it persists at, however, is very small. Taking these two facts into account, and doing some sensitivity analysis, I conclude that it is likely that models without capital overestimate the scale of responses to a noise shock. In alternative models with only labor, my results should carry over, and we would expect the model’s economy to have less volatility resulting from noise shocks if they were to include capital.

My model is based off of Lorenzoni [61] wherein he uses the labor-only version as an example to build intuition of his full-fledged model of dispersed information. His model of dispersed information also contains labor-only production, but with significant information frictions he finds that noise can potentially account for a sizable amount of volatility. He uses his simple model as an explanation of what mechanism is at play in his more opaque model. By this logic, the results found in my model should carry over if I impose a similar framework. A version of his model was also applied more recently in Blanchard et al.[17], and they find that noise shocks play a significant role in short-run fluctuations using a structural parametrization of their model. Angeletos and La’O [6],[7] have also explored the role and implications of noise shocks using a model comparable to Lorenzoni’s example model.
News shocks, as in Beaudry and Portier [14], [15], are related to noise shocks but should be differentiated. In particular, the idea of news shocks involves actual information about future productivity. On the other hand, noise shocks are completely extraneous to the economy.

A more recent paper by Barsky and Sims [13] also included endogenous capital along with habit. However, this paper was largely concerned on the impact of news vs. noise on confidence innovations and how these innovations lead to aggregate fluctuations. They find that noise does not seem to play a significant role in aggregate fluctuations. The only way that it can is through essentially raising nominal rigidities to unrealistic amounts. My paper, instead, looks specifically at the role of capital in models of noise shocks. It does not contain complicating factors such as habit or complex interest rate rules because I wanted it to be a clean exercise. In doing so, we can explicitly see what capital may do in comparable models (including Barsky and Sims [13]), and get a better understanding of the role of capital.

The concept of noise shocks owes its origins to initial thinkers such as Keynes who dubbed them “animal spirits”. It was more formally developed later in the works of Phelps et al. [76] and Lucas [63] in the form of expectational errors. The existence of “animal spirits” also gained further exposure in the 1990-1991 recession where discussions such as Blanchard [16] believed some sentiment outside of the system caused agents to respond in a way that curbed consumption.

Other related papers include alternative theories of information restrictions which include: sticky information introduced by Mankiw and Reis [66] and rational inattention as in Sims [87] and more recently developed in Maćkowiak and Wiederholt.
Work has also been done on the welfare consequences of imperfect information in the works of Morris and Shin [68] and Angeletos and Pavan [8].

Section 2 of my paper explains the model and discusses the learning process. Section 3 details calibration, methodology, and my solution. I then analyze a positive permanent productivity shock and a positive noise shock to my model and discuss the intuition for the observed responses. In Section 4, I compare my benchmark model with the model without capital and further elaborate on the role of capital. Section 5 has a very brief discussion of possible extensions, and Section 6 concludes the paper.

2.2 The Model

I use a New Keynesian model with a representative household and Calvo [20] pricing. It is largely based off of the simple model presented in Lorenzoni [61] with the principle differences being varying risk aversion, capital, and convex adjustment costs. The model’s technology process requires me to detrend the model. As such, it is more intuitive to first introduce the technology and shock process.

2.2.1 Technology

The representative household will consume a composite good, $C_t$, that is composed of a unit mass of intermediate goods, each indexed by $j \in [0,1]$. $C_t$ is created by a competitive final good firm that chooses inputs to minimize costs and meet demand of the household. Intermediate good firm $j$ produces according to:

$$Y_{j,t} = A_t^{1-\alpha}K_{j,t}^\alpha N_{j,t}^{1-\alpha}$$  \hspace{1cm} (2.1)

$A_t$ is a labor-augmenting technology process and is defined as:

$$A_t = X_t exp(\eta_t)$$  \hspace{1cm} (2.2)
Current technology is composed of a temporary component, \( \eta_t \), distributed iid \( N(0, \sigma_\eta^2) \) and a permanent component, \( X_t \), defined as:

\[
X_t = X_{t-1} e^{\epsilon_t}
\]  

(2.3)

The permanent component is a unit root process with \( \epsilon_t \) being distributed iid \( N(0, \sigma_\epsilon^2) \).

The household and firms can observe \( A_t \) but not \( X_t \). Therefore to induce stationarity, we must detrend by the observable variable \( A_t \). The process by which households form expectations about the permanent component will be discussed later in the paper. I define \( \tilde{A}_t \) as:

\[
\tilde{A}_t = \frac{A_t}{A_{t-1}} = \frac{X_t}{A_{t-1}} e^{\eta_t} = \tilde{X}_t e^{\eta_t}
\]  

(2.4)

where \( \tilde{X}_t = \frac{X_t}{A_{t-1}} e^{\epsilon_t} = e^{\epsilon_t - \eta_{t-1}} \)  

(2.5)

Let any variable with a tilde above it represent the detrended version of the original variable (e.g. \( \tilde{Y}_t = \frac{Y_t}{A_{t-1}} \)). For the purpose of exposition, however, I will present the model in its non-detrended form.

### 2.2.2 Representative Household

There is an infinitely-lived, representative household that maximizes their sum of expected discounted utility.

\[
E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_{t+1}^{1-\sigma}}{1-\sigma} - A_t^{1-\sigma} \frac{\omega}{1+\varsigma} N_t^{1+\varsigma} \right]
\]  

(2.6)

Where \( \beta \) is the discount factor, \( \sigma \) is the relative risk aversion, \( \omega \) the level of disutility of labor, \( \varsigma \) the inverse of the Frisch elasticity of labor, and \( N_t \) are hours worked. Maximizing the above is subject to a budget constraint and no-ponzi condition.

\[
P_tC_t + Q_tB_{t+1} + P_tI_t + P_t \frac{\phi}{2} \left( \frac{I_t}{K_t} \right)^2 K_t = B_t + W_t N_t + R^K_t K_t + \int_0^1 \Pi_{j,t} dj
\]  

(2.7)

75
\[
\lim_{T \to \infty} \beta^T B_T = 0 \quad (2.8)
\]

Where \( \Pi_{j,t} = (P_{j,t} Y_{j,t} - R^k_{t} K_{j,t} - W_t N_{j,t}) \)

\( Q_t \) is the nominal price of one-period bonds \( B \) at time \( t \), \( W_t \) nominal wage at time \( t \), \( P_t \) aggregate price index at time \( t \), \( P_{j,t} \) price set by firm \( j \) at time \( t \), and \( R^k_t \) is the nominal rental rate of capital at time \( t \). Capital faces depreciation at rate \( \delta \), and any changes in the capital stock faces a convex adjustment cost with level of costs determined by \( \phi \).\textsuperscript{46} The price index is defined as:

\[
P_t = \left( \int_0^1 P_{j,t}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}} \quad (2.9)
\]

Where \( \gamma \) is the elasticity of substitution between intermediates. Capital accumulation evolves according to:

\[
I_t = K_{t+1} - (1 - \delta)K_t \quad (2.10)
\]

### 2.2.3 Firm’s Problem

Through the final good firm’s optimality condition, we get the following demand for intermediates:

\[
Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\gamma} Y_t \quad (2.11)
\]

Where \( Y_t \) is output of the final good firm.

Intermediate good firm \( j \) faces Calvo-pricing and can adjust its price in a given period with probability \( 1 - \theta \), otherwise it maintains its previous period’s price. If intermediate firm \( j \) is able to change its price, it does so to maximize:

\[
E_t \sum_{\tau=0}^{\infty} \theta^\tau D_{t+\tau|t} \left[ P_{j,t+\tau} Y_{j,t+\tau} - W_{t+\tau} N_{j,t+\tau} - R^k_{t+\tau} K_{j,t+\tau} \right] \quad (2.12)
\]

\textsuperscript{46}Convex adjustment costs were added as a necessity to guarantee determinacy in a New Keynesian model with endogenous capital. More details can be found in Dupor [36] and Section 4.
s.t. \( P_{j,t+\tau} = P^*_t \), which is the optimal price at time t, (2.1), and (2.11)

\( D_{t+\tau|t} \) is the stochastic discount factor and is defined as \( D_{t+\tau|t} = \beta^\tau \left( \frac{C_{t+\tau}}{C_t} \right)^{-\nu} \). This, along with wage, rental rate of capital, and \( P_t \), are taken as given by the firm. A given intermediate good firm also minimizes real costs in every period:

\[
\min_{N_{j,t}, K_{j,t}} \left\{ \frac{W_t}{P_t} N_{j,t} + \frac{R^k_t}{P_t} K_{j,t} \right\}
\]

subject to equations (2.1) and (2.11).

### 2.2.4 Monetary Authority

Following Lorenzoni [61], the interest rule is relatively simple. This was chosen to make the exercise as clean-cut as possible without other factors heavily influencing responses to noise shocks.\(^{47}\) Let \( R_t = \frac{1}{\nu} \) denote the nominal interest rate. Then monetary policy follows a simple Taylor rule:

\[
R_t = \frac{1}{\beta} \left( \frac{P_t}{P_{t-1}} \right)^\nu
\]

\( \nu \) is chosen by the monetary authority. In general, I assume that \( \nu > 1 \), implying the monetary authority has an active interest rate rule.

### 2.2.5 Aggregation

It can be shown that aggregate production follows the following process:

\[
Y_t = A^{1-\alpha}_t N^{1-\alpha}_t K^\alpha_t
\]

Also, final goods market clearing must satisfy:

\[
Y_t = I_t + C_t
\]

\(^{47}\)Noise shocks, as will be seen, depend on the real rate being below the level of expectations. Having a Taylor rule that’s responsive to output dampens the overall impact of noise shocks.
2.2.6 Linearization

Given our household problem and firm’s problem, I solve for the optimality conditions. Detrending the resulting optimality conditions if necessary, I then solve for the balanced growth path (BGP) and log-linearize the set of all equations that fully specify equilibrium dynamics in my economy.\(^{48}\)

Let an uncapitalized letter with a tilde represent the percent deviation from the BGP of a detrended variable (e.g. \(\tilde{c}_t = \ln\left(\frac{C_t}{A_{t-1}}\right) - \ln(\tilde{C})\)). Likewise, let an uncapitalized letter without a tilde represents the percent deviation from the BGP of an already stationary variable.

Combining the household labor-leisure condition and consumption decision yields:

\[
\tilde{c}_t + \varsigma n_t = \hat{w}_t
\]  

(2.17)

Where \(\hat{w}_t = \tilde{w}_t - p_t\) is the detrended real wage. Define \(\pi_t = p_t - p_{t-1}\) as inflation.

Taking the consumption optimality condition and combining it with optimal bond holdings creates the bond Euler equation:

\[
\tilde{c}_t = E_t\{\tilde{c}_{t+1}\} - r_t + E_t\{\pi_{t+1}\} + \tilde{a}_t
\]  

(2.18)

Combining consumption optimality and optimal capital holdings results in the capital Euler equation:

\[
\tilde{c}_t = E_t\{\tilde{c}_{t+1}\} - (1 - \beta(1 - \delta))E_t\{\tilde{r}_{t+1}^k\} - \phi \tilde{k}_t + (1 + \phi)\phi \tilde{k}_{t+1} - \beta \phi E_t\{\tilde{k}_{t+2}\} + (1 + \phi)\tilde{a}_t - \beta \phi E_t\{\tilde{a}_{t+1}\}
\]  

(2.19)

Let \(\tilde{r}_t^k = r_t^k - p_t\) and note that, unlike wages, this variable is stationary and does not require detrending.

\(^{48}\)Solving the BGP was trivial, so it will not be presented in the paper. Calculations are available upon request.
We can see in (2.18) and (2.19) how current consumption (and consequentially output) depends positively on expectations of future consumption, but negatively on the real interest rate, \( r_t - E_t\{\pi_{t+1}\} \).\(^{49}\) As will be discussed shortly, it is through these two equations that expectational shocks are able to affect output, consumption, and investment.

A standard result under Calvo-pricing is that, taking the firm’s optimal pricing condition, the aggregate evolution of prices, and properly detrending, one can acquire the following forward-looking New Keynesian Phillips curve:

\[
\pi_t = \beta E_t\{\pi_{t+1}\} + \Omega(\alpha \hat{r}_t^k + (1 - \alpha) \hat{w}_t - (1 - \alpha) \hat{a}_t) \quad (2.20)
\]

where \( \Omega \equiv \frac{(1 - \theta)(1 - \beta)}{\theta} \) \((2.21)\)

Log-linearizing capital accumulation:

\[
\delta \hat{k}_t = \hat{k}_{t+1} - (1 - \delta) \hat{k}_t + \hat{a}_t \quad (2.22)
\]

Log-linearizing monetary policy:

\[
r_t = \nu \pi_t \quad (2.23)
\]

Therefore, given \( \nu > 1 \), the real interest rate responds greater than one in response to inflation.

Using the final goods condition (2.16) and log-linearizing yields the standard result:

\[
\bar{y}_t = \frac{\tilde{C}}{Y} \bar{c}_t + \frac{\tilde{I}}{Y} \bar{I}_t \quad (2.24)
\]

Solving the aggregate firm’s cost minimization problem and log-linearizing yields aggregate input demand:

\[
n_t + \hat{w}_t = \hat{r}_t^k + \hat{k}_t \quad (2.25)
\]

\(^{49}\)Current consumption also depends negatively on expected future returns to capital.
Lastly, I log-linearize the aggregate production function:

\[
\tilde{y}_t = (1 - \alpha)\tilde{a}_t + \alpha \tilde{k}_t + (1 - \alpha)n_t
\] (2.26)

Equations (2.17)-(2.26) fully specify the equilibrium dynamics of my economy. I must now take into account that current output and pricing decisions depend on expectations of future productivity. A good indicator of future productivity would be the underlying permanent component of technology, \(X_t\), but this is unobservable to the household. The household therefore faces a signal extraction problem and must infer the underlying permanent productivity process, and they do so with the addition of a signal and using a Kalman filtering process.

### 2.3 Learning and Signals

Log-linearizing our detrended productivity process (2.4) and (2.5), and noting that \(\ln(\tilde{X}) = \ln(\tilde{A}) = 0\) in the BGP, gives us the following set of equations

\[
\tilde{a}_t = \tilde{x}_t + \eta_t
\] (2.27)

\[
\tilde{x}_t = \epsilon_t - \eta_{t-1}
\] (2.28)

The temporary shock, \(\eta_t\), makes it so that the household cannot directly determine the underlying permanent productivity process; what they observe may not be due to a change in permanent productivity but merely a one-period shock. To help facilitate noise shocks, the parameterized variance of \(\eta_t\) will be large relative to other variances to complicate the signal extraction problem.

The household observes \(\tilde{a}_t\) but not \(\tilde{x}_t\). With only \(\tilde{a}_t\), the household is unable to learn effectively. Therefore, they also receive a signal about the permanent component. A real-world example would be an announced estimate of last period GDP.
However, as is established in Faust et al. [39], these estimates can be subject to significant errors. These errors give rise to noise/expectational shocks. Explicitly, the household receives:

$$\tilde{s}_t = \tilde{x}_t + e_t$$

(2.29)

Where $e_t$ is distributed iid $N(0, \sigma_e^2)$. A shock to the $e_t$ term embodies an expectational shock: a shock affecting only beliefs, not the fundamentals. The household then forms beliefs about the underlying permanent productivity using Kalman filtering. Let $\tilde{x}_{t|t} = E_t\{\ln(X_t)\} - \ln(A_{t-1})$ define the expectations at time $t$ of the detrended productivity process. It can be shown that these expectations evolve according to:

$$\tilde{x}_{t|t} = \rho(\tilde{x}_{t-1|t-1} - \tilde{a}_{t-1}) + (1 - \rho)(\chi\tilde{s}_t + (1 - \chi)\tilde{a}_t)$$

(2.30)

where $\rho = \frac{1}{\sigma_x^2 + \sigma_e^2 + \sigma_\eta^2}$ and $\chi = \frac{1}{\sigma_e^2 + \sigma_\eta^2}$

One peculiarity to mention is that the response of expectations to noise shocks varies nonmonotonically with the variance of $e_t$. This is because at low variances the signal is too precise and therefore there is little effect. At higher variances it is too imprecise, which causes the weight on the signal $((1 - \rho)\chi)$ to be too low for the household to respond.

50 Faust et al. [39] found that quarterly growth rate revisions in G-7 nations were commonly more than one percentage point, annualized. Furthermore, they find that this forecasting error is generally unpredictable.

51 Using standard Kalman filtering techniques and a little bit of algebra achieves this result.

52 More detail can be found in Lorenzoni [61].
2.4 Results

Given an explicit evolution of expectations (2.30), my system of dynamic equations (2.17)-(2.26), and calibration/parameterization, I proceed by solving for explicit dynamics using the method of undetermined coefficients. Acquiring values for impulse responses of the detrended/log-linearized aggregates, I convert them to levels and then percentage deviations from prior to shock levels for both permanent technology and noise shocks.\textsuperscript{53}

2.4.1 Calibration/Parameterization

The model is a quarterly model. $\beta = .99$ is set to imply an annual real interest rate of .04. I then choose $\delta$ so that annual $\frac{I}{K}$ is equal to .088 in the BGP. $\alpha = .34$ implies that the percentage of labor income is about 66%. The elasticity of substitution, $\gamma$, is 9.5 and was chosen such that $\frac{K}{Y} = 2.37$ annually along the BGP. This value also implies that BGP firm markup above MC is about 12%. I set $\omega = 4$, resulting in BGP labor supply of .33.

The $\phi$ parameter was set to 6. At low levels of $\phi$, investment responds \textit{negatively} to a noise shock. This is in fact true for a value of 3. According to the literature, if anything, a noise shock should result in a positive investment response because, as we’ll see, it acts as a classical demand shock: increasing prices, output, labor, consumption, etc. I then decided on a value of 6 which indeed results in a positive investment response, though I do undertake some sensitivity analysis of $\phi$ later in the

\textsuperscript{53}I decided to not look at temporary shocks, $\eta_t$, as they’re largely extraneous for the purpose of the paper. If one wishes to view the impulse responses, they’re available upon request.
paper. The remaining parameters are set using values used by Lorenzoni. [61]. The Frisch elasticity of labor parameter, $\varsigma$, was set to .5. The parameter on the monetary authority’s response to inflation, $\nu$, was set to 1.5. Next, $\theta$–the Calvo-pricing parameter–is set to $\frac{2}{3}$, such that the average price duration is 9 months. Variances of the shocks were chosen such that the signal extraction problem is relatively difficult for individuals. A summary of them and other parameters can be seen in Table (2.1).

Notice that the standard deviation (SD) of the temporary shock, $\sigma_\eta$, is nearly 20 times larger than the SD of the permanent technology shock and five times larger than the SD of the noise shock. As mentioned previously, this complicates the signal extraction problem to help give rise to meaningful responses from noise shocks.

Next, given the variances for all the shocks, one can solve for $\sigma_x$ as the resulting positive solution to the following Ricatti equation:

$$(\sigma_e^2 + \sigma_\eta^2)(\sigma_x^2)^2 - (\sigma_e^2 \sigma_\eta^2 + \sigma_e^2 \sigma_x^2) \sigma_x^2 - (\sigma_e^2 \sigma_e^2 \sigma_x^2) = 0$$

Lorenzoni [61] chose these parameter values to give noise shocks a strong possibility to have significant effects and persistence. By the same token, I wish to give noise shocks a ripe environment to flourish.
The solution results in the value of .0152 for $\sigma_x$. This parameter, in conjunction with the parameters in Table (2.1), itemize all provided variables of the model. We can now move forward to solving the model.

2.4.2 Method of Undetermined Coefficients

A large benefit of using the method of undetermined coefficients is that one can see how changes in the state variables translate to changes in endogenous variables. Given that this paper is more of an expositional exploration of the effect of capital on noise shocks, this solution method provides a nice avenue for intuition.

2.4.3 Methodology

In this model, the state variables at time $t$ are $\tilde{x}_{t|t}$, $\tilde{a}_t$, and $\tilde{k}_t$. Given these variables, I then make a conjectures as to the evolution of detrended output and capital, as well as inflation:

\begin{align*}
\pi_t &= \psi_{\pi k} \tilde{k}_t + \psi_{\pi a} \tilde{a}_t + \psi_{\pi x} \tilde{x}_{t|t} \\
\tilde{y}_t &= \psi_{y k} \tilde{k}_t + \psi_{y a} \tilde{a}_t + \psi_{y x} \tilde{x}_{t|t} \\
\tilde{k}_{t+1} &= \psi_{k k} \tilde{k}_t + \psi_{k a} \tilde{a}_t + \psi_{k x} \tilde{x}_{t|t}
\end{align*}
\tag{2.31}
\tag{2.32}
\tag{2.33}

Denote $\psi$ as the $[9 \times 1]$ vector of the above $\psi$s, of which the ordering does not matter for my purposes.

Using equations (2.17)-(2.20) and (2.22)-(2.26), I reduce the system of nine equations to three equations. Doing so results in a system of state, current choice, and future expectation variables. Given this set of equations, I then plug in conjectures (31)-(33) to try and create a system of only the state variables to solve for $\psi$. As can be seen from the Euler equations, this implies that I have to plug in one-period ahead...
conjectures. That is, my final equations contain $E_t\{\tilde{y}_{t+1}\}$, $E_t\{\tilde{k}_{t+2}\}$, and $E_t\{\pi_{t+1}\}$, which I then replace as follows:

$$E_t\{\pi_{t+1}\} = \psi_{\pi k} \tilde{k}_{t+1} + \psi_{\pi a} E_t\{\tilde{a}_{t+1}\} + \psi_{\pi x} \tilde{x}_{t+1} | t$$  \hspace{1cm} (2.34)

$$E_t\{\tilde{y}_{t+1}\} = \psi_{y k} \tilde{k}_{t+1} + \psi_{ya} E_t\{\tilde{a}_{t+1}\} + \psi_{yx} \tilde{x}_{t+1} | t$$  \hspace{1cm} (2.35)

$$E_t\{\tilde{k}_{t+2}\} = \psi_{kk} \tilde{k}_{t+1} + \psi_{ka} E_t\{\tilde{a}_{t+1}\} + \psi_{kx} \tilde{x}_{t+1} | t$$  \hspace{1cm} (2.36)

At this stage, I still run into an issue. My system now contains period-ahead forecasts of state variables (and still an endogenous variable remains). Focusing first on the period-ahead forecasts, because of the specification of technology, I am able to bring back expectations of these future variables to today. It can be shown:

$$E_t\{\tilde{a}_{t+1}\} = \tilde{x}_{t+1} | t = \tilde{x}_{t} | t - \tilde{a}_{t}$$  \hspace{1cm} (2.37)

Therefore, we can replace any step-ahead expectations of future productivity and permanent productivity by equation (2.37), effectively reducing them to today’s state variables.

The next issue is that I still have have an endogenous variable ($\tilde{k}_{t+1}$). I deal with this by simply once again substituting it with conjecture (2.33). A reader may notice that doing so will result in a $\psi^2_{kk}$ in my system. This means that I will have two possible solutions.

Making the above adjustments to my original system of three equations, I get a system composed of only constants–determined and undetermined–and current state variables. For an arbitrary individual equation in my original system of three, I can

55For example, $E_t\{a_{t+1}\} = E_t\{x_{t+1} - x_t + \eta_{t+1} - \eta_t\} = -E_t\{\eta_t\}$ by the fact that $x_t$ follows a unit root process. Then $-E_t\{\eta_t\} = x_{t|t} - \bar{a}_t = \tilde{x}_{t|t} - \bar{a}_t$. The proof for $\tilde{x}_{t+1|t}$ is similar.
rearrange terms such that the following holds true:

\[ A' \psi \tilde{x}_{t|t} + B' \psi \tilde{y}_t + C' \psi \tilde{k}_t = 0 \]  

(2.38)

Where A, B, and C are \([9 \times 1]\) vectors composed of parameter values (and potentially \(\psi_{kk}\)). Given my model’s assumptions, for (2.38) to be true it must be that:

\[ A' \psi = B' \psi = C' \psi = 0 \]  

(2.39)

Doing this for all three of my equations in the system yields nine equations and nine unknowns whose solution is the vector \(\psi\) such that the equivalent of (2.39) is satisfied for all nine equations.

### 2.4.4 Solution

To tackle the issue of a nonlinear set of equations, I simply solve the model numerically.\(^{56}\) As for the issue of duplicity, only one vector \(\psi\) resulted in reasonable responses.\(^{57}\) Acquiring \(\psi\), I can then use (2.31)-(2.33) to see how changes in the state variables translate to endogenous choice. Doing so results in:

\[ \pi_t = -.028 \tilde{k}_t - .126 \tilde{a}_t + .153 \tilde{x}_{t|t} \]  

(2.40)

\[ \tilde{y}_t = .25 \tilde{k}_t + .445 \tilde{a}_t + .306 \tilde{x}_{t|t} \]  

(2.41)

\[ \tilde{k}_{t+1} = .967 \tilde{k}_t - .97 \tilde{a}_t + .003 \tilde{x}_{t|t} \]  

(2.42)

How can we interpret these numbers? At a glance, they may seem a bit counterintuitive, but then we must remind ourselves that these are detrended variables. For instance, when looking at \(\tilde{k}_{t+1}\) in (2.42), note that this is \(ln \left( \frac{K_{t+1}}{A_t} \right) - ln(K)\). Therefore,

\(^{56}\)A great resource for the undetermined coefficient solution methodology can be found in Christiano [25].

\(^{57}\)The other led to divergent behavior with no return to a BGP.
the fact that it responds negatively to \( \tilde{a}_t \) is clearly the result of detrending. If we were to expand the variables and do some cancelations, we would get the following more intuitive form:

\[
\ln(K_{t+1}) - \ln(\tilde{K}) = 0.03\ln(A_t) + 0.967(\ln(K_t) - \ln(\tilde{K})) + 0.003\ln(X_{t|t}) \tag{2.43}
\]

These equations represent what would happen in my model even under perfect information. With perfect information, I would have \( X_{t|t} = X_t \). Notice the very weak response to expectations, which gives us some intuition as to what we can expect when this economy faces a noise shock that only affects expectations. Also, if we were to have a true permanent technology shock that increases \( a_t \) and \( x_t \) in my benchmark, we would indeed expect a positive response to capital, but a response that is less than what the true level of productivity would dictate as expectations lag behind the true permanent productivity.

Fortunately, (2.40) and (2.41) maintain their general intuition due to the fact that they’re detrended by \( A_{t-1} \) and therefore any change of \( A_t \) is only reflected in the right hand side of the equation. We can see output, (2.41), responds strongly to expectations about the permanent productivity and to current productivity. The reason for the strong response to current productivity is clear. Why then does output respond positively to expectations? The answer is through the household’s bond and capital Euler equations, (2.18) and (2.19). If one substitutes consumption in these equations with (2.24), then one sees that current output depends positively on expectations of future output. The best judge for future output is one’s expectations of the underlying permanent productivity process and it’s through this that positive expectations lead to positive output responses.
Taking the relatively large output response to expectations in conjunction with a weaker capital response—as seen in (2.43)—we can then expect any output response to noise shocks must be largely consumption driven as investment responds only weakly.

From (2.40), inflation can be seen as responding to a sort of output gap between the underlying permanent productivity and the natural output directed by $A_t$. If expectations are high compared to current productivity, the household responds positively in current output through their Euler equations; more so than what the current level of productivity would entail. This in turn causes firms to face higher real marginal costs and leads them to increase prices. The opposite would hold true if expectations lagged behind current productivity.

Using (2.40)-(2.42) with (2.17)-(2.26), I can now fully characterize the equilibrium dynamics of my complete economy when exposed to a shock at time $t$. However, to ease intuition in future sections, I convert these detrended/log-linearized values into levels and then to percentage deviation from previous level values. I convert detrended variables to levels with the following equation, letting $F_t$ be the level of some arbitrary variable at time $t$ that’s detrended:

$$F_t = (\tilde{f}_t F + F)A_{t-1}$$

(2.44)

Where $F$ is some predetermined level.

### 2.4.5 Response to Permanent Technology Shocks

To have an explicit value for the pre-shock level of variables, I must make an assumption on $A_{t-1}$ and $X_{t-1}$. Without loss of generality, I assume the value of these variables is 1 and that the economy is on the BGP at time $t - 1$ and has been on the BGP since at least $t - 3$. For my future exposition I will assume $t = 3$. In other
words, in all my analysis henceforth, I assume the shock occurs in period 3, and the economy was on the BGP in periods $t = 0, 1, 2$.

I apply a positive shock of $\epsilon_t = .01$ at $t = 3$, a 1% shock to permanent TFP. This results in $\bar{x}_3 = \bar{s}_3 = \bar{a}_3 = .01$. However, expectations do not fully respond to this increase. Using equation (30), we can find the evolution of $\bar{x}_{3|3}$ given the above and noticing $\bar{x}_4 = \bar{s}_4 = \bar{a}_4 = 0$:

$$\bar{x}_{3|3} = (1 - \rho)(.01)$$

$$\bar{x}_{3+t|3+t} = -(\rho)^{t+1}(.01) \text{ where } t > 0$$

Using the value of the productivity shock and (2.45)-(2.46), I am able to derive impulse responses using equations (2.40)-(2.42) and (2.17)-(2.26). I then convert the responses into levels using (44) and compute percentage deviation from pre-shock levels. The resulting impulse responses for a permanent technology shock can be seen in Figure 2.1.

First, one can immediately notice that output only gradually reaches the level that would be implied by the actual productivity level. The slow output response is largely the consequence of typically gradual capital accumulation and not the signal extraction problem itself. I will elaborate and support this point further in the next section. We can see that most other aggregates have relatively strong initial responses, but capital responds sluggishly and continues to do so for the remaining periods, translating to the observed output response.

As the household initially increases capital far below its natural levels, initial responses in output are not factor driven but technology driven. The household initially decreases the amount of labor they supply. At the same time, wages and the
Figure 2.1: Impulse responses to a .01 permanent technology shock. The x-axes are quarter periods and the y-axes are percentage deviation from previous levels (e.g. .1=.1% difference).
rental rate of capital respond less than what productivity would suggest, implying—through the New Keynesian Philips curve—that firms first respond with a decrease in their prices. Firms respond as such due to marginal costs lagging behind the additional output a firm has from improved technology. Eventually, however, capital accumulation begins to lag behind the level of output desired and wages have increased such that agents decide to contribute more labor. This increase in labor and sluggish capital accumulation leads to a rapidly increasing marginal product of capital and consequentially high rental rate of capital, $R^k$. The abnormally high rental rate coupled with increasing wages, eventually leads price-changing firms to increase their prices, resulting in positive inflation. As the capital shortage is alleviated and labor supply decreases, the rental rate of capital begins to deflate, and inflation returns to its steady-state level.

The effects of a positive shocks are impressively persistent. Figure 2.2 shows that it takes roughly 120 periods until output converges to its new BGP. Again, this is due to the nature of capital adjustment in the face of convex adjustment costs.

### 2.4.6 Response to Noise Shocks

As with a permanent productivity shock, a noise shock of size $e_3 = .01$ will hit the economy in period 3, implying a 1% change in signal. This shock only changes $\tilde{s}_3$ to a value of .01: $\tilde{x}_t$ and $\tilde{a}_t$ will remain 0, $\forall t$ and $\tilde{s}_{3+t} = 0, \forall t > 0$. Using our evolution of expectations (2.30), we can then map what expectations will be at any future period under a noise shock:

$$\tilde{x}_{3+t|3+t} = \rho^{t-1}(1 - \rho)\chi(.01), \forall t \geq 0 \quad (2.47)$$
Figure 2.2: The same impulse responses as Figure (1), but the x-axis has been expanded to 150 periods.
Using the above information, I once again use the evolution of beliefs and the dynamical system to derive a series of impulse response functions in the same manner as the permanent technology shock. Figure 2.3 shows the resulting impulse responses.

Everything in the economy responds positively to a positive noise shock. Why? $A_t$ has not changed, but expectations of future productivity have increased. Increased expectations of future output, coupled with nominal rigidity leading to real interest
rates not adjusting quickly, lead to an increase in current output through the household’s Euler equations. Again, productivity has not changed, so to fuel this output increase, more labor and more capital is introduced. With increased output comes increased labor demand, increasing wages through the labor-leisure condition. The overwhelming majority of the input response is labor; capital in comparison responds only lightly. Because of this, the marginal product of capital increases, leading to an increase in the rental rate of capital. Firms able to change their price, facing higher real marginal costs (as wages and $R^k$ have increased but $A_t$ has not), adjust prices upwards leading to higher inflation. Once the household begins to learn that perhaps this was not a technology shock, they slowly decumulate capital.

Recall that in Section 2.4.4 I conjectured that the output response from a noise shock would be largely consumption driven. Figure 3 substantiates this claim as consumption responds with a roughly .07% increase while investment responds with only a .025% increase. One must also take into account that consumption makes a considerably larger component of output.\(^{58}\) Percent-for-percent, output responds more strongly to consumption than investment. Coupling this effect and that the actual percentage change for consumption is larger, we can safely conclude that the output response is predominantly consumption driven. Conversely, when a permanent shock occurs and expectations are joined with a current productivity increase, initial responses in output is from investment. Figure 2.1 shows us that aggregates initially have investment responding by 1.35% and consumption by .3%. Agents do not significantly change investment when they do not face observed productivity increasing, though they are still willing to increase consumption if only expectations

\(^{58}\)The ratio of investment to output in the BGP is .209.
change. This also helps to ultimately explain why noise shocks tend to be relatively transient.

The signal problem may also affect the observed persistence. As mentioned in Lorenzoni [61], the signal extraction problem presented in this economy is relatively simple. Lorenzoni [61] is only able to generate meaningful persistence using a model that has significantly larger information frictions than this model can afford. One thing of importance is that the general mechanism of his simple model is translated to his full-fledged model of dispersed information, which also only uses labor as a factor input. I can similarly expect the same translation would occur if I was to apply his same treatment. Then, as we will soon see, the incorporation of capital may dampen the responses that Lorenzoni [61] was achieving even in his model of dispersed information.

A Discussion of Scale

Before examining what may happen in my model when we vary the significance of capital, we should first ask ourselves if my model’s response to noise shocks is notable in itself. A comparison of the response of a noise shock (*)& versus a permanent productivity shock (+) in Figure 2.4 reveals that a noise shock of the same scale leads to a meek response in comparison to a true productivity shock. Upon impact of a noise shock, output responds by roughly .06%. The equivalent statistic for a permanent technology shock is roughly .5%—a response larger than eight times that of output to a noise shock. A noise shock’s persistence is also small compared to a permanent technology shock, as discussed earlier.

Is this lack of a significant response facilitated or hurt by removing capital? The answer is yes and no. The inclusion of capital unequivocally decreases the initial
Figure 2.4: Comparing a noise and permanent productivity shock.
response of the economy to noise shocks compared to the model without capital. However, because the household had misallocated an input that is not easily adjustable, the persistence of a noise shock is improved with capital and becomes even more persistent with an increase in adjustment costs.

2.5 Role of Capital

To what degree were the observed impulse responses due to imperfect information? Or, was it largely a consequence induced by capital itself? It turns out that the latter conjecture is the correct one. Perhaps the most telling evidence is a comparison of a positive permanent productivity shock in my benchmark (with noise) and that of its perfect information equivalent. Figure 2.5 compares the difference in capital accumulation between the two models, with the solid line representing perfect information and the other my benchmark. It is apparent that the differences are almost non-existent. An explanation can be seen in equation (2.43). The coefficient on expectations is .003. Comparing this with the coefficient on current–and observed–productivity (.03), we see that expectations play a very insignificant role in capital accumulation. Therefore, it is the typical prolonged accumulation of capital itself that ultimately resulted in the persistence observed in Figure 2.1, not the information separation problem.

The lack of response to expectations in turn leads to negligible responses to a noise shock. When a noise shock occurs, the largest increase in capital occurs at its onset–at this point, expectations of \( X_t \) is highest and then degrades as time progresses. Capital’s initial response would have been relatively small even if the household also faced an observed productivity increase, but even this is not the case. With only
Figure 2.5: A permanent productivity shock’s effects on capital in my benchmark and under perfect information. The y-axis is percentage deviation from each respective model’s BGP.
Changing Importance of Capital in Production

Figure 2.6: Output responses to a noise shock in my benchmark versus lesser importance of capital. The y-axis is percentage deviation from each respective model’s BGP.

a change in expectations, capital gives an underwhelming response for reasons explained in the previous paragraph. With an essentially fixed factor, any significant changes in output must then be met with varying labor. However, if capital plays significant role (i.e. if $\alpha$ is high) then labor will face significant diminishing returns to scale. Because of this, the household responds by increasing labor but doing so in a restrained amount. If capital’s role was lessened, then the degree to which we have diminishing returns in labor would be alleviated. If this logic is correct, then even with capital fixed this period, we would expect higher labor responses, leading to higher output. Evidence for this effect can be found in Figure 2.6, wherein I apply a 1% expectational shock and set $\alpha = .1$. In period 3, capital available for production is fixed at last period’s pre-shock selected levels. Therefore, any changes in output
are a direct result of changes in labor. Figure 2.6 shows that when $\alpha = .1$ (the solid line), output responds strongly to a noise shock in comparison to my benchmark (*). At this value of $\alpha$, the household has a weaker effect from diminishing returns to scale and thus are more willing to provide labor in response to a noise shock. I can then conclude that the returns to scale effect from capital, coupled with usable capital this period being quasi-fixed, restrains the household’s reaction to noise shocks. Thus, there would be little surprise if removing capital lead to larger scale responses from noise shocks.

**Capital vs. No Capital**

In this section, I make the assumption that $\alpha = 0$ and solve my model using the process outlined in Section 2.3.2, adjusted for the new assumption. I obtain the conjectured responses of inflation and output, the equivalent of (2.40) and (2.41):

\[
\pi_t = -0.184\tilde{a}_t + 0.184\tilde{x}_{t|t} \\
\tilde{y}_t = 0.278\tilde{a}_t + 0.723\tilde{x}_{t|t}
\]  

Equation (2.49) makes it very clear that we can expect a much larger response to output from expectations than in my benchmark model. The coefficient on $\tilde{x}_{t|t}$ is 2.4 times larger in (2.49) than (2.41). Applying a 1% noise shock to this no-capital model (NC) and comparing it with my benchmark in Figure 2.7 shows that the NC model produces a response that is 2.4 times larger than the benchmark model.

On impact of a permanent productivity shock in the NC model, output increases by .42%, implying the permanent productivity shock results in a response that is

\[59\text{As } \alpha = 0, \tilde{k}_t = 0, \forall t, \text{ thus the term is omitted.}\]
Figure 2.7: Comparison of responses of output to a noise shock with and without capital. The y-axis is percentage deviation from each respective model’s BGP.

initially about four times larger than the noise shock. By this measure, noise shocks seem to have a much larger possibility of being important in aggregate fluctuations. It should also be mentioned that the difference of a factor of four is considerably smaller than the eight of my benchmark!

We must then question the results of models that abstract from capital. Capital takes time to adjust. In turn, this constrains the degree to which increasing labor can result in increased output. Because of this, and that capital responds weakly to expectations, the overall economy will not respond as strongly when faced with noise shocks. Remove capital and this effect vanishes, letting agents freely respond to noise shocks as they face constant returns in labor.

60 A large reason why output response is initially smaller in the NC model is that labor responds more negatively upon impact.
When introducing capital, I also introduce convex adjustment costs. As explained in Dupor [36], the bond-capital no arbitrage condition in a New Keynesian model can lead to potential indeterminacies when using an active interest policy rule which indeed occurs in my case. This necessitates the convex adjustment costs. This may lead one to then ask: how sensitive are the above results to specifications of \( \phi \)? The results are somewhat sensitive to its value, but the general result still holds that the NC model dominates the capital model in the scale of noise shock responses.

**Sensitivity to \( \phi \)**

I look at my benchmark model under three \( \phi \) values: 6, 10, and 60. I will assume a value of \( \phi = 60 \) for discussion.\(^{61}\) Using my new assumption and applying the solution method, we get the following (2.40)-(2.42) equivalents:

\[
\pi_t = -.008\tilde{k}_t - .163\tilde{a}_t + .171\tilde{x}_{t|t} \tag{2.50}
\]
\[
\tilde{y}_t = .35\tilde{k}_t + .233\tilde{a}_t + .417\tilde{x}_{t|t} \tag{2.51}
\]
\[
\tilde{k}_{t+1} = .988\tilde{k}_t - .996\tilde{a}_t + .008\tilde{x}_{t|t} \tag{2.52}
\]

From (2.51) and (2.52), we can see that output and capital now respond more to expectations compared to the benchmark. The household doesn’t wish to move its capital choice in large amounts and therefore is willing to respond a little bit more strongly with changes in expectations to mitigate the risk of having to make large adjustments in the capital stock. The increased capital accumulation causes an increase in output. However, comparing the coefficient on expectations in equation

\(^{61}\)The intuition becomes very clear for the most extreme value as to how dynamics change when this parameter changes.
(2.51) to the same coefficient in (2.49), we see that output still responds considerably less to expectations than the NC model—even when capital adjustment costs are unrealistically high.

Figure 2.8 and Figure 2.9 show the different specifications and their respective output, capital, and investment responses when each model is faced with a noise shock of the same magnitude. One can see that increasing $\phi$ causes a stronger initial investment response, but the initial response changes very little when one moves from $\phi = 10$ (green, +) to $\phi = 60$ (red, solid). The starting point and rate of capital decumulation also differs as one moves to higher values of $\phi$: at lower values, the household begins decumulating at an earlier stage and overall rate of decumulation becomes small when $\phi = 60$. This slow rate of decumulation translates to greater persistence of output to the noise shock. These two facts give evidence to my claim that the household does not wish to be caught off guard when adjustment costs are restrictive; it chooses to slowly accumulate capital until it’s quite confident that this was merely a noise shock. At this point, decumulation is also quite costly. Capital then takes a prolonged amount of time to reach its proper levels.

Even in the most extreme example of $\phi = 60$, we see that the response of a noise shock in the NC model is still higher. Also, though more persistent, the level at which the noise shock persists is still negligible. The discrepancy between the NC and capital model becomes increasingly more large as the convex adjustment cost becomes less inhibitive. One can safely conclude that the conjecture of the paper still holds: capital inhibits the response of the economy to noise shocks.
Figure 2.8: Output response of different specifications to a 1% noise shock

Figure 2.9: Capital and investment responses to a noise shock of the capital model under different values of $\phi$. 

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2.6 Possible Extensions

The form of adjustment costs may affect results. For instance, a common form of adjustment cost is \( \phi \frac{I^2}{K_t} \). If we were to compare it to my specification of incurring a cost if the capital stock changes, we may expect that this leads to a greater initial response but less persistence. For the same value of \( \phi \), agents must incur a larger cost for the same amount of investment in the alternative form. This would likely act like an increase of \( \phi \) in my form and lead to a greater investment response. But recall that even at exceptionally restrictive costs, noise shocks still did not respond to the same extent as they would without capital. On the other hand, agents under the alternative form can choose to let the stock depreciate without incurring any cost. In my form, an agent wishing to decumulate capital has to incur a convex costs, perhaps causing them to decumulate more slowly and increasing the persistence of a noise shock.

Another question is the role of habit. It’s likely that habit would lead to a more dampened response to a noise shock. Recall that noise shocks are largely consumption driven. If we were to mitigate the consumption response, ceteris paribus, we would likely see less of an overall output response in the economy from a noise shock.

A last thought is to apply noise shock analysis to a menu cost model. A likely candidate for menu costs, that affords tractability, would be the continuous distribution of fixed adjustment costs proposed in Dotsey et al. [34]. State-dependent pricing gives a more realistic setting of analysis, and it is the author’s opinion that exploration would lead to interesting insights into firm behavior under a comparable information framework.
2.7 Conclusion

Capital seems to play an important role in how agents respond in an imperfect information economy. From the perspective of permanent shocks, because capital accumulation is gradual, this results in prolonged responses to permanent productivity shocks. Alternatively, when agents do not see any improvement in current technology but only get a positive signal, their response is largely consumption driven with very limited investment, causing noise shocks to lack persistence. To make a significant step towards capital accumulation, agents need a stronger signal in the form of current productivity.

Abstracting from capital gives a more extreme picture; agents are more willing to quickly adjust an input whose returns are realized today, especially when they receive constant returns in that input. Labor-only models then may show higher output volatility resulting from noise shocks due to lack of accounting for intertemporal input decisions. On the other hand, incorporating capital provides a more strenuous test to past models. If one happens to find significant responses using capital and a justifiably more complicated environment/signal extraction problem, then this would be strong supporting evidence that noise shocks are relevant.
Chapter 3: Sunspots vs. Fundamentals: An Experimental Analysis

3.1 Introduction

In a rational and perfect information world, observed economic fluctuations are direct reflections of the underlying fundamentals. As we move away from perfect information, the decisions we observe may look erratic or as though they are driven by, as Keynes put it, animal spirits. A possible explanation for this behavior is that agents are using information uncorrelated with economic fundamentals. In the literature, such information is known as a sunspot, and it may be used by agents either because they believe it contains some relevant economic information or it helps to coordinate expectations. However, because sunspots are disconnected from the fundamentals, they may be volatile or misleading and result in welfare-reducing outcomes.

This paper addresses two questions in an experimental setting. When might agents use sunspots? How would this potentially affect the economy? Past experiments analyzing sunspots were settings of largely coordination with no role for fundamental information. Unlike past experimental work, I also incorporate meaningful, but imperfect, information about the fundamentals and a tradeoff between following the
sunspot versus the fundamentals. My setting then allows me to examine the relationship between the precision of fundamental information and the willingness to follow a sunspot. I also have a treatment points to the type of sunspot agents may be most attracted to when there are multiple options available.

I undertake three treatments. The first treatment gives subjects informative but very noisy fundamental information. I assume fundamental information is much less noisy in the second treatment. Lastly, I undertake a treatment where fundamental information is as noisy as in the first treatment, but agents are exposed to a more complicated sunspot signal composed of two competing sunspots. These sunspots differ, though, in that one is fixed for the entire session (static) while the other is chosen in each period as in the other treatments (dynamic).

There are 17 periods in the experiment with the first two being practice periods. As an initial result, I find that regardless of which period or treatment, some agents will use the sunspot. However they are more likely to use sunspots if the sunspot signal is not complex.

Several recent papers suggest recessions are periods of high fundamental uncertainty or noisiness.\(^{62}\) By comparing my first and second treatment, I find that individuals are more likely to use the sunspot when fundamental information is noisy, as in recessions. In addition, if exposed to multiple sunspots as in the third treatment, I find that agents will ultimately coordinate on the dynamic option.

My results then suggest that times of high fundamental noise would predispose agents to use sunspots. Furthermore, if they have access to multiple sources of extrinsic information, they will most likely use the most volatile option. These results

\(^{62}\)See Bloom et al. [18], Patton and Temmermann [73], and Ilut and Schneider [54].
can then help to explain why recessions are periods of high volatility and are often associated with behavior disconnected from the fundamentals (e.g., bank runs or stock market crashes).

There is a rich theoretical literature beginning with Shell [82] and further founded in Azariadis [11] and Cass and Shell [20]. In general, sunspots have been theoretically shown to be a potentially large player in many different settings: Farmer and Guo [38] and Christiano and Harrison [26] examined the effect of sunspots on business cycles, Peck and Shell [74] looked at sunspots under a model of imperfect competition, and it’s even been analyzed as a source of bank runs in Gorton [49] and Peck and Shell [75]. Another related topic is correlated equilibria, which can actually be equivalent to sunspot equilibria under certain cases. A discussion of this relationship can be found in Forges and Peck [43].

There has been notably less development from the experimental side. Duffy and Fisher [35] examined whether people are willing to follow sunspots. They examined how presentation of the sunspot (clearly defined meaning vs. ambiguous) affects whether or not individuals follow them; they found that the meaning of the sunspot must be clearly understood to have any effect, and also examined how market structure may influence individual’s choices. Fehr et al. [40] redid the presentation such that there was clear risk ranking between potential market outcomes amongst all individuals. Individuals received public information and/or correlated private information that were both unrelated to the fundamentals. They showed that individuals will choose the risk-dominated sunspot equilibrium under certain conditions. Arifovic et al. [9] produce a macroeconomic setting where there are clear rankings in terms of payoffs between outcomes. They find that individuals are still willing to follow
sunspots though the sunspot may point to a low-payoff outcome, reflecting the potential pull of this type of information. As mentioned previously, there was no role for information on the fundamentals in any of these experiments. My experiment gives some insight as to economic conditions that may predispose individuals to use sunspots as well as the type of sunspot that attracts them.

3.2 Decision Task and Theory

In this section, I introduce the economic setting and the basic decision task of subjects. I end by discussing possible pure strategy equilibria, when they exist, and when one may be preferable to another.

3.2.1 Economic Setting

Let $S = \{G, I, K, M, O\}$ be the set of all possible states. At the start of every period a new true state, $s^* \in S$, is randomly chosen. Agent $i$ receives a noisy private signal, $\tilde{s}_i$, where

$$\tilde{s}_i = \begin{cases} 
    s^* & \text{with probability } \omega \\
    s \in S - s^* & \text{with probability } \frac{1-\omega}{4}
\end{cases}$$

Across all treatments, the signal is informative ($\omega > .2$). Individuals also gain access to a signal which I will call the sunspot, $\hat{s} \in S$. Representing a key quality of what it means to be a sunspot, the signal is uncorrelated with the true state.

Agent $i$ must choose to operate assuming a particular state. Call this choice $c_i$. In total, there are $N$ agents that make this choice simultaneously and without communication. An agent can then receive positive payoff in only two ways: their choice matches the true state ($c_i = s^*$) or they make the same choice as the majority.

\[63\] In reality, there may be some persistence to $s^*$. However, persistence would only serve an informational role in this environment, which the private signal already provides.

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of other agents. I will call these the fundamental ($\theta_f$) and coordination ($\theta_c$) payoffs, respectively.

In my experiment, I set $N = 3$. Therefore, the potential payoff of an agent in any given period is given by:

$$\Pi_i = I_{s^*} \theta_f + I_c \theta_c$$  \hspace{1cm} (3.1)

where

$$I_{s^*} = \begin{cases} 1 & c_i = s^* \\ 0 & \text{otherwise} \end{cases}$$

$$I_c = \begin{cases} 1 & c_i = c_j \text{ with } i \neq j \\ 0 & \text{otherwise} \end{cases}$$

In general, I will assume that $\theta_f > \theta_c$, or that the fundamental payoff is higher than the coordination payoff. This implies that agents would ultimately desire to acquire the fundamental payoff, but it may be difficult to do so when fundamental information is noisy.

### 3.2.2 Equilibria

There are three possible pure strategy equilibria: following one’s private signal (private information equilibrium), following the sunspot (sunspot equilibrium), or choosing to coordinate on the same state (focal point equilibrium). The last two are actually ex-ante equivalent as both entail forgoing private information to guarantee the coordination payoff. Because of this, I will ignore the last one and note that existence of a sunspot equilibrium implies the same of a focal point equilibrium.

**Definition 1** A sunspot equilibrium exists under risk neutrality if

$$\mathbb{E}(\Pi(c_i = c_{j \neq i} = \hat{s})) \geq \mathbb{E}(\Pi(c_i = \check{s}_i \text{ and } c_{j \neq i} = \hat{s})) \quad \forall j \neq i \in N.$$  

---

64 One can imagine such a situation in stock investment. An agent would ultimately want to invest in the most profitable firm, but they would also receive some payoff by buying a stock others are purchasing.
Therefore, conditional on everyone else choosing the sunspot, a sunspot equilibrium exists when the expected payoff of selecting the sunspot is at least as high as choosing to follow one’s private information. In situations where \( \hat{s}_i = \hat{s} \), the above definition is trivially satisfied, so I will focus on other information outcomes.

**Theorem 1** A sunspot equilibrium will always exist under risk neutrality if \( \frac{\theta_c}{\theta_f} \equiv \theta \geq \frac{5\omega - 1}{4} \) and \( c_j = \hat{s} \forall j \neq i \in N \).

The proof of Theorem 1 is trivial and based on Definition 1 holding when \( \hat{s}_i \neq \hat{s} \).

Theorem 1 alternatively says that a sunspot equilibrium will exist if either the coordination payoff is relatively high or if the private signal’s precision is sufficiently low. If everyone is coordinating on the sunspot, then following one’s private information will guarantee a loss of the coordination payoff. If this payoff is at least as high as the fundamental payoff \( (\theta \geq 1) \), then a sunspot equilibrium always exists regardless of precision.

An analogous definition and theorem exists for the private information equilibrium.

**Definition 2** A private information equilibrium exists under risk neutrality if \( E(\Pi(c_i = \hat{s}_i, c_j \neq i = \hat{s}_j \neq i)) \geq E(\Pi(c_i = \hat{s} \text{ and } c_j \neq i = \hat{s}_j \neq i)) \forall j \neq i \in N \).

**Theorem 2** A private information equilibrium will always exist under risk neutrality if \( \theta \geq \frac{5\omega - 1}{4(P(I_c = 1|c_i \neq \hat{s}_i) - P(I_c = 1|c_i = \hat{s}_i))} \) and \( c_j = \hat{s}_j \forall j \neq i \in N \).

In Theorem 2, \( P(I_c = 1|c_i \neq \hat{s}_i) \) is the probability one will acquire the coordination payoff conditional on not announcing their private signal (and everyone else announcing their private signal). \( P(I_c = 1|c_i = \hat{s}_i) \) is the same except one announces
their private signal. Because I assume private information is always informative \((\omega > .2\)), it can be shown that the denominator on the righthand side of Theorem 2 is always negative. Therefore, this condition is satisfied for any ratio of strictly positive payoffs, and the private information equilibrium always exists.

When would one equilibrium be preferable to another? Assuming risk neutrality, this occurs when the expected payoff of playing one equilibrium is higher than the other. Expected payoff is changing in both \(\omega\) and \(\theta\).

In my experiment, I set \(\theta_c = \$5\) and \(\theta_f = \$7\). For the implied value of \(\theta\) given by these values, a sunspot equilibrium will exist \(\omega < .771\). Figure 3.1 shows how the expected payoff of each equilibrium evolves as \(\omega\) varies and assuming \(\hat{s}_i \neq \hat{s}\).
If private information is sufficiently imprecise, the sunspot equilibrium is preferred as it guarantees the coordination payoff. As \( \omega \) becomes larger, ignoring one’s private information becomes more costly and the expected payoff of the sunspot equilibrium diminishes. Conversely, the expected payoff of the private information equilibrium rises as both payoffs become more likely with increased precision. The crossing point where the private information payoff becomes more preferred occurs at around 49% precision.

3.3 Treatments and Design

In this section, I describe the treatments and design for the experiment. I also outline key hypotheses that I will test using experimental results.

3.3.1 Three Treatments

I undertake three treatments in my design: weak sunspots (WS), strong sunspots (SS), and multiple sunspots (2S).

The SS treatment sets \( \omega \) to .25, implying that fundamental information is quite noisy and that the sunspot equilibrium’s expected payoff is notably higher than the private information equilibrium. As the name implies, the pull of utilizing sunspots should be strong in this treatment.

I weaken this pull in the WS treatment by raising \( \omega \) to .5. Recall from Figure 3.1 that this precision is about where the expected payoff of the private information equilibrium surpasses that of the sunspot equilibrium. If subjects are risk neutral or loving, they should strictly prefer the private information equilibrium, but risk aversion may imply announcing the sunspot is still preferable as it guarantees the coordination payoff.
In reality, sunspots may come in different flavors and there may be multiple options available. To explore the implications of this possibility, the 2S environment provides agents with two sunspots. The first sunspot is stationary and is provided at the beginning of the session. On the other hand, the second sunspot is dynamic and redrawn every period within the session. Private information’s precision is set to $\omega = .25$, as in the SS treatment, so agents would likely prefer to coordinate on the sunspot, but this treatment makes the task more difficult.

3.3.2 Design

The experiment was designed and implemented using z-Tree [41] software. A session has 17 periods, with the first two being practice periods that do not affect payoff. In the last 15 periods, only one period is randomly chosen for payment. As mentioned previously, $\theta_c =$5 and $\theta_f =$7. Individuals are randomly assigned to groups of six and are randomly rematched each period to form two subgroups of three.

The true state and private information is chosen through a subject’s computer. To emphasize the extrinsic nature of sunspots, the sunspot signal is drawn from an urn with five balls corresponding to the five states. Once a draw is made, it is posted on the board for all to see. In the WS and SS treatment, the sunspot is redrawn every period. The 2S experiment not only has a sunspot drawn every period, but also one that is drawn that the start of the experiment, and left on the board for the entire session.

Subjects are provided with instructions describing the decision task and the type of information available to them. The instructions give no suggestion as to how the

\[^{65}\text{This payoff method was chosen to prevent any income effects in decisions.}\]
information should be used. A sample copy of the instructions is given in Appendix B.

During the session, subjects also have access to a history table that provides past true states, their private information, their choice, how many individuals from their subgroup matched each other, and—conditional on two or more matching—what was the most chosen state. Again, subjects were not allowed to communicate for the duration of the experiment.

### 3.3.3 Hypotheses

There are four hypotheses that I wish to explore in this experiment. The first hypothesis is comparable to past experimental studies that established the existence of sunspot equilibria.

**Hypothesis 1** *Subjects will use sunspots across all treatments.*

Even though I do not advise individuals how to use the information they receive, the above hypothesis suggests that agents see a specific role for sunspots and exploit this role.\(^{66}\) In contrast to past experiments, agents also have a tension between coordination and acting in the true state. As my analysis of expected payoffs suggests, subjects will be less likely to use sunspots as private information becomes more informative, leading to my second hypothesis.

**Hypothesis 2** *Individuals are more likely to use sunspots when fundamental information is noisy.*

\(^{66}\)In contrast, other experiments (e.g., Duffy and Fisher [35] and Arifovic et al. [9]) first show individuals how the sunspot may be useful by forcing choices that follow the sunspot in trial periods.
Hypothesis 2 can directly relate to business cycles. In times when fundamental information is more imprecise, which several papers suggest is during recessions, agents are more likely to use sunspots. Put differently, welfare-reducing phenomena that seem irrational and require coordination such as bank runs or stock market crashes may be induced by information uncorrelated with the fundamentals.

Duffy and Fisher [35] and Fehr et al. [40] both conclude that a clear sunspot signal is necessary for a sunspot equilibrium to ever arise, which leads to Hypothesis 3.

**Hypothesis 3** A *more complicated sunspot structure will inhibit the usage of sunspots relative to a simpler structure.*

When there is not a clear agreement between agents on how to respond to a sunspot, the usefulness of sunspots as a coordination device decreases. So even in situations where fundamental information is noisy, it may be less risky to ignore sunspots completely if there is no knowledge on how to act.

If agents manage to find a sunspot and coordinate successfully on it, what type of signal amongst several would catch their attention? Fehr et al. [40] finds that individuals converge on the risk-dominated strategy of following a moving sunspot opposed to a fixed point. Given this result, I would expect agents will ultimately be attracted to a signal that is actively moving.

**Hypothesis 4** When exposed to multiple sunspots, conditional on following a sunspot, subjects will converge on the most volatile option.

Hypothesis 1 and Hypothesis 4 provide a suggestion for why we observe high volatility during recessions. Not only are individuals more predisposed to using
sunspots during times of high fundamental uncertainty, but they are also more likely to follow sunspots that move erratically.\footnote{Though interesting to speculate, I do not explore the reason why agents may be more attracted to volatile sunspots.}

### 3.4 Results

Each treatment has 36 observations comprising of six groups of six. Figure 3.2 shows the fraction of these 36 that chose the sunspot for each treatment, and how this fraction changes over the experiment’s 17 periods. For the 2S treatment, the fraction is for the dynamic sunspot signal.

There are several patterns that we can observe between treatments. The use of sunspots is generally increasing across all treatments as the experiment progresses. Do
we see the opposite for private information? Indeed, Figure 3.3 shows that the fraction of individuals using their private information is decreasing across all treatments. This is perhaps not surprising for the SS or 2S treatment, but the sunspot equilibrium seems to also have pull in the WS treatment. The logic is fairly straightforward. Initially, some risk averse individuals try to use the sunspot to guarantee the coordination payoff. Because there is a non-trivial set of individuals initially following the sunspot, \( P(I_c = 1|c_i = \bar{s}_i) \) becomes lower and \( P(I_c = 1|c = i = \hat{s}) \) higher, decreasing the value of choosing one’s private information relative to choosing the sunspot. As more agents converge on the sunspot, this further decreases the expected value of announcing one’s private information, and we know the sunspot equilibrium exists even at a precision of 50%. Therefore, across all treatments, sunspots seem to have
some pull though it is weaker when private information increases in precision. These findings support Hypothesis 1 and 2.

We can also see that the SS treatment has quick convergence (roughly period 3) in using the sunspot compared to the other treatments. Subjects can easily recognize the usefulness of the sunspot as a coordination mechanism. When fundamental information is noisy, the sunspot equilibrium is very attractive and a clear sunspot assures this equilibrium is reached quickly. It is likely that agents also understand how they can use the sunspot in the other two treatments but convergence is much slower either due to a complicated signal (2S) or lower expected payoff (WS).

The 2S treatment has the same private information precision as the SS treatment, yet, as proposed by Hypothesis 3, the more complicated sunspot structure prevents subjects from utilizing the sunspot effectively.

Were individuals more attracted to the dynamic sunspot? Figure 3.4 shows the portion of individuals that chose the fixed sunspot. In the trial periods some individuals were using this information, but the fixed sunspot became ignored as the session progressed.68

The figures seem to provide corroboration for my four hypotheses, but one issue with the raw data is that certain choices are counted twice. For example, sometimes private information and the sunspot are equal for certain subjects. Throwing out these observations would also result in a substantial loss of within subject information, especially for the three signal 2S treatment.

To test my hypotheses, I want to categorize each subject $i$ within a period, $t$, into three mutually exclusive strategy types, $k_{i,t} \in K = \{\text{Dynamic Sunspot (d)}, \text{Fixed}\}$.

---

68In period 13, two of the sessions had the dynamic sunspot equal the fixed sunspot which explains the unusual blip.
Sunspot (f), Private Information (p), Unknown (u).\textsuperscript{69} I will define what it means to be a certain type in period $t$. A central assumption in these definitions is that subjects have some consistency across periods in their strategy. I will start by defining what it means to be a particular type in period 1.

**Definition 3** Let $\hat{s}$ be the static sunspot. A subject $i$ is a p-type in period $t = 1$ if

\[
(c_{i,t} = s_{i,t} \neq \hat{s}_t \neq \hat{s}) \lor (c_{i,t} = s_{i,t} \land c_{i,t+1} = s_{i,t+1}) \lor (c_{i,t} = s_{i,t} \land c_{i,t+1} \neq \hat{s}_{i,t+1} \neq \hat{s}).
\]

The definition of a p-type more easily assigns a subject to p-types than any other type. For example, if a subject announces their private information in period 1 and announces their private information period 2, I assume they are p-types even if they also announced a sunspot in both periods.\textsuperscript{70} This assumption makes it unlikely that

\textsuperscript{69}The fixed sunspot type is only applicable in the 2S treatment.

\textsuperscript{70}In fact, even if they do an unusual choice in period 2 that does not equal either sunspot or private information, they are p-types if they announced their private information in period 1.
I incorrectly count a p-type as a d or f-type, stacking against Hypothesis 1. Because I use the same methodology across treatments, I can assume any difference between the fraction of types is induced by the treatment itself.

**Definition 4** A subject is an f-type in period $t = 1$ if she is not a period 1 p-type and $(c_{i,t} = \hat{s} \neq \hat{s}_t \neq s_{i,t}) \lor (c_{i,t} = \hat{s} \land c_{i,t+1} = \hat{s})$.

Similar to p-types. An f-type is defined as a robustness check to Hypothesis 4. If one announces the fixed sunspot in period 1 and period 2, it does not matter if the both sunspots aligned in these periods.\(^{71}\)

**Definition 5** A subject is a d-type in period 1 if she is not a period 1 p or f-type and $(c_{i,t} = \hat{s}_t \neq \hat{s} \neq s_{i,t}) \lor (c_{i,t} = \hat{s} \land c_{i,t+1} = \hat{s}_{t+1})$.

Assuming that one’s choice in period 1 matched at least one piece of information, it is least likely that a type will be misclassified as a d-type. It is only classified as a d-type if we can reasonably rule out other strategies. Lastly we have the u-types.

**Definition 6** A subject is a u-type in period 1 if $c_{i,t} \neq \hat{s}_t \neq \hat{s} \neq s_{i,t}$.

Therefore, an unknown type in period 1 is a subject that completely ignores all their information.

For periods $t > 1$, a subject is classified as a type based on their past period type $k_{i,t-1}$ and their current choice. As in the $t = 1$ case, I assign types in such a way to create the most false positives in p-types and the least false positives in d-types. I define all four types below.

\(^{71}\)This condition was actually never satisfied. Across all treatments, $\hat{s}_1 \neq \hat{s}$.
Definition 7 A subject $i$ is a $p$-type in period $t > 1$ if $(c_{i,t} = s_{i,t} \neq \hat{s}_t \neq \hat{s}) \lor (c_{i,t} = s_{i,t} \land k_{i,t-1} = p)$.

Definition 8 A subject is an $f$-type in period $t > 1$ if she is not a period $t$ $p$-type and $(c_{i,t} = \hat{s} \neq s_{i,t} \neq \hat{s}_t) \lor (c_{i,t} = \hat{s} \land k_{i,t-1} = f)$.

Definition 9 A subject is a $d$-type in period $t > 1$ if she is not a period $t$ $p$ or $f$-type and $(c_{i,t} = \hat{s}_t \neq \hat{s} \neq s_{i,t}) \lor (c_{i,t} = \hat{s} \land k_{i,t-1} = d)$.

Definition 10 A subject is a $u$-type in period $t > 1$ if she is not any other time $t$ type.

Applying these definitions to my data, Figure 3.5 shows the fraction of various types across different treatments. Major trends that we saw in the raw data are now much more clear: for all treatments private information (dynamic sunspot) types are decreasing (increasing) over time and individuals only prominently use the fixed sunspot in the trial periods. A couple of other notable observations are that unknown types are much more prominent in the 2S treatment, and if we combine both sunspot types in the 2S treatment, it is still below the total fraction of sunspot types in the SS treatment.

3.4.1 Testing Hypotheses

The observations must be independent to test my hypotheses. Unfortunately, there is not independence across subjects as the type of someone in my group of six also influences my type. Across groups I can say there is independence, so I collapse my 36 observations for each treatment into their groups. For each group, I compute the fraction of subjects that are type $k_{i,t}$ for the trial periods ($t = -1$ to 0), first five
Figure 3.5: Fraction of types across all treatments. 2S+f is fraction of d-types plus f-types in the 2S treatment.
pay periods ($t = 1$ to $5$), middle five ($t = 6$ to $10$), and last five periods ($t = 11$ to $15$).

Table 3.1 shows the means and standard errors of this statistic.

Hypothesis 1 states that individuals use sunspots. Therefore, I will test the null hypothesis that the fraction of sunspot types is equal to 0. Table 3.1 makes it clear that the weakest possibility of sunspot types is the WS treatment in the trial period. Even for the weakest sunspot response, the proportion of individuals using sunspots is significant using a Mann-Whitney test (p-value < 1%), and we can safely reject the null hypothesis regardless of treatment or period.

In order to determine whether more individuals follow the sunspot when fundamental information is noisy, I perform a one-sided t-test on whether the difference between average sunspot players in the SS treatment is larger than the WS treatment.

The first row of Table 3.2 shows the resulting p-values across my four subperiods, and we can see that all p-values are below 5%. Regardless subperiod, more subjects tend to follow sunspots when fundamental information is noisy.

If the sunspot structure itself is difficult to follow, Hypothesis 3 proposes that less individuals will use sunspots. I test this hypothesis by comparing the SS and 2S treatment and consider a subject using a sunspot if they are either a d or f-type. The second row of Table 3.2 shows the test’s resulting p-values. Subjects in both treatments are trying to figure out their strategy in the trial periods which rejects the hypothesis at the 10% level. Once the payoff periods begin, subjects in the less complicated sunspot structure quickly converge on using the sunspot while the 2S treatment lags behind as subjects try to figure out which signal is being used. Hypothesis 3 then seems likely to be true.
Table 3.1: Fraction of Types Across Treatments and Periods

<table>
<thead>
<tr>
<th>Type and Treatment</th>
<th>Periods -1-0</th>
<th>Periods 1-5</th>
<th>Periods 6-10</th>
<th>Periods 11-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS Treatment</td>
<td>.33 (.08)</td>
<td>.15 (.05)</td>
<td>.08 (.03)</td>
<td>.05 (.03)</td>
</tr>
<tr>
<td>WS Treatment</td>
<td>.67 (.1)</td>
<td>.59 (.11)</td>
<td>.47 (.12)</td>
<td>.37 (.12)</td>
</tr>
<tr>
<td>2S Treatment</td>
<td>.38 (.09)</td>
<td>.27 (.06)</td>
<td>.19 (.04)</td>
<td>.09 (.03)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Fraction d-types</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SS Treatment</td>
<td>.6 (.08)</td>
<td>.79 (.07)</td>
<td>.87 (.06)</td>
<td>.9 (.04)</td>
</tr>
<tr>
<td>WS Treatment</td>
<td>.31 (.11)</td>
<td>.35 (.12)</td>
<td>.49 (.14)</td>
<td>.59 (.14)</td>
</tr>
<tr>
<td>2S Treatment</td>
<td>.25 (.06)</td>
<td>.38 (.07)</td>
<td>.55 (.09)</td>
<td>.67 (.07)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Fraction f-types</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2S Treatment</td>
<td>.24 (.07)</td>
<td>.14 (.05)</td>
<td>.11 (.08)</td>
<td>.11 (.07)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Fraction u-types</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SS Treatment</td>
<td>.07 (.03)</td>
<td>.06 (.04)</td>
<td>.05 (.03)</td>
<td>.05 (.03)</td>
</tr>
<tr>
<td>WS Treatment</td>
<td>.03 (.02)</td>
<td>.06 (.05)</td>
<td>.04 (.04)</td>
<td>.03 (.03)</td>
</tr>
<tr>
<td>2S Treatment</td>
<td>.14 (.06)</td>
<td>.2 (.04)</td>
<td>.14 (.04)</td>
<td>.13 (.04)</td>
</tr>
</tbody>
</table>

Note: Fraction of types within groups across treatments and periods. N=6 for each treatment. Standard errors are in parentheses.

Table 3.2: Testing Hypotheses 2-4

<table>
<thead>
<tr>
<th>Hypothesis Test</th>
<th>Per. -1-0</th>
<th>Per. 1-5</th>
<th>Per. 6-10</th>
<th>Per. 11-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>m(d-type in SS)-m(d-type in WS)&gt; 0</td>
<td>.03</td>
<td>.01</td>
<td>.03</td>
<td>.02</td>
</tr>
<tr>
<td>m(d-type in SS)-m(d+f-type in 2S)&gt; 0</td>
<td>.17</td>
<td>.02</td>
<td>.01</td>
<td>.06</td>
</tr>
<tr>
<td>m(d-type in 2S)-m(f-type in 2S)&gt; 0</td>
<td>.44</td>
<td>.01</td>
<td>.00</td>
<td>.00</td>
</tr>
</tbody>
</table>

Note: p-values for various one-side t-tests. N=6. m() is the mean.
Lastly, I conjecture that subjects exposed to multiple sunspots will be attracted to the more volatile option, or the dynamic sunspot. As can be seen in the last row of Table 3.2, I cannot say that the fraction of f versus d-types is different during the trial period. Table 3.1 also confirms that the number of both types is fairly close on average in the trial period. However, the fixed sunspot was quickly left behind once the payoff periods begins, p-values become quite small, and I can safely conclude that subjects become more attracted to the dynamic sunspot.

3.5 Conclusion

Recessions are periods associated with high uncertainty and volatility. This experiment suggests that the origin of this volatility may not be increased variance of the fundamental, but a consequence of agents being more predisposed to using sunspots. A more complicated sunspot structure may inhibit their usage, but agents will tend to be attracted to the more volatile option. In conjunction, both results suggest sunspots become more important in fluctuations and can lead to higher observed variance when fundamental information is noisy.
Appendix A: Additional Tables and Figures

Table A.1: List of Sectors

<table>
<thead>
<tr>
<th>Sector</th>
<th>NAICS Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>23</td>
</tr>
<tr>
<td>Mining and Logging</td>
<td>21, 113</td>
</tr>
<tr>
<td>Durables</td>
<td>321, 327-339</td>
</tr>
<tr>
<td>Non-Durables</td>
<td>311-315, 322-326</td>
</tr>
<tr>
<td>Wholesale trade</td>
<td>42</td>
</tr>
<tr>
<td>Retail trade</td>
<td>44</td>
</tr>
<tr>
<td>Transportation, warehousing and utilities</td>
<td>22, 48-49</td>
</tr>
<tr>
<td>Information</td>
<td>51</td>
</tr>
<tr>
<td>Finance, insurance, real estate, rental, and leasing (FIRE)</td>
<td>52-53</td>
</tr>
<tr>
<td>Professional and business services</td>
<td>54-56</td>
</tr>
<tr>
<td>Educational services, health care, and social assistance</td>
<td>61-62</td>
</tr>
<tr>
<td>Leisure and hospitality</td>
<td>71-72</td>
</tr>
<tr>
<td>Other services (excluding government)</td>
<td>81</td>
</tr>
</tbody>
</table>

Note: Classification of a sector is based on employment data from CES by major industry. NAICS codes represent the mapping from CES to BEA data.
Table A.2: Weights for Average Employment Deviation

<table>
<thead>
<tr>
<th>Sector i</th>
<th>$w_{i,R}$</th>
<th>$w_{i,E}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>0.84</td>
<td>0.76</td>
</tr>
<tr>
<td>Mining and Logging</td>
<td>1.15</td>
<td>1.12</td>
</tr>
<tr>
<td>Durables</td>
<td>0.68</td>
<td>0.66</td>
</tr>
<tr>
<td>Non-Durables</td>
<td>0.76</td>
<td>0.69</td>
</tr>
<tr>
<td>Wholesale trade</td>
<td>0.98</td>
<td>1.1</td>
</tr>
<tr>
<td>Retail trade</td>
<td>1.17</td>
<td>0.89</td>
</tr>
<tr>
<td>Transportation, warehousing and utilities</td>
<td>1.03</td>
<td>0.78</td>
</tr>
<tr>
<td>Information</td>
<td>1</td>
<td>0.89</td>
</tr>
<tr>
<td>Finance, insurance, real estate, rental, and leasing (FIRE)</td>
<td>1</td>
<td>1.58</td>
</tr>
<tr>
<td>Professional and business services</td>
<td>2.05</td>
<td>1.57</td>
</tr>
<tr>
<td>Educational services, health care, and social assistance</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>Leisure and hospitality</td>
<td>1.09</td>
<td>0.98</td>
</tr>
<tr>
<td>Other services (excluding government)</td>
<td>1.45</td>
<td>1.06</td>
</tr>
<tr>
<td>Aggregate Employment</td>
<td>0.75</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Note: E is expansionary, and R is recessionary. The recessionary periods are +/- 3 months of NBER dated recessions. Expansionary are in between recessionary periods.
### Table A.3: Calibrated and Parameterized Values—Single Sector Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Target</th>
<th>Value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>4% annual interest rate</td>
<td>.9992</td>
<td>4%</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Sectoral Productivity Persistence</td>
<td>.9895</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Z volatility of .016</td>
<td>.00433</td>
<td>.016</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>Average SR of 3.1%</td>
<td>.75</td>
<td>2.93%</td>
</tr>
<tr>
<td>$\mu_\epsilon$</td>
<td>$\sum_{i=1}^{N} \epsilon_i g(\epsilon_i) = 1$</td>
<td>.8275</td>
<td>$\sum_{i=1}^{N} \epsilon_i g(\epsilon_i) = 1$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Average $Z_k$ correlation of .37</td>
<td>.85</td>
<td>-</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Average u of 6%</td>
<td>.00546</td>
<td>6.08%</td>
</tr>
<tr>
<td>$B$</td>
<td>Average JFR 45.5%</td>
<td>.096</td>
<td>45.5%</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$\epsilon$ persistence</td>
<td>0.085</td>
<td>Average $\epsilon$ duration of one quarter</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Bargaining power</td>
<td>0.6</td>
<td>Average steady-state wage 1.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Matching elasticity</td>
<td>0.6</td>
<td>-</td>
</tr>
<tr>
<td>$b$</td>
<td>Unemployment benefits</td>
<td>0.6</td>
<td>60% of steady-state wage</td>
</tr>
<tr>
<td>$c$</td>
<td>Vacancy costs</td>
<td>0.2</td>
<td>20% of steady-state wage</td>
</tr>
<tr>
<td>$\bar{\xi}$</td>
<td>3.2% monthly job-to-job transitions</td>
<td>0.16</td>
<td>3.2%</td>
</tr>
</tbody>
</table>

Note: Targets are the same as in the benchmark. Parameterized variables are kept the same. Costs related to sectoral reallocation ($\gamma$ and $\xi^h$) have no affect in the single sector version and are not included in the table. The correlation parameter $\tau$ is still included in productivity and kept at the benchmark’s value.
Figure A.1: Cyclical job reallocation rates compared to cyclical sectoral reallocation measure (adjusted to scale). Shaded areas are NBER dated recessions.
Appendix B: Sample Instructions

Introduction

Welcome and thank you for participating! I kindly ask you to refrain from talking to one another, looking at one another’s screen, and from using your cellphone. If you have a question, raise your hand and I will assist you.

Disclaimer: We prohibit any experiments involving deception at the Ohio State University Economics Lab. Therefore, everything in these instructions and on your screen is true.

This is an experiment to examine how individuals utilize varying types of information when making decisions. The Journal of Money, Credit, and Banking (JMCB), housed at The Ohio State University, has provided funding for this research.

All individuals are guaranteed at least $5 for participating. Within the experiment, you can earn additional money that depends on your performance. All monetary values in the experiment will be written in experimental currency units (ECU). The conversion rate is $.25 per ECU.

There will be 17 rounds in the experiment. The first 2 rounds are practice and will not affect your potential earnings. Of the remaining 15 rounds, one will be randomly chosen for payment at the end of the experiment. Because of this, make careful decisions in each round as that may be the round chosen for payment.
Decision Task and Setup

At the beginning of each round, you will be randomly matched with two other individuals to form a group of three; you will not know the identity of these individuals. It is unlikely that the two individuals you are matched with are the same as last round.

You—and everyone you are matched with—will be presented with five letter choices: G, I, K, M, and O. Each individual will then select one of these choices. You will not be allowed to communicate within your group.

Once all choices in a group are made, potential payoffs are calculated for each individual and this ends the round. Remember that only one of these rounds is chosen for payment, in which case you receive the potential payoff of that round.

You can potentially receive a payoff in two ways: choosing the "hidden" letter of that round (explained below) and/or choosing the same letter that at least one other person in your group chose.

At the beginning of each round, the computer chooses a letter randomly from G, I, K, M, and O. This is called the hidden letter and it is the same for all participants. Each letter is equally likely to be chosen, and the hidden letter in one round has no affect as to what hidden letter is chosen in a different round. You will not be told what the hidden letter was until all individuals make their decision. When the round ends, if your choice matched the hidden letter, you will receive a potential payoff of 28 ECUs.

**EXAMPLE:** Suppose you chose "I" and the hidden letter was "I", then you will receive 28 ECUs if that round is chosen for payment. On the other hand, if you
chose "K" and the hidden letter was "O", you will not receive the hidden letter payoff if that round is chosen for payment.

Note that acquiring the hidden letter payoff does not depend on another individual’s choice in your group. For the purpose of the experiment, the hidden letter payment will be called the X2 payment.

You can also receive a payoff if your choice matches at least one other individual’s choice in your group. If this occurs, you will receive a potential payoff of 20 ECUs.

**EXAMPLE:** Suppose you chose "M" and one or two others chose "M" as well, then you will receive 20 ECUs if that round is chosen for payment. Conversely, if you chose "G" and the two other individuals chose letters different than G, you will not receive this payoff if that round is chosen for payment.

For the purpose of this experiment, this payoff will be called the X1 payoff. Notice that it’s possible to get both potential payoffs if at least one other person chose the same letter as you and that letter happened to be the hidden letter.

Are there any questions?

**Information**

To help make your decision, you will receive two pieces of information at the beginning of each round.

The first is information that predicts what letter is the hidden letter. Each individual gets their own information about the hidden letter provided by the computer. This particular information, which we’ll call the private letter, is 25% accurate (on average, 1 out of 4 times, the private letter will be the hidden letter). What that means is as follows:
If the hidden letter is G, you have a 25% chance of observing G in your private letter and a 75% chance of observing either I, K, M, or O, each with equal chance.

For instance, suppose there are 4 consecutive rounds. The above accuracy implies that of those 4 rounds, one round will have the private letter equal to the hidden letter. The other three rounds will have some other random letter not equal to the hidden letter of that round.

However, this is an average and the actual number may vary. Because every individual receives their own private letter chosen by the above rule, different individuals can receive different private letters.

The second information that you will receive is what we’ll call the public letter. At the beginning of each period, a ball will be drawn from an urn. There are 5 such balls in the urn, each corresponding to a letter: G, I, K, M, or O. Whichever ball is drawn is then posted on the board, meaning that all individuals will see this letter, and then put back into the urn. Each letter has equal chance of being drawn. Notice that the hidden letter and private letter of a particular round has no impact as to what letter is drawn. A new ball is drawn each period and posted on the board.

In summation, the private letter is chosen by the computer for each individual and is 25% accurate (1 out of 4 on average is correct) in predicting the hidden letter. The public letter is drawn randomly from an urn each period and posted on the board for all to see.

How you utilize the above information is up to you.

At the end of every period, you will see your potential payoff for that round and a history of past rounds containing information on past hidden letters, your private letter, your choice, your potential payoffs, how many individuals matched letters from
your group in that period (i.e., 0 means no one matched each other’s choices in your
group that period, 2 means two, and 3 means everyone in your group match each
other’s choice), and—if more than two people chose the same letter—the most chosen
letter in your group for a particular period.

At the end of the experiment, you will be asked a brief questionnaire regarding your
strategy.

Are there any questions?
Bibliography


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