Design and Location Optimization of Electrically Small Antennas Using Modal Techniques

Dissertation

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By

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Abstract

In this dissertation, the Theory of Characteristic Modes is used as a framework for the design, optimization, and benchmarking of electrically small radiating systems. The foundation of this work is in the theory of Characteristic Modes, an eigenvalue equation of the Method of Moments impedance matrix $[Z]$, that leads to derive the fundamental radiation modes of arbitrary-shaped bodies. After an overview of small antenna theory, we derive a new method for computing the $Q$ factor of arbitrary-shaped radiating bodies using CMs using only the Method of Moments impedance matrix $[Z]$. Following this derivation, we present a new method for computing the fundamental limits on $Q$ (and thus bandwidth) for arbitrary-shaped antennas. As a by-product of this method, we extract the optimal current distribution as a function of antenna shape for design guidelines. We further extend this theory to find the $Q$ limits of arbitrary-shaped antennas and antenna-platform systems, subject to specific radiation pattern requirements.

In the second part of the thesis, we use the Theory of Characteristic Modes to optimize the location and excitation of single and multiple in-situ ESAs mounted on finite, sub-wavelength platforms as relates to unmanned aerial vehicles (UAVs). By properly analyzing the CMs of the supporting platform, we show that a complex, multivariate optimization problems can by radically simplified using CMs. Based on this capability, we present a new, systematic design methodology for location
optimization of small antennas on-board finite platforms. The approach is shown to drastically reduce the time, computational cost, and complexity of a multi-element in-situ antenna design, as well as providing significant performance improvements in comparison to a typical single-antenna implementations.
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Chapter 1: Introduction

Current and emerging wireless protocols drive an increasing need for reliable antenna solutions integrated within lightweight and multifunctional subwavelength mobile terminals. Often, these antenna dimensions are small fractions of the operational wavelength, and are known as electrically small antennas (ESAs) [1]. The challenges associated with ESAs have been well-documented and identified since the 1940s, among which are low efficiency, narrow bandwidth, and radiation patterns restricted to the superposition of Hertzian dipole modes [2]. In response to these well-known problems of ESAs, a significant effort in the literature has been made to benchmark optimal ESA performance, as well as to find optimal ESA designs (see Fig. 1.1 and [2] for an extensive historical review). Consequently, with the ever-increasing consumer and private sector demand for small size and high data rate devices, small antenna theory remains a topic of pertinent interest today.

One of the central difficulties in modern antenna design relates to the lack of effective modeling techniques and systematic design methodologies for ESAs, in particular for ESAs mounted on finite platforms (in-situ ESAs). Ordinary ESA designs typically require large parametric studies on both the antenna geometry as well as its interaction with the mounting platform, and/or optimization routines requiring up to hundreds of full-wave simulations for a convergent solution, with little physical
insight gained. Moreover, while a well-posed optimization setup can reduce overall computational cost, a poorly-formulated optimization problem risks convergence to sub-optimal local solutions, or impractical parameter sensitivity. Therefore, more efficient tools, design methodologies, as well as fundamental physical insights are highly sought in order to optimize ESA and in-situ ESA antenna designs, subject to any number of antenna parameter requirements.

1.1 Dissertation Overview

In this dissertation, our goal is to present a comprehensive performance benchmarking and design methodology for ESA systems, particularly for arbitrary-shaped ESAs and in-situ ESAs mounted on small, finite platforms. Here, we focus our attention on two main questions:

- *Synthesis problem*: Given an available physical space, what is the optimal antenna that can be realized (see Fig 1.2)?
Given an available volume, what is the ideal antenna? How can we realize an optimal design?

Best antenna inside a cubic volume?

Figure 1.2: Synthesis problem: Finding the optimal ESA within a set of physical constraints

- Which location maximizes bandwidth?
- Which location maximizes gain in a certain direction?
- What antenna best interacts with the platform?

Figure 1.3: Placement problem: Finding the optimal mounting location for an ESA

- Placement problem: Given a finite mounting platform, where are the optimal mounting locations for a given ESA (or set of ESAs) to enhance gain and/or bandwidth?

To address the synthesis (Fig. 1.2) and platform problems (Fig. 1.3), the backbone of our research is drawn from the Theory of Characteristic Modes (CMs), discovered concurrently by Garbacz [3] and Harrington [4]. Succinctly, these modes tell us the relevant surface currents contributing to the radiation from an arbitrary-shaped...
antenna/scatterer. Characteristic modes will allow us to decompose an arbitrary-shaped radiating structure into its dominant radiating modes, revealing the effective mechanisms of radiation unique to that particular shape, as well as insight regarding excitation of these optimal modes. With this knowledge, we will show that CMs can be used to formulate accurate ESA benchmarks and design insights, as well as a new systematic methodology for the design of antennas on-board arbitrary-shaped finite platforms.

The key contributions to the field of antenna analysis and design presented in this dissertation are as follows:

- **A new method for computing the quality factor $Q$ for arbitrary-shaped electrically small antennas**

  This method for $Q$ computation uses only the Method of Moments (MoM) [5] impedance matrix $[Z]$, as well as the characteristic modes derived from $[Z]$. Consequently, this new method requires no modification to the traditional MoM kernel for arbitrary shapes, and is thus easily interfaced with existing MoM solvers with minimal effort and computational overhead in comparison to the state-of-the-art.

- **Fundamental limits on $Q$ for arbitrary-shaped ESAs and in-situ ESAs**

  In conjunction with our new formula for $Q$, the theory of CMs will be used to find fundamental lower bounds on $Q$ (and thus, bandwidth) for arbitrary-shaped ESAs and in-situ ESAs. Lower bounds on $Q$ subject to single as well as multi-moded radiation will be presented, through the use of CMs.
A systematic methodology for the feed and location optimization of antenna systems on-board arbitrary shaped finite platforms

We will show that based on the CMs of the supporting platform, optimal mounting regions for electrically small antennas can be identified. With these locations known, the appropriate feed voltages needed to synthesize a prescribed weighted sum of CMs can be realized through the solution of a new matrix equation involving the platform’s CMs.

Chapter 2 begins with a review of ESAs, including the ESA $Q$-factor and its fundamental lower bound. Chapter 3 is a review of the Theory of Characteristic modes, including its physical interpretation as well as basic applications to current and pattern synthesis. With the necessary background material, Chapter 4 presents a new method for the computation of $Q$ for arbitrary-shaped antennas using CMs. Fundamental lower bounds on $Q$ subject to specific radiation constraints will be discussed for volumetric and planar ESAs, as well as small mounting platforms. Finally, Chapter 5 demonstrates a new, CM-based design methodology for the location and feed optimization of ESAs on-board finite platforms.
Chapter 2: Electrically Small Antennas

2.1 Introduction

In this dissertation, we focus our attention on the design, optimization, and placement of electrically small antennas (ESAs) both in isolation as well as on-board finite platforms. Thus, before proceeding, it is highly beneficial to review the fundamental aspects of electrically small antennas (ESAs), and illustrate their common properties and characteristics. Here, our focus will be on four major quantities associated with ESAs: directivity, efficiency, bandwidth, and $Q$. While all four quantities are crucial to any ESA design, we pay particular attention to the antenna quality factor $Q$, as we will see there exists a fundamental lower bound as a function of antenna electrical size. Furthermore, a lower bound on $Q$ implies an upper bound on the potential bandwidth of an ESA. As we will see, computation of the fundamental limits on $Q$ for arbitrary-shaped antennas is one of the primary focuses of Chapter 4.

2.2 Electrically Small Antennas - Definition

The first formal definition of an electrically small antenna can be traced back to Wheeler [1], who defined an ESA as one whose maximum dimension is less than $\lambda/2\pi$, a quantity he referred to as a "radianlength". This definition of an ESA can
be reformulated an antenna that satisfies the condition

\[ ka < 0.5 \]  \hspace{1cm} (2.1) \]

where \( k \) is the free space wavenumber, and \( a \) is the radius of the minimum circumscribing sphere of the antenna (see Fig. 2.1), which hereafter we refer to as the "Chu sphere". Like all antennas, ESAs are characterized through their radiation resistances, efficiencies, and bandwidths, all of which typically decrease as a function of electrical size \( ka \). Another commonly cited definition of an ESA is \( ka < 1 \) \cite{6}, which is also boundary between the near and far field radiation for a Hertzian dipole. We note that in this work, we do not remain dogmatic in using (2.1) to define an electrically small antenna system, in particularly when we consider ESAs mounted on sub-wavelength platforms, where the composite antenna-platform is the actual mechanism of radiation. We will see that antenna systems of this type (Chapter 5) still exhibit the essential qualities of an ESA (e.g. narrowband, low efficiency), and hence we proceed to use the relevant benchmarks and techniques derived in subsequent sections and chapters for analyzing these cases.

2.2.1 Directivity and Gain

Small, single-fed antennas typically exhibit the radiation pattern of a Hertzian dipole of directivity \( D = 1.76 \, dBi \), which remains constant as electrical size \( ka \) decreases \cite{2}. For dipole-type ESAs, this radiation pattern is identical to that of the \( TM_{10} \) spherical mode, while for loop-type ESAs, the radiation pattern is that of the \( TE_{10} \) spherical mode (see Fig 2.2) \cite{7}. However, appropriate superpositions of dipole and loop-type ESAs can yield antennas with directivities up to \( D = 4.76 \, dBi \) \cite{8}, Kwon \cite{9,10}, and Pozar \cite{11}. Theoretically, multi-feed ESAs circumscribed by a Chu
Figure 2.1: Chu sphere circumscribing an ESA with maximum dimension 2a

Figure 2.2: Spherical $TM_{10}$ or $TE_{10}$ mode power pattern with the region $0 < \theta < 90$, $0 < \phi < 90$ omitted for clarity
sphere of radius $a$ are able to radiate higher order spherical modes, at the cost of rapidly increasing $Q$ (and thus reactive near fields) to impractical values [12]. As a result, there exist no practical implementations of such types of ESAs (which are also classified as superdirective antennas [6,13]).

Like all antennas, ESAs in practice are subject to material and feed network losses. These losses are typically quantified through the antenna radiation efficiency $\eta$, given as

$$\eta = \frac{P_{\text{rad}}}{P_{\text{loss}}}$$  \hspace{1cm} (2.2)$$

where $P_{\text{rad}}$ is the power radiated by the antenna, and $P_A$ is the power accepted by the input terminals. Here, $\eta$ includes the losses in the both the antenna as well as matching network due to mismatch and materials. Finally, the radiation efficiency can be linked to the antenna directivity $D$ and realized gain $G$ through $G = \eta D$. For narrowband ESAs whose input impedance is represented by a series reactance and resistance, $\eta$ can be represented as

$$\eta = \frac{R_{\text{loss}}}{R_{\text{loss}} + R_{\text{rad}}} = \frac{R_{\text{loss}}}{R_A}$$  \hspace{1cm} (2.3)$$

where $R_A$ is the total antenna input resistance $R_{\text{loss}} + R_{\text{rad}}$. Harrington [8] demonstrated that losses increase dramatically as electrical size $ka$ decreases, a consequence of frequency-dependent conduction and dielectric losses within the antenna.

2.2.2 Quality Factor and Bandwidth for ESAs

The concept of a quality factor $Q$ is one which frequently arises in the fields of circuit theory [14], microwave resonators [15], 2nd order filters [16], and ESAs. The quality factor for a general resonating system is defined as [14]

$$Q = \omega \frac{\text{Average Energy Stored}}{\text{Energy Loss/second}} = \omega \frac{W_M + W_E}{P_{\text{loss}}}$$  \hspace{1cm} (2.4)$$
where $W_M$ and $W_E$ are the stored magnetic and electric energies, respectively, and $P_{\text{loss}}$ is the power lost in the resonator. Of course, for a resonant system, $W_M = W_E$, and thus

$$Q = \frac{\omega}{P_{\text{loss}}} \frac{2W_M}{P_{\text{loss}}} = \frac{2W_E}{P_{\text{loss}}}.$$  

(2.5)

However, and most importantly, there exists a direct relationship between the $Q$ of a resonant system and its bandwidth. Since ESA input impedances can often be approximated in terms of simple RLC circuits [1], we first review the $Q$ of 2nd order RLC circuits to illustrate the direct relationships between $Q$ and bandwidth, before discussing the $Q$ of ESAs.

Quality Factor for RLC Circuits

Wheeler [1] recognized that a small antenna radiating the single spherical $TE_{10}$ mode can be accurately represented as the series RLC combination of Fig. 2.3. We note the series capacitor represents the ideal tuning element in (2.4) which brings the antenna to resonance. Similarly, a small antenna radiating only a spherical $TM_{10}$ mode can be accurately represented by the parallel RLC combination as in Fig. 2.3, where the shunt inductor represents the ideal tuning element in (2.4) that brings the antenna to resonance. More complicated, high-$Q$ circuits can be accurately represented as a series (for $X'_{in}(\omega) > 0$) or parallel (for $X'_{in}(\omega) < 0$) RLC circuits within the neighborhood of their resonant frequencies [12].

For the series RLC circuit of Fig. 2.3, the input impedance is derived as

$$Z_{in} = R + j\omega L - \frac{j}{\omega C} = R + j\omega L \left( \frac{\omega^2 - \omega_0^2}{\omega^2} \right).$$  

(2.6)
where $\omega_0 = \frac{1}{\sqrt{LC}}$ represents the resonant frequency at which the input impedance is purely real. This resonance occurs when the average stored electric energy is equal to the average stored magnetic energy in the circuit. Using the general definition of $Q$ in (2.4) and recognizing the current is the same in all circuit components, we find that

$$Q = \frac{2\omega_0 W_M}{P_A} = \frac{2\omega_0 \left(\frac{1}{4}LI^2\right)}{\frac{1}{2}I^2R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$  \hspace{1cm} (2.7)$$

where $I$ is the current through the series RLC circuit in Fig. 2.3. The bandwidth of the series RLC circuit can be estimated after introducing the approximation

$$F(\omega) = \omega^2 - \omega_0^2 \approx F(\omega_0) + (\omega + \omega_0)F'(\omega_0) = 2\omega \Delta\omega$$ \hspace{1cm} (2.8)$$

valid for small $\Delta\omega = \omega - \omega_0$. With this Taylor series, (2.6) becomes

$$Z_{in} = R + j\omega L\Delta\omega$$ \hspace{1cm} (2.9)$$

From (2.9), it is then evident the 3dB points occur when

$$2L\Delta\omega_{3dB} = \pm R$$ \hspace{1cm} (2.10)$$
where $\omega_{3dB}$ is the difference between the 3dB frequency and resonant frequency. Using equations (1.6) and (1.9) we can now write

$$2L \Delta \frac{\omega_{3dB}}{\omega_0} = Q \cdot FBW_{3dB} = 1$$  \hspace{1cm} (2.11)

where $FBW_{3dB} = \Delta \omega_{3dB}/\omega_0$. From this result, we then have the relationship $FBW_{3dB} = 1/Q$. Fig. 2.4 depicts the impedance as a function of frequency for a typical series RLC circuit for various $Q >> 1$ values.

For the parallel RLC circuit of Figure 1.5(b), the input admittance is

$$Y_{in} = G + j\omega C - \frac{j}{\omega L} = G + j\omega C \left( \frac{\omega^2 - \omega_0^2}{\omega^2} \right)$$  \hspace{1cm} (2.12)

where $\omega_0$ is again the resonant frequency for which the input admittance is purely real. Using the general definition of $Q$ and recognizing for the parallel RLC circuit the voltage $V$ across each component is the same, the $Q$ for the parallel RLC circuit
Figure 2.5: Normalized impedance magnitude for a parallel RLC circuit near resonance

at resonance is found to be

\[ Q = \frac{2\omega_0 W_E}{P_A} = \frac{2\omega_0 \left( \frac{1}{4} CV^2 \right)}{\frac{1}{2} V^2 G} = \frac{\omega_0 C}{G} = \frac{1}{\omega_0 GL} \]  

(2.13)

From the dual nature of the series and parallel RLC circuits, the same bandwidth relations obtained in (2.11) hold for the parallel RLC circuit. Fig. 2.5 depicts the impedance as a function of frequency for a typical parallel RLC circuit having \( Q \gg 1 \).

**ESA Quality Factor**

As noted previously, ESAs are typically not self-resonant, and thus the definition of \( Q \) for a resonant system in (2.5) is often not applicable directly. However, as is conventional with ESA theory, one can always assume that the ESA can be tuned to resonance by an ideal reactance. Under this assumption, the ESA \( Q \) is given as [17]

\[ Q = \frac{2\omega}{P_A} \max(W_M, W_E) \]  

(2.14)
where $\omega$ is the radian frequency, $P_A$ is the power accepted by the antenna, and $W_M$ and $W_E$ are the stored magnetic and electric energies of the ESA, respectively. We further note that the power accepted by the antenna is related to the radiated power through $P_{rad} = \eta P_A$, where $\eta$ is the antenna efficiency described previously.

For an antenna with a single feed port, the $Q$ of an ESA can be computed using the feed-point impedance as [17]

$$Q \approx \frac{\omega}{2R_A} \sqrt{\left(\frac{dR_A}{d\omega}\right)^2 + \left(\frac{dX_A}{d\omega} + \frac{|X_A|}{\omega}\right)^2}$$

where $Z_A(\omega) = R_A(\omega) + jX_A(\omega)$ is the frequency-dependent feed point impedance of the antenna. Note that for an ESA represented by the RLC circuits of Fig. 2.3 at resonance, (2.15) becomes (2.7) for the series case and (2.13) for the parallel case.

In a manner similar to 2nd order RLC circuit networks, the antenna $Q$ can be related to the bandwidth at the feed point of the antenna, as [17]

$$Q(\omega) \approx \frac{2}{FBW_{3dB}(\omega)}$$

where $FBW_{3dB}(\omega)$ is the matched 3dB fractional bandwidth of the antenna, assuming the antenna is tuned to resonance with an ideal lumped reactance, and fed with a transmission line of characteristic impedance $Z_C = R_A$. Note that the factor of 2 in (2.16) in comparison to (2.11) is due to the assumption of a source impedance $R_A$.

**Fundamental Limits on $Q$ for ESAs**

As discussed in this chapter, due to their small size, ESAs are inherently plagued by poor radiation efficiency, narrow bandwidth, and a radiation pattern restricted to linear combinations of the 1st order spherical modes. Modern design challenges
concerning ESAs are primarily focused on techniques to improve efficiency and bandwidth (see [2]). To that effort, there has been a significant effort in the literature to quantify optimal ESA performance as a function of size and material parameters.

While (2.15) yields the $Q$ (and hence, bandwidth) of a particular ESA, a fundamental question arises: what is the smallest antenna $Q$ possible for an ESA? The first idea of a minimum possible $Q$ antenna was first explored in an approximate method by Wheeler [1]. Wheeler considered the limits of small cylindrical capacitive and inductive antennas in terms of stored to radiated power, dubbed the “radiation power factor” ($p_{e,m}$). Representing the ESA input impedance by a capacitor or inductor with a radiation resistance, Wheeler determined the radiation power factor to be

$$p_{e,m} = \frac{R_{rad}}{X_{C,L}} = \frac{\sigma_{e,m} V_{cyl}}{6\pi l^3}$$

where subscript e (m) represents a cylindrical capacitive (inductive) antenna, $X_{C,L}$ is the reactance of the capacitive or inductive antenna, $R_{rad}$ is the radiation resistance, $l = \lambda/2\pi$ is the radianlength, $V_{cyl}$ is the volume of the cylinder enclosing the ESA, and $\sigma$ is a material and structural shape factor. Although Wheeler provided formulas in the context of $p_{e,m}$, it can be seen that $p_{e,m} = 1/Q_{e,m}$ (see RLC circuits in Fig. 2.3). Thus, from these simplified capacitive and inductive antennas, Wheeler established a clear connection between antenna electrical size and operational bandwidth. That is, with decreasing $p_{e,m}$, a reduction in operational bandwidth is observed [1].

While Wheeler’s conclusions between $Q$ and antenna size remain valid, his cylindrical antenna model approximation represents a small class of ESA designs. Later, an exact method for computing the $Q$ limits of ESAs was proposed famously by Chu [12]. Chu concluded that the fundamental lower bound on $Q$ for an ESA circumscribed by a Chu sphere (see Fig. 2.1) would have to be one which had zero
energy within the Chu sphere, and radiated a purely a lowest-order spherical mode outside the Chu sphere. The fundamental lower bound on $Q$ can then be written mathematically as

$$Q_{\text{min}} = \frac{1}{n} \left( \frac{1}{(ka)^3} + \frac{n}{ka} \right),$$  \hspace{1cm} (2.18)

where $n = 1$ assuming the antenna excites $TM_{10}$ or $TE_{10}$ radiation, and $n = 2$ for $TM_{10}$ and $TE_{10}$ circularly polarized radiation.

While (2.18) provides the absolute lower bound on $Q$ for any ESA, one finds that the limit given in (2.18) is far lower than the $Q$ values computed for typical small antenna geometries using (2.15). To illustrate, Fig. 2.6 depicts the $Q$ of several common ESAs (each circumscribed by a Chu sphere of radius $a$) compared to the $n=1$ Chu limit in (2.18). Clearly, it is seen that the $Q$ for practical shapes that do not well conform to the sphere are often orders of magnitude larger than the Chu limit. Hence, for accurate performance benchmarking, the Chu limit offers little utility, and no insight on what an optimal non-spherically shaped antenna might be.

Following the work of Chu, and a goal to determine the $Q$ of non-spherical structures, numerous methods have been presented with the aim to efficiently compute (2.14) and, thus, find the $Q$ limits for non-spherical shapes. Towards this goal, Foltz [18] and Hansen [6] computed the $Q$ limits for antennas whose shape was represented by prolate and oblate spheroids, by using spheroidal wavefunctions to represent the fields outside the spheroid’s volume. More recent efforts to determine the minimum $Q$ for arbitrary-shaped geometries were reported by Gustafsson [19, 20] and Yaghjian [21], using static polarizabilities in the long-wavelength limit ($ka \to 0$). However, these formulations require approximations on antenna absorption efficiency [19], and on the form of stored energy [19,21]. As such, they still
do not provide insight on the optimal currents (and hence, optimal antenna designs) that imply a minimum $Q$. Alternatively, Vandenbosch [22, 23] presented a method for computing the $Q$ of arbitrary-shaped antennas using integrals involving the actual antenna currents and charges. He further demonstrated that the minimum $Q$ currents can be extracted by determining a singular solution to an integral equation [23]. However, as stated in [23], care in numerical precision must be ensured and root finding algorithms must be employed to yield an accurate solution. In Chapter 4, we present new method for computing accurate lower bounds on $Q$ for arbitrary-shaped PEC antennas, directly computed using the method of moments impedance matrix $[Z]$. A by-product of this approach is the knowledge of the optimal currents that minimize $Q$. 

Figure 2.6: $Q$ of several common ESAs computed at the feed point compared to the Chu limit $Q_{\text{min}}(n = 1)$ in (2.18)
Bode-Fano Limits on Matching Networks

Finally, we conclude by noting that in addition to fundamental limits on antenna $Q$, the Bode-Fano limits dictate the maximum bandwidth possible from an ideal matching network, given a specific load impedance [24]. For ESAs exhibiting a narrowband input impedance, the fundamental bounds on lossless passive matching networks were derived by Fano [24–26] as

$$FBW_{3dB} \cdot Q \leq \frac{\pi}{ln \left( \frac{1}{\Gamma_{max}} \right)}$$

(2.19)

where $FBW_{3dB}$ is the 3dB fractional bandwidth, $\Gamma_{max}$ is the maximum allowable reflection coefficient in the passband, and $Q$ is the quality factor of the load (antenna) to be matched. The interpretation of (2.19) is depicted in Fig. 2.7 for $B = 0.25$, 0.5, and 1, with $Q = \pi$. The fundamental limitation given by (2.19) indicates that greater bandwidth can only be achieved at the cost of increased maximum reflection coefficient (less realized gain).

2.3 ESAs mounted on finite platforms

The traditional antenna design process typically involves the construction and parameter optimization within an isolated or analytical environment, such as free space or an infinite ground plane. Following such a design, the optimized ESA is then placed on the mounting platform or vehicle. However, as is often seen in ESA design, the mounting platform is often times a fraction of the operational wavelength, introducing significant perturbations to the radiation characteristics of the ESA in isolation. Fig. 2.8 depicts the wingspan of several common Unmanned Aerial Vehicles (UAVs) in wavelengths for the low-HF band (2 to 10MHz). Such a band is highly
Figure 2.7: Bode limit on reflection coefficient $\Gamma$ for various $FBW_{3dB}$ values

useful for beyond line of sight (BLOS) communications by utilizing ionosphere reflections [27]. Here, it is seen that within the low-HF band, the wingspan for each case is less than a wavelength, and thus the traditional infinite ground plane or free-space approximations are no longer valid.
### Wingspan of common UAVs (in wavelengths)

<table>
<thead>
<tr>
<th></th>
<th>2 MHz</th>
<th>10 MHz</th>
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</thead>
<tbody>
<tr>
<td>Predator</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>PUMA</td>
<td>0.019</td>
<td>0.09</td>
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<tr>
<td>Gray Eagle</td>
<td>0.11</td>
<td>0.57</td>
</tr>
<tr>
<td>X-45</td>
<td>0.05</td>
<td>0.27</td>
</tr>
</tbody>
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Figure 2.8: Wingspan (in wavelengths) of several common UAVs in the low-HF band

To illustrate the effect that a sub-wavelength mounting platform has on an ESA, Fig. 2.9 depicts an electrically small monopole (λ/20 at 100MHz) mounted on three different locations of a rectangular PEC plate (λ/3 × λ/4 at 100MHz). Figs. 2.10 and 2.11 show the resulting input resistance and reactance for the monopole placed at each of the three mounting locations (center, side, and corner), as well as the monopole impedance when placed on an infinite ground plane. While for each case the input reactance remains nearly the same, a significant variation in the input resistance is seen for each of the mounting locations, with the corner providing the lowest Q solution. Thus, a strategic placement of an ESA on-board small mounting platforms, by considering the composite ESA and platform structure as the total radiating body, can provide dramatic performance improvements in terms of radiation parameters.
Figure 2.9: Electrically small monopole ($\lambda/20$ at 100MHz) with three mounting locations of a rectangular PEC plate ($\lambda/3 \times \lambda/4$ at 100MHz)

Figure 2.10: Input resistance for each monopole mounting location in Fig. 2.9
Figure 2.11: Input reactance for each monopole mounting location in Fig. 2.9

2.4 Summary

In this chapter, the essential quantities associated with ESAs were reviewed, including directivity, efficiency, bandwidth, and $Q$ as a function of electrical size $ka$. As stated in the introduction, one of the primary challenges in modern ESA design is the lack of effective modeling techniques and performance prediction markers. To that effort, we first consider the problem posed by the ESA synthesis (see Fig. 1.2). In observing Fig. 1.2, we hypothesize that if we are able to obtain a set of functions encapsulating the radiation properties of the available space, optimal antenna solutions could be realized from such a decomposition. To that effort, a solution exists: the Theory of Characteristic Modes (CMs) [4]. Succinctly, the CMs are the radiation eigenmodes of an arbitrary-shaped antenna. Furthermore and to our advantage, only a small number of modes are needed to describe the radiation properties in the electrically small regime. Thus, our task in this dissertation is to fully exploit the use of
CMs for ESA design. In the next chapter, we will review the Theory of Characteristic Modes, and their application to the benchmarking and design of electrically small radiating structures in Chapters 4 and 5.
Chapter 3: Theory of Characteristic Modes

3.1 Introduction

In Chapter 2, we alluded to the lack of effective modeling techniques and performance prediction markers for the modern ESA designer. Like the design of any physical system, further insights into the fundamental physics of the problem translate into effective design methodologies and optimization schemes. For example, when designing a waveguide feed, it is critical to have knowledge of the waveguide’s inherent frequency-dependent guiding mechanisms: its eigenmodes. As a result, knowledge of the solutions to the source-free wave equation satisfying the waveguide boundary conditions will quickly illustrate the optimal locations for a feed probe as a function of frequency. Furthermore, with the advent of sophisticated numerical solvers, finding the eigenmodes of arbitrary-shaped waveguide cross-sections becomes a trivial task, significantly speeding up the design process for a wide variety of problems.

Analogous to numerically determining the eigenmodes of arbitrary-shaped waveguide cross-sections, the Theory of Characteristic Modes (CMs), discovered concurrently by Garbacz [3] and Harrington [4], acts as the analogue for scattering (or equivalently, antenna) theory. Characteristic Modes is a numerical method, based on the Electric Field Integral Equation (EFIE) Method of Moments (MoM) impedance
matrix \([Z]\) \cite{5}, which decomposes radiation performance of an arbitrary-shaped conducting surface in terms of an orthogonal set of modal currents and modal eigenvalues. Furthermore, it will be seen that for electrically small and sub-wavelength scatterers (e.g. ESAs and their supporting platforms), only a small number of CMs are necessary to describe the radiation properties. By accounting for only the low-order (low stored energy) CMs, we are able to identify optimal radiation and feed conditions, the basis of which forms Chapters 4 and 5.

In this chapter, we will review the basics of CM theory, including its computation from \([Z]\), physical interpretation eigencurrents and eigenvalues, orthogonality relationships, and practical limitations. We will also demonstrate an example of using CMs to realize a bidirectional pattern at 10MHz using the CMs of unmanned aerial vehicle (UAV) model in FEKO \cite{28}.

### 3.2 Theory of Characteristic Modes

Consider an arbitrary-shaped perfectly conducting (PEC) antenna/scatterer characterized by the Electric Field Integral Equation (EFIE) moment method (MoM) impedance matrix \([Z]\) (of dimension \(N \times N\)). The characteristic modes of this antenna/scatterer are the solutions to the generalized eigenvalue equation \cite{4}

\[
[X] \{I_n\} = \lambda_n [R] \{I_n\}
\]  

(3.1)

where \([R]\) and \([X]\) are the real and imaginary parts of the symmetric EFIE MoM impedance matrix \([Z] = [R] + j[X]\), \(\{I_n\}\) is the \(n^{th}\) eigencurrent, \(\lambda_n\) is the \(n^{th}\) eigenvalue. Here, we assume that the CMs are arranged in ascending order (\(|\lambda_1| < |\lambda_2| < \cdots < |\lambda_N|\)).
Since \([R]\) and \([X]\) are symmetric, and assuming \([R]\) is formulated as positive definite, the eigenvalues and eigencurrents are real-valued, and the set of \(\{I_n\}\) constitutes an orthogonal basis with respect to the \([R]\), \([X]\), and \([Z]\) matrices on the antenna surface [4]. Thus, after normalizing each eigencurrent to radiate unit power, we have the following orthogonality relations [4]

\[
\frac{1}{2} \{I_n\}^T [R] \{I_m\} = \delta_{nm} \tag{3.2}
\]

\[
\frac{1}{2} \{I_n\}^T [X] \{I_m\} = \lambda_n \delta_{nm} = 2\omega (W_{M,n} - W_{E,n}) \tag{3.3}
\]

where \(W_{M,n}\) and \(W_{E,n}\) are the stored magnetic and electric energies for the \(n^{th}\) eigencurrent per unit radiated power, \(\delta_{nm}\) is the Kronecker delta function, superscript \(T\) denotes the transpose, and the 1/2 factor accounts for time-averaged quantities. One common figure of merit frequently seen in the literature is Modal Significance, given as

\[
MS_k = \frac{1}{\sqrt{1 + \lambda_k^2}} \tag{3.4}
\]

where \(MS_k\) is the Modal Significance of CM \(k\). From (3.3) is it seen that a CMs with \(MS_k = 0\) indicate a resonating mode, while CMs with large \(MS_k\) imply evanescent, stored energy modes.

Finally, we remark that in addition to current orthogonality over the antenna surface, the set of far-field patterns radiated by each eigencurrent \(\{I_n\}\), denoted as the set of eigenpatterns, \(F_n\), are also orthogonal over the far-field unit sphere \(S_\infty\) [4]

\[
\int_{S_\infty} F_n^* \cdot F_m d\Omega = 2Z_0 \delta_{nm} \tag{3.5}
\]

where \(d\Omega = \sin \theta d\theta d\phi\), and \(Z_0\) is the free-space wave impedance.

For double precision \([Z]\) matrix sizes on the order of up to \(10^4\) unknowns, (3.1) can be solved efficiently using MATLAB’s \textit{eigs} command [29], which is effectively a
wrapper for ARPACK [30] in FORTRAN. The \textit{eigs} command solves the generalized eigenvalue equation for a specified number of modes, beginning with the lowest-order (smallest CM $|\lambda_1|$) and ascending. Since ARPACK is an iterative method for computing a small number of eigenvalues and eigenvectors of large matrices, numerical computation of the entire CMs spectrum in (3.1) is not typically a practical approach. Our work has empirically shown that for common $[Z]$ matrices of electrically small bodies, computation of $|\lambda_k| > 10^6$ using \textit{eigs} leads to inaccurate eigenvalues and eigencurrents, including complex-valued entries (which should not be seen obtained given the positive definite and symmetric properties of (3.1)). For the computation of larger, more complex systems, higher precision as well as memory allocation methods provided by FORTRAN and direct access to the ARPACK library [30] offer a mitigation. However, CMs whose eigenvalues $|\lambda_k| > 10^6$ are largely evanescent, and would be undesirable and/or impossible to excite for most practical situations, as we will see in Chapter 4.

### 3.2.1 CMs of a Perfectly Conducting Sphere

To illustrate the application of Characteristic Mode theory, the lowest-order CMs of a PEC sphere (see Fig. 3.1) of radius $a$ are computed using the $[Z]$ matrix generated by FEKO [28] (with 786 triangles) as well as the \textit{eigs} command in MATLAB [29]. Figs. 3.2 and 3.3 depict the resulting eigencurrents and eigenvalues. From Fig. 3.2, it is clear that the CMs of the sphere are in the form of multipole moments. We also note that for the sphere, the eigenvalues are degenerate. That is, for each eigenvalue of the sphere, there exist multiple orthogonal eigencurrents. For example, $\lambda_1$ generates
three eigencurrents corresponding to the electric dipole moments oriented on the $x$, $y$, and $z$ (pictured in Fig. 3.2) axes.

![Sphere model in FEKO [28] with associated 786 triangle mesh.](image1)

**Figure 3.1:** Sphere model in FEKO [28] with associated 786 triangle mesh.

![First four distinct eigencurrents for the sphere of radius a supporting the radiation patterns of an electric dipole (CMs 1), magnetic dipole (CMs 2), electric quadrupole (CMs 3), and magenetic quadrupole (CMs 4).](image2)

**Figure 3.2:** First four distinct eigencurrents for the sphere of radius $a$ supporting the radiation patterns of an electric dipole (CMs 1), magnetic dipole (CMs 2), electric quadrupole (CMs 3), and magnetic quadrupole (CMs 4).
One of the benefits of CM theory is the ability to theoretically synthesize any desired antenna pattern by properly exciting the inherent current modes of the radiating body. Based on the orthogonality properties given in (3.2) and (3.3), the eigencurrents \( \{ \mathbf{I}_n \} \) can be used as a basis to represent any current \( \{ \mathbf{I}_0 \} \) on the antenna body in a least-squares sense [4] as

\[
\{ \mathbf{I}_0 \} \approx \sum_{n=1}^{M} \alpha_n \{ \mathbf{I}_n \} = \sum_{n=1}^{M} \frac{1}{2} \left( \frac{\{ \mathbf{I}_n \}^T \{ \mathbf{V}_0 \}}{1 + j\lambda_n} \right) \{ \mathbf{I}_n \}
\]  

(3.6)

where \( \alpha_n \) is

\[
\alpha_n = \frac{1}{2} \left( \frac{\{ \mathbf{I}_n \}^T \{ \mathbf{V}_0 \}}{1 + j\lambda_n} \right)
\]  

(3.7)

and is the \( n^{th} \) excitation coefficient for CM \( n \), and \( \{ \mathbf{V}_0 \} = [Z] \{ \mathbf{I}_0 \} \) is the incident tangential electric field generating \( \{ \mathbf{I}_0 \} \). Assuming the excitation delivers all its power
to the antenna, the far-field and radiation properties are accurately represented by the $M$ lowest-order CMs when $\sum_{n=1}^{M} |\alpha_n|^2 \approx P_{rad}$ - a feature typically seen in electrically small scatterers. Consequently, the characteristic modes approach is most well-suited for electrically small surfaces, where only a small number of dominant radiating modes are present.

It is important to recognize that not only do the eigencurrents $\{I_n\}$ exhibit orthogonality over the surface of the antenna, but their far-field patterns $F_n$ are orthogonal as well (see (3.5)). Hence, the far-field $F_0$ radiated by the current $\{I_0\} = \sum_{n=1}^{M} \alpha_n \{I_n\}$ is given by

$$F_0 = \sum_{n=1}^{M} \alpha_n F_n$$

(3.8)

where the weighting coefficients $\alpha_n$ can be computed over the far-field sphere as

$$\alpha_n = \frac{1}{2Z_0} \int_{S_\infty} F_n^* \cdot F_0 d\Omega$$

(3.9)

and are identical to those given in (3.7), assuming each eigencurrent and eigenpattern has been normalized to radiate unit power.

Recall that $\{I_n\}$ is a real current mode on the antenna surface, and $\lambda_n$ is the ratio of stored to radiated power for mode $n$. Although there exist $N$ eigencurrent and eigenvalue pairs in (3.1), modes associated with large $MS_k$ (insignificant CMs) cannot be radiated effectively, which limits the number of usable modes in a design. Modes with small $MS_k$ (significant CMs) are efficient radiators and can be used in the design process. This concedes a fundamental physical limitation with using CMs for pattern synthesis: due to a limited number of significant modes at a given frequency, only a subset of eigenpatterns can truly be used in a practical CM design when real feeds are used.
3.3.1 Synthesis of a Bidirectional Pattern Using UAV CMs

Here, we illustrate the use of the current and pattern synthesis equations in (3.6) and (3.8) to synthesize the two different bidirectional radiation patterns. First, we introduce a FEKO [28] PEC model in Fig. 3.4 of an unmanned aerial vehicle (UAV) operating at 10MHz, discretized by 1014 triangles. The UAV wingspan is approximately 14.7m, and the fuselage/body length is approximately 9.02m.

Fig. 3.5 depicts the modal significance of the UAV from 1-10MHz, and snapshots of the eigencurrents (Fig. 3.6) and eigenpatterns (Fig. 3.7) are shown at 10MHz. We note that for electrically small antennas/scatterers, the eigencurrents and eigenpatterns vary slowly as a function of frequency and typically retain their same general shape, prior to self-resonance [31, 32]. Hence, we can assume these snapshots for the eigencurrents and eigenpatterns are a reasonable representation across the entire band. Observing Fig. 3.5, it is seen that CMs 1 and 2, the dipole modes of the wing and fuselage, respectively (see Fig. 3.6) are the dominant radiating modes when the UAV is sub-wavelength. It will be seen in the next chapter that excitation of these dominant modes corresponds to broader potential bandwidth.

To demonstrate the use of (3.6) and (3.8), we examine the synthesis of two different bidirectional patterns using the UAV CMs, at a frequency of 10MHz using the first $M = 10$ CMs. Fig. 3.8 depicts the radiation pattern associated with a crossed set of Hertzian electric dipoles, phased 90° with equal magnitude. The resulting far-field pattern using the first $M = 10$ CMs is shown in Fig. 3.8, where the CM expansion coefficients are computed using (3.9), with the far-field approximated using $N_\theta = 91 \theta$ sample points and $N_\phi = 181 \phi$ sample points for the far-field sphere. The resulting CM coefficients for Fig. 3.8 are shown in Table 3.1, where it is obvious that the first
Figure 3.4: FEKO UAV model at 10MHz, with a wingspan of 14.7m and fuselage/body length of 9.02m

Figure 3.5: UAV modal significance from 1MHz - 10MHz
two UAV CMs - the dipole modes of the wing and body - dominate in the synthesized pattern. This is further reflected by observing the least-squared pattern error as a function of the number of CMs, shown in Fig. 3.10. Here we define the least-squared pattern error $LSQ$ as

$$LSQ = \frac{\int_{S_{\infty}} |F_{D} - F_{0}|^2 d\Omega}{\int_{S_{\infty}} |F_{D}|^2 d\Omega}$$

where $F_{D}$ is the desired pattern, and $F_{0}$ is the CM synthesized pattern given in (3.8). Here, it is seen that the synthesis of the bidirectional pattern is near-optimal after only the first two CMs. Thus, when using CMs in the design process, a judicious choice of the desired pattern (typically realized through elementary sources) can radically improve the performance when using the low-order, dominant modes.

We next examine a slightly modified case of the previous bidirectional example, in which we now rotate the x-axis dipole $45^\circ$ towards the z-axis, with the second dipole remaining the same (see Fig. 3.9). As before, the dipoles are phased $90^\circ$ from one another, and the resulting far-field pattern is identical to that of the previous case, with the radiation pattern tilted $45^\circ$ in elevation angle. As previously, the rotated bidirectional pattern is synthesized using (3.8) and (3.9), with the first $M = 10$ UAV CMs, where again using $M = 10$ CMs we see an accurate representation of the desired pattern. However, for this case, we observe that the dipole mode of the fuselage (CM 2) is no longer the secondary dominant contributor as in the previous case, but an appropriate linear combination of CMs 2, 3, 4, and 8 are needed in order to realize the $45^\circ$ tilted dipole. This is further reflected by observing the $LSQ$ as a function of the number of CMs used in Fig. 3.10, where at least $M = 4$ CMs are needed for a reasonable approximation to the desired pattern. However, we will see in the
following section, this will come at the cost of the potential bandwidth, as well as the number of degrees of freedom needed when exciting the UAV.

| $|\alpha_1|$ | 0.707 | 0.707 |
| $|\alpha_2|$ | 0.695 | 0.453 |
| $|\alpha_3|$ | 0.112 | 0.127 |
| $|\alpha_4|$ | 0.050 | 0.503 |
| $|\alpha_5|$ | $3.97 \times 10^{-3}$ | $4.00 \times 10^{-3}$ |
| $|\alpha_6|$ | 0.010 | 0.050 |
| $|\alpha_7|$ | $8.17 \times 10^{-3}$ | $7.83 \times 10^{-3}$ |
| $|\alpha_8|$ | $6.68 \times 10^{-3}$ | 0.113 |
| $|\alpha_9|$ | $4.83 \times 10^{-6}$ | $6.28 \times 10^{-5}$ |
| $|\alpha_{10}|$ | $7.77 \times 10^{-3}$ | 0.082 |
Figure 3.6: First $N = 9$ UAV eigencurrent magnitudes at 10MHz
Figure 3.7: First $N = 9$ UAV eigenpatterns (radiated by the eigencurrents of Fig. 3.6 at 10MHz)
Figure 3.8: (a) Crossed set of electric dipoles in the $xy$ plane, phased 90° apart with far-field pattern; (b) Synthesized far-field using the first $M = 10$ CMs of the UAV of Fig. 3.4

Figure 3.9: (a) Crossed set of electric dipoles, phased 90° apart and rotated 45° from the $xy$ plane with far-field pattern; (b) Synthesized far-field using the first $M = 10$ CMs of the UAV of Fig. 3.4
Figure 3.10: Least-squared pattern error (%) for the synthesized patterns of Fig. 3.8 and 3.9 as a function of the number of CMs $M$ used.

### 3.4 Summary

In this chapter, we introduced the Theory of Characteristic Modes - a numerical method which extracts the radiation eigenmodes unique to a particular conducting shape. To our advantage, in the electrically small regime, only a small number of radiating modes dominate the performance. The set of CMs for an arbitrary shape consists of its eigencurrents, eigenpatterns, and eigenvalues. The eigencurrents are real-valued currents conforming to the shape under consideration, which also exhibit orthogonality with respect to the impedance matrix. The far-field patterns radiated by each eigencurrent, denoted as eigenpatterns, are also orthogonal to one another over the far-field sphere. The eigenvalues, again real valued, represent the ratio of stored to radiated power for the particular CM, with the sign indicating whether the
CM is capacitive or inductive. Finally, we explored the primary use of CM theory - current and pattern synthesis. We demonstrated the synthesis of two different bidirectional patterns, as well as an analysis of the pattern accuracy as a function of the number of modes used.

With the necessary background knowledge of small antennas and CM theory, we now proceed to address the synthesis (Fig. 1.2) and placement (Fig. 1.3) problems for ESAs, using the CMs as the backbone of our work.
Chapter 4: Q Limits for Arbitrary-Shaped Antennas Using CMs

4.1 Introduction

As outlined in the previous section, Characteristic Modes (CMs) can be interpreted as the radiation eigenmodes of an antenna. In this chapter, our objective is to determine which current modes, when excited, will minimize $Q$ given in (2.14). Since computing characteristic modes is a numerical approach, we no longer need to restrict ourselves to finding the $Q$ of analytical geometries, and can generalize our approach to arbitrary-shaped ESAs. In the following analysis, we assume all antennas are composed of perfectly conducting (PEC) materials.

We begin this chapter by deriving a new formula for the $Q$ of a current distribution radiating in free space. This formula, in conjunction with characteristic mode (CM) theory, is subsequently used to compute the antenna $Q$ using only the surface currents associated with the EFIE Method of Moments (MoM) impedance matrix $[Z]$. Furthermore, we will exploit this formula to find the minimum bounds on $Q$ for arbitrary-shaped radiating bodies, as well as limits on $Q$ subject to a prescribed
radiation pattern. We conclude by computing the minimum $Q$ bounds of several volumetric and planar shapes, frequently encountered in antenna design. Comparisons with literature data are also provided when appropriate.

### 4.2 Quality Factor for an Arbitrary-Shaped Antenna

To find a formula for $Q$, we first consider an impressed current $J_S$ confined to an electrically small volume $V_A$, as shown in Fig. 4.1, and radiating in the free space volume $V_\infty$. For this configuration, Maxwell’s equations are given by

\begin{equation}
\nabla \times \mathbf{E} = -j\omega \mathbf{B}
\end{equation}

\begin{equation}
\nabla \times \mathbf{H} = j\omega \mathbf{D} + \mathbf{J}_S
\end{equation}
where \( D = \epsilon_0 E \) and \( B = \mu_0 H \). Poynting’s theorem for the system described in Fig. 4.1 then gives

\[
- \frac{1}{2} \int_{V_A} J_s^* \cdot E \, dv = \frac{1}{2} \oint_{S_\infty} E \times H^* \cdot \hat{r} r d\Omega + j \frac{\omega}{2} \int_{V_\infty} (\mu_0 |H|^2 - \epsilon_0 |E|^2) \, dv \tag{4.3}
\]

with \( d\Omega = \sin \theta d\theta \, d\Phi \). From (4.3), equations for the radiated power \( P_{rad} \) and stored energy difference \( \omega (W_M - W_E) \) are given by

\[
P_{rad} = - \frac{1}{2} \Re \int_{V_A} J_s^* \cdot E \, dv \tag{4.4}
\]

and

\[
\omega (W_M - W_E) = - \frac{1}{4} \Im \int_{V_A} J_s^* \cdot E \, dv . \tag{4.5}
\]

To evaluate (2.14), in addition to using (4.4) and (4.5), we require another independent equation energy balance equation in order to determine \( W_M \) and \( W_E \) uniquely. To accomplish this, the frequency derivatives of Maxwell’s equations can be used to yield an additional energy expression involving the radiated power and stored energies. Following [17], [22], and [33], and taking the frequency derivative of (4.1) and (4.2), we have

\[
\nabla \times E' = - j \omega B' - j B \tag{4.6}
\]

\[
\nabla \times H' = j \omega D' + j D + J_s' \tag{4.7}
\]

where the prime denotes \( \zeta' = \partial \zeta / \partial \omega \). Our goal is now to appropriately couple equations (4.1)-(4.2) and (4.6)-(4.7) to obtain the sum \( \omega (W_M + W_E) \). Dotting (4.6) with \( H^* \) and the conjugate of (4.2) with \( -E' \), we get

\[
(\nabla \times E') \cdot H^* = - j \omega B' \cdot H^* - j B \cdot H^* \tag{4.8}
\]

\[
-(\nabla \times H') \cdot E' = j \omega D^* \cdot E' - J_s^* \cdot E' . \tag{4.9}
\]
Two additional expressions can be derived by taking the dot product of the conjugate of (4.7) with \( \mathbf{E} \) and (4.1) with \(-\mathbf{H}'^*\) to yield
\[
(\nabla \times \mathbf{H}'^* ) \cdot \mathbf{E} = -j\omega \mathbf{D}'^* \cdot \mathbf{E} - j \mathbf{D}' \cdot \mathbf{E} + \mathbf{J}_s'^* \cdot \mathbf{E} \tag{4.10}
\]
\[
-(\nabla \times \mathbf{E}) \cdot \mathbf{H}'^* = j\omega \mathbf{B} \cdot \mathbf{H}'^* \tag{4.11}
\]
Next, upon summing equations (4.8)-(4.11), and applying the vector identity \( \nabla \cdot (\mathbf{X} \times \mathbf{Y}) = \mathbf{Y} \cdot (\nabla \times \mathbf{X}) - \mathbf{X} \cdot (\nabla \times \mathbf{Y}) \), we acquire the frequency derivative energy equation
\[
\nabla \cdot [\mathbf{E}' \times \mathbf{H}^* - \mathbf{E} \times \mathbf{H}'^* ]
\]
\[
= -j \left( \mu_0 |\mathbf{H}|^2 + \epsilon_0 |\mathbf{E}|^2 \right) - \mathbf{J}_s^* \cdot \mathbf{E}' + \mathbf{J}_s'^* \cdot \mathbf{E} \tag{4.12}
\]
Integrating (4.12) over the entire space and invoking the divergence theorem yields
\[
-Im \int_{V_A} (\mathbf{J}_s'^* \cdot \mathbf{E} - \mathbf{J}_s^* \cdot \mathbf{E}') \, dv
\]
\[
= \int_{V_\infty} \left( \mu_0 |\mathbf{H}|^2 + \epsilon_0 |\mathbf{E}|^2 \right) \, dv
\]
\[
+Im \int_{S_\infty} (\mathbf{E}' \times \mathbf{H}^* - \mathbf{E} \times \mathbf{H}'^*) \cdot \mathbf{r} \, r^2 d\Omega \tag{4.13}
\]
for the imaginary part of the resulting integral.

To cast (4.13) into a more useful form, note that as \( r \to \infty \)
\[
\mathbf{E}(r \to \infty) = \mathbf{E}_0 \frac{e^{-jkr}}{r} \tag{4.14}
\]
Upon substituting (4.14) into (4.13), and re-arranging terms we obtain
\[
-Im \int_{V_A} (\mathbf{J}_s^* \cdot \mathbf{E} - \mathbf{J}_s'^* \cdot \mathbf{E}') \, dv - \frac{2}{Z_0} Im \int_{S_\infty} \mathbf{E}_0' \cdot \mathbf{E}_0^* d\Omega
\]
\[
= 4 \left[ \int_{V_\infty} \left( \frac{\mu_0}{4} |\mathbf{H}|^2 + \frac{\epsilon_0}{4} |\mathbf{E}|^2 \right) \, dv - \frac{r}{c} \int_{S_\infty} \frac{|\mathbf{E}_0|^2}{2Z_0} \, d\Omega \right] \tag{4.15}
\]
As argued first by Collin [34] and subsequently by Fante [33], the bracketed term on the right hand side of (4.15) is the subtraction of the total radiated energy throughout
space from the total energy (stored + radiated) contained in the $E$ and $H$ fields. Consequently, we may conclude that this represents the total stored energy ($W_M + W_E$). From (4.15), we can see that the total stored energy is computed from the frequency dependence of the source reactance, as well as a far-field dispersion term.

To derive the sum ($W_M + W_E$), we multiply both sides of (4.15) by $\omega/4$

$$\omega(W_M + W_E) = -\frac{\omega}{4} \text{Im} \int_{V_A} (J_S^* \cdot E - J_S^* \cdot E') dv - \frac{\omega}{2Z_0} \text{Im} \oint_{S_\infty} E'_{ff} \cdot E_{ff}^* d\Omega$$  \hspace{1cm} (4.16)

Finally, combining (4.4), (4.5), and (4.16) into (2.14), we obtain the $Q$ expression

$$Q = \max \left( \frac{A + B \pm C}{\frac{1}{2} \text{Re} \int_{V_A} J_S^* \cdot E dv} \right)$$  \hspace{1cm} (4.17)

where $A$, $B$, and $C$ are defined as

$$A = \frac{\omega}{4} \text{Im} \int_{V_A} (J_S^* \cdot E - J_S^* \cdot E') dv$$  \hspace{1cm} (4.18)

$$B = \frac{\omega}{2Z_0} \text{Im} \oint_{S_\infty} E'_{ff} \cdot E_{ff}^* d\Omega$$  \hspace{1cm} (4.19)

$$C = \frac{1}{4} \text{Im} \int_{V_A} J_S^* \cdot E dv$$  \hspace{1cm} (4.20)

with $Z_0$ being the free-space wave impedance for the antenna represented by $J_S$ in Fig. 4.1. We note that (4.17) involves integrals over the current volume $V_A$ and far-field sphere $S_\infty$. Thus, it requires an accurate computation of the far-fields and their frequency derivatives. However, for small antennas described by the EFIE MoM impedance matrix $[Z]$, (4.17) can be simplified to involve only the antenna currents and $[Z]$ using the theory of characteristic modes.

As demonstrated in the previous section, using the characteristic modes given by (3.1), we are able to obtain a basis to represent any scattering current on the antenna and their radiated pattern in a least-squares sense. Additionally, these eigenmodes
can be used to simplify (4.17) for antennas described by the EFIE MoM impedance matrix \([Z]\), and for computing the \(Q\) limits. For antennas described by the EFIE MoM impedance matrix \([Z]\), (4.17) becomes

\[
Q = \max \left( \frac{\Im \left[ \left\{ I_S \right\}'^H \left[ \left\{ I_S \right\} - \left\{ I_S \right\}' \right] \right]}{+ \frac{\omega}{2 \pi} \int_{S_{\infty}} E_{\text{ff}}^* E_{\text{ff}} d\Omega \pm \frac{1}{4} \left\{ I_S \right\}'^H \left[ X \right] \left\{ I_S \right\}} \right)
\]

\[\frac{1}{2} \left\{ I_S \right\}'^H \left[ R \right] \left\{ I_S \right\} \]

where the superscript \(H\) denotes the Hermitian transpose. The matrix derivatives at frequency \(\omega\) in (4.21) are approximated as

\[
\left\{ I_S \right\}' \approx \left\{ I_S (\omega + \Delta \omega) \right\} - \left\{ I_S (\omega) \right\} \Delta \omega
\]

(4.22)

and

\[
\left( [Z] \left\{ I_S \right\} \right)' \approx \left( \frac{\left[ Z(\omega+\Delta \omega) \right] \left\{ I_S (\omega+\Delta \omega) \right\} \left[ \left[ Z(\omega) \right] \left\{ I_S (\omega) \right\} \right]}{\Delta \omega} \right)
\]

(4.23)

for small enough \(\Delta \omega\).

To obtain a convenient expression for \(Q\) in terms of MoM parameters only, we must evaluate the far-field integral in the numerator of (4.21). To accomplish this, we start by expanding \(E_{\text{ff}}\) in (4.21) in terms of eigenpatterns \(F_n\) as

\[
E_{\text{ff}} \approx \sum_{n=1}^{M} \alpha_n F_n
\]

(4.24)

where \(M\) is large enough such that \(\sum_{n=1}^{M} |\alpha_n|^2 \approx P_{\text{rad}}\). Assuming the eigenpatterns are approximately constant with frequency \((F_n' \approx 0)\) and invoking (3.5), the far-field term of (4.21) can be accurately represented as

\[
\frac{2}{Z_0} \int_{S_{\infty}} E_{\text{ff}}^* E_{\text{ff}} d\Omega \approx 4 \int M \sum_{n=1}^{M} \alpha_n^* \alpha_n
\]

(4.25)

where \(\alpha_k\) are the CM weighting coefficients given in (4.24), which are equivalent to the coefficients in (3.6). Inserting (4.25) into (4.21), we find

\[
Q \approx \max \left( \frac{\Im \left[ \left\{ I_S \right\}'^H \left[ \left\{ I_S \right\} - \left\{ I_S \right\}' \right] \right]}{+ \omega \int_{S_{\infty}} \sum_{n=1}^{M} \alpha_n^* \alpha_n \left\{ I_S \right\}'^H \left[ X \right] \left\{ I_S \right\}} \right)
\]

\[\frac{1}{2} \left\{ I_S \right\}'^H \left[ R \right] \left\{ I_S \right\}
\]

(4.26)
We can now use (4.26) to find $Q$ for arbitrary-shaped antennas supporting a known current $\{I_S\}$. Importantly, only the impedance matrix $[Z]$ is needed to compute $Q$.

Before proceeding further, we wish to comment on the approximation $\mathbf{F}_{n'} \approx 0$. This approximation is based on our empirical observation that for $(ka < 0.5)$, the eigencurrents and eigenpatterns vary slowly as a function of frequency. Also, for two nearby frequencies $\omega$ and $\omega + \Delta \omega$, the eigencurrents and eigenpatterns retain the same general shape [31] [32]. A simple verification check for this condition would be to ensure that $\{I_n(\omega)\}^T [Z(\omega)] \{I_n(\omega)\} \approx \{I_n(\omega + \Delta \omega)\}^T [Z(\omega)] \{I_n(\omega + \Delta \omega)\}$.

To validate (4.26), we examine the $Q$ factors for a center-fed wire dipole (length = 2m) and a wire loop antenna (diameter = 2m), each of wire diameter 2mm. Both of these antennas were circumscribed by a Chu sphere of radius $a = 1m$. We proceed to compare (4.26) to the known expression [17]

$$Q_{\text{imp}} \approx \frac{\omega}{2R_{in}} \sqrt{\left(\frac{dR_{in}}{d\omega}\right)^2 + \left(\frac{dX_{in}}{d\omega} + \frac{|X_{in}|}{\omega}\right)^2}.$$  \hspace{1cm} (4.27)

Fig. 4.2 compares the $Q$ of each antenna as computed using (4.26) and (4.27). Indeed, we observe extremely close agreement in both the electrically small $ka < 0.5$ and large $ka > 0.5$ regions.

Although we are able to compute the $Q$ of arbitrary-shaped antennas using (4.26), we can maximize the utility of (4.26) by noting that it is well-suited for computing the $Q$ of individual or weighted sums of characteristic modes. That is, if we know the lowest-order CMs (smallest $|\lambda_n|$) of an antenna, we can easily determine the minimum $Q$ by optimally exciting only the lowest-order capacitive and/or inductive CMs. This is due to the lowest-order CMs containing the smallest ratios of stored energy to radiated power (see (3.3)), thus minimizing (2.14). To demonstrate how CMs and (4.26) can be used to find $Q$ bounds, we next consider several examples.

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Figure 4.2: $Q$ factors for a center-fed wire dipole of 2m length and center-fed wire loop of loop diameter 2m computed using (4.26) and (4.27). A wire diameter of 2mm was used for both antennas. $M = 10$ CMs were used for both $Q_{Loop,CM}$ and $Q_{Dipole,CM}$. 
4.3 Fundamental Limits on ESAs and Mounting Platforms

We now consider application of (4.26) to find the fundamental limits on small antenna \( Q \). First, we will consider the fundamental limits on \( Q \) for single-feed ESAs, which can be applied to antennas in free-space as well as infinite ground planes. For a single-feed antenna, we typically do not have control over the radiation pattern, and thus the \( Q \) is primary parameter of concern. Afterwards, we will show the limits on \( Q \) subject to a given radiation pattern, an application which is particularly useful when considering the bandwidth limits of platform-mounted ESAs, in which the ESA size is much smaller than that of the platform and a certain gain or radiation pattern is needed.

4.3.1 Fundamental Limits on Single-feed ESAs

For practical single-feed electrically small antennas, whether operating in free-space or on an infinite ground plane, the lower bounds on \( Q \) can be classified into two categories: single-mode limits and self-tuned limits. The single-mode \( Q \) limits corresponds to the minimum possible \( Q \) for an antenna radiating a single characteristic mode. Mathematically, the single-mode limit \( Q_{\text{min,SM}} \) can be written as

\[
Q_{\text{min,SM}} = \min \left[ Q(\{I_s\} = \{I_n\}) \right] ; \ n = 1, 2, ...
\]  

(4.28)

where \( Q \) is computed using (4.26). For ESAs, the lowest-order CMs radiate Hertzian dipole patterns, a consequence of Chu’s directivity-\( Q \) limits [12]. Thus, the single-mode \( Q \) limit is a practical metric to consider, as it best represents the lowest obtainable \( Q \) for single-port ESAs mounted on electrically large conducting platforms.

On the other hand, the self-tuned \( Q \) limit corresponds to the self-tuning radiation of the antenna’s lowest-order capacitive and inductive CMs. It provides a smaller
than that of a single characteristic mode alone. This self-tuned $Q_{\text{min,ST}}$, can be written as

$$Q_{\text{min,ST}} = \min \{Q(\{I_S\} = \alpha_L \{I_L\} + \alpha_C \{I_C\})\} \quad (4.29)$$

where $\{I_L\}$ and $\{I_C\}$ are the lowest $Q$ inductive and capacitive CMs. The excitation coefficients $\alpha_L$ and $\alpha_C$ are chosen such that self-resonance is obtained. In the solution to (4.29), we define the CM with the larger $|\alpha_{C,L}|$ as the dominant CM, and the CM with the smaller $|\alpha_{C,L}|$ as the tuning CM. Although the self-tuned limit may not be practically realizable (see Thal [35]), it represents an absolute lower-bound on $Q$ for the geometry itself.

Before demonstrating the application of (4.26) in determining the $Q$ limits of arbitrary-shaped antennas, we consider the $Q$ limits of the well-known spherical antenna. This is done for validation purposes.

4.3.2 $Q$ Limits for Spherical ESAs

We first consider the $Q$ bound on a sphere of radius $a$, with the lowest-order CM eigencurrents and eigenvalues as depicted in Figs. 3.2 and 3.3, respectively, compared to the Chu limit in (2.18). From Fig. 3.3, it is obvious that the lowest-order capacitive (CMs 1) and inductive (CMs 2) CMs will have $Q$ values much smaller than CMs $n > 3$, due to the fact that their eigenvalue magnitudes are much smaller. The single-mode $Q$ limits for the sphere’s capacitive ($\{I_1\}$) and inductive ($\{I_2\}$) mode radiation is shown in Fig. 4.3. We observe that for $ka < 0.5$, the lower bound on $Q$ for spherical antennas supporting Hertzian electric dipole radiation is approximately

$$Q_{\text{min,SM,C}} \approx 1.5 Q_{\text{Chu}}(n = 1) \quad . \quad (4.30)$$
Similarly, the lower bound on $Q$ for the spherical antenna supporting Hertzian magnetic dipole radiation is approximately

$$Q_{min,SM,L} \approx 3 Q_{Chu}(n=1).$$

(4.31)

We note that (4.30) and (4.31) agree exactly with the work of Thal [36], and (4.30) agrees with Yaghjian [21] and Gustafsson [19] (assuming 50% absorption efficiency and gain=1.5) for $ka \to 0$.

As is well-known, appropriately exciting the lowest-order capacitive and inductive radiating modes for self-tuning can yield the minimum possible $Q$ for a geometry [12] (see (4.4)). That is, the numerator of (2.14) remains the same as the single-moded case, while the denominator increases due to the contribution of the tuning mode. The minimum possible $Q$ for the sphere is shown in Fig. 4.3, and the ratio of capacitive
Figure 4.4: Ratio of the capacitive CM to inductive CM power that achieves the spherical self-tuned \( Q \) limit in (4.32).

to inductive CM power for self-tuning is given in Fig. 4.4. As seen in Fig. 4.3, for \( ka < 0.5 \), the self-tuned limit is

\[
Q_{\text{min,ST}} \approx Q_{\text{Chu}}(n = 1) \quad (4.32)
\]

In this case, the capacitive (dominant) CM power is approximately 3dB greater than the inductive (tuning) CM (see Fig. 4.4). Such results again coincide with Thal [36]. Therefore, we can conclude that the absolute lower-bound of antennas with air-filled cores is dictated by (4.3).

### 4.3.3 \( Q \) Limits for Arbitrary-Shaped ESAs

Since the solutions to (3.1) are determined numerically, the characteristic mode technique is well-suited to compute the minimum \( Q \) bounds for arbitrary-shaped...
radiators. As an example, we examine the $Q$ limits of four shapes depicted in Fig. 4.5 and having an aspect ratio $AR$. In all cases, the circumscribing Chu sphere radius was kept at $ka = 0.2$. Here, we assume the origin has been placed at the center of the circumscribing Chu sphere, with the $x$, $y$, and $z$ directions as in Fig. 4.5(a). The aspect ratio $AR$ is defined as

$$AR = \frac{D_z}{D_x} = \frac{D_z}{D_y}$$

(4.33)

where $D_{x,y,z}$ are the dimensions along the $x$, $y$, and $z$ axes.

For the geometries of Fig. 4.5, the single-mode $Q$ limits of concern are the characteristic modes which excite $m^x$, $m^y$, $m^z$, and $p^z$ Hertzian dipole radiation. Here,
we define $p^k$ as the well-known $\sin^2(\theta)$ power pattern corresponding to a Hertzian electric dipole, with dipole moment oriented in the $k$-direction. Similarly, we define $m^k$ as the $\sin^2(\theta)$ power pattern corresponding to a Hertzian magnetic dipole, for a loop whose area is normal to the $k$-direction. Note that $m^x$, $m^y$, and $p^z$ radiation can be supported by a PEC ground plane at $z = 0$, while $m^z$ may be supported by a ground plane in the $y = 0$ plane. Further, due to object symmetry, the operation of $m^x$ is equivalent to that of $m^y$ for a given aspect ratio $AR$.

Figs. 4.6-4.8 depict the single-mode $Q$ limits for $p^z$, $m^z$, and $m^x$ radiation, along with the associated eigencurrents realizing these limits for $AR = 1$. From Fig. 4.6, it is evident that long and thin ($AR >> 1$) geometries have potentially wider bandwidth as capacitive electric-dipole antennas. Correspondingly, from Fig. 4.8, short and fat geometries ($AR << 1$) have potentially wider bandwidth as inductive magnetic-dipole antennas. Fig. 4.8 shows that the $m^x$ CM provides the widest operational bandwidth for $AR$ close to unity, since $Q$ increases rapidly outside $AR \approx 1$.

Table 4.1 summarizes the single-mode $Q$ results of Figs. 4.6-4.8. From Table 4.1, the cubic, cylindrical, and spheroidal single-mode $Q$ limit is that of $p^z$ CM (for the optimal $AR$). However, the single-mode $Q$ limit of the conical antenna is the $Q$ of the $m^z$ CM (for the optimal $AR$). Predictably, the sphere provides the minimum possible $Q$, while for other shapes the minimum $Q$ is associated with a certain $m^k$ or $p^k$ mode that varies as a function of $AR$. We note that the slight discrepancy between the $m^x$ and $m^z$ limits of the spheroid for $AR = 1$ is due to the mesh discretization.

Although the single-mode limits are the most practical to consider for single-port antennas, the self-tuned limit provides a powerful measure of the minimum possible $Q$ for certain antenna geometries which contain significant capacitive and inductive
Figure 4.6: Single-mode $p^z$ $Q$ limits for the geometries in Fig. 4.5, with the optimal $p^z$ currents for $ka = 0.2$.

Figure 4.7: Single-mode $m^z$ $Q$ limits for the geometries in Fig. 4.5, with the optimal $m^z$ currents for $ka = 0.2$. 
Figure 4.8: Single-mode $m^x$ Q limits for the geometries in Fig. 4.5, with the optimal $m^x$ currents for $ka = 0.2$.

CMs at small sizes (for example, a small loop antenna). Figs. 4.9 and 4.10 depict the self-tuned Q limits and associated self-tuning power ratios, respectively, for the antennas in Fig. 4.5. From Fig. 4.9, it is clear that using the lowest-Q capacitive and inductive CMs for self-tuning perturbs the optimal AR from their single-mode cases (Figs. 4.6-4.8), and can provide a significant Q reduction when the tuning CM contributes to radiation (see Table I). For example, for the shapes in Fig. 4.5, a significant reduction in Q for the self-tuned limit (in comparison to the single-mode limit) is observed in Fig. 4.9 for $AR << 1$, where the tuning CM contributes approximately 3dB less power than the dominant CM (see Fig. 4.10). However, as the aspect ratio increases beyond unity. This is because the tuning CM $Q$ becomes larger. Therefore, for $AR >> 1$ the tuning CM contributes negligible radiated power (large self-tuned power ratio, see Fig. 4.10), and the self-tuned limits effectively approach those of a single-mode.
Figure 4.9: Self-tuned $Q$ limits for the antennas in Fig. 4.5.

Figure 4.10: Self-tuning ratios to achieve the self-tuned $Q$ limit in Fig. 4.9.
Since the CMs obtained in (3.1) are in the form of orthogonal surface currents, (4.26) is well suited to examine the $Q$ limits of planar antenna geometries. Such planar antenna $Q$ limits can be used to find the lower bounds of low-profile ESAs mounted on electrically small ground planes. For the latter, the ESA acts primarily as a coupler to the radiating ground plane. The ground plane then serves to define the $Q$ limits of the structure.

To demonstrate the above, Figs. 4.12 and 4.13 depict the single-mode $Q$ limits for the $p^z$ and $m^x$ CMs of the planar geometries in Fig. 4.11 as a function of aspect ratio $AR = D_Z/D_Y$, with a fixed Chu sphere radius $ka = 0.2$. Both of these CMs can be supported by a PEC ground plane on $z = 0$. From Fig. 4.12, for the $p^z$ CM, we can see that the single-mode $Q$ limits are on the same order of magnitude as their volumetric counterparts, using far less occupied volume. As expected, the flat disk is the optimal planar use of the Chu sphere, and yields the minimum of the single-mode limits. Specifically, from Fig. 4.12, we find that the lower bound for a single-mode planar electric dipole antenna is (for $ka < 0.5$)

$$Q_{C,disk} \approx 3.6 \ Q_{Chu}(n = 1) \ .$$

Similarly, the lower bound for a single-mode planar magnetic dipole antenna (see Fig. 4.13) is given by

$$Q_{L,disk} \approx 7.4 \ Q_{Chu}(n = 1) \ .$$

Furthermore, the self-tuned limit for an electrically small disk is approximately

$$Q_{min,disk} \approx 2.48 \ Q_{Chu}(n = 1)$$

in which the inductive (tuning CM) power is 3dB below the capacitive (dominant CM) power. Comparison between (4.30)-(4.32) and their planar counterparts (4.34)-(4.36)
reveals that utilization of the full Chu sphere can potentially provide a $Q$ reduction factor of 0.4. That is, from (4.32) and (4.36), $Q_{\text{min, sphere}} = 0.4Q_{\text{min, disk}}$.

As a final verification, we compare our CM method to Gustafsson [19] and Vandenbosch [23] $Q$ limits on a planar rectangle as a function of aspect ratio $W/L$ (see Fig. 4.14). As before, $p^Z$ is computed from the lowest-order electric dipole CM at $ka = 0.2$, while the Gustafsson and Vandenbosch results are obtained from the low-frequency approximation of the rectangular plate. Similarly, for Gustafsson’s work an absorption efficiency $\eta = 0.5$ and directivity $D = 1.5$ are assumed. From Fig. 4.14, extremely close agreement is seen among the three different methods.

### 4.3.4 $Q$ Limits Subject to Radiation Pattern Requirement

In many practical scenarios, an ESA is required to be placed on a non-ideal mounting platforms, for which the radiation properties of the original ESA become radically altered from those in isolation (as was seen in Chapter 2). In this scenario, the ESA and platform combination becomes the mechanism of radiation. Furthermore, it has been shown that when comparably small perturbations (ESAs) are added to a

![Figure 4.11: Planar antenna geometries with aspect ratios $AR = D_z/D_y$: (a) Bowtie, (b) Ellipse, (c) Rectangle.](image)

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Figure 4.12: Single-mode $p^z$ $Q$ limits for the planar shapes in Fig. 4.11.

Figure 4.13: Single-mode $m^x$ $Q$ limits for the planar shapes in Fig. 4.11.
Figure 4.14: Single-CM $p^2$ $Q$ limit compared to Gustafsson [19] and Vandenbosch [23] as a function of $W/L$.

conducting body (platform), the lowest-order eigenmodes of the composite structure (ESA and platform) remain relatively constant in comparison to the unperturbed case (platform-only) [32]. Thus, in this case, if we assume the mounting platform exhibits a certain set of $Q$ limits, when adding ESAs (whose dimensions are much smaller than that of the platform) to the mounting platform, the composite ESA-platform structure $Q$ limits will remain approximately the same. Hence, we can find the $Q$ limits on platform-mounted ESAs by examining the $Q$ limits of the platform itself, when the platform excites a specified radiation pattern.

In this chapter (as well as Chapter 5), we will analyze the UAV model of Fig. 3.4 at 10MHz. Fig. 4.15 depicts the $Q$ for each UAV CM as a function of frequency. Here we see that similar to the modal significance case (Fig. 3.5), the lowest-order
CMs have the broadest potential bandwidth. Hence, the absolute lower bound on $Q$ for a single ESA mounted on the UAV would be that of CM 1 ($Q_1$), which would be the single-mode limit. Similarly, the self-tuned limit would be approximately that of $Q_1$, as the lowest-order inductive CM (CM 4), the aforementioned contributing would contribute very little to radiation when tuning.

However, for the aforementioned cases, the resulting radiation pattern may not be desirable, given the application. It is common for design specifications for platform-mounted antennas to call for a specific gain or polarization requirement, which may be incompatible to that of any single-mode or self-tuned limit. In this case, we need to examine the limits on $Q$ subject to a specific radiation pattern. Thus, in order to find the limits on $Q$ for a given radiation pattern, we first need to synthesize the radiation
pattern (within acceptable error tolerance) using (3.9) coefficients $\alpha_n$. Subsequently, the current $\{I_0\}$ which excites this radiation pattern is given as

$$\{I_0\} = \sum_{n=1}^{M} \alpha_n \{I_n\}.$$  (4.37)

where $\{I_n\}$ is the $n^{th}$ eigencurrent of the platform. We now state that, assuming the dimensions of the ESAs to be mounted on the platform are much smaller than the platform itself (e.g. $MS_n$ for the CMs introduced by the ESA are much smaller than the dominant platform CM $MS_n$), the $Q$ limits subject to the prescribed radiation pattern is $Q(\{I_0\})$ using (4.26), as the radiation contributed by the ESA (or ESAs) is minute in comparison to the that of the platform.

To demonstrate the use of (4.26) for finding the $Q$ limits subject to a radiation pattern, we again use the UAV example at 10MHz for three test cases shown in Fig. 4.16: Bidirectional, Rotated Bidirectional, and Unidirectional. We note that the Bidirectional and Rotated Bidirectional radiation cases are identical to those studied in the previous chapter, and the Unidirectional case corresponds to the radiation of a crossed set of in-phase electric and magnetic Hertzian dipoles oriented in the $xy$ plane with equal dipole moments. We finally remark that this Unidirectional case is the NVIS radiation pattern that will be synthesized in the following chapter using discrete monopole feeds.

Figs. 4.17 and 4.18 depict the $LSQ$ (as defined in (3.10)) and $Q$ limit, subject to the three different radiation pattern cases in Fig. 4.16, as a function of the number of CMs used. First, we examine the Bidirectional case. Observing Fig. 4.18, we see that in order to achieve less than 1% $LSQ$ error, only $M = 3$ CMs are needed to approximate the unidirectional pattern. In this case, the minimum bound on $Q$ for
the UAV is $Q_{\text{min,Bidirectional}} \approx 6$, which remains relatively constant as the number of modes increases.

Figure 4.16: Three desired patterns for the UAV at 10MHz: Bidirectional, Rotated Bidirectional, and Unidirectional.
Figure 4.17: Lower bound on $Q$ for the patterns of Fig. 4.16 radiated by the UAV in Fig. 3.4 at 10MHz

Figure 4.18: Least-squared apttern error for the patterns in Fig. 4.16 as a function of the number of CMs $M$ used in the expansion.
Second, we examine the Rotated Bidirectional Case. From Fig. 4.18, it is obvious that a larger number of CMs is needed in order to accurately represent the Rotated Bidirectional pattern, and the 1% \( LSQ \) threshold is met when using \( M = 10 \) CMs. However, the utility of this method is seen by observing the rate of increase in \( Q \) versus the rate of decrease in \( LSQ \) as a function of the number of CMs. From this method, a decrease in \( LSQ \) from 2.26% \((M = 4)\) to 0.39% \((M = 10)\) results in an increase in minimum possible \( Q \) from 69.0 \((M = 4)\) to 109.1 \((M = 10)\). In practice, utilizing the minimum number of CMs for acceptable error will be highly desirable in practice, as explicit control of fewer CMs would be needed, and thus reduces the number of feed antennas required. Finally, we examine the Unidirectional Case. Here, the 1% \( LSQ \) threshold is met at \( M = 6 \) CMs. Again, in a practical application, the trade between the pattern error and \( Q \) at \( M = 5 \) \((Q_{\text{min}} = 11.7, \ LSQ = 4.2\%)\) and \( M = 6 \) \((Q_{\text{min}} = 28.0, \ LSQ = 0.15\%)\) needs to be assessed, while keeping in mind that increasing the number of modes to represent a pattern requires a larger number of feeds.

As a final note, we remark that based on Chu [12] theory, the Bidirectional and Rotated Bidirectional cases would yield the same \( Q \) limit, but by utilizing the eigen-modes of the structure (the CMs) we are able to have a more accurate \( Q \) bound tailored to the particular scenario. We further reiterate that the usefulness of this approach is in the analysis of the \( Q - LSQ \) tradeoff, and its consequences on the number of modes used to realize the pattern using in-situ ESAs. From a complexity standpoint, it is highly desirable to control (and thus feed) the smallest number of CMs needed for the particular application.
4.4 Summary

A new method for the computation of ESA $Q$ was presented, which requires only the EFIE MoM impedance matrix $[Z]$ and the CMs of the excited current $\{J_0\}$. With this formula, the $Q$ bounds of arbitrary-shaped antennas were formulated in terms of the EFIE MoM impedance matrix, in terms of single-mode and self-tuned limits. In addition to the limits on $Q$, the optimal minimum $Q$ currents are known exactly and decomposed in terms of CM eigencurrents. This method was then extended to find the $Q$ limits of mounting platforms (e.g. UAV, humvee, etc), subject to a prescribed radiation pattern. We discussed the tradeoffs between the $Q$ and pattern error $LSQ$, which aid in providing design guidelines and suggestions for real mounted antennas.
Chapter 5: Design of In-Situ ESAs Using Platform CMs

5.1 Introduction

The typical ESA design process begins with a construction and optimization within an isolated or analytical environment, such as free space or an infinite ground plane. Following such a design, the optimized ESA is then placed on the mounting platform or vehicle. For the case where the mounting platform is electrically large, the ESA analysis and design methods described in Chapter 4 are highly useful to find optimal currents and thus clues into optimal design. Unfortunately, in many instances, the mounting platform can be on the order of a wavelength or below, and thus may induce large changes in the radiation characteristics of the ESA, requiring further co-optimization of the antenna in the presence of the platform. Such a scenario is frequently encountered with ESA mounted on platforms on the order of a wavelength or below, for example, HF/VHF antennas mounted on military vehicles and autonomous aircraft, as well as antennas for mobile consumer electronics. In spite of the increased design complexity, an optimization of the composite ESA-platform system can provide dramatic improvements in the antenna bandwidth and efficiency in comparison to the free space/infinite GP case. Such a result is due to the fact that
the platform coupling allows the antenna to have a larger electrical footprint, and thus increases bandwidth and efficiency [2].

Despite rapid advancements in commercial EM solvers and computational power, conventional design and optimization of such in-situ ESAs on real-life platforms remains a cumbersome task. Ordinary in-situ designs involve extensive parametric studies of the specific platform and/or optimization routines requiring up to hundreds of full-wave simulations for a convergent solution. Additionally, such approaches provide little physical insight into the antenna-platform coupling physics. Therefore, an effective methodology to model and predict in-situ antenna performance is needed. Such a fast methodology could then be used to restrict optimization routines for quick parameter fine-tuning, and accelerate the design cycle.

The idea of utilizing characteristic modes as an antennas synthesis method was first proposed by Newman [38], who examined the placement of a small loop antenna on-board a symmetric wire cross. Austin [39] considered a multi lumped-port excitation of a towel-bar antenna mounted on a Humvee, where the port voltages chosen to synthesize an NVIS pattern using the theory of characteristic modes (CMs). It was shown that by considering the radiation eigenmodes of the composite Humvee and towel-bar structure, an optimized NVIS pattern could be achieved in comparison to a single-feed design, at the cost of design complexity. However, methods were based on lumped port modeling, where maximization of a desired CM is simply achieved by placing the lumped port at the desired eigencurrent maxima. Recently, in an effort to analyze more realistic feed types, the theory of characteristic modes has gained significant attention in the assessment of the location-dependent effects of monopole and slot antennas mounted on finite rectangular platforms [40–44], which mimic the
standard cell-phone chassis. However, the results are for simple rectangular platforms and provide minimal theoretical rigor, giving little confidence in an optimizing antennas on more complex platforms and varying feed antenna types. Consequently, there is a lack of robust antenna-platform models and knowledge of optimal design criteria for using characteristic modes as an in-situ ESA design tool.

In this chapter, a systematic methodology for the location optimization of multi-element antenna systems on-board arbitrary shaped platforms is presented. We will show that based on the CMs of the mounting platform, optimal regions for electrically small antennas can be located. Subsequently, we will show that once these locations are found, the appropriate feed voltages to synthesize a prescribed radiation pattern can be realized through the solution of a simple matrix equation. As a first test to gain physical insight into antenna-platform coupling, we look at the interaction between the ESA near-fields and the platform’s characteristic modes.

5.2 In-Situ Electrically Small Antennas and Platform Characteristic Modes

A general coupled antenna-platform system whose operation is expressed through the symmetric electric field integral equation (EFIE) moment method (MoM) impedance matrix $[Z]$ (see [5]) can be written in block form as (see also Fig. 5.1)

$$
[Z] \{I\} = \begin{bmatrix}
Z_{AA} & Z_{AP} \\
Z_{PA} & Z_{PP}
\end{bmatrix}
\begin{bmatrix}
\{I_A\} \\
\{I_P\}
\end{bmatrix} = \begin{bmatrix}
\{V_A\} \\
\{V_P\}
\end{bmatrix}
$$

(5.1)

where $[Z_{AA}]$ and $[Z_{PP}]$ are the antenna and platform impedance matrices, $[Z_{AP}]$ and $[Z_{PA}]$ are the mutual antenna-platform coupling matrices. Additionally, $\{I_A\}$ and
{I_P}\) are the antenna and platform currents, respectively, while \{{V_A}\} and \{{V_P}\} are the antenna and platform voltages.

In this chapter, we consider the class of problems described by (5.1), with the antenna dimensions much smaller than those of the platform, and the platform dimensions on the order of a wavelength and below. As mentioned previously, this allows us to consider the platform as the dominant radiating mechanism, while the antenna acts as a feed to the radiating platform. We then consider the characteristic modes of the platform only, which we define as the platform characteristic modes (PCMs), with our first goal study how a single ESA feed illuminates the desirable PCMs optimally. The PCMs of the antenna-platform system in (5.1) can be found through the symmetric, frequency-dependent generalized eigenvalue problem [4]

\[
[X_{PP}] \{I_{P,n}\} = \lambda_{P,n}[R_{PP}] \{I_{P,n}\} \tag{5.2}
\]
where \([Z_{PP}] = [R_{PP}] + j[X_{PP}]\) is defined in Fig. 5.1, \(\lambda_{P,n}\) is the \(n^{th}\) platform eigenvalue (indexed in order of ascending magnitude), and \(\{I_{P,n}\}\) is the \(n^{th}\) platform eigencurrent. The set of platform eigencurrents, \(\{I_{P,n}\}\), form an orthogonal current basis on the platform structure in isolation. The platform eigenvalues \(\lambda_{P,n}\) are the ratios of stored to radiated power for the associated \(\{I_{P,n}\}\). The first \(M = 9\) dominant eigencurrents of a UAV model are displayed in Fig. 3.6 at a frequency of 10MHz.

We next wish to investigate antenna-platform coupling through the language of PCMs, using straight-wire monopole antennas as feeds.

5.3 Single Monopole Location Optimization on a UAV

As an initial effort to understand the antenna-platform coupling of Fig. 5.1, we consider the \(Q\) (computed at the monopole feed point using (4.27) and PCMs excited by varying a small wire monopole (height = \(\lambda/50\)) location on both the wing and fuselage of the UAV in Fig. 3.4 at 10MHz.

5.3.1 Single Monopole Location Optimization on UAV Wing

First, we consider the excitation of a small wire monopole (height = \(\lambda/50\)) placed on the wing of the UAV model, and vary the normalized mounting location \(y\) at 10MHz (see Fig. 5.2). The \(Q\) computed from the feedpoint of the monopole (using 4.27) as a function of location \(y\) is shown in Fig. 5.3. As expected, the monopole bandwidth is dramatically enhanced (in comparison to the infinite ground plane case) by a strategic choice of location. Here, we see that the \(Q\) of the monopole decreases monotonically as a function of location \(y\), and reaches its lowest value \((Q(y = 1) = 10.5)\) at the wing edge \(y = 1\). Note that as expected, this is below the single-mode \(Q\)
limit of the UAV, as dictated by $Q_1$ in Fig. 4.15 (see Chapter 4 for further discussion), due to the extra capacitive reactance introduced by the monopole.

To illustrate the relation between bandwidth enhancement (low $Q$) and feed point location, we compute the PCM expansion coefficients $|\alpha_n|$ (using (3.8) and (3.9)) as a function of location $y$ in Fig. 5.4. In Fig. 5.3, it was observed that the $Q$ decreased monotonically with $y$. This can be rationalized by observing the behavior of the PCM coefficients as a function of location. Here, at $y = 0.1$, it is evident that the ratio of $|\alpha_1|$ to the other PCM $\alpha_n$ where $n > 1$ is minimized. This means that at position $y = 0.1$, we are minimizing coupling to the lowest-order, broadband PCM 1 and maximizing the coupling to the higher-order, narrowband PCMs, which increase $Q$. On the contrary, as the monopole approaches the wing edge $y = 1$, coupling to PCM 1 is maximized while higher-order mode couplings (PCMs $n > 3$) are minimized, resulting in a decrease in $Q$. 
Figure 5.2: Monopole (height = λ/50) location variation along the normalized wing length $y$, for the UAV model of Fig. 3.4 at 10MHz
Figure 5.3: $Q$ computed using (4.27) for the monopole-UAV of Fig. 5.2 as a function of position $y$.

Figure 5.4: UAV PCM excitation coefficients for Fig. 5.2 at 10MHz.
5.3.2 Single Monopole Location Optimization on UAV Fuselage

For the second single-monopole case, we consider the excitation of the same small wire monopole (height = $\lambda/50$) now placed on the fuselage of the UAV model, and vary the normalized mounting location $x$ at 10MHz (see Fig. 5.5). The $Q$ computed from the feedpoint of the monopole (using 4.27 as a function of location $x$ is shown in Fig. 5.6. Again, there is large variance in the monopole $Q$ as a function of location, and strategic choice of the location can provide dramatic enhancement of performance in comparison to the monopole on an infinite ground. Here, $Q$ is minimized at the edges ($x = -1$ and $x = 1$) of the normalized mounting location, which is nearly two orders of magnitude smaller than the placement of the monopole at the center of the fuselage ($x = 0$).

Again, we illustrate the relation between $Q$ and feed point location, by computing the PCM expansion coefficients $|\alpha_n|$ (using (3.8) and (3.9)) as a function of location $x$ in Fig. 5.7. In Fig. 5.6, it was observed that the $Q$ was minimized at the front and back locations, $x = -1$ and $x = 1$, respectively. From Fig. 5.7, it is seen that the locations $x = -1$ and $x = 1$ coincide with the maximum excitation of CM 2 (second-most broadband CM), while minimizing the excitation of higher order CMs (CMs with $n > 2$). A rapid change in performance is seen as $x = 0$, where CM 2 is no longer excited effectively, and CM 4 dominates performance, resulting in the maximum feed point $Q$ (see Fig. 5.6), as CM 4 is a comparatively higher-order mode.
Figure 5.5: Monopole (height = \(\lambda/50\)) location variation along the normalized fuselage length \(x\), for the UAV model of Fig. 3.4 at 10MHz
Figure 5.6: $Q$ computed using (4.27) for the monopole-UAV of Fig. 5.5 as a function of position $x$.

Figure 5.7: UAV PCM excitation coefficients for Fig. 5.5 at 10MHz.
5.4 Optimal Coupling to Platforms for Monopole Antennas

Using two different scenarios for a single monopole antenna on-board a UAV, it was shown that the $Q$ and $PCMs$ had a direct correspondence to one another, where $Q$ was minimized when the monopole excited the lowest-order PCMs (CMs of the UAV), and minimized the excitation of the higher-order, reactive PCMs. Thus, given a platform for which the PCMs could be computed, it would be highly beneficial to predict (or at least come close to) the optimal monopole location \textit{a priori}. In this section, we will show that the optimal location for a monopole antenna to couple to a particular PCM of index $n$ is directly related to the resulting modal electric field $E_n$, normal to the surface of the platform.

5.4.1 Electric-Field Coupling for Monopole ESAs

First, consider the UAV of Fig. 3.4 with arbitrary surface equivalent current $J_{\text{UAV}}$, along with an infinitesimal unit current source $J_\delta(r') = \hat{a}_\delta(r')$, located at position $r'$ with polarization $\hat{a}$. For this system, reciprocity states

$$\int_{V_\delta} J_\delta(r') \cdot E_{\text{UAV}} dv' = \int_{V_{UAV}} J_{\text{UAV}} \cdot E_\delta dv$$

(5.3)

Given the fact that the UAV has a Method of Moments discretization, the surface reaction integral in (5.3) becomes (after integrating the left hand side of (5.3))

$$\hat{a} \cdot E_{\text{UAV}}(r') = \{I_{UAV}\}^T \{V_\delta\}$$

(5.4)

Now assume that the discretized surface equivalent current $\{I_{UAV}\} = \{I_n\}$, where $\{I_n\}$ is the $n^{th}$ eigencurrent of the UAV, (5.4) becomes

$$\hat{a} \cdot E_n(r') = \{I_n\}^T \{V_\delta\} = \frac{1}{2} (1 + j\lambda_n)$$

(5.5)
which can be rewritten as

\[ \alpha_n = \frac{1}{2} \frac{\hat{a} \cdot E_n(r')} {1 + j\lambda_n} \]  

(5.6)

Hence, in order to maximize the excitation of \(|\alpha_n|\), \(J_\delta\) must be oriented in the direction of the CM E-field when approaching the UAV surface. The denominator term \(1 + j\lambda_n\) illustrates usefulness of the modal significance figure of merit (see (3.4)). That is, even if the delta source is placed at the maximum \(|\hat{a} \cdot E_n|\) for CM n, if the eigenvalue \(|\lambda_n|\) is large enough, the CM excitation coefficient \(\alpha_n\) will remain difficult to couple power to (and therefore highly reactive). Thus, we conclude that when approximating the monopole antenna as a delta current source \(J_\delta\) oriented normal to the surface of the UAV, (5.6) indicates that the optimal locations for exciting a desired CM n are at the maxima of \(|\hat{n}_s \cdot E_n/(1 + j\lambda_n)|\), where \(\hat{n}_s\) is the unit normal vector to the UAV surface.

To validate our claim, we return to the case of the monopole location variation along the wing (see Fig. 5.8). From Fig. 5.3, it is seen that \(Q\) decreases monotonically as \(y\) increases, directly coinciding with the increase of the modal electric fields \(|\hat{n}_s \cdot E_n/(1 + j\lambda_n)|\) in Fig. 5.8. Furthermore, it is observed that the ratio between the Platform CM coefficients in Fig. 5.4 is approximately that of the ratio between the modal electric fields in Fig. 5.8, as predicted by (5.6) for a delta source.

Next, we investigate the case of the monopole location varied along the fuselage (see Fig. 5.5), and compute \(|\hat{n}_s \cdot E_n/(1 + j\lambda_n)|\) in Fig. 5.9 for the dominant CMs along y. By comparing Fig. 5.7 with Fig. 5.9, the monopole-UAV coupling mechanism via (5.6) can be clearly observed. First, at \(y = -1\), it is seen that the monopole achieves maximum coupling to both CM 2 and CM 3, due to the fact that both have their local \(|\hat{n}_s \cdot E_n/(1 + j\lambda_n)|\) maxima here. Furthermore, from at (5.6), \(|\alpha_2| > |\alpha_3|\) is due
to the fact that $|\lambda_2| < |\lambda_3|$. Next, at $y = 0$, we observe that the CM 2 and CM 3 $|\hat{n}_s \cdot E_n/(1 + j\lambda_n)|$ exhibit local minima, and hence their excitation based on (5.6) is minimized. On the contrary, coupling to CM 4 is maximized at $y = 0$, due to the existence of the local minima for CMs 2 and 3. Finally, at $y = 1$, CMs 2 and 3 dominate the maximum coupling again, with CM 4 suppressed.

To summarize, we have demonstrated that by approximating the monopole as a delta current source, optimal locations for maximizing the coupling to PCM $n$ can be easily identified through the maxima of $|\hat{n}_s \cdot E_n/(1 + j\lambda_n)|$. Similarly, if we wish to maximize the reaction term in (5.3) to a given surface equivalent current $J_0$, we would place a monopole at the location where $|\hat{n} \cdot E_0|$ is maximized. This will become the foundation for location optimization of multi-ESA systems, for synthesizing a desired far-field pattern.

5.4.2 Location and Feed Computation of In-Situ ESAs Using Platform CMs

In the previous section, we demonstrated that the optimal monopole location for maximizing the coupling to a particular UAV PCM $n$ is that of $|\hat{n}_s \cdot E_n/(1 + j\lambda_n)|$ where $E_n$ is the near E-field of PCM $n$, $\lambda_n$ is the $n^{th}$ PCM eigenvalue, and $\hat{n}_s$ is the UAV surface normal. By extension, the optimal monopole location for maximizing the coupling to a weighted sum of UAV PCMs is that of $max (|\hat{n}_s \cdot E|)$, where $E$ is the near E-field of the weighted sum of PCMs.

Thus, although the optimal monopole locations for a prescribed current can be located by analyzing the near E-field, a precise excitation of a desired current $\{I_0\}$ requires each monopole to be fed with an appropriate complex voltage. Using traditional design techniques, a large number of monopoles on-board a common platform
Figure 5.8: Normal electric field $|\hat{n} \cdot \mathbf{E}_n/(1 + j\lambda_n)|$ for UAV CMs 1-12 in Fig. 5.2 as a function of wing monopole location $y$.

Figure 5.9: Normal electric field $|\hat{n} \cdot \mathbf{E}_n/(1 + j\lambda_n)|$ for UAV CMs 1-12 in Fig. 5.5 as a function of fuselage monopole location $x$. 

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becomes an unwieldy and unpredictable optimization process. However, using the characteristic modes of the composite antenna-platform system (see Fig. 5.1), the feed voltages which excite a desired set of PCM coefficients \( \alpha_k \) can be realized exactly, providing a powerful first step in a larger design. We proceed to derive this result, a methodology which we refer to as Characteristic Mode Port Synthesis (CMPS).

When using monopole feeds, our goal is to excite a current \( \{I_0\} \) (or far-field \( \mathbf{E}_{ff} \)) written as a sum of CMs \( \{I_0\} = \sum_{n=1}^{M} \alpha_n \{I_n\} \). First, assuming we choose \( K \) monopole antennas to realize the \( \{I_0\} \), we formulate the EFIE MoM admittance equation [5] as

\[
[Y]\{V\} = \{I\}
\]

(5.7)

where \( [Y] = [Z]^{-1} \) and \( [Z] = [R] + j[X] \) are \( N \times N \), \( \{I\} \) and \( \{V\} \) are \( N \times 1 \), with \( N \) as the number of unknowns in the antenna-platform discretization. We also write the CM equation in (3.1) of the full antenna-platform system in matrix form as

\[
[X][S] = [\Lambda][R][S]
\]

(5.8)

where

\[
[\Lambda] = diag(\lambda_1, \lambda_2, \cdots, \lambda_M)
\]

(5.9)

and

\[
[S] = \{\{I_1\} \quad \{I_2\} \quad \cdots \quad \{I_M\}\}
\]

(5.10)

Noting that \( [S]\{\alpha\} = \{I_0\} \) The MoM equation in terms of the admittance matrix can then be written as

\[
[S]\{\alpha\} = [Y]\{V_0\}
\]

(5.11)
where \( \alpha = [\alpha_1 \alpha_2 \cdots \alpha_M]^T \). Next, both sides of (5.11) are premultiplied by \( \frac{1}{2}[S]^T[R] \) to yield

\[
\frac{1}{2}[S]^T[R][S] \{ \alpha \} = \{ \alpha \} = \frac{1}{2}[S]^T[R][Y] \{ V_0 \}
\]

(5.12)
in which we identify \( \frac{1}{2}[S]^T[R][S] \) as the matrix form of the orthogonality property in (3.2). Finally, we cast (5.12) into the matrix relation

\[
[M] \{ \tilde{V} \} = \{ \alpha \}
\]

(5.13)

with

\[
[M] = \left[ \left\{ [S]^T[R][Y] \right\}_{P(1)} \left\{ [S]^T[R][Y] \right\}_{P(2)} \cdots \left\{ [S]^T[R][Y] \right\}_{P(K)} \right] \]

(5.14)

where \( \left\{ [S]^T[R][Y] \right\}_{P(i)} \) is the \( P(i) \) column of the matrix \([S]^T[R][Y]\) associated with monopole voltage \( i \), where \( i = 1, 2, \cdots, K \) and

\[
\{ \tilde{V} \} = [V_1 V_2 \cdots V_K]^T
\]

(5.15)

where \( V_i \) in (5.15) are the complex monopole voltages we wish to compute. Here, \([M]\) is \( M \times K \), \( \{ \tilde{V} \} \) is \( K \times 1 \), and \( \{ \alpha \} \) is \( M \times 1 \). We note that if \( K = M \) (number of ports equals the number of CMs used in \( \{ I_0 \} \)), the desired coefficients \( \{ \alpha \} \) are synthesized exactly, assuming \([M]\) full rank. If \( K < M \), then the coefficients \( \{ \alpha \} \) can be computed using the pseudoinverse of \([M]\), which minimizes the least-squared error.

In this section, we derived the optimal locations and feed voltages for a multiple feed antenna-platform systems, using the characteristic modes of the platform (PCMs) as well as the characteristic modes of the composite antenna-platform system. We now follow by illustrating an example applying this methodology for the location and feed optimization of monopoles on-board a UAV radiating a Near Vertical Incident Skywave (NVIS) pattern at 10MHz, for which the UAV is sub-wavelength in size.
5.5 Synthesis of a NVIS Pattern Using Monopole Feeds

Antenna systems which exploit ionosphere reflections in the low-HF band (2-10 MHz) have remained highly useful in modern long-distance, beyond-line-of-sight (BLOS) communication links by utilizing near-vertical incident skywave (NVIS) radiation (see Fig. 5.10) [27]. Published designs of single-feed NVIS HF antennas for ground and airborne-based scenarios include the tilted whip [45] and the shorted and open towel-bar designs [45–47] to achieve unidirectional radiation along the zenith for ground-based platforms, or bi-directional radiation under airborne conditions. Although the aforementioned realizations achieve desirable results, it is natural to question whether these are optimal solutions, and if alternate designs which take further advantage of the mounting platform can be constructed for improvements in pattern, bandwidth, and efficiency. Thus, in this example, we consider the CMPS of a unidirectional NVIS pattern using the perfectly conducting FEKO [28] model of a UAV (see Fig. 5.11 and Fig. 5.12) at 10MHz. Here, we take a multi-feed approach to the problem, as we recognize from the UAV CM profile that unidirectionality is impossible with a single mounted ESA.

The desired pattern NVIS pattern is that of a crossed Hertzian electric and magnetic dipole pair in the xy-plane, also shown in Fig. 4.16. This pattern has a directivity of 3.2 dBi in the $\hat{z}$ direction. Table 5.1 depicts the coefficients $\alpha_n$ found using (3.9, so that the approximate NVIS pattern $F_0 = \sum_{n=1}^{M} \alpha_n F_n$, where $F_n$ corresponds to the radiation of the $n^{th}$ mode or CM current of the UAV.

The above pattern synthesis needs to be realized by wire monopoles placed the strategic locations on the UAV shown in Fig. 5.11 and Fig. 5.12. To synthesize this pattern, we consider $M = 6$ CMs and $K = 6$ monopole excitations. As already
Figure 5.10: NVIS TX-RX system utilizing ionosphere reflections in the low-HF band (2-10MHz) for long-distance communications.

Figure 5.11: Monopole mounting locations along the fuselage, wing, and tail of the UAV (isometric view).
Figure 5.12: Monopole mounting locations along the fuselage, wing, and tail of the UAV (top-down view).

Table 5.1: Normalized CM weighting coefficients |α_n| for the Unidirectional NVIS pattern in Fig. 4.16

| α_1  | 2.25 × 10^{-4} |
| α_2  | 1.00            |
| α_3  | 1.49 × 10^{-1} |
| α_4  | 7.47 × 10^{-2} |
| α_5  | 4.99 × 10^{-3} |
| α_6  | 2.07 × 10^{-1} |
noted, the CM currents \( \{I_n\} \) are known for the UAV. Therefore, it remains to find the appropriate excitation location to best approximate the \( \alpha_n \) coefficients that generate the needed mode currents. The six monopole feeds are placed on the regions of the UAV corresponding to the maximum normal electric near-field (Figs. 5.13-5.16) generated by \( \{I_0\} = \sum_{n=1}^{M} \alpha_n \{I_n\} \), for optimal capacitive coupling. The excitation voltages for each feed in Fig. 5.17 can then be computed using (5.13), and are shown in Table 5.2.

The resulting pattern of the six feed arrangement is shown in Fig. 5.18, with the CM excitation coefficients shown in Fig. 5.19. Although (5.13) realizes the desired \( M = 6 \) CM coefficients (to a scale factor) exactly, the primary drawback of this choice of location is that the radiation of higher-order modes (\( index > M \)) are not accounted for, and cause large perturbations in the overall radiation pattern performance. To

![Diagram](image)

**Figure 5.13**: Normal electric field \( |\hat{n} \cdot E_{\text{synth}}| \) for the synthesized NVIS pattern generated by current \( \{I_0\} \) along the fuselage.
Figure 5.14: Normal electric field $|\hat{n} \cdot \mathbf{E}_{\text{synth}}|$ for the synthesized NVIS pattern generated by current $\{I_0\}$ along the tail.

Figure 5.15: Normal electric field $|\hat{n} \cdot \mathbf{E}_{\text{synth}}|$ for the synthesized NVIS pattern generated by current $\{I_0\}$ along the wing.
Figure 5.16: Normal electric field $|\hat{n} \cdot E_{\text{synth}}|$ for the synthesized NVIS pattern over the UAV.

Figure 5.17: Optimal monopole placements for NVIS pattern, based on the normal E-field $\hat{n} \cdot E_{\text{synth}}$ in Fig. 5.16.

solve this problem, we must account for the neighboring (most dominant) higher-order modes, and find a new set of mounting locations which provide a significantly more accurate representation of the desired far-field pattern.
Figure 5.18: Gain for ideal NVIS unidirectional pattern in Fig. 4.16, with gain for six-feed UAV in Fig. 5.17 using the voltages $V_1$-$V_6$ given in Table 5.2.

Figure 5.19: CM coefficients of Fig. 5.17 using the voltages $V_1$-$V_6$ given in Table 5.2, compared to the ideal NVIS pattern in Fig. 4.16.
Table 5.1: Voltage excitations and resulting quality factor $Q$ for the six-monopole UAV in Fig. 5.17, exciting the unidirectional pattern (Fig. 4.16 at 10MHz, computed using (5.13) at the max E-field locations

| $V$  | $|V|$   | phase($V$) |
|------|---------|------------|
| $V_1$| 0.555   | $-11.15^\circ$ |
| $V_2$| 0.555   | $-11.15^\circ$ |
| $V_3$| 0.367   | $162.12^\circ$ |
| $V_4$| 1.000   | $2.18^\circ$   |
| $V_5$| 0.831   | $-14.71^\circ$ |
| $V_6$| 0.831   | $-14.71^\circ$ |

5.5.1 Location Optimization Using Higher-Order CMs

As seen in Fig. 5.18 and 5.19, the first $M = 6$ coefficients are synthesized exactly to a scale factor, but large excitation of higher-order CMs causes significant perturbations in the resulting far-field pattern, particularly CMs 7, 8, and 10. Therefore, in order to synthesize our desired NVIS pattern, it becomes necessary to account for these higher-order modes in the resulting location optimization.

Table 5.3 shows the $M = 12$ lowest-order eigenvalues of the UAV in Fig. 3.4. Recall that NVIS pattern was realized using $M = 6$ CMs of the UAV, with the highest-order CM 6 having an eigenvalue $\lambda_1 = 508.25$. Furthermore, the next highest-order mode, CM 7, has an eigenvalue magnitude of comparable value to that of CM 6, which implies that if the monopole voltage configuration has sufficient complexity to excite CM 6, then CM 7 can be easily excited as well, and must be accommodated in the analysis.

To that effort, we consider finding the optimal mounting locations which suppress of the neighboring highest-order CMs - in this case CMs 7-10, which are within an order of magnitude of the highest desirable CM 6. As was seen in Figs. 5.8-5.9, CMs
Table 5.3: First 12 eigenvalues for the UAV of Fig. 3.4 at 10MHz.

<table>
<thead>
<tr>
<th>CM</th>
<th>$\lambda_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.258</td>
</tr>
<tr>
<td>2</td>
<td>-2.33</td>
</tr>
<tr>
<td>3</td>
<td>-27.33</td>
</tr>
<tr>
<td>4</td>
<td>-234.97</td>
</tr>
<tr>
<td>5</td>
<td>386.05</td>
</tr>
<tr>
<td>6</td>
<td>508.25</td>
</tr>
<tr>
<td>7</td>
<td>-564.2</td>
</tr>
<tr>
<td>8</td>
<td>-715.12</td>
</tr>
<tr>
<td>9</td>
<td>821.36</td>
</tr>
<tr>
<td>10</td>
<td>-4052</td>
</tr>
<tr>
<td>11</td>
<td>-6099.5</td>
</tr>
<tr>
<td>12</td>
<td>8834.6</td>
</tr>
</tbody>
</table>

7-10 exhibit their own unique near-field profile along the possible mounting locations. We therefore wish to determine the locations which minimize coupling to CMs 7-10 as a whole. To do so, we introduce the summation

$$\Delta_{HOM} = \sum_{k=M+1}^{M'} |\hat{n}_s \cdot \mathbf{E}_k / (1 + j\lambda_k)|$$  \hspace{1cm} (5.16)

where $M$ is the number of CMs in the synthesis problem (in this case, $M = 6$), and $M'$ is the highest-order neighboring CM whose eigenvalue is within a decade of CM $M$, seen as $M' = 10$. To account for the higher-order mode coupling and its impact on the desired lowest-order CMs, we define the modified electric field $\tilde{\mathbf{E}}$ as

$$\tilde{\mathbf{E}} = \mathbf{E} / \Delta_{HOM}$$  \hspace{1cm} (5.17)

Hence, we can use the modified electric fields in (5.17) to determine the locations which minimize interaction with the undesirable higher-order modes which perturb the desired radiation pattern.
To illustrate the utility of the modified electric fields, Figs. 5.20-5.22 depict the first 6 CM modified near-fields when using $M = 6$ and $M' = 10$ in (5.16). For the wing case, it is seen that although coupling is maximized to CM 1 at the wing tip, this also coincides with significant coupling to the neighboring higher-order CMs. Thus, placement of a monopole at the wing tip would be a poor choice if the monopole feed system requires control over CM 1 due to the large presence of higher-order (but still radiating) CMs. On the contrary, from Fig. 5.20, it is seen that the neighboring higher-order modes have little effect on the fuselage.

Figs. 5.23-5.25 depict the modified NVIS E-field (using (5.17)), where clear distinctions between optimal mounting locations are seen in comparison to the maximum E-field cases in Figs. 5.13-5.15, whose results were discussed previously. Here, we

![Figure 5.20: Modified normal electric field $|\hat{n} \cdot \vec{E}_n/(1 + j\lambda_n)|$ for UAV CMs 1-6 in Fig. 5.12 as a function of monopole fuselage location $x$.](image-url)
Figure 5.21: Modified normal electric field $|\hat{n} \cdot \tilde{E}_n/(1 + j\lambda_n)|$ for UAV CMs 1-6 in Fig. 5.12 as a function of monopole tail location $y$.

Figure 5.22: Modified normal electric field $|\hat{n} \cdot \tilde{E}_n/(1 + j\lambda_n)|$ for UAV CMs 1-6 in Fig. 5.12 as a function of monopole wing location $y$. 
observe that in order to suppress the neighboring higher-order CMs, the optimal locations for the two monopoles on the wings is closest to the fuselage. Similar results are observed for the monopoles on the tail, while the fuselage monopole optimum locations remain the same.

The resulting mounting locations for minimization of the neighboring highest-order CMs is depicted in Fig. 5.26. Again, with the mounting locations chosen in Fig. 5.26, the CMPS method in (5.13) can be implemented, with the voltages given in Table 5.4. The resulting far-field and UAV CM decomposition is shown in Figs. 5.27 and 5.28, respectively, where appreciable suppression of the higher-order CMs 7-10 is achieved, albeit with CM 10 still present. As a result, the far-field pattern

![Graph](image)

Figure 5.23: Modified normal electric field $|\hat{n} \cdot \vec{E}_{\text{synth}}|$ for the synthesized NVIS pattern generated by current $\{I_0\}$ along the fuselage.
Figure 5.24: Modified normal electric field $|\hat{n} \cdot \tilde{E}_{\text{synth}}|$ for the synthesized NVIS pattern generated by current $\{I_0\}$ along the wing.

Figure 5.25: Modified normal electric field $|\hat{n} \cdot \tilde{E}_{\text{synth}}|$ for the synthesized NVIS pattern generated by current $\{I_0\}$ along the tail.
Figure 5.26: Optimal monopole placements for NVIS pattern, based on the modified normal E-field $|\hat{n} \cdot \tilde{E}_{\text{synth}}|$ in Figs. 5.23-5.25.

resembles the desired NVIS pattern far more closely than Fig. 5.18 when using the maxima of the modified electric fields.

Although the monopole mounting locations in Fig. 5.26 was shown to yield superior performance in terms of pattern accuracy, it is also important to examine the resulting $Q$ factors for Figs 5.17 and 5.26, as a measure of the potential bandwidth achievable when connected to a feed network. Table 5.5 shows the $Q$ factor for the ideal NVIS pattern along with the $Q$ computed using (4.26) for the mounting locations in Fig. 5.17 and 5.26. Here, it is seen that the increase in pattern accuracy when choosing Fig. 5.26 comes at the cost of an increased $Q$ factor, due to the decrease in the total amount of capacitive coupling to the UAV platform.

5.6 Summary

A new design methodology for in-situ ESAs was presented, based on the theory of characteristic modes. By partitioning the antenna and platform geometries, we demonstrated that optimal locations for small monopoles can be found readily by
Figure 5.27: Gain for ideal NVIS unidirectional pattern in Fig. 4.16, with gain for six-feed UAV in Fig. 5.26 using the voltages $V_1$–$V_6$ given in Table 5.4.

Table 5.4: Voltage excitations and resulting quality factor $Q$ for the six-monopole UAV in Fig. 5.17, exciting the unidirectional pattern (Fig. 4.16 at 10MHz, computed using (5.13) at the max E-field locations

| $V$  | $|V|$ | $\text{phase}(V)$     |
|------|------|-----------------------|
| $V_1$| 0.398| 4.35°                 |
| $V_2$| 0.398| 4.35°                 |
| $V_3$| 1.000| $-170.57^\circ$       |
| $V_4$| 0.075| $-94.85^\circ$        |
| $V_5$| 0.900| 11.02°                |
| $V_6$| 0.900| 11.02°                |

Table 5.5: $Q$ factors for the monopole mounting locations in Figs. 5.26 and 5.17, excited by the voltages in Tables 5.2 and 5.4, respectively.

<table>
<thead>
<tr>
<th>$Q_{\text{NVIS,ideal}}$</th>
<th>28.07</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{\text{NVIS,maxE}}$ (Table 5.2)</td>
<td>$1.41 \times 10^4$</td>
</tr>
<tr>
<td>$Q_{\text{NVIS,maxE}}$ (Table 5.4)</td>
<td>$2.18 \times 10^4$</td>
</tr>
</tbody>
</table>
identifying the regions where each modal E-field is at its maxima. This method of identifying the local E-field maxima is applicable for single as well as multiple ESAs on-board arbitrary-shaped platforms. We further demonstrated a new method for computing the monopole voltages subject to a radiation specification, denoted Characteristic Mode Port Synthesis (CMPS), which reduces the feed voltage problem optimization into a simple matrix equation. We applied this unidirectional NVIS pattern was realized on a UAV at 10MHz using six monopole feeds, illustrating the benefit multiple co-radiating feeds have over single-feed designs in terms of pattern diversity and customization.

Figure 5.28: CM coefficients of Fig. 5.26 using the voltages $V_1$-$V_6$ given in Table 5.4, compared to the ideal NVIS pattern in Fig. 4.16.
Chapter 6: Conclusion

One of the central difficulties faced when designing any antenna system is finding the appropriate starting point. While increasingly quick, sophisticated, and well-tested optimization routines provided by commercial EM solvers can facilitate the design process, often times they can become a crutch to the antenna engineer. An understanding and appreciation of the underlying problem’s physics will not only aid in the specific design at hand, but also provide valuable carryover knowledge to subsequent related antenna designs.

In this dissertation, the Theory of Characteristic Modes was used as a physically intuitive framework for the optimization and benchmarking of electrically small radiating systems. First, a new method for computing the $Q$ of arbitrary-shaped antennas was introduced. This formula can then be used to find the $Q$ limits of antennas, as well as their supporting platforms subject to specific radiation conditions. With these benchmarks, the antenna engineer will be able to quickly analyze and critique their designs against an optimal limit. Following a thorough analysis of fundamental limits, we analyzed the physics behind antenna-platform coupling mechanisms through the language of characteristic modes. We have shown that by careful understanding of
the underlying mechanisms of antenna-platform interaction, the in-situ antenna design process can be simplified, as well as radically hastened in comparison to traditional grid search or global optimizer solutions.

6.1 Future Work

The key contributions from this work open a wide variety of research topics for small antenna applications using Characteristic Modes. Below, we discuss future research topics related to this thesis, each of which would make major contributions to the field.

6.1.1 Fundamental Limits on $Q$ for Material Bodies

One major task in this field is the incorporation of dielectric and magnetic material parameters in the computation of $Q$ limits, pattern-$Q$ limits, as well as Characteristic Mode Port Synthesis (CMPS). To date, there are no publications on the topic of CMs for composite material-PEC bodies. Furthermore, an investigation of the current literature shows that virtually no progress has been made in the development of CMs for dielectric/magnetic-only bodies since the inaugural papers by Harrington [48] and Chang [49] in the 1970s.

Therefore, the first goal in the development of CMs for material-PEC bodies is finding a proper characteristic mode definition for material-PEC bodies, and physical interpretation of the results. As stated in [48], if we attempt to find the characteristic modes of a material body using (3.1), we will obtain a set of orthogonal currents and voltages. Unfortunately, a significant problem arises, as stated in [48] and [49]. The issue is that the eigenvalues $\lambda_n$ no longer are the ratio of stored to radiated power, as they are in the PEC-only definition. Hence, for material-PECs we require
modifications to existing formulae describing the complex power balance to correctly identify the lowest-order modes. Once the complex power balance relations and physical properties of material-PEC CMs known, the work of Chapters 4 and 5 can be extended to materials loaded bodies.

6.1.2 Realization of Practical Feed Networks for Multi-Feed Antennas

Characteristic Mode Port Synthesis (CMPS) was shown to be a highly efficient and physically intuitive methodology for realizing a wide variety of radiation patterns using the platform’s CMs. However, increasing the number of feed antennas comes at the cost of realization complexity. That is, when more feed antennas are needed, the power division and matching networks become increasingly larger and more complex, since the system of ESAs still needs to be fed by a single source. Therefore, there is significant interest in finding cost effective, efficient, and practically realizable power division and matching networks for the multi-antenna systems.

6.1.3 CMPS Methods for Different Antenna Shapes

In Chapter 5, our focused was entirely on feeding a UAV platform using small monopoles of height = \( \frac{\lambda}{50} \). Monopoles of this size on an infinite ground plane have exceedingly high \( Q \) factors, low \( R_{in} \) and high \( X_{in} \), making them an unattractive option in most scenarios. However, Chapter 5 demonstrated how the bandwidth and radiation of these small monopoles could be enhanced dramatically through proper interaction with the subwavelength mounting platform.

The results found in Fig. 5 are not theoretically restricted to monopoles only. For instance, the reciprocity method for finding optimal feed locations in Chapter 5.4.1
could be easily extended to magnetic sources (e.g. small loops), showing that the optimal locations for small loop antennas is that of the eigencurrent’s magnetic field, oriented in the same direction as the loop moment. Similarly, a mix of monopole (capacitive) and loop (inductive) could be used to realize a specified far-field pattern, particularly when the feed locations are in advantageous mounting locations for the rest of the system design (e.g. structural integrity, aerodynamics, etc.).

6.1.4 CMPS Higher-Order Mode Suppression

One of the central advantages introduced by Characteristic Mode Port Synthesis (CMPS) in Chapter 5 is that a prescribed set of CM coefficients $\alpha_k$ can be synthesized exactly. However, as was seen in Fig. 5.28, neighboring higher-order modes affect performance and subsequently disturb the target pattern. It would be instructive to devise location and voltage-dependent optimization methodologies to suppress these higher-order modes to improve pattern purity, either through global formal optimization procedures, or alternative CMPS formulations.
Bibliography


